Title: An approximate method to calculate the minimum efficient size of a Wave Energy Converter, including submerged devices

Report by J.D. Ainsworth

SUMMARY

The method is based on very simplified assumptions about the Wave Energy Converter (WEC) device, including that for reasonable efficiency the local water velocity should not be required to increase much above that existing in the absence of the WEC.

Bottom-mounted devices, or intermediate-submersion devices fixed relative to the bottom, have considerable attractions over surface-mounted devices, including the absence of navigational obstruction. However, this note ends with the gloomy conclusion that a reasonable compromise between efficiency and cost will be at least difficult for these, and only the surface devices (with all their disadvantages) seem practicable, unless a major break-through is obtained in reduction of cost per unit area of submerged devices.

UNDISTURBED CONDITIONS

For water of mean depth D, undisturbed by the presence of any device, the relation between wavelength and period $T$ is

$$T = \sqrt{\frac{2\pi \lambda}{g \tan h (2\pi h/\lambda)}}$$

(1)

At a fixed point of height $h$ above the bottom as fig. 1, the undisturbed r.m.s. pressure variation at wave frequency is:

$$P_{rms} = \frac{\sigma g H_{rms} \cosh (2\pi h/\lambda)}{\cosh (2\pi h/\lambda)}$$

(2)

where $\sigma$ = water density

$H_{rms}$ = r.m.s. wave height

$g = 9.81 \text{ m/s}$

The local water velocity at the fixed point may be considered to have r.m.s. vertical ($V_y$) and horizontal ($V_x$) velocity components at wave frequency given by:

$$V_x = \frac{2\pi H_{rms} \cosh (2\pi h/\lambda)}{T \sinh (2\pi h/\lambda)}$$

(3)
Component \( V_y \) is in phase with pressure variation, and \( V_y \) is at 90° in time-phase \( \phi \). The three quantities \( P_{\text{rms}} \), \( V_x \), and \( V_y \) are all maximum at the surface, and decrease fairly rapidly for increasing depth of the point below the surface, particularly for short wavelengths.

**Single WEC device as a sink**

Now consider a small WEC device, perhaps one of several, effectively compressible, at height \( h \) above the bottom. This may be imagined as an open-ended pipe of small entrance area, coupled to an infinite reservoir at mean local water pressure. Air may be used as an intermediate medium in the pipe, and may drive an air turbine coupled to an electrical generator to generate power; other arrangements are possible, including purely mechanical devices. If the device is at the surface then the infinite reservoir may be the atmosphere; the device is then an elementary version of the NEL Oscillating Water Column. For a submerged device a more convenient alternative to the infinite reservoir is to couple devices at \( \lambda/2 \), apart, though other spacings are possible. Initially consider the case where the turbine resistance is removed (and other losses neglected), so that the device becomes virtually an ideal sink. In this case the water velocity in the WEC entrance will be assumed, as an approximation, to be the same as the local undisturbed absolute velocity. (Because the entrance area is small this assumption should be approximately independent of the direction in which the entrance faces, or of the instantaneous direction of undisturbed flow, because of the case with which water can change direction if it has no excessive local velocity). Thus WEC entrance velocity can in this case be considered to have a complex value of

\[
V = V_x + j V_y
\]

(5)

where the symbol \( j \) indicates phase-time quadrature, taking undisturbed pressure as phase reference.

There is now an analogy to the electrical circuit of fig. 2 in which a source comprising alternating e.m.f. \( E \) behind an impedance \( Z \) supplies a load resistor \( R \).

The analogy is that e.m.f. \( E \) (the same as open-circuit voltage) corresponds to pressure:

\[
E = P_{\text{rms}}
\]

(6)

Short-circuit current corresponds to volume flow at the WEC entrance, with turbine removed:

\[
I_{sc} = EY = A(V_x + j V_y)
\]

(7)

where \( Y \) = complex output admittance = \( 1/Z \)

\[
A = \text{WEC entrance area}
\]
Let \( Y \) be expressed in the form \((G + jB)\).

