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REASONING ABOUT QUANTITIES AND CONCEPTS: STUDIES IN SOCIAL LEARNING

DOCTORAL THESIS

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A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy in the Edinburgh Computational Cognitive Science Laboratory Department of Psychology School of Philosophy, Psychology, and Language Sciences

January 2023
Abstract

School of Philosophy, Psychology & Language Sciences
Department of Psychology
Doctor of Philosophy

Reasoning about quantities and concepts: Studies in social learning
by Jan-Philipp Fränken

We live and learn in a ‘society of mind’. This means that we form beliefs not just based on our own observations and prior expectations but also based on the communications from other people, such as our social network peers. Across seven experiments, I study how people combine their own private observations with other people’s communications to form and update beliefs about the environment. I will follow the tradition of rational analysis and benchmark human learning against optimal Bayesian inference at Marr’s computational level. To accommodate human resource constraints and cognitive biases, I will further contrast human learning with a variety of process level accounts. In Chapters 2–4, I examine how people reason about simple environmental quantities. I will focus on the effect of dependent information sources on the success of group and individual learning across a series of single-player and multi-player judgement tasks. Overall, the results from Chapters 2–4 highlight the nuances of real social network dynamics and provide insights into the conditions under which we can expect collective success versus failures such as the formation of inaccurate worldviews. In Chapter 5, I develop a more complex social learning task which goes beyond estimation of environmental quantities and focuses on inductive inference with symbolic concepts. Here, I investigate how people search compositional theory spaces to form and adapt their beliefs, and how symbolic belief adaptation interfaces with individual and social learning in a challenging active learning task. Results from Chapter 5 suggest that people might explore compositional theory spaces using local incremental search; and that it is difficult for people to use another person’s learning data to improve upon their hypothesis.
Social learning is ubiquitous. From deciding which coffee shop to pick to finding the right PhD programme, people constantly consult the opinions of others to make informed decisions. This thesis makes a substantial effort to bridge the gap between formal modelling and multi-player experiments to study the nuances of human social learning dynamics. Across seven behavioural experiments and computational modelling, I examine how people combine their own observations and expectations with the beliefs and observations of their peers. I focus on two different learning domains. First, I study how people reason about environmental quantities, such as the proportion of red and blue marbles in an urn. The results from this line of work suggest that people are adaptive social learners that can navigate complex social learning dynamics through a mixture of healthy scepticism as well as sophisticated inference mechanisms. I also find that several people are misled by simple distortions in social evidence, resulting in inferences that are not supported by evidence. Overall, my work on reasoning about environmental quantities highlights the nuances of real social network dynamics and provides insights into the conditions under which we can expect collective success versus failures such as the formation of inaccurate worldviews. Second, I study how people form and adapt hypotheses in form of language-like concepts. As an example, consider a person who entertains the hypothesis ‘My neighbour’s cat Spike must have stolen my fresh milk’. The results from my work on concept inference suggest that people incrementally adapt their hypotheses as they encounter new evidence; and that it can be difficult for people to use another person’s observations to improve upon their hypotheses.
Declaration of Authorship

I, Jan-Philipp FRÄNKEN, declare that this thesis titled, 'Reasoning about quantities and concepts: Studies in social learning' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at The University of Edinburgh.

- Where any part of this thesis has previously been submitted for a degree or any other qualification at The University of Edinburgh or any other institution, this has been clearly stated.

- Where I have consulted the published work of others, this is always clearly attributed.

- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.

- I have acknowledged all main sources of help.

- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed: ____________________________________________

Date: ______________________________________________
“These and almost all your communications will be useless to me unless you can propose some practicable way or other of supplying me with Observations. I want not your calculations but your Observations only.”

Newton to Flamsteed, 1695
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I would like to thank my supervisor Neil Bramley for his incredible support during my PhD. This thesis would not exist without Neil’s intelligent and supportive supervision. Over the past three years, Neil and I have exchanged more than 13,000 messages and more than 540 files on slack. Neil helped me to stay motivated and focused while our offices were closed, he introduced me to the world of computational modelling, and he started the Edinburgh CoCoSci Lab through which I met my co-advisors and collaborators Adam Moore, Chris Lucas, Simon Valentin, and Nikos Theodoropoulos.

Chris and Simon, thank you for sharing your technical expertise and our engaging discussions. I am looking forward to further collaborations. Adam, thank you for your encouragement and your guidance throughout my PhD. And thanks to Nikos for teaching me how to code multi-player games. I am also grateful for the support from my colleagues Tia Gong and Bonan Zhao, as well as the rest of the Edinburgh CoCoSci Lab.

Finally, I would like to thank the German Academic Scholarship Foundation and the ESRC for funding my research.
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Dedicated to my friends Jim and Sandro.
The chapters in this thesis are based on the following research outputs:

**Chapter 2**


**Chapter 3**


**Chapter 4**


**Chapter 5**


PDFs, data, code, and experiments are linked in each chapter and are also available on my website via janphilippfranken.github.io.
Chapter 1

Introduction

1.1 Motivating Example

Imagine you are new to Edinburgh and in desperate need for coffee. Suppose a colleague $B$ recommends a local coffee shop called ‘Format coffee’.¹ For some reason, you might not have much faith in $B$’s taste in coffee and you are not sure whether you should pay the place a visit. Now suppose a second colleague $C$ also recommends Format coffee. Considering both recommendations, you are excited to give Format a go. But what if you then learned that $C$ has only heard about Format from $B$ but has not tasted Format’s coffee themselves? Learning that $C$ only heard about the coffee shop from $B$ makes $C$’s testimony non-independent, and in this case, practically worthless (see Fig. 1.1, for an illustration).

Suppose you ignored the dependency between your colleagues $A$ and $B$ above and decided to give Format coffee a go. Upon your arrival at Format, you discover that someone had broken into the coffee shop and demolished Format’s expensive coffee machine. Muddy footprints leading from the coffee shop to a plant shop across the street make you initially suspect the owner of the plant shop. Additionally,

¹Format coffee is a small coffee shop near the university which has great speciality coffee.
you observe that the plant shop’s owner just started to serve coffee from one of Format’s greedy competitors in front of the plant shop’s entrance. Considering these two pieces of evidence, you start to believe that ‘the plant shop owner must have demolished the coffee machine’. Driven by your hypothesis, you attempt to walk across the street to confront the plant shop’s owner. Just as you are about to exit Format, Andrew—the owner of Format coffee—bumps into you and shows you a picture of himself and the plant shop owner planting a number of Ethiopian coffee plants in Andrew’s garden. Shocked by this contradictory piece of evidence, you decide to revise your belief about the initial suspect. By combining all three pieces of evidence, you now come to the conclusion that ‘Format’s greedy competitor demolished the coffee machine’ and that the competitor must have deliberately placed the footprints to mislead the investigation.

The above example highlights two important social learning dynamics that are of focal interest to the present thesis. First, it involves reasoning about quantities—such as the quality of Format’s coffee—under consideration of statistical dependencies between social information sources. Second, it involves reasoning about concepts—such as forming an explanatory theory about what happened to Format’s coffee machine—under consideration of various sources of private and social evidence. Before studying each of the above learning scenarios in further detail across Chapters 2–4 (focus on quantities) and Chapter 5 (focus on concepts), we next provide a brief introduction to related work in social learning.

1.2 Theoretical framework

1.2.1 Observational learning

The study of social learning has a rich history in Psychology. According to Heyes (1994), theories of social learning broadly fall into three main categories: (1) stimulus enhancement, (2) observational conditioning, and (3) observational learning. Stimulus enhancement is a type of social learning in which a person draws a learner’s attention to a single stimulus, thereby increasing the learner’s perceived value of the stimulus (Hoppitt & Laland, 2008). As an example, imagine Andrew repeatedly uses a specific brand of oat milk (e.g. Moma). The enhanced exposure to Moma may increase your awareness of the brand, making it more likely for you
Chapter 1. Introduction

to chose Moma in the future. Despite its simplicity, stimulus enhancement can be an effective method for interacting with and learning from the environment, and its fidelity has been observed in a variety of species. For example, it has been shown that stimulus enhancement supports increased skill acquisition in greylag geese (Fritz, Bisenberger, & Kotrschal, 2000) and bumblebees (Avarguès-Weber & Chittka, 2014). Interestingly, stimulus enhancement has also been studied in robotics. Here, it has been shown that robots engaging in stimulus enhancing social interactions perform better as compared to robots engaging in individual exploration (Cakmak, DePalma, Arriaga, & Thomaz, 2009). The second category, observational conditioning, is closely related to classical Pavlovian conditioning. Specifically, observational conditioning refers to learning a relationship between two stimuli. For example, if you observe a colleague drinking coffee from Format (stimulus A) followed by observing them being happy (stimulus B), you may start visiting Format yourself. Similarly, if you observe a colleague experiencing nausea upon drinking Format’s coffee, you may start to avoid the coffee shop after all. Social learning through observational conditioning can be particularly useful when learning about dangerous stimuli, and it has been shown to promote learning of fear (Olsson & Phelps, 2007) as well as risk avoidance (Wisenden, Chivers, Smith, et al., 1997).

While stimulus enhancement and observational conditioning have contributed significantly to our understanding of social learning across species (see Heyes, 2012, for a review), we here focus on the third, and potentially richest category of social learning: Observational learning. Theories of observational learning are mainly concerned with two different learning strategies: Imitation and emulation (Tomasello, 1999). Imitation refers to the process of acquiring new behaviours and skills by copying or imitating another person’s actions. As an example, imagine Andrew hired a new barista to join Format’s team. The new barista might initially copy Andrew’s specific actions—grind coffee beans, steam milk, pulling a shot of espresso, mixing milk and espresso—to come up with their own creation. The process of imitating another person’s actions can be high-fidelity, enabling a learner to quickly master a specific task in a well-defined environment. On the other hand, imitation learning suffers from limited robustness: Learning how to make coffee by copying Andrew’s actions can be a successful strategy at Format’s coffee shop, but it may fail in other coffee shops with different coffee machines, coffee beans, or milk. Emulation, on the other hand, involves observation of an outcome or a behaviour followed by inferring a set of latent representations that
might have generated one’s observations (Tomasello, Davis-Dasila, CamaK, & Bard, 1987; Whiten & Ham, 1992). For example, if a new barista observes an outcome (e.g. coffee), they will not simply copy Andrew’s specific sequence of actions, but instead try to derive a generative ad hoc model of how the coffee was made. Such a model or method for reproducing an outcome can accommodate for variable environmental circumstances such as changes to the size of the coffee mug or using cow milk versus oat milk. As such, emulation can be more robust than imitation.

In the present experiments, we are interested in further studying the mechanisms and robustness of emulation in social learning. We will specifically focus on emulation in the context of belief inference: Inferring another person’s model of the world given a sequence of observations (c.f. C. Wu, Vélez, & Cushman, 2021). We will tackle this inference problem through the lens of a learner’s theory-of-mind of another person—i.e. a learner’s ability to explain another person’s observed behaviour with respect to the person’s latent beliefs, desires, and precepts (see Frith & Frith, 2005, for a short introduction). In line with a growing body of literature on Bayesian approaches to belief inference in social learning (e.g. Baker, Jara-Ettinger, Saxe, & Tenenbaum, 2017; Baker, Saxe, & Tenenbaum, 2009; Baker & Tenenbaum, 2014; Jara-Ettinger, Schulz, & Tenenbaum, 2020; Saxe & Houlihan, 2017), we will look at people’s behaviour in our experiments from the perspective of rational analysis (J. R. Anderson, 1991) and model inference as a utility-maximising process in which learners infer latent mental states by inverting a generative model of others’ communications and actions (see Jara-Ettinger, Gweon, Schulz, & Tenenbaum, 2016, for a review). We will compare this approach with a variety of process accounts and heuristics to better understand how people might find the best computation-accuracy trade-off in complex social learning settings.

1.2.2 Reasoning about quantities

In Chapters 2–4, we will explore the idea that people infer the latent mental states of their peers by inverting a generative model of others’ communications and action. In our experiments, we will focus on a toy scenario in which people reason about environmental quantities under consideration of statistical dependencies between social information sources. Our focus on dependencies in belief updating
is based on three observations. First, we observe that following events such as the 2016 US elections and the 2016 Brexit referendum, there has been a surge in studies examining the spreading of misinformation and belief polarisation on social media (e.g., Brady, McLoughlin, Doan, & Crockett, 2021; Brady, Wills, Jost, Tucker, & Van Bavel, 2017; Del Vicario et al., 2016; Finkel et al., 2020; Jasny & Fisher, 2019; Lazer et al., 2018; L. Schulz, Rollwage, Dolan, & Fleming, 2020; Vosoughi, Roy, & Aral, 2018). This interest has resulted in a number of simulation-based studies examining the spreading of misinformation and polarisation in idealised networks of Bayesian agents (e.g., Lewandowsky, Pilditch, Madsen, Oreskes, & Risbey, 2019; Madsen, Bailey, & Pilditch, 2018; Pilditch, Roozenbeek, Madsen, & van der Linden, 2022). A common argument across these studies is that if idealised populations can lead to polarisation or echo chamber formation, real social networks—which are inhabited by fallible human learners—might even be more vulnerable to polarisation and related phenomena.

Second, we observe that Bayesian models have become increasingly relevant for the formal study of social learning (FeldmanHall & Shenhav, 2019), covering domains such as iterated learning (Griffiths & Kalish, 2007), inverse planning (Baker et al., 2009; Jara-Ettinger et al., 2016; Ullman et al., 2009; S. Wu et al., 2021), source credibility (Hahn, Harris, & Corner, 2009; Harris, Hahn, Madsen, & Hsu, 2016), and sensitivity to statistical dependencies in social learning (Pilditch, Hahn, Fenton, & Lagnado, 2020; Whalen, Griffiths, & Buchsbaum, 2018; Xie & Hayes, 2022). Here, we are particularly interested in the latter: How do people accommodate the statistical dependencies that give rise to correlated belief reports in social networks. We note that the study of dependencies and correlated belief reports as well as the epistemic difficulties of testimony have previously been studied in both economics and philosophy. For recent highlights, we refer the reader to work on ‘correlation neglect’ in economics (Enke & Zimmermann, 2019) as well a the book ‘Testimony: A philosophical introduction’ (Shieber, 2015).

Considering the above findings, one could argue that (1) Bayesian models provide a good account of human social learning dynamics across a variety of empirical settings and (2) that assuming rationality in simulation-based studies might be a reasonable and ecologically valid assumption. While we find these arguments compelling, we also observe that previous empirical studies (e.g., Baker et al., 2017, 

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2One of them being my own work (Fränken & Pilditch, 2021) which I started during my MSc at UCL (2019) and completed during the second year of my PhD (2021). This work is not included in my PhD thesis due to a potential conflict of interest.
Chapter 1. Introduction

2009; Harris et al., 2016; Hawthorne-Madell & Goodman, 2019; Kleiman-Weiner, Sosa, Gershman, & Cushman, 2019; Pilditch et al., 2020; Whalen et al., 2018; S. Wu, Sridhar, & Gerstenberg, 2022) involve carefully calibrated and controlled experimental stimuli, typically requiring one participant to interact with one or more simulated players. Such controlled environments might simplify the learning problem, thereby inflating participants’ alignment with Bayesian model predictions. This might have implications for the ecological validity of simulation-based studies assuming Bayes-rational behaviours as well as the conclusions drawn from controlled empirical settings.

Motivated by our three observations, we designed a series of single-player and multi-player judgement tasks in which we compare Bayesian model predictions with human inferences. Specifically, we developed an iterated observe-communicate-respond paradigm in which participants have to combine their own observations with social communications from other players to update their beliefs and provide judgements. Here, other players can be either independent or dependent sources of information, and this dependency may or may not be known to participants. Our learning paradigm involves both simulated players and real players as well as varying levels of complexity (e.g., the number of players and belief updates). By contrasting people’s alignment with Bayesian model predictions across five experiments, we hope to provide new insights into the relevance of task complexity and experimental control as well as their potential implications for rationality assumptions in future empirical and simulation-based studies.

1.2.3 Reasoning about concepts

To explain what happened to Format’s coffee machine (see Section 1.1), we have formed an explanatory theory by combining various sources of private and social evidence. In contrast to the environmental quantities introduced in Section 1.2.2, such a theory is clearly more complex and expressive, and it further has symbolic structure. In Chapter 5, we will investigate how people adapt such complex symbolic concepts considering both their own actively gathered evidence as well as evidence gathered by someone else. Our work on symbolic belief adaptation is motivated by two observations. First, we note that studying social learning
in simplified estimation settings such as the learning paradigm covered in Chapters 2–4 does not accommodate many of our everyday life beliefs which are often systematic and compositional, so exhibiting language-like properties (Fodor, 1975; Goodman, Tenenbaum, Feldman, & Griffiths, 2008; Lake, Salakhutdinov, & Tenenbaum, 2015).

Second, we observe that inference in compositional theory spaces requires symbolic search, which is expensive (Ullman, Goodman, & Tenenbaum, 2012) and practically intractable from an AI perspective if a learner does not possess highly specialised domain knowledge (Ellis et al., 2021). Despite their inherent complexity, human learners can flexibly navigate compositional theory spaces, allowing them to construct and adapt symbolic hypotheses\(^3\) across a variety of learning domains, such as boolean concept learning (Goodman et al., 2008), learning about geometric concepts (Bramley, Rothe, Tenenbaum, Xu, & Gureckis, 2018; Piantadosi, Tenenbaum, & Goodman, 2012, 2016) and visual concepts (Lake et al., 2015), strategic games (Rothe, lake, & Gureckis, 2017; Z. Wang & lake, 2019), and list transformations (Rule, Piantadosi, & Tenenbaum, 2022; Rule, Tenenbaum, & Piantadosi, 2020).

Considering these two observations, we developed a social learning task inspired by Bramley et al. (2018) in which participants gather evidence about an unknown environmental rule (i.e. symbolic hypothesis) governing the behaviour of geometric objects in a two-dimensional environment. Using this task, we elicit free response judgements from participants allowing them to describe their best guesses about the unknown rule in words. We examine learning across two experiments, including a single-player version of the task and a multi-player version in which two participants engage in real time. Moreover, we develop a family of process models (c.f. Griffiths, Vul, & Sanborn, 2012) based on the idea that human learners might navigate intractable hypothesis spaces by means of stochastic local search, that is, by adapting their current beliefs via a sequence of local incremental tweaks as new data arrives (Bramley, Dayan, Griffiths, & Lagnado, 2017). Overall, we hope that the results from Chapter 5 provide novel insights into people’s ability to deal with symbolic search problems, and whether this ability interacts with the types of evidence they encounter (i.e. evidence gathered by themselves or by another learner).

\(^3\)Also referred to as mental causal programs (Chater & Oaksford, 2013).
1.2.4 Relationship with previous work

We note that our work on reasoning about quantities is closely related to previous work on *function learning*. In a generic function learning setting, a learner has to discover relationships between observed variables, such as the (usually linear) relationship between the amount of hours worked by a contractor (Variable A) and the amount of money earned by that same contractor (Variable B). Previous work on function learning suggests that people might learn such functional relationships through rule-based approaches—i.e. by comparing observations to a given family of functions (e.g. exponential functions) followed by fine-tuning the best matching function’s parameters. Alternatively, it has been suggested that people might induce functions from observations directly by forming associations between observed variables, also known as similarity-based theories of function learning (e.g. Busemeyer, Byun, Delosh, & McDaniel, 1997). More recently, rule-based and similarity-based approaches have been merged by rational (Bayesian) models of function learning (e.g. Griffiths, Lucas, Williams, & Kalish, 2008; Lucas, Griffiths, Williams, & Kalish, 2015), which have provided a good account of how people acquire functional relationships across a variety of classical function learning tasks (Mcdaniel & Busemeyer, 2005), multi-armed bandit problems (e.g. E. Schulz, Konstantinidis, & Speekenbrink, 2018), and pattern matching (e.g. E. Schulz, Tenenbaum, Duvenaud, Speekenbrink, & Gershman, 2017).

From a theoretical point of view, one could argue that function learning and our focus on reasoning about quantities tackle the same fundamental questions: (1) how do people acquire generative models of observations and (2) how can people use such generative models to make predictions about future observations. At their core, both approaches are a statistical method for making sense of observed data. The key difference is that in function learning, we ‘only’ have to learn the generative model (e.g. a Gaussian kernel) and the parameters of the model (e.g. mean and variance). In contrast, our work on reasoning about quantities involves both learning a generative model and its parameters (in our models, a Beta distribution with shape parameters alpha and beta) and assuming a mapping between the generative model and observations. Specifically, people do not know how others’ latent states map to their observed judgements, and instead have to make assumptions such as maximising over probabilities (Hawthorne-Madell & Goodman, 2019) or probability matching (Shanks, Tunney, & McCarthy, 2002). As such, it can be argued that our work on reasoning about quantities is a specific
case of function learning in which we have to accommodate for an additional mapping function between a latent generative model (i.e. another learner’s belief state) and observations (e.g. another learner’s judgements).

Similar to the intersection between function learning and reasoning about quantities, we argue that our work on concept learning can be seen as a subset of the literature on categorisation (J. Anderson, 1991; Davidoff, 2001; Medin & Schaffer, 1978; Rosch, 1999; Rosch & Lloyd, 1978). Here, our work is particularly related to recent rational models of categorisation (e.g. Griffiths, Canini, Sanborn, & Navarro, 2007; A. Sanborn, Griffiths, & Navarro, 2006; A. N. Sanborn, Griffiths, & Navarro, 2010). Categorisation involves organising observations (e.g. objects of different size and colour) into distinct groups based on shared properties and characteristics (e.g. group objects by size). As such, categorisation supports prediction about observations based on their category membership and it helps with the acquisition of structured knowledge representations. Our focus on reasoning about concepts requires both acquisition of structured representations and the formation of mental causal programs (Chater & Oaksford, 2013) that can describe relationships between acquired representations and arbitrary environmental categories (for example, whether the presence of the colour red indicates category membership). In our modelling framework (see Chapter 5 for details), we assume that people already have rich and structured representations of observed data. Therefore, and similar to related work on concept inference (Bramley et al., 2018; Zhao, Lucas, & Bramley, 2022), we support the idea that people discretize the perceptual world in distinct categories over which mental programs can operate. The ability to form such discrete categorise can thus be seen as a prerequisite for our work on reasoning about concepts. We discuss potential alternative representations in Section 5.9.3 and highlight limitations of this approach in the General Discussion. We next introduce our modelling framework.

1.3 Modelling framework

1.3.1 Bayesian inference

Bayesian models provide a formal framework for the study of human reasoning under uncertainty (e.g., Chater, Tenenbaum, & Yuille, 2006; Griffiths, Chater,
Kemp, Perfors, & Tenenbaum, 2010; Oaksford, Chater, et al., 2007). In a generic Bayesian setting, inference can be described as learning a set of optimal parameter values $\theta$ that maximise the likelihood $p_{\theta}(D)$ of a set of observations $D = \{d_i, i = 1...|D|\}$.

1.3.1.1 Individual learning

In the case of individual learning—where $d$ corresponds to samples or observations from the environment—parameter learning can be formalised straightforwardly by sequential application of Bayes’ rule:

$$p(\theta | d) \propto p(d | \theta)p(\theta)$$ (1.1)

where a learner’s posterior parameter (i.e. belief) estimate $p(\theta | d)$ corresponds to the normalised product between a learner’s prior belief estimate $p(\theta)$ and the likelihood $p(d | \theta)$ of observation $d$. The posterior serves as the prior for the next learning instance, allowing a learner to gradually improve upon their belief estimate as new data arrives. As an intuitive example, imagine that Andrew was able to repair Format’s coffee machine. Prior to tasting Format’s coffee, you are completely uncertain about whether you will like Format’s speciality coffee or not. After enjoying your first coffee, you revise your initial estimate to match your positive experience. Keeping your revised estimate in mind, you decide to get a second coffee the next day, again enjoying Format’s brew. You again revise your estimate to match your observation, so gradually arriving at a belief estimate which reflects your positive impression of Format’s coffee.

1.3.1.2 Social learning

In many instances, a learner may have no direct access to observational data. For example, imagine that Andrew did not manage to repair his coffee machine. In this case, you are unable to collect data (i.e. drink coffee) yourself. However, you can still rely on the communications from other people who were able to taste Format’s coffee prior to the machine’s breakdown. In such a social learning scenario, an

---

4For a concise introduction to Bayesian statistics, refer to Chapter 4 in Murphy (2022).
optimal learner can use the belief communications \( \theta_1, ..., \theta_n \) from \( n \) other people to update their own prior estimate \( p(\theta) \) using Bayes's rule:

\[
p(\theta | \theta_1, ..., \theta_n) \propto p(\theta_1, ..., \theta_n | \theta)p(\theta)
\]  

(1.2)

where \( p(\theta | \theta_1, ..., \theta_n) \) corresponds to a learner’s posterior estimate considering belief communications \( \theta_1, ..., \theta_n \) and \( p(\theta_1, ..., \theta_n | \theta) \) corresponds to the likelihood of other people’s belief communications under the learner’s prior.

Assuming that people’s belief communications are perfectly independent, transparent and trustworthy; and that people communicate beliefs using probability densities, Equation 1.2 is fairly easy to compute (see Chapter 2, for details). In reality, however, this is usually not the case. People often combine individual and social learning—they taste Format’s coffee and talk about whether they like Format’s coffee or not—thereby influencing each other’s beliefs and making them non-independent. Additionally, people’s belief revision strategies might differ, and so does their credibility and trustworthiness (Hahn et al., 2009; Harris et al., 2016). For example, an expert’s belief communication might have a much stronger influence on your own belief revision than the belief communication from a coffee novice. At the same time, an expert might be much harder to impress than a novice, meaning that an expert will change their own belief estimate very slowly, whereas a novice will form a positive impression of Format after having tasted only a small number of coffees. Finally, people usually do not communicate their beliefs using probability densities, but instead default to simple discrete judgements such as ‘Format’s coffee is good’ or ‘Format’s coffee is not good’. In Chapters 2–4, we will explore how people combine their own observations with belief communications from other people with a specific focus on dependencies between sources of information as well as their communication format. In Chapter 5, we focus on a concept learning problem in which a learner has direct access to the data collected by another learner, thereby reducing the social inference problem to an observational learning problem. In the next section, we will briefly summarise two Bayesian inference frameworks that form the basis of our analyses.
1.3.2 The present learning models

1.3.2.1 The beta-binomial model

In Chapters 2–4, we will use a conjugate beta-binomial model to describe how a Bayes optimal learner may evaluate their estimate $\theta$ about an environmental quantity using a combination of private and social evidence. In our beta-binomial framework, *private evidence* refers to a learner’s own observations from the environment (e.g., red and blue marbles sampled from an urn or good and bad coffees sampled from Format) which are unknown to other learners. *Social evidence* refers to other learners’ beliefs about the environment given their own private observations. We describe the communication format for social evidence separately in each chapter and here focus on a simple beta-binomial belief update given a single piece of private evidence $d \in \{0, 1\}$.

In a beta-binomial model, we can represent a learner’s prior belief estimate $p(\theta)$ as a beta distribution:

$$
p(\theta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha, \beta)} \quad (1.3)
$$

where $\alpha, \beta > 0$ are shape parameters and $B(\alpha, \beta)$ is the beta function which is defined as:

$$
B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}. \quad (1.4)
$$

The likelihood $p(d \mid \theta)$ of observing $d$ is given by a binomial distribution:

$$
p(d \mid \theta) = \binom{n}{k} \theta^k (1-\theta)^{n-k} \quad (1.5)
$$

where $\binom{n}{k}$ is the binomial coefficient and $\theta$ corresponds to the expected value $\mathbb{E}[\theta] = \frac{\alpha}{\alpha+\beta}$ under the prior. Given this setup, we can now compute a learner’s posterior belief $p(\theta \mid d)$ about the environment following observation of $d$ using Bayes’ rule:

\footnote{$\Gamma(\cdot)$ refers to the gamma function which is given by $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx$.}
The beta distribution is conjugate to the binomial likelihood. This means that both distributions are from the same family, and the posterior $p(\theta \mid d)$ can be computed analytically by plugging in the definitions of the prior and likelihood into Equation 1.6:

$$
p(\theta \mid d) = \frac{p(d \mid \theta)p(\theta)}{\int_{\theta} p(d \mid \theta)p(\theta)}.
$$

(1.6)

which simplifies to:

$$
p(\theta \mid d) = \frac{\binom{n}{k} \theta^{\alpha+k-1}(1-\theta)^{\beta+n-k-1}}{B(\alpha, \beta)}.
$$

(1.7)

As an example, imagine a learner samples a single red marble from an urn (i.e. $d = 1$). In this case, $n = k = 1$ (if they sampled a blue marble, $d = 0$, $n = 1$, and $k = 0$). Starting from a uniform prior $Beta(\alpha = 1, \beta = 1)$, we can now simply compute the posterior by adding our observation to the learner’s initial $\alpha$ and $\beta$ parameters using Equation 1.8: $Beta(\alpha = 1 + k, \beta = 1 + n - k)$ which simplifies to $Beta(\alpha = 2, \beta = 1)$. Given the conjugate property of the beta-binomial model, optimal Bayesian inference thus simplifies to adding private evidence observations to a learner’s prior $\alpha$ and $\beta$ parameters. Due to its simplicity, one could argue that the beta-binomial inference setup is also computationally feasible for humans. That is, if people are asked to update their beliefs in response to a small number of discrete observations, beta-binomial inference might be both optimal and resource rational (Griffiths et al., 2012; Lieder, Griffiths, Huys, & Goodman, 2018b).

### 1.3.2.2 Markov chain Monte Carlo

The conjugate property of the beta-binomial model allows for simple analytical updates of a learner’s belief. In many cases, however, the integral in Equation 1.6 can not be computed analytically. In Chapter 5, we study a symbolic inference setting in which a learner has to evaluate beliefs about the environment coming from an infinite set. Since marginalisation over this set is intractable and the conjugate
property does not apply, we approximate optimal Bayesian inference using Markov chain Monte Carlo (MCMC) sampling. This section provides a brief introduction to Markov chains and the Metropolis-Hastings (Hastings, 1970) algorithm used in our analyses. Further details covering the derivation of the prior and likelihood function are provided in Chapter 5.

**Markov chains** Markov chains are based on the Markov property:

$$p(h_{t+1} \mid h_t, h_{1:t-1}) = p(h_{t+1} \mid h_t)$$  \hspace{1cm} (1.9)

which states that the future ($h_{t+1}$) is independent of the past ($h_{1:t-1}$) given the present ($h_t$). Given this property, a Markov chain can be defined as the joint distribution over a sequence of $T$ hypotheses (i.e. states):

$$p(h_{1:T}) = p(h_1) \prod_{t=2}^{T} p(h_t \mid h_{t-1})$$  \hspace{1cm} (1.10)

where $p(h_t \mid h_{t-1})$ refers to the Markov kernel which specifies the transition dynamics between hypotheses.

**Metropolis Hastings algorithm** We can approximate an intractable posterior distribution over hypotheses by constructing a Markov chain in which the transition dynamics $p(h_t \mid h_{t-1})$ correspond to random proposals which are stochastically filtered against evidence using the Metropolis-Hastings (MH) algorithm. To make this concrete, imagine a learner initially entertains a symbolic hypothesis such as $h_t = \text{‘the plant shop owner must have demolished the coffee machine’}$. For simplicity, let us assume that the prior probability of this hypothesis is $p(h_t) = 0.25$ and the likelihood is $p(d \mid h_t) = 0.4$. Now imagine that the learner generates a new proposal hypothesis $h' = \text{‘Format’s greedy competitor demolished the coffee machine’}$ with $p(h') = 0.2$ and likelihood $p(d \mid h') = 0.6$. In this example, the new proposal has a lower prior probability, but can better account for the data $d$. Using the MH algorithm, a learner now computes acceptance probability $r$ for the new proposal as follows:
\[ r = \min(1, \frac{p(h') p(d \mid h')}{p(h_t) p(d \mid h_t)}) \] (1.11)

where \( \frac{p(h') p(d \mid h')}{p(h_t) p(d \mid h_t)} = \frac{0.2 \times 0.6}{0.4} = 1.2 \).

As we can see, \( r = 1 \), which means that overall, the new proposal \( h' \) provides a better fit for the data (its higher likelihood outweighing its lower prior). Under MH, we next sample a uniform number between zero and one \( u \sim U(0, 1) \) to determine whether the new hypothesis is accepted:

\[
h_{t+1} = \begin{cases} 
    h' & \text{if } u \leq r \text{ (accept)} \\
    h_t & \text{if } u > r \text{ (reject)}.
\end{cases}
\] (1.12)

Since \( r = 1 \), we accept \( h' \) deterministically in this case. If \( r < 1 \), we would accept \( h' \) stochastically depending on whether \( r \) was larger or smaller than \( u \). In practice, we only generate \( u \) if \( r < 1 \) to increase computational efficiency. To return to our example, our next hypothesis \( h_{t+1} \) in the Markov chain is now equal to the proposal \( h' = \text{‘Format’s greedy competitor demolished the coffee machine’} \).

Assuming that we just started (i.e. seeded) our chain with the first hypothesis \( \text{‘the plant shop owner must have demolished the coffee machine’} \), our chain now contains the two hypotheses ['the plant shop owner must have demolished the coffee machine', 'Format’s greedy competitor demolished the coffee machine']. If \( h' \) was rejected, our chain would correspond to ['the plant shop owner must have demolished the coffee machine', 'the plant shop owner must have demolished the coffee machine']. Given a reasonable proposal distribution\(^6\), if we were to run this chain for many steps, our distribution over hypotheses eventually converges to the true posterior distribution, thus approximating the behaviour of an optimal Bayesian learner. This setup allows us to model transition dynamics between the kind of symbolic hypotheses we study in Chapter 5\(^7\).

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\(^6\)We will contrast two different proposal distributions in Chapter 5.

\(^7\)For further details on MCMC, refer to Chapter 12 in Murphy (2023).
1.4 Overview of this thesis

This thesis summarises the research output of my time as a PhD student at The University of Edinburgh (2019–2022). In Chapter 2, I examine how people reason about the competency of two competing political candidates in a simple social learning paradigm. In Chapters 3–4, I extend the learning paradigm introduced in Chapter 2 by introducing a more complex social inference setup. Here, the task is to learn about the proportion of red and blue fish on an alien planet. Finally, I develop a more complex social learning task in Chapter 5 which goes beyond estimation of environmental quantities. Here, I focus on inductive inference with symbolic concepts and investigate how people search compositional theory spaces to form and adapt their beliefs; and how symbolic belief adaptation interfaces with individual and social learning in a challenging active learning task. Chapter 6 closes with a brief discussion of results and highlights limitations and directions for further work.
Chapter 2

Belief revision in a micro-social network: Modeling sensitivity to statistical dependencies in social learning

Preamble

This chapter includes the first of five experiments assessing people’s sensitivity to statistical dependencies in a social learning task that involves reasoning about environmental quantities. The work has been published as a conference paper at CogSci 2020 (Fränken, Theodoropoulos, Moore, & Bramley, 2020) and was conducted by myself under the supervision of Adam Moore and Neil Bramley. Nikos Theodoropoulos (second author) helped with the conceptualisation of the study as well as the design of the interface. I wrote a first draft of the paper and all co-authors helped with the writing and editing of the draft. Our code is available on GitHub via FrankenTheodoropoulosMooreBramley2020. A demo video of our task is available via OSF.
Abstract

In both professional domains and everyday life, people must integrate their own experience with reports from social network peers to form and update their beliefs. It is therefore important to understand to what extent people accommodate the statistical dependencies that give rise to correlated belief reports in social networks. We investigate adults’ ability to integrate social evidence appropriately in a political scenario, varying the dependence between the sources of network peers’ beliefs. Using a novel interface that allows participants to express their probabilistic beliefs visually, we compare participants against a normative Bayesian standard. We find that they distinguish the value of evidence from dependent versus independent sources, but that they also treated social sources as substantially weaker evidence than direct experience. The value of our elicitation methodology and the implications of our results for modeling human-like belief revision in social networks are discussed.

