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A circular cylinder in subsonic cross-flow at moderate Reynolds numbers: free and near-wall effects.

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Thesis submitted for the degree:
Doctor of Philosophy

University of Edinburgh
School of Engineering

June 15, 2022
Declaration

I declare that this thesis was composed by myself and that the work contained therein is my own, except where explicitly stated otherwise in the text.

(Jack Adrian Hanson)
This thesis could not have been brought into being without the precious support I have been so lucky to receive. In acknowledgement for his immense effort and vast contributions to my life now and going forwards, I must thank my supervisor, Sina Haeri, for his guidance and care as I have undertaken this work. My thanks also go to the academic staff who kicked the journey off in Liverpool when I was but an undergraduate. For their support of my interest in fluid mechanics, particular mention and my deep thanks to David Allanson and Mehdi Seddighi. To my close friends Ross Kennedy and Sarah Davidson for acting as proof-readers, coffee-sharers and rambling partners. To my parents, John and Aletha Hanson for shouldering the burdens, both emotional and financial, to get me this far. To my brothers, Theo, Ben and James for reminding me, despite the distance, what more there is to life, to not be too dour and to enjoy these simple things. Finally, my long-suffering wife, un-credited editor-in-chief and principle cheerleader, sorry, I promise I will not do another PhD. To my honoured colleagues, my dearest friends, my precious family and my beloved Sarah: Guma slàn sibh, gach latha a chì is nach fhaic mi sibh.
Abstract

The flow around a cylinder is a crucial engineering approximation and is well studied for a range of Reynolds numbers. However, the research into the effects of compressibility has lagged behind as a recent paper notes (Nagata et al., 2020) that the low sub-critical Reynolds numbers are unexplored. Furthermore, the investigation of the interactions between a cylinder and wall has been gaining further attention in the last 50 years, however, compressible research in this area is extremely limited. With the advent of thin-atmosphere flight where Reynolds numbers are low and flight speed is appreciable, there is a growing need for this fundamental engineering approximation to be properly resolved. This thesis tackles the range of $400 < \text{Re}_D < 800$ in a subsonic flow, $0.20 \leq \text{Ma} \leq 0.45$, for both the free and near-wall cases to fill the gaps identified in the literature for this critical range of Reynolds numbers. This is achieved using a high-order, quasi-spectral finite difference method to solve the compressible Navier-Stokes equations. Firstly, the case of a 3D cylinder in a free flow is investigated with good agreement with the incompressible literature at $\text{Ma} = 0.20$. At $\text{Re}_D = 400$ it was found that compressibility has a limited impact on the flow and the drag followed the 2D model. At $\text{Re}_D = 800$ however, increasing Mach number caused 2D flow features to strengthen and resulted in a 3 fold increase in shedding intensity as well as a significant increase in drag over what the model predicts. This is followed by a larger parametric study of the near-wall case for gap heights of 3 to 1 exposed to a boundary layer with thicknesses greater than 1 diameter. For $\text{Re}_D < 700$, the effects due to Mach number were limited and these effects were removed entirely upon immersion in the boundary layer. However, at $G/D = 1.5$, $\text{Re}_D = 800$, increasing the Mach number led to an 8.5% increase in cylinder drag, a strong negative lift and an 8% increase in the shedding frequency. It was shown that in this case, the wall had the combined effect with the Mach number of inducing strong circulation due to accelerated flow in the gap, further decreasing the base pressure and shortening the formation region. These effects were significant enough to cause the excitation of vortex shedding as well as the increased drag. Finally, this unique case was studied in 3D and it was found that there was a drag increase, a drop in base pressure and a shortening of the formation region due to wall proximity. However, these changes were smaller in comparison to the 2D results. Furthermore, there was no reversal in the lift, rather a stronger positive lift was generated. Although shedding intensity increased due to wall proximity, there was no discernible trend in the shedding frequency. It was demonstrated that the longer formation region in 3D was preventing larger pressure decreases. The longer shear layers move high speed fluid further downstream, keeping the circulation lower and preventing the low pressure in wake from interacting strongly with the rear of the cylinder.
Lay Summary

The flow around a cylinder is a fundamental engineering approximation and considerable effort has been put towards the study in incompressible flows. These are flows where the fluid speed is low with regards to the ambient speed of sound and the change in density is negligible. However, the study of compressible flows is less complete and a recent publication identified a gap in the knowledge where no experiments have been conducted. This gap is in the Reynolds numbers between 200 and 1000 where fluid viscosity and density are quite low. This range is studied using a highly accurate method based on finite differences. This thesis begins by filling in this gap with a focused study in this range for a cylinder in the middle of domain. It was found that at higher Reynolds numbers, the shedding of vortices became increasingly excited in contrast to what is seen in the incompressible case. Following this a further gap in knowledge was identified for a cylinder near a wall, an important engineering approximation that is similarly unexplored for the compressible case. This was studied in 2D first as this is more efficient for large parameter set. Generally, any effects of compressibility were removed when the cylinder was immersed in the boundary layer. However there was a special case when the Reynolds number was higher that cause larger drag, the appearance of negative lift and an increase in the frequency of vortex shedding. This case was shown to arise due to increased speeds in the gap between the cylinder and the wall. This case was then replicated in 3D to check whether there are any discrepancies due to the 2D approximation. It was demonstrated that although there were some initial similarities, for example, the proximity of the wall lead to some flow excitation. However, fundamental differences between the two flows result in reduced loads on the cylinder in 3D.
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<tr>
<td>$a$</td>
<td>Speed of sound, POD time coefficient.</td>
</tr>
<tr>
<td>$A$</td>
<td>Inlet area.</td>
</tr>
<tr>
<td>$A_*$</td>
<td>Throat area.</td>
</tr>
<tr>
<td>$c_P$</td>
<td>Specific heat capacity at constant pressure.</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Coefficient of drag.</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Coefficient of lift.</td>
</tr>
<tr>
<td>$C_P$</td>
<td>Coefficient of pressure.</td>
</tr>
<tr>
<td>$D$</td>
<td>Cylinder diameter.</td>
</tr>
<tr>
<td>$D'$</td>
<td>Wake width.</td>
</tr>
<tr>
<td>$e_t$</td>
<td>Total internal energy.</td>
</tr>
<tr>
<td>$E$</td>
<td>Convective terms of fluid flow equations.</td>
</tr>
<tr>
<td>$f$</td>
<td>Vector of values for differentiation.</td>
</tr>
<tr>
<td>$F$</td>
<td>Force.</td>
</tr>
<tr>
<td>$F$</td>
<td>Diffusive terms of fluid flow equations.</td>
</tr>
<tr>
<td>$J$</td>
<td>Determinant of the Jacobian matrix.</td>
</tr>
<tr>
<td>$J$</td>
<td>Jacobian matrix.</td>
</tr>
<tr>
<td>$L_F$</td>
<td>Length of the formation region.</td>
</tr>
<tr>
<td>$L_T$</td>
<td>Length of the transition region.</td>
</tr>
<tr>
<td>$L_z$</td>
<td>Length along the wall.</td>
</tr>
<tr>
<td>$n_i$ or $n$</td>
<td>Outward normal, tensor and vector form.</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of points in a discretization.</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure.</td>
</tr>
<tr>
<td>$P$</td>
<td>Derivative coefficient matrix.</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius.</td>
</tr>
<tr>
<td>$R$</td>
<td>Specific gas constant for air.</td>
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<tr>
<td>$R_{ij}$</td>
<td>Reynolds stress.</td>
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<td>$R$</td>
<td>Filter coefficient matrix.</td>
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<tr>
<td>$S_{ij}$</td>
<td>Rate-of-strain tensor.</td>
</tr>
<tr>
<td>$S$</td>
<td>Sponge zone forcing term.</td>
</tr>
<tr>
<td>$t$</td>
<td>Time.</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature.</td>
</tr>
<tr>
<td>$T$</td>
<td>Filter coefficient matrix.</td>
</tr>
<tr>
<td>$q$</td>
<td>Heat flux.</td>
</tr>
<tr>
<td>$Q$</td>
<td>Q-criterion.</td>
</tr>
<tr>
<td>$Q$</td>
<td>Rate terms of fluid flow equations or derivative coefficient matrix.</td>
</tr>
<tr>
<td>$u, v, w$</td>
<td>Velocities along the Cartesian coordinates.</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Cartesian coordinates.</td>
</tr>
</tbody>
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Greek

$\alpha, \beta$ Coefficients of compact finite difference scheme.
$\gamma$ Ratio of specific heats or coefficient of compact finite difference scheme.
$\Gamma$ Circulation.
$\delta$ Boundary layer thickness.
$\partial$ Partial derivative.
$\epsilon_{ijk}$ Lewi-Civita symbol.
$\theta$ Angle from the rear of the cylinder.
$\lambda$ POD energy.
$\Lambda$ Matrix of POD energies.
$\rho$ Density.
$\mu$ Dynamic viscosity.
$\nu$ Kinematic viscosity.
$\xi, \eta, \zeta$ General coordinates
$\sigma$ Sponge zone strength.
$\tau$ Stress tensor component.
$\phi$ POD spatial coefficient.
$\omega$ Vorticity.

Subscripts and superscripts

$\infty$ Free-stream/far-field conditions
base Conditions at the base of the cylinder.
prs. Related to a pressure term, i.e. pressure component of drag.
s Pertaining to a shear layer.
stag. Stagnation conditions
vis. Related to a viscous term, i.e. viscous component of drag.
$
$ Fluctuating component or derivative (only in Chapter 2).
$\hat{\cdot}$ Effective/adjusted length.
$\tilde{\cdot}$ Pertaining to the filtered values.

Dimensionless numbers and named constants

$Ma$ Mach number.
$Pr$ Prandtl number.
$Re$ Reynolds number based on a given length scale.
$Re_D$ Reynolds number with regards to cylinder diameter.
$Re_x$ Reynolds number with regards to wall length.
$S, S'$ Sutherland’s constant for air, prime indicates dimensionless form.
$St$ Strouhal number.
### Abbreviations and special functions

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>CFD</td>
<td>Computational Fluid Mechanics.</td>
</tr>
<tr>
<td>CFL</td>
<td>Courant–Friedrichs–Lewy.</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform.</td>
</tr>
<tr>
<td>NS</td>
<td>Navier-Stokes.</td>
</tr>
<tr>
<td>POD</td>
<td>Proper Orthogonal Decomposition.</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Squared.</td>
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<td>SVD</td>
<td>Singular Value Decomposition.</td>
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Chapter 1

Introduction

This chapter lays out the research question and sets it in the context of the current knowledge. The first section will provide a brief overview and a motivation for the work including the rough definition of the parameter space. The following literature review will cover the subject areas relevant to the problem and justify the chosen parameter space more concretely. Finally, the research plan and the outline for the remaining chapters is defined.

1.1 Overview and motivations

The flow around a circular cylinder is a fundamental engineering problem, being widely used to both study flow phenomena such as vortex shedding and as an approximation for more complicated geometries. Over the last a century a large body of research has been accumulated with particular focus on the incompressible regime. There has generally been less research into compressible flows and a bulk of that research concerned with transonic and supersonic flow for aerospace applications. Studies of fundamental compressible flow around a cylinder have received significantly less attention. This gap in the research was highlighted by Nagata et al. (2020), Figure 1.1 shows the current state of research as of Nagata et al. (2020). Figure 1.1 shows a clear gap between Reynolds numbers of 200 and 1000 where there is no available data.

This range of Reynolds numbers is important to resolve for two main reasons which shall be briefly covered in the following. Firstly, as the circular cylinder is such a fundamental engineering approximation it is important that the flow behaviour is well resolved for the compressible case. The available incompressible literature for $200 < \text{Re}_D < 1000$ identifies large changes in the flow dynamics despite the small range of Reynolds numbers. Norberg (2003) for example, demonstrated that there is substantial decay in the intensity of fluctuating forces on the cylinder as the Reynolds number increases. Unal and Rockwell (1988); Norberg (2003) both demonstrated that this decay is linked to an increase in the pressure at the rear of the cylinder. Both publications also identify that the increase in pressure is coupled to a drop in the drag force on the cylinder. For compressible flows, the wake pressure is coupled to drag on the cylinder by the Eckert-Weise effect (Kurosaka et al., 1987) and there may be interactions between viscosity and compressibility that remain unknown in this region.

The second justification is an emerging application for low Reynolds number, subsonic studies in aerospace. Désert et al. (2019) explained, for small objects in thin atmospheres, the Reynolds numbers fall between 100 and 10,000 with flight speeds generally being subsonic, $\text{Ma} = 0.7$. In the last five years particularly there has been
As this engineering application is emerging and gaining research attention, it is crucial that there is a complete literature of the fundamental flows available. Nagata et al. (2020); Canuto and Taira (2015) showed that there are some key effects due to compressibility when $Re_D > 150$ and $1000 \leq Re_D \leq 5000$ that deviate from established compressible models. The most important conclusion of these papers is that drag forces on the cylinder for these Reynolds numbers are under-predicted by the widely used Glauert transformation (Glauert, 1928). This indicates that the drag force is being affected by additional force that appears to be a function of both Mach and Reynolds number. This is unlike the bulk of the research at higher Reynolds numbers where drag is dependent on the Mach number, examples of this can be found in Murthy and Rose (1978); Zdravkovich (1997). Combined with the knowledge of the incompressible flow and it’s variability over such a short range of Reynolds numbers, it is paramount that this parameter space be explored.

A further problem that is worthy of investigation is the effect on the cylinder due to the proximity of a boundary layer formed due to the presence of a wall. As with the research for compressible flow around free cylinders this area of research has very few publications that consider compressibility to the authors knowledge. In addition, Doig (2014) identified that the near-wall case as an under explored area in compressible research despite feeling that this was an important area worthy of attention. The study of compressible flow around a near-wall cylinder has very little available literature, only two publications to the authors knowledge. These are the works of Dudley and Ukeiley (2012); Sugar-Gabor (2018), both of whom chose to focus on high Reynolds numbers in the transonic to supersonic flow regime.

Although focused on wings, a recent publication by Sanal Kumar et al. (2021)
showed that interactions of subsonic boundary layers lead to a choking effect between 
the boundary layer of the wall and that of the body. As before, at the range of Reynolds 
numbers considered in this thesis, it is unknown what the effects of wall proximity 
are as there is no available literature. The conclusion of the first research effort, the 
study of the free cylinder, will provide new information that will support the near-wall 
investigation. New knowledge of the compressible effects for the range of Reynolds 
numbers can be combined with the abundant literature on incompressible flows for 
near-wall cylinders to show the effects of Mach number on this flow regime.

In summary, because of the abundance of unique features over the short range of 
Reynolds numbers, it is important that this be investigated in the compressible case. 
To this end the Reynolds numbers should be between 400 and 800, the former to remove 
the effects of mode switching in the vortex street (see Jiang and Cheng (2017)) and the 
latter as it represents the end of the regions of large change in flow characteristics, see 
Unal and Rockwell (1988); Norberg (2003). Further, the Mach number is kept subsonic, 
this is key both with regards to engineering applications for small spacecraft and in 
order to reduce the number of confounding effects by not introducing shocks.

1.2 Behaviour of free cylinders

1.2.1 Fundamentals and early work

The early experimental work by Wieselsberger (1922) showed a correlation between 
Re_D and the drag for a wide range of Reynolds numbers. By using cylinders of varying 
diameter, they covered Reynolds numbers of 4 to 8 × 10^5. They identified a sudden 
drag loss in the range of 10^5 to 10^6, the so-called critical region which is book-ended 
by the “sub-“ and “post-critical” regions. Characteristic of the sub-critical region is 
fairly constant drag and shedding frequency for Re_D > 1000. However, in the region 
50 ≤ Re_D ≤ 1000, there is a series of changes in the cylinder drag that are coincident 
with fundamental changes in the vortex shedding. At a Reynolds number of about 50 
laminar shedding begins, characterised by purely two-dimensional behaviour. As the 
Reynolds number rises to around 150 a transition begins with turbulence occurring in 
detached vortices in the wake. This is followed by the transition to turbulence in the 
 shear layers that detach from the surface of the cylinder (Bloor 1964) at Re_D > 200. 
Zdravkovich (1997) identified 400 ≤ Re_D ≤ 800 as a range for which the shear 
layers are laminar on the cylinder surface and the transition to turbulence in the layers 
happens at a constant distance from the cylinder. This region is also characterised 
by growth in the formation length, denoted as L_F and defined by Jiang and Cheng (2017) 
as the distance from the cylinder centre to the point of zero velocity in the wake.

1.2.2 Vortex production

Early observations of the flow past bluff bodies led Kirchhoff, see Gurevich (2014), to 
develop a streamline theory focusing on the free shear layers, which they described 
as coherent surfaces emerging from the cylinder. These structures were visible in the 
experimental data and, due to limited measurements, Kirchhoff makes the incorrect 
assumption that the flow velocity in the free shear layers is equivalent to the free-
stream value. A further assumption was that the wake is at the ambient pressure 
which leads to the Kirchhoff theory under-predicting the drag on the cylinder. Roshko 
(1954) improved this model using a notched hodograph theory which, among other 
adjustments, provides a parameter for scaling the streamline velocity against that in
the free-stream. Increasing the speed in the streamlines was related to lower than reference pressures at the rear of the cylinder. The low pressure at the cylinder rear is referred to as the base pressure in the literature. This base pressure is often expressed using a coefficient of pressure defined by the following equation.

\[ C_P = \frac{P - P_\infty}{P_{stag} - P_\infty} \]  

(1.1)

\( P_\infty \) refers to free-stream conditions and \( P_{stag} \) refers to stagnation conditions. For most of the work considered in this literature review, the stagnation pressure is defined using the incompressible assumption as follows:

\[ P_{stag} = P_\infty + \rho(u^2 + v^2 + w^2) \]  

(1.2)

Roshko (1954) showed that the higher speed in the free-shear layers and the corresponding drop in pressure in the wake is fundamental to the development of vortices. In their experimental measurements they extracted the pressure in the stream-wise direction from the cylinder rear to a point downstream. They found that the pressure in the wake of a cylinder contains a region that is lower than the base pressure.

To investigate the effect of this low pressure region on the formation of vortices, they then interfered with it using solid boundaries. The first method was a splitter-plate from the rear of the cylinder to just over five diameters downstream, significantly increasing the pressure in the wake of the cylinder. They note that the periodic vortex formation is inhibited by the presence of the splitter-plate. They surmise that the minimum pressure in the wake close to the cylinder is required for vortices to form. This was demonstrated in a second experiment where a plate just over one diameter in length was used to interfere. Similarly to the splitter plate, it was placed parallel to the stream-wise direction and varied in distance downstream from the rear of the cylinder. The shedding frequency would drop and the base pressure would rise as the plate transited the region of minimum pressure in the near wake. As the distance between the leading edge of the plate and rear of the cylinder exceeds 2 diameters, the shedding frequency and the base pressure rapidly return to free-stream values. This indicates that the low pressure region in the near-wake of the cylinder is vital to the production of vortex shedding. This behaviour appears to be fundamental as Gerrard (1978) repeated this second experiment at \( Re_D = 150 \), two orders of magnitude lower and close to the maximum of the laminar shedding regime. They were able to reproduce the behaviour, at around a gap of two diameters, the plate would no longer interfere with vortex shedding and the parameters return to their normal values.

The model of Roshko (1954) was uncertain for \( Re_D < 1000 \) due to a lack of experimental results at the time. This range was tackled by Bloor (1964), who studied the formation of vortices and their transition to turbulence for \( Re_D > 200 \). Their study starts just after the onset of three-dimensional shedding at \( Re_D = 200 \), showing that there are low-frequency disturbances in the flow close to the cylinder. The large amplitude of low frequencies disturbances is diminished as vortices travel into the wake. The diminishing of this disturbance is used to define the length of the formation region, \( L_F \). Above \( Re_D = 400 \), they showed that high-frequency fluctuations appeared in the vortices and these intensify as the vortex moves downstream. Similar to the definition of \( L_F \), the appearance of high frequency noise is used to define \( L_T \), the length at which turbulent motion begins. For the range of Reynolds numbers considered in this thesis, the relationships are simple. Normalised by the cylinder diameter, \( L_T \) is around 1.5
and the formation region increases with Reynolds number from $L_F/D = 2.0$ to about 2.5. Unal and Rockwell (1988) would collect the results of Bloor (1964) along with other experimental results for a wide range of Reynolds numbers. As shown in Figure 1.2, the measurement of $L_F$ shows quite a wide range. This is due to the definition of $L_F$ as the choice of the threshold beyond which low frequency signal is considered negligible is subjective. Regardless, there is a clear relationship between the Reynolds number, base pressure and the formation length. As the Reynolds number increases from 400, the base pressures begin to rise and the formation length grows until $Re_D \approx 2000$. They showed that the velocity fluctuation amplitudes in the free shear layer, $\tilde{u}/U$, decrease with the change in base pressure across the range considered in this thesis. The fluctuations were recorded at two positions along the shear layer, one at a position of maximum $u$ and the other and the edge of the shear layer. They are shown to follow the trend of the base pressure closely until about $Re_D = 4000$, implying that the increase in base pressure leads to a decrease in fluctuation intensity.

More contemporary work has shown correlations between other measures of vortex
generation. Norberg (2003) showed that the fluctuations of the coefficient of lift, \( C'_L \), are diminished over the range of Reynolds numbers considered. Similar to the results for the shear layers by Unal and Rockwell (1988), they showed that the transition to 3 dimensional turbulence leads to a drop in \( C'_L \) coincident with increasing base pressure. More recently, Jiang and Cheng (2017) found good agreement with the data of Bloor (1964) for the growth of the formation region with Reynolds number. However, they improve the definition of \( L_F \), choosing the position in the wake where the stream-wise velocity is zero. This wake stagnation point changes the length of the formation region but the overall trend is in good agreement. In addition, they show a similar growth in the distance between free shear layers that is coincident with a drop in the velocity of the free shear layer for the range of Reynolds numbers of interest.

The modern observations agree with the model of Roshko, the flow speed in the shear layers and the base pressures are inversely related to one another. Furthermore, the evidence of Roshko (1954); Gerrard (1978); Unal and Rockwell (1988) show that the low pressure in the wake is vital to the formation of vortices. It is interesting to note the lack of such a correlation between \( St \) and \( C_{P,\text{base}} \), as shown in Figure 1.2c. This apparent independence is explained by Roshko (1954), who proposed a “wake Strouhal number”. The wake Strouhal number (\( St^* \)) is defined in terms of the speed ratio in the shear layer (\( U_s/U_\infty \)) and the distance between the shear layers (\( D'/D \)).

\[
St^* = St \frac{D'}{D} \frac{U_\infty}{U_s}
\]

They further define a relationship between the speed ratio in the shear layer, where \( U_s \) is the maximum speed in the layer, and the base pressure coefficient:

\[
\frac{U_s}{U_\infty} = \sqrt{1 - C_{P,\text{base}}}
\]

Eq. (1.4) shows that the ratio of the speed in the shear layer is directly dependent on the base pressure coefficient. As the base pressure coefficient is negative, this implies that the speed ratio increases as the base pressure decreases. Data provided by Williamson and Brown (1998); Jiang and Cheng (2017) confirms that this relationship holds for the range of Reynolds numbers considered here. Therefore the consistency seen in the Strouhal number for \( 400 \leq Re_D \leq 800 \) in Figure 1.2c is due to this relationship. As the Reynolds number increases, the wake size grows, the speed in the shear layers drops due to the increasing pressure and the Strouhal number is kept constant.

1.2.3 Effects of compressibility

It is widely understood that the effects of compressibility become important for \( \text{Ma} \geq 0.3 \). Early work by Stanton (1928) tested at a Reynolds number of \( 2.2 \times 10^4 \), varying the Mach number from the incompressible limit to 2.04. They discusses three key deviations from incompressible data due to compressibility.

- The stagnation pressure coefficient measured at the leading edge of the cylinder increases from a value of one at the incompressible limit to 1.64. This indicates that the incompressible model for stagnation pressure is under-predicting the effect due to increasing density.

- The minimum pressure at the rear of the cylinder decreases as the Mach number increases, creating a stronger pressure gradient across the cylinder, inducing an
increase in drag. However, past Mach one, this effect appears to invert, with higher base pressures at the rear that cause the drag to drop.

- The points at which pressure coefficient changes sign from positive to negative move towards the rear of the cylinder as the Mach number increases. The point of minimum pressure changes around as the cylinder transitions from a shockless state. As shocks are produced the position of minimum pressure moves upstream. These positions of minimum pressure become indistinguishable from the base pressure as \( \text{Ma} > 1 \).

Stanton was only able to capture two regimes of compressible flow, from shockless to intermittent shock formation at \( \text{Ma} < 0.8 \) to that with a bow shockwave at \( \text{Ma} > 1 \). The other states would be filled in by later research, see Zdravkovich (1997), which showed large changes in the drag behaviour due to the different shock regimes. In the range of Mach numbers considered in this thesis, the increase in stagnation pressure can be reliably approximated by a two-dimensional solution of the Glauert transformation (Glauert, 1928) by following equation.

\[
C_{p, \text{stag}}(\text{Ma}) = \frac{C_{p, \text{stag}}(\text{Ma} = 0)}{\sqrt{1 - \text{Ma}^2}} \tag{1.5}
\]

This rule accurately predicts the increase in stagnation pressure on the cylinder from Reynolds numbers from 200 to just before the drag-crisis. In 1937 a fairly large range of Reynolds numbers was covered by Lindsey (1938) using a wind-tunnel and cylinders of varying diameter. Firstly, they showed that the relationship between drag and the Mach number is non-linear, drag increases as an exponential function of the Mach number. This relationship between the drag and the Mach number also can be modelled using Eq. (1.5). Gowen and Perkins (1953) sampled a larger range of Mach numbers with a similar experimental set up as Lindsey. They showed that the cylinder drag follows the Glauert transform for the sub-critical range of Reynolds numbers. In the case of flow around a cylinder, the relationship is clear, the low pressure at the rear of the cylinder decreases as the stagnation pressure rises leading to increased drag. Viscous contributions are limited due to the subsonic flow not generating significant surface stresses. As such pressure is the dominating force on the cylinder, particularly as \( \text{Re}_D > 200 \) (see Henderson (1995)).

\[
P = \rho RT \tag{1.6}
\]

For compressible flows, effects on the pressure are coupled to density and temperature by the equation of state as in Eq. (1.6). A key effect of compressibility is to allow the changing of density along fluid streamlines, something which is not possible in a single-phase incompressible flow. Therefore, in the low pressure wake of a cylinder for example, there will be a defect in both the density and the temperature as the Mach number increases.

Eckert and Weise (1942) demonstrated that this is the case, increasing the Mach number was shown to decrease the temperature within the wake of the cylinder, they noted that this defect is by as much as 20 degrees Celsius at high speeds (\( \text{Ma} > 0.7 \)). Thomann (1959) studied this effect, known as the Eckert-Weise effect, by introducing a splitter plate behind the cylinder. Similar to that observed for incompressible flows by Roshko (1954), the temperature in the wake would increase with the increasing base pressure as vortex production was inhibited by the splitter plate. This indicated
to Thomann that the pressure, temperature and vortex shedding where intimately connected.

