HEAT TRANSFER IN PACKED BEDS
OF LOW TUBE/PARTICLE DIAMETER RATIO

by

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I declare that the work in this thesis is
my own, and that the thesis has been
composed by me

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ABSTRACT

An experimental investigation of heat transfer in packed beds of low tube/particle diameter ratio is reported and the most widely-used two-dimensional homogeneous continuum models are thoroughly tested by statistical methods using the experimental data obtained.

It is shown that the omission of axial dispersion effects leads to significant lack-of-fit in such models and to parameter estimates which vary systematically with bed depth. A model including axial dispersion is considered for each of two possible simplified downstream boundary conditions. This axially-dispersed model shows no lack-of-fit and yields depth-independent parameter estimates when the boundary condition is placed at infinity; when the alternative condition at bed exit is used this model shows little improvement over the model which omits axial dispersion.

A new theory for predicting the axial and radial effective thermal conductivities and the effective wall heat transfer coefficient is derived from a two-phase continuum model containing the essential underlying and independently measureable heat transfer processes. The theory gives good agreement with the results obtained in this work, in contrast with previously-existing theory, explains much of the confused literature data, and pin-points the remaining major areas of uncertainty.

NOTES

- (a) The units of all dimensioned quantities are given, in the MKS system, in the relevant section of the nomenclature. Any departures from these units are specified at the point of usage.
- (b) Literature references are given at the end of each chapter, where they are numbered; these numbers are underlined when cited in the text. Figures and equations are numbered consecutively in each chapter without regard to subsections. When one of these numbers is cited in the text it is prefixed with the chapter number in which it appears, except when cited within the same chapter e.g. eqn.(2.3) in any chapter except the second, where eqn.(3) would be used.
- (c) The main points of the work in chapters 1-3 were presented at the Fifth International Symposium on Chemical Reaction Engineering at Houston, Texas in March 1978 and published in the proceedings:
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1 PACKED BEDS AND MODELS

1.1 Introduction

A packed bed chemical reactor is an arrangement for carrying out gas-solid or liquid-solid chemical reactions on an industrial scale. The packed bed itself is an assembly of randomly-arranged particles held firmly in position within a reactor tube. The particles are often spherical and usually the same size. The reactant fluid is forced along the tube and flows in a random manner between and around the particles.

Packed bed reactors are most commonly used for catalytic reactions, in which case the particles are porous to give a high fluid-solid interfacial area, and are coated with the catalyst. Many industrially important catalytic reactions are strongly exothermic; some examples with typical reaction temperature and heat evolved per mole of key reactant are shown in table 1.

For such reactions the problem of heat removal is extremely important and has a great effect on design and operation of the packed bed. Build-up of excessive heat in the bed ('hot-spots') can lead to promotion of unwanted side reactions (complete combustion in cases (2) - (4) of table 1) and sintering of the catalyst.

	<u>Reaction</u>	<u>Reaction temp.</u> (°C)	$\Delta H_{25^\circ\text{C}}^\circ$ (kJ/mole)
1.	$\text{C}_6\text{H}_6 + 3\text{H}_2 \rightarrow \text{C}_6\text{H}_{12}$ benzene cyclohexane	150 - 260	- 206
2.	$\text{C}_8\text{H}_{10} + 3\text{O}_2 \rightarrow \text{C}_8\text{H}_4\text{O}_3 + 3\text{H}_2\text{O}$ o - xylene phthalic anhydride	340 - 450	- 1128
3.	$\text{C}_2\text{H}_4 + \frac{1}{2}\text{O}_2 \rightarrow \text{C}_2\text{H}_4\text{O}$ ethylene ethylene oxide	240 - 290	- 103
4.	$\text{C}_4\text{H}_8 + 3\text{O}_2 \rightarrow \text{C}_4\text{H}_2\text{O}_3 + 3\text{H}_2\text{O}$ butylene maleic anhydride	430 - 480	- 1134
5.	$\text{C}_2\text{H}_2 + \text{HCl} \rightarrow \text{C}_2\text{H}_3\text{Cl}$ acetylene vinyl chloride	110 - 180	- 103

Table 1 : Some industrial exothermic catalytic reactions

The form of packed bed used for these reactions is that of a long thin tube, one of many in a bundle embedded in a heat exchange medium. Such tubes are many hundreds of particle diameters long, but only a few particle diameters across, thus facilitating heat loss radially through the walls of the tube. The study of the heat transfer properties of such a bundle usually begins with the study of the properties of a single packed tube, a difficult enough problem in itself. In spite of the obvious importance of such studies, there has been little work done on such tubes of low tube/particle diameter ratio (1 - 3), in contrast to the much wider literature available for beds of high ratio, which are more suited to reactions where heat transfer considerations play a smaller part.

The designer of a packed bed reactor has the twin objectives of safety and efficient operation in mind. In order to achieve these such variables as flow rate, feed composition and temperature and wall temperature may be adjusted on the industrial scale reactor, although such 'real-life' experimentation may be costly and difficult. Other physical variables, such as reactor dimensions, must be specified before construction, and hence the designer must resort to laboratory experiment to determine their effect. However the laboratory measurements cannot be applied directly to industrial situations, as usually every feature of these measurements is at a value different from any likely industrial scale value, and bed lengths and flow velocities will be at lower orders of magnitude. This is the 'scale-up' problem.

The standard procedure, therefore, is to construct a mathematical model of the reactor, which may be experimentally tested under laboratory conditions, and which will give reliable predictions under industrial conditions. Mathematical models are usually constructed from consideration of the physical and chemical processes occurring in the reactor. Any such model can only hope to represent the more important processes occurring, and is inevitably a compromise between accurate description of the reactor and computational pragmatism. With the advent of high speed digital computers, however, more realistic models may be constructed and solved and more sophisticated methods of data analysis become available.

The decision as to which processes to incorporate in a model remains largely intuitive; however more recent work (3, 4) has suggested that statistical methods be used to determine the adequacy of a model with regard to experimental data. The incorporation of such methods should lead to more rigorous model building procedures.

The situation is further complicated by the inclusion in practically all models of certain constants, or parameters. Some of these parameters, e.g. bed voidage ϵ , may be measured directly from the apparatus; others, e.g. effective radial bed conductivity k_r , must be estimated from experimental data, and are usually then correlated empirically against other variables e.g. feed flow rate. These correlations cannot be safely extrapolated, however, and to date there is considerable disagreement between different studies. A preferable procedure would be to predict the parameters theoretically. This is rarely done, however, and the usefulness of the models is thereby limited.

A packed bed reactor model must reflect the following areas:

(a) Chemical kinetics e.g. reaction rate

(b) Physical processes e.g. fluid flow, heat and mass transport.

At the present state of the model-building art, it is usual to model various aspects of (a) and (b) separately, and combine the contributions to form the final model. As demonstrated above, heat transfer is one of the most important areas.

The aims of this work may thus be summed up as follows:

(i) To conduct an experimental investigation into the industrially important but often neglected case of heat transfer in packed beds of low tube/particle diameter ratio (low d_t/d_p).

(ii) To apply statistical methods to the selection of a model for this case using the data so found.

(iii) To provide theoretical predictions of the parameters to be used in the model selected.

1.2 Heat transfer models

There are several choices of model describing packed bed chemical reactor operation at steady-state, the usual industrial mode. Dynamic problems of start-up and reactor stability, although important, are beyond the scope of this work. The models have been described in some detail in various review publications (5 - 9) which also give references to the original papers; in view of this only a brief review is necessary here.

Since it is desired to model heat transfer as a separate process, those parts of the models relating to mass transfer and chemical reaction are omitted. The thermal properties of chemical reactions are well-established in the chemical literature, and few heat transfer experiments are carried out in the presence of reaction (10 is an example). How chemical reaction affects the physical heat transfer processes is a virtually unexplored area.

The physical heat transfer mechanisms believed to occur in packed beds are:

- (a) forced convection in direction of flow (axial direction)
caused by bulk flow of fluid.
- (b) particle-to-particle conduction via finite area contacts
and small pockets of stagnant fluid around each contact.
- (c) particle-to-wall conduction similarly to (b).
- (d) particle-to-fluid transfer via a fluid 'boundary layer'
at the particle surface, usually taken for convenience
to be of uniform thickness.
- (e) fluid-to-wall heat transfer via a wall 'boundary layer'.
- (f) molecular conduction through the fluid.
- (g) mixing in the radial direction (perpendicular to flow) due
to splitting and recombination of fluid streams.

- (h) mixing in axial direction similarly to (g).
- (i) apparent axial heat transfer due to the fluid moving at different velocities at different radial positions.
- (j) radiation between particle surfaces and between void spaces.

Natural convection effects are regarded as negligible. In laboratory experiments and for many industrial reactions (j) may also be neglected, as radiation is negligible below 400°C , although for higher temperatures it will play an important part.

No mathematical models yet developed account for each of these effects individually, indeed should such a model exist it would probably be too cumbersome for practical use; all models 'lump' different contributions into 'effective' processes.

Models may be classified into two groups, the stochastic and the deterministic. Virtually no examples of the former exist for heat transfer in packed beds, although single-phase mixing may be expressed in terms of a probability theory (11). Only deterministic models are considered here.

The first group of these are termed the finite-stage or cell models. These are the most closely related to physical reality, as they take explicit account of the discrete nature of the bed, regarding the void spaces as being a network of perfectly stirred tanks. A detailed account of the various types may be found in (9). An underlying assumption of the original model was that heat and mass transport are caused only by fluid mixing. While this may be the situation for mass transport, and hence the considerable success of the two-dimensional model in this area (12), it is true for heat transport only in the limiting case of extremely high flow rates. Attempts to remedy this deficiency (13) lead to a very much more complicated model.

The second group of models are termed dispersion or diffusion models. These are based on energy balances over differential volumes in the bed, in which the flux is assumed to be smooth, and hence Fourier's law may be used. As such they are indefensible physically, but are justified by their success. This is due to the fact that the important gradients in the bed are the 'long-range' ones associated with radial and axial transfer and bulk flow, and 'short-range' discontinuities are smoothed out over a tube radius. This line of reasoning applies for the case of high d_t/d_p , but is a little more dubious for low d_t/d_p . Previous work (3) has indicated that radial temperature profiles are smooth enough to justify this type of model.

A third type of model has been briefly tried, the wave model (14), but has not found acceptance, and there is now some doubt as to its formulation.

Dispersion models are the most widely used for heat transfer, and appear to be the easiest form in which to incorporate the desired heat transfer mechanisms. Accordingly only this type of model is considered in the rest of this work.

1.3 Dispersion models

As both fluid and solid are present in a packed bed, it seems natural to take account of this in the model used. There are two main forms of such 'two-phase' models available in the literature. The first assumes each particle to be isolated in a cell of fluid which is at the same temperature at all points on the particle surface. This model thus precludes particle-to-particle and particle-to-wall heat transfer. As at least the former of these is

believed to be important (1, 3) at low d_t/d_p , this model must be discarded.

The second model (called the quasi-continuum model) represents these modes at the cost of dispensing with the physical notion of particles. The solid and fluid phases are both regarded as continuous. This model was first proposed in a restricted form omitting axial transport in both phases by Olbrich (15). The full form has no analytic solution and has not been tested experimentally. This model is further discussed in Chapter 4.

The most common models in use are the pseudo-homogeneous models, where the bed is considered as a single phase. The average temperature of the particle is assumed to be the same as that of the gas that would be at the same position as its centre. The fluid and solid effects are represented by lumped 'effective' radial and axial conductivities, k_r and k_a .

Further assumptions made for simplicity are:

- (a) The physical properties of the fluid are constant in the bed (i.e. density ρ and specific heat c_p)
- (b) k_r and k_a are constants
- (c) The fluid velocity is constant and equal to an average value u .
- (d) The temperature is independent of angular position.

None of these assumptions is true, but making them gives models which are easier to solve. However experimental studies have found that there is a sharp decrease in radial conductivity near the wall, and this is usually idealised into a resistance at the wall, characterised by the effective heat transfer coefficient h_w .

There have been attempts to dispense with h_w and include a radially-varying velocity profile and radial conductivity; however

the problem of how to let k_r and u vary is a difficult one - they are usually tied to bed voidage. The best results have been obtained by Froment and Leroux (16).

The model obtained from a heat balance over a differential volume of packed bed under assumptions (a) - (d) has been termed the axially-dispersed plug flow (ADPF) model and has equation

$$G_{c_p} \frac{\partial T_b}{\partial z} = k_r \left(\frac{\partial^2 T_b}{\partial r^2} + \frac{1}{r} \frac{\partial T_b}{\partial r} \right) + k_a \frac{\partial^2 T_b}{\partial z^2} \quad (1)$$

The boundary conditions for (1) are discussed in section 1.4. A more commonly used simplification of (1) is obtained by neglecting the effective axial dispersion term, and is termed the plug-flow (PF) model :

$$G_{c_p} \frac{\partial T_b}{\partial z} = k_r \left(\frac{\partial^2 T_b}{\partial r^2} + \frac{1}{r} \frac{\partial T_b}{\partial r} \right) \quad (2)$$

Further simplifications to one-dimensional models are common, but for exothermic reactions the importance of radial gradients has been emphasised (17, 18).

The PF and ADPF models are the main subjects of the present work.

1.4 Boundary conditions

The appropriate boundary conditions for equations (1) and (2) depend upon the reactor geometry. In this work it is proposed to consider the scheme shown in figure 1, similar to that used by Gunn and Khalid (4) and Paterson (3).

Figure 1 : Schematic description of packed bed

Both sections (a) and (b) are packed with spherical particles of uniform size.

At the centre-line there is the symmetry condition for both models

$$r = 0 \quad : \quad \frac{\partial T_b}{\partial r} = 0 \quad (3)$$

At the wall

$$r = R : \quad -k_r \frac{\partial T_b}{\partial r} = \begin{cases} h_w (T_b - T_w) & z > 0 \\ h_w (T_b - T_0) & z < 0 \end{cases} \quad (4)$$

For the PF model (4) is restricted to the domain $z > 0$. This model also requires initial conditions at $z = 0$:

$$z = 0 : \quad T_b = f(r) \quad (5)$$

Many workers have taken $f(r) \equiv T_0$ as the inlet profile, others have measured at $z = 0$ and fitted for $f(r)$. The latter procedure is undoubtedly correct in the absence of a calming section; in the present case, assuming no axial dispersion, the former condition is correct.

For the ADPF model equation (1) must be regarded as shorthand for the pair of identical equations yielded by separate energy balances

in the two sections, made necessary by the form of equation (4). These are linked via continuity of temperature and flux at the adjoining plane, giving

$$\begin{aligned} T_b(0^+) &= T_b(0^-) \\ \frac{\partial T_b}{\partial z}(0^+) &= \frac{\partial T_b}{\partial z}(0^-) \end{aligned} \quad (6)$$

under the assumption of constant k_a in both sections.

For an upstream boundary condition the calming section is regarded as semi-infinite giving

$$T_b \rightarrow T_0 \text{ as } z \rightarrow -\infty \quad (7)$$

The downstream condition is much less certain. An entirely rigorous procedure would be to perform another energy balance over the unpacked exit section (not shown in figure 1) and introduce another three effective heat transfer parameters for that section. However the presence of the thermocouple cross (see chapter 2) disturbs the flow patterns and the equations would be of doubtful validity.

Simpler alternatives are to treat the test section as semi-infinite (ADPF (IBC) model) giving

$$T_b \rightarrow T_w \text{ as } z \rightarrow \infty \quad (8)$$

as was done by Gunn and Khalid (4) ; or, for shorter beds, to assume plug-flow in the space above the packing (ADPF (FBC) model) giving

$$z = L : \frac{\partial T_b}{\partial z} = 0 \quad (9)$$

Both possibilities are examined in chapter 3 where the solutions of the above models are displayed, and the abilities of the different models to predict experimental data are examined.

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2. EXPERIMENTAL APPARATUS, PROCEDURE AND RESULTS

2.1 Description of apparatus

The experimental part of this work is an extension to cylindrical packed beds of the preliminary work for annular beds done by Paterson (1.3). The apparatus was therefore broadly similar to his, and in some places identical.

The general experimental layout is shown in figure 1. The bed itself consisted of two tubular sections of internal diameter 70.8mm, insulated at the join by a sandwich of two rubber gaskets with a PVC gasket between them. Twelve thermocouples entered the bed radially through the PVC gasket.

The lower bed section, termed the calming section, was a pyrex tube which was packed but unheated. The packing rested on a steel grid which separated the calming section from a short unpacked section into which the air feed line ran. Two thermocouples measured the air feed temperature at the grid, entering radially at the grid gasket. One was at the bed centre and one at the wall.

The fluid used in the bed was air, supplied via a 12.7mm compressed air line and passing through a silica gel drying region. The air flow-rates were controlled by a gate valve followed by a parallel battery of rotameters, each with its own valve. Flow rates of up to 180 litres/minute at 25°C and 0-2 p.s.i.g. could be obtained.

The upper bed section, or test section, was a packed brass tube sitting inside an outer brass tube, thus forming an annulus. In the annular gap steam was forced under pressure, exiting at the bottom of the test section via a tap and piping leading to a cold water bath. The steam supply was controlled by a valve. Since it was under pressure it was necessary to measure the wall temperature.

Figure 1 : Schematic diagram of experimental apparatus

This was effected by six thermocouples soldered, within the annular gap, onto the test section wall. The first was 24mm above the test section inlet, the others following at 80mm intervals. Their leads emerged in the inner tube, but well above the highest packing used.

Both sections were packed with identical particles to provide a continuous length of packing.

The temperature distribution in the air stream leaving the packing was measured by Cr/Al thermocouples supported by a brass cross similar to that used by De Wasch and Froment (1) and shown in figure 2. This was supported by screwed rods from a cross with notched arms so as to fit the top of the bed tightly. The distance between support and measurement crosses was adjusted by nuts on the screwed rods.

The 8-armed brass cross held 32 thermocouples arranged as shown in figure 3. 4 thermocouples were on each arm, in the configurations denoted A and B. The numbers refer to the distances from bed centre, in mm. Hence 4 measurements at 90° intervals were taken at each of 8 radial positions ($r = 9, 11, 13, 18, 24, 29, 31$ and 33mm).

Also shown in figure 3 are the positions of the 12 thermocouples at the test section entrance (hereafter referred to as giving 'entrance' data). From these 2 measurements at 180° intervals were taken at each of 6 radial positions ($r = 12, 16, 23, 28, 31$ and 34mm).

The various thermocouples were connected, via selector boxes, to 'Comark' electronic thermometers, with an ice/water cold junction.

2.2 Design and operation

Paterson (1.3) considered some alternatives in both design and operation of the apparatus and his relevant conclusions may be summarised here.

Figure 2 : Thermocouple cross at bed exit, support rod and cross at top of bed tube.

Exit thermocouples

Entrance thermocouples

Figure 3 : Location of thermocouples

- (a) There was negligible heat conduction along the brass cross thermocouple wires.
- (b) Immersion of the thermocouples in the bed gave the same results as location just above the surface of the bed, except in the case of zero flow. This conclusion justifies measurement above the packing, since no stagnant bed measurements were made in the present work.
- (c) Embedding of thermocouples was not considered as in beds of low d_t/d_p measurements can then not be made near the wall, a severe restriction. Also the packing is distorted by the embedded thermocouples.
- (d) Inserting thermocouples radially through the wall at different axial positions leads inevitably to large heat fluxes along the thermocouples and thus to inaccurate temperature measurement.

Before construction of the bed the thermocouples were calibrated against a 0.1°C standard reference mercury thermometer, in a thermostatic water bath, over the range $20\text{--}100^{\circ}\text{C}$. Negligible difference was found between thermocouples on the same 'Comark' input, but different calibration curves were necessary for different inputs. The raw data obtained in the experiments were corrected by a computer program using cubic spline interpolation between the calibration points (2).

The positions of the thermocouples on the cross and in the bed were measured accurately and since they were 'araldited' firmly in position, and the cross fitted tightly in the bed, it was considered that there was no error in radial location of the thermocouples.

The calming section was packed first, the gasket 'sandwich' placed in position, and finally the test section bolted on, for each repacking of the bed. This was to avoid dropping the packing past

the entrance thermocouples, which might have damaged them. The rest of the packing was then trickled into the test section. The test section was taken to begin at the entrance thermocouples and the distance from these to the top of the bed measured. The distance from the top of the packing to the top of the bed was measured by a dipstick, at several positions, and the bed height found by subtraction. The average bed height was taken but the variation with position was small.

The thermocouple cross was then lowered into the bed until it rested just on or above the packing. The nuts on the screwed rods could be preset to achieve this so that the notches on the arms engaged the tube top. The steam was then turned on and the walls allowed to reach a steady temperature, before the air feed began. The desired volumetric flow rate could be obtained on the appropriate rotameter using previously existing calibration charts to give the rotameter setting.

The bed was then allowed to reach steady-state, which took between $1\frac{1}{2}$ and 3 hours, depending on its previous steady-state. Temperature measurements on representative thermocouples were made at regular intervals, and when these gave little or no change all the thermocouples were read. The flow rate was then adjusted or the bed repacked as appropriate and the procedure repeated.

In all, 4 different non-porous packings were used, at various bed depths and flow rates. For a given packing, the same flow rates were set at each bed depth. A set of tables showing the combinations of packing, flow rates, bed depths and run numbers giving the experimental order are presented in appendix B (tables B1-B4). Re was calculated using the properties of air at 25°C . The remainder of the appendix gives the corrected data obtained. The entrance and

exit temperatures are given in full; however only the corrected average is given for wall temperature T_w and feed temperature T_0 . Occasionally a thermocouple gave no reading due to a loose connection, thus some readings are missing.

All the measurements were originally taken by Dr.D.L.Cresswell over a period of 1½ years at various times. However for the 6.4mm ceramic beads and the 9.5mm steel balls the PVC entrance thermocouples gasket was removed, so that no entrance data were taken. The exit data obtained in these cases exhibited some peculiar behaviour, including some values in the bed less than the feed temperature T_0 , clearly an impossibility. Accordingly the bed was dismantled, the thermocouples re-calibrated and their positions checked. The entrance thermocouples were also restored. Quite different thermocouple calibration curves were obtained, for no obvious reason. Possibly long periods of non-use have some effect, either on the thermocouples or the 'Comark' thermometers.

As entrance data was required for these cases it was felt that the best course of action was to re-take the data for the two packings. This work and the re-calibration were carried out by the present writer. It is these runs which are given in appendix B and show none of the peculiarities of the previous data.

A further set-back occurred with the runs for $Re = 109$ with 6.4mm ceramic beads and $Re = 146$ with 9.5mm steel balls. These values required a setting at the upper limit of operation of the intermediate rotameter, which was found to be very hard to control accurately. The runs were repeated using an appropriate setting near the lower limit of the large rotameter, which gave much more consistent readings. Both sets of data are given in appendix B, the large rotameter runs being the last ones in each case. It was these latter which were used when the data were analysed.

Discounting repeats, 107 runs were finally taken. For some combinations of flow rate and bed depth the temperature profile was virtually flat at bed exit. In such cases no measurements were taken, accounting for some of the gaps in tables B1, B2 and B4.

2.3 Preliminary analysis of results

For each run the wall temperature measurements showed a spread over the six readings of less than 0.5°C in virtually all cases; thus T_w was taken as a constant and equal to the corrected average. Similarly the average of the two feed measurements was taken as T_0 , as these rarely differed by more than 0.2°C .

Typical bed exit profiles are shown in figure 4, taking the case $Re = 120$ for 9.5mm ceramic beads. The values of T_w for each run are also shown.

The following may be noted:

(a) There is a significant, but apparently random, variation of temperature with angular position. For the longer bed depths this variation can be nearly as much as the total radial temperature change. No existing models account for this phenomenon.

(b) The angular average temperature appears to be a fairly smooth function of radial position, an encouraging sign for the applicability of dispersion models.

(c) There is a reproducible 'hump' in the radial temperature profile roughly $0.5 d_p$ from the wall. It is particularly noticeable in shallow beds, but is smoothed out with increasing depth. A similar hump, although at a greater distance from the wall, was observed by De Wasch and Froment (1). Paterson (1.3) observed two humps, also further from the walls, in his annular bed. By allowing u and k_r to be functions of radial position, Leroux and Froment

Figure 4 : Typical observed temperature profiles

(1.16) have simulated these humps with the PF model, although at the cost of much computational complexity. Marivoet et al (3) have also simulated the hump. As the magnitude of these humps is small compared with the angular scatter, it is doubtful whether this feature is worth incorporating into a model.

An indication as to whether the IBC or FBC version of the ADPF model is the more appropriate may be obtained from examination of the entrance profiles from each bed depth at a given flow rate. The infinite condition would be consistent with inlet profiles independent of bed length, whilst with the finite condition a systematic effect would be expected. The angular average inlet temperature profiles corresponding to 9.5mm ceramic beads at $Re = 120$ are shown in dimensionless form in table 1, so that runs with different T_w and T_o may be compared.

	<u>r(mm)</u>					
<u>L(mm)</u>	12	16	23	28	31	34
51	0.04	0.06	0.06	0.14	0.20	0.25
102	0.04	0.06	0.07	0.15	0.20	0.24
152	0.04	0.06	0.08	0.16	0.20	0.24
204	0.04	0.07	0.10	0.16	0.20	0.24
254	0.04	0.06	0.08	0.16	0.20	0.25

Table 1 : Dimensionless entrance profiles

No systematic trend is evident, which tends to support the IBC version; however the FBC model will be tested further in chapter 3, as the above can hardly be termed conclusive.

2.4 Voidage studies

Although the bed voidage ϵ is not featured explicitly in the PF or ADPF models it is an important fundamental property of a packed bed, and is worth determining for later use.

The experimental method adopted was again that of Paterson (1.3). PVC cylinders were packed with polystyrene balls, dyed candle wax was poured into the voids and left to set. The bed was then sawn into slices in the radial plane a sufficient distance above the base for the ordering effects to be negligible (about $5 d_p$). Photographs were taken of the surface of each slice and with suitable enlargement a transparency of polar graph paper could be superimposed, so that at any radial position the fraction of the circumference occupied by void could easily be measured. This determined the 'line voidage'. Blum and Wilhelm (4) have shown that area voidage and volume voidage are equal, and it may be assumed that all three are equal.

This procedure was followed for two values of d_t/d_p and two slices of each. Figures 5 and 6 show a slice for each of the d_t/d_p values. The line voidages at various dimensionless radial positions are given in table 2 for the four slices, and the average of the slices for each d_t/d_p value is shown in figure 7.

Figure 5 : Wax model slice, $d_t/d_p = 5.58$

Figure 6 : Wax model slice, $d_t/d_p = 11.81$

$$d_t/d_p = 5.58$$

$$d_t/d_p = 11.81$$

y	ϵ (slice 1)	ϵ (slice 2)	ϵ (slice 1)	ϵ (slice 2)
0.07	0.44	0.33	0.39	0.19
0.13	0.33	0.50	0.31	0.19
0.20	0.40	0.62	0.52	0.41
0.27	0.49	0.42	0.41	0.33
0.33	0.28	0.35	0.30	0.32
0.40	0.30	0.31	0.42	0.56
0.46	0.43	0.30	0.26	0.32
0.53	0.52	0.42	0.38	0.49
0.60	0.62	0.70	0.31	0.42
0.67	0.46	0.31	0.42	0.42
0.74	0.23	0.19	0.24	0.28
0.80	0.26	0.22	0.38	0.38
0.86	0.35	0.26	0.33	0.38
0.93	0.62	0.52	0.28	0.26
1.00	0.96	0.86	0.92	0.92

Table 2 : Voidage measurements

The oscillatory nature of the voidage, with maxima and minima approximately d_p apart, confirms the results of other workers (5, 6).

The average voidages for the bed were determined from the curves in figure 7. It is also of interest to determine separately the 'centre-bed' (ϵ_c) and 'wall-region' (ϵ_w) voidages, obtained by excluding the region within $0.5 d_p$ of the wall for ϵ_c , and taking only that region for ϵ_w . The results are shown in table 3.

Figure 7 : Average void fractions

$\frac{d_t}{d_p}$	ϵ	ϵ_c	ϵ_w
5.58	0.42	0.39	0.50
11.81	0.38	0.36	0.51

Table 3 : Various voidages

The values of d_t/d_p used correspond roughly to the two extreme values used experimentally, which were 5.57 and 11.15. Previous studies of the variation of ϵ with d_t/d_p (7) indicate that the dependence is weak enough for the above correspondence to be good, and interpolation to be used for the third experimental d_t/d_p value, which was 7.43.

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3. MODEL DISCRIMINATION AND PARAMETER ESTIMATION

3.1 Angular temperature measurements

It has been pointed out in chapter two that considerable angular scatter exists in the bed exit temperature measurements. Since the magnitude of the scatter is much greater than can be attributed to experimental error, it must be concluded that non-homogeneity of the bed is the cause. It would be expected that a thermocouple positioned above a solid particle should give a different reading to one above a void space.

It is further remarked in chapter two that no existing model takes account of angular variation of temperature. It would thus appear that either the models presented in chapter one must be modified to include this phenomenon, or regarded as inadequate. The former alternative would probably involve stochastic modelling, a very under-developed area in heat transfer studies. The latter would cast doubt on any parameter estimates obtained with such a model.

The horns of this dilemma may be avoided, if not removed, by a point of view put forward by Paterson (1.3). In this approach the bed temperature T_b is regarded as an angle-averaged temperature, and the angular temperatures as error-affected measurements of this average. If the 'errors' can be shown to behave like true measurement errors then standard estimation methods may be used.

Paterson found his 'errors' to be independent, normally-distributed with zero mean and to have constant variance. Since his measurements were made on much the same apparatus as those in the present work, it might be expected that the same conclusions should hold. An examination of the data in appendix B indicates that this is the case, with the exception of constant variance. The variance decreases on

longer beds since the temperatures are near wall temperature and must lie in a narrower range.

The consequence of this observation is that weighted least squares (WLS) appears to be more applicable in this case than the more common unweighted version (OLS), in order to estimate the parameters. However in long beds the profile is nearly flat and so less sensitive to the parameters. Hence implementation of WLS leads to greatest weight being placed on those measurements which are least sensitive to the parameters, a situation which should be avoided.

For these reasons it was decided to employ OLS methods for parameter estimation.

3.2 Least squares estimation

The parameters are estimated by adjusting them so that the temperatures predicted by the model are in some sense 'close' to the observed temperatures. In least squares estimation the idea of 'closeness' is made more precise by requiring the sum of squares of the discrepancies between observed and predicted temperatures to be a minimum. Thus if there are a total of m readings, the problem is to minimise

$$S(\underline{x}) = \sum_{i=1}^m (T_{b,i}(\underline{x}) - T_{obs,i})^2 = \sum_{i=1}^m f_i(\underline{x})^2 \quad (1)$$

with respect to $\underline{x}$, the vector of p ($\leq m$) adjustable parameters. This involves searching through the allowed or feasible region of $\underline{x}$ to find the point with the smallest value of $S(\underline{x})$. The search is carried out by starting from an initial guess point and forming a sequence of points tending to the minimum.

The least squares method has been described fully in the relevant literature (1,2) and a comparison of search methods suitable for

finding the minimum of a sum-of-squares function is made in the second reference. More general methods are described in the mathematics literature (3). The specific methods used in this work are described in section 3.4.

In appendix B the data for a given packing are grouped by runs, each run representing a different flow rate and bed depth combination. For parameter estimation the runs at different bed depths for the same flow rate (i.e. dimensionless Reynolds number Re) are considered together. This is termed "overall" analysis. If there are N bed depths associated with a given Re , and a total of n readings missing, then fitting to exit and entrance data simultaneously requires setting

$$m = 44 N - n \quad (2)$$

in equation (1). The PF model, however, cannot be fitted to entrance data as the profile at $z = 0$ is specified as a boundary condition. The fact that the profiles in appendix B are not flat, as predicted by the assumptions of this model, may be regarded as an indication of inadequacy. The bulk of the data collected is at bed exit, however, and overall analysis using this data alone may be achieved setting

$$m = 32 N - n \quad (3)$$

The models may also be fitted to data at individual bed depths in order to detect any trend in the parameters with bed depth. This only involves setting $N = 1$ in (2) and (3), and is termed "depth-by-depth" analysis.

