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Three Essays on Social Institutions and Individual Behavior

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Doctor of Philosophy

The University of Edinburgh

2022
To my parents,

Hongyu Chen and Qiaoyun Lyu
This thesis studies the interactions between individual behavior and social institutions. The first chapter proposes a model of equilibrium switching in games with multiple strict Nash equilibria such as the evolution of social conventions. In our model, players are forward-looking but use level-k reasoning about their peers’ choices. We find that equilibrium switching is deterministic and gradual, with higher $k$ players switching first. In some cases, the final equilibrium is reached only after several other equilibria are visited, where each step is Pareto improving. We completely characterize the switching paths for all games with finite strict Nash equilibria, and a large class of level-$k$ populations. The model provides a unified explanation for puzzling equilibrium switching results reported in recent experimental studies, including the direction of transition, asynchronous participation, and alternations between strict Nash equilibria.

The second chapter empirically tests a theory of equilibrium transition among strict Nash equilibria, the sampling best response dynamic that predicts equilibrium transitions under inexact (inaccurate but unbiased) information of opponents’ behaviors. We design a quasi-continuous-time experiment in which a group of subjects plays a coordination game recurrently under either more or less accurate information. We observe that more accurate information facilitates efficiency-improving transitions among strict Nash equilibria than less accurate information, which is in contrast with the evolutionary theory but supports the models of strategic teaching. More accurate information about opponents’ behaviors induces more subjects to engage in persistent strategic deviations from inefficient Nash equilibria that can induce more subjects to deviate in the future, resulting in efficiency-improving equilibrium transitions. When information is less accurate, subjects’ choices are less responsive to changes in the information received. The slow response to the information either blocks or delays efficiency-improving equilibrium transitions.

The third chapter investigates the optimal fertility strategy and its impact on the fertility rate and parental investment under patriarchal institutions. We show that the optimal fertility strategy is a one-stage look-ahead strategy and represents a son-prefering stopping rule if there is a patriarchal institution in favor of sons over daughters. We further characterize the resulting demographic distribution of the possible sex combinations of children in a family. It suggests that, under the
optimal fertility strategy, females are disproportionately born into larger families, and therefore receive lower average investment than males. In addition, both under and over-reproduction exist in a patriarchal society, but the average fertility rate can be lower than in a gender-neutral society.
The first chapter aims to understand social change emerging from spontaneous and unorganized individual behavior. Social change often involves a large number of social members collectively switching away from an established institution, norm, or convention. A typical feature of social change is asynchronous timing of participation: some individuals participate in a social movement at an early stage, some at a later stage, creating a pattern of leader-follower. This leader-follower pattern is puzzling because individuals often find it optimal to participate only after enough others have done so.

The first chapter proposes a dynamic cognitive hierarchy (DCH) model to explain asynchronous participation in collective actions. The DCH model predicts that individuals have idiosyncratic thresholds for participation in collective action. Importantly, an individual’s threshold is inversely related to her level of strategic sophistication. By strategic sophistication, we mean an individual can engage in the following iterative thinking: I think that you think that I will do certain things under certain conditions. An individual of a higher level can do such iterative thinking in more steps. Therefore, more sophisticated individuals can foresee their participation at the early stage of a social movement will trigger less sophisticated individuals to participate in the future. With a correct societal composition of individuals with different levels of sophistication, social change will succeed. The first chapter identifies all such correct compositions and names them domino distribution.

The second chapter studies another factor, the information about social members’ behavior, that could affect social change. Such information is critical to social change because individuals’ decision on participation often depends on how many social members have participated in a social movement. An economics theory, the sampling best response dynamic, predicts that social change is more likely to happen if individuals have inexact information. The intuition is that individuals may form false beliefs that the majority of society has deviated from the established state even if only a small proportion has done so, and hence voluntarily deviate themselves. In comparison, in the case of exact information, individuals will abstain from joining a movement if initially, few have done so.
The second chapter tests this theory in a lab experiment. We recruit human subjects and place them in a simulated society with a prescribed inferior social state. Subjects decide whether to join a movement to transit away from the initial state to an alternative better state. In theory, it is optimal in the short run for subjects to not participate in a transition unless enough subjects have done so. Some groups of subjects are treated with less exact information, while some with more exact information. We find that groups with more exact information are more likely to transit to a better state, inconsistent with the theory of sampling best response dynamic. The reason for the failure of the theory is that human subjects are less responsive to information when they can only access sufficiently inexact information. That is subjects tend to disregard the information that the majority has switched away from the initial state.

On the other hand, the reason behind successful transitions under more exact information is consistent with the dynamic cognitive hierarchy model in the first chapter. The DCH model predicts that more sophisticated individuals are willing to initiate a transition to a better state because they believe less sophisticated individuals will follow them. So, it is crucial for individuals to have sufficiently exact information about others’ behavior. We find that subjects indeed are more likely to participate in a transition at early stages when information is more exact, and subjects with higher cognitive ability are more likely to do so.

The first two chapters focus on how individual behavior can result in macro social changes, but the third chapter studies how social institutions affect individual behavior. In particular, we study parents’ fertility choices in a patriarchal society. Patriarchal institutions often differentiate children based on sex, making sons superior and daughters inferior. In such cases, the sex of a child matter. Since the sex is random at birth and cannot be determined by parents ex-ante, parents can only make fertility decisions based on the sex composition of the children already born.

We show that parents’ optimal fertility strategy corresponds to a son-preferring stopping rule. That is parents tend to stop fertility when they have more sons and continue fertility when they have more daughters. This stopping rule makes females disproportionately born into families with more children, therefore receive less parental investment than males on average. In addition, the average fertility rate in a patriarchal society can surprisingly be lower than in a gender-neutral one. Intuitively, parents tend to have more children to have more sons in a patriarchal society, leading to a higher fertility rate. However, following the son-preferring stopping rule, parents
tend to have a low fertility rate when the first several born are sons, resulting in a lower average fertility rate. Finally, we find that the sex ratio in a patriarchal society is balanced without sex selection despite the son-preferring son-stopping rule. With sex selection, however, the male-to-female ratio can be greater than 1.
I am indebted to my supervisor Ed Hopkins and Mariann Ollár for their continuous support, guidance, and encouragement! Ed has followed every stage of my Ph.D. study. He always encourages me to work on my ideas, asking key insightful questions that often later lead to feasible and polished projects. Mariann as my secondary supervisor is always available to provide honest and constructive comments on my research. Their supervision provides me with a great opportunity to learn to be an independent researcher and extraordinary mentor.

Many other people have provided me with guidance during my Ph.D. study including Tatiana Kornienko, Zhi Li, and Andrew Clausen. Their time, efforts, and kindness are greatly appreciated. I want to express my sincere gratitude to Andrew Clausen in particular. He has provided many very valuable and constructive comments on my job market paper. In addition, I am also grateful to Paweł Gola, Dimitri Migrow, Ina Taneva, József Sakovics, Axel Gottfries, Patrick Julius, Kohei Kawamura, Alex Possajennikov for their brilliant suggestions on my research.

I am also thankful to my friends Yue Gu and Yaoyao Xu. Many of my research ideas originated from casual talks with them. Great thanks go to our professional staff in the School of Economics Joe Stroud, Kalina Charvala, Grace Oliver, and Quincy Sugiuchi for their assistance and organization. I also want to express my sincere gratitude to Tongsheng Chen. Without her help, I could never complete all the experimental sessions for my thesis in three months.

I would like to thank my cat Luno. Thank you for all the joys in life.

Last, I thank my parents for their unconditional love and support. They have indulged me with all the freedom to pursue a life that I cherish.
I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification except as specified.

The second chapter is a joint work with Zhi Li and Yaoyao Xu. I developed the research idea, contributed to the design of the experiment, conducted the experiment, contributed to the analysis of the results and wrote the paper.

Jianxun Lyu
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Chapter 1

A Theory of Equilibrium Switching

1.1 Introduction

Social conventions, norms, and institutions often emerge as a way of resolving miscoordination to facilitate efficiency. However, established social conventions often demonstrate a strong resilience to change even when they have become inefficient (Mackie, 2011; Bicchieri, 2005; Kinzig et al., 2013). The reason may be that transitions between conventions often involve collective participation. Individuals are willing to deviate from a social norm only if they believe enough other people will deviate. Therefore, individuals may have no incentive to deviate even if the status quo is inefficient (Arthur, 1989; Brock and Durlauf, 2001). This argument is justified by game-theoretical models, especially evolutionary game models, in which established inefficient conventions can be regarded as the payoff-dominated but locally stable equilibrium in coordination games under evolutionary dynamics.

However, several recent experiments find that players in multiplayer coordination games can switch away from the inefficient equilibrium to the efficient, which we call equilibrium switching. Smerdon et al. (2019b), Duffy and Lafky (2021a) find such equilibrium switching in fixed matching repeated games with global information, i.e., players can observe actions played by all other players in the group.1 Andreoni et al. (2021a) on the other hand find that equilibrium switching is also possible in random matching population games. In Lim and Neary (2016a), however, switches between two strict symmetric pure equilibria are common in random matching games with global information when there are two conflicting groups and neither group constitutes the majority.

1These results are more common in treatments with certainty about preferences. In treatments with preference uncertainty, equilibrium switching is rarer. The authors argue that this comparison supports the theory of pluralistic ignorance.
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These results are puzzling since strict Nash equilibria are locally stable in coordination games. We should not expect any equilibrium switching (at least in the short and medium run). In this paper, we answer this puzzle by building a novel dynamic cognitive hierarchy model (DCH) to study when a society becomes stuck in a bad equilibrium and when it can exit, i.e., equilibrium switching. The DCH model introduces heterogeneity in rationality among players as in cognitive hierarchy and level-\( k \) models (Stahl and Wilson (1995), Nagel (1995), Stahl (1996), Duffy and Nagel (1997), Camerer et al. (2004)). In those models, a level-\( k \) player best responds myopically to expected or past behaviors of level-(\( k-1 \)) or a combination of level-0 to level-(\( k-1 \)) players for all \( k > 0 \). In the DCH model, we make the important change and assume that level-\( k \) players (\( k > 0 \)) are forward-looking, i.e., they care about not only present but also future payoffs. High-level and forward-looking players are therefore farsighted, and they will take into account the effects of their current actions on the future plays of their opponents.

We show that, in the DCH model, level-\( k \) players will follow a cutoff strategy in two-action coordination games: they will deviate from the inefficient Nash equilibrium if the population state, i.e., the fraction of players playing certain actions, is above a cutoff, and the cutoff is inversely related to the cognitive level \( k \). The cutoff strategy implies that more sophisticated players deviate earlier from an inefficient Nash equilibrium than less sophisticated players. We show that under a broad range of distributions of cognitive levels, early deviations from high-level players have the domino effect: they can induce low-level players to deviate in the future. We characterize the set of such distributions of levels, which we call domino distribution, and we show that it is the sufficient and necessary condition for efficiency-improving equilibrium switching.

Next, we extend the equilibrium switching result in two-action games to general games with finite strict Nash equilibria. We establish a transitive binary relation domino dominance on the set of strict Nash equilibria and show that a Nash equilibrium is unstable if strictly domino dominated in the sense that a population will transit away from it. If there exists only one strict Nash equilibrium not strictly domino-dominated, it is also efficient. In addition, we show that a population will transit away from a strict Nash equilibrium to another if the latter is the most efficient equilibrium among those that iteratively domino dominate the former.

\(^2\)For example, all strict Nash equilibria are stable in deterministic evolutionary dynamics such as best response dynamic and replicator dynamic. However, equilibrium switching is possible in stochastic evolutionary dynamics in the long run where individuals play myopic best response with rare mistakes (Foster and Young, 1990; Kandori et al., 1993; Young, 1993).
Finally, we apply the DCH model to study equilibrium switching with disagreement. We consider cases where society is divided into two groups and they have opposite preferences over two strict Nash equilibria. We find that the majority group can always switch the equilibrium to their preferred one provided that the distribution of cognitive levels in the majority group is a domino distribution. Interestingly, a group does not need to be the majority to get society to its preferred equilibrium. When both groups have some sophisticated players, the sophistication of one group can be exploited by another one to reach the latter’s preferred equilibrium. Hence, it is surprising that for the former group, being less sophisticated is better. On the other hand, when both groups have enough sufficiently sophisticated players, an alternation cycle, i.e., the population persistently switches between two Nash equilibria, can emerge. This finding coincides with the alternation behaviors observed in Lim and Neary (2016a).

The DCH model is in-between of evolutionary game theory and equilibrium analysis in repeated games. The first one only focuses on non-strategic adaptive players, which predicts no equilibrium switching among strict Nash equilibria in at least the short and medium run. No players will initiate equilibrium switching since such deviations will result in immediate short-run losses. The equilibrium analysis such as the folk theorems in repeated games instead focuses on strategic farsighted players with common knowledge of rationality. In such a setting, every Nash outcome of the stage game can be supported by some subgame-perfect equilibrium strategies, and therefore no unique prediction or equilibrium switching. We can see the DCH model as a combination of the two approaches, and equilibrium switching/selection arises naturally because of the heterogeneity in strategic sophistication.

The idea that more sophisticated players can teach or manipulate myopic and adaptive players to play a better (at least for the former) equilibrium is not entirely new, e.g., the models of strategic teaching and manipulation Fudenberg and Levine (1989); Watson (1993); Camerer et al. (2002a,b), but the DCH model has a distinct feature that it applies to multiplayer games. The previous papers only study two-player games in which a sophisticated player can unilaterally teach myopic players to play the equilibrium preferred by the former. For the sophisticated player, there is no coordination problem or strategic uncertainty. However, their results do not apply to multiplayer games directly. In the latter case, it often requires more than one farsighted sophisticated player to jointly teach or manipulate myopic adaptive players. Therefore, there is a nontrivial second-order coordination problem embedded in the original game, and the results in the previous papers no longer hold. This paper
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extends the models of strategic teaching/manipulation, showing that heterogeneity in strategic sophistication can resolve the second-order coordination problem among farsighted players and yield a unique selection of the efficient equilibrium under certain conditions.

The underlying mechanism of equilibrium selection/switching in the DCH model is different from stochastic evolutionary dynamics proposed by Foster and Young (1990); Kandori et al. (1993); Young (1993) based on mutations/mistakes. First, mistakes or deviations from the myopic best response are intentional moves by farsighted players in the DCH model, and individual deviations from the myopic best response are correlated. While in the stochastic models, mistakes are exogenously specified by a stochastic choice model, and individuals make mistakes independently. Second, evolutionary selection based on stochastic stability often takes a very long time to happen as it is rare to have multiple simultaneous deviations in the same direction (Ellison, 2000). In contrast, the equilibrium selection (switching) predicted by the DCH model need not take a long time to realize as deviations in the DCH model are intentional and correlated so that they can accumulate in the same direction over time.

The DCH model also provides an alternative interpretation of apparently non-optimal choices played by human subjects in experiments. Existing theories such as QRE, classical level-\(k\), and cognitive hierarchy, or stochastic dynamics, often interpret non-optimal choices as unintentional mistakes, lack of understanding of games, or false (naive) prior beliefs. These interpretations tend to downplay players’ levels of sophistication, while the DCH model shows that mistakes could also be strategic and intentional.

This paper also contributes to the threshold models of collective action (Granovetter, 1978; Oliver et al., 1985; Macy, 1991). Threshold models are proposed to explain social changes and movements that have a typical feature of asynchronous timing in participation or leader-follower structure. In such models, individuals have idiosyncratic thresholds (or cutoffs) for participating in a social movement such that they will participate if the number of participants has reached their thresholds. However, the literature mainly attributes threshold participation to non-strategic personal traits such as heterogeneity in emotion, preference, and morality (Granovetter, 1978; Sandholm, 2001b; Alger and Weibull, 2017). An exception is Acemoglu and Jackson (2015) that argue farsighted prominent agents can strategically coordinate beliefs and facilitate social transitions. Acemoglu and Jackson (2015) impose the prominence exogenously but the DCH model shows prominent agents can emerge from heterogeneity in strategic sophistication.
mechanism that generates threshold models of participation without heterogeneity in such personal traits. In addition, the DCH model has additional predictions such as the direction of social movements, the possibility of alternation among social states, and the effect of population sizes on which the previous models are silent.

1.1.1 Examples

Before introducing the DCH model formally, we first present two examples to illustrate the main idea of the model. In the DCH model, high-level forward-looking players could choose to suffer the short-run loss of playing suboptimal actions in order to induce low-level players to switch to better equilibrium in the long run. The presentation of the model in this section is informal, with precise definitions deferred until subsequent sections.

In a (simplified) DCH model, players are characterized by a cognitive level $k$ with a decision rule that level-$k$ ($k > 0$) best responds to level-$(k - 1)$ by maximizing their long-run average payoff. Furthermore, level-0 players are non-strategic and assumed to best respond to their opponents’ latest choice.

**Example 1.1.** Consider a coordination game presented in Table 1.1 played repeatedly.

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>(4, 4)</td>
</tr>
<tr>
<td>R</td>
<td>(2, 0)</td>
</tr>
</tbody>
</table>

Table 1.1: $2 \times 2$ Coordination Game

There are two pure Nash equilibria of the stage game: the payoff-dominant equilibrium $(L, L)$ and the risk-dominant equilibrium $(R, R)$. Suppose player 1 is level-1 and player 2 is level-0, and the status quo is $(R, R)$, we can show that the equilibrium will be switched to $(L, L)$.

Notice that player 2 is level-0, so player 2 will play $R$ in period 1 since the myopic best response to her latest opponent’s play is $R$. However, player 1 is level-1, which means she will take into account the effect of her action in period 1 on future plays of player 2. If she plays $R$ in period 1, then player 2 will keep playing $R$ in the following periods. This gives her a long-run average payoff 3. However, if she plays

---

4The payoff function of a level-$k$ ($k > 0$) player is specified in the form of the limit of means.
1.1. Introduction

L in period 1, she can induce player 2 to play L in period 2 and afterward. This generates an average payoff 4. Therefore, player 1’s forward-looking best response in period 1 is L. Therefore, the payoff-dominant equilibrium will be reached even though the status quo is a strict and risk-dominant Nash equilibrium.

Example 1.1 is similar to strategic teaching models in Camerer et al. (2002a), Camerer et al. (2002b), which apply to two-person games with one player sophisticated and farsighted and another naive and myopic. The DCH model however applies to multiplayer games. This is illustrated by the following example.

Example 1.2. Consider the following coordination game

Table 1.2: 2 × 2 × 2 Coordination Game

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>Player 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>(4, 4, 4)</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td>R</td>
<td>(-1, 1, 1)</td>
<td>(2, 2, 1)</td>
</tr>
</tbody>
</table>

The game has two pure Nash equilibria the L equilibrium (L, L, L) and the R equilibrium (R, R, R). Note that the L equilibrium is Pareto optimal, therefore forward-looking players may have the incentive to switch equilibrium if the status quo is the R equilibrium.

To see how equilibrium switching can arise, suppose in period 0 the R equilibrium is played, and player 1, 2, 3 are level-0, 1, and 2 respectively. Note that for player 1, it is optimal to play L only after she observed that two other players played L. Since the status quo is the R equilibrium, level-0 will play R in period 1. For player 2, however, it is optimal to play L only after she observed one player played L. To see it clearly, suppose there is one player playing L in period t, and player 2 can figure out that by playing L herself in period t + 1, there will be two L observed by level-0 players in period t + 1, therefore she can manipulate level-0 players to play L in the future. Equilibrium then is expected to be switched to the L equilibrium. However, since the status quo is the R equilibrium, player 2 will play R in period 1.
Finally, player 3 will play $L$ in period 1 since by playing $L$ she can induce level-1 players to play $L$ in period 2, which further induce level-0 players to play $L$ in period 3 and afterward. This will generate a long-run average payoff higher than that in the status quo. The DCH model predicts that player 3 will initiate a transition to the $L$ equilibrium, followed by player 2, and finished by player 1.

Example 1.2 illustrates how equilibrium switching can be achieved by boundedly rational players in multiplayer games. It first requires enough high-level players to initiate the equilibrium switching and to keep deviating to induce low-level players to follow. Furthermore, it requires a proper distribution of levels. For example, if there is no level-2 or higher levels, then no one will initiate the transition; if a level-2 is present but not level-1, switching will also fail. In the subsequent section, we will characterize a set of distributions of cognitive levels called domino distributions under which equilibrium switching always succeeds.

The remainder of the paper is organized as follows. Section 1.2 presents the dynamic cognitive hierarchy model. Section 1.3 studies equilibrium switching in finite games with multiple strict Nash equilibria. In section 1.4, we discuss related literature with the DCH model. Section 3.4 concludes.

1.2 Dynamic Cognitive Hierarchy Model

A population of $M$ players plays a symmetric normal-form game recurrently in discrete time. $A = \{1, 2, ..., n, ..., N\}$ denotes the finite set of actions available to each player. An action profile $a \in A^M$ is an $M \times 1$ vector, where the $m$-th element $a_m$ is the action played by player $m$; $a_{-m}$ denotes the action profile of player $m$’s $M-1$ opponents. The stage payoff function is represented by a function $u : A^M \rightarrow \mathbb{R}$.

The set of best responses of player $m$ is denoted by

$$BR(a_{-m}) := \arg \max_{a_m \in A} u(a_m, a_{-m}) \subset A.$$  

---

5Let $X = \Delta(A) = \{x = (x_1, x_2, ..., x_N) : \sum_n x_n = 1\}$ be the set of mixed strategies, which can also be interpreted as the set of action distributions where $x_n$ is the fraction of players playing action $n$. In symmetric games where the role of players is identical, we can use the action distribution $x$ to represent $a_{-m}$. The corresponding payoff function can be represented by $F : A \times X \rightarrow \mathbb{R}$. 
1.2. Dynamic Cognitive Hierarchy Model

The dynamic cognitive hierarchy model is built on the cognitive hierarchy model proposed by Camerer et al. (2004), which is a generalization of all relevant structural level-\(k\) models (Stahl and Wilson, 1995; Nagel, 1995; Stahl, 1996). In such models, there is heterogeneity in players’ step of reasoning, or simply, a cognitive level \(k\). Players who engage in zero steps of reasoning are assumed to follow an exogenously specified non-strategic decision rule and are labeled level-0. The level-0 decision rule provides a benchmark for the decision-making of players with higher levels: Level-1 best responds to level-0 choices; level-2 best responds to a linear combination of level-1 and level-0 choices; and so on (level-\(k\) only best responds to level-(\(k-1\)) in level-\(k\) models). The specification of level-0 decision rule is therefore at the core of cognitive hierarchy and level-\(k\) models.

A widely used level-0 specification is uniform randomization over the action set. Cognitive hierarchy and level-\(k\) models were initially proposed to predict the initial play in repeated interactions or the play in one-shot simultaneous-move games where uniform randomization as a naive non-strategic choice seems plausible in this setting. But here we focus on dynamic adjustments of play in repeated interactions, hence naive players have the opportunity to adapt their behavior to opponents’ past choices. Therefore, in the DCH model, we adopt a well-studied and empirically salient decision rule myopic best response (mBR) as the level-0 decision rule, that is we assume a level-0 player plays a myopic best response to the latest choice of her opponents. Denote \(s^0_t\) the optimal play of a level-\(k\) player in period \(t\).

**Definition 1.1 (Level-0).** A level-0 player \(m\) follows myopic best response, i.e.,

\[
s^0_t \in BR(a_{m,t}^c) \text{ and } a^c_{m,t} = a_{m,t-1}.
\]

where \(a_{m,t-1}^c\) denotes the action profile of player \(m\)’s opponents in period \(t - 1\).

Our choice of mBR as the level-0 decision rule in a dynamic cognitive hierarchy model needs a careful justification. A level-0 decision rule should be non-strategic and behaviorally plausible (Crawford et al., 2008, 2013), and also salient across games (Hargreaves Heap et al., 2014). We argue that myopic best response satisfies these criteria in a dynamic setting. First, playing mBR to the latest opponents’ choice is nonstrategic as it does not require any iterative strategic thinking such as how my opponents will react to my choice. The nonstrategic thinking implies that level-0 players take opponents’ play to be exogenous, and our specification requires level-0 to make the rational choice against the exogenous environment. Or in another word, whenever there are alternative profitable choices, level-0 will take advantage of this opportunity. The most well-known example of this sort of behavior in economics
1.2. Dynamic Cognitive Hierarchy Model

would be price-taking in a competitive market where the price is in fact an aggregate result of individual behaviors. The mBR is also behaviorally plausible in a dynamic setting as it requires naive players to adapt to the current environment, i.e., it is the latest opponents’ choice that naive players should optimize against not the choice in some ancient histories.\(^6\)

Myopic best response is also a salient decision rule across games in at least repeated interactions. The concept of myopic best response dates back to Cournot’s duopoly model; Nash (1950) uses it to interpret Nash equilibrium as a result of mass action. Gilboa and Matsui (1991) propose the best response dynamic which later becomes a widely used approach to the dynamic evolution of behaviors of boundedly rational players in evolutionary game theory, behavioral and experimental studies. Experimental studies have found the salience of myopic best response across many classes of games.\(^7\) A much more subtle and striking support for the salience of mBR is perhaps the deviations from it. Experimental studies have found that human subjects deviate from mBR in almost all games, but deviations are infrequent and exhibit systematic patterns such as cost-dependence, serial correlation, directions, decreasing in time (Mäs and Nax, 2016; Lim and Neary, 2016a; Bilancini et al., 2020; Li et al., 2022). If mBR is not the underlying benchmark for human subjects’ decision-making, we would not observe such patterns.

Next, we define the high-level players \((k > 0)\) in the DCH model. In a cognitive hierarchy model, a level-\(k\) \((k > 0)\) player is assumed to hold a fixed belief that all her opponents are below \(k\). For example, Camerer et al. (2004) assume that the objective distribution of cognitive levels \(f(k)\) is a Poisson distribution, and a level-\(k\)

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\(^6\)One can of course use more complex adaptive learning rules as the level-0 decision rule, but this will not affect the main feature of the model. For example, level-0 can follow a fictitious play with a finite-period memory, i.e., they play the mBR to a weighted average of the last-several-period plays.

\(^7\)Examples include coordination games, cyclic games, hawk dove, public goods games, price dispersion under various treatments including synchronous and asynchronous move, perfect and imperfect information, random and fixed matching, symmetric and asymmetric payoffs, with and without networks (Friedman, 1996; Raymond Battalio et al., 2001; Cason et al., 2014; Mäs and Nax, 2016; Lim and Neary, 2016a; Reischmann and Oechssler, 2018; Bilancini et al., 2020; Cason et al., 2021; Li et al., 2022).
(k > 0) player has an accurate guess about the relative fractions of players who are below k. In the DCH model, we relax this assumption and do not specify any specific functional form of \( f(k) \). However, we require the subjective distribution of a level-\( k \) player \( g_k \) to satisfy the following assumption.

**Assumption 1.1 (Downward Connected Belief).** For all \( k > 0 \), the subjective distribution \( g_k(l) \) satisfies

\[
\text{supp}(g_k) = \{l, l + 1, \ldots, k - 2, k - 1\}
\]

for some integer \( 0 \leq l \leq k - 1 \).

Assumption 1.1 requires that the support of a level-\( k \)’s belief on opponents’ level is connected downward from \( k - 1 \), and implies that high-level players are optimistic about their opponents’ level of sophistication. For instance, the support of a level-3’s belief can be \{2\}, \{1, 2\}, or \{0, 1, 2\}; on the other hand, disconnected supports such as \{0, 2\} are ruled out. Assumption 1.1 is a generalization of all level-\( k \) beliefs in the literature. In level-\( k \) models, \( g_k(k - 1) = 1 \), and \( g_k(l) = 0 \) for all \( l < k - 1 \); in static CH model, \( g_k(l) \) is a truncated Poisson distribution.

In static CH and level-\( k \) models, players have fixed beliefs - once a \( f(k) \) and a model of \( g_k(l) \) are specified, a level-\( k \) player can work out the plays of all levels below \( k \) recursively starting from level-0 and plays the best response accordingly. In a dynamic environment where a game is played repeatedly, however, the assumption of fixed belief is not reasonable: it is possible for players to realize some players with levels not in the support of \( g_k(l) \). In addition, repeated interactions enable high-level players to potentially exploit or manipulate lower levels, i.e., players can demonstrate the feature of forward-looking. In the following, we extend the static CH and level-\( k \) models in these two directions.

First, level-\( k \) players are forward-looking, which requires them to take future payoffs into consideration. We assume that players’ payoff function takes the form of the limit of means, i.e., they maximize their long-run average payoff.

\[ g_k(l) = \frac{f(l)}{\sum_{l'=1}^{k-1} f(l')} \]

where \( g_k(l) \) is level-\( k \)’s subjective fraction of level-\( l \) with \( 0 \leq l < k \). It is important to bear in mind that level-\( k \) reasoning involves higher-order beliefs. For example, a level-2 player not only believes that her opponents consist of level-0 and level-1 players but also that level-1 players believe all other players are level-0. In fact, a level-\( k \) player has \( k \)-th order belief in the DCH model.

\[ g_k(l) = \frac{f(l)}{\sum_{l'=1}^{k-1} f(l')} \]
Definition 1.2. In any period $t_0$, a level-$k$ ($k > 0$) player $m$ formalizes an action plan $\{a^*_{mt}\}_{t=t_0}^\infty$ to maximize her long-run average payoff

$$\lim_{t \to \infty} \frac{\sum_{t=t_0}^t u(a_{mt}, a_{-m,t})}{t}.$$

Next, we consider how beliefs are updated in the DCH model. To do so, we need to clarify what exactly we refer to the cognitive level $k$. There are two competing (but not mutually exclusive) interpretations of cognitive levels in CH and level-$k$ models. The first interpretation is the maximal steps of iterative reasoning a player can do. That is, the level reflects a player’s ability, and the level-$k$ behavior observed in labs or fields is due to bounded ability. The alternative interpretation is that cognitive levels reflect players’ bounded beliefs instead of ability. A player can do infinite steps of iterative reasoning, but may only do $k$ steps in practice because she believes her opponents only do $k - 1$ steps. Experimental studies find evidence of both bounded ability and bounded beliefs, e.g., Jin (2021); Alaoui and Penta (2022).

In the DCH model, we assume cognitive levels to be the ability bound as it implies that a level-$k$ player is unable to understand behaviors above $k - 1$. We argue that this implication leads to a natural extension of adaptive learning of level-0 to level-$k$ players. The main idea is that due to her bounded ability, a level-$k$ player cannot understand behaviors resulting from higher-level reasoning when these behaviors are different from those resulting from lower-level reasoning, therefore they (level-$k$ players) are likely to adapt to the latest ‘unexpected’ behaviors (in conjunction with considerations of expected plays of the rest lower-level players). This is just like how level-0 players adapt to their opponents’ choices.\(^9\)

Note that a level-$k$ player can derive which actions will be played by players below $k$ in the next period. We call the set of these actions $k$-rationalizable set. In any period, if a choice is not in the $k$-rationalizable set, it is an unrationalizable choice. We assume that a level-$k$ player uses the same adaptive learning rule of

---

\(^9\)One could ask why a level-$k$ player after observing behaviors ‘impossible’ by lower-level players could not understand these behaviors were made by higher levels. Or more generally, why can not a level-$k$ player do one more step of reasoning? After all, the logic of doing $k + 1$ steps is not structurally different from doing $k$ steps. A theoretical justification appeals to Alaoui and Penta (2016) where the cognitive level of a player is endogenously determined in the cost-benefit analysis. Fix a game, a player’s ability of iterative reasoning is bounded due to the cost of iterative reasoning. Therefore, it is not that a level-$k$ player cannot do iterative reasoning more than $k$ steps, rather, it is just too costly to do so.
level-0, i.e., the latest unrationalizable choices are expected to be carried forward in the future. Hence, a level-$k$ player best responds to the expected action profile composed of the latest unrationalizable choices and the expected choices of the rest of lower levels deduced from $g_k$.\footnote{Throughout the paper, the level-$k$ subjective distribution $g_k$ is fixed with renormalization. Including the update of $g_k$ does not affect the main results in this paper if Assumption 1.1 holds. For example, any support-preserving belief updating rule such as Bayesian updating does not violate Assumption 1.1.}

Formally, denote $A^k_t$ the $k$-rationalizable set at the beginning of period $t$, i.e., the set of actions expected to be played in period $t$ by players below $k$. Denote $s^k_t$ the optimal play of level-$k$ players in period $t$, and $\{s^k_t\}$ the set of all possible optimal plays (since the optimal choice may not be unique), we have

$$A^k_t := \cup_{l \in \text{supp}(g_k)} \{s^l_t\}. \quad (1.1)$$

By the construction of $k$-rationalizable set, we have the following observation.

