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Mitigation of Nonlinear Receiver Effects in Modern Radar: Advanced Signal Processing Techniques

Euan Ward
Abstract

This thesis presents a study into nonlinearities in the radar receiver and investigates advanced digital signal processing (DSP) techniques capable of mitigating the resultant deleterious effects. The need for these mitigation techniques has become more prevalent as the use of commercial radar sensors has increased rapidly over the last decade. While advancements in low-cost radio frequency (RF) technologies have made mass-produced radar systems more feasible, they also pose a significant risk to the functionality of the sensor. One of the major compromises when employing low-cost commercial off-the-shelf (COTS) components in the radar receiver is system linearity. This linearity trade-off leaves the radar susceptible to interfering signals as the RF receiver can now be driven into the weakly nonlinear regime. Radars are not designed to operate in the nonlinear regime as distortion is observed in the radar output if they do. If radars are to maintain operational performance in an RF environment that is becoming increasingly crowded, novel techniques that allow the sensor to operate in the nonlinear regime must be developed. Advanced DSP techniques offer a low-cost low-impact solution to the nonlinear receiver problem in modern radar. While there is very little work published on this topic in the radar literature, inspiration can be taken from the related field of communications where techniques have been successfully employed.

It is clear from the communications literature that for any mitigation algorithm to be successful, the mechanisms driving the nonlinear distortion in the receiver must be understood in great detail. Therefore, a behavioural modelling technique capable of capturing both the nonlinear amplitude and phase effects in the radar receiver is presented before any mitigation techniques are studied. Two distinct groups of mitigation algorithms are then developed specifically for radar systems with their performance tested in the medium pulse repetition frequency (MPRF) mode of operation. The first of these is the look-up table (LUT) approach which has the benefit of being mode independent and computationally inexpensive to implement. The limitations of this communications-based technique are discussed with particular emphasis placed on its performance against receiver nonlinearities that exhibit complex nonlinear memory effects. The second group of mitigation algorithms to be developed is the forward modelling technique. While this novel technique is both mode dependent and computationally intensive to implement, it has a unique formalisation that allows it to be extended to include nonlinear memory effects in a well-defined manner. The performance of this forward modelling technique is analysed and discussed in detail.
It was shown in this study that nonlinearities generated in the radar receiver can be successfully mitigated using advanced DSP techniques. For this to be the case however, the behaviour of the RF receiver must be characterised to a high degree of accuracy both in the linear and weakly nonlinear regimes. In the case where nonlinear memory effects are significant in the radar receiver, it was shown that memoryless mitigation techniques can become decorrelated drastically reducing their effectiveness. Importantly however, it was demonstrated that the LUT and forward modelling techniques can both be extended to compensate for complex nonlinear memory effects generated in the RF receiver. It was also found that the forward modelling technique dealt with the nonlinear memory effects in a far more robust manner than the LUT approach leading to a superior mitigation performance in the memory rich case.
Lay Abstract

Radars are well-established sensors in the defence sector and they are beginning to see employment in a far wider range of applications (e.g. driverless cars and unmanned aerial vehicle, UAVs). Undoubtedly, they will be among the most used and important sensors going forward. The consumerisation of radar systems is drastically changing the way in which radars are being designed. By using COTS components companies can develop economically viable systems that can easily be productionised. There are many commercial and technological reasons for employing COTS components in modern radars but as they are not specialised radar technology, they leave the system vulnerable to interference.

A radar is a radio receiver much like an AM/FM radio. In the same way that AM/FM radios require amplifiers so that programmes can be heard, the weak signals captured by radar must be amplified so that targets can be detected. These amplifiers are designed to operate in a linear fashion so that the output signal is directly proportional to the input. If, however, a very large signal is received from an interference source the amplifiers behave in a nonlinear manner which distorts the signal, making targets harder to detect or your radio programme harder to hear. COTS amplifiers have considerably smaller linear regimes than their bespoke counterparts. In order to maintain an acceptable level of performance the amplifiers in modern radars must operate closer to their nonlinear regime than ever before. Operating so close to this regime poses a severe risk to the functionality of the radar as interference from other nearby radars can force the amplifiers into a nonlinear state. With the use of radar technologies growing exponentially, the radio frequency spectrum is becoming increasingly crowded. Modern radar systems must therefore evolve so that they can operate effectively in the nonlinear regime. This PhD aims to show that by employing advanced signal processing techniques radars can maintain operational capability while their receiver amplifiers are in a nonlinear state.

Within the radar field there is little published literature on the mitigation of nonlinear receiver effects. Encouragement can be found however in the related field of communications where solutions to the nonlinear receiver problem have been published. Importantly, if these communications-based solutions are to be employed successfully in modern radar, they must be redesigned in order to account for the considerable differences between the two systems. This research aims to overcome the nonlinear receiver problem in radar by drawing on the mitigation techniques established in the communication literature and by exploiting the exciting new techniques being developed in the field of signal processing. If modern radars are to be commercially viable in mass markets this nonlinear receiver problem must be overcome.
Acknowledgements

Firstly, I would like to say a huge thanks to my academic and industrial supervisors Prof. Bernard Mulgrew and Dr. David Greig for their guidance, patience and invaluable advice throughout this research project.

I gratefully acknowledge Leonardo for sponsoring the PhD project through the UK Engineering and Physical Sciences Research Council (EPSRC) CASE Studentship Scheme [Grant No. EPSRC EP/N509644/1]. Further thanks goes to the Royal Commission for the Exhibition of 1851 who part funded this research through an Industrial Fellowship.

I would also like to take this opportunity to thank the Leonardo team in Edinburgh and colleagues at the University of Edinburgh’s Institute for Digital Communications for their unwavering support both technical and otherwise throughout the research project. In particular I would like to thank Dr. Shahzad Gishkori whose technical advice was key in overcoming the challenges posed by the nonlinear compressive sensing work. Finally, special mentions to Dr. Gavin Halcrow, Dr. Neville Ramsey, Mr Patrick Corsar, Mr Andy Glass, Dr. Robin Collings, Dr. Fiona Muirhead, Dr. Claire Tierney and Dr. Scott Smith to whom I am particularly grateful for all of their insightful comments and feedback over the course of the PhD.
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# List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>DSP</td>
<td>Digital Signal Processing</td>
</tr>
<tr>
<td>COTS</td>
<td>Commercial Off-The-Shelf</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency-Division Multiplexing</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicles</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>MPRF</td>
<td>Medium Pulse Repetition Frequency</td>
</tr>
<tr>
<td>LUT</td>
<td>Look-Up Table</td>
</tr>
<tr>
<td>DBF</td>
<td>Digital BeamForming</td>
</tr>
<tr>
<td>NLEQ</td>
<td>NonLinear EQualization</td>
</tr>
<tr>
<td>RE</td>
<td>Receiving Element</td>
</tr>
<tr>
<td>LNA</td>
<td>Low Noise Amplifier</td>
</tr>
<tr>
<td>AM/AM</td>
<td>Amplitude-Amplitude conversion</td>
</tr>
<tr>
<td>AM/PM</td>
<td>Amplitude-Phase conversion</td>
</tr>
<tr>
<td>TOI</td>
<td>Third-Order Intercept</td>
</tr>
<tr>
<td>IMD</td>
<td>InterModulation Distortion</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>PBTS</td>
<td>PassBand Taylor Series</td>
</tr>
<tr>
<td>PBVS</td>
<td>PassBand Volterra Series</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>PRF</td>
<td>Pulse Repetition Frequency</td>
</tr>
<tr>
<td>RD</td>
<td>Range Doppler</td>
</tr>
<tr>
<td>PD</td>
<td>Probability of Detection</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast-time Fourier Transform</td>
</tr>
<tr>
<td>STFT</td>
<td>Slow-Time Fourier Transform</td>
</tr>
<tr>
<td>PRI</td>
<td>Pulse Repetition Interval</td>
</tr>
<tr>
<td>CPI</td>
<td>Coherent Processing Interval</td>
</tr>
<tr>
<td>CFAR</td>
<td>Constant False Alarm Rate</td>
</tr>
<tr>
<td>Acronym</td>
<td>Definition</td>
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<tr>
<td>---------</td>
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</tr>
<tr>
<td>CA-CFAR</td>
<td>Cell Averaging-Constant False Alarm Rate</td>
</tr>
<tr>
<td>PFA</td>
<td>Probability of False Alarm</td>
</tr>
<tr>
<td>PFAR</td>
<td>Probability of False Alarm Rate</td>
</tr>
<tr>
<td>FAR</td>
<td>False Alarm Rate</td>
</tr>
<tr>
<td>LAA</td>
<td>Local Area Average</td>
</tr>
<tr>
<td>CUT</td>
<td>Cell Under Test</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>CW</td>
<td>Continuous Wave</td>
</tr>
<tr>
<td>FMCW</td>
<td>Frequency Modulated Continuous Wave</td>
</tr>
<tr>
<td>HPRF</td>
<td>High Pulse Repetition Frequency</td>
</tr>
<tr>
<td>LO</td>
<td>Local Oscillator</td>
</tr>
<tr>
<td>BBTS</td>
<td>BaseBand Taylor Series</td>
</tr>
<tr>
<td>BBVS</td>
<td>BaseBand Volterra Series</td>
</tr>
<tr>
<td>DPD</td>
<td>Digital Pre/Post-Distortion</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
</tr>
<tr>
<td>VNA</td>
<td>Vector Network Analyser</td>
</tr>
<tr>
<td>CS</td>
<td>Compressive Sensing</td>
</tr>
<tr>
<td>NCS</td>
<td>Nonlinear Compressive Sensing</td>
</tr>
<tr>
<td>SAR</td>
<td>Synthetic-Aperture Radar</td>
</tr>
<tr>
<td>STAP</td>
<td>Space-Time Adaptive Processing</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
</tr>
<tr>
<td>LS</td>
<td>Least Squares</td>
</tr>
<tr>
<td>LLS</td>
<td>Linear Least Squares</td>
</tr>
<tr>
<td>RLS</td>
<td>Recursive Least Squares</td>
</tr>
<tr>
<td>IHT</td>
<td>Iterative Hard Thresholding</td>
</tr>
<tr>
<td>ME</td>
<td>Modelling Error</td>
</tr>
<tr>
<td>RCS</td>
<td>Radar Cross-Section</td>
</tr>
<tr>
<td>AOB</td>
<td>Angle-Off-Boresight</td>
</tr>
<tr>
<td>DUT</td>
<td>Device Under Test</td>
</tr>
<tr>
<td>AWG</td>
<td>Arbitrary Waveform Generator</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>ADC</td>
<td>Analogue to Digital Converter</td>
</tr>
<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
</tr>
<tr>
<td>LO</td>
<td>Local Oscillator</td>
</tr>
</tbody>
</table>
Nomenclature

\[ c \] speed of light
\[ L \] memory length
\[ \omega_c \] angular carrier frequency
\[ f_c \] carrier frequency
\[ \omega_i \] angular interference frequency
\[ f_i \] interference frequency
\[ P \] nonlinear forward model power
\[ h \] passband forward model coefficients
\[ g \] passband inverse model coefficients
\[ \tilde{h} \] baseband forward model coefficients
\[ \tilde{g} \] baseband inverse model coefficients
\[ t \] continuous time
\[ n \] baseband discrete-time samples
\[ m \] passband discrete-time samples
\[ \tau \] continuous-time delay lag
\[ l \] passband discrete-time delay
\[ i \] baseband discrete-time delay
\[ \tilde{x}_1 \] complex baseband input signal around carrier - desired signal only
\[ \tilde{x}_2 \] complex baseband input signal around interferer
\[ \tilde{x}_3 \] complex baseband input signal around carrier - desired signal and interferer
\[ \tilde{y} \] complex baseband output signal around carrier
\[ \tilde{y}_{p1} \] complex baseband output signal around carrier linear component
\[ \tilde{y}_{p2} \] complex baseband output signal around carrier quadratic component
\[ \tilde{y}_{p3in} \] complex baseband output signal around carrier cubic \textit{in-channel} component
\[ \tilde{y}_{p3out} \] complex baseband output signal around carrier cubic \textit{out-of-band} component
\[ x \] passband input signal
\[ y \] passband output signal
\( j \) imaginary unit

\( ln \) natural logarithm

\( dB \) decibel

\( \tau_r \) time-lag

\( K \) inverse nonlinear model power

\( b \) passband FIR filter coefficients

\( \tilde{w} \) baseband white Gaussian noise signal

\( \tau \) matrix transpose

\( \eta \) conjugate/Hermitian transpose

\( \nabla \) gradient operation

\( \kappa \) Fourier domain discrete-time samples

\( \tilde{X}_1 \) desired baseband signal Fourier coefficients

\( \Phi \) Fourier basis matrix

\( e \) error vector

\( C \) cost function

\( \mu \) least squares step parameter

\( \beta \) least squares iteration index

\( \tilde{x} \) baseband in-band input signal vector

\( \tilde{v} \) baseband desired radar signal vector

\( \tilde{s} \) baseband in-band interference signal vector

\( \tilde{X} \) forward model baseband input signal matrix

\( \tilde{Y} \) forward model baseband nonlinear output signal matrix

\( q \) slow-time PRI index

\( r \) range index

\( j \) regressor Doppler index

\( N \) number of fast-time samples

\( Q \) number of slow-time samples

\( R \) number of Range samples

\( J \) number of regressor Doppler samples

\( \Psi \) range projection matrix

\( \Theta \) NCS model regressor matrix

\( \hat{\theta} \) NCS model regressor vector
\( \sigma \) target radar cross-section
\( \sigma_c \) clutter cell backscatter coefficient
\( G_{rx} \) receive-side directivity
\( G_{tx} \) transmit-side directivity
\( G_{ti} \) interferer’s transmit-side directivity
\( P_{tx} \) radar transmit power
\( P_{rx} \) received target power
\( P_{ti} \) interference transmit power
\( P_{ri} \) received interference power
\( R_r \) range to target
\( R_i \) range to interferer
\( \Delta \phi_t \) target Doppler phase shift
\( v_D \) Doppler velocity
\( v_p \) platform velocity
\( v_t \) target velocity
\( \varphi_p \) platform’s azimuthal velocity direction
\( \theta_p \) platform’s elevation velocity direction
\( \varphi_c \) target’s azimuthal angle off boresight
\( \theta_c \) target’s elevation angle off boresight
\( \varphi_t \) target’s azimuthal velocity direction
\( \theta_t \) target’s elevation velocity direction
\( v_i \) interferer velocity
\( \varphi_i \) interferer’s azimuthal velocity direction
\( \theta_i \) interferer’s elevation velocity direction
\( \varphi_{ci} \) interferer’s azimuthal angle off boresight
\( \theta_{ci} \) interferer’s elevation angle off boresight
\( (u, v) \) antenna frame coordinates
\( f_{s1} \) passband sampling frequency
\( f_{s2} \) baseband sampling frequency
\( f_{IF} \) intermediate frequency
\( f_{LO} \) local oscillator frequency
\( \hat{H} \) Fourier transform of baseband Volterra kernel
\( \tilde{h} \) Vectorised baseband Volterra kernel coefficients

\( \tilde{V} \) BB Volterra observation matrix

\( \tilde{\sigma} \) BB Volterra series functionals

\( D \) Number of baseband Volterra terms

\( \epsilon \) RLS fading memory parameter

\( z \) signal delay

\( \tilde{\kappa} \) BB Volterra polyphase sub-kernel
Chapter 1

Introduction

1.1 Motivation

Radars are high performance sensors that have been employed successfully for decades in the defence, aviation and marine sectors. Their ability to detect multiple targets at large stand-off ranges even in inclement weather makes them extremely attractive sensors for a far broader range of applications than just those listed above. However due to size, weight and cost constraints they have traditionally failed to find wider adoption in other sectors. The development of low-cost commercial off-the-shelf (COTS) components has led to a revolution in the radar industry as companies can now mass-produce systems that are both compact and economically viable. The productionisation of modern radar systems has led to them being adopted by a multitude of new platforms and modern technologies e.g. driverless cars and unmanned aerial vehicles (UAVs). This has resulted in radars no longer being considered as a strictly defence sensor. Interest in modern radar has spread across a wide variety of public and commercial sectors [1, 2, 3, 4, 5] with many thinking they could become one of the most commonly used and important sensors of the future. While the development of COTS based technologies has unlocked the commercial potential of radar, they also pose a serious threat to the future of the modern sensor. The radio frequency (RF) spectrum is becoming increasingly crowded and with the use of radars set to increase dramatically over the next decade this problem is only going to get worse [6]. The level of ambient RF energy will quickly become intolerable for modern radar causing performance problems due to interference between systems. There are many very good commercial and technological reasons for employing COTS components in modern radars, but as they are not specialised radar technology they can leave the system vulnerable to unwanted interference effects.

The immunity of a radar system to interfering signals is effectively defined by the linearity of the RF amplifiers in its receiver [7, 8, 9]. These RF receiver amplifiers are designed to operate in a linear fashion so that the output signal is directly proportional to the input signal. The main purpose of the radar receiver is to amplify the signals captured by the antenna so that targets can be detected. If strong interference is also captured by the antenna then the amplifiers in the radar receiver can behave in a nonlinear manner making target detection very difficult for the sensor. As long as the active components
in the RF receiver remain in the linear regime, while the desired signal and the interference signal pass through the chain simultaneously, then the principle of superposition holds and the task of removing the interference effects can be seen as a linear signal separation problem [10]. Based on this approach, traditional radars were designed using bespoke RF components to very high linearity specifications. By having large linear regions the sensors could preserve their linear behaviour even in the presence of strong interference signals by simply backing-off the receiver gain [11, 10, 12, 13]. Importantly, the term backing-off is one that is used frequently in the radar literature and throughout this thesis to described the process where the receiver gain is adaptively reduced in the presence of strong input signals. In essence, by reducing the gain of the active components in the receiver the linear performance of the radar can be maintained even in the presence of strong interference signals which in turn, protects the system from harmful nonlinear distortion effects. While this simple mitigation technique results in some loss in sensitivity, critical operational performance is maintained until the interference passes. In the case of the modern mass-produced radar, one of the major compromises when employing low-cost COTS technologies in the RF receiver is front-end linearity [9, 14, 15]. Therefore, these modern sensors must operate at the top of their linear regions to achieve acceptable performance leaving no room to back-off in the presence of interference. Operating at the top of the linear region poses a severe risk to the functionality of these sensors as interference from other nearby radars can force the RF amplifiers in its receiver into a nonlinear state. The nonlinear amplifiers cause the interference and target signal to become tangled, as the principle of superposition no longer holds, resulting in the radar processor being unable to distinguish the true targets from the false targets generated in the nonlinear amplifiers. This nonlinear radar phenomenon is illustrated in the Figure 1.1 [11].

The multi-sensor mutual interference problem is well-known in the radar community with the development of an intelligent radar network being the most widely accepted proposed solution [5, 16]. This ambition idea is centred on the premise that the entire radar network is comprised of fully cognitive sensors [17]. By exploiting the cognitive radar’s ability to be agile within its environment, it is conceivable that the sensors could avoid interfering with each other if they all successfully communicate their physical, spectral and temporal locations across the network [18]. This would allow the sensors to minimise the interference between systems by operating on separate centre frequencies and at different time instances. While the idea of a network of radar sensors all working cognitively to avoid interfering with each other is an interesting one, there are more established techniques in the field of communications that could be employed to tackle this mutual interference problem before this ambition network idea comes to fruition. Communications systems have had to work effectively in a crowded RF environment since they were originally commercialised and have therefore had to develop a series of techniques to allow them operate in close proximity to one another. Some of these include:
1.1. Motivation

FIGURE 1.1: Comparison of receiver characteristics. \( p_1 \) input power of desired radar signal, \( P_1 \) output power of desired radar signal post receiver, \( p_2 \) input power of desired radar signal and interference signal, \( P_2 \) output power of desired radar signal and interference signal post receiver. The bespoke receiver 1.1a maintains linearity even in the presence of the interfering signal whereas the COTS receiver 1.1b is driven into the weakly nonlinear regime by the interference.

employing pilot sequences to uniquely identify each communications waveform and employing orthogonal frequency-division multiplexing (OFDM) techniques to maximise the full extent of the available spectrum. While mutual interference can be minimised by introducing cognition to radar systems and by employing these more established communications based techniques, some self-sufficient protection operating within each sensor is required if performance is to be maintained even in the worst interference scenario. Therefore, a more reliable solution must also be sought which will protect the performance of the radar sensor in the case where mutual interference cannot be avoided by the strategies discussed above. For modern radar this means tackling the nonlinear receiver problem.

With the use of consumer radars set to increase dramatically over the next decade modern mass-produced radar systems must evolve so that they can operate effectively in environments crowded with RF energy. Crucially, the technological advancements being developed for modern radar systems will mean that the sensor can employ sophisticated techniques in order to protect itself from unwanted interference effects. Some of the key technologies that the modern radar sensor will benefit from are as follows: a phased array antenna removing the need for a mechanically scanned system; a highly digitised
Chapter 1. Introduction

antenna allowing the radar to look multiple directions at once; adaptive waveform capabilities making it a multi-functional sensor; and finally, vastly increased processing power allowing sophisticated digital signal processing techniques to be employed in real time. Importantly, in this PhD we focus on the final point and aim to develop advanced signal processing solutions that will allow modern radars to maintain operational capability when their receiver amplifiers have been driven into a nonlinear state.

1.2 Thesis Research

The main aim of this PhD project is to design advanced signal processing techniques to mitigate front-end nonlinear effects in modern pulse-Doppler radar systems. However, before digital mitigation techniques could be designed the nonlinear behaviour of the radar receiver had to be modelled and understood in great detail. The principal task for a radar sensor is to make target detections and it achieves this by studying both the amplitude and phase of the returned signals. While amplitude distortion effects caused by front-end receiver nonlinearities have been studied in the radar literature, there is little work published on the characterisation of nonlinear phase effects. Since radars rely so heavily on the signal’s phase information as well as its amplitude information when making detections, any mitigation technique that neglects phase distortion effects will most likely be ineffective. This is a trend observed in the far less sensitive communications systems where digital nonlinear mitigation techniques are now having to be modified to account for nonlinear phase distortion effects in order to achieve the desired system performance [19, 20]. The requirement to capture nonlinear phase effects as well as nonlinear amplitude effects in the radar receiver model massively complicates the characterisation task as memory effects have to be introduced. Therefore, the work presented in chapter 3 details the development of a comprehensive mathematical model capable of capturing the complex nonlinear behaviour of the modern radar receiver in the baseband domain. Inspiration for the black-box behavioural model was taken from the communications literature with the nonlinear design centred around memory based Volterra series. Importantly, this allows the model to capture both amplitude and phase nonlinear distortion effects.

Once the mathematical framework for the nonlinear receiver model had been derived, the focus of the PhD research turned towards the development of digital mitigation techniques. For simplicity, the development of these mitigation techniques was targeted at the two most problematic nonlinear interference scenarios observed in modern radar. These scenarios are the out-of-band interference case, typically referred to as the cross-modulation scenario in the radar literature, and the in-band interference case. Both of these scenarios are illustrated diagrammatically in Figure 1.2. In both the out-of-band and the in-band interference cases, spurious harmonics are generated across the RF spectrum with direct distortion also caused to the desired radar channel. For the purposes of
this research, it is assumed that the spurious harmonics lie outside the desired channel and are therefore successfully filtered out further down the RF receive chain. Consequently, the task for the digital mitigation techniques is to correct for the direct distortion caused to the radar channel. Independent digital mitigation strategies were adopted for the two different nonlinear scenarios described above. In the case of the out-of-band interference scenario, the look-up-table (LUT) mitigation approach was studied. This is a technique popular in the related field of communications, however little work has been done to study its effectiveness in radar systems. For the in-band interference scenario, inspiration was taken from the signal processing literature with the mitigation solution designed around the concept of forward modelling techniques. While both mitigation techniques were designed using the input-output characteristics of our theoretical nonlinear receiver model, it was important that their performance was assessed as part of a full radar system. Therefore, a comprehensive radar simulator which incorporated the complex nonlinear receiver model described above was written as part of this project. By testing the nonlinear mitigation solutions in this simulated radar environment, the system level performance of each technique could be studied in great detail.

Nonlinear distortion effects can be generated in all radar modes of operation as they are driven by weak nonlinearities in the active components at the front-end of the RF receiver. However, in this thesis we focus on these nonlinear effects specifically for a medium pulse repetition frequency (MPRF) mode but that is not to say that the techniques studied could not be extended to other radar modes of operation. The MPRF radar mode is one of the most popular modes of operation for modern radar systems which is one of the main reasons why it is the primary focus in this thesis. The popularity of the MPRF mode stems from the fact that it can provide accurate measurements of both range and Doppler which is crucial for detecting targets and maintaining tracks. We focus on the MPRF mode of operation throughout this thesis not only because it is one of the most popular modes of operation for modern radar, but also for the following key reasons:

1. In a typical MPRF mode the radar is tasked with detecting low signal to noise ratio (SNR) targets in the presence of a strong clutter signal which can leave it susceptible to nonlinear effects.

2. The MPRF mode is intrinsically narrowband which is the main assumption that must be satisfied when employing baseband behavioural models to simulate the RF receiver.

3. The modelling of MPRF radar clutter is well understood making it the perfect environment to test the performance of the nonlinear mitigation solutions [21, 22, 23, 13, 10, 11, 24].

It is important to note that while we develop and test the digital mitigation solutions derived throughout this thesis in the MPRF radar environment, we make no assumptions
about the PRF during the derivations which means that there is no reason the techniques
would not work in any other pulse-Doppler mode of operation\(^1\).

A block diagram describing the layout of the thesis research is presented in Figure
1.3. The first block depicts the characterisation work conducted in chapter 3 which as
discussed previously, was essential for the development of the digital mitigation tech-
niques designed in chapters 4 and 5. Chapter 4 focuses on the out-of-band interference
scenario and studies the effectiveness of two separate LUT mitigation techniques. These
LUT mitigation techniques were explored first as they are mode independent and there-
fore offer the best chance at a universal solution to the nonlinear receiver problem in
radar. Particular emphasis was placed on the applicability of these algorithms to the case
where the forward receiver nonlinearity contained memory effects. Chapter 5 focuses on
the in-band interference scenario and presents a unique forward modelling mitigation
solution. Unlike the LUT techniques which are based on nonlinear inverse structures,
the forward modelling approach incorporates the model of the forward nonlinearity it-
selves into the signal processing. While this technique is by nature mode specific, it has a
unique mathematical formalisation that lends itself to the case where the forward non-
linearity contains memory.

1.3 Thesis Organisation

1.3.1 Chapter 3: Modelling Nonlinear Radar Receivers

A sophisticated behavioural modelling technique capable of capturing complex nonlin-
ear memory effects is developed for radar systems in chapter 3. The theoretical frame-
work for the technique is derived by mapping the passband Volterra series behavioural
model to the baseband domain through a unique cross-channel process. Importantly, it is
shown that the resultant baseband behavioural model accurately describes the nonlinear
coupling between the desired radar channel and an out-of-band interferer in the scene.
Furthermore, an advanced system identification procedure is developed for the baseband
model as part of this chapter. The accuracy of the behavioural model and corresponding
system identification procedure are tested in a comprehensive simulation environment.

1.3.2 Chapter 4: LUT Inverse Mitigation Techniques

Chapter 4 studies the effectiveness of the nonlinear equalization LUT approach in miti-
gating front-end nonlinearities in modern radar systems. Two specific communications
based LUT approaches are studied with their performance tested against the cross-modulation
scenario in a simulated MPRF radar environment. Detailed stochastic based performance
analysis is presented, with both algorithms tested against memoryless and memory rich

\(^1\)Some slight refinements may need to be applied in the case of a synthetic aperture radar (SAR) or space-
time adaptive processing (STAP) mode but fundamentally the theory should remain the same.
1.3. Thesis Organisation

(A) In-band interference scenario. Receiver nonlinearity driven by the presence of an interferer inside the desired radar channel. $f_c$ carrier frequency and $f_i$ interference frequency.

(B) Out-of-band interference scenario. Strong out-of-band interference couples with the desired radar channel causing unwanted nonlinear effects. $f_c$ carrier frequency and $f_i$ interference frequency.

Figure 1.2: Comparison of receiver performance in the presence of interference. The bespoke receiver (blue) has a high tolerance for inference signals whereas the COTS receiver (red) is susceptible to nonlinear distortion effects.
Chapter 1. Introduction

Nonlinear radar simulator
LUT mitigation techniques
Forward modelling mitigation techniques
Memoryless
Memory rich
Memoryless
Memory rich
Nonlinear black-box behavioral model
System identification techniques
Nonlinear radar simulator
LUT mitigation techniques
Memoryless
Memory rich
Forward modelling mitigation techniques
Memoryless
Memory rich
Chapter 3
Chapter 4
Chapter 5

Figure 1.3: Thesis layout. Nonlinear characterisation work covered in chapter 3 with the LUT mitigation research outlined in chapter 4. The forward modelling mitigation technique is studied in chapter 5. The nonlinear radar simulator is central to the research and is therefore touched upon in all of the main thesis chapters.
forward receiver nonlinearities. The second LUT table approach is extended to include nonlinear memory effects with its effectiveness in mitigating forward receiver nonlinearities with memory studied in detail.

1.3.3 Chapter 5: Forward Modelling Mitigation Techniques

A novel nonlinear compressive sensing (NCS) technique for radar systems is derived in chapter 5. Furthermore, its effectiveness in mitigating nonlinear receiver effects in radar is studied for the in-band interference scenario. The NCS technique is tested against both memoryless and memory rich receiver nonlinearities with its performance analysed in a simulated MPRF radar environment. As with the LUT approach, results from stochastic based performance analysis are presented with the robustness of the algorithm tested rigorously. The advantages and limitations of the algorithm are discussed in detail and the feasibility of implementing the NCS nonlinear mitigation technique in a real radar system is considered.

1.4 Contributions

1.4.1 Novelty

This thesis presents a comprehensive study into nonlinear receiver effects in modern radar. A complex black-box behavioural model capable of characterising the nonlinear nature of the radar receiver to extremely high levels is derived. Two of the most problematic nonlinear interference scenarios in radar are studied with specific digital mitigation solutions developed for each. The effectiveness of the most successful communications based digital mitigation technique is analysed in a radar environment with its performance tested against both memoryless and memory rich RF receiver nonlinearities for the first time. A novel forward modelling nonlinear mitigation technique is presented before being tailored specifically to radar systems. The performance of this unique nonlinear mitigation solution is studied in a complex radar environment for both the case where the RF receiver nonlinearity exhibits memoryless and memory rich behaviour.

Importantly, the specific novel contributions that this PhD makes to the literature are stated below with their exact locations within the thesis also indicated. These are as follows:

1. Chapter 3: the derivation of a novel cross-channel baseband Volterra series model for modelling the complex nonlinear memory behaviour of the radar receiver, see section 3.2.

2. Chapter 3: the development of a sophisticated noise-based identification procedure capable of accurately learning the full set of kernel coefficients that characterise the memory rich behaviour of the nonlinear model, see section 3.3.
Chapter 1. Introduction

3. Chapter 4: a new performance based analysis on the effectiveness of the communications-based LUT techniques in mitigating harmful cross-modulation effects in modern radar, see section 4.3.4.

4. Chapter 4: a new performance based analysis on the effectiveness of the classic memoryless mitigation techniques in compensating for complex nonlinear memory effects generated in the radar receiver, see section 4.4.3.

5. Chapter 4: the extension of the classic LUT technique to include cross-channel complex nonlinear memory effects, see section 4.4.1.

6. Chapter 4: a new performance based analysis on the effectiveness of the memory rich LUT technique in mitigating cross-modulation effects generated in a typical MPRF radar mode, see section 4.4.3.

7. Chapter 5: a unique analysis of input and output noise as part of the complex baseband nonlinear model, see section 5.2.2.

8. Chapter 5: the extension of the forward nonlinear transfer function in the NCS algorithm to include nonlinear memory effects, see section 5.2.3.

9. Chapter 5: the extension of the NCS algorithm to deal with the complex problem rather than the purely real problem studied in previous work, see section 5.2.4.

10. Chapter 5: the treatment of a unique compressive sensing problem that exploits signal sparsity in slow-time and tackles a complex baseband nonlinearity with memory in fast-time, see section 5.2.5.

11. Chapter 5: a new performance based analysis on the effectiveness of the NCS algorithm in mitigating complex nonlinear memory effects in the modern radar receiver, see section 5.4.

12. Chapter 5: an examination of the NCS algorithm’s mitigation performance against real MPRF radar data, see section 5.5.

1.4.2 Industrial Impact

The behavioural modelling technique presented in this thesis provides radar companies with a mechanism for characterising the complex nonlinear memory behaviour of the RF receiver. This is a capability that is likely to become more and more important as radars move to increasingly wide receiver bandwidths where memory effects become more significant. Understanding the unique memory behaviour of front-end radar amplifiers will benefit companies when designing modern low-cost radars as they attempt to make the task of mitigating the unwanted nonlinear receiver effects as straightforward as possible. The digital mitigation techniques presented in this thesis offer industry an insight
into some of the advantages and limitations of employing advanced signal processing techniques when try to mitigate harmful nonlinear receiver effects. Certainly, this thesis highlights the importance of characterising the radar receiver into the weakly nonlinear regime as well the significance of the modern radar’s ability to form guard channels.

1.5 Publications

The following section contains a list of publications for this research project. The first paper details the nonlinear modelling and characterisation work described in chapter 3. The second and third papers outline some of the work carried out in chapter 4 on the mitigation of nonlinear receiver effects by the LUT techniques. The final paper provides a comprehensive review of the nonlinear compressive sensing mitigation technique derived in chapter 5.


Chapter 2

Literature Review

2.1 Background Literature Review

The following sections provide a literature review for the thesis. While this research project is radar specific, much inspiration is taken from the related fields of communications and signal processing respectively. Therefore, the literature review is split accordingly into three categories: a radar section, a communications section and a signal processing section. Key papers by Mellor et al., Fischer et al. and Earl [8, 7, 25] describe the widely known nonlinear receiver problem in radar highlighting different scenarios and distortion effects respectively. While papers studying nonlinear receiver effects are sparsely published in the radar literature, it is an active area of research in the related field of communications. Inspiration for a more sophisticated nonlinear radar receiver model is taken from the communications literature where the research is becoming increasingly focused on complex nonlinear memory effects [15]. In turn, the inspiration for a universal digital mitigation solution is also taken from the communications literature where the technique has been successfully implemented by Zou et al. and Ma et al. [14, 26] for the cross-channel and in-channel cases respectively. The mathematical basis for the forward modelling mitigation technique is founded in the signal processing literature where Blumensath details his theory in [27] before Chen et al. apply it to a nonlinear communications system in [28].

2.1.1 Radar Literature Review

With radar systems still being a predominantly defence-based sensor, much of the state-of-the-art industrial research is not published in the open literature due to concerns regarding security and intellectual property. This is particularly true for data collected from real systems. However, with the development of COTS based RF technologies driving radar towards more mainstream sectors, the field has seen an influx of research contributions from less sensitive applications. This section aims to provide a short review of the relevant radar literature covering the most important contributions from both the defence orientated side as well as the more commercial side of the literature. Furthermore, it will provide a brief background for some of the fundamental radar concepts which are paramount for understanding the main research chapters of the thesis.
Chapter 2. Literature Review

Radar Receiver Amplifier Models

In the radar literature, nonlinear receiver effects are typically separated into two distinct types: amplitude-amplitude (AM/AM) conversion and amplitude-phase (AM/PM) conversion. The AM/AM mechanism aims to describe the nonlinear distortion applied to the returned signal’s amplitude while the AM/PM mechanism aims to describe the distortion applied to its phase. It is important to note that the nonlinearity is driven by the amplitude of the input signal which is why the input amplitude, along with the characteristics of specific nonlinearity, ultimately defines the nonlinear distortion applied to both the amplitude and phase of the output signal. This is reflected in the above terminology by the ‘AM’ appearing first in the names of both the AM-to-AM and AM-to-PM conversion effects. In essence, the AM/PM conversion describes the amount of undesired phase deviation (PM) that is caused to the output by amplitude variations (AM) at the input of the nonlinear system, with the AM/AM conversion capturing the deviation of the output amplitude. Typically, radar receiver engineers focus on characterising the AM/AM nonlinear effect as they are particularly interested in the generation of spurious harmonics that could degrade the overall dynamic range of the RF receiver. Therefore, most receiver amplifier models in the radar literature are built around the passband Taylor series [25, 29, 30] which is stated in its continuous-time form below (2.1).

\[ y(t) = \sum_{p=0}^{P} h_p x(t)^p \]  

In (2.1); \( x(t) \) denotes the real input signal to the amplifier model, \( y(t) \) denotes the real output and the real coefficients \( h_p \) describe the behaviour of a particular RF amplifier. Typically in the radar literature, the Taylor model is truncated to order \( P = 3 \) as it is widely accepted that the most dominate nonlinear effects can be capture by the linear \( p = 1 \), quadratic \( p = 2 \) and cubic \( p = 3 \) terms of the series [25, 29].

\[ y(t) = h_0 + h_1 x(t) + h_2 x(t)^2 + h_3 x(t)^3 \]  

(2.2)

It is convenient to describe the received signal \( x(t) \) as the real part (\( \mathbb{R}\{\cdot\} \)) of a complex baseband signal \( \tilde{x}_1(t) \) modulated on a (complex) carrier frequency \( f_c = \frac{\omega_c}{2\pi} \).

\[ x(t) = \mathbb{R}\{\tilde{x}_1(t)e^{j\omega_c t}\} = \frac{1}{2}\{\tilde{x}_1(t)e^{j\omega_c t} + \tilde{x}_1^*(t)e^{-j\omega_c t}\} \]  

(2.3)

The full spectrum of the signal at the output of the amplifier can then be revealed by substituting (2.3) into (2.2) and exploiting the usual properties of Fourier transforms. In
2.1. Background Literature Review

\[y(t) = h_0 + \frac{h_1}{2} \{\tilde{x}_1(t)e^{j\omega_c t} + \tilde{x}_1^*(t)e^{-j\omega_c t}\} + \frac{h_2}{4} \{\tilde{x}_1(t)e^{j\omega_c t} + \tilde{x}_1^*(t)e^{-j\omega_c t}\}^2 + \frac{h_3}{8} \{\tilde{x}_1(t)e^{j\omega_c t} + \tilde{x}_1^*(t)e^{-j\omega_c t}\}^3\] (2.4)

Multiplying out the brackets in (2.4) yields components at 0, ±ω_c, ±2ω_c, and ±3ω_c. The post amplifier RF filtering\(^1\) is then tasked with removing all signal components except those at ±ω_c, the carrier itself. Assuming the RF receiver provides adequate filtering, the output signal takes the following form:

\[\Re\{\tilde{y}(t)e^{j\omega_c t}\} = \frac{h_1}{2} \{\tilde{x}_1(t)e^{j\omega_c t} + \tilde{x}_1^*(t)e^{-j\omega_c t}\} + \frac{3h_3}{8} \{\tilde{x}_1(t)^2\tilde{x}_1(t)e^{j\omega_c t} + |\tilde{x}_1(t)|^2\tilde{x}_1^*(t)e^{-j\omega_c t}\}\] (2.5)

where \(\tilde{y}(t)\) is the baseband signal of the amplifier output at the carrier frequency. Subsequent downconversion and digitising stages in the RF receiver allow \(\tilde{y}(t)\) to be sampled before being passed to the radar processor where target detection algorithms are performed.

It is clear from (2.1) that the characterisation of the RF amplifier’s nonlinear behaviour effectively boils down to the identification of the model coefficients \(h_p\). In the case of the third order passband Taylor series model (2.2) there are four coefficients to identify, however it is apparent from (2.5) that only the linear \(h_1\) and cubic \(h_3\) coefficients appear in the final expression for the radar signal after RF filtering. The coefficient \(h_1\) describes the linear gain of the RF amplifier and is easy to measure through a single-tone test on the carrier frequency. The cubic coefficient \(h_3\) is measured by means of a two-tone test in what is typically referred to in the radar literature as a measurement of the third-order intercept (TOI) [25, 31, 32]. Consider an RF amplifier described by the passband Taylor model (2.2) subject to the two-tone test signal (2.6).

\[x(t) = \Re\{Ae^{j\omega_a t} + Be^{j\omega_b t}\} = \left\{\frac{A}{2}e^{j\omega_a t} + e^{-j\omega_a t}\right\} + \frac{B}{2}\left\{e^{j\omega_b t} + e^{-j\omega_b t}\right\}\] (2.6)

Where \(A\) and \(B\) represent the amplitude of each input tone with angular frequencies \(\omega_a\) and \(\omega_b\) respectively. Expanding out the brackets and removing terms that do not survive

\(^1\)Throughout this thesis we use the phrase RF filtering to mean filtering performed in the analogue domain.
the RF filtering as before, the resultant passband signal takes the following form:

\[
y(t) = \left\{ \frac{h_1 A}{2} + \frac{3h_3 A^3}{8} + \frac{3h_3 A B^2}{4} \right\} \{e^{j\omega_a t} + e^{-j\omega_a t}\} \\
+ \left\{ \frac{h_1 B}{2} + \frac{3h_3 B^3}{8} + \frac{3h_3 A^2 B}{4} \right\} \{e^{j\omega_b t} + e^{-j\omega_b t}\} \\
+ \frac{3h_3 A^2 B}{8} \{e^{j(2\omega_a - \omega_b)t} + e^{-j(2\omega_a - \omega_b)t}\} \\
+ \frac{3h_3 A B^2}{8} \{e^{j(2\omega_b - \omega_a)t} + e^{-j(2\omega_b - \omega_a)t}\}
\] (2.7)

By definition, the input TOI point corresponds to the input power of two equal strength tones such that the power contained in the intermodulation distortion (IMD) products equals that of the linear term at the output [25, 32]. This theoretical intercept point is depicted in Figure 2.1. Mathematically, the TOI is defined by setting the strength of the two input tones in (2.7) equal, \(A = B\), and equating the linear contribution \(h_1 A^2\) to that from each IMD product, \(\{2\omega_a - \omega_b\}\) and \(\{2\omega_b - \omega_a\}\). Thus, the cubic Taylor coefficient can be related to the input TOI and the linear gain \(h_1\) as follows:

\[
\frac{h_1 A}{2} = \frac{3h_3 A^3}{8} \\
\text{TOI} = \sqrt{\frac{4h_1}{3h_3}}
\] (2.8)

where the input TOI is defined as \(A\). Combining the passband receiver model (2.5) with the measured linear gain and TOI values for the RF receiver completes the typical non-linear characterisation analysis performed on the receive side of the radar. There is little reported in the radar literature on the characterisation of AM/PM conversion effects despite the fact that radar’s make target detections based on both the returned signal’s amplitude and phase. Characterising the nonlinear phase behaviour of a RF amplifier is challenging. Simplified AM/PM models that claim to capture the essence of the effect are often published in the related literature [33, 30, 34] and are frequently discussed in the radar field [35, 36]. These AM/PM conversion models aim to characterise the amplifier’s nonlinear phase behaviour by means of another passband model applied most often through a Hilbert transform to distort the signal’s phase [33, 30, 34].

The vast majority of nonlinear characterisation analysis performed in the radar literature is interested in the generation of spurious harmonics and is therefore purely focused on the amplitude and phase characteristics of the nonlinear receiver for spot input frequencies. However, as radar systems move to increasingly wider receiver bandwidths it is imperative that their nonlinear amplitude and phase behaviour is understood for complex received signals. While these simple AM/AM and AM/PM conversion models can provide some insight into the device’s nonlinear amplitude and phase behaviour for single input tones, they fundamentally fail when the concept of bandwidth is introduced [37, 34]. In reality, linear RF amplifiers do not display constant gain and zero phase
2.1. Background Literature Review

Figure 2.1: Definition of third order intercept point for nonlinear devices. Found by an equal power two-tone excitation experiment where the output power contained in the fundamental and IMD sideband frequencies are compared [32].

shift characteristics across the entire channel bandwidth but rather exhibit a frequency-dependent gain and phase shift behaviour. In purely linear systems the concept of bandwidth is well understood as the principle of superposition holds. However, this concept fails to translate to nonlinear systems where the principle of superposition breaks down. To understand the complex nonlinear behaviour of the device across the entire channel bandwidth we are forced to revert to the more fundamental concept of system memory [34, 15, 38]. An amplifier’s memory is intrinsically linked to the time constant of the device and relates the behaviour of the current output to previous inputs through the concept of a time delay. Sophisticated black-box models that contain memory effects are required to accurately model the true nonlinear behaviour of the amplifier. These have been studied sparingly in the radar literature but have been the focus of much research in the related field of communications.

Radar Detection Theory

A coherent pulsed-Doppler radar performs target detections in an MPRF radar mode by forming range-Doppler plots (RD) from bursts of received pulses. In standard radar processing the received signals from each pulse repetition interval (PRI) are stored down the columns of the received data matrix in what is referred to as the fast-time dimension. The term “fast-time” corresponds to the different time slots making-up the PRI with the precise time intervals set by the baseband sampling frequency. In the case where the radar transmits a single-tone waveform, the fast-time dimension directly provides the range information from the scene with the fast-time rows often referred to as range gates or range bins in the radar literature. If a more complex transmit waveform is employed, such as a
Chapter 2. Literature Review

Received data matrix configuration. Received PRI signals are stacked down the successive columns of the received data matrix before an STFT is applied along the rows to produce the RD plot.

Range-Doppler map displaying main-beam clutter centred on Doppler frequency $-3kHz$ and a single target located away from the clutter in the noise limited region at Doppler frequency $-8kHz$.

Figure 2.2: Illustration of range-Doppler radar processing.

chirp, a matched filter can be employed to convert the fast-time signals into range. The velocity information is stored in the returned signals’ phase and can be revealed for each range gate by jointly processing successive pulses across the coherent processing interval (CPI). In effect, the Doppler phase information from the scene is sampled by the coherent pulses returned within the CPI. By stacking each PRI signal down successive columns of the received data matrix such that their fast-time axes align, a fast-time Fourier transform (FFT) can be applied across the pulses to form the two-dimensional RD plot. The FFT applied across the pulses is often described as operating in the slow-time dimension as the CPI sampling interval is governed by the PRI rather than the considerably higher baseband sampling rate that defines the fast-time dimension. In turn, this particular FFT is referred to as the slow-time Fourier transform (STFT) in the radar literature. This simple yet effective radar processing [11] reveals the range and Doppler information from the scene ultimately allowing detection algorithms to find targets. It is important to note that throughout this thesis we carefully picked the properties of each simulated target in the scene so that no range or Doppler ambiguities were generated in the pulse-Doppler processing. This was done for simplicity, however if the targets did display range or Doppler ambiguities then this would not actually affect the performance of the modelling or mitigation techniques presented in this thesis. The range-Doppler processing described above is illustrated in Figure 2.2.

Once the RD plots have been generated, detection algorithms are employed to distinguish targets from the interference and noise in the scene. In an MPRF mode of operation the target detections are typically noise limited as the clutter is contained to a small Doppler section of the RD space. Therefore, in most MPRF radar detection algorithms the clutter is removed by a Doppler notch leaving only the noise limited regions of the RD space. There are many different types of detection algorithms in radar with the majority of them exploiting thresholding techniques to distinguish targets from the
2.1. Background Literature Review

Throughout this thesis we employ an adaptive cell-averaging constant false alarm rate (CA-CFAR) thresholding technique [11, 39, 40, 41] to make the target detections in the RD space. The algorithm registers a target detection when a cell from the RD space crosses the adaptive threshold. The “CFAR” part of the algorithm refers to the fact that the adaptive threshold is set to maintain a constant false alarm rate (FAR) across the entire RD detection space effectively normalising the threshold to account for any local interference not removed by the main-beam clutter notch. The “adaptive” and “CA” part of the algorithm account for how this threshold level is set across the map. The cell-averaging concept is illustrated in Figure 2.3a and relies on generating a local area average (LAA) for each cell in the RD map. The LAA average is calculated from a set of reference cells surrounding the cell under test (CUT) known as the training band. To protect against the LAA being biased by processing leakage from the CUT, a guard band is typically introduced as illustrated in Figure 2.3a. The particular shape of the training band can vary however each training cell will contain noise samples and some may also include interference as well. The “adaptive” part of the algorithm refers to how this inhomogeneous background is accounted for when the threshold level for the CUT is set. By introducing a second parameter as well as the LAA when setting the threshold level, the CFAR detector can accommodate backgrounds with statistical distributions beyond the Rayleigh distribution which is used to derive the classic CA-CFAR in noise. The Weibull distribution is characterised by two independent parameters and can therefore capture the statistical behaviour of more backgrounds than the Rayleigh distribution. The probability density function (PDF) and cumulative distribution function (CDF) function for the Weibull distribution $W$ are given by (2.9) and (2.10) respectively,

$$W_{PDF}(\chi) = \left\{ \frac{c}{b} \right\} \left\{ \frac{\chi}{b} \right\}^{(c-1)} e^{-\frac{\chi}{b}}; \quad \chi \geq 0$$  \hspace{1cm} (2.9)

$$W_{CDF}(\chi) = 1 - e^{-\frac{\chi}{b}}; \quad \chi \geq 0$$  \hspace{1cm} (2.10)

Where $b$ is the scale parameter, $c$ is the shape parameter and $\chi$ is the independent random variable. For the Weibull distribution the scale parameter roughly corresponds to the mean of the distribution with the shape parameter describing the gross shape of the distribution. Both the shape and the scale parameter affect the variance of the Weibull PDF. The single cell probability of false alarm (PFA) for a given threshold $T$ is calculated as,

$$P_{FA} = \int_{T}^{\infty} W_{PDF}(\chi) d\chi$$
$$= 1 - \int_{0}^{T} W_{PDF}(\chi) d\chi$$
$$= 1 - W_{CDF}(T)$$
$$= e^{-\left(\frac{T}{b}\right)^c}$$  \hspace{1cm} (2.11)
**Chapter 2. Literature Review**

(A) CA-CFAR detection algorithm configuration. Training band cells (TB) are used to calculate the local area average with the specified guard band providing protection against processing leakage from the cell-under test.

(B) Histogram plots of the training band cells which allow the adaptive CA-CFAR algorithm to account for backgrounds beyond the strictly noise limited case. Line plots show the Weibull functions for the stated shape factor fitted to the respective histogram data.

**Figure 2.3:** Illustration of the adaptive CA-CFAR detection algorithm employed throughout this thesis.

Hence, to maintain a specific $P_{FA}$ for a given Weibull distribution the detection threshold $T_{DS}$ is calculated as,

$$T_{DS} = b \left\{ - \ln \left\{ P_{FA} \right\}^{\frac{1}{c}} \right\}$$

(2.12)

where the scale parameter $b$ and the shape parameter $c$ are estimated from the statistics of RD data surrounding the CUT. In this thesis we assume that the scale parameter $b$ equals the LAA value for the CUT and the shape parameter is estimated by fitting a Weibull distribution to a histogram of the training band cells. The process of estimating the shape and scale parameters for the Weibull distribution is illustrated in Figure 2.3.

**Cross-modulation Radar Problem**

Cross-modulation is a term used in the field of radar to describe the distortion of the returned radar signal due to the presence of a strong out-of-band interferer in the RF receiver. This effect can occur when the desired signal and the strong interferer pass through the active components in the RF front-end simultaneously. The cross-modulation effect is captured by the PBTS model and can be revealed by substituting the radar input signal (2.13) into (2.2) before collecting only those terms that sit on the radar carrier frequency $\omega_c$.

$$x(t) = \Re \left\{ \tilde{x}_1(t)e^{j\omega_c t} + \tilde{x}_2(t)e^{j\omega_i t} \right\}$$

$$= \frac{1}{2} \left\{ \tilde{x}_1(t)e^{j\omega_c t} + \tilde{x}_1^*(t)e^{-j\omega_c t} + \tilde{x}_2(t)e^{j\omega_i t} + \tilde{x}_2^*(t)e^{-j\omega_i t} \right\}$$

(2.13)

If the interference frequency $\omega_i$ does not sit on half the carrier frequency, then the second order effects are removed by RF filtering and the final expression for the nonlinear output
around the carrier, \( R\{\tilde{y}(t)e^{j\omega_c t}\} \), takes the following form:

\[
R\{\tilde{y}(t)e^{j\omega_c t}\} = \left\{ \frac{h_1 \tilde{x}_1(t)}{2} + \frac{3h_3|\tilde{x}_1(t)|^2 \tilde{x}_1(t)}{4} + \frac{3h_3 \tilde{x}_1(t)|\tilde{x}_2(t)|^2}{4} \right\} e^{j\omega_c t} + \left\{ \frac{h_1 \tilde{x}_1^*(t)}{2} + \frac{3h_3|\tilde{x}_1(t)|^2 \tilde{x}_1^*(t)}{8} + \frac{3h_3 \tilde{x}_1(t)|\tilde{x}_2(t)|^2}{8} \right\} e^{-j\omega_c t}
\]

where \( \tilde{x}_1(t) \) is the complex baseband input signal centred on the carrier frequency \( \omega_c \) and \( \tilde{x}_2(t) \) denotes the complex baseband input signal centred on the interference frequency \( \omega_i \). In expression (2.14), the term \( \frac{3h_3 |\tilde{x}_1(t)|^2 |\tilde{x}_2(t)|^2}{4} \) describes the cross-modulation distortion observed in the nonlinear output. Importantly, the cross-modulation term appears in the expression for the nonlinear output regardless of the interference frequency \( \omega_i \). Mathematically this is due to the fact that the interference is effectively mixed down to 0Hz (DC) before being modulated back onto the carrier frequency by the final cubic multiplication in the expansion of the PBTS model [25].

Cross-modulation distortion can be generated in all radar modes of operation as it is driven by a front-end nonlinear effect, however it is most easily observed in the medium pulse repetition frequency (MPRF) environment. In a typical MPRF mode the radar is tasked with detecting low signal to noise ratio (SNR) targets in the presence of a strong clutter signal. If the out-of-band signal is strong enough and the receiver components are not linear enough then the interferer can couple into the desired channel, see Figure 1.2b. The specific cross-modulation effect most often described in the radar literature relates to the scenario where the interference signal is pulsed [8, 7, 42]. In this specific case, the nonlinear coupling will corrupt the desired signal by modulating the strong clutter return potentially masking small targets and ultimately reducing the sensor’s overall performance. This effect is explained well by Mellor et al. in [8] and Fischer et al. in [7]. In essence, the mismatch between the interferer’s pulse repetition frequency (PRF) and the radar’s PRF cause nonlinear repeats of the clutter to be spread across the Doppler dimension in a harmonic like fashion. If the PRF of the interferer and the radar are both known then the Doppler domain locations of these clutter repeats can be predicted [7]. The cross-modulation effect for a typical MPRF radar mode is illustrated in Figure 2.4.

**In-band Interference Radar Problem**

In contrast to the cross-modulation phenomenon, the in-band interference scenario describes the case where the RF front-end is driven into its weakly nonlinear regime by an interference signal that sits in the desired radar channel. This effect is of particular
interest to mass-produced radar systems which are typically employed by highly consumerised platforms e.g. driverless cars and UAVs. With so many of these identical low-cost radars operating in close proximity to each other, both spatially and spectrally, it is inevitable that mutual interference will occur [43, 4, 44]. Similar to the cross-modulation scenario, the in-band interference effect is captured by the PBTS receiver model described by (2.1). The specific term in the PBTS model that drives the harmful distortion in the in-band scenario can be revealed by following similar analysis to that performed for the cross-modulation case. Instead of separating the interference signal from the desired baseband signal as before, we assume that it sits within the radar channel bandwidth and is therefore captured by the baseband modulation signal centred on the carrier frequency. In this specific in-band scenario, we describe the carrier baseband channel by \( \tilde{x}_3(t) \) to distinguish it from the desired baseband radar signal \( \tilde{x}_1(t) \) which occupies the same region of the spectrum. Therefore, the input to the PBTS model is described by (2.15),

\[
x(t) = \Re\{\tilde{x}_3(t)e^{j\omega_c t}\}
\]

\[
= \frac{1}{2}\left\{\tilde{x}_3(t)e^{j\omega_c t} + \tilde{x}_3^*(t)e^{-j\omega_c t}\right\} \quad (2.15)
\]

where

\[
\tilde{x}_3(t) = \{\tilde{x}_1(t) + \tilde{x}_2(t)\} \quad (2.16)
\]
2.1. Background Literature Review

Passing (2.15) through the PBTS model (2.1) and removing the out-of-band harmonics, which are suppressed by the RF filtering in receiver, yields the final expression for the nonlinear radar output around the carrier $\Re\{\tilde{y}(t)e^{j\omega_c t}\}$.

$$
\Re\{\tilde{y}(t)e^{j\omega_c t}\} = \left\{ \begin{array}{l}
\left( \frac{h_1 \tilde{x}_3(t)}{2} + \frac{3h_3|\tilde{x}_3(t)|^2 \tilde{x}_3(t)}{8} \right) e^{j\omega_c t} + \left( \frac{h_1 \tilde{x}_3^*(t)}{2} + \frac{3h_3|\tilde{x}_3(t)|^2 \tilde{x}_3^*(t)}{8} \right) e^{-j\omega_c t} \\
= \Re\left\{ \left( \frac{h_1 \tilde{x}_3(t)}{2} + \frac{3h_3|\tilde{x}_3(t)|^2 \tilde{x}_3(t)}{8} \right) e^{j\omega_c t} \right\}
\end{array} \right. 
$$

(2.17)

It is clear from (2.17) that the in-channel nonlinear term $\frac{3h_3|\tilde{x}_3(t)|^2 \tilde{x}_3(t)}{8}$ drives the nonlinear distortion in the in-band interference scenario. We assume that when the interference signal $\tilde{x}_2(t)$ is not present in the radar scene, and therefore $\tilde{x}_3(t) = \tilde{x}_1(t)$, the contribution from the in-channel nonlinear term is negligible. In other words, the radar is considered to operate linearly until a strong interference signal, $\tilde{x}_2(t)$, is present in the scene and drives the contributions from the in-channel nonlinear term above the level of the noise floor.

While mutual interference is a much talked about topic in the field of radar, there have been relatively few papers on the in-band nonlinear scenario published in the available literature [29, 45, 12, 15, 9, 42, 10]. The papers that have been published typically focus on the suppression of IMD spurs generated in the RF receiver from a hardware perspective. However, in this thesis we are interested in the direct corruption of the desired radar signal caused by an in-band interferer driving the active components in the RF front-end into their nonlinear regime. We focus on the scenario where a strong continuous wave (CW) interferer occupies the same region of the spectrum as the returned radar signal. The CW interferer can therefore couple with the desired signal through the nonlinearity resulting in distortion effects appearing in the RD detection space which can mask potential targets. The most common in-band nonlinear distortion effect observed in the RD domain is clutter spectrum broadening. This effect is illustrated in Figure 2.5 for a typical MPRF radar mode. In the case of the linear receiver, Figure 2.5a, the CW interferer does not interact with the clutter spectrum and therefore the weak target remains undistorted. However, when the RF receiver characteristics become weakly nonlinear, Figure 2.5b, the clutter spectrum can couple with the CW interferer causing the clutter to spread across the Doppler spectrum ultimately masking weak targets.

Nonlinear Receiver Mitigation Techniques

In the scenario where interference has been captured by the radar antenna alongside the desired received signal, several mitigation strategies for front-end receiver nonlinearities have been proposed in the radar literature. As discussed previously, the traditional method of backing-off the receiver gain [10] so that linear receiver performance can be maintained and thus linear signal separation techniques can be applied, is not an option for modern radar systems. While hardware solutions are often proposed they frequently
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Figure 2.5: Illustration of in-band nonlinear scenario caused by a CW interferer in an MPRF radar mode. Doppler cuts through the range-Doppler detection space are displayed for the target range gate. Note, plots generated using the model and data from chapter 5.

(A) Linear receiver scenario. CW interferer, $-2kHz$, and clutter spectrum, $2kHz \rightarrow 10kHz$, do not interact resulting in the weak target, $-11kHz$, being undistorted.

(B) Weakly nonlinear scenario. CW interferer, $-2kHz$, causes the clutter spectrum, $2kHz \rightarrow 10kHz$, to spread across the Doppler dimension masking the weak target, $-11kHz$.

Contradict the very low-cost nature of the modern sensor itself. As a comparison, communications system designers are minimising hardware costs by deliberately designing their receivers to operate entirely in the nonlinear regime even in the absence of interference [9]. While this is not yet the case for modern radar receivers, it highlights the cost-driven design mindset required for a fully consumerised modern sensor. Of these hardware mitigation strategies, adaptive front-end pre-filtering is the most attractive to radar systems [46, 47]. The idea being that a tuneable notch filter could be combined with an active guard channel to effectively remove the problematic interference frequencies before they cause any harmful nonlinear effects in the radar receiver. While in theory this technique sounds promising, as it is relatively low-cost and does not drastically alter the design of the RF receiver, in principle it has several serious drawbacks. The most pertinent of these is the effect the filtering has on the receiver noise figure. In general, the first component in the receiver chain has the most significant effect on the total noise figure [11]. Therefore, placing a tuneable notch filter, which has poor noise figure characteristics, right at the front of the RF receive chain could have potentially devastating effects for the performance of the sensor even when interference is not present in the scene.

The popular architecture of next generation radars opens up the possibility of another type of nonlinear mitigation technique. In order for radar systems to be compatible with new platforms such as driverless cars and UAVs they must be compact and have no moving parts. In order to achieve these properties most companies design systems
2.1. Background Literature Review

with a fixed wide-beam transmit antenna and a phased array receive antenna. These phased array antennas allow the received beam to be steered electronically rather than mechanically. By electronically steering the antenna’s received beam the radar can place a directivity notch in the direction of the interfering signal while returns from the remaining field-of-view are still amplified. In doing so, the strong interference would see less gain after the beamforming network than the desired radar signal effectively nullifying the risk of any subsequent receiver amplifiers being driven into their nonlinear regimes. This is referred to as digital beamforming (DBF) techniques in the radar literature and is often discussed as an effective way of removing strong interference from the radar scene [29, 48, 44, 10]. While this technique could potentially be successful in mitigating back-end receiver nonlinearities, it does not address the more prevalent issue of front-end nonlinear effects. At the front-end, each individual element in the phased array acts as an independent receiver with a virtually isotropic receive-side directivity pattern. With DBF suppression not being available until after the RF manifold, no nonlinear protection is offered to the active components in the radar front-end. In short, the damage may already have been caused before the DBF techniques can attempt to prevent it.

