

The effect of High Tip Speeds on the
Performance of Aeroplane Propellers.

Thesis for D.Sc. (Engineering).

by

George P. Douglas B.Sc.



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THE EFFECT OF HIGH TIP SPEEDS ON AIRSCREW PERFORMANCE.

SUMMARY.

Thrust, Torque, Efficiency, Thrust Grading and Blade Twist have been measured for a model airscrew running at tip speeds up to 1300 ft./sec. i.e. 1.2 times the velocity of sound.

CONCLUSIONS.

- (1) There was found to be a considerable loss of efficiency when the blade tips were run at speeds exceeding 75% of the velocity of sound.
- (2) The lift coefficient of a blade section was found to increase rapidly as its velocity rose from 60 to 80% of the velocity of sound.
- (3) At high tip speeds the designed thrust distribution along the blade is liable to serious modification by blade twist.
- (4) As the velocity of a blade section approaches that of sound the centre of pressure moves back and the drag coefficient increases.
- (5) There should be no difficulty as regards strength in constructing wood airscrews to run at tip speeds up to the velocity of sound.

SYMBOLS USED IN REPORT.

T = Thrust.

Q = Torque.

γ = efficiency.

V = forward speed.

n = revolution per unit time.

D = diameter.

r = radius % diameter.

c = blade width % diameter.

ρ = density of air.

a = speed of sound in air.

$$k_T = \frac{T}{\rho n^3 D^4}$$

$$k_Q = \frac{Q}{\rho n^3 D^5}$$

$$J = \frac{V}{N D}$$

$(1+a)V$ = speed of flow through airscrew disc.

Θ = angle of blade chord to plane of rotation.

φ = angle of resultant wind speed to plane of rotation

$$\text{i.e. } \tan \varphi = \frac{J(1+a)}{2\pi r}$$

α = angle of incidence of blades.

k_L = lift coefficient.

k_D = drag coefficient.

Note:- r is not the radius.

The above rotation is practically that used by Prof. Bairstow in his book on "Applied Aerodynamics".

1. PRELIMINARY.

The present practice is to design aeroplane propellers to run at tip speed of 600 to 800 ft./sec.. For normal flying speeds up to 120 miles per hour, there is little object in exceeding these tip speeds owing to strength considerations and the fact that the fine angles required call for unnecessarily high accuracy in manufacture. For very high flying speeds (200 miles per hour) the question of the effect of increasing the above tip speeds becomes important; for racing purposes a tip speed of 1100 ft./sec. has actually been used.

Theoretical considerations and the published tests on projectiles show that there is a complete change in the type of flow as the velocity of sound in air is approached, but no data was available to guide the airscrew designer as to what effect this change in flow had upon the forces on the propeller blades. The present investigation was begun about three years ago to obtain this information, and it has been found possible to measure the variation of thrust, torque and efficiency of a two foot diameter model airscrew over a range of tip speed from 150 ft./sec. to 1300 ft./sec. i. e. up to 1.2 times the velocity of sound in air. In addition the variation of the thrust along the blade has been determined for this speed range, and by the application of the Lanchester - Prandtl vortex theory the variation of the lift of an aerofoil has been determined.

2. USE OF MODELS FOR HIGH SPEED INVESTIGATIONS.

For airscrew design, the aerodynamic properties of the aerofoil sections of which the blade is made up are obtained from wind channel tests of the aerofoil sections

sections. These tests are only applicable within certain limits. The most familiar use of models is for the prediction of ship resistance. The resistance of a ship depends primarily on the surface waves it makes, and in order that the wave motions of the ship and the model may be similar, it is necessary to tow the model at a speed which is $\frac{1}{\sqrt{\text{scale of model}}}$ of the ship's speed. This is called the "corresponding" speed and, as far as the ship's resistance depends on the surface waves, we have

$$\begin{aligned} \text{Resistance of ship} &= \left(\frac{\text{resistance of model}}{\rho' L'^2 V'^2} \right) \rho L^2 V^2 = \\ &= k_D \rho L^2 V^2 \end{aligned}$$

where ρ is the density, L any linear dimension and V the velocity, the dashed symbols referring to the model; " k_D " is the drag coefficient.

In the case of an airscrew or aerofoil we are not concerned with surface waves but the slight viscosity of the air and its compressibility each impose similar limitations. In order that the flow round a model, $\frac{1}{n}$ scale, shall be identical with that of the full scale blade, tested in the same fluid, considerations of viscosity require that the model be tested at a "corresponding" speed n times that of the full scale blade. (For different fluids the $\frac{LV}{\mu}$ of the model and full scale tests must be the same, μ being the kinematic viscosity of the fluid). This is rather inconvenient but the extensive "Scale effect" tests carried out at the Royal Aircraft Establishment have shown that for normal aerofoils, a wind channel model, tested at an LV not below that corresponding to a 6" chord at 60 ft./sec., gives practically the same force coefficients as an aeroplane with 6' chord flying at 150 ft./sec., a range of

$\frac{LV}{\mu}$ going considerably beyond that of a high speed propeller blade.

Compressibility requires that the ratio of the velocity of the test to the velocity of sound i.e. " $\frac{v}{a}$ " shall be the same for model and full scale. Compressibility is not important at speeds below 300 or 400 ft. per sec. but as we approach the velocity of sound it becomes the dominant factor in determining the nature of the flow. No model test can satisfy, simultaneously, both the compressibility and viscosity conditions, and all we can do is to test the model at the correct " $\frac{v}{a}$ " and by varying the scale of the model see if the force coefficients are independent of $\frac{LV}{\mu}$.

If the model and full scale flows are similar any corresponding force will be equal to (coefficient) $\times \rho L^2 V^2$. In the case of a complete airscrew we use the coefficients k_T and k_Q given by

$$\text{Thrust} = k_T \rho n^2 D^4$$

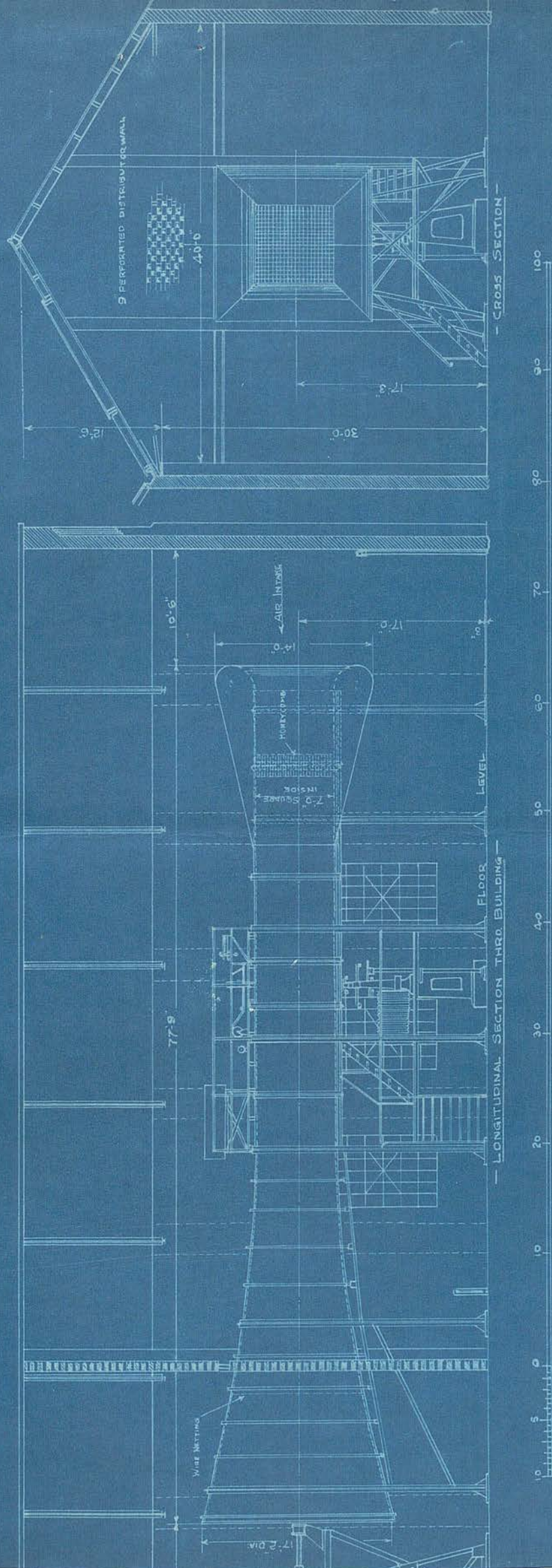
$$\text{Torque} = k_Q \rho n^2 D^5$$

3. OBJECT OF EXPERIMENTS.

The object of these experiments was to investigate experimentally the behaviour of an airscrew as the tip speed was increased from that used in ordinary model experiments up to that used for normal full scale airscrews, and to see what happened when the tip speed was further increased up to and beyond the velocity of sound. Prior to the present investigation two short experiments had been carried out at the Royal Aircraft Establishment. In the first* a full size airscrew was run on an electric motor at a fixed point. The torque coefficient increased rapidly at high tip speeds and when the blade tips attained

the

— GENERAL ARRANGEMENT OF N^o 2-7-0 WIND CHANNEL —



the velocity of sound the slip stream disappeared so the thrust must have practically vanished. An airscrew with very thick blades was used for these tests and in the subsequent work with a more normal airscrew these extraordinary results were not repeated.

For the second experiment a specially designed airscrew was fitted to an S.E.5 aeroplane with Viper engine. The engine was not sufficiently powerful, but by diving the aeroplane steeply a few measurements of thrust were made with the airscrew running at a tip speed just equal to the velocity of sound. As no more suitable engine was then available, the present investigation, for which the author was responsible, was put in hand.