2.1 Introduction

We live and learn in a “society of mind” (Minsky, 1988). This means that we form beliefs not just on the basis of our own observations (and prior expectations) but also based on the beliefs communicated by our neighbors in our social network. For instance, interview panel members will typically discuss job applicants even after having seen mostly the same application materials and interviews, making it difficult to distinguish individual panel members’ (prior) judgements of candidates’ abilities from collective judgements formed on the basis of the shared evidence. Similarly, imagine you have read about two political campaign strategies, each proposed by a different candidate. If initially you find both strategies equally compelling, resulting in uncertainty about which of the two candidates you would like to support, you might well seek out new information about the candidates by talking to friends. If your friends base their beliefs partially on reading the same articles, how should you weigh their opinions?

The above examples illustrate one source of statistical dependency between opinions in a social network: shared information originating from the same source. If we are to understand learning in a social world, we must understand how people deal with such statistical dependencies while integrating their direct observations from the environment with the communicated beliefs of their social network peers. Investigating how information spreads through social networks and how statistical dependencies affect the formation of people’s beliefs is thus a key issue for
cognitive science with implications for e.g., the study of misinformation and echo chambers (Bikhchandani, Hirshleifer, & Welch, 1992; Madsen et al., 2018; Watts, 2002; Whalen et al., 2018), the dynamics of micro-targeting (Madsen & Pilditch, 2018), or advocacy organisations’ attempts to shape public debate (Bail, 2016).

Here, we investigate how people integrate information based on statistical (in-)dependencies underlying the beliefs of three social network peers in a fictitious political context. We first introduce a simple Bayesian model of belief revision to account for the normative case. Building on previous work on sensitivity to shared information in social learning (Whalen et al., 2018), we compare three different conditions (see Fig. 2.1). The first (Fig. 2.1a) serves as baseline in a sense that statistical independence between the beliefs of network peers is induced by the cover story. In the second condition, independence is violated as participants are told that the three network neighbors form their beliefs sequentially, meaning that the belief of the neighbor that formed their belief last (neighbor C) contains all information gathered by the others (Fig. 2.1b). In a third condition, social network peers form beliefs on the basis of shared evidence (Fig. 2.1c). Dependencies between beliefs in condition 2 differ from dependencies in condition 3 in the sense that neighbor C is the most relevant source since their belief already incorporates the beliefs of the other neighbors. We report on a behavioral experiment that investigated how subjects update prior beliefs under these conditions.
2.1.1 Information cascades and probabilistic beliefs

Information cascades—spreading of beliefs through networks—can produce maladaptive collective outcomes even as agents incorporate information from network neighbors in individually rational ways (Bikhchandani et al., 1992). This inherent susceptibility of social networks towards information cascades is supported by research showing that information cascades occur in simulated social networks where agents are furnished with an individually rational cognitive architecture (Fränken & Pilditch, 2021). Similar results have been obtained from empirical analyses of social media data, which showed that users cluster into communities dominated by like-minded others, resulting in proliferation of unsubstantiated beliefs or conspiratorial thinking (Del Vicario et al., 2016).

Previous models of information cascades assumed that people settle on particular beliefs by maximizing over subjective probabilities, thus leading to degradation of the information transmitted (Bikhchandani et al., 1992; Pilditch, 2017). Additionally, simulation-based work by (Madsen et al., 2018; Pilditch, 2017) did not account for potential dependencies underlying the beliefs of network peers. The assumption of independent beliefs among network peers is frequently used in general models of opinion dynamics and consensus generation, where people’s belief revision processes have been modeled by combining their initial beliefs with the weighted average of neighboring beliefs (Hegselmann & Krause, 2002). Expanding this body of literature, recent empirical results have shown that people’s social learning strategies are adaptive—accounting for statistical dependencies underlying the beliefs in their social networks (Whalen et al., 2018). In addition to maximizing, Whalen et al. (2018) tested the assumption of “probability matching”, which assumes that people settle on beliefs stochastically, drawing particular conclusions proportional to the posterior probability of a belief (Shanks et al., 2002).

In the present work, we make neither assumption (matching or maximizing), instead empirically exploring a setting in which agents communicate their full probabilistic beliefs. Using probabilistic beliefs allows us to explore the influence of communicated certainty—defined here as the precision of the belief distribution—and its related probabilistic quantity confidence—the probability that a particular choice is correct. These are at the core notion of an rational agent (Fleming & Daw, 2017; Pouget, Drugowitsch, & Kepecs, 2016) and thus play a crucial role during
the integration of social information to update prior beliefs (see e.g., De Martino, Bobadilla-Suarez, Nouguchi, Sharot, & Love, 2017).

## 2.2 Normative framework

We explore a general sequential belief updating setting in which people first gather evidence by themselves (i.e. asocial information) before reporting their initial belief about the relative competence of two fictitious competing political candidates. Evidence comes in the form of binomial “performance tests” that result in either a 0 = loss or 1 = win for each candidate. We thus model evolving beliefs about the relative competence of candidate A over B using the beta probability distribution $X \sim \text{Beta}(\alpha,\beta)$. Following Bayes’ rule, the initial posterior probability of a belief or hypothesis $p(h)$, given asocial information, $d$, thus corresponds to the normalized product of the likelihood $p(d|h)$ and the prior $p(h)$:

$$p(h|d) \propto p(d|h)p(h). \tag{2.1}$$

In our computational model, we use an initially uniform Beta(1,1) prior which is conjugate to the binomial likelihood $\binom{n}{k} p^k (1-p)^{n-k}$, allowing us to model belief updating straightforwardly using the analytical posterior Beta($1+k, 1+n-k$). For example, observing data $D = \{0,1,1\}$ where $k = 2$ successes in $n = 3$ binomial trials, the posterior is $X \sim \text{Beta}(3,2)$ with a mean of $\frac{3}{5}$. This reflects the nature of subjects’ beliefs about the two candidates, which include an overall preference for a candidate (if mean $\frac{\alpha}{\alpha+\beta}$ is $<\text{ or } > .5$) and a measurement of certainty (precision of the beta distribution, given by $\frac{(\alpha+\beta)^2(\alpha+\beta+1)}{\alpha\beta}$). The model then also gives clear qualitative (directional) and quantitative predictions for how learners should update their belief upon observing the beliefs of other network neighbors depending on the condition. As in Equation 1, we use Bayes’ theorem to model how people should integrate the beliefs (i.e. social information) from their network neighbors $s$:

$$p(h|s_1,...s_n) \propto p(s_1,...s_n|h)p(h) \tag{2.2}$$

Assuming that the beliefs of neighbors are perfectly independent, transparent and trustworthy (Fig. 2.1a), the target’s posterior after incorporating the beliefs of
their peers should simply be a new beta distribution with the parameters $\text{Beta}(n_0 + k_A + k_B + k_C, n_0 + n_A + n_B + n_C - k_A - k_B - k_C)$ where $n_0$ and $k_0$ are from their prior. If neighbors are sequentially dependent, in the sense that A communicated their belief to B who then saw more data and communicated to C, the aggregated parameters of the posterior distribution should be based on the neighbor that formed their belief last (i.e. neighbor C in Fig. 2.1b). The normative posterior for the sequential case is thus equal to $\text{Beta}(n_0 + k_C, n_0 + n_C - k_C)$. Finally, if sources are dependent in the sense that their beliefs are based on at least partially shared information (Fig. 2.1c), the normative model provides an upper- and a lower bound for the revised posterior. The upper bound is equal to the independent case, and it assumes that none of the neighbors’ beliefs were influenced by the shared data (i.e. $D = \{\}$). Conceptually, this can be compared to a scenario in which panel members ignore all shared application materials and interviews, evaluating the candidate’s performance entirely based on their prior beliefs.

As we do not vary the parameters of peers between conditions in our experiment, the only source of variation in updating can be attributed to manipulating the dependencies between neighbors. Thus, the model lower bound is equal to subtracting the lowest $\alpha$ and $\beta$ parameters from the aggregate parameters. For the present experiment, we do not specify on how much neighbors were influenced by the shared data. Thus, assuming all possible combinations of overlap equi-probable, we model the normative impact of shared information on belief updating as having a magnitude intermediate between strictly independent information (higher magnitude) and sequentially updated beliefs (lower magnitude). Based on this framework (Fig. 2.1), we derived the following qualitative (directional) predictions: The difference between subjects initial- and revised posterior probabilistic beliefs will be smaller when the beliefs of social network peers are dependent as compared to the independent condition (1). The dependent case of sequentially updated beliefs will result in a smaller update of prior beliefs as compared to shared information (2).
2.3 Experiment

2.3.1 Participants

Participants (N = 79, range: 21–69 years, mean = 39.89, SD = 12.97, 35 female) were recruited and tested through Amazon’s Mechanical Turk. Participants were native English speakers based in the United States. They were paid $1.75 for their time (mean = 17.49 min, SD = 6.91 min).

2.3.2 Task description and measures

Participants imagined being a political consultant travelling around the US to help local branches of their political party decide between two fictitious competing candidates most suitable for public office. To do so, they imagined that they were travelling to three different cities, with two different candidates competing in each city. Prior to the main task, participants completed a short training phase and comprehension quiz to ensure that they understood how to provide their beliefs using the interface shown in Fig. 2.2. Specifically, participants used two response sliders to provide their full probabilistic beliefs, one controlling the mean of the density (belief slider) and one controlling the log precision (certainty slider). The response sliders ranged from 1–99; where a belief of 1 means full support for the left candidate, 50 is neutral, and 99 full support for the right candidate. A certainty of 1 is the lowest possible certainty and a certainty of 99 is the highest possible certainty. The resulting density was dynamically displayed to participants as they selected their response. The range of allowable values was restricted to ensure the belief function was concave (i.e. $\alpha \geq 1$ or $\beta \geq 1$).

Within each city, the order of steps was: (1) obtaining prior belief based on observing asocial evidence → (2) learning about the beliefs of social network neighbors under consideration of statistical (in)dependence → (3) providing final posterior belief. We set the scene using an uniform prior belief $X \sim \text{Beta}(1,1)$ telling subjects that the two competing candidates were tested on two initial tests (each winning one of them) prior to the arrival of participants. Participants then observed the performance of the two competing candidates across two additional independent selection tests assessing different qualifications not covered in the initial tests.
Following observation of asocial information, participants rated their prior belief and certainty in the relative suitability of the two competing political candidates for public office. The procedure was identical for each condition. After the initial assessment phase, subjects were shown the belief and certainty ratings of three social network neighbors (i.e. social information; see Fig. 2.3). The network neighbors were described as three locals that were likely voters from a subject’s political party who had learned about the candidates during debates in their local town-hall. Each city included a different cover story about the relationship between the three locals matching either statistical independence, sequential dependence, or shared information. After learning about the beliefs of locals and their relationship, subjects provided their final belief.

**Figure 2.2: Interface for rating belief and certainty.**

**Figure 2.3: Example beliefs of locals.**

### 2.3.3 Design and Procedure

We employed a within-subjects design with three levels (variations of network setups implied by differing cover stories). The three levels of our independent variable were: independent information, sequential dependence, and shared information. The two data points (i.e. test outcomes) used to parameterize subjects’
**Figure 2.4:** Summary of model predictions (columns 1-2) and behavioural results (columns 3-4) for each condition (rows 1–3). For our main analysis, we used the aggregate \( \mu \) and \( \sigma^2 \) parameters (plotted below each distribution) to make the interpretation of our predictions and results more intuitive. ** = \( p < 0.01 \) refers to the comparison of the (log) Jensen-Shannon Divergence from priors to posteriors between conditions (see model 4 in Table 2.2 and Fig. 2.5d for details).

Initial (neutral) prior beliefs, the resulting normative prior, and the parameters of the three locals are shown in Table 2.1. Parameter settings were constant across conditions, with the only source of variation being our independent variable. Resulting model predictions and sufficient statistics are summarised in Fig. 2.4 (columns 1-2). The order of conditions and the position of the candidate supported by locals (left/right) was randomized between participants. After completion of the main task, participants provided basic demographics (e.g., gender).

**Table 2.1:** Fixed parameters used across conditions.

<table>
<thead>
<tr>
<th>Data</th>
<th>Normative Prior</th>
<th>Parameters of Locals</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1,0}</td>
<td>B(2,2)</td>
<td>B(46.95), B(49.10), B(50.45)</td>
</tr>
</tbody>
</table>

### 2.3.4 Analysis

Our analysis has two parts. First, we compared the aggregate parameters of subjects’ posterior judgements between conditions to evaluate qualitative (i.e. directional) alignment with our model predictions. Thus, we first contrasted subjects’ posterior means (model 1) and variances (model 2) between conditions using two linear mixed-effects models with condition (i.e. social network set-up) as fixed
effect and subject as a random intercept. However, evaluating these separately may miss dependencies that exist in how participants updated these components of their beliefs. To address this, we computed the Jensen-Shannon Divergence $D_{JS}$ between priors and posteriors for each subject to determine whether the magnitude of updating prior beliefs differed by condition. $D_{JS}$ allows measurement of changes both in mean and variance between distributions through a single symmetric distance measure given by:

$$D_{JS}(P||Q) = \alpha D_{KL}(P||Q) + (1-\alpha)D_{KL}(Q||P)$$  \hspace{1cm} (2.3)

where $D_{KL}$ is the Kullback-Leibler Divergence, a standard asymmetric measure in information theory for measuring how much a probability density $P$ has moved compared to a reference distribution $Q$. By definition, $D_{KL} \geq 0$, being equal to 0 if and only if $P$ and $Q$ are identical. A limitation of $D_{KL}$ is its nonsymmetry, which is resolved by $D_{JS}$ if $\alpha = 0.5$. Having computed each subject’s $D_{JS}$, we fitted two additional mixed-effects models with condition as fixed effect and subject as random intercept to compare mean differences in $D_{JS}$ (model 3) and the log-transform of $D_{JS}$ (model 4). The reason for using log$D_{JS}$ in model 4 is that the distribution of log$D_{JS}$ residuals was closer to a normal distribution than the distribution of $D_{JS}$ residuals (which showed a skew to the right). All models were compared to a reduced model including only an intercept as predictor variable and subject as random effect. Models were implemented in R using the function \texttt{lmer()} from the package \texttt{lme4} (Bates, Mächler, Bolker, & Walker, 2015).

Following tests of our qualitative (directional) comparison between conditions, we compared subject and model performances across conditions to check in how far subjects aligned quantitatively with our normative framework. Therefore, we first computed $D_{JS}$ between subjects’ prior beliefs (Fig. 2.4, column 3) and the normative prior (Fig. 2.4, column 1) across conditions. Due to the skewed distribution of $D_{JS}$ for this comparison, we conducted a Wilcoxon signed rank test (non-parametric t-test; alternative hypothesis $> 0$) to check if subjects integrated the asocial information as predicted by our model. We also compared the difference between subjects’ prior mean and model prior mean and subjects’ prior variance and model prior variance (using two-sided Wilcoxon signed rank tests because the dependent variables did not follow a normal distribution). The three comparisons were repeated to contrast subjects’ posteriors with model posteriors.
2.4 Results

2.4.1 Sanity checks

Five subjects were removed from our analysis because they did not change the positions of sliders between their prior and posterior judgements (i.e. \( \log D_{JS} = -\infty \)), resulting in a final sample size of 74. Levene’s test revealed that the homogeneity of variance assumption was maintained for all four dependent measures (all \( p > 0.05 \)) used between models 1-4. Inspection of residual plots confirmed that the residual posterior means, posterior variances and \( \log D_{JS} \) residuals were normally distributed. For \( D_{JS} \), residuals showed skew to the right. Correlations between the three levels of our fixed effect (i.e. social network set-up) were moderate, ranging from \( \pm 0.470 \) to \( \pm 0.551 \). Comparing each model to its reduced version revealed that inclusion of social network set-up only contributed significantly to the proportion of explained variance in \( \log D_{JS} \) (see Table 2.2)\(^1\). The qualitative comparison in the remainder of this paper will thus focus on interpreting the results of model 4 (for completeness, regression coefficients for models 1-3 are reported in the next section and in Fig. 2.5).

Table 2.2: Model fits for each dependent variable (DV).

<table>
<thead>
<tr>
<th>Model</th>
<th>DV</th>
<th>BIC(_{\text{diff}})</th>
<th>( R^2_m )</th>
<th>( \chi^2 )</th>
<th>( p)-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \mu )</td>
<td>6.14</td>
<td>0.009</td>
<td>4.69</td>
<td>0.096</td>
</tr>
<tr>
<td>2</td>
<td>( \sigma^2 )</td>
<td>9.10</td>
<td>0.004</td>
<td>1.65</td>
<td>0.439</td>
</tr>
<tr>
<td>3</td>
<td>( D_{JS} )</td>
<td>6.40</td>
<td>0.012</td>
<td>4.52</td>
<td>0.105</td>
</tr>
<tr>
<td>4</td>
<td>( \log D_{JS} )</td>
<td>-1.43</td>
<td>0.032</td>
<td>12.27</td>
<td>0.002</td>
</tr>
</tbody>
</table>

2.4.2 Qualitative comparison

Fig. 2.4 summarises model predictions and aggregate parameters across subjects for our three experimental conditions. The directional shift from subjects’ prior distributions to their posteriors and the fact that subjects’ posterior distributions were more compressed than their priors suggested that social information resulted in increased belief and certainty ratings. As expected by our normative model, the

\(^1\)BIC\(_{\text{diff}}\) = BIC\(_{\text{full}}\) - BIC\(_{\text{intercept-only}}\); \( R^2_m \) = proportion of variance explained by the fixed effect (i.e. social network set-up).
results of model 4 showed that subjects changed their prior beliefs significantly less in the two dependent conditions as compared to the independent case (Fig. 2.5d; \( b = -0.639, t(148) = -3.07, p = 0.003 \) for sequentially updated beliefs and \( b = -0.641, t(148) = -3.08, p = 0.003 \) for beliefs based on shared information). This means that the magnitude to which people updated their beliefs (i.e. changed both the belief and certainty sliders) was in line with the predicted (directional) magnitude of our normative model. In other words, people were sensitive to differences in the statistical power of the information between the independent condition (larger statistical power implying stronger updating of prior beliefs) and the two dependent conditions (smaller statistical power implying smaller updating; see Fig. 2.4).

For the comparison between the two dependent conditions, no significant effects emerged (all \( ps > 0.05 \); see Fig. 2.4 and Fig. 2.5d). The results of models 1 (\( \mu \)) and 3 (\( D_{JS} \)) showed that the comparison between sequentially updated beliefs and independent beliefs was significant (\( b = -0.028, t(148) = -2.16, p = 0.032 \) for model 1; Fig. 2.5a and \( b = -1.703, t(148) = -2.02, p = 0.045 \) for model 3; Fig. 2.5c). For model 2 (\( \sigma^2 \)), no significant differences emerged (all \( ps > 0.05 \); Fig. 2.5b).
2.4.3 Quantitative comparison

Quantitative comparisons revealed that subjects’ prior distributions differed significantly from model priors in terms of $D_{JS}$ ($V = 25425, p < 0.001$). Inspection of Fig. 2.4 suggests that this difference might be driven by dissimilar variances, as subjects’ priors were less diffused than model priors. The results of further comparisons confirmed this observation, showing that the mean of subjects’ prior variance was significantly smaller than model prior variance ($V = 1607, p < 0.001$), despite finding no significant difference between prior means ($V = 13150, p = 0.652$, see Table 2.3). These findings might be attributed to an initial overestimation of certainty upon observing the outcomes of the candidates’ test trials. Specifically, the average parameters of subjects’ initial estimates of the candidates were equal to B(19.2, 19.1), which was 9.58 times higher in magnitude than the simple Bayesian model’s B(2.2).

Table 2.3: Means and SDs of the dependent variables (DV) used for quantitative comparison.

<table>
<thead>
<tr>
<th>Measure</th>
<th>DV</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>prior</td>
<td>$D_{JS}$</td>
<td>2.547</td>
<td>3.782</td>
</tr>
<tr>
<td>prior</td>
<td>$\mu_{subj} - \mu_{model}$</td>
<td>0.001</td>
<td>0.085</td>
</tr>
<tr>
<td>prior</td>
<td>$\sigma^2_{subj} - \sigma^2_{model}$</td>
<td>-0.027</td>
<td>0.023</td>
</tr>
<tr>
<td>posterior</td>
<td>$D_{JS}$</td>
<td>6.270</td>
<td>8.491</td>
</tr>
<tr>
<td>posterior</td>
<td>$\mu_{subj} - \mu_{model}$</td>
<td>0.033</td>
<td>0.132</td>
</tr>
<tr>
<td>posterior</td>
<td>$\sigma^2_{subj} - \sigma^2_{model}$</td>
<td>0.008</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Posterior contrasts revealed that model and subject distributions were significantly different from each other as measured by $D_{JS}$ ($V = 25425, p < 0.001$). For posteriors, this difference was driven by a mismatch both in terms of posterior means ($V = 17196, p < 0.001$) and variances ($V = 25183, p < 0.001$). These findings demonstrate that, overall, subjects changed their posterior means more than expected by the normative prediction (mainly due to a strong mismatch between posterior means in the sequential belief updating condition, see Fig. 2.4, row 2). Subjects’ average posterior variance was significantly larger than model posterior variance ($\sigma^2_{subj} = 0.01; \sigma^2_{model} = 0.002$). Compared to the average prior comparison, this finding might be attributed to the fact that subjects down-weighted the evidential value of social information obtained from their peers. The average model posterior parameters across conditions were equal to B(107.7, 58.4),
which was 3.55 times the magnitude of subjects average posterior parameters $B(31.8, 15.0)$.

2.5 Discussion and further work

We modeled a sequential belief updating process including a target agent (i.e. the participant) and three social network neighbors. The cover story describing the relationship between network peers was varied in three within-subject conditions to investigate the effects of three statistical (in)dependencies summarised in Fig. 2.1. Extending the findings of (Whalen et al., 2018), our behavioural results confirmed our prediction that people update their beliefs significantly less when the provided social information was coming from dependent sources (as compared to the independent case). Thus, our result shows that people are not simply combining their own beliefs with the communicated beliefs of their network neighbors. Rather, they are additionally sensitive to the origin of those beliefs and to what extent they are redundant. This contributes another empirical piece to the puzzle of how to counteract the spread of false consensus effects and information cascades, which have been suggested to occur in networks of agents forming their beliefs in individually rational ways (Bikhchandani et al., 1992; Pilditch, 2017).

We could not confirm whether people differentiated between the evidential signals of shared information and sequentially updated beliefs while revising prior beliefs (despite the trend matching the predicted pattern; see Fig. 2.4). This might be attributed to the context of the task: we assumed that subjects would learn about political candidates based on binomial “performance tests”; and we operationalized network peers as locals being likely voters from a subject’s political party that formed their beliefs based on attending debates in their local town-hall. A more abstract experimental paradigm, such as learning coordinating with others to estimate the proportion of blue vs. red marbles in an urn (a common paradigm used to study information cascades and sequential belief updating; see e.g., L. Anderson & Holt, 1997) which does not require such context specific assumptions might have resulted in a measurable difference between the two dependent conditions. Despite being unable to differentiate between the two dependent cases, our task provides a valuable contribution to the field of social learning in the context of (online) political belief formation (see e.g., Bond et al., 2012).
To address the limitation of context, further experiments need to test the ecological validity of the presented normative framework across a variety of scenarios. These might involve emotional decisions (e.g., in the context of moral dilemmas), rational tasks, such as business decisions, and more general scenarios that are abstract (e.g., urn-based tasks). An additional limitation of our work is that we assumed additive communication of evidence in the case of sequentially updated beliefs (see Fig. 2.1). Generally, a more formal explanation of how sources form their beliefs and a precise description of the computational processes underlying evidence accumulation between conditions are important issues that need to be addressed in further work.

Relative to our simple Bayesian account, quantitative comparison between model predictions and subjects suggested that subjects over-weighted the influence of asocial information while they under-weighted the influence of social information. This finding is in line with previous theoretical (Schöbel, Rieskamp, & Huber, 2016) and empirical (Nöth & Weber, 2003) work demonstrating that people are more influenced by their own private information as compared to social information, though in our case this might be an artifact of simulated social information (i.e., information coming from hypothetical social network members rather than actual ones). Moreover, we assumed that the shapes of the reported distributions reflect how subjects represent the distributions of their actual beliefs. If this assumption is not met, preferential weighting of asocial over social information might also be attributed to an initial misrepresentation (i.e., overestimation) of certainty. Thus, further work using the proposed interface could include a control measure of subjects’ ability to accurately report their certainty. This might involve an initial assessment of beliefs prior to engaging with any form of evidence.

To address some of the present limitations, we plan to replicate the current experiment across a variety of contexts where actual subjects come in pairs/triples to do the same task, enabling them to update beliefs dynamically. To generate a better understanding of whether people appropriately down-weight social evidence, further empirical work could also incorporate agent-based simulations contrasting normative accounts with competing models of social influence (e.g., Schöbel et al., 2016). This approach might enable measuring the empirical degree of information degradation due to correlated sources in realistic information networks. At the present stage, our finding suggests that people are able to understand and report probabilistic beliefs, which might be useful for calibrating belief parameters in
related agent-based models (ABMs) of echo chambers (Madsen et al., 2018) and scientific belief formation (Lewandowsky et al., 2019).

In summary, our results suggest that while a Bayesian framework provides a good qualitative account of how people update their beliefs based on social information coming from sources with different levels of independence, it cannot, in the current form, account for the relative weights that people assign to private and social information while updating their beliefs. We acknowledge that these findings might depend on the specific context of our experimental paradigm and plan further experimental validation of model predictions across alternative scenarios.
Chapter 3

Know your network: Sensitivity to structure in social learning

<table>
<thead>
<tr>
<th>Preamble</th>
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<tbody>
<tr>
<td>In Chapter 2, we found that when other players are simulated rational agents and network structures are explicitly provided, people seem to (qualitatively) accommodate social source dependence in a Bayesian fashion. In this chapter, we challenge this finding by increasing the difficulty of the learning setup in two ways: First, we include multiple belief updates. Second, we do not explicitly tell participants about dependencies between other players (i.e. the structure of the social network). Given the increased difficulty of this setup as well as our finding that participants’ were overly sensitive to private evidence using mean and variance sliders (see Fig. 2.4), we here revert to a discrete judgement scale to elicit participants’ responses (see Section 3.3.3, for details). The work in this chapter has been presented as a poster at CogSci 2021 (Fränken, Valentin, Lucas, &amp; Bramley, 2021) and was conducted by myself under the supervision of Chris Lucas and Neil Bramley. Simon Valentin (second author) helped with the conceptualisation of the study and the development and implementation of the structure learning algorithm presented in Section 3.3.4. I wrote a first draft of the paper and all co-authors helped with the writing and editing of the draft. Our code is available on GitHub via FrankenValentinLucasBramley2021. An interactive demo of our task is available here.</td>
</tr>
</tbody>
</table>
Abstract

To glean information from social networks, people must be adept at distinguishing real evidence from hearsay. Here, we investigate human inferences within a simulated social network. We introduce a novel social learning task in which participants infer their surrounding social network structure while also using social communications to aid judgements about the shared environment. We find that the majority of adults correctly identified independent and acyclic network structures, though they struggled to identify cyclic communications. Comparison of judgements with several social inference models supports a naïve social learning account that integrates social evidence with the learner’s own observations in a structure insensitive way. This suggests that people are capable of using communication patterns to identify social influences, but they are still misled by the distortions of evidence that these network dynamics can produce.

3.1 Introduction

The ability to communicate, and thereby combine individually gathered evidence and insights, is a key driver of the success of the human species (Henrich, 2017). Social learning enables us to acquire knowledge without direct experience, vicariously by observing the behavior of others (Bandura & McClelland, 1977), and via a variety of more deliberate and explicit forms of information exchange (Whalen et al., 2018). When it works, social learning is evolutionary adaptive (Laland, 2004; Rendell et al., 2011), allowing groups to outperform individuals in inference and estimation (Galton, 1907; Krafft, Shmueli, Griffiths, Tenenbaum, & Pentland, 2020) and transmit accumulated insights and wisdom, so “bootstrapping” future generations’ learning (Kleiman-Weiner et al., 2019).

One important predicate for the success of social learning is an ability to judiciously combine evidence observed directly from the environment, with reported or imputed beliefs of others. The latter can frequently be unreliable, incomplete, deceptive or redundant (Enquist, Eriksson, & Ghirlanda, 2007; Rendell et al., 2011). One component of this is the ability to distinguish between independently formed beliefs—such as those of independent witnesses to a crime—and statistically dependent and so partially redundant beliefs—such as those of witnesses who were together at the scene, or who conferred prior to testimony (Pilditch et al., 2020). A more extreme form of this dependence arises when receiving evidence from the same source multiple times without realizing that it has the same origin. This may occur when information is duplicated via several different proxies within
a social network, leading to the danger of systemic exaggeration of evidence. Unfortunately, anonymous online communication and social media appear to have created social network structures that can produce maladaptive behaviors, such as information cascades where unsupported or polarized beliefs spread through a population (Del Vicario et al., 2016; Madsen et al., 2018).

Recent work has explored human sensitivity to information redundancy in social learning (Fränken et al., 2020). This work finds that, at least when social network structure is known and beliefs are fully observed, people do not naively aggregate peers’ beliefs with their own, but are sensitive to differing levels of redundancy of social network neighbors depending on network structure. Specifically, participants substantially downweighted social evidence relative to direct evidence, moderated to the extent that the structure of their social network implied that the peers’ views were redundant. It has separately been suggested that people display sophisticated social learning mechanisms, allowing them to differentiate between the evidential value of beliefs coming from more or less independent sources (Whalen et al., 2018). Importantly, this previous work has focused on social inference under complete knowledge of communication pathways. It is an open question whether and to what extent people can infer network structure from patterns of communications over time, and whether they can use these inferred structural models to accurately integrate social evidence.

In this paper, we investigate whether people can use communications from two peers to (1) infer the micro-social network structure between them and (2) appropriately weigh peers’ communications when making a judgment about the shared environment. We study several qualitatively different micro-social network structures (Figure 3.1). In one, peers $B$ and $C$ do not communicate with one another,
making their beliefs independently valuable as evidence (Fig. 3.1a). In other conditions, one network peer influences the other in addition to signaling their belief to the target A, producing an asymmetric pattern of information redundancy in their communications (Fig. 3.1b&c). Extending previous work (Fränken et al., 2020; Whalen et al., 2018) we also explore scenarios where there is bidirectional communication between B and C (Fig. 3.1d), meaning peers’ beliefs can diverge chaotically from that licensed by the evidence either has observed directly.

The rest of the paper is organized as follows. We first introduce our novel task, then formalize the requisite probabilistic inference problem before laying out two competing accounts of social inference alongside two baseline models. We then report on a behavioral experiment in which we compare these models to participants’ inferences.

3.2 Our social learning task

We developed a novel learning task in which participants observe direct evidence as well as a series of communications from network peers relating to an environmental property. We look at social inference from the perspective of a target agent A (the participant in our task) who must infer the nature of a binomial property of the environment, and can use communications from their peers B and C alongside their own observations to do so. Critically, A receives communications from B and C, but does not initially know whether B and C also communicate with each other. A must infer this if they are to accurately identify the environmental property from peers’ communications. Analogous to standard “ball and urn” tasks (L. Anderson & Holt, 1997), participants are instructed that they are reasoning about the nature of a planet inhabited by strange space fish that can be either red or blue. The cover story establishes that it is important to work out whether the planet is inhabited predominantly by red or blue fish (essentially whether Bernoulli parameter \( \theta > .5 \) or \( \theta < .5 \)). Participants imagine traveling to five different space stations. At each space station they observe one fish themselves and observe simple communications from two locals (“crew members”; see Fig. 3.2a) whose relationship matches one of the social networks in Fig. 3.1. Participants ultimately make judgements about the social network structure of the communications on each space station (Fig. 3.2b) as well as a guess and corresponding confidence about whether the planet of origin has more red or blue fish (Fig. 3.2c).
Figure 3.2: Task Overview. a) Over the course of ten trials, participants observe communication signals from two crew members. At each trial, crew members’ current signals are displayed on the left side of the screen. After completion of a trial, crew members’ signals are added to the signal summary section on the right. Prior to the observation of crew members’ signals, participants observed one red fish which was displayed on the top right of the screen throughout the task. b) Following observation of signals, participants provided a forced-choice structure guess about the relationship between crew members. Possible structures include independence between crew members (crew members did not influence each other), acyclic relationships (one crew member influenced the other), and cyclic relationships (both crew members influenced each other; see Section 3.4.3). c) Following their structure guess, participants provided a forced-choice judgement about the planet travelled by the supplier (red versus blue) as well as a confidence rating ranging from 1 (very unconfident) to 3 (very confident).

3.3 Social network model

We represent the learning setting with a dynamic directed network \( G = \{V, E\} \) with both human and artificial agents represented as nodes \( \{A, B, C\} \in V \) and edges \( e_{ij} \in E \) indicating that a agent \( i \) “communicates” to agent \( j \). We model each artificial agent \( v \) as holding a private belief about the true proportion \( P(\theta)^t_v \) in the form of a Beta distribution with parameters \( \alpha^t_v \) and \( \beta^t_v \). We assume artificial agents retain this (belief) state from the previous time step (see Figure 3.3, gray arrows) and update it by integrating any social evidence \( s_t \) arriving from any parents in the graph \( \text{pa}(v) \) (black arrows) as well as any newly observed direct evidence \( e_t \) (dotted arrows).
3.3.1 Private evidence

Private evidence $e_t$ takes the form of occasional observed samples from the planet, allowing for straightforward increments of an agent’s $\alpha$ and $\beta - \alpha_t = \alpha_{t-1} + 1$ if $e_t = \text{red}$ and $\beta_t = \beta_{t-1} + 1$ if $e_t = \text{blue}$.

3.3.2 Social evidence

Agents communicate based on their private beliefs and these communications are received at the subsequent timestep. We here restrict ourselves to settings in which the human agent ($A$) does not communicate to anyone, and assume a simple communication format for artificial agents $B$ and $C$. Specifically, artificial agents always signal their belief to all their children in $G$ with a tuple $s = \{\text{direction, confidence}\}$ combining their best guess [$\text{red, blue}$]—and a confidence therein. Our simulated agents signal “red” if $\frac{\alpha}{\alpha + \beta} > .5$, “blue” if $\frac{\alpha}{\alpha + \beta} < .5$ and randomly signal either “red” or “blue” if their belief is perfectly neutral $\frac{\alpha}{\alpha + \beta} = 0.5$.

We model their associated (continuous) confidence signal as simply equal to the cumulative density for $P(\theta)_t$ falling on the requisite side of .5 and relative to a neutral baseline of .5—$F(1; \alpha^u_t, \beta^u_t) - F(.5; \alpha^v_t, \beta^v_t)$ if signaling “red” and $F(.5; \alpha^v_t, \beta^u_t) - .5$ if signaling “blue”.

For simplicity, we initially model the artificial agents as using a simple, graph agnostic way of incorporating this social evidence, simply taking this confidence $\in (0, .5)$ as a pseudocount and adding it to their $\alpha$ or $\beta$. For a concrete example,
consider network structure $B \rightarrow C$ (Fig. 3.4a, second panel). $B$ and $C$ start with a neutral Beta$(1,1)$ at $t=0$ and initially signal randomly with zero confidence. $B$ observes a blue fish at $t = 3$ leading to an updated belief state $P(\theta | t=3) = \text{Beta}(1,2)$ and consequently begins signaling $s_B \rightarrow \{\text{“blue”, .25}\}$ to $A$ and $C$ (direction: $\frac{1}{3} < .5$, confidence $F(0.5; 1,2) - .5 = 0.25$). At the subsequent time points this influences $C$ such that $P(\theta | t=4) = \text{Beta}(1,1.25)$, $P(\theta | t=5) = \text{Beta}(1,1.5)$ and so on.

Panel $B \rightarrow C$ in Fig. 3.4a shows how this scenario evolves following $B$’s observation at $t = 3$ and $C$’s observation of a red fish at $t = 6$. In spite of the evidence seen by $B$ and $C$ combined being neutral (one fish of either color) both $B$ and $C$ end up with a moderate belief that the planet contains more blue than red fish, essentially due to $B$’s earlier and sustained influence.

3.3.3 Discrete confidence

In our experiment, we opted to discretize visualizations and elicitations of confidence to the participant $A$ to align with the granularity of human qualitative judgements (Fleming & Daw, 2017). We thus visualized signals with a ternary confidence scale $[1: \text{low}, 2: \text{medium}, 3: \text{high}]$ and also used this scale to elicit participants’ judgements (Fig. 3.2c). Here, low confidence (visualized as a faintly colored circle with a “1”) corresponded to a continuous signal below $\frac{1}{6}$, medium confidence corresponded to a continuous confidence between $\frac{1}{6}$ and $\frac{1}{3}$, and high confidence (visualized as a saturated circle with a “3”) to a continuous signal larger than $\frac{1}{3}$.

