

WAVE FORCES ON CYLINDERS

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Thesis submitted for the Degree of Doctor of Philosophy

University of Edinburgh

1980



## ABSTRACT

A study is made of the wave loading on a partially submerged fixed horizontal circular cylinder. An extensive series of measurements of the horizontal and vertical forces on a 10cm diameter model cylinder, obtained with experimental equipment designed and built by the Wave Power Project group at the University of Edinburgh, is presented. These results are compared with a theoretical model for partially submerged bodies developed from Morison's equation by the introduction of a varying volume and, in the case of the vertical force, a buoyancy term. Values of the empirically determined inertia coefficient for each of the test cases are presented.

The reflection coefficients for a horizontal circular cylinder at several positions on or near the free surface are measured as a function of wave amplitude and wavelength. The data is used to calculate the mean horizontal forces acting on the cylinder which are then compared with experimental results.

The modifications to Morison's equation are developed further for irregular waves possessing a Gaussian wave elevation spectrum, enabling the horizontal and vertical force spectra to be calculated and compared with measurements in a wave tank.

A simple model of a long flexible floating pipe is then

presented and the dependence of bending moment on several important parameters is discussed with emphasis on applications to the wave extraction device known as the Salter duck.

### DECLARATION

This thesis has been composed by myself and, except where stated, the work contained is my own.

### Acknowledgements

The author would like to express his appreciation for the support and encouragement of Dr. C. A. Greated throughout the period of this work and also for the guidance and facilities afforded him by Mr. S. H. Salter and his associates. I am also indebted to Dr. J. Martin, Dr. T. S. Durrani and Dr. I. Daudpota for invaluable assistance on matters mathematical, to Mr. D. Molyneux for his constant technical support and to Mrs. J. M. Bateson who typed the manuscript. The receipt of a Science Research Council maintenance award covering the period of study is also gratefully acknowledged.

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CHAPTER 1

WAVE FORCES

## CHAPTER 1

### WAVE FORCES

#### 1.1 Introduction

The behaviour of man-made structures on and under the oceans has naturally been of very great interest for several thousand years, so it is surprising to find that the study of wave forces has only been developed in a systematic way in the past few decades. This recent upsurge in activity, both experimental and theoretical, stemmed from the development of oilfields in the Gulf of Mexico after the second world war. Since then, the discovery of oil in the North Sea, where more severe weather conditions are prevalent and the water much deeper, has meant that much research in this area of engineering is still being carried out in universities and institutes in various countries around the world, the most prominent of these being the United States of America, Britain, France, Japan, Norway, Denmark and Holland.

Before the advent of semi-permanent structures in the sea, most of the research that had been done on wave forces had focused on ships and harbours, and so naval architects are strongly represented in the literature devoted to marine loading. However, the experience gained in designing and building ships was not altogether sufficient for oilrigs. For instance, except during the towing stage of installation, oilrigs are fixed objects extending in the majority of cases to the sea bottom.

Designs varying from single tower platforms to multi-legged ones mean that wave loadings can be of several different types, even on the same platform.

It should be remembered that wave loading is not the only phenomenon to be taken into consideration in design work in an ocean environment, but that wind and currents are also present. They are, however, usually of secondary importance. These latter topics are discussed in Wiegel (1964) and Ippen (1966), while Hogben (1974) reviews the subject of wave loading and makes recommendations as to future work and areas of interest.

The most widely adopted approach to the problem of wave forces has been that involving the use of the so-called "Morrison equation". This equation was proposed nearly thirty years ago by Morrison, O'Brien, Johnson, and Schaaf (1950) for the prediction of wave forces on a vertical pile. The equation has been much used since then, and has been further developed for use in situations for which it was not originally intended. The basic idea behind the equation is that the wave force can be separated into two terms; one representing the inertial or accelerative forces and the other representing the drag or viscous forces which are dependent on the fluid velocity. These two terms are simply added to give the total force acting on a section of the pile. We now discuss this equation in greater detail.

## 1.2 Morison's Equation

### 1.2.1 The Force on a Vertical Pile

In their 1950 paper Morison et al. proposed that the horizontal in-line wave force,  $df_x$ , on a small segment,  $ds$ , of height  $dy$ , of a vertical pile in water of depth,  $h$ , (see figure 1.1), is given by,

$$df_x = [C_m \rho V \dot{u}_x + \frac{1}{2} C_d \rho A u_x |u_x|] dy, \quad (1.1)$$

where  $\rho$  is the density of water,  $V$  is the volume per unit length of the pile,  $A$  is the cross-sectional area of the pile transverse to the wave direction,  $u_x$  is the horizontal water particle velocity, and  $C_m$  and  $C_d$  are the inertial and drag coefficients respectively.  $C_m$  is sometimes referred to as the mass or added mass coefficient.

The most important assumption involved in equation (1.1) is that the pile is of sufficiently small diameter that the waves are not significantly altered by its presence, and that if the pile were removed the wave velocities and accelerations would be constant across the region previously occupied by it. A convenient point in this region (usually the centre) is taken for the calculation of  $u_x$  and  $\dot{u}_x$ . These quantities are functions of time, wavelength, wave amplitude, and water depth and so to obtain, for example, the maximum force on a surface piercing pile an integration over the length of the pile must be performed. The moment about the pile's base can

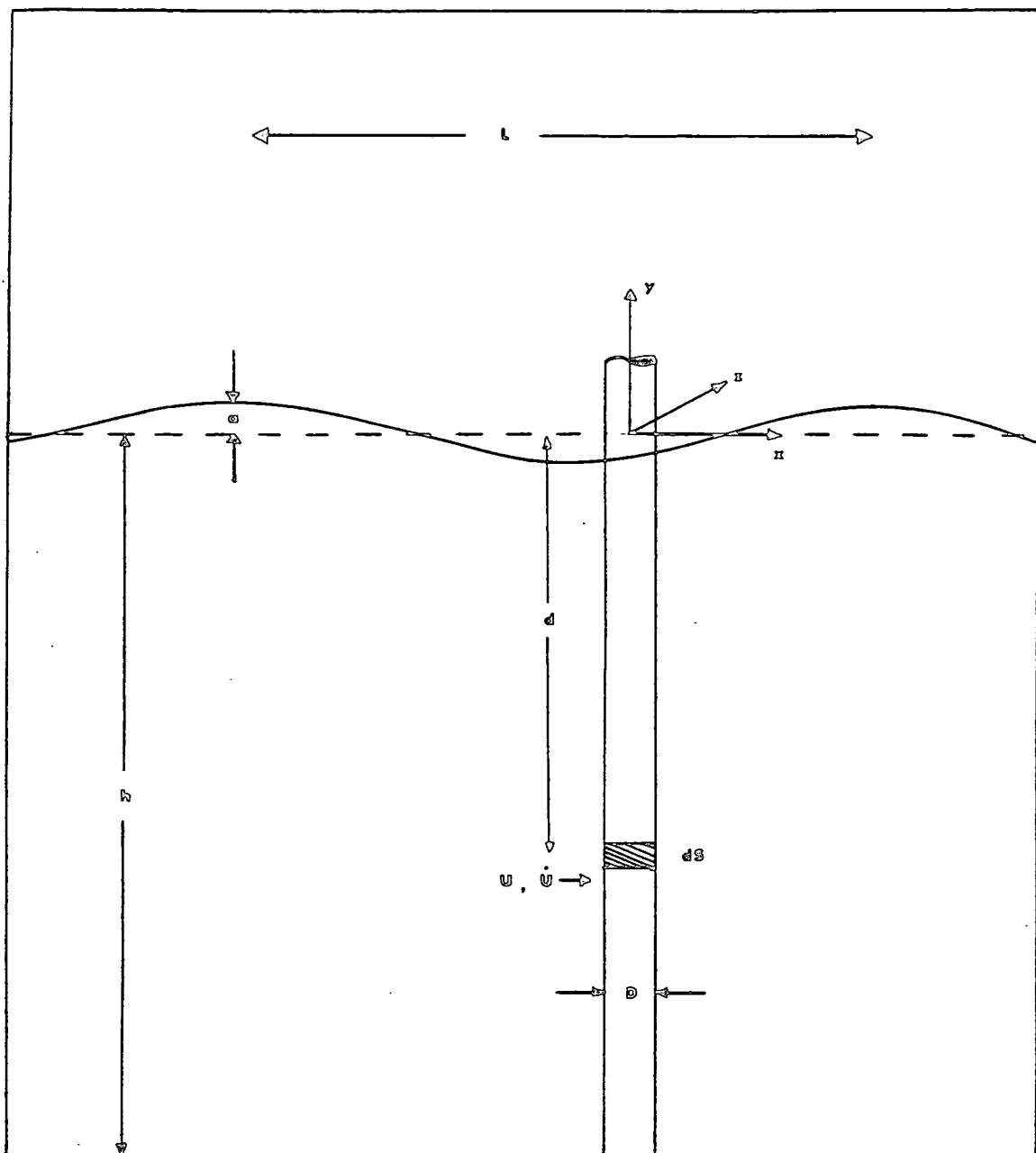


Figure 1.1 Schematic diagram showing the variables involved in Morison's equation.

be found from,

$$\int_{-h}^{\text{surface}} y df_x.$$

In their original paper Morison et al, only carried out the integration to the mean water level,  $y = 0$ , not to the actual surface,  $y = \eta(t)$ , asserting that the variation in moment arm caused by the changing length of the immersed pile would only be important in the case of breaking waves.

Although the use of equation (1.1) for predicting wave forces is a relatively recent proposal, the concepts of inertial and drag forces were established much earlier in the study of classical hydrodynamics.

### 1.2.2 Inertia Forces

The concept of an inertia force arose quite early in the study of the motion of objects placed in a fluid. Dubuat (1786) concluded that in order to describe the oscillation of spheres in water it was necessary to increase the mass of the sphere to account for the movement of the water. A similar conclusion was reached by Bessel (1826) in his study of the motion of pendula in a fluid. In this theory the object and the surrounding fluid are replaced by a single dynamical system having an inertia equal to the actual mass of the object plus the "added mass".

One method which has been used to treat the problem of the forces exerted on a fixed object in a moving fluid depends upon the object being at a sufficient distance below the free surface so that it can be considered to be in a fluid of infinite extent, and the flow is considered to be unidirectional. In a frictionless incompressible fluid of this type the inertia force,  $F_I$ , exerted on a submerged body may be expressed as (Milne-Thomson, (1968), p. 246),

$$F_I = (M_O + M_a) \ddot{U},$$

where  $M_O$  is the mass of the displaced fluid,  $M_a$  is the added mass which is dependent on the shape of the body and the flow field around it, and  $\ddot{U}$  is the acceleration of the undisturbed fluid in the region of the body with the body removed. The sum  $M_O + M_a$  is known as the "virtual mass". For a body of volume  $V$  the added mass is usually written in terms of an added mass coefficient,  $k$ , as follows,

$$M_a = k\rho V. \quad (1.2)$$

Therefore, the inertial force can be written,

$$\begin{aligned} F_I &= (1 + k)\rho V\ddot{U}, \\ &= C_m\rho V\ddot{U}, \end{aligned} \quad (1.3)$$

where  $C_m$  is the "inertia coefficient", sometimes referred to as the "mass coefficient".

The term  $\rho V \dot{U}$  in equation (1.3) is the resulting force obtained by integrating over the surface of the body the hydrodynamic pressure field due to the undisturbed incident wave. This force, which is usually termed the Froude-Krylov force is simply the force which in the absence of the body sustains the local acceleration of the fluid,  $\dot{U}$ . Thus the Froude-Krylov force is analagous to the buoyancy force in hydrostatics, where the resultant integrated pressure balances the weight of the fluid concerned (Archimedes principle).

The added mass coefficient,  $k$ , is a function of the size and shape of the body, the direction in which it is moved through the fluid with respect to an axis in the body, and the density and viscosity of the fluid. It can be calculated using inviscid incompressible irrotational flow theory by equating the rate of change of the total kinetic energy of the fluid and body to the rate at which the force driving the body does work. The body experiences a resistance to its motion of amount  $M_a \dot{U}$ . The added mass is the same whether the fluid is accelerating past the body or the body is accelerating in the fluid. The difference between the two situations is the appearance of the dynamic buoyancy force  $\rho V \dot{U}$  mentioned above in the case of the fluid moving past the body. In some papers, where added mass is defined to be  $1 + k$ , then it is indeed different for the two cases. Values of  $k$  for various shapes have

been calculated: e.g. for a sphere  $k = 0.5$  ( $C_m = 1.5$ ) and for a right circular cylinder  $k = 1.0$  ( $C_m = 2.0$ ). In the case of flat plates a nominal volume, such as a volume of revolution, is often used in performing the calculation.

An implicit assumption inherent in equation (1.3) is that the motion of the fluid can be characterised by a single acceleration,  $\ddot{U}$ , that is constant over the region of the body. Therefore, in the application of the added mass concept to the continuously varying accelerations (in time and space) of waves, we are restricted to situations where the body is small compared to the wavelength, so that we can consider the flow field to have an approximately constant acceleration over the region of the body. For larger bodies than this the incident wave will be considerably scattered and we must turn to diffraction theory for the evaluation of the forces. This will be discussed in a later section.

The formulation of the equation for the inertia force, given above involved the assumption that the object was in a fluid of infinite extent; an approximation that means that the object should be far from the free surface. The force depends only on the acceleration of the body and so ceases if the body moves with constant velocity. This gives rise to the well known theoretical prediction that a body moving in an infinite fluid (or one with rigid boundaries) with constant velocity experiences no drag force (D'Alembert's paradox). However, when an object

moves in a fluid near to a free surface the force on the object has two components; one in phase with the acceleration as before and one in phase with the velocity. If, for the sake of simplicity, we consider the body motion to be sinusoidal, then a train of waves moving outwards to infinity will be created and will carry away energy at a rate equal to the work done by the force component in phase with the velocity. This force component is usually described in terms of an "added damping" coefficient analagous to the added mass. The effect of the free surface on the added mass has been investigated by Murtha (1954) who measured the forces necessary to vibrate a circular cylinder vertically with very small amplitudes at a series of distances below the surface. The value of  $k$  varied from a minimum of 0.6 near the surface to 1.0 at a depth of only two diameters. Theoretical calculations of added mass for semi-immersed horizontal circular cylinders have been performed by Ursell (1949), Yu and Ursell (1961), and Ursell (1976). Ursell (1949) showed that for water of infinite depth the added mass coefficient was of order 1 over the greater part of the range of frequencies, but tended towards infinity at low frequencies. The extension by Yu and Ursell (1961) to water of finite depth contained errors that were corrected by Ursell (1976) who showed that the added mass in this case tended to a finite limit in long waves. A further discussion of added mass is given in Chapter 6 in connection with the dynamics of long beams near the free surface.

The damping term discussed above rises purely from the wave making action of an oscillating body, and so disappears in the case of a fixed object. The theory developed by Ursell and others is applicable only to a perfect fluid. In any real fluid there will be drag forces of viscous origin, the magnitude of which depends on a variety of parameters such as the shape and roughness of the body and the turbulence of the flow.

### 1.2.3 Drag Forces

It is the above mentioned drag forces which the second term in equation (1.1) attempts to model. The study of drag forces in a fluid initially concerned itself with bodies in uniform flow. The drag force was defined simply as the resultant fluid force acting on a body in a flow of constant velocity, or equivalently, the resistance experienced by a body moving with constant velocity through a still fluid.

Keulegan and Carpenter (1958) show through an analysis of the momentum equations of a real fluid that the drag force on an object of frontal area,  $A$ , in a stream of velocity  $U$  can be written in the form,

$$F_D = \frac{1}{2} C_d \rho A U^2, \quad (1.4)$$

where  $C_d$  is a function of the flow parameters. This analysis ignores the very important contribution to the drag force caused by the wake formed behind the cylinder.

The wake characteristics can be correlated to the flow parameters through the Reynold's number,  $R_e$ , defined by,

$$R_e = \frac{UD}{\nu}, \quad (1.5)$$

in which  $D$  is a typical length scale of the object and  $\nu$  is the kinematic viscosity, obtained by dividing the viscosity,  $\mu$ , of the fluid by its density,  $\rho$ . Morkovin (1964) gives a summary for a circular cylinder of the different regimes delineated by the Reynolds number, ranging from the laminar type flow for  $R_e < 1$ , through the twin vortex stage for  $3-5 < R_e < 30-40$ , the Karman range of alternate eddy shedding and the critical Reynolds number regime,  $2 \times 10^5 < R_e < 5 \times 10^5$ , to the post and transcritical regimes where turbulent separation occurs. For steady flow it has been found that the drag coefficient,  $C_d$  correlates well with Reynolds number, the most significant feature being a major transition in  $C_d$  over the critical regime as shown in figure 1.3a. Below this region the boundary layer is laminar and a periodic eddy wake is shed. Above it turbulence is established the wake contracts, is no longer periodic and the drag coefficient falls sharply. In order to use equation (1.4) for calculating the drag in waves it has to be slightly modified to ensure that it always has the correct sign; i.e. it is written as,

$$F_D = \frac{1}{2} C_d \rho A U |U|. \quad (1.6)$$

Although the drag coefficient for waves shows similar trends as a function of  $R_e$ , care must be taken when applying steady state values to wave flows because the flow is strongly influenced by two phenomena not present in steady flow: the elliptical orbital motion of the water particles and the fact that the wake is swept to and fro over the cylinder as the waves pass.

As well as the in-line drag force on a cylinder in uniform flow there is also a transverse component connected with the wake and the shedding of vortices, which is usually called the "lift" force. This force can under certain circumstances be of comparable magnitude to the in-line force. In waves, where we have motion in two directions simultaneously, the lift force due to motion in one direction will contribute to the force in the direction at right angles to this. This makes the analysis of the total force in terms of  $C_m$  and  $C_d$  difficult because in general it is not possible to separate the lift force from the drag and inertia forces. A more precise definition of the drag force would be: the component of fluid dynamic force on a body in line with and in phase with the velocity of the undisturbed flow. The lift force would then be defined as the component normal to the drag force.

#### 1.2.4 The Use of Morison's Equation

The most questionable step in the development of Morison's equation is the simple addition of the inertial

term and the drag term to give the total force. The admission of a drag force violates the assumptions upon which equation (1.3) was founded. The only justification that can be put forward is that, under a wide range of circumstances, the procedure results in sensible numbers that agree with experimental data. The drag and inertia terms interact; the inertia coefficient is modified by viscosity and the drag coefficient is increased over its steady state value because of the acceleration. Furthermore, in general the total force on an object depends not only on the instantaneous values of the velocity and acceleration, but on the entire time history of the motion. Because of these difficulties the use of theoretical values of  $C_m$  and  $C_d$  has been generally discarded in favour of empirical coefficients determined from fitting the force expression to experimental data.

Before we turn to the problem of evaluating the force coefficients we will discuss the water particle velocities and accelerations which must be computed. The most common method is to use the linear wave theory of infinitesimal waves derived from potential theory (Lamb (1952), Milne-Thomson (1968), Wiegel (1964), and Kinsman (1965)). This theory is fully described in the above references so only the principal results will be given here. The linear theory allows expressions to be developed for the water particle velocities and accelerations in terms of the parameters defining the surface wave elevation at the pile,  $\eta(t)$ , together with the water depth,  $h$ , and the

vertical position of the segment of the pile. If the wave elevation is given by,

$$\eta(t) = a \sin\left(\frac{2\pi t}{T}\right),$$

where  $a$  is the wave amplitude and  $T$  the wave period, the horizontal and vertical particle velocities and accelerations are given by,

$$u_x = \frac{2\pi a}{T} \frac{\cosh \frac{2\pi}{L}(y+h)}{\sinh \frac{2\pi h}{L}} \sin\left(\frac{2\pi t}{T}\right),$$

$$u_y = -\frac{2\pi a}{T} \frac{\sinh \frac{2\pi}{L}(y+h)}{\sinh \frac{2\pi h}{L}} \cos\left(\frac{2\pi t}{T}\right),$$

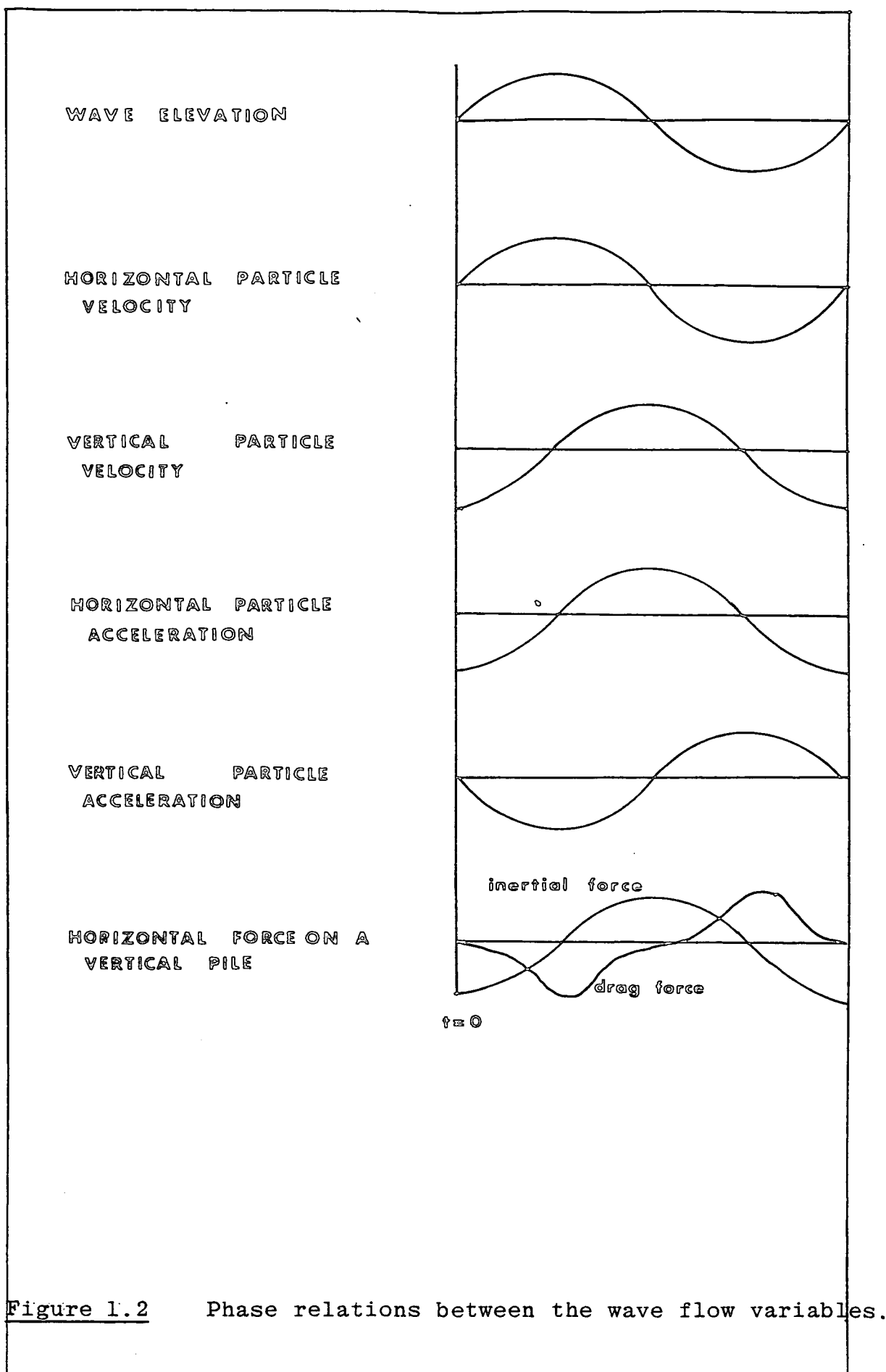
$$\dot{u}_x = -\frac{4\pi^2 a}{T^2} \frac{\sinh \frac{2\pi}{L}(y+h)}{\sinh \frac{2\pi h}{L}} \cos\left(\frac{2\pi t}{T}\right),$$

$$\dot{u}_y = -\frac{4\pi^2 a}{T^2} \frac{\sinh \frac{2\pi}{L}(y+h)}{\sinh \frac{2\pi h}{L}} \sin\left(\frac{2\pi t}{T}\right). \quad (1.7)$$

The wavelength of the wave,  $L$ , is related to the wave period via the dispersion relation,

$$\left(\frac{2\pi}{T}\right)^2 = \frac{2\pi g}{L} \tanh \frac{2\pi h}{L}.$$

Obviously, for a vertical pile only the horizontal



components are required. Figure 1.2 shows the relative form and phases of the parameters of interest. The general force terms have not been shown because in the case of an inclined cylinder the drag term has been shown to be of a slightly different form to equation (1.6). In fact it should be written as,

$$F_D = \frac{1}{2} C_d \rho A u_{x,y} |\underline{w}|, \quad (1.8)$$

where  $\underline{w}$  is the component of the velocity vector normal to the axis of the cylinder. This modification is discussed in Chapter 2 in connection with horizontal cylinders. However, for the horizontal force on a vertical pile equation (1.6) is still valid and is shown on figure 1.2.

As mentioned above the velocities and accelerations are a function of depth, and so the total force and moment about the pile bottom is found by integration over  $y$ . An exact integration, including the effect of the free surface, is given in Ippen (1966), p.349. It is also possible to calculate the phase of greatest moment and the position of the maximum force along the pile.

Several important conclusions can be derived from the above analysis. Firstly, the drag force diminishes more rapidly than the inertial force with distance below the surface. The maximum force does not occur simultaneously at all positions along the pile, but advances in phase from surface to bottom. It develops before the crest passes and this phase advance increases with the ratio of

the inertia term to the drag term. The relative importance of the inertial term increases with the ratio of pile diameter to wave height. A clearer insight into the relative size of these two terms is given by calculating the ratio of the maximum drag force to the maximum inertia force from Morison's equation. This is given by,

$$\frac{F_{D \max}}{F_{I \max}} = \frac{\frac{1}{2} C_d \rho A U_{\max}^2}{C_m \rho V U_{\max}} . \quad (1.9)$$

For the sinusoidal waves considered above  $U_{\max} = 2\pi a/T$  and  $\dot{U}_{\max} = 4\pi^2 a/T^2$ , and so for a circular cylinder,

$$\frac{F_{D \max}}{F_{I \max}} = \frac{C_d}{C_m} \frac{2 a}{\pi D} . \quad (1.10)$$

Since the inertial term is proportional to wave amplitude and drag to its square then a plot of force against this parameter is another way of deciding if either term is dominant.

As pointed out by Morison in his original paper waves of finite height depart from being sinusoidal and in fact are very nearly trochoidal. The orbit velocities are unsymmetrical, being greater in the forward direction than in the backward by 25% for a wave steepness ( $a/L$ ) of 0.05. Chakrabarti (1972, 1973 (a), 1974) employs Stokes fifth order theory for the calculation of the wave kinematics. Over a limited range of wave steepness and water depth this

gives improved results. This method relies on being able to write the surface elevation in the form,

$$\eta(t) = \eta_0 + \sum_{n=1}^5 A_n \sin \left[ 2\pi n \left( \frac{x}{L} - \frac{t}{T} \right) \right],$$

where  $\eta_0$  is the change in mean level and the coefficients  $A_n$  are amplitudes to be calculated. The formulae for particle velocities and pressures are written in a similar way.

The most direct approach to the evaluation of  $u_x$  and  $\dot{u}_x$  would be to take measurements of the flow field in the water (with or without the pile) using, for example, laser doppler anemometry (Lee, Greated, and Durrani (1974)), thus bypassing the limitations of any one wave theory. Nevertheless, linear wave theory remains as the most commonly used method of calculating these quantities and will be used in this work.

The evaluation of the coefficients  $C_m$  and  $\dot{C}_d$  can be done in one of several ways. Morison, et al (1950) noted that since in the linear theory  $u_x$  and  $\dot{u}_x$  differed in phase by  $90^\circ$ , there were particular points during the wave cycle at which either the inertial or the drag term was zero, while the other was a maximum, and this enabled him to calculate the remaining coefficient. The most serious difficulty associated with this method is the problem of finding the position where these phases occur. Errors in this can lead to quite large uncertainties in  $C_m$  or  $C_d$ , for example, if  $\eta(t) = a \cos(2\pi t/T)$ , the measured horizontal force on

a vertical pile could be written simply as,

$$F_{\text{measured}} = -\alpha \sin\left(\frac{2\pi t}{T}\right) + \beta \cos\left(\frac{2\pi t}{T}\right) \left| \cos\left(\frac{2\pi t}{T}\right) \right|.$$

The value of  $C_d$  would be found by finding the phase where  $2\pi t = 0$ , and then  $C_d = \beta / \rho A U_{\text{max}}^2$ . Now, if the phase was incorrect by 10% of one wave period say, then the calculated value of  $C_d$  would be found from,

$$\begin{aligned} C_d A U_{\text{max}}^2 &= -\alpha \sin\left(\frac{2\pi}{10}\right) + \beta \cos\left(\frac{2\pi}{10}\right) \left| \cos\left(\frac{2\pi}{10}\right) \right|. \\ &= -0.59\alpha + 0.65\beta \end{aligned}$$

So if the drag and inertia terms were of similar magnitude ( $\alpha = \beta$ ) then the calculated drag coefficient would be too small by a factor of 20.

Another method of finding the coefficients is to perform a least square fit of the equation to the experimental data. Since the equation is linear in  $C_m$  and  $C_d$  this is easy to do and means that the force over the entire wave period is taken into account, so unlike the method of phases, does not rely on only one data point. An important assumption in both these methods is that the value of the coefficients found will apply over the whole wave period. The first method finds values at particular points and the second results in an average value over the wave period. Neither of these procedures allow for the fact the  $C_m$  and  $C_d$  may be functions of time. In their analysis of the forces on cylinders and plates in an oscillating fluid Keulegan and Carpenter (1958) approached the problem of

calculating  $C_m$  and  $C_d$  in a different way. They expanded the force equation as a fourier series and were able to express the force coefficients in terms of the fourier coefficients. The inertial and drag terms were taken as the first two terms in the series. The analysis resulted in values of  $C_m$  and  $C_d$  which were weighted averages over a wave period as follows,

$$C_m = \frac{1}{\pi} \int_0^{2\pi} C_m(\theta) \sin^2 \theta d\theta,$$

$$C_d = \frac{1}{\pi} \int_0^{2\pi} C_d(\theta) |\cos \theta| \cos^2 \theta d\theta.$$

Keulegan and Carpenter also studied the variation of  $C_m$  and  $C_d$  during the wave cycle (i.e.  $C_m(\theta)$  and  $C_d(\theta)$ ) by an analysis of what they termed the remainder function, which was the difference between the computed and observed forces; in effect, the rest of the Fourier series. We will discuss their results in a later section.

### 1.3 Previous Work using Morison's Equation

Since Morison's equation was originally proposed for the calculation of forces on a pile with applications in the Gulf of Mexico, where, except in hurricanes and typhoons, the wave conditions were fairly moderate, the usual procedure was to use a "design wave". This single regular wave was chosen to simulate the worst possible wave that a structure was likely to encounter at sea. For this reason, and because measurements in the real ocean are very difficult to carry out, expensive, and time consuming, most of the early

experimental work on wave forces was carried out using regular waves in laboratory wave channels at a much smaller scale.

Morison et al (1950) reported average values of the coefficients for a vertical pile as,  $C_m = 1.5 \pm 0.2$  and  $C_d = 1.6 \pm 0.4$ . It was apparent in even the earliest experimental work that the coefficients showed wide scatter and that correlation with Reynolds number seemed to be far weaker than in the steady state case (Ippen (1966), figure 8.10 and Wiegel (1964), figure 11.8) and so its usefulness as a wave parameter is limited. In the context of waves, Reynolds number is usually computed from the maximum orbital velocity (which occurs at the free surface) and the pile diameter. For a vertical pile therefore, Reynolds number decreases from top to bottom and the correlation of an overall drag coefficient is perhaps somewhat meaningless. Another difficulty associated with the Reynolds number is that it is extremely difficult to produce prototype values of  $R_e$  in a laboratory wave channel, which is a limit on the practical usefulness of the data obtained in this way. Jen (1968) presents results of a laboratory study of inertia forces on a pile but emphasises that they are not directly applicable to prototype design. He concluded that the forces exerted by periodic waves of low steepness in relatively deep water were essentially inertial and his values of  $C_m$  ranged from 1.85 to 2.64 with an average of 2.04- very close to the potential theory value of 2.0. Wiegel, Beebe, and Moon (1957) give some of the first

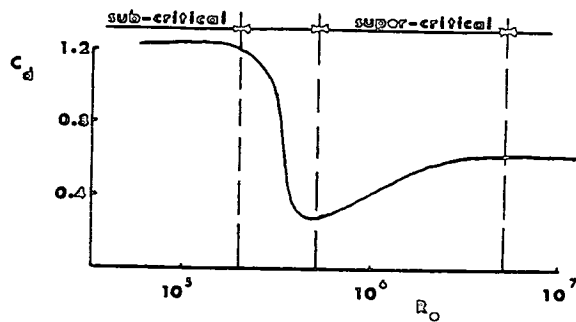


Figure 1.3(a) The variation of Reynolds number,  $R_e$ , with drag coefficient,  $C_d$ , in steady flow.

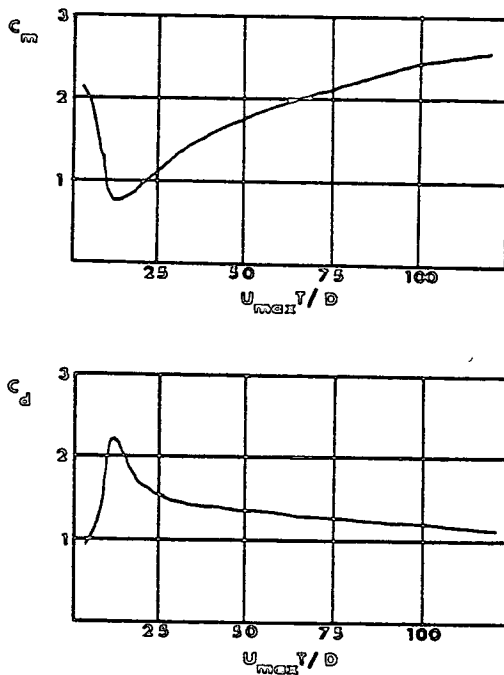


FIGURE 1.3(b)

The variation of Keulegan-Carpenter number,  $N_k$  with  $C_m$  and  $C_d$ .

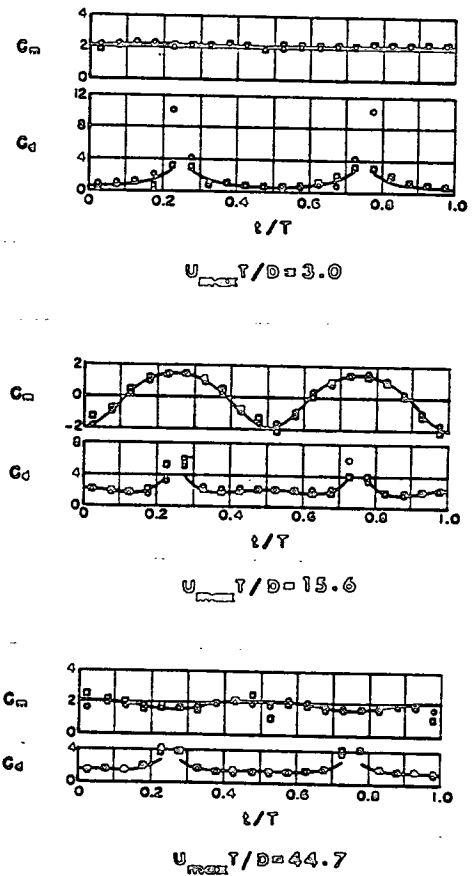


FIGURE 1.3(c)

The variation of  $C_m$  and  $C_d$  over a cycle (Keulegan and Carpenter (1958)).

extensive results of the wave forces exerted by ocean waves, but again, their data did not suggest much correlation with  $R_e$ . This was again brought out in Keulegan and Carpenter's paper (1958) on forces in an oscillating fluid. However, they did find that their values of  $C_m$  and  $C_d$  seemed to be related to a parameter,  $N_k$ , defined as,

$$N_k = \frac{U_{\max} T}{D} . \quad (1.11)$$

This has since become known as the Keulegan-Carpenter number and is referred to in their paper as the "period parameter". Figure 1.3b shows the variation of  $C_m$  and  $C_d$  with  $N_k$  for their results with cylinders. The data was taken in a rectangular basin with standing waves surging in it, so that at its centre the water would be a simple horizontal sinusoidal current. When the data in Wiegell, Beebe, and Moon (1957) is plotted in this fashion the empirical curves in figure 1.3b form an approximate upper bound for  $C_d$  and a lower bound for  $C_m$ . It is apparent from figure 1.3b that there is a critical value of  $N_k$ , at about 15-16, where the drag coefficient shows a sudden peak, and the inertial coefficient a sharp drop. We can shed some light on this by noticing that the Keulegan-Carpenter number is a measure of the water particle displacement ( $U_{\max} T$ ) relative to the size of the body. For large values of  $N_k$  the flow has time to approach a steady state situation with vortices being created. At lower values this is not allowed to happen and drag development may be negligible. If we use linear wave theory to calculate  $U_{\max}$  the  $N_k$  is

simply given by  $N_k = 2\pi a/D$ , a measure of the wave amplitude relative to the cylinder diameter. That  $N_k$  is a measure of the importance of the drag term can also be seen by returning to equation (1.10) which gives the relative size of the drag term to the inertial term in Morison's equation. This can be written in terms of  $N_k$  as,

$$\frac{F_{Dmax}}{F_{Imax}} = \frac{C_d}{C_m} \frac{N_k}{\pi^2}. \quad (1.12)$$

The variation of  $C_m$  and  $C_d$  over a cycle as studied by Keulegan and Carpenter is strongly influenced by this parameter. Figure 1.3c shows this variation at three different values of  $N_k$ . These results have been taken from Keulegan and Carpenter's original paper (1958). The same results given in Wiegel (1964, p.260) have been incorrectly labelled. The inertia coefficient is fairly constant throughout the cycle except at the critical value of  $N_k = 15-16$ . The drag coefficient also remains at its mean value except where the velocity goes through zero, where it increases considerably.

Garrison, Field and May (1977) were able to study the dependence of  $C_m$  and  $C_d$  over a much larger range of Reynolds number than had been attained by Keulegan and Carpenter. They also achieved independent control over  $R_e$  and  $N_k$  by conducting experiments, not in waves, but by oscillating a cylinder in still water. The period parameter could be chosen by altering the amplitude of the motion, and the Reynolds number varied by varying the speed of the driving

mechanism. Their results indicated that  $C_m$  and  $C_d$  were in fact strongly influenced by Reynolds number, except towards the high end of the range ( $Re > 2 \times 10^5$ ).

It would be wrong to place too much emphasis on the correct determination of  $C_m$  and  $C_d$  although the variations of these coefficients with flow parameters and phase has been the subject of much research in recent years. This may give the impression that Morison's equation is valid for any situation and that once the problem of determining the varying coefficients has been solved then that is the complete answer. Perhaps the correct line of reasoning would be to re-examine the problem physically as suggested by Lundgren et al (1979).

Only a small fraction of the available data on wave force and wave force coefficients has been discussed above. A recent comprehensive review of fluid loading on offshore structures has been given by Hogben et al (1977) who compiled an exhaustive list of coefficients and references pertaining to Morison's equation and its use. The overriding conclusion to be gained from an examination of this data is that the subject is one of uncertainty and guesswork, except in one or two narrowly defined areas.

#### 1.4 Other Methods of Wave Force Analysis

Although Morison's equation is the most commonly used method of predicting wave forces, there are others, some of which are more convenient and some that are more accurate in a given situation. The study of wave loading is often

approached via the method of dimensional analysis. The force is considered to be a function of the variables that are of influence and then these are grouped into non-dimensional parameters. The most usual variables to be taken into account are the fluid velocity and acceleration, density, and viscosity, gravity, and the geometry of the object. Of course, for waves the velocity and acceleration are taken to be functions of wave height, wavelength and water depth (wave period does not have to be included since it is related to these other variables through the dispersion relation). It was on the basis of dimensional analysis that Keulegan and Carpenter (1958) proposed that the period parameter would be a significant indicator to the size of the wave force. Similarly, Iversen and Balent (1951) attempted to model the net fluid resistance for a body in accelerating flow by writing,

$$F = \frac{1}{2} C_D A U^2,$$

where the overall force coefficient  $C$  was a function of Reynolds number, Froude's modulus, body shape and a correlation modulus  $\dot{U}D/U^2$ , (known since as Iversen's modulus), where  $D$  is a characteristic length of the body. Froude's modulus is a measure of the dynamical similarity between two situations where resistance is entirely due to gravity and is unaffected by viscosity. (In 1872, Froude produced experimental evidence to justify the separation of ship resistance into independent frictional and wave-making components). If viscous and gravity forces are negligible

then for similar shaped bodies the coefficient  $C$  will be the same in two situations provided that  $\dot{U}D/U^2$  is the same.

In his analysis of the wave forces on a submerged horizontal cylinder Schiller (1971) discarded Morison's equation in favour of a presentation of his experimental results entirely in terms of non-dimensional parameters, writing the maximum force as,

$$F_{\max}/(\rho g \frac{D^2}{4}) = f(\frac{\pi D}{L}, \frac{2L}{D}, \frac{2h}{D}, \frac{a}{D}). \quad (1.13)$$

He then presented graphs of this force as a function of each of these parameters in turn. The results of Herbich and Shank (1971) are plotted in a similar fashion.

Dalton, Hunt, and Hussain (1978) used a parametric analysis in their study of forces on an oscillating cylinder (cf Garrison, Field and May (1977)). They defined a force coefficient as  $C = F/(\frac{1}{2}\rho DU^2)$  and measured the dependence of  $C$  on instantaneous Reynolds number over one half cycle of cylinder motion, period parameter, and a third parameter,  $D/\sqrt{vT}$ . Sarpkaya (1976) has shown that the last parameter enables the data of Keulegan and Carpenter to be re-analysed and better understood.

The discussion of inertial forces in section 1.2.2 pointed out that the use of the added mass concept and therefore of Morison's equation was limited to situations where the pile was small compared to the wavelength. If this is not the case then considerable scattering of the

incident wave will take place and we must resort to diffraction theory for the calculation of wave forces. This method relies on potential theory where the force is found by integrating the pressure distribution around the object calculated from the velocity potential using Bernoulli's equation. The problem is that of determining the potential,  $\phi$ , which is written as the sum of the incident velocity potential (known) and the scattered velocity potential (unknown) as follows,

$$\phi = \phi_i + \phi_s. \quad (1.14)$$

$\phi_s$  is usually represented by a distribution of source potentials over the surface of the body with amplitudes that have to be found by solving Laplace's equation  $\nabla^2 \phi = 0$ , subject to suitable boundary conditions on the cylinder and at the free surface. Most commonly, infinitesimal waves are used where the boundary conditions on the free surface are linearised. MacCamy and Fuchs (1954) were the first workers to solve the problem of linearised wave forces on a vertical pile in water of finite depth using a diffraction theory developed from a solution for electromagnetic radiation diffraction (Morse (1948)). Havelock (1940) had previously done similar work for infinitely deep water. As part of their calculations MacCamy and Fuchs showed that when the cylinder became small in comparison with the wavelength, the force is equal to the inertial force predicted by Morison's equation provided the coefficient  $C_m$  was taken to be 2.0, so diffraction theory provides an

alternative method of calculating added masses. The interaction of waves with large submerged objects has been studied using diffraction analysis by Garrison and Rao (1971) and Garrison and Chow (1972). The numerical results presented in the second of these papers compared favourably with the closed form solution of MacCamy and Fuchs (1954). Although linearised diffraction theory has been used by most authors, extensions to fifth order Stokes theory have been done by Chakrabarti (1972, 1973(a), 1974), with applications to piles, halfcylinders, and hemispheres. In Britain, the National Maritime Institute has been deeply involved in waveloading and they have developed programs for computing the diffraction forces and added masses and dampings for use in other programs for predicting dynamic response of ocean structures, (Hogben and Standing (1975), Hogben (1977), Standing (1978)).

Diffraction theory neglects viscous forces and this may not always be justified. The conditions where this theory should be applied are defined by the ratio  $D/L$ , whereas the size of the drag term depends on the ratio  $a/L$ , (equation (1.10)). However, if  $5D > L \gg a$  then large diffraction forces are compatible with negligible drag, and for large structures in the ocean these inequalities hold in general.

It may be convenient, at this point, to review the various parameters that have been discussed and to summarise their relevance in the study of wave forces. Table 1.1 presents a summary of the salient features.

| PARAMETER                 | NOTATION | DEFINITION     | REMARKS                                                       |
|---------------------------|----------|----------------|---------------------------------------------------------------|
| Reynolds Number           | $R_e$    | $UD/\nu$       | In work on waves $U$ is usually the maximum orbital velocity. |
| Froude Number             | $N_f$    | $U/\sqrt{gD}$  |                                                               |
| Keulegan-Carpenter Number | $N_k$    | $UT/D$         | Equal to $2\pi a/D$ for linear waves                          |
| Iversen's Number          | $N_I$    | $\dot{U}D/U^2$ | Equal to $2\pi/N_k$                                           |
| Strouhal Number           | $N_s$    | $fD/U$         | Equal to $1/N_k$                                              |

Table 1.1 Important parameters in the study of wave forces.

Reynolds Number,  $R_e$ , is a measure of the dynamical similarity between two situations where viscosity is the most important influence. Above a certain critical value of  $R_e$  the flow becomes turbulent. It is still a matter of some debate whether or not  $R_e$  is a particularly useful criterion for wave forces, though some recent papers indicate that  $C_m$  and  $C_d$  are strongly influenced by it.

Froude Modulus,  $N_f$ , is a measure of dynamical similarity where resistance is entirely due to gravity. It is therefore important when the effects of the free surface are to be taken into account. Since it is difficult to achieve both Reynolds and Froude similarity between two flows the application of laboratory scale results to prototype situations is questionable unless one or other of the parameters is of little consequence.

Keulegan-Carpenter Number,  $N_k$ , is also called the "period parameter" and is a measure of the relative size of the water particle orbits and the cylinder diameter. It is thus an indicator of the transience of the flow so that for high values the velocity phase is sufficiently long for the drag to approximate to a steady state value. There exists a critical value of  $N_k$  ( $\approx 15-16$ ) in the region of which the drag coefficient increases sharply and the inertia decreases. For low values of the parameter inertial forces will dominate drag forces. It is sometimes referred to as the "amplitude parameter".

Iversen's Modulus,  $N_I$ , was proposed as a correlating

modulus for fluid resistance. It was a measure of the relative size of the inertial and drag term and, in fact is related to the Keulegan-Carpenter number by the formula,

$$N_I = 2\pi/N_K.$$

Strouhal Number,  $N_S$ , is associated with the spontaneous flow oscillations as in a flow past a cylinder, and is related to the frequency of the oscillating lift forces. The strouhal number is the reciprocal of the Keulegan-Carpenter number. The lift forces in waves can increase rapidly when the wave frequency equals the eddy shedding frequency, a situation known as "lock-in".

### 1.5 The Present Work

The oil crisis of 1973-4 highlighted the almost total dependence of developed nations like Britain and the United States of America on the availability of cheap fuel and energy. This motivated many research workers into examining the viability of other energy resources as possible alternatives in the event of the flow of crude oil being insufficient to meet demand, whether caused by political manoeuvres or simply the exhaustion of known supplies. The possibilities under investigation included solar power, wind, tidal and wave power. The climatic conditions prevailing in the seas off the north-west of Scotland showed promise that there might be sufficient energy present in a small enough area to make its extraction physically possible and economically feasible. One of the first methods put forward was that of Salter (1974, 1976) who proposed an

extraction mechanism now known as the "Salter Duck". Essentially this was a cam shaped segment which was allowed to rotate about a cylindrical "spine" placed near the sea surface. The shape of the duck was chosen so that the movement of the front would follow closely the motion of the water while the back part should have as small wave-making properties as possible, since considerable energy would be lost through this. It was envisaged that long lines of these ducks (known as a "string") would be placed facing the waves.

Clearly, in the design of such a structure an extremely detailed knowledge of the likely wave loading is needed. Rather than use previous data (which for horizontal cylinders in the free surface was extremely scarce) Salter adopted a philosophy of relying on the results of careful measurements made on model ducks at 150th the expected prototype scale. The results of the initial series of tests are detailed in Jeffrey, Richmond, Salter, and Taylor (1976). As well as measuring the forces on the ducks, extensive results were taken of the forces on a fixed horizontal cylinder for a variety of different wavelengths, wave amplitudes, and depths of immersion, and some of these are shown in figure 1.4. The horizontal forces are in general about twice the magnitude of the vertical forces which display a good deal more variation with phase, axis depth and frequency. Indeed the vertical force can show excitation at twice the wave frequency and in some cases is always negative (downwards). These phenomena cannot be predicted by Morison's equation

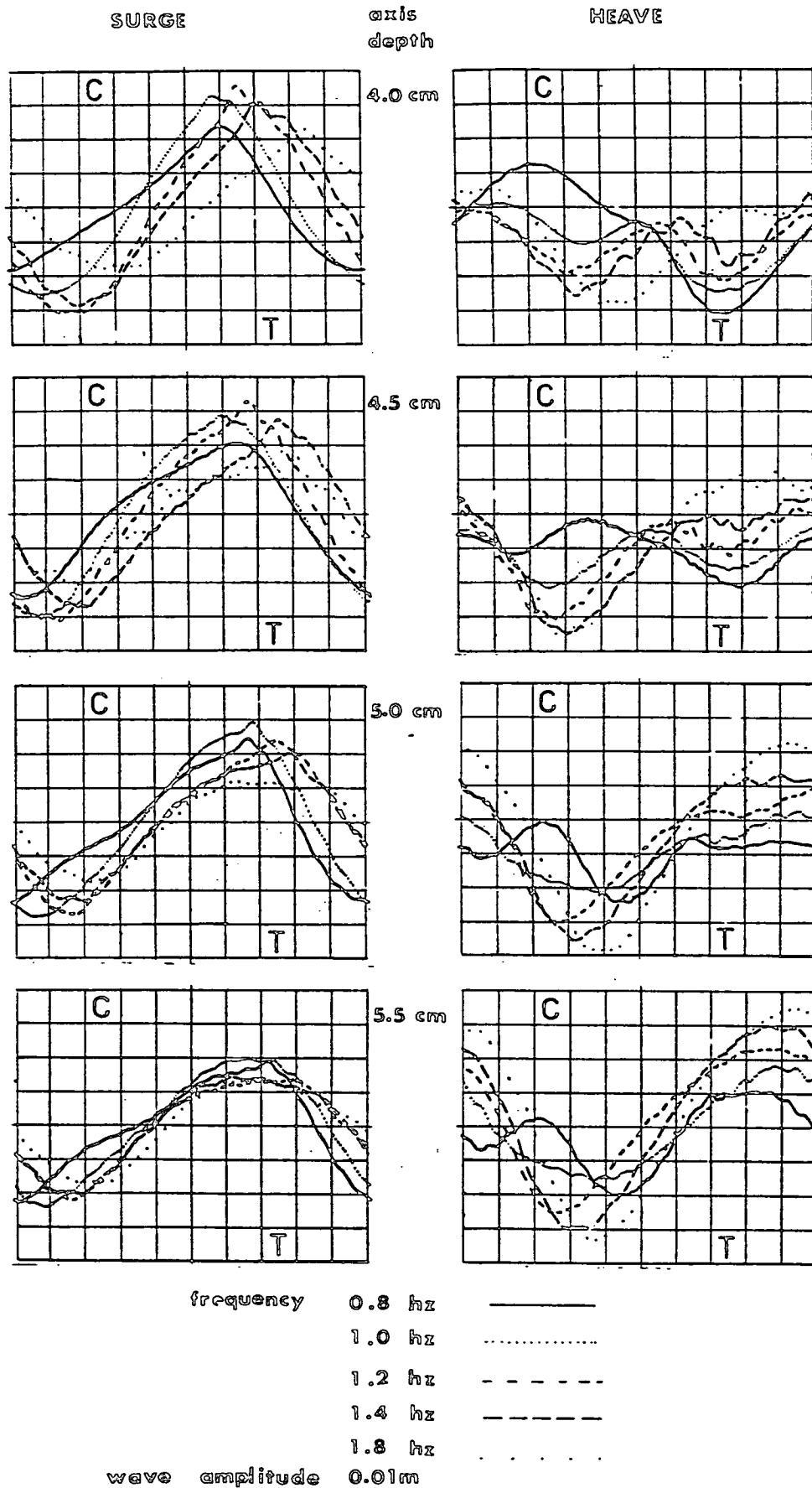


Figure 1.4      Some of the results of force measurements on a 10cm diameter horizontal cylinder taken by Salter (Jeffrey et al. (1976)).

as discussed in section 1.2. Although it is possible to determine  $C_m$  and  $C_d$  so that the peak or average forces are correctly given, this results in an extremely wide scatter in values, with even negative values of  $C_m$  being sometimes necessary (e.g. see figure 1.4, axis depth = 4 cm frequency = 0.8 hz). Their tests with different amplitudes suggested that the forces were proportional to wave height rather than its square, and so it seemed reasonable to suppose that the drag term would be small in comparison with the inertial term. The apparent futility of using Morison's equation led Salter to propose a peak force formula of the following form,

$$F = 2C_f \rho g a \ell D, \quad (1.15)$$

where  $\ell$  is the length of the cylinder. The force coefficient,  $C_f$  was assumed to be a function of  $L/D$ . Applying this equation to his data Salter found that  $C_f$  varied considerably less than did  $C_m$ . Equation (1.15) is very similar to the maximum amplitude of Morison's inertial term, and indeed the equivalence would be exact if  $C_f = C_m \frac{\pi^2}{4} \frac{D}{L}$ , (assuming that the deep water dispersion relation is valid).

The phenomena described in the previous paragraph arise from an interplay between inertial and buoyancy forces. The work presented in Chapter 2 attempts to show that it is possible to retain Morison's equation as a basis for force prediction for partially submerged horizontal cylinders by

making some simple modifications to it. An extension to the more realistic case of random seas is given in Chapter 4, where the description of the forces is recast in terms of power spectra, correlation functions and probability density functions. The reflections from the cylinder that were ignored in the application of Morison's equation in Chapter 2 are studied in Chapter 3, with the results being used in the prediction of mean horizontal forces. The experimental equipment and methods of analysis are discussed in Chapter 5. Once a knowledge of the forces on fixed ducks and cylinders has been obtained it is necessary to examine the dynamics of the entire string. In Chapter 6 preliminary results are given for the response of a long circular beam lying in the surface in terms of the bending moments along its length. An attempt is made to incorporate some of the ideas discussed in previous chapters. Some of the material presented in this thesis has formed the basis of two published papers (see Appendix III).

## CHAPTER 2

### PARTIALLY SUBMERGED CYLINDERS IN REGULAR WAVES

## CHAPTER 2

### PARTIALLY SUBMERGED CYLINDERS IN REGULAR WAVES

#### 2.1 Introduction

Morison's equation, equation (1.1) is strictly applicable only for the horizontal in-line force on a vertical pile of small cross-section compared to the wavelength, but it has, however, been generalised for use in other situations. The velocities and accelerations in equation (1.1) are components normal to the pile axis, and so the variables  $F$ ,  $u_x$ , and  $\dot{u}_x$  are vectors which can be treated as scalars. The tangential components of the velocity and acceleration are ignored in the prediction of the force. Borgman (1958) applied similar reasoning to the general case of a generally inclined cylinder and rewrote equation (1.1) as a vector equation, thus

$$\underline{F} = C_m \rho V \underline{\dot{w}} + \frac{1}{2} C_d \rho A w |\underline{w}|, \quad (2.1)$$

where  $\underline{w}$  and  $\underline{\dot{w}}$  are the normal velocity and acceleration vectors, (see figure 2.1a). That is, if  $\underline{u}$  and  $\underline{\dot{u}}$  are the water particle velocity and acceleration vectors and  $\underline{n}$  is the unit vector normal to the pile and in the plane formed by the pile axis and  $\underline{u}$ , then

$$\underline{w} = (\underline{u} \cdot \underline{n}) \underline{n}, \quad (2.2a)$$

$$\underline{\dot{w}} = (\underline{\dot{u}} \cdot \underline{n}) \underline{n}, \quad (2.2b)$$

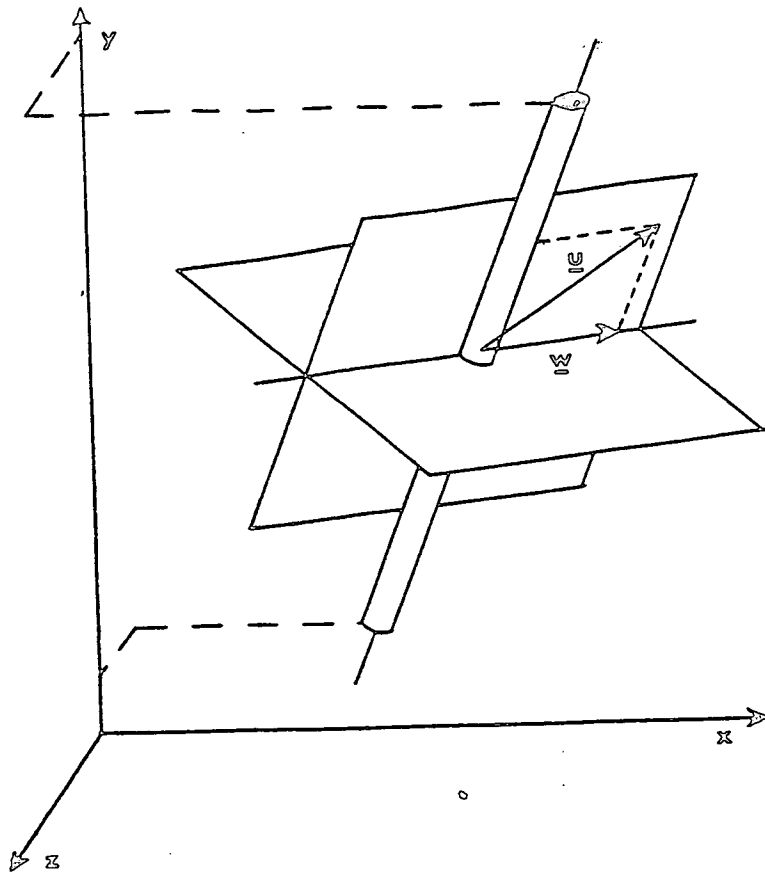


Figure 2.1(a) Vector components for an arbitrarily oriented pipe.

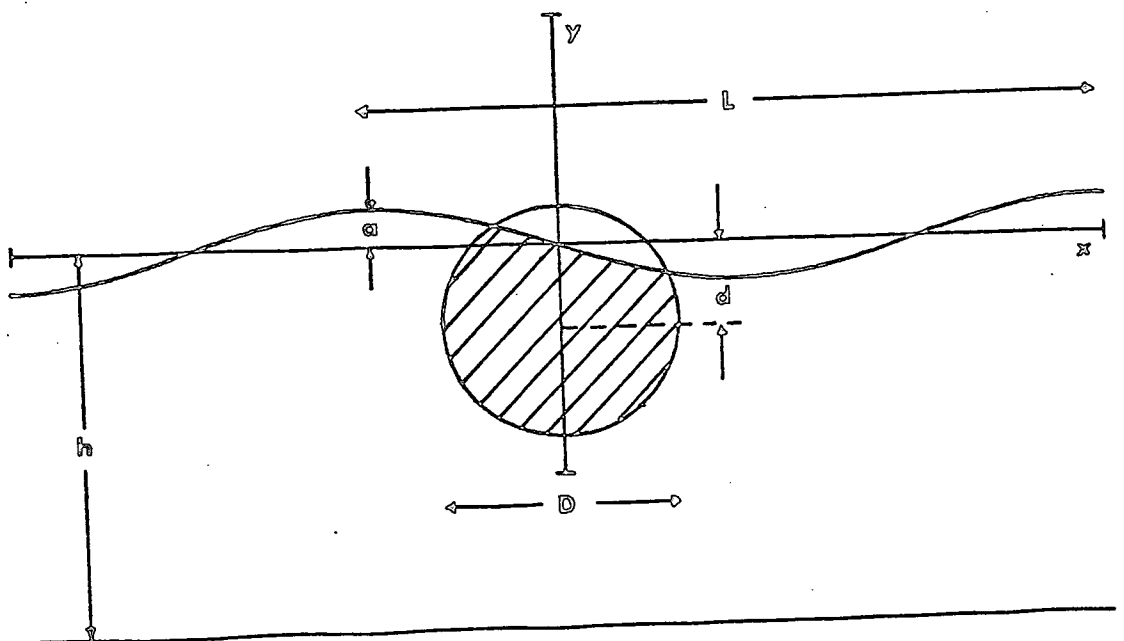


Figure 2.1(b) Schematic diagram for horizontal cylinder in waves

and  $\underline{n}$  is chosen so that  $\underline{u} \cdot \underline{n} \geq 0$ .