A "de Laval" type of air-turbine was used to drive a model airscrew in a 7 ft. channel. The No. 2 7 ft. channel at the Royal Aircraft Establishment was used for this work. It is fitted with a 200 HP motor and can give wind speeds from 25 to almost 100 miles/hr..

The force measuring mechanism fitted to the turbine was not sensitive for very small forces so special apparatus was fitted to a small electric motor for the low speed work.

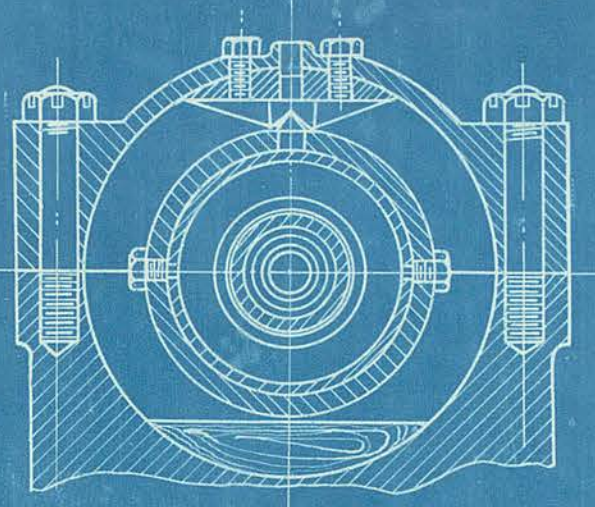
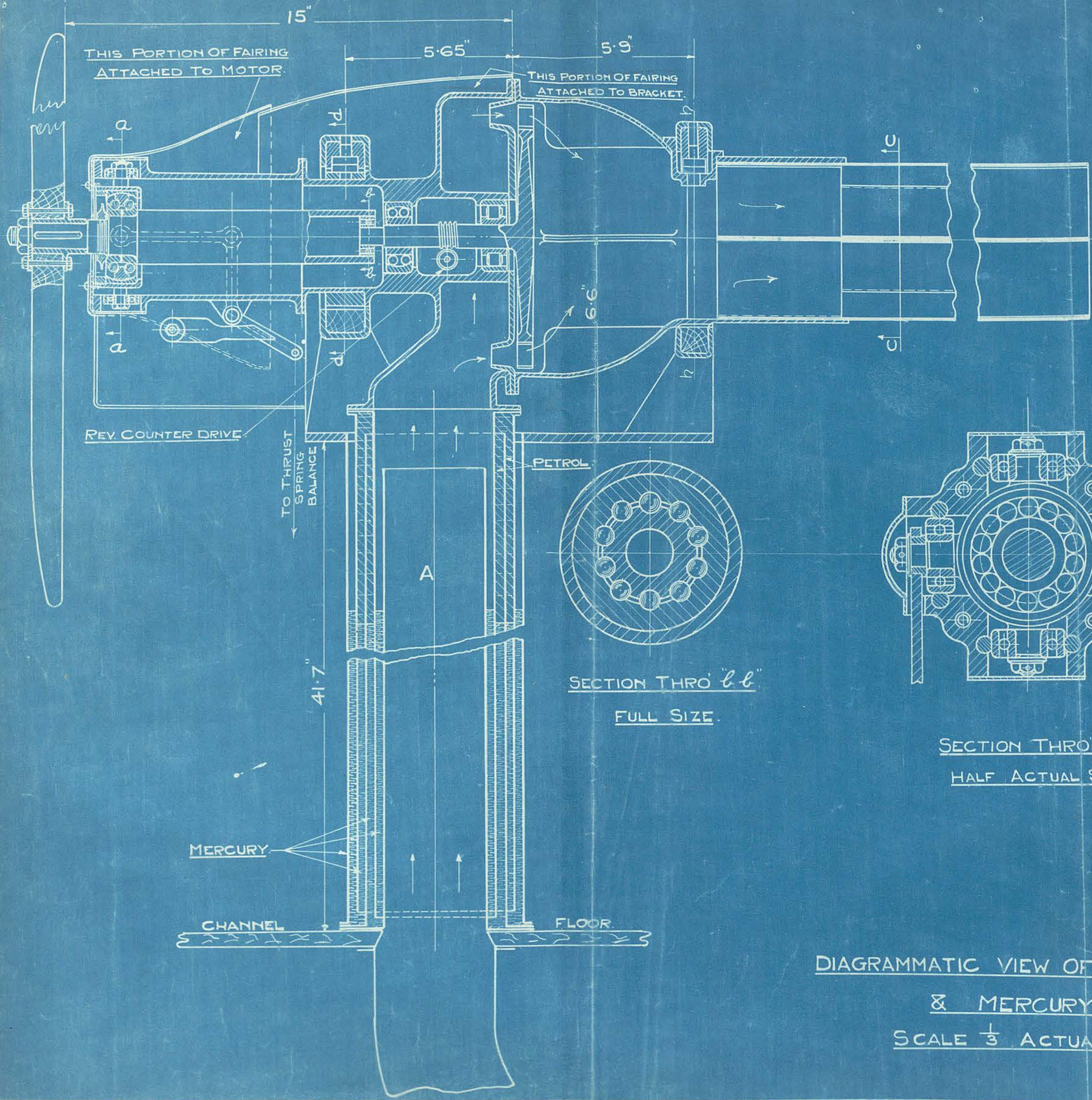
4. DESCRIPTION OF APPARATUS.

The air turbine used is shown in fig.1.

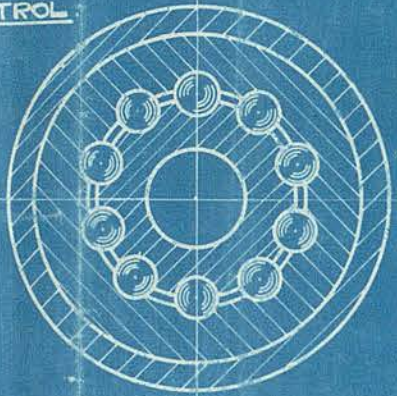
It was specially designed for the experiment by the staff of the Royal Aircraft Establishment engine drawing office, making use of a rotor which happened to be available.

The air was led in from underneath the motor, passed through the nozzles to the rotor and discharged axially.

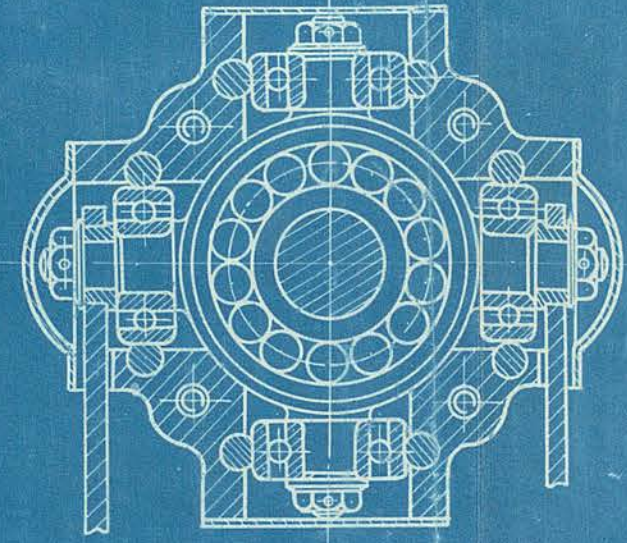
To measure thrust the shaft carrying the airscrew was mounted so that it was free to move axially, independent



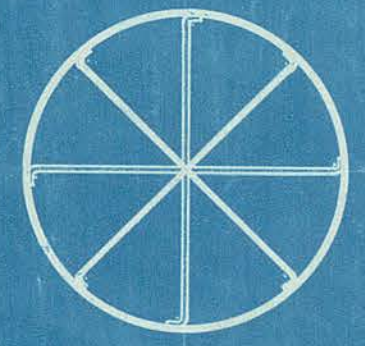
SECTION ON d.d.
(SIMILAR SECTION ON h.h)



SECTION THRO 'b.b.'
FULL SIZE.



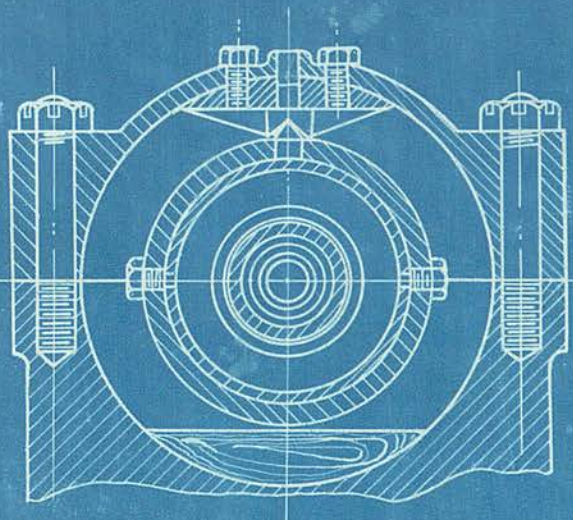
SECTION THRO 'a.a.'
HALF ACTUAL SIZE.



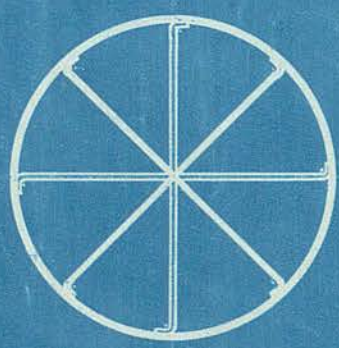
SECTION THRO C.C.

TO TORQUE
SPRING BALANCE.