3.3.4 Social inference models

We have outlined how we chose to model the evolving beliefs and communication signals of artificial agents $B$ and $C$ given evidence $e$ and a true network $G$. While this represents one possibility as to how our human agent will behave while playing the role of agent $A$, there are many possibilities and characterizing human behavior while embedded in these scenarios is our key empirical aim. We now lay out two alternatives that we will initially consider as accounts of how our participants may approach the task.
3.3.4.1 Naïve inference

The “Naïve” account simply follows the graph agnostic social inference mechanism we used to simulate the artificial agents above. Specifically, this account assumes that people will additively combine each communication signal from peers B and
Chapter 3. Sensitivity to structure in social learning

C with their own private evidence. This account ignores (1) the potential relationships between B and C (essentially assuming they are independent sources) and (2) the fact that signals may often convey evidence that has already been conveyed on an earlier trial.

3.3.4.2 Exact inference

We contrast na"ive social inference with a computationally idealized Bayesian model that inverts the social evidence received from both neighbors $s_{A\rightarrow A} = \{s_{B\rightarrow A}^{1:10}, s_{C\rightarrow A}^{1:10}\}$ to infer both the true underlying network $G \in \mathcal{G}$ jointly with the true fish observations of the other agents $o \in \mathcal{O}$:

$$P(G, o|s) = \frac{p(s|G, o)P(o|G)P(G)}{\sum_{G'\in\mathcal{G}} \sum_{o'\in\mathcal{O}} p(s|G', o')P(o'|G')P(G')}.$$  \hspace{1cm} (3.1)

Here, $\mathcal{G}$ just contains the four structures shown in Fig. 3.1 over which we assume an initially uniform prior. In order to provide a fair comparison to our human participants, we use the same discretized signals viewed by participants (Fig. 3.2a). For computational convenience, we also assume signaling and belief integration mechanisms of the artificial agents are known to the exact learner and further that possible sequences will always involve agents’ B and C sampling exactly one fish at random time points. However, this still results in a very large sequence space $\mathcal{O}$. Practically therefore, we use rejection sampling to compute marginals over graphs and the total blue and red fish observed by B and C.

To assess how reliably social network structure can be recovered from communication sequences like those in the current experiment, Table 3.1 shows the average marginal posteriors over $G$ for a set of random sequences sampled from each network.

For the present task, we selected sequences of communication signals for which the true network and the fish observed by the two artificial agents was strongly recoverable by this process (Figure 3.4a). As such, the Bayesian model’s prediction for $P(G|s)$ is always strongly peaked at the true graph and $p(\theta^A_{1:10}|s)$ is approximately equal to an “omniscient” inference based directly on the three fish truly sampled from the planet.
Table 3.1: Inferred posterior means over networks $G \in \mathcal{G}$

<table>
<thead>
<tr>
<th>Inferred</th>
<th>Indep.</th>
<th>$B \rightarrow C$</th>
<th>$C \rightarrow B$</th>
<th>$B \leftrightarrow C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indep.</td>
<td>0.67</td>
<td>0.13</td>
<td>0.17</td>
<td>0.03</td>
</tr>
<tr>
<td>$B \rightarrow C$</td>
<td>0.13</td>
<td>0.73</td>
<td>0.03</td>
<td>0.11</td>
</tr>
<tr>
<td>$C \rightarrow B$</td>
<td>0.23</td>
<td>0.03</td>
<td>0.72</td>
<td>0.02</td>
</tr>
<tr>
<td>$B \leftrightarrow C$</td>
<td>0.02</td>
<td>0.04</td>
<td>0.05</td>
<td>0.88</td>
</tr>
</tbody>
</table>

NB: For 30 sampled observation sequences of ten steps involving $B$ and $C$ observing one fish each.

3.4 Experiment

3.4.1 Participants

Participants ($N = 81$ native English speakers, $34.4 \pm 13.6$ years, 54 female, 1 non-binary) were recruited through Prolific (Palan & Schitter, 2018) and paid £1.00 + up to £2.50 performance bonus. The task took around 15 minutes.

3.4.2 Design and Procedure

Participants first completed an instruction phase in which they read the “space fish” cover story and learned about the communication signals used by the other agents. They then had to pass comprehension checks before facing five within-subject conditions, each with a different unknown social network structure. In each condition, participants first observed a fish themselves at $t = 0$, then clicked through a series of ten steps in which they would receive discretized signals from $B$ and $C$ (Fig. 3.2a). After this they made a forced choice judgment about the communication pattern (Fig. 3.2b), and then a further judgment about the nature of the planet and associated confidence (Fig. 3.2c) using a six point scale matching the six possible communications from $B$ and $C$.

At the end, participants were paid a bonus of £0.25 for each time they had selected a correct structure or planet. The condition order, fish color, planet and structure response options were randomized between participants. Additionally, the left/right positions of the crew members $B$ and $C$ were independently randomized between trials. After completing all trials, subjects provided basic demographics.

\footnote{Link to our task.}
3.4.3 Stimuli and model predictions

Fig. 3.4a shows the five learning conditions we included in the experiment. In each case, agent A (the participant) observes a fish at the start of the trial \((t = 0)\) and B and C each observe a fish some time later. To simplify visualization, Fig. 3.4 shows these sequences uncounterbalanced, such that A always initially encounters a red fish, agent B always encounters a blue fish, and agent C always encounters a red fish. The black and gray lines show the mean beliefs of artificial agents B and C over \(t = 1:10\). The yellow dashed line shows the model predictions for an agent A following the same na"ıve social integration approach as the artificial agents while the dashed green line shows the prediction under exact Bayesian inference.

We chose these sequences (Fig. 3.4) to exemplify several interesting network dynamics. For \(B \rightarrow C\), B’s observation precedes C’s, causing a primacy effect and correspondingly oversized influence of their observation on the final belief of C. For \(C \rightarrow B\), this pattern was reversed, with C’s observation preceding B’s. We also included two sequences for the \(B \leftrightarrow C\) network. In the inhibitory sequence (panel 4), B and C observed their evidence simultaneously and the subsequent communications serve to cancel one another out, approaching neutrality. In the excitatory sequence (panel 5), B receives their evidence earlier overriding C’s observation and creating an echo chamber in which both B and C become ever more confident that the planet is predominantly blue.

Based on the above sequences and observations of private evidence, we can derive directional predictions for a participant behaving like the na"ıve social learner (Fig. 3.4b). The exact Bayesian learner arrived at a \(p(\theta|t_{10})\) belief of approximately \(Beta(3,2)\) (plus counterbalancing) in all conditions.

3.4.4 Results and Discussion

Participants’ network structure judgements are shown in Fig. 3.5a. There was a significant relationship between condition and structure selection \((\chi^2(12) = 340.02, p < 0.001)\). The majority of participants correctly identified the independent and acyclic structures but frequently mistook the inhibitory cycle for independent and the excitatory cycle as acyclic. Planet judgements are shown in Fig. 3.5b. A Friedman’s test reveals that condition is a significant predictor of participants’ responses \((\chi^2(12) = 249.20, p < 0.001)\). Post-hoc pairwise Wilcoxon
Figure 3.5: a) Network structures selected by participants (x-axis) in each of our five experimental conditions. Y-axis corresponds to the percentage each of the four possible options was selected. Panel headings (Independent, $B \rightarrow C$, etc.) correspond to the true network structure in each condition. b) Average frequency with which participants’ (grey) selected each of the six discrete response options in our task (see Section 3.3.3). Naïve (orange, dotted) and exact (green, dotted) model predictions are shown on top of participants responses. Results show that while neither model did fully capture participants’ judgement patterns across conditions, the naïve account was closest to participants’ responses in condition $C \rightarrow B$ (Section 3.5).

Rank comparisons with Bonferroni-adjusted significance level $\alpha = 0.05$ show that all conditions significantly differ from each other on the 1–6 response scale (all $p$s < 0.005).

Thus, despite the network of agents encountering the same total evidence in every condition (i.e., two red and one blue fish), participants’ judgements differed substantially depending on the network structure and resultant communications. Of particular note, and in line with naïve social inference, only 35.8% favored the red planet for $B \rightarrow C$ and 32.1% for $B \leftrightarrow C_{\text{excite}}$. This suggests that an asymmetric or cyclic communication pattern combined with a single early (contradictory) source of evidence is enough to lead participants to judgements that contradict the overall balance of evidence and their own observations. However the response pattern for $B \rightarrow C$ is bimodal, with a substantial minority still favoring the red planet, in line with normative structure based inference or possibly with ignoring the social evidence altogether. This finding surprising on the face of it since a larger majority of participants correctly identified the network structure in $B \rightarrow C$ than actually used this to explain away the misleading communication signals.
Table 3.2: Model fits to participants’ planet judgements

<table>
<thead>
<tr>
<th>Data</th>
<th>Learner</th>
<th>BIC</th>
<th>$\tau$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full (405 trials)</td>
<td>Baseline</td>
<td>1451.3</td>
<td>-</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Non-Social</td>
<td>1401.7</td>
<td>3.99</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>Exact</td>
<td>1414.5</td>
<td>3.5</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Naïve</td>
<td>1265.7</td>
<td>6.24</td>
<td>50</td>
</tr>
<tr>
<td>Correct Structure (223 trials)</td>
<td>Baseline</td>
<td>799.1</td>
<td>-</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Non-Social</td>
<td>761.0</td>
<td>4.80</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>Exact</td>
<td>770.8</td>
<td>4.29</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>Naïve</td>
<td>697.1</td>
<td>6.04</td>
<td>35</td>
</tr>
</tbody>
</table>

$N =$ Number of participants best fit

3.5 Modeling participants’ planet judgements

We now assess the quantitative support for the naïve and exact models alongside two baselines. The first baseline model ("Non-social") is solely based on the participant’s privately observed fish, yielding a predicted posterior of $Beta(2,1)$ (with appropriate counterbalancing) for all conditions. The second baseline model ("Baseline") assigns uniform probabilities to all selections. We use maximum likelihood to fit models to participants’ planet judgements.

To fit each model $m \in M$, we converted the continuous posterior density $p(\theta)_{t=10}$ into six equal width bins $P(\theta)_{m}$ corresponding to the six direction and confidence combinations participants could select (Fig. 3.4b). We then fit a softmax function with temperature parameter $\tau$ to all but the “Baseline” model to convert these probability masses to choice probabilities. The likelihood that a participant selects response option $r \in R = \{1, 2, 3, 4, 5, 6\}$ on trial $t$ according to model $m$ is thus:

$$P(r)_{m}^t = \frac{e^{P(\theta)_{m}^t \tau}}{\sum_{r \in R} e^{P(\theta)_{m}^t \tau}}$$  \hspace{1cm} (3.2)

where $\tau \rightarrow 0$ indicates hard maximization over $P(\theta)_{m}^t$, while $\tau \rightarrow \infty$ indicates random responding. Table 3.2 compares the fitted models using BICs and reports the resultant $\tau$ value as well as the number of individuals best fit by each model.

We find that the naïve inference model provides the best overall BIC (1265.7) and the best fit for the largest number of participants (50/81). This finding supports our empirical results showing that participants’ judgements differed substantially
between conditions, suggesting that many participants naïvely combined communication signals from network peers with their own observations without consideration of structural dependencies (Fig. 3.5b). To further explore the impact of structure on participants’ judgements, we also contrasted trials during which participants selected the correct network structure (223 trials). Here, the naïve learner was again the best overall predictor of participants’ judgements with 35/79 participants best fit. For these trials, the performance of the exact inference model improved slightly, best accounting for 19/79 participants. This suggests that at least when structure inference is successful, responses from some participants reflect a more normative inference to the evidence behind the communications.

3.6 General Discussion

We investigated sensitivity to structural dependencies in combining social and private evidence. To do this, we introduced a novel task in which participants are embedded in a micro social network and use signals from network peers to aid judgements. Preliminary results suggest people can identify the presence and direction of communications between social network peers, but struggle to identify cyclic relationships and are easily led away from normative inference by peers’ repeated communications. Our modeling suggested relative structural insensitivity in the environmental judgements, with “Naïve” learning providing a better fit than structure sensitive “Exact” learning.

One puzzle was the finding that while the majority of participants correctly identified network structure $B \rightarrow C$, environmental judgements in this condition showed a bimodal pattern not characterized by either naïve integration of social evidence or structure sensitive normative inference taken alone. Overall, the naïve response patterns we found differ from previous results that suggest sophisticated accommodation of structure in social inference (Fränken et al., 2020; Pilditch et al., 2020; Whalen et al., 2018). However, the previous work provided explicit descriptions of dependencies and only considered a one-off communication. The current setting is more complex, involving uncertainty about communication channels and about when peers received evidence, making normative structure inference more computationally costly. Moreover, our results provide further evidence for the

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2 For this comparison, two participants were excluded as they did not provide any correct structure judgment(s).
ubiquity of information cascade effects and echo chambers in social network settings (Bikhchandani et al., 1992; Madsen et al., 2018), with some participants seemingly misled by the communications even while correctly identifying the network structure that produced them.

### 3.6.1 Limitations and Future Directions

While we think our network-embedded inference task provides an excellent test bed for studying social learning, we only had space to explore a few scenarios. Going forward, it would be informative to elicit multiple judgements to track participants’ beliefs over time and gain clearer insights into how repeated communication affects judgements. Another important limitation is that our assumed belief integration mechanism is implausibly simple. For instance, it is likely that if receiving repeated signals from the same source, real learners would be able to progressively downweight its impact. We also assume a simple way of integrating social evidence that might not line up with how people actually weigh the evidential value of communications relative to direct observation. Fitting a “social strength” parameter controlling the impact of social evidence is an obvious next step in developing the naïve account. Similarly, there are likely to be approximations and relaxations of Exact inference that provide a more computationally feasible structure sensitive account. Finally, we simplified the network setting by assuming that people have consistent access to peers’ communication signals and never communicate themselves. However, in reality we often actively select who to listen to as well as when and how often to communicate our own views, further complicating social inference. Further extensions could thus incorporate active learning (Gureckis & Markant, 2012), whereby participants selectively query network peers to obtain information and can insert their own communications into the network, opening the door to active and interventional social network structure discovery.
Chapter 4

Determinants of individual and collective success in human social learning

Preamble

The experiments in Chapters 2–3 suggest that people’s sensitivity to social source dependence is a function of task difficulty: In Chapter 2, we find that a Bayesian account could qualitatively capture participants’ inferences, while the harder task in Chapter 3 resulted in naïve inferences that departed qualitatively as well as quantitatively from Bayesian norms. Based on these insights, we now study learning in an extended version of the task presented in Chapter 3, including mixed human-artificial and all-human networks. Across three experiments, we focus on the role of simulated vs. real social learners and their implications for the accuracy of both human and Bayesian inferences. The work in this chapter is currently under review (Fränken, Valentin, Lucas, & Bramley, Under review) and was conducted by myself under the supervision of Chris Lucas and Neil Bramley. Similar to Chapter 3, Simon Valentin (second author) helped with the conceptualisation of the study and the development of the structure learning algorithm presented in section 4.5, as well as a code revision. I wrote a first draft of the paper and all co-authors helped with the writing and editing of the draft. Our code is available on GitHub via FrankenValentinLucasBramley2022. An interactive demo of Experiment 1 is available here.
Abstract

A key determinant of successful social learning seems to be the ability to partial out the unique information content of others’ communications. When someone’s testimony depends on others’ beliefs as much as it does on first-hand information, there is a danger of evidence becoming inflated or ignored. We show how, in principle, rational social learners can account for dependencies in social evidence by structure-sensitive Bayesian inference. We report on three multi-player experiments examining the dynamics of both mixed human–artificial and all-human mini-social networks. Our analyses suggest that the presence of a mixed population of both naïve and more sophisticated learners may be key to achieving robust information accumulation at the group level, even as individual-level beliefs are often misled by dependent social information. Overall, our results highlight the nuances of real social network dynamics and provide insights into the conditions under which we can expect collective success versus failures.

4.1 Introduction

Social learning is a key driver of the success of the human species (Henrich, 2017). It enables people to acquire knowledge vicariously, by observing the behaviour of others (Bandura & McClelland, 1977), and via more explicit forms of information exchange (Hawthorne-Madell & Goodman, 2019; Jern & Kemp, 2015; Lucas et al., 2014; S. Wu et al., 2021). When it works, social learning can be advantageous (Laland, 2004; Rendell et al., 2011), supporting efficient learning and rational behaviour at a population level (Krafft et al., 2020)—thereby enabling transmission of accumulated insights and wisdom to bootstrap future generations’ learning (Kleiman-Weiner et al., 2019). When it fails, however, social learning can result in poor collective outcomes, such as political polarisation (Finkel et al., 2020; Tokita, Guess, & Tarnita, 2021), information cascades and echo chambers (Bikhchandani et al., 1992; Del Vicario et al., 2016), inflation of extreme views (Navarro, Perfors, Kary, Brown, & Donkin, 2018), diffusion of moral content (Brady et al., 2017; Crockett, 2017), as well as financial bubbles and crashes (De Martino, O’Doherty, Ray, Bossaerts, & Camerer, 2013).

One important aspect of successful social learning is the ability to accurately evaluate the unique informational content of social communications coming from dependent network peers, that is, distinguish first-hand evidence from hearsay (Berg, 1993; Hahn, Hansen, & Olsson, 2020; Jönsson, Hahn, & Olsson, 2015; Pilditch et al., 2020; Whalen et al., 2018; Xie & Hayes, 2022). When peers have communicated
with one another, or may have based their beliefs on shared evidence, there is a
danger of evidence becoming inflated or ignored, leading to inaccurate or wrong
beliefs. For example, when two colleagues recommend a new local coffee shop, it
might seem like a ringing endorsement, until you learn that only one of the two
has actually been there while the second simply heard about it from the other and
thus has no first-hand evidence.

Bayesian models provide a rational framework for the study of social learning un-
der uncertainty (FeldmanHall & Shenhav, 2019). From a Bayesian perspective,
dependencies between peers determine how much weight we should place on what
each of them says. Specifically, rational social learners should use their knowl-
edge about who tends to communicate with whom—the structure of their social
network—to make inferences about the truth behind a claim. Similarly, rational
social learners should consider the timing and content of peers’ statements to in-
fer who is learning from whom, and when social communications are indicative
of new first-hand evidence. While recent work suggests that people are sensitive
to differing evidential values of social information implied by dependencies be-
tween network peers (Whalen et al., 2018), it has also been found that people fail
to accommodate dependencies in a rational (Bayesian) fashion when facing more
difficult learning problems, such as when social evidence is incomplete or seems
contradictory (Pilditch et al., 2020; Xie & Hayes, 2022). Importantly, previous
work has focused on fully-observed and one-shot learning settings. It remains an
open question whether people can recognise and accommodate social source de-
pendence in more naturalistic settings in which interactions are neither one-shot
nor fully-observed, such as when there are multiple communications from each
source and uncertainty about the social network structure.

In this paper, we examine how people combine first-hand evidence with social ev-
dence to make inferences about a property of their shared environment. While
social evidence can provide additional information, it can also lead to incorrect,
over- or under-confident inferences if dependencies between network peers are ig-
nored. We explore conditions with known and unknown social network structures,
that is, conditions in which learners either know who talks to whom or do not.
We examine both a setting in which we control the behaviour of all but one of the
learners (Experiment 1) and a more realistic social setting where all three learners
are human participants taking part in a real-time online multi-player game (Ex-
periments 2–3). To examine individual behaviour in this social inference setting,
we develop a Bayesian social learning framework capable of both naïve and socially sophisticated inferences. Our task distills several key dynamics of real social network interactions: (1) multiple communications from each source, (2) limited bandwidth communications (e.g. using like buttons or rating scales), and (3) uncertainty about the underlying social network structure. We study how people deal with social evidence and integrate it with first-hand evidence as seen through the lens of our Bayesian inference framework. Using agent-based simulations, we then validate the ramifications of both naïve and sophisticated inferences in simulated social networks.

4.2 General Methods

4.2.1 Task

We study how people reason about a shared environment (the nature of an alien planet) based on a sequence of private evidence (fish caught on the planet) and social evidence (other agents’ previous judgements), as illustrated in Fig. 4.1. Under our cover story, the participants have crash landed on one of two fictitious planets with different proportions of red and blue fish. As a variant of standard ‘ball and urn’ tasks (L. Anderson & Holt, 1997), participants are told that on the first planet, ‘Planet Red’, \( \frac{2}{3} \) of the fish are red, and \( \frac{1}{3} \) of the fish are blue. On the second planet, ‘Planet Blue’, proportions are reversed. Aside from the proportion of red and blue fish, the planets are indistinguishable.

At the beginning of the scenario, participants are told that they do not yet know which planet they have crashed on. Over the course of ten trials participants collect private evidence by fishing, occasionally catching either a red or blue fish and so learning about the proportion. Participants are tasked with making a series of sequential judgements about the identity of the planet using a 7-point scale ranging from ‘highly confident blue’ to ‘highly confident red’ (see Fig. 4.1b). Alongside their own observations, participants may see the previous judgements of up to two other agents. Thus, on each trial, participants may receive up to one piece of private evidence and zero, one or two pieces of social evidence. The frequency with which private evidence is provided by the environment (how often each agent catches fish) is unknown to participants.
Participants are incentivised to make correct judgements (and thereby also provide informative social evidence to other agents) by a small bonus of £0.15 on every trial in which their judgement is in the right direction of the scale. If participants have access to social evidence—that is, other agents’ previous judgements (on the same 7-point scale)—they have the potential to indirectly take into account the private evidence collected by the other agents. Importantly, participants never see other agents’ private evidence directly. While pieces of social evidence open the door to collective success, they need to be handled with care: If other agents can also see each other’s judgements, their judgements are likely to reflect not just their own private evidence but also their best guess about the evidence seen by the other agent. Identifying and accommodating such dependencies between other agents is key for accurately assessing the information content of social evidence about the environment, thereby distinguishing genuine information from rumours circulating in a social network. We next formalise this intuition through the lens of Bayesian inference.
Figure 4.2: Illustration of our inference models and the task interface. a) The Level-0 learner considers private evidence to evaluate beliefs about the environment and ignores social evidence. The naïve Level-1 learner considers private evidence and social evidence but ignores dependencies between network peers. The rational Level-2 learner extends Level-1 and evaluates social evidence with respect to dependencies between peers implied by the social network structure. Example illustration shows dependency $B \rightarrow C$, meaning that Agent C’s judgement depends on B’s earlier judgement. b) In our inference models, we represent participants’ beliefs about the proportion of red and blue fish as beta distributions $h \sim \text{Beta}(\alpha, \beta)$ and obtain discrete judgement probabilities for each of the seven response options using the cumulative density $I_x(\alpha, \beta)$. Given this simplified mapping between the latent Beta distribution and our discrete scale, our inference models can flexibly infer another agent’s $\alpha$ and $\beta$ parameters from the agent’s judgements. c) Example run of condition $B \rightarrow C$ at $t = 10$ from Experiment 1 with participant (agent A) and simulated agents B and C. Over the course of ten trials, a participant observes communication signals (i.e. judgements) from agents B and C. At each trial, a participant provides judgements and may additionally observe private evidence (red or blue fish). In the example shown above, a participant only observed one red fish during the second trial. Their red fish observation is displayed on the right side of the interface throughout the task. Compared to our previous experiment in Chapter 3 (see Fig. 3.2), we now collect a participant’s judgements at each time step, and not just at the end of a game. d) Interface used to elicit participants’ structure judgements in the second condition of Experiment 1 during which the network structure was Unknown. Participants had to make four forced-choice judgements about the potential relationships between each agent in the network. Incoming edges from agents’ B and C to A were fixed as participants could always see B’s and C’s judgements (Section 4.3.1.2).
4.2.2 Formal approach

We follow the tradition of rational analysis (J. Anderson, 2013) and develop three nested Bayesian social learning models capable of different levels of recursion (Fig. 4.2a) to describe ways a learner may evaluate beliefs $h$ about the proportion of red and blue fish on the planet. We model beliefs about the environment (the proportion of red and blue fish) $h$ as beta distribution $\text{Beta}(\alpha, \beta)$, with pseudo-counts $\alpha$ and $\beta$ for the number of red and blue fish, respectively. Discrete judgements $y \in \{3\text{-red}, ..., 3\text{-blue}\}$ are elicited using the 7-point scale shown in Fig. 4.1b. Judgement probabilities are provided by the cumulative density $I_x(\alpha, \beta)$ (Fig. 4.2b). The remainder of this section provides a brief description for each social learning model while technical details are provided in the Supplementary Information (SI) at the end of Chapter 4.

4.2.2.1 Level-0 learner

The simplest model, Level-0 learner, ignores social evidence (other agents’ previous judgements) and bases its belief $h$ only on its own private observations $d \in \{\text{red, blue}\}$.

4.2.2.2 Level-1 learner

Our second model bases its belief $h$ on private evidence $d$ and, where available, previous judgements from other agents. Level-1 naively assumes that other agents’ judgements are independent of one another. A problem with this assumption is that if structural dependencies exist (i.e. agents are non-independent; see Fig. 4.2a), Level-1 learning results in inaccurate or wrong beliefs about the environment.

4.2.2.3 Level-2 learner

Our third model extends Level-1 inference one recursive step, thus considering structural dependencies $g \in G$ between other agents (in the present experiments $G = \{\text{Independent} : [B \rightarrow A \leftarrow C]; B \rightarrow C: [A \leftarrow B \rightarrow C \rightarrow A]; C \rightarrow B: [A \leftarrow C \rightarrow B \rightarrow A]\}$; see Fig. 4.1d). For example, if a Level-2 agent A knows that agent C
can see agent B’s judgements ($B \rightarrow C$, Fig. 4.2c), they will infer less total private evidence from C if C’s judgements are plausibly the result of earlier judgements from B. By accommodating potential dependencies between other agents, assuming these other agents are also rational, a Level-2 learner can accurately evaluate the information content of received social evidence, thus preventing potential distortion of evidence.

4.3 Experiments

4.3.1 Experiment 1

4.3.1.1 Overview

We first examine a controlled setting with simulated rational network peers including known and unknown social network structures.

4.3.1.2 Methods

We recruited 146 participants (35.94 ± 15.52, 98 female, 48 male) using Prolific (Palan & Schitter, 2018). 47 participants were assigned to the first condition ($B \rightarrow C$), 50 participants were assigned to condition two (Unknown), and 49 participants were assigned to the third condition ($C \rightarrow B$). Participants were paid a base salary of 6.00/hr and a performance bonus of up to £1.50 (see Fig. 4.1b). The task took around 15 minutes. Prior to the start of the main task, participants completed instructions including a short training to familiarise them with the game environment and reward structure (see SI and Fig. 4.10 for details; full instructions and training are available through our online demo). Following completion of instructions, participants joined a lobby and registered with a username. At this stage, the two simulated agents were already registered and once participants completed their registration, the game started. During the main task, participants had 30 seconds for providing a planet judgement at each trial (overall, the main game lasted 300 seconds). If a participant failed to provide a judgement in time, we used their judgement from the previous trial as default selection and deducted the possible trial bonus from their overall bonus. If participants failed to provide
a judgement during the first trial, we defaulted to a judgement of ‘0’ (i.e. no preference for either planet).

In the first condition \((B \rightarrow C)\), participants were told that agent C could see agent B’s judgements (see SI and Fig. 4.10). Additionally, they were instructed that C always considered B’s evidence and their own sampled data when making a judgement, resulting in more informative judgements compared to B despite B being more confident. Participants were told that both B and C always provided accurate judgements and that they received the same bonus for providing correct responses as participants themselves. In the second experimental condition (Unknown), no structure information was provided to participants and instructions were adapted accordingly (Fig. 4.10). Here, participants provided a structure judgement about the network structure of their network at the end of the task (Fig. 4.2d). Incoming edges were provided to participants, resulting in four remaining edge selections (i.e. 16 network structures). Participants had 120 seconds for providing a structure judgement. If they failed to provide a judgement in time, they were redirected to the debriefing and removed from the analysis. We conditioned the predictions obtained from Level-2 inference on each participant’s structure selections (average edge selections are shown in Table 4.11 and Fig. 4.15). In the third condition \((C \rightarrow B)\), we flipped the relationship between B and C, now implying that B could see C’s evidence. Apart from this, the third condition was identical to condition one. Within each condition, the colour of A’s fish samples was randomised, and evidence for B and C were reversed accordingly. In our analysis, we focus on the task from the perspective of A sampling red fish observations. The full data set obtained from participants across conditions is provided in SI. Judgements for simulated agents B and C (Fig. 4.8) were selected to maximise Level-2’s predicted differences between conditions to test whether participants were sensitive to the structure manipulation. Specifically, we selected a sequence of observations implying different ground truths under Level-2 inference, resulting in a qualitatively different prediction (i.e. different colour) for condition \(B \rightarrow C\) compared to the other two conditions. In our behavioural analysis, we only include the last seven trials as given the specific simulated sequences, A’s judgements prior to \(t = 4\) were unaffected by the experimental manipulation.
Figure 4.3: Inferred beliefs \( h \sim \text{Beta}(\alpha = \text{number of red}, \beta = \text{number of blue}; \text{y-axis}) \) about the environment for each inference model at each trial of the task (x-axis). a) Experiment 1. Panel headings (\( B \rightarrow C \), Unknown, \( C \rightarrow B \)) correspond to the true network structure in each condition. Level-0 inference (grey, dotted) only considers A’s private evidence \( d_2 = 1 \) (one red fish during trial two) resulting in identical beliefs across conditions. Similarly, Level-1 inference (magenta, dotted) combines \( d_2 \) with inferred beliefs for B and C and ignores network structure, thus inferring identical beliefs across conditions. Level-2 inference (cyan, dotted) extends Level-1 by accommodating for dependencies between B and C implied by network structure, thus inferring more red fish in condition \( B \rightarrow C \) as compared to the other two conditions. b–c) Experiments 2–3.
4.3.1.3 Results

We first derive the expected patterns of inference for Agent A at Levels 0, 1 and 2. Recall Level 0–1 learners are insensitive to network structure, so making identical predictions for target agent A (participants) across conditions (Figs. 4.3a & 4.4a). Specifically, Level-0 learning only considers A’s private red fish catch at \( t = 2 \). The Level-1 learner naively assumes that B’s blue judgements and C’s red judgements are independent. Since B is more confident than C (Fig. 4.2c & Fig. 4.8), this leads to a preference for blue across conditions (Fig. 4.4a). Level-2 learning however leads to favoring the red planet in condition \( B \rightarrow C \) (average judgement across time-steps \( +0.594 \in \{-3\text{[highly confident blue]} - +3\text{[highly confident red]}\} \), see Table 4.8). Fig. 4.2c shows that C provided red judgements despite seeing blue judgements from B. Considering this dependency, a Level-2 learner infers that C must have observed multiple red fish to explain their tendency towards red despite the communications from B. Combining inferred red fish for C with A’s own catch, Level-2 learning thus arrives at an overall preference for red (Fig. 4.4a panel \( B \rightarrow C \)). For condition two (Unknown), Level-2 inferences were based on each participant’s selected network structure provided at the end of the game (Figs. 4.2d and 4.8). Level-2 reasoning here leads to a an average judgement favoring blue (mean = -0.509; see Fig. 4.4a panel Unknown). Level-2 inference applied to condition three predicts a negative (blue) average judgement (mean = -0.990) with the rationale that B must have observed multiple blue fish given their access to C’s red judgements (Fig. 4.4a panel \( C \rightarrow B \)).

To examine participants’ judgements, we first performed a Kruskal-Wallis (Kruskal & Wallis, 1952) analysis of variance (ANOVA) which revealed a main effect of structure manipulation (Kruskal’s \( H(2) = 13.73, p < 0.005, \epsilon^2 = 0.082, 95\% \) confidence intervals: \([-0.152, 0.274], [-0.763, -0.179], [-0.537, -0.058]\)). Pairwise Mann-Whitney (Mann & Whitney, 1947) \( U \) tests (one-sided) further confirmed this result: In line with the directional predictions derived from Level-2 inference, judgements in condition \( B \rightarrow C \) were significantly higher (i.e. more red; mean judgement: 0.0608, 95% confidence interval \([-0.152, 0.274]\)) as compared to the second condition with an unknown network structure (mean judgement: -0.471, 95% confidence interval \([-0.763, -0.179]\), standardised \( U \)-score: \( Z = 3.51, p < 0.001, \text{CLES} = 0.707; \) Fig. 4.5a). Similarly, contrasting condition \( B \rightarrow C \) and condition \( C \rightarrow B \) (mean judgement: -0.297, 95% confidence interval: \([-0.537, \)
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Figure 4.4: Average participant judgements and predicted model judgements (y-axis) at each trial of the task (x-axis). Model predictions were derived from inferred beliefs shown in Figure 3.3. a) Experiment 1. Level-0 and Level-1 inference make identical predictions across conditions given constant private and social evidence. Level-2 inference is additionally sensitive to dependencies between $B$ and $C$ and thus predicts red judgements in condition $B \rightarrow C$ and blue judgements in the other two conditions. b–c) Experiments 2–3. Including social evidence from real human players misled simulated Level-2 learner’s predictions in condition $B \rightarrow C$ of Experiment 3, resulting in a preference for blue at the final time step (see Section 4.3.3.3).

-0.058 confirmed Level-2’s qualitative predictions, with a higher average judgement in condition $B \rightarrow C$ compared to $C \rightarrow B$ (standardised $U$-score: $Z = 2.36$, $p < 0.01$, CLES = 0.640). Both effects were significant at a Bonferroni-corrected significance level of $\frac{0.05}{3}$. There was no difference between condition two and three (standardised $U$-score: $Z = -1.63$, $p > 0.05$, CLES = 0.405).

Overall, results from Experiment 1 show that when social evidence is provided by simulated rational agents, participants accommodated for dependencies between their social network peers and provide sensitive judgement patterns qualitatively in line with rational Level-2 reasoning.
4.3.2 Experiment 2

4.3.2.1 Overview

While Experiment 1 sheds a positive light on people’s ability to distinguish genuine evidence in social settings, its ecological validity is limited by its focus on inferences from communications by simulated rational agents. In contrast, more realistic dynamics between (potentially fallible) humans may challenge this optimistic perspective. To explore a more naturalistic setting, we next tested learning in a real social network with additional participants taking the roles of agents B and C.

Figure 4.5: Mean judgements across time steps in all Experiments. CIs show SEs. a) Experiment 1 ($N = 146$, agents B and C have fixed simulated behaviour). b) Experiment 2 ($N = 126$, all agents human, network structure provided) and c) Experiment 3 ($N = 129$, all agents human, network structure not provided).
4.3.2.2 Methods

We developed a client–server software using enabling real-time interactions between three randomly matched participants. Once matched, each triad was randomly assigned to one of the two between-subject conditions (Independent, B → C). Additionally, each participant was randomly assigned to one of the three agents (A, B, and C) within each triad and the position each player took on the screen was randomised. Overall we recruited 126 participants (25.82 ± 7.08, 70 female, 56 male) via Prolific. Participants were compensated as in Experiment 1 and the task was again timed for 15 minutes. 21 triads (63 participants) were assigned to the Independent condition and 21 triads (63 participants) were assigned to the B → C condition. Following (Fränken et al., 2021), we selected belief sequences resulting in strong directional differences between conditions at $t = 10$ for player A. Specifically, agents A and C always observed the same private evidence (i.e. one red fish at $t = 2$ and $t = 7$, respectively) while player B observed one blue fish at $t = 4$. As with Experiment 1, the colour of fish was randomised between triads, with A and C observing red in the red-flip version of the task and blue in the blue-flip version of the task (B always observed the opposite colour). We analyse the experiment from the perspective of A and C observing red. In contrast to Experiment 1, we only provided one illustration showing judgement exchanges between agents above the game interface (see Figs. 4.10 & 4.18). Mismatch between true fish observations and Level-2 predictions is a result of non-normative response patterns from real agents (Fig. 4.14).