Advanced signal processing techniques offer an alternative low-cost low-impact solution to the nonlinear receiver problem in radar. These digital mitigation techniques are made possible by advancements in the field of computer science which has allowed radar systems to benefit from vast amounts of processing power at comparatively low costs. The basic principle relies on having a well characterised RF receiver both in the linear and nonlinear regime as well as the capability of modern phased array antennas to form guard channels. Importantly, the guard channel will be configured to capture the raw interfering signal so that the radar is provided with information regarding the source of the nonlinear distortion. Therefore, when interference is present in the RF receiver the deleterious nonlinear effects can be mitigated digitally in the radar processor by sophisticated nonlinear equalization (NLEQ) algorithms. These techniques are extremely popular in related field of communications where they are applied extensively to the transmitter-side of the sensor and more recently to the receiver-side as well [49, 14, 26]. They offer a cheap mitigation solution to nonlinearities in the RF front-end that does not alter the compact design of the radar receiver itself. Undoubtedly, digital mitigation techniques will be central to the alleviation of harmful nonlinear receiver effects in modern radar. An illustration depicting the mitigation techniques discussed above is presented in Figure 2.6.

The digital mitigation of nonlinear receiver effects is an emerging area of research in the radar literature, however there is still relatively few papers published. As discussed above, most of the techniques proposed exploit a guard channel that can provide some information regarding the interfering signal causing the nonlinear distortion in the RF receiver. In [8], Mellor et al. present a relatively simple NLEQ technique aimed at suppressing cross-modulation effects in a high PRF (HPRF) mode. The technique uses a
Chapter 2. Literature Review

(A) Front-end notch filter nonlinear mitigation technique. Receiving element (RE) captures both the desired radar signal and the interference signal. The interference is suppressed by the notch filter before the signal hits any active components in the receiver which could cause harmful nonlinear effects.

(B) DBF nonlinear mitigation technique. Receiving element (RE) captures both the interference and the desired radar signal with both signals passing through the RF front-end to the RF manifold. DBF techniques are then used to place a directivity null in the direction of the interferer protecting active components after the RF manifold from going nonlinear.

(C) Advanced signal processing nonlinear mitigation techniques. Receiving element (RE) captures both the interference and the desired radar signal causing nonlinear distortion effects further down the receive chain. An active guard channel captures information about the interferer which is fed into the digital mitigation algorithms used to alleviate the deleterious nonlinear effects observed.

**Figure 2.6:** Illustration of potential nonlinear mitigation techniques in radar. Nonlinear distortion caused by the presence of an interferer with frequency \( f_i \) in the radar receiver. This thesis focuses on the development of digital mitigation techniques, Figure 2.6c.
linear guard channel to locate the sections of the received PRI signals corrupted by the interference. The corrupted segments are subsequently copied and placed into the correct fast and slow-time locations of a secondary received data matrix. RD plots for both the primary and secondary received data matrices are then generated before the synthetic RD map is multiplied by a complex scalar and subtracted from the measured RD map. The value for the complex scale factor is optimised via an iterative learning procedure in an adaptive filtering stage. This simple mitigation algorithm is detailed by two figures in the paper with the results section proclaiming to achieve 30dB of cross-modulation suppression through the adaptive cancellation method. In essence, this technique employs a linear cancellation procedure in an attempt to mitigate a fundamentally nonlinear effect. This simple technique will therefore be limited by the fact that the principle of superposition does not hold in the nonlinear regime.

Rabideau was one of the earliest researchers in the field of radar to publish studies into the mitigation of nonlinear receiver effects by digital signal processing techniques. His main research interests were in exploiting advanced DBF techniques to aid the mitigation of array nonlinearities in the front-end of the modern radar receiver [50, 51, 52, 29]. In [50, 51, 52], Rabideau describes a technique whereby the received signals for each channel are passed through a unique transform that allows the signals to be combined in the digital beamforming network without the correlated nonlinear distortion products adding constructively. The particular choice of known transform applied to each received channel, before demodulation and sampling, is dependent on the nonlinear effect being mitigated. In [50] a scheme is described where each received channel is demodulated by a different local oscillator (LO) frequency creating a modulating effect across the channels. Alternatively, in [52] Rabideau presents a technique where the unique transforms are achieved by applying independent phase shifts to each received channel in the array. Therefore, in the digital domain each channel signal will have a dominant linear term which can be made to coherently add in the beamformer by inverting each channel using the unique modification applied before the sampling process. Importantly, this inversion is only correct for the linear component of the output signal leading to the nonlinear artifacts in the signal decorrelating in the digital beamformer. This technique is primarily aimed at decorrelating nonlinear spurs generated in the radar output by front-end receiver nonlinearities. Unfortunately, the effectiveness of this technique in mitigating nonlinear distortion effects where the interference has physically corrupted the linear signal will be limited.

Of the NLEQ techniques studied more recently in the radar literature, the LUT approach is by far the most popular [53]. The simple idea behind this popular mitigation solution is that the nonlinear output can be corrected by digitally passing the raw time-domain signals through an inverse model of the forward nonlinearity. This is illustrated in Figure 2.7 where \( h_p \) describes the behaviour of the RF nonlinearity and \( g_k \) represents
Chapter 2. Literature Review

the impulse response of the inverse transfer function. Importantly, Rabideau does incorporate a LUT inverse transform into his DBF nonlinear decorrelation technique in [29]. However as with previous iterations of his technique, the “decorrelating linearization” method described in [29] is targeted at repairing the damage caused to the radar receiver’s dynamic range by nonlinear spurs generated in the RF front-end. Importantly, more generalised LUT nonlinear mitigation techniques are presented in the related field of communications where they are widely employed. These techniques are typically referred to as digital pre/post distortion (DPD) techniques in the communications literature and they will be discussed in detail in the communications section of this literature review. Additionally, the theoretical foundations underpinning the technique will be presented alongside the detailed discussions in the following section.

In this thesis, we develop the LUT mitigation solution specifically for the cross-modulation problem in radar and perform in-depth performance analysis on the technique for a typical MPRF mode of operation. Importantly, this is the first time that these LUT techniques have been applied to compensate for harmful interference-driven nonlinear effects in the radar receiver. Furthermore, the fact that the interferer and desired radar signal are both unknown to the sensor makes this problem fundamentally different from those studied previously in the communications literature, which is where the inspiration for this digital mitigation solution is founded. In addition to the fundamental scenario being completely different between the two systems, the processing applied in a pulse-Doppler radar is very different from what is typically implemented in a modern communications system making this study unique to radar systems. Importantly, in order for the digital mitigation solutions to be successful, they must be able remove the cross-modulation distortion from the radar detection space without also removing any weak targets that have been masked by the unwanted nonlinear effect. This is a non-trivial task as radars are extremely sensitive systems that detect targets by studying the phase and amplitude of the returned signals. In order to capture this highly sensitive behaviour in our analysis, we incorporated nonlinear memory effects into our study which allowed us to model the complex interactions between the different frequency components of the radar signal as it passes through the nonlinearity. Importantly, by considering these complex nonlinear memory effects in our cross-modulation analysis, we effectively make this research entirely radar specific as the simulated nonlinear effects are totally unique to radar systems. Crucially, this is unlike any research work that has been published previously in the radar literature.

2.1.2 Communications Literature Review

The current trend in the related field of communications is to reduce hardware costs by sacrificing front-end linearity in the RF receiver and preserving system performance by compensating for the deleterious effects in the digital domain. This is extremely relevant to the current trend in the field of radar where reducing the size and cost of the
RF receiver while maintaining system performance is a potential barrier for the development of a successful mass-producible system. While there are stark differences between radar and communications systems, much inspiration can be taken from the related communications literature. In this section, we outline the relevant literature in the field of communications and highlight the key differences that must be addressed if these communications-based techniques are to be successfully carried over to the radar domain.

**Passband Volterra Series**

The passband Volterra (PBVS) series is a powerful behaviour model used to characterise the nonlinear effects of passband amplifiers with memory. It is often described as a Taylor series with memory and has a unique linear in the parameters formalisation that makes it ideal for modelling weakly nonlinear RF systems. The Volterra series approach is a popular modelling technique in the communications literature due to its generalised nature and relatively intuitive formalisation of nonlinear memory. Mathematically, its continuous-time input-output relationship takes the form of a finite sum of multidimensional convolution integrals [34, 54, 55, 56], given by (2.18):

$$y(t) = \sum_{p=0}^{P} \left\{ \int_{-\infty}^{\infty} d\tau_1 \ldots \int_{-\infty}^{\infty} d\tau_p h_p(\tau_1, \ldots, \tau_p) \prod_{s=1}^{p} x(t - \tau_s) \right\}$$

(2.18)

where $x(t)$ and $y(t)$ are the real continuous-time input and output passband signals respectively. The convolution integrals result in the real input signal being delayed by successive time-lags $\tau$. These time-lags are intrinsically linked to the fundamental memory behaviour of the device through its bandwidth and time-constant characteristics [55, 15, 38]. The RF amplifier’s behaviour is therefore described by the full set of passband Volterra kernel functions $h_p(\tau_1, \ldots, \tau_p)$. This is illustrated by the simple diagram in Figure 2.8. As with the PBTS model (2.1), the first-order term $p = 1$ defines the linear response of the system with the subsequent higher-order kernels being linearly combined to form the full nonlinear black-box behavioural model. In order to identify the PBVS model for a real RF amplifier, the mathematical formalisation must first be translated.
from continuous-time to the discrete-time domain. Therefore, we must replace the integrations over continuous time-lags \( \tau \) in (2.18) with finite summations over Volterra taps the length of which are chosen to be \( L \) \[55, 34, 56, 57\]. The discrete-time formalisation of the PBVS model is displayed in (2.19) below,

\[
y[m] = \sum_{p=0}^{P} \left\{ \sum_{l_1=0}^{L-1} \cdots \sum_{l_p=0}^{L-1} \right\} h_p[l_1, \ldots, l_p] \prod_{s=1}^{p} x[m - l_s]
\] (2.19)

Importantly, in converting the PBVS model from the continuous-time domain to its discrete-time form, displayed in (2.19), we must apply the Nyquist-Shannon sampling theorem \[57, 58\]. While we do not provide a full treatment of the sampling theorem for a nonlinear system here, we go on to discuss this in great detail later in this thesis, see chapter 3 - specifically sections 3.2.2 & 3.3.3. Crucially, the inherent bandwidth expansion property of nonlinearities, see section 3.3.3, means that we have to be very careful when considering the input and output sampling rates for any nonlinear system. We use the phrase Volterra tap to describe a single term from the multidimensional sum over \([l_1, \ldots, l_p]\). In essence, the memory length \( L \) defines the amount of memory in the system with \([l_1, \ldots, l_p]\) defining each subsequent component’s passband delay lag. As with the continuous-time case, the specific behaviour of the nonlinear model is once again defined by a set of nonlinear functions \( h_p[l_1, \ldots, l_p] \) referred to in the discrete-time domain as the Volterra kernel coefficients. For clarity the mathematical structure of the third order Volterra term, \( p = 3 \), from the discrete-time PBVS model (2.19) is illustrated diagrammatically in Figure 2.9. Importantly, in the memoryless case where \( L = 1 \) the discrete-time PBVS model reproduces the discrete-time PBTS model exactly\(^2\). For completeness the discrete-time PBTS model is displayed in (2.20)\(^3\).

\[
y[m] = \sum_{p=0}^{P} h_p x[m]^p
\] (2.20)

The PBVS model is an exceptionally powerful black-box model capable of capturing

---

\(^2\)The names memoryless and Taylor are used interchangeably to describe the Taylor series behavioural models discussed.

\(^3\)The coefficients for the Taylor series behavioural models are distinguished from the full Volterra series behavioural models’ coefficients by omitting the continuous-time or discrete-time indices, \((\tau_1, \ldots, \tau_p)\) or \([l_1, \ldots, l_p]\), respectively.
complex nonlinear memory effects as well as the harmonic and *in-channel* nonlinear behaviour described by the simple PBTS model. However, for real RF amplifiers the full PBVS model is too large to identify as its complexity grows exponentially with nonlinear order \( P \) and memory length \( L \). The size of the PBVS model can be reduced significantly by truncating the nonlinear order \( P \) and limiting the amount of memory in the system \( L \). Additionally, symmetries in the nonlinear kernels can be exploited to reduce the number of Volterra coefficients further [56]. However, even with this drastic pruning the computational load of the model is still prohibitive for most communications-based applications. This is even more true for radar systems where the carrier frequency and therefore required passband sampling frequency are considerably higher than that for traditional communications systems. The vast majority of communications systems therefore drop the PBVS model down to the baseband domain where the significantly lower sampling rates and smaller model size make the Volterra based behavioural models far more straightforward to identify and simulate. Baseband communications based behavioural models are discussed in detail in the next section.

**Baseband Volterra Series**

Passband nonlinear models are the most general class of RF black-box behavioural models as they are full bandwidth and operate up at the carrier frequency of the device.

**Figure 2.9:** Illustration of \( p = 3 \) discrete-time PBVS term. The PB sample delays are denoted by \( z \) with the red lines indicating multiplication operations. The third order output \( y_{p3}[m] \) is calculated by summing all of the Volterra tap terms each of which is generated by following a new trace from top to bottom.
These generalised models are therefore capable of capturing both in-channel nonlinear distortion effects as well as the generation of spurious harmonics across the spectrum, see Figure 1.2. However, this modelling capability comes at a computational cost in the form of the Nyquist-Shannon sampling theorem which requires the sampling rate of the simulated model to be at least twice that of the highest harmonic product studied [57, 33]. For most communications systems these sampling rate requirements are far too high even if only the in-channel nonlinear effects are to be modelled. This is especially true for radar systems which have carrier frequencies that are considerably higher than those used for traditional communications systems. To reduce the computational load of these nonlinear receiver models they can be translated from the passband domain to the baseband domain by exploiting complex envelope notation and invoking the narrowband assumption [33, 34, 59]. In performing this transform the modelling capability of the baseband behavioural models are restricted to a small bandwidth around the carrier frequency specifically relating the input and output complex envelopes directly in the complex baseband domain. Therefore, in terms of the nonlinear distortion effects displayed in Figure 1.2 these baseband equivalent nonlinear behavioural models can capture the in-channel nonlinear distortion effects but not the generation of the spurious harmonics\(^4\).

The baseband equivalent Taylor series (BBTS) model can be generated for the in-band interference scenario by translating the continuous-time expression (2.17) to the discrete-time domain and directly relating the resultant baseband signals [59, 60, 34]. The BBTS model for the in-band interference scenario is stated in (2.21) below,

\[
\tilde{y}[n] = \tilde{h}_1 \tilde{x}_3[n] + \tilde{h}_3 |\tilde{x}_3[n]|^2 \tilde{x}_3[n]
\]

where the baseband linear and third-order in-band coefficients are given by \(\tilde{h}_1\) and \(\tilde{h}_{3,m}\) respectively (2.22),

\[
\tilde{h}_1 = \frac{h_1}{2}; \quad \tilde{h}_{3,m} = \frac{3h_3}{8}
\]

The derivation of the baseband equivalent Volterra series (BBVS) nonlinear model was first published by Benedetto \textit{et al.} in [59]. This paper had important implications for the field of communications where Volterra based behavioural models were previously too computationally expensive to employ. By restricting the PBVS’s modelling capabilities to a narrow bandwidth around the carrier frequency Benedetto succeeded in formulating a Volterra based behavioural model that is an identifiable size for real world communications and radar systems. Benedetto’s derivation of the BBVS model follows a similar structure to that for the BBTS model and is discussed in detail in chapter 3. The BBVS nonlinear model for the in-band interference scenario is stated below in (2.23) [59, 43].

\textit{In the communications literature the baseband equivalent nonlinear models are often referred to as low-pass equivalent nonlinear models.}
where $i$ denotes the baseband Volterra tap delay lag and $\tilde{h}_p[i_1, \ldots, i_{2p-1}]$ represents the baseband Volterra kernels. Importantly, unlike the memoryless BBTS model which has an identical formalisation to the in-channel terms of the PBTS model, the BBVS model is dependent on the relationship between the passband and baseband sampling rates through the delay tap variables $l$ and $i$. This has significant implications for the baseband Volterra kernels $\tilde{h}_p[i_1, \ldots, i_{2p-1}]$. Crucially, the memory of the nonlinear system is intrinsically linked to continuous-time lag $\tau$, and therefore the passband and baseband Volterra tap delays can be thought of as sampling the nonlinear impulse response of the device. This phenomenon is discussed in detail in chapter 3 and is illustrated in Figure 2.10. The BBVS model therefore captures less memory effects than the full PBVS model but still provides a powerful mechanism for incorporating weak nonlinear memory effects into the modelling of the RF receiver. In very literal terms the memory of a system is defined as the dependance the current output has on the previous inputs. Importantly however, throughout this thesis we use the concept of memory to fundamentally describe how the different frequency components of the input signal interact with each other as they pass through the nonlinear system. From a modelling perspective, it is this intrinsic link between bandwidth and memory that underpins the nonlinear response of the device for a complex input signal, and is therefore the essence of what these sophisticated Volterra based models are trying to characterise with their complex formalisation.

### Dual Channel Baseband Volterra Series

The development of tuneable dual-channel systems has been the focus of much research in the communications literature as they offer improved efficiency and reduced costs while still delivering flexible and dynamic networks. However, if multiple channels are transmitted simultaneously through a single high-power amplifier then significant cross-channel nonlinear effects will be observed in both channels at the transmitter output. In order to mitigate this unwanted effect, communications systems typically employ digital pre-distortion (DPD) techniques that modify the input signals to the power-amplifier such that the transmitted output channels are as previously designed. In order for this DPD technique to be successful, sophisticated nonlinear models that can capture cross-channel effects had to be developed. This led to a surge of research in the communications literature into the development of baseband behavioural models that could accurately model cross-channel effects [62, 49, 63, 20].
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The cross-channel BBTS model is derived by following a similar method to that employed to derived the BBTS in-band model in (2.21), except instead of feeding the PBTS model with a single channel complex envelope signal a dual-channel signal is used [62]. This simple analysis was done previously to reveal the cross-modulation effect in radar with the continuous-time passband signal described by (2.14). The cross-channel discrete-time BBTS model can therefore be found by equating the input and output complex envelope signals in (2.14) through the narrowband assumption before dropping the resultant expression into the discrete-time domain. The resultant cross-channel discrete-time BBTS expression is displayed in (2.24) below.

\[
\tilde{y}[n] = \tilde{h}_1 \tilde{x}_1[n] + \tilde{h}_{3,\text{in}} |\tilde{x}_1[n]|^2 \tilde{x}_1[n] + \tilde{h}_{3,\text{out}} \tilde{x}_1[n] |\tilde{x}_2[n]|^2
\]

(2.24)

where the baseband linear and third-order in-band coefficients are described by (2.22) and the baseband third-order out-of-band coefficient is given by \( \tilde{h}_{3,\text{out}} \):

\[
\tilde{h}_{3,\text{out}} = \frac{3h_3}{4}
\]

(2.25)

Similar to the BBTS in-band model in (2.21), the cross-channel case captures the nonlinear distortion caused to a narrow bandwidth around the carrier frequency but fails to model the generation of spurious harmonics across the spectrum, see Figure 1.2b. In effect, (2.24) describes the nonlinear coupling between a narrow bandwidth around the interferer and the desired radar channel. In the communications literature the dual-band transmitter is described using two BBTS models to capture the bi-directional coupling between the channels [62]. However for radar, the second BBTS model can be dropped as we are only interested in how the out-of-band interference corrupts the desired radar signal. More
recently there has been much work in the communications literature on extending the discrete-time BBTS model to include nonlinear memory effects [49, 63, 20]. The cross-channel BBVS model is hard to formulate in a compact manner however it is derived by Fehri et al. in [63, 20] by following a similar method to that employed for the cross-channel BBTS model. As with the in-band BBVS model, the memory effects captured by the discrete-time BBVS cross-channel model are not identical to those captured by the discrete-time PBVS model due to the differences between the baseband and passband sampling rates. This is discussed in detail in chapter 3 alongside the full derivation of the cross-channel BBVS model. Importantly, the development of the discrete-time cross-channel BBVS model allows weak nonlinear memory effects to be introduced to the cross-modulation scenario in radar.

Wiener and Hammerstein Models

The Volterra series is the most generalised form of nonlinear memory behavioural model, however its computational size makes it prohibitive for most communications-based applications. The power of the Volterra series is founded in its ability to capture both conventional and cross nonlinear memory terms with its complex formalisation. For many communications-based modelling applications the inclusion of all of these nonlinear cross-terms is overkill and therefore reduced forms of the Volterra series can be employed with equal effect. While these compact nonlinear memory models do not contain all of the Volterra series terms, it is common practice in the communications literature to still describe them using the full Volterra formalisation with the unused terms removed by setting the associated Volterra kernel coefficients to zero. It is important to remember that these reduced form models have traded-off modelling capability for computational efficiency and can therefore never recapture all of the effects contained in the full Volterra series model [64].

The generalised Wiener model is a special case of the Volterra series that is described by a linear filter followed by a memoryless/static nonlinearity [64, 54, 65, 33, 66], see Figure 2.11a. It is therefore one of the simplest nonlinear memory models to formulated and is stated for the passband case below in (2.26).

\[
y[m] = \sum_{p=1}^{P} h_p \left\{ \sum_{l=0}^{L-1} b[l] x[m-l] \right\}^p
\]  

(2.26)

where the linear finite impulse response (FIR) filter coefficients are described by \( b[l] \). Unlike the Volterra series, the Wiener model is nonlinear in the parameters due to the fact that its output \( y[m] \) depends nonlinearly on the linear filter coefficients \( b[l] \). This is a troublesome property as it makes the identification of these Wiener models more involved than for other linear in the parameters models. Additionally, it is less straightforward to
describe the Wiener model in terms of the Volterra series formalisation however it is still possible [65].

The Hammerstein model is the exact opposite of the Wiener model and is described by a memoryless/static nonlinearity followed by a linear filter [64, 65, 33, 66], see Figure 2.11b. The Hammerstein model has the desired property of being linear in the parameters, however unlike the Wiener model it does not capture any of the cross-term effects included in the Volterra model. It is therefore very limited in the nonlinear memory effects that it can model, although it does fit easily into the classic Volterra series formalisation. The Hammerstein model is described in full by (2.27) below,

\[ y[m] = \sum_{l=0}^{L-1} b[l] \sum_{p=1}^{P} h_p x^p[m - l] \]  

Typically, a higher class of Hammerstein model is employed in the communications literature referred to as the memory polynomial or parallel Hammerstein model [49, 64, 67]. Fundamentally, it follows the same structure as the traditional Hammerstein model except each nonlinear term sees a different linear filter before the signals are combined to form the final output, see Figure 2.11c. The memory polynomial maintains the linear in the parameters behaviour of the Hammerstein model and is described by (2.28).

\[ y[m] = \sum_{p=1}^{P} \sum_{l=0}^{L-1} h_p[l] \ x^p[m - l] \]  

The final model discussed is the Wiener-Hammerstein model which is generated by combining the Wiener, (2.26), and Hammerstein, (2.27), models to form (2.29) [65, 64, 66]. In (2.29), \( b_1 \) denotes the pre-filter coefficients while \( b_2 \) denotes the post-filter coefficients. The Wiener-Hammerstein model contains the most nonlinear memory effects of all the popular reduced form behavioural models and is described by a linear filter followed by a memoryless/static nonlinearity followed by another linear filter. The form of the Wiener-Hammerstein model is illustrated in 2.11d. As with the Wiener model, the Wiener-Hammerstein model is nonlinear in the parameters due to the linear filter stage preceding the memoryless/static nonlinearity.

\[ y[m] = \sum_{l_1=0}^{L-1} b_1[l_1] \sum_{l_2=0}^{L-1} \left\{ \sum_{p=1}^{P} b_2[l_2] \ x[m - l_1 - l_2] \right\}^p \]  

It is often helpful when working with the complex formalisation of the full Volterra model to consider the reduced form models discussed above and in particular, how their more intuitive parameters relate to Volterra kernel coefficients.
2.1. Background Literature Review

Nonlinear System Identification

The identification of a memoryless nonlinearity is a straightforward process and can be performed by a vector network analyser (VNA) through a combination of single-tone and two-tone tests as described in the “Radar Receiver Amplifier Models” section of this literature review. In contrast, the identification of a nonlinearity with memory is far more involved and has been the focus of much research in the communications literature [34, 33, 37, 68]. Fundamentally, the memory of a nonlinear system is intrinsically linked to its bandwidth [15, 34, 38]. Therefore, a single frequency test signal will never be able to induce complex memory effects in the nonlinear device. Additionally, it is important to remember that the principle of superposition does not hold in nonlinear systems and therefore, multiple spot frequency tests would provide no information regarding the device’s response to another input signal, single frequency or otherwise [34]. In order to stimulate memory effects in the device during the system identification process, frequency rich test signals must be used to characterise the nonlinearity. Note, we use the phrase frequency rich throughout this thesis to describe any signal that contains multiple frequency components. This frequency rich identification requirement is typically referred to as the persistent excitation condition. The persistent excitation condition states that the identification signal must contain at least as many frequency components as the nonlinear order $P$ [68, 69]. However, most identification procedures employ far richer tests signals to avoid ill-conditioning problems in the iterative learning procedure [33]. The most popular test signal for identifying nonlinearities with memory is white Gaussian noise as it is by nature frequency rich and can be used to characterise the entire

\[ x[m] \rightarrow \text{FIR} \rightarrow \text{PBTS} \rightarrow y[m] \]

\[ x[m] \rightarrow \text{PBTS} \rightarrow \text{FIR} \rightarrow y[m] \]

\[ x[m] \rightarrow h_1x[m] \rightarrow \text{FIR} \rightarrow y[m] \]

\[ x[m] \rightarrow h_2x[m]^2 \rightarrow \text{FIR} \rightarrow y[m] \]

\[ \vdots \]

\[ x[m] \rightarrow h_kx[m]^P \rightarrow \text{FIR} \rightarrow y[m] \]

\[ x[m] \rightarrow \text{FIR} \rightarrow \text{PBTS} \rightarrow \text{FIR} \rightarrow y[m] \]

\[ x[m] \rightarrow \text{FIR} \rightarrow \text{PBTS} \rightarrow \text{FIR} \rightarrow \text{FIR} \rightarrow y[m] \]

**Figure 2.11**: Illustration of reduced form nonlinear behavioural models with memory.
system bandwidth simultaneously [69, 33, 15, 70]. There are other methods to determine the memory coefficients of a nonlinear system such as using a nonlinear VNA to perform an X-parameter measurement [68], however these measurements are less popular in the communication literature as they are generally very expensive to conduct and require sophisticated measurement equipment. The identification of a nonlinearity with memory by means of a noise-based characterisation measurement is illustrated in Figure 2.12. The random amplitude and high bandwidth nature of the Gaussian noise guarantees that the entire spectrum and weakly nonlinear region are fully stimulated by the input test signal ensuring that the persistent excitation condition is satisfied. Typically, the nonlinear behavioural model coefficients would be identified from the measured input and output test signals through an iterative learning procedure [33, 15, 70].

**Memoryless DPD Techniques**

The digital mitigation of front-end nonlinearities has been the focus of much research in the communications literature [49, 63, 55, 9, 14, 15, 26]. These techniques were first developed to combat nonlinear distortion effects on the transmit side of the system, however they are now being employed on the receive side as well. When applied to the communications transmitter, they are typically referred to as digital pre/post-distortion (DPD) linearisation techniques and can be implemented either before or after the power amplifier nonlinearity depending on the system design [57, 55, 54]. In essence, the classic DPD technique aims to correct for the nonlinear distortion caused by the transmit-side power amplifier by digital inverting the forward nonlinear transfer function through a
tandem nonlinearity [54, 57, 55]. In the post-distortion setting, the tandem nonlinearity is applied after the RF receiver and can be thought of as inverting the forward nonlinearity to restore the desired linear performance. Whereas in the pre-distortion case, the tandem nonlinearity is applied before transmission and can be thought of as deliberately corrupting the input signal to the power amplifier such that the transmitted output is as original designed. In actual fact, these two operations are equivalent and are captured by the same theory of nonlinear inverse functions [54]. The classic DPD inverse for a memoryless passband forward nonlinearity, (2.20), is displayed in (2.30) where: $K$ defines the inverse nonlinear order, $\hat{x}[m]$ denotes the estimated linearised output and coefficients $g_k$ describes the inverse nonlinear transfer function. The inverse coefficients $g_k$ can be determined from the forward nonlinearity coefficients $h_k$ or alternatively they can be identified directly during the system characterisation process.

$$\hat{x}[m] = \sum_{k=0}^{K} g_k y[m]^k$$ (2.30)

Employing DPD techniques to correct for transmit-side power amplifier nonlinearities is common practice in modern communications systems. More recently however, there has been growing interest in applying analogous methods to correct for receive-side nonlinearities caused by the introduction of cheaper less-linear components in the RF front-end [14, 71, 26, 15, 9, 72, 19]. Much like the nonlinear problem is radar, receive-side nonlinearities can be induced in communications systems either by out-of-band signals in the scene [14] or alternatively in-band interference signals in the desired communication channel [71]. Additionally, the nonlinear behaviour of the communications receiver can often be by design; either to improve efficiency through concurrently receiving adjacent channels or simply to reduce hardware costs by choosing to always operate in the weakly nonlinear regime [26, 19, 15, 9, 72]. Initial research in the communications literature into the digital mitigation of receiver nonlinearities focused on the effectiveness of memoryless inverse transforms. In [14], Zou et al. present a memoryless inverse mitigation technique aimed at suppressing cross-channel nonlinear effects caused by a strong out-of-band blocker signal. The digital technique is implemented in the baseband domain but unlike the classic transmit-side DPD techniques, it aims recover the linear received signal by algebraically inverting the cross-channel forward nonlinear transfer function (2.24). Zou’s receive-side digital mitigation technique is stated in (2.31) where in this case; $\hat{x}_1[n]$ represents the estimated linear input signal to the RF receiver and $\tilde{w}[n]$ represents a white gaussian noise signal.

$$\hat{x}_1[n] = \frac{\tilde{y}[n]}{\left\{ h_1 + h_{3,\text{out}} |\tilde{x}_2[n] + \tilde{w}[n]|^2 \right\}}$$ (2.31)
Zou assumes in his derivation of (2.31) that the in-channel nonlinear effects in (2.24) are negligible and also that the strong out-of-band blocker signal $\tilde{x}_2[n]$ can be captured linearly by a secondary receive channel.

The in-channel nonlinear scenario is studied in detail by Ma et al. in [71, 26] with inspiration for their digital mitigation technique taken from the DPD strategies employed on the transmit side of the communications system. Similar to Zou’s cross-channel method, Ma et al. choose to adopt a memoryless formalisation with their in-channel mitigation strategy implemented in the baseband domain in [71] and in the passband domain in [26]. The technique aims to correct for the in-channel nonlinear distortion effects outlined in (2.21) by digitally applying a tandem inverse nonlinear function after the RF receiver. The basic baseband technique is outlined in (2.32) where $\hat{\tilde{x}}_3[n]$ represents the estimate of the linear receiver input signal including any unwanted in-channel interference.

$$\hat{\tilde{x}}_3[n] = \sum_{k=0}^{K} \tilde{g}_k \prod_{s=1}^{k} \tilde{y}^*[n] \prod_{d=k+1}^{2k+1} \tilde{y}[n]$$ (2.32)

In (2.32), $K$ represents the tandem inverse order and $\tilde{g}_k$ denotes the baseband nonlinear inverse model coefficients. Similar to the transmit-side DPD techniques, the inverse nonlinear coefficients in (2.32) must either be deduced from the respective forward model coefficients $\tilde{h}_p$ or they must be identified directly through a characterisation process. The identification of these receive-side inverse model coefficients is more challenging than for the transmit side due to the fact that the exact input signal to the front-end receiver nonlinearity is unknown.

**Memory DPD Techniques**

The power amplifiers employed on the transmit side of the communications system often exhibit nonlinear memory effects as they typically operate within the saturation region of their power operating curves to maximise efficiency [73, 33, 66, 67, 55, 37, 30]. This has led to the development of memory based DPD techniques that can correct for both memory and memoryless nonlinear effects induced by the highly nonlinear power amplifier [73, 55, 33, 49, 63, 74]. While the front-end amplifiers on the receive side of the communications system do not operate nearly as deep into the nonlinear regime as those on the transmit side, they can still exhibit weak nonlinear memory effects. Furthermore, these nonlinear memory effects will become more prevalent as modern communications receivers move to wider and wider bandwidths. This has led to increasing levels of research into the development of receive-side digital mitigation solutions that can correct for nonlinear memory effects in the RF front-end [15, 9, 19, 72]. While this is a relatively new area of research for the receive side of the communications system, as with the memoryless case, inspiration can be taken from the parallel transmit side where the techniques are far more established. Importantly however, caution must be taken when
carrying these transmit-side techniques to the receiver domain as the input signals and nonlinear memory effects observed are likely to be considerably different.

### 2.1.3 Signal Processing Literature Review

While inspiration for solutions to the nonlinear receiver problem in radar can be taken from the related field of communications, it is important that the effectiveness of alternative digital mitigation techniques are also explored. The main advantage of designing a novel digital mitigation solution from the ground up is that the technique can be truly system specific and therefore exploit the unique properties of radar. The theory underpinning the novel digital mitigation solution designed in chapter 5 of this thesis is founded in the signal processing literature. In particularly, the solution builds on the fundamental research conducted by Blumensath on extending the compressed sensing technique to include nonlinear observations [27]. In this section, we outline the relevant literature in the field of signal processing and briefly introduce some background concepts that are important for understanding the research work presented in chapter 5 of this thesis.

#### Linear Compressive Sensing

Compressive sensing (CS) is a signal processing technique that enables the recovery of a signal from far fewer samples than that predicted by the traditional Nyquist-Shannon sampling theory [75, 76, 77]. Importantly, the theory of CS does not violate the Nyquist-Shannon sampling theorem which states that: for a continuous-time signal to be perfectly recreated in the discrete-time domain it must be sampled at a rate at least twice that of the highest frequency component present in the signal [77]. In short, the CS problem can be thought of as trying to find a solution to an underdetermined linear system of equations. This signal recovery problem is fundamentally ill-posed as there are indefinitely many candidate signals that could fit the criteria. However, by incorporating some prior knowledge about the signal into the recovery process the problem is no longer ill-posed and can be solved [77]. The incorporation of this prior knowledge into the CS algorithm is typically referred to as the constraint step and can take many different forms depending on the known characteristic of the signal being exploited. Signal sparsity is the most popular property exploited in the CS recovery process as the majority of signals can be expressed in a convenient basis where the signal has a concise representation [75, 76, 77]. Crucially, if the correct signal is to be recovered by the CS process then the level of coherence between the chosen sparse basis and original sensing basis, where the samples are recorded, must be low [77]. This is referred to as the incoherence property in the signal processing literature and is satisfied for the popular time-frequency basis pair which is in fact maximal incoherent [77].
Compressive sensing ideas have been successfully applied to a wide variety of problems in radar signal processing such as inverse synthetic-aperture radar (SAR), interferometric SAR, moving target indication and direction of arrival estimation [10]. Let us start though, by considering a hypothetical linear compressive sensing problem in the context of radar. Initially we assume that the baseband input signal $\tilde{x}$ is adequately sampled and can be represented as a linear combination of $N$ frequency vectors $\{\phi[\kappa]\}_{\kappa=0}^{N-1}$ where each frequency vector is weighted by the corresponding complex Fourier coefficient $\tilde{X}[\kappa]$.

$$\tilde{x} = \frac{1}{\sqrt{N}} \sum_{\kappa=0}^{N-1} \phi[\kappa] \tilde{X}[\kappa] = \Phi \tilde{X} \quad (2.33)$$

The values of the complex Fourier coefficients $\tilde{X}$ can be found by multiplying both sides of (2.33) by the matrix inverse $\Phi^{-1}$,

$$\Phi^{-1} \tilde{x} = \{\Phi^{-1} \Phi\} \tilde{X}$$
$$\Phi^{-1} \tilde{x} = \tilde{X}$$
$$\Phi^H \tilde{x} = \tilde{X} \quad (2.34)$$

where the superscript $H$ in (2.34) denotes the “conjugate/Hermition transpose” operation\(^6\) which can be used in this case to describe the matrix inverse $\Phi^{-1}$ as the “Fourier” basis matrix $\Phi$ is designed to be orthonormal. Thus, $\Phi^H$ describes the discrete Fourier transform (DFT) operation with the inverse discrete Fourier transform (IDFT) operation equivalent to multiplying by the matrix $\Phi$. In order to make the generative model in (2.33) more realistic, we might assume that the signal is a linear combination of frequency vectors plus a noise vector $\tilde{w}$.

Let us now use this formalisation to consider the classic CS problem where our observed signal $\tilde{x}$ is missing some data and we therefore have less equations than unknowns i.e. our Fourier basis matrix $\Phi$ is “short and fat”. To proceed further, we must make some assumptions about the Fourier coefficients in the vector $\tilde{X}$ otherwise we are left with an indefinite number of possible solutions to the problem. We therefore assume that $\tilde{X}$ has a small number of strong coefficients with the rest being zero or very small in comparison\(^7\). In other words, vector $\tilde{x}$ is sparse in the frequency domain. This leads us to the more favourable situation where we have more equations than unknowns i.e. our Fourier basis matrix $\Phi$ is “tall and thin”. Importantly, when we have more equations than unknowns there is no exact solution to the problem as the left hand side of (2.33) will never be exactly equal to the right hand side. Somewhat counterintuitively, this is an advantageous property of the problem as it provides us with a generic model that includes the possibility of additive noise as well as lending itself to an optimisation-based solution. We can therefore formulate the problem as a standard least squares (LS) problem

\(^6\)Conjugate every element of the matrix, then transpose.

\(^7\)This is a reasonable assumption for many radar modes of operation.
2.1. Background Literature Review

where we want to choose $\tilde{X}$ such that the sum of the squared errors is minimised. The error vector $\tilde{e}$ is therefore defined as the difference between our estimate of the observed signal vector, $\Phi \tilde{X}$, and the observed signal vector itself, $\tilde{x}$.

$$\tilde{e} = \tilde{x} - \Phi \tilde{X}$$  \hspace{1cm} (2.35)

with the LS cost function defined as

$$C(\tilde{X}) = e^H e = (\tilde{x} - \Phi \tilde{X})^H (\tilde{x} - \Phi \tilde{X})$$ \hspace{1cm} (2.36)

In order to minimise the LS cost function, we must find the gradient of $C(\tilde{X})$ which involves differentiating (2.36) with respect to each element of $\tilde{X}$ before collecting the resultant derivatives together.

$$\nabla C(\tilde{X}) = \frac{\partial}{\partial \tilde{X}^H} C(\tilde{X}) = -\Phi^H (\tilde{x} - \Phi \tilde{X}) = -\Phi^H \tilde{e}$$ \hspace{1cm} (2.37)

Thus, the optimal solution, $\tilde{X}_{opt}$, is found when $\nabla(\tilde{X}) = 0$. This leads us to two popular classes of algorithm which are used for solving sparse systems of equations: greedy algorithms and iterative based algorithms. In both cases, the algorithm must identify the locations of the strongest Fourier coefficients as well as the power contained in each one. In this thesis, we are interested in the iterative based techniques as they have a significantly faster implementation than the greedy methods. These iterative refinement methods employ a steepest descent approach where the solution to the simultaneous equations is found indirectly [77, 76, 78, 75]. The algorithm can be broken down into three distinct steps after the first initialisation, “guess”, of $\tilde{X}^{(0)}$ is performed. The gradient step (2.38) involves calculating the error vector for the current iteration, $\beta$, before updating the estimate of the Fourier coefficients $\tilde{X}^{(\beta)}$ based on the LS formalisation previously outlined in (2.37),

error

$$\tilde{e}^{(\beta)} = \tilde{x} - \Phi \tilde{X}^{(\beta-1)}$$

update

$$\tilde{X}^{(\beta)} = \tilde{X}^{(\beta-1)} - \mu \nabla(\tilde{X}^{(\beta-1)})$$

$$= \tilde{X}^{(\beta-1)} + \mu \Phi^H (\tilde{x} - \Phi \tilde{X}^{(\beta-1)})$$

$$= \tilde{X}^{(\beta-1)} + \mu \Phi^H \tilde{e}^{(\beta)}$$ \hspace{1cm} (2.38)

where the step parameter $\mu$ is used to scale the rate of gradient descent. The second step involves imposing the chosen constraint which for this example is sparsity. Finally, step three involves invoking a stopping-criteria when the squared error, $\{\tilde{e}^{(\beta)}\}^H \{\tilde{e}^{(\beta)}\}$, is deemed to be small enough otherwise the algorithm repeats until this is the case.

While the CS method has been explained here using the classic undersampled signal scenario, CS techniques can be applied to adequately sampled signals so long as some known property of the signal can be exploit. As mentioned at the start of this section,
there are many applications other than subsampled signal recovery where CS techniques can provide performance enhancement.

**Forward Modelling**

In the context of this thesis, we can think of the *forward modelling* approach as the opposite of the *build-an-inverse* methodology described previously for the nonlinear receiver problem in radar. This means that instead of trying to recover the linear input signal by passing the corrupted output through an inverse model of the forward nonlinearity, we physically build the forward nonlinear receiver model into the signal processing itself. The problem of recovering the linear input signal can now be formulated as a nonlinear signal processing one where the signal of interest is observed through a nonlinear system. Crucially, the fact that the observations are intrinsically nonlinear means that traditional transform techniques cannot be used to infer the various signal components at the input. By setting the problem up as a sparse nonlinear optimisation problem the linear input signal can be estimated from the measured nonlinear output signal. Like most sparse signal processing problems iterative solutions which are signal dependent must be sought. In the case of radar, this signal dependence results in the technique being mode specific. Importantly, the forward modelling technique is illustrated in Figure 2.13 which can be compared with the illustration of the inverse mitigation technique displayed previously in Figure 2.7.

![Figure 2.13: Illustration of forward modelling digital mitigation technique. Input signal is estimated from the distorted output through an iterative learning procedure which incorporates knowledge of the forward nonlinearity.](image)

**Nonlinear Compressive Sensing**

Nonlinear sparse signal processing is in its infancy with only one major paper published on the topic by Blumensath [27]. Blumensath initially extended the ideas of linear compressive sensing (CS) to include nonlinear observations for the greedy based gradient pursuit method in [79], before publishing a more complete theorem for the iterative refinement approach in [27]. Crucially, he showed in [27] that a sparse or structured signal
observed through a nonlinear function could be recovered by the iterative hard thresholding (IHT) algorithm with constraints similar to those required in the linear setting. The standard IHT iteration used in the linear CS algorithm is shown in (2.38), however we recast it into Blumensath’s more generalised formalisation in (2.39) before extending it to include nonlinear observations, (2.40), as detailed in [27].

\[
x^{(\beta+1)} = P_A(x^{(\beta)} + \mu \Upsilon^*(y - \Upsilon x^{(\beta)}))
\] (2.39)

\[
x^{(\beta+1)} = P_A(x^{(\beta)} + \mu \Upsilon_{x(x^{(\beta)})}^*(y - \Upsilon(x^{(\beta)})))
\] (2.40)

In Blumensath’s formalisation, \( \Upsilon \) denotes a linear measurement operator used to sample a sparse input vector \( x \) which in turn produces output vector \( y \). By exploiting the sparse nature of \( x \), it is possible to recover \( x \) from \( y \) via the IHT algorithm by applying the sparse thresholding constraint \( P_A \). The sparse thresholding operator \( P_A(\ldots) \) can be thought of as a projection onto a general nonconvex constraint set \( A \), where signal \( x_A = P_A(x) \) minimises the sum of the squares observation error when formulated as a optimisation problem. In (2.40), the sampling process is converted from a linear operation to a nonlinear one where the mapping now takes a nonlinear form denoted by \( \Upsilon(x) \). The nonlinear mapping chosen for the proof in [27] is an affine Taylor series around point \( x^* \), such that \( \Upsilon(x) \approx \Upsilon(x^*) + \Upsilon_{x^*}(x - x^*) \) and \( \Upsilon_{x^*} \) is a linear operator that depends on \( x^* \). While Blumensath extends the linear measurement operator to be a nonlinear function in (2.40), he also restricts it’s modelling capabilities by assuming that \( \Upsilon(x) \) must have a linear approximation. Importantly, Blumensath’s proof holds under the strict condition that the system is not too nonlinear and therefore the error introduced in the linearisation is not too large i.e. that \( ||\Upsilon(x_A) - \Upsilon(x^{(\beta)}) - \Upsilon_{x_A}(x_A - x^{(\beta)})|| \) is small for large \( \beta \).

### 2.2 Chapter Specific Literature Review

In this section, we provide a brief literature review for each research chapter highlighting the key papers from across the fields of radar, communications and signal processing. Some of the papers referenced have already been introduced in the Background Literature Review section of this thesis while others are being introduced here for the first time. Importantly, unlike the previous section the discussions below are specific to the research presented in each individual chapter.

#### 2.2.1 Chapter 3: Modelling Nonlinear Radar Receivers

The passband Taylor series (PBTS) model, (2.1), is the most popular nonlinear receiver model employed in the field of radar [25, 29, 30]. The popularity of the model stems from
from its uniquely simple formalisation which allows it to accurately predict the spectral power and location of spurious harmonics generated within the nonlinear device. Furthermore, simple extensions to the PBTS model have been proposed which would allow it to not only capture AM-AM distortion effects but AM-PM conversion as well [30, 33, 34]. While these modified PBTS models may be able to capture some nonlinear phase effects, this modelling capability is fundamentally limited to single tone input signals. If the complex nonlinear behaviour of the RF front-end is be accurately modelled for the entire radar receiver bandwidth, the fundamental concept of system memory must be introduced [34, 15, 38]. The memory of a system is intrinsically linked to its bandwidth and therefore these nonlinear memory/phase effects are expected to become more important as receivers move to increasingly wider bandwidths. It is therefore thought that the memoryless PBTS model will struggle to accurately describe the in-channel nonlinear distortion effects observed in the modern radar receiver. This is a trend that has appeared in the related field of communications where the receiver bandwidths have been greatly expanded so that multiple communications channels can be jointly received by the system [9]. In order to accurately model the nonlinear distortion effects produced in the wideband communications receiver, advanced black-box behavioural models with memory have had to be developed. The most powerful of these is the passband Volterra series (PBVS) model which conveniently has a linear in the parameter formalisation that makes it ideally suited for modelling weakly nonlinear RF amplifiers [34, 56]. The PBVS model is described as a finite sum of multidimensional convolution integrals and was stated previously in its continuous-time and discrete-time form in (2.18) and (2.19) respectively. Importantly, the nonlinear behaviour of the Volterra series behavioural model is completely characterised by its full set of kernel coefficients.

While the PBVS model is capable of capturing all of the nonlinear distortion effects depicted in Figure 1.2, its use is prohibited for most radar simulators due to its large size and the fact that extremely high sampling rates are required to implement the model up at the radar passband. Therefore, in order to develop a Volterra based behavioural model that is suitable for radar systems, we must drop the PBVS model into a lower frequency domain. Inspiration for this reduced form Volterra model can be taken from the communications literature where the PBVS model is often employed in the baseband (BB) domain [33, 63, 20, 80]. The derivation of the baseband equivalent Volterra series (BBVS) model was first performed by Benedetto et al. in [59]. Importantly, during the derivation of the BBVS model the narrowband assumption is invoked which restricts its modelling capabilities to a small bandwidth around the carrier frequency. This results in the BBVS model being unable to describe the generation of nonlinear harmonics across the RF spectrum which in the case of [59], allows Benedetto to reduce the form of the model to the strictly odd-order terms. For this thesis, we are specifically interested in front-end nonlinearities which have been stimulated in the radar receiver by the presence of an unwanted interferer in the scene. We must therefore derive a radar specific BBVS
model that not only captures the in-band nonlinear effects described by the BBVS model in [59], but also models the nonlinear coupling between an out-of-band interferer and the desired radar channel. The development of a compact cross-channel BBVS model is a hot topic in the communications literature with the majority of the work focused on the concurrent transmission of multiple communications channels [63, 20, 80]. However, to the author’s knowledge no work has been published on the subject in the radar literature and particularly not related to the generation of front-end receiver nonlinearities.

2.2.2 Chapter 4: LUT Inverse Mitigation Techniques

Inspiration for the LUT mitigation techniques developed in this chapter has been taken from the related field of communications where they have been employed successfully for decades. Traditionally, these inverse mitigation techniques have been applied to the transmit side of the communications system in order to increase the efficiency of the front-end power amplifiers [55, 57]. More recently however, they have also been employed on the receive side of the system so that front-end linearity specifications can be relaxed and thus hardware cost can be reduced [26, 72, 9, 81, 15, 82]. While there are a multitude of publications available in the communications literature on the general inverse mitigation approach, for chapter 4 we are specifically interested in the current research trend which aims to develop multi-channel nonlinear equalization (NLEQ) techniques [14, 63, 20, 49, 74, 83, 19]. In a further effort to save costs and improve efficiency, communications systems are being designed to simultaneously process multiple nonlinear channels both on the transmit and receive side of the system. In the case of the wide-band communications receiver, this multi-channel NLEQ problem is very similar to the cross-modulation problem in radar. Therefore, the related digital signal processing solutions provide an excellent starting point for this study. We must be mindful however of the difference between the two systems when developing these multi-channel mitigation techniques for radar.

Two separate multi-channel NLEQ techniques are studied in this chapter both of which are borrowed from the communications literature. The first technique was published by Zou et al. in [14] where it is employed to specifically target a strong blocker signal in an adjacent communications channel. This technique was discussed in detail earlier in chapter 2 and is referred to as the direct inverse solution for the purposes of this thesis due to its mathematical formalisation. Importantly, Zou sets out the requirement for a separate receive path in [14] that is capable of linearly capturing the strong blocker signal which caused the nonlinear distortion in the desired communications channel. For radar systems this means using intelligent guard channels to accurately measure the scene for any interfering signals [10]. This fits with the popular phased array architecture of modern radar systems and is discussed in detail in the following section. The second mitigation technique studied in this chapter is taken from the transmit side of the communication literature where digital pre-distortion (DPD) methods are employed to
ensure that concurrently transmitted channels are outputted from the front-end power amplifier without any unwanted nonlinear effects. Unlike the first technique [14], which tries to correct for the nonlinear distortion by algebraically inverting the forward nonlinear transfer function, this alternative method attempts to recover the linear input signal by passing the corrupted outputs through a tandem nonlinearity. The fundamental mathematical theory underpinning this inverse mitigation approach is presented by Schetzen in [54, 84] with the unique multi-channel communications based formalisation presented in [63, 20, 49, 74]. For the purposes of this thesis, we refer to this LUT mitigation technique as the tandem inverse solution and unlike the direct inverse approach it can be extended to include complex nonlinear memory effects. This enhanced capability could prove critical as radars move to wider and wider receiver bandwidths where memory effects become significant.

Relatively few papers have been published in the radar literature on the feasibility of implementing LUT techniques to mitigate nonlinear distortion effects generated in the radar receiver [8, 29, 53]. In [8], the cross-modulation effect is suppressed in a high PRF mode by employing a linear guard channel to adaptively subtract the interfering signal from the nonlinear output. While this simple solution may seem similar to the direct inversion technique presented by Zou in [14], it is fundamentally different as it fails to account for the forward nonlinearity during the compensation procedure. More recently, several papers have studied the effectiveness of the tandem inverse technique in alleviating front-end nonlinearities in the radar transmit chain [38, 85]. However, unlike the communications literature very few papers have attempted to implement this technique on the receive side as well [29, 53]. The benefits of the tandem inverse technique for modern radar receivers were first reported by Kam et al. in [53] before its performance was studied as part of a hybrid mitigation scheme in [29], see section 2.1.1. Importantly, in this thesis we develop the LUT mitigation solution specifically for the cross-modulation problem in radar and perform in-depth performance analysis on the technique for a typical MPRF mode of operation.

During the course of this research project there has been growing interested in applying these NLEQ techniques to the receive side of the radar system. In [86], Peccarelli et al. implement the classic polynomial LUT inverse technique to mitigate both intermodulation and cross-modulation distortion effects caused by the presence of a strong adjacent channel. The paper focused on the strictly memoryless case and displays simulated results highlighting the effectiveness of the algorithm in suppressing both cross-modulation and intermodulation effects. Importantly, the publication of [86] coincided with the publication of the direct inversion research work presented in chapter 4 [87] - (Euan Ward and Bernard Mulgrew. “Mitigation of Cross-modulation Effects in Radar Receivers with Memory”. In: 2020 IEEE International Radar Conference (RADAR). 2020, pp. 856–859. DOI: 10.1109/RADAR42522.2020.9114661). Furthermore, the research presented in [86] does not include any analysis of nonlinear memory effects which is covered
2.2. Chapter Specific Literature Review


2.2.3 Chapter 5: Forward Modelling Mitigation Techniques

The memory of a system is intrinsically linked to its bandwidth [15, 38] and with modern radars moving to increasingly wider receiver bandwidths they can no longer be approximated as strictly memoryless systems. Importantly, it is shown in chapter 4 that as nonlinear memory effects are introduced to the radar receiver, memoryless LUT techniques become increasingly decorrelated significantly reducing their effectiveness. Therefore, in order for LUT techniques to be effective in mitigating nonlinear memory effects in the modern radar receiver, nonlinear inverse models with memory must be used. Extending the LUT approach to correct for nonlinear memory effects in radar is possible, as proven in chapter 4, however it is not yet known how robust this technique will be when applied to real wide-band radar systems. The identification of these inverse memory models is challenging for radar systems as unlike communications systems the signal returned to the receiver is both unknown to the sensor and potentially very diverse. For the very controlled communications systems, the literature suggests that compact inverse memory models can be learnt and implemented successfully through LUT techniques [54, 15]. However, in the case of radar where weak targets must be detected in the presence of clutter and interference, these memory inverse structures can become extremely complex and will often not have a closed form solution [54]. The functional form of such a memory inverse post-distorter is an open question in the radar literature as usually more memory and complexity is required in the inverse than the forward nonlinearity. Equally the nature of the signal required to train such a model is unclear [56]. While LUT techniques are an attractive solution to the memoryless nonlinear receiver problem, it is thought that they will not be universally applicable when memory effects become significant in the radar receiver.

Forward modelling offers an alternative solution to standard LUT techniques. The technique removes the need for a complex nonlinear inverse structure as instead the forward nonlinear model is integrated into the signal processing itself. Importantly, it is shown in chapter 4 that the extension of nonlinear behavioural models to included memory effects is well defined for the forward model but not so for the corresponding inverse structure. Designing a nonlinear mitigation solution around the forward nonlinear transfer function rather than the inverse, therefore allows the technique to be extended to include nonlinear memory effects in a well-defined manner. Crucially, the problem of recovering the desired input signal from the corrupted output must now be formulated as a nonlinear signal processing one where the observations themselves are intrinsically nonlinear. The theoretical basis for this forward modelling solution is founded in [27] where
Blumensath details a compressive sensing technique with nonlinear observations. By setting the problem up as a sparse nonlinear optimisation problem, Blumensath proved in [27] that the linear input signal could be recovered from the measured nonlinear output signal if the characteristics of the forward nonlinear transfer function are known. Additionally, the algorithm requires some prior knowledge of the input signal’s underlying structure in order for the IHT optimisation procedure to converge.

Since [27] was published in 2013, Blumensath’s IHT algorithm for NCS has been applied to a wide variety of different applications. In [28], Chen et al. apply the technique to a communications system for the first time with the aim of digitally correcting the nonlinear distortion from a front-end power amplifier leading to improved efficiency. The nonlinearity was described by means of a Rapp model and the NCS algorithm was shown to converge for both simulated and experimental data. This is a significant result as it proves that the NCS algorithm can be successfully employed to digitally mitigate nonlinear effects in a communications-based system. While Chen et al. employ Blumensath’s NCS technique to mitigate a transmit-side nonlinearity, Zhu et al. develop the technique further in [89] by applying it to the receive side of the system. The NCS technique in [89] focuses on a front-end LNA driven into its weakly nonlinear regime by the presence of an unwanted interferer in the spectrum. The LNA was modelled by means of a static cubic order Taylor series and Blumensath’s original IHT technique was tested alongside a novel gradient pursuit method. The gradient pursuit based NCS algorithm tested in [89] was adapted from a previously proposed version of the NCS technique presented by Blumensath in [79]. Crucially, Zhu et al. prove in [89] that the linear input signal can be recovered by both NCS techniques if the LNA is not too nonlinear and the Taylor series model parameters are known. While [28, 89] provides some encouragement, Blumensath’s NCS technique [27] must be developed specifically to exploit the unique behaviour of radar systems if it is to be successful in mitigating nonlinear effects generated in the radar receiver.
Chapter 3

Modelling Nonlinear Radar Receivers

3.1 Aim and Introduction

The radio frequency (RF) receiver effectively defines the sensitivity of the entire radar system. It is therefore imperative that its behaviour is characterised to a high degree of accuracy. For modern radar systems, which are expected to perform in an increasingly crowded RF environment, this in-depth characterisation must be extended beyond the linear regime as nonlinearities may be stimulated in the receiver front-end. This goes far beyond the standard characterisation measurements performed in traditional radar systems which are designed to operate deep within their linear regions. In order to accurately model the weakly nonlinear behaviour of the modern radar receiver, sophisticated nonlinear behavioural models will have to be developed. In this chapter, we derive a novel black-box behavioural model for radar systems that is capable of capturing the complex amplitude and phase effects generated by nonlinearities in the RF receiver. The sophisticated procedure required to identify these nonlinear behavioural models is discussed in detail before the modelling technique is tested in a radar specific simulation environment.

As discussed previously in chapter 2, nonlinear receiver effects are typically split up into two distinct types in the radar literature: amplitude-amplitude (AM-AM) conversion and amplitude-phase (AM-PM) conversion. The standard nonlinear models employed in radar systems usually focus on the AM-AM effects and in particular the generation of spurious harmonics across the spectrum, see Figure 1.2. This is due to the fact that RF receiver designers are predominantly interested in how front-end nonlinearities affect the receiver’s dynamic range. Therefore, the AM-PM phenomenon is typically ignored as the gross power and frequency of the nonlinear spurs can be accurately described by the purely AM-AM effects. While nonlinear harmonics generated at the front-end of the RF receiver can have serious consequences for the performance of the sensor, they can also be removed by RF filtering further down the receiver chain. In contrast, the front-end nonlinear distortion caused directly to the desired radar channel cannot be compensated for in hardware and must be mitigated in the digital domain. For these digital mitigation
Chapter 3. Modelling Nonlinear Radar Receivers

techniques to be successful in recovering the desired radar signal, they require an accurate model of the nonlinearity that caused the harmful distortion. Importantly, radar systems detect targets by studying both the amplitude and the phase of the signals returned to its receiver. The behavioural models used to describe the nonlinear distortion caused to the desired radar channel must therefore be able to capture both the AM-AM and AM-PM conversion phenomena. Crucially, this means dealing with the fundamental concept of system memory so that the nonlinear interaction between the different frequency components of the input signal can be accurately described by the behavioural model. In this thesis, we are interested in the in-channel nonlinear distortion effects caused by the presence of a strong interference signal in the radar scene, see Figure 1.2. The black-box behavioural models designed in this chapter must therefore be able to accurately model the nonlinear distortion effects generated by both in-band and out-of-band interference sources in the radar scene.

In this chapter, we derive a novel cross-channel BBVS model for modelling nonlinear radar systems, see section 3.2, and develop a sophisticated noise-based procedure to accurately identify the full set of kernel coefficients, see section 3.3. Importantly, this nonlinear modelling technique was the first of its kind to be published in the radar literature [90] - (Euan Ward and Bernard Mulgrew. “Baseband Equivalent Volterra Series for Modelling Cross-channel Nonlinear Distortion”. In: 2019 IEEE Radar Conference (RadarConf). 2019, pp. 1–4. DOI: 10.1109/RADAR.2019.8835647.) and is subsequently employed throughout this thesis to simulate the behaviour of the modern radar receiver.

3.2 Baseband Volterra Series Model

3.2.1 Dual Channel Model

In this section we derive the radar specific cross-channel BBVS model from the continuous-time PBVS model stated in (2.18). We focus on the scenario where an out-of-band interferer occupies half the carrier frequency of the radar as this is one of the most problematic interference scenarios for modern radar systems. This is due to the nonlinearity placing the second order harmonic of the interference signal right in the middle of the desired radar channel. Importantly, in order to perform this unique cross-channel mapping from the passband (PB) domain to BB domain, the even order terms in the Volterra series must be retained during the derivation of the BBVS model. Similar to the derivation of the classic BBVS model performed by Benedetto in [59], we must invoke the narrowband assumption which restricts the modelling capabilities of the BBVS model to a small bandwidth around the carrier frequency. However unlike the model in [59], the cross-channel nature of the derivation performed in this chapter means that the model also captures the nonlinear coupling between a narrow bandwidth around the interference frequency and the desired radar channel. We start by defining the PB radar input signal $x(t)$ in complex
3.2. Baseband Volterra Series Model

envelope form in (3.1) below;

\[ x(t) = \frac{1}{2} \left\{ \{ \tilde{x}_2(t) e^{j\omega c t} + \tilde{x}_2^*(t) e^{-j\omega c t} \} + \{ \tilde{x}_1(t) e^{j\omega i t} + \tilde{x}_1^*(t) e^{-j\omega i t} \} \right\} \]  

(3.1)

where \( \tilde{x}_1(t) \) and \( \tilde{x}_2(t) \) represent the complex baseband signals modulated onto the radar carrier frequency, \( \omega_c \), and interference carrier frequency, \( \omega_i \), respectively. As stated previously, we assume for this derivation that the interference signal has a carrier frequency which is exactly half that of the radar meaning \( \omega_i = \frac{\omega_c}{2} \) in (3.1).