Kurosaka et al. (1987) made significant progress in the understanding of the recovery factor with regards to flow past bluff bodies. They updated some incompressible vortex street models to include the Eckert-Weise effect. A key discovery was that the temperature defect and hence the pressure defect is related to the circulation of a vortex as it is generated. This relationship implies that lower temperatures at the rear of the cylinder produces stronger vortices. Recent 2D simulations by Aleksyuk (2022) confirm the models of Kurosaka et al. (1987) at Re\(_D\) = 1000 as well as providing visual evidence of the energy separation in vortices described in both Eckert and Weise (1942), Kurosaka et al. (1987). In addition to the modelling of the vortices, Kurosaka et al. (1987) also considered the lower pressures at the rear of the cylinder due to the temperature defect. They showed that the temperature defect was equal to the sum of the pressure drag on the cylinder and the increased entropy in the wake region due to increased thermodynamic work in the wake.

As the study of low Reynolds numbers is more modern, the literature is limited. In 2015, Canuto and Taira (2015) studied the subsonic regime for Reynolds numbers between 50 and 200. They show that the Glauert transformation breaks down at Re\(_D\) < 100, concluding that the increasing viscous forces begin to significantly contribute to the drag. They show that the Strouhal number is affected similarly, at Re\(_D\) ≥ 75 , increasing the Mach number to 0.45 reduces the St by 4%. However, at Re\(_D\) = 50, the Strouhal number is reduced by 9%. Nagata et al. (2020) showed that, for 1000 ≤ Re\(_D\) ≤ 5000, the stagnation pressure scales in agreement with the Glauert transformation. They show that as the Mach number increases, both the base pressure and the detachment angle decrease. However, this decrease in base pressure causes the cylinder drag to be higher than that predicted by the Glauert transformation. They also show that the trend in the Strouhal number identified by Canuto and Taira (2015) recurs for Reynolds numbers up to 3000. However, for Reynolds numbers above 3000, they show the opposite, increasing the Mach number increases the shedding frequency. This relationship between the Strouhal and Mach numbers disappears at higher Reynolds numbers. For example, over the range of \(8.3 \times 10^4 \leq \text{Re}_D \leq 5 \times 10^5\), Murthy and Rose (1978) showed that the Strouhal number is constant as the Mach number increases up to around Ma = 0.9.

### 1.3 Effects of wall proximity

This section will focus on the literature for the near-wall case and will be split into three major subject areas, with consideration given to the boundary layer thickness on the wall. Firstly, the forces on a near-wall cylinder followed by discussions of vortex shedding, closing with a brief discussion of the recent compressible studies.

Figure 1.3 shows a generic version of the near wall set up, where \(L\) is the wall length from the leading edge of the wall to the cylinder centre. The gap height \((G)\) is nondimensionalized by the cylinder diameter as is the boundary layer thickness \((\delta)\).

#### 1.3.1 Forces on the cylinder

An early measurement of the forces was by Roshko et al. (1975), who studied a cylinder approaching a wall around a Reynolds number of \(2 \times 10^3\) experimentally. The wind tunnel generated a boundary layer over the flat plate used to represent a wall all that had a thickness \((\delta)\), normalized by the diameter of the cylinder \((\delta/D)\), of 0.8. They found that the drag force would remain at a free stream value until the gap height
was less than one where the drag would fall to a minimum value as a cylinder touches the wall. They attribute the drop and drag to interference from the boundary layer however the short conference paper does not contain any additional investigation. The lift forces stays close to zero until the gap height is less than one where it becomes positive and inversely proportional to the gap height. Maximum lift is achieved when the cylinder is in contact with the wall with a value of 0.6.

Bearman and Zdravkovich (1978) studied a cylinder approaching a wall at \( \text{Re}_D = 4.5 \times 10^4 \). Although they did not collect any drag or lift data, they do provide a measurement of the base pressure at the rear of the cylinder. The authors also note the effect of the gap on the position of the front stagnation bubble. As the cylinder approaches the wall the front stagnation point moves from the centre-line towards the wall.

Zdravkovich (1985) experimented on a larger range of Reynolds numbers, \( 4.8 \times 10^4 \) to \( 3.0 \times 10^5 \). The effect of a variety of boundary layer thicknesses was investigated, generated by two different methods: rod tripping and mesh wire. This showed that there is a relationship between the drag and interference with the boundary layer. Once the cylinder is in the boundary layer the drag will drop significantly. Using the ratio of gap height to boundary layer thickness, \( G/\delta \), it was shown that the drop in drag happens as \( G/\delta < 1 \) for both types of boundary layer generation methods. The data of Roshko et al. (1975) was shown to also follow this pattern when adjusted to \( G/\delta \). This is contrasted by the lift results where the type of boundary layer that the cylinders exposed to has a significant effect on the lift to gap height relationship. Firstly the case of mesh wire generated boundary layer at sub-critical Reynolds numbers: the lift remains at zero as the cylinder approaches, then become negative as \( G/D = 1.5 \). When \( G/D < 0.5 \) the lift would become positive and inversely proportional to the gap height in a similar fashion to the results of Roshko et al. (1975). However at trans-critical Reynolds numbers, there was an additional pronounced positive lift generated when \( G/D > 1.5 \), before becoming negative. Finally for a range of Reynolds numbers in the rod generated boundary layers the negative lift disappeared. Instead the lifting forces are shown to be fairly stable until \( G/\delta < 1.0 \), then inversely proportional to the gap height similar to the results of Roshko et al. (1975).

Buresi and Lanciotti (1992) tested \( 0.86 \times 10^5 \leq \text{Re}_D \leq 2.77 \times 10^5 \) similar to Zdravkovich (1985), again investigating the effect on the drag and lift due to varying boundary layer thicknesses. Boundary layer thicknesses were varied, \( 0.1 \leq \delta/D \leq 1.1 \), using 3 different methods. The thinnest is naturally generated on the plate, an intermediate by a small rod and the thickest by a combination of larger rod and mesh.
They find similar effects in the drag to that of Zdravkovich (1985), however no lift reversal is observed. Instead, there is a delay in the critical gap height required for lift-off, when the lift rapidly becomes positive as seen in Roshko et al. (1975). Thicker boundary layers delay this effect, increasing \( \delta/D \) from 0.1 to 1.0 at \( \text{Re}_D = 2.76 \times 10^5 \) delayed the lift-off from a gap height of 0.8 to 0.2.

Lei et al. (1999) also did experimental studies of the effect of boundary layer thickness for Reynolds numbers of \( 1.30 \times 10^4 \leq \text{Re}_D \leq 1.45 \times 10^4 \), an order of magnitude lower than those of Zdravkovich (1985) and Buresti and Lanciotti (1992). The thicker boundary layers are being generated by a tripping rod. However, the drag data does not follow the pattern established by Zdravkovich (1985); Buresti and Lanciotti (1992). The authors provide two explanations for the discrepancy, firstly, the order of magnitude difference in the Reynolds numbers. Secondly, that the measurement is based on mid-span pressure instead of force-gauge measurements at ends of the cylinder as used in previous literature. Despite this, they find the negative lift when \( G/D < 1.5 \) with a positive lift generated when \( G/D \leq 0.5 \), similar to Zdravkovich (1985). In thin boundary layers, the behaviour is similar to previous works, lift is zero at large gap height and rises inversely proportional to gap height when \( G/D < 1.5 \). Lei et al. (1999) also collected other data, firstly the base pressure coefficient which they show follows the work of Bearman and Zdravkovich (1978) regardless of boundary layer thickness. They also show tracked the angular displacement of the front stagnation bubble. This showed that thicker boundary layers have the effect of delaying the rotation about the cylinder that was noted by Bearman and Zdravkovich (1978) as being due to decreasing gap height.

The boundary layer thickness and the gap height both have important roles to play in the forces on a cylinder in this configuration. In two separate publications Nishino et al. (2007, 2008), studied the effect on a cylinder with the boundary layer removed. A moving wall at the same speed as the flow removes everything except for a local boundary layer generated in the gap when the cylinder is within about one diameter. Comparing the data across their papers shows that the use of end plates on a cylinder has the same effect as applying periodic boundary conditions in their numerical studies. The removal of the boundary layer is shown to affect the drag in such that it is fairly consistent up to a gap height of around 0.5 where will suddenly drop from 1.3 to about .9 and settle again until the cylinder touches the wall. Despite this strange behaviour in the drag, the lift matches that behaviour for thin boundary layer problems showing a very strong dependency, where the lift is inversely proportional to the gap height.

### 1.3.2 Vortex shedding

Over the years two key issues in vortex shedding in near wall flows have arisen, firstly the cessation of vortex shedding for gap heights less than 0.3. Second is the apparent relationship between increasing base pressure for a fixed Reynolds number and a change in vortex shedding.

Taneda (1965) investigated the effect of wall proximity on the formation of vortices at \( \text{Re}_D = 170 \). They showed that the vortex street becomes increasingly single-sided as the cylinder approaches the wall. They noted the wave-number in the vortex street is decreasing for \( 0.1 \leq G/D \leq 0.6 \). In additions, at a gap height of 0.1, the vortex street is completely single-sided. The side of the cylinder that is closest to the gap is not shedding a vortex. They propose that the shedding on the underside is weakening as \( G/D \rightarrow 0.1 \), thus the wave-number increases. They also provide some limited data on the effect of wall proximity and the critical Reynolds number (\( \text{Re}_{D,\text{crit}} \)), that at which
shedding begins, that a gap height of 3 it’s around 60 rising to 150 at a gap height of 1.5.

Bearman and Zdravkovich (1978) also looked at the effect on the vortex street at \( Re_D = 4.5 \times 10^4 \), particularly the cessation of vortex shedding at small gap heights, what would become known as the “cessation problem”. They improve on previous research by providing power spectra of the vortex shedding for both the upper and lower separated shear layers. At larger gap heights, both power spectra have a single peak at the Strouhal number. This is fairly constant as gap height decreases, the peaks are of similar magnitude and the Strouhal number is fairly constant. However there is a change when \( G/D \leq 0.3 \), the peak in for the lower shear layer becomes indistinguishable from noise and the peak for the top side is noticeably weakened. Shedding is continuing from only the top separation point on the cylinder, a probable explanation for the decreased wave number in the wake observed by Taneda (1965).

Vortex shedding ceases completely when the cylinder is in contact with the wall.

Taniguchi and Miyakoshi (1990) performed a parametric study looking at both fluctuating lift and drag for a cylinder at \( Re_D = 9.4 \times 10^4 \) in a turbulent boundary layers of varying thickness. Tripping rods of varying diameter were used to create the boundary layers, exhibiting good agreement with the approximations (Coles, 1956). They find that boundary layer thickness has a significant impact on on the fluctuating forces. At \( \delta/D = 1.05 \) the fluctuations increase slightly around a \( G/D = 1.5 \) before dropping slightly at a gap height around 1, then rising to a secondary peak and descending as the cylinder touches the wall. However, decreasing the boundary layer thickness to 0.34 causes a larger first peak to appear around a gap height of 1. This is followed by a much more significant drop off where the fluctuating forces come close to zero as \( G/D = 0.3 \). From a gap height of 0.3 there is a small increase in \( C_{L}' \) then a final drop close to zero as the cylinder touches the wall. These second peaks are contained within the region where the cylinder is immersed in the boundary layer. They explain that the first trough in the data is when the bottom shear layers are coming into contact with the outer edge of the boundary layer. This mixing effect is removed as the cylinder is immersed the boundary layer and there is some recovery until contact with the wall. Similarly to the observations of Taneda (1965), they show a relationship between the critical gap height (\( G/D_{crit} \)) and the thickness of the boundary layer. Regular vortex shedding ceases when the lower shear layer on the cylinder is in contact with the outer edge of the turbulent boundary layer. Power spectra data showed that, as \( G/\delta \rightarrow 1.0 \), the defined single peak in the spectra begins to decrease in magnitude. This decrease continues to around a gap height of 0.3 where no distinguishable peak appears, similar to the observations of Bearman and Zdravkovich (1978). In addition, the Strouhal number is not greatly affected by the gap height regardless of the choice of boundary layer thickness, \( St \) is fairly consistent until \( G/D < 0.3 \) where shedding ceases due to wall interference.

Lei et al. (1999) tested \( 1.3 \times 10^4 \leq Re_D \leq 1.45 \times 10^4 \) with boundary layer thicknesses controlled with a tripping rod. Despite their range of boundary layer thicknesses containing the range of Taniguchi and Miyakoshi (1990), their results do not show the same behaviour. Instead, fluctuating coefficient of lift rises slowly to a peak near \( G/D = 0.8 \) before rapidly descending as the cylinder continues to approach the wall. Thinner boundary layer results show a more pronounced peak, with thicker boundary layers causing a weaker peak at higher \( G/D \). There is also a weak increase in \( C_{L}' \) when \( 0 \leq G/D \leq 0.3 \) but not the additional peak seen by Taniguchi and Miyakoshi (1990). The results are in agreement with the previous observations that vortex shedding ceases after a gap of 0.3. This is confirmed by power spectra that show the loss of a defined
peak in the signal for these small gaps indicating that vortex shedding has ceased. 

Lei et al. (2000) focused on studying $80 \leq \text{Re}_D \leq 1000$ numerically in a two dimensions. The domain produced natural boundary layers whose thickness is inversely proportional to the Reynolds number. The fluctuating coefficient of lift, when scaled against the free-stream reference data, show similar trends as the Reynolds number increases. At lower Reynolds numbers they see the cessation of vortex shedding at fairly large gap height, for example the results at $\text{Re}_D = 80$ stop shedding around $G/D = 1.2$. This minimum gap height decreases as the Reynolds number increases, tending towards 0.3 when $\text{Re}_D > 200$. For Reynolds numbers greater than 200, the scaled $C'_L$ overlap and the pattern is a slow rise to a peak around $G/D = 1.0$, followed by more rapid a decline close to zero before vortex shedding ceases entirely around a gap height of 0.3. Additionally, the Strouhal number depends more on gap height when $\text{Re}_D \leq 200$, a reduction of 10% before vortex shedding ceases. For $\text{Re}_D > 200$, $St$ values vary only little as $G/D \to 0$, an increase of 2% at $G/D = 1.5$. When $0.5 \leq G/D < 1.5$, there is a 10% drop followed with a small recovery before vortex shedding terminates completely at $G/D = 0.3$.

Rao et al. (2013) studied the effect of the gap height on $50 \leq \text{Re}_D \leq 200$. They used a moving wall in the same fashion as Nishino et al. (2007) and found that vortex cessation occurs later at lower Reynolds numbers than what is predicted by Lei et al. (2000). At a Reynolds number of 80 for example, vortex shedding ceases at a gap height of approximately 0.6. This supports the assertions of Taniguchi and Miyakoshi (1990), that the key relationship is between the gap height and the boundary layer thickness for a given Reynolds number.

1.3.3 Compressible effects near a wall

There is very limited work on the problem of compressible flows on a near wall cylinder. Dudley and Ukeiley (2012); Sugar-Gabor (2018) studied the interactions in super-sonic flows. Dudley and Ukeiley (2012) studied a cylinder immersed in a supersonic boundary layer at $Ma = 1.5$, $1.9 \times 10^4 \leq \text{Re}_D \leq 3.8 \times 10^4$. At these Mach numbers the shedding from the cylinder is very weak (Murthy and Rose, 1978) and they found that the lower speeds in the boundary layer served to shield the cylinder and caused shedding to resume. Sugar-Gabor (2018) studied $Ma = 1.5, 2.9$ at $\text{Re}_D = \{5.8 \times 10^5, 7.2 \times 10^5\}$ respectively in 2D. The wall boundary layer was removed using a moving wall, however the formation of local boundary layers was shown to be caused by interactions between a Mach stem and the wall.

The flow effects at lower Mach numbers are largely unexplored with some recent work by Sanal Kumar et al. (2021) being tangentially related. They showed that the decreasing area between boundary layers can encourage choked flow at low ground clearances between both wings and aircraft fuselage. Importantly, the effect appears to be 2D and occurs due to the bulk stream-wise motion. By assuming the isentropic compression laws (AMES Research Staff, 1953): reducing the flow area $A$ to a throat $A_*$, a region of choked flow will appear for a subsonic inflow. Therefore, taking the gap height, $G$, and the cylinder diameter $D$, the area ratio becomes:

$$\frac{A_*}{A} = \frac{G/D}{G/D + 0.5} \quad (1.7)$$

Where the inflow Mach number can be predicted by setting area ratio and rearranging Eq. 80 in AMES Research Staff (1953). This shows that, at $G/D = 1.5$, and inflow of $Ma = 0.43$ should produce a shock in the absence of any boundary layer.
interference. It remains an open question whether the choked flow condition will occur in the parameter space considered in this thesis.

1.4 Shortcomings of the two-dimensional approximation

The bulk of the numerical data contained in Chapter 4 uses a two-dimensional near-wall case to search the parameter space. There are some issues with this approximation, firstly, that the increase in base pressure seen in 3D is inverted in the 2D data. Henderson (1995) showed, in 2D, the base-pressure decreases with Reynolds number for the range considered. However, Mittal and Balachandar (1995) showed that the low pressure regions associated with fluid acceleration around the cylinder and shear layer detachment are quite similar at $Re_D = 525$. However, the low base pressures in 2D reduce the critical Mach number, $Ma_{crit} = 0.46$. In contrast, as the base pressure increases for $400 \leq Re_D \leq 800$ in 3D, it is reasonable to assume that $Ma < 0.6$ would be shock-less.

The second effect of the 2D approximation is on the shedding frequency. Jiang and Cheng (2017) showed that the extension of the recirculating region length is related to a flattening in the curve of Strouhal number. In 2D, the shedding frequency continues to rise with $St \propto \sqrt{Re_D}$ for $Re_D > 300$. However, the effect of the wall on Strouhal number appears to be fairly consistent regardless of 2 or 3D. The work of Shen and Wei (2018) also uses a natural boundary layer of comparable size to that of Lei et al. (2000) who reported similar changes in the Strouhal number at $Re_D = 200$ in 2 dimensions. Moreover, at $400 \leq Re_D \leq 1000$, the work of Lei et al. (2000) shows that the Strouhal number and the fluctuating coefficient of lift are functions of gap height, in agreement with the observations in 3D of Lei et al. (1999). Therefore, although the free-stream Strouhal numbers are different, the effect of the wall is comparable regardless of the 2D approximation.

A caveat is the case of $G/D < 1$ leading to a growing discrepancy between the 2 and 3D results. Nishino et al. (2008) compares 2D and 3D implementations at a Reynolds number of $4 \times 10^4$ and includes some experimental results from Nishino et al. (2007). The 3D results agree with experimental results, the 2D data shows a similar trend however, the drag is over predicted. The inaccuracies worsen significantly as $G/D < 1$, with the 2D data showing increasing drag as the cylinder approaches the wall. This is in conflict with the drag drop shown in the DES data and the experimental results.

Additional evidence for this deviation due to the 2D approximation is available for $Re_D < 200$ using three separate publications. Rao et al. (2013) studied a cylinder in 3D near a slip wall for $50 \leq Re_D \leq 200$. Yoon et al. (2007) studied a cylinder in 2D near a moving wall at $Re_D = \{100, 140, 180\}$. Shen and Wei (2018) studied Reynolds numbers of 150 and 200 in two dimensions with both full-slip and no-slip boundaries. These three studies agree that the drag increases when close to the no-slip wall, a feature of this setup at $Re_D \leq 200$. However, they begin to deviate when $G/D < 1.0$, the two dimensional studies show that drag drops as the cylinder becomes very close to the wall. Whereas, in 3D, Rao et al. (2013) shows that drag remains high even at very small gap heights $G/D = 0.025$.

In conclusion, the 2D approximation appears to be capable replicate the important 3D flow characteristics within the range of Reynolds numbers covered in this thesis. Furthermore, there is reason to believe that important flow features will be 2D. Primarily, the acceleration of the flow in the gap region can reasonably be said to be 2D. Due to the turbulent transition in the cylinder shear layers being around 1.5 diameters
downstream for the range of Reynolds numbers and the boundary layers on both the wall and the cylinder being laminar (see Section 2.3.1 for clarification), the contraction in the gap should be a laminar case. However, the differences reported in the literature impose two limitations on the studies, the first being a maximum Mach number of 0.45 due to the low pressures inducing transient shocks. Second, a minimum gap height of one due the large discrepancies between results.

1.5 Research questions and outline

In summary the literature review has shown that there are are clear holes in the existing data. Particularly with new applications arising in the last decade it is important that this highly variable, yet little studied, region of subsonic moderate Reynolds numbers is characterised. To this end this thesis fill in just three of the available gaps in the literature by resolving the following research questions.

What is the effect of increasing the Mach number for this range of moderate Reynolds numbers?

Firstly, as covered in Section 1.2.3, Nagata et al. (2020) identifies the range of moderate Reynolds numbers as unexplored in the compressible case. Furthermore, the cited articles make it clear that unexpected behaviour in the drag due to compressibility is expected in this region. Therefore the thesis will focus on the forces on the cylinder and their relation to compressibility. Furthermore, Section 1.2 explained that, in the incompressible case, this range of Reynolds numbers is characterised by large changes in the vortex shedding behaviour. Therefore it is also important that the compressible effects on, for example, the fluctuating coefficient of lift are well resolved.

What is the effect of increasing the Mach number when the cylinder is placed close to a wall boundary layer?

Secondly, Section 1.3 revealed that there is a growing interest in the effect of a plain boundary on the flow around a cylinder. Specifically that interactions with the boundary layer have marked effect on the loads and vortex production of a cylinder. Furthermore, Section 1.3.3 explained that, in a compressible flow, the changing area between the cylinder and the wall could cause a further acceleration of the fluid that would not be seen in an incompressible case. Section 1.3.3 also highlighted a paper that showed that boundary layers act to further reduce the area through which the flow can pass, increasing the acceleration. Accelerated fluid should produce areas of lower pressure close to the cylinder. Therefore it is important to ascertain what effects, if any, this change in the fluid has on the cylinder forces and vortex production?

With regards to the wall boundary layer, is the 2D approximation valid for this problem?

Finally Section 1.4 discussed the effects of the two-dimensional approximation and possible issues. Of particular interest is the discrepancy in formation region length between the 2D and 3D cases. It is important that some selected results of the two-dimensional near-wall case are taken for a further three-dimensional study. Here it is important to characterise the differences in the loads and vortex production due to the 2D assumption as well as appraise the validity of the 2D approach for this range of parameters.
Thesis outline

A significant gap has been identified in the knowledge of compressible effects at moderate Reynolds numbers in both the free and near wall case. In order to fill this gap, this thesis is arranged as follows:

- The high order, high resolution numerical method is explained along with the mesh development. This includes the validation of the mesh for the near wall case that improves orthogonality while mapping a circular cylinder to a flat wall. This chapter also covers the implementation of a proper orthogonal decomposition (POD) method that is used in subsequent chapters.

- Three dimensional data is provided in Chapter 3 for the range of Reynolds numbers showing the effects on drag and lift for a free cylinder. This data is to fill in the gap in the knowledge identified by Nagata et al. (2020) and provide free-stream reference data for the discussions in Chapters 4 and 5.

- A parametric study is performed in two dimensions to measure the effect of compressibility on near-wall problems. The data gathered is compared extensively to the incompressible literature, highlighting the key deviations due to compressible flow.

- A specific case of high drag and high shedding frequency due to wall proximity identified in Chapter 4 is studied in 3D. These results are compared to highlight the agreement between the 2 and 3D case. The conclusions of this chapter are used to make recommendations for future work.
Chapter 2

Numerical Methods and Implementation

This chapter covers the details of the numerical method, starting with the scaling of the compressible Navier-Stokes equations and a compact, conservative formulation is presented. This is followed by a discussion of the various numerical techniques for spatial discretization and time-marching, including the filtering used to perform implicit LES. In the third section mesh generation is discussed. A non-standard mesh is proposed and validated for the near-wall problem in 2 dimensions. Then, efficient 2D meshes are validated for the free cylinder case. These 2D free cylinder meshes are extruded into 3 dimensions and tested for convergence and domain independence. Finally, some key mathematical techniques are presented to aid later chapters.

2.1 Governing equations

The dimensionless compressible continuity equation and Navier-Stokes (NS) equations of momentum and energy are solved in a general curvilinear coordinate system. The cylinder diameter is used as the reference length scale. The reference time scale is defined using the length and the far-field speed of sound, $a_\infty$ where the subscript “$\infty$” is used to refer to far-field variables. Other variables are nondimensionalized as follows: density by $\rho_\infty$, temperature by $T_\infty$, dynamic viscosity by $\mu_\infty$ and the gas constant $R$ by the specific heat capacity at a constant pressure $c_{p,\infty}$. A reference pressure is derived from the other far-field values as $\rho_\infty a_\infty^2$.

The nondimensionalization of the NS equations yields the Reynolds, Mach and Prandtl numbers, which are represented by $Re_D$, $Ma$ and $Pr$ respectively. The working fluid is air, hence the Prandtl number is set to $Pr = 0.71$ and the ratio of specific heats, $\gamma$, is 1.4. The fluid obeys Sutherland’s law and the constant for air is $S = 110K$. From this point forward all parameters are dimensionless unless explicitly stated otherwise.

The NS equations can be written in a compact, conservative, general coordinate form using the dimensionless quantities as:

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial}{\partial \xi_i} \left( \frac{\partial \xi_i}{\partial x_j} \mathbf{E}_j - \mathbf{F}_j \right) = \frac{\mathbf{S}}{J}$$

(2.1)

where $i, j \in \{1, 2, 3\}$ and Einstein summation is assumed. The determinant of the Jacobian matrix $J$ is given by:
\[
J = \det(J) = \det \left[ \frac{\partial(x, y, z)}{\partial(\xi, \eta, \zeta)} \right] \tag{2.2}
\]

The term \( S \) is a body force added to enforce the far-field conditions, discussed in more detail in Section 2.2.3. In Eq. (2.1), \( Q \), \( E_j \) and \( F_j \) are given by:

\[
Q = [\rho, \rho u, \rho v, \rho w, \rho e]^T \tag{2.3}
\]

\[
E_j = [\rho u_j, \rho u u_j + \delta_{1j} P, \rho v u_j + \delta_{2j} P, \rho w u_j + \delta_{3j} P, (\rho e_t + P) u_j]^T \tag{2.4}
\]

\[
F_j = [0, \tau_{1j}, \tau_{2j}, \tau_{3j}, q_j + u_k \tau_{jk}]^T \tag{2.5}
\]

The stress tensor, heat fluxes and total internal energy are defined by:

\[
\tau_{ij} = \frac{\mu Ma}{Re_D} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \tag{2.6}
\]

\[
q_j = \frac{\mu Ma}{(\gamma - 1) Pr Re_D} \left( \frac{\partial T}{\partial x_j} \right) \tag{2.7}
\]

\[
e_t = \frac{P}{\rho(\gamma - 1)} + \frac{u^2 + v^2 + w^2}{2} \tag{2.8}
\]

Where the dynamic viscosity, \( \mu \), is calculated using the dimensionless form of Sutherland’s law:

\[
\mu = T^\frac{3}{2} \frac{1 + S' T}{T + S'q} \tag{2.9}
\]

where \( S' \) is the dimensionless constant for air in terms of the far-field temperature, \( T_\infty \), expressed in Kelvin as:

\[
S' = \frac{S}{T_\infty} \tag{2.10}
\]

2.2 Numerical techniques

The code used in this thesis is an in-house code that has been inherited from previous projects. This section shall cover the fundamentals of the code in detail with any modifications highlighted. The code has been used in numerous publications including high-resolution studies of aerofoil noise generation by [Kim et al., 2016].