Then by combining equations,

\[
G = \frac{2\pi A}{\sigma g T} \cdot \frac{1}{\tanh (2\pi h/\lambda)} \tag{8}
\]

\[
B = \frac{2\pi A}{\sigma g T} \cdot \frac{\tanh (2\pi h/\lambda)}{\tanh (2\pi D/\lambda)} \tag{9}
\]

The reactive component \( B \) is equal to \( G \) for a surface device in infinite depth, decreasing to zero for a bottom-mounted device \((h = 0)\).

Now assume that the load presented by the turbine is substantially resistive (i.e. it has negligible inertia effects due to long water columns) and that the effective resistance \( R_L \) (turbine constant) is adjustable.

The maximum power which can be extracted is when:

\[
R_L = \frac{1}{\sqrt{G^2 + B^2}} \quad \text{and is:}
\]

\[
\text{Max. power} = \frac{E^2 (G^2 + B^2)}{2\left[G + \sqrt{G^2 + B^2}\right]}
\]

\[
= 2 \frac{\sigma g T}{\text{RMS}} A \cosh (4\pi h/\lambda) \cdot \frac{T \sinh (4\pi D/\lambda)}{1 + \sqrt{1 + \tanh^2 (2\pi h/\lambda)}} \tag{10}
\]

(This result could of course be obtained without recourse to an electrical analogue if preferred). This solution assumes a linear system, with turbine pressure drop proportional to flow rate; real systems are not linear, but may be an approximation to this.

In the maximum power (matched) condition the ratio of actual r.m.s. pressure and flow to undisturbed values will be somewhat less than unity. The lowest ratio is 0.5 for a bottom-mounted device \((h = 0\), hence \(B = 0\)), and a little higher for other cases.

The available wave power is known to be:

\[
H_{\text{RMS}} \cdot \frac{2 \sigma g T}{4} \sqrt{\frac{H}{\pi}} \cdot \frac{4 \pi D}{\lambda} \cdot \text{cosh} (2\pi h/\lambda) \cdot \text{sinh} (4\pi D/\lambda) \tag{11}
\]

per unit width of wave front.

Now assume that the WEC is a line device extending indefinitely parallel to the wave front.
Let $L = \text{effective device length normal to the wave front}$. Then by replacing area $A$ by $L$ in (10), the maximum power is per unit width of wave front. Equate this to (11).

Then:

$$L_{\text{min}} = \frac{\lambda}{4\pi} \left[ \frac{4\pi D}{\lambda} + \sin h \left( \frac{4\pi D}{\lambda} \right) \right] \left[ 1 + \sqrt{1 + \tanh^2 \left( \frac{2\pi h}{\lambda} \right)} \right] \cosh \left( \frac{4\pi h}{\lambda} \right)$$ (12)

This is the minimum device length capable of giving (ideally) 100% efficiency with the given assumptions (mostly of no increase in local water velocity). For a surface device put $h = D$. For a bottom device $h = 0$. For an intermediate submerged device $h$ lies between zero and $D$. For a submerged device the calculated value of $L_{\text{min}}$ can be substantial, sometimes more than a wavelength, hence a single device would not then be effective. However it is in principle possible to subdivide total device length into a number of short devices spread over a total of $L_{\text{min}}$ normal to the wave front, each driving a separate turbine (or other device); the turbine time-phases will of course all be different. An example is shown in fig. 3.

**CALCULATION FOR PRACTICAL DEVICES**

**Surface device**

Fig. 4 shows calculated values of $L_{\text{min}}$, plotted against period $T$, for depths from 20m to 50m. As would be expected, the required length increases steadily with wavelength. The values of $L_{\text{min}}$ are always much smaller than a wavelength, hence a single device will be effective, provided it has a satisfactory mechanical reference (e.g. the spine of a Salter Duck). For shallow depth, a slightly shorter device can evidently be used, particularly for large wave periods, due to the greater power density.

Real devices such as the Salter Duck appear to have reasonable efficiency at a given wave period even if their length is less than this calculated value $L_{\text{min}}$; probably this means that for a highly streamlined device its effective length is longer than its real length.