We now derive the cross-channel BBVS model in full by substituting the radar input signal, (3.1), into the expression for the continuous-time PBVS model, (2.18), and gathering the resultant terms that lie on the radar carrier, \( e^{j\omega c t} \). It is assumed for the purposes of this thesis that the most dominant nonlinear effects can be captured by terms up to the cubic order of the PBVS model \([29, 25]\) and therefore (2.18) can be truncated to order \( P = 3 \). Furthermore, the linear-in-the-parameters formalisation of the PBVS model allows us to treat each set of Volterra terms separately during the derivation before linearly combining them in the BB domain to form our final expression for the BBVS model. This is displayed formally in (3.2) below,

\[ y(t) = y_{p_1}(t) + y_{p_2}(t) + y_{p_3}(t) \]  

(3.2)

where for this chapter \( y_{p_1}(t), y_{p_2}(t) \) and \( y_{p_3}(t) \) represent the linear, quadratic and cubic outputs from the PBVS model (2.18) respectively.

\[ y_{p_1}(t) = \int_{-\infty}^{\infty} d\tau_1 h_1(\tau_1) x(t - \tau_1) \]  

(3.3)

\[ y_{p_2}(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\tau_1 d\tau_2 h_2(\tau_1, \tau_2) x(t - \tau_1) x(t - \tau_2) \]  

(3.4)

\[ y_{p_3}(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 h_3(\tau_1, \tau_2, \tau_3) x(t - \tau_1) x(t - \tau_2) x(t - \tau_3) \]  

(3.5)

We start by considering the linear term in (3.2) where \( y_{p_1}(t) \) is described as a purely one-dimensional convolution operation (3.3). As mentioned previously, we can translate the PBVS model into the BB domain by substituting (3.1) into (3.2) and collecting those terms that exist on the radar carrier, \( e^{j\omega c t} \). Given that \( \omega_i = \frac{\omega_c}{2} \), it is unsurprising that the in-channel signal component corresponding to the linear term, \( y_{p_1}(t) \), is entirely
dependent on the desired radar input signal $\tilde{x}_1$. This is shown explicitly below,

$$y_{p_1}(t) = \int_{-\infty}^{\infty} d\tau_1 h_1(\tau_1) \frac{1}{2} \left\{ \{ \tilde{x}_2(t - \tau_1)e^{j\frac{\omega_c}{2}(t-\tau_1)} \} + \{ \tilde{x}_1(t - \tau_1)e^{j\omega_c(t-\tau_1)} \} \right\}$$

$$+ \{ \tilde{x}_2(t - \tau_1)e^{-j\frac{\omega_c}{2}(t-\tau_1)} \} + \{ \tilde{x}_1(t - \tau_1)e^{-j\omega_c(t-\tau_1)} \} \right\}$$

$$= \int_{-\infty}^{\infty} d\tau_1 h_1(\tau_1) \frac{1}{2} \left\{ \{ \tilde{x}_2(t - \tau_1)e^{j\frac{\omega_c}{2}t}e^{-j\frac{\omega_c}{2}t} \} + \{ \tilde{x}_2(t - \tau_1)e^{-j\frac{\omega_c}{2}t}e^{j\frac{\omega_c}{2}t} \} \right\}$$

$$+ \{ \tilde{x}_1(t - \tau_1)e^{j\omega_c(t-\tau_1)} \} + \{ \tilde{x}_1(t - \tau_1)e^{-j\omega_c(t-\tau_1)} \} \right\}$$

$$= e^{j\omega_c t} \int_{-\infty}^{\infty} d\tau_1 h_1(\tau_1) \frac{1}{2} \tilde{x}_1(t - \tau_1)e^{-j\omega_c t}$$

(3.6)

where the linear BB output signal $\tilde{y}_{p_1}(t)$ is given by,

$$\tilde{y}_{p_1}(t) = \int_{-\infty}^{\infty} d\tau_1 \tilde{h}_1(\tau_1) \tilde{x}_1(t - \tau_1)$$

(3.7)

In (3.7), $\tilde{h}_1(\tau_1)$ denotes the linear BB Volterra kernel which is constructed by collecting the terms in (3.6) that are not functions of the time parameter $t$. In essence, the BB Volterra kernels describe the mapping of the PB nonlinear impulse response to the BB domain and are used to characterise the behaviour of the BBVS model. Note, we derive expressions for the theoretical BBVS kernels in this section before the identification procedure required to learn them for a real-world nonlinearity is discussed in detail later in the chapter. In the linear case, the BB kernel $\tilde{h}_1(\tau_1)$ takes the following form;

$$\tilde{h}_1(\tau_1) = \frac{h_1(\tau_1)}{2}e^{-j\omega_c \tau_1}$$

(3.8)

We now consider the quadratic output term $y_{p_2}(t)$ from (3.2) and follow the exact same procedure as above to derive the BB equivalent form. Unlike the work previously published by Fehri et al. in [63, 20], we expect the contributions from the $P = 2$ term to be non-negligible due to the fact that $\omega_1 = \frac{\omega_c}{2}$ which results in the second harmonic sitting in the middle of desired radar channel. Substituting (3.1) into (3.4) we find;

$$y_{p_2}(t) = \int_{-\infty}^{\infty} d\tau_1 d\tau_2 h_2(\tau_1, \tau_2) \frac{1}{2} \left\{ \{ \tilde{x}_2(t - \tau_1)e^{j\frac{\omega_c}{2}(t-\tau_1)} \} + \{ \tilde{x}_2(t - \tau_1)e^{-j\frac{\omega_c}{2}(t-\tau_1)} \} \right\}$$

$$+ \{ \tilde{x}_1(t - \tau_1)e^{j\omega_c(t-\tau_1)} \} + \{ \tilde{x}_1(t - \tau_1)e^{-j\omega_c(t-\tau_1)} \} \right\}$$

$$+ \{ \tilde{x}_2(t - \tau_2)e^{j\frac{\omega_c}{2}(t-\tau_2)} \} + \{ \tilde{x}_2(t - \tau_2)e^{-j\frac{\omega_c}{2}(t-\tau_2)} \} \right\}$$

$$+ \{ \tilde{x}_1(t - \tau_2)e^{j\omega_c(t-\tau_2)} \} + \{ \tilde{x}_1(t - \tau_2)e^{-j\omega_c(t-\tau_2)} \} \right\}$$

(3.9)
3.2. Baseband Volterra Series Model

Expanding out the brackets in (3.9) and collecting only those terms that lie on the radar carrier leads to (3.10),

\[
y_{p2}(t) = e^{j\omega_c t} \int_{-\infty}^{\infty} d\tau_1 d\tau_2 \frac{h_2(\tau_1, \tau_2)}{4} \delta_2(t - \tau_1) \delta_2(t - \tau_2) e^{-j\frac{\omega_c}{2}(\tau_1 + \tau_2)}
\]  

(3.10)

As expected, only the second harmonic sits within the desired radar channel which means that the nonlinear contributions from the \( y_{p2}(t) \) term are entirely dependent on the BB interference signal \( \delta_2 \). The quadratic BB output signal \( \delta_{p2}(t) \) is therefore described by (3.11) with the corresponding second order BB Volterra kernel \( \hat{h}_2(\tau_1, \tau_2) \) given by (3.12).

\[
\delta_{p2}(t) = \int_{-\infty}^{\infty} d\tau_1 d\tau_2 \hat{h}_2(\tau_1, \tau_2) \delta_2(t - \tau_1) \delta_2(t - \tau_2)
\]  

(3.11)

\[
\hat{h}_2(\tau_1, \tau_2) = \frac{h_2(\tau_1, \tau_2)}{4} e^{-j\frac{\omega_c}{2}(\tau_1 + \tau_2)}
\]  

(3.12)

Finally we examine the cubic term \( y_{p3}(t) \) from (3.2) which we know from the analysis performed on the BBTS model in chapter 2, contains both in-channel and cross-channel nonlinear effects. Importantly, the process for deriving the BB equivalent form of \( y_{p3}(t) \) is exactly the same as for the other terms in the PBVS model and we therefore start by substituting (3.1) into (3.5).

\[
y_{p3}(t) = \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 h_3(\tau_1, \tau_2, \tau_3) \frac{1}{2} \left\{ \left\{ \tilde{x}_2(t - \tau_1) e^{j\frac{\omega_c}{2}(t - \tau_1)} + \tilde{x}_2(t - \tau_1) e^{-j\frac{\omega_c}{2}(t - \tau_1)} \right\} \right\}
\]

\[+ \left\{ \tilde{x}_1(t - \tau_1) e^{j\omega_c(t - \tau_1)} + \tilde{x}_1(t - \tau_1) e^{-j\omega_c(t - \tau_1)} \right\} \right\} \right\}
\]

\[
+ \left\{ \tilde{x}_1(t - \tau_2) e^{j\frac{\omega_c}{2}(t - \tau_2)} + \tilde{x}_1(t - \tau_2) e^{-j\frac{\omega_c}{2}(t - \tau_2)} \right\} \right\}
\]

\[
+ \left\{ \tilde{x}_2(t - \tau_3) e^{j\frac{\omega_c}{2}(t - \tau_3)} + \tilde{x}_2(t - \tau_3) e^{-j\frac{\omega_c}{2}(t - \tau_3)} \right\} \right\}
\]

\[+ \left\{ \tilde{x}_1(t - \tau_3) e^{j\omega_c(t - \tau_3)} + \tilde{x}_1(t - \tau_3) e^{-j\omega_c(t - \tau_3)} \right\} \right\}
\]

\[+ \tilde{x}_1(t - \tau_3) e^{-j\omega_c(t - \tau_3)} \right\}
\]

(3.13)
Expanding out the brackets and removing the terms that do not lie on the radar carrier \(e^{j\omega_c t}\) leads to:

\[
y_{p3}(t) = e^{j\omega_c t} \int \int \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \frac{h_3(\tau_1, \tau_2, \tau_3)}{8} \left\{ \right.
\]
\[
\tilde{x}_2^2(t - \tau_1) \tilde{x}_1(t - \tau_2) \tilde{x}_2(t - \tau_3) e^{-j\frac{\omega_c}{2} \{\tau_1 + 2\tau_2 + \tau_3\}} + \tilde{x}_1(t - \tau_1) \tilde{x}_2(t - \tau_2) \tilde{x}_2(t - \tau_3) e^{-j\frac{\omega_c}{2} \{2\tau_1 - \tau_2 + \tau_3\}} + \tilde{x}_2(t - \tau_1) \tilde{x}_2(t - \tau_2) \tilde{x}_2(t - \tau_3) e^{-j\frac{\omega_c}{2} \{\tau_1 + 2\tau_2 - \tau_3\}}
\]
\[
+ \tilde{x}_1(t - \tau_1) \tilde{x}_2(t - \tau_2) \tilde{x}_2(t - \tau_3) e^{-j\frac{\omega_c}{2} \{2\tau_1 + \tau_2 - \tau_3\}}
\]
\[
+ \tilde{x}_2(t - \tau_1) \tilde{x}_2(t - \tau_2) \tilde{x}_2(t - \tau_3) e^{-j\frac{\omega_c}{2} \{\tau_1 - \tau_2 + 2\tau_3\}}
\]
\[
+ \tilde{x}_1(t - \tau_1) \tilde{x}_2(t - \tau_2) \tilde{x}_1(t - \tau_3) e^{-j\frac{\omega_c}{2} \{\tau_1 + \tau_2 + 2\tau_3\}}
\]
\[
+ \tilde{x}_1(t - \tau_1) \tilde{x}_2(t - \tau_2) \tilde{x}_1(t - \tau_3) e^{-j\frac{\omega_c}{2} \{\tau_1 - \tau_2 - \tau_3\}}
\]
\[
+ \tilde{x}_1(t - \tau_1) \tilde{x}_1(t - \tau_2) \tilde{x}_1(t - \tau_3) e^{-j\omega_c \{\tau_1 + \tau_2 + \tau_3\}}
\]
\[
\left. \right\}
\]
\[
(3.14)
\]

Examining (3.14) it is apparent that there is a symmetry to the nonlinear terms in the expression with the bottom three terms resembling each other and the top six terms also looking very similar to each other as well. In actual fact, the bottom three terms are identical as their elements are invariant to permutation which is also separately true for the top six terms in the expression. Equation (3.14) can therefore be simplified by gathering the identical terms and apply the appropriate scale factors. This is displayed formally in (3.15) below,

\[
y_{p3}(t) = e^{j\omega_c t} \int \int \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \frac{h_3(\tau_1, \tau_2, \tau_3)}{8} \left\{ \right.
\]
\[
6 \tilde{x}_2(t - \tau_1) \tilde{x}_1(t - \tau_2) \tilde{x}_2(t - \tau_3) e^{-j\frac{\omega_c}{2} \{\tau_1 + 2\tau_2 - \tau_3\}}
\]
\[
+ 3 \tilde{x}_1(t - \tau_1) \tilde{x}_1(t - \tau_2) \tilde{x}_1(t - \tau_3) e^{-j\omega_c \{\tau_1 + \tau_2 + \tau_3\}}
\]
\[
\left. \right\}
\]
\[
(3.15)
\]

It is clear from (3.14) that \(y_{p3}(t)\) captures two separate nonlinear effects. We therefore choose to define the BB equivalent form of \(y_{p3}(t)\) with two independent expressions that describe the cubic in-channel, \(\tilde{y}_{p3in}(t)\), and cross-channel, \(\tilde{y}_{p3out}(t)\), nonlinear effects respectively. The first expression, (3.16), describes the in-channel nonlinear effect from (3.15) which is entirely dependent on the BB input signal, \(\tilde{x}_1\), centred around the radar carrier. Importantly, the behaviour of in-channel cubic nonlinearity is captured by the specific BB Volterra kernel \(\tilde{h}_{3\text{in}}(\tau_1, \tau_2, \tau_3)\) which is presented alongside the expression for \(\tilde{y}_{p3in}(t)\) in (3.17).

\[
\tilde{y}_{p3in}(t) = \int \int \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \tilde{h}_{3\text{in}}(\tau_1, \tau_2, \tau_3) \tilde{x}_1(t - \tau_1) \tilde{x}_1(t - \tau_2) \tilde{x}_1(t - \tau_3)
\]
\[
(3.16)
\]
This discrete-time framework was illustrated previously for the tions over Volterra taps the length of which define the amount of memory in the system. The process must replace the integrals over continuous-time delay-lags with finite summa-
discrete-time filter taps, $\tau$ with particular focus placed on the conversion of the continuous-time delay-lags, $\tau$. This general process was discussed at length in chapter 2 of this thesis of a front-end RF amplifier, it must be translated from continuous-time to the discrete-time domain. In order to employ the cross-channel nonlinear effect from (3.15) and is presented in its BB equivalent form in (3.18). Unlike the expression for $\tilde{y}_{p3in}(t)$, the cross-channel nonlinear output $\tilde{y}_{p3out}(t)$ depends on both $\tilde{x}_1$ and $\tilde{x}_2$. Comparing (3.18) with the memoryless BBTS model (2.14) from chapter 2, it is clear that the $\tilde{y}_{p3out}(t)$ output describes the classic cross-modulation effect in radar.

$$\tilde{y}_{p3out}(t) = \int \int \int_{-\infty}^{\infty} \int \int \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \tilde{h}_{3out}(t, \tau_1, \tau_2, \tau_3) \tilde{x}_2(t-\tau_1) \tilde{x}_1(t-\tau_2) \tilde{x}_2^*(t-\tau_3)$$  \hspace{1cm} (3.18)$$

$$\tilde{h}_{3out}(t, \tau_1, \tau_2, \tau_3) = \frac{6}{8} h_3(t, \tau_1, \tau_2, \tau_3) e^{-j \omega c (\tau_1 + \tau_2 - \tau_3)}$$  \hspace{1cm} (3.19)$$

It is important to note that unlike the output from $\tilde{y}_{p2}(t)$, it is not a requirement that the interferer has to have a carrier frequency equal to half that of the radar in order for $\tilde{y}_{p3out}(t)$ to be non-zero. Therefore, the BBVS model can still describe the cross-channel nonlinear coupling from an out-of-band interferer in the scene even when $\omega_i$ does not equal $\frac{\omega_c}{2}$. In essence, (3.18) captures the classic cross-modulation effect in radar described previously in chapter 2. Importantly however, in the case where the interferer does occupy half the carrier frequency of the radar, the cross-channel BBVS model takes its full form which is stated in (3.21) below.

$$\tilde{y}(t) = \tilde{y}_{p1}(t) + \tilde{y}_{p2}(t) + \tilde{y}_{p3in}(t) + \tilde{y}_{p3out}(t)$$  \hspace{1cm} (3.20)$$

$$\tilde{y}(t) = \int \int \int_{-\infty}^{\infty} \int \int \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \hat{h}_1(t, \tau_1) \tilde{x}_1(t-\tau_1) + \int \int \int_{-\infty}^{\infty} \int \int \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \hat{h}_2(t, \tau_1, \tau_2) \tilde{x}_2(t-\tau_1) \tilde{x}_2^*(t-\tau_2)$$

$$+ \int \int \int_{-\infty}^{\infty} \int \int \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \tilde{h}_{3out}(t, \tau_1, \tau_2, \tau_3) \tilde{x}_1(t-\tau_1) \tilde{x}_1(t-\tau_2) \tilde{x}_2^*(t-\tau_3)$$  \hspace{1cm} (3.21)$$

3.2.2 Discrete-Time Model

In order to employ the cross-channel BBVS model to characterise the nonlinear behaviour of a front-end RF amplifier, it must be translated from continuous-time to the discrete-time domain. This general process was discussed at length in chapter 2 of this thesis with particular focus placed on the conversion of the continuous-time delay-lags, $\tau_{pr}$, to discrete-time filter taps, $l_{pr}$, in the Volterra series framework. In short, the conversion process must replace the integrals over continuous-time delay-lags with finite summations over Volterra taps the length of which define the amount of memory in the system. This discrete-time framework was illustrated previously for the $y_{p3}$ term from the PBVS
model (2.19) in Figure 2.9. Before we present the cross-channel BBVS model (3.21) in its discrete-time form, we state the discrete-time PBVS model for the $P = 3$ case so that comparisons can be made between the two equivalent models in the PB and BB domains. The discrete-time PBVS model discussed in this chapter therefore takes the following form,

$$y[m] = \sum_{l_1=0}^{L-1} h_1[l_1] x[m - l_1] + \sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} h_2[l_1, l_2] x[m - l_1] x[m - l_2]$$

$$+ \sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} \sum_{l_3=0}^{L-1} h_3[l_1, l_2, l_3] x[m - l_1] x[m - l_2] x[m - l_3]$$

(3.22)

where $m$ is the current PB sample, $L$ is the memory length, $l_p$ is the PB sample delay and $h_1[l_1], h_2[l_1, l_2], h_3[l_1, l_2, l_3]$ represent the linear, quadratic and cubic discrete-time PBVS kernels respectively. Importantly, the time separation between successive PB samples is defined by the PB sampling rate, $f_{s_1}$, which due to the Nyquist-Shannon sampling theorem [91] must be set to be at least twice that of the highest frequency component that is required to be captured by the PBVS model. As discussed previously, this leads to the PBVS model demanding extremely high sampling rates even just to capture the in-channel nonlinear effects for a typical front-end radar amplifier.

By translating the PBVS model into the BB domain, the in-channel nonlinear behaviour of the RF front-end can be modelled at significantly lower sampling rates than $f_{s_1}$. Importantly, during the derivation of the BBVS model the narrowband assumption [59, 33, 34] is invoked which effectively defines the BB sampling frequency $f_{s_2}$. In the case of the cross-channel BBVS model derived above (3.21), the derivation employed a dual-channel complex envelope formalisation which means that the BB sampling frequency, $f_{s_2}$, effectively defines a coupled modelling bandwidth around the radar and interference carrier frequencies respectively. This is depicted in Figure 3.1 where the sampling requirements for the discrete-time PBVS model are also illustrated. In order to reflect the fact that the discrete-time BB samples and delays represent different time instances and periods to those in the PB domain, we adopt an alternative notation for the discrete-time BBVS model to that employed for PBVS model in (3.22). The discrete-time BBVS model therefore takes the following form where $n$ represents the current BB sample and $i_p$ denotes the BB sample delay.

$$\tilde{y}[n] = \sum_{i_1=0}^{L-1} \tilde{h}_1[i_1] \tilde{x}_1[n - i_1] + \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \tilde{h}_2[i_1, i_2] \tilde{x}_2[n - i_1] \tilde{x}_2[n - i_2]$$

$$+ \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \tilde{h}_{3,n}[i_1, i_2, i_3] \tilde{x}_1[n - i_1] \tilde{x}_1[n - i_2] \tilde{x}_1^*[n - i_3]$$

$$+ \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \tilde{h}_{3,ou}[i_1, i_2, i_3] \tilde{x}_2[n - i_1] \tilde{x}_2[n - i_2] \tilde{x}_2^*[n - i_3]$$

(3.23)
In (3.23); $\tilde{h}_1[i_1], \tilde{h}_2[i_1,i_2], \tilde{h}_{3i}[i_1,i_2,i_3], \tilde{h}_{3o}[i_1,i_2,i_3]$ represent the linear, quadratic and cubic in-channel and cross-channel discrete-time BBVS kernels respectively. It is important to note that the memory of a device is intrinsically linked to its bandwidth [15, 55, 38]. Therefore, as long as the BB sampling rate, $f_{s_2}$, is comparable to the bandwidth of the radar front-end then it is reasonable to assume that the BB sample delay, $i_p$, will be of an order similar to the true memory delay-time, $\tau_p$, of the device. This is discussed in more detail in the context of the BB Volterra kernels in the following section.

3.2.3 Baseband Volterra Kernels

In this section, we discuss the discrete-time BBVS kernels and consider how the BB sampling frequency affects the model’s ability to accurately represent the nonlinear impulse response of the RF device. Furthermore, we present a theoretical mapping of the Volterra kernel functions from the PB to BB domain and discuss the properties of individual kernel coefficients in the resultant BB structures.

As discussed in the previous section, the discrete-time cross-channel BBVS model (3.23) effectively describes the nonlinear coupling between a narrow bandwidth around an out-of-band interferer and a narrow bandwidth around the radar carrier. The spectral width of this modelling bandwidth is defined by the BB sampling frequency $f_{s_2}$ which in turn, determines the temporal separation between successive BB samples $n$ and the period of the BB memory delay taps $i_p$. Importantly, in the same way that we consider the complex envelope signals $\tilde{x}_1$ and $\tilde{x}_2$ to be sampled by $n$ at a rate of $\frac{1}{f_{s_2}}$, we can think of
the continuous-time nonlinear impulse response to also be sampled at the same rate by the BB Volterra filter taps $i_p$. This was illustrated previously in Figure 2.10 where the low sampling rate of the BBVS model means that the nonlinear device’s impulse response is sampled more coarsely than in the full PBVS case. While the full PBVS model may provide finer sampling of the nonlinear impulse response than its BB equivalent form, this does not necessarily mean that the nonlinear effects captured by both models are described more accurately in the PB domain. This is due to the fact that the memory of an RF amplifier is intrinsically linked to its bandwidth [15, 55, 38] and therefore, as long as the BB sampling rate, $f_{s_2}$, is matched to the RF bandwidth of the device then the nonlinear memory effects generated should be of an order similar to the BB delay lags, $i_p$. In other words, the full PBVS model effectively oversamples the nonlinear impulse response of the device with the BBVS model able to describe the same nonlinear behaviour with fewer memory samples.

The theoretical mapping of the PBVS Volterra kernels to the BB domain is somewhat academic as it is highly unlikely that a full PBVS model would be identified for a front-end radar amplifier only to be subsequently translated to the BB equivalent domain. Furthermore, the full PBVS kernels cannot be inferred from a measurement of the nonlinear impulse response in the BB domain as only those effects stimulated during the characterisation experiment can be accurately described by the resultant behavioural model. It therefore makes more sense to think of the BBVS kernels as sampling the continuous-time kernel functions described by (3.8), (3.12), (3.17) and (3.19). However, in this chapter we aim to prove that the cross-channel PB nonlinear effects can be accurately modelled in the BB domain. This in turn requires the identification of the cross-channel BBVS model to be studied in a comprehensive simulation environment. It is therefore important to provide a theoretical mapping linking the simulated PB nonlinearity to the BBVS kernels so that the accuracy of the identification procedure can be tested. For simplicity, we assume in this chapter that the discrete-time PBVS model (3.22) and cross-channel BBVS model (3.23) have equal memory lengths resulting in the Volterra filter taps $i_p$ and $l_p$ representing the same temporal delays. Importantly, we choose this memory tap width to be equal to one BB sample which is equivalent to a delay of $\frac{1}{f_{s_2}}$ in the time domain. This condition effectively forces the memory effects generated by the PBVS nonlinearity to be set by the bandwidth of the simulated device, defined by $f_{s_2}$, rather than the sampling rate of the PB domain. In order to achieve this in the simulation environment, the PB sampling frequency, $f_{s_1}$, must be set to be an integer multiple of the BB sampling frequency, $f_{s_2}$, so that the PB sample delay, $l_p$, is also an integer number of discrete-time samples, $l_p = \frac{f_{s_1}}{f_{s_2}} i_p$.

This is discussed further alongside the simulation architecture in the following section. However, given the conditions stated above the theoretical mapping of the discrete-time
3.2. Baseband Volterra Series Model

PBVS kernels to the BB equivalent domain can be written as follows\(^1\):

\[
\tilde{h}_1[i_1] = \frac{h_1[i_1]}{2} e^{-j\omega_c l_1}
\]

(3.24)

\[
\tilde{h}_2[i_1, i_2] = \frac{h_2[i_1, i_2]}{4} e^{-j\frac{\omega_c}{2} (l_1 + l_2)}
\]

(3.25)

\[
\tilde{h}_{3,\text{in}}[i_1, i_2, i_3] = \frac{3}{8} h_3[i_1, i_2, i_3] e^{-j\omega_c l_1 - l_3}
\]

(3.26)

\[
\tilde{h}_{3,\text{out}}[i_1, i_2, i_3] = \frac{3}{4} h_3[i_1, i_2, i_3] e^{-j\frac{\omega_c}{2} (l_1 + 2l_2 - l_3)}
\]

(3.27)

Now that the discrete-time cross-channel BBVS kernels have been defined, we can analyse the model’s unique kernel structure in detail. We start by considering the strictly memoryless case where \(L = 1\). Importantly, when there is no memory in the system the Volterra series models perfectly reproduce the equivalent Taylor series models in both the PB and BB domains. This characteristic holds true for the cross-channel BBVS model derived above, (3.23), which in the memoryless case reduces to the cross-channel BBTS model presented previously in (2.14). We therefore label the BBVS coefficients \(\tilde{h}_1[0]\), \(\tilde{h}_2[0, 0]\), \(\tilde{h}_{3,\text{in}}[0, 0, 0]\) and \(\tilde{h}_{3,\text{out}}[0, 0, 0]\) from (3.24), (3.25), (3.26) and (3.27) respectively as the memoryless kernel coefficients since they correspond to those terms in the model that have zero delay. Importantly, we differentiate the coefficients associated with the BBTS model, \(L = 1\), from the memoryless kernel coefficients described above by omitting the memory tap delay indices \(i_1\), \(i_2\) and \(i_3\) from the BBTS notation: \(\tilde{h}_1, \tilde{h}_2, \tilde{h}_{3,\text{in}}\) and \(\tilde{h}_{3,\text{out}}\). This is a convention that is employed throughout this thesis to distinguish the specific memoryless coefficients in the full Volterra series framework from the coefficients pertaining to the Taylor series behavioural models. Re-examining the cross-channel BBVS model (3.23) for the case where \(L > 1\), we observe that the dimensions of the Volterra kernels is dependent on the nonlinear order \(p\). This is illustrated in the context of the Volterra kernels in Figure 3.2 where the linear kernel takes the form of a vector, the quadratic kernel takes the form of a matrix and the cubic kernels are both data cubes. Furthermore, symmetries within the quadratic and cubic in-channel kernel structures allow the number of terms in the model to be truncated without any loss of generality [63, 20, 59]. This symmetric reduction is displayed formally in (3.28) and is also depicted diagrammatically in Figure

---

\(^1\)As mentioned previously, we define the memory length of the PBVS and BBVS models to be equal for this analysis. This turn means that the BBVS and PBVS discrete-time kernels have the same size explaining why their elements are both indexed using the BB Volterra tap index \(i_p\) in (3.24), (3.25), (3.26) and (3.27).
3.2.

\[ \bar{y}[n] = \sum_{i_1=0}^{L-1} \hat{h}_1[i_1] \bar{x}_1[n-i_1] + \sum_{i_1=0}^{L-1} \sum_{i_2=1}^{L-1} \hat{h}_2[i_1, i_2] \bar{x}_2[n-i_1] \bar{x}_2[n-i_2] + \sum_{i_1=0}^{L-1} \sum_{i_2=1}^{L-1} \sum_{i_3=0}^{L-1} \hat{h}_3_{in}[i_1, i_2, i_3] \bar{x}_1[n-i_1] \bar{x}_1[n-i_2] \bar{x}_1[n-i_3] + \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \hat{h}_3_{out}[i_1, i_2, i_3] \bar{x}_2[n-i_1] \bar{x}_1[n-i_2] \bar{x}_2[n-i_3] \]  

(3.28)

Comparing (3.23) with (3.28) it is clear that the quadratic and cubic in-channel summations have been truncated removing the symmetric terms from the resultant kernel structures. Finally, we label the terms in the higher order kernels, \( p > 1 \), depending on their locations relative to the kernels’ main-diagonal. The power of the Volterra series behavioural model stems from the so called off-diagonal terms in each kernel, which account for the nonlinear cross-talk between taps in the respective Volterra filters, see Figure 2.9. While these off-diagonal terms provide the Volterra series with its exceptional modelling power, they also account for the majority of the terms in the high order kernels and therefore the large computational cost associated with the behavioural model. It is for this reason that they are often removed from the black-box Volterra models employed in the communication literature, leaving only the on-diagonal elements to account for the nonlinear memory \([26, 64, 67, 49]\). Reduced forms of the Volterra series behavioural model were discussed in detail in chapter 2 of this thesis, with the Parallel Hammerstein model describing the case above where only the on-diagonal elements are retained in each of the higher order kernels. The structure of the higher order kernels for the cross-channel BBVS model derived in this chapter are illustrated in Figure 3.2.

3.3 Baseband System Identification

Now that the cross-channel BBVS model has been derived, we examine the system identification procedure required to identify the behavioural model for a front-end radar amplifier. As mentioned previously, the nonlinear behaviour of the BBVS model is completely characterised by the BB Volterra kernels: \( \hat{h}_1[i_1] \), \( \hat{h}_2[i_2, i_2] \), \( \hat{h}_3_{in}[i_1, i_2, i_3] \) and \( \hat{h}_3_{out}[i_1, i_2, i_3] \). The task for the identification procedure is to therefore learn the BBVS kernel coefficients that enables the black-box model to accurately predict the nonlinear output signal for a given set of receiver inputs. In this section, we develop a system identification procedure capable of characterising the complex nonlinear memory behaviour of the BBVS kernels and test the performance of the technique in a comprehensive simulation environment. Importantly, we verify that the BBVS model derived in the previous section perfectly captures the in-channel and cross-channel nonlinear effects described by the PBVS model.
3.3. Baseband System Identification

Additionally, we show that the theoretical BBVS kernel coefficients can be accurately predicted by means of a noise-based identification procedure.

In order to identify the BBVS kernel coefficients for an RF amplifier, a characterisation measurement must be performed to effectively train the behaviour of the black-box model. For radar systems, this training procedure will most likely take the form of an offline calibration stage where the RF amplifier’s weakly nonlinear regime is probed using predetermined input signals. By comparing the measured nonlinear output signal with the known input signals, the device’s nonlinear transfer function can then be learnt. This simple methodology is not too dissimilar to the approach already employed in radar systems, however for the BBVS model it must be performed in the BB domain with sophisticated training signals capable of capturing the complex nonlinear memory behaviour of the device. As always with the cross-channel BBVS model, we must think of it as describing the nonlinear coupling between a narrow bandwidth around the interferer and a narrow bandwidth around the radar carrier, see Figure 3.1. The signals of interest for the BB system identification are therefore the complex envelope signals \( \tilde{x}_1 \) and \( \tilde{x}_2 \) which for the nonlinear characterisation must be modulated onto their respective carriers before
being combined and passed through the RF amplifier. The resultant nonlinear output envelope $\tilde{y}$ must then be captured and returned to the BB domain so that the cross-channel BBVS model (3.28) can be identified. This BB identification procedure is illustrated for the cross-channel case in Figure 3.3 and Figure 3.4 below.

As mentioned previously, we are interested in using the cross-channel BBVS model (3.28) to characterise the nonlinear coupling between a narrow bandwidth around an interferer and a narrow bandwidth around the radar carrier. Importantly, after the nonlinear characterisation the BBVS model must be able to accurately predict the nonlinear distortion caused to the desired radar channel, $\tilde{y}[n]$, in response to any set of complex input signals, $\tilde{x}_1[n]$ and $\tilde{x}_2[n]$, within the model bandwidth, $f_{s2}$. In order to achieve this, the nonlinear transfer function of the RF amplifier must be identified for the entire model bandwidth across the chosen range of input powers. In the case of the memoryless nonlinear models currently employed in radar systems, this full bandwidth characterisation is typically achieved through a swept single-tone or two-tone measurement of the RF
amplifier, see chapter 2. While a similar swept-tone measurement could be envisaged for the BB system identification procedure, we must first consider whether an identification signal of this type would allow the BBVS model to characterise the complex nonlinear memory behaviour of the device. This is discussed in detail in the following section.

### 3.3.1 Persistent Excitation Condition

The memory of a system is intrinsically linked to its bandwidth. Therefore, as radar systems move to increasingly wide receiver bandwidths these memory effects are expected to become more and more significant [34]. As discussed previously, within the limits of the narrowband assumption [59, 33, 34] the cross-channel BBVS model is capable of modelling the complex nonlinear memory behaviour of an RF amplifier. This statement is only true however if the amplifier’s nonlinear impulse response is accurately described by the corresponding BBVS kernels. It is therefore imperative for the success of the model
that the full nonlinear impulse response of the device is captured during the identification procedure. This is made more challenging by the fact that the principle of superposition does not apply in the nonlinear case \[34\] which means that unlike linear system identification, the full behaviour of the device cannot be inferred from a subset of measurements. In other words, all of the nonlinear memory effects must be stimulated during the identification of the kernels if they are to be captured by the model. For wideband received signals this means capturing the interactions between frequency components as they pass through the device. Due to the inherent link between bandwidth and memory this is equivalent to identifying the memory effects of the amplifier.

In order to characterise the cross-channel BBVS model, known input signals must be used to stimulate the nonlinear effect of interest in the RF amplifier so that the corresponding BBVS kernels can be identified, see Figure 3.3 and Figure 3.4. While the form of the PB input signal was defined previously in (3.1), we must decide what modulation signals, \(\tilde{x}_1\) and \(\tilde{x}_2\), should be applied during the characterisation so that the PB nonlinearity can be accurately described by the BB behavioural model. Let us therefore start by considering the simple single-tone measurement typically employed in radar systems. We perform this analysis in the continuous-time domain and define our complex envelope signals as follows:

\[
\begin{align*}
\tilde{x}_1(t) &= A e^{j \omega_1 t} \\
\tilde{x}_2(t) &= B e^{j \omega_2 t}
\end{align*}
\]  

(3.29)

where \(\tilde{x}_1(t)\) is a sinusoidal signal with frequency \(\omega_1\) and amplitude \(A\); and \(\tilde{x}_2(t)\) is a sinusoidal signal with frequency \(\omega_2\) and amplitude \(B\). Importantly, we assume for this analysis that the amplitudes \(A\) and \(B\) are both real and the frequencies \(\omega_1\) and \(\omega_2\) both lie within the BB modelling bandwidth defined by \(f_{s_2}\). By feeding these complex envelope signals into our continuous-time expression for the cross-channel BBVS model (3.21), we can gain some insight into how they interact with the nonlinear kernels during the characterisation procedure. This is displayed formally in (3.30) below:

\[
\begin{align*}
\tilde{y}(t) &= \int_{-\infty}^{\infty} d\tau_1 \tilde{h}_1(\tau_1) \, A e^{j \omega_1 \{t-\tau_1\}} + \int_{-\infty}^{\infty} d\tau_1 d\tau_2 \tilde{h}_2(\tau_1, \tau_2) B e^{j \omega_2 \{t-\tau_1\}} B e^{j \omega_2 \{t-\tau_2\}} \\
&\quad + \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \tilde{h}_3(\tau_1, \tau_2, \tau_3) \, A e^{j \omega_1 \{t-\tau_1\}} A e^{j \omega_1 \{t-\tau_2\}} A e^{-j \omega_1 \{t-\tau_3\}} \\
&\quad + \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \tilde{h}_3(\tau_1, \tau_2, \tau_3) \, B e^{j \omega_2 \{t-\tau_1\}} A e^{j \omega_1 \{t-\tau_2\}} B e^{-j \omega_2 \{t-\tau_3\}} \\
&\quad + \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \tilde{h}_3(\tau_1, \tau_2, \tau_3) \, B e^{j \omega_2 \{t-\tau_1\}} B e^{j \omega_2 \{t-\tau_2\}} B e^{-j \omega_2 \{t-\tau_3\}}
\end{align*}
\]  

(3.30)
Expanding out the exponentials and collecting terms leads to,

\[
\tilde{y}(t) = Ae^{j\omega_1 t} \int_{-\infty}^{\infty} d\tau_1 \tilde{h}_1(\tau_1) e^{-j\omega_1 \tau_1} + B^2 e^{j2\omega_1 t} \int_{-\infty}^{\infty} d\tau_1 d\tau_2 \tilde{h}_2(\tau_1, \tau_2) e^{-j(\omega_2 \tau_1 + \omega_2 \tau_2)} + A^3 e^{j\omega_1 t} \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \tilde{h}_{3in}(\tau_1, \tau_2, \tau_3) e^{-j(\omega_1 \tau_1 + \omega_1 \tau_2 - \omega_1 \tau_3)} + B^2 A e^{j\omega_1 t} \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \tilde{h}_{3out}(\tau_1, \tau_2, \tau_3) e^{-j(\omega_2 \tau_1 + \omega_2 \tau_2 - \omega_2 \tau_3)}
\]

(3.31)

where \(\tilde{y}(t)\) represents the complex envelope signal outputted by the cross-channel BBVS model for the sinusoidal inputs (3.29). By taking the multi-dimensional Fourier transforms of the BB Volterra kernels in (3.31), we can reveal the frequency composition of the cross-channel BBVS model during the identification procedure. This is displayed formally in (3.33) below where the multi-dimensional Fourier transform of the \(p^{th}\) order Volterra kernels, \(\tilde{h}_p(\tau_1, \ldots, \tau_p)\), is described by the generalised frequency response function presented in (3.32) [59, 56].

\[
\tilde{H}_p(\omega_1, \ldots, \omega_p) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} d\tau_1 \cdots \int_{-\infty}^{\infty} d\tau_p \tilde{h}_p(\tau_1, \ldots, \tau_p) e^{-j(\omega_1 \tau_1 + \cdots + \omega_p \tau_p)}
\]

(3.32)

\[
\tilde{y}(t) = Ae^{j\omega_1 t} \tilde{H}_1(\omega_1) + B^2 e^{j2\omega_1 t} \tilde{H}_2(\omega_2, \omega_2) + A^3 e^{j\omega_1 t} \tilde{H}_{3in}(\omega_1, \omega_1, -\omega_1) + B^2 A e^{j\omega_1 t} \tilde{H}_{3out}(\omega_2, \omega_1, -\omega_2)
\]

(3.33)

Importantly in (3.33): \(\tilde{H}_1(\omega_1), \tilde{H}_2(\omega_2, \omega_2), \tilde{H}_{3in}(\omega_1, \omega_1, -\omega_1), \tilde{H}_{3out}(\omega_2, \omega_1, -\omega_1)\) denote the Fourier transforms of the Volterra kernels \(\tilde{h}_1(\tau_1), \tilde{h}_2(\tau_1, \tau_2), \tilde{h}_{3in}(\tau_1, \tau_2, \tau_3), \tilde{h}_{3out}(\tau_1, \tau_2, \tau_3)\) respectively. Examining the output \(\tilde{y}(t)\) in (3.33), it is clear that the phasor \(e^{j\omega_1 t}\) has a complex amplitude formed from a combination of complex frequency responses who magnitude and phase are dependent on both the amplitudes, \(A\) and \(B\), and angular frequencies, \(\omega_1\) and \(\omega_2\), of the input test signals (3.29). Furthermore, it is apparent from (3.33) that if the frequency of the interference test-tone, \(\omega_2\), was half that of the radar carrier path, \(\omega_1\), then the second order kernel would also contribute to the nonlinear characteristics of \(\tilde{y}(t)\) at frequency \(\omega_1\). Importantly, this frequency dependence leads to the cross-channel BBVS model having a nonlinear response with a non-trivial AM/PM characteristic at each frequency \(\omega_1\). As an aside, if the nonlinearity was strictly memoryless then the Volterra kernels would take the form of a simple impulse whose amplitudes are defined by the corresponding memoryless coefficients: \(\tilde{h}_1(\tau_1) = \tilde{\alpha}_1 \delta(\tau_1); \tilde{h}_2(\tau_1, \tau_2) = \tilde{\alpha}_2 \delta_2(\tau_1, \tau_2); \tilde{h}_{3in}(\tau_1, \tau_2, \tau_3) = \tilde{\alpha}_{3in} \delta_{3in}(\tau_1, \tau_2, \tau_3); \tilde{h}_{3out}(\tau_1, \tau_2, \tau_3) = \tilde{\alpha}_{3out} \delta_{3out}(\tau_1, \tau_2, \tau_3)\) where \(\delta_1, \delta_2\) and \(\delta_3\) represent linear, two-dimensional and three-dimensional impulses respectively. This would lead to (3.33) having a nonlinear response with no AM/PM behaviour.
as the amplitude of the output phasor $e^{j\omega_1 t}$ would have no dependence on the input frequencies $\omega_1$ and $\omega_2$ due to the fact that: $\tilde{H}_1(\omega_1) = \tilde{\alpha}_1; \tilde{H}_2(\omega_2, \omega_2) = \tilde{\alpha}_2; \tilde{H}_{3,n}(\omega_1, \omega_1, -\omega_1) = \tilde{\alpha}_{3,n}; \tilde{H}_{3,\text{out}}(\omega_2, \omega_1, -\omega_2) = \tilde{\alpha}_{3,\text{out}}$.

Like all Fourier transforms, the generalised frequency response function (3.32) is invertible. Therefore, $\tilde{H}_p(\omega_1, \ldots, \omega_p)$ can be related back to the desired BB time-domain kernels $\tilde{h}_p(\tau_1, \ldots, \tau_p)$ via the multi-dimensional inverse Fourier transform described by the inverse generalised frequency response function (3.34) \[59\].

$$\tilde{h}_p(\tau_1, \ldots, \tau_p) = \int_{-\infty}^{\infty} d\omega_1 \cdots \int_{-\infty}^{\infty} d\omega_p \tilde{H}_p(\omega_1, \ldots, \omega_p) e^{j(\omega_1 \tau_1 + \cdots + \omega_p \tau_p)} \tag{3.34}$$

Examining (3.34) it becomes clear that in order to perform this inverse transform we must know $\tilde{H}_p$ over the entire $p$ dimensional frequency space. Therefore, characterising our cross-channel BBVS model with simple sinusoidal input signals (3.29) leads to incomplete frequency domain kernels in (3.33) that have a single or, in the case of $\tilde{H}_{3,\text{out}}$, dual dependence on $\omega_1$ and $\omega_2$. This limited frequency dependence means that the inverse Fourier transform cannot be performed on the quadratic and cubic frequency domain kernels in (3.33). This result highlights the need for modulation signals with a richer frequency composition to be employed in the characterisation procedure so that the entire kernel frequency space is stimulated as the signals pass through the nonlinearity. It is clear from the (3.34) that in order to satisfy this condition for the $P^{\text{th}}$ order BBVS kernel, a minimum of $P$ independent tones must be present in the input modulations signals if the model is to be identified correctly \[59, 92, 68, 34, 33\]. This is effectively a statement of the persistent excitation condition which was discussed previously in chapter 2.

We have already stated above that in order to determine a $P^{\text{th}}$ order Volterra kernel we need a minimum of $P$ frequencies to be present in both input modulation signals $\tilde{x}_1$ and $\tilde{x}_2$. While this is true, an identification measurement performed using the minimum number of frequencies will only give us a slice through the $P$-dimensional Volterra kernel frequency space \[33\]. If we are to completely characterise all of the cross-channel BBVS kernels we must explore the full extent of their individual frequency spaces. By discretising the cross-channel BBVS model in (3.28) we set the number of frequency points we must visit to fully characterise our model \[92\]. We can choose to visit each of these frequency points by systematically sweeping each individual tone in the modulation signals or by modulating our inputs with random noise. In the communications literature, deterministic noise signals are often used to characterise the nonlinear behaviour of RF amplifiers as their high frequency content guarantees that the persistent excitation condition is satisfied during the identification procedure. \[80, 92, 33, 34, 56\]. Additionally, the random nature of the input signals’ instantaneous amplitude means that the gross input power does not need to be swept across the weakly nonlinear regime of the device. This is due to the fact that the random fluctuations of the noise will naturally excite the full weakly nonlinear region of the device during the characterisation procedure. This process was
illustrated previously in Figure 2.12 where the simultaneous noise identification samples are projected on top of the classic swept measurement.

### 3.3.2 Linear Least Squares Algorithm

Time-domain measurements provide the most complete method for characterising the nonlinear behaviour of a front-end radar amplifier as they can capture the device's genuine response to a frequency rich input signal. While swept-frequency/amplitude measurements are popular in the field of radar, they fundamentally fail to describe the complex interactions between different frequency components of the signal due to the fact that the principle of superposition does not hold in nonlinear systems. In contrast, given a rich enough input signal these memory driven interactions are automatically captured by a time-domain measurement of the system removing the need to infer any nonlinear behaviour from partial measurements of the kernels [34]. Therefore, in this thesis we choose to perform all nonlinear system identifications in the time-domain by means of a noise-based identification procedure. By choosing the modulation signals \( \tilde{x}_1[n] \) and \( \tilde{x}_2[n] \) to be deterministic random noise whose bandwidth is equal to the BB sampling frequency \( f_{s_2} \), we ensure that persistent excitation condition is adequately satisfied throughout the entire identification process [80]. Referring back to Figure 3.4, this means that the full modelling bandwidth of the cross-channel BBVS model is simultaneously characterised during the time-domain system identification by the independent noise waveforms modulated onto radar and interference carrier frequencies respectively. The task for the identification algorithm is to therefore identify the cross-channel BBVS model coefficients that most accurately map the input noise waveforms onto the measured nonlinear output signal, see Figure 3.4. Importantly, this identification process can be formulated as an optimisation problem where the solution is the Volterra coefficients that minimise the error between the modelled nonlinear output and the measured nonlinear output.

There are many different methods for determining the solution to an optimisation problem, however for the purposes of this thesis we choose to employ the linear least squares (LLS) algorithm to identify the kernel coefficients in (3.28) [91, 80, 63, 20]. The LLS algorithm was chosen for this analysis over other methods such as the Greedy approach as it is accurate, computationally efficient and it fits very well with the linear-in-the-parameters formalisation of the Volterra series model. Furthermore, the LLS algorithm performs well against Gaussian noise which we use throughout this thesis to model thermal noise at the input and output of the simulated RF nonlinearity. Crucially, the signal transforms used to derive the cross-channel BBVS model preserved the unique linear-in-the-parameters property of the black-box behavioural model. This in turn allows us to express (3.28) in the following matrix algebra (3.35) which importantly, is the correct form to compute the LLS solution.

\[
\hat{y} = \hat{V} \hat{h}
\] (3.35)
In (3.35), $\hat{V}$ represents the Volterra observation matrix of size $N \times D$ containing the cross-channel BBVS functionals $\hat{\sigma}^{[p]}_{\{i_1,i_2,i_3\}}[n]$.

$$\hat{V} = \left( \begin{array}{cccccc} \hat{\sigma}^{(1)}_{\{0\}}[1] & \cdots & \hat{\sigma}^{(1)}_{\{L-1\}}[1] & \hat{\sigma}^{(2)}_{\{0,0,0\}}[1] & \cdots & \hat{\sigma}^{(3)}_{\{L-1,L-1,L-1\}}[1] \\ \vdots & \ddots & \ddots & \vdots & \ddots & \vdots \\ \hat{\sigma}^{(1)}_{\{0\}}[N] & \cdots & \hat{\sigma}^{(1)}_{\{L-1\}}[N] & \hat{\sigma}^{(2)}_{\{0,0,0\}}[N] & \cdots & \hat{\sigma}^{(3)}_{\{L-1,L-1,L-1\}}[N] \end{array} \right)$$

(3.36)

where,

$$\hat{\sigma}^{(1)}_{\{i_1\}}[n] = \tilde{x}_1[n - i_1]$$

$$\hat{\sigma}^{(2)}_{\{i_1,i_2\}}[n] = \tilde{x}_2[n - i_1]\tilde{x}_2[n - i_2]$$

$$\hat{\sigma}^{(3)}_{\{i_1,i_2,i_3\}}[n] = \tilde{x}_1[n - i_1]\tilde{x}_1[n - i_2]\tilde{x}_1[n - i_3]$$

$$\hat{\sigma}^{(3_{out})}_{\{i_1,i_2,i_3\}}[n] = \tilde{x}_2[n - i_1]\tilde{x}_2[n - i_2]\tilde{x}_2[n - i_3]$$

(3.37)

Furthermore, in (3.35) $\tilde{h}$ denotes the Volterra kernel vector of length $D \times 1$ formed by concatenating all of the individual kernel coefficients from $\tilde{h}_1$, $\tilde{h}_2$, $\tilde{h}_{3_{in}}$ and $\tilde{h}_{3_{out}}$ into a column vector:

$$\tilde{h} = \left( \begin{array}{c} \tilde{h}^{(1)}_{\{0\}} \\ \vdots \\ \tilde{h}^{(1)}_{\{L-1\}} \\ \tilde{h}^{(2)}_{\{0,0\}} \\ \vdots \\ \tilde{h}^{(2)}_{\{L-1,L-1\}} \\ \tilde{h}^{(3)}_{\{0,0,0\}} \\ \vdots \\ \tilde{h}^{(3)}_{\{L-1,L-1,L-1\}} \\ \tilde{h}^{(3_{out})}_{\{0,0,0\}} \\ \vdots \\ \tilde{h}^{(3_{out})}_{\{L-1,L-1,L-1\}} \end{array} \right)$$

(3.38)

and $\tilde{y}$ is the output signal vector of length $N \times 1$ containing the BB nonlinear output samples $\tilde{y}[n]$

$$\tilde{y} = \left( \begin{array}{c} \tilde{y}[1] \\ \vdots \\ \tilde{y}[N] \end{array} \right)$$

(3.39)

Note, in the above expressions $N$ represents the total number of BB signal samples and the vector length $D$ has a size $\{ L + \{1 + L \frac{L}{2}\} + \{1 + L \frac{L}{2}\}L + L^3 \}$ in the case of the truncated cross-channel BBVS model (3.28).

The LLS solution to the problem defined by (3.35) is therefore,

$$\hat{h} = (\hat{V}^H \hat{V})^{-1} \hat{V}^H \tilde{y}$$

(3.40)

where $\hat{h}$ is the estimate of the cross-channel BBVS kernel coefficients (3.38) and $H$ is the
conjugate/Hermitian transpose [80, 63, 20]. Importantly, in order for the LLS algorithm to find the correct solution, an overdetermined system is required where the number of equations is greater than the number of unknowns [93]. This requirement manifests itself in the Volterra observation matrix $\tilde{V}$, which must be a full rank rectangular matrix where $N \geq D$ if the corresponding matrix $\tilde{V}^H \tilde{V}$ is to be invertible in (3.40). Ensuring that the columns of the Volterra observation matrix (3.36) are linearly independent throughout the entire identification process is challenging, particularly with nonlinear memory effects operating along the rows of matrix $\tilde{V}$. However, by choosing the input identification signals $\tilde{x}_1$ and $\tilde{x}_2$ to be independent noise signals the number of realisations in the system identification can be maximised. This in turn allows the full rank condition attached to matrix $\tilde{V}$ to be comfortably satisfied and the system identification to be optimised.

In this thesis, we choose to implement the LLS algorithm in a recursive fashion in order to achieve faster convergence and to limit the size of the matrices in (3.40). As discussed previously, depending on the memory length $L$ the Volterra series can have a very large number of terms and therefore a lot of training data is required to accurately identify its kernels. However, by employing the recursive least squares (RLS) algorithm [93, 91, 94] the solution to (3.35) can be re-formulated so that the estimate of the Volterra kernels, $\hat{h}$, can be updated iteratively allowing the input data length $N$ to be reduced to size $D$. The BB identification procedure can therefore be repeated with new randomly generated noise inputs until the RLS algorithm has converged to its minimum mean squared error solution. This type of iterative system identification is well suited for the nonlinear characterisation of modern radar systems as the large receiver bandwidths mean that capturing lots of training data in a single measurement could be seriously challenging for the analogue to digital converter (ADC). By performing lots of shorter nonlinear characterisation measurements the load can be taken off the measurement hardware without compromising the accuracy of the Volterra kernels estimate $\hat{h}$. The RLS algorithm employed in this thesis is presented in Table 3.1.

### 3.3.3 Polyphase Decomposition

In order to characterise the complex nonlinear behaviour of a front-end radar amplifier using the cross-channel BBVS model (3.28), the device’s nonlinear impulse response must be probed using noise modulation signals so that corresponding kernel coefficients can be accurately identified by the LLS algorithm in the BB domain. This sophisticated identification procedure was illustrated previously in Figure 3.3 and has been discussed in detail in the preceding sections. Importantly, before we can be confident that the BB system identification can successfully learn the correct Volterra kernel coefficients, we must first consider the response of the nonlinear device to a genuinely wideband input signal. In order to study this, we probe the continuous-time cross-channel BBVS model (3.21) with
Table 3.1: Summary of RLS algorithm used to estimate the cross-channel BBVS kernels (3.38) in the BB system identification procedure. The kernel estimate $\hat{h}$ is initialised for the first measurement iteration $\beta$ in line 3 before being updated for all future iterations on line 7. The effect previous measurement sets have on the current Volterra kernel estimate is controlled by the fading memory parameter $\epsilon$, which can take a value between 0 and 1.

The following complex envelope signals:

\[
\begin{align*}
\tilde{x}_1(t) &= A\{e^{(-j\omega_1 t)} + e^{(j\omega_1 t)}\} \\
\tilde{x}_2(t) &= A\{e^{(-j\omega_2 t)} + e^{(j\omega_2 t)}\}
\end{align*}
\]  

(3.41)

where $\{-\omega_1, \omega_1\}$ and $\{-\omega_2, \omega_2\}$ define the band edges of the carrier and interference modulation signals respectively. For the purposes of this analysis, we assume that both modulation signals $\tilde{x}_1$ and $\tilde{x}_2$ have the same bandwidth which is equal to the baseband
sampling frequency $f_{s2}$ and therefore $\omega_1 = \omega_2 = \pi f_{s2}$. Substituting (3.41) into our expression for the continuous-time cross-channel BBVS model (3.21) we find,

\[
\tilde{y}(t) = \int_{-\infty}^{\infty} d\tau_1 \tilde{h}_1(\tau_1) A \{ e^{-j\omega_1(t-\tau_1)} + e^{j\omega_1(t-\tau_1)} \}
+ \int_{-\infty}^{\infty} d\tau_1 d\tau_2 \tilde{h}_2(\tau_1, \tau_2) \left\{ B \{ e^{-j\omega_2(t-\tau_1)} + e^{j\omega_2(t-\tau_1)} \} + B \{ e^{-j\omega_2(t-\tau_2)} + e^{j\omega_2(t-\tau_2)} \} \right\}
+ \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \tilde{h}_{3_{\text{in}}}(\tau_1, \tau_2, \tau_3) \left\{ A \{ e^{-j\omega_1(t-\tau_1)} + e^{j\omega_1(t-\tau_1)} \} + A \{ e^{-j\omega_1(t-\tau_2)} + e^{j\omega_1(t-\tau_2)} \} \right\}
+ \int_{-\infty}^{\infty} d\tau_1 d\tau_2 d\tau_3 \tilde{h}_{3_{\text{out}}}(\tau_1, \tau_2, \tau_3) \left\{ B \{ e^{-j\omega_2(t-\tau_1)} + e^{j\omega_2(t-\tau_1)} \} + B \{ e^{-j\omega_2(t-\tau_2)} + e^{j\omega_2(t-\tau_2)} \} \right\}
\]

(3.42)

Expanding out the brackets and applying the generalised frequency response function (3.32), we can reveal the Fourier domain structure of the nonlinear output $\tilde{y}(t)$.

\[
\tilde{y}(t) = A \left\{ \tilde{H}_1(-\omega_1)e^{-j\omega_1 t} + \tilde{H}_1(\omega_1)e^{j\omega_1 t} \right\}
+ B^2 \left\{ \tilde{H}_2(-\omega_2, -\omega_2)e^{-j2\omega_2 t} + \tilde{H}_2(\omega_2, -\omega_2) + \tilde{H}_2(-\omega_2, \omega_2) + \tilde{H}_2(\omega_2, \omega_2)e^{j2\omega_2 t} \right\}
+ A^3 \left\{ \tilde{H}_{3_{\text{in}}}(\omega_1, -\omega_1, \omega_1)e^{j\omega_1 t} + \tilde{H}_{3_{\text{in}}}(\omega_1, -\omega_1, -\omega_1)e^{-j\omega_1 t} + \tilde{H}_{3_{\text{in}}}(\omega_1, -\omega_1, \omega_1)e^{j3\omega_1 t} + \tilde{H}_{3_{\text{in}}}(\omega_1, -\omega_1, -\omega_1)e^{-j3\omega_1 t} + \tilde{H}_{3_{\text{in}}}(\omega_1, \omega_1, -\omega_1)e^{j\omega_1 t} + \tilde{H}_{3_{\text{in}}}(\omega_1, \omega_1, \omega_1)e^{-j\omega_1 t} + \tilde{H}_{3_{\text{in}}}(\omega_1, \omega_1, -\omega_1)e^{j3\omega_1 t} + \tilde{H}_{3_{\text{in}}}(\omega_1, \omega_1, \omega_1)e^{-j3\omega_1 t} \right\}
+ B^2 A \left\{ \tilde{H}_{3_{\text{out}}}(\omega_2, \omega_1, -\omega_2)e^{j\omega_1 t} + \tilde{H}_{3_{\text{out}}}(\omega_2, -\omega_1, -\omega_2)e^{-j\omega_1 t} + \tilde{H}_{3_{\text{out}}}(\omega_2, -\omega_1, \omega_2)e^{j2\omega_2 - \omega_1} t + \tilde{H}_{3_{\text{out}}}(\omega_2, -\omega_1, -\omega_2)e^{-j(2\omega_2 - \omega_1) t} + \tilde{H}_{3_{\text{out}}}(\omega_2, -\omega_1, \omega_2)e^{j(2\omega_2 + \omega_1) t} + \tilde{H}_{3_{\text{out}}}(\omega_2, -\omega_1, -\omega_2)e^{-j(2\omega_2 + \omega_1) t} + \tilde{H}_{3_{\text{out}}}(\omega_2, \omega_1, \omega_2)e^{-j\omega_1 t} + \tilde{H}_{3_{\text{out}}}(\omega_2, -\omega_1, \omega_2)e^{j\omega_1 t} \right\}
\]

(3.43)

Examining equation (3.44) it is apparent that some of the energy outputted from the cross-channel BBVS model is distributed onto harmonics of the input modulation frequencies, $\{\pm \omega_1, \pm \omega_2\}$. This results in the nonlinear output signal $\tilde{y}(t)$ spanning a greater frequency extent than the BB inputs $\tilde{x}_1$ and $\tilde{x}_2$. If we now imagine that instead of specifying the input modulation signals as consisting of two independent tones, we define them to be
deterministic noise containing a full spectrum of tones out to the angular frequencies \( \pm \omega_1 \) and \( \pm \omega_2 \) respectively. In this case, the nonlinear output signal would be wider in bandwidth than the BB inputs. This effect is due to the intrinsic bandwidth expansion properties of the RF nonlinearity [33, 34] and is captured by both the PBVS and BBVS behavioural models\(^2\) [20, 57]. Importantly, the extent of the spectral regrowth is not dependent on the chosen modelling domain but is rather directly related to the order of the kernel that drives the nonlinear effect. In the context of this thesis, we have defined the modulation signals \( \tilde{x}_1 \) and \( \tilde{x}_2 \) to both have a bandwidth equal to \( f_{s_2} \) meaning that each nonlinear kernel generates an output signal with an expanded bandwidth equal to \( pf_{s_2} \), where \( p \) is the nonlinear kernel order. This poses a serious challenge for the BB system identification as in order to capture the full extent of the nonlinear output envelope \( \tilde{y}(t) \) it must be sampled at a significantly higher rate than the input noise envelopes \( \tilde{x}_1[n] \) and \( \tilde{x}_2[n] \). This nonlinear multirate system identification problem is illustrated in Figure 3.5 which expands on the BB system identification procedure presented previously in Figure 3.4.

In order for our cross-channel BBVS model to accurately characterise the PB nonlinearity, the nonlinear bandwidth expansion effects described above must be captured during the BB system identification procedure. Therefore, we must sample \( \tilde{y}(t) \) at three times the sampling rate of the input modulation signal, \( f_{s_2} \), if we are to satisfy the Nyquist-Shannon sampling theorem [91] regarding the output signal and capture the bandwidth expansion effects up to the cubic order. Sampling at this high rate successfully avoids aliasing effects in \( \tilde{y}[n] \) but introduces significant challenges for the system identification. Unfortunately, we cannot simply oversample the input signals and perform the identification of the cross-channel BBVS model at the high sampling rate for two key reasons. Firstly, the BB sample delay \( \frac{1}{f_{s_2}} \) is deliberately matched to the bandwidth of the input envelopes so that the BBVS tap delay width is intrinsically linked to the memory effects stimulated in the PB nonlinearity. If the input modulation signals \( \tilde{x}_1[n] \) and \( \tilde{x}_2[n] \) were to be translated to the high sampling rate, as illustrated in Figure 3.5, the BBVS memory taps would be decoupled from the input signal bandwidth resulting in more terms being required in the BBVS model to describe the same nonlinear memory effects. This could profoundly affect how identifiable the BBVS model is due to the fact that the number of terms in the Volterra series scales exponentially with memory length \( L \). Secondly, if we attempt to perform the system identification at the high sampling rate our LLS problem will become ill-conditioned causing the algorithm to tend towards numerically unstable solutions. This is due to the oversampling of the input signals \( \tilde{x}_1[n] \) and \( \tilde{x}_2[n] \) resulting in large sections of the identification space being unoccupied [33, 93, 91], see Figure 3.5. Multirate system identification techniques, specifically polyphase decomposition, must

\(^2\)It is important to note that for any system identification to be successful, we must assume that the input noise is marginal so that the input characterisation signal is know to a high degree of accuracy. We can therefore also assume that any noise magnification effects associated with the bandwidth expansion property of the nonlinearity will be negligible.
therefore be employed in order to successfully learn the kernels of the cross-channel BBVS model. Importantly, work published by Schwingshackl and Kubin in [95, 96, 97] extends the linear multirate theory [91, 93] so that the classic techniques can be applied to complex nonlinear filters.