2.2.1 Compact finite differences

The spatial derivatives of the governing equations are solved using a 4\(^{th}\)-order, high resolution compact finite difference scheme ([Turner et al., 2016]) with 6\(^{th}\) order compact filters ([Kim, 2010]). The central finite difference equation, solving for the first derivative, \( f' \), as a function of \( f \) at the interior points:

\[
\beta f'_{i-2} + \alpha f'_{i-1} + f'_i + \alpha f'_{i+1} + \beta f'_{i+2} = \frac{1}{\Delta x} \sum_{n=1}^{3} a_n (f_{i+n} - f_{i-n}) \tag{2.11}
\]
This scheme requires modifications at the boundaries to maintain the penta-diagonal matrix structure. The following equations are for the first and last three points of the solution matrix and the coefficients $\gamma$ here are defined in (Turner et al., 2016).

\[
f'_0 + \gamma_{01} f'_1 + \gamma_{02} f'_2 = \frac{1}{\Delta x} \sum_{n=1}^{6} a_{0n}(f_{i+n} - f_0) \tag{2.12}
\]

\[
\gamma_{10} f'_0 + f'_1 + \gamma_{12} f'_2 + \gamma_{13} f'_3 = \frac{1}{\Delta x} \sum_{n=0, \neq 1}^{6} a_{1n}(f_{i+n} - f_1) \tag{2.13}
\]

\[
\gamma_{20} f'_0 + \gamma_{21} f'_1 + f'_2 + \gamma_{23} f'_3 + \gamma_{24} f'_4 = \frac{1}{\Delta x} \sum_{n=0, \neq 2}^{6} a_{2n}(f_{i+n} - f_2) \tag{2.14}
\]

A system of linear equations can be created using the equations presented, the coefficients of which can be collected into matrices as:

\[
\mathbf{P} f' = \frac{1}{\Delta x} \mathbf{Q} f \tag{2.15}
\]

Where $f'$ contains the finite-difference approximations of the spatial derivative of $f$ for a given spacing $\Delta x$. The matrices are invertible, allowing the following:

\[
f' = \mathbf{P}^{-1} \mathbf{Q} f \tag{2.16}
\]

The spacing, $1/\Delta x$, is dropped here as the numerical schemes are optimised with the assumption of an equally spaced grid on which the solution is found. This is achieved through the transformation of the NS equations to general coordinates as described by Eq. (2.1). The coefficients of the numerical schemes are optimised for quasi-spectral performance, in this case the 0.1% error occurs at a wave-number of 0.839$\pi$. To maintain numerical stability and prevent spurious signals from being produced the results of the differentiation must be filtered. The filtering technique is similarly constructed to obtain filtered values, $\hat{f}$. The difference due to filtering, $\Delta f = \hat{f} - f$, is introduced to a similar method as used for the finite-difference calculations. The coefficients are marked with $*$ to indicate that they are specific to the filtered stencil. The formula for the interior points is:

\[
\beta^* \Delta f_{i-2} + \alpha^* \Delta f_{i-1} + \Delta f_i + \alpha^* \Delta f_{i+1} + \beta^* \Delta f_{i+2} = \sum_{n=1}^{3} a^*_n (f_{i+n} - 2f_i + f_{i-n}) \tag{2.17}
\]

The coefficients in this case are constrained such that the a target wave-number can be provided, $\kappa_C$ and the filters will adjust accordingly. The edge nodes are defined with a scheme similar to that of Eqs. (2.12) to (2.14):

\[
\hat{f}_0 + \gamma_{01}^* \hat{f}_1 + \gamma_{02}^* \hat{f}_2 = 0 \tag{2.18}
\]

\[
\gamma_{10}^* \hat{f}_0 + \hat{f}_1 + \gamma_{12}^* \hat{f}_2 + \gamma_{13}^* \hat{f}_3 = 0 \tag{2.19}
\]
\[ \gamma_{20} \Delta f_0 + \gamma_{21} \Delta f_1 + \Delta f_2 + \gamma_{23} \Delta f_3 + \gamma_{24} \Delta f_4 = \sum_{n=0, \neq 2}^{5} a_n^* (f_{i+n} - f_2) \quad (2.20) \]

These filter schemes can be arranged into a matrix equation:

\[ \mathbf{R} \hat{\mathbf{f}} = \mathbf{Tf} \quad (2.21) \]

Because the scheme in \( \mathbf{R} \) is penta-diagonal, the system of equations can be easily solved by forwards and backwards substitution. Therefore, the filters can be implemented as \( \mathbf{R}^{-1} \mathbf{T} \) into the calculation of the derivatives, yielding a complete solution:

\[ \mathbf{f}' = \mathbf{P}^{-1} \mathbf{Q} (\mathbf{I} + \mathbf{R}^{-1} \mathbf{T}) \mathbf{f} \quad (2.22) \]

In the simulation, the steps are separated and spatial derivatives are calculated first, then the filter scheme is applied once per time step to the variables in \( \mathbf{f} \), then these filtered variables are used in the next simulation step. This saves significant computational effort with no appreciable change in the accuracy, an appraisal of this short-cut. The full derivations of matrices \( \mathbf{P}, \mathbf{Q} \) are explained in [Kim (2007)], and the coefficients used in the difference scheme are in [Turner et al. (2016)]. Explanations of the filter cut-off, \( \kappa_C \) and the structure of \( \mathbf{R} \) and \( \mathbf{T} \) can be found in [Kim (2010)]. The value of \( \kappa_C = 0.87 \pi \) was found to be stable during initial tests and all simulations reported are run with this value.

Finally, as the code is developed for parallel use, the computational space is divided into sub-domains. The solution matrices are modified at these boundaries to take into account neighbouring information ([Kim, 2013]). This system preserves numerical accuracy across the boundary by way of a large stencil size. The product of this stencil, called a halo point, is then passed between cores using Message Passing Interface (MPI) using an asynchronous scheme as reported in [Haeri and Shrimpton (2012)].

### 2.2.2 Time marching

The times-marching is performed using a 4 stage Runge-Kutta scheme as described by [Kim and Sandberg (2012)]. The time step is chosen by setting a target CFL and using the following relationship to solve for \( \Delta t \)

\[ \text{CFL} = \max \left( \frac{\Delta t}{\Delta \xi_i} \left( |U_i| + a \sum_{j=1}^{3} \frac{\partial \xi_i}{\partial x_j} \right) \right) \quad (2.23) \]

In this equation, \( U_i \) indicates the local flow speed along the coordinate line which is summed with the local sound speed, \( a \). The sound speed is scaled by the length of the vector which follows the coordinate line. In this implementation, the general coordinates system is of uniform unit spacing and therefore \( 1/\Delta \xi_i \) reduces to 1, simplifying the expression for rearrangement.

### 2.2.3 Boundary conditions

All physical boundaries are treated using General Characteristic Boundary Conditions (GCBCs) derived fully with explanation of implementation in [Kim and Lee (2000, 2004)]. Briefly, the NS equations are cast into a one dimensional general form and then
converted to characteristic form. The dimension is chosen to be that which is normal to the boundary, the other fluxes are then absorbed into a source term. This one-dimensional form is used to solve for the characteristics which are then set by an imposed boundary condition. This modified form of the equations is then used to recover updated primitive variables and update the simulation variables using the relationships between the primitive and conservative variables. Non-reflecting conditions are applied at inlet/outlet (I/O) boundaries, preserving characteristics leaving the boundary and preventing any from entering the domain. At solid walls, incoming characteristics on the wall normal coordinate line are simply reflected to be outgoing, combined with a Drichlet non-slip condition to prevent wall penetration. Iso-thermal conditions are also applied to the walls to satisfy the solution of the energy equation. Finally, in 3D, the faces in $z$ are set to a periodic condition which uses the same technique as sub-block communication described earlier.

Sponge zones are a common treatment for I/O boundaries to prevent the generation of spurious noise (Colonius, 2004). The sponge zones are implemented with the following equation:

$$
S = -\sigma(x, y) \begin{cases} 
\rho - \rho_\infty \\
\rho u - \rho_\infty u_\infty \\
\rho v - \rho_\infty v_\infty \\
\rho w - \rho_\infty w_\infty \\
\rho e - \rho_\infty e_\infty 
\end{cases}
$$

Eq. (2.24) defines a body force based on the difference between the nodal value and the set far-field reference. The strength of the sponge zone, $\sigma$, is set as the specific problem requires, however a maximum value of $\sigma = 4$ is taken unless otherwise specified. The layout of the sponge zones will discussed in more detail in Section 2.3.

2.3 Mesh development and validation

2.3.1 Near-wall mesh

Initial meshes

A 2D near-wall mesh was developed first as it presented the greatest challenge and could be later extruded to produce a 3D near-wall mesh as needed. The GridPro software is used to generate a computational mesh with globally orthogonal and continuous grid lines using its elliptic solver. Unlike the case of a free cylinder, where the cylindrical coordinates system can be used, this case requires a conformal map from a cylinder to a flat boundary. This is because the code’s numerical accuracy and stability significantly improves when the coordinate lines are orthogonal and continuous. Prior work using less sensitive methods allowed for discontinuous interfaces in order to place a disk inside a rectangular domain, see Lei et al. (2000) for an example.

Using a rectangular outer boundary, regions of high skew developed in the mesh, particularly at the corners. These areas produced numerical noise that lead to a build up of spurious signals even when strong sponge zones were applied. Therefore, the rectangular domain was replaced with a super-ellipse defined by the following equation:

$$
\left| \frac{x-a}{b} \right|^n + \left| \frac{y-c}{d} \right|^n = 1,
$$

The domain centre is located at $(a, c)$ and the domain extent in $x$ and $y$ is $2b$ and
Figure 2.1: Sample domain with a cylinder at $G/D = 1.5$, showing the sponge zone strength, $\sigma$, in grey-scale. Inter-block boundaries are shown by the dashed lines and domain boundaries are solid. The red circle indicates a singularity in the boundary conditions across a coordinate line that is explained in Section 2.3.1.

$2d$ respectively. The power of the super-ellipse, $n$, is set to 32, high enough to ensure the wall remains flat from the leading to trailing edge. Sufficient flatness is defined as the gradient of the wall, $dy/dx$, being below $10^{-6}$. In addition, the use of a super-ellipse allows for rounded corners that are continuous, removing the issue caused by a rectangular domain.

Figure 2.1 shows the first domain generated around a cylinder at a gap height of 1.5 with a large super-ellipse as the outer boundary. During initial tests with $n = 32$, it was found that, due to the tight corners, a large domain led to the most relaxed solution. Maximum size in $x$ and $y$ of 50D could provide about 40 diameters of flat wall. Further testing revealed that GridPro allows points in the gap region to move to far apart, creating fairly large aspect ratios in the region of interest. It is believed that this is due to the meshing engine attempting to make the mesh as uniform as possible at all positions.

To remedy this, some additional surfaces were introduced to control the gap region. Here the bi-polar coordinates system is used to create a orthogonal, continuous surface from the cylinder to the wall as shown in Figure 2.2. These are defined using:

$$x = \alpha \frac{\sin \sigma}{\cosh \tau - \cos \sigma} \quad (2.26)$$

$$y = \alpha \frac{\sinh \tau}{\cosh \tau - \cos \sigma} \quad (2.27)$$

In these equations, $\tau$ describes concentric circles that grow around the foci and $\sigma$ describes the angle between the triangle formed between the foci and a point in the domain. The factor $\alpha$ describes the distance between the two foci and the origin. For each mesh, $\alpha$ is defined such so that a value of $\tau > 0$ describes the cylinder surface, ensuring that any choice of $\sigma$ will describe a coordinate line that is orthogonal to the cylinder surface.

These surfaces prevent GridPro from moving points outside of this region and effec-
Figure 2.2: Domain showing the hard control surfaces that were introduced. The effect on the mesh in the gap is shown. The mesh shown has every second point removed.

tively constrains the solution, maintaining a high resolution in the gap. Figure 2.2 also shows a second control surface connecting the cylinder to the leading and trailing edges. This surface was generated by a high resolution mesh that was allowed to fully relax. These coordinate lines were extracted, spline fitted in MATLAB, then re-imported as control surfaces. The control surface connecting the cylinder to the leading edge can be seen by the dashed lines in Figure 2.1.

The technique focuses as many cells as possible in the gap and near cylinder regions, out to about 3 diameters around the cylinder. However, the constraint creates large cell volumes in the far-field, limiting usefulness in cases where resolution of the vortex street is necessary. An additional problem is marked in the Figure 2.1 where the large control surface acts as the interface between two blocks. The block that contains the wall boundary condition (from \(-40 \leq x \leq 40\)) is tangential to I/O condition in the second block. The boundary conditions create noise a singular point is developed where conflicting information is being passed between blocks via the halo-points.

Figure 2.1 shows how the sponge zones are thickened to extend over this interface, in this case they terminate at 35 diameters up and downstream of the cylinder. This section of wall without sponge treatment is referred to as the effective wall length, \(L\). It is clear from Eq. (2.24) that the boundary layer will not grow as it would on a regular flat plate due to forcing at the edges. The full effects of this will be explored in Section 2.3.1. For the initial mesh validations, the sponge zones are thickened from the edge of the domain for 15 diameters and the maximum sponge zone strength is set to \(\sigma(x, y) = 4\).

Optimisation of mesh resolution

The first mesh developed was coarse, denoted as M1 and contains 17,280, as shown in Figure 2.3a. Refined meshes are generated by subdividing the existing cells between nodes. To assess the grid convergence, cells belonging to the initial coarse mesh are subdivided by 2, 3 and 4 new cells to produce progressively finer grids which are named M2, M3 and M4 respectively. An example of this refinement is shown in Figure 2.3b.
Figure 2.3: Effect of mesh subdivision for developing meshes of increasing resolution.
where a multiplier of 4 was applied. Grid convergence tests were performed on these grid levels with the following parameters: CFL = 0.8, Ma = 0.3, Re_D = 600.

Figure 2.4a shows the variation of the coefficient of pressure on the cylinder surface with increasing mesh multiplier. The coarsest mesh, M1, produces higher stagnation and base pressure compared to the other results. Increasing the resolution, Figure 2.4a shows that M2 is in agreement with M3 and M4 in the region of negative pressure but still over-predicts the front stagnation pressure as seen around θ = π. The pressures in the finest mesh, M4 (277,776 cells), show very little variation compared at M3 (156,006 cells), however incur a significant computational cost. Overall, the results with the M3 grid show a reasonable trade off between accuracy and computational cost. Furthermore, the mesh satisfies the minimum spacing of 0.005D normal to the cylinder surface, a constraint reported by Kim et al. (2010). Therefore, subsequent grids were developed using the M3 mesh as a template, maintaining similar minimum sizes and numbers of points.

Adopting the M3 mesh, time independence tests were carried out using CFL ∈ {0.5, 0.8, 0.9} and running the test mesh for the same Ma and Re_D for 500 time units. The results are presented in Figure 2.4b, showing some drift as the CFL is increased. Taking the FFT of these signals shows a difference in frequency of 0.06% between a CFL of 0.9 and 0.5 which is producing the drift observed. Because the change in the results is so small, any value CFL ≤ 0.9 is appropriate.

Effect of domain on boundary layer

Due to the use of sponge zones, the effective wall length (non-dimensional) becomes ˜L = 35, on which a boundary layer grows naturally. To assess the impact on the boundary layer, a set of 15 simulations were performed for Ma ∈ {0.30, 0.40, 0.45} and Re_D ∈ {400, 500, 600, 700, 800} using a gap height of G/D = 3.0. The cylinder interacts with the boundary layer even at this gap height as shown in Figure 2.5a. Cylinder stagnation pressure propagates upstream as shown by the line y = 0. The pressure increases by approximately 1% above the far-field pressure from presented with y = 45 line, until about x = −20. Therefore, being sufficiently undisturbed by the presence of the cylinder, this location x − ˜L = 15 was chosen for extraction of the boundary layer profile. The time-averaged stream-wise velocity was interpolated to produce a series of ˜u(x, y) profiles as shown in Figure 2.5b. A comparison to boundary layer theory will confirm the proper development of the boundary layers and allow estimation of the boundary layer thickness at the cylinder.

An equivalent flat plate Reynolds number is defined in terms of that of the cylinder in Eq. (2.28). The parameter, ˜x = x + ˜L, is introduced to recenter the coordinate system on the effective leading edge of the wall. The Re_˜x values are less that 5.6 × 10^4 for all Re_D considered in this work. This is well below the empirical transition (Schlichting, 2014) for a laminar flat plate boundary layer, Re_x ≪ 3 × 10^5 for the maximum effective wall length of 2L. Therefore, the Blasius laminar boundary layer theory (Schlichting, 2014) is used to define a universal wall-normal coordinate, ζ(x, y), as follows:

\[ \zeta = ˜y \left( \frac{˜x}{Re_D} \right)^{-1/2} \]  \hspace{1cm} \text{(2.29)}

Where ˜y = y + G/D + 1/2 is another adjustment to set ζ to zero on the wall. The
(a) Convergence of the cylinder surface pressure as the mesh density increases.

(b) Effect on the lift signal due to increasing CFL number using the M3 mesh.

Figure 2.4: Mesh and time independence testing at $Ma = 0.3$, $Re_D = 600$. 

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Figure 2.5: Formation of the boundary layer and pressure distributions with a cylinder present near the wall. a) Pressure distributions upstream of the cylinder at $G/D = 3.0$. b) Boundary layer profiles at $x - \tilde{L} = 15$. 
lines of $\bar{u}(x, y)$, examples of which are presented in Figure 2.5b, can now be transformed into lines of $\bar{u}(\zeta)$ based on Eq. (2.29).

Figure 2.6 shows a sample of the boundary layer profiles expressed as the ratio of local to the far-field stream-wise velocity as a function of the coordinate $\zeta$. The omitted Reynolds and Mach numbers vary smoothly between these results. By defining the boundary layer thickness, $\delta$, to be the distance at which $\bar{u}(\zeta)/U_\infty = 0.99$, we can extract the boundary layer thickness as a function of $\zeta$. Using this transformation, a thickening of the boundary layer due to increasing Mach number is observed. This effect is well documented in thermal boundary layers in the book by Schlichting [2014]. For $Ma = 0.30$, the boundary layer thicknesses average around $\zeta \approx 5.2$, thicker than the Blasius solution of $\zeta = 5$ when $u/U_\infty = 0.99$. Increasing the Mach number to 0.45 moves the average up to $\zeta = 5.55$.

Previous incompressible works use the boundary layer thickness at the cylinder centre location to investigate the effects on drag and lift. Table 2.1 shows the values of $\delta/D$ for the considered $Re_D$ and $Ma$ values at this position, $\bar{x} = L$ which is equivalent to $x = 0$. These were calculated by finding values of $\zeta$ as above and rearranging Eq. (2.29) to solve for $\delta = (y + h)$.

Table 2.1: Table of boundary layer thickness, $\delta/D$ at the cylinder centre ($x = 0$)

<table>
<thead>
<tr>
<th>$Re_D/Ma$</th>
<th>0.3</th>
<th>0.4</th>
<th>0.45</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>1.4760</td>
<td>1.5537</td>
<td>1.5926</td>
</tr>
<tr>
<td>500</td>
<td>1.3376</td>
<td>1.4071</td>
<td>1.4418</td>
</tr>
<tr>
<td>600</td>
<td>1.2527</td>
<td>1.3003</td>
<td>1.3320</td>
</tr>
<tr>
<td>700</td>
<td>1.1892</td>
<td>1.2332</td>
<td>1.2626</td>
</tr>
<tr>
<td>800</td>
<td>1.1398</td>
<td>1.1810</td>
<td>1.2085</td>
</tr>
</tbody>
</table>
Validation of near-wall meshes

Using the M3 mesh as a template, further meshes were developed for $1.0 \leq G/D \leq 3.0$ to correctly capture the effects shown by Lei et al. (2000). Their data shows a significant change in the fluctuating lift coefficient from 1.5 to 1.0. The fluctuating lift coefficient is the centred root-mean-squared of the lift, defined as:

$$C'_L = \left( \frac{\sum (C_L - \bar{C}_L)^2}{n} \right)^{1/2}$$  \hspace{1cm} (2.30)

These meshes were run for all gap-heights at $Ma = 0.2$ and $Re_D \in \{400, 600, 800\}$ for comparison to the data of Lei et al. (2000) for which good agreement is presented in Figure 2.7. Discrepancies in the $C'_L$ values appear, firstly at higher Reynolds numbers where the maximum error is $Err = 5.9\%$ at a gap height of 3.0. Secondly, at lower Reynolds numbers and $G/D < 1.5$ where the errors grow to a maximum of $Err = 13.6\%$. The differences in boundary layer thicknesses in our simulations versus the reference could explain the discrepancy. For example, the line of $Re_D = 400$ is fully immersed in the boundary layer at $G/D = 1.0$ where $G/\delta < 1$, contrast this with the results of Lei et al. (2000) where the boundary layer thickness is $0.68D$ and equivalent $G/\delta = 1.47$.

2.3.2 Free cylinder mesh

Compared to the near wall case, a mesh for a free cylinder is easier to generate and the cell count can be reduced by using cylindrical coordinates. As shown in the previous section, there is a minimum off wall spacing required to properly resolve the flow. However, some optimisations can be made in the tangential direction to reduce the number of points required for the cylindrical coordinates system defined by:
Figure 2.8: Mesh of $N_r, N_\theta = \{121, 260\}$ and clustering in both the radial and angular directions. The inset shows the mesh near the cylinder with every second point in each direction removed.

\begin{align*}
x &= r \cdot \cos(\theta) \\
y &= r \cdot \sin(\theta) \\
z &= z
\end{align*} \tag{2.31}

In order to improve accuracy with a limited number of points along both $r$ and $\theta$, a simple clustering equation is used (Farrashkhalvat and Miles, 2014). A coordinate line $\eta$ where $0 \leq \eta \leq 1$ with $n$ points placed equally along it can be biased towards $\eta = 0$ using the index, $i$ and a clustering parameter $a$.

\[
\eta(i) = \frac{(a + 1) + (a - 1)(\frac{a+1}{a-1})^{(1-\frac{i}{n})}}{(\frac{a+1}{a-1})^{(1-\frac{i}{n})} + 1} \tag{2.32}
\]

The effect of this clustering is shown in Figure 2.8. Cells are biased towards the cylinder surface along lines of constant $r$. Clustering is also used around $\theta = 0$ to improve the resolution in the wake. As the effect of the cylinder is not significant at greater than 20 diameters from the source, the maximum radius is set to $25D$ to allow some space for the sponge zone to be added. As the minimum off-wall spacing is known from the previous section, the number of radial points is set to 121. The number of angular points is varied $N_\theta \in \{132, 260, 400\}$.

Figure 2.9a shows no change in the coefficient of pressure due to angular resolution,
Figure 2.9: Change in key parameters for increasing angular resolution in 2D.
the stagnation pressure at the leading edge is 1 as expected and the base pressure at \( \theta = 0 \) is 1.70, close to the 1.64 of the incompressible work of Henderson (1995). Figure 2.9b shows that the drag and lift signals are in agreement, with a delay in the initial instability associated with shedding due to increasing cell count. These tests confirm that large aspect ratios can be allowed at the front of the cylinder with little effect on the solution. Therefore, highly clustered meshes can be used with little impact, with large aspect ratios at the front stagnation point of between 8 and 10. However, the lowest cell count is not satisfactory for a 3D wake. This is because of the changes to do with the re-circulation, as recorded by Jiang and Cheng (2017). According to their data, using the relationships for \( D'/D \) and \( LF \) at \( Re_D = 800 \), a region of \( 0.5 \leq x \leq 2.0 \) and \(-0.7 \leq y \leq 0.7 \) must be resolved. By increasing the bias to the rear of the cylinder and allowing the aspect ratio at the front of the cylinder to grow, a final mesh is generated using \( N_r, N_\theta = \{121, 200\} \).

3D cases are built by extruding this base mesh along the z-axis and applying periodic conditions in the span-wise direction. The span of the initial cylinder was 2 diameters, the minimum required to capture at least one pair of vortices in the \( xz \) plane (Mittal and Balachandar 1995). This principal vortex size was based by the work of Mansy et al. (1994) where the vortex size at \( Re_D = 400 \) is 1 diameter, dropping to 0.78 at \( Re_D = 800 \).

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The 2D mesh is equally spaced along the cylinder length \( L_z = 2 \), \( N_z \in \{35, 51, 101\} \), for a mesh size, \( \Delta z = \{0.0588, 0.04, 0.02\} \). The effect of the length, \( L_z \), is also considered with domains of 4D and 6D with a resolution of \( \Delta z = 0.05 \) as recommended by Jiang and Cheng (2017). Finally, a very high resolution mesh was created as a benchmark with a domain size of \( r_{max} = 40 \). This large mesh uses the most cells possible to ensure accuracy, \( N_r, N_\theta, N_z = \{251, 400, 501\} \) and minimum spacing of \( \Delta x, \Delta y, \Delta z = \{0.005, 0.005, 0.02\} \). These are all run at CFL = 0.9, Ma = 0.3 and \( Re_D = 800 \).

Figure 2.10a shows that the pressure distributions are similar regardless of the number of points included. However, Figures 2.10b and 2.10c show that the signals have a higher variance compared to the relatively smooth results at \( L_z = 10D \), implying that there is some effect due to the tightness of the domain.

Figure 2.11a shows that the results predict the pressure at the cylinder well, with some small variations again due to the turbulence. Comparing Figures 2.11b and 2.11c to Figures 2.10b and 2.10c shows that lengthening the domain removes this variance. The difference in the drag and the Strouhal number due to increasing \( L_z \), \( L_z = 4 \) and \( L_z = 10 \), is \( \approx 2\% \) in both cases. We choose the mesh of \( L_z = 4, \Delta z = 0.05 \) as it represents the least computational cost while correctly resolving other physical characteristics. The values of the clustering parameter in Eq. (2.32) used to generate the final mesh is 1.003 and 1.4 in the radial and angular directions respectively.

