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THE
ABSOLUTE THERMAL CONDUCTIVITY OF NICKEL.

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[WITH A PLATE.]

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"This paper has been composed by, and is a record of work done by myself

Signed Thomas C. Baillie

XII.—*The Absolute Thermal Conductivity of Nickel.*

By T. C. Baillie, M.A., B.Sc., Assistant Lecturer and Demonstrator in Physics, University College of North Wales, Bangor. (With a Plate.)

(Read 16th May 1898.)

§ 1. Introduction.—The experiments described in this paper were commenced with the view, not only of determining the absolute thermal conductivity of nickel, but also of comparing the results found by Forbes's and Ängstrom's methods for the same specimen. Although some readings were taken for Ängstrom's method, that part of the investigation was not completed, because it was found that the experimental errors — unavoidable, on account of the necessity of measuring rapidly changing temperatures—would be too great for the results to be of any value. The thermal conductivity of a portion of the bar of nickel used for Forbes's method was determined by a direct method involving the determination only of steady temperatures, and the results so obtained are given in the latter portion of this paper.

Forbes's Method.

§ 2. The Statical Experiment.—The nickel bar used was kindly lent by Dr Knott, being a piece about four feet long, which he had no immediate occasion to use for his own experiments on "The Strains produced in Iron, Steel, and Nickel Tubes in the Magnetic Field" (see *Trans.*, vol. xxxviii. part iii. No. 13). This bar was turned down so as to be of uniform circular section, and holes for thermometers were drilled into it by Messrs Aitken & Allan, Edinburgh. Four thermometer holes were drilled in each of the end portions of the bar, so as to leave a length of 19 inches intact for a tube required by Dr Knott at a later period.

The bar was set up in Professor Tait's private laboratory, with the same fittings, altered to suit the size of the nickel bar, as were used by Professor Tait, and afterwards by Dr Mitchell, in their experiments on thermal conductivities (see *Trans.*, 1878, xxx., and *Trans.*, 1887, xxxiii. part ii. p. 535). One end of the bar was fitted with white lead into a round hole in the side of a cast-iron pot, which was afterwards nearly filled with solder. This end of the bar was heated by a bunsen flame placed under the pot of solder. A constant temperature was maintained at this end of the bar by keeping the gas supplied to the bunsen burner at constant pressure by means of Professors Tait and Crum Brown's gas regulator. This regulator is like a small gasometer, one of the balancing weights of which, on descending, bends a piece of soft, flexible rubber tubing conducting the gas supply to it, so as to diminish the internal cross-section of the piece of soft tubing;

on the weight ascending, the cross-section of the tubing is increased. The nickel bar was protected by a double metal screen from thermal disturbances due to the heater. A constant stream of cold water was kept playing on the unheated end of the bar. The thermometers used were some of the Kew standard thermometers used by Professor TAIT and Dr MITCHELL. Temperatures above 200° C. were not used, and as the readings on the majority of the thermometers varied in the course of an experiment through a range of about 1° C., the thermometers were simply read by the naked eye to the nearest quarter of a degree. One could be quite certain of avoiding making any error due to parallax of more than that amount. A little mercury was put in each of the thermometer holes to give good thermal contact between the bulbs of the thermometers and the bar. No amalgamation of the nickel has ever been observed.

A steady state as regards the distribution of temperature along the bar was not reached in five hours from the time the gas at the heater was lighted, and the readings on the thermometers usually increased at the rate of about a quarter of a degree per hour for a few hours more. The following table shows the readings (uncorrected) obtained over an interval of nearly twenty-four hours on 18th July 1894 and the following morning. The gas was lighted at 8.30 A.M.

Time.	1	2	3	4	5	6	7	8	Temperature of Air.
12.30	175	130·5	100	78	26	23	20·5	18·2	19·0
1.30	177	133	102·5	80·5	28	25	22	19·1	19·1
2.0	177	133·3	103	81	28·9	25·4	22·5	19·5	19·1
2.15	176·5	133	103	81·5	29	25·8	22·5	19·7	19·1
2.45	176·5	133	103	81·5	29·4	26	23	20	19·1
6.0	177·8	134	104	82·2	30	26·5	23·2	20	19·1
6.30	178	134·3	104·5	82·5	30	26·5	23	20	19·2
7.0	178	134·3	104	82·5	30	26·5	23·1	20·1	19·2
7.30	177·5	134	104	82·3	30	26·5	23·2	20·1	19·4
8.0	178·5	134·5	104	82·5	30·1	26·7	23·4	20·2	19·3
8.30	178	134	104	82·2	30	26·6	23·1	20·0	19·3
9.0	178	134·3	104	82·3	30	26·5	23	20	19·4
9.30	178	134·3	104	82·5	30	26·5	23	20	19·4
10.0	178	134	104	82·5	30	26·5	23	20	19·3
11.20	179	135	105	83	30·1	26·6	23·1	20	19·4
1.30 a.m.	178	135	105	83	30·1	26·5	23·1	19·9	19·3
2.15	178	135	104·7	83	30·1	26·5	23·1	19·9	19·3
3.30	179·7	135·5	105	83·4	30·2	26·6	23	19·9	19·3
4.25	179	135·1	105	83·4	30·2	26·6	23	19·9	19·2
5.0	179	135	105	83·2	30·2	26·6	23	19·8	19·2
6.0	178·5	135	104·8	83	30	26·5	23	19·7	19·0

After several sets of readings had been taken, the bar was reversed, and heated at the other end. The thermometers were never shifted from their positions, except when the bar was reversed. They were read on each morning before the burner was lighted, and their readings on those occasions never differed by more than one-third of a degree. The thermometers were corrected for stem exposure by adding to the reading V. the product ·000113 V^2. This correction seems to me to be probably too high for some of the thermometers, but it is less than that which Dr MITCHELL applied to the same thermometers, viz., ·00016 V^2. The value of the stem correction which I have chosen is based on the results of experiments to be described later in connection with the other method of determining the conductivity.

The dimensions, etc., of the bar were as follows:—

Length of bar,	128·5 cm.
Mass of bar,	19·28 kilo.
Diameter of bar,	4·67 cm.
Density of nickel, *i.e.*, mass ÷ volume,	8·724 gms. per c.cm.
Specific gravity of a small piece cut off, determined by weighing in air and in water,	8·75

Distances of thermometer holes from one end of the bar:—

Number of Hole.	Distance to Centre of Hole.
1	14·75 cm.
2	23·02 „
3	31·34 „
4	39·73 „
5	88·72 „
6	97·02 „
7	105·26 „
8	113·59 „

The statical experiment was frequently repeated at the same and different temperatures, and the following table contains the readings chosen for calculation corrected for stem exposure. On 6th August and the following days the bar was heated at the opposite end to that at which it was heated on previous occasions. The table shows that any effect due to tarnishing of the surface during the few weeks occupied by these experiments is not noticeable.