Polyphase decomposition of Volterra kernels is the main principle developed by Schwingshackl and Kubin in [95] and it is this specific technique that we wish to exploit in order to identify our cross-channel BBVS model. The basic technique is illustrated in Figure 3.6 where the BB Volterra kernels are decomposed into sub-kernels \( \tilde{\kappa}_1, \tilde{\kappa}_2, \tilde{\kappa}_3 \) centred on different polyphase components. By performing three parallel, but smaller, system identifications a cross-channel BBVS model can be identified at the low sampling rate \( f_{s_2} \). This is a key result as it allows all of the nonlinear bandwidth expansion effects observed at the high sampling rate, \( 3f_{s_2} \), to be captured by a BBVS model identified at the low sampling rate, \( f_{s_2} \), therefore removing the ill-conditioning problem faced by the LLS algorithm. In essence, we must accept the fact that during the downsampling of the output signal to the low sampling rate there will be aliasing which causes the bandwidth expansion effects to be folded back into the LLS identification space \( \{-f_{s_2}/2 \rightarrow f_{s_2}/2\} \). Importantly, this aliasing operation does not lead to any loss of information as the aliased components
simply combine with rest of the signal through a complex addition which is purely dependent on the downsample phase. As the high sampling rate is three times greater than the low sampling rate in this case, the output signal can be downsampled with three different phases leading to three separate aliased outputs $\tilde{y}[1:3:3n], \tilde{y}[2:3:3n], \tilde{y}[3:3:3n]$. The key point however is to recognise that the bandwidth expansion effects and subsequent aliasing are naturally captured by the cross-channel BBVS model (3.28) when it is implemented at the low sampling rate $f_{s2}$. Therefore, each separate downsampled output can be accurately described by the cross-channel BBVS model so long as the BB input signals $\tilde{x}_1[n]$ and $\tilde{x}_2[n]$ are delayed by a temporal offset that is equivalent to the respective downsample phase. By matching the input signal delay to the downsample phase we ensure that the aliasing effects naturally implemented in the BBVS model are identical to those applied in the downsampling operation. Since the input sampling rate $f_{s2}$ is three times lower than the output sampling rate, this leads to three polyphase components whose temporal delays’ $\{z_1 = 0, z_2 = \frac{1}{3f_{s2}}, z_3 = \frac{2}{3f_{s2}}\}$ are a fraction of a BB input sample $\{\frac{1}{f_{s2}}\}$. In the case of the BB system identification, each polyphase component captures a different aliasing effect with the complete set of sub-kernels $\tilde{\kappa}_1, \tilde{\kappa}_2, \tilde{\kappa}_3$ collectively describing the full nonlinear bandwidth expansion effect at the high sampling rate $3f_{s2}$. This is illustrated in Figure 3.6.

### 3.3.4 Simulation Architecture

In order to prove that the theoretical results derived above are correct, it must be shown that our BBVS model can accurately capture all of the PBVS in-channel and cross-channel nonlinear distortion effects generated around the radar carrier due to an input signal of

\[\tilde{\kappa}_1, \tilde{\kappa}_2, \tilde{\kappa}_3\]

In the simulations performed in this chapter we choose to implement these fractional polyphase delays through the Fourier shift theorem.
the form described by (3.1). This verification was achieved by means of a comprehensive RF simulator written in the MATLAB programming language which is discussed in detail below. Importantly, the architecture of the RF simulation emulated the nonlinear characterisation measurement illustrated previously in Figure 3.3 with the nonlinearity described by means of the discrete-time PBVS model (3.22). In order for the PBVS model to be representative of a front-end radar amplifier it had to be placed around the radar carrier frequency $f_c$, which for the purposes of this thesis was chosen to be $10\text{GHz}$ so as to represent an X-band radar [13, 11, 21]. This unfortunately meant that the RF simulator had to employ an extremely high PB sampling rate, $f_{s_1}$, in order to accurately simulate the analogue domain. While it is imperative that the Nyquist-Shannon sampling theorem is satisfied when simulating the analogue domain in Figure 3.3, it is also important that the memory effects generated by PBVS model are of an order consistent with a real RF amplifier. It is well understood that the memory of a device is intrinsically linked to its bandwidth [15, 55, 38] and therefore, when employing the PBVS model to simulate the RF nonlinearity its memory tap delay width must be coupled to the bandwidth of the amplifier rather than the PB sampling frequency. We achieve this in the simulation environment through the BB sampling frequency $f_{s_2}$ which we use to define the receiver bandwidth of the RF amplifier. By setting the PBVS tap delay width, $l_p$, to be equal to a single BB sample interval, $\frac{1}{f_{s_2}}$, we successfully link the memory effects generated by the PBVS model to those produced by the BBVS model and by extension the bandwidth of the RF device. This was discussed in detail in section 3.2.3 where a theoretical mapping of the PBVS kernels to the BB domain was also presented. Importantly, in order for the theoretical expressions for the BBVS kernels to be valid the passband sampling frequency, $f_{s_1}$, had to be an integer multiple of the BB sampling frequency, $f_{s_2}$, so that the PB tap delay width, $l_p$, was also an integer number of discrete-time samples, $l_p = f_{s_1}f_{s_2}^{-1}$. In the case of the RF simulations performed in the chapter, the PB sampling frequency $f_{s_1}$ was therefore chosen to be $720$ times higher$^4$ than the BB sampling frequency $f_{s_2}$.

Now that the structure of the RF simulation environment has been defined, we must decide on the specific PB nonlinearity used to test the performance of the cross-channel BBVS model (3.28). As mentioned previously, we assume that the most dominant nonlinear effects can be captured by terms up to the cubic order of the series which allows us to model the RF nonlinearity using the PBVS model displayed in (3.22). Furthermore, we choose the PBVS model to have a memory length $L = 7$ so as to generate a wide variety of complex nonlinear memory effects that will adequately test the validity of BB identification procedure. Selecting such a large memory length however leads to the PBVS model being extremely computationally intensive at the high sampling rate due to (3.22) having an exorbitant number of terms, $L + L^2 + L^3 = 399$. Therefore in order to improve the efficiency of the RF simulator, the symmetric terms in the PBVS model (3.22) were

$^4$The reason the PB sampling frequency was chosen to be so much higher than the BB sampling frequency was to limit signal processing effects due to sampling rate mismatches in the RF simulator’s up-conversion and corresponding down-conversion routines.
removed through a similar methodology to that employed for BBVS kernels resulting in
the truncated PBVS model displayed below:

\[
y[m] = \sum_{l_1=0}^{L-1} h_1[l_1] x[m-l_1] + \sum_{l_1=0}^{L-1} \sum_{l_2=l_1}^{L-1} h_2[l_1,l_2] x[m-l_1] x[m-l_2] \\
+ \sum_{l_1=0}^{L-1} \sum_{l_2=l_1}^{L-1} \sum_{l_3=l_2}^{L-1} h_3[l_1,l_2,l_3] x[m-l_1] x[m-l_2] x[m-l_3]
\] (3.44)

It is important to note that by setting the memory length of our PBVS model to \( L = 7 \), we also set the memory length of our cross-channel BBVS model to be the same. As discussed previously, we have deliberately tied the memory of the PBVS model to the BB sampling frequency so that the nonlinear memory effects generated in the PB domain are intrinsically linked to the bandwidth of the input signal. This is reflected in the theoretical derivation presented above where the memory length parameter \( L \) is shared by both the PBVS (3.22) and BBVS (3.28) models respectively.

While the exact form of the PB nonlinearity implemented in the RF simulator has now been defined (3.44), we still need to decide on the specific PBVS kernel coefficients used to test the theoretical results presented above. Unfortunately, to the author’s knowledge there is nothing published in the available literature on the cross-channel memory behaviour of a typical RF amplifier. Therefore, in order to construct a challenging test scenario for the BBVS model identification, an arbitrary nonlinear impulse response had to be generated where the PBVS kernel coefficients were selected from a random distribution. Even though the PBVS model coefficients had to be arbitrary, some basic assumptions about the shape of the kernels could still be made in order to make the nonlinear impulse response more representative of a real RF amplifier. Firstly, it is assumed that the nonlinear kernel coefficients generally diminish with each subsequent memory term. This assumption is based on the radar receiver being casual with small relaxation times. Secondly, we assume that the on-diagonal memory coefficients are larger than their respective off-diagonal coefficients in the higher order kernels. This assumption is founded in the communications literature [15, 9] where to save computational complexity the Volterra series is typically reduced to the memory polynomial, see chapter 2. In short, the memory polynomial retains the more dominant on-diagonal coefficients and discards the weaker off-diagonal coefficients. The PBVS kernels chosen for this analysis therefore followed a Chebyshev filter response for the on-diagonal coefficients with the off-diagonal coefficients in the quadratic and cubic order kernels set to decay exponentially. This is illustrated for the PBVS kernel coefficients implemented in the RF simulator in Figure 3.7. Importantly, the gross power level of each PBVS kernel was selected so that they all contributed to the nonlinear output signal \( \tilde{y} \) when stimulated by an input signal of the form described by (3.1).

In order to verify that our BBVS model (3.28) can accurately capture all of the PBVS
3.3. Baseband System Identification

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
\includegraphics[width=\textwidth]{linear_kernel_coeffs.png}
\caption{Linear PBVS Kernel Coefficients}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\includegraphics[width=\textwidth]{quadratic_kernel_coeffs.png}
\caption{Quadratic PBVS Kernel Coefficients}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\includegraphics[width=\textwidth]{cubic_kernel_coeffs.png}
\caption{Cubic PBVS Kernel Coefficients}
\end{subfigure}
\caption{Amplitude representation of the Volterra kernel coefficients from the truncated PBVS model implemented in the RF simulator (3.44). Memoryless coefficient highlighted in green, on-diagonal coefficients highlighted in red and the off-diagonal coefficients highlighted in blue.}
\end{figure}

...in-channel and cross-channel nonlinear effects due to an out-of-band interferer at half the carrier frequency, we must simulate the BB input signals being passed through each nonlinear behavioural model independently and prove that they produce the same result for $\tilde{y}$. Before we can perform this comparison however, we must first identify the correct BBVS kernel coefficients. To do this we perform the multirate LLS identification described above using our input noise modulation signals, $\tilde{x}_1$ and $\tilde{x}_2$, and the corresponding PBVS output envelope, $\tilde{y}$, down-converted from the radar carrier, $f_c$. Importantly, we also produce a set of theoretical BBVS kernels defined by (3.24), (3.25), (3.26) and (3.27) respectively so that the mathematical derivation described above can be doubly verified in the simulation environment. Once the BBVS kernel coefficients have been identified, we generate a completely new set of input signals in order to perform the final model comparison. Similar to the identification routine, we choose to test the accuracy of our system identification using noise-based modulation signals as these will generate the most complex nonlinear memory effects and therefore allow us to probe the full extent of the BBVS models’ modelling capabilities.

Importantly, in the RF simulations the bandwidths of the input BB modulation signals $\tilde{x}_1$ and $\tilde{x}_2$ were chosen to be the same, $f_{x_2} = 100 MHz$, so that the maximum output
bandwidth was fully characterised by the cross-channel behavioural model. As mentioned previously, the BB system identification was tested against an arbitrary PBVS model of order $P = 3$ and memory length $L = 7$. This resulted in the total expanded bandwidth characterised by the BBVS model spanned $3f_{s_2} = 300\, MHz$ around the radar carrier $f_c = 10\, GHz$. In order to fully test the cross-channel modelling capabilities of our BBVS behavioural model, the out-of-band interference was centred on half the radar carrier frequency, $f_i = \frac{f_c}{2} = 5\, GHz$, so as to generate both even and odd order nonlinear effects in the PBVS output. Note, the simulation architecture discussed above is illustrated by a combination of Figure 3.3 and Figure 3.5. Testing the performance of the BB behavioural model in this unique interference scenario meant that the system identification had to successfully learn all of the kernel coefficients in (3.28) including those from $\tilde{h}_2[i_1, i_2]$. This required a significant amount of training data to enable the system identification to converge to the minimum squared error solution. Thus, in order to achieve faster convergence and minimise the size of the signal vectors produced in the PB domain, each BB system identification was performed with three times as many equations, $N$, as unknowns, $D$, so as to provide the learning algorithm with a significantly overdetermined system. This ensured that the RLS had enough information to accurately identify the BBVS kernel coefficients through varying levels of output noise [93].

3.3.5 Simulation Results

In this section we present the simulation results from the performance analysis of the cross-channel BBVS model discussed above. Once the BBVS kernel coefficients had been identified using the process described above, the RF simulator tested the accuracy the cross-channel BBVS model against the true nonlinear envelope down-converted from the PB domain. In order to perform this analysis, 100 completely new sets of input modulation signals were generated by the RF simulator which were then passed through the BBVS and PBVS behavioural models separately. The respective output envelopes were then averaged and compared in the frequency domain so that the accuracy of the BBVS model could be studied across the full expanded bandwidth, $\{-150\, MHz \rightarrow 150\, MHz\}$, of the nonlinear output signal $\tilde{y}$. By averaging the output spectrums in this way, the random fluctuations of the noise waveforms could be lessened allowing for a more fundamental comparison of the two models’ nonlinear behaviour. This analysis was done for the BBVS kernels in (3.28) both individually and collectively in order to check that the BB system identification successfully satisfies the persistent excitation condition in each of the $p$-dimensional kernel spaces. The results for the individual kernel analysis are displayed in Figure 3.8 where the amplitude spectrum of the BBVS output envelope
3.3. Baseband System Identification

(Figure 3.8: Comparison results from the individual kernel analysis. Averaged envelope spectrums from the BBVS model, blue, plotted on top of the averaged outputs from respective PBVS model, magenta. In all cases, it is clear that the BBVS model can accurately describe the complex memory behaviour of arbitrary PBVS model.)

is plotted on top of the corresponding output spectrum down-converted from the PB domain. Examining the results in Figure 3.8, it is clear that the BBVS model can accurately describe the complex in-channel and cross-channel nonlinear memory behaviour of the arbitrary PBVS nonlinearity. The noise-based identification procedure has successfully satisfied the persistent excitation condition for each individual kernel and in doing so, has characterised the nonlinear coupling between \( \bar{x}_1 \) and \( \bar{x}_2 \) through the PBVS model. It is important to note that while we use the output power spectral density here to illustrate the accuracy of the BBVS model, the complex phase behaviour of the PBVS nonlinearity is described just as well by the cross-channel BBVS model in the above simulations.

In order to study the robustness of BB system identification procedure in more detail, further simulation results were generated where the BBVS model was identified through varying levels of output noise. This was done for the full set of BBVS kernels in (3.28) using the following signal to noise ratios (SNR): \( \text{SNR}_{\text{out}} = 20\,\text{dB},\,30\,\text{dB},\,40\,\text{dB} \) and \( \infty\,\text{dB} \). The accuracy of these models was also tested alongside the theoretical BBVS kernels defined by: (3.24), (3.25), (3.26) and (3.27). Importantly, in order to generate the full bandwidth expanded output signal for the theoretical case, each polyphase sub-kernel had to
Chapter 3. Modelling Nonlinear Radar Receivers

Figure 3.9: Simulation results comparing the output signals from different cross-channel BBVS models identified through varying levels of output noise: SNR = 20dB (Green), SNR = 30dB (Gold), SNR = 40dB (red), SNR = ∞dB (blue), theoretical (Black). The BB system identification learns the BBVS kernel coefficients to a high degree of accuracy through the output noise in all cases.

be set equal to the theoretical BBVS kernels translated from the PB domain. In essence, the input delays attached to each polyphase component, see Figure 3.6, effectively unravels the aliasing effects produced by the theoretical kernels at the low sampling rate, \(f_s\). This allowed the theoretical signal’s fully expanded bandwidth to be compared with the other output signals at the high sampling rate, \(3f_s\). The results displayed in Figure 3.9 therefore compare the output envelopes \(\tilde{y}\) from the five separate BBVS models in response to exactly the same input modulation signals. The comparison takes the from \(10log10(S(f)) - 10log10(S_t(f))\) where: \(S(f)\) is the power spectral density of the signal outputted from BBVS model and \(S_t(f)\) is the corresponding PBVS power spectral density down-converted from the PB domain. As with the results presented in Figure 3.8, the output signals were averaged using 100 independently simulated datasets in order to reduce the random fluctuation of the noise waveforms and make the comparison easier.

Examining the results in Figure 3.9, it is clear that the cross-channel BBVS model can capture all of the bandwidth expansion effects generated by arbitrary PBVS nonlinearity. This is an important result as it suggests that multirate system identification techniques can be used to identify the nonlinear memory behaviour of BB behavioural models including the complex bandwidth expansion effects. Furthermore, it is shown in Figure 3.9 that the noise-based system identification procedure can successfully learn the BBVS kernel coefficients through significant levels of noise at the output of the RF nonlinearity.
3.4. Discussion and Summary

It is important to note that as the output SNR decreases, the model struggles to capture the nonlinear effects at the band edges of the expanded identification space. This is to be expected as the additive noise is considerably stronger than the weak nonlinear distortion effects in this region, see Figure 3.8c and Figure 3.8d. Finally, it is clear from the black curve in Figure 3.9 that the theoretical kernels perfectly describe the behaviour of the PBVS model in the baseband domain. This final simulation effectively verifies the mathematical derivation of the cross-channel BBVS model presented in this chapter.

3.4 Discussion and Summary

In this chapter, we presented a novel behavioural modelling technique for radar systems capable of describing the nonlinear coupling between the desired radar channel and an out-of-band interferer in the scene. The technique takes inspiration from the communication literature [59, 63, 20, 80] and was specifically focussed on modelling complex nonlinear memory effects in the BB domain. In order to derive this complex BB behavioural model, the desired effects from the universal passband Volterra series had to be dropped to the BB domain by invoking the narrowband assumption [59]. Importantly, the narrowband assumption was invoked through a dual-band complex envelope signal so that the resultant BB model described the nonlinear coupling between an out-of-band interferer and the desired radar channel. More specifically, the nonlinear scenario chosen for this analysis placed the interference signal on half the carrier frequency of the radar. This was done for two key reasons. Firstly, this is a particularly problematic interference scenario for modern radar systems as it provides the most direct path for the out-of-band energy to couple with the radar channel. Secondly, the derivation for this specific scenario leads to the most general form of the BB Volterra series model as it retains both even and odd order terms from the parent passband Volterra series. Once the novel cross-channel BBVS model had been derived, the focus of the research turned toward the development of a system identification technique capable of learning the memory rich Volterra kernels. This led to the derivation of the persistent excitation condition which states that; the characterisation signal must contain at least $P$ frequency components in each complex envelope signal if the behaviour of the $P^{th}$ order BBVS kernel is to be accurately identified. Digital noise waveforms were therefore employed in the BB system identification procedure as they are frequency rich and will naturally excite the weakly nonlinear region of the RF amplifier. However, this led to a multirate system identification problem due to the inherent bandwidth expansion properties of the nonlinearity causing the bandwidth of the output envelope to be larger than the bandwidth of the input modulation signals. In order to account for this phenomenon in the system identification procedure, polyphase decomposition techniques were exploited to enable the LLS identification algorithm to identify the BBVS kernel coefficients at the input sampling rate. This was verified through a series of complex simulations.
Importantly, the cross-channel BBVS model derived above is used throughout the remainder of this thesis to simulate complex nonlinear memory effects in modern radar systems and to develop sophisticated nonlinear mitigation techniques. While the full form of the BBVS model in (3.28) describes the scenario where the out-of-band interferer sits on half the radar carrier frequency, the model can be used to describe other out-of-band or in-band interference scenarios by removing the insignificant terms for that specific case. Importantly however, the fundamental narrowband assumption must always be observed when the BBVS model is employed which limits its modelling capabilities to a narrow bandwidth around the interferer and the radar respectively. The research presented in this chapter could be extended in future work by employing the modelling techniques discussed above to characterise the nonlinear behaviour of a real front-end radar amplifier. Measurement results of this type would provide radar system designers with invaluable insight into the nonlinear memory behaviour of the modern radar receiver. Furthermore, this in-depth knowledge of the radar receiver could then be used to influence the choice of nonlinear digital mitigation technique employed in the radar processor. This is discussed in detail in the following chapters which specifically focuses on the digital mitigation of harmful nonlinear effects generated in the modern radar receiver.
Chapter 4

LUT Inverse Mitigation Techniques

4.1 Aim and Introduction

One of the main research objectives of this thesis is to develop advanced digital signal processing solutions to combat the nonlinear receiver problem in modern radar. We start by focusing on the LUT inverse mitigation technique as it is by far the most popular and successful solution employed in the related field of communications. In principle, the LUT technique is a very attractive solution for modern radar systems as it is mode-independent and relatively computationally inexpensive to implement. Despite this its effectiveness has been studied sparingly in the radar literature. In this chapter we develop the LUT inverse mitigation technique for radar systems and present a comprehensive review of its performance against both memoryless and memory rich front-end receiver nonlinearities.

Cross-modulation is one of the most problematic nonlinear interference effects in radar as it can be generated in the RF receiver regardless of the relationship between the radar’s and the corresponding interferer’s centre frequencies. We therefore focus on the development of cross-channel LUT mitigation techniques in this chapter which are capable of compensating for harmful cross-modulation effects. Cross-modulation distortion is discussed in detail in chapter 2 but in short; the effect describes the case where strong out-of-band interference couples with the desired radar channel through a front-end nonlinearity causing unwanted distortion in the radar output. Importantly, it was shown in chapter 3 that this nonlinear coupling could be accurately described in the baseband (BB) domain by the cross-channel BBVS black-box behavioural model. By building a comprehensive radar simulator around the cross-channel black-box receiver model the LUT techniques could be developed and tested against simulated cross-modulation effects. While the frequency independent nature of the cross-modulation phenomenon means that it affects all radar modes of operation, its harmful distortion effects will be felt most profoundly in the popular MPRF mode where low SNR must be detected in the presence of strong clutter. The cross-channel LUT techniques developed in this chapter were therefore targeted at the MPRF mode of operation, however that does not mean that the general mitigation technique is strictly limited to this case. One of the most problematic cross-modulation scenarios observed in the MPRF radar mode corresponds to
the case where the interference signal is pulsed [8, 7]. In this specific case, the nonlinear coupling provides a mechanism for the pulsed interferer to modulate the strong MPRF clutter causing nonlinear repeats to be generated across the Doppler spectrum. This common interference effect was illustrated previously for the MPRF radar mode in Figure 2.4, where the specific PRF mismatch between the pulsed interferer and the radar determines the Doppler locations of the nonlinear repeats. It is clear from Figure 2.4 that for the LUT mitigation technique to be successful in restoring the radar’s sensitivity back to the linear case, it must reduce the level of the nonlinear distortion in the range-Doppler (RD) detection space without also removing the potential targets from the scene. This is a daunting task particularly when we consider nonlinear memory effects which as discussed previously are likely to become increasingly prevalent in modern radars as they move to wider and wider receiver bandwidths. Importantly, by designing the MPRF radar simulator around the BBVS black-box receiver model derived in chapter 3 the performance of the LUT techniques developed in this chapter could be tested against both memoryless and memory rich forward nonlinearities. The architecture of this MPRF radar simulator is discussed in detail in the following section.

Importantly, the work presented in this chapter was the first of its kind to be published in the radar literature. In [87] - (Euan Ward and Bernard Mulgrew. “Mitigation of Cross-modulation Effects in Radar Receivers with Memory”. In: 2020 IEEE International Radar Conference (RADAR). 2020, pp. 856–859. DOI: 10.1109/RADAR42522.2020.9114661) the direct inverse approach is applied to both the memoryless and memory rich cross-modulation scenario for a typical MPRF mode with in-depth performance analysis results presented. This work was then extended by the publication of [88] - (Euan Ward and Bernard Mulgrew. “Memory NLEQ Techniques to Mitigate Cross-modulation Effects in Radar”. In: 2021 IEEE Radar Conference (RadarConf21). 2021, pp. 1–6. DOI: 10.1109/RadarConf2147009.2021.9455301) which studied the tandem inverse approach and in particular how memory terms could be included in the inverse structure to compensate for complex memory effects generated in the forward nonlinearity.

In this chapter, the LUT technique is developed to mitigate cross-modulation effects in radar systems. In addition to performing in depth performance analysis on the standard memoryless LUT solutions, see section 4.3.4, the effectiveness of these techniques is studied in the case where the RF receiver exhibits complex nonlinear memory effects, see section 4.4.3. Furthermore, the cross-channel tandem inverse technique is extended to include nonlinear memory terms, see section 4.4.1, with its performance in mitigating complex nonlinear memory effects studied extensively for the MPRF radar mode, see section 4.4.3. Finally, the feasibility of implementing these cross-channel mitigation solutions in modern radar is discussed in detail with particular emphasis place on the identification of the memory rich tandem inverse structure.
4.2 MPRF Radar Simulator

In order to develop and test the novel LUT mitigation techniques designed in this chapter, a comprehensive radar simulator had to be constructed that was capable of simulating a variety of different interference scenarios. The radar simulator was written in the MATLAB programming language and was designed to be highly configurable so as to capture as many complex nonlinear effects as possible. For the purposes of this chapter, the MATLAB model was set to operate in the BB domain so that the entire RF receive chain could be simulated by the BBVS black-box behavioural model derived in chapter 3. For convenience the full cross-channel BBVS model is restated below in (4.1).

\[
\tilde{y}[n] = \sum_{i_1=0}^{L-1} \hat{h}_1[i_1] \tilde{x}_1[n-i_1] + \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \hat{h}_2[i_1, i_2] \tilde{x}_2[n-i_1] \tilde{x}_2[n-i_2] + \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \hat{h}_{3,in}[i_1, i_2, i_3] \tilde{x}_1[n-i_1] \tilde{x}_1[n-i_2] \tilde{x}_2[n-i_3] + \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \hat{h}_{3,out}[i_1, i_2, i_3] \tilde{x}_2[n-i_1] \tilde{x}_1[n-i_2] \tilde{x}_2[n-i_3] + \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \hat{h}_{3,3-in}[i_1, i_2, i_3] \tilde{x}_1[n-i_1] \tilde{x}_1[n-i_2] \tilde{x}_1[n-i_3]
\] (4.1)

We assume for this analysis that the most dominant cross-channel nonlinear effects can be captured by terms up to the cubic order of the Volterra series [29], which allows the nonlinear receiver output \(\tilde{y}[n]\) to be accurately described by (4.1). As in the previous chapter: \(\tilde{x}_1[n]\) represents the desired BB input signal centred on the carrier frequency, \(\tilde{x}_2[n]\) describes the interference BB input signal centred on the interference frequency and \(L\) denotes the memory length which defines the amount of memory in the system. Furthermore, the variables \(\hat{h}_1[i_1], \hat{h}_2[i_1, i_2], \hat{h}_{3,in}[i_1, i_2, i_3] \text{ and } \hat{h}_{3,out}[i_1, i_2, i_3]\) represent the linear, quadratic, third order in-channel and third order cross-channel BBVS kernels respectively.

By performing the simulations in the BB domain, the radar simulator could model the nonlinear distortion caused directly to the desired radar channel but not the generation of any spurious harmonics across the RF spectrum, see Figure 1.2. This was due to the fact that the narrowband assumption was invoked during the derivation of the BBVS model in chapter 3 and therefore, the modelling capabilities of the radar simulator were restricted to a small bandwidth around the carrier frequency (as a general rule of thumb this assumption is satisfied when the bandwidth of the signal is no greater than a tenth of the carrier frequency). While a full passband simulation would capture both types of nonlinear distortion, the sampling rates required for such an analysis are extremely high which results in the computational cost of the simulations becoming unmanageable. This is particularly true when complex nonlinear memory effects are introduced to the radar receiver model. It is conceivable that the nonlinear harmonics may survive the RF filtering and fall back into the radar channel further down the RF receive chain. However, this is a secondary effect and for this thesis we are specifically interested in the front-end
nonlinear distortion caused directly to the desired radar channel. We must therefore assume for the BB simulations that the RF filtering adequately suppresses any unwanted harmonics generated by the nonlinear amplifiers in the RF front-end. By performing the simulations in the BB domain, the sampling rate of the radar simulator could be drastically reduced and the BBVS model could be used to incorporate complex nonlinear memory effects into the analysis. Crucially, it was shown in chapter 3 that both the in-channel and cross-channel nonlinear distortion effects introduced by BBVS model perfectly match those caused to the desired radar channel by the full PBVS model, see Figure 3.9.

It is important to note however that the memory tap delay width of the BBVS model, \( i \), is directly related to the simulations’ sampling frequency and therefore independent from PB impulse response. In order to ensure that the memory effects generated by the BBVS model matched those from the passband RF receiver, the sampling frequency of the BB simulations was set equal to the specified radar receiver bandwidth. As discussed in chapter 3, the memory of a system is intrinsically linked to its bandwidth. Therefore, by defining the BB sampling frequency in this way we ensured that the memory effects generated in radar simulator were always reference to the impulse response of the front-end nonlinearity regardless of the mode/scenario simulated. The simple design of the BB radar simulator is illustrated diagrammatically in Figure 4.1.

Once the interference scenario and radar scene had been defined, the radar simulator generated the input signals for the BBVS receiver model on a burst-by-burst basis. This is illustrated in Figure 4.1 where the linear guard channel is also displayed for completeness. In order to simulate the receiver input for each PRI, the radar simulator generated returns from every specified artefact in the scene before linearly combining them to form the desired radar signal \( \tilde{x}_1[n] \). Importantly, the interference signal \( \tilde{x}_2[n] \) was generated
separately from the desired radar signal $\tilde{x}_1[n]$ so that they could be combined subsequently in the radar receiver through the BBVS nonlinear model described above. For the analysis performed in this chapter, an airborne radar was configured to operate in a look-down scenario so as to generate main-beam clutter and a single ground-based target was placed in the scene. Furthermore, the radar was set to operate in an MPRF mode with a pulse repetition frequency of 30kHz and a corresponding receiver bandwidth of 33MHz. Note, the MPRF radar mode was chosen for this analysis due to the reasoning given previously in chapter 1. For simplicity we set the simulated radar to transmit a single-tone waveform which had a pulse-width equal to 1.1% of the PRI and a BB offset frequency of 11MHz.

The target signal for each successive PRI was constructed by placing the returned pulses into the correct temporal positions before applying the respective amplitude scale factors and phase shifts. The amplitude of each returned target pulse was defined by the specified radar cross-section (RCS) through the radar range equation [13, 10, 11] stated in (4.2) below.

$$P_{rx} = \frac{P_{tx} G_{rx}(u,v) G_{tx}(u,v) \sigma c^2}{\{4\pi\}^3 f_c^2 R_t^4} \quad (4.2)$$

where: $P_{rx}$ represents the power returned to the radar from the target, $P_{tx}$ denotes power transmitted by the radar, $G_{rx}(u,v)$ and $G_{tx}(u,v)$ describe the receive and transmit side directivities respectively, $\sigma$ denotes the target RCS, $c$ is the speed of light, $f_c$ is the radar carrier frequency and $R_t$ represents the slant range to the target$^1$. Importantly, both the receive and transmit side directivities are dependent on the target’s angle off boresight (AOB) hence the dependence on the antenna frame coordinates $u$ and $v$. For the simulations performed in this thesis, we were more interested in the relative powers at the front-end of the radar receiver rather than the absolute powers as the receiver characteristics were used to control the strength of the nonlinear effects observed at the radar output. Crucially, the relative power of the returned target signal could be easily controlled by the RCS parameter $\sigma$ which for this analysis was assumed to follow a Swerling 0 fluctuation model [13, 10, 11, 21].

The phase shift applied to each returned target signal consisted of the initial waveform phase, the transmitted antenna phase referenced to the target’s AOB and the individual Doppler phase shift which was determined by the pre-defined radar scene. As the radar platform and ground based target were both moving in scene, both relative velocities had to be accounted for in the Doppler phase shift calculation. The expression for the Doppler phase shift, $\Delta \phi_t$, corresponding to each returned target pulse is displayed in (4.3) below.

$$\Delta \phi_t = \left\{ \frac{4\pi f_c v_D}{c} \right\} \left\{ \text{PulseNumber} - 1 \right\} \frac{1}{\text{PRF}} \quad (4.3)$$

$^1$We assumed for this analysis that atmospheric losses were negligible resulting in them being omitted from the radar range equation.
where the Doppler velocity, $v_D$, is calculated as follows,

$$v_D = v_p \cos(\varphi_p - \varphi_c) \cos(\theta_c - \theta_p) + v_t \cos(\varphi_t - \varphi_c) \cos(\theta_c - \theta_t)$$  \hspace{1cm} (4.4)$$

In (4.4): $v_p$ represents the platform velocity; $v_t$ represent the target velocity; $\varphi_p$ and $\varphi_t$ denote the azimuthal direction of travel relative to the antenna boresight for the platform and target respectively; $\theta_p$ and $\theta_t$ denote the elevation direction of travel relative to the horizontal axis for the platform and target respectively; $\varphi_c$ represents the target’s azimuthal AOB; $\theta_c$ represents the depression angle to the target referenced to the horizontal axis\(^2\). Importantly, the ability of the nonlinear mitigation techniques to recover weak targets from the cross-modulation distortion could be probed by controlling the radar target’s relative Doppler velocity and slant range. The simulated radar scenario is illustrate in Figure 4.2 where the radar main-beam is projected onto the ground generating a series clutter returns.

In order to accurately model the clutter returns for a standard MPRF mode, the radar simulator divided the ground up into individual cells in range and azimuth before calculating the returns from each one as if they were independent point targets. The clutter cells were defined by concentric rings in range known as isorange rings and were further divided by spokes of constant azimuth [11, 21, 24, 23]. For the simulations performed in this chapter, we assumed that the main-beam clutter was dominant and therefore only those clutter cells that were illuminated by the main-beam when it was projected onto

\(^2\)The depression angle is the angle between the horizontal axis containing the radar and the line from the radar to the target. In the MATLAB radar simulator angles below the horizontal axis adopted a negative sign convention.
the ground were used to calculate the resultant clutter returns. This is illustrated diagrammatically in Figure 4.2 where the extent of the main-beam was defined by its 3dB beam-width. As with the radar target modelling described above, the first step in the clutter simulation process was to calculate the slant range to every individual clutter patch on the ground and then place each corresponding pulse at the correct temporal location in the received PRI. The power returned from each clutter cell was then calculated using the same radar range equation as for the target case (4.2), with the exception of the RCS parameter $\sigma$ which was replaced by the backscatter coefficient of the cell $\sigma_c$. Much like the target RCS, the clutter backscatter coefficient $\sigma_c$ quantifies the individual clutter patch reflectivity and was therefore related to the cross-sectional area of each cell on the ground. In order to accurately simulate the typical clutter observed in an MPRF radar mode, the backscatter coefficients had to mirror the natural variations in the terrain reflectivity across the whole clutter area. Therefore, each backscatter coefficients encompassed a unique RCS sampled from a random distribution of reflectivities. For the simulations presented in this thesis, a Weibull distribution was used to model the clutter variations for a typical MPRF radar mode [21, 22, 23, 11] where the distribution’s shape and scale parameters were related to the pre-defined terrain type. Note, we chose to use the Weibull distribution throughout this thesis to model the amplitude fluctuations of the MPRF clutter as it allows different terrain types to be simulated which meant that the digital mitigation techniques could be tested against a wide variety of clutter spectrums. Additionally, the Weibull distribution is one of the most popular distribution in the field of radar for simulating MPRF ground clutter [21].

Similar to the target case, the phase attributed to each returned clutter patch signal comprised of the initial waveform phase, the transmitted antenna phase, the Doppler phase shift and finally the terrain dependent phase fluctuation. The transmitted antenna phase was dependent on each clutter cell’s AOB and was computed from the directivity pattern of the transmit antenna. Each clutter cell’s Doppler phase shift was calculated using (4.3) and (4.4) where: $v_t$ was set equal to zero as the ground clutter was assumed to be stationary, $\varphi_c$ equalled the individual clutter cell’s AOB and $\theta_c$ equalled the specific clutter cell’s depression angle referenced to the horizontal axis. Finally, the terrain dependent phase shift was sampled from a uniform phase distribution so as to resemble the typical characteristics of MPRF radar ground clutter [21, 22, 23, 11]. The stochastic nature of the radar simulator meant that the clutter signals returned during each successive PRI varied according to the statistical amplitude and phase distributions of the pre-defined clutter terrain. An example RD plot for the radar scenario described above is displayed in Figure 4.3 where the radar receiver was configured to be strictly linear.

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3While the radar receiver was modelled as a single receiving element, the simulated transmit antenna consisted of a phased array of transmitting elements that produced a highly directive beam pattern.

4See chapter 2 for more information on the Weibull distribution.
4.2.1 Cross-modulation Scenario

In order for the MPRF radar simulator to generate the standard cross-modulation effect described in chapter 2, the interference signal $\tilde{x}_2[n]$ had to be pulsed [8, 7]. We therefore assumed for the purposes of this chapter that the interferer transmitted a pulsed single-tone waveform whose relative power was strong enough to drive a nonlinearity in the simulated radar receiver. In the case where the radar is operating on an airborne platform in a look-down scenario, the most likely source of interference is from a ground-based radar system operating at a much closer stand-off range than the desired target location. The interferer was therefore placed at half the distance to the radar than the clutter/target range and was configured to have PRF of 54kHz. As with the generation of the desired radar signal $\tilde{x}_1[n]$, the first step in the interference simulation procedure was to place the interfering pulses at the correct temporal locations in the received PRI. Unlike the desired signal case $\tilde{x}_1[n]$, the power contained in the interference pulses was determined by the single sided radar range equation (4.5) as the signal only had to travel one way from the interferer to the radar.

$$P_{ri} = \frac{P_t G_{rz}(u, v)G_{t}(u, v)c^2}{\{4\pi\}^3 f_t^2 R_i^2}$$  \hspace{1cm} (4.5)
In (4.5): $P_{ti}$ and $P_{ri}$ denote the transmitted and received interference power respectively, $f_i$ represents the interferer carrier frequency, $G_{ti}(u, v)$ denotes the interferer’s transmit-side directivity and $R_i$ denotes the range to the interference platform. Since the interference signal was not generated through a scattering phenomenon, the RCS parameter $\sigma$ was dropped from (4.5) and therefore the strength of $\tilde{x}_2[n]$ was controlled in the simulation by the transmitted interference power $P_{ti}$ and directivity $G_{ti}(u, v)$. The phase of the received interference signal comprised of the transmitted interference waveform phase and the Doppler phase shift. As both the interferer and the radar platform were moving in the simulation, the Doppler phase shift applied to $\tilde{x}_2[n]$ was calculated using (4.3) and (4.4) where the target angle and velocity parameters now represented those from the interference platform, $[v_i, \varphi_i, \theta_i, \varphi_c, \theta_c]$.

Regarding the characteristics of the simulated radar receiver described by (4.1), we started by making the assumption that the radar operated in the linear regime when the out-of-band interference was not present in the scene. Therefore, the contributions from the cubic in-channel term $\tilde{h}_{3_{in}}[i_1, i_2, i_3]$ had an insignificant effect on the nonlinear output $\tilde{y}[n]$ as they sat below the level of the noise floor. We also assumed for this analysis that the pulsed interference signal did not occupy half the carrier frequency and therefore the contributions from the second order term could be considered to be negligible as well. This allows the quadratic term to be omitted from the expression for the BBVS receiver model (4.1). The cross-channel nonlinear effect we are interested in was therefore completely described by the $\tilde{h}_{3_{out}}[i_1, i_2, i_3]$ term in (4.1) which we now refer to as the cross-modulation term. Additionally, it is important to note that the narrowband assumption intrinsic to the BBVS model does not affect the radar simulator’s ability to capture the cross-modulation scenario for a typical MPRF radar mode. This is due to the fact that the MPRF mode of operation is itself intrinsically narrowband in nature. An example RD plot for the cross-modulation scenario described above is displayed in Figure 4.4 where the radar receiver was configured to be strictly memoryless.

Examining the nonlinear RD plot displayed in Figure 4.4, it is clear that the radar simulator can reproduce the classic cross-modulation phenomena presented in the radar literature [8, 7]. Furthermore, by comparing Figure 4.3 and Figure 4.4 it is apparent that the presence of the out-of-band interferer in the scene has caused the simulated radar’s sensitivity to be drastically reduced. The radar target has been masked by the nonlinear repeats of the main-beam clutter which has effectively made it undetectable to the sensor. The task for the digital LUT mitigation techniques is to therefore restore the linear performance of the sensor by removing the nonlinear distortion effects generated by the cross-modulation term in (4.1). For this to be the case, they must untangle the nonlinear distortion from the desired radar target so that it may be recovered while the unwanted interference is removed.
Chapter 4. LUT Inverse Mitigation Techniques

Figure 4.4: Cross-modulation RD plot outputted from the MPRF radar simulator. The radar target at \(-6.3kHz, -1.3km\) has been masked by the nonlinear harmonics of the main-beam clutter \(-3.0kHz \rightarrow -0.7kHz\) spread across the Doppler spectrum. Example plot generated for the case where the nonlinearity was strictly memoryless (BBTS case).

4.2.2 The Role of the Guard Channel

In order for the cross-channel digital mitigation techniques to have any chance of removing the cross-modulation effects observed in the radar output, they require information regarding the interference that caused the nonlinear distortion in the first place. The ability of modern phased array radars to form guard channels must therefore be exploited to capture the raw interfering signal. It is reasonable to assume in radar that an active guard channel could be backed-off sufficiently so that the strong out-of-band interference driving the cross-modulation effect could be captured in a linear fashion. In essence, the reduced gain of the backed-off receiver allows the guard channel to capture the strong interference signal accurately with the weak radar signal falling below the noise. This is illustrated in Figure 4.1 where the interference signal \(\tilde{x}_2[n]\) is fed into the guard channel receiver but the desired signal \(\tilde{x}_1[n]\) is not. The linear guard channel measurement is denoted by \(\tilde{z}[n]\) and is therefore described by (4.6) below,

\[
\tilde{z}[n] = \tilde{x}_2[n] + \tilde{w}[n]
\]  

(4.6)

where \(\tilde{w}[n]\) denotes additive white Gaussian noise. While the performance of the linear guard channel is defined by its SNR in (4.6), it is by no means the only limiting factor that
will affect the performance of the digital mitigation techniques. In [8], Mellor et al. highlight the challenges of trying to exploit information from a linear guard channel when digitally compensating for cross-modulation effects generated in the radar receiver. Of the potential problems identified in [8], timing difference between the measured interference pulses and the actual interference pulses proved to be the most troublesome. This was also highlighted in the case of the communications receiver by Zou et al. in [14] where an adjacent channel is employed to isolated a strong blocker signal. The solution proposed by Zou in [14] involved performing a robust synchronization procedure where the communications transmitter was asked to transmit a known sequence of training signals that could be used to identify any timing offsets between the channels. While this is a fine solution for communications systems, where the signals received by the sensor are both known and highly configurable, it is less suitable for radar systems that have little knowledge of the incoming signal. However, if modern radar systems employ phased array antennas it is conceivable that the linear guard channel could be synchronised during operation by an adjacent element in the array. The basic idea is similar to that proposed by Zou in [14] except the training signals would be transmitted into the guard channel through mutual coupling with a neighbouring element in the array. While this concept of self-calibration is becoming popular in the radar literature [98, 99], it is still very important to understand the robustness of any digital mitigation technique to both guard channel noise and timing offsets.

### 4.3 Mitigation of Memoryless Nonlinearities

LUT mitigation techniques offer the most attractive digital solution to the nonlinear receiver problem in radar as they are mode independent and computation inexpensive to implement. While the technique is well established in the communications literature, little research has been conducted on its performance in radar systems. We start our analysis of the problem by investigation the strictly memoryless case where the BBVS model described in (4.1) is reduced to the BBTS model by setting $L = 1$.

$$
\tilde{y}[n] = \tilde{h}_1 \tilde{x}_1[n] + \tilde{h}_2 \tilde{x}_2[n] + \tilde{h}_{3_{in}} \tilde{x}_1[n] \tilde{x}_1^*[n] + \tilde{h}_{3_{out}} \tilde{x}_2[n] \tilde{x}_1[n] \tilde{x}_2^*[n] 
$$

(4.7)

In (4.7): $\tilde{h}_1$, $\tilde{h}_2$, $\tilde{h}_{3_{in}}$ and $\tilde{h}_{3_{out}}$ represent the linear, quadratic, cubic in-channel and cubic cross-channel Taylor BB coefficients respectively. As discussed in chapter 2, the memory of a system is intrinsically linked to its bandwidth [15, 38] and therefore memoryless LUT techniques may be sufficient to mitigation cross-modulation effects generated in traditional narrowband radar systems. In this section, we study the performance of two separate communication based LUT techniques in mitigating cross-modulation distortion generated by a strictly memoryless nonlinearity in the RF receiver.

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Note the BBTS coefficients are distinguished from the BBVS coefficients by dropping the dependence on $[i_1, i_2, i_3]$ in the kernel formalisation.
4.3.1 Direct Inversion

The first communications technique studied in this section is based on the work previously published by Zou et al. in [14] which attempts to correct for the cross-modulation distortion by mathematically inverting the forward nonlinearity. While most dual-channel NLEQ techniques developed in the communications literature target the concurrent reception of multiple adjacent channels [26], the technique designed by Zou specifically addresses the cross-modulation problem. By directly inverting the forward nonlinear transfer function Zou develops a simple LUT solution the performance of which is not constrained by the power of the out-of-band interferer. This is a property that is extremely attractive for radar systems where the power of the potential signals received from interferers is so unpredictable. Importantly, the technique is designed specifically to mitigate distortion effects caused by a memoryless receiver nonlinearity and therefore its formalisation cannot be extended to include nonlinear memory effects. For radar systems, the LUT mitigation technique would be implemented in the radar processor and would be applied to the raw received signals before any range-Doppler processing is performed.

The simple mitigation technique designed by Zou et al. [14] attempts to directly invert the forward nonlinear model to obtain an expression for the uncorrupted input $\tilde{x}_1[n]$. In doing this derivation, Zou assumes that the system is operating linearly when the interferer is not present in the scene and therefore contributions from the third order in-channel term can be considered to be negligible compared to those from the linear term. This is a sensible assumption for radar systems which are designed to operate within their linear regimes. Additionally, we assume for this analysis that the interferer does not occupy half the carrier frequency of the radar and therefore contribution from the quadratic term $\tilde{h}_2$ can also be considered to have an insignificant effect on the nonlinear output $\tilde{y}[n]$. The task for the mitigation algorithm is to therefore remove the unwanted nonlinear distortion effects caused by the cross-modulation term $\tilde{h}_{3,\text{out}}$ in (4.7). Thus, the final expression for the direct inversion approach can be found by removing the unnecessary terms $\tilde{h}_{3,\text{in}}$ and $\tilde{h}_2$ in (4.7) before rearranging the expression to find the desired input signal $\tilde{x}_1[n]$.

$$\hat{\tilde{x}}_1[n] = \frac{\tilde{y}[n]}{\{\tilde{h}_1 + \tilde{h}_{3,\text{out}}|\tilde{z}[n]|^2\}}$$ (4.8)

In (4.8), the desired input signal is estimated, $\hat{\tilde{x}}_1[n]$, through the direct inversion of the nonlinear output, $\tilde{y}[n]$, where the strong interference signal, $\tilde{x}_2[n]$, is measured separately through a linear guard channel, $\tilde{z}[n]$. In short, the estimation procedure relies on the source of the cross-modulation being captured accurately by means of a linear guard channel and the memoryless coefficients of the forward model being known. As discussed previously, if the interference signal is strong enough to cause a nonlinear effect in the RF receiver it will most likely be much stronger than the returned radar signal and therefore a simple guard channel could be backed-off to linearly capture it. With regards to
4.3. Mitigation of Memoryless Nonlinearities

the forward model coefficients, Zou et al. outline a nonlinear parameter estimation technique in [14] which aims to actively characterise the nonlinear channel during operation. This technique is not applicable to radar systems as it utilises the predetermined transmitted pilot symbols unique to communication signals. It must therefore be assumed that the memoryless coefficients of the forward model can instead be identified offline using the standard test techniques discussed in chapter 2. Both of these requirements seem reasonable for modern radar systems.

4.3.2 Tandem Inverse

The second nonlinear mitigation technique studied in this section is based on the tandem inverse approach which is the most popular digital mitigation solution in the communication literature [26, 71, 15, 9, 83, 72, 81]. In short, the technique attempts to identify an inverse nonlinear structure that when placed in tandem with the forward receiver nonlinearity reproduces the desired linear input signal. The tandem inverse solution was initially extended to compensate for cross-channel nonlinear effects in the transmit side of the communications literature [57, 63, 20], however more recently it been applied to the receive side as well [71, 83]. An illustration of the mitigation technique is presented for the memoryless case in Figure 4.5, with the expression for the nonlinear inverse implemented in this section displayed in (4.9) below.

\[
\hat{x}_1[n] = \bar{g}_1 y[n] + \bar{g}_2 \bar{z}[n] \bar{z}[n] + \bar{g}_{3\text{in}} y[n] \bar{y}[n] \bar{y}[n] + \bar{g}_{3\text{out}} \bar{z}[n] \bar{y}[n] \bar{z}[n]
\]  

(4.9)

where \( \bar{g}_1, \bar{g}_2, \bar{g}_{3\text{in}} \) and \( \bar{g}_{3\text{out}} \) represent the linear, quadratic, cubic in-channel and cubic cross-channel inverse BBTS coefficients respectively. The identification of the inverse nonlinear coefficients that provide the best estimate of the desired input signal \( \hat{x}_1 \) will be discussed in detail in the following section. However, we can gain some insight into the process and the effectiveness of the technique by firstly studying the theoretical solution. As mentioned previously, we assume for the cross-modulation scenario studied in this chapter that contributions from the quadratic \( \bar{h}_2 \) and cubic in-channel \( \bar{h}_{3\text{in}} \) terms in the forward nonlinearity are negligible. The memoryless forward nonlinearity for the
cross-modulation scenario can therefore be simplified to (4.10) below:

\[ \tilde{y}[n] = \tilde{h}_1 \tilde{x}_1[n] + \tilde{h}_{3_{\text{out}}} \tilde{x}_2[n] \tilde{x}_1[n] \tilde{x}^*_2[n] \]  

(4.10)

with the corresponding LUT inverse structure given by,

\[ \hat{\tilde{x}}_1[n] = \tilde{g}_1 \tilde{y}[n] + \tilde{g}_{3_{\text{out}}} \tilde{z}[n] \tilde{y}[n] \tilde{z}^*[n] \]  

(4.11)

substituting (4.10) into (4.11) and expanding out the brackets we find,

\[ \hat{\tilde{x}}_1[n] = \tilde{g}_1 \tilde{h}_1 \tilde{x}_1[n] + \tilde{g}_{3_{\text{out}}} \tilde{h}_{3_{\text{out}}} \tilde{x}_1[n] \tilde{x}^*_2[n] \]

\[ + \tilde{g}_{3_{\text{out}}} \tilde{h}_1 \tilde{z}[n] \tilde{x}_1[n] \tilde{z}^*[n] + \tilde{g}_{3_{\text{out}}} \tilde{h}_{3_{\text{out}}} \tilde{z}[n] \tilde{x}_2[n] \tilde{x}_1[n] \tilde{x}^*_2[n] \tilde{z}^*[n] \]  

(4.12)

Now if we consider the case where there is no noise on the linear guard channel, \( \tilde{z}[n] = \tilde{x}_2[n] \), we can reduce (4.12) to the following form (4.13),

\[ \hat{\tilde{x}}_1[n] = \tilde{g}_1 \tilde{h}_1 \tilde{x}_1[n] + \{ \tilde{g}_1 \tilde{h}_{3_{\text{out}}} + \tilde{g}_{3_{\text{out}}} \tilde{h}_1 \} |\tilde{x}_2[n]|^2 |\tilde{x}_1[n]|^2 \tilde{x}_1[n] \]

(4.13)

Choosing the inverse coefficients \( \tilde{g}_1, \) and \( \tilde{g}_{3_{\text{out}}} \) to minimise contributions from the unwanted third order cross-modulation term, we are left with the final expression for the desired signal estimate \( \hat{\tilde{x}}_1[n] \) (4.14),

\[ \hat{\tilde{x}}_1[n] = \tilde{x}_1[n] - \frac{\tilde{h}_{3_{\text{out}}}}{\tilde{h}_1} \frac{\tilde{h}_{3_{\text{out}}}}{\tilde{h}_1} |\tilde{x}_2[n]|^2 |\tilde{x}_2[n]|^2 \tilde{x}_1[n] \]

(4.14)

where the theoretical inverse coefficients are given by (4.15),

\[ \tilde{g}_1 = \frac{1}{\tilde{h}_1}; \quad \tilde{g}_{3_{\text{out}}} = -\frac{\tilde{h}_{3_{\text{out}}}}{\tilde{h}_1} \frac{\tilde{h}_{3_{\text{out}}}}{\tilde{h}_1} \]

(4.15)

It is clear from this simplified memoryless analysis that the LUT technique will only fully restore the performance of the radar to the linear case if the quintic term generated in the tandem inverse structure sits below the level of the noise floor. While the theoretical inverse coefficients derived above can be used to compensate for cross-modulation effects generated in the radar receiver, there may exists coefficients that find a better balance between cancelling the cross-modulation term \( \tilde{h}_{3_{\text{out}}} \) and minimising the deleterious effects from the higher-order harmonics. An alternative indirect approach can be adopted whereby the inverse coefficients are learnt directly in an offline calibration stage. By identifying the inverse structure rather than the forward nonlinearity itself, the performance of the mitigation technique can be greatly enhanced. This is discussed further in the following section.
4.3. Mitigation of Memoryless Nonlinearities

4.3.3 Simulation Architecture

Once the radar scene and pulsed interferer characteristics were defined, the behaviour of the cross-modulation distortion generated in the simulation was governed entirely by the specific choice of nonlinear kernel coefficients. In both the BBTS and BBVS case, the gross level of the cross-modulation distortion produced in the radar receiver was set by the choice of nonlinear coefficients for the memoryless case; \( \tilde{h}_1, \tilde{h}_{3in} \) and \( \tilde{h}_{3out} \). As discussed previously, the linear coefficient \( \tilde{h}_1 \) was set to the value of the RF receiver gain, while the third order \emph{in-channel} coefficient \( \tilde{h}_{3in} \) was subsequently chosen such that the nonlinear distortion effects sat just below the level of the noise floor when the interference was not present in the scene. The level of the memoryless \emph{cross-channel} coefficient \( \tilde{h}_{3out} \) was selected to maximise the cross-modulation distortion generated in the radar output while keeping the quintic term produced in the tandem inverse structure (4.15) below the level of the noise floor. By defining the strength of the \emph{cross-channel} coefficient \( \tilde{h}_{3out} \) in this way, we ensured that the cross-modulation distortion generated in the simulated receiver was within the operating region of the tandem LUT digital mitigation technique. Importantly, this allowed for a fair comparison between the two memoryless LUT techniques. The complex BBTS nonlinear coefficients used for the simulations performed in this chapter are displayed in Table 4.1 where each coefficients’ phase was selected randomly. Furthermore, a frequency domain comparison of the contribution from each term in the BBTS is presented in Figure 4.6. Examining Figure 4.6 it is clear that when the out-of-band interference is present in the scene, the contribution from the \( \tilde{h}_{3out} \) term dominate the nonlinear effect and are at level that will corrupt the linear gain output from \( \tilde{h}_1 \).

<table>
<thead>
<tr>
<th>BBTS Kernel Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{h}_1 )</td>
</tr>
<tr>
<td>( \tilde{h}_2 )</td>
</tr>
<tr>
<td>( \tilde{h}_{3in} )</td>
</tr>
<tr>
<td>( \tilde{h}_{3out} )</td>
</tr>
</tbody>
</table>

\text{Table 4.1: List of the BBTS kernel coefficients implemented in the MPRF radar simulator for the LUT performance analysis.}

For the purposes of this thesis, we assume that the forward nonlinear coefficients were constant for the full coherent integration time of the radar and therefore, once identified the LUT inverses could simply be applied to every returned pulse in the burst. For the direct LUT approach, the inverse is defined by (4.8) and since we have assumed that the forward model coefficients are known to the radar no further identification procedure is required for this method. While it was shown in (4.15) that the memoryless coefficients
for the tandem inverse structure (4.11) can also be found theoretically from the forward model coefficients, in this chapter we choose to identify them by means of the indirect learning procedure. For a real radar system this indirect learning procedure would take the form of an off-line calibration stage where the inverse coefficients that minimised the error between the estimated input signal and the known input test signal are selected. In this case, we employ a noise identification procedure similar to that described in chapter 3 however the simple swept tone frequency test outlined in chapter 2 would also be sufficient for the strictly memoryless case. Unlike the theoretical approach, the indirect learning procedure will balance suppressing the third order distortion generated in the forward model as well as the quintic distortion generated in the inverse model when identifying the coefficients. This leads to better mitigation performance of the tandem inverse technique. RD plots showing the memoryless LUT mitigation techniques applied to the BBTS cross-modulation scenario illustrated in Figure 4.4 are displayed in Figure 4.7a and 4.7b below.

4.3.4 Simulation Results

In order to analyse the performance of the LUT mitigation techniques for a typical MPRF radar mode in detail, statistical probability of detection (PD) analysis was performed in the RD domain on a single radar target. To form each RD plot the MPRF radar simulator generated 256 received pulses based on the pre-defined radar scene before passing them through the BBTS receiver model (4.10) and the linear gain model. A further two datasets were generated by applying the direct inverse and tandem inverse LUT mitigation techniques to the nonlinear receiver output. In all simulations noise was added to the received input signals, SNR = 25dB, as well as to the output of the receiver, SNR = 25dB.
4.3. Mitigation of Memoryless Nonlinearities

and additionally to the linear guard channel, SNR = 25\,dB. Simulation results for a single burst showing the cross-modulation distortion generated from the BBTS nonlinear receiver and the subsequent correction by the memoryless LUT techniques are displayed in Figure 4.4 and Figure 4.7 respectively. It is clear from Figure 4.7 that both LUT mitigation techniques have reduced the cross-modulation distortion observed in the simulated radar output. However, the performance of the LUT inverse solutions can be analysed in much more detail by studying the results of the stochastic based PD analysis. In addition to the main PD plots presented, results showing how guard channel noise and guard channel delay affect the performance of the LUT techniques are also displayed.

In order to generate the PD curves shown below, an adaptive cell-averaging constant false alarm rate (CA-CFAR) thresholding technique [11] that maintained a constant probability of false alarm rate of 0.2 was applied in all cases\(^6\). The CA-CFAR algorithm is described in detail in chapter 2 but in short, the algorithm sets the detection threshold based on the statistical behaviour and power of the cells neighbouring the target cell. Thus, the simulated radar displays maximum sensitivity when its receiver operates in its linear regime as this is when the cells surrounding the target cell are strictly noise limited.

---

\(^6\)Note, the FAR of 0.2 was chosen arbitrarily for the purposes of this analysis and it is not supposed to be representative of the FAR used by a real pulse-Doppler radar system. We are really only interested in the relationship between the different PD curves as this will inform us about how much sensitivity has been lost due to the nonlinear effect and then restored by the different mitigation techniques. Therefore, so long as the FAR remains the same for all cases then its exact value does not really matter for this comparative analysis.
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When cross-modulation is observed in the radar output the distortion causes the detection threshold to increase and thus the sensitivity of the sensor to decrease. Therefore, if the LUT techniques are to be successful in restoring system sensitivity they must reduce the level of cross-modulation distortion observed in the RD domain without also removing the target. For this analysis, the sensitivity of the simulated radar was studied in the different nonlinear scenarios by varying the input SNR of the desired radar target. RD plots for 400 bursts were used to estimate the PD value at each data point. We start by analysing the performance of the direct inverse approach where the full set of PD results are displayed in Figure 4.8.

\[ \text{Figure 4.8: Probability of detection analysis results for the direct inverse LUT technique. The mitigation solution performs very well in the memoryless cross-modulation scenario restoring the simulated radar's sensitivity back to the linear gain case. As the level of guard channel noise or delay is increased the performance of the technique degrades. Note, the delay offset is displayed as a percentage of the interference pulse-width.} \]

\[ \text{Throughout this thesis each PD value was determined from 400 bursts as this provided an accurate estimate of each data point while balancing the considerable time required to run the full set stochastic based simulations.} \]
4.3. Mitigation of Memoryless Nonlinearities

<table>
<thead>
<tr>
<th>Guard Channel Noise (dB)</th>
<th>Recovered Performance (%)</th>
<th>Guard Channel Delay (%)</th>
<th>Recovered Performance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR = ∞</td>
<td>100</td>
<td>Delay = 0</td>
<td>100</td>
</tr>
<tr>
<td>SNR = 25</td>
<td>100</td>
<td>Delay = 12</td>
<td>70</td>
</tr>
<tr>
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<tr>
<td>SNR = 7</td>
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<td>Delay = 37</td>
<td>42</td>
</tr>
<tr>
<td>SNR = 0</td>
<td>17</td>
<td>Delay = 49</td>
<td>36</td>
</tr>
<tr>
<td>SNR = -5</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.2:** Snapshot results for PD = 0.7 from Figure 4.8 showing the recovered performance by the Direct BBTS mitigation technique as a percentage. 100% represents the case where the mitigation technique has fully restored the radar performance back to the linear case.