Figure 2.12 shows the ratio of the average cell length to the Kolmogorov micro-scale for the final 3D mesh. Here the value of \( \eta \) is the Kolomogorov micro-scale,

\[ \eta = 5(\nu^3/(1 + Ma_t)\epsilon)^{0.25}. \]

The multiplier of 5 is added as a benchmark length to properly resolve the necessary scales (Laizet et al. 2015 [Turner and Kim 2020]). The dissipation of turbulent kinetic energy, \( \epsilon \), takes the maximum value of \( 2\nu S_{ij}S_{ij} \) where \( S_{ij} \) is the rate of strain of the fluctuating components of velocity. The additional factor of \((1 + Ma_t^2)\) is introduced to take account of turbulent dilation due to compressibility. For subsonic flows, Sarkar et al. (1991) showed that dilation term is approximately \( Ma_t^2 \) of the solenoidal strain rate. The value of the solenoidal part is taken at the edge of the shear layers where there is a transition to turbulence based on the values of Bloor (1964) as a guide. The largest value of \( \epsilon \) in the shear layer across the parameter space was found for \( Ma = 0.45 \), \( Re_D = 800 \) and therefore, the mesh is designed around
Figure 2.10: Change in key parameters with increasing cell count along $L_z = 2D$ compared to the high resolution results. Line legend applies to all figures.
Figure 2.11: Change in key parameters with the increasing domain size compared against the high resolution results. Line legend applies to all figures.
Figure 2.12: Ratio of cell dimension to five times the Kolmogorov micro-scale (η) showing that wake region, particularly where turbulent transition occurs is well resolved. The dissipation of turbulent kinetic energy, ϵ, is calculated for Ma = 0.45, Re_D = 800.

this case. The other parameter combinations do not require as fine a mesh resolution. This claim is supported by the dimensional analysis of the equation for the micro-scale reducing to η = 5Re_D^{-3/4}, lower Reynolds numbers will produce large values and therefore require lower resolutions. Figure 2.12 confirms that this mesh can capture the turbulent behaviour in the near wake as J^{1/3}/η < 1 up to a distance of 3 diameters downstream. This well resolved region contains both the formation and transition lengths (Bloor 1964) as well as the detachment into the wake at 2 ≤ x ≤ 3. As the 2D mesh for the near-wall case has smaller cell volumes in this important region it can be extruded using the same parameters with no loss in accuracy.

2.4 Analysis techniques

Throughout the remaining chapters, a variety of derived quantities will be used repeatedly. These are covered here to aid the reader and shall simply be invoked in the text. Section 2.4.1 will cover the scaling and manipulation of fluid variables by well established techniques such as the mean and coefficient of pressure. Section 2.4.2 will cover the proper orthogonal decomposition in detail with attention paid to the type of POD used and the inner product attached to the vector space to generate the correlation tensor.

2.4.1 Scaling and derived quantities

The coefficient of pressure is defined using the isentropic relations (AMES Research Staff 1953):

\[ C_P = \frac{P - P_\infty}{P_{stag.} - P_\infty} \]  

(2.33)

This corrects for the effects due to compressibility that the regular definition cannot.
The denominator is normally \( \rho U^2 \) as in Eq. (1.2), which is the stagnation energy of the flow assuming no change in density. The change in density is accounted for in the following definition of the stagnation pressure.

\[
P_{\text{stag.}} = P_\infty \left(1 + \frac{\gamma - 1}{2} M_\infty^2 \right)^{\frac{\gamma - 1}{\gamma}}
\]  

(2.34)

The vorticity is defined by the curl of the velocity field, the general definition is as follows:

\[
\omega_i = \varepsilon_{ijk} \frac{\partial u_j}{\partial x_k}
\]  

(2.35)

This planar vorticity is a good way to find vortices in 2D simulations. In 3D however, especially for initial identification, the Q criterion is preferred. Q criterion is formally defined as regions where the second invariant of the velocity gradient tensor is positive. A complete discussion of this criterion can be found in Kolár (2007).

\[
Q = \left( \frac{\partial u_i}{\partial x_i} \right)^2 - \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} > 0
\]  

(2.36)

As in the study of incompressible fluids, the first term, \( \partial u_i^2 / \partial x_i \) representing the divergence of the flow is ignored for calculations within this thesis. This is because, as will be discussed in Section 3.1.1, the divergence in the wake and vortex street is near-zero and hence this term can be removed.

Forces on the cylinder surface are expressed as the components of the surface integral of the stress tensor:

\[
F_i = \int_{\partial S} \sigma_{ij} n_j
\]  

(2.37)

Where,

\[
\sigma_{ij} = \tau_{ij} - \delta_{ij} P
\]  

(2.38)

\( \partial S \) represents the closed loop of the cylinder surface. The components of \( F \) can be time-averaged, then normalised to yield the coefficients of drag and lift. In this case, the parameters \( \rho_\infty, a_\infty \) and \( D \) are dimensionless reference quantities with value of unity.

\[
C_D = \frac{2 F_x}{\rho_\infty (a_\infty M_a)^2 D}
\]  

(2.39)

\[
C_L = \frac{2 F_y}{\rho_\infty (a_\infty M_a)^2 D}
\]  

(2.40)

An important parameter derived using \( C_L \) is the fluctuating coefficient of lift. This is the centred root-mean-square (RMS) of the lift signal. The root-mean-square is defined as:

\[
\text{RMS}(C_L(t)) = \sqrt{\frac{\sum_{i=1}^{n} C_L^2(t_i)}{n}}
\]  

(2.41)

Centring is achieved by removing the mean in time of \( C_L \), \( \bar{C_L} = \langle C_L(t) \rangle \), from the signal before calculating the RMS. The final derived quantity is the variance of
Figure 2.13: FFT of the first four oscillating modes of a POD analysis of a free cylinder in 3D at \( \text{Re}_D = 800 \) and \( \text{Ma} = 0.45 \).

The fluctuating components of velocity. This is proportionate to the fluctuating kinetic energy and is defined for \( u \) as:

\[
\frac{u'^2(x)}{U_\infty^2} = \frac{1}{U_\infty^2} \langle (u(x, t) - \langle u(x, t) \rangle)^2 \rangle
\]  

(2.42)

Where the angled brackets indicate a mean in time, \( t \), and a normalisation has been added using the square of the reference speed.

2.4.2 Proper orthogonal decomposition

Proper orthogonal decomposition (POD) is a method to extract prominent flow features to shed some light on complex interactions. This technique was discussed recently in the study of vortex induced vibrations of near-wall cylinders by Li et al. (2017). The technique is known to have limitations, particularly when features of interest have a low signal-to-noise ratio or when the flow is highly turbulent. In the case of the Reynolds and Mach numbers for this thesis POD is still a reasonable technique. Primarily, there is transition to turbulence in the wake of the cylinder, however, the shear layers on the cylinder surface are completely laminar. Therefore in the evaluation of forces on the cylinder, a main research question of this thesis, are known to be regular and dominated by a single frequency. Unal and Rockwell (1988) is a good example, showing that the principle frequency of the lift force is much greater in amplitude than those of other flow features.

In their paper Sieber et al. (2016) demonstrate the improved performance of a spectral POD over the regular POD, showing that the regular POD failed to capture the correct frequency for a respective spatial mode. However, as shown in Figure 2.13, the POD captures the behaviour and the peaks are large with respect to any numerical noise.

Some additional issues using POD where identified for Reynolds numbers less than 200, however, this is due to a unique feature of that range. Wherein, the behaviour of
the vortex street is defined by mode switching. This means that large windows of time and large data-sets are required to properly resolve the flow as it moves from one mode to another. It is well known however that for Reynolds numbers above about 250 this effect dissipates, see Jiang and Cheng (2017), and the flow is dominated by a single mode of vortex shedding.

When applied, the POD separates spatial and temporal modes of a data set and orders them in a quantitative manner. This allows discussion of the spatial components (modes $\phi^k(x)$) and their fluctuations in time (coefficients $a^k(t)$) with a sample size, $K$, large enough such that:

$$u(x, t) \approx \sum_{k=1}^{K} a^k(t) \phi^k(x)$$ (2.43)

In this work the POD implementation is based on the “snapshot method” of Sirovich as described by Cordier and Bergmann (2002) to solve Eq. (2.43). Correlation tensors are created using the standard inner product for a scalar field which in turn induces an $L^2$ norm. For the snapshot method, the correlations are found over the spatial coordinate as the number of points in space is much larger than the number in time. The spatial correlation tensor is then split into its components using the spectral method. The original data set, a matrix $Q$ which has dimensions $N_s \times N_t$ for the total number of spatial points and samples in time respectively, is decomposed by SVD into three separate matrices:

$$Q = U \Lambda V$$ (2.44)

The spatial modes $\phi^k(x)$, are stored in the columns of the $N_s \times N_s$ matrix $U$. Time coefficients $a^k(t)$ are in the rows of the $N_t \times N_t$ matrix $V$. The time coefficients in $V$ are the eigenvectors of the spatial correlation tensor. The diagonal matrix $\Lambda$ – of size $N_s \times N_t$ – contains the modal energies, $\sqrt{\lambda^k}$, where $\lambda$ are the eigenvalues of the spatial correlation tensor. The nature of $\Lambda$ shows that the resolution of the method, hence the range of $k$, is limited by $\min(N_s, N_t)$. In this case we are limited by $N_t$, the column and row indexes greater than $N_t$ of $U$ and $\Lambda$ respectively are equal to zero. For later analysis of the POD contours, it is possible to normalise POD data by reference values. A generic reference quantity, $q$, which could be the reference pressure $P_\infty$ for example, can scale a mode, $\phi^k(x)$, according to the following:

$$\tilde{\phi}^k(x) = \frac{\phi^k(x)}{\phi^k_{ref}}$$ (2.45)

Where the reference value $\phi^k_{ref}$ is defined as:

$$\phi^k_{ref} = \frac{q}{\bar{a}^k \sqrt{\lambda^k}}$$ (2.46)

The value of $\bar{a}^k$ is computed using the signal $a^k(t)$ and has to be non-zero. In later analysis we will take the root-mean-square (RMS) of the signals as this constant. To validate this approach, Figure 2.14 shows the scaling of the first mode of pressure for $G/D = 1.5$, $Ma = 0.45$ and $Re_D = 800$ compared to the $C_P$ extractions presented earlier. Here the values of $C_P$ are calculated by extracting $\phi^1$ of pressure from the surface of the cylinder and scaling the reference pressure, $P_\infty$, using Eq. (2.46). Comparison of the lines shows that this mode alone is enough to effectively reconstruct the pressure on the cylinder with a maximum error $< 2\%$. 38
Figure 2.14: Reconstruction of $C_p$ using parameter scaling of POD data matched to a time averaged $C_p$ distribution. Data is taken from Chapter 4.
Chapter 3

Free cylinder in subsonic cross-flow at moderate Reynolds numbers

In this chapter, the 3D simulations are performed on a mesh of $\xi, \eta, \zeta = 121, 200, 81$ and a cylindrical domain with an outer radius $r = 25.5$ and a span of $L_z = 4$. The results of the parametric study on a 3D cylinder in a subsonic flow are presented and discussed for $Ma = \{0.20, 0.30, 0.35, 0.40, 0.45\}$ at $Re_D = \{400, 600, 800\}$. The drag and lift data and derived quantities are discussed in detail. Some key deviations from the incompressible literature are identified in the data. These deviations are explored using both time-averaged fields and the POD method. The conclusion of the parametric study then leads into a discussion of the validity of 2D testing for compressible flows.

3.1 Results and discussions

3.1.1 Sample flow fields

Figure 3.1 shows the divergence of velocity at $Ma = 0.45$ and Reynolds numbers of 400 and 800. The accelerated fluid travelling around the front of the cylinder is strongly diverged in comparison to that of the wake. This gives confidence that the following Q-criterion figures properly capture the fluid motion despite the removal of the divergence term in Section 2.4.

Figure 3.2 shows contours of Q-criterion in the wake behind the cylinder for increasing Mach and Reynolds number. The figures show the free shear layers rolling up in the wake of the cylinder as well as some detached vortices moving downstream. All figures are selected to represent a similar state in vortex production with the roll-up of a clockwise vortex and the detachment of the anticlockwise vortex into the wake. Figure 3.2a and Figure 3.2b show the effect of increasing the Reynolds number at $Ma = 0.20$. At $Re_D = 400$, Figure 3.2a shows clear structures that are distinct from one another, vortex cores in the wake are surrounded by “fingers” that are associated with $\omega_z$, the vorticity on the stream-wise axis. Increasing the Mach number at $Re_D = 400$ increases the strength of vortex structures with no clear change in the formation as shown in Figure 3.2c.

These structures are less distinct at $Ma = 0.20$, $Re_D = 800$, as shown in Figure 3.2b. The clockwise vortex roll-up is present, however the detaching vortex appears to be dissipating in the wake. Moreover, any vortex structures similar to the discrete vortex
(a) $Ma = 0.45$, $Re_D = 400$.

(b) $Ma = 0.45$, $Re_D = 800$.

Figure 3.1: Divergence of velocity at $Ma = 0.45$, $Re_D = 400$, 800
Figure 3.2: Q-criterion in the wake of the cylinder for $Ma = \{0.20, 0.45\}$, $Re_D = \{400, 800\}$. The iso-surfaces are set to 0.005, 0.010 and 0.015.
cores seen in Figure 3.2a cannot be identified in the wake in Figure 3.2b. However, increasing the Mach number causes these structures to reappear, Figure 3.2d shows the vortex cores in the wake and a more coherent structure associated with the detaching anticlockwise vortex.

Figure 3.2 shows that there is a coherent shear layer emanating from the cylinder surface which is relatively 2D until transition occurs downstream. The main component of this coherent shear layer is the xy-plane vorticity that is shown in Figure 3.3. At Ma = 0.20, Re_D = 800, as shown in Figure 3.3b the shear layers are much longer than those seen at Re_D = 400 in Figure 3.3a. In both figures, the recirculating fluid appears to bound between the strong layers of shear and the centre of the re-circulation pattern appears to be roughly located at the termination of the shear layer. This observation holds as the flow speed increases, however, the overall length of the shear layers decreases. This is less noticeable at Re_D = 400, but is pronounced at Re_D = 800 with the point with the focal points of the recirculating fluid moving about half a diameter upstream.

3.1.2 Forces of the cylinder

The trends observed in the flow fields should be reflected in the loads on the cylinder. Figure 3.4 shows the time history of the drag and lift for a range of Mach and Reynolds
Figure 3.4: Drag and lift signals at set Reynolds numbers showing the various characteristics due to change in Mach number.
(a) FFT of the lift signals at $Ma = \{0.20, 0.35, 0.45\}$ $Re_D = 600$. Here the frequencies have been converted to Strouhal numbers.

(b) Principal frequencies extracted from the FFT for all Mach and Reynolds numbers. There is a general trend for the drag to drop as Reynolds number increases, in line with observations of the drag curve in 3D (Wieselsberger 1922). Increasing the Mach number appears to cause the average drag to increase and makes the fluctuations more intense at all Reynolds numbers. This effect is particularly noticeable at $Re_D = 800$ shown in Figure 3.4c. Here the drag curve is almost flat at $Ma = 0.2$ and the lift signal is of lower amplitude compared to higher Mach numbers. The weak oscillations in $C_D$ and the low amplitude of $C_L$ at $Ma = 0.20$ and $Re_D = 800$ compared to other Reynolds numbers further indicates that the shedding behaviour is weakened as the Reynolds number increases.

Figure 3.5a shows the FFT of the lift signals for increasing Mach numbers at $Re_D = 600$. The signals are dominated by a single frequency, denoted as the principal frequency, which is used to define the Strouhal number for each flow case. The conversion to Strouhal numbers is based on the free-stream flow speed:
(a) Dimensionless shedding frequency, $St$, with the min/max of the reference data of Unal and Rockwell (1988).

Figure 3.6: Change in key measures of vortex shedding due to increasing Mach number.

(b) Fluctuating coefficient of lift, $C'_L$.

Figure 3.6: Change in key measures of vortex shedding due to increasing Mach number.

$$St = \frac{f}{U_\infty D} \tag{3.1}$$

Where the free-stream flow speed is defined using the Mach number at the inlet, $U_\infty = a_\infty Ma$. When adjusted for the effect of flow speed the results cluster to a fixed value, indicating little impact of increasing Mach number on the shedding frequency. The collapse of the data seen in Figure 3.5a is reflected in linear trend of the principal frequency shown in Figure 3.5b. The frequency change is dependent solely on Mach number as changes due to the Reynolds number are limited (Jiang and Cheng, 2017; Unal and Rockwell, 1988).

The Strouhal numbers for all parameter combinations are shown in Figure 3.6a where the dash-dotted lines represent the maximum and minimum values of $St$ reported by Unal and Rockwell (1988). There is a general decreasing trend as the Mach number increases, however the results are mostly contained within the limits of the incompressible literature. A second measure the vortex shedding is the fluctuating co-
The forces on the cylinder and the associated vortex shedding are known to be influenced by the pressure distribution, in particular the stagnation on the upstream face and the low pressure region in the wake (Roshko 1954, Henderson 1997). The additional effect of the compressible flow is a further increase in the stagnation pressure for a given speed due to the change in density. This increase in the stagnation pressure is well predicted for a wide range of Reynolds numbers by the isentropic stagnation model (AMES Research Staff 1953). Figure 3.7a shows the agreement of the ratio of stagnation to ambient pressure at increasing Mach number with the model. Figures 3.7b
Figure 3.8: Characterisations of the pressure distributions on the cylinder. Legend shared between figures.

to 3.7d shows the pressure distributions on the cylinder, presented as the coefficient of pressure, showing the stagnation point at $\theta = \pi$. The figures only show half of the distribution as the cylinder is in an undisturbed flow and the curves are therefore symmetric about the y-axis. In agreement with Figure 3.7a the stagnation pressures shown at $\theta = \pi$ in Figures 3.7b to 3.7d stay close to unity for increasing Mach number. Figure 3.7d shows that the change due to increasing Mach number is weak at $Re_D = 400$, and most of the data follows the quasi-incompressible case of $Ma = 0.20$. At $Ma = 0.45$ there is a drop in the base pressure, visible at $\theta < 1.0$. This drop in base pressure is more pronounced at $Re_D \geq 600$ as shown in Figures 3.7c and 3.7d. Figure 3.7d also shows that base pressure is decreasing at lower Mach numbers and the results are more spread out than at $Re_D \leq 600$.

Figure 3.8a shows the base pressure on the cylinder for the entire range of Mach and Reynolds numbers, including extra data that was generated for the $Re_D = 800$ case. Figure 3.8a shows that the base pressure is decreasing as the Mach number increases, however, the non-linear pattern seen in the coefficient of pressure is not as
Figure 3.9: The point of flow stagnation in the wake and the base pressure at $Re_D = 800$ with additional data points showing the correlation between the two parameters.

obvious here. This indicates that the relatively small changes in the base pressure are being accentuated by the computation of the coefficient of pressure. Moreover, the figure shows that the pressures at the base follow a similar trend regardless of Reynolds number, being within $\leq 2\%$ of one another. As shown in Figure 3.8b, the difference between the Reynolds numbers is the drop in pressure across the rear of the cylinder. This figure shows the effect of the pressure distributions by taking the integral of the absolute value of the pressure:

$$\int |C_P| \, d\theta \quad (3.2)$$

This integral can take account of both drops in the base pressure and increases in the stagnation should they occur. In this case, Figure 3.7a shows that the stagnation pressure is stable, therefore this integral take account of the pressure drops over the rear of the cylinder. Figure 3.8b shows that the pressure is fairly consistent at $Re_D = 400$ as the Mach number increases. This stability is reflected in the clustering of pressure curves around a stable base pressure coefficient shown in Figure 3.7d. At $Re_D = 600$, there is a deviation at $Ma = 0.45$ which again reflects the large pressure change shown in Figure 3.7c. At $Re_D = 800$ the change is more pronounced, as $Ma > 0.20$, the curve shows a marked increase in the integral, reflective that the decrease in the pressure across the rear seen in Figure 3.7d. The results at $Re_D = 800$ follow the trend of $Re_D = 600$ as the Mach number increases, including the uptick at $Ma = 0.45$. As will be demonstrated in the following sections, these small changes have a marked effect on the flow field as the Mach number increases.

This non-linear effect in the coefficient of pressure, due to the small variations shown in Figure 3.8a, is further demonstrated by the additional data. $Re_D = 800$, with additional data at $Ma = \{0.250, 0.275, 0.325, 0.425\}$, is presented in Figure 3.9. The length of the formation region $L_F$ is also presented here, defined as the distance from the cylinder centre to the wake stagnation point. The wake stagnation point is where, excluding the position of zero velocity on the cylinder rear, the flow speed is zero in the wake along the centre-line. At $Ma = 0.20$ the base pressure is within
2.5% of the incompressible value of predicted by Henderson (1995) and the formation length is within 2% of the data of Jiang and Cheng (2017). The base pressure drops significantly between Ma = 0.20 and Ma = 0.30, accompanied by similar contraction of the formation region. This is followed by an increase in pressure and an elongation of the formation region up to Ma = 0.35, before both decrease rapidly to an global minimum at Ma = 0.45. The data shows that the formation region length is strongly coupled to the base pressure and is sensitive to the relatively small changes presented in Figure 3.8a. Across the Reynolds numbers, the overall decrease in base pressure between Ma = 0.20 and Ma = 0.45 is about 13.5%. However, at Re_D = 800 the formation region contracts by around 35% whereas the other Reynolds numbers only experience a 12% contraction. This in conjunction with the large increase in shedding intensity suggests a unique effect due to higher Mach numbers at Re_D = 800.

Another effect of these pressure differences across the cylinder is the induced drag. Figure 3.10 shows the total drag on the cylinder and the pressure and viscous components for all Ma and Re_D. As the Mach number increases, Figure 3.10a shows that the drag on the cylinder increases inline with the isentropic model for Ma ≤ 0.4 at Re_D = 400. At Ma = 0.45 the drag is under-predicted by the isentropic model with the discrepancy growing as the Reynolds number increases. As Reynolds number increases above 200, the drag decreases for an incompressible flow and the pressure differential over the cylinder is responsible for most of the drag (Henderson, 1995). The deviations in the drag are attributed in particular to the decreasing pressures across the rear of the cylinder, shown in Figures 3.7b to 3.7d, which induces the increase in pressure drag seen in Figure 3.10b. This is for two reasons, firstly, the stagnation pressure is in agreement with the predictions of the isentropic model and therefore predicts that drag should similarly follow the model. Secondly, the viscous component of the drag shown in Figure 3.10c is increasing weakly with Mach number, a more obvious increase at higher Reynolds numbers. However, the viscous contributions are small and confirm that the change in the pressure across the rear of the cylinder, characterised by Figure 3.8b, is the dominant effect and causes the increase in drag.

Canuto and Taira (2015) showed that the isentropic model predicts the compressible drag well at Re_D = 200. However Nagata et al. (2020) found, for Re_D ≥ 1000, that the drag increase due to Mach number does not follow the isentropic model. The data presented here shows that there is a transition, from Re_D = 400 where the behaviour mostly follows the model, where as Re_D = 800 diverges substantially. Therefore, the subsequent analysis will focus mainly on the Re_D = 800 case with comparisons to the Re_D = 400 results as these appear to represent a transition between two different behaviours.

3.1.3 Compressible effects on the formation region

Due to the strong coupling between the base pressure and the formation region length, it is important to further investigate the dynamics in the wake. Incompressible experiments and models have shown the behaviour of the shear layers is intimately related to the wake and base pressures.

Nagata et al. (2020) proposed that a change in the shear layers, both the separation angle and the maximum distance between the layers, was related to the deviation of the drag from the model. For the range of Reynolds numbers covered here, Figure 3.11a shows the separation angles of the flow based on the positions of minimum pressure either side of the base. Take Figure 3.7c for example, the position of minimum pressure is located around θ = 1.8 for all Mach numbers. Figure 3.11a shows that the maximum
Figure 3.10: Drag forces on the cylinder for parameter space. The solid black lines in Figure 3.10a are the isentropic model zeroed using the incompressible drag from Wieselsberger (1922).
(a) Angle of minimum pressure based on the cylinder surface pressure distributions. Filled symbols are the angle of the minimum at $\theta > 0$, empty symbols are at $\theta < 0$.

(b) $\omega_z$ extracted across a line of constant $x = 0.5$ for increasing Mach number at $Re_D = 600$.

Figure 3.11: Characterisations of the separated shear layer.
Figure 3.12: The ratio of the speed in shear layer against the approximation based on the base pressure.

angle change, from $Ma = 0.20$ to $Ma = 0.45$, is about 1 degree between the extrema of Mach number with the change in angle being small until $Ma > 0.35$. However, there is no noticeable change in the distance between shear layers to accompany this change in angle. The distance between shear layers, $D'$, is taken to be the distance between peak vorticity extracted on a line of constant $x$, $(x = 0.5)$. The distance between the shear layers is normalised by the diameter and denoted as $D'/D$. Figure 3.11b shows the distributions of vorticity extracted along the line. There is no clear change in the shear layer due to increasing Mach number. Taking the positions of peak vorticity the value of $D'/D = 1.22$, in agreement with the data of Jiang and Cheng (2017), $D'/D = 1.21$. This agreement with Jiang and Cheng (2017) is further found for all $Ma$ and $Re_D$ tested. In contrast to the findings of Nagata et al. (2020), the dominant effect in the wake is the drop in pressure within the formation region and not a change in the shear layers that bound it. This is further evidenced in Figure 3.11b, showing that increasing the Mach number is strengthening a region of shear at the rear of the cylinder where $|y| < 0.5$.

The model of Roshko (1954) shows that the speed ratio in the shear layers is related directly to the base pressure on the cylinder, see Eq. (1.4). Figure 3.12 shows the ratio between maximum speed in the shear layer extracted from the domain against the maximum speed calculated using the base pressures and Eq. (1.4). Figure 3.12 shows a discrepancy of maximum 2.5% at $Ma = 0.45$ and $Re_D = 400$. However, as most of the points are within 2% of unity, Figure 3.12 demonstrates that the model of Roshko (1954) works well to explain the behaviour of the base pressure in a compressible case.

Figure 3.13 and Figure 3.14 show the effect of increasing the Mach number from 0.20 to 0.45 for all the Reynolds numbers tested. At low speeds, Figure 3.13 shows that the wake bubble moves away from the cylinder as the Reynolds number increases. In addition, the overall pressure in the wake bubble is increasing with Reynolds number, reflecting the results of the incompressible literature.

Increasing the Mach number however leads to a decrease in the wake pressure in agreement with the model of Kurosaka et al. (1987). Again, the pressures are generally lower in the $Re_D = 400$ case as shown in Figure 3.14a. However, comparing Figure 3.14c
Figure 3.13: Time averaged pressure fields near the cylinder at $Ma = 0.20$. Contour values apply to all sub-figures.
Figure 3.14: Time averaged pressure fields near the cylinder at Ma = 0.45. Contour values apply to all sub-figures.
Figure 3.15: Normalised $P$ and $u$ presented for varying Mach number at $\text{Re}_D = 800$. Pressure is normalised against the base pressure for each Mach number.

to Figure 3.13c, it is clear that the low pressure in the wake has advanced substantially upstream. Furthermore, it is clear that lower pressures from the formation region are interacting with the rear of the cylinder more strongly at $Ma = 0.45$ compared to $Ma = 0.20$. This interaction would explain, not only the drop in base pressure, but the general decrease in pressure along the back of the cylinder shown in Figure 3.7d.