Date of Experiment.	Numbers of Thermometer Holes.								Temperature of Air.
	1	2	3	4	5	6	7	8	
July 12	150·5	114·8	90·2	72·3	28·9	26·0	23·2	20·6	20·2
„ 13	180·6	135·0	103·5	81·7	28·9	25·4	22·5	19·7	19·4
„ 16	181·7	135·6	104·4	82·1	29·3	26·0	22·8	20·0	18·8
„ 17	181·0	136·0	104·9	82·8	29·8	26·1	23·0	20·0	19·1
„ 18	181·6	136·0	105·2	83·2	30·1	26·5	23·0	20·1	19·3
„ 27	69·8	57·0	47·7	40·8	22·0	20·7	19·2	18·0	18·5
„ 30	116·7	100·1	72·8	59·9	26·5	24·1	22·0	20·0	18·9
„ 31	116·6	99·9	72·5	59·6	26·5	24·4	22·4	20·5	19·1
August 1	72·5	59·4	49·4	42·3	23·0	21·8	20·4	19·0	19·0
„ 2	67·5	55·4	46·8	40·3	22·7	21·5	20·4	19·1	18·6
„ 3	67·5	55·3	46·7	40·3	23·9	23·0	22·4	21·9	18·7
„ 6	150·3	114·2	89·7	71·3	28·5	25·7	23·1	20·9	18·9
„ 7	198·8	147·9	113·8	89·2	31·6	27·9	24·5	21·4	19·2
„ 8	109·3	85·9	69·5	57·0	25·9	23·7	21·6	19·5	18·9
„ 9	70·0	57·4	48·3	41·2	22·8	21·4	20·0	18·8	18·3
„ 10	165·3	124·9	97·6	77·6	29·8	26·6	23·8	21·1	18·9
„ 13	163·5	123·2	96·0	75·6	28·3	25·3	22·7	20·0	18·4

§ 3. Reduction of the Readings.—The equation for the conduction of heat in a bar, each part of which is at a steady temperature, is :—

$$\mathrm{KA}\frac{d^2\theta}{dx^2}=\mathrm{E}p\theta,$$

where K is the thermal conductivity, A the cross-section, E the emissivity, and p the perimeter of a part of the bar, at temperature θ above the surrounding air, and at a distance x from some fixed point in the axis of the bar. Since K, A, E, and p are either constants or functions of θ only, it follows that $\frac{d^2\theta}{dx^2}$ is a function of θ only, and therefore the value of $d^2\theta/dx^2$ for any given value of θ should be the same, no matter which of the sets of readings it is derived from. This affords a means of testing the concordance of the various sets of readings. The determination of $d^2\theta/dx^2$ directly—by drawing a curve representing θ as a function of x, taking the tangents at various points, and thus getting another curve showing $d\theta/dx$ as a function of x; and from this, by a similar process, another showing $d^2\theta/dx^2$ as a function of x, and using the first and last curves to get $d^2\theta/dx^2$ as a function of θ—does not give $d^2\theta/dx^2$ with sufficient accuracy. A common proceeding is to find suitable values of the constants in some empirical equation representing θ as a function of x, and to differentiate the equation to obtain $d\theta/dx$. The following method of reducing the readings obtained in the statical experiment was adopted after trying others. Curves were made from the sets of readings on the first four thermometers only, in which log. θ was shown as a function of x. The gradient of these curves increased, but not very rapidly, with log. θ, and therefore $d(\log.\ \theta)/dx$ increased as θ increased. The curves were drawn by a lath, to the ends of which couples were applied so as to give it the slight curvature necessary to make the curve produced by its means pass in close proximity to each of the four points given by the corrected readings of the thermometers. It was noticed that the value of $d(\log.\ \theta)/dx$ was practically the same, for the same value of θ, for all curves. A new curve was then constructed, in which $d(\log.\ \theta)/dx$ was shown as a function of θ. The different points found on this curve lay very approximately in a straight line—that is to say, $\frac{d}{d\theta}\left(\frac{d(\log.\ \theta)}{dx}\right)$ was practically constant. The equation to the statical curves must then be of the form $\frac{1}{bc}\log.\frac{\theta}{\theta+b}=x+\mathrm{B}$, where b and c have the same value for each curve. The simplicity of this method of finding the average values of $d^2\theta/dx^2$ for all sets of readings was what led to its adoption. In any case, $d(\log.\ \theta)/dx$ does not vary so rapidly as $d\theta/dx$, and it is therefore easier to get $d\theta/dx$ with accuracy, when using graphical methods, by multiplying $d(\log.\ \theta)/dx$ by θ, than it is to get $d\theta/dx$ directly. The values found for the constants in the above equation were $c=\cdot0000505$, and $b=670$. The value of $d^2\theta/dx^2$ is $c^2(2\theta+b)(\theta+b)\theta$. The following table contains the values of $d^2\theta/dx^2$ calculated, not from the expression just given, but from the numbers found in the curve, using the formula

$$\frac{d^2\theta}{dx^2} = \theta \frac{d\,(\log.\,\theta)}{dx} \left\{ \frac{d\,(\log.\,\theta)}{dx} + \theta \frac{d}{d\theta}\left(\frac{d\,(\log.\,\theta)}{dx} \right) \right\}.$$

The temperatures given in the tables are actual temperatures, not temperature excesses. They have been got by adding 19—about the average temperature of the air during the experiments—to the numbers used in the curves which were differences of temperature between the bar and the air.

Temperatures.	$\frac{d\,(\log.\,\theta)}{dx}$	$\frac{d^2\theta}{dx^2}$	Temperatures.	$\frac{d\,(\log.\,\theta)}{dx}$	$\frac{d^2\theta}{dx^2}$
40	·0335	·0239	110	·0370	·138
50	·0340	·0369	120	·0375	·159
60	·0345	·0509	130	·0380	·181
70	·0350	·0661	140	·0385	·205
80	·0355	·0823	150	·0390	·230
90	·0360	·0997	160	·0395	·256
100	·0365	·1183	170	·0400	·284
			180	·0405	·313
			190	·0410	·344
			200	·0415	·376

§ 4. THE COOLING EXPERIMENT.—For this experiment a short bar turned down from a left-over portion of Dr KNOTT'S nickel bars was used. It was a piece of the same rod as the bar used in the statical experiment: it was turned down in the same way, at the same time, and to the same diameter, as was found by careful measurement. The length of the cooling bar was 21·55 cm., and as its diameter was 4·67 cm., the surface exposed at the ends was 9$\frac{3}{4}$ per cent. of the whole surface. This involves an increase in the rates of cooling of about ten per cent. This is a serious drawback in these experiments. It has been allowed for by diminishing the observed rates of cooling in the ratio of the whole area of the cooling bar to the area of the curved portion only. It is possible that in air the emissivity of a vertical surface is, *ceteris paribus*, greater than the average emissivity of a curved cylindrical surface of the same diameter. As there is heat lost from the ends of the cooling bar, there must be some fall of temperature between the centre and the ends. I have given up all attempts at making allowance for this. The best way of meeting difficulties of that kind is to make the end correction negligible altogether. The bar was heated over a row of bunsen burners, without the previous warming necessary to avoid "sweating." The bar was heated pretty rapidly, and turned round rapidly while being heated, and very little moisture condensed upon it. A Kew mercury thermometer was used to measure the rate of cooling of the short bar which was heated to about 250° C. Readings were not taken until the bar had cooled for some time with the thermometer in position, since the distribution of temperature in the thermometer itself is at first irregular. This is discussed very fully by Professor TAIT in his paper already referred to. The thermometer was observed through the telescope of a cathetometer, and the time at which the top of the mercury column passed each degree division mark was noted by looking at that instant at the dial of a watch. After some practice it was found easy to note the time of such transits to within a couple of seconds from the position of the seconds' hand, without paying much attention to the divisions round the dial. The time of transit set down for each degree division was late by the time taken to look from the thermometer to the watch, but as this is small and affects each reading, it is of no consequence. When the cooling became comparatively slow, as it did below 100° C., it was possible to see the top of the mercury column disappear behind a degree division mark, note the time, and have the eye in position at the telescope again in time to see the top of the column reappear on the under edge of the division mark.