Examining the PD results in Figure 4.8a, we firstly observe that the curve for the memoryless BBTS receiver outputs fall off at a much higher SNR than the desired linear receiver output curve. This reaffirms the fact that the cross-modulation distortion generated in the nonlinear receiver has reduced the overall sensitivity of the simulated radar. Importantly, the direct inverse LUT technique performs well when applied to the BBTS receiver output and completely restores the system sensitivity back to the linear case, see BBTS - direct BBTS curve. Note, while it appears that the mitigated output (BBTS - direct BBTS) outperforms the Linear case in Figure 4.8a, this is not in fact the case and any variations between the two PD curves is down to the random fluctuations associated with the stochastic based analysis. Studying the results in Figure 4.8b it is clear that the technique is robust against reasonable levels of guard channel noise but degrades quite quickly after a certain SNR level is reached. This is to be expected as the technique relies on an accurate measure of the interfering signal in order to perform the nonlinear correction. In Figure 4.8c, the delay offset between the true interference signal and the measured interference signal is displayed as a percentage of the interfering signal’s pulse-width. It is clear from the Figure 4.8c that having good synchronisation between the guard channel and the main receive path is just as important as ensuring that the receiver noise levels are low. While the mitigation technique can handle some slight timing offsets, if they become too large then the algorithm will apply the nonlinear correction to the incorrect samples in the received PRIs resulting in much poorer mitigation performance. Importantly, a snapshot of the key results from Figure 4.8 are presented in Table 4.2 so that they can be interpreted more easily.
We now study the results from the tandem inverse LUT mitigation technique which are displayed in Figure 4.9 and Table 4.3. As with the direct inverse technique, the tandem inverse approach performs very well in the memoryless case, see BBTS - Tandem BBTS curve. It is important to note however that unlike the direct inverse solution, this LUT technique has an operational range and if the power of the out-of-band interference becomes too strong then its mitigation performance will degrade sharply. Nevertheless, if the interfering source does not drive an unmanageable higher-order effect in the tandem inverse structure then it can provide excellent mitigation performance in the cross-modulation scenario. Interestingly, while the technique appears to have a similar tolerance for timing offsets in the guard channel as the direct inverse approach (see Table 4.2), it is more sensitive to guard channel noise. It is highly likely that this increased noise susceptibility is due to the higher-order harmonic generated in the tandem inverse structure which has been suppressed in this scenario by the indirect learning procedure. One of the main advantages of the tandem inverse technique over its direct inverse counterpart is the fact that it can be extended to include nonlinear memory effects. This unique property is studied in detail in the following section.

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>Recovered Performance (%)</th>
<th>Guard Channel Delay (%)</th>
<th>Recovered Performance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞</td>
<td>93</td>
<td>Delay = 0</td>
<td>93</td>
</tr>
<tr>
<td>25</td>
<td>93</td>
<td>Delay = 12</td>
<td>54</td>
</tr>
<tr>
<td>15</td>
<td>64</td>
<td>Delay = 25</td>
<td>29</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>Delay = 37</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>Delay = 49</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 4.3: Snapshot results for PD = 0.7 from Figure 4.9 showing the recovered performance by the Tandem BBTS mitigation technique as a percentage. 100% represents the case where the mitigation technique has fully restored the radar performance back to the linear case.

### 4.4 Mitigation of Memory Rich Nonlinearities

Radars are highly sensitive systems that detect targets by studying both the amplitude and phase of the signals returned to its receiver. Due to this sensitivity and the importance of the phase measurement when detecting targets, it is thought that memory effects cannot be ignored when the radar receiver operates in its nonlinear regime. This is a
4.4. Mitigation of Memory Rich Nonlinearities

Trend that has appeared in the related field of communications as systems have moved to wider receiver bandwidths and the linearity specifications of the RF front-end have been squeezed to save costs [15, 9, 83, 19]. As modern radar systems begin to follow a similar evolution in their effort to become a highly commercialised sensor, it is imperative that digital mitigation techniques capable of compensating for nonlinear memory effects in the radar receiver are developed. Memoryless LUT mitigation techniques still offer the most attractive solution to the problem as they are computationally efficient and by far the most straightforward to implement. However, it is not yet known how effective these digital mitigation techniques will be when nonlinear memory effects are introduced to the radar receiver. Furthermore, if they should prove to be ineffective then more sophisticated digital mitigation solutions capable of compensating for nonlinear memory effects
must be developed.

### 4.4.1 BBVS Inverse

The extension of LUT mitigation technique to include nonlinear memory effects is a non-trivial task and has been the focus of much research in the related field of communications [15, 9, 49, 63, 20]. Of the cross-channel LUT techniques described above, (4.8) and (4.11), only the tandem inverse approach can be extended to compensate for nonlinear memory effects in the RF receiver. The concept behind this technique is described by the theory of $p^{th}$ order inverses [54, 84] which states that; if an inverse to the nonlinearity exists, then the system can be inverted by connecting another memory rich nonlinear system in tandem. This basic concept has been applied successfully to compensate for complex nonlinear memory effects in the communications receiver [15, 9] and has been extended to compensate for cross-channel nonlinearities in the baseband domain in [49, 63, 20]. In this section, we follow a similar approach to that implemented in [63, 20] and exploit the BBVS formalisation to define our cross-channel tandem inverse structure. The memory rich LUT technique applied in this section is therefore defined by (4.16) below,

$$
\hat{x}_1[n] = \sum_{i_1=0}^{K-1} \tilde{g}_1[i_1] \tilde{y}[n-i_1] + \sum_{i_1=0}^{K-1} \sum_{i_2=0}^{K-1} \sum_{i_3=0}^{K-1} \tilde{g}_{3_{\text{out}}}[i_1, i_2, i_3] \tilde{z}[n-i_1] \tilde{y}[n-i_2] \tilde{z}^*[n-i_3]
$$

(4.16)

where $\tilde{g}_1[i_1]$ and $\tilde{g}_{3_{\text{out}}}[i_1, i_2, i_3]$ represent the linear and third order cross-channel inverse kernel coefficients respectively and $K$ denotes the memory length of the inverse structure. Importantly, the required functional form of a Volterra based nonlinear equaliser is an open question in the literature. This is covered by the theory of $p^{th}$ order inverses [54, 84] where there is the possibility that the tandem inverse structure requires a higher-order form than the forward nonlinearity. While this could very well be the case for the nonlinearities generated in the radar receiver, extending the inverse BBVS model to be of an order greater than the forward model (4.1) would lead to an inverse with an unmanageable number of terms. We therefore restrict the tandem inverse to have an order no greater than the that defined by the forward model, in this case a cubic (4.16). While this will restrict the performance of the LUT mitigation technique, it is hoped that the inverse (4.16) will still provide enough nonlinear memory terms to compensate for the memory rich cross-modulation effect. It is possible to find the theoretical inverse coefficients for the full BBVS model through a similar process to that performed for the memoryless case above [54, 84]. However, the calculations can become extremely complex and are unlikely to yield the best results. Alternatively, an indirect approach can be adopted whereby the inverse coefficients are learnt directly in an offline calibration stage. This indirect learning approach is discussed in detail for the memory case in the following section.
4.4.2 Simulation Architecture

The BBVS model (4.1) implemented in the MPRF radar simulator Figure 4.1 is capable of simulating the subtle nonlinear phase and amplitude distortion effects present in the radar receiver. Additionally, the well-defined formalisation of the BBVS model allows the complex nonlinear behaviour of the simulated device to be controlled with high precision. This is an uncommon property in nonlinear behavioural models with memory which made the BBVS model the ideal choice for this study. In order to preserve the previously defined nonlinear scenario once memory effects were introduced, each BBVS kernel was normalised so that the respective power contributed from each group of terms in the BBVS model (4.1) was equivalent to that from the BBTS case. This is somewhat illustrated in Figure 4.10 where the temporal power contributed from each BBVS kernel matches that from the BBTS case displayed in Figure 4.6. Importantly, by comparing the plot in Figure 4.10 with the outputs displayed in Figure 4.6 it is clear that the additional memory terms in the BBVS model have produced a significantly more complex response from all of the individual kernel functions. This explains why the peak spectral power of each kernel output in Figure 4.6 does not perfectly match those in Figure 4.10 as more frequency components are contributing to the strength of the time domain signal in the BBVS case.

To the author’s knowledge there is nothing published in the available literature on the cross-channel memory behaviour of a typical RF amplifier. The complex memory coefficients for the forward model were therefore selected from a random distribution in order to generate a challenging nonlinear scenario. Importantly however, the kernel normalisation ensured that the introduction of memory effects only ever subtly changed the
structure and phase of the cross-modulation generated rather than the gross effect. The forward BBVS model was chosen to have a memory length of \( L = 3 \) for the simulations in this chapter as it is thought that the impulse response of the passband nonlinearity will decay quickly when sampled in the baseband domain. Furthermore, it is assumed that contribution from the memory polynomial coefficients, \( i_1 = i_2 = i_3 \), dominate those from the cross-terms in the BBVS model as this is what is typically observed in the communications literature [15, 9]. The magnitude of the cross-channel BBVS coefficients therefore followed a Chebyshev response on the main-diagonal \(^8\) with the associated cross-terms decaying exponentially in all directions. This is illustrated in Figure 4.11 where the memory polynomial coefficients are highlighted. Furthermore, a RD plot showing the cross-modulation distortion generated from the simulated BBVS forward receiver model described above is displayed in Figure 4.12. Comparing the RD plot for the BBVS receiver model with that from BBTS model displayed in Figure 4.4, it is clear that introducing the memory terms to the nonlinearity has not altered the gross level of the cross-modulation distortion but rather subtly changed its structure and phase.

![Figure 4.11: Illustration of the cubic cross-channel BBVS kernel coefficients implemented in the MPRF radar simulator. Highlighted coefficients correspond to those for the memory polynomial model where \( i_1 = i_2 = i_3 \).](image)

Once the random nonlinear coefficients were selected for the forward BBVS model, (4.1), the simulation identified the BBVS inverse coefficients (4.16) by means of the indirect learning approach. It is well understood that the nonlinear coefficients of the forward model can be identified by stimulating the RF nonlinearity with a deterministic random noise signal, see chapter 3. Probing the forward nonlinearity with noise ensures that the persistent excitation condition is satisfied and thus the complex memory behaviour of the device is captured. However in the case of the inverse model, the coefficients of the

\(^8\)In the Volterra framework the memory polynomial coefficients lie on the main-diagonal of the nonlinear kernels.
4.4. Mitigation of Memory Rich Nonlinearities

Figure 4.12: Example RD plot for the BBVS cross-modulation scenario outputted from the MPRF radar simulator. The radar target at \{-6.3kHz, -1.3km\} has been masked by the nonlinear harmonics of the main-beam clutter \(-3.0kHz \rightarrow -0.7kHz\) spread across the Doppler spectrum.

tandem nonlinearity must be identified by a signal that has already passed through the forward nonlinearity. It is therefore impossible to guarantee that the persistent excitation condition is being satisfied in the tandem nonlinearity and by extension, that all of the nonlinear memory effects are being capture by the identification procedure. Nevertheless, a deterministic random noise signal provides the best chance of successfully identifying the inverse coefficients directly which is why it is employed in the indirect learning approach.

Unlike the strictly memoryless case, we cannot assume that the tandem inverse structure will have a memory length equal to that of the forward nonlinearity. This complicates the process of constructing an effective LUT memory inverse as its form is dependent on the particular forward nonlinearity observed in the radar receiver. While there is no strict relationship between the inverse memory length \(K\) and the forward memory length \(L\) for nonlinear systems, linear system theory can provide some useful insights into the problem [93, 91]. If we consider a finite impulse response (FIR) filter, it will only have an inverse structure with memory length equal to its own if the forward FIR filter coefficients are minimum phase. If the FIR filter coefficients are non-minimum phase, then the inverse structure must contain more memory than the forward filter and for best results the inverse coefficients should be identified against a delayed version of the
target signal. While the concept of minimum phase does not extend to the nonlinear coefficients, it is fair to assume that the forward nonlinearity will not be well behaved as the front-end RF amplifier has not been designed to operate in its weakly nonlinear regime. For the purposes of this thesis, we therefore choose to implement an NLEQ inverse with memory length double\(^9\) that of the forward nonlinearity, \(K = 5\). For the simulations performed in this chapter, the indirect learning approach was used to identify the BBVS inverse coefficients against a target signal that had been delayed by half the inverse memory length. Furthermore, the inverse coefficients were learnt by means of the linear least squares algorithm through a procedure very similar to that outlined in chapter 3.

### 4.4.3 Simulation Results

Similar to the BBTS nonlinear scenario studied previously, the performance of the memory rich LUT mitigation technique was analysed through in-depth statistical PD analysis. As before, the MPRF radar simulator generated 256 received pulses before passing them through the BBVS nonlinearity (4.1) and the linear gain model. A further three datasets were generated by applying the two memoryless LUT techniques, (4.8) and (4.11), and the memory rich LUT mitigation technique, (4.16), to the BBVS receiver output. As a comparison, example RD plots for the three mitigation techniques applied to the BBVS scenario are displayed in Figure 4.13. In all of the mitigated RD plots, the level of the BBVS cross-modulation distortion has been reduced from that displayed in Figure 4.12. However, it is also clear that the BBVS tandem inverse technique has performed best in both reducing the level of the nonlinear interference in the scene and recovering the target. In order to truly understand the ability of these mitigation techniques to restore the simulated radar’s system sensitivity back to the linear gain case, we need to study the full PD results. As with the PD results presented previously, a CA-CFAR algorithm was employed in this case to maintain a PFAR of 0.2 in all scenarios. Furthermore, all of the receiver noise levels were once again set to \(\text{SNR} = 25\,\text{dB}\) and RD plots for 400 bursts were used to estimate the PD value at each data point. The final PD results for the BBVS cross-modulation scenario are presented in Figure 4.14 where the results displayed in Figures 4.14b and 4.14c study the effect of guard channel noise and delay on the performance of the memory rich LUT technique respectively. Importantly, a snapshot of the key results from Figure 4.14 are presented in Table 4.4 and Table 4.5 so that they can be interpreted more easily.

Examining the PD results in Figure 4.14, the first thing we observe is that despite the strength of the memoryless and BBVS nonlinearities being matched in each received PRI, see Figures 4.10 and 4.6, the resultant PD curves do not lie on top of each other. This rather surprising result is not due to any gross power variation between the cross-modulation generated in the RD domain, but rather reflects subtle phase effects introduced by the

\(^9\)For the simulations the memory length was defined using a slightly different convention to that presented in this thesis, hence the slight inconsistency here.
4.4. Mitigation of Memory Rich Nonlinearities

Mitigation of Memory Rich Nonlinearities

In all of the example RD plots, the level of the BBVS cross-modulation distortion displayed in Figure 4.12 has been reduced. However, it is also clear that the BBVS tandem inverse technique has performed the best in this scenario, see Figure 4.13a.

This seriously complicates that task of recovering the desired radar target from the unwanted cross-modulation. In other words, the cross-modulation distortion generated by the complex BBVS nonlinearity has not just masked the target but rather fundamentally corrupted it. While the memoryless LUT techniques performed well when applied to the BBTS receiver output, both techniques fail to provide much performance improvement when memory effects are introduced to the nonlinear receiver model, see BBVS - Direct BBTS curve and BBVS - Tandem BBTS entries in Table 4.4. This is a significant result as it shows that subtle changes caused to the cross-modulation distortion by the introduction of nonlinear memory effects, can decorrelate the memoryless LUT mitigation techniques. The fact that this is the case for both memoryless mitigation
Chapter 4. LUT Inverse Mitigation Techniques

techniques suggests that this decorrelation is not specific to the technique employed but rather a universal phenomenon. It is therefore clear that if the nonlinearities in the radar receiver exhibit significant memory effects, then for any digital mitigation techniques to be successful they must correct for the distortion caused by the nonlinear memory terms in the BBVS model as well as the memoryless terms. This makes the mitigation task exponentially harder than for the strictly memoryless case.

<table>
<thead>
<tr>
<th>Performance Comparison Figure 4.14a (PD = 0.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mitigation Technique</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>BBVS - Tandem BBVS</td>
</tr>
<tr>
<td>BBVS - Direct BBTS</td>
</tr>
<tr>
<td>BBVS - Tandem BBTS</td>
</tr>
</tbody>
</table>

Table 4.4: Snapshot results for PD = 0.7 from Figure 4.14a showing a comparison of the performance of the various mitigation techniques in the BBVS case. 100% represents the case where the mitigation technique has fully restored the radar performance back to the linear case.

<table>
<thead>
<tr>
<th>BBVS - Tandem BBVS (PD = 0.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guard Channel Noise (dB)</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>SNR = ∞</td>
</tr>
<tr>
<td>SNR = 25</td>
</tr>
<tr>
<td>SNR = 15</td>
</tr>
<tr>
<td>SNR = 10</td>
</tr>
<tr>
<td>SNR = 8</td>
</tr>
<tr>
<td>SNR = 6</td>
</tr>
</tbody>
</table>

Table 4.5: Snapshot results for PD = 0.7 from Figure 4.14b & 4.14c showing the recovered performance by the Tandem BBVS mitigation technique as a percentage. 100% represents the case where the mitigation technique has fully restored the radar performance back to the linear case.

Re-examining the results in Figure 4.14 and Table 4.4, we observe that introducing nonlinear memory effects to the tandem inverse model can drastically improve the effectiveness of the mitigation technique in the scenario where the radar receiver exhibits
4.4. Mitigation of Memory Rich Nonlinearities

Figure 4.14: Probability of detection analysis results for the memory rich cross-modulation scenario. Memoryless NLEQ technique performs well in the BBTS case but fails when significant memory effects are introduced to the forward nonlinearity. The performance of the LUT technique can be greatly enhanced in the BBVS case by extending the inverse structure to include nonlinear memory terms. As the level of guard channel noise or delay is increased the performance of the memory rich mitigation technique degrades. Note, the delay offset is displayed as a percentage of the interference pulse-width.
nonlinear memory effects, see BBVS - Tandem BBVS curve. The additional memory terms in the inverse LUT structure are able to untangle the complex memory effects introduced by the forward BBVS nonlinearity and restore the system performance almost back to the linear case. Furthermore, the technique is shown to be robust against reasonable levels of guard channel noise and delays as shown by Figures 4.14b and 4.14c respectively with a snapshot of the results also displayed in Table 4.5. This is an important result as it shows that the LUT inverse technique could provide a universal solution to the nonlinear receiver problem in radar that is both mode independent and capable of correcting for complex nonlinear memory effects. However, to fully understand the robustness of these LUT mitigation techniques in a real radar system more work to study the nonlinear memory behaviour of the radar receiver must be conducted. It was clear from further simulations with slightly different BBVS coefficients that the effectiveness of the memory rich LUT inverse mitigation technique was very dependent on the complexity of the forward nonlinear transfer function. While there were nonlinearities where the memory rich LUT technique performed considerably better than the standard memoryless techniques, there were also nonlinearities with complex nonlinear memory terms whose distortion effects could be successfully compensated for by varying the phase of the memoryless cross-channel inverse coefficient, $\tilde{g}_{3(out)}$. It is therefore likely that the choice of digital mitigation technique will depend on the specific nonlinear characteristics of the radar receiver and the source of the interference causing the unwanted distortion.

4.5 Discussion and Summary

In this chapter, we presented a study into the effectiveness of the LUT mitigation technique in correcting for cross-modulation distortion generated in the radar receiver. The work focused on the classic cross-modulation scenario for a typical MPRF radar mode where a pulsed out-of-band interferer stimulates a nonlinearity in the RF receiver. The research started by examining the mitigation performance of memoryless LUT techniques in the case where the receiver nonlinearity was strictly memoryless. It was observed that for this particular case, the memoryless mitigation techniques performed exceptionally well and completely restored the system sensitivity back to the linear case. Significant nonlinear memory effects were then introduced to the receiver nonlinearity and it was shown that the effectiveness of the memoryless LUT solutions were drastically diminished in comparison to the strictly memoryless case. This is an important result as it suggests that if the forward nonlinearity exhibits significant memory effects then the digital mitigation techniques must correct for the nonlinear memory otherwise their effectiveness will be considerably reduced. The implications of this result are likely to become more profound as modern radar systems move to wider receiver bandwidths where the memory effects can no longer be considered to be negligible. The latter half of this chapter focused on the development of a LUT mitigation solution capable of correcting for
4.5. Discussion and Summary

receiver nonlinearities with memory. By extending the tandem inverse approach to include nonlinear memory terms, it was shown that the technique could compensate for the subtle amplitude and phase effects which decorrelated the memoryless LUT techniques. Crucially, this allowed the digital mitigation technique to recover the majority of the sensitivity lost due to the memory rich nonlinearity (62% from Table 4.4).

The LUT digital mitigation techniques provides an attractive solution to the nonlinear receiver problem in radar that is both mode independent and computationally efficient to implement. In the case where the receiver nonlinearity is strictly memoryless, the LUT technique is capable of completely restoring the system performance back to the linear gain case. Furthermore, if nonlinear memory effects prove to be significant in the modern radar receiver then the LUT technique can be extended to provide a simple yet effective method for mitigating the complex nonlinear distortion generated. While the above statement is true, it is also important to note that in the memory rich case the performance of the LUT inverse technique is very much dependent on the complexity of the forward nonlinear transfer function. As mentioned previously, the theory of $p^{th}$ order inverses [54, 84] states that an inverse to a nonlinear transfer function does not always exist. Furthermore, even if an inverse does exist its exact form and the range of input powers over which it is defined is often ambiguous. The identification of a memory rich inverse nonlinearity is therefore an extremely challenging task particularly for radar systems where the source of the interference can be so diverse. More work needs to be done to understand the true nonlinear memory behaviour of the radar receiver before it is decided whether memory rich LUT techniques are suitable for a real radar system.

The research presented in this chapter could be extended in future work by studying the performance of the LUT technique for more memory rich receiver nonlinearities. This would benefit the understanding of when the memoryless solutions fail and when the memory rich technique provide performance improvements. Ideally this would be done using real coefficients from measured receiver nonlinearities. The inverse identification procedure could be developed further so that it successfully learns more inverse memory structures and ensures that they are robust against a wide variety of interference effects. This would most likely require a better understanding of whether the persistent excitation condition is being satisfied in the inverse during the noise identification procedure. Finally, the tandem inverse technique could be developed for the in-band interference scenario where the power of the interference signal driving the nonlinearity can make the identification of a memory inverse extremely challenging.
Chapter 5

Forward Modelling Mitigation Techniques

5.1 Aim and Introduction

The key aim of this chapter is to develop a forward modelling technique that can mitigate nonlinear distortion effects generated in the radar receiver. The forward modelling solution provides an alternative approach to the LUT inverse mitigation techniques described in chapter 4. The novel technique builds on the work previously published by Blumensath in [27] which was described in detail in chapter 2.

The novel NCS forward modelling technique derived in this chapter is targeted specifically at the in-band interference scenario for radar. However, that is not to say that the generalised NCS theory could not be extended to the cross-modulation problem studied in chapter 4. The in-band case was chosen for this analysis over the cross-modulation scenario as the addition of a secondary interference channel would lead to a far more complex formalisation. The in-band interference scenario is equally problematic for modern radar and was discussed in detail in chapters 1 and 2. As discussed previously, the wide bandwidth nature of modern radar systems means that the NCS algorithm must be designed to compensate for both memoryless and memory rich nonlinear receiver effects. Therefore, the black-box receiver model chosen for this analysis was the in-band BBVS model, (2.23), which was carried over to radar systems in chapter 3. The NCS algorithm is therefore tasked with recovering the baseband linear input signal \( \tilde{x}_3[n] \) from the distorted in-channel nonlinear output signal \( \tilde{y}[n] \), where all of the analysis is performed in the discrete-time baseband domain. For this analysis we chose the specific in-band interference to be a CW signal that is strong enough to drive the front-end receiver amplifiers into their weakly nonlinear regime. This common nonlinear radar scenario is detailed in chapter 2 with the deleterious effects illustrated for a typical MPRF mode in Figure 2.5. For the NCS algorithm to restore system sensitivity back to the linear case, it must reduce the strength of the unwanted clutter broadening effect such that the desired target can be recovered. While the introduction of nonlinear memory effects is not expected to change the gross level of clutter broadening generated, we do expect it to subtly change the structure and phase of the nonlinear distortion observed. Importantly, it was shown in
In chapter 4 that these subtle differences can decorrelate the classic memoryless LUT techniques used to restore system performance. We anticipate this also being the case for the forward modelling technique and therefore, the NCS algorithm must be formulated to incorporate complex nonlinear memory effects if it is to be successful in modern radar.

In this chapter, the NCS algorithm is extended and applied to mitigate nonlinear effects in the radar receiver caused by the presence of an in-band interferer. In addition to introducing this novel nonlinear mitigation technique to the field of radar, the following key contributions are presented in this chapter:

(i) Input and output noise are addressed as part of the complex baseband nonlinear model, see section 5.2.2.
(ii) The forward nonlinear function in the NCS algorithm is extended to include nonlinear memory effects, see section 5.2.3.
(iii) Unlike previous work that deals with the purely real problem, the NCS algorithm is extended to the complex case, see section 5.2.4.
(iv) A unique CS problem that exploits signal sparsity in slow-time and tackles a complex baseband nonlinearity with memory in fast-time is addressed, see section 5.2.5.
(v) The NCS algorithm is shown to mitigate nonlinear memory effects in the radar receiver for the first time, see section 5.4.
(vi) The NCS algorithm is tested against real radar data for the first time, see section 5.5.


5.2 Nonlinear Compressive Sensing

5.2.1 System Model

Before the novel NCS technique can be derived, we must first set-out the radar specific linear signal model that fundamentally underpins the unique formalisation of the algorithm. As mentioned previously, the scenario of interest for this chapter is where an in-band interferer drives the radar receiver into its weakly nonlinear region causing the desired signal to be distorted. We therefore describe the baseband input signal $\tilde{x}_3[n]$ as a linear combination of the desired radar signal $\tilde{x}_1[n]$ and the baseband in-band interference signal $\tilde{x}_2[n]$.

$$\tilde{x}_3[n] = \tilde{x}_1[n] + \tilde{x}_2[n]$$  (5.1)
5.2. Nonlinear Compressive Sensing

However, to keep the notation as simple as possible we drop the signal subscripts for this chapter so that the protected baseband symbol \( \hat{x} \) now specifically denotes the in-channel baseband input signal \( \hat{x}_3 \) with the carrier baseband output signal denoted by \( \hat{y} \) as before. Additionally, we recast (5.1) into vector notation in (5.2) where \( \hat{x} \) denotes the baseband input signal \( \hat{x}_3 \) with the carrier baseband output signal denoted by \( \hat{y} \) as before.

\[
\hat{x} = \hat{v} + \hat{s}
\]

In the above expression \( \hat{x} \), \( \hat{v} \) and \( \hat{s} \) represent column vectors of length \( N \) consisting of fast-time signal samples \( \hat{x}_n \), \( \hat{v}_n \) and \( \hat{s}_n \) respectively\(^1\). Importantly, the desired radar signal \( \hat{v} \) consists of a linear combination of target, clutter and noise signals while we choose signal \( \hat{s} \) to be a CW interferer. For simplicity we assume that the radar transmits a single-tone pulsed signal and the CW interferer is ever-present in the received PRI. Each received PRI signal is indexed with the subscript \( \hat{x}_q \) and is concatenated into a received signal matrix of size \( N \times Q \) labelled \( \hat{X} = [\hat{x}_1 \hat{x}_2 \ldots \hat{x}_Q] \), see Figure 2.2. The corresponding output matrix \( \hat{Y} \) is constructed similarly by concatenating each subsequent PRI output \( \hat{y}_q \) from the black-box receiver model into an \( N \times Q \) matrix as follows \( \hat{Y} = [\hat{y}_1 \hat{y}_2 \ldots \hat{y}_Q] \). The nonlinearity is therefore applied down the columns of matrix \( \hat{X} \) to produce matrix \( \hat{Y} \) acting in what is referred to as the fast-time dimension\(^2\).

In this chapter we once again focus on the MPRF mode of operation for modern radar. The standard pulse-Doppler radar processing used to reveal targets in a MPRF mode was discussed previously in chapter 2 [11, 10]. Typically, a matched filter is applied down the columns of \( \hat{Y} \) to convert the raw fast-time signals into range before a discrete-time Fourier transform is applied across the received PRIs to reveal the Doppler behaviour of the artefacts in the scene. Importantly for this analysis, we assume that the radar transmits a single-tone waveform resulting in the matched filter operation being redundant as fast-time is directly equivalent to range in our received signal matrix. The simple radar processing therefore consists of a DFT applied along the rows of \( \hat{Y} \) in what is referred to as the slow-time dimension leading to the operation being labelled the slow-time Fourier transform (STFT). By performing the STFT on the received data the weak targets can be separated from the clutter in Doppler and due to their coherent nature can also be pulled above the noise floor. It is in this two dimensional range-Doppler (RD) space that target detection analysis is typically performed, see chapter 2\(^3\).

---

\(^1\)The subscripts are used in this chapter to denote vector and matrix element indices.

\(^2\)A single matrix element is denoted by a non-bold symbol with dual subscripts, \( \hat{x}_{\{n,q\}} \). Furthermore, specific rows and columns of the data matrices are denoted by a bold symbol with a single subscript that indicates the matrix dimension that has been pulled out; \( \hat{x}_n \) denotes a vector corresponding to row \( n \) of matrix \( \hat{X} \) of size \( 1 \times Q \) and \( \hat{x}_q \) denotes a vector corresponding to column \( q \) of matrix \( \hat{X} \) of size \( N \times 1 \).

\(^3\)Subscripts \( n \) and \( q \) are used to denote the fast-time and slow-time dimensions of the received data matrices respectively.
Importantly, in this section we define a two-dimensional linear signal model to effectively describe the simple MPRF radar processing operation outlined above. We start by considering the input signal vector $\tilde{x}_q$ for the $q^{th}$ PRI which consists of a linear combination of received pulses from both the radar scene and the interference source. In the following formalisation, $\tilde{x}_q$ denotes the signal corresponding to column $q$ of matrix $\tilde{X}$ and contains a series of fast-time signal samples $n$. As discussed previously, in standard pulse-Doppler radar processing a filter is applied down each received PRI in order to convert fast-time, $n$, into range, $r$. This is illustrated formally for a single PRI signal in (5.3).

$$\tilde{x}_q = \sum_{r=1}^{R} \psi_r \tilde{x}'_{(r,q)} = \Psi \tilde{x}'_q$$

(5.3)

where $\psi_r$ represents column vector $r$ of the fast-time filter matrix $\Psi$ size $N \times R$ which can take the form of a Toeplitz convolution matrix; and $\tilde{x}'_q$ denotes the range converted signal vector for PRI $q$ size $R \times 1$. Now that the received PRI signals have been converted from fast-time into range, their underlying structure can be studied in the slow-time dimension which acts along the rows of matrix $\tilde{X}'$ where $\tilde{X}' = [\tilde{x}'_1 \ \tilde{x}'_2 \ \ldots \ \tilde{x}'_Q]$. Note in this chapter we adopt the convention whereby row vectors are denoted by the underline accent e.g. $\tilde{x}'_r = [\tilde{x}'_{(r,1)} \ \tilde{x}'_{(r,2)} \ \ldots \ \tilde{x}'_{(r,Q)}]$. Let us therefore examine the slow-time signal vector for a single range gate $r$. The observed signal vector $\tilde{x}'_r$ can be represented as a linear combination of $J$ frequency vectors $\{\phi_j\}_{j=1}^{J}$ where each frequency vector is weighted by the corresponding complex coefficient $\tilde{\theta}_{(r,j)}$. Therefore, the row vector $\tilde{\theta}_r$ where $\tilde{\theta}_r = [\tilde{\theta}_{(r,1)} \ \tilde{\theta}_{(r,2)} \ \ldots \ \tilde{\theta}_{(r,J)}]$ contains a set of Fourier coefficients that describes the slow-time behaviour of the scatterers at a particular range gate $r$. This is displayed formally in (5.4) below,

$$\tilde{x}'_r = \frac{1}{\sqrt{J}} \sum_{j=1}^{J} \tilde{\theta}_{(r,j)} \phi_j = \tilde{\theta}_r \Phi$$

(5.4)

where $\Phi$ is a frequency basis matrix of size $J \times Q$, i.e, comprising the row vectors $\phi_j$. By combing (5.3) and (5.4) we can define the two-dimensional linear signal model (5.5) which effectively describes the simple radar processing operation applied in a pulse-Doppler radar. In (5.5), the Fourier vectors $\tilde{\theta}_r$ have been concatenated in range to form the matrix $\tilde{\Theta}$ of size $R \times J$, e.g. $\tilde{\Theta} = [\tilde{\theta}_1^T \ \tilde{\theta}_2^T \ \ldots \ \tilde{\theta}_R^T]^T$ where $^T$ denotes the matrix transpose operation.

$$\tilde{X} = \Psi \tilde{\Theta} \Phi$$

(5.5)

In the signal model described by (5.5), $\Psi$ represents the range projection matrix that maps range to fast-time, $\Phi$ describes the slow-time Fourier basis matrix and $\tilde{\Theta}$ denotes the corresponding signal model regressors. As discussed previously, we assume that the radar transmits a single-tone waveform and therefore fast-time is equivalent to range. This assumption simplifies the structure of the range projection matrix $\Psi$ to be an identity.
matrix of size $N \times R$ where $N = R$. If the radar were to transmit a more complex waveform then the range projection matrix $\Psi$ would need to be updated to represent the new mapping from range to fast-time. In contrast, the Fourier basis matrix has a size $J \times Q$ and takes the same form as that described in chapter 2 where each column of $\Phi$ represents an independent frequency vector\(^4\). When dealing with the radar signal model (5.5), it is often convenient to adopt the elementwise formalisation stated in (5.6) below.

\[
\tilde{x}_{\{n,q\}} = \sum_j \left\{ \sum_r \Psi_{\{n,r\}} \tilde{\theta}_{\{r,j\}} \right\} \Phi_{\{j,q\}}
\]

(5.6)

In (5.6), the subscripts $n, q, r, j$ are used to denote the elements of the corresponding matrix\(^5\). Additionally, the subscripts are used to label the dimensions of each specific matrix with: $n$ representing the fast-time dimension, $q$ indicating the slow-time dimension, $r$ denoting the range dimension and $j$ representing the Doppler dimension. Examining (5.6), it is clear that matrix $\Psi$ converts range to fast-time and matrix $\Phi$ converts Doppler to slow-time resulting in the modelling regressors being mapped from $\tilde{\theta}_{\{r,j\}}$ to $\tilde{x}_{\{n,q\}}$.

To further understand the basis for the two-dimensional radar signal model, (5.5), we can combine these two basis matrices into a single transformation matrix by means of the Kronecker product denoted by the symbol $\otimes$. This results in the vectorised radar signal model, (5.7), where $vec(.)$ denotes the vectorisation operation which stacks matrices in a column-wise manner.

\[
vec(\tilde{X}) = \{ \Phi^T \otimes \Psi \} vec(\tilde{\Theta})
\]

(5.7)

In (5.7), $vec(\tilde{X})$ is a column vector of size $NQ$, $vec(\tilde{\Theta})$ is a column vector of size $RJ$ and $\{ \Phi^T \otimes \Psi \}$ is a matrix of size $NQ \times RJ$ where $^T$ denotes the matrix transpose operation. The structure of the Kronecker transformation matrix $\{ \Phi^T \otimes \Psi \}$ is displayed in Figure 5.1 alongside the structure of the range projection matrix $\Psi$ and Fourier basis $\Phi$. Examining Figure 5.1c it is apparent that the full support structure for the radar signal model is a block matrix where each block consists of a Fourier basis element $\Phi_{\{j,q\}}$ multiplied by the identity matrix $\Psi$. Considering the fact that the Kronecker transformation matrix is applied to $vec(\tilde{\Theta})$ it is clear that this matrix multiplication represents an IDFT operation applied along the rows of matrix $\Theta$. While we do not employ the vectorised signal model (5.7) in our derivation of the NCS algorithm, picturing the radar signal model in this way can be very helpful when trying to understand the basis support structure which fundamentally underpins the NCS radar theory. Furthermore, by considering the inverse version of the vectorised signal model (5.7) we can gain a better insight into the meaning of the signal model regressor matrix $\tilde{\Theta}$. The inverse vectorised signal model is stated in

\(^4\)The Fourier basis matrix is defined by the DFT operation and was discussed in the Linear Compressive Sensing section of chapter 2.

\(^5\)When indexing elements from a matrix, a lowercase symbol is used if the variable represents a set of signals while an uppercase symbol is used if it represents a basis/transformation matrix. For example $\tilde{x}_{\{n,q\}}$ denotes elements from the parent signal matrix $\tilde{X}$ while $\Phi_{\{j,q\}}$ denotes elements from the parent basis matrix $\Phi$. 
(A) Range projection matrix size $N \times R$. Equal to the identity matrix when the radar transmits a single-tone waveform causing fast-time and range to be equivalent.

(b) Fourier basis matrix size $J \times Q$. Formed by performing a normalised IDFT down the columns of the identity matrix. Each column therefore corresponds to a separate Doppler frequency vector.

(c) Kronecker transformation matrix size $NQ \times RJ$. Block matrix representation resulting in a matrix that performs an IDFT operation along the rows of $\tilde{\Theta}$. Notice that the $\Phi$ matrix indices have been flipped due to the transpose operation in the Kronecker product $\{\Phi^T \otimes \Psi\}$.

**Figure 5.1**: Illustration of radar signal model basis. The structures of $\Psi$ and $\Phi$ are depicted in Figures 5.1a and 5.1b respectively. The structure of the Kronecker transformation matrix $\{\Phi^T \otimes \Psi\}$ is illustrated in Figure 5.1c.
(5.8) below where $^H$ denotes the matrix Hermitian/conjugate transpose.

$$\{\Phi^T \otimes \Psi\}^H vec(\tilde{X}) = vec(\tilde{\Theta})$$  (5.8)

The inverse Kronecker transformation matrix $\{\Phi^T \otimes \Psi\}^H$ has a similar structure to the forward Kronecker transformation matrix, see Figure 5.1c, except instead of performing an IDFT along the rows of matrix $\Theta$ it performs the DFT operation along the rows of matrix $\tilde{X}$. Importantly, this DFT operation is exactly the same for each subsequent range gate as each slow-time signal $\tilde{x}_n$ is described by the same Fourier basis support $\Phi$. Thus, it is apparent from (5.8) that the inverse signal model effectively defines the STFT operation described in the previous section. It therefore follows that the model regressors themselves describe the entire range-Doppler detection space, the structure of which was discussed previously for the in-band interferer scenario in chapter 2 and is illustrated for the linear case in Figure 5.2 below.

![Figure 5.2: Illustration of linear range-Doppler map for the in-band interference scenario. Target located at $\{-6.2kHz, -1.3km\}$, clutter located between $-3.0kHz$ and $-0.7kHz$ Doppler and the CW interferer has a Doppler frequency of $11.2kHz$. The regressor matrix $\tilde{\Theta}$ describes the entire RD detection space.](image)

Finally, we define the nonlinear observation model (5.9) which relates the measured nonlinear output matrix $\tilde{Y}$ to the linear regressor matrix $\tilde{\Theta}$ by combining the radar signal model (5.5) with the specific black-box nonlinear receiver model denoted by function $\Gamma(\cdot)$.

$$\tilde{Y} = \Gamma(\tilde{X}) = \Gamma(\Psi\tilde{\Theta}\Phi)$$  (5.9)
By integrating this nonlinear observation model into the radar processing, the problem of recovering the linear input signal $\tilde{X}$ is equivalent to estimating the model regressor matrix $\tilde{\Theta}$. Importantly, this allows prior knowledge of the radar detection space to be incorporated into the estimation process. The problem can therefore be setup as a sparse nonlinear optimisation one where the linear regressor matrix $\tilde{\Theta}$ can be estimated from the measured nonlinear output matrix $\tilde{Y}$. Like most solutions to sparse signal processing problems, the NCS algorithm is signal dependent and has an iterative formalisation. The algorithm’s sparsity constraint and resultant signal dependence are discussed in detail at the end of this section once the iterative step has been derived in full. We start however by considering the effect of introducing noise to the nonlinear observation model in the following section.

### 5.2.2 Dealing with Noise

When dealing with noise in the nonlinear observation model we consider two independent noise sources: input noise $\tilde{U}$ and output noise $\tilde{W}$. The former is additive noise present at the input of the receiver. The latter is additive noise present at the output of the receiver. For linear systems these could be used interchangeably [11, 21, 13], however due to the nonlinearity in the observation model we must start by treating these noise sources separately. Therefore, $\tilde{U}$ might be thought of as representing the background noise from the radar scene (not including clutter or interference) with $\tilde{W}$ representing the thermal noise in the RF receiver. If we assume that the black-box behavioural model (2.23) accurately captures the forward nonlinearity, then the output noise $\tilde{W}$ can be considered to be genuinely additive and uncorrelated from sample to sample. It is therefore straightforward to address by simply tagging the $N \times Q$ noise matrix $\tilde{W}$ to the nonlinear output matrix $\tilde{Y}$. It is important to note that the output noise term $\tilde{W}$ may also be viewed as accounting for un-modelled terms in the in-band nonlinear behavioural model (2.23) i.e. terms beyond the $P^{th}$ one. In which case, the output noise will be signal dependent and most likely correlated from sample to sample in the fast-time dimension $n$.

The input noise is more challenging to deal with in the case of the nonlinear observation model as unlike the linear case, we cannot assume that each output sample is a linear combination of signal plus noise. To study this further we follow the input noise through the memoryless in-band BBVS forward nonlinear model described by (2.23) where $L = 1$. The full matrix form of the linear signal model with input noise is shown below. The input noise is represented by $\tilde{U}$ and is a matrix of size $N \times Q$.

$$\tilde{X} = \Psi \tilde{\Theta} \Phi + \tilde{U} \quad \text{(5.10)}$$

For mathematical simplicity we continue with an elementwise formalisation where each dimension is labelled using the previously defined subscripts. Thus, the elementwise
signal model takes the following form:

\[
\tilde{x}_{\{n,q\}} = \sum_j \left\{ \sum_r \Psi_{\{n,r\}} \tilde{\theta}_{\{r,j\}} \right\} \Phi_{\{j,q\}} + \tilde{u}_{\{n,q\}} \tag{5.11}
\]

The memoryless observation model is formed by combining the elementwise formalisation of (5.9) with the memoryless BBVS black-box behavioural model, (2.23). For simplicity, the scale factor \( \frac{1}{2} \left( \frac{2^{p+1}}{2^{2p}} \right) \) in BBVS model (2.23) has been absorbed into the nonlinear kernel coefficients \( \tilde{h}_{2p+1} \) for this analysis\(^6\). Thus, the elementwise form of the memoryless nonlinear observation model is as follows,

\[
\tilde{y}_{\{n,q\}} = \sum_{p=0}^P \tilde{h}_{2p+1} \prod_{s=1}^p \tilde{x}_{\{n,q\}} \prod_{d=p+1}^{2p+1} \tilde{x}_{\{n,q\}} + \tilde{w}_{\{n,q\}} \tag{5.12}
\]

Substituting (5.11) into (5.12) leads to,

\[
\tilde{y}_{\{n,q\}} = \sum_{p=0}^P \tilde{h}_{2p+1} \left\{ \sum_j \left\{ \sum_r \Psi_{\{n,r\}} \tilde{\theta}_{\{r,j\}} \right\} \Phi_{\{j,q\}} + \tilde{u}_{\{n,q\}} \right\}^{2p} \tag{5.13}
\]

For what might be called the signal limited case, \( \sum_j \left\{ \sum_r \Psi_{\{n,r\}} \tilde{\theta}_{\{r,j\}} \right\} \Phi_{\{j,q\}} \) >> \( \tilde{u}_{\{n,q\}} \), we find that \( \left| \tilde{x}_{\{n,q\}} \right|^{2p} \approx \left| \sum_j \left\{ \sum_r \Psi_{\{n,r\}} \tilde{\theta}_{\{r,j\}} \right\} \Phi_{\{j,q\}} \right|^{2p} \). Thus, (5.13) can be reduced to the following form:

\[
\tilde{y}_{\{n,q\}} = \sum_{p=0}^P \tilde{h}_{2p+1} \left\{ \sum_j \left\{ \sum_r \Psi_{\{n,r\}} \tilde{\theta}_{\{r,j\}} \right\} \Phi_{\{j,q\}} \right\}^{2p} \tag{5.14}
\]

and expanding out the brackets,

\[
\tilde{y}_{\{n,q\}} = \sum_{p=0}^P \tilde{h}_{2p+1} \left\{ \sum_j \left\{ \sum_r \Psi_{\{n,r\}} \tilde{\theta}_{\{r,j\}} \right\} \Phi_{\{j,q\}} \right\}^{2p} \sum_j \left\{ \sum_r \Psi_{\{n,r\}} \tilde{\theta}_{\{r,j\}} \right\} \Phi_{\{j,q\}} \tag{5.15}
\]

\[^6\text{The coefficients for the BBTS model are distinguished from those for the full BBVS model by omitting the delay tap indices \{i_1, \ldots, i_{2p+1}\} from the notation.}\]
If we now re-define the signal model to remove the noise term $\tilde{u}_{n,q}$ as in (5.6), the associated memoryless nonlinear observation model becomes:

$$\tilde{y}_{n,q} = \sum_{p=0}^{P} \tilde{h}_{2p+1} |\tilde{x}_{n,q}|^{2p} + \tilde{u}_{n,q} \sum_{p=0}^{P} \tilde{h}_{2p+1} |\tilde{x}_{n,q}|^{2p} + \tilde{w}_{n,q}$$  \hspace{1cm} (5.16)

Examining (5.16) it is clear that in the signal limited case the associated input noise term is separate from the dominant nonlinear response described by (5.12). This strictly additive nature allows the input noise term to be absorbed by the output noise term $\tilde{w}_{n,q}$. We rename $\tilde{w}_{n,q}$ as system modelling error as it now encompasses both output and input noise terms. In essence, the system modelling error captures any deviation, both noise or otherwise, from the black-box behavioural model output and the true nonlinear output. Importantly, this novel approach for treating the noise in the nonlinear observation model will be validated in the Simulation Architecture section of this chapter.

### 5.2.3 IHT Algorithm for NCS

Now that the linear signal model has been defined in (5.5) we derive the IHT algorithm’s iterative step in full. As with the signal noise analysis above, we employ an elementwise formalisation for simplicity. By combining the BBVS expression (2.23) with the nonlinear observation model (5.9) we rewrite the nonlinear output signal $\tilde{y}_{n,q}$ in terms of the elementwise nonlinear function $\tilde{F}_{n,q,p,\{i_1,\ldots,i_{2p+1}\}}$:

$$\tilde{y}_{n,q} = \Gamma(\tilde{x}_{n,q}) + \tilde{w}_{n,q}$$

$$= \sum_{p=0}^{P} \left( \sum_{i_1=0}^{L-1} \cdots \sum_{i_{2p+1}=0}^{L-1} \tilde{h}_{2p+1,\{i_1,\ldots,i_{2p+1}\}} \tilde{F}_{n,q,p,\{i_1,\ldots,i_{2p+1}\}} \right) + \tilde{w}_{n,q}$$ \hspace{1cm} (5.17)

where $\tilde{F}_{n,q,p,\{i_1,\ldots,i_{2p+1}\}}$ takes the generalised BBVS form,

$$\tilde{F}_{n,q,p,\{i_1,\ldots,i_{2p+1}\}} = \prod_{s=1}^{p} \tilde{x}_{n-i_s,q} \prod_{d=p+1}^{2p+1} \tilde{x}_{n-i_d,q}$$ \hspace{1cm} (5.18)

In (5.17) and (5.18), $i$ denotes the baseband sample delay as previously defined and we assume that the scale factor $\frac{1}{2} \left[ \binom{2p+1}{p} / 2^{2p} \right]$ in (2.23) has been absorbed into the nonlinear kernel coefficients $\tilde{h}_{2p+1,\{i_1,\ldots,i_{2p+1}\}}$ as in the previous section. As with any CS problem the task for the NCS IHT algorithm is to recover the desired signal $\tilde{x}_{n,q}$ from the corrupted output signal $\tilde{y}_{n,q}$. If we assume that the modelling error $\tilde{w}_{n,q}$ is small then we may define the observation error $\tilde{e}_{n,q}$ as follows:

$$\tilde{e}_{n,q} = \tilde{y}_{n,q} - \sum_{p=0}^{P} \left( \sum_{i_1=0}^{L-1} \cdots \sum_{i_{2p+1}=0}^{L-1} \tilde{h}_{2p+1,\{i_1,\ldots,i_{2p+1}\}} \tilde{F}_{n,q,p,\{i_1,\ldots,i_{2p+1}\}} \right)$$ \hspace{1cm} (5.19)
with the corresponding least squares (LS) cost function, $C$, defined in the usual manner (5.20).

$$C = \sum_q \sum_n \tilde{e}^*_\{n,q\} \tilde{e}_\{n,q\}$$  \hspace{1cm} (5.20)

By seeking an iterative gradient-based solution, the above cost function can be minimised with respect to the unknown linear signal model parameters $\tilde{\Theta}$ subject to any constraints on $\tilde{\Theta}$ e.g. sparsity. Unlike Blumensath’s IHT algorithm for NCS [27] which dealt with the strictly real case, the cost function in (5.20) has been derived in the baseband domain and is therefore complex in nature. This is potentially problematic for the IHT algorithm as the usual mathematical concept of derivation to obtain a gradient is only defined for real numbers [93, 101]. An operational solution for the optimization of real cost functions as a function of a complex vector was developed by Brandwood in [102] and is applied here to deduce the gradient step. It is worth noting, that the technique developed by Brandwood is extended by Amin et al. in [103] in what is referred to as the Wirtinger flow method. Importantly, the Wirtinger flow technique developed in [103] is fundamentally based around Brandwood’s initial theory and is just a more advanced solution to the complex-gradient problem for neural networks. This more sophisticated technique is unlikely to provide any measurable advantage\(^7\) for the problem case tackled in this thesis which is why we apply Brandwood’s original theorem from [102] here. This will require a $1 \times J$ row vector of co-factors for each range gate regressor vector $\tilde{\theta}_r$.

$$\frac{dC}{d\tilde{\Theta}_r} = \begin{bmatrix} \frac{\partial C}{\partial \tilde{\theta}_\{r,1\}} & \frac{\partial C}{\partial \tilde{\theta}_\{r,2\}} & \cdots & \frac{\partial C}{\partial \tilde{\theta}_\{r,J\}} \end{bmatrix}$$  \hspace{1cm} (5.21)

and associated gradient vector,

$$\nabla_C(\tilde{\Theta}_r) = \left\{ \frac{dC}{d\tilde{\Theta}_r} \right\}^*$$  \hspace{1cm} (5.22)

Two equations define the gradient step in the iterative learning procedure:

$$\tilde{\theta}_r^{(\beta)} = \tilde{\theta}_r^{(\beta-1)} - \mu \nabla_C(\tilde{\Theta}_r^{(\beta-1)})$$  \hspace{1cm} (5.23)

and a constraint step which is applied to the entire $R \times J$ regressor matrix $\tilde{\Theta}^{(\beta)}$.

$$\tilde{\Theta}^{(\beta)} = \begin{bmatrix} \tilde{\Theta}_1^{(\beta)} & \tilde{\Theta}_2^{(\beta)} & \cdots & \tilde{\Theta}_R^{(\beta)} \end{bmatrix}^T$$

$$\tilde{\Theta}^{(\beta)} \leftarrow P_A(\tilde{\Theta}^{(\beta)})$$  \hspace{1cm} (5.24)

\(^7\)A study into whether any convergence advantages could be gained by applying the Wirtinger flow solution to the complex-gradient problem instead of Brandwood’s original technique could be an interesting piece of future work.
In (5.23) and (5.24), \( P_A(.) \) denotes the radar constraint operation, \( \mu \) represents the step parameter and \( \beta \) denotes the IHT iteration number. These steps are applied repeatedly until the LS error \( C \) is suitably small at which point the algorithm can be considered to have converged. Obviously in the full formalisation of the IHT algorithm for NCS, the regressor matrix \( \tilde{\Theta}^{(\beta)} \) must be updated as part of a loop over range gate \( r \). This is illustrated in Table 5.1 where the full algorithm is presented.

Blumensath showed in [27] that the IHT algorithm can theoretically recover a sparse signal observed through a nonlinear function under similar assumptions to those made in the linear CS case. However, Blumensath assumes that the nonlinear observation has a proper linear approximation which effectively projects it back into a linear function. In [28] and [89], the NCS algorithm is extended to exploit knowledge of the nonlinear function through which the input signal is observed. While the nonlinear functions in [28] and [89] are more complicated than that in [27], both are still strictly real and have a purely memoryless formalisation. For the NCS algorithm to be extended to the complex BBVS case, we must consider the complex gradient calculation more carefully.

### 5.2.4 Calculating the Gradient

In order for the IHT algorithm to converge to the correct solution, the least squares cost function \( C \) must be minimised with respect to the unknown model regressor matrix \( \tilde{\Theta} \) subject to the chosen radar constraint \( P_A(.) \). The gradient step (5.23) therefore plays a crucial role in the iterative learning procedure as it constantly steers the algorithm towards the minimum of the cost function. The calculation of the gradient is non-trivial in the NCS algorithm as the desired signal has been observed through a complex nonlinear function. Let us start by considering a single term \( \frac{\partial C}{\partial \tilde{\theta}_{(r,j)}} \) from the vector of co-factors described by (5.21). As with the previous sections in this chapter, we adopt an elementwise formalisation for mathematical simplicity.

\[
\frac{\partial C}{\partial \tilde{\theta}_{(r,j)}} = \frac{\partial}{\partial \tilde{\theta}_{(r,j)}} \left\{ \sum_q \sum_n \tilde{e}^*_{(n,q)} \tilde{e}_{(n,q)} \right\} = \sum_q \sum_n \left\{ \tilde{e}^*_{(n,q)} \frac{\partial}{\partial \tilde{\theta}_{(r,j)}} \{ \tilde{e}_{(n,q)} \} + \frac{\partial}{\partial \tilde{\theta}_{(r,j)}} \{ \tilde{e}^*_{(n,q)} \} \tilde{e}_{(n,q)} \right\}
\]

Substituting (5.19) in for \( \tilde{e}_{(n,q)} \) we find,

\[
\frac{\partial C}{\partial \tilde{\theta}_{(r,j)}} = -\sum_q \sum_n \left\{ \tilde{e}^*_{(n,q)} \sum_{p=0}^{P} \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \tilde{h}_{(2p+1,(i_1,...,i_2p+1))} \frac{\partial}{\partial \tilde{\theta}_{(r,j)}} \{ \tilde{F}^*_{(n,q,p,(i_1,...,i_2p+1))} \} \right\} + \tilde{e}_{(n,q)} \sum_{p=0}^{P} \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \tilde{h}_{(2p+1,(i_1,...,i_2p+1))} \frac{\partial}{\partial \tilde{\theta}_{(r,j)}} \{ \tilde{F}^*_{(n,q,p,(i_1,...,i_2p+1))} \} \]
\]

\[(5.25)\]

\[(5.26)\]
### IHT Algorithm for NCS

1. **for** $\beta = 1 : \beta_{\text{max}}$

2. **if** $\beta = 1$

3. $\tilde{\Theta} = \Psi^H \tilde{Y} \Phi^H$

4. **elseif** $\beta > 1$

5. $\tilde{X} = \Psi \tilde{\Theta} \Phi$

6. $\tilde{E} = \tilde{Y} - \Gamma(\tilde{X})$

7. $C = \text{vec}(\tilde{E})^H \text{vec}(\tilde{E})$

8. **if** $\beta > 2$

9. **if** $C^{(\beta)} > C^{(\beta-1)}$

10. **break**

11. **end**

12. **end**

13. **for** $r = 1 : R$

14. \[
\frac{dC}{d\tilde{\theta}_r} = \left[ \frac{\partial C}{\partial \tilde{\theta}_{(r,1)}} \quad \frac{\partial C}{\partial \tilde{\theta}_{(r,2)}} \quad \ldots \quad \frac{\partial C}{\partial \tilde{\theta}_{(r,J)}} \right]^{T}
\]

15. $\nabla C(\tilde{\theta}_r) = \left[ \frac{dC}{d\tilde{\theta}_r} \right]^{T}$

16. $\tilde{\theta}_r = \tilde{\theta}_r - \mu \nabla C(\tilde{\theta}_r)$

17. **end**

18. $\tilde{\Theta} = \left[ \tilde{\theta}_1^T \quad \tilde{\theta}_2^T \quad \ldots \quad \tilde{\theta}_R^T \right]^T$

19. **end**

20. $\hat{\Theta} = P_A(\tilde{\Theta})$

21. **end**

### Table 5.1: Summary of IHT algorithm for NCS

The algorithm runs for a maximum of $\beta_{\text{max}}$ iterations or at least until it has converged to a minimum-error solution where the next iteration increases the LS error value $C$, see line 10. The model regressor matrix $\tilde{\Theta}$ is initialised during the first iteration of the algorithm using the measured nonlinear output matrix $\tilde{Y}$, see line 3, before being updated in all subsequent iterations. The error matrix $\tilde{E}$ is calculated in line 6 from the current estimate of the linear input signal matrix $\tilde{X}$ before being used to compute the least square error in line 7. The NCS gradient is calculated in lines 13-18 before the radar constraint $P_A(\cdot)$ is applied in line 20 completing the iterative learning procedure.
By applying the chain rule of differentiation to (5.26) we can expand the expression to the following form,

$$\frac{\partial C}{\partial \theta_{(r,j)}} =$$

$$- \sum_q \sum_n \left\{ e^*_q \{ n, q \} \sum_{p=0}^{n} \left\{ \sum_{i_1=0}^{L-1} \cdots \sum_{i_{2p+1}=0}^{L-1} \right\} \hat{h}_{(2p+1, \{ i_1, \ldots, i_{2p+1} \})}$$

$$\left\{ \left\{ \frac{\partial \hat{F}_{(n,q,p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \hat{x}_{\{n-i_1,q\}}} \frac{\partial \hat{x}_{\{n-i_1,q\}}}{\partial \theta_{(r,j)}} + \frac{\partial \hat{F}_{(n,q,p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \hat{x}_{\{n-i_1,q\}}} \frac{\partial \hat{x}_{\{n-i_1,q\}}}{\partial \theta_{(r,j)}} \right\}$$

$$+ \ldots + \left\{ \left\{ \frac{\partial \hat{F}_{(n,q,p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \hat{x}_{\{n-i_2p+1,q\}}} \frac{\partial \hat{x}_{\{n-i_2p+1,q\}}}{\partial \theta_{(r,j)}} + \frac{\partial \hat{F}_{(n,q,p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \hat{x}_{\{n-i_2p+1,q\}}} \frac{\partial \hat{x}_{\{n-i_2p+1,q\}}}{\partial \theta_{(r,j)}} \right\} \right\}$$

$$+ \hat{e}_{(q,n)} \sum_{p=0}^{n} \left\{ \sum_{i_1=0}^{L-1} \cdots \sum_{i_{2p+1}=0}^{L-1} \hat{h}_{(2p+1, \{ i_1, \ldots, i_{2p+1} \})}$$

$$\left\{ \left\{ \frac{\partial \hat{F}_{(n,q,p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \hat{x}_{\{n-i_1,q\}}} \frac{\partial \hat{x}_{\{n-i_1,q\}}}{\partial \theta_{(r,j)}} + \frac{\partial \hat{F}_{(n,q,p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \hat{x}_{\{n-i_1,q\}}} \frac{\partial \hat{x}_{\{n-i_1,q\}}}{\partial \theta_{(r,j)}} \right\}$$

$$+ \ldots + \left\{ \left\{ \frac{\partial \hat{F}_{(n,q,p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \hat{x}_{\{n-i_2p+1,q\}}} \frac{\partial \hat{x}_{\{n-i_2p+1,q\}}}{\partial \theta_{(r,j)}} + \frac{\partial \hat{F}_{(n,q,p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \hat{x}_{\{n-i_2p+1,q\}}} \frac{\partial \hat{x}_{\{n-i_2p+1,q\}}}{\partial \theta_{(r,j)}} \right\} \right\} \right\}$$

(5.27)

As discussed previously, the derivative of a complex function of a complex variable does not exist in general. However, using the theory developed in [102] it can be defined to give a gradient for a real function $C$ of a complex variable $\hat{\theta}_{(r,j)}$ if $\tilde{\theta}_{(r,j)}$ is treated as a constant and $\frac{\partial \theta_{(r,j)}}{\partial \theta_{(r,j)}} = 0$. Therefore, in the context of the NCS algorithm:

$$\tilde{x}_{\{n-i_{2p+1},q\}} = \sum_j \left\{ \sum_r \Psi_{\{n-i_{2p+1},r\}} \tilde{\theta}_{(r,j)} \right\} \Phi_{\{q,j\}}$$

(5.28)

and,

$$\tilde{x}^*_n_{2p+1},q = \sum_j \left\{ \sum_r \Psi^*_{\{n-i_{2p+1},r\}} \tilde{\theta}^*_n_{2p+1},r \right\} \Phi^*_{\{q,j\}}$$

(5.29)

Additionally, since $\frac{\partial \tilde{x}^*_n_{2p+1},q}{\partial \theta_{(r,j)}} = 0$ then $\tilde{x}^*_n_{2p+1},q$ can also be treated as a constant for other partial derivatives when applying the chain rule. By applying these simple rules to
(5.27) the form of the gradient expression can be simplified drastically.

\[
\frac{\partial C}{\partial \theta_{(r,j)}} = \]

\[
-\sum_q \sum_n \left\{ \tilde{e}_{(q,n)}^* \sum_{p=0}^{P} \left( \sum_{i_1=0}^{L-1} \cdots \sum_{i_{2p+1}=0}^{L-1} \right) \right\} \tilde{h}_{(2p+1,\{i_1,\ldots,i_{2p+1}\})} \left\{ \frac{\partial \tilde{F}_{(n\cdot q\cdot p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \tilde{x}_{(n\cdot i_1,q)}} \Psi_{(n-i_1,r)} \right\} \Phi_{(q,j)}
\]

\[
+ \cdots + \frac{\partial \tilde{F}_{(n\cdot q\cdot p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \tilde{x}_{(n-i_{2p+1},q)}} \Psi_{(n-i_{2p+1},r)} \Phi_{(q,j)} \right\}
\]

\[
+ \tilde{e}_{(q,n)} \sum_{p=0}^{P} \left( \sum_{i_1=0}^{L-1} \cdots \sum_{i_{2p+1}=0}^{L-1} \right) \tilde{h}_{(2p+1,\{i_1,\ldots,i_{2p+1}\})} \left\{ \frac{\partial \tilde{F}_{(n\cdot q\cdot p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \tilde{x}_{(n\cdot i_1,q)}} \Psi_{(n-i_1,r)} \right\} \Phi_{(q,j)}
\]

\[
+ \cdots + \frac{\partial \tilde{F}_{(n\cdot q\cdot p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \tilde{x}_{(n-i_{2p+1},q)}} \Psi_{(n-i_{2p+1},r)} \Phi_{(q,j)} \right\}
\]

(5.30)

Important in (5.30), each slow-time signal \( \tilde{x}_n \) is described by the same Fourier basis support \( \Phi \) and therefore matrix \( \Phi_{(q,j)} \) is not delayed in any of the terms in the above expression for the gradient. This unique property of the radar signal model allows us to simplify (5.30) significantly by taking the Fourier basis matrix \( \Phi_{(q,j)} \) out as a constant from the sum over memory taps. The final expression for the gradient therefore takes the following form,

\[
\frac{\partial C}{\partial \theta_{(r,j)}} = \]

\[
-\sum_q \Phi_{(q,j)} \sum_n \left\{ \tilde{e}_{(q,n)}^* \sum_{p=0}^{P} \left( \sum_{i_1=0}^{L-1} \cdots \sum_{i_{2p+1}=0}^{L-1} \right) \right\} \tilde{h}_{(2p+1,\{i_1,\ldots,i_{2p+1}\})} \left\{ \frac{\partial \tilde{F}_{(n\cdot q\cdot p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \tilde{x}_{(n\cdot i_1,q)}} \Psi_{(n-i_1,r)} \right\} \]

\[
+ \cdots + \frac{\partial \tilde{F}_{(n\cdot q\cdot p,\{i_1,\ldots,i_{2p+1}\})}}{\partial \tilde{x}_{(n-i_{2p+1},q)}} \Psi_{(n-i_{2p+1},r)} \Phi_{(q,j)} \right\}
\]

(5.31)

To help us understand the expression for the gradient let us briefly consider the linear FIR filter scenario where \( P = 0 \) and thus \( \tilde{F}_{(n\cdot q\cdot 0,\{i_1\})} = \tilde{x}_{(n-i_1,q)} \). In this case the gradient expression (5.31) takes the following form,

\[
\frac{\partial C}{\partial \theta_{(r,j)}} = -\sum_q \Phi_{(q,j)} \sum_n \left\{ \tilde{e}_{(q,n)}^* \right\} \left\{ \sum_{i_1=0}^{L-1} \tilde{h}_{(1,\{i_1\})} \Psi_{(n-i_1,r)} \right\} \]

(5.32)
In the strictly linear case, we know that the equivalent of a FIR filter operation has been applied in fast-time before an STFT is applied across the slow-time dimension to reveal the range-Doppler information from the scene. Examining (5.32), it is clear that these two operations are mirrored in the expression for the gradient. As before, the STFT operation is represented by \( \sum_q \Phi_{\{q,j\}} \{ \ldots \} \) and therefore \( \sum_n \left\{ \hat{e}_{\{n,q\}} \left\{ \sum_{i_1=0}^{L-1} \tilde{h}_{\{1,\{i_1\}\}} \Psi_{\{n-i_1,r\}} \right\} \right\} \) represents the FIR filter operation applied down the fast-time dimension \( n \). Interestingly, the FIR filter operates on the error matrix \( \hat{e}_{\{n,q\}} \) where in matrix notation \( \{ \sum_{i_1=0}^{L-1} \tilde{h}_{\{1,\{i_1\}\}} \Psi_{\{n-i_1,r\}} \} \) represents the Toeplitz filter matrix. It therefore follows that in the full BBVS case, the gradient expression must perform a Volterra based filter operation on the respective error matrices which is exactly what is observed in (5.31). Crucially, the delayed nonlinear functions in (5.31) provide a mechanism in the gradient for the IHT algorithm to compensate for complex nonlinear memory effects.