In general the normal velocity vector and acceleration vector will not be in the same line, and so the sum of an inertial term and a drag term must be taken to be a vector sum. Chakrabarti <sup>et al.</sup>/(1975) has shown that for many years the wrong expression has been used to predict the forces on a horizontal cylinder. Most workers had used the following equations,

$$F_x = C_m \rho V \dot{u}_x + \frac{1}{2} C_d \rho A u_x |u_x|, \quad (2.3a)$$

$$F_y = C_m \rho V \dot{u}_y + \frac{1}{2} C_d \rho A u_y |u_y|. \quad (2.3b)$$

Chakrabarti demonstrates that this is incorrect and that the forces should be written,

$$F_x = C_m \rho V \dot{u}_x + \frac{1}{2} C_d \rho A u_x |\underline{w}|, \quad (2.4a)$$

$$F_y = C_m \rho V \dot{u}_y + \frac{1}{2} C_d \rho A u_y |\underline{w}|. \quad (2.4b)$$

As  $\underline{w}$  involves both  $u_x$  and  $u_y$  it can be seen that the force in the x direction involves the velocity in the y direction and vice versa. It is the drag term only that is affected, the inertial one remaining the same. The values of  $C_m$  and  $C_d$  will therefore be slightly modified, and care must be taken when interpreting other work to check which set of equations (2.3 or 2.4) have been employed. For a vertical cylinder  $|\underline{w}| = |u_x|$  and so equation (2.3a) is correct.

When Morison's equation is used to predict the total force on a vertical pile the integration over depth is usually carried from the bottom to the mean water level not the actual water level. This means that forces due to the fluctuations of the free surface are neglected. The relative size of this effect depends on the ratio of wave amplitude to wavelength (since short waves are "concentrated" nearer the surface than are long waves because the exponential decay with depth depends on the wavelength) and to some extent also on the ratio of wave amplitude to water depth. In their application of diffraction theory MacCamy and Fuchs (1954) estimated the error in the force due to surface fluctuations. For  $d/L = 0.4$  and  $a/L = 0.02$  the error was less than 2%. This theory assumes, however, that there is no drag. The relative size of the drag term in Morison's equation increases as the free surface is approached so the contribution of drag to the forces caused by surface fluctuations is important in general.

Tung (1975) treats the effects of surface fluctuations on the statistical properties of waves and wave forces. He employs the Heaviside unit function to describe the wave kinematics.

The study of wave forces on horizontal cylinders has not been given anywhere near as much attention as that of piles, even though the use of horizontal members in ocean structures is widespread. What little work has been done

has concentrated in the main on the modelling of pipelines and so attention has been focussed on horizontal cylinders on or near the seabed (Schiller (1970), Norman (1977)). Schiller's work is a parametric study of wave forces and he does not use Morison's equation. Norman has studied drag and lift forces caused by vortex shedding.

A great deal of effort has been put into diffraction theory for submerged objects, usually in an attempt to model wave forces on large submerged tanks (Garrison and Rao (1971), Garrison and Chow (1972), Chakrabarti (1973), Chakrabarti and Naftzger (1974)). Herbich and Shank (1971) conducted a series of experimental studies on a variety of submerged objects, and Muellenhoff and Slotta (1971) concentrated on the forces on a submerged circular horizontal cylinder.

An important method of investigation extensively employed is to study the forces on a cylinder, not in waves, but in a sinusoidally oscillating fluid, e.g. in a U-tube, (Sarpkaya (1975, 1976)). Emphasis is usually placed on vortex shedding and the variation of inertial, drag, and lift coefficients with Keulegan-Carpenter number and Reynolds number. Care is needed when extending these results to waves because motions are in two directions at once, not just one. A closely related technique to this is to oscillate a body in still water and measure the forces, (Hamann and Dalton (1971), Dalton, Hunt and Hussain (1978)). These forces can then be related to the problem

of a fixed body in a moving fluid in a simple way. Because of the proximity of the walls in oscillating water tunnels results obtained by this method are chiefly applicable to objects placed near the seabed.

For horizontal cylinders at or near the free surface variations in the waterline are obviously going to be of great importance. Despite the apparent difficulties associated with objects on the free surface, the work presented in this chapter attempts to show that a modified version of Morison's equation can predict the forces due to a broad range of regular waves on a partially submerged cylinder with accuracy; both with regard to the detailed variation of the force over a wave period and also to average and peak forces.

## 2.2 Modifications to Morison's Equation

### 2.2.1 General

We shall be concerned with the horizontal and vertical wave forces on a horizontal circular cylinder placed at or near the free surface in a train of one dimensional regular waves (assumed to be sinusoidal). Figure 2.1b shows a schematic diagram of the situation together with the relevant parameters.

As the study of wave forces was prompted by the need for knowledge of loads on wave energy devices the wave regime of interest was the region between drag dominated flow on one hand and flow where diffraction was the most important factor on the other. This region can be defined by two

parameters; the relative wave amplitude,  $a/D$ , and the ratio of cylinder diameter to wavelength,  $D/L$ . Reynolds number is also an important criterion for defining a flow region. Table 2.1 lists the smallest and the largest values of these parameters used in the present work.  $R_e$  has been calculated in the manner described in Chapter 1 using linear wave theory.

A comparison of the figures in Table 2.1 with the diagram on p.519 of Hogben et al (1977) shows that in some cases diffraction effects might be important but in general they are small enough to be dealt with using the added mass concept. Nath and Harleman (1970) and Garrison and Rao (1971) also discuss the relative size of the drag and the inertial terms concluding that for small values of  $a/D$  then the drag will be negligible.

If we use equation (1.12) to compare the drag and inertia terms, then we can see that for the largest values of  $a/D$  used drag will be important. However for full scale ducks and for most of the measurements presented here  $a/D$  will be much smaller and drag will be small. Except for the extreme waves the Reynolds number is well below critical, which is to be expected for experiments conducted in a Laboratory wave tank. The problem of scaling will be discussed later. We shall aim to modify Morison's equation in a physically plausible way such that we need not resort to a numerical calculation for the force using diffraction theory, as it seems at first sight we might have to.

| PARAMETER                              | SMALLEST VALUE  | LARGEST VALUE   |
|----------------------------------------|-----------------|-----------------|
| Wave amplitude (a)                     | 0.005m          | 0.050m          |
| Wavelength (L)                         | 0.482m          | 4.344m          |
| Water depth (h)                        | 0.60m           | 0.60m           |
| Cylinder diameter (D)                  | 0.016m          | 0.100m          |
| Reynolds Number ( $R_e$ )              | $3 \times 10^2$ | $6 \times 10^5$ |
| Keulegan-Carpenter<br>Number ( $N_k$ ) | 0.31            | 19.67           |
| a/D                                    | 0.05            | 3.13            |
| L/D                                    | 4.82            | 271.00          |
| a/L                                    | 0.001           | 0.104           |
| L/h                                    | 0.80            | 7.23            |

Table 2.1 Values of the parameters defining the wave regime considered in this work.

### 2.2.2 Horizontal Wave Forces

Equation (2.4a) gives the force on a submerged horizontal cylinder. Referring to the comments in the paragraph above we firstly discard the drag term as being small. We next replace the constant volume,  $V$ , in the inertial term by the varying volume,  $V(t)$ . Intuitively the force should depend on how much the cylinder is submerged, and so  $V(t)$  is simply the displaced volume of water at any instant. Hence,

$$F_x = C_m \rho V(t) \ddot{u}_x. \quad (2.5)$$

If the body is a simple shape  $V(t)$  can be written down as a analytical function. This will be done below for the case of a circular cylinder in waves of small steepness.

### 2.2.3 Vertical Wave Forces

In the case of vertical wave forces there is an additional modification to make because the varying waterline gives rise to an additional varying buoyancy force proportional to the change in displaced volume. This buoyancy force,  $F_b$ , is given simply by,

$$F_b = \rho g (V(t) - V_0). \quad (2.6)$$

Here,  $V_0$  is the initial displacement of the cylinder giving rise to the static buoyancy. Since we are only interested in the forces due to the passage of the wave

we subtract this force. The total vertical force is now given by,

$$F_y = C_m \rho V(t) \dot{u}_y + \rho g(V(t) - V_0). \quad (2.7)$$

We will now show through a simple case how equation 2.7 can predict the interesting phenomena that were described in Chapter 1.

Consider a semi-submerged square block of side  $b$ , and volume  $V$ , (see figure 2.2a). If we assume that the water-line is always horizontal and that the wave elevation at  $x = 0$  is given by  $\eta(0,t) = b/2 \sin 2\pi t/T$ , then

$$V(t) = V/2(1 + \sin(2\pi t/T)), \text{ and } V_0 = V/2$$

Using linear wave theory we can write (Wiegel (1964), chap.2)

$$\dot{U}_y = A \sin \frac{2\pi t}{T},$$

where  $A$  is constant in time. Hence we can write

$$F_y = B(-\sin \frac{2\pi t}{T}) (1 + \sin \frac{2\pi t}{T}) + C \sin \frac{2\pi t}{T},$$

where  $B$  and  $C$  are also constants. If we have chosen the parameters such that  $B = C$ , then

$$F_y = -B (\sin^2 \frac{2\pi t}{T}).$$

Since  $B$  is always positive then the resultant force is as shown in figure 2.2b Obviously the effects introduced by

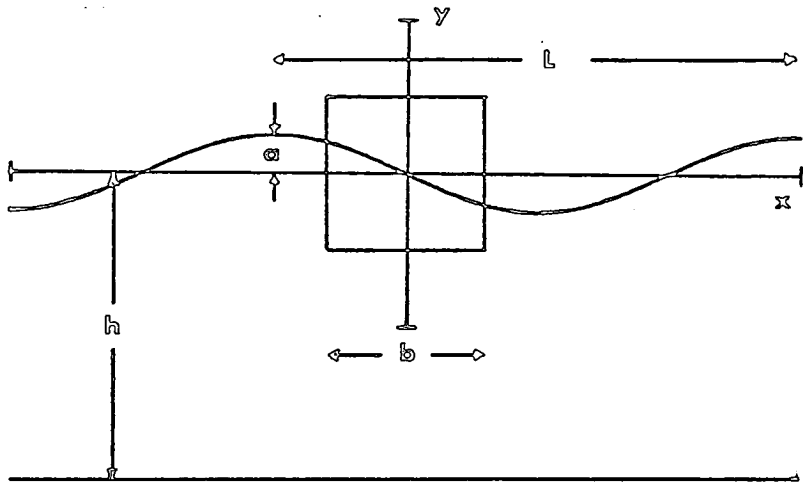


Figure 2.2(a) Schematic diagram for square section cylinder in waves

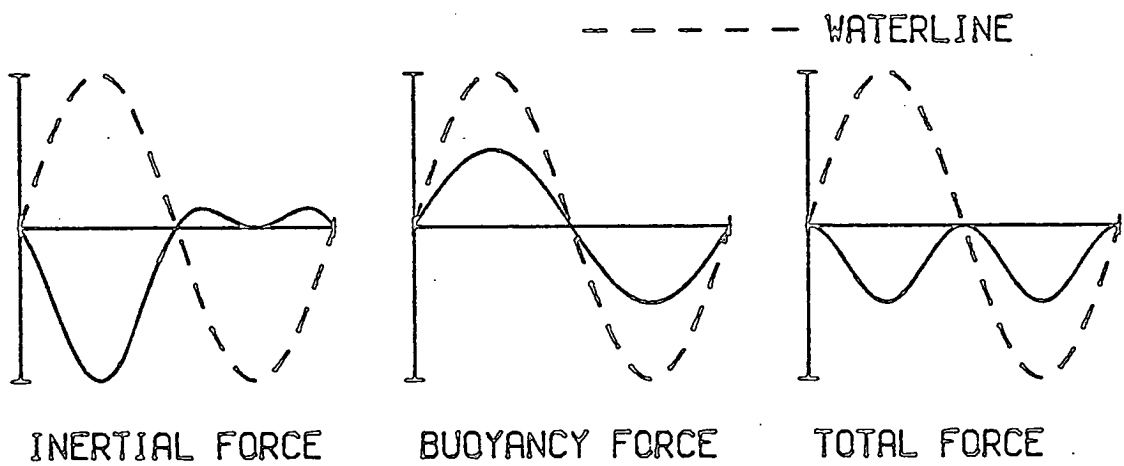


Figure 2.2(b) Form of inertia and buoyancy forces for square cylinder

the sharp edges have been neglected, but the force does show the characteristics that we are looking for.

#### 2.2.4 Forces for a Circular Cylinder

We will now apply equations (2.5) and (2.7) to the case of a circular cylinder, restricting our attention in the first instance to situations where the waterline stays within the limits of the cylinder. That is when,

$$-D/2 + d \leq \eta(t) \leq D/2 + d, \quad (2.8)$$

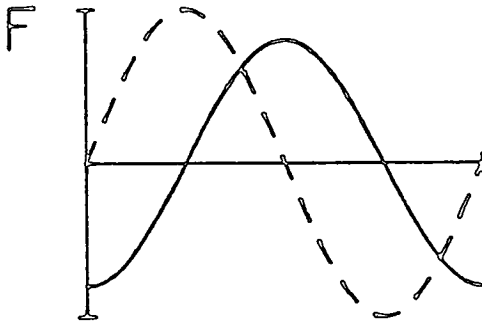
where  $\eta(t)$  is the wave elevation at  $x = 0$ , and  $d$  is the cylinder axis depth referred to the still waterline, (positive = upwards), as shown in figure 2.2.

The displaced volume,  $V(t)$ , is given simply by the area of the cross-hatched section times the cylinder length,  $\ell$ . This area can be approximated by the area under the horizontal line  $\eta(x,t) = \eta(0,t) = \eta(t) = a \sin(\frac{2\pi t}{T})$ , provided the wave steepness is small, which is in accord with the assumption of small  $a/L$ , (see Table 2.1). This area is given by a simple integration as a function of time by,

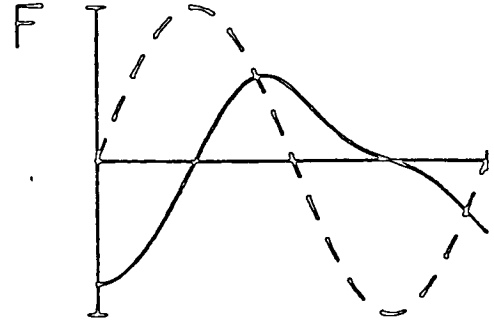
$$A(t) = \frac{D^2}{8} \left( \pi + \frac{4}{D}(-d + \eta(t)) \left[ 1 - \frac{4}{D^2}(-d + \eta(t))^2 \right]^{\frac{1}{2}} + 2 \sin^{-1} \left[ \frac{2(-d + \eta(t))}{D} \right] \right). \quad (2.9)$$

We must now choose an appropriate wave theory for calculating  $\dot{u}_x$ . Linear wave theory has been employed

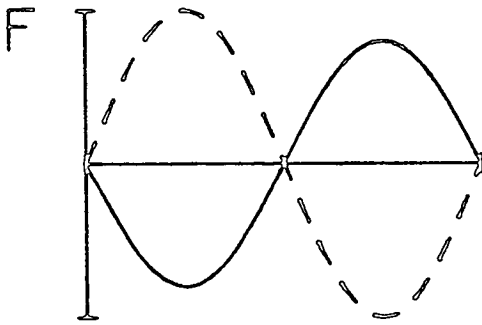
--- WATERLINE



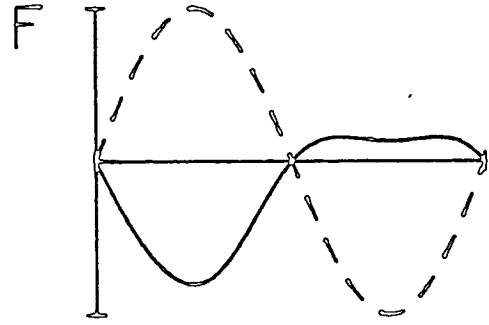
HORIZONTAL FORCE FOR  
TOTALLY SUBMERGED  
CYLINDER



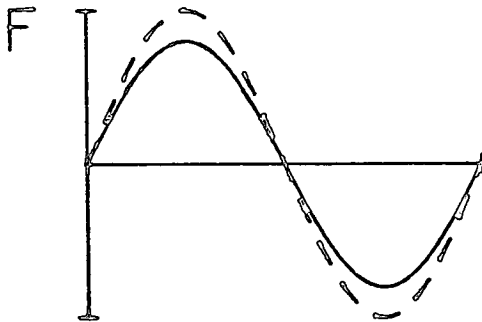
HORIZONTAL FORCE FOR  
PARTIALLY SUBMERGED  
CYLINDER



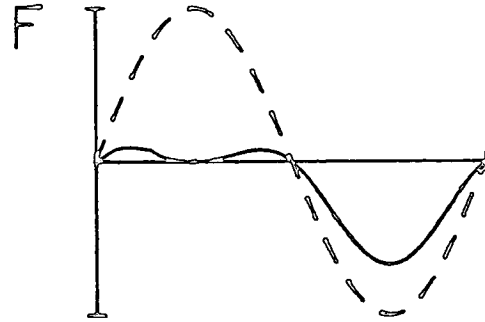
VERTICAL FORCE FOR  
TOTALLY SUBMERGED  
CYLINDER



INERTIAL FORCE FOR  
PARTIALLY SUBMERGED  
CYLINDER



BUOYANCY FORCE FOR  
PARTIALLY SUBMERGED  
CYLINDER



TOTAL VERTICAL  
FORCE FOR PARTIALLY  
SUBMERGED CYLINDER

Figure 2.3 Form of relevant forces on a  
circular cylinder

extensively in the past (Wiegel (1964), chap. 4) and is used here for two reasons. Firstly, the wave steepnesses involved are all small and secondly the errors are considered to be smaller than those due to the neglect of diffraction forces. Dean (1970) compares various wave theories as to their relative validity in different wave regimes. His comparisons are made on the goodness of fit to the free surface boundary conditions. It should be noted that the theory which fits these conditions most accurately may not be the one which predicts water particle velocities and accelerations the best.

If  $\eta(0,t) = a \sin(\frac{2\pi t}{T})$  then the linear theory gives the horizontal water particle acceleration as,

$$\dot{u}_x = -\frac{4\pi^2 a}{T^2} \frac{\sinh \left[ \frac{2\pi(y+h)/L}{\sinh \left[ \frac{2\pi h}{L} \right]} \right] \cos(\frac{2\pi t}{T})}{\sinh \left[ \frac{2\pi h}{L} \right]} \quad (2.10)$$

where we take  $y$  to be the vertical position of the cylinder axis,  $d$ . If  $2\pi(y+h)/L \gg 1$  then we can approximate the  $\sinh$  terms by exponentials and use the deep water dispersion relation,  $L = gT^2/2\pi$ , to give,

$$\dot{u}_x = -\frac{2\pi g}{L} \exp\left(\frac{2\pi d}{L}\right) a \cos\left(\frac{2\pi t}{T}\right). \quad (2.11)$$

Similarly, the vertical water particle acceleration is,

$$\dot{u}_y = -\frac{2\pi g}{L} \exp\left(\frac{2\pi d}{L}\right) a \sin\left(\frac{2\pi t}{T}\right). \quad (2.12)$$

We note for future reference that equations (2.11) and (2.12) can be written as,

$$\dot{u}_x = - \left[ \frac{2\pi g}{L} \right]^{\frac{3}{2}} \exp \left( \frac{2\pi d}{L} \right) \dot{\eta}(t) \quad (2.13a)$$

$$\dot{u}_y = - \frac{2\pi g}{L} \exp \left( \frac{2\pi d}{L} \right) \eta(t). \quad (2.13b)$$

The initial displaced volume,  $V_0$ , is given by,

$$V_0 = \frac{D^2}{8} \left[ \pi + \frac{4}{D} (-d) \left( 1 - \frac{4d^2}{D^2} \right)^{\frac{1}{2}} + 2 \sin^{-1} \left( -\frac{2d}{D} \right) \right] \quad (2.14)$$

We can transform the force equations into a more useful form by defining some dimensionless parameters as follows:

|                         |                                                         |
|-------------------------|---------------------------------------------------------|
| relative force          | $F'_{x,y} = F_{x,y} / (\rho g \frac{\pi D^2}{4} \eta),$ |
| relative wave elevation | $\eta' = \eta/D,$                                       |
| relative wave amplitude | $a' = a/D,$                                             |
| relative wavelength     | $L' = L/D,$                                             |
| relative axis depth     | $d' = d/D. \quad (2.15)$                                |

The dimensionless variable  $F'$  measures the ratio of the wave force to the weight of water displaced by a totally submerged cylinder in still water, i.e. the static buoyancy.

Substituting the above equations for  $\dot{u}_x$ ,  $\dot{u}_y$ ,  $V(t)$  and  $V_0$  into equations (2.6) and (2.7) it is possible to write expressions for the horizontal and vertical forces in terms of the above relative parameters.



parameters,

$$F'_x = -C_m \frac{a'}{L'} \exp \left( \frac{2\pi d'}{L'} \right) [\pi + 4(-d' + \eta(t)) [1 - 4(-d' + \eta'(t))^2]^{\frac{1}{2}} + 2\sin^{-1} 2(-d' + \eta'(t))] \cos \frac{2\pi t}{T} \quad (2.16a)$$

$$F'_y = -C_m \frac{a'}{L'} \exp \left( \frac{2\pi d'}{L'} \right) [\pi + 4(-d' + \eta(t)) [1 - 4(-d' + \eta'(t))^2]^{\frac{1}{2}} + 2\sin^{-1} 2(-d' + \eta'(t))] \sin \frac{2\pi t}{T} + \frac{1}{2\pi} [4(-d' + \eta'(t)) \times [1 - 4(-d' + \eta'(t))^2]^{\frac{1}{2}} + 2\sin^{-1} 2(-d' + \eta'(t)) + 4d'(1 - 4d'^2)^{\frac{1}{2}} - 2\sin^{-1} (-2d')] \quad (2.16b)$$

Equations (2.16) are only valid when the inequality equation (2.8) is satisfied. We can overcome this restriction by introducing a function  $G(\eta)$ , dependent on the wave elevation,  $\eta(t)$ . That is, we write,

$$F'_x = C_m \rho G(\eta) \ddot{u}_x / (\rho g \frac{\pi D^2 \ell}{4}), \quad (2.17a)$$

$$F'_y = [C_m \rho G(\eta) \ddot{u}_y + \rho g (G(\eta) - V_0)] / (\rho g \frac{\pi D^2 \ell}{4}) \quad (2.17b)$$

where

$$\begin{aligned} G(\eta) &= V_T, \eta > (D/2) + d, \\ &= V(t), d - (D/2) < \eta < d + (D/2), \\ &= 0, \eta < -(D/2) + d, \end{aligned} \quad (2.17c)$$

where  $V_T$  = the total volume of the cylinder.

This means that when the cylinder is totally submerged equations (2.16) reduce to the normal inertial term in Morison's equation and if the cylinder is ever uncovered the force is a simple constant force acting downwards.

### 2.3 Experimental Results

We have so far made no comments on the appropriate choice of the inertial coefficient,  $C_m$ . These values have to be determined empirically from the experimental data. As discussed in Chapter 1 there are several ways of doing this. The one selected here is the least square method, which is described in more detail in Chapter 5. This technique provides average values of  $C_m$  over a wave period. No attempt was made to analyse the variations during a cycle as done by Keulegan and Carpenter (1958).

A series of experiments was carried out using the equipment designed and built by Edinburgh Wave Power Team. The apparatus is described in Chapter 5, so only a brief description will be given here.

Monochromatic waves were produced by an absorbing wave-maker at one end of a narrow tank (8m by 30cms) at the other end of which was a beach. A rig held the various cylinders (ranging in size from 1.6cm to 10cm) in the water at different depths and forces were measured using strain gauges.

Figures 2.4 to 2.23 show the experimental relative force

curves over one wave period for a variety of different relative wave amplitudes, wave lengths and axis depths. Also shown in each figure are the theoretical relative forces computed from equations (2.17) using the value of  $C_m$  determined by the least squares fitting program. These values of  $C_m$  are tabulated in Table 2.2 together with the corresponding values of the wave parameters, and values of a phase coefficient,  $\delta$ , used to obtain a better fit to the experimental data. The introduction of this second parameter allows slight errors in determining the relative phase between the wave profile and the forces to be corrected for in the least squares program (see Chapter 5).

In all of the figures the phase is such that  $t = 0$  corresponds to the waterline passing upwards through  $y = 0$ .

Figures 2.4 to 2.7 show the horizontal forces for different values of  $a', L'$  and  $d'$  respectively. More data were taken of the vertical forces than the horizontal ones because of the interesting interplay between inertial and buoyancy forces.

Figures 2.8 to 2.14 show the forces on a cylinder of diameter 10cms. The effects of scale were investigated in Figures 2.18 to 2.23. Figures 2.18 to 2.21 show curves for four different cylinders in waves of constant absolute amplitude, wavelength and at a constant axis depth. Figures 2.22 and 2.23 show the forces on different cylinders in

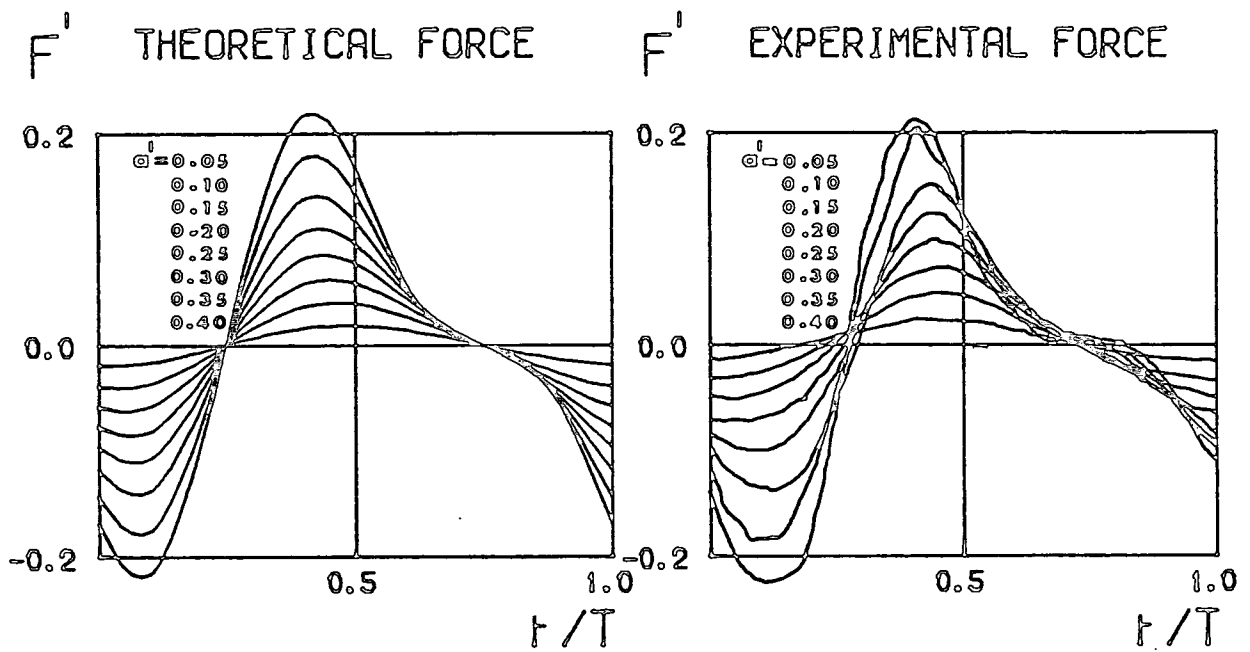


Figure 2.4 Relative horizontal forces for various relative wave amplitudes,  $d'=0.0$ ,  $L'=16$ .

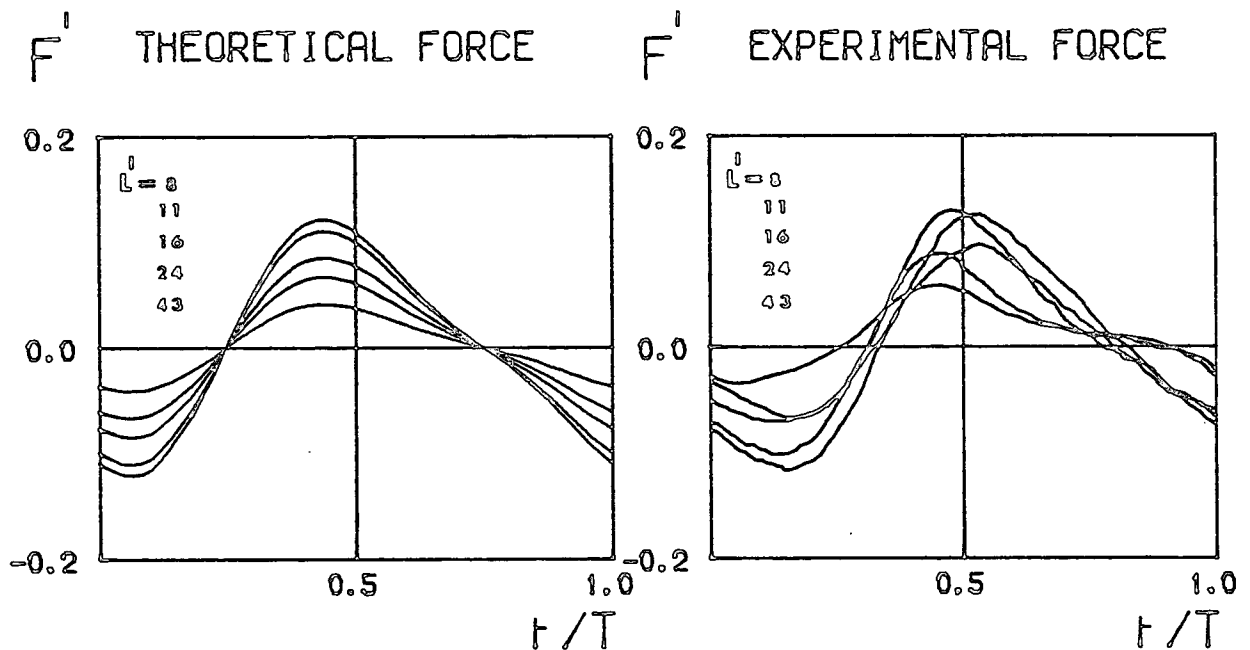


Figure 2.5 Relative horizontal forces for various relative wavelengths,  $d'=0.0$ ,  $a'=0.2$ .

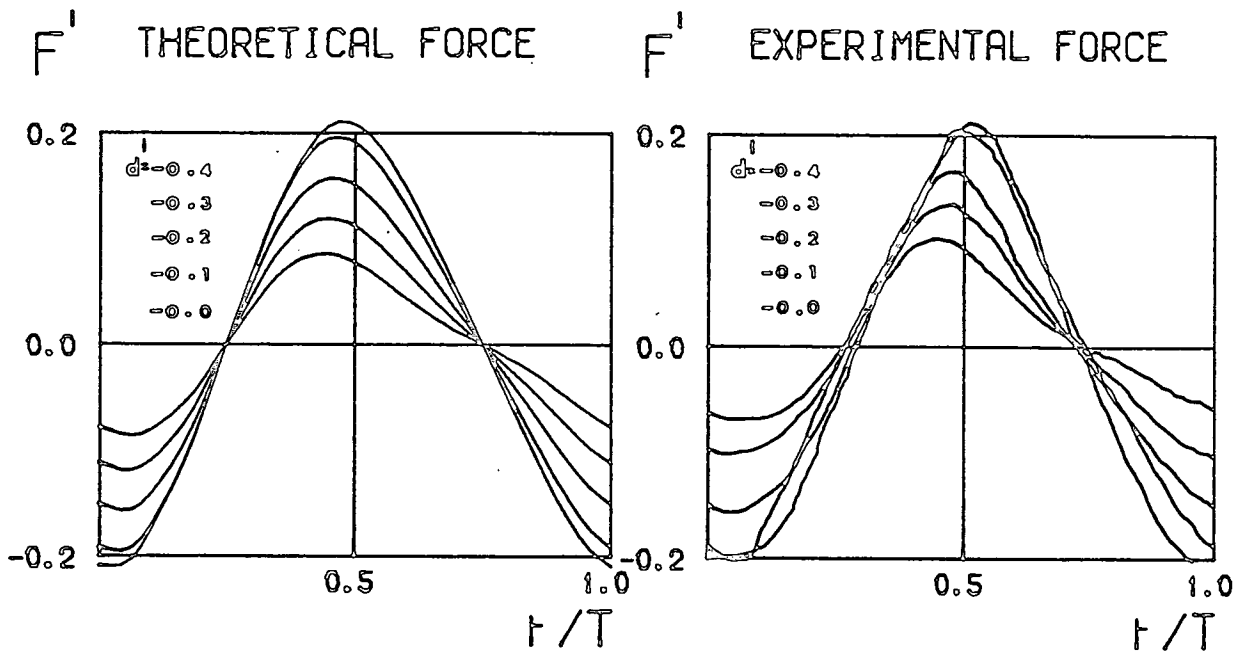


Figure 2.6 Relative horizontal forces for various relative axis depths,  $a'=0.2$ ,  $L'=16..$

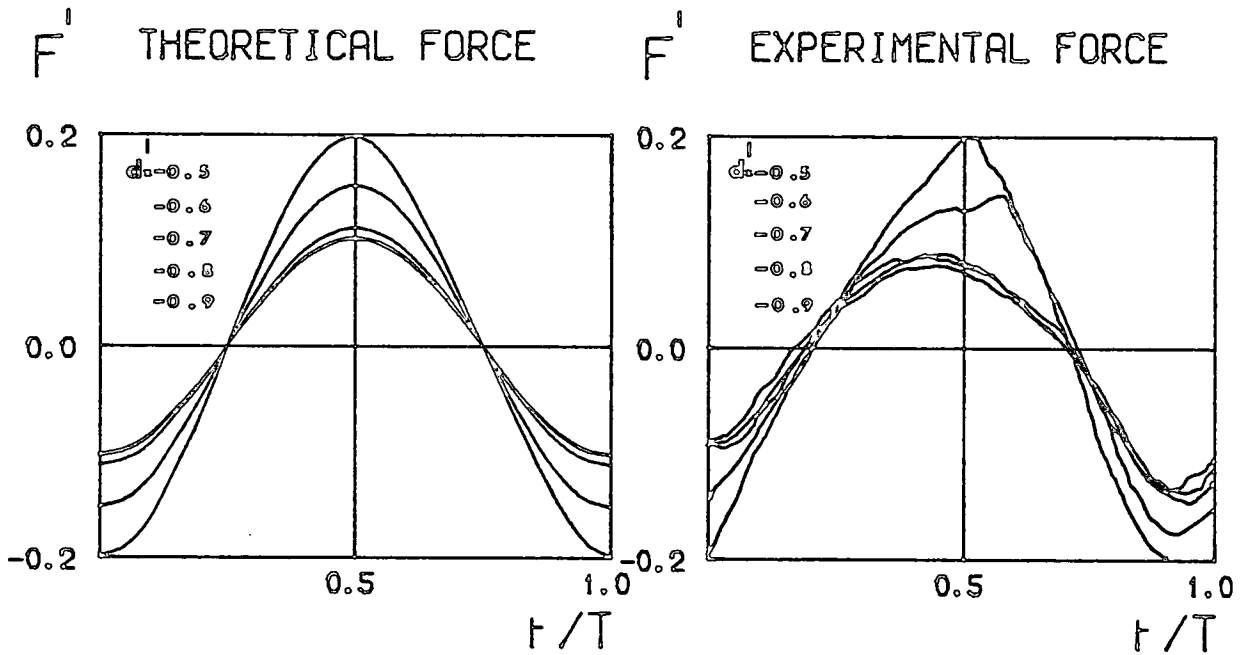


Figure 2.7 Relative horizontal forces for various relative axis depths,  $a'=0.2$ ,  $L'=16.$

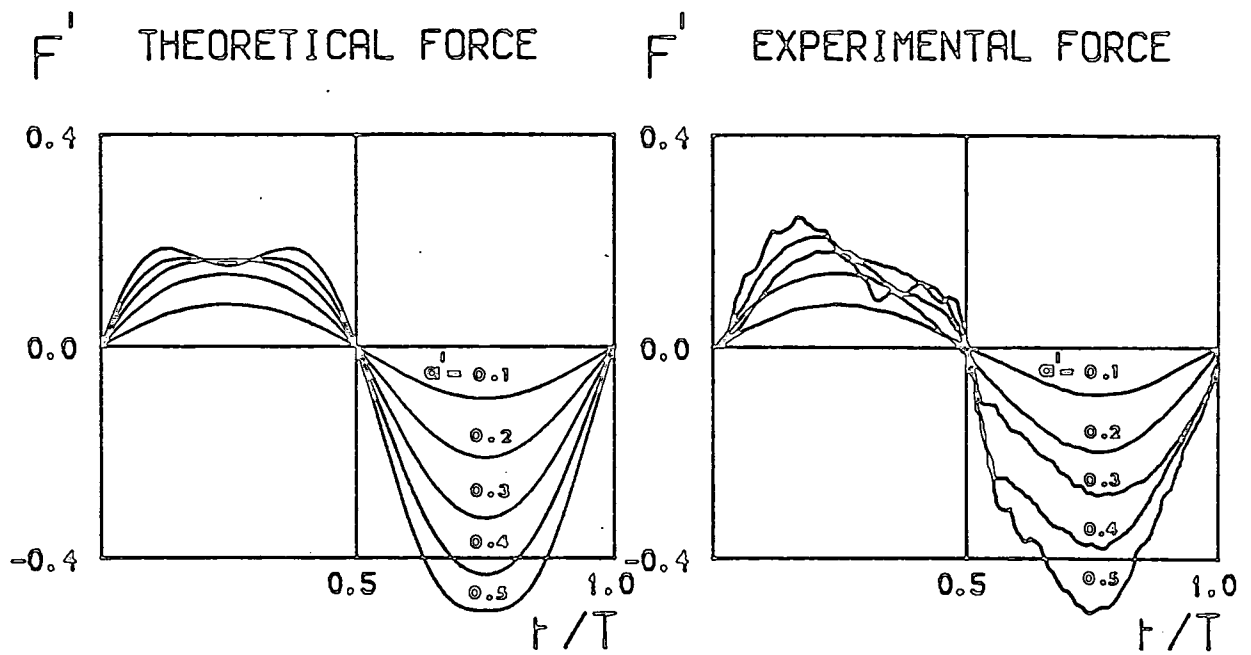


Figure 2.8 Relative vertical forces for various relative wave amplitudes,  $d' = 0.0$ ,  $L' = 16$ .

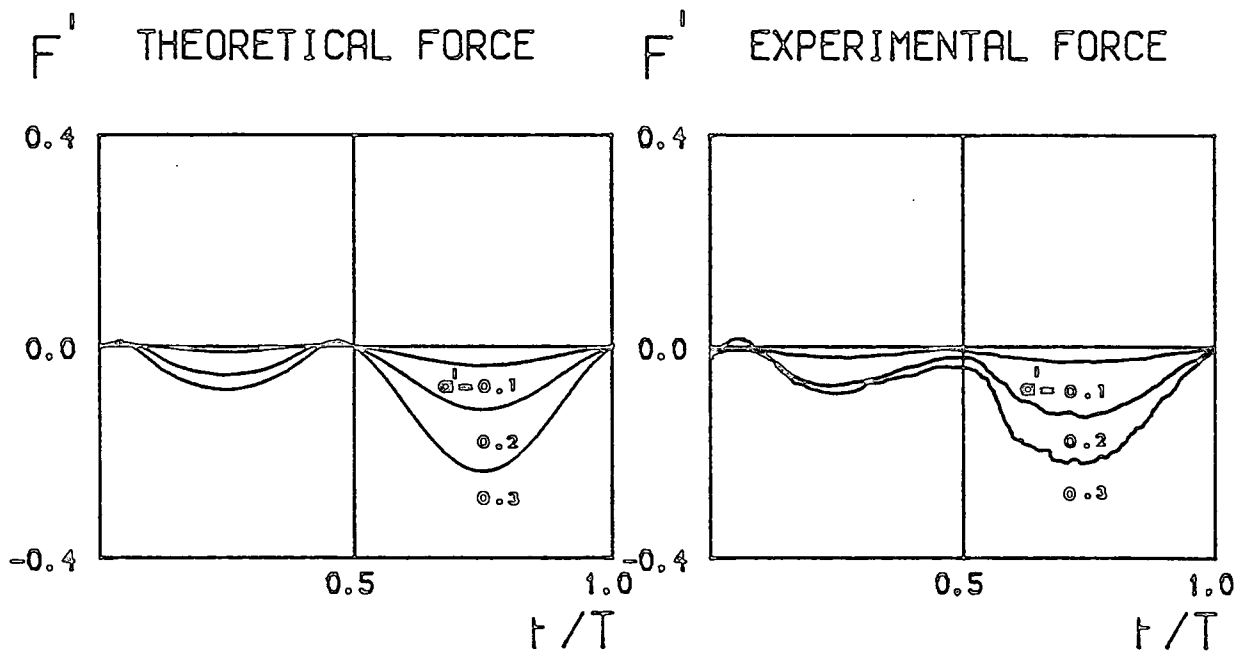


Figure 2.9 Relative vertical forces for various relative wave amplitudes,  $d' = -0.4$ ,  $L' = 16$ .

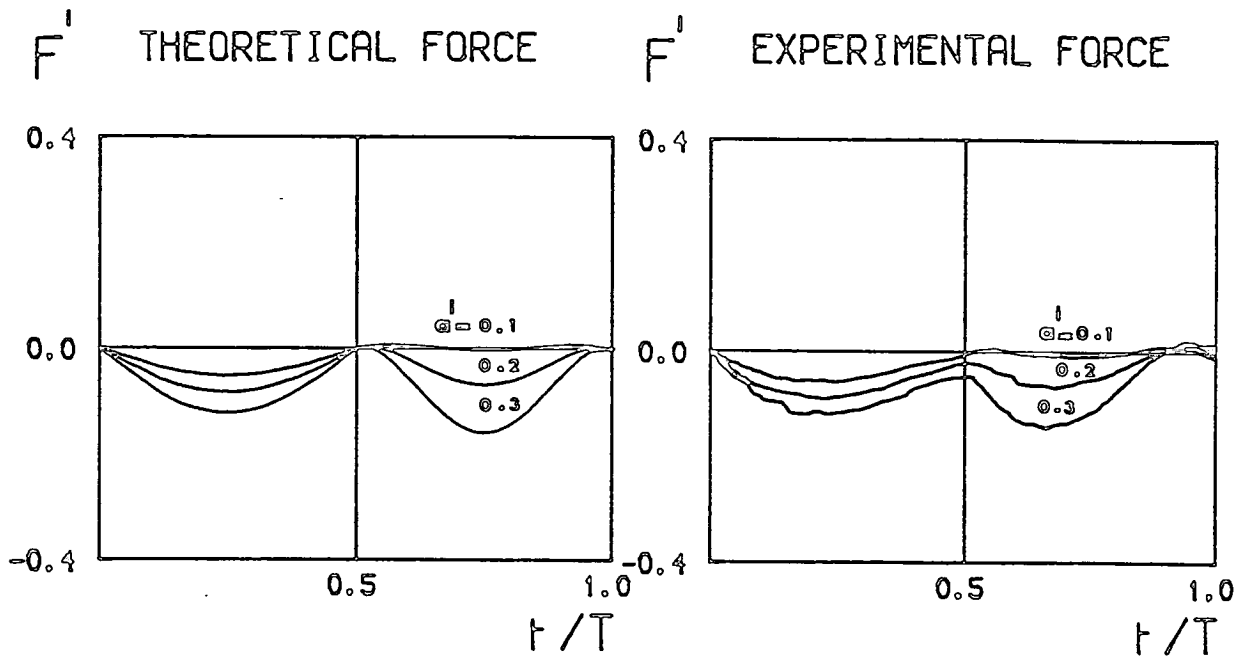


Figure 2.10 Relative vertical forces for various relative wave amplitudes,  $d' = -0.5$ ,  $L' = 16$ .

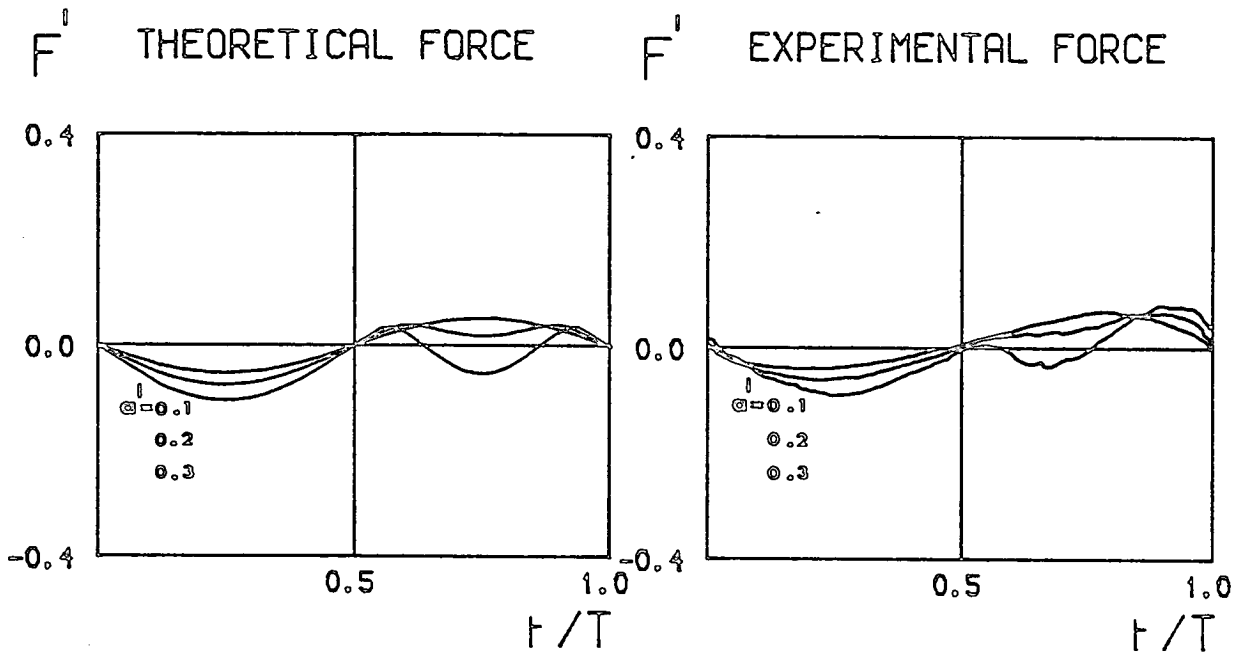


Figure 2.11 Relative vertical forces for various relative wave amplitudes,  $d' = -0.6$ ,  $L' = 16$ .

| CORRESPONDING<br>FIGURE | RELATIVE<br>WAVELENGTH | RELATIVE<br>WAVE<br>AMPLITUDE | RELATIVE<br>DEPTH | $C_m$ | $\delta$<br>(rad) |
|-------------------------|------------------------|-------------------------------|-------------------|-------|-------------------|
| Figure 2.4              | 16                     | 0.05                          | 0.0               | 1.85  | 0.28              |
| "                       | 16                     | 0.10                          | 0.0               | 1.95  | 0.16              |
| "                       | 16                     | 0.15                          | 0.0               | 1.93  | 0.05              |
| "                       | 16                     | 0.20                          | 0.0               | 1.92  | -0.05             |
| "                       | 16                     | 0.25                          | 0.0               | 1.91  | -0.12             |
| "                       | 16                     | 0.30                          | 0.0               | 1.95  | -0.27             |
| "                       | 16                     | 0.35                          | 0.0               | 2.04  | -0.41             |
| "                       | 16                     | 0.40                          | 0.0               | 2.10  | -0.38             |
| Figure 2.5              | 43                     | 0.2                           | 0.0               | 2.59  | -0.14             |
| "                       | 24                     | 0.2                           | 0.0               | 2.36  | -0.38             |
| "                       | 16                     | 0.2                           | 0.0               | 1.92  | -0.05             |
| "                       | 11                     | 0.2                           | 0.0               | 1.74  | -0.34             |
| "                       | 8                      | 0.2                           | 0.0               | 1.40  | -0.56             |
| Figure 2.6              | 16                     | 0.2                           | 0.0               | 1.93  | -0.08             |
| "                       | 16                     | 0.2                           | -0.1              | 2.31  | -0.10             |
| "                       | 16                     | 0.2                           | -0.2              | 2.72  | -0.19             |
| "                       | 16                     | 0.2                           | -0.3              | 3.13  | -0.30             |
| "                       | 16                     | 0.2                           | -0.4              | 3.21  | -0.30             |
| Figure 2.7              | 16                     | 0.2                           | -0.5              | 3.02  | 0.25              |
| "                       | 16                     | 0.2                           | -0.6              | 2.40  | 0.20              |
| "                       | 16                     | 0.2                           | -0.7              | 1.70  | 0.48              |
| "                       | 16                     | 0.2                           | -0.8              | 1.74  | 0.62              |
| "                       | 16                     | 0.2                           | -0.9              | 2.00  | 0.67              |

Table 2.2a

Values of the empirically determined inertia coefficient,  $C_m$ , and the phase coefficient,  $\delta$ , for the horizontal forces; variation with wave amplitude, frequency and depth,  $D=0.10m$ .

| CORRESPONDING<br>FIGURE | RELATIVE<br>WAVELENGTH | RELATIVE<br>WAVE<br>AMPLITUDE | RELATIVE<br>DEPTH | $C_m$ | $\delta$<br>Rad. |
|-------------------------|------------------------|-------------------------------|-------------------|-------|------------------|
| Figure 2.8              | 16                     | 0.1                           | 0.0               | 1.86  | 0.01             |
| "                       | 16                     | 0.2                           | 0.0               | 1.86  | -0.03            |
| "                       | 16                     | 0.3                           | 0.0               | 1.87  | -0.18            |
| "                       | 16                     | 0.4                           | 0.0               | 1.88  | -0.11            |
| "                       | 16                     | 0.5                           | 0.0               | 1.73  | -0.01            |
| Figure 2.9              | 16                     | 0.1                           | -0.4              | 1.85  | 0.03             |
| "                       | 16                     | 0.2                           | -0.4              | 1.56  | 0.05             |
| "                       | 16                     | 0.3                           | -0.4              | 1.31  | 0.08             |
| Figure 2.10             | 16                     | 0.1                           | -0.5              | 1.57  | 0.15             |
| "                       | 16                     | 0.2                           | -0.5              | 1.28  | 0.26             |
| "                       | 16                     | 0.3                           | -0.5              | 1.25  | 0.31             |
| Figure 2.11             | 16                     | 0.1                           | -0.6              | 1.66  | 0.03             |
| "                       | 16                     | 0.2                           | -0.6              | 1.18  | -0.04            |
| "                       | 16                     | 0.3                           | -0.6              | 1.10  | 0.12             |
| Figure 2.12             | 43                     | 0.2                           | 0.0               | 2.77  | 0.06             |
| "                       | 24                     | 0.2                           | 0.0               | 1.97  | 0.01             |
| "                       | 16                     | 0.2                           | 0.0               | 1.99  | 0.03             |
| "                       | 11                     | 0.2                           | 0.0               | 1.62  | -0.08            |
| "                       | 8                      | 0.2                           | 0.0               | 1.45  | -0.31            |
| Figure 2.13             | 43                     | 0.2                           | -0.4              | 2.17  | 0.02             |
| "                       | 24                     | 0.2                           | -0.4              | 1.78  | -0.14            |
| "                       | 16                     | 0.2                           | -0.4              | 1.49  | -0.07            |
| "                       | 11                     | 0.2                           | -0.4              | 1.47  | -0.23            |
| "                       | 8                      | 0.2                           | -0.4              | 1.56  | -0.10            |

Table 2.2b

Values of the empirically determined inertia coefficient,  $C_m$ , and the phase coefficient,  $\delta$ , for the vertical forces; variation with wave amplitude and frequency at different depths,  $D = 0.10m$ .

| CORRESPONDING<br>FIGURE | RELATIVE<br>WAVELENGTH | RELATIVE<br>WAVE<br>AMPLITUDE | RELATIVE<br>AXIS<br>DEPTH | $C_m$ | $\delta$<br>Rad. |
|-------------------------|------------------------|-------------------------------|---------------------------|-------|------------------|
| Figure 2.14             | 16                     | 0.1                           | 0.3                       | 3.51  | 0.03             |
| "                       | 16                     | 0.1                           | 0.2                       | 2.69  | 0.06             |
| "                       | 16                     | 0.1                           | 0.1                       | 2.31  | 0.06             |
| "                       | 16                     | 0.1                           | 0.0                       | 1.99  | 0.03             |
| Figure 2.15             | 16                     | 0.1                           | -0.1                      | 1.85  | 0.03             |
| "                       | 16                     | 0.1                           | -0.2                      | 1.64  | 0.07             |
| "                       | 16                     | 0.1                           | -0.3                      | 1.58  | -0.01            |
| "                       | 16                     | 0.1                           | -0.4                      | 1.85  | 0.03             |
| Figure 2.16             | 16                     | 0.2                           | 0.3                       | 2.44  | 0.07             |
| "                       | 16                     | 0.2                           | 0.2                       | 2.24  | -0.04            |
| "                       | 16                     | 0.2                           | 0.1                       | 2.03  | -0.04            |
| "                       | 16                     | 0.2                           | 0.0                       | 1.87  | -0.04            |
| Figure 2.17             | 16                     | 0.2                           | -0.1                      | 1.81  | -0.07            |
| "                       | 16                     | 0.2                           | -0.2                      | 1.72  | -0.09            |
| "                       | 16                     | 0.2                           | -0.3                      | 1.78  | -0.14            |
| "                       | 16                     | 0.2                           | -0.4                      | 1.56  | -0.11            |

Table 2.2c Values of the empirically determined inertia coefficient,  $C_m$ , and the phase coefficient,  $\delta$ , for the vertical forces; variations with axis depth for two different amplitudes,  $D = 0.10m$ .

| CORRESPONDING<br>FIGURE | CYLINDER<br>DIAMETER | WAVELENGTH | WAVE<br>AMPLITUDE | AXIS<br>DEPTH | $C_m$ | $\delta$<br>Rad. |
|-------------------------|----------------------|------------|-------------------|---------------|-------|------------------|
|                         | (m)                  | (m)        | (m)               | (m)           |       |                  |
| Figure 2.18             | 0.016                | 1.6        | 0.01              | 0.00          | -3.62 | 0.08             |
| "                       | 0.030                | 1.6        | 0.01              | 0.00          | 2.71  | -0.03            |
| "                       | 0.049                | 1.6        | 0.01              | 0.00          | 2.25  | 0.09             |
| "                       | 0.100                | 1.6        | 0.01              | 0.00          | 1.86  | 0.01             |
| Figure 2.19             | 0.016                | 1.6        | 0.02              | 0.00          | -1.66 | 0.06             |
| "                       | 0.030                | 1.6        | 0.02              | 0.00          | 1.67  | 0.00             |
| "                       | 0.049                | 1.6        | 0.02              | 0.00          | 2.21  | 0.03             |
| "                       | 0.100                | 1.6        | 0.02              | 0.00          | 1.86  | -0.03            |
| Figure 2.20             | 0.016                | 1.6        | 0.01              | -0.04         | 0.84  | -0.25            |
| "                       | 0.030                | 1.6        | 0.01              | -0.04         | 1.23  | -0.29            |
| "                       | 0.049                | 1.6        | 0.01              | -0.04         | 1.47  | -0.04            |
| "                       | 0.100                | 1.6        | 0.01              | -0.04         | 1.85  | 0.03             |
| Figure 2.21             | 0.016                | 1.6        | 0.02              | -0.04         | 1.09  | -0.30            |
| "                       | 0.030                | 1.6        | 0.02              | -0.04         | 1.35  | -0.30            |
| "                       | 0.049                | 1.6        | 0.02              | -0.04         | 1.54  | -0.17            |
| "                       | 0.100                | 1.6        | 0.02              | -0.04         | 1.56  | 0.05             |

Table 2.2d Values of the empirically determined inertia coefficient,  $C_m$ , and the phase coefficient,  $\delta$ , for the vertical forces, variation with cylinder diameter.

| CORRESPONDING<br>FIGURE | CYLINDER<br>DIAMETER<br>(metres) | RELATIVE<br>WAVELENGTH | RELATIVE<br>WAVE<br>AMPLITUDE | RELATIVE<br>AXIS DEPTH | $C_m$ | $\delta$<br>Rad. |
|-------------------------|----------------------------------|------------------------|-------------------------------|------------------------|-------|------------------|
| Figure 2.22             | 0.016                            | 28                     | 0.2                           | 0.0                    | 0.23  | -0.01            |
| "                       | 0.030                            | 28                     | 0.2                           | 0.0                    | 2.27  | -0.04            |
| "                       | 0.049                            | 28                     | 0.2                           | 0.0                    | 2.45  | 0.05             |
| "                       | 0.100                            | 28                     | 0.2                           | 0.0                    | 2.08  | 0.10             |
| Figure 2.23             | 0.016                            | 28                     | 0.2                           | 0.0                    | -1.38 | 0.17             |
| "                       | 0.030                            | 28                     | 0.2                           | 0.0                    | -0.02 | -0.10            |
| "                       | 0.049                            | 28                     | 0.2                           | 0.0                    | 0.98  | 0.02             |
| "                       | 0.100                            | 28                     | 0.2                           | 0.0                    | 1.18  | 0.05             |

Table 2.2d (Continued)

Values of the empirically determined inertia coefficient,  $C_m$ , and the phase coefficient,  $\delta$ , for the vertical forces, variation with cylinder diameter.

waves of constant relative amplitude, wavelength and axis depth.

The variations of  $C_m$  with  $a'$ ,  $L'$ , and  $d'$  for horizontal forces are plotted in figure 2.24. Values of  $C_m$  for vertical forces for different  $a'$ ,  $L'$ , and,  $d'$  are shown in figures 2.25 to 2.27. Those for different diameters are given in Figure 2.28, (constant  $a$ ,  $L$ , and  $d$ ) and figure 2.29, (constant  $a'$ ,  $L'$ , and  $d'$ ).

Since peak forces and root mean square forces are of particular interest these are given in figure 2.30 for horizontal forces and figure 2.31 for the vertical forces.

#### 2.4 Discussion

The modifications to Morison's equation appear to work extremely well in the case of horizontal forces. The forces rise linearly with wave amplitude (figure 2.4) and decrease with wavelength as expected. The phase of the position of maximum force decreases as the amplitude increases, and the wave force becomes more asymmetric whilst the original form of Morison's equation predicts a symmetric inertial term. The fitted coefficient,  $C_m$ , is almost constant for different wave amplitudes but shows a steady increase with wavelength, (figure 2.24), which would suggest that the dependence on wavelength is not as strong as  $L^{-1}$  but more as  $L^{-\frac{1}{2}}$  perhaps. The variations of the forces with axis depth (figures 2.6 and 2.7) show the widest variation of  $C_m$  and the forces peak at about  $d' = 0.4$ , corresponding to the peak in  $C_m$ . It may be that the

assumptions involved, such as a horizontal waterline, are weakening the dependence on axis depth in the theoretical equation. For the largest axis depths ( $d' = -0.7, -0.8, -0.9$ ) the cylinder is always submerged and the original formula would be sufficient.

It is the vertical forces which show the most interesting results as can be seen by the variation with amplitude at several different axis depths, figures 2.8 to 2.11. For the larger axis depths the force is always negative and shows strong tendencies towards a periodicity of twice the wave period. Considering the approximations and assumptions involved the modified force equation gives a remarkably good fit. The corresponding values of  $C_m$  are shown in figure 2.25 where it can be seen that they decrease with increasing wave amplitude and also with increasing depth. The experimental curves show an asymmetry in the crest which is not predicted by the theory. This is probably due to the interaction between the cylinder and the waves which gives rise to reflections and also causes the waves to break on the cylinder.

Figures 2.12 and 2.13 show the variation of the vertical force with wavelength at two axis depths,  $d' = 0.0$  and  $d' = -0.4$ . The forces increase with increasing wavelength for the zero axis depth which is in sharp contrast to the corresponding figure for the horizontal force where they decreased (figure 2.4). The reason for this is that the vertical force is a sum of an inertial term which does

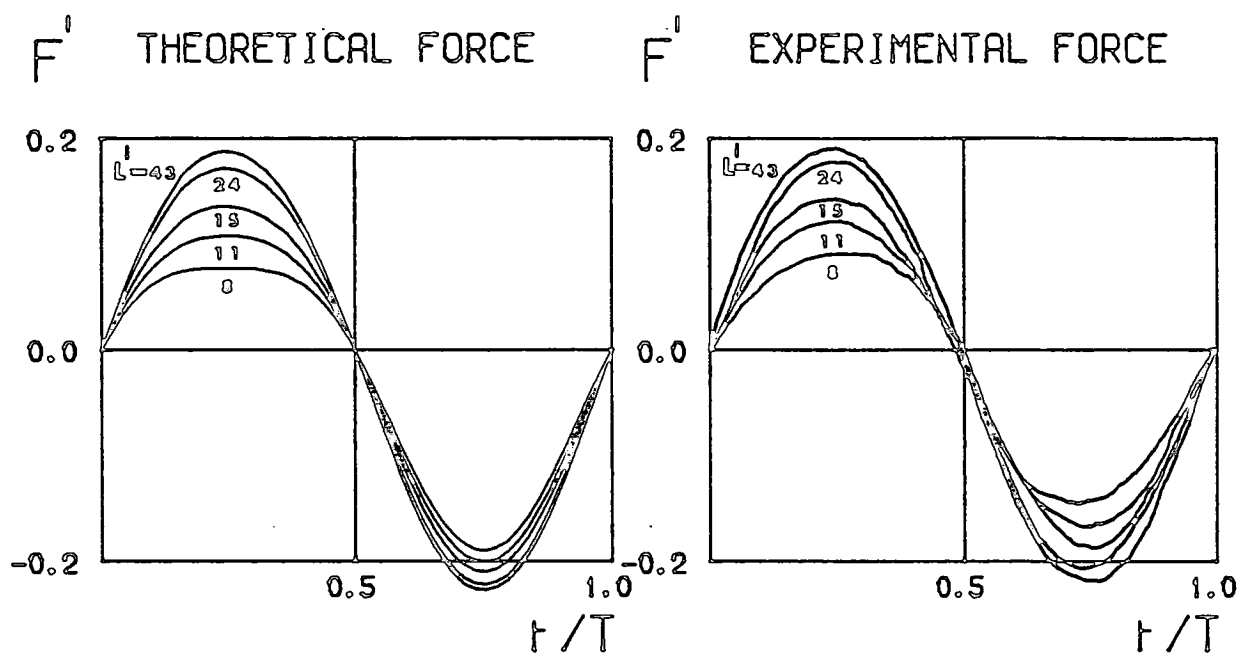


Figure 2.12 Relative vertical forces for various relative wavelengths,  $d'=0.0$ ,  $a'=0.2$ .

