LOGO, MATHEMATICS AND UPPER PRIMARY SCHOOL CHILDREN

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Declaration

This thesis was written by myself and contains original work which I have carried out.

ABSTRACT

This study was set up to assess the contribution that a computer modelling approach using the language LOGO could make to the quality of mathematics learning in primary school children. Following a constructivist theory of mathematical learning it is argued that many problems children have with their mathematics results from instrumental learning without understanding, rather than relational learning. LOGO was developed, in part, to provide a learning environment for children to investigate mathematical ideas and thus develop their own understanding. Previous research has not provided much evidence that this happens, nor specified what mathematical learning could be expected to take place and what pedagogic approach could bring it about. Other questions relating to the maturity of the children and their aptitude for programming have similarly been neglected. This study was set up to identify the mathematical ideas intrinsic to Turtle Geometry and to explore the conditions under which this learning could best be fostered.

The study was carried out in three phases. The first phase considered the constraints of maturity and the need to program on the learning of 9 and 11 year old children. The second phase of the study followed up the programming of the older children, to see what mathematics they were encountering, and what sort of activities encouraged them to think mathematically. Pre and post tests were used to identify the mathematical learning which was taking place. In Phase III a control group

was used to identify the particular mathematical learning which could be attributed to LOGO experience, and to assess the transfer of mathematical learning from the LOGO context to novel problem solving.

The first two phases revealed considerable mathematical activity intrinsic to Turtle Geometry. The need to learn some simple programming apparently did not present a barrier to mathematical investigation. The test results in the third phase showed that the children had deepened their understanding of angles, variables and general process aspects of mathematics through using LOGO. The performance of the children on the computers was monitored and was found to be revealing of their current mathematical understanding.

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CHAPTER 1

INTRODUCTION

Recent surveys and research have shown that contemporary mathematics education in Britain is not being very successful. Extensive work on secondary school mathematics carried out at Chelsea College (Hart 1981) has shown that one third of pupils leaving school have only a rudimentary understanding to show for their five years of mathematics learning. The Cockcroft Report (1982) also highlighted the lack of confidence of many adults in using the mathematical knowledge which they did possess. This report recommended that more attention in schools should be paid to the development of mathematical understanding and strategies, rather than to the computational skills which have made up the main body of traditional mathematics teaching.

There are two likely reasons why in the past children have learned mainly arithmetic calculation skills in their mathematics lessons. Apart from their practical use in the days before calculators, such skills were both easy to teach and easy to test. Understanding of mathematical concepts and strategies are difficult to teach directly and were assumed to develop through computational practise. The Chelsea College studies (Hart 1981) indicate that this assumption is not valid.

Much modern thinking about mathematics learning traces back to the work of Piaget. His theories on the development of knowledge popularised the idea that practical exploratory activity provides the necessary basis for the acquisition of new mathematical concepts. This is tacitly accepted in most modern theories of mathematics learning, and resulted in much discussion in the 1960's on the merits of discovery learning.

It has become evident from the research work undertaken in the last two decades that there are qualitatively different types of mathematical learning (Egan and Greeno 1972; Skemp 1976; Skemp 1979; Davis and McKnight 1979). Skemp (1979) classified the two main learning outcomes as "instrumental learning" and "relational learning". The former implies that a new cognitive structure has been produced, which can be applied in a certain context, but which is not necessarily related to any other existing structures. The latter implies the development of connections relating new material to existing structures. These relational structures give greater flexiblity and enable the learner to think about the problem and recognise its application in new contexts.

This thesis argues that the difference in learning outcomes is important in illuminating the difficulties which children have in mathematics. Instrumental and relational learning are both necessary for successful mathematical performance. However, for the solution of novel problems, the flexibility intrinsic to relational learning is essential. Within this theoretical framework the findings of Hart (1981) suggest that many children are learning mathematics only instrumentally. They are thus able to perform calculations with ease, but not discuss the type of calculation required for a particular problem. The latter involves thinking about mathematical processes rather than doing them. As will be explained in Chapter 2, this meta-level ability to think about processes is an intrinsic part of relational learning. The lack of confidence in adults reported by Cockcroft (1982) can also be attributed to the same cause. Instumental learning is useful only in a context which is similar to the one in which it was learned. Adults are thus not able to apply their learning to problems presented in an unfamiliar form. The problem of current mathematical teaching can therefore be interpreted as one of improving the quality of learning by promoting relational learning.

Following the Piagetian school, a variety of different learning aids were developed in the 1960's to give children experience of manipulating mathematical objects. Their advantages and limitations for developing relational learning are assessed in Chapter 2. As computers have become available in schools their use as modelling tools for mathematics learning was suggested. In particular the computer language LOGO was developed with mathematics learning in mind (Feurzeig et al 1969). It was claimed that children could become active mathematicians by investigating mathematical ideas with computers, rather than being told about mathematics (Papert 1972), thus developing their understanding and strategies. Their active involvement in their own learning, it was claimed, would produce a qualitatively different outcome. Chapter 3 puts forward in detail how model building activities using the LOGO programming language could contribute towards mathematical education. The features of LOGO, as opposed to other computer languages, which make it particularly suitable for this use are described. The way children may get involved in LOGO learning activities and the type of mathematics learning intrinsic in them is then discussed.

The study reported below was set up to assess the contribution that a LOGO approach could make to the quality of mathematics learning in primary school children. Previous studies using LOGO, reviewed in Chapter 4, have only been partially successful, though this particular issue about the quality of learning has not been considered. Previously, researchers investigated the effects of LOGO experience on general mathematical performance and problem solving behaviour. We shall argue that neither of these are particularly appropriate areas for investigation. Many practical issues about the use of LOGO also arise from previous research, relating in particular to:-

- the teaching approach chosen;
- the level of expertise in programming required;
- the effects of ability and developmental levels of children on their use of LOGO for mathematical learning.

Unlike previous studies the current study is on the quality of learning which could be attained through using LOGO. However, other practical considerations raised by earlier research have also been investigated as they arose.

The study was carried out in three phases. The main purpose of the first phase, described in Chapter 5, was to investigate the constraints placed by ability and developmental maturity on the child's use of LOGO in learning mathematics. Two classes, one of 9 year olds and the other of 11 year olds, took part. Each worked in small groups with the floor turtle for one hour each week, over the period of one school term. Their ability to program the floor turtle and to learn mathematics from it was assessed by short tests. The results suggested that no child was prevented from developing mathematical concepts by an inability to program.

In Phase II of the study, described in Chapter 6, the same class of 11 year old children continued to use LOGO for two further terms, using TI99/4a microcomputers. This second phase was used to identify what particular mathematical learning could be attributed to LOGO experience. For this, pre and post tests of mathematical understanding were used. The LOGO work of the children throughout the year was also monitored. The results showed that children developed a good understanding of angles and shape. There was evidence from monitoring the children's performance that some were learning to think about mathematical processes, and a few showed improved understanding of variables after using LOGO.

The third phase was the main part of the study. This is reported in Chapters 7 and 8. It was designed to go further in teasing out the mathematical learning taking place, particularly with respect to process aspects of mathematics, and to assess the transfer of this learning to normal school mathematics. Two further classes, each of thirty two 11 year old children, were involved in this part of the study, one as the LOGO class and the other as control. Tests of understanding of angles, variables and process aspects of mathematics, involving generalisation, abstraction and use of information in novel problems, were used. Though initially both classes were matched on mathematics attainment and IQ, the LOGO group performed better on all the post-experiment tests. Records of the LOGO work done by the children were used diagnostically, to assess the quality of learning. Case study material of the children's programming throughout the year was analysed and is discussed in Chapter 8. Chapter 9 describes other research findings on:

- learning programming
- the role of the teacher
- organizational aspects of the approach used.

The final chapter assesses the findings of this study in the field of mathematics education.

CHAPTER 2

MATHEMATICS LEARNING

2.1 Mathematics in Britain.

The Cockcroft Committee inquired into the teaching of mathematics in schools. In the early part of its report it discusses findings of several studies of the practical application of mathematics in everyday life (Cockcroft 1982).

One study by Sewell, for the Advisory Council for Adult and Continuing Education, found many adults were incapable of handling mathematics in their normal lives and used complex methods to get around and hide their inability (op.cit. para 16-30). On considering what adults need to know, it was found to be limited and to relate to:-

- money,
- measurement,
- understanding of timetables and charts,
- calculations associated with any of the above.

There was also a need for people to understand approximations and to be able to use them, but the Cockcroft Committee concluded that the greatest need was for people to "have sufficient confidence to make effective use of whatever mathematical skills and understanding they possess..." (op. cit. para 34).

Two other studies were carried out in Bath and Nottingham on the actual use of mathematics in industry (op. cit. para 59-66). They also concluded that estimation and approximations were very important, for checking the reasonableness of answers, choosing appropriate measuring scales and selecting the required degree of accuracy (op. cit. para 78).

The section of the Cockcroft Report on primary education states that a wide spread of understanding exists among children of the same age (Hart 1981, op.cit. para 341). There is also a wide variety of attitudes towards mathematics, from those who see it as providing a means of explaining and controlling the environment, to others who fail to see any relevance at all outside the classroom (op. cit. para 346).

Too much time has been spent in the repetitive practice of processes which children already understand (HMSO 1978, op. cit. para 302), and the results of this practice were illustrated in the research carried out at Chelsea College (Hart 1981). Children, when asked to explain how to approach a simple mathematical problem, found more difficulty in choosing which strategy to adopt than in doing the calculation. It seems that though they can use mathematical techniques they do not understand the purpose or application of them, and have insufficient confidence to use their skills.

The Cockcroft Committee recommended that practical work should be carried out throughout the primary years (op. cit. para 289) and that mathematical explorations and investigations should be encouraged (op. cit. para 321). Children should be given opportunities to learn general problem solving strategies:-

-seeing patterns in results;

-making conjectures and testing them out;

-looking for simpler problems to give a lead;

-persisting in exploring a problem;

-working with others, and communicating (op. cit. para 323).

They concluded this section with the admission that little is known about how to promote these activities, or to what extent strategies and processes for problem solving can be taught.

2.2 Theories of mathematical development.

Much modern work on mathematical learning is based on Piagetian ideas. According to Piaget, a child extends his understanding of the physical world by fitting new experience into his mental model of reality which he has built up from the past (Piaget 1970). The model of reality or cognitive structure is adjusted to allow for new information through processes of assimilation and accomodation, and the very existence of the cognitive structure provides motivation to explore the environment and learn. As new information is encountered which cannot be assimilated into the existing model it sets up a disequilibrium in the system which slowly builds up. Equilibrium is restored only by a fundamental change in the underlying model, and so the child moves to the next stage in his cognitive development.

The idea of developmental stages is an important part of the theory, but Piaget's approach was that of an epistemologist, looking primarily at the development of knowledge and understanding in particular areas, rather than at the total development of the child. In the theory, the developmental stages are characterised by distinctive behavioural and reasoning patterns. At first children pass through a sensori-motor stage where they learn through physical exploration of the world around them. Next they enter the pre-operational stage where mental activity is dominated by their immediate perceptions. At the pre-operational stage children are unable to consider two dimensions simultaneously, or appreciate the conservation of physical quantities, being misled by appearances. Around the age of 7 or 8 years they enter the concrete operational stage and are able to conserve quantities, follow simple logic and manipulate ideas when presented in a fairly concrete form. The next stage, that of formal operations, is not attained until the age of 12 or older. They can then consider hypothetical outcomes and explain phenomena in a scientific way. The stages are not arrived at at fixed ages, but depend on the maturity and experience of each child. However, the order in which they occur is invariant (Piaget 1970).

Piaget stimulated much research, particularly on the stages of development. Different theoretical schools have resulted from this research. Some neo-Piagetians have postulated underlying mechanisms, such as the biological development of memory capacity, as leading to the surface effects of developmental stages as described by Piaget (Pascual-Leone ,1970, Case 1982). Others have found the concept of stages too elusive and dependent on the tests used to measure them, whilst recognising the important contribution Piaget made to the understanding of learning processes (Biggs 1980).

Several implications for the teaching of mathematics can be drawn from the Piagetian school:-

> Practical activity and experience are necessary forerunners to the development of cognitive structures.
>
> The experience and material must be at the appropriate conceptual level for the child to relate it to his existing model of reality, so the learning environment should be structured for the child and involve activity for which he can see the purpose.
>
> The child must become aware of contradictions in his existing model in order to develop beyond it. Thus making mistakes and using the information from them is an important part of learning.

Attempts have been made to relate Piagetian developmental stages directly to children's performance in mathematics and science, both theoretically and practically. The results seem to depend on the method chosen to measure developmental stages. Where scientific tasks similar to those used originally by Piaget in his interviews have been used, fairly consistent results have been obtained (Shayer, Kuchmann and Wylam 1976). An attempt to measure the stages in a mathematical context by researchers at Chelsea College (Hart 1981) was unsuccessful, as children were found to be inconsistent on different tasks.

A theoretical approach was taken by Collis (1975) in which he examined the content and structure of mathematical problems, to determine their level of difficulty. Each was defined as concrete or abstract. Concrete content refers to small numbers which are within the intuitive grasp of children, whereas large numbers or algebraic symbols are considered to be formal. A problem is said to have concrete structure if it contains only first order relationships, such as finding the average speed from distance and time measurements, whereas second order relationships, e.g. comparison of ratios or acceleration, have formal structure.

Collis them associated these problem difficulty levels with Piagetian levels in the following manner:-

concrete content and structure .... early concrete operations

formal content with concrete structure .. late concrete operations

concrete content with formal structure ... early formal operations

formal content and structure .... late formal operations

Studies which have looked at children's performance in science and mathematics have generally concluded that the main difficulty arises with the need for formal operational thinking (Lunzer 1973, Lovell 1974, Shayer 1978). Collis found that children could succeed with problems of formal structure typically only at the age of 14 years, whereas they could cope with formal content in concrete structure problems at 10 or 11 years. One particular difficulty which rendered a problem structure "formal" was the lack of closure. Children could cope with a problem in several steps, as long as each step could be completed before the next was tackled. When this was not the case, the problem was "unclosed". Both Collis (1975) and Lunzer (1973) described this problem. A common finding was that children had a high success rate with problems in the form

$$17 - 8 = ?$$

but were much less successful with the unclosed form

$$17 - ? = 8$$

The first is a simple subtraction problem but the second requires some rearangement before the simple subtraction algorithm can be applied.

2.3 Instrumental and relational learning.

Other researchers have developed models of mathematical thinking, starting from positions other than the Piagetian one. Skemp has put forward a theory of intelligence, though particularly interested in mathematical learning, based on an information processing model (Skemp 1979a). In this theory learning takes place at two levels. To distinguish between the two Skemp uses the term "director systems". The first level is concerned with actions carried out in the real world. These operations are under the direction of director system delta one. The second level director system, delta two, acts not in the real world, but on the contents of delta one. In the context of mathematical learning, delta one carries out the mathematical process, and delta two reflects on the process carried out. Delta two is therefore responsible for the selection of processes used by delta one, and for building the connections in knowledge and relating new information to previously acquired knowledge.

Related to the operations of these two director systems Skemp defines two learning outcomes; instrumental learning and relational learning. Instrumental learning takes place within the delta one system, for instance, when new mathematical techniques are learned through practice. This newly acquired knowledge can be applied to situations similar to the one in which it was learned, but may fail to be applied in a different context. It stands as an isolated unit of knowledge, unrelated to other conceptual structures. As such, it is not particularly meaningful, and thus is liable to be forgotten.

Relational learning takes place through the actions of delta two. New information is assimilated into existing conceptual structures, and thus related to previously acquired knowledge. The knowledge is then no longer isolated and limited in context, but can be applied with flexibility. Because it has thus become part of a larger chunk of knowledge, it becomes more meaningful and so is less likely to be forgotten. Relational learning is also concerned with the processes and strategies used in delta one, i.e. it has a meta-level component where the learner is thinking about mathematical thinking. The development of relational learning takes longer than instrumental learning, and is typically accompanied by a time of reflection, which may occur before or after the problem solving activity (Davis and McKnight 1979, Thompson 1984).

The distinction between instrumental and relational learning would seem to be quite useful in illuminating the processes going on when a person is learning mathematics. Both types of learning are required for successful performance. Mathematical problem solving, however, almost always requires relational understanding.

A different theoretical model of mathematical thinking has been put forward by Davis (Davis and McKnight 1979, Davis

1980). His work has been based on Artificial Intelligence models, using such concepts as "frames", (Minsky 1975). Davis has attempted to apply this theoretical work to a mathematical context. Though starting from a different theoretical viewpoint from Skemp, he also makes the important distinction between the two types of learning outcomes. Davis uses the terms "sequential learning" and "gestalt learning". Sequential learning is essentially the same as instrumental learning. It is acquired by rote, and consists of chains of actions which are fairly inflexible. Gestalt learning is similar to relational learning, in that the knowledge forms part of an interconnected whole. It allows much more flexibility of thinking. Davis postulates a pattern matching mechanism which recognises a mathematical problem as a particular type and retrieves the appropriate frame. This frame then guides the search for relevant information in the problem. Relational understanding, within this theoretical model, can thus be diagnosed from the way a learner uses information in a problem. Similarly his ability to think about and choose different strategies is also indicative of relational understanding.

These models, though from a different theoretical basis, are quite consistent with the Piagetian model of learning, and share the emphasis on the need for manipulation of mathematical objects prior to the construction of new cognitive structures. The findings of Collis (1975) and Lunzer (1973) on the difficulty caused by lack of closure in a problem can be explained simply in terms of instrumental and relational understanding. The sequential problem

$17 - 8 = ?$

requires only instrumental learning as the information is presented in the usual order for carrying out a learned subtraction computation. The unclosed form

$17 - ? = 8$

requires an understanding of the meaning of subtraction, i.e. a relational understanding. In general formal operational problems, as outlined by Collis (1975), require relational understanding for their solution, whereas children can be quite successful at concrete operational problems using only instrumental learning. The findings of Hart (1981) that one third of children in secondary schools were not getting beyond an elementary level of mathematical understanding also suggests that these children are only learning instrumentally. Their performance then depends heavily on memory, tends to involve fixed sequences of activity, and breaks down if the sequence is broken or the information is given in a slightly different form. They are thus successful at highly practised mechanical computations, but typically find word problems, and problem solving in general, difficult.

2.4 Factors which affect the learning outcome.

As the problems of mathematics teaching are seen to stem from the need to develop relational learning, factors which may affect the quality of learning must be considered. These fall into three different but interrelated categories:-

- the teaching approach;
- the student's personality, motivation and perception of the learning task; and
- the educational materials used.

Following the research work stimulated by the Piagetian school in the early 1960's, much interest was shown in "discovery learning". This has been interpreted in different ways by different researchers, some of it being heavily dependent on the use of particular materials, while other work used different approaches with the same material. Several studies in the late 1960's (Land and Bishop 1967 - 70, Worthen 1968) showed that a guided discovery approach to learning mathematics, by presenting examples in a structured sequence rather than stating the underlying principle, gave children a superior recall of the information at a later time, better transfer to other problems, and better problem solving strategies. Only on immediate recall did the exposition approach, stating the rule first and then relating all examples to it, perform better than the discovery approach.

Similar results were obtained by Mayer, Egan and Greeno investigating the quality of learning resulting from the two approaches (Mayer and Greeno 1972, Egan and Greeno 1972). They concluded that relational learning was less likely to occur with the exposition approach. In discovery learning they suggested that the learner's existing knowledge must be invoked in order to make sense of the new knowledge, thus making it more meaningful.

Anthony (1973) pointed out that discovery learning only produced better results when the discovery was successful. He analysed the results of some earlier studies and showed that their overall inconsistencies were caused by a failure to separate the successful from the unsuccessful discoverers. Scandura (1969) also demonstrated the negative effects of unsuccessful discovery learning.

It seems likely that personality differences may also affect suitability for some types of discovery approaches. Trown (1970) found that extroverts responded better to a discovery approach than to one where the rules were thrust upon them. In a later study (Trown and Leith 1975) he found that the converse was true of highly anxious pupils. This finding was also confirmed by Bennett (1976) looking at the performance of pupils in formal and informal classrooms. It is hypothesised that the important factor affecting performance in these studies was the amount of uncertainty in the learning situation. Pask (1975) also looked at tolerance for uncertainty amongst pupils, whilst investigating their learning styles. He concluded that comprehension, or wholist learners could tolerate higher levels of uncertainty than serial learners. When designing learning situations to promote relational learning, these personality variations in tolerance for uncertainty and the need for the discovery to be successful must to be borne in mind.

Other differences between students affecting their quality of learning relate to their own perception of the learning task. It is clear from the work carried out by Marton and Saljo (1976) that children and students may in fact choose the manner in which they learn something. A student learning for an examination or test may choose an instrumental approach, though capable of relational learning. The instrumental approach is quicker to complete, and may give superior accuracy and speed for immediate recall (Egan and Greeno 1972).

Other factors which may affect the learning outcome relate to the learning materials used. Throughout the history of mathematics teaching a variety of different tools and aids have been invented to facilitate investigative approaches, i.e. to promote relational learning. The tools were often developed as concrete embodiments of specific abstract mathematical ideas with which children frequently had difficulty. Some of these, the number line for directed numbers or shaded areas for fractions, involve only pencil and paper. Others use special equipment, such as the abacus, Dienes' blocks and Cuisenaire rods.

The choice of material and the way it is used have been the subject of research over the last twenty years. Some materials have been found to be inappropriate in certain circumstances. Bell (1976) found that work with a spike abacus to illustrate addition in base 5 caused confusion with weaker pupils. They were concerned that the discarded rings from the first spike should be put somewhere when the total of 5 was reached. In this case preoccupation with the physical apparatus distracted attention from the mathematical principles of place value. Similarly, use of squared paper as a concrete embodiment for the essentially abstract task of multiplication of decimals was found to be of little use (Bell 1983). It is important that any aid should not be intrusive and distract from the mathematical content.

Dienes (1960) showed in his experimental work that analysis of mathematical ideas can arise "either out of construction or out of rule-based play". He also stressed the need for perceptual variability, stating the principle that to build a schema or understand a mathematical structure effectively one must meet it in a number of perceptually different situations to identify its purely structural properties. He found that if pupils worked too long exclusively on one particular embodiment, they began to attach importance to mathematically irrelevant features.

Following Dienes' work several researchers have looked at the number of embodiments to be used together. Wheeler (1972) showed that children who had used three or four different aids for adding and subtracting two-digit numbers were better able to extend the method to multi-digit ones. Children using only two aids did equally well on two-digit problems, but showed less ability to transfer the principle to the multi-digit test.

Experience of several embodiments appears to facilitate transfer to other situations, giving evidence for relational learning. Incidental features of particular embodiments are eliminated by this approach and it may also assist in the assimilation of different facets of the conceptual structure highlighted by different embodiments.

2.5 Summary of Chapter 2.

The limitations of current mathematical education in Britain, as reported in the Cockcroft Report and from the extensive Chelsea College studies of mathematics in secondary schools, have been reviewed, together with some theories of mathematical learning. The shortcomings were found to relate to the quality of mathematical learning of a substantial minority of children. These children are apparently learning only instrumentally. They are able to perform mathematical calculations within the classroom, but unable to apply their knowledge to problem solving, or to any practical situations outside. The need then is to develop teaching methods to encourage relational learning in mathematics.

A review of the research on learning methods and styles led to the following conclusions:

1) A guided discovery learning approach, allowing children to find rules from structured examples, rather than stating the rule first and then giving the examples, is more likely to encourage relational learning.

2) The personality of a child is likely to affect his response towards discovery learning approaches. Some children have a low tolerence for uncertainty in a learning task, and so require sufficient structure to ensure the success of their investigations.

3) The purpose of the work should be clear to the children to improve their chances of developing relational understanding.

4) Educational aids used should be flexible and varied, illustrating mathematical principles, not distracting from them.

CHAPTER 3

COMPUTER MODELLING

It is argued in this thesis that computers can be used to provide excellent educational tools for developing relational learning in mathematics. Since the advent of small computers, they have been used in a variety of different ways in education and training. Any interactive computer work has at least two advantages over traditional class work:-

- The computer can be used to give immediate feedback on a task. This is not usually available from a busy class teacher, or from most other investigative apparatus.

- The computer responses are neutral and uncritical. This encourages the child to experiment and thus gain confidence, without fear of failure.

As a result, most children find work with computers quite highly motivating, though not all programs used in schools are sound in educational principles.

For mathematics work, the computer has been used in a number of different ways, from drill and practise programs to automatic theorem provers. Howe argues that one of the most fruitful educational uses of a computer, consistent with the aim of developing relational learning is as a tool for model building

(Howe and Ross 1981).

3.1 Model building.

In this approach to learning the pupil attempts to build a model of a particular system, thereby making explicit his understanding of that system. He needs to be provided with materials and a tool kit with which to build his models. This should contain simple elements which can easily be assembled into component sections, which in turn can be put together into the model. It should be easy to change elements which do not fit correctly, and modify the model to make it more efficient. The model elements should at all times fit together logically and consistently, and themselves be of a general character so that they can be fashioned into many different sorts of models in order that the tool kit should be flexible, (Howe etal 1984).

Computers can be used to provide such model building tools, with the previously mentioned advantages of giving immediate responses when used interactively. They can be used with an enormous range of software, and thus have a potential range of application far outweighing that provided by other forms of concrete learning aids, such as building blocks and Meccano. This makes them very attractive educationally. However, their actual application depends heavily on the software employed and the manner of its use.

The LISP-based computer language LOGO was designed for model building activities by novices, and has all the necessary attributes for providing a flexible tool kit. It is a procedural language which allows the user to break down the problem into smaller components, develop procedures for the solution of each one separately, and then combine them to form the problem solution. Each procedure can be built from a small set of primitive commands and, once built, can be used within any other procedure. This gives the language great flexibility and extensibility.

LOGO is claimed to be particularly suitable for model building in the field of mathematics, (Papert 1972). A set of domain specific procedures can be provided for a child to use as a tool kit, for example for shape transformations or fractions. He would not require any sophisticated programming knowledge to use them. As he gains in understanding he can begin to develop his own procedures and work at a conceptually more demanding level. Thus the tools given to the user for model building can be designed to fit and extend his current state of knowledge. Through using such a tool kit a child can investigate and reflect on mathematical ideas, thus acting as a creative mathematician. Such experience is claimed to enable him to gain an appreciation of the logic and beauty of mathematics, giving him access to powerful ideas, whilst still working within the limits of his understanding (Papert 1980).

The LOGO modelling activities envisaged by Papert fulfil the requirements for a mathematical learning environment suggested by the Piagetian school; i.e. it is based on practical activity, working interactively with the computer; using material at an appropriate level to relate to existing knowledge and extend it; and seeing contradictions and learning from mistakes, as debugging is an integral part of model building in LOGO.

3.2 Why LOGO?

As LOGO is a computer language it could be argued that any other language could be used as a medium for learning mathematics. It is not a new idea to use computers for this purpose; studies have been done using BASIC (Johnson 1979, Sorketal 1975, Hart 1982) and APL (Iverson 1972) claiming positive results. There are, however, important differences in the structure of different proramming languages and in the philosophy underlying their use as educational tools.

Some studies have used programming as a structural analogy to mathematics (Banaetal 1981, Schmitt 1975). The hierarchical structure of definitions and theorems in mathematics is modelled by procedures and subprocedures using previously defined operations. Programming is thought to be good training in that it requires a disciplined approach and emphasises the logical and lawful structure of the material. An algorithm design approach, where a student is asked to write a program to model a particular mathematical process, involves articulation of thinking which may lead to greater understanding.

Papert's LOGO philosophy is rather different in that the emphasis is placed on discovery learning by the child in the novel learning environment. Though also providing a rigorous system, the turtle geometry subset of LOGO can be used to build designs with very little programming knowledge. LOGO's suitability for providing model building tool kits is based on the following properties. It is:-

- interactive,
- procedural,
- extensible,
- flexible,
- user friendly, and
- easy to learn.

Other computer languages are less suitable for this type of application as they lack one or more of these attributes. BASIC can claim to be interactive and easy to learn, but most available versions of the language are not procedural nor extensible. A BASIC program must be written as one complete item, including any subroutines within it. Following the working of such a program and debugging it can therefore be quite difficult. Each of the separate subprocedures making up a LOGO procedure, on the other hand, can be tested and debugged separately; it is generally easier to follow its workings as it may be only a few lines long. BASIC also lacks the facility to do recursion simply, and therefore requires a more complex and less elegant programming style to achieve the same objective.

Other commonly available procedural languages such as PASCAL are non-interactive, which makes them less suitable as learning environment. Though also providing a rigorous system, the turtle geometry subset of LOGO can be used to build designs with very little programming knowledge. LOGO's suitability for providing model building tool kits is based on the following properties. It is:-

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- easy to learn.

Other computer languages are less suitable for this type of application as they lack one or more of these attributes. BASIC can claim to be interactive and easy to learn, but most available versions of the language are not procedural nor extensible. A BASIC program must be written as one complete item, including any subroutines within it. Following the working of such a program and debugging it can therefore be quite difficult. Each of the separate subprocedures making up a LOGO procedure, on the other hand, can be tested and debugged separately; it is generally easier to follow its workings as it may be only a few lines long. BASIC also lacks the facility to do recursion simply, and therefore requires a more complex and less elegant programming style to achieve the same objective.

Other commonly available procedural languages such as PASCAL are non-interactive, which makes them less suitable as first languages for beginners to learn. Comparisons of LOGO with other readily available languages are given by Harvey (1982) and O'Shea (1983). A language with a somewhat different structure not mentioned in these reviews, is SMALLTALK. This was developed with educational purposes in mind, to provide a system for the storage, retrieval and organisation of information. It is particularly suitable for describing classifications and simulations. It has been used with some success by 12 and 13 year olds in the US for learning geometry (Goldberg 1978), but the children using it all had previous experience with BASIC. SMALLTALK has the disadvantages that it is more difficult to learn than LOGO, having some uncomfortable syntax. As it only runs on expensive machines, it is not readily available. Other languages have been developed with educational uses in mind, such as COMAL, SOLO and PROLOG, each of which has areas of application to which it is particularly suited. However, these are generally less oriented towards mathematics.

3.3 The LOGO language.

The LOGO programming language was developed by researchers in Artificial Intelligence about 15 years ago (Feurzeig et al 1969). An important feature of the language which makes it appealing to children is the in-built turtle geometry system which provides:-

- a creative drawing device, requiring little programming knowledge to operate;

- a system through which mathematical ideas can be investigated; and
- an attractive introduction to more sophisticated programming ideas.

Children can be introduced to LOGO programming through turtle geometry using the floor turtle, a small robot carrying a pen, which moves about the floor in response to commands from a button box. This is simple enough for a young child of 6 or 7 years of age to operate. The turtle can move forwards or backwards or turn on the spot to left or right. Each of the commands takes an input to specify the distance moved (in turtle units) or the amount of rotation (in degrees). Thus the commands:-

```
FORWARD   50
RIGHT     90
FORWARD   50
RIGHT     90
FORWARD   50
RIGHT     90
FORWARD   50
RIGHT     90
```

will cause the turtle to draw a square of side length 50 units, and return to its starting position. The commands are body centred thus a command of LEFT 30 will cause the turtle to change its current heading by 30 degrees in an anti-clockwise direction. Commands to make a new drawing must take into account the current state of the turtle, that is, its position and heading at the start. It is argued that body centred commands are most natural to children (Papert 1980), as their own movements and intuitive use of left and right are also relative to their own position. They therefore find it easy to identify with the turtle movements.

The button box for driving the floor turtle, built in the AI Department at Edinburgh University, has facilities for repeating single commands, and for building three simple procedures using three procedure buttons.

BUILD	FORWARD	LEFT	7	8	9
END	BACK	RIGHT	4	5	6
HOOT	LIFT	DROP	1	2	3
REPEAT	A	B	C	0	STOP
					COMMAND

Layout of the Edinburgh Button Box.

To enter a command, such as FORWARD 50, the child must press the button labelled "FORWARD" followed by "5", "0" and "COMMAND". Each button lights up when pressed, so the child can check the entry before pressing COMMAND. LIFT and DROP operate the pen and do not take any inputs. REPEAT takes two inputs; the first is the number of times the command is to be repeated, and the second is the command, with its input, if it requires one. Procedures are built on the three buttons labelled A, B and C. To start building a procedure on Button A the child must press the buttons "BUILD", "A" and "COMMAND". The BUILD light remains on while all the commands in the procedure are entered. The procedure is completed with the command "END". It can then be run by pressing "A" and "COMMAND". It can also be used as a subprocedure within another procedure to be built on Button B or C.

Thus one side of a square, involving a forward movement followed by a right angle turn, can be built as a procedure. This procedure replaces the individual move and turn commands. Now a square can be built, either using the repeat facility, or within another procedure. For example, the procedure "SIDE" can be built onto Button A, as follows:-

```
BUILD SIDE (Button A)
   FORWARD  70
   LEFT     90
END
```

A square can then be drawn using the REPEAT command with this procedure on Button A.

```
REPEAT  4  SIDE
```

A procedure to draw a square, "SQUARE", can be built onto Button B using the "SIDE" procedure which is already on Button A.

```
BUILD SQUARE (Button B)
   SIDE
   SIDE
   SIDE
   SIDE
END
```

If a new procedure is built onto a particular button, then it overwrites any previous procedure on that button. If a different square is built on Button A, and a triangle on Button B, the two can be used together in a third procedure to draw a house on

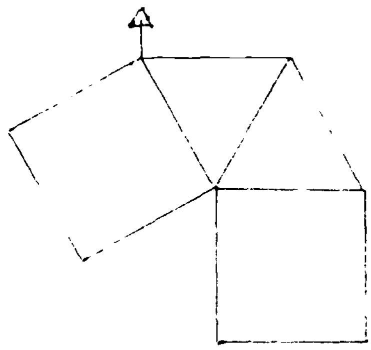
Button C.

```
BUILD TRIANGLE (Button B)
   FORWARD  70
   RIGHT   120
   FORWARD  70
   RIGHT   120
   FORWARD  70
   RIGHT   120
END
```

```
BUILD HOUSE (Button C)
   SQUARE
   TRIANGLE
END
```

Patterns can be built up by repeating procedures;

```
BUILD HOUSEPATTERN
   SQUARE
   TRIANGLE
   LIFT
   FORWARD 70
   RIGHT   60
   DROP
END
```



```
REPEAT 6 HOUSEPATTERN
```

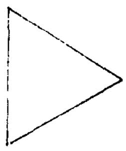
Tail recursion, where the procedure calls itself, can also be used in designs. In this case the STOP button must be used to halt the procedure.

```
BUILD CIRCLE
   FORWARD 5
   LEFT    5
   CIRCLE
END
```

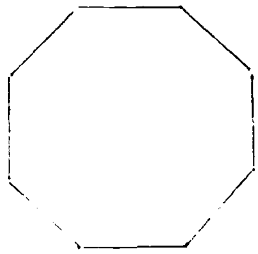
Using the full implementation of LOGO on a microprocessor more sophisticated turtle geometry can be developed with procedures taking inputs, arithmetic processes, tests and stopping conditions.

```
TO POLYGON :SIDES
   REPEAT :SIDES [FORWARD 50 RIGHT 360/:SIDES]
END
```

POLYGON 3

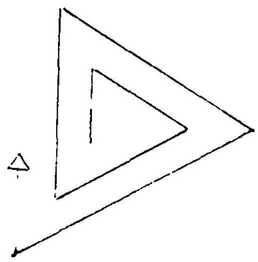


POLYGON 8



This procedure is to draw a polygon with any required number of sides. The exterior angle turned in drawing the polygon is calculated in the procedure by dividing 360 by the number of sides, and the complete drawing is made by repeating the two commands which make up one side the required number of times.

```
TO TRISPI :LENGTH
   IF :LENGTH > 50 THEN STOP
   FORWARD :LENGTH
   RIGHT       120
   TRISPI :LENGTH + 5
END
```



TRISPI 15

This procedure uses tail recursion to draw a triangular spiral. Each time the procedure is called, a larger value of :LENGTH is used, until its value exceeds 50 when the procedure is stopped.