**Bottom-mounted devices**

Fig. 5 shows values of $L_{\text{min}}$ for this, in depths from 10m to 30m. These characteristics are quite different to those for surface devices, because the power density on the bottom decreases rapidly at small wave periods, so that the required device length increases rapidly as period decreases, depending on depth. For large values of $L_{\text{min}}$, it is obvious that a single basic device will be ineffective. However, it is then possible to subdivide the device into several much shorter elements.

Fig. 3 shows as an example a possible version of the Russell bottom-mounted membrane device, in which four devices each of length say $\lambda/8$ are spaced by $\lambda/8$. These are coupled in pairs, by turbines, each pair being at $\lambda/2$ pitch to provide a mutual reference. This arrangement can in theory give high efficiency provided the effective total membrane length satisfies eqn. 12. It is possible that "effective length" can be taken to include the gaps between membranes, provided they are not too large; e.g. with gap length...
equal to membrane length, the average local water flow velocity will be about equal to the undisturbed value \( \bar{V} \). Another requirement is that the available vertical diaphragm movement shall be sufficient to pump the appropriate amount of air at "rated" power.

With bottom-mounted devices, tide height variation (about 8.4m from MLWS to MLWS near the U.K.) is a major problem. Thus if efficient operation is required over the principal wave spectrum range for the U.K. (period about 7s to 15s) then with a shipping clearance of 8m, the tide depth at mean tide will be about 12m, and the effective device length required is about 180m, which will be rather expensive. This length is greater than one wavelength over most of the relevant spectrum, hence an arrangement such as Fig. 3 (which is also broadly tuned to a particular wavelength) cannot be used directly; probably a greater number of basic devices will be required, say 9 devices each 10m long, with 10m spacing, coupled by means which effectively gives broader tuning.

If shipping clearance can be permitted to be smaller, then a shorter effective device length can be used, though in such shallow water there may be some bottom loss in the few km in front of the WEC's, depending on the nature of the bottom (large boulders, hard sand, etc.).

Intermediate submergence devices:

Fig. 6 shows calculated results for a device fixed at 20m above the bottom, and for water of depths 5m to 20m over the device (i.e. total depth 25m to 40m). As for the bottom-device, tide height variation must be allowed for; however losses due to bottom roughness in front of the device are likely to be small for this case, so choice of depth will be more concerned with shipping clearance (assuming this is a requirement).

Efficient operation should evidently be possible for a device of effective length 180m in a mean tidal depth of about 12m over the device (32m to bottom) giving shipping clearance of 8m at MLWS in zero waves. This is practically the same length as for the bottom-device, and may be similarly uneconomic.

If the effective device height could be changed slowly with tide height to maintain constant clearance, say 10m, then an effective length of 100m could be used; this appears to be impracticable except for a moored floating device with at least some surface protrusion.

CONCLUSIONS

Surface devices require expensive moorings and flexible electrical cables, have difficult access, and form a major navigational obstruction, but appear to be the only type of WEC likely to be efficient at reasonable cost.

Bottom-mounted WEC's will require such a shallow depth to obtain efficiency in a reasonable size that tide height variation will be a major problem.

The same applies to intermediate-depth WEC's mounted at a fixed height above the bottom, if shipping clearance must be guaranteed. If such a device could have its height adjusted slowly with tide, then it could be more attractive, but the problem of engineering this at a reasonable cost is formidable.
APPENDIX:

Comment on "Proposal for a sea-bed Wave Energy Converter"

by J.D. Ainsworth

This proposal comprised a set of inverted air-boxes fixed submerged at some distance above the bottom, driving air turbines, but was based on an error in calculation. In fact, from the results above, the arrangement as proposed would be so far below the surface that its efficiency would be much too low, particularly for short wave periods. It could be improved by using longer legs, in shallower water than given in the proposal, but the coupling air pipes and turbines would have to be mounted under the air boxes instead of over, so as to give navigational clearance; they would then flood rather easily in excessive waves.

