

NON-DESTRUCTIVE TESTING
OF
CONCRETE PILES
USING
THE SONIC ECHO
AND
TRANSIENT SHOCK METHODS

BY

HON-FUNG CYRIL CHAN

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DECLARATION

It is declared that this thesis has been composed by the author. The work and results reported in this thesis were carried out solely by him under the supervision of Dr. M.C. Forde, unless otherwise stated.

Edinburgh, May 1987

H.F.C. CHAN

To My Parents

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Abstract

The purpose of this project was to investigate and to improve the two most popular methods of Non-Destructive Testing of piles. Both the sonic-echo and the transient shock methods are dynamic methods that make use of the properties of stress wave propagation in piles, however, analyses are performed in different domains.

Theoretical aspects of waves in rod-like structures were studied to obtain a sound understanding of the two testing methods. Testing and analysis techniques were investigated with the aim of ensuring that necessary information could be extracted from the test results and then interpreted correctly. The instrumentation system was constantly upgraded and improved in order to provide a fast and reliable system both for experimental and site testing. Simulation techniques, in the time domain and in the frequency domain, were developed to help the understanding of the convolution effect on the time trace and the coupling effect on the vibration spectrum respectively. Large-scale model piles with built-in defects were constructed in order that the various testing methods could be verified. The experimental programme was found to be an extremely valuable exercise which will aid the interpretation of site results. Finally, site piles were tested in order to confirm the versatility as well as reveal the limitations of the different methods.

As a result of this study, a successful combination of the sonic-echo and transient shock methods has been achieved. The instrumentation system has been developed in such a way that a single test result will allow information to be extracted both in the time and frequency domains. The Edinburgh method, using filtered spectrum and cepstrum analysis, is a significant improvement in the interpretation of pile test results.

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CHAPTER 1

INTRODUCTION

Piles may be defined as structural members, wholly or partially buried in the ground, which receive load at their upper ends and transmit that load at depth to the substrata¹. They are sometimes required to resist uplift or lateral loads transmitted to them by the superstructure.

As with other structural members, it is of great importance that their design criteria are met. This may be assured by testing the structural member in question. Two aspects of testing are of concern to piles. They are structural integrity and load bearing capacity. Given nothing but the protruding head of a pile, it is impossible to inspect the pile directly for integrity. Under such a situation, the logical procedure is to load test the pile and assume that satisfactory behaviour means a sound pile². Load testing a pile is really a test of the support given by the pile-soil interaction, but gives no information on the quality, dimensions and installation of the pile. The load test is primarily a method of establishing the short-term load-settlement characteristic of the pile. Integrity testing on the other hand is primarily a quality check. As most piles are buried and not accessible for visible inspection, the quality check is by indirect methods³.

Traditional load testing, in various forms, is both time consuming and expensive. On a large site, only a few percent of the piles, either pre-selected or chosen at random, will be load-tested. As a consequence of large modern buildings and structures, large diameter bored piles are frequently used. The risk of defects is particularly important with the large diameter bored pile which takes the place of several conventional smaller piles in a group⁴. It has become increasingly expensive and less feasible to load test as many piles as an engineer would wish, in order to obtain a reasonable degree of confidence on the piled foundation. As a result, only a few piles are load-tested and this

leads one to ask the question whether the test results are a good representation of the average quality of the piles on the site.

No. of piles not meeting specification (x)	No. of piles tested (n)	Probability of selecting at least one low grade pile (x)
2	2	0.0398
2	5	0.0980
2	10	0.1909
5	2	0.0980
5	5	0.2304
5	10	0.4162
10	2	0.1909
10	5	0.4162
10	10	0.6695

Table 1.1

where N = number of piles on site
 = 100 in this case
 x = number of defective piles
 n = number of piles tested
P(x) = Probability of selecting at least one low grade pile

$$= \sum_{r=1}^x \frac{\binom{x}{r} \binom{N-x}{n-r}}{\binom{N}{n}}$$

Table 1.1 illustrates the probability of detecting a defective pile from a population of one hundred piles with different numbers of assumed defective piles in the group. From this table, it is evident that to achieve a reliable assurance of the satisfactory quality of the group of piles, a large number of

piles has to be tested. Due to the uncertainty of pile quality, piled foundations are usually over-designed in order to achieve a high factor of safety. However, if a quick and relatively cheap integrity testing method were available to check the quality of all, or a majority, of the piles on a site and load testing were performed selectively on piles which showed doubt under integrity test, then greater assurance of the satisfactory performance of the foundation could be achieved. This would lead to a better and more economic design of the foundation.

By far the most important consequence of a poor foundation is the collapse of the superstructure on top of it. Several cases of disasters have been reported in the past.^{5,6} Some of these events of failures were traced to problems of integrity rather than subsoil variation. If reliable integrity tests had been carried out on these piles, then the probability of these disasters occurring would have been minimized.

Although integrity testing can be a very useful tool for pile quality control, it should not be regarded as a substitute for load testing. The integrity test is not designed to produce a value of the load bearing capacity of a pile, although, in some systems, a certain amount of information relating to the pile/soil system may be obtained. Pile integrity testing has been under research for almost twenty years and several methods have been evolved. None of them can completely fulfil the Construction Industry Research and Information Association recommendation⁷:

"Development of relatively simple, reliable and inexpensive methods of establishing the structural integrity of piles, with particular reference to methods which do not require special means of instrumentation to be incorporated in the piles at the time of construction. Over-sensitivity of equipment should be discouraged, since the existence of minute defects in piled foundation is inevitable."

The two most successful and commercially available methods so far are the sonic-echo method and the transient shock method. Both have their advantages and disadvantages. Encouraging results have been obtained by both methods but there remains much scope for improvement and further development of the methods. Better understanding of dynamic behaviour and vibration response of concrete piles will definitely help to improve interpretation of test results. Both the sonic-echo method and transient shock method can be classified as dynamic tests, although analysis is carried out in the time domain for the former method and in the frequency domain for the latter method. With the advent of portable micro-computers, powerful signal analysers and other sophisticated electronic instruments, capable of more complex analysis, the work at Edinburgh University has combined these two methods to provide a comprehensive technique for assessing the quality of piles on site.

CHAPTER 2

PILE FOUNDATION, DEFECTS AND TESTING

2.1 PILE TYPES

The two basic methods of installing piles are well known, namely driving into the ground, or excavation of the ground, usually by boring, and filling the void with concrete. Piles can therefore be broadly classified as displacement or non-displacement types according to their installation methods. They can be further subdivided on the basis of mode of installation in the case of displacement piles, and on the basis of pile formation and pile diameter for non-displacement types. A detailed classification of different pile types is shown in Figure 2.1. A full discussion on pile types and methods of construction can be found in references.^{1,3,8}

2.1.1 Displacement Piles

Displacement piles, normally referred to as driven piles, may be divided into two main types:

1. Totally Preformed piles
2. Driven cast-in-place piles

A preformed pile is formed on the surface and then driven into the ground to such a depth that it will support the required load. Tubular or solid sections are used. The hollow tubular types may be formed from steel or concrete, and the solid sections from steel, timber or concrete; the latter are precast and may be prestressed. A preformed pile has the great advantage that it may be inspected and checked as a sound structural member before it is driven into the ground. However, care must be taken to ensure the pile is not damaged due to high stress during driving.

A driven cast-in-place pile is formed by driving a temporary steel lining tube, closed at the lower end with a detachable shoe or a plug into the ground to the required depth. Concrete is then poured into the tube before or whilst the steel casing is withdrawn.

2.1.2 Non-Displacement Piles

These piles are formed by excavating the soil from the ground. The pile bore is formed either by percussive or rotary means. It is normal practice to line the borehole with a temporary casing through unstable or water-bearing ground. Sometimes permanent casing is used. In certain conditions the borehole may be supported by a bentonite suspension. There are three types of non-displacement piles:

1. Bored cast-in-place
2. Partially-preformed
3. Grout-intruded

In bored cast-in-place piles, a completed pile bore is filled with concrete. If a temporary lining tube is used, it is withdrawn from the bore as concreting proceeds or when it is completed. If water or a bentonite slurry is used to support the side of the hole instead of casing, then a tremie pipe is normally used to place the concrete.

In partially preformed non-displacement piles, hollow precast concrete sections are lowered into the completed pile bore. The bore may be lined to the full length if necessary. The central hole of the precast sections is filled with cement grout as the casing is withdrawn in stages and the annulus

between the precast units and the subsoil is grouted up to form a solid pile keyed into the surrounding strata. This type of pile may sometimes be referred as prestcore system. The advantage of this system is that quality control on the precast units can be undertaken before they are lowered into the bore hole.

In grout-intruded piles, the borehole is formed by means of a hollow-centred continuous flight auger. No lining tubes are required because the auger and soil remain in the hole until concreting commences. Intrusion grout is pumped under pressure through the central hole of the auger and the spoil may be removed or left to mix with the grout to form the pile body.

2.2 CONSTRUCTIONAL PROBLEMS ASSOCIATED WITH PILED FOUNDATIONS

Piled foundations are a rapid and economical method of foundation construction. Piles are commonly used in poor ground conditions such as soft, loose and water-bearing soils. It is under these conditions that constructional difficulties arise and lead to defects in piles. Sometimes the situation is aggravated by the highly competitive state of the market in specialist piling work. Some firms may be led to make promises both in the pile quality and job completion time, based upon an over-optimistic appraisal of the ground conditions.⁹ Apart from that, inadequate site investigations and poor workmanship during construction can also lead to pile faults.

2.2.1 Preformed Piles

If precast concrete piles are not properly cured during manufacturing, shrinkage cracks may be formed. Lifting premature piles can also cause cracking. Piles may be damaged as a result of improper handling during transport. The formation of cracks can lead to corrosion of the reinforcement.

Fortunately, such defects can be detected easily by visual inspection before driving.

Any defects which occur during driving are potentially more serious, since they may remain undetected. Damage can be caused by over stress either in compression or in tension. Breakage may occur during driving with possibly the lower portion pushed out at an oblique angle. The problem of heave may lead to the formation of tensile cracks in the pile shaft.

2.2.2 Cast-In-Place Piles

Due to its inaccessibility for quality control by visual inspection, pile integrity is of more importance with cast-in-place piles. Piles cast on site suffer from the lack of form work to contain the wet concrete mass unless a permanent casing is used. Concrete is poured directly into the borehole and the shape of the pile thus formed is dictated by the shape of the borehole. Defects such as reduction in cross-sectional area may occur in adverse ground conditions if precautionary procedures have not been taken appropriately.

2.2.2.1 Problems Associated With Boring

Overbreak may be defined as the removal of material outside the nominal pile periphery during formation of the pile bore and may result ^{from} local cavitation. Overbreak has been reported to be one of the most significant problems associated with pile defects.⁷ These defects are also influenced by the placing of the temporary steel liners and reinforcing cage and the workability of the concrete mix.

Overbreak is usually formed when boring in unstable or weak water-bearing strata. It is essential in these conditions to support the sidewall

with a temporary steel casing and that the cutting edge of the casing is driven below the base of the advancing bore. If for some reasons, the pile bore is advanced in front of the casing, overbreak may be formed as a result of material from the sidewall falls into the borehole. The overbreak may be subsequently sealed by the advanced casing.

Fleming¹⁰ pointed out that the cavity outside the casing can cause detrimental defects to the pile shaft, especially if it is water-filled. Figure 2.2 shows the mechanism of the formation of the defects. As the temporary casing is extracted after concreting, concrete of high workability may slump into the cavity with the formation of necking above a bulbous projection. In the extreme case, where a large water-cavity is involved, slumping of large quantities of concrete may result in a complete discontinuity in the pile shaft, as shown in Figure 2.3. If low workability concrete and a dense reinforcing cage are used, then a temporarily stable column of concrete will remain. However, the ingress of water into the concrete can cause segregation with the annulus of concrete outwith the reinforcement slumping into the cavity. Such a defect is shown in Figure 2.4.

With a dry cavity, the effect of defects is not as detrimental. Concrete may slump into the cavity with the formation of a bulbous projection and extra concrete has to be added to top up the pile. The only danger is that debris can be dragged into the pile shaft resulting in lower quality concrete at this level. A pile with a bulbous projection is shown in Figure 2.5.

2.2.2.2 Problems associated with casing

Several types of defect are associated with the withdrawal of the steel liner tubes of driven cast-in-place piles and the temporary steel casings of

bored piles. The problems are mainly related to the workability and head of concrete placed within the temporary steel liners prior to their extraction.⁷

Complete or partial separation of the pile shaft can occur during the extraction of temporary casing if friction between the concrete and the casing is too great. This can happen if the concrete is not placed within an hour after mixing or the casing is not extracted within two hours after concreting.⁴ The situation is aggravated by the use of an unsuitable concrete mix, dirty or dented casings. A pile with partial separation in the shaft is shown in Figure 2.6.

2.2.2.3 Problems associated with reinforcement cage

Closely spaced reinforcement may prevent the outflow of low slump concrete which may result in a pile whose reinforcement has little or no cover. Figure 2.7 illustrates a pile whose concrete fails to penetrate the reinforcement cage.

2.2.2.4 Problems associated with ground water

In water-bearing ground, it is usual to prevent sidewall collapse and ingress of water by employing a temporary steel casing. A perfectly formed concrete shaft may be achieved if care is taken during concreting and extraction of the temporary casing. However, once the casing is removed the concrete may be under both physical and chemical attack from the groundwater. The worst situation is created by a strong and rapid flow of groundwater, along steep interfaces between permeable strata and cohesive soils or between made ground and glacial till. This may result in leaching out of the cement and washing of the aggregate^{7,9}. Figure 2.8 shows an example of pile shaft erosion by groundwater.

Another problem may arise if the pile cut-off level is below ground level. The situation is shown in Figure 2.9.¹¹ High pressure facilitates groundwater penetration into the pile shaft resulting in poor quality concrete near the top.¹¹

2.2.2.5 Problems due to fallen debris

It is not unusual for small block-like portions of rock and soil materials to dislodge from the sides of partially lined boreholes and fall into the base of the bore. If the debris has not been removed before concreting, a pile with reduced base resistance may result.

Apart from material fallen from the sidewall, debris such as small items of boring equipment, footwear and cement bags, have also been found in cast-in-place concrete piles.⁷ Figure 2.10 shows the smooth nearly horizontal plane of separation in a concrete pile shaft formed by a cement bag.

2.2.2.6 Pile defects

Although there seem to be a number of problems associated with piled foundations, the defects produced are quite similar even though the causes may be different. Results of extensive investigation into the types and causes of defects in cast-in-place piles, together with preventative measures have been given in a C.I.R.I.A. Report.⁷ More recently Sliwinski and Fleming¹² have tried to summarize the main defects affecting the integrity of piles. They are:

- "Defective concrete; segregated or of inferior strength.
- Incomplete concrete section, or cavities within the pile.
- Inclusion of foreign material, soil lumps, slurry, etc. within the body of the pile.

- Disturbance or loosening of the founding layer at and below the pile base.
- Displacement of the reinforcement cage.
- Incorrect pile dimensions; for example, a short pile, or a pile of reduced diameter."

Cementation Piling and Foundation Ltd. have carried out pile integrity test using the sonic-echo method. Their results of testing in 1981 and 1982 are reproduced in Table 2.1^{3,12}, which shows that a majority of pile defects were caused after construction by breaking down to cut off level or by site traffic. It is also to be noted that faults in shafts like contamination, necking, and voids are most frequently found in the top part of the piles.

YEAR	1981	1982
Number of piles tested	5000	4550
Number of piles to show faults	73	88
Soil contamination 0-2m	24%	5%
Soil contamination 2-7m	9%	9%
Poor quality concrete	6%	3%
Voids adjacent to pile shaft	3%	2%
Damage subsequent to construction	58%	80%
Total percentage of piles with defects	1.5%	1.9%
Percent failure due to construction defects	0.6%	0.4%

(After Cementation Piling & Foundations Ltd.^{3,12})

Table 2.1

2.3 METHODS OF PILE TESTING

There are two distinct functions of pile testing. Firstly, a pile is tested to check whether the subsoil will support the load being transmitted to it by a

correctly designed and well constructed pile. Secondly, the pile is tested to check whether the workmanship of an installed pile is satisfactory.¹ Pile testing methods can be classified as load test, dynamic test and integrity test.

2.3.1 Load Test

A static load test is the most direct procedure to check the load-bearing capacity and the performance of a pile under load. The reasons for carrying out a load test may be¹³:

1. "To serve as a proof test to ensure that failure does not occur before a selected proof load is reached, this proof load being the minimum required factor (usually 1.5) times the working load. This test may be referred to as a working load test.
2. To determine the ultimate bearing capacity as a check on the value calculated from dynamic or static approaches, or to obtain backfigured soil data that will enable other piles to be designed. This test may be regarded as a preliminary test performed on pre-selected piles to obtain design information.
3. To determine the load-settlement behaviour of a pile, especially in the region of the anticipated working load. This test is used to check the performance of the pile-soil system.
4. To indicate the structural soundness of the pile."

However, Sliwinski and Fleming¹² have pointed out that the use of static loading test as a check on integrity seems wasteful. It is costly and may only be applied on a few piles with a low probability of discovering defective work. Besides some serious defects may not be found by load testing.

The three most common procedures for load test are maintained loading test, constant-rate-of-penetration test, and method of equilibrium. In maintained loading test, the load is applied in stages. At each stage the load is

maintained constant until the resulting settlement of the pile virtually ceases before applying the next increment. This is a relatively slow procedure as the time required for the settlement to cease may be quite long. The constant-rate-of-penetration test was designed to quicken the test. Instead of a maintained load, the load is increased continuously such that the rate of penetration of the pile into the soil is constant. Another procedure designed to increase the speed of the load test is the method of equilibrium. Basically the procedure is similar to that of the maintained loading test, but the load applied at each stage is slightly greater than the required load and it is subsequently reduced to the desired value so that the rate of pile settlement is increased.

2.3.2 Dynamic Test

The estimation of ultimate pile loads using information gathered during pile driving may be regarded as a crude method of pile testing. In this method, the driving formula, such as the Engineering News Record and Hiley formulae, relates ultimate load capacity to pile set (the vertical movement per blow of the driving hammer) and assumes that the driving resistance is equal to the load capacity of the pile under static loading¹³. The assumption of rigid body motion by the driving formula leads to the discrepancies between predicted ultimate load capacity by using the formula and the measured values. A relatively recent improvement in the estimation of load capacity by dynamic methods has resulted from the use of the wave equation to examine the transmission of compression waves down the pile. This wave-equation approach takes account of the fact that each hammer blow produces a stress wave that propagates down the pile at the speed of sound, so that the entire length of the pile is not stressed simultaneously, as assumed in the conventional dynamic formulae.

The wave equation may be derived as:

$$\frac{\partial^2 u}{\partial t^2} = c \frac{\partial^2 u}{\partial x^2} \pm R \quad (2.1)$$

where u = longitudinal displacement
 c = propagation velocity
 t = time
 x = direction of longitudinal axis
 R = soil resistance term

The derivation of this equation and a detailed discussion of wave theory appropriate to stress wave propagation in piles are given in Chapter 4. Equation (2.1) represents an over-simplified dynamic model of a pile. If soil stiffness, soil damping and boundary conditions are to be incorporated into the analysis, a closed-form solution will become impossible. A common procedure adopted by many researchers is to discretize the pile into many lumped components, where each component may have its own parameters. Figure 2.11 shows an idealised pile. This idealization makes the computation of solutions very difficult and finite difference methods have to be employed to solve the wave equation. With the advent of powerful digital computers, Smith¹⁴ was able to develop a dynamic pile model to determine the pile set for a given ultimate pile load. Subsequently Case Western Reserve University^{15,16} and T.N.O.^{17,18} in the Netherlands have both modified Smith's model for use in predicting the load bearing capacity of piles. A very complex model can be formed, if necessary, by the addition of further elements. However, establishing realistic parameters for the elements is very difficult.

The Case Western Reserve University developed the Case Pile Wave Analysis Program (CAPWAP) which relates the ultimate dynamic resistance of the pile with the force and velocity at the pile top as:^{16,19}

$$R_u = \frac{1}{2} \left[F(t_{\max}) + F\left(t_{\max} + \frac{2L}{c}\right) \right] + \frac{Mc}{2L} \left[v(t_{\max}) - v\left(t_{\max} + \frac{2L}{c}\right) \right] \quad (2.2)$$

where R_u = ultimate dynamic resistance
 $F(t)$ = force at the pile top at time t
 $t_{\max}$ = time associated with the first relative maximum in the force and velocity
 $v(t)$ = particle velocity at the pile top at time t
 M = mass of the pile
 L = length of the pile
 c = propagation velocity

Frequently the ultimate dynamic resistance is assumed to be the sum of the static resistance and a viscous component as:^{3,16,19}

$$R_u = R_s + J \cdot v_b \quad (2.3)$$

where R_s = static resistance
 J = soil damping constant
 v_b = particle velocity at base
 $= 2v(t_{\max}) - EAR_u/c$

Equation (2.3) allows the static failure load to be estimated from dynamic tests on a pile. The reliability of the estimation depends very much on the value of the soil damping constant J . Several researchers^{20,21,22,23} have reported problems in choosing the value of parameter J . Balthans and Fruchtenicht²² stated that even within the range of empirically meaningful damping factors of 0.05 to 0.2 for sandy soils computed static resistance may vary by up to 50%. Apart from soil properties, the damping factor is also dependent on the pile types — driven or cast-in-place piles. The irregular cross-sectional area of a cast-in-place pile may have a significant effect on the damping constant. More research on the damping factors and improvement in the pile-soil model may result in a better prediction of bearing capacity. If the method is calibrated against static load tests, reasonably reliable predictions of ultimate bearing capacity may be made.³

The Institute T.N.O. in Netherlands has also undertaken ^{an} extensive research programme into dynamic pile testing and produced a wave analysis program called the TNO-Wave program. Basically, the T.N.O. method is similarly to the CAPWAP analysis, the response of the pile is monitored by a strain gauge and an accelerometer fixed on the pile top. T.N.O. has modified the method for testing cast-in-place piles, a pile is loaded dynamically by dropping a mass, normally not exceeding 2000kg, from varying height onto the pile head. The pile response is then fitted into a micro-computer for analysis. It was claimed that the friction force and end reaction force resulting from the dynamic load can be obtained.²⁴

However, Liang²⁵ showed that an equation describing the particle displacement along the pile shaft had been erroneously derived in the T.N.O. method. This is a very important equation since it is used to determine the point resistance of a pile.

2.3.3 Integrity Tests

Traditional load testing of piles, though reliable, is both expensive and time-consuming, and has lead Civil Engineers to look for alternative methods in order to obtain a higher reliability of the pile performance. Whilst integrity testing of piles cannot be regarded as a substitute for load testing, it has an important role to play in assessing the quality of piles. In the past, integrity of any structural members was usually assessed by visual inspection. For a structural unit buried in soil like a pile, visual inspection is only possible if the pile is excavated or samples from the pile are taken by drilling or coring. The earlier methods of pile integrity tests were associated with visual inspection, with differences in how a sample was extracted or a particular part of the pile was made accessible for direct inspection. The advantage of visual check on

pile is direct and conclusive. No expert interpretation is necessary. However it suffers from the fact that all these extraction or excavation methods are relatively expensive and time-consuming, and in some cases it may become impossible to include the tested pile in the foundation. Over the last twenty years, with the rapid development of micro-computers and other electronic equipment, indirect methods of pile integrity test have become available. Most methods are related to the dynamic properties of a pile whilst some methods make use of the differences in electric conductance of the pile body and its surrounding soil. Others consider the absorption of radioactive substances by the pile concrete. Integrity testing methods are listed below as:

1. Excavation
2. Exploratory boring and drilling
3. Closed circuit television methods and caliper logging
4. Integral compression method
5. Acoustic method
6. Seismic method (Sonic-echo method)
7. Dynamic response methods
8. Receptance method
9. Dynamic load method
10. Electrical methods
11. Radiometric methods

The first three methods are based on visual inspection, method 4 may be regarded as a variation of the load test, methods 5 to 9 are related to

dynamic properties of a pile, method 10 is an electrical method and the last method is associated with the absorption properties of radioactive matter by the pile. A brief description and the costs of some methods will be given in the following section. Advantages and disadvantages of individual systems will be discussed.

2.4 REVIEW OF METHODS OF INTEGRITY TESTING

2.4.1 Excavation

Excavation of piles for inspection is not strictly an integrity test, although it will reveal obvious external defects. Soil surrounding a doubtful pile is removed and part or the whole pile is exposed for inspection. Usually excavation is used to confirm the existence of pile defects or to identify the cause of failure in the instance where a pile has failed a load test or other integrity test.

Advantages:

- positive identification of defects
- cause of pile failure may be established
- non-specialist interpretation

Disadvantages:

- expensive if depth of excavation is deeper than 1.2m, when temporary support is required²⁶
- slow and may interfere with adjacent work
- not suitable for closely packed piles
- reduces bearing capacity of friction piles

2.4.2 Exploratory Drilling Coring

Drilling and coring the pile shaft may be employed as a direct means of inspection. By carefully monitoring of the flushing media, indications as to the homogeneity of the pile shaft can be deduced.²⁶ Coring is more expensive and slower than drilling, however, the core obtained allows a direct assessment of the quality of pile concrete.

Advantages:

- direct identification of pile faults
- non-specialist interpretation
- concrete strength can be obtained by testing cores
- the drilled holes may be used for other tests

Disadvantages:

- can be very expensive, especially with diamond coring
- time consuming
- only faults along the drilling or coring paths will be detected

2.4.3 Closed Circuit Television Methods And Caliper Logging

On a site where extensive percussion drilling has been carried out, a proportion of the holes could be selected for quick television scanning to reveal defects at the immediate vicinity of the hole. Submersible cameras and lamps may be lowered down the hole and the signal received are shown on a monitor or recorded on a video tape recorder.²⁶

Alternatively, caliper logging can be used to check the diameter of the borehole. Voids and moderate size discontinuities can be detected by lowering a three-armed probe down the hole.²⁶

Advantages:

- quick and relatively low cost if the core hole is already available
- easy interpretation
- pre-selection is not necessary

Disadvantages:

- only defects intersected by the drilled hole are detected
- can be very expensive if a drilled hole is not available

2.4.4 Integral Compression Method

This method, developed by Moon²⁷, checks the integrity of a pile by the application of a compressive force over a length of the pile through internally cast and recoverable rods or cables. If the pile is significantly weakened by any form of fault this becomes apparent by a downward movement of the top in the case of a fault near the pile head, or an upward movement of the lower regions in the case of a fault near the base.²⁶ Figure 2.12 illustrates the method.

Advantages:

- effect of defects is appraised in a practical way
- direct immediate interpretation of result on site

Disadvantages:

- expensive as it costs an additional 12% of the cost of the pile²⁶
- position and type of defect cannot be determined
- provision of cable ducts may cause congestion of steel reinforcement
- not all defects which could be identified using other techniques may be identified by this method

2.4.5 Acoustics Methods (Sonic Coring)

This method, developed by the Centre Experimental de Recherche et d'Etudes du Batiment et des Travaux Publics (C.E.B.T.P), involves the transmission of continuous sonic pulses between several vertical tubes cast into a pile.^{3,26} As a transmitter and a receiver are lowered slowly down two tubes, the pile shaft is scanned and the transmission time gives an indication of the pile quality or any discontinuities across the transmission path (See Figure 2.13.). Alternatively, a single-hole test may be employed using a combined transmitter/receiver probe. The transmitter and receiver are separated by an acoustic insulator to prevent direct transmission. Concrete adjacent to the probe is scanned for quality and discontinuities.

Advantages:

- high accuracy of fault location
- fairly quick test
- may be employed on piles with drilled hole

Disadvantages:

- requires pre-selection as tubes have to be installed during pile construction
- addition of tubes may cause congestion of steel reinforcement
- specialist interpretation is necessary

Cost:

- estimated at around £1 to £2 per metre excluding installation of tubes.³

2.4.6 Seismic Method (Sonic-echo Method)

This method was developed independently in the early 1970's by C.E.B.T.P in France ^{28,29}, T.N.O.³⁰ in Holland, and by the Illinois Institute of Technology in the U.S.A.^{31,32} Experimental study of the method has also been carried out by the University of Edinburgh.³³ The basic principle of the method involves sending a sonic pulse down the pile shaft and monitoring the time of a reflection signal. From the echo time and a knowledge of the velocity of sound in concrete, the pile length or the position of a defect may be estimated. A detailed description and subsequent modification of the method will be presented in Chapter 3.

Advantages:

- very quick as up to 50 piles may be tested daily
- minimal pile head preparation is needed
- no pre-selection

Disadvantages:

- not suitable for jointed piles
- specialist interpretation is required

Cost:

- approximately £25 per pile

2.4.7 Dynamic Response Method

The transient response of a pile to a single shock has been investigated by Dvorak^{34,35} who recorded two types of vibration by a vibrograph. The fundamental mode (large amplitude and low frequency) is thought to be related to the pile stiffness offered by the pile-soil system. A secondary oscillation (low amplitude and high frequency), not always discernable, is dependent on the pile length and the quality of concrete. This method is very crude since it cannot indicate the location and the type of defect. The method is not commercially available in the U.K.

A more sophisticated dynamic response method has been developed by the C.E.B.T.P.^{28,36,37} A pile is continuously excited to a steady state at varying frequencies. As with the transient response method, the dynamic behaviour at low frequencies is assumed to be related to the pile stiffness and higher frequency oscillations related to the pile itself. This method will be studied in detail in Chapter 3.

Advantages:

- moderate cost method which does not require pre-selection
- pile/soil stiffness at low stress may be estimated
- pile length and location of major discontinuities may be computed

Disadvantages:

- specialist interpretation is required
- quite time-consuming as some preparation of and fixing of vibrator to pile head are needed

Cost:

- for sites of 100 piles or more, cost may be of the order of £50 per pile³

2.4.8 Receptance Method

The receptance method of pile integrity testing is basically a resonant vibration technique in which a pile is excited to its first few modes of vibration. The testing procedure is very similar to that of the steady state vibration method described in the previous sub-section. However, in the receptance method the resonant frequencies are used for analysis instead of the dynamic response of the pile. The receptance theory of a vibrating system will be presented in Chapter 7.

The first application of the receptance theory on non-destructive testing of structures has been reported by Adam et al.³⁸ The natural

frequencies of a damaged bar, shown in Figure 2.14, can be expressed as:

$$\frac{1}{k} = \beta_{xx} + \gamma_{xx} \quad (2.4)$$

where k = spring stiffness of the damaged zone
 β_{xx}, γ_{xx} = direct receptances of the undamaged sections

If the expressions of direct receptances of the undamaged sections are substituted into equation (2.4) and simplified, an equation relating the location of the damaged zone and the natural frequencies can be found as:

$$\frac{EA}{k} = \frac{1}{\lambda} \left[\cot \lambda x + \cot \lambda (l-x) \right] \quad (2.5)$$

where E = Elastic modulus of the bar
 A = cross-sectional area of the bar
 λ = angular frequency (ω) divided by the wave velocity (c)
 l = length of the bar
 x = distance of damaged zone from one end of the bar

Assuming the propagation velocity and the bar length are known and the resonant frequencies of the first three modes of the damaged bar can be determined from testing, the right-hand side of equation (2.5) can then be plotted against position x , as in Figure 2.14. The intersections of the curves for the different modes indicate the possible locations of damage. Once the position of damage has been identified, the stiffness k , assumed to be a measure of the severity of the damage, can be computed from equation (2.5), provided the Young's modulus and the cross-sectional area of the bar are known. If k is infinite, there is no damage, while decreasing values of k indicate increasing damage.

Lilley³⁹ has successfully extended the receptance method into the testing of precast model piles with various built-in defects. For the bulb/neck type defect, a quadratic equation can be formed as⁴⁰:

$$a(A_2/A_1)^2 + b(A_2/A_1) - 1 = 0 \quad (2.6)$$

where A_1 = cross-sectional area of the pile shaft
 A_2 = cross-sectional area of the defect
 a, b = coefficients related to the characteristics of the pile and the soil, and also the frequency of vibration of the pile system.

Very good agreement has been obtained between predicted and actual positions of damage for model piles with only one damaged zone. For piles with two or more damaged zones, the modal curves become so confused that a reliable determination of the damaged locations is not possible. Furthermore, this method suffers from the fact that different formulations of natural frequency are required for different numbers and different types of defects. For example, equations (2.5) and (2.6) are only valid for piles with a single crack and a bulb/neck type defect respectively. A major drawback of this method is that the identification of defects is based on a very slight change in the resonant frequencies between a defective pile and a normal pile. Clearly this requirement is difficult to be met in testing site piles, either due to the effect of damping or variation in the response of the testing instruments.

The method is in the development stage and no results of testing cast-in-place piles have been reported. Further development of this method is currently being undertaken by Newcastle University.⁴⁰

Advantages:

- determine locations and types of defects in precast piles

- estimation of the extent of faults may be possible

Disadvantages:

- equipment set-up is time-consuming
- specialist interpretation is required
- difficulty in formulation of natural frequencies of piles and the effect of heavy damping may eventually rule out the possibility of testing in-situ piles

2.4.9 Dynamic Load Method

The dynamic testing method discussed in section 2.3.2 purports to yield information about the pile integrity as well as the static load capacity. The velocity and force records obtained from the pile top can be used to locate any horizontal discontinuity or change in cross-sectional area. The velocity record can be used alone for checking the integrity of the pile and its interpretation is similarly to the sonic-echo method with the advantage that the impact force is larger in the dynamic load method. Attempts have been tried to make use of both velocity and force records to locate and quantify the defect.^{16,18,19,41}

Advantages:

- severity of defects may be evaluated in favourable conditions
- pile load capacity can be estimated
- very efficient for driven piles

Disadvantages:

- moderately time-consuming in the case of testing cast-in-place piles
- expert interpretation is required
- pre-selection is necessary as this method is fairly expensive
- limitations as per the sonic-echo method

Costs:

- estimated to be £250 to £1000 or more per pile, depending on the number of piles to be tested³

2.4.10 Electrical Methods

Electrical methods , in various forms³, usually involve the setting up of an electrical circuit between the pile reinforcement and an electrode buried in soil some distance away from the pile. Pile integrity may be deduced from the measurement of various electrical properties in the circuit. One of the most promising methods, based on measurement of resistance to earth (see Figure 2.15), has been studied by the University of Edinburgh.^{42,43}

Advantages:

- cheap and quick method
- may be used on low cut-off piles
- test can be performed on freshly cast piles

Disadvantages:

- location and types of defects may not be identified
- specialist interpretation
- can only indicate integrity of reinforced part of a pile
- defects inside reinforcing cage cannot be detected
- test results dependent on age or batch of concrete
- require the resistivity of concrete to be significantly higher than that of the surrounding soils
- mis-interpretation may arise in the case of piles buried in soil strata of varying resistivity

Cost:

- estimated to be around £10 per pile for a site with more than 20 piles³

2.4.11 Radiometric Methods

The idea of testing pile integrity by radiometric methods is very similar to that of the sonic-coring methods. In this case, radiation energy is utilized instead of acoustic energy. The degree of absorption of radiation energy is related to the density of concrete. In the single hole test, a radiation emitter is separated from a detector by a lead shield to prevent direct transmission, as shown in Figure 2.16. Pile integrity can be deduced from monitoring the radiation which is deflected after leaving the source by collisions with particles in the pile materials.^{3,26,44} Alternatively, a twin-hole test may be employed as in the sonic-coring method.

Advantages:

- quick and low cost method if drilled holes are available
- can be performed on freshly cast concrete
- fairly sensitive to voids and inclusions

Disadvantages:

- special safety precautions required
- expensive and requires pre-selection if drilled holes are not available
- specialist interpretation

Cost:

- between £2 to £5 per pile, excluding cost of forming the inspection hole³

2.5 CONCLUSIONS

Pile faults can be reduced by carefully conducted site investigations, better supervision and workmanship during construction. However, defective piles will inevitably occur due to unforeseeable situations. Pile testing is therefore necessary to ensure that a pile has been correctly designed and properly constructed. A pile has to fulfill its requirements both in load bearing capacity and in settlement. Load testing is too expensive and time consuming to be applied to every pile on a site. It is therefore highly desirable to have a cheap and effective NDT technique which allows blanket testing to be carried out on all or a majority of the piles on a site.

Altogether 11 different methods of non-destructive testing have been reviewed in this chapter. Their advantages and disadvantages have been discussed. In general, methods that require pre-selection and preparation are less acceptable to the piling industry. Among these methods, the sonic-echo and the transient shock methods are most promising and can be carried out quickly on a site. These two methods are thus chosen for further study in this project.

CHAPTER 3

THE SONIC-ECHO AND DYNAMIC RESPONSE METHODS

3.1 REVIEW OF THE SONIC-ECHO METHOD

The increasing use of large diameter bored cast-in-place piles in the late 1960s had prompted civil engineers to look for reliable and economical methods to assess the structural integrity of piles. One of the earliest methods that has received much attention is the sonic-echo method. The basic principle behind this method is relatively simple. An impulse is introduced to the pile top either by dropping a mass onto the pile head or striking it with a hammer. This impulse will then travel, in the form of a stress wave, down the pile shaft with the velocity of sound in concrete. Any major defects, such as enlargement, necking and discontinuities, can be detected as an echo with its reflection time corresponding to the time required for a stress wave to travel twice from the pile top to the position of the defects. Hence, if the propagation velocity of the stress waves and the reflection time of an echo are both known, the position of the defect can be computed as:

$$l = C_l t_r / 2 \quad (3.1)$$

where l = distance of defect measured from pile top
 C_l = propagation velocity of stress wave in pile
 t_r = reflection time of the echo

Normally, a transducer is mounted on the pile top to monitor the propagation of the stress wave and the reflection time is then estimated from a trace of the pile head movement. Figure 3.1 illustrates the general equipment set-up of the sonic-echo method.

The simplicity of the idea of sonic echo method and the relative easiness of equipment set-up encouraged research bodies in many different countries to investigate and develop the technique for pile integrity testing during the early 1970s. In the following sections, the research findings of some

research organizations on the sonic-echo method will be reported.

3.1.1 Illinois Institute Of Technology

Steinbach and Vey^{31,32}, of the Illinois Institute of Technology (I.I.T) in the U.S.A, had carried out a feasibility study of the stress wave propagation for pile integrity testing. Their equipment consisted of:

- steel ball or rod as a hammer
- an electrical circuit serves as a triggering system
- accelerometer
- storage oscilloscope
- polaroid camera

Preliminary experiments on aluminium and concrete bars showed that it was possible to detect abrupt changes in cross-sectional area and discontinuities in the bars. A number of reflections, due to the bouncing forth and backward of the pulse between the two ends of a bar, were observed with a freely suspended perfect bar. Subsequent field tests on precast piles showed that in many cases more than one reflection can be obtained.

The most important problem with this method is that surface waves are usually generated on the pile top which may interfere with the propagation of longitudinal waves and hence makes the identification of the reflection signal difficult. Attempts have been made to use a low-pass filter in order to reduce the effect of surface wave interference upon the reflected signal. However, it is sometimes difficult to determine the cut-off frequency required to minimize surface noise but at the same time not distort the longitudinal pulse. Another problem reported was the high attenuation of reflection signals encountered in testing cast-in-place piles.

For a perfectly circular pile head, the surface wave velocity (C_r) was estimated on the assumption that the time interval (Δt) between two consecutive surface wave peaks, corresponds to the time required for a surface wave to travel a distance of one diameter (d), that is:

$$d = C_r \Delta t \quad (3.2)$$

or $C_r = d / \Delta t \quad (3.3)$

The longitudinal wave velocity (C_l) was then computed from the following expression:

$$C_l = k C_r \quad (3.4)$$

where k = proportional constant

The value of k is given in Chapter 4.

Figure 3.2³² plots the pile head acceleration against time for a 7 feet long precast concrete pile.

3.1.2 C.E.B.T.P

C.E.B.T.P. in France has actively engaged in developing methods for pile integrity testing. Several methods such as sonic-echo, sonic-logging, vibration testing and transient shock methods have been developed.

Among these methods, sonic-echo was the first method to be investigated by C.E.B.T.P. Basically, the equipment used is similar to that of the Illinois Institute of Technology with the exception that a geophone is used to monitor the pile head velocity instead of acceleration. Signal processing techniques employed are²⁸:

1. signal averaging to reduce noise

2. exponential gain amplifier to compensate for the loss of energy due to soil damping
3. low-pass filter to minimize surface noise
4. deconvolution filter to reduce the effect of signal fluctuation

Figure 3.3 shows the effects of these signal processing techniques on a velocity trace.

A few problems with this method were reported. They included surface wave interference, difficulty in fixing transducers onto pile head and high attenuation of signals which in some cases resulted in no echoes being detected. For these reasons, C.E.B.T.P. recommended this method to be employed on piles with length less than 15 metres.

3.1.3 T.N.O

For the last twenty years, T.N.O. in the Netherlands has been continuously involved in the development of sonic-echo method of pile integrity testing. At the time of commencing this research project in October 1983, the T.N.O. equipment^{24,30,45}, as shown in Figure 3.4, comprised:

- hammer with a triggering transducer
- accelerometer and pre-amplifier
- signal processor
- storage oscilloscope
- polaroid camera

The signal processing unit comprises of an integrator, a bandpass filter and an exponential gain amplifier. During a test, the accelerometer is hand-pressed onto the pile head to monitor its movement. Signals obtained by the

accelerometer will be integrated twice to displacement, amplified exponentially and then displayed on the oscilloscope.

More recently, T.N.O. modified its testing equipment and produced a "Foundation Pile Diagnostic System", which is a portable measuring computer using a motorola 68000 processor dedicated to pile testing (see Figure 3.5)⁴⁶. With this system, a digital approach to signal processing techniques, such as integration, amplification and filtering, has been adopted⁴⁷. Signals in terms of pile head velocity rather than displacement have been used for interpretation⁴⁸. T.N.O.'s latest system is based upon an IBM AT portable clone.⁴⁸

3.1.4 Edinburgh University

Between 1978 to 1981, Fegen³³ of Edinburgh University undertook an extensive experimental programme to study several important aspects of the sonic-echo method. The test programme included signal generation by steel rods, effects of types and locations of transducers on signals, identification of reflection signals with different base conditions and built-in defects such as saw-cuts and protrusions.

The experiment was carried out mainly on a 10m long beam with some modifications at different stages. Equipment used was similar to those adopted by other organizations, however, no signal processing techniques had been employed.

Figure 3.6 shows a typical acceleration trace obtained on the 10m long perfect beam. In most cases promising results regarding the identification of various built-in defects were obtained. However, interpretation of the acceleration trace became impossible when the length of the beam was reduced to 8m and two large protrusions were added onto it. This finally

modified beam, now referred to as Beam 1 is shown in Figure 3.7(a).