Advanced mathematical applications of turtle geometry are described by Abelson and diSessa (1981). It is, however, only a subset of the LOGO language. The full implementation includes powerful list processing capabilities, where lists can contain numbers, words or other lists, and variables do not have to be declared.

3.4 How LOGO is used by children learning mathematics.

Most children begin with turtle geometry as this is simple and very rewarding. After being introduced to the simple drawing commands, they can begin to make their own designs and gain an understanding of the body centred commands. Then they can be introduced to the idea of procedures, as teaching the turtle to remember some commands, and through doing so, gain some fundamental programming knowledge. The way they progress from there depends on the learning situation they find themselves in, and the amount of structure there is in it.

A child placed in an unstructured learning environment will typically adopt one of three approaches (Solomon 1982):-

- MICRO He may operate at the micro level, using direct commands one at a time to develop a design, with some vague idea of what he would like to draw.

- MACRO He may draw a simple shape and, with no fixed end point in mind, develop the shape, moving, repeating and rotating it, and varying the inputs to it, or otherwise changing it.

- PLANNER He may decide on a definite end point and plan how to get there.

The first approach is often an earlier stage before the child moves on to the second or third methods. It may be necessary for him to gain confidence and understanding of the system at this micro-level.

The second approach can involve a great deal of discovery learning, as questions of how a particular effect is produced are encountered when trying to reproduce it under slightly different conditions. This provides a ready made environment for Dienes' rule-based play (1960).

The third approach stresses logical thinking and the general problem attack skills of breaking down a complex system into simpler units. It may also provide strong motivation for necessary calculations and bring out erroneous assumptions which can then be corrected.

There are some problems which may occur if the child is placed in a completely unstructured learning environment. He may choose to stay at the micro level, particularly if he is not aware of elementary programming concepts, and so make little progress, (the lost in the jungle syndrome).

A structured learning environment may define both the mathematical topics to be investigated and the approach to be used in the investigation. One approach is to require the pupil to write a procedure to carry out a particular mathematical process, the "algorithm design" method. This was used in some of the earlier studies with other programming languages and it encourages a planner approach. du Boulay (1980) has argued that, where the main objective is the learning of particular mathematical topics, this approach is inappropriate, in that it makes heavy demands on programming skills, and serves little purpose as an algorithm which can be programmed must already be well understood. However, the planner approach, for its own sake, can be used more appropriately in the field of geometry, where the student is asked to program a geometric design and must therefore identify the constituent parts and the relations between them.

A different structured approach is to demonstrate a concept and give the child the opportunity to investigate it. Thus children learning about functions and graphs may be given certain procedures relevant to this work. For instance, they could be given a function rule program which produces the value of y for a given value of x, and plots it on the axes. Pupils can then run this program until they can guess what the rule is, and then inspect the program to check their answer, and change it to produce a different function.

This is a "concept demonstration" approach. Advanced programming skills are not required of the pupil, as the program is given, but understanding of the basic mathematical relationships between input and output of the function rule is necessary. The pupil, changing the procedure to produce different functions, is operating at the macro level and may discover rules about, and deepen his understanding of, linear functions.

The drawbacks to this approach stem from the problem of designing the task to suit the learning needs of the child. The task may be too easy or too difficult for him. Since the learning outcome is predetermined he is also not given the opportunity to act creatively (the package tour syndrome).

For the development of relational learning in mathematics the macro level of investigation may be the most productive, but the task needs to be sufficiently loosely structured to allow the child to direct his own work as a creative mathematician. Some structure is necessary to reduce the uncertainty for some children and encourage the macro level investigations. This can be provided by specifying the general field for investigation, making available "tool kit" procedures and programming concepts useful in this field, and leaving the learner to work on loosely defined projects (the travel guide approach).

3.5 Mathematical learning through Turtle Geometry.

A great deal of mathematics naturally occurs within turtle geometry. Abelson and diSessa have explored the possibilities of it for illustrating relativity, but have not put them into practise in an educational environment. As most turtle movements require numerical inputs, at the simplest level the turtle can be used to develop the concept of number as linear or angular displacement. For young children simple procedures can be built, first requiring no inputs, and then using numbers between 1 and 10, as the commonly used values between 30 and 300 are outside their intuitive grasp (Collis 1975).

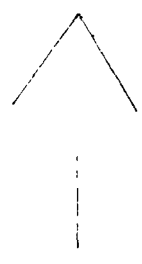
The concept of angle as amount of turning, and the additive and inverse properties of numbers as angles, can also be developed through using the turtle. This approach contrasts strongly with the traditional introduction to angles in geometry as the size of corners in plane figures. The latter gives a static picture of an angle as something given, whereas the turtle emphasizes the dynamic process aspects.

A non-geometric concept which may also be developed through turtle geometry is that of a variable. Children gain concrete experience of variables by using inputs in procedures and subprocedures, from which they may develop the concept. For instance they may use a simple procedure, such as HAT taking an input to determine the size of the drawing.

```
TO HAT :SIZE
   RIGHT    30
   BACK    :SIZE
   FORWARD :SIZE
   LEFT     60
   BACK    :SIZE
   FORWARD :SIZE
   RIGHT    30
END
```

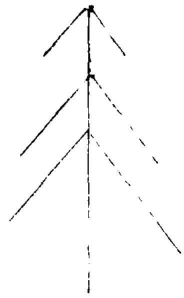
This procedure can then be used to draw an arrow with the shaft of the arrow twice as long as the head.

```
TO ARROW :SIZE
   FORWARD :SIZE * 2
   HAT     :SIZE
END
```



It could also be made into a firtree, using a recursive procedure.

```
TO FIRTREE :SIZE
   IF :SIZE < 5 THEN STOP
   FORWARD :SIZE
   HAT     :SIZE
   FIRTREE :SIZE - 5
END
```



Each of these implies a more sophisticated idea of a variable. In the HAT procedure only a straight substitution is needed. The design can be tried out first using direct commands, and then :SIZE put in in place of the value (25 in this case) each time.

```
RIGHT   30
BACK    25
FORWARD 25
LEFT    60
BACK    25
FORWARD 25
RIGHT   30
```

The arrow requires a more abstract use of SIZE as a number which can be operated on, and it also involves an idea of proportionality. The shaft of the arrow must always be twice as long as the head, and this is the way of expressing this relationship mathematically using the variable SIZE. The FIRTREE procedure involves a still more abstract use of the variable, not just in place of one number, but to represent different numbers as the recursive procedure is run.

The concept of variable is generally found to be difficult for many children, as the way it is introduced in secondary mathematics classes tends to lead to an instrumental understanding of it as a direct substitute for a number, (Hart 1981, Kuchemann 1978). Turtle geometry provides a concrete analogy for the concept, which may then prove to be more successful in the development of relational understanding.

3.6 Summary of Chapter 3.

This chapter has discussed computer modelling as an approach to learning mathematics, and the relative advantages of the LOGO language as a medium for this modelling. The type of activities which children are likely to get involved in when using LOGO are considered, including the different approaches which could be used. The likely learning outcomes of each approach have also been discussed. Finally some of the mathematical concepts which are intrinsic to turtle geometry, and could possibly be developed, have been mentioned.

CHAPTER 4

PREVIOUS STUDIES

4.1 Research questions

Until 1980 LOGO had to be run on mainframe computers, so there were few studies prior to that time. Since then it has become available on several microcomputers and so is more widely used, particularly in schools in the US. Some research studies have been done and much research has started in the last few years and is still in progress. Below we look at completed studies to shed light on four particular research issues:-

- What effects do different approaches have on the learning process and outcome?

- What evidence is there that children can make cognitive gains through using LOGO?

- What level of programming expertise is required for different learning processes and outcomes?

- How do the ability and maturity of children affect the cognitive gains they might make?

Three main approaches have been used in studies using LOGO. Most early studies used unstructured "discovery" learning methods, and often took a case study approach. The aims of such studies were widespread, looking at the complete learning environment, assessing attitudes of children towards autonomous learning and mathematics, as well as their cognitive gains in general problem solving skills. These studies were characterized by the absence of any formal programming instruction.

The second approach has been a structured one, with specific and limited aims in mathematical learning or problem solving. The assumption underlying these studies has been that LOGO programming provides a novel way of learning mathematics. Some elementary programming knowledge is a prerequisite for using this approach, and thus formal programming instruction was given in the early part of the studies, often as a separate learning unit before the knowledge was applied to mathematical problems.

The third approach, taken in more recent studies, has been between the two extremes, and can be classed as guided discovery learning.

4.2 Unstructured learning studies.

The Syracuse LOGO project (Statz 1972) was the earliest large scale study to be carried out. It had the ambitious aims of assessing attitudes, abilities and problem solving skills of children over a wide age range, after using LOGO. Several different teaching and classroom approaches were used, but these were most often unstructured, in that systematic programming instruction was not given. Much of the programming time was spent not using turtle graphics, as teletype terminals were used, and turtle graphics was only available through a peripheral floor turtle.

Many children in this study were considered to be unsuccessful, in that they did not maintain a positive attitude to programming work. Some problems may have been caused by the particular circumstances in which the children were working, as in some cases other activities were going on which the children would rather have been involved in. There was some correlation between developmental levels and achievement in programming, but the study gave no support to the notion that LOGO programming assists in the development of general problem solving skills. The tasks used to assess this were in no way related to programming activities, including the solution of anagrams and the "Tower of Hanoi" logic problem. Another criticism which can be made of the study is that it tried to look at too many interrelated factors and was thus unable to give any clear results.

The Brookline LOGO project (Papert et al 1979) was one of the most extensive studies carried out at MIT. It aimed to look at the complete learning environment provided through using LOGO, to assess the advantages of it. The researchers made no claims that the study was a representative educational experience, but set out to show what was possible under "ideal" learning conditions. Sixteen 11 to 12 year old children were studied as they worked with LOGO for a total of 25 hours each over seven weeks. The children chose their own projects and worked on them with the minimum of guidance, though several experimenters were present at all times. A case study was written on each child. An analysis of the children's work was used to identify the mathematical concepts which they had used. These included qualitative and quantitative notions of length and angle, and the composition and inverse group properties of numbers. Evidence for the development of mathematical thinking and creative work was given.

The sixteen children were of a wide spread of abilities and two of the less able ones in the study did not in fact learn to program at all. It was also found with several of the more successful children that some of the embedded mathematical concepts which they had used were not "discovered" by them, as they were not made explicit. The detail given of the children's work is quite illuminating, but the study has been criticized for its lack of objective measures. The amount and quality of intervention by the researchers is also not recorded, and may have had more influence on the learning outcomes than is admitted.

Two unstructured studies in the US were carried out using pre-school children working with the floor turtle (Perlman 1976, Gregg 1978). Both were concerned with children's ability to understand turtle tasks, rather than to assess the effect of this experience on any other abilities. They used a button box interface, and Perlman also used a slot machine through which children could write procedures by selecting command cards and posting them in order in the machine. They both reported that children had difficulty mapping the concepts onto the button box, and found the left/right differentiation particularly hard. Gregg concluded that there were developmental stages associated with the acquisition of understanding of turtle tasks. Perlman reported that some children kept repeating the same commands over and over until they became bored, and did not experiment with other possibilities.

Recently Pea and Kurland and their associates at Bank Street College Centre for Children and Technology in New York have begun to look critically at LOGO programming in the classroom, taking a multidisciplinary approach to its evaluation. They have looked at the effects on social interaction within the classroom and on children's understanding of programming, and have considered the transfer of cognitive skills to other areas of the curriculum. Most of this work involved children taken from two classes of 8 to 9 year olds and 11 to 12 year olds who used a discovery learning approach to LOGO for one year. They had computer contact time of between 20 and 30 hours each.

Children's understanding of recursion was investigated by asking them to think out loud as they worked out what moves the turtle would make as a recursive procedure was run. They then tried it out to see if their predictions were correct. Many children were found to have an incorrect mental model of recursion as a looping process, and this model persisted even when they were shown contradictory effects. The researchers concluded that it would be more effective to teach particular programming control structures rather than let the children discover them, as they were clearly not gaining a correct understanding (Kurland and Pea 1983).

They also investigated the hypothesis that learning to program assists children in the development of general planning skills. This has been frequently cited as a reason for teaching programming, but rarely tested. A classroom planning activity was devised, which involved the scheduling of various classroom chores. This was used as a test of planning ability in two versions; one a direct pencil and paper test, and the other a computerized task in which the child had to give commands to a robot to carry out the tasks. Neither form of the planning task showed any difference between children who had used LOGO programming for a year and a control group (Pea and Kurland 1983). In their conclusions, the researchers discuss the use of guided instruction in addition to discovery learning for the development of thinking skills.

Various workers have criticised this research on the grounds that it is not testing problem solving skills, but problem formulation ability. Nevertheless some claims have been made that LOGO experience, at least in the planner mode, contributes to problem solving through practise in problem decomposition. This was an attempt to provide a relevant problem to test the children´s planning ability on. There is however little similarity in content between programming and this chore sceduling task. The points made by the researchers that different results may have been obtained had a guided discovery approach been used remain to be tested.

Another recent study, using an unstructured approach, was carried out in France (Berdonneau and Dumas 1982). Ten year old children, in quite large groups of seven or eight, were working on the computer, using LOGO in an unstructured way. Each group was carrying out its own project. One group decided to investigate number work. They were reported to have used addition, subtraction and division, but did not use multiplication because they did not "discover" that * was the infix symbol for it. This research summarises the folly of using a strictly unstructured approach. It is clearly possible to "discover" meaningless facts, such as arbitrary keyboard symbols, but this has little educational value and may distract from meaningful learning.

4.3 Structured studies.

One of the early structured studies was carried out by Milner (1973), working with 10 year olds. His research was set up to test the hypothesis that children, trained in LOGO, could use this medium to acquire the concept of a variable. He used a CAI program to train the children in LOGO initially, and then used a variety of approaches and projects for the children to develop their understanding. Pre and post tests were used and supported the hypothesis. Subjective evidence was also given to indicate that the children's problem solving techniques were enhanced and personal expression of ideas fostered.

Two short ten week studies were reported by Hartley (1980) in which LOGO was used to assist the learning of directed number and fractions. Pre and post tests, taken by the LOGO group and a control group, showed significant superiority for the LOGO group on all measures of achievement. They were also claimed to be better able to express themselves and give accurate and explicit instructions on how to operate with fractions and how to draw a particular figure. The results showed a bimodal distribution, however, which indicated that not all the children were able to perform the abstract reasoning tasks required in fractions work, even after their LOGO experience.

Howe, O'Shea and Plane (1980) did a much more extensive study with 11 to 13 year old boys learning mathematics through LOGO programming. They spent the first year of the study working through a primer to learn programming, and then in the second year studied the entire year's syllabus of mathematics through that medium. The mathematics was presented in the form of worksheets, using mainly a concept demonstration or algorithm design approach. The boys, who were chosen as being particularly weak in mathematics, showed an improved performance on standard mathematics attainment tests, in comparison with their classmates who had not used LOGO. They also showed an increased willingness and confidence in discussing mathematics problems. This work could be criticized for being unrepresentative of normal educational environments. The group of boys using LOGO was quite small and they received a great deal of personal attention throughout the two year duration of the study. This alone could account for the improvements in confidence and willingness to discuss mathematical problems.

du Boulay (1978) used LOGO to help trainee primary teachers who were weak in mathematics to overcome their difficulties. He also used a structured approach, training the students in LOGO programming before using it in projects designed to help them overcome their particular difficulties. Although the students benefited from the experience, in that programming showed them the necessity for clear thinking and explicit language usage, they were discouraged by having to learn programming which they felt was not relevant to their problems. It was also difficult to design projects which were both directed towards their mathematical difficulties and at the right level of representation. Very often more time and effort was spent on the programming part of the problem than on the mathematics. du Boulay advocates the use of a concept demonstration approach, particularly for weak students, in that it focuses on the mathematics and demands less programming skills than an algorithm design approach.

These structured studies have all explicitly taught programming and then used more structured material to teach specific mathematical topics through LOGO programming. In most studies the effectiveness of this teaching was assessed with mathematical performance tests, and they all claimed to have achieved the learning objectives. This type of approach can be criticized for the lack of justification for the use of computers in the first place. Though comparisons have been made with control groups, it is not clear in any of the studies that the quality of learning achieved was dramatically superior to that obtained by more usual classroom methods, or sufficient to justify the use of the computers. The mathematics pursued in all cases was very conventional and part of the existing curriculum in schools, and thus made no contribution towards curriculum development. It could be said that the potential learning "revolution" available through child directed exploratory learning was being wasted, as the equipment was used to achieve ends which had been achieved in many different ways before.

4.4 Guided discovery learning approaches.

Studies in this category did not set out to teach programming before applying the knowledge to mathematical learning, but gave some instruction on programming notions throughout the study. The learners worked on projects of their own design, or suggested to them, so that all pupils encountered and approached certain learning problems.

A recent study in Edinburgh attempted to integrate LOGO into normal mathematics classes in the first two years of secondary education (Howe et al 1984). Three classes were used as the experimental group and another three in the same year were used as contol. Pre and post tests of mathematical attainment were given.The LOGO work was presented in the form of worksheets, demonstrating a concept and then developing it. A lot of programming knowledge was not required, as an example was usually given to follow. Initially this work was designed to fit in with, and complement the on going class work, which was following a modular syllabus, changing topics every two weeks. In the second year of the study this policy was altered, and instead of one LOGO worksheet being designed to follow each of the topics, separate LOGO modules were used in place of some of the existing modules. Problems with the timetabling and the limited number of machines resulted in the pupils having much less time on the computers than anticipated, and the pressure of the examination system and tests for early setting of mathematics classes affected the performance of both the pupils and the teachers. Nevertheless, a significant overall improvement in the mathematical performance of the girls using LOGO was noted at the end of the year. There was a relative improvement shown by the experimental group boys who, at the start of the project gave a significantly poorer performance than the control group boys. By the end of the study this difference had disappeared.

A recent study in Canada was less linked to external mathematical topics, but considered the logical, geometrical and programming notions intrinsic to turtle geometry, (Shultz et al 1984). Eight of these concepts were identified prior to the study which used a guided discovery approach to ensure that each of the children encountered them in the course of their LOGO work. The eight concepts were:-

geometry - rotation
- translation

logic - AND
- OR
- NOT
- IF

programming - recursion
- variable

Thirty seven children from grades 5 to 7 (10 to 13 year olds) were used as the experimental group, while 55 other children from the same grades in two other schools acted as control. The experimental group spent 30 sessions of 45 mins each using Apple LOGO. Two sets of tests were given to the experimental group, one inside the LOGO context and the other outside, but only the latter was given to the control group. The hypothesis that LOGO experience accelerates cognitive development was tested for, using the results of these tests. The LOGO group performed equally well on both tests showing that their understanding could extend beyond the immediate LOGO context. They also did significantly better than the control group, supporting the general hypothesis of accelerated cognitive development.

An on-going study in Australia is taking a similar guided discovery approach, by setting certain tasks of drawings for the children to copy, as well as encouraging them to work on their own projects (Clarke and Chambers 1984). The investigators are looking at individual differences in learning behaviour, and the transfer of general problem solving skills to other learning situations.

4.5 The learning approach.

The type of approach taken clearly had a considerable effect on the outcome of each study. In the structured studies the effects of LOGO work were predicted and tested for within a narrow band of traditional mathematical learning. These were generally successful. In most of the unstructured studies, in which there was no formal testing, some children were considered more successful than others, and some, in the terms of the research, could be regarded as failures. These studies revealed three problems:-

- Some children, for reasons of personality or maturation, made little progress in investigating the LOGO learning environment (Perlman 1976, Statz et al 1972 Papert et al 1979).

- Where particular learning outcomes were predicted, many children did not have the experience from which this learning was likely to stem. Thus, planning skills were predicted to arise from LOGO programming, but this is only a reasonable prediction if the children are working in the "planner" mode (Pea and Kurland 1983). In free unstructured studies some children persevered with a particular mode, and were furthermore ignorant of the advantages or educational aims of using different modes.

- Even in cases where children used, say, "macro" mode to investigate mathematical ideas, some children remained unaware of the embedded mathematical and programming concepts (Papert et al 1979, Kurland and Pea 1983). The investigation work may have been valuable experience, but it was not sufficient in itself to achieve all the desired learning outcomes.

Where particular learning aims are set up, a guided discovery approach may be the most profitable, to ensure that each child encounters the learning experience without removing all initiative from him.

4.6 Cognitive gains from using LOGO.

Many studies support the notion that children may make gains in understanding of mathematical concepts through LOGO experience (Papert et al 1979; Howe et al. 1984; Shultz et al 1984). Some also suggest that learning logical concepts and programming notions also occurs, but the style of presentation of the learning material seems to be an important factor in this. Some researchers considered that a certain amount of formal instruction in programming was desirable, to avoid misconceptions arising (Pea and Kurland 1983).

The claims for the development of problem solving skills remain unsupported, though the testing of these has not been very satisfactory. Statz used tasks quite unrelated to LOGO experience, following the general problem solving model of Newell and Simon which stressed the strategies common to all problem solutions (Newell and Simon 1972). Later problem solving models (Simon and Simon 1978) recognized the importance of a specific knowledge base. The difference between a novice and an expert solving a problem lies not only in the different strategies used, but in the choice and order in which the strategies are tried. The expert is able to limit his search of the problem space because of his underlying knowledge of the problem area. The novice, following the best general problem solving heuristics available, is unlikely to find the most efficient strategy in an area with which he is unfamiliar. From this perspective it seems unrealistic to expect programming in LOGO to assist in the solution of anagrams as there are no shared specific skills between them. A more appropriate test, perhaps related to mathematical problem solving, might have given different results.

The study by Pea and Kurland (1983) also gave no positive results, although they made an attempt to design a problem planning task relevant to the children and not completely unlike the type of problem decompostion which is used in LOGO programming in the "planner" mode. Here it is likely that the failure was due to the programming approach adopted, rather than to the testing procedure. The children learned LOGO in an unstructured environment, where they were free to chose what mode they wished to use. It is likely that few children spontaneously adopted a "planner" approach, and thus did not practise any problem solving skills.

4.7 Level of programming expertise.

This question has not been considered in many research studies, though some have assumed that programming ability was necessary in order to use LOGO to study mathematical topics. du Boulay took this approach with trainee teachers, but many of his students found the programming as difficult as the mathematics, and saw it as irrelevant to their problems. Howe (1980) successfully used the notion of a "virtual machine" to give children a mental framework in which to structure their programming knowledge. Kurland and Pea (1983), who did not give the children any formal programming instruction, found the concepts they did discover were often wrong. Following this experience they prepared a discussion paper on the role of programming in learning through LOGO, (Pea and Kurland 1983a). In this they stressed the need for research to address the questions:-

- How is programming learned?
- What programming skill level is required to obtain different learning outcomes?
- What is the relationship between the cognitive constraints on learning to program and the cognitive consequences?

They suggest that the claims that children aquire higher cognitive skills of planning, problem solving heuristics and reflectiveness, through programming are based on the belief that spontaneous experience with powerful symbolic systems will have beneficial cognitive consequences (the Latin syndrome). There is little research to support these claims. They discuss the possible transfer of learning from different levels of programming skill, and conclude that little general problem solving skill could be expected before the children had reached a fairly sophisticated level of programming ability. Their own research shows that after 50 hours of programming even the best 25% of their pupils were not at such a stage.

Their approach could be criticized as being too limited to a computer science outlook, stressing the need for children to learn programming. It nevertheless provides an interesting contrast to some of the claims of other researchers. The questions they raise need to be seriously considered and the extent to which cognitive gains can be made, through using turtle geometry as an investigative tool kit without requiring sophisticated programming, needs to be assessed.

4.8 Ability and maturity of children.

This question has not been directly approached in many studies. However, various ones have shown that the more able children gain rather more from their experience than the poorer students. The results have thus formed a bimodal distribution

(Howe 1980, Hartley 1980, Papert et al 1979) The question of maturity arose in studies with preschool children (Gregg 1978, Perlmann 1976) and in the Syracuse LOGO project (Statz et al 1972) It seems likely that, even with the use of concept keyboards designed to suit the child's ability, the logical thinking required in the building and use of procedures will prevent a pre-operational level child from learning to program successfully.

4.9 Summary of Chapter 4.

A variety of studies on learning through a LOGO environment have been reviewed, revealing little general agreement on the approach or learning outcomes. Some studies were based on unstated assumptions about the need for programming knowledge, while others discounted it. The role of the experimenter or teacher in the studies has often been ignored. Between them they have made and tested some ambitious claims on possible cognitive effects of LOGO experience.

In general, the expected learning outcomes have been illdefined, and there has been little attempt to explain how, and under what conditions different learning can be achieved.

There is a need for more fundamental research on children's activities in using LOGO, to specify the logical and mathematical concepts which could be made accessible to them through these activities, and the pedagogical approaches which could best promote learning.

CHAPTER 5

THE STUDIES - PHASE I

5.1 Research design.

This was action research designed to investigate what mathematics could be learned through using LOGO, and to see if relational learning could be fostered through this medium. The first two phases of the study were a formative evaluation, observing children using LOGO to identify the mathematical ideas they were using. The third phase of the study was to assess the transfer of this learning outside LOGO experience.

Throughout these studies the aim was to develop mathematical ideas rather than produce expert LOGO programmers. Nevertheless some programming was required in order that the children could be in control of the computers, rather than the other way round. The programming and mathematical ideas were integrated and presented in a structured way through the use of worksheets. The aim was to allow the children to investigate various mathematical ideas within their programming ability at that time, so that the programming requirements should not significantly interfere with the development of mathematical thinking.

The worksheets were designed to introduce new programming ideas one at a time, and informally develop the mathematical content at the same time. They were modified whilst in use. In the first and second studies, seven different groups of four children worked with the computers for one hour each week, so if any difficulties were found with the first groups changes were made for the later ones. The tests used in the first two phases were to give some indications of likely mathematical effects of LOGO experience. The development of suitable tests was difficult, partly because this involved the prediction of likely outcomes of using LOGO, and partly because the expected outcomes in depth and quality of understanding were hard to assess with any performance measure.

Children of quite a wide range of ability took part in the studies. Only one set of worksheets was prepared initially for all children, with the idea that each child could work at his own pace and so keep within his own level of understanding. In the first phase of the study, the experimenter was present at all times. This meant that she could monitor the performance of the children, and give additional help and explanation when required. The groups of four children who worked on the computers at the same time were chosed to be of similar ability level, so that they were likely to progress through the worksheets at about the same rate.

Analyses of the performances of children in the first two phases were used to determine likely learning outcomes of the LOGO experience. The approach was then modified for the third phase of the study, to promote what was seen as desirable learning outcomes, and to test for their transfer to normal school mathematics.

5.2 Phase I study - Teaching method.

The preliminary study was carried out to introduce pupils to the LOGO programming language through the use of the floor turtle. The main aim of this part of the study was to assess the effects of children's cognitive developmental levels on their ability to use LOGO, and to see what contribution the experience made to their mathematical thinking.

The floor turtle was used with two mixed ability classes, a Primary 5 class (9 year olds) and a Primary 6 class, (at the end of the school year when they were mostly 11 years old). In both cases the pupils were taken out of the classroom to work with the floor turtle in small groups of three or four pupils at a time for about one hour each week. They were given work cards to work through, each introducing a new programming or mathematical idea, but were also encouraged to follow their own ideas and make up their own designs. The programming was therefore introduced to them in a structured way. This work continued for one term with each class (at least 8 weeks).

The decision to use a guided discovery approach was taken after carrying out some preliminary unstructured investigations, with different children, and considering the literature. In the preliminary work observations were made of children working on their own projects. It was found that they could give the impression of understanding when in fact they were not always in full control of the turtle. On several occasions a child produced a beautiful pattern by accident, but could neither repeat it another time, nor explain why the effect was produced (Finlayson 1983). It was thought that if they were required to draw some given shapes, as well as create their own, this would give some indication of their understanding, and prompt them to think about how certain effects were produced.

The workcards followed the development of programming ideas:-

- direct drawing commands
- simple procedures
- repeat commands
- subprocedures
- tail recursion.

A summary of the workcard contents in given in Appendix I. The pupils worked under the supervision of the experimenter, who also kept a record of their work. Each child was working individually, but ideas were often shared and discussed. The children also worked out their designs on paper before trying them out. This was very useful, particularly with the button box, as there was no visible record of what commands had been entered when building a procedure. It also gave a permanent record of a child's work.

5.3 Methods of evaluation.

The progress of each child in the two classes was monitored by the experimenter and at the end of the study they took a short turtle performance test to assess how well they understood the programming concepts, if they would spontaneously use sophisticated techniques, and what mathematics they had learned from their experience. They were also given a standardised mathematics attainment test.

To assess the Piagetian developmental levels they were given various tests administered to the whole class. The "Classification Activities" (Gal-Choppin 1979) were used. These were based on the premise that in order for a child to attain a level of concrete operations, he must be able to classify material in more than one dimension. There were four activities in all using mainly pictorial material which had to be matched or sorted by particular attributes. These activities were judged to be suitable for the younger age group. Two of Shayer, Wylam, Adey and Kuchmann's scientific tasks, based on the original Piagetian work were also used (Wylam and Shayer 1980). The first one, on spatial relationships, was used with both age groups, and the second one, on volume and heaviness, was used with only the older children. These two tasks were graded in terms of Piagetian levels.

The standardised mathematics attainment tests used were NFER Mathematics Attainment Test B1 for the younger pupils, and Test C1 for the older ones.

In the turtle performance test the 9 year old pupils were asked to draw the simple shape given below:

to build a procedure to draw a square or triangle, and then to make any pattern using the shape they had built. They were also asked the following questions in an informal interview:-

- What determines the size of the drawing? (this was to differentiate the forward input from the angle input.)

- What difference would it make if all left and right turns were changed over in a turtle drawing? (the mirror image of the original shape would be produced.)

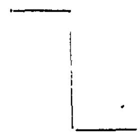
- They were asked to comment on the relationship between the angle and the shape it creates, with respect to the drawing they had actually done.

The 11 year old children did a similar performance test. They were actually tested before the younger ones and were not asked the questions. Instead, after doing the same drawing tasks, they were asked to build a 'V' and to repeat it three times, and to draw a 10 or 12 sided polygon, with the help of a sheet showing the commands for 3, 4 and 5 sided polygons.

5.4 Results.

The monitoring of the turtle work of individual children revealed considerable differences in understanding

In the turtle performance test the 9 year old pupils were asked to draw the simple shape given below:



to build a procedure to draw a square or triangle, and then to make any pattern using the shape they had built. They were also asked the following questions in an informal interview:-

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5.4 Results.

The monitoring of the turtle work of individual children revealed considerable differences in understanding between the 9 year olds and the 11 year olds, although they had spent a similar amount of time using the turtle. It also showed no great differences in their programming ability at this level. A total of 54 children were tested and almost evey one could operate the button box and run procedures. Eleven of the older children spontaneously built procedures in the course of creating patterns, compared with four of the younger ones, but the real differences lay in their mathematical understanding.

The following implicit mathematical ideas were encountered in the turtle work:-

1) Additive and inverse properties of number as linear or angular displacement.

```
FORWARD 20
 FORWARD 50
                are equivalent to FORWARD 70

 FORWARD 50
 BACK    30
                are equivalent to FORWARD 20

 LEFT    30
 LEFT    60
                are equivalent to LEFT    90

 LEFT    60
 RIGHT  120
                are equivalent to RIGHT   60
```

2) Important differentiation of inputs to forward (and backward) commands, and to turns, left and right. When repeating pairs of forward and turn commands, the forward input determines the size of the figure, whereas the angle input determines the shape. This leads to the realization that the shape of a figure is fundamentally determined by the angles used in the figure, and thus that all rectangular figures, for instance, have the common property of "rectangularity".

REPEAT 4 [FORWARD 90 RIGHT 90] --- a square

REPEAT 4 [FORWARD 120 RIGHT 90] --- a larger square

REPEAT 4 [FORWARD 90 RIGHT 120] --- not a square at all.

3) Mirror image effect. If all left and right turns in a drawing are changed over, then the mirror image is produced. (This is a special effect of the equivalence and inverse properties of left and right turns, which can be further developed with the introduction of negative numbers, but these were not available on the button box). A similar related rule, which follows as a consequence of inverse and angle properties, is that a simple closed shape is formed by making all the turns in the same direction.

4) The conservation of shape. If a certain ordered set of commands is used to draw a shape, it will always draw the same shape and cannot be used to draw any other. Furthermore, a different ordered set of commands cannot be used to produce an identical shape

(excluding mirror image effects).

5) Any regular pattern is founded on an underlying rule; that is, in all mathematical systems the behaviour of the elements is always lawful. In turtle geometry many of the patterns are based on one complete turn being 360 degrees.

5.5 Primary 6 turtle performance.

There was no evidence in the Primary 6 turtle work of children confusing the angle and forward inputs, or of not understanding the conservation of shape, though this was not tested. Quite often if a child had turned left instead of right he would correct this mistake in two stages.

LEFT 90
RIGHT 90
RIGHT 90

and when using estimation to find an angle would use several turns in the same direction

RIGHT 90
RIGHT 20
RIGHT 10

but on repeating the drawing would usually simplify these commands, showing a good understanding of equivalence values.

All these children, except one with particular spatial difficulties, had a good understanding of mirror images. This one boy had problems differentiating left and right turns and in the turtle performance test attempted to draw a closed shape, a triangle, using alternate left and right commands. He did not recognise that simple closed shapes are produced by turning consistently in one direction.