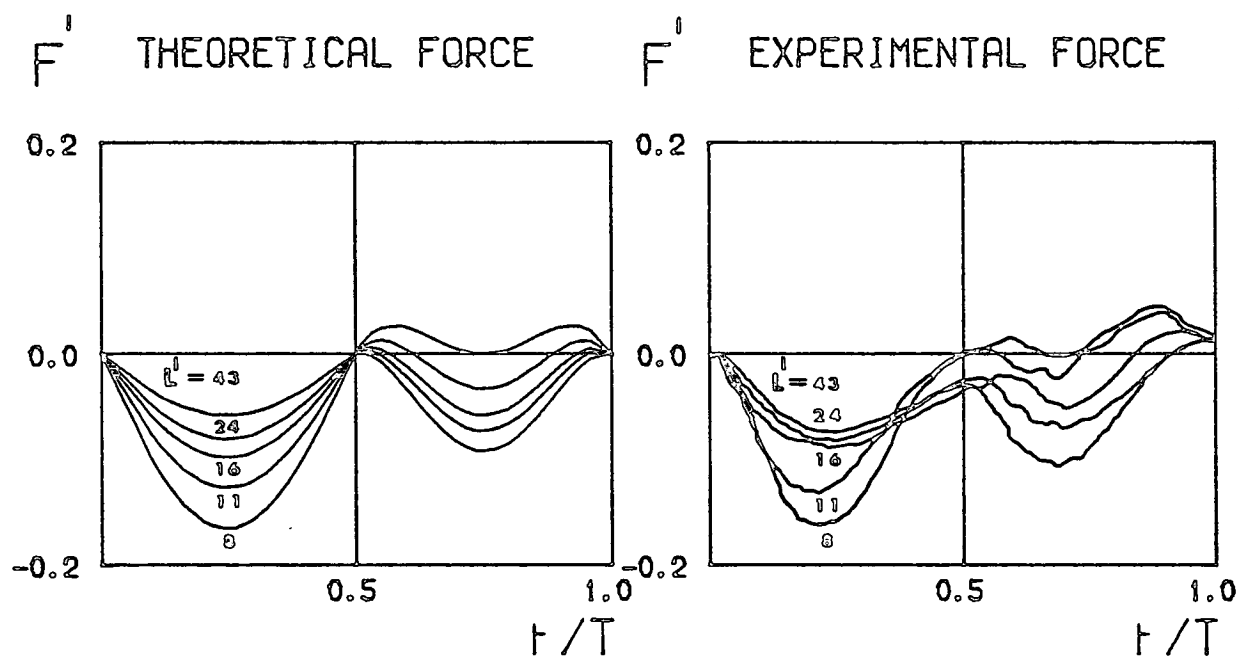


Figure 2.13 Relative vertical forces for various relative wavelengths,  $d'=-0.4$ ,  $a'=0.2$ .

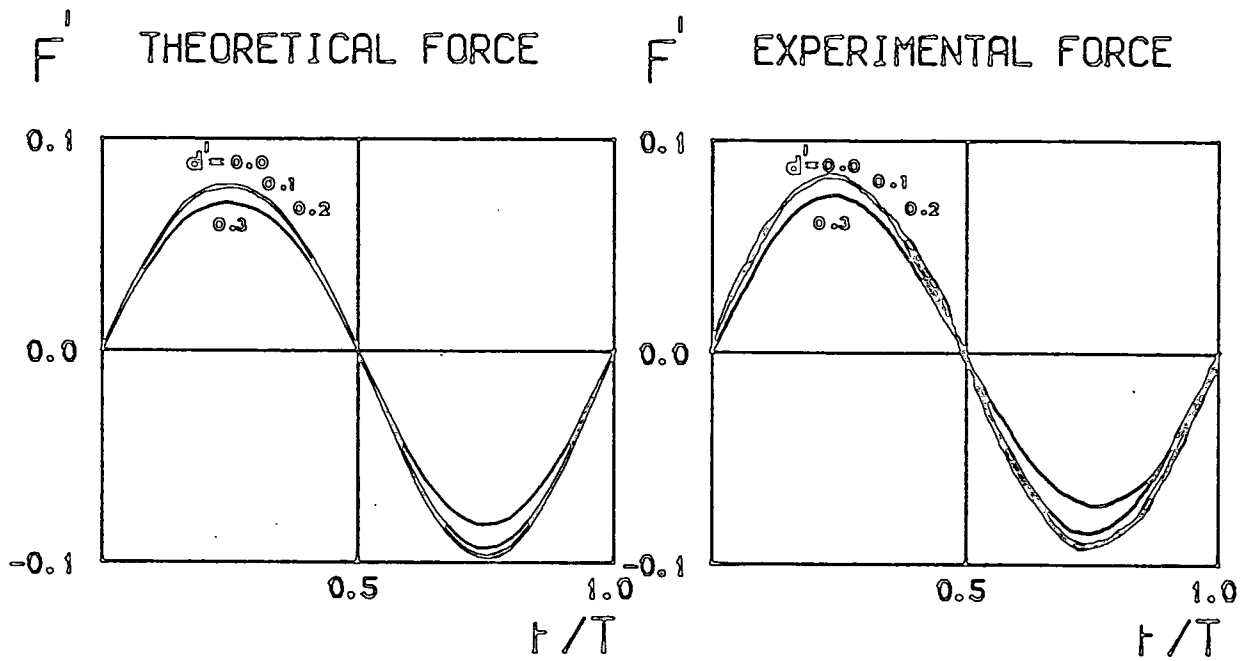


Figure 2.14 Relative vertical forces for various relative axis depths,  $a'=0.1$ ,  $L'=16$ .

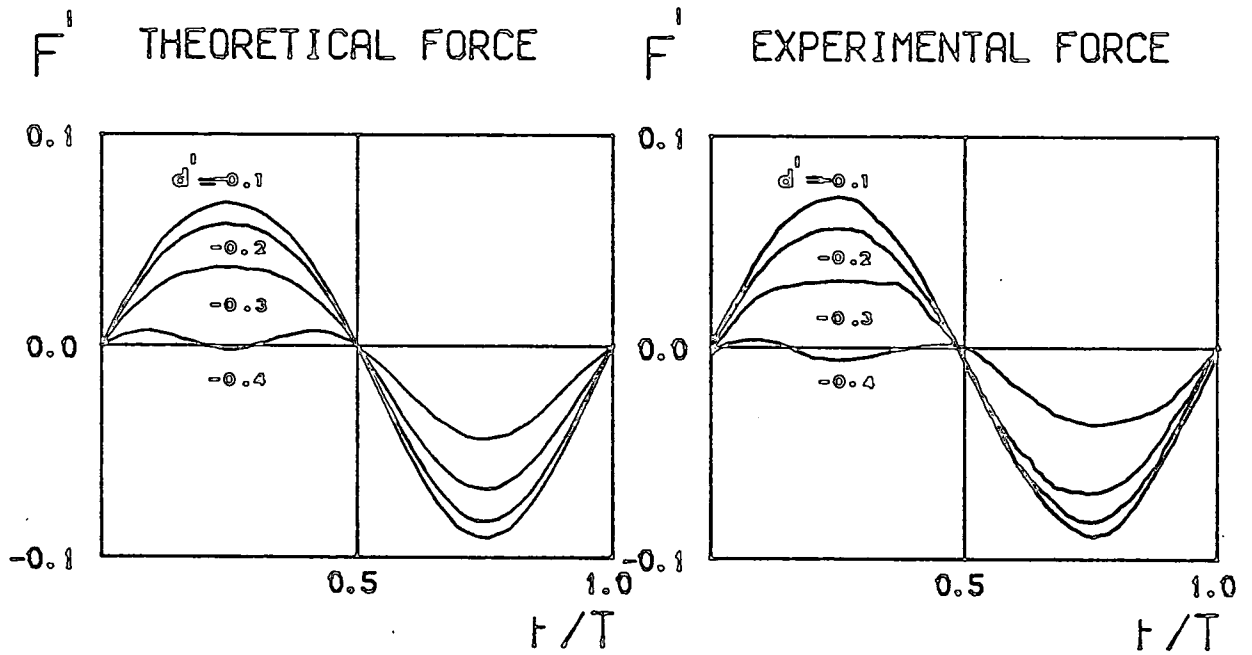


Figure 2.15 Relative vertical forces for various relative axis depths,  $a'=0.1$ ,  $L'=16$ .

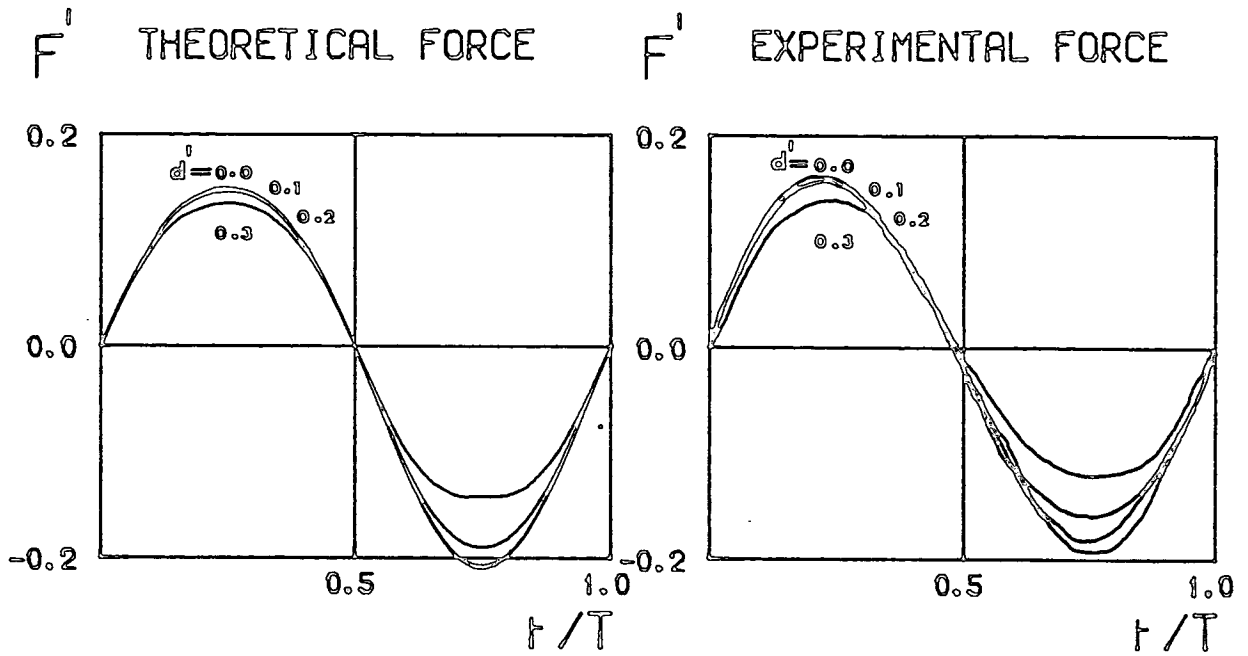


Figure 2.16 Relative vertical forces for various relative axis depths,  $a'=0.2$ ,  $L'=16$ .

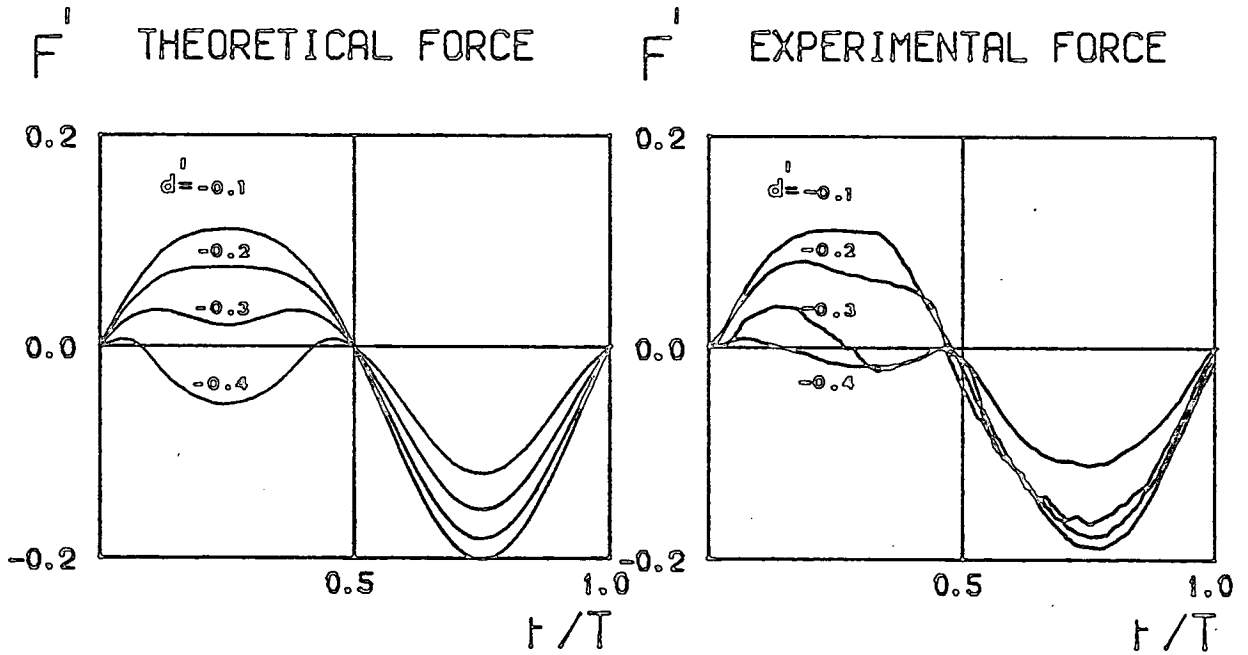


Figure 2.17 Relative vertical forces for various relative axis depths,  $a'=0.2$ ,  $L'=16$ .

decrease with wavelength and a buoyancy term which is independent (as far as is known). Since the buoyancy term dominates for zero axis depth the overall force will actually increase as the inertial term decreases. In figure 2.13 where the cylinder is nearly submerged ( $d' = -0.4$ ) buoyancy and inertia are more equal and in the crest of the wave the forces again decrease with wavelength. These contrasting situations have led to confusion in the past over the exact relationship between peak vertical forces and wavelength for partially submerged cylinders. Overall there appears to be less variation of force with  $L/D$  than expected and this enabled Salter (1976) to propose his equation for peak forces.

The curves in figures 2.14 to 2.17 show the variation of vertical force for relative axis depths ranging from 0.3 to - 0.4, at two different wave amplitudes. In general buoyancy dominates and so although the inertial term increases as more of the cylinder is submerged the overall force decreases. The inertia coefficients in this case increase as the cylinder comes further out of the water. It should be noted that if Morison's original equation is used in a situation where buoyancy dominates then negative values of  $C_m$  would result in every case. This difficulty is overcome by the modifications presented here.

Before we can justify the use of the modified equation for prototype situations we must demonstrate that the

predicted forces scale properly. We could just argue that the modifications introduced do not scale differently to the original equation and that Morison's original equation has been often used at full scale with success. However, we will present more data to give added confidence to these predictions.

Firstly we will show that the equation can predict forces to a good degree of accuracy for different size cylinders in the same wave environment, i.e. the same absolute wave amplitude, wavelength, and axis depth. Figures 2.18 to 21 show the forces on four cylinders ranging from 0.016m to 0.100m in diameter in four different situations. As can be seen the agreement is still good although the values of  $C_m$  vary quite widely. The two negative values arise because buoyancy was very dominant and so large changes in  $C_m$  produce little change in the overall force. The calculated buoyancy force was slightly larger than the actual force and so large negative values of  $C_m$  resulted. It may seem strange at first that the forces decrease with increasing size, but it must be remembered that it is still the relative wave force that is being plotted, not the absolute force, which does increase.

The second way of testing the modified equation for correct scaling is to notice that if the relative parameters,  $a'$ ,  $d'$ , and  $L'$  in equations (2.16) are kept constant while  $D$  is varied then the relative force should remain the same for all diameters, assuming constant values of  $C_m$ . Figures

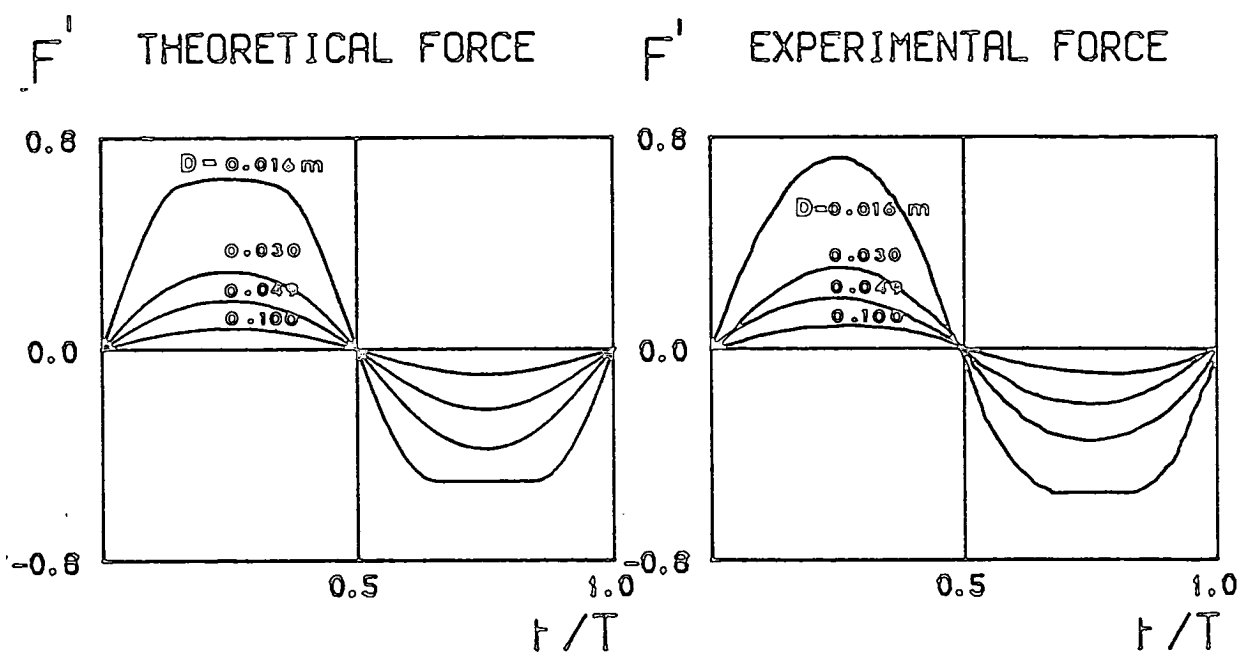


Figure 2.18 Relative vertical forces for cylinders of various diameters,  $a = 0.01\text{m}$ ,  $d = -0.00\text{m}$ ,  $L = 1.6\text{m}$

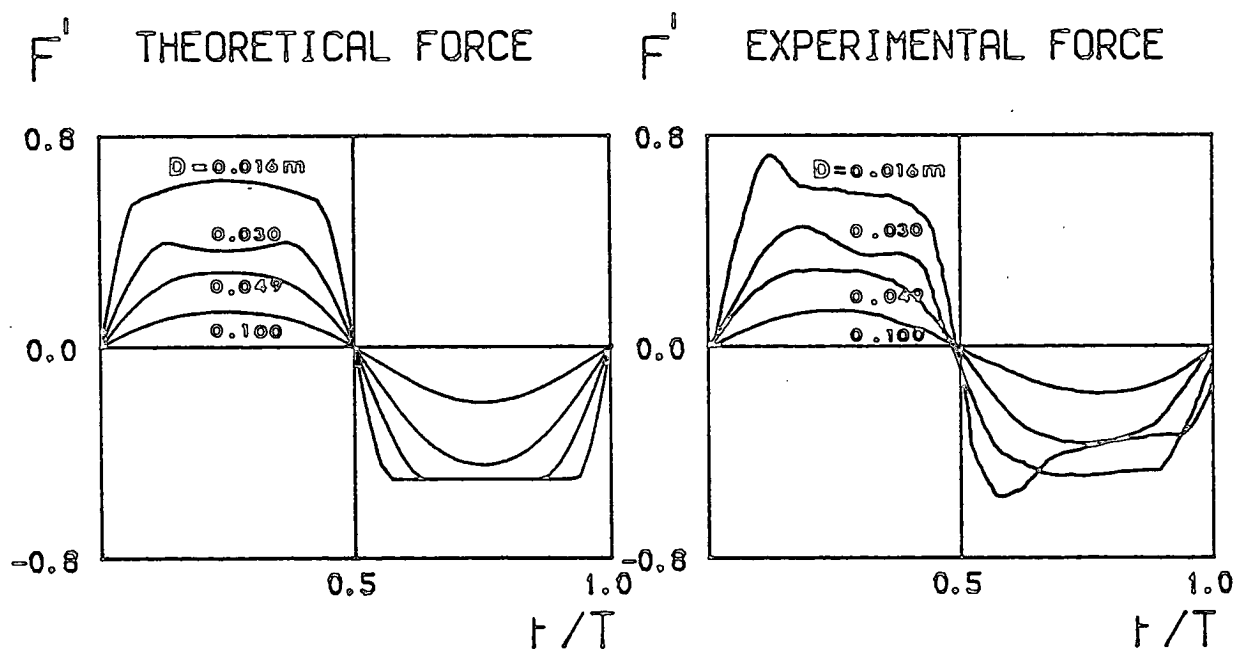


Figure 2.19 Relative vertical forces for cylinders of various diameters,  $a = 0.02\text{m}$ ,  $d = -0.00\text{m}$ ,  $L = 1.6\text{m}$ .

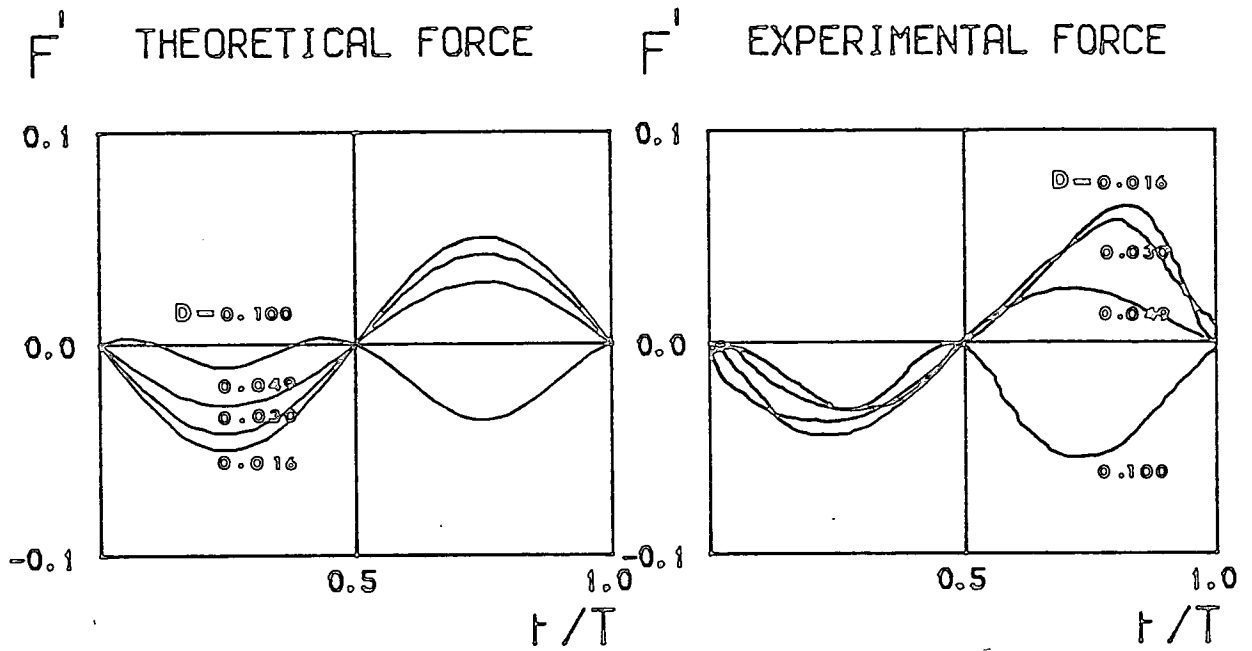


Figure 2.20 Relative vertical forces for cylinders of various diameters,  $a = 0.01\text{m}$ ,  $d = 0.04\text{m}$ ,  $L = 1.6\text{m}$ .

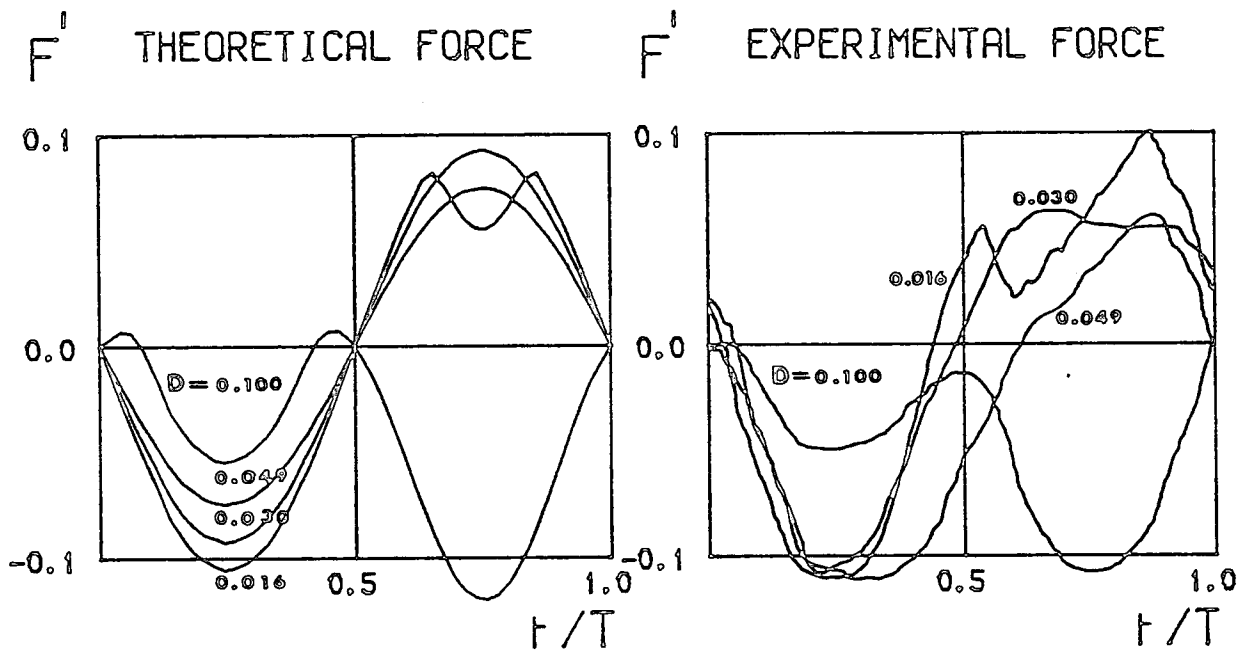


Figure 2.21 Relative vertical forces for cylinders of various diameters,  $a = 0.02\text{m}$ ,  $d = -0.04\text{m}$ ,  $L = 1.6\text{m}$ .

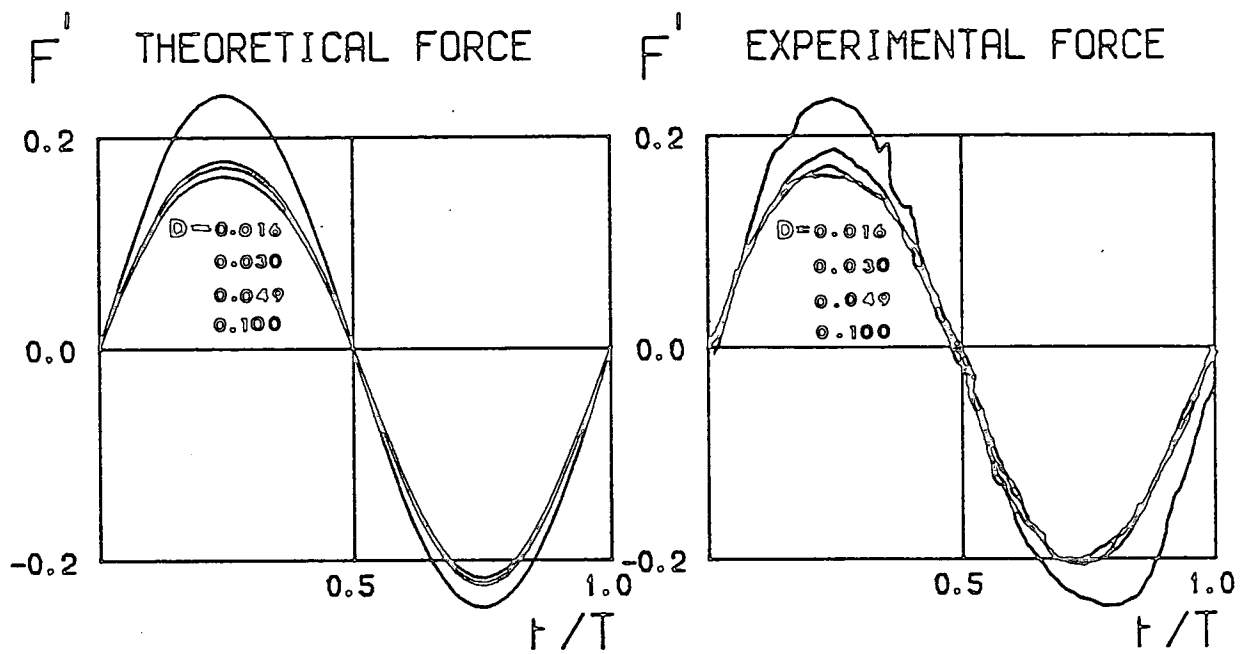


Figure 2.22 Relative vertical forces for cylinders of various diameters,  $a' = 0.2$ ,  $d = 0.0$ ,  $L' = 2.8$ .

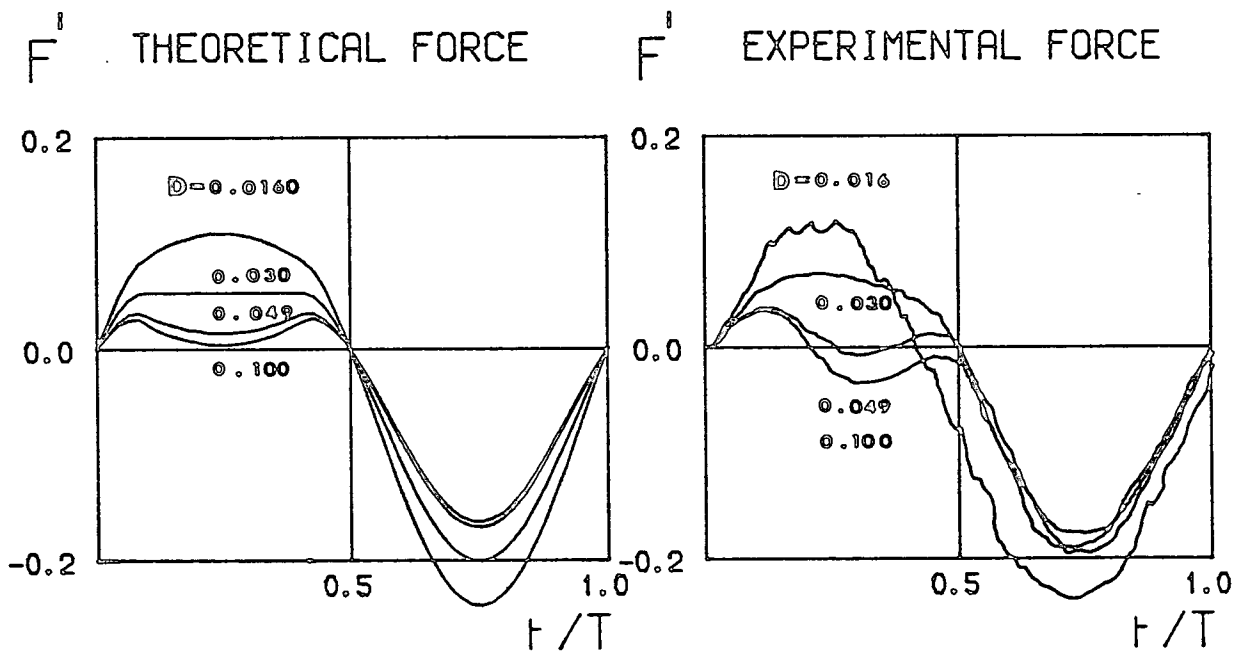


Figure 2.23 Relative vertical forces for cylinders of various diameters,  $a' = 0.2$ ,  $d' = -0.4$ ,  $L' = 2.8$ .

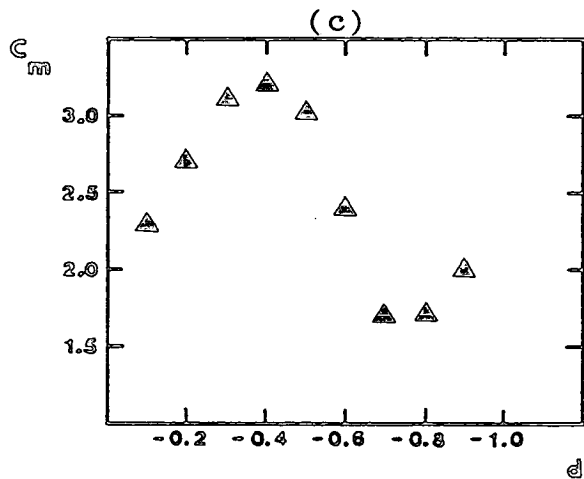
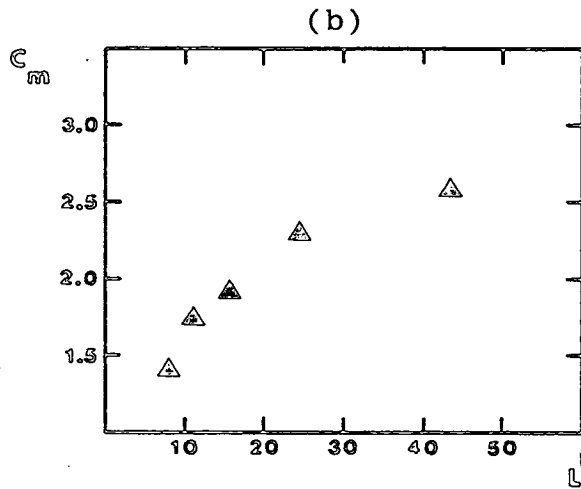
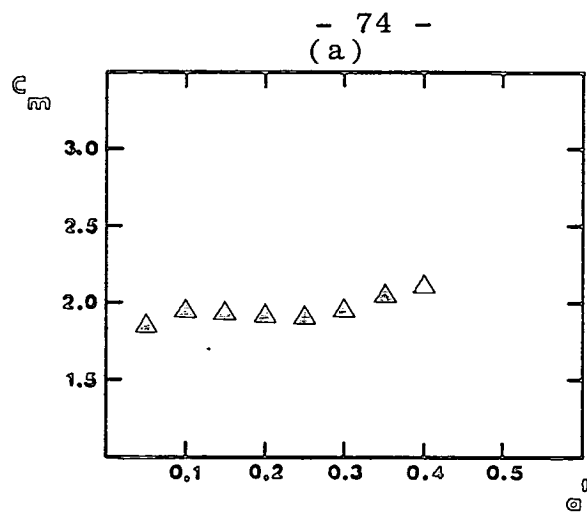


Figure 2.24 Values of the inertia coefficient,  $C_m$   
for the horizontal forces presented in  
figures 2.4 to 2.7.

- (a) Variation with relative wave amplitude,  $d'=0.0$ ,  $L'=16$ .  
 (b) Variation with relative wavelength,  $d'=0.0$ ,  $a'=0.2$ .  
 (c) Variation with relative axis depth,  $a'=0.2$ ,  $L'=16$ .

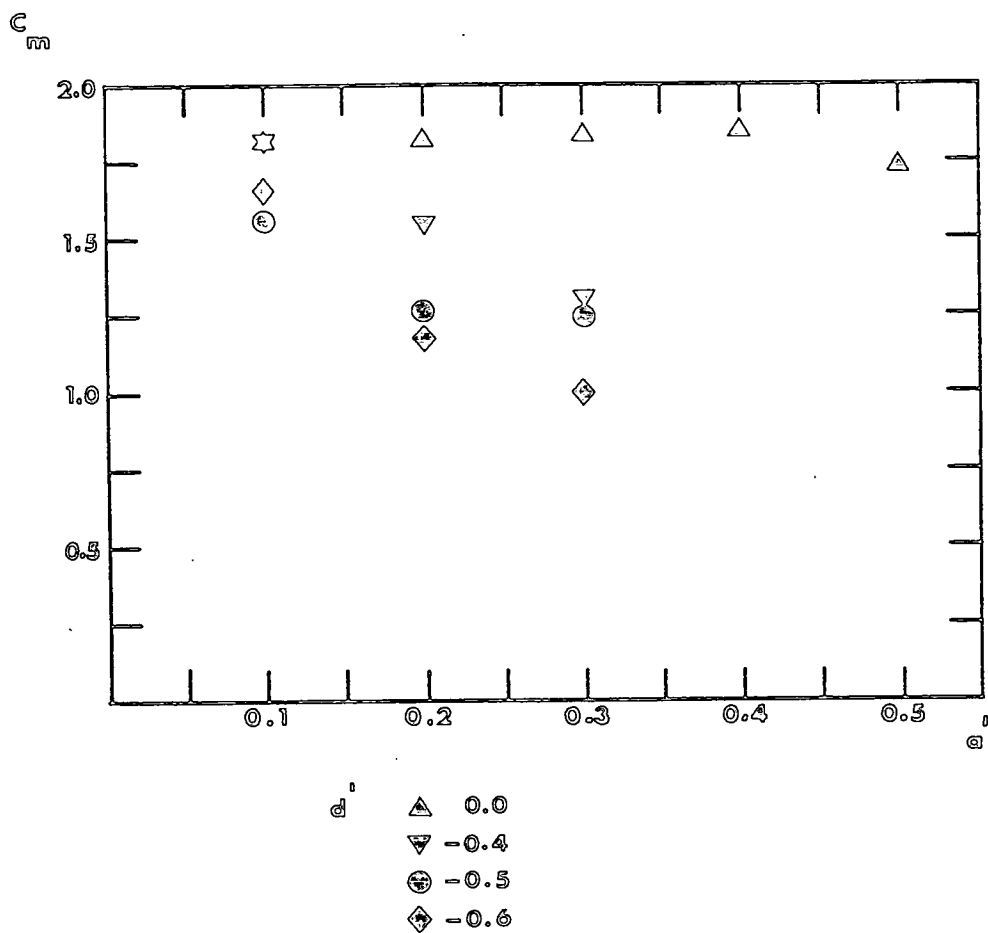


**Figure 2.25** Values of the inertia coefficient,  $C_m$ , for the vertical forces presented in figures 2.8 to 2.11, variation with relative wave amplitude at different axis depths,  $L' = 16$ .

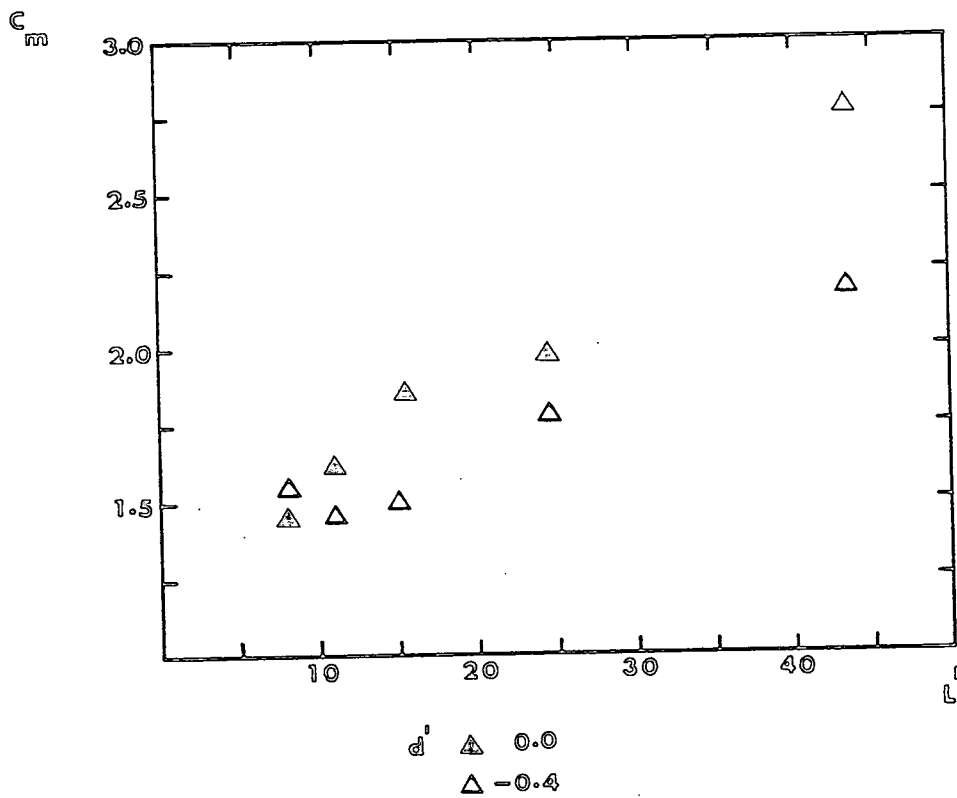


Figure 2.26 Values of the inertia coefficient,  $C_m$ , for the vertical forces presented in figures 2.12 and 2.13, variation with relative wavelength for two relative axis depths,  $a' = 0.2$ .

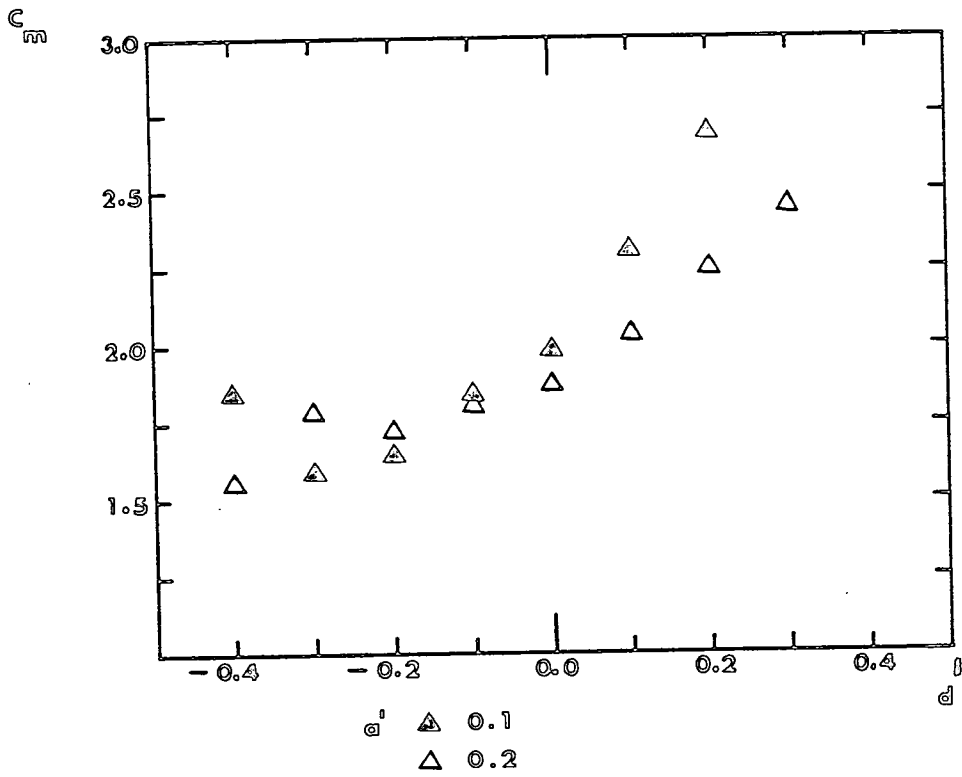


Figure 2.27 Values of the inertia coefficient,  $C_m$ , for the vertical forces presented in figures 2.14 to 2.17, variation with relative axis depth for two relative amplitudes,  $L' = 16$ .

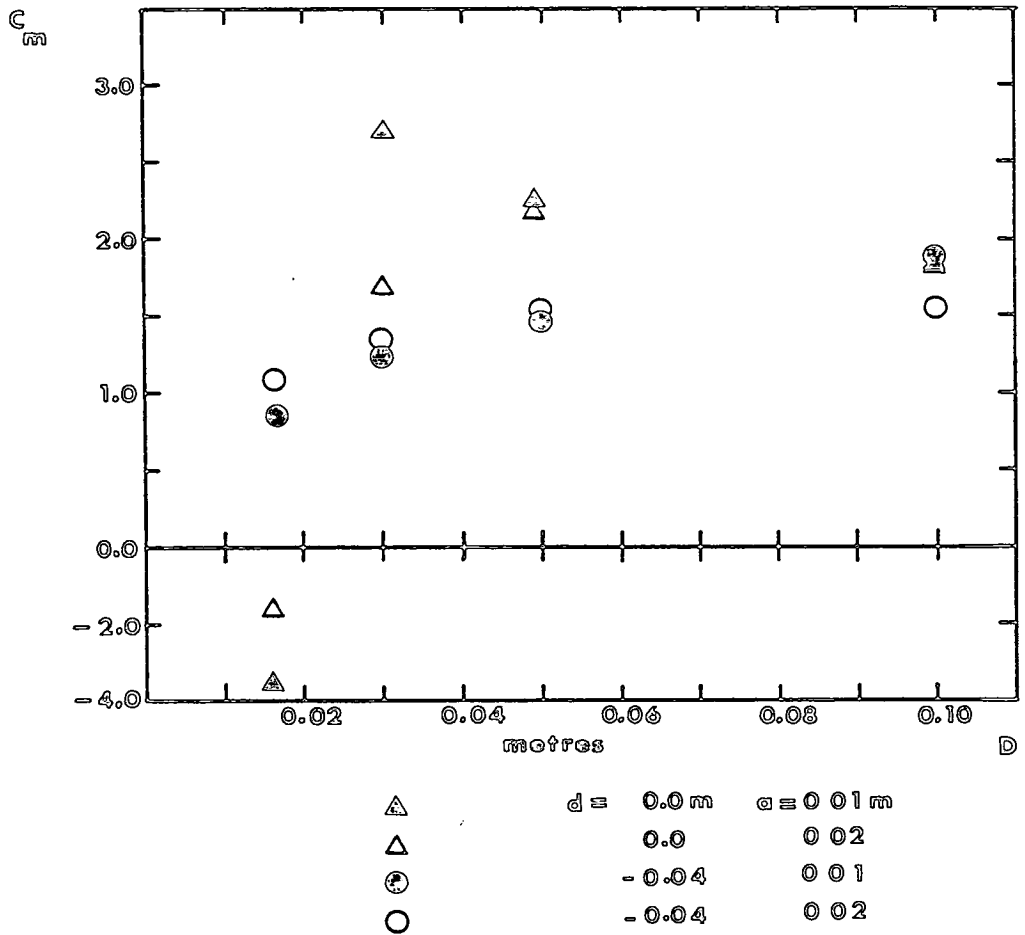


**Figure 2.28** Values of the inertia coefficient,  $C_m$ , for the vertical forces presented in figures 2.18 to 2.21, variation with cylinder diameter at constant absolute wave amplitude, wavelength and axis depth.

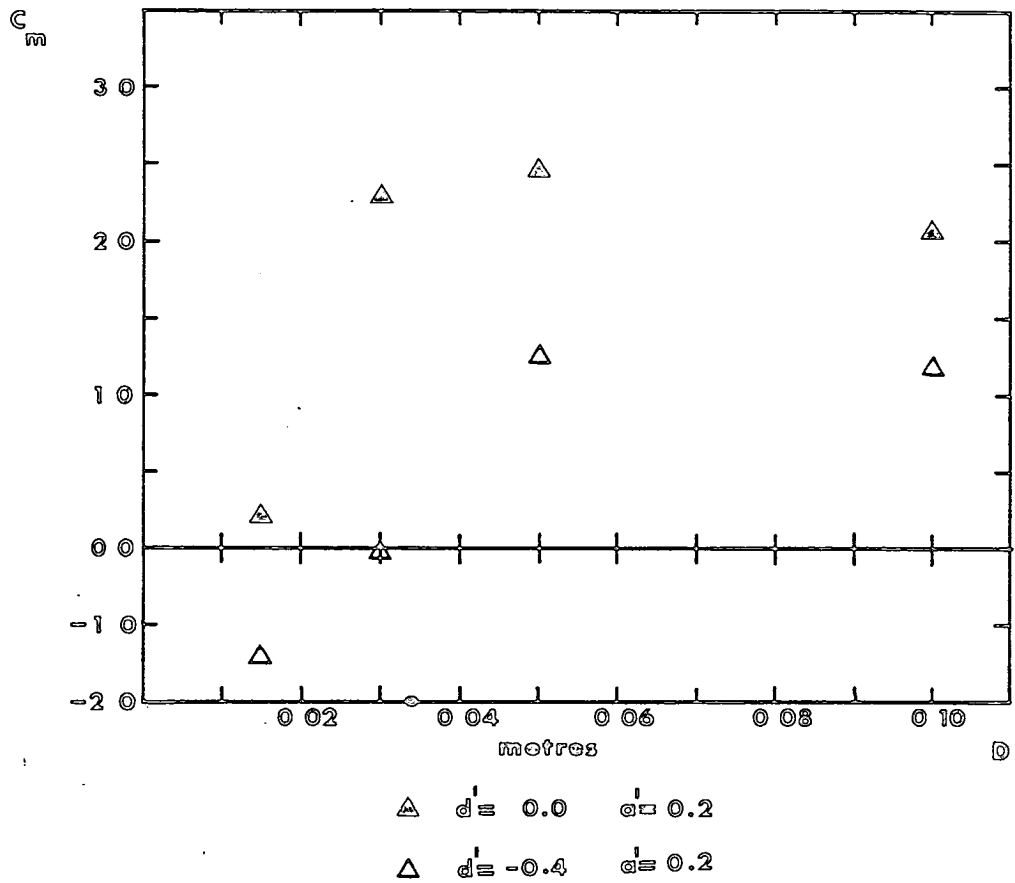


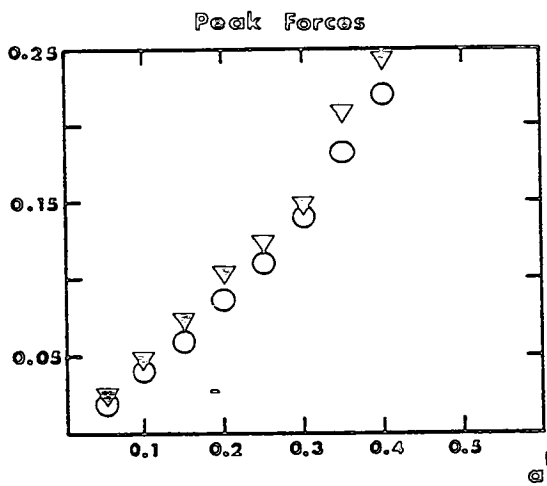
**Figure 2.29** Values of the inertia coefficient,  $C_m$ , for the vertical force presented in figures 2.22 and 2.23 variation with cylinder diameter at constant relative wave amplitude, wavelength and axis depth.

2.22 and 2.23 show the theoretical and experimental results where  $a'$ ,  $d'$ , and  $L'$  have been kept constant. Ideally, the curves should be identical and values of  $C_m$  (figure 2.29) be all the same. There is sufficient similarity between the individual curves and between the experimental and theoretical ones to enable one to propose that the equations could be used at full scale.

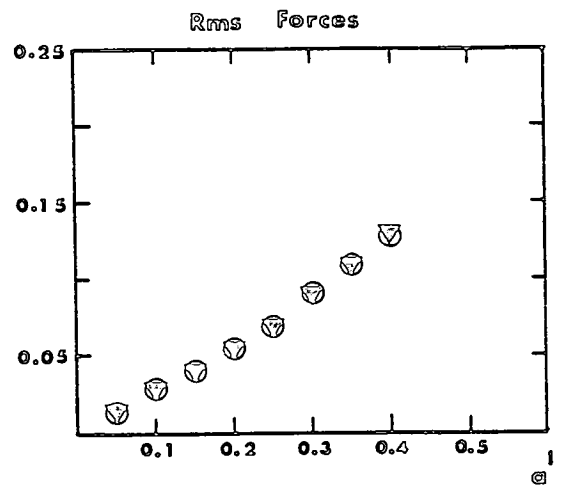
To show the modified equation's usefulness the peak and root mean square (r.m.s.) forces for different relative wave amplitudes, wavelengths and axis depths are shown in figure 2.30 for the horizontal force and figure 2.31 for the vertical force. As would be expected, r.m.s. forces are predicted better than peak forces. Both peak and r.m.s. forces follow the same trends for each varied parameter.

It would be useful to know the relative sizes of the buoyancy and inertial terms for the vertical forces. To estimate this we could calculate the ratio of the root mean square inertial force to the root mean square buoyancy force; the averages being taken over one cycle. Table 2.3 lists the numerical calculation of this ratio for each case, using the best fit value of  $C_m$ .

It would be extremely difficult to integrate the square of each term in equations (2.16) to arrive at an explicit expression for this ratio, but we can make some headway if we consider small amplitude waves. Referring to figure 2.1b we can approximate  $V(t)$  by writing it as the sum of



○ experimental



▽ theoretical

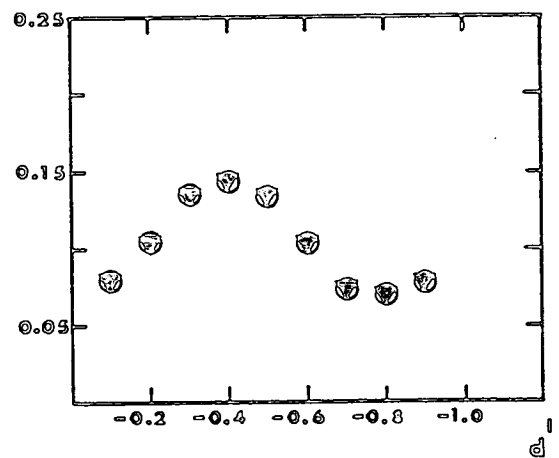
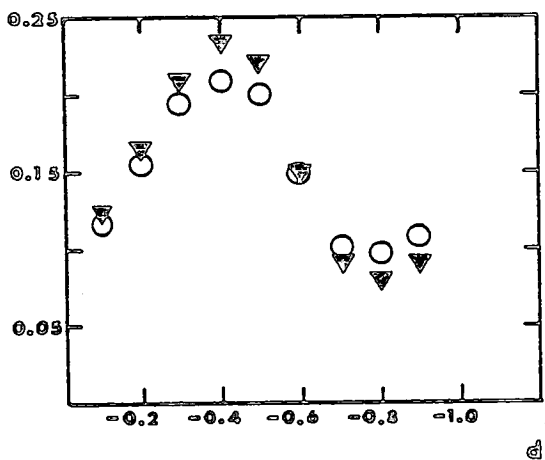
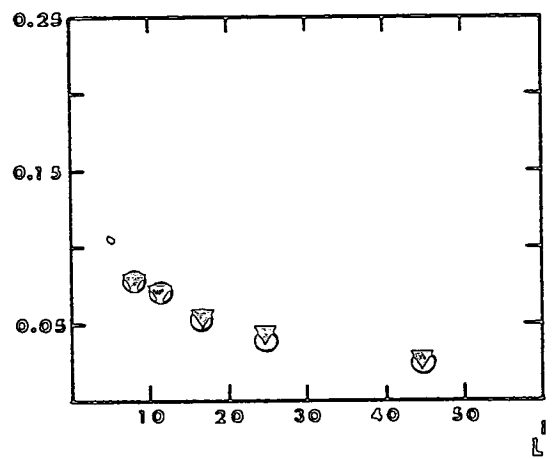
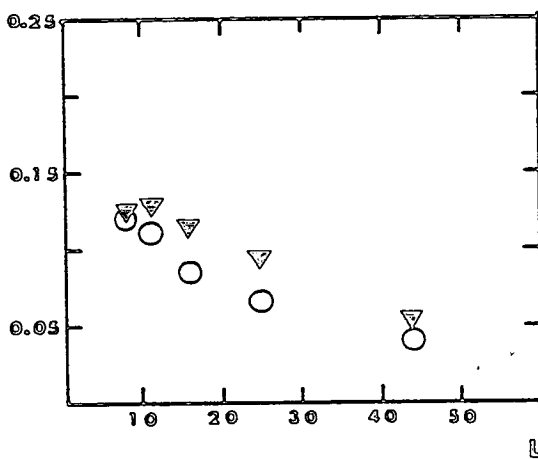


Figure 2.30 Peak and root mean square horizontal forces, variations with relative wave amplitude, wavelength and axis depth.

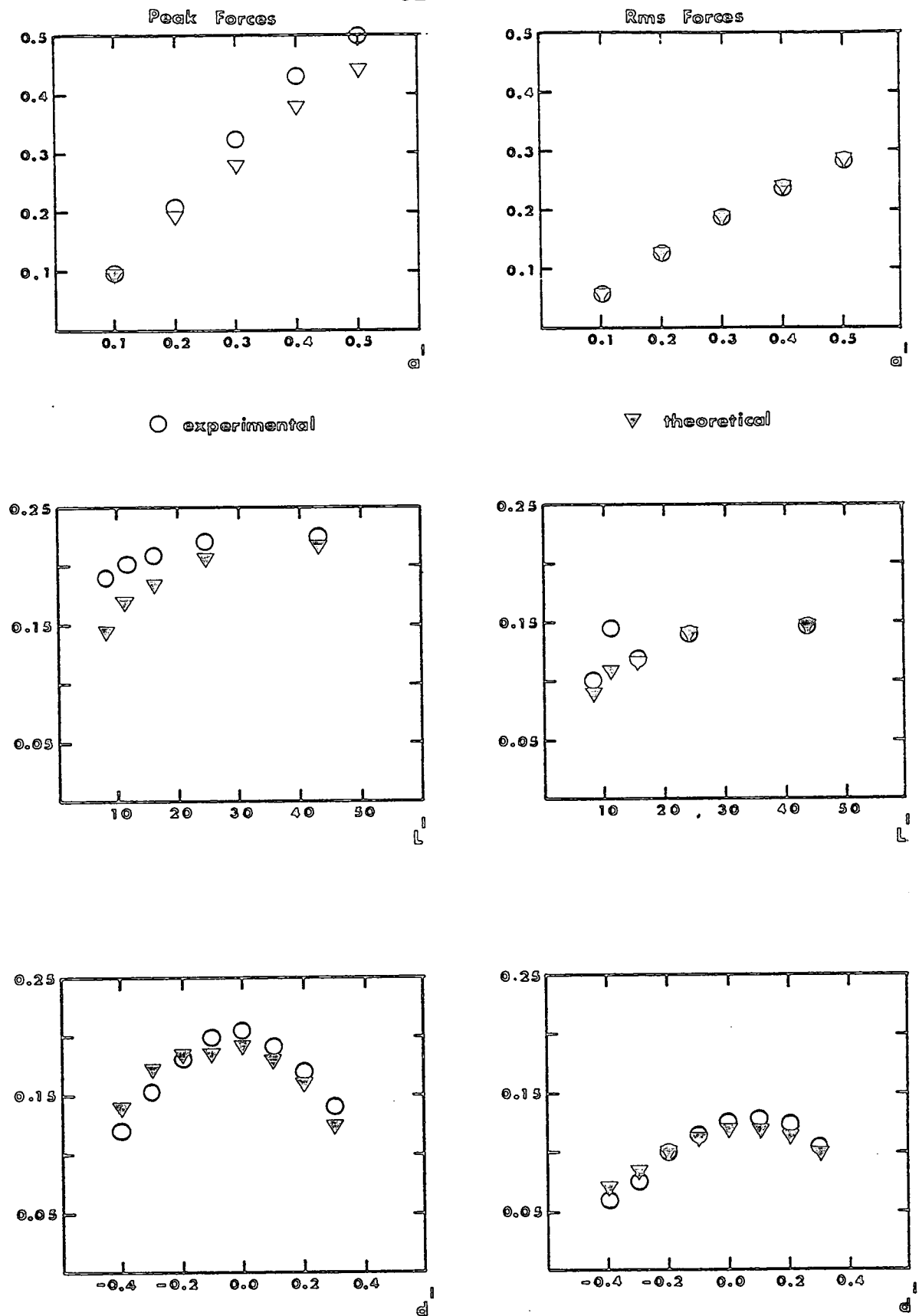


Figure 2.31 Peak and root mean square vertical forces, variations with relative wave amplitude, wavelength and axis depth.

the initial displacement in still water plus an estimate of the fluctuating part, as follows,

$$V(t) = V_0 + aD'\ell \sin\left(\frac{2\pi t}{T}\right), \quad (2.18)$$

where 
$$D' = D\left(1 - \frac{4d^2}{D^2}\right)^{\frac{1}{2}},$$

It can be shown that the resulting ratio is given by,

$$R = C_m \frac{2\pi}{L} \exp \frac{2\pi d}{L} \left[ \frac{V_0^2}{D'\ell} + a^2 \right]^{\frac{1}{2}}. \quad (2.19)$$

For the particular case  $d = 0$ , this simplifies to,

$$R = C_m \frac{2\pi}{L} \left[ \left(\frac{\pi D}{8}\right)^2 + a^2 \right]^{\frac{1}{2}}. \quad (2.20)$$

As an example, take  $a = 0.01\text{m}$ ,  $D = 0.10\text{m}$ ,  $L = 1.6\text{m}$  and  $C = 1.86$ , then  $R = 0.30$  to be compared with the experimental value of  $R = 0.33$  taken from Table 2.3.