From his experimental results, Fegen³³ drew the following conclusions:

1. "The reflection time is unaffected by any variation in equipment design or operation
2. Signal period can be increased by reducing the impact velocity
3. Signal to noise ratio becomes less favourable with increasing impact velocity
4. location of transducer affects the degree of noise picked up from pile head oscillations
5. Pile base conditions can affect signal amplitude and period and change the reflection time
6. A fixed end and an increase in cross-sectional area both produce an inverted echo signal."

3.1.5 Comment

The review of the sonic-echo method of the four organizations has led to the following comments:

1. Equipment and testing procedures are very similar amongst the four research organizations.
2. There is a different preference on the matter of choosing the type of motion for interpretation. Acceleration was chosen by both Steinbach and Fegen, velocity was chosen by C.E.B.T.P and T.N.O. Displacement had been used in the past by T.N.O.
3. Fixing a transducer onto the pile head may sometimes create a problem if the pile top is not trimmed and cleaned.
4. Surface wave interference was reported to be the major problem with the method. At the moment, filtering seems to

be the best procedure to reduce its effect.

5. For a long pile, high attenuation of signal due to soil damping may result in inconclusive interpretation of results.
6. Sonic-echo is a promising method for pile integrity testing. Encouraging results have been obtained from precast and cast-in-place piles.

3.2 PRELIMINARY INVESTIGATION OF THE SONIC-ECHO METHOD

3.2.1 Instrumentation

From the last section, it is clear that apart from the latest T.N.O. equipment the most common signal display unit is an analog storage oscilloscope. Although an oscilloscope is portable and quite easy to operate, it suffers from the fact that a permanent record of the result can only be obtained by taking a Polaroid photograph. Other problems with this type of instrumentation are:

1. Only very crude estimate of reflection time is possible by measuring the trace with a scale rule.
2. Basic signal processing techniques such as filtering, integration and exponential amplification have to be applied before the signal is displayed.
3. As a result of 2, an expert must be on site to supervise the testing procedures. Time is also wasted on getting acceptable parameters for the basic signal processing techniques by trial and error.
4. No post-capture signal processing is allowed.

5. Reproduction of signal trace is only possible by photocopying.

Due to the above problems, the author of this thesis adopted a different instrumentation, referred to here as the Edinburgh University Phase I instrumentation. The instrumentation, shown in Figure 3.8. comprises of:

1. plastic-tip hammer
2. accelerometer
3. analog oscilloscope
4. analog to digital transient recorder (Data Laboratory 902)
5. micro-computer (Hewlett Packard 85)

During a test, an accelerometer may be fixed onto the pile head with grease, the signal obtained will be passed to be displayed on the oscilloscope and at the same time to the transient recorder. If the signal is acceptable then it may be down-loaded from the transient recorder to the micro-computer. No signal processing has been done so far and the signal on the cathode-ray-tube (C.R.T) of the micro-computer should be the same as displayed on the screen of the oscilloscope. Signal processing can be performed by calling up the appropriate parts of the software. Alternatively, the data may be stored in a tape if it is decided to interpret the result in the laboratory. The testing procedures are illustrated in Figure 3.9.

3.2.2 Data Acquisition And Signal Processing Software

Computer software for data acquisition and signal processing has been developed by the author for the sonic-echo method. The following features are included in the software named "Signal":

1. Data entry modes — keyboard, transient recorder and tape. Entry from keyboard provides data for a quick preliminary check on different routines of the program. Fresh data obtained from testing can be down-loaded directly from the transient recorder into the computer. Previously stored data in tapes can be re-entered into the computer for analysis.
2. Adjustment for pre-triggering — a usual problem associated with many oscilloscopes and some data acquisition devices is the loss of information before triggering. Ideally, a data capturing device should be triggered exactly at the moment a signal is generated, however, in practice it is not possible. If the trigger-level is set too low, the device may be triggered by unwanted noise. On the other hand, if the trigger-level is too high, the device may not be triggered or if it is triggered, information before triggering cannot be captured. Many devices, such as the transient recorder (DL902), provide a pre-triggering facility which allows a certain amount of data before triggering to be included in a captured trace. Therefore, the exact arrival of a signal can be pin-pointed from the trace and unwanted parts of the trace before signal arrival can then be removed.
3. Base-line correction — another advantage of using the pre-triggering mode of the transient recorder is that the information on base-line deviation, usually caused by a constant direct current, can be obtained. The existence of a DC-term will have a significant effect on integration. It must therefore be removed before any integration is performed on a signal trace.
4. Dynamic range — this is the ratio between the amplitude of the impact signal and the amplitude of the reflection signal. For the transient recorder (DL902), the amplitude resolution is one part in 256 (8 bits), that is any sample will

have a value between -128 to +127. To make maximum use of the 8-bit analogue to digital conversion, it is important to ensure that the impact signal is as close as possible to the full scale setting of the transient recorder. This can be done by getting the absolute maximum value of a digitized trace to be near 127.

5. Integration — normally, the pile head response to an impact is monitored by an accelerometer. The program has included a subroutine which carries out single or double integration to convert an acceleration trace into a velocity or displacement trace.
6. Exponential amplification — exponential decay of signal due to pile skin friction has been assumed in the program. To facilitate the interpretation of a reflection signal, a trace can be amplified exponentially so that a signal which comes from a long distance will be amplified more strongly. Usually, the amplification factor bears no real relation with the soil properties, and an assumed value is used in order to amplify a decaying trace so that a very weak return signal may be identified.
7. Output facilities — at any stage of signal processing, the result can be plotted on the C.R.T. of the micro-computer for viewing or a hard copy can be printed as a record.
8. Data storing — data before or after signal processing can be stored on tapes for later analysis.

3.2.3 Techniques To Deal With Surface Oscillation

It has been mentioned that the interference of surface oscillations on the pile top is a major problem of the sonic-echo method. Surface oscillations are not as quickly attenuated as the longitudinal waves which travel along the

pile shaft. A transducer positioned on the pile top is under the greatest influence of the surface oscillations and a heavily damped longitudinal signal may sometimes be masked by the surface oscillations to such an extent that direct and clear interpretation of the sonic-echo trace is impossible. There are basically two ways to deal with the problem. The first one is to avoid generating surface oscillations, and the second one is to reduce its effects on the sonic-echo trace. In the following subsections, the author's experience with some techniques to deal with surface oscillation interference will be reported.

3.2.3.1 Signal averaging

It is assumed that, for both perfectly circular and irregular pile heads, the surface oscillations will differ in phase and amplitude according to the positions of impact and transducer, and the reflection signal will arrive to the pile top at a definite time. Therefore, signal averaging can be used to reduce the effects of the surface waves and reinforce the reflection signal.

Figure 3.10 shows the result of signal averaging applied on two traces obtained from testing Beam 1 (refer to Figure 3.7(a)). When the averaged trace is compared with each of the individual traces, it is clear that the reflection signal is enhanced while the interference of the surface wave is reduced. From the above result, it seems that signal averaging is quite promising. However, no definite conclusion can be drawn since the reflection signal in each individual trace is already quite dominant.

Extensive experiments have been performed on Beam 2, shown in Figure 3.7(b), in order to investigate the effects of impact and transducer positions on the signal traces. The beam was 6m long with a protrusion at its mid-length. Transducer and impact positions have been arranged so as to

cover as many as possible combinations of the positions (see Figure 3.11). Altogether, ten traces have been obtained with different arrangements of positions. The test results are included in Figure 3.12(a)-(j).

Due to the interference of high frequency large amplitude surface oscillations, most of the individual traces are very difficult to interpret. Different extents of surface wave interference were observed from the traces with one or two particular position arrangements less affected by the oscillations. It seems that better results were obtained when the impact position was at the centre of the pile top and the transducer was positioned between the centre and the edge of the pile top. Results of signal averaging, shown in Figure 3.12(k), of the ten traces show a slight improvement, but not sufficient to reduce the surface wave effect. For this reason, it may be concluded that signal averaging cannot be used solely to tackle surface wave interference.

3.2.3.2 Integration

Sonic-echo traces in one of the three forms of motions, namely acceleration, velocity and displacement, have been used by different research organizations for the interpretation of pile integrity. Integration has to be performed at times to convert a trace from one form of motion to another. There seem to be preferences with some organizations in choosing a particular type of motion for interpretation, however, nobody has ever suggested the use of integration for reducing surface wave interference.

From the author's experience, it was found that the high frequency oscillations in an acceleration trace can be averaged out if the trace is integrated to velocity. A numerical model will help to illustrate this point. Figure

3.13(a), (b) and (c) are the ideal sonic-echo traces, without surface wave interference, in the three forms of motions respectively. In each trace, the presence of an echo can be identified without difficulty. In Figure 3.13(d), (e) and (f), high frequency large amplitude slowly decaying surface waves are shown. Figure 3.13(g) is the result of combining (a) and (d), which shows that the surface oscillations have obscured the echo to such an extent that its identification is not possible. The acceleration trace is subsequently integrated to velocity, as shown in Figure 3.13(h), in which the echo can be identified with confidence. Further integration to displacement, as in Figure 3.13(i), shows no improvement in the trace.

The ability of integration to reduce surface wave effects is very much dependent upon the difference between the longitudinal signal frequency and the surface wave frequency. In the numerical model, the longitudinal signal frequency is 1 KHz and the surface wave frequency is 5KHz.

An experimental result from Beam 1 is shown in Figure 3.14, in which the velocity trace is no doubt easier to interpret than the acceleration and displacement traces.

3.2.3.3 Filtering

Frequency filtering has been used with some success by C.E.B.T.P., I.I.T. and T.N.O. in order to reduce the effects of surface wave interference on the sonic-echo traces. The first two organizations used a low-pass filter and the last institute used a band-pass filter. No information with regards to how a cut-off frequency should be selected has been mentioned, except that I.I.T. gave an approximate relationship between the surface wave frequency and the pile diameter.

The theoretical background of surface waves will be studied in detail in Chapter 4 and it will be shown that the frequency of the fundamental mode of pile top oscillations can be estimated as:

$$f = C_r/d \quad (3.5)$$

where f = frequency of fundamental mode
 C_r = velocity of surface wave
 d = diameter of pile

Figure 3.15 plots the fundamental frequencies of surface waves against different pile diameters as estimated from equation (3.5).

A short concrete cylinder, of length 0.91m and diameter 0.24m, has been cast to verify this estimation. Because the cylinder was very short a number of large amplitude end reflections were obtained which made the identification of surface waves very difficult, as shown in Figure 3.16(a). In order to investigate surface waves in such a short cylinder, it is inevitable that the transient recorder has to be overloaded in order to obtain surface waves of sufficient amplitude. In Figure 3.16(b), surface waves can be identified as disturbances in between two overloaded end reflections. If the time period of these disturbances are measured, it can be shown that the fundamental frequency is in good agreement ^{with} the estimated value.

The estimation has also been verified on a site pile of length 8m and diameter 450mm. This pile was chosen particularly for testing because its pile head was very smooth and circular. A record of the surface waves is shown in Figure 3.17(a), in which the time range was deliberately set to very short so as to exclude any base reflection. A frequency analysis performed on the record showed the existence of a very dominant frequency at 4883Hz (see Figure 3.17(b)). Assume the ratio between the velocities of surface wave and

longitudinal waves to be 0.55 and the velocity of longitudinal wave to be 4000m/s. The fundamental frequency of the surface oscillations can be estimated from equation (3.5) as:

$$\begin{aligned} f &= \frac{0.55 \times 4000 \text{ m/s}}{0.45\text{m}} \\ &= 4889 \text{ Hz} \end{aligned}$$

This estimation is in extremely good agreement with the frequency obtained from site testing.

From the above findings, a low-pass filter with its cut-off frequency set to below the fundamental frequency of the surface wave is an effective measure to reduce the surface wave interference. Steinbach^{31,32} showed examples of improved signal traces by filtering. Setting the cut-off frequency of a filter according to the pile diameter suffers from the drawback that most pile heads on site are not circular. In this case estimation of cut-off frequency becomes impossible. For this reason, a new way of surface wave filtering, making use of the Edinburgh University Phase I Instrumentation has been developed for pile heads of *any* shape.

The disadvantage of the traditional sonic-echo instrumentation is that post-capture signal processing is not possible. The cut-off frequency of an electronic filter has to be set before a test is carried out. This type of filtering, by making use of an electronic device, is usually referred as analogue filtering. With the flexibility of the Edinburgh University Phase I Instrumentation, digital filtering after signal capture becomes possible. In the Edinburgh approach to filtering, a time trace will be converted by Fast Fourier Transform (FFT) to its spectrum. The spectrum is then analysed for any high dominant frequency and the cut-off frequency is set just below it by replacing spectral above the

cut-off frequency with zeroes.

The ability of digital filtering in reducing surface wave interference has been tested with a numerical model. A simulated trace, as shown in Figure 3.18(a), comprises an impact signal, an echo and an exponential decaying surface wave of frequency 3516Hz. After FFT digital filtering was applied with the cut-off frequency set at 3516Hz, as shown in Figure 3.18(c), the time trace was then re-constructed by Inverse Fast Fourier Transform (IFFT), as shown in Figure 3.18(d). The effect of surface wave has been reduced to such an extent that the echo can be identified readily. The fluctuation at the end of the record is due to truncation error of FFT. This error will be dealt with in Chapter 5.

This approach to surface wave filtering has been successfully applied to the averaged trace (refer to Figure 3.12(k)) obtained from Beam 2. After digital filtering, the echoes can be identified readily, as shown in Figure 3.19. It is worth noting that the beam has a square cross-section hence analogue filtering, which requires an estimation of cut-off frequency from pile diameter, is not applicable in this situation.

As a basic requirement for any form of filtering, there must be a significant difference between the frequencies of the longitudinal pulse and of the surface wave. A knowledge of the impact signal frequency is therefore beneficial to the application of filtering.

3.2.3.4 Down-hole excitation

From the investigation of surface wave motion in Chapter 4, it will be clear that at a depth of one diameter, the vertical displacement of a surface wave is 20% less than that on the top (see Figure 4.2). If a hole of approximately one diameter deep is made available on a pile head, then a

signal could be generated down the hole, as shown in Figure 3.20. By separating the levels of the impact point and the transducer position, the concrete mass around the hole could add extra stiffness to suppress surface wave motion and hence significantly improve a signal trace. However, this down-hole excitation technique, also suggested by Steinbach³¹, suffers from the drawback that a hole must be drilled before testing.

3.2.3.5 Low frequency excitation

Integration and filtering may be regarded as the best solutions for the reduction of surface wave effect. Both techniques require a significant difference in the frequencies between the impact signal and the surface wave. Since surface waves on pile heads are generally associated with high frequency oscillations, it is therefore reasonable to argue that a low frequency excitation will reduce the amount of surface wave being excited and at the same time provide the most favourable situation in which integration and filtering are more effective in reducing surface interference.

As can be seen from Figure 3.15, the fundamental frequency of surface oscillations for a pile with diameter less than one metre is greater than 2 KHz. A hammer blow with its dominant frequency well below 2 KHz is an ideal impact signal which will only generate surface oscillations of a relatively small amplitude. Steinbach³¹ and Fegen³³ have both suggested the use of different lengths of steel rods, falling from varying heights, to control the frequency content of a signal. However, none of them have successfully considered its implication on reducing surface waves. Furthermore, a steel bar is not as convenient as a hand-held hammer for signal generation because of the difficulty in catching the bar after it has been rebounded from the pile top.

The amplitudes and frequency contents of several types of hammer have been investigated by striking the hammers against a long concrete column with the signals picked up by an accelerometer. In general, the following conclusions with regard to different hammer tip materials can be drawn:

1. steel hammer — can be used to generate large amplitude signal with dominant frequency at around 2 KHz. Concrete on the surface may be damaged if a very hard blow is applied.
2. Plastic-tipped hammer — medium amplitude signal with dominant frequency at around 1 KHz. Concrete is not damaged even with a hard blow.
3. Soft-rubber-tipped hammer — the amplitude is too low to be picked up by an accelerometer. Hence dominant frequency cannot be estimated.

The dominant frequency is computed as the reciprocal of the time duration of a signal, that is, the contact time between the hammer and the concrete surface. The amplitude and the dominant frequency of a signal are both determined by the approach velocity of a blow and the elasticity of the materials in contact. A high velocity blow by a hard-material-tipped hammer will produce a signal with large amplitude and high dominant frequency. From experience, it has been found that the plastic-tipped hammer, shown in Figure 3.8, is a compromise for signal amplitude and dominant frequency.

3.3 REVIEW OF THE DYNAMIC RESPONSE METHODS

The dynamic response of piles subjected to sinusoidal force excitation

has been investigated by C.E.B.T.P. for integrity testing as early as in the late 1960s. Over the years, two methods of excitation have evolved, namely the vibration method and the transient shock method. These dynamic response methods differ from the sonic-echo method in that the vibrational response of a pile is monitored for investigation rather than the propagation of stress waves in a pile. As a result, interpretation of dynamic response methods is carried out in the frequency domain instead of the time domain.

Basically, interpretation of results for the two methods of excitations are the same. The differences between the two methods are mainly on how a pile is excited to its resonances and in the way of signal preparation for interpretation. In the vibration method, a pile is excited at a number of pre-determined frequencies by a vibrator and the results obtained can be plotted immediately for analysis. In the transient method, a pile is excited by a short impulse and the results obtained have to be converted from the time domain to the frequency domain before analysis.

3.3.1 Vibration Testing Method

The vibration testing method was originally developed by Paquet²⁸ and Briard³⁶ of C.E.B.T.P. in the late 1960s. It was first introduced to Britain in 1973 by Gardner and Moses⁴⁹ to test bored piles formed in laminated clays. A detailed description of the method was presented in a paper by Davis and Dunn (1974)³⁷.

Figure 3.21 illustrates the vibration testing equipment, which comprises a velocity transducer, a sine wave generator, an electrodynamic vibrator, a force transducer and a force amplitude regulator. Before a test is carried out, the pile head has to be made approximately horizontal and

preferably level with the surrounding ground surface. Steel plates are then fixed to the pile head with a epoxy resin so that the vibrator unit and the velocity transducer can be mounted on top of them.

The pile head is excited sinusoidally by the electrodynamic vibrator with frequency varied from about 20Hz to 1000Hz. The amplitude of excitation is regulated to a fixed value (f_0) and the pile head response, in terms of particle velocity (v_0), at steady state is measured by a velocity transducer.

A plot of the amplitude of mechanical admittance ($|v_0/F_0|$) against the frequency of excitation is shown in Figure 3.22. It was claimed that a number of parameters derived from the graph can be used to check pile integrity and to predict pile behaviour under load^{36,37,50,51}.

Briard³⁶ expressed the factor of soil damping effect per unit length of pile as:

$$\sigma = \frac{\rho' \beta'}{\rho_c V_c r} \quad (3.6)$$

where $\sigma' =$ soil damping factor (m^{-1})
 $\rho' =$ bulk density of soil (kg/m^3)
 $\beta' =$ velocity of propagation of transverse wave in soil (m/s)
 $\rho_c =$ density of concrete (kg/m^3)
 $V_c =$ velocity of plane wave propagation in concrete (m/s)
 $r =$ radius of pile (m)

The geometrical mean value of the amplitude of mechanical admittance (N) in the higher frequency region of the response curve can be expressed as:

$$N = \frac{1}{\rho_c V_c A_c} \quad (3.7)$$

where $A_c =$ cross-sectional area of pile

The maximum value P and the minimum value Q of the curve are related to the soil damping factor as:

$$P = N \coth(\sigma L) \quad (3.8)$$

$$Q = N \tanh(\sigma L) \quad (3.9)$$

where L = length of pile (m)

From equations (3.8) and (3.9) N can be expressed as the geometric mean of P and Q as:

$$N = \sqrt{(PQ)} \quad (3.10)$$

and the damping effect σL may be expressed as:

$$\coth(\sigma L) = \sqrt{(P/Q)} \quad (3.11)$$

If the propagation velocity v_c is known, the frequency interval (Δf) between harmonics can be used to estimate the length of a pile (L) as:

$$L = \frac{V_c}{2\Delta f} \quad (3.12)$$

In the case of a major defect in the pile, such as a complete discontinuity, the length estimated by equation (3.12) will indicate the position of the defect instead of the actual pile length. Using equations (3.7) and (3.12), the pile mass (M_p) can be computed as:

$$M_p = LA_c \rho_c = \frac{1}{2\Delta f N} \quad (3.13)$$

For the dynamic behaviour of the pile/soil system, a value of the pile head dynamic stiffness can be obtained from:

$$E' = 2\pi/m \quad (3.14)$$

where E' = dynamic stiffness measured from pile head
 m = gradient of the initial part of the curve

The dynamic response curves obtained from real piles on a site are seldom as simple as the theoretical curve for a perfect pile. In some situations it may become almost impossible to compute all the parameters as from equation (3.6) to equation (3.14). Attention is therefore usually paid on the more important parameters such as the effective length of a pile as in equation (3.12) and the dynamic stiffness of pile head in equation (3.14).

It can be argued that at low frequencies, the pile/soil system is vibrating as a unit and the dynamic stiffness can be used as an indication of the performance of the system under load. However, at high frequencies the pile is vibrating on its own and hence the frequency interval can be used to calculate the resonating length of the pile. Information regarding the fixity of a pile base can be obtained by considering the position of the first resonant frequency. A value which is equal to the frequency interval of the resonating length is an indication of very poor base fixity. A value which is equal to half of the frequency interval is an indication of a very rigid base. For intermediate elastic base, the first resonant frequency will have a value in between the above two cases (see Figure 3.23).

3.3.2 Transient Shock Method

The transient shock method can be regarded as a natural progression in the development of the vibration test. It is simply an easier and more practical way of obtaining the same results by subjecting the pile head simultaneously to a continuous frequency spectrum excitation, hence saving the trouble of repeatedly vibrating the pile at different individual frequencies. The equipment includes a load cell, a velocity transducer, an oscilloscope, a microprocessor and an X-Y graph plotter^{50,52}. Figure 3.24 shows the equipment for the shock test.

During a test, the pile head is struck with a hammer via a centrally placed load cell which measures the magnitude and the shape of this dynamic force. A hammer blow applied in this way contains energy over a wide frequency spectrum. The transient vibrational response of the pile is picked up by the velocity transducer (geophone). A signal processing technique, the Fourier Transform, converts the force and velocity traces into their spectra. A graph of mechanical admittance against frequency is obtained by dividing the velocity spectrum by the force spectrum. A schematic diagram of the test procedures is shown in Figure 3.25. Test results and interpretation are in every way identical to the vibration method.

3.4 PRELIMINARY INVESTIGATION OF THE DYNAMIC RESPONSE METHODS

With the view of extending the Edinburgh Phrase I instrumentation to test piles by the shock method, it became necessary to replace the original plastic-tipped hammer with an instrumented hammer, which was capable of measuring the input force. A hammer with a built-in load cell to measure the impact force was adopted (see Figure 3.26). Several tips of different hardness are provided to control the frequency content of the impact force.

The most important information obtained from dynamic response test can be listed as:

1. Dynamic stiffness of pile head
2. Base fixity of the pile
3. Effective length of the pile

Although a number of publications on the dynamic response method are available, insufficient work has been reported on explaining how the different

parameters obtained from a mechanical admittance graph relate to the pile behaviour under excitation. It is therefore considered that an investigation would be beneficial to the understanding of interpretation and limitations of the method.

3.4.1 Dynamic Stiffness

A much simplified mathematical model of a foundation pile can be formed as shown in Figure 3.27. The dynamic behaviour of the system under a sinusoidal force excitation can be expressed in mathematical form as:

$$M\ddot{x} + c\dot{x} + kx = F\sin\omega t \quad (3.15)$$

where M = pile mass
 c = damping factor of the system
 k = stiffness of the system
 F = amplitude of excitation force
 ω = angular frequency of excitation force
 $= 2\pi f$

The frequency transfer function in terms of mechanical admittance, $H(\omega)$, can be obtained through Laplace Transform on equation (3.15) as⁵³:

$$H(\omega) = \frac{\dot{x}(\omega)}{F(\omega)} = \frac{i\omega}{(k - M\omega^2) + i\omega c} \quad (3.16)$$

where $\dot{x}(\omega)$ = velocity spectrum of the response
 $F(\omega)$ = force spectrum of excitation

The amplitude of the mechanical admittance can be shown as:

$$|H(\omega)| = \frac{\omega}{\sqrt{\{(k - M\omega^2)^2 + \omega^2 c^2\}}} \quad (3.17)$$

From this equation, the dynamic stiffness of the pile/soil system at very low

frequency can be approximated as:

$$k = 2\pi/m \quad (3.18)$$

where m = initial gradient of a mechanical admittance plot
against frequency
 $= |H(\omega)|/f$

3.4.2 Base Fixity

A knowledge of the degree of base fixity is very important especially in the case of end-bearing piles. This information may be obtained by observing the sequence of the resonant frequencies. When a pile, which is embedded in a very firm base is excited, it is expected that a vibration node to be developed at the pile toe. The first resonant frequency is given by:

$$f_1 = c/4L \quad (3.19)$$

where c = propagation velocity
 L = pile length

Higher harmonics are expected to occur at $3f_1$, $5f_1$, $7f_1$, and so on. This is a sequence of odd-numbered harmonics in increasing order. If on the other hand, a pile is embedded into a very soft material, then an anti-node would be developed at the pile toe and the first pile resonant frequency is therefore:

$$f_1 = c/2L \quad (3.20)$$

In this case, an even-numbered harmonic sequence is expected as $2f_1$, $4f_1$, $6f_1$, and so on.

It is worth-mentioning that the first resonant frequency in the low frequency range may not always occur at the position as suggested by equations (3.19) and (3.20), even for a rigidly fixed or free-end pile. At low

frequencies, the stiffness of the pile/soil system will have a greater influence on the position of the first resonance. It may therefore be necessary to examine the harmonic sequence in order to determine the base fixity.

3.4.3 Effective Length

In the last section, it was shown that at the higher frequency range, a sequence of resonant harmonics may be developed in such a way that the difference between two successive harmonics is:

$$\Delta f = c/2L \quad (3.21)$$

where L can be the pile length or the distance measured from the pile top to a complete discontinuity in the pile shaft. :

Equation (3.21) is applicable to piles of regular cross-sectional area, embedded either in a rigidly fixed or an infinitely compressible base. For bases of intermediate stiffness, further investigation is necessary to provide more information on how the degree of fixity will affect the resonant harmonics. In the case of piles with a change in cross-sectional area, it is doubtful whether a sequence of resonant harmonics with a regular frequency interval will ever develop.

Experiments undertaken upon Beam 1 show that the resonant frequencies are not at a regular interval apart (see Figure 3.28). The resonant frequencies for this beam, which comprises two enlargements, are given in Table 3.1.

The irregularity of the frequency interval between the resonant frequencies suggests that it may not be possible to estimate the positions of the enlargements from a mechanical admittance plot. More model beams with

different built-in defects are required before a definite conclusion can be drawn.

Resonant frequency (Hz)		Frequency interval (Hz)	
End 1	End 2	End 1	End 2
29	34	166	166
195	200	147	142
342	342	322	210
664	552		

Table 3.1

3.5 CONCLUSIONS

The review of the sonic-echo and transient shock methods have suggested that both techniques are feasible in pile integrity tests. The interpretation of sonic-echo test results is direct and straightforward, especially in determining the length of a pile but no information on the pile/soil interaction may be obtained. The transient shock method requires more advanced instrumentation as well as sophisticated analysis techniques. The dynamic stiffness estimated from this method indicates the interaction between the pile/soil system. The methods are complementary to one another and any instrumentation system which allows both tests to be performed is certainly advantageous.

As a result of the preliminary investigations on the two methods, a number of conclusions can be drawn. They are:

1. It is advantageous to develop^{an} instrumentation system in such a way that test results for both methods can be obtained and allow

post-capture analysis.

2. Interference from surface wave oscillations on the time signal can be reduced by the following techniques:

- Signal averaging — the amplitudes of the out-of-phase surface wave oscillations may be reduced.
- Integration — a very effective way of reducing high frequency surface wave oscillations. It was found that the best improvement was obtained when an acceleration trace was integrated to a velocity trace.
- Filtering — a very useful technique but it is very difficult to predict the cut-off frequency in the case of a non-circular pile head. Post capture filtering is preferred.
- Down-hole excitation — may be very useful but a hole of approximately one diameter deep has to be drilled. Hence, it is not a very practical method.
- Low-frequency excitation — the most effective and practical method of avoiding the interference from surface wave oscillations. The frequency content of an impact signal can be controlled by using hammer tips of different materials. More detail on this aspect will be reported in Chapter 6.

3. The calculation of the dynamic stiffness value, using the gradient of the initial slope of a mechanical admittance curve, has been verified by assuming a much simplified mathematical lump-mass model.
4. The degree of base fixity may not always be predicted from the position of the first resonant frequency. Investigation of this topic based on computer simulations will be reported in Chapter 7.
5. The position of a defect and the effective length of a pile cannot always be identified from a mechanical admittance plot. Experiments performed on Beam 1 and Beam 2, both of which have bulbs as built-in defects, produced mechanical admittance curves which were associated with irregular frequency intervals. Further investigations on model piles with different built-in defects are required.

To further develop both methods, it is necessary to have knowledge of:

1. A physical understanding of the phenomena under observation.
2. A sound background of signal processing and analysis techniques.
3. A basic understanding of the instrumentation system, and of how the system is affecting the experimental results.

These will be discussed individually in the following three chapters.

CHAPTER 4

WAVE THEORY

4.1 WAVES IN AN UNBOUNDED ELASTIC MEDIUM

In an unbounded isotropic elastic medium two types of waves can propagate. They are termed dilatational and distortional. In many practical situations, these two waves are frequently referred to as longitudinal waves and transverse (or shear) waves. In a plane dilatational wave the particle motion is along the direction of propagation, whilst in a plane distortional wave it is perpendicular to the direction of propagation. A compression wave and a rotation wave can therefore be regarded as dilatational and distortional in nature respectively.

As there are a number of treatises^{54,55} on bodily waves, it is not proposed to pursue a rigorous proof of the wave nature. Instead some classical wave equations are quoted. By considering the forces acting on a small rectangular parallelepiped and the stress-strain relationship of an elastic medium, Kolsky⁵⁴ expressed the particle motion within an isotropic elastic solid as:

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial x} + \mu \nabla^2 u, \quad (4.1)$$

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial y} + \mu \nabla^2 v, \quad (4.2)$$

$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial z} + \mu \nabla^2 w, \quad (4.3)$$

where $\Delta = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

ρ = density

λ = Lamé's constant

μ = modulus of rigidity

u, v, w = displacements in x, y , and z directions respectively in Cartesian coordinates
 $\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}$ = components of strain in Cartesian coordinates
 t = time

The modulus of elasticity (E) and the Poisson's ratio (ν) of a material are related to the Lamé's constant λ and the modulus of rigidity μ as:

$$E = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu} \quad (4.4)$$

$$\nu = \frac{\lambda}{2(\lambda+\mu)} \quad (4.5)$$

If both sides of equations (4.1), (4.2) and (4.3) are differentiated with respect to x, y and z respectively and added together:

$$\rho \frac{\partial^2 \Delta}{\partial t^2} = (\lambda+2\mu) \nabla^2 \Delta \quad (4.6)$$

This is a wave equation which shows that the dilatation Δ is propagated through the medium with velocity $[(\lambda+2\mu)/\rho]^{0.5}$.

If both sides of equations (4.2) and (4.3) are differentiated with respect to z and y respectively, and eliminate Δ by subtracting, the following equation can be obtained:

$$\rho \frac{\partial^2}{\partial t^2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) = \mu \nabla^2 \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$

or

$$\rho \frac{\partial^2 \omega_x}{\partial t^2} = \mu \nabla^2 \omega_x \quad (4.7)$$

where ω_x is the rotation about the x -axis. Similar equations may be obtained

for ω_y and ω_z . Hence, from equation (4.7), it can be seen that rotation propagates with velocity $(\mu/\rho)^{0.5}$.

It may be shown that a plane wave propagated in an isotropic elastic medium must travel with either one of the two above velocities. Without loss in generality, assume that a plane wave propagates positively along the x-axis with velocity c , without distortion and with negligible attenuation. After a time t the perturbation will have been transferred by a distance ct . Thus the displacements u , v , and w will be functions of a single parameter $\psi=x-ct$. Assuming an arbitrary function f such that $u=f(x-ct)$, which describes the displacement u at position x after time t , then differentiating both sides twice with respect to t and x respectively, two equations can be obtained as:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial \psi^2} \quad \text{and} \quad \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial \psi^2} \quad (4.8)$$

Similar equations can be written for v and w as:

$$\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial \psi^2} \quad \text{and} \quad \frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial \psi^2} \quad (4.9)$$

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial \psi^2} \quad \text{and} \quad \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial \psi^2} \quad (4.10)$$

Since the perturbation depends only on a function of $(x-ct)$, the differential coefficients for u , v and w with respect to y and z are all zero.

Substituting all these differential coefficients accordingly into equations (4.1), (4.2) and (4.3) produces:

$$\rho c^2 \frac{\partial^2 u}{\partial \psi^2} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial \psi^2}, \quad (4.11)$$

$$\rho c^2 \frac{\partial^2 v}{\partial \psi^2} = \mu \frac{\partial^2 v}{\partial \psi^2}, \quad (4.12)$$

$$\rho c^2 \frac{\partial^2 w}{\partial \psi^2} = \mu \frac{\partial^2 w}{\partial \psi^2} \quad (4.13)$$

These equations can only be satisfied if $c^2 = (\lambda + 2\mu)/\rho$ and $\partial^2 v / \partial \psi^2$ and $\partial^2 w / \partial \psi^2$ are zero, or $c^2 = \mu/\rho$, and $\partial^2 u / \partial \psi^2 = 0$. The first case corresponds to a longitudinal wave travelling at velocity $c_l = [(\lambda + 2\mu/\rho)]^{0.5}$, and the second case to a transverse wave travelling at velocity $c_t = [\mu/\rho]^{0.5}$. This completes the proof that a plane wave can only travel with either dilatational or distortional velocity.

Using equations (4.4) and (4.5), the velocities of longitudinal and shear waves in an unbounded elastic medium can be rewritten as:

$$c_l^2 = \frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)} \quad (4.14)$$

$$\text{and } c_t^2 = \frac{E}{2(1+\nu)\rho} \quad (4.15)$$

And the ratio (k_1) between the velocities of transverse waves and longitudinal wave in an unbounded medium is:

$$k_1^2 = \frac{1-2\nu}{2(1-\nu)} \quad (4.16)$$

From Neville⁵⁶, the dynamic Poisson's ratio of concrete is 0.24. Substituting this value into equation (4.16), the value of k_1 is found to be

0.5849.

4.2 WAVES IN AN ELASTIC HALF-SPACE

4.2.1 Rayleigh Surface Wave

In the last section, it was shown that only two types of wave — dilatational and distortional, can be propagated in an unbounded elastic medium. However, in an elastic half-space there is a third type of wave, called Rayleigh wave, which can propagate on the surface of the medium. This wave, which is similar to gravitational surface water waves, was first investigated by Lord Rayleigh⁵⁷.

Kolsky⁵⁴ considered the propagation of a sinusoidal plane wave along the boundary of an elastic half-space, and attempted to find a solution of the equations of motion (4.1), (4.2) and (4.3) which satisfies the condition that the boundary is free from stress. He expressed the horizontal and vertical displacements of the surface wave as:

$$u = Af[e^{-qz} - 2qs(s^2 + f^2)^{-1}e^{-sz}]\sin(pt - fx) \quad (4.17)$$

$$w = Aq[e^{-qz} - 2f^2(s^2 + f^2)^{-1}e^{-sz}]\cos(pt - fx) \quad (4.18)$$

$$\text{with } (q/f)^2 = 1 - \alpha^2 k_2^2 \quad (4.19)$$

$$(s/f)^2 = 1 - k_2^2 \quad (4.20)$$

where A, p = amplitude and angular frequency of sinusoidal excitation
 f = wavenumber
 $= 2\pi/\Lambda$
 Λ = wavelength
 s, q = attenuation factors which control the rate of wave decay with depth z
 $\alpha = (1 - 2\nu)/2(1 - \nu)$.
 ν = Poisson's ratio

and the velocity of the surface wave can be found from:

$$k_2^6 - 8k_2^4 + (24 - 16\alpha^2)k_2^2 + 16(\alpha^2 - 1) = 0 \quad (4.21)$$

where k_2 = ratio between the velocity of a surface wave and that of a shear wave

Substitute $\nu=0.24$ for concrete into equation (4.21) and simplify:

$$k_2^6 - 8k_2^4 + 18.526k_2^2 - 10.5264 = 0 \quad (4.22)$$

Solving by computer, the ratio between the velocity of a surface wave and that of a shear wave is found to be:

$$k_2 = \pm 0.9178, \pm 1.7383 \text{ or } \pm 2.0338 \quad (4.23)$$

This implies there are 6 values for the velocity of a surface wave, whereas of course only one of them is physically acceptable. From equation (4.20), the following inequality can be derived:

$$(s/f)^2 = 1 - k_2^2 \geq 0$$

Hence $k_2^2 \leq 1$

$$\text{or } -1 \leq k_2 \leq +1 \quad (4.24)$$

The only valid solution for k_2 is 0.9178, since it must be less than 1 and must be positive.

Using the result of equation (4.16), the ratio (k_3) between the velocities of surface waves and longitudinal waves in an unbounded concrete medium is 0.5368.

Hence if $\nu = 0.24$ for concrete, then $k_2 = 0.9178$. If these two values are substituted into equations (4.19) and (4.20), then $q/f = 0.8437$ and $s/f = 0.3970$. From equation (4.17), the horizontal displacement of surface waves on the

plane boundary, i.e. $z=0$, can be expressed as:

$$u_o = Af[1-2qs(s^2+f^2)^{-1}]\sin(pt-fx) \quad (4.25)$$

and the horizontal displacement below the boundary relative to that on the surface is:

$$\frac{u}{u_o} = \frac{e^{-qz}-2qs(s^2+f^2)^{-1}e^{-sz}}{1-2qs(s^2+f^2)^{-1}} \quad (4.26)$$

Substituting the values of q/f and s/f from above into this equation and simplifying, the ratio of horizontal displacement at a depth z to that on the surface is:

$$u/u_o = 2.3736(0.4301^{2\pi z/\Lambda} - 0.5787 \times 0.6723^{2\pi z/\Lambda}) \quad (4.27)$$

Similarly, it can be found from equation (4.18) that the vertical displacement at any depth z relative to that on the surface is:

$$\frac{w}{w_o} = \frac{e^{-qz}-2f^2(s^2+f^2)^{-1}e^{-sz}}{1-2f^2(s^2+f^2)^{-1}} \quad (4.28)$$

Substituting the calculated values of q/f and s/f into this equation, the following expression is obtained:

$$w/w_o = 1.3742(1.7277 \times 0.6723^{2\pi z/\Lambda} - 0.4301^{2\pi z/\Lambda}) \quad (4.29)$$

Figures 4.1 and 4.2 show the relative horizontal and vertical displacements, with the change of depth, as a function of wavelength. It can be seen from Figure 4.1 that horizontal displacement decreases gradually with depth until its direction is reversed at a depth of about 0.2 wavelength. This reversed

displacement increases with depth up to about 0.4 wavelength and gradually dies out. The vertical displacement behaves differently. From Figure 4.2, it shows that as depth increases the amplitude of vibration first increases, reaches a maximum at a depth of 0.074 wavelengths, and then decreases monotonically. At a depth of one wavelength, $z=\lambda$, the amplitude has fallen to 0.189 of its value on the surface.

Equations (4.17) and (4.18) can be used to understand the particle motion of surface waves. These equations describe the horizontal and vertical displacements of a particle at position (x,z) at time t . If parts of the equations are rewritten as functions of the depth z and expressed as:

$$u = f(z)\sin(pt-fx) \quad (4.30)$$

$$w = g(z)\cos(pt-fx) \quad (4.31)$$

then, squaring both sides, it can be shown that:

$$[u/f(z)]^2 + [w/g(z)]^2 = 1 \quad (4.32)$$

which is the equation of an ellipse whose principal axes lie along the x and z axes; these are horizontal and vertical respectively. Thus the locus of each particle is an upright ellipse, centred on the particle's equilibrium position. Whether the major axis is along the surface or normal to it, depends upon the magnitudes of the functions $f(z)$ and $g(z)$. With Poisson's ratio of concrete, $\nu=0.24$, it is found that the major axis is normal to the surface. For particles at the surface, $z=0$, the ratio between the major and minor axes of the ellipse is 1.457.

4.2.2 Wave System At Surface Of Half-Space Generated By A Point Source

So far it has been shown that the velocities of longitudinal, shear and surface waves are $c_l = [(\lambda + 2\mu)/\rho]^{0.5}$, $c_t = [\mu/\rho]^{0.5}$ and $c_r = k_3 c_l$. It is clear that longitudinal waves travel with the fastest velocity which is at least twice that of shear waves, whilst surface waves are the slowest travelling slightly slower than shear waves.

When an impulse is exerted on the surface of a semi-infinite medium, energy is transmitted in terms of longitudinal, shear and surface waves away from the source. The bodily waves, longitudinal and shear, propagate radially outward from the source along a hemispherical wave front whilst the surface wave propagates radially outward along a cylindrical wave front. These waves spread out to cover an increasingly large volume of the medium and their energy density is decreased proportional to the volume they cover. This effect which leads to the decrease in displacement amplitude is called geometrical damping. Ewing, Jardetzky and Press⁵⁸ showed that the amplitude of body waves decreases inversely proportional to the distance r from the source, except along the surface where the amplitude decreases inversely proportional to the distance squared. The amplitude of surface waves decreases inversely proportional to the square root of distance. Based upon the above geometrical damping law and the velocities of these waves, Woods⁵⁹ produced a diagram of the wave system in the near field of the source. Figure 4.3 shows the basic features of this wave field. The shaded zones along the wave fronts for the body waves indicate the relative amplitude of particle displacement, as a function of the dip angle, as calculated by Miller and Pursey⁶⁰, and Hirona⁶¹.

The above system considers the effects in the near field. As this near

field effect spreads out radially, another wave system is set up in the far field. The wave system gradually develops a characteristic form, marked by three salient features travelling with velocities proper to longitudinal, shear and Rayleigh waves respectively. Lamb⁶² described in detail the surface motion that occurs at large distances from the point source. The comparatively small disturbance caused by the arrival of longitudinal and shear waves is termed as minor tremor, which is followed by a much larger main shock when the surface wave arrives. The horizontal and vertical displacements of surface particles are shown separately in Figure 4.4. Lamb proved the above results mathematically in 1903 but it was not until 1959 that Goodier, Jahsman and Ripperger⁶³ verified it experimentally. This was a very important experiment as it confirmed the different velocities of the waves mentioned.