Figure 3.15 shows extractions made from the rear of the cylinder to 3D into the wake, enough to capture the formation region as per Bloor (1964). Figure 3.15a shows the wake pressure normalised by the base pressures, see Figure 3.8a, taken from rear of the cylinder. By normalising in this way, it is clear that increasing the Mach number is causing the wake pressure to decrease further than what can be accounted for by the drop in base pressure. At $\text{Re}_D = 800$, the special case at $Ma = 0.30$ is visible, the drop in wake pressure is similar to that at $Ma = 0.35$. However, the position of minimum pressure is further upstream in the $Ma = 0.30$ case, reflective of the contraction in the formation region demonstrated in Figure 3.9. The coupling between the base pressure and the formation length is further demonstrated in Figure 3.15b where the x-axis is zeroed on the rear of the cylinder at $x = r$ and then normalised by the zeroed formation region length $L_F - r$. These show that the position of minimum pressure is fairly constant, at around 75% of the zeroed formation length.

Figure 3.15c shows little change in the stream-wise velocity when the effect of increased Mach number is taken into account. Although the position of minimum speed moves with respect to the formation region size, the minimum speed appears to be
uncoupled from the deviations in the minimum wake pressure. Therefore, the velocity profiles in the formation region are only dependent on wake pressures in so far as the pressure dictates the formation region length. This is demonstrated in Figure 3.15d where the velocity profiles collapse onto one another with a position of minimum $u$ just upstream of the minimum pressures at about 60% of the zeroed formation length.

Figure 3.16 shows that the trend in the minimum wake pressure follows similar trend between Reynolds numbers to that seen in the base pressure. The decreases in the minimum wake pressure appear to also be weakly dependent on Reynolds number. This indicates that the general decreasing trend in the base and minimum wake pressures are a function of the Mach number. Furthermore, the growing disparity between base and minimum wake pressure is a function of the Mach number, a purely compressible effect.

However, these combined effects are coincident with changes in the cylinder drag coefficient that cannot be accounted for by the isentropic model and are dependent on the Reynolds number. The Reynolds number also appears to be an important factor in the shedding intensity, characterised by $C'_L$, which also shows a non-linear relation to the Mach number.

The generation of lower wake pressures is coincident with increasing vortex shedding intensity at $Re_D = 800$. A measure of the intensity of vortex shedding is the normalised $v'^2$ which is shown in Figure 3.17. This describes the fluctuating stream-normal velocity associated with the development of a vortex. Figure 3.17a shows the stability of the generating structures as the Mach number increases at $Re_D = 400$. At $Re_D = 800$, raising the Mach number increased the fluctuating coefficient of lift, Figure 3.17b shows that this is coincident with the increase in the variance within the formation region. The line between $0.75 \leq x \leq 2.00$ demonstrate the sensitivity of vortex formation to the small pressure changes, with a significant increase in the variance between $Ma = 0.20$ and $Ma = 0.30$. The variance then decreases slightly as the Mach number increases to 0.40, the variance then increasing to the maximum at $Ma = 0.45$. Deeper into the wake, at $x = 3.0$, the variances cluster closer to one another which, along with the increased activity at $x = 1.50$, indicates that the significant flow changes are occurring.
Figure 3.17: Change in $v'$ on lines at $x = \{0.00, 0.50, 0.75, 1.00, 1.50, 2.00, 3.00\}$
in the formation region.

\( \sigma^2 \) is also important in vortex generation, representing the fluctuating momentum fed from the shear-layers into the formation region (Unal and Rockwell 1988). The trends of Unal and Rockwell (1988) for the \( \sigma^2 \) and \( C_{p,\text{base}} \) follow the trend of \( C'_{L} \) shown by Norberg (2003). Figure 3.18 shows that the stream-wise fluctuations within the shear layers near the cylinder \( (x < 2) \) intensify with the Mach number at \( Re_D = 800 \). Figures 3.17 and 3.18 demonstrate that the trend of increased vortex shedding intensity at \( Re_D = 800 \) is reflected in the fluctuating fields of \( v \) and \( u \). The values are fairly constant at \( Re_D = 400 \), with limited changes in the variance and a fairly stable \( C'_{L} \) as the Mach number increases. At \( Re_D = 800 \) however, the fluctuations are close to zero at \( Ma = 0.20 \). As the Mach number increases, the general trend is an increase in the variance at positions known to be critical to vortex shedding. This shows that, despite the base and minimum wake pressures being dependent on Mach number, the effect of these changing static fields varies by Reynolds number.
Figure 3.19: The first four oscillating modes of pressure at $Ma = 0.30$ and $Re_D = 800$. The modes have been scaled by both the energy and $RMS(a(t))$ for comparison, the contours are locked at $\pm 0.001$.

3.1.4 Coherent structures and vortex generation

At $Ma = 0.45$, $Re_D = 800$, Figure 3.15a shows that the minimum wake pressure is upstream of the maximum variance of $v$ which is located at the end of the formation region ($x = L_F$) in Figure 3.17b. However, positions of lower pressure are not collocated with the variance of $u$, implying that a fluctuating pressure field that is removed due to averaging may be responsible. The POD procedure covered in Chapter 2 is used to separate spatial and temporal modes of pressure. In this case, the pressure data is provided as an input without first removing the average, causing the first mode to be equivalent to the standard time-averaged pressure field. In this particular case, the subsequent modes are all oscillatory about zero, representing the different fluctuating components of the pressure in time.

Figure 3.19 shows the first four oscillating modes of the pressure field, the first mode has been removed as it is static and equivalent to the time-averaged pressure. All of the modes have been scaled according to Section 2.4.2 and the contours are locked at $\pm 0.001$. This means that these structures highlighted by the contours are of comparable contribution to the overall flow solution. Figure 3.19a shows that the second mode has a strong pair of alternate sign on the top and bottom of the cylinder which are fairly
even along the length. There is a second, fairly even distribution in the wake of the cylinder, with greater 3D effects on and a general reducing in size of structures deeper into the wake.

Figures 3.19b to 3.19d show that modes 3, 4 and 5 have limited interaction with the cylinder compared to the second mode. Figure 3.19b shows a pair of laminar structures of alternate sign similar to those in mode 2. However, these first appear in the wake of the cylinder with only very weak distributions on the cylinder surface. 3D effects again appear on structures deeper into the wake and the structures generally shrink in size more readily than those in mode 2. Figures 3.19c and 3.19d have smaller laminar structures but these are only alternating in the wake direction.

As the second mode appears to interact with the cylinder the most, this is considered further in Figure 3.20 as the Mach number increases. These modes show that increasing the Mach number is causing an even distribution of fluctuating pressure to appear on the top and bottom of the cylinder. At Ma = 0.20 these distributions are absent which is due to the generally weaker fields close to the cylinder surface. At higher Mach numbers, the structures close to the cylinder appear to be fairly laminar with limited variation over the span.

Figure 3.20 shows that increasing the Mach number causes the fields to generally become stronger, this is seen as an increase in the size of the iso-surface between
Figures 3.20 and 3.20d for example. This increase is also reflected in the modal energies which are shown for the extrema of Reynolds number in Figure 3.21. Figure 3.21 shows that the modal energy associated with modes other than the first is increasing with the Mach number. The first mode is a special case of a time stable mode, analogous to the averaged $P$ field where the energies are very similar regardless of Mach and Reynolds number. The difference in energies in subsequent modes indicates that these are particularly sensitive to changes in Mach and Reynolds number. At low Mach numbers, the difference in energy for modes 2 and 3 is about $1/2$ comparing $Re_D = 800$ to $Re_D = 400$. Increasing the Mach number causes the energies to increase in general as well as closing the gap between energies due to Reynolds number.

Key flow characteristics are contained in the second mode of pressure and can be reconstructed, for this example the case of $Ma = 0.45$ and $Re_D = 800$ is used. The fluctuations in the coefficient of lift can be reconstructed using a single mode. Limiting the matrices created by the POD procedure to the second mode of pressure (column/row 2 in the matrices), an approximation of the complete data-set can be created. The mean field in mode one is discarded as the calculation of $C'_L$ subtracts the average from the data before calculating the RMS, rendering the inclusion of mode one redundant. The temporal average of these reconstructed pressure fields is then followed by integration of pressure forces on the cylinder surface. The result of this is shown in Figure 3.22, compared against the pressure component of the lift signal taken from the simulation. Calculating a value of $C'_L$ for each gives 0.4549 and 0.4459 for the raw data and the reconstruction respectively. There is a difference of 2% between the raw data and the reconstruction. This is due to the coarseness of the data as well the removal of some of the features of the lift signal which are contained in other modes. The pressure component of the RMS coefficient of lift is the dominant contributor at around > 90% of the total lift signal compared to the viscous component depending on Reynolds number. The values of $C'_L$ are also higher than those recorded in Figure 3.6b. This is because the signal captured is fluctuating more strongly at this particular range of time. The PODs presented later include additional data recorded over 300 time units, enough time for these effects to dissipate.
Figure 3.22: Reconstruction of the pressure component of the lift signal by only using the second POD mode of pressure. Data presented is for the \( \text{Ma} = 0.45, \text{Re}_D = 800 \) case.

Figure 3.23: Mid-span slice of the second mode of pressure for varying Mach and Reynolds numbers.
Figure 3.23 shows the second mode of pressure at the mid-span of the cylinder. The modes are anti-symmetric over the x-axis and hence only half is presented. At \( \text{Ma} = 0.20 \), comparing Figures 3.23a and 3.23b, increasing the Reynolds number weakens the fluctuating field on the cylinder surface. When combined with the fact that the modal energy decreases by 50% between \( \text{Re}_D = 400 \) to 800, this weakened field in Figure 3.23b explains the low fluctuating coefficient of lift at \( \text{Ma} = 0.20, \text{Re}_D = 800 \).

Increasing the Mach number causes the fluctuations to become stronger at both Reynolds numbers. At \( \text{Re}_D = 400 \), Figure 3.23c shows that the distributions on the cylinder surface are stronger and that the region of high fluctuation just downstream \( (x \approx 1.0) \) has moved upstream compared to a similar structure in Figure 3.23a. At \( \text{Re}_D = 800 \), the change in the distributions on the cylinder is similar to the \( \text{Re}_D = 400 \) case. Figure 3.23d shows that the fluctuations at the top of the cylinder and those in the close to \( x = 1.0 \) have increased in strength substantially. In addition, the increase in modal energy for the second mode is 42% due to the increase in Reynolds number. This means that the distributions at \( \text{Re}_D = 800 \) contribute significantly more to the complete solution \( \text{Ma} = 0.45 \).

Figure 3.20 shows that the distributions close to the cylinder are largely even over the span as well as being fairly laminar. This implies that there may be a fundamentally 2D structure close to the cylinder that is influencing the behaviour. Figure 3.24 shows the effect on the xy-plane Reynolds stress. In essence, identifying the fluctuating rotating parts of the flow as \( R_{12} = \rho u'v' \). This is normalised using the square of the reference velocity \( U_\infty \) to remove the effect of increasing flow speed. This Reynolds stress...
is chosen because the prominent structures identified in Figure 3.2 are mostly described by the xy-plane vorticity. Figure 3.24 reflects this, the regions of stress closest to the rear of cylinder are collocated with the recirculating time averaged flow shown in Figure 3.3. Figures 3.24a and 3.24c show that increasing the Mach number at Re$_D$ = 400 does little to change the underlying structures. However, Figure 3.24d shows that the increasing Mach number at Re$_D$ = 800 causes the structures to become significantly stronger and advance upstream.

The effects of the reintroduction of quasi-2D flow features at Re$_D$ = 800 due to increasing Mach number can be seen comparing the results in Figure 3.25. In this figure, the vorticity in the xy-plane, $\omega_z$, is normalised by $D/U_\infty$ and the surfaces represent $\omega_z D/U_\infty = \pm \varepsilon$. The data was chosen to represent a position of maximum $C_L$, where a low pressure region is generated at the top of the cylinder associated with negative (clockwise) vortex roll up. These flow fields are from the same files used to generate the fields in shown in Figures 3.2b and 3.2d respectively. At Ma = 0.45, shown in Figure 3.25b, the shear layer emanating from the top of cylinder is rolling up tightly at position A. Compare this to position A in Figure 3.25a where the shear layer roll-up is less pronounced and occurs further downstream from the cylinder. Finally, comparing between the figures at position B, the detaching positive vortex is more coherent along the span in the Ma = 0.45 case. At Ma = 0.2, the vortex sheet appears to break apart into the flow field downstream and lacks the smooth layers feeding back under the core of the vortex at point A.

### 3.2 Comparison to 2D data.

Figure 3.26 shows the difference in the second mode of pressure between 2D and 3D simulations at Re$_D$ = 800 due to increasing flow speed. The 2D data is shown in the solid contour, the contour lines show the 3D data. The contour lines show the development of the higher fluctuation intensity with increasing Mach number. Figure 3.26a shows that, in 3D, the fluctuations are very weak in comparison to Figure 3.26b. The distributions on the cylinder top, the minimal contour (contour number 8 in Figure 3.26a)
Figure 3.26: Comparison of the scaled second mode of pressure in 2 and 3D at $Re_D = 800$ with increasing Mach number. The filled contours are the 2D data, the lines are the 3D.
number 9 in Figure 3.26a, is five times stronger in Figure 3.26b. Comparing the positions of maximum fluctuation, the pressure field does not show the same advancement upstream seen in Figure 3.24.

So far, some quasi-2D states have been identified for the range of Reynolds numbers at higher Mach and Reynolds numbers. The next chapter uses a 2D near-wall case as an approximation to be more computationally efficient. However, the validity of the 2D approximation is an issue for which there is no reference data for compressible flow at these Mach numbers. As such, the data collected in this chapter is compared to that generated using the 2D version of the code. The extrema of Reynolds numbers are used for comparison as the \( \text{Re}_D = 400 \) appears to be very stable with increasing Mach number and \( \text{Re}_D = 800 \) is relatively sensitive in comparison. Furthermore, the results at \( \text{Re}_D = 600 \) show that the transition between these two cases is related to the Reynolds number alone. The comparisons will focus on the distributions across what will be the gap region as it has the smallest cross sectional area and will therefore induce the largest possible change due to compression. Any effects due to the wall boundary layer are assumed to be 2D as per discussions in Section 2.3.1.

Figure 3.27 shows the stream-wise velocity contours for increasing \( \text{Ma} \) and \( \text{Re}_D \) numbers in both 2D and 3D. The 2D and 3D data are calculated as the time-average of stable simulations, the 3D data is additionally averaged across the span to create 2D slice for comparison. The general trends are, firstly, good agreement upstream
of the stagnation point, indicating that the fluid deceleration is well resolved in 2D. Secondly, growing discrepancies as fluid travels around the cylinder into the wake region. In particular, the 2D data shows flow reversal post-separation and low speed fluid is concentrated much closer to the rear of the cylinder in comparison to the 3D data. These differences are well understood in the literature and are due to the 2D approximation causing an additional circulating fluid region (see Mittal and Balachandar (1995) for example).

However, as previously mentioned, the concern is the gap region and how the flow across that region may differ from 2 to 3D. Therefore, it is the region of low pressure and fluid acceleration around the cylinder that will have the greatest impact on the flow in the gap. At the top of the cylinder, the boundary layers are quite similar across plots, especially at $Re_D = 400$ comparing Figure 3.27a and Figure 3.27c. More variation in the regions of accelerated flow is present at $Re_D = 800$. In Figure 3.27b, at $Ma = 0.30$, the high speed flow is thinner and longer in comparison to the similar contour level in 2D, contour 11. Increasing to $Ma = 0.45$, shown in Figure 3.27d, causes this region to thicken and come closer to that of the 2D simulation in size and position.

To demonstrate similarity, Figure 3.28 highlights the discrepancies across the gap region using line extractions from the top of the cylinder, $(x, y) = (0.0, 0.5)$, to $(0.0, 3.5)$. Variables are presented with normalised values to aid in comparison between plots. Comparing the plots the velocity distributions are very similar, with higher speeds in
the shear layer seen in the 2D tests. This discrepancy is greatest when $Ma = 0.45$, $Re_D = 800$, at 7%. However, the discrepancy decays further away from the cylinder to $< 1\%$ at $y = 1.5$, which is the $y$ position equivalent to a gap height of one. Despite the increase in the speed there is little effect on the boundary layer thickness. Choosing the edge of the boundary layer to be the position of maximum speed, the variation due to the 2D approximation in boundary layer thicknesses is $< 0.1\%$. The normal velocity $v$ is weakly affected relative to $u$, except in the case of $Ma = 0.30$, $Re_D = 800$. Compared to $u$ however, $v$ contributes less to the overall speed at a given point. Moreover, any discrepancies decay before $y = 1.5$, therefore changes to the flow in the gap due to $v$ is negligible.

Fluid density is also weakly affected by the 2D approximation and deviations are low. The largest discrepancy is found at $Ma = 0.45$, $Re_D = 800$ where the maximum change in the density is 2% lower in the 2D case. In comparison to other parameters however, larger differences are found for the pressure in all combinations Mach and Reynolds numbers. Lower pressures are generated on the cylinder surface in the 2D case, especially at $Ma = 0.45$, $Re_D = 800$ where the maximum discrepancy is 7.3%. This is expected however, due to the constraint of 2D dimensions base pressures decrease substantially \cite{Henderson1995} in comparison to the 3D results. Despite the large changes in pressure on the surface, the discrepancy also diminishes to less than 1% at $y = 1.5$. In conclusion, the distributions of key flow variables are similar between tests when $y \geq 1.5$. This is gives confidence that the flow in the gap will not be unduly affected by the 2D approximation for all the gap heights tested in Chapter 4.

### 3.3 Conclusions

In general, the effect of raising the Mach number is to decrease pressure at the rear of the cylinder and the advance of the wake up-stream. The base pressure was shown to follow the model of \cite{Roshko1954} with a direct relationship between increased speed in the shear layers and a decrease in the base pressure. In addition, the length of the formation region was shown to be strongly coupled to the base pressure specifically. Furthermore, the formation length is sensitive to small changes in the base pressure. Despite the movement in the wake however, at $Re_D = 400$ any changes in the loads and shedding behaviour due to these decreased wake pressures are limited. At $Re_D = 400$, shedding intensity is fairly constant and the increase in drag obeys the isentropic rule.

There are however, significant changes in the behaviour at $Ma = 0.45$ as the Reynolds number is increased. At $Re_D = 800$ in particular, the decreasing base pressure is tied to a significant contraction of the formation region compared to the length at low speeds. At $Re_D = 800$ the contraction is $\approx 35\%$ in contrast to $\approx 12.5\%$ for $Re_D \leq 600$. The advancement of the low pressure bubble in the wake leads to a further drop in pressure along the rear of the cylinder that accounts for the increase in the drag that could not be resolved with the isentropic model.

Coupled to this pressure decrease is the restoration of vortex shedding at $Re_D \geq 600$ when $Ma > 0.35$. At $Re_D = 800$ in particular, the fluctuating coefficient of lift is increased threefold with respect to the incompressible reference and strong planar Reynolds stresses reappear as the Mach number is raised. In the incompressible literature, see \cite{MittalBalachandar1995}, the weakening of this plane Reynolds stress is connected to the reduced shedding intensity as the Reynolds number is raised from 200 to 800. Moreover, the decrease in shedding intensity as the Reynolds number increases for an incompressible flow was tied by \cite{UnalRockwell1988, Norberg2003} to
decreases in the fluctuating velocity fields in the shear layers.

In compressible flows, stronger vortex production is tied to increasing pressure defect by the model of Kurosaka et al. (1987) and decreasing base pressure follows the relationship of Roshko (1954). Therefore, increasing the Mach number for \( \text{Re}_D \geq 600 \) advances the formation region closer to the rear of the cylinder due to the lower pressures generated at the base by increased flow speed. This lower pressure closer to the cylinder encourages stronger vortex generation which was shown by the marked increase in the plane stresses, the fluctuating velocity fields and the restoration of the fluctuating pressure field.

The validity of the 2D approximation was also discussed based on the data collected. It was shown that, due to the laminar nature of the shear layers on the cylinder surface, the disagreement between the 2D and 3D cases was negligible at a distance from the cylinder representative of the smallest gap height under consideration in Chapter 4.
Chapter 4

Near wall studies in 2D

This chapter covers the results of a 2D parametric study of a cylinder approaching a wall. Inflow Mach numbers are $0.20 \leq Ma \leq 0.45$ and the Reynolds numbers are $\{400, 500, 600, 700, 800\}$. The gap heights, $G/D$, are $\{1.0, 1.3, 1.4, 2.0, 3.0\}$, run using sponge zones of $\sigma = 4$ and these are extended from the boundary such that the effective wall length is $35D$ as discussed prior in Chapter 2. There is an exception, for the case of $G/D = 1.5$, where the effective wall length is $25D$ due to a systematic error. This shorter wall length results in a thinner boundary layer and the scaled parameters are adjusted accordingly.

4.1 Initial fields and signal analysis

Figures 4.1 and 4.2 show the coefficient of pressure, $C_P$, and the vorticity, $\omega$, respectively for three gap heights $G/D \in \{1, 1.5, 3\}$ at $Ma = 0.45$ and $Re_D = 600$. Figures 4.1 and 4.2 show the general trends that will be further discussed in the following sections. These images are produced at a time of minimal lift and are therefore in a comparable state. Figures 4.1a and 4.2a show the limited impact of the wall on the flow for $G/D = 3.0$. The vortex street is evenly distributed and pressure bubbles associated with these vortices appear to be equal in magnitude. The pressure distribution around the underside of the disk is not obviously affected by the wall, however, there is a weak negative vortex on wall indicating a limited interaction.

Decreasing the gap height to $G/D = 1.5$, Figures 4.1b and 4.2b show interactions with the boundary layer and the wall. The wake is pointed up away from the wall and the vortices are travelling downstream in pairs. The low pressures associated with shear layer roll-up cannot propagate freely into the domain and instead spread out over the wall. In the gap, the weak negative vortex has become stronger and a positive vortex has appeared upstream. In the wake, the positive vortex that has been shed, at $x = 4$ in Figure 4.2b, is interacting weakly with the boundary layer. This interaction is collocated with a region of higher pressure in Figure 4.1b. The pressure located at the vortex centre is higher than that of the companion negative vortex. Deeper into the wake, the discrete pairs of vortices are rotating clockwise as they travel downstream.

At $G/D = 1.0$ ($G/\delta = 0.751$), the cylinder is now partially immersed in the boundary layer. Figure 4.1c shows the tail to be further off centre with the pressure bubbles in pairs. The pairs are spaced further apart and higher pressures appear in the spaces in-between. The vortices on the wall are stronger in comparison to Figure 4.2b and an additional positive vortex has appear downstream of the gap. Comparing Figure 4.2c to Figure 4.2a, the vorticity field close to the cylinder does not change with wall proxim-
Figure 4.1: Contours of $C_p$ for a similar stage of the vortex shedding, varying $G/D$ at $Ma = 0.45$ and $Re_D = 600$. 
Figure 4.2: Contours of $\omega$ for a similar stage of the vortex shedding, varying $G/D$ at $Ma = 0.45$ and $Re_D = 600$. 

(a) $G/D = 3.0$

(b) $G/D = 1.5$

(c) $G/D = 1.0$
ity, however there are stronger interactions post-separation. Figure 4.1c shows that the pressure associated with a positive vortex is higher than that of the companion vortex. This discrepancy in size and magnitude appears as there is a stronger interaction between the pair and the boundary layer. Further in the wake, the vortices are rotating about a shared axis, the rate of rotation is higher that of Figure 4.2b.

Figures 4.1 and 4.2 showed the fields at a position of minimum lift coefficient, taken directly from the simulations. These forces in the x and y directions, with a conversion to drag and lift coefficients, are presented for further analysis.

Figures 4.3 and 4.4 show the drag and lift in time for the same parameters as Figures 4.1 and 4.2. The G/D = 3.0 results show no clear interactions with the wall. The lift signals do not vary between gap heights of 3.0 and 1.5, however, drag shows some change when G/D ≤ 1.5. At a gap height of 1.5, shown in Figures 4.3b and 4.4b, there are some changes in the drag signal. Starting during a trough in the lift cycle and a corresponding position of maximum drag in Figure 4.3b and advancing in time, the peak of the drag signal coincident with a positive cycle of the lift is diminished compared to the first. This then leads into a trough that represents the minimum drag before the pattern repeats. Moreover, increasing the Mach number at this gap height causes this effect to become more pronounced with little change in the lift.

Figures 4.3c and 4.4c show that a further reduction in the gap height results in a stronger suppression during the peak lift at low speed. Figure 4.3c shows that the amplitude of the weaker cycle (described by $C_{D}C - C_{D}B$) has been reduced to 24.86% of the maximum amplitude ($C_{D}A - C_{D}D$). Figure 4.4c shows an inversion of the behaviour seen at G/D = 1.5, increasing Mach number in Figure 4.4c recovers some of the amplitude in the drag as $(C_{D}A - C_{D}D)/(C_{D}C - C_{D}B) = 32.56%$. The amplitude recovery due to increasing the Mach number is about 31%. To investigate these changes in amplitude further, the signals are transformed into the frequency domain using a Fast Fourier Transform (FFT).

Figure 4.5 shows the effect on the frequency of drag and lift in two cases of increasing the flow speed at G/D = 3.0 and G/D = 1.0. At the largest gap height, shown in Figure 4.5a, the classical behaviour of a free cylinder is observed. The drag and lift signals produce one peak each, with the drag at double the frequency of the lift. The lift frequency will be useful in further discussions, we shall refer to it as the “primary frequency”.

Figure 4.5b shows the drag and lift relationship at G/D = 1.0 discussed earlier with Figures 4.3c and 4.4c. The proximity to the wall causes a second peak in the drag at the primary frequency. This second peak is of comparable magnitude to that of the first at low speed. At Ma = 0.45 the first peak is shown to increase in magnitude, about 29% relative to the second. This effect mirrors the earlier discussions in time, in Figure 4.4c, as the recovery of the signal amplitude between points $C_{D}B$ and $C_{D}C$ in Figure 4.4c.

Taking the lift frequencies next, both cases in Figure 4.5 show that increasing the Mach number increases the primary frequency. In Figure 4.5a, for example, the primary frequency is increased from 0.0697 to 0.1055. Converting these to Strouhal numbers ($St = f/UD$ where $U = u_\infty Ma$) however, results in 0.232 and 0.234 respectively, showing little change when the effect of flow speed is removed.

Figures 4.6 and 4.7 show that the shedding characteristics are dependant on gap-height and Reynolds number for fixed Mach numbers. Figure 4.8 shows more detail of the effect of Mach number on shedding characteristics for Reynolds numbers of 400 and 800 at fixed gap heights. Both sets of figures show, at G/D = 3.0, increasing the Mach number decreases the fluctuation intensity, especially as Ma > 0.3. When G/D = 1.5,
Figure 4.3: Lift and drag signals plotted over 100 time units for varying $G/D$ at $Ma = 0.30$, $Re_D = 600$. 