When estimates of the parameters are found, it is necessary to give some idea of how good these estimates are, by giving either the variances of the parameters or the associated confidence intervals at an appropriate level of significance.

The $m \times p$ matrix $J_{ij}(\underline{x}) = \left[\frac{\partial f_i}{\partial x_j} \right]_{\underline{x}}$ is called the Jacobian matrix.

If this is available as a by-product of minimisation, or can be calculated, then letting $H = [2J^T J]^{-1}$ and S be the sum of squares, both calculated at the solution $\hat{x}$, an estimate of the variance of the i^{th} parameter x_i is

$$\text{var}(x_i) = \frac{s}{m-p} H_{ii} \quad (4)$$

and of the covariance of x_i and x_j is

$$\text{cov}(x_i, x_j) = \frac{s}{m-p} H_{ij} \quad (5)$$

If x^* is the true solution then the $100(1-\beta)$ confidence interval on $\hat{x}$ is

$$x_i - \sqrt{\text{var}(x_i)} t_{(\beta/2, m-p)} < x_i^* < x_i + \sqrt{\text{var}(x_i)} t_{(\beta/2, m-p)} \quad (6)$$

$i = 1, \dots, p$

where $t_{(\beta/2, m-p)}$ is the $100\beta/2$ percentage point of the t -distribution with $m-p$ degrees of freedom.

Hence the approximation to the Jacobian usually available after minimisation may be used to compute approximate confidence intervals, if the model used is adequate.

3.3 Solution of model equations

The analysis of the solutions to the model equations is facilitated if they are made dimensionless. Thus the equations for the ADPF models become

$$\frac{\partial \theta}{\partial x} = \frac{1}{Pe_A} \frac{\partial^2 \theta}{\partial x^2} + \frac{1}{Pe_R} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{y} \frac{\partial \theta}{\partial y} \right) \quad (7)$$

for $0 < y < 1$ and $-\infty < x < \infty$ (IBC model)

$-\infty < x < L/R$ (FBC model)

Centre-line boundary condition : $\frac{\partial \theta}{\partial y} = 0 \quad (8)$

$$\text{Wall boundary condition : } \frac{\partial \theta}{\partial y} + \text{Bi } \theta = \begin{cases} \text{Bi} & x > 0 \\ 0 & x < 0 \end{cases} \quad (9)$$

$$\text{Upstream boundary condition : } \theta = 0 \text{ at } x = -\infty \quad (10)$$

$$\text{Downstream boundary condition: } \theta = 1 \text{ at } x = \infty \text{ (IBC model)} \quad (11a)$$

$$\frac{\partial \theta}{\partial x} = 0 \text{ at } x = L/R \text{ (FBC model)} \quad (11b)$$

The special case arising from neglect of axial dispersion, the PF model, is obtained from (7) by letting $\text{Pe}_A \rightarrow \infty$:

$$\frac{\partial \theta}{\partial x} = \frac{1}{\text{Pe}_R} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{y} \frac{\partial \theta}{\partial y} \right) \quad (12)$$

for $0 < y < 1$ and $x > 0$

The radial boundary conditions (8) and (9) are unaltered apart from restriction to the range $x > 0$. The downstream boundary condition may be dispensed with as the equation is now of parabolic type, and the upstream condition modified to be $\theta = 0$ at $x = 0$ (13)

It should be noted that Pe_A and Pe_R are based on tube radius R for convenience in these equations; later discussion uses Pe_a and Pe_r based on particle diameter d_p , the more usual form.

The above model equations are easily solved by either Laplace transform or separation of variables technique to give:

(a) ADPF (IBC) model

$$\theta(x,y) = \begin{cases} 1 - \sum_{n=1}^{\infty} \frac{\text{Bi}(1+\beta_n) J_0(\lambda_n y)}{(\text{Bi}^2 + \lambda_n^2) \beta_n J_0(\lambda_n)} \exp \left\{ \frac{\text{Pe}_A x (1-\beta_n)}{2} \right\} & x > 0 \\ \sum_{n=1}^{\infty} \frac{\text{Bi}(\beta_n - 1) J_0(\lambda_n y)}{(\text{Bi}^2 + \lambda_n^2) \beta_n J_0(\lambda_n)} \exp \left\{ \frac{\text{Pe}_A x (1+\beta_n)}{2} \right\} & x < 0 \end{cases} \quad (14)$$

(b) ADPF (FBC) model

$$\theta(x,y) = \begin{cases} 1 - \sum_{n=1}^{\infty} \frac{Bi(1+\beta_n) J_0(\lambda_n y)}{(Bi^2 + \lambda_n^2) \beta_n J_0(\lambda_n)} \left[\exp \left\{ \frac{Pe_A x (1-\beta_n)}{2} \right\} \right. \\ \left. + \frac{\beta_n - 1}{\beta_n + 1} \exp \left\{ \frac{Pe_A (\beta_n (x-2L/R) + x)}{2} \right\} \right] & x > 0 \\ \sum_{n=1}^{\infty} \frac{Bi(\beta_n - 1) J_0(\lambda_n y)}{(Bi^2 + \lambda_n^2) \beta_n J_0(\lambda_n)} \left[\exp \left\{ \frac{Pe_A x (1-\beta_n)}{2} \right\} \right. \\ \left. - \exp \left\{ \frac{Pe_A (\beta_n (x-2L/R) + x)}{2} \right\} \right] & x < 0 \end{cases} \quad (15)$$

(c) PF model

$$\theta(x,y) = 1 - \sum_{n=1}^{\infty} \frac{2Bi J_0(\lambda_n y)}{(Bi^2 + \lambda_n^2) J_0(\lambda_n)} \exp \left\{ - \frac{\lambda_n^2 x}{Pe_R} \right\} \quad (16)$$

The λ_n are the roots of the characteristic equation

$$\lambda_n J_1(\lambda_n) - Bi J_0(\lambda_n) = 0 \quad (17)$$

$$\text{and } \beta_n = \sqrt{1 + 4\lambda_n^2 / (Pe_A Pe_R)}$$

For the PF model an alternative form of the solution may be obtained, useful for small values of x :

$$\theta(x,y) \approx 1 - \left[\frac{2 Bi \sqrt{x}}{\sqrt{y} Pe_R} \operatorname{ierfc} \left\{ \frac{1-y}{2\sqrt{x/Pe_R}} \right\} + \frac{4 Bi x}{\sqrt{y} Pe_R} \times \right. \\ \left. i^2 \operatorname{erfc} \left\{ \frac{1-y}{2\sqrt{x/Pe_R}} \right\} \left(\frac{1}{8y} + \frac{3}{8} - Bi \right) + \dots \right] \quad (18)$$

Similar expressions for the ADPF models are not easily obtained.

Bed exit profiles may be obtained from the truncated series solutions for $x > 0$ in equations (14), (15) and (16). It is not practicable to examine remainder terms from the truncated series due to the complexity of the expressions involved. It is easier to perform

numerical experiments taking 'worst case' values of the parameters. It should be borne in mind that experimental variation in the temperatures measured is large, and hence high accuracy in the analytical solutions is unnecessary. Also the use of these solutions for parameter estimation, where they will be evaluated many times, indicates that as few terms as possible should be used.

It is easily shown that for all models the worst case of convergence occurs for high Pe_R and low x . For the ADPF models the worst case is also for low Pe_A . Bi , however, influences λ_n through equation (17) and so affects each term in the series less straight-forwardly. Hence, it is not easy to determine worst-case values of Bi and a practical range must be covered, 0.5-10 being taken in this work. Taking $Pe_R = 200$, $Pe_A = 0.5$ and $x = 0.3$ gives practical extremes for the experimental conditions covered.

Each solution was evaluated at the 8 bed exit radial measurement points, using up to 50 terms, and examined after the addition of each term. A Newton-Raphson iteration was used to find the λ_n from equation (17) to 6 decimal places. In all cases the solutions agreed to 4 decimal places (0.01°C) well before the 50th term was added, and this solution was taken to be 'exact'. An accuracy requirement of 0.1°C , or 3 decimal places in dimensionless solution, was taken as a reasonable compromise between speed and accuracy. This was achieved in all cases by taking 8 terms of equation (16) and 15 terms for each of equations (14) and (15).

A special case required for the ADPF models is $x = 0$. In this case the solution was barely determined to 1°C (2 decimal places) by 50 terms, and was still not determined by 200 terms. This is due to the absence of the exponential term in x , which 'damps out' the Bessel function oscillations quickly for $x > 0$. Since the only alternative

is a numerical solution which would be slower and of probably no greater accuracy, a 50-term analytic solution may be regarded as a reasonable approximation to profiles at $x = 0$.

3.4 Computation of parameter estimates

The subroutine chosen from the Edinburgh Regional Computing Centre's NAG library (4) to minimise $S(\underline{x})$ in equation (1) takes advantage of the sum of squares form of $S(\underline{x})$. It is iterative and uses a strategy similar to that applied by Powell (5) to systems of nonlinear equations together with a version of the step selection procedure devised by Marquardt (6). Although the partial derivatives $\partial f_i / \partial x_j$ are not required to be supplied by the user, they are assumed to exist and be continuous, and are approximated by finite differences by the subroutine. On exit the sum of squares S and the Jacobian J are available, calculated at the best point.

The iteration is stopped when it is predicted that $S(\underline{x})$ is within "user-specified" accuracy of the minimum. This accuracy must be carefully chosen, since too large a value and the parameter estimates may be poor if $\underline{x}$ changes quickly near the minimum, too small a value and convergence to a minimum may not occur. In the latter case the parameter estimates over the last few iterations should be examined. If there is little change then it may not be necessary (or possible) to obtain the required accuracy, and the 'best' estimates accepted. For this reason it is advisable to print out the parameter values and $S(\underline{x})$ after each iteration.

An ever-present hazard in nonlinear minimisation is that the minimum found is local and not global. To guard against this, a preliminary examination of the sum-of-squares surface $S(\underline{x})$ was made for selected cases over a physically reasonable range of $\underline{x}$, where

$\underline{x} = (Pe_r, Bi)$ for the PF model and $\underline{x} = (Pe_a, Pe_r, Bi)$ for the ADPF models. Only two terms were used in the analytic series to keep the amount of computation practical.

The contours of the surface near the global minimum are shown in figure one for two cases of overall analysis with the PF model. Their shapes confirm those found by De Wasch and Froment (2.1) and Tsang et al (7). From these contours it appears that Pe_r is best determined at lower Re and Bi at higher Re.

Although no other local minima were found on the sum-of-squares surface, there appear to be several near the global minimum. Accordingly several different starting points were taken within the innermost unbroken contour. As the same solution was reached in each case, it is concluded that the 'minima' in figure 1 are the result of misinterpretation of too coarse a mesh by the contour-drawing routine used.

As a final check, a conjugate-gradient method (8) devised by Powell(9) was used with widely scattered starting values for the PF model. It always confirmed the global minimum found by the main minimisation routine.

The starting-values yielded by the grid search proved to be good estimates of the solution point in all cases, and convergence was fairly rapid. For "depth-by-depth" analysis the "overall" analysis solutions were used as starting points. These were also used for the ADPF models and an initial estimate for Pe_a taken to be 0.1, to prevent "overshoot" into $Pe_a < 0$, where the analytic solution breaks down. However, some contours for these models are shown for interest in figures 2 and 3, in the form of sections through the three-dimensional surface. It is seen that the ADPF(IBC) model determines Pe_a better than the ADPF(FBC) model.

Figure 1 : PF model contours, overall analysis

12.7 mm ceramic beads

Figure 2 : ADFP(IBC) model contours, overall analysis

Pe = 536 12.7 mm ceramic beads

Figure 3 : ADPF(FBC) model contours, overall analysis

Re = 536, 12.7 mm ceramic beads

Due to the experimental difficulties described in chapter two, only the data for 12.7mm and 9.5mm ceramic beads were available for analysis for some time. The full range of proposed models (ADPF(IBC), ADPF(FBC) and PF) was therefore fitted to these data only. The model discrimination work described in section 3.5 was then carried out, and as a result of this only the ADPF(IBC) model was fitted to the 6.4mm ceramic beads and 9.5mm steel balls data.

The results of the analysis are given in appendix A in tables A5-A37. The PF model was fitted in both "overall" and "depth-by-depth" modes to the exit data. The ADPF models were fitted both "overall" and "depth-by-depth" to the exit and entrance data simultaneously, and "overall" to the exit data alone. It proved impossible to obtain convergence when fitting a three-parameter model to exit data from a single bed depth. In some other cases, especially for long bed depths, no convergence was also obtained. In all these cases large changes in the parameters produced little or no change in $S(\underline{x})$.

Since the ADPF(IBC) model gives one entrance profile regardless of bed depth, equation (1) was rewritten in terms of dimensionless temperatures so that runs of different T_0 and T_w could be included in the "overall" analysis of this model with exit and entrance data. This has no effect on the parameter estimates obtained.

In the tables each parameter estimate is accompanied by a 95% confidence interval, calculated from equations (4)-(6). The appropriate statistic for a single bed depth is

$$t_{(0.025,30)} = 2.04 \quad (19)$$

while for "overall" analysis $m-p$ may be taken as ∞

$$t_{(0.025,\infty)} = 1.96 \quad (20)$$

From this information the parameter variances are easily calculated from the confidence intervals and so are omitted from the tables.

The dimensionless parameters were actually the ones estimated; from these the physical parameters were easily obtained using the formulae

$$k_r = (Re)(Pr) k_g / Pe_r \quad (21)$$

$$k_a = (Re)(Pr) k_g / Pe_a \quad (22)$$

$$h_w = k_r Bi/R \quad (23)$$

where k_g is conductivity of air at 25°C, and $Pr = 0.72$.

To obtain their variances and covariances the formulae (21) - (23) were expanded as Taylor series about the true parameter values, linearised, and the estimates substituted for the true values. This process leads to the formulae:

$$\text{var}(k_r) \approx (k_r/Pe_r)^2 \text{var}(Pe_r) \quad (24)$$

$$\text{var}(k_a) \approx (k_a/Pe_a)^2 \text{var}(Pe_a) \quad (25)$$

$$\begin{aligned} \text{var}(h_w) \approx & (k_r/R)^2 \text{var}(Bi) + (Bi/R)^2 \text{var}(k_r) \\ & - 2 k_r Bi/(R^2 Pe_r) \text{cov}(Pe_r, Bi) \end{aligned} \quad (26)$$

$$\text{cov}(k_a, k_r) \approx (k_r/Pe_a)^2 \text{cov}(Pe_a, Pe_r) \quad (27)$$

$$\text{cov}(k_r, h_w) \approx (Bi/k_r) \text{var}(k_r) - k_r^2/(R Pe_r) \text{cov}(Pe_r, Bi) \quad (28)$$

$$\text{cov}(k_a, h_w) \approx (Bi/R) \text{cov}(k_a, k_r) - k_a k_r/(R Pe_a) \text{cov}(Pe_a, Bi) \quad (29)$$

From these the approximate 95% confidence intervals for the physical parameters were obtained.

Thus the physical parameters and confidence intervals are also given in appendix A.

Correlation coefficients between the parameters were also obtained from the covariances and a selection of these, in cases where the models were adequate, is also given in appendix A.

3.5 Model discrimination

One consequence of the point of view adopted in section 3.1 is that for each flow rate measurements made at the same radial and axial positions, but at different angular positions, may be regarded as 'replicates'. Thus the exit data at each depth give four replicate sets of eight measurements, and the entrance data give two replicate sets of six measurements.

Hence, a first test of model adequacy is to perform a conventional analysis of variance and F-test for each minimisation along the lines indicated by Kittrell (10). It should be noted, however, that the F-test applies strictly to the case of a model linear in its parameters only. For a nonlinear model it must be interpreted as an indication of the adequacy of fit, to be supported by other tests.

The pure-error sum of squares is calculated in general from

$$SSPE = \sum_{1}^N \left(\sum_{1}^8 \sum_{1}^4 (T_{\text{obs}} - T_{\text{av}})^2 + \sum_{1}^6 \sum_{1}^2 (T_{\text{obs}} - T_{\text{av}})^2 \right) \quad (30)$$

where indices are omitted for clarity and the (different) averages are taken over each set of 4 and 2 replicate measurements respectively.

For "depth-by-depth" analysis $N=1$ and for entrance data omitted the second term is also omitted. For the ADPF(IBC) model the $2N$ entrance profiles are regarded as replicates, rather than N sets of 2 replicates as for the ADPF(FBC) case. This requires a different formula:

$$SSPE = \sum_{1}^N \sum_{1}^8 \sum_{1}^4 (\theta_{\text{obs}} - \theta_{\text{av}})^2 + \sum_{1}^6 \sum_{1}^{2N} (\theta_{\text{obs}} - \theta_{\text{av}})^2 \quad (31)$$

The mean square pure error is given by

$$MSPE = SSPE/n_2 \quad (32)$$

$$\text{where } n_2 = m - (\text{no. of averages found}) \quad (33)$$

MSPE is an unbiased estimate of the error variance; as this appears to vary weakly with bed depth MSPE must be interpreted as

estimating the depth-averaged error variance for the "overall" analysis case.

The mean lack-of-fit sum of squares is given by

$$\text{MSLF} = (S - \text{SSPE}) / n_1 \quad (34)$$

$$\text{where } n_1 = m - p - n_2 \quad (35)$$

If the model is adequate and linear in the parameters then MSLF is an independent unbiased estimator of error variance; if the model is either inadequate or nonlinear then it is a biased estimator.

Hence a test of model inadequacy may be stated

$$F_c = \text{MSLF} / \text{MSPE} > F_{0.05}(n_1, n_2) \quad (36)$$

The results of this test for each fit of a model to data are given in tables A1-A4 of appendix A.

The most stringent test of the models is provided by the case of "overall" analysis, i.e. fitting to several bed depths simultaneously. The case of no entrance data is given in table A1. The PF model was clearly rejected in 10 out of 11 cases, with values of $F_c / F_{0.05}$ between 1.4 and 8. The ADPF(FBC) model gave similar results, also being clearly rejected 10 times. The ADPF(IBC) model, however, was clearly rejected in only one case out of twenty-three, with typical ratio values of 0.4-1.0. Hence little or no significant lack-of-fit was shown by this model.

The inclusion of entrance data, given in table A2, led to rejection of both ADPF models in nearly all cases, except for the 9.5mm ceramic beads data.

In the case of "depth-by-depth" analysis, the PF model fitted to exit data showed significant lack-of-fit in only two cases out of 47. The corresponding analysis with ADPF models was not possible, entrance data had to be included. The models gave the same results, being rejected in 7 out of 47 cases; this was probably the result of the entrance data.

In general, model fit was worst at low Re , probably due to the difficulty of measuring the development of the radial temperature profile, since at low Re the bed attained wall temperature within a few d_p of the entrance.

Since all models exhibited no significant "lack-of-fit" when fitted to individual bed depths, it would appear that radial profiles are adequately represented and that the overall lack-of-fit to bed exit data occurs in the axial direction. This is made clearer by plotting residuals against bed depth at the same radial position. An example for 9.5mm ceramic beads at $Re = 281$ and $r = 9\text{mm}$ is shown in figure 4. The inadequacy of the PF and ADPF(FBC) models is shown by the systematic trend of their residuals, indicating their inability to represent the axial profiles correctly. This is understandable in the case of the PF model, as axial dispersion is omitted; for the ADPF(FBC) model it would appear that the finite boundary condition affects the axial profiles adversely, supporting the observations of section 2.3. There is no obvious trend in the ADPF(IBC) model residuals.

The inability of either ADPF model to fit most of the entrance data is not so easily explained. Examination of the profiles in appendix B shows that in some cases there is great scatter in the 'replicates'; possibly one or two "wild" measurements are having an undue effect as only 12 measurements were taken. Also the disturbance in the packing due to the presence of the thermocouples may have adverse effects.

This first test of model adequacy points to the ADPF(IBC) model as being the only one capable of representing the data adequately. A typical fit of this model to the angular average bed exit data "overall" is shown in figure 5.

Figure 4 : Residual plot, 9.5 mm ceramic beads

Re = 281, r = 9 mm

Figure 5.: Fit of ADPF(IBC) model to angular average temperatures, 9.5 mm ceramic beads, $Re=120$

A second test of the models is provided by graphically examining the depth dependence of the parameters. This is done in figures 6-21, where the results of tables A7-A10 and A26-A37 of appendix A are presented, to illustrate this dependence, when present. A key to the symbols used is given in table 1 :

<u>Symbol</u>	<u>L(12.7mm ceramic)</u>	<u>L(9.5mm ceramic)</u>
O	51mm	51mm
△	102mm	102mm
▽	178mm	152mm
□	204mm	204mm
⊙	356mm	254mm

Table 1 : Key to figures 6-21

In these figures parameter values corresponding to cases of non-fit by the models are omitted.

Figures 6 and 8 show that k_r decreases strongly with bed depth when the PF model is used. In figure 8 the 102mm and 152mm values are the same, as are the 204mm and 254mm data, hence only one set of symbols is shown in each case. The wall coefficient, h_w , also appears to vary with depth and is shown in figures 7 and 9 to pass through a minimum. The depth-dependence of the parameters in the PF model was also noted by De Wasch and Froment (2.1)

When the ADPF(IBC) model is used to analyse the data, figures 10 and 13 show that there is no dependence on bed depth of k_r . This is clearest in figure 13; all the cases of non-fit occurred for 12.7mm ceramic beads, so there are few values in figure 10. The situation is not so clear-cut for h_w , there appears to be some weak dependence still in figure 14, with a shallow minimum; however this dependence is not significant within the scatter of the data. As a check figures 12 and 15 show there is no depth dependence of k_a .

Figure 6 : Depth dependence of k_r - 12.7 mm ceramic beads, PF model

Figure 7 : Depth dependence of h_w - 12.7 mm ceramic beads, PF model

Figure 8 : Depth dependence of k_r - 9.5 mm ceramic beads, PF model

Figure 9 : Depth dependence of h_w - 9.5 mm ceramic beads, PF model

Figure 10 : Depth dependence of k_r - 12.7 mm ceramic beads, ADPF(IBC) model

Figure 11 : Depth dependence of h_w - 12.7 mm ceramic beads, ADPF(IBC) model

Figure 12 : Depth dependence of k_a - 12.7 mm ceramic beads, ADPF(IBC) model

Figure 13 : Depth dependence of k_r - 9.5 mm ceramic beads, ADPF(IBC) model

Figure 14 : Depth dependence of h_w - 9.5 mm ceramic beads, ADPF(IBC) model

Figure 15 : Depth dependence of k_a - 9.5 mm ceramic beads, ADPF(IBC) model

Figure 16 : Depth dependence of k_r - 12.7 mm ceramic beads, ADPF(FBC) model

Figure 17 : Depth dependence of h_w - 12.7 mm ceramic beads, ADPF(FBC) model

Figure 18 : Depth dependence of k_a - 12.7 mm ceramic beads, ADPF(FBC) model

Figure 19 : Depth dependence of k_r - 9.5 mm ceramic beads, ADPF(FBC) model

Figure 20 : Depth dependence of h_w - 9.5 mm ceramic beads, ADPF(FBC) model

Figure 21 : Depth dependence of k_a - 9.5 mm ceramic beads, ADPF(FBC) model

Virtually the same results hold for the ADPF(FBC) model.

Figure 16 suggests that k_r might be depth-dependent, but this is not supported by figure 19, although the values are spread more than for the IBC case. These observations support the idea that the omission of axial dispersion causes depth-dependence of parameters, but that the finite boundary condition is not realistic and causes lack-of-fit.

De Wasch and Froment correlated their data from the longest bed used, seeking to minimise axial dispersion effects. Comparison of figures 8 and 13 shows reasonable agreement for k_r ; however the parameters become difficult to determine at longer bed depths and this procedure cannot be recommended.

Since the literature parameter values were all obtained using the PF model on beds of different lengths this behaviour of this model may be a significant factor in explaining some of the scatter in literature correlations.

From the above tests it is concluded that only the ADPF(IBC) model can be accepted as giving reasonable agreement with the bed exit data. Accordingly only the parameter estimates given by this model in "overall" analysis (tables A13-A16 in appendix A) are used in the rest of this work.

Finally it may be noted from tables A11, A12 and A17 that there is a considerable degree of correlation between k_r and h_w , whereas k_r and k_a appear to be independent, as do k_a and h_w . This lends some weight to the idea that h_w is no more than an empirical parameter needed in the model to account for a decreasing k_r near the wall, and may have no real physical basis.

3.6 Empirical correlation of parameters

The radial Peclet number Pe_r and the Biot number Bi were correlated out of curiosity against flow rate in the form of Re , although it was felt that yet more empirical correlations would be of little use. The axial Peclet numbers were so scattered as to make correlation useless; they increased with Re in rough agreement with the values of Votruba et al. (14).

An idea of the spread in the literature data for Pe_r may be obtained from table 2. In this table the results of each reference for packings nearest those of the present work are presented; the correlations have all been reduced to a common form, taking $Pr = 0.72$ for air where necessary. Earlier authors used one correlation for all packings; later different conductivity packings were given separate correlations and more recently De Wasch and Froment included dependence on d_t/d_p .

The present data were correlated by standard linear regression methods (15) and gave the correlations:

$$(Pe_r)^{-1} = 0.11 + 20.64 (Re)^{-1} \quad \text{for ceramic beads} \quad (37)$$

$$(Pe_r)^{-1} = 0.06 + 57.87 (Re)^{-1} \quad \text{for steel balls} \quad (38)$$

All the ceramic beads data were well correlated by a single line as shown in figure 22; 95% confidence intervals are also shown.

Of more interest is the Biot number, since no literature correlations exist for this parameter. The effective wall heat transfer coefficient h_w is invariably correlated in the form of a wall Nusselt number $Nu_w = h_w d_p / k_g$. The scatter of correlations for this parameter is vast, too vast to be presented here. Instead it is interesting to note that after some heuristic searching, all the present Biot number values were correlated by the single curve

$$Bi (d_p/d_t)^{1/2} = 5.73 (Re)^{-0.262} \quad (39)$$

as shown in figure 23.

<u>Authors</u>	<u>Ref.</u>	<u>Packing</u>	<u>d_t/d_p</u>	<u>Correlation</u>	<u>Re range</u>
Calderbank & Pogorski	11	all	all	$(Pe_r)^{-1} = 0.15 + 15.6(Re)^{-1}$	50 - 2000
Plautz & Johnstone	12	all	all	$(Pe_r)^{-1} = 0.11 + 38.2(Re)^{-1}$	100 - 2000
Yagi, Kunii & Wakao	13	glass beads	10	$(Pe_r)^{-1} = 0.08 + 11.1(Re)^{-1}$	0 - 800
			4.8	$(Pe_r)^{-1} = 0.06 + 11.1(Re)^{-1}$	0 - 800
Agnew & Potter	1.1	steel cylinders	10	$(Pe_r)^{-1} = 0.19 + 20.8(Re)^{-1}$	25 - 350
		glass spheres	4.5	$(Pe_r)^{-1} = 0.17 + 6.9(Re)^{-1}$	25 - 350
De Wasch & Froment	2.1	iron oxide cylinders	10.4	$(Pe_r)^{-1} = \frac{0.13}{1+46(d_p/d_t)^2}$	200 - 900
				$+ 18.4(Re)^{-1}$ $= 0.09 + 18.4(Re)^{-1}$	

Table 2 : Some literature correlations for effective radial thermal conductivity

Figure 22 : Empirical correlation of radial Peclet numbers

Figure 23 : Empirical correlation of Biot numbers

Since Biot numbers for steel packing are similar to those for ceramic packing, yet k_r is some 30-50% greater, these results show that h_w is dependent on particle conductivity k_p . This implies a significant amount of heat transferred by wall-solid contacts. This could explain some of the scatter in published correlations for h_w using packings of different conductivity.

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4. THEORETICAL PREDICTION OF EFFECTIVE HEAT TRANSFER PARAMETERS

4.1 Introduction

It was suggested in section 1.1 and later in section 3.4, that the empirical formulae usually obtained from studies of the present kind are rather unsatisfactory, due to the dangers of extrapolation outside the range of laboratory conditions, and also to the great scatter in the results of different workers. In chapter three two possible explanations for this scatter were noted: (a) model inadequacy leading to systematic bias in the parameter estimates; (b) failure to include dependence on variables such as particle conductivity in the correlating formulae. The former cause may be eliminated by statistical testing of models fitted to the data; the latter cause is more difficult, as the "effective" parameters estimated "lump" together several processes, thus denying the engineer the use of physical intuition. A prohibitive amount of experimentation would be required to determine the effect of every variable.

For these reasons it is desirable to employ a theoretical approach which allows prediction of the effective parameters. By "theoretical approach" is meant the development of a model relating effective heat transfer parameters to the essential underlying and independently measurable heat transfer processes, so that no empirical or adjustable constants are involved. The remainder of the present work is concerned with such a development.

Previous work in this area is relatively scarce, and has for the most part followed the lines begun by Argo and Smith (1) for the prediction of effective radial conductivity from a physical model of contacting spheres in the bed. This approach has been successful in

predicting effective radial stagnant conductivity k_r^0 (2,3), and the extension to flow conditions, assuming additivity of static and dynamic contributions, has been successful for beds of reasonably large d_t/d_p (3.13), although less so for beds of low d_t/d_p (1.1). A general approach, applicable to all the parameters, is, however, lacking.

A different approach is proposed here, in which a two-phase diffusional model is considered, which contains many, if not all, of the basic heat transfer parameters which are believed to be effective and which can be determined experimentally. Relationships between the parameters of this model and those of the ADPF(IBC) model are obtained by "matching" their solutions in a sense defined later. This procedure deals with all the effective parameters simultaneously; the relationships found are compared with experimental results in chapter 5.

4.2 Two-phase continuum model

The two-phase model considered here is similar to that proposed by Olbrich (1.15) and considered briefly in section 1.3. It would appear to be the natural extension to two phases of homogeneous continuum models, in that each phase is regarded as a homogeneous continuum. Thus different temperature profiles in solid and fluid phases are allowed.

This approach may appear contradictory, in that the solid particles in the bed are essentially discrete, having, in the case of spheres, no more than negligibly small finite area contacts. However, if the "pockets" of stagnant fluid surrounding each contact are included as part of the "solid" phase, rather than the fluid, then it is possible to regard this "solid" phase as a continuum, with an

effective solid conductivity. This conductivity is further interpreted in a later section.

The main reason for preferring this model is that it allows for particle-to-particle heat transfer, and also particle-to-wall heat transfer. These have been shown to be important, at least for beds of low d_t/d_p , both in the present work and by Paterson (1.3).