However, from the figures calculated in this note, it is unlikely that such an arrangement could be efficient, except with an uneconomically large size (plan area), while maintaining navigational clearance at low water.
Fig. 1

Fig. 2. Electrical analogue

Fig. 3. Russell bottom-mounted membrane WEC's
Minimum effective device length for surface devices

Fig. 4

Date: 6th February, 197
Minimum effective device length for intermediate-submersion devices mounted at 20m above the bottom.
1. This is an interim note for circulation in TAGG only, and will be included in part in a general study report later.

2. Comments are invited please.

3. The electrical system is exactly suited also for generation by wind-power. (The idea of using synchronised a.c. generators for windmills is in my opinion just as fatuous as it is for wave-power.)
OUTLINE OF PROPOSED ELECTRICAL CONTROL CHARACTERISTICS FOR WAVE ENERGY CONVERTERS

By J. D. Ainsworth

Summary

This note summarises the principal characteristics of the control of the electrical transmission system proposed for Wave Energy Converters (WEC's) in WEESC78GT42, using a.c. generators, diode rectifiers in series, h.v.d.c. cable, and shore inverter, with some modifications.
1. INTRODUCTION

This note assumes an electrical system generally as WESC78GT42, but with some changes as follows:

(a) It is now thought to be more economical to design rectifiers so that the sum of their d.c. voltage ratings per pole is say twice the inverter voltage rating, on the assumption that conditions giving a total power capability per pole at any instant of more than 50% of its total rectifier power rating will be rare, and the resulting energy loss will be acceptable.

(b) The concept of a "power capability" signal per WEC, connected in a feedback loop via the mechanical side is introduced, for optimisation of each WEC output.

(c) As a consequence of (a) operation will normally be at about rated current for all loadings, and manual power reduction will normally be by inverter voltage reduction, not by reducing its current order.

(d) Parallel operation of rectifier poles requires further study to determine statistical collection loss in various wave conditions.

2. CONTROL SYSTEM FOR ONE WAVE ENERGY CONVERTER (WEC)

Figure 1 shows a block diagram of part of the local control for one WEC. This comprises:

(a) Current limit, by subtracting a current limit setting signal from a current signal.

(b) Voltage limit ditto.

(c) Field current limit ditto.

(d) Power limit by subtracting a power capability signal from a power signal (via 3-phase a.c. multipliers).

(e) The most positive of the error signals from (a) - (d) controls generator field voltage.

Figure 2 shows the d.c. voltage/current characteristic ABCDEFG of one WEC. The voltage (8H) and current (9H) limits are normally fixed at values equal to equipment rating (taken here as 5 MW, 10 kV, 500 A peak per WEC referred to diode rectifier output). In addition the field current limit will cause the effective main voltage limit to fall at low generator speed.

The principal electrical control locally available is the power capability signal. If the power capability of the WEC mechanical system (including e.g. turbine) is known at a given instant then this signal should normally be set equal to it. Operation can then
occur anywhere on the rectangular hyperbola CDE as convenient to the electrical system, without affecting turbine torque.

Obviously the power capability signal must in practice be arranged inside a feedback loop. This depends on the mechanical arrangement, and is not discussed in detail here. However the prime function of this loop would be to maximise power capability at every instant, qualified in that where power can be drawn temporarily from turbine + generator inertia, it is power averaged over many seconds which is to be maximised. As a simple example, assuming an air device, if measured air velocity were constant over at least a few tens of seconds, then optimum turbine speed is known (e.g. from electrical look-up tables) and can be held accurately constant at this value by deriving the power capability signal in an integral loop from measured speed; this would be directly relevant to a windmill. For a wave-power device the situation is more complex due to variations in the wave cycle, and the requirement to match the WEC main structure. (Probably lowest possible inertia is desirable for maximum mean power, but not for lowest power fluctuations).

It is important to note that the response time of the generator field to the power capability signal, hence change of generator electrical torque, will be a fraction of a second (due to use of field forcing) which is almost infinitely fast compared with a wave period. It is this facility for individual optimum power matching which gives the d.c. link method its major advantage over any method which relies on natural characteristics of electrical generators (particularly obvious with synchronous a.c. generators, but true also of some other arrangements).