4.3 WAVES IN AN ROD-LIKE STRUCTURE

Since a pile is normally a slender structure, the study of wave propagation in rods or beams is appropriate. Bodily waves such as longitudinal, torsional and flexural waves can be propagated within these structures and surface waves can be generated on both ends depending upon the method of excitation. As far as pile testing is concerned, longitudinal and surface waves are most important. Torsional waves are seldom generated as the impulse is always applied perpendicularly onto the top end, whilst flexural waves can easily be avoided if the impulse is applied near the centre of the top end so that there is no eccentric loading.

Longitudinal and surface waves in a bar can have many differences from those in an unbounded medium. Clearly, any differences between them are due to the boundary effects.

4.3.1 Longitudinal Waves In An Infinitely Long Rod Structure

4.3.1.1 Elementary theory

Consider an element of a length of rod where a pulse is propagating in the x-direction, as shown in Figure 4.5. The sum of forces in the x-direction, neglecting inertia forces, can be written as:

$$-\sigma A + \left(\sigma + \frac{\partial \sigma}{\partial x} dx \right) A = \rho A dx \frac{\partial^2 u}{\partial t^2} \quad (4.33)$$

where σ = the longitudinal stress
 A = cross-sectional area of the rod
 ρ = density of the rod
 u = displacement in the x-direction

The above equation can be simplified as:

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \quad (4.34)$$

Assuming elastic wave propagation, the stress can be related to the Young's modulus and the longitudinal strain as follows:

$$\sigma = E \frac{\partial u}{\partial x} \quad (4.35)$$

and
$$\frac{\partial \sigma}{\partial x} = E \frac{\partial^2 u}{\partial x^2} \quad (4.36)$$

Then combining equations (4.34) and (4.36),

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2}$$

$$\text{or} \quad \frac{\partial^2 u}{\partial t^2} = c_o^2 \frac{\partial^2 u}{\partial x^2} \quad \text{with} \quad c_o = \sqrt{[E/\rho]} \quad (4.37)$$

which is the elementary wave equation with c_o as the bar velocity of the longitudinal wave in a bounded media.

Using equation (4.14) and assuming $\nu = 0.24$, the ratio (k_4) between the bar velocity and the longitudinal velocity in an unbounded concrete medium is 0.9211. And the ratio (k_5) between the surface wave velocity and the bar velocity is 0.5828. The solution of equation (4.37) may be written as:

$$u(x,t) = f(x-c_o t) + g(x+c_o t) \quad (4.38)$$

where f and g are arbitrary functions depending upon the shape of the pulse. f can be regarded as a pulse propagating in the x -direction. It can be seen that after time Δt , the wave is repeated as x is displaced by $c_o \Delta t$. That is, $f(x-c_o t)$ is equal to $f[(x+c_o \Delta t)-c_o(t+\Delta t)]$. Hence the wave is moving with velocity c_o without change in shape, as shown in Figure 4.6. Similarly, it can be seen that g is a pulse propagating in the negative x -direction with velocity c_o .

The bar velocity, $c_o = (E/\rho)^{0.5}$, which is independent of frequency, suggests that waves of all frequencies will travel with that velocity without distortion. This is only true if plane transverse sections of the bar remain plane during the passage of the stress waves, and the stress acts uniformly over each section. It is not an exact description of wave propagation in a bar since it does not include the radial motion, and narrowing and thickening of the bar when the wave passes due to Poisson's effect as shown in Figure 4.7. This latter effect will cause a non-uniform distribution of stress across the sections of the bar, and plane transverse sections will become distorted.

4.3.1.2 Exact theory

According to Redwood⁶⁴, the exact solution of a sinusoidal wave propagation in an infinitely long cylindrical bar was first investigated by Pochhammer and Chree separately. An infinitely long bar was assumed due to the difficulty in solving the wave equations subject to the boundary conditions at the intersection between the curved surface and the end. The Pochhammer-Chree treatment gave a frequency equation as:

$$K_o^2 K_t \frac{J_0(K_t a)}{J_1(K_t a)} - \frac{\omega^2}{2ac_t^2} + \left(\frac{\omega^2}{2c_t^2} - K_o^2 \right) \frac{J_0(K_l a)}{K_l J_1(K_l a)} = 0 \quad (4.39)$$

where K_o = propagation constant

$$K_t = \sqrt{[(\omega/c_t)^2 - K_o^2]}$$

$$K_l = \sqrt{[(\omega/c_l)^2 - K_o^2]}$$

a = radius of the bar

ω = angular frequency

$J_m(K_l a)$ = Bessel's function of the first kind of order m .

From this equation the phase velocity for sinusoidal waves of any frequency along an infinitely long cylinder may be obtained. The term phase velocity is used here instead of wave velocity simply because a wave of combined frequencies will no longer travel as a whole and its velocity of propagation depends on frequency. The phase velocity is defined as the rate of travel of a point of fixed phase angle. In other words, it is the wave velocity for a well-defined wavelength. Theoretically any wave can be broken down by Fourier Transform into components of different frequencies and these components will travel along a bar with phase velocities according to their frequencies. Therefore a wave travelling along a bar will change in shape and

this effect of wave distortion is called dispersion.

Although equation (4.39) has existed for more than a century, it was not until the 1930s that the exact solution was computed successfully. This is mainly due to the complex nature of the equation. The problem has been studied by Bancroft⁶⁵ and Davies⁶⁶. Bancroft has presented the most comprehensive analyses and his results of phase velocity for the first mode of vibration as a function of radius of bar and wavelength are plotted (curve 2) along with the solution of the elementary theory (curve 1) in Figure 4.8.

4.3.1.3 Approximate theory

Due to the difficulties in solving the wave equation (4.39), many researchers have attempted to produce approximate equations either by considering a solution of the wave equation for large wavelength vibration or by taking radial motion and shearing strain and stress into account. These approximations were based on the assumptions that plane sections should remain plane and that the axial stress should be uniform over the section.

According to Kolsky⁵⁴, Bessel functions in equation (4.39) can be expanded in terms of power series as:

$$J_0(K_t a) = 1 - (K_t a)^2/4 + (K_t a)^4/64 - \dots$$

$$J_1(K_t a) = (K_t a)/2 - (K_t a)^3/16 + \dots$$

If the ratio of the radius and wavelength is sufficiently small, the second and higher powers of $(K_t a)$ can be neglected, hence $J_0(K_t a) = 1$ and $J_1(K_t a) = K_t a/2$. Putting these approximate values into the wave equation and solving, the longitudinal wave velocity is obtained as $(E/\rho)^{0.5}$. This is the same as the case when the wavelength is infinitely greater than the radius and then the

assumptions for elementary theory are fully satisfied.

Love⁶⁷ included the second power of the series into the wave equation and obtained the phase velocity as:

$$\frac{c_p}{c_o} = 1 - \nu^2 \pi^2 \frac{a^2}{\Lambda^2} \quad (4.40)$$

This equation was also derived by Rayleigh⁵⁷ from considerations of the energy associated with the lateral motion of the bar. The solution for this equation is also plotted in Figure 4.8 as curve 3.

Love⁶⁷ on the other hand developed an approximate equation by applying Hamilton's principle, that the total energy integrated between fixed time is zero, to an energy expression containing a transverse inertia term. A differential equation which includes a correction term for the inertia of cross section is obtained as:

$$\rho \left(\frac{\partial^2 u}{\partial t^2} - (\sigma r)^2 \frac{\partial^4}{\partial x^2 \partial t^2} \right) - E \frac{\partial^2 u}{\partial x^2} = 0 \quad (4.41)$$

Apart from the second term in the equation, it is exactly the same as the elementary wave equation (4.37). Solution of this equation is plotted as curve 4 in Figure 4.8, which shows it to be better approximation than the elementary theory.

There are many other approximate equations for this problem, the most frequently used approximation has been given by Mindlin and Herrmann^{68,69}, who took shearing strains and stress into account. These stresses inevitably accompany the propagation of a wavefront, especially when the wavelengths are small. Their results are plotted as curve 5 in Figure 4.8.

It is clear from Figure 4.8 that the approximate theories reported are more accurate with low values of the ratio of radius and wavelength and become inaccurate when the wavelengths are small. This is due to the fact that whilst some of these approximations have attempted to take account of radial motion of and shear stress in the cross section, they are still required to satisfy the basic assumptions that plane sections shall remain plane and that the axial stress shall be uniform over the section. These assumptions can be satisfied only when the wavelength is very large compared to the radius of the bar. The distribution of amplitudes of vibrations in the cross-section of a cylindrical bar is reproduced from Malecki⁵⁵ in Figure 4.9, which shows that as the wavelength increases for a bar of a fixed radius, the amplitudes of vibrations become more non-uniform over the cross section. However at large wavelengths the assumptions, that plane sections remain plane and axial stress is uniform, are almost satisfied.

4.3.2 Longitudinal Waves In Bars Of Other Cross-Section

The study of longitudinal wave propagation in rectangular bars and cylindrical shells may have practical value in the testing of precast piles and shell piles.

The elementary theory of cylindrical bars described in the last section is applicable to bars of any uniform cross section so long as the wavelength is large compared with the lateral dimensions of the bar, but fails when the wavelength is small. For short wavelength propagation an exact analysis may be required. The exact analysis of rectangular bars is exceedingly complex. Here only approximate treatments are presented. Chree⁷⁰ derived an approximate expression for phase velocity of waves travelling in elliptical and rectangular bars as:

$$\frac{c_p}{c_o} = 1 - 2v^2\pi^2 \frac{I}{\Lambda^2} \quad (4.42)$$

where I = second moment of cross section about the axis of the bar
(This term may be misleading, it is the radius of gyration rather than the second moment of area.)

= $(a^2+b^2)/4$ for a elliptical bar with axes a and b .

= $(a^2+b^2)/3$ for rectangular bar with sides a and b .

In the special case of an elliptical bar with equal major and minor axes ($a=b$), that is a circular bar, equation (4.42) becomes exactly as equation (4.40). Morse^{71,72} has found experimentally that a square bar gives the same dispersion curve as a cylindrical bar, if the ratio of the diameter of the cylinder to the side of the square is 1.13. Under this condition both bars have the same cross sectional area.

The exact theory of waves in shells has been developed by Mirsky and Herrmann⁷³. This very complicated frequency equation can be found in Redwood⁶⁴. The dispersion curve of shells is very similar to that of cylindrical bars. In general, at large wavelengths, the elementary theory of cylindrical bars holds for shells as well. However during the construction of shell piles, concrete is used to fill up the centres of the shells. It is therefore reasonable to treat shell piles as any ordinary piles so long as the acoustic properties of concrete in the centre are not too different from those of the shells.

4.4 PULSE PROPAGATION IN BARS OF FINITE LENGTHS

In the last section, longitudinal sinusoidal wave propagation in an infinitely long bar has been discussed. The elementary theory only applies to waves of infinitely large wavelengths, all approximate theories are true for long wavelengths compared to the lateral dimensions of the bar. The exact

frequency equation may be used to obtain the phase velocity for sinusoidal waves of any frequency along an infinitely long cylinder. However, these solutions are not exact for a finite length bar, since the conditions that the ends are free from traction have not been included in deriving the equation. For a slender bar like a pile, which has a large length to diameter ratio, the exact solution may still be valid.

In the case of single pulse propagation in a bar structure, the results of the exact theory cannot be readily applied, since such a pulse can only be analysed into sinusoidal components in terms of a Fourier integral. It is therefore expected that the pulse will undergo distortion due to different phase velocities of its Fourier components. Components of lower frequencies travel with faster velocities than those of high frequencies, as from Figure 4.8. Since a pulse does not travel with a unique phase velocity, it is common to refer to the velocity of pulse propagation as group velocity. The group velocity of a pulse may be defined as the velocity of points of highest amplitude. Theoretically, the distorted shape of the pulse can be calculated by adding the components at some further points down the structure. Hsieh and Kolsky⁷⁴ have successfully confirmed this approach by experiment. Their results, shown in Figure 4.10, illustrate clearly the characteristics of pulse propagation seen in Davies' work⁶⁶ — the slowing of the pulse rise, and the oscillations on the top of the pulse. This is due to the fact that high frequency components in the original pulse travel slower and display themselves at a later time. Independent experimental work on pulse propagation in cylindrical bars has been carried out by Ripperger⁷⁵, who observed that:

- "the general shape of the main pulse is not seriously changed as it travels along the bar.
- the changes in shape which do take place consist chiefly of a

reduction in amplitude, a widening of the base of the pulse, and a decrease in the slope of the leading edge, with increasing distance from the impact end. These effects are all reduced as the pulse duration is increased.

- the shorter the pulse the more noticeable is the disturbance which follows it. For the shorter pulse the disturbance becomes quite large and complex."

4.5 END RESONANCE OF CYLINDRICAL BAR

The subject matter of end resonance of cylindrical bars has not received as much attention as longitudinal waves due to the difficulty of solving wave equations subject to the intersecting boundary conditions of the end and the curved surface of a cylinder. The end resonance was first identified when Oliver⁷⁶ was investigating pulse propagation in a cylindrical bar. He reported in his paper that, in addition to the propagating wave, waves of a resonant nature associated with the ends of the rod were identified. These may be thought of as the result of constructive interference of surface waves on the end of the rod.

When a pulse is applied to the centre of one end of a rod, most of the energy goes into the longitudinal wave. A small fraction in a narrow frequency band will remain at the end as resonance. Its energy will gradually leak down the rod as a longitudinal wave. A similar end resonance will be set up at the other end on arrival of the main pulse.

Oliver's suggestion of the particle motion at the end of the rod during the end resonance is reproduced in Figure 4.11. The motion is symmetrical about the axis of the rod. When the centre is compressed, the edge heaves up and closes in towards the centre. When the centre heaves up, the edge is depressed and expands away from the centre. This is due to the Poisson's

ratio effect. As mentioned before, it would be very difficult to have an exact solution for this problem due to the intersecting boundary conditions. An approximate solution may be possible if radial shear stress is ignored, and hence the particle motion could be simplified as in Figure 4.12. This mode of motion is similar to surface motion of contained water. The wavelength (Λ) of the surface resonance is therefore the diameter (d) itself. For a bar of surface wave velocity (c_r), the frequency (f) is given by:

$$f\Lambda = c_r$$

or $f = c_r/d$ (4.43)

Steinbach and Vey³² calculated the surface wave velocity by dividing the diameter by the time period of surface oscillations. This approach is similar to the above approximation. Their result for the surface wave velocity of concrete was found to be in close agreement with equation (4.43).

4.6 REFLECTION AND TRANSMISSION OF PULSES AT BOUNDARIES

In the following discussion about reflection and transmission of pulses, it is assumed that the elementary theory of longitudinal propagation is valid. That is, the pulses are travelling non-dispersively with the bar velocity c_o . This requires a small ratio of radius of the bar to the pulse main wavelength.

4.6.1 Reflection From Fixed And Free Ends

Consider a pulse $u=f(x-c_o t)$ travelling down a rod of length L with a fixed end. At the boundary $x=L$, a reflected pulse $g(x+c_o t)$ is formed. Hence the particle displacement is $u=f(L-c_o t)+g(L+c_o t)$, and the particle velocity is $v=-c_o f'(L-c_o t)+c_o g'(L+c_o t)$. At a fixed boundary, particle displacement and velocity must vanish. Therefore, $g(L+c_o t)=-f(L-c_o t)$ and $g'(L+c_o t)=f'(L-c_o t)$. The

stress at the boundary can be found by equation (4.35) as:

$$\begin{aligned}
 \sigma &= E(\partial u / \partial x) \\
 &= E[f'(L - c_0 t) + g'(L + c_0 t)] \\
 &= 2Ef'(L - c_0 t)
 \end{aligned}
 \tag{4.44}$$

Hence for a fixed-end rod, the reflected pulse must be of the same sense as the incident pulse (i.e. compression pulse being reflected as compression, tension pulse being reflected as tension), and the stress at the boundary is twice of the corresponding value when the pulse is travelling along the rod.

In the case of a free-end rod, the stress at the boundary must vanish.

Hence,

$$\begin{aligned}
 f'(L - c_0 t) + g'(L + c_0 t) &= 0 \\
 \text{or } g'(L + c_0 t) &= -f'(L - c_0 t)
 \end{aligned}
 \tag{4.45}$$

And the particle velocity at the boundary is given by:

$$\begin{aligned}
 v &= -c_0 f'(L - c_0 t) + c_0 g'(L + c_0 t) \\
 &= -2c_0 f'(L - c_0 t)
 \end{aligned}
 \tag{4.46}$$

Therefore, the reflected pulse is in the opposite sense to the incident pulse (compression reflected as tension and vice versa) and the particle velocity at the free boundary is doubled.

In order to understand the physical implications of the above mathematics, the mechanism of a single compression pulse propagating in a rod will be investigated. A particle within the compressed zone where the pulse is propagating will push another particle in front. There is kinetic energy due to the particle motion and potential energy since the bar is at the same

time under stress. It can be shown that the kinetic energy is equal to the potential energy. For simplicity, a rectangular pulse is considered, then the mass m of particles within the compression zone attains a uniform velocity of v_1 . Both kinetic and potential energy are $mv_1^2/2$ and the total energy in the system is therefore mv_1^2 . When the pulse reaches a free end, two events happen there. Firstly, the compression pulse is gradually reflected as a tension pulse. When half of the pulse is reflected, there is no overall strain in the bar and the mass of particles under motion is $m/2$. These particles are under the combining effect of the incident compression pulse and the reflected tension pulse, that is, a push and pull situation. Since there is no overall strain, then the original potential energy must have transformed into kinetic energy. If the particle velocity at the free end is v_2 , then by conservation of energy, the following expression can be obtained:

$$(m/2)v_2^2/2 = mv_1^2$$

or $v_2 = 2v_1$ (4.47)

It shows that the particle velocity at a free end is doubled. Figure 4.13 and Figure 4.14 show separately the process of a half-sine pulse propagating in a bar with a fixed and a free end. The corresponding particle motion at the top end, in terms of displacement, velocity and acceleration, are shown in Figures 4.15 and 4.16.

4.6.2 Transmission And Reflection From A Boundary Of Discontinuity

Reflection of a pulse from a fixed or free end represents an ideal situation only. Any structure, except in vacuum, must be in contact with some other types of materials. Strictly speaking, the ends of a suspended bar are not free from stress and vibration energy will be leaked into air. When a pulse

strikes a boundary, part of its energy is transmitted and the remainder reflected. The partition of transmitted and reflected energy is dependent upon the characteristic impedance and the cross-sectional area of the materials concerned. The characteristic impedance of a material is defined as the ratio between the stress at any point and its particle velocity. Consider a pulse with displacement $u=f(x-c_0t)$ travelling in the x-direction. Its derivatives with respect to t and x respectively are:

$$\frac{\partial u}{\partial t} = c_0 f'(x-c_0t) \quad (4.48)$$

$$\frac{\partial u}{\partial x} = f'(x-c_0t) \quad (4.49)$$

Combining these two equations, the strain can be expressed as:

$$\epsilon = \frac{\partial u}{\partial x} = - \frac{1}{c_0} \frac{\partial u}{\partial t} \quad (4.50)$$

Hence the stress is:

$$\sigma = E\epsilon = - \frac{E}{c_0} \frac{\partial u}{\partial t}$$

or $\sigma = - \rho c_0 \frac{\partial u}{\partial t} \quad (4.51)$

which is an equation relating the stress to the particle velocity. Therefore the characteristic impedance of the material is $z=\rho c_0$.

Consider a pulse incidents upon a surface which separates two materials of different characteristic impedances and different cross-sectional areas as shown in Figure 4.17. At the interface, two conditions must be

satisfied:

1. Forces on both sides must be equal
2. Particle velocity must be continuous

Two equations can be obtained from the above conditions as:

$$(\sigma_i + \sigma_r)A_1 = \sigma_t A_2 \quad (4.52)$$

$$\text{and} \quad v_i - v_r = v_t \quad (4.53)$$

where

- σ_i = stress of the incident pulse
- σ_r = stress of the reflected pulse
- σ_t = stress of the transmitted pulse
- A_1 = area of section 1
- A_2 = area of section 2
- v_i = particle velocity in the incident pulse at the interface
- v_r = particle velocity in the reflected pulse at the interface
- v_t = particle velocity in the transmitted pulse at the interface

Note that if the reflected pulse is to have a stress of the same sense as the incident pulse, the particle velocity must have the opposite sign. Using the definition of characteristic impedance, equation (4.53) can be written as:

$$\frac{\sigma_i}{\rho_1 c_1} - \frac{\sigma_r}{\rho_1 c_1} = \frac{\sigma_t}{\rho_2 c_2}$$

$$\text{or} \quad (\sigma_i - \sigma_r)z_2 = \sigma_t z_1 \quad (4.54)$$

Solving equations (4.35) and (4.37) together:

$$r = \frac{\sigma_r}{\sigma_i} = \frac{A_2 z_2 - A_1 z_1}{A_2 z_2 + A_1 z_1} \quad (4.55)$$

$$\text{and} \quad t = \frac{\sigma_t}{\sigma_i} = \frac{2A_1 z_2}{A_2 z_2 + A_1 z_1} \quad (4.56)$$

where r and t are the coefficients of reflection and transmission respectively.

In an elastic medium, it is expected that the energy of the incident pulse will be equal to the total energy in the reflected and transmitted pulses. As mentioned earlier in the last section the potential energy is the same as the kinetic energy in a travelling pulse. In the case of a rectangular pulse, the energy in the incident, reflected and transmitted pulses is therefore $m_1(\sigma_i^2/z_1^2)$, $m_1(\sigma_r^2/z_1^2)$, and $m_2(\sigma_t^2/z_2^2)$ respectively, where:

$$\begin{aligned} m_1 &= \text{mass of particles under stress in the first section} \\ &= A_1 \rho_1 c_1 t \\ m_2 &= \text{mass of particles under stress in the second section} \\ &= A_2 \rho_2 c_2 t \\ \sigma_i^2, \sigma_r^2, \sigma_t^2 &= \text{mean square values of stresses} \end{aligned}$$

The total energy after reflection and transmission is:

$$\begin{aligned} &= m_1 \left(\frac{\sigma_r^2}{z_1^2} \right) + m_2 \left(\frac{\sigma_t^2}{z_2^2} \right) \\ &= m_1 \left(\frac{r^2 \sigma_i^2}{z_1^2} \right) + \left(\frac{A_2 z_2}{A_1 z_1} \right) m_1 \left(\frac{t^2 \sigma_i^2}{z_2^2} \right) \\ &= m_1 \left(\frac{\sigma_i^2}{z_1^2} \right) r^2 + \left(\frac{A_2 z_1}{A_1 z_2} \right) t^2 \end{aligned} \quad (4.57)$$

By using equations (4.55) and (4.56), it can be shown that the term $[r^2 + (A_2 z_1 / A_1 z_2) t^2]$ equals 1. Therefore energy is conserved after the incident pulse strikes the boundary.

4.6.2.1 Discontinuity in characteristic impedances

If the areas on both sides of the boundary are equal, equations (4.55) and (4.56) become:

$$r = \frac{z_2 - z_1}{z_2 + z_1} \quad (4.58)$$

$$t = \frac{2z_2}{z_2 + z_1} \quad (4.59)$$

There are three cases of interest with respect to the above two equations:

1. Case 1: $z_1 \ll z_2$ Material 2 is acoustically much harder than material 1. Equations (4.58) and (4.59) reduce to $r=1$ and $t=2$. This indicates that the amplitude of the reflected pulse equals the amplitude of the incident pulse and the amplitude of the transmitted pulse is twice the amplitude of the incident pulse. Both reflected and transmitted pulses are in the same sense as the incident pulse.
2. Case 2: $z_1 = z_2$ Both materials have the same characteristic impedances. Equations (4.58) and (4.59) reduce to $r=0$ and $t=1$. The incident pulse passes straight through into the second material as if it is still in the first.
3. Case 3: $z_1 \gg z_2$ Material 2 is acoustically much softer than material 1. Equations (4.58) and (4.59) reduce to $r=-1$ and $t=0$. All of the energy of the incident pulse is reflected back into material 1. None of the energy is transmitted into material 2. The incident and reflected pulses have opposite senses.

4.6.2.2 Discontinuity in cross-sectional areas

If the characteristic impedances on both sides of the boundary are equal, equations (4.55) and (4.56) become:

$$r = \frac{A_2 - A_1}{A_2 + A_1} \quad (4.60)$$

$$\text{and } t = \frac{2A_1}{A_2 + A_1} \quad (4.61)$$

From equations (4.60) and (4.61), it may be seen that if there is an increase in area, i.e. $A_2 > A_1$, the incident pulse is reflected without change in sense. If there is a decrease in area, i.e. $A_2 < A_1$, the reflected pulse will have an opposite sense to the incident pulse. On the other hand, the transmitted pulse always has the same sense as the incident pulse.

Although equations (4.58), (4.59), (4.60) and (4.61) are derived from the same principles, it is expected that when it comes to application the pair of equations (4.60) and (4.61) for the change in cross-sectional area will lead to larger errors than their counterpart for the change in characteristic impedance. The first reason for this is that the original equations (4.55) and (4.56) are based upon the elementary wave theory which requires plane wave propagation. This requirement can be easily violated in the case of cross-sectional area change. The greater the change in area, the more severe the violation. When a longitudinal plane wave strikes a boundary of discontinuity in area, a plane wave cannot remain plane especially around the edges. In the case of a large increase in area, wave energy will be radiated hemi-spherically down into the second medium through the interface. The second reason is violation of the force equilibrium condition at the interface as defined by equation (4.52). If the change in cross-sectional area is large, it is difficult to see how an area remote from the interface can contribute to force balance on the interface in a dynamic situation. Equation (4.52) will be acceptable only if the difference between A_1 and A_2 is not too large.

Ripperger and Abramson⁷⁷ have performed experiments on the reflection and transmission of elastic pulses in a bar with a discontinuity in

cross section. They found that the discrepancy between predicted and measured results was too great to be attributed to experimental error. However, they concluded that equations (4.60) and (4.61) give sufficiently accurate prediction for many purposes.

4.7 CONCLUSIONS

1. As far as dynamic pile testing is concerned, longitudinal waves and surface waves are most important.
2. Elementary wave theory is sufficient for both circular and non-circular piles, provided the wavelength of the longitudinal pulse generated is much greater than the diameter or the lateral dimensions of the piles.
3. If the above requirement is fulfilled, no significant signal distortion caused by dispersive effects will occur.
4. A long wavelength pulse is generated if the frequency content of the pulse is concentrated on the lower frequency region. A sonic technique is thus more appropriate.
5. Since a foundation pile is a rod-like structure embedded in soils, the propagation velocity of stress waves in a pile should be between the bar velocity and the longitudinal wave velocity in an unbounded medium.
6. When a longitudinal pulse strikes a fixed boundary, a reflected pulse of the same sense as the incident pulse will be generated. At a free boundary, the reflected pulse will have an opposite sense as the incident pulse.
7. When a longitudinal pulse strikes a boundary of discontinuity, either in characteristic impedance or in cross-sectional area, a reflected pulse and a transmitted pulse will be created. The reflected pulse will have the same sense as the incident pulse if the boundary is associated with an increase in characteristic impedance or in

cross-sectional area. The sense of the transmitted pulse will be the same as the incident pulse regardless of the nature of the boundary.

8. Surface wave oscillations are confined to the top of a pile. At a depth of one wavelength, the amplitude of vertical displacement is reduced to less than 20 percent of its value on the surface.
9. The dominant frequency of the surface wave oscillations on a circular pile head can be approximated by dividing the surface wave velocity by the pile diameter.
10. Using the various equations in this chapter and by assuming the Young's modulus $E = 38.4 \text{ KN/mm}^2$, density $\rho = 2400 \text{ kg/m}^3$ and the Poisson's ratio $\nu = 0.24$ for concrete, the velocities of different types of waves can be computed as:

Unbounded longitudinal wave velocity $c_l = 4343\text{m/s}$

Unbounded transverse wave velocity $c_t = 2540\text{m/s}$

Bar velocity $c_o = 4000\text{m/s}$

Surface wave velocity $c_r = 2331\text{m/s}$

The ratios between these velocities are:

$$k_1 = c_t/c_l = 0.5849$$

$$k_2 = c_r/c_t = 0.9178$$

$$k_3 = c_r/c_l = 0.5368$$

$$k_4 = c_o/c_l = 0.9211$$

$$k_5 = c_r/c_o = 0.5828$$

These calculations are based on the above assumed properties of concrete. Whilst the density and the Poisson's ratio of concrete are not too variable, the Young's modulus of concrete can have a wide range of values due to changes in concrete strength. Different results may be obtained for concrete of different properties. Thus the above calculations are not absolute but serve as a guideline to

the velocities of different types of waves and their ratios.

CHAPTER 5

TESTING AND ANALYSIS TECHNIQUES

5.1 FOURIER ANALYSIS

Normally in structural testing, input and output signals in terms of time history are recorded. Although these time histories can yield important information about the structural behaviour, it is sometimes useful to convert information from the time domain into the frequency domain. The process that transforms information from one domain to the other is called Fourier Analysis. It should be stressed that no extra information will be gained by performing a Fourier Analysis. The idea of Fourier Analysis is to present the information in such a way that it is easy to interpret and facilitate solutions to the problems. An excellent discussion on Fourier Analysis and its application can be found in Brigham⁷⁸ and Thrane⁷⁹.

:

5.1.1 Fourier Series Of A Periodic And Continuous Signal

There are two types of periodic signals — sinusoidal and complex periodic. For sinusoidal excitation, a structure may be excited at a single frequency. For complex periodic excitation, the system is excited by a signal composed of a series of sinusoidal signals whose higher frequencies are an integer multiple of the frequency of the repetition rate of the signal. A sinusoidal signal and a complex periodic signal are shown in Figure 5.1 separately. For periodic signals, only one period of the time signal has to be specified and included in the Fourier Transform. The time signal can be reconstructed from the spectrum by inverse Fourier Transform. The mathematical relationship between a Fourier pair of a signal of period T as shown in Figure 5.2 is:

$$G(f) = \frac{1}{T} \int_{-T/2}^{T/2} g(t) e^{-j2\pi ft} dt$$

$$g(t) = \sum_{f=-\infty}^{\infty} G(f)e^{j2\pi ft} \quad (5.1)$$

where $G(f)$ is a frequency series
 $g(t)$ is the periodic time signal
 $f = k(1/T)$
 k is an integer from $-\infty$ to ∞

Notice the periodic and continuous time signal is transformed into a discrete frequency spectrum, and vice versa.

5.1.2 Fourier Transform Of Non-Periodic Continuous Signal

For a non-periodic continuous signal, the Fourier integral has to be extended over all time, $-\infty < t < +\infty$, to produce a continuous frequency spectrum also extending over all frequencies, $-\infty < f < +\infty$, as shown in Figure 5.3. The Fourier pair are related to each other by:

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft} dt$$

$$g(t) = \int_{-\infty}^{\infty} G(f)e^{j2\pi ft} df \quad (5.2)$$

This represents an ideal transform, which in principle could be applied to all time signals. However it is impossible in practice to have a knowledge of the time signal extending over all time, except a relatively short transient signal.

5.1.3 Discrete Fourier Transform

The increasing use of digital instruments, such as the Nicolet digital oscilloscope and the Bruel and Kjaer signal analyser introduced in chapter 6, has resulted in an increased emphasis on a discrete version of Fourier Transform. Fourier Series and Continuous Fourier Transform, described in the

last two subsections for periodic and non-periodic signals, are both not suitable for the transform of a time-limited non-periodic digital signal into its frequency spectrum. For such a signal, the Discrete Fourier Transform DFT has to be used. The Discrete Fourier Transform equations are as follows:

$$G(k) = \frac{1}{N} \sum_{n=0}^{N-1} g(n)e^{-j2\pi nk/N}$$

$$g(n) = \sum_{k=0}^{N-1} G(k)e^{j2\pi nk/N} \quad (5.3)$$

There are three important aspects of a real time analog signal that must be investigated in relation to Discrete Fourier Transform. Firstly, the analog signal must be properly sampled. Secondly, the effect of time limitation or block length on the Fourier Transform. Thirdly, picket-fence effect due to sampling in the frequency domain.

5.1.3.1 Aliasing effect

The link between the analog signal and the corresponding discrete signal is provided by what is known as the sampling theorem. According to Stremler⁸⁰, the sampling theorem can be stated simply as follows:

"A real-valued band-limited signal having no spectral components above a frequency of B Hz can be determined uniquely by sampling it with a time interval of less than 1/2B seconds."

In other words, the signal must be sampled with a sampling frequency of at least twice the highest frequency component in the time signal. Under such conditions, an analog signal can be reconstructed completely from a set of uniformly spaced discrete samples in time.

If the sampling frequency is not high enough for a particular signal,

frequency components higher than the sampling frequency will be mis-sampled as if they are of a lower frequency content. When the sampled signal is transformed into its spectrum, these components will be folded back along the sampling frequency spectral into the lower frequency region. This is normally referred to as aliasing effects.

The effects of aliasing on pile testing results can be tackled in two ways. Firstly, a pile should be excited at frequencies within the region of interest. From the result of preliminary studies in Chapter 3, frequency below 2 KHz will be sufficient. This will require a sampling frequency of not less than 4 KHz, that is, at least twice higher than the maximum frequency of interest. Secondly, to exclude higher frequencies due to non-linearity from distorting the frequency spectrum, an anti-aliasing filter should be used to attenuate those frequency components outside the band of interest. Anti-aliasing filters have been used in the analysis of both experimental and practical results in Chapter 8 and Chapter 9.

5.1.3.2 Leakage effect

When a signal is sampled, there must be a time limitation or block length for the number of samples to be taken. In other words, a continuous signal has to be truncated at some point. Truncation of a signal normally causes error in the frequency analysis. Additional frequency components will be included in the spectrum of the signal. This error is termed as leakage. Leakage in the frequency analysis can be avoided completely in the case of a periodic signal if the time limitation is set to match with an exact multiple period of the signal. For non-periodic signals leakage will inevitably occur due to truncation of the signal by the time limitation. However, the effect of leakage on the spectrum of the signal can be reduced by appropriate use of

different types of windows. The problem of leakage will be dealt with in detail in section 5.2.

5.1.3.3 Picket-fence effect

So far only the real time signal is digitized, however, for a complete digital computation both Fourier Transform pairs must be in digital form. Note a discrete time signal may not necessarily give a discrete spectrum (see Figure 5.2). A separate digitization process in the frequency domain is thus necessary. The digitization of a continuous frequency signal is called frequency sampling. Usually the frequency signal is digitized with a frequency interval $\Delta f = 1/T$, where T is the time limitation or the time length of the record. Sampling in the frequency domain is associated with the so-called Picket-fence effect, since sampling a continuous signal is like observing the signal through a picket-fence. An ideal sampling is one that correctly picks up the spectral lines from the continuous frequency signal. If for some reason, the frequency interval Δf is not spaced at the correct positions of a spectrum, error may be included into the analysis. For example, the correct maximum value of a very peaked component in the spectrum might not be picked up, but instead only the lower value on the slope of the peak is taken. An example of picket-fence effect is shown in Figure 5.6(b).

5.1.3.4 Example of discrete Fourier Transform

The graphical development of a Fourier Transform pair is shown in Figure 5.4. Different time functions involved in the process are shown side by side with their frequency spectra.

The signal with its spectrum are shown in Figure 5.4(a). The sampling function in the time domain is shown in Figure 5.4(b), which is a series of unit

impulses spaced at a time interval of T . The digitized signal shown in Figure 5.4(c) is the result of multiplying the time signal $h(t)$ by the sampling function $\Delta_o(t)$. It is worth mentioning that multiplication in the time domain is equivalent to convolution in the frequency domain. The concept of convolution is outlined in sections 5.4.4 and 5.4.7. Time limitation of the signal record is represented by a rectangular window of duration T_o , which will not alter the signal within the window but truncates the signal outside the window as shown in Figure 5.4(e). The frequency sampling function is shown in Figure 5.4(f), whose Fourier pair are the inverse of the time sampling function pair. Finally, the time signal is reconstructed from the spectrum via inverse Fourier Transform as shown in Figure 5.4(g).

This exercise serves as a very good example to illustrate the problems associated with the effects of aliasing and leakage. The aliasing effect can be seen in the spectrum shown in Figure 5.4(c) as overlapping of frequency components near the vicinity of frequency $1/2T$. This problem can be solved by increasing the sampling rate, that is smaller sampling interval T , to such an extent that no overlapping of frequency components occurs. The leakage effect manifests itself as a ripple on the spectrum as shown in Figure 5.4(e). This is caused by truncating the time signal with a time limitation of T_o . The error can be reduced if a longer time record is taken.

5.1.4 Fast Fourier Transform

Fast Fourier Transform is an algorithm which enables the calculation of Discrete Fourier Transform to be carried out in a very efficient way. Hence FFT is a calculation scheme which enables DFT to be computed with a minimum time. FFT is widely adopted in many analysis instruments, for instance, the Nicolet digital oscilloscope and the Bruel and Kjaer signal analyzer

described in Chapter 6. The FFT algorithm is due to Cooley-Tukey's formulation. A detailed discussion of the FFT algorithm and its comparison with DFT can be found in Thrane⁷⁹. Aside from the algorithm itself, the interpretation of the FFT is the same as that for the DFT. From an application point of view it is of little interest to know how the spectra are calculated, with or without FFT, since the results are the same. However, it is interesting to know the saving in computation time when FFT is used and its requirements in signal sampling. Computation of DFT of a time signal of N samples requires N^2 multiplications and the resulting computation time becomes excessive when N becomes large, as multiplications are the most time consuming operations. Using the FFT algorithm the number of multiplications can be reduced to $N \log_2 N$, assuming N is a power of two. In the typical case of $N=2048$, the reduction in calculation time is more than 186. The requirement of FFT is that the number of samples N must be a power of 2, i.e. $N=2^r$, r any integer. This imposes a restriction on the record length of a signal. The restriction on the number of samples is not serious in practice as long as a weighting function is used appropriately on the signal.

5.2 WEIGHTING FUNCTIONS

It has been mentioned earlier that leakage in the frequency analysis will almost occur when Discrete Fourier Transform is used. In the case of a periodic signal, leakage may be caused by the truncation of the signal at other than an exact multiple of the period which results in a sharp discontinuity in the time domain, or equivalently side-lobes in the frequency domain. Side-lobes are the secondary lobes, other than the mainlobe in the spectrum of a truncation or weighting function (see Figures 5.5 and 5.7). These side-lobes are responsible for the additional frequency components which distorts the

spectrum. Leakage does not occur in the frequency analysis of a periodic signal if the truncation time equals to an integer multiple of the period of the signal. However, for non-periodic signals, leakage will inevitably occur. A remedy is to choose a window function which truncates the signal and whose Fourier Transform has low sidelobes. This has been a topic of much research and no ideal solution exists. Nevertheless, a proper use of the window function can reduce the effect of leakage to an acceptable level. In the following section, a few commonly available window functions are discussed along with their applications.

5.2.1 Rectangular Weighting Function

A rectangular weighting function, as shown in Figure 5.5⁸¹, is the block length of the record itself where samples within the window are given a weighting unit of one and those outside the window are suppressed to zero. This window function is only suitable for periodic signals where the window length matches with an integer multiple of the period. Under this situation, the artificial infinitely long periodic signal assumed by the DFT is exactly the same as the original signal, hence the DFT is an exact perfect analysis of the signal. It should be mentioned that the rectangular window is generally not a good weighting function due to its large sidelobes. However, when it is used in a periodic signal there is no picket-fence effect. The spectrum is sampled at the peak of the mainlobe which gives exactly the signal frequency and at the 'zeros' between the sidelobes leaving the sidelobes unsampled, as shown in Figure 5.6(a). If a periodic signal is truncated by a rectangular window at other than an integer multiple of the period, leakage in the frequency domain will still occur. The effect of truncation at other than a multiple of the period is to create a sharp discontinuity in the artificial periodic signal assumed by DFT.

The introduction of these sharp changes in the time domain will result in the inclusion of additional frequency components in the frequency domain (see Figure 5.6(b)).

5.2.2 Hanning Weighting Function

For a non-periodic signal, there will be picket-fence effect. A rectangular weighting function is not recommended for such a signal due to its large sidelobes in the frequency domain. As a basic requirement, a weighting function should be able to remove discontinuity in the time domain caused by truncation at other than an integer multiple of the period. One such weighting function is the Hanning Window as shown in Figure 5.7. The Hanning function is simply one period of a sinusoid having a length equal to the block length T , but lifted up so that it starts and stops at zero. When a Hanning window is applied to a time signal, leakage in the frequency analysis will be reduced since the window has the effect of decreasing the amplitude at the beginning and the end of the block length to zero, thereby eliminating any discontinuities. However, it should be kept in mind that the Hanning window is not a perfect weighting function since it distorts the time signal.

An example of the effect of a Hanning window on DFT analysis of a truncated sine-wave is shown in Figure 5.8. Figure 5.8(a) shows the result of truncating a sine-wave at an exact multiple of the period. There is no leakage effect and the discrete Fourier Transform gives exactly the same result as the continuous Fourier Transform. Figure 5.8(b) illustrates the effects of leakage in the spectrum when the sine-wave is truncated at a half integer multiple of the period. It will be noticed that additional frequency components have crept into the spectrum. The truncation error is reduced when a Hanning window is applied to the signal, as shown in Figure 5.8(c). However, due to signal

distortion by the Hanning window, the discrete Fourier Transform does not give an exact frequency analysis of the sine-wave signal.

5.2.3 Transient Weighting Function

Both rectangular and Hanning weighting functions are not suitable for short transient or impulse signals. When a transient signal is sampled, it might only occupy a small fraction of the total block length, leaving the rest of the record filled up with noise. It is therefore desirable to have a weighting function which captures the transient event but excludes noise beyond it. The transient window, as shown in Figure 5.9, may be regarded as a combination of a shortened rectangular window and a Hanning window. The flat top in the central part of the window retains the feature of a rectangular window which will not distort the signal. The sloping sides at the beginning and the end of the window have a similar smoothing effect on discontinuities as a Hanning window. A transient window can be positioned anywhere along the recorded length and its window width can be adjusted to capture any transient event. This is particularly useful on an impulse signal to increase the signal to noise ratio and a very short transient window may sometimes be referred as a force window.

5.2.4 Exponential Weighting Function

For an exponentially decaying signal record which has not decayed to zero by the end of the block length, as shown in Figure 5.10, an exponential weighting function has to be applied to the signal to suppress it to zero. The Hanning window is unsuitable in this case since it also suppresses the beginning of the signal, which contains most energy. Figure 5.11 shows an exponential window which consists of a leading half cosine taper to unity,

followed by an exponential decay with decay constant T . The decay rate can be adjusted to force a signal to zero before the end of the block length in order to avoid truncation error. When an exponential window is used, the extra added decay will increase the apparent damping of the signal, and hence decrease the amplitude of the resonant peak.