**Observation 1.1.** If an action cannot be rationalized by level-$k$ players, it also cannot be rationalized by players below $k$, i.e.,

$$A^k \subseteq A^{k+1} \text{ for } k \in \{0, 1, 2, 3, \ldots\}.$$  

This feature is obvious and guarantees that level-$k$ players can keep tracking the choice of players below $k$ in a dynamic setting.

Upon observing the action profile $a_{t-1}$ in period $t-1$, we can deduce the optimal play of level-$k$ players $s^k_t$ for all $k$ in period $t$. First, for a level-$k$ player, the latest unrationalizable choice profile, denoted as $\tilde{a}^k_{t-1}$, is expected to be carried forward:

$$\mathbb{E}(\tilde{a}^k_{t+j}) = \tilde{a}^k_{t-1}$$

for any non-negative integer $j$. Second, the future expected optimal choice profile of the rest lower levels by a level-$k$, denoted as $\tilde{s}^k_{t+j}$, can be derived from $g_k$ given a sequence of the level-$k$ player’s own action choices. That is, the future optimal choice of lower levels is a function of the level-$k$ player’s own future choice.

At the beginning of a period $t$, a level-$k$ player’s expected action profile of opponents in a future period $t+j$ is denoted as $a_{-k,t+j}$. Fix a feasible choice sequence $(a_t, a_{t+1}, \ldots)$ of a level-$k$ player from period $t$, there exists a corresponding sequence of expected action profile of her opponents $(a_{-k,t}, a_{-k,t+1}, \ldots)$. A level-$k$
player then will pick the optimal choice sequence

\[(a_t^*, a_{t+1}^*, \ldots)\]

that maximizes her expected long-run average payoff, and the optimal play of a level-\(k\) player in period \(t\) \(s^k_t \equiv a^*_t\). Note that the optimal play \(s^k_t\) in any period \(t\) is determined upon observing the latest (actual) action profile \(a_{t-1}\). Therefore, we call \(s^k_t\) the optimal strategy of level-\(k\) players that specifies an action in period \(t\) to the action profile of her opponents in period \(t-1\).

For a given cognitive distribution \(f(k)\), the dynamic of play can be aggregated to a compact form. Let \(x \in X = \Delta(A)\) be a distribution of action, i.e., population state, where its \(n^{th}\) element \(x_n\) denotes the fraction of players playing action \(n\). Denote \(x(a)\) the distribution of action corresponding to action profile \(a\), and \(\sigma^k\) the mixed strategy representation of \(s^k\), i.e., \(\sigma^k_a = 1\) if \(s^k = a\). The optimal strategy now can be described as a function

\[\sigma^k : X \rightarrow \Delta(A)\]

Aggregating individual plays we obtain the following sophisticated best response dynamic (sBRD) of population state \(x\):

\[\Delta x_t = \sum_k f(k)\sigma^k(x_{t-1}(a_{t-1})) - x_{t-1}.\]

In the DCH model, only Nash equilibria stable under the sophisticated best response dynamic are expected to be played. Of course, the stability of equilibria, therefore the resulting equilibrium refinement or selection, depends on the distribution of cognitive levels. It is worth noting that if all players are level-0, we obtain the standard best response dynamic in Gilboa and Matsui (1991) as a special case of sBRD.

### 1.3 Equilibrium Switching

In this section, we apply the dynamic cognitive hierarchy model to study transitions among Pareto-ranked strict Nash equilibria. We first consider the simplest strategic environment where there are only two strict Nash equilibria. A population of \(M\) players chooses between two actions \(\{1, 2\}\), and \(x \in [0, 1]\) denotes the aggregate behavior of a typical player’s \(M - 1\) opponents, which is the fraction of action 2 among her \(M - 1\) opponents. The payoff of a typical player who plays action \(a\) is given by \(F_a(x)\).
1.3. Equilibrium Switching

**Assumption 1.2.** $F_1(x)$ is decreasing in $x$ and $F_2(x)$ is increasing in $x$. In addition, $F_1(0) > F_2(0)$, $F_1(1) < F_2(1)$, and $F_1(0) < F_2(1)$.

Assumption 1.2 implies that: first, players have the incentive to conform to the majority; second, there are the two homogeneous population states $e_1$ where all players choose action 1 and $e_2$ where all players choose action 2 that are strict Nash equilibria; and third, $e_2$ is Pareto superior to $e_1$. We are interested in the following question: Under what conditions transitions between the two equilibria will happen, and if so, in what direction. Without loss of generality, assume the initial population state to be $e_1$, we study how can a population transit away from $e_1$ to $e_2$.

Note that there exists a solution to equation $F_1(x) = F_2(x)$ in $(0, 1)$. Denote the solution $x^* = T_{12}$. For level-0 players, it is easy to see that they follow a cutoff strategy with a cutoff $T_{12}$, i.e.,

$$s^0_t = \begin{cases} 2 & \text{if } x_{t-1} \geq T_{12}, \\ 1 & \text{otherwise.} \end{cases} \quad (1.2)$$

We call $T_{12}$ the tipping point of transitions from $e_1$ to $e_2$. It means that given the initial state $e_1$, it requires that at least $T_{12}$ fraction of players among $M-1$ opponents deviating from $e_1$ to induce level-0 players to switch from action 1 to action 2. The tipping point essentially reflects the difficulty of transitions from $e_1$ to $e_2$. Next, we show that high-level players follow a similar cutoff strategy where the cutoff population proportion is lower for more sophisticated players.

**Proposition 1.1.** The level-$k$ players’ optimal strategy is a cutoff strategy such that

$$s^k_t = \begin{cases} 2 & \text{if } x_{t-1} \geq T_{12} - \frac{k}{M-1}, \\ 1 & \text{otherwise.} \end{cases}$$

**Proof.** See Appendix A. ■

Proposition 1.1 shows that the optimal strategy of a level-$k$ player is a cutoff decision rule that includes level-0’s as a special case. The cutoff $T_{12} - \frac{k}{M-1}$ measures the requirement for a level-$k$ player to deviate from $e_1$. The intuition behind Proposition 1.1 is that sophisticated players whose beliefs satisfy Assumption 1.1 are confident in the sophistication of their opponents, and they will take a short-run loss to initiate an equilibrium switching to the efficient state because they believe that their deviations to the efficient equilibrium will be followed by others.

**Observation 1.2.** The cutoff $T_{12} - \frac{k}{M-1}$ is decreasing in players’ level of sophistication $k$, and increasing in population size $M$. 

Observation 1.2 suggests that a more sophisticated player is more likely to deviate from mBR in a small population. If $k$ is sufficiently high, the cutoff will be less than 0, which implies that sufficiently high-level players will be the initiators of equilibrium switching from $e_1$ to $e_2$. On the other hand, population size also plays an important role: the cutoff increasing in $M$ suggests that transitions are easier in smaller populations such as households and workplaces.

**Observation 1.3.** Deviations from myopic best response made by high-level players ($k > 0$) are directed from the inefficient equilibrium $e_1$ to the efficient equilibrium $e_2$, not vice versa.

This observation follows the fact no player will deviate from mBR if the mBR is the efficient action. Observation 1.3 suggests that transitions among strict Nash equilibria if happen, are directed from the inefficient Nash equilibrium to the efficient one. This implication is consistent with experimental evidence. For example, Raymond Battalio et al. (2001); Lim and Neary (2016a) find human subjects on average have a tendency to deviate from a bad equilibrium to a better one under random matching and perfect information about population play $x$. Similar results are also found in Li et al. (2022) under fixed matching and imperfect information.

It is clear that if there are enough high-level players, switchings from $e_1$ to $e_2$ can happen. The next proposition characterizes a set of cognitive distributions under which equilibrium switching can actually happen. Recall that $f(k)$ (pmf) is the population distribution of cognitive levels. Let $\text{supp}(f) \subseteq \{0, 1, 2, 3, 4, ..., K\}$, where 

$$K \equiv \lceil(M - 1)T_{12} \rceil,$$

and $f(K)$ represents the fraction of players weakly above $K$.

**Definition 1.3** (Domino Distribution). A distribution of cognitive levels $f(k)$ is a domino distribution with respect to an equilibrium pair $(e_1, e_2)$ if

$$\sum_{l=k}^{K} f(l) \geq \frac{M - 1}{M} T_{12} - \frac{k - 1}{M}$$

for $k \in \{1, 2, ..., K\}$.

In the definition of domino distribution, the left-hand side of the inequality $\sum_{l=k}^{K} f(l)$ measures the total mass of players weakly above $k$. The inequality ensures that level-$k$ players’ cutoff will be reached for all $k$ if players above $k$ deviate from $e_1$. The next proposition states that a domino distribution is both sufficient and necessary for efficiency-improving transitions between two strict Nash equilibria in the DCH model.

\[\text{sup} \frac{x}{x} \text{ is the least integer that is no less than } x \text{ for } x \in \mathbb{R}.\]

\[\text{We can also define domino distributions with respect to } (e_2, e_1) \text{ similarly.}\]
1.3. Equilibrium Switching

**Proposition 1.2.** Suppose $e_2$ is Pareto superior to $e_1$, a population will switch from Nash equilibrium $e_1$ to $e_2$ if and only if the cognitive distribution $f(k)$ is a domino distribution with respect to the equilibrium-pair $(e_1, e_2)$. However, a population will not switch from $e_2$ to $e_1$ regardless of $f(k)$.

**Proof.** See Appendix A. ■

The equilibrium switching is stepwise in the DCH model because different levels have different cutoffs. Heterogeneity in strategic sophistication makes players with different levels believe they are the pivotal player in the equilibrium switching at different population states. Therefore, even though it requires joint deviations from the bad equilibrium in multi-player games, there is no coordination problem among sophisticated players. The name of domino distribution comes from the similarity between stepwise transitions and the game of dominoes: early deviations from high-level players have a domino effect: they trigger low-level players to deviate in the future; if a significant fraction of middle-level players is missing, the equilibrium switching would fail as if the distance of two adjacent domino cards is too long.

The concept of domino distribution lies in the center of Proposition 1.2, but its definition depends on particular games and equilibrium pairs at consideration. The next two propositions state the relationships among different domino distributions for a fixed equilibrium pair; and different equilibrium pairs for a fixed cognitive distribution.

**Proposition 1.3.** For an equilibrium pair $(e, e')$ with a tipping point $T$ and a domino distribution $f(k)$

1. a cognitive distribution $f'(k)$ is also a domino distribution of $(e, e')$ if $f'(k)$ first-order stochastically dominates $f(k)$;
2. the pair has a minimal domino distribution $f(k)$ such that for any cognitive distribution $f'(k)$, it is a domino distribution of $(e, e')$ if and only if it first-order stochastically dominates $f$, where the minimal domino distribution $\bar{f}$ is given by

\[
\bar{f}(k) = \begin{cases} 
\frac{M-[(M-1)T]}{M} & \text{if } k = 0, \\
\frac{1}{M} & \text{if } k \in \{1, 2, 3, \ldots, \lceil (M-1)T \rceil \}, \\
0 & \text{otherwise.}
\end{cases}
\]

**Proof.** See Appendix A. ■
The first part of Proposition 1.3 states a sufficient condition for a distribution to be domino with respect to a pair. The second part instead states the sufficient and necessary condition. The minimal domino distribution of an equilibrium pair allocates its probability mass to lower levels as much as possible, and it reflects the minimal requirement on strategic sophistication for equilibrium switching.

Proposition 1.4. For a distribution on non-negative integers \( f(k) \) that is a domino distribution of an equilibrium pair with a tipping point \( T \),

1. it is also a domino distribution with respect to any equilibrium pair with a tipping point \( T' \) if \( T' \leq T \);
2. it admits a maximal tipping value \( \bar{T}(f) \in [0,1] \) such that \( f(k) \) is a domino distribution for any equilibrium pair with a tipping point \( T' \) if and only if \( T' \leq \bar{T}(f) \).

Proof. See Appendix A.  ■

Proposition 1.4 is a counterpart of Proposition 1.3 by fixing a cognitive distribution. With Proposition 1.3 and 1.4, we can obtain interesting implications even if we do not know the exact cognitive distribution of a group. For example, if a group has shown its ability to switch for an equilibrium pair \((e_i, e_j)\) with a tipping point \( T \) (i.e., switch from \( e_i \) to \( e_j \)), we can expect that it can do so for any equilibrium pair with a tipping point \( T' < T \). On the other hand, if a group did not switch from a strict equilibrium \( e_i \) to a better alternative \( e_j \) given the tipping point \( T \), we should not expect it to switch for any other pairs with tipping points greater than \( T \).

1.3.1 A General Model

The equilibrium switching results in two-action games can be extended to finite normal-form games with multiple strict Nash equilibria. In this subsection, we consider a symmetric normal-form game with strategic complementarity. The game has an action set \( A = \{1, 2, 3, ..., n, ..., N\} \), and is played by a population of \( M \) players. Denote \( X = \{x = (x_1, x_2, ..., x_n, ..., x_N) : \sum_{n=1}^{N} x_n = 1\} \) the set of action distributions among a typical player’s \( M - 1 \) opponents, and the payoff function is given by \( F : X \rightarrow \mathbb{R}^N \), where the \( n^{th} \) element \( F_n(x) \) is the payoff to action \( n \). Further denote \( e_n \in X \) as the action distribution where all players play action \( n \), which is equivalent to that any player’s \( M - 1 \) opponents play action \( n \).

Assumption 1.3. \( F_n(x) \) is increasing in \( x_n \) and decreasing in \( x_i \) for \( i \neq n \). In addition, \( F_n(e_n) > F_i(e_n) \) for \( i \neq n \), and \( F_n(e_n) < F_{n+1}(e_{n+1}) \).
1.3. Equilibrium Switching

Assumption 1.3 again imposes strategic complementarity to the game. Therefore, the set of strict Nash equilibria is \( \{ e_n : n \in A \} \). There is also a Pareto ranking over the set of strict Nash equilibria: \( e_{n+1} \) is Pareto superior to \( e_n \). We are interested in equilibrium switching among those strict Nash equilibria.

The best response region (BR region) of a strict Nash equilibrium \( e_n \) is denoted \( BR_n \subseteq X \) that satisfies

\[
F_n(x) \geq F_i(x) \text{ for all } i \neq n \text{ if } x \in BR_n.
\]

That is action \( n \) is a myopic best response to action distributions in its BR region. Recall the definition of level-0 players, their optimal strategy can then be written as

\[
s_t^1 = n \text{ if } x_{t-1} \in BR_n.
\]

Next we define the tipping point between two strict Nash equilibria whose BR regions intersect. Let \( BR_{ij} := BR_i \cap BR_j \) be the intersection of BR regions of strict Nash equilibrium \( e_i \) and \( e_j \). Let \( || \cdot ||_1 \) be the \( L_1 \) distance, i.e., \( ||x||_1 = \sum_{n=1}^{N} |x_n| \).

**Definition 1.4.** A tipping point of an equilibrium pair \( (e_i, e_j) \) is an action distribution \( T_{ij} \in X \) such that

\[
T_{ij} \in \arg \min_{x \in BR_{ij}} ||x - e_i||_1.
\]

The tipping point of the pair \( (e_j, e_i) \) is denoted \( T_{ji} \) and is defined similarly. Note that there might be many tipping points for an equilibrium pair. A tipping point \( T_{ij} \) is one of the closest action distributions to \( e_i \) in \( BR_{ij} \). That is if a population wants to transit from \( e_i \) to the best response region of \( e_j \), the line that connects \( e_i \) and a \( T_{ij} \) is one of the shortest paths. Figure 1.1 illustrates examples of the set of tipping points (denoted as red points) in a three-action game.

Let \( d_{ij} := 1/2 ||T_{ij} - e_i||_1 \), then \( K \equiv (M - 1)d_{ij} \) is the minimal number of deviations from \( e_i \) required to reach the best response region of \( e_j \).

**Definition 1.5.** A distribution of cognitive levels \( f(k) \) is a domino distribution with respect to an equilibrium-pair \( (e_i, e_j) \) if

\[
\sum_{l=k}^{K} f(l) \geq \frac{M - 1}{M} d_{ij} - \frac{k - 1}{M} \text{ for } k \in \{1, 2, ..., K\}.
\]
\[ F(x) = \begin{pmatrix} 20 & 12 & 8 \\ 12 & 24 & 20 \\ 0 & 16 & 28 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \]

Figure 1.1: An illustration of tipping points in a three-action coordination game \( F(x) \). The equilateral triangle is simplex in \( \mathbb{R}^3 \), which is the set of all action distributions. \( BR_i \) denotes the best response region of Nash equilibrium \( e_i \), and the two segments in the simplex are boundaries between BR regions. Tipping points are plotted in red. The set of tipping points: \( \{ T_{12} \} = \{ (x_1, x_2, x_3) : x_1 = 0.4 \} \), \( d_{12} = 0.4 \); \( \{ T_{23} \} = \{ (0, 0.5, 0.5) \} \), \( d_{23} = 0.5 \).

The definition of domino distribution in multi-action games is similar to the one in binary-action games. The only difference is replacing \( T_{ij} \) with \( d_{ij} \). To obtain a complete prediction on equilibrium switching, we abstract away from the details of a game and focus on the relationship between its strict equilibria. We first define a (incomplete) binary relation over the set of strict Nash equilibria.

**Definition 1.6 (Domino Dominance).** A strict Nash equilibrium \( e_j \) (strictly) domino dominates a strict Nash \( e_i \) if

1. \( BR_{ij} \) is non-empty;
2. \( f(k) \) is a domino distribution with respect \((e_i, e_j)\);
3. \( F_j(e_j) \geq (>)F_i(e_i) \).

**Corollary 1.1.** A population will switch away from a strict Nash equilibrium if it is strictly domino dominated. On the other hand, a population will not switch away from a strict Nash equilibrium if it is not domino dominated.

**Proof.** See Appendix A. \( \blacksquare \)
Corollary 1.1 establishes a connection between (in)stability of strict Nash equilibria and *(strict)* domino dominance - strict domino dominance implies instability while (non-) domino dominance implies stability. This result is a direct application of Proposition 1.2. However, the corollary is silent on to which equilibrium the population will switch. The reason is that a strict equilibrium may have multiple adjacent Pareto-superior equilibria.

To fully characterize equilibrium switching, we need further look at the relationship between equilibria with disjoint BR regions. Intuitively, if a strict Nash equilibrium \(e_i\) strictly domino dominates \(e_{i-1}\), and \(e_{i+1}\) strictly domino dominates \(e_i\), we may expect that a population will transit from \(e_{i-1}\) to \(e_{i+1}\). Therefore, it is natural to extend the domino dominance to an iterative version.

**Definition 1.7 (Iterative Domino Dominance).** A strict Nash equilibrium \(e_j\) iteratively *(strictly)* domino dominates a strict Nash \(e_i\) if there exists a finite chain of strict Nash equilibria \(\{e_i, e^{n+1}, e^{n+2}, ..., e^{n+L}, e_j\}\) such that

1. \(e^n\) (strictly) domino dominates \(e_i\);
2. \(e^{n+1}\) (strictly) domino dominates \(e^{n+L-1}\);
3. \(e^j\) (strictly) domino dominates \(e^{n+L}\).

**Proposition 1.5.** A population will switch from \(e_i\) to \(e_j\) if and only if

1. \(e_j\) iteratively domino dominates \(e_i\);
2. \(e_j\) is Pareto superior to \(e_i\);
3. \(e_j\) is the most efficient equilibrium among all equilibria that iteratively domino dominates \(e_i\).

**Proof.** See Appendix A. ■

Proposition 1.5 states the sufficient and necessary condition for both instability and stability. To better understand the result, we define a weighted directed graph to represent iterative domino dominance.

**Definition 1.8 (NE Graph).** The NE graph of a game with a set of strict equilibria \(SE\) is a weighted directed graph \(G = (V, E)\) such that

1. the set of nodes \(V = SE\);
2. the set of edges \(E\) consists of all ordered pairs of strict equilibria \((e_i, e_j)\) where
   - \(e_j\) is Pareto superior to \(e_i\),
   - \(BR_{ij}\) is non-empty;
3. the weight of an edge \((e_i, e_j)\) is given by \(d_{ij}\).
1.3. Equilibrium Switching

Figure 1.2: An example of NE graph and domino graph with mutual domino dominated equilibria.

Fix a cognitive distribution, equilibrium switching is feasible along some edges, but not all of them. We can delete edges where the population fails in equilibrium switching, i.e., domino dominance does not hold. We call the resulting graph the \textbf{domino graph} of the game.

Proposition 1.5 implies that a strict Nash equilibrium $e_i$ is unstable if and only if there exists a directed path connecting $e_i$ to another Pareto superior equilibrium $e_j$ in the domino graph. Below we provide some examples to illustrate Proposition 1.5.

\textbf{Example 1.3.} Consider a game with three strict Nash equilibria $e_1$, $e_2$, $e_3$, and the equilibrium payoffs satisfy $F_1(e_1) = F_2(e_2) < F_3(e_3)$. Further, suppose the cognitive distribution $f(k)$ of the population admits a maximal tipping value $\bar{T}(f) = 0.6$. Figure 1.2(a) and 1.2(b) show the NE graph and the domino graph of the game respectively. The domino graph tells us that $e_1$ is domino dominated by $e_2$, and is iteratively domino dominated by $e_3$ which is Pareto superior to $e_1$; $e_2$ is domino dominated by $e_1$ and strictly domino dominated by $e_3$. By Proposition 1.5, both $e_1$ and $e_2$ are unstable and a population will transit to $e_3$ following the path

$$e_1 \rightarrow e_2 \rightarrow e_3 \text{ or } e_2 \rightarrow e_3.$$ 

The fact that $e_1$ and $e_2$ are connected in both directions does not imply alternations between the two. In fact, transitions will never happen between $e_1$ and $e_2$ if there is no possibility of further transitions to a Pareto superior Nash equilibrium. This is why both $e_1$ and $e_3$ are not strictly domino dominated, but only $e_3$ is stable. It also illustrates why being strictly dominated is sufficient but not necessary for instability of a strict Nash equilibrium as stated in Corollary 1.1.
Example 1.4. Consider a game with a set of strict Nash equilibria \( \{ e_1, e_2, e_3, e_4 \} \), and the equilibrium payoffs satisfies \( F_1(e_1) < F_2(e_2) < F_3(e_3) < F(e_4) \). Further, suppose the cognitive distribution \( f(k) \) of the population admits a maximal tipping value \( \bar{T}(f) = 0.6 \). Figure 1.3(a) and 1.3(b) show the NE graph and the domino graph of the game respectively. Note that in this game, both \( e_3 \) and \( e_4 \) are not domino dominated, and both iteratively domino dominate \( e_1 \) and \( e_2 \). By Proposition 1.5, either from \( e_1 \) or \( e_2 \), the population will only transit to \( e_4 \). It is obvious because \( e_4 \) yields a higher equilibrium payoff than \( e_3 \). Therefore, despite that the population is capable of switching to \( e_3 \), it will never happen since there is a better alternative. On the other hand, \( e_3 \) is still stable in the sense that if a population is initially at \( e_3 \), the population will not transit away from it.

The graph representation provides a way to quickly determine whether an equilibrium \( e_j \) iteratively domino dominates \( e_i \), and if so, through which path. Denote \( C_{ij} \) an ordered chain of equilibria from \( e_i \) to \( e_j \) in the NE graph, and \( \mathcal{T}(C_{ij}) \) the set of all tipping points of the adjacent equilibrium pairs in the chain. For a tipping point \( T \in \mathcal{T}(C_{ij}) \), we can obtain the corresponding \( d(T) \), the minimal fraction of deviations required for equilibrium switching for a pair with a tipping point \( T \). Let \( \mathcal{D}(C_{ij}) \) be the set of all \( d(T) \) for all \( T \in \mathcal{T}(C_{ij}) \). Given a chain \( C_{ij} \), the number \( D(C_{ij}) := \max(\mathcal{D}(C_{ij})) \) reflects the minimal requirement on the cognitive distribution for \( e_j \) iteratively domino dominating \( e_i \) through this chain. That is to say, among all chains connecting \( e_i \) to \( e_j \), we only need to focus on the chain with the smallest \( D(C_{ij}) \), denoted by \( D^*_{ij} \).

Corollary 1.2. Fix a cognitive distribution \( f(k) \) with an associated maximal tipping value \( \bar{T}(f) \).
1.3. Equilibrium Switching

Figure 1.4: An example of the NE graph and domino graphs. The number next to an edge is the minimal fraction of deviations $d$ required for a directed transition. Panel (a) shows all finite chains among strict Nash equilibria; panels (b) and (c) are the domino graphs under cognitive distributions $f_1(k)$ and $f_2(k)$ respectively, where the associated maximal tipping values are $\bar{T}(f_1) = 0.6$ and $\bar{T}(f_2) = 0.5$.

1. a strict equilibrium $e_j$ iteratively domino dominates $e_i$ if and only if $D^*_{ij} \leq \bar{T}(f)$;
2. an equilibrium switching from $e_i$ to $e_j$ is feasible through a finite chain $C_{ij}$ if and only if $D(C_{ij}) \leq \bar{T}(f)$.

Proof. See Appendix A. □

The following example illustrates the above results.

Example 1.5. Consider a symmetric four-action coordination game with a set of strict Nash equilibria $\{e_1, e_2, e_3, e_4\}$. Figure 1.4(a) shows all finite chains among strict Nash equilibria, and the number next to an edge is the minimal fraction of deviations $d$ required for equilibrium switching. We can see that there are three finite chains connecting $e_1$ to $e_4$: $C_1 = \{e_1, e_2, e_3, e_4\}$, $C_2 = \{e_1, e_3, e_4\}$, $C_3 = \{e_1, e_2, e_4\}$. From Figure 1.4(a), we can easily obtain

$$\mathcal{D}(C_1) = \{0.3, 0.4, 0.6\} \quad \mathcal{D}(C_2) = \{0.4, 0.7\} \quad \mathcal{D}(C_3) = \{0.4, 0.5\}$$

$$D(C_1) = \max(\mathcal{D}(C_1)) = 0.6, \quad D(C_2) = \max(\mathcal{D}(C_2)) = 0.7, \quad D(C_3) = \max(\mathcal{D}(C_3)) = 0.5.$$

Consider two cognitive distributions $f_1(k)$ and $f_2(k)$ with the maximal tipping value $\bar{T}(f_1) = 0.6$ and $\bar{T}(f_2) = 0.5$ respectively. Figure 1.4(b) shows the domino graph under $f_1(k)$ and 1.4(c) shows the domino graph under $f_2(k)$. For $f_1(k)$, it is a domino distribution for all adjacent equilibrium pairs except $(e_1, e_3)$ whose tipping point is $T_{13} = 0.7$. Therefore, by deleting the edge $e_1e_3$, we obtain the domino graph.
under $f_1(k)$ in which there are two finite chains $C_1$ and $C_3$ which connect $e_1$ to $e_4$. It is easy to verify Corollary 1.2: $D(C_3) < D(C_1) < ar{T}(f_1) < D(C_2)$, therefore $C_1$ and $C_3$ are the two feasible switching paths. Similarly, under $f_2(k)$, we have $D(C_3) < ar{T}(f_2) < D(C_1) < D(C_2)$, therefore only $C_3$ is feasible.

**Proposition 1.6.** If there exists only one strict Nash equilibrium that is not strictly domino dominated, it iteratively strictly domino dominates all other strict Nash equilibria, and it is also the efficient Nash equilibrium.

**Proof.** See Appendix A.

Proposition 1.6 establishes a connection between domino dominance and efficiency. It indicates a unique selection of the efficient equilibrium provided that there are enough sophisticated players. If there are no sophisticated players at all, then all strict Nash equilibria are locally stable as in best response dynamics. For the intermediate case, the DCH model provides a refinement of strict Nash equilibria. The refined set of strict equilibria in general consists of equilibria with one of or both the following properties:

1. a relatively large BR region (in any direction),
2. the highest equilibrium payoff compared to most of its adjacent equilibria.

### 1.3.2 Equilibrium Switching with Disagreement

In reality, people’s interests may not be aligned and may have different preferences over a set of social states. Examples include the Battle of the Sexes and Hawk-Dove. In the presence of disagreement, it is even more interesting to talk about equilibrium switching: about which group will get the society to play the equilibrium they prefer. We focus on cases where there are two groups who have opposite preferences over two strict Nash equilibria.

Consider a population of $M$ players playing a two-action coordination game with an action set $A = \{1, 2\}$. There are two groups $\{1, 2\}$, and a group $i$’s share in the population is denoted $q_i$, which is public information to players. Let $x$ denote the fraction of action 2 in a population of $M - 1$ players. The payoff to action 1 and 2 played by a member in group $i$ is denoted $F^i_1(x)$ and $F^i_2(x)$.

**Assumption 1.4.** The payoff function $F^i(x)$ satisfies the following conditions:

1. **Strategic Complementarity** $F^i_1(x)$ is decreasing in $x$, and $F^i_2(x)$ is increasing in $x$ for all $i$;
2. **Single-Crossing** $F^i_1(0) > F^i_2(0)$ and $F^i_1(1) < F^i_2(1)$;
3. Disagreement in Efficiency \( F_1^1(0) > F_2^1(1) \) and \( F_1^2(0) < F_2^2(1) \).

The first condition in Assumption 1.4 makes that all members of the society have incentives to coordinate on the same action, and the single-crossing property of payoff functions suggests that there exist two pure strict Nash equilibria that are homogeneous states \( e_1 \) and \( e_2 \). The third condition basically says that two groups have opposite preferences over the two states: group 1 prefers \( e_1 \) to \( e_2 \) while group 2 prefers \( e_2 \) to \( e_1 \).

We first look at the case where all players are level-0. Let \( T_i \) be the solution to \( F_i^1(x) = F_i^2(x) \). A member of group \( i \)'s optimal strategy is simply

\[
s^0(x) = \begin{cases} 
1 & \text{if } x < T_i \\
2 & \text{if } x \geq T_i 
\end{cases}
\]  

Therefore \( T_i \) is the tipping point for members of group \( i \) to choose action 2 voluntarily. We assume that \( T_2 \) is lower than \( T_1 \). It is natural that group 2 has a lower tipping point since group 2 prefers \( e_2 \) over \( e_1 \).

**Proposition 1.7.** Suppose all players are level-0. There are two locally stable symmetric Nash equilibria \( e_1 \) and \( e_2 \). In addition, if \( q_2 \in (\frac{M-1}{M}T_2 + \frac{1}{M}, \frac{M-1}{M}T_1) \), there exists a locally stable asymmetric Nash equilibrium \( e_a = (q_1, q_2) \) where all members of group 1 play action 1 and all members of group 2 play action 2.

**Proof.** See Appendix A. \( \blacksquare \)

**Observation 1.4.** The BR regions of \( e_1 \) and \( e_2 \) depend on the existence of the asymmetric Nash equilibrium.

1. The BR region of \( e_1 \) is \([0, T_2] \), and the BR region of \( e_2 \) is \((T_2, 1] \) if \( q_2 \geq \frac{M-1}{M}T_1 \);
2. the BR region of \( e_1 \) is \([0, T_1] \), and the BR region of \( e_2 \) is \((T_1, 1] \) if \( q_2 \leq \frac{M-1}{M}T_2 + \frac{1}{M} \);
3. The BR region of \( e_1 \) is \([0, T_2] \), the BR region of \( e_2 \) is \((T_1, 1] \), and the BR region of the asymmetric Nash equilibrium \( e_a = (q_1, q_2) \) is \((T_1, T_2) \) if \( q_2 \in (\frac{M-1}{M}T_2 + \frac{1}{M}, \frac{M-1}{M}T_1) \).

Figure 1.5 depicts the dynamics of coordination games with disagreement in a population of level-0 players. Observation 1.4 tells us the tipping points for transitions from a Nash equilibrium to another. Without loss of generality, we consider the equilibrium switching from \( e_1 \) to \( e_2 \). Note that only group 2 wants to reach \( e_2 \), while group 1 prefers to stay at \( e_1 \). We first focus on cases where the asymmetric Nash equilibrium does not exist. The tipping point \( T_{12} = T_2 \) if \( q_2 \geq \frac{M-1}{M}T_1 \), and \( T_{12} = T_1 \) if \( q_2 \leq \frac{M-1}{M}T_2 \). Note that \( T_2 < T_1 \), if \( q_2 \leq \frac{M-1}{M}T_1 \), the size of group 2 is too
1.3. Equilibrium Switching

\[ (a) \quad q_2 \geq \frac{M-1}{M} T_1 \]

\[ (b) \quad q_2 \leq \frac{M-1}{M} T_2 + \frac{1}{M} \]

\[ (c) \quad q_2 \in \left( \frac{M-1}{M} T_2 + \frac{1}{M}, \frac{M-1}{M} T_1 \right) \]

Figure 1.5: Best response dynamics (level-0) of coordination games with disagreement. The segment \(e_1 e_2\) denotes the simplex in \(\mathbb{R}\) which is the domain of \(x\). \(e_1\) and \(e_2\) denote the two symmetric Nash equilibria respectively and \(e_a\) denotes the asymmetric Nash equilibrium.

small to switch the population from \(e_1\) and \(e_2\). Too see it clearly, if all members of group 2 play 2, \(x = \frac{M q_2}{M-1} \leq T_2\), which is still in the BR region of \(e_1\), and all members of group 1 will still play action 1. On the other hand, if \(q_2 \geq \frac{M-1}{M} T_1\), group 2 will have enough members to potentially switch the equilibrium.