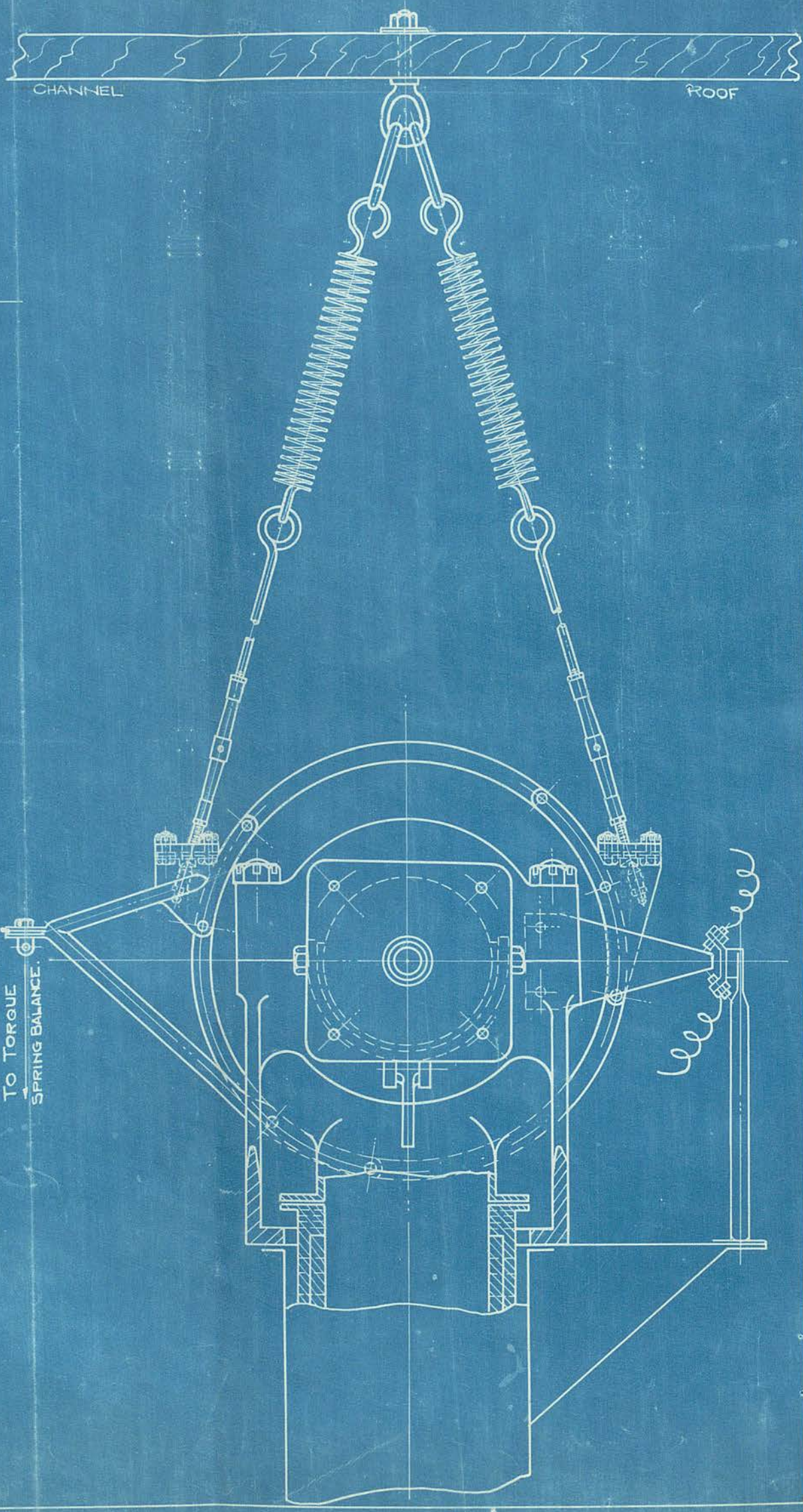
DIAGRAMMATIC VIEW OF HIGH SPEED AIR MOTOR
& MERCURY SEAL.
SCALE $\frac{1}{3}$ ACTUAL SIZE.



SECTION ON d.d.
(SIMILAR SECTION ON h.h.)



SECTION THRO.C.C.



CHANNEL

ROOF

TO TORQUE
SPRING BALANCE.

"a.a."
SIZE.

HIGH SPEED AIR MOTOR

SEAL.

SIZE.

of the shaft carrying the rotor. The torque was transmitted from the one shaft to the other by means of grooves filled with balls (Section bb. fig 1.). A linkage connected the airscrew shaft to a spring balance under the channel. Torque was measured by reaction and, as designed, the turbine casing was carried in two large bearings one ball and one roller. (Section dd, hh). An arm connected to the outside of the casing was connected to a spring balance under the channel. The rate of rotation was measured by a tacheometer driven by a reduction gear off the rotor shaft.

The air to drive the turbine was supplied by a compressing plant about 400 ft. away, using an existing pipe line. The turbine gave up to 60 BHP with a pressure of 35lbs/sq". The compressors were of ample power but owing to the excessive losses in the pipe line the required pressure could not be maintained. Pressure gauges were fitted at various points along the pipe line, simultaneous readings taken of the pressures, and the portions of the pipe line showing exceptional pressure drop were relaid. The effect of varying the number of nozzles was tried and it was found that five gave the most economical result, this being the minimum number to give the required power with the maximum permissible pressure.

As the torque of the airscrew was being measured by its reaction on the turbine casing, great precautions were necessary to ensure that the torque measured was entirely due to the airscrew. It was necessary -

- (1) to introduce the compressed air into the turbine without applying a torque to the casing;
- (2) to get rid of the exhaust without torque;

(3) to ensure that the slipstream from the airscrew did not itself apply a torque to the casing.

Taking these in the reverse order, to overcome (3), a streamline cover was fitted to the front portion of the turbine so that the slipstream did not come in contact with the turbine casing.

(2) Since the exhaust is discharged axially the condition for no torque is that the exhaust be discharged without rotation. This was achieved by fitting a series of baffle plates in the exhaust tube (fig. 1). The repetition of a few tests with a less number of baffle plates showed that there was no error on this account.

(1) The introduction of the compressed air was the most difficult problem. The inlet pipe was 4" diameter and pressures up to 35 lbs./sq. " were required. Preliminary experiments were made using heavy rubber tubing, but when the pressure was applied the tubing did not stretch uniformly and bulged out to one side. Another form of coupling was tried consisting of a length of steel tube attached by two short lengths of rubber tube, but this did not give any better results. The maximum torque to be measured was of the order of 25 ft. lbs. so it appeared that a rubber coupling would have to be so heavy to overcome its instability that the accuracy of the torque measurements would be small.

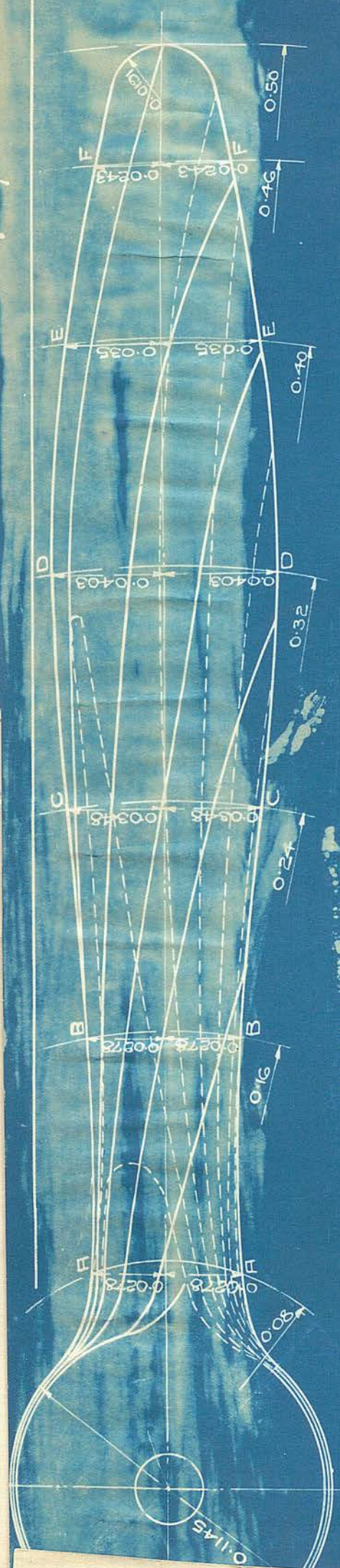
The construction of a mercury seal appeared to be the only hopeful solution. The constructional difficulties were considerable as the clearances had to be small, and to prevent action by the

mercury the seal had to be made of steel and all joints welded. To maintain a pressure of 35 lbs./sq " a head of 80 inches of mercury was required, and it was found most convenient to have two concentric seals in series, each giving a 40 inch head. To connect the inner and the outer seals, petrol was used. Thin oil was tried, but it proved unsatisfactory as it formed an emulsion with the mercury. The seal worked very well, but required great care in use as if the pressure were applied too rapidly it blew all the mercury out of the seal. On one occasion when this happened the author noticed that the absence of the mercury made no appreciable difference to the power given by the turbine. Apparently the loss in head of the air in going round the various corners was sufficient to maintain the pressure with very slight leakage. For future work it is proposed to develop a seal on these lines.