In terms of relative parameters equation (2.20) can be written as,

$$R = C_m \frac{2\pi}{L} \left[ \left(\frac{\pi}{8}\right)^2 + (a')^2 \right]^{\frac{1}{2}}. \quad (2.21)$$

So at other scales this ratio remains constant if the relative parameters are kept the same.

The derivation of equation (2.21) assumes that the waves do not overlap the cylinder. If they do then the inertial

| CORRESPONDING<br>FIGURE | RELATIVE<br>WAVELENGTH | RELATIVE<br>WAVE<br>AMPLITUDE | RELATIVE<br>AXIS<br>DEPTH | R        |
|-------------------------|------------------------|-------------------------------|---------------------------|----------|
| Figure 2.8              | 16                     | 0.1                           | 0.0                       | 0.31     |
| "                       | 16                     | 0.2                           | 0.0                       | 0.33     |
| "                       | 16                     | 0.3                           | 0.0                       | 0.37     |
| "                       | 16                     | 0.4                           | 0.0                       | 0.42     |
| "                       | 16                     | 0.5                           | 0.0                       | 0.44     |
| Figure 2.9              | 16                     | 0.1                           | -0.4                      | 0.80     |
| "                       | 16                     | 0.2                           | -0.4                      | 0.66     |
| "                       | 16                     | 0.3                           | -0.4                      | 0.51     |
| Figure 2.10             | 16                     | 0.1                           | -0.5                      | 1.49     |
| "                       | 16                     | 0.2                           | -0.5                      | 0.85     |
| "                       | 16                     | 0.3                           | -0.5                      | 0.67     |
| Figure 2.11             | 16                     | 0.1                           | -0.6                      | $\infty$ |
| "                       | 16                     | 0.2                           | -0.6                      | 2.58     |
| "                       | 16                     | 0.3                           | -0.6                      | 1.18     |
| Figure 2.12             | 43                     | 0.2                           | 0.0                       | 0.18     |
| "                       | 24                     | 0.2                           | 0.0                       | 0.22     |
| "                       | 16                     | 0.2                           | 0.0                       | 0.35     |
| "                       | 11                     | 0.2                           | 0.0                       | 0.41     |
| "                       | 8                      | 0.2                           | 0.0                       | 0.50     |
| Figure 2.13             | 43                     | 0.2                           | -0.4                      | 2.88     |
| "                       | 24                     | 0.2                           | -0.4                      | 1.51     |
| "                       | 16                     | 0.2                           | -0.4                      | 1.01     |
| "                       | 11                     | 0.2                           | -0.4                      | 0.61     |
| "                       | 8                      | 0.2                           | -0.4                      | 0.41     |
| Figure 2.14             | 16                     | 0.1                           | 0.3                       | 0.27     |
| "                       | 16                     | 0.1                           | 0.2                       | 0.28     |
| "                       | 16                     | 0.1                           | 0.1                       | 0.28     |
| "                       | 16                     | 0.1                           | 0.0                       | 0.30     |
| Figure 2.15             | 16                     | 0.1                           | -0.1                      | 0.37     |
| "                       | 16                     | 0.1                           | -0.2                      | 0.39     |
| "                       | 16                     | 0.1                           | -0.3                      | 0.48     |
| "                       | 16                     | 0.1                           | -0.4                      | 0.80     |
| Figure 2.16             | 16                     | 0.2                           | 0.3                       | 0.27     |
| "                       | 16                     | 0.2                           | 0.2                       | 0.29     |
| "                       | 16                     | 0.2                           | 0.1                       | 0.30     |
| "                       | 16                     | 0.2                           | 0.0                       | 0.33     |
| Figure 2.17             | 16                     | 0.2                           | -0.1                      | 0.37     |
| "                       | 16                     | 0.2                           | -0.2                      | 0.42     |
| "                       | 16                     | 0.2                           | -0.3                      | 0.55     |
| "                       | 16                     | 0.2                           | -0.4                      |          |

Table 2.3

Values of R, the ratio of the r.m.s. vertical inertia force to the r.m.s. buoyancy force.

force increases while the buoyancy force stays constant, and hence  $R$  increases. If the cylinder is uncovered then slamming would occur and this phenomenon is outside the scope of the present work.

In the study of waves the correct choice of force coefficient is always the most difficult problem. Hogben et al. (1977) give a comprehensive survey of all the known literature concerning inertia and drag coefficients. The data is categorised according to which wave regime is under consideration by parameters such as relative wave height  $a/D$  and Reynolds number. The data in this work would belong to the low Reynolds number and low Keulegan-Carpenter number in the inertia dominated region. The results of the fitting procedure yield values of  $C_m$  ranging between 1.0 and 3.0 (ignoring the few negative results). This range of values is much smaller than that which would result from a fit to Morison's original equation. If one assumes on the basis of the experimental results for different diameters that the equation does scale properly, then one could use the values here for prototype designs. In the case of vertical forces which are buoyancy dominated the choice of  $C_m$  has less influence on the total force and it is recommended that the potential theory value of  $C_m = 2.0$  be used.

The procedure described in this chapter for the evaluation of wave forces on a partially-submerged horizontal circular cylinder could be improved if necessary

by the inclusion of a drag term modified in a similar way. Values of the drag coefficient  $C_d$  would also have to be calculated. By using numerical integration the method could be extended to shapes other than the circular cylinder, and similarly the approximation used here, that the waterline is always horizontal, could be lifted. It should be noted, however, that this approximation is consistent with the use of linear wave theory for the evaluation of the accelerations, and the error introduced by it will be of the same order. Therefore, if the actual waterline is used in the calculation of  $V(t)$  then a higher order wave theory should also be used.

## CHAPTER 3

### WAVE REFLECTIONS AND MEAN HORIZONTAL FORCES

## CHAPTER 3

### WAVE REFLECTIONS AND MEAN HORIZONTAL FORCES

#### 3.1 Wave Reflections

The use of Morison's equation for the prediction of wave forces takes no account of the interaction between the cylinder and the waves. The flow field is assumed to be the same as if the cylinder were not there. The cylinders discussed in this work, however, are of a size where considerable reflections occur and it is remarkable that the modified equation presented in Chapter 2 gives the agreement that it does. Although a great deal of work has been done on wave reflection much of it is inapplicable to the cylinders we are interested in. Dean (1945) calculated the reflection and transmission coefficients for a plane vertical barrier extending from deep water (assumed infinite) to a point below the free surface. He showed that unless the barrier was very close to the surface (i.e. the gap is very small compared to the wavelength) the reflection was small. Ursell (1947) performed similar calculations for barriers extending from the free surface to a depth,  $d$  below, and also for some other positions. In a later paper Dean (1948) showed that the reflection coefficient for a submerged fixed circular cylinder was zero, the only effect introduced by the obstacle being a phase change between the incident and transmitted waves. The same problem was treated by Ursell (1950) in a more rigorous fashion and a uniqueness proof given. In all

these treatments two-dimensional linearised potential theory is used so the results are only valid for small wave steepnesses. Ursell's method for submerged cylinders was employed by Ogilvie (1963) to several specific problems including the calculation of the forces on a fixed cylinder. He also discussed the allied problem of a cylinder forced to oscillate in otherwise calm water, where the problem to be solved is the determination of the added mass and damping. Smith (1965) approached the case of surface waves over a submerged cylinder in a way which enabled the condition on the free surface to be completely satisfied and that on the cylinder only to an approximation. The advantage of this being that the error can be calculated. Previous methods using successive approximation techniques only gave the degree to which one approximation differed from the following one, and not the actual error.

Most of the analytical work on partially submerged cylinders has been concerned with short wave asymptotic solutions. The general problem of the motion of floating bodies has been solved by John (1949, 1950), but few specific cases have been evaluated in detail. Dean and Ursell (1959) studied the reflected wave amplitudes and forces on a fixed semi-immersed circular cylinder, and Barakat (1970) calculated the forces and reflection coefficients using both Neumann and Dirichlet boundary conditions on the surface of the cylinder. Although he was able to write analytical expressions for these quantities the two methods did not give consistent

results. The scattering of waves by surface obstacles in the limit of short wavelengths has been discussed by Ursell (1961), and again is worked out in detail for a fixed semi-immersed circular cylinder. Calculations for rectangular obstacles performed by Mei and Black (1969) for water of finite depth were done numerically using a variational formulation. The most notable feature of this work is the oscillation of the reflection coefficient with wavelength caused by the interaction between the ends of the obstacle, and this oscillation increased as its length increased. The scattering of short surface waves from a partially submerged cylinder has been considered by Alker (1977) using the method of matched asymptotic expansions developed by Van Dyke (1964) as applied to other problems involving short surface waves by Leppington (1972, 1973(a), (b)), and Ayad and Leppington (1977). The latter paper derives a short wave solution for a rectangular scatterer similar to that discussed by Mei and Black (1969) for longer wavelengths. The scattering of waves from partially submerged cylinders is difficult in the case of arbitrary wavelengths because of the beach effects at the surface. It is easier to treat scatterers that intersect the free surface at right angles, and for this reason diffraction from rectangular objects, vertical piles, and semi-immersed cylinders has received much more attention. A review of water wave diffraction is given by Newman (1972) and Mei (1978) who discusses numerical methods.

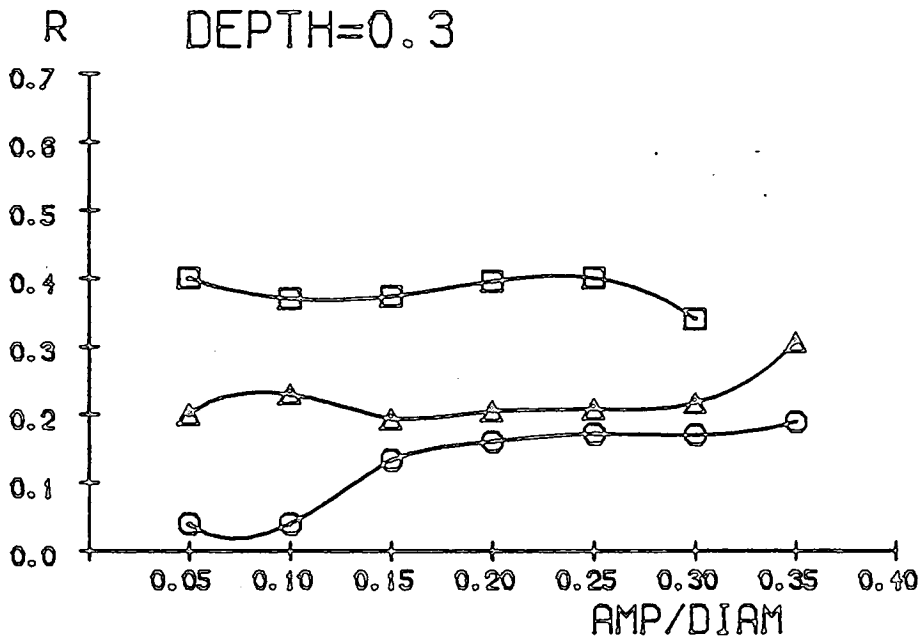
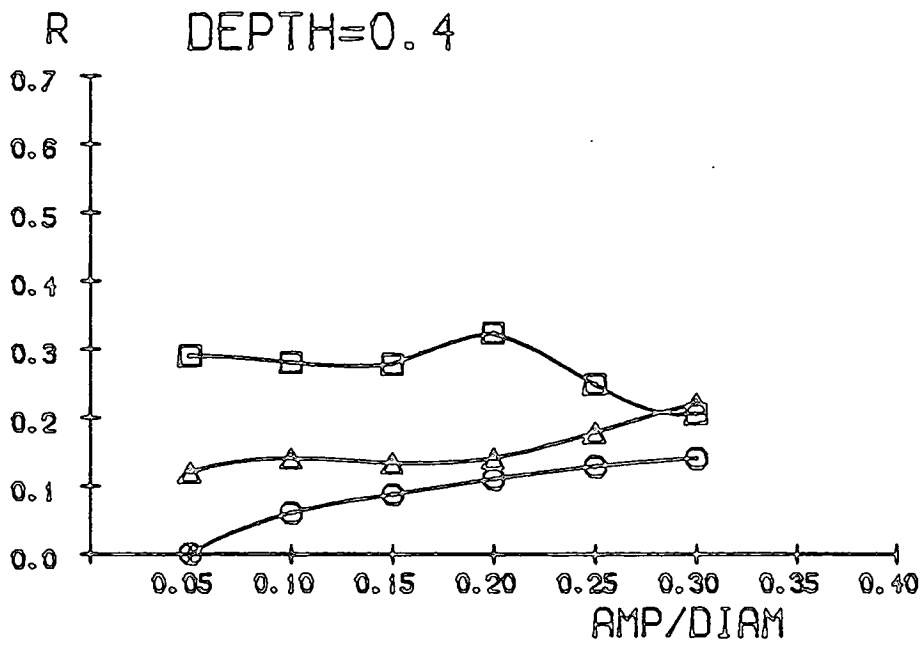
Another approach to the diffraction problem is often

made via the study of the forced oscillation of bodies in otherwise calm water. Advantage can then be taken of a set of reciprocity relations known as the Haskind relations (Haskind (1957)). The main result of these relations is that the force on an object can be computed without a knowledge of the diffraction potential. Indeed, for two-dimensional waves a knowledge of the amplitude and phase of the outgoing radiated waves due to forced motion in calm water is sufficient to calculate the force on a fixed body in the presence of incident waves. This method is used by Ursell (1948) who calculates the waves produced by a heaving circular cylinder on water of finite depth together with the added mass. The results are extended to water of finite depth by Yu and Ursell (1961) and compared with experiment. As mentioned in Chapter 1 it is customary to describe the force on a harmonically oscillating body in terms of the two parameters known as the added mass and the added damping. The force component in phase with the body's acceleration is associated with the added mass and that in phase with the velocity with the added damping. The added damping is related to the "amplitude ratio", which is defined as the ratio of the wave amplitude at infinity to the amplitude of oscillation of the cylinder, by equating the work done by the cylinder to the energy carried to infinity by the waves. Bolton and Ursell (1973) use the Haskind relations to compute the vertical wave force on an infinitely long semi-immersed cylinder in an oblique sea.

The only measured reflection coefficients known to the present writer for horizontal circular cylinders are those given by Dean and Ursell (1959) for the semi-immersed case, and by Zdravkovich and Cullasy (1978) who were interested in the attenuation properties of various groups of submerged horizontal cylinders. This lack of data prompted the work presented in this chapter, where the reflections from circular cylinders were studied for various axis depths, wavelengths, and wave amplitudes. The equipment used to do this is described in Chapter 5. The reflections were measured by placing wave gauges in front of the cylinder at a distance apart equal to one quarter wavelength.. It is shown in Chapter 5 that by searching up and down the tank for a position where the difference between the two gauges is maximised the incident wave is given by the average of the two signals and the reflected wave by half the difference.

Figures 3.1 to 3.7 show the reflection coefficients for a 10cm diameter horizontal circular cylinder at fourteen different axis depths and for three wave frequencies, 0.75hz, 1.00hz, and 1.25hz, the corresponding wavelengths being 2.78m, 1.56m, and 1.00m (deep water). The coefficients were measured as a function of wave amplitude over a range from 0.5cm to 3.5cm. In the figures the wavelength, wave amplitude and axis depth have been non-dimensionalised by dividing by the cylinder diameter, as in Chapter 2. An axis depth of -0.5 therefore corresponds to a cylinder that is just beneath the surface. Positive axis depths mean

that the cylinder axis is above the still waterline, but the results for these situations cannot be compared to Alker (1977) because we are considering long waves. The reflection coefficients on the whole remain fairly constant over the range of amplitudes considered, although there is a tendency for them to increase with amplitude for the longer waves and decrease with the shorter ones. The coefficients are larger for the shorter waves than for the longer ones, which is to be expected since for shorter waves the water motion is "concentrated" nearer to the surface and will be reflected more easily. The long waves tend to pass under the cylinder with a change in phase being the most noticeable effect. The change in phase has not been measured in this work. The variation with axis depth shows a peak at about 0.3-0.4 diameters. For greater depths the cylinder is totally submerged for the smaller amplitudes and the coefficients are considerably reduced in agreement with the Dean (1945) prediction of zero reflection for such situations. The curves presented here could be used to predict reflection coefficients for full scale cylinders providing similar values of relative amplitude, wavelength and axis depth were appropriate. There appears to be no readily available theoretical data of either analytical or numerical origin for comparison with the results given here. One possible line of enquiry might be to propose an empirical equation based on the above measurements. The writer's first tentative results show that some degree of success can be achieved by writing,



L/D      □ 10  
         △ 16  
         ○ 28

Figure 3.1 Reflection coefficients for a horizontal circular cylinder, variation with relative wave amplitude, wavelength and axis depth.

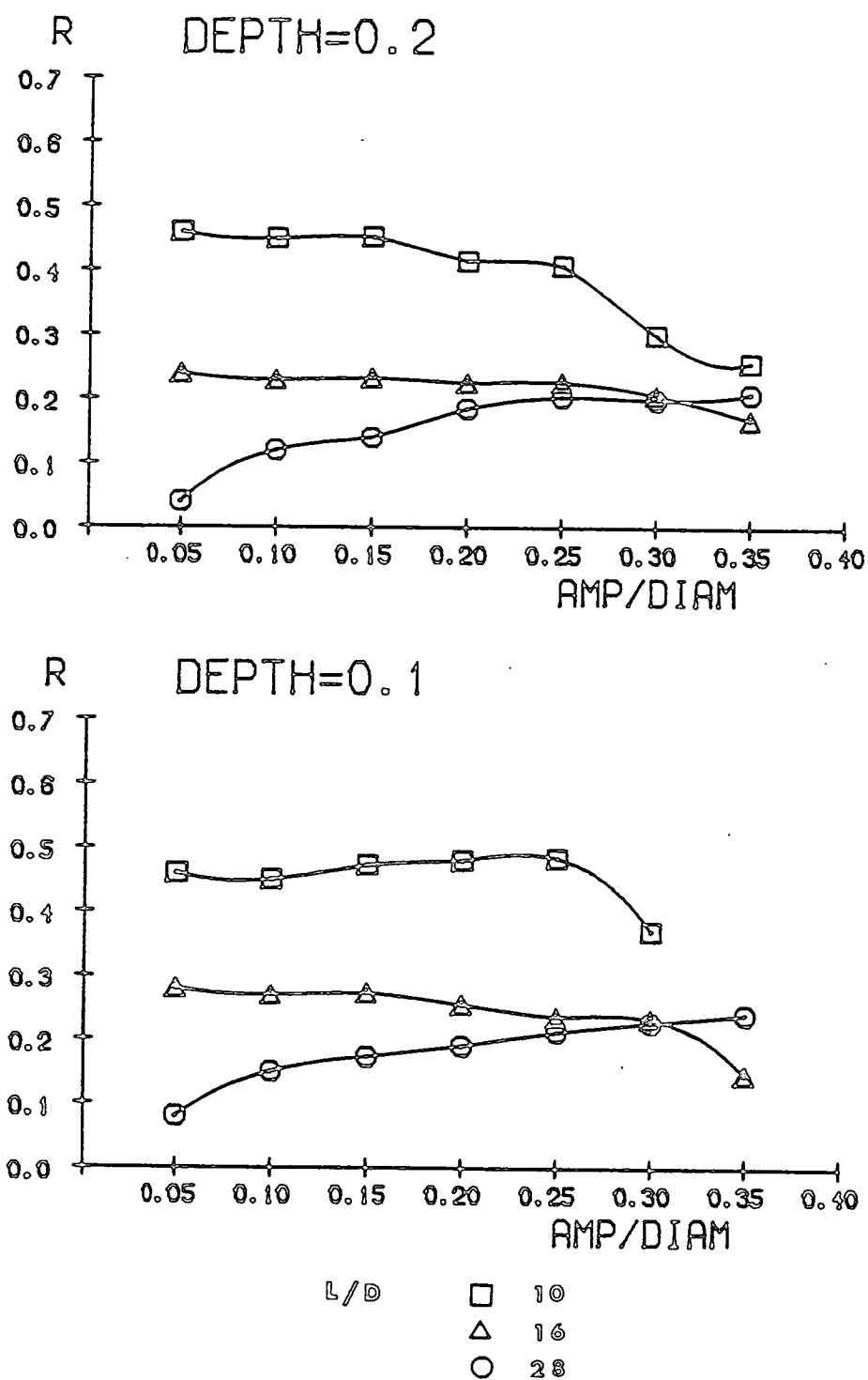


Figure 3.2 Reflection coefficients for a horizontal circular cylinder, variation with relative wave amplitude, wavelength and axis depth.

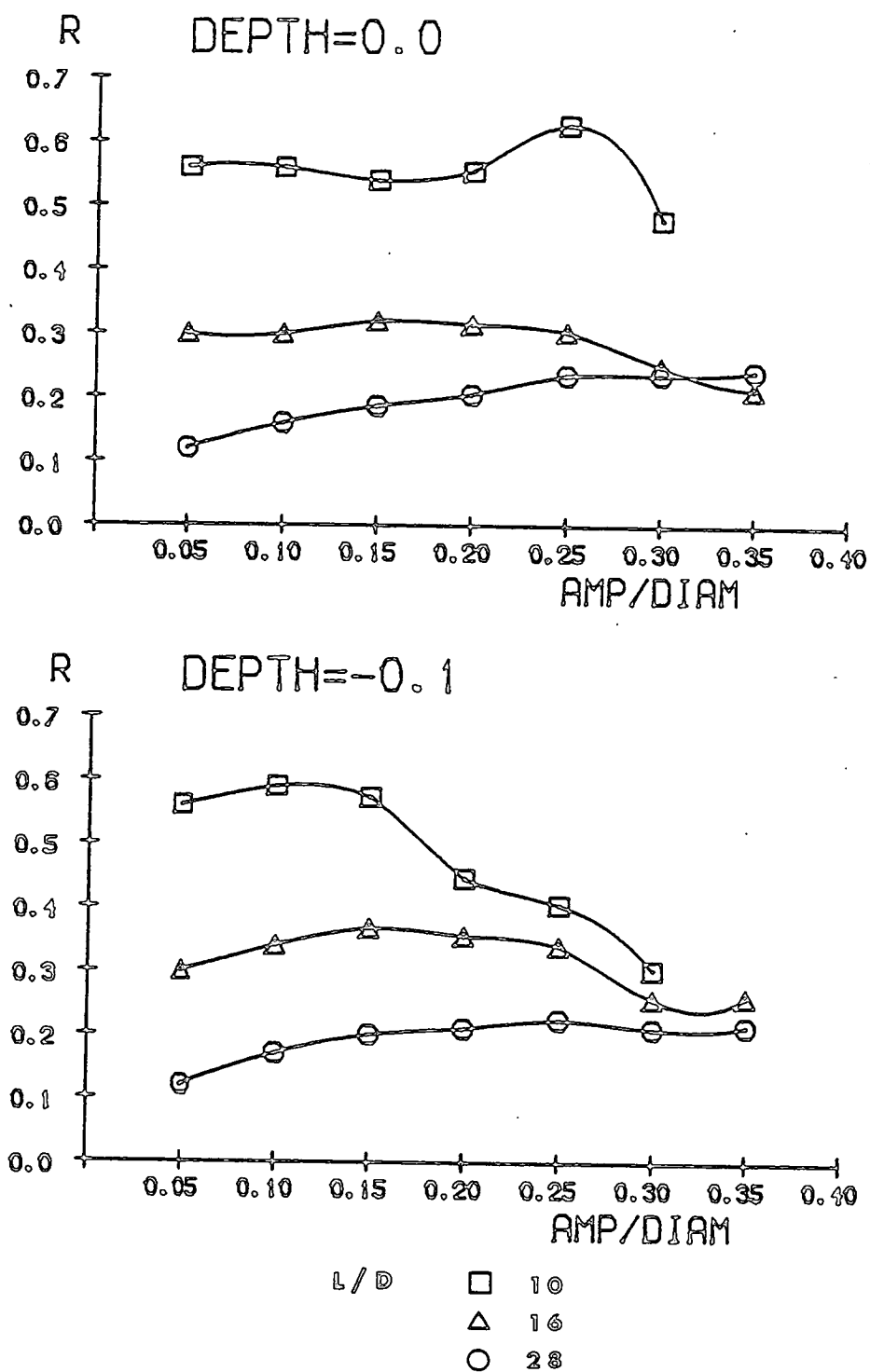
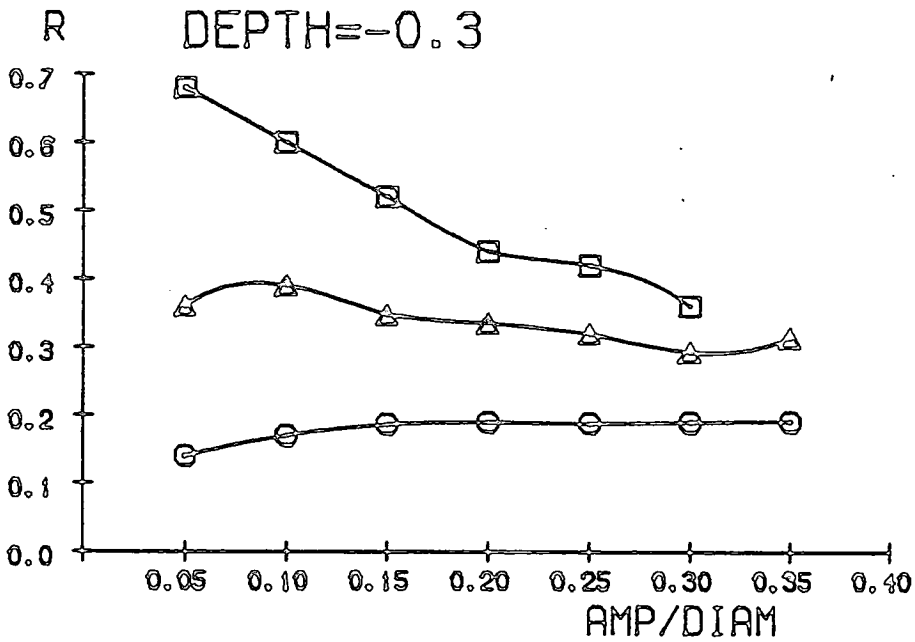
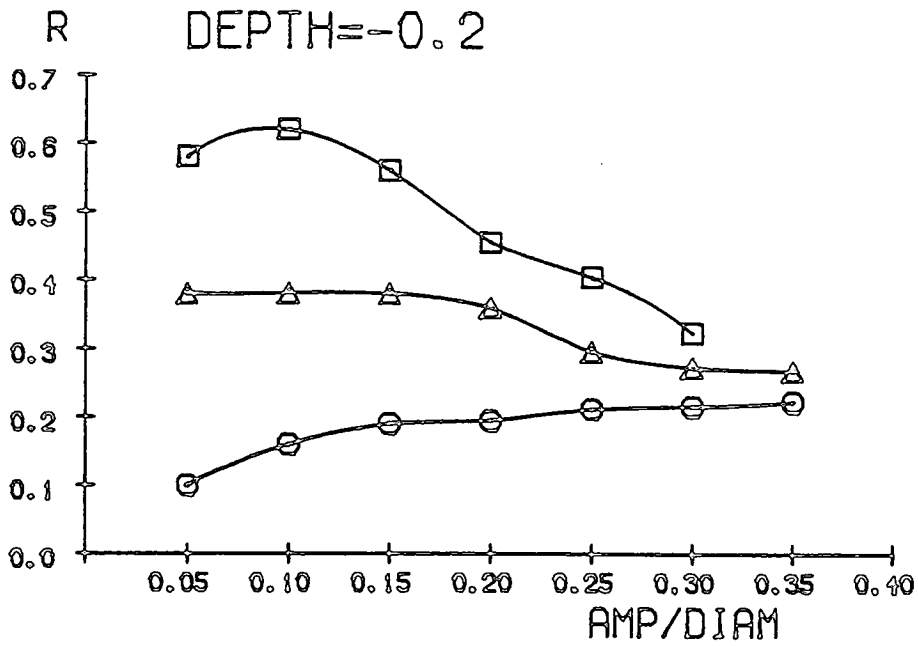


Figure 3.3 Reflection coefficients for a horizontal circular cylinder, variation with relative wave amplitude, wavelength and axis depth.



L/D    □    10  
          △    16  
          ○    28

Figure 3.4 Reflection coefficients for a horizontal circular cylinder, variation with relative wave amplitude, wavelength and axis depth.

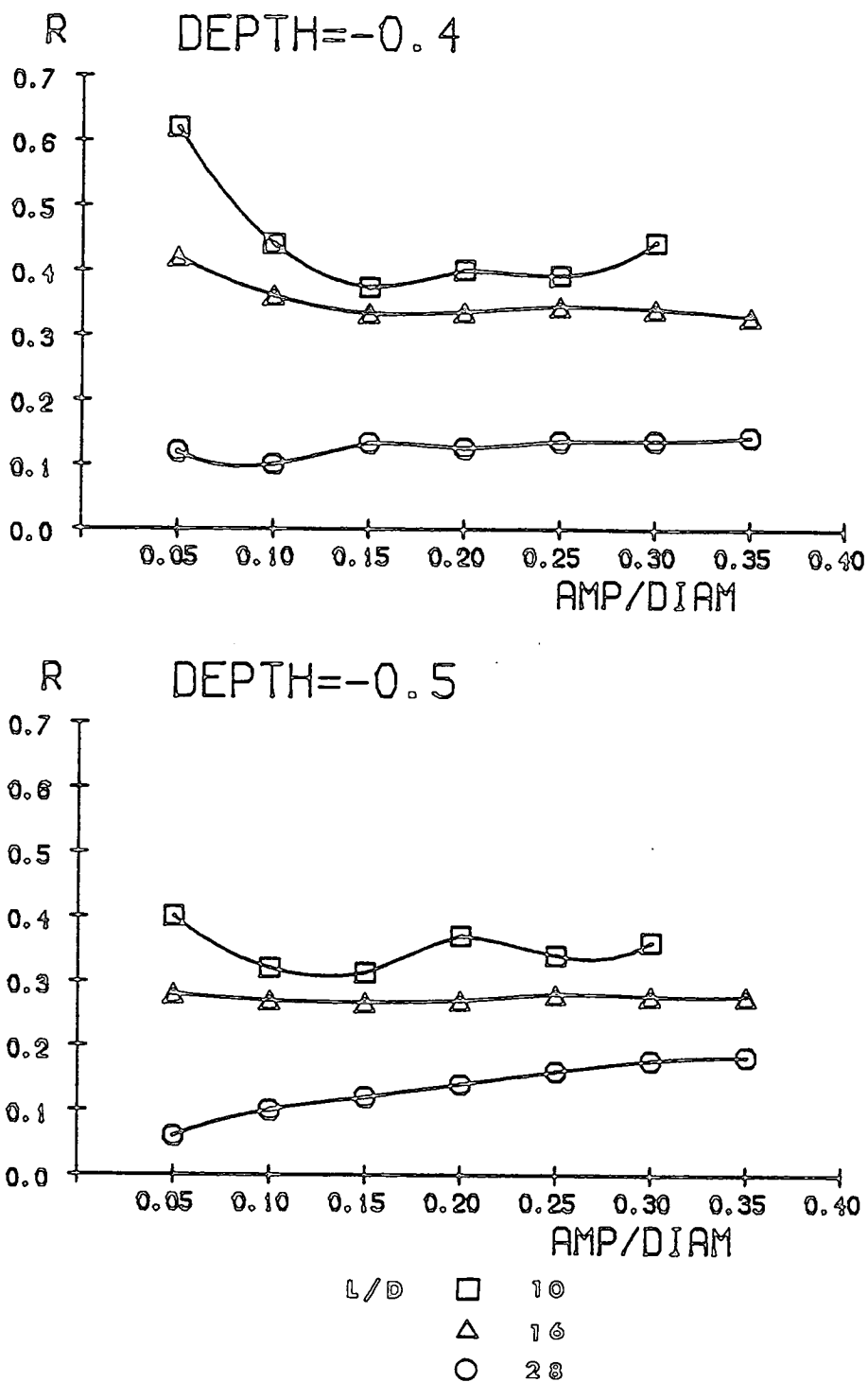
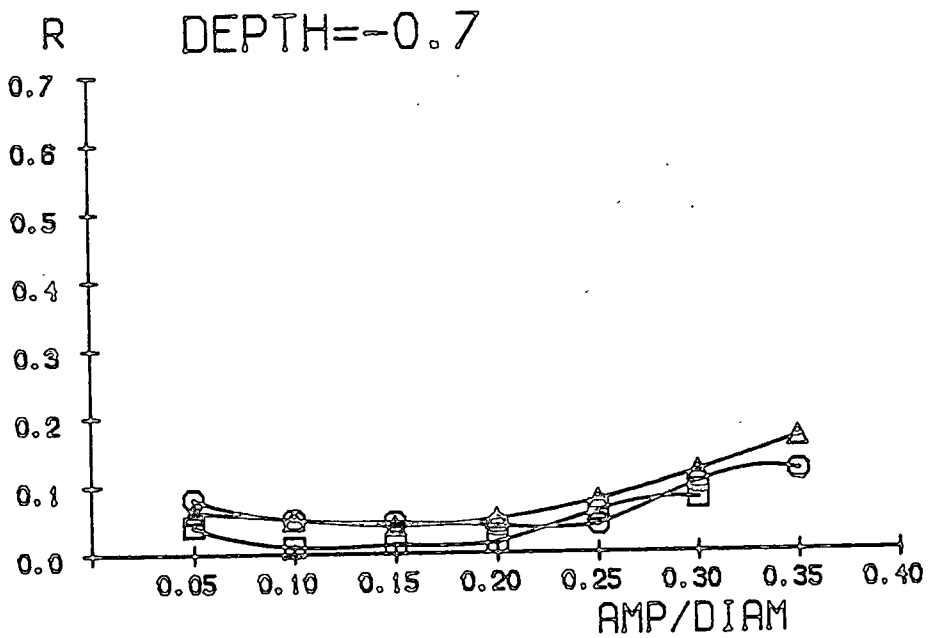
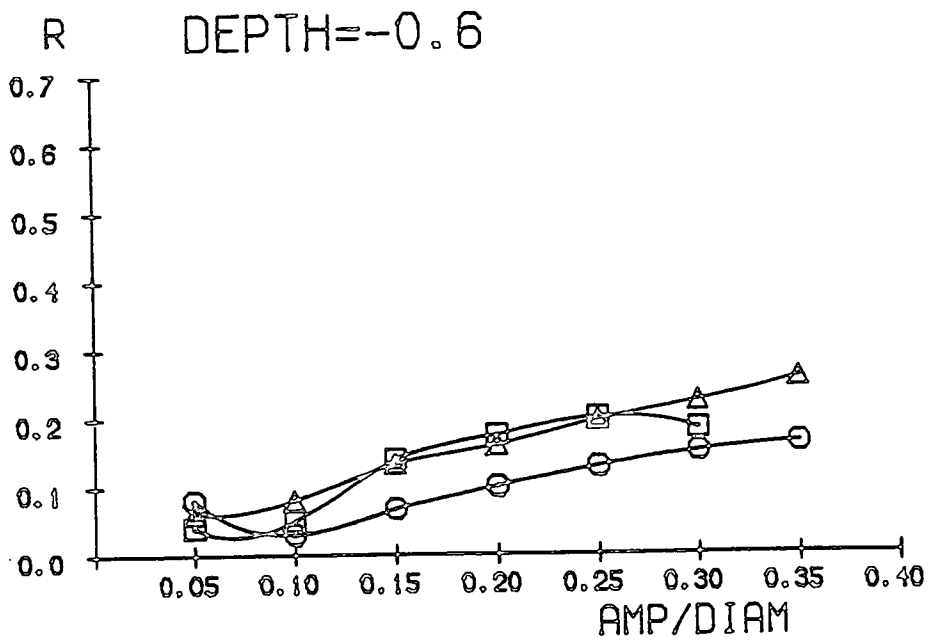
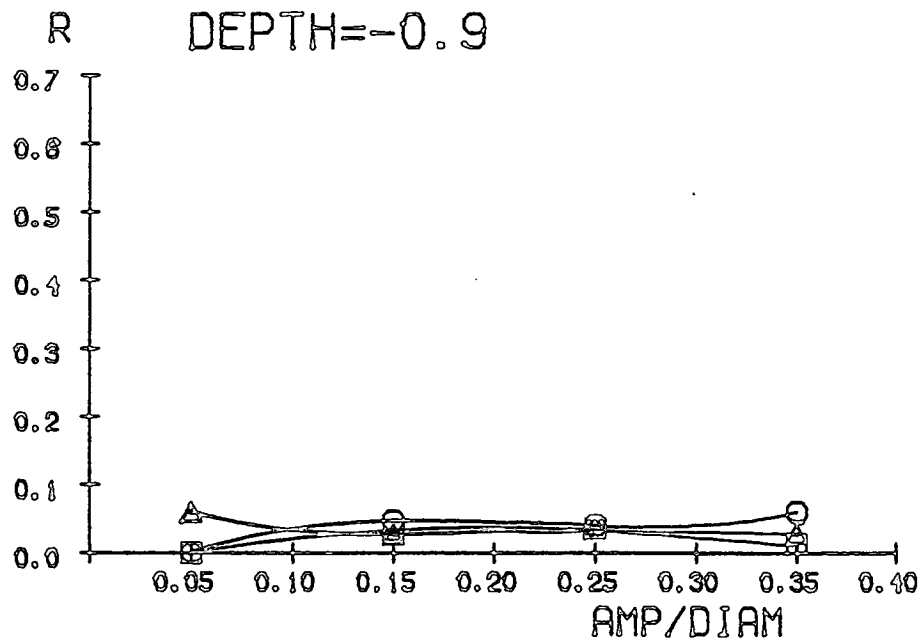
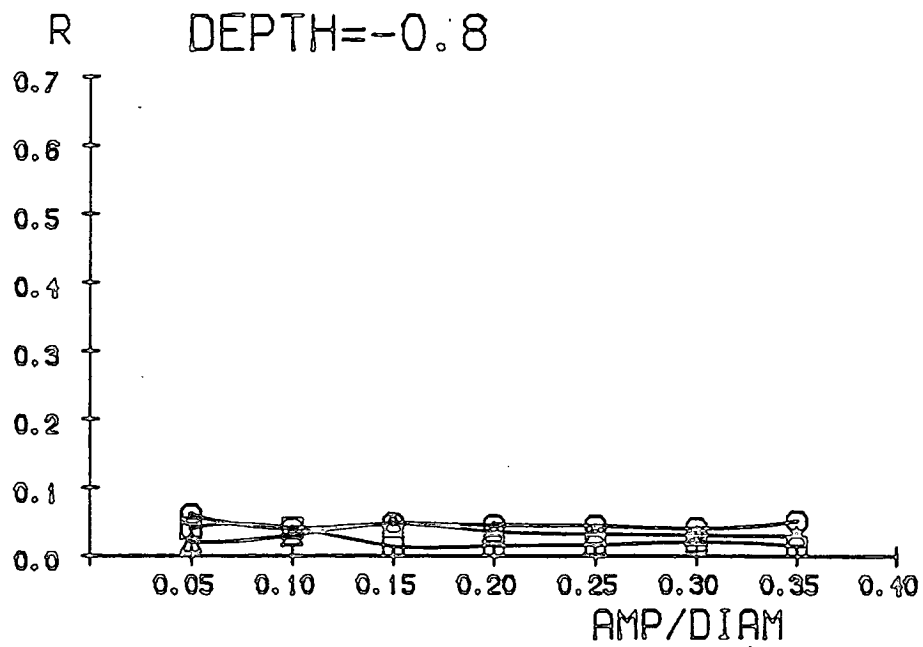


Figure 3.5 Reflection coefficients for a horizontal circular cylinder, variation with relative wave amplitude, wavelength and axis depth.



L/D    □    10  
         △    16  
         ○    28

Figure 3.6 Reflection coefficients for a horizontal circular cylinder, variation with relative wave amplitude, wavelength and axis depth.



L/D    □    10  
         △    16  
         ○    28

Figure 3.7 Reflection coefficients for a horizontal circular cylinder, variation with relative wave amplitude, wavelength and axis depth.

$$R = f(a/D, a/L, L/D)$$

The most suitable choice of  $f$  is still under investigation.

### 3.2 Mean Horizontal Forces

The mean horizontal forces exerted by waves on floating or submerged bodies has been discussed by Longuet-Higgins (1976). He shows that it is possible to calculate the mean force by considering the momentum flux of the waves across a plane normal to the wave direction. The general expression for the horizontal momentum flux across such a plane is,

$$\int_{-h}^{\eta} (p + \rho u^2) dy, \quad (3.1)$$

where  $p$  is the hydrodynamic pressure. If we subtract the flux in the absence of waves (caused by the hydrostatic pressure  $p_0$ ) and take averages over a wave period, the excess flux due to the waves is,

$$\int_{-h}^{\eta} (p + \rho u^2) dy - \int_{-h}^0 p_0 dy \quad (3.2)$$

If there are no reflected waves and the wave elevation has zero mean this excess is the "radiation stress" discussed by Longuet-Higgins and Stewart (1964). The same concept had been treated by Lundgren (1963) under the name of "wave thrust", in order to explain some phenomena of waves such as surf-beats and wave-induced currents. If we consider the momentum flux across two planes one on either

side of an object that can reflect the waves, then by assuming that the average horizontal momentum of the water is conserved the difference in the two fluxes must equal the mean horizontal force on the body. Longuet-Higgins shows that this is given by,

$$F = \frac{1}{2} \rho g (a_i^2 + a_t^2 - a_r^2) \left( 1 + \frac{2kh}{\sinh 2kh} \right), \quad (3.3)$$

where  $a_i$ ,  $a_t$ , and  $a_r$  are the incident, transmitted and reflected wave amplitudes respectively. If we assume that there is no energy absorption by the reflector then by conservation of energy,

$$a_i^2 = a_t^2 + a_r^2 \quad (3.4)$$

and therefore,

$$F = \frac{1}{2} \rho g a_t^2 \left( 1 + \frac{2kh}{\sinh 2kh} \right) \quad (3.5)$$

This force is always positive and acts in the direction of wave travel.

Longuet-Higgins' experimental results showed that the measured mean forces were a good deal smaller than the calculated ones for non-breaking waves and that for breaking waves the force was negative (seawards). A possible explanation for this in terms of the "wave set-up" was suggested, together with the contribution of the second harmonic to the mean force.

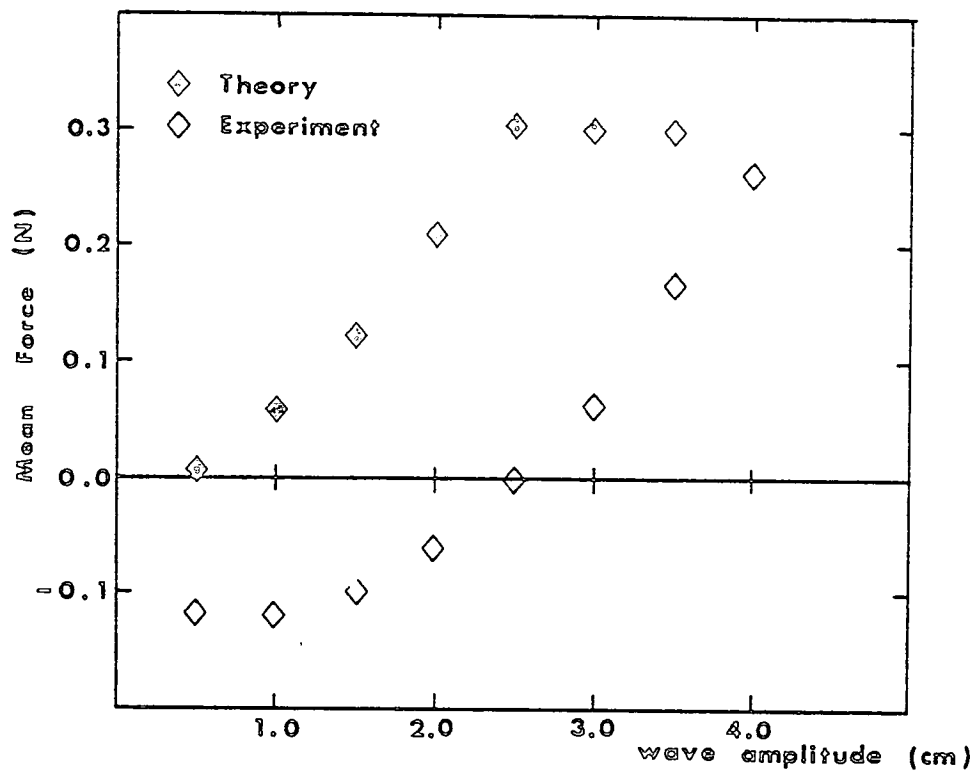


Figure 3.8(a) Mean horizontal forces on a horizontal circular cylinder, variation with wave amplitude.

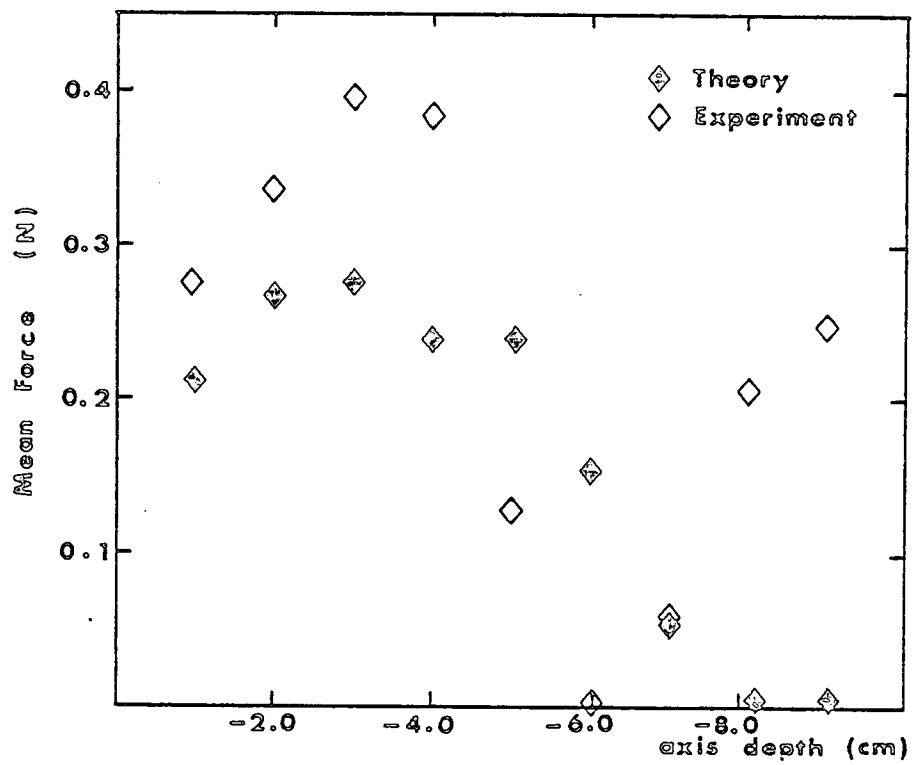


Figure 3.8(b) Mean horizontal forces on a horizontal circular cylinder, variation with axis depth.

The reflection coefficients presented above have been used to calculate the mean horizontal force on a 10cm cylinder using equation (3.5). The results are plotted in figures 3.8 as a function of wave amplitude for a semi-immersed cylinder and as a function of axis depth for constant amplitude. In all cases the wave frequency was 1.0hz. On the same figures the measured values, computed from the curves given in Chapter 2 (figures 2.4 and 2.6), are also given.

The first surprising result is that the negative forces occur for small wave amplitudes not large ones. In fact, the double beach effect introduced by the cylinder causes the small waves to break, and the larger ones go by with less interaction. The dependence of the mean force on axis depth is roughly similar for experimental values and for the theoretical ones, except for the two deepest cases where zero reflection should result in no mean force. It should be emphasised at this point that the mean horizontal forces are small compared to the maximum loads exerted by the waves and therefore of less importance. They are needed, however, in the design of the mooring system for the duckstring, but in this case the mean forces caused by the momentum change introduced by the power take-off also have to be considered.

## CHAPTER 4

### PARTIALLY SUBMERGED CYLINDERS IN IRREGULAR WAVES

## CHAPTER 4

### PARTIALLY SUBMERGED CYLINDERS IN IRREGULAR WAVES

#### 4.1 Introduction

So far, we have only discussed the wave forces induced by regular sinusoidal waves that are unidirectional and long-crested. It need hardly be said that the real ocean is by no means regular and the results obtained previously must be examined and further developed so that they may be applied to a more realistic situation. Two approaches are commonly used to this end. In the first an irregular ocean environment is characterised by an extreme or 'design' wave, which should represent the worst type of wave to be expected, and then peak forces are calculated using Morison's equation as outlined in the first chapter. This method relies on the correct choice of the wave profile and a wave theory for the predictions of the relevant velocities and accelerations. The second method is to treat the problem as a stochastic process rather than a deterministic one and then to invoke statistical theory to calculate meaningful quantities such as force probability distributions, spectra, correlation functions and the various averages and moments. The large literature already developed for noise, random walks and other phenomena can be utilised towards this end with very little modification.

The problem can be stated as follows: given a statistical description of the ocean surface in terms of the wave elevation, water particle velocities and accelerations, how

can we derive corresponding properties of the wave force acting on a particular structure or part of a structure?

The first difficulty is one of providing the initial data about the ocean itself. The most commonly measured parameter is the wave height, and for longcrested unidirectional seas it is relatively easy to measure the wave spectrum, for example. However, for three dimensional seas this becomes much more difficult though some measurements have been taken (Ewing (1975)). Theoretical wave spectra have been proposed based on physical mechanisms of energy transfer between wind and waves. For example, the Pierson-Moskowitz spectrum uses wind speed as a characterising parameter to predict wave spectra for fully developed seas. Spectrum models based on experimental work include the JONSWAP (Joint North Sea Wave Project) spectrum and the Mitsuyasu spectrum (see Hasselmann et al (1976) and Mitsuyasu (1975)).

A commonly used model for the sea surface is the Gaussian one which considers the sea to be a linear summation of wavelets with a random phase (uniformly distributed between  $-\pi$  and  $\pi$ ) and an amplitude related to the energy content of the waves in that direction and that frequency, (Pierson (1955)). For unidirectional waves with a narrow band spectrum, Longuet-Higgins (1952) showed that the wave heights followed a Rayleigh distribution law, a fact supported by experimental evidence, except in shallow water. Most of the statistical work on wave forces has been

concerned with Morison's equation. Pierson and Holmes (1965) derived the probability distribution (p.d.f.) for the wave force on the basis of the wave elevation being a stationary Gaussian process. Borgman (1964) calculated from Morison's equation the p.d.f. for the maximum total force on a pile together with the cumulative distribution function (c.d.f.) and various other statistical parameters such as the kth moment. He assumed in his derivation that the sea surface was a narrow band Gaussian process which could be represented as

$$\eta(t) = V_t \cos (\omega_o t + \phi_t),$$

where  $V_t$  and  $\phi_t$  are slowly changing random variables. In his paper three methods are given for the calculation of the two empirical coefficients  $C_m$  and  $C_d$ . These are the method of moments, the method of maximum likelihood, and a graphical procedure. The coefficients are considered to be constant with depth and wave frequency and it has not been proved that the three methods give consistent results.

The work by Pierson and Holmes was extended by Borgman (1967a) who derived the p.d.f. and moment generating function for the wave force together with the covariance for the forces at two space-time points. This theory was then applied to a practical problem (Borgman (1967b)). By using the first term of a series approximation it was shown that the drag term in Morison's equation can be linearised. It is possible to derive the force spectrum from the correlation function using a Fourier Transform (see Newland (1975) p.42).

Starting with a knowledge of the wave elevation spectrum one calculates the water particle velocity and acceleration spectra (assuming linear wave theory) which are then Fourier transformed to provide the corresponding correlation functions. These are the input data for calculating the force correlation function (see Borgman (1967b), p.134).

All these procedures are very time consuming and if possible short cuts should be sought. For example, one could measure the water particle velocity and acceleration correlation functions directly using Laserdoppler anemometry, thus overcoming the limitations of a particular wave theory.

Another method for calculating wave forces as a function of time is given by Wheeler (1970) using a linear filter technique for wave profiles of arbitrary shape and length.

A series of papers by Tung (1975(a), (b), (c)) developes expressions for statistical quantities of wave kinetics and wave force allowing for the effect of the free surface fluctuations (ignored in the previous papers). By employing the Heaviside unit function he shows that the velocity and acceleration are no longer Gaussian processes and that the above quantities are altered, especially in the region of the free surface. The mean values of the horizontal component of fluid particle velocity and corresponding wave force are shown to be non-zero except at points far below the mean water level.

Tickell (1977) developes a multivariate p.d.f. for the loading on a number of structural members in a longcrested random sea.

The work presented in this chapter developes expressions for force correlations, spectra and averages for small waves using an approximate version of the modified Morison's equation given in chapter 2. As usual the assumption that the sea surface is a Gaussian process will be used. In order to show the main points of interest clearly and to obtain explicit expressions for the above quantities it will be assumed that the wave spectrum is also a Gaussian. This is a totally different assumption from the previous one and quite independent of it. The extension to more realistic spectra will be discussed later.

The same force measuring equipment that was used for regular waves was employed to obtain experimental data for comparison with some of the theoretical results derived below. The methods of data collection and analysis are fully described in chapter 5. The results are for two dimensional (i.e unidirectional) waves only.

#### 4.2 The Wave Elevation Spectrum

As stated above we shall, for the purposes of this work, assume that the wave elevation is a Gaussian process and that the wave elevation spectrum is also a Gaussian. More precisely, we assume that the autocorrelation function for  $\eta(t)$  is given by,

$$R_{\eta}(\tau) = S_0 \exp\left(-\frac{\sigma^2 \tau^2}{2}\right) \cos(\omega_0 \tau), \quad (4.1)$$

where  $\tau$  = lag interval,  $S_0$  = mean square wave elevation, and  $\sigma$  and  $\omega_0$  define the spectral width and the mean frequency respectively. Note that we assume that the process is stationary in time and space and for the purposes of comparison with data the usual assumption of ergodicity will be used (see Newland (1975), pp. 19-20).

We will follow Newland (p.42) in defining the corresponding two-sided wave spectrum,  $S(\omega)$ , as,

$$S_{\eta}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{\eta}(\tau) e^{-i\omega\tau} d\tau, \quad (4.2)$$

where  $\omega$  is the angular frequency in radians per second.  $S_{\eta}(\omega)$  and  $R_{\eta}(\tau)$  are a Fourier transform pair with  $R_{\eta}(\tau)$  given by,

$$R_{\eta}(\tau) = \int_{-\infty}^{\infty} S_{\eta}(\omega) e^{i\omega\tau} d\omega. \quad (4.3)$$

For  $R_{\eta}(\tau)$  given by equation (4.1) the transform gives,

$$S_{\eta}(\omega) = S_0 (8\pi \sigma^2)^{-\frac{1}{2}} \left[ \exp \frac{-(\omega - \omega_0)^2}{2\sigma^2} + \exp \frac{-(\omega + \omega_0)^2}{2\sigma^2} \right]. \quad (4.4)$$

The one-sided spectrum,  $G_{\eta}(\omega)$  is defined for  $\omega \geq 0$  as,

$$\begin{aligned}
 G_{\eta}(\omega) &= 2 S_{\eta}(\omega), \\
 &= \frac{1}{\pi} \int_{-\infty}^{\infty} R_{\eta}(\tau) e^{-i\omega\tau} d\tau, \\
 &= \frac{2}{\pi} \int_0^{\infty} R_{\eta}(\tau) \cos(\omega\tau) d\tau, \quad (4.5)
 \end{aligned}$$

since  $R_{\eta}(\tau)$  is an even function.

#### 4.3 Correlation Function and Spectrum of the Vertical Force

The modified force equations presented in chapter 2 are much too complicated to give a simple derivation of a force spectrum. We shall therefore restrict ourselves to a simpler situation and introduce a few approximations to enable a direct attack on the problem to be made. We shall assume that the irregular waves are small so that although there is a finite probability of the waves covering or uncovering the cylinder any effects due to this are small. If the waves are small compared to the cylinder we can write the volume term,  $V(t)$ , as the sum of the initial displaced volume plus a small fluctuating part, so that,

$$V(t) = V_0 + D' \eta(t), \quad (4.6)$$

where  $D' = 2(D^2/4 - d^2)^{1/2}$ , is the width of the cylinder at  $y = 0$ .

Hence the vertical force is given by,

$$F_y = C_m \rho (V_o + D' \ell \eta(t) \dot{u}_y) + \rho g D' \ell \eta(t). \quad (4.7)$$

Using the expression for the vertical water particle acceleration, (equation (2.12)).

$$\dot{u}_y = -\frac{2\pi g}{L} \exp\left(\frac{2\pi d}{L}\right) \eta(t),$$

we can therefore write,

$$F_y = A\eta(t) + B\eta^2(t), \quad (4.8)$$

a quadratic in  $\eta(t)$ , where

$$A = -C_m \rho g V_o \frac{2\pi}{L} \exp\left(\frac{2\pi d}{L}\right) + \rho g D' \ell$$

$$B = -C_m \rho g D' \ell \frac{2\pi}{L} \exp\left(\frac{2\pi d}{L}\right). \quad (4.9)$$

This method of writing the force in terms of the wave elevation depends on the wave elevation being sinusoidal. We shall try and justify using the same equations when we consider  $\eta(t)$  to be a random variable.

The assumption that  $\eta(t)$  is a Gaussian variable involves considering  $\eta(t)$  to be a sum of sinusoidal waves, i.e.

$$\eta(t) = \sum_i a_i \sin(\omega_i t + \delta_i), \quad (4.10)$$

where  $a_i$  is the amplitude of each component wavelet,  $\omega_i$  the angular frequency and  $\delta_i$  the phase. If we assume that the

total force,  $F$ , is the linear sum of the forces due to each component then,

$$\begin{aligned} F &= \sum_i \{Aa_i \sin(\omega_i t + \delta_i) + B(a_i \sin(\omega_i t + \delta_i))^2\} \\ &= A \sum_i \sin(\omega_i t + \delta_i) + B \sum_i [a_i \sin(\omega_i t + \delta_i)]^2. \end{aligned} \quad (4.11)$$

We wish to be able to write the total force as,

$$\begin{aligned} F &= A\eta(t) + B\eta^2(t) \\ &= A \sum_i a_i \sin(\omega_i t + \delta_i) + B[\sum_i a_i \sin(\omega_i t + \delta_i)]^2. \end{aligned} \quad (4.12)$$

Clearly, equations (4.11) and (4.12) would be equivalent if,

$$[\sum_i a_i \sin(\omega_i t + \delta_i)]^2 = \sum_i [a_i \sin(\omega_i t + \delta_i)]^2.$$

Now

$$\begin{aligned} [\sum_i (a_i \sin(\omega_i t + \delta_i))]^2 &= \sum_i \sum_j a_i \sin(\omega_i t + \delta_i) a_j \sin(\omega_j t + \delta_j), \\ &= \sum_i [a_i \sin(\omega_i t + \delta_i)]^2 + \\ &\quad \sum_{i \neq j} a_i \sin(\omega_i t + \delta_i) a_j \sin(\omega_j t + \delta_j). \end{aligned}$$

In computing the force correlation we will be interested only in average values of  $F$ , therefore, since the phases and amplitudes are randomly distributed and the  $a_i$ 's and  $a_j$ 's uncorrelated the second term will average out to zero

and equation (4.12) is justified even for random  $\eta(t)$ .

It should be noted that the above discussion depends on the coefficients  $C_m$  and  $C_d$  being constant for each frequency. The results in chapter 2 suggest that this is not true, but this fact will be neglected here.