**Proposition 1.8.** A level-\(k\) player in group 2 follows a cutoff strategy such that

\[ s^k_i = \begin{cases} 
2 & \text{if } x_{i-1} \geq T_2 - \frac{k}{M-1} \text{ and } q_2 \geq \frac{M-1}{M} T_1, \\
1 & \text{otherwise.}
\end{cases} \]

**Proof.** See Appendix A. □

Denote \(f^i(k)\) the cognitive distribution within group \(i\).

**Definition 1.9.** A distribution \(f^2(k)\) is a domino distribution with respect to the equilibrium-pair \((e_i, e_j)\) if

\[ q_2 \sum_{l=k}^{K} f^2(l) \geq \frac{M - 1}{M} d_{ij} - \frac{k - 1}{M} \text{ for } k \in \{1, 2, ..., K\}. \]

**Proposition 1.9.** A population will switch from \(e_1\) to \(e_2\) if

1. \(q_2 \geq \frac{M-1}{M} T_1\);
2. \(f^2(k)\) is a domino distribution with respect to \((e_1, e_2)\), i.e., \(d_{ij} = T_2\).

**Proof.** See Appendix A. □

Proposition 1.8 and 1.9 suggest three factors required for equilibrium switching with disagreement. First, it requires the group that wants to switch equilibrium should be large enough \((q_2 \geq \frac{M-1}{M} T_1)\). Second, the group that wants to switch needs to have enough sophisticated members to initiate switching (domino distribution). Last, the sizes of groups in society should be public information so that high-level players are willing to initiate switching.
Next, we consider the case where the asymmetric Nash equilibrium exists.

**Observation 1.5.** The asymmetric Nash equilibrium is Pareto inferior to $e_1$ and $e_2$.

Observation 1.5 suggests that both groups do not want to stay at the asymmetric equilibrium in the long run. So the existence of the asymmetric equilibrium may deter potential equilibrium switching if the society has a probability to reach the asymmetric equilibrium. On the other hand, it also makes alternations between the two symmetric equilibria possible provided that both groups are capable of moving the society away from the asymmetric Nash equilibrium.

Note that the BR regions of $e_1$ and $e_2$ are disjoint, to switch equilibrium from $e_1$ to $e_2$ (or from $e_2$ to $e_1$), the population must first reach BR regions of the asymmetric Nash equilibrium. Let $e_a$ denote the asymmetric Nash equilibrium. If $f^2(k)$ is a domino distribution with respect to the pair $(e_1, e_a)$, then group 2 can switch the equilibrium from $e_1$ to $e_a$ where all members of group 2 play action 2. To further switch equilibrium from $e_a$ to $e_2$, it can be done only by group 1 that prefers $e_1$. Under what conditions will group 1 switch the equilibrium from $e_a$ to $e_2$? First, group 1 can switch equilibrium from $e_a$ to $e_2$, and second, group 2 cannot switch equilibrium from $e_a$ to $e_1$. If that is the case, group 1 will switch the equilibrium since $e_2$ is Pareto superior to $e_a$ for group 1, as illustrated in Figure 1.6(a).

**Corollary 1.3.** For $q_2 \in (\frac{M-1}{M}T_2 + \frac{1}{M}, \frac{M-1}{M}T_1)$, a population will switch from $e_1$ to $e_2$ only if

1. $f^2(k)$ is a domino distribution with respect to $(e_1, e_a)$;
2. $f^1(k)$ is a domino distribution with respect to $(e_a, e_2)$;
3. $f^2(k)$ is not a domino distribution with respect to $(e_a, e_1)$.
1.3. Equilibrium Switching

Corollary 1.3 suggests that information about the sophistication of a group is important. Note that, if there are not many high-level players in group 1 so that $f^1(k)$ is not a domino distribution with respect to $(e_a, e_2)$, group 2 will not switch equilibrium from $e_1$ to $e_a$ in the first place. This suggests that it may be better for a group to be naive in some cases.

**Corollary 1.4.** For $q_2 \in (\frac{M-1}{M} T_2 + \frac{1}{M}, \frac{M-1}{M} T_1)$, persistent alternations between the two symmetric Nash equilibria $e_1$ and $e_2$ can arise only if

1. $f^2(k)$ is a domino distribution with respect to $(e_1, e_a)$ and $(e_a, e_1)$;
2. $f^1(k)$ is a domino distribution with respect to $(e_a, e_2)$ and $(e_2, e_a)$.

Corollary 1.4 implies that when both groups are sufficiently sophisticated, an alternation cycle may emerge as a form of mutual compromise as shown in Figure 1.6(b). A notable feature of the cycle arising from sophistication is that the time spent on the two symmetric equilibria should be significantly longer than the duration of transitions. This will guarantee that both groups can profit from the alternation cycle.

The results of equilibrium switching with disagreement can be extended to asymmetric games with multiple strict Nash equilibria. The following example illustrates the results using a two-population random matching Hawk-Dove game.

**Example 1.6.** Consider two populations (or groups) randomly matched to play a Hawk-Dove game as shown in the following table, where $v/2 - c < 0$, and players 1 and 2 are randomly drawn from population 1 and 2 respectively. Let $x$ and $y$ denote the fraction of $H$ in population 1 and 2 respectively. Note that for $y < v/2c$ ($x < v/2c$), it is optimal for level-0 players in population 1 (population 2) to play $H$, otherwise it is optimal for them to play $D$. There exists two stable strict Nash equilibria $e_1 = (1, 0)$ and $e_2 = (0, 1)$ if all players are level-0, and population 1 prefers $e_1$ while population 2 prefers $e_2$.

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>$(v/2 - c, v/2 - c)$</td>
</tr>
<tr>
<td>$D$</td>
<td>$(v, 0)$</td>
</tr>
<tr>
<td></td>
<td>$(0, v)$</td>
</tr>
<tr>
<td></td>
<td>$(v/2, v/2)$</td>
</tr>
</tbody>
</table>

**Table 1.3: Hawk-Dove**

Let us consider equilibrium switching from $e_1$ to $e_2$ (as shown in the left panel of Figure 1.7). Since population 1 prefers $e_1$, no players from population 1 are willing to deviate. However, if there are enough high-level players in population 2 such that they can move the $y$ to be greater than $v/2c$, level-0 players from population
1.3. Equilibrium Switching

Figure 1.7: Equilibrium switching in Hawk-Dove games. The horizontal (vertical) axis is the fraction of hawks in the population 1 (2). $e_1 = (1, 0)$ and $e_2 = (0, 1)$ are two pure Nash equilibria, and the dotted minor diagonal is the boundary between BR regions of $e_1$ and $e_2$ with the upper (lower) triangle being the BR region of $e_2$ ($e_1$). The state $(v/2c, v/2c)$ is the unstable mixed equilibrium, and $x = v/2c$ and $y = v/2c$ are the set of tipping points for equilibrium pair $(e_2, e_1)$ and $(e_1, e_2)$ respectively. Blue arrows represent the deviations from sophisticated players in one population and red arrows represent the follow-up best response of level-0 players.

1 will play $D$. If population 1 does not have enough high-level players, this will further make $x < v/2c$, which means level-0 players from population 2 will play $H$. Equilibrium will be switched to $e_2$ where all players from population 1 play $D$ and all players from population 2 play $H$. On the other hand, if both populations are sufficiently sophisticated, an alternation cycle can emerge (as shown in the right panel of Figure 1.7).
1.3. Equilibrium Switching

1.3.3 Discounted Utility

In previous sections, we have assumed that level-$k$ players have preferences in the form of the limit of the means. In such a setting, high-level players completely disregard short-run loss during a switch. We now consider cases where high-level players have a $\delta$-discounted utility function. In this setting, high-level players care about the duration of transitions. Intuitively, if a potential switch is expected to take a very long time, the short-run loss may eventually outweigh the future gain so that high-level players will not initiate a switch in the first place.

The duration of a transition can be affected by how fast players learn about their opponents’ behaviors. Suppose level-0 players now use the average play of the most recent $W$ periods of their opponents as their predictions about their opponents’ behaviors in the next period. It is well-known that the higher $W$ is, the lower the learning speed. A lower learning speed then implies that the duration of transitions will be relatively long. High-level players who deviated earlier would suffer more losses as they will play a suboptimal action for a longer time. The losses during transitions may outweigh the future gains. Hence, the optimal strategy of high-level players may be mimicking the myopic best response, therefore no cutoff strategy and equilibrium switching.

**Observation 1.6.** Fix a discounting rate $\delta$, and an adaptive learning rule with $W$-period memory.

1. Given $\delta$, there exists an upper bound $\bar{W}$, such that for $W < \bar{W}$, Proposition 1.1 and 1.2 hold;
2. given $W$, there exists a lower bound $\bar{\delta}$ such that for $\delta > \bar{\delta}$, Proposition 1.1 and 1.2 hold.

Another factor that may affect the incentive of equilibrium switching is the payoff structure of the stage game. Changes in payoffs not only affect tipping points but also gains and losses incurred during a switch. Raymond Battalio et al. (2001) find that in coordination games with the same tipping point, deviations from mBR can also be affected by what they call optimization premium $\eta$, a function of payoff parameters. They find the probability of deviations from mBR decreasing in $\eta$ given the tipping point fixed.

---

13Examples include finite-period fictitious play, reinforcement learning, or any type of adaptive learning
1.3. Equilibrium Switching

We show in the following that the optimization premium effect can also be deduced for the DCH model with discounted utility. Building on Raymond Battalio et al. (2001), we define the parameter of optimization premium in a multiplayer two-action coordination game as follows. Let $x$ be a scalar that represents the fraction of action 2, and tipping point $T$ be the solution to equation $F_1(x) = F_2(x)$. The loss of deviations is

$$G(x) := |F_1(x) - F_2(x)|$$

$$= |G(x) - G(T)|$$

$$\approx |G'(T)|(x - T)|.$$

The parameter of optimization premiums is $\eta := |G'(T)|$. Notice that high $\eta$ implies high losses due to deviations from mBR at all levels of $x$. In the DCH model with discounted utility, the loss during transitions is increasing in $\eta$.

**Observation 1.7.** Fix a discounting rate $\delta$ and memory length $W$, there exists an upper bound $\bar{\eta}$, such that for all $\eta < \bar{\eta}$, Proposition 1.1 and 1.2 hold.

Discounted utility has a very important implication on switching failure as discussed above. Notice that with discounted utility, deviated high-level players will turn to play the myopic best response quickly when the cognitive distribution is not a domino distribution as in this case, no player will follow their deviations.

1.4 Discussions

The dynamic cognitive hierarchy model has several interesting implications for individual behavior and macro social changes. For example, the DCH model implies a new source of suboptimal behaviors that have been observed in experimental studies and in the real world; it also suggests a strong relationship between the observability of choices and strategic play. In this section, we discuss these implications and related literature.

1.4.1 Suboptimal Choices

Experimental studies often find that human subjects do not always play the myopic best response to the information they access. Many models of stochastic choices have been proposed in which players predominantly play the myopic best response but occasionally deviate from it. Examples include uniform deviation (Young, 1993)
1.4. Discussions

where all non-optimal choices are played with the same probability; payoff-dependent
deviation (Blume, 1993; Myatt and Wallace, 2003) where deviations involving higher
payoff loss compared to the myopic best response are less likely to happen. These
models often see these deviations as nonstrategic mistakes and attribute suboptimal
choices to cognitive limitations such as the lack of understanding of games or (ra-
tional or behavioral) inattention. The DCH model, however, suggests that there is
another type of deviation which are strategic.

The DCH model predicts that the two types (strategic and nonstrategic) of
deviations have three very different features. First, strategic deviations are less
payoff-dependent than nonstrategic ones as high-level players tend to deviate in
the early stage of equilibrium switching which often incurs the highest payoff loss.
Second, strategic deviations should have a higher level of serial correlation. This is
because strategic deviations are often made to manipulate or teach low-level players,
therefore they have to be persistent so that low-level players can catch up. The third
feature is the direction of deviations. The DCH model predicts that a sophisticated
player often deviates in the efficiency-improving direction, i.e., they deviate from
inefficient states to the efficient state. However, deviations from less sophisticated
players can be in both directions considering the possibility of mistakes.

The existing experimental results on deviations from the myopic best response
are mixed, although they are still consistent with the predictions of the DCH model.
For example, Raymond Battalio et al. (2001) find that deviations from the myopic
best response are payoff-dependent, but subjects also tend to deviate to the efficient
Nash equilibrium on average. On the other hand, Lim and Neary (2016a) find that
deviations in their experiment are not payoff-dependent. In addition, deviations
are directed to the efficient equilibrium and highly serial correlated. Both studies
did not differentiate between strategic and nonstrategic deviations, which could be
the reason for the mixed results. Indeed, Li et al. (2022) find results supporting
this conjecture. They differentiate deviations by directions and find that deviations
from mBR to the efficient action are less correlated with the payoff loss than those
to inefficient actions. In addition, they also find that subjects with higher level-k
reasoning are more likely to persistently deviate from mBR to efficient action.
1.4. Discussions

Strategic deviations made by high-level players in the DCH model rely on crucially the observability of choices. If these deviations cannot be observed by other players at all, high-level players have no incentive to deviate. The experimental results in Li et al. (2022) also support this implication. They find that subjects with more accurate information about opponents’ choices are more likely to engage in strategic deviations persistently.

1.4.2 Equilibrium Selection

The DCH model relates to other models of equilibrium selection: risk-dominance and payoff-dominance (Harsanyi and Selten, 1988); stochastic stability (Foster and Young, 1990; Kandori et al., 1993; Young, 1993); global games (Carlsson and Van Damme, 1993). Harsanyi and Selten (1988)’s axiomatic approach provides two complementary selection principles: risk dominance and payoff dominance. A series of experimental studies have focused on testing the relative salience of the two principles, but the results are mixed (Van Huyck et al., 1990, 1991; Rankin et al., 2000; Schmidt et al., 2003; Blume and Ortmann, 2007). The DCH model predicts the stability of equilibria that are not domino dominated. Note that domino dominance has ingredients from both principles - a strict equilibrium that is not domino dominated is either (locally) efficient among its adjacent equilibria or its best response region is sufficiently large. Here the size of best response regions (or basin of attraction) is a reflection of risk dominance - in $2 \times 2$ games, the two concepts are equivalent. Therefore, the mixed experimental results on the relative salience of the two principles may be a result of the difference in the cognitive distributions of subject groups.

The selection or refinement result of the DCH model arises from intentional deviations made by sophisticated players. This is in contrast with the theory of evolutionary selection based on stochastic stability (Foster and Young, 1990; Kandori et al., 1993; Young, 1993). Stochastic stability predominantly selects the risk-dominant equilibrium as the selection result arises from the assumption that players occasionally deviate from the myopic best response so that a population could escape from the BR region of a strict Nash equilibrium if enough deviations happen simultaneously. With the probability of deviations converging to zero in the time limit, a population is most likely to be fixed at the Nash equilibrium with the largest BR region. The evolutionary selection also takes a very long time to happen as it is
rare to have multiple simultaneous deviations in the same direction (Ellison, 2000). In contrast, the equilibrium selection (switching) predicted by the DCH model need not take a long time as intentional deviations accumulate in the same direction over time.

The theory of global games also yields cutoff strategies and equilibrium selection, but the DCH model is fundamentally different. First, there exists uncertainty about the economic fundamentals in global games. Players observe private signals about the underlying state of the world, and the cutoff strategy is in terms of signals. In the DCH model, however, there is no uncertainty, the cutoff is a fraction of players playing certain actions, and the equilibrium selection/switching is deterministic. In addition, the solution concept of global games is static, while equilibrium switching in the DCH model is gradual with predictions on the timing of individual participation.

1.4.3 Large-scale Social Changes

The DCH model implies that the minimal level of sophistication required for equilibrium switching, $\lceil (M - 1)T \rceil$, increases with the population size $M$. This casts a doubt on whether the DCH model is relevant in large-scale social changes such as political protests and revolutions. The reason is that experimental studies on bounded rationality, e.g., Stahl and Wilson (1995); Nagel (1995); Stahl (1996); Costa-Gomes et al. (2001); Costa-Gomes and Crawford (2006), often find that human subjects are generally below level 5. However, Alaoui and Penta (2016) propose a theory that the level of sophistication is endogenously determined by economic incentives: If the economic gain and loss are relatively high, players may choose to be more sophisticated. Alaoui and Penta (2022) find experimental evidence supporting this argument.

Another factor that eases the population-size problem is social organization. In reality, social members are often organized into formal and informal social groups where group members move together as a unity. Examples include political parties and unions of workers. Such social organizations could significantly reduce the level required for social changes. We can imagine that a large population is partitioned into several groups, and the game of social change is played only by these groups. One of the most famous revolutionaries Vladimir Ilich Lenin once wrote (Lenin, 1935) “Give me an organization of revolutionaries, and I will overturn Russia.” An example is the 1979 Iranian revolution in which the nationwide protests against the Pahlavi royal house were successfully initiated by geographically isolated groups of radicals.
and clergies who were, however, unified through the network of mosques across the country. A retrospective speech by Hojat al Islam Khamene’i, then President of the Republic, on 4 June 1986 emphasized the important role of the nationwide mosque network in the revolution (Vakili-Zad, 1990)

...a need for an organization and to educate the public. A movement needs organization, ideology and qualified leaders...

### 1.5 Concluding Remarks

In this paper, we introduce a novel dynamic cognitive hierarchy model of repeated games. The DCH model generates predictions on transitions among strict Nash equilibria and predominantly selects the payoff-dominant equilibria. The DCH model combines the idea of strategic manipulation and heterogeneity in cognitive ability so that the more sophisticated players can manipulate the plays of the less sophisticated. Equilibrium switching requires a proper distribution of cognitive levels. One important feature embedded in such distributions is that the minimal level of sophistication required for equilibrium switching increases with population size. The DCH model also yields interesting implications on suboptimal behaviors observed in experimental studies. Suboptimal behaviors are not necessarily mistakes - they could also be strategic plays intended to manipulate other players. The DCH model predicts that the two types of suboptimal behaviors (strategic plays and nonstrategic mistakes) should have very different statistical features on direction, serial correlation, and payoff dependence which have been observed in lab experiments.
Chapter 2

Inexact Information and Equilibrium Transition

2.1 Introduction

Understanding whether and how agents can move out of bad equilibria to more efficient ones is critical for the improvement of social welfare. Recent advancements in the evolutionary game theory show that sufficiently “inexact” (i.e., inaccurate but unbiased) information about opponents’ behaviors can facilitate spontaneous transitions among strict Nash equilibria (Ellison, 1997; Sandholm, 2001a; Oyama et al., 2015; Sawa and Wu, 2021) (hereafter the inexact information models). The intuition is that agents may form false beliefs that the majority of a population has deviated from an established equilibrium even if only a small proportion has done so, and hence voluntarily deviate from a strict equilibrium. However, the transitions in these models rely on two key assumptions: first, agents best respond to signals regardless of accuracy; and second, agents are myopic so that they only care about their immediate but not long-term gains or losses. It is critical to know whether human subjects follow such assumptions and whether the inexact information can facilitate efficiency-improving transitions among strict Nash equilibria empirically. Answering these questions is very much relevant in the era of social media where information is easier to be transmitted and manipulated.

This paper looks into the role of information about opponents’ behaviors in equilibrium transitions in a quasi-continuous-time lab experiment. We investigate whether it is more or less accurate information that facilitates equilibrium transitions and how agents manage to do so if a transition succeeds. We design the experiment and derive theoretical predictions based on sampling best response dynamics (sBRD) (Oyama et al., 2015). In sBRD, a population of agents play a coordination game
recurrently, observe a private random sample of the population, and then play a
myopic best response to their private samples. In our experiment, 14 subjects play a
three-action coordination game with three strict equilibria different in efficiency and
the initial state being the least efficient equilibrium. We control the information by
varying the sampling size $s$, that is, $s = 2$ vs. 7. The theoretical prediction of sBRD
is that the population (group) transits away from the least efficient equilibrium to
the most efficient one under the treatment of $s = 2$ while to the medium-efficient
one or staying at the status quo under the treatment of $s = 7$.

This paper finds novel patterns of equilibrium transitions with inexact informa-
tion that differ from the sBRD predictions. Although groups are observed to transit
away from the least efficient equilibrium in all of the sessions, transitions to the most
efficient equilibrium are more frequent under $s = 7$ than $s = 2$. Specifically, all of the
session groups under $s = 7$ transited to the efficient equilibrium, while only one half
under $s = 2$ transited to the efficient one with the other half to the medium-efficient
equilibrium. The reason for the inconsistency between our experimental results and
the theoretical predictions is twofold.

First, we find that subjects’ responses do depend on the accuracy of the in-
formation: subjects tend to be less responsive to changes in signals with a smaller
sampling size. The probability of playing an action is observed to increase with the
fraction of that action included in subjects’ samples under both treatments in our
experiment, but subjects under $s = 7$ adjust the probability more promptly than
those under $s = 2$. The less and slower responsiveness associated with inaccurate
signals results in more deviations from myopic best responses that point to a less
efficient equilibrium (downward deviations hereafter) under $s = 2$, which delay or
prevent transitions from the medium-efficient equilibrium to the most efficient one.
Second, we find that more accurate information induces more long-term strategic
behaviors, resulting in more successful transitions to the efficient Nash equilibrium
due to more persistent deviations that point to a more efficient equilibrium (upward
deviations hereafter) under $s = 7$ than $s = 2$.

The long-term strategic behaviors of persistent upward deviations induced by
more accurate information in our results support the models of strategic teaching
(Camerer et al., 2002c; Naidu et al., 2010; Lyu, 2022). In these models, forward-
looking agents teach myopic ones to play the efficient equilibrium by persistent
upward deviations from an inefficient equilibrium. Lyu (2022) shows in theory that
sufficiently accurate information is necessary for strategic upward deviations since
forward-looking agents are willing to do so only if the opponents can observe such de-
viations and also forward-looking agents can observe whether the opponents indeed follow to deviate in the future. Moreover, we find upward deviations are less correlated to the expected immediate payoff loss than downward deviations, indicating that farsighted subjects think beyond the immediate loss of upward deviations.¹

Our results on the correlation between deviations and the payoff loss of deviations also contribute to the experimental literature on noisy suboptimal choices in games. For example, Battalio et al. (2001); Bilancini et al. (2020) find that deviations from the myopic best response on average are cost-dependent, but Lim and Neary (2016b) find that deviations are not cost-dependent. The mixed experimental evidence may be explained by the differentiation between upward and downward deviations found in our experiments. Downward deviations may be mistakes or arise from inattention, and are strongly affected by the cost of deviations; while upward deviations are more likely to be intentional and hence are less sensitive to the cost.

This paper has a methodological contribution to the design of evolutionary game experiments. The initial population state plays a key role in experiments of equilibrium transitions (or evolutionary games in general). In contrast with the approach of changing preferences used necessarily to induce subjects to play certain actions (e.g., Smerdon et al., 2019a; Duffy and Lafky, 2021b; Andreoni et al., 2021b), we propose a simple protocol that places all subjects initially at a given population state. We find this protocol is effective in a quasi-continuous-time experiment where subjects make choices asynchronously and a population is less likely to jump away from the assigned initial state.

This paper also relates to a growing literature on experimental studies of social changes. Smerdon et al. (2019a), Duffy and Lafky (2021b), and Andreoni et al. (2021b) study the social changes with perfect and imperfect information on individuals’ private preferences over two alternative social states in discrete-time lab experiments. More information about opponents’ preferences is found to be beneficial for efficiency-improving social changes, since imperfect information may hinder social changes by invoking pluralistic ignorance - individuals are not sure whether the majority of society prefers the status quo or the alternative state. We find inexact information about opponents’ behaviors generates similar effects in our experiment when subjects have common knowledge about preferences.

¹Our finding also implies that the equilibrium transitions observed in our experiment are different from those arising from random mistakes as in the models of stochastic stability (Foster and Young, 1990; Young, 1993; Kandori et al., 1993).
2.1. Introduction

It is worth noting the difference between our research and the experimental literature on the role of information in equilibrium selection based on global games (Heinemann et al., 2004; Anctil et al., 2004; Cornand, 2006; Cabrales et al., 2007; Van Huyck et al., 2018). The latter focuses on testing the prediction of static equilibrium plays, where subjects receive exogenously pre-specified imperfect (private or public) signals about a payoff-relevant state. We investigate transition dynamics where subjects are uncertain about the opponents’ current behaviors and the received signals depend on the choices of their opponents.

The rest of the paper is organized as follows. Section 3.2 presents the experimental design, hypotheses, and implementation. In section 3.3, we discuss the main findings of the experiment. The last section concludes.

2.2 Experimental Design and Hypotheses

2.2.1 The Game

In the experiment, we simulate a population of agents randomly matched to a two-person coordination game recurrently. The coordination game has an action set $S = \{1, 2, 3\}$ and the payoff matrix is given by

$$G = \begin{pmatrix}
20 & 12 & 6 \\
12 & 24 & 18 \\
0 & 14 & 30
\end{pmatrix},$$

where $G_{jk}$ corresponds to the payoff of action $j \in S$ when the opponent plays action $k \in S$. $X = \{x \in \mathbb{R}^{|S|} : \sum_k x_k = 1\}$, the simplex in $\mathbb{R}^{|S|}$, is the set of population states (or mixed strategies); for each population state $x \in X$, $x_k$ is the fraction of players choosing action $k \in S$; $e_k$ for each $k \in S$ represents the population state where all players use action $k$. In the random matching environment, the expected payoffs of actions are given by a function $F(x) = Gx \in \mathbb{R}^{|S|}$, where the $k$th element of $F(x)$ is the expected payoff to action $k$ at population state $x$.

The game has three strict Nash equilibria of $e_1$, $e_2$, and $e_3$, ranked by efficiency in ascending order. The question is whether and how a population can transit from $e_1$ to a more efficient state. If players have the full information of the population state $x$ and play a myopic best response to $x$, the evolution of population state can
be modeled by the standard Best Response Dynamic (BRD) (Gilboa and Matsui, 1991). The three monomorphic states $e_1$, $e_2$, $e_3$ are locally stable under BRD (Figure 2.1(a)). Thus, a transition away from $e_1$ is unlikely to happen with the full information if all players are myopic.

Note that the game used in the experiment has no fully mixed Nash equilibrium. Therefore, the best response regions of the three actions (or the basins of attraction of strict Nash equilibria) only intersect one by one, as shown in Figure 2.1(a). It suggests that to transit away from $e_1$ to $e_3$, a population must first transit to $e_2$. In addition, it is worth noting that along the edge $e_1e_2$, the tipping point of transitions from $e_1$ to $e_2$, $T_{12}$, is 0.4, i.e., it requires more than 40% of the population to play action 2 to transit from $e_1$ to $e_2$. Similarly, along the edge $e_2e_3$, the tipping point is roughly 0.45. Therefore, the transition from $e_2$ to $e_3$ is more difficult than that from $e_1$ to $e_2$.

When agents only observe a finite sample of the population and thus play a myopic best response (mBR) to the empirical distribution of actions in the sample, the population might be able to transit to a more efficient equilibrium under the sampling Best Response Dynamics (Oyama et al., 2015). Figure 2.1(b) and 2.1(c) illustrate the sampling dynamics of the game with a sampling size of $s = 2$ and $s = 7$.

---

\(^2\)Phase diagrams in this paper are plotted in Dynamo developed by Franchetti and Sandholm (2013).
2.2. Experimental Design and Hypotheses

Table 2.1: Transition probabilities from $e_1$ to $e_2$, $T_{12} = 0.4$, $N = 14$, one-shot deviation from action 1 to 2.

<table>
<thead>
<tr>
<th># of Deviations</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = 2$</td>
<td>0.838</td>
<td>0.971</td>
<td>0.992</td>
<td>0.999</td>
<td>1.000</td>
</tr>
<tr>
<td>$s = 7$</td>
<td>0.016</td>
<td>0.192</td>
<td>0.614</td>
<td>0.902</td>
<td>0.989</td>
</tr>
</tbody>
</table>

Table 2.2: Transition probabilities from $e_2$ to $e_3$, $T_{23} = 0.45$, $N = 14$, one-shot deviation from action 2 to 3.

<table>
<thead>
<tr>
<th># of Deviations</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = 2$</td>
<td>0.823</td>
<td>0.961</td>
<td>0.995</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$s = 7$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.014</td>
<td>0.105</td>
<td>0.363</td>
</tr>
</tbody>
</table>

$s = 7$ respectively. When $s = 2$, all of the interior trajectories converge to $e_3$. When $s = 7$, the stability of the dynamics is similar to the full-information case, but the basin of attraction of $e_1$ becomes smaller, suggesting an easier transition from $e_1$ to $e_2$ than with the full information.\(^3\)

The theoretical predictions above cannot be directly applied to a lab environment where the population size is relatively small. The reason is that sampling BRD can be seen as a deterministic approximation of a stochastic process when the population size is sufficiently large. When the population size is small, e.g., small groups in lab experiments, there are non-negligible stochastic components that may affect transition probabilities. We estimated transition probabilities in small populations under different sampling sizes. Tables 2.1 and 2.2 present the transition probabilities in 1000 simulations from $e_1$ to $e_2$ and from $e_2$ to $e_3$ for the population size $N = 14$ given the number of one-shot deviations respectively. The estimations are generally consistent with the predictions of sampling BRD with one exception that it is possible to transit from $e_1$ to $e_2$ with $s = 7$. However, it is still unlikely to transit from $e_2$ to $e_3$ under $s = 7$ unless there are sufficiently many deviations.

2.2.2 Testable Hypotheses

We derive the testable hypotheses from the theoretical predictions of sampling BRD with $e_1$ as the initial state. Hypothesis 1 describes the equilibrium transitions resulting from transition probabilities under the two sampling sizes presented in Tables 2.1 and 2.2.

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\(^3\)In fact, for the game used in this paper, all strict Nash equilibria are locally stable for $s > 2$, and $e_3$ is almost globally stable for $s = 2$. The choice of the two sampling sizes 2 and 7 is further determined by the differences in the transition probabilities under the two sampling sizes (see Table 2.1 and 2.2).
2.2. Experimental Design and Hypotheses

**Hypothesis 1.** Transitions among the three strict Nash equilibria:

(a) The population will transit away from $e_1$ more frequently under $s = 2$ than under $s = 7$.

(b) The population will transit to $e_3$ more frequently under $s = 2$ than under $s = 7$.

Hypothesis 2 tests whether subjects play myopic best response given the inexact information regardless of the accuracy of the information determined by the sampling size.

**Hypothesis 2.** Subjects best respond to the empirical distribution of actions in their private samples regardless of the sampling size.

Hypothesis 2 has an important implication on equilibrium transition. If the rate of best response depends on the accuracy of the information, the equilibrium transition predicted by sBRD may fail to appear. The reason is that the information indicating an action corresponding to a more efficient equilibrium as the mBR is rare at the early stage of transitions. If subjects do not best respond to such information immediately, it will require more persistent efficiency-improving costly deviations from mBR to initiate a transition.

2.2.3 Experimental Design and Implementation

In each session, a group of 14 subjects played the game as described above for 80 periods with a sampling size of either $s = 2$ or $s = 7$. Each period lasted for 10 seconds. At the beginning of a period, every subject received a private signal, which consists of $s$ random draws of their opponents’ action choices in the last period with replacement (see Figure B.12 in Appendix B.4 for the interface). Hereafter, we denote a signal by $\hat{x} = (s_1/s, s_2/s, s_3/s)$, where $s_k$ is the sampled number of action $k$ in the signal with $\sum s_k = s$. Upon receiving the signals, subjects decided whether to choose an action or not in the current period. The decisions in the last period were carried over if no decision was made. That is, subjects were free to change their previous decisions in a period in the sense that the decision-making in our design was in the quasi-continuous environment and thus asynchronous. The

---

4Here $\hat{x}$ is the empirical distribution of actions in the received random sample. It is also an unbiased estimator of population state $x$. 
population state and payoff were updated at the end of each period. Importantly, neither the feedback of the subjects’ own payoff nor that of their opponents’ was provided during the game to avoid potential confounding effects - the payoff feedback might be perceived as complementary information for decision-making.