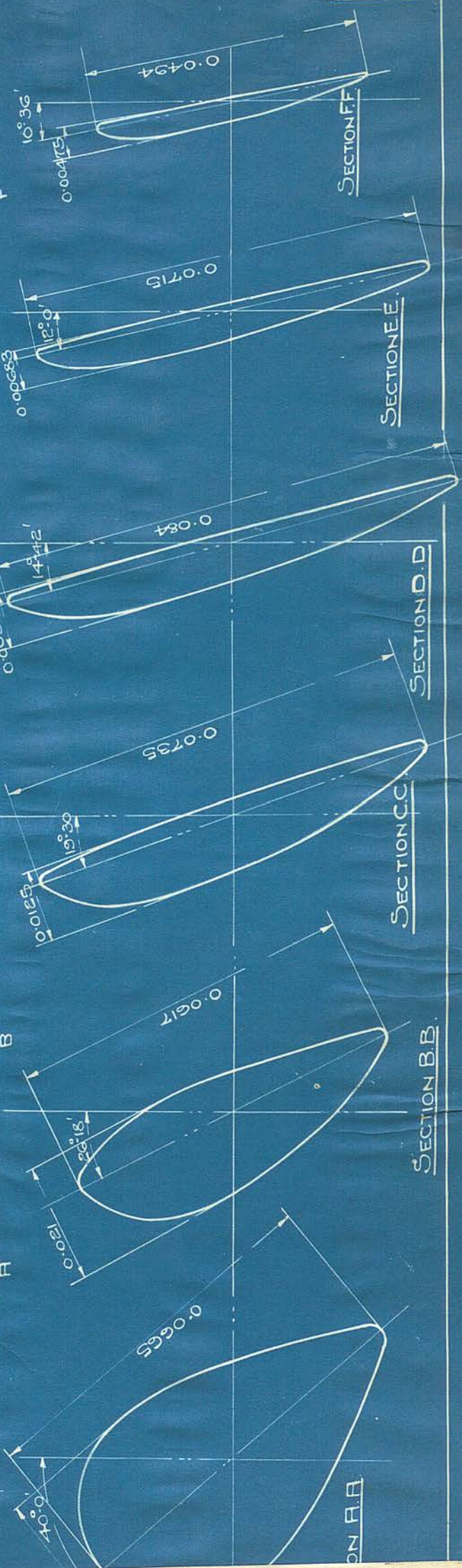
With the mercury seal the condition that the incoming air shall not apply a torque to the turbine casing is that the axis of the moving portion of the seal be exactly vertical. To obtain this a stop was fixed on the casing to prevent the airscrew rotating, and the turbine was tilted until no change in torque reading was observed when the compressed air pressure was applied.

The scheme by which the turbine casing was carried in two large bearings proved unsatisfactory. The bearings were not sufficiently free owing to the distortion of the casing under load; also the very slight rotation permitted by the ^{torque measuring} apparatus was insufficient to maintain the axis of the rollers in their proper direction. The bearings were replaced by the knife edges shown in the figure and torque was measured by reaction about an axis, parallel

FIG. 2.



A ALL LINEAR DIMENSIONS ARE EXPRESSED IN TERMS OF THE DIAMETER.



parallel to that of the turbine, but a few inches above it. As the force on the turbine due to the compressed air was much greater than the weight of the turbine, the knife edges were inverted and the springs shown were used to keep the turbine in its knife edges when the air pressure was removed.

The bearing at AA which takes up the thrust up to 150 lbs. tended to heat up at the high speeds. Several designs were tried and the type shown in the diagram was evolved. The thrust measuring mechanism was very satisfactory. Both thrust and torque mechanisms floated between electric contacts connected to small lamps under the channel. In both cases the mechanism was calibrated by the direct application of load to the airscrew shaft.

For the very low speeds test, a lighter and more sensitive apparatus was made up using a $\frac{1}{2}$ HP electric motor. The motor was mounted on knife edges to measure torque by reaction and was also freely suspended from the roof to measure thrust. To screen the motor itself from the wind it was cased in a fairing which represented the main outline of the turbine.

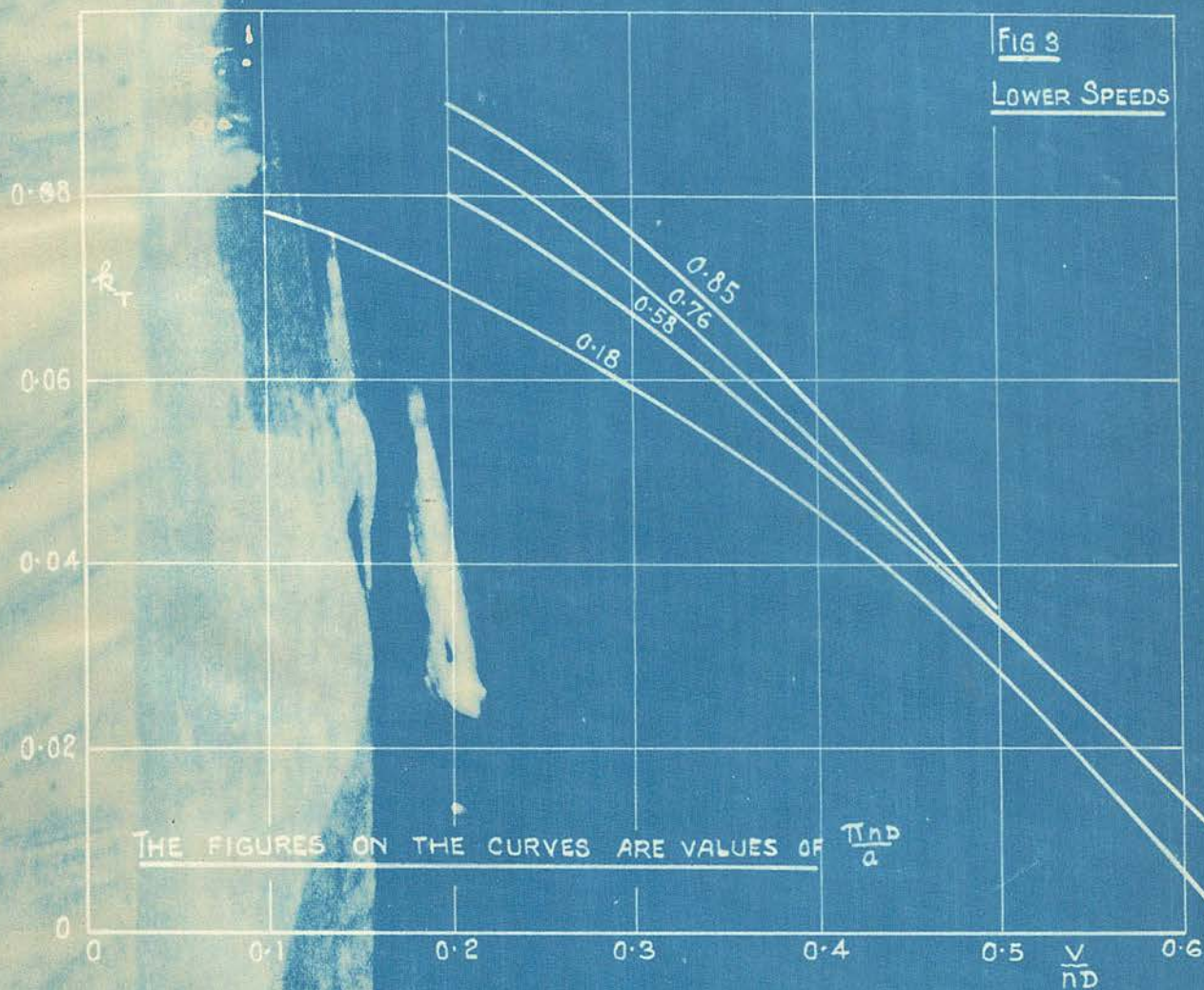
5. AIRSCREW.

Fig. 2. gives the leading dimensions and angles of the airscrew tested. In the drawing all dimensions are given in terms of the diameter. The airscrew has two blades of symmetrical plan form and a slight forward tilt. The model tested was two feet in diameter. The wood used was American Black Walnut.