5.2.5 The Proper Use Of Weighting Functions For Pile Testing

Four types of weighting functions with their special effects on DFT of different signals have been discussed. Apart from these four windows, there are other types of windows such as Flat-Top, Kaiser-Bess, etc. Some of these windows are provided by spectrum analysers. Some spectrum analysers even allow users to define their own window functions for special signals.

TYPE OF SIGNAL	BLOCK LENGTH	SUGGESTED WEIGHTING FUNCTION
Periodic signal	an integer multiple of periods	rectangular
Periodic signal	not an integer multiple of periods	Hanning
Impulse or short transient	long compares with signal	force
Random impact	----	Hanning
Exponential decay	short compares with signal	exponential
Exponential decay	long compares with signal	transient

Table 5.1

Following the above discussion of different weighting functions, it is clear that windows have to be used carefully on different signals in order to

reduce the effects of truncation error. Halvorsen and Brown⁸², Sohaney and Nieters⁸³ have investigated the usage of different windows. Their results are summarized in Table 5.1 above.

The proper use of weighting functions is very important in pile testing, especially when an numerical output is required. For instance, the computation of the dynamic stiffness value from a mechanical admittance plot. The use of a weighting function has to be considered separately for the excitation signal (input to the structure) and the vibrational response of the structure (output from the structure). Excitation techniques will be discussed in the next section.

As far as dynamic pile testing is concerned, the most popular excitation method is impact excitation. If this method of excitation is adopted, a very short transient (or force) window should be used on the input signal to exclude noise beyond the vicinity of the impulse from contaminating the frequency spectrum.

The choice of weighting functions on the output signal is mainly dependent upon the degree of damping in the signal. For an underdamped signal, such as the experimental results to be reported in Chapter 8, an exponential window should be used to suppress the signal to zero before the end of the time record. This will have the effect of reducing truncation error. However, it must be borne in mind that this type of window will introduce extra damping to the signal. In the case of a heavily damped signal, such as the site results to be reported in Chapter 9, a short transient window is more appropriate. Truncation error is no longer a major problem since the signal is damped to zero before the end of the record. A rectangular window may be used for this type of signal, however, a short transient window has the advantage that noise can be excluded from the analysis. The effects of different

weighting functions on the dynamic stiffness value of a site pile will be investigated in Chapter 9.

5.3 EXCITATION TECHNIQUES FOR STRUCTURAL TESTING

In structural testing, a well controlled and measureable excitation method is highly desirable since the amount and quality of information that can be extracted from structural response depends very much on the input to the structure. The choice of excitation method depends amongst other things upon the test application, non-linear behaviour of the system and time available for the analysis.

A controlled and measureable input signal for structural testing can either be excitation by a shaker or an impact hammer as shown in Figure 5.12. There are a number of different types of excitation signals. The common ones are:

1. Random Noise Excitation
2. Pseudo-Random Excitation
3. Periodic Impulse Excitation
4. Periodic Random Excitation
5. Sinusoidal Excitation
6. Impact Excitation
7. Random Impact Excitation

A signal generator and an amplifier are needed to control the shaker in the first five methods. The last two methods only require an impact hammer. Herlufsen⁸⁴ gave a very detailed account of the first six excitation signals. Here only the main features are discussed.

5.3.1 Random Noise Excitation

Reynolds⁸⁵ classified observed time histories of signals as either deterministic or random. The difference between a deterministic and a random signal is that the former can be expressed by explicit mathematical relations whilst the latter cannot.

A random signal can be defined as a continuous type of signal which never repeats itself. An example of a random signal is shown in Figure 5.13. If random noise excitation is used for testing, it is preferable to have a constant spectral density in the frequency range of interest. That is, the autospectrum of the excitation signal should be flat in this frequency range and the random signal is called a band limited white noise signal. This can be easily achieved by filtering and modulating the original broad band white noise signal from the generator. The dynamic range in the analysis is therefore increased since the structure is not excited by frequencies outside the analysis bandwidth. Since the random signal is continuous and does not repeat itself, it is impossible to capture a complete period of the signal. At some time later the signal has to be truncated to fit the block length in the analysis. This will cause a truncation error in the analysis. Truncation error can be reduced by applying a smooth weighting function (such as the Hanning window) onto the captured signal. This, however, will cause leakage in the spectral estimate. Random noise excitation gives the best linear fit to the structural response because the structure is excited by frequencies of random amplitudes and random phases.

5.3.2 Pseudo-Random Excitation

A pseudo-random signal such as shown in Figure 5.14, is a deterministic and periodic signal which is made up of segments of repeating

random signal. It has energy only at discrete frequencies $f=k(1/T)$, where T is the period length and K is an integer. A pseudo-random signal is specially designed for DFT analysis, whose time period T is matched to the block length in the analysis. One record length of the signal is sufficient to contain all the information in the signal. Rectangular weighting functions should be used and there will be no leakage in the spectral estimates if there is perfect matching between the record length and the period length T . The main advantages in using pseudo-random excitation are no leakage in the analysis, the excitation frequencies can easily be shaped into the range of interest and only a few averages are needed. However, pseudo-random signals are only satisfactory for linear systems since non-linearities in the system will also be excited by the periodic nature of the signal.

5.3.3 Periodic Impulse Excitation

A periodic impulse signal with its autospectrum is shown in Figure 5.15. It is made up of short impulses of one sample time which repeat with a period T . The impulse is so short that the spectrum is nearly flat in the baseband frequency range. It has the same advantage as a pseudo-random signal in avoiding leakage in analysis if a rectangular weighting function is applied on the signal to match with the block length. Again due to its periodic nature, periodic impulse excitation is not suitable to be used on non-linear systems. The high crest factor, ratio of peak to RMS level, means that the system might be excited by high amplitude levels beyond its range of linear behaviour.

5.3.4 Periodic Random Excitation

The periodic random excitation as shown in Figure 5.16 is specially designed to combine the advantages of random and pseudo-random signals. It is made up of several independent pseudo-random sequences of length T which repeat themselves a couple of times. The first two periods are for the transient response of the system after changing sequence. The last period is then used for steady state analysis. There will be no leakage in analysis if a rectangular window is used and the block length matches with the period T . It will also give the best linear estimate of the system, as the amplitudes and phases of the sequences are independent. The only disadvantage is that it is time-consuming since the system response has to be tailored to extract the steady state vibration.

5.3.5 Sinusoidal Excitation

Sinusoidal excitation has been widely used in structural testing for many years. Sinewaves can either be stepped or swept through the range of frequency interest. Davis and Dunn³⁷ used stepped sinewaves for pile testing in the 1970's. A constant vibration level can be maintained by making use of the output level in a servo loop to control the input level to the exciter. The advantages of this method of excitation are well controlled input level, high signal to noise level, and low crest factor. Leakage in analysis can be avoided if a rectangular weighting function is matched to complete periods of the sinewave. The main disadvantages are that non-linearities may be excited by periodic excitation and it is slow compared to other methods where all the frequencies are excited and analysed simultaneously.

5.3.6 Impact Excitation

Impact Excitation, which is sometimes referred to as the Impulse or Tap method, is a very popular and convenient transient excitation technique. It deserves particular attention because, for a wide range of structures, it is the simplest and fastest technique for obtaining good estimates of the required frequency response function. When an impulse is applied to a structure by an impact hammer, energy is transferred to the structure in a very short period of time. A typical input force signal is shown in Figure 5.17. The shape of the force signal depends on a number of factors such as hammer tip stiffness, mass of hammer, dynamic properties of the testing structure and the impact velocity. The bandwidth of the force spectrum depends on the time length of the signal, which in turn depends on the contact time between the impact hammer and the structure. As a general rule, the shorter the contact time, the higher the cut-off frequency of the signal, i.e wider bandwidth of the force spectrum. Impact excitation will be investigated in more detail in Chapter 6. Leakage in analyses can be reduced if a correct weighting function is used. The transient window is most suitable and it has the advantage in improving the signal to noise level. However, impact excitation is not suitable for non-linear systems due to its high crest factor. The advantages of using impact excitation are:

1. It is very fast, only a few averages are needed.
2. Minimum preparation work compare with other methods using a shaker as no elaborate fixing of a shaker is required.
3. No heavy power generator is needed to supply power to the impact hammer.
4. Easy to use in the field as no signal generator and amplifier

are needed.

5.3.7 Random Impact Excitation

Random impact excitation as shown in Figure 5.18 is specially designed to combine the advantages of random and impact excitation. Impacts are randomly spaced in each time record. It allows a small impact hammer to input more energy into a large structure and hence speeds up the modal test. Only a few averages are required for a frequency response analysis. According to Sohaney and Nieters⁸³ a Hanning window should be used to avoid truncating the signal at the end of the block length to reduce leakage in the frequency domain. Better control especially in the impact strength is necessary in order to avoid one overshooting impact spoiling the whole time record.

5.3.8 Summary Of Excitation Methods And Recommendations For Pile Testing

Herlufsen⁸⁴ has produced a table comparing advantages and disadvantages of the first six excitation methods. The table is reproduced in Table 5.2 with comments on random impact excitation added in.

TYPE OF SIGNAL	LEAKAGE IN ANALYSIS	BEST LINEAR FIT OF NON-LINEAR SYSTEM	CREST FACTOR	SIGNAL TO NOISE RATIO	CONTROL OF EXCITATION BANDWIDTH	SPEED
Random	yes	yes	medium	fair	good	fast
Pseudo-random	no	no	medium	good	good	very fast
Periodic impulse	depends on weighting functions	no	high	poor	limited (no zoom)	very fast
Periodic random	no	yes	medium	good	good	slower than random
Sine	can be avoided	no	low	good	good	slow
Impact	depends on weighting functions	no	high	poor	limited (no zoom)	very fast
Random impact	depends on weighting functions	no	high	poor	fair	very fast

Table 5.2

Although there are a number of excitation methods, the pile testing industry so far has experience with only two of the methods — the sinusoidal excitation and the impact excitation methods. In the early 1970's, the sinusoidal excitation method was used by C.E.B.T.P. for vibration testing of piles⁸⁵. The test method and the equipment used has been introduced in Chapter 3. The drawbacks of this type of excitation for pile testing are:

1. Bulky and heavy instruments, such as an electrodynamic exciter, sine-wave generator, amplifier and signal controlling or monitoring units, are required. Testing may be slowed down due to shifting of instruments around a site.
2. Preparation work is needed to trim a pile head to level and to attach an exciter onto it using epoxy resin.

3. The test is carried out slowly as a pile has to be excited at predetermined individual frequencies within the range of interest.
4. In order to speed up a test, the frequency interval between individual excitation frequencies can be increased. However, this may lead to a very poor resolution and thus an unacceptable tolerance in the prediction of the pile length. Using the instrumentation system reported in Chapter 6 and the impact excitation technique, the frequency resolution can be limited to as low as 2Hz for a frequency range of up to 1.6 KHz..

In 1979 C.E.B.T.P. changed their excitation method to impact excitation⁵². A load cell was fixed to the pile head and an impact signal was generated by striking it with a hand-held hammer. This is not the best arrangement for impact testing as time will be wasted fixing the load cell to the pile head. A better technique is to use an instrumented hammer which basically is a hand-held hammer with built-in electronics and a load cell to monitor the impact signal. The impact characteristics of two instrumented hammers will be tested in Chapter 6.

The choice of excitation techniques for pile testing should be considered in three areas. Firstly, the method must be capable of exciting a pile so that its integrity can be revealed by the structural response. Secondly, the instruments used must be robust and portable in order to facilitate testing in a rough environment. Bulky or heavy instruments should be avoided. Thirdly, the speed of testing a pile must be fast in order that as many piles as possible can be tested in one visit to the site.

Technically, the best excitation method for pile testing is periodic

random excitation, as suggested by Table 5.2. However, since an electrodynamic exciter is required for this method, it suffers from most of the drawbacks as in the sinusoidal excitation method. Using such a method of excitation, the testing cannot proceed fast enough for the pile testing industry to justify its use.

The impact and the random impact excitation methods are recommended for pile testing. The advantages of these methods have been discussed in sections 5.3.6 and 5.3.7. The experimental work and the site pile testing to be reported in Chapter 8 and Chapter 9 are based on impact excitation. More work should be done on random impact excitation to further develop the method for pile testing. This technique may be found useful in the testing of heavily damped piles in the sense that more energy can be input to a pile by multiple impacts.

5.4 ANALYSIS TECHNIQUES

There are a number of signal processing techniques which can be used for the analysis of structural response due to different excitation methods. Generally, these techniques can be classified either as time domain analysis or frequency domain analysis. Basically, time domain analysis yields information about pulse reflection while frequency domain analysis gives information about the vibrational response of the structure.

5.4.1 Time History

Inspection of time history traces was the fundamental approach to the sonic-echo method of pile testing in the 1970's. A photograph of the response of a pile head caused by a hammer blow was usually taken from an oscilloscope. This type of instrumentation prevented a detailed analysis being

carried out on the integrity of the piles. A trace was usually inspected for any sign of a reflection pulse and the reflection time was estimated from the photograph according to its scale. Today, time history inspection is often used as the first step in signal analysis. Apart from identification of the reflection pulse, it can also yield information on the quality of the site test and decisions can be made up on whether the trace should be rejected or accepted for further analysis.

5.4.2 Enhanced Time History

The idea of Enhanced Time History is to examine and average a signal, which is contaminated by random noise or interference, many times in order to reduce the noise level whilst the true signal is enhanced. An essential requirement is that the signal must be repeatable, and that there must be a triggering device which can accurately synchronize the commencement of the signal in time. According to Hamilton⁸⁶, if the noise is truly random, the signal to noise ratio improves as the square root of the number of repetitions. If a signal is repeated 25 times the improvement will be by a factor of 5. Enhanced Time History is useful in reducing noise and surface wave interference in the sonic–echo method. It is argued that the echo will be reflected at the same time regardless of the position of the transducer on the pile head whilst surface oscillations will be picked up out of phase depending on the transducer position and the point of impact excitation. Therefore signal averaging can be applied to a number of signals obtained either by varying the transducer position or the impact position. Experimental work on the use of enhanced time history has been reported in Chapter 3.

5.4.3 Signal Filtering

Signal filtering is a major step to reduce the interference of noise and surface waves on a time trace so that an echo can be more easily identified. This can be done either by an analog or digital filter, depending upon the instrumentation system. If digital filtering is used, an analog signal has to be converted into a digital signal and then transformed into the frequency domain. The unwanted frequencies are removed to produce a filtered spectrum which is subsequently transformed back to the time domain. It is good practice to record an analog signal without filtering since filtering may remove useful information as well as noise. Filtering can be applied upon the signal after it has been inspected. Experimental work on signal filtering has been reported in Chapter 3.

5.4.4 Impulse Response Function

Figure 5.19 shows a linear system which outputs a signal $b(t)$ when it is excited by an input signal $a(t)$. The relationship between the output and input can be expressed as:^{84,85}

$$\begin{aligned} b(t) &= h(t)*a(t) \\ &= \int_{-\infty}^{\infty} h(\tau)a(t-\tau)d\tau \end{aligned} \quad (5.4)$$

where $b(t)$ is the convolution of $h(t)$ and $a(t)$. The impulse response function $h(t)$ describes the system in the time domain. Physically the impulse response function can be regarded as the response signal from the system caused by a unit impulse input signal at time zero. It can be used for the identification of reflection pulses for pile testing. If the input pulse is not dispersed by the pile, reflection pulses will show up as narrow peaks in the impulse response

function at points corresponding to the delay times.

5.4.5 Auto-Correlation Function

Often when analyzing a signal it is useful to know the general dependence of the values of the signal at one instant in time to the values at another instant in time. This is particularly useful in estimating the delay time of an echo. The Auto-Correlation Function may be used for this purpose. The mathematical definition of Auto-Correlation Function of a signal $a(t)$ is as follows:^{84,85,87}

$$R_{aa}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T a(t)a(t+\tau)dt \quad (5.5)$$

where τ is a time delay of the signal. The practical interpretation of Auto-Correlation Function $R_{aa}(t)$ is to what degree the time signal $a(t)$ is similar to a displaced version of itself as a function of the time displacement t . The best correlation will be at $t=0$, i.e no time delay, when the signal resembles itself. Sometimes it is desirable to normalize the Auto-Correlation Function with respect to the best correlation $R_{aa}(0)$. When this is done $\rho_{aa}(\tau)$ is called the Auto-Correlation Coefficient Function where

$$\rho_{aa}(\tau) = \frac{R_{aa}(\tau)}{R_{aa}(0)} \quad (5.6)$$

If the signal is a broadband signal and an echo with a time delay of τ_0 exists in the signal, the auto-correlation function will peak at $\tau=\tau_0$ (apart from at $\tau=0$) and the value of the Auto-Correlation Coefficient Function at τ_0 , $\rho_{aa}(\tau_0)$, will give a measure of the relative strength of the echo.

5.4.6 Cross-Correlation Function

The Cross-Correlation Function can be used to describe the dependence of one signal upon another. The mathematical definition is very similar to that of the Auto-Correlation Function as:^{84,85,87}

$$R_{ab}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T a(t)b(t+\tau)dt \quad (5.7)$$

where τ is the time delay. It can be seen from equation (5.5) that the Cross-Correlation Function $R_{ab}(\tau)$ gives a measure of the degree of similarity between the time signal $a(t)$ and a displaced version of another time signal $b(t)$ as a function of the time displacement τ . If the input $a(t)$ and output $b(t)$ of a system are broadband and the propagation is non-dispersive, then the Cross-Correlation Function $R_{ab}(\tau)$ will peak at a time delay corresponding to an echo, if such an echo exists in the output signal $b(t)$. Excellent discussions and examples on the identification of propagation path can be found in Bendat and Piersol⁸⁷.

5.4.7 Cepstrum Analysis

Cepstrum Analysis is a relatively new technique for decomposing a composite signal of unknown multiple wavelets overlapping in time. It can be used for signal extraction, echo detection and removal of a convolution-type system. Research on cepstrum analysis has been conducted in the last twenty years^{88,89,90,91,92}. There are mainly two types of cepstra — power cepstrum and complex cepstrum. The power cepstrum of a signal is the power spectrum of the logarithm of the power spectrum of that signal. The complex cepstrum of a signal is defined as the inverse Fourier Transform of the logarithm of the Fourier transform of that signal. Note that both processes transform a time

signal into a frequency spectrum and then back to a spectrum in the time domain. Hence a cepstrum is a time domain analysis technique. According to Kemerait and Childers⁸⁹, power cepstrum is most efficient in echo detection whilst complex cepstrum is invaluable in signal extraction and signal deconvolution — removal of echoes from a signal. Here only power cepstrum will be discussed since it is more appropriate for ^{the}sonic-echo pile testing method. A block diagram of power cepstrum processing is shown in Figure 5.20.

In order to explain the ability of power cepstrum in echo detection, an example of a single additive echo as in Figure 5.21 will be examined. The output signal can be expressed as the result of convolution of the input signal $a(t)$ with the system impulse response function $h(t)$, i.e $b(t)=a(t)*h(t)$. In this particular case only one echo is involved, the output signal can be written as follows:

$$b(t) = a(t)+ra(t-\tau) \quad (5.8)$$

The power spectrum of $b(t)$ can be expressed as:

$$F_b(\omega) = F_a(\omega)(1+r^2+2r\cos\omega\tau) \quad (5.9)$$

Applying the logarithmic function on both sides of equation (5.9) gives:

$$\log F_b(\omega) = \log F_a(\omega)+\log(1+r^2+2r\cos\omega\tau) \quad (5.10)$$

This is the major procedure in power cepstrum analysis in order to separate the echo effect from the input signal power spectrum $F_a(\omega)$. It is important to introduce the log series expansion as follows:

$$\log(1+x) = x-x^2/2+x^3/3-x^4/4+..... \quad -1 < x \leq 1 \quad (5.11)$$

If the reflection coefficient r is much less than unity, the second term in equation (5.10) can be approximated by $2r\cos\omega\tau$. Thus, the log power spectrum of the output signal is made up of the log power spectrum of the input signal with a nearly cosinusoidal ripple $2r\cos\omega\tau$. The ripple will usually be obscured by irregularities in the log power spectrum itself. This situation is similar to a periodic signal embedded in noise. It is common to use the power spectrum to detect the periodic phenomena. The power spectrum of the log power spectrum, i.e. power cepstrum will have a peak at quefreny corresponding to the echo arrival time. Quefreny in the time domain may be thought as the counterpart of frequency in the frequency domain.

5.4.8 Spectrum Analysis

There are two types of spectrum — instant spectrum and auto-spectrum. Instant spectrum is the result of the Fourier Transform of a single record of a time signal. If there are a few records of the time signal, averaging can be performed on their spectra to obtain an auto-spectrum. The advantage of auto-spectrum over instant spectrum is on the improvement of signal to noise ratio due to its averaging effect. The use of spectrum for structural response analysis is to identify the resonance of the structure when it is excited. This information may not be easily recognized in a complicated time trace.

5.4.9 Liftered Spectrum Analysis

Liftered spectrum may be regarded as an edited version of a spectrum. Liftering in the time domain has the same purpose as filtering in the frequency domain. Unwanted parts of a signal are liftered in the time domain and a spectrum is reformed from what remains. Liftered spectrum has to be used in

conjunction with a cepstrum for liftering purpose. The primary use of liftering is in connection with signals containing more than one harmonic family. Short pass and long pass lifter can be used on cepstrum to separate the harmonic families which then can be viewed separately in the Liftered Spectrum.

5.4.10 Frequency Response Function

Consider the ideal system shown in Figure 5.22 with input, output and impulse response function as $a(t)$, $b(t)$ and $h(\tau)$ respectively. According to section 5.4.4, the output signal can be written in terms of convolution as $b(t)=h(\tau)*a(t)$. A similar relationship can also be written for the system in the frequency domain by making use of the Convolution Theorem, which states that a convolution process in one domain is equivalent to the multiplication process in the other domain. Hence, here the output $B(f)$ and input $A(f)$ in the time domain is related by the following expression as:

$$B(f) = H(f) \cdot A(f) \quad (5.12)$$

where $H(f)$ is referred to as the Frequency Response Function. Alternatively, the Frequency Response Function $H(f)$ can be regarded as the Fourier Transform of the Impulse Response Function $h(\tau)$. Both $h(\tau)$ and $H(f)$ describe the system characteristics but in different domains. As mentioned before, no information will be gained or lost in transforming from one domain to the other. However, interpretation of the Frequency Response Function is easier for the understanding of the structure's dynamic characteristics.

If the input and output signals are measured during a structural test, the system's Frequency Response Function can be calculated as:

$$H(f) = \frac{B(f)}{A(f)}$$

$$\text{or } H(f) = \frac{G_{AB}(f)}{G_{AA}(f)} \quad (5.13)$$

where $G_{AB}(f) = \hat{A}(f) \cdot B(f)$
 = cross-spectrum of the input and output signals

$G_{AA}(f) = \hat{A}(f) \cdot A(f)$
 = auto-spectrum of the input signal

$\hat{} = \text{complex conjugate}$

Equation (5.13) gives the Frequency Response Function for an ideal system, i.e. one without noise contamination. In a practical situation, presence of noise in the signals will make equation (5.13) unsuitable for the calculation of Frequency Response Function. Elliott and Mitchell^{93,94} suggested two different ways of calculation of Frequency Response Function according to whether the noise contamination is in the input or output signal. Consider the system shown in Figure 5.23 with true input and output signals $u(t)$ and $v(t)$ which are contaminated with uncorrelated noise $n(t)$ and $m(t)$. The measured input signal will be $a(t)=u(t)+n(t)$ whilst the measured output signal will be $b(t)=v(t)+m(t)$. The true Frequency Response Function of the system is:

$$H(f) = \frac{V(f)}{U(f)} = \frac{G_{uv}}{G_{uu}} = \frac{G_{vv}}{G_{vu}} \quad (5.14)$$

This equation cannot be computed, however, because the measured signals are $a(t)$ and $b(t)$ instead of $u(t)$ and $v(t)$. Any methods of calculation must make use of the measured signals. The first suggested method is:

$$H_1(f) = \frac{G_{AB}}{G_{AA}} \quad (5.15)$$

It can be shown that G_{AB} equals to G_{uv} since the noise is uncorrelated with the signals, and the autospectrum G_{AA} equals to the sum of G_{uu} and G_{mm} . Equation (5.15) can be written as:

$$H_1 = \frac{G_{uv}}{G_{uu}+G_{mm}} = \frac{H(f)}{1+G_{mm}/G_{uu}} \quad (5.16)$$

This shows that $H_1(f)$ calculation gives the true $H(f)$ if there is no noise in the input, i.e. $G_{mm}=0$. The second method of calculation is:

$$H_2(f) = \frac{G_{BB}}{G_{BA}} \quad (5.17)$$

Again using the same argument for $H_1(f)$, equation (5.17) can be written as:

$$\begin{aligned} H_2(f) &= [G_{vv}+G_{nn}]/G_{vu} \\ &= H(f)+G_{nn}/G_{vu} \\ &= H(f)+[G_{vv}/G_{vu}] \cdot [G_{nn}/G_{vv}] \\ &= H(f)(1+G_{nn}/G_{vv}) \end{aligned} \quad (5.18)$$

This shows that $H_2(f)$ calculation gives the true $H(f)$ if $G_{nn}=0$, i.e. no output noise. For a system with noise in both measured input and output signals, it is clear from equations (5.16) and (5.18) that $|H_1(f)|$ will underestimate $|H(f)|$ due to noise at the input and that $|H_2(f)|$ will overestimate $|H(f)|$ due to the noise at output. Therefore $|H_1(f)|$ and $|H_2(f)|$ serve as lower and the upper bounds for the true Frequency Response Function $|H(f)|$ as:

$$|H_1(f)| \leq |H(f)| \leq |H_2(f)| \quad (5.19)$$

A major advantage of using the Frequency Response Function is that it can be used to check the linearity of a system. This is done by the Coherence

Function $\gamma^2(f)$ of the signals $a(t)$ and $b(t)$ which on a scale from 0 to 1 measures the degree of linear relationship between two signals at any given frequency f . Its mathematical definition is:

$$\begin{aligned}\gamma^2(f) &= H_1(f)/H_2(f) \\ &= 1/[(1+G_{mm}/G_{uu}) \cdot (1+G_{nn}/G_{vv})]\end{aligned}\quad (5.20)$$

The Coherence Function can detect the presence of uncorrelated noise, but it cannot distinguish between input and output noise. As can be seen from equation (5.19), if the lower bound and upper bound are very close to the true Frequency Response Function, then the Coherence Function approaches unity. Coherence is poor if $|H_1(f)|$ is very different from $|H_2(f)|$. Using the Coherence Function, Bendat and Piersol⁸⁷ were able to identify the sources of errors in vibration experiments performed on a cantilever beam and a panel structure.

Structural frequency response testing, also known as modal analysis, is the application of the Frequency Response Function on structural testing to determine the resonance characteristics of the structure. Usually one of the excitation methods discussed in section 5.3 will be used. Input signal in terms of force is measured and the output signal (structural response) in terms of motion (acceleration, velocity or displacement) is also measured. The type of motion measured depends on the type of transducer being used. Table 5.3 shows different forms of Frequency Response Function according to the type of motion measured.

FREQUENCY RESPONSE FUNCTION (H_1 or H_2)	SPECIFIC NAME OF TESTING
$\frac{\text{Acceleration}}{\text{Force}}$	inertance
$\frac{\text{Velocity}}{\text{Force}}$	mobility or mechanical admittance
$\frac{\text{Displacement}}{\text{Force}}$	dynamic compliance
$\frac{\text{Force}}{\text{Acceleration}}$	dynamic mass
$\frac{\text{Force}}{\text{Velocity}}$	mechanical impedance
$\frac{\text{Force}}{\text{Displacement}}$	dynamic stiffness

Table 5.3

5.4.11 Analysis Techniques Adopted In This Project

The objective of this project is to develop a pile testing method which makes use of the advantages of analysing a test result in both domains. Analysis techniques adopted in this project therefore involve the various techniques described earlier in both domains.

In the time domain analysis, a time record is firstly checked for the quality of the site test. Signal averaging is then performed to suppress any spurious noise. The enhanced time record is inspected for any reflection signals. Finally, cross-correlation or auto-correlation is used to confirm the time position of the echo. Since a very small time interval ($10\mu\text{s}$ - $20\mu\text{s}$) is normally used for sampling in order to reveal fine details in a signal, an echo peak may spread out to such an extent that the signal can no longer be classified as a broadband signal. Although this is not the best situation to use the correlation functions, the reflection time of an echo can still be estimated. However, care must be taken not to mis-interpret every multiple reflection as

an individual echo. Cross-correlation is to be preferred as the interpretation is more straightforward whilst auto-correlation may be affected by multiple reflections in the signal. Signal filtering is seldom used as high frequency surface oscillations can be successfully avoided by low frequency excitation.

In the frequency domain analysis, the frequency response function (in terms of mechanical admittance) is firstly inspected for any regular frequency interval. The dynamic stiffness is calculated from the initial slope of the curve. Auto-spectrum and the coherence function are used to check the quality of the site test.

If there is any query in the interpretation in any one of the domains, either due to multiple reflections or vibration coupling, then cepstrum and liftered spectrum analyses should be used.

Examples of this analysis will be given in Chapter 8 for laboratory built models and in Chapter 9 for site piles.

5.5 THE EDINBURGH APPROACH TO NON-DESTRUCTIVE PILE TESTING

Basically there are three major aspects of development, namely excitation technique, instrumentation and analysis technique. Impact excitation is adopted for its quick and almost preparation free advantage. Instrumented hammers with built-in load cell are used. The Edinburgh Instrumentation System, described in the next chapter, is designed to provide test results for analysis in both domains. A revolutionary technique, combining the advantages of both the sonic-echo and the transient shock methods, using liftered spectrum and cepstrum is the most significant development in ^{the} analysis. Examples of analysis using this new technique will be given in Chapter 8 and Chapter 9.

CHAPTER 6

INSTRUMENTATION AND DEVELOPMENT

6.1 INTRODUCTION

The rapid development of micro-processors and advances in modern dynamics measurement equipment in the last decade have allowed the instrumentation for pile testing to be constantly improved and upgraded. Recent technology in instrumentation enables quicker and more reliable tests to be performed on site and at the same time allows more sophisticated analysis to be carried out to extract information which was not possible in the past.

Preliminary studies on both the sonic-echo and the transient shock methods in Chapter 3 have suggested that both methods are feasible for pile integrity testing. There are advantages and disadvantages with both methods, however, information obtained in one method can be complementary to the other. In order to make the best use of both techniques, it is intended in the development of instrumentation systems that results from both test methods can be obtained by one system. The Edinburgh Phase I Instrumentation System described in Chapter 3 has clearly demonstrated this possibility.

6.2 GENERAL CONSIDERATION OF INSTRUMENTATION SYSTEMS

In modern dynamics measurement, it is seldom that only one piece of instrumentation is involved. On many occasions, a number of instruments are combined together to form a system in such a way that vibrational response can be transduced, converted, analysed and finally displayed⁹⁵. A generalised instrumentation system is shown in Figure 6.1. Apart from these instruments, a signal generator may also be needed to provide excitation to the structure to be tested.

In many instances, the first step in the observation of structural

response is to convert it to an equivalent electrical response. This mechanical-to-electrical conversion is called trans⁵duction and the instrument used is a transducer. The electrical output from a transducer usually has to undergo operations like amplification, windowing, filtering, differentiation or integration etc. prior to analysis. These operations may be referred to as signal conditioning. Conditioning units may include amplifiers, filters and integrators. After conditioning, the electrical waveform will be suitable for analysis. This may involve one or more of the techniques described in Chapter 5 in order to extract information about the structure. Analysis can be performed either by a signal analyser or a micro-computer. The final operation of the instrumentation system is to display the analysed result, usually on a screen, for interpretation.

In order to obtain reliable test results, it is important that an operator should have a sound knowledge of the properties of the instrumentation system, especially the transfer function, linearity and dynamic range of the system⁹⁵. Figure 6.2 shows an example of a system transfer function which comprises an amplitude response and a phase response. It is clear that the transfer function does not have an equal weight for all frequencies. Components of very low and very high frequencies are expected to be distorted with only components falling within the flat passband of the system unaffected. Similarly, this problem can also happen in the time domain if the system has been used outside its linear operating region. As illustrated in Figure 6.3, overloading a system causes non-linear response and hence the creation of spurious frequency components in the spectrum. This can only be avoided by making sure the system is operated within its linear range. As far as the linear range is concerned, there is only one upper limit such that the system is not overloaded. However, any instrumentation system is bound to be affected by inherent electrical noise and any weak input signal will certainly be

contaminated. The dynamic range of a system is therefore determined by the maximum signal amplitude that can be reliably handled by the system and the inherent electrical noise floor of the system, as shown in Figure 6.4.

6.3 THE EDINBURGH PHASE II INSTRUMENTATION SYSTEM

Although the Edinburgh Phase I Instrumentation System is adequate for data acquisition and analysis, it suffers from the drawback that the whole system is too bulky and heavy to be moved around a construction site. Apart from that, the micro-computer (Hewlett Packard 85) is too slow in storing data and carrying out analysis. An American-made digital oscilloscope was adopted to take the place of the micro-computer, transient recorder and analog oscilloscope in the Phase I system. The upgraded Edinburgh Phase II Instrumentation now comprises mainly:

1. An instrumented hammer
2. An accelerometer
3. Conditioning units for hammer and accelerometer outputs
4. A digital oscilloscope

A general description of the instruments follows.

6.3.1 Instrumented Hammer⁹⁶

The hammer consists of an integral, integrated-circuit piezoelectric (ICP), quartz force sensor mounted on the striking end of the hammer head. The sensing element functions to transfer impact forces into an electrical signal for display and analysis. The ICP sensor is powered by a 27 volts D.C. power unit.

The hammer is a single, integral unit. The striking end of the hammer has a threaded hole for a variety of impact tips. Tips of different hardness allow a variation of pulse width and frequency content of the force. The hammer impulse consists of a nearly constant force over a broad frequency range and is therefore capable of exciting all resonances in that range. The amplitude and frequency content of the force impulse is determined by the hammer mass, length, material and velocity at impact. The frequency content is generally controlled by the impact cap material while the energy content is governed by the velocity at impact.

Two types of hammers (PCB Piezotronics models 0862B20 and 086B50) are available as shown in Figure 3.26. The short sledge hammer (weight 3lb.) is usually used for laboratory work and for testing shorter piles on site. For long and heavily damped piles, the large sledge hammer (weight 12lb.) may be required. Specifications and calibration certificates of the hammers are given in Appendices A.1, A.2 and A.3..

6.3.2 Accelerometer^{97,98}

A piezoelectric accelerometer is an electromechanical transducer that generates an electrical output when subjected to mechanical shock or vibration. Over a wide frequency range and dynamic range its electrical output is directly proportional to the acceleration of the vibration applied at its base.

It is important to make sure the resonant frequency of the accelerometer is not excited by confining all measurements to the linear portion of the frequency response. Broch⁹⁹ suggested that the upper frequency limit for measurements to be set to one-third of the accelerometer resonance frequency so that vibration components measured at this limit will be in error

by no more than +12% (~1dB).

When an accelerometer is used to measure shocks and transient vibrations, particular attention must be paid to the overall linearity of the system otherwise transients will be distorted. The overall linearity of the measuring system can be limited at low and high frequencies by phenomena known as zero shift and ringing respectively. These effects are shown graphically in Figure 6.5.^{98,99} Zero shift is caused by phase non-linearities in the piezoelectric element of the accelerometer retaining charge after being subjected to very high level shocks. This effect can be minimized by avoiding relatively high shocks to the accelerometer. Ringing occurs when the measured transient contains high frequency components which excite the accelerometer at its resonant frequency. To reduce this effect, i.e. keeping amplitude errors to within 5% , it is necessary to ensure that the resonant frequency is at least 10 times greater than the most dominant frequency of a pulse.

The output signal generated by a piezoelectric accelerometer is developed across an extremely high electrical impedance and is of very low power content. For this reason, some accelerometers are manufactured with built-in amplifiers. The PCB Piezotronics model 308B15 accelerometer falls into this category. Its specifications are given in Appendix A.4.

6.3.3 Conditioning Units^{100,101}

Two conditioning units are needed separately to power the instrumented hammer and the piezoelectric accelerometer.

The PCB Piezotronics model 480D06 battery power unit supplies a 27 volt D.C. power to the instrumented hammer and at the same time functions as a signal amplifier with a gain of 1, 10 or 100.

The PCB Piezotronics model 480A08 integrating power unit supplies a 18 volt D.C. power to the piezoelectric accelerometer and also functions as an integrator. It provides broad band analog output signals for either acceleration or velocity. However, particular attention has to be paid to the low frequency components of a signal since all electronic integrators are limited by a certain low frequency cut-off, below which no integration takes place. Figure 6.6⁹⁹ illustrates signal distortion after single and double integration. Specifications of the two units are given in Appendices A.5 and A.6.

6.3.4 Digital Oscilloscope¹⁰²

The Nicolet 4094 digital oscilloscope (see Figure 6.7) performs several functions, including data acquisition, storing data, analysing and displaying the results on a screen. As mentioned before, it takes the place of a micro-computer, a transient recorder and an analog oscilloscope. The improved portability allows more piles to be tested on a site.

6.3.4.1 Data acquisition

The oscilloscope has a data memory capacity of about 16k, in other words the machine is capable of handling 15872 data points. This compares favourably with the Phase I instrumentation of 2k memory (2048 words). If two-channel sampling is required, one for the force impulse and another for structural response, then the total memory capacity of the oscilloscope is equally shared between the two signals with each one having a capacity of 8k (~7936 words).

The intervals between sample points can be selected to suit the method of pile testing, a faster sampling rate (10 μ s per point) for sonic-echo method and a slower rate (100 μ s per point) for transient shock method. Hence,

a pile is tested at least twice with different sampling rates to cover the two test methods.

The dynamic range of the oscilloscope is 12 bit, that is the ratio between the smallest and the largest components in a sampled trace can be as much as 1 to 4096. Theoretically, this means that if the dynamic range is fully utilized an echo signal with amplitude as low as $1/4096$ of the impulse signal can still be distinguished from the rest of the trace. This represents a significant improvement of a factor of 16 in the vertical resolution when compared with the previous instrumentation with a dynamic range of 8 bit. Practically, the dynamic range is affected by the inherent noise floor of the instrument as mentioned earlier in section 6.2.

6.3.4.2 Storing data

Using the built-in disk recorders, data can be stored permanently on floppy diskettes. Up to forty 8k records can be stored on one of the two diskettes, ready for instant recall and display.

The increased speed and disk storage of the digital oscilloscope compared favourably with the tape storage of the HP85 in the previous instrumentation. This ensures that pile tests can be carried out much faster.

6.3.4.3 Analysis

A data manipulation program disk and a waveform analysis program disk are supplied with the digital oscilloscope. Functions such as integration, division, Fast Fourier Transform, Inverse Fast Fourier Transform and windowing etc. can be performed on the signals. This enables an acceleration trace to be integrated to velocity trace for sonic-echo interpretation and mechanical

admittance computation from force and velocity signals.

6.3.4.4 Displaying

Signal traces and analysis results, for example mechanical admittance, can be displayed on a 5 inch screen with a vertical or horizontal expansion up to 256 times. A number of traces can be displayed at the same time for comparison. A hard copy of any trace can be obtained by plotting the data on a digital plotter connected to the oscilloscope.

6.4 THE EDINBURGH PHASE III INSTRUMENTATION SYSTEM

After a period of experimentation with the Nicolet digital oscilloscope, it was found that the instrument was excellent for the sonic-echo analysis but less convenient for the more complicated transient shock analysis. Sonic-echo traces can be analysed by using the front panel pushbutton controlled built-in functions. For the transient shock analysis, a number of programs have to be called up from the diskettes and hence slow down interpretation. Apart from that the digital oscilloscope is heavy and relatively delicate for use on site. Additionally, a heavy generator is needed to power the oscilloscope and there is danger that the noise created by the generator may cause interference with the vibration signals. In essence the Nicolet digital oscilloscope is a very high resolution time domain analyser with specific application to problems where very long transient responses are analysed. For example, sonic testing of stone masonry bridges.

A modification of the existing instrumentation system with special attention paid to portability and speed of analysis became necessary. The Edinburgh Phase III instrumentation system is considered to be a solution, The system shown in Figure 6.8 consists of:

1. Instrumented hammer
2. Accelerometer with built-in preamplifier
3. Power and conditioning units
4. Analog frequency modulated (FM) tape recorder
5. Real-time signal analyser (Bruel and Kjaer 2034)
6. Desktop computer (Hewlett Packard 9816)
7. Peripheral devices such as printers and plotters.

In view of the increasing number of instruments used for pile testing, an NDT laboratory was set up at the University of Edinburgh. When site testing is required, items 1-4 of the instrumentation system will be taken out to site with the remaining items left behind in the laboratory. All the instruments used on site are relatively light-weight and are powered by batteries. The tape recorder has a built-in rechargeable battery which can be used continuously for up to 5 hours. With this arrangement portability is no longer a problem for site testing.

The analog tape recorder¹⁰³ (Data Acquisition type DA 1444-4-4), runs on high quality laboratory standard C90 cassettes. The frequency range of the recorder is from DC up to 10KHz if it is run at a speed of 38cm/s. A switched meter is provided to monitor input or replay signal levels for each channel. The instrument is supplied with standard plug-in modules whose sensitivity switches allow signal levels to be attenuated or amplified. Once the signal levels for both the force impulse and the dynamic response have been adjusted to an acceptable level, pile testing can proceed quickly. The tape recorder being analog has the advantage that one test is sufficient for both the

sonic-echo and the transient shock methods. The analog signals can be replayed and digitized with an appropriate sampling rate for the two methods by the signal analyser.

The Bruel and Kjaer type 2034 real-time signal analyser¹⁰⁴ is the main feature of the whole instrumentation system. This two-channel FFT analyser based on a dual processor architecture enabling it to perform complex signal processing in real-time. Figure 6.9 shows the operation and some functions available in the analyser. Most signal processing functions described in Chapter 5 can be carried out in a fraction of a second, for example, a mechanical admittance plot can be obtained on the screen at the speed of pushing a button. The analyser has a frequency range of DC up to 25.6KHz with a resolution of 801 lines. The dynamic range is 80dB, equivalent to a ratio of 1 to 10000, which is over twice that of the Nicolet 4096 digital oscilloscope. Up to 20 measurement and display set-ups can be stored in a non-volatile memory. Any of the different time domain and frequency domain functions can be displayed on a 12 inch screen which is clearly an advantage when compared with the 5 inch screens of other analysers and oscilloscopes. Alternatively, two functions can be displayed, one on the upper part and the other on the lower part of the screen. Functions displayed on the screen can be down loaded to the micro-computer (HP9816) through an IEEE interface. Hard copy of any functions can be obtained from a plotter connected either to the analyser or to the micro-computer.