To set $e_1$ as the status quo, we employed a novel design of assigned actions - action 1 was assigned to every subject as the default option before the game started.\(^5\) Note that the signals in the first period were based on the default state $e_1$, and thus all the subjects received a signal $(1, 0, 0)$, which indicates that action 1 is the myopic best response. As a result, myopic subjects would always play action 1 and thus be stuck at $e_1$ if there is no disturbance such as mistakes or intentional deviations from mBR. Further, the asynchronous move combined with the 10-second time limit makes abrupt large deviations from the default state practically difficult in the first period, so the state quo is expected to be sustained. Thus, the default option design can ensure $e_1$ as the state quo.\(^6\)

The accumulated payoff of all the 80 periods was paid. At the end of the experiment, subjects answered a paid survey, including self-report demographic questions, cognitive reflection tests (Frederick, 2005; Toplak et al., 2011), social preferences (Falk et al., 2016, 2018), incentivized risk preferences (Eckel and Grossman, 2002; Reynaud and Couture, 2012) and level-k thinking using the 11-20 game (Arad and Rubinstein, 2012).

The experiment was conducted in the Finance and Economics Experimental Laboratory at Xiamen University in China using oTree (Chen et al., 2016). A total of 112 subjects were recruited from Xiamen University for the 8 sessions. Each subject only participated in one session, and each session lasted for about 45 minutes. The instructions were read aloud in front of the subjects.\(^7\) Subjects had to pass a comprehension quiz and had an unpaid 10-period training session before proceeding to the game with payments. The average earnings were 26 CNY ($\approx$ 4.02 USD) per subject, close to doubled students’ hourly wage rate.\(^8\) For summary statistics of our data set, see Appendix B.2 Table B.2.

---

\(^5\)Subjects were informed that they would be assigned an action and only know which action was assigned to themselves right before the game started.

\(^6\)Our results show that by using a default option every session stayed inside of basin of attraction of $e_1$ in the first period.

\(^7\)A copy of instructions translated in English is in Appendix B.3.

\(^8\)The hourly students’ wage rate at Xiamen University was 18 CNY at the time of the experiment.
2.3 Results

2.3.1 Transitions

Figure 2.2 plots the evolution of the fractions of players choosing each action in each session, and it provides visual evidence on novel patterns of equilibrium transitions.

Result 1. Both inexact information treatments induce transitions away from the least efficient equilibrium to a more efficient equilibrium; the transitions to the most efficient equilibrium $e_3$ are more frequent with more accurate information of $s = 7$ compared to $s = 2$.

Figure 2.2: Evolution of fractions of actions by sessions.

Figure 2.2 shows that the fraction of action 3 (the red curve) is close to 100% at the end of the game in all of the sessions under $s = 7$; while under $s = 2$, only two groups (Sessions 1 and 2) transited to $e_3$ with the other two (Sessions 3 and 4) to $e_2$. In particular, there exists one session under $s = 7$ (Session 5) that the group transited away from $e_1$ to $e_3$ directly and rapidly without passing through the neighborhood of the medium-efficient equilibrium $e_2$.

\footnote{Here transitions to an equilibrium include cases where populations transit to a small neighborhood of the equilibrium.}
2.3. Results

Figure 2.3: Distance to the equilibrium each session arrived at the end of the session. The reference lines are the estimated structural breaks.

To quantify the equilibrium transitions in Figure 2.2, we plot the distance of the population state to the final equilibrium reached over time with estimated structural breaks (Figure 2.3), which suggest trends the sessions have at different stages. A significant trend combined with the final arrival at an equilibrium (or neighborhood of an equilibrium) can be interpreted as a convergence to the equilibrium arrived. Table B.3 columns (1) to (8) report OLS regressions of distance on time with the estimated breaks respectively for the eight sessions and all support the convergence to the equilibrium reached toward the end of each session. For example, Session 6 has three breaks in periods 20, 49 and 70 - the distance of population state to $e_3$ is large and relatively stable before period 70 but it has a significant decreasing trend since period 70, indicating convergence to the most efficient equilibrium $e_3$ in the end (in Table B.3 column 6, the coefficient $t_4$ of the time trend after period 70 has a significantly negative coefficient, indicating the distance of population state to the final equilibrium identified is decreasing in time).

---

10 Structural breaks which are points of time that a data series has abrupt changes are estimated based on Ditzen et al. (2021). Both breaks in trend and constant are allowed in the test and estimation of structural breaks in distance series.

11 Note that the reversion in Session 8, i.e., the group slightly moves away from the equilibrium reached, is not significant. Both trend and constant in the final segment of the session (after the last break) are not significant (column (8) of Table B.3), indicating the distance to $e_3$ that Session 8 has is not significantly different from zero, and thus the reversion is not significant. Since $e_3$ Pareto dominates the other two equilibria, it is expected that no group would deviate from $e_3$ once it is reached and the result would be more robust in sessions with more periods.
2.3. Results

Result 1 does not support Hypothesis 1 which predicts more transitions to $e_3$ under a smaller sampling size of $s = 2$ and hence raises two questions that sampling best response dynamics fail to explain: first, why all of the groups transited to $e_3$ under $s = 7$, contradicting the prediction of none transiting to $e_3$ under $s = 7$; and second, why some of the groups failed to transit from $e_2$ to $e_3$ within 80 periods under $s = 2$, in contrast to the prediction of all transiting to $e_3$ under $s = 2$. To answer these two questions, we focus on investigating the differences in individual behaviors across the treatments, specifically the deviations from mBR to signals in the remainder of the section.

2.3.2 Deviations from mBR to Signals

We define an action choice of subject $i$ in period $t$ that is not mBR to the private signal as a deviation. We find that there is a significant portion of deviation behaviors at the beginning of the game under both treatments, and the deviation rate has a decreasing trend over time.$^{12}$ However, we find no significant difference in the aggregate deviation rates across the treatments according to two-sided Mann-Whitney test ($0.229$ vs. $0.240$, $p = 0.4472$). This observation motivates us to look at the directions of deviations, the differences of which may favor different equilibrium transitions.

Direction of Deviations

The deviations towards a more efficient equilibrium, as pointed out in Section 2.2.1, are of significant importance for efficiency-improving transitions. On the other hand, deviations toward a less efficient equilibrium would hamper such transitions. Consider an example where the signals indicate action 2 as the mBR. In this example, deviations to action 3 obviously favor transitions to $e_3$ while deviations to 1 do not. That is, the direction of deviations matters. Formally, we define the direction of deviations as follows.

**Definition 2.1.** Given a private signal $\hat{x}$, an action $a \in S$ is called an upward (downward) deviation from the myopic best response $a^*(\hat{x}) \in S$ if $a > a^*(\hat{x})$ ($a < a^*(\hat{x})$).

$^{12}$In Appendix B.2, we plot the deviate rate over time by session (Figure B.2). The figures point out the importance of the deviations in the transitions - the deviation rate rises when the transition starts and falls when a Nash equilibrium is reached. It also explains the reversion in the aggregate deviation rate at the treatment level in the last 20 periods, as in some sessions the transitions happened at the end of the game.
2.3. Results

Table 2.3: Proportion of each type of deviations by treatment

<table>
<thead>
<tr>
<th></th>
<th>s = 2</th>
<th>s = 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>from 2/3 to 1</td>
<td>2.05</td>
<td>-</td>
</tr>
<tr>
<td>from 3 to 1</td>
<td>5.46</td>
<td>4.74</td>
</tr>
<tr>
<td>Down</td>
<td></td>
<td></td>
</tr>
<tr>
<td>from 3 to 2</td>
<td>60.72</td>
<td>21.73</td>
</tr>
<tr>
<td>from 2 to 1</td>
<td>8.19</td>
<td>25.35</td>
</tr>
<tr>
<td>Subtotal</td>
<td>76.41</td>
<td>51.81</td>
</tr>
<tr>
<td>Up</td>
<td></td>
<td></td>
</tr>
<tr>
<td>from 1 to 2</td>
<td>3.22</td>
<td>4.83</td>
</tr>
<tr>
<td>from 2 to 3</td>
<td>18.91</td>
<td>40.48</td>
</tr>
<tr>
<td>from 1 to 3</td>
<td>1.46</td>
<td>2.88</td>
</tr>
<tr>
<td>Subtotal</td>
<td>23.59</td>
<td>48.19</td>
</tr>
<tr>
<td>Total</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Based on the definitions of upward and downward deviations, we have the following result on the direction of deviations.

**Result 2.** There are significantly more upward deviations and fewer downward deviations under a larger sampling size $s = 7$ than a smaller one $s = 2$.

As reported in Table 2.3, at the treatment level, there are significantly more upward deviations (48.19% vs. 23.59%) and fewer downward deviations (51.81% vs. 76.41%) out of all the deviations under $s = 7$ than $s = 2$ according to $\chi^2$ test ($p < 0.001$). In particular, treatment $s = 2$ has more downward deviations from action 3 to 2 (60.72% vs. 21.73%, $\chi^2$ test $p < 0.001$) and fewer upward deviations from action 2 to action 3 (18.91% vs. 40.48%, $\chi^2$ test $p < 0.001$) than treatment $s = 7$.

The result on the direction of deviations is consistent with the transition result that more transitions to $e_3$ under a larger sampling size. Figure 2.4 illustrates the number of deviations by direction in each session over time, where upward (downward) deviations are highlighted in red (blue). The first pattern to notice is that there are more downward deviations and fewer upward deviations under treatment $s = 2$ than $s = 7$. This pattern confirms Result 2. In addition, focusing on upward deviations, the deviation $1 \rightarrow 2$ is more frequent at the early stage while $2 \rightarrow 3$ is more frequent at the later stage of the game. It is consistent with the

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\[13\text{If taking best responses into account, in treatment s = 2 there are 77.10\% of best responses, 17.50\% of downward deviations, and 5.40\% of upward deviations. While the proportions are 75.96\%, 12.46\% and 11.58\% respectively in treatment s = 7. Still, the difference is significant at the 1\% level according to \(\chi^2\) test (}p < 0.001).\]
2.3. Results

Figure 2.4: Direction of deviations over time by session. Upward (downward) deviations are in red (blue). Under treatment \( s = 2 \), there exists one signal \( (1/2, 0, 1/2) \) admitting both action 2 and 3 as the best response. Deviations conditional on this signal are labeled by the legend ‘from 2/3 to 1’.

observation that populations often first transit from $e_1$ to a neighborhood of $e_2$, then to $e_3$. Such deviation pattern can be explained by the negative correlation of a deviation play and its associated payoff loss, which will be discussed in section 2.3.5 in detail: For example, at the early stage of the game when more subjects play action 1, a deviation $1 \rightarrow 3$, in fact, results in a higher payoff loss than a deviation $1 \rightarrow 2$, therefore, more upward deviations are from 1 to 2 at the early stage. Such deviation pattern also results in a hump-shape distribution of deviations over time as a (attempted) transition to the most efficient equilibrium $e_3$ is split into two steps (first $e_1$ to $e_2$ then $e_2$ to $e_3$). The only exception is Session 5, in which the population directly transited from $e_1$ to $e_3$.

**Treatment Effects on Deviation Frequency by Direction**

One possible explanation for the significant treatment difference in upward and downward deviations in quantity is that there exists a significant difference in the frequencies of upward and downward deviation plays across treatments. Here we examine this possibility using fractional logit models.

Let us first denote $Down_i$ and $Up_i$ as the frequency of downward and upward deviations of subject $i$:

$$Down_i = \frac{D_i}{TD_i}, \quad Up_i = \frac{U_i}{TU_i},$$

where $D_i$ and $U_i$ denote the number of periods subject $i$ played downward and upward deviations, respectively; $TD_i$ and $TU_i$ denote the total periods of downward and upward deviations available to $i$, respectively. Note that $TD_i$ and $TU_i$ are in general less than 80. For example, if a subject $i$ observes a signal in period $t$ that indicates action 1 as mBR, then a downward deviation is not an option for subject $i$ in period $t$.\(^\text{14}\)

\(^{14}\)See Figure B.3 in Appendix B.2 for the chances that subjects in each session had to play downward and upward deviations. The chances to deviate downward have little difference across sessions, while the chances to deviate upward vary a lot, indicating the importance of the accommodation of available chances.
2.3. Results

Figure 2.5: Cumulative distributions of frequency of upward and downward deviations.

Figure 2.5 shows the empirical cumulative distributions of $Up_i$ and $Down_i$ by treatment. On average, subjects under $s = 2$ seem to play downward deviations more often than subjects under $s = 7$, while the opposite holds for upward deviations.\(^\text{15}\) Both Mann-Whitney test ($p = 0.005$) and Kolmogorov-Smirnov test ($p = 0.006$) suggest the significant difference in $Down_i$ across treatments, but not in $Up_i$ (Mann-Whitney test $p = 0.0684$, Kolmogorov–Smirnov test $p = 0.153$). It indicates that subjects do not play upward deviations more often under $s = 7$ than $s = 2$.

To further verify the findings, we run fractional logit regressions using the following specification:

$$DV_i = \beta_0 + \beta_1 s_{7i} + \beta_2 Female_i + \beta_3 Economics_i + \beta_4 Degree_i + \beta_5 Age_i + \beta_6 nCRT_i + \epsilon_i \quad (2.1)$$

\(^{15}\)Note there is one obvious outlier in the distribution of downward deviations for $s = 7$, which is equal to 0.86. The outlier is omitted from the analysis.
where the dependent variable $DV_i$ is either $Down_i$ or $Up_i$, $s7_i$ is a dummy variable, taking the value of 1 if subject $i$ was in the treatment $s = 7$ and 0 otherwise, and $nCRT_i$ is a variable for cognitive ability.\textsuperscript{16} It is measured by the number of correctly answered seven questions from CRT normalized by the seconds spent on the questions scaled up by 100.\textsuperscript{17} $\epsilon_i$ is the error term. The standard errors are clustered at the session level. The regression results are reported in Table 2.4.

The regression results support the discussion above. We find a significant negative effect of the larger sampling size on the frequency of downward deviations ($s7_i$ in columns (3) and (4) of Table 2.4), but no significant effect on the frequency of upward deviations ($s7_i$ in columns (1) and (2)). Interestingly, high-cognition and male subjects tend to play upward deviations significantly more often and downward deviations less often. Additionally, subjects studying economics also deviate upward at a significantly higher frequency. The results are robust after controlling for the social preference and the level-$k$ behavior (see columns (1) and (2) of Table B.4 in Appendix B.2).

Result 3. Subjects under the treatment $s = 7$ play downward deviations significantly less often than $s = 2$. However, the sampling sizes have no significant effect on the frequency of upward deviations.

2.3.3 Forward-looking Players

In this subsection, we investigate why there are more upward deviations in quantity under $s = 7$ but no difference in the frequency of upward deviations across treatments. The expected number of upward deviations depends on both the probability of individual deviations and the number of subjects who deviate. Since the probability (the frequency of upward deviations) does not differ significantly across treatments, a conjecture is that subjects who are willing to play upward deviations are different. Strategic teaching models, (e.g., Camerer et al., 2002c; Lyu, 2022),

\textsuperscript{16}One could also use mixed logit models with a categorical dependent variable taking three different values for best responses, downward and upward deviations respectively. As shown in Table B.5 of the Appendix B.2, the results are qualitatively the same. It is important to note that one should not use multinomial logit due to the violation of the assumption of fixed choice set and thus IIA by design, since upward deviations and downward deviations are not always available, while mixed logit can deal with the issue.

\textsuperscript{17}The normalized CRT instead of the raw CRT score is used due to few variations in the raw score with a mean of 5.50 and a standard deviation of 1.45. The normalized CRT has a mean of 2.41 and a standard deviation of 1.40. The two measures show a reasonably high correlation at the 1\% significance level, with a Pearson’s correlation coefficient of 0.6167 and a Spearman’s rho of 0.6981. See Figure B.4 for the relationship between the two measures in Appendix B.2.
Table 2.4: Test of treatment effect on deviation frequency by direction.

<table>
<thead>
<tr>
<th></th>
<th>$U_{pi}$</th>
<th></th>
<th></th>
<th>$D_{ni}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$s_{ti}$</td>
<td>0.227</td>
<td>0.543</td>
<td>-0.481**</td>
<td>-0.577***</td>
</tr>
<tr>
<td></td>
<td>(0.567)</td>
<td>(0.482)</td>
<td>(0.240)</td>
<td>(0.224)</td>
</tr>
<tr>
<td>$Female_{i}$</td>
<td>-0.598*</td>
<td>0.549**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.327)</td>
<td>(0.215)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Economics_{i}$</td>
<td>0.537***</td>
<td>0.073</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.156)</td>
<td>(0.278)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$nC_{RTi}$</td>
<td>0.150**</td>
<td>-0.141**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.061)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Degree_{i}$</td>
<td>-0.691</td>
<td>0.178</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.457)</td>
<td>(0.311)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Age_{i}$</td>
<td>0.108</td>
<td>-0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.051)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.334**</td>
<td>-1.656**</td>
<td>-1.516***</td>
<td>-1.937***</td>
</tr>
<tr>
<td></td>
<td>(0.528)</td>
<td>(0.835)</td>
<td>(0.130)</td>
<td>(0.770)</td>
</tr>
</tbody>
</table>

Notes: Fractional logit regressions. Standard errors are clustered at the session level in parentheses. Fewer observations in columns (2) and (4) are due to missing values in $Female_{i}$ and $Age_{i}$. One outlier is omitted for columns (3) and (4) (see footnote 15). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

provide a theoretical rationale for this conjecture, which states that forward-looking players may find it optimal to deviate from a bad equilibrium to a better one at the early stage of games in order to teach myopic players to deviate in the future. In particular, Lyu (2022) shows the importance of the observability of actions for such strategic deviations.
2.3. Results

To test the argument, we first identify subjects who are forward-looking, namely who would like to play upward deviations intentionally in order to transit to a better equilibrium. To separate intentional deviations from those by mistakes, we focus on the persistent upward deviations (e.g., Lim and Neary, 2016b). We define an upward deviation by subject $i$ in period $t$ to be strategic if she also plays an upward deviation in period $t + 1$.\(^{18}\)

We classify subjects into three types based on their observed behaviors: myopic players, forward-looking players (henceforth FL players), and others. The classification rule is summarized as follows (Table 2.5):

- **Myopic Players** play best responses for at least 85% of time with no strategic deviation, i.e., no persistent upward deviation;
- **FL Players** play best responses and upward deviations together for 85% of the time with at least one strategic deviation;
- **Others** who cannot be classified into the two types above.

The threshold of 85% is chosen according to the 95% confidence interval $(0.13, 0.17)$ of the mean time of downward deviations in the 80 periods, which is set to allow for mistakes.\(^{19}\)

The requirement for the classification of FL players is not very strict in the sense that subjects will be classified as FL players if they reveal their type by only one strategic deviation. That is, it is not required that FL subjects play strategic deviations whenever possible. Moreover, it implicitly allows FL subjects to swap between upward deviations and best responses - one might stop deviating upward if she believes transitions would be unsuccessful. As demonstrated in Figure 2.5, few subjects deviate upward all the time. Further, we do not restrict subjects to upward deviations to the same action in two consecutive periods: subjects could deviate upward to action 2 and subsequently to action 3 if the transition to $e_2$ is perceived to be successful at some point.\(^{20}\)

\(^{18}\)Note that there exist scenarios where the one-shot upward deviation is not a mistake but cannot be identified as being strategic, because the choice of upward deviations may not be available in two consecutive periods given the signals. Therefore, our measure provides a lower bound of the number of strategic deviations.

\(^{19}\)All of the downward deviations are considered as mistakes here. However, whether we consider down deviations as mistakes does not affect the identification of FL players who are defined by strategic deviations together with best responses. In the next subsection, we will show that downward deviations are caused partly by the inaccuracy of signals.

\(^{20}\)One might argue that a smaller sample size would lead to smaller persistent chances for upward deviations, resulting in fewer subjects identified as FL players. In our data set, there are two such subjects. One is identified as ‘others’ due to too many downward deviations while the other is myopic with 7.5% of the time on one-shot upward deviations. The results do not change if the subject is classified as an FL player.
Table 2.5: Classification of subject types. Count (Percent)

<table>
<thead>
<tr>
<th>Classification Rule</th>
<th>FL Player</th>
<th>Myopic Player</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR + S + NS-Up ≥ 85%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S ≥ 1</td>
<td>3 (21.43%)</td>
<td>6 (42.86%)</td>
<td>5 (35.71%)</td>
</tr>
<tr>
<td>BR ≥ 85%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S = 0</td>
<td>4 (28.57%)</td>
<td>2 (14.29%)</td>
<td>8 (57.14%)</td>
</tr>
<tr>
<td>None of the above</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Session 1, s = 2, e_3 | 3 (21.43%) | 6 (42.86%)    | 5 (35.71%) |
| Session 2, s = 2, e_3 | 4 (28.57%) | 2 (14.29%)    | 8 (57.14%) |
| Session 3, s = 2, e_2 | 3 (21.43%) | 1 (7.14%)     | 10 (71.43%)|
| Session 4, s = 2, e_2 | 4 (28.57%) | 0 (0.00%)     | 10 (71.43%)|
| Total s = 2           | 14 (25.00%)| 10 (17.86%)   | 32 (57.14%) |

| Session 5, s = 7, e_3 | 5 (35.71%) | 8 (57.14%)    | 1 (7.14%)  |
| Session 6, s = 7, e_3 | 9 (64.29%) | 2 (14.29%)    | 3 (21.43%) |
| Session 7, s = 7, e_3 | 4 (28.57%) | 2 (14.29%)    | 8 (57.14%) |
| Session 8, s = 7, e_3 | 7 (50.00%) | 2 (14.29%)    | 5 (35.71%) |
| Total s = 7           | 25 (44.64%)| 15 (26.78%)   | 16 (28.57%)|

Notes: BR: best response; S: strategic deviation; NS-Up: nonstrategic upward deviation; NS-Down: nonstrategic downward deviation. The threshold 85% is chosen according to the 95% confidence interval (0.132, 0.181) of the mean downward deviation time. Each session is labeled by the treatment and the final equilibrium reached.

As shown in Table 2.5, 44.64% of the subjects are classified as FL players when $s = 7$, significantly greater than the proportion 25% when $n = 2$ (Fisher’s exact test $p = 0.047$). Note that on average the percentage of FL players when $s = 7$ is very close to the tipping point for the transition from $e_2$ to $e_3$, namely 45% ($t$ test $p = 0.9577$), which could explain the transitions to $e_3$ when $s = 7$. On the other hand, notice that a large number of subjects are identified as ‘Others’ under sampling size $s = 2$ compared to $s = 7$ (57.14% vs. 28.58%). Those subjects are classified as ‘Others’ due to a high downward deviation rate (26% on average). However, they still play mBR for most of the time in the experiment - the average rate of mBR of Others is 68.65%.

The treatment effect of sampling size on the probability of a subject being identified as a forward-looking player is estimated by the following logit model:

$$FL_i = \beta_0 + \beta_1 s_7i + \beta_2 Female_i + \beta_3 Economics_i + \beta_4 Degree_i + \beta_5 Age_i + \beta_6 nCRT_i + \epsilon_i.$$  

(2.2)

---

21See Figure B.9 in Appendix B.2 for the time spent on each kind of behavior across subject types at the individual level.
Table 2.6: Test of treatment effect on identified FL players.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FL_i$</td>
<td>0.884***</td>
<td>1.119**</td>
</tr>
<tr>
<td>$s7_i$</td>
<td>(0.314)</td>
<td>(0.464)</td>
</tr>
<tr>
<td>$Degree_i$</td>
<td>0.110</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.589)</td>
<td></td>
</tr>
<tr>
<td>$Female_i$</td>
<td>-0.260</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.481)</td>
<td></td>
</tr>
<tr>
<td>$Economics_i$</td>
<td>-0.054</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.524)</td>
<td></td>
</tr>
<tr>
<td>$nCRT_i$</td>
<td>0.468***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.131)</td>
<td></td>
</tr>
<tr>
<td>$Age_i$</td>
<td>0.020</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.128)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.099***</td>
<td>-2.983*</td>
</tr>
<tr>
<td></td>
<td>(0.102)</td>
<td>(1.791)</td>
</tr>
<tr>
<td>$N$</td>
<td>112</td>
<td>108</td>
</tr>
</tbody>
</table>

Notes: Logit regressions. Standard errors clustered at session level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Fewer observations in column (2) due to missing values in $Female_i$ and $Age_i$.

The dependent variable $FL_i$ is a dummy taking the value of 1 if subject $i$ is identified as an FL player, and 0 otherwise. Table 2.6 presents two model specifications, with model (1) including only the treatment dummy of sampling size $s7_i$ and model (2) further controlling for individual characteristics of gender, major, and normalized CRT score, given the significant effects of individual characteristics on the frequency of upward deviations. The standard errors are clustered at the session level.

As shown in Table 2.6, there is a significant positive treatment effect of sampling size on the probability of a subject being identified as a forward-looking player under both specifications. Further, a subject with higher cognitive ability is also more likely to be a forward-looking player, but the effects of other individual characteristics such as gender and major that were found to have significant effects on the frequency of upward deviations are not significant on strategic deviations. Table B.4 in Appendix B.2 presents consistent results after controlling for patience, social preferences and level-k reasoning.
2.3. Results

Result 4. A larger sampling size induces significantly more subjects to play strategic deviations (or persistent upward deviations) than a smaller sampling size. Subjects with higher cognitive ability are more likely to play strategic deviations.

Result 4 supports the conjecture based on strategic teaching models. Lyu (2022) suggests that if players have sufficiently accurate information about their opponents’ choices, forward-looking players with higher cognitive levels (measured by level-k reasoning in Lyu (2022)) will strategically deviate from an inefficient equilibrium at the early stage of a game to induce lower-cognitive-level players to deviate in the future so that the population can reach a more efficient equilibrium. This is because, with inaccurate information such as a sampling size 2, strategic deviations made by higher-cognitive-level players are less likely to be observed by lower-level players. Therefore, higher-level players will not play upward deviations in the first place in this case. The regression results in Table B.4, Appendix B.2 provide additional evidence of this argument with social preferences and level-k reasoning controlled. We find that the ability of higher level-k reasoning also has a significant positive effect on strategic deviations.

Another channel through which the sampling size may affect strategic deviations is the existence of a free-riding problem when the sampling size is small. When \( s = 2 \), forward-looking players who are aware of the effect of a single upward deviation predicted by sBRD may free-ride from the very beginning if they believe there exist other FL players. As a result, they would play mBR and not be identified as FL players. When the sampling size is large, the incentive to free ride is weakened as more upward deviations are required for successful efficiency-improving transitions.

2.3.4 Responsiveness to Signals

In this subsection, we explore the potential channel through which the sampling size affects the frequency of downward deviations. Subjects under the treatment \( s = 2 \) may consider the signals from only two random draws to be non-representative of the true population state. In our experiment, transitions that occur generally follow the path of \( e_1 - e_2 - e_3 \), so a slow response to changes in signals often results in downward deviations. To see this, all subjects initially play the inefficient action \( e_1 \) until they make a change. Downward deviations occur whenever subjects who receive a signal indicating action 2 (or 3) as the mBR do not best respond to the
Figure 2.6: Predicted probability of choosing each action given a signal based on estimated equation (2.3) reported in Table B.6. Signal2 and Signal3 are the fractions of action 2 and 3 in a signal respectively.

signal immediately. Here we test whether subjects perceive signals with \( s = 2 \) less accurate than \( s = 7 \) so that they are less responsive to an increase of the fraction of one action in the signals (see Figure B.8 of Appendix B.2 for the subjects’ action choices given each signal across treatments).

We run the following mixed logit regression of \( \text{Action}_{it} \), that is, the chosen action of a subject \( i \) in period \( t \), on the treatment of the sampling size and the signals:

\[
\log \frac{Pr(\text{Action}_{it} = j)}{Pr(\text{Action}_{it} = 2)} = \beta_{0j} + \beta_{1j} s_{i} + \beta_{2j} \text{Signal2}_{it} + \beta_{3j} s_{i} \times \text{Signal2}_{it} + \beta_{4j} \text{Signal3}_{it} + \beta_{5j} s_{i} \times \text{Signal3}_{it} + \Gamma X_{i} + \epsilon_{itj} \tag{2.3}
\]

where action 2 is the base outcome and \( j \) indicates either action 1 or 3. The variable \( \text{Signal2}_{it} (\text{Signal3}_{it}) \) is the fraction of action 2 (3) observed in the signal by subject \( i \) in period \( t \); \( X_{i} \) is a set of control variables as used in equation (2.1) and \( \epsilon_{itj} \) is the error term. The interaction terms of the sampling size and the signals are included to test the effects of the sampling size on the subjects’ perceptions of the signals’ accuracy. The standard errors are clustered at the session level.

Figure 2.6 illustrates the predicted probabilities of choosing each action given the fractions of actions 2 and 3, i.e., Signal2 and Signal3, respectively by treatment based on the estimates of equation (2.3). It is clear that the probability of choosing an action increases with the fraction of the action in the signals. However, subjects’ responsiveness to signals indeed differs across treatments - an increase in the fraction
of an action in signals has a larger positive effect on the probability of the action to be chosen under $s = 7$ than $s = 2$ for all three actions. (Also see the average marginal effects of the independent variables, reported in Table B.7 in Appendix B.2.)

**Result 5.** Subjects under the smaller sampling size treatment $s = 2$ are less responsive to the changes in signals than those under the larger sampling size treatment $s = 7$.

In Appendix B.1, we also discuss whether inertia (i.e., the tendency to play the same action over time regardless of changes in signals) is an alternative channel that results in downward deviations, of which we find no supporting evidence.

### 2.3.5 Payoff Loss Dependent Deviations

In this subsection, we investigate how a deviation play is affected by the associated payoff loss, i.e., the expected payoff difference between the mBR action and the non-mBR action. Blume (1993); Myatt and Wallace (2003) propose a cost-dependent noise model where deviations involving a higher payoff loss are less likely to happen. However, the existing experimental results are mixed. Battalio et al. (2001); Mäss and Nax (2016); Hwang et al. (2018); Bilancini et al. (2020) find that deviations from mBR are cost-dependent, while Lim and Neary (2016b) find that deviations are not cost-dependent. Our experimental results show that deviations are cost-dependent, but there is a significant difference in cost-dependence between deviations with different directions.

**Result 6.** Deviation plays have a significant negative association with the expected payoff loss. Upward deviations are less sensitive to the payoff loss than downward deviations.
2.3. Results

Figure 2.7: Scatter plots of deviation rate and associated payoff loss given signals by direction.

Figure 2.7 plots the rate of deviation by direction and its associated payoff loss given each signal.\(^{22}\) Note that both downward and upward deviation rates tend to decrease with the payoff loss. Further, Spearman’s rank order tests suggest a significant negative relationship between the deviation play and the payoff loss: Spearman’s \(\rho_{\text{down}} = -0.9072, p < 0.01\) for the downward deviation and \(\rho_{\text{up}} = -0.5635, p < 0.01\) for the upward deviations.

More importantly, the test results suggest that the upward deviations are less correlated to the payoff loss than the downward deviations \((\Delta \rho = 0.3437, p = 0.037)\). As we have shown, FL players are critical for transitions between strict Nash equilibria. These FL players are willing to bear some short-run losses due to upward deviations in order to obtain long-run gains. That is, upward deviations tend to be affected by payoff losses less than downward deviations.\(^{23}\)

2.3.6 Payoffs

In this subsection, we look at the realized payoffs in games. Transitions to the most efficient equilibrium \(e_3\) are expected to increase payoffs. In the context of a coordination game, the payoff is also affected by the transition duration, i.e., the longer periods of the population state being unsettled, the lower the payoff will

\(^{22}\)If there exist two possible actions for deviations, the one with a higher payoff is used, as the action associated with the lower payoff was chosen much less often - it consists of 1.71% of the data set and is not included for analysis. In Figure B.7 of Appendix B.2, we show that the results with the whole data set are qualitatively the same.

\(^{23}\)Restricting data to FL players gives Spearman’s \(\rho = -0.3697, p = 0.048\) for upward deviations. Though \(\rho\) is smaller than when all the players are included, the difference is not significant \((\Delta = -0.1938, p = 0.463)\).
Table 2.7: Transition duration by session.

<table>
<thead>
<tr>
<th>Session</th>
<th>Transition to $e_2$</th>
<th>Transition to $e_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, $s = 2$</td>
<td>-</td>
<td>36</td>
</tr>
<tr>
<td>2, $s = 2$</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td>3, $s = 2$</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td>4, $s = 2$</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>5, $s = 7$</td>
<td>-</td>
<td>6</td>
</tr>
<tr>
<td>6, $s = 7$</td>
<td>-</td>
<td>74</td>
</tr>
<tr>
<td>7, $s = 7$</td>
<td>-</td>
<td>71</td>
</tr>
<tr>
<td>8, $s = 7$</td>
<td>-</td>
<td>65</td>
</tr>
</tbody>
</table>

be. We define the transition duration from the initial state $e_1$ to the reached final equilibrium as the number of periods it takes for a population to reach and stay inside the basin of attraction of the equilibrium till the end of the game. For example, if a population state $x$ stays in the basin of attraction of the final equilibrium reached from the period $T + 1$ to the end of the game, that is, the population state $x$ is not in the basin of attraction in period $T$, the transition duration is $T$.