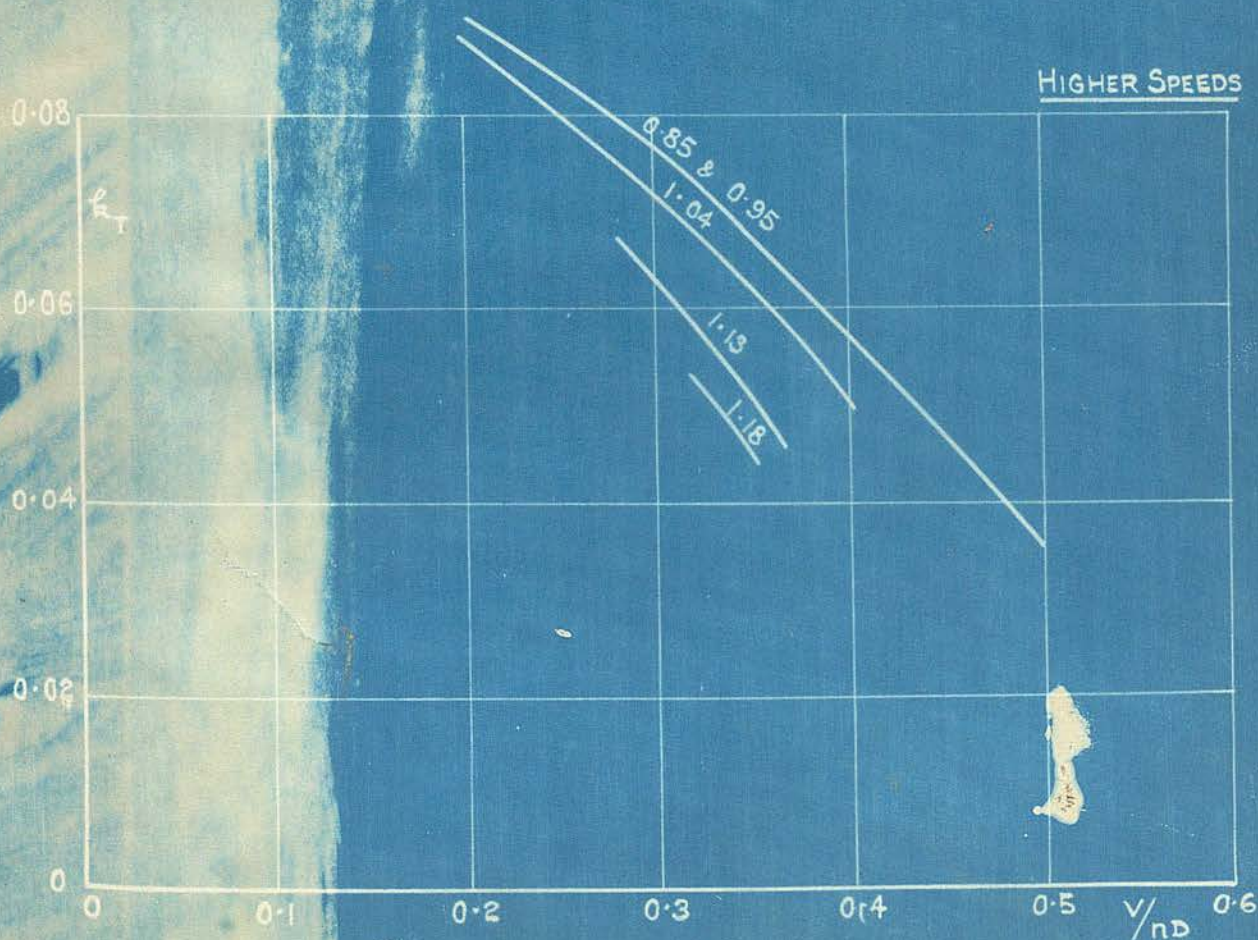
THRUST

FIG 3

LOWER SPEEDS



HIGHER SPEEDS



6. THRUST, TORQUE AND EFFICIENCY.

The results of the thrust and torque measurements are given in Figs. 3 and 4. The actual observations are given in tables ~~to 3~~, but in the figures smooth curves have been drawn and the points omitted to avoid confusion. Non-dimensional coefficients are used in all cases,

$$\text{The thrust coefficient, } k_T = \frac{\text{Thrust}}{\rho n^2 D^4}$$

$$\text{The torque coefficient, } k_Q = \frac{\text{Torque}}{\rho n^2 D^5}$$

where ρ is the mass of unit volume of air, n the rotational speed of the airscrew in revolutions per second and D is the diameter of the airscrew.

Thrust and torque coefficients are plotted against $\frac{V}{nD}$ which is the advance per revolution in diameters. The numbers on the curves show the ratio of the tip speed of the airscrew to the velocity of sound i.e. $\pi nD/a$ where "a" is the velocity of sound.

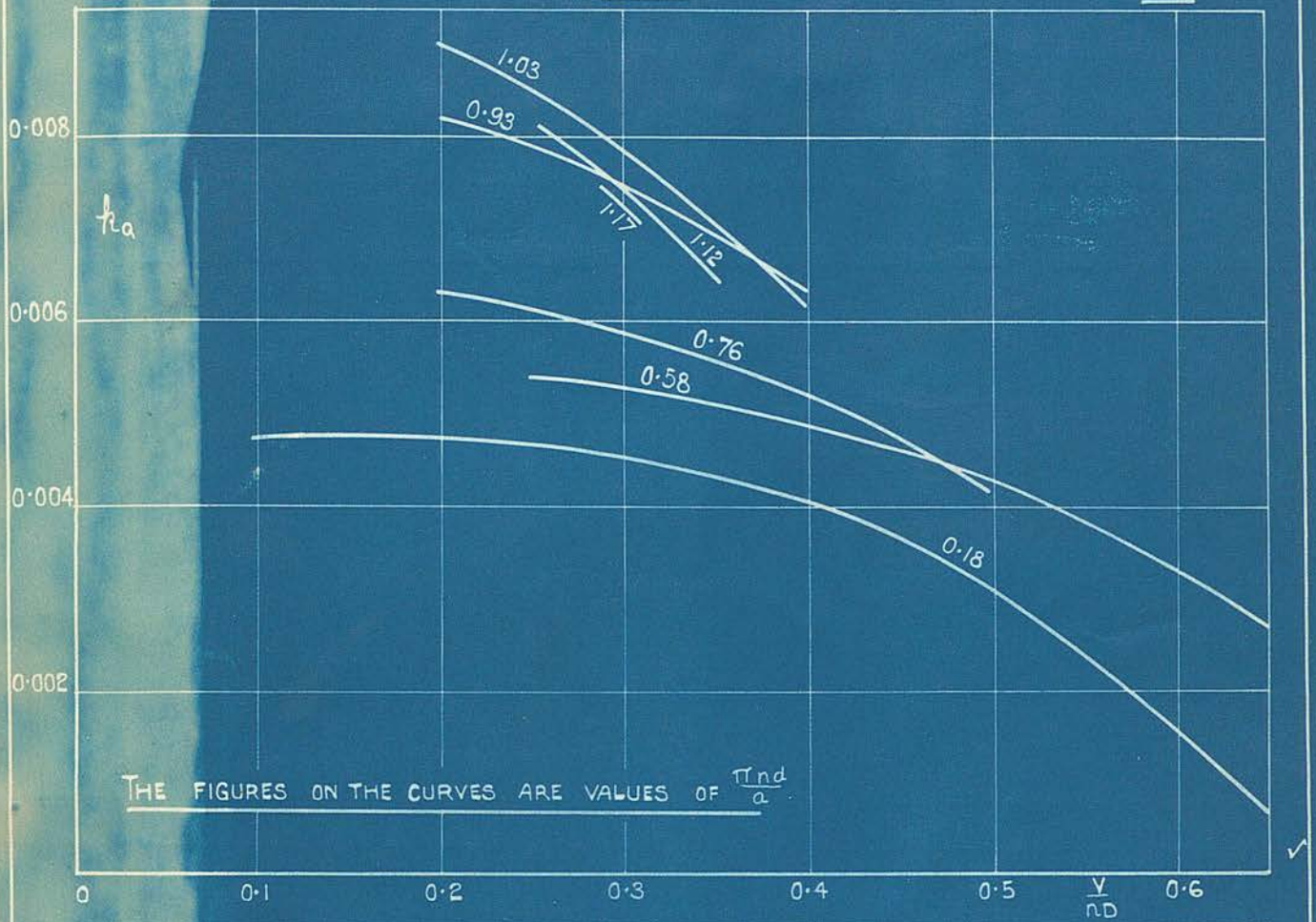
It will be noted that at the highest tip speed it has only been possible to obtain measurements for a very limited range of $\frac{V}{nD}$ owing to limitations of channel speed and turbine power. Fig. 3. shows that at constant $\frac{V}{nD}$ the thrust coefficient increases steadily with tip speed until we reach 0.85 of the velocity of sound. It then remains fairly constant until the velocity of sound is attained, after which it falls off rapidly.

The torque coefficient (Fig. 4) rises steadily with speed attaining a maximum when the tip speed is just over the velocity of sound.

$$\begin{aligned} \text{The efficiency} &= \frac{\text{Thrust} \times \text{velocity}}{\text{Torque} \times 2 \pi n.} \\ &= \frac{k_T \rho n^2 D^4 V}{2 \pi n k_Q \rho n^2 D^5} \\ &= \frac{1}{2 \pi} \times \frac{k_T}{k_Q} \times \frac{V}{nD} \end{aligned}$$