6.5 CALIBRATION OF THE SYSTEM

As far as the sonic-echo method is concerned, system calibration is not necessary since interpretation of the sonic-echo test is based on relative amplitudes rather than absolute values. The identification of an echo signal and

the measurement of its corresponding reflection time are the two most important features of the method. However, an accurate system calibration is absolutely essential to the transient shock method if a meaningful value of dynamic stiffness is required. The force impulse and the structural response both have to be calibrated. As mentioned earlier in this Chapter, these two dynamic quantities are inevitably transduced to electrical quantities. Calibration as an inversion process is needed to convert the electrical quantities back to dynamic quantities. Basically, there are two approaches to calibration — theoretical and experimental.

6.5.1 Theoretical Calibration

Most vibration instruments have to undergo a series of rigorous calibration tests just before delivery and the test results in ^{the} form of charts, diagrams or statements will be supplied with a specification document to the purchaser. If several items are included in an instrumentation system, then theoretical calibration can be carried out by following the individual calibration test result or specification of each instrument. The theoretical calibration of the Edinburgh Phase III instrumentation system will be considered. Figure 6.10 outlines the instrumentation set up and the sensitivity of each instrument provided by the manufacturers.

If the system calibration is calculated from these sensitivities for both channels, then the calibration values can be input to the calibration entry of the measurement set up field of the Bruel and Kjaer signal analyser. This in turn will give out the correct dynamic quantities on any signal traces displayed on the screen.

As an example of theoretical calibration, assume the gains on the

conditioning units and the tape recorder to be unity, that is to say the signal level is not altered when it is recorded. The sensitivity of the hammer is given as 0.18 mV per Newton and it will remain unchanged throughout the system. Therefore the calibration of channel A is 0.18 mV per Newton. Using the same argument the calibration of channel B is given by the sensitivity of the accelerometer as 10.19 mV per m/s^2 , provided the acceleration signal is not integrated. When the signal is integrated to velocity the sensitivity of the accelerometer conditioning unit has to be used as the calibration of channel B. In this case, the value is 39.37 V per m/s.

The above calibration is based on a unit gain throughout the testing procedures. In the case of signal level being altered by a gain factor other than unity the calibration value has to be adjusted correspondingly in order to bring the signal back to its original level.

6.5.2 Experimental Calibration

If a facility is available to produce a known dynamic output, experiments can be performed to provide calibration values of a system. This can be done by subjecting the system to a known input and measure the corresponding output. A calibration value is obtained by dividing the output (electrical quantity) by the known input (dynamic quantity). An example of the experimental calibration of the Phase III instrumentation should clarify this point.

6.5.2.1 Structural response calibration

This was carried out with the help of a Bruel and Kjaer type 4294 Calibration Exciter¹⁰⁵(see Figure 6.11). It is a pocket-size, completely self-contained vibration reference source, intended for rapid calibration and

checking of piezoelectric accelerometers. The calibrator embodies an electromagnetic exciter driven by a stabilised oscillator at a frequency of 159.2 Hz (1000rad^{-1}). Servo feedback via a small accelerometer on the underside of the vibration table is used to maintain a constant vibration level of 10 ms^{-2} (rms 14.14 ms^{-2} peak) $\pm 3\%$. This reference signal may additionally be used for velocity calibration of 10 mms^{-1} (rms 14.14 mms^{-1} peak) $\pm 4\%$.

To calibrate the system experimentally, the accelerometer was screwed onto the exciter and the signal thus generated was fed to the conditioning unit, the tape recorder. Finally a sinusoidal wave of amplitude 0.148 Volts was displayed on the analyser screen (see Figure 6.12(a)). The acceleration calibration of the system can be obtained as:

$$\begin{aligned}\frac{\text{output}}{\text{input}} &= \frac{0.148\text{ volts}}{14.14\text{ ms}^{-2}} \\ &= 10.47\text{ mv per ms}^{-2}\end{aligned}\tag{6.1}$$

If this value is compared with the theoretical one ($10.19\text{ mv per ms}^{-2}$), a difference of 2.75% is obtained, which falls within the error bound ($+3\%$) of the calibration exciter.

The procedure was then repeated in exactly the same manner as above except that the acceleration signal was integrated for velocity calibration. A sinusoidal wave of amplitude 0.543 volt was obtained as shown in Figure 6.12(b). The velocity calibration of the system is therefore:

$$\begin{aligned}\frac{\text{output}}{\text{input}} &= \frac{0.543\text{ volts}}{14.14\text{ mms}^{-1}} \\ &= 38.4\text{ v per ms}^{-1}\end{aligned}\tag{6.2}$$

This compares favourably with the theoretical value of 39.37 v per m/s, that is a difference of 2.46% which again falls within the error bound ($\pm 4\%$) of the calibration excitor.

6.5.2.2 Force excitation calibration

The experimental calibration of the instrumented hammer is not as simple as the accelerometer calibration because of the difficulties in getting a known force input to the hammer. The calibration is further complicated by the fact that the force measured by the force transducer is not the true force which is input to the structure. The force actually measured during an impact is the force across the load cell. The force input to the structure, on the other hand, is the force between the hammer tip and the test structure⁸⁴ (see Figure 6.13). What happens on impact is that the force developed at the hammer/structure interface brakes the entire hammer mass whilst the crystal element experiences only a portion of the actual impact force, about 70 to 95 percent depending on the hammer configuration, which stops the mass of the head and extender. It does not sense the component of force related to the impact cap and seismic mass.

Referring to Figure 6.13, several equations regarding the dynamic behaviour of different parts of the system can be written as:

$$F_a = M_s a_s \quad (6.3)$$

$$F_a - F_m = M_t a_h \quad (6.4)$$

$$F_m = (M_h - M_t) a_h \quad (6.5)$$

where F_a = actual force input to structure
 F_m = measured force
 M_h = effective mass of hammer plus tip
 M_t = effective mass of tip
 M_s = effective mass of structure
 a_s = acceleration of structure

a_h = acceleration of hammer

The effective mass takes into account the material characteristics of the tip and the hammer. It is defined as a rigid mass which would generate the same force when acted upon by the same linear acceleration⁸². Eliminating a_h from equations (6.4) and (6.5) and solving gives the ratio of the actual force to the measured force as:

$$\frac{F_a}{F_m} = \frac{M_h}{M_h - M_t} \quad (6.6)$$

This expression is exactly the same given by Halvorsen and Brown⁸².

The manufacturer of the instrumented hammer has suggested two methods of calibration⁹⁶. The first method involves suspending a known mass in air, striking it through the centre of gravity with the hammer and measuring the acceleration of the mass by attaching an accelerometer onto it. The second method is quite similar to the first one in principle but instead of striking the mass with the hammer a known mass is allowed to drop vertically onto a fixed stationary hammer. Due to the mass loading effect of the crystal described above, the first method (referred to as the moving hammer method) gives a different calibration constant for each hammer configuration. The second method (referred to as the moving mass method), on the other hand, checks the factory calibration of the transducer, since in this process the seismic mass and cap assembly does not move appreciably.

In view of the simplicity of the moving hammer method, it was adopted for routine calibration and performance testing of the instrumented hammer. The procedure involved striking a pendulous mass with the hammer and measuring the acceleration of the mass with an attached accelerometer.

Both force and acceleration signals are then passed unchanged via the conditioning units to the signal analyser, of which the calibration field of channel A (force input) is set to 1 volt per volt and that of channel B (acceleration input) is set according to the sensitivity of the accelerometer. Since the acceleration of the pendulous mass is known from channel B, the actual impact force can be calculated from the well-known equation $F=ma$ and the calibration constant of the force excitation can be obtained by dividing the voltage output of channel A with the impact force F . A test has been performed on the short sledge hammer with a black plastic cap, and the result is shown in table 6.1.

(x) Channel A 1 volt/volt (mv)	(a) Channel B 10.19mv/ms ⁻² (ms ⁻²)	(y) m=3.698kg F=ma (N)	(x/y) Force calibration constant (mv/N)
213.90	343	1268.41	0.17
236.90	324	1198.15	0.20
243.80	363	1342.37	0.18
169.97	254	939.29	0.18
232.30	281	1039.14	0.22
138.23	196	724.81	0.19
70.38	106	391.99	0.18
230.00	288	1065.02	0.22
234.60	319	1179.66	0.20
194.81	247	913.41	0.21
			Ave. = 0.195

Table 6.1

Bearing in mind the simplistic limitations of the test, the average force calibration constant of 0.195 mv/N compares favourably with the

manufacturer's suggested value of 0.18 mv/N. Apart from the mass loading effect, the discrepancy might also be due to the fact that it was not always possible to strike the mass through the line of centre of gravity, hence the mass was set into rotational motion as well as translational motion. Thus the accelerometer might not have picked up the true acceleration caused solely by the translational motion.

6.6 AMPLITUDE AND SPECTRUM OF AN IMPACT FORCE

One of the most essential features of the dynamic structural response testing is the force input to the test structure since for a linear structure the response output is dependent on what is input to it. For example, the natural resonance of a structure only occurs when it is excited by a force at the natural frequency. It is therefore important to know the amplitude and spectrum of the impact force that can be generated by the instrumented hammer.

An impact signal can be obtained by striking a concrete block with the hammer. A typical impact signal, using the black plastic cap supplied, is shown along with its spectrum in Figure 6.14. The impact force extends from DC to more than 1.6 KHz, with an approximate drop of 10 dB in amplitude.

The procedure has been repeated for the small and the large instrumented hammers using different hammer tips. Figure 6.15 shows the force signals obtained with the small hammer. The autopower spectra for force signals are given separately in Figures 6.16 and 6.17 for the small and large hammers.

From the results, the following conclusions about the magnitude and the frequency content of the instrumented hammers can be drawn:

1. magnitude is directly proportional to the mass of the hammer and the impact velocity, and is inversely proportional to the tip hardness.
2. frequency content is directly proportional to the impact velocity and the tip hardness, but is inversely proportional to the impact duration and the mass of the hammer.

The useful frequency ranges of the two hammers with different tips are suggested in table 6.2. Variations in the values are expected in application due to changes in impact velocity and hardness of material to be tested.

Hammer	Short sledge hammer						Large sledge hammer			
Material of cap	metal		plastic		rubber		plastic		rubber	
	steel	aluminium	black	red	brown	grey	black	red	brown	grey
Suggested useful range (Hz)	5k	4k	1.6k	1k	0.6k	0.6k	1.2k	1k	0.6k	0.5k

Table 6.2

6.7 SOFTWARE DEVELOPMENT

The ability of the Bruel and Kjaer signal analyser to carry out real-time analysis is acheived by hardware computation. Functions can be executed from the keyboard with the results being displayed on the screen near instantaneously. Editing the functions is not possible as all operations are carried out in hardware. However, data can be transferred from the analyser to a micro- computer through IEEE interfacing, and if necessary additional analysis can be performed using computer software.

The analyser is connected to a Hewlett Packard 9816 desk-top computer with a 2.6 MByte of memory and twin 720k 3.5 inch disk drives.

Interfacing software has been supplied by Bruel and Kjaer, which allows data to be transferred and plotted on to an HP digital plotter. The program has been modified subsequently by the author of this thesis to incorporate the facilities thought to be useful in the interpretation of the sonic-echo and transient shock test results. The additional facilities are:

1. printer output
2. integration
3. computation of dynamic stiffness
4. side-band cursor

6.7.1 Printer Output

Although the original interfacing software allowed results to be plotted on an HP plotter, the process was found to be quite time-consuming. If a very high quality graph was not necessary, then a fast printing facility, which produces reasonable quality prints, was most desirable in order to reduce the time required for analysis. This could be extremely useful if a large number of piles had been tested.

For this reason, an HP Thinkjet printer was linked to the computer and a subroutine to control the printing process was added to the interfacing software. Two sizes of prints are available. A large size A4 print and a reduced size of about 1/3 A4 print can be selected by the user.

6.7.2 Integration

As the most popular transducer used for vibration measurement is an

accelerometer, an accurate integration process is essential if clearer interpretation requires to be conducted in other measurand such as velocity and displacement. It has been shown in Chapter 3 that velocity traces are by far the most favorable as far as sonic–echo interpretation is concerned whilst in the transient shock method structural response in terms of particle velocity is a requirement in obtaining mechanical admittance graphs.

Integration of a signal can be carried out in two ways — electronically or digitally. As mentioned earlier in section 6.3.3, most electronic integrators are less effective at their low frequency ranges near D.C., digital integration may be an alternative.

Mathematically a correct integration process is one that sums up the area under a curve. This definition may sometimes be insufficient for signal processing, especially in the case of a signal displaced by a DC-term. DC-shift due to slight fluctuation of electrical properties of instruments is a common problem of signal capturing. Powerful signal analyser such as the Bruel and Kjaer instrument performs a DC adjustment before integrating a signal (see Figure 6.18). Integration is therefore carried out with reference to the adjusted baseline such that there is no DC-term. This type of integration is very useful for a relatively long signal which comprises at least a few cycles of vibration. For this reason it is suggested that this type of integration is appropriate for the transient shock method. However, its use on sonic–echo traces is doubtful since most sonic–echo traces are relatively short and may comprise one or at most two echoes. For this type of signal, a DC adjustment will only distort the integrated signal. A digital integration process which sums up the area under a curve regardless of DC term has been incorporated by the author of this thesis into the Bruel and Kjaer interfacing software (see Figure 6.18).

6.7.3 Dynamic Stiffness Calculation

The dynamic stiffness of a structure can be obtained from a mechanical admittance plot as:

$$E' = 2\pi/m \quad (6.7)$$

where E' = dynamic stiffness
 m = gradient of the initial linear part of the curve

The hand calculation of dynamic stiffness from a graph may involve inspecting the initial part of the curve for any linear relationship between the mechanical admittance and frequency, fitting a straight line by a scale rule, calculating the gradient of the straight line and finally working out the dynamic stiffness by equation (6.7). This procedure may become very tedious and time-consuming if there are a large number of pile test results to be analysed.

Since the mechanical admittance function on the signal analyser can be transferred to the micro-computer, an automatic process of dynamic stiffness computation becomes possible. A subroutine has been incorporated into the interfacing software for this purpose. After asking a user to inspect the curve and input the interested frequency range, the computer will perform a least squares straight line fit to the interested range, calculate the dynamic stiffness value and display it on the screen. It is worth-mentioning that the correctness of the computed dynamic stiffness value is dependent upon the accuracy of calibration.

6.7.4 Side-Band Cursors

The signal analyser has provided the facility of fitting a series of side-band cursors onto the resonance peaks of a mechanical admittance plot.

The cursors can be spaced at a selected distance apart in order to locate the harmonic peaks. This helps the calculation of effective length of a pile by identifying the frequency interval between resonances. Unfortunately this piece of information cannot be passed from the signal analyser on to the micro-computer for plotting. A subroutine has been therefore incorporated into the interfacing software to allow side-band cursors to be plotted on the mechanical admittance function on the computer. (see Figure 6.19)

6.8 COMMENTS AND CONCLUSIONS

1. An instrumentation system, based on a portable tape recorder for signal capturing and a sophisticated signal analyser for analysis, has been developed. The Edinburgh Phase III Instrumentation System allows test results from both the sonic-echo and the transient methods to be obtained from one test.
2. Custom software for the HP 9816 micro-computer has been developed to facilitate the interpretation of test results. Probably the most useful software is for the automatic computation of the dynamic stiffness values. The computation is based on a least squares straight line fit to the initial slope of a mechanical admittance curve.
3. The T.N.O. digital signal processing techniques basically include filtering, integration and exponential amplification. These techniques are very useful for the sonic-echo analysis. However, more information relating to a pile's integrity may be obtained if advanced analysis techniques such as the correlation functions and the cepstrum analysis are used. The correlation functions and the cepstrum analysis are provided with the Edinburgh Phase III Instrumentation System and their applications will be illustrated in Chapter 8 and Chapter 9.
4. C.E.B.T.P.'s instrumentation system is based on a Hewlett Packard 3582A spectrum analyser, which is specially designed to obtain the

transfer function (frequency response function) of a system. The main advantage of this instrument for pile testing is its portability. Interpretation can be carried out on a site in order to give an immediate assessment of a pile's integrity. However, this instrument suffers from the fact that only three types of weighting function are available. They are flat-top window, Hanning window and uniform window. Strictly speaking, these windows are not appropriate for the transient signals obtained from a transient shock test. As far as this test method is concerned, the main drawback of the analyser is its limited frequency resolution. The HP 3582A spectrum analyser is an 8 bit instrument whilst the Bruel and Kjaer signal analyser is a 12 bit instrument. Furthermore, other advanced analysis techniques, such as the liftered spectrum analysis, are not provided with the HP analyser.

5. It is certainly advantageous to have an instrumentaion system which allows a pile's integrity to be assessed by both test methods.

CHAPTER 7

COMPUTER SIMULATION

7.1 INTRODUCTION

Computer simulation is a very useful technique to aid the understanding and interpretation of pile testing results. Although a physical model is the best way to represent a real structure, the cost of construction and the time spent on building are considerable compared with a numerical model. It is normal practice to build as few physical models as necessary to represent different situations. At the same time numerical models are often constructed in order that results from both modeling methods can be compared. When sufficient confidence is gained by comparing both models, numerical modeling may be used on its own to simulate other cases.

Since pile testing analysis can be conducted either in the time domain or the frequency domain, simulation methods in both domains have to be investigated. Two methods of simulation in each domain will be introduced in this chapter, their advantages and limitations will be discussed.

7.2 TIME DOMAIN SIMULATION

Before attempting any simulation method, it is necessary to have a critical understanding of the problems involved in the interpretation of practical results. One of the main difficulties in using the sonic-echo testing method is multiple reflections caused by a change in mechanical impedance.

A signal trace can be regarded as a time series made up of reflected events together with various interfering waves and noise. The desired reflected events are the primary reflections, which are waves reflected by a change in acoustic impedance. An important type of undesired interfering wave is multiple reflection, which is the result of continuous leaking of vibration energy from a section of different acoustic impedance back to the top of the pile.

7.2.1 Simulation By The Method Of Convolution

7.2.1.1 The wavelet model

In the sonic-echo method of pile testing, transducers record the response of the pile head to an impulse-like sonic source, which is denoted as the source wavelet. The recorded signal is representative of the reflected energy received at the transducer as a function of time and constitutes a time series. When a wave encounters an interface between two sections of different cross-sectional area within a pile, part of its energy is reflected and the rest is transmitted. If there are several such interfaces within the pile length, the input signal will undergo a series of reflections and transmissions. For every encounter, new reflection and transmission wavelets will be generated. Each wavelet generated can be treated as a new source wavelet which will undergo reflection and transmission when it encounters an interface. As a result of many such encounters, the received time series on the top of the pile can be considered as a sum of amplitude-scaled and time-delayed wavelets. The amplitude scale factors are dependent upon the changes in cross-sectional area and the changes in acoustic impedance at the interface and the time delays dependent upon the distance travelled and wave propagation velocity. In mathematical terms, if the input signal is denoted by $a(t)$, then the received time series $b(t)$ can be expressed as¹⁰⁶:

$$b(t) = \sum_{n=1}^{\infty} h_n a(t - \tau_n) + n(t) \quad (7.1)$$

where h_n = amplitude scale factor
 τ_n = time delay
 n_t = additive noise or surface wave interference

Equation (7.1) represents a model for the sonic-echo method of pile testing.

This model is widely used in other areas of research such as communications, speech, radar, and biomedical data processing where one considers the analysis of time series of multiple overlapping wavelets in a noisy environment. From equation (7.1), if the input wavelet $a(t)$ is known and the noise $n(t)$ is excluded, then a mathematical model can be built up provided the amplitude scale factor h_n and time delay T_n are known.

7.2.1.2 Wavelet concept of multiple reflection

Without loss of generality, consider the pile in Figure 7.1, which comprises a length with normal cross-sectional area A_1 , and an enlargement of area A_2 . Assume the acoustic properties of the concrete are the same throughout the pile length. The coefficients of reflection and transmission for a wave going from section 1 into section 2 are denoted by $r_{12}=r$ and $t_{12}=1-r$ which can be obtained from equations (4.60) and (4.61) respectively. Table 7.1 summarizes the coefficients of reflection and transmission of waves travelling up or down the pile, and the results are indicated on the reflection and transmission rays of Figure 7.1.

DIRECTION	AREA	COEFFICIENT OF REFLECTION r	COEFFICIENT OF TRANSMISSION t
Section 1 ↓ Section 2	A_1 A_2	$r_{12} = \frac{A_2 - A_1}{A_2 + A_1}$ $= r$	$t_{12} = \frac{2A_1}{A_2 + A_1}$ $= 1 - r$
Section 2 ↓ Section 3	A_2 A_1	$r_{23} = \frac{A_1 - A_2}{A_2 + A_1}$ $= -r$	$t_{23} = \frac{2A_2}{A_2 + A_1}$ $= 1 + r$
Section 1 ↑ Section 2	A_1 A_2	$r_{21} = \frac{A_1 - A_2}{A_2 + A_1}$ $= -r$	$t_{21} = \frac{2A_2}{A_2 + A_1}$ $= 1 + r$

Table 7.1

When an incident wave $I(t)$ strikes interface 1, the wave is partly reflected and partly transmitted. The amplitudes of reflected and transmitted waves depend on the coefficients of reflection and transmission. The reflected wave travels back to the top of the pile whilst the transmitted wave undergoes a series of reflections and transmissions. The reflected signal $R(t)$ can be visualised as the result of a combination of a series of wavelets each of which is delayed by $\tau=2l/c$, i.e the two-way travel time within the enlargement. Similar arguments can be applied to the transmitted signal. If the amplitudes and time delays are taken into account, then equations for the reflected and transmitted wavelets can be formulated as follows:

$$R(t) = rI(t) - r(1-r^2)I(t-\tau) - r^3(1-r^2)I(t-2\tau) - \dots \quad (7.2)$$

$$T(t) = (1-r^2)I(t) + r^2(1-r^2)I(t-\tau) + r^4(1-r^2)I(t-2\tau) + \dots \quad (7.3)$$

where $I(t)$ = incident wave
 $R(t)$ = reflected wave
 $T(t)$ = transmitted wave
 r = coefficient of reflection

$$r = \frac{A_2 - A_1}{A_2 + A_1}$$

 l = length of enlargement
 τ = delay for each reflection or transmission
 $= 2l/c$

It can be seen from equation (7.2) that the first component of the reflection signal is of the same sense as the incident wave but the subsequent wave components are of the opposite sense. Also from equation (7.3), the transmitted signal comprises components all of the same sense as the incident wave. A conclusion which can be drawn from the above observation is that if the incident wave $I(t)$ is of a compression nature, then the reflected signal $R(t)$ is made up firstly of a compression component and then modified by a series of tension waves, whilst the transmitted signal is made up of a series of compression waves.

7.2.1.3 Examples of modelling by summation of wavelets

Computer simulation has been attempted on Beam 2 (refer to Figure 3.7(b)). The shape and dimensions of the beam are shown with the ray diagram of the reflected and transmitted wavelets in Figure 7.2. For clarity, only the first component of each reflected and transmitted wavelet is shown, subsequent multiple reflections within the overbreak are omitted. From Figure 7.2, it can be seen that the output signal $b(t)$ is made up of the input signal $a(t)$, the first and second reflections from the overbreak, and the reflection from the end of the beam. Reflected and transmitted wavelets can be calculated from equations (7.2) and (7.3). If the time delay for each reflection is found by dividing the two-way distance by the wave propagation velocity, then the output signal $b(t)$ can be calculated. A program was written for this problem and the simulated results are shown in Figure 7.3. From the results of experimental work, the input signal $a(t)$ was chosen to be a half-sine impulse. Figure 7.3(a) shows the input signal $a(t)$ with the first reflection wavelet $R_1(t)$. The second reflection wavelet $R_2(t)$ is included in Figure 7.3(b). The transmitted wavelet $T_2(t)$ is shown in Figure 7.3(c). The overall output signal $b(t)$ is shown in Figure 7.3(d). A systematic development of the output signal $b(t)$ from the various wavelets is shown in Figure 7.4. Experimental results obtained from testing on the beam are shown in Figure 7.5(a). A comparison between simulated and experimental results demonstrates that the simulation process is successful in the time domain.

7.2.1.4 The convolution model

If the output signal $b(t)$ is digitized into a time series, then equation(7.1) can be reformed as¹⁰⁶:

$$b(k\Delta t) = \sum_{n=1}^{\infty} h_n a[(k-n)\Delta t] + n(k\Delta t) \quad (7.4)$$

$$\text{or} \quad b_k = \sum_{n=1}^{\infty} h_n a_{k-n} + n_k \quad (7.5)$$

Equation (7.5) expresses the model in a convolution form. The received time series b_k can be regarded as the convolution of the input wavelet a_k with the impulse response of the pile head h_n plus an additive noise sequence n_k . A simple convolution model has been shown in Figure 5.21. The impulse response function of a linear system depends only on the acoustic properties and the physical dimensions of the system, independent of the input signal. If the input signal and the impulse response of the system are known, then the convolution model can be constructed by equation (7.5). The input signal can be defined as desired. The most common types are half-sine impulse, triangular impulse, rectangular impulse, and unit impulse. The impulse response which can be found by a process called deconvolution if the input and output series are known. Deconvolution, which is a reverse process of convolution, is commonly used by seismologists to extract information about the earth's stratification from reflection seismograms. As an example, the simulated input and output time series of Beam 2 in Figure 7.3 are deconvoluted to obtain the impulse response of the beam. This was undertaken using another computer program (Deconvolut) written by the author and the result obtained is shown in Figure 7.5(b). Once the impulse response function $h(t)$ of a system is available, output time series $b(t)$ can be obtained by convolution of the impulse response function $h(t)$ with any desired input time series $a(t)$. For example, the output time series $b(t)$ for a rectangular input signal $a(t)$ on the beam has been obtained by using a computer program (Convolute), which is based on equation (7.5). The result is shown in Figure 7.5(c). Alternatively, the impulse response

function $h(t)$ of a system can be obtained directly by the method described in section 7.2.1.2 with a unit impulse as the input signal. Figure 7.6 shows the impulse response function $h(t)$ of a two-section structure. This is a relatively simple structure which can be modelled by wavelet summation since it has only one interface. For structures with more interfaces, it is very tedious to keep track of the multiple reflections, and a better method of simulation is required.

7.2.2 Simulation By The Method Of Characteristics

The method of characteristics is an essential tool for the theoretical study of one-dimensional stress waves propagation in bars. Its application is particularly simple in the case of systems which can be described by partial differential equations that involve just two independent variables, for example, one space co-ordinate and time¹⁰⁷. The method has increasingly found a wide application in many branches of engineering science, especially those which involve mechanics of solids and fluids. When applied to pile testing, the differential equations are integrated along the two set of characteristics. This amounts to separating the solution into two waves, one travelling in the downward direction through the pile, and the other travelling in an upward direction. The pile is divided into a number of sections along its length, each section corresponds to a time step such that the wave propagates from one section to another with a constant velocity.

7.2.2.1 Formulation of characteristic equations

The differential equation of wave propagation in a bar can be found by considering an elemental section of length dx in which the stress wave is propagating as shown in Figure 7.7. Summing the longitudinal forces acting on

the element gives:

$$F + \rho A dx \frac{\partial v}{\partial t} = F + \frac{\partial F}{\partial x} dx + F_r dx$$

$$\text{or} \quad \frac{\partial F}{\partial x} + F_r = \rho A \frac{\partial v}{\partial t} \quad (7.6)$$

where F = axial force
 F_r = frictional force per length of bar
 v = particle velocity
 ρ = density of bar
 A = cross-sectional area of bar
 dx = length of element

If frictional force is neglected, then equation (7.6) can be simplified as:

$$\frac{\partial F}{\partial x} = \rho A \frac{\partial v}{\partial t} \quad (7.7)$$

This equation can be easily converted into the one-dimensional elastic wave equation as stated in equation (4.37). Equation (7.7) can be written in finite difference form as:

$$\frac{F_{x+1,t+1} - F_{x,t}}{\Delta x} = \rho A \frac{V_{x+1,t+1} - V_{x,t}}{\Delta t} \quad (7.8)$$

The time step Δt can be chosen such that $|\Delta x / \Delta t| = c$, where c is the velocity of the longitudinal elastic wave. Depending upon the direction of wave propagation, the characteristic equation may be expressed as:

$$(F_{x\pm 1,t+1} - F_{x,t}) \pm \rho A c (V_{x\pm 1,t+1} - V_{x,t}) = 0 \quad (7.9)$$

The characteristic equations are integrated along the characteristic lines drawn in the x - t plane, and their gradient is given by the characteristic direction, as shown in Figure 7.8. Consider the point $(x-1,t)$ with characteristic direction c^+ . Replacing (x,t) by $(x-1,t)$ in equation (7.9) gives:

$$(F_{x,t+1} - F_{x-1,t}) + Z(V_{x,t+1} - V_{x-1,t}) = 0 \quad (7.10)$$

Similarly for the point $(x+1,t)$ with characteristic direction c^- , gives:

$$(F_{x,t+1} - F_{x+1,t}) - Z(V_{x,t+1} - V_{x+1,t}) = 0 \quad (7.11)$$

where $Z = \rho A c$. Thus if the axial forces F and particle velocities V at points $(x-1,t)$ and $(x+1,t)$ are known, then equations (7.10) and (7.11) can be solved simultaneously to obtain the axial force and particle velocity at point $(x,t+1)$ as:

$$F_{x,t+1} = \frac{1}{2}[F_{x-1,t} + F_{x+1,t} + Z(V_{x-1,t} - V_{x+1,t})] \quad (7.12)$$

$$V_{x,t+1} = \frac{1}{2}[(F_{x-1,t} - F_{x+1,t})/Z + V_{x-1,t} - V_{x+1,t}] \quad (7.13)$$

Equations (7.12) and (7.13) are also applicable to other points along the rod. Hence, there exists an integration process for each section of the rod, provided that the initial conditions of the rod are known.

7.2.2.2 Boundary conditions

The two ideal boundary conditions commonly encountered in the discussion of wave propagation in a bar are free and fixed ends. As discussed in Chapter 4, at a free end the axial force must vanish, whilst particle velocity is zero at a fixed end. As an example, suppose a pile is free at the top end and is fixed at the base. Assume the pile is divided into N sections and hence $N+1$ nodes. Boundary equations can be derived as follows:

$$F_{1,t+1} = 0 \quad (7.14)$$

$$V_{1,t+1} = V_{2,t} - F_{2,t}/Z \quad (7.15)$$

$$F_{N+1,t+1} = F_{N,t} + ZV_{N,t} \quad (7.16)$$

$$V_{N+1,t+1} = 0 \quad (7.17)$$

A computer program was written to simulate the problem of one-dimensional

wave propagation in the pile under the above boundary conditions. Equations (7.12) to (7.17) were incorporated in the program to calculate the new state of motion along the pile. Particle velocities on the pile top at every time step were gathered to obtain the output signal and the result was then plotted in Figure 7.9. The features in the signal show close agreement with the one dimensional elastic wave theories discussed in Chapter 4.

7.2.2.3 Modification for discontinuity

As mentioned earlier in section 4.6.2, discontinuity can be either due to a change in cross-sectional area, or a change in characteristic impedance, or both. In general, all these changes can be described in terms of the mechanical impedance $Z=\rho Ac$. By using this definition, equations (7.10) and (7.11) can be rewritten to deal with discontinuity. The modified equations are:

$$(F_{x,t+1}-F_{x-1,t}) + Z_L(V_{x,t+1}-V_{x-1,t}) = 0 \quad (7.18)$$

$$(F_{x,t+1}-F_{x+1,t}) - Z_R(V_{x,t+1}-V_{x+1,t}) = 0 \quad (7.19)$$

where Z_L and Z_R are the mechanical impedances on the left-hand and right-hand sides of the boundary. Again, solving them simultaneously gives the axial force and particle velocity on the boundary at a time step later as:

$$F_{x,t+1} = [F_{x-1,t}-F_{x+1,t}+Z_L V_{x-1,t}+Z_R V_{x+1,t}]/(Z_L+Z_R) \quad (7.20)$$

$$V_{x,t+1} = [Z_R F_{x-1,t}+Z_L F_{x+1,t}+Z_L Z_R (V_{x-1,t}-V_{x+1,t})]/(Z_L+Z_R) \quad (7.21)$$

7.2.2.4 Examples of simulation by the method of characteristics

A program was written to simulate Beam 2 (refer to Figure 3.7(b)). The simulated result, shown in Figure 7.10, is near identical to the result modelled by the convolution method (see Figure 7.3). Another program was written to

simulate Beam 1 (refer to Figure 3.7(a)). This beam proved almost impossible to be modelled by the convolution method due to too many multiple reflections from the two overbreaks. The simulated result obtained by the method of characteristics and the experimental result are shown in Figures 7.11 and 7.12 respectively. The general features of the simulated result resembled the experimental result with good agreement. The main difference is on the relative amplitudes of the wavelets. This is due to the violation of the assumptions of one-dimensional elastic wave propagation and the ignoring of frictional losses.

7.3 COMMENTS ON THE TIME DOMAIN SIMULATION METHODS

Basically the difference between simulations by the method of convolution and the method of characteristics stems out from the fact that the former is an analytical method whilst the latter is a mathematical method.

In the method of convolution a thorough understanding of the properties of stress wave propagation is needed. A model is formed by keeping track of multiple reflections within a structure. This is a very tedious process even for a relatively simple structure, and the method becomes inefficient in dealing with structures with many interfaces. However, it is a very rewarding exercise in the understanding of the interaction of stress wave propagation in a multi-section structure. The knowledge gained from such an exercise is very useful in the interpretation of sonic-echo traces.

Simulation by the method of characteristics is a more systematic way of modelling. It is a very efficient method and can be easily programmed for structures with many interfaces. Since it is a mathematical method, it does not require a sound background knowledge of stress wave propagation. It has the advantage that damping effects can be incorporated into the wave equation if

required.

Since both methods are based on one-dimensional idealization and the elementary wave theory, differences in simulated and measured results are to be expected.

7.4 FREQUENCY DOMAIN SIMULATION

In section 7.2, two methods of simulation in the time domain have been introduced. Relatively short time histories of the structural transient behaviour were investigated in terms of one-dimensional elastic wave propagation. In this section, two methods of simulation for the vibrational behaviour of different systems will be discussed. The two methods are the receptance method and the four-pole techniques. Again, damping is ignored for simplicity of simulation. However, attention will be concentrated on finding the resonant frequencies of different systems. In both methods, the system in question will be divided into several subsystems, or lumped components. The vibration response of each subsystem is then investigated and the equation describing the vibrational behaviour of the whole system will be formulated by linking all the subsystems together. The most commonly encountered subsystems are mass, spring and rod. If damping is included in the analysis, then a damper unit may be introduced. There are two simple mathematical models for the damping in a system — viscous or hysteretic damping. In the following analysis, damping is ignored because the treatment for undamped vibration, which is much simpler than damped vibration, is quite sufficient for finding the resonance of a system.

7.4.1 Receptance Model

The concept of receptance is based on the linear response of an elastic system to an exciting force. A comprehensive treatment of the basic principles of the receptance theory has been given by Bishop and Johnson¹⁰⁸. A vibration technique of NDT pile testing based on the receptance theory is currently under investigation by Lilley.⁴⁰ Receptance of a system can be defined as the ratio of the displacement amplitude (U) to the amplitude of an harmonic excitation force (F). If a harmonic force $Fe^{i\omega t}$ acts at some point on a linear system such that the system undertakes an harmonic motion with the same frequency ω , then its displacement at the point of force application is given by:

$$u = Ue^{i\omega t} \quad (7.22)$$

On the other hand, for a linear system under forced vibration, its displacement is proportional to the exciting force as in the following equation:

$$u = \theta Fe^{i\omega t} \quad (7.23)$$

where θ is a constant dependent on the dynamic properties of the system. From equations (7.22) and (7.23), it is clear that $\theta=U/F$, hence according to the earlier stated definition of receptance, θ is in fact the receptance of the linear system.

The receptance of a system can be classified either as direct or cross receptance¹⁰⁹ according to the point where the displacement is measured. Direct receptance refers to receptance calculated with the displacement taken at the same point of application of force. Cross receptance is calculated with displacement taken at a point other than the point of force application. In order to distinguish between direct and cross-receptances, and to specify the points

at which receptances are to be calculated, subscripts are added to the receptance symbol θ . The cross receptance θ_{xy} means that the displacement is taken at point x with the disturbing force applied at point y . A direct receptance therefore should have two identical subscripts and may be denoted as θ_{yy} .

7.4.1.1 Receptance of a single system

As mentioned earlier there are three basic types of single system — mass, spring and rod. The calculation of receptance of these systems will be presented.

For a rigid mass M under an exciting force $Fe^{i\omega t}$ as shown in Figure 7.13, its equation of motion is :

$$M\ddot{u} = Fe^{i\omega t} \quad (7.24)$$

Substituting $u = Ue^{i\omega t}$ from equation (7.22) into equation (7.24) gives:

$$-M\omega^2 U = F \quad (7.25)$$

That is to say the receptance is:

$$\theta = \frac{U}{F} = - \frac{1}{M\omega^2} \quad (7.26)$$

Note that there is no difference between the direct and cross receptance of a rigid mass simply because the mass is regarded as a point mass.

The equation of motion of a massless spring of length l with stiffness k under an harmonic excitation force $Fe^{i\omega t}$ as shown in Figure 7.14 is given by:

$$u = \frac{xFe^{i\omega t}}{lk} \quad (7.27)$$

Using equations (7.22) and (7.27), the receptance at any point x along the spring is:

$$\theta = \frac{x}{l} \cdot \frac{1}{k} \quad (7.28)$$

The equation of motion of a cylindrical bar as shown in Figure 7.15 is given by:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (7.29)$$

The solution for a free-free bar can be expressed as:

$$u = - \frac{\cos \lambda x}{AE \lambda \sin \lambda l} Fe^{i\omega t} \quad (7.30)$$

Hence, the receptance at any point along the bar can be expressed as:

$$\theta = - \frac{\cos \lambda x}{AE \lambda \sin \lambda l} \quad (7.31)$$

Using the same argument but with different boundary conditions, the receptance at any point along a fixed-free beam is found to be:

$$\theta = \frac{\sin \lambda x}{AE \lambda \cos \lambda l} \quad (7.32)$$

Table 7.2 summarises the direct and cross receptances of the three basic single systems. It should be noted that for a symmetrical system with the same type of support on both sides, the cross receptance satisfies the principle

of reciprocity in that the cross receptance is the same when the point of response measured and the point of force application are interchanged. i.e

$$\theta_{xy} = \theta_{yx}$$

TYPE OF SYSTEM	BOUNDARY CONDITIONS	DIRECT RECEPTANCE (θ_{xx})	CROSS RECEPTANCE (θ_{xy})
Mass	no constraints along direction of motion	$-1/M\omega^2$	$-1/M\omega^2$
Spring	fixed-free	$1/k$	x/lk
Rod	free-free	$-\cot\lambda l/AE\lambda$	$-\cos\lambda x/AE\lambda\sin\lambda l$
	fixed-free	$\tan\lambda l/AE\lambda$	$-\sin\lambda x/AE\lambda\cos\lambda l$

Table 7.2

7.4.1.2 Receptance of composite system

Consider the two single systems, shown in Figure 7.16, which could be connected at co-ordinate 2 to obtain a two-unit composite system. For system A, the displacements at co-ordinates 1 and 2, due to force applied at co-ordinate 2, can be written as $u_1 = A_{12}F_2$ and $u_2 = A_{22}F_2$ respectively. A_{12} and A_{22} being the direct and cross receptances of system A. The displacements at co-ordinate 2 when forces are applied at co-ordinates 2 and 3 separately are given by $u_2 = -B_{22}F_2$ and $u_2 = B_{23}F_3$ respectively. If a composite system is formed by connecting system A and system B together, then force equilibrium and compatibility conditions at co-ordinate 2 must be satisfied. The following equations can be written for the response of the composite system when a disturbing force F_3 is applied at co-ordinate 3.

$$u_1 = A_{12}F_2 \tag{7.33}$$

$$u_2 = A_{22}F_2 \quad (7.34)$$

$$u_2 = B_{23}F_3 - B_{22}F_2 \quad (7.35)$$

$$u_3 = B_{33}F_3 - B_{32}F_2 \quad (7.36)$$

By definition, the direct and cross receptances of the composite system are $\theta_{33}=u_3/F_3$ and $\theta_{13}=u_1/F_3$. Eliminating u_2 from equations (7.34) and (7.35), one obtains $F_2=B_{23}F_3/(A_{22}+ B_{22})$. Substituting this result into equations (7.33) and (7.36), the direct and cross receptances can be expressed as:

$$\theta_{13} = \frac{A_{12}B_{23}}{A_{22}+B_{22}} \quad (7.37)$$

$$\theta_{33} = B_{33} - \frac{B_{23}B_{32}}{A_{22}+B_{22}} \quad (7.38)$$

For a three-unit composite system, the first two units can be combined to form a single system by the method mentioned above. After this is done, the same method can be used again on the combined system with the remaining unit to calculate the receptance of the composite system. The receptances of a three-unit composite system are found to be:

$$\theta_{14} = \frac{A_{12}B_{23}C_{34}}{(A_{22}+B_{22})(B_{33}+C_{33})-B_{23}B_{32}} \quad (7.39)$$

$$\theta_{44} = C_{44} - \frac{(A_{22}+B_{22})C_{34}C_{43}}{(A_{22}+B_{22})(B_{33}+C_{33})-B_{23}B_{32}} \quad (7.40)$$

Repeated application of this method can lead to the results of receptances for a composite system of more single units.