Table 2.7 reports the transition durations from the initial state $e_1$ to $e_2$ or $e_3$ by session. The transition duration under $s = 7$ is on average longer than that under $s = 2$, regardless of the final equilibrium reached. Under $s = 7$, the four sessions transited to $e_3$ with the transition durations of 6, 74, 71, 65 periods, respectively. Under $s = 2$, sessions 1 and 2 transited to $e_3$ using 36 and 20 periods, respectively, and sessions 3 and 4 transited to $e_2$ using 60 and 9 periods, respectively. Thus, even though more sessions reach the most efficient equilibrium $e_3$ under $s = 7$, whether subjects can earn more under $s = 7$ than $s = 2$ is indeterminate.

The average of accumulated individual total payoffs is shown in Figure 2.8 by treatment and the equilibrium reached. Let $\pi(s, e)$ denote the average accumulated individual payoff from groups that transited to equilibrium $e$ under treatment $s$. We find that

$$\pi(2, e_3) > \pi(7, e_3) > \pi(2, e_2).$$

That is, subjects earned the highest payoff in groups that transited to $e_3$ under $s = 2$ ($\pi(2, e_3) = 2006.96$ tokens), which is significantly greater than those under $s = 7$ also transiting to $e_3$ ($\pi(2, e_3) = 1770.78$ tokens) by one-sided Mann-Whitney test ($\pi(2, e_3) > \pi(7, e_3), p < 0.001$), while subjects in groups that transited to $e_2$ under $s = 2$ earned the lowest payoff ($\pi(2, e_2) = 1638.35$ tokens $< \pi(3, e_3)$ by one-sided Mann-Whitney test, $p = 0.072$).
2.3. Results

Figure 2.8: Mean of realized individual payoffs (accumulated over 80 periods) by treatment and the final equilibrium reached.

The difference in the accumulated payoffs can be explained by transition durations and the final equilibrium reached. For example, groups transited to the most efficient equilibrium $e_3$ under $s = 2$ spent less time on transitions than those under $s = 7$, suggesting that $\pi(2, e_3) > \pi(7, e_3)$. For groups that transited to different equilibria, those transiting to $e_3$ have a higher payoff than those transiting to $e_2$, e.g., $\pi(2, e_3) > \pi(2, e_2)$, and $\pi(7, e_3) > \pi(2, e_2)$. The ranking in accumulated payoffs is robust to any longer time of the game play. If the game is played for another $n > 0$ periods, the accumulated payoffs will satisfy

$$\pi(2, e_3) + 30n > \pi(7, e_3) + 30n > \pi(2, e_2) + 20n.$$ 

Result 7. The average accumulated individual payoffs in the game have the following ranking:

$$\pi(2, e_3) > \pi(7, e_3) > \pi(2, e_2).$$

Result 7 has an interesting implication. More accurate information ensures transitions to the best equilibrium but involves a longer transition duration and a higher transition payoff loss. On the other hand, less accurate information involves a shorter transition duration but does not guarantee the final efficiency.
2.3. Results

Figure 2.9 shows average payoffs by subjects’ type. Under $s = 2$, all of the three types received similar payoffs regardless of the final equilibrium reached (Mann-Whitney test $p > 0.05$ for all of the pairwise comparisons). In contrast, under $s = 7$, myopic subjects received the highest payoffs (1980.39), followed by the FL subjects (1761.04) and the rest (1589.49) (Mann-Whitney test $p < 0.05$ for all of the pairwise comparisons).

**Result 8.** Under the treatment $s = 2$, subjects of the three types received similar payoffs regardless of the equilibria finally reached. Under the treatment $s = 7$, myopic subjects received significantly higher payoffs than FL subjects, followed by the others.

This result has important policy implications on the tradeoff between welfare equality and efficiency when designing an information structure or institutions. For example, a small sampling size such as $s = 2$ can induce equal payoffs among subjects but it does not guarantee a transition to the more efficient equilibrium.

\[\text{Accumulated Game Payoff}\]

<table>
<thead>
<tr>
<th>s=2. Transit to e2</th>
<th>s=2. Transit to e3</th>
<th>s=7. Transit to e3</th>
</tr>
</thead>
<tbody>
<tr>
<td>FL 1631.67</td>
<td>FL 2027.14</td>
<td>FL 1761.04</td>
</tr>
<tr>
<td>Myopic 1646.39</td>
<td>Myopic 2017.13</td>
<td>Myopic 2005.53</td>
</tr>
<tr>
<td>Others 1640.29</td>
<td>Others 1989.83</td>
<td>Others 1591.77</td>
</tr>
</tbody>
</table>

\[\text{Figure B.10 for payoffs over time by session.}\]

\[\text{Recall that subjects types are defined by patterns of deviation and mBR play, and deviations tend to involve payoff loss. Thus one might expect payoffs to differ by type under both treatments. The equal payoff across types under } s = 2 \text{ could be explained by the small sampling size with which the signals are less representative of the true population state, thus mBR is less likely to be empirically optimal (and a deviation is more likely to be profitable). See Figure B.10 for payoffs over time by session.}\]
2.4 Conclusion

This paper provides an experimental investigation of the evolutionary game models which predict transitions among strict Nash equilibria under inexact information about opponents’ behaviors. In our lab experiment, a population of 14 subjects played a three-action coordination game recurrently for 80 periods with two informational treatments: the sampling size \( s = 2 \) for the less accurate information and \( s = 7 \) for the more accurate information. We observe that populations in all of the sessions transited away from the least efficient equilibrium under the two sampling sizes, but more sessions transited to the most efficient equilibrium under \( s = 7 \) than \( s = 2 \), which is in contrast with the theory.

We find that the larger sampling size \( s = 7 \) induces significantly more subjects to strategically play upward deviations that can lead the population to a more efficient equilibrium, although there is no significant treatment effect on the probability of such upward deviation play. On the other hand, the failure in transitions to the efficient equilibrium under \( s = 2 \) arises from the fact that subjects are less elastic to changes in their private signals due to the less accurate signals, leading to significantly more downward deviations, which either delay or block the transitions.

Our results on the equilibrium transitions and the underlying individual behaviors suggest that human subjects tend to disregard inaccurate signals despite the fact that the signals are unbiased. More importantly, human subjects are more likely to exhibit farsighted strategic behaviors when information is relatively more accurate. One insight from our results is that researchers and policy-makers may take these two types of behaviors into consideration when designing any information-related mechanisms or institutions. Finally, we find that although accurate information can induce transitions to the efficient equilibrium, it also involves a higher welfare loss during the transitions. In our experiment, subjects under \( s = 7 \) did not earn significantly more payoffs than \( s = 2 \) due to longer transition durations. Moreover, under \( s = 7 \), myopic subjects received higher payoffs than forward-looking subjects, followed by others; while under \( s = 2 \) three types earned equal amounts. Therefore, a small sampling size might be desirable when equality is preferred over efficiency.
Chapter 3

Optimal Fertility Choices under Patriarchal Institutions

3.1 Introduction

Parents who live in a patriarchal society may have incentives to continue having births until a certain number of sons are born, or the family size reaches an upper bound. These are often referred to as differential stopping behaviors or son-prefering stopping rules. Empirical studies have found evidence that such stopping rules exist in a wide range of regions such as Asia, Africa, and North America (Larsen et al., 1998; Dahl and Moretti, 2008; Barcellos et al., 2014; Kugler and Kumar, 2017; Altindag, 2016; Chen et al., 2019). Son-prefering stopping rules have important social and economic implications. For example, Downey (2006); Dang and Rogers (2009); Altindag (2016) among others find that females are disproportionately born into larger families (i.e., families with more children) under son-prefering stopping rules, hence receive less investment than males on average. Basu and De Jong (2010) show that females are also more likely to be born before males which may affect females’ welfare through birth order effects (Dayioglu et al., 2009; De Haan, 2010; Hotz and Pantano, 2015).

Although son-prefering stopping rules have important social and economic impacts, the existing literature remains silent on their explicit form and properties. Previous theoretical studies, e.g., Yamaguchi (1989), Seidl (1995), and Basu and De Jong (2010), often exogenously specify a stopping rule, then investigate its socioeconomic effects. For example, Basu and De Jong (2010) assumes that parents “continue childbearing till they attain a desired ‘target’ number of sons, $k$, or hit a ceiling for the maximum number of children, $N$, with $k \leq N$”. However, a stopping

---

1This is true even if there is no intra-household discrimination, i.e., parental investments in females and males are same within a household.
rule is, in general, a fertility strategy that parents choose to follow. That is to say, a stopping rule should be endogenously deduced from parents’ utility maximization; whether it is son-preferring or not is a property of the strategy. In this sense, the exogenous stopping rules can be misspecified.

In this paper, we address this issue by formally characterizing the optimal stopping rule. We extend the classical models of fertility proposed by Becker and Lewis (1973) and Becker and Tomes (1976) by nesting a stochastic sequential decision problem into parents’ static utility maximization problem. In the classical models, parents choose the optimal quantity and quality (parental investment) of their children at the same time by maximizing their utility. However, the fertility outcome is stochastic due to the asymmetry in sons and daughters in a patriarchal society, and therefore parents cannot choose how many sons and daughters to have ex-ante. In this paper, we assume that the fertility decision takes place sequentially: parents choose between two actions Continue and Stop. If they choose Continue, one more child is born; if they choose Stop, the fertility process ends and parents make consumption-investment decisions based on the realized fertility outcome. The solution to our model is a fertility strategy that assigns an action in \{Continue, Stop\} to each possible sex composition of children, combined with the optimal consumption and parental investments.

Under the assumption that parents’ preference (defined as a function of consumption) and investment technology are convex, we first show that the optimal fertility strategy is a one-step look-ahead strategy, and it is a son-preferring stopping rule if the rate of return on sons is higher than that on daughters. By son-preferring stopping rule, it means parents tend to stop fertility when they have more sons but continue fertility when they have more daughters. Specifically, the optimal fertility strategy can be stated as a set of sex compositions that we call the optimal stopping set. Parents will continue to have children until the sex composition reaches the optimal stopping set.

We characterize the distribution of the possible sex combinations of children in a family under the optimal fertility strategy and show that the covariance between family size and the number of daughters in a family is non-negative. The non-negative correlation implies that daughters are on average born into larger families than sons, which coincides with empirical findings that early-born daughters are positively correlated with larger family sizes (Larsen et al., 1998; Dahl and Moretti, 2007). Formally, we define a sex composition of children to be a tuple \((K_s, K_d)\), where \(K_s\) and \(K_d\) are the number of sons and daughters in a family respectively.
In addition, the sex ratio induced by the optimal fertility strategy is balanced despite parents following a son-preferring stopping rule. We also derive the population distributions of parental investment in sons and daughters under the optimal strategy, and show that the optimal investment in sons per capita first-order stochastically dominates that in daughters.

The endogenous determination of the optimal fertility strategy in this paper allows us to study how changes in exogenous variables such as patriarchal institutions affect parents’ fertility and parental investment decisions. A counter-intuitive result is the coexistence of over- and under-reproduction in a patriarchal society (with the fertility rate in a gender-neutral society as the benchmark). Intuitively, parents in a patriarchal society will over-reproduce to have a desirable number of sons, and this should lead to a higher average fertility rate. The coexistence of over- and under-reproduction however makes a lower average fertility rate possible in a patriarchal society. We find that a small increase in the intensity of patriarchal institutions may bring down the average fertility rate in a society when the intensity itself is at a low level.

Lastly, we incorporate the possibility of sex selection and show the robustness of the optimal fertility strategy as a son-preferring stopping rule. The main property of the optimal fertility strategy that parents tend to stop fertility when they have more sons remains in the presence of sex selection. Therefore, the non-negative correlation between family size and the number of daughters in a family still holds, followed by the stochastic dominance of parental investment in sons over daughters. The sex ratio in a patriarchal society however is not necessarily balanced when sex selection is optional.

The results in this paper are rather general. The sequential fertility decision problem in the presence of asymmetry in sexes has been studied by Rosenblum (2013); Hazan and Zoabi (2015), which adopt the same nested framework that parents make fertility decisions first, then consumption and parental investment decisions based on the fertility outcome. Both Rosenblum (2013) and Hazan and Zoabi (2015) assume a specific utility function and investment function, then derive some properties of the fertility outcome induced by the optimal fertility strategy under additional conditions on model parameters. However, this paper explicitly characterizes the optimal fertility strategy and derives the resulting distribution and properties of fertility outcome without assuming specific functional forms of utility and investment technology.
3.1. Introduction

This paper is also different from another dynamic stochastic model of fertility where uncertainty often comes from the stochastic survival of infants or random shocks on exogenous variables such as earning risk (Wolpin, 1984; Arroyo and Zhang, 1997; Sommer, 2016). These models mainly focus on the life-cycle fertility choices such as timing and spacing decisions, and parents make contingent plans for all choice variables. Our model instead focuses on another source of uncertainty, the (patriarchal) asymmetry in sexes, and its effects on fertility and parental investment decisions. A difference is that consumption and investment are still one-off decisions in our model, i.e., they are made after the fertility process. This simplification allows us to derive closed-form results that in general can not be obtained in those fully dynamic models.

The rest of the paper is organized as follows. Section 3.2 describes the sequential fertility decision model and some concepts used to solve the model. Section 3.3 presents the main results. The last section concludes with remarks.

3.2 Model

We consider a two-period decision problem faced by parents in the benchmark model. Parents are assumed to be rational and self-regarding, i.e., they care only about their own utility, which is given by:

$$u(c_1) + \beta u(c_2),$$

where $c_t$ is the consumption in period $t \in \{1, 2\}$, and $\beta$ measures relative weight on utility in period 2. In period 1, parents are endowed with income $I$, and they make a fertility decision and allocate the income to $c_1$, and investment in each child. In period 2, parents are in old age. For simplicity, we assume parents cannot save for consumption in period 2, instead, $c_2$ can only be paid for by transfers from their children, who are adults in period 2 and earn money by working. The children’s future income depends on investments made by parents $z \in \mathbb{R}^+$ and an investment function $f(z)$. That is:

$$I_i = h_i = f(z_i),$$

where $h_i$ is the level of the human capital of child $i$. We assume a convex preference and an investment technology.

**Assumption 3.1.** Utility function $u : R_+ \rightarrow R$ and investment function $f : R_+ \rightarrow R$ are strictly concave. In addition, $f'(0) = \infty$, $f'(\infty) = 0$. 
We denote a social institution or norm as a tuple $(\alpha_s, \alpha_d)$ that prescribes the different obligations of sons and daughters for supporting parents when parents are old. For example, in China, daughters after married do not count as family members from the parents’ perspective, hence are not responsible for supporting the latter. Sons, on the other hand, are obliged to support their parents when the latter are old. In this example, $(\alpha_s, \alpha_d)$ represents the social institution, where $\alpha_s$ ($\alpha_d$) is the share of the income of sons (daughters) that should be transferred to parents in period 2. We assume $\alpha_s > \alpha_d$ without loss of generality, which means that the institution is patriarchal. There are many other interpretations of the model. For example, $\alpha$ can be interpreted as parents’ belief in the return rate of human capital in the labor market, which also generates the same asymmetry in the sexes. Alternatively, we can interpret the model such that parents are altruistic toward children, and $\alpha$ measures the intensity of altruism. Lastly, we also assume sex selection is not an option and parents can give birth to only one child at a time, i.e., twins, triplets, etc. are excluded.\footnote{In subsection 3.3.3, we extend the benchmark model to include sex selection.}

Sex composition of children matters due to the asymmetry in sexes. Since the sex of birth is stochastic, it is therefore impossible for parents to choose an exact ‘optimal’ sex composition ex-ante. We, therefore, depart from the literature (Becker and Lewis, 1973; Becker and Tomes, 1976) and assume parents choose fertility sequentially. We see the fertility decision as an extensive-form game where parents play against Nature whose strategy is programmed to be the mixed strategy $(\frac{1}{2}, \frac{1}{2})$ over the set $\{\text{Son, Daughter}\}$. In this game, parents can choose an action from the following action set $\{\text{Continue}(C), \text{Stop}(S)\}$ given the current sex composition of the children already born. If parents choose $S$ at a sex composition $(K_s, K_d)$, the fertility process ends and the sex composition in the family is determined. Parents then allocate their income between consumption and investment given the determined sex composition. A game tree representation of the sequential decision problem is shown in Figure 3.1.

When the fertility process stops, given the realized sex composition $(K_s, K_d)$, the parents’ decision problem is:
Figure 3.1: A game tree representation. P stands for parents and N for Nature. Nature always plays the mixed strategy \((\frac{1}{2}, \frac{1}{2})\) over the action set \(\{\text{Son, Daughter}\}\), and parents can observe the sex of a child only after Nature moves. At each node of parents, if they choose \(S\), the game ends and parents receive utility \(V(K_s, K_d)\) for a \((K_s, K_d)\).

\[
\max_{c,z} U = u(c_1) + \beta u(c_2)
\]

s.t. \[c_1 + \sum_{i=0}^{K_s} z_i + \sum_{j=0}^{K_d} z_j + (K_s + K_d)\delta \leq I,\]
\[c_2 \leq [\alpha_s \sum_i f(z_i) + \alpha_d \sum_j f(z_j)],\]

where \(i(j)\) represents son (daughter) \(i (j)\), and \(\delta\) is the cost per birth. Note that \(\delta\) can be seen as a price of a child and does not include the subsequent parental investment.
3.2. Model

It is easy to see from the model that the final realized sex composition will affect parents’ consumption-investment decision. Given a sex composition \((K_s, K_d)\), we can obtain the indirect utility

\[ V(K_s, K_d) \equiv \max_{c_1, c_2} u(c_1) + \beta u(c_2) \]

as a function of \((K_s, K_d)\). Note that \(V(K_s, K_d)\) is the value (indirect utility) of stopping at \((K_s, K_d)\) and does not include the option value of having more children. The interdependence of fertility and consumption-investment decisions draws a distinction between our model and models with exogenous stopping rules.

The solution to the fertility decision problem is not a tuple of optimal amounts of sons and daughters; it is instead an optimal fertility strategy (or policy) specifying the decision on whether or not to continue fertility at each sex composition \((K_s, K_d)\).

**Definition 3.1.** A fertility strategy is an action plan \(s\), which specifies a specific action to each sex composition \((K_s, K_d)\), i.e.,

\[ s : \mathbb{N}^2 \to \{C, S\}. \]

**Definition 3.2.** A fertility strategy \(s\) is finite if there exists an integer \(K \in \mathbb{N}\) such that \(s(K_s, K_d) = S\) for all \((K_s, K_d) \in \{(K_s, K_d) : K_s + K_d \geq K\}\).

We say the problem is finite if all feasible strategies are finite.

**Observation 3.1.** The decision problem faced by parents is finite.

Observation 3.1 follows the fact of the existence of a budget constraint. Due to the fixed cost of having a child \(\delta\), the maximum number of children parents can have is \([\frac{I}{\delta}]\). Observation 3.1 indicates that there exists a finite number of feasible strategies, therefore the optimal fertility strategy that gives parents the highest expected utility ex-ante must exist. Next, we introduce another concept that helps obtain the optimal strategy.

**Definition 3.3.** A stopping point is a sex composition \((K_s, K_d)\) if the best response of parents at \((K_s, K_d)\) is \(S\). Similarly, a non-stopping point is a sex composition \((K_s, K_d)\) if the best response of parents at \((K_s, K_d)\) is \(C\).

It is easy to see a strategy is never optimal if it specifies an \(S\) at some non-stopping points.

**Definition 3.4.** A point (sex composition) \((x_s, x_d)\) is directly reachable from another point \((y_s, y_d)\) if starting from \((y_s, y_d)\), the probability of reaching \((x_s, x_d)\) by reproducing one more child is positive. The point \((y_s, y_d)\) is then called a direct preceding point of \((x_s, x_d)\).
Note that a sex composition has at most two direct preceding points. For example, sex composition \((3, 4)\) has two direct preceding points \((2, 4)\) and \((3, 3)\), while \((0, 4)\) has one and only one direct preceding point \((0, 3)\). Denote \(\mathbb{K}\) the set of all stopping points.

**Definition 3.5.** The lower bound set \(\mathbb{K}^*\) of the set of stopping points \(\mathbb{K}\) consists of all stopping points with at least one direct preceding point not in \(\mathbb{K}\).

If a stopping point has all of its direct preceding points in the set of stopping points \(\mathbb{K}\), it will not be reached under the optimal fertility strategy. On the other hand, if a stopping point has a preceding point not in \(\mathbb{K}\), it may be reached with a positive probability under the optimal fertility strategy. In another word, all of the sex compositions of children we would expect to observe in the data are in the defined lower bound set \(\mathbb{K}^*\) of \(\mathbb{K}\), while the other sex compositions in \(\mathbb{K}\) we would never observe despite being stopping points.

In the following, we start with parents’ consumption-investment decision for a given sex composition, then derive the optimal fertility strategy. We show that all sex compositions in the defined lower bound \(\mathbb{K}^*\) are stopping points that can be realized with a positive probability under the optimal fertility strategy.

### 3.3 Results

#### 3.3.1 Optimal Fertility Strategy

We first look at the utility maximization problem for any given sex composition \((K_s, K_d)\). Substitute the budget constraints into the utility function of parents, we get the first-order conditions

\[
\begin{align*}
\frac{\partial U}{\partial z_i} &= -u'(c_1) + \beta \alpha_s f'(z_i) u'(c_2) = 0, \\
\frac{\partial U}{\partial z_j} &= -u'(c_1) + \beta \alpha_d f'(z_j) u'(c_2) = 0.
\end{align*}
\]

Note that parents only differentiate between sons and daughters, not within daughters or sons. Hence, all sons (daughters) will receive the same level of investment \(z_s\) \((z_d)\) from parents. From first-order conditions, we can obtain

\[
\alpha_s f'(z_s) = \alpha_d f'(z_d).
\] (3.1)
Equation (3.1) requires that the optimal investment in sons and daughters should make the same marginal returns to parents. We can easily derive \((z^*_s, z^*_d)\) as a function of \((K_s, K_d)\). Let \(K \equiv K_s + K_d\) be the family size.

**Proposition 3.1.** If there is a patriarchal institution, i.e., \(\alpha_s > \alpha_d\), the optimal investment \((z^*_s, z^*_d)\) have the following properties,

1. **Intra-Household Discrimination:** \(z^*_s > z^*_d\);
2. **Sibling Rivalry Effect:** \(\partial z^*_i / \partial K_s < 0\) and \(\partial z^*_i / \partial K_d > 0\) in the extensive domain \(\mathbb{R}^2_+\) conditional on family size \(K\) being fixed for \(i \in \{s, d\}\).

**Proof.** See Appendix C. ■

The first part of Proposition 3.1 is evident - the optimal investment in each son is higher than that in each daughter due to patriarchal institution \(\alpha_s > \alpha_d\). The second part says that conditional on family size, both sons and daughters benefit from having more sisters than brothers. It is intuitive since parents tend to allocate more investment to sons, i.e., more sons result in higher intensity of sibling rivalry within households.

With the optimal consumption and investment, the indirect utility function \(V(K_s, K_d)\) is determined for a given sex composition \((K_s, K_d)\), which is the utility obtained by stopping fertility at \((K_s, K_d)\). Let

\[
\Delta V_s(K_s, K_d) := V(K_s + 1, K_d) - V(K_s, K_d),
\]
\[
\Delta V_d(K_s, K_d) := V(K_s, K_d + 1) - V(K_s, K_d)
\]
denote the marginal gains from having an additional son and daughter at a sex composition \((K_s, K_d)\) respectively.

**Lemma 3.1.** Under Assumption 3.1, we have \(\partial^2 V / (\partial K_i \partial K_j) < 0\) for \(i, j \in \{s, d\}\) if

\[
\frac{\partial V}{\partial K_s} + \frac{\partial V}{\partial K_d} < 0
\]

in the extensive domain \(\mathbb{R}^2_+\).

**Proof.** See Appendix C. ■
Note that Inequality (3.2) holding at a sex composition \((K_s, K_d)\) implies that reproducing one more child from \((K_s, K_d)\) then stopping at \((K_s + 1, K_d)\) or \((K_s, K_d + 1)\) is not optimal, but it does not suggest that it is optimal for parents to stop fertility at \((K_s, K_d)\) because it may be optimal to reproduce more children starting from \((K_s, K_d)\). Lemma 3.1 excludes this possibility: Inequality (3.2) holding at \((K_s, K_d)\) also implies that it holds at all sex compositions that can be reached with a positive probability by reproducing one or more children from \((K_s, K_d)\).

**Proposition 3.2.** Under Assumption 3.1, the optimal fertility strategy is

\[
s(K_s, K_d) = \begin{cases} 
C & \text{if } \Delta V_s(K_s, K_d) + \Delta V_d(K_s, K_d) \geq 0, \\
S & \text{otherwise}.
\end{cases}
\]

**Proof.** See Appendix C. ■

The optimal fertility strategy is a one-stage look ahead strategy - for any sex composition of the children already born, parents only need to compare the (expected) utility of stopping right now and of stopping after reproducing one more child. If the latter is higher, then it is optimal to continue fertility at the current sex composition; otherwise, it is optimal to stop.

We call a stopping point that can be reached with a positive probability under the optimal fertility strategy an optimal stopping point, and the set of all optimal stopping points the optimal stopping set.

**Corollary 3.1.** Under Assumption 3.1, the optimal stopping set is the lower bound \(K^*\) of the set of stopping points \(\mathbb{K}\).

Corollary 3.1 establishes the fact that the lower bound \(K^*\) of the set of all stopping points is the optimal stopping set. Next, we characterize the properties of the optimal stopping set \(K^*\), and provide an algorithm to find it.

**Lemma 3.2.** Given a patriarchal institution \(\alpha_s > \alpha_d\), the indirect utility function \(V(K_s, K_d)\) has the following properties:

1. \(\Delta V_s > \Delta V_d\) for all \((K_s, K_d)\);
2. \(\partial \Delta V_i / \partial K_s < 0\) in the extensive domain \(\mathbb{R}_+^2\) for \(i \in \{s, d\}\) conditional on family size being fixed.

**Proof.** See Appendix C. ■
Lemma 3.2 suggests that: first, the marginal value of having one more son is higher than that of having one more daughter; second, conditional on family size, the marginal value of having one more child (son or daughter) for families with more sons is lower than for those with more daughters. This suggests that parents who already have many sons have a stronger incentive to stop fertility than those having more daughters.

Combining Lemma 3.1-3.2, we can characterize the optimal stopping set. Let $x = (x_s, x_d) \in \mathbb{N}^2$ be a sex composition, where $x_s$ ($x_d$) is the number of sons (daughters).

**Proposition 3.3** (Optimal Stopping Set). Given a patriarchal institution, i.e., $\alpha_s > \alpha_d$, the optimal stopping set $K^*$ has the following properties:

1. **All-Daughter Family**: $\tilde{K}_d = \max_{x \in K^*} \{x_d\}$, and $(0, \tilde{K}_d) \in K^*$;
2. **All-Son Family**: $K_s = \max_{x \in K^*} \{x_s\}$, and $(K_s, 0) \in K^*$;
3. **Largest and Smallest Family Sizes**: $\tilde{K}_d \geq x_s + x_d$ and $K_s \leq x_s + x_d \forall x \in K^*$;
4. **Optimality Criterion**: for all $x = (x_s, x_d) \in K^*$,
   $$(x_s, x_d - 1) \in K^* \text{ if and only if } (x_s + 1, x_d - 1) \notin K^*,$$
   or equivalently,
   $$(x_s, x_d - 1) \notin K^* \text{ if and only if } (x_s + 1, x_d - 1) \in K^*.$$

**Proof.** See Appendix C. ■

Proposition 3.3 first describes several properties of the optimal stopping set and provides an algorithm (the last term) to find it. The first two items tell us that there are two and only two single-sex families in the optimal stopping set, the all-daughter family and the all-son family. The third item says that the all-daughter family has the highest fertility rate while the all-son family has the lowest. Starting from any optimal stopping point, for instance, the all-daughter family $(0, \tilde{K}_d)$, applying the optimality criterion, we can easily find all other optimal stopping points. The

---

$^4$The all-daughter sex composition $(0, \tilde{K}_d)$ can be obtained from the following conditions:

$$\Delta V_s(0, K_d - 1) + \Delta V_d(0, K_d - 1) \geq 0 \text{ and } \Delta V_s(0, K_d) + \Delta V_d(0, K_d) < 0.$$

Or in the extensive domain:

$$\frac{\partial V}{\partial K_s} \bigg|_{K_s=0} + \frac{\partial V}{\partial K_d} \bigg|_{K_s=0} = 0.$$
optimal stopping set must admit an upward-stair shape (or a horizontal shape if the difference in return rates is sufficiently small) as shown in Figure 3.2, which itself indicates a son-preferring stopping rule, i.e., parents tend to stop earlier when they already have many sons.

With the optimal stopping set, we can derive a joint distribution of sex compositions $F(k_s, k_d)$ from the optimal stopping set $K^*$. The joint distribution $F(k_s, k_d)$ can be seen as either the ex-ante distribution of the fertility outcome of a family or the ex-post distribution of the realized fertility outcome in society.\(^5\)

**Proposition 3.4.** The distribution of sex composition $F : \mathbb{N}^2 \rightarrow [0, 1]$ (probability mass function) is given by

1. $\text{Supp}(F) = K^*$;

\(^5\)In reality, heterogeneity is pervasive: individuals may have different income levels or different regions may have different social institutions. Therefore, the results in the optimal stopping set derived here are better interpreted as a population average (of different social groups).
2. for any \((K_s, K_d) \in K^*\), let \(K = K_s + K_d\) and \(k_1 = \min\{K_s, K_d\}\), \(k_2 = \min\{K_s - 1, K_d\}\),

\[
F(K_s, K_d) = \begin{cases} 
\binom{K}{k_1} \left(\frac{1}{2}\right)^K & \text{if } (K_s, K_d - 1) \notin K^*, \\
\binom{K - 1}{k_2} \left(\frac{1}{2}\right)^K & \text{otherwise};
\end{cases}
\]

3. \(\text{Cov}_F(K, K_d) \geq 0\), i.e., the covariance between family size and the number of daughters in a family is non-negative;
4. the male to female ratio is 1, or equivalently, \(\mathbb{E}_F(K_s) = \mathbb{E}_F(K_d)\).

Proof. See Appendix C.

Proposition 3.4 characterizes the joint distribution of sex compositions under the optimal fertility strategy. An implication of the optimal strategy à la a son-prefering stopping rule is that females are disproportionately born into larger families (more siblings) compared to males. The non-negative correlation between family size and the number of daughters in a family therefore implies differential parental investments. However, the sex ratio in a population is balanced. As we will discuss in the next section, the sex ratio can only be unbalanced when sex selection is optional.

Corollary 3.2. The distribution of parental investment \(z_i\) for \(i \in \{s, d\}\) is given by probability mass function \(G_i\) such that

1. \(\text{Supp}(G_i) = \{z_i^*(K_s, K_d) : (K_s, K_d) \in K^*\}\);
2. \(G_i(z_i(K_s, K_d)) = F(K_s, K_d)\);
3. \(z_s\) first-order stochastically dominates \(z_d\).

The optimal stopping set \(K^*\) contains information about the distribution of sex compositions as well as the induced distributions of parental investment in females and males. However, without specifying an exact form of the utility function and investment function, it is impossible to derive the exact optimal stopping set and conduct comparative statics. Hence in the next section, we specifically consider a case with a logarithmic utility function and a square root investment function and study how patriarchal institution \(\alpha\) affects fertility outcome and parental investments.

3.3.2 Fertility Rate and Parental Investment

This subsection focuses on how patriarchal institution \((\alpha_s, \alpha_d)\) affects fertility rate and parental investment using the following specification \(u = \log(c)\) and \(f(z) = \sqrt{z}\). The optimal profile \((c, z)\) given \((K_s, K_d)\) and the indirect utility function \(V(K_s, K_d)\) are
3.3. Results

1. **optimal investment:**

\[
\begin{align*}
    z_s^* &= \alpha^2 z_d^*, \\
    z_d^* &= \frac{\beta(I - \delta(K_s + K_d))}{(2 + \beta)(\alpha^2 K_s + K_d)}
\end{align*}
\]

where \(\alpha = \frac{\alpha_s}{\alpha_d}\).