In our analysis, we looked at the network from the perspective of agent A who can see both B’s and C’s judgements while manipulating the relationship between B and C. In the Independent condition, B and C do not communicate, while in the B → C condition, C can see B’s earlier judgements in the same way that A can see both B’s and C’s earlier judgements. We held the private evidence constant across conditions, with A observing one red fish at $t = 2$ (as in Experiment 1), B observing one blue fish at $t = 4$, and C observing one red fish at $t = 7$. The time points for private evidence observations were based on an experiment in (Fränken et al., 2021) with simulated agents that aimed to maximise predicted differences between experimental conditions under Level-1 learning (see SI).
4.3.2.3 Results

We derived two behavioural predictions from our inference models. First, we expected participants assigned to C to provide higher average judgements in the Independent condition compared to condition B → C in which they could see the previous judgements from B who observes a blue fish. Second, we predicted that a structure sensitive participant A would understand that C in the B → C condition could see B’s previous judgements, and thus accommodate this potential dependency when inferring C’s likely observations. Thus, Level-2 inference predicts little difference in A’s mean judgements across conditions while Level-1 predicts a higher average judgement in the Independent condition (i.e. more skewed to red) compared to condition B → C. We did not expect any systematic differences between conditions for B since they could only see their own private evidence. Beliefs $h$ for each inference model and resulting predictions are shown in Fig. 4.3b and Fig. 4.4b (see Table 4.9 for details).

We first performed a Mann-Whitney $U$ test (one-sided) to contrast C’s judgements across conditions. Confirming our predictions, average judgements were skewed more to red in the Independent condition (mean judgement = 0.561, 95% confidence interval: [0.414, 0.709]) as compared to condition B → C (mean judgement = 0.047, 95% confidence interval: [-0.350, 0.446]) in which they could see B’s previous judgements reflecting their observation of a blue fish (standardised $U$-score: $Z = 1.82$, $p = 0.031$, CLES = 0.665; Fig. 4.5b, Agent C). Agent A’s judgements did not differ significantly between conditions despite this difference in B’s communications (95% confidence intervals: [-0.197, 0.616], [-0.532, 0.494]; standardised $U$-score: $Z = 0.70$, $p > 0.05$, CLES = 0.564; Fig. 4.5b, Agent A).

Importantly, the nature of social evidence in Experiment 2 was different from the simulated rational social evidence provided in Experiment 1 because real peers B and C provided noisy judgements that differed across triads and often diverged from ‘rational’ response patterns (see Fig. 4.14). As such, Level-2 predictions deviated from an evidence-omniscient guess about the true state of the environment (Fig. 4.3b and Fig. 4.4b). On the other hand, participants’ judgements were strikingly robust to variations in responses from real network peers despite showing a tendency towards smaller judgements in condition B → C (see Individual-level cross validation below for further details).


4.3.3 Experiment 3

4.3.3.1 Overview

In Experiment 2, we assumed that each peer had complete access to the structure of their social network (i.e. who sees whose judgements). However, in real social interactions, we often have either no or only limited knowledge about other people’s precise communication histories. To assess how this might affect inferences in the current learning problem, we finally studied a setting with real network agents and unknown social network structures (excepting that each agent could always see who was communicating directly to them).

4.3.3.2 Methods

In Experiment 3, we had participants make judgements about the structure of the unobserved portion of the network they had been in, and used participants’ structure guesses to drive Level-2 predictions at the subject level. Inferred beliefs and predictions of our three inference models were similar to Experiment 2 apart from variations due to participants’ response patterns and structure judgements see (Fig. 4.3c and Fig. 4.4c).

Participants were 129 adults (25.22 ± 7.04 years, 61 female, 68 male) recruited through Prolific and paid as in Experiments 1–2. Procedures were identical to Experiment 2 with the only exception being the omission of the structure hint throughout the task. Overall, 22 triads (63 participants) were assigned to the Independent condition and 21 triads (63 participants) were assigned to the $B \rightarrow C$ condition. After completing planet judgements, participants provided structure judgements using the same interface used to elicit structure judgements in the second condition of Experiment 1.

4.3.3.3 Results

Recalling C can see B’s judgements in $B \rightarrow C$, a Mann-Whitney $U$ tests (one-sided) contrasting C’s judgements between conditions confirmed model predictions (both Level-1 and Level-2 predictions were identical for Agent C), with a higher (redder) average judgement (mean = 0.500, 95% confidence interval: [0.214, 0.785]) in the
**Independent** condition than \( B \rightarrow C \) (mean = -0.076, 95% confidence interval: [-0.404, 0.252]; standardised \( U \)-score: \( Z = 3.01, p < 0.01, \text{CLES} = 0.769; \) Fig. 4.5c, Agent C). For A’s judgements, there were again no significant differences between conditions (95% confidence intervals: [-0.280, 0.735], [-0.210, 0.515]; standardised \( U \)-score: \( Z = 0.486, p > 0.05, \text{CLES} = 0.544; \) Fig. 4.5c, Agent A).

Crucially, Level-2 inferences in Experiment 3 deviated from the true state of the environment (Fig. 4.3c and Fig. 4.4c). This is partially a result of some participants making inaccurate inferences about the network structure (Figs. 4.15–4.17 and Table 4.12) but also due to the presence of inconsistent response patterns from real network peers (Fig. 4.14). As shown in Fig. 4.4c, participants judgements aligned closer with the ground truth (ultimately favouring the red planet) than Level-2 predictions (weakly favoring the blue planet), suggesting that participants’ judgements were less affected by structure uncertainty or noisy social evidence.

![Figure 4.6: Cross validation results showing number of participants (\( N_{\text{Best}} \)) best predicted for each inference model. ‘Sticky’ versions include a fitted mixture weight \( \pi \) to account for anchoring effects (see main text). ‘Level’ is abbreviated by L. a-c) Experiments 1–3. Striped patterns in b) indicate that predictions for Level-1 and Level-2 were identical.](image-url)
4.3.4 Individual-level cross validation

To accommodate differences in response patterns between simulated rational and real network peers as well as different structure guesses between participants, we next sought to develop a more detailed account of how well each variant of our Bayesian learning models can predict inferences on an individual subject level. For this analysis, we considered two families of inference models: The first family predicted a participant’s judgement sequence \( y \) using the cumulative density of their belief \( h \) using a softmax function with inverse temperature parameter \( \tau \) to turn predictions into response probabilities. Based on an extensive literature on order effects, suggesting that sequential judgements are often anchored to one another (Bramley et al., 2017; Dasgupta, Schulz, & Gershman, 2017; Hogarth & Einhorn, 1992; Lieder, Griffiths, Huys, & Goodman, 2018a), we also consider a second variant (‘sticky’) of the first family in which each response probability corresponds to a mixture of soft-max model predictions and a tendency towards sticking with the previous judgement, with mixture weight \( \pi \).

In addition to deriving predictions for each participant using our Bayesian learners, this analysis thus involved fitting the free parameters \( \tau \) and \( \pi \). We evaluated the predictive accuracy of each fitted model using leave-one-out cross validation (LOOCV). Using LOOCV, we split our data sets into training sets of size \( N - 1 \) and test sets of size 1 (‘left-out’ participant) for each experiment. The number of participants best predicted by each model across conditions and experiments is summarised in Fig. 4.6. In Experiment 1, this analysis suggests predominantly (naïve) structure insensitive inferences with a bias towards sticking with the previous judgement (Level-1\textsubscript{sticky}) across conditions, which best predicted 48% of participants overall (Fig. 4.6a). Additionally, a substantial proportion of participants (24% across conditions) were best characterised by the sophisticated structure sensitive Level-2\textsubscript{sticky} competitor.

Overall, these results support and go beyond the findings from our behavioural analysis, suggesting that a nontrivial number of participants were able to account for dependencies between informants in line with Level-2’s predictions, while the majority were better described by a structure insensitive naïve account.

For Experiment 2, individual model fitting again supported predominantly structure insensitive inference with a bias towards sticking close to earlier judgements
(Level-1\textsubscript{sticky}) in condition two (B → C), best predicting 62% of participants compared to only 10% best accounted for by structure sensitive Level-2\textsubscript{sticky} (Fig. 4.6b). This was a notably smaller percentage compared to Experiment 1. Note that in the Independent condition of Experiment 2, Level-1 and Level-2 inference predictions coincide, and thus claim an equal number of subjects. In Experiment 3, Level-1\textsubscript{sticky} best predicted 41% of participants in condition one (Independent), which was again higher than Level-2\textsubscript{sticky} (23%; see Fig. 4.6c). In condition two (B → C), Level-1 and Level-2 inference shared the same proportion of participants best predicted (29%). Results for participants allocated to the positions of agents B and C in Experiments 2–3 are provided in SI.

### 4.3.5 Simulations

Results from individual model fitting suggest the presence of both structure sensitive (Level-2) and structure insensitive (Level-1) social inference, with the latter appearing more dominant across experiments. To demonstrate the ramifications of pure populations of either learning level, we next simulated a population of structure sensitive learners and contrasted their average inferences with a simulated population of structure insensitive naïve learners. We held environmental evidence constant across simulations, with A and B observing one red piece of evidence and C observing one blue piece of evidence at the start of a simulation. For simulations, we assumed agents were stochastic in their communications, essentially probability matching from their posterior belief (Shanks et al., 2002). For each simulated agent A, we computed the difference ('bias') between their beliefs \( h \) about the environment in the structurally dependent B → C and C → B networks. Simulations ran for five time steps. Further details including a worked example are available through our online repository. Fig. 4.7 shows the results of these simulations for both Level-1 and Level-2 learners, revealing a systematic bias towards red (blue) in condition B → C (C → B) for structure insensitive Level-1 learning, and no systematic bias for Level-2 learners that correct for dependencies between network peers.
Sensitivity to Structure in Social Learning

The results of our first experiment show that a substantial proportion of people’s inference models. Overall, our method is a promising approach for understanding how people form beliefs from sequential observations and social testimony. In three increasingly realistic experiments, we aim to understand sensitivity to sequential dependencies in dynamic environments. We find that a substantial proportion of people are able to account for such dependencies.

4.4 Discussion

Effective social learning depends on being able to infer the evidence behind others’ beliefs and what they choose to communicate. In particular, it is important to understand when communications contain new first-hand information. By contrasting social learning accounts of different levels of sophistication across three behavioural experiments, we show that groups of adults can make inferences that combine individual observations effectively and in line with recursive Bayesian inference, but that are also surprisingly robust to the presence of more and less sophisticated peers, and uncertainty about communication structure. Our individual-level modelling reveals that while a substantial minority of participants exhibit hallmarks of recursive social inference, the majority are better described as learning more naively, seeming to lack sensitivity to dependencies between their peers beyond a generally healthy skepticism about the value of communications relative to first-hand evidence. We show how naïve inference nevertheless results in inaccurate individual beliefs about the environment determined by the order in which evidence arrives. Together, these results suggest that human social learning involves a mixed population of both naïve and more sophisticated social learners and that this permits reasonable aggregation of information at the group level, even as individual-level belief trajectories remain susceptible to social distortion.
We focused on a naturalistic setting involving multiple communications between peers embedded in either a known or unknown social network structure, extending previous work (Fränken et al., 2020; Jönsson et al., 2015; Madsen et al., 2018; Whalen et al., 2018) that was restricted to simulations or artificial social peers. Even in our all-human communications setting, a substantial minority of people were able distinguish novel information from re-circulating information, shown by them being best captured by our Level-2 inference model. Sensitivity to the redundancy of repeated signals from the same person over multiple occasions is a critical social ability that is essential for use of social evidence in the real world. This ability is especially relevant in the context of polarised online political discourse, where anonymity can challenge functioning of our source-novelty detection (Lees & Cikara, 2020; McCarty, Poole, & Rosenthal, 2016). For example, when competing candidates spread (dis-)information to win voters (Kurvers et al., 2021), they count on the fact that many will fail to recognise when many apparent viewpoints stem from a small number of points of origin.

The dominance of Level-1 reasoning across our experiments and conditions suggests that the ability to adjust for redundancy of repeated signals is the exception not the rule. This aligns with previous findings suggesting people struggle to appropriately integrate evidence from dependent sources (Pilditch et al., 2020; Xie & Hayes, 2022). Combining group- and individual-level analyses, our results thus suggest that adaptive group inferences might depend on the presence of a few more sophisticated social learners, especially at critical positions within a social network. At the same time, the naïve Level-1 account highlights a generic social learning mechanism that can account for maladaptive dynamics like information cascades (Bikhchandani et al., 1992) and the formation and persistence of echo chambers (Jasny, Waggle, & Fisher, 2015). Social media, by design, seems to exacerbate these phenomena by selectively connecting people to those who have similar beliefs, making it near impossible for individuals to judge the novelty or evidential weight of the social evidence they are exposed to. We note that our learning models can be applied to analyse a range of related problems, such as the increasing occurrence of moral outrage on social media (Brady et al., 2021).

More generally, our results challenge the idea that social inference can be adequately modelled through the lens of the rationality principle (Gershman, Horvitz,
& Tenenbaum, 2015; Oaksford et al., 2007) or a naïve utility calculus (Jara-Ettinger, Gweon, Tenenbaum, & Schulz, 2015). Specifically, results from Experiments 2–3 demonstrate that assuming others are more rational than they actually are can also result in systematic errors. For example, simulated Level-2 learners exposed to the real human communications in Experiment 3 made overconfident and directionally wrong judgements (Fig. 4.3b–c) while human learners playing this role in the network performed rather better on average (Fig. 4.4c). This shows that people have more nuanced assumptions about other people than the infinitely rational ‘homo economicus’ common to idealised accounts. This seems to allow for more balanced inferences accommodating for the fact that other people are fallible but also that they are frequently predictably and helpfully naïve.

Moving forward, it would be interesting to study settings with shared incentives (e.g. rewarding individuals based on the group’s performance), or opposing incentives (e.g. based on their performance relative to the group). This could provide insights into how people use communications to influence others, and also to probe others’ beliefs, competencies and social connections. This would open the door to incorporating theories of active learning (Coenen, Nelson, & Gureckis, 2019) and metacognition (Fleming & Daw, 2017) to capture how people can learn socially despite uncertainty about peers’ rationality, motivations and social network structure.

To summarise, we showed that social learning involves a mixture of sophisticated social learners—who account for dependencies between peers’ communications—and naïve social learners—who treat their peers as independent albeit not entirely trustworthy information sources. Our analyses provide new insights into how and when adaptive group inferences arise, suggesting these depend on healthy levels of skepticism about others’ communications and the presence of some group-members capable of engaging in recursively sophisticated social inferences. Our results further highlight the dominance of naïve learners that can explain the formation and persistence of pockets of maladaptive population dynamics and outcomes such as information cascades or echo chambers. Finally, we argued that prior accounts of social inference depend on overly strong assumptions of rationality of peers, meaning they may have limited utility in capturing the dynamics of naturalistic social learning. Comprehensive accounts of social cognition in the wild should take population heterogeneity and inferential naïvety seriously as features of human social learning.
4.5 Supplementary Information

4.5.1 Additional modelling details

4.5.1.1 Judgement probabilities

Judgement probabilities for each of the seven discrete judgements \( y \) were derived from the cumulative density of a learner’s beta distribution in each of seven equally sized bins \( I_x(\alpha, \beta) \). Specifically, we specify an observation model that maps belief \( h \) to a probability of observing judgement \( y \) by computing the area under the beta density associated with \( y \):

\[
p(y|h) = \int_{y/k}^{y/(k-1)} \text{Beta}(x; \alpha, \beta)dx
\]

where \( k = 7 \) corresponds to the number of possible discrete judgements used in the present task and each specific judgement \( y \) determines the lower and upper bound of the area under the curve associated with this judgement (e.g. 3-blue \( \rightarrow y = 1 \), lower bound=0, upper bound = \( \frac{1}{7} \); 3-red \( \rightarrow y = 7 \), lower bound=\( \frac{6}{7} \), upper bound = \( \frac{7}{7} \)).

4.5.1.2 Level-0 learner

Level-0 inference functions as an asocial baseline that disregards social evidence and evaluates beliefs \( h \sim \text{Beta}(\alpha, \beta) \) about the proportion of red and blue fish on the planet by sequential application of Bayes’ rule at each trial \( p(h | d_t) \propto p(d_t | h)p(h) \) where \( p(h | d_t) \) is the posterior belief about the true proportion of red and blue fish on the planet. Private evidence provided by the environment on each trial might be a red fish \( d_t = 1 \), a blue fish \( d_t = -1 \) or no fish \( d_t = \emptyset \). Whenever there is a new piece of evidence, a learner updates their belief \( h \) about the environment using a conjugate update of the beta distribution:

\[
\alpha_t = \begin{cases} 
\alpha_{t-1} + 1 & \text{if } d_t = 1 \\
\alpha_{t-1} & \text{if } d_t = -1
\end{cases} \quad \beta_t = \begin{cases} 
\beta_{t-1} + 1 & \text{if } d_t = -1 \\
\beta_{t-1} & \text{if } d_t = 1
\end{cases}
\]
During trials in which no private evidence was observed (i.e. \( d_t = \emptyset \)), \( \alpha_t \) and \( \beta_t \) remained identical to their respective values from the previous time step.

### 4.5.1.3 Level-1 learner

Level-1 inference uses each agent’s most recent judgement \( y_{t-1}^{(i)} \) to compute a posterior probability for each agent’s most recent private evidence observations \( p(d_{t-1}^{(i)}) \) given the agent’s previous belief \( h_{t-2}^{(i)} \):

\[
p(d_{t-1}^{(i)} \mid h_{t-2}^{(i)}, y_{t-1}^{(i)}) \propto p(y_{t-1}^{(i)} \mid h_{t-2}^{(i)}, d_{t-1}^{(i)}) p(d_{t-1}^{(i)})
\]  

(4.3)

where \( h_{t-2}^{(i)} \) corresponds to agent \( i \)’s own beta-distributed belief about the planet colour attribute prior to incorporating previous private evidence \( d_{t-1}^{(i)} \). To account for uncertainty about agent \( i \)’s unobserved private evidence, Level-1 inference updates \( i \)’s belief parameters \( \alpha \) and \( \beta \) at each time step using weighted pseudo-counts obtained from Equation 4.3 using filtering. For example, if Level-1 infers a posterior probability of \( p(d_{t-1}^{(i)} = 1 \mid h_{t-2}^{(i)}, y_{t-1}^{(i)}) = 0.85 \), a Level-1 learner will set \( i \)’s latent belief parameter \( \alpha_{t-1}^{(i)} \) equal to \( \alpha_{t-2}^{(i)} + 1 \times 0.85 \). Importantly, a Level-1 learner assumes that agents form beliefs independently of one another, that is, agents can never observe each other’s judgements. Given this assumption, Level-1 inference can now update their own belief \( h \) about the colour attribute of the planet at each time step by conditioning jointly on their current private evidence \( d_t \) and inferred previous beliefs \( \in \{h_{t-1}^{(1)}, ..., h_{t-1}^{(n)}\} \) from \( n \) independent agents:

\[
p(h \mid d_t, h_{t-1}^{(1)}, ..., h_{t-1}^{(n)}) \propto p(h_{t-1}^{(1)}, ..., h_{t-1}^{(n)} \mid d_{t-1}^{(1)}, ..., d_{t-1}^{(n)}) p(d_t \mid h) p(h).
\]  

(4.4)

Here, \( p(h \mid d_t, h_{t-1}^{(1)}, ..., h_{t-1}^{(n)}) \) corresponds to the learner’s posterior belief about the colour attribute of the planet and \( p(h_{t-1}^{(1)}, ..., h_{t-1}^{(n)} \mid d_{t-1}^{(1)}, ..., d_{t-1}^{(n)}) \) are the inferred beliefs for \( n \) agents based their own (unobserved) private evidence from the previous time step obtained from Equation 4.3.
4.5.1.4 Level-2 learner

By neglecting structural dependencies through the assumption that agents form beliefs independently of one another, Level-1 inference can result in wrong inferences about the environment that are not supported by the true private evidence available to each agent. Level-2 inference thus extends Level-1 by one recursive step, considering whether agents could see each other’s judgements during previous time-steps. This was implied by the network structure $g$ of the environment. Here, the arrow from B to C implies that agents B and C are structurally dependent, such that C can observe B’s previous judgements while B cannot observe C’s previous judgements. This means that inferences about C’s belief $h$ are dependent on whatever C themselves inferred about B’s belief at the previous time step, which might have influenced their most recent judgement in addition to potential private evidence observed by C. Level-2 inference corrects for such dependencies by recursively evaluating agents’ beliefs to build a joint probability distribution over the potential private evidence observed by each agent which can then be marginalised over. Formally, Level-2 inference can be expressed as:

$$p(h_{t-1}^{(1)}, ..., h_{t-1}^{(n)}, d_{t-1}^{(1)}, ..., d_{t-1}^{(n)}, g) = \sum_{g \in G} p(h_{t-2}^{(1)} | h_{t-1}^{(1)}, ..., h_{t-2}^{(i-1)}, d_{t-1}^{(i)}, g) p(g).$$

(4.5)

Here, $p(g)$ corresponds to the probability of a given network structure $g \in G$ (Fig. 1d) and $p(h_{t-2}^{(i)} | h_{t-1}^{(1)}, ..., h_{t-2}^{(i-1)}, d_{t-1}^{(i)}, g)$ is computed by a recursive function which runs back in time until a termination condition is reached\(^1\). In the present learning problems, $g$ was provided to the learner (either through experimental manipulations or by eliciting structure judgements from participants; see Fig. 4.2d), allowing to drop the marginalisation over structures $G$.

We assumed that agents started the task with a uniform prior belief $h \sim \text{Beta}(\alpha_{t_0} = 1, \beta_{t_0} = 1)$, meaning that there was no preference for selecting either of the two planets prior to the observation of private or social evidence. Additionally, we assumed that agents were reliable communicators that always reported evidence with

\(^1\)This termination condition can be (1) finding an agent that has no parents (i.e. an independent agent) or (2) the start of the game if there are no independent agents. For an implementation, see agent.py in our online repository.
the highest probability given their belief $h$. This assumption was in place across all experiments except condition three in Experiment 1 in which judgements by agent B corresponded to the second most probable judgement under $h$ during later judgements to enforce consistency between judgement sequences across conditions (resulting in a minor mismatch between the inferred state of the environment and the true state of the environment; see Fig. 4.3a, panel Unknown). This mismatch had no additional implications for our analysis. Following computation of discrete probabilities for each judgement $y$ given $h$, we computed the expected loss for each possible judgement:

$$L_y = \mathbb{E}_y((y - \bar{y})^2)$$

followed by a softmax-function with inverse temperature parameter $\tau$ to obtain final choice probabilities for each $y$:

$$p(y) = \frac{e^{L_y \tau}}{\sum L_y e^{L_y \tau}}.$$  \hfill (4.7)

We set $\tau = 10$ (high reliability as in Hawthorne-Madell and Goodman (2019)), resulting in a strong preference for selecting judgements with higher probability given observations. For the sticky variant of our inference models, we included an additional mixture weight $\pi$ to assess the weight that participants placed on their previous judgement $p(y_{-1})$:

$$p(y) = \pi p(y) + (1 - \pi)p(y_{-1}).$$

We obtained values for $\pi$ and $\tau$ by joint maximum likelihood optimisation using L-BFGS approximation implemented in SciPy (Virtanen et al., 2020).

### 4.5.1.5 Structure Learning

We inferred structure selections shown in Tables 4.11 & 4.12 using a normative (Bayesian) change-based judgement approach with one free parameter $\theta = 0.9$ corresponding to the execution accuracy, that is, the probability with which the structure learner executes a deterministic policy to update their belief $p(g)$ in a
given network structure. The structure learner predicted a judgement change from $t = -1$ to $t = 0$ conditional on potential changes in other agents’ judgements from $t = -2$ to $t = -1$. The structure learner combined changes in evidence from more than one other agent (here written as target agent $v$’s parents $pa(v)$) using an additive combination function:

$$\Delta_{y_{-2} \rightarrow y_{-1}} = \sum_{i \in pa(v)} \Delta_{y_{-2} \rightarrow y_{-1}}.$$

(4.9)

Given the above assumptions, the posterior probability of a specific network structure underlying sequences of judgements from $n$ agents is given by:

$$p(g|y_{1:t-1}, ..., y_{n:t-1}) \propto p(y_{1:t-1}, ..., y_{n:t-1}|g) p(g)$$

(4.10)

where $p(g)$ is the prior probability of the structure (uniform in the present context) and $p(y_{1:t-1}, ..., y_{n:t-1}|g)$ is the likelihood of judgement sequences given $g$. For a given agent $v$ and structure $g$, if $pa(v) = \{}$ or $\Delta_{y_{-2} \rightarrow y_{-1}} = 0$, the likelihood that $v$ keeps their previous state (i.e. $\Delta_{y_{-1} \rightarrow y_{t}} = 0$) is given by $\theta + \frac{1-\theta}{k}$ where $k = 7$ is the number of possible responses in our experiments. If $\Delta_{y_{-2} \rightarrow y_{-1}} > 0$, the likelihood of a positive change for $v$ is given by $\frac{\theta}{r_{v}+1} + \frac{1-\theta}{k}$ where $r_{v}$ corresponds to $v$’s previous judgement . Finally, if $\Delta_{y_{-2} \rightarrow y_{-1}} < 0$, the likelihood of a negative change is given by $\frac{\theta}{r_{v}+1} + \frac{1-\theta}{k}$.

### 4.5.2 Stimuli selection

For Experiment 1, we selected judgement sequences for B and C that maximised both the predicted differences for Level-2 inferences between conditions and predicted differences between Level-1 inference and Level-2 within each condition. When generating stimuli, we assumed that agents B and C were fully rational and reliable, meaning that their provided judgements corresponded to those with the highest posterior probability under their respective beliefs $h$. The resulting stimuli for trials in which A received a red fish at $t = 2$ are shown in Fig. 4.8. The selected stimuli made it difficult to infer a potential relationship between B and C in the second condition (Unknown) of Experiment 1 with no provided structure.
knowledge. Specifically, to maximise differences between condition one and two, the selected sequence had to be as uninformative as possible about relationships between B and C to avoid that participants inferred the same structure as in condition one (B \rightarrow C). To test the degree to which the selected judgements were informative about possible structures under normative inference, we present inferred edge probabilities in Table 4.11 (see Structure Learning, for further details). Here, the average frequency with which a normative structure learner selected the edge B \rightarrow C was only 3%, which was deemed sufficiently low for the manipulation. For simplicity, we kept A’s observations constant across all experiments (one fish observation at \( t = 2 \)). In our analysis and figures, we report results from the perspective of a participant A receiving a red fish observation at \( t = 2 \). During the actual experiment, this observation was randomised (either red or blue), and judgements for B and C were flipped accordingly.

4.5.3 Supplementary Materials

4.5.3.1 Experiment 1

Instructions were identical in condition one and three during which explicit structure hints were provided (Fig. 4.10). In condition two, instructions page 3 included the sentence:

‘While it will be clear whose evidence you can see, you can’t be sure what the other players can see. That is, you cannot be sure whether player 2 can see player 3’s evidence or if player 3 can see player 2’s evidence or if either player can see your own evidence. Below is an illustration showing one possibility. Here, Chris can see the evidence of both Neil and Simon. Simon sees Neil’s judgement but can’t see Chris’. Neil can see neither Chris’ nor Simon’s evidence.’

in place of the text shown in Fig. 4.10c. Additionally, the fourth sentence in the summary on instructions page 5 (Fig. 4.10e) was replaced by:
‘You can’t be sure if the other players can see each other’s judgments or if they can see your judgments.’

and comprehension quiz question five (Fig. 4.10f) was replaced by:

‘You can’t be sure if the other players are seeing each other’s judgements or if they can see your own judgements.’

Furthermore, condition two had no additional hints about the relationships between B and C during the main task (no text description or arrows suggesting judgement transmissions). In addition to providing a structure hint through the graphical illustrations shown in Fig. 4.2c, condition one and two of Experiment 1 included a written structure hint:

‘Important: Agent \{C, B\} always considers both Agent \{B, C\}’s previous judgments and her own catches. This means that \{C, B\} is more informative than \{B, C\}, even when she is less confident.’

combined with an additional explicit illustration (Fig. 4.18 that was displayed throughout the task.

4.5.3.2 Experiment 2

Instructions and procedures in Experiment 2 were identical to condition one and condition three in Experiment 1. The only difference was that now all agents corresponded to real participants. Additionally, the structure hint was less explicit compared to Experiment 1. Specifically, the hint in Experiment 2 was restricted to the small illustration shown in Fig. 4.18 without additional written hints or arrows in the ‘TRIAL SUMMARY’ section shown in Fig. 4.2c.
4.5.3.3 Experiment 3

Instructions and procedures in Experiment 3 were identical to condition two in Experiment 1: no structure hints during the game and a structure judgement question after the main task was completed (Fig. 4.2d).
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Figure 4.8: Stimuli (judgements for B and C) used in Experiment 1 during trials in which participants (allocated to the position of agent A) received a red fish as their own private evidence at \( t = 2 \). We randomised the colour of private evidence received at \( t = 2 \) between participants. If participants received a blue fish, the colour of the judgements for B and C were flipped accordingly (e.g. ‘3–blue’ → ‘3–red’).

Figure 4.9: a) Experiment 2 (network structure provided): Inferred beliefs \( h \) for agent B and C. For B, inferred beliefs were identical across the two network conditions (Independent, B → C) as they did not see other agents’ judgements. b) Experiment 3 (network structure unknown): Inferred beliefs \( h \) for agent B and C. As with Experiment, B’s inferred beliefs were identical across the two network conditions (Independent, B → C) as they did not see other agents’ judgements.
Figure 4.10: Instructions from condition one (B → C) in Experiment 1. a) Cover story. b) Task instructions. c) Description of network structure. d) Experiment procedure. e) Instructions summary. f) Comprehension quiz. For the Unknown condition in Experiment 1 and both conditions in Experiment 3 (in which no knowledge of the network structure was provided), the wording in the instructions was adjusted accordingly: ‘You can directly see...’ → ‘You can’t be sure if the other players can see each other’s judgments or if they can see your judgments’ and ‘...you will be able to see which judgments the other players can see...’ → ‘While it will be clear whose judgments you can see, you can’t be sure what the other players can see. That is, you cannot be sure whether player 2 can see player 3’s judgments or if player 3 can see player 2’s judgments or if either player can see your own judgments...’ Further details including experiment code are available on GitHub at FrankenValentinLucasBramley2022.
Figure 4.11: Raw data from Experiment 1 condition one (B → C) with simulated rational agents B and C. Cells marked with (R) indicate that a response was simulated (for B and C, all responses were simulated). During trials in which a participant (A) failed to provide a response in time, we defaulted to the participant’s previous judgement. If a participant failed to provide a judgement during the first trial, we defaulted to ‘0’ (neutral) as their initial response.

Figure 4.12: Raw data from Experiment 1 condition two (Unknown) with simulated rational agents B and C.
Figure 4.13: Raw data from Experiment 1 condition three ($C \rightarrow B$) with simulated rational agents B and C.

Figure 4.14: Raw data from Experiments 2–3 with real participants allocated to each agent in the network. a) Data from Experiment 1. Cells marked with (R) correspond to trials during which a participant failed to provide a response in time. During these trials, we defaulted to the participant’s previous judgement. If a participant failed to provide a judgement during the first trial, we defaulted to ‘0’ (neutral) as their initial response. b) Data from Experiment 2.
Figure 4.15: Participants (A) structure judgements during Experiment 1 condition two (Unknown). Green edges correspond to edges that were provided to participants as they could always see B’s and C’s judgements. See Table 4.11 for further details.
Figure 4.16: Participants (A) structure judgements during Experiment 3 condition one (Independent). Green edges correspond to edges that were provided to participants as they could always see B’s and C’s judgements. See Table 4.12 for further details.

Figure 4.17: Participants (A) structure judgements during Experiment 3 condition two (B → C). Green edges correspond to edges that were provided to participants as they could always see B’s and C’s judgements. See Table 4.12 for further details.
Figure 4.18: Additional structure hint shown to participants in Experiment 2.
### Table 4.1: Experiment 1: Model performances across conditions for agent A.

\(N\) = number of subjects best predicted by leave-one-out cross validation. \(\tau\) and \(\pi\) correspond to median values.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Model</th>
<th>(-\log\text{Error})</th>
<th>(N)</th>
<th>(\tau)</th>
<th>(\pi)</th>
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### Table 4.2: Experiment 2: Model performances across conditions for agent A.

\(N\) = number of subjects best predicted by leave-one-out cross validation. \(\tau\) and \(\pi\) correspond to median values.

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<th>(\tau)</th>
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Table 4.3: Experiment 2: Model performances across conditions for agent B. 
$N$ = number of subjects best predicted by leave-one-out cross validation. \( \tau \) and \( \pi \) are median values.

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<td>18</td>
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Table 4.4: Experiment 2: Model performances across conditions for agent C. 
$N$ = number of subjects best predicted by leave-one-out cross validation. \( \tau \) and \( \pi \) correspond to median values.