The polygon question on the performance test was to assess the children's ability in finding underlying rules. They had not been taught the "polygon rule", but some of the quicker groups had spent some time playing with polygons. Thirteen of the twenty eight pupils were able to draw the required polygon, and eight of them did so by building a two line procedure for one side and repeating it.

Only four primary 6 pupils were not successful in the earlier part of the test. These were the above mentioned boy, and three other children who had difficulties with finding the right magnitude of angles to use.

5.6 Primary 5 turtle performance.

Twenty-six children were tested, and although they had no difficulty in using the button box and building procedures, many of them found the logical thinking underlying the turtle work quite difficult. In the early sessions it was quite common for children to confuse the forward and angle inputs. A child would successfully draw a square using just the two commands

```
FORWARD 50
RIGHT   90
```

four times over. Then when asked to draw a larger square, he would increase the angle input as well as the forward one. Five of the children were still unable to differentiate these effects at the end of term.

Seven children were not clear about the mirror image effects of changing over left and right turns. The most common difficulty seemed to be caused by overgeneralizing, or not paying attention to the orientation of the drawing, in that they would predict that an indentical shape would be drawn, not the mirror image.

Many more of these younger children had difficulties with the conservation of shape. The children at this stage were dealing with angles between 0 and 180 degrees, and had not encountered the case of

LEFT Q degrees = LEFT Q + 360 degrees

LEFT Q degrees = RIGHT 360 - Q degrees.

One time a boy drew a square using the commands alternating a forward amount with LEFT 90 four times. He then attempted to draw a staircase using identical commands.

In another incident two boys were discussing their work as they drew polygons. They had both drawn a square using REPEAT 4 times and the angle of 90 degrees, and a triangle using REPEAT 3 times and the angle 120. The first boy said he could draw a 6 sided figure, and used REPEAT 6 and the angle 60 degrees to do so. The other, not to be outdone, said he could draw a 10 sided figure, and chose to use REPEAT 10 and the angle 90.

SQUARE	REPEAT 4	FORWARD 50	RIGHT 90
TRIANGLE	REPEAT 3	FORWARD 50	RIGHT 120
HEXAGON	REPEAT 6	FORWARD 50	RIGHT 60
DECAGON	REPEAT 10	FORWARD 50	RIGHT ?

Both were surprised when a square was drawn, and discussing the error decided that he should have used 60 degrees instead because this had worked for the hexagon. This was difficult in that the problem had two dimensions, and to a certain extent they were distracted by the REPEAT command which they had correctly realized was related to the number of sides. What they did not see was that the two angles they should not have used were 90 and 60 degrees, because they already knew that 90 draws a square and 60 draws a hexagon, and therefore could not draw any other shape.

In the final turtle performance test eleven 9 year old children fell down on the conservation of shape. Having agreed that a square was drawn by repeating the two commands FORWARD some quantity and RIGHT 90, they confidently stated that a triangle could also be drawn using only these two same commands. Some also said that squares could be drawn using different angles, such as 20 or 60 degrees. In the interview, care was taken to check that this was a genuine lack of understanding, not just a poor explanation of a deeper understanding.

	Primary 6 n=28	Primary 5 n=27
fail to differentiate forward and angle inputs	0	5
fail mirror image effect	1	7
fail conservation of shape	0	11
spontaneous use of procedures	11	4
able to use polygon rule	13	nt

Table 1. Summary of Turtle Performance.

5.7 LOGO performance of children at different levels of development.

The assessment of developmental levels was not satisfactory, as there was quite a wide variety of performance on the different measures. The Shayer, Wylam, Adey and Kuchmann Task II was chosen for the assessment of the older group, as giving the most internally consistent results in an area fairly closely related to mathematics. As it was not considered suitable for the younger children because of the scientific content, it was not possible to make a direct comparison between the groups, as had been originally intended. The classification task results were used with the younger children to give some indication of their cognitive level, but these results were found to be highly correlated with mathematical ability as measured by the standardised mathematics attainment test (Spearman's rank correletion coefficient r = 0.84 between Activity 4 and Test B1). The results of the Shayer Task I on perception of horizontal and vertical lines bore little relation to any of the other tests used, and so was excluded in the assessment.

The Shayer task II was graded in terms of Piagetian levels. The results for the primary 6 children spread over a fairly narrow range. Three of the pupils were classified at the 2a level, early concrete operations, and four at the 2b level, late concrete operation, with two pupils at the early formal level, 3a.

The remaining nineteen pupils were in transition between levels 2a and 2b. All the children were rated as very good, good, moderate or poor on their turtle performance. When these performance ratings were compared with developmental levels there appeared to be some correspondence between the two. The four poor performers were all at lower developmental levels, and the very good ones at higher levels.

	3a	2b	2a/b	2a	
VG	2	1	5	0	8
G	0	2	7	1	10
M	0	1	4	1	6
P	0	0	3	1	4
	2	4	19	3	28

Table 2. Comparison of Pigetian levels and programming performance of 11 year olds.

Applying Spearman's rank correlation coefficient to this data yields r = 0.23 which is not significant at the .05 level.

It was not possible to classify the primary 5 children in the same way into Piagetian levels, as the classification task used did not give results in this form. The results on this task were weakly correlated with turtle performance (Spearman's rank correlation coefficient r = 0.39). The turtle performance was also correlated with the standard mathematics score (r = 0.34), but the classification score and mathematics score were highly correlated (r = 0.84). Details of the performance of the Primary 5 and Primary 6 children are given in Appendix 2.

The primary 6 children had had two more years at school in comparison with the primary 5's. In this time they were formally introduced to some geometry and covered more general mathematics. In the absence of reliable developmental measures the effects of this particular experience cannot be distinguished from their general cognitive development. The combined effect made considerable difference to children's ability to use the turtle, and perhaps to what they learned from it.

The older children seemed more able to apply logical reasoning to the turtle tasks, and hence develop concepts of angle and shape quite quickly. The fact that eleven of the younger children failed in the conservation of shape after eight weeks of using the turtle may be an indication of their lack of logical thinking, as much as their need for more practical experience. As the primary 5 children were geometrically naive, the turtle work gave interesting insights to the way in which turtle geometry concepts are developed.

5.8 Development of turtle geometry concepts.

Concept of Angle.

The first requirement for the development of this concept is the ability to differentiate between the input to a FORWARD (or BACK) command and the input to a RIGHT (or LEFT) command. The former causes a change in the position of the turtle with respect to the board, whereas the latter changes the turtle's orientation only. Children who found difficulty differentiating between the two typically made errors when reducing the size of a figure. The tendency was to reduce the angle input as well as the forward one. This was a very common early bug, but later most children avoided it.

Most children then went through the stage of using fixed size angles (usually 90 degrees, as this was the example given to them) with an intuitive recognition of the inverse relationship between LEFT 90 and RIGHT 90. Very few children remained at this stage, but on the final performance task one boy, asked to draw a triangle using 120 degrees, used the commands

```
FORWARD  120
RIGHT     90
```

which illustrates this problem.

The next stage is the recognition of angles of varying amounts, without fully accepting the conservation of angle or its additive properties. Children would use apparently random inputs when drawing a regular figure, failing to recognise identical angles.

The conservation of angle is then developed. This is the recognition that a given input to a RIGHT (or LEFT) command will always produce the same amount of turning. The inverse of this is that the same input to a LEFT (or RIGHT) command will produce the same amount of turning in the opposite direction. The conservation of shape can only be developed when the concept of angle is fully understood.

Analysis of the records of children's work throughout the term also revealed the type of mathematical strategies the children were developing as they worked with the turtle. These were more obvious with the 9 year old children, as this type of geometric work was completely new to them. They were not therefore bringing any prior geometric knowledge with them to the task.

In discussing the work of individual children, initials have been used as a means of identification.

5.9 Strategies used by primary 5 children.

One of the most productive activities the primary 5 children did while using the floor turtle was to build short two line procedures consisting of a forward command and a turn command, and to repeat each procedure as many times as necessary to make a closed pattern. The results can be classified under the headings of stars, polygons or circles.

Stars: - Some groups of children mainly produced stars. None investigated the underlying patterns, but some generalizations were made. For example, Mi generalized a 'star angle' of 144 degrees to a different context, to draw a V shape. He then repeated the V and recreated the star. Ad recognised that in a 10 pointed star the procedure is repeated 10 times. He did not go on to state the formal rule or to test it out, perhaps because he did not know what other angles to use, but he seemed to realise the generality of his finding.

Polygons: Some children pursued patterns in polygons. For example Ai and Tm used their earlier investigations to relate the number of sides of a polygon to the angle turned:-

SIDES	ANGLE
3	120
4	90
6	60
8	45

They generalized the rule that the larger the number of sides the smaller the angle, and from this they were able to make successive approximations to find the required angle for a 5 sided polygon. They also recognised the halving and doubling effect relating the 4 and 8 sided figures and the 3 and 6 sided ones. They extended this to find the angle for a 10 sided polygon by halving that required for a pentagon. They did not recognize the significance of 360 degrees nor did they formalize the rule.

Two children came across instances where the same polygon had been drawn using two different angles. Sa noticed a hexagon drawn by a member of her group using RIGHT 300, and knew she could draw one using RIGHT 60. She recognized that there must be some relationship between the two angles, but was not able to discover it. Another pupil, Tm, saw a square drawn with LEFT 270 as well as LEFT 90. He concluded that there was a multiplicative relation between them

90 X 3 = 270

and then tested out the hypothesis that the same result would be obtained with other multiples of 90. He tried

90 X 2 = 180

but was disappointed. Several other children recognized the possibility of different angles drawing the same shape, but were not able to give examples or explain why (Ga Pa and Lu).

Circles: Though turtle geometry circles are indeed polygons, and recognized as such by the children, they were investigated in a different way. Several groups set out to draw a family of circles of different sizes. They all recognized that small angles were required (less than 20 degrees), and most also saw that smaller circles were made by reducing the forward input, thus generalizing the size rule to circles. However a common early error was to reduce both the forward and the turn inputs, producing circles of roughly the same size.

FORWARD 3 RIGHT 5

FORWARD 6 RIGHT 10

Some children recognized the inverse relationship between the two inputs. For example, Ka and Cr both stated that the turn had to be bigger than the forward input, introducing a comparative element. Also Lu and Mi were able to specify that a small circle required a small forward input and a large turn input.

Some children also investigated the number of repeats required for different circles. For example, prior to doing the polygon work, Tm tried out several circles.

FORWARD 10	RIGHT 10	REPEAT 36
FORWARD 10	RIGHT 5	REPEAT 72
FORWARD 10	RIGHT 3	(too big for the board)
FORWARD 3	RIGHT 3	REPEAT 120

At this stage he saw no relation between the number of repeats and the angle used. He took a fairly systematic approach, which was unusual among these children, holding the FORWARD input constant while changing the angle. He changed it only when the drawing became too big to fit on the board, and then while changing it, kept the angle constant.

Lu and Mi also investigated the number of repeats used with different circles. They saw that the pattern of repeats was inversly related to the angle.

RIGHT 1	REPEAT 360
RIGHT 10	REPEAT 36
RIGHT 5	REPEAT 72
RIGHT 20	REPEAT 18

They were able to produce the correct number of repeats required for a given angle, but were not able to formalize the rule, nor to see the significance of 360 degrees.

In general children were not very systematic about their investigations, with the exception of Tm. Mi systematically increased the forward input by 10 each time and used alternate left and right turns, but the angle inputs were seemingly random. Tm was also unusual in formulating and testing out his hypothesis, though Ai, Lu and Mi also did this somewhat intuitively. It is interesting to note that Mi and Lu were not particularly high achievers at school.

5.10 Conclusions to Phase I study.

Some correlation was shown between the performance with the turtle and cognitive developmental levels for both the primary 6 and primary 5 children. However there was no evidence of a threshold developmental level required for the turtle task. All children were able to operated the button box to build and run procedures. Among the younger children, most of those with low scores on the standard mathematics test and classification activities also had difficulty with understanding the underlying mathematical ideas, but some did surprisingly well, (see Appendix 2)

The difficulties encountered by the younger children proved much less of a problem to the primary 6s. Four of these did not perform at a satisfactory level on the turtle test, and in these cases the problem lay in estimating the magnitude of the angles. As the developmental level assessment was unsatisfactory, it was not possible to make direct comparisons between the children of different ages. However those tests which both classes did take did not clearly differentiate between them. In the light of this, it seems likely that the superior performance of the older children was attributable as much to their formal education and experience, as to their cognitive developmental level.

The primary 5 children showed quite a wide spread of understanding which enabled the development of turtle geometry concepts to be traced. They had not previously learned any formal geometry and so had little prior knowledge to bring to the tasks. Some needed a greated amount of time than others to develop these fundamental concepts.

The primary 6 children had about the same amount of time with the turtle, but brought to the task some understanding of the concepts of angle and right-angle; (many also apparantly brought the idea that a triangle contains angles of 45 degrees.) They were able to apply these concepts to the turtle tasks, and so make much faster progress towards recognising generalisable mathematical rules.

5.11 Summary of Chapter 5

The first phase of the study was described. This was a preliminary study using the floor turtle with two different groups of children, aged 9 and 11, to investigate the mathematical notions implicit in turtle geometry and the limitations imposed by maturity on children's ability to develop these notions.

The results showed that turtle geometry is a good diagnostic tool for illuminating the thinking processes being used by each child, showing his lack of understanding of angles and logical relations, for instance, in a way not possible in normal class work. There were considerable differences in understanding between the two age groups, but these differences were not detected on Piagetian developmental level tests, so could not be strictly attributed to developmental stages.

Observations of the children as they investigated angles gave some insight into the way in which an understanding of angle, as amount of rotation, was being developed. Some indications were also given of mathematical strategies which children were capable of using.

The results of this phase were used in the design of worksheets for the second phase of the study, to present mathematically rich topics for investigation.

CHAPTER 6

PHASE II STUDY.

6.1 Objectives

The purpose of this phase was to give children of a range of abilities an extended period of time using the full LOGO language, rather than just the floor turtle; to observe what mathematics they were using and to assess any changes in mathematical understanding which might result from this experience. It was also to see how their understanding of programming developed, and if this limited their mathematical investigations. The same twenty seven Primary 7 pupils (11 year olds) who had previously been involved in the Phase I study with the floor turtle, whilst they were in Primary 6, took part in the second phase.

6.2 Method - a) Equipment.

Four Texas Instruments TI99/4a microcomputers were installed in a separate room in the school, for this part of the study. These machines run TI LOGO, which is a similar dialect to the APPLE and TERRAPIN LOGO's, but has rather more limitations. Several features of this version of the language and the hardware used restricted the mathematical work which could be carried out. In particular numerical work was limited to integers. Not only did this limit the use of arithmetic procedures, as, for instance, division would return a whole number only, discarding the remainder, but it also affected turtle graphics. The ARC procedures available in other LOGOs for drawing arcs and circles of given radius were not available, because the calculations using pi could not be carried out. Many of the editing features of the machine use the function key, thus most keys have three operations. The children learned to cope quite quickly with these. However, the QUIT key caused difficulties. The effect of QUIT is to switch the machine off, erasing all the working memory. As it is located on the same button as the plus sign, one activated using the function key and the other using shift, the button could not be disabled. No disc drives were available for these machines, so any saving of procedures had to be done using tape. This was very time consuming, and often unsuccessful, so in practise the tapes were hardly used. This had an important effect of "modularizing" the work within a session. Children kept written records of their procedures throughout the year, which they would use to refer back to, but they did not continue with any particular investigation over a number of weeks.

As with the floor turtle, the pupils were withdrawn from normal class lessons for one hour each week, in groups of four, to work individually on the computers. Worksheets were used to introduce the essential elements of programming. Pre and post tests were given at the beginning and end of the year, to assess the changes in the children's mathematical understanding. Their programming work was monitored throughout the term and a final LOGO performance test was also given.

It was originally envisaged that these Primary 7 pupils would have most of a school year working with the full LOGO language, amounting to between 30 and 40 hours. In practice the average amount of time spent was 23 hours per child.

b) The worksheets

A guided discovery learning approach was used to introduce the necessary programming ideas to the children. It was found to be desirable for a child to explore his own projects, but also to try some set tasks. These challenge him to show that he can really control the computer and check on his understanding, whereas the former exploration provides strong motivation and can be quite revealing of how the child is thinking.

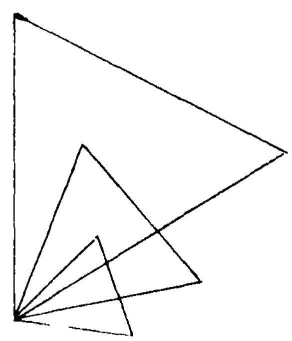
During the first half of the year the children were working with angles, using direct commands, building simple procedures and making patterns by repeating and rotating procedures. They were introduced to subprocedures, but did not spend a lot of time using them, and to negative numbers and Cartesian co-ordinates through using the SX and SY commands to set the position of the turtle on the screen. Each child was working individually and free to go at his own pace, but the interaction within each of the groups of four children working at one time tended to keep a group working together.

After about 10 weeks most of the children began using inputs in procedures. A few children did not have the programming competence for this and continued with simple procedures. Others found that this work became increasingly difficult. In the third term they all spent a few weeks exploring Sprites and pursued more projects of their own choosing.

6.3 Assessment.

The performance of the children using LOGO was monitored throughout the experimental period by the experimenter, and they also did a LOGO performance test at the end of the study. In the LOGO test the children were asked to :-

"Build a procedure to draw this pattern. You may want to use subprocedures and inputs. When you have done that, change your procedure to draw the mirror image of the pattern."



The problem has four component processes;-

analysis - breakdown of the pattern into three equilateral triangles of different sizes, rotated about a point;

construction - of one or more triangle procedures;

assembly - of the three triangles with small rotations into a superprocedure;

mirror image - recognition of at least one way to produce a mirror image.

The most elegant procedure would be a recursive one, possibly using a triangle subprocedure:-

```
TO PAT X
   IF :X < 20 THEN STOP
   TRI :X
   RIGHT 20
   PAT :X - 20.
END

TO TRI X
   REPEAT 3 [FORWARD :X  RIGHT 120]
END
```

The mirror image could be produced by changing all the right turns to left turns or by changing the initial position and turning angle between each triangle from right to left.

Mathematical understanding.

The effects on mathematical understanding are subtle ones which are rather difficult to assess with normal performance tests. Children can perform well on such tests without a great deal of understanding. For this reason, where possible, test items were selected from the Chelsea Mathematics Project (Hart 1981) as these were designed to assess understanding rather than computational ability. Three different aspects of the effect on mathematics were considered:-

(1) the effect on directly related mathematical skills from turtle geometry;

- angle estimation
- reflections and rotations

(2) ability to generalize and abstract principles in novel problem situations; (evidence for relational learning).

- use of the concept of variable in algebraic problems
- problem solving strategies in pattern recognition problems;

(3). general mathematical performance.

The reflections and rotations test was adapted from the Chelsea test (Hart 1981). The Chelsea College studies reported that mathematical problems could usefully be considered as being at different levels of difficulty. Large statistical samples were used to investigate these levels and cluster analysis was performed on the results. The material presented to children in the first five years of secondary school, on any particular mathematical topic, was found to fall into four or five levels of difficulty. The children in this study were at the younger end of the range covered by the Chelsea work, so an attempt was made to choose questions from the published tests which would be suitable for this age. Only questions which fell unambiguously into one of the first three levels of difficulty were chosen. For the reflections and rotations test, a total of sixteen questions were taken; five from level 1, six from level 2 and five from level 3. The angle estimation test was devised from one used in the Brookline project (Papert et al 1979). Examples of these tests are given in Appendix 3.

The test of the use of variables was adapted from the Chelsea test on generalized arithmetic in the same way as the reflections and rotations, choosing sixteen items from the first three levels of difficulty (see Appendix 3c). The problem solving strategies were taken from suggestions in the APU study (1982)(see Appendix 3d), and standardized tests of mathematical attainment C1 and EF , published by NFER, were used to assess general mathematical performance. Some of these tests were also used in the third phase of the study.

6.4 Results

As there was some delay in obtaining the equipment, some weeks were lost at the beginning of the study. As a result, the average amount of time spent per child on the machines was only 23 hours, over a period of eight months. In addition they had already spent eight to ten weeks using the floor turtle in Phase I. Changes in performance on the pre and post tests were used to indicate possible mathematical learning effects. The final LOGO test and records of the work throughout the year were used to assess programming performance.

a) LOGO performance on angles.

All the pupils spent a considerable amount of time investigating angles. In the final LOGO performance test five of the twenty five children had difficulty in writing a simple procedure to draw a triangle. They could not remember the correct angle to use for an equilateral triangle. Throughout the year, two of them had consistently had problems caused by associating 45 degrees with equilateral triangles. Had these two been asked to draw a different shape they might have been successful. For the other three, the fact that it was a case of remembering indicated that they did not have sufficient understanding to work out what was required, or even to use a successful trial and error method. They could all build the procedure once they knew what angle to use, showing that the problem lay in their mathematical understanding, not with the programming.

Results on tests related to angles

Overall the class showed a considerable improvement in their performance on the reflections and rotations test. Thirteen out of twenty six improved their scores on the test by 4 or more points (25%). A further two children had initial high scores and were unable to show any improvement. The three pupils who had difficulties with the angles for the triangles in the LOGO performance test showed no improvement on any of the tests. On the estimation of angles test fourteen out of twenty five children scored 6 or more out of a possible 8, and only two pupils scored less than 4. This indicates that most children had a reasonable understanding of angles. The girls performed rather better than the boys on this. Details of the results are given in Appendix 4

b) LOGO performance with inputs.

Many children found this area of investigation too conceptually demanding, and in the final LOGO test, in which it was clearly appropriate to build a procedure with an input, only ten of the twenty five chose to do so. The others preferred to employ the simpler but less economical techniques developed in the angle investigations. Two of the successful ten had other difficulties with the programming, but the remaining eight seemed quite competent operating at this level.

Two girls, Sa and Ly were both competent at building procedures with inputs but showed only a partial understanding of variables. In the performance test Sa built a procedure with an input SIDE and then built a second one using SIDE - 10

```
TO TRIANGLE SIDE
   FORWARD  :SIDE
   RIGHT  120
   FORWARD  :SIDE
   RIGHT  120
   FORWARD  :SIDE
   RIGHT  150
END
```

```
TO TRI SIDE
   FORWARD  :SIDE - 10
   RIGHT  120
   FORWARD  :SIDE - 10
   RIGHT  120
   FORWARD  :SIDE - 10
   RIGHT  150
END
```

These procedures each incorporated a turn of RIGHT 30 in the last command. Sa used the two together to draw the complete pattern.

```
TO TRIAN
   TRIANGLE  60
   TRI  50
   TRI  40
END
```

When discussing what she had done she appeared unaware of the complete redundancy of the second TRI procedure, although she was able to use it with two different inputs, and so could have used just one procedure with three inputs for the pattern.

Ly was sitting at the next computer and quite correctly built a triangle procedure with an input and used it to draw the pattern, not however building a superprocedure for it.

```
TRI 60
RIGHT 20
TRI 40
RIGHT 20
TRI 20
```

When asked to put the pattern into a procedure, she then began to copy Sa and built two redundant procedures using :SIDE - 20 and :SIDE - 40, incorporating the turning angle into the triangle procedures, and then used these in her final pattern procedure.

```
TO PATTERN
   TRI 70
   TRIA 70
   TRIAN 70
END
```

They had both previously used recursion with a reducing input, and it is likely that they were remembering this as a "sophisticated technique" without fully understanding its application. This is an example of "style conscious" programming found in earlier studies (Howe 1980).

Overall, the pre and post tests on understanding of variables were remarkably consistent and showed little change. However, considerable improvements on this test were made by three individual children, all from among the eight who successfully used inputs in the final test. This suggests that they had grasped the concept of variable from their experience. The test on generalization was given only as a post test, and thus yielded little information. It was included mainly as a pilot for this type of test to be used in the next phase.

There were no tests associated with Sprites, and most children did not spend a lot of time working with them. It was felt that at this level of cognitive development the Sprites were very entertaining but tended to distract attention from underlying rules and principles, rather than providing useful illustrations for them.

c) General mathematical performance

One of the standardized mathematics attainment tests, Test EF, was found to give inaccurate standardization. This had been found in a previous study with secondary children (Howe et al 1984). In that study it was possible to restandardize the test, as a large sample of children was involved. This was not considered feasible in this study, involving only twenty seven pupils. However, as the age difference within the class was small and the purpose of the test was to make comparisons within pupils, it was decided to use the raw scores on both tests, rather than the standardized ones.

The previous study with LOGO in secondary school mathematics showed some improvement in the general mathematical performance of girls after using LOGO (Howe et al 1984). A similar effect was found in this study, in that three pupils showed marked improvements on the mathematical attainment tests, all of whom were girls, (see Appendix 4). A further four pupils maintained their level of performance, against a general trend of lower scores on the second (harder) test. Three of these were girls. This improvement was also commented upon during the year by the class teacher. She noticed the improved confidence and motivation of these girls in mathematics class work. As the mathematics attainment test contained relatively few questions directly related to LOGO experience, it seems likely that this is an affective rather than cognitive result of the computer work. Success in using LOGO boosted the confidence of the girls in their own abilities in a mathematical domain, which then carried over to their classroom performance. (See Appendix 4b).

6.5 Monitoring of children's work.

It would have been desirable to use either dribble files to give a full record of the children's work or videos to record the work on the screen, but in this small study neither was available. Incomplete records of the children's work were obtained from their own records of procedures which they would write down at the end of each session and from observations. Some interesting activities were observed, often when the children had bugs in their programming.

It was possible to see when children were making generalizations, applying something learned in one context to another, or seeing underlying patterns in, for instance, polygons. The LOGO work was seen to contain numerous instances for generalizing and abstracting rules, so interventions were used to prompt the children into exploring some of them.

Observations also revealed when children were not applying knowledge in different contexts, as for instance in the problem of drawing a house from a square and a triangle. This was initially conceived as an example in using subprocedures, but the problem of interfacing the two shapes revealed unexpected inabilities of children to estimate angles with successive approximations. This particular problem is discussed in detail in Chapter 8. Such incidents, though not recorded in depth during the second phase of the study, illustrated the type of mathematical activities the children could be involved in, and were used in the design of the third phase.

Children were rated as very good, good, moderate or poor on their programming ability from these observations and from the LOGO performance test. Both boys and girls appeared to be equally keen to use the computers, irrespective of their programming ability. Some differences were noted in their approach. Some boys appeared to be very "product oriented" and wanted to draw spectacular patterns, but were unconcerned about how these were produced. They sometimes copied procedures from other people, spent more time using Sprites, and were often working beyond their understanding. Girls also copied procedures from one another, but in general spent more time working through the worksheets.

6.6 Discussion of Phase II results.

The results of the phase II study were reviewed before the third phase was set up, in the framework of the research questions:

1) What mathematics had been learned?

2) What level of programming expertise was reached or required?

3) What effect did different ability levels make on the children's progress?

1) Mathematical learning.

A review of the results of the pre and post tests on mathematical understanding showed that seventeen out of the twenty six pupils improved on at least some of their scores. The main area for improvement was in reflections and rotations where thirteen children had 25% higher scores on the post tests. Three pupils also showed improvements of 4 or more points out of a possible 16 on the algebra test. They each changed from a moderate to a high score, indicating that an understanding of the concept of variable had been gained. Seven pupils showed relative improvements in their general mathematical performance. Of the remaining nine pupils, two had very good initial scores and could not therefore register any improvements. The other seven showed no overall improvements.

The results are summarized in the table below

AREA OF IMPROVEMENT	GIRLS n=14	BOYS n=12	TOTAL n=26
Reflections and rotations.	7	6	13
Variables.	2	1	3
General mathematics.	6	1	7
None (high scores)	1	1	2
None	2	5	7

Table 3. Improvements between pre and post test performance.

In making assessments of improvements of pupils, comparisons were made between each particular pupil and the rest of the class. There was a remarkable consistency on most tests between pre and post scores, so the improvements were easily detected. However, children do develop at different rates and comparisons within children over a period of several months do not distinguish effects of natural development from learning from other sources.

2) What level of programming expertise was reached or required?

The final performance test illustrated the children's understanding of programming concepts. Twenty out of twenty-four pupils used procedures in the test:

7 used one single procedure for the whole figure;

5 used three separate procedures, one for each triangle;

8 used one triangle procedure with an input.

Some children who had used inputs revealed the incompleteness of their understanding by building redundant procedures. None of the children was able to use recursion. Of the four children who did not build procedures, three were judged to be capable of doing so, but had a poor grasp of angles.

The fourth boy was not competent at building procedures at the end of the year. In fact he stopped using the worksheets after the first term, as they were too difficult for him. His span of attention was very short, and when typing in commands he would sometimes forget to press ENTER and so would not see any effect. All the other children could build procedures, but about half of them had difficulty using inputs or subprocedures. When a procedure was given on the worksheet, they would type it in and use it, without necessarily understanding it.

From the results of the performance test and from the monitoring throughout the year, eleven pupils were rated as poor or moderate at LOGO programming. Surprisingly, five of them still improved their scores on reflections and rotations and had good scores on angle estimations.

Many pupils spent only a brief time using inputs in procedures. Some worked quite slowly and so did not get to these worksheets, and others found the work difficult and so retreated to using simple procedures. The three pupils who were found to have gained an understanding of variables were all rated as good on LOGO performance, and had all spent some time using inputs in procedures.

These results suggest that different things can be gained from LOGO at different levels of programming expertise. Children using turtle geometry in the form of direct commands, with perhaps some simple procedures, were apparently able to develop the concept of angle through this experience. This is also supported by the results of the Phase I study. The cognitive demands of operating at this level are quite low.

At a more advanced level, work with inputs in procedures may help with the development of the concept of variable. It is also possible that mathematical strategies can be developed, though the only evidence for this is in the monitoring records of the children.

Based on these results, an analysis of the levels of programming expertise, and the cognitive demands and likely mathematical gains, is given below.

Level 1 Direct commands and simple procedures in turtle geometry.

Little cognitive demand;

Applications:

investigation of number as linear and angular displacement;

inverse operations;

concepts of angle and shape;

use of information;

approximations.

Level 2 Procedures with inputs and subprocedures.

More demanding, requires logical thinking;

Applications:

concept of variable;

planning through decomposition;

pattern recognition and generalizations;

Level 3 Recursion and conditionals.

Yet more demanding;

Applications:

design of algorithms;

logic concepts;

general problem solving heuristics.

No child in this study was working at level 3 in programming.

3) What effect does the child's ability level have on his learning from the LOGO experience?

The indicator of ability level used in this study was the mathematics attainment test. On this the five pupils who were rated as poor or moderate in LOGO performance, and showed no improvement in their post test scores, were ranked 10, 17, 18, 21, and 27 in the class. They were all boys. In addition, one girl, rated as good in LOGO performance, showed no improvements on the tests. The poorer programmers and those who apparently got least out of the experience were generally amongst the mathematically less able children. The converse, however, was not true. The six girls who showed improvements in general mathematics were from the middle and bottom sections of the class. They were initially ranked 11, 13, 15, 20, 22 and 26.

6.7 Conclusions:

This phase II study lent support to the notion that gains in mathematical understanding could be made from using turtle geometry, without requiring a high level of programming ability. From the results and observations made, children seemed to be making gains in understanding as long as they were in control of the learning process, and not attempting too difficult projects. The apparent difference between the less able boys and girls may relate to the relative tendency of these boys to attempt ambitious things, and therefore work beyond their understanding, in comparison with the more conservative girls. A similar finding, that boys are more likely to be overambitious than girls and thus get into difficulties, was found by Hoyles (1985).

The findings of this phase of the study were carefully considered when setting up the main study. The action research to this point clearly showed that children were learning mathematics during the time they were using LOGO. It was not however clear how much of this learning was directly related to LOGO experience, and how much to other factors such as schooling and natural development. In Phase II the children were in an experimental situation; they were withdrawn from class and received some individual attention from the experimenter while working on the computers. In order to assess the generality of the findings, it was decided to make two major changes in the study for the third phase; first to put the computer work within a normal classroom, under the direction of the class teacher, and second, to use a control group of children not using LOGO to determine what mathematical learning could be attributed to LOGO experience.

In order to observe the progress of the children, with minimum interference, it was decided that the experimenter should be present in the classroom only one day each week. Thus the children would be working much of the time without direct supervision. In anticipation of this the worksheets were changed slightly to provide all the programming information which the children might need at each stage. They were also made more open ended, to encourage the children to explore their own ideas. The children were put into pairs to work on the computers. During phase II some children did work together, though on separate computers, and this seemed to be quite productive. It was also thought that two children together were less likely to come to an impasse than one on his own.

The test results of Phase II showed that children changed their understanding of angles and reflections and rotations. They suggested the possibility that children might learn to generalize the idea of a variable from using inputs in procedures. Other development of mathematical strategies could have taken place, but was not adequately tested for in this phase. Thus it was decided to extend the tests to include mathematical strategies used in novel problem solving tasks. If it were found that children were learning mathematical strategies from their LOGO work, then this would provide important evidence that the quality of learning was relational. An instrumental understanding of angles, that is, being able to associate particular numbers with particular shapes, is of little use to a child. However a relational understanding can contribute to further geometric work, and to his comprehension of mathematical systems.

6.8 Summary of Chapter 6

In the second phase of the study 11 year old children used a full LOGO over the period of twenty three weeks. The development of their mathematical understanding was assessed by means of pre and post tests on particular concepts related to turtle geometry, and on general mathematics. Their work on the computers was observed throughout the study and their understanding of programming assessed from these observations and from a final performance test.

Three quarters of the children were found to have made significant improvements in some of their test scores. Over half the class improved on understanding of angles, though some of these children remained poor at programming. Substantial improvements in general mathematical performance were shown by seven pupils, six of whom were girls. Five children were rated poor at programming and showed no improvements in any of their test scores. They were all boys. It is suggested that this failure of the boys may be related to their tendency to attempt work at a level beyond their understanding.

CHAPTER 7

PHASE III STUDY

7.1 Objectives

The objectives of the third phase of the study were:-

- To replicate the second phase study in a normal classroom.

- To make further investigations into the quality of mathematical learning taking place, in particular in the area of mathematical processes.

- To test for the transfer of learning from the LOGO environment to normal school mathematics.

The second phase of the study was carried out under experimental conditions, in which the children were withdrawn from normal lessons to learn LOGO with the experimenter. For the third phase it was decided to put work into a normal classroom, to minimize the effects of experimenter interaction with the children. The original idea was that the work would be under the general direction of the class teacher. The experimenter would be present only part of the time to observe and take notes on the children's work.