It should be noticed that the model presented here is more general than that of Olbrich, in that axial dispersion in both phases is included. The assumptions made in setting up the ADPF(IBC) model are retained. The resulting differential heat balances yield:

Fluid phase

$$k_{rf} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + k_{af} \frac{\partial^2 T}{\partial z^2} - ah (T-t) = Gc_p \frac{\partial T}{\partial z} \quad (1)$$

Solid phase

$$k_{rs} \left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} \right) + k_{as} \frac{\partial^2 t}{\partial z^2} + ah (T-t) = 0 \quad (2)$$

with boundary conditions

$$\frac{\partial T}{\partial r} = \frac{\partial t}{\partial r} = 0 \quad \text{at } r = 0 \quad (3)$$

$$- k_{rf} \frac{\partial T}{\partial r} = \begin{cases} h_{wf} (T - T_w) & z > 0 \\ h_{wf} (T - T_o) & z < 0 \end{cases} \quad \text{at } r = R \quad (4)$$

$$- k_{rs} \frac{\partial t}{\partial r} = \begin{cases} h_{ws} (t - T_w) & z > 0 \\ h_{ws} (t - T_o) & z < 0 \end{cases} \quad \text{at } r = R \quad (5)$$

$$t, T \rightarrow T_o \quad \text{as } z \rightarrow -\infty \quad (6)$$

$$t, T \rightarrow T_w \quad \text{as } z \rightarrow \infty \quad (7)$$

$$T(0^+) = T(0^-) ; \quad t(0^+) = t(0^-) \quad (8)$$

$$\frac{\partial T}{\partial z} (0^+) = \frac{\partial T}{\partial z} (0^-) ; \quad \frac{\partial t}{\partial z} (0^+) = \frac{\partial t}{\partial z} (0^-) \quad (9)$$

The boundary conditions for this model have been chosen to suit the bed geometry of the present study. The parameters in the model are interpreted and discussed in section 4.5. As usual it is convenient to render the equations dimensionless:

Fluid phase

$$\frac{1}{Pe_{RF}} \left(\frac{\partial^2 T^*}{\partial y^2} + \frac{1}{y} \frac{\partial T^*}{\partial y} \right) + \frac{1}{Pe_{AF}} \frac{\partial^2 T^*}{\partial x^2} - \frac{N_F}{Pe_{RF}} (T^* - t^*) = \frac{\partial T^*}{\partial x} \quad (10)$$

Solid phase

$$\left(\frac{\partial^2 t^*}{\partial y^2} + \frac{1}{y} \frac{\partial t^*}{\partial y} \right) + K \frac{\partial^2 t^*}{\partial x^2} + N_S (T^* - t^*) = 0 \quad (11)$$

and the boundary conditions become

$$\frac{\partial T^*}{\partial y} = \frac{\partial t^*}{\partial y} = 0 \quad \text{at } y = 0 \quad (12)$$

$$\frac{\partial T^*}{\partial y} + Bi_f T^* = \begin{cases} Bi_f & x > 0 \\ 0 & x < 0 \end{cases} \quad \text{at } y = 1 \quad (13)$$

$$\frac{\partial t^*}{\partial y} + Bi_s t^* = \begin{cases} Bi_s & x > 0 \\ 0 & x < 0 \end{cases} \quad \text{at } y = 1 \quad (14)$$

$$t^*, T^* \rightarrow 0 \quad \text{as } x \rightarrow -\infty \quad (15)$$

$$t^*, T^* \rightarrow 1 \quad \text{as } x \rightarrow \infty \quad (16)$$

$$T^*(0^+) = T^*(0^-) ; \quad t^*(0^+) = t^*(0^-) \quad (17)$$

$$\frac{\partial T^*}{\partial x}(0^+) = \frac{\partial T^*}{\partial x}(0^-) ; \quad \frac{\partial t^*}{\partial x}(0^+) = \frac{\partial t^*}{\partial x}(0^-) \quad (18)$$

It is the dimensionless equations (10) - (18) which are solved in 4.3.

4.3 Approximate solutions

Olbrich (1.15) obtained a (complicated) exact analytical solution of the two-phase model by neglecting axial conduction terms in both phases. This procedure cannot be followed here as matching with an

axially-dispersed model is required. To date, a totally-determined analytic solution of the full model does not appear possible.

Also, since matching of models usually means equating the first terms of respective infinite series solutions, itself an approximate procedure, it is preferable to derive approximate solutions of the models at the outset.

The radial derivatives in equations (10) - (18) are first discretised by a one-point collocation technique, as described by Villadsen and Michelson (4). The collocation point is chosen, after consideration of the form of the wall boundary conditions (13) and (14), to be the zero of the first degree symmetric Legendre polynomial

$$P_1^{(0,0)}(y^2) = 2y^2 - 1 \quad (19)$$

Denote this by $y_1 (= 1/\sqrt{2})$ and further let $y_2 = 1$; then write

$$T^*(x, y_i) = T_i^*(x) \quad \text{and} \quad t^*(x, y_i) = t_i^*(x).$$

Then, suppressing arguments,

$$\left(\frac{\partial^2 T^*}{\partial y^2} + \frac{1}{y} \frac{\partial T^*}{\partial y} \right)_{y=y_1} = -8 T_1^* + 8 T_2^* \quad (20)$$

$$\left(\frac{\partial T^*}{\partial y} \right)_{y=1} = -4 T_1^* + 4 T_2^* \quad (21)$$

(20) is substituted into (10), and T_2^* eliminated via the wall boundary condition (13) and (21) to give

$$\begin{aligned} \frac{1}{Pe_{AF}} \frac{d^2 T_1^*}{dx^2} - \frac{dT_1^*}{dx} - \left(\gamma + \frac{N_F}{Pe_{RF}} \right) T_1^* + \frac{N_F}{Pe_{RF}} t_1^* \\ = \begin{cases} -\gamma & (x > 0) \\ 0 & (x < 0) \end{cases} \end{aligned} \quad (22)$$

$$\text{where } \gamma = 8 Bi_f / (Pe_{RF} (Bi_f + 4)) \quad (23)$$

Similarly equation (11) becomes

$$\eta \frac{d^2 t_1^*}{dx^2} - \left(\alpha + \frac{N_S}{8} \right) t_1^* + \frac{N_S}{8} T_1^* = \begin{cases} -\alpha & (x > 0) \\ 0 & (x < 0) \end{cases} \quad (24)$$

$$\text{where } \eta = K/8 \text{ and } \alpha = Bi_s / (Bi_s + 4) \quad (25)$$

The analytical solution of this pair of coupled boundary-value problems requires the roots of a 4th order polynomial, which must be found numerically due to the complexity of the parameters. An exact solution to (22) and (24) is possible under certain restrictive assumptions

$$(i) \quad T_1^* = t_1^*$$

$$\text{or} \quad (ii) \quad K = 0$$

Neither of these is acceptable, and it is preferable to develop an approximate solution via perturbation methods. The perturbation parameter is η , which is approximately 0.125. Hence the solution is approximated by perturbing away from the solution obtained under assumption (ii) above.

$$\text{Let } t_1^*(x, \eta) = \sum_{m=0}^{\infty} \eta^m \psi_m(x) \quad (26)$$

Substituting (26) into (24) and equating coefficients of like powers of η yields

$$\psi_0 = \begin{cases} \left(\frac{N_S}{8} T_1^* + \alpha \right) / \left(\frac{N_S}{8} + \alpha \right) & x > 0 \\ \frac{N_S}{8} T_1^* / \left(\frac{N_S}{8} + \alpha \right) & x < 0 \end{cases} \quad (27)$$

$$\begin{aligned} \psi_m &= \psi_{m-1}'' / (N_S/8 + \alpha) & m \geq 1 \\ &= \frac{N_S}{8} T_1^{*(2m)} / (N_S/8 + \alpha)^{m+1} \end{aligned} \quad (28)$$

If (26) is truncated after $k+1$ terms and substituted into (22), an order $2k$ ODE in T_1^* results. Hence, it is practical to consider only $k=1$, giving

$$\frac{1}{Pe'_{AF}} \frac{d^2 T_1^{\circ}}{dx^2} - \frac{dT_1^{\circ}}{dx} - \delta T_1^{\circ} = \begin{cases} \delta & (x > 0) \\ 0 & (x < 0) \end{cases} \quad (29)$$

where

$$\frac{1}{Pe'_{AF}} = \frac{1}{Pe_{AF}} + \frac{\eta N_S N_F}{8 Pe_{RF} (\frac{N_S}{8} + \alpha)^2} \quad (30)$$

$$\delta = \gamma + \frac{\alpha N_F}{Pe_{RF} (\frac{N_S}{8} + \alpha)} \quad (31)$$

The solution of equation (29) is straightforward; the constants of integration being obtained via the boundary conditions at $x = \pm \infty$ and the continuity conditions at $x = 0$:

$$T_1^{\circ}(x) = \begin{cases} 1 - \frac{1+\Delta}{2\Delta} \exp \left\{ \frac{Pe'_{AF} x (1-\Delta)}{2} \right\} & x > 0 \\ \frac{\Delta-1}{2\Delta} \exp \left\{ \frac{Pe'_{AF} x (1+\Delta)}{2} \right\} & x < 0 \end{cases} \quad (32)$$

$$\text{where } \Delta = \sqrt{1 + \frac{4}{Pe'_{AF}} \delta} \quad (33)$$

The fluid-phase approximate solution is found by interpolation between T_1° and T_2° :

$$T^{\circ}(x,y) = 2(1-y^2) T_1^{\circ}(x) + (2y^2-1) T_2^{\circ}(x) \\ = \begin{cases} 1 + \frac{\{Bi_f(y^2-1)-2\}}{Bi_f+4} \cdot \frac{1+\Delta}{\Delta} \exp \left\{ \frac{Pe'_{AF} x (1-\Delta)}{2} \right\} & (x > 0) \\ \frac{\{Bi_f(y^2-1)-2\}}{Bi_f+4} \cdot \frac{1-\Delta}{\Delta} \exp \left\{ \frac{Pe'_{AF} x (1+\Delta)}{2} \right\} & (x < 0) \end{cases} \quad (34)$$

It is easy to obtain the solid-phase approximate solution, if desired, by back-substitution of (32) into the truncated (26).

A similar procedure is used to obtain an approximate solution to the ADPF(IBC) model, equations (3.7) - (3.11a). This time the application

of the one-point collocation technique results in a single boundary-value problem, which is easily solved analytically. The procedure is routine, so only the solution is given here:

$$\theta(x,y) = \begin{cases} 1 + \left\{ \frac{Bi(y^2-1)-2}{Bi+4} \right\} \left(\frac{1+\beta}{\beta} \right) \exp \left\{ \frac{Pe_A x(1-\beta)}{2} \right\} & x > 0 \\ \left\{ \frac{Bi(y^2-1)-2}{Bi+4} \right\} \left(\frac{1-\beta}{\beta} \right) \exp \left\{ \frac{Pe_A x(1+\beta)}{2} \right\} & x < 0 \end{cases} \quad (35)$$

$$\text{where } \beta = \sqrt{1 + \frac{32Bi/(Bi+4)}{Pe_A Pe_R}} \quad (36)$$

4.4 Model matching

There are doubtless many ways in which the two approximate solutions found in section 4.3 may be matched. Possibly the best guide as to which to choose is to consider the experimental situation used to determine the parameters. In the present case, as in many other studies, the radial temperature profile of the fluid leaving the bed was measured by an array of thermocouples suspended above the bed. This would seem to suggest a simple matching of the homogeneous model solution with that of the fluid phase in the two-phase model.

Some workers have measured the central axis temperature profile by inserting thermocouples radially through the wall (5, 1.1). The exit and entrance mixing-cup temperatures were also measured. This suggests using some average temperature from the two-phase model, although it is not clear how the averaging should be carried out. This is also the more dubious experimental procedure, as noted in section 2.2.

The first procedure is employed here, and comparison of equations (34) and (35) shows that the two approximate solutions may be matched

exactly at all points (x,y) by the following relationships

$$Bi = Bi_f \quad (37)$$

$$Pe_A = Pe'_{AF} \quad (38)$$

$$\beta = \Delta \quad (39)$$

Relationships (38) and (39) may be expanded in terms of the original dimensionless parameters and the more usually-defined Peclet numbers, and after a little manipulation become

$$\frac{1}{Pe_a} = \frac{1}{Pe_{af}} + \frac{K N_F / N_S}{Pe_{rf} \left(1 + \frac{8 Bi_s}{N_S (Bi_s + 4)} \right)^2} \quad (40)$$

$$\frac{1}{Pe_r} = \frac{1}{Pe_{rf}} \left(1 + \frac{N_F \left(\frac{Bi_f + 4}{Bi_f} \right)}{8 + N_S \left(\frac{Bi_s + 4}{Bi_s} \right)} \right) \quad (41)$$

These relationships are the most comprehensive so far derived for predicting the homogeneous continuum model parameters, and show how complicated is the synthesis of the effective parameters from the more basic ones. The terms on the right-hand sides of equations (37), (40) and (41) can be estimated from existing correlations, a review of which is given in the next section, so that no adjustable constants are involved.

4.5 Relevant heat transfer correlations

The correlations available in the literature for estimation of the basic heat transfer parameters required in the above prediction formulae are reviewed in this section

(a) Fluid phase radial Peclet number (Pe_{rf})

This group can be estimated from radial mixing experiments, since a heat/mass analogy implies $Pe_{rf} = Pe_{rm}$. At low Re molecular

diffusion dominates, while at higher values (say $Re > 40$) turbulent mixing is the dominant mechanism and Pe_{rm} , and hence Pe_{rf} , tends to a constant value $Pe_{rm}(\infty)$.

Gunn (6) suggests the interpolating correlation

$$\frac{1}{Pe_{rm}} = \frac{1}{Pe_{rm}(\infty)} + \frac{0.67 \epsilon}{(Re)(Sc)} \quad (42)$$

based on the work of Pryce (7) in the range $1 < Re < 420$. This also fits the results of Hiby (8) well, although it does not account for the rather weak maximum in the observed Pe_{rm} vs. Re relationship.

Gunn takes $Pe_{rm}(\infty) = 11$ in (42), with no dependence on (d_t/d_p) . Earlier work by Fahien and Smith (9), however finds a dependence given by

$$Pe_{rm}(\infty) = A \{1 + 19.4 (d_p/d_t)^2\} \quad (43)$$

with $8 \leq A \leq 10$. Olbrich and Potter (10) also use this form, but take $A = 7$ for their experiments at low d_t/d_p . Closer examination of the results is required to resolve this disagreement.

Olbrich and Potter use the form (43) in imitation of the earlier work of Fahien and Smith; they present a graph justifying their decision and shown in figure 1.

Figure 1 : Graph of Olbrich and Potter

The curve they fitted is shown by the full line, and depends strongly on the point at $d_t/d_p = 4$; they give no confidence intervals and it is impossible to know if the strong upward trend is significant, possibly the weaker dependence shown by the dashed line is just as good a fit.

The greater part of the results of Fahien and Smith are point estimates of Pe_{rm} , which must be considered unreliable as they rely on differentiating the radial data twice. Examination of the average Peclet numbers presented in their paper shows (i) only one set of results is clearly separated from the others; (ii) the distinction between sets of data at different d_t/d_p is less than a reasonable estimate of confidence intervals would be. Their results are also suspect in that axial dispersion was ignored; Roemer et al (11) found this neglect to result in over-estimation of Pe_{rm} by 10-15%.

Gunn and Pryce used acceptable statistical methods to analyse their data, and also included axial dispersion; however the ranges of Re covered by their different results at each d_t/d_p do not overlap sufficiently for definite conclusions to be drawn.

It would appear that equation (43) must be regarded with suspicion; Fahien and Smith did not use (d_t/d_p) ratios of sufficient severity to test their correlation adequately. It may be that there is a dependence of $Pe_{rm}(\infty)$ on d_t/d_p , although a weaker one. As the majority of published data lies in the range 8-12, the best that can be done at present is to take $Pe_{rm}(\infty) = 10$ in equation (42).

(b) Fluid phase axial Peclet number $-(Pe_{af})$

Similarly to (a), this group can be estimated from axial mixing experiments so that $Pe_{af} = Pe_{am}$. The bulk of the literature data for axial mixing in gases lies in the region of $(d_t/d_p) > 12$. For

this range the semi-theoretical equation of Edwards and Richardson (12,13) appears adequate

$$\frac{1}{Pe_{am}} = \frac{0.73 \epsilon}{(Re)(Sc)} + \frac{0.5}{1 + \epsilon \beta / (Re Sc)} \quad (44)$$

β is determined experimentally as $\beta = \begin{cases} 9.7 & \text{(air-argon)} \\ 14.1 & \text{(helium-nitrogen)} \end{cases}$

A similar correlation is due to Hiby (8)

$$\frac{1}{Pe_{am}} = \frac{0.67 \epsilon}{(Re)(Sc)} + \frac{0.65}{1 + 7 \sqrt{\frac{\epsilon}{ReSc}}} \quad (45)$$

which has been shown by Gunn (6) to give good agreement with the results of Pryce (7) and Evans and Kenney (14). However, (44) predicts $Pe_{am}(\infty) = 2$, in agreement with the random walk theory of Aris and Amundson (15), in which the interstitial voids act as perfect mixers.

For $(d_t/d_p) < 12$, however, this theory is not applicable, and recently Hsiang and Haynes (16) have presented results for $1.4 \leq (d_t/d_p) \leq 8$ in which $Pe_{am}(\infty)$ appears to be in the region of 0.5 - 1.0; however their experiments do not cover high enough flow rates in most cases to ensure that the limiting value has been reached. They explain this augmented axial dispersion by transfer to and from a fast moving wall stream, but can present only empirically fitted formulae for their data. Confirmation and extension of these results to higher Re will be necessary before reliable correlations can be developed for this (d_t/d_p) range.

For $1 \leq (d_t/d_p) \leq 1.4$ the wall streams break down and the voids again behave like perfect mixers. In this region the results of Scott et al (17), supported by some runs of Hsiang and Haynes, confirm that

$Pe_{rm}(\infty) = 2$ and that equation (44) may again be used.

(c) Effective solid phase conductivities (k_{rs} , k_{as})

As noted in section 4.2, the solid phase conductivities are not equal to the pellet conductivity (k_p) as the "solid" includes stagnant fluid.

Olbrich (1.15) reports work of Ikeda et al (18) which established that $k_{as} = k^0$, the stagnant bed conductivity. This is supported by Littmann and Sliva (19), who determined k_{as} under flow conditions and found their average value to agree with static measurements by Preston (20). Kunii and Smith (2) and Yagi et al (21) found $k_r^0 = k_a^0 = k^0$. Thus it seems reasonable to take $k_{rs} = k_{as}$ and to estimate this value by omitting the fluid phase conduction term from one of the many prediction formulae for k^0 .

A review of these formulae has been given by Zehner and Schlünder (22), and they also present their own, which appears to be more general than the others, as it is valid for $0 < \epsilon < 1$. It also has various limiting properties, given in the paper, which are not all possessed by any other formula. They consider a unit cell in the bed divided into fluid and solid continuous phases based on voidage, rather than the usual "two-spheres" model of Kunii and Smith (2). This seems more appropriate for the present studies, and as Zehner and Schlünder correlate virtually all stagnant bed data to present to within 30% by their equation, it will be used here.

It should be noted that no formulae for k^0 are truly theoretical; all have at least one adjustable parameter to be determined from data.

Most experiments on heat transfer are carried out under conditions of temperature and pressure which allow radiation and finite area contacts to be neglected. Then subtracting the fluid conduction term gives

$$k^o = \frac{2 \sqrt{1-\epsilon}}{1-B(k_g/k_p)} \left[\frac{(1-k_g/k_p)^B}{(1-Bk_g/k_p)^2} \ln \left(\frac{k_p}{Bk_g} \right) - \frac{B+1}{2} - \frac{B-1}{1-Bk_g/k_p} \right] \quad (46)$$

$$\text{where } B = C \left(\frac{1-\epsilon}{\epsilon} \right)^{10/9} \quad (47)$$

$$\text{and } C = \begin{cases} 1.25 & (\text{spheres}) \\ 1.4 & (\text{crushed particles}) \\ 2.5 & (\text{cylinders/Raschig rings}) \end{cases}$$

[Note that for $Bk_g/k_p = 1$ equation (46) is not defined;

$$k^o = \sqrt{1-\epsilon} \left(\frac{1 + 2B^3 - 3B^2}{3(B-1)^2} \right) \quad (48)$$

must be used instead.]

It is interesting to note that the fluid conduction term is given by Zehner and Schlünder by analogy from measured stagnant diffusivities as $(1 - \sqrt{1-\epsilon}) k_g$ ($\approx 0.23 k_g$ for $\epsilon = 0.4$). This is less than the expected value of ϵk_g due to the fraction of fluid associated with the solid phase. This value, however compares closely with the values $0.67 \epsilon k_g$ ($\approx 0.27 k_g$ for $\epsilon = 0.4$) and $0.73 \epsilon k_g$ ($\approx 0.29 k_g$ for $\epsilon = 0.4$) obtained by extrapolating equations (42) and (44) to stagnant conditions. It may be surmised that the factors 0.67 and 0.73, usually labelled "tortuosity factors", are no more than corrections for the intra-solid-phase fluid.

(d) Wall heat transfer coefficients (h_{wf} , h_{ws})

The fluid-to-wall heat transfer coefficient h_{wf} is obtained from mass transfer experiments using a heat and mass transfer analogy, since direct determination from heat transfer experiments is made difficult due to simultaneous heat transfer between the wall and the solid.

Yagi and Wakao (5) measured the dissolution rate of 2 - naphthol coated on the inner wall of the tube to water flowing through the bed. By analogy this gives

$$Nu_{wf} = \begin{cases} 0.60 (Pr)^{0.33} (Re)^{0.5} & (1 < Re < 40) \\ 0.20 (Pr)^{0.33} (Re)^{0.8} & (40 < Re < 2000) \end{cases} \quad (49)$$

The validity of this correlation for low Re must be in doubt as axial dispersion was neglected in analysing the data.

Kunii et al (23) report results of Kunii and Suzuki (24) giving

$$Nu_{wf} = \frac{0.06}{\epsilon_w^2} (Pr)^{0.33} (Re)^{0.75} \quad (Re > 100) \quad (50)$$

where ϵ_w is "near-wall" voidage, determined in section 2.4. No experimental details are reported.

Olbrich and Potter (10) measured the vapourisation of mercury into a nitrogen stream. They had to consider also simultaneous axial and radial gas mixing in the voids as well as accounting for the pressure drop across the bed, side-effects absent in the work of Yagi and Wakao. They obtained

$$Nu_{wf} = 8.9 (Pr)^{0.33} (Re)^{0.34} \quad (100 < Re < 3000) \quad (51)$$

substantially different from the other workers.

There is little, if any, published data on h_{ws} . Olbrich (1.15) has given a theoretical lower bound by considering hexagonally close-packed spheres:

$$Bi_s \geq 2.12 R/d_p \quad (52)$$

In practice Bi_s has little effect on the equations when greater than 10, so unless R/d_p is very small it is permissible to set $Bi_s = \infty$.

(e) Interphase heat transfer coefficient (h)

The more commonly correlated interphase heat transfer coefficient h_{fs} is related to particle surface temperature, whereas the smoothing of solid temperature variations in the present two-phase model requires that h be related to average particle temperature, and is consequently a lumping of transfer across a film and conduction through the pellet.

The appropriate lumping was first shown by Stuke (25):

$$\frac{1}{h} = \frac{1}{h_{fs}} + \frac{d_p/\beta}{k_p} \quad (53)$$

where $\beta = 10, 8$ and 6 for spheres, cylinders and slabs respectively, and d_p is the sphere or cylinder diameter or slab thickness. This lumping has been confirmed by later work (26).

The available correlations for h_{fs} may conveniently be considered in two groups

(i) High Re correlations

For a moderate to high Reynolds number the results of many studies are in approximate agreement and several correlations averaging a lot of data have been presented. Some are given in table 1; that of Handley and Heggs is widely-used and is therefore adopted in this work.

<u>Author(s)</u>	<u>Ref.</u>	<u>Correlation</u>	<u>Re range</u>
Handley & Heggs	27	$Nu_{fs} = \frac{0.255}{\epsilon} (Pr)^{0.33} (Re)^{0.665}$	100 - 6000
Thodos	28	$Nu_{fs} = \frac{0.357}{\epsilon} (Pr)^{0.33} (Re)^{0.641}$	3 - 1000
Aerov & Umnik	29	$Nu_{fs} = \frac{0.395}{\epsilon} (Pr)^{0.33} (Re)^{0.64}$	30 - 10^5
Vortmeyer	30	$Nu_{fs} = \frac{0.288}{\epsilon} \frac{Re}{1.22 Re^{0.35} - 1.9}$	> 60

Table 1 : Correlations for Nu_{fs}

(ii) Low Re correlations

There is considerable disagreement in the literature over the values of Nu_{fs} in the range $Re < 10$, with estimates from both theory and experiment differing by up to two orders of magnitude. In particular, the limiting value as $Re \rightarrow 0$ is in dispute. A summary of the results available is given by Nelson and Galloway (31), who also present a theory predicting $Nu_{fs} \rightarrow 0$ as $Re \rightarrow 0$. The most recent paper at the time of writing is by Gunn (32), which updates this summary, and also uses his probabilistic model to give $Nu_{fs} \rightarrow 3.54$ as $Re \rightarrow 0$, in reasonable agreement with the numerical results of Sorensen and Stewart (33), giving $Nu_{fs} \rightarrow 3.9$ as $Re \rightarrow 0$.

Experimental results range from $Nu_{fs} \rightarrow 0$ from Kunii and Suzuki (34), to $Nu_{fs} \rightarrow 12.4$, inferred from the mass transfer work of Miyauchi et al (35). There is some doubt about the results giving $Nu_{fs} \rightarrow 0$ as axial dispersion was omitted from the analysis of results (36).

Any choice of correlation is clearly arbitrary; however Littman and Sliva (19) did careful heat transfer experiments using the two-phase quasi-continuum model to analyse their results, and give a correlation which appears to extend the Handley-Heggs correlation to lower Re .

$$Nu_{fs} = 0.42 + 0.35 (Re)^{0.8} \quad (0.6 < Re < 13) \quad (54)$$

This equation is used when needed in the present work.

4.6 Accuracy of approximate solutions

The validity of the parameter relationships (37), (40) and (41) must ultimately stand or fall by their agreement with reliable experimental results. However some indirect indications may be obtained by examining whether the approximate solutions used are close to the exact solution of the two-phase model, and by comparison of

the exact solutions of the one-and two-phase plug-flow models.

It is convenient to examine the approximate solution to the axially-dispersed two-phase model in two stages:

- (i) collocation approximation to radial derivatives
 - (ii) perturbation solution of collocated equations
- (i) Collocation approximation

This can be studied in isolation by considering the one-phase model and comparing the approximate solution (35) with the exact analytic solution (3.14). This is done for a typical case in figure 2; 50 terms of (3.14) were used in all cases. It is clear that the approximate solution is poor at bed entrance but improves rapidly with increasing bed depth, being within 2% of the exact solution at a depth of $10 d_p$.

The poor approximation at bed entrance is due to the quickly-changing profile near the wall, caused by the discontinuous wall temperature. The Biot number is critical for determining this effect, and for larger Bi the poor approximation will persist for larger values of x.

Villadsen and Michelson (4) have proposed a one-point "optimal" method based on choosing the collocation point as a function of Bi, to allow for the change in type of the wall boundary condition (3.9) with varying Bi. In the present case this involves setting

$y_1 = \sqrt{\frac{1}{3} \frac{Bi+6}{Bi+4}}$. Repeating the analysis leads to much more complicated parameter relationships for the effective Peclet numbers:

$$\frac{1}{Pe_a} = \frac{1}{Pe_{af}} + \frac{K N_F / N_S}{Pe_{rf} \left(1 + \frac{6 Bi_s (Bi_s + 4)}{N_S \{3(Bi_s + 4) + Bi_s (Bi_s + 3)\}} \right)} \quad (55)$$

Figure 2 : Comparison of 1-point collocation and exact solutions of one-phase model

$$\frac{1}{Pe_r} = \frac{1}{Pe_{rf}} \left[1 + \frac{N_F \left(\frac{3(Bi+4) + Bi(Bi+3)}{Bi(Bi+4)} \right)}{6 + N_S \left(\frac{3(Bi_s+4) + Bi_s(Bi_s+3)}{Bi_s(Bi_s+4)} \right)} \right] \quad (56)$$

There is little difference numerically between these and (40) and (41); the improvement to the radial approximation is also small. The refinement does not justify the extra complication for this case.

(ii) Perturbation solution

An analysis of the truncation error in the perturbation expansion solution of equations (22) and (24) would be extremely difficult. However these equations may be solved for specific values of the parameters by finding the roots of their characteristic equation numerically. This solution may then be compared with that given by the perturbation technique.

By examining the truncated series for t_1^* "worst-case" conditions regarding accuracy are anticipated for small N_S . This occurs for small (d_t/d_p) , large (k_{rs}/k_g) and small Nu_{fs} i.e. small Re . The values taken are therefore $(k_{rs}/k_g) = 18$ and $(d_t/d_p) = 7.5$; Re is varied.

Equations (22) and (24) may be combined to give

$$\begin{aligned} \frac{K}{Pe_{AF}} \frac{d^4 T_1^*}{dx^4} - K \frac{d^3 T_1^*}{dx^3} - \left[\frac{N_S+8}{Pe_{AF}} + \frac{K}{Pe_{RF}} \left(N_F + \frac{8 Bi_f}{Bi_f+4} \right) \right] \frac{d^2 T_1^*}{dx^2} \\ + (N_S+8) \frac{dT_1^*}{dx} + \frac{8}{Pe_{RF}} \left[\frac{(N_S+8) Bi_f}{Bi_f+4} + N_F \right] T_1^* = \begin{cases} \frac{8}{Pe_{RF}} \left[\frac{(N_S+8) Bi_f}{Bi_f+4} + N_F \right] & (x > 0) \\ 0 & (x < 0) \end{cases} \quad (57) \end{aligned}$$

where Bi_s is taken as ∞ .

Case (a) $Re = 50$

From the correlations of section 4.5: $Pe_{RF} = 34.9$, $Pe_{AF} = 8.18$,
 $Bi_f = 3.97$ and extending the Handley-Heggs correlation slightly
 $N_S = 21.94$, $N_F = 102.1$

The characteristic equation is

$$0.12 D^4 - D^3 - 6.70 D^2 + 29.94 D + 26.82 = 0 \quad (58)$$

The roots are found graphically and after some algebra the solution is

$$T_1^* = \begin{cases} 1 + 0.033 e^{-6.2x} - 0.89 e^{-0.8x} & x > 0 \\ 0.15 e^{4.1x} - 0.0085 e^{11.1x} & x < 0 \end{cases} \quad (59)$$

$$t_1^* = \begin{cases} 1 - 0.088 e^{-6.2x} - 0.662 e^{-0.8x} & x > 0 \\ 0.259 e^{4.1x} + 0.002 e^{11.1x} & x < 0 \end{cases} \quad (60)$$

The perturbation solution gives, for T_1^* and $x > 0$,

$$T_1^* = 1 - 0.88 e^{-0.77x} \quad (61)$$

Comparison of relevant expressions for T_1^* shows extremely good agreement, shown in figure 3. It is interesting to note that the solid and fluid phase temperatures given in equations (59) and (60) from the "exact" solution are significantly different.

Case (b) $Re = 1$

It is unlikely that heat transfer parameters will be examined using the relationships in this work at $Re < 1$.

The characteristic equation is

$$0.13 D^4 - D^3 - 16.31 D^2 + 10.16 D + 121.23 = 0 \quad (62)$$

and the solutions are:

$$T_1^* = \begin{cases} 1 + 0.062 e^{-7.7x} - 0.64 e^{-2.7x} & x > 0 \\ 0.416 e^{2.9x} + 0.0036 e^{15.6x} & x < 0 \end{cases} \quad (63)$$

Figure 3 : Comparison of perturbation and analytical solutions of the collocated two-phase model equations

$$t_1^* = \begin{cases} 1 - 0.002 e^{-7.7x} - 0.505 e^{-2.7x} & x > 0 \\ 0.487 e^{2.9x} + 0.0003 e^{15.6x} & x < 0 \end{cases} \quad (64)$$

and the perturbation solution is

$$T_1^* = 1 - 0.69 e^{-1.95x} \quad x < 0 \quad (65)$$

This comparison is also shown in figure 3. Agreement is not so good in this case, for small x , but is still reasonable. It should be noted that the correlation used for Nu_{fs} has a large effect at such low Re , the larger the value taken as the asymptotic limit of Nu_{fs} as $Re \rightarrow 0$, the better the approximation.

PF models

As it is observed (37) that equations (37) and (41) are unchanged if axial dispersion is omitted from the models matched in section 4.4, a test of these relationships is provided by choosing values of the two-phase plug-flow model parameters, using (37) and (41) to obtain the effective (PF model) parameters from them, and comparing the radial profiles predicted by each model. Analytical solutions are available for both one-phase (equation 3.16) and two-phase (1.15) models, and comparisons may be made at bed depths such that the first term of the series solutions suffices.

This is shown in figure 4 for the parameter values given there; agreement is reasonable, the gradient near the wall in the fluid phase is especially well matched by the one-phase model. It is interesting to note a slight inflection in the fluid-phase profile, which a one-point collocation could not reproduce.