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<td>0.815</td>
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<td>Level(_0)-sticky</td>
<td>9.246</td>
<td>17</td>
<td>1.387</td>
<td>0.410</td>
</tr>
<tr>
<td></td>
<td>Level(_1)-sticky</td>
<td>8.360</td>
<td>17</td>
<td>1.213</td>
<td>0.410</td>
</tr>
<tr>
<td></td>
<td>Level(_2)-sticky</td>
<td>8.360</td>
<td>17</td>
<td>1.213</td>
<td>0.410</td>
</tr>
<tr>
<td>B (\rightarrow) C</td>
<td>Level-2</td>
<td>17.297</td>
<td>1</td>
<td>0.245</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Level(_0)-sticky</td>
<td>16.455</td>
<td>2</td>
<td>0.307</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Level(_1)-sticky</td>
<td>16.455</td>
<td>2</td>
<td>0.307</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Level(_2)-sticky</td>
<td>12.805</td>
<td>12</td>
<td>0.212</td>
<td>0.542</td>
</tr>
<tr>
<td></td>
<td>Level(_2)-sticky</td>
<td>12.805</td>
<td>12</td>
<td>0.212</td>
<td>0.542</td>
</tr>
</tbody>
</table>
Table 4.5: Experiment 3: Model performances across conditions for agent A. $N =$ number of subjects best predicted by leave-one-out cross validation. $\tau$ and $\pi$ correspond to median values.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Model</th>
<th>-LogError</th>
<th>$N$</th>
<th>$\tau$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>19.459</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Level-0</td>
<td>17.401</td>
<td>1</td>
<td>0.192</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Level-1</td>
<td>15.817</td>
<td>2</td>
<td>0.342</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Independent</td>
<td>Level-2</td>
<td>16.687</td>
<td>2</td>
<td>0.273</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Level$_0$-sticky</td>
<td>12.908</td>
<td>3</td>
<td>0.174</td>
<td>0.544</td>
</tr>
<tr>
<td></td>
<td>Level$_1$-sticky</td>
<td>11.849</td>
<td>9</td>
<td>0.361</td>
<td>0.520</td>
</tr>
<tr>
<td></td>
<td>Level$_2$-sticky</td>
<td>12.244</td>
<td>5</td>
<td>0.303</td>
<td>0.538</td>
</tr>
</tbody>
</table>

Table 4.6: Experiment 3: Model performances across conditions for agent B. $N =$ number of subjects best predicted by leave-one-out cross validation. $\tau$ and $\pi$ correspond to median values.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Model</th>
<th>-LogError</th>
<th>$N$</th>
<th>$\tau$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>19.459</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Level-0</td>
<td>16.640</td>
<td>4</td>
<td>0.236</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Level-1</td>
<td>16.392</td>
<td>2</td>
<td>0.311</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B $\rightarrow$ C</td>
<td>Level-2</td>
<td>16.844</td>
<td>3</td>
<td>0.278</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Level$_0$-sticky</td>
<td>14.296</td>
<td>3</td>
<td>0.256</td>
<td>0.383</td>
</tr>
<tr>
<td></td>
<td>Level$_1$-sticky</td>
<td>13.607</td>
<td>6</td>
<td>0.397</td>
<td>0.357</td>
</tr>
<tr>
<td></td>
<td>Level$_2$-sticky</td>
<td>13.691</td>
<td>6</td>
<td>0.383</td>
<td>0.374</td>
</tr>
</tbody>
</table>
Table 4.7: Experiment 3: Model performances across conditions for agent C. 
$N =$ number of subjects best predicted by leave-one-out cross validation. $\tau$ and $\pi$ correspond to median values.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Model</th>
<th>-LogError</th>
<th>$N$</th>
<th>$\tau$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>19.459</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Level-0</td>
<td>14.463</td>
<td>4</td>
<td>0.627</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Level-1</td>
<td>14.463</td>
<td>4</td>
<td>0.627</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Independent</td>
<td>Level-2</td>
<td>14.463</td>
<td>4</td>
<td>0.627</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Level$_0$-sticky</td>
<td>11.737</td>
<td>16</td>
<td>0.330</td>
<td>0.578</td>
</tr>
<tr>
<td></td>
<td>Level$_1$-sticky</td>
<td>11.737</td>
<td>16</td>
<td>0.330</td>
<td>0.578</td>
</tr>
<tr>
<td></td>
<td>Level$_2$-sticky</td>
<td>11.737</td>
<td>16</td>
<td>0.330</td>
<td>0.578</td>
</tr>
<tr>
<td>Random</td>
<td>19.459</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Level-0</td>
<td>16.095</td>
<td>1</td>
<td>0.319</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Level-1</td>
<td>14.338</td>
<td>5</td>
<td>0.476</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B $\rightarrow$ C</td>
<td>Level-2</td>
<td>14.338</td>
<td>5</td>
<td>0.476</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Level$_0$-sticky</td>
<td>14.341</td>
<td>3</td>
<td>0.237</td>
<td>0.390</td>
</tr>
<tr>
<td></td>
<td>Level$_1$-sticky</td>
<td>13.049</td>
<td>11</td>
<td>0.408</td>
<td>0.357</td>
</tr>
<tr>
<td></td>
<td>Level$_2$-sticky</td>
<td>13.049</td>
<td>11</td>
<td>0.408</td>
<td>0.357</td>
</tr>
</tbody>
</table>

Table 4.8: Experiment 1: Average judgements for each inference model across experimental conditions. Average was based on judgements starting at $t = 4$ which corresponded to the first judgement affected by the experimental manipulations.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Model</th>
<th>Average judgement ($t_{4:10}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B $\rightarrow$ C</td>
<td>Level-0</td>
<td>1.143</td>
</tr>
<tr>
<td></td>
<td>Level-1</td>
<td>-0.725</td>
</tr>
<tr>
<td></td>
<td>Level-2</td>
<td>0.594</td>
</tr>
<tr>
<td>Unknown</td>
<td>Level-0</td>
<td>1.143</td>
</tr>
<tr>
<td></td>
<td>Level-1</td>
<td>-0.725</td>
</tr>
<tr>
<td></td>
<td>Level-2</td>
<td>-0.509</td>
</tr>
<tr>
<td>C $\rightarrow$ B</td>
<td>Level-0</td>
<td>1.143</td>
</tr>
<tr>
<td></td>
<td>Level-1</td>
<td>-0.725</td>
</tr>
<tr>
<td></td>
<td>Level-2</td>
<td>-0.990</td>
</tr>
</tbody>
</table>
### Table 4.9: Experiment 2: Average judgements for each inference model across experimental conditions. Average was based on judgements starting at $t = 6$ which corresponded to the first judgement affected by the experimental manipulations.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Model</th>
<th>Average judgement ($t_{6:10}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td>Level-0</td>
<td>1.143</td>
</tr>
<tr>
<td></td>
<td>Level-1</td>
<td>0.404</td>
</tr>
<tr>
<td></td>
<td>Level-2</td>
<td>0.404</td>
</tr>
<tr>
<td>B → C</td>
<td>Level-0</td>
<td>1.143</td>
</tr>
<tr>
<td></td>
<td>Level-1</td>
<td>0.171</td>
</tr>
<tr>
<td></td>
<td>Level-2</td>
<td>0.397</td>
</tr>
</tbody>
</table>

### Table 4.10: Experiment 3: Average judgements for each inference model across experimental conditions. Average was based on judgements starting at $t = 6$ which corresponded to the first judgement affected by the experimental manipulations.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Model</th>
<th>Average judgement ($t_{6:10}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td>Level-0</td>
<td>1.143</td>
</tr>
<tr>
<td></td>
<td>Level-1</td>
<td>0.154</td>
</tr>
<tr>
<td></td>
<td>Level-2</td>
<td>0.009</td>
</tr>
<tr>
<td>B → C</td>
<td>Level-0</td>
<td>1.143</td>
</tr>
<tr>
<td></td>
<td>Level-1</td>
<td>-0.060</td>
</tr>
<tr>
<td></td>
<td>Level-2</td>
<td>-0.121</td>
</tr>
</tbody>
</table>

### Table 4.11: Experiment 1: Average structure guesses provided in condition two with unknown network structure. ‘Inferred’ corresponds to posterior edge probabilities derived in the Structure Learning section. Adjacency matrix shows average frequency of edge selections (read as from row to column)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0.84</td>
<td>0.41</td>
</tr>
<tr>
<td>Participant B</td>
<td>1</td>
<td>0</td>
<td>0.22</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.37</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>0.50</td>
<td>0.09</td>
</tr>
<tr>
<td>Inferred B</td>
<td>1</td>
<td>0</td>
<td>0.03</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.21</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 4.12: Experiment 3: Average structure guesses provided across conditions. ‘Inferred’ corresponds to posterior edge probabilities derived in the Structure Learning section. Adjacency matrix shows average frequency of edge selections (read as from row to column).

<table>
<thead>
<tr>
<th>Condition</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0.62</td>
<td>0.67</td>
</tr>
<tr>
<td>Independent</td>
<td>B</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1</td>
<td>0.57</td>
</tr>
<tr>
<td>Participant</td>
<td>A</td>
<td>0</td>
<td>0.65</td>
</tr>
<tr>
<td>B → C</td>
<td>B</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1</td>
<td>0.60</td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>0.33</td>
<td>0.31</td>
</tr>
<tr>
<td>Independent</td>
<td>B</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1</td>
<td>0.15</td>
</tr>
<tr>
<td>Inferred</td>
<td>A</td>
<td>0</td>
<td>0.29</td>
</tr>
<tr>
<td>B → C</td>
<td>B</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Chapter 5

Algorithms of adaptation in inductive inference

Preamble

So far, we have investigated people’s sensitivity to statistical dependencies when reasoning about environmental quantities (Chapters 2–4). In this chapter, we depart from reasoning about simple quantities and instead focus on symbolic hypotheses (i.e. concepts), which are integral to many of the problems we encounter in everyday life (see section 1.2.3 and section 5.2, for details). Reasoning about concepts faces the hard problem of symbolic search, meaning that we can not rely on simple analytical computation of a Bayesian posterior. Here, we will default to a sampling-based approach to approximate the behaviour of an optimal Bayesian learner and discuss alternative approaches in section 5.9.2. Given the increased complexity of the symbolic hypothesis space, we simplify the social learning setting and do not include dependencies between other players in our learning task. Instead, we focus on a richer model of individual cognition and compare several competing ideas on how a learner might search a symbolic hypothesis space. On the social learning side, we will investigate whether a learner can combine their own actively gathered data with the learning data gathered by another player to improve upon their hypothesis. The work in this chapter has been published in Cognitive Psychology (Fränken, Theodoropoulos, & Bramley, 2022) and was conducted by myself under the supervision of Neil Bramley. Nikos Theodoropoulos (second author) helped with the implementation of the multi-player learning task. I wrote a first draft of the paper and all co-authors helped with the writing and editing of the draft. Our code is available on GitHub via FrankenThedoropoulos-Bramley2022. An interactive demo of our task is available here.
Abstract

We investigate the idea that human concept inference utilizes local adaptive search within a compositional mental theory space. To explore this, we study human judgements in a challenging task that involves actively gathering evidence about a symbolic rule governing the behavior of a simulated environment. Participants learn by performing mini-experiments before making generalizations and explicit guesses about a hidden rule. They then collect additional evidence themselves (Experiment 1) or observe evidence gathered by someone else (Experiment 2) before revising their own generalizations and guesses. In each case, we focus on the relationship between participants’ initial and revised guesses about the hidden rule concept. We find an order effect whereby revised guesses are anchored to idiosyncratic elements of the earlier guess. To explain this pattern, we develop a family of process accounts that combine program induction ideas with local (MCMC-like) adaptation mechanisms. A particularly local variant of this adaptive account captures participants’ hypothesis revisions better than a range of alternative explanations. We take this as suggestive that people deal with the inherent complexity of concept inference partly through use of local adaptive search in a latent compositional theory space.

5.1 Introduction

One of the defining aspects of being human is the ability to flexibly generate and adapt ideas and hypotheses. For example, we readily come up with possible faults if our car breaks down, or plausible maladies when we feel unwell, and we will readily refine and update these hypotheses as we gather more evidence. Our ideas often combine familiar objects, concepts, and relations, making them symbolic, amenable to communication but also to decomposition and piecewise adaptation. As a lighthearted example, suppose you are a detective trying to solve a murder at an old country house. Muddy footprints leading from the back door to the body might initially make you suspect the gardener. However, if your subsequent investigation reveals the gardener has an alibi, this may lead you to consider who else could be the wearer of the muddy boots. Perhaps it was the butler, whose shoes appear suspiciously newly polished. Next, supposing your assistant (pursuing their own line of reasoning) discovers some muddy boots hidden in the attic. This may make you reason that the murderer made the tracks deliberately to misdirect suspicion. Combining you assistant’s evidence with your own observations, you can piece together the true order of events, in which the butler and gardener conspired to kill their boss. One way to describe the detective’s cognition in this toy scenario is as generating and dismissing a
procession of independent explanatory hypotheses. However, another intuitive explanation is that it involves a process of sequential refinement or adaptation of a focal working hypothesis as different forms of evidence arrive. The muddy tracks are indeed part of the story, but further investigation reveals their relationship with the murder to be indirect, and the final hypothesis is arrived at after a sequence of revisions and edits.

Symbolic concept induction can be framed, at the computational level (Marr, 1982), as a top-down Bayesian inference over an infinite latent compositional theory space (Bramley et al., 2018; Goodman et al., 2008; Piantadosi et al., 2016). At the heart of these approaches is the idea that the mind leverages a “probabilistic language of thought”—roughly a system of conceptual primitives and stochastic rules for how they can be combined. Such models capture the fact that human creative hypothesis generation appears to exhibit “infinite use of finite means”, seemingly requiring that it is based in a system with language-like properties of systematicity and compositionality (Fodor, 1975; Lake et al., 2015).

In practice, fully normative inference in compositional theory spaces is almost always intractable. Compositional hypothesis spaces are notoriously thorny to work with (Ullman et al., 2012), with approximation involving extensive sampling even when reasoning in “small worlds” (Savage, 1954). One naïve approach is to assume a learner starts with a sufficiently expressive generative concept grammar (Bramley et al., 2018) and uses this to generate a large number of prior samples or “candidate hypotheses” that can then be weighted and filtered based on their fit to evidence. Depending on the grammar’s expressivity, this procedure might require generating very large numbers of samples to ensure sufficient coverage of the space. Provided the true hypothesis is among those generated, it will gradually stand out from the alternatives as evidence arrives. Here, previous work on rule-based concept learning has proposed a rule-plus-exception account (RULEX), characterizing human inferences as utilizing a cascade of search strategies, starting with exact search over simple rules and, if necessary, progressing to search over more complex rules or allowing for more exceptions until a termination condition is met (Nosofsky, Palmeri, & McKinley, 1994). However, this appears to be an inefficient and cognitively implausible approach in general. Sampling from a concept grammar expressive enough to cover all plausible hypotheses even for a narrow domain will rarely produce the ground truth by chance.
Alternatively, a more promising approach may be to use stochastic search to explore the posterior directly. As an approximation to Bayesian inference, one can use Markov Chain Monte Carlo (MCMC) to stochastically adapt a working hypothesis in light of an underlying concept grammar and the relevant evidence, so directly entertaining a chain of autocorrelated hypotheses with each step biased towards whatever local alternatives have the better combination of prior plausibility and fit to evidence. A sufficiently long chain of considered hypotheses will approximate the posterior distribution. This approach does not require a learner to have already generated their final hypothesis before seeing evidence, but does produce order effects and local minima in which a learner’s current hypothesis constrains what new hypotheses they can—or are likely to—conceive of next.

While a variety of particle filtering, MCMC and hybrid approaches have been used to construct normative predictions in recent symbolic concept induction research (Goodman et al., 2008; Piantadosi et al., 2016; Yang & Piantadosi, 2022), there is yet to be a systematic exploration of whether any of these approximation schemes provide a compelling process account of human adaptation. That is, whether any variants of these algorithms reproduce human patterns of successes and failures in online inductive inference settings. One of the most ubiquitous non-normative response patterns are order effects in which sequential judgements are dependent, typically anchored to one another and thus inheriting errors and biases (Hogarth & Einhorn, 1992). Recently, these have been held as indicative of stochastic cognitive search in quantity judgements (Lieder et al., 2018a), conditional probability judgements (Dasgupta et al., 2017) and in causal structure inference (Bramley et al., 2017). Thus, we see the MCMC-like search as a promising algorithmic candidate for understanding the adaptation of compositional hypotheses in cognition.

The goal of the current work is to investigate whether there is support for the “adaptation as MCMC-like search” proposal in an experimental setting that mimics the problem of generating and refining scientific theories. We focus on how people update their beliefs given two different sources of evidence: (1) evidence collected actively by the learner in further investigating their hypothesis (Experiment 1)—similar to the detective’s own investigations in our earlier example and (2) evidence gathered by another learner (Experiment 2)—analogous to the insights the detective draws from their assistant’s observations. Self-generated evidence comes with both benefits and costs (Markant & Gureckis, 2014). The key benefit of self-directed learning is an ability to target one’s own hypotheses and
uncertainty, for instance performing tests designed to distinguish between possibilities that have occurred to you. For a fully normative learner, able to consider and weight all possibilities, this will generally supercharge learning (Markant & Gureckis, 2014). However, the potential costs of self-generated data stem from the limits of the learner’s current ideas. To the extent that the learner fails to consider plausible alternatives (as is very likely in our task) they may also fail to generate tests that adequately challenge the hypotheses they have generated. This can lead to a learning trap (Rich & Gureckis, 2018; Settles, 2009), where testing patterns driven by ruling between hypotheses considered by the learner limit their progress to discovering other hypotheses outside this set.

We will measure to what extent participants’ self-generated tests suggest confirmatory testing of their initial hypothesis, and consider the consequences of these testing patterns in the degree of change between their initial and revised guesses, and between initial and final accuracy.

The rest of paper is organized as follows. First, we briefly review related models of concept learning and motivate the probabilistic language of thought framework used to model inductive symbolic inference as program induction. We then summarize recent work proposing process-level accounts of learning in related settings to motivate our algorithmic hypothesis. We sketch the inductive learning task and introduce the formal modeling framework. We then report on two experiments that vary the kind of additional evidence participants receive between their initial and revised judgements. We first unpack judgment patterns descriptively and then analyze them through the lens of our modeling framework. We end by discussing limitations and avenues for future work.

5.2 Background

5.2.1 Language of thought

The idea that some of the most characteristically human forms of reasoning are driven by a symbol-and-rule-based conceptual system—a language of thought (LOT)—dates at least as far back as (Fodor, 1975). The key idea is that creative thought, imagination and problem solving often seem to require compositionality and systematicity analogous to that enabled by a system of symbols and rules for their
combination – that is, a language. For example, if we have a primitive concept of a “triangle” and of the property “blue” we can then imagine the compound concept of a “blue triangle”, and could do so without having experienced this combination directly. With just a few more conceptual primitives, including some basic operators allowing quantification and logic, it becomes possible to generate, imagine or entertain an arbitrarily large space of concepts—i.e., to hypothesize that “all blue triangles are larger than any red triangles” and so forth. Despite the fact that these are the aspects of human intelligence that have proven hardest to synthesize, the original language of thought hypothesis fell out of favor along with much of so-called “good old fashioned AI” with the rise of probabilistic rational models and neural networks (Chater & Oaksford, 1990). However, recent work has rehabilitated the LOT idea by marrying it with computational approaches to symbolic inference under uncertainty (Piantadosi & Jacobs, 2016; Yang & Piantadosi, 2022). Specifically, the idea that higher level cognition uses a LOT has been combined with probabilistic context free grammars (PCFGs) to model how a cognizer might stochastically generate hypotheses that can cover an infinite latent hypothesis space. Roughly, a PCFG consists of a set of stochastic production rules that can be used to iteratively compose—and potentially to adapt—hypotheses by (re)combining primitives. The set of production rules and their probabilities implicitly specify a probability distribution over the infinite space of possible hypotheses.

5.2.2 Learning as program induction

By combining LOT-like symbolic representations with a stochastic grammar for generating hypotheses or “programs”, human learning can be framed as a form of program induction (Bramley et al., 2018; Calvo & Symons, 2014; Chater & Oaksford, 2013; Lake, Ullman, Tenenbaum, & Gershman, 2017; Piantadosi et al., 2016; Romano et al., 2018; Rothe et al., 2017; Rule, Schulz, Piantadosi, & Tenenbaum, 2018). Program induction approaches model learning as a stochastic search over a space of potential symbolic program-like representations (Rule et al., 2020). Work by (Goodman et al., 2008) demonstrated the utility of program induction for modeling human learning, introducing a rational model of symbolic concept learning which provided a good description of adults’ generalizations across a Boolean concept learning task. More recent extensions of this idea have demonstrated that symbolic programs combining observed features with first-order logic provide good
coverage of human inferences about categories of geometric objects (Piantadosi et al., 2016). In another paper, (Lake et al., 2015) demonstrate that a model that combines symbolic representations of concepts with a Bayesian fitness criterion outperforms deep-learning benchmarks in a one-shot classification task involving handwritten characters from real and invented languages. Moreover, the model introduced by (Lake et al., 2015) had the ability to generate novel examples that were indistinguishable from humans across several “visual Turing tests”.

In addition to accounting for people’s inferences and generalizations, program induction approaches to number learning have provided insights into how children might abduct a general understanding of natural numbers via bootstrapping (Piantadosi et al., 2012). Using bootstrapping to learn difficult concepts has also been explored in related work modeling people’s acquisition of functional concepts such as sorting or summing lists of inputs from a program induction perspective (Rule et al., 2018). Asking questions to actively resolve uncertainty is another important component of human intelligence. Here, work by (Rothe et al., 2017) and (Z. Wang & lake, 2019) leveraged program induction ideas to develop a probabilistic model of open-ended question generation about the positions of two-dimensional objects in an ambiguous game environment. The authors used program induction to characterise this as involving a search for high-information–low-complexity queries with respect to resolving uncertainty about object positions.

A common theme across the above examples is that program induction involves stochastic generation of alternatives which are evaluated against one another using some fitness criterion. At a computational level, the generation of hypotheses is traditionally conceived as fully top-down problem, meaning that new programs are generated from a prior and only then evaluated against environmental stimuli. In a recent paper (Bramley et al., 2018) questioned whether fully top-down generation is a plausible cognitive account. Specifically, the authors compared a top-down Bayesian approach against a partially bottom-up, instance-driven generation (IDG) procedure during which encountered environmental stimuli inspire and partly shape the generation of new alternatives. Though further work is needed to validate the IDG proposal, initial results supported the notion that people use a partly bottom-up and datS-inspired mechanism for generating hypotheses. More recently, program induction has been proposed as a broad account of learning under the “child as a hacker” metaphor, which emphasizes the inevitable incompleteness of any one learner’s coverage of the compositional space
of possible world-models or theories, and evokes process-level considerations such as the learner’s intrinsic motivation and curiosity as drivers for progress and discovery (Rule et al., 2020).

### 5.2.3 Process models

Bayesian models have been successful at characterizing many of the computational problems faced by cognitive agents, and frequently also characterizing aggregate patterns of human judgements in domains including causal structure learning (Gopnik et al., 2004; Griffiths & Tenenbaum, 2005, 2009), social learning (Feldman Hall & Shenhav, 2019) and concept learning (Goodman et al., 2008; Piantadosi et al., 2016). An idealized Bayesian agent reasons about a hypothesis space \( H \) on the basis of data \( D \) and arrives at an updated (posterior) belief \( P(H \mid D) \) by computing the normalized product of their prior belief \( P(H) \) and the likelihood of each hypothesis of producing the data \( P(D \mid H) \):

\[
P(H \mid D) = \frac{P(D \mid H)P(H)}{\sum_{H'} P(D \mid H')P(H')}. \tag{5.1}
\]

However, direct application of Bayes’ theorem is frequently intractable in naturalistic settings, and while collective behavior often appears noisily normative, individual cognizers are much more fallible (Jones & Love, 2011). This means that solutions achieved by real agents must be approximate and the best they can do is to strike a balance between computational cost and accuracy. Process accounts attempt to model how cognition uses algorithmic approximations to deal with intractable problems under consideration of limited computational resources (Griffiths et al., 2012; A. N. Sanborn et al., 2010; van Rooij, Blokpoel, Kwisthout, & Wareham, 2019). This section summarizes relevant work on process models of inference under uncertainty. While machine learning research has developed a plethora of approaches to approximating inference under uncertainty, two major flavors are those that use sampling, and those that use variational approximations (A. N. Sanborn, 2017). In the current work we focus on the potential use of sample-based approximations in cognition, in particular particle-based approximations and Markov Chain Monte Carlo (MCMC) methods.
5.2.3.1 Particle approximations

Particle filters are a class of Monte Carlo algorithm that can approximate Bayesian inference where data arrives sequentially. A particle can be thought of as a single candidate hypothesis from the probability distribution in question. Filtering (reweighting and resampling) ensembles of particles as new data becomes available allows for the collection of particles to approximate the Bayesian posterior (A. N. Sanborn et al., 2010). Several particle based process accounts of behavioral data found that individual participants are best fit as maintaining only a few particles—essentially a small set of current working hypotheses—in associative learning (Daw & Courville, 2008), categorization (A. N. Sanborn et al., 2010), and binary decision making (Vul, Goodman, Griffiths, & Tenenbaum, 2014). Across these domains, particle accounts help explain the paradoxical tendency for aggregate response patterns to reflect Bayesian norms while individual judgements are noisy and diverse.

An extreme version of particle filtering relevant to the present paper is the proposal that a learner’s current hypothesis can be represented by a single particle that is resampled from the posterior if it becomes inconsistent with new data (and kept otherwise). This idea has been formalized as the “Win–Stay, Lose–Sample” (WSLS) algorithm (Bonawitz, Denison, Gopnik, & Griffiths, 2014), and has been shown to provide good agreement when compared with the performance of adults and preschoolers in several causal learning tasks (Bonawitz, Denison, Gopnik, & Griffiths, 2014; Bonawitz, Denison, Griffiths, & Gopnik, 2014). Importantly, while WSLS provides a promising starting point for understanding human reasoning, it does not provide a recipe for resampling hypotheses in the case that the learner decides to reject their current hypothesis. We now consider recent work addressing the question of how human learners search for new hypotheses. In particular, we explore the idea that this may involve adapting an existing hypothesis rather than constructing a new one from scratch.

5.2.3.2 Adaptive search

To model sampling and adaptation of particles, we turn to algorithms for sequential local search. Such algorithms generate sequences of (usually autocorrelated) hypotheses via a simple stochastic transition mechanism that is easier to compute than generating a new proposal from scratch. Two prominent approaches
are Markov Chain Monte Carlo (MCMC) and hill climbing. Hill climbing can be achieved by proposing changes at random and accepting them only if they improve the previous hypothesis. This is good for rapidly improving a hypothesis, but limits the search space meaning that, in multimodal spaces such as a compositional theory space, the learner will tend to get stuck in a local optimum; better than any of the adjacent hypotheses but worse than the global optimum (Ullman et al., 2012). Here, we are specifically interested in the idea that human hypothesis adaptation might work a little like MCMC sampling, where proposed changes are accepted stochastically even if they do not improve the hypothesis, allowing the learner to escape local optima but still arrive more often at higher probability hypotheses. Because, by design an MCMC chain’s stationary distribution is equal to the true posterior distribution, MCMC sampling asymptotically approximates the true posterior avoiding the need to evaluate an intractable normalization constant $P(D) = \sum_H P(H, D)$, and allowing a sufficiently long chain to end in an unbiased sample from the posterior.

5.2.3.3 MCMC-like search and anchoring

The proposal that stochastic local search forms a core component of hypothesis learning has been explored by (Dasgupta et al., 2017), who modeled human estimation of environmental conditional probabilities (e.g., “what is the probability that an image containing a table also contains a [chair/computer/curtain]”) as involving an MCMC-like search with a limited number of iterations. Thus, their model provides a computational rationale for a variety of known cognitive biases, including subadditivity (Fox & Tversky, 1998), superadditivity (Sloman, Rottenstreich, Wisniewski, Hadjichristidis, & Fox, 2004) and anchoring and adjustment effects (Hogarth & Einhorn, 1992). The idea that people’s hypothesis inferences are anchored by their initial ideas and adjusted by subsequent adaptations of these has been investigated by (Lieder et al., 2018a). Specifically, (Lieder et al., 2018a) explored the proposal that cognizers adjust quantitative estimates (e.g., about the arrival time of a bus) away from an arbitrary initial seed or anchor via an MCMC-like process. They find that participants’ judgements’ degree of dependence on the anchor is broadly resource rational in the sense of striking a good balance between computational cost and accuracy.
In the present work, we build on the literature on MCMC-like search, focusing on the algorithmic mechanisms underlying potential adaptations to an anchor. We build on a recent process account of online causal structure learning (Bramley et al., 2017). (Bramley et al., 2017) found support for the idea that people update their causal structure beliefs in a probabilistic setting in a local and incremental way, considering just a few local edits when surprising evidence arrives. Their account assumes participants hold just a single global causal structure hypothesis at any time point, rather than a distribution over the full hypothesis space. Belief updates were modeled as involving very short MCMC Gibbs sampling chains where each new proposal might add, remove, or re-orientate a single causal connection in the light of the most recent data and conditional on the rest of the current hypothesis. In this way, the new hypotheses learners arrived at were those that accounted for the latest evidence they had seen while remaining anchored to their previous judgment. The previous judgment thus acted as a stand in for a prior in the absence of complete memory for the older data making the model a kind of “single particle, particle filter”. This account outperformed a noisily normative account, WSLS, and a number of heuristic alternatives in capturing participants’ judgment patterns. The best fitting parameters were consistent with adaptive search chains of just a few steps, with search behavior falling somewhere between MCMC and hill climbing. In the present work we extend this framework, taking it from the large but finite hypothesis space of possible causal graphs to the formally infinite space of possible concepts expressible in a LOT.

5.2.4 Search and positive testing

Our secondary objective is to examine our process account under consideration of different forms of additional evidence. Specifically, we considered two forms of evidence: (1) additional evidence gathered by the learner and (2) evidence gathered by another learner. Evidence gathered actively via a learner’s own directed actions (Gureckis & Markant, 2012; Settles, 2009) is one of the most fundamental forms of data in individual cognition. In principle, well chosen actions or “experiments” performed by an active learner can improve their learning efficiency relative to passive observation because they can target areas of subjective uncertainty (Bramley, Lagnado, & Speekenbrink, 2015; Steyvers, Tenenbaum, Wagenmakers, & Blum, 2003). However, they can create pitfalls and learning traps given the bounded nature of our cognition (Rich & Gureckis, 2018). For example, active learning
without consideration of the full hypothesis space is likely to result in confirmatory testing biases in which a learner performs new experiments that seek to demonstrate the behavior predicted by their favored working hypothesis but that fail to distinguish it from possible alternatives that the learner has not considered (Klayman & Ha, 1987; Wason, 1960).

5.3 Task

![Image of the inductive learning task interface](image)

**Figure 5.1:** Overview of the inductive learning task. a) Interface for generating and testing arrangements of cones including initial positive example. Star at the top-right indicates category membership (yellow fill = arrangement follows hidden rule). b) Example sequence of 7 tests generated by a learner using interface shown in a). The first example test was provided to all participants. c) Generalization phase: Participants decided which of a set of new scenes are rule following by clicking on them (grey background = selection). Hidden rule in the above example is \( \exists(x_1 : = (x_1, \text{red}, \text{color}), \mathcal{X}) \) (“there is a red cone”).

We adapt an environment introduced by (Bramley et al., 2018) in which participants perform experiments to discover a hidden rule hypothesis. The environment involves a two-dimensional box in which learners can place and arrange triangular objects called “cones” of a range of colors and sizes (Figure 5.1a). Objects in the environment are governed by realistic simulated physical laws and certain combinations and arrangements of objects produce a causal effect when tested. A star shape at the top-right with yellow fill indicates category membership (i.e., whether the arrangement follows the hidden rule/produces the causal effect). The learner’s goal is to infer the hidden rule that determines the circumstances under which the effect occurs. For example, if the true rule is that “there is a small red cone” any scene that includes a small red cone will produce the causal effect. We probe learning by asking and incentivizing learners to (1) generalize by labelling a set of novel scenes as rule following or not (Figure 5.1c) and (2) provide an explicit written guess of the unknown rule concept. In our experiments, we will probe
participants’ learning twice: first after a they complete an initial active learning phase in which they gather evidence $D_{init}$, and again after completion of a second learning phase in which they either gather additional evidence $D_{rev}$, themselves or observe additional evidence gathered by another learner (Figure 5.2; see Methods for details).

5.3.1 The grammar

There are any number of possible grammars and choices of primitives capable of expressing the rules we explore in this task. However, following (Piantadosi et al., 2016), we adopt an expressive grammar that combines the basic object features of size $\in \{\text{small, medium, large}\}$, color $\in \{\text{red, blue, green}\}$, orientation $\in \{\text{up, down, left, right}\}$ and grounded $\in \{\text{yes, no}\}$ with first order (predicate) logic and use lambda abstraction (Church, 1932) to bind assertions to one or multiple variables. Table 5.1 contains the full grammar. “There is a small red cone” can be written as $\exists(\lambda x_1: \land (= (x_1, \text{small}, \text{size}), (= (x_1, \text{red}, \text{color})), \mathcal{X})$ which can be read as “there exists a cone (or element) $x_1$ in the set of all cones in a rule-following scene $\mathcal{X}$, such that $x_1$ has the size small and color red”. Arbitrarily complex hypotheses can be constructed by binding more than one variable. For example, “the largest cone is red” can be expressed as $\exists(\lambda x_1: \forall (\lambda x_2: \land (= (x_1, \text{red}, \text{color}), > (x_1, x_2, \text{size})), \mathcal{X}), \mathcal{X})$. Pragmatically, we also include a relational primitive of “contact” that can hold between any pairs of cones. We will use our grammar to translate participants’ explicit written guesses of the unknown rule concept into lambda abstraction representations used for formal analysis.

<table>
<thead>
<tr>
<th>There exists an object $x_i$ such that...</th>
<th>$\exists(\lambda x_i: \mathcal{X})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>For all $x_i$...</td>
<td>$\forall(\lambda x_i: \mathcal{X})$</td>
</tr>
<tr>
<td>There exist(s) ${&lt;, &gt;, =}$ $N$ objects $x_i$ such that...</td>
<td>$N{&lt;, &gt;, =}(\lambda x_i: N, \mathcal{X})$</td>
</tr>
<tr>
<td>Feature $f$ of $x_i$ has value ${&lt;, &gt;, \leq, \geq, =}$ to $v$</td>
<td>${&lt;, &gt;, \leq, \geq, =}(x_i, v, f)$</td>
</tr>
<tr>
<td>Feature $f$ of $x_i$ is ${&lt;, &gt;, \leq, \geq, =}$ to feature $f$ of $x_j$</td>
<td>${&lt;, &gt;, \leq, \geq, =}(x_i, x_j, f)$</td>
</tr>
<tr>
<td>Relation $r$ between $x_i$ and $x_j$ holds</td>
<td>$\Gamma(x_i, x_j, r)$</td>
</tr>
<tr>
<td>Booleans ${\text{and, or, not}}$</td>
<td>${\land, \lor, \neg}(x)$</td>
</tr>
</tbody>
</table>

Note that $\{<, >, \geq, \leq\}$ comparisons only apply to the feature size.
5.3.2 Production rules

5.3.2.1 Sampling from the prior

To generate symbolic hypotheses relevant to our inductive problem—that is, possible rules governing when the causal effect occurs—we use a probabilistic production process capable of producing all grammatical compositions of the primitives relevant for the learning setting. In practice, we simulate this production process by iteratively rewriting subparts of a string until a termination condition is reached (Bramley et al., 2018; Goodman et al., 2008; Manning & Schutze, 1999; Piantadosi et al., 2016) but are agnostic about how such a process could be implemented neurally (see SI for technical details).
5.3.2.2 Prior probabilities

By design, our production grammar embodies a prior preference for simplicity. That is, the production process will terminate in a short and simple hypothesis more frequently than more complex hypotheses. For example, there is a geometrically decreasing probability of generating hypotheses with a larger number of bound variables—half of the hypotheses produced make reference to only a single subset of the objects in a scene $x_1$ (i.e., have only one quantifier), a quarter refer to two subsets $x_1, x_2$, 12.5% to three subsets $x_1, x_2, x_3$, and so on. The prior probability of a given expression can be derived from the product of the probabilities of the sequence of productions that created it. This product inevitably decreases with the length of productions. Following the notation in (Goodman et al., 2008), we write the prior generation probability for hypothesis $h$ as:

$$P(Deriv_h | \mathcal{G}, \tau) = \prod_{s \in Deriv_h} \tau(s)$$

(5.2)

where $\mathcal{G}$ refers to our grammar, $\tau$ refers to the production probabilities, $s \in Deriv_h$ refers to the productions in the derivation, and $\tau(s)$ to the probability of each production. While derivations can vary in length, each derivation ultimately arrives at a termination production at which no further productions are sampled (see Figure 5.3a and Table 5.5 for further details). One complicating detail is that there are typically multiple ways of realizing a given semantics in syntax. Consider again the concept “there is a small red cone” which can be written as $\exists(\lambda x_1 : (=(x_1, \text{small}, \text{size}), =(x_1, \text{red}, \text{color})), \mathcal{X})$. Though this rule is syntactically distinct from $\exists(\lambda x_1 : (=(x_1, \text{red}, \text{color}), =(x_1, \text{small}, \text{size})), \mathcal{X})$, both expressions have the same meaning. That is, they will return the same truth value for any scene. The number of semantically equivalent expressions increases as an inverse function of the prior generation probability of a rule (essentially increasing with rule complexity, Equation 5.2). We thus derive a polynomial adjustment that we later use to approximate the total prior generation probability of a semantic rule class from the generation probability of a specific syntactic exemplar from that class (further details on prior generation probabilities are provided in SI).
Figure 5.3: a) Visualization of the top down rule generation process using our concept grammar. A rule is generated by starting at “Start” and following outgoing edges stochastically, in each case replacing the non-terminal symbol (capital letter with color matching the arrow) with the string at the arrow’s target. The process naturally terminates when no non-terminal symbols remain (see Table 5.5 for further details). b) Illustration of different adaptation mechanisms. Example rule $h_{\text{init}} \triangleright \exists(\lambda x_1 \cdot (x_1, \text{red, color}), X)$ (“there is a red cone”; top) and example “Tree Regrowth” (TR) and “Tree Surgery” (TS) adaptations (bottom; see main text and SI for further details).

5.3.3 Likelihood

We assume a simple likelihood function that allows for some stochasticity in the behaviour of each rule (cf. Goodman et al., 2008; Lewis, Perez, & Tenenbaum, 2014), such that the likelihood that a hypothesized rule $h$ correctly labels a set of scenes $D$ is a decreasing exponential function of its number of mispredictions

$$P(D \mid h, b) \propto e^{-bF}$$

(5.3)

where $F$ corresponds to the cardinality of the set of scenes $\{d \in D \mid h(d) \neq T(d)\}$ predicted by the rule that do not match the true observed labels $T$. $b$ is an inverse temperature parameter such that as $b \to \infty$, the likelihood of a scene producing stars when the true rule says it will not, or not producing stars when the rule says it will, approaches zero.
5.4 Models of adaptation

This section introduces the specific models of hypothesis adaptation we will compare to human inference patterns. We focus on predicting participants’ revised explicit rule guesses $h_{\text{rev.}}$, which were elicited after participants collected both initial $D_{\text{init.}}$ and revised evidence $D_{\text{rev.}}$ (see Figure 5.2 and Methods).