The children were to work in pairs, rather than individually, as this was expected to be more productive especially in the absence of direct supervision. The worksheets were changed a little from phase II, both to prompt more mathematically fruitful activities, and to provide all the necessary information to the children in the absence of direct assistance. The children were not expected to spend all their time working from worksheets, but were encouraged to follow their own investigations. The worksheets were there to introduce new programming notions, to encourage them to go beyond their current knowledge, and to advance their understanding.

In order to distinguish what learning was due to LOGO experience, and what due to the effects of schooling and maturation over the year, it was decided to carry out tests of mathematical understanding on both the LOGO group and a control group of children, of the same age, who did not do any LOGO work. Some tests of mathematical processes of generalization, abstraction and use of information were given to both groups at the end of the year, in addition to those on understanding of angles, reflections and rotations and variables, used in Phase II.

7.2 Method

The third phase of the study was carried out in the year following the second phase. Two parallel mixed ability classes in one school were taken as experimental and control groups. They each had thirty two pupils between the ages of 10.5 and 11.5 years at the beginning of the year. Both classes were given pre-tests at this time; a non-verbal general intelligence test NFER Test DH, and a mathematical attainment test, NFER Test

C1. The results of these tests are given in Appendix 6a. There was no significant difference between the classes on either measure. It would have been desirable to take half of each class to form the experimental group, in order to control for the effect of different class teachers, but this was not possible in the school setting. Thus, the whole of one class was taken as the experimental group, and the other was used as the control group.

The control class had substantial use of the school's two BBC microcomputers over the next two terms. The children used various packages from the Micro Primer set suitable to their age group, covering a range of different, not necessarily mathematical, topics. They neither learned any programming, nor used LOGO. The four Texas Instruments TI 99/4a microcomputers which ran LOGO were installed in the experimental classroom. Each pair of pupils in the experimental class had two or more sessions each week on the computer. This continued for 28 weeks. In the previous term the experimental class also spent five weeks using the floor turtle in preparation for the study.

The selection of the pairs of children to work together was done by the class teacher. They were chosen as being of similar ability levels, and prepared to work together. Usually the children were of the same sex and in the same mathematics group. Most classwork was organized in groups, so work on the computers became one of the usual group activities.

The experimenter was present in the classroom one morning each week to monitor the progress of the children. She also gave brief class sessions occasionally, to clarify particular teaching points, such as the use of the LOGO editor, reading error messages and debugging.

Details of the contents of the worksheets are given in Appendix 5. The programming notions in them are summarized below:

Programming Notions	Worksheets
direct drawing	1 2 3
simple procedures	4 5
use of REPEAT	4 5 8
subprocedures	6 7 8
state transparency	7 22
SX SY	9 10
inputs in procedures	11 14
inputs in subprocedures	12 13 15
multiple inputs	16

Additional elements included in later worksheets were

tail recursion	17
simple conditionals	17
changing inputs	17 18 19
random numbers	20 21
accepting keyboard input	20

There were twenty two worksheets in all. In the twenty eight weeks of the study most pupils reached Worksheet 16 which introduced the use of two or more inputs in a procedure. Then many of the slower pupils jumped to Worksheet 22, missing out 17 to 21 which contained a lot of new syntax and programming notions. Only a few pupils covered most of the sheets, and none did them all. The programming notions covered by most pupils were fairly elementary, extending only as far as the use of subprocedures with inputs.

The Tests Used.

Many of the tests used in this part of the study were the same as those used in Phase II. All the tests were also given to the control class. At the beginning of Phase III both classes were given the mathematics attainment test C1, used in Phase II, and a non-verbal IQ test, NFER Test DH. These were to check for initial differences between the classes. The post tests given during the third term included:-

reflections and rotations test (Chelsea 1)

estimation of angles

variables (Chelsea II)

mathematical attainment (NFER Test EF).

These tests were all used in Phase II. In addition a new test of mathematical strategies was used in place of the earlier one. This was adapted from the work of Bell, Shiu and Horton at Nottingham (1981). They published several tests to evaluate process aspects of mathematics in connection with their teaching programme. Several items from their tests were chosen and put together into two separate papers, one of three questions and one of four. A short explanation of what was required was given at the beginning of each question. Copies of these tests are given in Appendix 3e.

The testing programme was carried out at the beginning of the third term in four separate sessions on consecutive weeks. Each session, of one or two tests, lasted for about one hour. Both classes were given the same tests on the same mornings, one at 9.30 and one at 11 a.m. The order in which the tests were given was reversed, so that each class had two tests at the earlier time and two at the later. The wording used in the explanation of each test was kept as exact as possible, whilst ensuring that the children understood the nature of the tasks.

The effect of the class teacher could not be removed in the design of the study, so the final mathematical attainment test was used to assess this factor. The test contained questions on work which would have been covered during the year in the class lessons. Each question was classified by mathematical content, in the manual. Some of these questions, on shape and angle, could be expected to relate to the LOGO work done by the experimental class, while others showed no direct connection to this work.

Of the other concepts and strategies tested for, only the reflections and rotations topics had been covered in class lessons during the year. Neither class had encountered any algebra, nor had they received any instruction directed towards mathematical strategies. Both classes were using the same text books and syllabus.

7.3 Results on mathematical skills related to turtle geometry.

7.3a) Reflections and Rotations ... Test Chelsea 1

The results of this test are given in Table 4.

Level	Exp. Gp.	Control Gp.
3	16	12
2	12	10
1	6	5
0	1	2
Total	32	29

Table 4 Results of Chelsea 1 Reflections and Rotations

Overall, this test was very well done by both classes, with no statistically significant difference in performance between the experimental and control groups. It was marked by the level of question that the pupils could successfully answer, having to score 3 or more at level 1 to pass that level, 4 or more at level 2 to pass it (having passed level 1), and 3 or more at level 3 (having already passed the other two levels).

7.3b) Estimation of Angle

This was not a test of recognition of particular angles, for which a great deal of practise would be required, but of estimating the relative sizes of angles from a given one, involving knowledge of the additive and inverse properties of angles. Children with a good understanding of angles, without a lot of estimation practise, should have been able to use the example given to make reasonable estimates of the other angles. The scores given were 2 points for an answer within 10% of the actual angle, and 1 point for an answer outside this range but within the critical area, for example, an estimate greater than 90 degrees for an obtuse angle. The data is given in Table 5.

Score	Exp. Gp.	Control Gp.	
6 - 8	21	6	27
3 - 5	10	7	17
0 - 2	1	16	17
Total	32	29	61

χ^2 =22.02 2 d.f. Significant at .001 level one tailed test.

Table 5 Estimation of Angle

A chi squared test on the results showed that the experimental group scored significantly better than the control class. Over half the control group scored less than 3 out of a possible 8, whereas two thirds of the experimental group scored 6 or more.

7.4 Results on general mathematical development.

7.4a) Concept of a Variable . Chelsea II Test of Generalized Arithmetic.

Although the experimental group had used variables as inputs to procedures, most of the time they were given meaningful names such as SIDE or ANGLE. Consequently, the LOGO pupils were no more exposed to standard algebraic notation than were the control group. The purpose of this test was to see if the use of inputs in procedures contributed to the understanding of variables when used in a different context.

This Chelsea II test was also marked in levels of understanding, in exactly the same way as the Chelsea I paper. The data is given in Tables 6 and 6a.

Level	Exp. Gp.	Control Gp.
3	6	2
2	11	5
1	15	17
0	0	3
Total	32	27

Table 6 Variables Chelsea II

The data was grouped into high and low levels in order to give sufficient numbers in each cell to use a chi squared test. This was significant at the .05 level, showing that significantly more of the LOGO group children were able to use the concept of variable, than the control group.

Level	Exp. Gp.	Control Gp.	
2 - 3	17	7	24
0 - 1	15	20	35
	32	27	59

$\chi^2 = 3.43$ 1 d.f. Significant at the .05 level 1 tailed test.

Table 6a Variables

As a check on this result, a Mann Whitney U test on the actual scores of the Chelsea II test was also used. This confirmed the significance of the result ($z = 2.10$ with a probability of p 0.018 on a one tailed test.)

7.4b) Mathematical Strategies.

The seven questions on the two papers of mathematical strategies each had a slightly different emphasis. Five of these, A,C,E,F and G were to do with generalization and abstraction. They required the continuation of a pattern, sometimes the recognition of particular instances or non-instances of that pattern, and then the formulation of the underlying rules. Some patterns were familiar, as with A number sentences, and C odd and even numbers, whereas others dealt with novel situations. Each of the other two questions dealt with different strategies; B was on the use of information for making inferences, and D involved explanation and a certain amount of "reversible thinking", working logically back through the process.

7.4b.1 Generalization in novel problems, questions E F and G.

The research hypothesis was that each group would be equally capable of continuing the pattern after the explanation, but that the LOGO group would be better able to generalize and formulate the rules underlying the patterns. The questions were marked as recommended by Bell, Shiu and Horton, to identify the responses to the different sections.

Question E Move Along.

The results of question E are given in tables 7a, 7b and 7c. 7a gives the results of the pattern continuation part of the question, whilst 7b and 7c show the results on the formulation of each of the rules.

Scores	Exp. Gp.	Cont. Gp.	Total
5 - 6	17	17	34
3 - 4	5	4	9
0 - 2	8	10	18
	30	31	61

χ^2 is not significant at .05 level

Table 7a E - pattern continuation scores

Scores	Exp. Gp.	Cont. Gp.	Total
1 - 2	13	10	23
0	17	21	38
	30	31	61

χ^2 is not significant at the .05 level.

Table 7b E - rule 1

Scores	Exp. Gp.	Cont. Gp.	Total
1 - 2	8	6	14
0	22	25	47
	30	31	61

X^2 is not significant at the .05 level.

Table 7c E - rule 2

There was no significant difference between the scores on the pattern continuation questions. On the rule questions the experimental group appeared to be slightly better than the control group, but the difference did not reach significance.

Question F Arrow.

This question involved the movement of an arrow according to two rules which were labelled P and Q. One involved a quarter turn and the other a half turn. Successive P and Q movements were applied to the arrow and the children were asked to predict its final position. One part of the question was again concerned with pattern continuation; understanding the movements and how they were applied. The rest of the question was on finding and stating rules for predicting the final position after a large number of movements, without going through each one. Tables 8a, 8b, 8c and 8d give the results of different parts of the question.

Score	Exp. Gp.	Cont. Gp.	Total
2	25	25	50
1	5	6	11
	30	31	61

χ^2 is not significant at .05 level.

Table 8a F - pattern continuation

Scores	Exp. Gp.	Cont. Gp.	Total
4+	15	6	21
0 - 3	15	25	40
	30	31	61

$\chi^2 = 5.06$ 1 d.f. significant at the .025 level 1 tailed test.

Table 8b F - Rules P and Q

Scores	Exp. Gp.	Cont. Gp.	Total
2 - 3	14	9	23
0 - 1	16	22	38
	30	31	61

$\chi^2 = 1.34$ not significant at .05 level.

Table 8c F - Rule P

Scores	Exp. Gp.	Cont. Gp.	Total
2 - 3	16	6	22
0 - 1	14	25	39
	30	31	61

$\chi^2 = 6.23$ 1 d.f. significant at .01 level 1 tailed test.

Table 8d F - Rule Q

On question F there were no differences between the two groups on the continuation of the pattern, shown in Table 8a. The experimental group were better at identifying and stating the underlying rules. This finding was significant at the .01 level for rule Q.

Question G Roofs

The first part required understanding of the pattern of symbols used to represent the shape of a roof, and the second part involved the explanation of the rules underlying the pattern. The data is given in tables 9a 9b 9c and 9d. Table 9a gives the results on the continuation of the pattern; Table 9b gives results on both rules, and the other two tables look at each of the rules separately.

Score	Exp. Gp.	Cont. Gp.
2	28	26
1	2	5
	30	31

Table 9a Question G Roofs, pattern continuation.

There was no significant difference between the two groups.

Score	Exp. Gp.	Cont. Gp.	
1+	20	13	33
0	10	18	28
	30	31	61

$\chi^2 = 2.83$ Significant at the .05 level 1 tailed test.

Table 9b Question G Roofs, Rule 1 + Rule 2

Score	Exp. Gp.	Cont. Gp.	
2+	14	5	19
0 - 1	16	26	42
	30	31	61

$\chi^2 = 5.28$ significant at .025 level 1 tailed test.

Table 9c Question G Roofs, Rule 1

Score	Exp. Gp.	Cont. Gp.	
1+	11	7	18
0	19	24	43
	30	31	61

$\chi^2 = 0.86$ not significant.

Table 9d Question G Roofs, Rule 2

The responses of the two groups on the first part were very similar, and of a high level, showing that they understood the task. However, the experimental group was superior at finding and stating the underlying rules.

Using the chi squared test the experimental group's performance was significantly better on R1 (.025 level) and R1 + R2 (.05 level). There were no differences between the groups on their ability to give explanations or to use formal algebraic symbols. Both these additional parts of the question were poorly done. This was to be expected in children of this age, having learned no formal algebra.

7.4b.2 Generalization in familiar problems, questions A and C

Questions on topics which looked familiar were thought more likely to attract intuitive answers. This happened in question A, consisting of a pattern of number sentences. These were simple addition of two numbers to make a third, in the ratios 2 : 3 : 5.

2 + 3 = 5

4 + 6 = 10

6 + 9 = 15

8 + 12 = 20

Almost all the children in both groups recognized the "adding on" rules of the columns, rather than the ratio rule of each sentence, and evidently used this strategy in the first three parts of the question to continue the pattern. The fourth part of the question gave only the final number of the pattern and so was more difficult to do in this way. It required working backwards, involving more understanding of the relations between the numbers in each sentence. The results are given in Table 10a and 10b.

Score	Exp. Gp.	Cont. Gp.	
4	20	11	31
0 - 3	11	19	30
Total	31	30	61

$\chi^2 = 3.56$ 1 d.f. significant at .05 level 1 tailed test.

Table 10a A. Number sentences, first four parts.

When the experimental and control groups were compared on these first four parts of the question, more of the experimental group were successful on all four parts, significant at the .05 level.

The remaining parts of the question involved the statement of the underlying rules of the number sentences.

Score	Exp. Gp.	Cont. Gp.	Total
5 +	10	3	13
2 - 4	14	14	28
0 - 1	7	13	20
Total	31	30	61

X^2 = 5.61 2 d.f. significant at the .05 level 1 tailed test.

Table 10b Rules underlying the number sentences A

Again the performance of the children in the experimental group was superior to the control group. This was significant at the .05 level.

Question C Odd and Even

This question was also on a familiar topic, the addition of odd and even numbers. The data is given in Table 11a, 11b and 11c.

Score	Exp. Gp.	Cont. Gp.	
1	26	22	48
0	5	8	13
	31	30	61

This gave no significant differences.

Table 11a Question C Odd and Even, pattern continuation.

Score	Exp. Gp.	Cont. Gp.	
3 - 4	13	11	24
0 - 2	18	19	37
	31	30	61

This gave no significant differences.

Table 11b Question C Odd and Even, rule for odd numbers.

Score	Exp. Gp.	Cont. Gp.	Total
3 - 5	8	2	10
0 - 2	23	28	51
Total	31	30	61

$\chi^2 = 2.80$ 1 d.f. significant at .05 level 1 tailed test.

Table 11c Rules for Odd and Even number combinations.

This was the least well done of all the generalization questions as children in both groups were uncertain of what to do when asked to make investigations. Some rules for the combinations of odd numbers were found by about half the pupils but there was no difference in performance between the two groups on this. Far fewer children extended the investigation successfully to mixtures of odd and even numbers. Here the experimental group were better.

Question B Football

This question was on the use of information only. It consisted of three statements about team performances from which children had to say which team got most points, and which statements were necessary to determine this.

a) Both teams have the same number of draws.

b) Team A has beaten team B twice.

c) Team B has won two more games than team A.

The data is given in Table 12.

Score	Exp. Gp.	Cont. Gp.	Total
4 - 5	24	16	40
0 - 3	7	14	21
Total	31	30	61

$\chi^2 = 2.92$ 1 d.f. Significant at .05 level 1 tailed test.

Table 12 Football question B

The experimental group again scored better overall than the control group. The most common error was to discount the information about the draws.

Question D Game of 25

This is a game played by two people, to add up numbers between 1 and 6. They take it in turns to chose a number and add it on to the total. The winner is the first to get to 25. The children are shown that the first person to get to 18 can always win. The first part of the question required an explanation of this fact. The scores on explanations are given in Table 13a.

Score	Exp. Gp.	Cont. Gp.	Total
2 - 3	23	12	35
0 - 1	8	18	26
	31	30	61

χ^2 = 5.96 1 d.f. significant at .01 level 1 tailed test.

Table 13a Question D, Game of 25, Explanations.

The experimental group was better at giving explanations. This finding was significant at the .01 level.

The second part asked the children to find other winning points, by extending the argument for 18 backwards. Bell described it as a combination of abstration and generalization, the recognition of chunking or curtailment of argument. The results are given in Table 13b.

Score	Exp. Gp.	Cont. Gp.	Total.
1+	9	3	12
0	22	27	49
	31	30	61

χ^2 = 2.39 not significant at .05 level

Table 13b Game of 25, Rules underlying it.

This was found to be very difficult for most children. Only 5 boys and 4 girls from the experimental group, and 3 boys from the control group were able to do some of it. These were so few that the differences were not significant.

Conclusions from the generalization tests.

Overall, the experimental group was found to be consistently better at generalization and abstraction of underlying rules than the control group, taking a .05 level of significance. There is some indication that they were better at identification of relevant information, from question B, and explanation of processes from question D as well. As hypothesised, they were no better at the continuation of patterns, showing that the difference lay in their ability to generalize from the information given, rather than in understanding the question.

7.5 Results on general mathematical attainment - Test EF

The purpose of giving the post test on mathematical attainment was to give some measure of the effect of the teacher on class performances. As there was no significant difference between the groups at the beginning of the year, it was hypothesised that if there was no overall difference between the two teachers, in terms of the success rate of their pupils, there should be no difference in performance on mathematical questions not related to the LOGO experience, at the end of the year. This argument does however discount the possibility of motivational effects of LOGO work found in phase II. A difference on LOGO questions could then be attributed to the experimental treatment. Table 14a shows the results of the mathematics attainment test EF.

Score	Exp. Gp.	Cont. Gp.	Total
35+	22	11	33
34-	8	17	25
	30	28	58

χ^2 = 5.53 significant at the .02 level 2 tailed test.

Table 14a Mathematics attainment test EF

The results of the test EF indeed showed the experimental group to be superior. Analysis of the contents of the questions in the test revealed two broad topic areas in which the experimental group's superiority lay. One of these, as predicted, was LOGO related factors, contained in 17 questions with the following breakdown:

angle and shape - 12 questions

approximations - 3 questions

algebra - 2 questions

The other topic area was fractions, decimals and percentages, accounting for 11 questions, leaving 32 remaining questions. Analysis of scores in these three topic areas are given in Tables 14b,14c and 14d.

Score	Exp. Gp.	Cont. Gp.	Total
14 - 17	20	6	26
10 - 13	5	11	16
0 - 10	5	11	16
	30	28	58

χ^2 = 13.16 2 d.f. significant at .01 level 2 tailed test.

Table 14b LOGO related questions (17)

Score	Exp. Gp.	Cont. Gp.	Total
9 - 11	15	7	22
6 - 8	14	9	23
0 - 5	1	12	13
	30	28	58

χ^2 = 13.23 2 d.f. significant at .01 level 2 tailed test.

Table 14c Fractions, decimals and percentages (11 questions)

Score	Exp. Gp.	Cont. Gp.	Total
21 - 32	7	4	11
15 - 20	16	9	25
0 - 14	7	15	22
	30	28	58

χ^2 = 5.62 2 d.f. not significant at .05 level 2 tailed test.

Table 14d Remaining questions (32)

The result on Table 14c fractions, decimals and percentages is a "teacher factor" and related to the fact that these topics had just been revised with the children in the experimental class. The remaining questions show a slight superiority towards the experimental class, but this does not reach significance. There is no reason to suppose that the differences found in the other post tests are caused by the effects of the different teachers. It could also be argued that a general improvement in mathematics would occur through increased motivation, as a result of LOGO experience. Previous work has shown LOGO to improve attitudes and confidence towards mathematics which may affect performance on a general test.

7.6 Discussion of Test Results.

1. Angle and Shape

The experimental group was significantly superior, at the .01 level, in angle estimation, but there was no difference between the groups on the test of reflections and rotations. This was the one topic which had been taught in class, and was very well done by both classes.

2. Variables

The experimental group was superior in understanding and using variables, at the .05 level.

3. Mathematical strategies.

Five questions dealt with generalizations and abstraction of underlying rules. The early part of each question involved the continuation of a pattern: there was no difference between the groups on these parts. In four out of the five questions the experimental group was significantly better (.05) at deriving the underlying rules. The other two questions, testing ability to use information and explanations of a finding, were also significantly better done by the experimental group, (.05).

Discussion.

The evidence gathered suggests that through using LOGO children's mathematical understanding in the areas of angles, variables and mathematical strategies, had improved. On the initial testing there was no difference in mathematical attainment or IQ between the LOGO and control groups; there was no difference in their performance on the earlier part of the generalization questions also, showing that they were equally able to understand the task. So it seems likely that the improvement in mathematics had taken place over the year in which they were using LOGO.

The two classes did have different teachers, and this apparently made a difference in the final mathematics attainment test, so this difference cannot be completely ruled out as an explanation of the test results. However, the topics of generalization and variables were not specifically taught to either class, so this is thought to be unlikely. The one topic which was included in the mathematics syllabus, reflections and rotations, was very well done by all children.

Evidence for relational as opposed to instrumental learning is given by the results of the mathematical strategies tests. Within the given theoretical framework, the ability to apply mathematical knowledge to novel problems implies that a relational understanding has been developed. The LOGO children showed an ability to structure new problems and find the underlying rules, which was not shown by the control class children.

An illustration of the difference in quality between the instrumental and relational learning is shown in the results of the angles test. Almost all the LOGO group were able to recognize angles as measurement of turning, obeying the normal laws of addition and subtraction of integers, and were thus able to make an estimation of one angle from the size of another. It is suggested that they would also be able to cope with arbitrary units; if told the original angle was 4 dobs instead of 40 degrees, they would be able to give estimates for the other angles in dobs.

Most of the children in the control group, on the other hand, though they had learned about angles in the formal syllabus, were unable to make these estimations. This suggests that their learning about angles had been instrumental. The nature of angles was not clearly understood, so the addition and subtraction of them was not recognised. They may have learned to associate particular shapes with particular labels, thus a right angle is associated with the label "90 degrees". Some labels are more common than others so, in the absence of relational understanding, they just tried to attach common labels to the odd shapes they were faced with.

The results of the variables test showed that over half the LOGO children were able to answer questions at the second level of difficulty, or above, compared with a quarter of the control children. Neither class had learned any formal algebra. This finding supports the notion that the work done with inputs to procedures gave LOGO children a foundation for understanding the use of variables in different contexts.

As the variable problems the children met were novel both in form and content, success with them is again indicative of relational learning. Mathematically bright children would be expected to develop relational understanding, and seven children in the control group appear to have done so. According to the pretests the two classes were essentially similar in composition, thus the result shows that LOGO experience has enabled some of the less bright children also to develop relational understanding.

The conclusion to be reached from these results is that experience with LOGO does enable children to develop relational understanding of angle and variable, and thus to develop their awareness of mathematical strategies. Almost all LOGO children showed a good understanding of angles, but they were not all equally successful with variables. This may reflect the fact that whereas all children spent time using angles some of them had little time using inputs in procedures. Those who apparently gained most from the experience were the ones in the middle band of mathematical ability.

7.7 Summary of Chapter 7.

The third phase of the study was set up to verify the results indicated by the second phase, that children were able to develop their mathematical thinking through using LOGO programming. In order to assess what learning could be attributed to the programming experience, and what to other factors, a control group of children who did not use LOGO, also took the tests of mathematical understanding.

The test results showed the LOGO group to be superior on estimation of angles, use of variables, and generalization, abstraction and the use of information. There were no differences in IQ or mathematics attainment between the two groups at the beginning of the study. By the end of the study there were also no differences in tests of mathematical work covered in mathematics classes.

CHAPTER 8

THE QUALITY OF MATHEMATICAL LEARNING THROUGH LOGO

The test results indicated that the children were developing relational understanding of angles and variables, from their LOGO work, and through this were learning about mathematical processes. We then looked at the observations of their problem solving activities to see if such learning could be detected, and to gain more information on how it came about.

Evidence for relational learning is given by the way a child can apply knowledge learned in one context to another. Davis's work suggests that it can also be detected by the way a child is able to use available information in a problem. According to his theory, if the child understands some area of mathematics in a relational way, then he will have built up a flexible mental representation of it, which can be called a schema. The schema can be thought of as a skeleton solution to problems of a particular type, with variable "slots" to be filled from the information contained in any particular problem.

When the child recognises the nature of a problem, he retrieves the appropriate schema for it and attempts to match the information contained in the problem with the schema variable "slots". When he is successful, this confirms his choice of schema, and he goes on to solve the problem. Thus the possession of a relational understanding guides his search for relevant information. Conversely, without a relevant schema, the child may be unable to sort out and make use of the information contained in a problem.

The sort of schemas which might be developed through using turtle geometry can be postulated, and the absence of them would be indicated by particular performance bugs. The most common ones are:-

1) Angles - recognizing the extension of integer arithmetic to angular as well as to linear displacement.

The absence of this schema, and attempts to use a rule-based approach in angle problems, is illustrated by certain bugs;

a. inappropriate guesses at angles

b. inability to recognize and carry out addition and subtraction of angles

c. inability to estimate angles through successive approximations.

2) Shapes - recognizing that the angle input determines the shape, whereas the forward input determines the size of a figure.

The absence of this schema is shown by the attempted application of the same angle to draw two or more different shapes, or the use of different angles to draw the same shape.

3) Polygons - recognition that the turtle makes a complete turn of 360 degrees in drawing a regular polygon, so the number of sides and the angle used are inversely related.

4) The cyclical nature of angular measurement - recognition that all angles greater than 360 degrees can be represented by other angles between 0 and 360, and that all left turns can be represented by equivalent right turns and vice versa.

5) Variables - recognizing the use of a word or a letter to represent any number, as input to a procedure.

Examples of each of these schemas, or the lack of them will be given in turn.

8.1 Angle schema.

This is the schema for which there is most evidence, because it is fundamental to turtle geometry.

The addition and subtraction of linear amounts was readily recognized by all children from very early on.

FORWARD 20 FORWARD 30 would be simplified to FORWARD 50

This was not the case with angles. Most children did soon pick this up, but in the final test one boy (Sh) made no attempt to simplify the angles he was using. He drew the three triangles of the test using direct commands, and then tried to copy the commands into a procedure. He used several groups of angles in the direct commands;

```
RT 90          RT 50          RT 90
RT 90          RT 240         RT 90
LT 20          RT 120         RT 390
LT 20
```

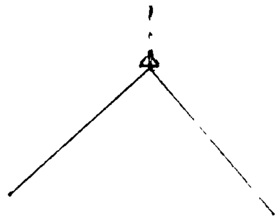
He copied these into his procedure, without simplifying any of them. He also made one addition in the FORWARD inputs.

```
FORWARD  1
FORWARD 14
```

This was simplified to FORWARD 15 for the procedure.

Two other boys (H and Jn), towards the end of the second term, also used multiple angles in their procedure LEGS.

```
TO LEGS
   RT  90
   RT  45
   FD  40
   BK  40
   RT  90
   FD  40
   BK  40
   RT  90
   RT  45
END
```



The symmetry was apparently recognized, but no attempt was made to add the two RIGHT commands at the beginning and end of the procedure.

In a slightly different context, two girls (J and G) failed to recognize that left and right turns of the same magnitude cancel each other out. This was at the end of the first term, after 14 weeks of LOGO work. They built a two line procedure and then used it to draw a 12 sided polygon.

```
TO  M
  FD  20
  RT  30
END

REPEAT 12 [ M ]
```

Then they tried a variation on this.

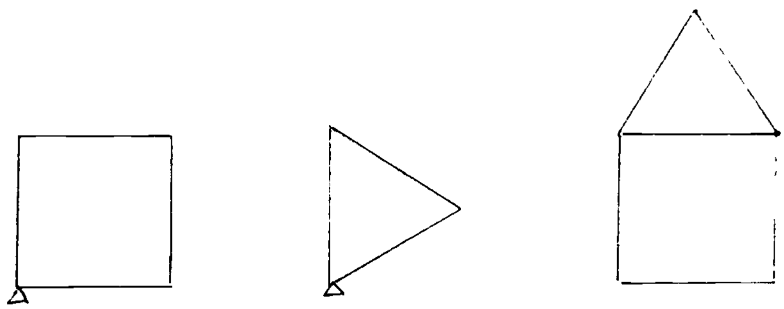
```
REPEAT 12 [ M  RT  30 ]
```

This drew a hexagon which they were very pleased with. They then tried another variation.

```
REPEAT 12 [ M  RT  30  LT  30 ]
```

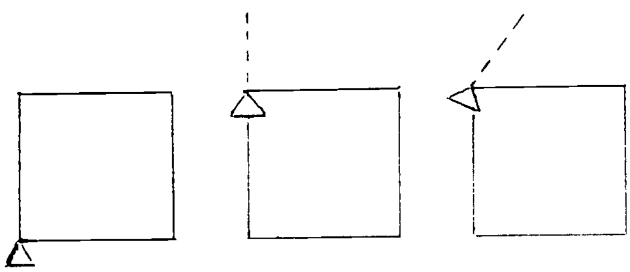
This again drew a 12 sided polygon, since it was effectively the same as the first one, but they were amazed at this result.

The inability to estimate angles with successive approximations was first noted in the phase II study, with the problem of putting a triangle on top of a square to build a house.



Putting the two together involved moving the turtle from the final position of the square, to the correct starting position for the triangle. This could be done in a variety of ways, but the most common way was to move forward the length of the side, and to turn right:

```
SQUARE :SIDE
FORWARD :SIDE
RIGHT 30
TRIANGLE :SIDE
```



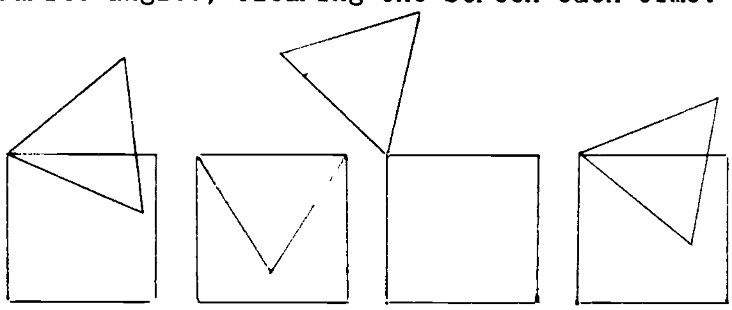
Most children had difficulty with finding the correct magnitude of angle. Very few tried to work it out theoretically, but used a trial and error approach instead, guessing 45 degrees to start with.

At this point a large number of children made no interpretation of the resulting drawing, which clearly showed that the turn was too great, but made further guesses of an apparently random selection of favourite angles, clearing the screen each time. For example :-

RIGHT 90

LEFT 45

RIGHT 70



By discussing the resulting drawing with the child, it was possible to bring his attention to the information which the bug contained. He could then make a more suitable guess, based on this information.

In the third phase study many children.showed the same difficulty in finding the correct angle and the same neglect of the information contained in the trials. The children were prompted to talk about what they were doing, and it was found that they were all able to make linear estimates of too much or too little and could apply the technique to angles when it was suggested. It is suggested that they were not making successive approximations with angles, not because they did not know the technique, but because they did not recognize that the technique applied in this case. Angular displacement was thought of as a different sort of mathematical object to linear displacement as they had not developed a relational understanding of it. Abstracting relevant information during the solution of a problem depends on the existence of an appropriate schema. Where this exists, the match of input information to schema variable "slots" can then be made. The schema thus guides the information search, and the match of information confirms the suitable application of the schema. This was clearly not happening in this case. Where only learned sequences are available, then there is no principle to enable the choice between them to be made, except perhaps the general psychological factors of frequency and recency. Thus the children without angle schemas made unrelated guesses at angles, using the more familiar values of 90 and 45 degrees more often than others.

Some quite able children were apparently using instrumental knowledge of angles at around the 10th and 11th weeks of using LOGO, but later constructed angle schemas. Faced with finding an unknown angle, two top maths group girls, L and J, made six or seven unrelated guesses and made no attempt to gain information by examining the drawing, other than to decide if the angle was correct or not.

One boy working on his own, Ke, tried to find the top angle in drawing a zed. He ignored the symmetry of the problem, and tried out the series of angles

125 115 180 90 45 180 90

It was pointed out to him that he was trying the same angles again. He stopped for a moment then decided that the required angle had to be between 125 and 180, and correctly chose 135. It is interesting to note that the actions he eventually gained the information from occurred some minutes before he made the decision, suggesting that he was reflecting on his earlier findings while trying out other values.

Two other boys, D and F, showed interesting interaction as they worked on the problem of drawing a 10 sided polygon. The angles they used at first were 45 and 20.

F "the RT is wrong, it should be between 45 and 20"

D (types in 50)

F "try more than 90"

D (types in 120 ... neither showed any recognition that they could have expected a triangle to be drawn)

D (types in 75 ... this gave an approximate 5 sided figure)

F "5 is half 10 so double it"

D (types in 150)

(At this stage it was suggested that they should write down the angles they knew which drew particular shapes. They wrote down the angles for 3, 4 and 8 sided figures.)

D "it needs to be smaller than 45 to give it more room" (types in 35 ... this left a small gap)

D (types in 30 ...gave a larger gap)

D (types in 37, and then finally types in 36 and. completes the 10 sided polygon).

They were working with an unclear notion of angles, made up of several sequences, feeling around without any clear structure to their search, though F had stated the correct range for the angle at the beginning. They had some half-formed ideas on the relation between the number of sides and the angle, but needed to reflect on their experience in order to develop the schema.

This work contrasts with that of two other boys, C and Am, who had developed both angle and polygon schemas. In the third week of using LOGO they were developing patterns by rotating shapes. They built a triangle and a rectangle for these patterns, and then decided to do a pentagon and heptagon. They calculated the angles in their heads, using an approximation of 51 degrees for the heptagon, built the figures and then made patterns from them in a very short time. The existence of these schemas can be hypothesised from the ease and fluency with which the boys approached the problem.

Many children were seen both before and after their understanding of angles had developed. Very few had apparently not got a relational understanding of angle by the end of the year. For most children, even ones who scored well on the mathematics attainment test, it was several weeks before they were able to recognise the additive and inverse properties of angles. Some were well into the second term before they were able to use this knowledge in their problem solving.