Integrating the above gradient expression with the rest of the IHT algorithm described in Table 5.1 completes the iterative step of the NCS technique for the complex BBVS case. However, for the IHT algorithm to minimize the cost function with respect to the unknown linear signal model parameters \( \hat{\Theta} \), the constraint step (5.24) must be applied effectively at each iteration. The nature of this radar constraint is discussed in detail in the next section.

### 5.2.5 Radar Constraint step

The choice of the constraint applied at step (5.24) will be signal dependent and therefore unique to the radar’s mode of operation. For the purposes of this paper we focus on the MPRF mode but the algorithm is not limited to this case. As mentioned previously, our unique formalisation of the NCS algorithm means that vector \( \hat{\theta}_r \) represents the Doppler spectrum at range gate \( r \). We choose to exploit the sparsity constraint here as for the MPRF mode the dominant clutter is typically limited to a small region of the Doppler spectrum [21]. Furthermore, we expect the spectral behaviour of the clutter to be largely consistent across multiple range gates [21]. Before invoking the sparse threshold constraint \( P_A(\cdot) \) we must first consider the unique formalisation of the NCS algorithm.

In the NCS algorithm, the memory of the BBVS model acts down the fast-time dimension while the sparse nature of the MPRF radar signal, which we wish to exploit, exists in the slow-time dimension. In the case where the nonlinearity has memory, the estimate \( \hat{\theta}_r^{[\beta]} \) not only depends on \( \hat{\theta}_r^{[\beta]} \) but also \( \hat{\theta}_{r-1}^{[\beta]} \) to \( \hat{\theta}_{r-(L-1)}^{[\beta]} \). Therefore, if the gradient (5.31) is to be calculated correctly in the memory case then every range regressor vector \( \hat{\theta}_r \) must be updated simultaneously at each iteration. This is displayed formally in (5.24) where the threshold constraint \( P_A(\cdot) \) is applied to the full regressor matrix \( \hat{\Theta}^{[\beta]} \). Importantly, this is where we exploit our knowledge of the radar mode in the choice of sparsity level set. The dependency between the range regressor vectors explains why the problem’s
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Complexity is intrinsically linked to the memory of the fast-time nonlinearity. This complexity ultimately manifests itself in the constraint step of the algorithm and can lead to the NCS algorithm requiring more iterations to converge to the correct solution.

There are a multitude of different ways to apply the sparse threshold holding constraint step in (5.24), however for this analysis we choose to employ Blumensath’s hard thresholding method⁸ [104, 27]. This is illustrated in Table 5.2 where the hard thresholding algorithm implemented in this chapter is presented in full. In short, the hard thresholding constraint $P_A(\tilde{\Theta}^{(\beta)})$ represents a nonlinear operation where all but the $A$ largest (in magnitude) elements for each range regressor vector $\tilde{\theta}_r$ are set equal to zero.

While the number of retained regressors is predefined by the specified Doppler sparsity level for the MPRF radar scene, the locations of these $A$ regressors are independently determined for every range regressor vector $\tilde{\theta}_r$ at each iteration $\beta$. Crucially, this allows the hard thresholding algorithm to identify artifacts in the scene such as targets, inhomogeneous clutter and interference that do not exist across all range gates. For best results the Doppler sparsity level should be set to be slightly lower than the true sparsity level of the MPRF radar scene to provide the hard thresholding algorithm with some leeway when determining the locations of the strongest regressors in each range gate vector $\tilde{\theta}_r$.

---

**Table 5.2:** Summary of hard thresholding algorithm that implements the radar constraint step $P_A(\tilde{\Theta}^{(\beta)})$ at every iteration $\beta$ of the NCS algorithm. Where the $\text{sort}(.)$ function outputs a vector of size $1 \times J$ that contains the indices of $\tilde{\theta}_r$ arranged in ascending order by the magnitude of the elements.

---

⁸This hard thresholding constraint can be related to applying an $\ell_1$-norm penalty in the iterative refinement procedure.
Chapter 5. Forward Modelling Mitigation Techniques

5.3 NCS Simulation Architecture

In order to test the performance of the novel NCS algorithm, the MPRF radar simulator discussed in chapter 4 was adapted to capture the in-band interference scenario and the NCS algorithm was configured to mitigate the unwanted distortion effects. In short, the comprehensive radar simulator simulated the pre-defined radar scene for a standard MPRF mode before passing the raw time-domain signals through the black-box nonlinear receiver model described by (5.17) and (5.18). The MPRF simulator was set-up specifically to generate the problematic in-band nonlinear effects detailed in chapter 2 and was configured to operate entirely in the BB domain. Once the radar scene and CW interferer characteristics were defined, the structure of the in-band nonlinear distortion effects produced in the simulation were entirely governed by the particular nonlinear transfer function chosen. Furthermore, the gross level of the nonlinear distortion generated in the simulated RF receiver was determined by the magnitude of the specific nonlinear coefficients chosen for the memoryless case; \( \hat{h}_1 \) and \( \hat{h}_{3in} \). The magnitude of the linear coefficient \( \hat{h}_1 \) was set equal to the RF receiver gain, while the magnitude of the third order in-channel coefficient \( \hat{h}_{3in} \) was subsequently selected so that the unwanted nonlinear effects sat just below the level of the noise floor when the CW interferer was not present in the scene. In specifying the forward nonlinearity in this manner, we can assume that the simulated radar operates within its linear region when the interference is not present in the scene. Additionally, in order to ensure that the same strength of nonlinear effect was being generated in the radar receiver when memory effects were introduced to the problem, each BBVS kernel was normalised so that the respective power contributed from each group of terms in the BBVS model (5.17) was equivalent to that from the strictly memoryless case.

As with the previous chapters in this thesis, we assume that the dominant nonlinear receiver effects can be captured by terms up to the cubic order of the BBVS model [25, 29]. For the in-band nonlinear scenario, this corresponds to the case where the black-box receiver model, (5.17) and (5.18), has a nonlinear order of \( P = 1 \). We can therefore simplify the expression for the forward nonlinearity to the following form,

\[
\tilde{y}_{\{n,q\}} = \sum_{i_1=0}^{L-1} \tilde{h}_{\{1,\{i_1\}\}} \tilde{F}_{\{n,q,0,\{i_1\}\}} + \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \tilde{h}_{\{3\in,\{i_1,i_2,i_3\}\}} \tilde{F}_{\{n,q,1,\{i_1,i_2,i_3\}\}}
\]  

(5.33)

where,

\[
\tilde{F}_{\{n,q,0,\{i_1\}\}} = \tilde{x}_{\{n-i_1,q\}}
\]

\[
\tilde{F}_{\{n,q,1,\{i_1,i_2,i_3\}\}} = \tilde{x}_{\{n-i_1,q\}}^* \tilde{x}_{\{n-i_2,q\}} \tilde{x}_{\{n-i_3,q\}}
\]

(5.34)
Furthermore, substituting (5.34) into (5.31) and setting \( P = 1 \) allows the generalised expression for the NCS gradient to be drastically simplified for this specific case,

\[
\frac{\partial C}{\partial \theta_{\{r,j\}}} = 
- \sum_q \Phi_{\{q,j\}} \sum_n \left\{ \tilde{e}_{\{n,q\}} \sum_{i_1=0}^{L-1} \left\{ \tilde{h}_{\{1,\{i_1\}\}} \right\} \left\{ \frac{\partial \{ \tilde{x}_{\{n-i_1,q\}} \}}{\partial x_{\{n-i_1,q\}}} \right\} \Psi_{\{n-i_1,r\}} \right\} 
+ \tilde{e}_{\{n,q\}} \sum_{i_1=0}^{L-1} \left\{ \tilde{h}_{\{1,\{i_1\}\}} \right\} \left\{ \frac{\partial \{ \tilde{x}_{\{n-i_1,q\}} \}}{\partial \tilde{x}_{\{n-i_1,q\}}} \right\} \Psi_{\{n-i_1,r\}} 
+ \tilde{e}_{\{n,q\}} \sum_{i_1=0}^{L-1} \left\{ \tilde{h}_{\{3,n,\{i_1,i_2,i_3\}\}} \right\} \left\{ \frac{\partial \{ \tilde{x}_{\{n-i_1,q\}} \tilde{x}_{\{n-i_2,q\}} \tilde{x}_{\{n-i_3,q\}} \}}{\partial \tilde{x}_{\{n-i_1,q\}}} \right\} \Psi_{\{n-i_1,r\}} 
+ \tilde{e}_{\{n,q\}} \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \left\{ \tilde{h}_{\{3,n,\{i_1,i_2,i_3\}\}} \right\} \left\{ \frac{\partial \{ \tilde{x}_{\{n-i_1,q\}} \tilde{x}_{\{n-i_2,q\}} \tilde{x}_{\{n-i_3,q\}} \}}{\partial \tilde{x}_{\{n-i_1,q\}}} \right\} \Psi_{\{n-i_1,r\}} 
+ \tilde{e}_{\{n,q\}} \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \left\{ \tilde{h}_{\{3,n,\{i_1,i_2,i_3\}\}} \right\} \left\{ \frac{\partial \{ \tilde{x}_{\{n-i_1,q\}} \tilde{x}_{\{n-i_2,q\}} \tilde{x}_{\{n-i_3,q\}} \}}{\partial \tilde{x}_{\{n-i_1,q\}}} \right\} \Psi_{\{n-i_1,r\}} 
(5.35)

Performing the partial differentiation and collecting terms results in (5.36) which is the final expression for the NCS gradient implemented in the MPRF simulator.

\[
\frac{\partial C}{\partial \theta_{\{r,j\}}} = 
- \sum_q \Phi_{\{q,j\}} \sum_n \left\{ \tilde{e}_{\{n,q\}} \sum_{i_1=0}^{L-1} \left\{ \tilde{h}_{\{1,\{i_1\}\}} \right\} \Psi_{\{n-i_1,r\}} \right\} 
+ \tilde{e}_{\{n,q\}} \sum_{i_1=0}^{L-1} \left\{ \tilde{h}_{\{1,\{i_1\}\}} \right\} \left\{ \frac{\partial \{ \tilde{x}_{\{n-i_1,q\}} \}}{\partial \tilde{x}_{\{n-i_1,q\}}} \right\} \Psi_{\{n-i_1,r\}} 
+ \tilde{e}_{\{n,q\}} \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \left\{ \tilde{h}_{\{3,n,\{i_1,i_2,i_3\}\}} \right\} \left\{ \frac{\partial \{ \tilde{x}_{\{n-i_1,q\}} \tilde{x}_{\{n-i_2,q\}} \tilde{x}_{\{n-i_3,q\}} \}}{\partial \tilde{x}_{\{n-i_1,q\}}} \right\} \Psi_{\{n-i_1,r\}} 
+ \tilde{e}_{\{n,q\}} \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} \left\{ \tilde{h}_{\{3,n,\{i_1,i_2,i_3\}\}} \right\} \left\{ \frac{\partial \{ \tilde{x}_{\{n-i_1,q\}} \tilde{x}_{\{n-i_2,q\}} \tilde{x}_{\{n-i_3,q\}} \}}{\partial \tilde{x}_{\{n-i_1,q\}}} \right\} \Psi_{\{n-i_1,r\}} 
(5.36)

Importantly, the matrix rotations denoted by \( \Psi_{\{n-i_2p+1,r\}} \) in the expression for the NCS gradient were performed via a linear shift rather than a circular one. This simple extension meant that the simulations were more realistic than the theoretical NCS algorithm which is why it was implemented in all of the simulations presented in this chapter. In
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essence, the linear shift introduces edge effects to the problem as not all of the range regressors vectors, $\tilde{\theta}_r$, that affect the gradient are updated as part of the IHT iteration. As a comparison, we also test the performance of the memoryless NCS algorithm, $L = 1$, in mitigating the BBTS model output as well as that from the BBVS case. The memoryless in-band forward nonlinearity was defined previously in (2.21) and is restated below,

$$\tilde{y}_{\{n,q\}} = \tilde{h}_1 \tilde{x}_{\{n,q\}} + \tilde{h}_{3_{in}} |\tilde{x}_{\{n,q\}}|^2 \tilde{x}_{\{n,q\}} \tag{5.37}$$

with the corresponding gradient expression taking the following form,

$$\frac{\partial C}{\partial \theta_{\{r,j\}}} = -\sum_q \Phi_{\{q,j\}} \sum_n \left\{ \tilde{e}_{\{n,q\}} \left\{ \tilde{h}_1 \Psi_{\{n,r\}} + 2 \tilde{h}_{3_{in}} \{ \tilde{x}^*_{\{n,q\}} \tilde{x}_{\{n,q\}} \} \Psi_{\{n,r\}} \right\} \right. 
+ \tilde{e}_{\{n,q\}} \tilde{h}_{3_{in}} \{ \tilde{x}^*_{\{n,q\}} \tilde{x}^*_{\{n,q\}} \} \Psi_{\{n,r\}} \right\} \tag{5.38}$$

While the magnitude of the linear gain and the nonlinear distortion generated in the forward black-box receiver model were fixed in the simulation, the particular choice of complex BB kernel coefficients were subject to change. It is understood that introducing nonlinear memory effects is not expected to change the gross level of the distortion observed at the output of the radar receiver but rather subtly change its structure and phase. The NCS algorithm was therefore tested against a wide variety of different nonlinear transfer functions with varying degrees of memory in order to gain a comprehensive understanding of its performance. Crucially, the nonlinear models tested in the radar simulation had to fit into the Volterra series framework otherwise the above expression for the NCS gradient (5.36) becomes invalid, see chapter 2. The specific nonlinear models tested in the simulation were therefore: the BBTS model, the BBVS model, the BB Hammerstein model and the BB parallel Hammerstein model. For each nonlinear model listed above, the radar simulator was configured to perform statistical convergence analysis on the algorithm’s mitigation performance was tested against randomly generated sets of kernel coefficients.

While this broad convergence analysis is important, it is also important to determine the true performance of the NCS algorithm for an MPRF radar mode with a realistic black-box nonlinear receiver model. As discussed previously, there is a severe lack of published data in the available literature on the memory behaviour of front-end receiver amplifiers. One of the very few papers that has published such data is [15] by Vansebrouck et al. In [15], the authors’ employ a quintic, $P = 2$, parallel Hammerstein model with memory length $L = 5$ to describe a wideband communications receiver centred on 250MHz. It is clear from the data published in [15] that the third-order nonlinear effects are far more dominant than the fifth order distortion generated which allows us to truncate Vansebrouck’s nonlinear model to order $P = 1$. Furthermore, to implement the nonlinear model described in [15] in our MPRF radar simulator we must translate the passband nonlinear impulse response from a centre frequency of 250MHz to baseband.
In performing this translation, we convert the real PB coefficients to complex BB coefficients and disregard the even order terms in the model as they fall outside the bandwidth of the desired receiver channel. The final forward nonlinear model implemented in the MPRF radar simulator was therefore a BB parallel Hammerstein model with coefficients equal to those displayed in Table 5.3. In terms of the full Volterra formalisation, (5.17), these coefficients correspond to the on-diagonal elements of the respective Volterra kernels, i.e. where \( i_1 = i_2 = i_3 \), with all other off-diagonal elements set equal to zero. Note, in order to distinguish this specific nonlinear model from other BBVS models employed in this paper we refer to it as the “BBVS-[15]” model as it was derived from the parallel Hammerstein model in [15]. An example RD plot outputted from the MPRF radar simulator that corresponds to the above parallel Hammerstein model is displayed in Figure 5.3 with the corresponding desired linear output displayed previously in Figure 5.2. The task for the NCS algorithm is to restore the simulated radar’s target detection performance, which in essence is equivalent to recovering the linear RD plot displayed in Figure 5.2 from the corrupted RD plot displayed in Figure 5.3.

<table>
<thead>
<tr>
<th>BBVS Kernel Coefficients for PD Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Kernel</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>( \tilde{h}_{1,{0}} )</td>
</tr>
<tr>
<td>( \tilde{h}_{1,{1}} )</td>
</tr>
<tr>
<td>( \tilde{h}_{1,{2}} )</td>
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<tr>
<td>( \tilde{h}_{1,{3}} )</td>
</tr>
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<td>( \tilde{h}_{1,{4}} )</td>
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</table>

**Table 5.3:** List of the BBVS kernel coefficients implemented in the MPRF radar simulator for the NCS performance analysis. The normalised coefficients are scaled in the simulation by the magnitude of the respective memoryless coefficient to generate the desired level of nonlinear distortion. All coefficients not listed in Table 5.3 were set equal to zero in the BBVS-[15] model.

In order to study the performance of the novel NCS algorithm for a typical MPRF radar mode in detail, statistical probability of detection (PD) analysis was performed in the RD domain on a single target. Similar to the PD analysis performed in chapter 4, the MPRF radar simulator generated input receive data matrices, \( \tilde{X} \), consisting of 256 received pulses based on a pre-defined radar scene. The simulation then generated three output matrices, \( \tilde{Y} \), by passing the input data through the following black-box receiver models: the BBVS-[15] model, the corresponding BBTS model and finally the linear gain model. In all cases, both input and output noise were added to the received signals with
the SNR level referenced to the input so that the output noise level was not biased by the specific nonlinear transfer function chosen, $\text{SNR}_{\text{noise}} = 35\text{dB}$. A further three data sets were then generated by applying the memory rich NCS algorithm to the BBVS-[15] receiver output and by applying the memoryless NCS algorithm to both the BBVS-[15] and BBTS outputs respectively. Simulation results for a single burst illustrating the nonlinear distortion generated from the BBVS-[15] model and the subsequent correction by the memory rich NCS algorithm are displayed in Figures 5.3 and 5.4 respectively. It is clear from Figure 5.4 that the memory rich NCS algorithm has performed well in restoring the simulated radar’s performance back to the desired linear case. However, the performance and robustness of the algorithm can be studied in much more detail through the stochastic based PD analysis. It is important to recognise at this point that the noise assumptions invoked during the derivation of the NCS algorithm have not been implemented in the radar simulator. Therefore, the fact that the NCS algorithm has recovered the correct solution in Figure 5.4 clearly validates the noise approach detailed in section 5.2.2.

As well as performing the PD analysis on the scenarios discussed above, further results were generated to probe the robustness of the algorithm for varying levels of modelling error, $\tilde{\omega}_{(n,q)}$. While all of the simulations conducted had some degree of modelling
error due to the input and output noise added to the receive data matrices, more in-depth analysis was required to fully understand the limitations of the algorithm. The NCS algorithm is fundamentally designed around the forward black-box receiver model and therefore, its performance is limited by how well the nonlinear behaviour of the RF receiver has been characterised. For a real radar system, it is highly likely that this nonlinear system identification will be performed offline and it is therefore reasonable to assume in the simulations that the forward nonlinearity has been identified to a high degree of accuracy. However, as discussed previously it is important to understand how much modelling error the algorithm can tolerate before it fails. To perform this modelling error analysis as part of the simulation, the MPRF radar simulator was configured to identify the forward nonlinearity for each burst through varying levels of output noise. The accuracy of the forward nonlinear coefficients was therefore dependent on the level of the output noise through which they were learnt. The linear least squares (LLS) algorithm was used to learn the nonlinear coefficients by means of a noise identification procedure, see chapter 3, with the residual error value providing an accurate measure of the modelling error in the simulation.
5.4 Simulation Results

5.4.1 Convergence Analysis

For the NCS convergence analysis, the mitigation algorithm was tested against different classes of randomly generated nonlinear transfer functions in order to study its overall convergence properties both in the memoryless and memory rich case. While this stochastic based convergence analysis will not inform us about the effectiveness of the NCS algorithm for all nonlinear radar receivers, as there are infinitely many, it can provide us with a better understanding of the specific nonlinear characteristics that will prove problematic for convergence. We use the percentage of successful convergence as a measure of the algorithm’s efficacy, with a 100% convergence defining the situation where the NCS algorithm always recovers the weak target from the corrupted RD map, see Figures 5.3 and 5.4. The success of the mitigation technique was determined for each realisation by means of local area average (LAA) target thresholding algorithm combined with a residual error thresholding technique. The NCS modelling error was strictly limited by the receiver output noise in this case, $\text{SNR}_{\text{noise}} = 50\text{dB}$, as there was no input noise and the nonlinear coefficients were set equal to the true forward model coefficients in the NCS algorithm. The convergence results for the NCS algorithm are displayed in Table 5.4 with every convergence percentage value determined from 100 realisations of the simulation each with a unique nonlinear transfer function.

When dealing with nonlinear transfer functions, the concept of monotonicity is a fundamental one. In the case of a static nonlinearity, the transfer function is considered strictly monotonic if there exists a one-to-one mapping between the inputs and the outputs of the nonlinear model [57]. Importantly, if a function is strictly monotonic then there exists a unique inverse function that maps the outputs back onto the inputs. If however this one-to-one mapping does not exist, then the nonlinear transfer function is described as being non-monotonic and is therefore non-invertible. In the context of the NCS algorithm, non-monotonicity of the forward nonlinearity manifests itself in the least squares cost function whereby the ambiguity generates multiple local minima for the algorithm to converge to. In other words, there is more than one solution to the problem. This is most easily observed for the BBTS case in the above results table. We must be very careful when interpreting these convergence results as the nonlinear transfer functions generated in the simulation are not representative of a real RF receiver. However, it is clear from the results that if the memoryless forward nonlinearity is strictly monotonic then the NCS algorithm will converge to the correct solution. In the case where the memoryless nonlinearity is non-monotonic, the NCS algorithm’s ability to recover the correct solution is dependent on the particular forward nonlinear transfer function through which the desired input signal was observed.

In the case where memory is introduced to the forward nonlinearity, we have to be careful with how we interpret the concept of monotonicity which is only defined for
5.4. Simulation Results

### NCS Algorithm Convergence Results

<table>
<thead>
<tr>
<th>Nonlinear Model</th>
<th>Nonlinearity Monotonicity</th>
<th>Successful Convergence %</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBTS</td>
<td>Strictly Monotonic</td>
<td>100</td>
</tr>
<tr>
<td>BBTS</td>
<td>Non-monotonic</td>
<td>49</td>
</tr>
<tr>
<td>BB Hammerstein</td>
<td>Strictly Monotonic</td>
<td>100</td>
</tr>
<tr>
<td>BB Hammerstein</td>
<td>Non-monotonic</td>
<td>53</td>
</tr>
<tr>
<td>BB Parallel Hammerstein</td>
<td>Strictly Monotonic</td>
<td>100</td>
</tr>
<tr>
<td>BB Parallel Hammerstein</td>
<td>Non-monotonic</td>
<td>89</td>
</tr>
<tr>
<td>Full BBVS</td>
<td>N/A</td>
<td>83</td>
</tr>
</tbody>
</table>

**Table 5.4:** List of NCS convergence results. 100 individual nonlinear transfer functions were used to generate the successful convergence value in each case. All simulations were performed with a nonlinearity of cubic order, \( P = 1 \), and those with memory had a length \( L = 4 \).

one-dimensional functions. We start with the Hammerstein nonlinear model as it the simplest extension from the BBTS model to the memory case. The Hammerstein model is described by a memoryless nonlinearity followed by a FIR filter and therefore its monotonicity is entirely defined by its static nonlinearity. Examining the convergence results for the BB Hammerstein model we find that they reflect that of the BBTS case. This makes sense as the monotonicity of the forward nonlinearity is defined in exactly the same way as the BBTS model. Importantly, the NCS algorithm has been shown to converge to the correct solution consistently in the case where the forward nonlinearity exhibits nonlinear memory effects. Taking this analysis one step further, the Hammerstein model is extended to the parallel Hammerstein model by applying an individual FIR filter to each term in the static nonlinearity, see chapter 2 for more details. In essence, the parallel Hammerstein model consists of \( L \) static nonlinear functions that can be thought of as acting on individual taps of a FIR filter. Therefore, the nonlinear transfer function is considered strictly monotonic if all of the \( L \) static nonlinearities are themselves strictly monotonic. In this case, the convergence of the NCS algorithm is guaranteed which is indicated by the successful convergence percentage of 100% in Table 5.4. Furthermore, we define the non-monotonic case for the parallel Hammerstein model to be when any one of the \( L \) static nonlinearities stop being strictly monotonic. Interestingly, the successful convergence
percentage is much higher in this case than in the standard Hammerstein and BBTS cases suggesting that the NCS algorithm can tolerate some degree of non-monotonicity in its nonlinear memory terms. Finally, we consider the full BBVS model where unfortunately the nonlinear cross-terms mean that we cannot define the concept of monotonicity in this case. Examining the successful convergence percentage for the full BBVS case we observe that the NCS algorithm can successfully recover the desired input from the corrupted output even when the forward nonlinearity exhibits complex nonlinear memory effects. While the general convergence of the NCS algorithm cannot be guaranteed in this case, the final result highlights the capabilities of this novel mitigation technique.

5.4.2 Radar PD Analysis

For the PD performance analysis of the NCS algorithm, the MPRF radar simulator employed an adaptive cell-averaging constant false alarm rate (CA-CFAR) thresholding technique to study the simulated radar’s detection performance in the different scenarios. The CA-CFAR technique was discussed in detail in chapter 2 but in short, the algorithm was configured to maintain a specified probability of false alarm rate (PFAR) which it achieved by studying the power and statistical behaviour of the RD cells in the neighbourhood of the target cell. The simulated radar therefore exhibits maximum sensitivity when its receiver operates in the linear regime as in this case the cells that surround the target cell in the RD map are strictly noise limited. For the nonlinear scenarios, the level of the unwanted distortion was set by the memoryless coefficients, as discussed previously, with the BBVS-[15] model coefficients described by those in Table 5.3. Importantly, for this analysis both the BBTS and BBVS-[15] nonlinearities had strictly monotonic forward transfer functions. When the CW interferer is introduced to the radar scene, the unwanted clutter broadening effect drives the target detection threshold up and consequently reduces the sensitivity of the sensor. Thus, for the NCS algorithm to restore the system sensitivity of the radar back to the linear case it must reduce the level of distortion in the RD detection space without removing the potential targets from the scene. To study the sensitivity of the radar in the different scenarios, the input SNR of the radar test target was varied with each SNR value forming a single data point in the respective PD curves. The simulation results for the PD analysis are displayed in Figure 5.5 with the CA-CFAR algorithm set to maintain a constant PFAR of 0.2. For each data point in Figure 5.5, RD maps for 400 bursts were used to estimate the PD value.

Examining the simulation results in Figure 5.5 and Table 5.5, the first thing that we observe is that the PD curves for the BBTS and BBVS-[15] receiver outputs fall off at a much higher input SNR value than that for the linear gain case. This reflects the loss of system sensitivity experienced by the simulated radar in the case where its RF receiver operates in the nonlinear regime. Furthermore, the PD curves for the BBTS and BBVS-[15] cases almost lie on top of each other indicating that the gross level of the nonlinear effect has not been significantly altered by the introduction of nonlinear memory effects.
5.4. Simulation Results

![Graph showing PD results for the NCS algorithm.](image)

**Figure 5.5:** Performance PD results for the NCS algorithm. Ideal system sensitivity illustrated by linear receiver curve, red; Memoryless nonlinear receiver curve, black; Memory rich nonlinear receiver curve, magenta; Memoryless nonlinear receiver output corrected by memoryless NCS algorithm, blue; Memory rich nonlinear receiver output correct by memory rich NCS algorithm, green; Memory rich nonlinear receiver output corrected by memoryless NCS algorithm.

<table>
<thead>
<tr>
<th>Mitigation Technique</th>
<th>Recovered Performance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBTS - NCS BBTS</td>
<td>109</td>
</tr>
<tr>
<td>BBVS - NCS BBVS</td>
<td>98</td>
</tr>
<tr>
<td>BBVS - NCS BBTS</td>
<td>11</td>
</tr>
</tbody>
</table>

**Table 5.5:** Snapshot results for PD = 0.7 from Figure 5.5 showing a comparison of the performance of the various NCS mitigation techniques. 100% represents the case where the mitigation technique has fully restored the radar performance back to the linear case.

In fact, the slight deviation between the two nonlinear curves is not due to any variation in the strength of the clutter broadening effect, but rather reflects subtle phase changes introduced by the nonlinear memory that alters the relationship between the nonlinear distortion and the test target. Examining the blue curve in Figure 5.5, it is clear that the memoryless NCS algorithm has succeeded in fully restoring the sensitivity of the simulated radar in the case where the nonlinear RF receiver is strictly memoryless. This
is a significant result as it confirms that the NCS technique can be successfully employed to mitigate nonlinear receiver effects in modern radar systems. It is important to note the superior performance of the mitigated BBTS case over the linear case in Figure 5.5. Unlike the LUT analysis presented in chapter 4, the deviation between the two curves cannot simply be explained by the natural error of the stochastic based PD analysis as the variations are both consistent and too large. The slightly improved performance is therefore thought to be due to the compressive sensing algorithm out-performing the CA-CFAR detection algorithm. In essence, the NCS algorithm is a detection algorithm as it is identifying the locations of strong scatters in the scene. We choose to project the final regressor matrix back into the RD domain and then perform the CA-CFAR algorithm used in the linear case for our comparative analysis. However, if the NCS algorithm has been successful in recovering the target then the RD cells surrounding the target will effectively be denoised leading to almost perfect CA-CFAR performance. For the linear case, there is still noise surrounding the target cell which will lead to some inevitable inaccuracies in the adaptive threshold setting and thus the slight deviation. Examining the PD results in Figure 5.5, the NCS algorithm obviously reduces the noise effects around the target cell better in the mitigated BBTS case than in the mitigated BBVS case. This is to be expected as the complexity is much less in the BBTS case than in the BBVS case.

Interestingly, it is clear from Figure 5.5 that when the memoryless NCS technique is applied to the memory rich BBVS-[15] output the mitigation algorithm fails to restore any of the simulated radar’s lost performance. Much like the memoryless LUT techniques discussed in chapter 4, the memoryless NCS algorithm is effectively decorrelated by the subtle amplitude and phase effects introduced by nonlinear memory terms in the forward receiver model. The fact that the memoryless NCS algorithm is ineffective in this case provides further proof that; if the RF receiver exhibits nonlinear memory effects then the corresponding mitigation technique, whatever that might be, must fundamentally compensate for the memory effects if it is to be successful. In the case of the NCS mitigation technique, the result of introducing complex memory effects to the algorithm’s formalisation is displayed in Figure 5.5, see the green curve. Clearly, introducing the memory terms to the NCS algorithm has drastically improved the mitigation technique’s performance in the BBVS-[15] scenario with the simulated radar’s system sensitivity more or less restored back to the linear gain case. This was fundamentally due to the expression for NCS gradient (5.36) which incorporated complex nonlinear memory effects so that each iteration of the IHT algorithm was correctly pointed towards the minimum of the cost function. This is a significant result as it shows that the novel NCS algorithm provides a forward modelling framework capable of compensating for complex nonlinear memory effects generated in the modern radar receiver.
5.4.3 Modelling Error Analysis

In this section of the simulation results we examine the robustness of the NCS algorithm to varying levels of modelling error. As discussed previously, the amount of modelling error in the PD simulations was governed by the input/output noise of the simulated radar receiver as well as the error in the forward model coefficients implemented in the NCS algorithm. While the SNR ratio was fixed at 35dB for both the input and output noise in all simulations, the level of uncertainty in the NCS forward model coefficients was varied. The PD analysis performed by the MPRF radar simulator was therefore exactly the same as that described in the previous section, with the exception that the NCS algorithm was applied multiple times on each burst using different sets of forward model coefficients. We focus on the scenario where the BBVS-[15] model is described by the BB parallel Hammerstein model in Table 5.3 and is mitigated by the corresponding memory rich NCS algorithm. The results are displayed in Figure 5.6 with each PD data point estimated from 400 bursts and the CA-CFAR set to maintain a constant PFAR of 0.2 as before. In Figure 5.6, the PD curves referenced by the label ME denote those that have been corrected by the NCS algorithm with the corresponding modelling error (ME) stated in the brackets. The first ME term in the legend brackets corresponds to the residual error from the LLS estimation procedure and the second term corresponds to the error norm between the estimated forward model coefficients and the true forward model coefficients.

<table>
<thead>
<tr>
<th>ME (dB)</th>
<th>Recovered Performance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-\infty, -\infty]$</td>
<td>98</td>
</tr>
<tr>
<td>$[-10, -18]$</td>
<td>67</td>
</tr>
<tr>
<td>$[-9, -17]$</td>
<td>52</td>
</tr>
<tr>
<td>$[-6, -14]$</td>
<td>24</td>
</tr>
<tr>
<td>$[-4, -13]$</td>
<td>2</td>
</tr>
</tbody>
</table>

**Table 5.6:** Snapshot results for PD = 0.7 from Figure 5.6 showing the recovered performance by the NCS BBVS mitigation technique as a percentage. 100% represents the case where the mitigation technique has fully restored the radar performance back to the linear case.

Examining the PD result in Figure 5.6 and Table 5.6, it is clear that modelling error in the forward black-box receiver model can have quite a profound effect on the mitigation performance of the NCS algorithm. This makes sense as the mitigation technique is
Chapter 5. Forward Modelling Mitigation Techniques

Figure 5.6: Modelling error PD analysis for the NCS algorithm. Desired linear gain performance curve shown in red with the corresponding BBVS-[15] nonlinear performance curve shown in magenta. PD curves corresponding to the NCS mitigated outputs are reference by their modelling error (ME) level from low to high respectively: black, blue, green, cyan and gold. The ME brackets hold two measures of the NCS modelling error with $-\infty dB$ denoting the case where the true nonlinear coefficients are employed in the NCS algorithm.

fundamentally designed around the forward nonlinearity. Encouragingly, the NCS algorithm can tolerate a reasonable amount of modelling error in the forward model coefficients before a significant drop in performance is observed. This drop-off in performance of the NCS algorithm appears to occur quite sharply around $ME = [-10dB, -18dB]$, which suggests that beyond this point the iterative learning procedure struggles to consistently identify the minimum of the cost function and therefore the desired solution. Despite this, the NCS algorithm can still provide noticeable performance improvements from the unmitigated BBVS-[15] case even when the modelling error is significantly distorting the IHT estimate on each iteration. When the level of modelling error becomes too strong for the NCS algorithm, $ME = [-4dB, -13dB]$ for this case, the mitigation technique fails to provide any performance improvements from the corrupted nonlinear output. As discussed previously, it is expected that the nonlinear behaviour of the RF receiver will be characterised off-line in a lab-based experiment. This should allow the level of modelling error in the NCS algorithm to be kept reasonably low as the nonlinear identification procedure can be trained with substantial amount of measured experimental data. It is clear however that if the nonlinear characteristics of the RF receiver were to vary slightly during operation, then the NCS algorithm would be able to cope with the
resultant modelling error introduced.

5.5 Trials Data Results

Regardless of how sophisticated a radar simulator is it can never fully replicate all of the complex effects observed in a real-world system. It is therefore extremely important when designing novel digital signal processing techniques that they are tested as part of a physical trials system after they have been suitably developed in the simulation environment. While this is equally true for the novel NCS algorithm derived above, designing and conducting a dual radar trial capable of testing the performance of the NCS mitigation technique is a considerable task which is beyond the scope of this work. However, to study the performance and robustness of the NCS algorithm further, the novel mitigation technique was tested against real radar trails data that had been digitally corrupted by a nonlinearity and a synthetically generated CW interferer. The trials data used for this analysis was collected by Leonardo’s PicoSAR system as part of a flight trial where the radar was setup to operate in an MPRF mode with a downward looking configuration. The range, Doppler and power values of the input trials data were normalised for this analysis so that they matched those from the MPRF radar simulator, however some additional information about the trial that was originally conducted is listed below to provide some context for the analysis. By performing this normalisation, the same nonlinear scenario studied in the previous results section could be digitally stimulated for the trials data case. Thus, the forward nonlinearity took the form of a BBVS-[15] model with coefficients equal to those listed in Table 5.3. The final results from the trials data analysis are displayed in Figure 5.7 with mitigation outputs from both the memoryless and memory rich NCS algorithm presented. In both cases, the forward model coefficients employed in the NCS algorithms were learnt by means of a noise identification procedure which allowed additional modelling error to be introduced to the memory rich case. This was done in exactly the same way as in the previous section, with the BBVS forward model coefficients identified through a heightened level of output noise leading to uncertainty being introduced to the NCS algorithm. The residual LLS forward modelling error attributed to the memory rich case was therefore $-14\,dB$ which compared to $-4\,dB$ for the memoryless NCS case\footnote{Unlike in the memory rich case, the BBTS model identified for the memoryless NCS algorithm was learnt through no output noise.}.

For reference, the PicoSAR radar is a pulsed radar system operating in the X-band frequency range with a vertically oriented polarisation and a power of roughly 150W. It has two operational channels which allows it to perform monopulse measurements, however for this analysis only one of the channels was used to generate the pulse-Doppler data that was ultimately fed into the nonlinear simulator. Importantly, during the trial the PicoSAR radar operated with a chirped waveform and it aimed to detect a pick-up...
truck which was driving down an aeroplane runway in a controlled manner. Therefore, the raw trials data had to be pulse compressed before it was normalised and fed through the digital nonlinearity in order for it to be compatible with the nonlinear simulator described above. By performing this pulse-compression operation we could effectively assume that the data had been collected using a single-tone waveform which importantly fits with the NCS algorithm derived above. Finally, we synthetically added two extra targets to the trials data-set so that we could analyse the mitigation performance of the NCS algorithm more easily. These synthetic targets were deliberately added to resemble the ground-moving target already in the scene but importantly had slightly different range and velocity characteristics. The final results from the trials data analysis are displayed in Figure 5.7 below.

Examining the trials data results from Figure 5.7 we firstly observe that the ground clutter in the linear RD map, between $-4kHz \rightarrow 5kHz$ Doppler, is noticeably more complex than the simulated clutter generated in the MPRF radar simulator. This is useful as it will test the performance of NCS algorithm against a less uniform clutter spectrum where the sparsity may not be at exactly the same Doppler locations in each range gate. Importantly, there are three distinct radar targets in the trials data RD map located around $-9kHz$ Doppler which can be used as a marker to gauge the performance of the NCS mitigation technique. In Figure 5.7b, these targets are no longer distinguishable from the radar background as they have become entangled with the nonlinear clutter broadening effect stimulated by the presence of the synthetic CW interferer in the scene, $10.5kHz$ Doppler. It is clear from Figure 5.7c that the memoryless NCS algorithm is incapable of correcting for the nonlinear memory effects in the radar receiver resulting in the radar targets still being indiscernible from the background. Examining the final result in Figure 5.7d, we observe that introducing memory terms to the NCS algorithm allows it to unscramble the complex nonlinear memory effects generated in the RF receiver and ultimately restore the sensor’s performance almost back to the linear case. Crucially, the test targets previously lost in the nonlinear clutter broadening effect are now clearly identifiable above the RD background.

It is clear from all of the results presented above that it is the behaviour of the nonlinear transfer function across the input power range that defines the mitigation performance of the NCS algorithm rather than the specific radar scenario. This is shown to be the case for the trials results in Figure 5.7 where the NCS algorithm performs well against a complex clutter background and is supported by the extra results presented in the Appendix A. Importantly, the additional results in Appendix A demonstrate that the NCS algorithm performs well against a wide variety of different nonlinearities as well as in the Frequency Modulated CW (FMCW) interference scenario. These results highlight the robustness of the NCS algorithm to other more complicated sources of interference and showcase the effectiveness of the forward modelling technique in mitigating complex nonlinear receiver effects in modern radar.
5.5. Trials Data Results

**Figure 5.7**: Trials data RD results for the NCS algorithm. Ground clutter located between $-4 \text{kHz} \rightarrow 5 \text{kHz}$ Doppler with the synthetically added CW interferer located at $10.5 \text{kHz}$ Doppler. The strongest targets located around $-9 \text{kHz}$ Doppler become masked by the clutter *broadening* effects when the receiver nonlinearity is introduced. NCS mitigation technique successfully recovers strongest targets located around $-9 \text{kHz}$ Doppler only when memory terms are introduced to the NCS algorithm.
Chapter 5. Forward Modelling Mitigation Techniques

5.6 Discussion and Summary

In this chapter, we presented a novel forward modelling mitigation technique designed to digitally compensate for complex nonlinear memory effects in the radar receiver. The technique builds on the work previously presented by Blumensath in [27] and was specifically targeted at the in-band interference scenario in radar where a CW interferer drives the RF receiver into its nonlinear regime. In order for the forward modelling technique to restore the system sensitivity in radar, Blumensath’s nonlinear compressive sensing framework had to be extended to represent the unique processing applied in modern radar systems. Furthermore, the NCS algorithm was expanded beyond those results published in [28] and [89] so that complex nonlinear memory effects could be mitigated by the forward modelling technique for the first time. The novel NCS algorithm was tested extensively by means of a comprehensive MPRF radar simulator that operated entirely in the baseband domain and was capable of simulating sophisticated nonlinear receiver effects. The convergence properties of the NCS algorithm were studied for a wide variety of nonlinear transfer functions with the convergence of the algorithm guaranteed in the case where the forward nonlinearity was strictly monotonic. In addition, the NCS algorithm was shown to successfully mitigate deleterious effects from both memoryless and memory rich receiver nonlinearities in a MPRF radar mode through in-depth probability of detection of analysis. Furthermore, this impressive mitigation performance was shown to hold for real-world MPRF radar data and for significant levels of modelling error built into the forward model of the NCS algorithm. If nonlinear memory effects prove to be significant in the modern radar receiver then forward modelling techniques offer a digital signal processing solution that is not built around the inverse nonlinear transform. While it is clear from the convergence analysis that some of the fundamental limitations of trying to invert a nonlinear transfer function still manifest themselves in this novel forward modelling approach, the unique formalisation of the NCS algorithm provides a more explicit framework to compensate for complex nonlinear memory effects than the standard LUT mitigation techniques discussed in chapter 4.

The research presented in this chapter could be expanded in future work by studying the convergence and mitigation properties of the NCS algorithm for a wider variety of nonlinear radar scenarios. The algorithm’s theoretical framework could be developed for the cross-channel interference scenario as well as for other operational radar modes beyond the MPRF mode. The performance of the mitigation technique should be properly tested as part of a real-world radar trial where an interfering RF source drives the radar receiver into its weakly nonlinear regime and therefore generates harmful effects in the resultant RD detection space. Finally, the convergence properties of the NCS algorithm could be studied in more detail to see if the algorithm could be consistently steered towards the global minimum of the LS cost function in the case where the forward nonlinear transfer function is non-monotonic. More specifically, a study into whether employing the Wirtinger flow technique for finding the complex-gradient provides any
convergence improvements in the non-monotonic case would be very interesting.
Chapter 6

Conclusion

6.1 Research Project Conclusions

The aim of this thesis was to study nonlinearities in the radar receiver and to develop advanced digital signal processing techniques capable of mitigating the resultant deleterious effects. With the use of commercial radar systems set to increase dramatically over the next decade it is imperative that the modern sensors are able to operate efficiently in a crowded RF environment. In order to achieve this, they must learn to compensate for nonlinearities in their receiver front-ends’ stimulated by mutual interference in the scene. However, before any nonlinear mitigation strategies could be developed the nonlinear behaviour of the RF receiver had to be studied in great detail. This was done in chapter 3 of this thesis with the modelling results fed into a sophisticated radar simulator so that advanced digital mitigation techniques could be suitably developed and tested.

The first class of digital mitigation solutions studied in this thesis was the Look-up Table technique which has seen much success in the related field of communications. Importantly, the LUT technique was developed in chapter 4 of this thesis to specifically target the cross-modulation problem in radar with its performance tested in the MPRF radar environment. This was done for the case where the cross-modulation distortion was generated by a memoryless front-end receiver nonlinearity and also for the case where nonlinear memory effects become significant. Finally, a forward modelling digital mitigation solution was derived in chapter 5 of this thesis that built on the nonlinear compressive sensing techniques first reported in the signal processing literature. The novel forward modelling technique was developed specifically for the in-band interference scenario in radar and was target at the popular MPRF mode of operation. Similar to the LUT techniques derived in chapter 4, the forward modelling technique addressed the case where the front-end receiver nonlinearity was strictly memoryless as well as the case where it exhibits complex nonlinear memory effects. While the structure of this thesis has been briefly summarised above, the specific conclusions found in each chapter are discussed in detail in the following three sections.
6.1.1 Chapter 3: Modelling Nonlinear Radar Receivers

In this chapter, a novel cross-channel baseband Volterra series behavioural model was derived that is capable of capturing the complex nonlinear coupling between the desired radar channel and an out-of-band interferer in the scene. Inspiration for the technique was taken from the related communications literature with the model designed specifically to allow complex nonlinear memory effects to be modelled in the baseband domain. This could prove to be a very important capability as modern radar systems move to increasingly wide receiver bandwidths where memory effect become more significant. It is important to note that only by dropping the Volterra series to the BB domain can its considerable modelling power be utilised by radar systems whose high carrier frequencies typically prohibit its use. Once the BBVS model had been derived, the focus of the research work turned towards the identification of the Volterra kernel coefficients that characterise the model’s nonlinear impulse response. It was shown that in order to capture the complex memory behaviour of the nonlinear device, the BBVS model must be identified using a frequency rich input signal so as to satisfy the persistent excitation condition. A unique noise-based identification procedure was therefore developed that allows the nonlinear memory behaviour of the device to be characterised including the bandwidth expansion effects. In order to achieve this, the identification procedure had to overcome a multirate system identification problem by employing sophisticated polyphase decomposition techniques.

6.1.2 Chapter 4: LUT Inverse Mitigation Techniques

Chapter 4 studied the effectiveness of the LUT digital mitigation technique in correcting for cross-modulation distortion generated in the radar receiver. The work focussed on the classic cross-modulation scenario for a typical MPRF radar mode where a pulsed out-of-band interferer stimulates a nonlinearity in the RF receiver. The mitigation performance of two separate LUT techniques were studied in this chapter both of which were carried over from the related field of communications. The first mitigation technique to be tested was the direct inversion approach which was first proposed by Zou et al. in [14]. This simple mitigation solution aimed to mathematically invert the memoryless forward nonlinear transfer function by exploiting the modern radar’s ability to form guard channels. Importantly, it was shown that in the case where the forward receiver nonlinearity did not exhibit nonlinear memory effects then Zou’s simple LUT technique could successfully restore the system performance back to the desired linear case. However, in the case where nonlinear memory effects did become significant in the radar receiver the direct inversion approach was decorrelated drastically reducing its effectiveness. The second mitigation technique to be studied in this chapter was the tandem inverse approach which attempted to correct for the unwanted nonlinear receiver effects by digitally passing the corrupted output through a second nonlinear transfer function. Similar
6.1. Research Project Conclusions

to the direct inversion approach, this nonlinear equalisation technique required accurate information about the source of the interference from a linear guard channel. While the tandem inverse approach could successfully recover the desired linear performance in the case where the receiver nonlinearity was memoryless, much like the direct inverse approach it also failed when nonlinear memory effects were introduced to the forward nonlinear transfer function. This is an important result as it suggests that in order for the digital mitigation techniques to recover linear system performance in the case where the radar receiver exhibits significant nonlinear memory effects, they must directly correct for the memory terms in the forward nonlinearity. The implications of this result are likely to become more profound as modern radar systems move to wider receiver bandwidths where the memory effects can no longer be considered to be negligible. Finally, the tandem inverse technique was extended to include nonlinear memory terms and it was shown that the method could be used to correct for complex nonlinear memory effects generated in the radar receiver. The LUT technique provides a mode-independent solution to the nonlinear receiver problem in radar that performs particularly well in the memoryless scenario. However, if nonlinear memory effects do become significant in the radar receiver then alternative more robust digital mitigation solution will likely need to be sought.

6.1.3 Chapter 5: Forward Modelling Mitigation Techniques

The final chapter of this thesis developed a novel forward modelling mitigation technique to digitally compensate for harmful nonlinear effects generated in the radar receiver. The technique builds on the theory of nonlinear compressive sensing first published by Blumensath in [27] and was specifically targeted at the in-band interference scenario in radar where a CW interferer drives the RF receiver into its nonlinear regime. Importantly, Blumensath’s NCS theory had to be redesigned to exploit the unique processing applied in modern radar systems. Furthermore, the theoretical framework for the NCS technique had to be expanded so that complex nonlinear memory effects could be mitigated by the forward modelling approach for the first time. The BBVS model was employed as part of the MPRF radar simulator to allow the mitigation capabilities of the NCS algorithm to be tested against sophisticated receiver nonlinearities with memory. In depth convergence analysis of the NCS algorithm found that successful convergence could be guaranteed in the case where the forward nonlinearity was strictly monotonic. Importantly, this was not only shown to be the case for the memoryless nonlinearity but for the memory rich case as well. This is a significant result as unlike the LUT mitigation techniques presented in chapter 4, the forward modelling solution provides a well-defined method for determining whether the nonlinear distortion can be properly mitigated or not. In addition to the convergence analysis, the mitigation performance of the NCS algorithm was tested in a simulated MPRF radar mode through extensive probability of detection analysis. It was shown that the forward modelling technique could completely restore the simulated
radar’s detection performance back to the desired linear case for both the memoryless and memory rich nonlinear receiver scenarios. Importantly, this analysis was performed using a measured nonlinear behavioural model from a communications receiver with the excellent mitigation performance shown to hold for: significant levels of modelling error, through complex real-world clutter and in other more sophisticated in-band interference scenarios. While the forward modelling technique does not offer a mode-independent solution to the nonlinear receiver problem in radar like the LUT approach, it does provide a far more robust formalism for compensating for complex nonlinear memory effects. This could prove to be an extremely important capability as modern radars move to wider and wider receiver bandwidths where memory effects become more significant.

6.2 Future Research

This thesis has provided the first in-depth look at the nonlinear receiver problem in radar and has developed a variety of advanced digital mitigation solutions that could be employed by the modern system to try and compensate for the harmful nonlinear effects. With the RF spectrum becoming increasingly crowded it is imperative that radars are able to protect themselves against both in-band and out-of-band interference signals. While it is highly likely that the modern radar will exploit its ability to work cognitively as part of a wider network of sensors to avoid interference, each sensor must still have some self-sufficient protection if performance is to be maintained even in the worst interference scenarios. In order to achieve this, the sensor’s RF receiver must be characterised extremely well both in the linear and weakly nonlinear regimes. A sophisticated nonlinear behavioural modelling technique was designed in chapter 3 of this thesis that will provide radar systems with a mechanism for characterising the complex nonlinear memory behaviour of their RF front-ends. This could prove to be a crucial capability as the modern sensor moves to increasingly wide receiver bandwidths where memory effects become more significant. While the simulation results presented in chapter 3 highlight the power of the BBVS model, it would be extremely useful in future work if the behavioural modelling technique could be employed to characterise the nonlinear behaviour of a real front-end radar amplifier. Measurement results of this type would provide radar designers with invaluable insight into the nonlinear memory behaviour of the modern radar receiver which in turn could influence the choice of nonlinear digital mitigation technique applied in the radar processor.

The LUT digital mitigation technique provides an attractive solution to the nonlinear receiver problem in radar that is both mode independent and computationally efficient to implement. The results from chapter 4 demonstrated the effectiveness of the standard LUT techniques in mitigating memoryless receiver nonlinearities in radar, however they
6.2. Future Research

also show that the technique quickly becomes decorrelated as memory effects are introduced to the forward nonlinear transfer function. While it is possible to expand the tandem inverse approach to compensate for complex nonlinear memory effects generated in the RF receiver, the identification of such an inverse model is extremely challenging for radar systems. More work needs to be done to study the performance of the LUT techniques against a broad range of memory rich receiver nonlinearities and other interference scenarios. This would benefit the understanding of when the memoryless solutions become decorrelated and when the memory rich techniques provide performance improvements. Furthermore, the inverse identification procedure could be developed further so that it successfully learns more inverse memory models and ensures that they are robust against a wide variety of interference effects. Finally, the tandem inverse technique could be developed for the in-band interference scenario where the power of the interference signal driving the nonlinearity can make the identification of an effective inverse structure extremely challenging.

The novel NCS algorithm derived in chapter 5 offers an alternative solution to the nonlinear receiver problem in radar. While the forward modelling technique is mode dependent and considerably more computationally intensive to implement than other NLEQ techniques, it offers a far more robust method for dealing with complex nonlinear memory effects than the standard LUT approach. This is due to the fact that the mitigation technique was specifically designed around the forward nonlinear transfer function rather than the inverse as is the case for the other NLEQ techniques. Importantly, it was shown in chapter 5 that if the forward nonlinear transfer function is strictly monotonic then the NCS algorithm is guaranteed to recover the desired linear performance regardless of the amount of memory in the system. This analysis was done using the MPRF radar simulator operating in an in-band interference scenario, however it would be useful in future work to study the convergence and mitigation properties of the NCS algorithm for a wider variety of nonlinear radar scenarios. To achieve this, the algorithm’s theoretical framework could be developed for the cross-channel interference scenario as well as for other operational radar modes beyond the MPRF mode. While the performance of the NCS algorithm was shown to be robust against modelling error and inhomogeneous clutter, a real-world radar trial should be conducted to properly test the performance of the mitigation technique. Finally, more work needs to be done to study the non-convergence properties of the NCS algorithm to see if anything can be done to improve the technique’s mitigation performance in the case where the forward nonlinearity is not strictly monotonic.
Appendix A

Extra Results from Chapter 5

Extra results to support the research presented in chapter 5 on the forward modelling nonlinear mitigation technique. Importantly, the mitigation performance of the NCS algorithm is tested in both the CW and FMCW inference scenario and against a variety of measured nonlinearities.

A.1 Convergence Analysis

<table>
<thead>
<tr>
<th>Nonlinear Model</th>
<th>Monotonicity</th>
<th>Successful Convergence %</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBTS</td>
<td>Strictly Monotonic</td>
<td>100</td>
</tr>
<tr>
<td>BBTS</td>
<td>Non-monotonic</td>
<td>58</td>
</tr>
<tr>
<td>BB Hammerstein</td>
<td>Strictly Monotonic</td>
<td>100</td>
</tr>
<tr>
<td>BB Hammerstein</td>
<td>Non-monotonic</td>
<td>52</td>
</tr>
<tr>
<td>BB Parallel</td>
<td>Strictly Monotonic</td>
<td>100</td>
</tr>
<tr>
<td>Hammerstein</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BB Parallel</td>
<td>Non-monotonic</td>
<td>82</td>
</tr>
<tr>
<td>Hammerstein</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full BBVS</td>
<td>N/A</td>
<td>40</td>
</tr>
</tbody>
</table>

Table A.1: List of NCS convergence results for the FMCW interference scenario. 100 individual nonlinear transfer functions were used to generate the successful convergence value in each case. All simulations were performed with a nonlinearity of cubic order, $P = 1$, and those with memory had a length $L = 4$. 
It is clear by comparing the results in Table A.1 with the convergence results presented previously in Table 5.4 that changing the interference scenario does not alter the effectiveness of the NCS algorithm. The FMCW interference scenario was chosen for this analysis as it is an extremely popular waveform for modern radar systems meaning it is also one of the most likely signals to cause mutual interference in the crowded RF environment. It could also be easily simulated using the existing radar simulator. Importantly, unlike the simple CW interferer the FMCW waveform has a bandwidth which for this analysis was chosen to be equal to the simulated bandwidth of the radar receiver. Performing this type of convergence analysis further tests the robustness of the NCS algorithm as the high bandwidth nature of the FMCW interferer will generate more sophisticated nonlinear memory effects in the receiver. This is due to the intrinsic link between bandwidth and system memory [15, 38]. It is therefore very encouraging that the NCS algorithm still performs as effectively against this more challenging interferer. While the two sets of results are extremely similar, it is important to note the large discrepancy between the Full BBVS convergence value in the FMCW results displayed here (40%) and the CW convergence results presented in chapter 5 (83%). It is thought that this discrepancy reflects the increased problem complexity caused by the additional interference bandwidth in the FMCW case. In the case of the CW interferer, the BBVS model can generate complex nonlinear memory effects in the radar output however the lack of bandwidth in the signal driving the nonlinearity means that the contributions from a lot of the memory cross-terms will be very similar. Importantly, when bandwidth is introduced to the interference signal, as in the FMCW case, the contributions from each of these nonlinear cross-terms becomes more unique effectively leading to a more complicated nonlinear output from the same nonlinear transfer function. It is therefore thought that the drop in successful convergence in the FMCW case is driven by the fact that the NCS algorithm must now correct for all of the nonlinear cross-terms leading to the possibility of the problem becoming too complex for the NCS algorithm to solve. While we tend not consider monotonicity in the BBVS case, we can think of this increased problem complexity as being analogous to the additional interference bandwidth stimulating more non-monotonic effects in the BBVS model which in turn leads to lower levels of convergence.

A.2 Trials Data Results

In this section we test the performance of the NCS algorithm against measured nonlinear models from the transmit-side of the communications literature in both the CW and FMCW interference scenarios. Importantly, we use the same raw trials data for this extra analysis as in the Figure 5.7. We start by testing the NCS algorithm in the in-band FMCW interference scenario using the Vansebrouck nonlinearity [15] described by Table 5.3. In order to provide a comparison for this analysis we present the FMCW plots alongside results from the CW interference case which are similar to those presented previously.
A.2. Trials Data Results

in chapter 5. In all of the plots presented in this section the ground clutter is located between $-4kHz \rightarrow 5kHz$ Doppler with the synthetically added CW/FMCW interferer located at $-4.5kHz$ Doppler. Furthermore, the strongest targets located around $-10kHz$ Doppler become masked by the clutter broadening effects when the receiver nonlinearity is introduced. In all of the figures presented below the plots are labelled as follows: desired linear input (top left), corrupted nonlinear output (top right), corrupted output mitigated by memoryless NCS algorithm (bottom left), corrupted output mitigated by memory rich NCS algorithm (bottom right). Examining the results for the Vansebrouck nonlinearity in Figures A.1 and A.2 it is clear that the NCS algorithm can successfully restore system performance back to the linear case equally well in both interference scenarios. This agrees with the extra convergence analysis results presented above in Table A.1 which found the NCS algorithm to work equally well for both interference scenarios. Furthermore, it is clear from the bottom two plots in both figures that the targets can only be successfully recovered from the corrupted output if the NCS algorithm compensates for the memory terms in the forward nonlinear transfer function. This mirrors the results presented previously in chapter 5.