The correlation function for the vertical force is defined by,

$$R_y(\tau) = \langle F_y(t) F_y(t + \tau) \rangle, \quad (4.13)$$

where the angular brackets denote the averaging operator,  $E$ . Using the notation,  $\eta(t) = \eta_1$ ,  $\eta(t + \tau) = \eta_2$ , together with the force formula, equation (4.8), gives

$$\begin{aligned} R_y(\tau) &= \langle (A\eta_1 + B\eta_1^2)(A\eta_2 + B\eta_2^2) \rangle, \\ &= A^2 \langle \eta_1 \eta_2 \rangle + AB \langle \eta_2 \eta_1^2 \rangle + AB \langle \eta_1 \eta_2^2 \rangle \\ &\quad + B^2 \langle \eta_1^2 \eta_2^2 \rangle. \end{aligned} \quad (4.14)$$

Since we are assuming that  $\eta(t)$  is a Gaussian process the third order moments are zero (see Bendat and Piersol (1971)) and we can expand the fourth order moments as, (Middleton, (1960) p.343),

$$\begin{aligned} \langle \eta_1^2 \eta_2^2 \rangle &= \langle \eta_1^2 \rangle \langle \eta_2^2 \rangle + 2 \langle \eta_1 \eta_2 \rangle^2, \\ &= S_o^2 + 2R_\eta^2(\tau). \end{aligned} \quad (4.15)$$

This gives,

$$R_y(\tau) = A^2 R_\eta(\tau) + B^2 S_o^2 + 2B^2 R_\eta^2(\tau) \quad (4.16)$$

Substituting for  $R_\eta(t)$  we have

$$R_y(\tau) = A^2 S_o \exp\left(\frac{-\sigma^2 \tau^2}{2}\right) \cos \omega_o \tau + B^2 S_o^2 \exp(-\sigma^2 \tau^2) \times \\ \cos^2 \omega_o \tau + B^2 S_o^2 \quad (4.17)$$

The one-sided force spectrum is then derived from

$$G_y(\omega) = \frac{2}{\pi} \int_0^\infty R_y(\omega) \cos(\omega \tau) d\tau. \quad (4.18)$$

Neglecting terms centred on negative frequencies this results in,

$$G_y(\omega) = A^2 S_o (2\pi\sigma^2)^{-\frac{1}{2}} \exp \frac{-(\omega - \omega_o)^2}{2\sigma^2} \\ + B^2 S_o^2 (4\pi\sigma^2)^{-\frac{1}{2}} \exp \frac{-(\omega - 2\omega_o)^2}{4\sigma^2} \\ + B^2 S_o^2 (4\pi\sigma^2)^{-\frac{1}{2}} \exp \frac{-\omega^2}{4\sigma^2} \\ + 2B^2 S_o^2 \delta(\omega), \quad (4.19)$$

where  $\delta(\omega)$  is the dirac delta function

The net result, therefore, consists of four terms: an

impulse term due to the fact that  $F_y(t)$  does not have zero mean, and three terms with peaks at zero,  $\omega$ , and  $2\omega$  respectively. The two side peaks have a 'width'  $\sqrt{2}$  times that of the central peak. The spectrum could be normalised by dividing by  $R_y(0)$ , so that it has unit area.

Some Useful Averages. We can predict the mean, the mean square, and the variance for the vertical force. The mean is given by,

$$\begin{aligned}\mu_y &= \langle F_y(t) \rangle = \langle A n(t) + B n^2(t) \rangle, \\ &= B S_0.\end{aligned}\tag{4.20}$$

The mean square is given by,

$$\begin{aligned}\langle F_y^2(t) \rangle &= R_y(0) \\ &= A^2 S_0 + 3B^2 S_0^2\end{aligned}\tag{4.21}$$

Hence the variance,  $\sigma_y^2$  is given by,

$$\begin{aligned}\sigma_y^2 &= \text{mean square} - (\text{mean})^2 \\ &= A^2 S_0 + 2B^2 S_0^2.\end{aligned}\tag{4.22}$$

#### 4.4 The Correlation Function for the Horizontal Force

We can make the same approximation to the formula for the varying volume as we did in the case of the vertical force. Hence,

$$F_x = C\dot{\eta}(t) + G\eta(t) \dot{\eta}(t), \quad (4.23)$$

since we have no buoyancy term and the horizontal acceleration is  $90^\circ$  out of phase with the wave elevation.

The constants C and G are given by,

$$C = -C_m \rho g V_o \frac{2\pi}{T} \exp \frac{2\pi d}{L},$$

and

$$G = -C_m \rho g D' l \frac{2\pi}{T} \exp \frac{2\pi d}{L}. \quad (4.24)$$

The correlation function,  $R_x(\tau)$  is then,

$$R_x(\tau) = \langle (C\dot{\eta}_1 + G\eta_1\dot{\eta}_1)(C\dot{\eta}_2 + G\eta_2\dot{\eta}_2) \rangle, \quad (4.25)$$

$$= C^2 \langle \dot{\eta}_1 \dot{\eta}_2 \rangle + G^2 \langle \eta_1 \eta_2 \dot{\eta}_1 \dot{\eta}_2 \rangle, \quad (4.26)$$

since third order moments are zero.

Using the expansion for fourth order moments again, we have,

$$\begin{aligned} R_x(\tau) &= C^2 \langle \dot{\eta}_1 \dot{\eta}_2 \rangle + G^2 \langle \eta_1 \eta_2 \rangle \langle \dot{\eta}_1 \dot{\eta}_2 \rangle \\ &= G^2 \langle \eta_2 \dot{\eta}_1 \rangle \langle \eta_1 \dot{\eta}_2 \rangle + G^2 \langle \eta_1 \dot{\eta}_1 \rangle \langle \eta_2 \dot{\eta}_2 \rangle. \end{aligned} \quad (4.27)$$

Now we use the following useful facts (see Newland (1975) and Bendat and Piersol (1971)):

$$\langle \eta_1 \dot{\eta}_2 \rangle = \frac{\partial R_{\eta}(\tau)}{\partial \tau} = - \langle \dot{\eta}_1 \eta_2 \rangle \quad (4.28a)$$

$$\langle \dot{\eta}_1 \dot{\eta}_2 \rangle = - \frac{\partial^2 R_{\eta}(\tau)}{\partial \tau^2}, \quad (4.28b)$$

$$\text{and} \quad \langle \eta_1 \dot{\eta}_1 \rangle = \langle \eta_2 \dot{\eta}_2 \rangle = 0 \quad (4.28c)$$

We can now write  $R_X(\tau)$  as follows,

$$R_X(\tau) = -C^2 \frac{\partial^2}{\partial \tau^2} R_{\eta}(\tau) - G^2 R_{\eta} \frac{\partial^2 R_{\eta}(\tau)}{\partial \tau^2} - G^2 \left( \frac{\partial R_{\eta}}{\partial \tau} \right)^2. \quad (4.29)$$

Differentiating equation (4.1) with respect to  $\tau$  once and twice and substituting the results into equation (4.28) gives,

$$\begin{aligned} R_X(\tau) = & -C^2 S_0 \exp\left(-\frac{\sigma^2 \tau^2}{2}\right) \left[ (\sigma^4 \tau^2 - \sigma^2 - \omega_0^2) \cos^2 \omega_0 \tau \right. \\ & \left. + 2\omega_0 \sigma^2 \tau \sin \omega_0 \tau \right] - G^2 S_0^2 \exp(-\sigma^2 \tau^2) \times \\ & \left[ (\sigma^4 \tau^2 - \frac{1}{2} \sigma^2 - \omega_0^2) \cos 2\omega_0 \tau + (\sigma^4 \tau^2 - \frac{1}{2} \sigma^2) \right. \\ & \left. + 2\sigma \tau \omega_0 \sin 2\omega_0 \tau \right]. \end{aligned} \quad (4.30)$$

The Fourier transform of equation (4.30) to obtain the horizontal force spectrum is fairly complicated, but can be determined if one remembers one of the results from Fourier transform theory. Namely, if  $Y(\omega)$  is the Fourier transform of  $X(\tau)$  then the fourier transform of  $\tau^n X(\tau)$  is  $d^n Y(\omega)/d\tau^n$ , for  $n$  even. A similar theorem holds for odd powers of  $\tau$ .

Reference is made to Erdelyi et al (1954) for tables of Fourier transforms.

The mean of the horizontal force is zero and the mean square is given by,

$$R_x(0) = S_o(C^2 + G^2 S_o)(\omega_o^2 + \sigma^2). \quad (4.31)$$

#### 4.5 The Crosscorrelation between Horizontal and Vertical Forces.

It is often useful to determine the interrelation between the vertical and the horizontal forces. One way of doing this for random waves is to calculate the crosscorrelation between the two. This is defined as,

$$R_{xy}(\tau) = \langle F_x(t) F_y(t + \tau) \rangle \quad (4.32)$$

Substituting in the formulae for the forces gives,

$$R_{xy}(\tau) = \langle (A\eta_1 + B\eta_1^2)(C\dot{\eta}_2 + G\eta_2\dot{\eta}_2) \rangle \quad (4.33)$$

$$= -AC \frac{\partial R_{\eta}(\tau)}{\partial \tau} - 2BG R_{\eta}(\tau) \frac{\partial R_{\eta}(\tau)}{\partial \tau}, \quad (4.34)$$

using the properties of averages given above (equations (4.28)). The final expression for the crosscorrelation is then,

$$\begin{aligned} R_{xy}(\tau) = & ACS_o(\omega_o \sin \omega_o \tau + \tau \sigma^2 \cos \omega_o \tau) \exp(-\frac{\sigma^2 \tau^2}{2}) \\ & + BGS_o^2(\omega_o \sin \omega_o \tau + 2\sigma^2(1+\cos 2\omega_o \tau) \exp(-\sigma^2 \tau^2)) \end{aligned} \quad (4.35)$$

If we require the other crosscorrelation,  $R_{yx}(\tau)$ , we can use the fact that  $R_{yx}(\tau) = R_{xy}(-\tau)$ . Note that the crosscorrelation is not an even function of  $\tau$  unlike the autocorrelation.

#### 4.6 The Probability Density Function and Distributions

The probability distribution of a random variable  $X$  is the function  $P_X(x) = \text{Prob}\{X \leq x\}$  defined for any number  $x$ , from  $-\infty$  to  $\infty$ . The probability density function is defined as,

$$p_X(x)dx = \text{Prob}\{x \leq X \leq x + dx\} \quad (4.36)$$

It is evident that

$$P_X(x) = \int_{-\infty}^x p_X(x)dx \quad (4.37)$$

and that,

$$p_X(x) = dP_X(x)/dx. \quad (4.38)$$

Our aim in this section is to discuss the probability functions for the wave force in relation to the density and distribution of the wave elevation,  $\eta(t)$ . We shall use as our model of vertical wave force equation (4.8),

$$F_y = A\eta(t) + B\eta^2(t).$$

Since we are assuming that  $\eta(t)$  is a Gaussian process with zero mean, we know that it has a p.d.f. given by,

$$p_{\eta}(\eta) = (2\pi\sigma_{\eta}^2)^{-\frac{1}{2}} \exp \left[ -\frac{\eta^2}{2\sigma_{\eta}^2} \right], \quad (4.39)$$

where  $\sigma_{\eta}^2$  is the variance of  $\eta(t)$  and, since the mean is zero, is equal to  $S_0$ , the mean square (cf. equation (4.1)). The distribution of  $\eta(t)$  is therefore,

$$P_{\eta}(\eta) = \int_{-\infty}^{\eta} p_{\eta}(x') dx' \quad (4.40)$$

$$= \frac{1}{2} + \operatorname{erf} \left( \frac{\eta}{\sigma_{\eta}} \right). \quad (4.41)$$

To find the p.d.f. of  $F_y$  we must first solve the equation,

$$F_y = A\eta(t) + B\eta^2(t) = g(\eta),$$

for  $\eta(t)$  in terms of  $F_y$ . This has the two real roots,  $\eta_1$  and  $\eta_2$ , given by,

$$\eta_1 = \frac{-A + \sqrt{A^2 + 4BF_y}}{2B},$$

and

$$\eta_2 = \frac{-A - \sqrt{A^2 + 4BF_y}}{2B},$$

where  $F_y$  must be greater or equal to  $-A^2/4B$  for the solutions to be real.

The p.d.f. of  $F_y$  is then given by (Papoulis (1965), p.126),

$$p_y = \frac{p_{\eta}(\eta_1)}{|g'(\eta_1)|} + \frac{p_{\eta}(\eta_2)}{|g'(\eta_2)|} \quad (4.43)$$

where  $g'(\eta) = dg(\eta)/d\eta$ .

$$\text{Since } g'(\eta_1) = A + 2B\eta_1 = \sqrt{A^2 + 4BF_y},$$

$$\text{and } g'(\eta_2) = A + 2B\eta_2 = -\sqrt{A^2 + 4BF_y},$$

$$\text{then } p_y(y) = (A^2 + 4BF_y)^{-\frac{1}{2}} \left[ p_{\eta} \left( \frac{-A + \sqrt{A^2 + 4BF_y}}{2B} \right) \right. \\ \left. + p_{\eta} \left( \frac{-A - \sqrt{A^2 + 4BF_y}}{2B} \right) \right] U(y + \frac{A^2}{4B}), \quad (4.44)$$

where  $U(y + A^2/4B)$  is the unit step function. Substitution of  $p_{\eta}$  gives,

$$p_y(y) = [(A^2 + 4BF_y)(2\pi\sigma_{\eta}^2)]^{-\frac{1}{2}} \{ \exp - [(\frac{-A + \sqrt{A^2 + 4BF_y}}{2B}) / 2\sigma_{\eta}^2] \\ + \exp - [(\frac{-A - \sqrt{A^2 + 4BF_y}}{2B}) / 2\sigma_{\eta}^2] \} U(y + A^2/4B) \quad (4.45)$$

The distribution function can then be derived from equation (4.37). The p.d.f. is determined solely by the parameters A, B, and  $S_0$ .

Unfortunately, we cannot follow the same line of reasoning and derive the probability density function for the horizontal force using equation (4.23) as a starting point, because this equation has an infinite number of

solutions for  $\eta(t)$  in terms of  $F_x$ , arising from the constant of integration.

#### 4.7 Experimental Results

A series of measurements was taken of the forces exerted by pseudo-random waves on a 10 cm diameter circular cylinder rigidly held at various depths by the same rig that was used for regular wave tests. The production of the random driving signal to the wavemaker by the superposition of a finite number of discrete oscillators of variable amplitude is described in Chapter 5 together with the methods of data collection and analysis. The oscillator amplitudes were set so that the driving signal was as near to a Gaussian form as was possible with only a finite number of frequencies. It was assumed that the transfer function between the drive and the waves was uniform and that the gain control was linear. The gain was set so that the waves were small compared to the "wavelength" of the waves and the cylinder diameter, That is, the probability of breaking waves being present was very small.

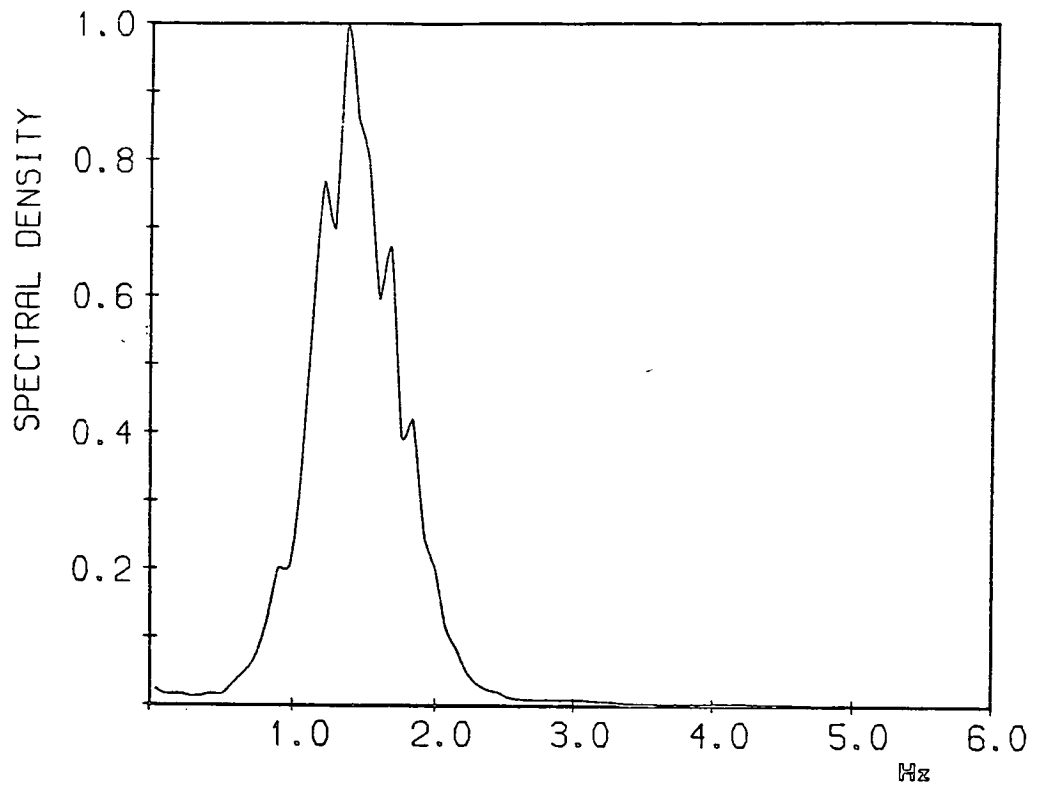
A 5 minute recording of the wave elevation with the cylinder removed was taken using a magnetic tape recorder. This was later analysed to give the mean-square wave height and a wave spectrum from which the mean frequency  $\omega_0$  and the spectral width  $\sigma$  were deduced. With the cylinder in position, the horizontal and vertical forces were recorded simultaneously over a period of 5 minutes, for relative axis depths ranging from -0.10 to -0.80.

Figure 4.1 shows the experimentally measured wave height spectrum together with the gaussian spectrum to which it is an approximation. All the spectra presented here have been normalised to unit height for convenience. The measured spectral width,  $\sigma$ , has been used to calculate figure 4.1(b) and also the theoretical force spectra. The experimental spectra for the horizontal forces are shown in figures 4.2(a) to (d). The calculated spectra are not given because the author is of the opinion that the information to be gained from the theoretical spectrum would not be worth the effort involved in fourier transforming the horizontal force correlation function (equation 4.30), given that the experimental spectra do not show any secondary peaks of significance. It is suggested that it would perhaps be sufficient to approximate the horizontal force spectrum by another Gaussian with mean frequency  $\omega_0$ , width  $\sigma$  and total area given by equation (4.31), remembering that the area under a power spectrum is equal to the mean square of the variable in question, with the area under a Gaussian

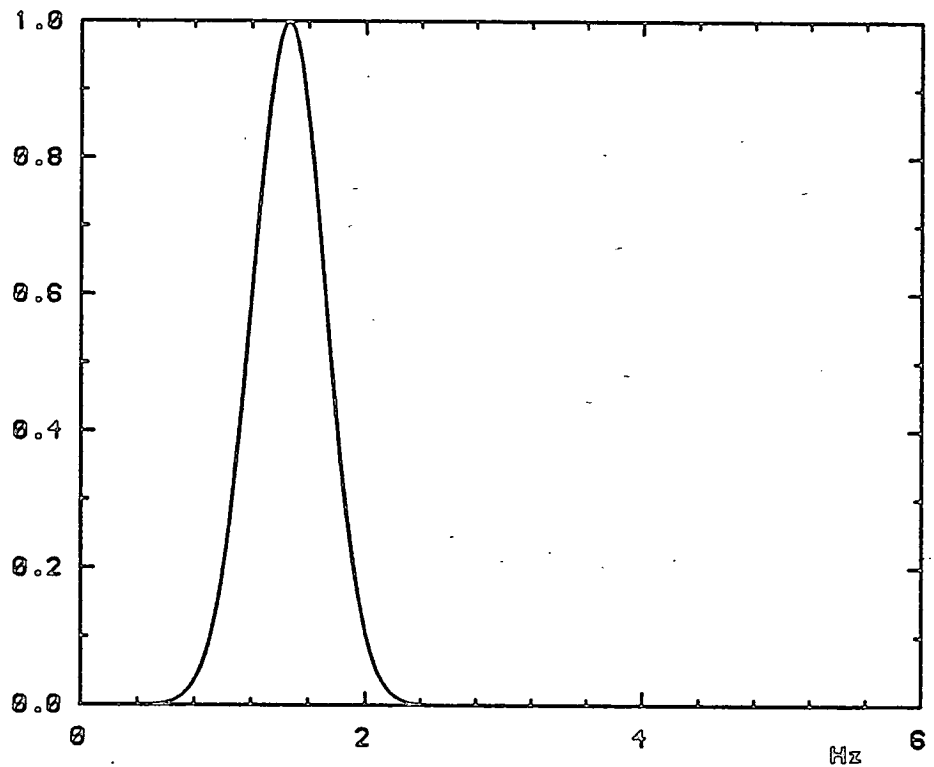
$$S_x = \exp \frac{-(\omega - \omega_0)^2}{2\sigma_x^2}$$

equal to  $(2\pi\sigma_x^2)^{\frac{1}{2}}$ .

Figures 4.3(a) to (h) show the force spectra for the vertical force. As can be seen, the side peaks only occur over a small range of relative axis depths with a peak somewhere between -0.35 and -0.45. Figure 4.4 shows the



(a)



(b)

Figure 4.1 (a) The experimental wave elevation spectrum used in the tests.

(b) The gaussian spectrum to which the above is an approximation.

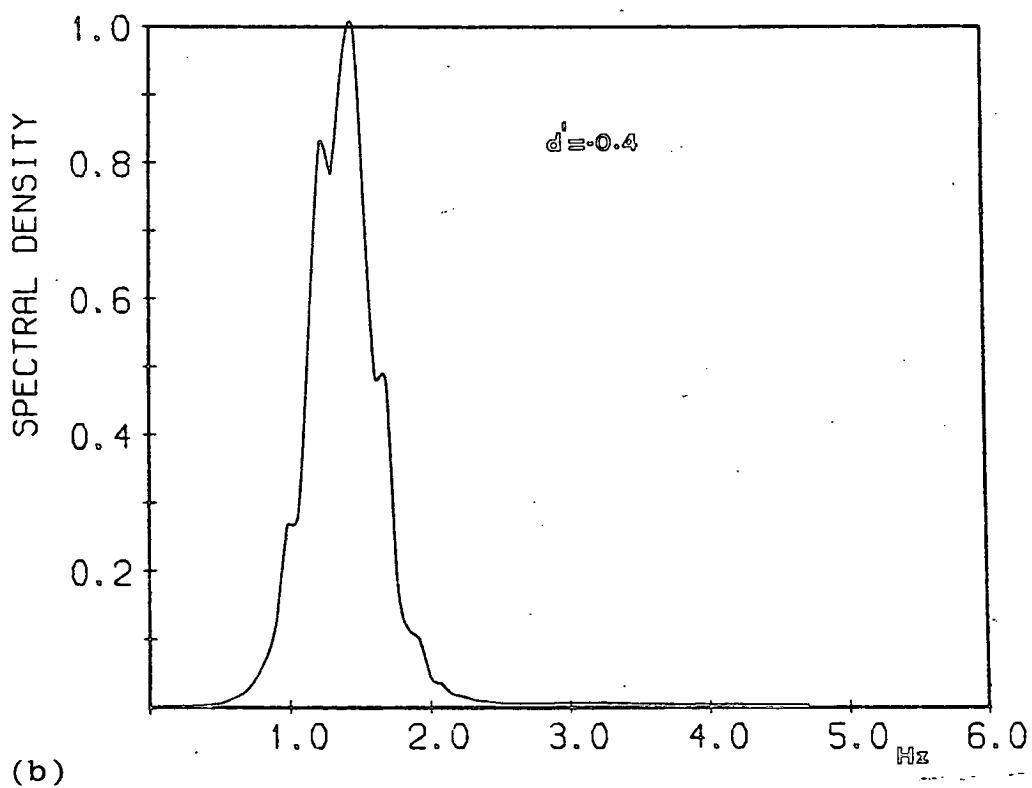
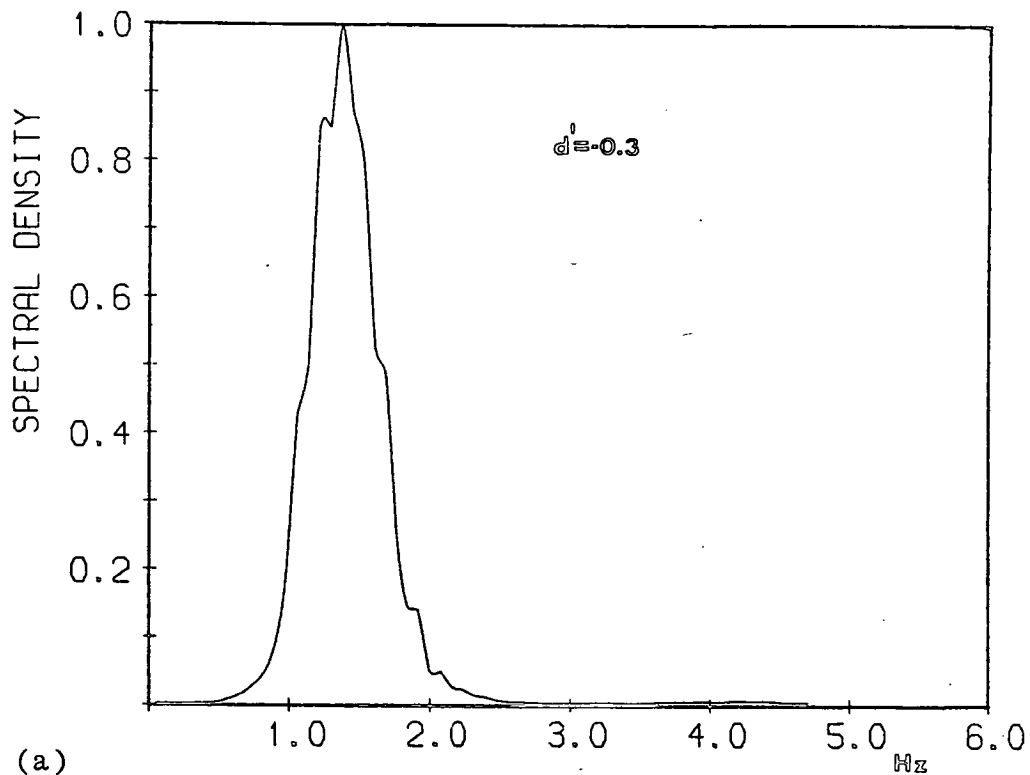


**Figure 4.2** The experimental horizontal force spectra for four axis depths.

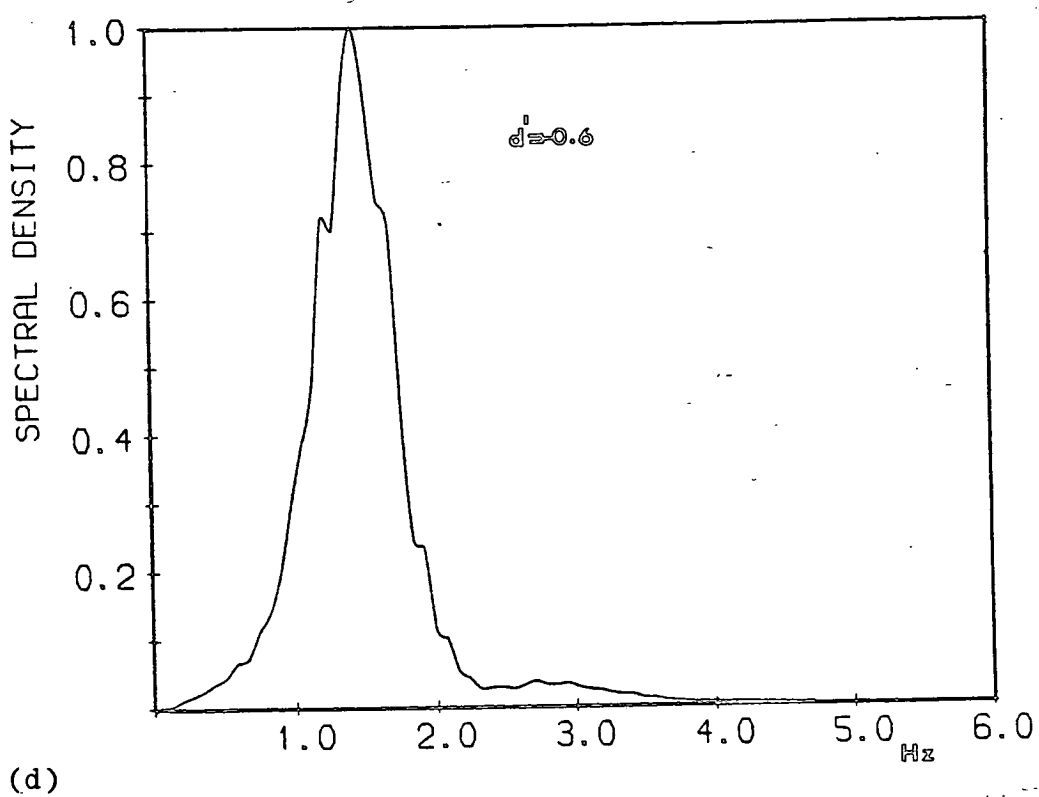
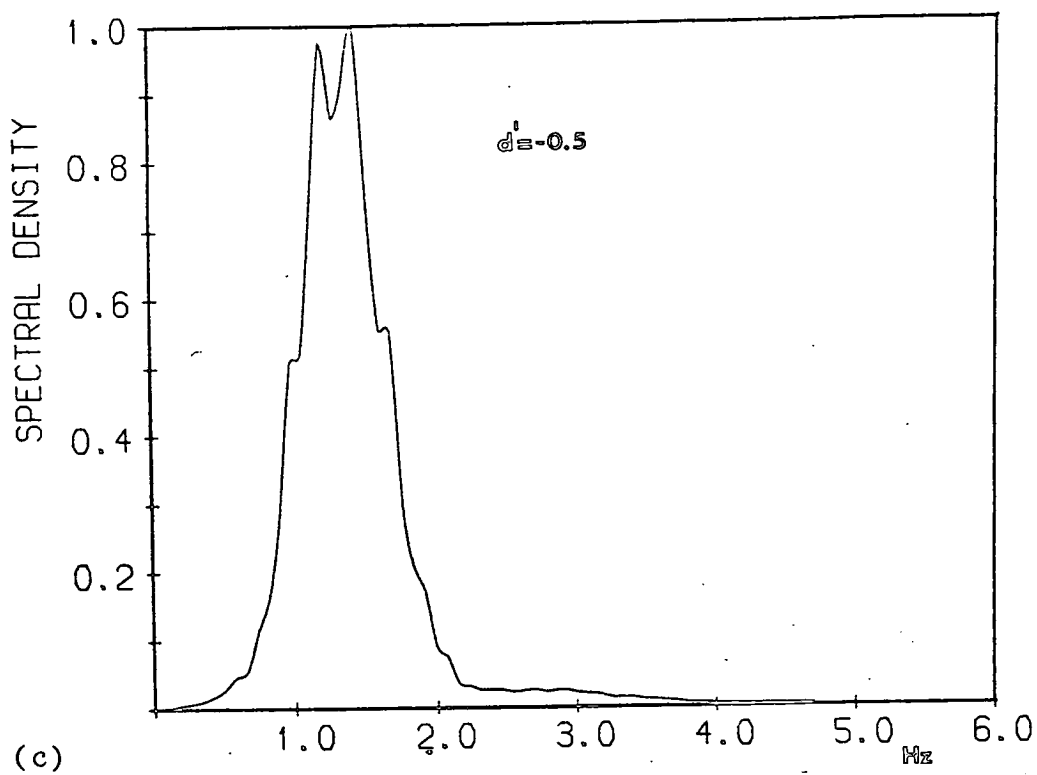


Figure 4.2 The experimental horizontal force spectra for four axis depths.

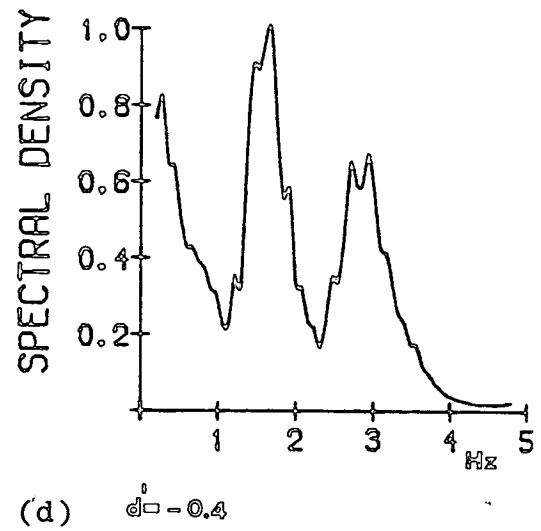
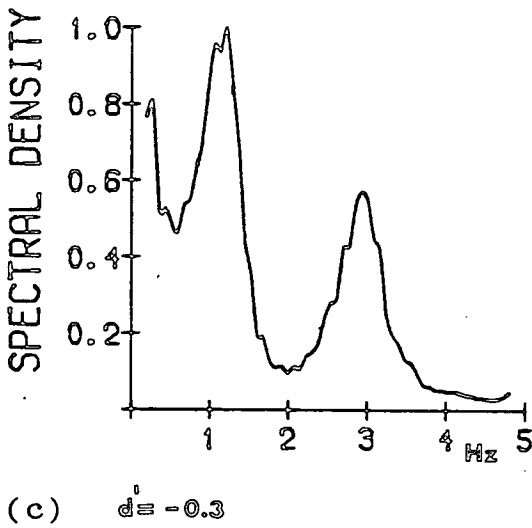
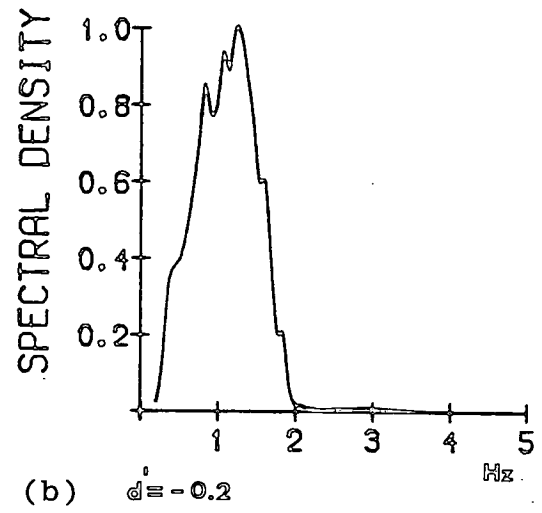
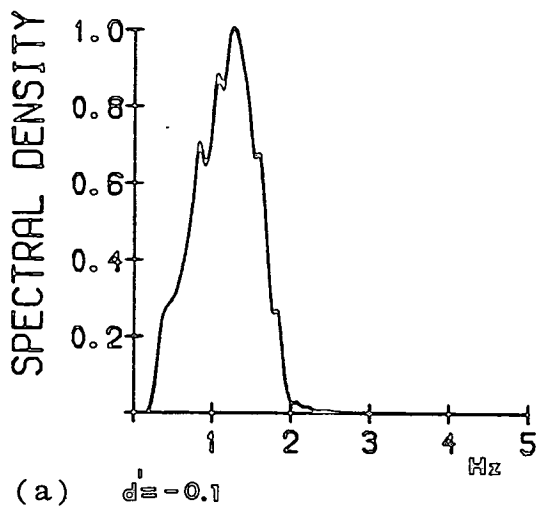


Figure 4.3 The experimental vertical force spectra for eight axis depths.

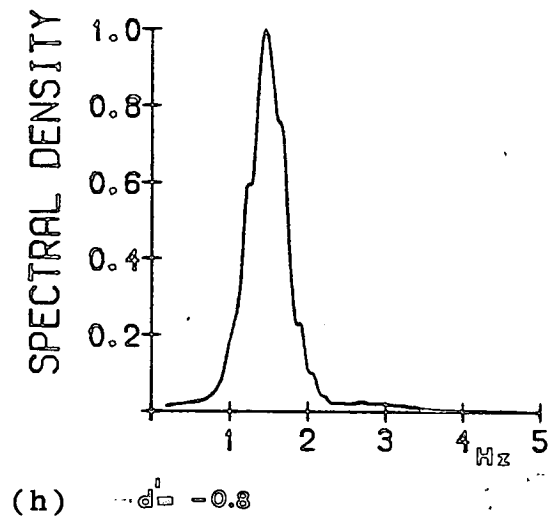
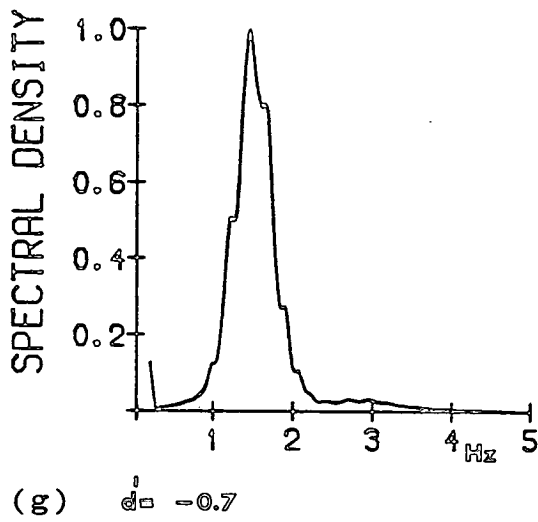
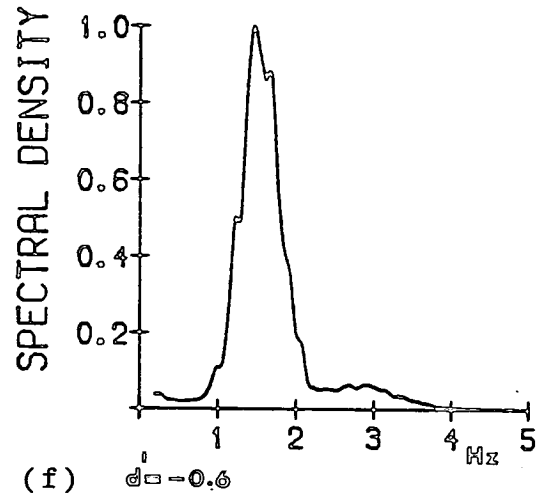
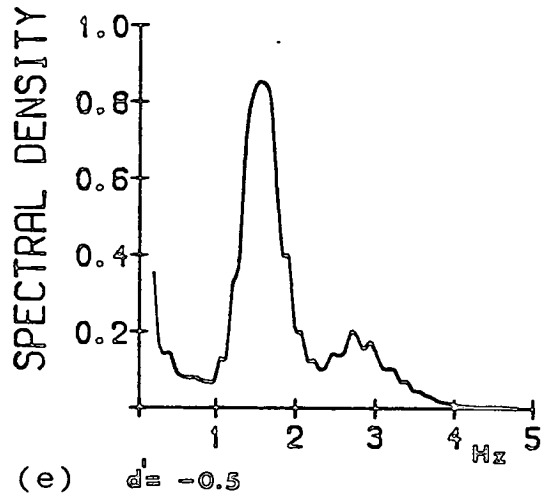


Figure 4.3 (cont'd)

The experimental vertical force spectra for eight axis depths.

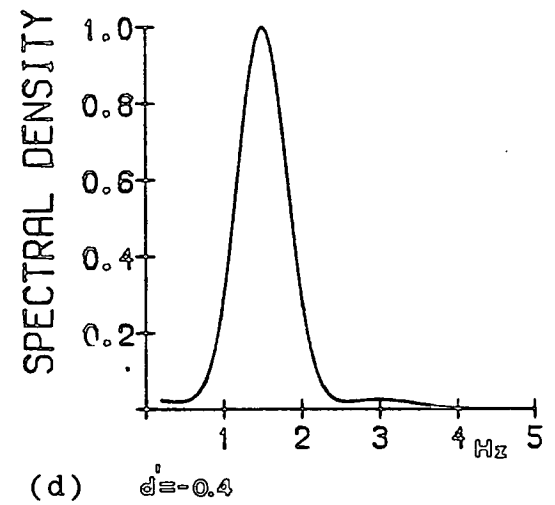
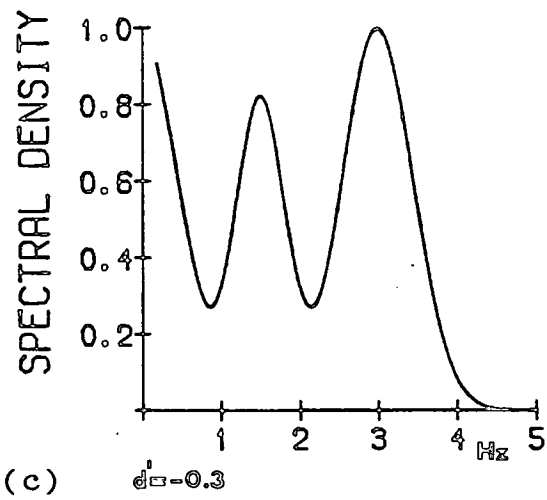
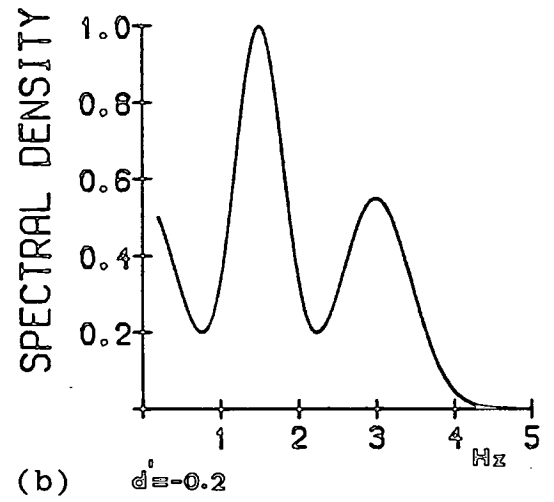
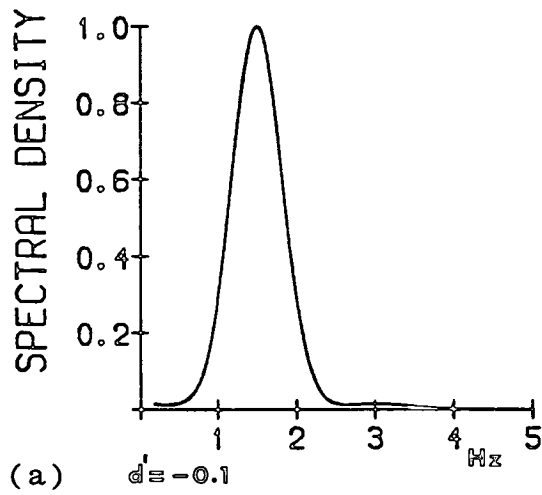


Figure 4.4 The theoretical vertical force spectra for four axis depths.

theoretical spectra for the first four of these depths, since the method of analysis would not be applicable for the more deeply submerged cases. The coefficients A, B, C, and G contained the variable, L, which in the case of regular waves is simply the wavelength. For the purposes of computing the above coefficients the wavelength corresponding to the mean frequency  $\omega_0$  has been used. Again the side peaks are present only over a short range of depths, but they reach a maximum at a smaller submergence than those in the experiments. An inspection of equation (4.19) shows that the ratio of the side peaks to the central one is given approximately by  $A^2/\sqrt{2} B^2 S_0$ . Since this is a function of the squares of the determining parameters it will be sensitive to error arising from both the approximations involved in the analysis giving A and B, and also to the measurement of the mean square wave elevation. The potential theory value of the inertia coefficient,  $C_m = 2.0$ , has been used in all the theoretical calculations.

The crosscorrelation between the horizontal and vertical forces gives a measure of the interdependence of the two variables and, important in the case of forces, enables the relative phase between them to be estimated. Figure 4.5 presents the experimental and theoretical cross correlations for two different axis depths: one where the cylinder is under the surface and one where it is semi-immersed. In the first instance the forces are well represented by Morison's equation as it originally stands, and the results show that the

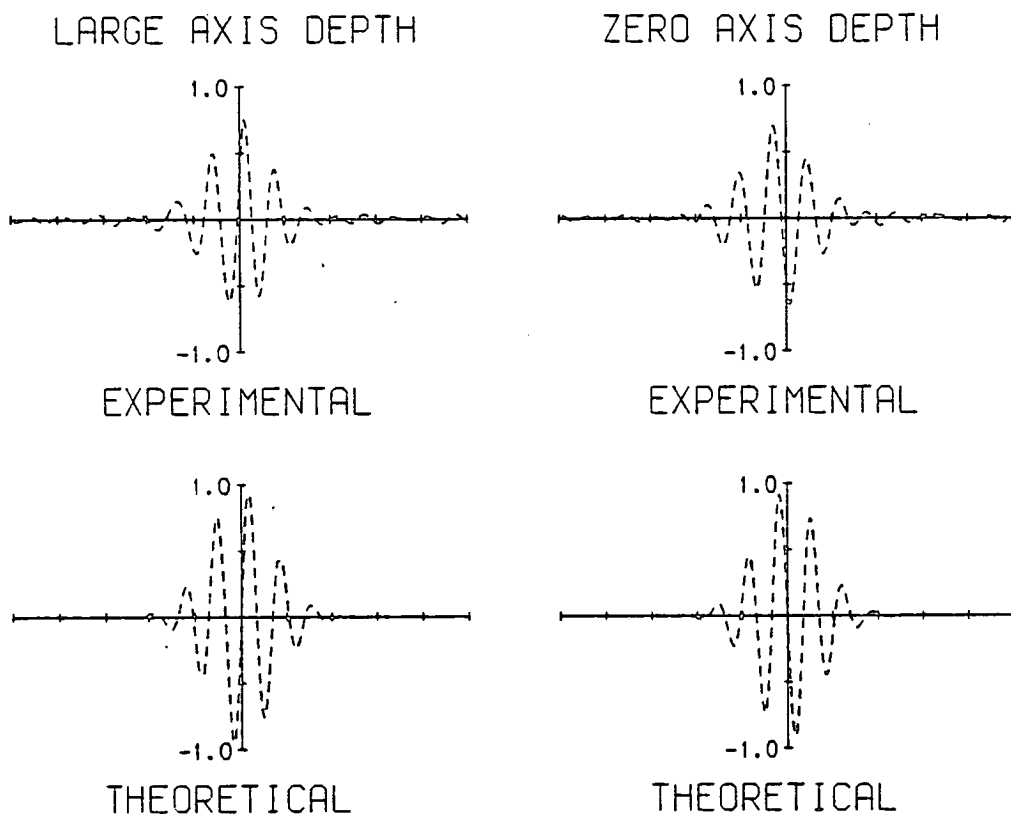


Figure 4.5 The crosscorrelation between the horizontal and vertical forces for two axis depths.

horizontal force leads by  $90^\circ$ . When the cylinder lies in the surface, however, the vertical force is dominated by buoyancy, and the horizontal force then lags by  $90^\circ$ . This switch is convincingly demonstrated, both experimentally and theoretically, in figure 4.5.

## CHAPTER 5

### EXPERIMENTAL EQUIPMENT, DATA COLLECTION AND ANALYSIS

## CHAPTER 5

### EXPERIMENTAL EQUIPMENT. DATA COLLECTION AND ANALYSIS

In this chapter we will describe the experimental rig used for measuring the forces on a cylinder, the methods of recording data, and the subsequent analysis of the results. All the equipment was designed and built by the Edinburgh Wave Power Team of the Department of Mechanical Engineering, the University of Edinburgh, as part of their programme of research into the extraction of energy from sea waves.

#### 5.1 The Experimental Equipment

A narrow wave tank (see figure 5.1a) was constructed from a framework of Dexion with walls of float glass, and was 9.14m (30ft) long, 0.33m (13ins) wide (internally), and 0.8m (32ins) deep. These dimensions were chosen so that the scale of the models was 1/150th of the expected prototype scales. The tank was lit from underneath so that the waves could be easily seen. Waves were produced at one end of the tank by an 'absorbing' wavemaker. By this term we mean that the wavemaker was able, to a large extent, to absorb any waves travelling towards it. This is an obligatory requirement when working on wave energy projects since otherwise reflections from the energy extracting device are rereflected from the wavemaker and therefore increase the incident wave, giving rise to an efficiency for the device that is too high. The absorbing is done electronically by feeding back into the force input that drives the wavemaker a signal that is proportional to its velocity. An external



Figure 5.1(a) The narrow wave tank.

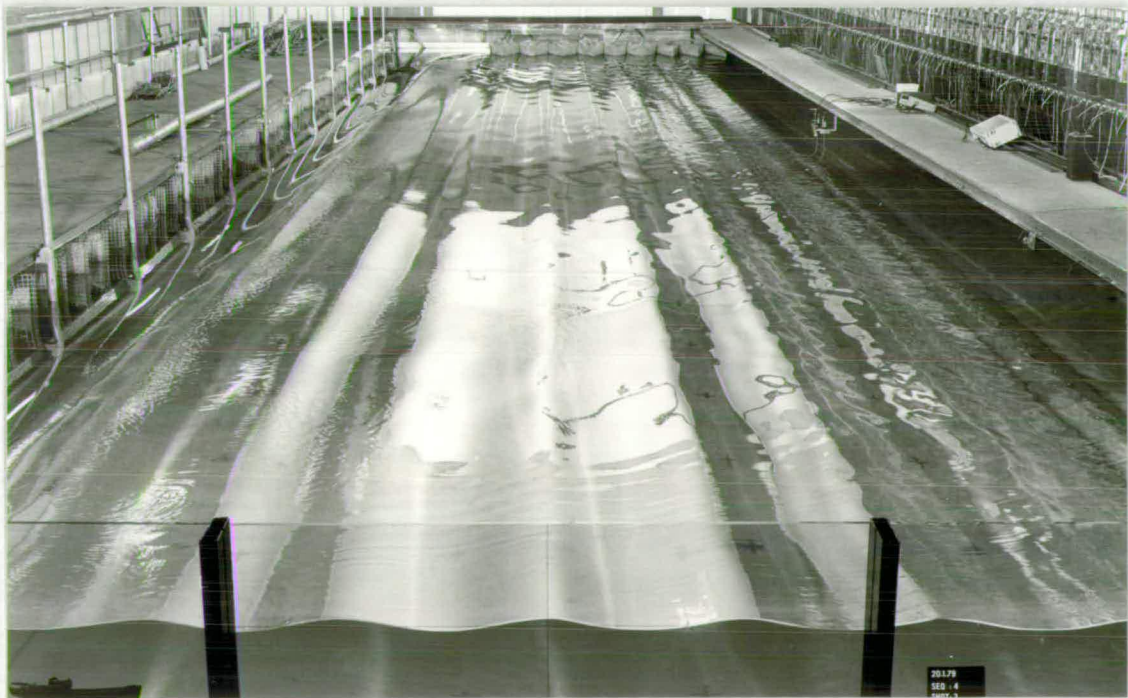


Figure 5.1(b) The wide wave tank.

signal added to the drive makes waves without affecting the absorption. A force feedback system is also used to give better response at low signal levels and to improve absorption. Force is sensed by a piezo-electric transducer mounted on the wavemaker. The force voltage is compared with the input voltage and any difference is fed to the power amplifier driving the motor. This changes the force voltage until the difference is zero. With infinite gain in the loop the force is constrained so that it always equals the input. In practice a gain of about twenty was possible before instability occurred. The displacer of the wavemaker was a flat wedge made of light alloy hinged at the bottom. A watertight seal was made with membrane stretched across the front of the flap so that there was no water to the rear. This was done to reduce the power consumption (since no waves are created behind the wavemaker) and to make feedback control easier. The resulting hydrostatic pressure on the flap is then balanced by a spring attached to the top of the displacer by a wire which is wrapped round a pulley on the spindle of the drive motor (surplus computer disc drives were used). The spring rate is chosen such that it will cause a resonance at the most convenient frequency. The absorption will not be equally efficient at all frequencies but can be tuned electronically to give the best overall response, see figure 5.2. Figures 5.3 and 5.4 show the transfer function of the wavemaker and a plot showing the linearity of the gain. As with all narrow wave tanks there are certain frequencies at which

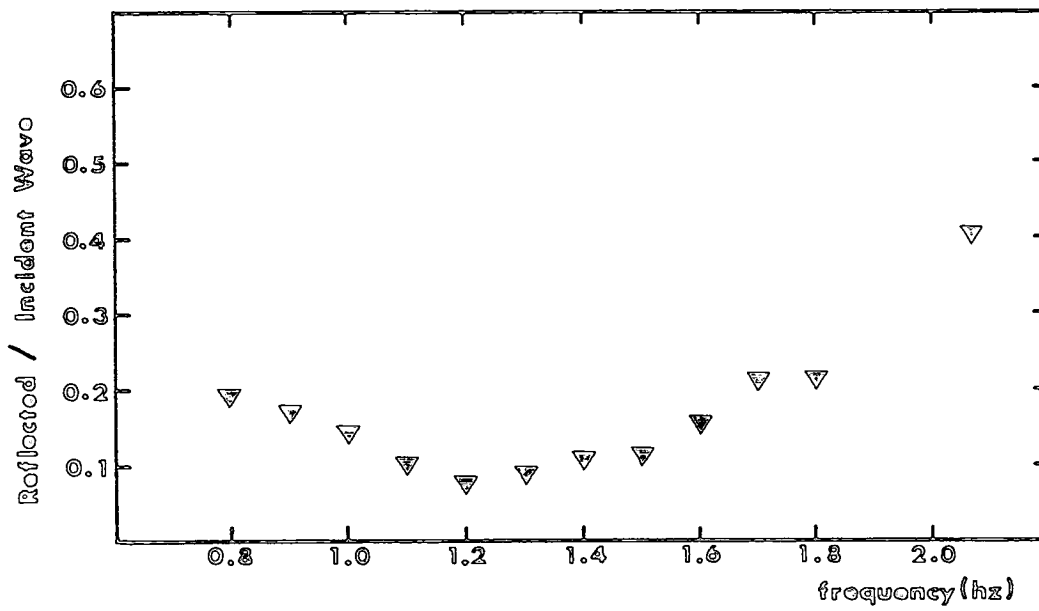


Figure 5.2 Wavemaker reflection.

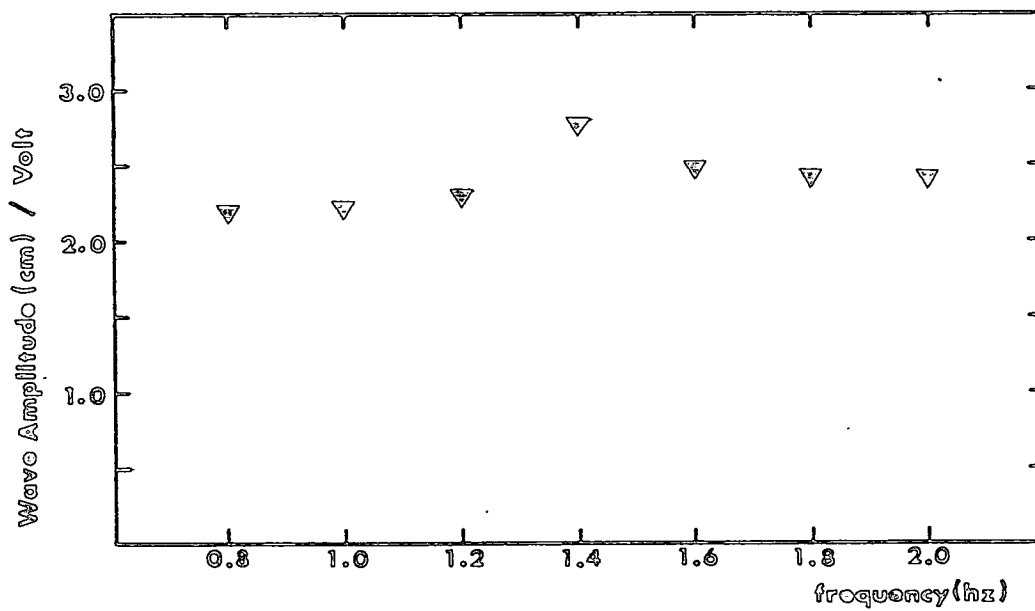


Figure 5.3 Transfer function of the wavemaker.

large sideways oscillations occur (when the wavelength equals twice the width of the tank) and any measurements become impossible. This happened for the present tank at a frequency of 1.6Hz. Since frequencies of this order and above represent quite short wavelengths at full scale no measurements were taken above 1.4Hz for regular waves.

At the other end of the tank from the wavemaker a beach was positioned to absorb as much wave energy as possible so that end reflections could be neglected in efficiency calculations and force measurements. Instead of the more usual sloping beach, which requires a considerable amount of room, a vertical wedge of 'Expamet' was packed into a cage with the density increasing to the rear. Expamet, which is designed for use as a filter material, is made from thin sheets of metal with a pattern of slits which is pulled out and corrugated. This beach works very well except for very low amplitude waves, see figure 5.5.

The signal source used to drive the wavemaker sinusoidally was usually a transfer function analyser. This has a variable output and can measure input signals in terms of amplitude and phase or real and imaginary parts relative to the output. So when measuring a signal from a wave gauge, for example, very stable readings are possible. The first ten harmonics of a signal can be measured with the same instrument.

The side ways oscillations mentioned above also occur

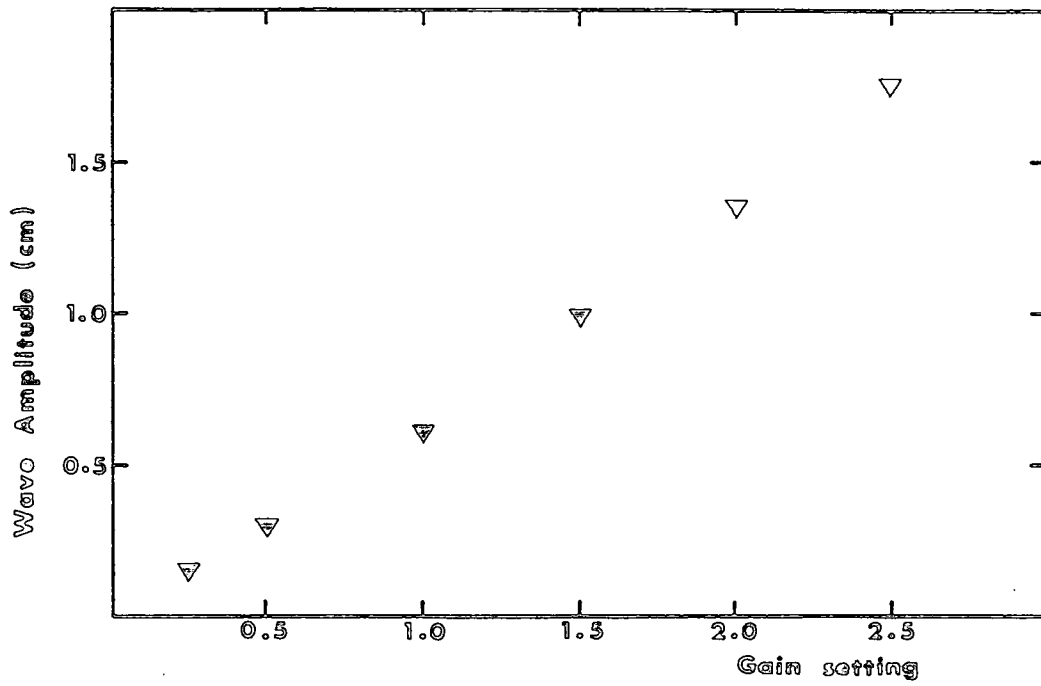


Figure 5.4 Linearity of the gain.

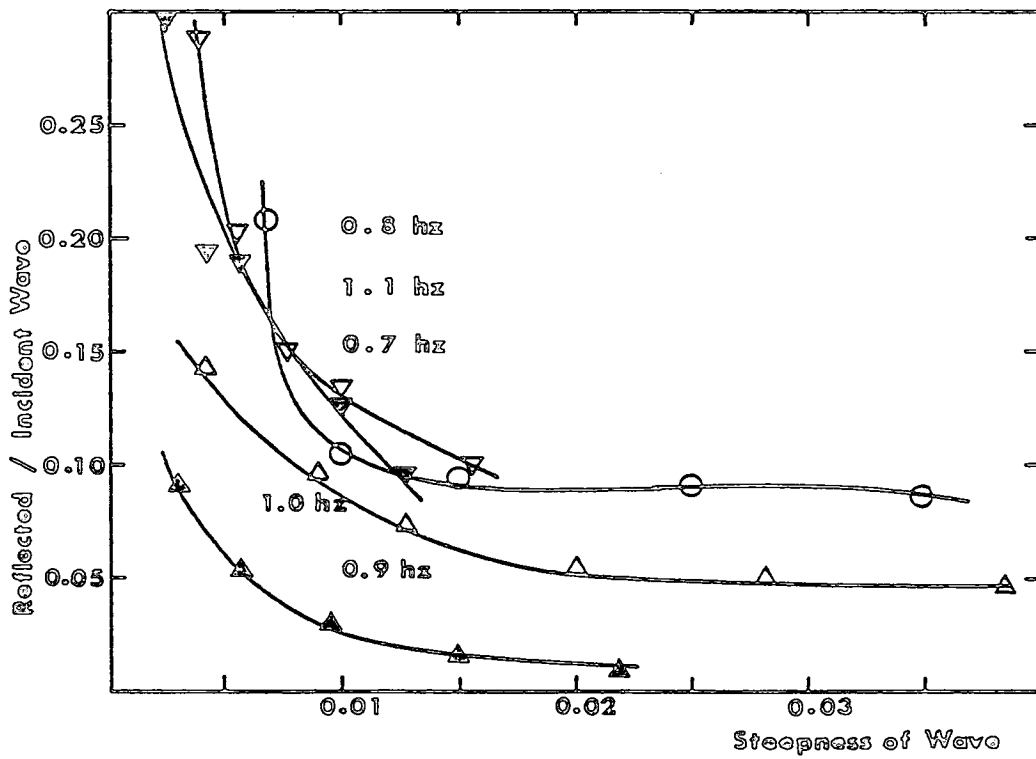


Figure 5.5 Beach reflection.

to a lesser extent at other frequencies and so any method of measuring waveheight based on one position only would give erroneous results. For this reason the wave gauges used were of the float type, stretching across the whole width of the tank, and so measured an average waveheight. The floats were initially made of rolled paper waterproofed with polyurethane. After a long period they began to leak and so for later experiments, circular cylinders of expanded polystyrene, cut using a hot wire, were employed. The floats were constrained to move vertically by a linkage at each end which was attached to the movement of a microammeter. This results in a signal proportional to the velocity of the float which was integrated to yield displacement. The signal was then demodulated and calibrated to give a wave amplitude in centimetres (1 volt = 1 cm). A wave gauge of this type cannot tell which direction the wave is travelling in, and so is unable to distinguish between incident and reflected waves. To surmount this problem two wave gauges were used in tandem, since it is then possible to determine the amplitude of the incident wave and also the reflected wave. The procedure is to place the gauges one quarter of a wavelength apart and then search up and down the tank for a position where the difference of the two signals is maximised. The reflected wave amplitude is then half this difference and the incident wave amplitude is the average of the two signals. The method assumes that the waves add linearly and it is trivial to prove that this relationship is correct

for total reflection when a standing wave of twice the incident wave amplitude would be formed. The present writer has been unable to find a reference giving the proof for arbitrary reflection and since the method was used extensively in Chapter 3 we present one now.