§ 5. Reduction of the Cooling Readings.—The method employed for reducing these readings was as follows:—On paper ruled in squares temperatures were plotted as *abscissae*, and the times (in seconds) taken by the bar to cool through one degree were plotted as ordinates corresponding to the mean temperatures for those degrees: thus, for example, the ordinate corresponding to the temperature 102·5° C. was the observed time taken by the bar to cool from 103° C. to 102° C. The advantage of this method of reduction is the simplicity of correcting for errors of observation, &c. Suppose, for example, that the time set down for the transit across the 175 degree division mark is too late, the time noted for cooling from 176° to 175° is too great, and the amount by which it is unduly increased is deducted from the time of cooling from 175° to 174°; but the average time of cooling for a range including 176° to 174° is not affected by the supposed error. An error in graduation, by which one of the division marks is displaced, produces a similar effect. The ordinates would, if there were no errors of any kind, increase in length continuously as the temperature diminishes. If the curve formed by the ends of the ordinates is not continuous, all that is necessary is to make a continuous curve by reducing the lengths of those ordinates which are obviously too long and increasing the lengths of adjacent ordinates, and *vice versa*, so as to keep the sum total of the lengths of all the ordinates constant. This treatment will get rid of the effects of errors such as those considered above. The way in which this was carried out was to form a new curve in which for each reading was substituted the average of the five nearest readings. This gave a curve which was smooth but with small "waves" along it. A mean curve was then drawn by means of a lath planed thinner towards one end so as to produce the necessary variation of curvature along it. The ordinate at any temperature of the curve so constructed is the reciprocal of the rate of cooling at that temperature.

A cooling experiment was done alongside of the statical experiment on several days, until the surface of the cooling bar was thought to be just perceptibly dimmer than that of the long bar. It was confidently expected that the repeated heating of the short bar, especially as "sweating" was not entirely avoided, would affect the surface and increase its emissivity. The readings taken show that each time the bar was heated its emissivity was increased before the tarnish on the surface became even perceptible. The following table will show this, and it provides the means of allowing for it.

Date of Experiment.	Time of Cooling from 200° to 150° in Seconds.	Temperature of Air.	Time of Cooling from 150° to 100° in Seconds.	Temperature of Air.	Time of Cooling from 100° to 70° in Seconds.	Temperature of Air.	Time of Cooling from 70° to 40° in Seconds.	Temperature of Air.
1894								
July 12	1375	20·4°	2210	20·3°	2295	20·3°	5125	20·2°
„ 13	1355	19·5°	2175	19·3°	2280	19·3°		
„ 17			2087	19·1°	2193	19·1°	4795	19·1°
„ 18			2105	19·1°				
August 2	1348	18·8°	2150	18·7°	2201	18·6°	4660	18·55°
„ 3					2191	18·7°	4503	18·7°
„ 7			2178	18·3°	2214	18·5°		
„ 8	1341	19·2°	2149	19·2°	2233	19·1°	4740	19·1°

The table also shows irregularities in the cooling, due possibly to changes in the state of the atmosphere, or to variations in the unavoidable draughts of the second order of magnitude. The readings obtained on 8th August gave the best curves, and they have been used to determine the rates of cooling given in a later table. A reduction of 2 per cent. was made in the rates of cooling found, in order to allow for the increase in the emissivity which had taken place by 8th August. A correction is necessary in the cooling experiment for the exposed stem of the thermometer. In the statical experiment the stem correction was made by adding to the observed reading V. the product $\cdot 000113\ V^2$. Using the same form of correction in the cooling experiment, let V. be the observed, and θ the true temperature, then since $\theta = V. + \cdot 000113\ V^2.$, the true rate of cooling $d\theta/dt$ is equal to $(1 + \cdot 000226\ V.)$ times the apparent rate of cooling dv/dt got from the curves in which stem exposure is not allowed for. Since the thermometer parts with its heat to the cooling bar, its temperature must always during the cooling be higher than that of the bar; in other words, the thermometer lags behind the bar by an amount depending on the rate of cooling. No attempt has been made to allow for that in these experiments. Some mercury was put into the hole for the thermometer to give good thermal contact between the bar and the thermometer. The following table gives the rates of cooling of the short bar found after applying the corrections referred to above, except that due to lag, and that due to loss of heat at the ends of the bar.

Temperature.	Rate of Cooling.	Temperature.	Rate of Cooling.	Temperature.	Rate of Cooling.
40	·00362	90	·0149	150	·0308
50	·00556	100	·0174	160	·0336
60	·00770	110	·0200	170	·0363
70	·01005	120	·0225	180	·0392
80	·01250	130	·0251	190	·0423
		140	·0279	200	·0455

§ 6. SPECIFIC HEAT OF NICKEL.—The determination of the specific heat of the nickel has been found by far the most troublesome part of these experiments. A portion of the cooling bar about 2·5 inches long had a hole drilled into it to receive the thermometer. A little mercury was put into the hole along with the thermometer. It was then heated and allowed to cool. At some instant the temperature was noted just as it was let fall into a large calorimeter, and the heat given out by the nickel was measured in the ordinary way by the method of mixtures. This was repeated at the same and different temperatures, and the results were not quite concordant, but indicated that the specific heat increased with temperature. As it was not quite certain that the temperature in all parts of the interior of the piece of nickel was that of the mixture when the readings were taken nickel turnings were tried. Several pieces of the turnings made in turning down the nickel bars were tied together with a short piece of thread, whose mass was negligible, and heated in the inner chamber of a double cylinder of copper containing glycerine between the cylinders. This was heated to over 200°C. and packed up with cotton wool in a wooden case provided with a contrivance for opening a slide at the bottom and allowing the nickel to fall into the calorimeter at the moment of opening. The calorimeter used was a small glass beaker of suitable dimensions. With it the cooling correction was smaller than with a copper calorimeter of the same size. It was hoped that as the heater cooled very slowly it would be safe to assume that the temperature of the turnings after being in the heater some time would be that of the thermometer whose bulb was inserted amongst them. As the heater cooled, determinations of the specific heat could be done on the same day at lower and lower temperatures. It was found that the sets of determinations obtained on separate days did not agree no matter how long the nickel was kept in the heater, and as all the quantities involved could be measured within 1 per cent., and the correction for cooling was only about 1 per cent., the heater was regarded as the cause of the irregularities.

In some subsequent experiments the nickel turnings were heated in a steam jacket of the usual laboratory pattern, and results agreeing within 2 per cent. were obtained when the nickel was in the heater for not less than two hours. As the heat required to raise the temperature of 1 gramme of water is not constant, but varies in a manner depending on the thermometer used in the calorimeter, closer agreement than this is not to be expected. The specific heat thus obtained was higher than that got for the same temperature from the large mass cut off the cooling bar. At the same time there is *no* reason for supposing that the specific heat would *not* be affected by the nickel being cut up and distorted as it is in the form of turnings.