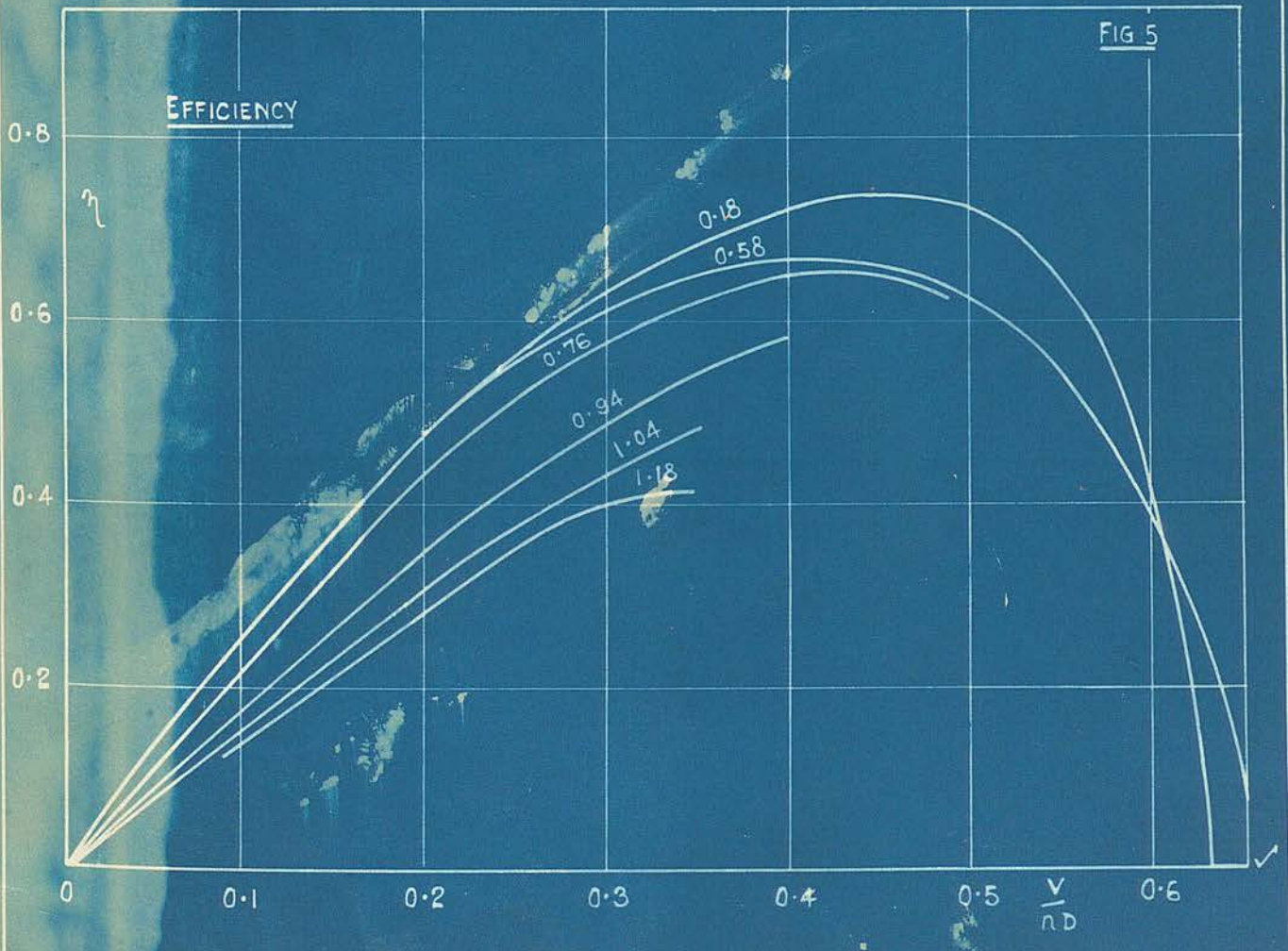
TORQUE

FIG 4



THE FIGURES ON THE CURVES ARE VALUES OF $\frac{\pi nd}{a}$

FIG 5



This has been calculated from the thrust and torque values, and is given in fig. 5. It appears that, for the airscrew tested, the efficiency falls off continuously as the tip speed increases, but the falling off is very slight if the tip speed does not exceed 75% of the velocity of sound.

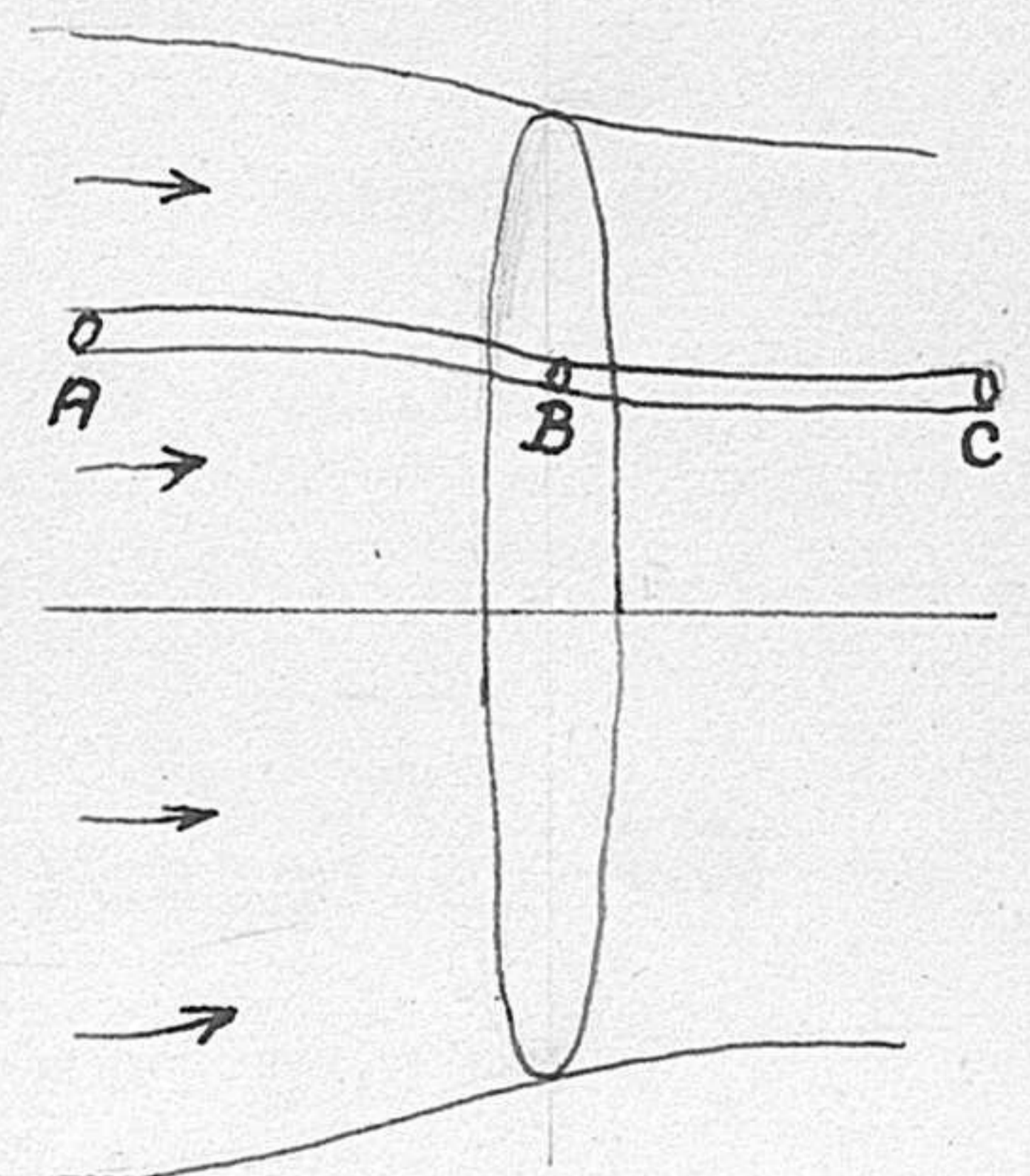
7. DISTRIBUTION OF THRUST ALONG THE BLADE.

From the thrust coefficient curves in fig. 3 we can see the general effect of high speeds on the thrust, but we shall obtain a much clearer idea of what is happening if we can find out how the thrust varies along the blade. Even at the highest tip speeds attained only 15% of the blade is travelling above the velocity of sound so that any special effects occurring above this speed would be partially concealed by the fact that the remainder of the blade was behaving in a more normal manner. Mr. McKinnon Wood drew the author's attention to a method originally used by Dr. Staunton which offers a very convenient way of obtaining the thrust distribution.

Following Froude's theory we regard the circle in which the airscrew is rotating as a disc of discontinuous pressure.

For any stream tube passing through the airscrew disc, the total energy at C will be equal to the total energy at A plus the pressure energy added at B. A pitot tube measures

the total energy in the stream tube so all we have to do is to fix up pitot tubes at A and C and the difference in



head

head will give us the pressure due to the thrust of the blade. The pressure will be constant for any given radius.

If P be the value of this pressure at radius x the total pressure over the elementary annulus at this radius will be $P \times 2\pi x dx$. This must be equal to the thrust, dT , given by the portions of the blades rotating in this annulus. -

$$\text{hence } dT = P \times 2\pi x dx.$$

$$\text{or } \frac{dT}{dx} = P \times 2\pi x$$

Since $T = k_T \rho n^2 D^4$ and $r = \frac{x}{D}$, we can transform the above into the non-dimensional form

$$\frac{d k_T}{dr} = \frac{P}{\rho n^2 D^4} \times 2\pi r.$$

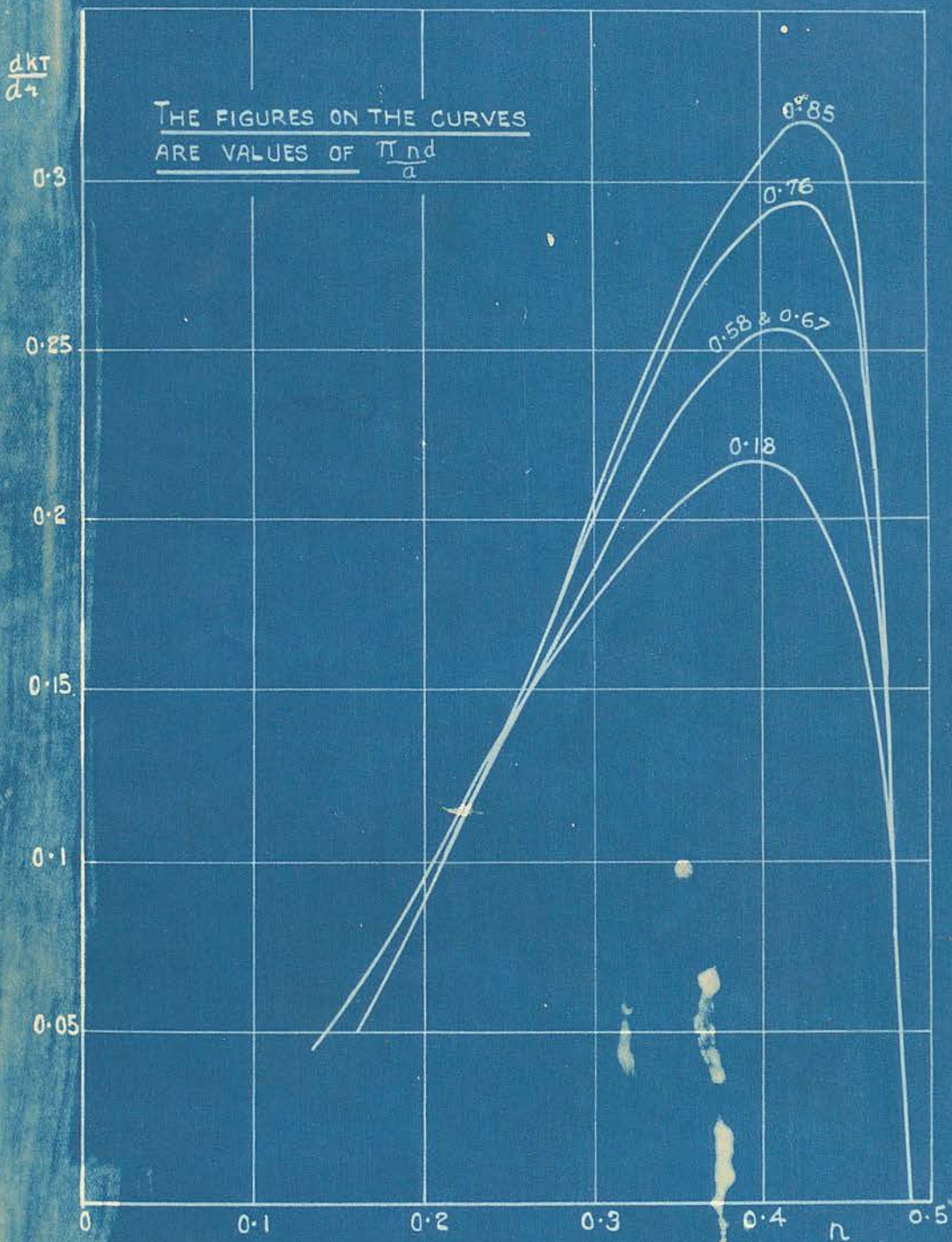
$\frac{d k_T}{dr}$ gives the thrust loading on the blade at any radius.