2. **optimal consumption:**

\[
\begin{align*}
    c_1^* &= \frac{2}{2 + \beta} (I - \delta(K_s + K_d)), \\
    c_2^* &= \alpha_s K_s \sqrt{z_s^*} + \alpha_d K_d \sqrt{z_d^*}
\end{align*}
\]

3. **indirect utility function:**

\[
V(K_s, K_d) = A + \frac{\beta}{2} \log(\alpha_s^2 K_s + \alpha_d^2 K_d) + \frac{2 + \beta}{2} \log(I - \delta(K_s + K_d)),
\]

where \(A\) is a constant.

Suppose \(\alpha_0\) is the rate of return on investment in a child (son or daughter) without any patriarchal distortion. Let \(\Delta \alpha \geq 0\) denote a spread, such that \(\alpha_s = \alpha_0 + \Delta \alpha\) and \(\alpha_d = \alpha_0 - \Delta \alpha\). We say that such a patriarchal institution \((\alpha_s, \alpha_d)\) is mean-preserving. In reality, most patriarchal institutions differentiate sexes in this way. One formal or informal right delegated to men often means that women do not share the same right. Intuitively, such patriarchal institutions will result in a higher fertility rate as parents who follow a son-prefering stopping rule would over-reproduce just to have more sons. However, we show that the average fertility rate in a patriarchal society is actually lower than that in a gender-neutral society. Before proceeding to this result, we present a useful observation.

**Observation 3.2.** There exists a partition \(\Omega^\alpha\) of \(\Delta \alpha \in [0, \alpha_0)\) such that for all \(\Omega_i^\alpha \in \Omega^\alpha\), the optimal stopping set, hence the fertility rate, is unaffected by a change in \(\Delta \alpha\) for \(\Delta \alpha \in \Omega_i^\alpha\).

This observation comes from the fact that the fertility outcome is discrete while the spread of patriarchal institution is a continuous variable. Therefore, small changes in patriarchal institution would not affect the stopping set. But if the spread varies across the boundary of two adjacent subsets of \(\Omega^\alpha\), the optimal stopping set will change. Similar observations also hold for other exogenous continuous variables such as income and cost per birth.
3.3. Results

To obtain insights on how a spread $\Delta \alpha$ may affect fertility rate, without loss of generality, let us consider the indirect utility function on the extended domain $\mathbb{R}_+^2$. Suppose $H(\Delta \alpha) := \partial V/\partial K_s + \partial V/\partial K_d$. A sex composition $(K_s, K_d)$ that is an optimal stopping point given a spread $\Delta \alpha$ satisfies

$$H(\Delta \alpha) = 0.$$  \tag{3.3}

If a change in $\Delta \alpha$ results in $H(\Delta \alpha) < 0$ at $(K_s, K_d)$, it is optimal to stop before reaching $(K_s, K_d)$, i.e., the fertility rate decreases; similarly, if a change in $\Delta \alpha$ results in $H(\Delta \alpha) > 0$, $(K_s, K_d)$ becomes a non-stopping point, i.e., fertility rate increases.

With our specification,

$$H(\Delta \alpha) = \frac{\beta}{2A_1}[(\alpha_0 + \Delta \alpha)^2 + (\alpha_0 - \Delta \alpha)^2] - \frac{\delta(2 + \beta)}{A_2},$$

where $A_1 = (\alpha_0 + \Delta \alpha)^2K_s + (\alpha_0 - \Delta \alpha)^2K_d$, $A_2 = I - \delta(K_s + K_d)$. The derivative of $H(\Delta \alpha)$ with respect to $\Delta \alpha$ at an optimal stopping point $(K_s, K_d)$ is

$$\frac{dH}{d\Delta \alpha} \bigg|_{H(\Delta \alpha)=0} = \frac{1}{A_1} \{2\beta\Delta \alpha - \frac{\delta(4 + 2\beta)}{A_2}[K_s(\alpha_0 + \Delta \alpha) - K_d(\alpha_0 - \Delta \alpha)]\}$$

$$= \frac{1}{A_1}[(2\beta - B(K_s + K_d))\Delta \alpha - B(K_s - K_d)\alpha_0], \tag{3.4}$$

where $B = \delta(4 + 2\beta)/A_2$. Note that the right-hand side of Equation (3.4) can be either positive or negative depending on sex compositions. For example, suppose $2\beta - B(K_s + K_d) > 0$, at an optimal stopping point $(K_s, K_d)$,

$$\frac{dH}{d\Delta \alpha} < 0,$$

if $K_s > K_d$ and $\Delta \alpha < \frac{\alpha_0B(K_s - K_d)}{2\beta - B(K_s + K_d)},$

$$\frac{dH}{d\Delta \alpha} \geq 0$$

otherwise.

In this case, at an optimal stopping point $(K_s, K_d)$ with $K_s > K_d$, when the spread $\Delta \alpha$ is low, a small increase in $\Delta \alpha$ leads to $H(\Delta \alpha) < 0$, which means families who have more sons now will stop fertility before reaching $(K_s, K_d)$ in response to the increase in $\Delta \alpha$. \footnote{The inequality $2\beta - B(K_s + K_d) > 0$ in general holds when $I \gg \delta(K_s + K_d)$. If $2\beta - B(K_s + K_d) < 0$, $dH/d\Delta \alpha < 0$ whenever $K_s > K_d$ regardless of the value of $\Delta \alpha$, also indicating that fertility would stop earlier in response to an increase in $\Delta \alpha$.} On the other hand, families with more daughters will increase their...
fertility rates. Therefore, it is possible that an increase in the patriarchal spread may result in a lower average fertility rate than in a gender-neutral society (i.e., $\Delta \alpha = 0$). However, when the spread $\Delta \alpha$ is sufficiently high, an increase in $\Delta \alpha$ will increase the average fertility rate for sure.

Figure 3.3 illustrates the preceding analysis. Figure 3.3(a) shows that when the patriarchal spread $\Delta \alpha = 0$, there is no differentiation in sexes, and the optimal amount of children is 2. However, when $\Delta \alpha$ increases to 0.05 as in Figure 3.3(b), parents choose to stop fertility immediately after they have a son, but $\Delta \alpha = 0.05$ is too small to induce parents to continue fertility when they have two daughters despite that they prefer sons. As $\Delta \alpha$ further increases to 1 in Figure 3.3(c), the differential between sons and daughters becomes large enough so that parents will continue fertility when they have two daughters. These examples suggest that the effects of patriarchal institutions on fertility and parental investment are non-monotone. In the following, we present some simulation results.

The fertility rate in a family with a fertility outcome $(K_s, K_d)$ is $K = K_s + K_d$. Denote $P(K)$ the population distribution of the family fertility rate under the optimal fertility strategy.

$$P(K) = \sum_{\{(K_s, K_d) \in K^* : K_s + K_d = K\}} F(K_s, K_d),$$
where $F(K_s, K_d)$ is the population distribution of sex compositions under the optimal fertility strategy. The average (or aggregate) fertility rate and the spread of fertility rate are defined to be the expectation of $K$, $E_P(K)$, and the standard deviation of $K$, $\sigma_P(K)$.

Figure 3.4 illustrates this seemingly counter-intuitive result that a mean-preserving patriarchal spread in the rate of return can result in a lower average fertility rate. In fact, there are two opposite effects of a mean-preserving spread on fertility behaviors. First, as the spread increases, sons become more attractive while daughters are less attractive, which makes reproduction riskier, and parents would stop fertility early once they have a son (or a small number of sons). This will decrease the fertility rate. Second, as the spread keeps increasing, parents who follow a son-preferring stopping rule would like to have more children in order to have more sons. This tends to increase the average fertility rate. The simulation results in Figure 3.4 further show that despite the fertility rate increasing with the spread when the spread is high, it will not return to the level of a gender-neutral society, i.e., $\Delta \alpha = 0$.

We summarize the simulation results about the fertility rate in the following observation.
3.3. Results

![Graphs](image)

(a) Standard deviation of fertility rate
(b) Parental investment

Figure 3.5: The effects of patriarchal institution on the standard deviation of fertility rate and parental investment. \( I = 55, \delta = 7, \alpha_0 = 0.3, \beta = 0.75, \Delta \alpha \in [0, 0.28] \).

**Observation 3.3.** A mean-preserving patriarchal spread in the rate of return can result in a lower average fertility rate than that of a gender-neutral society. In addition, the average fertility rate first drops and then weakly increases as the spread increases but never returns to the level with zero spread (if the spread \( \Delta \alpha \) is bounded by \( \alpha_0 \)).

Although a spread can decrease the average fertility rate, it also can result in a higher spread in the fertility rate, which means that parents with identical socioeconomic conditions may have very different fertility outcomes in a patriarchal society. Figure 3.3 shows several optimal stopping sets under different patriarchal spread \( \Delta \alpha \). It is easy to see as \( \Delta \alpha \) increases, the optimal stopping set (consisting of black nodes) is elongated. On the one hand, families with more daughters will over-reproduce to have a son; on the other hand, families who have a son will stop immediately. We capture this effect by the standard deviation of the fertility rate in a family. Figure 3.5(a) shows that the standard deviation of fertility rate is weakly increasing in the spread \( \Delta \alpha \).

**Observation 3.4.** An increase in the mean-preserving patriarchal spread \( \Delta \alpha \) can weakly result in a higher spread in fertility rate.

The combination of Observations 3.3 and 3.4 suggests the coexistence of a low aggregate fertility rate and over-reproduction.

Next, we study the effects of a mean-preserving spread on parental investment. The optimal investment is a function of \( \Delta \alpha \) and optimal stopping points \( (K_s, K_d) \), which are also a function of \( \Delta \alpha \). Hence we have

\[
\frac{dz^*}{d\Delta \alpha} = \frac{\partial z^*}{\partial \Delta \alpha} + \frac{\partial z^*}{\partial K_s} \frac{dK_s}{d\Delta \alpha} + \frac{\partial z^*}{\partial K_d} \frac{dK_d}{d\Delta \alpha}.
\]

**direct effect**
**indirect effect**
3.3. Results

The total effect of a spread $\Delta \alpha$ on the optimal investment can be decomposed into the direct effect and the indirect effect. A spread $\alpha$ first affects optimal investment by altering rates of return on sons and daughters, i.e., the direct effect; it can also affect fertility outcome and therefore affect the optimal investment, i.e., the indirect effect. The direct effect is quite straightforward: an increase in $\Delta \alpha$ will result in higher investment in sons and lower investment in daughters. For example, with our specification of the utility function and investment function,

$$\frac{\partial z_s^{*}}{\partial \Delta \alpha} = \frac{4\alpha K_d (I - \delta (K_s + K_d))}{(2 + \beta) (\alpha^2 K_s + K_d)^2} \frac{\alpha_0}{(\alpha_0 - \Delta \alpha)^2} > 0,$$

$$\frac{\partial z_d^{*}}{\partial \Delta \alpha} = -\frac{4K_s (I - \delta (K_s + K_d))}{(2 + \beta) (\alpha^2 K_s + K_d)^2} \frac{\alpha_0}{(\alpha_0 - \Delta \alpha)^2} < 0.$$

However, the indirect effect is not monotonic. As discussed earlier in this subsection, a low $\Delta \alpha$ may bring down the aggregate fertility rate, which means children will have fewer siblings, therefore may receive more investments from parents regardless of sex. Figure 3.5(b) illustrates this possibility: when the average fertility rate drops in response to an increase in $\Delta \alpha$ around 0.045, the parental investments in sons and daughters jump up; while in other cases, the parental investments in sons and daughters drop since the average fertility rate increases with the spread.

3.3.3 Sex Selection

It is unrealistic not to consider the possibility of sex selection when parents have a son preference. In this subsection, we study some properties of the optimal stopping set in the presence of sex selection. We show that the optimal stopping set is still upward stair-like in the presence of sex selection, therefore the non-negative correlation between family size and the number of daughters remains. However, the male-to-female ratio need not be 1.

We assume sex selection is costly, with a fixed cost $\gamma$ measured in utility. By sex selection, we mean that parents can guarantee to get a son or daughter by paying a fixed cost $\gamma$. It can be interpreted as that, compared to the present value of lifetime income $I$, the monetary cost of sex selection is negligible. Therefore, the cost of selection will not affect the fertility decision by decreasing the disposable income.\(^7\) We may interpret $\gamma$ as the psychological loss arising from sex selection. Hence, at an optimal stopping point $(K_s, K_d)$, parents’ ex-post utility will be a form

\(^7\)In other words, we are imposing separability on parents’ utility function. A similar assumption is also used in Bhaskar (2011).
3.3. Results

of $V(K_s, K_d) - n\gamma$, where $n$ is the number of sex selections conducted along the path to $(K_s, K_d)$. Due to the fixed cost $\gamma$, it is also easy to see that it must be daughters who are selectively aborted if sex selection happens. Hence, for a non-stopping point $(K_s, K_d)$, the immediate gain of conducting sex selection at it will be $\Delta V_s(K_s, K_d) - \gamma$.

Denote a set $X$ in the absence (presence) of sex selection as $X(NSS)$ ($X(SS)$), e.g., $K^*(SS)$ represents the optimal stopping set in the presence of sex selection.

Definition 3.6. For any two sets of sex compositions $A_1$ and $A_2$, $A_1$ is reachable from $A_2$ if, for any sex composition $(x_s, x_d) \in A_1$, there is at least one sex composition $(y_s, y_d) \in A_2$ such that by reproducing a non-negative number of children from $(y_s, y_d)$, the sex composition reaches $(x_s, x_d)$ with a positive probability.

Lemma 3.3. The optimal stopping set in the presence of sex selection $K^*(SS)$ is reachable from the optimal stopping set in the absence of sex selection $K^*(NSS)$.

Proof. See Appendix C.

Lemma 3.3 essentially implies that a non-stopping point in the absence of sex selection can not be a stopping point in the presence of sex selection, but not vice versa. This is intuitive since regardless of the possibility of sex selection, continuing without sex selection is better than stopping at a sex composition that is a non-stopping point in the absence of sex selection.

Lemma 3.4. It is optimal to continue fertility at a sex composition $(K_s, K_d) \in K(NSS)$ if and only if $\Delta V_s(K_s, K_d) \geq \gamma$.

Proof. See Appendix C.

Lemma 3.4 provides the sufficient and necessary conditions to continue (or equivalently stop) fertility at sex compositions that are stopping points in the absence of sex selection. These two lemmas imply that if the cost of sex selection is high, the optimal stopping set in the presence of sex selection ($K^*(SS)$) is always a subset of that in the absence of sex selection ($K^*(NSS)$).

To see it clearly, if the cost of sex selection $\gamma$ satisfies $\gamma \geq \Delta V_s(K_s, K_d)$ for all $(K_s, K_d) \in K^*(NSS)$, all stopping points in the absence of sex selection will still be stopping points in the presence of sex selection, i.e., $K^*(SS) \subseteq K^*(NSS)$. In another word, when the cost of sex selection is high enough, it will not result in a higher fertility rate.

The two preceding lemmas imply that the optimal stopping set in the presence of sex selection cannot be downward stair-like, therefore still represents a son-prefering stopping rule.
Proposition 3.5. Given a patriarchal institution, i.e., \( \alpha_s > \alpha_d \), and an option of sex selection, for any sex composition \((K_s, K_d)\) in the optimal stopping set \(K^*(SS)\), a sex composition \((y_s, y_d)\) is not in \(K^*(SS)\) if

\[
y_s + y_d > K_s + K_d, \text{ and } y_s > K_s.
\]

Proof. See Appendix C. ■

Proposition 3.5 states a weak version of the optimality criterion in Proposition 3.3. It indicates a non-negative correlation between family size and the number of daughters in a family under the optimal fertility strategy. That is that females are still disproportionately born into larger families when sex selection is optional. Another thing to note is that the male-to-female ratio under the optimal fertility strategy in the presence of sex selection is greater than 1 if sex selection ever is conducted. This is not surprising since the sex ratio is 1 if there is no sex selection while sex selection always increases the probability of giving birth to a son.

Corollary 3.3. With the option of sex selection, the population demographics have the following properties under the optimal fertility strategy,

1. \( \text{Cov}(K, K_d) \geq 0 \), where \( K = K_s + K_d \) for a sex composition \((K_s, K_d)\) \(\in K^*(SS)\);
2. the male to female ratio is weakly greater than 1, i.e., \( \mathbb{E}(K_s) \geq \mathbb{E}(K_d) \).

Finally, we summarize our results regarding sex selection in the following numerical example.
3.3. Results

Example 3.1. Suppose $u = \log(c)$, $f(z) = \sqrt{z}$, and $I = 50$, $\delta = 5$, $\beta = 0.7$, $\alpha_s = 0.2$, $\alpha_d = 0.15$. As shown in Figure 3.6(a), the optimal stopping set without sex selection is

$$K^*(NSS) = \{(0, 3), (1, 2), (1, 1), (2, 0)\}.$$ 

The optimal fertility strategy that results in the optimal stopping set is:

$$s^*(K_s, K_d) = \begin{cases} 
\text{Continue} & \text{if } (K_s, K_d) \in \{(0, 0), (0, 1), (1, 0), (0, 2)\}, \\
\text{Stop} & \text{otherwise.} 
\end{cases}$$

It is easy to verify that the male-to-female ratio is 1, i.e.,

$$E(K_s) = E(K_d) = \frac{9}{8}.$$ 

However, if sex selection is possible at a cost $\gamma = 0.05$, the new optimal stopping set is shown in Figure 3.6(b):

$$K^*(SS) = \{(1, 1), (2, 0)\} \subset K^*(NSS).$$

The optimal fertility strategy now is

$$s^*(K_s, K_d) = \begin{cases} 
\text{Continue with SS} & \text{if } (K_s, K_d) \in \{(0, 1)\}, \\
\text{Continue with NSS} & \text{if } (K_s, K_d) \in \{(0, 0), (1, 0), (0, 2)\}, \\
\text{Stop} & \text{otherwise.} 
\end{cases}$$

Note that despite that it is optimal to continue fertility without sex selection at sex composition $(0, 2)$, it will never be reached following the optimal fertility strategy. The reason is that it is optimal to conduct sex selection at its only preceding point $(0, 1)$ as shown in Figure 3.6(b). Again, one can verify that the male-to-female ratio is greater than 1 when sex selection is possible:

$$E(K_s) = \frac{5}{4} > E(K_d) = \frac{3}{4}.$$ 

3.4 Conclusion

This paper studies a stochastic sequential model of fertility with discriminatory preferences. We show that, with convex preference and investment technology, patriarchal institutions that are in favor of sons generate a one-step look ahead son-preferring fertility strategy (or stopping rule). We fully characterize the optimal
fertility strategy and the induced optimal stopping set. We then derive demographic results induced by the optimal fertility strategy such as the population distribution of sex compositions, the non-negative correlation between family size and the number of daughters, and the sex ratio. In particular, females are more likely born into larger families so the population distribution of investment in males first-order stochastically dominates that of females in a patriarchal society. These findings are robust to the introduction of sex selection. Another interesting implication of the optimal fertility strategy is that the average fertility rate in a patriarchal society can be lower than in a gender-neutral society.

Finally, it is worth noting that the approach this paper takes to derive the optimal fertility strategy can be applied to a broad range of optimization problems which involves two stages where the first stage is a stochastic sequential decision problem, whose outcome affects the second-stage (static) decision. One example is costly information acquisition, where decision-makers first decide when to stop acquiring information, e.g., good or bad signal, and the final signal composition would later affect another decision problem such as investment and verdict decisions.
Proof of Proposition 1.1. We prove Proposition 1.1 by induction. We start with level-1 players. Given the optimal strategy of level-0 players

$$s^0_t(x_{t-1}) = \begin{cases} 2 & x_{t-1} \geq T_{12}, \\ 1 & x_{t-1} < T_{12}, \end{cases}$$

it is easy to see that the 1-rationalizable set

$$A^1_t = \begin{cases} \{2\} & x_{t-1} \geq T_{12}, \\ \{1\} & x_{t-1} < T_{12}, \end{cases}$$

Hence for $0 < x_{t-1} < T_{12}$, level-1 players believe those who played action 2 in period $t - 1$ will play action 2 in period $t$ and onward; those who played action 1 are level 0.

Suppose $x_{t-1} \geq T_{12} - 1/(M - 1)$. Consider an action sequence of a level-1 player $\{2\}^\infty_t$, she believes for any non-negative integer $t$

$$x_{t+l} \geq x_{t-1} + \frac{1}{M - 1} \geq T_{12}.$$ 

Therefore, she believes level-0 will play action 2 from period $t+1$, and the equilibrium will be switched to $e_2$, yielding the highest possible long-run average payoff $F_2(1)$. That is, the optimal choice of level-1 players in period $t$ is action 2.

Suppose $x_{t-1} < T_{12} - 1/(M - 1)$. Consider an action sequence of a level-1 player $\{2\}^\infty_t$, she believes

$$x_{t+l} = x_{t-1} + \frac{1}{M - 1} < T_{12}.$$ 

That is, she believes her choice of action 2 will not make level-0 players play action 2 in the future. The long-run average payoff would be $F_2(x_{t-1} + 1/M - 1)$. 

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Consider another action sequence of a level-1 player \( \{1\}_t^\infty \), she believes

\[
x_{t+l} = x_{t-1} < T_{12}.
\]

The long-run average payoff is \( F_1(x_{t-1}) \). Recall that \( F_1(x) \) \((F_2(x)) \) is decreasing \((\text{increasing})\) in \( x \), and \( F_1(x) > F_2(x) \) for \( x < T_{12} \), we have

\[
F_1(x_{t-1}) > F_1(x_{t-1} + \frac{1}{M-1}) > F_2(x_{t-1} + \frac{1}{M-1}).
\]

Note that the long-run average payoffs of any other action sequences are a convex combination of \( F_1(x_{t-1}) \) and \( F_2(x_{t-1} + 1/(M-1)) \). Therefore, the optimal choice of level-1 players in period \( t \) is action 1. To summarize, the optimal strategy of level-1 players is

\[
s^1_t(x_{t-1}) = \begin{cases} 
2 & x_{t-1} \geq T_{12} - \frac{1}{M-1}, \\
1 & \text{otherwise}.
\end{cases}
\]

Suppose the proposition holds for up to level \( k - 1 \). Due to Assumption 1.1 on the level-\( k \) subjective distribution, a level-\( k \) player expects that if \( x_{t-1} \geq T_{12} - k/(M-1) \), her unilateral deviation from action 1 to action 2 from period \( t \) onward will induce all players below \( k \) to play action 2 within the next \( k \) periods from period \( t \): She expects that all level-(\( k - 1 \)) to play action 2 in period \( t + 1 \); at least all level-(\( k - 1 \)) and level-(\( k - 2 \)) players to play action 2 in period \( t + 2 \); at least all level-(\( k - 3 \)) to level-(\( k - 1 \)) to play action 2 in period \( t + 3 \); ...; at least all players to play action 2 in period \( t + k \). This leads the level-\( k \) player to believe that her long-run average payoff is \( F_2(e_2) \), the highest payoff she could obtain.

If \( x_{t-1} < T_{12} - k/(M-1) \), it is easy for a level-\( k \) player to realize that her unilateral deviation from action 1 to action 2 cannot induce players below \( k \) to play action 2 in the future. The long-run average payoff of playing action 2 is less than playing action 1 since she expects that \( x_{t+l} < T_{12} \). Therefore, it is optimal for level-\( k \) players to play action 1 if \( x_{t-1} < T_{12} - k/(M-1) \). This completes the proof. ■

**Proof of Proposition 1.2.** We first prove sufficiency. In order to switch from \( e_1 \) to \( e_2 \), it requires at least \( \lceil (M-1)T_{12} \rceil \) players above level 0. Since the distribution of levels \( f(l) \) is a domino distribution, i.e.,

\[
\sum_{l=k}^K f(l) \geq \frac{M-1}{M} T_{12} - \frac{k-1}{M} \quad \text{for} \quad k \in \{1, 2, \ldots, K\}.
\]
Let \( k = 1 \), we have
\[
M \sum_{l=1}^{K} f(l) \geq (M - 1)T_{12}.
\]
Hence the number of players above level 0 is weakly greater than \( \lceil (M - 1)T_{12} \rceil \). That is, we have enough sophisticated players to succeed in equilibrium switching.

Recall from Proposition 1.1 that a level-\( k \) player plays 2 if \( x \geq T_{12} - \frac{k}{M-1} \). The following part shows that the inequality condition imposed on \( f(k) \) creates a sequence of deviations that ensures the condition \( x \geq T_{12} - \frac{k}{M-1} \) for all \( k \) along a transition from \( e_1 \) to \( e_2 \). Recall that \( K = \lceil (M - 1)T_{12} \rceil \), and \( f(K) \) represents the proportion of players with a level no lower than \( K \). Note that \( T_{12} - \frac{K}{M-1} \leq 0 \), and \( x \geq 0 \), the optimal strategy of those players is \( s^K_t(x_{t-1}) = 2 \) for all \( t \).

Now let us consider the decision of level-(\( K - 1 \)) players. Given players above \( K - 1 \) play 2, we have \( x \geq \frac{Mf(K)}{M-1} \). From the inequality we have
\[
f(K) \geq \frac{M-1}{M} T_{12} - \frac{K-1}{M},
\]
\[
\Rightarrow x \geq \frac{Mf(K)}{M-1} \geq T_{12} - \frac{K-1}{M-1}.
\]
Hence deviations from 1 to 2 by level-\( K \) will induce level-\( K - 1 \) to play 2 in the future under domino distributions. Repeating the process, it is easy to see deviations from action 1 to action 2 by players weakly above \( k \) in period \( t \) will induce at least level-(\( k - 1 \)) to play 2 in the future for all \( k > 0 \).

Next we prove necessity. If equilibrium switching from \( e_1 \) to \( e_2 \) succeeds, it means that after players above \( k - 1 \) switched from action 1 to action 2, the cutoffs of some players below \( k \) must be reached. Further, notice that if a level-\( l \)'s cutoff is reached, then the thresholds of all levels above \( l \) are also reached. This means after players above \( k - 1 \) switched, at least the cutoff of level-(\( k - 1 \)) is reached, i.e.,
\[
\frac{M}{M-1} \sum_{l=k}^{K} f(l) \geq T_{12} - \frac{k-1}{M-1}.
\]
Rearranging the above inequality, it easy to verify that \( f(l) \) is a domino distribution.

Finally, it is easy to see from the optimal strategy of level-\( k \) players that level-\( k \) players will not deviate from efficient equilibrium \( e_2 \) to \( e_1 \), hence no equilibrium switching from \( e_2 \) to \( e_1 \) regardless of whether \( f(k) \) is a domino distribution with respect to \((e_2, e_1)\). ☐
A. Appendix for Chapter 1

**Proof of Proposition 1.3.** For the first part, consider any binary-action game with two strict Nash equilibria $e$ and $e'$. Let $f(l)$ be a distribution of cognitive level, $T$ be the associated tipping point of the pair $(e, e')$, $K \equiv \lceil (M - 1)T \rceil$ and $f(K)$ denote the fraction of players with a level weakly higher than $K$. Since $f(l)$ is a domino distribution of the pair, it satisfies

$$\sum_{l=k}^{K} f(l) \geq \frac{M - 1}{M} T - \frac{k - 1}{M} \text{ for } k \in \{1, 2, ..., K\}.$$ 

If $f'(l)$ first-order stochastically dominates $f(l)$,

$$\sum_{l=k}^{K} f'(l) \geq \sum_{l=k}^{K} f(l) \geq \frac{M - 1}{M} T - \frac{k - 1}{M} \text{ for } k \in \{1, 2, ..., K\}.$$ 

Therefore, by definition, $f'(l)$ is also a domino distribution of the pair $(e, e')$.

For the second part, consider any binary-action game with two strict Nash equilibria $e$ and $e'$. Let $f'(l)$ be a distribution of cognitive level, $T$ be the associated tipping point of the pair $(e, e')$, $K \equiv \lceil (M - 1)T \rceil$ and $f'(K)$ be the fraction of players with a level weakly higher than $K$. It is easy to verify that $f$ is a domino distribution of the pair $(e, e')$. The sufficiency is obvious by Proposition 1.3. Next, we prove the necessity. Denote

$$H(k) \equiv \frac{M - 1}{M} T - \frac{k - 1}{M}.$$ 

If $f'(l)$ is a domino distribution of the pair, it satisfies

$$\sum_{l=k}^{K} f'(l) \geq H(k). \tag{A.1}$$

Note that

$$H(k) - \sum_{l=k}^{K} f(l) = \frac{(M - 1)T - k + 1}{M} - \frac{K - k + 1}{M} = \frac{(M - 1)T - \lceil (M - 1)T \rceil}{M} \in (-\frac{1}{M}, 0],$$

$$\Rightarrow |H(k) - \sum_{l=k}^{K} f(l)| \in [0, \frac{1}{M}).$$
Further note that the minimal increment in the fraction of cognitive levels is \(1/M\). Therefore, \(\sum_{l=k}^{K} f(l)\) is the only possible value of the fraction of some cognitive levels. Inequality (A.1) then implies

\[
\sum_{l=k}^{K} f'(l) \geq \sum_{l=k}^{K} f(l),
\]

i.e., \(f\) first-order stochastically dominates \(\bar{f}\).

**Proof of Proposition 1.4.** We first prove the first part. If a distribution of cognitive level \(f(l)\) is a domino distribution of an equilibrium pair with a tipping point \(T\), it satisfies

\[
\sum_{l=k}^{K} f(l) \geq \frac{M - 1}{M} T - \frac{k - 1}{M} \text{ for } k \in \{1, 2, ..., K\}.
\]

Note that the right-hand side of the inequality is decreasing in \(T\), therefore, it is also a domino distribution of any equilibrium pair with a tipping point less than \(T\).

For the second part, consider a distribution \(f(l)\) with a finite support \(\text{supp}(f) \subset \mathbb{N}\). For any \(k \in \text{supp}(f)\) and \(k \neq 0\), the following equation defines a function \(T(k)\),

\[
\sum_{l \geq k} f(l) = \frac{M - 1}{M} T - \frac{k - 1}{M},
\]

where \(M\) is a positive integer, represent the population size. Let

\[
\bar{T}(f) := \min_{k \in \text{supp}(f)} (T(k)).
\]

Then the distribution must satisfy

\[
\sum_{l \geq k} f(l) \geq \frac{M - 1}{M} \bar{T}(f) - \frac{k - 1}{M} \text{ for any positive } k \in \text{supp}(f),
\]

\[
\sum_{l \geq k} f(l) = \frac{M - 1}{M} \bar{T}(f) - \frac{k - 1}{M} \text{ for some positive } k \in \text{supp}(f).
\]

It is obvious from here that \(f(l)\) is a domino distribution of an equilibrium pair with a tipping point \(T'\) if and only if \(T' \leq \bar{T}(f)\).
Proof of Corollary 1.1. Corollary 1.1 is a direct result of Proposition 1.2. For the first part, consider any strict Nash equilibrium $e_i$, and suppose there exists exactly one strict Nash equilibrium $e_j$ strictly domino dominates $e_i$. To apply Proposition 1.2, we define a new coordination game with an action set

$$\{D (deviate), C (conform)\},$$

where $D$ represents deviation from $e_i$ in the direction of $T_{ij} - e_i$, and $C$ represents playing action $i$. The new game has two strict Nash equilibria that corresponds to $e_i$ and $e_j$. Without loss of generality, we say $e_i$ and $e_j$ are the two strict Nash equilibria of the new game. Since $e_j$ is Pareto superior to $e_i$, and the tipping point for transitions from $e_i$ to $e_j$ is $d_{ij}$, and $f(k)$ is a domino distribution with respect to $(e_i, e_j)$. By Proposition 1.2, a population will transit from $e_i$ to $e_j$. The second part is obvious by the definition of domino dominance and Proposition 1.2.

Proof of Proposition 1.5. We first prove the sufficiency. We first prove a population will switch away from $e_i$. The first condition implies that there exists an ordered chain of strict Nash equilibria connecting $e_i$ to $e_j$, and along the chain, efficiency is weakly improving and domino dominance holds. If domino dominance along the chain is strict, Corollary 1.1 implies that $e_i$ as well as all other equilibria except $e_j$ in the chain is unstable. To see why $e_j$ is stable, note that the third condition implies that $e_j$ is not domino dominated. If not, there exists another equilibria $e_{j+1}$ domino dominated $e_j$, then $e_{j+1}$ must also iteratively domino dominates $e_i$, and $F_{j+1}(e_j) \geq F_j(e_j)$. A contradiction with the third condition. Next to see why the population will switch to $e_j$, note that for any other strict Nash equilibrium $e_l$ that iteratively domino dominates $e_i$, the third condition implies that the equilibrium payoff $F_j(e_j) > F_l(e_l)$. That is equilibrium $e_j$ yields the highest long-run average payoff, therefore, payoff-maximizing players will switch the population to $e_j$.