8.2 Understanding of shape

This schema involves the notion that the shape of a closed figure is fundamentally determined by the angles in it, thus the common element of rectangularity, for example, can be abstracted from all rectangular figures. In rotations, translations, reflections and enlargements, the angles within figures remain constant. The shape schema is also fundamental to the definition of regular polygons; a square can be any size, but all sides must be the same length and all angles must be 90 degrees. Similarly, regular triangles, hexagons and octagons can be defined as having all sides equal and all external angles 120, 60 and 45 degrees respectively.

In turtle geometry children spend a lot of time drawing and rotating regular polygons, from which experience the shape schema can be constructed. The following discussion is generally in the context of drawing regular polygons and stars, using the repeat of a two line procedure specifying forward and angle inputs. It is commonly found that the rule

90 degrees will draw a square

occurs earlier than the negative statement

90 degrees will not draw any other shape.

The first is a sequence or rule which can be learned, recited and used successfully in the context of drawing squares. The second is suggestive of the existence of a shape schema.

The development of the shape schema clearly depends on the prior construction of an angle schema, for unless the child has a clear notion of the consistency of angular measurement, he cannot relate angle to shape. Indications of the lack of a shape schema are given by the child using a familiar angle, or one just previously used for a different shape, when trying to draw something else.

This was first noted with the Primary 5 children in the preliminary study reported in Chapter 5, when two boys chose deliberately to use angles they had just used in other figures when trying to draw a different one. They argued that because these angles worked for the other shapes, they should also work for the new one. This tendency to use familiar angles in unfamiliar problems was quite common with Primary 5 children. Also the older Primary 7 participants occasionally showed this bug, particularly in the first term of using LOGO.

Two girls, K and Lu, used 60 degrees to make a three spoke pattern, and then tried to use 60 degrees again, immediately after, to do a six spoke pattern. K and S, when changing a spiral procedure to draw a square spiral after drawing a triangular one, did not recognize that the angle of 120 degrees had to be replaced by one of 90 degrees.

Other children showed that they had acquired this schema, even when they did not know the angles associated with particular shapes. D and F, though struggling with the angle schema in the first term, by midway through the second term had a good shape schema, and successfully used successive approximations to find the required angle to draw a particular shape. Two girls, J and R, who had also had difficulties in the first term, in the second one showed their understanding of the unique angle - shape relationship in drawing required patterns using appropriate angles.

8.3 Understanding of polygons and 360 degrees.

These schemas relate a complete turn to 360 degrees and the drawing of a simple polygon as making one complete turn. The angle turned and number of sides of the figure are then related, and angles greater than 360 degrees can all be seen to be represented by ones between 0 and 360.

It was quite common for children to learn the rule that one complete turn is 360 degrees, without developing the schema. Quite early on both Mi and K could calculate the angle required for a polygon with a given number of sides, but when the problem was taken out of this particular context they were both lost. MI tried drawing a circle by repeating the commands

```
FD 1   RT 10
```

He chose to repeat it 360 times because there were 360 degrees in a circle. K initially had no idea how to draw a circle, but eventually drew one using

```
REPEAT 360  [ FD 1  RT 1 ]
```

This was too big, so she first changed the repeat to 270, then looked up in her book and found the procedure

```
REPEAT  36 [ FD 1  RT 10 ]
```

She then tried to draw a semicircle, and halved the number of repeats, but was not sure whether to half the angle as well. Then on meeting the same problem the following week, with the same circle procedure, she decided to half the forward input in order to draw a semicircle.

She was clearly operating on a rule based system, not a schematic understanding. She knew the rule of 360 degrees in a complete turn, and had also some rule of the form

if you want half the size, half the input

but was unable to specify which input was to be halved. She therefore applied guesswork to the problem, sometimes getting it right, and sometimes getting it wrong, but not learning anything in the process.

Davis (1980) describes such behaviour in terms of early frames which tend to overgeneralize, and are inappropriately recalled. Thus Mi's bug of using repeat 360 for a circle, regardless of the angle used in it, would be a frame built from the first experience of drawing a circle with FD 1 RT 1. Similarly, K could be described as having an undiscriminated frame relating size and input. In this analysis the terms "rules" or "sequences" are used for such rigid early frames, as they are seen as being qualitatively different from schemas. The rules stand in isolation, and there is no way of determining which of several ones should be applied first. This is not the case for schemas. They possess structure relating them to other schemas, and therefore allow a flexibility in thinking to match the problem.

One strange phenomena observed with several children was the fascination with large numbers, and the use of numbers greater than 360 as input to angles. It was not clear if this was a deliberate tactic, in order to chose a number at random, or if there was a lack of understanding of the cyclical nature of angular measurement.

One pair of boys, Ke and Ar, used a large forward input and a large turn input in a recursive "random" dot procedure.

```
TO K
  FD 1
  RT 789
  PU
  FD 567
  PD
  FD 1
  K
END
```

The wrap-around facility of the screen was also being employed here, as the maximum width of the screen was only about 200 units. Other people (usually boys) used large numbers for angles when using procedures such as STAR and POLYSPI with angle inputs, (see worksheets in Appendix 5).

8.4 Understanding of variables

The way in which variables were introduced was through the use of inputs, developed first for one command, then extended to their use for any command. Children then met changing inputs, and the idea that arithmetic operations could be performed on them. This introduced two difficulties; first the idea that the value of an input can change, and second the acceptance of the algebraic notation :SIDE + 2 as valid.

Most children found the original substitution of a word :SIDE for a number not too difficult to cope with. When they had to apply this to other commands, this caused some problems. Only a few children used inputs with changing procedures, as most uses of this also involved recursion, increasing the difficulty of both the mathematics and the programming. The variable schema being developed by most children involved just the interchange between words and numbers within a procedure. Some children learned the syntax rules, as a sequence to give a desired output, whereas others build a schema which allowed them to use and adapt the idea of inputs to new problems.

Evidence for the schema is given in the spontaneous extension of the idea of inputs to other commands, and the recognition of the need to match the number of inputs required with the number supplied. The absence of the schema is shown by the inability to extend the idea, and the mismatch of numbers and inputs, by either specifying mock inputs which serve no purpose, or not giving sufficient inputs when trying to run a procedure. (The latter is a common bug with experienced programmers, but is quickly recognized. In the absence of the input schema, children are unable to see what the bug is caused by.)

Several children spontaneously extended the idea of an input to angles or to the SETX command (Ro, C). Others, when asked to build a procedure with a variable called ANGLE used the commands

```
FD :ANGLE
RT 90
```

and when the rectangle procedure was introduced, requiring two inputs for the width and length of the rectangle, three pairs of boys called for help because their procedures would not run. They did not see that, having used two different inputs names, they would have to specify values for both inputs in order for the procedure to run. This was an early bug which was later overcome.

The most difficult work involved the use of two unknowns in a procedure, the relation between them being known. This occurred in the general diamond procedure, as the two exterior angles used in any rhombus add up to 180 degrees. Only a few children were given this problem to tackle, and it was suggested that they should try to find angles which they could use to draw diamonds, by trial and error. Having found several pairs of them, they were asked about the relation between them, and then shown the notation

```
RT :ANGLE
RT 180 - :ANGLE
```

which could be used in a procedure.

Some of the same children were also asked to build a general procedure to draw a polygon with any number of sides. The common solution to this problem was a recursive procedure specifying the angle rather than the number of sides.

```
TO POLY ANGLE
  FD 50
  RT :ANGLE
  POLY :ANGLE
END
```

They all knew that the number of sides could be found from the angle by dividing it into 360, and could follow the working of procedures of the form

```
REPEAT 5 [ FD  50   RT  360/5 ]

REPEAT 8 [ FD  50   RT  360/8 ]
```

but the next abstracting step of specifying the procedure for an N-sided polygon was very difficult.

```
TO POLY  N
  REPEAT :N [ FD 50  RT 360/:N ]
END
```

The difficulty seemed to stem from the use of formal notation, rather than a lack of understanding of the generalization. Understanding of variables was assessed in the post tests, and over half the LOGO class were able to answer questions at or above the second level of difficulty, although many of them had not spent a lot of time using inputs in procedures.

8.5 Conclusions.

There is not enough information in the observed records to state unequivocally which schemas each child had built. This information does however illustrate when the understanding was not present. Most of the incidents quoted were from the less mathematically able children. The observations are consistent with the theories of different qualities of learning, and confirm the test results that most of these children gained a relational understanding of angle from their LOGO work, but only half of them gained an understanding of variable.

The type of errors and difficulties children have illustrate very well, to an observer, their current level of understanding. Weir has used this approach with learning disabled children to give an insight into their particular abilities and disabilities (Weir 1981). This would seem to be equally productive within a normal classroom. Emphasis has recently been given to the possibility of children monitoring their own learning processes, and thus learning how to learn from observing themselves (Flavell 1979). This study has not looked at the children's perceptions of what they are learning through LOGO, but this is a possible extension which could be developed in the future.

8.6 Summary of Chapter 8

Observations of the children's work were analysed to illustrate the level of understanding of the children at that point in time. The type of errors which would occur in the absence of five particular schemas were predicted and illustrated.

These were the notions of angle, shape, the polygon rule, the cyclical nature of angles and variables.

CHAPTER 9

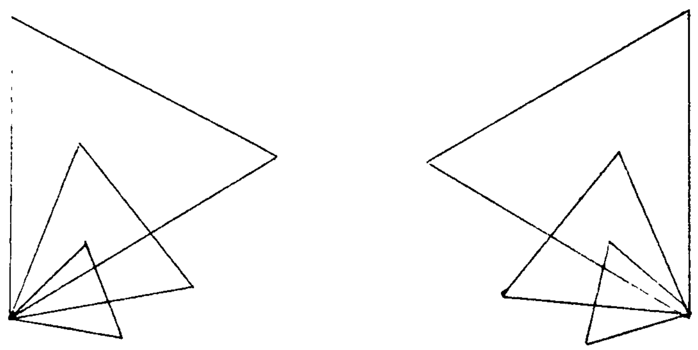
ASSESSMENT OF PROGRAMMING ABILITY

Though the prime aim of the study was to look at the development of mathematic thinking through using LOGO, some other issues were raised by earlier research, to which these studies could contribute. Programming ability was found not to be an obstacle for children investigating mathematics but it is interesting to look at the way children do learn to program. Pea and Kurland (1983) suggested that, if not taught correctly, they may pick up incorrect ideas, while other researchers have ignored this factor. The observations made during the Phase III study were used to look at the development of children's programming ability, their debugging strategies, and the common bugs which frequently occurred. Other organizational aspects of the studies; the role of the teacher; the compatibility of working pairs; and the amount of time spent on the computers have also been considered.

9.1 Assessment of programming ability.

Two tests of programming ability were given, one after about twelve weeks and one at the end of the twenty eight week LOGO period. The first test required the use of direct commands, the building of a simple procedure and the drawing of a ten sided polygon. Thirty children were tested. Three pairs had difficulty in completing the test, two of these with angle estimation, systematic thinking and debugging, and the remaining pair with editing commands, as well as the above problems.

The second test was the same one used with the Phase II study, requiring the children to build a procedure to draw a pattern of three triangles of different sizes rotated about one corner, and then to produce the mirror image of it.



Of the six pupils with problems in the first test, one had left the school, and one was absent so he was not tested. The remaining four, as one pair and two individuals, were completely successful. They all built one triangle procedure with an input as a subprocedure for the main pattern. A summary of the performance of the whole class on the second test is given in Table 15 below.

$$\begin{tikzcd} \text{Successful} & \text{Used input} & \text{Turning angle only} \\ 22 \arrow[r] \arrow[ddr] & 18 \arrow[r] \arrow[ddr] & 6 \\ & & \\ & \text{No input} & \text{Changed triangle} \\ & 4 & 12 \end{tikzcd}$$

Table 15. Performance of Phase III pupils on the triangles test.

The remaining eight children exhibited the following weaknesses:

- mirror image by rotation bug, (one pupil);

- repeat/recursion bug, (one pair);
- state transparency bug, (one pair);
- not using triangle procedure, (three pupils, as one pair and one single).

In all twenty two children succeeded in building a procedure to draw the pattern and a second procedure to produce the mirror image of the pattern. Eighteen of these used a triangle procedure with an input as a subprocedure in the main pattern. The other two pairs used quite different approaches. One pair, C and A, built a total of six separate subprocedures for the two patterns and then two procedures each using three of the subprocedures to draw the required pattern and its mirror image; the other, M and R built one long procedure for each pattern. Of the eighteen very successful children, only three pairs saw that the mirror image could be drawn by changing the turning angles only. The rest changed each triangle from a right turning one to a left turning one.

Of the remaining eight, one, H, succeeded in the first part of the pattern, using one long procedure, but thought the mirror image could be achieved by rotations. Two pairs built triangle subprocedures with inputs, but had bugs they could not deal with. One pair, A and E, had a repeat/recursion bug

```
TO TRI :SIDE
   FORWARD :SIDE
   RIGHT 120
   REPEAT 3 [ TRI :SIDE ]
END
```

The other pair, L and G, had state transparency problems and unnecessary inputs to their triangle procedure:

```
TO TRIANGLE :ANGLE :SIDE
   FORWARD :SIDE
   RIGHT 120
   FORWARD :SIDE
   RIGHT 120
   FORWARD :SIDE
END
```

When their bugs were removed, both pairs showed difficulty in "seeing" the spatial pattern and linking the three triangles with appropriate turns.

One pair of girls, S and K, used a mixture of procedures and direct commands, and having built a triangle using 120 degrees then tried to draw a second one using different angles. They built a procedure to draw the smallest triangle, then used this procedure followed by direct commands to add the turn between triangles and draw the second one. After adding the second turn they recognized that they were drawing just another triangle for the final one, so edited the triangle procedure to change the forward values, then used the changed procedure to draw the largest triangle.

The remaining pupil, Sh, produced the drawing using direct commands. Only the first triangle was equilateral, and he used trial and error unsystematically for the rest. He copied down the commands as he went along and then attempted to build them as one long procedure.

Only one pair of boys, A and K, attempted to use a recursive approach and they could not remember the correct syntax for what they wanted to do, so abandoned the attempt. None of the pairs of pupils had had sufficient exposure to recursion to be able to incorporate it in their work, so all were programming at an elementary level, as was predicted from the analysis of the worksheets completed.

Conclusion.

These results show that all the pupils, bar three, were able to build and use procedures with inputs when they were appropriate. This is not to say that all pupils understood the same amount of programming, rather that most children were able to reach a particular criterion level. Some were much quicker at finding solutions to the programming problems than others, but ability levels, as measured by the mathematical attainment tests, did not prevent children from learning elementary programming. This is a similar finding to that in Phase II, though overall the standard of programming in Phase III was higher. This must, in part, reflect the increased amount of time the children spent using the computers in the later study; 32.5 hours as opposed to 23 hours in Phase II. There were other differences in the way the studies were carried out which may also have had a bearing on this result. They are discussed below.

9.2 Debugging and the use of information.

One of the common justifications for advocating the use of programming in education is that debugging fulfills important educational purposes.

- It attacks the right / wrong dichotomy. Different approaches are shown to be acceptable, and different results can be interesting and give new ideas for problem solutions. It is however important to understand where the results have come from, so that they can be repeated if required.

- Debugging gives good exercise in learning from mistakes. The mistaken thinking is immediately shown up. It may be a design fault, an arithmetic error, or a programming misunderstanding. The misconceived notions can be corrected, and so the internal mental model is amended and developed.

- It gives practise in the use of available information, which is a fundamental skill in all problem solving.

As the experimenter was not present a lot of the time, and the class teacher was often busy working with other groups, the children on the computers often devised their own debugging strategies. They did not all have the above educational advantages. The observed strategies are listed below.

1) The widely reported adaptation of a drawing to fit the bug. This is the most common first approach reported by Howe (1980) and Noss (1983), that children adapted the design to something else rather than correcting the bug.

2) With direct drawing, some children cleared the screen using

the CS command and started again. This was distressingly often used by the less confident children, who, after using it several times in a session, ended up with nothing to show for their efforts, because each time the drawing went wrong.

3) With direct commands, the PE command was used to rub out an incorrect line. Some children were very happy with this approach and drew on the screen rather as they would use a sketch pad, rubbing out any wrong lines as they went along. The snag was that the line must be rubbed out straight away. If it was left until later it was often difficult to place the turtle in exactly the right position.

4) A more sophisticated version of the sketching technique was used extensively by two boys to put diagonals into rectangles. They tried out the line first with the pen up, and then drew it when it had been seen to be correct.

5) With procedures the most primitive debugging method was to use the command QUIT, erase all procedures and start again. This was used by one boy every time he wanted to change a procedure, because he did not know how to edit. It was used by others when frustrated, often having messed up a procedure by trying to use CS and other direct commands while in the editor, and not knowing how to get out.

6) With procedures some children rubbed out the entire contents of the editor, often including the name of the procedure and the END command. Some did this even when the procedure worked well, in order to build a new procedure. (Five of the pupils in the

class had computers at home before the LOGO study started, and some of these had done some BASIC programming. It is likely that this erasing of earlier procedures might relate to this experience.)

7) The "correct strategy" of reading the error message, and editing the line was gradually developed. This required an understanding of error messages to find the bug, and an ability to use the editor to correct it.

8) With a long complicated procedure with many errors, one useful strategy occasionally employed was to leave the editor, erase the procedure using the command ERASE, and to build a new one.

When persistent negative debugging strategies, such as using QUIT, had been observed, a fifteen minute session on reading error messages and debugging procedures was given to the whole class.

9.3 Common bugs in children's programming.

Again, because the children were working largely without supervision, some persistent bugs occurred in their programming, which remained undetected, or ignored, by them. These mostly occurred when children were building procedures with subprocedures or inputs.

a) Unintentional recursion, through using REPEAT inside a procedure.

This was a common bug when children were first learning to use REPEAT in procedures. This was caused by a lack of knowledge of the correct syntax for REPEAT.

```
TO CIRCLE
   FD 1
   RT 1
   REPEAT 360 [ CIRCLE]
END
```

This bug was not obvious to the children, because often they were imprecise about the required number of repeats for a circle, and so would use 100, or 1000, or some other arbitrary number, and stop the procedure when the circle was complete. As a result the children made no effort to correct it. They required an understanding of recursion to recognize why this procedure did not work as they intended.

b) One very common bug arose when children tried to make up patterns or super-procedures from procedures which were not state transparent. This happened with Worksheet 7, making a flag from a square, (see Appendix 5). When it was understood that the bug stemmed from the final position of the turtle, it was quite common to see children using the command HOME to correct it. This approach was however inadequate and resulted in other bugs occurring when the drawing was moved to a different position or orientation. Too often children tried to correct a bug of this type at the level of the super-procedure, rather than recognizing it as arising in the sub procedure. They needed to have an understanding of turtle state for this to be meaningful to them.

c) Declaring variables.

When children began to use variables, there was often confusion in declaring the variable, because on the TI machine when they typed

```
TO SQUARE SIDE
```

to get into the editor the SIDE was ignored, so only

```
TO SQUARE_
```

was shown in the editor. The SIDE had then to be typed in again. The error message "SIDE HAS NO VALUE AT ...." was difficult for the children to understand.

d) Procedure and variable names.

When passing variables in subprocedures, it is necessary to use the correct variable and procedure names. This is fairly obvious when a child has a correct mental model of the process, but some children showed confusion about this, particularly when similar names were used for the procedure and subprocedure.

Worksheet 8 gave examples of a procedure to draw a hexagon using a subprocedure to draw one side. Children confused the names of the procedures using two different subprocedure names.

```
TO HEX.BIT
   FD 20
   LT 60
END

TO HEXAGON
   REPEAT 6 [ HEXAGON.BIT ]
END
```

For some children this may just have been a slip in concentration; for others it indicates a spurious connection between a procedure and its subprocedure.

Another problem arose in the use of variable names. The first procedures with inputs all used the input to vary the size of the drawing, and so the majority were called SIDE. On Worksheet 13 they were given three examples of recursive polygon procedures

```
TO SQUARE          TO POLY6          TO POLY3
   FD 50              FD 50             FD 50
   RT 90              RT 60             RT 120
   SQUARE             POLY6             POLY3
END                END               END
```

then asked to build a general polygon procedure with an input ANGLE. A variety of errors were produced:

```
TO POLY ANGLE        TO POLY ANGLE        TO POLYANGLE
   FD :ANGLE            FD :SIDE             FD 50
   RT 90                RT 60                RT :ANGLE
   POLY :ANGLE          POLY :ANGLE          POLYANGLE
END                  END                  END
```

It is probably worth using purely arbitrary names occasionally, for both procedures and variables, to overcome these problems.

Some of these bugs undoubtably arose because of the inadequacies of the worksheets which pushed the children into using techniques they did not fully understand. The analysis of bugs was used during the study to ascertain which elements of programming required overt teaching. Three short class sessions were then given, on editing, debugging and subprocedures and recursion. The experimenter intervened to point out other bugs which were not being detected by the children, as they occurred.

These findings support those of Pea and Kurland (1983), that recursion and other programming notions must be explained fully. They also illustrate the need for a correct mental model of the processes carried out by the computer, for children to understand the use of variables, particularly in subprocedures. Though some of the early unstructured research studies suggest that children learn to program without direct instruction, these findings suggest that this is highly unlikely.

9.4 Organisational factors in LOGO learning.

Due to the way the LOGO work was organized in the classroom, children spent differing amounts of time using the computers, from two hours per week for some to forty minutes per week for others. There was also a difference in the amount of supervision that each pair received. Some children had their main computer sessions at other times than when the experimenter was present, and so received very little direct supervision, whereas others were seen every week. A third difference was in the placing of children in pairs to work on the machines. Some of these pairs were more permanent than others. This factor was not considered in any detail in this study, but from monitoring of performance it was felt to be important for successful learning. Various pointers to suitable pairings could be gleaned from the available information.

9.5 Analysis of time spent using the computers.

In total, the children in the LOGO group spent twenty eight weeks in two terms using the computers. They had also spent five weeks in the previous term investigating turtle graphics with the floor turtle, but this is not included in this analysis.

The children recorded the amount of time they spent each week in the back of their notebooks. Many of these records are incomplete, but the completed weeks give some indication of the time each child spent. Weekly averages for each term have been estimated from these. (See Appendix 6). Each week most children used the computer for one session of about forty minutes while the experimenter was there. Otherwise they used the machines when there was a choice of activity, or in free time while other children were involved in extra curricular activities, such as skiing, choir or the school play. Some children were heavily involved in these pursuits, and had correspondingly less time available for extra computer work, so there is considerable variation in the hands-on times.

The average total time spent per child using the computer was 32 hours 30 minutes, over a period of twenty eight weeks. This gives an average time of 1 hour 10 minutes per week. In fact more time was spent in the first term than the second, partly due to the number of other encroaching activities.

	Total	Weeks	Mean
Term 1	21 hrs	15	1 hr 24
Term 2	11 hr 30 min	13	53 min

Table 16a Hands-on time in Phase III.

There were also considerable differences between boys and girls in the amount of time they spent. On average the girls used the computers only two thirds of the amount of time that the boys did. This was consistent over both terms.

	Mean time per week	
	Boys	Girls
Term 1	1 hr 43 min	1 hr 4 min
Term 2	1 hr 4 min	47 min

Table 16b Mean hands-on time for boys and girls.

As there were quite large variations between children, they were each assessed as high (> 90 min), medium, and low (< 60 min) users, from the first term estimates. The distribution of boys and girls in these groups is then shown in Table 16c.

	Boys	Girls	
High > 90 min	8	0	8
Med 60 - 90 min	6	9	15
Low < 60 min	1	6	7
	15	15	30

Table 16c Distribution of boys and girls in high and low user categories.

This difference reflects the fact reported by the teacher that the boys would often choose to use the machines rather than do other activities in free time. Many girls seemed to be reluctant to use them apart from at the set times, even when the boys were out of the room on some other activity. The time recorded by the boys is likely to be an underestimate as they were unreliable at keeping the records.

The time data does not bring out the changes in attitude which were noticeable among several pupils. Three particular children registered a dislike for video games in an early questionnaire to find out about home use of computers, and were a little reluctant to use LOGO. They were all mathematically able, but did not initially recognise the potential of LOGO. By the end of the second term they were all very keen and using the machines at least as often as other children. On the time records, the average time per week for these children did not drop from the first term to the second, though overall the hands-on time for the second term was only two thirds that of the first.

Performance differences

As the girls spent less time than the boys using LOGO, it is interesting to compare their scores on the tests of skills gained from this experience. When the results of the tests of Generalizations, Algebra and Angle estimations were considered, all of which showed significant superiority for the experimental group over the control, the boys and girls in the experimental group had done equally well.

Score	Boys	Girls
> 30	7	6
< 30	9	7

Table 17a Total generalization score

Score	Boys	Girls
12 +	3	3
8 +	9	8
2 +	5	4

Table 17b Algebra score

Score	Boys	Girls
6 +	11	10
2 +	6	6

Table 17c Angle estimation score

There are no differences between boys and girls evident in this data.

Rather than looking at general sex differences, more information ought to be obtained from a comparison of performance of the high and low users, irrespective of sex. The scores of the eight high users and seven low users were compared on the same tests.

Score	High users	Low users
>30	2	4
<30	6	3

Table 18a Generalization score by high and low users.

Score	High users	Low users
12 +	0	1
8 +	3	2
2 +	5	4

Table 18b Algebra score by high and low users.

Score	High users	Low users
6 +	6	4
2 +	2	3

Table 18c Angles estimation score by high and low users.

The low users actually scored better on the generalization test, and the high users on the angles test, but non of these differences are significant. It seems likely that the performance of children on these tests is relatively independent of the amount of time they spent using the computers. This suggests that what the children did on the machines was more important than the amount of time they spent on them. All the extra time was spent without supervision, and there is no systematic record of what the children did then.

9.6 Supervision during LOGO work; the role of the teacher.

The experimenter was present for only one morning per week. Most of this time the children's LOGO work was monitored and particular buggy episodes were recorded on paper. There was little intervention, but where bugs occurred which were not acted upon by the children, they were pointed out and discussed. Most other comments were generally given for encouragement and occasionally challenges were set up.

Though the level of intervention was deliberately low during the monitoring period, it was nevertheless quite important. In the early weeks some children were rarely seen as they had their LOGO sessions on other days. Two main problems arose with unsupervised worksheet use. One was that some programming mal-rules were learned and remained undetected for some weeks. The second problem was related more to the attitude of the children towards LOGO work, and their perception of the purpose of it. Different pupils clearly had different ideas of why they were learning LOGO, and their immediate objectives ranged from getting through the worksheets as fast as possible to creating the most elaborate original pattern. Few saw the principal objective as discovering and extending mathematical patterns and detecting their underlying rules. Small group discussions, held every two weeks perhaps, could have been most useful in overcoming both of these problems.

Intervention in the LOGO work can play a crucial role in setting up the learning environment. The following gives an example of intervention between the experimenter and two girls in the early weeks of using LOGO. They had not yet learned to build procedures, but had learned the REPEAT command and were using it to investigate circles.

The girls drew a circle using

```
REPEAT 180 [ FD 2  LT 2 ]
```

prompt 1 - suggest they try to draw circles of different sizes.

They try various other inputs.

prompt 2 - ask "why repeat 180 times?"

The girls give a description rather than an explanation.
They say they have no idea how to make a bigger circle.

prompt 3 - ask "how do you draw a bigger square?"

Reply "make the numbers bigger - Oh I know!"
They first alter the number of repeats, then comment "no that is not right". Then they try

```
REPEAT 180 [ FD 4  LT 4 ]
```

The circle comes out the same size, but the turtle goes round it twice.

prompt 4 - ask "why does the turtle go round twice?"

This causes some discussion and they show an understanding of the need to reduce the repeat, but do not give a clear explanation of it. They try

```
REPEAT 90 [ FD 4  LT 4 ]
```

This is still wrong size. They try

```
REPEAT 90 [ FD 10  LT 10 ]
```

They say they have tried everything, but it does not work, so they want to go on to play with Sprites.

prompt 5 - say "you can't have tried everything, or you would have succeeded"

They try

```
REPEAT 180 [ FD 1  LT 1 ]
```

This draws a semicircle, so they do it again to give the complete circle, but it is still the same size. Then they try

```
REPEAT 180 [ FD 1  LT 2 ]
```