7.4.1.3 Natural frequencies of composite system

It has been mentioned that the purpose of frequency domain simulation is to find out the natural frequencies of different systems. It will be shown in this section how to obtain the natural frequencies from the receptances of a composite system. One technique for doing this is to imagine that the excitation force F vanishes for free vibration whilst the displacement u remains finite. Under such a situation, the reciprocal of receptance is zero, i.e. $\Delta = 1/\theta = 0$. $\Delta = 0$ is usually referred to as the frequency equation of a vibrating system. The roots of this equation are the desired natural frequencies. For a two-unit system, the natural frequencies may be obtained from either equation (7.37) or equation (7.38) as:

$$\Delta = A_{22} + B_{22} = 0 \quad (7.41)$$

As an example, consider the spring and mass system in Figure 7.17. The necessary information on receptance of a single system may be selected from Table 7.2 and substituted into equation (7.41) to give:

$$\frac{1}{k} - \frac{1}{M\omega^2} = 0 \quad (7.42)$$

Solving this equation gives the natural frequency of the system as $\omega = \sqrt{(k/M)}$. This solution can also be obtained by solving the differential equation which describes the motion of the system⁸⁵. In this case, the equation of motion is:

$$M\ddot{u} + ku = Fe^{i\omega t} \quad (7.43)$$

The trial solution for motion with the applied frequency, is $u = Ue^{i\omega t}$ and this gives:

$$U = \frac{F}{k - M\omega^2} \quad (7.44)$$

That is to say the direct receptance of the mass-spring system is:

$$\theta = \frac{1}{k - M\omega^2} \quad (7.45)$$

which confirms the natural frequency as $\omega = \sqrt{k/M}$.

Only one natural frequency is obtained for this two-unit system since there is only one degree of freedom for the system. As a rule of thumb, the number of natural frequencies of a system equals the degrees of freedom minus the number of constraints of the system⁹⁹. Hence a pendulum swinging in an x-y plane but constrained in a circular arc has only one natural frequency.

From the example shown above, it seems that solving the equation of motion of a system may be a much easier way to obtain the natural frequency of a system. However, to set up the equation of motion for a multi-unit system can be very difficult and the task of solving the differential equations may render the method inefficient for simulation. On the other hand, the method of receptance is very straightforward once the individual receptances for each single system are known. This advantage makes the method of receptance a very good technique for the simulation of composite systems.

7.4.1.4 Simulation of real structures by the receptance method

Beam 1 (refer to Figure 3.7(a)) can be simulated as a five-unit system. Following the procedures described in section 7.4.1.2, the frequency equation for a five-unit system, A, B, C, D, E, may be derived as:

$$\Delta = [(C_{xx}+D_{xx})(D_{xx}+E_{xx})-D_{xy}D_{yx}][(A_{xx}+B_{xx})(B_{xx}+C_{xx})-B_{xy}B_{yx}] - (A_{xx}+B_{xx})(D_{xx}+E_{xx})C_{xy}C_{yx} = 0 \tag{7.46}$$

The normal sections of the beam can be simulated as rod-structures with the two overbreaks simulated as point masses. Once again the appropriate receptance expressions for these subsystems can be obtained from Table 7.2, and then substituted into equation (7.46) to form a frequency equation for the structure. A program was written to solve a five-unit system frequency equation. The first four resonant frequencies obtained were shown, side by side, with the experimental results in Table 7.3.

----- RESONANT FREQUENCY (Hz) -----		
Simulated	Experimental	
	end 1	end 2
200	210	210
349	352	358
563	562	-----
703	-----	696

Table 7.3

7.4.2 Four-Pole Techniques

Simulation by the Four-Pole techniques is one of the best methods for vibration analysis. A complete introduction to the techniques with fully explained examples can be found in Clark¹¹⁰. The approach of the Four-Pole techniques to vibration analysis is quite similar to that of the receptance method. Both methods require the evaluation of some basic relationship between vibration response of a single system or lumped component with the exciting force. A complex system can then be created by patching many basic

elements together, and its dynamic characteristics can be evaluated by considering force equilibrium and compatibility conditions at the junctions between individual elements. The structural response taken in the Four-Pole techniques is velocity rather than displacement as in the receptance method. In the receptance method, one is looking for the receptance of a system. The receptance describes the relationship between displacement and the exciting force. Here, in the Four-Pole techniques, one has to determine the Four-Pole parameters, which is the name applied to the four coefficients α_{11} , α_{12} , α_{21} , α_{22} in the following equations:

$$F_1 = \alpha_{11}F_2 + \alpha_{12}v_2 \quad (7.47)$$

$$v_1 = \alpha_{21}F_2 + \alpha_{22}v_2 \quad (7.48)$$

Those equations are the performance equations of the mechanical system shown in Figure 7.18. The equations can be written in matrix form as:

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} F_2 \\ v_2 \end{bmatrix} \quad (7.49)$$

The system in Figure 7.18 can be any combination of some basic systems such as mass, spring, rod and dampers. Any set of units may be used in equation (7.49) provided they are consistent. It can be seen that the parameters α_{11} and α_{22} are dimensionless while α_{12} has the dimensions of force over velocity and α_{21} the dimensions of velocity over force.

7.4.2.1 Four-Pole parameters of basic components

The general procedures for determining the four-pole parameters are to utilize the equations of motion of a particular system in order to describe it in the form of simple algebraic or differential equations. These equations are

then converted into the form of equation (7.49) and the four-pole parameters would be readily identified.

As an example, suppose the elastic system in Figure 7.18 to be a rigid mass M . Utilizing the concept of perfect rigidity and Newton's law of motion one can describe the motion of the rigid mass as:

$$v_1 = v_2 \quad (7.50)$$

$$F_1 - F_2 = M \frac{dv_1}{dt} = M \frac{dv_2}{dt} \quad (7.51)$$

If the exciting force is sinusoidal, then the structural response in terms of velocity is:

$$v_1 = v_2 = v e^{i\omega t} \quad (7.52)$$

$$\text{and } \frac{dv_1}{dt} = \frac{dv_2}{dt} = i\omega v e^{i\omega t} \quad (7.53)$$

Substituting equations (7.52) and (7.53) into equations (7.50) and (7.51) accordingly, an equation of motion in the same form of the matrix equation (7.49) can be written as:

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1 & M\omega i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_2 \\ v_2 \end{bmatrix} \quad (7.54)$$

Hence, the four-pole parameters of a rigid mass can be identified as $\alpha_{11}=1$, $\alpha_{12}=m\omega i$, $\alpha_{21}=0$ and $\alpha_{22}=1$.

The four-pole parameters of other basic components can also be determined as in the same way. Clark's¹¹⁰ results for both end free components are summerized in Table 7.4.

TYPE OF COMPONENT	$\alpha_{11}(=\alpha_{22})$	α_{12}	α_{21}
Mass	1	$M\omega i$	0
Spring	1	0	$i\omega/k$
Rod	$\cos\phi$	$iZ\sin\phi$	$(i/Z)\sin\phi$
Viscous damper	1	0	$1/c$

Table 7.4

where

- M = mass of rigid mass
- ω = angular frequency of exciting force
- k = spring stiffness
- $\phi = \omega l/c_o$
- Z = AE/c_o
- l = length of rod
- A = cross-sectional area
- c_o = bar velocity
- c = resistance of damper

7.4.2.2 Series-connected composite system

Composite systems can be easily created by connecting elements together at their input and output junctions. The four-pole equations for each of the individual elements, that make up the series-connected composite system in Figure 7.19, are still valid. They are:

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \alpha_{11}^{[1]} & \alpha_{12}^{[1]} \\ \alpha_{21}^{[1]} & \alpha_{22}^{[1]} \end{bmatrix} \begin{bmatrix} F_2 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} F_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} \alpha_{11}^{[2]} & \alpha_{12}^{[2]} \\ \alpha_{21}^{[2]} & \alpha_{22}^{[2]} \end{bmatrix} \begin{bmatrix} F_3 \\ v_3 \end{bmatrix}$$

...

$$\begin{bmatrix} F_n \\ v_n \end{bmatrix} = \begin{bmatrix} \alpha_{11}^{[n]} & \alpha_{12}^{[n]} \\ \alpha_{21}^{[n]} & \alpha_{22}^{[n]} \end{bmatrix} \begin{bmatrix} F_{n+1} \\ v_{n+1} \end{bmatrix} \quad (7.55)$$

This set of individual equations can be combined to form a four-pole equation for the composite system and hence the corresponding parameters can be identified.

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \alpha_{11}^{[1]} & \alpha_{12}^{[1]} \\ \alpha_{21}^{[1]} & \alpha_{22}^{[1]} \end{bmatrix} \times \dots \times \begin{bmatrix} \alpha_{11}^{[n]} & \alpha_{12}^{[n]} \\ \alpha_{21}^{[n]} & \alpha_{22}^{[n]} \end{bmatrix} \begin{bmatrix} F_{n+1} \\ v_{n+1} \end{bmatrix} \quad (7.56)$$

The four-pole parameters of the composite system are therefore:

$$\beta_{ij} = \alpha_{ij}^{[1]} \times \alpha_{ij}^{[2]} \times \dots \times \alpha_{ij}^{[n]} \quad (7.57)$$

where

$$\beta_{ij} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}$$

The natural frequencies of a composite system can be evaluated from the four-pole equation of that system. As an example, reconsider the mass-spring system in Figure 7.17. The overall performance equation of the mass-spring system can be written as:

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1 & M\omega i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ i\omega/k & 1 \end{bmatrix} \begin{bmatrix} F_3 \\ v_3 \end{bmatrix}$$

or

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1-M\omega^2/k & M\omega i \\ i\omega/k & 1 \end{bmatrix} \begin{bmatrix} F_3 \\ v_3 \end{bmatrix} \quad (7.58)$$

To find the frequency equation of the system, the exciting force is put to zero for natural vibration and the boundary condition at the rigid mass support suggests that velocity at the base is also zero. Putting these requirements into

equation (7.58) and extracting the top equation from the matrix system gives:

$$(1-M\omega^2/k)/F_3 + M\omega i \cdot (0) = 0 \quad (7.59)$$

The natural frequency is found to be $\sqrt{(k/M)}$, which is the same as calculated earlier by the other methods.

7.4.2.3 Parallel-connected composite system

A parallel-connected composite system is shown in Figure 7.19, which has all the input junctions of the individual elements connected to a single input and their output junctions to another single output point. It should be noted that all their input junctions move with a common velocity and all their output junctions with a common velocity, and the input and output forces are the sum of the input and output forces of the individual components. According to Clark¹¹⁰, the overall four-pole parameters β_{ij} of a composite system, formed by n single systems of parameters α_{ij} , can be written as:

$$\beta_{11} = A/B, \quad \beta_{12} = AC/B-B$$

$$\beta_{21} = 1/B, \quad \beta_{22} = C/B \quad (7.60)$$

where

$$A = \sum_{i=1}^n \left[\frac{\alpha_{11}^{[i]}}{\alpha_{21}^{[i]}} \right]$$

$$B = \sum_{i=1}^n \left[\frac{1}{\alpha_{21}^{[i]}} \right]$$

$$C = \sum_{i=1}^n \left[\frac{\alpha_{22}^{[i]}}{\alpha_{21}^{[i]}} \right]$$

As an example, the earlier mass-spring system is re-considered but with a damper element connected in parallel with the spring as shown in

Figure 7.20. The necessary four-pole parameters α_{ij} of the individual elements can be obtained from Table 7.4. By using equation (7.60), the four-pole parameters γ_{ij} of the combined spring-damper subsystem are found to be $\gamma_{11}=1$, $\gamma_{12}=0$, $\gamma_{21}=1/[(k/i\omega)+c]$ and $\gamma_{22}=1$. This subsystem is then connected in series with the rigid mass M , hence the overall four-pole parameters α_{ij} can be obtained by multiplying the parameters of the rigid mass with those of the spring-damper subsystem correspondingly. The performance equation of the composite system is:

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{M\omega i}{(k/i\omega)+c} & M\omega i \\ \frac{1}{(k/i\omega)+c} & 1 \end{bmatrix} \begin{bmatrix} F_2 \\ v_2 \end{bmatrix} \quad (7.61)$$

Since the structure is fixed onto a rigid foundation, the velocity V_2 must vanish and a frequency response function in terms of mechanical admittance can be written as:

$$H(\omega) = \frac{v_1}{F_1} = \frac{i\omega}{(k-M\omega^2)+i\omega c} \quad (7.62)$$

The amplitude of the mechanical admittance can be shown as:

$$\left| \frac{v_1}{F_1} \right| = \frac{\omega}{\sqrt{[(k-M\omega^2)^2 + \omega^2 c^2]}} \quad (7.63)$$

This equation is exactly the same as equation (3.17), which has been used to verify the computation of the dynamic stiffness value of a pile.

The natural frequency of this mass-spring-damper system can be found such that the amplitude of mechanical admittance is a maximum. This

corresponds to the case when the denominator of equation (7.63) is a minimum. Once again the natural frequency is found to be $\sqrt{k/M}$. Hence the natural frequency of this composite system, with or without a damper element, is still the same.

For a column standing on a compressible base, it can be modelled as shown in Figure 7.21. The rod element represents the beam itself, the spring element represents the stiffness of the foundation and the damper to absorb energy dissipated into the foundation.

7.4.2.4 Examples of four-pole techniques simulation

The objective of the following simulation is to predict the resonant frequencies of the model piles — Beam 1 and Beam 2 (see Figure 3.7). As a start, the problem is simplified by ignoring the influences of the surrounding soils and base stiffness on the dynamic behaviour of the piles. This simplification is justified for the following models since the structures are not buried.

Beam 2 can be modelled as a three-rod system each having the same acoustic and dynamic properties. The only difference between the sub-systems is in the cross-sectional areas. A computer program has been written to evaluate the overall four-pole parameters Δ_{ij} of the series-connected system. The performance equation is formulated as:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \quad (7.64)$$

It is assumed that at natural frequencies, the exciting force must vanish whilst the structural response V_1 is finite. The frequency equation may be formed

dependent upon the end conditions of the beam. If the beam is fixed at the output point 2, then the velocity V_2 must vanish. This condition can only be satisfied if $\Delta_{11}=0$. If on the other hand, the output end is free as in the actual case, then the force F_2 must vanish. This gives the frequency equation as $\Delta_{12}=0$. The result of simulation is shown along with the experimental result in Table 7.5. Apart from the missing third resonant frequency from simulated results, other values of resonant frequencies obtained are very close to the experimental results.

RESONANT FREQUENCY (Hz)	
Simulated	Experimental
315	284
521	536
-----	644
943	962
1094	1096
1561	1556

Table 7.5

Beam 1 is simulated as a five-element component connected in series. Both ends of the beam are free, hence the frequency equation used is $\Delta_{12}=0$ as before. The resonant frequencies obtained are very close to those obtained by the method of receptance. The results are tabulated in Table 7.6.

RESONANT FREQUENCY (Hz)			
Simulated		Experimental	
Four-pole Techniques	Receptance Method	End 1	End 2
200	200	210	210
350	349	352	358
560	563	562	-----
700	703	-----	696

Table 7.6

So far the two illustrated examples have been concerned with rod and mass elements only. Here it is intended to simulate a structure with spring elements. A case study is taken from the experimental testing of damaged aluminium bars by Adams et al³⁸. In the experiment, the frequencies of the first three modes of vibration were taken for an undamaged aluminium bar of length 0.4321m. After that, the bar was damaged at 0.26m from one end by two saw cuts on opposite sides of the bar, each cut removing about 15 percent of the cross-sectional area. The frequencies of the damaged bar were then measured. The results of testing are reproduced in Table 7.7.

MODE OF VIBRATION	RESONANT FREQUENCY (Hz)	
	undamaged	damaged
1	5140	5688
2	11474	11430
3	17200	17135

Table 7.7

It was suggested that the damaged bar may be modelled with two rod elements connected together by a spring element in series. The spring element

is used to represent the stiffness of the damaged zone. The spring constant k is equal to the product of the Young's modulus E and the cross-sectional area A divided by the length of damage. Theoretically, the damaged zone should be simulated as a mass-spring subsystem. However, the mass being so tiny that it is ignored. Using the test results and a formulation based on the receptance method, Adams et al were able to pin-point the position of the saw cut and produced an EA/k value which is assumed to be related to the severity of damage. In this particular case, the EA/k value was found to be $4.4 \times 10^{-3} \text{m}$.

Four-pole techniques simulation was performed on the bar based upon information on the lengths of the two undamaged sections and the stiffness k of the damaged zone. The Young's modulus was assumed to be 70KN/mm^2 , the area to be $10 \text{mm} \times 10 \text{mm}$ and the velocity of propagation in aluminium bar to be 4950m/s . The stiffness k was found to be 1.591MN/mm for the damaged zone. The results of simulation are shown in Table 7.8, with the percentage difference between simulated results and experimental results enclosed in brackets.

MODE OF VIBRATION	RESONANT FREQUENCY (Hz)	
	undamaged	damaged
1	5739 (-0.02%)	5687 (-0.02%)
2	11479 (+0.04%)	11438 (+0.07%)
3	17218 (+0.10%)	17160 (+0.15%)

Table 7.8

7.5 COMMENTS ON THE FREQUENCY DOMAIN SIMULATION METHODS

The principles of the two simulation methods are very similar. The vibration response of an individual system due to a sinusoidal excitation is firstly evaluated. More complex systems can then be created by patching a number of individual systems together.

The Four-Pole Techniques have a number of advantages over the receptance method. Formulation using the four-pole techniques is more straightforward which allows a simpler way of programming. Patching of systems, whether series-connected or parallel-connected, can be dealt with using standard formulae. Therefore the techniques can be extended efficiently to simulate a pile/soil system which involves a lot of branching to cover the interaction between the pile body and the surrounding soils. This extension will be reported in the next section.

One dimensional idealization and linear behaviour are assumed in both methods. Therefore, a slight disagreement between simulated and practical results is to be expected.

7.6 MECHANICAL ADMITTANCE SIMULATION

In this section a comprehensive description of mechanical admittance simulation using the four-pole techniques will be given. Simulation of mechanical admittance on pile-soil interaction has been outlined in the past by Davis and Dunn.³⁷ They tackled the problem via electric analogue instead of dealing directly with the mechanical components (see Figure 7.22). Civil Engineers may find it easier to understand the lump-mass simulation than the electrical analogue. Since theories of lumped-mass modelling of mechanical systems are fully developed, as indicated by the receptance method and the

four-pole techniques, it is therefore logical to tackle the problem by lumped-mass simulation.

A realistic model for a foundation pile is shown in Figure 7.23. The pile body is modelled by a number of rigid masses with internal friction modelled by springs k_p and material damping by the dampers c_p . The spring k_b and damper c_b at the base are used to model the stiffness and damping of the foundation. Soil stiffness and damping are represented by spring k_s and damper c_s . Technically, the problem can be solved by the four-pole techniques using a step-by-step combination of parallel-connected and series-connected components. The calculation would be very tedious and time-consuming. Whilst numerical simulation can be done by a computer, the main difficulty with such a model is choosing the correct stiffness and damping values. The stiffness value and the internal damping coefficient of the pile itself may be estimated with confidence by well-controlled laboratory experiments. The only problem lies with the soil parameters which are almost impossible to estimate with reasonable accuracy. Even though reliable soil parameters can be obtained, they are highly variable and dependent upon soil plasticities or density, grading, moisture content and stress history.

7.6.1 Parameters For Dampers And Springs

As for many simulation processes, the most critical part lies in choosing the appropriate parameters for the individual components. Davis and Dunn³⁷ suggested expressions for the parameters for dampers and springs. They are related either to the pile, the surrounding soils or the base properties. These expressions are listed below as:

$$M = \rho A l \quad (7.65)$$

$$k_p = EA/l \quad (7.66)$$

$$c_p = 0 \quad (7.67)$$

$$k_s = \pi G l \quad (7.68)$$

$$c_s = 2\pi r l \rho' \beta' \quad (7.69)$$

$$k_b = 1.84rE_b/(1-\nu^2) \quad (7.70)$$

$$c_b = 0 \quad (7.71)$$

where

- M = sectional mass of pile
- k_p, c_p = internal stiffness and damping coefficient due to pile
- k_s, c_s = external stiffness and damping coefficient due to surrounding soil
- k_b, c_b = base stiffness and radiation damping
- ρ = density of pile
- A = cross-sectional area of pile
- l = sectional length of pile
- G = shear modulus of surrounding soil
- r = radius of pile
- ρ' = density of surrounding soil
- β' = shear wave velocity of surrounding soil
- E_b = Young's modulus of base material
- ν = poisson's ratio of base material

The coefficients of internal damping (c_p) and radiation damping (c_b) are normally equated to zero because the damping they offer is very low compared to the damping due to the surrounding soils.

7.6.2 Computing Algorithm

The computation of the mechanical admittance of a complex lumped-mass system such as the one shown in Figure 7.23 is not as straightforward as using the series and parallel connections of the four-pole techniques. The inclusion of soil effects on the overall dynamic behaviour of the whole system creates a problem. Since the dynamic output of the soil components is not connected back to the pile body, it becomes impossible to use the parallel connection technique. This may be the reason why so far no work on the lumped-mass simulation of the pile-soil interaction has been

reported , though other indirect means of tackling the problem such as electric analogue have.

A careful study of the situation represented in Figure 7.23 shows that it is possible to solve the problem purely in terms of a lumped-mass system. This, however, would require a slight manipulation of the side-branching of the soil components. In Figure 7.24 the output points of the soil components are all connected up to the rigid base upon which the pile is supposed to be supported. This alteration will not change the dynamic behaviour of the whole system, yet it allows the system to be tackled by series and parallel connection techniques step by step. The computation algorithm is listed below as:

```

Set overall system matrix  $[\alpha] = [1]$ 
 $[\alpha] = [\alpha] \times B[i]$ 
For i=n downto 1
 $[\alpha] = [\alpha] \times P[i]$ 
 $[\alpha] = [\alpha] * S[i]$ 
 $[\alpha] = [\alpha] \times M[i]$ 
next i

```

where $[\alpha]$ = overall system matrix
 $[1]$ = identity matrix
 $\times$ = series connection computation
 n = number of discretized sections
 $*$ = parallel connection computation
 $B[i]$ = base stiffness and damping matrix (see Figure 7.24)
 $P[i]$ = pile stiffness and damping matrix (see Figure 7.24)
 $S[i]$ = soil stiffness and damping matrix (see Figure 7.24)
 $M[i]$ = pile mass matrix (see Figure 7.24)

Following the algorithm one would arrive the expression for the overall system matrix as defined by:

$$\begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} F_n \\ v_n \end{bmatrix} \quad (7.72)$$

where F_1, v_1 = response at pile top
 F_n, v_n = response at pile bottom

7.6.3 Testing Of The Lumped-Mass Model

The accuracy of the simulation model is largely dependent upon the discretization process. The greater the number of elements means longer computation time, but on the other hand a smaller number of elements may result in a large discrepancy in the calculation. To test convergency a rod was simulated as a lumped-mass system and the result compared with those obtained by treating the rod as a continuum. It was found that the degree of discretization required to keep the discrepancy within an acceptable level is related to the frequency range of interest. A high degree of discretization is needed to cover high frequencies. If the highest frequency of interest is 2500Hz for a concrete rod of velocity 4000m/s, then the shortest wavelength in the rod is 1.6m ($4000/2500$). A discretization of 0.1m per element will be quite adequate since it allows 16 elements to describe the vibration at the highest frequency.

A program (Mech_Admit) based on the earlier discussed theory and algorithm was written to simulate the dynamic behaviour of piles (see Appendix B). In order to test the correctness of the program two previous simulations by Davis and Dunn were made with the program. Very good agreement was obtained between the simulations (see Figures 7.25 and 7.26). Note that the dynamic stiffness value was also computed automatically by the program. This demonstrates that the lumped-mass simulation is as successful as the electric analogue.

7.6.4 Typical Example Of Interpretation

A perfect model pile, shown in Figure 7.27, of length 25m was simulated by the program (Mech_Admit). The input information and the result of

simulation are given below.

Input information:

NUMBER OF SUBSYSTEMS= 1

File Properties

Diameter= 1000.000 mm

Length= 25.00 m

Density= 24.000 KN/m-cube

Evalue= 38.400 KN/mm-square

Damping constant= 0 Ns/m-square

Soil Properties

Soil Density= 18.240 KN/m-cube

Soil Shear Wave Velocity= 200.000 m/s

Base Properties

Evalue= 1.992 KN/mm-square

Poisson's ratio= .25

Simulation results:

FREQUENCY INTERVAL= 10 Hz

FREQUENCY RANGE= 0 Hz to 1000 Hz

RESONANCE FREQUENCY (Hz)	MECHANICAL ADMITTANCE (m/s/N)
100	1.22939843706E-7
170	1.32114075226E-7
240	1.34514656266E-7
320	1.3570550662E-7
400	1.3619075384E-7
480	1.36368493492E-7
560	1.36380404272E-7
640	1.36286232586E-7
710	1.36221378587E-7
790	1.36378919362E-7
870	1.36473465184E-7
950	1.36509331588E-7

A graphical plot of the result, shown in Figure 7.28, shows that the resonant frequency interval (80Hz) matches with the expected value for a pile of 25m with velocity 4000m/s.

This example is used to check the validity of the theoretical interpretation of the vibration testing method (refer to section 3.3.1). According to equations (3.6), (3.7) and (3.13), the following parameters can be obtained from the information of the pile-soil system as:

$$\begin{aligned}\sigma &= 0.076\text{m}^{-1} \\ N &= 1.3263 \times 10^{-7} \text{ m/s/N} \\ M_p &= 47123.89 \text{ kg}\end{aligned}$$

Again according to equations (3.11), (3.10) and (3.13) the above parameters can be calculated from the simulation results as:

$$\begin{aligned}\sigma &= 0.077\text{m}^{-1} \\ N &= 1.3066 \times 10^{-7} \text{ m/s/N} \\ M_p &= 47123.89 \text{ kg}\end{aligned}$$

This shows a very good agreement between the theoretical and simulated values for the various parameters. Hence the theoretical interpretation given in section 3.3.1 is validated.

7.6.5 Simulation To Study Various Aspects Of The Pile/Soil System

In the following subsections the effects of the surrounding soils, the base properties and the influence of the top part of the pile being exposed above the ground on the mechanical admittance and the dynamic stiffness will be investigated. The model piles are classified according to their length/diameter ratios as friction or end-bearing piles. As far as this simulation is concerned, friction piles refer to those with a length/diameter ratio of 20 and end-bearing piles refer to those with a length/diameter ratio of 10. All model piles are perfect with straight shafts and assumed to be made up of good

quality concrete of velocity 4000m/s. Throughout this simulation the shear wave velocity of the surrounding soil is chosen to be 50m/s – 200m/s, as suggested by Davis.¹¹¹

7.6.5.1 Effect of soil resistance

This investigation has concentrated on models of friction piles since these piles are designed in such a way that the load is transferred to the surrounding soils. Two situations were studied. The first one corresponds to piles sited on loose materials, with longitudinal velocity of 100m/s, and the surrounding soil shear wave velocity being varied from 100m/s to 150m/s (see Figures 7.29 and 7.30). The second situation corresponds to piles sited on dense materials, with longitudinal velocity of 4000m/s (velocity of concrete), and the surrounding soil shear wave velocity being varied from 50m/s to 150m/s (see Figures 7.31, 7.32 and 7.33).

In both cases, it can be seen that the dynamic stiffness value increases as the shear wave velocity of the surrounding soil increases and offers stiffer support to the pile. The difference between the peak and the trough of the mechanical admittance curve decreases, indicating a heavier damping, as the soil becomes stiffer. The materials at the base also affect the dynamic behaviour, the stronger is the material, the stiffer is the pile. In all simulations, regardless of the soil resistance and base properties, the frequency interval ($\Delta f = 200\text{Hz}$) remains unchanged and agrees well with the pile length (10m).

7.6.5.2 Effect of base fixity

The effect of base fixity on the dynamic behaviour of a pile is best studied with an end-bearing pile since this type of pile is designed to transfer

load to its base. Two sets of results are available. The first set, shown in Figures 7.34, 7.35, 7.36, 7.37 and 7.38, corresponds to a pile embedded into very soft surrounding soils, with shear wave velocity of 50m/s. The second set, shown in Figures 7.39, 7.40, 7.41, 7.42 and 7.43, corresponds to a pile embedded into a stronger surrounding soil, with shear wave velocity of 100m/s. In both sets the longitudinal wave velocity of the base material was varied from as low as 100m/s to 4000m/s and its effects on the dynamic behaviour of the pile/soil system were studied.

The overall conclusion that can be drawn from these two sets of results is that the dynamic stiffness of an end-bearing pile increases as the base becomes stronger. It is interesting to find that as the fixity of the pile base is increased, the fundamental frequency of the pile-soil system shifts from the case of a free-end pile to that of a fixed-end pile. In Figure 7.34 the harmonic series is associated with wavelengths of $2L$, $4L$, $6L$ and so on, which corresponds to the vibration of a freely-suspended rod of length L . This type of response is expected from a pile which is supported by a very soft surrounding soil and a very compressible base. The stiffness cannot be computed since the initial slope is negative.

In section 7.4.5.1 it has been concluded that the dynamic stiffness of a friction pile increases as the base stiffness is increased. This may not always be true if the surrounding soil itself is so stiff that the pile is insensitive to the base stiffness. An example may help to clarify this point. A set of simulation results is obtained for friction piles embedded in a very stiff soil, with shear wave velocity of 200m/s. The base longitudinal velocity is varied from 10m/s to 10000m/s (see Figures 7.44, 7.45, 7.46, 7.47, 7.48, 7.49, 7.50 and 7.51). Some of these velocities may be fictitious, yet they serve the purpose of creating an

ideal end condition. From the diagrams it can be seen that the fundamental frequency shifts according to the base fixity but in no case has it approached near enough to the situation of a fixed-end for a pile to be interpreted as such. Theoretically, the fundamental frequency for these piles with a fully fixed end is 100Hz. The nearest fundamental frequency at 140Hz was obtained in Figure 7.51 when a fictitious base longitudinal velocity of 10000m/s was used. For this reason it is not recommended to determine the fixity of a pile base from the position of the fundamental frequency as suggested by Davis and Dunn.³⁷ The dynamic stiffness, varying from 1.32199MN/mm to 1.42934MN/mm is not sensitive to the base fixity even for a significant change in the base longitudinal velocity. The frequency interval (200Hz) for all the cases, however, agrees very well with the pile length.

7.6.5.3 Effect of part of a pile exposed above soil

The effect of the top part of a pile exposed above soil was studied with friction piles of 0.5m diameter and 10m in length. As a control model, a fully-embedded pile was simulated and the result plotted in Figure 7.46. The same pile was modelled again, first with a section of 0.5m exposed and then with a section of 1m exposed. The results are plotted in Figures 7.52 and 7.53. Comparing the results shows that the stiffness of the pile decreases as a longer length of the pile is exposed above ground. The mechanical admittance curve becomes less steady and appears to be enclosed by a secondary envelope. The effect is expected to a lesser extent for an end-bearing pile embedded into a soft surrounding soil since this type of soil offers very little skin friction force.

7.6.6 Simulation To Study The Effects Of Variations In Cross-Sectional Area

In the following subsections the effects due to changes in cross-sectional area will be studied. The three most common types of defect (necking, overbreak and a combination of the two) will be considered. This investigation will be concentrated with a 10m long pile with a diameter of 0.5m supported by stiff soils (shear wave velocity = 200 m/s). Defects of 1m long are introduced at the mid-section of the model piles.

7.6.6.1 Effects of necking

In Figure 7.54 the mechanical admittance curves of two defective piles and a normal pile of the same length are plotted. The reduced diameters of the defective piles are 300mm and 400mm, which represent an area reduction of 64 percent and 36 percent respectively. In the first case, the area reduction is so great that the pile behaves almost like a broken pile of 5m long. In the second case, two sets of harmonics can be seen, one corresponds to a defect at a depth of 5m ($\Delta f_1=400$ Hz) whilst the other one corresponds to the full length of the pile ($2\Delta f_2=400\text{Hz}$). Note Δf_2 is an averaged value and it is clear from the graph that the differences in frequency between peaks are no longer in a regular interval. This illustrates that interpretation of defective piles may not be as simple as identifying regular frequency intervals from a mechanical admittance plot and calls for more complicated analysis techniques such as liftered spectrum in conjunction with cepstrum.

The fundamental frequencies of both necked piles are slightly lower than those of the normal pile (see Table 7.10). The precise positions and the natures of the defects, however, cannot be derived from these frequencies.

Apart from distortion of resonance peaks, a slight decrease in dynamic stiffness value is also noticed, which of course would depend on the extent of the defect and also on the soil properties.

7.6.6.2 Effect of overbreak

Figure 7.55 shows the mechanical admittance curves of two defective piles and a normal pile. All the piles are of 10m long and are embedded in the same soil strata. The first defective pile has an overbreak of diameter of 600mm which accounts for a 44 percent increase in cross-sectional area. The second defective pile has an overbreak of diameter of 700mm, corresponding to an area increase of 96 percent.

Basically, the interpretation is quite similar to that of the necked piles. The position of the overbreak and the pile length can be estimated by identifying different sets of harmonics in the curve. The fundamental frequency, however, yields no information about the position and the nature of the defect. If only one curve is given, it would be almost impossible to determine whether the defect is associated with a reduction or an increase in area. Again this demonstrates the need of a better analysis technique, not just in identifying the frequency interval but also in distinguishing a neck from an overbreak.

Strictly speaking an overbreak should be regarded as a defect since it does not conform with the pile design specification. However, structurally an overbreak may be an advantage to the pile performance. This is revealed in the increase in the dynamic stiffness value of an overbreak pile. In fact this advantage has been utilized in the construction of underreamed piles which may be thought as a pile with an overbreak at the bottom.

7.6.6.3 Effect of a necked area above an overbreak

This is one of the most common types of pile defect which has been described in Chapter 2. In the simulation, two different degrees of damage are investigated. The first one with a necked area of diameter of 400mm above an increase diameter of 600mm. The second one is more substantially damaged, with a reduced diameter of 300mm above an increased diameter of 700mm. In both cases the lengths of necking and overbreak are each 500mm. Simulation results of these two cases with a normal pile are plotted in Figure 7.56.

In the case of a smaller defect, its position and the pile length can be estimated from the frequency intervals. In the case of a larger defect, only its position can be estimated but not the overall length. Again, the first resonant frequency fails to precisely indicate the position or the nature of the defect.

A drop in dynamic stiffness is also found with both defective piles. The increase in dynamic stiffness caused by an overbreak therefore cannot compensate for the decrease caused by a comparable reduction in area of a necked section.

7.6.7 Simulation To Study The Effects Of The Position Of A Defect

In this section, the effects of the position of a defect on the dynamic behaviour of the pile/soil system will be investigated. The study will be concentrated on a 10m long pile with a diameter of 0.5m. The soil and the base conditions are similar to those reported in the last subsection. Necking with diameter 400mm and length 1m long are introduced to the pile at different positions. Altogether 5 cases of simulation have been carried out, each of which corresponds to a defect centred at 3m, 4m, 5m, 6m and 7m as measured from the pile top respectively.

The simulated mechanical admittance curves of the 5 cases are shown in Figures 7.57, 7.58, 7.59, 7.60 and 7.61 separately. Two sets of harmonics can be identified in each of the curves. The frequency intervals of these harmonic families are given in Table 7.11, with Δf_1 and Δf_2 corresponding to the position of the defect and the pile overall length respectively. Results for a normal pile are also included in the table for comparison. Using the assumed concrete velocity and the identified frequency intervals, the defect positions can be estimated as at 3.45m, 3.57m, 5.13m, 5.41m and 3.45m respectively for the 5 cases. Apart from the last case, all the estimated positions are acceptable when compared with the actual positions. Due to difficulties in separating the harmonic families by visual inspection, mis-interpretation may arise as indicated in the last case (see Figure 7.61 and Table 7.11).

The change in the first resonant frequency is not sensitive enough to the position of a defect (see Table 7.11). Again it demonstrates that the first resonant frequency is dependent upon the pile/soil interaction rather than merely on the pile length. Therefore, it is not recommended to determine the position or the nature of a defect from the first resonant frequency.

The extent of the defects, which are the same for the 5 cases, cannot be determined from the simulated results. The position of the defect, however, influences the dynamic stiffness value (see Table 7.11). The dynamic stiffness value decreases as the position of the defect becomes nearer to the pile top. This confirms that a defect near the pile top is potentially more dangerous than one near the bottom in the case of friction piles.

7.6.8 Summary Of Results And Conclusions

The effects of soil resistance, base fixity and length of exposed pile on the dynamic behaviour of the pile/soil system are summarized in Table 7.9.

Figure number	Pile code	Pile type (l/d ratio)	Soil properties : shear wave velocity (m/s)	Base fixity : longitudinal velocity (m/s)	Length exposed (m)	Fundamental frequency (Hz)	Frequency interval Δf	Dynamic stiffness E (MN/mm)
7.29	B100S100	(20)	100	100	0	50	200	0.54829
7.30	B100S150	(20)	150	100	0	200	200	0.97067
7.31	Soil50	(20)	50	4000	0	100	190	0.70509
7.32	Soil100	(20)	100	4000	0	100	190	0.85597
7.33	Soil150	(20)	150	4000	0	120	190	1.11173
7.34	EBase100	(10)	50	100	0	20	200	---
7.35	EBase1000	(10)	50	1000	0	50	200	1.14902
7.36	EBase2000	(10)	50	2000	0	80	190	1.96164
7.37	EBase3000	(10)	50	3000	0	90	190	2.34848
7.38	EBase4000	(10)	50	4000	0	90	190	2.51338
7.39	EB100S100	(10)	100	100	0	30	200	1.02523
7.40	EB1000S100	(10)	100	1000	0	50	200	1.41043
7.41	EB2000S100	(10)	100	2000	0	80	190	2.05486
7.42	EB3000S100	(10)	100	3000	0	90	190	2.56552
7.43	EB4000S100	(10)	100	4000	0	90	190	2.71444
7.44	Base10	(20)	200	10	0	210	200	1.32199
7.45	Base100	(20)	200	100	0	210	200	1.32233
7.46	Base1000	(20)	200	1000	0	240	190	1.37168
7.47	Base2000	(20)	200	2000	0	270	190	1.39191
7.48	Base3000	(20)	200	3000	0	150	190	1.40201
7.49	Base4000	(20)	200	4000	0	150	190	1.40668
7.50	Base5000	(20)	200	5000	0	140	190	1.42796
7.51	Base10000	(20)	200	10000	0	140	200	1.42934
7.52	T500	(20)	200	1000	0.5	240	200	1.32075
7.53	T1000	(20)	200	1000	1.0	230	200	1.17829

length of pile = 10m
diameter of pile = 500mm
velocity of concrete = 4000m/s

Table 7.9

Conclusions drawn from the above simulation exercise are:

1. It is not always possible to determine the base fixity of a

- pile by the fundamental frequency. The fundamental frequency is also dependent upon the pile/soil interaction rather than merely on the base fixity.
2. In the cases of perfect piles the frequency interval is governed by the length of the pile, regardless of the soil conditions.
 3. In general the dynamic stiffness increases as the strength of the surrounding soils and the base material increases. However, in the case of a very stiff surrounding soil the dynamic stiffness is insensitive to the increase in the strength of the base material.
 4. The length of the exposed part of a pile also has an effect on the dynamic stiffness. A longer length exposed means lesser support to the pile and thus a lower stiffness value.

The effects of variation of cross-sectional area on the dynamic behaviour of the pile/soil system are listed in Table 7.10.

Figure number	Pile code	Necking		Overbreak		Fundamental frequency (Hz)	Frequency interval (Hz)	Dynamic stiffness E (MN/mm)
		diameter	position	diameter	position			
7.54 7.55 7.56	Normal500	---	---	---	---	240	200	1.484
7.54	Neck400	400mm	4.5m-5.5m	---	---	220	200,400	1.451
7.54	Neck300	300mm	4.5m-5.5m	---	---	200	400	1.401
7.55	Over600	---	---	600mm	4.5m-5.5m	240	200,400	1.509
7.55	Over700	---	---	700mm	4.5m-5.5m	250	200,400	1.530
7.56	N400O600	400mm	4.5m-5.0m	600mm	5.0m-5.5m	230	200,400	1.476
7.56	N300O700	300mm	4.5m-5.0m	700mm	5.0m-5.5m	220	400	1.451

length of pile = 10m
nominal diameter of pile = 500 mm
velocity of concrete = 4000m/s
soil properties: shear wave velocity = 200m/s
base fixity: longitudinal velocity = 1000m/s

Table 7.10

The following conclusions can be drawn regarding the simulated

defective piles:

1. In all the cases, the positions of defects can be estimated.
2. The overall pile length can be identified if the extent of the defect is small, say, less than 50 percent change in cross-sectional area.
3. The nature of a defect cannot always be determined.
4. The extent of a defect cannot be determined.
5. The fundamental frequency yields no information on the position or the nature of a defect.
6. The dynamic stiffness is affected by the type of defect. It decreases with a reduction in area and increases with an increase in area. In the case of a combination of the two types of defect, the decrease in stiffness due to a necked area outweighs the increase due to an overbreak even though the extents of change in area of the two defects are comparable.
7. The interpretation was based on the idea of inspecting the spectrum and finding out individual sets of harmonics. In many cases, a spectrum has been distorted to such an extent that an averaged frequency interval has to be used to calculate the pile length. This approach may not always work, especially in the case of a highly complex spectrum comprising several sets of harmonics.
8. Better analysis techniques are therefore required to decouple a vibration spectrum so that individual sets of harmonics can be identified. The filtered spectrum and cepstrum analyses, as introduced in Chapter 5, can be used for this purpose. Their applications to experimental and practical results will be demonstrated in Chapter 8 and Chapter 9 separately.

The effects of the position of a defect on the dynamic pile/soil system are tabulated in Table 7.11.

Figure number	Pile code	Necking		Fundamental frequency (Hz)	Frequency interval (Hz)		Dynamic stiffness E (MN/mm)
		diameter	position		Δf_1	Δf_2	
7.57	N400A3M	400mm	2.5m-3.5m	230	580	193	1.417
7.58	N400A4M	400mm	3.5m-4.5m	220	560	187	1.436
7.59	Neck400	400mm	4.5m-5.5m	220	390	195	1.451
7.60	N400A6M	400mm	5.5m-6.5m	230	370	185	1.462
7.61	N400A7M	400mm	6.5m-7.5m	240	580	193	1.471
7.55	Normal500	---	---	240	---	200	1.484

length of pile = 10m
nominal diameter of pile = 500mm
velocity of concrete = 4000m/s
soil properties: shear wave velocity = 200m/s
base fixity: longitudinal velocity = 1000m/s

Table 7.11

Conclusions drawn from this simulation are:

1. The position of a defect may be approximately identified by visual inspection of the mechanical admittance curve for sets of harmonics. However, mis-interpretation may arise in the case of a very complex spectrum.
2. The fundamental frequency cannot be used to determine the position of a defect.
3. For the same extent of defects, a difference in the dynamic stiffness value has been obtained for different positions of defects. Smaller dynamic stiffness value is obtained if a defect is near the pile top. Defects at the top few metres of a pile are therefore potentially more dangerous than those near the bottom, in the case of a friction pile.
4. The identification of sets of harmonics from a mechanical admittance curve by visual inspection may result in a mis-leading interpretation. More reliable, unambiguous analysis techniques, such as liftered spectrum and cepstrum, are thus required.