For the necessity, if a population switches from $e_i$ to $e_j$, the first condition is obviously true. Further note that the equilibrium payoff of $e_j$ should be strictly higher than $e_i$, i.e., $e_j$ is Pareto superior to $e_i$. The last condition also holds. If not, there exists another strict Nash equilibrium $e_l$ that iteratively domino dominates $e_i$, and yields a higher long-run average payoff than $e_j$. This implies that a sufficiently high-level player can adopt an action sequence that leads the population to $e_l$. A contradiction with the assumption of payoff maximization.
Proof of Corollary 1.2. It is obvious by the construction of $D(C_{ij})$ and $D^*_{ij}$, and Proposition 1.4. ■

Proof of Proposition 1.6. Since there exists only one strict Nash equilibrium, denoted as $e^*$, that is not strictly domino dominated. For any other strict equilibrium, there must exist at least one strict equilibrium that strictly domino dominates it. In addition, note that there are finite strict Nash equilibria. Together these two facts imply that $e^*$ strictly iteratively domino dominates any other strict equilibrium. Corollary 1.5 then implies Proposition 1.6.

To see it clearly, pick an arbitrary strict equilibrium $e_i$, and suppose that $e^*$ does not strictly iteratively domino dominates $e_i$. First note that $e_i$ is strictly domino dominated ($sDD$) by at least one strict equilibrium, denoted as $e_{i+1}$. That is to say, for every $e_{i+k}$ which is not $e^*$, is $sDD$ by another strict Nash equilibrium, denoted as $e_{i+k+1}$. This forms a chain of strictly iterative domino dominance, and $e^*$ is not in the chain. Since there are finite strict Nash equilibria. There must exists $e_j$ and $e_l$ in the chain such that $e_j$ strictly iteratively domino dominates $e_l$ and $e_l$ strictly domino dominates $e_j$. By the definition of strict domino dominance, we have $F_j(e_j) > F_l(e_l)$ and $F_l(e_l) > F_j(e_j)$, a contradiction. Therefore, $e^*$ strictly iteratively domino dominates all other strict equilibria. In addition, by the definition of domino dominance, $e^*$ is the efficient Nash equilibrium. ■

Proof of Proposition 1.7. It is obvious that $e_1$ and $e_2$ are two pure Nash equilibria since $F_1^1(e_1) > F_2^1(e_1)$ and $F_2^2(e_2) > F_1^2(e_2)$ based on the strategic complementarity and the single crossing assumptions. Next, if $q_2 \in (\frac{M-1}{M}T_2 + \frac{1}{M}, \frac{M-1}{M}T_1)$, it easy to verify that $e_a = (q_1, q_2)$ is also a pure Nash equilibrium. At state $e_a$, all members of group 1 play 1 and all members of group 2 play 2. For members of group 1, since the fraction of players among $M - 1$ players is $\frac{Mq_1}{M-1} < T_1$, $F_1^1(e_a) > F_2^1(e_a)$, i.e., it is optimal for members of group 1 to play 1 at $e_a$. For members of group 2, the fraction of players among $M - 1$ players is $\frac{Mq_2}{M-1} > T_2$, hence $F_2^2(e_a) > F_1^2(e_a)$, i.e., it is optimal for members of group 2 to play 2 at $e_a$. This completes the proof. ■

Proof of Proposition 1.8. The proof is similar to the proof of Proposition 1.1. The only difference between the two is the condition

$$q_2 \geq \frac{M - 1}{M}T_1.$$
Note that this inequality simply implies that if all members of group 2 plays 2, then it is optimal for group 1 to play 2, but not vice versa. Therefore, switching from $e_1$ to $e_2$ solely depends on the group 2. Therefore, level-$k$ players in group 2 will have the same strategies as in the Proposition 1.1. ■

**Proof of Proposition 1.9.** The proof is similar to the proof of Proposition 1.2 since inequality $q_2 \geq \frac{M-1}{M} T_1$ suggests that equilibrium switching from $e_1$ to $e_2$ solely depends on group 2. ■
Appendix B

Appendix for Chapter 2

B.1 Does the Inertia Cause Downward Deviations

In the main section 2.3.4, we investigate the different perceptions of and thus the responsiveness to signals with two sampling sizes as the mechanism driving the downward deviations. Here we study whether subjects are inertial while they should have caught up with changes in the signals so that their action choices are classified as downward deviations. Specifically, if subjects were inertial, they would choose the same action regardless of the changes in the signals.

Let $\text{inertia}_i$ take the value of 1 if $\text{Action}_i = \text{Action}_{i,t-1}$, and 0 otherwise. We describe an action in period $t$ by subject $i$ as an inertial choice if $\text{inertia}_i = 1$. It is important to note that inertial choice is a description of behaviors, which may or may not be caused innate tendency of inertia of subjects. Figure B.1 depicts the number of periods that a subject played mBR, downward and upward deviations, grouped by $\text{inertia}_i$, where each bar corresponds to a subject. Among the downward deviations, there is a noticeable proportion of inertial choices, indicating that downward deviations might be caused by innate tendency of inertia.

We further test the inertia hypothesis by logit regressions of $\text{inertia}_i$. The explanatory variables include $s_{it}$, OwnSignal$_{it}$ which is the fraction of the action chosen in period $t-1$ by subject $i$ in the signal at the beginning of period $t$, $\text{DSignal}_{it}^{-}$ which is the dummy variable taking the value of 1 if the fraction of the action chosen in period $t-1$ in a signal decreases, the interaction term $\text{DSignal}_{it}^{-} \times \text{OwnSignal}_{it}$, and the same set of controls as in the main paper. The standard errors are clustered at the session level. Model (1) in Table B.1 reports the basic estimates and model (2) includes control variables. We say a signal in period $t$ is good if $\text{DSignal}_{it}^{-} = 1$ in the sense that this signal encourages subjects to use the action chosen in the last period because $\text{DSignal}_{it}^{-} = 1$ indicates more players are playing the previously chosen action; otherwise, a signal is bad.
B.1. Does the Inertia Cause Downward Deviations

Figure B.1: The number of periods of each type of behaviors, per subject, sorted by downward deviations. Each bar corresponds a subject in a session. A choice is labelled by ‘Inertia’ if $Action_{it} = Action_{i,t-1}$, ‘notInertia’ otherwise.

Models (1) and (2) both support that subjects adjust their action choices to changes in signals, suggesting that subjects do not have the trait of innate inertia. First, the significant negative effect of $DSignal_{it}$ on the probability of playing the same action indicates that subjects would like to switch away from the previous action if the signal at the current period is bad. Second, the significant positive effect of $OwnSignal_{it}$ on the probability of playing the same action indicates that subjects increase the probability playing the same action in the last period if more other subjects are believed to play the action. In addition, the size of the effect of $OwnSignal_{it}$ is significantly smaller when signals are bad. That is, the effects of good and bad signals are asymmetric: subjects over-react to bad signals than to good signals.

Since two out of all sessions transit to the medium-efficient equilibrium $e_2$ instead of the most efficient equilibrium $e_3$, we further look at whether subjects are on average more reluctant to switch away from action 2 than from other actions. Building on models (1) and (2) in Table B.1, models (3) and (4) further include action dummies in period $t-1$ and their interaction terms with $OwnSignal_{it}$. Compared to action 2, choosing action 3 in period $t-1$ does not significantly change the probability of choosing itself again in period $t$; while choosing action 1 in period $t-1$ significantly decreases the probability (of choosing itself in period $t$). Thus, the switch rates away from the previous action choices are the same when the previous action choices are action 2 and 3; however, if the previous action choice is action 1,
subjects switch away from it more often compared to the case where the previous action is action 2. The interaction terms of action dummies and $OwnSignal_t$ are not significant, i.e., the effects of $OwnSignal_t$ on the probability of choosing the same action as in the last period are the same for all action choices.

**Result 9.** Subjects do not have innate tendency to play the same action over time when the effect of signals is controlled, and thus downward deviations are not caused by inertia. On the other hand, subjects on average switch away from action 1 more often than action 2, but there is no difference in the rates of switching away from action 2 and 3.
Table B.1: Panel logit regressions. Test of Inertia.

<table>
<thead>
<tr>
<th>Inertia_{it}</th>
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<th>(3)</th>
<th>(4)</th>
</tr>
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<tbody>
<tr>
<td>s7_{i,t}</td>
<td>0.235**</td>
<td>0.305***</td>
<td>0.312***</td>
<td>0.361***</td>
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<tr>
<td></td>
<td>(0.108)</td>
<td>(0.090)</td>
<td>(0.116)</td>
<td>(0.112)</td>
</tr>
<tr>
<td>DSignal_{it}</td>
<td>-0.361**</td>
<td>-0.356**</td>
<td>-0.423***</td>
<td>-0.418***</td>
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<tr>
<td></td>
<td>(0.144)</td>
<td>(0.145)</td>
<td>(0.135)</td>
<td>(0.140)</td>
</tr>
<tr>
<td>OwnSignal_{it}</td>
<td>3.763***</td>
<td>3.727***</td>
<td>3.562***</td>
<td>3.542***</td>
</tr>
<tr>
<td></td>
<td>(0.244)</td>
<td>(0.235)</td>
<td>(0.226)</td>
<td>(0.226)</td>
</tr>
<tr>
<td>DSignal_{it} × OwnSignal_{it}</td>
<td>-0.701**</td>
<td>-0.737**</td>
<td>-0.562*</td>
<td>-0.600**</td>
</tr>
<tr>
<td></td>
<td>(0.304)</td>
<td>(0.308)</td>
<td>(0.296)</td>
<td>(0.299)</td>
</tr>
<tr>
<td>Action1_{i,t−1} × OwnSignal_{it}</td>
<td>-0.842*</td>
<td>-0.864</td>
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</tr>
<tr>
<td></td>
<td>(0.469)</td>
<td>(0.526)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Action3_{i,t−1} × OwnSignal_{it}</td>
<td>0.293</td>
<td>0.265</td>
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<tr>
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<td>(0.495)</td>
<td>(0.508)</td>
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<td></td>
</tr>
<tr>
<td>Action1_{i,t−1}</td>
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<td>-0.443**</td>
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<tr>
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<td>(0.184)</td>
<td>(0.206)</td>
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<td></td>
</tr>
<tr>
<td>Action3_{i,t−1}</td>
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<td>0.036</td>
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<td>(0.195)</td>
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</tr>
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<td>Degree_{i}</td>
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<td>(0.458)</td>
<td>(0.492)</td>
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</tr>
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<td>Female_{i}</td>
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<td></td>
<td>(0.321)</td>
<td>(0.347)</td>
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<td></td>
<td>(0.134)</td>
<td>(0.152)</td>
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</tr>
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<td>nCRT_{i}</td>
<td>0.199**</td>
<td>0.200**</td>
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<td>(0.084)</td>
<td>(0.087)</td>
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</tr>
<tr>
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<td>(0.074)</td>
<td>(0.078)</td>
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<td></td>
</tr>
<tr>
<td>Constant</td>
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<td>-1.214**</td>
<td>-0.210</td>
<td>-1.191*</td>
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<tr>
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<td>(0.092)</td>
<td>(0.576)</td>
<td>(0.146)</td>
<td>(0.622)</td>
</tr>
<tr>
<td>N</td>
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<td>8532</td>
<td>8848</td>
<td>8532</td>
</tr>
</tbody>
</table>

Notes: Session clustered standard errors in parentheses. * p < 0.1, ** p < 0.05, *** p < 0.01. Fewer observations in columns (2) and (4) are due to missing values in Female_{i} and Age_{i}.
B.2 Additional Tables and Figures

Figure B.2: The deviation rate over time by session with transitions plotted at the background. In general, the deviation rate is high at the beginning of the game, then decreases to a low level as the population reached an equilibrium (or the neighborhood of an equilibrium). In the sessions transiting to $e_2$ and $e_3$ in sequence, the rise in the deviation rate is evident again when the population starts to transit to $e_3$ followed by another decrease. Thus, the deviation rate usually has a hump shape from the beginning to the end of the transitions. It provides a solid ground to examine the direction of deviations discussed in the main text of the paper.
Figure B.3: The number of periods when two types of deviations are available. Error bars represent standard deviations.
Figure B.4: Relationship between raw CRT and normalized CRT. There is significant positive correlation between the two CRT scores: The Pearson’s correlation coefficient is 0.6167**, and the Spearman’s $\rho$ is 0.6981***.
Figure B.5: Downward Deviation

Figure B.6: Upward Deviation

Figure B.7: Scatter plots of deviation rate and associated payoff loss given signals (full data set). Both upward and downward deviations are negatively correlated with the associated payoff loss: The Spearman’s $\rho$ is $-0.9042^{\ast\ast\ast}$ for downward deviation and $-0.6882^{\ast\ast\ast}$ for upward deviation. Furthermore, upward deviation is significantly less correlated with the payoff loss than downward deviation ($\Delta \rho = 0.2160^{\ast}$).
B.2. Additional Tables and Figures

Figure B.8: The proportion of action choices given each signal.
Figure B.9: The number of periods of BR, downward and upward deviations played by every subject, sorted by downward deviations. The red horizontal reference line is 15% of time on downward deviations. Each subject is labeled by her type: F = FL player, M = Myopic player, O = Others.
Figure B.10: Realized payoffs of subjects playing mBR, downward and upward deviations in each period.
### Table B.2: Summary statistics. Mean (SD).

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<th>Session</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Total</th>
<th>N</th>
</tr>
</thead>
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<tr>
<td>Female</td>
<td>0.43</td>
<td>0.57</td>
<td>0.71</td>
<td>0.93</td>
<td>0.46</td>
<td>0.85</td>
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<td>0.71</td>
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<td>109</td>
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<tr>
<td>(0.51)</td>
<td>(0.51)</td>
<td>(0.47)</td>
<td>(0.27)</td>
<td>(0.52)</td>
<td>(0.38)</td>
<td>(0.48)</td>
<td>(0.47)</td>
<td>(0.47)</td>
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</tr>
<tr>
<td>Degree</td>
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<td>3.07</td>
<td>3.00</td>
<td>3.93</td>
<td>3.00</td>
<td>3.79</td>
<td>3.36</td>
<td>4.00</td>
<td>3.39</td>
<td>112</td>
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<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.47)</td>
<td>(0.00)</td>
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<td>(0.63)</td>
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<td>Economics</td>
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<td>0.07</td>
<td>0.50</td>
<td>0.00</td>
<td>0.07</td>
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<td>(0.51)</td>
<td>(0.51)</td>
<td>(0.50)</td>
<td>(0.27)</td>
<td>(0.52)</td>
<td>(0.00)</td>
<td>(0.27)</td>
<td>(0.36)</td>
<td>(0.47)</td>
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</tr>
<tr>
<td>Age</td>
<td>19.93</td>
<td>20.29</td>
<td>20.08</td>
<td>24.14</td>
<td>20.43</td>
<td>23.29</td>
<td>21.00</td>
<td>24.00</td>
<td>21.66</td>
<td>111</td>
</tr>
<tr>
<td>(1.07)</td>
<td>(1.73)</td>
<td>(1.12)</td>
<td>(2.96)</td>
<td>(1.45)</td>
<td>(4.10)</td>
<td>(4.40)</td>
<td>(1.47)</td>
<td>(3.07)</td>
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<td></td>
</tr>
</tbody>
</table>

**Cognitive Reflection Test**

- Q1: 1.00 (0.00) 0.93 (0.00) 1.00 (0.00) 0.93 (0.00) 1.00 (0.00) 1.00 (0.00) 1.00 (0.00) 0.98 (0.00) 112
- Q2: 0.64 (0.50) 0.43 (0.51) 0.50 (0.52) 0.71 (0.47) 0.71 (0.47) 0.71 (0.47) 0.43 (0.47) 0.60 (0.47) 112
- Q3: 0.93 (0.27) 0.79 (0.43) 0.71 (0.47) 0.79 (0.47) 0.79 (0.47) 0.79 (0.47) 0.71 (0.47) 0.81 (0.47) 112
- Q4: 1.00 (0.00) 0.93 (0.27) 0.93 (0.27) 0.93 (0.27) 1.00 (0.00) 0.86 (0.00) 0.79 (0.00) 0.93 (0.00) 112
- Q5: 0.93 (0.27) 0.71 (0.50) 0.93 (0.47) 0.79 (0.43) 0.79 (0.43) 0.79 (0.43) 0.86 (0.43) 0.79 (0.43) 112
- Q6: 1.00 (0.00) 0.86 (0.36) 0.79 (0.43) 0.93 (0.27) 0.93 (0.27) 0.93 (0.27) 0.79 (0.27) 0.79 (0.27) 0.82 (0.27) 112
- Q7: 0.86 (0.36) 0.57 (0.47) 0.57 (0.43) 0.43 (0.50) 0.57 (0.50) 0.57 (0.50) 0.57 (0.50) 0.57 (0.50) 0.57 (0.50) 112
- Q1-Q7: 6.36 (0.84) 5.64 (1.50) 5.21 (1.58) 5.43 (1.45) 5.50 (1.34) 5.43 (1.16) 5.71 (1.14) 4.71 (1.86) 5.50 (1.45) 112

**Seconds spent on CRT**

- 268.14 (85.32) 237.00 (102.93) 238.43 (96.85) 301.36 (98.14) 259.00 (102.63) 286.00 (104.47) 310.14 (107.65) 298.21 (118.45) 274.79 (102.70) 112

**(Q1-Q7)/Seconds\(\times 100\)**

- 2.74 (1.32) 2.95 (1.70) 2.78 (1.88) 2.04 (1.14) 2.61 (1.58) 2.24 (1.13) 2.13 (1.01) 1.76 (0.94) 2.40 (1.39) 112

**Risk**

- 4.93 (2.92) 3.86 (3.25) 5.07 (3.17) 3.29 (2.61) 4.57 (3.01) 4.79 (3.42) 5.07 (2.97) 4.93 (3.36) 4.56 (3.06) 112

**Trust**

- 5.29 (2.76) 5.36 (2.53) 5.57 (2.03) 5.86 (2.03) 6.00 (2.42) 6.00 (2.49) 5.29 (2.29) 6.93 (2.23) 5.79 (1.59) 112

**Patience**

- 17.10 (8.19) 14.44 (7.96) 14.95 (7.24) 12.05 (8.30) 15.44 (7.81) 14.56 (7.60) 15.18 (8.21) 13.08 (8.14) 14.60 (7.82) 112

**Negative Reciprocity**

- 4.05 (1.59) 4.44 (1.87) 4.14 (2.11) 4.92 (2.21) 5.52 (1.66) 4.34 (2.04) 4.35 (1.83) 4.46 (2.04) 4.53 (1.92) 112

**Positive Reciprocity**

- 13.33 (2.72) 13.20 (2.29) 13.93 (1.83) 15.06 (1.26) 13.35 (2.70) 13.81 (3.26) 14.92 (3.13) 14.11 (3.32) 13.96 (2.48) 112

**Altruism**

- 0.72 (0.97) 0.81 (0.78) 0.83 (0.58) 1.04 (0.54) 0.69 (0.72) 0.98 (0.46) 0.98 (0.46) 0.82 (0.79) 0.82 (0.71) 112

**Level k**

- 3.93 (2.40) 5.00 (2.72) 4.21 (3.12) 5.43 (3.03) 5.35 (2.23) 5.86 (2.63) 5.71 (2.95) 4.71 (3.36) 4.79 (3.07) 112

Notes: Female takes missing value if subjects chose to not say. Degree = 1 if below college, = 2 if with college degree, = 3 if undergraduate, = 4 if postgraduate (taught), = 5 if postgraduate (research). One observation of Age is replaced by missing value due to unreasonably high value (i.e., 222). See Frederick (2005); Toplak et al. (2011) for CRT, Eckel and Grossman (2002); Reynaud and Couture (2012) for risk preferences, Falk et al. (2016, 2018) for social preferences, Arad and Rubinstein (2012) for level \(k\).
Table B.3: OLS of distance on time allowing for breaks in trend and constant.

<table>
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<th>Distance</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
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<td>t1</td>
<td>0.015***</td>
<td>-0.035***</td>
<td>0.001</td>
<td>-0.039***</td>
<td>-0.091***</td>
<td>-0.006</td>
<td>-0.040**</td>
<td>0.017***</td>
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<tr>
<td></td>
<td>(0.002)</td>
<td>(0.005)</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.010)</td>
<td>(0.004)</td>
<td>(0.015)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>t2</td>
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<td>0.003</td>
<td>0.000</td>
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<td>-0.001</td>
<td>0.022***</td>
<td>-0.000</td>
<td>-0.004</td>
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<tr>
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<td>(0.011)</td>
<td>(0.015)</td>
<td>(0.007)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.015)</td>
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<tr>
<td>t3</td>
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<td>-0.000</td>
<td>-0.024***</td>
<td>-0.046***</td>
<td>0.005</td>
<td>-0.038***</td>
<td>-0.099***</td>
<td>-0.004</td>
</tr>
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<td>(0.009)</td>
<td>(0.001)</td>
<td>(0.005)</td>
<td>(0.006)</td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.013)</td>
<td>(0.015)</td>
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<tr>
<td>t4</td>
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<td>-0.014**</td>
<td>-0.005</td>
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<td>0.133***</td>
<td>-0.013**</td>
<td>-0.004</td>
<td>(0.015)</td>
</tr>
<tr>
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<td>(0.008)</td>
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<td>(0.015)</td>
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<td>(0.053)</td>
<td>(0.049)</td>
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<td>(0.019)</td>
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<td>(0.077)</td>
<td>(0.429)</td>
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<td>5.748***</td>
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<td>3.769***</td>
<td>0.901***</td>
<td>2.263***</td>
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<tr>
<td></td>
<td>(0.343)</td>
<td>(0.059)</td>
<td>(0.296)</td>
<td>(0.459)</td>
<td>(0.211)</td>
<td>(0.177)</td>
<td>(0.465)</td>
<td>(0.465)</td>
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<td>constant4</td>
<td>0.076</td>
<td>1.391***</td>
<td>0.430</td>
<td>9.750***</td>
<td>-5.885***</td>
<td>0.835***</td>
<td>2.325***</td>
<td>-6.780***</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.481)</td>
<td>(0.598)</td>
<td>(0.808)</td>
<td>(0.736)</td>
<td>(0.262)</td>
<td>(0.298)</td>
<td>(0.922)</td>
</tr>
<tr>
<td>constant5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant6</td>
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<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Notes: Columns (1)-(8) are regressions of eight sessions respectively. Independent variables t1-t6 and constant1-constant6 are separated time trends and constants based on estimated structural breaks. Standard errors in parentheses. * p < 0.1, ** p < 0.05, *** p < 0.01.
Table B.4: Robustness test of treatment effects.

<table>
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<tr>
<th></th>
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<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_{pi}$</td>
<td>$Down_{pi}$</td>
<td>$FL_{pi}$</td>
</tr>
<tr>
<td>s7</td>
<td>0.493</td>
<td>-0.522**</td>
<td>0.918*</td>
</tr>
<tr>
<td></td>
<td>(0.492)</td>
<td>(0.223)</td>
<td>(0.501)</td>
</tr>
<tr>
<td>Degree</td>
<td>-0.577</td>
<td>0.170</td>
<td>0.250</td>
</tr>
<tr>
<td></td>
<td>(0.610)</td>
<td>(0.314)</td>
<td>(0.570)</td>
</tr>
<tr>
<td>Female</td>
<td>-0.590**</td>
<td>0.614***</td>
<td>-0.442</td>
</tr>
<tr>
<td></td>
<td>(0.301)</td>
<td>(0.211)</td>
<td>(0.404)</td>
</tr>
<tr>
<td>Economics</td>
<td>0.580***</td>
<td>0.161</td>
<td>-0.207</td>
</tr>
<tr>
<td></td>
<td>(0.167)</td>
<td>(0.279)</td>
<td>(0.546)</td>
</tr>
<tr>
<td>nCRT</td>
<td>0.154*</td>
<td>-0.108*</td>
<td>0.414**</td>
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<tr>
<td></td>
<td>(0.086)</td>
<td>(0.064)</td>
<td>(0.165)</td>
</tr>
<tr>
<td>Age</td>
<td>0.083</td>
<td>-0.006</td>
<td>-0.041</td>
</tr>
<tr>
<td></td>
<td>(0.096)</td>
<td>(0.055)</td>
<td>(0.144)</td>
</tr>
<tr>
<td>Risk</td>
<td>0.041</td>
<td>-0.020</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>(0.047)</td>
<td>(0.032)</td>
<td>(0.099)</td>
</tr>
<tr>
<td>Trust</td>
<td>0.033</td>
<td>-0.042</td>
<td>0.175**</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.041)</td>
<td>(0.077)</td>
</tr>
<tr>
<td>Patience</td>
<td>-0.013*</td>
<td>-0.015**</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>Negative Reciprocity</td>
<td>0.000</td>
<td>-0.032</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.038)</td>
<td>(0.121)</td>
</tr>
<tr>
<td>Positive Reciprocity</td>
<td>-0.015</td>
<td>-0.009</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.014)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>Altruism</td>
<td>0.160</td>
<td>0.171**</td>
<td>-0.106</td>
</tr>
<tr>
<td></td>
<td>(0.207)</td>
<td>(0.075)</td>
<td>(0.278)</td>
</tr>
<tr>
<td>Level K</td>
<td>0.062**</td>
<td>-0.002</td>
<td>0.137*</td>
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<tr>
<td></td>
<td>(0.032)</td>
<td>(0.039)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.943</td>
<td>-1.511*</td>
<td>-5.597**</td>
</tr>
<tr>
<td></td>
<td>(1.212)</td>
<td>(0.782)</td>
<td>(2.528)</td>
</tr>
</tbody>
</table>

Notes: Columns (1) and (2) are fractional logit regression and column (3) is logit regression. Clustered standard errors in parentheses. One outlier is omitted for column (2) (see footnote 15). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. 
Table B.5: Average treatment effects computed based on mixed logit regressions with categorical dependent variable $UpDownBR_t$ taking three values for best responses, downward and upward deviations.

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<td>Down</td>
<td>BR</td>
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<tr>
<td>$s7$</td>
<td>-0.081***</td>
<td>0.068**</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>.Signal2</td>
<td>-0.285**</td>
<td>0.319***</td>
</tr>
<tr>
<td></td>
<td>(0.115)</td>
<td>(0.097)</td>
</tr>
<tr>
<td>.Signal3</td>
<td>-0.243**</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>(0.114)</td>
<td>(0.098)</td>
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<tr>
<td>Female</td>
<td>0.069***</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.000</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Degree</td>
<td>0.036</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.053)</td>
</tr>
<tr>
<td>Economics</td>
<td>0.001</td>
<td>-0.032</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>nCRT</td>
<td>-0.017**</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Risk</td>
<td>-0.002</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Trust</td>
<td>-0.005</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Patience</td>
<td>-0.002*</td>
<td>0.003**</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Negative Reciprocity</td>
<td>-0.004</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Positive Reciprocity</td>
<td>-0.000</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Altruism</td>
<td>0.019*</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Level K</td>
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<td>-0.006</td>
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<tr>
<td></td>
<td>(0.006)</td>
<td>(0.005)</td>
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<tr>
<td>N</td>
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<td>20759</td>
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</table>

Notes: The session clustered standard error is used. The outlier is excluded (see footnote 15). Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. 
## B.2. Additional Tables and Figures

Table B.6: Robustness test of treatment effects on the responsiveness to signals.

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<th></th>
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<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
<td>(2)</td>
<td></td>
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<tr>
<td>s7</td>
<td>1.443***</td>
<td>0.503</td>
<td>1.572***</td>
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</tr>
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<td></td>
<td>(0.457)</td>
<td>(0.335)</td>
<td>(0.542)</td>
<td>(0.335)</td>
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</tr>
<tr>
<td>Signal2</td>
<td>-3.817***</td>
<td>-0.821***</td>
<td>-3.974***</td>
<td>-0.814***</td>
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</tr>
<tr>
<td></td>
<td>(0.283)</td>
<td>(0.192)</td>
<td>(0.232)</td>
<td>(0.189)</td>
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</tr>
<tr>
<td>s7×Signal2</td>
<td>-0.827</td>
<td>-0.481</td>
<td>-0.788</td>
<td>-0.567*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.895)</td>
<td>(0.308)</td>
<td>(0.954)</td>
<td>(0.339)</td>
<td></td>
</tr>
<tr>
<td>Signal3</td>
<td>-2.547***</td>
<td>3.341***</td>
<td>-2.709***</td>
<td>3.409***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.551)</td>
<td>(0.264)</td>
<td>(0.530)</td>
<td>(0.332)</td>
<td></td>
</tr>
<tr>
<td>s7×Signal3</td>
<td>-0.987</td>
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<td>-0.911</td>
<td>0.431</td>
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<td>(0.435)</td>
<td>(0.886)</td>
<td>(0.464)</td>
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<td>0.342</td>
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<td>(0.707)</td>
<td>(1.022)</td>
<td>(0.627)</td>
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<tr>
<td>Female</td>
<td>-0.627</td>
<td>-0.892***</td>
<td>-0.422</td>
<td>-0.848***</td>
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</tr>
<tr>
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<td>(0.475)</td>
<td>(0.302)</td>
<td>(0.443)</td>
<td>(0.278)</td>
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</tr>
<tr>
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<td>0.418</td>
<td>0.346</td>
<td>0.467*</td>
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</tr>
<tr>
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<td>(0.310)</td>
<td>(0.261)</td>
<td>(0.265)</td>
<td>(0.267)</td>
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</tr>
<tr>
<td>nCRT</td>
<td>0.002</td>
<td>0.160***</td>
<td>0.078</td>
<td>0.154**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.160)</td>
<td>(0.061)</td>
<td>(0.153)</td>
<td>(0.061)</td>
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</tr>
<tr>
<td>Age</td>
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<td>-0.045</td>
<td>0.007</td>
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</tr>
<tr>
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<td>(0.222)</td>
<td>(0.117)</td>
<td>(0.173)</td>
<td>(0.107)</td>
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</tr>
<tr>
<td>Risk</td>
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<td>0.061</td>
<td>-0.065</td>
<td>0.044</td>
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</tr>
<tr>
<td>Trust</td>
<td>-0.183***</td>
<td>-0.009</td>
<td>(0.053)</td>
<td>(0.041)</td>
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</tr>
<tr>
<td>Patience</td>
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<td>-0.010</td>
<td>(0.030)</td>
<td>(0.008)</td>
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</tr>
<tr>
<td>Negative Reciprocity</td>
<td>-0.022</td>
<td>0.030</td>
<td>(0.055)</td>
<td>(0.068)</td>
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</tr>
<tr>
<td>Positive Reciprocity</td>
<td>-0.036</td>
<td>-0.010</td>
<td>(0.042)</td>
<td>(0.034)</td>
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</tr>
<tr>
<td>Altruism</td>
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<td>-0.057</td>
<td>(0.285)</td>
<td>(0.124)</td>
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</tr>
<tr>
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<td>0.047</td>
<td>(0.075)</td>
<td>(0.036)</td>
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</tr>
<tr>
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<td>(0.954)</td>
<td>(0.783)</td>
<td>(1.230)</td>
<td>(0.895)</td>
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</tr>
</tbody>
</table>

Notes: Mixed logit regressions. Action 2 is used as base outcome. Standard errors are clustered at session level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. 

$N$ 25920 25920
Table B.7: Average marginal effects computed based on regressions in Table B.6.