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Figure 4 : Comparison of one- and two-phase PF model ,
 profiles, using equations (37) and (41)
 to obtain effective parameters

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5. COMPARISON OF THEORY WITH EXPERIMENTAL RESULTS

5.1 Selection of data

As was shown in chapter three, model inadequacy may lead to an artificial spread in the data. When possible experimental inadequacies are added to this, it is clear that to expect the relationships developed in chapter four to predict all current literature data would be unreasonable; some selection procedure must be implemented.

Ideally the following criteria should be met by acceptable data:

(a) The data should be obtained through experimentation suitably designed to reliably determine the parameters to be estimated.

(b) A statistically adequate model should be employed in the analysis of the data.

(c) Parameter estimation methods which are statistically sound in the face of experimental error should be used.

Unfortunately the rigorous application of these criteria would often result in the only acceptable data being that of the present work. To avoid this undesirable situation a more detailed inspection of the literature is required, and results will often be included where the effects of violation of one or more of the above criteria are judged to be minimal. One conclusion reached during this selection is that the effective radial Peclet number appears to be "robust", in that reasonable estimates are obtained even under highly unfavourable experimental conditions. The apparent wall Biot number and the effective axial Peclet number are, on the other hand, "sensitive" to experimental conditions.

For effective radial Peclet numbers, the results of Gunn and Khalid (1.4), which include axial conduction effects, supply sufficient

reliable data on which to examine the theory at low and intermediate Re , in spite of their estimating three parameters at a single bed depth, which was not found possible in the present work. These are supplemented by the present results, and at high Re the results of Kunii et al (4.23) may be taken, as axial conduction effects can be neglected (this is verified by the improved fit of the PF model at higher Re in table A1).

Measurement of effective axial Peclet numbers has usually proceeded in one of two ways:

(i) experiments in adiabatic packed beds i.e. no radial conduction present

(ii) joint estimation with Pe_r and Bi from radial temperature profiles at bed exit.

The latter method, used in the present work and by Gunn and Khalid, tends to give unreliable estimates of Pe_a , as this parameter has little effect on exit profiles. Hence only data obtained by the former method are accepted (3.14, 4.21 and 4.36).

The wall heat transfer coefficient h_w is commonly used in the formation of two dimensionless groups. It appears in dispersion models in the form of the Biot number Bi ($= h_w R/k_r$) representing the ratio of resistance inside the bed to that at the wall. In spite of appearing naturally in this form, h_w has been presented by most workers in the form of the Nusselt number Nu_w ($= h_w d_p/k_g$). Li and Finlayson (1) have recently presented a comprehensive review of the literature data on Nu_w , and have shown a large amount of scatter, particularly at low-to-moderate Re .

Since the PF model, used in all the studies cited in (1), leads to depth-dependent estimates of h_w , Li and Finlayson postulate a "length effect" as being responsible for this scatter. They then

accept or reject data according to whether the method of analysis used yields "asymptotic" values of h_w , or not. As a result they only accept data determined from measurements made at the exit of long tubes, as in the method of Yagi and Wakao (4.5).

The results of chapter three, however, have shown the depth-dependency of h_w to be due to the use of an inadequate model, in which axial conduction is neglected. Since Li and Finlayson present no evidence for the existence of a "length effect" in packed beds, but invoke this entity in order to be able to reject unwanted data, their analysis cannot be considered valid, and a fresh examination must be made.

First note that the data of some workers (2.3) is difficult to obtain other than in the review of Li and Finlayson, from which Biot numbers could not be obtained, or experimental procedure examined; this data is not considered here.

The results of Hawthorn et al (1.10) are for chemical reaction taking place on the wall; unfortunately, since this is the only data available with a reaction present, no thermal conductivities were measured and Biot numbers were not obtainable.

Of the remaining work a substantial body may be criticised on the adequacy of experimental procedure and the techniques used for analysis. It is shown in section 5.3 that in the range $40 \leq Re \leq 400$ the group $Bi (d_p/R)$ is in the range 0.7 - 1.2. Since the results of chapter three show that Biot numbers greater than 10 cannot be reliably estimated using the PF model (a conclusion supported by Li and Finlayson), data in this Re range for which $(d_t/d_p) > 20$ must be excluded.

A method of data analysis used by earlier workers (e.g. Plautz and Johnstone (3.12), Coberley and Marshall (4)) involved numerical

differentiation of data to obtain approximations to the derivatives appearing in the model equations. This method is notoriously unreliable, especially for estimating second derivatives. In addition it produces biased parameter estimates, since systematic truncation errors are introduced into the approximation of derivatives.

A further widely-used method of analysis is that of Yagi and Wakao (4.5), which was also used by Agnew and Potter (1.1). They took several measurements of bed temperature along the central axis at different positions (T_{bc}), and also measured mean exit gas temperature. They assumed $T_w = 100^\circ\text{C}$ and did not actually measure it. Taking the first term of (3.16) gives

$$\frac{T_w - T_{bc}}{T_w - T_o} = \frac{2 \text{ Bi } \exp \{ -z d_p \lambda_1^2 / R^2 \text{ Pe}_r \}}{(\text{Bi}^2 + \lambda_1^2) J_0(\lambda_1)} \quad (1)$$

Bi and Pe_r may be calculated simultaneously from the slope of the straight-line portion of the semi-log plot of $T_w - T_{bc}$ versus z , together with the ratio of temperature differences at bed exit

$$\frac{T_w - T_{em}}{T_w - T_{ec}} = 2 \frac{J_1(\lambda_1)}{\lambda_1} \quad (2)$$

and the characteristic equation

$$\lambda_1 J_1(\lambda_1) = \text{Bi } J_0(\lambda_1) \quad (3)$$

However Bi alone may be obtained from (2) and (3) only, requiring mean and centre-line bed exit temperatures. Thus this method may be applied to data in the present work, if the exit profiles are plotted graphically and extrapolated to wall and centre-line, giving T_{ec} directly and T_{em} by numerical integration. A comparison may then be effected with the results of the non-linear least squares method

<u>Packing</u>	<u>Re</u>	<u>L</u> <u>(mm)</u>	<u>T_w</u>	<u>T_{ec}</u>	<u>T_{em}</u>	$\left(\frac{T_w - T_{em}}{T_w - T_{ec}} \right)$	<u>λ_1</u>	<u>Bi</u> <u>(Y-W)</u>	<u>Bi</u> <u>(NLLS)</u>
12.7mm		51	106.6	32.0	42.3	0.8619	1.075	0.681	1.599
ceramic beads	536	178	107.0	53.2	64.0	0.7992	1.310	1.116	2.132
		356	105.9	80.5	85.8	0.7913	1.340	1.186	2.606
	429	178	106.8	57.0	68.1	0.7930	1.333	1.170	2.453
	291	"	106.9	64.0	73.8	0.7716	1.408	1.360	2.874
	140	"	106.4	83.5	88.2	0.7948	1.326	1.153	4.614
9.5mm	373	204	103.6	53.5	65.1	0.7685	1.417	1.384	2.878
ceramic beads	281	"	103.8	58.5	69.7	0.7528	1.469	1.535	3.253
	231	"	104.3	60.5	72.0	0.7374	1.520	1.702	3.379
	181	"	104.0	67.0	77.1	0.7270	1.552	1.814	3.576
	120	"	104.4	76.0	83.5	0.7359	1.525	1.719	3.572

Table 1 : Comparison of Yagi and Wakao's estimation method and nonlinear least squares.

(NLLS) used with the PF model; this is shown in Table 1. From this it appears that the appealingly simple procedure of Yagi and Wakao gives estimates of Bi with a large negative bias. Clearly criterion (c) is being violated, and results obtained using this method should be rejected.

Results obtained by Ziolkowski (5) are also rejected, although least squares estimation was used, on consideration of experimental procedure. The thermocouples were inserted vertically into the bed; which disturbs the packing and can lead to channelling. Quinton and Storrow (6) determined their parameters from typical radial temperature profiles of 3°C temperature rise, which can be regarded as flat within the scatter of the data.

The results of Gunn and Khalid, when re-computed, give the only Biot numbers which increase with Re . Why this should be is not clear, and their data is suspended pending further elucidation; possibly the effects which prevented three-parameter minimisation from a single bed depth in chapter three are here making themselves felt.

The acceptable data is considered to be that of Kunii et al (4.23) since at such high Re virtually all resistance in the bed will be at the wall, allowing good determination of Bi . Also the "non-homogeneity" of the bed will have small effect and more accurate measurements may be made, thus minimising the bias due to the method of estimation used. The only objection to the data of De Wasch and Froment (2.1) is the use of the PF model; however, if the results from all the bed depths are used, they should span the "correct" results and at least give an idea of the qualitative trend of the data.

5.2 Effective radial Peclet number

The vast bulk of the literature data on this parameter concerns heat transfer experiments carried out in large diameter beds of small particles, for which $(d_t/d_p) \rightarrow \infty$. The case of heat transfer in relatively narrow tubes, for which typically $4 \leq (d_t/d_p) \leq 12$, which is one of considerable practical importance, has received insufficient attention.

The relationship for Pe_r may be written in the form

$$\frac{1}{Pe_r} = \frac{1}{Pe_{rf}} + \frac{k_{rs}}{k_g} (Re Pr)^{-1} \left(\frac{Bi_f + 4}{Bi_f} \right) \left[\frac{8}{N_S} + \frac{Bi_s + 4}{Bi_s} \right]^{-1} \quad (4)$$

$$\text{where } N_S = \frac{1.5 (1-\epsilon) (d_t/d_p)^2}{\frac{k_{rs}}{k_g} \left\{ \frac{1}{Nu_{fs}} + \frac{0.1}{k_p/k_g} \right\}} \quad (5)$$

Note first that as $(d_t/d_p) \rightarrow \infty$, $N_S \rightarrow \infty$ and it may be deduced from equations (4.10) - (4.18) that $Bi_f = Bi_s$ in this limit. Thus (4) reduces to the familiar limiting form

$$\frac{1}{Pe_r} = \frac{1}{Pe_{rf}} + \frac{k_{rs}}{k_g} (Re Pr)^{-1} \quad (6)$$

This form of correlation has been used with considerable success (3.13). Figure 1 shows the data of Kunii et al and Gunn and Khalid; a key to symbols is given in table 2. Equation 6 is shown to predict the Peclet numbers to within 25% for non-metallic packings such as glass and celite, having an average $(k_{rs}/k_g) = 7$, and using $Pe_{rm}(\infty) = 10$. This is to be expected for the results of Gunn and Khalid (1.4) since most of their data involves high (d_t/d_p) . The data of Kunii et al (4.23) was taken at high Re so that the second terms of equations (4) and (6) are dominated by the first, which ensures a reasonable fit. Equation (6) is less successful for metallic packings. In these cases (k_{rs}/k_g) is larger, perhaps 15-18, and to

Figure 1 : Comparison of prediction formulae for Pe_r

maintain a large N_S requires increased (d_t/d_p) or very much increased Re .

<u>Symbol</u>	<u>Packing</u>	<u>d_t/d_p</u>	<u>Workers</u>
○	6.8mm Nickel	14.0	Gunn & Khalid
□	6.4mm Steel	14.9	"
▽	1.0mm Lead	95.2	"
△	7.0mm Lead	13.6	"
▽	6.0mm Glass	15.9	"
⊕	0.5mm "	190.5	"
⊙	1.15mm "	82.8	"
□	3.0mm "	31.8	"
○	28mm Celite	5.0	Kunii et al
△	42mm "	3.3	"

Table 2 : Key to figure 1

For the present data, at low (d_t/d_p) and moderate flow rate, equation (6) would not be expected to be so successful, and this is confirmed in figures 2 and 3. The 95% confidence intervals are shown for this data and the symbols are the same as for figure 4. For data at such (d_t/d_p) ratios (in range 5-12) the general equation (4) must be used. The best agreement with the data is found for $Bi_s = \infty$, and in the absence of data on h_{ws} , it is recommended that this value be used in computing Pe_r . The data in figure 2 indicates that particle diameter has only a weak effect on Pe_r and the data can be adequately predicted by a single curve for the intermediate (d_t/d_p) ratio (= 7.5) and taking $Pe_{rm}(\infty) = 10$. For the ceramics $(k_{rs}/k_g) = 8.2$ and was measured directly using Sehr's method (7,8). For steel balls a value of k_p was taken from standard tables (9) and equation (4.46) used. Bi_f was calculated using equations (12) and (4.37)

Figure 2 : Comparison of prediction formulae for Pe_r - nonmetallic packing

Figure 3 : Comparison of prediction formulae for Pe_r - metallic packing

In figure 1 is also shown the prediction of equation (4) for the metallic packings at lower (d_t/d_p) (13.6 - 14.9) and moderate Re (50 - 400). The deviation of this data from the prediction of equation (6) was particularly bad. The values (d_t/d_p) = 14 and (k_{rs}/k_g) = 18 were taken. The prediction now lies within the data; however the data has a greater slope, as is the case with virtually all Gunn and Khalid's data. It is not clear why this should be; it is encouraging, however, that equation (4) predicts the magnitude of the data well, and is a definite improvement over equation (6).

Finally, note that as $Re \rightarrow 0$ equation (6) becomes

$$\frac{k_r^0}{k_g} = 0.67\epsilon + \frac{k_{rs}}{k_g} \quad (7)$$

the familiar form for stagnant conductivity. The behaviour of equation (4), however, hinges directly on the behaviour of Nu_{fs} and Nu_{wf} as $Re \rightarrow 0$. Since this is far from well-established, no conclusion can be reached about equation (4).

5.3 Apparent wall Biot number

From equation (3.37):

$$\begin{aligned} Bi &= Bi_f = \frac{h_{wf} R}{k_{rf}} \\ &= \left(\frac{h_{wf} d_p}{k_g} \right) \left(\frac{R}{d_p} \right) \frac{Pe_{rf}}{(Re Pr)} \end{aligned} \quad (8)$$

$$= Nu_{wf} \left(\frac{R}{d_p} \right) \frac{Pe_{rf}}{(Re Pr)} \quad (9)$$

Now for $Re > 40$ $Pe_{rf} = Pe_{rm}(\infty)$ and a constant value of $Pe_{rm}(\infty) = 10$, was found to be adequate for prediction of Pe_r in section 5.2.

Further, present correlations for Nu_{wf} involve only Re and Pr. Hence,

for a given fluid, data plotted as $Bi(d_p/R)$ versus Re should fall onto a single line for $Re > 40$. This is in agreement with Li and Finlayson (1), although they found empirically

$$Bi(d_p/R) = \text{const.} \quad (10)$$

There are three choices of correlation for Nu_{wf} given in section 4.5. Substituting (4.49) - (4.51) into equation (9) and taking $Pr = 0.72$, $\epsilon_w = 0.5$ and $Pe_{rm}(\infty) = 10$ gives, respectively

$$Bi(d_p/R) = \begin{cases} 7.48 Re^{-0.5} & (Re < 40) \\ 2.49 Re^{-0.2} & (Re > 40) \end{cases} \quad (11)$$

$$Bi(d_p/R) = 3.00 Re^{-0.25} \quad (Re > 100) \quad (12)$$

$$Bi(d_p/R) = 110.91 Re^{-0.66} \quad (100 < Re < 3000) \quad (13)$$

Figure 4 shows equations (11) - (13) compared with the data from the present work. Clearly there is little to choose between (11) and (12), both lying a little below the data; equation (13) does not appear to give reasonable estimates and can be rejected. It is interesting to note the similarity of Re dependence between equation (12) and the empirical correlation found in section 3.6, equation (3.39):

$$Bi(d_p/d_t)^{\frac{1}{2}} = 5.73 Re^{-0.26}$$

Although equation (11) appears to fit the data in figure 4 a little better than equation (12), the latter gives a better fit to all the acceptable data, and is therefore the equation used in figure 5 where this data is shown. A key is given in table 3. It is seen that equation (12) gives a good prediction at moderate and high Re , and that the data do indeed fall onto one line.

Figure 4 : Comparison of predictions for Bi using different correlations for Nu_{wf}

Figure 5 : Comparison of Biot number predictions with experimental results

<u>Symbol</u>	<u>Packing</u>	<u>$\frac{d_t}{d_p}$</u>	<u>Workers</u>
Celite spheres	5.00	Kunii et al (<u>4.23</u>)
"	3.33	"
V ₂ O ₅ cylinders	17.24	De Wasch & Froment (<u>2.1</u>)
ceramic spheres	5.57	Present work
"	7.43	"
"	11.15	"
steel balls	7.43	"

Table 3 : Key to figures 5 and 6

At low Re (< 40) nothing definite can be said as only one correlation for h_{wf} is available, that of Yagi and Wakao (4.5). Their analysis omitted axial dispersion, which has been shown by Gunn and de Souza (3.36) to be important at low Re in the analagous problem of determining h_{fs} . Experimental data on Bi at low Re will be difficult to obtain as short bed depths must be used to give significant radial profiles, leading to difficulties, not the least of which is which downstream boundary condition to use in the axial dispersion model, a decision which becomes important for short beds.

If the data in figure 5 are re-plotted as Nusselt numbers (Nu_w), as shown in figure 6, the spread noticed by Li and Finlayson is again present. This may be accounted for as follows:

$$\begin{aligned}
 Nu_w &= \frac{h_w d_p}{k_g} = \left(\frac{h_w R}{k_r} \right) \left(\frac{d_p}{R} \right) \left(\frac{Re Pr}{Pe_r} \right) \\
 &= Bi \left(\frac{d_p}{R} \right) \left(\frac{Re Pr}{Pe_r} \right)
 \end{aligned} \tag{14}$$

Substituting for Bi from equation (9) gives

$$Nu_w = \frac{Nu_{wf} Pe_{rf}}{Pe_r} \tag{15}$$

Figure 6 : Comparison of Nusselt number predictions with experimental results

Now Nu_{wf} depends only on Re and Pr , and Pe_{rf} may be taken as constant. In section 5.2 it was shown that Pe_r depends on Bi_f , Bi_s , Nu_{fs} , (d_t/d_p) , (k_{rs}/k_g) and Re . Hence Nu_w must also depend on these variables, and it should be expected that data plotted as Nu_w vs. Re will be scattered, if they involve different packings and tube sizes. The apparent "scatter" is no more than a failure to take these variables into account, as hinted in section 3.6.

In figure 6 curves are shown corresponding to the cases of $d_t/d_p = \infty$ with low conductivity packing, and $d_t/d_p = 6$ with metallic packing. These reproduce the trend of the data well, and account for almost all the spread. In figures 4 to 6 the predictions appear consistently slightly below the data at low and moderate Re ; possibly the correlation used is a little inaccurate in this region.

5.4 Effective axial Peclet number

The data indicated as acceptable in section 5.1 are presented in figure 7 ; a key is given in table 4. The results show considerable scatter, typically of an order of magnitude, at all Re . There is no clear distinction between metallic and non-metallic packings; however, the results of Votruba et al (3.14), in all but two cases having $4 \leq (d_t/d_p) \leq 12$, lie fairly consistently beneath the rest, which were mostly in the range $d_t/d_p > 20$.

Figure 7 : Axial Peclet number data

<u>Symbol</u>	<u>Packing</u>	<u>d_t/d_p</u>	<u>Workers</u>
Various	3.8 - 12, 19.7, 57.8	Votruba et al. (3.14)
6.0mm Glass	8.3	Yagi et al (4.21)
2.6mm "	19.2	"
0.9mm "	55.6	"
4.8mm Steel	10.4	"
4.8mm "	14.2	"
3.0mm "	22.7	"
1.5mm Lead	45.3	"
0.5mm Glass	190.0	Gunn & de Souza (4.36)
1.2mm "	79.0	"
2.2mm "	43.3	"
3.0mm "	31.8	"
6.0mm "	15.9	"
3.2mm Steel	29.8	"
6.3mm "	15.1	"
0.8mm Lead	119.1	"

Table 4 : Key to figure 7

After some re-arrangement the relationship for axial Peclet numbers is

$$\frac{1}{Pe_a} = \frac{1}{Pe_{af}} + \frac{k_{as}}{k_g} (Re Pr)^{-1} \left(1 + \frac{8 Bi_s}{N_S (Bi_s + 4)} \right)^{-2} \quad (16)$$

where N_S is given in equation (5).

(a) Large (d_t/d_p)

In the limit $d_t/d_p \rightarrow \infty$, $N_S \rightarrow \infty$ and the limiting form of the equation is

$$\frac{1}{Pe_a} = \frac{1}{Pe_{af}} + \frac{k_{as}}{k_g} (Re Pr)^{-1} \quad (17)$$

If equation (4.44) for Pe_{am} is used and substituted into (17):

$$\frac{1}{Pe_a} = \frac{0.73\epsilon}{Re Pr} + \frac{0.5}{1 + \epsilon\beta/(Re Pr)} + \frac{k_{as}}{k_g} (Re Pr)^{-1} \quad (18)$$

and $\beta = 9.7$ for air as fluid.

Note that as a consequence of the choice of correlation for Pe_{am} the following limiting relationships hold:

$$Pe_a \rightarrow 2 \text{ as } Re \rightarrow \infty \quad (19)$$

$$k^0/k_g = 0.73 \epsilon + k_{as}/k_g \quad (20)$$

If the alternative equation (4.45) is used for Pe_{rm} the limits are

$$Pe_a \rightarrow 1.54 \text{ as } Re \rightarrow \infty \quad (21)$$

$$k^0/k_g = 0.67 \epsilon + k_{as}/k_g \quad (22)$$

Taking an average $\epsilon = 0.4$, a band is formed in figure 7 by the limiting cases of $k_{as}/k_g = 6$ (line A, non-metallics) and $k_{as}/k_g = 18$ (line B, metallics); the majority of the data of Yagi et al and Gunn and de Souza lie within this band, which uses equation (18). This is as expected, as this data mainly represents high (d_t/d_p) values. There is, however, some deviation from the band at higher Re . The results of Gunn and de Souza are very scattered in this area; the results of Yagi et al, on the other hand, appear to systematically deviate, in particular the 4.8mm steel spheres, with $(d_t/d_p) = 10.4$. The low (d_t/d_p) results of Votruba et al lie beneath the band, and appear to have a slower increase with increasing Re .

Within the band the data for metallic packings lie on or near line B, while those for non-metallic lie nearer line A. Hence particle conductivity would appear to produce a secondary ordering on the data.

(b) Small (d_t/d_p) --

In this case N_s may not be large and the general equation (16) may have to be used. For this general formula, which is simpler than that for Pe_r , it is possible to work out the conditions in terms of Re , (d_t/d_p) and (k_p/k_g) under which the simplified form (17) gives unacceptable errors. This has been done for experiments using air as fluid ($Pr = 0.72$), taking an average $\epsilon = 0.4$ and setting an error tolerance of 5% in an estimate of magnitude 2. The results are shown in figure 8, taking $Bi_s = \infty$ (worst-case) and using the Handley-Heggs correlation (4.27) for Nu_{fs} , supplemented by that of Lüttman & Sliva (4.19) at low Re .

To use figure 8, the line applicable to particle/fluid conductivity ratio (k_p/k_g) should be chosen. Then with one of Re , (d_t/d_p) given, the range of the other for which (17) may be used with less than 5% error is that which gives points above and to the right of the (k_p/k_g) line.

From figure 8, it is seen that the discrepancies between the predictions of equation (17) and the data of Votruba et al for low (d_t/d_p) in figure 7 cannot be accounted for by using equation (16) instead of equation (17).

The explanation for the discrepancies is readily provided by the results of Hsiang and Haynes (4.16) as discussed in section 4.5. Their anomalous decreases in Pe_{am} as Re increases, for $1.4 < (d_t/d_p) < 8$, suggest that the commonly taken constant asymptote $Pe_{am}(\infty)$ should be regarded as a function of (d_t/d_p) . Unfortunately their experiments were not conducted at high enough Re to allow definite conclusions as to the form of the function; indeed they were able to fit an empirical correlation giving $Pe_{am} \rightarrow 0$ for some finite Re .

Figure 8 : Criterion for use of eqn.(17) for prediction of axial Peclet numbers

Inspection of the literature reveals an acute shortage of work in the Re and (d_t/d_p) ranges of interest here, and the work of Hsiang and Haynes remains uncorroborated.

Indirect support, however, comes from consideration of the empirical formulae employed by heat transfer workers. The results of Gunn and de Souza (4.36) were too scattered to permit correlation, however Yagi et al (4.21) fitted their results well with a formula which may be written in the present notation after some re-arrangement as

$$\frac{1}{Pe_a} = \frac{0.73\epsilon}{Re Pr} + \frac{1}{Pe_a(\infty)} + \frac{k_{as}}{k_g} (Pr Re)^{-1} \quad (23)$$

where their average value of $Pe_a(\infty)$ is 1.33.

Votruba et al (3.14) arrived at the empirical correlation

$$\frac{1}{Pe_a} = \frac{0.73\epsilon}{Re Pr} + \frac{k_{as}}{k_g} (Re Pr)^{-1} + \frac{14.5/d_p}{\left(1 + \frac{C_3}{(Re Pr)}\right)} \quad (24)$$

where d_p is in mm and the dependence of $Pe_a(\infty)$ upon (d_t/d_p) is evident. Using these results a speculative relationship is shown in figure 9.

Figure 9 : Speculative (d_t/d_p) dependence of $Pe_{am}(\infty)$

However this relationship cannot be recommended, and reliable prediction of Pe_a at high Re must await further work on Pe_{am} .

(c) Prediction of Pe_a at low Re

The previous discussion is possibly of more academic than practical interest, however, as axial conduction is more important at low Re . Unfortunately another area of doubt arises here, since figure 8 usually requires that the general equation (16) be used, and there is no definitive correlation for Nu_{fs} available at low Re .

To show the influence of this parameter, the data of Votruba et al for $1 < Re < 10$ and low (d_t/d_p) is plotted in figure 10, making no distinction between different particle sizes. Two extreme cases of correlation for Nu_{fs} are considered.

$$(i) \quad Nu_{fs} = 12 \quad (\text{determined experimentally by Miyauchi et al (4.35)})$$

$$(ii) \quad Nu_{fs} = 0.032 Re \quad (\text{theoretical prediction, Nelson and Galloway (4.31)})$$

For case (i) extremes of particle conductivity are taken as for figure 7 to give a band (curves A and B). For each of A and B there are two lines, the full line representing equation (16) and the dotted line equation (17).

Since the data represented asymptote to values of $Pe_a(\infty)$ between 0.15 and 0.5, an average $Pe_a(\infty) = 0.3$ was taken and Pe_{am} obtained from an appropriately modified version of equation (4.44).

It is seen that either band represents the data reasonably, and that the differences between equations (16) and (17) under case (i) are not large; possibly they may be regarded as giving the same results.

Under case (ii), however, equation (16) becomes $Pe_a = Pe_{af}$, as shown by curve C, which is clearly unrealistic.

Figure 10 : Prediction of axial Peclet numbers at low Re

Possibly the best choice for predicting Pe_a at low Re is to use equation (17), as the errors involved are small compared with those made if an inappropriate choice for Nu_{fs} is used.

(d) Theoretical model of Vortmeyer and Schaeffer

An alternative theoretical prediction formula has been obtained by Vortmeyer and Schaeffer (10). They matched one-and two-phase models of adiabatic beds, assuming equality of second axial temperature derivatives in the two phases. It is easy to show that this implies equality of temperature in the two phases using the present boundary conditions, which is an unjustifiable result.

Vortmeyer and Schaeffer obtained the result

$$\frac{1}{Pe_a} = \frac{k_{as}}{k_g} (Re Pr)^{-1} + \frac{Re Pr}{6 (1-\epsilon) Nu_{fs}} \quad (25)$$

In a further paper by Vortmeyer (4.30) this was shown to predict the data of Gunn and de Souza at lower Re . However at low Re the first term in equation (25) dominates and good agreement is guaranteed, providing the appropriate (k_{as}/k_g) value is chosen. On the other hand, as $Re \rightarrow \infty$ equation (25) gives $Pe_a \rightarrow 0$ which does not agree with experimental data. Equation (25) cannot be regarded as a reliable prediction formula.

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CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

The work presented in this thesis may conveniently be divided into two parts : (i) model testing and parameter estimation (chapters 1 to 3); (ii) theoretical prediction of effective heat transfer parameters (chapters 4 and 5).

In the first part of the thesis, statistical methods were applied to the analysis, via homogeneous continuum models, of experimental data obtained from a low d_t/d_p packed bed heat exchanger. These statistical methods discriminated clearly amongst the models, showing that the omission of axial conduction in such models leads to a systematic lack-of-fit of the model and to biased parameter estimates. The 'depth-dependence' of the effective heat transfer parameters was shown to be due to this omission, rather than other physical effects, as suggested in the literature (5.1).

The above results show that while physical intuition must be used to determine and simplify engineering models, it is essential that these are then justified by statistical analysis. If this step is omitted, as is usually the case in chemical engineering studies, the parameter estimates obtained from an inadequate model may be at best biased, at worst meaningless. The need for rigorous analysis is highlighted by the fact that the inclusion of axial conduction changes the magnitude of predicted bed exit temperature profiles very little, as shown by the difficulty in estimating effective axial Peclet numbers. This observation is often used to justify the omission of axial conduction; however, only statistical analysis can detect the small but systematic changes in profile. It is a matter of conjecture in how many other areas inadequate models are being used; the parameters may not always show such convenient 'symptoms' of bias.

As a corollary to the above, it must also be emphasised that parameter estimates, even if obtained from an adequate model, should always be accompanied by estimates of variance or confidence intervals. Without these it is impossible to know if trends in the parameters are significant. This is again rarely done in chemical engineering literature.

It was also shown in this part of the work that the choice of boundary conditions is as important to a model as the main equation. The inclusion of axial conduction effects leads to the problem of choosing a simplified downstream boundary condition. The assumption of plug-flow in the space above the packing was shown to be inadequate both by statistical rejection of the model and by a separate test for any change in entrance profile with different bed lengths. A boundary condition placed at infinity was compatible with the observed lack of change.

Finally, in this part of the work, it should be stressed that the model which passed the statistical tests predicts only angle-averaged temperatures. The inclusion of angular dependence is a task for the future; at present only probabilistic models appear to show any hope in this direction.

In the second part of the work a new theory for prediction of the effective heat transfer parameters appearing in homogeneous continuum models was presented. This theory relates the effective parameters to more basic underlying heat transfer processes which may be independently measured and thus estimated from literature correlations.

The development of this theory required the solution to a two-phase quasi-continuum model containing the basic heat transfer parameters. Previous solutions to this model had involved finding at least the eigenvalues of the model equations numerically, so that

no totally-determined analytic solution existed. In this work an approximate analytic solution was found, using a one-point collocation-perturbation method, which was shown to give accurate fluid-phase temperatures away from bed entrance. In particular the perturbation method gave pleasing results. The collocation method is a familiar numerical tool; this application shows its versatility in deriving qualitative aspects of a solution, often more important in engineering situations than precise numerical answers.

The effective parameters were then related to the basic ones by matching one-phase and fluid-phase approximate solutions to the respective models. The relationships found were complex, especially for the effective Peclet numbers. The assumption of additivity of fluid and solid phase conductivities, until now a standard one in the literature, was shown to be a special case applicable only in the case of large d_t/d_p beds; for small d_t/d_p beds, as in the present study, wall and interphase effects must be included.

Besides showing the correct 'lumping' procedures for the effective parameters, the relationships also do much to explain the scattered literature data. In particular, the scatter in the correlation of the wall heat transfer coefficient, in the form of a Nusselt number versus Reynolds number, was explained as a failure to include other influencing factors. A simpler correlation in the form of a Biot number was shown by the theory to be preferable.

Although the parameter estimates of this work, and certain of the literature data, were predicted accurately by the theory, final testing has not yet been possible due to two factors. The first is the regrettable conclusion that much literature data is unreliable, due either to poor experimental design or poor analysis of results.

The latter reason is the chief cause; much data has been analysed using inadequate models or poor parameter estimation techniques.

The 'short-cut' methods used in the pre-computer era are not rigorous in the face of experimental errors; the safest course is to discard all data obtained using them. As good experimental data are costly and time-consuming to obtain, more thought should be given to their analysis.