The values of the specific heat given below were found from a solid piece of the nickel weighing nearly 100 grammes, and as a glass calorimeter could not be used with so large a mass, a copper calorimeter was made of thin sheet copper, the depth of it being 5 inches, and the diameter $2\frac{1}{4}$ inches. There was a slight recess along one side to accommodate the thermometer and a flange round the lip by which it was suspended in the interior of a large copper vessel which protected it from draughts in the room. The thermometer used in the calorimeter was one of DUCRETET's précision thermometers divided into tenths of degrees centigrade, and it was read by means of a telescope fixed a little distance off, hundredths of a degree being estimated by eye. The steam heater was used in the ordinary way, but in order to get determinations at different temperatures the same heater was used with methylated spirits instead of water. A tube was arranged to conduct all the spirits which condensed in the apparatus back to the boiler by a pipe leading in at the bottom of it. Only a small portion of the spirits was distilled off, as the flame of the burner heating the boiler was so arranged that very little vapour was formed over and above that required to produce heat enough by its condensation to maintain the heater at a uniform temperature. The boiling point rose by less than one degree in the course of a day on account of loss by distillation of the more volatile constituents of the spirits. The top of the chamber in the heater, in which the nickel was suspended while being heated, was closed by a large cork in which were two holes, one letting in the thermometer which indicated the temperature of the nickel, and another through which passed a fine wire supporting the nickel. The bottom of the chamber was closed by a slide padded with cotton wool. This slide was drawn aside when the nickel was dropped into the calorimeter, an arrangement being made for doing all this with great rapidity. The wire which had supported the nickel in the heater remained attached to it, and the end of it, which was usually found projecting out of the mouth of the calorimeter, was at once seized and the contents of the calorimeter stirred by moving the nickel up and down and to and fro in the water. While this was being done the thermometer was being watched through the telescope. Two persons were thus necessary. No correction has been made for the thermal equivalent of the work done in stirring, as it has been assumed to be negligible. It was impossible to observe if stirring the water produced an appreciable quantity of heat as under all circumstances its effect was quite obscured by the disturbances produced by other causes.

The correction for cooling was applied in the following way:—The observer at the telescope had in his hands a stop watch, with two hands so arranged that, by pressing one stop, they would start together; pressing another stop made one of them remain where it was at the instant of pressing; a second press made it overtake and go on as before with the other hand. In this way, the time at which the thermometer indicated any reading could be noted to a fraction of a second, the reading being written down subsequently; and a fresh reading could be then taken in the same way. A curve was then plotted from the readings thus obtained, with temperatures as ordinates, and times as *abscissae*. The part of the curve corresponding to times after the maximum temperature had been reached was produced backwards to the axis of zero time, and in this way the temperature which the calorimeter must have cooled from, had its rise of temperature been instantaneous, was found. This is very nearly the temperature which would have been reached if the calorimeter had not lost any heat at all. This correction is probably too much by something less than one-half per cent. An example of such a curve of correction for cooling is given in fig. 1.

At temperatures over 100° C. another form of heater was used. An iron tube was surrounded by a conical-shaped iron chamber riveted on to it, and mercury was put in the space between. This was heated by a circular gas burner, the flame of which was regulated by the volume of the mercury, which, on expanding above a certain limit, cut off all the gas, except what found its way through a small by-pass. The by-pass was arranged to allow just sufficient gas through it to keep the burner lighted, and thus to save the trouble of lighting the gas when the mercury had contracted sufficiently. The arrangement was similar to that shown at E in fig. 2. The flame rose and fell about ten times in a minute. The temperature at any one place in the heater was very steady after it had been in action for an hour or two, but the temperature near the top of the inner tube was 2 or 3 degrees lower than that of the hottest part. The thermometer used for reading the temperature of the nickel in this heater was put in so that the bulb touched the nickel. The nickel dropped into the calorimeter through the centre of the ring burner. The inner tube of the heater was prolonged below the burner. Corrections for stem exposure have been applied to the readings of the thermometers. It was possible to obtain only an approximation to the stem corrections on account of the manner in which the thermometers were placed, with some part of their stem at unknown temperatures; but as the correction is only 2 per cent. in the greatest instance, they are probably accurate enough.

The following tables give the data obtained in the last sets of experiments done :—

SET I.

Mass of nickel, ...	99·3 grammes.
Mass of water in calorimeter, . . .	156·7 ,,
Water equivalent of calorimeter, etc., . .	3·5

Date of Experiment.	Initial Temp. of Water.	Corrected Temp. of Mixture.	Rise of Temp. of Water.	Initial Temp. of Nickel.	Fall of Temp. of Nickel.	Log. Ratio Rise of Temp. of Water to Fall of Temp. of Nickel.
1897 July 7	16·94	21·87	4·93	99·1	77·2	$\bar{2}$·8052
,, ,,	18·80	23·65	4·85	99·2	75·2	$\bar{2}$·8072
,, ,,	19·98	24·83	4·85	99·3	74·5	$\bar{2}$·8135
,, ,,	20·88	25·72	4·84	99·4	73·7	$\bar{2}$·8173
,, 8	18·65	23·54	4·89	99·4	75·9	$\bar{2}$·8091
,, ,,	19·60	24·46	4·86	99·4	74·9	$\bar{2}$·8121
,, ,,	19·12	23·99	4·87	99·4	75·4	$\bar{2}$·8101
Arithmetic mean of observed values	19·14	24·01	4·87	99·3	75·3	$\bar{2}$·8107

Average value of specific heat, ·104.

SET II.

Mass of nickel, ...	99·3 grammes.
Mass of water in calorimeter, . . .	126·7 ,,
Water equivalent of calorimeter, etc., . .	3·3

Date of Experiment.	Initial Temp. of Water.	Corrected Temp. of Mixture.	Rise of Temp. of Water.	Initial Temp. of Nickel.	Fall of Temp. of Nickel.	Log. Ratio Rise of Temp. of Water to Fall of Temp. of Nickel.
1897						
July 9	17·10	23·13	6·03	99·4	76·3	$\bar{2}$·8978
,, ,,	18·10	24·08	5·98	99·4	75·3	$\bar{2}$·8999
,, ,,	19·40	25·34	5·94	99·4	74·1	$\bar{2}$·9040
,, ,,	19·88	25·72	5·84	99·5	73·8	$\bar{2}$·8983
,, 10	18·19	24·28	6·09	99·6	75·3	$\bar{2}$·9078
,, ,,	20·25	26·12	5·87	98·6	72·5	$\bar{2}$·9083
Arithmetic mean of observed values	18·82	24·78	5·96	99·3	74·55	$\bar{2}$·9027

Average value of specific heat of nickel, ·105.

SET III.

Mass of nickel, ...	99·3 grammes.
Mass of water in calorimeter, . . .	126·7 ,,
Water equivalent of calorimeter, etc., . .	3·3

Date of Experiment.	Initial Temp. of Water.	Corrected Temp. of Mixture.	Rise of Temp. of Water.	Initial Temp. of Nickel.	Fall of Temp. of Nickel.	Log. Ratio Rise of Temp. of Water to Fall of Temp. of Nickel.
1897						
July 13	21·05	25·27	4·22	77·3	52·0	$\bar{2}$·9093
,, ,,	22·27	26·31	4·04	77·5	51·2	$\bar{2}$·8971
,, ,,	23·80	27·82	4·02	78·1	50·3	$\bar{2}$·9026
,, 14	21·01	25·23	4·22	78·2	53·0	$\bar{2}$·9010
,, ,,	22·13	26·20	4·07	77·5	51·3	$\bar{2}$·8995
,, ,,	23·55	27·51	3·96	77·5	50·0	$\bar{2}$·8987
Arithmetic mean	22·30	26·39	4·09	77·7	51·3	$\bar{2}$·9015

Average value of specific heat of nickel, ·105.

SET IV.

Mass of nickel, ...	99·3 grammes.
Mass of water in calorimeter, . . .	156·7 ,,
Water equivalent of calorimeter, etc., . .	3·5

Date of Experiment.	Initial Temp. of Water.	Corrected Temp. of Mixture.	Rise of Temp. of Water.	Initial Temp. of Nickel.	Fall of Temp. of Nickel.	Log. Ratio Rise of Temp. of Water to Fall of Temp. of Nickel.
1897						
July 20	21·60	29·75	8·15	145·7	116·0	$\bar{2}{\cdot}8467$
„ „	22·80	30·93	8·13	146·4	115·5	$\bar{2}{\cdot}8475$
„ 21	20·31	28·73	8·42	147·3	118·6	$\bar{2}{\cdot}8511$
„ „	21·12	29·32	8·20	147·3	118·0	$\bar{2}{\cdot}8419$
„ „	21·90	30·08	8·18	147·3	117·2	$\bar{2}{\cdot}8438$
„ 22	18·78	27·35	8·57	148·1	120·8	$\bar{2}{\cdot}8510$
„ „	18·99	27·50	8·51	148·8	121·3	$\bar{2}{\cdot}8461$
„ „	19·63	28·05	8·42	149·1	121·0	$\bar{2}{\cdot}8425$
„ 23	21·06	29·43	8·37	149·0	119·6	$\bar{2}{\cdot}8449$
Arithmetic mean	20·69	29·02	8·33	147·67	118·65	$\bar{2}{\cdot}8462$

Average value of specific heat of nickel, ·113.