In order to test the robustness of the NCS algorithm further, its performance was tested against nonlinear behavioural models with memory from the transmit-side of the communications literature. While the availability of measured data on the nonlinear memory behaviour of front-end power amplifiers is still limited, it is certainly more common than for front-end receiver amplifiers. We must be cautious however as transmit-side power amplifiers are typically driven a lot further into their nonlinear regimes than a receiver side amplifier could ever be expected to operate. Therefore, there is a high chance that the measured nonlinear memory behaviour of these front-end power amplifiers will not be representative of a weakly nonlinear radar receiver. It is still interesting however to test the performance of the NCS algorithm using measured nonlinear data and in more challenging scenarios. The nonlinear transfer functions used for this extra analysis were taken from the paper by Ku et al. [67] which similar to the Vansebrouck paper [15] modelled the measured nonlinearity using a BB Parallel Hammerstein model. We specifically used the measured nonlinear models with unit delays from Ku’s Table III in this section as these fitted best with the framework already built into our radar simulator. As with the Vansebrouck nonlinearity, we choose to test the performance of the NCS algorithm against each Ku nonlinearity in both the CW and FMCW interference scenarios in order to provide a complete set of results.

Examining the trials results for the Ku nonlinearities it is clear that the NCS algorithm performs well in all cases. The test targets around $-10kHz$ Doppler are clearly visible in all of the mitigated NCS BBVS outputs (bottom right) indicating that the NCS algorithm can restore system performance back to the desired linear case for both the CW and FMCW interference scenarios. Importantly, this further validates the extra convergence results presented above.
A.2.1 Vansebrouck Nonlinearity: P=1, L=5, BB Parallel Hammerstein Model

<table>
<thead>
<tr>
<th>BBVS Kernel Coefficients for Vansebrouck Nonlinearity</th>
<th>Linear Kernel</th>
<th>Cubic Kernel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{h}_{1{0}}$</td>
<td>${1 + 0j}$</td>
<td>$\tilde{h}_{3{0,0}}$</td>
</tr>
<tr>
<td>$\tilde{h}_{1{1}}$</td>
<td>${0}$</td>
<td>$\tilde{h}_{3{1,1}}$</td>
</tr>
<tr>
<td>$\tilde{h}_{1{2}}$</td>
<td>${0}$</td>
<td>$\tilde{h}_{3{2,2}}$</td>
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<tr>
<td>$\tilde{h}_{1{3}}$</td>
<td>${0}$</td>
<td>$\tilde{h}_{3{3,3}}$</td>
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<tr>
<td>$\tilde{h}_{1{4}}$</td>
<td>${0}$</td>
<td>$\tilde{h}_{3{4,4}}$</td>
</tr>
</tbody>
</table>

Table A.2: Vansebrouck Nonlinearity. All other coefficients set equal to zero.

Figure A.1: Extra trials data results: Vansebrouck nonlinearity - CW interference scenario.

(A) Trials RD map for the linear gain output.

(B) Trials RD map for the BBVS-[15] nonlinear output.

(C) Trials RD map for the BBVS-[15] output mitigated by the memoryless NCS algorithm.

(D) Trials RD map for the BBVS-[15] output mitigated by the memory rich NCS algorithm.
A.2. Trials Data Results

Figure A.2: Extra trials data results: Vansebrouck nonlinearity - FMCW interference scenario.

(A) Trials RD map for the linear gain output.

(B) Trials RD map for the BBVS-[15] nonlinear output.

(C) Trials RD map for the BBVS-[15] output mitigated by the memoryless NCS algorithm.

(D) Trials RD map for the BBVS-[15] output mitigated by the memory rich NCS algorithm.
A.2.2 Ku Nonlinearity 1: P=1, L=1, BB Parallel Hammerstein Model

<table>
<thead>
<tr>
<th>BBVS Kernel Coefficients for Ku Nonlinearity 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Kernel</td>
</tr>
<tr>
<td>( \hat{h}_{1,{0}} )</td>
</tr>
<tr>
<td>cubic Kernel</td>
</tr>
<tr>
<td>( \hat{h}_{3_m,{0,0,0}} )</td>
</tr>
<tr>
<td>( {0.9798 - 0.2887j} )</td>
</tr>
<tr>
<td>( {-0.2901 + 0.4350j} )</td>
</tr>
</tbody>
</table>

Table A.3: Ku Nonlinearity 1. All other coefficients set equal to zero.

(A) Trials RD map for the linear gain output.

(B) Trials RD map for the BBTS nonlinear output.

(C) Trials RD map for the BBTS output mitigated by the memoryless NCS algorithm.

Figure A.3: Extra trials data results: Ku nonlinearity 1 - CW interference scenario.
A.2. Trials Data Results

Figure A.4: Extra trials data results: Ku nonlinearity 1 - FMCW interference scenario.
Appendix A. Extra Results from Chapter 5

A.2.3 Ku Nonlinearity 2: P=1, L=2, BB Parallel Hammerstein Model

<table>
<thead>
<tr>
<th>BBVS Kernel Coefficients for Ku Nonlinearity 2</th>
</tr>
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<tbody>
<tr>
<td>Linear Kernel</td>
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<tr>
<td>$\tilde{h}_{{1,0}}$</td>
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<tr>
<td>${0.7969 - 0.4685j}$</td>
</tr>
<tr>
<td>$\tilde{h}_{{1,1}}$</td>
</tr>
<tr>
<td>${0.2142 + 0.1832j}$</td>
</tr>
<tr>
<td>Cubic Kernel</td>
</tr>
<tr>
<td>$\tilde{h}<em>{{3</em>{m,{0,0}}}}$</td>
</tr>
<tr>
<td>${-0.2340 + 0.4487j}$</td>
</tr>
<tr>
<td>$\tilde{h}<em>{{3</em>{m,{1,1}}}}$</td>
</tr>
<tr>
<td>${-0.0527 - 0.0203j}$</td>
</tr>
</tbody>
</table>

Table A.4: Ku Nonlinearity 2. All other coefficients set equal to zero.

Figure A.5: Extra trials data results: Ku nonlinearity 2 - CW interference scenario.
A.2. Trials Data Results

Figure A.6: Extra trials data results: Ku nonlinearity 2 - FMCW interference scenario.
A.2.4 Ku Nonlinearity 3: P=1, L=3, BB Parallel Hammerstein Model

<table>
<thead>
<tr>
<th>Linear Kernel</th>
<th>Cubic Kernel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{h}_{{1,{0}}}$</td>
<td>${1.4045 + 0.8149j}$</td>
</tr>
<tr>
<td>$\tilde{h}_{{1,{1}}}$</td>
<td>${-1.0554 - 2.3300j}$</td>
</tr>
<tr>
<td>$\tilde{h}_{{1,{2}}}$</td>
<td>${0.6497 + 1.2759j}$</td>
</tr>
</tbody>
</table>

Table A.5: Ku Nonlinearity 3. All other coefficients set equal to zero.

Figure A.7: Extra trials data results: Ku nonlinearity 3 - CW interference scenario.
A.2. Trials Data Results

(Figure A.8: Extra trials data results: Ku nonlinearity 3 - FMCW interference scenario.)
A.2.5 Ku Nonlinearity 4: P=1, L=4, BB Parallel Hammerstein Model

BBVS Kernel Coefficients for Ku Nonlinearity 4

<table>
<thead>
<tr>
<th>Linear Kernel</th>
<th>Cubic Kernel</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{h}_{1,{0}} )</td>
<td>{0.7411 − 0.0234j}</td>
</tr>
<tr>
<td>( \tilde{h}_{1,{1}} )</td>
<td>{0.9881 + 0.2705j}</td>
</tr>
<tr>
<td>( \tilde{h}_{1,{2}} )</td>
<td>{−1.5036 − 1.4375j}</td>
</tr>
<tr>
<td>( \tilde{h}_{1,{3}} )</td>
<td>{0.7795 + 0.9569j}</td>
</tr>
</tbody>
</table>

**Table A.6:** Ku Nonlinearity 4. All other coefficients set equal to zero.

(Figure A.9: Extra trials data results: Ku nonlinearity 4 - CW interference scenario.)
A.2. Trials Data Results

(Figure A.10: Extra trials data results: Ku nonlinearity 4 - FMCW interference scenario.)
Appendix B

Publications and Conferences


Abstract—Mutual interference is a significant challenge for modern radars that is becoming increasingly problematic as the sensor is adopted onto new platforms. If nonlinearities are present in the radar receiver, interference signals may cause intermodulation and cross-modulation effects. This paper shows that cross-channel nonlinear distortion effects can be modelled accurately using a baseband equivalent Volterra series. The specific cross-channel baseband Volterra Series we derived from the full passband Volterra series is presented in the System Model section of this paper. Complex computer simulations were used to verify that all of the relevant nonlinear distortion effects are captured by this new behavioural model. The architecture of these simulations is outlined in this paper with particular emphasis on the techniques designed to identify the cross-channel bandwidth expansion effects caused by the nonlinearity. As a result, the derived model can capture all nonlinear distortion effects of passband amplifiers can be accurately characterised using behavioural models based on baseband Volterra series. This result is of particular interest to radar systems as it will allow the nonlinear effects of receiver amplifiers to be characterised in greater detail.

I. INTRODUCTION

Radars are well established sensors in the defence, aviation and marine sectors however size, weight and cost constraints restrict the number of systems deployed. This often means that systems based on older, lower performance technology are kept in service for many years. By exploiting commercial off-the-shelf (COTS) components companies can now produce compact economically viable systems that can easily be productionised. This has resulted in modern radars being adopted in a variety of new platforms and consumer technologies e.g. driverless cars and unmanned aerial vehicles (UAVs). The radio frequency (RF) spectrum is becoming increasingly crowded and with the use of radars set to increase dramatically over the next decade the level of ambient RF energy will quickly become intolerable for the sensors. This crowded RF environment may cause performance problems for modern mass-produced radars due to interference between systems. This multi-sensor mutual interference problem can somewhat be addressed if each individual radar operates as part of a wider network of sensors. By exploiting the modern radar’s ability to be frequency agile it is conceivable that the sensors could avoid interfering with each other if they all successfully communicate their location across the radar network [1]. While communicating over a radar network will undoubtedly be central to the successful operation of multiple radars in close proximity to one another a more reliable solution must also be sought. A self-sufficient system that operates within each radar is needed if the sensors are to maintain performance even in the worst interference scenario. The immunity of a radar system to interfering signals is defined by the linearity of the RF amplifiers in its receiver [2], [3]. Unfortunately, one of the major compromises when employing COTS components in the radar receiver is system linearity. The modern mass-produced radar will therefore be far more susceptible to interfering signals than its bespoke predecessor. Having a low tolerance to interfering signals means that the RF amplifiers in the radar receiver may be forced to operate in a nonlinear state when interference signals are present. The potential effects of operating a radar with a nonlinear receiver are illustrated in Fig. 1. In contrast to the linear receiver, the out-of-band interference signals can now couple into the desired channel through the nonlinearity causing significant distortion. If modern radars are to maintain operational capability in a crowded RF environment it is imperative that the nonlinear behaviour of their receiver amplifiers is understood in great detail.
In this paper a behavioural modelling technique that will allow the complex nonlinear response of RF amplifiers to be characterised in great detail is presented. This paper develops on the work previously published by Zou et al. [4] which modelled cross-channel nonlinear distortion effects for communication systems. The modelling technique derived in [4] exploits certain characteristics that are unique to communication systems which allows a Taylor series approach to be employed. For these nonlinear modelling techniques to be transferred to a radar system the more general Volterra series approach must be adopted. The Volterra series is often described as a Taylor series with memory and was first reduced series approach must be adopted. The Volterra series is often described in [5] has been used extensively to model in-channel nonlinear effects for communication systems [6], [7] but it must be re-derived if it is to capture cross-channel distortion effects.

The specific cross-channel BBVS we derived is presented in Section II. In Section III we present the architecture of the simulations used to validate our behavioural model. Particular emphasis is placed on the system identification techniques designed to allow the full nonlinear bandwidth expansion effects to be captured. Finally, the simulation results are presented and discussed in Section IV.

II. SYSTEM MODEL

The passband Volterra series is a powerful behaviour model used to characterise the nonlinear effects of passband amplifiers with memory. Mathematically it takes the form of an infinite sum of multidimensional convolution integrals, given by (1):

$$y(t) = \sum_{p=1}^{\infty} \left\{ \int_{-\infty}^{\infty} dt_1 \cdots \int_{-\infty}^{\infty} dt_p h_p(\tau_1, \ldots, \tau_p) \prod_{r=1}^{p} x(t - \tau_r) \right\}$$

(1)

where $x(t)$ and $y(t)$ are the real continuous-time input and output passband signals respectively. The convolution integrals result in the real input signal being delayed by successive time lags $\tau_r$. The RF amplifier’s behaviour is completely characterised by the full set of passband Volterra kernels $h_p(\tau_1, \ldots, \tau_p)$. This is illustrated by the simple diagram in Fig. 2.

Fig. 2. Illustration of PBVS.

The first-order term $p = 1$ defines the linear response of the system with the subsequent higher order kernels being linearly combined to form the full nonlinear behavioural model. The most dominant nonlinear system effects can be captured by the first three summations in (1). Even after truncating the Volterra model to its dominant terms it is still far too large to be identified for an RF amplifier and must therefore be reduced further. By invoking the narrowband assumption we restrict our modelling capabilities to a small bandwidth around the carrier but we succeed in reducing our behavioural model to an identifiable size. This narrowband assumption is central to the derivation of the traditional BBVS as it allows Benedetto et al. to neglect all of the even order Volterra terms [5]. Therefore the BBVS model derived in [5] is capable of modelling the in-channel nonlinear distortion effects around the carrier but fails to capture any cross-channel effects. By keeping the second order term in the mapping of the PBVS model to the baseband domain it is shown that all of the cross-channel nonlinear distortion effects can be captured by the BBVS model. To demonstrate this, a BBVS model centred on the carrier was derived from an input signal with a component centred on the carrier and an out-of-band interferer at half-the-carrier. This input signal is represented in complex envelope notation in equation (2):

$$x(t) = \frac{1}{4} [\tilde{x}_1 \exp(j\omega_0 t) + \tilde{x}_1^* \exp(-j\omega_0 t)] + [\tilde{x}_2 \exp(j2\omega_0 t) + \tilde{x}_2^* \exp(-j2\omega_0 t)]$$

(2)

where $\tilde{x}_1$ and $\tilde{x}_2$ denote the complex baseband signal components modulated on half-the-carrier ($\omega_0 = 2\pi f_s$) and the carrier ($2\omega_0 = 4\pi f_s$) respectively. Inserting (2) into our reduced PBVS expression and using a methodology similar to that used by Enzinger et al. in [6] we arrive at a purely baseband input-output relation (3).

$$\tilde{y}(t) = \int_{-\infty}^{\infty} dt_1 \tilde{h}_1(\tau_1) \tilde{x}_2(t - \tau_1)$$

$$+ \int_{-\infty}^{\infty} dt_1 dt_2 \tilde{h}_2(\tau_1, \tau_2) \tilde{x}_1(t - \tau_1) \tilde{x}_1(t - \tau_2)$$

$$+ \int_{-\infty}^{\infty} dt_1 dt_2 dt_3 \tilde{h}_3(\tau_1, \tau_2, \tau_3)$$

$$\tilde{x}_1(t - \tau_1) \tilde{x}_2(t - \tau_2) \tilde{x}_1(t - \tau_3)$$

$$+ \int_{-\infty}^{\infty} dt_1 dt_2 dt_3 \tilde{h}_3(\tau_1, \tau_2, \tau_3)$$

$$\tilde{x}_2(t - \tau_1) \tilde{x}_2(t - \tau_2) \tilde{x}_2(t - \tau_3)$$

(3)

Where $\tilde{y}(t)$ is the baseband output signal centred on the carrier frequency ($2f_s$). Our final expression for $\tilde{y}(t)$ (3) distorts the baseband modulation on the carrier due to the traditional in-band nonlinear effects captured by the baseband kernels $\tilde{h}_1(\tau_1)$ and $\tilde{h}_3(\tau_1, \tau_2, \tau_3)$ as well as the new cross-channel nonlinear effects from half-the-carrier captured by the baseband kernels $\tilde{h}_2(\tau_1, \tau_2)$ and $\tilde{h}_3(\tau_1, \tau_2, \tau_3)$. For the input signal $x(t)$ (2) the BBVS model derived will distort the modulation around the carrier in exactly same way as the full PBVS model.

III. SIMULATION ARCHITECTURE

In order to prove the theoretical results derived above are correct it must be shown that our BBVS model can capture all of the PBVS in-channel and cross-channel nonlinear distortion effects around the carrier for an input signal $x(t)$ (2). This
was done by means of a computer simulation the architecture of which is illustrated in Fig. 3. In order to perform the simulation our continuous-time Volterra series expressions had to be transformed into causal discrete-time filters [6]. To do this the infinite integrals are converted into summations over Volterra taps the length of which are chosen to be \( M \). These Volterra taps define the number of previous input samples that can affect the current output sample \( n \). In essence, they characterise the memory of the system and therefore \( M \) defines the overall size of the Volterra kernel vectors. An arbitrary PBVS model was selected for the simulation. The BBVS kernels must be learnt by feeding our simulation with a complex baseband input signal and capturing the resultant complex baseband output signal that has been distorted by our full PBVS model. While we know the form of our input signal \( x[n] \) from (2) we must decide what modulation signals, \( \tilde{x}_1[n] \) and \( \tilde{x}_2[n] \), we must apply to fully characterise our baseband behavioural model. In general, the numbers of tones in the modulation signal required to identify a Volterra kernel scales with the Volterra kernel order \( p \) and the Volterra tap length \( M \) [8]. To adequately satisfy this persistent excitation condition [9] the modulation signals \( \tilde{x}_1[n] \) and \( \tilde{x}_2[n] \) were both chosen to be deterministic random noise [6]. Before we can compare the output from our BBVS model with the true carrier modulation output from the arbitrary PBVS model we must first identify the BBVS kernels. To do this we perform a linear least square (LLS) identification [7], [10] using our input modulation signals, \( \tilde{x}_1 \) and \( \tilde{x}_2 \), and our downsampled PBVS output signal \( \tilde{y}[n] \). This identification task is made more challenging by the intrinsic bandwidth expansion properties of the nonlinearity, proof of which is omitted due to page restrictions and will be submitted in a fuller paper in due course. We are faced with a nonlinear multirate system identification problem as the second and third order Volterra kernels cause the output modulation bandwidth to expand by a factor of two and three respectively. For our BBVS model to be successful these bandwidth expansion effects had to be captured.

In order to satisfy the Nyquist theorem regarding the output signal we must sample \( \tilde{y}[n] \) at three times the sampling rate of the input signal. Sampling at this high rate successfully avoids aliasing effects in \( \tilde{y}[n] \) but introduces significant challenges for the system identification. Unfortunately, we cannot simply oversample the input signals and perform the identification of the BBVS model at the high sampling rate for two key reasons. Firstly, the number of terms in the Volterra series scales exponentially with sampling rate resulting in our BBVS being too large to identify at the high sampling rate. Secondly, our LLS algorithm becomes ill-conditioned if we attempt to perform the system identification at the high sampling rate. The oversampling of the input signals, \( \tilde{x}_1[n] \) and \( \tilde{x}_2[n] \), causes the LLS algorithm to tend toward numerically unstable solutions as large sections of the identification space are occupied [9], [11]. Multirate system identification techniques, specifically polyphase decomposition, must be employed to learn the BBVS model. Schwinshackl's paper [12] extends the linear multirate theory so that the classic techniques can be applied to nonlinear filters. Polyphase decomposition of Volterra kernels is the main principle developed by Schwinshackl in [12] and it is this specific technique that we exploit to identify our BBVS model. The basic technique is illustrated in Fig. 4 where the Volterra kernels are decomposed into subkernels \( (\kappa_1, \kappa_2, \kappa_3) \) centred on different polyphase components. By performing three parallel but smaller system identifications a BBVS model can be identified at the low sampling rate. This is a key result as it allows all of the nonlinear bandwidth expansion effects observed at the high sampling rate to be captured by a BBVS model identified at the low sampling rate.

IV. SIMULATION RESULTS

In the simulations, the bandwidth of the input baseband modulation signals, \( \tilde{x}_1[n] \) and \( \tilde{x}_2[n] \), were chosen to be the same \((100 MHz)\) so that the maximum bandwidth around the carrier was characterised \((300 MHz)\). The arbitrary PBVS, to be identified by our BBVS model, had Volterra order \( p = 3 \)

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**Fig. 3.** Simulation Architecture used to learn and test the BBVS model. For the learning mode the PBVS output was passed to BBVS output. For the testing mode the BBVS output was compared with PBVS output.

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**Fig. 4.** Nonlinear multirate system identification model. Polyphase subkernels identified in parallel from uniquely delayed (\( z_1, z_2, z_3 \)) versions of the input signals and separate decompositions of the output signal.

and was chosen to have a memory tap length $M = 6$. Three separate BBVS models all of the form illustrated in Section II and with memory tap length also $M = 6$ were identified for the arbitrary PBVS. Each separate BBVS model was identified for a different signal to noise ratio (SNR) at the output of the PBVS nonlinearity. The number of input samples used for the identification was three times the number of unknown parameters [11]. The results display in Fig. 5 below compares the output signals $\hat{y}[n]$ for the three separate BBVS models when each is fed with exactly the same input modulation signals, $\tilde{x}_1[n]$ and $\tilde{x}_2[n]$. The comparison takes the form $10 \log_{10}(S(f)) - 10 \log_{10}(|S_m(f)|)$ where $S(f)$ is the power spectral density of the signal at the amplifier output and $S_m(f)$ is the power spectral density of the signal at the output of the Volterra model using the estimated parameters.

By displaying our results in this manner the accuracy to which our BBVS can model the carrier distortion effects induced by the PBVS for input signal $x(t)$ (2) can be observed. Additionally, the accuracy to which we can identify our BBVS. Behavioural model through different levels of output noise can be studied. It is clear from Fig. 5 that our BBVS can capture the full nonlinear behaviour of the PBVS including the bandwidth expansion effects. The identification algorithm successfully learns the BBVS kernels through the noise in all cases. As the SNR is decreased the model struggles to capture the nonlinear bandwidth expansion effects out at the identification band edges. This is to be expected as the additive noise is considerably stronger than the weak nonlinear distortion effects in this region.

![Simulations results. Comparison of output signals from BBVS models identified through different levels of noise. Green SNR=20dB, Red SNR=40dB and Blue SNR=0dB.](image)

V. CONCLUSION

In this paper, we presented a compact behavioural modelling technique based on the baseband Volterra series and have shown that it is capable of capturing all of the cross-channel and in-channel nonlinear effects contained in the full passband Volterra series model. This was demonstrated for an input signal with a component centred on the carrier frequency and an out-of-band interferer centred on half the carrier frequency. With the RF spectrum becoming increasingly crowded it is imperative that radars are able to protect themselves against interference signals. This can only be achieved if the sensors are extremely well characterised both in the linear and nonlinear regime. The behavioural modelling technique presented in this paper will allow modern radar receivers to be characterised in greater detail.

ACKNOWLEDGMENT

This work was supported by the UK Engineering and Physical Sciences Research Council (EPSRC) [Grant No. EPSRC EP/N509644/1], Leonardo through the CASE Studentship Scheme and the Royal Commission for the Exhibition of 1851.

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Abstract—Modern mass produced radar systems offer a wide variety of advantages over their bespoke predecessors however to achieve these benefits the linearity of their RF receivers is often sacrificed. This compromise leaves these modern sensors susceptible to unwanted cross-modulation effects and with the RF spectrum becoming increasingly crowded this problem is only going to become more pertinent. If mitigation techniques are to be developed they must correct for the nonlinear distortion applied to both the amplitude and phase of the returned radar signal. In order to accurately model nonlinear phase effects in the radar receiver we must consider memory. This paper studies how introducing nonlinear memory in the radar receiver affects the cross-modulation distortion generated. It also presents a simple cross-modulation mitigation technique developed in the communications literature and applies it to the radar cross-modulation problem. As a result the simple communications based mitigation technique successfully corrects for cross-modulation distortion generated from a memoryless nonlinear radar receiver. However, it is ineffective when nonlinear memory effects are introduced. This is a significant result as it suggests that if the nonlinearities generating cross-modulation effects in modern radar are found to have memory then sophisticated memory mitigation techniques will have to be developed.

I. INTRODUCTION

Cross-modulation is a term used in the field of radar to describe the distortion of the returned radar signal due to the presence of a strong out-of-band interferer in the radio frequency (RF) receiver. This effect can occur when the desired signal and the strong interferer pass through the active components in the RF front-end simultaneously. Cross-modulation is most easily observed in the medium pulse repetition frequency (MPRF) environment which is why it is the focus of this study. In a typical MPRF mode the radar is tasked with detecting low signal to noise ratio (SNR) targets in the presence of a strong clutter signal. If the out-of-band signal is strong enough and the receiver components are not linear enough then the interferer can couple into the desired channel. This nonlinear coupling will corrupt the desired signal by modulating the strong clutter return potentially masking small targets and ultimately reducing the sensor’s overall performance. Traditional radars protect themselves against this unwanted effect by choosing to operate further down the linearity curve when in the presence of strong interference so that this nonlinear coupling cannot occur. Some modern radars cannot afford to employ this mitigation technique as their RF receivers are constructed from low-cost commercial-off-the-shelf components. While these components have allowed companies to mass produce radar systems in an economically viable manner the sensor’s front-end receiver linearity has been compromised. To achieve acceptable performance these modern commercial radars will have to operate at the top of their linear regions leaving no margin to back-off in the presence of interference. A more sophisticated cross-modulation mitigation technique will have to be adopted if these low-cost sensors are to maintain performance in the presence of interference. There is very little published work in the radar literature on the mitigation of nonlinear receiver effects [1], [2]. However, a comprehensive survey of the wider literature revealed that successful solutions to the dual-channel nonlinear receiver problem have been published in the related field of communications [3], [4]. This paper presents a study into the effectiveness of a typical communications based mitigation technique in an MPRF radar environment.

The specific communications technique studied in this paper was based on the work previously published by Zou et al. [4] which attempts to correct for the cross-channel nonlinear distortion by building a mathematical inverse of the forward nonlinearity. The method employed in [4] is specifically designed to mitigate nonlinear distortion caused by an amplifier with no memory effects. While this memoryless assumption appears to be widely accepted for communications systems we cannot be confident that it will hold for a typical radar system. Radars are far more sensitive than communications systems with the received signals being both unknown to the sensor and more complicated than the typical communications signal. This suggests that more sophisticated nonlinear models that include memory effects must be employed when simulating the characteristics of the nonlinear radar receiver. In this paper the performance of the nonlinear memoryless mitigation technique discussed above was tested against the complex Baseband Equivalent Volterra Series (BBVS) nonlinear model derived in [5]. The Volterra series is often described as a Taylor series with memory and is capable of modelling complex nonlinear...
memory effects. By incorporating the BBVS model into a simulated MPRF radar test environment the mitigation technique’s effectiveness in correcting cross-modulation effects in a typical radar system was studied in detail. The effect of introducing nonlinear memory to the cross-modulation problem has not previously been studied in the radar literature. Additionally, this paper develops on the work previously published in [4] by applying Zou’s mitigation technique to a nonlinearity with memory specifically in a radar system.

The cross-channel BBVS implemented in the MPRF radar simulator is presented alongside the formalisation of the communications based mitigation technique in Section II. In Section III we outline the overall architecture of the MPRF radar simulator used to test the mitigation technique. Particular emphasis is place on the shape of the chosen nonlinearity and the simulated scenario used to test the effectiveness of the mitigation algorithms. Finally, the simulation results are presented and discussed in Section IV.

II. System Model

The particular type of Volterra model we employed in our MPRF simulator was the low-pass equivalent Volterra series [6] which was rederived for cross-channel nonlinear distortion in radar systems in [5]. The second order term contained in the model of [5] has been omitted for the purposes of this paper as we are interested in the more general cross-modulation effect contained in the third order term. Hence forth we refer to this specific Volterra series as the BBVS model and it is presented in its discrete time form in (1).

\[
\hat{y}[n] = \sum_{i_1=0}^{M} \hat{h}_1[i_1] \hat{x}_1[n - i_1] + \ldots
\]

\[
\sum_{i_1=0}^{M} \sum_{i_2=0}^{M} \sum_{i_3=0}^{M} \hat{h}_{3,\text{in}}[i_1, i_2, i_3] \hat{x}_2[n - i_1] \hat{x}_1[n - i_2] \hat{x}_2[n - i_3] + \ldots
\]

\[
\sum_{i_1=0}^{M} \sum_{i_2=0}^{M} \sum_{i_3=0}^{M} \hat{h}_{3,\text{out}}[i_1, i_2, i_3] \hat{x}_1[n - i_1] \hat{x}_1[n - i_2] \hat{x}_1[n - i_3]
\]

(1)

The BBVS model combines the desired input baseband signal, \( \hat{x}_1[n] \), centred on the carrier frequency and the input baseband interference signal, \( \hat{x}_2[n] \), centred on the interference frequency to form the nonlinear baseband output signal, \( \hat{y}[n] \), also centred on the carrier frequency. The BBVS makes the assumption that the input signals are narrowband which restricts its modelling capabilities to a small bandwidth around the carrier frequency. However, within this bandwidth the BBVS model captures all of the theoretical nonlinear effects contained in the passband Volterra model caused by the presence of an interference signal anywhere in the spectrum. It is assumed for this analysis that the interferer does not occupy the desired channel and is therefore filtered out once it has caused the front-end nonlinear distortion. The Volterra tap length, \( M \), defines the number of previous input samples that can affect the current output sample, \( n \). In essence \( M \) defines the amount of memory in the system. The variables \( \hat{h}_1[i_1], \hat{h}_{3,\text{in}}[i_1, i_2, i_3], \hat{h}_{3,\text{out}}[i_1, i_2, i_3] \) in (1) represent the linear, third order cross-channel and third order in-channel Volterra kernels respectively. These kernels completely characterise both the in-channel and cross-channel nonlinear distortion effects of the simulated radar receiver in the MPRF simulator.

The task for the mitigation algorithms is to remove the unwanted nonlinear distortion effect caused by the third order cross-channel term described in (1) characterised by \( \hat{h}_{3,\text{in}}[i_1, i_2, i_3] \). The mitigation technique designed by Zou et al. [4] attempts to mathematically invert the forward model to obtain an expression for the uncorrupted input \( \hat{x}_1[n] \). By reducing (1) to the memoryless case, \( M = 0 \), and rearranging the expression a theoretical estimate of the desired input signal \( \hat{x}_1[n] \) can be found.

\[
\hat{x}_1[n] = \frac{\hat{y}[n]}{(h_1[0] + \hat{w}[n][\hat{x}_1[n] + \hat{w}[n]]^2)}
\]

(2)

In doing this derivation Zou assumes that the system is operating linearly when the interferer is not present and therefore contributions from the third order in-channel term are negligible compared to those from the linear term. This is a sensible assumption for radar systems which are designed to operate within their linear regimes. In short, the technique relies on the interferer being captured separately by a linear guard channel and the memoryless coefficients of the forward model being known. Firstly, if the interference signal is strong enough to cause a nonlinear effect in the RF receiver it will most likely be much stronger than the returned radar signal and therefore a simple guard channel could be backed-off to linearly capture it. The additive white Gaussian noise associated with the linear guard channel is denoted by \( \hat{w}[n] \) in (2). With regards to the forward model coefficients, Zou et al. outline a nonlinear parameter estimation technique in [4] which aims to actively characterise the nonlinear channel during operation. This technique is not applicable to radar systems as it utilizes the predetermined transmitted pilot symbols unique to communication signals. It must therefore be assumed that the memoryless coefficients of the forward model can be identified offline using standard test techniques.

III. Simulation Architecture

In order to test the theoretical mitigation algorithm outlined in Section II against the BBVS nonlinear model a nonlinear radar simulator had to be developed. The algorithm was tested against an MPRF radar mode for two key reasons. Firstly, the MPRF mode is one of the most popular modes of operation for modern radar systems. Secondly, the MPRF mode is intrinsically narrowband which as discussed previously is one the main assumption that must be satisfied when employing the BBVS nonlinear model. The MPRF simulator operates down at baseband and generates a returned radar signal corresponding to a chosen radar scene. This signal is then passed through the BBVS model which represents the entire nonlinear radar receiver. The particular choice of BBVS kernel coefficients governed the nonlinear behaviour
of the simulated receiver. While the choice of these kernel coefficients had to be arbitrary, due to the lack of available data, reasonable assumptions about the general shape of the coefficients could be made to make the simulated nonlinear effects more realistic. First of all, the simulated radar had to have a discernible linear region which the receiver operates in when the interference signal is not present. This assumption set the general level of the third order nonlinear kernel from which both the *in-channel* coefficients, \( h_{3,n}[i_1, i_2, i_3] \), and *cross-channel* coefficients, \( h_{3,n_2}[i_1, i_2, i_3] \), were derived. It is difficult to predict the nonlinear memory behaviour of the kernels as very little has been published in the available literature however several basic assumptions can be made to contain the complexity of the nonlinearity. Firstly, it is assumed that the nonlinear kernel coefficients generally diminish with each subsequent memory term. This assumption is based on the radar receiver being casual with small relaxation times. Secondly, it is assumed that the on-diagonal memory coefficients are larger than their respective cross-coefficients. This assumption is founded in the communications literature where to save computational complexity the Volterra series is reduced to the memory polynomial [3]. In short, the memory polynomial retains the more dominant on-diagonal coefficients and discards the weaker cross-coefficients. The BBVS model chosen for this paper had a memory length \( M = 6 \) with the on-diagonal memory coefficients following a Chebyshev filter response and the off-diagonal cross-coefficients decaying exponentially. The \( h_{3,n_2}[i_1, i_2, i_3] \) BBVS kernel coefficients used for this paper are displayed in Fig. 1.

There are many different operational scenarios where cross-modulation can cause problematic effects in the radar receiver. However, for the purposes of this paper we focus on the classic case where a pulsed single-tone interferer, \( \tilde{x}_2[n] \), modulates the radar’s own clutter returns masking a genuine target. This is the same scenario as that studied by Mellor et al. in [1]. The PRF of the interferer and the radar were mismatched resulting in the cross-modulation distortion having a harmonic like behaviour across the Doppler dimension. The MPRF simulator generated three outputs corresponding to the linear, Taylor and BBVS RF receivers for the specified radar scene. In all cases the desired radar signal, \( \tilde{x}_1[n] \), remained constant. For the linear case the full BBVS receiver model was used but the out-of-band interferer was removed from the scene. For the nonlinear receiver simulations the out-of-band interferer was present but the set of nonlinear kernel coefficients used to define the simulated RF receiver varied. In the BBVS case the receiver model consisted of the full set of nonlinear kernel coefficients whereas in the Taylor case the model was reduced so that it consisted of only the Taylor coefficients. This is illustrated in Fig. 1. The range-Doppler plots for the linear and BBVS outputs are displayed in Fig. 2a and Fig. 2b respectively. The output for the Taylor case is extremely similar to that of the BBVS output which is why it has not been displayed here. A more sophisticated comparison of these outputs is performed in the results section of the paper. However, it is clear from Fig. 2a and Fig. 2b that the simulated radar’s detection capability has been affected by the cross-modulation distortion as the target is no longer in a noise limited region. By comparing the simulated cross-modulation results with those already published in the radar literature [1], [2] it is clear that the MPRF simulator correctly reproduces the classic cross-modulation effects. The mitigation technique (2) was applied on each simulated pulse after the nonlinearity but before any coherent processing. The SNR for the simulation was defined at the input to the nonlinearity with the linear guard channel SNR set separately to a value of 20\( \text{dB} \).

### IV. Simulation Results

In order to study the performance of the communications based mitigation technique (2) for an MPRF radar mode in
detail, statistical probability of detection (PD) analysis was performed on the radar target. This was done for the linear, Taylor, BBVS, Mitigated Taylor and Mitigated BBVS test cases with the results displayed in Fig. 3. The detection algorithm applied a cell-averaging constant false alarm rate (CA-CFAR) thresholding technique [7] that maintained a constant probability of false alarm rate of 0.20 in all test cases. In effect the CA-CFAR algorithm adapts the detection threshold depending on the power and statistical behaviour of the range-Doppler cells surrounding the target cell. For the nonlinear test cases the cross-modulation causes the detection space around the target to no longer be noise limited, see Fig. 2a and Fig. 2b. In turn this results in the target detection threshold being raised and the sensitivity of our simulated radar decreasing. By varying the input SNR of the target the sensitivity of the simulated radar was studied in detail. For each data point in Fig. 3 200 bursts were used to estimate the PD value.

Fig. 3: Probability of detection analysis results.

As expected, both the Taylor and BBVS PD curves in Fig. 3 fall off at a much higher input SNR value than for the linear case. This confirms that the cross-modulation distortion has significantly affected the simulated radar’s ability to detect the target. Observing the Mitigated Taylor curve in Fig. 3 it is clear that the communications based mitigation technique (2) has succeeded in restoring the simulated radar’s detection performance to that of the desired linear case. It is important to note that as the SNR of the linear guard channel, $w[n]$, is decreased the mitigation algorithm becomes less effective but still provides some improvement. Comparing the Mitigated BBVS curve with BBVS curve in Fig. 3 it is apparent that the communications based mitigation technique (2) has been ineffective in restoring the simulated radar performance back to the desired linear case. While the simulated cross-modulation distortion generated from the Taylor and BBVS receivers appear to be very similar in the range-Doppler domain and have very similar PD curves, the additional memory terms cause subtle nonlinear effects that become important when trying to mitigate the unwanted distortion. The memoryless mitigation technique has effectively been decorrelated by the additional memory terms in the nonlinear BBVS model. For any mitigation technique to be successful in this case it must correct for the distortion caused by nonlinear memory terms in the BBVS model as well as the memoryless terms. This makes the mitigation task exponentially harder than for the memoryless case.

V. CONCLUSION

In this paper, we presented a study into the cross-modulation effects in a radar receiver with memory and analysed the performance of a simple communications based mitigation technique. By simulating the cross-modulation distortion for an MPRF radar mode we found that introducing memory to the nonlinear receiver model didn’t appear to affect the target detection performance of the system when compared with the memoryless case. However, when the memoryless mitigation technique was applied to both sets of data the importance of the nonlinear memory terms became apparent. While the technique worked well in the memoryless nonlinear receiver case it was ineffective in the case where the radar receiver included nonlinear memory effects. If nonlinear memory effects prove to be important in modern radar receivers then sophisticated cross-modulation mitigation techniques that can capture nonlinear memory effects will have to be developed.

ACKNOWLEDGMENT

This work was supported by the UK Engineering and Physical Sciences Research Council (EPSRC) [Grant No. EPSRC EP/N509644/1], Leonardo through the CASE Studentship Scheme and the Royal Commission for the Exhibition of 1851.

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Memory NLEQ Techniques to Mitigate Cross-modulation Effects in Radar

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Abstract—Cross-modulation is a well-known phenomenon in the field of radar and is an effect that is becoming increasing problematic for the modern sensor due to the overcrowding of the radio frequency (RF) spectrum. The low-cost design of modern mass-produced radar systems means that digital mitigation techniques are preferable to an expensive hardware solution. This paper presents a study into the effectiveness of the nonlinear equalisation (NLEQ) technique which is one of the most popular digital mitigation solutions in the related field of communications. Importantly, this study extends the NLEQ technique to include nonlinear memory effects for the first time in the radar literature and presents detailed probability of detection (PD) analysis on its effectiveness in the cross-modulation scenario. As a result, the NLEQ inverse structure must include memory effects if it is to be successful in correcting for cross-modulation distortion generated from a forward receiver nonlinearity with significant memory. Additionally, it is shown that system performance can be largely restored in the case where the forward receiver nonlinearity has significant memory by identifying a memory rich NLEQ inverse. This is a significant result as it shows that NLEQ mitigation techniques provide a mode independent framework that can compensate for nonlinear memory effects in the radar receiver.

I. INTRODUCTION

Of the potential nonlinear interference effects that could be generated in the radar receiver, cross-modulation distortion is the most problematic as it is not frequency specific [1]–[4]. The term cross-modulation is used to describe the scenario where strong out-of-band interference couples with the desired radar channel through a front-end nonlinearity causing unwanted distortion in the radar output. Cross-modulation distortion can therefore be generated in all radar modes of operation. However, it is most easily observed in the medium pulse repetition frequency (MPRF) environment where low signal to noise ratio (SNR) targets must be detected in the presence of a strong clutter signal. The specific cross-modulation effect most often described in the radar literature relates to the scenario where the interference signal is pulsed [1], [2]. In this specific case, the nonlinear coupling will corrupt the desired signal by modulating the strong clutter returns potentially masking small targets and ultimately reducing the sensor’s overall performance. In essence, the mismatch between the interferer’s pulse repetition frequency (PRF) and the radar’s PRF cause nonlinear repeats of the clutter to be spread across the Doppler dimension in a harmonic like fashion. The cross-modulation effects for a typical MPRF radar mode and pulsed interferer is illustrated in Fig. 1.

Advanced signal processing techniques offer an alternative low-cost low-impact solution to the nonlinear receiver problem in radar. The basic principle relies on having a well characterised RF receiver both in the linear and nonlinear regime as well as the capability of modern phased array antennas to form guard channels. Importantly, the guard channel must be configured to capture the raw interfering signal so that the radar is provided with information regarding the source of the nonlinear distortion. Of these advanced signal processing techniques, NLEQ is the most attractive to radar systems as it is mode-independent and computationally inexpensive to implement. In short, NLEQ is a universal mitigation technique where the forward nonlinear effects can be corrected by digitally passing the distorted output through an inverse of the receiver nonlinearity. These NLEQ techniques are extremely popular in related field of communications as they offer a cheap mitigation solution to front-end nonlinearities without altering the compact design of the receiver itself [5].

Several papers in the radar literature have studied the feasibility of implementing NLEQ techniques in the radar processor.
to alleviate distortion effects caused by interference in the radar receiver [3], [6], [7]. In [3] Peccarelli et al. implement the classic polynomial NLEQ inverse technique to mitigate both intermodulation and cross-modulation distortion effects caused by the presence of a strong adjacent channel. The paper focused on the strictly memoryless case and displays simulated results highlighting the effectiveness of the algorithm in supressing both cross-modulation and intermodulation effects. In [6], Rabideau extends the NLEQ formalisation to include memory effects before implementing a memoryless NLEQ inverse as part of a nonlinear decorrelation technique. Rabideau’s technique focused on a simplified in-band scenario and was shown to be effective in reducing the strength of nonlinear spurs generated by two equal strength test tones.

Extending the nonlinear receiver problem to include memory effects is important for understanding the subtleties of how the desired signal’s amplitude and phase are actually corrupted by the front-end nonlinearity. This is particularly true as radar systems move to increasingly wider bandwidth where memory effects become more significant. The effect of introducing nonlinear memory to the cross-modulation problem was studied in [7] as well as the effectiveness of a simple memoryless inverse mitigation technique. The mitigation technique employed in [7] was first published by Zou et al. in [8] where unlike the typical NLEQ techniques the memoryless forward nonlinearity is mathematically inverted to produce a theoretical inverse function. It was shown in [7] that if the forward nonlinearity contained significant nonlinear memory effects then memoryless inverse mitigation techniques will become decorrelated and therefore less effective. In this paper, the effectiveness of the classic polynomial NLEQ technique in mitigating cross-modulation effects generated from nonlinearities with memory is studied before the NLEQ technique itself is extended to include memory as well. The complex memory NLEQ technique will be described in full before its performance is tested against the cross-modulation scenario with memory. The performance of the NLEQ technique, either memoryless or otherwise, has not previously been studied for the cross-modulation scenario in the radar literature. Additionally, the paper extends the understanding of how these complex memory NLEQ techniques might be identified and implemented in a modern radar system.

The cross-channel nonlinear model used to generate the memory cross-modulation effects studied in this paper is introduced alongside the NLEQ mitigation techniques in section II. In section III the simulation architecture designed to test the performance of the mitigation techniques in an MPRF radar mode is presented. Particular emphasis is placed on the structure and identification of the complex memory NLEQ inverse implemented. Finally, the results are displayed in section IV with the performance of both the memoryless and memory rich NLEQ techniques discussed in detail.

II. SYSTEM MODEL

The Volterra series is a powerful black-box behavioural model that can be used to characterise nonlinearities with memory [9]. For the purposes this paper we employ the baseband equivalent Volterra series (BBVS) model from [9] to describe the front-end nonlinearity driving the cross-modulation distortion in the radar receiver. We assume that the most dominant cross-channel nonlinear effects can be captured by terms up to the polynomial order of the series [6] which allows the full BBVS model to be pruned to the third order. The specific cross-channel form of the BBVS model used in this paper is presented in its discrete-time form below (1). The quadratic term has been omitted from (1) as we assume that the pulsed interference signal does not occupy half the carrier frequency and therefore the contributions from the second order term can be considered to be negligible.

$$
\tilde{y}[n] = \sum_{i_1=0}^{M_1} \tilde{h}_1[i_1] \tilde{x}_1[n-i_1] \\
+ \sum_{i_1=1}^{M_1} \sum_{i_2=i_1}^{M_1} \tilde{h}_{3,in}[i_1, i_2, i_3] \tilde{x}_1[n-i_1] \tilde{x}_1[n-i_2] \tilde{x}_1[n-i_3] \\
+ \sum_{i_1=0}^{M_1} \sum_{i_2=0}^{M_1} \tilde{h}_{3,out}[i_1, i_2, i_3] \tilde{x}_2[n-i_1] \tilde{x}_1[n-i_2] \tilde{x}_2[n-i_3]
$$

(1)

In (1); $\tilde{x}_1$ describes the desired baseband input signal centred on the carrier frequency, $\tilde{x}_2$ describes the interference baseband input signal centred on the interference frequency and $M_1$ denotes the memory length which defines the amount of memory in the system. In the derivation of the BBVS model the narrowband assumption is invoked which restricts the modelling capabilities of the BBVS model to a small bandwidth around the carrier frequency. This restriction does not affect the BBVS model’s ability to capture the cross-modulation scenario for a typical MPRF mode as the MPRF mode of operation is intrinsically narrowband. In (1) the variables $\tilde{h}_1[i_1]$, $\tilde{h}_{3,in}[i_1, i_2, i_3]$ and $\tilde{h}_{3,out}[i_1, i_2, i_3]$ represent the linear, third order in-channel and third order cross-channel Volterra kernels respectively. For the purposes of this study we assume that the Volterra kernels described above completely characterise the in-channel and cross-channel nonlinear behaviour of the radar receiver for a typical MPRF mode of operation. The $\tilde{h}_{3,in}[i_1, i_2, i_3]$ term in (1) is referred to as the cross-modulation term and captures the nonlinear effect we are interested in for this paper. We make the assumption that the radar operates in the linear regime when the out-of-band interference is not present in the scene. Thus, the contributions from the in-channel nonlinear term $\tilde{h}_{3,in}[i_1, i_2, i_3]$ have an insignificant effect on the output $\tilde{y}[n]$ as they sit below the level of the noise floor. The task for the digital NLEQ mitigation techniques is to therefore restore the linear performance of the sensor by removing the nonlinear distortion caused by the cross-modulation in term (1).

In order for the NLEQ techniques to have any chance of mitigating the cross-modulation effects observed in the radar output, they require information regarding the interference that caused the nonlinear distortion in the first place. The ability of modern phased array radars to form guard channels must
therefore be exploited to capture the raw interfering signal. It is reasonable to assume that an active guard channel could be back-off sufficiently so that the strong out-of-band interference driving the cross-modulation effect could be captured in a linear fashion. The linear guard channel measurement \( \hat{z}[n] \) is described by (2) below:
\[
\hat{z}[n] = (\hat{x}_2[n] + \hat{w}[n])
\]
where \( \hat{w}[n] \) denotes additive white gaussian noise. The full memory NLEQ inverse is described by the theory of \( p^{th} \) order inverses [10] and is stated for our cross-channel BBVS forward nonlinearity in (3). The basic principle behind the \( p^{th} \) order inverse theory states that if an inverse to the nonlinearity exists then the system can be inverted by connecting another nonlinear system in tandem [10].
\[
\hat{x}_1[n] = \sum_{i_1=0}^{M_2} \hat{g}_1[i_1] \hat{y}[n - i_1] + \sum_{i_1=0}^{M_2} \sum_{i_2=0}^{M_2} \hat{g}_3_{out}[i_1, i_2, i_3] \hat{z}[n - i_1][\hat{y}[n - i_2] \hat{z}^*[n - i_3]]
\]
In (3); \( \hat{x}_1 \) represents the estimate of the desired radar signal after the inverse function, \( M_2 \) denotes the memory length of the inverse, \( \hat{g}_1[i_1] \) and \( \hat{g}_3_{out}[i_1, i_2, i_3] \) represent the linear and third order cross-channel inverse kernel coefficients respectively. The process of identifying these inverse kernel coefficients will be discussed in detail in section III, however we can gain some insight by firstly studying the memoryless forward nonlinearity in (4),
\[
\hat{y}[n] = \hat{h}_1 \hat{x}_1[n] + \hat{h}_3_{out} \hat{x}_2[n] \hat{x}_1[n] \hat{x}_2^*[n]
\]
with the corresponding NLEQ inverse structure given by,
\[
\hat{x}_1[n] = \hat{g}_1 \hat{y}[n] + \hat{g}_3_{out} \hat{z}[n] \hat{y}[n] \hat{z}^*[n]
\]
substituting (4) into (5) and expanding out the brackets we find,
\[
\hat{x}_1[n] = \hat{g}_1 \hat{h}_1 \hat{x}_1[n] + \hat{g}_1 \hat{h}_3_{out} \hat{x}_2[n] \hat{x}_1[n] \hat{x}_2^*[n] + \hat{g}_3_{out} \hat{h}_1 \hat{z}[n] \hat{x}_1[n] \hat{z}^*[n] + \hat{g}_3_{out} \hat{h}_3_{out} \hat{z}[n] \hat{x}_2[n] \hat{z}[n] \hat{x}_1[n] \hat{z}^*[n]
\]
Now if for this simplified memoryless analysis we consider the case where there is no noise on the linear guard channel, \( \hat{z}[n] = \hat{x}_2[n] \), we can reduce (6) to the following form (7),
\[
\hat{x}_1[n] = \hat{g}_1 \hat{h}_1 \hat{x}_1[n] + (\hat{g}_1 \hat{h}_3_{out} + \hat{g}_3_{out} \hat{h}_1) |\hat{x}_2|^2 \hat{x}_1 + \hat{g}_3_{out} |\hat{x}_2|^2 \hat{x}_1 \hat{x}_2^*[n] \hat{z}^*[n]
\]
Choosing the inverse coefficients \( \hat{g}_1 \) and \( \hat{g}_3_{out} \) to minimise contributions from the unwanted third order cross-modulation term we are left with the final expression for the desired signal estimate \( \hat{x}_1 \) (8),
\[
\hat{x}_1[n] = \hat{x}_1[n] - \frac{\hat{h}_3_{out}}{\hat{h}_1} |\hat{x}_2|^2 \hat{x}_1 \hat{x}_2^*[n] \hat{z}^*[n]
\]
where the theoretical inverse coefficients are given by (9),
\[
\hat{g}_1 = \frac{1}{\hat{h}_1}; \quad \hat{g}_3_{out} = -\frac{\hat{h}_3_{out}}{\hat{h}_1}
\]
It is clear from this simplified memoryless analysis that the NLEQ technique, memoryless or otherwise, will only fully restore the performance of the radar to the linear case if the cubic term generated in the tandem inverse structure sits below the level of the noise floor. It is possible to perform a similar analysis as above to find the theoretical inverse coefficients for the full \( p^{th} \) order memory structure [10], however the calculations can become extremely complex and are unlikely to yield the best results when noise effects are re-introduced. Alternatively, an indirect approach can be adopted whereby the inverse coefficients are learnt directly in an offline calibration stage. This indirect learning approach is discussed in more detail in section III.

III. SIMULATION ARCHITECTURE

The principal aim of this paper is to study the performance of memoryless and memory rich NLEQ mitigation techniques against a forward receiver nonlinearity with memory. In order to test this, a comprehensive radar simulator was developed that simulated the radar scene for a typical MPRF mode before passing the raw time-domain signals through the nonlinear receiver model described by (1). The MPRF simulator operated entirely in the baseband domain and was configured to generate the classic cross-modulation effect described in section I [1], [2]. Once the radar scene and pulsed interferer characteristics were defined, the behaviour of the cross-modulation distortion generated in the simulation was governed entirely by the specific choice of nonlinear kernel coefficients. The gross level of the cross-modulation distortion produced in the radar receiver was set by the choice of nonlinear coefficients for the memoryless case; \( h_1, h_3_{out} \) and \( h_3_{in} \). The linear coefficient \( h_1 \) was set to the value of the RF receiver gain, while the third order in-channel coefficient \( h_3_{in} \) was subsequently chosen such that the nonlinear distortion effects sat just below the level of the noise floor when the interference was not present in the scene. The level of the memoryless cross-channel coefficient \( h_3_{out} \) was selected to maximise the cross-modulation distortion generated in the radar output while keeping the cubic term produced in the tandem inverse structure, (9), below the level of the noise floor. By defining the strength of the cross-channel coefficient \( \hat{h}_3_{out} \) in this way, we ensured that the cross-modulation distortion generated in the simulated receiver was within the operating region of the memoryless NLEQ digital mitigation technique.
In order to preserve this nonlinear scenario once memory effects were introduced, each BBVS kernel was normalised so that the respective power contributed from each group of
terms in the BBVS model (1) was equivalent to that from the memoryless case. To the author’s knowledge there is nothing published in the available literature on the cross-channel memory behaviour of a typical RF amplifier. The complex memory coefficients for the forward model were therefore selected from a random distribution in order to generate a challenging nonlinear scenario. Importantly however, the kernel normalisation ensured that the introduction of memory effects only ever subtly changed the structure and phase of the cross-modulation generated rather than the gross effect. The forward BBVS model was chosen to have a memory length of $M_1 = 2$ for the simulations in this paper as it is thought that the impulse response of the passband nonlinearity will decay quickly when sampled in the baseband domain. Furthermore, it is assumed that contribution from the memory polynomial coefficients dominate those from the cross-terms in the BBVS model as this is what is typically observed in the communications literature [5]. The amplitude of the cross-channel BBVS coefficients therefore followed a Chebyshev response on the main-diagonal\(^1\) with the associated cross-terms decaying exponentially in all directions. A range-Doppler (RD) plot showing the cross-modulation distortion generated from the simulated BBVS forward receiver model described above is displayed in Fig. 2.

![Simulated cross-modulation RD plot](image)

**Fig. 2:** Simulated cross-modulation RD plot for the BBVS forward receiver nonlinearity. Cross-modulation generates nonlinear repeats of the main-beam clutter $\{-3kHz \rightarrow -0.7kHz\}$ which masks the target at $\{-8kHz, -1.37km\}$.

Unlike the strictly memoryless case, we cannot assume that the NLEQ inverse structure will have a memory length equal to that of the forward nonlinearity. This complicates the process of constructing an effective NLEQ memory inverse as its form is dependent on the particular forward nonlinearity observed in the radar receiver. While there is no strict relationship between the inverse memory length $M_2$ and the forward memory length $M_1$ for nonlinear systems, linear system theory can provide some useful insights into the problem [11]. If we consider a finite impulse response (FIR) filter, it will only have an inverse structure with memory length equal to its own if the forward FIR filter coefficients are minimum phase. If the FIR filter coefficients are non-minimum phase, then the inverse structure must contain more memory than the forward filter and for best results the inverse coefficients should be identified against a delayed version of the target signal. While the concept of minimum phase does not extend to the nonlinear coefficients, it is fair to assume that the forward nonlinearity will not be well behaved as the front-end RF amplifier has not been designed to operate in its weakly nonlinear regime. For the purposes of this paper, we therefore choose to implement an NLEQ inverse with memory length double that of the forward nonlinearity, $M_2 = 2$. Additionally, we assumed that the forward nonlinear coefficients were constant for the full coherent integration time of the radar and therefore once identified the NLEQ inverse could simply be applied to every returned pulse in the burst.

Once the random nonlinear coefficients were selected for the forward BBVS model, (1), the simulation identified the NLEQ inverse coefficients, (3), by means of the indirect learning approach. For a real radar system this would take the form of an off-line calibration stage where the inverse coefficients that minimised the error between the estimated input signal and the known input test signal are selected. It is well understood that the nonlinear coefficients of the forward model can be identified by stimulating the RF nonlinearity with a deterministic random noise signal [9]. Probing the forward nonlinearity with noise ensures that the persistent excitation condition is satisfied and thus the complex memory behaviour of the device is captured. However, in the case of the inverse model the coefficients of the tandem nonlinearity must be identified by a signal that has already passed through the forward nonlinearity. It is therefore impossible to guarantee that the persistent excitation condition is being satisfied in the tandem nonlinearity and by extension, that all of the nonlinear memory effects are being captured by the identification procedure. Nevertheless, a deterministic random noise signal provides the best chance of successfully identifying the inverse coefficients directly which is why it is employed in the indirect learning approach.

For the simulations in this paper, the indirect learning approach was used to identify the NLEQ inverse coefficients against a target signal that had been delayed by half the inverse memory length $M_2$. The inverse coefficients were learnt by means of the linear least squares algorithm through a procedure very similar to that outlined in [9]. Additionally, the memoryless NLEQ inverse coefficients were also selected by the indirect learning approach rather than the theoretical method described in section II. Unlike the theoretical identification, the indirect learning approach will balance suppressing the third order distortion generated in the forward model as well as the cubic distortion generated in the inverse model when identifying the coefficients. This leads to better performance of the NLEQ algorithm. A RD plot showing the memory NLEQ mitigation technique applied to the BBVS cross-modulation scenario illustrated in Fig. 2 is displayed below in Fig. 3.

**IV. SIMULATION RESULTS**

In order to analyse the performance of the NLEQ mitigation technique for a typical MPRF radar mode in detail, statistical\(^1\)In the Volterra framework the memory polynomial coefficients lie on the main-diagonal of the nonlinear kernels.
PD analysis was performed in the RD domain on a single radar target. To form each RD plot the MPRF radar simulator generated 256 received pulses based on the pre-defined radar scene before passing them through the three radar receiver models: the memory BBVS nonlinearity (1), the corresponding memoryless BBTS nonlinearity (4) and the linear gain model. A further three data sets were generated by applying the memory rich NLEQ mitigation technique (3) to the BBVS receiver output and by applying the memoryless NLEQ mitigation technique (5) to both the BBVS and BBTS nonlinear receiver outputs respectively. In all simulations noise was added to the received input signals, $SNR = 25dB$, as well as to the output of the receiver, $SNR = 25dB$, and additionally to the linear guard channel, $SNR = 25dB$. Simulation results for a single burst showing the cross-modulation distortion generated from the BBVS nonlinear receiver and the subsequent correction by the memory rich NLEQ mitigation technique are displayed in Fig. 2 and Fig. 3 respectively. It is clear from Fig. 3 that the NLEQ mitigation technique has reduced the cross-modulation distortion observed in the simulated radar output. However, the performance of the NLEQ inverse can be analysed in much more detail by studying the results of the stochastic PD analysis. Where the results displayed in Fig. 5 and Fig. 6 study the effect of guard channel noise and delay on the performance of the memory rich NLEQ technique respectively.

In order to generate the PD curves shown in Fig. 4, 5 & 6 an adaptive cell-averaging constant false alarm rate (CA-CFAR) thresholding technique [12] that maintained a constant probability of false alarm rate of 0.2 was applied in all cases. In short, the adaptive CA-CFAR algorithm sets the detection threshold based on the statistical behaviour and power of the cells neighbouring the target cell. Thus, the simulated radar displays maximum sensitivity when its receiver operates in its linear regime as this is where the cells surrounding the target cell are strictly noise limited. When cross-modulation is observed in the radar output the distortion causes the detection threshold to increase and thus the sensitivity of the sensor to decrease. Therefore, if the NLEQ techniques are to be successful in restoring system sensitivity they must reduce the level of cross-modulation distortion observed in the RD domain without also removing the target. For this analysis, the sensitivity of the simulated radar was studied in the different nonlinear scenarios by varying the input SNR of the desired radar target. RD plots for 400 bursts were used to estimate the PD value at each data point in Fig. 4.

Examining the PD results in Fig. 4, we firstly observe that the curves for the memoryless and memory rich BBVS receiver outputs fall off at a much higher SNR than the desired linear receiver output curve. This reaffirms the fact that the cross-modulation distortion generated in the nonlinear receivers have reduced the overall sensitivity of the simulated radar. Interestingly, despite the strength of the memoryless and BBVS nonlinearities being matched in each received PRI the resultant PD curves do not lie on top of each other in Fig. 4. This rather surprising result is not due to any gross power variation between the cross-modulation generated in the RD domain, but rather reflects subtle phase effects introduced by the nonlinear memory terms that have altered the relationship between the cross-modulation and the target. In other words, the cross-modulation distortion generated by the complex BBVS nonlinearity has not just masked the target but rather fundamentally corrupted it.

The memoryless NLEQ inverse technique performs well when applied to the memoryless receiver output and completely restores the system sensitivity back to the linear case. However, when the memoryless NLEQ inverse is applied to the memory rich BBVS receiver output we observe that the mitigation technique fails to restore most of the sensitivity lost due the cross-modulation. This confirms the main result from [7] which showed that the subtle changes caused to the cross-
modulation by the introduction of nonlinear memory effects can decorrelate the memoryless mitigation technique. Showing this is also the case for the NLEQ inverse techniques as well as the technique studied in [7], proves that the decorrelation of the memoryless mitigation methods is not specific to the technique employed but is rather a universal phenomenon.

Finally, it is clear from Fig. 4 that introducing nonlinear memory effects to the NLEQ inverse model can drastically improve the effectiveness of the mitigation technique in the scenario where the radar receiver exhibits nonlinear memory effects. The additional memory terms in the inverse NLEQ structure are able to untangle the complex memory effects introduced by the forward BBVS nonlinearity and restore the system performance almost back to the linear case. Furthermore, the technique is shown to be robust against reasonable levels of guard channel noise and delays as shown by Fig. 5 & 6 respectively. This is an important result as it shows that the NLEQ inverse technique could provide a universal solution to the nonlinear receiver problem in radar that is both mode independent and capable of correcting complex nonlinear memory effects. However, to fully understand the robustness of these NLEQ mitigation techniques in a real radar system more work to study the nonlinear memory behaviour of the radar receiver must be conducted. It was clear from further simulations with slightly different BBVS coefficients that the effectiveness of the memory NLEQ inverse technique was dependent on the complexity of the forward nonlinear transfer function.