The second factor is the necessity of awaiting further work on the basic heat transfer correlations. For some of these the situation is less certain than could be wished, for others it is not at all certain. It is appropriate to conclude this study by mentioning the most urgent areas requiring clarification:

- (a) axial mass transfer at low d_t/d_p (3-12); corroboration of the work of Hsiang and Haynes (4.16) and extension to larger Re to determine limiting values.
- (b) interphase heat or mass transfer at low Re (< 10)
- (c) wall-to-fluid mass transfer at low Re (< 100) for gases.
- (d) wall-to-solid heat transfer.

NOMENCLATURE

a	specific interfacial surface area	(m^{-1})
c_p	fluid specific heat	$(kJ/kg^{\circ}C)$
d_p	pellet diameter	(m)
d_t	tube diameter	(m)
D_{af}	axial fluid diffusivity	(m^2/s)
D_{rf}	radial fluid diffusivity	(m^2/s)
$f_i(\underline{x})$	residual temperature (eqn. 3.1)	$(^{\circ}C)$
G	superficial mass flow rate	(kg/m^2s)
h	apparent interphase heat transfer coefficient	$(w/m^2^{\circ}C)$
h_w	apparent wall heat transfer coefficient	$(w/m^2^{\circ}C)$
h_{wf}	wall-fluid heat transfer coefficient	$(w/m^2^{\circ}C)$
h_{ws}	wall-solid heat transfer coefficient	$(w/m^2^{\circ}C)$
h_{fs}	fluid-solid heat transfer coefficient	$(w/m^2^{\circ}C)$
k°	stagnant bed conductivity	$(w/m^{\circ}C)$
k_a	axial effective conductivity	$(w/m^{\circ}C)$
k_a°	stagnant bed axial effective conductivity	$(w/m^{\circ}C)$
k_r	radial effective conductivity	$(w/m^{\circ}C)$
k_r°	stagnant bed radial effective conductivity	$(w/m^{\circ}C)$
k_{af}	axial conductivity of fluid phase	$(w/m^{\circ}C)$
k_{as}	axial conductivity of solid phase	$(w/m^{\circ}C)$
k_{rf}	radial conductivity of fluid phase	$(w/m^{\circ}C)$
k_{rs}	radial conductivity of solid phase	$(w/m^{\circ}C)$
k_g	molecular conductivity of fluid	$(w/m^{\circ}C)$
k_p	pellet conductivity	$(w/m^{\circ}C)$
L	length of packed test section	(m)
r	radial co-ordinate	(m)

R	tube radius	(m)
S	sum-of-squares at 'solution'	$(^{\circ}\text{C})^2$
S(<u>x</u>)	sum-of-squares at <u>x</u>	$(^{\circ}\text{C})^2$
t	solid phase temperature	$(^{\circ}\text{C})$
T	fluid phase temperature	$(^{\circ}\text{C})$
T _b	bed temperature (in one-phase models)	$(^{\circ}\text{C})$
T _O	temperature of calming section wall	$(^{\circ}\text{C})$
T _w	temperature of test section wall	$(^{\circ}\text{C})$
T _{bc}	bed temperature at $r = 0$	$(^{\circ}\text{C})$
T _{ec}	bed exit temperature at $r = 0$	$(^{\circ}\text{C})$
T _{em}	mixing-cup temperature at bed exit	$(^{\circ}\text{C})$
T _{av}	average replicate temperature	$(^{\circ}\text{C})$
T _{obs}	experimentally measured temperature	$(^{\circ}\text{C})$
u	superficial fluid velocity	(m/s)
z	axial co-ordinate	(m)

Dimensionless parameters

Bi	apparent wall Biot number, $h_w R/k_r$
Bi _f	fluid-wall Biot number, $h_{wf} R/k_{rf}$
Bi _s	solid-wall Biot number, $h_{ws} R/k_{rs}$
F _c	calculated F-statistic, eqn.(3.36)
K	ratio of solid conductivities, k_{as}/k_{rs}
m	number of observations in a minimisation
n ₁	degrees of freedom associated with MSLF
n ₂	degrees of freedom associated with MSPE
N	number of bed depths at a particular flow rate
N _F	interphase heat transfer group, $aR^2 h/k_{rf}$

N_S	interphase heat transfer group, aR^2h/k_{rs}
Nu_w	apparent wall Nusselt number, $h_w d_p/k_g$
Nu_{fs}	fluid-solid Nusselt number, $h_{fs} d_p/k_g$
Nu_{wf}	fluid-wall Nusselt number, $h_{wf} d_p/k_g$
p	number of parameters estimated in a minimisation
Pe_a	effective axial Peclet number, $Gc_p d_p/k_a$
Pe_A	effective axial Peclet number (based on R), $Gc_p R/k_a$
Pe_r	effective radial Peclet number, $Gc_p d_p/k_r$
Pe_R	effective radial Peclet number (based on R), $Gc_p R/k_r$
Pe_{af}	axial fluid Peclet number, $Gc_p d_p/k_{af}$
Pe_{AF}	axial fluid Peclet number (based on R), $Gc_p R/k_{af}$
Pe'_{AF}	modified axial fluid Peclet number, eqn.(4.30)
Pe_{rf}	radial fluid Peclet number, $Gc_p d_p/k_{rf}$
Pe_{RF}	radial fluid Peclet number (based on R), $Gc_p R/k_{rf}$
Pe_{am}	axial mass Peclet number, ud_p/D_{af}
Pe_{rm}	radial mass Peclet number, ud_p/D_{rf}
Pr	Prandtl number, $\mu c_p/k_g$
Re	Reynolds number, Gd_p/μ
Sc	Schmidt number, $\mu/\rho D$
t^*	dimensionless solid temperature, $(t-T_0)/(T_w-T_0)$
t_1^*, t_2^*	values of t^* at collocation points y_1, y_2
T^*	dimensionless fluid temperature, $(T-T_0)/(T_w-T_0)$
T_1^*, T_2^*	values of T^* at collocation points y_1, y_2
x	normalised axial co-ordinate, z/R
y	normalised radial co-ordinate, r/R
y_1, y_2	collocation points ($y_2=1$)

Greek symbols

α	defined in eqn.(4.25)
β	defined in eqn.(4.36)
β_n	defined after eqn.(3.17)
γ	defined in eqn.(4.23)
δ	defined in eqn.(4.31)
Δ	defined in eqn.(4.33)
ϵ	bed voidage
ϵ_c	'centre-bed' voidage
ϵ_w	'near-wall' voidage
η	perturbation parameter, K/8
θ	dimensionless bed temperature, $(T_b - T_0)/(T_w - T_0)$
θ_{av}	dimensionless average of replicate temperatures
θ_{obs}	dimensionless observed temperature, $(T_{obs} - T_0)/(T_w - T_0)$
λ_n	eigenvalues of one-phase models, eqn.(3.17)
μ	viscosity of fluid (kg/ms)
ρ	density of fluid (kg/m ³)
$\rho_{A,B}$	correlation coefficient between parameters A and B

Abbreviations

ADPF(IBC)	axially-dispersed plug-flow model (infinite boundary condition)
ADPF(FBC)	axially-dispersed plug-flow model (finite boundary condition)
MSLF	mean lack-of-fit sum-of-squares
MSPE	mean pure error sum-of-squares
NLLS	nonlinear least squares
OLS	ordinary (i.e. unweighted) least squares
PF	plug-flow model

SSPE pure error sum-of-squares

WLS weighted least squares

Vectors and matrices

H approximation to inverse Hessian of $S(\underline{x})$

$J_{ij}(\underline{x}), J$ approximation to Jacobian of $S(\underline{x})$

$\underline{x}$ vector of parameters to be estimated

Symbols which are standard mathematical notation (e.g. $J_0(x)$, $J_1(x)$ the Bessel functions of order 0,1) have been omitted from this list. Also some symbols which appear only once and are defined on their appearance are omitted; this latter allows some 'general-purpose' symbols to be used more than once in different contexts.

APPENDIX A

Tables of F-tests and parameter estimates

ADEQUACY OF MODELS

Overall Analysis

Table A1

<u>Packing</u>	<u>d_p</u> (mm)	<u>Re</u>	<u>n₁</u>	<u>n₂</u>	<u>————— F_c/F_{0.05} —————</u>		
					<u>PF</u>	<u>ADPF</u> (IBC)	<u>ADPF</u> (FBC)
Ceramic	12.7	536	29	93	1.57	0.42	1.15
		430	37	117	2.61	0.48	2.03
		290	37	118	2.40	0.67	1.86
		140	29	94	2.61	0.68	2.05
		36	29	94	8.01	5.81	8.19
Ceramic	9.5	373	37	119	0.92	0.50	0.87
		281	37	120	1.54	0.64	1.47
		231	37	120	1.43	0.53	1.25
		181	37	120	2.04	0.51	1.55
		120	37	120	3.40	0.78	2.64
Ceramic	6.4	63	21	72	5.17	1.03	3.79
		274	37	119	-	0.89	-
		206	37	119	-	0.81	-
		171	37	119	-	0.91	-
		138	37	118	-	0.91	-
		109	37	118	-	0.57	-
		73	37	118	-	1.05	-
Steel	9.5	45	37	118	-	0.26	-
		468	37	119	-	0.82	-
		356	37	119	-	0.74	-
		224	37	119	-	0.70	-
		146	29	95	-	0.82	-
		77	21	71	-	0.46	-

ADEQUACY OF ADPF MODELSOverall AnalysisTable A2Entrance Data Included

<u>Packing</u>	<u>d_p</u> (mm)	<u>Re</u>	<u>n₁</u>	<u>n₂</u>	<u>F_c/F_{0.05}</u>		<u>n₁</u>	<u>n₂</u>	<u>F_c/F_{0.05}</u>	
					<u>ADPF</u>				<u>ADPF</u>	
					(IBC)				(FBC)	
Ceramic	12.7	536	35	130	0.97		53	112	1.26	
		430	43	165	1.53		67	141	2.01	
		290	43	167	1.84		67	143	2.00	
		140	35	132	1.86		53	114	2.26	
		36	35	132	4.91		53	114	6.62	
Ceramic	9.5	373	43	172	0.58		67	149	0.59	
		281	43	174	0.69		67	150	0.92	
		231	43	174	0.57		67	150	0.77	
		181	43	174	0.58		67	150	0.96	
		120	43	174	0.94		67	150	1.56	
		63	27	102	1.39		39	90	2.20	
Ceramic	6.4	274	43	173	3.20		-	-	-	
		206	43	173	3.36		-	-	-	
		171	43	173	3.62		-	-	-	
		138	43	172	3.51		-	-	-	
		109	43	168	3.48		-	-	-	
		73	43	172	3.96		-	-	-	
		45	43	172	3.23		-	-	-	
Steel	9.5	468	43	173	1.74		-	-	-	
		356	43	173	1.92		-	-	-	
		224	43	173	1.76		-	-	-	
		146	35	133	2.62		-	-	-	
		77	27	101	0.71		-	-	-	

ADEQUACY OF MODELSDepth-by-depth AnalysisTable A3Entrance Data Included In ADPF Models12.7 mm Ceramic Beads

<u>Re</u>	<u>L</u> (mm)	$F_c/F_{0.05}$			$F_c/F_{0.05}$		
		<u>n₁</u>	<u>n₂</u>	<u>PF</u>	<u>n₁</u>	<u>n₂</u>	<u>ADPF</u> (IBC) <u>ADPF</u> (FBC)
536	51	6	23	0.23	11	28	0.27 0.32
	102	6	23	0.24	11	28	0.38 0.40
	178	6	23	0.14	11	27	0.26 0.26
	356	6	24	0.27	11	29	1.12 1.13
430	51	6	23	0.18	11	28	0.24 0.29
	102	6	23	0.25	11	28	0.44 0.46
	178	6	23	0.15	11	27	0.32 0.32
	254	6	24	0.09	11	29	1.57 1.58
	356	6	24	0.37	11	29	2.07 2.09
290	51	6	23	0.21	11	28	0.29 0.34
	102	6	23	0.28	11	28	0.51 0.53
	178	6	24	0.10	11	29	0.41 0.42
	254	6	24	0.25	11	29	3.44 3.48
	356	6	24	0.43	11	29	3.31 3.32
140	51	6	23	0.23	11	28	0.39 0.44
	102	6	23	0.15	11	27	0.61 0.72
	178	6	24	0.12	11	29	0.45 0.46
	356	-	-	-	11	29	2.41 2.33
36	51	6	23	0.72	11	28	1.43 1.58
	102	6	23	0.52	11	28	0.87 0.77
	178	-	-	-	-	-	- -
	356	-	-	-	-	-	- -

ADEQUACY OF MODELSDepth-by-depth AnalysisTable A4Entrance Data Included In ADPF Models9.5 mm Ceramic Beads

<u>Re</u>	<u>L</u> (mm)	$F_c/F_{0.05}$			$F_c/F_{0.05}$		
		<u>n₁</u>	<u>n₂</u>	<u>PF</u>	<u>n₁</u>	<u>n₂</u>	<u>ADPF</u> (IBC) <u>ADPF</u> (FBC)
373	51	6	24	0.80	11	30	0.31 0.33
	102	6	23	0.08	11	29	0.10 0.10
	152	6	24	0.44	11	30	0.32 0.32
	204	6	24	0.39	11	30	0.28 0.28
	254	6	24	0.34	11	30	0.26 0.26
281	51	6	24	0.84	11	30	0.34 0.36
	102	6	24	0.19	11	30	0.13 0.13
	152	6	24	0.41	11	30	0.30 0.30
	204	6	24	0.36	11	30	0.25 0.25
	254	6	24	0.28	11	30	0.21 0.21
231	51	6	24	0.84	11	30	0.37 0.39
	102	6	24	0.21	11	30	0.15 0.15
	152	6	24	0.26	11	30	0.20 0.20
	204	6	24	0.44	11	30	0.29 0.29
	254	6	24	0.10	11	30	0.09 0.09
181	51	6	24	1.12	11	30	0.58 0.63
	102	6	24	0.15	11	30	0.13 0.14
	152	6	24	0.35	11	30	0.28 0.28
	204	6	24	0.51	11	30	0.32 0.32
	254	6	24	0.14	11	30	0.13 0.13

Table A4 (contd.)

<u>Re</u>	<u>L</u> (mm)	$F_c/F_{0.05}$			$F_c/F_{0.05}$		
		<u>n₁</u>	<u>n₂</u>	<u>PF</u>	<u>n₁</u>	<u>n₂</u>	<u>ADPF</u> (IBC) <u>ADPF</u> (FBC)
120	51	6	24	1.57	11	30	0.86 0.92
	102	6	24	0.49	11	30	0.42 0.44
	152	6	24	0.24	11	30	0.25 0.25
	204	6	24	0.27	11	30	0.19 0.19
	254	6	24	0.16	11	30	0.21 0.22
63	51	6	24	0.88	11	30	0.90 0.97
	102	6	24	0.49	11	30	0.71 0.73
	152	6	24	0.16	11	30	0.32 0.33

PF MODEL PARAMETERS

Overall Analysis

Table A5

12.7 mm Ceramic Beads

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
536	5.38	4.88 - 5.89	2.19	1.98 - 2.39
430	5.28	4.84 - 5.72	1.63	1.50 - 1.77
290	4.56	4.20 - 4.91	1.30	1.20 - 1.40
140	2.95	2.67 - 3.23	0.96	0.87 - 1.06
36	1.72	1.59 - 1.84	0.43	0.40 - 0.46

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
536	1.97	1.69 - 2.25	121.9	114.5 - 129.4
430	2.34	2.02 - 2.66	107.8	101.0 - 114.6
290	2.54	2.20 - 2.89	93.4	87.4 - 99.5
140	3.01	2.47 - 3.55	82.2	74.4 - 89.9
36	10.27	6.72 - 13.83	124.2	89.9 - 158.5

PF MODEL PARAMETERS

Overall Analysis

Table A6

9.5 mm Ceramic Beads

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
373	5.76	5.54 - 5.99	1.42	1.36 - 1.48
281	5.19	4.98 - 5.41	1.19	1.14 - 1.24
231	4.98	4.78 - 5.17	1.02	0.98 - 1.06
181	4.28	4.11 - 4.46	0.93	0.89 - 0.97
120	3.43	3.27 - 3.58	0.77	0.73 - 0.80
63	2.09	1.96 - 2.21	0.67	0.63 - 0.71

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
373	2.94	2.73 - 3.14	117.8	113.6 - 122.1
281	3.26	3.00 - 3.51	109.5	104.8 - 114.1
231	3.48	3.22 - 3.75	100.2	96.0 - 104.4
181	3.55	3.26 - 3.84	93.1	88.9 - 97.4
120	3.95	3.57 - 4.34	85.8	80.9 - 90.7
63	4.55	3.90 - 5.22	86.0	78.3 - 93.7

PF MODEL PARAMETERS

Depth-by-depth Analysis

Table A7

12.7 mm Ceramic Beads

<u>Re</u>	<u>L</u> (mm)	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
536	51	3.55	3.07 - 4.04	3.32	2.86 - 3.77
	102	4.24	3.50 - 4.98	2.76	2.28 - 3.25
	178	6.23	5.27 - 7.18	1.76	1.49 - 2.04
	356	6.80	6.13 - 7.48	1.61	1.45 - 1.78
430	51	3.31	2.88 - 3.73	2.60	2.27 - 2.94
	102	3.97	3.31 - 4.63	2.17	1.81 - 2.53
	178	5.85	4.97 - 6.73	1.47	1.25 - 1.69
	254	5.64	5.18 - 6.10	1.53	1.40 - 1.65
	356	6.55	5.95 - 7.14	1.31	1.19 - 1.43
290	51	3.06	2.68 - 3.44	1.93	1.69 - 2.17
	102	3.82	3.22 - 4.41	1.55	1.31 - 1.80
	178	5.37	4.61 - 6.13	1.10	0.95 - 1.26
	254	4.90	4.50 - 5.29	1.21	1.11 - 1.31
	356	6.06	5.55 - 6.57	0.98	0.90 - 1.06
140	51	2.32	2.04 - 2.59	1.23	1.08 - 1.37
	102	2.88	2.45 - 3.30	0.99	0.84 - 1.14
	178	4.19	3.49 - 4.88	0.68	0.57 - 0.79
	356	-	-	-	-
36	51	1.60	1.48 - 1.71	0.46	0.43 - 0.49
	102	2.22	1.98 - 2.45	0.33	0.30 - 0.37
	178	-	-	-	-
	356	-	-	-	-

PF MODEL PARAMETERS

Depth-by-depth Analysis

Table A8

12.7 mm Ceramic Beads

<u>Re</u>	<u>L</u> (mm)	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
536	51	1.60	1.31 - 1.88	149.7	138.0 - 161.4
	102	1.67	1.25 - 2.09	130.4	116.9 - 143.9
	178	2.13	1.60 - 2.66	106.3	94.5 - 118.0
	356	2.61	2.12 - 3.09	118.8	108.3 - 129.3
430	51	1.90	1.56 - 2.24	139.6	127.9 - 151.4
	102	1.92	1.43 - 2.41	117.8	104.6 - 130.9
	178	2.45	1.82 - 3.09	101.9	89.4 - 114.4
	254	2.18	1.88 - 2.48	94.0	88.4 - 99.7
	356	3.03	2.48 - 3.59	112.6	102.0 - 123.3
290	51	2.28	1.84 - 2.71	124.4	112.8 - 136.0
	102	1.93	1.47 - 2.40	84.8	75.9 - 93.7
	178	2.87	2.10 - 3.64	89.5	77.2 - 101.8
	254	2.78	2.35 - 3.20	94.9	87.7 - 102.2
	356	3.97	3.18 - 4.77	109.7	96.7 - 122.6
140	51	2.87	2.26 - 3.49	99.6	88.2 - 110.9
	102	2.52	1.85 - 3.18	70.3	61.4 - 79.2
	178	4.61	2.64 - 6.59	88.6	64.7 - 112.4
	356	-	-	-	-
36	51	8.94	6.25 - 11.63	116.0	89.0 - 143.0
	102	16.05	3.55 - 28.54	150.4	48.5 - 252.2
	178	-	-	-	-
	356	-	-	-	-

PF MODEL PARAMETERS

Depth-by-depth Analysis

Table A9

9.5 mm Ceramic Beads

<u>Re</u>	<u>L</u> (mm)	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
373	51	4.28	3.90 - 4.65	1.91	1.75 - 2.08
	102	5.42	4.92 - 5.91	1.51	1.38 - 1.65
	152	5.46	4.96 - 5.96	1.50	1.36 - 1.64
	204	5.88	5.31 - 6.46	1.39	1.26 - 1.53
	254	5.98	5.59 - 6.38	1.37	1.28 - 1.46
281	51	3.78	3.50 - 4.06	1.64	1.52 - 1.76
	102	4.82	4.45 - 5.20	1.28	1.18 - 1.38
	152	4.86	4.42 - 5.31	1.27	1.16 - 1.39
	204	5.23	4.70 - 5.76	1.18	1.06 - 1.30
	254	5.27	4.85 - 5.69	1.17	1.08 - 1.27
231	51	3.76	3.48 - 4.03	1.35	1.25 - 1.45
	102	4.56	4.20 - 4.92	1.11	1.02 - 1.20
	152	4.59	4.21 - 4.97	1.10	1.01 - 1.20
	204	5.25	4.75 - 5.75	0.96	0.87 - 1.06
	254	5.39	5.03 - 5.76	0.94	0.88 - 1.00
181	51	3.07	2.91 - 3.23	1.29	1.23 - 1.36
	102	4.03	3.72 - 4.34	0.98	0.91 - 1.06
	152	4.08	3.73 - 4.42	0.97	0.89 - 1.06
	204	4.60	4.18 - 5.03	0.86	0.78 - 0.94
	254	4.86	4.48 - 5.24	0.82	0.75 - 0.88

Table A9 (contd.)

<u>Re</u>	<u>L</u> (mm)	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
120	51	2.57	2.44 - 2.71	1.02	0.97 - 1.08
	102	3.18	2.95 - 3.42	0.83	0.77 - 0.89
	152	3.37	3.08 - 3.66	0.78	0.71 - 0.85
	204	3.78	3.37 - 4.20	0.70	0.64 - 0.77
	254	4.05	3.65 - 4.46	0.65	0.58 - 0.72
63	51	1.75	1.66 - 1.83	0.80	0.76 - 0.83
	102	2.25	2.08 - 2.42	0.62	0.57 - 0.66
	152	2.45	2.20 - 2.71	0.57	0.51 - 0.63

PF MODEL PARAMETERS

Depth-by-depth Analysis

Table A10

9.5 mm Ceramic Beads

<u>Re</u>	<u>L</u> (mm)	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
373	51	2.42	2.16 - 2.69	130.9	124.6 - 137.3
	102	2.89	2.48 - 3.30	123.4	114.4 - 132.3
	152	2.65	2.23 - 3.06	112.2	103.3 - 121.2
	204	2.88	2.36 - 3.40	113.2	102.4 - 123.8
	254	3.03	2.64 - 3.41	117.1	109.2 - 124.9
281	51	2.77	2.49 - 3.04	127.7	121.5 - 133.9
	102	3.25	2.81 - 3.70	117.8	109.0 - 126.5
	152	2.79	2.33 - 3.24	100.1	91.6 - 108.5
	204	3.01	2.42 - 3.60	100.3	90.0 - 110.6
	254	3.11	2.62 - 3.61	103.1	94.3 - 111.8
231	51	2.88	2.58 - 3.18	109.7	104.1 - 115.2
	102	3.38	2.90 - 3.86	105.9	97.6 - 114.2
	152	2.95	2.49 - 3.40	91.9	84.4 - 99.4
	204	3.53	2.82 - 4.24	96.1	85.2 - 107.0
	254	3.80	3.23 - 4.38	100.9	92.0 - 109.7
181	51	2.71	2.50 - 2.92	99.0	95.2 - 102.8
	102	3.58	3.04 - 4.11	99.5	90.9 - 108.1
	152	3.19	2.65 - 3.72	87.7	79.7 - 95.7
	204	3.79	3.00 - 4.58	92.4	81.2 - 103.5
	254	3.99	3.26 - 4.72	92.2	82.2 - 102.2

Table A10 (contd.)

<u>Re</u>	<u>L</u> (mm)	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
120	51	3.30	3.00 - 3.59	95.2	90.6 - 99.9
	102	3.57	3.04 - 4.10	83.4	76.3 - 90.4
	152	3.81	3.08 - 4.55	84.1	74.7 - 93.6
	204	3.97	2.95 - 4.98	78.0	66.2 - 89.8
	254	4.86	3.55 - 6.17	89.1	73.7 - 104.5
63	51	4.07	3.65 - 4.49	91.4	85.7 - 97.0
	102	4.45	3.63 - 5.28	77.7	68.7 - 86.6
	152	5.92	3.97 - 7.87	94.7	73.2 - 116.3

PF MODEL CORRELATION COEFFICIENTS

Depth-by-depth Analysis

Table All

12.7 mm Ceramic Beads

<u>Re</u>	<u>L</u> (mm)	<u>$\rho_{Pe,Bi}$</u>	<u>ρ_{k_r,h_w}</u>
536	51	0.91	-0.33
	102	0.94	-0.61
	178	0.95	-0.74
	356	0.98	-0.93
430	51	0.91	-0.42
	102	0.95	-0.67
	178	0.96	-0.79
	254	0.98	-0.88
	356	0.98	-0.94
290	51	0.91	-0.52
	102	0.95	-0.69
	178	0.96	-0.85
	254	0.98	-0.93
	356	0.99	-0.96
140	51	0.93	-0.72
	102	0.97	-0.84
	178	0.98	-0.94
	356	-	-
36	51	0.96	-0.93
	102	0.98	-0.97
	178	-	-
	356	-	-

PF MODEL CORRELATION COEFFICIENTS

Depth-by-depth Analysis

Table A12

9.5 mm Ceramic Beads

<u>Re</u>	<u>L</u> (mm)	<u>$\rho_{Pe,Bi}$</u>	<u>ρ_{k_r,h_w}</u>
373	51	0.90	-0.23
	102	0.90	-0.50
	152	0.93	-0.67
	204	0.94	-0.77
	254	0.96	-0.85
281	51	0.89	-0.34
	102	0.90	-0.61
	152	0.94	-0.73
	204	0.96	-0.84
	254	0.97	-0.88
231	51	0.89	-0.36
	102	0.90	-0.64
	152	0.94	-0.76
	204	0.95	-0.84
	254	0.97	-0.90
181	51	0.89	-0.43
	102	0.91	-0.70
	152	0.95	-0.82
	204	0.97	-0.90
	254	0.97	-0.92

Table A12 (contd.)

<u>Re</u>	<u>L</u> (mm)	<u>$\rho_{\text{Pe, Bi}}$</u>	<u>$\rho_{\text{k}_r, \text{h}_w}$</u>
120	51	0.89	-0.67
	102	0.94	-0.80
	152	0.96	-0.89
	204	0.97	-0.92
	254	0.98	-0.95
63	51	0.92	-0.77
	102	0.96	-0.90
	152	0.98	-0.96

ADPF(IBC) MODEL PARAMETERS

Overall Analysis

Table A13

12.7 mm Ceramic Beads

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
536	6.37	5.83 - 6.90	1.85	1.69 - 2.01
430	6.06	5.66 - 6.46	1.42	1.32 - 1.52
290	5.07	4.74 - 5.41	1.17	1.09 - 1.24
140	3.36	3.09 - 3.62	0.85	0.78 - 0.91
36	1.28	1.10 - 1.47	0.57	0.49 - 0.66

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
536	2.37	2.06 - 2.68	123.9	116.6 - 131.2
430	2.85	2.51 - 3.18	114.2	107.5 - 120.9
290	3.04	2.67 - 3.41	100.4	94.1 - 106.6
140	4.15	3.43 - 4.88	99.4	88.8 - 110.0
36	15.74	8.88 - 22.60	254.7	141.8 - 367.6

<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
536	0.38	0.28 - 0.47	31.4	23.1 - 39.7
430	0.28	0.22 - 0.33	30.9	25.0 - 36.8
290	0.38	0.30 - 0.46	15.8	12.4 - 19.2
140	0.30	0.22 - 0.38	9.5	7.0 - 12.0
36	0.29	0.14 - 0.43	2.5	1.3 - 3.8

ADPF(IBC) MODEL PARAMETERSOverall AnalysisTable A149.5 mm Ceramic Beads

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
373	6.12	5.87 - 6.37	1.34	1.28 - 1.39
281	5.59	5.36 - 5.81	1.11	1.06 - 1.15
231	5.30	5.11 - 5.50	0.95	0.92 - 0.99
181	4.58	4.43 - 4.74	0.87	0.84 - 0.90
120	3.64	3.52 - 3.77	0.72	0.70 - 0.75
63	2.17	2.08 - 2.26	0.64	0.61 - 0.67

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
373	3.12	2.91 - 3.34	118.1	114.0 - 122.1
281	3.56	3.29 - 3.82	111.2	106.7 - 115.6
231	3.80	3.53 - 4.06	102.4	98.4 - 106.4
181	3.96	3.68 - 4.24	96.9	93.0 - 100.9
120	4.57	4.20 - 4.93	93.2	88.6 - 97.8
63	5.73	5.08 - 6.38	103.6	95.3 - 111.9

<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
373	0.70	0.48 - 0.93	11.6	7.9 - 15.3
281	0.51	0.38 - 0.64	12.1	9.3 - 15.0
231	0.58	0.45 - 0.72	8.7	6.6 - 10.7
181	0.49	0.39 - 0.58	8.1	6.5 - 9.7
120	0.40	0.34 - 0.47	6.5	5.4 - 7.5
63	0.35	0.28 - 0.42	4.0	3.2 - 4.8

ADPF(IBC) MODEL PARAMETERS

Overall Analysis

Table A15

6.4 mm Ceramic Beads

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
274	5.42	5.20 - 5.64	1.01	0.97 - 1.05
206	4.81	4.63 - 4.99	0.86	0.82 - 0.89
171	4.47	4.31 - 4.63	0.76	0.74 - 0.79
138	4.03	3.88 - 4.17	0.68	0.66 - 0.71
109	3.43	3.32 - 3.54	0.63	0.61 - 0.65
73	2.55	2.46 - 2.63	0.57	0.55 - 0.59
45	1.74	1.70 - 1.80	0.51	0.50 - 0.53

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
274	4.19	3.89 - 4.50	119.3	114.5 - 124.2
206	4.58	4.25 - 4.91	110.9	106.2 - 115.6
171	5.02	4.64 - 5.39	108.2	103.2 - 115.1
138	5.48	5.04 - 5.92	105.6	100.2 - 110.9
109	6.29	5.77 - 6.81	112.3	106.1 - 118.6
73	5.85	5.35 - 6.35	94.6	89.2 - 99.9
45	6.62	6.08 - 7.15	95.7	90.2 - 101.1

<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
274	1.00	0.46 - 1.54	5.5	2.5 - 8.4
206	0.55	0.39 - 0.71	7.5	5.3 - 9.7
171	0.46	0.36 - 0.56	7.4	5.8 - 9.1
138	0.36	0.29 - 0.42	7.7	6.2 - 9.2
109	0.27	0.23 - 0.30	8.1	7.0 - 9.1
73	0.30	0.26 - 0.35	4.8	4.1 - 5.5
45	0.27	0.24 - 0.30	3.3	3.0 - 3.6

ADPF(IBC) MODEL PARAMETERS

Overall Analysis

Table A16

9.5 mm Steel Balls

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
468	5.01	4.82 - 5.21	2.06	1.98 - 2.14
356	4.23	4.07 - 4.39	1.86	1.79 - 1.93
224	3.11	2.98 - 3.23	1.59	1.53 - 1.65
146	2.46	2.33 - 2.58	1.32	1.25 - 1.39
77	1.20	1.10 - 1.30	1.41	1.30 - 1.53

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
468	3.40	3.15 - 3.64	197.7	190.0 - 205.4
356	3.47	3.21 - 3.72	181.9	174.5 - 189.3
224	3.90	3.57 - 4.23	175.1	166.4 - 183.9
146	3.72	3.31 - 4.12	138.5	129.8 - 147.2
77	4.84	4.09 - 5.59	193.2	168.3 - 218.0