Consider a wave incident on an object that partially reflects it back again. We can write the incident wave as

$$y_i = a_i \exp i(kx - \omega t), \quad (5.1)$$

where  $a_i$  is the incident wave amplitude,  $k = 2\pi/L$ ,  $\omega = 2\pi \times$  wave frequency, and it is understood that the real part of the right hand side should be taken. We can write a similar expression for the reflected wave,

$$y_r = a_r \exp i(-kx - \omega t + \phi_r), \quad (5.2)$$

where  $\phi_r$  is the phase change of the reflected wave. The total wave is therefore,

$$y = \exp(-i\omega t) [a_i \exp(ikx) + a_r \exp i(-kx + \phi_r)]. \quad (5.3)$$

If we place a wave gauge at position A such that  $x_A = x$ , and one at position B where  $x_B = x + L/4$ , then the amplitude of the signal at A is,

$$Y_A = a_i \exp ikx + a_r \exp -i(kx - \phi_r), \quad (5.4)$$

therefore,  $|Y_A|^2 = Y_A Y_A^*$ , where  $*$  denotes complex conjugate,

$$= a_i^2 + a_r^2 + 2a_i a_r \cos(2kx + \phi_r). \quad (5.5)$$

Similarly,  $|Y_B|^2 = a_i^2 + a_r^2 + 2a_i a_r \cos(2k(x + \frac{L}{4}) + \phi_r)$

$$= a_i^2 + a_r^2 - 2a_i a_r \cos 2kx + \phi_r. \quad (5.6)$$

Hence,  $(|Y_A| - |Y_B|)^2 = |Y_A|^2 + |Y_B|^2 - 2|Y_A||Y_B|,$

$$= 2a_i^2 + 2a_r^2 - 2\sqrt{(a_i^2 + a_r^2)^2 + 4a_i^2 a_r^2 \cos^2(2kx + \phi_r)}$$

and so  $(|Y_A| - |Y_B|)_{\max}^2 = 2a_i^2 + 2a_r^2 - 2\sqrt{(a_i^2 + a_r^2)^2 - 4a_i^2 a_r^2}$

$$= 4a_r^2.$$

Therefore

$$a_r = \left( \frac{|Y_A| - |Y_B|}{2} \right)_{\max}$$

A similar line of reasoning shows that

$$a_i = \left( \frac{|Y_A| + |Y_B|}{2} \right)_{\min}$$

and the method is proven.

The rig for measuring forces is shown in figure 5.6. The cylinder is mounted on two vertical arms which themselves are joined to horizontal arms. The connections are made of phosphor bronze tubing on which strain gauges



Figure 5.6 The force measuring rig.

are mounted in pairs. Similar connections fix the horizontal arms to the mounting. The strain gauges measure the torque acting about these two points and it is a simple matter to relate them to the vertical and horizontal forces acting at the cylinder axis. These computations were done by circuits on board the rig and a calibrated signal was sent to the instrument desk. The calibration was done with a system of pulleys and known weights using the sign conventions of; positive = upwards for the vertical force and positive = seawards (away from the beach) for the horizontal force.

The force rig was mounted on a support which allowed it to be moved in and out of the water, and the resulting static buoyancy force was corrected for electronically before any measurements were taken.

A Tectronix 4051 computer was used to collect the data. The signal from any one of the input channels could be selected, sampled at 256Hz over one wave period and digitised. The data was then stored on a magnetic tape cartridge for subsequent analysis. In order to start the sampling always at the same phase of the wave the analogue-to-digital convertor was synchronised to the transfer function analyser producing the wavemaker drive. The relative phase between the two could be altered and the following procedure was adopted to achieve the correct starting phase for each measurement (i.e., at  $t = 0$  the waterline at the cylinder axis is passing through  $y = 0$ ,

travelling upwards). A wave gauge was placed at the position of the cylinder axis with the cylinder removed and for each frequency to be used in the experiments the relative phase between the drive and the sampler was changed to give the necessary starting phase. These values were noted and used when the cylinder was replaced. An oscilloscope system, also synchronised to the transfer function analyser and triggered at the same phase as the digitiser, allowed the signals to be viewed directly and the phase adjusted.

## 5.2 Data Analysis for Regular Waves

The fitting of the coefficient,  $C_m$ , was done on the Tectronix computer and the force averages computed. Our model for wave forces does not allow for any phase shift in the inertial or buoyancy forces with respect to the wave elevation. That is, for a sinusoidal wave elevation the inertial force should always be a negative sine wave (modified by  $V(t)$ ), and the buoyancy force a positive sine wave. The horizontal inertial force should be a negative cosine. The actual measurements showed slight variations in phase which would lead to quite large variations in the fitted values of  $C_m$ . To overcome this problem a phase coefficient was introduced into the theoretical equations and was allowed to vary to give the best fit. So, instead of writing  $2\pi t/T$ , we write  $2\pi t/T + \delta$  everywhere in the equations.

The least square fitting procedure finds the values of

$C_m$  and  $\delta$  that minimise the quantity

$$\sum_i (Y_i - F_i)^2$$

where  $Y_i$  are the observed data points and  $F_i$  the corresponding theoretical ones. The inclusion of the phase coefficient means that the resulting equations are now non-linear in the two coefficients and it is therefore not possible to obtain an explicit solution and an iterative technique must be adopted. The method used was taken from Pennington (1970), pp 421-426. The starting values for  $C_m$  and  $\delta$  were taken to be 2.0 and zero respectively. The iteration converged in all cases and was stopped when the change on the values of  $C_m$  and  $\delta$  was less than 0.1%. The necessary derivatives were calibrated from analytical expressions for  $\frac{\partial F}{\partial C_m}$  and  $\frac{\partial F}{\partial \delta}$ . For shapes where this is not possible a numerical approximation for the derivatives could be used instead.

### 5.3 Data Analysis for Irregular Waves

The method for measuring forces for irregular waves differed from the above in two ways. The input signal to the wavemaker was produced by a spectrum which gave a 'comb' spectrum using twenty oscillators at closely spaced frequencies with variable amplitudes mixed in random phase. The frequencies ranged from 0.5Hz to 2.5Hz in nineteen logarithmic steps, i.e.

$$f_n = f_1 \times 5^{\frac{(n-1)}{19}}$$

Thus the oscillator frequencies are not an exact multiple of some lower frequency and the sequences do not repeat. The disadvantage of this is that the energy density is not constant if the individual amplitudes are all made the same. The unequal spacing means that there will be more energy at the lower end of the spectrum than at the higher. This problem could be lessened by multiplying each amplitude by a suitable correction factor.

The mean frequency for the spectrum was chosen to be 1.5Hz\* and so the gain of the oscillator whose frequency was nearest to this was set to its maximum (the gain control was split into twelve discrete steps). The other oscillators were then set so that each amplitude was given by,

$$\text{amplitude} = A \exp \left[ \frac{-(f-f_o)^2}{2\sigma^2} \right],$$

where A is the gain of the oscillator of frequency  $f_o$ ,  $\sigma$  was chosen to be 0.4Hz to give a fairly narrow spectrum, and f is the frequency of the oscillator concerned. After each oscillator had been set the overall gain control was adjusted so that the waves produced were of a reasonable size. This assumes that the transfer function between the input signal and the wave height is flat, see figure 5.3.

The forces produced by the irregular waves were recorded on a four channel F.M. tape recorder in analogue form, together with a record of the wave height with the

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\* The amplitudes of the components of the random wave were all small and lateral oscillations were negligible.

cylinder removed. These records were digitised at 10Hz which was twice the frequency thought to be the upper limit of the range of interest. The data was then transferred to a digital computer for analysis. This was carried out using a program package developed by Allan (1978) to compute the power spectral density function of a general time series,  $x(t)$ . The definition of the spectrum can be written not only as the fourier transform of the autocorrelation function of  $x(t)$  but also as

$$S_x(\omega) = \lim_{T \rightarrow \infty} \frac{2}{T} E[|X(\omega, T)|^2], \quad (5.7)$$

where

$$X(\omega, T) = \int_0^T x(t) \exp i \omega t dt. \quad (5.8)$$

Until the advent of the fast fourier transform power spectra were usually computed from the correlation function. However, the Cooley-Tukey algorithm has meant that it is now easier to find the spectrum by direct transformation of the original time series.

The discrete form of equation (5.8) is,

$$X(\omega, T) = \sum_{n=0}^N x_n \exp[i\omega n \Delta t] \Delta t$$

where  $\Delta t^{-1}$  is the digitising sample rate and,

$$x_n = x(n\Delta t), \quad n=0, 1, \dots, N$$

and  $T = N\Delta t$ .

The function  $X(\omega, T)$  is usually evaluated using equally spaced values of frequency,  $\omega_k$ , such that,

$$\omega_k = k\Delta\omega, \text{ where } \Delta\omega = \frac{2\pi}{T}, k = 1, 2, \dots, \frac{N+1}{2}.$$

Thus 
$$X(\omega_k, t) = \sum_{n=0}^N x_n \exp [i k_n \omega_k \Delta t] \Delta t$$

$$= \sum_{n=0}^N x_n \exp \left[ 2\pi i \frac{kn}{N} \right] \Delta t.$$

An inconsistent estimator of the power spectral density is then

$$\hat{S}_k(\omega_k t) = \frac{2}{T} \left| \sum_{n=0}^N x_n \exp \left[ 2\pi i \frac{kn}{N} \right] \Delta t \right|^2.$$

By the term consistent we mean that for any  $\epsilon > 0$

$$\lim_{N \rightarrow \infty} \text{Prob} \left[ \left| \hat{S}_k - S \right|^2 > \epsilon \right] = 0.$$

An unbiased estimator is one for which

$$E(\hat{S}) = S.$$

An unbiased consistent estimator of the power spectral density of a Gaussian process is not known.

A well known problem associated with numerical fourier analysis is that of 'leakage', (see Bendat and Piersol

(1971)). The method of leakage reduction used here was that of frequency smoothing, and the Goodman-Enochson-Otnes window functions were used in computing all the spectra. The standard error associated with the spectral estimates was reduced by both moving and fixed frequency averaging. The accuracy of the final estimates can be approximated by assuming the data to be Gaussian and uncorrelated. For estimates which are frequency averaged over  $\ell$  points the normalised error is

$$\epsilon_r = \left(\frac{1}{\ell}\right)^{\frac{1}{2}}$$

and the number of degrees of freedom is  $2\ell$ . When frequency smoothing precedes the averaging the effective number of degrees of freedom is reduced by a factor,  $\beta$ , which for the window function used here is 0.78. This effective number can be used to place confidence limits on the spectral estimates for a given confidence level. An approximate formula for the confidence factor,  $F_c$  is given by,

$$F_c = \exp \left[ \frac{2.3b}{10\sqrt{k-1}} \right]$$

where  $k$  is the effective number of degrees of freedom and  $b = 20$  for a confidence level of 90%. Error bars can be applied to the final estimates, and for final estimates  $s_k$  the upper and lower bounds are,

$$s_k/\sqrt{F_c} \quad \text{and} \quad s_k/\sqrt{F_c}.$$

Since the actual data is correlated these equations serve only as a guide to the actual errors. The spectra presented here were calculated from time series of 2048 points spaced 0.1 seconds apart. The estimates were smoothed using a moving average of 5 and a fixed average of 16. The effective number of degrees of freedom was therefore  $0.78 \times 5 \times 16 = 62.4$ , giving  $\sqrt{F}_l = 1.34$ , and a frequency resolution of 0.39Hz. In retrospect it must be concluded that the record lengths were too short for really accurate results.

#### 5.4 Measurement of Bending Moments

The measurements of bending moments presented in Chapter 6 were taken by the Edinburgh Wave Power Team in their wide wave tank. This facility is described in detail in Jeffrey et al. (1978, vol.3), but since this report is not widely available a brief description will be given here.

The wide tank was 27.5m wide, 11m long, and contained water to a depth of 1.2m. Along the width of the tank a bank of wavemakers, each similar to the one in the narrow tank, was able to produce directional seas by suitable choice of the relative phase between one wavemaker and the next. Each wavemaker had independent frequency, phase, and amplitude controlled by a microprocessor. Facing the wavemakers at the opposite side of the tank there was a beach composed of a line of wedges filled with Expamet.

The model spine was constructed from seven 3m sections of P.V.C. pipe (Marley RS 4514) with extra flexible tubing inside to control the static buoyancy. One of these sections instrumented with three sets of strain gauges along its length, each set consisting of two pairs of gauges set at right angles to each other. This section was moved to different places along the spine when required. The spine was lightly moored to prevent drift during the tests. A tendency for the spine to roll meant that it was not possible to measure horizontal and vertical moments separately as originally intended, so the modulus of the total moment was measured instead.

## CHAPTER 6

### BENDING MOMENTS IN PARTIALLY SUBMERGED PIPES

## CHAPTER 6

### BENDING MOMENTS IN PARTIALLY SUBMERGED PIPES

#### 6.1 The Dynamics of Ocean Structures

The cylinders in the previous chapters were rigidly fixed and inflexible, and therefore we did not discuss their dynamics. We shall now extend the work to the problem of modelling the interaction between a long flexible pipe and surface waves. Since we are interested in applying the results to Salter ducks we will restrict our attention to pipes on or near the surface. The ducks will be mounted on a 'backbone' whose rigidity will be chosen to suit the conditions. Information about the bending moments and deflections along the backbone is therefore required so that this can be done.

We will now discuss the equation of motion for such a structure, with emphasis on the physical interpretation of each term, (reference is made to Hogben (1974) and Standing (1978) for further details). The movements of the pipe will be assumed small so that the equation is linearised. The total force acting on a section of the pipe can be split into two types; the fluid forces and the external forces. The fluid forces can also be separated into two parts; the forces acting on the moving object in still water and the forces acting on a fixed object in moving water. These forces, which we shall call the radiation and scattering forces respectively, are calculated separately and then added linearly to give the total fluid force. The scattering

part consists of the Froude-Krylov force,  $F_k$ , due to the undisturbed pressure field and the term representing the scattering or diffraction of the incident wave,  $F_d$ . Motion dependent forces are represented by added mass and damping matrices. These are calculated by forcing the structure to move in each mode in turn and the force required to do this is split into components in phase and in quadrature with the motion. These components are then divided by the structures acceleration and velocity respectively to give the added mass and damping. A viscous drag term can be added to the total force together with a stiffness term representing varying buoyancy. To clarify our ideas we will write down the resulting equations for a structure with one degree of freedom,  $x$ . The scattering term is,

$$F_s = F_d + F_k, \quad (6.1)$$

which may be written as

$$F_s = C_s F_k, \quad (6.2)$$

in which  $C_s = (F_d + F_k)/F_k$ , is a diffraction coefficient. The radiation force,  $F_r$ , is written in terms of the added mass and damping coefficients,  $P$  and  $Q$  as,

$$F_r = -P\dot{x} - Qx. \quad (6.3)$$

The viscous drag is written in the usual form as  $1/2 C_d \rho A u_x |u_x|$

and so the total force is,

$$F = -P\ddot{x} - Q\dot{x} + C_S F_k + \frac{1}{2} C_d \rho A u_x |u_x|, \quad (6.4)$$

and the equation of motion for  $x$  is,

$$(M + P)\ddot{x} + Q\dot{x} = F_{\text{external}} + C_S F_k + \frac{1}{2} C_d \rho A u_x |u_x| \quad (6.5)$$

where  $M$  is the inertial mass of the structure. For the vertical force a linear buoyancy stiffness term can be written as  $-S_b x$  and added to equation (6.5). The external forces can include mooring stiffnesses, damping, wind loading, and flexural forces. The scattering force term,  $F_d$ , includes the effect of local disturbance, usually described in terms of an added mass coefficient,  $k$ , and also the scattering or diffraction of the incident wave. If the wavelength is much smaller than the body then the second of these terms is negligible and so  $F_d = k \rho V \dot{u}_x$ . For long waves  $F_k \doteq \rho V \dot{u}_x$  and so,

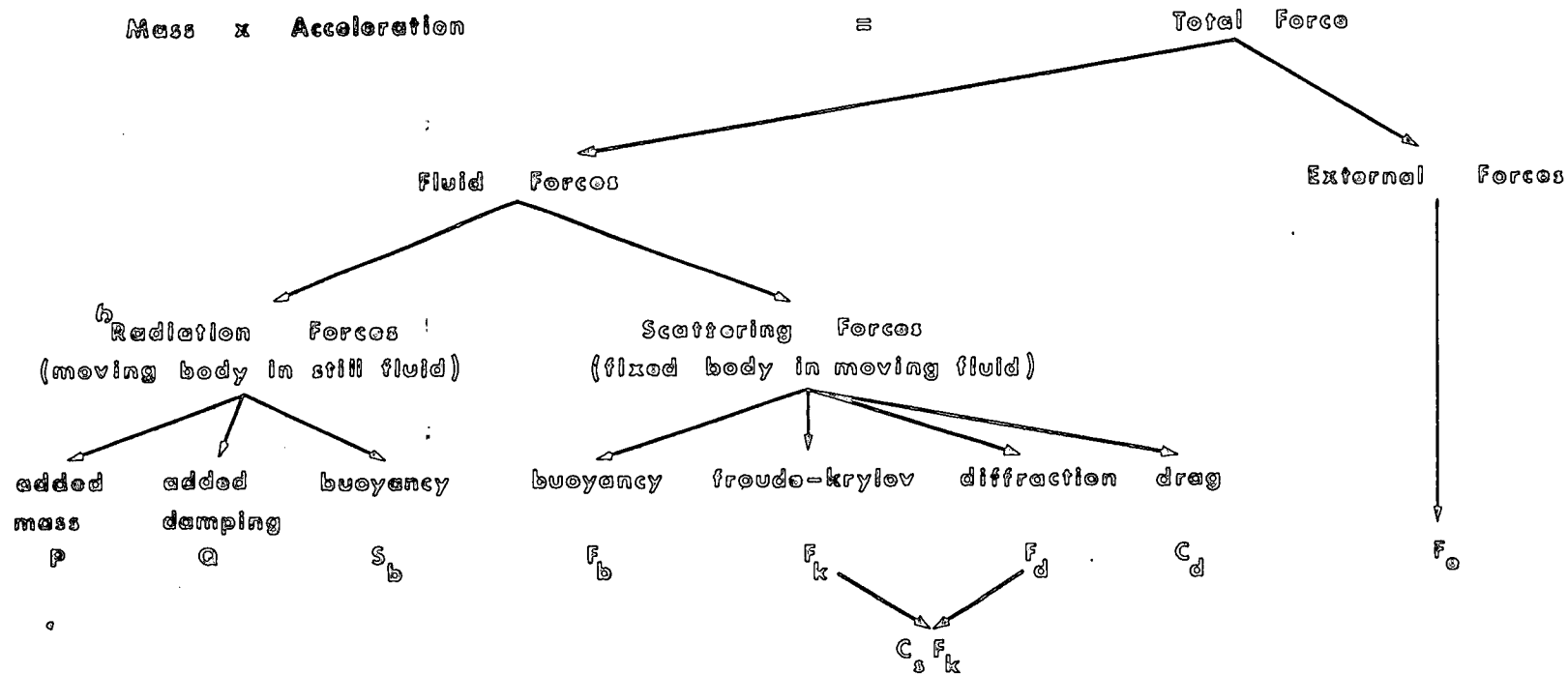
$$F_S = (1 + k) \rho V \dot{u}_x. \quad (6.6)$$

The equation of motion is therefore,

$$(M + P)\ddot{x} + Q\dot{x} + S_b x = F_{\text{external}} + F_{\text{wave}} = F(t) \quad (6.7)$$

where  $F_{\text{wave}} = C_m \rho V \dot{u} + C_d \rho A u_x |u_x|$ , i.e. Morison's equation.

Figure 6.1 shows the evolution of equation (6.7).



$$(M + P) \ddot{q} + Q \dot{q} + S_b q = C_s F_k + \frac{1}{2} C_d \rho A U_q |U_q| + F_b + F_o$$

Figure 6.1 The linearised equation of motion for a one dimensional system.

Equation (6.5) serves as the basis for a program written for computing wave loads on large fixed bodies (Hogben and Standing (1975)), which has been further developed for the prediction of the dynamics of a body able to move (Standing (1978)). The method is easily extended to structures with more than one degree of freedom by assuming the motions are small and the equations are still linear. Standing discusses the use of the program for predicting added mass and damping for one or two wave energy devices where energy extraction is modelled by simple springs and dampers. The solution of the equations is carried out numerically for each individual case.

## 6.2 Long Horizontal Pipes.

We shall model the backbone of the duckstring by a one dimensional pipe that can flex both vertically and horizontally. Several assumptions will be made so that we can arrive at an explicit solution for the response and bending moments in the pipe. The only reference to similar work the present author has been able to find is a paper by Guilloud and Vignat (1979) which was concerned with the possibility of towing large lengths of pipe at sea (so that most of the welding could be carried out on land). They considered the vertical displacement of a pipe held beneath the surface by floats. Since they included a drag term in the equation, it could not be solved analytically, and they resorted to a finite element method of computation. They did attempt a direct analysis by linearising the drag

term and looking for a steady state solution in the form of a travelling wave. This required that the pipe be of infinite length, which was a reasonable approximation in their case. They were thus able to compute response and bending moments for the pipe. The experimental results presented were in rough agreement with both these methods.

The present work differs in that we will be concerned with a pipe that lies on the free surface and we will attempt to find a solution which incorporates end effects (i.e. the pipe will be of finite length).

Consider a pipe of length  $2l$  that floats so that it is nearly awash (see figure 6.2). A train of regular waves of wavelength  $L$ , angular frequency,  $\omega$ , and amplitude  $a$ , is incident at an angle  $\theta$  to the normal to the pipe. We will assume that the mooring is so light that it can be neglected and that the only external force is the flexural force term associated with the elasticity of the pipe. The pipe is assumed to have two degrees of freedom, one vertically and one horizontally (transverse to the pipe). The elastic force term is given by (see Morse (1948), p.153)

$$F_{\text{external}} = -EI \frac{\partial^4 y}{\partial x^4}, \text{ in the vertical direction,}$$

and

$$= -EI \frac{\partial^4 z}{\partial x^4}, \text{ in the horizontal direction,}$$

where  $E$  is the Young's modulus of the pipe and  $I$  the moment



of inertia about its central axis. The actual backbone will have a lumped distribution of elasticity, but for the purposes of this work we will assume that it is constant along the pipe's length. The moment of inertia is also assumed to be constant. Equation (6.7) then reads,

$$EI \frac{\partial^4 y}{\partial x^4} + (M+P_y) \ddot{y} + Q_y \dot{y} + S_b y = F_y(t), \quad (6.8a)$$

for the vertical forces and,

$$EI \frac{\partial^4 z}{\partial x^4} + (M+P_z) \ddot{z} + Q_z \dot{z} = F_z(t), \quad (6.8b)$$

where M is now the mass per unit length of the pipe. Note that the added mass and damping are not necessarily the same for each direction. The problem to be faced now is the choice of  $F(t)$ , which represents the wave-induced forces. The use of Morison's equation, (equation 1.1), seems inappropriate in the light of the evidence of Chapter 2, so we shall therefore investigate two other possibilities.

1. Salter's force equation (equation 1.15)
2. The approximate version of the modified Morison's equation presented in Chapter 4.

The first of these suggests that we write,

$$F_y(t) = C_y \rho g D a \cos (kx - \omega t), \quad (6.9a)$$

and 
$$F_z(t) = C_z \rho g D a \sin (kx - \omega t) \cos \theta, \quad (6.9b)$$

where  $D$  is the pipe diameter and  $k = 2\pi/\lambda$ .  $\lambda$  is the distance between two crests on the pipe (see figure 6.2) which we will term the 'crestlength'. It is therefore a measure of the angle of incidence (since  $\lambda = L/\cos \theta$ ) and varies between  $L$  for waves running along the pipe to infinity for  $\theta = 90^\circ$ . In the latter situation the bending moments will be zero and the pipe will just heave and surge as a rigid body.  $C_y$  and  $C_z$  are force coefficients as defined by Salter (Jeffrey et al (1976)). We will assume that the total bending moment,  $M_T$ , at a position,  $x$ , will have an amplitude given by,

$$|M_T| = \left[ |M_y|^2 + |M_z|^2 \right]^{\frac{1}{2}},$$

in which  $M_y$  and  $M_z$  are the moments in the vertical and horizontal directions respectively and we can solve for them separately. For this particular choice of  $F(t)$  equations (6.8) can be written,

$$\frac{\partial^4 y}{\partial x^4} + \alpha_1 \frac{\partial^2 y}{\partial t^2} + \beta_1 \frac{\partial y}{\partial t} + \gamma_1 y = K_1 \cos(kx - \omega t), \quad (6.10a)$$

and

$$\frac{\partial^4 z}{\partial x^4} + \alpha_2 \frac{\partial^2 z}{\partial t^2} + \beta_2 \frac{\partial z}{\partial t} = K_2 \sin(kx - \omega t), \quad (6.10b)$$

where

$$\begin{aligned} \alpha_1 &= (M + P_y)/EI, \\ \alpha_2 &= (M + P_z)/EI, \end{aligned}$$

$$\beta_1 = Q_y/EI,$$

$$\beta_2 = Q_z/EI,$$

$$\gamma_1 = S_b/EI,$$

$$K_1 = C_y \rho g Da,$$

and  $K_2 = C_z \rho g Da \cos \theta.$

Having found the solutions to equations (6.10) the moments  $M_y$  and  $M_z$  are derived from,

$$M_y = -EI \frac{\partial^2 y}{\partial x^2}, \quad (6.11a)$$

and  $M_z = -EI \frac{\partial^2 z}{\partial x^2}. \quad (6.11b)$

Since equations (6.10) are of fourth order there will be four constants of integration in the final solution. These must be determined from a suitable choice of boundary conditions. As the string of ducks will not be rigidly fixed at any position it was decided to model the pipe as having free ends. This means that the forces and bending moments at either end are zero and so the boundary conditions to be satisfied are:

1.  $\frac{\partial^2 y}{\partial x^2}$  and  $\frac{\partial^2 z}{\partial x^2} = 0$ , at  $x = \pm l$  (moments zero).
2.  $\frac{\partial^3 y}{\partial x^3}$  and  $\frac{\partial^3 z}{\partial x^3} = 0$ , at  $x = \pm l$  (forces zero).

The solutions of equations (6.10) are similar so we will concentrate in detail only on the first. We now look at the equation,

$$\phi_{xxxx} + \alpha \phi_{tt} + \beta \phi_t + \gamma \phi = K \exp i(kx - \omega t), \quad (6.13)$$

where the subscripts denote partial derivatives. If  $\phi$  is a solution of this equation with  $\phi_{xx} = \phi_{xxx} = 0$ , at  $x = \pm \ell$ , then

$$y = \text{Re}(\phi)$$

satisfies equation (6.10a) with  $y_{xx} = y_{xxx} = 0$ , at  $x = \pm \ell$ . Instead of a travelling wave (which would not satisfy the boundary conditions) we try a more general solution of the form,

$$Y = \phi(x)e^{-i\omega t}. \quad (6.14)$$

This means that we are assuming a steady state solution and that all transients have died away. Substitution of the above into equation (6.13) results in,

$$\phi_{xxxx}(x) + (\gamma - \alpha\omega^2 - i\omega\beta)\phi(x) = Ke^{-i\omega t}. \quad (6.15)$$

$\phi(x)$  satisfies the same boundary conditions as  $\phi$ , namely,

$$\phi_{xx}(x) = \phi_{xxx}(x) = 0, \text{ at } x = \pm \ell.$$

As equation (6.15) is a linear fourth order differential equation a general solution will be of the form,

$$\phi(x) = \sum_{n=0}^3 A_n \exp(i\omega_n x),$$

where the  $\omega_n$  are the four complex roots of the complex

number  $\lambda' = (i\omega\beta + \alpha\omega - \gamma)$ . A particular integral for the equation can be found by trying the solution,  $\phi(x) = A \exp(ikx)$ . Substitution of this shows that  $A = K/((k)^4 - \lambda')$  and therefore the complete solution of equation (6.15) is,

$$\phi(x) = \sum_{n=0}^3 A_n \exp(i\omega_n x) + \frac{K}{k^4 - \lambda'}. \quad (6.16)$$

Differentiating this equation two and three times respectively gives the four boundary conditions:

$$\begin{aligned} - \sum_{n=0}^3 A_n \omega_n^2 \exp(i\omega_n x) &= \frac{k^2 K \exp(ikx)}{k^4 - \lambda'}, \\ - \sum_{n=0}^3 A_n i \omega_n^3 \exp(i\omega_n x) &= \frac{ik^3 K \exp(ikx)}{k^4 - \lambda'}. \end{aligned} \quad x = \pm l \quad (6.17)$$

In general  $A_n$ ,  $\omega$  and  $\lambda'$  are complex so that we have to solve equations (6.17) for the real and imaginary parts of  $A_n$ . This involves the solution of an eight by eight system of equations since the real and imaginary parts of  $A_n$  cannot be decoupled. It would be tedious to derive explicit expressions for the solutions and so the method adopted was to invert the coefficient matrix numerically within the computer program. When the real and imaginary parts of  $A_n$  have been found then the real and imaginary parts of  $\phi(x)$  can be computed ( $\phi_r(x)$  and  $\phi_i(x)$ ). The bending moment amplitude is then,

$$\begin{aligned}
 |M_y|^2 &= (EI)^2 \left| \frac{\partial^2 y}{\partial x^2} \right|^2, \\
 &= (EI)^2 \left| \frac{\partial^2}{\partial x^2} \operatorname{Re}(\phi) \right|^2, \\
 &= (EI)^2 \left| \frac{\partial^2}{\partial x^2} \operatorname{Re}(\phi(x)e^{-i\omega t}) \right|^2, \\
 &= (EI)^2 \left| \frac{\partial^2}{\partial x^2} (\phi_r \cos \omega t + \phi_i \sin \omega t) \right|^2, \\
 &= (EI)^2 \left[ \left( \frac{\partial^2 \phi_r}{\partial x^2} \right)^2 + \left( \frac{\partial^2 \phi_i}{\partial x^2} \right)^2 \right]^{\frac{1}{2}}. \quad (6.18)
 \end{aligned}$$

The line of reasoning for  $|M_z|^2$  is similar except that the coefficients are different and the deflection in the z-direction is given by,

$$z = \operatorname{Im}(\phi).$$

The second possibility for the wave force function,  $F(t)$ , that we shall investigate is the approximate version of the modified Morison's equation used in Chapter 4. Recalling that we approximated the varying volume  $V(t)$  by  $V + aDL \cos(\frac{2\pi t}{T})$  then the horizontal (normal to the pipe) and the vertical forces per unit width are given by,

$$\begin{aligned}
 F_z &= C_m \rho g V_0 \frac{2\pi a}{L} \exp \frac{2\pi d}{L} \cos \theta \sin(kx - \omega t) \\
 &+ C_m \rho g D' \frac{2\pi a^2}{L} \exp \frac{2\pi d}{L} \cos \theta \sin(kx - \omega t) \cos(kx - \omega t),
 \end{aligned}$$

and

$$F_y = \left[ -C_m \rho g V_o \frac{2\pi a}{L} \exp \frac{2\pi d}{L} + \rho g a D' \right] \cos (kx - \omega t) \\ + \left[ -C_m \rho g D' \frac{\pi a^2}{L} \exp \frac{2\pi d}{L} \right] \cos(kx - \omega t) \cos 2(kx - \omega t), \quad (6.19)$$

where  $V_o$  is the volume of displaced water (with no waves) per unit length of pipe. These equations can be written as follows:

$$F_z = K_3 \sin (kx - \omega t) + K_4 \sin 2(kx - \omega t),$$

and  $F_y = K_5 \cos (kx - \omega t) + K_6 \cos 2(kx - \omega t) + K_7, \quad (6.20)$

where  $K_3 = C_m \rho g V_o \frac{2\pi a}{L} \exp \frac{2\pi d}{L} \cos \theta,$

$$K_4 = C_m \rho g D' \frac{\pi a^2}{L} \exp \frac{2\pi d}{L} \cos \theta,$$

$$K_5 = -C_m \rho g V_o \frac{2\pi a}{L} \exp \frac{2\pi d}{L} + \rho g a D',$$

$$K_6 = -C_m \rho g D' \frac{\pi a^2}{L} \exp \frac{2\pi d}{L},$$

$$K_7 = K_6.$$

The coefficient,  $K_7$ , represents a mean vertical force acting on the beam. This term arises in the case of a rigidly fixed cylinder, but here we are assuming that the pipe is free to move and so the effect of this will be to produce a small displacement such that the mean force is cancelled by an increase in buoyancy. We will assume that

the wave force is given by the remaining terms.

The resulting differential equations will be similar to equations (6.10) except that there are extra terms on the right hand side. We again proceed by looking at the equation,

$$\phi_{xxxx} + \alpha\phi_{tt} + \beta\phi_t + \gamma\phi = K' \exp i(kx-\omega t) + K'' \exp i2(kx-\omega t). \quad (6.21)$$

If  $\phi'$  is a solution of,

$$\phi_{xxxx} + \alpha\phi_{tt} + \beta\phi_t + \gamma\phi = K' \exp i(kx-\omega t), \quad (6.22)$$

and  $\phi''$  is a solution of,

$$\phi_{xxxx} + \alpha\phi_{tt} + \beta\phi_t + \gamma\phi = K'' \exp i2(kx-\omega t), \quad (6.23)$$

then  $\phi = \phi' + \phi''$  is a solution of equation (6.20).  $\phi'$  and  $\phi''$  are found in exactly the same way as before by trying solutions of the form

$$\begin{aligned} \phi' &= \phi'(x)e^{-i\omega t} \\ \text{and} \quad \phi'' &= \phi''(x)e^{-i2\omega t} \end{aligned} \quad (6.24)$$

The horizontal and vertical moments are then found by taking the imaginary and real parts of  $\phi = \phi' + \phi''$  and the root mean square moment (over one wave period) is calculated as follows. Let the real and imaginary parts of  $\phi'(x)$  and  $\phi''(x)$  be  $R'(x)$ ,  $I'(x)$ ,  $R''(x)$ , and  $I''(x)$ . Then the

horizontal moment is given by,

$$\begin{aligned}
 M_z &= EI \frac{\partial^2}{\partial x^2} [\text{Im}(\phi_z)], \\
 &= EI \frac{\partial^2}{\partial x^2} [\text{Im}\{(R_z'(x) + i I_z'(x))e^{-i\omega t} \\
 &\quad + (R_z''(x) + i I_z''(x))e^{-i2\omega t}\}], \\
 &= EI \frac{\partial^2}{\partial x^2} [I_z'(x)\cos \omega t - R_z'(x)\sin \omega t \\
 &\quad + I_z''(x)\cos 2\omega t - R_z''(x)\sin 2\omega t]. \quad (6.25)
 \end{aligned}$$

The vertical moment is found in a similar way to be,

$$\begin{aligned}
 M_y &= EI \frac{\partial^2}{\partial x^2} [\text{Re}(\phi_y)], \\
 &= EI \frac{\partial^2}{\partial x^2} [R_y'(x)\cos \omega t - I_y'(x)\sin \omega t \\
 &\quad + R_y''(x)\cos 2\omega t + I_y''(x)\sin 2\omega t]. \quad (6.26)
 \end{aligned}$$

Note that the values of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $K'$ , and  $K''$  are different for the vertical moment but the method is the same. The root mean square moment is found by averaging  $M_T^2 = M_z^2 + M_y^2$  over one wave period, i.e.,

$$M_{r.m.s} = \int_0^T = \frac{2\pi}{\omega} M_z^2 + M_y^2 dt.$$

This is given by,

$$M_{r.m.s} = \frac{EI}{\sqrt{2}} \left[ \frac{\partial^2}{\partial x^2} (R_z'^2 + I_z'^2 + R_z''^2 + I_z''^2 + R_y'^2 + I_y'^2 + R_y''^2 + I_y''^2) \right] \quad (6.26)$$

The modulus of the bending moment is calculated rather than the vertical and horizontal components so that a comparison of the results can be made with the small amount of data available (Jeffrey et al (1978), vol.1). The wave power team measured the bending moments in a 21m long pipe (see Chapter 5 for experimental details) in regular and irregular waves. For the regular waves they measured the bending moments in the middle of the pipe as a function of wave frequency and crest length (angle of incidence), the root mean square being taken because the pipe tended to roll. Their results are reproduced in figure 6.3. There are several features about these curves that we would like to be able to predict, the most obvious of these being the general magnitude of the moments. The variation with crestlength shows a peak between 10 and 12 metres which corresponds with crestlength being equal to half the length of the pipe. There are also several peaks of the bending moment as a function of frequency which we ought to be able to predict. It is possible to calculate the natural frequencies of a pipe with free ends if we neglect the effects of friction. These frequencies are given by Morse (1948, p.162) as,

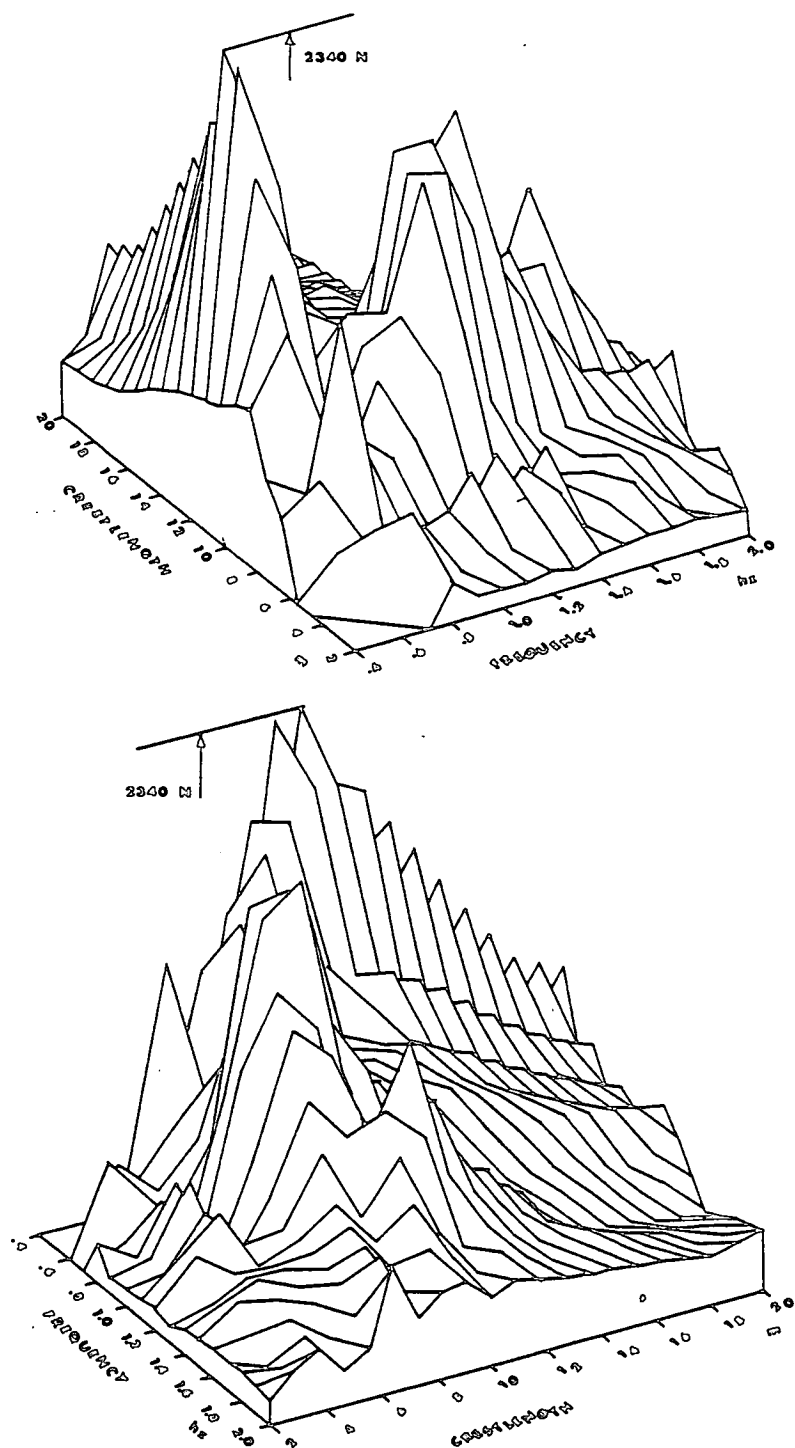


Figure 6.3 (a)

Experimental root mean square total bending moments at the centre of a 21m long pipe (0.083m diameter), variation with frequency and crestlength, wave amplitude = 0.01m. The bending moments on this and all subsequent figures have been normalised by dividing by the root mean square wave amplitude.

(a) without computer smoothing.

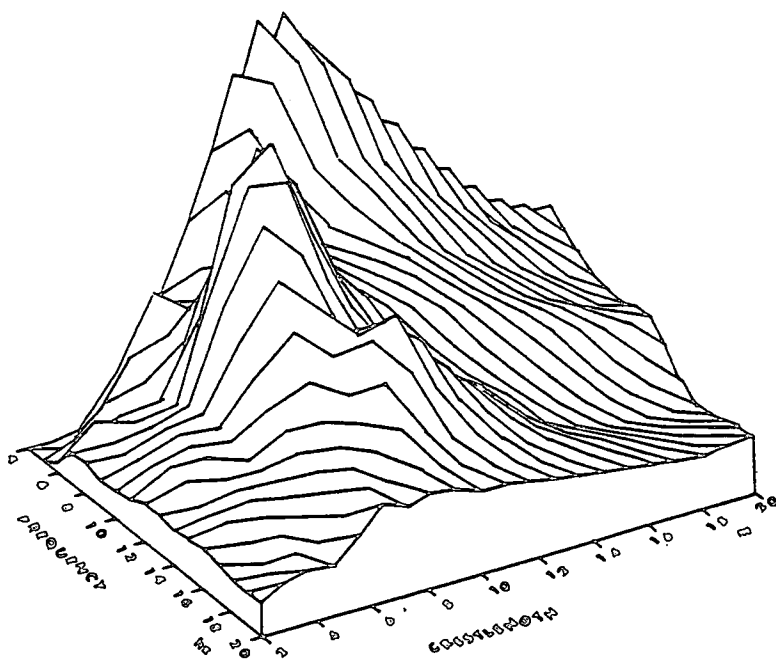
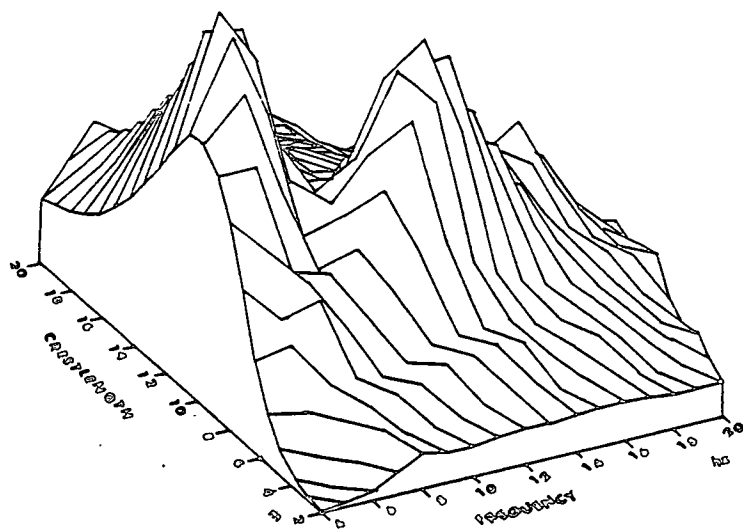


Figure 6.3 (b) Experimental root mean square total bending moments at the centre of a 21m long pipe (0.083m diameter), variation with frequency and crestlength, wave amplitude = 0.01m.  
(b) with computer smoothing.

$$\omega_n = \frac{\pi}{8\ell} \sqrt{\frac{EI}{M}} \beta_n^2,$$

where  $\beta_n \doteq (n + \frac{1}{2})$ . This also neglects the added mass effect which could be corrected by replacing  $M$  by  $M + M_a$ . The resonant frequencies will not be significantly changed by the inclusion of damping, provided that it is fairly small. The correct prediction of the resonant frequencies therefore depends, in the main, on the correct value of added mass. The inclusion of added mass and damping in the dynamical equation presents the most serious problem to be overcome. It is well known that these parameters are highly frequency dependent, which would mean that the general equation of motion of a pipe in water has coefficients that are functions of frequency. Therefore, it is impossible to solve for the natural frequencies explicitly, since the coefficients are not known until the problem has been solved. Forced motion, however, can be treated if we assume, as we did above, that all transient motion has died away and a solution with the same periodicity as the driving force is assumed. There are two possible ways of calculating the added mass and damping. The first would be to measure the two necessary force components in a wave tank for the frequencies of interest, as done by Murtha (1954) for heaving motion. The second method would be to calculate these quantities numerically. For the purposes of this work it was decided to adopt the latter procedure and rely on data produced by a two

dimensional program developed at the C.E.G.B. laboratories at Marchwood\*, and written using the method of Frank (1967). The results of Ogilvie (1963) for the added mass and damping are not applicable because the cylinder is not totally submerged. The general problem of the harmonic oscillation of a rigid body on a free surface has been discussed by Kim (1965) who used a Green's function to reduce the problem to that of solving an integral equation, which he did numerically. The results are only applicable to bodies whose centre of gravity is at the free surface.

Table 6.1 presents values of these coefficients for a horizontal cylinder of diameter 0.083 m at two axis depths,  $d = 0$  and  $d = -D/2$ . It is evident that both added mass and damping vary widely with frequency, especially in the  $d = 0$  case, and that there are times when negative added mass coefficients occur, this possibility being pointed out by Ogilvie (1963). The small negative added dampings are artefacts of the computation and can be equated to zero. Since only a very limited amount of added mass and damping data was available to the present author it was not possible to use the correct values for every frequency tested by Salter (Jeffrey et. al. (1978)). A program was not readily available for the calculation of these parameters and time did not allow for one to be written. The theoretical results presented in this chapter refer to the

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\*The author is indebted to R. Jeffreys and B.M. Count of the C.E.G.B. for the added mass and damping coefficients presented here.

| RELATIVE<br>AXIS<br>DEPTH | FREQUENCY<br>(Hz) | SURGE<br>ADDED<br>MASS<br>(kg) | SURGE<br>ADDED<br>DAMPING<br>(kg/s) | HEAVE<br>ADDED<br>MASS<br>(kg) | HEAVE<br>ADDED<br>DAMPING<br>(kg/s) |
|---------------------------|-------------------|--------------------------------|-------------------------------------|--------------------------------|-------------------------------------|
| -0.50                     | 0.28              | 12.5                           | 0.8                                 | 7.4                            | -0.7                                |
|                           | 0.55              | 17.4                           | 39.1                                | 8.8                            | 3.3                                 |
|                           | 0.83              | 15.2                           | 475.9                               | 10.6                           | 65.5                                |
|                           | 1.10              | -6.8                           | 330.7                               | 7.9                            | 230.9                               |
|                           | 1.38              | -3.9                           | 84.2                                | 2.1                            | 277.7                               |
|                           | 1.66              | -0.3                           | 4.8                                 | -0.2                           | 195.8                               |
|                           | 1.93              | 1.5                            | 6.7                                 | -0.2                           | 102.7                               |
|                           | 2.21              | 2.1                            | 23.8                                | 0.7                            | 39.4                                |
|                           | 2.49              | 2.3                            | 26.9                                | 1.5                            | 8.6                                 |
|                           | 2.76              | 2.4                            | 19.7                                | 2.1                            | -1.2                                |
| 0.00                      | 0.28              | 2.8                            | 0.00                                | 6.7                            | 42.0                                |
|                           | 0.55              | 3.2                            | 2.6                                 | 3.8                            | 65.5                                |
|                           | 0.83              | 3.6                            | 18.5                                | 2.5                            | 73.7                                |
|                           | 1.10              | 3.2                            | 56.4                                | 1.9                            | 71.3                                |
|                           | 1.38              | 2.0                            | 85.7                                | 1.7                            | 62.4                                |
|                           | 1.66              | 1.2                            | 90.0                                | 1.6                            | 50.2                                |
|                           | 1.93              | 0.8                            | 83.3                                | 1.7                            | 37.7                                |
|                           | 2.21              | 0.6                            | 73.1                                | 1.9                            | 26.2                                |
|                           | 2.49              | 0.5                            | 63.4                                | 2.0                            | 20.9                                |
|                           | 2.76              | 0.5                            | 54.0                                | 2.1                            | 13.9                                |

Table 6.1      The added mass and damping per unit length  
for a 0.083m diameter horizontal circular  
cylinder for a number of frequencies and  
two depths.

bending moments in a model pipe, as used in the series of experimental tests the results of which are presented in figure 6.3. The root mean square bending moment (over a wave cycle) as a function of position along the pipe and crestlength is shown in figure 6.4. These moments have been calculated using the first set of force equations (equations (6.9)). The bending moments have been normalised by dividing by the root mean square wave amplitude. The frequency used was 0.83 Hz and the added mass and damping were taken from Table 6.1. The force coefficients  $C_{fx}$  and  $C_{fy}$  were taken to be 1.0 and 0.5 respectively, on the basis of Salter's analysis of his force measurements for a duck using equation (1.15). The EI value used was the measured value for the model spine of  $2400 \text{ Nm}^3$ . As expected the bending moments reach a peak over the range covered when the crestlength is half the length of the pipe, as shown up in the experimental tests of figure 6.3. The most significant feature of figure 6.4 is that the bending moments are not symmetrical about the centre of the pipe as might be anticipated. This asymmetry is introduced by the added damping term and increases with it. Symmetry returns if the damping is zero as demonstrated in figure 6.5. This asymmetry causes the bending moments at the downwave end to be larger than at the other end. This phenomenon could be very significant but has yet to be confirmed experimentally. One of the most important requirements of the design of the duckstring is a knowledge of the influence of the parameter, EI, on

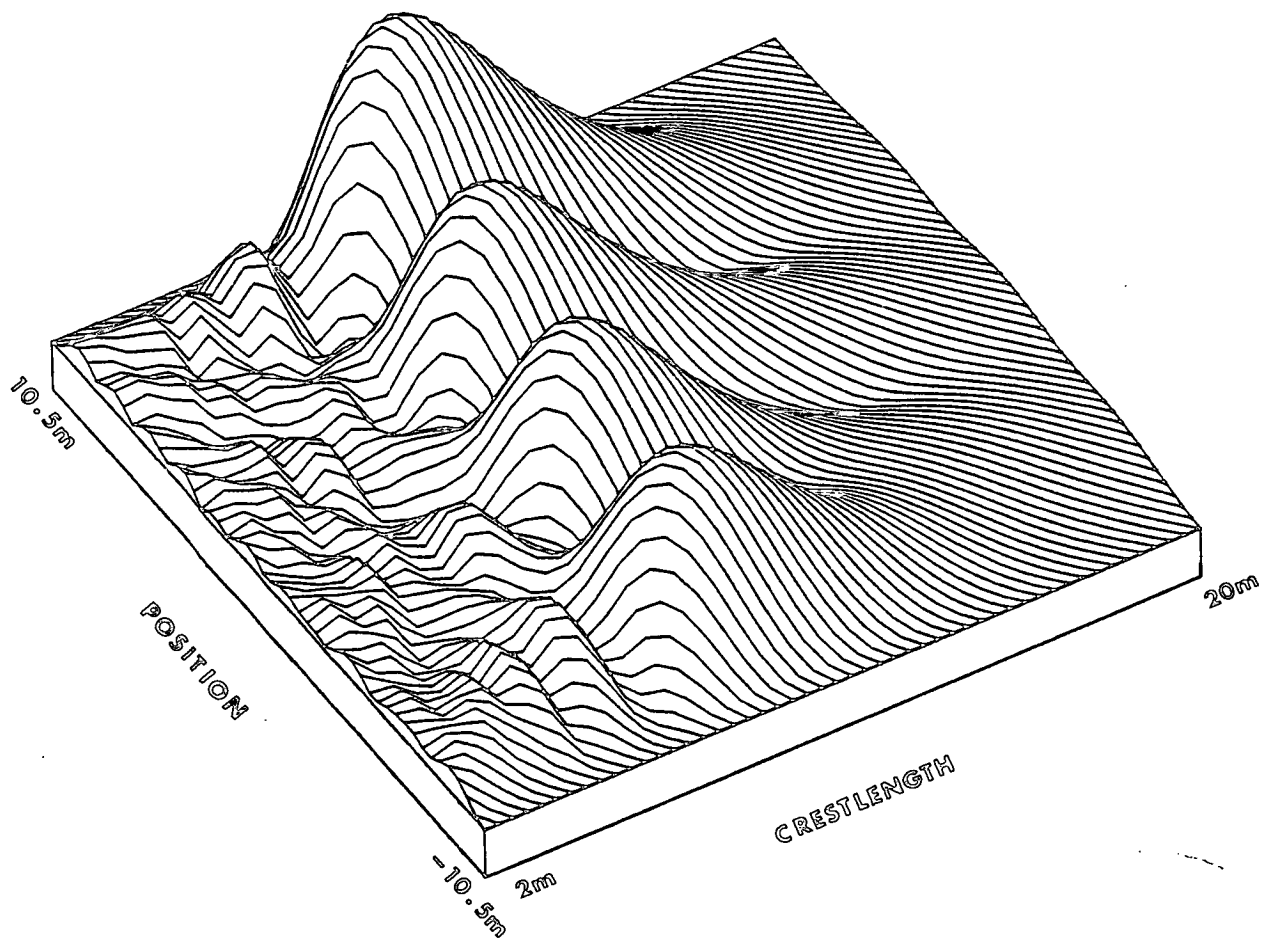


Figure 6.4

Theoretical bending moments, using Salter's force equation, along a 2lm pipe as a function of crestlength.

|                     |   |              |
|---------------------|---|--------------|
| Wave frequency      | = | 0.83 Hz      |
| Wave amplitude      | = | 0.01 m       |
| Surge added mass    | = | 15.2 kg/m    |
| Surge added damping | = | 475.9 kg/m/s |
| Heave added mass    | = | 10.6 kg/m    |
| Heave added damping | = | 65.5 kg/m/s  |

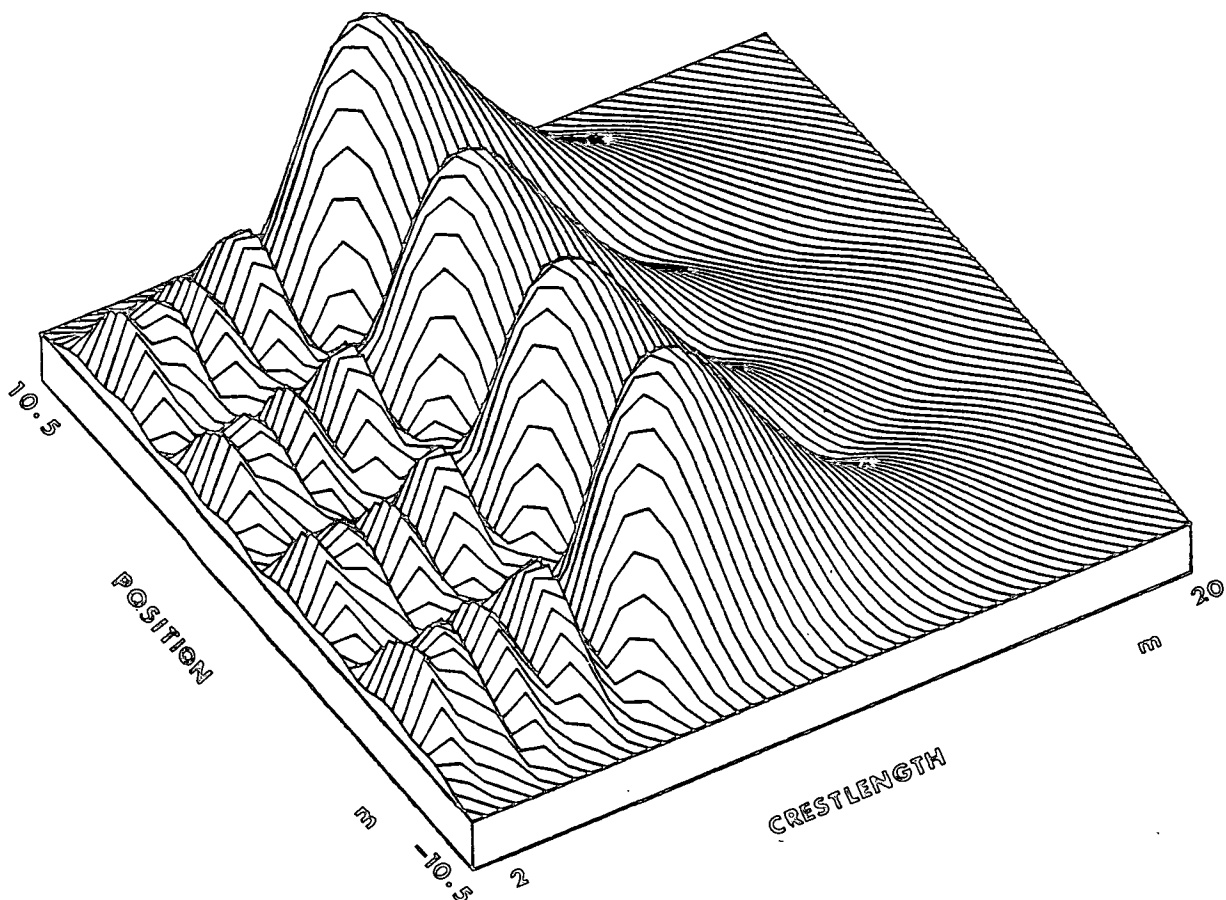


Figure 6.5

Theoretical bending moments, using Salter's force equation, along a 21m pipe as a function of crestlength.

|                     |   |            |
|---------------------|---|------------|
| wave frequency      | = | 0.83 Hz    |
| wave amplitude      | = | 0.01 m     |
| surge added mass    | = | 15.2 kg/m  |
| surge added damping | = | 0.0 kg/m/s |
| heave added mass    | = | 10.6 kg/m  |
| heave added damping | = | 0.0 kg/m/s |

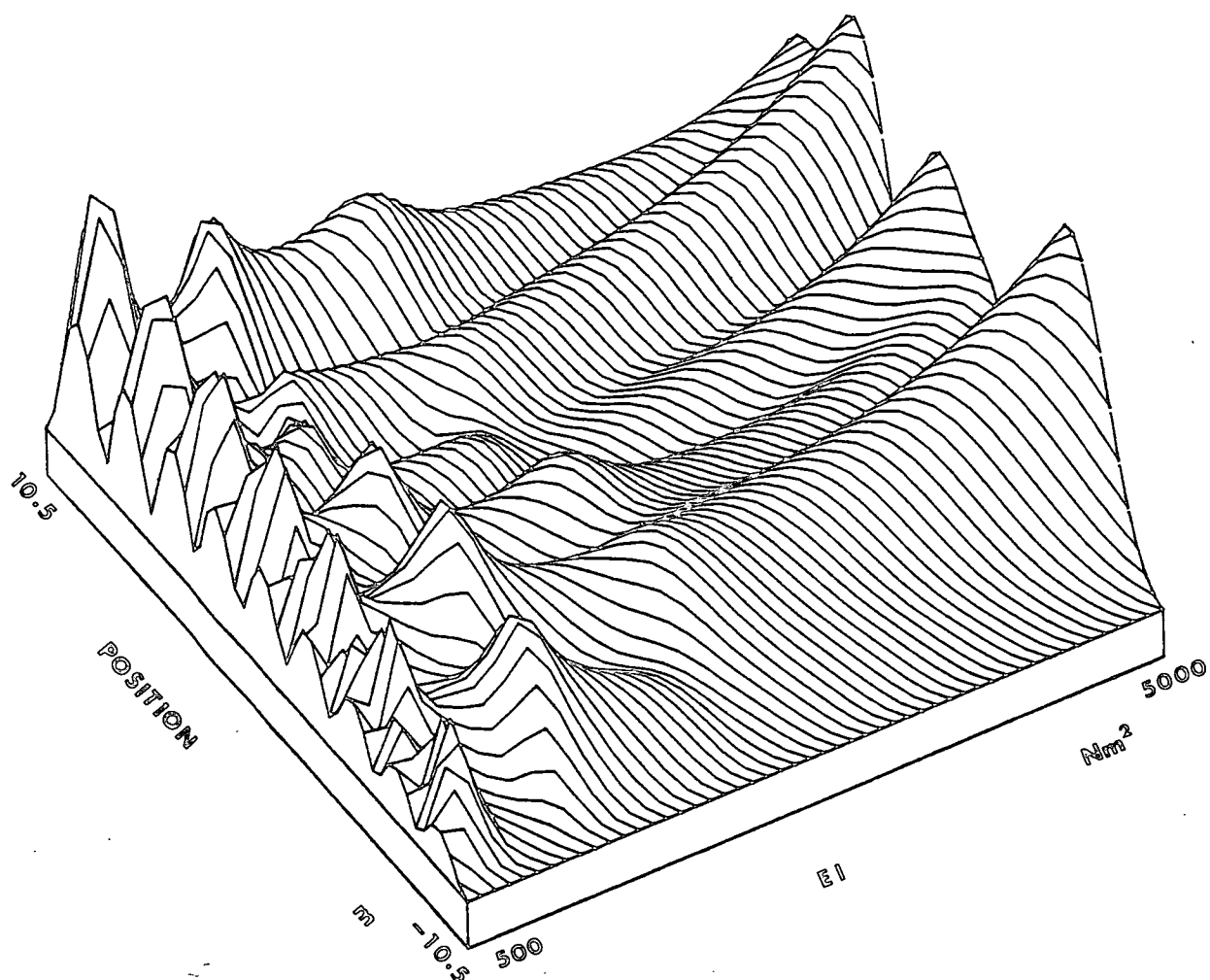


**Figure 6.6**

Theoretical bending moments using Salter's force equation, along a 21m pipe as a function of EI

|                     |   |              |
|---------------------|---|--------------|
| wave frequency      | = | 0.83 Hz      |
| wave amplitude      | = | 0.01 m       |
| crestlength         | = | 5.00 m       |
| surge added mass    | = | 15.2 kg/m    |
| surge added damping | = | 475.9 kg/m/s |
| heave added mass    | = | 10.6 kg/m    |
| heave added damping | = | 65.5 kg/m/s  |

the bending moments. This parameter could not be varied on the model and so, at the time of writing, the only guidance available is that to be gained from the theoretical model outlined above. Figure 6.6 shows the variation of bending moment with EI over a range including the value of the model pipe. The crestlength used was 5m and the wave frequency, 0.83Hz. The moments are fairly constant over the middle of the range but increase quite rapidly for the higher stiffnesses.