5.4.1 Normative simulations

We first consider a practically unbounded Bayesian learner that functions as a normative benchmark for our model comparisons. This model approximates a posterior distribution over the hypothesis space at each phase of the experiments and marginalizes over this posterior to make generalizations and guesses as to the true hidden rule. The posterior over rules after seeing initial evidence $D = \{D_{\text{init.}}\}$ or a combination of initial and revised evidence $D = \{D_{\text{init.}}, D_{\text{rev.}}\}$ is given by:

\[
P(H \mid D) \propto P(D \mid H)P(H) \tag{5.4}
\]

and the marginal posterior probability that a newly observed scene $s$ is rule following (across all possible rules) corresponds to:

\[
P(s \mid D) = \sum_{h \in H} P(s \mid h)P(h \mid D). \tag{5.5}
\]

In practice, these equations cannot be evaluated directly because $H$ is infinite. However, the intractable posterior $P(H \mid D)$ over rules and predictive distributions over selections for new scenes $P(s \mid D)$ can be approximated in various ways. Here, we use an MCMC chain with a “Tree Regrowth” proposal distribution (Goodman et al., 2008) to generate large posterior samples (see SI for further details of this approximation scheme). For constructing normative predictions, we used sufficiently numerous and lengthy MCMC chains to erase the patterns of anchoring on the chain-seed, since these are integral to our process ideas. Thus, the normative learner predicts that participants’ initial ($h_{\text{init.}}$) and final rule judgements ($h_{\text{rev.}}$) are independent conditional on the evidence.
5.4.2 Win–Stay, Lose–Sample

We next consider an idealized manifestation of Win–stay, lose–sample (WSLS, Bonawitz, Denison, Gopnik, & Griffiths, 2014). Under WSLS, the learner samples their initial hypothesis $h_{\text{rev.}}$ from $P(H \mid D_{\text{init.}})$. Then upon exposure to additional data $D_{\text{rev.}}$, WSLS keeps the current hypothesis with a probability tied to $h_{\text{init.}}$’s ability to account for the new data, or else samples a new hypothesis. Concretely, the chance WSLS draws a new sample is the complement of the likelihood of the new data given the old hypotheses:

$$1 - P(D_{\text{rev.}} \mid h_{\text{init.}}, b).$$

(5.6)

This means that hypotheses which provide a good fit to the additional set of test scenes are likely to be kept (even if other hypotheses would also provide a good fit), and the worse the initial hypothesis does, the more likely the learner is to resample from the posterior distribution $P(H \mid D_{\text{init.}}, D_{\text{rev.}})$ which here leverages the standard likelihood ratio comparison (see Equation 5.13 in SI). We note that this behavior predicts an all or none dependence on the initial hypothesis $h_{\text{init.}}$: Either $h_{\text{rev.}}$ will be stay the same as $h_{\text{init.}}$, with probability $P(D_{\text{rev.}} \mid h_{\text{init.}}, b)$, or it will be an unbiased posterior sample $P(H \mid D_{\text{init.}}, D_{\text{rev.}})$ with probability $1 - P(D_{\text{rev.}} \mid h_{\text{init.}}, b)$. Though WSLS predicts an order effect in which the use of posterior distribution over revised hypotheses is conditioned on the fitness of $h_{\text{init.}}$ to the new data, it is not yet a candidate process model, because it provides no recipe for how a learner resamples in the cases they decide to do so.

5.4.3 Hypothesis adaptation as local search

Our central hypothesis is that people update their beliefs through incremental adaptation. This means that we do not expect just all-or-none dependence between a learner’s $h_{\text{init.}}$ (initial rule guess) and $h_{\text{rev.}}$ (revised rule guess). We rather expect a graded dependence reflecting some form of local (MCMC-like) search for promising alternatives (Bramley et al., 2017). We now introduce two process models under which adaptation is modeled as a limited local MCMC search for alternatives under consideration of the full evidence $D = \{D_{\text{init.}}, D_{\text{rev.}}\}$ available to a learner after completion of the revised learning phase of our task. The
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first process model uses the established proposal distribution used for rule search, known as “subtree-regeneration” or “Tree Regrowth” (Goodman et al., 2008). The second process model uses a novel, and more local variant, that we name “Tree Surgery” (for details see Figure 5.3b and SI). We nest the two proposals within the WSLS framework, meaning that $h_{\text{rev.}}$ will be identical to $h_{\text{init.}}$ with probability $(D_{\text{rev.}} \mid h_{\text{init.}}, b)$, or it will be a local adaptation of $h_{\text{init.}}$ obtained by the respective local search procedure with probability $1 - P(D_{\text{rev.}} \mid h_{\text{init.}}, b)$. The local search procedures implemented by our process models differ from WSLS in terms of their mechanistic implementation of editing hypotheses when the initial rule guess proves sufficiently inconsistent with new evidence to motivate a change.

While WSLS resamples a new hypothesis from scratch, our process accounts *adapt* the initial hypothesis $h_{\text{init.}}$. In other words, the generation of new proposals is conditioned on a learner’s initial guess $h_{\text{init.}}$ and the data $D = \{h_{\text{init.}}, h_{\text{rev.}}\}$. As such, adaptation is modeled by stochastic transitions from $h_{\text{init.}}$ to $h_{\text{rev.}}$ involving a short sequence of edits, potentially reflecting an anchoring process in which the initial hypothesis acts as a stand in for a prior (Bramley et al., 2017).

### 5.4.3.1 Tree regrowth (TR-Learner)

Our first process learner implements edits to an existing hypothesis $h_{\text{init.}}$ by regrowing random subtrees from the overall hypothesis tree structure. Tree Regrowth (TR) works by sampling a random node or edge of the original hypothesis and regrowing a new subtree below. Figure 5.3b illustrates one potential step of a TR process, targeting the edge between $\exists$ and $\ = $, resulting in the new rule “there exists a small cone that is oriented upright”. Here, the TR-Learner essentially replaces $=(x_1, \text{red}, \text{color})$ with $B$ and runs the grammar until termination (Figure 5.3a). Diverging from idealized WSLS and normative simulations, we assume a TR-Learner uses only a few $k$ regrowth steps to adapt their prior hypothesis without losing it altogether. Following (Bramley et al., 2017), we model a learner’s variability in “search length” $k$ on a given trial (i.e., the length of the MCMC chain) by sampling $k$ from a shifted Poisson distribution with an average search length of $\lambda + 1$:

$$P(K = k \mid \lambda) = \sum_{i=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k_i!} + 1. \quad (5.7)$$
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Considering an initial hypothesis \( h_{\text{init.}} \) and evidence \( D = \{ D_{\text{init.}}, D_{\text{rev.}} \} \), the TR-Learner adapts an initial rule \( k \) times, starting with \( h_0 = h_{\text{init.}} \) and ending with \( h_k = h_{\text{rev.}} \), where each \( h_{i+1} \) is generated by an MCMC step. As such, a tree can be modified up to \( k \) times, depending on how often a new modification was accepted by means of Equation 5.13.

5.4.3.2 Tree surgery (TS-Learner)

While Tree Regrowth is a convenient form of MCMC for machine learning applications, due to its reuse of the general PCFG hypothesis generation mechanism, it is less clear whether it would be a good candidate adaptation mechanism for cognition. Tree Regrowth tends to produce hypotheses that differ substantially from their predecessors. The higher up the tree each regrowth process is initiated, the more of the original tree is overwritten. Indeed, each update overwrites an average of half of the tree, and within a few steps the updated hypothesis is likely to bear little resemblance to the starting point, typically retaining only its top level quantification but changing what features and assertions they pertain to. Intuitively, human hypothesis adaptation is often considerably more “surgical” than this. For instance, it seems natural that adaptations to the hypothesis shown in Figure 5.3b might result in a more specific hypothesis such as that there needs to be a small red cone. This kind of change amounts to introducing a branch conjunctively to the hypothesis while retaining its descendants. Alternatively, a proposal might remove a part of a more complex rule. For example, if one started with the rule “there is a red lying on its left hand side” and new evidence revealed that an upward standing red cone is rule following, one might then simplify the rule to “there is a red” assuming that color is the only relevant feature. An even simpler surgical edit to a rule could be to replace one boolean with another (i.e., replace an \( \& \) with an \( \lor \) ). This might be appropriate when new evidence suggests that being “red” and “lying on its left hand side” are individually sufficient features. In addition to targeting Booleans, we assume TS edits can also involve simple replacements of quantifiers with no change in the predicate or scope. For example, the rule “there is a small” — \( \exists (\lambda x_1 := (x_1, \text{small}, \text{size}), X) \) — might simply be adapted to become “all are small” \( \forall (\lambda x_1 := (x_1, \text{small}, \text{size}), X) \) or “there is exactly one small” \( \exists! (\lambda x_1 := (x_1, \text{small}, \text{size}), 1, X) \) (see SI for further technical details).
As with the TR-Learner, our Tree Surgery account adapts $h_{init}$ by proposing $k$ edits under consideration of $D = \{D_{init}, D_{rev}\}$ and asymptotically produces a new sample hypothesis from the posterior. The difference between the TR-Learner and TS-Learner is in the locality of proposed edits. The TS-Learner implements “surgical” local proposals while keeping the rest of the tree fixed as far as possible. The set of possible replacements for a given node are those that maintain compatibility with the type signatures of the components above and below in the tree. We note that both methods generate new proposals in a a top-down fashion, meaning that novel adaptations are suggested at random and not inspired by particular observations but their acceptance probability is shaped by their relative fit to the evidence. An example of bottom-up data-driven search has been explored by (Bramley et al., 2018).

### 5.4.3.3 Alternative accounts

To assess how competitive our adaptation proposal is, we pit it against the rules-plus-exception (RULEX) model (Nosofsky et al., 1994). RULEX has been used to model human learning in a number of concept learning studies, including classic experimental results by (Medin & Schaffer, 1978). RULEX works by applying a cascade of increasingly complex and approximate search procedures as required, to arrive at a symbolic rule. Traditionally it has been used in a propositional “Boolean logic” setting where rules do not require bound variables, but we here extend it to handle our setting. In brief, RULEX begins by searching for a rule referring to a single feature (exact search). If exact search succeeds, meaning that a rule is generated that correctly classifies all data points, RULEX adopts that rule permanently and terminates. Otherwise, RULEX continues by either performing an imperfect search over simple rules or a new exact search over conjunctive rules (i.e., rules pertaining to two features combined with a Boolean). For the imperfect search it again terminates if this search succeeds, where the success criterion is now that a rule is found that makes less than a threshold number of misclassifications. If the conjunctive search succeeds it terminates and adopts the conjunctive rule. If imperfect or conjunctive search fails (i.e., too many incorrect classifications), RULEX switches to the other type of search (for details, see Nosofsky et al., 1994). Finally, if both fail, RULEX searches for an imperfect simple or conjunctive rule, lowering its threshold for termination to allow for more and more misclassifications.
(i.e., “exceptions”) until a rule is found that passes the bar. Misclassifications are finally added to the final rule as listed disjunctive “exceptions”

RULEX does not work “out of the box” on our task because our setting requires predicate logic and further involves disjunctive as well as conjunctive rules (see Table 5.2). That is, rules involve quantifications like “all” and “there exists” and can refer to multiple subsets of objects in the environment potentially combining several conjunctions, disjunctions and negations. To accommodate this larger possibility space, we had our implementation of RULEX search over rules sampled from our hypothesis space constrained to either simple rules (no conjunctions or disjunctions but allowing arbitrary quantification and negation) or conjunctive and disjunctive rules (those containing exactly one conjunction or one disjunction again allowing for arbitrary quantification and negation). Just as with our other process models, we assume RULEX’s ability to search is limited. Therefore, we assume that a RULEX learner considers a finite number of hypotheses sampled from an appropriately restricted prior at each stage, with the number sampled from a shifted Poisson distribution just as with our other process models (Equation 5.7). We provide further details our our implementation of RULEX in the Model Comparison section and SI.

5.5 Overview of experiments

We study human learning across two experiments. For each experiment, our primary analysis compares participants’ generalizations and explicit rule guesses against those of a large posterior sample. We then investigate quantitatively which of our competing process accounts is best at finding participants’ revised rule guesses $h_{\text{rev}}$. To foreshadow, our model comparison finds support for several of these models but best supports the idea that participants’ revised rule guesses $h_{\text{rev}}$ are best predicted by means of a local, TS-like adaptation to their initial rule guesses $h_{\text{init}}$. 
5.6 Experiment 1

5.6.1 Methods

5.6.1.1 Participants

Ninety Participants (age 36.8±10.5 years [M±SD]) were recruited from Amazon’s Mechanical Turk (hit approval rate ≥ 95%) and paid $4.50 upon completion + [0.05, $2.00] bonus based on the accuracy of their revised generalizations. We were able to translate 248 of the total 450 free responses (each trial included two explicit rule guesses, one initial guess, and one revised guess) into our grammar.\footnote{The other free responses were either ambiguous (e.g., “I saw this one had cones inside the other and there was one more cone in this group”, $N = 92$) or nonsensical (e.g., “very nice task”, $N = 110$).} We thus use all 450 trials for our primary analyses but only the 248 trials with unambiguous initial and revised rule guesses during quantitative model comparisons.

5.6.1.2 Software

Both experiments were implemented using a javascript port of the Box2D physics game engine and ran in participants’ browsers. Scenes were displayed and constructed using an interactive 800 by 500 pixel iframe window embedded in the full screen task interface. A demo version of Experiment 1 is available here.

5.6.1.3 Cover story

Following (Bramley et al., 2018), we used a minimal “alien planet” cover story to frame the task in both Experiments. In it, participants act as space scientists tasked with working out the laws (rules) governing why particular combinations of alien objects (“cones”) produce alien forms of radiation. The instruction phase included five examples of already discovered radiation (shown in Figure 5.4) which served to orientate participants to the relevant features of the scenes.
5.6.1.4 Stimuli

For the main phase of both experiments, participants investigated five different forms of radiation (i.e., different test rules) varying in content and prior generation probability in random order (see Table 5.2).

5.6.1.5 Procedure

**Initial learning phase**  For each rule, participants first underwent a learning phase in which they observed one rule-following scene and subsequently created and tested seven scenes of their own (see Figures 5.1-5.2). Participants could add cones using the buttons shown at the bottom of Figure 5.1a, remove cones via right-clicking on them or by holding left-click on them. They could rotate cones counterclockwise and clockwise using “Z” and “X” respectively. Using the “Test” button, participants could then check whether the constructed scene followed the underlying rule or not and move on to their next test. Upon pressing “Test”, scenes that followed the rule would be overlaid with a graphic of stars shooting upward and the message “This arrangement DOES emit {name of wave} waves!” was displayed while for non-rule-following scenes no stars would appear and the
Table 5.2: Five unknown test rules used during the main experiment.

<table>
<thead>
<tr>
<th>Name</th>
<th>Rule</th>
<th>Prior Probability</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Zeta</td>
<td>(\exists (\lambda x_1: = (x_1, \text{red, color}), X)) (“there is a red cone”)</td>
<td>(\tau_{\text{color}})</td>
<td><img src="chart" alt="Zeta" /></td>
</tr>
<tr>
<td>(2) Iota</td>
<td>(\forall (\lambda x_1: = (x_1, \text{blue, color}), 1, X)) (“there is exactly one blue cone”)</td>
<td>(\tau_{\text{color}})</td>
<td><img src="chart" alt="Iota" /></td>
</tr>
<tr>
<td>(3) Upsilon</td>
<td>(\exists (\lambda x_1: \neg (=(x_1, \text{upright, orientation})), X)) (“no cone is upright”)</td>
<td>(\tau_{\text{orientation}})</td>
<td><img src="chart" alt="Upsilon" /></td>
</tr>
<tr>
<td>(4) Kappa</td>
<td>(\exists (\lambda x_1: \land (=(x_1, \text{small, size}), =((x_1, \text{blue, color}), X)) (“there is a small blue cone”)</td>
<td>(\tau_{\text{size}} \times \tau_{\text{color}})</td>
<td><img src="chart" alt="Kappa" /></td>
</tr>
<tr>
<td>(5) Omega</td>
<td>(\forall (\lambda x_1: \lor (=(x_1, \text{small, size}), =((x_1, \text{blue, color}), X)) (“all cones are small or blue”)</td>
<td>(\tau_{\text{size}} \times \tau_{\text{color}})</td>
<td><img src="chart" alt="Omega" /></td>
</tr>
</tbody>
</table>

message “This arrangement DOES NOT emit \{name of wave\} waves!” appeared. Messages were displayed for one second.

**Initial test phase** After the (active) learning phase was a test phase. In this, participants were first asked to make generalization predictions about which of eight additional scenes were rule following (Figure 5.1c). The test set for each rule contained four rule-following and four non-rule-following scenes selected from a large set of scenes created with a random scene generator and was the same for every participant. The true number of rule-following and not-rule-following scenes in this set was unknown to participants. Participants had to select at least one scene as rule following and fewer than all eight. While labelling test scenes, participants could see the summary of their initial test scenes (Figure 5.1b). The positions of the rule-following and non-rule-following scenes were randomized between participants and trials. Participants then provided a free-text typed guess of the hidden rule (i.e., their initial hypothesis \(h_{\text{init}}\) about the possible cause(s) of radiation). The response box required at least 15 characters. Underneath the textbox were bullet points asking participants to be as specific and unambiguous as possible and reminded them that the hidden rule could be related to any of the properties of cones and was unaffected by the previously created scenes.

**Subsequent learning and final test phases** Following the initial learning and generalization phase, participants were given the opportunity to perform seven
additional experiments to discover the underlying rule. The procedure was identical to their initial active learning phase. Thereafter, participants were asked to revise their generalizations under consideration of the additional active learning data. Their initial selections were displayed during this revised generalization phase (Figure 5.2). Finally, participants provided a second written response of their best guess of the hidden rule $h_{rev}$. Participants were paid a performance bonus of 5 cent for each correctly selected test scene (max. bonus $= 5 \text{ cent} \times 8 \text{ selections per trial} \times 5 \text{ trials} = 2.00$). We did not provide performance feedback between trials (i.e., participants did not receive feedback about the accuracy of their generalizations or explicit rule guesses).

5.6.1.6 Analysis strategy

**Primary analysis** We will first report participants’ generalization accuracy comparing it against that expected under rules sampled from the posterior.$^2$ Next, we assess how frequently participants and posterior-sampled hypotheses line up with the correct rule. Finally, we examine explicit changes to rule elements between initial ($h_{init}$) and revised ($h_{rev}$) rule guesses to motivate our adaptation proposal.

**Free response coding** To analyze participants’ explicit guesses about the rule in detail, we had two human coders convert participants’ text responses into the first order logic and lambda abstraction format we use in our grammar wherever this was possible to do unambiguously. The human coders were trained on the grammar and blind to the relevant ground truths and instructed to preserve the syntactic structure and order used by the participant as far as possible. Coder 1 translated all free text responses into the grammar and Coder 2 independently checked and re-coded a random 15% of the translated rules. Inter-rater agreements were 0.93 and 0.97 for Experiments 1 and 2, each higher than the 0.7 heuristic benchmark for adequacy (Krippendorff, 2018). Further details on our coding scheme and worked examples of the translation process are provided in SI.
We ran a linear mixed-effects regression using revision (init. = 0, rev. = 1) as fixed predictor. The participants' free text responses and corresponding lambda-abstraction translations were analyzed. Across all trials excluding ambiguous explicit rule guesses, participants and the posterior sample from normative simulations identified the correct rule. Results for Experiment 2 refer to 450 trials including ambiguous explicit rule guesses. Encoded trials correspond to the subset of trials during which participants provided rule guesses that could be translated into our grammar. Results show that both participants' and posterior sample's accuracy improved from initial to revised generalizations for all rules. Proportion (y-axis) of rule guesses that could be translated into our grammar. Results show that both participants' and posterior sample's correct rule guesses improved from the initial to the revised learning phase (apart from the disjunctive rule “All cones are small or blue”, which was never identified by participants). Probability (y-axis) with which participants performed conditional changes on rule elements between their initial and revised rule guesses (Section 5.6.2.3). Results show that participants engaged in fewer conditional changes than the posterior sample. Average number (y-axis) of added and removed rule elements (x-axis) between initial and revised rule guesses (Section 5.6.2.3). Results show that participants had a slight preference for adding new elements to their initial rule guesses, while the posterior sample was more likely to remove rule elements than add.

**Figure 5.5:** Summary of the results from Experiment 1. a) Results of our analysis on generalization accuracy (Section 5.6.2.1). Y-axis corresponds to the average accuracy for each initial and revised rule guess split by rule for both participants and a posterior sample generated with normative simulations (x-axis). All trials include trials that could not be translated into our grammar. Encoded trials correspond to the subset of trials during which participants provided rule guesses that could be translated into our grammar. Results show that both participants' and posterior sample’s accuracy improved from initial to revised generalizations for all rules. b) Proportion (y-axis) of rule guesses that could be translated into our grammar. Results show that both participants’ and posterior sample’s correct rule guesses improved from the initial to the revised learning phase (apart from the disjunctive rule “All cones are small or blue”, which was never identified by participants). Probability (y-axis) with which participants and posterior sample changed specific rule elements (e.g. Quantifiers or Booleans; x-axis) between their initial and revised rule guesses (Section 5.6.2.3). Results show that participants were less inclined to change rule elements as compared to posterior sample. d) Heat maps show probability with which participants and posterior sample changed rule elements conditional on changing other rule elements between initial and revised rule guesses (Section 5.6.2.3). For example, given a change to a quantifier (y-axis), the left panel shows how often participants changed either quantifiers, booleans, equalities, features, or feature values (x-axis). Results show that participants engaged in fewer conditional changes than the posterior sample. e) Average number (y-axis) of added and removed rule elements (x-axis) between initial and revised rule guesses (Section 5.6.2.3). Results show that participants had a slight preference for adding new elements to their initial rule guesses, while the posterior sample was more likely to remove rule elements than add.
5.6.2 Results

5.6.2.1 Generalization accuracy

Participants correctly labelled 62.6±12.6% of the generalization scenes after the initial learning phase, and this increased to 66.8±16.1% after the second learning phase (Figure 5.5a). Generalization accuracy thus improved significantly from phase one to phase two, as shown by a linear mixed-effects regression using revision as fixed predictor of accuracy and including random intercepts for both participants and rule types ($\beta = 0.042, p < 0.001$). Generalization accuracy for simulated normative posterior guesses based on the data generated by participants showed a similar pattern going from 66.9±7.9% in phase one to 79.5±11.3% when based on all the learning evidence in phase two ($\beta = 0.126, p < 10^{-10}$). The pattern of the results for the 248 trials with encodeable free responses was similar to that for the full dataset, with participants’ generalization accuracy increasing from 70.5±14.0% to 77.1±15.8% ($\beta = 0.066, p < 0.001$) and 69.7±9.7% to 82.5±9.6% ($\beta = 0.128, p < 10^{-15}$) for posterior sampled hypotheses given the same evidence.

5.6.2.2 Rule guesses

We next analyzed the frequency with which participants and posterior samples resulted in rule guesses semantically identical to the ground truth. Overall, participants guessed the rule correctly 32.3±24.0% of the time after the initial learning phase and 45.2±29.2% after the revised learning phase. This was commensurate with the probability of a normative posterior sample lining up exactly with the ground truth in 28.7±17.3% after the initial learning phase and 54.5±20.7% after revision. Figure 5.5b breaks this down by rule, revealing that participants’ guesses and posterior samples were similarly accurate for rules 1–3, while for rule 4 (“there is a small blue cone”), participants were more likely to guess correctly than posterior sampling. For rule 5 (“all cones are small or blue”), no participant guessed correctly with their initial ($h_{init}$) or revised ($h_{rev}$) guess, while posterior samples did occasionally land on the correct rule for the revised guess based on all data $P(H | D_{init}, D_{rev})$. Table 5.3 shows that participants’ rules were more complex.

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2For this we used the best fitting likelihood shape parameter $b$; see Model Comparison, for details.

3We established semantic identity by checking that the guessed or sampled rule labelled 500 randomly sampled test scenes identically to the ground truth.
Table 5.3: Mean (±SD) log prior generation probability and fit (loglikelihood) for participants’ guesses and posterior rule samples using $b = 5$.

<table>
<thead>
<tr>
<th>Exp.</th>
<th>Learner</th>
<th>$\log P(h_{init})$</th>
<th>$\log P(h_{rev.})$</th>
<th>$\log P(D_{init} \mid h_{init}, b)$</th>
<th>$\log P(D_{init,rev.} \mid h_{rev.}, b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. 1</td>
<td>Participants</td>
<td>$-3.61 \pm 0.60$</td>
<td>$-3.56 \pm 0.62$</td>
<td>$-4.64 \pm 5.97$</td>
<td>$-10.22 \pm 10.71$</td>
</tr>
<tr>
<td></td>
<td>Posterior Sample</td>
<td>$-2.71 \pm 0.12$</td>
<td>$-2.90 \pm 0.20$</td>
<td>$-1.53 \pm 0.87$</td>
<td>$-2.97 \pm 1.76$</td>
</tr>
<tr>
<td>Exp. 2</td>
<td>Participants</td>
<td>$-3.58 \pm 0.53$</td>
<td>$-3.58 \pm 0.56$</td>
<td>$-5.51 \pm 5.28$</td>
<td>$-13.94 \pm 10.37$</td>
</tr>
<tr>
<td></td>
<td>Posterior Sample</td>
<td>$-2.71 \pm 0.11$</td>
<td>$-2.93 \pm 0.19$</td>
<td>$-1.49 \pm 1.01$</td>
<td>$-3.19 \pm 1.86$</td>
</tr>
</tbody>
</table>

Prior generation probabilities are based on the polynomial adjustment (see Equation 5.9).

5.6.2.3 Explicit rule changes

To assess whether there is qualitative support for our incremental hypothesis adaptation proposal, we next investigated whether participants’ exhibit patterns of anchoring between their initial and revised guesses. Following previous work on anchoring effects (Bramley et al., 2017; Dasgupta et al., 2017; Lieder et al., 2018a), we hypothesized that participants would tend to retain elements of their initial rules while adapting others, leading to non-normatively high levels of similarity between initial and revised guesses. Of course there are normative reasons to expect similarity between initial and revised guesses—often the additional data will not overturn the earlier guess—as well as cases where we expect a complete change—such as when the new evidence leads to a completely new hypothesis being favored. Thus, as well as low overall change to hypotheses, we also expect anchoring to manifest in small proportions of conditional changes. That is, if learners tend to tweak just a few elements in their initial hypothesis to arrive at their revised hypothesis, we expect that conditioning on there being a change in one element of a participant’s hypothesis, there should be a relatively low probability of other elements also changing compared to a change pattern between two independent posterior samples.

To explore the relationship between initial and revised guesses, we visualize the absolute proportion of change in each rule element type (Figure 5.5c) as well the degree of change conditional on other changes (Figure 5.5d) comparing against independent posterior sampling in each case (for details on how we calculated...
conditional changes, see SI). This reveals that participants revise their hypotheses much more locally, making few changes overall and having much lower conditional change probabilities – i.e., less tendency to changing multiple elements in combination. We note that while the absolute amount of change between an initial and revised posterior sample depends on the strength of the prior and likelihood function, we would still expect participants’ heatmap pattern of “internal” conditional changes to resemble that of normative sampling if their revised guess is independent of their initial guess. However, we see a markedly different pattern in Fig. 5.5d with very low conditional change probabilities across the board for participants. Finally, we separately consider the number of added and removed rule elements. Figure 5.5e shows that, on average, participants removed slightly more rule elements than they added going into their final judgment, while independent posterior samples have the opposite pattern, tending to increase in complexity in phase two when more evidence is available, excepting the number of quantifiers (and hence bound variables). The general increase in complexity makes sense since more evidence licences more complex theories (with patterns in the data playing a relatively greater role and the prior preference for simpler rules having relatively less impact). The smaller number of quantifiers might reflect that revised normative samples also better lined up with the true conjunctive and disjunctive rules (“there is a small blue cone”, “all cones are small or blue”) which, in this experiment, only ever required a single quantifier (Figure 5.5b).

### 5.6.3 Discussion of Experiment 1

In Experiment 1, we saw that participants improved the quality of their guesses after gathering additional active learning data. This was true in terms of the accuracy of their generalizations, but also in terms of how often their explicit rule guesses matched the ground truth. Participants’ performance in terms of generalization and correct guesses was broadly commensurate with that of posterior sampling. However, a closer analysis of explicit rule changes suggests that participants’ pairs of guesses were substantially anchored to one another. We take this as qualitative support for our idea that participants often revise their hypothesis through limited local adaptation starting from their initial hypothesis in the light of the additional learning data.
Chapter 5. *Adaptation in inductive inference*  

5.7 Experiment 2 - Dyadic yoked learning

5.7.1 Methods

5.7.1.1 Participants

Forty-two participants (making up 21 dyads; age 35.30±10.51 years [M±SD]) were recruited from Amazon’s Mechanical Turk (hit approval rate ≥ 95%) and paid as in Experiment 1. To enable real-time interaction between paired participants, we used Socket.IO. Similar to Experiment 1, not all of the 210 free responses could be unambiguously translated into our grammar. Trials with ambiguous (N = 45) or nonsensical (N = 38) rule guesses were only included in the primary analysis and model comparisons are based on the 127 encoded trials.

5.7.1.2 Procedure

The initial active learning and generalization phases were identical to Experiment 1. However, after this, participants were shown the active learning data produced by their learning partner, and vice versa. As with Experiment 1, participants then had the opportunity to provide updated generalizations and an updated guess about the true rule. Both the participants’ own learning data and initial selections and the experiments performed by their partner were displayed during revision (see Figure 5.2). Our analyses of Experiment 2 will follow the same structure as Experiment 1.

5.7.2 Results

5.7.2.1 Generalization accuracy

Participants correctly labelled 65.5±11.8% of the generalization scenes after phase one and this increased to 68.5±11.9% after seeing their partner’s data (Figure 5.6a). Unlike Experiment 1, this increase was not statistically significant (β = 0.029, p = 0.154). However, the predictive accuracy of normative posterior samples increased from 69.8±7.6% to 82.2±6.6% (β = 0.124, p < 10^{-10}) indicating that the additional data was informative in principle. Similarly, for the 127 trials in which
participants made two unambiguous rule guesses, initial generalization accuracy 73.0±11.7% did not differ from final accuracy (74.3±11.9%; \( \beta = 0.013, p = 0.561 \)), while the posterior sample improved from 70.5±9.3% to 82.3±8.6% (\( \beta = 0.118, p < 10^{-69} \)).
5.7.2.2 Rule guesses & anchoring

Replicating the pattern from Experiment 1, we found that participants showed commensurate overall accuracy as posterior samples. In Experiment 2, participants’ rule guesses aligned with the ground truth in 28.3±18.1% of the time after the initial learning phase and 31.5±20.0% after revision, while the posterior sample had a match of 30±19.7% after the initial learning phase and 50±20.1% after revision (Figure 5.6b). Participants again outperformed posterior sampling for rule 4 (“there is a small blue cone”) but this time only with their initial guess. They again failed to guess rule 5 correctly for either initial or revised responses. Unlike Experiment 1, participants showed little improvement from initial to revised guesses, with the proportion guessing correctly improving only for rule 2 (“there is exactly one blue cone”). The pattern of prior generation probabilities and rule fits matched the findings from Experiment 1 with participants again producing somewhat more complex and less well fitting guesses than would be expected under normative posterior sampling (Table 5.3).

For participants, both the unconditional changes (Figure 5.6c) and conditional changes (Figure 5.6d) to rule elements between their initial and revised guesses were lower than in Experiment 1 and with a different pattern to the independent posterior sampling pattern. This is again consistent with our hypothesis that revisions are made through limited local edits. Compared to Experiment 1, participants were also less likely to add or remove rule elements. To the extent that they did, they tended to remove more Booleans, equalities and features than add them while the pattern for the normative learner was almost identical to Experiment 1, showing more additions than removals overall (Figure 5.6e).

5.7.3 Positive testing and hypothesis adaptation

We next sought to better understand why participants’ accuracy increased in Experiment 1 while there was no significant difference as a function of revision in Experiment 2. We hypothesized based on past work (Klayman & Ha, 1987; Wason, 1960) that participants frequently generate tests that are likely to be rule following under their current hypothesis. Thus, we compared the proportion of such positive tests (i.e., test scenes following participants’ initial explicit rule guess $h_{init}$) during the revised learning phase across experiments using a binomial mixed-effects
regression with Experiment (1 vs 2) as fixed effect predicting test scene (scene follows $h_{\text{init.}} = 1$, scene does not follow $h_{\text{init.}} = 0$) including random intercepts for both participants and rules. This revealed a significant decrease in the proportion of positive tests from $68.5 \pm 15.4\%$ in Experiment 1 to $53.5 \pm 23.7\%$ in Experiment 2 ($\beta = -0.784$, $p < 0.001$). This confirms that actively gathered data in the phase two of Experiment 1 indeed bears hallmarks of confirmatory or positive testing, while by design this does not hold in Experiment 2.

5.7.4 Discussion of Experiment 2

In Experiment 2, we found that seeing additional active learning data coming from a partner did not allow participants to improve the quality of their generalizations or their guesses. Though the average pattern of results was similar to Experiment 1 (i.e., revised generalization accuracy was higher than initial generalization accuracy), looking at accuracy per rule for encoded trials showed that participants’ revised generalization accuracy for the rule “there is a red cone” and the rule “there is a small blue cone” was slightly lower than their initial generalization accuracy for these rules. Our analysis of rule guesses provided further insight into the differences between experiments. Participants’ guesses only improved for 1 of the 5 rules, suggesting that seeing active learning data collected by someone else did not benefit them in the same way as additional active learning data had in Experiment 1. Our analysis of explicit rule changes suggested even greater degrees of anchoring than that found in Experiment 1 with participants making very few changes to their hypotheses on average when seeing their partners’ data despite this data being valuable from a normative perspective.

5.8 Model comparison

In order to quantitatively examine our conjecture that participants locally adapted their initial hypotheses to form their revised hypotheses, we now compare a range of computational and process level models to participants’ guesses and generalizations. We compared seven models in total, fitting one or two parameters in each case using a coarse grid search, and using the Bayesian Information Criterion (BIC, Schwarz et al., 1978) as our metric of model quality. The models we
considered were: (1) \( h_{\text{init}} \) as a baseline, (2) Posterior Sampling (PS), (3) Maximum a posteriori (MAP), (4) Win–Stay, Lose–Sample (WSLS), local adaptation by (5) Tree Regrowth (TR) and (6) Tree Surgery (TS), and (7) a modified form of RULEX that we refer to as RULEX* in our results.

We hypothesize that a local adaptation model (TR or TS) will better capture participants’ revised judgements than the models that treat each guess as independent (Posterior Sampling or MAP). Furthermore, we hypothesize that the kinds of edits participants make may be better characterized by a local “Tree Surgery” (TS) than the more mobile “Tree Regrowth” (TR) algorithm. We also hypothesized that participants do not always stick with their initial hypothesis (\( h_{\text{init}} \)), will diverge from Win–Stay, Lose–Sample’s all-or-none dependence pattern, and also from a heuristic RULEX*-style search.

### 5.8.1 Parameters

For each model, we fit a likelihood parameter with a coarse grid search \( b \in \{1, 3, 5, 7, 9\} \). This grid corresponds to a likelihood with which a rule mislabels a given scene that ranges from a highly stochastic \( e^{-1} = 0.37 \) to a practically deterministic \( e^{-9} = 0.0001 \); see Equation 5.3). For TR, TS and RULEX* we assume each search is performed for a finite number of steps \( k \sim \text{Poisson}(\lambda) + 1 \) (see Equation 5.7), where we examine \( \lambda \in \{0.5, 0.75, 1, 3, 10\} \).

### 5.8.2 Evaluating the models

We first considered two computational level accounts: Posterior Sampling, and selection of the MAP hypothesis. Since the posterior probability of \( h_{\text{rev}} \) is independent of \( h_{\text{init}} \) conditional on the data, we can estimate the likelihood of drawing \( h_{\text{rev}} \) as an independent posterior sample by simply counting how frequently a posterior sample based on all learning data faced by a participant exactly matches the participant’s revised guess \( h_{\text{rev}} \). If \( h_{\text{rev}} \) did not appear at all in a sample of 10,000 posterior hypotheses, we fall back on an \( \epsilon \) likelihood of \( \frac{1}{10,000} \).

For the MAP, we identify the most likely hypothesis (or joint most likely hypotheses) in a sample of 10,000 and assign a likelihood of 1 if \( h_{\text{rev}} \) was equal to the MAP hypothesis or \( \epsilon \). We examine this choice in SI.
for trials in which the MAP hypothesis was not equal to $h_{\text{rev}}$. Both these models have one free parameter $b$ controlling the impact of mislabeling on the likelihood of rules (see Equation 5.3).