7.7 CONCLUSIONS

1. Time domain simulation by summation of wavelets, the convolution model and the method of characteristics have been investigated in this chapter. Detailed investigation of the effects of multiple reflections on the time traces has been undertaken on a real structure (Beam 2). Simulations by summation of wavelets allow a step-by-step study of the various wavelets produced by multiple reflections from different interfaces. It is a very rewarding exercise in the understanding of the situation in which a stress wave impinges a boundary of different acoustic properties. This method, however, is not very efficient in dealing with multi-interface structures. The method of characteristics was therefore used to simulate the double-protrusion beam (Beam 1) and the simulation was successful. For both beams, the nature of the defects (overbreak) can be determined from the simulated traces. The method of characteristics can be further developed into simulating practical results obtained from site tests if external damping and frictional forces are taken into account. The study as a whole shows that multiple reflections are the major obstacles to sonic-echo interpretation. Techniques such as cepstrum analysis may be useful in deconvoluting the effects of reflection overlapping. The application of cepstrum analysis on laboratory built model piles will be illustrated in Chapter 8.
2. Frequency domain simulation can be carried out using the receptance method or the four-pole techniques. The four-pole techniques are preferred as far as computer programming is

concerned. Both methods give the resonant frequencies of a vibrating structure. Very good agreement between simulated and experimental results have been obtained for Beam 1 and Beam 2. The methods are thus very useful in checking the dynamic behaviour of underdamped structures. If external damping and stiffness are included into the analysis, the four-pole techniques can be extended to simulate a pile/soil system.

3. Mechanical admittance simulation of pile/soil systems have been carried out by assuming a lump-mass model and solving the systems by the four-pole techniques. A number of different aspects as regard to the conditions of the pile/soil systems have been thoroughly investigated. Summary of results and conclusions have been given in the last section. By far the most important conclusion is that for a defective pile the mechanical admittance curve is normally distorted by at least two sets of vibration harmonics. Interpretation is therefore based on the identification of individual sets of harmonics by visual inspection. Mis-leading results may be obtained for a very complex spectrum. Better techniques, such as lifted spectrum analysis, are required to decouple a distorted spectrum so that interpretation can be carried out confidently.
4. As a result of the simulation work in this chapter a new approach to dynamic pile testing, making use of information obtained from both domains, can be adopted. The procedures of analysing pile test results may be listed as follows:
 - a. Analyse results in the time domain. If a defect is identified, its nature can be determined from the

phase information.

- b. Analyse results in the frequency domain. Calculate the dynamic stiffness value. The state of a pile can be revealed from its mechanical admittance curve. A curve associated with a regular frequency interval shows that the pile is perfect, otherwise the pile is defective.
- c. If a pile is found to be defective, an advanced analysis technique, such as filtered spectrum and cepstrum analyses, should be performed to identify the position and the nature of the defect.

These procedures will be adopted to analyse test results in the next two chapters.

CHAPTER 8

EXPERIMENTAL PROGRAMME

8.1 INTRODUCTION

The discussion on pile faults in Chapter 2 suggested that many pile defects are associated with a change of pile geometry from design shape. The most common anomalies are protrusions, neckings, cracks and inclusions. Pile performance and bearing capacity may be adversely affected if these defects occur in a pile.

So far no definite conclusions can be drawn about the ability of the two test methods on the identification of these anomalies. Although encouraging results obtained by the two methods had been reported in the past, there are still some doubts regarding the sonic-echo method in distinguishing a protrusion from a necking¹¹², equally scepticism still exists about the transient shock method in the identification of the effective length (see section 3.4.3).

An experimental programme was therefore designed with a view to investigating the ability of the two methods to identifying the major anomalies. In addition, the newly developed Edinburgh Method, using liftered spectrum and cepstrum analysis, will also be tested. It was believed that such an exercise would also be beneficial to the interpretation of site results.

8.2 EXSITING MODELS

Over the last few years two beams have been built at the University of Edinburgh for experimental work on non-destructive pile testing. One beam has double-protrusions along its length of 8m and the other one has a single protrusion in the middle of its 6m length (see Figures 8.1 and 8.2). The dimensions of the two beams can be found in Figures 3.7(a) and (b) respectively. Due to difficulties in constructing cylindrical models, both beams

are rectangular in cross-section.

8.3 NEW MODELS

Since the two existing models are rectangular in cross-section and both have protrusions as built-in defects, it was felt that cylindrical models with other anomalies would be useful. Cylindrical models are more realistic since bored piles are circular in cross-section.

Initially three model piles of length 5.5m were constructed, two of them were perfect with a uniform cross-sectional area running through their entire lengths and the third one had built-in necking along a section of its shaft. In order to have as many combinations of pile geometry as possible, the three model piles were tested and then modified at different stages. Figures 8.3, 8.4 and 8.5 show the life history of the models.

8.3.1 Structural Design Of Models

Originally the 3 model piles were designed to be perfect beams 5.3m long instead of the finally constructed length of 5.5m. One structural design was adopted for all the beams regardless of the slight difference in geometry. Since these model piles were not real piles and hence were not expected to carry the type of loading the actual piles would carry, the models were designed as circular beams subject to their dead load due to handling. These models were actually oversized to allow subsequent modification of the model geometry, for instance, the addition of a lump of concrete to a shaft as a protrusion.

Design calculation and reinforcement details are given in Appendices C1 and C2.

8.3.2 Concrete Mix Design Of Models

The design was aimed at producing a high workability concrete of slump 60–180mm with compressive strength 25 N/mm² at 28 days. Sieve analyses was performed on the fine aggregate available in the Department of Civil Engineering and the results showed that it should be classified as within grading zone 3 (see Appendix C3).

Several different mix designs were attempted and the trial mixes showed that low workability was always the problem. The problem was eventually traced to the fine aggregate used. The sand had been left outside the department uncovered for at least two years and as a result of this a large amount of organic matter such as tree roots and weeds had deposited and grown in it.

During mixing a large proportion of water would be absorbed by the organic material, hence producing a concrete mix of inadequate slump as well as poor strength. New fines were ordered and tests on trial mixes showed that workability (average slump = 150mm) and cube strength were both up to the design requirements. Mix design calculations are shown in Appendix C4.

8.3.3 Construction Of The Models

A plastic cylindrical mould of internal diameter 300mm and length 5500mm was provided by Civiltech NDT Ltd. To facilitate the dismantling of the mould without damaging it so that it could be re-used, the mould was cut into four sections, two upper half sections and two lower half sections (see Figure 8.6). During construction the cuts were sealed with strong tape to prevent leakage and the sections held together by specially made steel bands and wooden clamps, see Figure 8.7.

The reinforcement cages were made by tying a number of circular rings, at a fixed distance apart, to the six longitudinal bars. Figure 8.8 shows a completed steel cage which was later used for the construction of a normal model pile. For the necked pile a special technique was required within the cage to control the reduced area. This involved fixing a section of plastic pipe of a smaller diameter to that of the mould into the centre of the cage, with a circular hollow wooden shutter at each end (see Figure 8.9).

The lower section of an assembled mould with a reinforcement cage in position is shown erected prior to concreting in Figure 8.10. Once concrete filled up near the top of the lower section, the upper section was assembled and jointed to the lower section. Concreting started again from the very top of the mould until it filled up to the required level. Due to the danger of leakage of concrete from the sealed cuts, a concrete vibrator was only used for the necking model, as a high workability concrete was used. The mould was dismantled, from the model 48 hours after concreting, to be used again for another model. Each model was wrapped with a polythene sheet to prevent moisture loss from the surface and cured in this condition for at least 28 days. Figure 8.11 shows the first stage of the three completed model piles.

8.3.4 Modifications To The Models

A progressive circular saw cut was introduced to model pile 1 to simulate a cracked pile (refer to Figure 8.3(b)). Figure 8.12 shows the cutting operation, in which the model was cut to a depth from 10mm to 40mm in steps of 10mm. The cut was achieved by resting the model on two roller supports and rotated at an even rate against the blade of the cutter. The maximum cut depth achieved was 40mm as the model snapped into two pieces, as shown in Figure 3.18, under its own weight when deeper cut was attempted.

A circular bulb was added to each of the models as a further variation in the pile geometry (refers to Figures 8.3(c), 8.4(b) and 8.5(b)). Figure 8.3(c) simulated the situation where a protrusion is formed on a pile. Figure 8.4(b) simulated a more realistic case in which the concrete out with the reinforcement cage has slumped, resulting in the formation of a necked area above a protrusion. Figure 8.5(b) simulated a pile with an enlarged base, that is, a belled pile. The moulds and supports for the construction of bulbs are shown in Figure 8.14. The modified models are shown in Figure 8.15.

Clearly the method used in the casting of all the model piles has resulted in 'perfect' or uniform/symmetrical anomalies, and the validity of such models should therefore be examined. The protrusion on model 1 was roughened (refers to Figure 8.3(d)) with a Kango hammer. The roughened protrusion is shown in Figure 8.16.

The final modification was performed on model pile 3 (refers to Figure 8.5(c)). Segmental cuts, as shown in Figure 8.17, were made on the model to simulate a pile damaged by excessive bending or foreign material inclusion from the sides. The cut area was increased from approximately 5% to 15% in increments of 5%. Altogether 3 cuts were made and the model snapped into two parts when a fourth cut was attempted.

8.4 MODEL PILE WITH INCLUSION

This model was cast in order to confirm the belief that it is possible to detect an inclusion of foreign material into a pile.

The model pile, shown in Figure 8.18, is 3.7m long with a diameter of 300mm. The inclusion was made by inserting a half metre length of stacked circular polystyrene discs into the reinforcement cage (see Figure 8.19). The

inclusion was held in position by a triangle of wire at both ends as shown in Figure 8.20.

8.5 ANALYSIS OF THE EXISTING BEAMS (refers to Figures 8.1, 8.2 and 3.7)

8.5.1 Sonic-echo Interpretation Of Beam 1

Tests were performed on both ends of the beam, using the 3lb short sledge instrumented hammer with the black plastic cap.

Figure 8.21 shows the measurement setup and the velocity trace obtained by testing from End 1 of the beam. Due to the short distance between the first overbreak and End 1, there is no clear time lag between the end of the impulse signal and the commencement of the echo, making it very difficult to determine the reflection time. Nevertheless, an echo from the overbreak, at a distance of 1.9m from End 1, is just discernable with a reflection time of 1.083ms. The velocity of the stress wave in the beam is thus estimated to be 3500 m/s.

The velocity trace obtained from End 2 is shown in Figure 8.22. Since the distance of the first overbreak from End 2 is longer (3.9m), there is a clear time lag between the impulse signal and the echo signal. The reflection time is found to be approximately 2.227ms which confirms with earlier estimated propagation velocity.

In both tests, only the position of the first overbreak can be determined, no information about the second overbreak and the length of the beam can be derived. This can be explained by the fact that the overbreaks represent a substantial increase in cross-sectional area (1125 percent) and therefore most of the wave energy was reflected, leaving a small amount

transmitted through. Notice that the echoes in both tests are inverted indicating an increase of area (overbreak) rather than a reduction (necking).

8.5.2 Transient Shock Interpretation Of Beam 1

The force and velocity signals obtained from End 1 are both given in Figure 8.23. A transient window was applied to the impact force to eliminate noise and an exponential window was used on the velocity trace to reduce the effect of truncation errors. Details of the windows can be found in the measurement setup in Figure 8.24, which also shows the frequency response function (mechanical admittance curve) averaged over 10 samples.

Since the beam is not buried in soil, it is expected that very little damping would be encountered and hence the sharply peaked resonance obtained in the curve. However, due to the vibration coupling of the two overbreaks with the beam, the vibration pattern becomes very complicated and it is impossible to interpret the result.

8.5.3 The Edinburgh Interpretation Of Beam 1

The very complex nature of the frequency response function calls for a better analysis technique which would ideally be capable of separating the coupling effect of the various parts of the beam on the spectrum. A significant development in the Edinburgh University NDT analysis technique is the use of liftered spectrum in conjunction with the cepstrum. The liftered spectrum and the cepstrum of the above result are illustrated in Figure 8.25. In the cepstrum, a dominant cepstral at time 1.09ms shows a strong reflection from the first overbreak. This is again confirmed in the liftered spectrum when 11 elements, 2 elements more than the dominant cepstral, are allowed to pass the shortpass lifter of the cepstrum. A fundamental frequency at 472 Hz and a regular

frequency interval of 845Hz are obtained in the filtered spectrum. Using the frequency interval and assuming a velocity of 3500m/s, the centre of the overbreak is identified at 2.07m from End 1. Since the resonant frequencies are in an odd-numbered sequence (f , $3f$, $5f$, $7f$ and so on), the anomaly at 2.07m from End 1 is associated with an increase in area.

Similar analysis and interpretation can be applied to the test result obtained from End 2 of the beam (see Figures 8.26 and 8.27). In this case the cepstral at 2.19ms and the frequency interval of 432.5Hz both indicate the position of the overbreak at 3.9m from End 2. It is also worth-noting that the frequency response functions obtained from different ends of the beam are slightly different. This is due to the fact that the beam is not symmetrical along its mid-point.

The dynamic stiffness, computed in Figure 8.28, is very low simply because the beam was lying freely on the ground during testing.

8.5.4 Sonic-echo Interpretation Of Beam 2

In this analysis, the enhanced time (signal averaging) technique has been used. The averaged velocity trace and its cross-correlation with the hammer blow are both included in Figure 8.29. The overbreak at 2.7m from both ends is revealed by an inverted reflection signal at a time of 1.480ms, giving a velocity of propagation of approximate 3650ms.

A time domain simulation of stress wave propagation in the beam (see Figure 8.30 and refers to section 7.2.1.3) shows that the large reflection from 3ms to 4ms is a combination of a second reflection from the overbreak and the end reflection of the beam. Using cross-correlation the reflection time from the end of the beam is estimated as 3.250ms. Assuming a velocity of 3650m/s the

length of the beam is found to be 5.93m.

8.5.5 Transient Shock Interpretation Of Beam 2

The frequency response function (mechanical admittance against frequency) of Beam 2 is shown in Figure 8.31, with the upper diagram in linear scale and the lower diagram in dB scale. Once again the usual vibration interpretation fails to interpret the result. However, when plotted in dB scale, there appear to be sets or families of harmonics with one group having a frequency interval of 600Hz.

Computer simulation of the dynamic response of the beam, based on a lump-mass idealisation, indicates that there are two sets of harmonics with frequency intervals of 600Hz and 300Hz (see Figure 8.32). Using the earlier assumed velocity of 3650m/s, the centre of the overbreak and the length of the beam are found to be 3.04m and 6.08m respectively.

8.5.6 The Edinburgh Interpretation Of Beam 2

Since there are sets of harmonics in the frequency response function, it would therefore be interesting to test the ability of liftered spectrum and cepstrum in decoupling of them and to reveal the individual sets of harmonics. Figure 8.33 shows the result of allowing elements up to the first reflection of the overbreak to pass the lifter. The liftered spectrum is dominated by a set of harmonics of frequency interval of 610Hz, with the first one (the fundamental frequency) at 320Hz. The odd-numbered harmonic sequence (f , $3f$, $5f$,...) indicates the anomaly as an overbreak. A further attempt has been tried to allow elements up to the reflection from the end of the beam to pass the lifter and this resulted in a harmonic sequence of interval 305Hz, beginning with 304Hz. This is an even-numbered sequence (f , $2f$, $4f$ and so on) indicating a

free end. These two frequency intervals compare favourably with the values obtained earlier based on computer simulation. The dynamic stiffness is computed as 0.05473MN/mm as in Figure 8.35.

Note in this beam the overbreak represents a 73 percent increase in cross-sectional area, a much smaller increase compared to Beam 1. For this reason a substantial amount of wave energy can pass through the overbreak to reach the other end, making identification of the overbreak and the end of beam possible in both the time and the frequency domains. However, determination of the end of beam is not as clear as of the overbreak due to the effect of multiple reflections from the overbreak. The arrival of the end reflection in the time domain is obscured by the second reflection from the overbreak. The frequency interval associated with the end of the beam in the lifted spectrum is not very regular due to distortion by harmonic sequence of the overbreak. This problem may be solved if a bandpass lifter is available to eliminate the elements associated with the overbreak reflections. However, this facility is not provided with the Bruel and Kjaer analyser and would in many cases produce a too artificial analysis.

8.6 ANALYSIS OF NEW MODELS

All the models were tested when they were either suspended from a crane or were lying freely on the ground. In most testing the short sledge (3lb) instrumented hammer, with the black plastic cap, was used with excitation on the top end of the models unless otherwise specified.

Acceleration signals were recorded on tape and hence digital integration was required during analysis to obtain velocity response.

The objective of the following analysis is to determine the positions

and the type of defects. Since the models were not buried in soil, it is not intended here to calculate the dynamic stiffness value and for this reason the calibration of the system is ignored.

8.6.1 Model 1 (refers to Figure 8.3)

8.6.1.1 Stage 1

The measurement setup for the sonic-echo analysis of this perfect model and the captured hammer blow are both given in Figure 8.36. The acceleration and velocity traces are shown in Figure 8.37, from which the propagation velocity (3450 m/s) can be computed over an average of 9 end reflections.

The measurement setup for transient shock analysis and the acceleration trace are given in Figure 8.38. A transient or force window was applied to the hammer blow to reduce noise and an exponential window was used on the acceleration trace to reduce truncation error.

The frequency response function of 8 averages, plotted in linear and dB scales, is given in Figure 8.39. The even-numbered harmonic sequence gives the correct length of the free-ended model.

Figure 8.40 shows the force spectrum and the coherence function of the force and acceleration signals. It can be seen from the graphs that coherence is very poor above 1200Hz and that is attributed to the drop in impact energy of the black plastic tipped hammer beyond that range.

8.6.1.2 Stage 2

Figures 8.41, 8.42, 8.43 and 8.44 are the acceleration and velocity traces of the model corresponding to circular cuts of depth 10mm, 20mm, 30mm and 40mm respectively. The reduction in the cross-sectional area produced by these cuts are 12.89 percent, 24.89 percent, 36 percent and 46.22 percent separately.

In all the traces, a very strong reflection from the end of the model is noticed but no indication of the cuts can be obtained.

A similar experiment was performed by Steinbach and Vey^{31,32} on an aluminium bar, with circular cuts corresponding to an area reduction of 44.14 percent, 78.35 percent and 96 percent individually. They concluded that the first cut was apparently not deep enough to affect the form of the signal record whilst the latter two cuts could be identified.

The frequency response function of 9 averages, given in Figure 8.45, again only reveals the length of the model but not the position of the cuts.

8.6.1.3 Stage 3

The position of the added overbreak can be determined from the first echo in the acceleration and velocity traces of Figure 8.46.

The recorded traces were re-sampled with a slower sampling rate for transient shock analysis. The acceleration trace and the frequency response function are both included in Figure 8.47. Vibration interpretation is not possible without the use of liftered spectrum and cepstrum.

8.6.1.4 Stage 4

Similar traces and frequency response function (see Figure 8.48 and Figure 8.49) were obtained after the overbreak had been roughened to an irregular shape by a Kango hammer. Again the distorted frequency response function requires the use of liftered spectrum and cepstrum for its interpretation (see Figure 8.50 and Figure 8.51).

8.6.2 Model 2 (refers to Figure 8.4)

8.6.2.1 Stage 1

The necking represents a 71.5 percent reduction in area. Its position is revealed by the large reflection in the time traces of Figure 8.52. The frequency response function in Figure 8.53 cannot be interpreted. However, using liftered spectrum and cepstrum the position of the necking at 3.17m from the top has been successfully identified (see Figure 8.54).

The test was repeated with a steel-tipped hammer on the bottom end of the model. The steel tip was used since the necked area is at a closer distance from the bottom. A clear time lag between the impulse and the echo was obtained in Figure 8.55. A very complicated frequency response function was obtained, as shown in Figure 8.56. Subsequent analysis using liftered spectrum and cepstrum gave the necking as 1.63m from the bottom end of the model (see Figure 8.57).

8.6.2.2 Stage 2

A steel tipped hammer was used to strike the top end of the model in order to give a longer time lag between the impulse and the echo in the time domain, as well as allowing a higher frequency range for transient shock

analysis. Both the time domain and frequency domain analyses, shown in Figures 8.58 and 8.59 are successful in identifying the necked area. However, the newly added overbreak cannot be revealed by either method.

The above test was repeated on the bottom end of the model and the results are shown in Figures 8.60 and 8.61. The overbreak reflection is recognised in the time trace just before the arrival of a strong echo from the necking. From filtered spectrum and cepstrum analysis the positions of the overbreak and necking are estimated as 1.46m and 1.76m respectively (see Figure 8.62 and Figure 8.63).

8.6.3 Model 3 (refers to Figure 8.5)

8.6.3.1 Stage 1

The dynamic behaviour of this perfect model is manifested in both the time and frequency domains, shown in Figures 8.64 and 8.65. There is no difficulty in interpreting the results.

8.6.3.2 Stage 2

The base of the model was enlarged with an increase of 177.8 percent in area. This is revealed in the time traces, shown in Figure 8.66, as a small inverted echo just before the arrival of the very strong free-end reflection.

The frequency response function, in Figure 8.67, is slightly distorted. However, the fitted side-band cursors still indicate the full length of the model. This gives rise to the argument that whilst the sonic-echo analysis correctly suggested the existence of an enlarged base, the transient shock analysis failed to do so.

It is therefore interesting to attempt liftered spectrum and cepstrum analysis to see whether this technique can be used to determine the full length of the model as well as indicating the existence of an enlarged end. Figures 8.68 and 8.69 show that the technique is successful in revealing both.

A test was performed from the enlarged end of the model and the results are plotted in Figures 8.70, 8.71 and 8.72. In all the figures, the full length of the model can be easily determined. If the enlarged end is imagined as a pile cap, that would suggest both the sonic-echo and transient shock methods could be used to test piles after construction of the pile cap.

8.6.3.3 Stage 3

The time traces for cut 1, cut 2 and cut 3 (refer to Figure 8.17) are given in Figures 8.73, 8.74 and 8.75 respectively. The enlarged free-end of the model can be recognised from all the traces but the cut position is not readily seen. Careful inspection of the velocity traces reveals a dip at about 2.4ms which agrees well with the position of the cuts. The dip becomes steeper as the cut area is progressively increased.

The frequency response function, in Figure 8.76, gives the full length of the model as 5.54m. Using liftered spectrum and cepstrum the enlarged end can be estimated as at 5.00m from the top and the full length of the model as 5.7m (see Figures 8.77 and 8.78). However, both the frequency response function and the liftered spectrum analysis both failed to indicate the existence of the cuts.

8.7 ANALYSIS OF THE INCLUSION MODEL (refer to Figure 8.18)

The inclusion model was tested using the 3lb short sledge hammer. The black plastic tip was fitted for testing the top end and the steel tip fitted for testing the bottom end. Testing occurred within a week of concrete casting. It was expected that the propagation velocity of stress waves in the model would be quite low as the concrete had not been given sufficient time to develop its strength. The inclusion, made up of polystyrene, accounted for a 55.56 percent reduction in concrete cross-sectional area.

8.7.1 Test From The Top End Of The Inclusion Model

The acceleration and velocity traces are shown in Figure 8.79. The inclusion is identified as a same phase reflection at 1.251ms. The bottom end reflection can be approximately estimated as at 2.4ms. The bottom end reflection is confused by the second reflection from the inclusion. The propagation velocity can be computed as 3037m/s, which is low when compared with the velocity for concrete of normal strength.

Using this velocity, the frequency response function in Figure 8.80 gives a length of 3.37m, which is shorter than the full length of the model 3.76m.

8.7.2 Test From The Bottom End Of The Inclusion Model

The time traces obtained from testing the bottom end with the steel-tipped hammer are given in Figure 8.83. In the velocity time trace, the first and second reflections from the inclusion can be seen prior to the arrival of the echo at 2.456ms from the top end. The velocity of the stress wave propagation is found to be 3013m/s, which agrees well with the earlier

estimated velocity.

The frequency response function in Figure 8.84 fails to indicate either the position of the inclusion or the full length of the model.

Figure 8.85 shows the liftered spectrum and the cepstrum obtained by allowing the first reflection from the inclusion to pass the shortpass filter. Using the velocity of 3013m/s and frequency interval of 1160Hz, the inclusion can be estimated to be at 1.3m from the bottom end. The even-numbered harmonic sequence obtained due to the reduction in area by the inclusion will be noted.

The above analysis was repeated but with elements up to the end reflection being allowed to pass the lifter. The result, shown in Figure 8.86, is another even-numbered sequence with a frequency interval of 385Hz, from which the full length of the model is found to be 3.9m. This is slightly longer than the actual length. This discrepancy of 5 percent can be traced to the fact that while the lifter has effectively eliminated the effects of all elements beyond the end reflection, it is not possible to exclude the distortion caused by the first and second reflections from the inclusion. As mentioned earlier, this problem can only be solved by using a bandpass lifter instead of a shortpass lifter. However, if such a bandpass lifter is used, there is a danger that the result of analysis may become too artificial.

8.8 SUMMARY OF EXPERIMENTAL RESULTS AND CONCLUSIONS

Test number	Model	Test position	Feature	Positive Identification		
				Sonic-echo	Transient shock	Edinburgh method
1	Beam 1	End 1	1st overbreak	yes	no	yes
			2nd overbreak	no	no	no
			full length	no	no	no
2	Beam 1	End 2	1st overbreak	yes	no	yes
			2nd overbreak	no	no	no
			full length	no	no	no
3	Beam 2	Top	overbreak	yes	no	yes
			full length	yes	no	yes
4	Model 1 (Stage 1)	Top	full length	yes	yes	yes
5	Model 1 (Stage 2)	Top	circular cut	no	no	no
			full length	yes	yes	yes
6	Model 1 (Stage 3)	Top	overbreak	yes	no	yes
			full length	possible	no	yes
7	Model 1 (Stage 4)	Top	rough overbreak	yes	no	yes
			full length	possible	no	yes
8	Model 2 (Stage 1)	Top	necking	yes	no	yes
			full length	possible	no	yes
9	Model 2 (Stage 1)	Bottom	necking	yes	no	yes
			full length	no	no	no
10	Model 2 (Stage 2)	Top	necking	yes	no	yes
			overbreak	no	no	no
			full length	no	no	no
11	Model 2 (Stage 2)	Bottom	overbreak	yes	no	yes
			necking	possible	no	yes
			full length	no	no	no
12	Model 3 (Stage 1)	Top	full length	yes	yes	yes
13	Model 3 (Stage 2)	Top	enlarged base	yes	no	yes
			full length	possible	yes	yes
14	Model 3 (Stage 2)	Bottom	pile cap	possible	no	no
			full length	yes	yes	yes
15	Model 3 (Stage 3)	Top	segmental cut	possible	no	no
			enlarged base	yes	no	yes
			full length	possible	yes	yes
16	Model 4	Top	inclusion	yes	no	yes
			full length	possible	possible	yes
17	Model 4	Bottom	inclusion	yes	no	yes
			full length	yes	no	yes

Table 8.1

The six structures, comprising two existing beams and four new models, offered altogether 17 different combinations of tests. This was made possible by testing structures at various stages of modification and from different ends. The 17 tests have been interpreted by the sonic-echo method, the transient shock method and the newly developed Edinburgh Method (using filtered spectrum in conjunction with cepstrum analysis). A comparison of the three methods on the identification of the various built-in defects in each structure and its full length is given in Table 8.8. The following conclusions can be drawn from the experimental study undertaken upon undamped piles:

1. For undamped perfect models, the full length can be accurately determined by the sonic-echo method, the transient shock method and the Edinburgh method.
2. In all the experiments with cuts, circular or segmental, none of the above methods can positively identify their existence.
3. Defects such as overbreak, necking and inclusion can be identified by both the sonic-echo method and the Edinburgh method.
4. The transient shock method could not identify either the position of a defect or the full length of a defective model, except in the case of cuts where the full length of the model could be determined. Vibration coupling has constantly distorted the frequency response function, making it non-interpretable.
5. The sonic-echo method is the most effective in identifying the first defect. However, the position of subsequent defects or the full length cannot be determined confidently.
6. The Edinburgh method is the most promising. Not only were the first defects (except cuts) in all the tests identified, but subsequent defect or the model full length were determined

accurately, provided that the first defect was not too large.

CHAPTER 9

CASE STUDIES OF SITE PILES

9.1 INTRODUCTION

In this chapter a number of case studies of tests on site piles will be presented, with the intention of investigating and verifying the various techniques used in checking the integrity of piles. With knowledge gained from the experimental work undertaken upon model piles in the laboratory a degree of confidence exists in the interpretation of data collected on site.

The case studies were chosen to cover normal piles, defective piles and non-interpretable piles. A velocity of 4000m/s is assumed throughout the analyses. This will of course only give an approximation to the pile length as well as the position of defects. If a more reliable velocity is required, transmission test can be performed on concrete cubes or core samples of the piles. However, it is difficult to obtain a realistic estimate of velocity for a population of piles as the concrete in the cast-in-situ pile may:

1. come from more than one 'ready-mix' truck
2. be subject to curing at different temperature due to depth
3. be tested at differing ages across a site

9.2 CASE 1 (Leith of Edinburgh)

A total of 10 piles were tested on a visit to this site, using the Phase II Instrumentation System (see section 6.3). Only time domain analyses will be reported since this type of instrumentation was not very efficient in frequency domain analysis. The piles were 500mm in diameter and were supposedly 8m to 9m long.

A typical hammer blow and a velocity trace of a pile are included in

Figure 9.1. A distinct reflection signal of inverted phase can be observed in the velocity trace. Cross-correlation and auto-correlation, shown separately in Figures 9.2 and 9.3, confirm that the reflection time is 4.10ms. Assuming a velocity of 4000m/s for the concrete, this pile is estimated to be 8.2m long, which compares favourably with the constructed length. The phase-inverted echo suggested that the material at the pile base is acoustically stiffer than the pile concrete, indicating a good fixity of the pile. The piles were constructed through fill to dense sand.

9.3 CASE 2 (Portobello, Edinburgh)

Four tension piles of 1000mm diameter and length between 8m to 9m were tested, using Phase III instrumentation. Due to the massive area of the piles, it was decided to use the 12lb large sledge hammer for testing in order that more impact energy could be injected into the piles.

The hammer blow and the measurement setup of a pile for sonic-echo analysis are shown in Figure 9.4. An impact force of very large amplitude (50KN) has been obtained by striking the pile top with the sledge hammer. Because of the very high energy input and the low slenderness ratio of the pile, a very good result was obtained. Ten velocity traces were averaged using the enhanced time technique. The enhanced trace and its cross-correlation with the impact force are given in Figure 9.5. Several echoes can be observed from the averaged velocity trace and the first reflection time obtained from cross-correlation is 4.165ms. The pile length can be approximated to be 8.33m by assuming a velocity of 4000m/s. The in phase echoes indicate soft material at the pile base. The tension piles were therefore designed to carry load by friction along their sides.

The frequency response function of the pile and the measurement setup for transient shock analysis are both included in Figure 9.6. A very regular interval of 238Hz exists, indicating the pile is perfect with a length of 8.40m. The force spectrum and its coherence function with the velocity spectrum are given in Figure 9.7. It can be seen that above 1170Hz coherence is very poor, coinciding with the low force energy input from the hammer. Therefore interpretation should not be extended beyond this cut-off frequency as the frequency response function becomes unstable.

Liftered spectrum and cepstrum analysis, though not necessary in this case, confirm that the earlier interpretation is correct (see Figure 9.8). Figure 9.9 compares the differences in the liftered spectrum resulted from allowing and restricting the reflection cepstral to pass the shortpass lifter.

During the computation of dynamic stiffness value, two particular problems were encountered. The first one was caused by the lower frequency limit of electronic integration (see section 6.7.2). According to the specifications of the PCB electronic integrator, the lower frequency limit is 10Hz, below which no integration will be performed. The non-linear part of the frequency response function was therefore ignored and the dynamic stiffness value was calculated from the linear part above 10Hz (see Figure 9.10).

The second problem was associated with the use of weighting functions. Table 9.1 compares the effects of different weighting functions on the dynamic stiffness value. According to section 5.2.4 an exponential window is most appropriate for a lightly damped response which has not decayed to zero by the end of the analyser time record in order to reduce truncation error. Applying such a weighting function will inevitably increase the damping of the structural response. This artificial damping will have a significant effect upon

the value of dynamic stiffness, as demonstrated in Table 9.1. Since most site test results are heavily damped, no truncation error will occur and a rectangular weighting function is the correct window to be used. However, if the signal to noise ratio is to be increased, a transient window is recommended in order to eliminate noise effects at the end of the response (see Figure 9.11).

Weighting function	Weighting constant $T_w(\text{ms})$	Artificial damping $\sigma_w(\text{rad/s})$	Dynamic stiffness $E(\text{MN/mm})$
Exponential	10	100	5.0
	15	66.67	4.0
	50	20	1.7
	100	10	1.6
	500	2	1.5
Rectangular	---	0	1.3
Transient	40	0	1.3

Table 9.1

9.4 CASE 3 (St, Enochs Square of Glasgow)

A friction pile of 600mm diameter and length 23.7m was tested, using both the 3lb short sledge and the 12lb large sledge hammers. The pile had a reinforcing cage of 7m in length at the head of the pile.

The velocity traces obtained by the two hammers are shown in Figure 9.12 and Figure 9.13 separately. The time domain analysis on both tests show that the existence of an overbreak at a depth of 6.56m and the base of the pile at 18.89m. The pile is regarded defective due to its shorter length, but not because of the existence of an overbreak.

The transient shock analysis of this pile reported below is based on tests using the large 12lb sledge hammer. The frequency response function and its measurement setup are given in Figure 9.14. A quite regular frequency interval was obtained which corresponded to the position of the overbreak. It can be predicted from the figure that the overbreak was of quite a large size since a large fundamental frequency peak was obtained. This peak is mainly due to resonance of the pile shaft above the overbreak but not of the pile/soil interaction. The frequency response function, however, gives no indication of the pile length. The spectrum above 1000Hz should not be interpreted as will be apparent from an inspection of the force spectrum and the coherence function in Figure 9.15.

Using the shortpass lifter the liftered spectrum and the cepstrum, shown in Figure 9.16, indicate the correct position of the overbreak. Figure 9.17 shows the effects of allowing and restricting elements thought to be associated with the end reflection to pass the shortpass lifter. It appears that a regular frequency interval corresponding to the overall pile length exist when elements associated with the end reflection are allowed to pass. A longpass lifter, which allows elements from a specified point to the end of record to pass, has been tried successfully to reveal the end of the pile (see Figure 9.18).

Finally, the dynamic stiffness of this pile was found to be 0.71147 MN/mm as shown in Figure 9.19.

9.5 CASE 4 (St. Vincent Street of Glasgow)

The time domain analysis of a pile on this site is shown in Figure 9.20. An overbreak at a depth of 5.58m was revealed in the enhanced time record, however it is uncertain about the cause of the second reflection between 5ms

to 6ms. It could be a second reflection from the overbreak or the end reflection of the pile.

The frequency response function, shown in Figure 9.21, has a quite regular interval which suggests the pile length to be 11.11m. The position of the overbreak, however, cannot be determined.

Using a longpass lifter in liftered spectrum and cepstrum analysis, it was found that the second reflection was probably related to the pile end and suggested the pile to be 11.11m long (see Figure 9.22).

The dynamic stiffness was found to be 0.20093MN/mm as calculated in Figure 9.23.

9.6 CASE 5 (Rosyth of Scotland)

The velocity trace and the frequency response function for a 22.8m long pile tested on this site are shown in Figure 9.24. The response is heavily damped to such an extent that no echo can be visualized in the time domain and no regular frequency interval has been established in the frequency domain. The liftered spectrum and cepstrum analysis also fails to produce any useful information. The pile test result is therefore classified as non-interpretable. The dynamic stiffness of the pile, shown in Figure 9.25, is 1.87147MN/mm. This may be thought of as the only information obtained from the test.

9.7 CONCLUSIONS

From the five case studies on site piles, the following conclusions can be drawn:

1. Pile defects at a shallow depth can best be identified by the sonic-echo method. The method can distinguish an overbreak from a necking.
2. The transient response method can be used to predict the state of a pile. If a regular frequency interval is established in the frequency response function (mechanical admittance plot), it is either associated with the position of a defect or the overall length of the pile. This is dependent upon the extent of the defect.
3. If a defect exists in a pile, the vibration spectrum may be distorted to such a degree that no regular frequency interval will establish. In such a situation, the lifted spectrum and the cepstrum analysis should be attempted to separate the coupled spectrum.
4. A shortpass lifter can be useful in the identification of shallow defects whilst a longpass lifter may be used to identify the pile length or defects at a deep level.
5. For a heavily damped pile, it may become impossible to interpret the test results by any one of the methods. The only indication of the state of a piled foundation may be the dynamic stiffness from a transient shock test.
6. Due to the non-linearity properties of the electronic integrators, the computation of the dynamic stiffness value should be based upon admittances above the lower frequency cut-off limit of an integrator.
7. Weighting functions should be applied appropriately onto the test results if a meaningful dynamic stiffness value is required. For site piles, a transient window is recommended.
8. The coherence function and the force autospectrum can be used to check the quality of a test result.

CHAPTER 10

CONCLUSIONS AND RECOMMENDATIONS

10.1 CONCLUSIONS OF PROJECT

The purpose of this project was to investigate and to improve the two most popular methods of Non-Destructive Testing of piles — the sonic-echo and transient shock methods.

Specific conclusions have been given at the end of each chapter wherever it was felt appropriate. From the experimental and site work, undertaken in this project, the following main conclusions can be drawn regarding the various testing methods:

1. Surface waves can be effectively reduced by digital filtering, integration and low-frequency excitation. The latter two techniques are strongly recommended.
2. Necking and overbreak can be distinguished in the time domain.
3. The sonic-echo method is best for the identification of shallow defects.
4. Base fixity of a pile can be determined from a time trace.
5. Multiple reflections caused distortion of pulses and lead to non-interpretable results. Auto-correlation and cross-correlation can sometimes be useful in this situation but must be used cautiously.
6. The transient shock method can be used to indicate whether a pile is good or defective. However, if a defect does exist, coupling of different vibration modes may render the result non-interpretable.
7. The proper use of weighting function in the transient shock method is very important especially when a numerical output is required. The dynamic stiffness value varies

significantly with the type of weighting function being applied. For a heavily damped test result, the transient window is more appropriate.

8. The frequency content of the excitation force can be tailored, using different hammer tips, to excite a pile in the range of interest.
9. The quality of a captured signal should be checked by using autospectrum and the coherence function prior to carrying out analysis.
10. The Edinburgh Method is a significant improvement in the analysis of test results which combines the advantages of the sonic-echo and transient shock methods. Its power in decomposing convoluted time traces and decoupling distorted spectra has been successfully tested on the model piles, and its application has been verified in testing site piles.
11. Fine cracks/cuts may not be identified by any one of the methods, unless the reduced area is quite substantial.
12. All these dynamic testing methods, however, suffer from the fact that if heavy damping is encountered, either due to stiff soils or a large slenderness ratio, the test results may become non-interpretable.
13. Debate occurs in the testing industry upon the relative superiority of the sonic-echo method and the transient shock method. Theoretically, the two methods have been shown to be different branches of the same family, linked together by a relationship called Fourier Transform. The family being the dynamic testing of piles. Since the testing procedures and instrumentation for the two methods are very similar, it can be argued that no extra information can be created when test results in one domain are transformed into the other domain. This point was demonstrated when

dealing with a heavily damped signal. When no identifiable echo was obtained in the time trace due to heavy damping, no harmonics with a regular interval could be expected in the frequency spectrum.

The simulation work in the frequency domain has yielded several important conclusions regarding the transient shock method:

1. It is not always possible to determine the base fixity of a pile by the fundamental frequency. The fundamental frequency is also dependent upon the pile/soil interaction rather than merely on the base fixity.
2. In the cases of perfect piles the frequency interval is governed by the length of the pile, regardless of the soil conditions.
3. The nature of a defect cannot always be determined.
4. The extent of a defect cannot be determined.
5. The fundamental frequency yields no information on the position or the nature of a defect.
6. The interpretation was based on the idea of inspecting the spectrum and finding out individual sets of harmonics. In many cases, a spectrum has been distorted to such an extent that an averaged frequency interval has to be used to calculate the pile length.
7. The above approach may not always work, especially in the case of a highly complex spectrum comprising several sets of harmonics. Mis-leading interpretation was obtained in one case of the simulation exercise. More reliable, unambiguous analysis techniques, such as filtered spectrum and cepstrum, should be used in the case of a complicated spectrum.

10.2 RECOMMENDATIONS FOR FUTURE WORK

The dynamic testing of piles using the sonic–echo and the transient shock methods has been extensively investigated in this thesis and has resulted in a better instrumentation system and an improved analysis technique. However, there are still a number of aspects which require further development in order that the various methods can be used more effectively to test site piles. Recommendations for future work are:

1. More work is required to accurately detect cracks in piles.
2. In the present thesis, work is mainly directed to the dynamic behaviour of undamped model piles, their interaction with surrounding soils has largely been neglected. Future work should concentrate on the study of pile/soil interaction. The correlation between dynamic and static stiffness could be investigated by testing short model piles embedded in different soil types.
3. The effect of soil damping is the major obstacle to the various testing methods. Easier ways of injecting higher impact energy into piles is desirable. One of the excitation techniques described in section 5.3.7 as Random Impact Excitation seems promising and deserves more attention in the future development of dynamic testing of piles.
4. The computer simulation programmes developed in this project, in both domains, should be further refined following with site testing. Pattern recognition in the time domain may be useful in the determination of the types of defect. Work should be directed towards obtaining representative parameters of the pile/soil model.

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APPENDIX

A.1 Manufacturer's specifications of the impulse hammers

SPECIFICATIONS

IMPULSE HAMMER

REVISIONS

SHEET 1 OF 1

MODEL NO		086820	086850
FREQUENCY RANGE	kHz	1.0	0.50
HAMMER RANGE (5V OUTPUT)	$1b_f(N)$	5000 (22000)	5000 (22000)
HAMMER SENSITIVITY (APPROX)	mV/ $1b_f$	1.0 (0.23)	1.0 (0.23)
RESONANT FREQUENCY	kHz	12	2.7
HAMMER MASS	$1b(kg)$	3.0 (1.4)	12 (5.4)
HEAD DIAMETER	in(cm)	2.0 (5.1)	3.0 (7.6)
TIP DIAMETER	in(cm)	2.0 (5.1)	3.0 (7.6)
HANDLE LENGTH	in(cm)	11.5 (29.2)	31.5 (80.0)
CONNECTOR (COAXIAL)	thd	10-32	10-32
CABLE (COAXIAL)	MOD NO	007T20	007T20
TIP SUPERSOFT	MOD NO	084A60	084A30
TIP SOFT	MOD NO	084A61	084A31
TIP MEDIUM	MOD NO	084A62	084A32
TIP HARD	MOD NO	084A63	084A33
CASE	MOD NO	001A11	081A16

MOD NO 081805 STUD (1)
MOD NO 007T20 CABLE (1)
MOD NO 070A02 ADAPTOR (1)
MOD NO 007T05 CABLE (1)

SUPPLIED ACCESSORIES:

APP'D	QWL	1/19/83
ENGR	REL	1/17/83
SALES	SL	1/19/83

SPEC No.