(a) $G/D = 3.0$. 

(b) $G/D = 1.5$. 

(c) $G/D = 1.0$. 

Figure 4.3: Lift and drag signals plotted over 100 time units for varying $G/D$ at $Ma = 0.30$, $Re_D = 600$. 

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Figure 4.4: Lift and drag signals plotted over 100 time units for varying $G/D$ at $Ma = 0.45$, $Re_D = 600$. 

(a) $G/D = 3.0$

(b) $G/D = 1.5$

(c) $G/D = 1.0$
Figure 4.5: FFT of drag and lift at two different gap heights showing variation due to Mach number at Re_D = 600. Right scale is $C_D$, the left hand side is $C_L$. 
Figure 4.6: Fluctuating coefficient of lift and Strouhal number dependent on the gap-height.
there is a change in behaviour that is related to the Reynolds number. Figure 4.8a shows that the values of $C'_L$ trend upwards with Mach number at $Re_D = 400$. At $Re_D = 800$ however, Figure 4.8c shows the positive trend tends to a maximum around $Ma = 0.40$ where further increasing Mach number causes $C'_L$ to drop. When $G/D < 1.5$ the overall value of $C'_L$ decreases but maintains a positive relationship with $Ma$ even as the cylinder is immersed in the boundary layer.

Figure 4.8b and Figure 4.8d show the dependency of $St$ on Mach number. Comparing these figures with Figure 4.7 $St$ appears independent of Mach number at $G/D = 3.0$. At this gap-height there is no effect from the wall as the values here are close to the values found by Lei et al. (2000) and match the 2D data of Jiang and Cheng (2017). Values of $St$ follow the incompressible results for $Ma \leq 0.40$ where the shedding frequency is dependent on the gap height. At this speed and for any fixed $Re_D$, $St$ increases slightly as the gap height decreases, to a maximum value of $St$ when $G/D = 1.5$, before decreasing substantially.

At $G/D = 1.5$, increasing the Mach number increases shedding frequency. Especially at $Re_D = 800$ where there is a maximum increase of 8.5% compared to results at $G/D = 3.0$. Comparing Figure 4.8d to Figure 4.8b increasing the Reynolds number makes this effect more pronounced. Figure 4.8d shows that increased $St$ when $Ma > 0.3$ starts at $G/D = 2.0$ and is sustained until $G/D = 1.0$ Figure 4.7c shows that this effect dissipates as the Reynolds number decreases with a small peak at $Re_D = 400$. This is seen in Figure 4.8b where a weak peak forms at $G/D = 1.5$ and $Ma > 0.40$, before the effect disappears, coincident with boundary layer immersion.

Figure 4.9 show the time averaged drag and lift as a function of gap height divided by boundary layer thickness, $G/\delta$, calculated using values of $\delta$ from Table 2.1. This adjustment is made to aid the analysis as Buresti and Lanciotti (1992); Lei et al. (1999) and Zdravkovich (1985) showed that for “thick boundary layers” (defined as $\delta/D \geq 1$) the drag and lift effects are dependent on this ratio.

There are two general trends at large gap heights (when $G/\delta \geq 2.0$) similar to results in the literature. Firstly, there is a weak positive lift for all Mach and Reynolds numbers. Secondly, increasing drag with Mach number at these larger gap heights can be accounted for by the increasing stagnation pressures (AMES Research Staff, 1953) on the cylinder as:

$$C_D \propto \left[1 + Ma^2 \left(\frac{\gamma - 1}{2}\right)\right]^\frac{\gamma}{2(\gamma - 1)}$$  \hspace{1cm} (4.1)\

This relationship is shown by the black lines in Figure 4.11a and Figure 4.11c. The incompressible formula of Henderson (1995) gives a reference drag, this is used as the coefficient of proportionality for Eq. (4.1). Figure 4.11a and Figure 4.11c show that the drag at $G/D = 3.0$ is well predicted by the isentropic law. When $Ma < 0.35$, drag and lift remain fairly constant until the cylinder comes close to the boundary layer, when $G/\delta \leq 1.25$ which is similar behaviour to the incompressible research Lei et al. (1999). It is around this gap height where the shear layers on the cylinder are touching the outer surface of the boundary layer. Figure 4.9a and Figure 4.10a both show a decreasing trend as the gap height decreases below $G/\delta \leq 1.25$, with drag falling and lift reversing from being weakly upwards to more strongly downwards.

Increasing the Mach and Reynolds numbers causes some deviations from these trends at $1.0 \leq G/\delta \leq 1.5$, an example at $Ma = 0.45$ is shown in Figure 4.9c. Here the cylinder experiences increasing drag as the Reynolds number increases. At $Re_D = 800$ Figure 4.11c shows that the drag deviates as $Ma > 0.3$ compared to both the data
Figure 4.7: Fluctuating coefficient of lift and Strouhal number dependent on the gap-height.
Figure 4.8: Fluctuating coefficient of lift and Strouhal number for $Re_D = \{400, 800\}$. The dependence of lift and St on Ma for fixed gap heights is evident.
Figure 4.9: Dependence of the lift and drag forces on the gap-height to boundary layer thickness ratio.
Figure 4.10: Dependence of the lift and drag forces on the gap-height to boundary layer thickness ratio.
Figure 4.11: The dependence of drag and lift coefficient on Ma for fixed $G/D$ and $Re_D = \{400, 800\}$.

at $G/D = 3.0$ and the approximation. At $Ma = 0.45$ the drag peaks at about 8.5% above the reference value. Figure 4.10c shows at the lift this region of $G/\delta$ varies from positive to negative with increasing Reynolds number. Figure 4.11d shows that the onset of negative lift is delayed up to around $Ma = 0.40$. The lift then recovers for all Mach numbers as $G/\delta \leq 1.25$, the lines of $G/D \leq 1.4$ follow similar trends. The specific change in behaviour between $1.0 < G/\delta < 2.0$ will be covered in detail in later sections.

Figure 4.9 shows that the drag is reduced for $G/\delta < 1.0$ for all Mach numbers and the data shows a proportional relationship for all Reynolds numbers. Figure 4.11a and Figure 4.11c show the compressible effect is not completely removed by immersion, rather the overall drag drops. Figure 4.10 shows that the magnitude of negative lift is dependent on both the Mach and Reynolds number. Compare Figure 4.11d to Figure 4.11b at $G/D = 1.0$. For $Ma \leq 0.25$ the negative lift is similar, however as the Mach number increases the lift decreases. The rate of this decrease is tied to Reynolds number, higher $Re_D$ experiencing more negative lift at higher Mach numbers. This effect can also be seen in Figure 4.10 as the Mach number increases, the results for $G/D = 1.0$ spread out.

In summary, these basic statistics of the flow show that there is a sudden change in the loads on the cylinder at $G/D = 1.5$ as $Ma > 0.40$. These appear to occur when
the cylinder sufficiently close to a wall yet not immersed in a boundary layer. The best example shown is of $Re_D = 800$ at gap heights around 1.5 where significant increases in drag and shedding frequency are coincident with a sudden loss of lift. The $Re_D = 400$ results are a good comparison as they are shown to be dependent mostly on gap height and boundary layer thickness. Importantly, they do not show the large deviations in drag, lift and shedding frequency experienced by higher Reynolds numbers. From this point forwards, analysis will focus on this special case of increased drag and shedding at $G/D = 1.5$, $Ma = 0.45$ and $Re_D = 800$, with a comparison to the $Re_D = 400$ results.

### 4.1.1 Analysis of pressure forces

Figure 4.12 shows the independent contributions of pressure and viscous forces to the drag and lift coefficients. These are calculated by separating the pressure and viscous terms in the stress tensor in Eq. (2.37). Pressure is the dominant contributor to the total drag and lift, for example, the pressure contribution to the time-averaged drag is between 87% and the 92%. The variation in this range is due to the increasing Reynolds number and agrees with the work of Henderson (1995) who found similar contributions for an incompressible flow around a free cylinder.

Figure 4.13 shows the variation of the coefficient of pressure on the surface of a cylinder approaching the wall from $G/D = 3.0$. The increments of $G/D$ in each figure were chosen such that their corresponding values of $G/\delta$ are similar. For example, at a Mach number of 0.45 the gap heights of 1.3 and 1.0 for Reynolds numbers of 400 and 800 respectively have $G/\delta$ approx 0.87. In addition, as $G/D$ decreases from 3 to 2 at $Re_D = 400$, $G/\delta = 1.88$ down to $G/\delta = 1.25$ crosses the value of $G/\delta = 1.47$ for $G/D = 1.5$ at $Re_D = 800$.

The front stagnation point is the peak around $\theta = \pm \pi$ and the rear stagnation is close to $\theta = 0$. Pressure distributions that have been shown in the literature for higher Reynolds numbers in the sub-critical range, the pressure curve has two minima with a fairly flat pressure distributed over the rear quadrant of the cylinder ($-1.2 \leq \theta \leq 1.2$). However, for the range of Reynolds numbers considered, a third minimum appears around $\theta = 0$. This third minimum is known to be a product of the two-dimensional approximation for this range of Reynolds numbers and was shown by Henderson (1997) and Mittal and Balachandar (1995). The base pressure in 2D is defined the same way as for 3D, taking the minimum pressure close to $\theta = 0$.

Figure 4.13a shows the results at $Re_D = 400$ where, at gap heights of 3.0 and 2.0, there is a decrease in pressure at the rear of the cylinder as the Mach number increases.
Figure 4.13: The variations of $C_p$ distribution on the cylinder surface. See Figure 1.3 for the definition of $\theta$. 
This is expected based on the literature for compressible flow around a cylinder where the points of inflection move downstream and base pressure decreases (Zdravkovich, 1997). Further decreasing the gap height to where the cylinder is immersed in the boundary layer increases the base pressure. It appears that the effects of Mach number are being removed on the side of the cylinder that is closest to the boundary layer ($-2 \leq \theta \leq 0$).

Increasing the Reynolds number to 800, shown in Figure 4.13b, causes some key deviations. Firstly at $G/D = 3.0$ and independent of Mach number, the pressure distributions are weakly asymmetric around $\theta = 0$, favouring the underside of the cylinder. Lowering the gap height to 1.5 when $Ma = 0.30$, the pressure distribution follows that of the quasi free stream case. However, increasing the Mach number to 0.45 causes a large decrease in the pressure favouring the negative side of the cylinder. The bias towards the negative side of the cylinder and greater magnitude of the drop at the rear are coincident with the sudden negative lift and large drag increase show in Figures 4.9 and 4.10. The final set of lines show an immersed case, however the effect of Mach number is not completely removed. It appears that stronger low pressure is still produced at the rear of the cylinder in this case with a bias towards the top side. Contrasted with Figure 4.13a, this effect of the Mach number appears dependent on the Reynolds number.

To further compare the effects of $G/D$, $Ma$ and $Re_D$ on the pressure fields, take lines of constant $y$ at select locations downstream of the cylinder for cases of matching $G/D$. The time-averaged coefficient of pressure for a selection of positions from the rear of the cylinder to 3 diameters downstream are shown in Figure 4.14. The general trend is an decrease in minimum pressure when the Reynolds number increases. The difference in minimum pressure between Reynolds numbers is smallest at $x=1.50$ where, both upstream and downstream of this point, the pressure differences grow in magnitude.

The first thing to consider is a changing gap height at $Re_D = 400$ where very little variation is seen in the results. There is some drift deeper into the wake, particularly obvious when $x \geq 2$ comparing the $y$ positions of minimum pressure. When the cylinder
is closer to the wall at $G/D = 1.5$, points of minimum pressure are to be shifted away from the wall. There is limited variation due to Mach number, the most obvious deviations are deep in the wake, for example when $x = 3.0$, there is a small variation in pressure at the top side of the curve.

Contrast these observations to the $Re_D = 800$ results where there is a pronounced decrease in pressure close to the cylinder with increasing Mach number. The decrease in pressure appears to recover as $x$ increases into the wake around $x = 1.5$, results cluster together irrespective of Mach number. Continuing towards $x = 3.0$ a discrepancy in the pressures appears with two characteristics. Firstly the minimum pressure decreases with increasing Mach number. Secondly there is a bias appearing at $Ma = 0.45$, where there are lower pressures maintained on the bottom side of the curve ($y < 0$). The deviations in the minimum pressure can be quantified as an angle of deflection for the wake. Taking the $y$ position of the minimum pressure in the distribution and the prescribed $x = 3$ we can define some deflection angle as $\phi = \arctan(y/x)$.

In the $Ma = 0.45$, $Re_D = 400$ case where we see that the drift due to a change in gap height from 2 to 1.5 is 2.7 to 4.62 degrees respectively. There is a slight change due to increase in Mach number when $G/D = 1.5$, where the angle of deflection increases from 4.24 to 4.62. Although it appears that decreasing gap height and increasing Mach number act to push the wake off of the centre-line, contradictory results occur when $Re_D = 800$. Firstly, the angle of deflection at $Ma = 0.30$ is 3.46 degrees, larger than what is seen at $G/D = 2.0$, $Re_D = 400$. Comparing the Reynolds numbers when $G/D = 1.5$ shows this angle is smaller, moreover, raising the Mach number increases this discrepancy, the deflection decreases to 1.93 degrees.

4.2 Analysis of a select case supported by POD

Here the case of high drag and high shedding frequency, at $G/D = 1.5$, $Ma = 0.45$ and $Re_D = 800$, is selected to discuss some general observations of the POD technique for the near wall case. Further analysis of this case is provided in subsequent sections.

Figure 4.15 shows the time coefficients and energies of a POD decomposition for the case under consideration. The signals in Figures 4.15a to 4.15c are taken from the rows of $V$ for the first three modes. Figure 4.15d shows the energy ($\sqrt{\Lambda}$) for the first 25 modes. A total energy can also be defined for the POD: as the sum of all diagonal elements of $\Lambda$, in this case the decomposition was performed using 401 samples in time. Only the first 25 modes are considered here as their sum contains 99.9% of all the energy. As shown in Figure 4.15d, the fields of $P$ and $u$ are dominated by a single mode, the first mode of pressure comprising over 98% of the available energy. Similarly, the first mode of $u$ comprises just over 94% of the energy. Furthermore, as shown in Figure 4.15a and Figure 4.15b respectively, the first mode for both $P$ and $u$ is stable in time.

For both $P$ and $u$, the next two modes are oscillatory and appear to be $\approx \pi/2$ out of phase with one another. FFT analysis of these oscillations supports the previous assertion of the lift signal being a fundamental frequency. This is shown in Figure 4.16 where the first two oscillating modes are at the primary frequency. Looking at the frequency associated with the peak in the FFT amplitude, the phase difference is indeed $\approx \pi/2$. Furthermore, the signals average to zero in time indicating that the stable mode of $P$ is the dominant factor in the time averaged drag and lift. These observations are universal for $P$ and $u$ across this data set, the stable mode, $\sqrt{\Lambda}$, is always the dominant contributor to the field energy, referred to hence as the principal
Figure 4.15: Temporal modes and energies for $G/D = 1.5$, $Ma = 0.45$ and $Re_D = 800$.

Figure 4.16: The FFT analysis of the first 2 oscillating signals of $P$ from Figure 4.15a. The amplitude and the phase difference between the two signals are presented.
Figure 4.17: Spatial modes of $P$, $u$ and $v$ for the case of $G/D = 1.5$, $Ma = 0.45$ and $Re_D = 800$. The red line in some figures shows $\tilde{\varphi}^k = 0$. Each of the following sub-figure labels is the values of the white and black lines respectively. a) $P$, mode 1, (0.79, 1.10), b) $u$, mode 1, (−0.025, 1.30), c) $v$, mode 1, (−0.6, 0.6), d) $P$, mode 2, (−0.05, 0.05), e) $u$, mode 2, (−0.55, 0.55), f) $v$, mode 2, (−0.7, 0.7), g) $P$, mode 3, (−0.03, 0.05), h) $u$, mode 3, (−0.3, 0.3), i) $v$, mode 3, (−0.35, 0.35).

POD analysis of $v$ is provided in Figure 4.15c indicating that the modes of $v$ have different characteristics compared to $P$ and $u$ modes. The first two modes shown in Figure 4.15d are matched to the oscillating signals in Figure 4.15c. These comprise about 34% of the total energy each which is followed by a cluster of modes, referring to consecutive modes of similar energy. As seen in Figure 4.15d, modes 3 to 7 are about $\approx 5\%$ total energy each. Mode 3 in this case is fairly stable, see Figure 4.15c however the rest of cluster is oscillatory.

Although not used in the following discussions, it is interesting to note that modes of $v$ change as the cylinder gets closer to the wall ($G/D < 1.3$). The stable signal’s weak oscillations presented in Figure 4.15c become more significant, and $v$ becomes dominated by 3 modes in a cluster, each $\pi/2$ out of phase with each other and oscillating about zero.
Figure 4.17 shows the first three spatial modes of \( P, u \) and \( v \), the contour values presented are scaled by the modal energy, \( \sqrt{\lambda} \) and the RMS of \( a_k(t) \). This scaling technique will be used throughout the remaining discussions unless otherwise stated. A deeper discussion of these spatial modes will be performed in later sections. However, there are key observations for \( P, u \) and \( v \) as follows:

- Firstly, the stable mode of pressure in Figure 4.17a shows that lower pressures are generated on the underside of the cylinder in agreement with observations of Figure 2.14. Comparing the distributions above and under the cylinder, the presence of the wall appears to cause lower pressures around the bottom of the cylinder indicated by the white line. The effect of compressibility is also visible in the region enclosed by the black line at a contour value of 1.1. Here the maximum contour value at the front stagnation point is 1.149, corresponding to the compressible stagnation pressure of 0.8208 (using Eq. (2.34)). The oscillating modes two and three, Figure 4.17d and Figure 4.17g respectively, show that the wall affects the fluctuating pressure as well. Comparing the black and white line in Figure 4.17d shows an imbalance, in this case the contour values are lower on the bottom side of the cylinder compared to the top. Figure 4.17g also shows an imbalance, particularly in the region where vortices are being generated. As indicated by the black and white contour lines, there is an imbalance across the centre-line. On the bottom side, there is a large region of positive pressure of greater magnitude than the bubbles around it and connected to the positive bubble on the top of the cylinder.

- The first contour of \( u \) clearly shows the effect of the boundary layer, the high speed region at underside of the cylinder is confined, producing a larger region of higher speed as show by comparing the black contour lines either side of the cylinder. This confinement appears to have an impact on the re-circulation region which is uneven with stronger negative velocities in the bottom right quadrant indicated by white lines. The two oscillatory modes are shown in Figure 4.17e and Figure 4.17f. These modes both show a pair of structures of opposite sense that appear to originate from the cylinder surface, for example those highlighted by black and white lines in Figure 4.17e. Further downstream, there are pairs of blobs of alternating sign where the vortex street, again following the contour lines in Figure 4.17h. Both modes show a sight imbalance in the structures that emanate from the cylinder, Figure 4.17e showing some bias to the top side of the cylinder. Interestingly Figure 4.17h not only shows an inverse of this bias, favouring the bottom side, but stronger regions of negative value downstream, comparing the size of the black and white contour lines for pairs across the centre-line. It also appears that the modes are staggered, in the third mode for example, blobs of high absolute magnitude (large values of \( \phi^3(x) \)) appear in positions where the second mode is close to zero (small values of \( \phi^2(x) \)). The oscillatory modes also show some evidence of wall interference, for example compare positions \((x = 2.5, y = \pm 1)\), contour lines appear to be confined closer to each other by the presence of the wall as well as being offset in \( x \). Taking the line \( x = 2.5 \), and matching pairs of opposite sign across the centre-line, it is clear that matching contour values move upstream on the bottom side.

- The first two modes of \( v \), shown in Figure 4.17c and Figure 4.17f are associated with oscillatory time signals and exhibit similar behaviour to what was observed in the second and third mode of \( u \). Blobs appear to be staggered in such a way
that the second mode has points of maximum absolute magnitude in positions where the first mode is close to zero. Similarly modes of $u$ there is interference from the wall in modes 1 and 2, but the effect appears to be weaker distortions are only obvious in contour lines that are close to zero. However, the time-stable third mode displayed in Figure 4.17h shows some effects due to the wall, firstly upstream of $x = 0$, the negative values of $v$ are clearly confined due to the wall. This confinement however appears to have only a limited effect on the distributions at the front of the cylinder which are fairly even. Secondly, downstream of the cylinder, the blob of positive $v$ is greater in a magnitude than the corresponding negative. There is also a clear bias to the positive side as the zero-line in red is elevated off centre.

4.2.1 Time-stable fields and static forces

As discussed in Section 4.1.1 there are significant variations in forces and vortex shedding characteristics between the extrema of Reynolds numbers when the $G/D = 1.5$ and $Ma = 0.45$. The first question to be answered is the origin of the increase in time-averaged drag and lift forces. As these effects are shown to be pressure related, the principal mode of pressure is shown in Figure 4.18 for $Re_D \in \{400, 800\}$.

The increase in Reynolds number has significantly altered the pressure field. Near the cylinder, the increase in $Re_D$ has caused the rear bubble to move closer to the cylinder surface with relatively low pressures spreading over the cylinder rear. Deeper into the wake, it is clear that the increase in Reynolds number has decreased the pressure
in the wake and the angle at which the wake is offset relative to the centre-line, which are in agreement with the \( C_P \) results discussed in Figure 4.14.

Furthermore, the POD reveals other changes in pressure that were noted in the discussion of \( C_P \) extractions. The increase in \( C_P \) around \( x = 1.50 \) is visible in both plots. Upstream, the region close to the cylinder decreases in pressure as is expected. Due to the POD considering the whole domain, the technique also captures features that the line extractions missed. For example, there is a minimum pressure the wake region dependant on the Reynolds number. Increasing the value of \( \text{Re}_D \) from 400 to 800 causes the bubble of minimum pressure to move upstream from around \( x \approx 5 \) to \( x \approx 3.5 \), qualitatively similar to contractions in the wake, as well as decreasing in pressure by 8.98%. This distribution implies deeper wake cooling not seen in the 3D results in Chapter 3.

With this understanding of the pressure field, we move on to the static velocity field of \( u \) (mode 1), for this comparison, modes of \( v \) are discarded. POD analysis showed that the vast majority of the flow energy in \( v \) is contained in oscillatory modes that average to zero in time (95% of all energy is associated with modes of this type).

Figure 4.19 shows the effect of Reynolds number on the principal mode of \( u \) at a gap height of 1.5 and Mach number of 0.45. There are two clear differences between the figures, firstly near the cylinder in Figure 4.19b, where the pressure changes in the re-circulation region mentioned earlier are collocated to areas of higher stream-wise velocity. Comparing values of \( W_1 \) and \( W_2 \) shows a 19.1% and 15.3% increase in stream-wise velocity with the increase in Reynolds number. Figure 4.19b also shows that this increase in speed results in an overall shortening of the region compared to Figure 4.19a. Increasing the Reynolds number also causes changes in the higher speed flow generated on the underside of the cylinder. These changes appear to be due to the boundary layer both up and downstream of the gap. By increasing the Reynolds number, hence thinning the boundary layer, allows higher speed fluid into the gap. Comparing the points \( E_1 \) and \( E_2 \) between the figures, the increase is 7.89% and 6.60% at each point respectively. The higher Reynolds number case also has as a larger area of high speed flow across the gap, particularly noticeable in the neighborhood of point \( V \). Downstream from the gap, thinning the boundary layer allows the gap flow to maintain higher speeds into the wake.

In the region on the cylinder where uneven pressures are seen in Section 4.1.1 between points \( V \) and \( R \) a shearing region is created due to velocity reversal. Despite the changes in the size of the re-circulation region, the effect close to the cylinder due to the boundary layer is weaker. Although the region of high speed flow around \( V \) is larger in size in Figure 4.19b compared to Figure 4.19a, the contour values are very similar, varying by less than 2%. Therefore, as was also shown in Figure 4.17b, the presence of the boundary layer is not accelerating the fluid significantly, rather acting to maintain higher speeds over a larger region. Comparing values at \( R \) between figures does show that flow reversal occurs in both cases and becomes slightly stronger with Reynolds number, from a values \(-4.08\%\) at \( \text{Re}_D = 400 \) to a \(-7.19\%\) when \( \text{Re}_D = 800 \).

In order to quantify the previous claims about the flow, the measure of circulation, \( \Gamma \), is calculated about the cylinder. Here, we take concentric circles about the origin of increasing radius from the cylinder surface \((r = 0.5)\) to the wall \((r = G/D + 0.5)\) as the closed paths of integration, \( \partial S \) in the equation:

\[
\Gamma(r) = \int_{\partial S(r)} \mathbf{v} \cdot d\mathbf{l} \tag{4.2}
\]
Figure 4.19: Variation in the principal mode of $u$ due to a change in the Reynolds number for $G/D = 1.5$ and $Ma = 0.45$. 
Where $v_i$ is the velocity vector and $dl_i$ is a length element of the closed loop. The results of this analysis are shown in Figure 4.20 for some key cases. Because the integration surfaces are concentric circles about the origin, these results are interpreted as the “bulk rotation” of fluid around the cylinder. Due to how the surfaces are parameterized, a positive circulation represents an anticlockwise rotation. The axes are also scaled to aid visualisation, where the surface radius $r$ is centred on the cylinder surface and scaled by the gap-height. The circulation $\Gamma$ is scaled by $1/2\pi r$ to compensate for the increasing surface size at large gap heights. At $G/D = 3.0$ the interactions between the cylinder and the wall are negligible as the circulation for both Reynolds numbers is close to zero near the cylinder. Closer to the wall, $0.6 \leq r/G \leq 1.0$, it is clear that the boundary layer begins to have an effect resulting in a net negative circulation.

The effect of boundary layer thickness at lower gap-heights is shown in two ways, firstly by taking cases $G/D = 2.0$, $Re_D = 400$ and $G/D = 1.5$, $Re_D = 800$. The effect of wall proximity and the boundary layer is to create a region of increased positive circulation between the cylinder and the boundary layer at higher Reynolds number. Furthermore the wall appears to be important in this case as the larger gap height produces a lower positive circulation for the same flow velocity.

Secondly, varying the Reynolds number at matched gap heights, $G/D = 1.5$, the regions of higher speed noted in the previous discussions of the stable mode of $u$ are co-located with areas of increased circulation. These regions show the effect of boundary layer thickness on the problem where, at $Re_D = 400$, Figure 4.20 shows that circulation increases slightly with wall proximity. In comparison to $Re_D = 800$ however, the thinner boundary layer interferes less with the higher speed flow in the gap producing higher circulation.

### 4.2.2 Oscillatory fields and vortex shedding

POD analysis of pressure and stream-wise velocity showed that the majority of the flow is described by a single, high energy, time stable mode of each variable. The time dependent nature of vortex shedding means that we should pick a point in time, $t$, and
evaluate the spatial modes, $\phi^k(x)$ in the context of the changes in $a^k(t)$. In order to pick a position in time, we start with the lift curve, a natural choice as the Strouhal number is defined using this signal.