<table>
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<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<td>(0.028)</td>
<td>(0.018)</td>
<td>(0.032)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Signal2</td>
<td>-0.219***</td>
<td>0.269***</td>
<td>-0.050**</td>
<td>-0.220***</td>
<td>0.272***</td>
<td>-0.052***</td>
</tr>
<tr>
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<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.021)</td>
<td>(0.018)</td>
<td>(0.017)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Signal2 s = 2</td>
<td>-0.172***</td>
<td>0.223***</td>
<td>-0.051**</td>
<td>-0.166***</td>
<td>0.218***</td>
<td>-0.052**</td>
</tr>
<tr>
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<td>(0.025)</td>
<td>(0.023)</td>
<td>(0.022)</td>
<td>(0.027)</td>
<td>(0.027)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Signal2 s = 7</td>
<td>-0.256***</td>
<td>0.310***</td>
<td>-0.054*</td>
<td>-0.268***</td>
<td>0.323***</td>
<td>-0.056*</td>
</tr>
<tr>
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<td>(0.014)</td>
<td>(0.022)</td>
<td>(0.029)</td>
<td>(0.015)</td>
<td>(0.025)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Signal3</td>
<td>-0.246***</td>
<td>-0.265***</td>
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<td>-0.248***</td>
<td>-0.270***</td>
<td>0.518***</td>
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<td>(0.021)</td>
<td>(0.017)</td>
<td>(0.011)</td>
<td>(0.024)</td>
<td>(0.019)</td>
</tr>
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<td>Signal3 s = 2</td>
<td>-0.177***</td>
<td>-0.316***</td>
<td>0.493***</td>
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<td>(0.021)</td>
<td>(0.030)</td>
<td>(0.019)</td>
<td>(0.024)</td>
<td>(0.037)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Signal3 s = 7</td>
<td>-0.295***</td>
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<td>0.514***</td>
<td>-0.309***</td>
<td>-0.211***</td>
<td>0.520***</td>
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<td>(0.030)</td>
<td>(0.036)</td>
<td>(0.025)</td>
<td>(0.032)</td>
<td>(0.039)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Degree</td>
<td>0.030</td>
<td>0.016</td>
<td>-0.046</td>
<td>0.021</td>
<td>0.000</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.118)</td>
<td>(0.068)</td>
<td>(0.045)</td>
<td>(0.097)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>Female</td>
<td>-0.017</td>
<td>0.122***</td>
<td>-0.104***</td>
<td>-0.006</td>
<td>0.108***</td>
<td>-0.102***</td>
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<tr>
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<td>(0.027)</td>
<td>(0.044)</td>
<td>(0.039)</td>
<td>(0.025)</td>
<td>(0.037)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>Economics</td>
<td>0.001</td>
<td>-0.052**</td>
<td>0.050</td>
<td>0.010</td>
<td>-0.062***</td>
<td>0.053</td>
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<tr>
<td></td>
<td>(0.021)</td>
<td>(0.023)</td>
<td>(0.039)</td>
<td>(0.020)</td>
<td>(0.021)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>nCRT</td>
<td>-0.003</td>
<td>-0.017**</td>
<td>0.020***</td>
<td>0.001</td>
<td>-0.019**</td>
<td>0.018**</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.010)</td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.010)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.005</td>
<td>-0.002</td>
<td>0.007</td>
<td>-0.003</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.020)</td>
<td>(0.011)</td>
<td>(0.008)</td>
<td>(0.017)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Risk</td>
<td>-0.003</td>
<td>-0.005</td>
<td>0.008*</td>
<td></td>
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<tr>
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<td>(0.006)</td>
<td>(0.005)</td>
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</tr>
<tr>
<td>Trust</td>
<td>-0.010***</td>
<td>0.007</td>
<td>0.002</td>
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<tr>
<td>Negative Reciprocity</td>
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<td>-0.002</td>
<td>0.004</td>
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<td>(0.008)</td>
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<tr>
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<td>0.002</td>
<td>-0.001</td>
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<tr>
<td></td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.004)</td>
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<tr>
<td>Altruism</td>
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<td>0.002</td>
<td>-0.010</td>
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<td></td>
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<tr>
<td>Level K</td>
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<td>0.006</td>
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</table>

Notes: Standard errors in parentheses. * p < 0.1, ** p < 0.05, *** p < 0.01.
B.3 Instructions

Please read the following instructions carefully. To ensure that you fully understand the instructions, you will be asked to take a comprehension quiz after reading the instructions. While you are answering the quiz, please feel free to refer to the paper instructions. It will be followed by a training stage to help you get familiar with the interface. If you have any questions, please raise your hand so an experimenter would come to help you in person. Please silence your phone and any other electrical devices. Communication with other participants during the experiment is not allowed. Your cooperation is much appreciated.

The Rule

In this experiment, you will earn tokens by playing a 14-player game. At the end of the experiment, the tokens you earn will be converted into money as will be described below. The greater are your token earnings, the greater are your money earnings.

All the players in the game will choose one out of three available actions, 1, 2, and 3. Every player’s payoff (in tokens) will depend on his/her own action choice and the other players’ action choices in a group. Each player’s payoff possibilities are the same, and will be presented in a payoff table, similar to the one in Table B.8 below.

\[
\begin{array}{c|ccc}
\text{Fractions} & x_1 & x_2 & x_3 \\
\hline
\text{Your Actions} & & & \\
1 & 30 & 0 & 13 \\
2 & 9 & 22 & 0 \\
3 & 10 & 6 & 24 \\
\end{array}
\]

Table B.8: Payoff Table Example

The table rows are labeled by your choice (1, 2 and 3) and the columns are labeled by the fraction of the other players in your group (excluding you) choosing each action:

- \(x_1\) is the fraction of players choosing action 1;
- \(x_2\) is the fraction of players choosing action 2;
- \(x_3\) is the fraction of players choosing action 3;
Each table entry represents a token payoff given the indicated choice and fraction. Suppose that all the other players in your group choose action 1, thus we have fractions $x_1 = \frac{12}{13}$, $x_2 = 0$, and $x_3 = 0$. If you choose action 1, you will get 30 tokens, the number in the first row and first column. If all the other players choose action 2 instead of action 1, the three fractions are $x_1 = 0$, $x_2 = \frac{13}{13}$, and $x_3 = 0$ respectively. Now by choosing action 1, you will get 0 token (the number in the first row and second column). Similarly, if all the other players choose action 3, you will get 13 tokens by choosing action 1 (the number in first row and third column).

Now let’s suppose that among the other 13 players in your group, there are 4 players choosing action 1, 4 choosing action 2 and 5 choosing action 3. Thus the fractions are $x_1 = \frac{4}{13}$, $x_2 = \frac{4}{13}$, and $x_3 = \frac{5}{13}$. Your payoff of choosing action 1 now is the weighted average of all the entries in the first row: $14.23$ tokens = $30$ tokens $\times \frac{4}{13} + 0$ tokens $\times \frac{4}{13} + 13$ token $\times \frac{5}{13}$.

How your payoff is calculated when you choose action 1 therefore can be generalised as follows:

$$30 \text{ tokens} \times x_1 + 0 \text{ token} \times x_2 + 13 \text{ tokens} \times x_3$$

Similarly, if you choose action 2 (see the entries in the second row), your payoff will be

$$9 \text{ tokens} \times x_1 + 22 \text{ tokens} \times x_2 + 0 \text{ token} \times x_3;$$

if you choose action 3 (see the entries in the last row), your payoff will be

$$10 \text{ tokens} \times x_1 + 6 \text{ tokens} \times x_2 + 24 \text{ tokens} \times x_3.$$

### Your Decisions

In the experiment, you will play a 14-player game as described above for 80 periods. Every period will last for 10 seconds.

You will be matched anonymously with counterparts, some or all of the participants in today’s experiment. The group members will stay the same throughout the whole experiment.

Before the game starts, every player in a group will be assigned an action choice, out of three available actions. You will be informed of the action assigned to you after the game starts.

### Periods

At the beginning of each period, you will be provided with one piece of information:
B.3. Instructions

- We will use random procedure to select one player among the other 13 players in your group (excluding you) for $s$ times, and show you their action choices from the last period. Note:
  - Each time, the selection is from the whole group but excluding you.
  - That is, it is possible the same player in your group might be selected more than once. As the result it is possible that all of these $s$ randomly selected decisions might come from the same player, however the chances of this are relatively small.
  - Every player in a group will receive a piece of such information. The number of selection times is the same for everyone. However, the information may differ across players in your group, and you will only see your information but not others’ information.
  - You will be included in the random selection for the information to others.

Figure B.11 shows an example of the information with $n = 3$. Blue represents action 1, green is action 2, orange is action 3. The horizontal axis is the period. Here the stacked column is the information to you at the beginning of period 12: among 3 times of selections, every action is observed once, namely $\frac{1}{3}$ labelled on the column. Note: the more players choosing an action, then the action is more likely to be observed in each selection.

![Figure B.11: Information Example ($s = 3$)](image)

After you review the information provided, you can choose one from three actions. You have to click ‘Submit’ button to successfully submit your choice. You can also decide to not make any choice. If you do not choose before a period ends, your action in the last period will be carried over. You can only make at most one choice in each period. At the end of a period, we will calculate each player’s payoff of the current period using the fractions based on the players’ actions at this moment.
The procedure is the same in every period. Your payoff for playing the game will be sum of payoffs across all the periods. You will not be informed of the payoffs until the end of the experiment.

Note: The information shown to you at the beginning of the first period is based on players’ assigned actions. The payoff table and the selection times are same for every player in your group and stays the same in all the periods. Before game starts, we will inform everyone of the selection times $s$. When and which action to choose are up to you. It is in your best interest to make decisions which you believe would give you the most tokens.

Survey
You will answer a survey after finishing the game.

Your Earnings
You will earn ¥5 for showing up for the experiment.

The tokens you earn in the game will be converted into yuan. 120 tokens = ¥1.

At last, you will earn some amount of money for answering the survey. The details will be made clear as it comes.

Your total earnings of participating in this experiment will be the following:

Total Earnings = ¥5 for showing up + token(s) earned in games/120 + earnings for survey
B.4 Screenshot of Game Interface

游戏（正式）

第 4/80 期 剩余时间： 0:05

当前策略： 1
请从下列策略中选择一个：

- ○ 1
- ○ 2
- ○ 3

提交

Figure B.12: Interface
Proof of Proposition 3.1

Proof. The first part is obvious. For the second part, denote $m \equiv (f')^{-1}$, hence $m' < 0$. Let $\alpha \equiv \frac{\alpha_s}{\alpha_d}$ then $z_d^* = m(\alpha f'(z_s^*))$. We can rewrite $c_1$ as $\hat{I} - K_s z_s - (K - K_s)m(\alpha f'(z_s))$ where $\hat{I} \equiv I -(K_s + K_d)\delta$ is a constant, and $c_2$ as $\alpha_s K_s f(z_s) + \alpha_d (K - K_s) f(m(\alpha f'(z_s)))$. Hence the first-order derivative with respect to $z_s$ indicates an implicit function

$$g(z_s, K_s) \equiv -u'(c_1) + \beta \alpha_s f'(z_s) u'(c_2) = 0.$$ 

By implicit function theorem, we have

$$\frac{\partial z^*_s}{\partial K_s} = - \frac{\partial g/\partial K_s}{\partial g/\partial z^*_s},$$

conditional on $K$ being a constant. Notice that $u$ and $f$ are strictly concave, and $m' < 0$, $\alpha_s > \alpha_d$, $z_s > z_d$, hence

$$\frac{\partial g}{\partial K_s} = -u''(c_1)(-z_s + z_d) + \beta \alpha_s f'(z_s) u''(c_2) (\alpha_s f(z_s) - \alpha_d f(z_d)) < 0,$$

$$\frac{\partial g}{\partial z^*_s} = -u''(c_1)(-K_s - (K - K_s)m' \alpha f''(z_s)) + \beta \alpha_s u'(c_2) f''(z_s) +$$

$$\beta \alpha_s f'(z_s) u''(c_2) (\alpha_s K_s f'(z_s) + \alpha_d (K - K_s) f'(z_d)m' \alpha f''(z_s)) < 0.$$ 

Therefore, $\frac{\partial z^*_s}{\partial K_s} < 0$. Notice that $K_d = K - K_s$ is decreasing in $K_s$, implying $\frac{\partial z^*_s}{\partial K_d} > 0$. Also notice

$$z_d = m(\alpha f'(z_s)) \Rightarrow \frac{dz_d}{dz_s} > 0,$$

hence $\frac{\partial z^*_s}{\partial K_s} < 0$ and $\frac{\partial z^*_s}{\partial K_d} > 0$ conditional on $K$ being a constant. \qed
Proof of Lemma 3.1

Proof. Scenario 1: $\partial V / \partial K_s < 0$

By Proposition 3.2, we have $\partial V / \partial K_d < \partial V / \partial K_s < 0$, which ensures $\partial V / \partial K_s + \partial V / \partial K_d < 0$. Notice that

$$
\frac{\partial V}{\partial K_s} = u'(c_1)(-\delta - z_s) + \beta u'(c_2) \alpha_s f(z_s) < 0
$$

$$
\Rightarrow f(z_s) < f'(z_s)(\delta + z_s).
$$

The last step comes from the first-order condition $u'(c_1) = \beta \alpha_s f'(z_s) u'(c_2)$. Similarly, we also have

$$
f(z_d) < f'(z_d)(\delta + z_d).
$$

We first focus on the sign of

$$
\frac{\partial^2 V}{\partial K_s^2} = u''(c_1) \frac{\partial c_1}{\partial K_s}(-\delta - z_s) + \beta u''(c_2) \alpha_s f(z_s) \frac{\partial c_2}{\partial K_s}.
$$

Case 1: $\partial c_2 / \partial K_s \geq 0$

In this case, we only need to show $\frac{\partial c_1}{\partial K_s} < 0$. Notice that

$$
\frac{\partial c_1}{\partial K_s} = -\delta - z_s - K_s \frac{\partial z_s}{\partial K_s} - K_d \frac{\partial z_d}{\partial K_s}
$$

$$
\frac{\partial c_2}{\partial K_s} = \alpha_s f(z_s) + \alpha_s K_s f'(z_s) \frac{\partial z_s}{\partial K_s} + \alpha_d K_d f'(z_d) \frac{\partial z_d}{\partial K_s}.
$$

Further notice that

$$
\frac{\partial c_2}{\partial K_s} = \alpha_s f(z_s) + \alpha_s K_s f'(z_s) \frac{\partial z_s}{\partial K_s} + \alpha_d K_d f'(z_d) \frac{\partial z_d}{\partial K_s}
$$

$$
< \alpha_s f'(z_s)(\delta + z_s + K_s \frac{\partial z_s}{\partial K_s} + K_d \frac{\partial z_d}{\partial K_s})
$$

$$
< \alpha_s f'(z_s)(-\frac{\partial c_1}{\partial K_s}).
$$

Hence $\frac{\partial c_2}{\partial K_s} \geq 0$ implies that $\frac{\partial c_1}{\partial K_s} < 0$. 

Case 2: \( \partial c_2 / \partial K_s < 0 \)

\[
\frac{\partial^2 V}{\partial K_s^2} = u''(c_1) \frac{\partial c_1}{\partial K_s} (-\delta - z_s) + \beta u''(c_2) \alpha_s f(z_s) \frac{\partial c_2}{\partial K_s} < (\delta + z_s) (\alpha_s \beta u''(c_2) f'(z_s) \frac{\partial c_2}{\partial K_s} - u''(c_1) \frac{\partial c_1}{\partial K_s}) = -(\delta + z_s) \beta \alpha_s f''(z_s) u'(c_2) \frac{\partial z_s}{\partial K_s} + \beta \alpha_s f''(z_s) u'(c_2) \frac{\partial z_s}{\partial K_s} = H(K_s, K_d).
\]

The inequality comes from Equation (C.1), and the last step is due to the fact that first-order condition in the consumption-investment decision holds for any given sex compositions, i.e.,

\[
u'(c_1) = \beta \alpha_s f'(z_s) u'(c_2) \Rightarrow u''(c_1) \frac{\partial c_1}{\partial K_s} = \alpha_s \beta u''(c_2) f'(z_s) \frac{\partial c_2}{\partial K_s} + \beta \alpha_s f''(z_s) u'(c_2) \frac{\partial z_s}{\partial K_s}.
\]

Notice that \( \text{sign}(H) = \text{sign}(\partial z_s / \partial K_s) \). If we can show \( \partial z_s / \partial K_s < 0 \), then proof is completed. Again, by the first-order condition

\[\alpha_s f'(z_s) = \alpha_d f'(z_d),\]

\( z_s \) and \( z_d \) move in the same direction. Suppose an increase in \( K_s \) increases \( z_s \), it also increases \( z_d \), which means that \( c_2 \) must be increasing. This contradicts the assumption in this case. Hence \( \partial^2 V / \partial K_s^2 < 0 \). Similarly, one can prove that \( \partial^2 V / (\partial K_i \partial K_j) < 0 \) for \( i, j \in \{s, d\} \).

Scenario 2: \( \partial V / \partial K_s > 0 \)

To ensure \( \partial V / \partial K_s + \partial V / \partial K_d < 0 \), we must have \( \partial V / \partial K_d < 0 \). Notice that \( \partial c_2 / \partial K_s > 0 \). If not, then \( \partial c_2 / \partial K_s < 0 \) implies that \( \partial z_i / \partial K_s < 0 \). From the first-order condition,

\[u'(c_1) = \beta \alpha_s f'(z_i) u'(c_2),\]

immediately we know that \( \partial c_1 / \partial K_s < 0 \). This contradicts with \( \partial V / \partial K_s > 0 \). Similarly, \( \partial V / \partial K_d < 0 \) implies that \( \partial z_i / \partial K_d < 0 \). Next, we prove \( \partial^2 V / \partial K_s^2 < 0 \), and one can prove \( \partial^2 V / (\partial K_i \partial K_j) < 0 \) for \( i, j \in \{s, d\} \) similarly.
Case 1: \( \partial c_1/\partial K_s \leq 0 \)

\[
\frac{\partial^2 V}{\partial K_s^2} = u''(c_1) \frac{\partial c_1}{\partial K_s} (-\delta - z_s) + \beta u''(c_2) \alpha_s f'(z_s) \frac{\partial c_2}{\partial K_s} \\
< u''(c_1) \frac{\partial c_1}{\partial K_s} (-\delta - z_d) + \beta u''(c_2) \alpha_d f(z_d) \frac{\partial c_2}{\partial K_s} \\
= \frac{\partial^2 V}{\partial K_d \partial K_s} < 0.
\]

Case 2: \( \partial c_1/\partial K_s > 0 \)

Notice that \( f(z_s) > f'(z_s)(\delta + z_s) \) due to \( \frac{\partial V}{\partial K_s} > 0 \).

\[
\frac{\partial^2 V}{\partial K_s^2} = u''(c_1) \frac{\partial c_1}{\partial K_s} (-\delta - z_s) + \beta u''(c_2) \alpha_s f'(z_s) \frac{\partial c_2}{\partial K_s} \\
< u''(c_1) \frac{\partial c_1}{\partial K_s} (-\delta - z_d) + \beta u''(c_2) \alpha_d f'(z_s)(\delta + z_s) \frac{\partial c_2}{\partial K_s} \\
= (\delta + z_s)(-u''(c_1) \frac{\partial c_1}{\partial K_s} + \beta \alpha_s f'(z_s)u''(c_2) \frac{\partial c_2}{\partial K_s}) \\
= -(\delta + z_s)\beta \alpha_s f''(z_s)u'(c_2) \frac{\partial z_s}{\partial K_s} \\
= H(K_s, K_d) < 0.
\]

This completes the proof. ■

Proof of Proposition 3.2

Proof. By Lemma 3.1, it is easy to see that for any sex composition \( (K_s, K_d) \), if it is not optimal to reproducing one more child, i.e.,

\[
\frac{\partial V}{\partial K_s} + \frac{\partial V}{\partial K_d} < 0,
\]

it is optimal to stop at \( (K_s, K_d) \). On the other hand, it is always optimal to choose \textit{Continue} if

\[
\frac{\partial V}{\partial K_s} + \frac{\partial V}{\partial K_d} \geq 0.
\]

Lemma 3.1 further implies that in the extensive domain, the set of all stopping points \( K \) is connected with genus 0, which means that it has a unique lower bound which is also (path-)connected (in the extensive domain). Therefore, the lower bound set \( K^* \) must coincide with the optimal stopping set. ■
Proof of Lemma 3.2

Proof. By envelope theorem,

\[ \frac{\partial V}{\partial K_i} = -u'(c_1)(\delta + z_i) + \beta \alpha_i u'(c_2)f(z_i) \]

\[ \Rightarrow \frac{\partial V}{\partial K_s} - \frac{\partial V}{\partial K_d} = u'(c_1)(z_d - z_s) + \beta u'(c_2)(\alpha_s f(z_s) - \alpha_d f(z_d)) \]

\[ \Rightarrow \frac{\partial V}{\partial K_s} - \frac{\partial V}{\partial K_d} > 0 \iff \frac{\alpha_s f(z_s) - \alpha_d f(z_d)}{z_s - z_d} > \frac{u'(c_1)}{\beta u'(c_2)} = \alpha_s f'(z_s) = \alpha_d f'(z_d). \]

The last two equalities are due to the first-order condition. Notice that

\[ \frac{\alpha_s f(z_s) - \alpha_d f(z_d)}{z_s - z_d} > \alpha_s f'(z) \]

By intermediate value theorem, we have

\[ \frac{\alpha_s f(z_s) - \alpha_d f(z_d)}{z_s - z_d} > \alpha_s f'(z) \]

The last inequality is due to the fact that \( z_d < z < z_s \) and \( f \) is concave. Hence

\[ \frac{\partial V}{\partial K_s} > \frac{\partial V}{\partial K_d}. \]

In the following we prove \( \partial \Delta V_i/\partial K_s < 0 \) conditional on \( K_s + K_d \) being a constant.

Step 1. Deriving \( \partial^2 V_i/\partial K_s^2 \)

Conditional on \( K \equiv K_s + K_d \) being a constant,

\[ \frac{\partial^2 V_i}{\partial K_s^2} = -u''(c_1)(\delta + z_s)(z_d - z_i) + u'(c_1)(-\frac{\partial z_i}{\partial K_s}) + \beta u'(c_2)\alpha_i f'(z_i)\frac{\partial z_i}{\partial K_s} \]

\[ + \beta \alpha_i f(z_i)u''(c_2)\frac{\partial c_2}{\partial K_s} \]

\[ = -u''(c_1)(\delta + z_s)(z_d - z_i) + \beta \alpha_i f(z_i)u''(c_2)\frac{\partial c_2}{\partial K_s}. \]

Recall our aim is to show that \( \partial \Delta V_i/\partial K_s < 0 \). Notice that in the right-hand side, the first part

\[ -u''(c_1)(\delta + z_s)(z_d - z_i) \leq 0 \]
since \( z_d < z_s \), and the sign of the second part

\[
\text{sign}(\beta \alpha_i f'(z_i)u''(c_2) \frac{\partial c_2}{\partial K_s}) = -\text{sign}(\frac{\partial c_2}{\partial K_s}).
\]

Therefore, the proof is completed if \( \frac{\partial c_2}{\partial K_s} > 0 \).

**Step 2. Show** \( \frac{\partial c_2}{\partial K_s} > 0 \) **conditional on family size by contradiction**

Note that the first-order condition is

\[
-u'(c_1) + \beta \alpha_i f'(z_i)u'(c_2) = 0
\]

\[\Leftrightarrow \beta \alpha_i f'(z_i)u'(c_2) = u'(c_1).\]

By Proposition 3.1, conditional on \( K = K_s + K_d \) being a constant, an increase in \( K_s \) results in a decrease in \( z_i \), which means that \( f'(z_i) \) is increasing in \( K_s \) (conditional on \( K \) being a constant). Suppose now that \( c_2 \) is decreasing in \( K_s \) as well, then the left-hand side is increasing in \( K_s \), which means that \( c_1 \) has to be decreasing in \( K_s \) conditional on \( K \) being a constant. Since both \( c_1 \) and \( c_2 \) are decreasing, \( V(K_s, K_d) \) is decreasing in \( K_s \) conditional on family size. However, \( \partial V/\partial K_s > \partial V/\partial K_d \) at any sex composition, which implies that \( V(K_s, K_d) \) is increasing in \( K_s \) conditional on \( K \) being a constant. Contradiction. Hence \( c_2 \) is increasing in \( K_s \) conditional on \( K \) being a constant. This completes the proof.

**Proof of Proposition 3.3**

**Proof.** We first prove the optimality criterion. By Lemma 3.2, \( V(K_s, K_d) \) satisfies \( \partial \Delta V_i/\partial K_s < 0 \) conditional on \( K_s + K_d \) being fixed. This means

\[
\Delta V_i(K_s, K_d) < 0 \Rightarrow \Delta V_i(K_s + 1, K_d - 1) < 0. \tag{C.3}
\]

Therefore, if a sex composition \( (x_s, x_d) \) is a stopping point, any sex composition \( (y_s, y_d) \) satisfying \( y_s > x_s \) and \( y_s + y_d = x_s + x_d \) is also a stopping point. Suppose \( (x_s, x_d) \) is an optimal stopping point, i.e., \( (x_s, x_d) \in K^* \).

We first prove the necessity. Equation (C.3) implies that \( (x_s + 1, x_d - 1) \) is a stopping point. To determine whether \( (x_s + 1, x_d - 1) \in K^* \), we only need to look at if its two direct preceding points \( (x_s, x_d - 1) \) and \( (x_s + 1, x_d - 2) \). If \( (x_s, x_d - 1) \in K^* \), Equation (C.3) again implies that \( (x_s + 1, x_d - 2) \) is a stopping point, therefore, \( (x_s + 1, x_d - 1) \) is not in \( K^* \). For the sufficiency, if \( (x_s + 1, x_d - 1) \notin K^* \), both of its two direct preceding points must be stopping points. Because \( (x_s, x_d) \) is an
optimal stopping point, at least one of its two direct preceding points, \((x_s - 1, x_d)\) and \((x_s, x_d - 1)\) must not be a stopping point. Since \((x_s, x_d - 1)\) is a stopping point, \((x_s - 1, x_d)\) must not be a stopping point. This suggests that \((x_s - 1, x_d - 1)\) is not a stopping point. If not, Lemma 3.1 implies that \((x_s - 1, x_d)\) is a stopping point. Further note that \((x_s - 1, x_d - 1)\) is a direct preceding point of \((x_s, x_d - 1)\), therefore, \((x_s, x_d - 1)\) is an optimal stopping point, i.e., \((x_s, x_d - 1) \in K^*\).

It is easy to see that there exists a all-daughter and a all-son family denoted as \((0, k_d)\) and \((k_s, 0)\) respectively in the optimal stopping set \(K^*\). Applying the optimality criterion from \((0, k_d)\), we can obtain all optimal stopping points. Note that, in the application of the optimality criterion, the family size \(K_s + K_d\) is weakly decreasing and the number of sons \(K_s\) is weakly increasing. Therefore, \(k_d = \bar{K}_2 = \max_{x \in K^*} \{x_d\}\), and \(k_s = \bar{K}_1 = \max_{x \in K^*} \{x_s\}\).

**Proof of Proposition 3.4**

**Proof.** The first statement of the proposition is obvious. We focus on the other three results.

**Result 2:** For the second part, recall that a sex composition \((K_s, K_d)\) has at most two preceding points, \((K_s - 1, K_d)\) and \((K_s, K_d - 1)\). We can characterize two types of sex compositions in the optimal stopping set based on their preceding points. The first-type has all of their preceding points not in \(K^*\), and for the second-type, one of their preceding points is not in \(K^*\). Recall that if a sex composition \((K_s, K_d)\) is a stopping point, all \((x_s, x_d)\) with \(x_s + x_d = K_s + K_d\) and \(x_s > K_s\) are stopping points. Therefore, for a second-type sex composition \((K_s, K_d)\), if it exists, its preceding point \((K_s - 1, K_d)\) must not be a stopping point. Otherwise, \((K_s, K_d)\) is not in \(K^*\).

Let us first consider a first-type \((K_s, K_d) \in K^*\). Since all of their preceding points are non-stopping points, it implies that to reach \((K_s, K_d)\), parents will give birth to \(K = K_s + K_d\) children, and \(K_s\) of them are sons. Therefore,

\[
F(K_s, K_d) = \binom{K}{k_1} \left(\frac{1}{2}\right)^K
\]

where \(k_1 = \min\{K_s, K_d\}\).
To reach a second-type \((K_s, K_d)\), one must first reach \((K_s - 1, K_d)\). Under the optimal fertility strategy, after parents have reached \((K_s - 1, K_d)\), the probability of reaching \((K_s, K_d)\) is \(\frac{1}{2}\). Therefore,

\[
F(K_s, K_d) = \binom{K - 1}{k_2} \left(\frac{1}{2}\right)^K
\]

where \(k_2 = \min\{K_s - 1, K_d\}\).

**Result 3:** The non-negative correlation between family size and the number of daughters in a family is a direct result of the optimality criterion in Proposition 3.3. To see it clearly, if all optimal stopping points have the same family size, \(\text{Cov}_F(K, K_d) = 0\) as there is no variance in family size \(K\). The optimality criterion implies that for any optimal stopping point \((x_s, x_d)\), any sex composition \((y_s, y_d)\) satisfies

\[
y_s + y_d > x_s + x_d \quad \text{and} \quad y_d \leq x_d
\]

are not in the optimal stopping set \(K^*\). Therefore, \(\text{Cov}_F(K, K_d) \geq 0\).

**Result 4:** The last statement on the sex ratio under the optimal fertility strategy follows the Martingale Stopping Theorem. Denote \(X_n\) to be the difference between the number of sons and daughters after having \(n\) births. The random process \(\{X_n\}\) is a martingale. It is obvious that \(E(X_n) = 0\) for any integer \(n\). Note that the optimal fertility strategy \(s^*\) implies that parents stop fertility when they reach a sex composition in the optimal stopping set. Further note that the sex composition of a family always reaches the optimal stopping set in finite births. Let \(X_{s^*}\) be the difference between the number of sons and daughters at a sex composition stopped according to \(s^*\). The Martingale Stopping Theorem implies that \(E(X_{s^*}) = E(X_n) = 0\). This completes the proof.

**Proof of Lemma 3.3**

**Proof.** We prove the lemma by contradiction. It is easy to see that a sex composition that is not a stopping point in the absence of sex selection cannot be a stopping point in the presence of sex selection. If not, let \((K_s, K_d)\) be such a sex composition. Then it is optimal to stop at \((K_s, K_d)\) in the presence of sex selection. On the other hand, \((K_s, K_d)\) is not a stopping point in the absence of sex selection, i.e., \(\Delta V_s(K_s, K_d) + \Delta V_d(K_s, K_d) > 0\). Hence, although it is not optimal to conduct sex
selection, it is also not optimal to stop at \((K_s, K_d)\), which means that \((K_s, K_d)\) cannot be in \(K^*(SS)\). A contradiction. Therefore any point in \(K^*(SS)\) is reachable from a point in \(K^*(NSS)\) under the optimal fertility strategy in the presence of sex selection, but not vice versa. ■

**Proof of Lemma 3.4**

Proof. Let \((K_s, K_d)\) be a stopping point in the absence of sex selection, i.e., \((K_s, K_d) \in K(NSS)\). The sufficiency is obvious. We prove the necessity that is \(\Delta V(K_s, K_d) \geq \gamma\) if it is optimal to continue fertility at sex composition \((K_s, K_d)\). Note that the necessity is equivalent to the following statement: it is optimal to stop fertility at \((K_s, K_d)\) if \(\Delta V(K_s, K_d) < \gamma\). Since \((K_s, K_d)\) is a stopping point in the absence of sex selection, reproducing any positive number of children without sex selection starting from \((K_s, K_d)\) is not optimal. In addition, Lemma 3.1 tells us that \(\Delta V_i\) is decreasing in \(K_j\) for \(i, j \in \{s, d\}\) in \(K(NSS)\). Therefore, any sex composition \((x_s, x_d)\) reachable from \((K_s, K_d)\) by reproducing any positive number of children satisfies

\[
\Delta V_s(x_s, x_d) < \gamma.
\]

Hence, reproducing any positive number of children with sex selection starting from \((K_s, K_d)\) is also not optimal. The preceding analysis further suggests that reproducing any positive number of children starting from \((K_s, K_d)\) with or without sex selection at any birth is not optimal. This completes the proof. ■

**Proof of Proposition 3.5**

Proof. Suppose \((K_s, K_d) \in K^*(SS)\). Consider a sex composition \((x_s, x_d)\) satisfying that \(x_s + x_d = K_s + K_d\) and \(x_s \geq K_s\). We claim that \((x_s, x_d)\) is a stopping point with and without the option of sex selection.

From Lemma 3.3, all sex compositions in \(K^*(SS)\) must be stopping points without the option of sex selection, therefore \((K_s, K_d)\) is an optimal stopping point in the absence of sex selection, i.e.,

\[
\Delta V_s(K_s, K_d) + \Delta V_d(K_s, K_d) < 0.
\]

From Proposition 3.1, the above inequality implies

\[
\Delta V_s(x_s, x_d) + \Delta V_d(x_s, x_d) < 0. \quad (C.4)
\]
Inequality (C.4) implies that \((x_s, x_d)\) is also a stopping point in the absence of sex selection, i.e., \((x_s, x_d) \in \mathbb{K}(NSS)\). Since \((K_s, K_d) \in \mathbf{K}^*(SS)\), we have

\[
\Delta V_s(K_s, K_d) - \gamma < 0.
\]

Proposition 3.1 then implies

\[
\Delta V_s(x_s, x_d) - \gamma < 0.
\]

By Lemma 3.4, \((x_s, x_d)\) must also be a stopping point in the presence of sex selection.

Note that any sex compositions \((y_s, y_d)\) satisfying \(y_s + y_d > K_s + K_d\) and \(y_s > K_s\) can only be reached from some \((x_s, x_d)\) satisfying \(x_s + x_d = K_s + K_d\) and \(x_s \geq K_s\) which are stopping points in the presence of sex selection, therefore, they are not in the optimal stopping set \(\mathbf{K}^*(SS)\). ■
Bibliography


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