The average value of the specific heat of the nickel turnings for a range varying from just under 100° C. to about 20° C. was about ·11. This shows that either the thermal capacity is altered in the process of disintegration, or that there is some error in the determination depending upon the size of the pieces employed. The latter I believe to be the case. During the month of June, I did several determinations at the same time with a bundle of copper washers, and with the piece of nickel referred to. After a few trials I found the mass of copper (114·8 grammes) which had the same thermal capacity as the 99·3 grammes of nickel. I tried to discover a difference between the rate of rise of temperature in the calorimeter when the copper was employed from that when the nickel was used. The difference in the times taken to reach the maximum reading was only about six seconds, the whole time being about one minute to one and a quarter. Probably the lag of the thermometer behind the calorimeter obscured the greater part of the actual difference.

The effect of not receiving all the heat from the nickel would be to make the apparent specific heat less than the true specific heat. This error would obviously be greater at low temperatures than at high temperatures, and thus would make the apparent specific heat increase more rapidly with temperature than the true specific heat actually does. Probably this is the real reason why the specific heats of carbon and silicon—so-called bad conductors of heat—have been found to be much lower at ordinary temperatures than that expected from DULONG and PETIT's law of constant atomic heats, whereas at very high temperatures their specific heats are much greater and nearly great enough to fulfil the law. Errors of this kind are reduced to a minimum by using WATERMAN's calorimeter. A description of this apparatus, and a short discussion of the determinations of specific heats is given in a paper by WATERMAN in the *Physical Review* (vol. iv. No. 3) for December 1896.

§ 7. It seems to me a disadvantage of FORBES's method that its accuracy has to depend on that of the determination of specific heat. While I have no confidence in the values found for the specific heat of the nickel, I give the values of the conductivity found by using them. I hope to be able at a future time to supplant these figures by others which can be relied on.

The following table gives the values of the ratio of the conductivity to the specific heat after applying the end correction to the rates of cooling given in a previous table, and the values of the conductivity using the values of the specific heat in the adjacent column. No corrections have been applied for changes of the dimensions of the nickel with temperature as these are really negligible.

Temperature.	Ratio of Conductivity to Specific Heat.	Specific Heat.	Conductivity.
40	1·19	·098	·118
50	1·19	·102	·121
60	1·19	·105	·125
70	1·20	·108	·130
80	1·20	·111	·133
90	1·18	·114	·135
100	1·16	·118	·137
110	1·14	·121	·138
120	1·12	·124	·139
130	1·09	·127	·139
140	1·07	·130	·140
150	1·06	·134	·142
160	1·04		
170	1·01		
180	·99		
190	·97		
200	·96		

§ 8. EXPERIMENTAL AND OTHER ERRORS IN FORBES'S METHOD.—The sources of error may be classified as follows :—

Statical Experiment.

(1) Thermometric errors.

(2) Errors in reduction of results, for example in differentiating the temperature curve.

(3) Want of uniformity or regularity in the substance or surface of the FORBES bar.

Cooling Experiment.

(4) Radiation from ends of bar.

(5) Lag of thermometer behind bar due to gradient of temperature necessary to cause flow of heat from thermometer to bar.

(6) Thermometric errors.

(7) Errors of observation in taking cooling readings.

(8) Errors in reduction of rate of cooling from these readings.

(9) Difference in the emissive powers of the surfaces of the cooling bar and statical bar. Some of the causes of such differences may be—
  - (*a*) tarnish.
  - (*b*) differences in the amount of polish.
  - (*c*) difference in the surroundings or in the state of the atmosphere during the cooling and statical experiments.
  - (*d*) differences in the radiation due to the temperature of the cooling bar always falling while that of the statical bar is steady.

(10) Errors in the determination of the specific heat. Of these the chief are—
  - (*a*) the specimen used for this may not be a fair average specimen.
  - (*b*) want of uniformity in its temperature when put into the calorimeter.
  - (*c*) the calorimeter not receiving the whole of the heat supposed to be given out from the specimen.
  - (*d*) changes in the thermal capacity of water with temperature as measured by the thermometer used (or, in the case of ice calorimeters, errors in the value of the latent heat or other constants used).

Of these there is little difficulty in arranging the errors from (1), (2), (4), (6), (7), (8) to be small by using proper care and suitably arranging the apparatus or the bars used. Serious error from (3) could be detected by taking a sufficient number of properly varied sets of readings. Small errors due to (9) (*b*) and (9) (*c*) are difficult to avoid and it is impossible to discover their existence. (10) (*c*) is the most serious cause of error in the ordinary method of mixtures. There is probably some error from this cause even in BUNSEN'S calorimeter, as it usually gives lower values than other methods. (10) (*d*) is unavoidable in all thermometric thermal measurements. (5) and (9) (*d*) are inherent to the method and are not to be avoided by the use of thermo-electric junctions instead of thermometers. It is also impossible to estimate the errors arising therefrom. In ÅNGSTROM'S method errors from (10) affect the result in the same way, and as all the temperatures measured are varying temperatures, errors of the same sort as (5) and (9) (*d*) may occur. ÅNGSTROM'S method is unreliable on other grounds. It is essentially based on the assumption that the ratio of the conductivity to the emissivity is constant.

The values of the conductivity of copper found by Professor TAIT were (reduced to C.G.S. units) for good conducting copper $1{\cdot}08\ (1+{\cdot}0013t)$; for bad conducting copper $\cdot 71\ (1+{\cdot}0014t)$. The ratio of these values is independent of nearly every source of error mentioned, and yet Dr STEWART (vide *Trans. Roy. Soc.*, 1893, p. 569) found $1{\cdot}12\ (1-{\cdot}001t)$; while KIRCHHOFF and HANSEMANN (*vide* WIEDEMANN'S *Annalen*, 9, p. 1; 13, p. 406) found $\cdot 51\ (1+{\cdot}0057t)$ both for pure copper. One has doubts about believing that the wide range of variations of these values is due only to differences in the specimens of metal used. I, therefore, determined to find the conductivity of the nickel I had used by a method with fewer sources of error.

Direct Method.

§ 9. THE APPARATUS.—The method of determining thermal conductivity by direct measurement of the rate of flow of heat and gradient of temperature is that adopted in the following experiments. This method was used long ago by CLEMENT and by PÉCLET (vide *Ann. de Chimie et de Physique*, 3ᵉ tom. ii. p. 107, 1841), and in their hands did not yield satisfactory results as they did not measure the temperatures of the metal itself, but it has been used with success by E. H. HALL, who utilised the metal experimented upon as one of a thermo-electric couple to measure its own gradient of temperature (vide *Pro. American Academy*, vol. xxxi. p. 271).

In the present investigation one end of the nickel bar used for FORBES's method was cut off, and an extra thermometer hole was drilled into it. Its surface was repolished. The dimensions were as follows :—

Diameter, from 4·660 to 4·667 cm.
Length, 42·55 cm.
Density, 8·75 grammes per c.cm.