Without an extensive knowledge of the added mass and damping coefficients we cannot predict with accuracy the variation of bending moment with frequency. One possible recourse might be to assume constant values of these parameters. Figures 6.7 to 6.9 show the variation of bending moment along the spine with frequency. In each case the added mass and damping have been kept constant using values taken from Table 6.1 for three different frequencies.

The angle of the waves has been kept constant at  $45^{\circ}$  and so the crestlength is equal to 1.41 times the wavelength. In all three cases there is a very large resonance when the crestlength is equal to half the length of the pipe. It must be remembered that the assumptions on which the theoretical model is based will break down long before bending moments of these magnitudes are reached and the spine will no longer behave elastically but will enter the plastic region. However, the results do serve to show

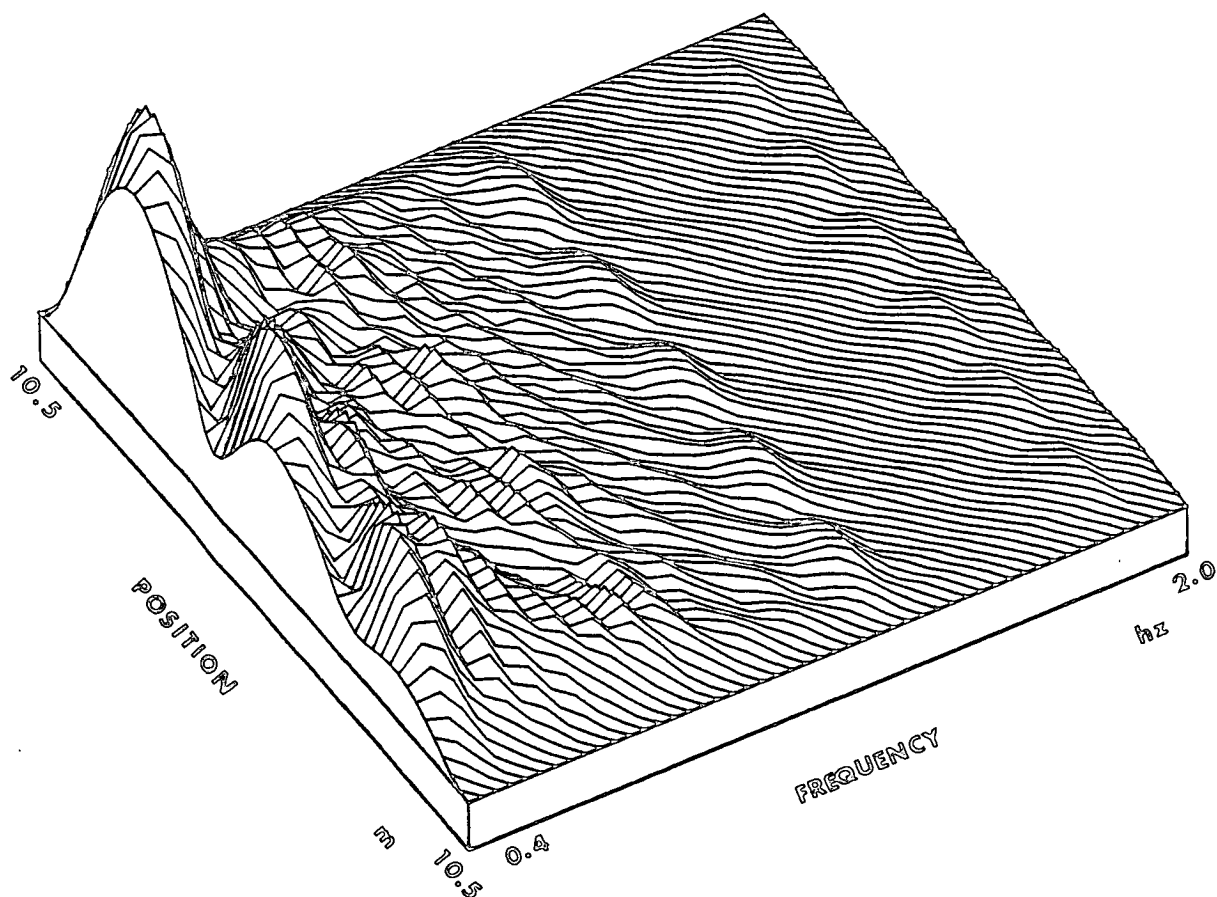


Figure 6.7

Theoretical bending moments, using Salter's force equation, along a 21m pipe as a function of frequency.

crestlength =  $\lambda/2$  wavelength

wave amplitude = 0.01 m

surge added mass = 15.2 kg/m

surge added damping = 475.9 kg/m/s

heave added mass = 10.6 kg/m

heave added damping = 65.5 kg/m/s

(0.83 Hz)

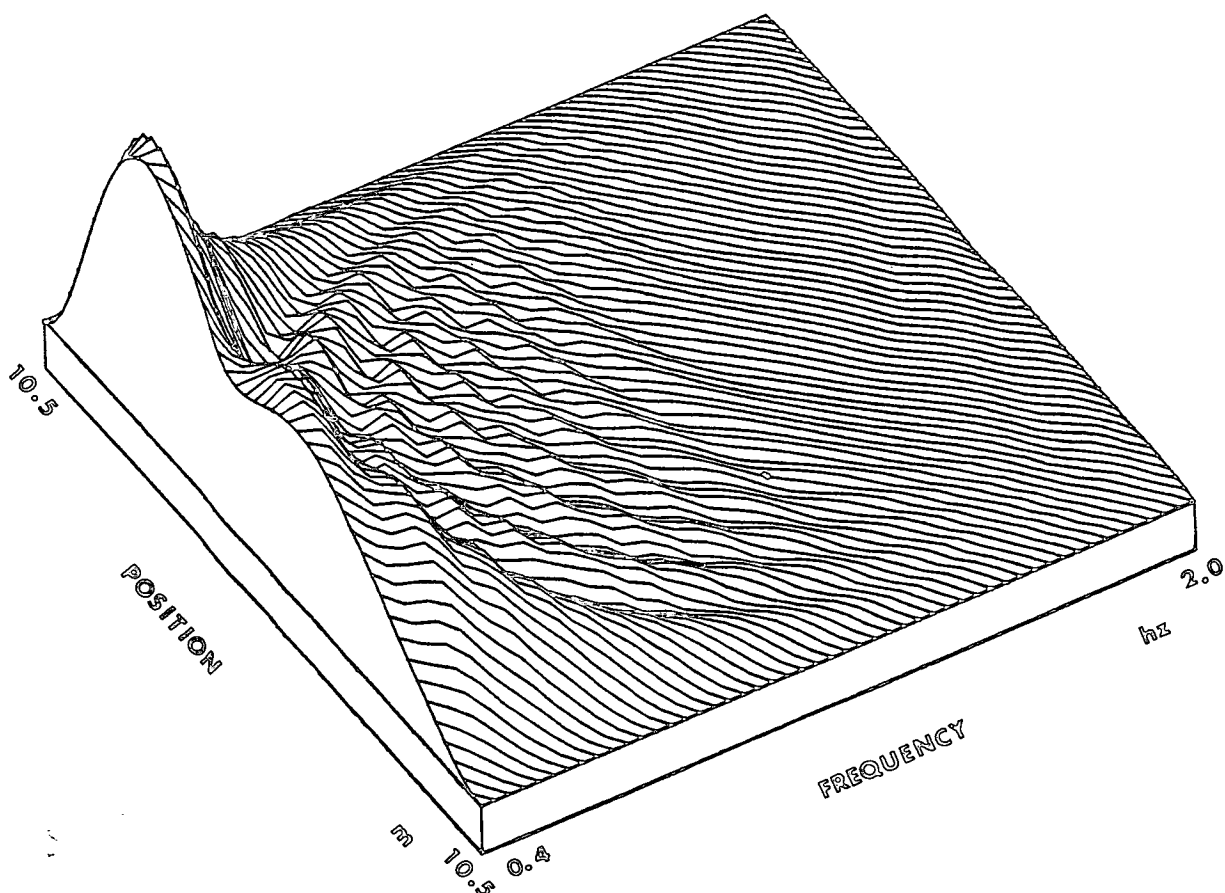


Figure 6.8

Theoretical bending moments, using Salter's force equation, along a 21m pipe as a function of frequency.

crestlength =  $\sqrt{2}$  wavelength

wave amplitude = 0.01 m

surge added mass = 3.8 kg/m

surge added damping = 84.2 kg/m/s

heave added mass = 2.1 kg/m

heave added damping = 277.6 kg/m/s

(1.38 Hz)

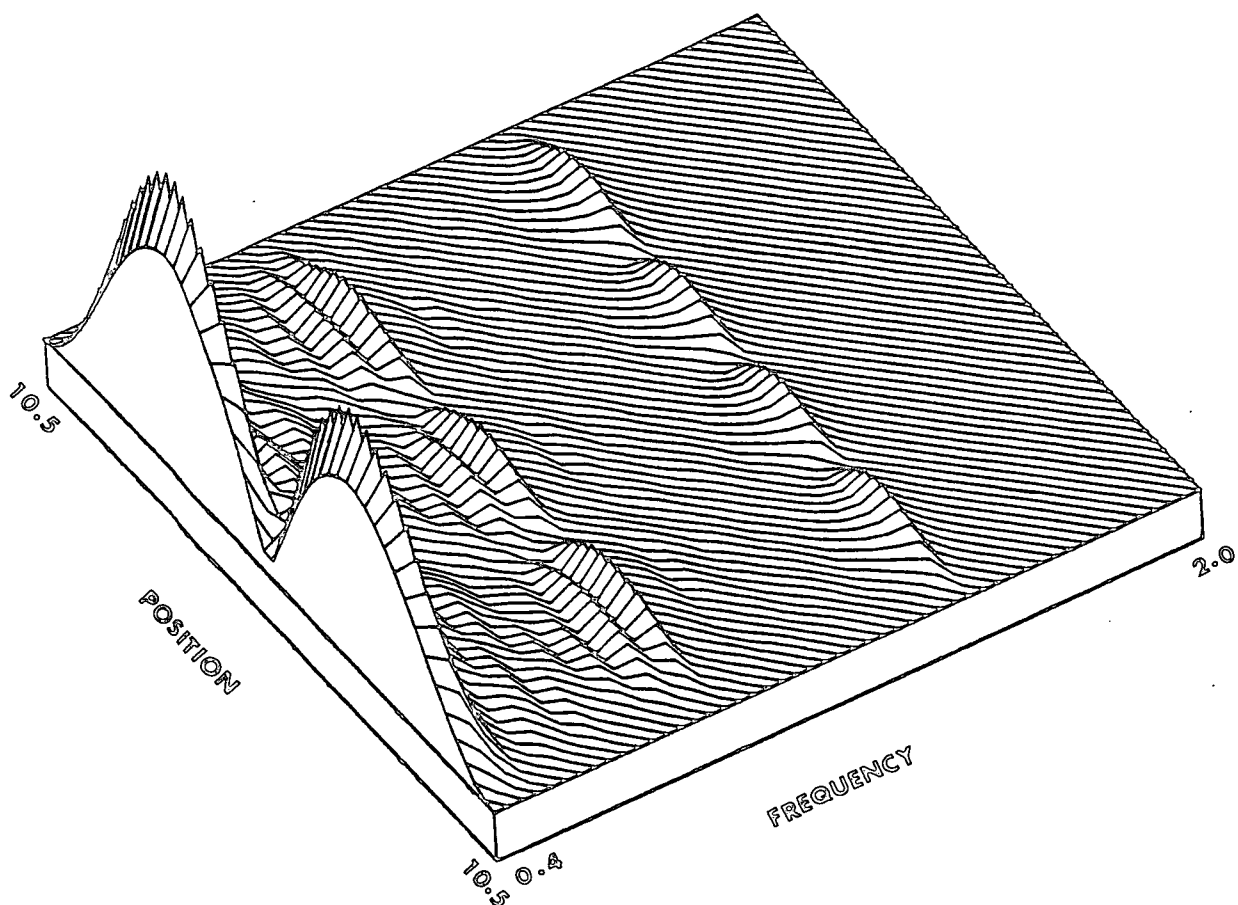


Figure 6.9

Theoretical bending moments, using Salter's force equation, along a 21m pipe as a function of frequency.

|                     |                         |           |
|---------------------|-------------------------|-----------|
| crestlength         | = $\sqrt{2}$ wavelength |           |
| wave amplitude      | = 0.01 m                |           |
| surge added mass    | = 1.5 kg/m              | (1.93 Hz) |
| surge added damping | = 6.7 kg/m/s            |           |
| heave added mass    | = -0.1 kg/m             |           |
| heave added damping | = 102.7 kg/m/s          |           |

when the moments become large and that again there is an asymmetry about the centre of the spine. The frequency range covered was chosen to match the equivalent range likely to be encountered at full scale - remembering that frequency scales to the power of  $-\frac{1}{2}$ . The same three sets of added mass and damping have been used to compute the bending moments in figures 6.10 to 6.12 which give the variation of the bending moment at the centre of the pipe as a function of frequency and crestlength - which therefore corresponds to the experimental data shown in figure 6.3. It can be seen that the added masses and dampings are crucial to predicting accurate bending moments. There is perhaps sufficient similarities between figure 6.3 and figures 6.10 to 6.12 to suggest that if the correct added masses and dampings could be inserted for each frequency then the method will give reasonable results, with respect to both magnitude and variation with crestlength and frequency. Until this is possible only overall trends can be predicted using 'guess' values of added mass and damping which are kept constant. At the time of writing there are two other approaches to the problem of bending moments in the duck spine which are being investigated. The first is a research program being carried out by Lloyds Shipping Register under a contract from the Department of Energy. Their approach is also an analytical one in which the hydrodynamics coefficients are first calculated using a program developed at the National Maritime Institute and then used in a finite element

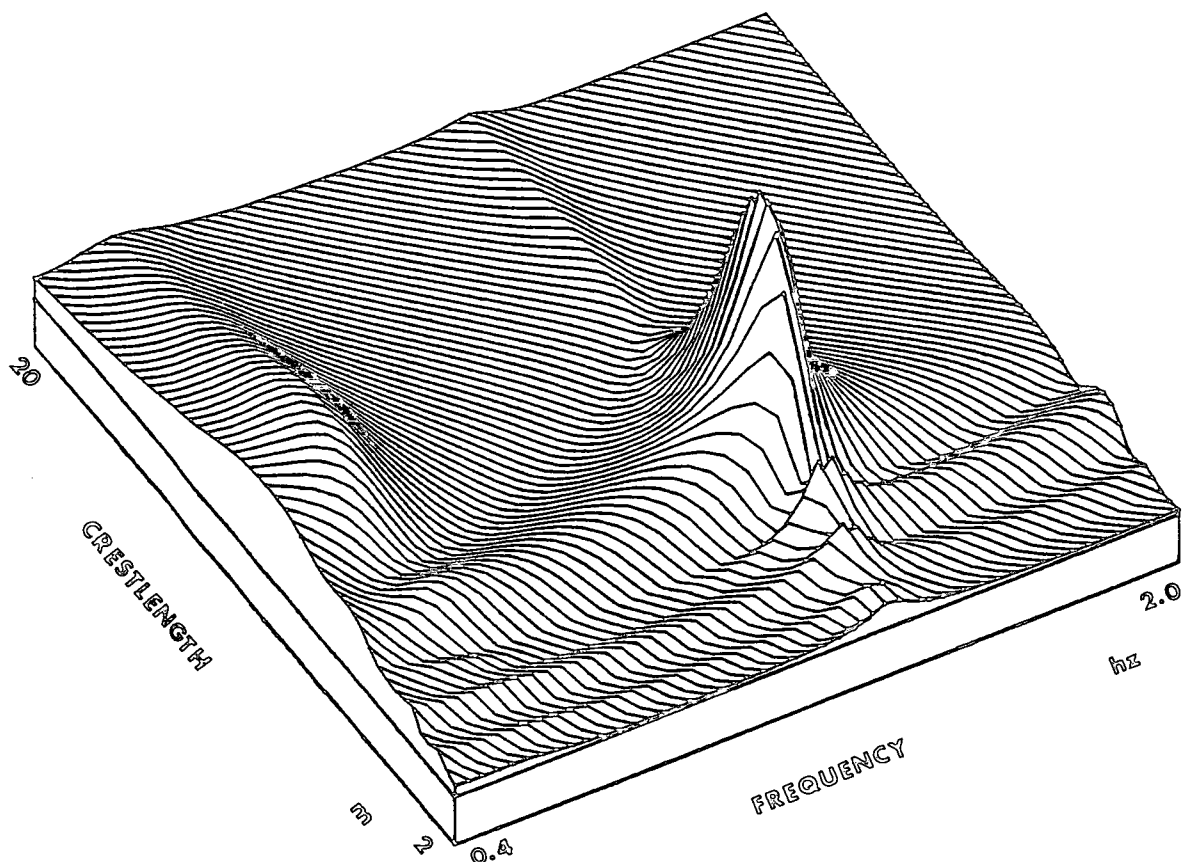


**Figure 6.10** Theoretical bending moments, using Salter's force equation, at the centre of a 21m pipe as a function of frequency and crestlength.

|                     |                          |
|---------------------|--------------------------|
| wave amplitude      | = 0.01 m                 |
| surge added mass    | = 15.2 kg/m              |
| surge added damping | = 475.9 kg/m/s (0.83 Hz) |
| heave added mass    | = 10.6 kg/m              |
| heave added damping | = 65.5 kg/m/s            |

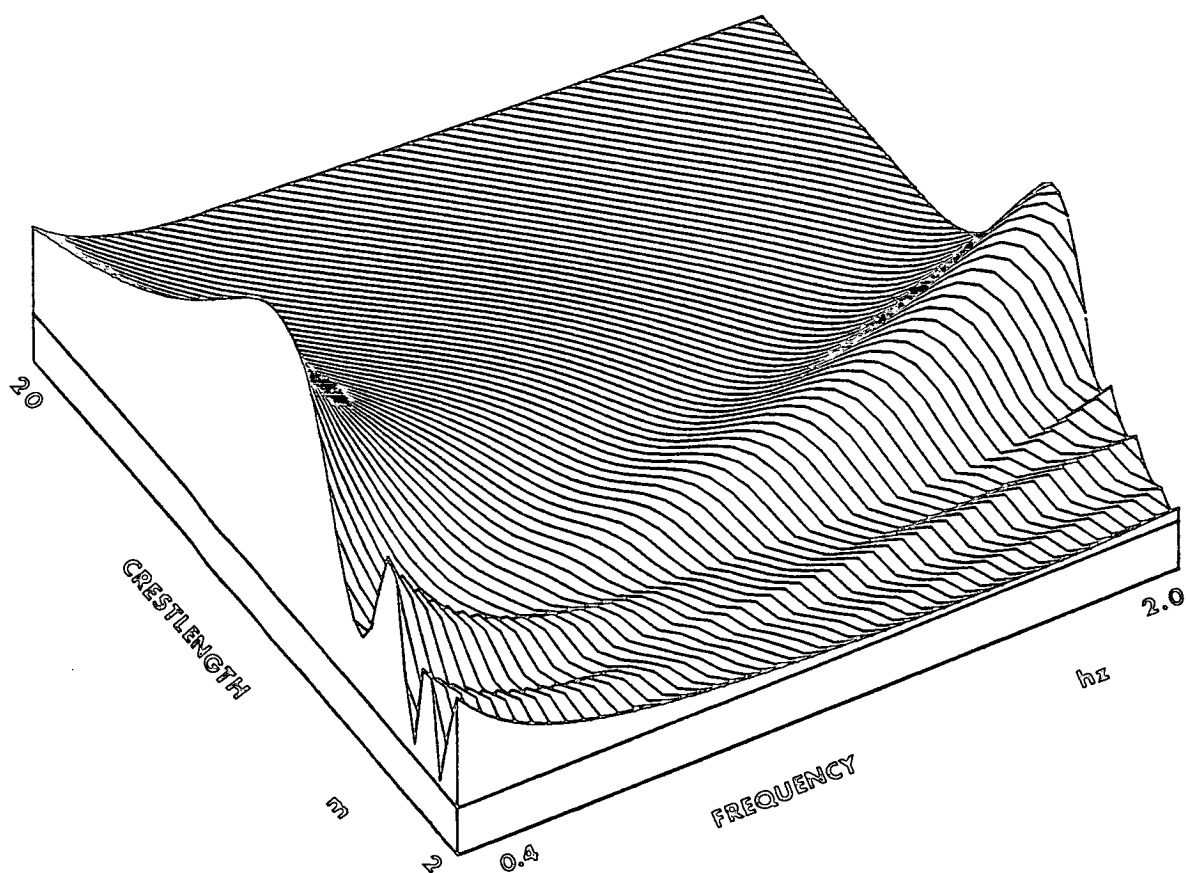


Figure 6.11

Theoretical bending moments using Salter's force equation, at the centre of a 21m pipe as a function of frequency and crestlength.

|                     |                         |
|---------------------|-------------------------|
| wave amplitude      | = 0.01 m                |
| surge added mass    | = 3.8 kg/m              |
| surge added damping | = 84.2 kg/m/s (1.38 Hz) |
| heave added mass    | = 2.1 kg/m              |
| heave added damping | = 277.6 kg/m/s          |

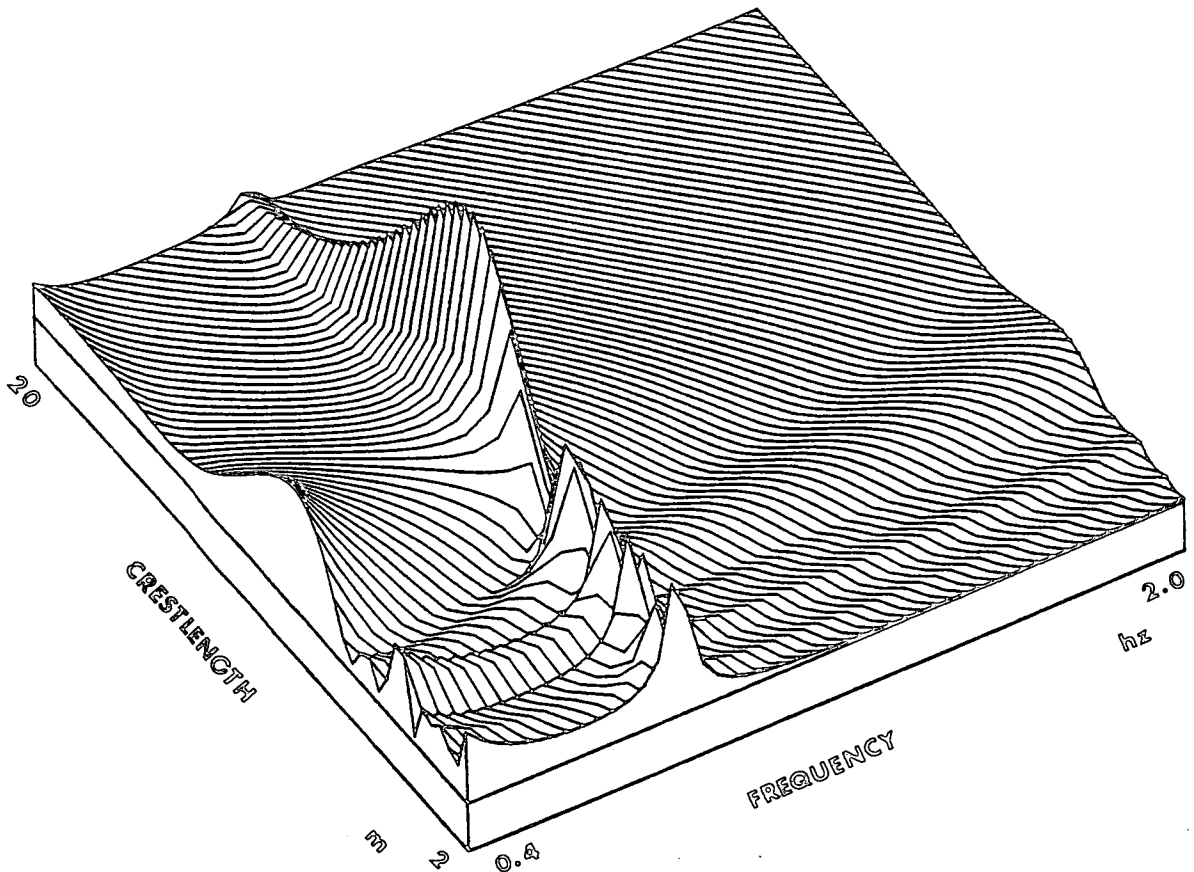


Figure 6.12      Theoretical bending moments using Salter's force equation, at the centre of a 21m pipe as a function of frequency and crestlength.

|                     |   |                           |
|---------------------|---|---------------------------|
| wave amplitude      | = | 0.01 m                    |
| surge added mass    | = | 1.5 kg/m                  |
| surge added damping | = | 6.7 kg/m/s      (1.93 Hz) |
| heave added mass    | = | -0.1 kg/m                 |
| heave added damping | = | 102.7 kg/m/s              |

structural dynamics program to derive the bending moments. This research has not been completed yet and no results are expected until 1980. Its main disadvantage is that it is extremely costly in computing time and therefore there are economic restrictions in the number of frequencies which can be studied. Eventually, this method will be able to model the ducks and the effect of power take-off as well as the spine alone, something which has not been attempted in the one dimensional model presented in this chapter. The second investigation is an experimental one being undertaken by the Edinburgh Wave Power Team. They are constructing a string of model ducks on which they will be able to vary all the parameters of interest, including rigidity. The string will not have constant rigidity all the way along, but will have joints between each duck whose stiffness can be altered at will. Therefore in the actual model  $EI$  will not be constant along the string but will have a lumped distribution. It will be possible to measure the bending moments at all the joints and investigate the variation of these with wave amplitude, frequency and rigidity. One possibility being considered is to have active joints whose stiffness will be controlled by the prevailing wave conditions. Power take-off equipment may also be used in the joints - in a manner akin to the Cockerall wave raft system of energy extraction. This would require major modification to be made to the theoretical model proposed here. The reason for pursuing the present line of enquiry is that some results are

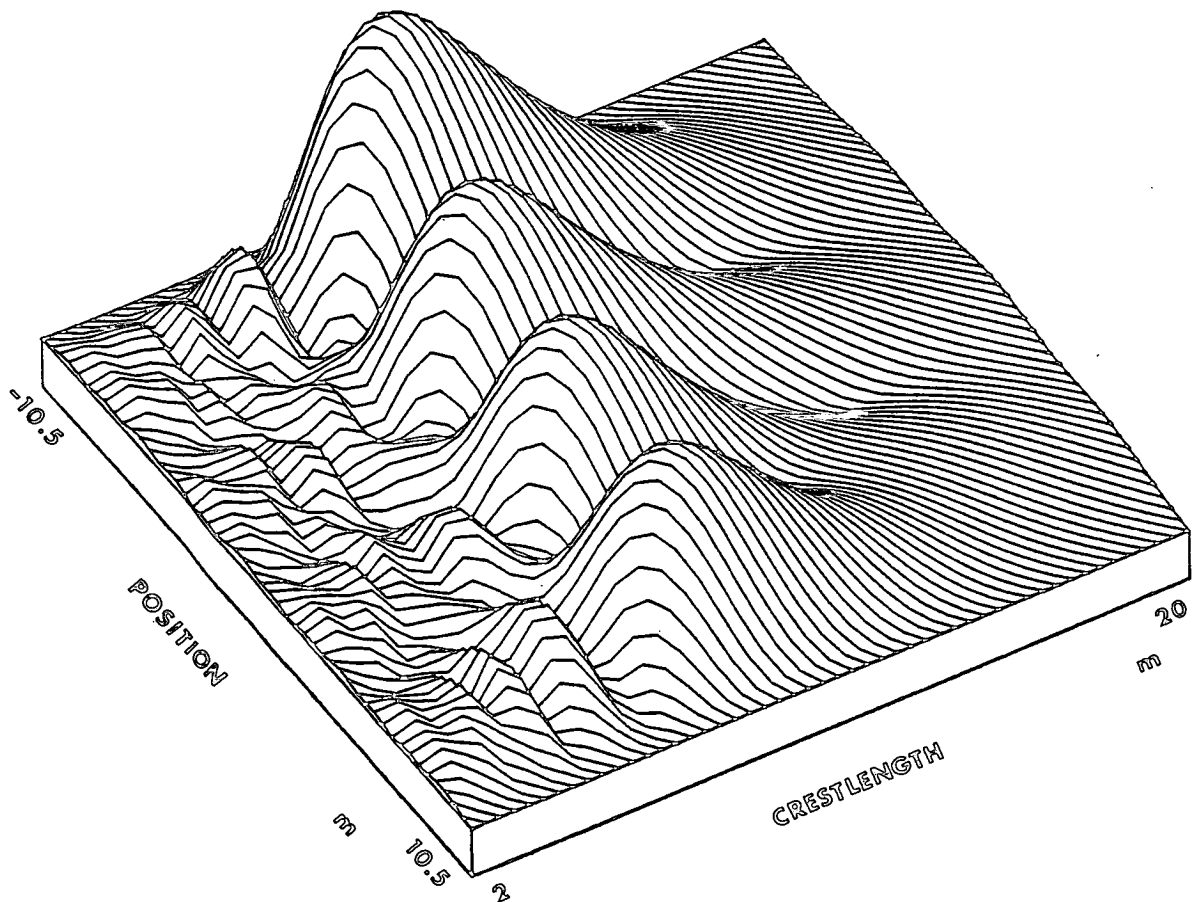


**Figure 6.13** Theoretical bending moments, using the modified Morison's force equation, along a 21m pipe as a function of crestlength.

|                     |                |
|---------------------|----------------|
| wave frequency      | = 0.83 Hz      |
| $C_m$               | = 2.0          |
| wave amplitude      | = 0.01 m       |
| surge added mass    | = 15.2 kg/m    |
| surge added damping | = 475.9 kg/m/s |
| heave added mass    | = 10.6 kg/m    |
| heave added damping | = 65.5 kg/m/s  |

required very quickly and the other two investigations will not produce any data for several months. It will then be possible to compare all three methods and perhaps combine them to present the most useful picture of the problem.

Figure 6.13 shows the variation of bending moment vs crestlength and position using the other set of force equations (equation 6.18). The frequency was 0.83 Hz and the coefficient  $C_m$  was taken to be the potential theory value of 2.0 for convenience. The similarity with the corresponding figure using Salter's force equations (equations (6.9)) was such that no further results were plotted, as it was considered that the problem of added mass and damping was more crucial than that of finding the correct force equation.

## CHAPTER 7

### GENERAL DISCUSSION AND CONCLUSIONS

## CHAPTER 7

### GENERAL DISCUSSION AND CONCLUSIONS

#### 7.1 Summary

Research into the extraction of energy from ocean waves requires a thorough knowledge and understanding of the loads likely to be imposed on the particular device under consideration. The most important source of these loads is the waves themselves and this thesis addresses itself to that problem with particular reference to the wave power system proposed by Salter at the University of Edinburgh. This design, whose main component consists of a cam-shaped segment called a duck, presents considerable difficulties in assessing the wave forces, mainly associated with the fact that the entire structure lies on the free surface. Most of the previous work on wave forces has concerned itself with vertical piles and cylinders extending from the seabed.

Research into the forces on horizontal cylinders has been scarce and has always dealt with totally submerged bodies like pipelines or with the phenomenon known as wave slamming. The prediction of wave forces on a pile is usually done using Morison's equation: a formula which separates the wave force into an inertial term and a drag term. Empirically determined coefficients, the inertia coefficient and the drag coefficient, are used in calculating the force. The variation of these coefficients has been the subject of intense research for thirty years.

For partially immersed bodies like the Salter duck free surface effects play an important part in determining the wave loads. Initial work with Morison's equation for a fixed horizontal cylinder produced unsatisfactory results when compared to experimental data, with a wide scatter in the values of the coefficients, an inaccurate representation of the force variation over a wave cycle and conflicting trends when the wavelength is varied. It is suggested that for bodies with the dimensions envisaged for a wave energy device the drag term in Morison's equation can be neglected and that suitable modifications to the equation, incorporating the varying buoyancy force on a body on the free surface and the effect of a varying displacement on the loads, enable a much more accurate prediction of wave forces to be made with the scatter in the values of the inertia coefficient considerably reduced and the conflictions resolved. Extensive measurements have been presented to justify the model and to allow the values of the inertia coefficient to be calculated for different values of the parameters of interest.

The modified theory has been extended to the prediction of force spectra. It is shown that the buoyancy force interacts with the inertial force creating secondary harmonics in the total vertical force on a horizontal cylinder which appear as a peak in the vertical force spectrum at twice the mean frequency of the wave spectrum. Some simple assumptions have been made enabling a theoretical force spectrum to be derived for comparison

with measurements taken in a wave tank. Some other statistical quantities have also been derived from the new equations.

In the application of Morison's equation it is an implicit assumption that the waves are not significantly affected by the presence of the cylinder. Although agreement of the theoretical forces with experimental measurements was generally good, observation of the waves as they passed the cylinder revealed significant reflections which may account for some of the differences between observed and predicted forces. This interaction was studied in a series of experiments where the reflections from a horizontal circular cylinder were measured. Although these results could not be incorporated in a direct way into Morison's equation they were used to calculate the mean horizontal forces on the cylinder according to a theory developed from the conservation of wave momentum flux by Longuet-Higgins. Comparison with experimental mean forces showed that this theory can only account in part for the actual mean drift forces acting on the cylinder.

One of the most serious problems facing the designers of the wave energy system is the behaviour of the long "backbone" which will support an array of ducks. A simple approach to this has been made here where the motion and bending moments of a long flexible pipe have been studied. A simple model has been devised based on

a one-dimensional beam theory where the wave forces are the source of the exciting function. The fluid-structure interaction is incorporated by the use of added mass and damping terms in the equation of motion and a simplified wave load equation based on the earlier work with fixed cylinders is used. This method relies on a detailed knowledge of the added mass and damping coefficients which are strongly dependent on frequency. The most striking feature of the predictions is that the bending moments are not symmetric about the centre of the backbone but increase on the downwave side and this asymmetry increases with increased added damping. This has yet to be confirmed experimentally but may be of extreme importance in the design.

## 7.2 Future Work

During the course of this study several lines of research showed promise of further development but it was not possible to devote time to all of them. Although the modifications to Morison's equation reduced the scatter in the values of the inertia coefficient there still remain significant variations for which explanations should be sought. Extensions of the method to arbitrarily shaped bodies and higher order theories also offer opportunities for more work. Alternatively, the Morison approach could be abandoned entirely for partially submerged bodies and a numerical model be developed starting with the basic equations of fluid mechanics and incorporating the free

surface effects from the beginning. A general solution to the wave reflection problem is still to be discovered even in linear theory. Non-linear interactions provide further difficulties which have to be overcome. The simple theory presented here for irregular waves has limitations but it should be possible to extend the work so that it has wider application, although this would probably have to be done numerically rather than analytically.

The area in which there is widest scope for future research is the motion of a floating flexible pipe of the dimensions of a wave energy device. The model developed here has to be extended to include, among other things, a variable elasticity, the effect of power take-off, three-dimensionality of the backbone and the forces exerted by the mooring system. The finite element method of analysis widely used by engineers for both structural and fluid mechanical problems is likely to be most suitable for this research.

## APPENDIX I

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## APPENDIX II

### NOTATION

NOTATION

|       |                                                  |
|-------|--------------------------------------------------|
| $a$   | wave amplitude.                                  |
| $a_i$ | incident wave amplitude; amplitude of a wavelet. |
| $a_r$ | reflected wave amplitude.                        |
| $a_t$ | transmitted wave amplitude.                      |
| $a'$  | relative wave amplitude.                         |
| $A$   | cross-sectional area of a pile; a constant.      |
| $A_n$ | coefficient in expansion of $\eta(t)$ .          |
| $b$   | dimension of square body.                        |
| $B$   | a constant.                                      |
| $C$   | a constant.                                      |
| $C_d$ | drag coefficient.                                |
| $C_f$ | Salter's force coefficient.                      |
| $C_m$ | inertia coefficient.                             |
| $C_s$ | $(F_d + F_k)/F_k$ .                              |
| $C_y$ | vertical force coefficient.                      |
| $C_z$ | horizontal force coefficient.                    |
| $d$   | axis depth.                                      |
| $d'$  | relative axis depth.                             |
| $D$   | cylinder diameter; general body dimension.       |
| $D'$  | width of cylinder at $y = 0$ .                   |
| $E$   | Young's modulus; averaging operator.             |
| $f$   | frequency.                                       |
| $f_x$ | wave force on a vertical pile.                   |
| $F$   | fluid force on a body.                           |
| $F_b$ | buoyancy force.                                  |

|               |                                                |
|---------------|------------------------------------------------|
| $F_c$         | confidence factor.                             |
| $F_d$         | diffraction force.                             |
| $F_k$         | Froude-Krylov force.                           |
| $F_r$         | radiation force.                               |
| $F_s$         | scattering force.                              |
| $F_x$         | horizontal wave force on a cylinder.           |
| $F_y$         | vertical wave force on a cylinder.             |
| $F_z$         | horizontal wave force on a long pipe.          |
| $F_D$         | drag force.                                    |
| $F_I$         | inertial force.                                |
| $F'_x$        | relative horizontal force on a cylinder.       |
| $F'_y$        | relative vertical force on a cylinder.         |
| $g$           | acceleration due to gravity.                   |
| $G$           | a constant.                                    |
| $G_y(\omega)$ | one-sided vertical force spectrum.             |
| $G(\eta)$     | window function.                               |
| $h$           | water depth.                                   |
| $I$           | moment of inertia.                             |
| $k$           | added mass coefficient.                        |
| $K$           | a constant.                                    |
| $K_1 - K_7$   | see page                                       |
| $l$           | length of cylinder or pipe; general dimension. |
| $L$           | wavelength.                                    |
| $L'$          | relative wavelength.                           |
| $M$           | mass.                                          |
| $M_a$         | added mass.                                    |
| $M_o$         | displaced mass.                                |
| $M_z$         | horizontal bending moment.                     |

|                    |                                                         |
|--------------------|---------------------------------------------------------|
| $M_y$              | vertical bending moment.                                |
| $M_T$              | total bending moment.                                   |
| $\underline{n}$    | unit vector normal to a pile.                           |
| $N_f$              | Froude number.                                          |
| $N_k$              | Keulegan-Carpenter number.                              |
| $N_I$              | Iversen's modulus.                                      |
| $N_S$              | Strouhal number.                                        |
| $p$                | hydrodynamic pressure.                                  |
| $p_o$              | hydrostatic pressure.                                   |
| $p(x)$             | probability density function.                           |
| $P$                | a constant.                                             |
| $P(x)$             | probability distribution function.                      |
| $Q$                | a constant.                                             |
| $R$                | ratio of r.m.s. buoyancy to r.m.s. inertia force.       |
| $R_e$              | Reynolds number.                                        |
| $R_x(\tau)$        | correlation function of the horizontal wave force.      |
| $R_y(\tau)$        | correlation function of the vertical wave force.        |
| $R_{xy}(\tau)$     | crosscorrelation of the horizontal and vertical forces. |
| $R_{\eta}(\tau)$   | correlation function of the wave elevation.             |
| $S_o$              | mean square wave elevation.                             |
| $S_b$              | buoyancy stiffness.                                     |
| $S_{\eta}(\omega)$ | two-sided wave elevation spectrum.                      |
| $S$                | spectral estimator.                                     |
| $t$                | time.                                                   |

|                              |                                                       |
|------------------------------|-------------------------------------------------------|
| $T$                          | wave period.                                          |
| $\underline{u}$              | water particle velocity vector.                       |
| $u_x$                        | horizontal water particle velocity.                   |
| $u_y$                        | vertical water particle velocity.                     |
| $\dot{u}_x$                  | horizontal water particle acceleration.               |
| $\dot{u}_y$                  | vertical water particle acceleration.                 |
| $U$                          | general velocity of a body.                           |
| $\dot{U}$                    | general acceleration of a body.                       |
| $V$                          | volume.                                               |
| $V_o$                        | initial immersed volume in static water.              |
| $\underline{W}$              | component of velocity vector normal to cylinder axis. |
| $x$                          | horizontal coordinate.                                |
| $y$                          | vertical coordinate.                                  |
| $y_i$                        | incident wave.                                        |
| $y_r$                        | reflected wave.                                       |
| $y_t$                        | transmitted wave.                                     |
| $z$                          | horizontal coordinate.                                |
| $\alpha, \alpha_1, \alpha_2$ | constants.                                            |
| $\beta, \beta_1, \beta_2$    | constants.                                            |
| $\gamma_1$                   | constant.                                             |
| $\delta$                     | phase coefficient.                                    |
| $\delta_i$                   | phase of a wavelet.                                   |
| $\delta(\omega)$             | dirac delta function.                                 |
| $\epsilon_r$                 | normalised error of a spectral estimate.              |
| $\eta$                       | wave elevation.                                       |
| $\eta_o$                     | mean change in water level.                           |
| $\eta_1$                     | $\eta(t)$ .                                           |

|            |                                                         |
|------------|---------------------------------------------------------|
| $\eta_2$   | $\eta(t + \tau)$                                        |
| $\theta$   | angle between regular waves and a long pipe.            |
| $\lambda$  | crestlength.                                            |
| $\mu$      | viscosity.                                              |
| $\mu_y$    | mean vertical force.                                    |
| $\nu$      | kinematic viscosity.                                    |
| $\nu_n$    | natural frequencies of a pipe.                          |
| $\pi$      | pi - mathematical constant.                             |
| $\rho$     | density of water.                                       |
| $\sigma$   | width of wave spectrum.                                 |
| $\sigma_x$ | mean square horizontal wave force.                      |
| $\sigma_y$ | mean square vertical wave force.                        |
| $\tau$     | time lag.                                               |
| $\phi$     | velocity potential; general function.                   |
| $\phi_i$   | incident velocity potential; imaginary part of $\phi$ . |
| $\phi_r$   | real part of $\phi$ ; phase change of a reflected wave. |
| $\phi_s$   | scattered velocity potential.                           |
| $\phi_t$   | phase change of a transmitted wave.                     |
| $\omega$   | angular frequency.                                      |
| $\omega_o$ | mean angular frequency of a spectrum.                   |

APPENDIX III

PUBLISHED PAPERS

A G DIXON, T S DURRANI  
and C A GREATED

Wave Force Statistics for Partially Submerged Horizontal  
Cylinders

#### SUMMARY

When regular waves are incident on fixed cylinders that are only partially submerged it is found that the vertical force can oscillate at twice the wave frequency because of interplay between inertial and buoyancy forces. When irregular waves are incident, secondary peaks appear in the vertical force spectrum at twice the mean wave frequency. This Paper presents preliminary results for a theoretical spectrum based on a force equation derived from Morison's equation by a few simple modifications. The wave elevation is assumed to be a Gaussian process and to have a Gaussian spectrum. Some expressions for other statistical parameters are also given.

#### 1. INTRODUCTION

The subject of wave force statistics has received a great deal of attention over the past fifteen years or so. The basic problem can be stated as follows. Given the statistical properties of the sea surface, what are the corresponding properties acting on objects placed in the sea? Borgman (1972) reviews and summarises statistical predictions for forces on piles using the famous Morison equation, together with linear wave theory. Expressions for various statistical parameters such as probability densities, moments, covariances and spectra are given, with special emphasis on narrow band spectra. A discussion of the estimation of force coefficients occurring in Morison's equation using statistical methods is also given.

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Papers by Tung (1975a, 1975b, 1976) extend this work by considering the effects of surface fluctuations, that is, those effects deriving from the fact that parts of the pile are sometimes submerged, sometimes not. The Heaviside unit function is employed to rederive many of the statistical properties.

These surface fluctuations are of particular importance for partially submerged fixed horizontal cylinders. Dixon et al (1978) show that, in this case, Morison's equation is not at all suitable for predicting wave forces when regular waves are incident. This is particularly so in the case of vertical force. They show how it is possible to modify the equation to give much better results. The present Paper presents results for the spectra of the forces using the modified equation. Also the cross correlation between vertical and horizontal forces is discussed.

The method is to assume that the wave elevation is a Gaussian process and can be described using a Gaussian spectrum (two entirely different assumptions). Some approximations are made to allow the force equations to be written as simple functions of wave elevation. The force correlation function can then be derived and hence the spectrum. Expressions for the mean and mean square forces are also given. Experimental results are then compared to the theory.

## 2. THE FORCE EQUATIONS

Morison's equation states that the horizontal force per unit length acting on a pile is the sum of an inertial and a drag term, viz;

$$F_h = C_M \rho V \frac{\partial U_h}{\partial t} + \frac{1}{2} C_D \rho A |U_h| U_h \quad (1)$$

where  $F_h$  = horizontal force per unit length acting on a vertical pile of volume  $V$  per unit length, and  $U_h$  = horizontal component of the fluid velocity in the region of the pile and is a function of depth.

Eqn. 1 can be used to predict vertical and horizontal forces on a horizontal cylinder providing it is always submerged. For vertical forces the vertical component of the fluid velocity,  $U_v$ , should be used.

Fig. 1(a) shows the vertical force acting on a fixed horizontal cylinder of diameter 0.1m and length 0.295m, at an

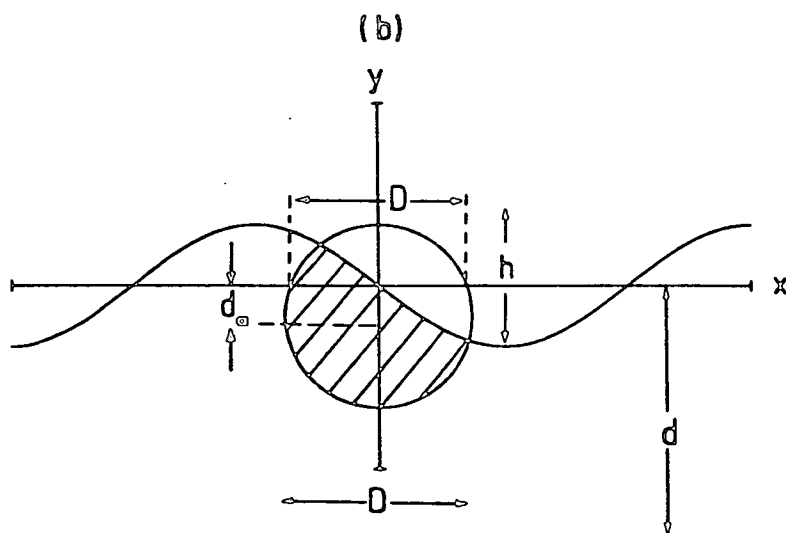
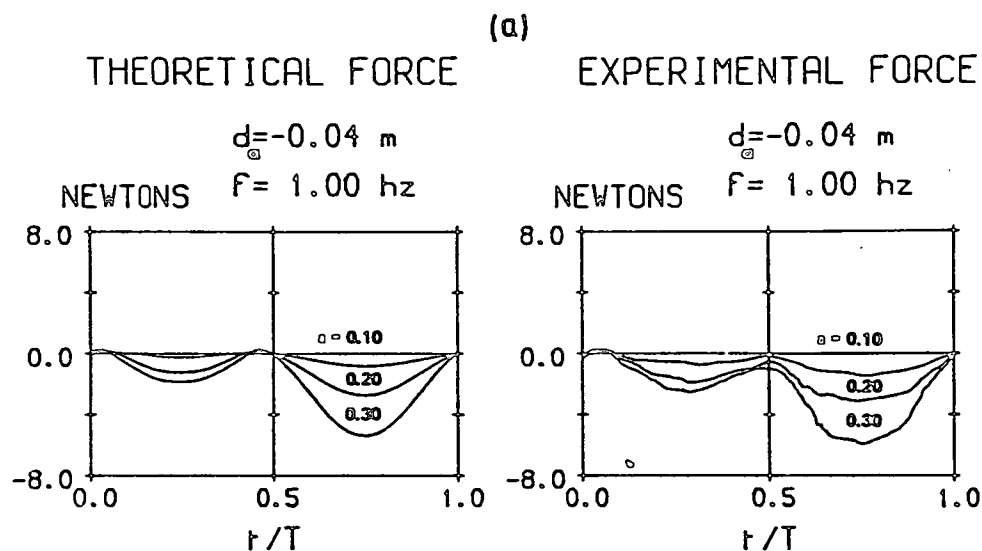


FIG. 1 Schematic Diagram

axis depth w.r.t. the still waterline of 0.04 m. The waves are regular of period 1 second, wavelength 1.54m. It is evident that Morison's equation will not predict these curves as it stands. The modifications are described fully by Dixon et al (1978).

The main factors affecting the vertical force are the varying buoyancy and the varying volume of water acting on the cylinder. We introduce three alterations to Eqn. 1;

1. Discard the drag term since it is small compared with the buoyancy and inertial forces. Garrison and Rao (1971 ) and Nath and Harleman (1970) discuss the relative magnitude of the drag and inertial terms;
2. Introduce a varying volume  $V(t)$  instead of a fixed volume. This will be the total volume of water displaced at any instant;
3. Add a varying buoyancy term,  $\rho g V(t)$ .

Since we are only interested in the forces caused by the waves, we subtract the initial buoyancy force,  $\rho g V_0$ , where  $V_0$  is the volume of water displaced in still water, and  $g$  is the acceleration due to gravity. The force equation can now be written as,

$$F_v = C_M \rho V(t) \frac{\partial u}{\partial t} + \rho g (V(t) - V_0) \quad (2)$$

In order that we can write  $V(t)$  as a simple function we restrict our attention to waves which neither cover the cylinder nor uncover it at any time. If we assume that the waterline is always horizontal across the cylinder and oscillates sinusoidally we can write the wave elevation ( $\eta$ ) as

$$\eta(x,t) = \eta(0,t) = \eta(t) = \frac{h}{2} \sin \frac{2\pi t}{T}$$

The x-axis is assumed to be along the still water line and y is the positive upwards.

It is then easy to calculate  $V(t)$  as a function of axis depth,  $d_a$ ; cylinder diameter,  $D$ ; cylinder length,  $\ell$ ; and  $\eta(t)$ .

We will give an expression here for  $V(t)$  which is approximately correct for small wave amplitudes. Referring to Fig. 1b,  $V(t)$  is given as the sum of the initial displaced volume plus a small fluctuating part, so that

$$V(t) = V_0 + D' \ell \frac{h}{2} \sin \frac{2\pi t}{T}$$

where  $D' = 2(D^2/4 - d_a^2)^{1/2}$  is the width of the cylinder at  $y = 0$ . Hence,

$$F_v = C_M \rho (V_0 + D' \ell \frac{h}{2} \sin \frac{2\pi t}{T}) \frac{\partial U_v}{\partial t} + \rho g D' \ell \frac{h}{2} \sin \frac{2\pi t}{T}$$

Linear wave theory gives the vertical water acceleration as

$$\frac{\partial U_v}{\partial t} = A' \frac{h}{2} \sin \frac{2\pi t}{T}$$

where

$$A' = - \frac{2\pi g \sinh 2\pi(d_a + d)/L}{L \sinh 2\pi d_a/L}$$

For deep water we can write,

$$A' = - \frac{2\pi g}{L} \exp \frac{2\pi d_a}{L}$$

Combining these expressions gives

$$F_v = A \eta(t) + B \eta^2(t) \quad (3)$$

which is a quadratic in  $\eta(t)$ , where

$$A = -C_M \rho g V_0 \frac{2\pi}{L} \exp \frac{2\pi d_a}{L} + \rho g D \ell ; \quad B = -C_M \rho g D \ell \frac{2\pi}{L} \exp \frac{2\pi d_a}{L}$$

For the horizontal force we can write a similar expression except that we have no buoyancy term and the acceleration is out of phase by  $90^\circ$  with the wave elevation.

This gives,

$$F_h = C \frac{\partial \eta(t)}{\partial t} + D \eta(t) \frac{\partial d(t)}{\partial t} \quad (4)$$

where,

$$C = -C_M \rho g V_o \frac{2\pi}{T} \exp \frac{2\pi d_a}{L}$$

and,

$$D = -C_M \rho g D_l \frac{2\pi}{T} \exp \frac{2\pi d_a}{L}$$

Eqns. 3 and 4 form the basis of the rest of this Paper.

Dixon et al (1978) give a more exact expression for  $F_v$ . This was used to compute the theoretical curves in Fig. 1a. It is too complicated to use directly in the prediction of force spectra.

### 3. IRREGULAR WAVES AND THE FORCE SPECTRA

We will now assume that the waves are irregular and that the wave elevation is a Gaussian process. In addition we will assume that the wave elevation spectrum is also Gaussian. More precisely, we assume that the autocorrelation function for  $\eta(t)$  is given by

$$R_\eta(\tau) = S_o \exp \left( -\frac{\sigma^2 \tau^2}{2} \right) \cos \omega_o \tau$$

where  $\tau$  = lag interval;  $S_o$  mean square elevation;  $\sigma$  and  $\omega_o$  define the spectral width and the mean frequency respectively.

The corresponding two-sided spectrum  $S_\eta(\omega)$  is defined by,  $S_\eta(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_\eta(\tau) e^{-i\omega\tau} d\tau$  which, for  $R_\eta(\tau)$  given above, yields,

$$S_\eta(\omega) = S_o (8\pi\sigma^2)^{-\frac{1}{2}} \left\{ \exp \frac{-(\omega-\omega_o)^2}{2\sigma^2} + \exp \frac{-(\omega+\omega_o)^2}{2\sigma^2} \right\} \quad (5)$$

The one-sided spectrum  $G_\eta(\omega)$  is defined, for  $\omega \geq 0$ , as  $G_\eta(\omega) = 2 S_\eta(\omega)$ . If the spectrum is fairly narrow, we can neglect the second term in Eqn. 5 and write

$$G_{\eta}(\omega) = S_0 (2\pi\sigma^2)^{\frac{1}{2}} \exp \left\{ \frac{-(\omega - \omega_0)^2}{2\sigma^2} \right\} \quad (6)$$

### 3.1 Correlation function and spectrum of the vertical force

In the following derivation we are assuming that, although there is a finite probability of the waves covering or uncovering the cylinder, any effects are small. The correlation function is defined as

$$R_y(\tau) = \langle F(t) F(t+\tau) \rangle \quad (7)$$

where the brackets denote the averaging operator. Strictly speaking this should be an ensemble average but as usual we will assume ergodicity and take time average over record lengths. Using the notation,  $\eta(t) = \eta_1$ ,  $v(t + \tau) = \eta_2$ , together with the force formula, Eqn. 3, gives

$$\begin{aligned} R_y(\tau) &= \langle (A\eta_1 + B\eta_1^2)(A\eta_2 + B\eta_2^2) \rangle \\ &= A^2 \langle \eta_1 \eta_2 \rangle + AB \langle \eta_2 \eta_1^2 \rangle + AB \langle \eta_1 \eta_2^2 \rangle + B^2 \langle \eta_1^2 \eta_2^2 \rangle \end{aligned}$$

Since we are assuming that  $\eta(t)$  is a Gaussian process (See Borgman, 1972, p.149), the third order moments are zero and we can expand the fourth order moment as

$$\begin{aligned} \langle \eta_1^2 \eta_2^2 \rangle &= \langle \eta_1^2 \rangle \langle \eta_2^2 \rangle + 2 \langle \eta_1 \eta_2 \rangle^2 \\ &= S_0^2 + 2R_{\eta}^2(\tau) \end{aligned}$$

This gives

$$R_y(\tau) = A^2 R_{\eta}(\tau) + B^2 S_0^2 + 2B^2 R_{\eta}^2(\tau) \quad (8)$$

Substituting for  $R_{\eta}(\tau)$  we have

$$R_y(\tau) = A^2 S_o \exp\left(-\frac{\sigma^2 \tau^2}{2}\right) \cos \omega_o \tau + B^2 S_o^2 e^{-\sigma^2 \tau^2} (\cos 2\omega_o \tau + 1) + B^2 S_o^2$$

The one-sided force spectrum is then derived from

$$G_y(\omega) = \frac{2}{\pi} \int_0^\infty R_y(\tau) \cos \omega \tau d\tau$$

Neglecting terms centred on negative frequencies this results in

$$G_y(\omega) = A^2 S_o (2\pi\sigma^2)^{-\frac{1}{2}} \exp \frac{-(\omega - \omega_o)^2}{2\sigma^2} + B^2 S_o^2 (4\pi\sigma^2)^{-\frac{1}{2}} \exp \frac{-(\omega - 2\omega_o)^2}{4\sigma^2} + \\ + B^2 S_o^2 (4\pi\sigma^2)^{-\frac{1}{2}} \exp \frac{-\omega^2}{4\sigma^2} + 2B^2 S_o^2 \delta(\omega) \quad (9)$$

The net result, therefore, consists of four terms; an impulse term due to the fact that  $F_y(t)$  does not have zero mean and three terms with peaks at zero,  $\omega_o$  and  $2\omega_o$  respectively. The two sided peaks have a 'width'  $\sqrt{2}$  times that of the central peak. The spectrum could be normalised by dividing by  $R_y(0)$ , so that it has unit area.

### 3.2 Some useful averages

We can predict the mean, the mean square and the variance for the vertical force. The mean is given by

$$\mu_y = \langle F_y(t) \rangle = \langle A\eta(t) + B\eta^2(t) \rangle \\ = BS_o \quad (10)$$

The mean square is given by

$$\langle F_y^2(t) \rangle = R_y(0) \\ = A^2 S_o + 3B^2 S_o^2 \quad (11)$$

Hence the standard deviation  $\sigma_y^2$  is given by

$$\sigma_y^2 = \text{mean square} - (\text{mean})^2 = A^2 S_o + 2B^2 S_o^2 \quad (12)$$

### 3.3 Correlation function of the Horizontal Spectrum

The correlation function of the horizontal force is

$$R_x(\tau) = \langle (B\eta_1' + C\eta_1\eta_1')(B\eta_2' + C\eta_2\eta_1') \rangle \quad (13)$$

where the prime denotes the partial derivative w.r.t. time. Following along similar lines to the derivation for the vertical force correlation, and using the following useful facts,

$$\begin{aligned} \langle \eta_1\eta_2' \rangle &= \frac{\partial R(\tau)}{\partial \tau} = -\langle \eta_1'\eta_2 \rangle \\ \langle \eta_1'\eta_2' \rangle &= -\frac{\partial^2 R(\tau)}{\partial \tau^2} ; \quad \langle \eta_1\eta_1' \rangle = 0 \end{aligned}$$

we arrive at

$$\begin{aligned} R_x(\tau) &= A^2 S_0 \exp(-\sigma^2 \tau^2) \{ (\omega_0^2 + \sigma^2 - \sigma^4 \tau^2) \cos \omega_0 \tau - 2\sigma^2 \omega_0 \tau \sin \omega_0 \tau \} + \\ &+ B^2 S_0^2 \exp(-\frac{\sigma^2 \tau^2}{2}) \{ (\omega_0^2 + \sigma^2 - \sigma^4 \tau^2) \cos 2\omega_0 \tau - 2\sigma^2 \omega_0 \tau \sin 2\omega_0 \tau + \frac{(\sigma^2 - 2\sigma^4 \tau^2)}{2} \} \end{aligned} \quad (14)$$

The spectrum could be derived as usual. However, experimental data do not show any significant peaks in the horizontal force spectrum other than the central one. So one could simply write an equation for the spectrum which had the form of a Gaussian such that the area underneath had the value of the mean square force,  $R_x(0)$ .