Figure 4.21 shows the lift signal at $Ma = 0.45$ and $Re_D = 800$ for two gap heights. Firstly, the signals are obviously very stable in time, the drift observed is due to the difference in shedding frequency. The inset shows the positions of maximum $C_L$ and a position of 45% of this maximum, after the peak. The values of $t$ for gap heights of 3.0 and 1.5 are 808.0 and 808.5 respectively. These positions are useful as they represent a similar “state” and are available in the data from the simulations.

Figure 4.22 shows contours of $C_P$ and $\omega$ for the values of $t$ prescribed. Comparing the figures it is clear that a reduction in gap height has two main effects. Firstly, Figure 4.22a and Figure 4.22c show lower pressures generated near the bottom right quadrant of the cylinder, with a corresponding higher pressure generated just downstream. The effect of the wall in this case appears to be both a decrease in the low pressure at the bottom right of the cylinder and the increase in the high pressure surrounding it, particularly directly downstream.

Secondly, looking at the vorticity field in Figure 4.22d there is the beginning of a positive vortex already rolling up close to the cylinder before the complete detachment of the negative vortex. This beginning of a positive vortex is on top of a low pressure bubble at the bottom right hand side of the cylinder, visible in Figure 4.22c. This area is experiencing significantly lower pressures compared to the corresponding region in Figure 4.22a. The higher pressures in Figure 4.22d are collocated with the positive shear layer on the cylinder in Figure 4.22b. In addition, this shear layer’s roll-up is less advanced compared to that seen in Figure 4.22a. Comparing the larger pressure bubble directly behind the cylinder, collocated to the negative vortex in both pairs of figures, there is higher pressure in the near-wall case. In conclusion, the closeness of the wall in this case is causing uneven pressures during vortex generation and the premature roll-up of an underside vortex.

Figure 4.21: The coefficient of lift in time for a change in gap height when $Ma = 0.45$ and $Re_D = 800$. Inset shows a zoomed-in portion of the curve around the position of maximum positive lift.
Figure 4.22: Contours of $C_P$ and $\omega$ for different gap heights when $Ma = 0.45$ and $Re_D = 800$. 
Figure 4.23: Extracted values of $C_p$ and $\omega$ at a probe in the domain for a change in the gap height when $Ma = 0.45$ and $Re_D = 800$. 

(a) $G/D = 3.0$

(b) $G/D = 1.5$
Figure 4.24: The normalised variance of $u$ and $v$ for the near-cylinder and wake regions.

To attempt to measure this effect, a probe is placed in the domain at $(1.0, 0.0)$, from which values of $C_P$ and $\omega$ are extracted and shown in Figure 4.23. Figure 4.23a shows this quasi-free-stream state when $G/D = 3.0$, the period of peak vorticity magnitude is associated with relatively low pressure, then an increase in pressure follows the reversal of vorticity in the formation region. Decreasing gap height to 1.5 however, shown in Figure 4.23b, results in a different behaviour. The pressures present during formation of a negative signed vortex are higher than at $G/D = 3.0$, with a larger spike in pressure during the transition to positive vorticity.

The lines in Figure 4.23 show that there is an alternating vorticity field around one diameter downstream of the cylinder. Therefore, there should be fluctuations in $u$ and $v$ around this region to produce this vorticity. The variance of the velocity is used to investigate the effect of the parameters further. Figure 4.24 shows the normalised variance for the velocity components for a selection of gap heights, Mach and Reynolds numbers. Starting with $u'^2$, close to the cylinder at $x = 0.75$, fairly even profiles are observed except for the case $G/D = 1.5$, $Ma = 0.45$ and $Re_D = 800$ where the variance on the top side of the cylinder is noticeably larger than it is on the bottom. Furthermore, $u'^2$ curves are grouped into two distinct clusters based on the Reynolds number. Moving downstream to $x = 1.00$, for $Re_D = 400$, $u'^2$ remain quite independent from the gap height or Mach number. However, a slight decrease in the variation in the case of $G/D = 1.5$ and $Ma = 0.30$ on the bottom ($y \leq 0$) is observed.

There are greater changes at $Re_D = 800$, particularly when raising the Mach number. At $G/D = 3.0$, the variance is similar on the bottom ($y \leq 0$) for both Mach
numbers, however, there is a small increase in the variance on the top ($y \geq 0$) with the increase in Mach number. Reducing gap height to $G/D = 1.5$ causes larger deviations with Mach number. At $y < 0$, increasing the Mach number causes the variance to drop. At $y > 0$ however, increasing the Mach number recovers the variance and moves the peak away from the centre line.

Further downstream at $x = 1.50$, the structure is similar for all choices of $G/D$, Ma and Re_D. The exception being $G/D = 1.5$, Ma = 0.45 and Re_D = 800, where there are drops in peak variance either side of the centre line relative to the other results. Note that scales on the graphs are different, fluctuations of $u$ are weaker compared to those of $v$ and appear to decay in a short distance downstream from the cylinder.

Moving on to the variance of $v$, starting close to the cylinder rear at $x = 0.75$, the results form two clear bands that are varied by Reynolds number. There is a change at $G/D = 1.5$, Re_D = 400, raising the Mach number causes an increase in variation at $y > 0$. Raising the Reynolds number increases this effect noticeably and causes it to appear at $y < 0$ as well.

This increase in $v'^2$ for these cases is maintained when $x = 1.00$. The effect at Re_D = 400 is still weak in comparison to that Re_D = 800 which are shown in the black box. Here, at $G/D = 3.0$ and Re_D = 800, increasing Mach number causes a drop in the variance. Approaching the wall at low speed causes the variance to increase slightly relative to the $G/D = 3.0$ results. Finally, raising the Mach number to Ma = 0.45 at this gap height causes a large increase in variance. The difference in peak variance caused by this change in gap height when Ma = 0.45 and Re_D = 800 is $\approx 45\%$.

Further downstream at $x = 1.50$, there is banding that is related to both Ma and Re_D. Increasing Ma and Re_D independently causes an increase in the variance, with limited effect due to gap height. Therefore, in the analysis using the POD, a strong structure in $v$ around $x = 1.00$ is predicted as this is the position of the greatest variance.

### 4.2.3 Identification of fluctuating regions by POD

In order to identify the formation of vortices with the POD, a relationship between time coefficients $a^k(t)$ and the lift signal is necessary. The first step in creating this connection is to show a direct correlation between the oscillating modes and the statistical methods used earlier.

Figure 4.25 shows the first and second modes of $v$ for four of the cases that were captured in Figure 4.24 at a constant speed Ma = 0.45. The first column shows the effect at Re_D = 400, at a gap height of 3 the distributions are even about the centre-line and there are no clear impacts of the wall on the near-cylinder field. Approach to the wall produces limited change, the deflection of the vortex street is noticeable when the cylinder is at a gap height of 1.5. However, there does not appear to be a change in the magnitude of each bubble when compared to corresponding positions in the field at $G/D = 3.0$.

Following the centre-line at $x = 0.75$ and $x = 1.00$, the contours are quite low in magnitude, reflecting the generally lower variance seen earlier. Note, in Figure 4.25c, the sign of the second contour has reversed, represented by the change from solid to dashed lines. Analysis of the time coefficients, $a^k(t)$, when $G/D = 3.0$ showed that mode 1 leads mode 2. This is reversed when $G/D = 1.5$, meaning that the inverted contour values compensate for the change in the lead/lag of the signals.

Figure 4.25b and d show the results for Re_D = 800, firstly at $G/D = 3.0$, the bubbles are mostly even about the centre-line close to the cylinder. Some small deviations can
Figure 4.25: Contours of the first and second mode of \( v \) for four cases. The first mode is in colour and the second is overlaid as lines.
be seen, following contour line number 9 at around \( x = 1.0 \) for example, it is clear that there is a weak effect on this bubble from the wall side \( (y \leq 0) \). Decreasing the gap height however produces a large increase in the normal velocity close to the cylinder where \( x \leq 1.0 \). Firstly in mode 1, there is an increase in magnitude close to the cylinder where much lower values are present with comparison to a larger gap height. Secondly, comparing the line number 17 in both plots around \( x = 1.0 \), it is clear that the approach to the wall has increased the magnitude of that bubble in mode 2.

The values from these modes can be converted to variance and compared to the extracted data given in Figure 4.24. Given that there is a time stable mode with a non-zero time average, mode 3 in the case of POD of \( v \), the modes presented here can be added (based on the linearity of POD and the use of \( \sqrt{\mathbf{a}^k(t)^2} \) as a scaling factor) and directly compared with the variance.

Figure 4.26 shows the sum of the squares of first two modes \( v \), the act of squaring converting them to components of the variance, compared against the extracted data that was shown earlier. It is obvious that these two modes describe the fluctuating field well, recovering the variance with a maximum error of 2.8%. This gives confidence that the POD modes can be directly interrogated to explain the features of the flow.

Therefore, we start by comparing the structures of \( v \) in Figure 4.25 and Figure 4.25 with their respective pressure fields shown in Figure 4.18. It is clear that, when \( \text{Re}_D = 800 \), the lower pressure regions are collocated with more intense fluctuations of \( v \). The fluctuations of \( v \) are critical to the vortex generation as the this is feeding momentum through entrainment into the vortex generation region following the descriptions of Gerrard (1966). Furthermore, the literature has shown a strong connection between the lower pressures close to the cylinder and an increase in shedding frequency. However, the spatial coefficients of \( v \) are fairly even about the centre-line and their respective time coefficients average to zero in time. Therefore, fluctuations of \( v \) supply the vortices with stream-normal momentum equally, in other words, a positive vortex gains as much from fluctuations in \( v \) as a negative vortex. Therefore, the early roll-up of the positive vortex seen in the field data is due to the imbalance in the static
4.2.4 Fluctuating pressure fields

The fluctuating modes of pressure can be investigated in a similar manner to those of \( v \). The fluctuating components of \( P \), mode 2 is shown in Figures 4.17d and 4.17g for example, can be summed together to reform the data-set. Similar to \( v \), the fluctuations close to the cylinder can be reconstructed using the second and third modes of pressure. At \( G/D = 1.5 \), \( Ma = 0.45 \), \( Re_D = 800 \), two modes will reproduce a fluctuating load on the cylinder. This is found by rebuilding the pressure field based on modes 2 and 3 using Eq. (2.44), then integrating the pressure forces using Eq. (2.37). The RMS of this reconstructed signal has an error of < 0.3% compared to the raw data taken from the simulation. Further analysis shows that mode 2 is the largest contributor, over 99.9%, the same as the fluctuations found on the surface in Chapter 3. Similarly small errors are found for other test cases, showing that the second mode contains the fluctuating field on the cylinder surface.

Figure 4.27 shows the effect on the spatial component of mode 2 due to wall proximity and changing Reynolds number at \( Ma = 0.45 \). Figure 4.27a shows \( G/D = 3.0 \) with fluctuations on the cylinder surface even across the centre line. These fluctuations from the surface appear to protrude into the wake as well, roughly following the free shear layers. Figure 4.27b shows that the presence of the wall increases the fluctuation intensity in the gap region compared to the fluctuations on the free side. In addition, a stronger region appears in the wake directly downstream of the gap structure, around \((2, -0.5)\). Figure 4.27c shows the effect of decreasing the Reynolds number at \( G/D = 1.5 \) and \( Ma = 0.45 \). Firstly, the decrease in \( Re_D \) causes the fluctuations to become both weaker and more spread out from the cylinder surface. In Figure 4.27b at the points \((0, \pm 0.5)\), there is an increase in the fluctuation intensity of 15% due to the increase in Reynolds number. Further comparing Figure 4.27c to Figure 4.27b, the distortions in the region around \( x = 2 \) are less pronounced. Moreover, the region close to the cylinder rear is more even over the centre-line at \( Re_D = 400 \).

Deeper into the wake, Figure 4.23 showed that a low pressure is developed close to the cylinder rear, at \( x = 1.0 \), during the production of a positive vortex. The analysis of figures in this section has shown that the combination of \( G/D = 1.5 \) and \( Re_D = 800 \) will produce strong, uneven fluctuations at the cylinder rear. However, the overall contribution of an individual spatial mode is affected by the modal energy and temporal coefficients. By reconstructing the pressure at the probe point using the POD modes, the dominating contribution can be identified. Figure 4.28 shows the the fluctuations in time of the pressure at the probe point based on different modes. Mode 1 has been removed as it is time-stable and, due to the linear nature of the POD, the remaining modes can be viewed as fluctuations about the mean. Modes 4-5 and 6-9 are grouped together as they are of similar modal energy.

Comparing the summation of modes 2-9 to the signals in Figure 4.23, the modal reconstruction can capture the large features with some loss of small details. Converting these fluctuations to \( C_p \) values and adding the mean (mode 1) will recover the signals in Figure 4.23. At \( Re_D = 400 \), comparing between Figure 4.28a and Figure 4.28c, decreasing gap height has created large contributions from mode 3 to the signal. From a peak of \( \pm 0.002 \) to \( \pm 0.03 \), this mode has come to dominate the pressure field, and the effect is clear in the ensemble signal, encouraging the low pressures that were discussed in the previous section. There is an inverse effect on the modes 4 to 5 due to decreasing gap height, where the peak drops from \( \pm 0.019 \) to 0.010. This decrease is much smaller in
Figure 4.27: The second mode of pressure presented for three cases that are being compared.

(a) $G/D = 3.0$, $Re_D = 800$

(b) $G/D = 1.5$, $Re_D = 800$

(c) $G/D = 1.5$, $Re_D = 400$
Figure 4.28: Reconstruction of the fluctuating field of $P$ based on summation and isolation of modes. Data is probed at the point $(1.0, 0.0)$ in the domain.

Comparison to the increase in mode 3 and therefore, in order to identify the contributing spatial structures, mode 3 is singled out for further analysis.

Figure 4.29 shows the effect on the spatial component of mode 3 due to wall proximity and Reynolds number at $Ma = 0.45$. Figure 4.29a shows that mode, at $G/D = 3.0$ and $Re_D = 800$, is fairly even along the centre-line with strong distributions of pressure in the immediate wake region. Contrast this with the Figure 4.29b where the gap height decreases to $G/D = 1.5$. The wall effects an increase in the fluctuation intensity across the gap and at the rear of the cylinder. There is a structure of high strength that originates from the wall and extends to a peak in the wake downstream of the cylinder. This structure is associated with the strong fluctuations at the top right of the cylinder as both fields are positive, indicating that they are in phase with one another in time. Note that comparing a negative spatial value to a positive one would result in the respective signals being $\pi/2$ out of phase. This is due to the negative sign inverting the associated temporal mode which is equivalent to a $\pi/2$ phase shift. Therefore, a detaching negative vortex that would be associated with higher pressures at the top right of the cylinder would experience an increase in pressure at the formation site around $(1.0, 0.0)$. This explains both the increased pressure noted in Figure 4.22d and the reason for the fluctuations in pressure seen in Figure 4.23b.

The difference due to Reynolds number at $G/D = 1.5$ is seen in Figure 4.29c. Regions of high positive fluctuations downstream of the gap with a strong concentration of fluctuations around $(1.5, -0.6)$. Comparing the wake regions around $0.5 \leq x \leq 2$ shows that increasing Reynolds number controls the influence of this wall effect. At $Re_D = 400$, the region is more balanced in the immediate wake around $x = 1.0,$
Figure 4.29: The third mode of pressure presented for three cases that are being compared.
corroborated by the weaker contributions of mode 3 at this point in Figure 4.28d. At Re_D = 800, this region of positive fluctuation associated with the wall effect crosses the wake and joins to the positive fluctuations at the top right of the cylinder. More generally, fluctuations at the cylinder rear are stronger than those in the Re_D = 400 case. These effects compound to produce the strong response from mode 3 seen in Figure 4.28d.

4.3 Conclusions

A parametric investigation of the subsonic flow around a cylinder interacting with a wall boundary layer has been completed using a two-dimensional approximation. There are some general trends, first of which is at large gap heights, G/D ≥ 2, there is very little impact from the presence of the wall for all Mach numbers. Only a a weak positive lift force that is known in the incompressible literature.

Secondly, as the the cylinder approaches the boundary layer there is a slight increase in the fluctuating coefficient of lift before immersion. There is a further increase in the fluctuating coefficient of lift before immersion due to an increase Mach number which is tied to generally lower pressures experienced at the base of the cylinder. In addition, this trend is stronger at higher Reynolds numbers where the increase in fluctuating coefficient due to Mach number is larger.

The final trend is due to immersion in the boundary layer, the changes in the cylinder surface coefficient of pressure due to compressibility disappear. As the cylinder is exposed to slower fluid from the boundary layer there is a general increase in static pressure. This causes base pressure to increase significantly, hence the drag force and the fluctuating coefficient of lift both decrease substantially. When immersed the changes due to Mach number, such as increased drag, are removed with Reynolds number being the only differentiating factor.

A unique flow case was identified at G/D = 1.5, Re_D = 800 and Ma = 0.45. Increasing the flow speed from Ma = 0.30 created a region of high speed flow in the channel between the cylinder and the boundary layer. This led to higher circulation around the cylinder, pressure decreases over what was seen at G/D = 3.0 and a substantial advancement of the wake up stream. This advancement of the wake upstream was noted in Chapter 3 where increasing Mach number decreases the base-pressure which was shown to be directly coupled to the formation region length.

In the 2D study however, the proximity of the low-pressure position in the wake to the rear of the cylinder caused the pressure on the rear to drop substantially with a bias towards the wall, resulting in a reversal of lift from weakly positive to more strongly negative. This was coincident with a large increase in the drag force and the Strouhal number, both by around 8.5% with reference to the Ma = 0.30 case.

From the static fields of P and u, it is clear that the acceleration of fluid in the gap between the cylinder and wall boundary layer as the Mach number increases is decreasing the pressure in the wake by the reasoning of Kurosaka et al. (1987). In addition, the model of Roshko (1954), which was confirmed for compressible flows in Chapter 3, predicts that the increased speed in the shear layer is decreasing the base pressure with a bias towards the gap.

This substantial decrease in pressure in the formation region was tied directly to an increase in the fluctuations of υ which were identified using both well established methods and POD. A mode of the POD of υ was shown represent the largest contributor to the υ′^2. Furthermore, this field was shown to be fairly even in space, providing
momentum for vortices equally. However an imbalance in the vortex production was identified, indicating another factor at play.

By POD analysis of the pressure field, it was shown that increasing the Mach number strengthened a fluctuating pressure distribution downstream of the gap with a bias to the formation of an underside vortex. This indicated that the differences noted between Mach numbers at \( G/D = 1.5 \) and \( \text{Re}_D = 800 \) were due to these imbalances in the POD modes. The static \( u \) and \( v \) field generated a larger positive circulation of fluid, encouraging the lower pressures, hence higher flow speeds during the formation of an underside vortex. Conversely, this combination of factors adversely affects the formation of the topside vortex, reducing speeds and increasing the pressure.

In the \( \text{Re}_D = 400 \) case by contrast, the boundary layers were found to be too thick for these structures to appear. When the cylinder was close enough to the wall for a channel to form in the \( \text{Re}_D = 800 \) the cylinder was already immersed in the outer edge of the boundary layer. This removed any compressible effects in the same manner as described just previously. Furthermore, the longer formation region at \( \text{Re}_D = 400 \), due to the 2D approximation, reduced the interaction between the low pressure wake and the base pressure compared to that at \( \text{Re}_D = 800 \).
Chapter 5

Near wall study in 3D

Section 4.3 suggests that a cylinder at $G/D = 1.5, \text{Ma} = 0.45$ and $\text{Re}_D = 800$ will experience an increase in drag, reversal of lift and an increase in shedding frequency and intensity. This effect was linked to the shortening of the wake such that there is a significant decrease in base pressure. Due to the 2D approximation however, the undisturbed formation length is shorter than an equivalent 3D case. This chapter provides the data for a 3D run of the special case identified in the 2D data at $\text{Ma} = 0.45$ and $\text{Re}_D = 800$ and an additional run at $\text{Ma} = 0.30$, $\text{Re}_D = 800$ for comparison. The mesh is an extrusion of the $G/D = 1.5$ grid used in Chapter 4. The extruded length is $L_z = 4$ and $N_z = 81$. The 3D near-wall cases are compared to the data for a free cylinder provided in Chapter 3 and some key deviations due to the wall are identified. Following this, the 3D near-wall simulations are compared to the 2D data from Chapter 4 where some similarities in the flow-fields and coherent structures are discussed.

5.1 Near-wall results

5.1.1 Sample flow-fields and key statistics

Figure 5.1 shows the flow fields for the 3D, near-wall case. The results are chosen to represent a similar state: anti-clockwise vortex separating, clockwise vortex generating. The Q-criterion is normalised by $D/U_\infty^2$ and the iso-surface levels are set to $\{1/3, 2/3, 1\}$. The slice in the background displays the data at the domain edge, $z = -2.0$. The coefficient of pressure is used here and the limits are set to $\{-2, 1\}$. The wall and cylinder are represented by the grey shaded regions. At $\text{Ma} = 0.3$, Figure 5.1a shows shedding that similar to what was seen in the free-case where the vortex sheets are breaking up and dissipating in during formation. This dissipation continues as the vortices detach into the wake. The region downstream of the separating anti-clockwise vortex is what remains of a fully separated clockwise vortex. It is surrounded by structures that are associated with diffusion of momentum from a vortex due to rotation in the yz plane ($\omega_z$). It is difficult to identify complete structures in the vortex street due to wall interactions that will be covered in more detail later.

Increasing the Mach number appears to have a similar effect to what was seen in the free case. At $\text{Ma} = 0.45$, shown in Figure 5.1b the vortex roll-up is more coherent and the fully detached clockwise vortex is more obvious. The vortex roll-up is also closer to the rear of the cylinder, with larger, stronger structures present. However, there is dissipation downstream with increased interactions from the wall, similar to the low speed results. The 2D data showed a deep wake of coherent vortices was rotated up away from the wall, in 3D however, it appears that these structures break down
Figure 5.1: Normalised Q-criterion (iso-surfaces) and coefficient of pressure (slice) in the $G/D = 1.5$, $Re_D = 800$ case for 2 different Mach numbers.
Table 5.1: Table of key characteristics for the free and near-wall simulations in 3D. Relevant calculations are performed in the following sections.

<table>
<thead>
<tr>
<th>Ma</th>
<th>C_P_stag.</th>
<th>C_P_base.</th>
<th>L_F</th>
<th>Wake angle (°)</th>
<th>C_D</th>
<th>C'_L</th>
<th>St</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td>0.30</td>
<td>1.0250</td>
<td>-0.9605</td>
<td>1.6354</td>
<td>N/A</td>
<td>1.1585</td>
<td>0.2582</td>
</tr>
<tr>
<td>0.45</td>
<td>1.0275</td>
<td>-1.0270</td>
<td>1.5425</td>
<td>N/A</td>
<td>1.2987</td>
<td>0.3666</td>
<td>0.2044</td>
</tr>
<tr>
<td>G/D = 1.5</td>
<td>0.30</td>
<td>1.0545</td>
<td>-0.9910</td>
<td>1.5640</td>
<td>2.2609</td>
<td>1.2047</td>
<td>0.2916</td>
</tr>
<tr>
<td>0.45</td>
<td>1.0727</td>
<td>-1.0727</td>
<td>1.4840</td>
<td>1.4920</td>
<td>1.3486</td>
<td>0.4096</td>
<td>0.2159</td>
</tr>
</tbody>
</table>

Figure 5.2: Comparison of drag and lift signals for 3D cases showing the effects of increasing Mach number and wall proximity.

quickly. As such, the comparisons will focus on the region close to the cylinder within 3 diameters from the centre.

5.1.2 Forces on the cylinder in 3D

Figure 5.2 shows the drag and lift signals for free and near-wall 3D simulations. The drag signals show the small increase due to the wall. Table 5.1 shows the time-averaged drag in the near-wall case is greater than the free case, 3.98% and 3.84% for Ma = 0.30 and 0.45 respectively. In the free case lift is negligible, however, in the near-wall case the lift is around 0.045 for both Mach numbers. This is in agreement with the incompressible research where the lift on the cylinder is weakly positive at G/D = 1.5. Figure 5.2 also shows the increase in the amplitude of the lift signal in each near-wall case. The increase due to wall proximity is about 12% for both Mach numbers. This increase in the fluctuating coefficient of lift is quite large compared to results in the incompressible literature. Chapter 3 demonstrated significant deviation due to compressibility that was related to the overall drop in the rear pressure, an effect that will be covered later. Finally, an FFT of the lift signal provides the principal frequency which is converted to a Strouhal number. The values of St show some significant variation, however the collected data of Unal and Rockwell (1988) shows large variations in the Strouhal number in turbulent flow and there is no compelling trend in the near-wall data when compared to that presented in figure 3.5.

In Chapter 3, increasing the Mach number caused a general decrease in the base
pressure on the cylinder which is associated with the increase in drag and shedding intensity at $Re_D = 800$. Furthermore, Chapter 4 demonstrated that the near-wall case had the effect of further decreasing the base-pressure in 2D. Figure 5.3 shows the effect of the wall in 3D where there is a drop in base pressure at both Mach numbers due to the wall. The drop in pressure is biased to the underside of the cylinder similarly to the 2D tests in Chapter 4. The decrease in the minimum pressures around $\theta = -1.8$ is 2.4% and 3.6% for $Ma = 0.30$ and 0.45 respectively and this decrease is maintained across the region of $-1.8 \leq \theta \leq 1.0$.

This effect, though qualitatively similar to the pressure distributions seen in 2D, is weaker by comparison to the data in Chapter 4. This indicates that the increase in drag on the cylinder in 3D is not purely produced by the drop in base pressure. Around $\theta \geq -\pi$, Figure 5.3 shows that the stagnation pressure is both moving off-centre and increasing in value in the 3D, near-wall case. For both Mach numbers, the shift is $-2.6$ degrees which is in agreement with Lei et al. (1999) for thick boundary layer cases. The increase in stagnation pressure is 2.8% and 5.3% for $Ma = 0.30$ and 0.45 respectively. The combination of the this rotation of the stagnation point and general increase in stagnation pressure is sufficient to produce to the positive lift and the increase in the drag. Taking the integral of the absolute value of the pressure coefficient as in Eq. (3.2) provides a measure of the total area of the of the pressure distribution. This quantifies the overall change due to the interaction of the wall by taking account of both the increase in the stagnation pressure and the decrease pressure across the cylinder rear. The change due to the wall on the overall pressure distribution is 3.7% and 4.2% for each Mach number respectively. As previously mentioned in Chapters 3 and 4, pressure dominates the drag and this overall change in the distribution agrees with the trend of increasing drag due to the wall. Furthermore, the decrease in the pressure over the rear of the cylinder explains the increase in the fluctuation intensity as well based on the relationships shown in chapter 3.
Figure 5.4: Coefficient of pressure and normalised velocity components averaged along the span of the cylinder for the near-wall case. Results for $Ma = 0.30$ and $0.45$ are presented in the left and right columns respectively. Contour values are consistent for the same variable.
5.1.3 Analysis of static fields

Figure 5.4 shows the effect of the increase in speed on the coefficient of pressure and velocity components for the near-wall cases. Figure 5.4b shows that the increase in flow speed causes the pressure to drop in the wake and this bubble of low pressure to move upstream. The positions of low pressure also show that the wake rotated away from the wall slightly. The \((x, y)\) position of the minimum pressure is used to calculated the wake deflection recorded in Table 5.1 as \(\phi = \arctan(y/x)\). Similarly to the 2D case, increasing the flow speed at near the wall is causing the deflection angle to decrease. In the 2D case, increasing the speed decreases the wake deflection from 3.46 to 1.93 degrees. In 3D, the change in angle follows the same trend but is less pronounced. The 3D case at \(Ma = 0.30\) has a lower angle of deflection in comparison to the 2D test at 2.29 degrees, which drops to 1.49 at \(Ma = 0.45\).