No. of Thermometer Hole.	Distance in Centimetres from end at which the Rate of Flow of Heat was measured.
1	2·84
2	11·20
3	19·54
4	27·88
5	36·17

A shorter length would have sufficed, and it would have been an advantage to have made more thermometer holes. The bar was fitted up so that one end could be kept at any constant high temperature, while a flow of water could be kept cooling the other, the rise of temperature of the water and the mass of water passing per unit of time being measured. These data were sufficient to measure the rate at which heat left the end of the bar. The gradient of temperature at any point is given by the tangent to the curve drawn from the readings given by the thermometers.

A slide bench was erected in front of the table carrying the apparatus, and was arranged to carry a telescope which could be raised or lowered in a vertical line, and at the same time moved to and fro along the bench which was placed parallel to the axis of the nickel bar. The thermometers used in the bar were some of Professor TAIT's Kew thermometers from the same stock as those used in the FORBES bar, and they were placed so as to hang vertically, this being tested by a plumb line. As the telescope could only move so as to be always horizontal, parallax was avoided. When the telescope was adjusted so that one of the thermometers was in focus, all were in focus for that same adjustment, which was never altered. The readings were estimated by eye to the nearest tenth of a degree.

A diagram representing a vertical section through the axis of the bar is given in fig. 2. The heater was the cast-iron pot, J, which was used with the FORBES bar. The bar was fixed into the circular hole (at K) in the side of it with red lead. A Jena glass flask, F, with a fairly long neck was filled to the bottom of the neck with mercury and put into the pot; and the space, H, between was filled nearly full of mercury. To prevent mercury leaking through the cast-iron pot it was previously lined with pipe-clay, a paste of pipe-clay and water being painted in with a brush and allowed to dry. Into the neck of the flask was fitted a glass piece made as shown in fig. 2. The arrows show the path taken by the gas to reach the burner, and the temperature was kept constant by the mercury cutting off the gas supply at E on reaching a certain temperature. The by-pass, B, was opened to allow a full supply of gas while the heater was being warmed up to the proper temperature, the mercury in the flask being allowed to run over at D, which was at all other times closed. When the desired temperature was reached the by-pass, B, was nearly closed, enough gas being allowed to pass through it to keep the Argand burner, G, from going out. This gas regulator worked so well that a thermometer hung in the same place in the pot of mercury showed no variation exceeding one-tenth of a degree centigrade during a whole day.

At first a large steel cap was fitted on the end of the bar, with mercury inside it, the idea being to make it at once the heater and the regulator. It showed a steady, slow rise of temperature, and, although there was no visible leakage, in a few days fine drops of mercury were seen on the iron tray placed under the burner to catch the mercury in case of accident. No leakage of mercury could be noticed from it even under greater pressure from the inside while standing cold, and therefore the mercury must have leaked through pores too small to be noticed while the flame played upon them. The variation of the temperature of cut-off is a very delicate test of such leakage.

The rate at which heat was given out at the other end of the bar was obtained by measuring the rise of temperature and the rate of flow of the stream of water which played on the end of the bar. A brass cap, M, was fitted on the end of the bar, the water entering the space between it and the bar having its temperature measured at O, and the temperature of the water leaving the bar was measured at P. The thermometers at O and P were Anschutz thermometers graduated in fifths of a degree centigrade. The rate of flow of the water was found by observing the time taken to fill the flask, Q, of known capacity to the fiducial mark. The water was supplied at constant level from a chamber, S, containing the well-known inverted bottle device, R. Distilled water was used, but great difficulty was found in keeping the rate of flow regular until the plan was tried of making the outlet of a piece of glass tubing drawn out fine and broken off at the capillary portion. With this improvement the flow was very uniform, and the temperature of the water (at O) reaching the bar was also very steady, but the temperature of the water leaving the bar (at P) varied. When the water had been once used it was cooled by being put in the inner chamber of a double copper tank, while cold tap water was circulating in the outer chamber surrounding it. The same water was thus used over and over again.

The order of taking readings was as follows:—1°, the thermometers in the bar; 2°, the temperature of the cold water (at O) going to the bar; 3°, the time at which the empty flask, Q, was put to catch the overflowing water; 4°, the temperature of the water leaving the bar (at P) was read every half-minute while the flask was filling; 5°, the time the flask was exactly filled to the fiducial mark; 6°, the temperature of the water entering the cap at O; 7°, the thermometers in the bar. All these readings varied little in the course of one evening, and the rate at which heat was given out at the end of the bar varied within 2 per cent. The following table gives the readings (uncorrected) taken on 6th January 1898.

Average Temperature of Water.	Average Rate of Flow of Heat in Calories per Second.	Temperature of Air.	Temperatures of Holes in Bar.				
			No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
21·7	4·85	13·2	39·6	61·65	88·35	122·2	168·2
		13·2	39·6	61·7	88·4	122·4	168·1
21·8	4·86	13·2	39·6	61·7	88·4	122·4	168·1
		13·3	39·8	61·8	88·5	122·45	168·2
21·85	4·88	13·3	39·65	61·9	88·55	122·5	168·25
		13·3	39·65	61·8	88·5	122·55	168·45
21·86	4·83	13·3	39·65	61·8	88·5	122·55	168·45
		13·3	39·7	61·85	88·6	122·6	168·35
Mean 21·8	4·86	13·3	39·65	61·8	88·5	122·45	168·25

§ 10. Correction of the Thermometers.—The corrections of the thermometers were not found in the ordinary way, as there is always more or less doubt attached to any allowance that may be made for stem exposure on account of the impossibility of knowing the exact distribution of temperature along the stem of the thermometer. The method adopted was simple and allowed the testing to be carried out with the thermometers in as nearly as possible the same circumstances as they are in during the experiments.

The thermometers were tested at three different temperatures, at 0° C., about 100° C., and about 218° C. At 0° C., the correction was found by hanging the thermometers in a vertical position with their bulbs, and as much of their stem as was under the surface of the bar, embedded in powdered ice washed with distilled water. At the other two temperatures the apparatus, a vertical section through the centre of which is shown in fig. 3, was used. A piece of brass tubing of nearly the same diameter as the bar of nickel was cut into three lengths, E, F, and G, and these were brazed together as shown. The end, H, was closed, and three small tubes, A, B, and C, were brazed in the middle piece F. These tubes were about half full of Wood's alloy. The piece E was half-filled with water which was heated by the burner D, which was adjusted until just a small quantity of water vapour escaped at the open end K. The tubes, A, B, and C, were of about the same depth as the thermometer holes in the bar, and during the test the thermometers were suspended in a vertical position with their bulbs near the bottom of the tubes. Asbestos screens were fitted up at L and M to shield the thermometers from the disturbing effects of the burner on the one side and the escaping vapour on the other. The arrangement is just that of a modified reflux condenser.

The thermometers were suspended with their bulbs in the mercury of the heater (at H in fig. 2), and the temperature of the heater was gradually raised to about 100° C., when the regulator was adjusted to act. The thermometers were left there under these conditions from morning till evening. As readings were always taken in the evenings, while the heater was set working in the mornings, the thermometers were never read until they had been at the same temperature for several hours. It was therefore thought necessary to keep the thermometers the same length of time at 100° C. before testing them at that temperature, so as to allow the glass to take on the same set that it had in the bar at the same temperature. It is possible that if an ordinary mercury thermometer is kept for hours at some temperature before it is read, its reading at the same temperature on some other occasion will only be the same after it has remained at that temperature for some hours.