In practice, matching of mechanical systems (including turbines and WEC prime structures) can only be approximate by open-loop methods, hence fine trimming by self-optimisation over periods of say 5 minutes should add a few percent to efficiency.

3. OPERATION OF ONE D.C. POLE

Consider the simplest installation, comprising a number N of WEC/rectifier units connected in series to form one pole, then via d.c. cable/line to a 250 kV, 500 A inverter (i.e. no paralleled rectifier poles). The first question is the value of N. For 10 kV (max.) per unit, and 250 kV inverter, a value of N = 50 is proposed, i.e. voltage redundancy factor 2. This means that if every WEC has its maximum power capability of 5 MW, then only half of this can be used (i.e. total 125 MW, equal to inverter rating). If power capability of all WEC's is half or less of peak rating, then it can all be fully used.

The redundancy factor obviously directly affects cable and inverter cost per peak MW of WEC rating (or vice versa). The value of 2 is a guess; perhaps experience will show that some other value would be better.
Now consider the overall $V_d/I_d$ characteristic of a rectifier pole at a given instant, assuming random distribution of power capabilities. Figure 1, Curve A is for the general case where some REC capabilities extend up to their peak rating. The full power output is only available at rated current (500 A) since for lower d.c. current some REC's will be voltage limited. Curve B is where no capability exceeds 50% of its peak rating, hence full power can be obtained for any d.c. current from 250 A to 500 A. Curve C is an intermediate case, and D a low power case.

In general, operation permanently at rated current (set by the inverter) will evidently guarantee that all the available power is collected, as far as the electrical system is concerned, i.e. the system is a constant-current scheme (as distinct from conventional a.c. systems, which are constant-voltage, parallel-connected).

Operation at lower d.c. current may in general give some collection power loss because more REC's will reach their voltage limits; however this is a statistical matter, and e.g. at relatively low average power, the average collection loss will be small, down to e.g. 50% current. Saving of $I^2R$ loss at low current may push optimum current down a little, but is unlikely to be a major effect.

The conclusion here is that it is easy to operate a single-pole scheme with low collection loss.

4. OPERATION WITH PARALLELED RECTIFIER POLES

Consider four 500 A rectifier pole groups paralleled on to a 2000 A inverter. Obviously the pole voltages must be identical, referred to the paralleling point, but currents could be different.

There is now a virtual certainty of some electrical collection loss, even with inverter current constant at 2000 A. This is because poles having a low total power capability will tend to reduce the common voltage, so that some REC's within poles having higher total capability will reach their voltage limits.

No ideal solution can be seen to this problem (other than avoiding paralleling, by sub-dividing the inverter or by using 2000 A rectifiers all in series; both will be expensive, and the latter will have poor security). However it seems probable that average collection power loss may well be small and probably acceptable; this is a statistical problem, and has not yet been solved.

5. MANUAL REDUCTION OF POWER

If the inverter maximum voltage is reduced by control via a constant-voltage loop to a value below the "natural" rectifier pole voltage at 550 A (1.1 p.u.) in a given wave condition then all rectifiers will operate in current limit at 550 A, and the received power per pole is simply 550 A x inverter voltage setting.
Power can be adjusted to any value from total capability down to zero.

This is a method for either reducing power by a large amount (if necessary for receiving system requirements) or for reducing total power fluctuations by cutting off peaks above a set value (at the expense of losing some average power).

6. PROTECTION

If the inverter is shut down by block-and-bypass or is set to very low power then d.c. voltage will be about zero, and all WEC’s will operate at current limit (550 A). The WEC’s cannot distinguish this from a cable fault, hence must all take action by shutting off after say 1 minute.

Unsolved problems are at what voltage per WEC should shut-down be initiated, how to re-start automatically, and whether (and how) to provide manual start and stop control effectively at the WEC’s.

J. D. Ainsworth
Fig. 1. Generator control system

Fig. 2. $V_d/I_d$ characteristic of one WEC/rectifier unit
Fig. 3. $V_d/I_d$ characteristics of one rectifier pole