As a baseline, we also considered whether participants might be described as simply keeping $h_{\text{init}}$, (resulting in a likelihood of 1 if $h_{\text{rev}} = h_{\text{init}}$, or of $\epsilon$ otherwise). We then also considered Win–Stay, Lose–Sample (WSLS). For this, we take a mixture between predicting a match with a participant’s initial guess (likelihood of 1 if $h_{\text{rev}} = h_{\text{init}}$, and $\epsilon$ otherwise) with mixture weight $P(D_{\text{rev}} \mid h_{\text{init}}, b)$ and a predicted selection of an independent posterior sample as above, with mixture weight $1 - P(D_{\text{rev}} \mid h_{\text{init}}, b)$. This model also had one free parameter $b$ affecting both the chance of re-sampling and the shape of the posterior being resampled from.

For the TR and TS local adaptation models, we estimated the likelihood of a participant’s $h_{\text{rev}}$ by starting with a participant’s initial rule guesses $h_{\text{init}}$ and running many MCMC adaptation chains. We used inverse binomial sampling (van Opheusden, Acerbi, & Ma, 2020) to estimate the reciprocal of the likelihood that a given chain with proposal distribution (TR or TS) and mean search length $\lambda + 1$ would terminate in that participant’s $h_{\text{rev}}$. Essentially, this involved tracking how many chain attempts it took on average before a chain ended on a participant’s judgment. For each combination of $b$ and $\lambda$, we simulated each adaptation algorithm repeatedly until it either produced a participant’s final rule guess exactly or reached a time-out threshold (which we set to 100 to maintain computational tractability). We repeated this process 100 times for each participant’s guess and averaged the resulting $\frac{1}{n_{\text{attempts}}}$ values as the likelihood of generating $h_{\text{rev}}$, taking timeouts as zeros and falling back on a likelihood of $\epsilon$ if $h_{\text{rev}}$ was never generated. We nested the TR and TS proposals within WSLS. That is, we assumed participants updated their hypothesis with probability $1 - P(D_{\text{rev}} \mid h_{\text{init}}, b)$ or left it as it was initially with probability $P(D_{\text{rev}} \mid h_{\text{init}}, b)$.

Finally, we based our implementation of RULEX on (Navarro, 2005). Our RULEX* has several search phases: 1. Exact search (searching for a single feature rule that perfectly classifies all data points), 2. imperfect simple search (searching for a single feature rule that classifies most data points correctly, but now allowing for some exception(s)), and 3. conjunct-disjunct search (searching for a rule including either one conjunction or one disjunction that classifies most data points correctly, but allowing for some exception(s)). Whether imperfect simple or conjunct-disjunct
search is performed after exact search is assumed to be random. In our task, we take simple rules to be any that refer to exactly one feature of the environment and contain no conjunctions or disjunctions. For example, “there is a red cone” or “there is exactly one blue cone” are simple in this sense. We take conjunctive rules to be those that include exactly one conjunction, such as “there is a small blue cone” or “there is a red and exactly one blue”. Since our set of test rules also included a disjunctive rule (“all cones are small or blue”, see ground truth 5 in Table 5.2), we allow our RULEX* implementation to also generate disjunctive rules directly during the conjunct-disjunct search phase rather than requiring it to generate a conjunction of negations (e.g., “all cones are not not small and not blue”).

Since RULEX has no mechanism for sampling datS-consistent rules directly, we conceived of search as generating hypotheses uniformly without replacement from the prior restricted to the complexity class and evaluating these against the evidence. For each stage of search, if one or more of the considered hypotheses meet the termination condition, this is selected (uniformly if there are several front runners) as the rule chosen by the algorithm (see SI, for further details). There are 528 simple rules that can be generated from our grammar, and these contain our ground truths 1–3. There are at least 557,568 five possible conjunctive and disjunctive rules containing our ground truth 4 and ground truth 5. We allowed RULEX* a variable search capacity $k = \lambda + 1$ as with TR- and TS-Learners, again evaluating this at $\lambda \in \{0.5, 0.75, 1, 3, 10\}$. Note that if we allowed a large search capacity, such as $\lambda = 1000$, RULEX* would always find simple rules 1–3 provided they are unique among simple rules consistent with the evidence, but would rarely find conjunctive or disjunctive rules due to the much larger search space. To increase its competitiveness, we further assumed RULEX* started with participants’ initial rule guesses $h_{\text{init}}$. That is, RULEX* included $h_{\text{init}}$ in its initial search phase (regardless of $h_{\text{init}}$’s complexity) and terminated with $h_{\text{init}}$ deterministically if $h_{\text{init}}$ accounted for all the evidence. If $h_{\text{init}}$ failed during exact search, RULEX* discarded $h_{\text{init}}$ and proceeded with its other search phases. We simulated 10,000 such search attempts for each trial of the experiment and counted how many terminated in $h_{\text{rev.}}$ with $\frac{N}{10,000}$ resulting as our estimate of the likelihood of RULEX* and falling back to an $\epsilon = \frac{1}{10,000}$ likelihood if no search attempt terminated in $h_{\text{rev.}}$.

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557,568 can be seen as a lower bound estimate of a hypothesis space consisting of $528^2$ conjunctive rules and $528^2$ disjunctive rules. Since we allow for arbitrary negations and quantifications such as “all cones are not not small and not blue” the actual hypothesis space is larger than the lower bound.
5.8.3 Modeling Results and Discussion

Table 5.4: Best fitting model performances for participants’ revised rule guesses ($h_{\text{rev}.}$).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Model</th>
<th>Mean $BIC_{h_{\text{rev}.}}$</th>
<th>$N_{\text{Best},h_{\text{rev}.}}$</th>
<th>$b$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h_{\text{init}}.$</td>
<td>8.988</td>
<td>22.17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Exp. 1</td>
<td>Posterior Sample (PS)</td>
<td>9.817</td>
<td>6.00</td>
<td>5.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Maximum a posteriori (MAP)</td>
<td>11.164</td>
<td>4.17</td>
<td>7.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Win–stay, lose–sample (WSLS)</td>
<td>7.158</td>
<td>7.17</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>WSLS with Tree Regrowth (TR)</td>
<td>6.368</td>
<td>12.17</td>
<td>3.0</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td><strong>WSLS with Tree Surgery (TS)</strong></td>
<td><strong>5.917</strong></td>
<td><strong>15.17</strong></td>
<td><strong>5.0</strong></td>
<td><strong>10.0</strong></td>
</tr>
<tr>
<td></td>
<td>RULEX*</td>
<td>9.202</td>
<td>2.17</td>
<td>-</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>$h_{\text{init}}.$</td>
<td>5.657</td>
<td>16.37</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Exp. 2</td>
<td>Posterior sample (PS)</td>
<td>11.403</td>
<td>2.00</td>
<td>5.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Maximum a posteriori (MAP)</td>
<td>12.367</td>
<td>1.17</td>
<td>7.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Win–stay, lose–sample (WSLS)</td>
<td>6.573</td>
<td>0.37</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>WSLS with Tree Regrowth (TR)</td>
<td>4.473</td>
<td>3.37</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td><strong>WSLS with Tree Surgery (TS)</strong></td>
<td><strong>4.065</strong></td>
<td><strong>10.37</strong></td>
<td><strong>1.0</strong></td>
<td><strong>1.0</strong></td>
</tr>
<tr>
<td></td>
<td>RULEX*</td>
<td>9.220</td>
<td>2.37</td>
<td>-</td>
<td>3.0</td>
</tr>
</tbody>
</table>

We evaluated all models by fitting all data points at each of our parameter grid search values. Table 5.4 reports the BIC for the best fitting setting of each model, expressed per trial to accommodate the fact that we have different numbers of data points for different participants (since not all participants provided unambiguous guesses for all five rules). Figure 5.7 shows the by-trial likelihood of each model generating $h_{\text{rev}.}$. Results under all considered values of $\lambda$ and $b$ are provided in SI Tables 5.7–5.12. Across both experiments, the TS-Learner performed best in terms of its overall BIC. At the individual level $h_{\text{init}}.$ best fits the largest number of participants in both experiments reflecting the frequent occasions in which participants retained their initial hypothesis. However, excepting these participants,
Tree Surgery best fit the largest proportion of the remaining participants (32% and 59%).

In Experiment 1, the best fitting $\lambda$ for both TR and TS was 10, at the upper end of the range of our grid search. However this substantially outperforms WSLS which follows an identical procedure but with a 10,000 step search (to obliterate anchoring). In Experiment 2, participants only changed their initial rule guess $h_{\text{init}}$ on 39 out of 127 trials compared to 121 out of 248 trials in Experiment 1. Capturing this generally decreased willingness to change, TS and TR had thus best fitting average search parameters of $\lambda = 1$ for Experiment 2. Increasing search length resulted in a tradeoff between an increased likelihood of finding $h_{\text{rev}}$ during trials in which participants changed their initial rule guess $h_{\text{init}}$ during revision and a decreased likelihood assigned to trials in which participants stuck with $h_{\text{init}}$ despite the presence of local and better alternatives (see Figure 5.8). In sum, results from our model comparison revealed that participants’ revised rule guesses were best identified by TS-like adaptation to their initial rule guesses across experiments.

### 5.8.4 Predicting generalizations

Having considered a range of models of the generation of $h_{\text{rev}}$, we finally consider whether $h_{\text{rev}}$, or adaptive search more generally, can explain participants’ generalizations. While we considered end-to-end versions of all the models we compare above, we also considered $h_{\text{init}}$ and $h_{\text{rev}}$ as potentially directly predicting participants’ generalizations. Consistent with our general thesis that inductive generalizations are driven by learners’ symbolic belief about the rule, $h_{\text{rev}}$ was by far the single best predictor of revised generalizations across both experiments. See SI and Table 5.6 for details.

### 5.9 General Discussion

We explored how people learn symbolic rules in a challenging inductive learning task that admits a practically unbounded compositional hypothesis space. We specifically focused on the relationship between participants’ initial and revised guesses about a hidden symbolic rule. We elicited free text guesses as well as
Figure 5.8: a) Experiment 1. b) Experiment 2. Both panels show mean likelihood of $h_{\text{rev}}$ (shading, darker = more likely) for different values of $\lambda$ and $b$. “No Change” subsets out trials during which participants did retain their initial guess (i.e., $h_{\text{init}} = h_{\text{rev}}$), “Change” subsets trials where $h_{\text{init}} \neq h_{\text{rev}}$.

generalizations of predictions to a set of new scenes. Across two experiments, we examined the difference between participants’ initial and revised guesses asking whether we can account for these as the result of adaptive search under limited computational capacity.

Overall, accuracy was close to that which would be expected under posterior sampling or probability matching. However, participants explicit rule guesses were also more complex and less able to account for all the learning data than posterior samples on average. Further diverging from a normative account, we found that participants’ revised hypotheses were anchored to their initial hypotheses, containing a much larger overlap of syntactic elements than would be expected under independent sampling or an all-or-none approach like Win–Stay, Lose–Sample. We
speculated this could be due to participants considering a short sequence of local edits to their initial hypotheses in the light of new evidence.

To capture participants’ guesses in this scheme, we encoded them in lambda calculus and proposed that updated hypotheses were discovered through a short MCMC-like search in which local edits are proposed and accepted according to a fitness function combining prior probability and fit to evidence. We compared an established MCMC proposal distribution (Tree Regrowth) with a stickier variant (Tree Surgery) and compared this account against a set of competitors and benchmarks. We found participants’ judgements lined up closest with our Tree-Surgery model in both experiments, suggesting they considered a small number of “surgical” elements that replaced part of their earlier guess while minimally disturbing the rest of their rule structure.

5.9.1 Search and positive testing

Participants’ self-generated tests in phase two of Experiment 1 were subject to a positive testing bias, being much more likely than chance to be rule following under their initial rule guess $h_{init}$. In Experiment 2, phase two introduced learning data from another participant, presumably driven by confirmation of some other hypothesis not yet considered by the learner. At the same time, participants adapted their guesses more in Experiment 1 and improved in accuracy in Experiment 1 but not Experiment 2. We think a compelling explanation for this pattern is that learners’ selected what scenes to test in order to distinguish between their current hypothesis and local alternatives that occurred to them, so actively supporting a process of incremental search of the sort captured by our TR and TS models. Since another person’s learning data is less likely to be diagnostic about these particular local alternatives, learners in Experiment 2 were less able to use it to adapt and improve their guesses.

While we find this explanation satisfying, we cannot rule out the possibility that yoked learners in Experiment 2 performed worse in phase two than participants in Experiment 1 because they did not process the additional evidence as deeply as they would have had they gathered it themselves trial-by-trial (Ruggeri, Markant, Gureckis, & Xu, 2016). Another complicating factor in Experiment 2 could be that participants may have tried to use theory of mind inference, for instance trying to reverse engineer their partner’s guess from their testing patterns. This is
Chapter 5. *Adaptation in inductive inference*  

5.9.2 Limitations and further work

Local search is an important aspect of higher level cognition (Hart et al., 2020; Hills, Todd, & Goldstone, 2008; Hills et al., 2015). Here, we attempted to reverse engineer people’s likely sequences of mental search steps as they grappled with a challenging inductive learning task. While our analyses go significantly beyond past work, our analyses are limited in a number of respects. First, we elicited explicit rule guesses and generalizations after completion of each learning phase, that is, after participants gathered a set of learning data. To allow for a more granular investigation of incremental hypothesis revision over time, future extensions of our work might focus on a learning setting in which learners provide explicit rule guesses after each individual data point. This would unlock the possibility of a closer exploration of how hypotheses shift scene by scene, and how this may relate to the process of active scene construction.

One central assumption of our local adaptation proposal was that people build and edit a single focal hypothesis during compositional inference rather than simultaneously considering many possibilities. Although our analysis showed participants’ responses were sequentially dependent, there were cases where participants’ responses changed completely. This could mean that people entertain more than one option and are able to switch when one starts to outperform the other. People might also sometimes consider their current candidate to be a “dead end” and generate a new hypothesis from scratch (Speekenbrink & Shanks, 2010). A model that allowed learners to have several hypotheses or “particles” in play at a time could help to capture these patterns but is beyond the scope of this project (Daw & Courville, 2008; A. N. Sanborn et al., 2010).

A third limitation is that the present anchoring between $h_{\text{init}}$ and $h_{\text{rev}}$ does not necessarily implicate a causal role of the initial rule guess $h_{\text{init}}$. It could also be
a consequence of primacy, for instance if a participant only attends to the first few scenes. It might also reflect an incomplete processing of all the evidence, as focusing on only a subset of the potentially relevant features or relations. Such factors might lead to similarity between hypotheses over and above that which is normatively justified by the data but not because the later hypothesis is adapted from the first. This might provide an alternative explanation for why the TS-Learner, which naturally produces the strongest dependence between initial rule guesses ($h_{\text{init}}$) and revised rule guesses ($h_{\text{rev}}$) for the same search length was the best predictor of $h_{\text{rev}}$. We do not find this explanation particularly compelling, for one because we considered a range of search length values, and also because recency could also result in the opposite pattern, with $h_{\text{rev}}$ resulting in too little resemblance to $h_{\text{init}}$. Finally, an incomplete focus on the features would be expected to produce similarity across all judgements, not just those within a learning problem which we did not see.

Investigating how participants generated their initial guesses in the first instance is another interesting question for further research. Specifically, while our PCFG framework provides a recipe for generating hypotheses a priori, it is inherently inefficient and a computationally implausible account of top-down hypothesis generation. Here, we refer to recent work using an instance driven approach to generation (Bramley et al., 2018), which suggests that patterns in observed environmental features might directly inspire the generation of new hypotheses that could then be refined and adapted through a local search process such as the one described in the present work. Further contrasting the present learning architectures with alternative models of human concept learning such as the generalization and exemplar learning algorithm SEQL (Kuehne, Forbus, Gentner, & Quinn, 2000; Skorstad, Gentner, & Medin, 1988) or ALIGN (McLure, Friedman, & Forbus, 2010) could be another interesting avenue for further work.

Finally, we note that sample-based approaches represent just one family of approximation to intractable probabilistic inference problems. Variational approaches, that convert inference to an optimisation over some approximation family, have also been explored in a number of recent treatments of bounded cognitive computation. Future work could try to pit variational and sample based approximation against one another as competing accounts of symbolic concept induction (A. N. Sanborn, 2017).
5.9.3 Alternative representations

In the present work, we had participants describe their rule guesses in natural language and then translated them into first order logic with references to features of the scenes. As such, we assumed that learners intuitively represent their concepts symbolically and in terms of the features and relations we included in our grammar. However, not all concepts are expressible in logic, and statistical concepts are hard to express in natural language. Subsymbolic feature-similarity based representations such as those of exemplar or prototype models might provide an alternative approach to predicting participants’ generalizations in the present learning problem (Medin & Schaffer, 1978; Nosofsky, 1986; Rosch, 1999). On that view, linguistic descriptions of reasoning processes might taken to be “retrospective confabulations” primarily serving communication rather than capturing the underlying representation (Dennett, 1981). However, these accounts provide no explanation for how participants readily generated an explicit, symbolically structured rule guess when asked to do so. Additionally, the finding that participants’ revised rule guesses were the single best predictor of their generalizations suggests that the simplest explanation for both the guesses and the generalizations is that the guesses drove the generalizations.

5.9.4 Beyond rule complexity and fit

We modeled inductive inference as a form of program induction. That is, we assumed that participants beliefs progressed via a process involving construction and adaptation of programs (here, “rules”) guided by their ability to explain data (Flener & Schmid, 2008; Rule et al., 2018). While program induction is proving to be a powerful framework for synthesizing human-like concept generation (Goodman et al., 2008; Lake et al., 2015; Lewis et al., 2014; Piantadosi et al., 2016) and other learning domains (Rothe et al., 2017; Ullman et al., 2012; Yang & Piantadosi, 2022), it is computationally intensive and serves to highlight how tough some of the inference problems solved by humans really are (Ullman et al., 2012). As such, it seems likely that individual humans chart incomplete paths through such learning settings, employing various hacks, inductive biases, heuristics and approximations along the way (Rule et al., 2020). We see local adaptation as a critical part of this picture, but also note that the perspective invites consideration
of other mechanisms for bootstrapping and active search with concomitant cognitive parameters such as motivation and curiosity alongside the search capacity we highlight here.

5.9.5 Conclusions

The present paper explored the idea that human inductive inference involves adaptation of symbolic hypotheses via sequential local changes in the light of evidence. Across two experiments that varied the source of learners’ evidence, we showed a local adaptation mechanism provided a better account of participants’ hypothesis revisions than a set of benchmarks and competitors. We extend on previous work that explored local adaptation in the large but finite space of causal graphs (Bramley et al., 2017) by studying inferences in a setting with an essentially infinite compositional theory space. Moreover, by examining self-generated and yoked learning side by side, we were able to implicate confirmatory testing as potentially subserving a process of local adaptation. Overall, our results suggest that people deal with the inherent complexity of concept inference in part through use of compositional local search.

5.10 Supplementary Information

S-1 and S-2 provide further technical details on the production process and computation of prior rule generation probabilities. Details of the MCMC approximation scheme and implementation of “Tree Regrowth” and “Tree Surgery” are provided in S-3 and S-4. S-5–S-7 provide details on our implementation of RULEX, the free response coding scheme used to translate participants’ written rule guesses into symbolic rules, and the computation of conditional rule changes. Additional modeling details and supplementary results are provided in S-8.

5.10.1 S-1: Production process

In our production process, a hypothesis starts with a single nonterminal symbol \(S\). We use capital letters as non-terminal symbols. Each production rule replaces a non-terminal symbol with one of the options written in the requisite
row of Table 5.5. The process continues until no non-terminal symbols remain. We assume for simplicity that all possible rewrites are equally probable with the exception of feature selections and value expansions (see S-2). For example, since there are three possible expansions for $S$ each could be selected with probability $\frac{1}{3}$. The string resulting from this process is guaranteed to be a grammatical statement asserting something about the rule following scenes. Branching resulting from the $A$ and $B$ rewrites can result in an arbitrary number of nested Boolean functions and other complex relationships. For example, the hypothesis $\exists (\lambda x_1: = (x_1, small, size), \mathcal{X})$ (“there is a small cone”) is produced by the following sequence of productions:

1. $S \rightarrow \exists (\lambda x_1: A, \mathcal{X})$
2. $\exists (\lambda x_1: A, \mathcal{X}) \rightarrow \exists (\lambda x_1: B, \mathcal{X})$
3. $\exists (\lambda x_1: B, \mathcal{X}) \rightarrow \exists (\lambda x_1: D, \mathcal{X})$
4. $\exists (\lambda x_1: D, \mathcal{X}) \rightarrow \exists (\lambda x_1: = (x_1, G), \mathcal{X})$
5. $\exists (\lambda x_1: = (x_1, G), \mathcal{X}) \rightarrow \exists (\lambda x_1: = (x_1, small, size), \mathcal{X})$.

The (syntactic) prior generation probability of the hypothesis (Equation 5.2) is given by the product of each of the productions that went into creating it:

$$\frac{1}{3} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{5} \times \tau_{\text{size}} = \frac{\tau_{\text{size}}}{90}$$  \quad (5.8)$$

where $\tau$ refers to the product of the probabilities for feature selection $size$ and value expansion $small$.

As there exists an infinite amount of possibilities for realizing a semantic in syntax, we used a rule’s estimated semantic generation probability in our analysis instead of working with the syntactic prior. This was given by fitting $\kappa$ in the following function

$$P(\text{Deriv}_h \rightarrow \text{semantic}) = P(\text{Deriv}_h \mid \mathcal{G}, \tau)^\kappa$$  \quad (5.9)$$

6In the present work, we limit the number of quantifiers to 3 for computational convenience.
Table 5.5: Prior Production Process.

<table>
<thead>
<tr>
<th>Productions</th>
<th>Start</th>
<th>Bind additional</th>
<th>Expand</th>
<th>Function</th>
<th>Numeric feature</th>
<th>Numeric feature</th>
<th>Feature</th>
<th>Feature</th>
<th>Relation</th>
<th>Boolean</th>
<th>Inequality</th>
<th>Quantifier</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S \rightarrow \exists (\lambda x_i : A, X) \land (\lambda x_i : A, X) \land (\lambda x_i : A, K, X)$</td>
<td>$A \rightarrow B$</td>
<td>$B \rightarrow D$</td>
<td>$D \rightarrow I(x, E)$</td>
<td>$I(x_i, x_j, F)$</td>
<td>$=(x_i, G)$</td>
<td>$=(x_i, x_j, H)$</td>
<td>$\Gamma(x, x_j, J)$</td>
<td>$E \rightarrow$</td>
<td>$F \rightarrow$</td>
<td>$G \rightarrow$</td>
<td>$H \rightarrow$</td>
<td>$K \rightarrow$</td>
</tr>
<tr>
<td></td>
<td>$N(\lambda x_i : A, K, X)$</td>
<td>$S$</td>
<td>$C(B, B)$</td>
<td>$=(B)$</td>
<td>$value, feature$</td>
<td>$value, feature$</td>
<td>$value, feature$</td>
<td>$value, feature$</td>
<td>$feature$</td>
<td>$\land \lor$</td>
<td>$\leq \geq &gt; &lt;$</td>
<td>$\leq \geq$</td>
<td>$\in {1, 2, 3}$</td>
</tr>
</tbody>
</table>

Note: Context-sensitive aspects of the grammar: Bound variable(s) sampled uniformly without replacement from set; expressions requiring multiple variables censored if only one.

Figure 5.9: Relationship between cluster’s median syntactic rule complexity (x-axis) and estimated semantic complexity of rules in cluster (y-axis). Data correspond to 95,912 clusters obtained from a sample of 1,000,000 synthetic rules generated from the grammar.

to a large sample of $N$ synthetic rules in which syntactically distinct rules were clustered based on their meaning (i.e., semantics), and then dividing the size of each cluster by the total size of the synthetic sample. The semantic complexity (cluster size / total size of synthetic sample) was then compared against the median syntactic complexity $P(Deriv_h | G, \tau)$ from each cluster. Figure 5.9 illustrates the relationship between a cluster’s median log complexity / log prior generation probability (x-axis) and the estimated log semantic generation probability for each rule in the cluster (y-axis). In total, we simulated $N = 1,000,000$ synthetic rules from our grammar, evaluated each rule against 500 test scenes, resulting in 95,912 distinct clusters following grouping of rules that provided the same predictions for the set of test scenes. Fitting Equation 5.9 to the synthetic rule set shown in Figure 5.9 resulted in $\kappa = 0.486$. 
5.10.2 S-2: Value and feature weights

In this section, we outline the derivation of production weights for values and features. In previous work (Bramley et al., 2018), feature values were sampled uniformly from their support. For example, the values of the feature color $\in \{\text{red, blue, green}\}$ would be sampled with probabilities $\phi_{\text{color}} = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$. For each feature $i$, the specific feature values $v_i$ thus follow a multinomial distribution $v_i \sim \text{Mult}(n_i, \phi_i)$ where $n_i$ refers to the number of value expansions for feature $i$ (e.g., $n_{\text{color}} = 3$ given $v_{\text{color}} = \{\text{red, blue, green}\}$). In our modeling analysis, we weighted value expansion probabilities for each feature based on the content and selections of the scenes generated by the learner during the training phase, whose labels are thus known.

For each feature $i$, we thus counted the number of occurrences of values $v_{ij}$ between the average rule following scene and the average non-rule following scene. For example, the average rule following scene might include 2.3 red cones, while the average non-rule following scene might include 0 red cones given a hypothesis such as $\exists(x_1:=(x_1, \text{red, color}), X)$ (“there is a red one”). The value expansion $j[\text{red}]$ of feature $i[\text{color}]$ thus receives a weight of 2.3. Formally, the score for all specific feature values $v_{ij}$ is calculated as follows:

$$v_{ij} = \frac{1}{|S||r|} \sum_{k=1, v_i \in S||r}^{v_i} I_{v_{ij}}(x_k) - \frac{1}{|S||\neg r|} \sum_{k=1, v_i \in S||\neg r}^{v_i} I_{v_{ij}}(x_k). \quad (5.10)$$

Here, $I_{v_{ij}}(x_k)$ is an indicator function returning 1 if the value of cone $x_k$ is equal to feature value $v_{ij}$. $S||r$ refers to the set of test scenes consistent with a rule while $S||\neg r$ refers to the set of test scenes not consistent with a rule. Importantly, the above equation only accounts for positive influences of feature values on eliciting rule-following scenes (e.g., if the rule was “there is a red cone”, it would assign higher scores towards the generation of rules mentioning red cones than cones of another color). However, there might be instances in which the absence of a specific feature is critical to a rule. Consider for example the rule “no cone is upright”. In this case, a scene belongs to the set of rule following scenes $S||r$ if no cone object is standing upright, meaning that Equation 5.10 might result in a negative value for $v_{ij}$. To address this issue, the learning procedure is sensitive to the presence and absence of features, flipping the value of $v_{ij}$ if there is no cone $x_k$ that is equal to $v_{ij}$ in the set of rule following scenes:
Chapter 5. Adaptation in inductive inference

\[ v_{ij} = \begin{cases} v_{ij} & \text{if } \sum_{k=1, v_i \in S} I_{vij}(x_k) > 0 \\ -v_{ij} & \text{if } \sum_{k=1, v_i \in S} I_{vij}(x_k) = 0 \end{cases} \]  \hspace{1cm} (5.11)

Having computed values for each \( v_{ij} \) we add \( \frac{1}{n_i} \) to each \( v_{ij} \) (initial uniform weight prior to observing scenes) and then renormalize scores such that each \( \phi_i = \sum_{j=1}^{n_i} v_{ij} = 1 \). To determine the relevance of each feature \( i \) on its own, we further weighted the vector over feature probabilities \( \psi_i \). Specifically, the probability \( \phi \) of sampling a feature \( i \) was given by the normalised difference between the feature’s value with the highest probability and the feature’s value with the lowest probability:

\[ \psi_i = \max(\phi_i) - \min(\phi_i). \]  \hspace{1cm} (5.12)

That way, production probabilities were “inherited” from value to feature level. We added \( \frac{1}{n} \) to each \( \psi_i \) where \( n \) refers to the number of features used in the present task. Finally \( \psi \) was renormalized such that \( \psi = \sum_{i=1}^{n} \psi_i = 1 \).

5.10.3 S-3: Inference with Tree regrowth MCMC

We used Tree Regrowth MCMC also referred to as the “Rational Rules Model” (Goodman et al., 2008) with long chains (10,000 samples per chain) to generate normative posterior samples. At each iteration of this approximation, a non-terminal node \( n \) of the current hypothesis \( h \) is selected and deleted. Tree Regrowth then removes all branches below \( n \) and regrows the tree below \( n \) according to the same stochastic rules used to derive the initial hypothesis \( h \). The probability \( r \) that a new proposal \( h' \) replaces \( h \) at the next time step is then given by:

\[ r = \min(1, \frac{P(D \mid h', b)}{P(D \mid h, b)} \times \frac{P(h' \mid G, \tau)}{P(h \mid G, \tau)}). \]  \hspace{1cm} (5.13)

At each step of the iteration, proposal \( h' \) is accepted if \( r \leq x \leftarrow \text{Uniform}(0,1) \). If a proposal is rejected, the current hypothesis is retained \( h_{t+1} = h \).
5.10.4 S-4: Tree surgery as an alternative MCMC proposal distribution

The TR-Learner uses the same established Tree Regrowth mechanism to adapt \( h_{\text{init}} \), that we used to generate normative posterior samples (see S-3). The difference between normative posterior samples and TR is that TR uses short chains (length parameterised by \( \lambda \)) that are seeded with \( h_{\text{init}} \). This section provides further details on the algorithmic implementation of the TS-Learner shown in Figure 5.3b which uses an alternative proposal distribution to adapt \( h_{\text{init}} \). Compared to the more mobile TR-Learner, TS retains rule elements when resampling alternatives, thus promoting more local or “surgical” edits to a hypothesis. As an example, consider an initial rule \( h_{\text{init}}: \exists (\lambda x_1 := (x_1, \text{red}, \text{color}), X) \). After translating this rule into a list-like representation, we get:

\[
[\exists, [\lambda x_1 :=, [x_1, \text{red}, \text{color}], X]].
\]  \hspace{1cm} (5.14)

We can extract all non-terminal and terminal productions from this rule recursively:

\[
\text{productions} = [S, A, B, D, = (x_i, G), \text{red}, \text{color}].
\]  \hspace{1cm} (5.15)

The TS-Learner now targets a random non-terminal in \( n \in \{ S, A, B, D \} \) present in \( h_{\text{init}} \). The selected non-terminal is replaced with a random proposal from the set of available productions \( \in \mathcal{G} \). For example, the TS-Learner could pick \( S \) and resample the random proposal \( \forall \). The TS-Learner then replaces the initial \( \exists \) with the newly generated proposal, and after transforming the list back into the initial rule representation we get:

\[
\forall (\lambda x_1 := (x_1, \text{red}, \text{color}), X).
\]  \hspace{1cm} (5.16)

In the above example, the TS-Learner preserves all parts of \( h_{\text{init}} \) except for the quantifier \( \exists \), while the TR-Learner would have resampled all components of the hypothesis following the quantifier, thus increasing the chance of arriving at a very different hypothesis (see also Figure 5.3b). The TS-Learner can also apply more complex edits to an initial hypothesis that involve sampling of additional
features and values not present in the initial hypothesis $h_{\text{init}}$. For example, if the TS-Learner targets the non-terminal $B$ from the set of available non-terminals, we might resample the expansion $C(B, B)$. After running $G$, the TS-Learner might then come up with a new proposal $h'$ of the form:

$$
\exists (\lambda x_1: \land (=(x_1, \text{small}, \text{size}), =(x_1, \text{upright}, \text{orientation})), X) \quad (5.17)
$$

where $C \rightarrow \land$ and the additional $B \rightarrow D \rightarrow =((x_i, G) \rightarrow (x_1, \text{size}) \rightarrow (x_1, \text{small}, \text{size})$.

Involving a conjunction during adaptation while maintaining the initial components of $h_{\text{init}}$ is another demonstration of the TS-Learner’s emphasis on resolving local uncertainty. A current limitation of our TS-approach is that it did not allow for changes in the number quantifiers resulting from a selection of $A$. In this case, the TS-Learner followed the same steps as the TR-Learner.

### 5.10.5 S-5: Our RULEX implementation

RULEX traditionally has nine parameters: a lower bound $\lambda$ (minimum number of steps a rule is maintained as long as its performance is above lax criterion $\Phi_L$), an upper bound $\mu$ (maximum number of steps a rule is maintained as long as its performance is above lax criterion $\Phi_L$), lax or performance criterion $\Phi_L$ (minimum proportion of correct classifications required by rule), imperfect acceptance criterion $\Phi_I$ (determines whether a simple rule surviving upper bound $\mu$ is accepted permanently), a conjunctive acceptance criterion $\Phi_C$ (determines whether a conjunctive rule surviving upper bound $\mu$ is accepted permanently), a branching probability $\beta$ (determines probability with which RULEX continues with either imperfect or conjunctive search following incorrect classification of a data point during exact search), storage probability $\sigma$, capacity parameter $\gamma$ ($\sigma$ and $\gamma$ are responsible for computing the probability of storing a current rule), and decision error $\epsilon$ (probability with which a rule makes the opposite response, thus making it probabilistic). Further details on parameter definitions can be found in (Navarro, 2005). Fitting the majority of these parameters is not feasible in the current context, since we are forced to use brute force sampling to estimate likelihoods of particular guesses and marginal likelihoods of particular generalizations. Thus we endeavored to somewhat simplify the original RULEX procedure and to use default or previously supported values wherever possible. We here set $\beta$ to 0.5,
meaning that our RULEX (referred to as RULEX*) has equal chance of moving to imperfect or conjunct-disjunct search if exact search fails. For both imperfect and conjunct-disjunct search, we set the lower bound to \( \lambda = 0 \), meaning that a rule generated by either imperfect or conjunct-disjunct search was disregarded immediately if its performance was below the lax criterion \( \Phi_L \) which was initially set to 1 (i.e., a maximum of one incorrect classifications of the 16 training scenes). Once a rule met the requirements of the lax criterion, RULEX* adopts the rule permanently (i.e., \( \mu = 1 \)) with perfect memory (i.e., \( \sigma = 1 \) and \( \gamma = 1 \)). This means that all accepted rules generated during search are stored. If at the imprecise search stage, no rule meets the current lax criterion, we gradually increased the number of accepted incorrect classifications (i.e., updating the lax criterion up to a maximum of 16 incorrect classifications), repeating either imperfect or conjunct-disjunct search with \( \beta = 0.5 \). For simplicity, we set the decision error \( \epsilon \) for each single rule guess to zero. We note that our resultant rule predictions are still probabilistic as we average 10,000 sampled RULEX* runs to obtain marginal likelihoods of guesses and generalizations.

5.10.6 S-6: Coding free responses

Natural language free responses were systematically translated into lambda abstraction representations by identifying rule elements and rearranging them into the lambda abstraction format. For a concrete example of a free-text response, consider the guess “A blue triangle makes Iota waves” provided by a random participant from Experiment 1. This rule can be translated into the lambda abstraction representation format by identifying the quantifier “A blue triangle” \( \rightarrow \exists \), checking the number of bound variables (one bound variable), identifying Booleans and equalities (no Booleans, one equality: “blue triangle” \( \rightarrow \) “triangle = blue”), and finally, features (“color”) and values (“blue”). The resulting lambda representation is then: \( \exists (\lambda x_1 : =(x_1, blue, color), X) \). Importantly, the order in which participants entered rules was preserved during translation, meaning that two semantically equivalent rules (“there is a small red”, “there is a red that is small”) resulted in two syntactically different structures to match participants’ free responses as closely as possible. In our grammar, we only included a single relational operator (“contact”). However, other potential relational operators were identified during encoding of free responses (e.g., “stacked” or “pointing at each other”). Rules including such operators were excluded from our analysis, which
primarily focused on non-relational operators. If participants provided multiple rules for a trial, we used their first rule in our analysis (this happened in less than 5% of trials). Finally, when translating quantifiers, we looked for words such as “one” or “two”. If participants wrote e.g., “just a single red”, this would have resulted in the same translated rule as “there needs to be one red”. All rules and translated responses have been uploaded to GitHub.