The burner D was lit, and after the testing apparatus had been at 100° C. for some time, one of the thermometers was taken out of the mercury heater and quickly put into tube A. After the first two or three occasions, it was found easy to do this so dexterously that the reading on the thermometer did not fall more than 2° in the interval. After it had been in A for some time, during which the reading was constant, it was rapidly transferred to B, and by and by to C. The thermometers were hung up vertically by means of a plumb line, and the readings taken with the telescope. It was found that when E was too full of water, even when it was just over half-full, the readings in A, B, and C were not alike. When that was the case, the thermometer was left in one of the tubes until enough of the water had evaporated. The barometer was read sometime during the test and the true temperature of the bulb of the thermometer found from Regnault's tables. The difference between the observed reading on the thermometer and the temperature of the water vapour gave the whole correction at that temperature, the graduation correction and the stem exposure correction being thus lumped together. The same thing was gone through for each of the thermometers.

The same sort of process was repeated with the same apparatus, after the water had been dried out and naphthalene put in its place. Pure naphthalene was used, and as the boiling point of pure naphthalene has been determined on the air thermometer scale by Crafts, and has been found to be very constant, it is as satisfactory a "fixed" point on the scale of temperatures as one can wish for. The total correction of each of the thermometers was thus found at the temperature of the boiling point of naphthalene. The graduation corrections on the Kew thermometers used were known to be small, and hence it was only to be expected that an expression of the form $a+bt^2$ would represent the correction. This expression suited the values of the corrections found for all the thermometers except one to within a fifth of a degree, but the value of b was not the same in all cases, as it varied from ·00008 to ·000115. Curiously enough, b was smaller for those thermometers graduated up to 300° C. than for those which could not read above 220° C. The corrections at 0° C. were zero for most of the thermometers. One read ·55° too low, but that was due to a small particle of the mercury having been shaken up into the top of the stem—probably during transit—from which it could not be again dislodged. It was on the strength of these results that $\cdot000113t^2$ was used to give the stem correction in the FORBES bar experiments.

The following table gives the corrected mean readings obtained from the last three experiments, together with the values of the conductivity calculated from them.

Date of Experiment.	Temp. of Air.	Corrected Mean Temperatures of Holes in Bar.					Temp. at End of Bar.	Gradient at End of Bar.	Mean Temp. of Water.	Flow of Heat in Calories per Second.	Conductivity at End of Bar.
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.					
31/12/97	14·8	39·3	74·1	115·9	168·1	242·7	28·5	3·66	16·8	8·12	·130
4/1/98	13·2	47·0	79·0	118·9	170·0	243·2	37·7	3·23	22·1	7·225	·131
6/1/98	13·3	40·5	62·3	89·4	123·9	170·3	34·3	2·08	21·8	4·86	·136

§ 11. THEORY OF THE METHOD.—Let K be the conductivity, θ the temperature, X the distance from some fixed point on the axis of the bar, of a section of the bar of area A, across which H units of heat pass in unit of time, then

$$\mathrm{KA}\frac{d\theta}{dx}=\mathrm{H}.$$

Corresponding values of θ, H, and $\frac{d\theta}{dx}$ are given in the above table for the end section of the bar whose cross-section is 17·1 square centimetres (diameter is 4·663 cms.). The values of H given are subject to two corrections: (1) a correction for heat lost by radiation from the brass cap; (2) correction for the changes in the thermal capacity of unit mass of water with temperature. An estimate of the former error shows that it never exceeded 1 per cent., so that it is probable that these corrections combined do not exceed 2 per cent. They are rather smaller than the corresponding corrections in a specific heat determination.

The values of $\frac{d\theta}{dx}$ are liable to error from two sources: (1) thermometric errors in the temperature of the nearest thermometer hole; (2) arithmetical or geometrical errors in differentiating the temperature curve. Errors from both of these causes would have been reduced by having more thermometer holes, and what discordance there is between the values of the conductivity found from the three sets of readings given above is probably mostly due to errors in estimating $d\theta/dx$. Differences amounting to 2 or 3 per cent. are only to be expected. All these sources of error effect FORBES's method,—and, of course, also ÅNGSTROM's—but to these are added in FORBES's method all those arising from the cooling experiment.

The measurements referred to only determine the conductivity at temperatures somewhat above that of the air, but the conductivity could be found in a similar manner at other temperatures (such as slightly over 100° C., by allowing the water in the cap to be evaporated into steam). Also, by using an electrical heater, the heat supplied at the hot end (subject to corrections for radiation) could be measured and the gradient of temperature at that end. Such experiments, however, were not carried out in this case, because it was seen, in the manner described below, that the conductivity varied little with temperature.

§ 12. CHANGE OF CONDUCTIVITY WITH TEMPERATURE.—Before the brass cap was fitted on the end of the bar for the experiments just described readings were taken with the bar losing heat only by radiation. After the distribution of temperature became steady, the heat which passed any cross-section of the bar was lost by radiation from the rest of the bar beyond. The following table gives the temperatures obtained, thermometric corrections being applied.

Temperature of Air.	Corrected Mean Temperatures of Holes in Bar.				
	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
13·6	63·85	68·9	78·7	95·35	119·5
8·4	83·55	91·0	107·35	134·15	174·4
14·3	102·7	112·75	132·8	167·35	219·1
9·2	110·15	121·8	146·2	185·4	248·3

Curves were drawn from these readings, and differentiated. By supposing the bar prolonged by an amount slightly over the length of the radius, and producing the temperature curve to that point, one obtained the curve which would suit the bar if no heat had been lost from the end, at which place $d\theta/dx$ would then be zero. From the first set of readings the value of $d\theta/dx$ at the section which had the temperature 120·65° C. was found to be 3·55, and its distance from the point at which $d\theta/dx$ vanished was 38 centimetres. The average excess of the temperature of those 38 centimetres of the bar over the temperature of the surrounding air was 67·45°. This gives the following relation :—

$$\mathrm{KA} \times 3{\cdot}55 = \mathrm{E}p \times 67{\cdot}45 \times 38\,,$$

where K is the conductivity at 120·65° C. and E the average emissivity under the conditions referred to. From the set of readings obtained on 31st December 1897, and given on page 19, the gradient at 120·65° C. was found to be 5·66, the gradient at 63·2° C. to be 4·33 ; and the distance between the points at these two temperatures was 11·72 centimetres. The average excess of the temperature of those 11·72 centimetres of the bar over the temperature of the surrounding air was 76·0°. If we *assume* the average emissivity to be the *same* in these two cases, we find that 1·23 is the value of that part of the gradient which is required to account for the heat lost by cooling over the 11·72 centimetres in the latter instance. For if

$$\mathrm{KA} \times 3{\cdot}55 = \mathrm{E}p \times 67{\cdot}45 \times 38\,,$$

then

$$\mathrm{KA} \times 1{\cdot}23 = \mathrm{E}p \times 76{\cdot}0 \times 11{\cdot}72\,.$$

If we deduct 1·23 from the gradient, 5·66, at 120·65° C. in the latter experiment (date 31st December 1897), we find the gradient (the remaining 4·43) which would cause the same heat to pass the cross-section at 120·65° C. as passes the cross-section at 63·2° C. with its gradient at 4·33. In other words 4·43 and 4·33 would be corresponding values of the gradients at 120·65° and 63·2° respectively if no heat were lost by radiation from the bar. The conductivities at these two temperatures are inversely as these numbers. This shows a diminution of conductivity of 2½ per cent., with a rise in temperature of about 60°. This is within the limits of experimental error. The assumption that the average emissivities for temperature excesses of 67·45° and 76° are the same is not likely to be correct. The emissivity in the latter case will be greater, probably by something of the order of 2 per cent. The effect of the increase of emissivity with temperature will be to reduce the apparent diminution of conductivity with rise of temperature, and might even change it into an increase, but in any case it would be very small and within the limits of experimental error.