<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
468	0.73	0.52 - 0.94	14.2	10.1 - 18.3
356	0.77	0.55 - 0.98	10.3	7.4 - 13.2
224	0.51	0.41 - 0.62	9.7	7.7 - 11.6
146	0.95	0.62 - 1.28	3.4	2.2 - 4.6
77	0.22	0.16 - 0.28	7.6	5.5 - 9.8

ADPF(IBC) MODEL CORRELATION COEFFICIENTS

Overall Analysis

Table A17

<u>Packing</u>	<u>d_p</u> (mm)	<u>Re</u>	<u>ρ_{k_a, k_r}</u>	<u>ρ_{k_a, h_w}</u>	<u>ρ_{k_r, h_w}</u>
Ceramic	12.7	536	-0.34	0.04	-0.68
		430	-0.22	0.15	-0.76
		290	-0.18	0.18	-0.80
		140	-0.11	0.38	-0.77
		36	0.88	0.62	0.21
Ceramic	9.5	373	-0.44	0.03	-0.64
		281	-0.41	0.12	-0.70
		231	-0.34	0.12	-0.74
		181	-0.28	0.17	-0.78
		120	-0.14	0.24	-0.81
		63	-0.01	0.42	-0.75
Ceramic	6.4	274	-0.51	0.07	-0.58
		206	-0.36	0.01	-0.64
		171	-0.35	0.13	-0.70
		138	-0.34	0.13	-0.72
		109	-0.24	0.14	-0.78
		73	-0.14	0.15	-0.79
		45	-0.05	0.21	-0.79
Steel	9.5	468	-0.31	0.09	-0.74
		356	-0.24	0.15	-0.79
		224	-0.09	0.26	-0.82
		146	-0.12	0.27	-0.84
		77	0.60	0.70	-0.06

ADPF(FBC) MODEL PARAMETERS

Overall Analysis

Table A18

12.7 mm Ceramic Beads

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
536	4.25	2.60 - 5.90	2.77	1.69 - 3.85
430	3.45	1.59 - 5.30	2.50	1.15 - 3.84
290	2.89	1.34 - 4.44	2.05	0.95 - 3.14
140	1.77	0.33 - 3.22	1.61	0.30 - 2.92
36	0.08	-1.98 - 2.15	8.66	-201 - 219

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
536	3.89	2.60 - 5.17	304.2	103.2 - 505.2
430	4.77	3.25 - 6.30	336.4	66.0 - 606.9
290	5.06	3.48 - 6.63	292.6	60.5 - 524.7
140	6.06	3.44 - 8.67	274.8	-50.7 - 600.3
36	8.50	2.90 - 14.1	2079	-5 ₁₀ 4 - 5 ₁₀ 4

<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
536	0.07	-0.01 - 0.15	171.0	-19.6 - 361.5
430	0.06	-0.01 - 0.12	154.5	-29.3 - 338.3
290	0.07	-0.01 - 0.15	88.2	-14.6 - 190.9
140	0.10	-0.07 - 0.28	27.3	-18.4 - 73.0
36	0.02	-0.47 - 0.51	38.5	-954 - 1032

ADPF(FBC) MODEL PARAMETERS

Overall Analysis

Table A19

9.5 mm Ceramic Beads

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
373	5.78	5.56 - 6.01	1.42	1.36 - 1.47
281	5.18	4.97 - 5.40	1.19	1.14 - 1.24
231	4.93	4.73 - 5.13	1.03	0.98 - 1.07
181	4.04	3.79 - 4.29	0.98	0.92 - 1.04
120	2.78	2.35 - 3.20	0.95	0.80 - 1.09
63	0.77	-0.08 - 1.62	1.81	-0.19 - 3.81

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
373	3.19	2.88 - 3.50	127.7	117.8 - 137.7
281	3.63	3.23 - 4.04	122.3	110.4 - 134.2
231	4.13	3.66 - 4.60	119.7	106.8 - 132.7
181	4.76	4.12 - 5.40	132.3	111.1 - 153.5
120	6.38	5.16 - 7.59	170.7	118.2 - 223.2
63	9.61	6.09 - 13.1	491.4	-205.9 - 1188

<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
373	0.95	0.08 - 1.82	8.6	0.7 - 16.5
281	0.80	0.11 - 1.49	7.7	1.1 - 14.3
231	0.53	0.20 - 0.87	9.5	3.5 - 15.4
181	0.28	0.13 - 0.42	14.4	6.8 - 22.1
120	0.15	0.06 - 0.24	17.4	6.9 - 27.8
63	0.05	-0.03 - 0.14	27.1	-18.1 - 72.4

ADPF(IBC) MODEL PARAMETERS

<u>Overall Analysis</u>			<u>Table A20</u>	
<u>12.7 mm Ceramic Beads</u>			<u>Entrance Data Included</u>	
<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
536	6.50	6.03 - 6.96	1.81	1.68 - 1.94
430	6.15	5.79 - 6.51	1.40	1.32 - 1.48
290	5.34	5.04 - 5.65	1.11	1.04 - 1.17
140	3.61	3.36 - 3.85	0.79	0.74 - 0.84
36	1.63	1.54 - 1.73	0.45	0.42 - 0.48
<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
536	2.48	2.20 - 2.76	127.1	120.3 - 133.9
430	2.93	2.63 - 3.24	115.9	109.9 - 121.9
290	3.36	2.99 - 3.74	105.3	98.8 - 111.8
140	4.70	3.97 - 5.42	104.8	94.8 - 114.7
36	50.03	5.98 - 94.1	635.9	109.6 - 1162
<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
536	0.49	0.42 - 0.56	24.2	20.8 - 27.8
430	0.45	0.40 - 0.50	19.2	17.0 - 21.4
290	0.47	0.42 - 0.52	12.6	11.2 - 14.0
140	0.47	0.42 - 0.52	6.1	5.4 - 6.7
36	0.49	0.46 - 0.52	1.5	1.4 - 1.6

ADPF(IBC) MODEL PARAMETERS

Overall Analysis

Table A21

9.5 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
373	6.21	6.00 - 6.42	1.32	1.28 - 1.36
281	5.60	5.41 - 5.79	1.10	1.07 - 1.14
231	5.38	5.20 - 5.54	0.94	0.91 - 0.97
181	4.60	4.46 - 4.75	0.86	0.84 - 0.89
120	3.69	3.57 - 3.81	0.71	0.69 - 0.74
63	2.26	2.16 - 2.36	0.62	0.59 - 0.64

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
373	3.19	3.00 - 3.38	118.8	115.0 - 122.6
281	3.60	3.37 - 3.83	112.2	108.2 - 116.2
231	3.90	3.65 - 4.14	103.6	99.9 - 107.4
181	4.01	3.75 - 4.27	97.6	93.9 - 101.3
120	4.70	4.35 - 5.05	94.6	90.3 - 99.0
63	6.20	5.45 - 6.96	107.8	99.0 - 116.7

<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
373	0.61	0.54 - 0.68	13.5	12.0 - 15.0
281	0.54	0.48 - 0.60	11.4	10.3 - 12.6
231	0.54	0.49 - 0.59	9.4	8.5 - 10.2
181	0.50	0.46 - 0.54	7.9	7.3 - 8.6
120	0.47	0.44 - 0.51	5.6	5.2 - 6.0
63	0.44	0.41 - 0.48	3.1	2.9 - 3.4

ADPF(IBC) MODEL PARAMETERS

Overall Analysis

Table A22

6.4 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
274	6.12	5.78 - 6.46	0.89	0.84 - 0.94
206	5.32	5.04 - 5.60	0.77	0.73 - 0.82
171	4.90	4.64 - 5.15	0.70	0.66 - 0.73
138	4.34	4.12 - 4.56	0.63	0.60 - 0.66
109	3.58	3.45 - 3.72	0.60	0.58 - 0.62
73	2.69	2.54 - 2.83	0.54	0.51 - 0.57
45	1.82	1.73 - 1.92	0.49	0.46 - 0.52

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
274	4.84	4.33 - 5.36	122.0	114.2 - 129.8
206	5.30	4.70 - 5.89	115.8	107.7 - 123.9
171	5.86	5.17 - 6.56	115.3	106.4 - 124.1
138	6.44	5.64 - 7.25	115.2	105.6 - 124.8
109	7.17	6.44 - 7.90	121.5	112.8 - 130.2
73	7.36	6.26 - 8.46	112.7	100.8 - 124.5
45	9.47	7.76 - 11.2	131.0	113.3 - 148.7

<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
274	0.18	0.15 - 0.20	30.6	26.2 - 35.1
206	0.16	0.14 - 0.18	25.8	22.6 - 29.0
171	0.15	0.13 - 0.17	22.2	19.4 - 25.0
138	0.14	0.13 - 0.16	19.0	16.9 - 21.2
109	0.16	0.14 - 0.17	13.7	12.6 - 14.9
73	0.14	0.13 - 0.15	10.6	9.7 - 11.5
45	0.13	0.12 - 0.14	7.1	6.5 - 7.7

ADPF(IBC) MODEL PARAMETERS

Overall Analysis

9.5 mm Steel Balls

Table A23

Entrance Data Included

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
468	5.19	4.94 - 5.43	1.99	1.90 - 2.09
356	4.38	4.16 - 4.59	1.80	1.71 - 1.88
224	3.12	2.97 - 3.28	1.58	1.50 - 1.66
146	2.50	2.36 - 2.63	1.30	1.23 - 1.37
77	1.21	1.12 - 1.31	1.40	1.29 - 1.51

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
468	3.57	3.25 - 3.90	201.0	190.9 - 211.0
356	3.71	3.56 - 4.07	188.3	178.1 - 198.4
224	4.12	3.68 - 4.55	183.8	172.6 - 195.1
146	4.10	3.62 - 4.58	150.6	140.2 - 161.0
77	5.17	4.20 - 6.14	204.3	179.9 - 228.7

<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
468	0.34	0.31 - 0.38	30.2	27.2 - 33.2
356	0.32	0.29 - 0.35	24.3	22.1 - 26.6
224	0.30	0.28 - 0.33	16.2	15.0 - 17.5
146	0.36	0.32 - 0.39	9.1	8.3 - 9.9
77	0.21	0.19 - 0.22	8.2	7.5 - 8.8

ADPF(FBC) MODEL PARAMETERS

<u>Overall Analysis</u>			<u>Table A24</u>	
<u>12.7 mm Ceramic Beads</u>			<u>Entrance Data Included</u>	
<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
536	5.81	5.35 - 6.26	1.94	1.78 - 2.09
430	5.62	5.23 - 6.00	1.53	1.43 - 1.64
290	4.82	4.50 - 5.13	1.23	1.15 - 1.31
140	3.01	2.77 - 3.25	0.94	0.87 - 1.02
36	1.33	1.27 - 1.39	0.55	0.53 - 0.58
<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
536	2.62	2.28 - 2.96	143.3	134.1 - 152.5
430	3.20	2.80 - 3.60	138.5	129.2 - 147.8
290	3.67	3.21 - 4.14	127.6	118.4 - 136.7
140	4.91	4.04 - 5.78	131.0	116.4 - 145.6
36	63.20	19.18 - 107.2	988.4	333.2 - 1643
<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
536	0.58	0.47 - 0.68	19.5	16.0 - 23.1
430	0.57	0.48 - 0.66	15.1	12.8 - 17.4
290	0.55	0.48 - 0.62	10.7	9.4 - 12.1
140	0.59	0.52 - 0.66	4.8	4.2 - 5.4
36	0.62	0.58 - 0.66	1.2	1.1 - 1.3

ADPF(FBC) MODEL PARAMETERS

Overall Analysis

Table A25

9.5 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
373	5.77	5.57 - 5.98	1.42	1.37 - 1.47
281	5.17	4.98 - 5.37	1.20	1.15 - 1.24
231	4.95	4.78 - 5.13	1.02	0.99 - 1.06
181	4.24	4.09 - 4.39	0.94	0.90 - 0.97
120	3.35	3.22 - 3.48	0.79	0.76 - 0.82
63	1.92	1.81 - 2.03	0.73	0.69 - 0.77

<u>Re</u>	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
373	3.29	3.08 - 3.50	131.9	127.4 - 136.5
281	3.74	3.47 - 4.01	126.3	121.1 - 131.4
231	4.09	3.81 - 4.38	118.2	113.4 - 123.1
181	4.30	3.98 - 4.62	113.9	108.9 - 119.0
120	5.06	4.60 - 5.52	112.4	106.1 - 119.0
63	6.48	5.50 - 7.46	133.2	119.5 - 147.0

<u>Re</u>	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
373	0.66	0.58 - 0.74	12.4	10.9 - 14.0
281	0.61	0.54 - 0.68	10.1	8.9 - 11.2
231	0.59	0.53 - 0.66	8.5	7.6 - 9.4
181	0.56	0.50 - 0.61	7.1	6.5 - 7.8
120	0.55	0.50 - 0.60	4.8	4.4 - 5.2
63	0.54	0.48 - 0.59	2.6	2.4 - 2.9

ADPF(IBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A26

12.7 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
536	51	6.30	4.98 - 7.63	1.87	1.48 - 2.26
	102	5.43	5.43 - 5.43	2.16	2.16 - 2.16
	178	6.79	5.81 - 7.77	1.62	1.38 - 1.85
	356	7.18	6.47 - 7.88	1.53	1.38 - 1.68
430	51	5.66	4.58 - 6.74	1.52	1.23 - 1.81
	102	4.93	4.07 - 5.79	1.75	1.44 - 2.05
	178	6.37	5.47 - 7.27	1.35	1.16 - 1.54
	254	5.94	5.37 - 6.51	1.45	1.31 - 1.59
	356	7.17	6.48 - 7.86	1.20	1.08 - 1.32
290	51	4.82	4.00 - 5.63	1.23	1.02 - 1.44
	102	4.72	3.94 - 5.49	1.26	1.05 - 1.46
	178	5.79	5.02 - 6.56	1.02	0.89 - 1.16
	254	5.38	4.81 - 5.94	1.10	0.99 - 1.22
	356	7.30	6.70 - 7.90	0.81	0.74 - 0.88
140	51	3.16	2.74 - 3.58	0.90	0.78 - 1.02
	102	3.20	2.72 - 3.67	0.89	0.76 - 1.02
	178	4.42	3.81 - 5.04	0.64	0.55 - 0.73
	356	4.33	3.95 - 4.70	0.66	0.60 - 0.72
36	51	1.51	1.39 - 1.62	0.49	0.45 - 0.52
	102	1.82	1.65 - 1.99	0.40	0.37 - 0.44
	178	-	-	-	-
	356	-	-	-	-

ADPF(IBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A27

12.7 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
536	51	2.56	1.98 - 3.14	135.3	122.4 - 148.2
	102	2.09	2.09 - 2.09	127.3	127.3 - 127.3
	178	2.37	1.81 - 2.93	108.3	96.4 - 120.2
	356	3.10	2.50 - 3.70	133.9	120.4 - 147.5
430	51	3.08	2.37 - 3.78	132.0	119.0 - 145.0
	102	2.43	1.78 - 3.07	119.7	105.0 - 134.4
	178	2.81	2.11 - 3.51	107.1	93.8 - 120.4
	254	2.49	2.07 - 2.92	101.8	93.6 - 110.1
	356	4.13	3.20 - 5.07	140.1	121.3 - 158.8
290	51	3.59	2.74 - 4.43	124.5	110.8 - 138.3
	102	2.45	1.82 - 3.07	86.8	76.6 - 97.0
	178	3.43	2.52 - 4.35	99.2	84.6 - 113.8
	254	3.82	2.93 - 4.71	118.9	103.0 - 134.8
	356	8.99	6.00 - 12.0	206.0	153.7 - 258.4
140	51	4.38	3.27 - 5.50	111.5	95.3 - 127.6
	102	3.22	2.31 - 4.13	81.0	68.8 - 93.1
	178	7.21	3.92 - 10.5	131.0	88.1 - 174.0
	356	87.9	-104 - 280	1633	-1837 - 5103
36	51	18.4	9.40 - 27.5	254.2	146.6 - 361.8
	102	68.5	-80.4 - 217	780.4	-850 - 2410
	178	-	-	-	-
	356	-	-	-	-

ADPF(IBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A28

12.7 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
536	51	0.57	0.36 - 0.77	20.8	13.3 - 28.3
	102	0.44	0.44 - 0.44	26.4	26.4 - 26.4
	178	0.60	0.30 - 0.89	18.5	9.4 - 27.5
	356	0.59	0.48 - 0.70	18.6	15.3 - 22.0
430	51	0.54	0.36 - 0.72	15.9	10.5 - 21.3
	102	0.49	0.29 - 0.68	17.6	10.5 - 24.8
	178	0.51	0.31 - 0.71	16.9	10.2 - 23.7
	254	0.56	0.45 - 0.68	15.3	12.2 - 18.3
	356	0.57	0.48 - 0.67	15.0	12.6 - 17.4
290	51	0.58	0.38 - 0.78	10.2	6.8 - 13.6
	102	0.47	0.30 - 0.64	12.6	8.0 - 17.3
	178	0.44	0.31 - 0.58	13.3	9.3 - 17.3
	254	0.54	0.45 - 0.63	10.9	9.1 - 12.8
	356	0.59	0.51 - 0.66	10.1	8.7 - 11.5
140	51	0.55	0.39 - 0.71	5.2	3.7 - 6.7
	102	0.46	0.34 - 0.58	6.2	4.5 - 7.8
	178	0.50	0.39 - 0.60	5.8	4.5 - 7.0
	356	0.90	0.80 - 1.00	3.2	2.8 - 3.5
36	51	0.57	0.50 - 0.64	1.3	1.1 - 1.4
	102	0.46	0.42 - 0.51	1.6	1.4 - 1.7
	178	-	-	-	-
	356	-	-	-	-

ADPF(IBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A29

9.5 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
373	51	6.48	5.78 - 7.18	1.26	1.13 - 1.40
	102	6.41	5.78 - 7.04	1.28	1.15 - 1.40
	152	5.94	5.37 - 6.50	1.38	1.25 - 1.51
	204	6.12	5.57 - 6.67	1.34	1.22 - 1.46
	254	6.00	5.58 - 6.43	1.36	1.27 - 1.46
281	51	5.68	5.13 - 6.22	1.09	0.98 - 1.19
	102	5.73	5.25 - 6.20	1.08	0.99 - 1.17
	152	5.23	4.77 - 5.68	1.18	1.08 - 1.28
	204	5.37	4.88 - 5.87	1.15	1.04 - 1.26
	254	5.25	4.80 - 5.70	1.18	1.08 - 1.28
231	51	5.62	5.08 - 6.17	0.90	0.81 - 0.99
	102	5.37	4.93 - 5.81	0.94	0.86 - 1.02
	152	4.91	4.52 - 5.30	1.03	0.95 - 1.11
	204	5.36	4.87 - 5.84	0.94	0.86 - 1.03
	254	5.34	4.95 - 5.72	0.95	0.88 - 1.02
181	51	4.64	4.27 - 5.00	0.86	0.79 - 0.92
	102	4.63	4.25 - 5.00	0.86	0.79 - 0.93
	152	4.28	3.93 - 4.64	0.93	0.85 - 1.00
	204	4.60	4.18 - 5.02	0.86	0.78 - 0.94
	254	4.78	4.38 - 5.19	0.83	0.76 - 0.90

Table A29 (contd.)

<u>Re</u>	<u>L</u> (mm)	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
120	51	3.67	3.39 - 3.95	0.72	0.66 - 0.77
	102	3.52	3.26 - 3.78	0.75	0.69 - 0.80
	152	3.43	3.13 - 3.73	0.77	0.70 - 0.83
	204	3.72	3.32 - 4.12	0.71	0.63 - 0.78
	254	4.02	3.64 - 4.41	0.65	0.59 - 0.72
63	51	2.15	2.02 - 2.29	0.65	0.60 - 0.69
	102	2.29	2.09 - 2.48	0.61	0.56 - 0.66
	152	2.43	2.16 - 2.71	0.57	0.51 - 0.64

ADPF(IBC) MODEL PARAMETERSDepth-by-depth AnalysisTable A309.5 mm Ceramic BeadsEntrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
373	51	3.49	3.06 - 3.92	124.7	118.2 - 131.1
	102	3.41	2.92 - 3.90	123.2	113.7 - 132.6
	152	2.88	2.50 - 3.26	112.1	105.0 - 119.2
	204	3.02	2.51 - 3.53	114.0	103.1 - 125.0
	254	3.07	2.65 - 3.48	118.2	109.6 - 126.8
281	51	4.08	3.58 - 4.58	125.4	118.4 - 132.4
	102	3.94	3.39 - 4.48	120.0	110.7 - 129.2
	152	3.02	2.55 - 3.49	100.9	92.6 - 109.2
	204	3.15	2.58 - 3.71	102.2	92.4 - 112.1
	254	3.18	2.64 - 3.72	105.7	96.1 - 115.3
231	51	4.28	3.75 - 4.81	108.8	102.8 - 114.7
	102	4.08	3.49 - 4.67	108.7	99.6 - 117.8
	152	3.22	2.76 - 3.68	93.8	86.6 - 101.0
	204	3.71	3.00 - 4.42	99.0	88.0 - 109.9
	254	3.88	3.27 - 4.50	104.1	94.5 - 113.7
181	51	4.08	3.66 - 4.51	98.8	93.9 - 103.8
	102	4.27	3.62 - 4.91	103.5	94.1 - 112.8
	152	3.50	2.92 - 4.07	91.5	83.1 - 99.9
	204	3.97	3.17 - 4.78	96.8	85.3 - 108.3
	254	4.18	3.36 - 5.01	98.1	86.6 - 109.6

Table A30 (contd.)

<u>Re</u>	<u>L</u> (mm)	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
120	51	5.00	4.38 - 5.62	101.3	94.4 - 108.2
	102	4.22	3.58 - 4.87	89.2	81.0 - 97.3
	152	4.28	3.43 - 5.13	92.7	81.5 - 103.8
	204	4.32	3.23 - 5.40	86.2	73.0 - 99.3
	254	5.74	4.18 - 7.30	106.0	86.7 - 125.3
63	51	6.01	5.10 - 6.91	109.6	98.9 - 120.3
	102	5.52	4.22 - 6.83	94.8	79.7 - 109.8
	152	8.91	5.08 - 12.7	143.7	96.9 - 190.5

ADPF(IBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A31

9.5 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
373	51	0.67	0.53 - 0.82	12.1	9.5 - 14.8
	102	0.66	0.44 - 0.89	12.4	8.1 - 16.6
	152	0.57	0.38 - 0.77	14.3	9.4 - 19.2
	204	0.53	0.35 - 0.71	15.5	10.2 - 20.7
	254	0.54	0.39 - 0.69	15.2	11.0 - 19.5
281	51	0.64	0.51 - 0.76	9.7	7.8 - 11.6
	102	0.56	0.41 - 0.72	11.0	8.0 - 14.0
	152	0.53	0.37 - 0.69	11.6	8.2 - 15.1
	204	0.49	0.35 - 0.63	12.7	9.1 - 16.4
	254	0.52	0.40 - 0.63	12.0	9.3 - 14.7
231	51	0.64	0.50 - 0.77	8.0	6.3 - 9.6
	102	0.55	0.40 - 0.70	9.2	6.7 - 11.6
	152	0.48	0.36 - 0.60	10.5	7.9 - 13.1
	204	0.46	0.33 - 0.59	11.0	7.9 - 14.1
	254	0.54	0.43 - 0.64	9.4	7.6 - 11.3
181	51	0.56	0.47 - 0.66	7.0	5.9 - 8.2
	102	0.53	0.40 - 0.65	7.5	5.8 - 9.3
	152	0.47	0.36 - 0.57	8.5	6.6 - 10.4
	204	0.48	0.37 - 0.58	8.4	6.5 - 10.2
	254	0.48	0.40 - 0.56	8.3	6.8 - 9.7

Table A31 (contd.)

<u>Re</u>	<u>L</u> (mm)	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
120	51	0.53	0.44 - 0.62	5.0	4.2 - 5.8
	102	0.49	0.39 - 0.58	5.4	4.3 - 6.5
	152	0.49	0.40 - 0.58	5.4	4.4 - 6.4
	204	0.45	0.36 - 0.54	5.9	4.7 - 7.0
	254	0.49	0.42 - 0.57	5.4	4.5 - 6.2
63	51	0.46	0.39 - 0.52	3.0	2.6 - 3.5
	102	0.45	0.38 - 0.53	3.1	2.6 - 3.6
	152	0.48	0.40 - 0.55	2.9	2.4 - 3.4

ADPF(FBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A32

12.7 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
536	51	4.14	3.55 - 4.73	2.84	2.44 - 3.25
	102	4.40	3.65 - 5.15	2.66	2.21 - 3.12
	178	6.10	5.21 - 6.99	1.80	1.54 - 2.06
	356	6.81	6.13 - 7.50	1.61	1.45 - 1.77
430	51	3.71	3.20 - 4.21	2.32	2.01 - 2.64
	102	3.95	3.30 - 4.60	2.18	1.82 - 2.54
	178	5.65	4.84 - 6.46	1.52	1.30 - 1.74
	254	5.51	4.97 - 6.06	1.56	1.41 - 1.71
	356	6.80	6.13 - 7.48	1.26	1.14 - 1.39
290	51	3.29	2.87 - 3.70	1.80	1.58 - 2.03
	102	3.76	3.17 - 4.36	1.57	1.33 - 1.82
	178	5.08	4.38 - 5.78	1.16	1.00 - 1.32
	254	4.99	4.46 - 5.53	1.19	1.06 - 1.31
	356	6.95	6.37 - 7.54	0.85	0.78 - 0.92
140	51	2.21	1.94 - 2.48	1.29	1.13 - 1.44
	102	2.61	2.21 - 3.01	1.09	0.92 - 1.26
	178	3.98	3.40 - 4.56	0.72	0.61 - 0.82
	356	4.18	3.92 - 4.44	0.68	0.64 - 0.72
36	51	1.18	1.08 - 1.27	0.62	0.57 - 0.68
	102	1.61	1.52 - 1.69	0.46	0.43 - 0.48
	178	-	-	-	-
	356	-	-	-	-

ADPF(FBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A33

12.7 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
536	51	2.24	1.80 - 2.68	179.9	163.6 - 196.2
	102	2.10	1.57 - 2.64	158.1	139.2 - 176.9
	178	2.35	1.80 - 2.90	119.4	105.8 - 133.1
	356	3.10	2.50 - 3.70	141.1	127.0 - 155.2
430	51	2.67	2.13 - 3.21	175.1	155.6 - 194.6
	102	2.33	1.72 - 2.93	143.4	124.4 - 162.3
	178	2.78	2.09 - 3.46	119.4	104.1 - 134.6
	254	2.49	2.07 - 2.91	109.6	100.9 - 118.4
	356	4.13	3.20 - 5.06	147.6	128.2 - 167.0
290	51	3.15	2.49 - 3.81	160.4	141.9 - 179.0
	102	2.34	1.76 - 2.92	103.9	91.4 - 116.5
	178	3.39	2.49 - 4.29	111.6	95.1 - 128.0
	254	3.82	2.94 - 4.71	128.1	111.2 - 145.1
	356	9.04	6.07 - 12.0	217.6	163.4 - 271.8
140	51	3.88	2.97 - 4.79	141.0	121.1 - 161.0
	102	3.11	2.27 - 3.95	95.6	82.5 - 108.7
	178	7.17	3.91 - 10.4	144.8	98.3 - 191.2
	356	78.5	32.3 - 124	1509	648.8 - 2370
36	51	16.1	8.82 - 23.3	283.6	176.1 - 391.1
	102	88.7	3.25 - 174	1146	82.0 - 2210
	178	-	-	-	-
	356	-	-	-	-

ADPF(FBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A34

12.7 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
536	51	0.78	0.54 - 1.02	15.2	10.4 - 20.0
	102	0.54	0.34 - 0.74	21.7	13.6 - 29.9
	178	0.66	0.37 - 0.95	16.6	9.3 - 23.9
	356	0.62	0.52 - 0.73	17.7	14.6 - 20.7
430	51	0.73	0.54 - 0.92	11.7	8.7 - 14.8
	102	0.59	0.33 - 0.85	14.6	8.1 - 21.1
	178	0.57	0.37 - 0.78	15.0	9.6 - 20.5
	254	0.61	0.49 - 0.72	14.2	11.5 - 16.8
	356	0.60	0.51 - 0.70	14.2	12.0 - 16.4
290	51	0.78	0.55 - 1.00	7.6	5.4 - 9.8
	102	0.58	0.39 - 0.76	10.2	6.9 - 13.6
	178	0.50	0.37 - 0.64	11.7	8.6 - 14.9
	254	0.58	0.49 - 0.68	10.2	8.6 - 11.8
	356	0.62	0.54 - 0.70	9.6	8.4 - 10.8
140	51	0.73	0.55 - 0.91	3.9	2.9 - 4.9
	102	0.56	0.42 - 0.70	5.1	3.8 - 6.4
	178	0.55	0.44 - 0.66	5.2	4.1 - 6.2
	356	0.94	0.85 - 1.03	3.0	2.7 - 3.3
36	51	0.70	0.63 - 0.78	1.0	0.9 - 1.2
	102	0.54	0.49 - 0.59	1.4	1.2 - 1.5
	178	-	-	-	-
	356	-	-	-	-

ADPF(FBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A35

9.5 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
373	51	4.91	4.51 - 5.31	1.67	1.53 - 1.80
	102	5.59	5.11 - 6.07	1.46	1.34 - 1.59
	152	5.37	4.90 - 5.83	1.53	1.39 - 1.66
	204	5.65	5.13 - 6.16	1.45	1.32 - 1.58
	254	5.64	5.23 - 6.05	1.45	1.35 - 1.56
281	51	4.26	3.95 - 4.57	1.45	1.34 - 1.56
	102	4.90	4.53 - 5.26	1.26	1.17 - 1.36
	152	4.70	4.29 - 5.11	1.32	1.20 - 1.43
	204	4.93	4.46 - 5.41	1.25	1.13 - 1.37
	254	4.92	4.48 - 5.36	1.26	1.14 - 1.37
231	51	4.22	3.92 - 4.53	1.20	1.11 - 1.29
	102	4.59	4.24 - 4.93	1.10	1.02 - 1.19
	152	4.38	4.03 - 4.74	1.16	1.06 - 1.25
	204	4.90	4.43 - 5.37	1.03	0.93 - 1.13
	254	5.02	4.64 - 5.39	1.01	0.93 - 1.08
181	51	3.39	3.19 - 3.59	1.17	1.10 - 1.24
	102	3.95	3.65 - 4.24	1.01	0.93 - 1.08
	152	3.83	3.50 - 4.15	1.04	0.95 - 1.13
	204	4.23	3.87 - 4.63	0.94	0.85 - 1.03
	254	4.48	4.08 - 4.87	0.89	0.81 - 0.96

Table A35 (contd.)

<u>Re</u>	<u>L</u> (mm)	<u>Pe_r</u>	<u>95% Conf. Int.</u>	<u>k_r</u>	<u>95% Conf. Int.</u>
120	51	2.68	2.52 - 2.84	0.98	0.92 - 1.04
	102	2.99	2.77 - 3.22	0.88	0.81 - 0.94
	152	3.09	2.81 - 3.37	0.85	0.77 - 0.93
	204	3.42	3.04 - 3.80	0.77	0.68 - 0.86
	254	3.77	3.40 - 4.15	0.70	0.63 - 0.77
63	51	1.58	1.48 - 1.68	0.88	0.82 - 0.94
	102	1.96	1.78 - 2.15	0.71	0.64 - 0.78
	152	2.20	1.95 - 2.46	0.63	0.56 - 0.70

ADPF(FBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A36

9.5 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
373	51	3.23	2.87 - 3.59	152.3	144.6 - 159.9
	102	3.33	2.87 - 3.79	137.8	127.5 - 148.1
	152	2.85	2.43 - 3.27	122.8	113.4 - 132.2
	204	3.00	2.50 - 3.49	122.9	112.2 - 133.5
	254	3.06	2.64 - 3.48	125.5	116.6 - 134.5
281	51	3.74	3.33 - 4.16	153.5	144.5 - 162.5
	102	3.82	3.30 - 4.33	136.0	125.5 - 146.4
	152	2.99	2.53 - 3.45	111.1	102.1 - 120.1
	204	3.13	2.57 - 3.69	110.7	100.2 - 121.2
	254	3.16	2.62 - 3.70	112.2	102.1 - 122.4
231	51	3.92	3.49 - 4.36	132.8	125.2 - 140.3
	102	3.96	3.40 - 4.51	123.3	113.6 - 133.0
	152	3.19	2.72 - 3.66	104.0	95.7 - 112.2
	204	3.68	2.98 - 4.39	107.4	95.8 - 119.1
	254	3.87	3.26 - 4.49	110.4	100.3 - 120.4
181	51	3.71	3.36 - 4.06	122.6	116.4 - 128.8
	102	4.14	3.53 - 4.75	117.7	107.4 - 128.1
	152	3.45	2.89 - 4.02	101.2	91.8 - 110.7
	204	3.95	3.15 - 4.75	104.7	92.5 - 116.9
	254	4.17	3.37 - 4.97	104.5	93.0 - 116.0

Table A36 (contd.)