The mean of the horizontal force is zero and the mean square is given by

$$R_x(0) = S_0 (A^2 + B^2 S_0) (\omega_0^2 + \sigma^2) \quad (15)$$

### 3.4 The crosscorrelation between horizontal and vertical forces

The crosscorrelation is defined as

$$R_{xy}(\tau) = \langle F_h(t) F_v(t+\tau) \rangle$$

Substituting in the formula for  $F_h$  and  $F_v$  and using the properties of average stated above gives

$$R_{xy}(\tau) = AC \frac{dR_{\eta}}{d\tau} + 2BDR_{\eta} \frac{dR_{\eta}}{d\tau} \quad (16)$$

and we can use  $R_{xy}(-\tau) = R_{yx}(\tau)$  if we require the other crosscorrelation. Using the expression for  $R_{\eta}$  results in

$$R_{xy}(\tau) = -ACS_0 (\omega_0 \sin \omega_0 \tau + \tau \sigma^2 \cos \omega_0 \tau) \exp\left(-\frac{\sigma^2 \tau^2}{2}\right) + \\ -BDS_0^2 \{\omega_0 \sin 2\omega_0 \tau + 2\sigma^2 (\cos 2\omega_0 \tau + 1)\} \exp(-\sigma^2 \tau^2) \quad (17)$$

#### 4. EXPERIMENTAL EQUIPMENT AND RESULTS

The experimental results in this Paper were taken using equipment designed and built by the Wave Power Research team in the Department of Engineering at Edinburgh University.

The wave tank was 0.3m wide and 9.14m long. The cylinder was mounted on a rig in a fixed position, but its depth of submersion could be altered. Strain gauges were used to measure the torque about two points. From these signals the vertical and horizontal forces were calculated and calibrated electronically. A system of pulleys and known weights was used for the calibration. The signals were recorded on magnetic tape and were digitised for later analysis by computer. The wave elevation was also recorded from a float wave gauge. Waves were produced using a flat plate absorbing wavemaker. To create a random sea with an approximately Gaussian spectrum, twenty oscillators whose frequency ranged in 19 logarithmic steps from 0.5 Hz to 2.5 Hz were mixed in random phase and the amplitude of each oscillator was set following a Gaussian distribution centred on 1.5 Hz. Two spectra were used, one wider than the other.

For each spectrum the vertical and horizontal forces were recorded for a series of different axis depths. The wave elevation was recorded with the cylinder out of the water. Estimates for the spectral density of each force and the wave elevation were calculated using standard Fast Fourier Transform techniques. The crosscorrelation between the two forces was also estimated and then the normalised covariance was calculated.

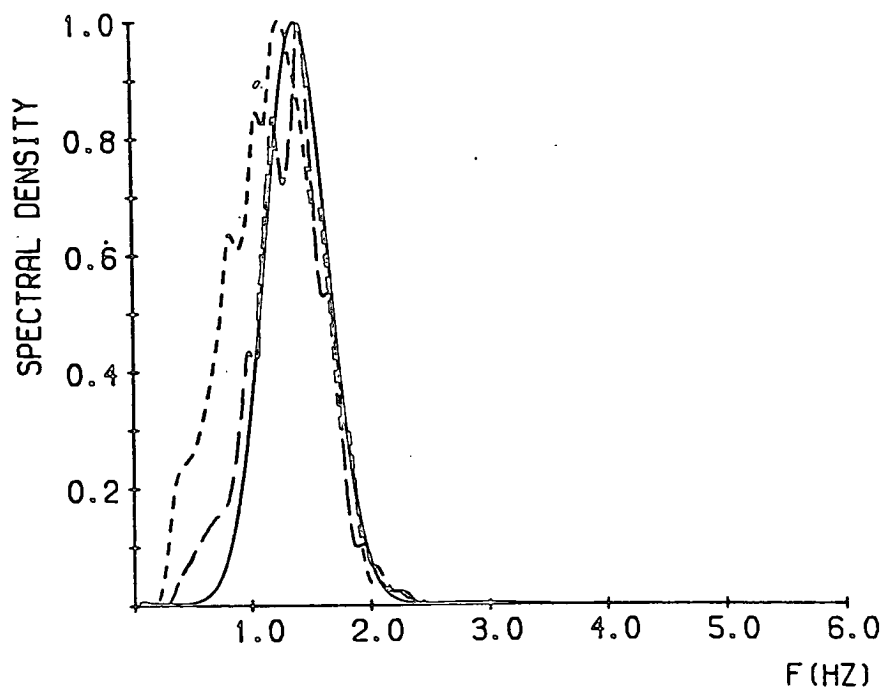


FIG. 2

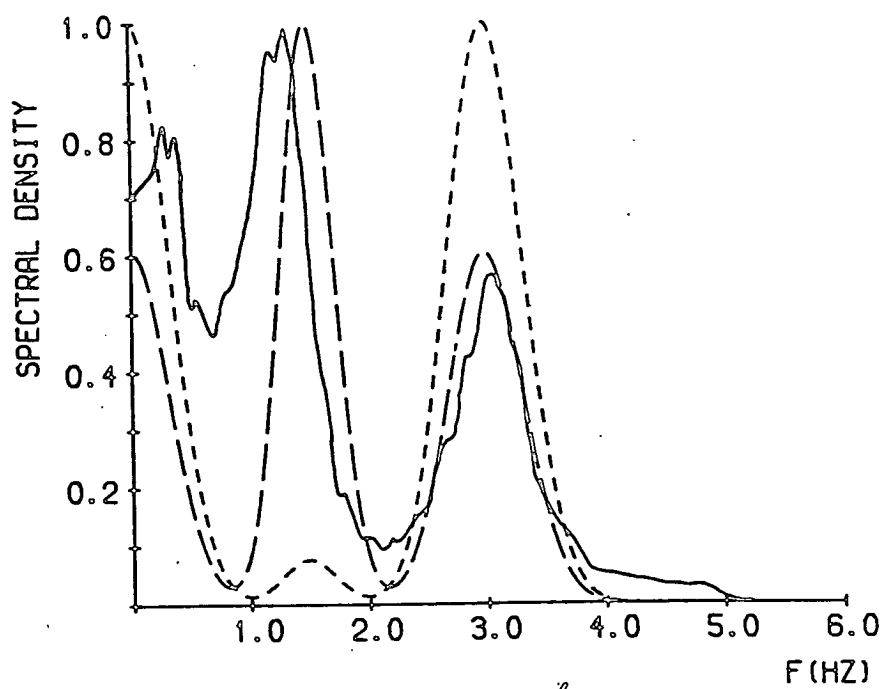


FIG. 3

The normalised covariance,  $\rho_{xy}$  is defined by

$$R_{xy} = (R_{xy} - \mu_x \mu_y) / \sigma_x \sigma_y$$

where  $\mu_x$ ,  $\mu_y$  and  $\sigma_x$ ,  $\sigma_y$  are the relevant means and standard deviations.

## 5. DISCUSSION

The ratio of each of the two side peaks to the central one in Eqn. 9 is given approximately by  $S_0 B^2 / \sqrt{2} A^2$ . The three parameters  $A$ ,  $B$  and  $S_0$  enable us to determine whether or not there will be significant forces acting at twice the average wave frequency. As the ratio depends on the squares of the coefficients and the mean square wave elevation, it is very sensitive to slight changes in them, so they have to be determined accurately. The approximations leading to the force spectrum given in Eqn. 9 are very crude so an exact matching with experimental results is not to be expected. The force equation is most accurate for zero axis depth. For this depth both theory and experiment show that the central peak is dominant (Fig. 2). In order to see large side peaks in the experimental force spectrum one has to allow the waves to be large enough that they swamp the cylinder at times. The appearance of the peaks is also sensitive to axis depth, only appearing when  $d_a/D \approx -\frac{1}{2}$ . In their present form the equations in this Paper do not cover this situation. Fig. 3a shows an experimental spectrum for  $d_a/D = -0.30$ . One can put the relevant numbers into Eqn. 9 to see what happens (Fig. 3b). The central peak has disappeared because for this combination of axis depth and mean wave elevation the coefficient,  $A$ , is nearly zero. However, merely by changing  $d_a/D$  in the theoretical equation to  $-0.33$  we have a much better fit (Fig. 3c). Work is continuing to improve the equations so they are not so sensitive to the input parameters.

The cross correlation is useful for showing the relative phase of the horizontal and vertical forces. For large axis depths the vertical force is mostly inertial and the horizontal force leads by  $90^\circ$ . This can be seen in Fig. 4a both experimentally and theoretically (Eqn. 17). For small axis depths, buoyancy dominates and then the horizontal force lags behind by  $90^\circ$  because vertical inertial forces and buoyancy forces are in antiphase. Again this can be seen in Fig. 4b.

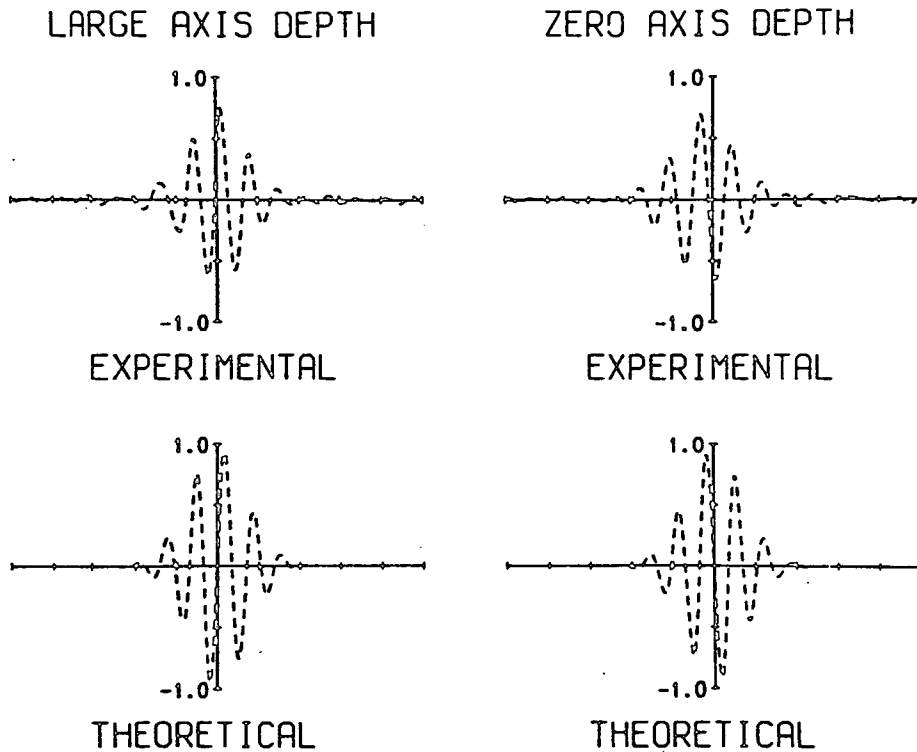


FIG. 4 Crosscorrelation between Horizontal and Vertical Forces  $-5 < \tau < 5$  (secs)

#### 6. ACKNOWLEDGEMENTS

The authors would like to thank Mr. Steven Salter of the Department of Mechanical Engineering, Edinburgh University, for the use of experimental equipment and to the Department of Energy who financed the building of it. A.G. Dixon acknowledges the receipt of an S.R.C. studentship covering the period of this work.

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# JOURNAL OF THE WATERWAY PORT COASTAL AND OCEAN DIVISION

## WAVE FORCES ON PARTIALLY SUBMERGED CYLINDERS

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### INTRODUCTION

The subject of waves and wave forces has received a great deal of attention over the past 30 yr from both experimental and theoretical points of view. A great deal of this has been due to the rapidly expanding field of offshore engineering. Hogben (12) reviews the subject. The particular topic treated here, wave forces acting on a partially submerged fixed cylinder, arose from a series of experiments undertaken by the wave power research group in the University of Edinburgh's Department of Mechanical Engineering. They had carried out extensive measurements of horizontal and vertical forces acting on submerged and partially submerged cylinders when a train of two-dimensional regular waves is incident normally. Their results showed some interesting phenomena in the case of vertical (heave) forces. Under certain circumstances the heave force showed strong tendencies towards acting at twice the wave frequency and also was often negative (downwards) for the entire wave cycle. These results were difficult to fit to any known theory for wave forces and seemed to imply a negative value of the inertial force coefficient,  $C_M$ . From a physical point of view the effects seemed to arise from an interplay between buoyancy and inertial forces.

The purpose of the present work is to find an equation to predict the vertical forces in these situations and to compare the theoretical expression with some experimental results taken with the same equipment as used by the wave energy group. This is done by modifying existing equations: the results are fairly

Note.—Discussion open until April 1, 1980. To extend the closing date one month, a written request must be filed with the Editor of Technical Publications, ASCE. This paper is part of the copyrighted Journal of the Waterway, Port, Coastal and Ocean Division, Proceedings of the American Society of Civil Engineers, Vol. 105, No. WW4, November, 1979. Manuscript was submitted for review for possible publication on June 2, 1978.

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satisfactory despite the simplifying assumptions introduced. It is shown that a constant positive value of  $C_M = 2.0$  may be used with good results.

### THEORY OF WAVE FORCES

Wave force theories can be divided into two groups. The first being the two-dimensional potential theory group. The papers by Havelock, Dean, Ursell, Ogilvie, and others cover this field (see Refs. 1, 2, 8, 10, 18, and 19). They are inconvenient in two respects. Firstly, their use implies all the assumptions of potential theory, i.e., inviscid, irrotational two-dimensional flow. Secondly, the solutions are series whose coefficients have to be found by solving an infinite number of equations in an infinite number of unknowns. Even approximating with only a few terms requires much tedious numerical work.

The second group proposes force equations that are simpler but have less mathematical rigor behind them. They employ empirical coefficients to fit the equations to the data. The most widely used equation is one put forward by Morison, et al. (15) in 1950. Many other workers have since used this as a theoretical basis (3,11,17,20). The equation was originally proposed for the horizontal force per unit length acting on a vertical pile placed in the waves. It assumes that the diameter of the pile is small so that the water particle velocity and acceleration is essentially constant across the region of the diameter if the pile were not there. Then the force is calculated as the sum of an inertial (accelerative) and a drag (velocity dependent) term with corresponding coefficients, i.e.,

$$F_h = C_M \rho V \frac{\partial U_h}{\partial t} + \frac{1}{2} C_D \rho A |U_h| U_h \quad \dots \dots \dots (1)$$

in which  $F_h$  = horizontal force per unit length acting on a vertical pile of volume  $V$  per unit length;  $\rho$  = the fluid mass density;  $C_M$  = coefficient of inertia;  $U_h$  = horizontal component of fluid velocity in the region of the pile and is a function of depth;  $C_D$  = drag coefficient;  $A$  = the projected area per unit length of the pile normal to the waves; and  $t$  = time. There are several ways in which the coefficients can be determined. Morison chose to fit the equation to experimental data at points where one of the terms is known to be zero, i.e., when  $U_h$  or  $\partial U_h / \partial t$  is zero. Alternatively, Keulegan and Carpenter (13) show how to calculate the coefficients using Fourier series analysis. This method gives average values for the coefficients over the wave cycle. It is well known however that the coefficients vary during one wave period. It is also possible to use a least squares fitting technique to find  $C_M$  and  $C_D$ . This also yields average values for  $C_M$  and  $C_D$ . Borgman (3) and Chakrabarti (7) show that the force equation should be modified slightly if it is to be used for inclined or horizontal cylinders. For horizontal cylinders the horizontal force is given by

$$F_h = C_M \rho V \frac{\partial U_h}{\partial t} + \frac{1}{2} C_D \rho A |W| U_h \quad \dots \dots \dots (2)$$

in which  $W$  = the normal fluid particle velocity. The vertical force is given by

$$F_v = C_M \rho V \frac{\partial U_v}{\partial t} + \frac{1}{2} C_D \rho A |W| U_v \dots \dots \dots (3)$$

These equations apply only to cylinders that are always totally submerged. However there are many cases in offshore engineering where horizontal members are placed in the so-called "splash zone." Most attention in this region has been focussed on slamming effects and breaking waves. These topics, although important, are not considered here. Wiegel (21) examines the use of Eq. 1 with linear wave theory.

Modifications.—We will now modify Eq. 3 in several ways to try and arrive at an expression that will predict the phenomena described in the introduction, at the same time trying to give plausible physical arguments for doing so.

Firstly we will discard the drag term involving  $C_D$ . Garrison and Rao (8) have concluded that for large values of the object size to wave amplitude ratio then viscous effects are negligible. In the experiments in this paper this value had a minimum of two and was more usually around 10.

The greatest difficulty lies in the fact that in order to use Eq. 3, the cylinder should always be totally submerged. An attempt to account for the cylinder being only partly immersed can be made by using a varying volume in the inertial term, i.e., the inertial force depends on the volume of liquid displaced by the cylinder, which is a function of time. So now we have for the vertical force,  $F_v$

$$F_v = C_M \rho V(t) \frac{\partial U_v}{\partial t} \dots \dots \dots (4)$$

Next we must include a term that describes the varying buoyancy force exerted on the cylinder by the varying water line. This is given by the weight of fluid displaced by the cylinder and acts upwards, i.e.,

$$F_b = \rho V(t) g \dots \dots \dots (5)$$

in which  $F_b$  = buoyancy force; and  $g$  = the acceleration due to gravity and we have neglected centrifugal forces (see the following). Since we require only the force exerted by the passage of the wave, we must subtract the initial buoyancy in still water; thus

$$F_b = \rho g [V(t) - V_o] \dots \dots \dots (6)$$

in which  $V_o$  = the initial immersed volume.

So now for the total force,  $F$ , we have adding Eqs. 4 and 6

$$F = C_M \rho V(t) \frac{\partial U_v}{\partial t} + \rho g [V(t) - V_o]; \quad F = F_{inertial} + F_{buoyancy} \dots \dots \dots (7)$$

If we consider the case where the water line in the vertical plane passing through the cylinder axis is given by  $\eta(o,t) = a \sin (2\pi t/T)$ , in which  $a$  = the wave amplitude, (and  $T$  = the wave period, then the inertial term in Eq. 3 looks like Fig. 1(a). Introducing a varying volume decreases the force in the trough and the inertial force now looks like Fig. 1(b). The buoyancy term has the shape given in Fig. 1(c). We are, of course, neglecting the fact that the waves are reflected by the cylinder and the water elevation is changed from its

undisturbed state, being of smaller amplitude on one side than the other where an antinode forms.

**Simple Case.**—We now show through a simple case how the force could become totally negative and act at twice the wave frequency.

Consider the square block of side  $b$  and volume  $V_T$  (Fig. 2). If we assume that the water line is always horizontal and that the wave elevation is given by  $\eta(o,t) = (b/2) \sin (2\pi t/T)$ , then  $V(t) = (V_T/2) [1 + \sin (2\pi t/T)]$ . We also have  $V_o = V_T/2$ .

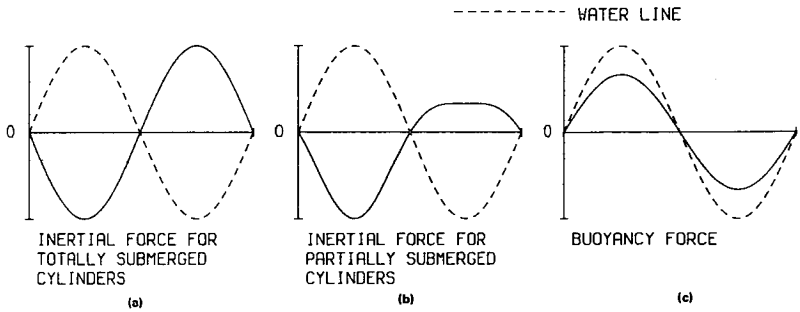


FIG. 1.—Vertical Force Terms over One Wave Period

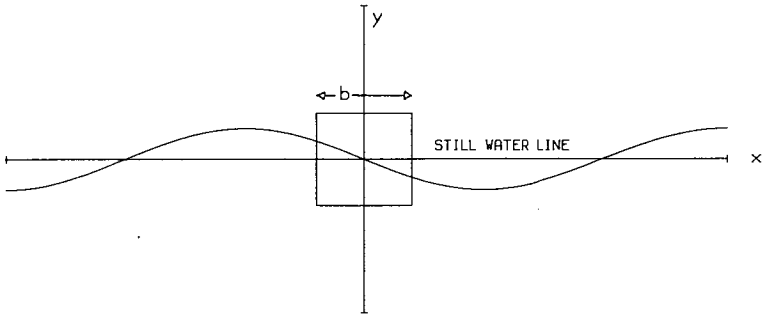


FIG. 2.—Definition Sketch for Simple Case

Using linear wave theory we can write [see Wiegel (21), chapter 2]

$$\frac{\partial U_v}{\partial t} = -A \sin \frac{2\pi t}{T} \dots \dots \dots (8)$$

in which  $A$  = constant in time. Thus

$$F = B \left( -\sin \frac{2\pi t}{T} \right) \left( 1 + \sin \frac{2\pi t}{T} \right) + C \sin \frac{2\pi t}{T} \dots \dots \dots (9)$$

in which  $B$  and  $C$  = constants (in time). If we have chosen the parameters such that  $B \approx C$ , (Fig. 3), then  $F = B [-\sin^2 (2\pi t/T)]$ . Since  $B$  is always positive then the resultant force is as shown in Fig. 3 and has the required

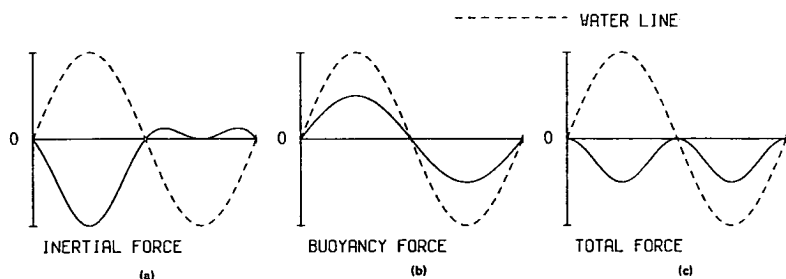


FIG. 3.—Component Terms and Total Force in Modified Equation (Eq. 5) over One Wave Period

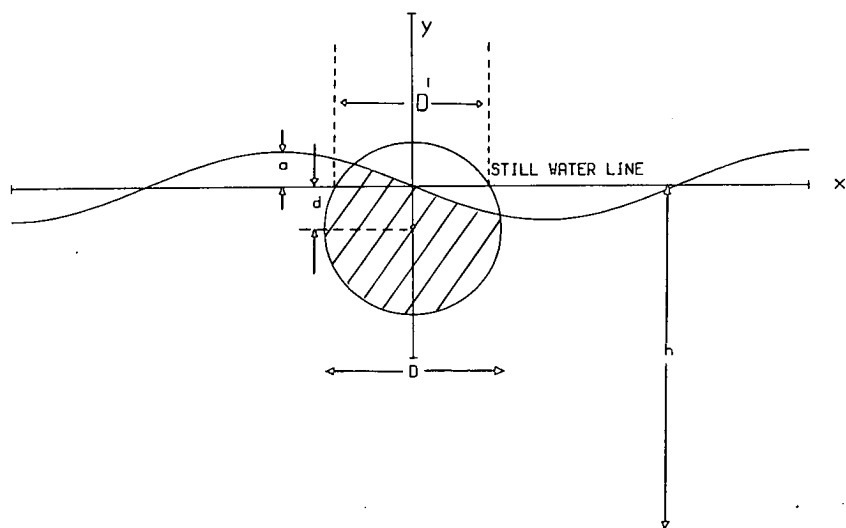


FIG. 4.—Definition Sketch for Cylinder

properties. The condition that  $B \approx C$  is not difficult to satisfy, as will be seen later. Obviously the effects introduced by the sharp edges have been neglected.

**Circular Cylinder.**—We now calculate the force on a circular cylinder using linear wave theory.

First consider the case where the water line stays within the cylinder limits. Referring to Fig. 4 the displaced volume,  $V(t)$ , is given by the area of the crosshatched section times the cylinder length ( $l$ ). This area can be approximated by the area under the line  $\eta(x, t) = \eta(o, t) = \eta(t) = a \sin(2\pi t/T)$  providing the wave steepness is small, which is in accordance with the assumption of a small wave amplitude to wavelength ratio,  $a/L$ .

This area is given by a simple integration as a function of time by

$$A(t) = \frac{D^2}{8} \left( \pi + \frac{4}{D} [-d + \eta(t)] \left\{ 1 - \frac{4}{D^2} [-d + \eta(t)]^2 \right\}^{1/2} \right)$$

$$+ 2 \sin^{-1} \frac{2[-d + \eta(t)]}{D} \Bigg) \dots \dots \dots (10)$$

in which  $d$  = the axis depth and  $D$  = the cylinder diameter. The initial area is then given by  $A_o = A(o)$ ; thus

$$A_o = \frac{D^2}{8} \left[ \pi + \frac{4}{D} (-d) \left( 1 - \frac{4d^2}{D^2} \right)^{1/2} + 2 \sin^{-1} \frac{2(-d)}{D} \right] \dots \dots \dots (11)$$

To keep  $A(t)$  real we must have  $4[-d + \eta(t)]^2 \leq D^2$ . The vertical water particle acceleration is given in linear wave theory (according to Eq. 8)

$$\frac{\partial U_v}{\partial t} = -\frac{4\pi^2 a}{T^2} \frac{\sinh \left[ \frac{2\pi(y+h)}{L} \right]}{\sinh \left( \frac{2\pi h}{L} \right)} \sin \frac{2\pi t}{T} \dots \dots \dots (12)$$

in which  $h$  = the water depth; and we use  $y = d$  as the axis depth (see Fig. 4). If  $2\pi(y+h)/L \gg 1$  and  $2\pi h/L \gg 1$  then we can approximate the  $\sinh$  terms by exponentials and use the deep water dispersion relation,  $L = gT^2/(2\pi)$ , giving

$$\frac{\partial U_v}{\partial t} = -\frac{2\pi g}{L} \exp \left( \frac{2\pi d}{L} \right) \eta(t) \dots \dots \dots (13)$$

Now we have for the total vertical force on the cylinder, substituting for  $\partial U_v/\partial t$ ,  $V(t)$  and  $V_o$

$$\begin{aligned} F = & -C_M \frac{2\pi g}{L} \exp \frac{2\pi d}{L} \eta(t) \frac{\rho l D^2}{8} \left( \pi + \frac{4}{D} [-d + \eta(t)] \left\{ 1 - \frac{4}{D^2} [-d \right. \right. \\ & + \eta(t)]^2 \Bigg\}^{1/2} + 2 \sin^{-1} \frac{2[-d + \eta(t)]}{D} \Bigg) + \rho g \frac{ID^2}{8} \left( \frac{4}{D} [-d \right. \\ & + \eta(t)] \left\{ 1 - \frac{4}{D^2} [-d + \eta(t)]^2 \right\}^{1/2} + \sin^{-1} 2 \frac{-d + \eta(t)}{D} \\ & \left. - \frac{4}{D} (-d) \left( 1 - \frac{4d^2}{D^2} \right)^{1/2} - 2 \sin^{-1} \frac{-2d}{D} \right) \dots \dots \dots (14) \end{aligned}$$

Eq. 14 can be transformed into a more useful form by defining some dimensionless parameters as follows: (1) Relative force  $F' = F/[\rho g (\pi D^2 l/4)]$ ; (2) relative wave elevation  $\eta'(t) = \eta(t)/D$ ; (3) relative wave amplitude  $a' = a/D$ ; (4) relative wavelength  $L' = L/D$ ; and (5) relative axis depth  $d' = d/D$ .

The dimensionless variable  $F'$  measures the ratio of the wave force to the weight of water displaced by a totally submerged cylinder in still water. Thus Eq. 14 now reads

$$F' = -C_M \frac{\eta'(t)}{L'} \exp \frac{2\pi d'}{L'} \left( \pi + 4 [-d' + \eta'(t)] \left\{ 1 - 4 [-d' + \eta'(t)]^2 \right\}^{1/2} \right.$$

$$\begin{aligned}
 &+ 2 \sin^{-1} 2 [-d' - \eta'(t)] \Big) + \frac{1}{2\pi} \Big( 4 [-d' + \eta'(t)] \{1 \\
 &- 4 [-d' + \eta'(t)]^2\}^{1/2} + 2 \sin^{-1} 2 [-d' + \eta'(t)] \\
 &+ 4d' (1 - 4d'^2)^{1/2} - 2 \sin^{-1} (-2d') \Big) \dots \dots \dots (15)
 \end{aligned}$$

The main parameters in Eq. 15 are  $a'$ ,  $L'$ , and  $d'$ . Various sets of results using different values of these parameters were calculated from Eq. 15 and are shown in Figs. 5 and 6. The phase is such that the  $x$  origin is a positive going zero crossing of the wave elevation at the cylinder. The parameters are chosen such that the water surface stays within the cylinder limits, i.e.,

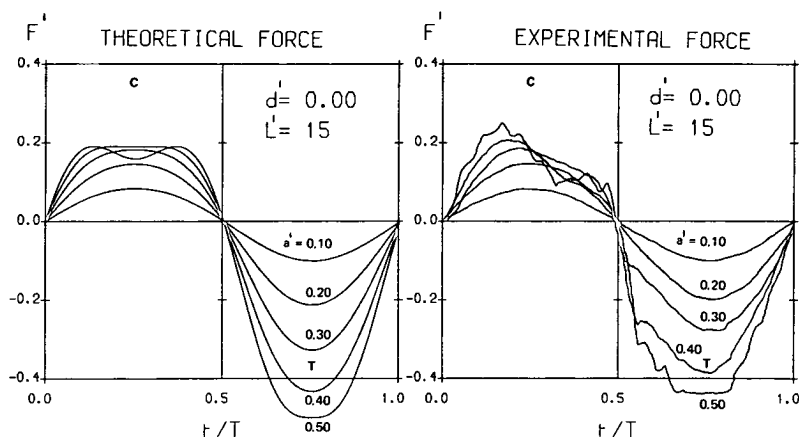


FIG. 5.—Vertical Force for Constant Axis Depth and Frequency and Various Wave Amplitudes (See Table 1 for Values of  $C_M$ )

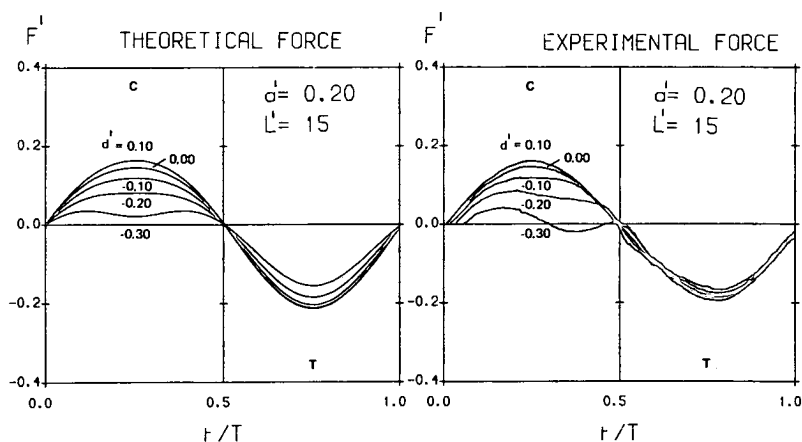


FIG. 6.—Vertical Force for Constant Wave Amplitude and Frequency and Various Axis Depths (See Table 1 for Values of  $C_M$ )

$$a' + |d'| < \frac{1}{2} \dots \dots \dots (16)$$

A value of  $C_M$  for each curve was calculated using a least squares fitting program. Table 1, Col. 4, shows the various values for all the test cases presented in this paper. Since the results of the fit would be sensitive to slight changes in phase of the experimental data a second coefficient was included to give

TABLE 1.—Force Coefficients and Root-Mean-Square Relative Forces for Test Cases

| Relative wave-length<br>(1) | Relative amplitude<br>(2) | Relative axis depth<br>(3) | $C_M$<br>(4) | Experimental root-mean-square force<br>(5) | Theoretical root-mean-square force (best fit)<br>(6) | Theoretical root-mean-square force ( $C_M = 2.0$ )<br>(7) |
|-----------------------------|---------------------------|----------------------------|--------------|--------------------------------------------|------------------------------------------------------|-----------------------------------------------------------|
| 15.62                       | 0.10                      | 0.00                       | 1.86         | 0.062                                      | 0.063                                                | 0.061                                                     |
| 15.62                       | 0.20                      | 0.00                       | 1.86         | 0.121                                      | 0.126                                                | 0.122                                                     |
| 15.62                       | 0.30                      | 0.00                       | 1.87         | 0.172                                      | 0.185                                                | 0.190                                                     |
| 15.62                       | 0.40                      | 0.00                       | 1.88         | 0.221                                      | 0.238                                                | 0.233                                                     |
| 15.62                       | 0.50                      | 0.00                       | 1.73         | 0.264                                      | 0.284                                                | 0.274                                                     |
| 15.62                       | 0.20                      | 0.10                       | 2.03         | 0.121                                      | 0.128                                                | 0.129                                                     |
| 15.62                       | 0.20                      | 0.00                       | 1.86         | 0.121                                      | 0.126                                                | 0.122                                                     |
| 15.62                       | 0.20                      | -0.10                      | 1.81         | 0.112                                      | 0.115                                                | 0.109                                                     |
| 15.62                       | 0.20                      | -0.20                      | 1.72         | 0.101                                      | 0.099                                                | 0.090                                                     |
| 15.62                       | 0.20                      | -0.30                      | 1.78         | 0.087                                      | 0.074                                                | 0.068                                                     |
| 15.62                       | 0.10                      | -0.40                      | 1.85         | 0.031                                      | 0.017                                                | 0.016                                                     |
| 15.62                       | 0.20                      | -0.40                      | 1.56         | 0.070                                      | 0.061                                                | 0.058                                                     |
| 15.62                       | 0.30                      | -0.40                      | 1.31         | 0.118                                      | 0.116                                                | 0.112                                                     |
| 15.62                       | 0.10                      | -0.50                      | 1.57         | 0.030                                      | 0.026                                                | 0.034                                                     |
| 15.62                       | 0.20                      | -0.50                      | 1.28         | 0.054                                      | 0.051                                                | 0.067                                                     |
| 15.62                       | 0.30                      | -0.50                      | 1.25         | 0.089                                      | 0.093                                                | 0.107                                                     |
| 15.62                       | 0.10                      | -0.60                      | 1.66         | 0.039                                      | 0.037                                                | 0.045                                                     |
| 15.62                       | 0.20                      | -0.60                      | 1.18         | 0.042                                      | 0.042                                                | 0.077                                                     |
| 15.62                       | 0.30                      | -0.60                      | 1.10         | 0.052                                      | 0.057                                                | 0.101                                                     |
| 43.39                       | 0.20                      | 0.00                       | 2.77         | 0.147                                      | 0.148                                                | 0.156                                                     |
| 24.41                       | 0.20                      | 0.00                       | 1.97         | 0.139                                      | 0.141                                                | 0.141                                                     |
| 15.62                       | 0.20                      | 0.00                       | 1.86         | 0.121                                      | 0.126                                                | 0.122                                                     |
| 10.85                       | 0.20                      | 0.00                       | 1.62         | 0.107                                      | 0.114                                                | 0.101                                                     |
| 7.97                        | 0.20                      | 0.00                       | 1.45         | 0.090                                      | 0.102                                                | 0.081                                                     |

another degree-of-freedom for the fitting procedure. The inclusion of a phase coefficient requires that a nonlinear least squares program be used. Iterations continued until the change in the residuals was less than 0.1%.

**Centrifugal Forces.**—In calculating the buoyancy in Eq. 7 we have neglected the fact that the weight of the water is more in the trough and less in the crest because of centrifugal forces. It will be shown that no correction is necessary because the coefficient,  $C_M$ , adjusts itself to include this effect.

The force acting radially inwards on a particle travelling in a circle of radius  $r$  would be given by  $m\omega^2 r$ , in which  $m$  = the mass of the particle; and  $\omega$  = its angular frequency. For deep water waves the water at depth  $d$  travels in circles of radius  $a \exp(2\pi d/L)$ . Thus

$$m\omega^2 r = \rho V(t) \frac{2\pi g}{L} a \exp \frac{2\pi d}{L} \dots \dots \dots (17)$$

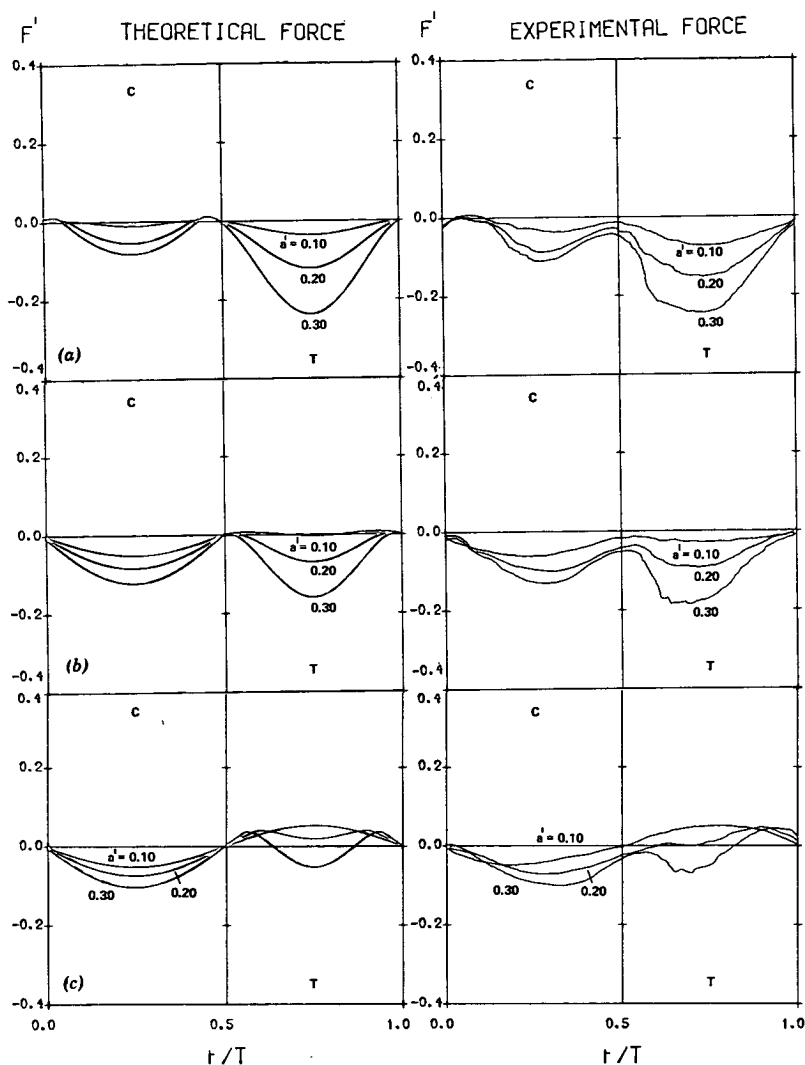


FIG. 7.—Vertical Force for Constant Axis Depth, Constant Frequency, and Various Wave Amplitudes: (a)  $d' = 0.40$ ,  $L' = 15$ ; (b)  $d' = 0.50$ ,  $L' = 15$ ; (c)  $d' = 0.60$ ,  $L' = 15$  (See Table 1 for Values of  $C_M$ )

The component in the vertical direction is therefore given by

$$(m \omega^2 r)_v = -\rho V(t) \frac{2 \pi g}{L} a \exp \frac{2 \pi d}{L} \sin \frac{2 \pi t}{T}$$

Except for a factor of  $C_M$  this is of the same form as Eq. 14. By redefining  $C_M$  to be  $C_M + 1$ , we arrive back at the same result. Since  $C_M$  is determined experimentally the correction is included.

The need for the correction arises because in waves we have motion in both horizontal and vertical directions simultaneously. Therefore the force in one direction is affected by the motion in the other. Morison's equation strictly should only be used for unidirectional accelerating flows. This effect leads to values of  $C_M$  being larger for vertical wave forces than for other flows.

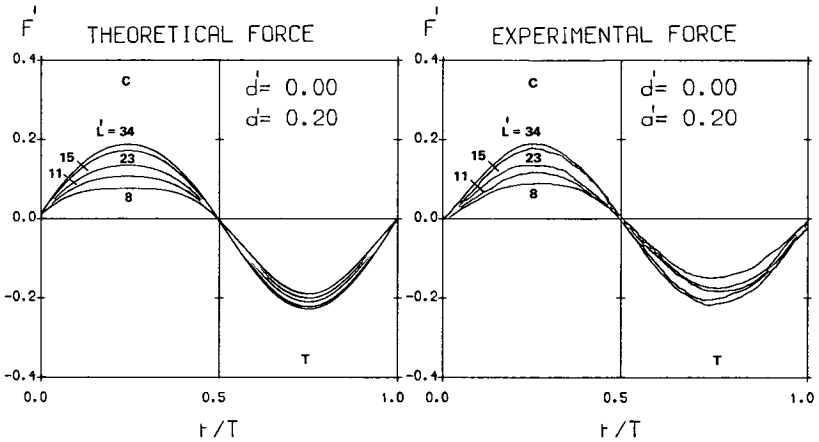


FIG. 8.—Vertical Force for Constant Axis Depth, Constant Wave Amplitude, and Varying Frequency (See Table 1 for Values of  $C_M$ )

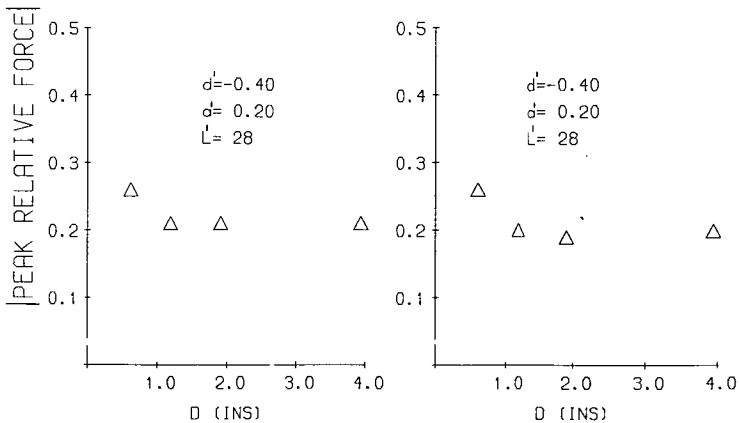


FIG. 9.—Variation of  $|F'|_{\max}$  with Diameter at Two Depths (1 in. = 25.4 mm)

**General Case.**—Eq. 14 is only valid when the condition of Eq. 16 is satisfied. This restriction can be overcome by introducing a function,  $G(\eta)$ , dependent on the water elevation,  $\eta(t)$ , i.e., we write Eq. 7 as

$$F = C_M \rho G(\eta) \frac{\partial U_v}{\partial t} + \rho g [G(\eta) - V_o] \dots \dots \dots (18)$$

in which  $G(\eta) = V_T$ ,  $\eta > D/2 + d = V(t)$ ,  $-D/2 + d \leq \eta \leq D/2 + d - o$ ,  $\eta < -D/2 + d$ , where  $V_T$  = the total volume of the cylinder.

This means that when the cylinder is totally submerged, Eq. 18 reduces to the normal Morison's equation, Eq. 4. If the cylinder is ever totally uncovered the force is a simple constant force acting downwards and for cases in between Eq. 18, it reverts back to Eq. 14.

The curves in Fig. 7, show the results for various values of relative wave amplitude and axis depth. Variation with relative wavelength is shown in Fig. 8, and with cylinder diameter in Fig. 9. The corresponding values of  $C_M$  are in Table 1.

EXPERIMENTAL INVESTIGATION

**Regular Waves.**—To check the accuracy of the modified equations, measurements were taken using the same values of  $a'$ ,  $d'$ , and  $L'$ , as were used in



FIG. 10.—Cylinder Mounting and Force Measuring Rig

the calculations. The experimental rig was designed and built by the wave power team under a contract from the Department of Energy.

A neutrally buoyant light alloy cylinder is mounted on two arms (See Fig. 10). On each side of the mounting vertically above the cylinder axis are two force transducers consisting of strain gages on a thin-walled phosphor bronze torque tube. They respond to horizontal forces. Another pair to the left respond to both vertical and horizontal forces. Some analogue electronic computation yields the two forces separately. The system is calibrated with weights. The signals were sent via an analogue to digital converter to a graphics system that drew the experimental forces curves in Figs. 5, 6, 7, and 8.

The force was measured over one cycle starting at the point where the water line would have crossed the still water level at the front edge of the cylinder going upwards, in the absence of the model. The wave amplitude was measured by a pair of float gages placed at a distance of one-quarter of a wavelength apart. The averaged sum of the two signals automatically corrects for any reflections from the cylinder.

The frequency was set by the electronic drive to the absorbing hinged plate wavemaker. Reflections from the end of the tank were less than 5% and the force measurements were accurate to within 1% of the largest force measured. The initial force on the cylinder in still water was electronically subtracted before the force measurements were taken.

## ANALYSIS

Figs. 5 and 6 show the experimental and theoretical forces on the cylinder with varying relative amplitude and axis depth, respectively;  $C$  and  $T$  denote the wave crest and trough. The parameters were chosen to satisfy Eq. 16. In each set two of the parameters are held constant while the third one varies. Considering the assumptions and approximations the agreement is remarkably good over a wide range of the parameter values. Not surprisingly, as the relative wave amplitude increases, the theory becomes inapplicable and agreement worsens. The variation with amplitude shows that for zero depth, buoyancy dominates with the amplitude of the force increasing with wave height. Also the mean force over a period becomes increasingly more negative with distortions beginning to occur in the crest for higher amplitudes. The experimental curves show an asymmetry in the crest not predicted by the equation. This is considered in the following. For the results in Fig. 5 the cylinder was semi-immersed—a situation dealt with by Ursell, and Bolton and Ursell (19,2). This theory would predict only first order buoyancy effects, and not the frequency doubling occurring here.

A first glance at Fig. 5 with Morison's equation in mind would lead to negative values of  $C_M$ , a difficulty that is overcome by the modification here. The variation of heave force with relative axis depth is shown in Fig. 6. The axis depth varies from +0.10 to -0.3, the relative wavelength and wave amplitude being kept constant. As the depth increases, more of the cylinder is covered, and the inertial term grows, thus decreasing the overall force. Again there is asymmetry in the experimental curves but the overall agreement is still good.

Fig. 7 shows results using Eq. 18. It is in this region that the most interesting phenomena lie. As can be seen, the forces are almost always entirely negative

and act at twice the wave frequency. The 0.2 curve on Fig. 7(a) shows this most clearly (according to Fig. 3). An interesting effect on most of the experimental curves is that the force never even reaches zero, implying that there is some small steady downward force present not accounted for by Eqs. 14 and 18. As the amplitude grows the agreement becomes worse as is to be expected, but the general trends are still predicted. The variation of heave force with cylinder diameter has to be treated with care. If all the relative parameters  $a'$ ,  $d'$ , and  $L'$  are kept constant, then Eq. 15 predicts that the force per unit volume will be constant and so the total force will rise as the square of the

TABLE 2.—Keulegan-Carpenter Numbers ( $N_{KC}$ ) and Ratios of Root-Mean-Square Inertial Force/Root-Mean-Square Buoyancy Force ( $R$ ) for Test Cases

| Relative wavelength<br>(1) | Relative amplitude<br>(2) | Relative axis depth<br>(3) | $N_{KC} = 2a/D$<br>(4) | $R$<br>(best fit)<br>(5) | $R$<br>( $C_M = 2.0$ )<br>(6) |
|----------------------------|---------------------------|----------------------------|------------------------|--------------------------|-------------------------------|
| 15.62                      | 0.10                      | 0.00                       | 0.6                    | 0.31                     | 0.33                          |
| 15.62                      | 0.20                      | 0.00                       | 1.3                    | 0.33                     | 0.35                          |
| 15.62                      | 0.30                      | 0.00                       | 1.9                    | 0.37                     | 0.39                          |
| 15.62                      | 0.40                      | 0.00                       | 2.5                    | 0.42                     | 0.44                          |
| 15.62                      | 0.50                      | 0.00                       | 3.1                    | 0.44                     | 0.50                          |
| 15.62                      | 0.20                      | 0.10                       | 1.3                    | 0.30                     | 0.30                          |
| 15.62                      | 0.20                      | 0.00                       | 1.3                    | 0.33                     | 0.35                          |
| 15.62                      | 0.20                      | -0.10                      | 1.3                    | 0.37                     | 0.41                          |
| 15.62                      | 0.20                      | -0.20                      | 1.3                    | 0.42                     | 0.49                          |
| 15.62                      | 0.20                      | -0.30                      | 1.3                    | 0.55                     | 0.61                          |
| 15.62                      | 0.10                      | -0.40                      | 0.6                    | 0.80                     | 0.86                          |
| 15.62                      | 0.20                      | -0.40                      | 1.3                    | 0.66                     | 0.85                          |
| 15.62                      | 0.30                      | -0.40                      | 1.9                    | 0.51                     | 0.78                          |
| 15.62                      | 0.10                      | -0.50                      | 0.6                    | 1.49                     | 1.89                          |
| 15.62                      | 0.20                      | -0.50                      | 1.3                    | 0.85                     | 1.33                          |
| 15.62                      | 0.30                      | -0.50                      | 1.9                    | 0.67                     | 1.07                          |
| 15.62                      | 0.10                      | -0.60                      | 0.6                    | $\infty$                 | $\infty$                      |
| 15.62                      | 0.20                      | -0.60                      | 1.3                    | 2.58                     | 4.38                          |
| 15.62                      | 0.30                      | -0.60                      | 1.9                    | 1.18                     | 2.15                          |
| 43.39                      | 0.20                      | 0.00                       | 1.3                    | 0.18                     | 0.13                          |
| 24.41                      | 0.20                      | 0.00                       | 1.3                    | 0.22                     | 0.23                          |
| 15.62                      | 0.20                      | 0.00                       | 1.3                    | 0.35                     | 0.35                          |
| 10.85                      | 0.20                      | 0.00                       | 1.3                    | 0.41                     | 0.51                          |
| 7.97                       | 0.20                      | 0.00                       | 1.3                    | 0.50                     | 0.67                          |

diameter. This fact is shown in Fig. 9. Here, the modulus of the largest relative force,  $F'$ , over one cycle, has been plotted for two different axis depths. This is fairly constant over a wide range of  $D$ .

Variation of force with diameter is not as simple if  $a'$ ,  $d'$ , and  $L'$  are not kept constant. If  $L'$  is too small then there will be large phase changes across the cylinder leading to some force cancellation. Thus, forces will rise more slowly with diameter than expected. The exponential decay of the water motion with depth will also lead to smaller forces for large cylinders. Note that for very small cylinders drag forces will become comparable with inertial forces

and ought not be discarded. An important quantity relevant to the relative size of drag and inertial forces is the Keulegan-Carpenter number ( $N_{KC}$ ), sometimes referred to as the "period parameter" (see Ref. 13). This is defined as  $U_o T/D$ , in which  $U_o$  = the amplitude of the velocity variation. For linear waves this reduces to  $2\pi a/D$ . At low values of  $N_{KC}$  drag becomes negligible compared to inertial forces. For values greater than about  $\pi$  its presence is noticeable and must be taken into account. The Keulegan-Carpenter number for each test case is given in Table 2, Col. 4.

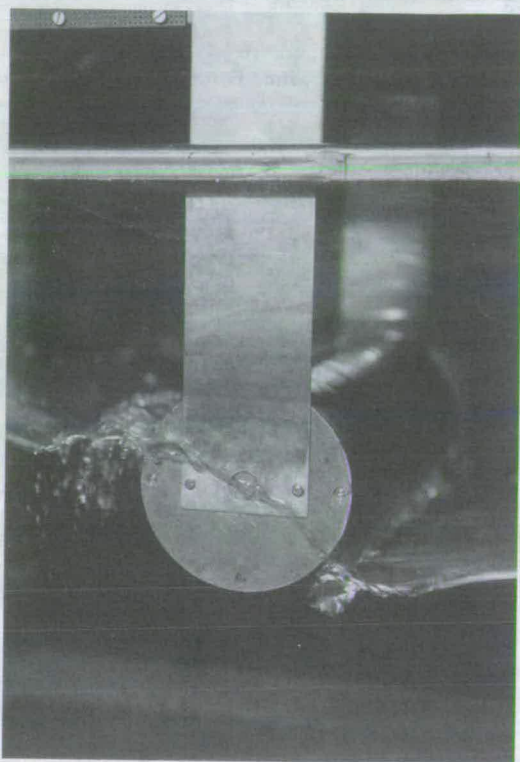


FIG. 11.—Wave Profile around Cylinder for Large Amplitude

Variation with relative wavelength does not have any direct effect on buoyancy forces. The effect on the total heave force is that for  $L' > 6d'$  the inertial term decreased with larger  $L'$  and so the force in the crest is made more positive and in the trough more negative. These features are shown in Fig. 8. For waves of large amplitude the steepness of the waves cause a large variation of the water level across the cylinder (see Fig. 11). Fig. 11 shows the wave at a point where  $t \approx 0.4T$ . The waves are interacting with the cylinder causing more of it to be uncovered than is assumed in the theory. This leads to a larger downward force giving rise to the asymmetry seen in some of the results. It would be very difficult to account for this in Eq. 13.

As can be seen from Table 1, there is a fairly wide spread in the values

calculated for  $C_M$ . The general trend is that the coefficient decreases for increasing amplitude, frequency, and axis depth. This poses the problem of deciding what value to use in a given situation. The suggestion put forward here is that the value derived from potential flow theory for a completely submerged cylinder could be used, i.e.,  $C_M = 2.0$ . To give a little justification for this choice we shall look at the root-mean-square force. Table 1, Cols. 5-7, shows the experimental root-mean-square relative force together with the theoretical root-mean-square relative force for two values of  $C_M$ ; Col. 6 is for the "best fit" value, and Col. 7 is for  $C_M = 2.0$ . In general it can be seen that the two theoretical columns differ from each other by only a few percent. Agreement is best where buoyancy forces dominate. In these cases it is obvious that the overall force will be less sensitive to changes in  $C_M$ . It would be useful to know what the relative sizes of the buoyancy and inertial terms are. To estimate this it would be instructive to calculate the ratio of the root-mean-square inertial force to the root-mean-square buoyancy force; the averages being taken over one wave cycle. Col. 5 in Table 2 gives the numerical calculation of the ratio for each case, using the best fit value for  $C_M$ . Table 2, Col. 6, gives the ratio for  $C_M = 2.0$ .

It would be extremely difficult to integrate the square of each term in Eq. 13 to arrive at an explicit expression for this ratio. We can make some headway, however, if we consider small amplitude waves. Referring to Fig. 4 we can approximate  $V(t)$  by writing

$$V(t) = V_o + a D' l \sin \frac{2\pi t}{T} \dots \dots \dots (19)$$

in which  $D' = D [1 - (4d^2/D^2)]^{1/2}$ .

It can be shown that the resulting ratio is given by

$$R = C_M \frac{2\pi}{L} \exp \left( \frac{2\pi d}{L} \right) \left[ \left( \frac{V_o}{D' l} \right)^2 + a^2 \right]^{1/2} \dots \dots \dots (20)$$

For the particular case  $d = 0$  this simplifies to

$$R = C_M \frac{2\pi}{L} \left[ \left( \frac{\pi D}{8} \right)^2 + a^2 \right]^{1/2} \dots \dots \dots (21)$$

For small values of  $a$  this gives very close agreement to the numerical calculations. We can write Eq. 21 in terms of the relative parameters  $a'$  and  $L'$

$$R = C_M \frac{2\pi}{L'} \left[ \left( \frac{\pi}{8} \right)^2 + (a')^2 \right]^{1/2} \dots \dots \dots (22)$$

So at larger scales this ratio remains constant if the relative parameters are kept constant. The derivation of Eq. 20 assumes that the waves do not overlap the cylinder. If they do then the inertial force increases while the buoyancy force remains constant and thus  $R$  increases. If the waves ever uncover the cylinder, then slamming would occur. This phenomenon is outside the scope of this paper.

The results presented here are not meant to be the final answer to the problem. The equations are more an empirical model than a fluid dynamical solution.

A complete treatment of this problem, using say potential theory, has not yet been achieved. The model could be improved in several ways. For example, some account of reflections could be incorporated, or a higher order wave theory could be used. Nevertheless, even as it stands, the method is considerably better than a blind application of Morison's equation (Eq. 1) to the situation.

## CONCLUSIONS

It has been shown that for partially submerged cylinders varying buoyancy can play as large a part as inertial forces. The interplay between inertia and buoyancy leads in certain situations to entirely negative heave forces which act at twice the wave frequency. Morison's equation (Eq. 1) has been adapted to predict the effects for regular waves by introducing a varying volume and a buoyancy term. For small values of wave amplitude (Keulegan-Carpenter number) and wave steepness the agreement with experiment is made fairly good by fitting the inertial coefficient to each of the experimental curves. Although there is a spread in the values of  $C_M$  obtained, it is possible to retain a good deal of accuracy using the theoretical value for a submerged cylinder,  $C_M = 2.0$ , as a design coefficient in all cases. The considerations applied to a cylinder could also be used for other objects.

## ACKNOWLEDGMENTS

The writers would like to thank the Department of Energy who financed the experimental investigations discussed in this paper. The first writer acknowledges the receipt of a Science Research Council award during the period of this study.

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## APPENDIX II.—NOTATION

The following symbols are used in this paper:

|                       |   |                                                                    |
|-----------------------|---|--------------------------------------------------------------------|
| $A$                   | = | constant in time;                                                  |
| $A(t)$                | = | cross-sectional area of submerged part of cylinder, $A_o = A(o)$ ; |
| $a$                   | = | wave amplitude;                                                    |
| $a'$                  | = | relative wave amplitude, $a/D$ ;                                   |
| $B$                   | = | constant in time;                                                  |
| $b$                   | = | length of side of square block;                                    |
| $C$                   | = | constant in time;                                                  |
| $C_D$                 | = | coefficient of drag;                                               |
| $C_f$                 | = | overall force coefficient;                                         |
| $C_M$                 | = | coefficient of inertia;                                            |
| $D$                   | = | cylinder diameter;                                                 |
| $d$                   | = | cylinder axis depth below still water level;                       |
| $d'$                  | = | relative axis depth, $d/D$ ;                                       |
| $F$                   | = | total vertical force on object;                                    |
| $F'$                  | = | relative force, $F/(\rho g V_T)$ ;                                 |
| $F_b$                 | = | buoyancy force;                                                    |
| $F_v$                 | = | vertical force neglecting drag and buoyancy;                       |
| $F_{\text{inertial}}$ | = | inertial term in force equation;                                   |
| $F_{\text{buoyancy}}$ | = | buoyancy term in force equation;                                   |
| $f$                   | = | wave frequency;                                                    |

- $G(\eta)$  = function of water elevation defined in Eq. 16;  
 $g$  = acceleration due to gravity;  
 $h$  = water depth;  
 $L$  = wave length;  
 $L'$  = relative wavelength,  $L/D$ ;  
 $l$  = length of cylinder;  
 $R$  = root-mean-square inertial force/root-mean-square buoyancy force;  
 $T$  = wave period;  
 $t$  = time;  
 $U_h$  = horizontal water particle velocity;  
 $U_v$  = vertical water particle velocity;  
 $V$  = submerged volume of object, volume per unit length of pile;  
 $V_o$  = initial submerged volume of object;  
 $V_T$  = total volume of object;  
 $x$  = horizontal coordinate;  
 $y$  = vertical coordinate;  
 $\eta$  = wave profile;  
 $\rho$  = mass density of water; and  
 $\omega$  = angular frequency.

#### 14996 WAVE FORCE ON PARTIALLY SUBMERGED CYLINDER

**KEY WORDS:** Buoyancy; Coefficients; Cylinders; Drag; Energy; Inertial forces; Loads (forces); Ocean engineering; Wave energy; Wave pressure; Waves

**ABSTRACT:** Morison's equation for wave force is adapted to predict the force exerted by regular waves on a partially submerged horizontal cylinder. In particular, the vertical force equation is modified by introducing a buoyancy term and a varying volume after first discarding the drag term. Some experimental data taken in a 30-cm wide tank are used to calculate a value of the inertial coefficient  $C_M$  using a least squares fitting procedure. Although a spread is found in the values of  $C_M$ , it is suggested that the theoretical value for a submerged cylinder,  $C_M=2.0$ , could be used with some success.

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