The following tables give two sets of data obtained from the curves drawn from the corrected readings already given in tabular form.

Corresponding Values of			Average Temperature Excess.	Corresponding Values at another Section of Bar of		
x	θ	$d\theta/dx$		x	θ	$d\theta/dx$
0	63·2	0	67·45	38·0	120·65	3·55
8·73	63·2	4·33	76·0	20·45	120·65	5·66
0	108·5	0	125·0	30·0	188·7	6·00
18·23	108·5	5·335	130·8	30·55	188·7	8·13

From these are deduced the following :—

	θ	$d\theta/dx$
Corresponding values of θ and $d\theta/dx$ which would be found if no heat were lost from surface of bar,	63·2 120·6	4·33 4·43
Ditto,	108·5 188·7	5·335 5·55

These figures indicate a diminution of conductivity of the amount ·000066 per rise of temperature of 1° C. The conductivity cannot fall so much as this, and in any case the change of conductivity with temperature is within the limits of error of such experiments up to a temperature of 200° C.

§ 13. Conclusion.—The conductivity of nickel found by the direct method is ·132. There is some doubt about the third figure after the decimal point, and that figure is the only one affected by changes of temperature up to 200°C. It is interesting to note that the value of the specific heat of nickel found by using nickel turnings, viz., ·11, would, if multiplied by the ratio of the conductivity to the specific heat at the mean temperature 60°, give a result in exact agreement with the above. It should, however, be stated that the specimen of nickel showed slight fissures. These were not serious enough to affect the readings sufficiently to make it noticeable in the appearance of the temperature curves, and the readings obtained from the Forbes bar do not show irregularities from such a cause. The nickel used was also very pure. I am much indebted to my colleague, Mr F. V. Dutton, for analysing it for me with the following result:—

Analysis of Nickel.

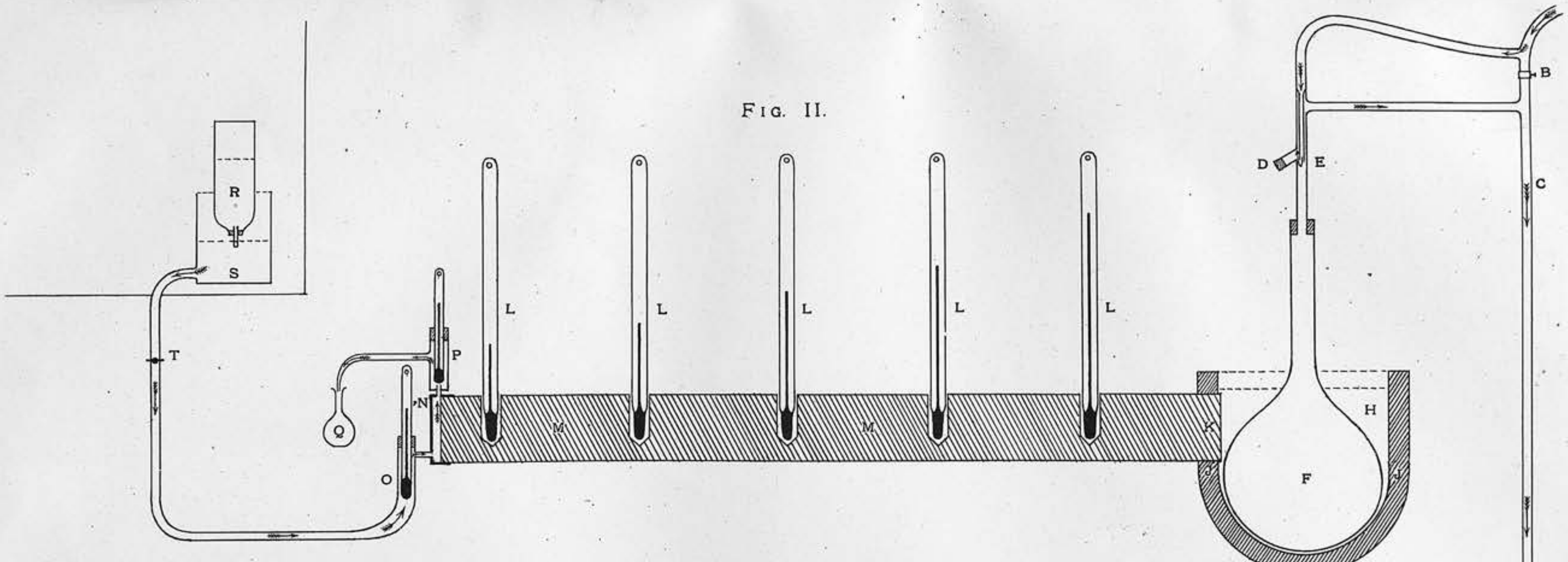
Manganese,	1·63	per cent.
Magnesium,	0·28	,,
Iron,	0·75	,,
Nickel,	97·22	,,
Total,	99·88	

The Forbes's method experiments were carried out in Edinburgh University Physical Laboratory; the other method was done in the Physical Laboratory of the University College of North Wales, and from time to time the work was carried on partly in Edinburgh, partly in Bangor. I have to thank Professor Tait and Professor Gray for affording me every facility in carrying out these determinations.

EXPLANATION OF FIGURES.

Fig. 2. A. Gas supply.
B. By-pass.
C. Tube leading gas to burner.
D. Opening for letting out mercury to regulate temperature of cut-off.
E. Place at which mercury acts on gas supply.
F. Glass flask containing mercury.
G. Argand gas burner.
H. Mercury.
J. Cast-iron pot.
K. End of bar heated.
L. Thermometers.
M. Bar of Nickel.
N. Brass cap.
O. Water inlet with thermometer.
P. Water outlet with thermometer.
Q. Flask for measuring rate of flow of water.
R. Inverted bottle
S. Water tank } on reduced scale.
Fig. 3. A, B, C. Tubulures for thermometers containing Wood's alloy.
D. Gas burner.
E. Chamber of water or naphthalene.
K. Open end of apparatus.
L, M. Asbestos screens.

FIG. II.



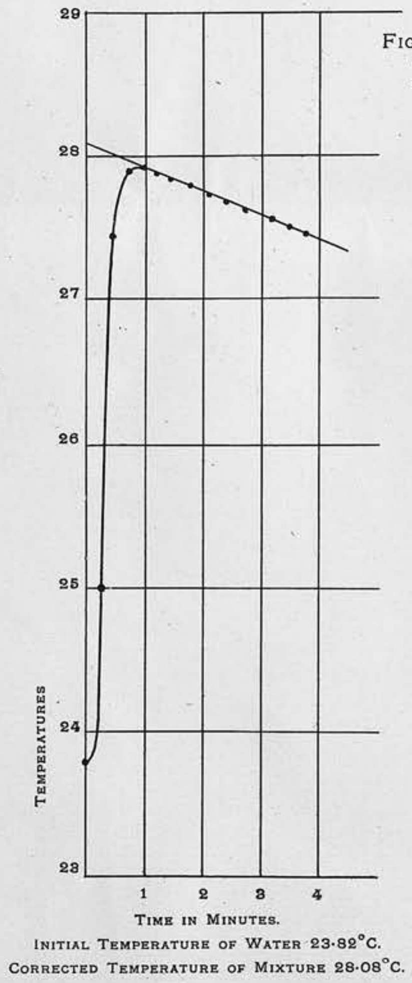


FIG. I.—CURVE ILLUSTRATING THE METHOD OF APPLYING THE CORRECTION FOR COOLING IN SPECIFIC HEAT DETERMINATIONS.

FIG. III.