5.10.7 S-7: Conditional change probabilities

We expected the probability with which participants changed rule elements (quantifiers, Booleans, equalities, features, values) given a change to a specific element (e.g., quantifier change) to be smaller compared to the probability with which rule elements were changed between initial and revised posterior samples. As an example of how we calculated conditional change probabilities, consider a change in quantifiers from “exists” to “exactly” (e.g., “there is a red” → “there is exactly one red”). In this example, the root of the hypothesis tree, the quantifier, changed. However, references to the color feature and value red are retained therefore there are no conditional changes in feature and value, and no opportunity for additional changes in Booleans (since there are none) or quantifiers (since the only one was changed already). On the other hand, going from “there is a blue” to a revised guess of “there is exactly one large red” involves considerably more changes. Conditioning on the quantification change, we now see that additionally the color feature value has changed (from blue to red) and several additional elements have been added (+1 Boolean, +1 equality, +1 feature, and +1 value). Such additions are shown in the “Number of Added and Removed Elements” panel in Figure 5.5e and Figure 5.6e). Several rules included multiple elements of the same class (e.g., two Booleans), which is why the diagonal entries of Figure 5.5d and Figure 5.6d are not equal to 1.0. Diagonal entries reflect the average frequency with which changes to one element result in changes to other elements of the same class. As an example, consider the expression “one that is small and red lying on its left hand side” → “one that is small or red lying on its left hand side”. Here, on a minimal change construal, the conjunction “and” has changed to the disjunction “or”. The change probability of Booleans conditional on this change “and” → “or” is thus \( \frac{1}{2} \), since one of the two original Booleans has changed (\{and, and\} → \{or, and\}).
5.10.8 S-8: Additional modeling details

5.10.8.1 Modeling generalizations

We fit all models to predict participants’ generalizations using a single a decision noise parameter $\omega$. Specifically, the probability that participant $p$ selected scene $s$ during trial $t$ as rule following was modeled as the average over a model’s softmaxed predictions:

$$P(\text{choose}_{pt} = s) = \frac{1}{N} \sum_{t} \frac{e^{P(s)_{pt}/\omega}}{e^{P(s)_{pt}/\omega} + e^{(1-P(s)_{pt})/\omega}}. \quad (5.18)$$

$\omega \to 0$ indicates hard maximization over $P(s)$, while $\omega \to \infty$ indicates random responding. We fit $\omega$ to the full set of responses for all models using maximum likelihood estimation. For generating normative predictions, we again used the posterior predictive distributions over the posterior samples used earlier for analyses. To generate generalization predictions for the TR-Learner and TS-Learner blind to participants’ revised rules we ran 1000 independent chains starting from participants’ initial rules, each time keeping the final edit. For RULEX*, we computed the average over 10,000 softmaxed rule predictions using the same bag of rules used to predict $h_{\text{rev}}$. We further included a random response baseline that has no parameters and simply assigns a 0.5 probability to each generalization selection. Table 5.6 shows best fitting results of participants’ generalizations.

We note that while RULEX* fits generalizations better than TR and TS for both experiments we do not think its win is meaningful since it does so with a minimally small search parameter $\lambda = .5$ meaning it has almost zero chance of generating a better alternative than $h_{\text{init}}$. RULEX* here outperforms $h_{\text{init}}$ by being minimally adaptive, using $h_{\text{init}}$ to make predictions whenever it is consistent with all the learning data or making essentially random predictions otherwise. End to end TR-Learner, TS-Learner and RULEX* accounts all outperform the predictions of independent Posterior Sampled rules or the MAP in predicting the revised generalizations, but do not come close to matching the predictions of the participants’ own reported $h_{\text{rev}}$.

7To generate predictions for the posterior sample, the softmax was applied only once using the average prediction across the 10,000 rules in the sample.
Table 5.6: Best fitting model performances for participants’ revised generalizations (Gen.).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Model</th>
<th>Mean BIC</th>
<th>b</th>
<th>λ</th>
<th>ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. 1</td>
<td>Random Baseline</td>
<td>11.090</td>
<td>-</td>
<td>-</td>
<td>∞</td>
</tr>
<tr>
<td></td>
<td>h_{init.}</td>
<td>8.810</td>
<td>-</td>
<td>-</td>
<td>0.861</td>
</tr>
<tr>
<td></td>
<td>h_{rev.}</td>
<td>6.777</td>
<td>-</td>
<td>-</td>
<td>0.576</td>
</tr>
<tr>
<td></td>
<td>Posterior Sample (PS)</td>
<td>9.144</td>
<td>7.0</td>
<td>-</td>
<td>0.820</td>
</tr>
<tr>
<td></td>
<td>Maximum a posteriori (MAP)</td>
<td>9.355</td>
<td>9.0</td>
<td>-</td>
<td>0.999</td>
</tr>
<tr>
<td></td>
<td>Win–stay, lose–sample (WSLS)</td>
<td>9.012</td>
<td>7.0</td>
<td>-</td>
<td>1.189</td>
</tr>
<tr>
<td></td>
<td>WSLS with Tree Regrowth (TR)</td>
<td>8.407</td>
<td>5.0</td>
<td>3.0</td>
<td>0.636</td>
</tr>
<tr>
<td></td>
<td>WSLS with Tree Surgery (TS)</td>
<td>8.350</td>
<td>5.0</td>
<td>3.0</td>
<td>0.622</td>
</tr>
<tr>
<td></td>
<td>RULEX*</td>
<td>8.272</td>
<td>-</td>
<td>0.5</td>
<td>0.388</td>
</tr>
<tr>
<td>Exp. 2</td>
<td>Random Baseline</td>
<td>11.090</td>
<td>-</td>
<td>-</td>
<td>∞</td>
</tr>
<tr>
<td></td>
<td>h_{init.}</td>
<td>8.729</td>
<td>-</td>
<td>-</td>
<td>0.840</td>
</tr>
<tr>
<td></td>
<td>h_{rev.}</td>
<td>7.862</td>
<td>-</td>
<td>-</td>
<td>0.696</td>
</tr>
<tr>
<td></td>
<td>Posterior Sample (PS)</td>
<td>9.469</td>
<td>5.0</td>
<td>-</td>
<td>0.900</td>
</tr>
<tr>
<td></td>
<td>Maximum a posteriori (MAP)</td>
<td>9.541</td>
<td>7.0</td>
<td>-</td>
<td>1.051</td>
</tr>
<tr>
<td></td>
<td>Win–stay, lose–sample (WSLS)</td>
<td>9.120</td>
<td>5.0</td>
<td>-</td>
<td>1.236</td>
</tr>
<tr>
<td></td>
<td>WSLS with Tree Regrowth (TR)</td>
<td>8.437</td>
<td>7.0</td>
<td>1.0</td>
<td>0.653</td>
</tr>
<tr>
<td></td>
<td>WSLS with Tree Surgery (TS)</td>
<td>8.413</td>
<td>9.0</td>
<td>0.75</td>
<td>0.654</td>
</tr>
<tr>
<td></td>
<td>RULEX*</td>
<td>8.268</td>
<td>-</td>
<td>0.5</td>
<td>0.395</td>
</tr>
</tbody>
</table>

5.10.8.2 Supplementary modeling results

Tables 5.7-5.12 include modeling results for each parameter combination of $\lambda$ and $b$ with $\epsilon = \frac{1}{10,000}$ for trials during which a model failed to identify $h_{rev.}$. $N_{\text{Found}_{h_{rev.}}}$ corresponds to the number of trials during which $h_{rev.}$ was identified by a model. For example $N_{\text{Found}_{h_{rev.}}}$ = 134 means that during 134 out of the total 248 trials in Experiment 1, a posterior sample of size 10,000 contained at least one rule semantically identical to $h_{rev.}$. “Gen. Accuracy” refers to the accuracy of each model’s generalization selections and “Freq. True Rule” to the probability of the model guessing the ground truth correctly. We also provide best fitting modeling results for $h_{rev.}$ using $\epsilon = \frac{1}{100,000}$ to demonstrate that the ranking of our models was not a function of $\epsilon$. These are shown in Table 5.13. As expected, fits for each model are worse as trials in which $h_{rev.}$ was not found have now a log probability of $\log(\frac{1}{100,000})$ instead of $\log(\frac{1}{10,000})$. In our main results (Table 5.4), we use $\epsilon = \frac{1}{10,000}$ as 10,000 corresponded to the total number of rules simulated by each model during our analysis of $h_{rev.}$.
Table 5.7: Experiment 1. Normative modeling results for different values of $b$
(248 Trials).

<table>
<thead>
<tr>
<th>$b$</th>
<th>Model</th>
<th>Mean $BIC_{\text{rev.}}$</th>
<th>$N_{\text{found}}$</th>
<th>$MeanBIC_{\text{Gen.}}$</th>
<th>Gen. Accuracy</th>
<th>Freq.</th>
<th>True Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Normative</td>
<td>11.307</td>
<td>134</td>
<td>10.941 ($\omega = 2.331$)</td>
<td>0.626</td>
<td>0.180</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MAP</td>
<td>12.724</td>
<td>77</td>
<td>10.361 ($\omega = 1.568$)</td>
<td>0.755</td>
<td>0.403</td>
<td></td>
</tr>
<tr>
<td></td>
<td>WSLS</td>
<td>7.158</td>
<td>181</td>
<td>10.415 ($\omega = 2.257$)</td>
<td>0.743</td>
<td>0.400</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Normative</td>
<td>10.567</td>
<td>121</td>
<td>10.000 ($\omega = 1.097$)</td>
<td>0.772</td>
<td>0.423</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MAP</td>
<td>12.129</td>
<td>85</td>
<td>10.001 ($\omega = 1.280$)</td>
<td>0.810</td>
<td>0.488</td>
<td></td>
</tr>
<tr>
<td></td>
<td>WSLS</td>
<td>8.809</td>
<td>170</td>
<td>9.634 ($\omega = 1.502$)</td>
<td>0.821</td>
<td>0.525</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Normative</td>
<td>9.817</td>
<td>127</td>
<td>9.434 ($\omega = 0.896$)</td>
<td>0.825</td>
<td>0.545</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MAP</td>
<td>11.238</td>
<td>97</td>
<td>9.603 ($\omega = 1.086$)</td>
<td>0.842</td>
<td>0.589</td>
<td></td>
</tr>
<tr>
<td></td>
<td>WSLS</td>
<td>10.058</td>
<td>171</td>
<td>9.226 ($\omega = 1.287$)</td>
<td>0.858</td>
<td>0.616</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Normative</td>
<td>10.037</td>
<td>123</td>
<td>9.144 ($\omega = 0.820$)</td>
<td>0.838</td>
<td>0.560</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MAP</td>
<td>11.164</td>
<td>98</td>
<td>9.363 ($\omega = 1.001$)</td>
<td>0.862</td>
<td>0.613</td>
<td></td>
</tr>
<tr>
<td></td>
<td>WSLS</td>
<td>11.843</td>
<td>167</td>
<td>9.012 ($\omega = 1.189$)</td>
<td>0.866</td>
<td>0.627</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Normative</td>
<td>10.244</td>
<td>120</td>
<td>9.196 ($\omega = 0.837$)</td>
<td>0.825</td>
<td>0.522</td>
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<tr>
<td></td>
<td>MAP</td>
<td>11.535</td>
<td>93</td>
<td>9.355 ($\omega = 0.999$)</td>
<td>0.841</td>
<td>0.560</td>
<td></td>
</tr>
<tr>
<td></td>
<td>WSLS</td>
<td>13.326</td>
<td>167</td>
<td>9.036 ($\omega = 1.202$)</td>
<td>0.858</td>
<td>0.601</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.8: Experiment 1. TR- and TS results for different values of $\lambda$ and $b$ (248 Trials).

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$b$</th>
<th>Model</th>
<th>Mean $BIC_{\text{rev}}$</th>
<th>$N_{\text{Found}}$</th>
<th>Mean $BIC_{\text{Gen}}$</th>
<th>Gen. Accuracy</th>
<th>Freq.</th>
<th>True Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1</td>
<td>TR</td>
<td>6.920</td>
<td>192</td>
<td>8.481 ($\omega = 0.651$)</td>
<td>0.710</td>
<td>0.326</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>TS</td>
<td>6.445</td>
<td>191</td>
<td>8.437 ($\omega = 0.646$)</td>
<td>0.711</td>
<td>0.329</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>TR</td>
<td>6.955</td>
<td>190</td>
<td>8.507 ($\omega = 0.685$)</td>
<td>0.714</td>
<td>0.327</td>
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<td>9</td>
<td>TR</td>
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<td>8.527 ($\omega = 0.635$)</td>
<td>0.726</td>
<td>0.345</td>
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<tr>
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Table 5.9: Experiment 1. RULEX* results for different values of $\lambda$ (248 Trials).

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<th>$\lambda$</th>
<th>Model</th>
<th>Mean $BIC_{hrev}$</th>
<th>$N_{\text{Found}_{hrev}}$</th>
<th>Mean $BIC_{\text{Gen}}$</th>
<th>Gen. Accuracy</th>
<th>Freq. True Rule</th>
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<tbody>
<tr>
<td>0.5</td>
<td>RULEX*</td>
<td>9.964</td>
<td>170</td>
<td>8.272 ($\omega = 0.388$)</td>
<td>0.676</td>
<td>0.323</td>
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<td>0.75</td>
<td>RULEX*</td>
<td>9.870</td>
<td>172</td>
<td>8.341 ($\omega = 0.394$)</td>
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<td>0.323</td>
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<td>172</td>
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Table 5.10: Experiment 2. Normative results for different values of $b$ (127 Trials).

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<th>Model</th>
<th>Mean $BIC_{hrev}$</th>
<th>$N_{\text{Found}_{hrev}}$</th>
<th>Mean $BIC_{\text{Gen}}$</th>
<th>Gen. Accuracy</th>
<th>Freq. True Rule</th>
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<td>Normative</td>
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<td>10.866 ($\omega = 1.776$)</td>
<td>0.634</td>
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<tr>
<td>1</td>
<td>MAP</td>
<td>13.527</td>
<td>34</td>
<td>10.349 ($\omega = 1.523$)</td>
<td>0.766</td>
<td>0.409</td>
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<tr>
<td></td>
<td>WSLS</td>
<td>6.573</td>
<td>103</td>
<td>10.248 ($\omega = 1.932$)</td>
<td>0.744</td>
<td>0.367</td>
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<td>Normative</td>
<td>12.027</td>
<td>51</td>
<td>9.755 ($\omega = 0.955$)</td>
<td>0.796</td>
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<tr>
<td>3</td>
<td>MAP</td>
<td>12.947</td>
<td>38</td>
<td>9.687 ($\omega = 1.107$)</td>
<td>0.843</td>
<td>0.559</td>
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<td>WSLS</td>
<td>10.926</td>
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<td>9.367 ($\omega = 1.329$)</td>
<td>0.832</td>
<td>0.532</td>
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<tr>
<td>Normative</td>
<td>11.403</td>
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<td>9.469 ($\omega = 0.900$)</td>
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<tr>
<td>5</td>
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<td>12.657</td>
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<td>9.314 ($\omega = 1.332$)</td>
<td>0.865</td>
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### Table 5.11: Experiment 2. TR- and TS results for different values of $\lambda$ and $b$

(127 Trials).

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<th>$N_{Found_{rev}}$</th>
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<th>Gen. Accuracy</th>
<th>Freq.</th>
<th>True Rule</th>
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<td>4.615</td>
<td>106 8.519 ($\omega = 0.650$)</td>
<td>0.713</td>
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<tr>
<td>1</td>
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<td>109 8.481 ($\omega = 0.647$)</td>
<td>0.715</td>
<td>0.288</td>
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<td>3</td>
<td>TR</td>
<td>4.711</td>
<td>105 8.490 ($\omega = 0.667$)</td>
<td>0.716</td>
<td>0.287</td>
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<td>TS</td>
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<td>0.718</td>
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<td>0.5</td>
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<td>107 8.471 ($\omega = 0.670$)</td>
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<td>0.719</td>
<td>0.289</td>
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| 0.75      | TR  | 4.495 | 107 8.510 ($\omega = 0.641$) | 0.712             | 0.290            |
| 0.75      | TS  | 4.108 | 109 8.463 ($\omega = 0.636$) | 0.715             | 0.288            |
| 3         | TR  | 4.707 | 106 8.472 ($\omega = 0.657$) | 0.715             | 0.287            |
| 3         | TS  | 4.268 | 106 8.437 ($\omega = 0.651$) | 0.718             | 0.290            |
| 1         | TR  | 4.657 | 105 8.470 ($\omega = 0.663$) | 0.716             | 0.287            |
| 1         | TS  | 4.250 | 105 8.442 ($\omega = 0.657$) | 0.719             | 0.290            |
| 7         | TR  | 4.669 | 104 8.453 ($\omega = 0.662$) | 0.720             | 0.287            |
| 7         | TS  | 4.245 | 107 8.443 ($\omega = 0.659$) | 0.719             | 0.290            |
| 9         | TR  | 4.621 | 106 8.444 ($\omega = 0.661$) | 0.717             | 0.288            |
| 9         | TS  | 4.255 | 107 8.413 ($\omega = 0.654$) | 0.720             | 0.291            |

| 1         | TR  | 4.473 | 110 8.506 ($\omega = 0.633$) | 0.711             | 0.287            |
| 1         | TS  | 4.065 | 110 8.461 ($\omega = 0.628$) | 0.714             | 0.289            |
| 3         | TR  | 4.656 | 108 8.475 ($\omega = 0.652$) | 0.715             | 0.288            |
| 3         | TS  | 4.190 | 108 8.447 ($\omega = 0.647$) | 0.718             | 0.290            |
| 5         | TR  | 4.635 | 107 8.449 ($\omega = 0.653$) | 0.716             | 0.288            |
| 5         | TS  | 4.199 | 107 8.447 ($\omega = 0.652$) | 0.719             | 0.291            |
| 7         | TR  | 4.679 | 105 8.437 ($\omega = 0.653$) | 0.717             | 0.288            |
| 7         | TS  | 4.192 | 106 8.442 ($\omega = 0.653$) | 0.719             | 0.291            |
| 9         | TR  | 4.657 | 106 8.444 ($\omega = 0.655$) | 0.717             | 0.288            |
| 9         | TS  | 4.263 | 105 8.420 ($\omega = 0.649$) | 0.719             | 0.291            |

| 1         | TR  | 4.656 | 109 8.630 ($\omega = 0.620$) | 0.707             | 0.289            |
| 1         | TS  | 4.155 | 110 8.561 ($\omega = 0.609$) | 0.711             | 0.292            |
| 3         | TR  | 4.615 | 109 8.560 ($\omega = 0.641$) | 0.713             | 0.291            |
| 3         | TS  | 4.297 | 108 8.504 ($\omega = 0.627$) | 0.717             | 0.296            |
| 5         | TR  | 4.650 | 108 8.518 ($\omega = 0.640$) | 0.715             | 0.291            |
| 5         | TS  | 4.316 | 106 8.489 ($\omega = 0.631$) | 0.718             | 0.297            |
| 7         | TR  | 4.740 | 106 8.503 ($\omega = 0.639$) | 0.716             | 0.292            |
| 7         | TS  | 4.232 | 107 8.458 ($\omega = 0.628$) | 0.719             | 0.297            |
| 9         | TR  | 4.687 | 107 8.503 ($\omega = 0.643$) | 0.716             | 0.291            |
| 9         | TS  | 4.243 | 107 8.434 ($\omega = 0.628$) | 0.719             | 0.297            |

| 1         | TR  | 5.209 | 110 8.979 ($\omega = 0.669$) | 0.704             | 0.292            |
| 1         | TS  | 4.781 | 110 8.911 ($\omega = 0.645$) | 0.707             | 0.297            |
| 3         | TR  | 5.495 | 111 8.914 ($\omega = 0.693$) | 0.714             | 0.297            |
| 3         | TS  | 4.963 | 109 8.802 ($\omega = 0.665$) | 0.719             | 0.309            |
| 10        | TR  | 5.479 | 109 8.857 ($\omega = 0.689$) | 0.717             | 0.230            |
| 10        | TS  | 4.995 | 107 8.746 ($\omega = 0.663$) | 0.721             | 0.311            |
| 7         | TR  | 6.130 | 107 8.815 ($\omega = 0.684$) | 0.718             | 0.300            |
| 7         | TS  | 4.993 | 108 8.705 ($\omega = 0.656$) | 0.722             | 0.312            |
| 9         | TR  | 6.178 | 107 8.790 ($\omega = 0.681$) | 0.717             | 0.230            |
| 9         | TS  | 5.063 | 107 8.689 ($\omega = 0.658$) | 0.723             | 0.311            |
Table 5.12: Experiment 2. RULEX* results for different values of $\lambda$ (127 Trials).

<table>
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<th>$\lambda$</th>
<th>Model</th>
<th>Mean $BIC_{h_{rev}}$</th>
<th>$N_{found_{h_{rev}}}$</th>
<th>Mean $BIC_{Gen}$</th>
<th>Gen. Accuracy</th>
<th>Freq. True Rule</th>
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<tr>
<td>0.5</td>
<td>RULEX*</td>
<td>9.964</td>
<td>86</td>
<td>8.268 ($\omega = 0.395$)</td>
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<td>8.324 ($\omega = 0.401$)</td>
<td>0.671</td>
<td>0.285</td>
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<td>8.404 ($\omega = 0.410$)</td>
<td>0.673</td>
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<tr>
<td>3</td>
<td>RULEX*</td>
<td>9.220</td>
<td>93</td>
<td>8.958 ($\omega = 0.520$)</td>
<td>0.679</td>
<td>0.286</td>
</tr>
<tr>
<td>10</td>
<td>RULEX*</td>
<td>9.265</td>
<td>85</td>
<td>9.298 ($\omega = 0.665$)</td>
<td>0.691</td>
<td>0.289</td>
</tr>
</tbody>
</table>

Table 5.13: Model performances for participants’ revised rule guesses ($h_{rev}$) with $\epsilon = \frac{1}{100,000}$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Model</th>
<th>Mean $BIC_{h_{rev}}$</th>
<th>$N_{Best_{h_{rev}}}$</th>
<th>$b$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h_{init}$</td>
<td>11.234</td>
<td>21.17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Exp. 1</td>
<td>Posterior Sample (PS)</td>
<td>12.064</td>
<td>5.00</td>
<td>5.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Maximum a posteriori (MAP)</td>
<td>13.949</td>
<td>4.17</td>
<td>7.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Win–stay, lose–sample (WSLS)</td>
<td>8.403</td>
<td>7.17</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>WSLS with Tree Regrowth (TR)</td>
<td>7.185</td>
<td>13.17</td>
<td>3.0</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td><strong>WSLS with Tree Surgery (TS)</strong></td>
<td><strong>6.753</strong></td>
<td><strong>15.17</strong></td>
<td><strong>5.0</strong></td>
<td><strong>10.0</strong></td>
</tr>
<tr>
<td></td>
<td>RULEX*</td>
<td>10.724</td>
<td>3.17</td>
<td>-</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>$h_{init}$</td>
<td>7.071</td>
<td>15.37</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Exp. 2</td>
<td>Posterior sample (PS)</td>
<td>13.977</td>
<td>1.00</td>
<td>5.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Maximum a posteriori (MAP)</td>
<td>15.449</td>
<td>1.17</td>
<td>7.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Win–stay, lose–sample (WSLS)</td>
<td>7.443</td>
<td>0.37</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>WSLS with Tree Regrowth (TR)</td>
<td>5.089</td>
<td>4.37</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td><strong>WSLS with Tree Surgery (TS)</strong></td>
<td><strong>4.681</strong></td>
<td><strong>11.37</strong></td>
<td><strong>1.0</strong></td>
<td><strong>1.0</strong></td>
</tr>
<tr>
<td></td>
<td>RULEX*</td>
<td>10.453</td>
<td>2.37</td>
<td>-</td>
<td>3.0</td>
</tr>
</tbody>
</table>
Chapter 6

General Discussion

6.1 Summary

This thesis started with a simple learning paradigm examining how people reason about environmental quantities with dependent information sources (Chapter 2). Our work was motivated by three observations: (1) misinformation and political polarisation are important topics (e.g., Lazer et al., 2018) that have been investigated by simulation-based studies in which agents are assumed to be rational Bayesian learners (e.g., Madsen et al., 2018), (2) Bayesian social learning models have become increasingly relevant in empirical settings, with several studies showing good fits between human inferences and Bayesian model predictions (e.g., Whalen et al., 2018), and (3) previous empirical settings often involve controlled multi-player environments with carefully calibrated stimuli. Results from Chapter 2 supported previous findings, suggesting that a Bayesian account can provide qualitatively accurate behavioural predictions about people’s sensitivity to social source dependence. However, given the simplicity of our initial learning setup, which—similar to previous studies (e.g., Hawthorne-Madell & Goodman, 2019; Whalen et al., 2018)—included simulated players, known dependencies, as well as a single-shot belief update, we suspected that the correspondence between model predictions and human inferences might not accurately capture how people reason about dependencies in more realistic social learning settings, such as social networks with multiple real players. Therefore, we decided to gradually increase the difficulty of our learning task across Chapters 3–4 and included multiple belief updates, known and unknown dependencies, as well as multi-player experiments
in which three participants engaged in real time. As expected, changing the difficulty of the learning setup resulted in a weaker alignment between Bayesian and human inferences, suggesting that the majority of people were better characterised by a structure insensitive, naïve (‘Level-1’) process account (Chapter 3–4). Additionally, results from Chapter 4 revealed that human learners were more robust to the noisy social evidence coming from real social network peers, while the performance of our computational level ‘Level-2’ account dropped. Taken together, the results from Chapters 2–4 challenge the idea that social inference can be simply modelled through the lens of the rationality principle (Gershman et al., 2015; Oaksford et al., 2007) or a naïve utility calculus (Jara-Ettinger et al., 2015), and thus suggest that future accounts of social inference—both simulation-based and empirically validated—might have to consider inferential naïveté as an important feature, and not just a limitation of human social learning dynamics. We refer to this empirical lesson as the ‘blessing of naïvety’: Human social learners with limited computational resources might navigate complex learning situations with heterogeneous social actors through a simplified (i.e. naïve) model of other actors’ inference behaviours.

We motivated Chapter 5 of this thesis by the observation that explanatory theories form a key component of human intelligence. From explaining what happened to our coffee machines to describing the colour and shape of a banana, people use symbolically structured, language-like expression (Fodor, 1975; Lake et al., 2015). Considering this observation, we wanted to extend our rational analysis of social learning to a more challenging inference setup involving symbolic search for good explanations of observed data. Inspired by recent work on inductive concept inference (Goodman et al., 2008; Piantadosi & Jacobs, 2016; Piantadosi et al., 2012, 2016; Yang & Piantadosi, 2022) and process accounts of causal theory change (Bramley et al., 2017), we designed a new social learning task adapted from (Bramley et al., 2018) in which up to two human players provided written explanations of their best rule guesses across two learning phases. We found that participants performance was close to the performance of a normative, computational-level account. However, we also observed that participants written explanations were more complex than normative responses and further anchored to idiosyncratic elements of initial explanations. We argued that this could be due to participants considering a short sequence of local edits to their initial hypotheses in the light of new evidence. Finally, we observed that another
person’s learning data might be difficult to combine with a short sequence of local incremental edits, as reflected by participants’ decreased performance in the two-player version of our task. In summary, the results from Chapter 5 suggest that the local incremental search hypothesis—also referred to as ‘Neurath’s ship’ metaphor (Bramley et al., 2017)—might provide a good account of how people search symbolic (i.e. compositional) theory spaces.

### 6.2 Limitations

An important limitation of our work is that the ‘Level-2’ Bayesian learner introduced in Chapters 3–4 simplified the inference problem by assuming that all learners use the same inference mechanism to reason about others. For example, it was assumed that each learner is fully reliable, meaning that a learner’s responses always corresponded to the judgement with the highest probability given their belief. Real learners, however, were much more variable, as shown by inter-individual differences in response patterns (see Fig. 4.14). This suggests that different learners might alternate between strategies, including maximising over probabilities (Hawthorne-Madell & Goodman, 2019), probability matching (Shanks et al., 2002), and potentially even more sophisticated approaches considering aspects such as the trust and expertise of a source (Hahn et al., 2009; Harris et al., 2016). Failing to account for inter-individual differences in response strategies is likely one reason for the weak performance of the ‘Level-2’ account across the last two experiments of Chapter 4. Unfortunately, including inter-individual differences during inference is hard from a computational perspective, as it requires a learner to have a theory of mind model of another person’s inference behaviours. Such theory of mind models can quickly result in a recursive inference problem, requiring a learner to anticipate that another person may also make inferences about the learner themselves, a problem known as ‘level-k thinking’ or ‘level-k reasoning’ (see e.g., Camerer, Ho, & Chong, 2004; Hula, Vilares, Lohrenz, Dayan, & Montague, 2018). In the present thesis, we relaxed the level-k component w.r.t. to inter-individual differences in reasoning strategies, and instead focused on sensitivity to dependencies across multiple time steps, which required recursive evaluation w.r.t. to time. A natural extension of Chapter 4 would thus allow for recursion across both time and individual minds, that is, allowing a learner to not only account for dependencies between information sources, but also account for differences in the
reasoning strategies used by other learners. Moving forward, our current Level-2 learner could thus benefit from using e.g., a Poisson distribution to model the recursion depth of other learners’ reasoning strategies (Camerer et al., 2004; Oey, Schachner, & Vul, 2022), thereby becoming more flexible w.r.t. inter-individual differences in reasoning strategies. We expect that such a ‘Level-3’ approach could improve the accuracy and fit of the present Level-2 learner.

A limitation of our work on concept inference refers to the observation that the hypothesis adaptation models presented in Chapter 5—in their current form—only work in our learning task (or a very similar modification). Specifically, our models required explicit provision of a hand-crafted domain specific language (see Table 5.1). While this narrow approach was suitable considering the nature of our investigation (i.e., studying incremental search in a single domain), it is important that future work considers more general approaches to search which account for the fact that human learners can flexibility switch between a variety of concept learning domains (e.g., Lake et al., 2017; Piantadosi & Jacobs, 2016; Zhao, Bramley, & Lucas, 2022; Zhao, Lucas, & Bramley, 2022). Here, it might be helpful to seek inspiration from recent work on neurosymbolic program synthesis (Ellis et al., 2021; Tang & Ellis, 2022) as well as discrete (Van Den Oord, Vinyals, et al., 2017) and causal (Schölkopf et al., 2021) representation learning, which could inform more flexible learning architectures capturing human cross-domain concept inference. An extension of the present adaptation accounts could thus focus on modelling human hypothesis search with a more flexible domain specific language (e.g., Ellis et al., 2021; Zhao, Bramley, & Lucas, 2022), and potentially further accommodate for lower level perceptual components that give rise to the kind or representations we studied in our learning tasks (see e.g., Eslami et al., 2016; Mao, Gan, Kohli, Tenenbaum, & Wu, 2019; R. Wang, Mao, Gershman, & Wu, 2020, for examples).

6.3 Implications

Results from Chapters 2–4 suggest that while a substantial minority of participants exhibit hallmarks of rational social inference, the dominant inference pattern is more naïve and sticky, seeming to lack accommodation of communicative dependencies between peers. This finding challenges the assumptions made in related agent-based models of information cascades and echo chamber formation.
Specifically, recent agent-based models typically model belief formation in idealised, homogeneous populations of equally rational agents (e.g. Bikhchandani et al., 1992; Fränken & Pilditch, 2021; Lewandowsky et al., 2019; Madsen & Pilditch, 2018; Pilditch, 2017). A common argument from this line of work is as follows: ‘If echo chambers can emerge in such idealised conditions [populations of equally rational agents], it is reasonable to assume they will also emerge with less ideal agents, and perhaps more efficiently and at a faster rate’ (Madsen et al., 2018, p. 2). Similarly, it has been stated that: [...] ‘networks of rational agents are intrinsically susceptible to high levels of localized clustering (i.e. echo-chambers)’ [...] (Pilditch, 2017, p. 948). Results from our work suggest that real populations of social reasoners are far from homogeneous. Instead, our results suggest the presence of a mixed population of both rational Level-2 reasoners and naïve Level-1 reasoners. Additionally, the robustness of naïve inference in Experiments 2–3 of Chapter 4 suggests that in more naturalistic settings, where fully rational inference can be expensive, some degree of naïvety might be adaptive after all. As such, extensions of our work might focus on incorporating the present empirically informed population heterogeneity into previous agent-based models such as Madsen et al. (2018). Here, an important prediction from our work is that with limited computational resources, a heterogeneous population of both naïve and rational learners might be more robust to the formation of echo chambers and inaccurate inferences than the idealised homogeneous populations studied in previous simulations (see also Fränken & Pilditch, 2021). To validate this prediction, further work could simulate the behaviours of different proportions of Level-1 and Level-2 learners and test under which conditions population heterogeneity and Level-1 inference can be adaptive.

In Chapter 5, we found evidence for local adaptive search in a challenging concept learning task. This finding contributes another piece to the puzzle of how human learners face intractable inference problems. Specifically, it suggests that following a local search strategy with incremental revisions allows people to efficiently update beliefs as new information becomes available, thereby gradually improving upon their predictions about the world around them. As shown by Experiment 1 in Chapter 5, this approach can be very fruitful, as it allows someone to reuse elements that worked well in the past while simultaneously responding to changes in the environment. Thus, we argue that our local adaptive account shares important features with interleaved learning approaches in the domain of category learning. Similar to local adaptation and incremental revision, interleaved learning supports
reuse of previously acquired skills to supercharge learning of new concepts (see e.g. Carvalho & Goldstone, 2015). Moreover, our account is closely related to previous work on relational reasoning and analogy (e.g. Doumas, Hummel, & Sandhofer, 2008) as well as adaptor grammars (e.g. Zhao, Bramley, & Lucas, 2022) where concept inference leverages ‘chunking’ of previously acquired concepts during the acquisition of new ones. One potential shortcoming of our local adaptive account is that radical changes to the environment—such as the learning data gathered by someone else in the second experiment of Chapter 5—might be hard to combine with a learner’s current hypothesis. In the context of social learning, which often involves aggregation of evidence collected by multiple (potentially different) learners, our model thus predicts that the success of an individual learner depends on how close others’ beliefs are to the learner’s own assumptions about the environment. Ideas that falls outside the orbit of a learner’s current beliefs might be hard to interpret. At the same time, if other people’s beliefs are too close to a learner’s current ideas, this might not allow for much improvement. An important avenue for future work could thus focus on comparing the performance of local adaptive search in a learning environment where learners are initially matched with closely related peers and then gradually encounter learning data from peers that are increasingly different from a learner’s own beliefs and hypotheses (see §, for a related simulation-based example). Here, it will be interesting to test whether people with shorter search-length parameters (Section 5.8.) are more prone to polarisation than their peers.

6.4 Conclusions

In summary, this thesis examined how people reason about quantities and concepts across seven behavioural experiments. Additionally, we developed a variety of process accounts such as the naïve (‘Level-1’) inference account presented in Chapters 3–4 and our local incremental search account presented in Chapter 5. Chapter 2 showed that people are sensitive to dependencies in social learning provided that people are explicitly told about dependencies (i.e. the structure of their social network). In Chapter 3, we challenged this idea and increased the difficulty of our social learning environment by incorporating multiple belief updates as well as uncertainty about dependencies between sources of information.
Here, we find that the majority of people failed to align with a Bayesian account and instead followed a more naïve approach were social communications are taken at their face value irrespective of potential dependencies underlying these communications. Chapter 4 extended our analyses of dependencies by including multi-player experiments in which three participants engaged in real time. The results from this work further support our naïve learning account, which was the best predictor of participants’ judgements across experiments. Importantly, we also showed that assuming some degree of naivety might be adaptive at a group level, as demonstrated by the finding that people’s inferences were more robust to distortions in the social evidence (e.g., erratic response patterns) compared to a Bayesian learner which assumed the same degree of rationality across participants.

Chapter 5 focused on individual and social learning in a symbolic hypothesis space, examining how people deal with the inherent complexity of concept inference considering both active learning data gathered by themselves as well as learning data gathered by another player. Results from this chapter show that a process model focusing on local incremental search might provide a reasonable account for online human concept inference. Moreover, we find that learning data gathered by someone else might be less informative for local search compared to the learning data gathered by another learner. Taken together, we show that (1) human inferential naivety might be an important feature of adaptive social learning when facing communications from multiple, potentially dependent heterogeneous actors; and (2) that people might deal with the inherent complexity of concept inference via a sequence of local incremental tweaks as new data arrives.
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