In this paper, we presented a study into the effectiveness of the NLEQ inverse mitigation technique in correcting cross-modulation effects in radar. The paper focused on nonlinear memory effects and presented how the classic NLEQ inverse technique could be extended to compensate for forward nonlinearities with memory. By simulating the cross-modulation effect for a typical MPRF radar mode it was shown that the memoryless NLEQ inverse was effective in restoring system sensitivity for the memoryless case but failed when nonlinear memory effects were introduced to the forward receiver model. Importantly, it was shown that these nonlinear memory effects could be compensated for if memory is introduced to the inverse NLEQ structure. If nonlinear memory effects prove to be significant in the modern radar receiver then the inverse NLEQ technique provides a simple and effective method for mitigating the harmful cross-modulation distortion generated.

V. CONCLUSION

Fig. 5: Guard channel noise PD results for the memory rich NLEQ technique.

In this paper, we presented a study into the effectiveness of the NLEQ inverse mitigation technique in correcting cross-modulation effects in radar. The paper focused on nonlinear memory effects and presented how the classic NLEQ inverse technique could be extended to compensate for forward nonlinearities with memory. By simulating the cross-modulation effect for a typical MPRF radar mode it was shown that the memoryless NLEQ inverse was effective in restoring system sensitivity for the memoryless case but failed when nonlinear memory effects were introduced to the forward receiver model. Importantly, it was shown that these nonlinear memory effects could be compensated for if memory is introduced to the inverse NLEQ structure. If nonlinear memory effects prove to be significant in the modern radar receiver then the inverse NLEQ technique provides a simple and effective method for mitigating the harmful cross-modulation distortion generated.

ACKNOWLEDGMENT

This work was supported by the UK Engineering and Physical Sciences Research Council (EPSRC) [Grant No. EPSRC EP/N509644/1], Leonardo through the CASE Studentship Scheme and the Royal Commission for the Exhibition of 1851.

REFERENCES

Compressive Sensing Technique for Mitigating Nonlinear Memory Effects in Radar Receivers

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Receiver nonlinearities pose a serious risk to the functionality of modern radars as they can compromise the sensor’s immunity to interfering signals. With the radio frequency (RF) spectrum becoming increasingly crowded, it is now more important than ever that the sensor can maintain system performance when exposed to mutual interference. In this article, we present a nonlinear compressive sensing (NCS) solution, which, unlike the standard nonlinear equalization (NLEQ) techniques, is designed around the forward nonlinearity rather than the inverse. Importantly, in this article, the NCS theory is extended to include nonlinear memory. Furthermore, a radar-specific formalization is derived, which allows the nonlinear optimization to exploit the unique properties of pulsed-Doppler radar processing. As a result, the NCS solution can successfully restore system sensitivity back to the linear case when in-band interference drives the radar receiver into its nonlinear regime. Additionally, it is shown that the technique can consistently mitigate complex nonlinear memory effects generated in the RF receiver. This is a significant result as it proves that forward modeling techniques are a viable alternative to NLEQ. This is of particular importance to radar systems as they provide a far more explicit formalization to mitigate nonlinear memory effects.

I. INTRODUCTION

The multisensor mutual interference problem is well known in the radar community with the development of an intelligent radar network being the most widely accepted proposed solution [1]. The future of modern radars will undoubtedly involve a network of cognitive sensors that are frequency agile so that mutual interference is minimized [2]. However, some self-sufficient protection operating within each sensor is required if performance is to be maintained even in the worst interference scenario. In the case where an interferer is present in the radar’s desired channel, the linearity of the radio frequency (RF) amplifiers in the receiver front-end effectively defines the system’s immunity to the unwanted signal [3]–[5]. As long as the active components in the RF receiver remain in the linear regime, while the desired signal and the interference signal pass through the chain simultaneously, then the principle of superposition holds, and the task of removing the interference effects can be seen as a linear signal separation problem [6].

In an effort to save costs, modern radar systems are being designed from commercial-off-the-shelf RF components, which have significantly poorer linearity specifications than their bespoke counterparts [5], [7], [8]. These modern sensors must, therefore, operate at the top of their linear regions in order to achieve acceptable performance. Crucially, this leaves no room to back-off the receiver in the presence of interference. Operating at the top of the linear region poses a serious risk to the functionality of these sensors as interference from other nearby radars can force the RF amplifiers in its receiver into a nonlinear state. With the use of consumer radars set to increase dramatically over the next decade, modern mass-produced radar systems must evolve so that they can operate effectively beyond a purely linear regime.

The diagram in Fig. 1 illustrates the potential nonlinear effects an in-band interferer can cause if a radar’s receiver linearity cannot be preserved. In contrast to the linear receiver, where the interferer and the desired signal are amplified separately, the nonlinearity causes the desired signal to become distorted as well as generating spurious harmonics across the spectrum [3]–[5], [10]–[12]. This article presents a novel digital signal processing solution that is capable of mitigating the in-channel nonlinear distortion caused by the presence of an in-band interferer. It is assumed for the purposes of this article that any spurious harmonics generated by the nonlinearity are successfully filtered out further down the RF receiver chain and are, therefore, omitted from this study.

While little work has been published on the mitigation of nonlinear receiver effects in the radar literature [4], [5], [10], [11], [13]–[15], it is an area of active research...
The fact that the observations are intrinsically nonlinear means that traditional transform techniques cannot be used to infer the various signal components. Sparse signal processing offers a forward modeling solution. By setting the problem up as a sparse nonlinear optimization problem, the linear input signal can be estimated from the measured nonlinear output signal. Like most sparse signal processing problems, iterative solutions that are signal dependent must be sought. In the case of radar, this signal dependence results in the technique being mode specific. Nonlinear sparse signal processing is in its infancy with only one major article published on the topic [21]. In [21], Blumensath showed that a sparse or structured signal observed through a nonlinear function could be recovered by the iterative hard thresholding (IHT) algorithm with constraints similar to those required in the linear setting. Importantly, Blumensath’s proof holds under the condition that the system is not too nonlinear, and therefore, the error introduced in the linearization is not too large.

Since the article presented in [21] was published in 2013, Blumensath’s IHT algorithm for nonlinear compressive sensing (NCS) has been applied to a wide variety of different applications. Chen et al. [22] apply the technique to a communications system for the first time with the aim of digitally correcting the nonlinear distortion from a front-end power amplifier leading to improved efficiency. The nonlinearity was described by means of a Rapp model, and the NCS algorithm was shown to converge for both simulated and experimental data. This is a significant result as it proves that the NCS algorithm can be successfully employed to digitally mitigate nonlinear effects in a communication-based system. While Chen et al. employ Blumensath’s NCS technique to mitigate a transmit-side nonlinearity, Zhu et al. develop the technique further in [23] by applying it to the receive side of the system. The NCS technique in the article presented in [23] focuses on a front-end low-noise amplifier (LNA) driven into its weakly nonlinear regime by the presence of an unwanted interferer in the spectrum. The LNA was modeled by means of a cubic order Taylor series, and Blumensath’s IHT technique was tested alongside a novel gradient pursuit method. Crucially, Zhu et al. prove in [23] that the linear input signal can be recovered by the NCS technique if the LNA is not too nonlinear and the Taylor series model parameters are known. While Chen et al. [22] and Zhu et al. [23] provide some encouragement, Blumensath’s NCS technique [21] must be developed specifically to exploit the unique behavior of radar systems if it is to be successful in mitigating nonlinear effects generated in the radar receiver.

In this article, the NCS algorithm is extended and applied to mitigate nonlinear effects in the radar receiver caused by the presence of an in-band interferer. In addition to introducing this novel digital mitigation technique to the field of radar, the following key contributions are presented.

1) Input and output noise are addressed as part of the complex baseband (BB) nonlinear model (see Section III-B).

Fig. 1. Illustration of nonlinear distortion effects caused by an in-band interferer. Figure adapted from the article presented in [9].
2) The forward nonlinear function in the NCS algorithm is extended to include nonlinear memory effects (see Section III-C).

3) Unlike previous work that deals with the purely real problem, the NCS algorithm is extended to the complex case (see Section III-D).

4) A unique compressive sensing (CS) problem that exploits signal sparsity in slow time and tackles a complex BB nonlinearity with memory in fast time is addressed (see Section III-E).

5) The NCS algorithm is shown to mitigate nonlinear memory effects in the radar receiver for the first time (see Section V).

6) The NCS algorithm is tested against real radar data for the first time (see Section VI).

The rest of this article is organized as follows. In Section II, the interference scenario and the system model used to describe the nonlinear RF receiver are presented. In Section III, the novel NCS algorithm is derived in full before the simulation used to test the performance of the technique is discussed in Section IV. Simulated and experimental results are displayed in Sections V and VI respectively, with the performance of the NCS algorithm discussed in detail. Finally, Section VII concludes this article.

II. SYSTEM MODEL

The Volterra series is a powerful black-box behavioral model capable of modeling complex nonlinear memory effects. It is often described as a Taylor series with memory and has a unique linear-in-the-parameters formalization that makes it ideal for modeling weakly nonlinear RF amplifiers [8]. Mathematically, it is described as a sum of multidimensional convolutions, and in this article, we employed it in its complex BB form [24] for computational efficiency. Importantly, when deriving the BB equivalent form of the Volterra series (BBVS) [9], [11], [24], the narrowband assumption is invoked, which restricts its modeling capabilities to a small bandwidth around the carrier frequency. Referring to the nonlinear scenario illustrated in Fig. 1, this means that only the in-channel distortion effects can be captured by the model. We must, therefore, assume that any spurious harmonics generated by the passband nonlinearity are successfully filtered out further down the RF receive chain. The full form of the BBVS model employed in this article is described as follows:

\[
y_{[n]} = \sum_{p=0}^{P} \left( \sum_{i_1=0}^{L-1} \cdots \sum_{i_{p+1}=0}^{L-1} \right) \prod_{x=1}^{P} k_{[i_{p+1}, \ldots, i_{2p+1}]} \prod_{d=p+1}^{2p+1} k_{(n-i_d)} \quad (1)
\]

where \(y_{[n]}\) denotes the current BB output sample, \(x_{[n]}\) represents the current BB input sample, and \(P\) specifies the nonlinear model order. The BBVS model [9], [11], [24] described above takes a purely odd-order form as the distortion generated from the even-order terms falls outside the desired channel [24] and can, therefore, be ignored. Importantly, each power term in (1) describes a sum over Volterra taps. We use the phrase Volterra tap here to describe a single term from the multidimensional sum over \([i_1, \ldots, i_{2p+1}]\). In essence, the memory length \(L\) defines the amount of memory in the system with the terms \([i_1, \ldots, i_{2p+1}]\) defining each subsequent component’s BB sample delay. The specific behavior of the nonlinear model is defined by a set of nonlinear coefficients \(k_{[i_1, \ldots, i_{2p+1}]}\) referred to as the Volterra kernel coefficients in the case of the BBVS model. Importantly, in the memoryless case where \(L = 1\), the BBVS model reproduces the baseband Taylor series (BBTS) model exactly. The complexity of the BBVS model grows exponentially with nonlinear order \(P\) and memory length \(L\); however, even for small values of \(P\) and \(L\), the BBVS model can capture complex nonlinear memory effects. For the purposes of this article, we simplify the entire radar RF receiver to be a single amplifier stage and choose to model its behavior using one of the nonlinear black-box behavioral models discussed above.

As mentioned previously, Fig. 1 illustrates the scenario of interest, where the presence of an in-band interferer drives the radar receiver into the weakly nonlinear region causing the desired signal to be distorted. We, therefore, describe the BB input signal \(x(t)\) as consisting of the desired radar signal \(v(t)\) and the in-band interference signal \(s(t)\) as displayed in (2) below:

\[
x(t) = v(t) + s(t) \quad (2)
\]

Importantly, the desired radar signal \(v(t)\) consists of a linear combination of target, clutter, and noise signals, while we choose signal \(s(t)\) to be a continuous-wave (CW) interferer. For simplicity, we assume that the radar transmits a single-tone pulsed signal, and the CW interferer is ever present in the received pulse repetition interval (PRI). The discrete-time input for the black-box receiver model (1) is, therefore, formed by sampling the continuous-time signal \(x(t)\) through the Nyquist–Shannon sampling theorem [25] and stacking the fast-time signal samples \(x_q\) into a column vector \(x\) of length \(N\). Each received PRI signal is indexed with the subscript \(x_q\), for \(q = 1, \ldots, Q\), and is concatenated into a received signal matrix of size \(N \times Q\) labeled \(X = [x_1, x_2, \ldots, x_Q]\) in what is referred to as the slow-time dimension. The corresponding output matrix \(Y\) is constructed similarly by concatenating each subsequent PRI output \(y_q\) from the black-box receiver model into an \(N \times Q\) matrix as follows: \(Y = [y_1, y_2, \ldots, y_Q]\). The nonlinearity is, therefore, applied down the columns of matrix \(X\) to produce matrix \(Y\) acting in what is referred to as the fast-time dimension.\(^5\)

\(^5\)The coefficients for the BBTS model are distinguished from those for the full BBVS model by omitting the delay-tap indices \([i_1, \ldots, i_{2p+1}]\) from the notation.

\(^6\)A single matrix element is denoted by a nonbold symbol with dual subscripts, \(x_{[i_1, \ldots, i_{2p+1}]}\). Furthermore, specific rows and columns of a matrix are denoted by a bold lowercase symbol with a single subscript that indicates the matrix dimension that has been pulled out. Importantly, we use a tilde accent to distinguish row vectors from column vectors in the notation. Thus, \(x_{\tilde{n}}\) denotes a vector corresponding to row \(n\) of matrix \(X\) of size \(1 \times Q\) and \(x_{\tilde{q}}\) denotes a vector corresponding to column \(q\) of matrix \(X\) of size \(N \times 1\).
Fig. 2. Example RD plot for the linear in-band interference scenario. The target [−6.2 kHz, 1.3 km] is clearly visible in the noise limited region away from the clutter at −3.0 kHz → −0.7 kHz and the CW interferer at 11.2 kHz Doppler.

In this article, we focus on airborne pulsed Doppler radar and, more specifically, the medium pulse repetition frequency (MPRF) mode [6], [25], [26] for several key reasons:

1) The MPRF mode is one of the most popular modes of operation for modern radar as it provides both range and velocity measurements of the scene.

2) In a typical MPRF mode, the radar is tasked with detecting low-signal-to-noise ratio (SNR) targets in the presence of a strong clutter signal, which can leave it susceptible to nonlinear effects.

3) The MPRF mode is intrinsically narrowband, which as mentioned previously is the main assumption that must be satisfied when employing BB behavioral models to simulate the RF receiver.

4) The unique characteristic of the clutter in the MPRF mode makes it well suited to the NCS mitigation technique.

Typically, in an MPRF radar mode, pulse-Doppler processing is employed to detect targets in the scene. In standard pulse-Doppler radar processing [6], [25], a matched filter is applied down the columns of the Y to convert the raw fast-time signals into range before a discrete-time Fourier transform is applied across the received PRI to reveal the Doppler behavior of the artifacts in the scene. By performing this simple processing on the received data, the weak targets can be separated from the clutter in Doppler and, due to their coherent nature, can also be pulled above the noise floor.

It is in this range-Doppler (RD) space that target detection analysis is typically performed. An example RD plot for the in-band interference scenario in the case where the RF receiver operates in its linear regime is displayed in Fig. 2.

In the case where an interferer drives a nonlinearity in the RF receiver, the resulting distortion generated in the RD domain can have devastating effects on the radar’s target detection performance. The most common in-band nonlinear distortion effect observed in the RD space involves the broadening of the main-beam clutter [14]. This clutter broadening effect can result in weak targets being masked by the nonlinear distortion. Importantly, the introduction of nonlinear memory effects is not expected to change the gross nonlinear effect described above but rather subtly change the structure and phase of the nonlinear distortion observed. While these subtle differences might not be enough to alter the target detection performance from the memoryless case, they can decorrelate the memoryless mitigation techniques [11].

III. NONLINEAR COMPRESSIVE SENSING

A. Signal Model

In this section, we derive the novel NCS algorithm in full. We start by considering the input signal vector \( x_q \) for the \( q \)th PRI, which consists of a linear combination of received pulses from both the radar scene and the interference source. As mentioned previously, \( x_q \) denotes the signal corresponding to column \( q \) of the matrix \( X \) and contains a series of fast-time signal samples \( n \). In standard pulse-Doppler radar processing, a matched filter is applied down each received PRI in order to convert fast time, \( n \), into range, \( r \). This is illustrated formally for a single PRI signal as

\[
x_q = \sum_{r=1}^{R} \psi^\dagger_r x_r = \Psi^\dagger_q x_q
\]

where \( \psi^\dagger \) represents column vector \( r \) of the fast-time filter matrix \( \Psi \) size \( R \times R \), which can take the form of a Toeplitz convolution matrix, and \( x^\dagger_q \) denotes the range converted signal vector for PRI \( q \) size \( R \times 1 \). Now that the received PRI signals have been converted from fast time into range, their underlying structure can be studied in the slow-time dimension, which acts along the rows of matrix \( X^\dagger \), where \( X^\dagger = [x^\dagger_1, x^\dagger_2, \ldots, x^\dagger_Q] \). Let us, therefore, examine the slow-time signal vector for a single range gate \( r \). The observed signal vector \( x^\dagger_r \) can be represented as a linear combination of \( J \) frequency vectors \( [\phi^\dagger_{(r,j)}]^T \), where each frequency vector is weighted by the corresponding complex coefficient \( \theta_{(r,j)} \). Therefore, the row vector \( \hat{\theta}_r \), where \( \hat{\theta}_r = [\theta_{(r,1)}, \theta_{(r,2)}, \ldots, \theta_{(r,J)}] \), contains a set of Fourier coefficients that describes the slow-time behavior of the scatterers at a particular range gate \( r \). This is displayed formally in as

\[
x^\dagger_r = \frac{1}{\sqrt{J}} \sum_{j=1}^{J} \theta_{(r,j)} \hat{\phi}_j = \hat{\theta}_r \Phi
\]

where \( \Phi \) is a frequency basis matrix of size \( J \times Q \) comprising the row vectors \( \hat{\phi}_j \). By combining (3) and (4), we can define the 2-D linear signal model (5), which effectively describes the simple radar processing operation applied in a pulse-Doppler radar. In (5), the Fourier vectors \( \hat{\theta}_r \) have been concatenated in range to form the matrix \( \Theta \) of size \( R \times J \), e.g., \( \Theta = [\hat{\theta}_1^T, \hat{\theta}_2^T, \ldots, \hat{\theta}_R^T]^T \), where \( ^T \) denotes the matrix transpose operation.

\[
X = \Psi \Phi \Theta
\]

Henceforth, the matrices in (5) are referred to as follows: \( \Psi \) represents the range projection matrix that maps range to fast time, \( \Phi \) describes the slow-time Fourier basis matrix,
and $\Theta$ denotes the corresponding signal model regressors. Importantly, in this article, we assume that the radar transmits a single-tone waveform resulting in the matched filter operation being redundant as fast time is directly equivalent to range in our received signal matrix. This assumption simplifies the structure of the range projection matrix $\Psi$ to be an identity matrix of size $N \times R$, where $N = R$. If the radar were to transmit a more complex waveform, then the range projection matrix $\Psi$ would need to be updated to represent the new mapping from range to fast time. In contrast, the Fourier basis matrix has a size $J \times Q$ and is constructed so that each column of the orthonormal matrix $\Phi$ represents an independent frequency vector. When dealing with the radar signal model (5), it is often convenient to adopt the elementwise formalization stated as follows:

$$x_{n,q} = \sum_{j} \left\{ \sum_{r} \psi_{n,r} \theta_{r,j} \right\} \phi_{j,q}$$ (6)

In (6), the subscripts $n, q, r,$ and $j$ are used to denote the elements of the corresponding matrix. Additionally, the subscripts are used to label the dimensions of each specific matrix with $n$ representing the fast-time dimension, $q$ indicating the slow-time dimension, $r$ denoting the range dimension, and $j$ representing the Doppler dimension. Importantly, it is clear from (6) that the model regressors themselves describe the entire RD detection space.

Finally, we define the nonlinear observation model (7), which relates the measured nonlinear output matrix $Y$ to the linear regressor matrix $\Theta$ by combining the radar signal model (5) with the specific black-box nonlinear receiver model denoted by function $\Gamma(\cdot)$, as follows:

$$Y = \Gamma(X) = \Gamma(\Psi \Theta \Phi + U)$$ (7)

By integrating this nonlinear observation model into the radar processing, the problem of recovering the linear input signal $X$ is equivalent to estimating the model regressor matrix $\Theta$. Importantly, this allows prior knowledge of the radar detection space to be incorporated into the estimation process. The problem can, therefore, be set up as a sparse nonlinear optimization one, where the linear regressor matrix $\Theta$ can be estimated from the measured nonlinear output matrix $Y$. Like most solutions to sparse signal processing problems, the NCS algorithm is signal dependent and has an iterative formalization. The algorithm’s sparsity constraint and resultant signal dependence are discussed in detail at the end of this section once the iterative step has been derived in full. We start, however, by considering the effect of introducing noise to the nonlinear observation model in the following section.

B. Dealing With Noise

When dealing with noise in the nonlinear observation model, we consider two independent noise sources: input noise $U$ and output noise $W$. The former is additive noise present at the input of the receiver. The latter is additive noise present at the output of the receiver. For linear systems, these could be used interchangeably [25], [26]; however, due the nonlinearity in the observation model, we must start by treating these noise sources separately. Therefore, $U$ might be thought of as representing the background noise from the radar scene, with $W$ representing the thermal noise in the RF receiver. If we assume that the black-box behavioral model (1) accurately captures the forward nonlinearity, then the output noise $W$ can be considered to be genuinely additive and uncorrelated from sample to sample. It is, therefore, straightforward to address by simply tagging the $N \times Q$ noise matrix $W$ to the nonlinear output matrix $Y$.

It is important to note that the output noise term $W$ may also be viewed as accounting for unmodeled terms in the in-band nonlinear behavioral model (1), i.e., terms beyond the $P$th one, in which case the output noise will be signal dependent and most likely correlated from sample to sample in the fast-time dimension $n$.

The input noise is more challenging to deal with in the case of the nonlinear observation model, as unlike the linear case, we cannot assume that each output sample is a linear combination of signal plus noise. To study this further, we follow the input noise through the memoryless forward nonlinear model described by (1), where $L = 1$. The full matrix form of the linear signal model with input noise is shown as follows. The input noise is represented by $U$ and is a matrix of size $N \times Q$.

$$X = \Psi \Theta \Phi + U$$ (8)

For mathematical simplicity, we continue with an elementwise formalization, where each dimension is labeled using the previously defined subscripts. Thus, the elementwise signal model takes the following form:

$$x_{n,q} = \sum_{j} \left\{ \sum_{r} \psi_{n,r} \theta_{r,j} \right\} \phi_{j,q} + u_{n,q}$$ (9)

The memoryless observation model is formed by combining the elementwise formalization of (7) with the BBTS black-box behavioral model from (1), where $L = 1$. Thus, the elementwise form of the memoryless nonlinear observation model is as follows:

$$y_{n,q} = \sum_{p=0}^{P} h_{2p+1} |x_{n,q}|^{2p} x_{n,q} + w_{n,q}$$ (10)

Substituting (9) into (10) leads to

$$y_{n,q} = \sum_{p=0}^{P} \sum_{j} \left\{ \sum_{r} \psi_{n,r} \theta_{r,j} \right\} \phi_{j,q} + u_{n,q} |^{2p} \sum_{j} \left\{ \sum_{r} \psi_{n,r} \theta_{r,j} \right\} \phi_{j,q} + u_{n,q} + w_{n,q}$$ (11)

For what might be called the signal limited case, $|\sum_{j} (\sum_{r} \psi_{n,r} \theta_{r,j}) \phi_{j,q}| >> |u_{n,q}|$, we find that...
For each range gate regressor as in (6), the associated memoryless nonlinear observation model becomes

\[
y_{n,q} = \sum_{p=0}^{P} h_{2p+1} \sum_{j} \left( \sum_{r} \Psi_{n,r} \theta_{r,j} \right) \Phi_{j,q} + u_{n,q} + w_{n,q} \tag{13}
\]

and expanding out the brackets, we obtain

\[
y_{n,q} = \sum_{p=0}^{P} h_{2p+1} \left( \sum_{j} \left( \sum_{r} \Psi_{n,r} \theta_{r,j} \right) \Phi_{j,q} \right) x_{n,q}^{2p} + u_{n,q} + w_{n,q} \tag{14}
\]

If we now redefine the signal model to remove the noise term \(u_{n,q}\) as in (6), the associated memoryless nonlinear observation model becomes

\[
y_{n,q} = \sum_{p=0}^{P} h_{2p+1} \sum_{j} \left( \sum_{r} \Psi_{n,r} \theta_{r,j} \right) x_{n,q}^{2p} + w_{n,q} \tag{15}
\]

Now that the linear signal model has been defined in (5), we derive the IHT algorithm’s iterative step in full. As with the input noise analysis above, we employ an elementwise formalization for simplicity. By combining the BBVS expression (1) with the nonlinear observation model (7), we rewrite the nonlinear output signal \(y_{n,q}\) in terms of the elementwise nonlinear function \(F_{n,p,q,(i_1,...,i_{2p+1})}\)

\[
y_{n,q} = \sum_{p=0}^{P} \left( \sum_{i_{2p+1}=0}^{L-1} \ldots \sum_{i_1=0}^{L-1} \right) h_{2p+1,1}(i_1,...,i_{2p+1}) F_{n,p,q,(i_1,...,i_{2p+1})} + u_{n,q} \tag{16}
\]

where \(F_{n,p,q,(i_1,...,i_{2p+1})}\) takes the generalized BBVS form

\[
F_{n,p,q,(i_1,...,i_{2p+1})} = \prod_{s=1}^{P} x_{n-i_{2s-1}}^{2p+1} \tag{17}
\]

with the corresponding least squares (LS) cost function, \(C\), defined in the usual manner, as follows:

\[
C = \sum_{q} \sum_{n} \epsilon_{n,q}^* \epsilon_{n,q} \tag{18}
\]

By seeking an iterative gradient-based solution, the above cost function can be minimized with respect to the unknown linear signal model parameters \(\Theta\) subject to any constraints on \(\Theta\), e.g., sparsity. Unlike Blumensath’s IHT algorithm for NCS [21], which dealt with the strictly real case, the cost function in (18) has been derived in the BB domain and is, therefore, complex in nature. This is potentially problematic for the IHT algorithm as the usual mathematical concept of derivation to obtain a gradient is only defined for real numbers [27],[28]. An operational solution for the optimization of real cost functions as a function of a complex vector was developed by Brandwood [29] and is applied here to deduce the gradient step. This will require a \(1 \times J\) row vector of cofactors for each range gate regressor vector \(\theta_r\)

\[
\frac{dC}{d\theta_r} = \left[ \frac{\partial C}{\partial \theta_{r,1}} \frac{\partial C}{\partial \theta_{r,2}} \ldots \frac{\partial C}{\partial \theta_{r,J}} \right] \tag{19}
\]

and associated gradient vector

\[
\nabla C(\theta_r) = \left[ \frac{dC}{d\theta_r} \right]^* \tag{20}
\]

Two equations define the gradient step in the iterative learning procedure

\[
\theta_r^{(k+1)} = \theta_r^{(k)} - \mu \nabla C(\theta_r^{(k)}) \tag{21}
\]
Algorithm 1: IHT Algorithm for NCS. The algorithm runs for a maximum of $k_{\text{max}}$ iterations or at least until it has converged to a minimum-error solution; see line 10. The model regressors $\Theta$ are initialized during the first iteration (see line 3) before being updated in all subsequent iterations. The error matrix $E$ is calculated in line 6 from the current estimate of the input signal matrix $X$ before being used to compute the LS error in line 7. The NCS gradient is calculated in lines 13–18 before the radar constraint $P_A(.)$ is applied in line 20.

Input: $k_{\text{max}}, \Psi, Y, \Phi, \Gamma, R, \mu, P_A$

1: for $k = 1 : k_{\text{max}}$ do
2: \hspace{1em} if $(k = 1)$ then
3: \hspace{2em} $\Theta = \Psi^H Y \Phi^H$ \hspace{0.5em} Initialization
4: \hspace{1em} else
5: \hspace{2em} $X = \Psi \Theta \Phi$
6: \hspace{2em} $E = Y - \Gamma(X)$ \hspace{0.5em} Calculate error matrix
7: \hspace{2em} $C = \text{vec}(E)^H \text{vec}(E)$ \hspace{0.5em} Compute LS error
8: \hspace{2em} if $(k > 2)$ then
9: \hspace{3em} if $(C(k) > C(k-1))$ then
10: \hspace{4em} break \hspace{0.5em} Convergence condition
11: \hspace{3em} end if
12: \hspace{2em} end if
13: \hspace{2em} for $r = 1 : R$ do
14: \hspace{3em} \hspace{0.5em} $\frac{dC}{d\theta} = \begin{bmatrix} \frac{dC}{d\theta_1} & \frac{dC}{d\theta_2} & \cdots & \frac{dC}{d\theta_R} \end{bmatrix}$
15: \hspace{3em} $\nabla_C(\theta_r) = \left( \frac{dC}{d\theta_r} \right)^*$ \hspace{0.5em} Calculate NCS gradient
16: \hspace{3em} end for
17: \hspace{2em} $\Theta = \begin{bmatrix} \tilde{\theta}_1^T & \tilde{\theta}_2^T & \cdots & \tilde{\theta}_R^T \end{bmatrix}^T$
18: \hspace{2em} end if
19: $\Theta = P_A(\Theta)$ \hspace{0.5em} Apply radar constraint
20: end for

Output: Estimated $\Theta$

and a constraint step, which is applied to the entire $R \times J$ regressor matrix $\Theta^{(k)}$

$$\Theta^{(k)} = \begin{bmatrix} \tilde{\theta}_1^{(k)} & \tilde{\theta}_2^{(k)} & \cdots & \tilde{\theta}_R^{(k)} \end{bmatrix}^T$$

$$\Theta^{(k)} \leftarrow P_A(\Theta^{(k)})$$

(22)

In (21) and (22), $P_A(.)$ denotes the projection operator that enforces the radar sparsity constraint, $\mu$ represents the step parameter, and $k$ denotes the IHT iteration number. These steps are applied repeatedly until the LS error $C$ is suitably small at which point the algorithm can be considered to have converged. Obviously, in the full formalization of the IHT algorithm for NCS, the regressor matrix $\Theta^{(k)}$ must be updated as part of a loop over range gate $r$. This is illustrated in Algorithm 1, where the full algorithm is presented. Importantly, the function vec(.) on line 7 of Algorithm 1 denotes the vectorization operation, which stacks matrices in a columnwise manner and $^T$ denotes the matrix Hermitian/conjugate transpose.

Blumensath showed in [21] that the IHT algorithm can theoretically recover a sparse signal observed through a nonlinear function under similar assumptions to those made in the linear CS case. However, Blumensath assumes that the nonlinear observation has a proper linear approximation, which effectively projects it back into a linear function. In [22] and [23], the NCS algorithm is extended to exploit knowledge of the nonlinear function through which the input signal is observed. While the nonlinear functions in [22] and [23] are more complicated than that in [21], both are still strictly real and have a purely memoryless formalization. For the NCS algorithm to be extended to the complex BBVS case, we must consider the complex gradient calculation more carefully.

D. Calculating the Gradient

In order for the IHT algorithm to converge to the correct solution, the LS cost function $C$ must be minimized with respect to the unknown model regressor matrix $\Theta$ subject to the chosen radar constraint $P_A(.)$. The gradient step (21), therefore, plays a crucial role in the iterative learning procedure as it constantly steers the algorithm toward the minimum of the cost function. The calculation of the gradient is non-trivial in the NCS algorithm as the desired signal has been observed through a complex nonlinear function. Let us start by considering a single term $\frac{\partial C}{\partial \theta_{[r,j]}}$ from the vector of cofactors described by (19). As with the previous sections in this article, we adopt an elementwise formalization for mathematical simplicity.

$$\frac{\partial C}{\partial \theta_{[r,j]}} = \sum_q \sum_n \left[ \frac{\partial}{\partial \theta_{[r,j]}} \left( e_{[r,q],n} e_{[n,q]}^* \right) \right]$$

Substituting (17) into (23) for $e_{[n,q]}$, we find

$$\frac{\partial C}{\partial \theta_{[r,j]}} = \sum_q \sum_n \left[ e_{[r,q],n}^* \sum_{p=0}^{L-1} \sum_{i_p=0}^{L-1} h_{[2p+1],[i_1,\ldots,i_{p+1}]} \right]$$

$$+ e_{[n,q]} \sum_{p=0}^{L-1} \sum_{i_p=0}^{L-1} h_{[2p+1],[i_1,\ldots,i_{p+1}]}$$

(24)
By applying the chain rule of differentiation to (24), we can expand the expression to the following form:

\[
\frac{\partial C}{\partial \theta_{(r,j)}} = \sum_{q} e_{[q,n]} \sum_{p=0}^{P} \left\{ \sum_{i=0}^{L-1} \sum_{t_{2p+1}}^{L-1} h_{2p+1,i} \right\} \left\{ - \sum_{i=0}^{L-1} \sum_{t_{2p+1}}^{L-1} \frac{\partial F_{[q,n,p,i],\theta_{(r,j)}}}{\partial \theta_{(r,j)}} \frac{\partial F_{[q,n,p,i],\theta_{(r,j)}}}{\partial \theta_{(r,j)}} \right\} + \frac{\partial F_{[q,n,p,i],\theta_{(r,j)}}}{\partial \theta_{(r,j)}} \frac{\partial F_{[q,n,p,i],\theta_{(r,j)}}}{\partial \theta_{(r,j)}}
\]

(25)

As discussed previously, the derivative of a complex function of a complex variable does not exist in general. However, using the theory developed in [29], it can be defined to give a gradient for a real function \( C \) of a complex variable \( \theta_{(r,j)} \) if \( \theta_{(r,j)} \) is treated as a constant and \( \frac{\partial \theta_{(r,j)}}{\partial \theta_{(r,j)}} \equiv 0 \). Therefore, in the context of the NCS algorithm, we have

\[
\frac{\partial C}{\partial \theta_{(r,j)}} = -\sum_{q} e_{[q,n]} \sum_{p=0}^{P} \left\{ \sum_{i=0}^{L-1} \sum_{t_{2p+1}}^{L-1} h_{2p+1,i} \right\} \left\{ - \sum_{i=0}^{L-1} \sum_{t_{2p+1}}^{L-1} \frac{\partial F_{[q,n,p,i],\theta_{(r,j)}}}{\partial \theta_{(r,j)}} \frac{\partial F_{[q,n,p,i],\theta_{(r,j)}}}{\partial \theta_{(r,j)}} \right\} + \frac{\partial F_{[q,n,p,i],\theta_{(r,j)}}}{\partial \theta_{(r,j)}} \frac{\partial F_{[q,n,p,i],\theta_{(r,j)}}}{\partial \theta_{(r,j)}}
\]

Integrating the above gradient expression with the rest of the IHT algorithm described in Algorithm 1 completes the iterative step of the NCS technique for the complex BBVS case. However, for the IHT algorithm to minimize the cost function with respect to the unknown linear signal model parameters \( \Theta \), the constraint step (22) must be applied effectivity at each iteration. The nature of this radar constraint is discussed in detail in the next section.

E. Radar Constraint Step

The choice of the constraint applied at step (22) will be signal dependent and, therefore, unique to the radar’s mode of operation. For the purposes of this article, we focus on the MPRF mode, but the algorithm is not limited to this case. As mentioned previously, our unique formalization of the NCS algorithm means that vector \( \hat{\theta}_r \) represents the Doppler spectrum at range gate \( r \). We choose to exploit the sparsity constraint here as for the MPRF mode, the dominant clutter is typically limited to a small region of the Doppler spectrum [26]. Furthermore, we expect the spectral behavior of the clutter to be largely consistent across multiple range gates [26]. Before invoking the sparse threshold constraint \( P_{\Delta}(.) \), we must first consider the unique formalization of the NCS algorithm.

In the NCS algorithm, the memory of the BBVS model acts down the fast-time dimension, while the sparse nature of the MPRF radar signal, which we wish to exploit, exists in the slow-time dimension. In the case where the nonlinearity has memory, the estimate \( \hat{\theta}_r^{(k)} \) depends on not only \( \hat{\theta}_r^{(k)} \) but also \( \theta_r^{(k-1)} \) to \( \theta_r^{(k-L-1)} \). Therefore, if the gradient (27) is to be calculated correctly in the memory case, then every range regressor vector \( \Theta_r \) must be updated simultaneously at each iteration. This is displayed formally in (22), where
Algorithm 2: Hard Thresholding Algorithm that implements the radar constraint step $P_A(\Theta^{(k)})$ at every iteration $k$ of the NCS algorithm, where the sort(.) function outputs a vector of size $1 \times J$ that contains the indices of $\theta_j$ arranged in ascending order by the magnitude of the elements.

Input: $\Theta$, $R$, $J$, $\mathcal{A}$

1: for $r = 1 : R$ do
2: \[ \bar{\alpha} = \text{sort}(\theta_j) \] Order regressor indices
3: for $j = 1 : |J - \mathcal{A}|$ do
4: \[ \beta = \alpha_{(j)} \]
5: \[ \theta_{(\bar{\alpha}[j])} = 0 \] Zero weakest $\mathcal{A}$ regressors
6: end for
7: end for

Output: $\Theta$

specifically to generate the problematic in-band nonlinear effects detailed in Section II and was configured to operate entirely in the BB domain. Once the radar scene and CW interferer characteristics were defined, the structure of the in-band nonlinear distortion effects produced in the simulation was entirely governed by the particular nonlinear transfer function chosen. Furthermore, the gross level of the nonlinear distortion generated in the simulated RF receiver was determined by the magnitude of the specific nonlinear coefficients chosen for the memoryless case: $h_1$ and $h_3$. The magnitude of the linear coefficient $h_2$ was set equal to the RF receiver gain, while the magnitude of the third order coefficient $h_3$ was subsequently selected so that the unwanted nonlinear effects sat just below the level of the noise floor when the CW interferer was not present in the scene. In specifying the forward nonlinearity in this manner, we can assume that the simulated radar operates within its linear region when the interference is not present in the scene. Additionally, in order to ensure that the same strength of nonlinear effect was being generated in the radar receiver when memory effects were introduced to the problem, each BBVS kernel was normalized so that the respective power contributed from each group of terms in the BBVS model (15) was equivalent to that from the strictly memoryless case.

For this analysis, we assume that the dominant nonlinear receiver effects can be captured by terms up to the cubic order of the BBVS model [10], [12]. For the in-band nonlinear scenario, this corresponds to the case where the black-box receiver model, (15) and (16), has a nonlinear order of $P = 1$. We can, therefore, simplify the expression for the forward nonlinearity to the following form:

\[
y[n,q] = \frac{1}{L-1} \sum_{i_1=0}^{L-1} F_{n,q,1,i_1} x[n-i_1,q] + \sum_{i_2=0}^{L-1} \sum_{i_3=0}^{L-1} h[3,i_2,i_3] F_{n,q,1,i_2,i_3} x[n-i_2,q] x[n-i_3,q] \quad (28)
\]

where

\[
F_{n,q,0,0} = x[n-i_q,q] F_{n,q,1,i_2,i_3} = x[n-i_1,q] x[n-i_2,q] x[n-i_3,q] \quad (29)
\]

Furthermore, the generalized expression for the NCS gradient can be drastically simplified for this specific case by: substituting (29) into (27), setting $P = 1$, performing the partial differentiation, and collecting terms. The final expression for the NCS gradient implemented in the MPRF radar simulator, therefore, takes the following form:

\[
\frac{\partial \Phi}{\partial \theta_{(i)}} = -\sum_q \Phi_{(i,q)} \left\{ e_{n,q} \left( \sum_{i_1=0}^{L-1} h[3,i_1,i_2] x[n-i_1,q] x[n-i_2,q] \right) \right\}
\]

IV. SIMULATION ARCHITECTURE

In order to test the performance of the novel NCS algorithm derived above, an MPRF simulator capable of simulating in-band nonlinear distortion effects with memory had to be developed. In short, the comprehensive radar simulator simulated the predefined radar scene for a standard MPRF mode before passing the raw time-domain signals through the black-box nonlinear receiver model described by (15) and (16). The MPRF simulator was set up...
Importantly, the matrix rotations denoted by $\Psi_{[p \rightarrow 2p+1, r]}$ in the above expression for the NCS gradient were performed in the simulations via a linear shift rather than a circular one. This simple extension meant that the simulations were more realistic than the theoretical NCS algorithm, which is why it was implemented for all of the results presented in this article. In essence, the linear shift introduces edge effects to the problem as not all of the range regressors, $\theta_r$, that affect the gradient are updated as part of the IHT iteration.

While the magnitude of the linear gain and the nonlinear distortion generated in the forward black-box receiver model were fixed in the simulation, the particular choice of complex BB kernel coefficients was subject to change. It is understood that introducing nonlinear memory effects is not expected to change the gross level of the distortion observed at the output of the radar receiver but rather subtly change its structure and phase. The NCS algorithm was, therefore, tested against a wide variety of different nonlinear transfer functions with varying degrees of memory in order to gain a comprehensive understanding of its performance. Crucially, the nonlinear models tested in the radar simulation had to fit into the Volterra series framework; otherwise, the above expression for the NCS gradient (30) becomes invalid. The specific nonlinear models tested in the simulation were, therefore, the BBVS model, the BBVS model, the BB Hammerstein model, and the BB parallel Hammerstein model [8], [31]. For each nonlinear model listed above, the radar simulator was configured to perform statistical convergence analysis on the NCS algorithm, whereby the algorithm’s mitigation performance was tested against randomly generated sets of kernel coefficients.

While this broad convergence analysis is important, it is also important to determine the true performance of the NCS algorithm for an MPRF radar mode with a realistic black-box nonlinear receiver model. Unfortunately, there is a severe lack of published data in the available literature on the memory behavior of front-end receiver amplifiers. One of the very few papers that has published such data is [8]. In [8], Vansebrouck et al. employ a quintic, $P = 2$, parallel Hammerstein model with memory length $L = 5$ to describe a wideband communications receiver centered on 250 MHz. It is clear from the data published in [8] that the third-order nonlinear effects are far more dominant than the fifth-order distortion generated, which allows us to truncate Vansebrouck’s nonlinear model to order $P = 1$. Furthermore, to implement the nonlinear model described in [8] in our MPRF radar simulator, we must translate the passband nonlinear impulse response from a center frequency of 250 MHz to BB. In performing this translation, we convert the real passband coefficients to complex BB coefficients and disregard the even-order terms in the model as they fall outside the bandwidth of the desired receiver channel. The final forward nonlinear model implemented in the MPRF radar simulator was, therefore, a BB parallel Hammerstein model with coefficients equal to those displayed in Table I. In terms of the full Volterra formalization, (15), these coefficients correspond to the off-diagonal elements of the respective Volterra kernels, i.e., where $i_1 = i_2 = i_3$, with all other off-diagonal elements set equal to zero. Note that, in order to distinguish this specific nonlinear model from other BBVS models employed in this article, we refer to it as the “BBVS-[8]” model as it was derived from the parallel Hammerstein model in [8]. An example RD plot outputted from the MPRF radar simulator that corresponds to the above parallel Hammerstein model is displayed in Fig. 3 with the corresponding desired linear output displayed in Fig. 2.

In order to study the performance of the novel NCS algorithm for a typical MPRF radar mode in detail, statistical probability of detection (PD) analysis was performed in the RD domain on a single target. On each burst, the MPRF radar simulator generated input receive data matrices, $X$, consisting of 256 received pulses based on a predefined radar scene. The simulation then generated three output matrices, $Y$, by passing the input data through the following black-box receiver models: the BBVS-[8] model, the corresponding BBTS model, and finally the linear gain model. In all cases, both input and output noise was added to the received signals with the SNR level referenced to the input so that the output noise level was not biased by the specific nonlinear transfer function chosen, $SNR_{noise} = 35$ dB. A further three datasets were then generated by applying the memory-rich NCS algorithm to the BBVS-[8] receiver output and by applying the memoryless NCS algorithm to both the BBVS-[8] and BBTS outputs respectively. Simulation results for a single burst illustrating the nonlinear distortion generated from the BBVS-[8] model and the subsequent correction by the memory-rich NCS algorithm are displayed.
in Fig. 3 and Fig. 4 respectively. It is clear from Fig. 4 that the memory-rich NCS algorithm has performed well in restoring the simulated radar’s performance back to the desired linear case. However, the performance and robustness of the algorithm can be studied in much more detail through the stochastic-based PD analysis. It is important to recognize at this point that the noise assumptions invoked during the derivation of the NCS algorithm have not been implemented in the radar simulator. Therefore, the fact that the NCS algorithm has recovered the correct solution in Fig. 4 clearly validates the noise approach detailed in Section III-B.

As well as performing the PD analysis on the scenarios discussed above, further results were generated to probe the robustness of the algorithm for varying levels of ME, \( \mathcal{W}_{[n,q]} \). While all of the simulations conducted had some degree of ME due to the input and output noise added to the receive data matrices, more in-depth analysis was required to fully understand the limitations of the algorithm. The NCS algorithm is fundamentally designed around the forward black-box receiver model, and therefore, its performance is limited by how well the nonlinear behavior of the RF receiver has been characterized. For a real radar system, it is highly likely that this nonlinear system identification will be performed offline, and it is, therefore, reasonable to assume in the simulations that the forward nonlinearity has been identified to a high degree of accuracy. However, it is important to understand how much ME the algorithm can tolerate before it fails. To perform this ME analysis as part of the simulation, the MPRF radar simulator was configured to identify the forward nonlinearity for each burst through varying levels of output noise. The accuracy of the forward nonlinear coefficients was, therefore, dependent on the level of the output noise through which they were learnt. The linear least squares (LLS) algorithm was used to learn the nonlinear coefficients by means of a noise identification procedure [9] with the residual error value providing an accurate measure of the ME in the simulation.

V. SIMULATION RESULTS

A. Convergence Analysis

For the NCS convergence analysis, the mitigation algorithm was tested against different classes of randomly generated nonlinear transfer functions in order to study its overall convergence properties in both memoryless and memory-rich cases. While this stochastic-based convergence analysis will not inform us about the effectiveness of the NCS algorithm for all nonlinear radar receivers, as there are infinitely many, it can provide us with a better understanding of the specific nonlinear characteristics that will prove problematic for convergence. We use the percentage of successful convergence as a measure of the algorithm’s efficacy, with a 100% convergence defining the situation where the NCS algorithm always recovers the weak target from the corrupted RD map (see Figs. 3 and 4). The success of the mitigation technique was determined for each realization by means of local area average target thresholding algorithm combined with a residual error thresholding technique. The NCS ME was strictly limited by the receiver output noise in this case, \( \text{SNR}_{\text{noise}} = 50 \, \text{dB} \), as there was no input noise and the nonlinear coefficients were set equal to the true forward model coefficients in the NCS algorithm. The convergence results for the NCS algorithm are displayed in Table II with every convergence percentage value determined from 100 realizations of the simulation each with a unique nonlinear transfer function.

When dealing with nonlinear transfer functions, the concept of monotonicity is a fundamental one. In the case of a static nonlinearity, the transfer function is considered strictly monotonic if there exists a one-to-one mapping between the inputs and the outputs of the nonlinear model [32]. Importantly, if a function is strictly monotonic, then there exists a unique inverse function that maps the outputs back onto the inputs. If, however, this one-to-one mapping does not exist, then the nonlinear transfer function is described as being non-monotonic and is, therefore, non-invertible. In the context of the NCS algorithm, non-monotonicity of the forward nonlinearity manifests itself in the LS cost function, whereby the ambiguity generates multiple local minima for the algorithm to converge to. In other words, there is more than one solution to the problem. This is most easily observed for the BBTS result in Table II. We must be very careful when interpreting the convergence results.

<table>
<thead>
<tr>
<th>Nonlinear Model</th>
<th>Monotonicity</th>
<th>Successful Convergence %</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBTS</td>
<td>Strictly Monotonic</td>
<td>100</td>
</tr>
<tr>
<td>BBTS</td>
<td>Non-monotonic</td>
<td>49</td>
</tr>
<tr>
<td>BB HAMMERSTEIN</td>
<td>Strictly Monotonic</td>
<td>100</td>
</tr>
<tr>
<td>BB HAMMERSTEIN</td>
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<td>53</td>
</tr>
<tr>
<td>BB PARALLEL HAMMERSTEIN</td>
<td>Strictly Monotonic</td>
<td>100</td>
</tr>
<tr>
<td>BB PARALLEL HAMMERSTEIN</td>
<td>Non-monotonic</td>
<td>89</td>
</tr>
<tr>
<td>FULL BBVS</td>
<td>N/A</td>
<td>83</td>
</tr>
</tbody>
</table>

100 individual nonlinear transfer functions were used to generate the successful convergence value in each case.
in Table II as the nonlinear transfer functions generated in the simulation are not representative of a real RF receiver. However, it is clear from the results that if the memoryless forward nonlinearity is strictly monotonic, then the NCS algorithm will converge to the correct solution. In the case where the memoryless nonlinearity is non-monotonic, the NCS algorithm’s ability to recover the correct solution is dependent on the particular forward nonlinear transfer function, through which the desired input signal was observed.

In the case where memory is introduced to the forward nonlinearity, we have to be careful with how we interpret the concept of monotonicity, which is only defined for 1-D functions. We start with the Hammerstein nonlinear model as it is the simplest extension from the BBTS model to the memory case. The Hammerstein model is described by a memoryless nonlinearity followed by a finite-impulse response (FIR) filter, and therefore, its monotonicity is entirely defined by its static nonlinearity. Examining the convergence results for the BB Hammerstein model, we find that they reflect that of the BBTS case. This makes sense as the monotonicity of the forward nonlinearity is defined in exactly the same way as the BBTS model. Importantly, the NCS algorithm has been shown to converge to the correct solution consistently in the case where the forward nonlinearity exhibits nonlinear memory effects. Taking this analysis one step further, the Hammerstein model is extended to the parallel Hammerstein model by applying an individual FIR filter to each term in the static nonlinearity [8]. In essence, the parallel Hammerstein model consists of $L$ static nonlinear functions that can be thought of as acting on individual taps of an FIR filter. Therefore, the nonlinear transfer function is considered strictly monotonic if all of the $L$ static nonlinearities are themselves strictly monotonic. In this case, the convergence of the NCS algorithm is guaranteed, which is indicated by the successful convergence percentage of 100% in Table II. Furthermore, we define the non-monotonic case for the parallel Hammerstein model to be when any one of the $L$ static nonlinearities stop being strictly monotonic. Interestingly, the successful convergence percentage is much higher in this case than in the standard Hammerstein and BBTS cases suggesting that the NCS algorithm can tolerate some degree of non-monotonicity in its nonlinear memory terms. Finally, we consider the full BBVS model, where, unfortunately, the nonlinear cross terms mean that we cannot define the concept of monotonicity in this case. Examining the successful convergence percentage for the full BBVS case, we observe that the NCS algorithm can successfully recover the desired input from the corrupted output even when the forward nonlinearity exhibits complex nonlinear memory effects. While the general convergence of the NCS algorithm cannot be guaranteed in this case, the final result highlights the capabilities of this novel mitigation technique.

B. Radar PD Analysis

For the PD performance analysis of the NCS algorithm, the MPRF radar simulator employed an adaptive cell-averaging constant false alarm rate (CA-CFAR) thresholding technique to study the simulated radar’s detection performance in the different scenarios. The CA-CFAR technique is discussed in detail in [25], but, in short, the algorithm was configured to maintain a specified probability of false alarm rate (PFAR), which is achieved by studying the power and statistical behavior of the RD cells in the neighborhood of the target cell. The simulated radar, therefore, exhibits maximum sensitivity when its receiver operates in the linear regime as, in this case, the cells that surround the target cell in the RD map are strictly noise limited. For the nonlinear scenarios, the level of the unwanted distortion was set by the memoryless coefficients as discussed previously with the BBVS-[8] model coefficients described by those in Table I. Importantly, for this analysis, both the BBTS and BBVS-[8] nonlinearities had strictly monotonic forward transfer functions. When the CW interferer is introduced to the radar scene, the unwanted clutter broadening effect drives the target detection threshold up and consequently reduces the sensitivity of the sensor. Thus, for the NCS algorithm to restore the system sensitivity of the radar back to the linear case, it must reduce the level of distortion in the RD detection space without removing the potential targets from the scene. To study the sensitivity of the radar in the different scenarios, the input SNR of the radar test target was varied, with each SNR value forming a single data point in the respective PD curves. The simulation results for the PD analysis are displayed in Fig. 5 with the CA-CFAR algorithm set to maintain a PFAR of 0.2. For each data point in Fig. 5, RD maps for 400 bursts were used to estimate the PD value.

Examining the simulation results in Fig. 5, the first thing that we observe is that the PD curves for the BBTS and BBVS-[8] receiver outputs fall off at a much higher input SNR value than that for the linear gain case. This reflects the loss of system sensitivity experienced by the simulated radar.
in the case where its RF receiver operates in the nonlinear regime. Somewhat surprisingly, the PD curves for the BBTS and BBVS-[8] cases do not lie on top of each other in Fig. 5 despite the strength of the nonlinearities being matched in the MPRF radar simulator. This slight deviation between the two curves is not due to any variation in the strength of the clutter broadening effect but rather reflects subtle phase changes introduced by the nonlinear memory that alters the relationship between the nonlinear distortion and the test target. Examining the blue curve in Fig. 5, it is clear that the memoryless NCS algorithm has succeeded in fully restoring the memory of the simulated radar in the case where the nonlinear RF receiver is strictly memoryless. This is a significant result as it confirms that the NCS technique can be successfully employed to mitigate nonlinear receiver effects in modern radar systems. Interestingly, while the memoryless NCS technique is applied to the memory-rich BBVS-[8] output, the mitigation algorithm fails to restore any of the simulated radar’s lost performance. Much like the memoryless NLEQ techniques discussed in [11], the memoryless NCS algorithm is effectively decorrelated by the subtle amplitude and phase effects introduced by nonlinear memory terms in the forward receiver model. This confirms that if memory effects are significant in the RF receiver, then memory-rich mitigation techniques must be employed if system sensitivity is to be restored.

In the case of the NCS mitigation technique, the result of introducing complex memory effects to the algorithm’s formalization is displayed in Fig. 5; see the green curve. Clearly, introducing the memory terms to the NCS algorithm has drastically improved the mitigation technique’s performance in the BBVS-[8] scenario with the simulated radar’s system sensitivity more or less restored back to the linear gain case. This was fundamentally due to the expression for the NCS gradient (30), which incorporated complex nonlinear memory effects so that each iteration of the IHT algorithm was correctly pointed toward the minimum of the cost function. This is a significant result as it shows that the novel NCS algorithm provides a forward modeling framework capable of compensating for complex nonlinear memory effects generated in the modern radar receiver.

C. ME Analysis

In this section of the simulation results, we examine the robustness of the NCS algorithm to varying levels of ME. As discussed previously, the amount of ME in the PD simulations was governed by the input/output noise of the simulated radar receiver as well as the error in the forward model coefficients implemented in the NCS algorithm. While the SNR ratio was fixed at 35 dB for both the input and output noise in all simulations, the level of uncertainty in the NCS forward model coefficients was varied. The PD analysis performed by the MPRF radar simulator was, therefore, exactly the same as that described in the previous section, with the exception that the NCS algorithm was applied multiple times on each burst using different sets of forward model coefficients. We focus on the scenario where the nonlinearity is described by the BB parallel Hammerstein model in Table I, BBVS-[8], and is mitigated by the corresponding memory-rich NCS algorithm. The results are displayed in Fig. 6 with each PD data point estimated from 400 bursts and the CA-CFAR set to maintain a PFAR of 0.2 as before. As discussed previously, the ME is given by the residual error for the LLS noise identification procedure.

Examining the PD result in Fig. 6, it is clear that the ME in the forward black-box receiver model can have quite a profound effect on the mitigation performance of the NCS algorithm. This makes sense as the mitigation technique is fundamentally designed around the forward nonlinearity. Encouragingly, the NCS algorithm can tolerate a reasonable amount of ME in the forward model coefficients before a significant drop in performance is observed. This drop-off in performance of the NCS algorithm appears to occur quite sharply around ME = −10 dB, which suggests that beyond this point, the iterative learning procedure struggles to consistently identify the minimum of the cost function and, therefore, the desired solution. Despite this, the NCS algorithm can still provide noticeable performance improvements from the unmitigated BBVS-[8] case even when the ME is significantly distorting the IHT estimate on each iteration.

VI. TRIALS DATA RESULTS

In order to study the performance and robustness of the NCS algorithm further, the novel mitigation technique was tested against real radar trials data that had been digitally corrupted by a nonlinearity and a synthetically generated frequency-modulated CW (FM/CW) interferer. The FM/CW interference scenario was chosen for this analysis as it is an extremely popular waveform for modern radar systems [2], meaning that it is also one of the most likely signals to cause mutual interference in the crowded RF environment. Importantly, unlike the simple CW interferer, the FM/CW waveform has a bandwidth, which for this analysis was chosen to be equal to the simulated bandwidth of the radar.
receiver. Performing this type of analysis will further test the robustness of the NCS algorithm as the high bandwidth nature of the FMCW interferer will generate more sophisticated nonlinear memory effects in the simulated nonlinearity. This is due to the fact that the memory of a system is intrinsically linked to its bandwidth [8], [20]. The trials data used for this analysis was collected by Leonardo’s experimental AEXAR system as part of a flight trial where the radar was setup to operate in an MPRF mode with a downward looking configuration. The range, Doppler, and power values of the input trials data were normalized for this analysis so that they matched those from the MPRF radar simulator. By performing this normalization, the same nonlinear scenario studied in the previous results section could be digitally stimulated for the trials data case. Thus, the forward nonlinearity took the form of the BBVS-[8] model with coefficients equal to those listed in Table I. The final results from the trials data analysis are displayed in Fig. 7 with mitigation outputs from both the memoryless and memory-rich NCS algorithm presented. In both cases, the forward model coefficients employed in the NCS algorithms were learnt by means of a noise identification procedure, which importantly allowed additional ME to be introduced to the memory-rich case. The residual LLS forward ME attributed to the memory-rich case was, therefore, $-14$ dB, which compared to $-6$ dB for the memoryless NCS case.

Examining the trials data results from Fig. 7, we first observe that the ground clutter in the linear RD map, between $-4$ kHz to $5$ kHz Doppler, is noticeably more complex than the simulated clutter generated in the MPRF radar simulator. This is useful as it allows the performance of the NCS algorithm to be tested against a less uniform clutter spectrum, where the sparsity may not be at exactly the same Doppler locations in each range gate. Importantly, there are three distinct radar targets in the trials data RD map located around $-9$ kHz Doppler, which can be used as a marker to gauge the performance of the NCS mitigation technique. In Fig. 7(b), these targets are no longer distinguishable from the radar background as they have become entangled with the nonlinear clutter broadening effect stimulated by the presence of the FMCW interferer in the scene, $-4.5$ kHz Doppler. It is clear from Fig. 7(c) that the memoryless NCS algorithm is incapable of correcting for the nonlinear memory effects in the radar receiver resulting in the radar targets still being indiscernible from the background. Examining the final result in Fig. 7(d), we observe that introducing memory terms to the NCS algorithm allows it to unscramble the complex nonlinear memory effects generated in the RF receiver and ultimately restore the sensor’s performance almost back to the linear case. Crucially, the test targets previously lost in the nonlinear clutter broadening effect are now clearly identifiable above the RD background. This is an extremely encouraging result as it highlights the effectiveness of the NCS algorithm in mitigating complex nonlinear receiver effects generated by modern radars in sophisticated clutter and interference scenarios.

![Trials data RD plots. (a) Linear gain output. (b) Nonlinear BBVS-[8] output. (c) BBVS-[8] output mitigated by memoryless NCS. (d) BBVS-[8] output mitigated by memory-rich NCS. Ground clutter located between $-4$ kHz to $5$ kHz Doppler with the synthetically added FMCW interferer located at $-4.5$ kHz Doppler. The mitigation technique successfully recovers the strongest targets located around $-9$ kHz Doppler only when memory terms are introduced to the NCS algorithm.](image)

VII. CONCLUSION

In this article, we presented a novel forward modeling technique designed to digitally compensate for complex nonlinear memory effects in the radar receiver. The technique builds on the work previously presented by Blumensath [21] and was specifically targeted at the in-band interference scenario in radar. In order for the forward...
modeling technique to be successful, Blumensath’s NCS framework had to be extended to represent the unique processing applied in modern radar systems. Furthermore, the NCS algorithm had to be expanded beyond those results published in [22] and [23] so that complex nonlinear memory effects could be mitigated by the NCS technique for the first time. The novel NCS algorithm was tested extensively by means of an MPRF radar simulator that was capable of simulating sophisticated nonlinear receiver effects. The convergence properties of the NCS algorithm were studied for a wide variety of nonlinear transfer functions with the convergence of the algorithm guaranteed in the case where the forward nonlinearity was strictly monotonic. Importantly, more work needs to be done to fully understand the convergence behavior of the NCS algorithm in the non-monotonic case; however, a full analysis of this problem is beyond the scope of this article.

In addition to the convergence analysis, the NCS algorithm was shown to successfully mitigate deleterious effects from both memoryless and memory-rich receiver nonlinearities in an MPRF radar mode through in-depth PD analysis. This impressive mitigation performance was shown to hold for real-world MPRF radar data and for significant levels of ME built into the forward model of the NCS algorithm. If nonlinear memory effects prove to be significant in the modern radar receiver, then forward modeling techniques offer a digital signal processing solution that is not built around the inverse nonlinear transform. Crucially, the unique formalization of the NCS algorithm provides a more explicit framework to compensate for complex nonlinear memory effects than the standard NLEQ mitigation techniques.

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