086-2200-80

A.2 Calibration certificate of the short sledge hammer, Model 20



PCB CALIBRATION CERTIFICATE

IMPULSE FORCE HAMMER

Model No. 006B20

Customer: _____

Serial No. 1214

Range 0-5000 lb

Linearity error <2.0 %

Invoice No.: _____

Discharge Time Constant 2000 s

Output Impedance 100 ohms

Output Bias 9.40 volts

Traceable to NBS through 737/234009-85 Calibration Specification MIL-STD 45662

Initials Tm Date: 1-17-85

Accelerometer: Model No. 308M147 Serial No. 12325 Sens. 100.0 mV/g

Pendulous Test Mass 116.1 lb (52.7K gram) including accelerometer

Hammer Sensitivity:

CONFIGURATION	Tip	MEDIUM (RED)		
	Extender	NONE		
SCALING FACTOR (SENSITIVITY RATIO)	lb/g	122		
		(SEE NOTE)		
Accel/Force	(N/ms ⁻²)	55.0		
	mV/lb	0.82		
HAMMER SENSITIVITY	(mV/N)	0.18		

Above data is valid for all supplied tips.

NOTES:

- The sensitivity ratio (Sa/Sf) is the scaling factor for converting structural transfer measurements into engineering units. Divide results by this ratio.
- Each specific hammer configuration has a different sensitivity. The difference is a constant percentage, which depends on the mass of the cap and tip assembly relative to the total mass of the head. Calibrating the specific hammer structure being used automatically compensates for mass effects.

Effective mass ----- with 302A07 attached and vinyl capped plastic tip.

A.3 Calibration certificate of the large sledge hammer, Model 50



PCB CALIBRATION CERTIFICATE

IMPULSE FORCE HAMMER

Model No. 086B50

Customer: _____

Serial No. 1240

Range 0-5000 lb

Linearity error <2.0 %

Invoice No.: _____

Discharge Time Constant 2000 s

Output Impedance 100 ohms

Output Bias 12.5 volts

Traceable to NBS through 737/234009-85 Calibration Specification MIL-STD 45662

Initials TH Date: 1-30-85

Accelerometer: Model No. 300M147 Serial No. 12325 Sens. 100.0 mV/g

Pendulous Test Mass 116.1 lb (52.7K gram) including accelerometer

Hammer Sensitivity:

CONFIGURATION	Tip	MEDIUM (RED)		
	Extender	NONE		
SCALING FACTOR (SENSITIVITY RATIO)	lb/g	93.5		
		(SEE NOTE)		
Accel/Force	(N/ms ⁻²)	42.1		
	mV/lb	1.07		
HAMMER SENSITIVITY	(mV/N)	0.24		

Above data is valid for all supplied tips.

NOTES:

- The sensitivity ratio (Sa/Sf) is the scaling factor for converting structural transfer measurements into engineering units. Divide results by this ratio.
- Each specific hammer configuration has a different sensitivity. The difference is a constant percentage, which depends on the mass of the cap and tip assembly relative to the total mass of the head. Calibrating the specific hammer structure being used automatically compensates for mass effects.

Effective mass

with 302A07 attached and vinyl capped plastic tip.

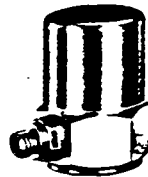
A.4 Specifications of the accelerometer (Model 308B15)

LOW-IMPEDANCE, VOLTAGE-MODE ACCELEROMETER Model 308B15



with built-in amplifier

- high sensitivity of 100mV/g
- shock protected to 5000 g
- stable quartz element
- insensitive to cable length or motion
- drives long coaxial or 2-wire cables
- ground isolated
- hermetically sealed



See Optional Model
below Specifications.



Typical System Kit
Model K308B15
Includes Battery Power Unit,
Cables and Accessories

Model 308B15 is a smaller, lighter weight version of the popular Model 308B Accelerometer; now available with hermetic sealing and ground isolation. Optional, nonisolated Model 308B14 is offered at lower cost.

Model 308B15, a modern, low-impedance, high-precision quartz accelerometer, routinely measures shock and vibratory motion in both laboratory and rough industrial environments on machine tools, machinery, vibrators, impact testers, vehicles, trains, boats, buildings, bridges, containers and other structures.

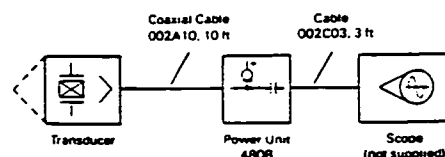
Containing a sensitive, proven quartz element and advanced, low-noise microelectronic circuitry, this precision instrument measures the acceleration aspect of vibratory motion to very low amplitudes and frequencies. Acceleration relates directly to force causing the motion.

Quartz accelerometers install by clamping the base surface in intimate contact with a precision machined surface or mounting pad on the structure of the test object with an elastic beryllium-copper stud. An optional magnetic base functions well up to 1000 Hz. The RTV boot supplied provides mechanical protection and transient thermal isolation. Electrical connection is usually by means of a single coaxial cable which conducts both signal and power, or 007A Series ribbon wire cable which is customer-repairable.

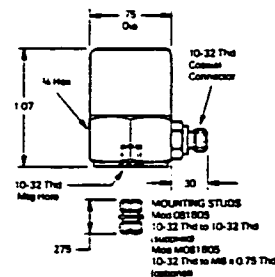
To operate the accelerometer, you need only a low cost battery or line power/signal conditioner as shown below in the typical system. A complete line of low cost single and multi-channel power units, with or without gain, is available. True RMS meter monitors like the 487A05, and 487A06 with high set point, also contain the power circuit for these accelerometers. The basic power circuit is now incorporated in many FFT analyzers.

SPECIFICATIONS:	Model No.	308B15
Range (for $\pm 5V$ output)	g	± 50
Resolution	g	.001
Useful Overrange	g	70
SENSITIVITY ($\pm 2\%$)	mV/g	100
Resonant Frequency (mtd)	Hz	25000
Frequency Range ($\pm 5\%$)	Hz	1-3000
Frequency Range ($\pm 10\%$)	Hz	0.7-5000
Discharge Time Constant	s	0.5
Amplitude Linearity	%FS	1.0
Polarity		positive
Output Impedance	ohm	100
Output Bias	V	+8 to 13
Overload Recovery	μs	10
Transverse Sensitivity (max)	%	5.0
Strain Sensitivity	g/ $\mu in/in$	.05
Temperature Range	$^{\circ}F$	-100 to +250
Temp Coefficient	%/ $^{\circ}F$	.03
Vibration (max)	g peak	1000
Shock (max)	g	5000
Structure		upright
Size (hex x height)	in	.75 x 1.10
Sealing		hermetic
Case Material		SS
Weight	gm	55
Connector (microdot)	coaxial	10-32
Ground Isolation	ohm	10^8
Excitation (Const Current)	mA	2 to 20
Voltage to Current Regulator	VDC	+18 to 28
Optional Model: (lower cost) Nonisolated & epoxy sealed		308B14

Typical Systems: K308B15 battery power kit is shown below. Also available as GK308B15 with gain. Recharge and long life external battery pack options available.



DIMENSIONS



A.5 Specifications of battery power unit (Model 480D06)

AMPLIFYING BATTERY POWER UNIT Model 480D06



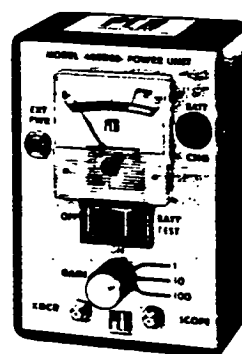
- amplifies signals X1, 10 or 100
- operates from 27V extending range of many ICP transducers to 10 volts
- includes connector for external power option
- includes connector for charger option

For use with low impedance (ICP) piezoelectric transducers, model 480D06 functions to power the transducer electronics, amplify the signal, debias the output signal and indicate normal or faulty system operation. It is a combination power supply and signal amplifier.

New model 480D06 features ± 27 volt power, provided by three standard 9 volt alkaline transistor radio batteries connected in series, to extend the dynamic range of many ICP transducers to 10 volts output. Other features include an external DC power jack for use with optional long life (1 year) battery pack model 073A05 and a battery charge jack for use with Model 488B Battery Charger / NiCd Battery option. If this option is ordered with the 480D06, NiCd instead of alkaline batteries will be provided. A decoupling capacitor located behind the output connector removes DC bias on the output signal and provides a drift free A.C. mode of operation. The self-test meter circuit, monitoring bias on the signal, indicates normal operation (green), short (red), or open circuit (yellow). It also checks battery voltage (± 27 V) when the rocker switch is depressed to the right. Circuits are housed in a shielded plastic case with a metal panel. Connectors are coaxial micro 10-32 jacks. Options and accessories include the following by model number:

480D09 - Power Unit with BNC connectors
480D06/488B - Power Unit with NiCd batteries and charger
488B - Charger with 3 NiCd batteries
073A09 - NiCd rechargeable battery
073A05 - Long life external battery pack

Sensor kits including Model 480D06 Power Unit simplify specifying and ordering. These kits contain the sensor (transducer), sensor cable (10 ft. long), power unit, scope cable (3 ft. long terminated in BNC) and accessories. To specify, add prefix GK to the sensor model number and add prefix cost to the sensor price. Use prefix GKR to specify kit with NiCd battery and charger option, e.g. GKR302A Accelerometer Kit.



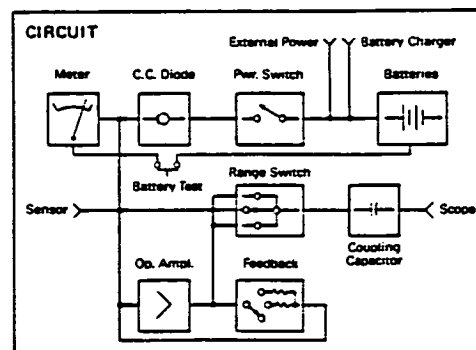
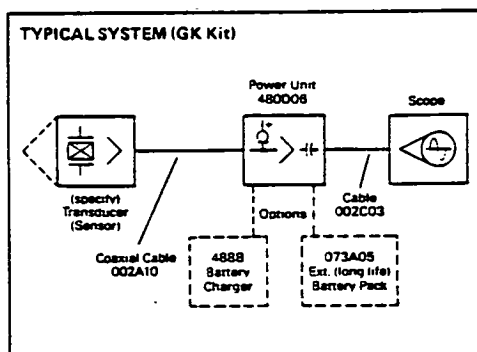
Model 488B Battery Charger with 3 NiCd Batteries (optional)



Model 073A05 long life, external Battery Pack (optional)

SPECIFICATIONS: Model No.		480D06
Transducer Excitation	VDC	± 27
Excitation Current	mA	2
Voltage Gain (selectable)	x	1, 10, 100
Coupling Capacitor	μ F	22
Frequency Range ($\pm 5\%$) ⁽¹⁾	Hz	0.15 to 100 000
Output Signal	V	± 10 , -5
Noise Output (PK to PK) X1	mV	0.2
X10	mV	2
X100	mV	20
D.C. Offset (into 1 Megohm)	mV	30
Connectors	micro	10-32
Meter	VFS	27
Size (L. W. H)	in	1.8 x 2.9 x 4
Weight	oz (gm)	15 (425)
Batteries (three 9V alkaline)	V	27
Battery Life	hr	40

(1) Low frequency is specified into 1 megohm load



A.6 Specifications of integrating power unit (Model 480A08)



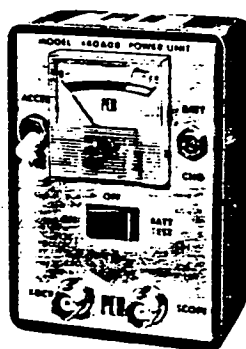
info-gram
interim data sheet

NEW!

INTEGRATING POWER UNIT model 480A08

PROVIDES EITHER ACCELERATION OR VELOCITY OUTPUT
POWERS VOLTAGE MODE ACCELEROMETERS
ELIMINATES BIAS ON OUTPUT
MONITORS NORMAL OR FAULTY OPERATION
OPERATES FROM TWO 9V BATTERIES

The Model 480A08 Integrating Power Unit, for use with PCB Quartz Accelerometers with built-in amplifiers, provides broad band analog output signals for either acceleration or velocity. The sensitivity for either output is dependent upon the accelerometer as shown in the specifications. Other features include constant current excitation to power the accelerometer, and meter for monitoring sensor bias and battery condition. The meter does not measure velocity or acceleration. Although normally supplied with alkaline batteries, a battery/charge jack is provided for charging optional rechargeable NiCd batteries.



SPECIFICATIONS: Model No.		480A08
Sensor Excitation (CC)	mA	2
Acceleration Output	gain	1
Velocity Output	mV/in/s	
100 mV/g Accelerometer		1000
10 mV/g Accelerometer		100
Frequency Range, Velocity	Hz	10-10 000
Acceleration*	Hz	.05-100 000
Output Coupling Capacitor	uF	10
DC Offset (max)*	mV	30
Batteries (two 9v)	V	18
Alkaline (std)	hr	100
NiCd (option)	hr	60
Battery/Input Cond. Monitor	VFS	20
Connectors (Input & Output)		BNC
Size (WxLxH)	in	2.9x4.0x1.9
Weight (including batteries)	oz	20
*Into one megohm load		
Optional Model: With Model 4888 Charger and NiCd batteries		480A08/4888

B Listing of mechanical admittance simulation program (Mech-Admit)

```

10      !
20      !   FOUR-POLE TECHNIQUES "MECH_ADMIT" TO SOLVE COMPOSITE SYSTEM
30      !
40      !   DEVELOPED BY H.F.C.CHAN   (9/9/85)
50      !   MODIFIED BY H.F.C.CHAN   (24/7/86)
60      !   FURTHER MODIFIED BY H.F.C.CHAN (22/10/86)
70      !
80      OPTION BASE 1
90      COM M(10),Pilesk(10),Piledc(10),Soilsk(10),Soildc(10),Admittance(2100),N1
100     !
110    ! GENERAL INFORMATION
120    !
130    DISP "NUMBER OF GROUPED SYSTEMS";
140    INPUT N1
150    PRINTER IS 708
160    PRINT USING "27A,2D";"NUMBER OF GROUPED SYSTEMS= ",N1
170    FOR I=1 TO N1
180    PRINTER IS 1
190    PRINT USING "35A,2D,2A";"ENTER INFORMATION OF GROUPED SYSTEM",I," : "
200    PRINT USING "15A";"PILE PROPERTIES"
210    CALL Mass(I,M(*))
220    CALL Spring_damper(I,Pilesk(*),Piledc(*))
230    PRINT USING "15A";"SOIL PROPERTIES"
240    CALL Spring_damper(I,Soilsk(*),Soildc(*))
250    NEXT I
260    DISP USING "$,70A";"END CONDITIONS :--- 1)BOTH ENDS FREE, 2)END 1 FREE A
NO END 2 FIXED"
270    INPUT Endcondition
280    PRINT " "
290    PRINT " "
300    IF Endcondition=1 THEN
310    PRINT "END CONDITION :--- BOTH ENDS FREE"
320    ELSE
330    PRINT "END CONDITION :--- END 1 FREE AND END 2 FIXED"
340    END IF
350    ON KEY 0 LABEL "RESON. FREQ." GOTO 420
360    ON KEY 1 LABEL "END " GOTO 910
370    GOTO 350
380    !
390    !   SOLVE EQUATION
400    !
410    DISP " "
420    DISP USING "$,42A";"ENTER Frequency Start AND Frequency Finish"
430    INPUT Fsta,Ffin
440    DISP "ENTER Frequency Interval";
450    INPUT Fi
460    DISP USING "$,15A";"PROCESS RUNNING"
470    PRINT " "
480    PRINT " "
490    PRINT " "
500    PRINT USING "20A,2D,3A";"FREQUENCY INTERVAL=","Fi," Hz"

```

```

510     PRINT USING "20A,5D,7A,5D,3A";"FREQUENCY RANGE= ",Fsta," Hz to ",Ffin,"
      Hz"
520     PRINT USING "25A,5X,30A";"RESONANCE FREQUENCY (Hz) ", "MECHANICAL ADMITTA
      NCE (m/s/N)"
530     Admittance(1)=1.E+308
540     Admittance(2)=1.E+308
550     N2=ABS((Ffin-Fsta)/Fi)+1
560     FOR Nf=1 TO N2
570     F=Fsta+(Nf-1)*Fi
580     IF F=0 THEN
590         Admittance(Nf+2)=1.E+308
600         GOTO 670
610     END IF
620     Admittance(Nf+2)=FNAdmiquation(F,N1,Endcondition,M(*),Pilesk(*),Piledc(*
      ),Soilsk(*),Soildc(*))
630     PRINT F,Admittance(Nf+2)
640     IF ABS(Admittance(Nf+1))>ABS(Admittance(Nf)) AND ABS(Admittance(Nf+1))>A
      BS(Admittance(Nf+2)) THEN
650         PRINT USING "10X,5D,18X,K";F-Fi,Admittance(Nf+1)
660     END IF
670     NEXT Nf
680     DISP " "
690     BEEP
700     DISP "END OF RANGE"
710     WAIT 5
720     DISP USING "$,22A";"STORE DATA :--- Y or N"
730     INPUT Q$
740     IF Q$="Y" THEN
750         DISP USING "$,15A";"ENTER FILE NAME"
760         INPUT A$
770         OUTPUT 2 USING "$,8,K";255,"K" ! Clear Screen
780         DISP "STORING DATA";
790         CREATE 80AT A$,100
800         ASSIGN 80A TO A$
810         IF Admittance(3)=1.E+308 THEN Admittance(3)=Admittance(4)
820         FOR Nf=1 TO N2
830             OUTPUT 80A:Admittance(Nf+2)
840         NEXT Nf
850         OUTPUT 2 USING "$,8,K";255,"K" ! Clear Screen
860         ASSIGN 80A TO *
870         DISP "DONE"
880         BEEP
890     END IF
900     GOTO 350
910     DISP " "
920     DISP USING "$,14A";"END OF PROGRAM"
930     END
940     !
950     !
960     !
970     !
980     !
990     SUB Mass(I,M(*))
1000     PRINTER IS 708
1010     DISP USING "$,23A,2D,10A";"ENTER Volume of SYSTEM ",I," in m-cube"
1020     INPUT Volume
1030     DISP USING "$,24A,2D,15A";"ENTER Density of SYSTEM ",I," in KN/m-cube"
1040     INPUT Density
1050     M(I)=(Volume*Density*1.E+3)/9.81 ! UNIT: Kg

```

```

1060 PRINT " "
1070 PRINT USING "7A,2D,3X,6A";"SYSTEM ",I,"(Mass)"
1080 PRINT USING "5X,9A,3D,3D,7A";"Volume= ",Volume," m-cube"
1090 PRINT USING "5X,9A,5D,0,10A";"Density= ",Density," KN/m-cube"
1100 PRINT USING "5X,7A,5D,2D,3A";"Mass= ",Volume*Density," KN"
1110 PRINTER IS 1
1120 SUBEND
1130 DEF FNAdmiquation(F,N1,Endcondition,M(*),Pilesk(*),Piledc(*),Soilsk(*),Soil
dc(*)
1140 DIM Element1(2,4),Massalpha(2,4),Pilealpha(2,4),Soilalpha(2,4)
1150 !
1160 ! GENERATE COMPLEX IDENTITY MATRICE
1170 !
1180 Element1(1,1)=1
1190 Element1(1,2)=0
1200 Element1(1,3)=0
1210 Element1(1,4)=0
1220 Element1(2,1)=0
1230 Element1(2,2)=0
1240 Element1(2,3)=1
1250 Element1(2,4)=0
1260 !
1270 ! CALCULATE FOUR-POLE PARAMETERS
1280 !
1290 FOR I=N1 TO 1 STEP -1
1300 CALL Sprdamfe(I,F,Soilsk(*),Soildc(*),Soilalpha(*))
1310 CALL Sprdamfe(I,F,Pilesk(*),Piledc(*),Pilealpha(*))
1320 CALL Massfe(I,F,M(*),Massalpha(*))
1330 CALL Series(Element1(*),Pilealpha(*))
1340 CALL Parallel(Element1(*),Soilalpha(*))
1350 CALL Series(Element1(*),Massalpha(*))
1360 NEXT I
1370 !
1380 ! SELETE ADMITTANCE EQUATION ACCORDING TO END CONDITIONS
1390 !
1400 IF Endcondition=1 THEN
1410 Dummya=Element1(2,3)*Element1(1,3)+Element1(2,4)*Element1(1,4)
1420 Dummyb=Element1(2,4)*Element1(1,3)-Element1(2,3)*Element1(1,4)
1430 Dummyc=Element1(1,3)^2+Element1(1,4)^2
1440 ELSE
1450 Dummya=Element1(2,1)*Element1(1,1)+Element1(2,2)*Element1(1,2)
1460 Dummyb=Element1(2,2)*Element1(1,1)-Element1(2,1)*Element1(1,2)
1470 Dummyc=Element1(1,1)^2+Element1(1,2)^2
1480 END IF
1490 Amplitude=SQR(Dummya^2+Dummyb^2)/Dummyc
1500 RETURN Amplitude
1510 FNEND
1520 !
1530 !
1540 !
1550 SUB Massfe(I,F,M(*),Alpha(*))
1560 DIM Alpha(2,4)
1570 Alpha(1,1)=1
1580 Alpha(1,2)=0
1590 Alpha(1,3)=0
1600 Alpha(1,4)=2*PI*F*M(I)

```

```

1610 Alpha(2,1)=0
1620 Alpha(2,2)=0
1630 Alpha(2,3)=1
1640 Alpha(2,4)=0
1650 SUBEND
1660 SUB Spring_damper(I,Sk(*),Dc(*))
1670 PRINTER IS 708
1680 DISP USING "$,32A,2D,10A";"ENTER Spring constant of SYSTEM ",I," in MN/m
1690 INPUT Sk(I)
1700 DISP USING "$,32A,2D,10A";"ENTER Damper constant of SYSTEM ",I," in Ns/m"
1710 INPUT Dc(I)
1720 PRINT " "
1730 PRINT USING "7A,2D,3X,15A";"SYSTEM ",I,"(Spring-Damper)"
1740 PRINT USING "5X,17A,5D,3D,6A";"Spring constant= ",Sk(I)," MN/mm"
1750 PRINT USING "5X,17A,5D,3D,6A";"Damper constant= ",Dc(I)," Ns/m"
1760 Sk(I)=Sk(I)+1.E+9 ! UNIT: N/m
1770 PRINTER IS 1
1780 SUBEND
1790 !
1800 !
1810 !
1820 SUB Sprdampe(I,F,Sk(*),Dc(*),Alpha(*))
1830 W=2*PI*F
1840 Denominator=Sk(I)^2+(W*Dc(I))^2
1850 Alpha(1,1)=1
1860 Alpha(1,2)=0
1870 Alpha(1,3)=0
1880 Alpha(1,4)=0
1890 Alpha(2,1)=W^2*Dc(I)/Denominator
1900 Alpha(2,2)=W*Sk(I)/Denominator
1910 Alpha(2,3)=1
1920 Alpha(2,4)=0
1930 SUBEND
1940 SUB Series(Element1(*),Alpha(*))
1950 !
1960 ! COMPLEX MATRIX MULTIPLICATION
1970 !
1980 Element2(1,1)=Alpha(1,1)*Element1(1,1)-Alpha(1,2)*Element1(1,2)+Alpha(1,
1990 Element2(1,2)=Alpha(1,1)*Element1(1,2)+Alpha(1,2)*Element1(1,1)+Alpha(1,
2000 Element2(1,3)=Alpha(1,1)*Element1(1,3)-Alpha(1,2)*Element1(1,4)+Alpha(1,
2010 Element2(1,4)=Alpha(1,1)*Element1(1,4)+Alpha(1,2)*Element1(1,3)+Alpha(1,
2020 Element2(2,1)=Alpha(2,1)*Element1(1,1)-Alpha(2,2)*Element1(1,2)+Alpha(2,
2030 Element2(2,2)=Alpha(2,1)*Element1(1,2)+Alpha(2,2)*Element1(1,1)+Alpha(2,
2040 Element2(2,3)=Alpha(2,1)*Element1(1,3)-Alpha(2,2)*Element1(1,4)+Alpha(2,
2050 Element2(2,4)=Alpha(2,1)*Element1(1,4)+Alpha(2,2)*Element1(1,3)+Alpha(2,
2060 FOR J=1 TO 2
2070 FOR K=1 TO 4
2080 Element1(J,K)=Element2(J,K)
2090 NEXT K
2100 NEXT J
2110 SUBEND

```

```

2120 !
2130 !
2140 !
2150 SUB Parallel(Element1(*),Alpha(*))
2160 REAL Renum,Imnum,Reden,Imden,Redummy,Indummy
2170 REAL Rea,Ima,Reb,Imb,Rec,Imc,Re,Im
2180 FOR J=1 TO 2
2190 FOR K=1 TO 4
2200 System(1,J,K)=Element1(J,K)
2210 System(2,J,K)=Alpha(J,K)
2220 NEXT K
2230 NEXT J
2240 ! Calculate coefficient A
2250 Rea=0
2260 Ima=0
2270 FOR I=1 TO 2
2280 Renum=System(I,1,1)
2290 Imnum=System(I,1,2)
2300 Reden=System(I,2,1)
2310 Imden=System(I,2,2)
2320 CALL Div(Renum,Imnum,Reden,Imden,Re,Im)
2330 Rea=Rea+Re
2340 Ima=Ima+Im
2350 NEXT I
2360 ! Calculate coefficient B
2370 Reb=0
2380 Imb=0
2390 FOR I=1 TO 2
2400 Renum=1
2410 Imnum=0
2420 Reden=System(I,2,1)
2430 Imden=System(I,2,2)
2440 CALL Div(Renum,Imnum,Reden,Imden,Re,Im)
2450 Reb=Reb+Re
2460 Imb=Imb+Im
2470 NEXT I
2480 ! Calculate coefficient C
2490 Rec=0
2500 Imc=0
2510 FOR I=1 TO 2
2520 Renum=System(I,2,3)
2530 Imnum=System(I,2,4)
2540 Reden=System(I,2,1)
2550 Imden=System(I,2,2)
2560 CALL Div(Renum,Imnum,Reden,Imden,Re,Im)
2570 Rec=Rec+Re
2580 Imc=Imc+Im
2590 NEXT I
2600 ! Form parallel matrice elements
2610 CALL Div(Rea,Ima,Reb,Imb,Re,Im)
2620 Element1(1,1)=Re
2630 Element1(1,2)=Im
2640 Redummy=Re*Rec-Im*Imc
2650 Indummy=Re*Imc+Im*Rec

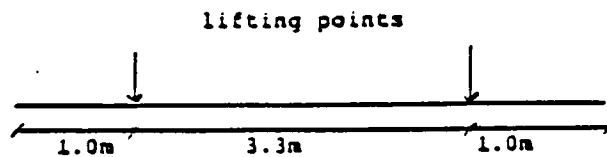
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2660   Element1(1,3)=Redummy-Reb
2670   Element1(1,4)=Imdummy-Imb
2680   CALL Div(1,0,Reb,Imb,Re,Im)
2690   Element1(2,1)=Re
2700   Element1(2,2)=Im
2710   CALL Div(Rec,Imc,Reb,Imb,Re,Im)
2720   Element1(2,3)=Re
2730   Element1(2,4)=Im
2740   SUBEND
2750   !
2760   !
2770   !
2780   SUB Div(Renum,Imnum,Reden,Imden,Re,Im)
2790     A=Renum*Reden+Imnum*Imden
2800     B=Imnum*Reden-Renum*Imden
2810     C=Reden^2+Imden^2
2820     Re=A/C
2830     Im=B/C
2840   SUBEND

```

C.1 Design calculations of model piles



Loading:

$$\text{Loading} = \frac{\pi \cdot 3^2 \cdot 24}{4} = 1.7 \text{ KN/m}$$

$$\text{Max Load} = 1.4 \cdot 1.7 = 2.9 \text{ KN/m}$$

Bending Moment:

$$\begin{aligned} \text{Cantilever } M &= 2.9 \cdot \frac{1}{2} \cdot 1^2 = 1.45 \text{ KNm} \\ SF &= 2.9 \cdot 1 = 2.90 \text{ KNm} \end{aligned}$$

$$\text{Span } M = \frac{2.9 \cdot 3.3^2}{8} = 3.95 \text{ KNm}$$

(neglect cantilever reduction)

$$SF = \frac{2.9 \cdot 3.3}{2} = 4.80 \text{ KN}$$

$$A_{st} = \frac{\pi \cdot 10^2}{4} = 78.5 \text{ mm}^2 \quad f_{cu} = 25$$

$$f_c = f_b \quad f_y = 425$$

$$0.4 f_{cu} b x + 0.82 \cdot f_y A_s = 0.87 f_y A_s$$

$$(0.4 \cdot 25 \cdot 100 x) + (0.72 \cdot 425 \cdot 78.5) = 0.87 \cdot 425 \cdot 78.5$$

$$\Rightarrow x = 5 \text{ mm}$$

Bending:

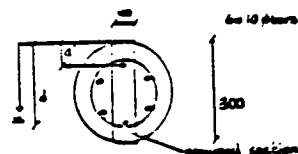
$$\begin{aligned} M &= <0.4 \cdot f_{cu} b x (d - x/2) + 0.72 f_y A_s (d - d')> 10^{-6} \\ &= <0.4 \cdot 25 \cdot 100 \cdot 5 (255 - 5/2) + 0.72 \cdot 425 \cdot 78.5 (255 - 45)> 10^{-6} \\ &= 6.5 \text{ KNm} > 3.95 \text{ KNm} \quad \text{O.K.} \end{aligned}$$

Shear:

$$v = 4.8 \cdot 10^3 = 0.19 \text{ N/mm}^2 < 0.35 \text{ N/mm}^2 \quad \text{O.K.}$$

$$S_v = \frac{A_{sv}}{0.002b} = \frac{3 \cdot \pi \cdot 6^2 / 4}{0.002 \cdot 100} = 282 \text{ mm}$$

$$\text{or } 12 \cdot 10 = 120 \text{ mm} \quad \text{say } 200 \text{ mm}$$



Adopt: Grade 25 Concrete

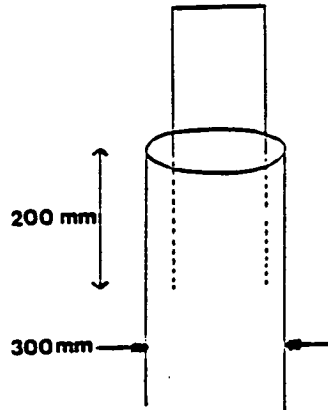
6 * 10mm diameter high yield bars symmetrically placed.

Cover to reinforcement 40mm.

6mm diameter mild steel stirrups (links) @ 200mm

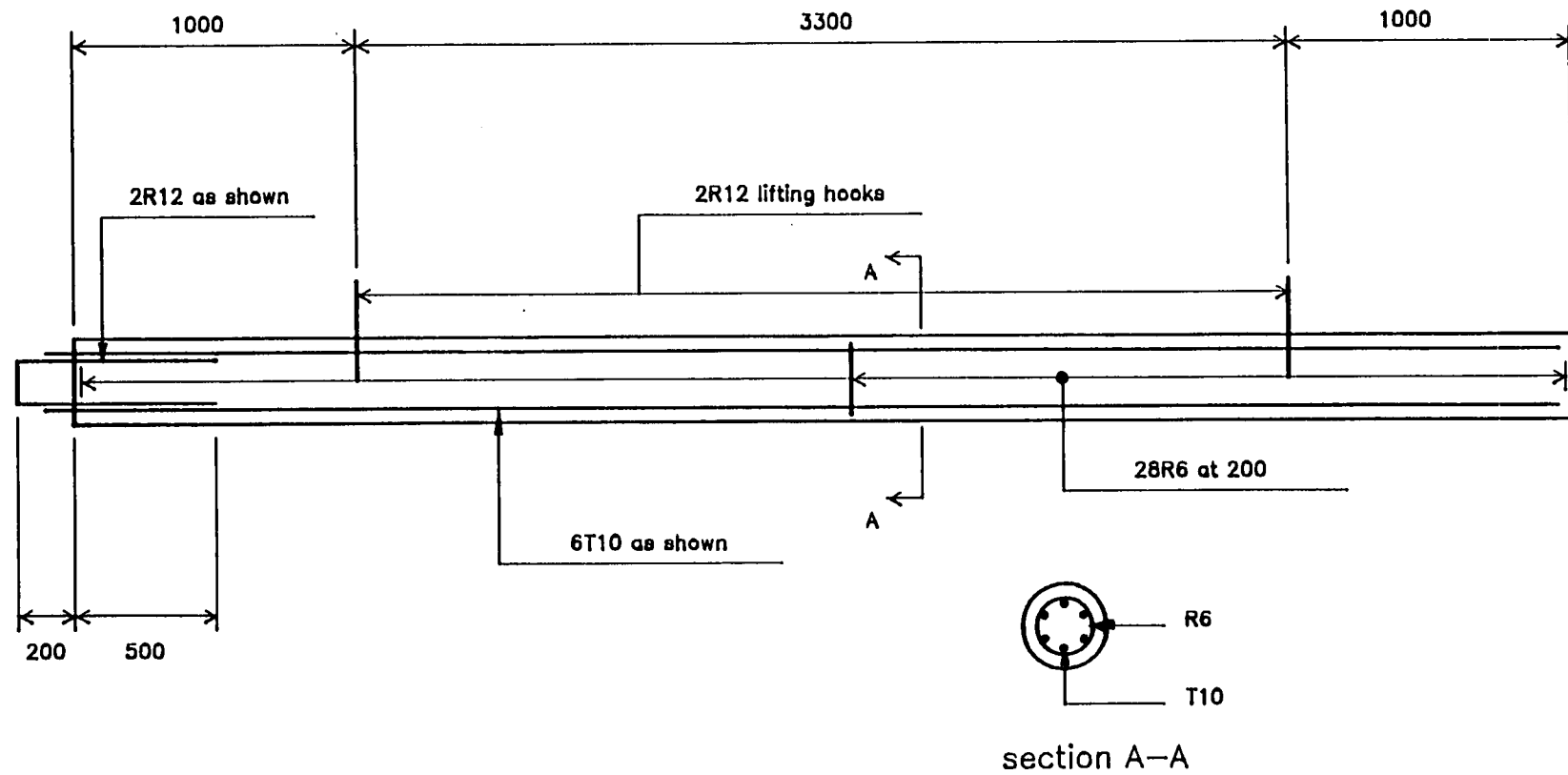
Bond Stress:

2 lifting hooks tied at right angles to each other (12mm ϕ)

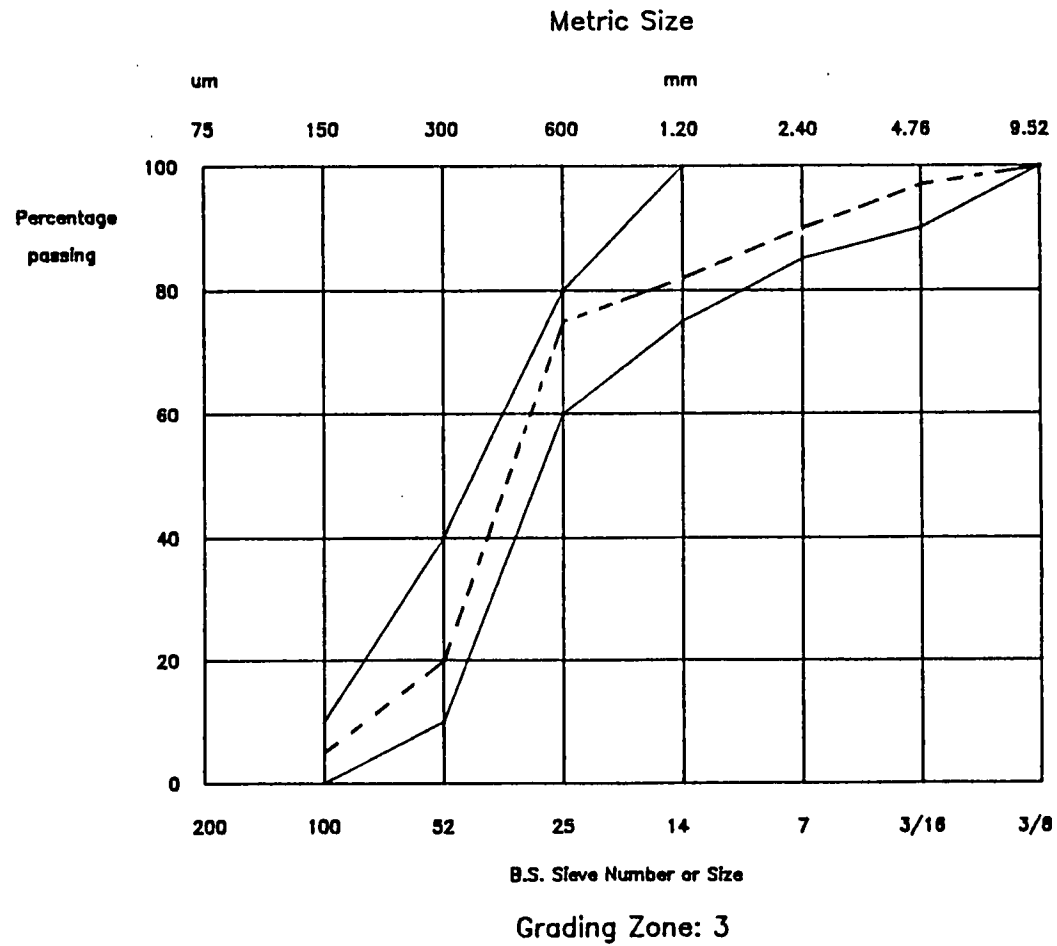


$$\begin{aligned}\text{Weight of column} &= \frac{\pi \times 0.3^2 \times 5.5}{4} = 24.3 \text{ KN} \\ &= 9.45 \text{ KN} \\ \text{Bonded area of one hook} &= 2 \times 300 \times \pi \times 12 \text{ mm}^2 \\ &= 22619.5 \text{ mm}^2 \\ \text{Bond Stress} &= \frac{9.45 \text{ KN}}{22619.5 \text{ mm}^2} \\ &= 0.42 \text{ N/mm}^2 \\ &< 1.4 \text{ N/mm}^2 \quad \text{permissible} \\ &\quad \text{(Table 22. P57 CPI10. Plain bar in} \\ &\quad \text{tension. Concrete Grade 25)}\end{aligned}$$

C.2 Details of reinforcement bars for the model piles



C.3 Grading chart of the fine aggregate used for model construction



C.4 Concrete mix design calculations for the model piles

Concrete mix design form

Stage	Item	Reference or calculation	Values		
1	1.1 Characteristic strength	Specified	COMPRESSIVE 25 N/mm ² at 28 days		
			Proportion defective 5 per cent		
	1.2 Standard deviation	Fig 3	N/mm ² or no data 8.0 N/mm ²		
	1.3 Margin	C1	$(k = 1.64) 1.64 \times 8.0 = 13.1$ N/mm ²		
	1.4 Target mean strength	C2	$25 + 13.1 = 38.1$ N/mm ²		
	1.5 Cement type	Specified	OPC/SRPC/RHPC		
	1.6 Aggregate type: coarse Aggregate type: fine		uncrushed uncrushed		
	1.7 Free-water/cement ratio	Table 2, Fig 4	0.52		
	1.8 Maximum free-water/cement ratio	Specified	—		
			— Use the lower value		
2	2.1 Slump or V-B	Specified	Slump 60-180 mm or V-B — s		
	2.2 Maximum aggregate size	Specified	20 mm		
	2.3 Free-water content	Table 3	195 kg/m ³		
3	3.1 Cement content	C3	$195 + 0.52 = 375$ kg/m ³		
	3.2 Maximum cement content	Specified	— kg/m ³		
	3.3 Minimum cement content	Specified	250 kg/m ³ — Use if greater than Item 3.1 and calculate Item 3.4		
	3.4 Modified free-water/cement ratio		—		
4	4.1 Relative density of aggregate (SSD)		2.6 known/assumed		
	4.2 Concrete density	Fig 5	2360 kg/m ³		
	4.3 Total aggregate content	C4	$2360 - 375 - 195 = 1790$ kg/m ³		
5	5.1 Grading of fine aggregate	BS 882	Zone 3		
	5.2 Proportion of fine aggregate	Fig 6	31.4 to 37.4 say 35 per cent		
	5.3 Fine aggregate content	C5	$1790 \times 0.35 = 626.5$ kg/m ³		
	5.4 Coarse aggregate content		$1790 - 626.5 = 1163.5$ kg/m ³		
Quantities		Cement (kg)	Water (kg or l)	Fine aggregate (kg)	Coarse aggregate (kg)
per m ³ (to nearest 5 kg)		375	195	626.5	1163.5
per trial mix of 0.05 m ³		18.75	9.75	31.33	58.18

Items in italics are optional limiting values that may be specified.

1 N/mm² = 1 MN/m² = 1 MPa.

OPC = ordinary Portland cement; SRPC = sulphate-resisting Portland cement; RHPC = rapid-hardening Portland cement.

Relative density = specific gravity.

SSD = based on a saturated surface-dry basis.

LIST OF PUBLISHED WORK

1. M.C.Forde, H.F.C.Chan and A.J.Batchelor, *Acoustic and Vibration NDT Testing of Piles in Glacial Tills*, Proceedings of the International Conference on Construction in Glacial Tills and Boulder Clay, held 12-14 March 1985 at the Caledonian Hotel, Edinburgh.
2. M.C.Forde, H.F.C.Chan and A.J.Batchelor, *Interpretation of Non-destructive Tests on Piles*, Proceedings of the Second International Conference on Structural Faults and Repair, held 30 April-2 May 1985 at the Institution of Civil Engineers, London.
3. M.C.Forde and H.F.C.Chan, *Integrity Assessment of Concrete Piles*, presented at the Concrete Society Materials Research Seminar, held 7-8 July 1986 at Dundee University.
4. H.F.C.Chan, M.C.Forde and A.J.Batchelor, *Developments In Transient Shock Pile Testing*, presented at the International Conference on Foundations and Tunnels, 24-26 March 1987 at the University of London.