<u>Re</u>	<u>L</u> (mm)	<u>Bi</u>	<u>95% Conf. Int.</u>	<u>h_w</u>	<u>95% Conf. Int.</u>
120	51	4.50	3.99 - 5.01	124.8	116.5 - 133.2
	102	4.11	3.49 - 4.73	102.0	92.8 - 111.2
	152	4.24	3.40 - 5.08	101.9	90.0 - 113.9
	204	4.29	3.21 - 5.38	93.3	79.4 - 107.3
	254	5.69	4.15 - 7.23	112.1	92.2 - 132.0
63	51	5.41	4.65 - 6.17	134.4	122.0 - 147.0
	102	5.40	4.12 - 6.67	107.9	91.4 - 124.4
	152	8.69	5.04 - 12.3	154.8	106.4 - 203.1

ADPF(FBC) MODEL PARAMETERS

Depth-by-depth Analysis

Table A37

9.5 mm Ceramic Beads

Entrance Data Included

<u>Re</u>	<u>L</u> (mm)	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
373	51	0.82	0.66 - 0.98	10.0	8.0 - 11.9
	102	0.75	0.55 - 0.94	11.0	8.1 - 13.9
	152	0.63	0.43 - 0.83	13.0	8.9 - 17.1
	204	0.57	0.40 - 0.75	14.3	9.9 - 18.7
	254	0.57	0.45 - 0.70	14.3	11.2 - 17.4
281	51	0.78	0.64 - 0.92	7.9	6.6 - 9.2
	102	0.64	0.48 - 0.80	9.6	7.2 - 12.0
	152	0.59	0.42 - 0.75	10.5	7.6 - 13.4
	204	0.53	0.39 - 0.67	11.7	8.6 - 14.8
	254	0.55	0.46 - 0.64	11.2	9.4 - 13.0
231	51	0.78	0.64 - 0.92	6.5	5.3 - 7.6
	102	0.63	0.48 - 0.78	8.0	6.1 - 9.9
	152	0.54	0.41 - 0.66	9.4	7.2 - 11.6
	204	0.50	0.38 - 0.62	10.1	7.6 - 12.6
	254	0.57	0.47 - 0.67	8.9	7.3 - 10.4
181	51	0.71	0.61 - 0.81	5.6	4.8 - 6.4
	102	0.61	0.47 - 0.74	6.5	5.1 - 8.0
	152	0.52	0.44 - 0.61	7.6	6.4 - 8.8
	204	0.52	0.41 - 0.62	7.7	6.1 - 9.3
	254	0.51	0.42 - 0.60	7.8	6.4 - 9.1

Table A37 (contd.)

<u>Re</u>	<u>L</u> (mm)	<u>Pe_a</u>	<u>95% Conf. Int.</u>	<u>k_a</u>	<u>95% Conf. Int.</u>
120	51	0.67	0.57 - 0.77	3.9	3.4 - 4.5
	102	0.56	0.46 - 0.66	4.7	3.8 - 5.5
	152	0.54	0.45 - 0.64	4.8	4.0 - 5.7
	204	0.49	0.40 - 0.58	5.4	4.4 - 6.4
	254	0.52	0.44 - 0.60	5.0	4.3 - 5.8
63	51	0.58	0.51 - 0.66	2.4	2.1 - 2.7
	102	0.52	0.44 - 0.60	2.7	2.3 - 3.1
	152	0.52	0.44 - 0.60	2.7	2.3 - 3.0

APPENDIX B

Experimental temperature measurements

(All temperatures in °C)

RUN NUMBER-FLOW RATE-BED DEPTH CORRESPONDENCE

12.7 mm Ceramic Beads

Table B1

	L (mm)				
	51	102	178	254	356
536	1	16	6	-	11
430	2	17	7	22	12
290	3	18	8	21	13
140	4	19	9	-	14
36	5	20	10	-	15

9.5 mm Ceramic Beads

Table B2

	L (mm)				
	51	102	152	204	254
373	6	7	13	19	24
281	1	8	14	20	25
231	2	9	15	21	26
181	3	10	16	22	27
120	4	11	17	23	28
63	5	12	18	-	-

RUN NUMBER-FLOW RATE-BED DEPTH CORRESPONDENCE.

6.4 mm Ceramic Beads

Table B3

		L (mm)				
		29	80	141	208	267
Re	274	1	8	15	22	29
	206	2	9	16	23	30
	171	3	10	17	24	31
	138	4	11	18	25	32
	109	5,36	12,37	19,38	26,39	33,40
	73	6	13	20	27	34
	45	7	14	21	28	35

9.5 mm Steel Balls

Table B4

		L (mm)				
		54	112	163	213	259
Re	468	1	6	11	16	20
	356	2	7	12	17	21
	224	3	8	13	18	22
	146	4,23	9,24	14,25	19,26	-
	77	5	10	15	-	-

12.7 mm CERAMIC BEADS

Run number		1	$T_0 = 23.4$		$T_w = 105.1$		
24.8	25.3	26.6	30.6	37.3	42.4		
24.7	25.0	25.6	30.5	35.6	-		
33.3	34.1	37.0	38.6	36.5	48.5	61.7	62.4
33.9	34.5	35.7	37.4	43.8	61.4	47.4	67.2
35.4	35.6	37.7	37.7	44.9	-	59.0	65.0
33.6	33.8	36.0	41.5	50.2	45.8	56.6	55.6

Run number		2	$T_0 = 22.4$		$T_w = 104.9$		
24.9	25.8	26.9	32.1	38.8	44.2		
24.8	25.2	25.8	31.7	37.2	-		
34.6	35.8	39.4	41.5	38.4	52.0	65.7	67.3
35.5	36.8	38.0	40.5	46.4	66.0	50.7	71.5
37.6	38.0	39.6	39.6	49.6	-	62.8	68.4
35.6	35.5	39.1	44.0	55.0	49.5	63.9	59.9

Run number		3	$T_0 = 23.0$		$T_w = 106.0$		
25.7	26.8	28.2	35.2	41.7	46.7		
25.7	26.6	26.9	33.8	40.1	-		
38.3	40.0	44.5	46.9	41.7	58.7	71.6	74.2
39.6	41.4	43.3	46.1	51.1	71.8	55.7	78.3
41.5	42.1	44.0	43.3	58.0	-	69.5	73.6
39.8	39.2	44.6	48.3	62.1	54.5	67.6	69.3

Run number		4	$T_0 = 22.7$		$T_w = 105.6$		
28.6	30.6	32.5	39.7	46.8	51.9		
28.6	30.1	30.6	38.2	45.7	-		
49.4	53.8	55.6	59.5	51.1	73.5	80.5	85.6
50.9	51.3	54.9	60.2	61.6	74.6	64.8	83.9
52.9	52.9	55.5	55.7	71.6	-	79.1	81.1
52.6	49.5	55.2	51.9	75.4	66.7	79.9	84.2

Run number		5	$T_0 = 22.6$		$T_w = 105.3$		
38.8	40.1	42.8	50.0	56.1	58.5		
37.7	39.7	41.1	49.7	55.2	-		
76.1	79.1	80.9	83.3	86.6	95.6	97.2	101.7
78.2	79.4	82.5	84.1	87.3	94.1	91.4	100.4
80.3	81.2	81.2	80.1	89.9	-	97.7	100.5
81.3	80.0	82.6	82.4	93.7	93.0	97.9	101.9

Run number			6		$T_0 = 23.3$	$T_w = 105.4$		
23.8	24.8	25.0	-	36.4	42.7			
23.5	24.0	25.0	30.5	34.8	-			
-	60.0	50.0	62.9	63.5	75.1	73.5	82.3	
61.0	52.1	58.8	59.0	70.7	76.2	85.9	87.4	
62.1	62.4	67.4	66.2	72.9	75.9	81.7	80.9	
57.8	50.2	55.7	58.9	60.8	70.4	67.0	72.5	

Run number			7		$T_0 = 23.3$	$T_w = 105.3$		
24.8	25.9	26.7	-	38.6	46.5			
24.4	25.3	26.7	32.7	37.4	-			
-	64.8	54.8	67.5	67.9	78.8	77.2	85.6	
65.7	56.8	63.3	63.8	74.9	81.1	88.4	92.6	
66.3	66.5	71.4	70.3	77.4	79.7	85.5	84.9	
62.4	55.2	59.7	63.7	66.0	74.5	72.0	76.2	

Run number			8		$T_0 = 22.9$	$T_w = 105.4$		
26.3	27.7	28.6	33.8	41.5	51.1			
25.7	26.7	28.0	35.1	39.9	-			
61.5	70.3	61.2	73.0	74.2	82.9	81.8	89.8	
71.7	62.5	69.0	69.9	79.1	85.5	89.5	96.4	
72.5	73.0	77.2	76.2	82.1	84.8	89.9	90.9	
69.5	62.0	65.7	69.8	72.2	79.8	77.8	81.7	

Run number			9		$T_0 = 22.8$	$T_w = 104.9$		
29.8	31.6	32.6	39.4	45.8	54.5			
28.7	29.2	31.4	41.2	44.1	-			
77.3	86.4	77.8	88.7	86.9	92.1	93.3	100.1	
88.4	79.5	84.9	83.4	92.4	95.4	99.3	101.7	
86.5	88.7	89.3	90.8	94.3	94.4	97.2	101.9	
84.8	78.3	82.9	86.0	86.5	93.2	89.2	93.0	

Run number			10		$T_0 = 22.3$	$T_w = 105.0$		
43.7	45.6	46.8	51.8	57.5	60.8			
43.4	42.4	45.5	53.7	56.4	-			
104.5	105.6	104.8	106.3	106.7	107.5	107.8	108.3	
106.1	105.3	106.0	105.7	107.2	107.8	108.3	108.6	
105.4	106.2	106.2	106.4	107.0	107.3	108.0	108.2	
106.0	104.4	105.8	105.9	105.9	107.1	107.1	108.0	

Run number		11		$T_0 = 20.5$		$T_w = 104.4$	
21.9	23.0	23.6	27.7	34.7	41.1		
21.9	22.3	23.6	28.7	33.1	-		
82.1	79.5	83.5	85.0	87.5	91.8	93.6	97.0
82.5	84.6	84.2	83.2	91.4	90.8	95.2	95.3
82.1	80.3	81.9	82.6	86.2	91.8	91.1	94.2
82.5	82.0	83.7	82.4	84.5	89.0	88.1	91.0

Run number		12		$T_0 = 20.8$		$T_w = 105.1$	
22.9	23.9	24.8	29.5	36.8	44.6		
22.3	23.2	24.8	30.6	35.6	-		
86.5	84.1	87.7	89.2	91.3	95.2	96.6	99.8
86.9	88.8	87.9	87.5	93.1	93.9	98.0	98.0
86.5	84.7	86.2	86.9	90.5	94.9	94.6	97.1
86.7	86.3	88.0	86.6	88.5	93.0	92.4	94.8

Run number		13		$T_0 = 20.5$		$T_w = 105.3$	
23.8	25.7	26.6	31.6	39.6	48.4		
23.5	24.8	26.2	33.4	38.4	-		
92.6	90.2	93.1	94.8	96.1	99.0	100.3	102.8
92.7	94.5	93.1	93.1	95.8	97.9	100.8	101.4
92.2	90.7	92.1	93.0	95.8	98.8	99.0	99.8
92.4	92.1	93.5	92.5	94.4	97.5	97.7	99.1

Run number		14		$T_0 = 21.0$		$T_w = 105.1$	
28.7	30.1	31.1	36.6	44.7	53.5		
27.0	27.6	29.7	39.4	42.9	-		
103.4	101.7	103.7	104.5	104.7	105.8	106.2	107.3
103.2	103.9	103.8	103.6	104.0	106.3	106.0	107.3
102.9	101.8	102.7	103.9	104.6	105.4	105.4	105.7
102.9	103.0	103.3	102.8	104.5	105.0	106.0	106.1

Run number		15		$T_0 = 20.8$		$T_w = 105.2$	
38.8	41.4	42.1	47.3	54.7	57.3		
37.6	38.4	41.1	50.6	53.2	-		
106.3	106.0	106.3	106.6	106.8	107.7	107.8	108.2
106.1	106.3	106.3	106.4	107.1	106.9	107.9	107.6
106.3	105.7	106.3	106.2	106.6	107.2	107.5	107.6
106.4	106.2	106.3	106.2	106.9	107.3	107.6	107.6

Run number			16		$T_0 = 23.5$	$T_w = 106.2$		
24.9	25.8	27.7	35.1	38.5	48.9			
25.3	26.7	27.7	31.4	37.7	-			
52.5	49.2	55.5	49.2	60.6	76.0	74.8	85.2	
50.8	49.5	-	51.1	59.4	55.7	77.9	65.6	
43.2	46.6	43.5	46.2	56.9	54.4	67.0	62.0	
51.3	47.9	51.5	53.0	55.9	65.3	64.8	77.1	

Run number			17		$T_0 = 23.2$	$T_w = 106.2$		
24.9	25.8	27.6	35.9	39.1	50.5			
25.2	26.7	27.6	31.6	38.7	-			
58.0	53.7	61.5	53.5	64.6	79.4	78.0	89.1	
55.1	53.3	-	54.7	64.8	59.4	81.8	72.9	
45.9	50.8	46.4	50.5	60.7	59.3	71.6	67.4	
54.7	52.0	55.3	58.0	60.2	69.9	68.6	81.2	

Run number			18		$T_0 = 23.6$	$T_w = 107.1$		
25.7	26.7	28.7	37.5	40.1	51.8			
26.2	27.8	28.7	32.8	39.7	-			
60.4	55.5	62.7	54.9	66.1	80.7	79.1	90.1	
57.3	55.4	-	56.7	66.7	62.6	82.8	74.6	
48.1	52.9	48.6	52.7	62.3	62.1	73.0	70.5	
56.8	54.3	56.9	59.6	62.3	71.2	70.5	82.1	

Run number			19		$T_0 = 23.5$	$T_w = 106.2$		
28.7	30.7	32.7	42.6	45.1	56.3			
29.7	31.7	32.9	37.4	45.8	-			
70.7	69.9	76.2	69.0	78.2	90.5	89.6	97.4	
73.1	73.1	-	71.8	81.6	73.5	90.8	85.9	
63.8	68.4	65.5	73.7	77.5	78.6	88.0	82.0	
68.6	67.2	63.3	73.1	77.0	81.1	78.4	90.0	

Run number			20		$T_0 = 23.2$	$T_w = 105.4$		
39.6	40.9	45.6	54.0	55.1	61.2			
40.6	43.8	43.6	49.0	56.2	-			
95.7	94.8	97.2	95.3	99.0	102.8	103.7	105.6	
96.3	96.0	-	97.0	101.2	99.3	104.7	104.7	
93.7	95.3	94.0	95.8	98.6	99.1	103.5	103.0	
94.9	93.7	94.3	96.3	96.3	101.8	100.6	104.6	

Run number		21	$T_0 = 23.9$		$T_w = 106.2$		
26.7	27.8	29.5	34.6	40.8	52.5		
27.2	28.7	29.6	34.1	40.6	-		
84.7	83.4	85.9	85.8	88.2	94.6	94.3	98.8
83.9	87.2	84.1	89.5	92.8	94.3	96.0	95.8
84.9	87.2	85.2	87.1	92.8	92.2	95.7	96.9
85.9	84.4	86.0	85.7	88.7	91.4	92.6	97.6

Run number		22	$T_0 = 24.2$		$T_w = 106.4$		
25.8	26.6	27.8	30.7	37.8	47.8		
25.9	27.3	27.9	31.5	37.3	-		
75.1	74.2	76.5	78.0	78.7	89.3	88.6	93.0
74.4	78.3	75.6	81.0	86.2	86.6	91.1	89.5
75.9	78.4	76.7	78.2	86.0	83.7	90.0	88.5
76.3	75.1	76.7	79.1	80.4	85.2	86.1	92.4

9.5 mm CERAMIC BEADS

Run number		1	$T_0 = 27.2$		$T_w = 103.3$		
28.2	28.7	32.6	34.6	35.6	37.6		
28.2	29.6	28.2	33.1	39.1	42.8		
35.6	35.9	35.8	39.8	48.0	61.1	60.9	68.8
34.9	35.6	36.7	40.0	49.1	62.2	59.0	73.6
33.6	35.6	35.9	39.1	44.2	56.7	58.9	65.9
33.7	35.6	36.1	39.8	47.4	57.1	63.5	65.4

Run number		2	$T_0 = 23.9$		$T_w = 104.6$		
24.9	25.3	29.6	31.6	33.1	35.6		
24.9	26.3	24.9	30.6	37.1	40.8		
32.8	33.8	33.0	38.8	46.8	61.2	60.9	69.7
32.0	32.7	33.7	37.9	48.5	60.9	59.9	74.4
31.0	33.5	33.3	36.4	42.0	54.9	57.8	65.4
31.3	33.3	33.8	38.0	46.0	57.2	61.2	65.7

Run number		3	$T_0 = 25.1$		$T_w = 104.5$		
26.3	27.2	31.6	34.1	36.6	38.6		
26.3	28.7	25.8	33.1	40.1	44.3		
37.3	38.1	37.4	44.1	52.1	66.4	65.4	72.8
36.0	36.6	38.0	42.0	47.6	59.8	63.6	70.6
35.0	37.4	38.0	42.0	47.6	59.8	63.6	70.6
35.2	38.3	38.5	43.6	50.4	62.2	64.3	68.8

Run number		4		$T_0 = 25.3$		$T_w = 104.3$	
28.7	29.6	34.1	37.1	40.1	41.6		
28.2	31.1	26.7	35.6	43.6	48.2		
43.9	43.9	43.4	51.5	58.0	70.9	70.9	78.1
40.9	43.7	43.9	47.4	58.2	71.0	69.1	82.5
39.7	42.5	43.5	49.1	54.7	66.3	69.8	76.3
41.4	44.4	44.7	51.0	58.7	68.1	68.3	78.1

Run number		5		$T_0 = 26.0$		$T_w = 104.2$	
32.6	34.6	40.6	44.6	46.7	48.2		
32.6	35.6	32.6	42.6	49.8	54.4		
61.1	60.6	58.9	65.5	70.8	82.4	81.6	89.6
54.9	55.6	60.7	63.3	71.9	80.4	79.3	88.7
53.3	56.0	59.1	65.7	70.0	78.8	80.0	85.8
57.4	58.0	58.5	65.3	72.2	80.1	81.6	88.2

Run number		6		$T_0 = 28.7$		$T_w = 103.9$	
29.1	29.6	32.6	34.6	34.6	37.6		
29.1	29.6	30.6	33.1	38.6	41.2		
34.7	35.5	34.9	39.3	46.1	57.9	58.1	65.6
34.0	34.6	35.5	38.4	46.2	57.3	57.4	71.1
33.2	35.0	34.7	36.5	41.7	53.4	56.3	60.7
33.6	34.6	35.1	37.6	45.7	54.0	57.0	63.2

Run number		7		$T_0 = 29.1$		$T_w = 104.1$	
29.6	30.1	33.1	35.6	35.1	39.1		
29.6	30.1	29.1	33.6	39.1	42.6		
41.2	39.1	42.3	48.4	56.8	73.3	69.6	78.8
41.2	41.9	44.1	46.9	64.8	69.3	73.0	72.5
42.6	-	45.1	50.3	55.7	61.5	67.0	66.9
45.0	46.1	47.2	51.2	53.4	68.4	70.5	76.1

Run number		8		$T_0 = 27.5$		$T_w = 104.3$	
28.2	29.1	34.6	35.6	35.6	39.6		
28.2	29.6	28.2	33.6	39.6	43.6		
44.2	40.8	45.8	51.6	60.7	75.6	72.3	81.9
43.9	44.8	47.9	48.6	64.7	75.4	73.7	83.0
44.6	48.7	48.2	53.7	58.8	65.4	70.6	70.9
47.7	49.1	49.8	54.4	57.0	70.3	72.8	79.4

Run number		9	$T_0 = 28.7$		$T_w = 104.5$		
29.6	30.6	36.1	37.1	37.6	41.1		
29.6	31.1	29.6	35.1	41.6	45.5		
46.8	43.4	48.4	54.3	62.7	77.9	74.5	84.3
46.5	47.8	51.0	51.4	66.7	77.3	75.8	84.7
47.1	51.5	51.2	56.7	61.6	68.0	73.2	72.3
50.7	52.9	53.3	56.8	60.2	73.5	75.0	82.3

Run number		10	$T_0 = 26.7$		$T_w = 104.2$		
28.7	29.6	34.6	37.6	37.6	40.6		
28.7	30.1	28.2	35.1	41.6	45.1		
50.0	45.4	51.0	56.2	64.5	80.4	75.5	85.8
49.0	51.9	53.2	55.6	73.1	78.6	78.9	84.1
49.0	54.7	53.9	60.0	64.7	70.5	75.3	76.4
54.2	56.3	57.2	58.8	63.2	76.4	78.1	85.3

Run number		11	$T_0 = 23.3$		$T_w = 103.9$		
26.7	28.0	33.1	36.8	37.6	40.6		
26.5	28.9	25.8	33.6	41.6	45.7		
54.4	50.8	57.6	64.1	69.3	81.2	78.9	90.1
54.8	57.5	57.2	63.4	75.4	80.6	79.9	86.4
58.7	60.9	61.9	65.8	69.7	77.1	76.6	82.9
60.6	60.0	62.5	65.7	67.4	77.9	81.0	90.0

Run number		12	$T_0 = 24.0$		$T_w = 103.8$		
31.1	32.8	38.6	44.6	44.6	46.7		
30.6	33.8	25.8	40.6	47.7	50.0		
71.5	67.2	72.9	78.2	82.8	88.9	88.4	95.5
72.7	75.7	75.0	78.3	84.9	90.0	89.2	95.0
75.6	76.8	77.9	81.6	82.4	88.0	88.7	92.8
76.2	75.5	79.0	79.1	82.7	90.7	90.3	96.8

Run number		13	$T_0 = 23.3$		$T_w = 103.8$		
23.9	24.4	31.1	31.6	29.6	33.1		
23.9	24.9	23.9	28.2	33.6	37.1		
47.0	46.1	50.0	59.6	57.9	67.5	78.1	72.3
44.3	52.0	49.5	59.5	61.3	72.1	76.7	83.7
49.3	52.3	54.6	57.1	65.8	65.5	76.1	74.7
47.5	44.5	55.0	47.3	64.4	64.7	76.3	75.8

Run number		14	$T_0 = 23.0$		$T_w = 103.8$		
24.2	24.9	31.8	32.6	30.9	34.2		
23.9	25.3	23.9	29.1	35.1	38.4		
50.9	50.9	53.5	63.3	62.1	70.6	80.9	75.9
48.4	56.5	54.1	63.7	65.1	75.7	79.3	86.4
53.8	57.9	59.2	61.7	69.7	68.9	79.4	77.9
51.9	48.9	59.7	51.8	67.7	68.9	79.4	79.0

Run number		15	$T_0 = 23.1$		$T_w = 103.9$		
24.9	25.8	31.6	33.1	32.6	35.6		
24.7	26.3	24.4	30.6	37.1	40.8		
55.5	56.4	56.6	66.0	64.4	73.6	80.9	78.3
51.7	58.4	57.1	64.9	67.4	79.3	80.1	88.3
56.5	58.5	63.0	63.7	73.1	72.3	81.5	80.0
54.6	52.6	62.7	54.6	69.2	73.2	82.0	82.0

Run number		16	$T_0 = 23.1$		$T_w = 103.9$		
25.3	26.3	33.6	34.6	34.6	37.6		
25.3	26.9	24.4	32.1	39.1	42.3		
60.3	63.5	62.3	70.7	69.6	75.5	85.0	82.1
56.5	63.4	62.1	69.6	71.3	81.4	84.0	90.8
63.0	65.5	68.9	69.7	76.8	77.0	84.6	84.2
59.4	58.4	68.0	60.4	74.2	79.1	85.3	86.9

Run number		17	$T_0 = 23.8$		$T_w = 103.0$		
26.7	28.5	35.9	37.6	37.6	40.6		
26.6	29.2	25.3	35.1	42.1	45.8		
71.8	75.1	73.0	79.2	78.4	85.3	88.8	90.5
73.3	70.1	69.0	75.0	77.6	85.4	88.7	93.5
72.3	74.5	76.5	79.9	83.3	86.6	88.4	92.8
69.8	68.9	76.2	68.2	80.5	86.7	89.7	91.5

Run number		18	$T_0 = 23.6$		$T_w = 102.9$		
31.1	33.1	40.8	43.6	44.1	46.7		
30.8	34.1	25.8	40.6	48.2	51.4		
89.5	90.5	90.5	93.3	92.6	96.9	97.1	100.0
84.0	87.3	85.4	89.6	90.6	95.5	96.9	99.7
88.8	89.0	91.0	92.2	94.1	95.6	96.6	97.6
84.1	87.2	88.5	86.3	92.9	94.5	97.6	98.8

Run number		19	$T_0 = 24.0$		$T_w = 103.0$		
25.3	25.5	32.1	32.6	30.6	34.1		
25.1	25.8	24.9	29.1	34.6	37.4		
56.7	64.0	64.2	68.2	70.9	82.3	87.4	86.6
57.2	60.2	59.2	61.5	70.3	72.6	80.7	81.1
55.2	56.3	58.0	62.4	67.9	73.6	80.8	78.9
55.9	57.5	55.2	59.5	68.2	65.7	79.5	82.6

Run number		20	$T_0 = 24.0$		$T_w = 103.2$		
25.5	26.3	33.6	33.6	32.1	35.6		
25.5	26.7	25.3	30.4	36.6	39.3		
62.3	68.8	70.4	72.9	75.2	84.1	90.0	88.9
61.8	65.1	64.9	65.9	74.9	77.6	84.7	83.6
60.7	58.6	63.4	67.2	73.1	77.1	84.0	82.1
61.3	62.4	60.0	64.1	72.8	69.6	82.8	85.1

Run number		21	$T_0 = 22.8$		$T_w = 103.7$		
24.6	25.3	35.1	33.6	32.1	35.1		
24.4	25.8	24.1	30.1	37.1	40.4		
63.8	70.2	69.6	76.0	77.3	86.6	91.8	90.7
65.1	66.9	63.3	66.6	77.4	78.3	87.4	84.7
61.7	65.4	65.4	70.0	74.6	81.4	85.4	89.3
63.0	64.0	62.3	66.6	75.8	71.4	85.3	87.2

Run number		22	$T_0 = 22.9$		$T_w = 103.4$		
25.2	25.8	35.6	35.1	33.8	36.6		
24.9	26.7	24.4	31.6	38.6	41.4		
69.6	75.9	74.7	80.0	81.1	89.0	94.1	93.6
71.0	72.5	68.8	72.2	81.9	85.7	90.3	90.2
68.5	71.6	71.3	75.2	79.9	83.3	88.9	90.4
69.3	71.1	69.7	72.6	79.9	75.5	87.9	90.1

Run number		23	$T_0 = 21.6$		$T_w = 103.8$		
25.1	26.5	36.6	36.1	35.2	38.6		
24.9	27.5	23.5	33.1	40.1	43.3		
77.5	82.0	82.9	86.6	85.9	93.8	97.2	97.2
77.8	79.1	74.9	79.5	87.7	90.2	94.6	93.6
76.8	78.7	79.1	82.4	85.5	89.5	92.7	93.3
78.5	80.1	80.7	79.9	86.3	80.7	90.0	95.9

Run number		24	$T_0 = 23.0$		$T_w = 103.5$		
23.9	24.4	34.8	30.9	29.6	33.1		
23.9	24.9	23.9	28.2	33.6	36.3		
64.9	66.7	69.1	69.3	76.6	84.4	85.6	89.8
67.8	66.7	69.3	72.8	76.4	79.2	87.1	83.8
63.4	65.3	65.0	70.2	72.7	83.4	83.7	86.4
63.2	66.4	69.6	69.9	76.2	75.7	86.9	87.7

Run number		25	$T_0 = 23.3$		$T_w = 103.4$		
24.4	24.9	36.1	32.1	31.1	34.1		
24.4	25.3	24.4	29.6	35.6	39.3		
69.1	71.8	73.0	73.0	80.1	87.5	88.3	92.5
74.8	73.7	74.8	77.1	79.3	82.6	89.1	85.6
68.5	73.5	69.8	78.1	77.2	86.1	87.4	88.7
69.0	69.6	72.9	74.8	80.7	79.4	90.4	91.2

Run number		26	$T_0 = 22.5$		$T_w = 103.8$		
23.9	24.9	33.6	31.6	31.6	34.6		
23.9	24.9	23.7	29.6	36.4	39.4		
72.8	74.0	76.9	75.9	82.4	88.7	89.7	95.0
75.3	71.8	76.0	77.9	82.9	87.2	90.3	91.0
71.0	76.4	71.6	82.2	81.7	88.9	87.9	90.6
69.6	72.7	73.7	77.7	83.2	82.2	91.8	92.6

Run number		27	$T_0 = 23.1$		$T_w = 104.3$		
25.2	25.8	36.1	34.0	34.6	37.1		
24.9	26.4	24.5	32.0	39.1	42.0		
79.7	79.6	83.1	81.1	85.7	92.9	93.7	97.2
81.3	77.5	80.9	81.3	85.6	89.8	92.1	94.1
76.2	81.9	77.4	84.9	85.9	90.7	92.0	93.1
74.4	77.4	76.2	82.1	86.4	86.5	94.4	96.3

Run number		28	$T_0 = 23.5$		$T_w = 103.9$		
26.7	28.2	34.6	37.8	37.9	40.6		
26.7	29.1	25.5	34.6	42.3	46.1		
88.2	87.4	91.0	89.7	92.2	97.2	97.2	100.7
88.9	85.8	88.8	88.4	91.3	94.3	95.0	98.8
85.2	88.5	86.5	92.1	93.7	95.7	97.6	97.2
83.6	88.0	83.4	90.0	92.6	91.3	98.5	99.3

6.4 mm CERAMIC BEADS

Run number		1	$T_0 = 22.6$		$T_w = 101.0$		
25.1	25.4	39.3	37.9	32.8	40.5		
25.2	26.0	30.0	30.0	39.6	34.9		
25.7	26.0	26.0	25.0	31.7	35.3	45.9	46.0
25.1	24.5	24.9	25.8	27.1	38.6	37.7	50.8
24.6	24.2	24.9	26.0	30.6	30.9	38.9	39.7
24.3	25.8	25.4	25.9	30.7	40.6	41.6	50.3

Run number		2	$T_0 = 22.2$		$T_w = 101.1$		
25.3	25.9	41.1	38.5	35.3	42.4		
25.4	26.7	31.8	32.2	42.3	36.9		
26.6	27.4	27.7	26.2	34.5	39.1	51.5	51.2
26.0	25.2	26.0	27.6	28.7	41.3	40.9	54.4
25.4	24.0	26.3	27.8	33.2	33.8	42.6	43.0
25.1	27.1	26.7	27.4	32.9	43.2	46.2	52.2