and succeed in drawing a smaller circle. They are very pleased with it and go on to draw a bigger circle using

```
REPEAT 180 [ FD 3  LT 2 ]
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By the end of this session the girls were able to recognise that the size of a circle depends on the relationship between the size of the forward and turn inputs. The number of repeats also depends on the inputs used. They were probably not able to state explicitly the total turtle trip theorem, but had some elementary understanding of it.

This was a rich learning situation, but the prompts form an important part of it. The functions they performed were:

1) widen the task to general circles;

2) and 4) draw attention to the repeat factors which, because they are not necessary for drawing circles, are often ignored. One technique commonly used by other children is to use REPEAT 1000 and stop the procedure when the circle is complete.

3) suggest generalization from earlier experience, to prevent stalling.

5) persevere with the task which might otherwise be abandoned just short of success.

Had the prompts not been made, most probably these girls would have stopped their investigation and gone on to play with Sprites, as other children were doing. Had this happened, this learning episode would have been less successful, and so less motivating in the future. They might have returned to investigate circles at a later time, and succeeded with them, but the impact of .the learning might have been less. As it was the experience was successful and they learned a lot from it. They were more likely to return to it, because they were successful.

9.7 Success in LOGO, pairing and amount of supervision.

Ten particular children were infrequently seen at the beginning of the year. Subsequent observation of their LOGO performance revealed that five were generally successful and apparently thinking mathematically, whereas the other five were less successful. The immediate difference between the two groups lay in their mathematical ability. The more able children could do work without supervision, whereas the less able apparently needed more attention, particularly in the early weeks. There were other differences. Some worked in constant pairs most of the time, whereas others changed partners quite frequently. None of the less successful ones was happily paired, two of them partnering particularly difficult pupils, and two of them having no fixed partners at all. Only one out of the other five was unpartnered, and she probably did less well than might have been predicted from her mathematics score. Usually the other four worked as two pairs and both made very good progress.

Although good ability in mathematics generally predicted good LOGO performance, the converse was not true. Some children who scored badly in the mathematics pre-test were eventually very successful in LOGO work. They were distinguished from the less successful by having compatible partners, so that both were required to contribute to the joint activity on the computer. Some of these worked gradually through the worksheets, always staying within their comprehension level. They gradually came to understand the programming and mathematical notions involved throughout the two terms. Others showed no apparent understanding until sometime in the second term, when their experience suddenly became meaningful. The ones who apparently gained least from the experience were those who, on finding a difficulty with a worksheet, would leave it and go onto the next one (H and Jn, S and K, M and R).

It is suggested that the supervisor is less necessary when the children are working in consistent pairs, because each child acts as supervisor for his partner. More evidence on how children work together on the computers, the nature of their interaction, and the learning effects of their work is currently being investigated by Hoyles et al (1985). Such close and constant observation was outside the scope of this particular study.

9.8 Summary of Chapter 9

Other results of the Phase III study were examined, concerning programming and organizational matters, and their likely effect on the learning outcomes. Overall the children in

Phase III achieved a higher level of programming ability than those in Phase II, and more children reached the criterion level of being able to use inputs in procedures and subprocedures, when necessary. The type of bugs which occurred in the children's work were discussed, together with the common debugging techniques used. It was concluded that some formal instruction in programming was desirable to prevent the children from picking up incorrect rules.

As the children could choose the amount of time they spent on the computers, this was analysed. It was found that boys spent on average half as much time again as girls. There is no record of what they did in this additional time, and there were also no noticeable differences in either programming or mathematical understanding between the boys and the girls. The role of the supervisor was mentioned as contributing to the children's perception of the purpose of the LOGO work. Pairing the pupils to work on the machines was generally considered to be an improvement on working individually, partly because the partners could provide some of the stimulation between them which a child on his own might otherwise require from a supervisor.

CHAPTER 10

CONCLUSIONS

This study set out to look at LOGO as a computer modelling approach to learning mathematics. The main purpose was to investigate the quality of learning which could be achieved. This involved assessing the mathematics which could be made available to children through LOGO. It also considered the constraints on exploiting this learning situation. The children were observed using LOGO and tests were used to assess the transfer of learning from the LOGO environment to other areas of mathematics. The effects of ability and maturity as well as the need to learn programming were considered. Other factors which affected the learning environment were also discussed.

10.1 Mathematical learning.

All three phases of the study revealed that a lot of mathematics was intrinsic to turtle geometry. The first two studies were used to try out different tasks to identify activities which were likely to exploit this learning potential. The investigation of circles and polygons, for example, was found to be particularly fruitful, not only for investigating geometric notions concerned with angle and shape, but also in developing mathematical strategies.

Evidence that children were learning such things from their LOGO investigations came from the test results in Phase

III. Though initially performing at the same level in mathematics as the control group, after using LOGO for twenty eight weeks the experimental group gave superior results on tests of angle estimation, variables and generalizations. Because of the nature of the tests these results support the notion that children were constructing new mental schemas of angle and variable from their LOGO work, and through this learning were also developing mathematical strategies which could be applied to other topic areas.

LOGO, while providing the learning environment for developing schemas, was also shown to be useful as a diagnostic tool, to enable the thinking process of the child to be observed. As the children worked they were interacting with the computer and making explicit their understanding and assumptions. Thus the type of bugs which occurred illustrated their thinking. By careful observation of their interactions, the existence of schemas could be detected. Following the performance of the same children over time also enables the process which they pass through in developing new schemas to be analysed. This makes a contribution to our knowledge both of how particular schemas may be built by different children, and how particular children develop their mathematical understanding.

Analysis of the children's work in the Phase III study revealed how their thinking developed during the experimental period, particularly in the understanding of angle which was basic to other mathematical notions in turtle geometry and investigated by all children.

10.2 Transfer of understanding to a non-LOGO context.

It is always difficult to assess transfer in learning. It is essential that the tests used should be appropriate, and with an educational study there are so many uncontrolled factors involved that may interfere with the final outcome. This study was not a classically controlled experiment. The control group was used as some measure of natural development and learning over time, but the different class teachers and the difference in amount of time spent using the computers were clearly confounding factors. Nevertheless, the test results indicate that the LOGO children did learn transferable knowledge from their experience in the areas of mathematical strategies of generalization and abstraction of rules, important in problem solving, and in basic algebra.

The evidence from the two LOGO performance tests and the records of children's progress shows that, while some children progressed steadily throughout the year, others were well into the second term before they began to make sense of the LOGO experience in terms of mathematical learning. It could be argued that one or two children were still to get to this point at the end of the study. The test results indicate that almost all the children had constructed angle schemas, and about half of them had constructed some variable schema by the time of testing. It seems reasonable to expect that with continuing LOGO experience more children would gain this understanding in time.

The development of the variable schema provides a foundation for formal algebra. The concrete analogy of inputs in procedures and understanding of their operation gives children an experiential base to refer back to when being introduced to more formal notation. This is a most encouraging finding, as understanding of variables is found to be very difficult for the majority of children (Hart 1981). The persistence of this effect would need to be tested by following the mathematical development of these children over several years. Similarly, the meta-level learning of mathematical strategies through LOGO experience is an interesting finding, suggesting that qualitatively different learning outcomes in mathematics can be obtained. It is not clear how much this result could be attributed to the particular approach to LOGO learning used in this study, and how much it is a general finding of using LOGO to investigate mathematics. Theoretically, the latter should be the case, but this would need to be replicated on a larger scale. Again, the long term effects of the experience would also need to be investigated in a follow up study.

10.3 The level of programming required.

It seems that the need to learn programming did not pose any constraints on mathematical learning. From the first phase of the study it was found that the mathematical notions were generally more difficult than the programming ones. Almost all children could build and run simple procedures, which was all that was required of them at the time. Even children whose programming was least developed were able to use direct commands to investigate and build up an understanding of angles. Also they were able to investigate directed number without requiring any programming knowledge. This was not emphasized in this study, but Thompson (1984) has successfully used LOGO-type tools specifically for this purpose.

The level of programming reached by most of the children in the second and third phases of the study was not high. In Phase II, after 23 hours about one third of the class were able to use procedures with inputs when appropriate. In Phase III, after 32.5 hours two thirds of the class reached this level. They were also familiar with the use of subprocedures. Most of the remaining pupils in both classes could build and use simple procedures, though in each class there was one child who failed to reach this standard. A certain amount of instruction was found to be desirable to ensure that children did not pick up incorrect notions of programming.

10.4 Ability and maturity.

Developmental levels clearly affect what children are likely to learn from their experience, because each child starts from a different base of knowledge and abilities. Although the tests of developmental levels gave no reliable results, comparison of the 9 year old and 11 year old children revealed that the older ones were better able to exploit the learning situation. The 11 year old age group was chosen for the final study because they were approaching the stage of more formal and abstract mathematics. The logical reasoning abilities they possessed probably enabled them to get more out of the investigating experience than younger children would have done.

There was a wide ability spread in the classes using LOGO. Some children in all three phases of the study were less successful at developing their mathematical ideas than others. Most of them were amongst the less able in the class, but other children of similar ability were very successful using LOGO.

10.5 Unmeasured effects of LOGO experience.

There were several important effects of LOGO experience which were not measured in this study. The two class teachers of the second and third phases both made spontaneous comments on the increased confidence and motivation towards classroom mathematics shown by many of the children. In general, success in LOGO work seemed to have convinced the children that they were able to do mathematics. They were therefore more prepared to try new problems, and to spend time working something out which they would previously have left as too difficult. This was most noticeable among children who had previously performed badly in mathematics, but also affected the attitude of others, encouraging them to look for underlying rules and therefore expend more effort finding a solution. This effect was also noted in earlier studies (Howe 1980, Milner 1973).

The motivational effect showed up in the general mathematics attainment scores of some of the children. In Phase II, seven children showed general improvements, of whom six were girls, but in the third phase this sex difference was not in evidence. Of the ten children who showed similar improvements in mathematics in the later study, six were boys and four girls.

It seems that the important factors which determine success from LOGO work might relate to the way children perceive the task. In both phases the pupils who showed least improvement were ones who tended to skip worksheets and leave things which they found difficult. Some boys, in particular, tended to act as though they were in competition with other boys for producing spectacular patterns. By contrast most girls were more conformist and more conscientious in finishing the projects they had started.

It is suggested that for a child to be successful in developing mathematical thinking through using LOGO, he must feel himself to be in full control of the computer and understand how the different effects are created i.e. he must work within his level of understanding. The importance of this was recognised at the end of the second phase, so some effort was made by both the experimenter and the class teacher to prevent the boys in the third phase from getting ahead of themselves. This may have contributed to the improved results in the third phase. However, more research needs to be done on the child's perception of the task to verify this.

10.6 Evaluation of the contribution of LOGO to mathematics education.

It might be argued that other approaches could be used to teach each of the elements, angle, variables and strategies, to the control group, to obtain the same results. It is not obvious that as good results would have been achieved. The LOGO approach has various advantages which make a unique contribution to the possible quality of learning. Classroom methods for mathematics teaching all too often result in rule-based learning rather than schematic learning, as was found by the Chelsea College work, the surveys of adult's mathematical abilities and the results of the angle test in this study. Mathematical aids and investigative approaches are therefore called for, particularly to get over difficult concepts such as variable. Mathematical strategies are even more difficult to teach, and again necessitate an investigative approach.

When the children in Phase III were using LOGO in the classroom they were drawing pictures and patterns, setting and solving problems, such as how to get a roof onto a house, and investigating the geometry of angles. In some activities, such as drawing polygons, they may have been looking for patterns, making conjectures and testing them out, applying knowledge gained in one field to another. Throughout they were working with partners, discussing their next moves and justifying their ideas. In fact they were investigating mathematical problems in the way recommended by the Cockcroft Committee (1982).

As an investigative approach LOGO has distinct advantages over most others. Once set up, it runs without requiring enthusiasm generated by the teacher. Strong motivation is produced by the LOGO work itself, and continues by the interactions between children using it. It has additional confidence boosting attributes because of the immediate and uncritical interaction between the learner and the computer. The third advantage of LOGO over other investigative techniques is that it provides a suitable learning environment for children of all abilities.

The results of this study indicate that a LOGO approach to mathematical learning could make a strong contribution to education in primary and early secondary schools. It may provide an opportunity for children to learn mathematics through building schemas, rather than learning rules, thus contributing fundamentally to their mathematical thinking abilities, and at the same time providing a diagnostic tool for the teacher to monitor her children's understanding.

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APPENDIX 1

CONTENTS OF THE WORKCARDS USED WITH THE FLOOR TURTLE

1. Elementary commands of FORWARD, BACKWARD, LEFT and RIGHT. Used to build letter shapes with 90 degree turns.

2. The same commands used to draw squares of different sizes, and other square shapes.

3. Introduction of 45 degrees in making rounded letter shapes.

Whilst working on these three worksheets the children were:

a) becomming familiar with the button box;

b) getting commands in the correct order, for example using

RIGHT 90

FORWARD 20

instead of putting the commands together by mistake as

RIGHT 20

c) learning to check left and right turns by walking out the turtle moves, (playing turtle).

4. Triangles: Using 120 degree turns and making patterns out of triangles.

5. Building simple procedures on the three procedure buttons.

6. Procedures: building squares and triangles and making patterns by rotating them.

Using these workcards the children built many procedures, to learn how the procedure is built and how it can be used. They found that procedures remain to be used over again until they are overwritten. They also began making patterns by rotating shapes in a regular manner.

7. The REPEAT button. Using repeat with procedures to make patterns. The very simple procedure, composed of one forward command and one turn, was used to great effect using repeat until it formed a closed pattern. Some children investigated the number of repeats required for patterns using different angles; others tried to build circles and stars of different sizes.

8. Drawing spirals.

9. and 10. Introduction to polygons.

The last three cards were not used by all pupils as they were found to be too directive. Instead most children spent longer time using repeat with simple procedures and tried out drawing projects of their own.

APPENDIX 2a

DETAILED SCORES OF PRIMARY 5 CHILDREN

CHILD	B1	ACT 4	INPUT	MIRROR	SHAPE	TURTLE
Sa	130	8	1	1	1	G
Ai	127	8	1	1	1	G
Da	123	7	1	1	1	VG
Du	121	7	1	1	1	G
Ad	121	7	1	1	1	M
Ga	119	4	1	1	1	G
Tm	118	7	1	1	1	VG
Nk	116	7	1	1	0	M
Ch	115	7	1	1	1	VG
Mi	111	4	1	1	1	VG
Jo	108	6	0	1	0	P
Kt	107	6	1	1	1	G
Cf	106	4	1	1	1	VG
Ps	103	7	1	1	1	G
Ru	101	5	0	0	0	P
Kh	99	6	1	1	1	G
Pa	97	5	1	1	1	VG
Cd	94	5	0	0	0	P
Ha	94	3	0	0	0	P
Gr	92	6	1	1	1	VG
Su	90	4	1	1	0	M
Lu	89	2	1	0	0	M
Ma	86	1	1	1	0	M
Pw	72	2	1	0	0	P
Nc	72	2	nt	nt	nt	P
St	nt	7	1	0	0	P
Zo	nt	2	0	0	0	P

B1 Standardized mathematics attainment test B1.

ACT 4 Score out of 8 on classification activity 4.

INPUT Ability to differentiate between the effects of inputs to forward and turn commands: 1 - can do so; 0 - cannot do so. nt - not tested.

MIRROR Ability to recognise the mirror image effect of reversing left and right commands.

SHAPE Understands the conservation of shape.

TURTLE General rating on turtle performance: VG - very good; G - good; M - moderate; P - poor.

APPENDIX 2b

DETAILED SCORES OF PRIMARY 6/7 PUPILS USING THE FLOOR TURTLE.

	CHILD	C1	ACT 4	SKW	TURTLE
GIRLS	Su	130	6	2a/b	G
	Me	121	8	2a/b	G
	Ca	111	5	2a/b	VG
	Sa	114	6	2b	G
	Ly	113	4	2a/b	M
	Sn	118	5	2a/b	G
	Va	116	2	2a/b	VG
	An	112	5	2a/b	G
	Sk	108	6	2a/b	VG
	Ka	107	7	2a/b	VG
	Ke	112	6	2a	P
	Pk	104	5	2a/b	M
	Ad	97	6	2a/b	G
	Lo	101	5	2a/b	G
	Mi	104	6	2a/b	G
BOYS	Mn	134	8	3a	VG
	Ja	120	7	2b/3a	VG
	Sm	118	4	2a/b	M
	Ga	115	6	2b	M
	Gh	115	8	2b	G
	Ro	111	7	2a/b	M
	Kh	104	4	2a	M
	Ia	111	3	2a/b	P
	Si	104	5	2a/b	VG
	Po	108	4	2a	G
	Pc	94	5	2a/b	P
	Jo	81	4	2a/b	P
	Mh	nt	6	2b	VG

C1 score on standardized mathematics attainment test C1

ACT 4 score out of 8 on classification activity 4 (Gal-Choppin)

SKW score on Shayer scientific tasks II.

TURTLE general turtle performance: VG - very good; G - good; M - moderate; P - poor.

APPENDIX 2c

COMPARISON BETWEEN PRIMARY 6/7 AND PRIMARY 5 CHILDREN ON TWO TESTS OF DEVELOPMENTAL LEVELS.

Shayer Task I on spatial relationships.

Piagetian levels	2A	2A/2B	2B	Total
Primary 5	4	10	12	26
Primary 6/7	4	8	16	28
	8	18	28	54

Comparison based on Activity 4 (Gal-Choppin)
Number of children having the given score.

Score	Primary 5	Primary 6/7
1	1	0
2	4	1
3	1	1
4	4	5
5	3	7
6	4	8
7	8	3
8	2	3
	27	28

No significant differences between the two groups are discernible from this data.

APPENDIX 3

The following are examples of the tests used in phase II and phase III of the study to assess mathematical understanding. They are adapted from published tests to suit the ability levels of the children in the study. The administration and marking of the tests followed the guide lines set out by their authors.

- Appendix 3a - Chelsea Test I on reflections and rotations

- Appendix 3b - Estimation of Angles

- Appendix 3c - Chelsea Test II on algebra and variables

- Appendix 3d - Generalization test used in phase II only

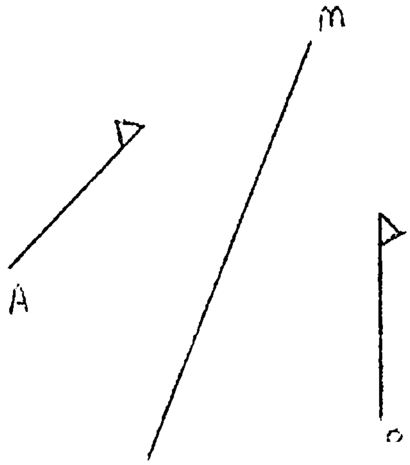
- Appendix 3e - Two generalization papers used in phase III.

NAME ....   DATE ....

Imagine folding this paper along the line m.

the flag A would go exactly onto the flag B.

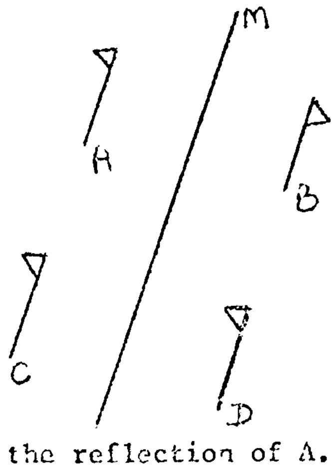
We are going to say
A is REFLECTED onto B, and
B is the REFLECTION of A,
and we are going to call
the line m the MIRROR LINE.



Practice Question.

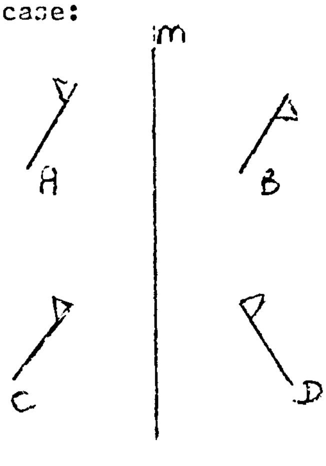
For each of these diagrams say whether B is the reflection of A and whether D is the reflection of C.

Draw a circle round the YES or the NO in each case:



B is the reflection of A. YES NO

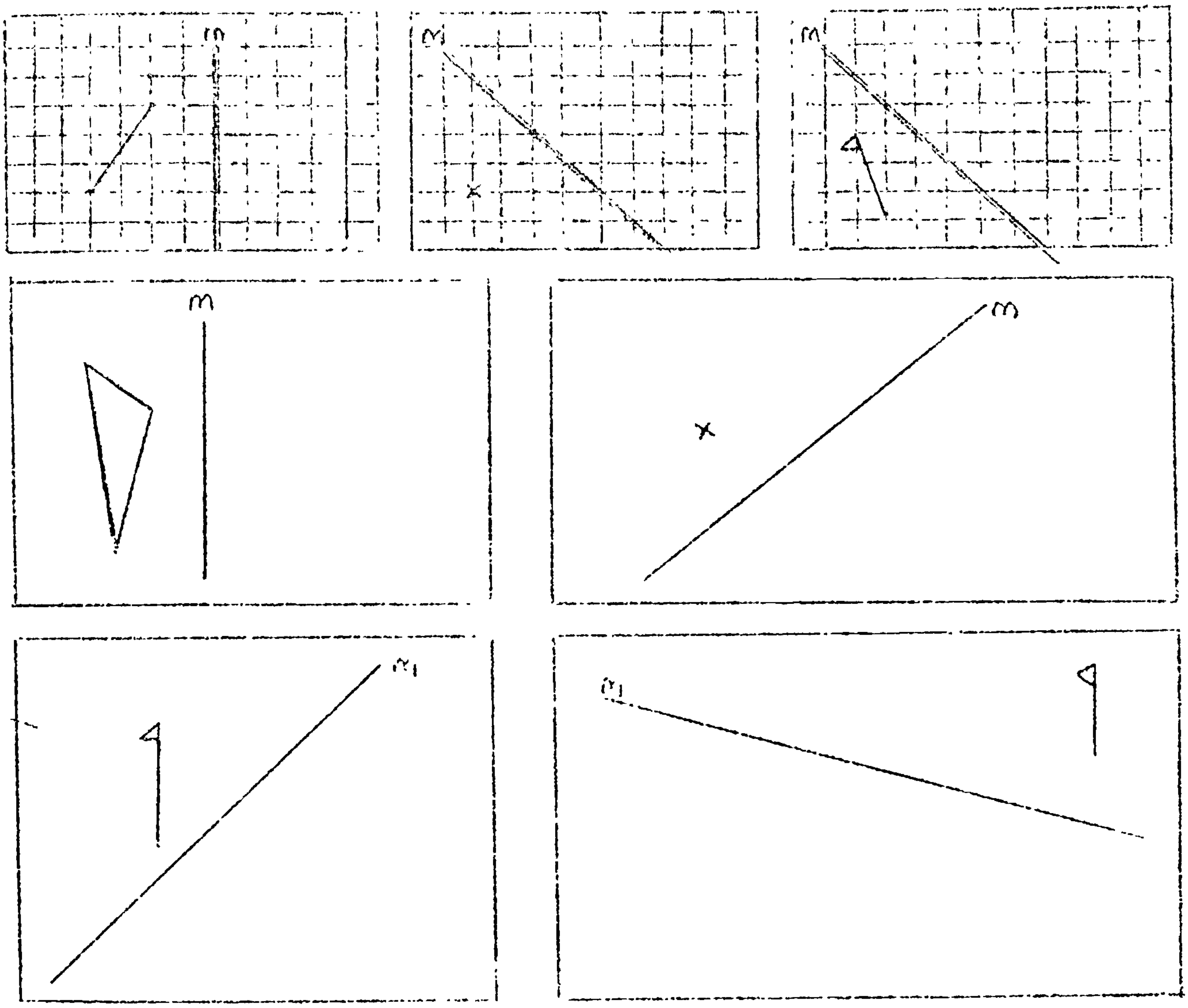
D is the reflection of C. YES NO



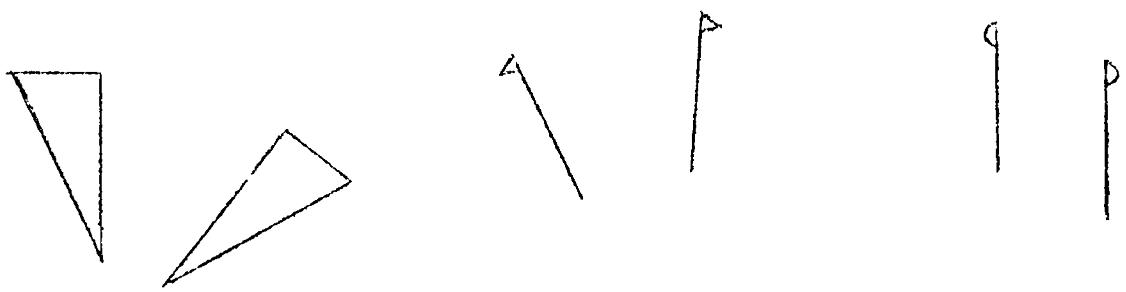
B is the reflection of A. YES NO

D is the reflection of C. YES NO

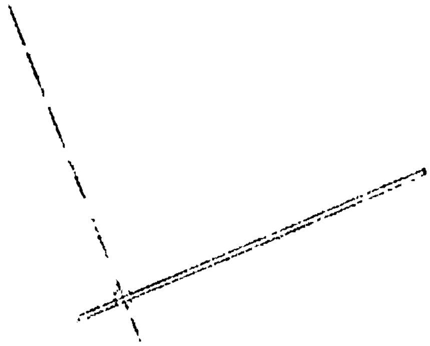
1. Reflect in each mirror line m, and draw your answers freehand.
   DO NOT USE A RULER.



2. Sketch the mirror lines. If you think there is no mirror line, say so.



In this diagram imagine the solid line is a stick; imagine we put a pin through the dot and imagine we now ROTATE the stick through a ¼ turn anticlockwise



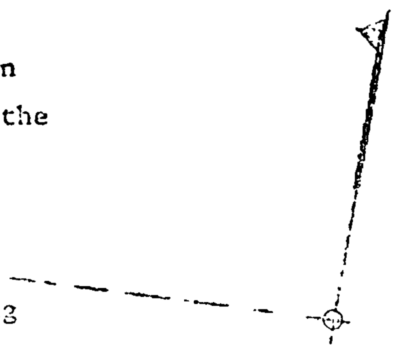
then the stick will end up on the broken line

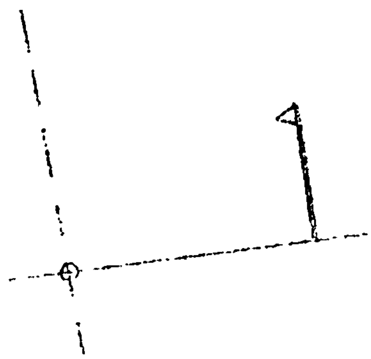
Practice Question.

Rotate each of these sticks through a ¼ turn anticlockwise, using the dot as the "centre of rotation".

Sketch where each stick ends up.

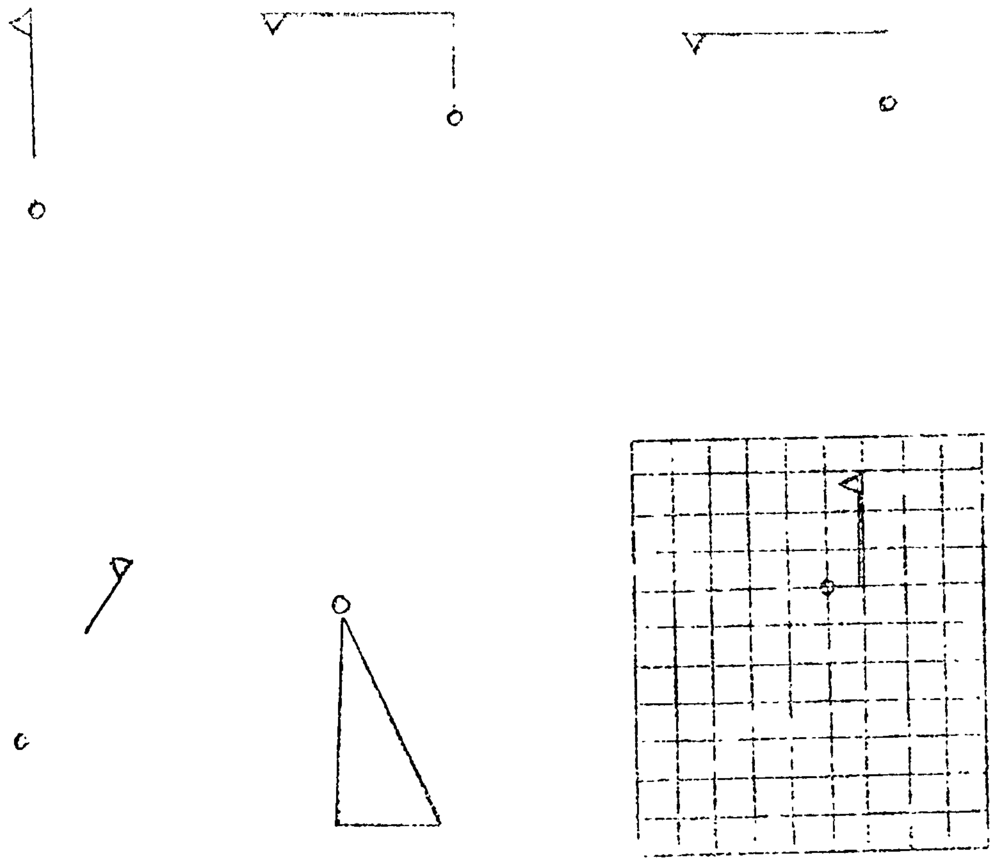
Rotate each of these flags through a ¼ turn anticlockwise, using the dot as the centre of rotation.



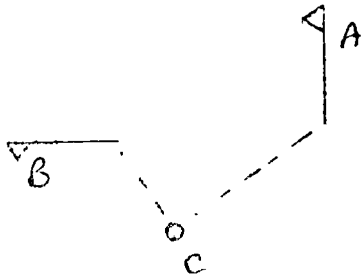


Sketch where each flag ends up.

3. Each of these shapes is to be rotated through a ½ turn anticlockwise using each dot as the centre of rotation.
Sketch where each shape ends up. (DO NOT USE A RULER.)



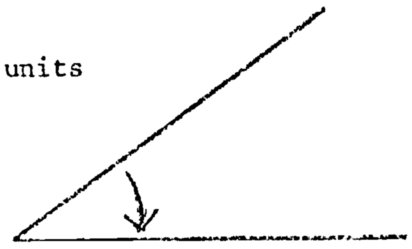
4. This flag has been rotated through a ¼ turn anticlockwise, so A ends up at B.



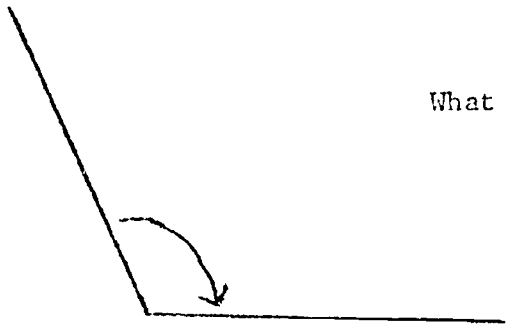
Explain why C is NOT the centre of rotation.

TASK B

If this angle is 40 units

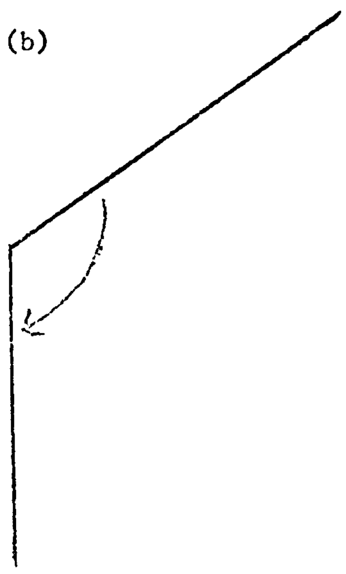


(a)

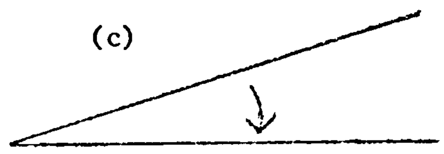


What would you estimate these to be?

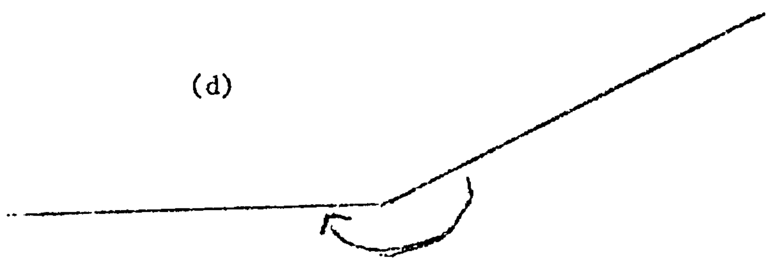
(b)



(c)



(d)



CHELSEA TEST 2

NAME ____________________

DATE ____________________

Trial Items.

1. What number does n + 4 stand for if n = 3 ________

   What number does 5n stand for if n = 2 ________

   What number does p + 3 stand for if p = 1 ________

   What number does 3a stand for if a = 3 ________

2.

x --> 3x	y --> y + 3	n --> 7n
2 --> 6	5 --> 8	2 -->
5 -->	4 -->	
p -->	x --> .	

1. 3 added to n can be written as $n + 3$

Add 3 on to each of these

5 _____

$n + 4$ _____

$2n$ _____

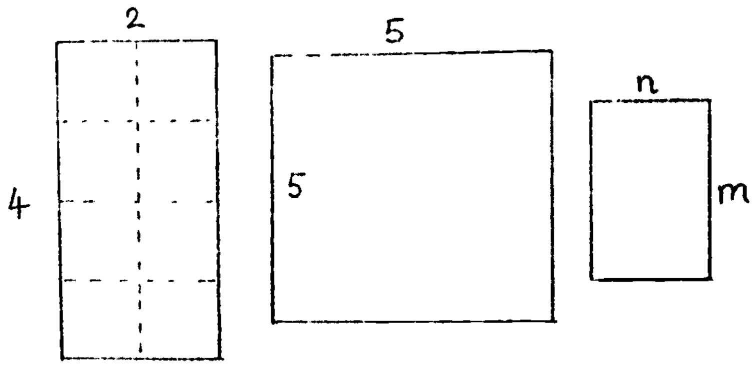
2. If $a + b = 47$ $a + b + 2 =$ _____

If $n - 246 = 763$ $n - 247 =$ _____

If $e + f = 8$ $e + f + g =$ _____

3. What can you say about a if $a + 5 = 8$ _____

4. What are the areas of these shapes ?

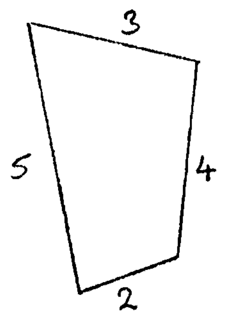


_____ _____ _____

5. What can you say about p if $p = q + 3$

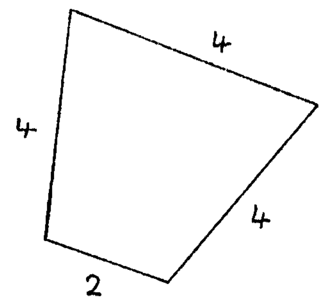
and $q = 2$ _____

6.



Tne perimeter of this shape

is 5 + 3 + 4 + 2 = 14

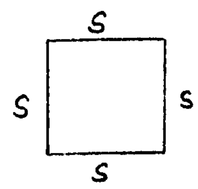


Work out the perimeter

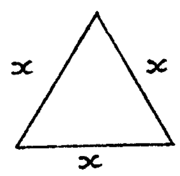
of this shape _______

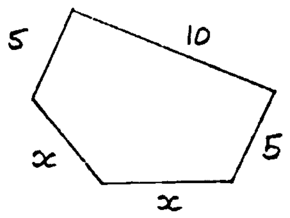
7. Tnis square has sides of length s

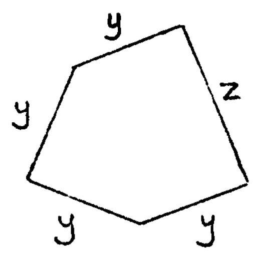
so its perimeter is 4s



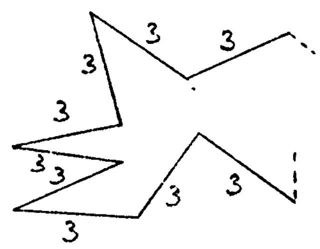
Write down the perimeter of each of these shapes







Part of this figure is not drawn.
There are n sides altogether, all of length 3.



8. What can you say about r if $r = s + t$

and $r + s + t = 30$ _____

What can you say about p if $p + q = 12$

and p is less than q _____

NAME ----------

DATE ----------

A1.

0 + 1 = 1
1 + 2 = 3
2 + 3 = 5
3 + 4 = 7
4 + 5 = 9
5 +   =
  + 7 =
  +   =
  +   =

Continue this adding pattern by writing the missing numbers in the blank spaces.

A2. Write down anything you notice about the <u>answers</u> to the additions in each line of the pattern.

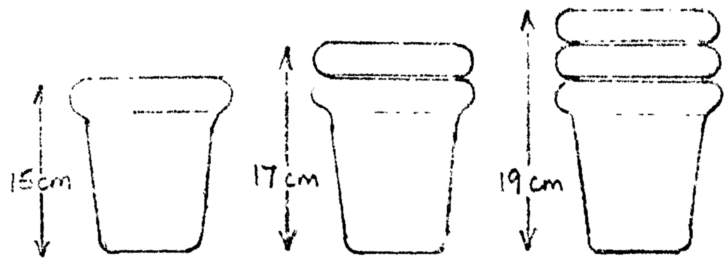
What do you notice about the other two numbers in each line?

A3. John says that this line belongs to the same pattern. Is he right? Explain. 14 + 16 = 30

A4. This line belongs to the pattern ___ + ___ = 57

Find the missing numbers and write them in. Show your working.

B1. A gardener has some flower pots which are exactly alike. They are 15 cm tall and can be stacked like this:



How high will a stack of 4 pots be?

How high will a stack of 5 pots be?

B2. What is the largest number of pots in a stack that will stand upright between two shelves which are 34 cm apart?

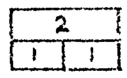
B3. A shelf has space for 5 stacks of pots and room for o.. no more than 28 cm high.
What is the largest number of pots that can be placed on the shelf?

B4. The gardener wants to store 100 pots on another shelf which has room for objects no more than 30 cm high. How many stacks of pots will be needed if each stack is as high as possible.

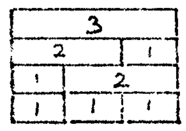
You have a lot of number rods of lengths 1, 2, 3, 4, 5 and 6 cm.

A length of 1 cm can be made in only 1 way.

A length of 2 cm can be made in 2 ways.



A length of 3 cm can be made in 4 ways.



How many ways can a length of 4 cm be made up?
Draw them all and write down how many there are.

C2. Write down how many ways you think there will be of making up a length of 5 cm. How did you get your answer?

C3. Write down how many ways you could make up a length of 6 cm and the rule you used.

GENERALIZATION

Name

Class

Date

A. Number sentences

2	+	3	=	5		Write two more number sentences to show how the pattern continues.
4	+	5	=	10		
6	+	9	=	15		
8	+	12	=	20		

Here are two number sentences taken from the same pattern. Can you finish them?

16 + ... = ...

... + ... = 60

29 + ... = ...

Tim says this number sentence is taken from the same pattern, but Jane says he is wrong. Who is right? How do you know?

Is there a rule to tell you what numbers can be in the last place? Describe it.

If you know the first number in one of the number sentences in the pattern can you find the others?

If you say yes, explain how to find the other numbers.

B Football

At the end of the football season the following facts are known about two teams D and C.

a) Both teams have the same number of draws.

b) C has beaten D twice.

c) D has won two more games than C.

(Assume the old scoring applies: Win (2 points); Draw (1 point); Lose (0 points).)

1. Which of D and C finished with more points? ☐

2. Tick each one of (a) (b) (c) which you need to say this and put a cross against any you did not need to know to answer the question.

Tick or Cross

(a) ☐

(b) ☐

(c) ☐

C. Odd and Even

O stands for any odd number.

E stands for any even number.

Write down what you get in the following addition sums.

O + O =

E + E =

E + O =

O + O + O =

O + O + O + O =

O + O + E + O =

O + O + O + O + O =

Investigate what you will get if you add together various numbers of odd numbers. Write down what you find out.

Extend your investigation to cover a mixture of even and odd numbers being added together. Write down your conclusions.

D. Game of 25

RULES.

Player 1 picks a number from the list 1, 2, 3, 4, 5, 6.

Player 2 picks a number from the same list and adds it on.

The game continues like this with each player taking turns. The player who makes the total of 25 is the winner.

1) Explain why the player who makes the total 18 can always win. (Give your reasons in full.)

2) Find all other numbers (less than 18) from which a player can always win.
Show your working and explain your reasons.
If you think there are none, explain why.

3) Can the player going first always win?
Explain why or why not.

GENERALIZATION 2

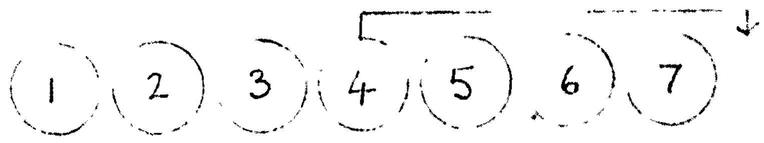
NAME:

CLASS:

DATE:

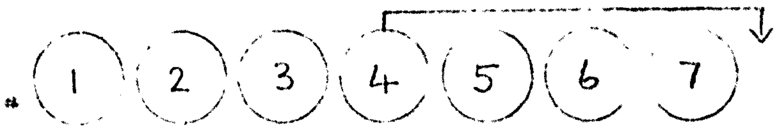
E. Move along

Here is a row of counters. In each move the counter in the middle moves to the right hand end.

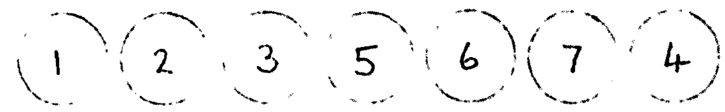


Look at these moves

Start position

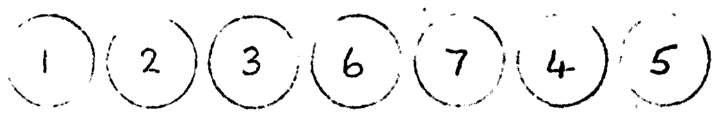


After 1 move



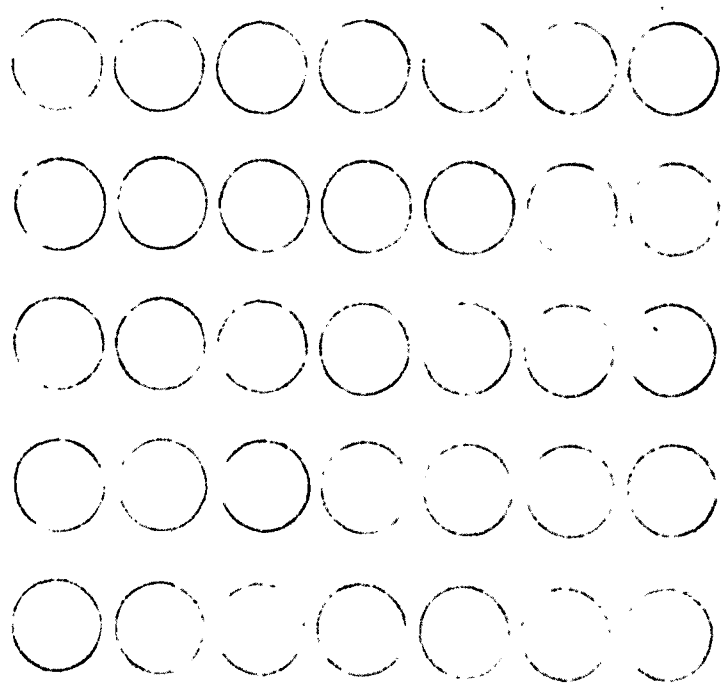
Counter number 4 has moved.

After 2 moves



Counter number 5 has moved.

Find out what happens if you continue to move the counters in the same way. Use this space to draw the positions.



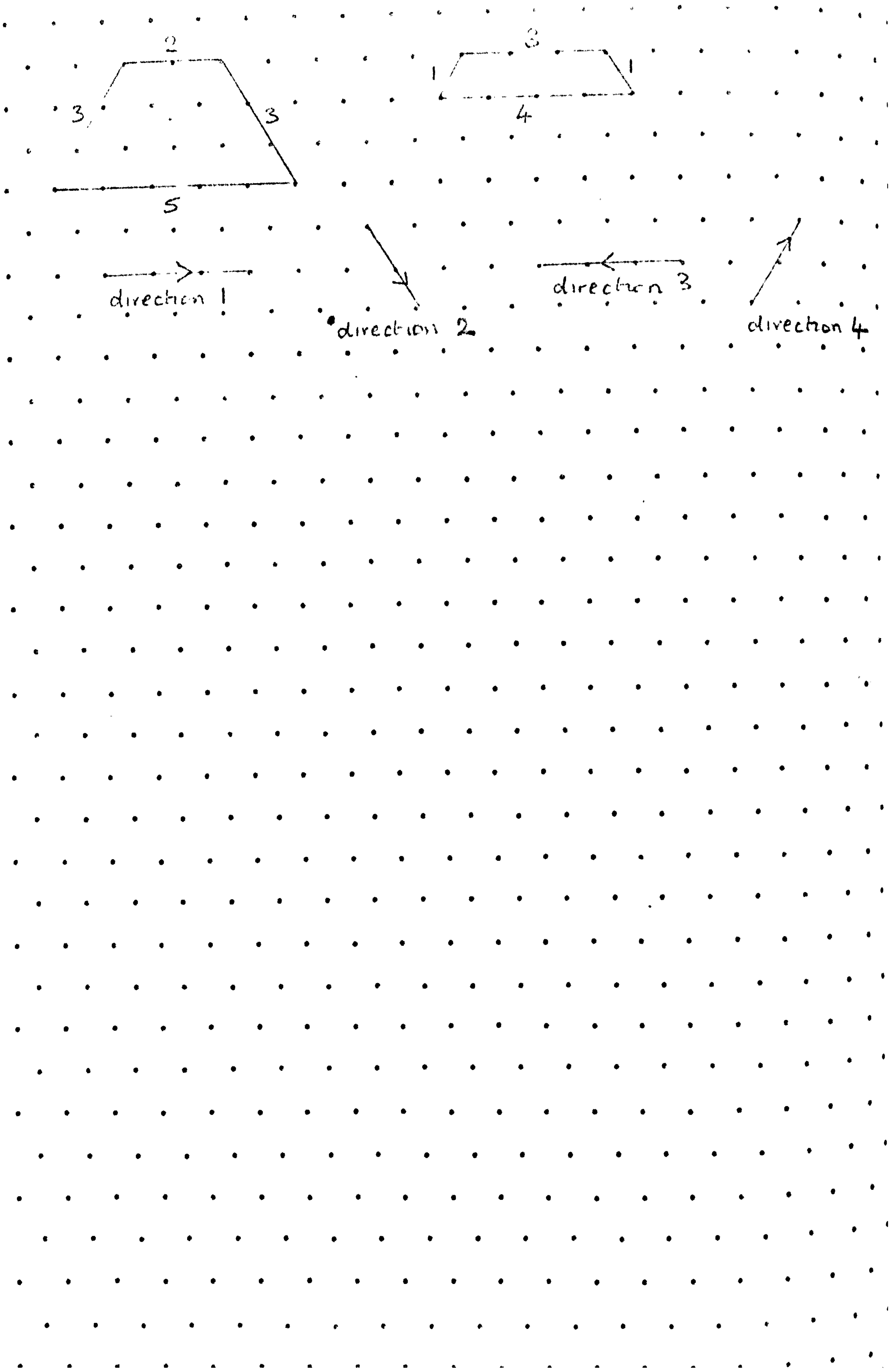
Four children are asked to continue making moves in the same way. When they are asked to stop they each write down the position they have reached.

John's position	1	2	3	6	7	4	5
Susan's position	1	2	4	5	6	7	3
David's position	1	2	3	7	4	6	5
Mary's position	1	2	3	4	5	6	7

If you think any of the children made a mistake say what is wrong with the position they have written down.

Can you see any rules which tell you whether a position is possible or not?

If so state them as clearly as you can.


2
3
3
5
3
1
1
4
direction 1
direction 2
direction 3
direction 4

G. Roofs

Roofs can be drawn in different shapes and sizes, using the dots provided. The first one drawn above is a 2 3 5 3. The second is a 3 1 4 1.

(The first number tells you how many units to draw in direction 1, the second in direction 2, the third in direction 3 and the fourth in direction 4.)

1) Draw a 2 2 4 2 and a 4 1 5 1.

2) Try to draw a 3 2 5 1 and a 1 4 3 4.
Explain what happens.

Use the dots for trials in the following questions.

3) Find some rules which enable you to tell for any four numbers, without drawing, whether you can draw a roof from them or not. Explain why these rules work.

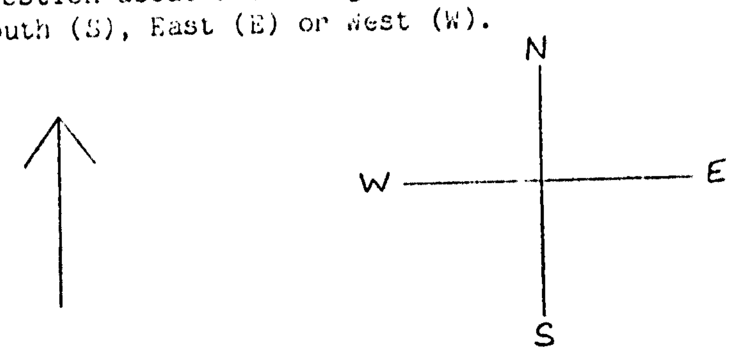
RULES:

EXPLANATIONS:

4) If we call the four numbers a, b, c and d respectively, can you express your rules more simply?

F. ARROW

This is a question about a turning arrow. It is free to point North (N) South (S), East (E) or West (W).



Two changes P and Q can be made to this arrow.

P turns it round so that it points in the opposite direstion.

Q turns it through a quarter turn in a clockwise direction.

It always starts pointing North.

If we do PQ (P then Q) to the arrow. After doing P it points S. Then we do Q which turns it from S to W. So PQ has turned it from N to W. (The second move is done from where the first move leaves the arrow).

a) Which way does the arrow point after doing Q^2 (Q twice)?

b) Which way does the arrow point after doing the following changes?

(1) QP

(2) Q^3P

(3) $PQPQ^2P^2Q$

(4) $P^3Q^2P^{19}Q^{25}$

c) Can you see any rules for solving problems like (b) part 4 without having to do all the movements?
State these rules as clearly as you can.

APPENDIX 4a

PHASE II RESULTS

NAME	REF & ROT pre	REF & ROT post	ANGLE	VARIABLE pre	VARIABLE post	MATHS pre	MATHS post
GIRLS							
Su	14	15	7	9.5	8	96	85
Me	11	12	6	6	11*	88	82
Ca	13	10	5	7	11*	72	88*
Sa	6	10*	6	10	11	78	83*
Ly	11	16*	8	10	9	76	73
Sn	8	12*	8	4	6	86	70
Va	12	9	6	8.5	8	84	73
An	2	4	4	6	7	76	75*
Ka	2	11*	8	5	7	64	73*
Ke	9	13*	nt	7	nt	74	57
Pk	4	3	7	7	5	58	63*
Ad	3	9*	6	6	7	46	65*
Lo	6	11*	4	4.5	5	52	50
Mi	8	7	6	5	5	64	33
BOYS							
Mn	15	14	4	12	12	98	97
Ja	9	14*	6	7	13*	88	90*
Sm	10	14*	6	11	10	84	78
Ga	7	12*	5	8	8	80	73
Gh	10	10	4	8.5	7	82	77
Ro	9	5	5	5.5	7	74	70
Kh	7	4	4	3	5	62	55
Ia	1	6*	0	6	7	72	60
Si	7	11*	5	6.5	7	60	52
Po	10	8	6	8	6	66	50
Pc	3	12*	6	3	5	44	nt
Jo	3	3	0	1	0	24	17

REF & ROT scores out of 16 on tests of reflections and rotations. (Chelsea I)

ANGLE score out of 8 on angle test.

VARIABLE score out of 16 on test of variables. (Chelsea II)

MATHS percentage raw scores on attainment tests C1 (pre) and EF (post).

* showing improvement in post test scores.

APPENDIX 4b

IMPROVEMENTS IN MATHEMATICS ATTAINMENT OF PHASE II GIRLS.

A comparison of the scores on the EF test on LOGO related questions and other questions was made for the 7 pupils who showed relative improvements on this test. The average percentage correct were

LOGO questions	85.7%
non-LOGO questions	73.4%

This gives a ratio of 1.167 for success rates of LOGO against non-LOGO questions.

For comparison, the results of the control group in the Phase III study on the LOGO and non-LOGO questions was taken.

Control group

LOGO questions	60.5%
non-LOGO questions	48.2%

This gives a ratio of 1.256 for success rates of LOGO against non-LOGO questions.

This shows that the LOGO children in Phase II did relatively LESS well on the LOGO questions than the control group children.

It thus seems likely that the improvements in their mathematical performance was due to a general motivating effect on all areas of mathematics.

APPENDIX 5

WORKSHEET CONTENTS

This gives a summary of the contents and purpose of each worksheet in introducing new programming notions or illuminating particular mathematical concepts or strategies. The full set of worksheets used in Phase III of the study is also given below.

(1) Introduction to simple drawing commands with suggestions for the drawing of "square" designs.

LOGO: Introduction to the keyboard and simple commands.

Maths: Estimation of size and angles.

(2) Additional drawing comands and drawing practise, including the rubbing out of lines.

LOGO: Elementary debugging of drawings.

(3) Introduction of the command HOME with drawing practise and an exercise in using angles of different magnitudes.

Maths: Conservation of angle, additive properties of angles, and the equivalence of LEFT Q and RIGHT 360 - Q turns.
The relationship between the number of regular spokes in a wheel and the angle between them.

(4) The building of simple procedures is introduced step by step with an example. Suggestions are then made for children to build their own procedures and to use the REPEAT command.

LOGO: Introduction to the editor and REPEAT command.

This was designed to bring out the following points.

a. Once built, a procedure can be run at any time.

b. More than one procedure can be used at the same time.

c. REPEAT can take more than one command.

Maths: Introducing rotations of state transparent figures to make patterns.

(5) Procedure building practise to reinforce the above points. Suggestions for simple shapes to be built as procedures are given.

Maths: Polygons are introduced, giving practise at estimating angles and relating the angles turned to the number of sides in the figure.

(6) Procedures and sub-procedures. An example is given of how part of a drawing can be built as a subprocedure and then used to make the whole design.

LOGO: Introducing subprocedures.

Maths: Angles and rotations. Use of successive approximations to find required angles in a rotated pattern.

(7) More subprocedures, illustrating the possibility of changing the subprocedure only to give a different final result.

LOGO: To show the dependance of a procedure on the subprocedure used. Also illustrated the desirability of using state transparent procedures.

(8) Polygons, using the subprocedure to draw one side in the polygon procedure.

LOGO: Use of subprocedures. Use of REPEAT within a procedure.

Maths: Polygon rule relating the angle turned to the number of sides of a polygon.

(9) Introducing SX as an absolute positioning on the screen.

Maths: Discovery of negative numbers to denote displacement to the left of a central point.

(10) Squares of different sizes, as a build up to the use of inputs in procedures.

LOGO: to show the need for inputs.

Maths: Cartesian coordinate system with positive and negative directions in two dimentions.

(11) Procedures with inputs. The syntax is explained with an example and then exercises given to practise using inputs.

LOGO: Introduction to inputs in procedures.

(12) Practise with inputs in building a house procedure.

LOGO: Passing inputs from subprocedures to the procedure.

Maths: Generalization of inputs to any shape, and systematic thinking to discriminate between the constant and variable dimensions.

(13) Stars, using a tail recursive procedure to investigate the effect of different angles.

LOGO: Use of inputs for angle as opposed to displacement commands. Introduction to tail recursion.

Maths: Generalization of inputs to any dimension. Investigation of angles and discovery of equivalent left and right turning angles. Generalization of the underlying rules.

(14) Arcs and circles.

Maths: Investigation of circles of different sizes. The size of a circle depends on the forward input and varies inversly with the size of the angle used. The perimeter of the circle is determined by the forward input and the number of repeats used (which in turn depends on the angle used). A semi-circle requires half the number of repeats, and a 90 degree arc requires one quarter of the total number of repeats to draw the complete circle. The angle between the tangent and the radius of a circle is 90 degrees.

(15) Subprocedures with inputs, using a basic shape as a subprocedure in several other designs.

Maths: To extend the idea of a variable by showing that arithmetic operations can be performed on the inputs to procedures. Introducing ratio and proportion and showing that proportions can be maintained through the use of multiplicative factors.

(16) Procedures with more than one input, using a rectangle and rhombus as examples of figures with two independently variable dimensions.

LOGO: To illustrate that more than one input can be used in a procedure.

Maths: the geometry of the rectangle and rhombus, and angle properties of parallel lines. The use of variables in more complex algebraic forms, as when using the command RIGHT 180 - :ANGLE. Estimation of angles and generalization of underlying laws.

(17) Spirals, using a tail-recursive procedure ,with changing inputs and stopping conditions.

LOGO: Introduction to stopping conditions and changing inputs in recursive procedures.

Maths: Extension of the idea of a variable to represent any number which can change in value while the procedure is being run. Recognition of a spiral as formed by regular incremental increases in the length of the side. Use of limiting < > conditions in control statements.

(18) Variables and the MAKE statement used in an arithmetic context of printing out multiplication tables and counting.

LOGO: Syntax for printing variable names and values and the MAKE statement, changing the value were introduced.

Maths: Elaboration and practise in the use of variables with arithmetic operations.

(19) Polyspi, the use of three variables in a tail recursive procedure.

Maths: Seperation of variables, and generalization of angle and shape rules to a new context.

(20) Random numbers.

LOGO: Extension of conditional statements and introduction of question and answer syntax.

Maths: Further practise in the use of variables.

(21) Random numbers in drawing.

Maths: use of variables and proportions.

(22) Tiling patterns

Maths: Use of successive approximations to find unknown angles, or generalization of angle knowledge to a new context. Discovery of repeated patterns underlying each tiling pattern, to generalize a common strategy to use on all examples.

Worksheet 1, an introduction to the TI 99 /4A machine.

USE OF THE KEYBOARD AND DIRECT COMMANDS.

Type TELL TURTLE then press "enter"

If you make a typing mistake, such as a wrong spelling or missing out a space, you can correct it using two keys at once, the function key and key 3. This is labelled ERASE on the strip along the top of the keyboard.

Try out different drawing instructions to the turtle. Just use the normal commands:-

FD forward
BK backward
LT left
RT right

don't forget to follow each with a number and the "enter" key, and leave a space between the instruction and the number.

e.g. FD 30 enter

Three other commands you can use are

PU pen up
PD pen down
CS which clears the screen and puts the turtle back in the middle pointing upwards.

Try out some simple drawings:-

draw a house
draw the initials of your name
draw a staircase
make up your own design.

Try out the erase keys and the PU PD and CS commands.

See how big the screen is and try to keep your drawings on it.

Worksheet 2.

MORE DRAWING COMMANDS

So far you have used:-

FD	forward	PU	pen up
BK	back	PD	pen down
RT	right	CS	clear screen
LT	left		

There are some other pen commands:

PE	pen erase	rubs out along the path the turtle travels.
PR	pen reverse	draws where there is no line, but rubs out where there is one.

Use the commands below to move the turtle over to the left of the screen.