The movement of the low pressure in the wake appears to be tied to the position of minimum stream-wise velocity. Figure 5.4d shows that the increase in Mach number causes the low-speed region in the wake to move upstream and closer to the centre-line, reflecting the change in formation length and wake deflection respectively. The effect of the gap is also clear, similar to the 2D case, a high speed core is produced in the gap due to the presence of the wall. Downstream of the high speed core there is a complementary increase in the normal velocity, as shown in Figures 5.4e and 5.4f. In general, the effect of the wall appears to be the increase in the velocities both within and directly downstream of the gap. Unlike the 2D simulations however, the normalisation reveals that there is little change in the velocity distributions with the increasing Mach number.

Figure 5.5 shows extractions of the stream-wise velocity and the ratio of pressure against the base pressure from the rear of the cylinder to 3 diameters into the wake. The line used for interpolation is offset from the centre-line in the near-wall cases by the angle of wake deflection in order to properly capture the minimum pressure in the wake. Figure 5.5a shows that the effect of the wall is to advance the position of \(x(u = 0)\) to move upstream and that the minimum speed in the wake is lower in the near-wall case at both Mach numbers. The position of zero velocity in the wake is used to define the formation region length as in Chapter 3. Figure 5.5b shows that, similarly to the observations of Chapter 3, the velocities largely collapse with some variation in the minimum velocity within the formation region. This implies that, although the formation region is moved off centre, it is still dominated by the relationship between base pressure and formation length.

This relationship is reflected in Figure 5.5c where the contraction in the formation region is particularly obvious at \(Ma = 0.45\). For both Mach numbers the change in the wake minimum pressure is quite limited, about 0.5\% when adjusted for the effect of drop in base pressure. Figure 5.5d confirms that the fundamental relationship in the formation region is that of the base-pressure and the formation length. Furthermore, the normalisation demonstrates that the effect of the wall is to rotate this fundamental structure without changing the underlying relationship.

Figure 5.6 shows the variance of \(u\) and \(v\) for both the free and near-wall cases. In Chapter 3 it was shown that the decrease in base pressure at \(Re_D = 800\) was encouraging increased shedding behaviour which was tied to these fluctuating fields. It was further shown that the largest fluctuations occur along the detached shear layer in the case of \(u'^2\) and then around the formation length \((x = L_F)\) for \(v'^2\).

Figure 5.6a shows further increases in the fluctuations due to the wall for both Mach numbers and that these increases are greater in the top shear layer. Comparing the
Figure 5.5: Normalised stream-wise velocity and base pressure ratio extractions along the wake for the 3D free and near-wall cases. The second column shows an additional scaling of the wake coordinate by the formation region length, $L_F$. 
Figure 5.6: Line extractions of the variances of $v$ and $u$ showing the change due to flow-speed and wall-proximity in the 3D case. Lines are extracted for specific regions that are deemed to be critical in Chapter 3.
Figure 5.7: The xy plane Reynolds stress for free and near-wall cases at both Mach numbers.

variance in the near wall case to that seen in 2D in Figure 4.2,b although the overall magnitude of the fluctuations is weaker, the bias towards the top shear layer is similar. This increase in the variance appears to dissipate in 3D by \( x = 1.25 \), the differences between cases becoming smaller. This is also seen in the 2D data where the extractions were clustered about one another at \( x = 1.5 \).

Further similarities to the 2D data are shown by Figure 5.6b. The variance of \( v \) is higher in the near-wall case at \( Ma = 0.45 \) in comparison to the free case. However, this is also true for the \( Ma = 0.30 \) case, whereas in 2D there was a drop in the variance due to the wall. Furthermore, this difference between Mach numbers in 3D persists deeper into the wake, comparing the \( Ma = 0.30 \) cases at \( x = 1.5 \) in contrast to those at \( Ma = 0.45 \). At \( x = 1.75 \) and outside the largest formation region (see Table 5.1) the maximum variances become similar between cases with the only difference in the curves being due to wake deflection.

As discussed in Chapter 3, increasing the Mach number had a regularising effect on the flow. It increased the strength of 2D flow features, demonstrated by the xy-plane Reynolds stresses, shown for the near-wall cases in Figure 5.7. Here the Reynolds stresses are normalised by \( D^2/U_\infty^2 \). At \( Ma = 0.30 \), the effect of the lower wake pressures is reflected in the increased Reynolds stress in Figure 5.7b compared to Figure 5.7a. Particularly, the imbalance that appears in the negative region around \( x = 2 \), compared to the corresponding positive region. The distribution has also moved upstream by 0.169\( D \) in line with movement of the respective positions of minimum pressure. At
Figure 5.8: Drag and lift signals at a variety of gap heights at \( \text{Ma} = 0.45, \text{Re}_D = 800 \). 

\( \text{Ma} = 0.45 \), Figure 5.7d shows that the near-wall case experiences a decrease in the positive Reynolds stresses around \( x = 2 \) compared to the free case in Figure 5.7c. However, the imbalance across the centre-line seen in Figure 5.7b is present with a further decrease in the negative region compared to that in Figure 5.7c. This bias towards the distributions on the free side of the centre-line follows the trend seen in the variances in Figure 5.6.

5.2 Comparisons to the 2D data.

In 2D, the \( \text{Ma} = 0.45 \) case was shown to be of particular interest, exhibiting significant changes in drag, lift and shedding frequency. Therefore, this section will focus on the 3D effects at \( \text{Ma} = 0.45 \) in comparison to the 2D data from Chapter 4.

5.2.1 Drag and lift forces

Figure 5.8 shows the drag and lift curves for 2 and 3D simulations at \( \text{Ma} = 0.45 \) comparing for a change in gap height. Here the 2D results at a gap height of three are used as it is a convenient, large data set and has been shown to be independent of the wall. The increase in mean drag due to raising the Mach number is visible as an offset between the signals in both the 2 and 3D cases. However, the change in the amplitude in 2D is not present in the 3D data. This is due to the difference in the formation length where strong fluctuations in the shear are present at the rear of the cylinder in 2D. In 3D these regions of shear are downstream of the cylinder and therefore the interactions are much weaker. Similarly, the increase in amplitude of the lift signal is pronounced in 2D, less so in the 3D data, which is due to the large differences in the fluctuating pressures as covered in Chapters 3 and 4.

It is established that the variation in drag is dominated by pressure forces and related to the base pressure for both 2D and 3D cases. Decreases in which are linked to increased shedding intensity in Chapters 3 and 4. Figure 5.9 shows the pressure distributions on the cylinder surface for a 2 and 3D in free and near-wall cases at

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Ma = 0.45. The 2D near-wall case shows a large region of low pressure being generated at the bottom-right quadrant of the cylinder, \( \theta < 0 \). This lower pressure extends to the separation of the shear layer where it then recovers to match the free result. There is also an increase in the pressure at the top of the cylinder, the action of these pressure changes combined is the increase in drag and the reversal of lift from weakly positive to strongly negative. In 3D the pressure also decreases compared to the free stream, however, the decrease is weaker compared to the 2D data.

Similarly, Figure 5.10 shows that a weaker version of the behaviour in 2D is visible in the 3D data. The minimum pressures around \( \theta = 1.8 \) are fairly constant as the Mach number increases, however the pressure between \(-1.8 < \theta < 1.0\) is lower in the Ma = 0.45 case for both 2 and 3D data. Around \( \theta = 1.8 \) in 2D, raising the Mach number causes the pressure to increase in this region. This change is not reflected in the 3D cases where the pressures remain largely similar.

5.2.2 Flow fields

In Chapter 4, a characteristic of the near wall flow is positive circulation generated in the velocity fields due to the wall. The largest contributor to this circulation is the increased stream-wise speed due fluid acceleration in the gap. Figure 5.11 shows extractions of the stream-wise velocity from the gap into the wake at the cylinder rear at \( G/D = 1.5 \) for both Mach numbers. When adjusted for flow speed the largest differences appears to be due to the 2D approximations as \( x > 0.5 \). Between \(-1.5 \leq y \leq -0.5\) the flow speed decreases in the 2D case as \( x \) increases. In the 3D cases however, high speeds are maintained past the formation length (\( x > L_F \)). By the relationships of Roshko (1954), this explains the further drop in pressure in the wake due to the increased speeds in the shear layer. Deeper into the wake, at \( x = 3.0 \) the results begin to collapse as the 3D flows return to the free-stream speed.

In the region \(|y| \leq 0.5\) there are additional discrepancies between the 2 and 3D results. At \( x = 0.50 \), the 2D data shows the additional regions of flow reversal that are associated with the additional re-circulation bubble that occurs due to the 2D approximation. As \( x > 0.5 \), the effect of the different formation lengths is visible.
Figure 5.10: Cylinder surface pressure distributions from 2D and 3D near-wall tests for both Mach numbers at $\text{Re}_D = 800$.

Figure 5.11: Extractions of $u/U_\infty$ at $x = \{0.50, 1.00, 1.50, 2.00, 3.00\}$ for different set ups at $\text{Re}_D = 800$. 
following the centre-line at $y = 0$. At $x = 1.0$ for example, both 2D flows are weakly positive and the 3D flows are equally weakly negative.

Focusing on the $Ma = 0.45$ case, Figure 5.12a shows that the higher speeds are maintained much deeper into the wake in the 3D cases due to the elongated formation region compared to the 2D cases. This high speed channel is being maintained in the gap formed between the lower free shear layer and the wall boundary layer. In the 2D case the combination of a shorter formation region and lower pressures at the base draw the flow into the rear region. These low pressures are shown in Figure 5.12c, the light blue in the filled contour is the minimum pressure in the 3D wake that is much higher and further downstream in comparison to that of the 2D simulation. A further change in the flow due to this low pressure in 2D is also shown in Figure 5.12b where the positive normal velocity downstream of the gap is closer to the cylinder and uneven over the centre-line. In 2D, this indicates that the higher speed flow from the gap is being drawn into the formation region close to the cylinder rear. In 3D, the regions of $v$ are more even over the centre-line and are comparatively further downstream. However the overall mechanism appears to be similar: higher speed fluid from the gap flows downstream and is drawn into the formation region due to the lower pressures in both 2 and 3D simulations.

Although the mechanism by which momentum is transported from the accelerated fluid due to the gap to the formation region is qualitatively similar, a quantitative measure of the effect is the circulation around the cylinder. Figure 5.13 shows a normalised circulation for both Mach numbers in 2 and 3D. Here the circulation is normalised by $1/2\pi r_a\infty Ma$ to remove changes due to both increasing contour perimeter and flow speed, however the integral surfaces are generated in the same fashion as in Chapter 4. As mentioned in Chapter 4, stronger positive circulation is being generated in the 2D case as the flow speed increases. In comparison to the 2D case, the 3D case at $Ma = 0.30$ has lower circulation. This is due to the longer formation length moving the strong $v$ fields downstream to $r \geq 1.5$, lessening their contribution to the circulation. Increasing in flow speed in 3D improves the circulation close to the cylinder and moves the peak circulation further into the gap. This higher circulation deeper in the gap around $r/G = 0.5$ corresponds to the stable distributions of $v$, where there is a higher positive distribution that has moved upstream. In comparison to the 2D data however, the larger formation length still prevents significant circulation from occurring.

### 5.3 Conclusions

There are some similarities between the 2D and 3D cases, primarily that fluid entering the gap between the boundary layers on the cylinder and on the wall is accelerated. Furthermore, this acceleration is correlated with lower pressures in the wake region. However there are 3 key differences due to the inherently 3D nature of the flow:

- The drag increase which is seen in the two-dimensional case as the Mach number increases is not replicated in three dimensions; in fact the drag changes are relatively small.
- No reversal of lift in the three-dimensional case, the lift is consistently positive regardless of Mach number.
- No significant change in the shedding frequency, although the individual Strouhal numbers are different, they are well within in the limit of known Strouhal numbers in the literature.

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Figure 5.12: Comparison of the flow variables averaged in both time and the z-axis between the 2 and 3D tests at $G/D = 1.5$ and $Ma = 0.45$. Filled contour is the 3D case, lines is the 2D, negative values not including zero are represented by dashed lines.
Figure 5.13: Comparison of the normalised circulation for a range of 2 and 3D near-wall tests.

The reason that the drag is generally not as high in the 3D case is due to the lower pressures at the base of the cylinder. In addition, increasing the Mach number does not lead to a significant decrease in the base pressure in comparison to that seen in two dimensions. This appears to be due entirely to the large difference in base pressure that is known to be due to the 2D approximation as demonstrated by Henderson (1997); Mittal and Balachandar (1995). This effect is notable at \( \text{Ma} = 0.45 \) where lower pressures are generated in the wake, however the formation region is significantly further away from the rear of the cylinder. Because of this distance, the low pressure generated at the formation region does not interact with the base pressure in the way seen in 2D.

Pressure distributions also explain the persistent positive lift in 3D which is again due to the effect of the 2D approximation. As described in Section 1.3.1, in 3D there is a general trend as the cylinder approaches the wall, the stagnation point moves around cylinder surface with a bias towards the wall side. This effect is not replicated in the 2D tests where the stagnation bubble is generally stable at the gap heights tested here. Although the surface pressure distributions showed that there was a slight bias towards the wall in the base pressure, the rotation of the stagnation point produces a much more significant positive lift on the cylinder in 3D.

The stability in the shedding frequency in 3D can also be explained by the two-dimensional approximation. In 2D the shedding frequency is strongly coupled to the base pressure. During the transition to 3D structures in the wake however, the shedding frequency becomes uncoupled from the base pressure above Reynolds number of approximately 300. Although the shedding frequency does change somewhat as shown in Table 5.1, these are well within the upper and lower bounds for the Strouhal number at this Reynolds number. Again, due to 3D effects, the pressures at the rear of the cylinder are too high to have an effect on the shedding frequency.

Despite the disagreement close to the cylinder, there were some similar effects downstream that were found in the POD. As shown in Figure 5.14, some of the the two-dimensional structures recur in the otherwise 3D simulation. Of interest in particular
Figure 5.14: Scaled POD modes of pressure in 3D at $Ma = 0.45$. 

(a) Mode 2. 

(b) Mode 3.
is the large region strong distributions in Figure 5.14b. Between one and two diameters downstream there is a strong distribution of fluctuating pressure that appears to be interacting with the wall. These oscillating modes in 3D make up a similar contribution of the total modal energy and the structures in the spatial modes are of similar amplitudes to those in 2D. This means that when reconstructing the flow Fields from the POD the structure has a similar influence on the final solution in both cases.

In summary, some similarities in structures in the wake notwithstanding, these tests have shown that the two dimensional approximation is not appropriate for this range of Reynolds numbers. The difference between in the shear layer lengths and the length of the formation region means that the effects seen in two dimensions cannot be replicated. Although there is acceleration along the shear layers in 3D, the positions of highest speed are much deeper in the wake compared to those in 2D. In conjunction with this pressures are generally higher in the wake in the three-dimensional case which leads to an overall lessening of vortex strength. Therefore, the much lower pressures seen in both the static and transient 2D solution cannot be replicated in any way in 3D.
Chapter 6

Conclusions and recommendations

In all, two parametric studies and an exploratory investigation have been performed into the compressible effects of a subsonic flow, $0.2 \leq Ma \leq 0.45$ around a cylinder at $400 \leq Re_D \leq 800$ both in the free stream and near a wall. In general it was found that the effects on the flow field are largely dependant on the thermodynamics, specifically the Eckert-Weise effect as explained by Kurosaka et al. (1987). This compressible effect lead to decreasing base pressures as the Mach number increased and this relationship was pronounced at a Reynolds number of 800.

During the near-wall tests it was shown that these changes in the wake had significant implications for the interactions between the cylinder and a boundary layer. Specifically the inversion of the relationship between the Reynolds number and both the base pressure and the length of the formation due to the two-dimensional approximation. This lead to significant change in behaviour when select results in two dimensions, the causes for which were related to significantly shortened formation regions, were tested in 3D.

6.1 Revisiting the research questions.

What is the effect of increasing the Mach number for this range of moderate Reynolds numbers?

Chapter 3 presented the first parametric study, focused on the forces and vortex generation of a free cylinder in 3 dimensions, an as yet unexplored parameter space (see Nagata et al. (2020)). The study showed good agreement with the available literature at $Ma = 0.20$, however significant deviations were shown as the Mach number increased, especially at $Re_D = 800$.

It was found that the base pressure followed a downward trend with increasing Mach number, being only weakly dependent on the Reynolds number by comparison. Furthermore, it was found that the model of Roshko (1954) holds for compressible flows over this range. Minimum wake pressures also decreased with Mach number, however at a higher rate than the base pressure causing the two to diverge especially as $Ma > 0.3$.

The decrease in base pressure was shown to have different effects at different Reynolds numbers. This was first characterised by the total change in pressure coefficient over the cylinder as per Eq. (3.2). At $Re_D = 400$, increasing the Mach number
was shown to have little impact, the distribution of surface pressure coefficient was stable, except the case of a slight increase in the drag above what is predicted by the Glauert transformation at Ma = 0.45.

At $Re_D = 800$ however, the drag forces and shedding intensity increased substantially as the Mach number was increased. The drag was shown to be consistently higher than that predicted by the Glauert transformation and this was tied to the decreases in the overall pressure at the cylinder rear.

The base pressure was shown to be coupled to the length of the formation region, decreasing the base pressure would prompt a contraction in the length of the formation region. At $Re_D \leq 600$, this shortening is limited to $\approx 12.5\%$, at $Re_D = 800$ the contraction is $\approx 35\%$. It was further demonstrated that the base pressure and formation length were important in the characterisation of the flow. Normalisation by these parameters causes extracted flow data along the wake to collapse onto one another, suggesting a form of universal behaviour whose specific dimensions are controlled by base pressure.

The effect of Mach number on vortex shedding was also shown to vary by Reynolds number. Using the xy-plane Reynolds Stress at $Re_D = 400$ showed that the flow had strong 2D features at low speeds that did not change significantly with the Mach number. The stability of these features is coincident with a stable fluctuating fields of $u$ and $v$ for all Mach numbers. These fields feed momentum into the vortices and in turn cause fluctuations in the lift that are stable as the Mach number increases.

At $Re_D = 800$ however, in the incompressible case, this plane Reynolds stress is known to be weak and this weakness is a principle reason for the diminished fluctuations found in the wake. We showed that increasing Mach number was linked to a significant increase in the strength of this plane Reynolds stress which was connected to the reactivation of the fluctuations in the shear layers and a three fold increase in the shedding intensity. This increase in the shedding intensity was matched to another 2D flow feature using the POD method. The POD revealed a high energy mode of fluctuating pressure that could be used in isolation to reconstruct the majority of the fluctuating lift signal.

Taking the reasoning of Kurosaka et al. (1987) and Roshko (1954), an explanation of this effect at higher $Re_D$ and Ma was formulated. Decreases in the base pressure due to increased shear layer speed lead to wake contraction. This causes the wake pressure to come closer to the rear of the cylinder. This wake pressure is also decreasing due to the Eckert-Weise effect as the Mach number increases. These low pressures are increasing vortex strength due to the restoration of the 2D flow features characterised by the fluctuating fields.

Following the discovery of this restorative effect of increasing Mach number on 2D flow features, it was proposed that the parametric study of the near-wall effects be undertaken in 2D to improve efficiency. Some further investigations revealed that the key flow variables had a discrepancy less that 1% at a distance of 1 diameter from the cylinder surface, perpendicular to the flow direction. This distance corresponds to the smallest gap height tested and confirmed that the effect of the 2D flow approximation would be minimal across the gap, only varying significantly within the shear layer on the cylinder.
What is the effect of increasing the Mach number when the cylinder is placed close to a wall boundary layer?

Chapter 4 covered the parametric investigation of a 2D cylinder at a distance between 3.0 and 1.0 from a plane wall with a boundary layer. These tests were performed on a mesh developed specifically for this use case and validated at Ma = 0.20 against an incompressible reference. The study showed that raising the Mach number would increase drag and reverse the lift, from weakly positive to more strongly negative, at ReD > 600. It was shown that this effect occurred within a specific range of gap heights and was further found to depend on the ratio of the gap height to boundary layer thickness. If this ratio was greater than about 1.5, the cylinder would be too far from the wall for the flow around the cylinder to be sufficiently accelerated. Should the ratio be less than unity, it was shown that immersion in the boundary layer would remove any compressible effects.

The case of ReD = 800 was identified as being of particular interest in that raising the Mach number at a gap height of 1.5 caused the drag and the shedding frequency to increase by 8.5%. A much greater deviation than those shown for lower Reynolds numbers at the same gap height and flow speed. By a combination of well established methods it was shown that the proximity at the wall induced strong circulation at Ma = 0.45, ReD = 800 as well as a significant decrease in pressure at the rear of the cylinder, with a bias towards the wall.

POD was used to identify flow features by similar methods to that seen in Chapter 3. The cylinder surface coefficient of pressure and fluctuating coefficient of lift were both recovered using POD modes of the pressure field. This allowed for the identification of important flow features by coupling them directly to more fundamental analyses. Crucially, it was shown that the low pressure in the wake was close enough to the cylinder at Ma = 0.45, ReD = 800 to be significantly interacting with the base pressure.

The coincident increase in the variance of \( v \) at Ma = 0.45, ReD = 800 with the decrease in base pressure was investigated in a similar fashion. POD modes of \( v \) showed that a strongly oscillating region contained in a single mode was collocated to the minimum wake pressure. Furthermore, this single mode of \( v \) could be used in isolation to recover the majority of the variance of \( v \) in the wake. This indicated that the variance and hence the vortex generation was dependent on this minimum wake pressure.

By applying the understanding of Kurosaka et al. (1987), this effect of pressure defect is linked to the thermodynamic behaviour of compressible fluids. Accelerated fluid and increase circulation are leading to a further defect in the pressure at the rear of the cylinder. In addition, the already short formation length due to the two-dimensional approximation causes low wake pressures to interact significantly with the cylinder itself.

With regards to the wall boundary layer, is the 2D approximation valid for this problem?

Chapter 4 further investigated the features identified in Chapter 4 using a 3D test at \( G/D = 1.5, \text{ Ma} = \{0.30, 0.45\} \) and ReD = 800. This study showed some similar flow features arise in 3D, particularly the formation of a higher speed flow in the gap between the cylinder and boundary layer. Pressure extractions on the cylinder showed that the wall effect was qualitatively similar, causing generally lower pressures at the rear of the cylinder with a bias towards the near wall side. The proximity of the wall also caused a further contraction of the formation region at both Mach numbers which
matched this drop in pressure.

However, these effects where shown to be significantly weaker than those seen in the 2D case. Although an increase in the drag was found, it was shown to be 3.5% for both Mach numbers. This does not agree with the predictions of the 2D tests where the Ma = 0.30 case was largely unchanged and the Ma = 0.45 case experienced an 8.5% increase in the drag. Moreover, the increase in the 3D drag was shown to be due to a different mechanism to that found in 2D. An effect known for 3D incompressible flow is the anti-clockwise rotation of the stagnation point which in turn increases the stagnation pressure at a gap height of 1.5. This rotation was present and its effect caused a stronger positive lift instead of the lift reversal seen in 2D.

Finally, the increase in the Strouhal number in the 2D cases could not be seen. In 3D, the decrease in base pressure is also coincident with an increase in the fluctuating coefficient of lift, however, there was no clear trend in the shedding frequency. This is explained by the incompressible literature where in two dimensions the Strouhal number continues to increase. In 3D however, successive transitions in the flow due to changes in the length of both formation and wake regions as well as the transition to turbulence act in concert to maintain a stable shedding frequency.

Despite the disagreement between the 2 and 3D data, it was shown that some similar features in the vortex shedding appear. Importantly, the increase in the shedding intensity in the 3D cases is matched by increases in the variance of the velocity fields as in the 2D case. Additionally, the increase in the variance of $v$ occurs near the position of minimum wake pressure as predicted by the 2D data.

However, the increased formation length and generally lower speeds in the 3D cases caused the circulation to be weaker compared to the 2D data for a gap height of 1.5. As the base pressure and formation length are coupled, the higher base pressures in 3D are preventing the strong interactions between the cylinder rear and the low pressures in the wake seen in 2D. In summary, the effects of compressibility conform to the reasoning and models of prior research, what is key here is the effect of the 2D approximation alone. This approximation removes vital 3D features and results in radically different behaviour.

### 6.2 Recommendations for further work

The non-linear behaviour over this region of Reynolds numbers that is known from the incompressible literature combined with the revelation of key flow features being dependent on the Mach number indicate a number of future research questions.

Firstly, the effect of the Reynolds and Mach number should be further explored for the free cylinder case with more refinement in both parameters as well as an increase in the maximum Mach number. The data herein suggests that the base and minimum wake pressures are diverging as the Mach number increases and that this effect is independent of Reynolds numbers in this range. It appears that this relationship to the Mach number may be a simple power law, the definition of which would produce useful models for engineering applications.

Secondly, it was demonstrated that increasing the Mach number at $Re_D = 800$ strengthens 2D flow features as well as significantly increasing the shedding intensity. This is important for the calculation of the load due to vibration. Increasing the Mach number causes significant deviation from the incompressible data, leading to greatly under-predicting the fluctuating loads.

Finally, the near wall case requires further investigation as there is very limited
compressible research available in this area. The incompressible literature on wall interaction gives some direction:

- The effect of the boundary layer should be properly investigated by the use of varying thicknesses as well as the complete removal of the boundary layer. This will provide some insight into the fundamental flow features of the near-wall case. Furthermore, the case of no boundary layer will have more general engineering applications, for example the landing of a small spacecraft.

- Higher speeds should be investigated, particularly for the range of Reynolds numbers up to $Ma = 0.7$ which is the upper limit suggested by Désert et al. (2019) to have an engineering application.

- Smaller gap heights should be investigated, particularly in the case of no boundary layer being present as the data suggests that boundary layers remove compressible effects. There is incompressible research to suggest that small gap heights ($G/D \ll 1.0$) cause significant changes in the flow and there is only one paper to the authors knowledge that tackles this in 3D.
Bibliography