Run number		3	$T_0 = 22.3$		$T_w = 101.4$		
25.6	26.3	41.9	38.5	37.2	43.7		
25.7	27.2	33.3	34.1	43.7	38.3		
27.5	28.3	28.7	27.2	36.3	41.9	52.7	54.9
27.0	25.9	26.7	29.2	29.9	43.6	44.4	57.3
26.2	24.0	27.3	29.3	35.2	35.6	46.1	44.2
26.1	28.2	27.7	28.7	34.6	45.0	50.3	53.7

Run number		4	$T_0 = 22.0$		$T_w = 101.5$		
26.0	26.9	42.9	39.6	39.1	45.0		
26.1	28.2	35.6	36.5	45.0	39.7		
29.0	29.6	30.4	28.9	38.5	46.4	53.6	59.3
28.3	27.0	28.0	30.8	31.2	46.5	48.3	60.0
27.6	-	28.9	30.9	38.6	37.8	49.2	47.3
26.9	29.9	29.2	30.5	37.1	46.9	55.5	56.1

Run number		5	$T_0 = 22.0$		$T_w = 101.1$		
26.7	27.6	43.4	40.8	40.2	45.9		
26.7	29.4	37.1	37.8	45.9	40.5		
30.8	31.2	31.8	30.6	41.1	49.8	56.4	62.5
30.4	28.4	29.5	32.3	33.1	50.0	51.0	62.6
29.1	-	30.9	32.4	41.1	40.8	52.5	51.5
27.2	31.8	30.4	32.6	39.6	49.4	57.6	60.0

Run number		6		$T_0 = 21.7$		$T_w = 101.1$	
28.3	30.0	45.3	43.0	43.5	47.6		
28.2	32.5	40.7	41.2	49.4	44.1		
35.1	37.0	36.4	35.3	47.9	58.1	65.4	69.3
34.4	32.5	34.7	37.1	38.3	54.7	52.9	68.4
33.3	-	35.9	37.7	49.7	47.9	59.1	56.5
29.4	36.8	35.1	38.1	46.6	55.7	62.1	70.0

Run number		7		$T_0 = 21.6$		$T_w = 101.2$	
32.1	34.7	47.9	46.6	46.6	50.9		
31.8	37.7	44.1	44.3	54.3	47.1		
41.3	43.7	43.8	42.9	55.7	66.7	72.9	76.6
40.7	38.8	42.9	44.6	46.2	61.9	62.0	75.9
39.1	-	43.3	46.0	56.7	57.2	68.6	65.7
30.3	44.4	41.9	46.0	53.4	64.9	68.8	76.4

Run number		8		$T_0 = 20.1$		$T_w = 102.1$	
22.6	22.9	37.8	33.6	29.4	34.8		
22.7	23.3	26.9	26.9	37.9	33.8		
25.9	26.2	27.9	30.2	37.7	54.2	53.8	73.5
27.0	28.7	28.1	36.9	43.0	59.9	71.2	73.5
27.5	-	30.0	34.5	47.5	51.4	62.3	59.4
26.5	27.2	26.4	31.2	44.5	52.8	59.5	61.0

Run number		9		$T_0 = 20.0$		$T_w = 101.5$	
23.1	23.6	39.7	36.2	32.1	37.7		
23.2	24.1	28.8	29.0	41.4	35.7		
28.3	28.7	30.8	33.2	42.3	58.0	58.5	76.4
29.7	31.6	31.4	40.6	46.0	64.4	74.0	76.8
30.4	-	32.7	38.2	51.5	55.5	65.0	63.1
29.2	29.8	29.2	33.8	47.8	57.5	62.7	64.5

Run number		10		$T_0 = 19.9$		$T_w = 101.4$	
23.4	24.1	41.1	37.7	34.4	39.4		
23.5	24.8	30.5	30.7	43.0	37.2		
30.4	31.0	33.1	35.9	45.2	64.3	62.8	78.7
32.0	34.4	34.7	44.0	48.1	66.8	76.3	78.7
33.4	-	34.8	41.1	54.2	59.3	68.3	66.9
31.6	31.9	31.7	36.0	50.2	61.3	64.8	69.3

Run number		11		$T_0 = 19.8$		$T_w = 101.5$	
23.9	24.5	42.3	39.5	37.2	41.3		
23.8	25.4	32.2	32.8	44.3	38.5		
33.5	33.9	36.2	38.9	49.2	68.9	67.5	81.2
35.5	37.7	37.3	46.4	51.6	69.7	78.8	80.5
36.8	-	38.4	44.0	57.0	61.6	72.6	70.6
34.6	35.0	34.7	39.7	54.1	65.8	69.1	76.5

Run number		12		$T_0 = 19.8$		$T_w = 101.4$	
24.9	25.7	43.9	41.4	40.1	43.3		
24.7	27.4	35.4	36.2	46.3	40.9		
38.9	40.1	41.9	45.2	54.9	74.6	73.6	81.9
42.0	43.6	43.4	53.9	56.9	75.5	81.7	82.6
42.7	-	45.1	49.6	62.3	67.4	77.4	75.3
40.1	39.4	39.1	45.0	59.7	72.1	73.7	81.2

Run number		13		$T_0 = 19.7$		$T_w = 101.3$	
25.8	27.0	44.9	42.6	42.0	44.8		
25.6	29.0	37.9	38.2	47.9	42.7		
44.1	45.6	46.1	51.1	59.3	76.2	77.8	82.6
47.8	47.8	48.4	57.7	62.2	78.4	83.8	85.4
47.6	-	49.9	55.0	65.5	70.0	78.9	78.0
44.7	44.5	44.6	49.7	64.1	74.2	77.2	82.7

Run number		14		$T_0 = 19.6$		$T_w = 101.3$	
29.2	31.2	47.1	45.6	45.3	48.5		
28.7	34.2	42.4	42.4	52.2	46.1		
57.9	60.6	59.9	65.6	72.2	82.7	84.8	88.8
60.9	62.0	62.6	69.1	73.0	85.2	89.4	90.4
60.8	-	63.5	69.6	75.9	80.4	85.7	86.0
57.5	58.3	58.5	62.8	74.4	81.3	83.3	87.0

Run number		15		$T_0 = 26.9$		$T_w = 102.7$	
28.8	29.6	51.3	48.6	40.3	44.4		
28.8	29.9	38.1	34.8	40.8	40.1		
40.3	41.6	43.6	45.6	62.2	71.1	85.2	82.2
40.9	41.3	45.4	46.7	59.8	68.7	75.6	78.3
41.5	43.3	44.3	52.3	59.0	75.6	73.3	80.8
41.7	41.4	41.4	48.2	59.8	73.8	79.2	81.2

Run number		16		$T_0 = 27.0$	$T_w = 103.0$		
29.6	31.0	53.7	50.4	42.4	46.1		
29.8	31.5	40.6	37.2	42.9	42.8		
45.7	46.8	48.5	50.4	67.3	74.0	87.8	84.8
45.5	46.2	49.6	51.8	63.7	72.8	78.5	80.8
46.2	48.1	49.2	57.7	64.9	78.1	76.8	83.8
46.3	46.0	45.5	53.4	63.9	77.5	83.3	84.5

Run number		17		$T_0 = 27.2$	$T_w = 103.0$		
30.5	31.9	54.8	51.8	43.7	47.4		
30.5	32.7	41.8	38.9	44.2	44.8		
49.0	50.0	52.0	53.6	68.4	77.4	89.7	86.7
48.4	49.4	52.6	55.5	65.8	76.1	80.4	83.2
49.0	51.3	52.2	61.3	68.3	79.9	78.8	85.8
49.3	49.2	48.5	56.8	67.4	79.8	86.2	86.6

Run number		18		$T_0 = 27.4$	$T_w = 102.3$		
31.2	33.1	55.9	53.2	44.7	49.0		
31.3	33.8	42.8	40.2	45.2	46.3		
53.3	54.0	57.0	57.9	71.4	80.7	90.7	88.0
52.9	53.7	56.3	59.4	68.8	78.5	82.1	85.7
53.7	55.4	56.3	65.3	71.3	82.7	79.0	87.8
53.6	53.6	52.6	59.9	69.6	81.6	87.8	87.9

Run number		19		$T_0 = 26.1$	$T_w = 102.7$		
31.5	34.1	57.2	55.5	46.0	52.7		
31.6	34.7	44.4	42.4	46.9	48.8		
61.0	62.0	65.3	67.7	77.8	84.7	90.2	91.2
60.4	61.2	63.9	66.2	72.2	83.1	86.4	89.0
61.5	63.4	64.1	73.3	77.5	85.8	84.6	91.2
62.1	61.8	62.0	65.6	77.6	83.8	90.5	92.0

Run number		20		$T_0 = 25.9$	$T_w = 102.9$		
34.1	37.1	60.1	58.1	48.3	56.2		
34.4	37.8	46.3	44.3	49.4	50.9		
70.5	71.7	73.9	76.0	83.1	88.7	94.4	94.6
70.0	70.5	72.6	74.3	79.0	88.1	90.7	92.9
70.9	72.3	73.0	79.7	83.7	89.1	88.3	93.7
71.2	71.3	70.9	74.4	82.9	88.5	93.9	94.7

Run number		21	$T_0 = 24.4$			$T_w = 103.0$		
37.5	40.3	62.3	60.9	51.8	59.7			
37.5	41.1	48.7	46.4	53.2	53.1			
80.8	82.3	83.2	84.6	89.0	94.0	97.6	97.6	
80.8	80.8	82.3	83.3	86.8	93.0	94.7	96.8	
81.0	82.0	82.8	86.4	89.8	93.0	93.6	96.5	
80.9	81.3	80.7	83.0	88.9	93.6	97.1	97.8	

Run number		22	$T_0 = 26.7$			$T_w = 102.9$		
29.2	30.2	50.9	49.1	40.4	45.0			
29.4	30.3	39.9	35.4	41.4	39.7			
53.6	56.8	58.2	60.9	67.3	76.4	77.9	83.9	
55.7	52.5	58.0	56.5	71.2	72.0	78.6	79.5	
47.9	54.0	55.1	64.3	71.5	77.8	79.0	86.0	
48.7	51.5	50.9	61.1	69.8	76.2	76.8	82.6	

Run number		23	$T_0 = 26.9$			$T_w = 103.0$		
29.5	30.9	53.6	50.8	42.4	46.6			
29.8	31.2	41.7	37.4	43.2	42.9			
59.6	61.3	64.4	65.4	71.7	80.8	83.1	86.2	
59.9	58.1	63.7	62.6	75.8	75.9	80.4	83.4	
53.6	60.7	61.2	68.1	76.0	80.9	82.8	88.7	
54.2	57.0	56.5	66.8	72.3	80.0	80.8	87.0	

Run number		24	$T_0 = 27.1$			$T_w = 102.6$		
30.2	32.1	54.9	52.0	43.5	47.9			
30.5	32.2	42.7	39.0	44.3	44.9			
62.9	64.6	67.3	68.5	74.0	83.5	85.9	88.0	
62.7	61.4	66.8	65.2	78.0	77.9	81.8	85.2	
57.1	64.3	65.8	70.7	78.4	82.7	85.6	90.6	
57.8	60.4	59.9	70.0	75.5	81.9	83.4	88.8	

Run number		25	$T_0 = 27.1$			$T_w = 102.3$		
31.0	33.3	56.2	53.7	44.6	50.0			
31.3	33.6	43.6	40.5	45.4	46.8			
70.8	69.5	71.7	72.9	77.7	86.5	88.7	90.7	
67.3	66.3	71.4	69.7	80.5	80.8	84.2	86.9	
61.0	68.7	68.0	76.2	80.6	85.8	89.6	92.5	
62.3	65.1	63.9	75.0	73.3	84.6	86.8	89.1	

Run number		26	$T_0 = 25.5$		$T_w = 102.9$		
32.4	35.0	58.0	55.7	46.0	52.8		
32.7	35.3	45.0	42.4	46.9	48.9		
74.1	75.0	77.4	78.3	82.1	88.5	91.0	92.6
74.0	72.9	76.0	76.4	85.5	87.0	88.1	90.8
67.6	74.0	73.1	80.4	85.0	88.5	89.9	94.7
69.8	73.4	70.6	79.8	81.4	88.1	89.8	91.5

Run number		27	$T_0 = 25.8$		$T_w = 102.9$		
33.9	36.8	59.4	57.6	47.6	55.3		
34.2	37.4	46.3	43.8	48.6	50.5		
80.6	81.5	83.0	84.2	87.0	91.6	94.0	95.0
80.4	79.4	81.9	82.3	89.6	90.2	92.2	94.7
76.1	80.7	80.4	85.3	89.4	92.1	93.0	96.9
77.3	80.0	78.4	84.4	89.0	91.5	93.3	94.6

Run number		28	$T_0 = 23.9$		$T_w = 102.1$		
35.4	38.9	55.2	59.5	51.0	59.1		
35.4	39.6	46.9	45.2	51.5	53.0		
90.6	91.0	91.0	92.8	94.0	96.9	98.7	98.9
90.6	91.1	91.4	91.2	94.9	96.3	97.9	98.8
88.8	90.5	91.6	93.4	95.6	97.5	98.6	100.1
89.2	90.7	91.3	92.2	95.9	97.3	97.9	99.8

Run number		29	$T_0 = 24.6$		$T_w = 102.0$		
27.5	28.6	47.0	48.3	39.5	44.6		
27.8	28.7	37.4	34.1	40.4	39.3		
59.9	63.0	62.7	65.4	74.2	79.4	88.4	85.3
61.2	60.4	65.9	60.3	74.5	76.0	81.4	83.5
61.2	62.3	62.1	62.4	72.5	78.3	79.2	85.5
59.5	60.2	61.1	67.3	72.0	82.0	81.0	88.6

Run number		30	$T_0 = 24.5$		$T_w = 101.8$		
27.9	29.2	47.6	49.6	41.4	46.0		
28.2	29.6	39.4	35.9	42.1	41.9		
65.9	68.2	67.4	70.8	78.2	82.4	90.2	87.1
66.0	66.5	69.9	65.5	78.2	80.2	83.0	87.2
67.4	66.9	67.3	67.6	75.5	80.9	80.8	87.4
64.5	64.9	65.9	71.6	75.2	84.4	82.9	90.4

Run number		31		$T_0 = 24.4$		$T_w = 101.7$	
28.1	29.9	47.8	50.7	42.4	47.2		
28.4	30.0	40.3	36.9	43.1	43.8		
69.5	71.0	71.1	74.2	80.4	84.9	91.0	89.0
69.5	70.9	73.8	68.8	81.4	82.4	84.9	89.7
71.3	70.2	71.1	71.3	77.7	82.8	82.4	88.9
67.8	68.5	69.4	74.7	77.8	86.0	84.6	91.8

Run number		32		$T_0 = 24.2$		$T_w = 102.3$	
28.6	30.5	48.0	51.9	43.7	48.8		
28.8	31.0	41.4	38.5	44.1	45.6		
74.7	74.2	78.7	77.7	82.7	86.7	93.2	91.6
74.0	74.0	77.4	73.5	85.1	87.7	87.1	91.0
74.8	74.3	75.3	74.2	80.9	84.5	84.7	90.4
72.0	74.3	74.0	78.7	80.9	87.5	87.5	93.0

Run number		33		$T_0 = 23.3$		$T_w = 102.2$	
27.9	30.5	47.3	52.7	44.8	50.9		
27.9	30.9	42.0	39.7	45.5	47.8		
84.6	82.4	87.3	84.4	86.5	89.2	95.5	93.9
81.7	82.9	82.4	82.6	91.2	91.4	92.7	96.1
84.3	83.9	84.8	84.7	88.5	91.6	91.5	95.4
81.7	81.9	81.4	85.3	86.0	91.9	92.1	95.8

Run number		34		$T_0 = 23.5$		$T_w = 102.0$	
29.8	32.7	48.8	54.7	46.3	53.5		
29.9	33.3	43.5	41.4	46.7	49.3		
89.9	88.1	91.9	89.9	91.0	93.9	97.8	96.9
88.0	89.1	88.4	88.0	94.4	95.1	95.7	98.1
90.3	90.5	90.3	90.0	92.3	94.7	94.7	97.5
88.4	87.7	88.3	90.5	91.0	96.1	95.0	97.9

Run number		35		$T_0 = 23.3$		$T_w = 102.3$	
35.0	38.5	52.7	59.2	50.9	59.2		
35.0	39.1	46.7	45.0	51.5	52.7		
96.0	95.7	97.2	96.6	97.3	100.0	101.1	100.8
95.8	96.5	96.0	95.7	98.2	99.6	100.4	101.2
96.7	96.9	96.6	96.8	97.9	99.8	98.9	101.1
96.4	95.4	96.4	96.9	97.5	100.0	99.5	101.3

Run number			$T_0 = 23.4$		$T_w = 103.0$		
27.8	29.6	44.6	-	44.9	48.3		
27.4	31.2	41.4	38.6	45.6	45.4		
30.8	31.9	34.4	33.5	43.1	53.0	61.0	73.0
30.9	33.8	33.3	36.2	38.0	54.2	-	66.5
25.6	32.5	32.0	36.4	46.2	54.7	60.5	62.6
32.1	32.5	33.2	38.8	44.4	59.6	64.4	70.7

Run number			$T_0 = 23.7$		$T_w = 103.2$		
29.0	30.7	43.5	-	45.4	48.9		
28.6	32.4	42.3	39.5	46.1	43.9		
44.1	45.6	51.0	50.7	62.2	72.0	77.5	73.8
40.2	45.4	49.0	49.3	62.8	63.9	78.0	87.0
41.2	48.5	41.2	53.7	56.2	66.7	71.7	84.5
50.4	41.7	47.3	49.0	62.1	72.3	70.0	78.2

Run number			$T_0 = 23.7$		$T_w = 102.3$		
29.5	31.3	43.9	-	45.5	48.4		
29.0	33.0	42.5	39.6	46.3	44.1		
56.1	57.6	57.0	61.6	72.9	81.2	84.3	85.9
54.3	57.3	58.6	61.7	75.2	79.2	85.5	84.4
51.4	58.9	60.1	67.0	74.9	82.0	81.2	86.9
59.9	59.9	61.6	63.9	73.8	87.9	84.6	93.0

Run number			$T_0 = 23.9$		$T_w = 102.3$		
29.8	31.6	42.4	-	45.6	48.5		
29.2	33.3	43.1	39.7	46.3	44.0		
73.2	71.9	75.6	78.2	81.7	89.7	95.7	93.4
71.1	74.8	72.1	76.0	80.1	87.3	88.5	92.4
71.5	72.8	75.7	75.9	81.7	87.6	85.4	92.8
71.3	75.6	72.2	74.4	83.7	84.8	92.0	92.4

Run number			$T_0 = 23.9$		$T_w = 101.6$		
29.9	31.5	41.7	-	45.6	48.4		
29.2	33.3	43.5	39.6	46.3	44.3		
79.0	81.8	81.6	83.6	87.7	93.4	97.2	97.3
80.7	81.5	83.0	83.6	86.9	90.8	91.2	96.0
78.0	79.7	78.1	82.3	88.3	90.7	92.6	93.6
80.5	80.3	81.2	83.1	85.5	92.4	91.5	95.7

9.5 mm STEEL BALLS

Run number		1	$T_0 = 23.0$		$T_w = 105.1$		
26.1	26.5	42.0	32.7	32.4	37.2		
26.1	26.9	32.0	29.1	38.5	36.3		
33.0	39.0	35.2	36.9	47.1	61.9	62.5	71.7
34.2	32.6	36.3	38.0	46.0	52.0	55.2	53.5
33.0	-	33.7	36.4	44.9	45.1	57.8	65.6
32.6	31.5	34.8	37.5	45.1	49.9	57.9	62.5

Run number		2	$T_0 = 23.5$		$T_w = 102.1$		
26.9	27.3	42.9	33.8	34.5	39.2		
26.9	27.8	33.9	30.6	40.6	37.6		
35.9	42.4	38.2	40.0	50.2	63.8	65.2	73.4
36.4	35.1	39.0	41.0	49.1	54.4	57.7	55.6
35.5	-	36.3	39.2	48.6	47.7	60.8	68.9
34.5	33.5	37.0	39.8	46.9	53.2	59.9	65.7

Run number		3	$T_0 = 23.4$		$T_w = 102.1$		
29.2	31.4	44.4	37.2	39.6	43.1		
28.8	30.3	38.0	34.2	44.2	41.4		
44.4	51.8	47.2	47.5	61.5	70.3	75.0	79.9
44.5	43.1	49.0	49.7	57.8	61.8	66.4	64.9
44.4	-	45.7	49.3	58.9	54.9	69.6	77.8
42.4	40.1	45.8	49.8	52.7	63.8	65.6	76.6

Run number		4	$T_0 = 23.9$		$T_w = 101.7$		
32.4	32.6	46.4	41.2	42.6	45.4		
31.5	33.3	41.6	37.4	46.8	44.3		
52.0	57.7	55.7	57.3	69.3	77.3	83.0	85.9
51.3	51.2	52.0	56.9	64.8	69.5	73.6	74.3
53.3	-	54.7	56.7	68.3	61.8	76.7	81.3
44.4	47.5	57.0	56.5	60.7	72.9	71.5	84.1

Run number		5	$T_0 = 23.8$		$T_w = 101.7$		
41.9	41.9	51.4	47.2	49.0	51.9		
40.6	41.6	46.9	42.8	54.5	50.5		
75.1	72.6	76.1	70.6	79.2	88.6	88.4	94.9
67.6	71.9	68.0	73.8	77.2	83.6	83.8	86.9
70.3	-	70.4	72.8	80.0	77.5	84.8	88.1
66.0	65.0	71.8	71.2	75.1	83.7	81.5	89.9

Run number		6		$T_0 = 24.5$	$T_w = 101.3$		
28.3	28.9	45.0	46.3	35.0	38.9		
28.3	29.4	34.0	31.1	41.6	37.2		
46.7	45.7	47.9	51.4	63.6	66.9	77.2	79.5
45.5	46.6	48.2	50.5	53.4	70.2	73.4	81.6
41.6	47.5	47.3	51.8	56.6	73.1	64.5	82.1
49.0	47.6	51.8	59.3	66.6	77.5	72.6	81.7

Run number		7		$T_0 = 24.3$	$T_w = 101.5$		
28.9	29.4	45.3	47.0	36.8	40.2		
28.8	29.8	35.5	32.3	42.9	38.8		
52.2	51.4	53.3	56.5	67.2	71.2	80.2	82.3
51.3	51.4	53.5	55.4	57.8	74.3	76.9	84.0
46.4	52.8	52.3	57.1	61.6	76.0	69.1	84.1
53.0	53.1	55.7	64.7	71.0	81.1	75.9	84.5

Run number		8		$T_0 = 22.6$	$T_w = 103.0$		
28.1	29.0	44.1	48.4	39.0	41.6		
27.6	29.7	36.7	33.0	44.9	42.5		
65.0	62.3	63.4	66.1	72.2	79.2	83.6	90.0
62.8	64.9	64.3	66.3	67.6	83.5	85.3	88.9
57.4	63.2	60.0	68.7	71.0	84.3	78.6	90.1
62.5	65.2	64.7	75.7	79.0	88.6	83.8	91.1

Run number		9		$T_0 = 22.8$	$T_w = 103.0$		
33.9	35.7	47.7	53.1	44.8	45.7		
33.2	36.4	42.9	38.9	49.6	46.9		
79.2	77.8	78.5	81.7	83.6	87.7	92.1	94.8
78.2	79.0	78.5	78.7	82.4	92.0	93.4	94.9
73.1	78.3	75.9	80.6	82.5	91.1	86.6	95.6
76.7	79.8	78.6	83.9	88.7	95.1	91.9	96.9

Run number		10		$T_0 = 22.8$	$T_w = 102.8$		
41.3	42.6	52.1	58.9	51.1	50.6		
40.6	42.3	46.1	43.5	55.7	51.3		
88.2	88.2	89.1	90.3	92.0	94.5	97.6	98.8
88.1	88.8	88.2	88.6	91.6	97.5	97.9	99.0
85.4	88.7	87.5	89.9	91.4	96.4	94.2	99.0
87.4	88.0	88.1	92.1	95.5	98.8	97.3	100.1

Run number		11		$T_0 = 24.4$		$T_w = 103.6$	
27.8	28.2	41.5	46.8	34.9	38.0		
27.7	28.9	33.8	30.9	42.0	37.6		
59.1	58.7	63.0	67.2	77.3	78.2	82.8	86.2
60.1	56.2	64.2	64.3	71.9	77.3	84.5	85.0
56.5	57.3	58.8	64.9	74.6	78.4	81.9	82.9
54.8	60.9	58.7	68.0	72.1	84.7	82.0	88.4

Run number		12		$T_0 = 23.8$		$T_w = 103.0$	
27.9	28.8	44.5	47.3	36.5	39.0		
27.8	29.4	34.8	32.1	43.1	39.1		
66.3	65.4	69.0	70.5	78.4	81.5	86.4	88.7
65.9	60.7	68.4	67.2	76.0	80.7	86.0	87.0
64.3	64.2	63.5	69.2	78.7	81.4	84.2	85.8
61.9	64.7	65.6	74.9	75.2	86.9	85.3	90.4

Run number		13		$T_0 = 23.8$		$T_w = 103.0$	
30.2	31.1	46.2	49.6	40.4	42.6		
29.5	32.2	38.5	35.3	45.8	43.2		
78.6	77.6	79.4	80.7	86.2	89.8	92.3	93.8
78.0	73.8	78.5	78.0	83.9	88.1	92.1	91.9
76.5	76.2	75.5	80.0	85.5	88.7	90.4	92.3
75.7	76.9	78.6	83.3	83.6	92.5	91.3	95.6

Run number		14		$T_0 = 24.0$		$T_w = 103.0$	
33.8	35.1	48.0	52.6	43.5	44.9		
32.6	35.9	41.9	38.4	48.5	47.0		
86.7	85.8	87.4	87.9	93.2	95.1	96.9	97.2
86.5	82.7	86.5	86.4	90.0	92.9	96.9	95.9
84.7	84.7	84.0	88.1	90.4	93.8	94.0	96.7
86.2	85.9	87.4	89.6	89.6	96.5	95.5	98.7

Run number		15		$T_0 = 24.1$		$T_w = 103.0$	
42.4	43.9	54.7	60.2	50.9	51.8		
41.8	43.3	46.8	44.3	57.1	52.4		
97.8	96.8	98.0	98.2	99.6	100.8	101.7	102.0
96.9	95.7	97.3	96.8	98.8	99.6	101.3	101.1
96.5	96.8	96.6	97.9	99.4	100.7	100.5	101.7
96.4	96.7	97.0	97.9	98.1	101.1	101.2	102.3

Run number		16		$T_0 = 25.8$		$T_w = 103.0$	
29.5	30.2	45.0	46.2	36.8	39.6		
29.6	30.9	35.7	33.2	42.8	38.5		
65.7	67.3	69.2	70.2	75.6	82.2	84.6	87.4
66.2	64.5	68.5	71.8	79.7	85.4	84.4	87.7
68.1	69.7	71.4	77.5	79.3	86.9	84.8	90.9
67.9	71.3	70.1	72.8	80.8	85.5	88.4	93.6

Run number		17		$T_0 = 26.0$		$T_w = 103.0$	
30.6	31.7	46.2	47.9	39.3	41.8		
30.6	32.5	37.9	34.9	44.4	40.6		
71.7	73.8	75.2	76.0	80.4	86.4	87.8	90.8
72.1	71.0	74.4	77.6	83.7	89.1	87.6	91.0
74.3	75.4	77.0	81.7	83.4	90.6	88.4	93.7
74.0	77.0	75.5	78.2	84.7	89.0	91.3	96.1

Run number		18		$T_0 = 25.6$		$T_w = 103.0$	
33.2	34.8	48.0	49.9	42.6	44.3		
33.1	35.6	41.2	38.2	47.3	44.7		
82.4	84.5	85.3	86.5	88.7	93.0	93.7	95.0
83.0	82.1	84.7	86.0	91.6	94.9	94.8	96.0
83.2	85.6	86.5	89.6	90.8	95.7	94.8	97.9
84.1	86.5	85.5	87.1	91.2	93.8	96.0	99.3

Run number		19		$T_0 = 25.1$		$T_w = 103.0$	
38.0	40.2	51.3	54.6	46.4	47.4		
37.5	40.0	44.7	42.0	51.7	48.8		
93.8	95.1	95.7	96.0	97.3	99.4	99.0	100.1
94.0	93.9	95.2	95.6	98.8	100.2	100.7	101.1
94.9	96.6	96.1	97.5	97.9	99.9	100.4	101.8
95.6	96.4	96.2	96.7	98.1	100.0	101.1	102.6

Run number		20		$T_0 = 24.7$		$T_w = 102.3$	
27.6	28.2	43.5	46.3	34.7	37.1		
27.7	28.9	33.6	31.0	41.4	37.6		
74.6	75.7	73.3	78.0	78.7	87.3	89.6	87.7
74.8	77.3	75.9	79.9	82.3	87.2	88.0	92.8
73.1	79.0	75.3	80.2	81.7	87.8	87.3	92.1
74.4	75.8	73.4	77.7	81.3	87.3	88.6	92.0

Run number		21	$T_0 = 24.7$		$T_w = 102.3$		
29.1	29.8	44.7	47.7	37.8	39.8		
29.2	30.7	36.4	33.4	43.3	39.5		
80.6	81.2	79.2	82.6	84.0	90.7	92.8	91.1
80.6	82.8	81.2	84.7	86.5	90.8	91.3	95.4
78.9	84.0	80.8	84.6	86.4	91.3	91.0	94.6
80.2	81.3	79.3	82.7	86.0	90.6	92.0	94.2

Run number		22	$T_0 = 24.9$		$T_w = 101.8$		
31.0	32.4	45.7	49.9	41.4	42.9		
30.9	33.5	39.7	36.4	45.7	43.7		
89.6	89.9	88.5	90.4	92.8	96.1	97.9	97.2
89.6	91.1	89.9	92.6	93.3	96.1	96.5	98.7
87.8	91.3	89.3	91.5	93.7	96.7	96.6	98.2
89.6	90.5	88.9	90.9	92.8	95.5	97.2	97.9

Run number		23	$T_0 = 24.5$		$T_w = 102.1$		
32.1	33.8	39.8	-	43.5	46.2		
32.4	35.7	41.7	44.2	44.3	42.2		
43.6	49.5	43.5	51.1	60.0	78.3	71.8	87.3
56.6	48.6	58.6	51.6	59.3	65.6	72.5	76.0
49.4	47.5	47.9	56.9	59.3	69.4	66.2	74.8
46.9	49.4	49.1	48.6	62.9	75.2	70.2	-

Run number		24	$T_0 = 22.9$		$T_w = 102.2$		
29.4	31.0	40.1	-	42.3	44.9		
30.7	33.3	41.9	41.0	44.2	39.1		
72.5	71.7	74.4	78.5	83.6	88.3	88.9	93.2
73.6	71.0	74.6	74.2	82.1	80.9	87.1	89.9
69.3	70.4	70.1	80.2	83.0	88.3	88.7	91.4
71.4	72.0	71.2	70.1	81.2	88.2	88.3	93.8

Run number		25	$T_0 = 22.8$		$T_w = 102.2$		
29.0	30.9	40.1	-	42.2	45.0		
30.7	33.2	41.8	41.0	44.3	39.1		
82.4	85.6	84.3	86.7	90.1	93.0	92.6	97.4
82.8	85.6	85.7	87.1	88.0	92.9	95.1	97.0
84.5	83.5	87.4	86.4	92.8	94.5	96.6	96.1
84.6	86.1	86.9	87.3	89.6	93.8	93.8	97.5

Run number		26	$T_0 = 23.2$		$T_w = 102.4$		
30.0	31.9	42.3	-	42.9	45.5		
31.3	33.2	42.1	41.6	44.8	39.7		
91.0	91.8	91.6	93.8	94.0	96.7	96.9	98.7
91.0	89.2	90.6	90.7	92.0	93.4	97.5	96.2
88.2	89.6	89.6	90.8	93.9	96.2	95.8	97.4
88.3	90.2	89.3	91.4	93.8	96.0	94.2	98.1