```
TELL TURTLE
PU
RT  90
BK  80
PD
```

Now draw a dotted line across the screen starting.

```
FD  10
PU
FD  10
PD  .....   you finish it off.
```

When you have done that, use PR to reverse the line.

Then use PE to rub it out.

Worksheet 3

DRAWING COMMANDS AND ANGLES.

So far you have used:-

FD	forward	PU	pen up	PE	pen erase
BK	back	PD	pen down	PR	pen reverse
LT	left	CS	clear screen		
RT	right				

Now try using the command " HOME " after you have done some drawing, e.g.

```
TELL TURTLE
FD 30
RT 90
FD 40
HOME

RT 90
FD 50
HOME
```

"HOME" always brings the turtle back to the centre of the screen pointing upwards.

Now draw the spokes of a wheel

```
CS
RT 30
FD 40
HOME

RT 60
FD 40
HOME
```

Keep on increasing the angle the turtle turns right by 30 degrees until you have drawn all the spokes . (There should be 12 spokes).

Now type PR and make the turtle turn left to rub out all the spokes.

```
PR
LT 30
FD 40
HOME
```

Can you draw a wheel in the same way which has just 8 spokes?

Worksheet 4

SIMPLE PROCEDURES.

To build a procedure on the TI machine you must choose a name for the procedure and type

TO followed by the name.

e.g. to build a procedure to draw the letter "T" the name TEE can be used.

TO TEE press enter

the screen changes and you are now in the "edit mode". At the top of the screen it will say

```
TO TEE_
END
```

The little line after the word TEE is the cursor which shows where the next thing you type in will be placed.

Press enter again to move the cursor to the next line.

Now type in the commands to draw the letter T, following each command by "enter" so that each is on a different line. The screen should then look like this:-

```
TO TEE
FD 40
LT 90
BK 20
FD 40
END
```

To get out of the edit mode, press Function 9 (Back). The screen then changes back to show the turtle.

To run your procedure, just type

TEE

You can use TEE as many times as you like.

Build a procedure to draw a step. Call it STEP.

TO STEP press enter

press enter again, then type in your commands.

When you have finished press Function 9 (back).

Type STEP to run your procedure.

You can use STEP and TEE together. Try it.

e.g.
STEP
TEE
TEE
STEP

You can also use REPEAT. The commands to be repeated must be written in square brackets which are typed using Function R and Function T.

Type REPEAT 4 [STEP] enter

REPEAT 3 [TEE] enter

REPEAT 3 [TEE LT 90] enter

Repeat can take more than one command.

Write your own procedure to draw a knot and repeat it several times.

Make patterns by repeating one of your procedures followed by an angle.

Worksheet 5

Build a procedure to draw:-

the letter L

the letter Z

an open box

a rectangle

a six sided polygon (hexagon)

an eight sided polygon (octagon)

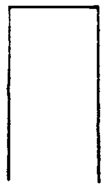
a ten sided polygon (decagon)

Call each one by a suitable name (e.g. EL, ZED etc)

Use REPEAT with each one to make patterns.

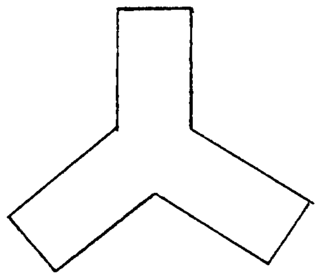
Worksheet 6 PROCEDURES AND SUB-PROCEDURES.

Write a procedure to draw an arm.
Call it ARM.



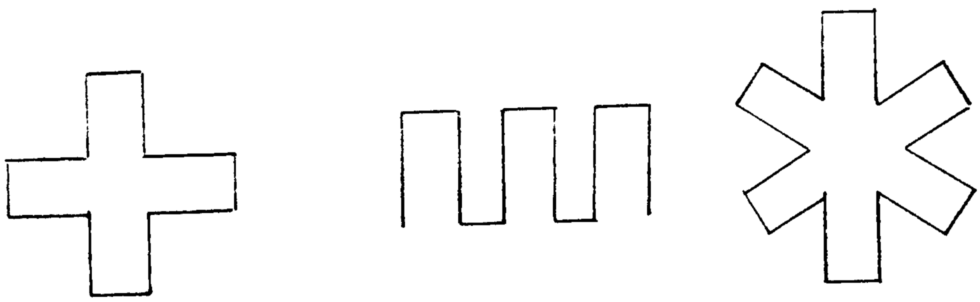
ARM can be used as a sub-procedure in a procedure to draw this shape.

```
TO MAN
  ARM
  LT 60
  ARM
  LT 60
  ARM
  LT 60
END
```



Type this in and try it out.

Now build procedures to draw the shapes below, using ARM as a subprocedure.

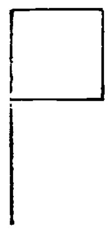


Try out your own designs, using ARM, or another simple shape as a subprocedure.

Worksheet 7

FLAG is a procedure built using the sub-procedure SQUARE.

```
TO FLAG
  FD  20
  SQUARE
  BK 20
END
```

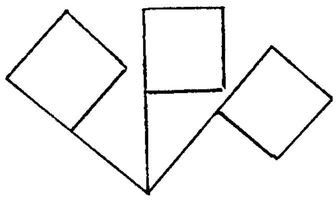


Build a procedure called SQUARE to draw a square of side 20, <u>with the turtle finishing in the same position as it started</u>.

Run it.

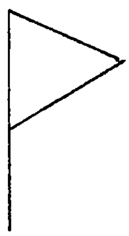
Now type in the procedure to draw a FLAG, and try it out.

Draw some patterns with your FLAG.



Write a procedure to draw a NEWFLAG (all one word) using a triangle as subprocedure. You must also write the procedure for the triangle before you try to run it.

Draw patterns with this newflag.



Worksheet 8 POLYGONS

To draw a hexagon the turtle turns 6 times through an angle of 360 ÷ 6 degrees. So each angle is 60 degrees.

The commands to build a hexagon are:-

```
TO HEXAGON
   FD 20
   LT 60
   FD 20
   LT 60
   FD 20
   LT 60
   FD 20
   LT 60
   FD 20
   LT 60
   FD 20
   LT 60
END
```

This is a lot of commands to type in, but they are the same two commands repeated six times.

A much quicker way to write the procedure is using REPEAT. You can build a procedure just using these two commands:-

```
TO HEX.BIT
   FD 20
   LT 60
END
```

This can now be used as a subprocedure to draw a hexagon.

```
TO NEW.HEXAGON
   REPEAT 6  [ HEX.BIT ]
END
```

HEX.BIT is a subprocedure in your procedure NEW.HEXAGON.

Try this out, and make patterns with your hexagons, and then try building a SQUARE.BIT as a subprocedure for a SQUARE and do other subprocedures like this to draw lots of different polygons with 3, 5, 8, 10, 12, 18, 20, 30, 36, 72 sides.

Worksheet 9   SX

Type in a procedure to draw one of the polygons you did last week. Run it.

Type SX 50
and then run your polygon procedure again.
Type SX 100
and run the polygon procedure again.

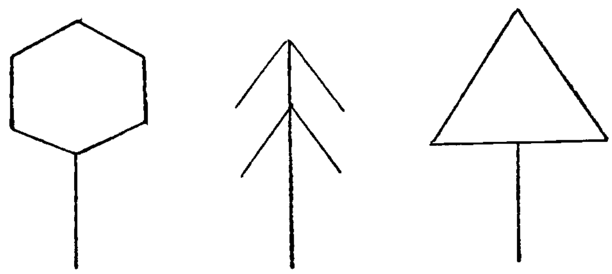
Try out different values of SX less than 100, and see what happens.

Can you use SX to put the turtle on the left hand side of the screen using only values of SX less than 100?

Clear the screen.

Now see if you can use SX to draw a row of polygons right across the screen.

Build a procedure to draw a tree, any style you like, but make it quite small. Make the turtle finish in exactly the same position as it started.



Now use SX to draw a row of trees.

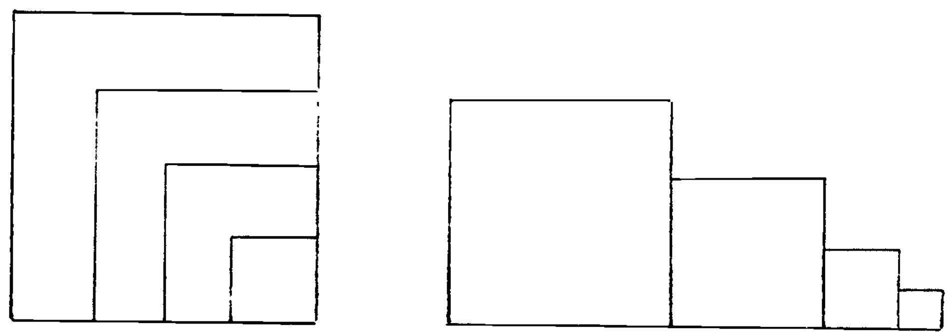
When you finish this you can try drawing a row of houses, or houses and trees, or a row of any other shape you like.

Worksheet 10

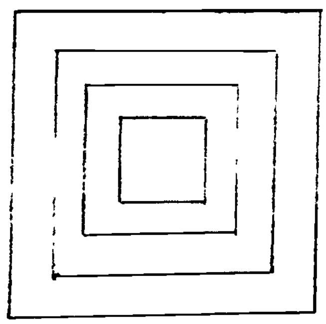
Build 5 different procedures to draw 5 squares of sidelength
8, 16, 24, 32, 40 units
Each square must leave the turtle in the same position as it started.

Each procedure will have to have a different name, e.g. WEESQ, SMALLSQ, MIDDLESQ, etc. or even S1, S2, S3, etc.

Use SX with the procedures to make the patterns below.



You can also use SY in the same way as SX to move the turtle up or down on the screen.



Worksheet 11 PROCEDURES WITH INPUTS.

Instead of having to build 5 different procedures to draw 5 squares of different sizes, it would be more convenient to have a general "square" procedure which takes an input to specify its size.

The command FORWARD is a general command which takes an input to specify how far the turtle should move forward.

We can build the general square procedure in this way:-

Type in TO SQUARE

When you are in the editor you must add the input to the top line

TO SQUARE SIDE_

Now instead of giving the command FD 50 for a side of 50 we must say "Forward the value of the sidelength". This is written as :-

```
FD :SIDE
```

The : means "the value of". So the whole procedure will look like this.

```
TO SQUARE SIDE
   FD :SIDE
   RT 90
   FD :SIDE
   RT 90
   FD ;SIDE
   RT 90
   FD :SIDE
   RT 90
END
```

Or using REPEAT the procedure would look like this.

```
TO SQUARE SIDE
   REPEAT 4 [ FD :SIDE RT 90 ]
END
```

Type one of these in then press Function 9 (Back) to get out of the editor.

To draw a square using this procedure you must always give it an input. Try

```
SQUARE 20
SQUARE 100
SQUARE 67
```

When you type in SQUARE 20, the procedure reads 20 for all the places where you have put :SIDE.

Use this procedure with different inputs to draw the patterns on Worksheet 10.

Worksheet 12

You could build a square of any size using SQUARE.BIT as a subprocedure.

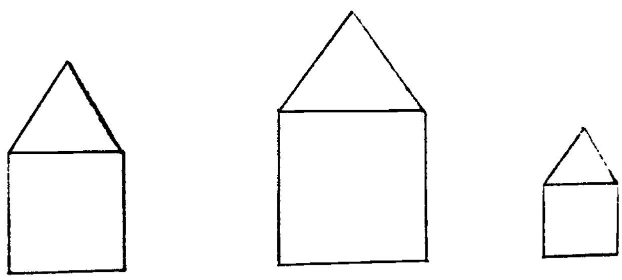
```
TO SQUARE.BIT  SIDE
   FD  :SIDE
   RT  90
END

TO SQUARE  SIDE
   REPEAT 4 [ SQUARE.BIT  :SIDE ]
END
```

Try it out.

In the same way you can build a general procedure to draw a triangle of any size. Do this. Use these procedures to draw flower patterns by rotating squares or triangles of different sizes.

When you have a procedure to draw a square of any size, and one to draw a triangle of any size, put them together in another procedure to draw a house, and use it to draw a row of houses of different sizes.



Worksheet 13 STARS

Build a procedure to draw a square.

```
TO SQUARE
FD 50
RT 90
SQUARE
END
```

Now run the procedure by typing SQUARE. You will have to press Fn. 9 (back) to stop it.

What happens is this.

```
SQUARE        SQUARE        SQUARE        SQUARE
FD 50         FD 50         FD 50         FD 50
RT 90         RT 90         RT 90         RT 90
```

It will go on until the computer memory is full, unless you stop it with fn 9 (back).

Now build a 6 sided polygon in the same way:-

```
TO POLY6
FD 50
RT 60
POLY6
END
```

Run it. Then write a procedure for a 3 sided polygon and draw it.

You can draw any polygon if you use the ANGLE as an input to the procedure.

TO POLY ANGLE ... you finish it off.

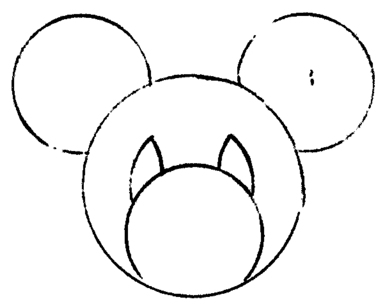
Use this procedure to investigate the shapes made using lots of different angles between 60 and 300 degrees. Write down the angle and what shape is drawn each time, and pick out any angles which give exactly the same shape. Why do they do it?

Worksheet 14 Circles and arcs.

Write procedures to draw :-

a large circle
a medium sized circle
a small circle

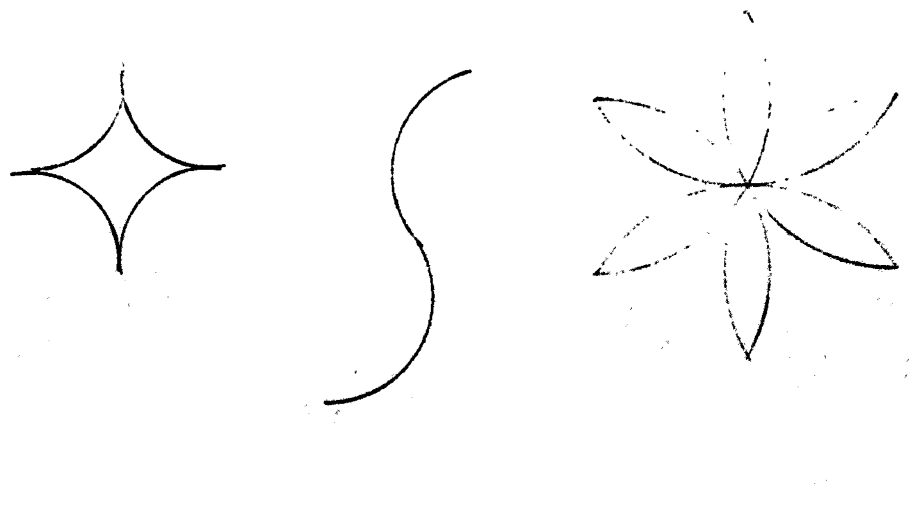
Use them to draw pictures or patterns.



A 20 sided polygon can be used as a circle. Write a procedure to draw one which can be any size.

Write another procedure to draw an arc of the circle, that is to go a quarter of the way round. Give this procedure an input too, so it can be any size.

This arc can be used to draw interesting patterns, such as the fourstar, petal, and flower below. Try some of these designs, and some of your own.



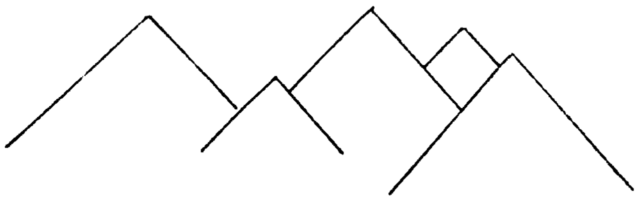
Worksheet 15   SUBPROCEDURES WITH INPUTS.

Here is a procedure to draw a hat of any size

```
TO HAT SIZE
RT 30
BK :SIZE
FD :SIZE
LT 60
BK :SIZE
FD :SIZE
RT 30
END
```

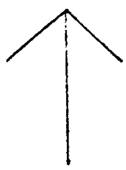
Type this in and run it a few times.

Draw a mountain range.

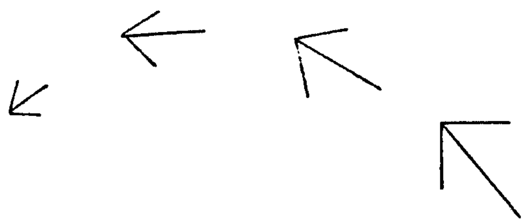


HAT can be used as a subprocedure to draw an arrow

```
TO ARROW SIZE
FD :SIZE * 2
HAT :SIZE
BK :SIZE * 2
END
```

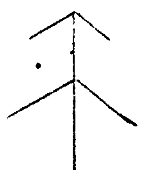


Type in the arrow procedure. Try it out. Draw a path of arrows.



Write a procedure to draw a tree, using HAT or ARROW as subprocedures.

Use SX and SY to make a forest of trees.



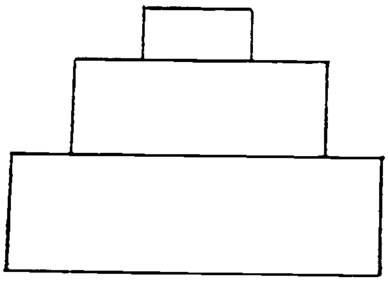
Worksheet 16 PROCEDURES WITH TWO INPUTS.

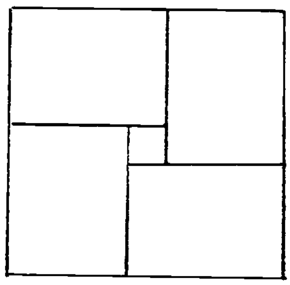
A procedure can take more than one input. A rectangle has a different length and width, and both can be varied. Both inputs must be typed on the title line of the procedure.