In making use of this idea we can take advantage of the fact that the position of the pitot tube at A is not important as the pitot pressure should be constant in front of the airscrew disc. The stream tubes curve after passing through the airscrew, and we require to know at what radius the stream tube which arrived at C passed through the airscrew disc. In the above formula " x " is the radius of B. To meet this difficulty small neat pitot tubes were used and mounted as close behind the blade as possible so that the radius at C would not differ appreciably from that at B.

The battery of pitot tubes arranged behind the airscrew is shown in Photo 1.* The pitots and one pitot tube set in front of the airscrew were connected to vertical glass tubes containing coloured water, and

THRUST GRADING LOWER SPEEDS

$$\frac{v}{nD} = 0.34$$



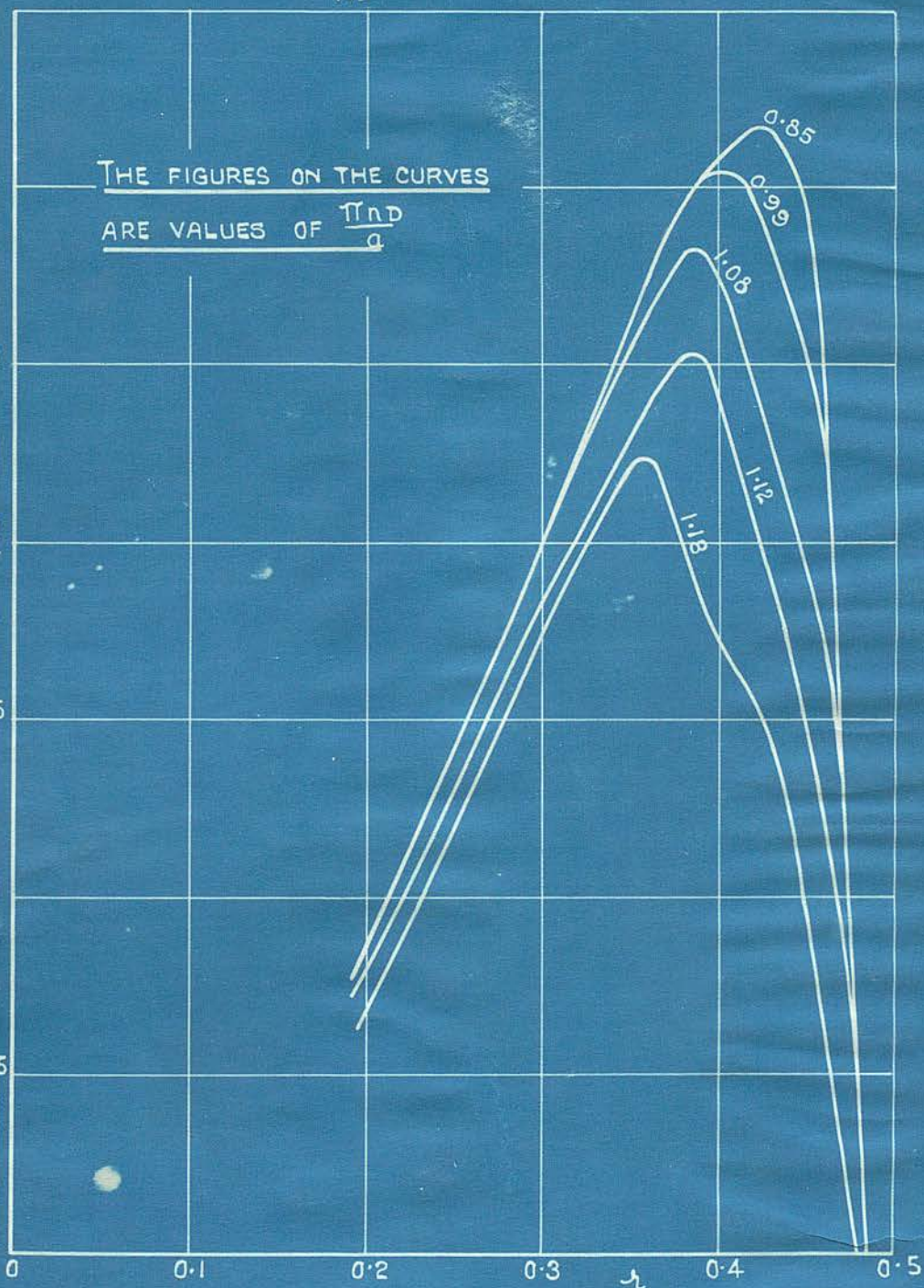
THRUST GRADING HIGHER SPEEDS

$$\frac{V}{nD} = 0.34$$

$\frac{dkt}{dr}$

0.3
0.25
0.2
0.15
0.1
0.05

THE FIGURES ON THE CURVES
ARE VALUES OF $\frac{\pi n D}{a}$



readings of pressure were obtained by photographing the columns in the tubes against a background of squared paper. For the low speed tests a sensitive tilting monometer was employed.

The results of the pitot exploration are given in table 4 and in figs. 6 and 7 they have been plotted out for $\frac{V}{nD} = 0.34$. Each curve shows how the thrust is distributed along the blade at a given tip speed. If there were no change in the flow pattern at the different speeds all the curves would, of course, coincide. Actually the coefficients increase fairly steadily, as in fig. 3, until the tip speed reaches 0.85 of the velocity of sound; a further increase of speed causes a falling off which travels inwards from the tip. Owing to the play in the thrust measuring mechanism the gap between the blade and the pitot tube could not be reduced below 0.25 inch; this introduces a slight uncertainty just at the tip.

The area enclosed by each curve i.e.

$$\int_{r=0}^{r=0.5} \frac{dk_T}{dr} dr$$

ought to be equal to the measured thrust obtained in the first test.

Fig. 8 shows a cross plot from fig. 3 at $\frac{V}{nD} = 0.34$ and also shows the thrust obtained by integrating the thrust

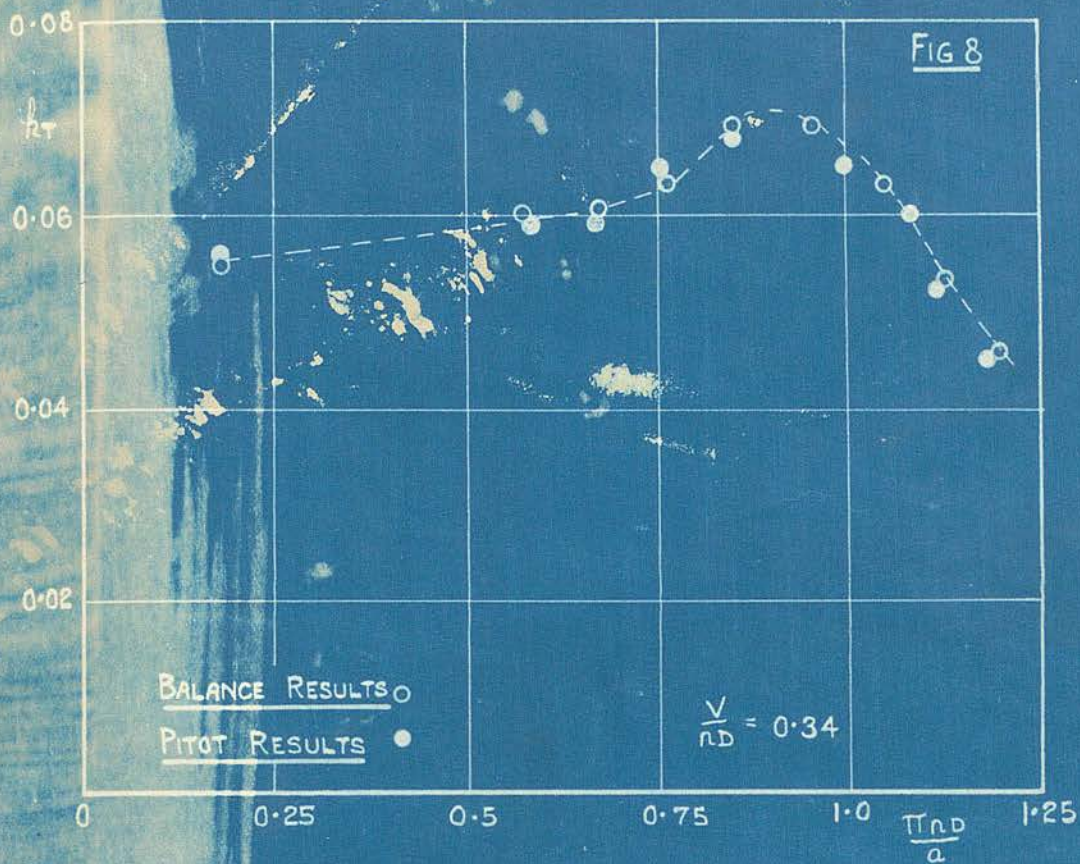
distribution curves. The results are in good agreement up to the highest tip speeds.