```
TO RECTANGLE WIDE LONG
FD :WIDE
RT 90
FD :LONG
```

You finish off the procedure and then run it with different inputs.

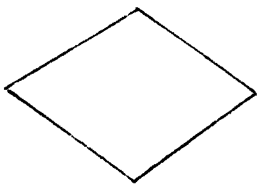
Use the RECTANGLE procedure to draw the patterns below, and make up your own.





A diamond (rhombus) is a squashed square. Its sides are all the same length, and it has two pairs of equal opposite angles.

Build a procedure to draw a diamond that has two inputs, SIDE and ANGLE. Make flower patterns using your diamond procedure.



Build a procedure to draw a regular polygon which can be any size and have any number of sides.

Worksheet 17 SPIRALS

Type in this procedure.

```
TO TRISPI SIDE
FD :SIDE
RT 120
TRISPI :SIDE+2
END
```

Try TRISPI 10. What does the +2 do in the procedure?

Change the 2 for another number and see what happens.

This procedure is called TRISPI because it is a triangular spiral. Write another procedure to draw a :-

square spiral SQUARESPI

hexagonal spiral HEXSPI

Recursive procedures like these will not stop unless you press fn 9 (back), or until the computer runs out of ink.

You can add a stopping condition to a procedure to say " When the side becomes bigger than 50 stop. " This is written in LOGO as

```
IF :SIDE > 50 THEN STOP
```

the ">" sign means "greater than". Put this line into your procedure for the spiral hexagon. It must go in before the procedure calls itself again. Usually stopping conditions are put in as the first line to make sure they are read each time the procedure is run.

Make a pattern by repeating your procedure several times. Change the stopping condition to 40 instead of 50 and make another pattern. See how many different patterns you can make in this way.

Add stopping conditions to your other spiral procedures, and make patterns by repeating the procedures and changing the stopping conditions.

Worksheet 18   Variables and the MAKE command

Type in these commands and see what happens.

```
MAKE "FRED  42
PRINT [ FRED]
PRINT :FRED

MAKE "X  13 + 17
PRINT [ X ]
PRINT :X
PRINT :X - 10

PRINT :FRED
MAKE "FRED  :FRED + 16
PRINT :FRED
```

FRED and X in these examples are the names of variables. Any word or letter can be used as a variable name.

The variable is given a value in a MAKE statement. This can be a completely new value ... MAKE "SIDE 20 ... or it can change an existing value
... MAKE "SIDE :SIDE + 1 ...

When talking about the NAME of a variable in a MAKE statement the quote sign " is used just in front of it.
When talking about the VALUE of a variable, the number represented by that name, the sign : is used just in front of it.

Here is a procedure to count up in threes starting at any given number and stopping at 60.

```
TO COUNT  X
   IF  :X > 60 THEN STOP
   PRINT :X
   WAIT 20
   MAKE "X  :X + 3
   COUNT  :X
END
```

Try this out and change bits of the procedure to check how it works.

Write your own procedures to:-

a) Count up in 5's and stop at 100.

b) Count down from 10 to 0

c) Print out the 7 times table. (You want a variable which will go up in one's and print out this number times 7 each time.)

d) Print out any times table you want (using two inputs).

Worksheet 19   POLYSPI

Type in this procedure

```
TO POLYSPI   DISTANCE   ANGLE   CHANGE
  FD  :DISTANCE
  RT  :ANGLE
  MAKE "DISTANCE  :DISTANCE + :CHANGE
  POLYSPI  :DISTANCE  :ANGLE  :CHANGE
END
```

It has three inputs, Distance, Angle and Change, so it must be given three numbers.

Try   POLYSPI 10 61 3

See what different designs you can create and keep a note of the inputs you used for the most interesting ones.

Find out what difference each of the inputs makes to the design.

Worksheet 20 Random numbers

Type in PRINT RANDOM

```
        PRINT RANDOM * 10
        REPEAT 8 [PRINT RANDOM * 10 + RANDOM]

        MAKE "X  RANDOM
        PRINT :X
        PRINT RANDOM
        MAKE "Y RANDOM
        PRINT :X   PRINT :Y
```

Each time you use RANDOM it will give a different number, so if you want to use the same number again you must give it a NAME, such as X or Y in these examples. Once X has been given a value, it keeps the same value until it is changed with another MAKE statement.

RANDOM only gives a number between 0 and 9, so to get a random number between 0 and 99, you must have one random number for the tens and one for the units. If you are using this a lot you can build a little procedure to give you the numbers

```
TO  RANDOM99
   OUTPUT RANDOM * 10 + RANDOM
END
```

Try it out.
Write another procedure to give random numbers between 0 and 999, and try it out.

Here is a procedure using RANDOM to test your tables.

```
TO TABLE.TEST
   MAKE "X  RANDOM
   MAKE "Y  RANDOM
   PRINT SE SE :X [TIMES] :Y
   MAKE "ANSWER  FIRST READLINE
   IF :ANSWER = :X * :Y THEN PRINT [ THAT'S RIGHT ] ELSE PRINT
        SE [NO THE CORRECT ANSWER IS ] :X * :Y
   TABLE.TEST
END
```

Try this out. Then you can change the procedure to make it a bit better. Put in a variable COUNT which increases by one each time the procedure is run, and use this to stop the recursion after 10 questions.

Use this procedure as a model for a new procedure to test addition of two numbers between 0 and 99 (use RANDOM99 for this).

The really ambitious can try to make a guessing game. The computer works out a random number which you have to guess. Each guess receives a hint to say if it is too big or too small, so it must be a recursive procedure until the right answer is found.

Worksheet 21 Random numbers in drawings.

You can use random numbers in your drawing commands with the turtle.

To produce a simple random pattern you just need a forward command and a turn command, to right or left, giving each one a random input.

```
TO RANDRAW
   FORWARD  RANDOM * 5
   LEFT  RANDOM * 10
   RANDRAW
END
```

Try it out and change it.

You can also use RANDOM with SX and SY. The procedure SKY will put one star in the sky at a random position.

```
TO SKY
   MAKE "X RANDOM * 10
   MAKE "Y RANDOM * 10
   MAKE "P RANDOM
   IF :P < 5  THEN SX -:X ELSE SX :X
   SY :Y
   STAR
END
```

The variable P is given a random value, so it can be any number between 0 and 9. Half the time it should be between 0 and 4, and half the time it should be between 5 and 9. When it has a value 0,1,2,3 or 4 SX is made negative, and when it has a value 5,6,7 8 or 9 SX has a positive value.

To run this procedure you will have to build the subprocedure STAR.

Build another procedure to put 20 stars in the sky, using SKY as a subprocedure.

Try out some procedures of your own using RANDOM in drawing.

Change the RANDRAW procedure to repeat the same two commands several times.

Use RANDOM to change the pen colour using SC RANDOM.

Use something like the sky procedure to put enemy planes in the sky, then write a SHOOT procedure which will remove each plane it is targeted on (using SX SY and PE).

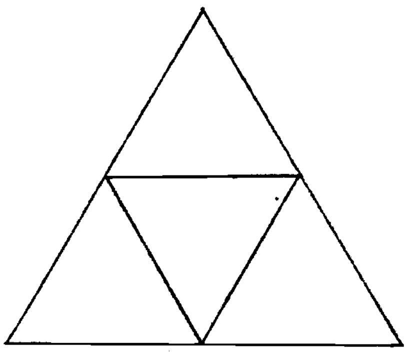
Worksheet 22 Tiling Patterns

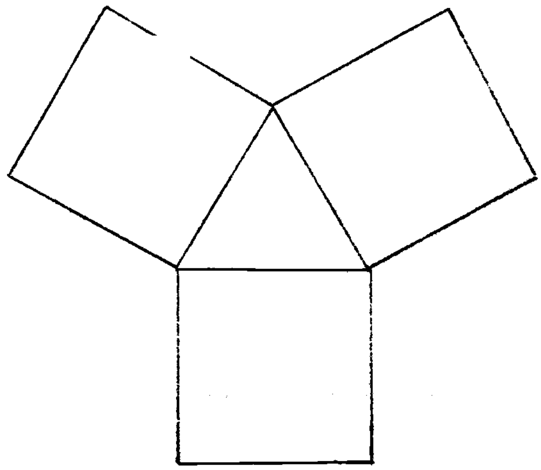
Write a procedure called SHAPES which will draw any regular polygon when you give it an input specifying the number of sides it must have. Make each side 20 units long.

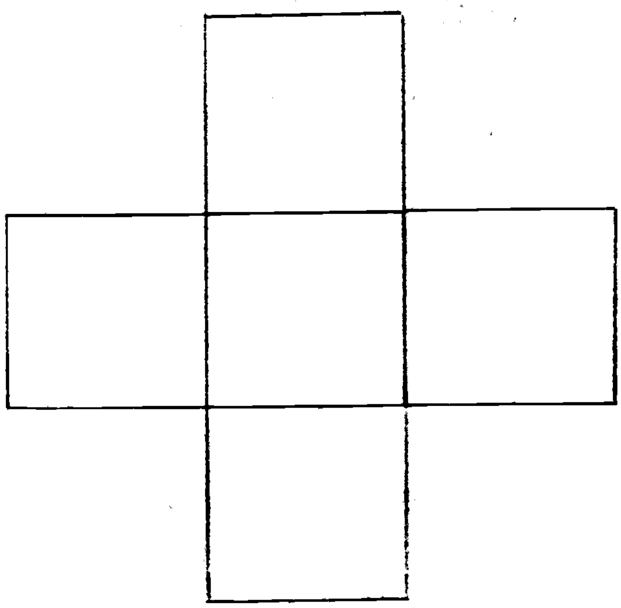
```
TO SHAPES N
```

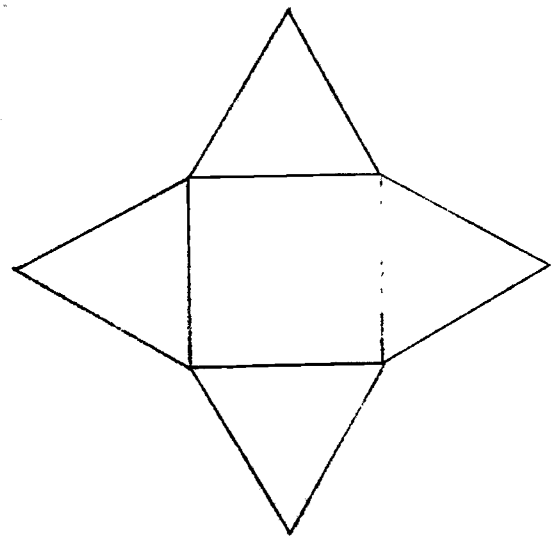
Here are a lot of patterns which can be built from your SHAPES procedure. Try to draw them, and then when you have worked out how to do it, build a different procedure for each pattern.

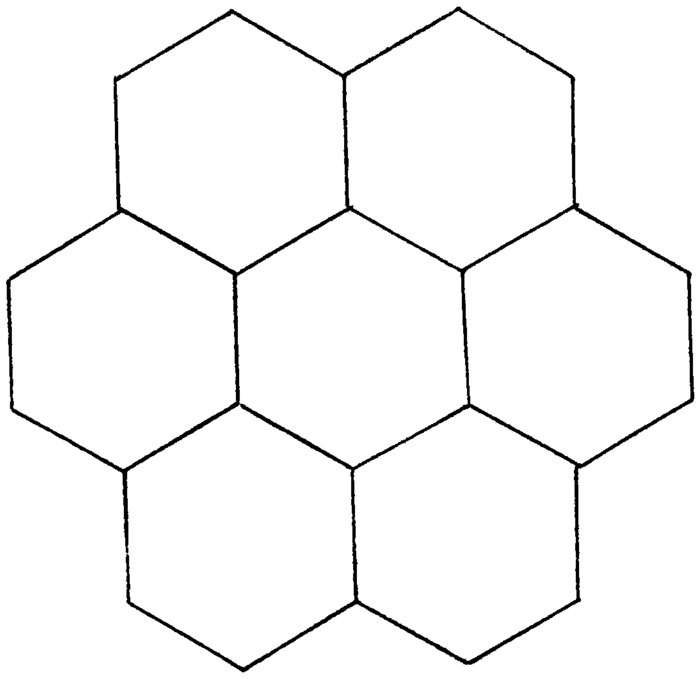
You should be able to use a similar approach for each one.

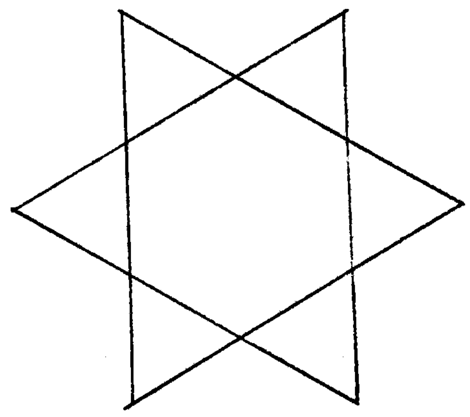


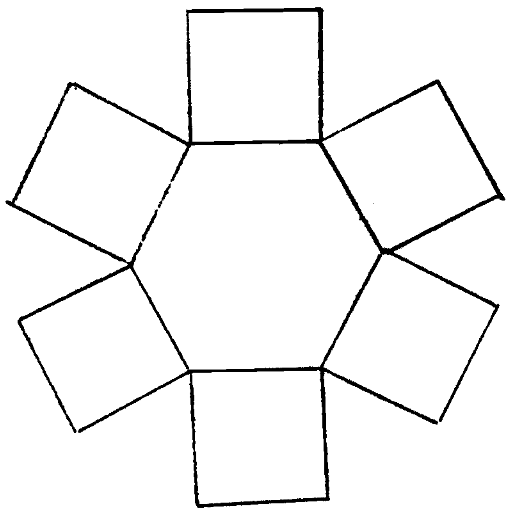


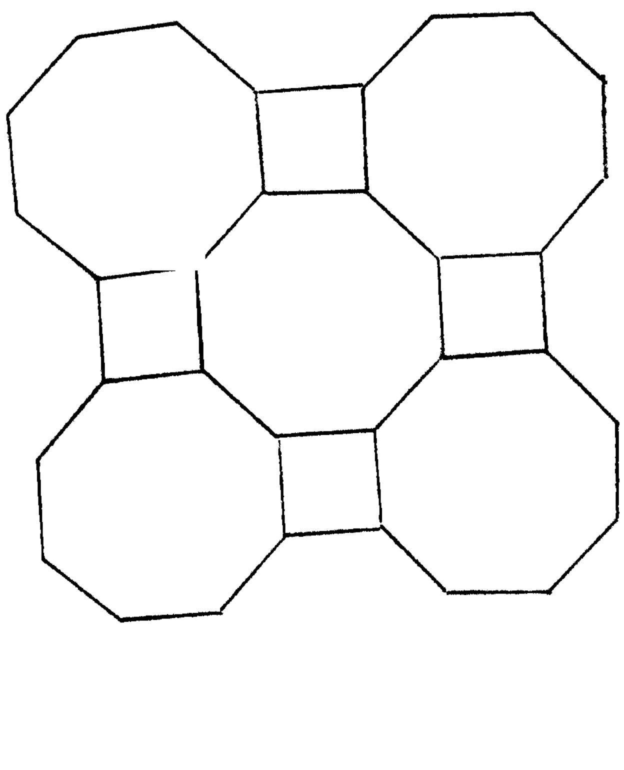












APPENDIX 6a

STANDARDISED AND PERCENTAGE SCORES FOR EXPERIMENTAL GROUP ON NFER TESTS DH (non-verbal intelligence), C1 and EF (pre and post mathematics attainment tests).

GIRLS	DH	C1	C1%	EF%	(L)	(F)	(R)
An	127	103	60	78	15	10	22
Ef	118	128	94	68	15	8	18
Ad	117	107	68	78	17	11	19
Ky	124	124	92	78	16	10	21
Aj	124	118	84	75	15	10	20
Be	117	114	82	77	15	8	23
Gi	97	103	60	50	14	5	11
Lu	100	100	58	45	9	8	10
J	93	84	30	33	8	6	6
R	108	104	64	70	16	7	19
Ju	96	114	80	70	15	9	18
Ly	111	111	78	70	13	10	19
K	107	102	56	63	14	8	16
Su	85	97	52	45	7	10	10
Sj	113	106	66	65	14	9	16
BOYS							
Am	105	115	84	70	15	8	19
C	113	119	88	87	17	11	24
Ar	118	113	80	70	16	9	17
Ke	104	125	92	80	15	11	22
Br	138	129	96	92	17	11	27
Ma	127	121	88	88	17	11	25
Cl	105	105	68	57	12	7	15
Ro	102	111	76	75	16	11	18
D	101	108	70	60	14	10	12
F	93	100	54	47	10	6	12
H	92	90	38	37	7	6	9
Jn	107	102	60	58	12	8	15
Jm	101	121	90	nt			
Tz	108	99	50	62	12	8	17
Mi	98	100	54	52	7	7	17
Rt	117	98	50	63	15	6	17
Sh	85		8	nt			

(L) score on the 17 LOGO related questions; (F) score on 11 fractions, decimals and percentages questions; (R) score on the remaining 32 questions.

APPENDIX 6b

STANDARDISED AND PERCENTAGE SCORES FOR CONTROL GROUP ON NFER TESTS DH (non-verbal intelligence), C1 and EF (pre and post mathematics attainment tests).

GIRLS	DH	C1	C1%	EF%	(L)	(F)	(R)
	92	82	28	17	5	1	4
	88	71	14	nt			
	86	84	30	37	7	5	10
	112	104	60	nt			
	91	98	54	53	10	7	15
	92	98	52	37	9	3	10
	106	114	80	67	14	9	17
	114	116	84	62	11	8	18
	105	103	64	37	8	3	11
	125	112	76	72	12	11	20
	120	121	90	78	13	10	24
	118	105	66	57	12	8	14
	123	105	64	55	11	8	14
	116	111	74	60	12	8	16
	101	95	46	nt			
	96	105	68	40	5	5	14
BOYS							
	108	98	48	53	13	4	15
	nt	nt		28	7	6	4
	92	94	48	30	5	3	10
	91	97	48	45	11	2	14
	100	87	32	35	7	4	10
	108	110	72	70	13	10	19
	88	99	52	35	7	6	8
	102	98	52	25	4	5	6
	112	112	78	62	14	8	15
	107	96	52	37	7	3	12
	97	94	44	43	10	3	13
	119	104	66	70	14	7	21
	133	134	98	85	17	10	24
	136	138	100	90	16	11	27
	115	113	74	68	14	9	18
	101	111	74	nt			

(L) score on the 17 LOGO related questions; (F) score on 11 fractions, decimals and percentages questions; (R) score on the remaining 32 questions.

APPENDIX 6c

SCORES FOR EXPERIMENTAL GROUP ON CHELSEA I (reflections and rotations), ANGLE ESTIMATION AND CHELSEA II (variables).

GIRLS	CHELSEA I L1	L2	L3	T	Level	ANGLE	CHELSEA II L1	L2	L3	T	Level
An	5	4	2	11	2	6	4	1	1	6	1
Ef	5	6	4	15	3	6	5	6	3	14	3
Ad	5	6	4	15	3	5	4	5	1	10	2
Ky	5	6	5	16	3	6	4	5	5	14	3
Aj	5	6	4	15	3	7	5	6	4	15	3
Be	5	5	5	15	3	8	4	3	1	8	2
Gi	4	1	2	7	1	4	3	0	0	3	1
Lu	5	4	2	11	2	3	4	2	1	7	1
J	4	4	2	10	2	3	3	1	0	4	1
R	4	3	2	9	2	7	5	0	0	5	1
Ju	4	3	1	8	2	6	5	5	0	10	2
Ly	4	4	4	12	3	6	5	2	2	9	2
K	5	3	1	9	2	6	4	4	0	8	2
Su	4	2	3	9	2	3	3	1	0	4	1
Sj	4	2	3	9	2	8	4	4	0	8	2
BOYS											
Am	4	4	1	9	2	7	4	2	1	7	1
C	5	5	2	12	2	8	4	3	2	9	2
Ar	5	5	5	15	3	8	5	1	2	8	1
Ke	2	3	1	6	1	5	3	5	2	10	2
Br	5	6	5	16	3	8	5	5	5	15	3
Ma	5	5	5	15	3	7	5	4	5	14	3
Cl	5	5	0	10	2	5	3	0	1	4	1
Ro	5	5	5	15	3	7	4	5	3	12	3
D	4	4	4	12	3	6	5	2	2	9	2
F	3	3	0	6	1	4	3	1	0	4	1
H	1	1	1	3	1	0	4	1	0	5	1
Jn	5	3	3	11	2	4	4	5	2	11	2
Jm	5	6	3	14	3	7	4	3	1	8	2
Tz	5	3	0	8	1	8	3	0	0	3	1
Mi	2	2	0	4	1	7	4	2	1	7	1
Rt	5	5	5	15	3	7	4	1	1	6	1
Sh	2	3	3	8	1	4	3	1	0	4	1

L1,L2,L3 - levels of question; T - total score out of 16; level - level of question at which the child is successful; Angle - score out of 8.

APPENDIX 6d

SCORES FOR CONTROL GROUP ON CHELSEA I (reflections and rotations), ANGLE ESTIMATION AND CHELSEA II (variables).

GIRLS	CHELSEA I L1	L2	L3	T	Level	ANGLE	CHELSEA II L1	L2	L3	T	Level
	1	2	0	3	1	2	nt				
	nt										
	2	2	1	5	1	1	4	1	1	6	1
	4	2	2	8	2	2	5	3	1	9	2
	5	4	3	11	2	1	3	0	0	3	1
	4	2	2	8	2	4	2	2	0	4	1
	5	3	2	10	2	2	5	3	0	8	1
	5	4	4	13	3	1	4	1	0	5	1
	3	3	3	9	2	4	5	2	0	7	1
	5	4	3	12	3	1	5	4	0	9	2
	5	6	4	15	3	6	5	4	2	11	2
	5	4	2	11	2	1	3	2	0	5	1
	5	5	5	15	3	6	3	1	0	4	1
	5	6	3	14	3	3	4	1	0	5	1
	4	5	4	13	3	3	3	1	0	4	1
	2	2	2	6	1	0	4	3	1	8	2
BOYS											
	3	3	5	11	3	6	4	2	0	6	1
	4	1	2	7	1	3	1	0	0	1	0
	1	1	0	2	0	2	2	0	0	2	.0
	3	4	1	8	2	0	4	0	0	4	1
	0	0	0	0	0	1	2	0	0	2	0
	5	3	4	12	3	7	3	1	0	4	1
	5	4	3	12	3	0	5	3	1	9	2
	2	2	1	5	1	2	4	2	0	6	1
	5	5	4	14	3	5	4	0	1	5	1
	3	3	3	9	2	2	nt				
	3	3	3	9	2	1	3	2	0	5	1
	3	5	5	13	3	5	4	4	5	13	3
	5	6	4	15	3	8	4	5	5	14	3
	nt										
	4	3	2	9	2	7	4	3	0	7	1
	nt										

L1,L2,L3 - levels of question; T - total score out of 16; level - level of question at which the child is successful; Angle - score out of 8.

APPENDIX 6e

SCORES FOR EXPERIMENTAL GROUP ON GENERALIZATION QUESTIONS A,B,C,D.

GIRLS	A			B	C				D		
	p	r	t	t	p	r1	r2	t	p	r	t
An	4	5	9	1	1	3	1	5	3	7	10
Ef	4	4	8	5	1	4	0	5	2	1	3
Ad	4	2	6	4	1	4	3	8	3	0	3
Ky	4	5	9	4	1	4	4	9	3	2	5
Aj	nt										
Be	4	6	10	4	1	4	0	5	3	4	7
Gi	4	2	6	4	1	0	0	1	0	0	0
Lu	4	4	8	4	1	1	1	3	3	0	3
J	2	3	5	4	1	0	0	1	0	0	0
R	4	4	8	4	1	2	0	3	2	0	2
Ju	4	5	9	5	1	3	3	7	3	0	3
Ly	4	3	7	4	1	4	3	8	3	0	3
K	4	2	6	3	0	0	0	0	1	0	1
Su	4	2	6	4	1	0	0	1	0	0	0
Sj	4	4	8	3	1	0	0	1	3	0	3
BOYS											
Am	3	2	5	4	1	0	0	1	2	0	2
C	4	7	11	4	1	4	4	9	3	3	6
Ar	4	5	9	4	1	4	3	8	2	4	6
Ke	2	6	8	5	1	4	5	10	2	0	2
Br	4	6	10	3	1	4	0	5	3	2	5
Ma	2	1	3	2	1	4	4	9	2	5	7
Cl	3	5	8	4	1	2	1	4	2	0	2
Ro	4	4	8	4	1	4	0	5	3	1	4
D	2	1	3	4	1	1	1	3	3	0	3
F	2	1	3	4	1	1	0	2	0	0	0
H	4	2	6	5	0	0	0	0	0	0	0
Jn	4	3	7	1	0	0	1	1	2	0	2
Jm	4	5	9	4	0	0	0	0	2	0	2
Tz	3	1	4	5	0	1	0	1	2	0	2
Mi	2	0	2	4	1	1	1	3	1	0	1
Rt	2	1	3	0	1	1	0	2	3	0	3
Sh	2	1	3	4	1	0	0	2	0	0	0

p - pattern continuation; r - rules; t - total

APPENDIX 6f

SCORES FOR CONTROL GROUP ON GENERALIZATION QUESTIONS A,B,C,D.

GIRLS	A			B	C				D		
	p	r	t	t	p	r1	r2	t	p	r	t
	2	2	4	4	0	0	0	0	1	0	1
	nt										
	2	0	2	5	1	0	0	1	1	0	1
	1	0	1	4	1	0	0	1	1	0	1
	2	1	3	2	0	0	0	0	0	0	0
	0	0	0	4	0	0	0	0	1	0	1
	4	5	9	5	1	1	1	3	2	0	2
	3	4	7	1	1	4	2	7	2	0	2
	2	4	6	4	1	4	0	5	1	0	1
	4	4	8	3	1	4	0	5	3	0	3
	3	4	7	3	1	4	0	5	3	0	3
	2	2	4	4	1	4	0	5	1	0	1
	4	4	8	3	1	1	1	3	1	0	1
	4	0	4	4	1	4	1	6	3	0	3
	4	1	5	4	0	4	0	4	2	0	2
	2	1	3	4	0	0	0	0	1	0	1
BOYS											
	4	3	7	3	1	4	1	6	1	0	1
	0	0	0	5	0	0	0	0	0	0	0
	0	0	0	3	1	1	0	2	0	0	0
	0	0	0	4	1	0	0	1	2	0	2
	2	1	3	3	0	0	0	0	1	1	2
	4	4	8	4	1	4	2	7	0	0	0
	2	2	4	2	1	1	1	3	0	0	0
	2	1	3	3	1	0	0	1	0	0	0
	3	4	7	3	0	2	0	2	3	0	3
	4	1	5	1	1	0	0	1	0	0	0
	4	2	6	3	1	2	1	4	0	0	0
	4	4	8	1	1	2	1	4	3	0	3
	3	5	8	5	1	4	5	10	3	5	8
	4	6	10	4	1	4	4	9	3	0	3
	3	4	7	4	1	0	0	1	2	5	7
	nt										

p - pattern continuation; r - rules; t - total

APPENDIX 6g

SCORES FOR EXPERIMENTAL GROUP ON GENERALIZATION QUESTIONS E,F,G.

GIRLS	E				F				G			
	p	r1	r2	t	p	r1	r2	t	p	r1	r2	t
An	5	2	2	9	2	3	3	8	2	1	1	4
Ef	6	0	0	6	2	3	3	8	2	2	2	6
Ad	5	2	0	7	2	0	3	5	2	2	2	6
Ky	nt											
Aj	5	2	1	8	2	3	3	8	2	1	3	6
Be	4	2	0	6	2	1	3	6	2	2	2	6
Gi	3	0	0	3	2	0	0	2	1	0	0	1
Lu	2	0	0	2	2	0	0	2	2	0	0	2
J	2	0	0	2	2	0	0	2	2	0	0	2
R	6	0	0	6	2	1	0	3	2	1	0	3
Ju	3	0	1	4	2	0	0	2	2	2	0	4
Ly	5	2	0	7	2	3	2	7	2	2	0	4
K	5	0	0	5	2	1	0	3	2	1	0	3
Su	4	0	0	4	2	0	1	3	2	2	1	5
Sj	6	0	0	6	2	2	2	6	2	0	0	2
BOYS												
Am	6	2	0	8	2	0	0	2	2	0	0	2
C	6	2	0	8	2	3	3	8	2	3	3	8
Ar	6	1	2	9	2	2	2	6	2	2	0	4
Ke	6	2	0	8	1	0	0	1	2	2	2	6
Br	6	2	1	9	2	3	2	7	2	0	1	3
Ma	6	2	0	8	2	3	3	8	2	1	1	4
Cl	nt											
Ro	5	2	1	8	2	3	3	8	2	2	1	5
D	4	0	0	4	1	0	0	1	2	1	0	3
F	2	0	0	2	2	0	0	2	1	0	0	1
H	1	0	0	1	1	0	0	1	2	0	0	2
Jn	1	0	1	2	2	3	3	8	2	2	1	5
Jm	6	2	1	9	2	3	3	8	2	2	0	4
Tz	5	0	0	5	1	0	0	1	2	0	0	2
Mi	1	0	0	1	1	3	2	6	2	1	0	3
Rt	2	0	0	2	2	2	2	6	2	0	0	2
Sh	2	0	0	2	2	0	0	2	2	0	0	2

p - pattern continuation; r1, r2 - rules; t - total

APPENDIX 6h

SCORES FOR CONTROL GROUP ON GENERALIZATION QUESTIONS E,F,G.

GIRLS	E p	E r1	E r2	E t	F p	F r1	F r2	F t	G p	G r1	G r2	G t
	1	0	0	1	1	0	0	1	2	0	0	2
	0	0	0	0	1	0	0	1	2	0	0	2
	2	0	0	2	1	0	0	1	2	0	0	2
	6	0	0	6	2	0	0	2	2	0	0	2
	2	0	0	2	2	0	0	2	2	0	0	2
	3	0	0	3	2	0	0	2	2	0	0	2
	6	2	0	8	2	1	0	3	2	1	0	3
	6	2	0	8	2	3	3	8	2	2	2	6
	6	0	0	6	2	2	2	6	2	0	0	2
	6	0	0	6	2	0	0	2	2	1	0	3
	6	2	2	10	2	3	0	5	2	2	2	6
	4	0	0	4	2	0	0	2	2	1	0	3
	6	0	0	6	2	0	0	2	2	1	0	3
	6	0	0	6	2	2	0	4	2	1	0	3
	5	2	0	7	2	0	0	2	2	0	0	2
	3	0	0	3	1	0	0	1	0	0	0	0
BOYS												
	6	2	0	8	2	1	0	3	2	0	0	2
	0	0	0	0	1	0	0	1	0	0	0	0
	2	0	0	2	2	0	0	2	0	0	0	0
	3	0	0	3	2	0	0	2	2	0	0	2
	0	0	0	0	1	0	0	1	0	0	0	0
	6	1	1	8	2	3	2	7	2	1	0	3
	2	0	0	2	2	0	0	2	2	1	0	3
	2	0	0	2	2	0	0	2	1	0	0	1
	5	0	1	6	2	2	1	5	2	1	1	4
	2	0	0	2	2	0	0	2	2	0	0	2
	5	2	0	7	2	0	0	2	2	0	0	2
	6	2	1	9	2	4	3	9	2	2	1	5
	6	2	2	10	2	4	3	9	2	2	3	7
	6	2	2	10	2	3	3	8	2	2	3	7
	5	0	0	5	2	0	0	2	2	0	0	2
	nt											

p - pattern continuation; r1, r2 - rules; t - total

APPENDIX 6i

AVERAGE TIME PER WEEK SPENT ON THE COMPUTER
(in hours and minutes)

GIRLS	TERM 1 (15 weeks)	TERM 2 (13 weeks)
An	.52	.52
Ef	1.08	.39
Ad	1.08	.51
Ky	1.20	1.00
Aj	1.10	.46
Be	.38	.43
Gi	.55	nr
Lu	1.06	nr
J	.50	.45
R	1.28	.40
Ju	1.12	.27
Ly	.42	.35
K	1.21	.45
Su	.35	.55
Sj	1.30	1.10
BOYS		
Am	2.45	nr
C	1.35	nr
Ar	1.14	.42
Ke	1.48	1.47
Br	1.18	nr
Ma	.53	.45
Cl	1.13	1.20
Ro	1.04	.57
D	2.18	nr
F	2.18	nr
H	nr	nr
Jn	1.11	nr
Jm	1.25	nr
Tz	2.35	nr
Mi	1.56	1.15
Rt	2.12	.42
Sh	nr	nr

nr - no record of the amount of time spent