Fig. 8 shows similar cross plots for torque and efficiency from Figs. 4 and 5.

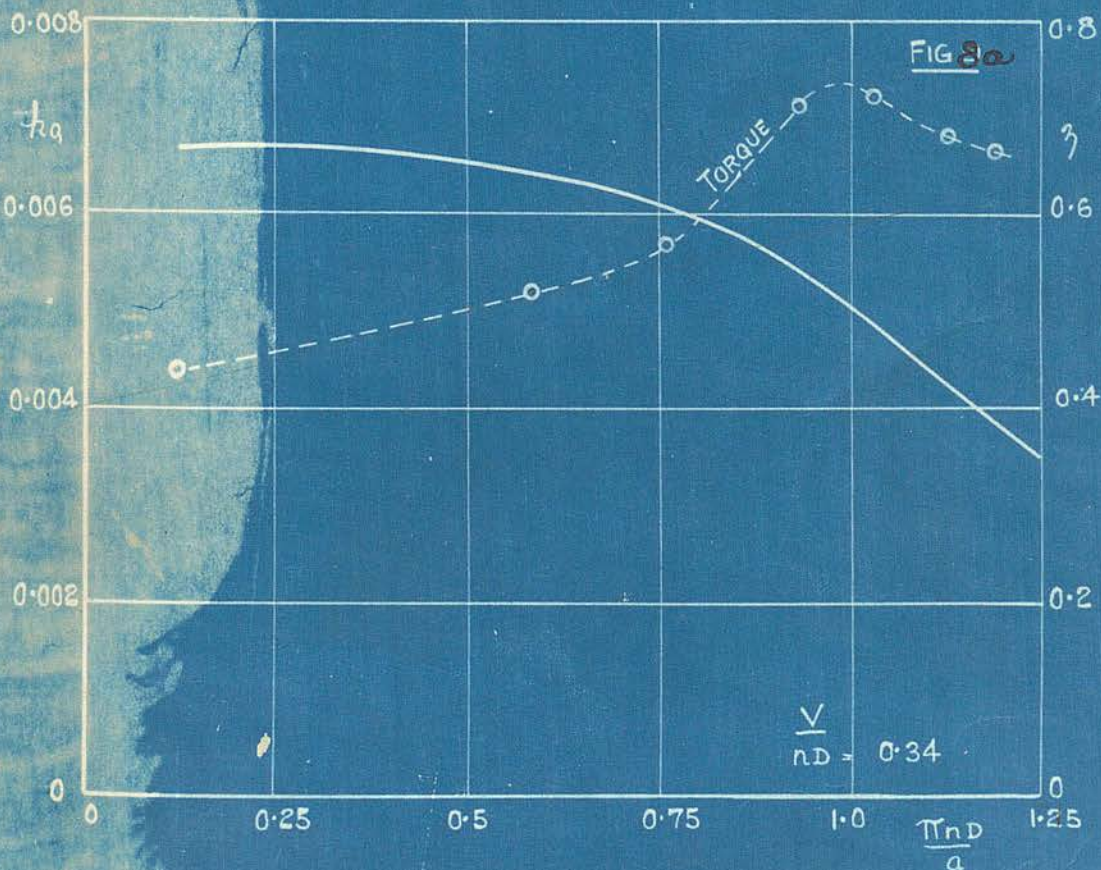
8. MEASUREMENT OF TWIST.

The question of the collapse of the thrust on the portion of the blade travelling above the velocity of sound (fig. 7) required further investigation. With the results of the previous experiments (Section 3)

THRUST



TORQUE AND EFFICIENCY



in mind there was a tendency to regard it as wholly due to aerodynamic speed effect but the possibility of blade twist could not be excluded. To clear up the matter it was decided to measure the blade twist.

As any object attached near the tip of the blade would be subject to a centrifugal force of 50,000 g the attachment of a mirror did not seem promising. The photographing of the blade from the side when normal to the camera axis was also considered. The width of the image would give a measure of the twist. The method would show the deformation of the whole blade in one photograph but any lack of clearness due to vibration would render the interpretation of the results very doubtful.

The method finally adopted was to fix up a camera with its axis pointing radially along the blade and to photograph a chord line which was illuminated electrically (as with the oscilloscope) as it passed in front of the camera. The method involved a high degree of synchronism in the electrical illumination and it was necessary to take a separate photograph to determine the twist at each radius, but the results obtained were very satisfactory and with care the twist could be measured to 0.1° . Photograph No. 1 shows the airscrew mounted on the turbine in the wind channel with the photographic apparatus in position. Two of the chord lines at which the twist was measured can be seen on each blade on the airscrew. The camera is shown at A and at B are the illuminating spark gaps.



Photo. 1.

FIG 9

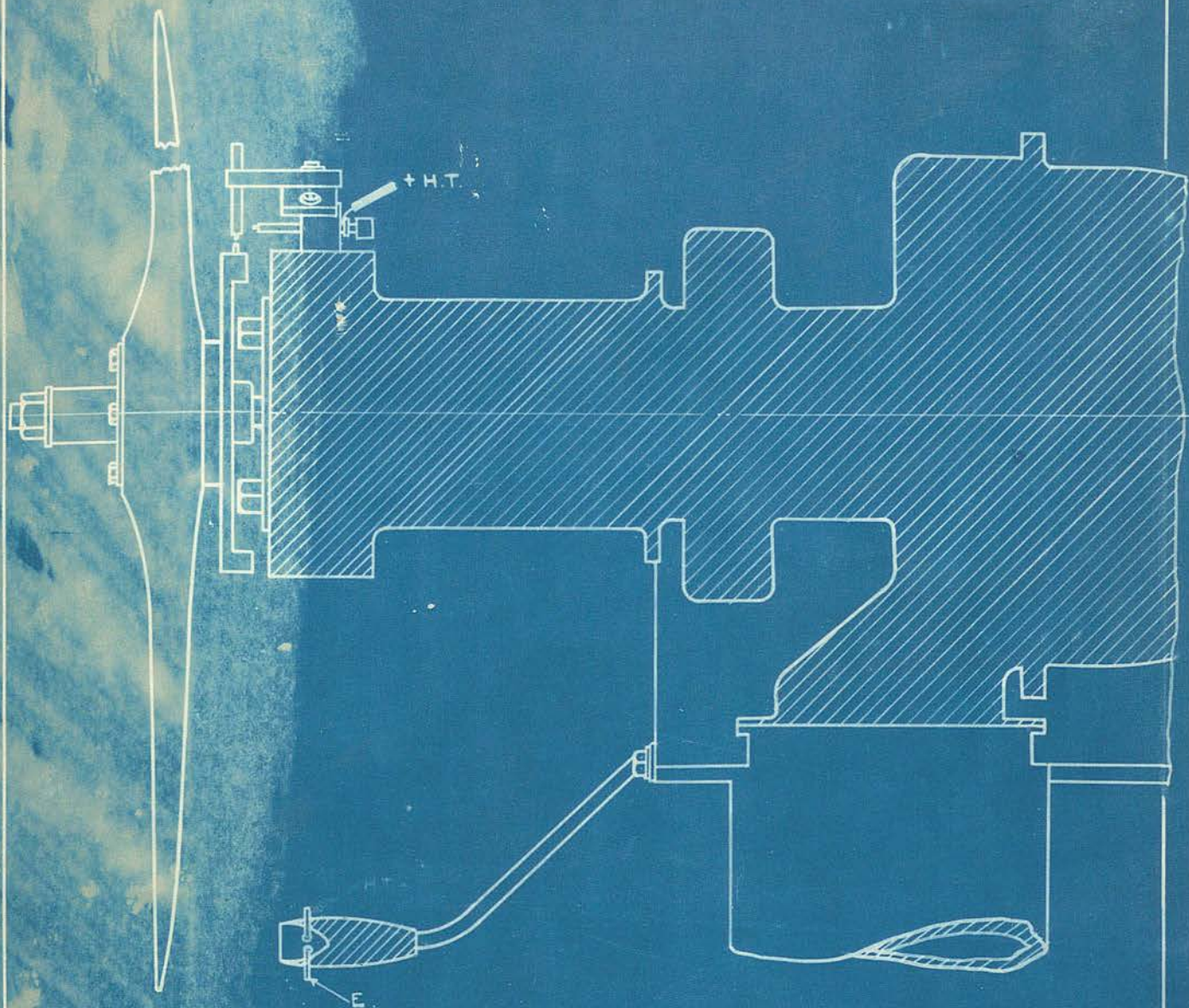
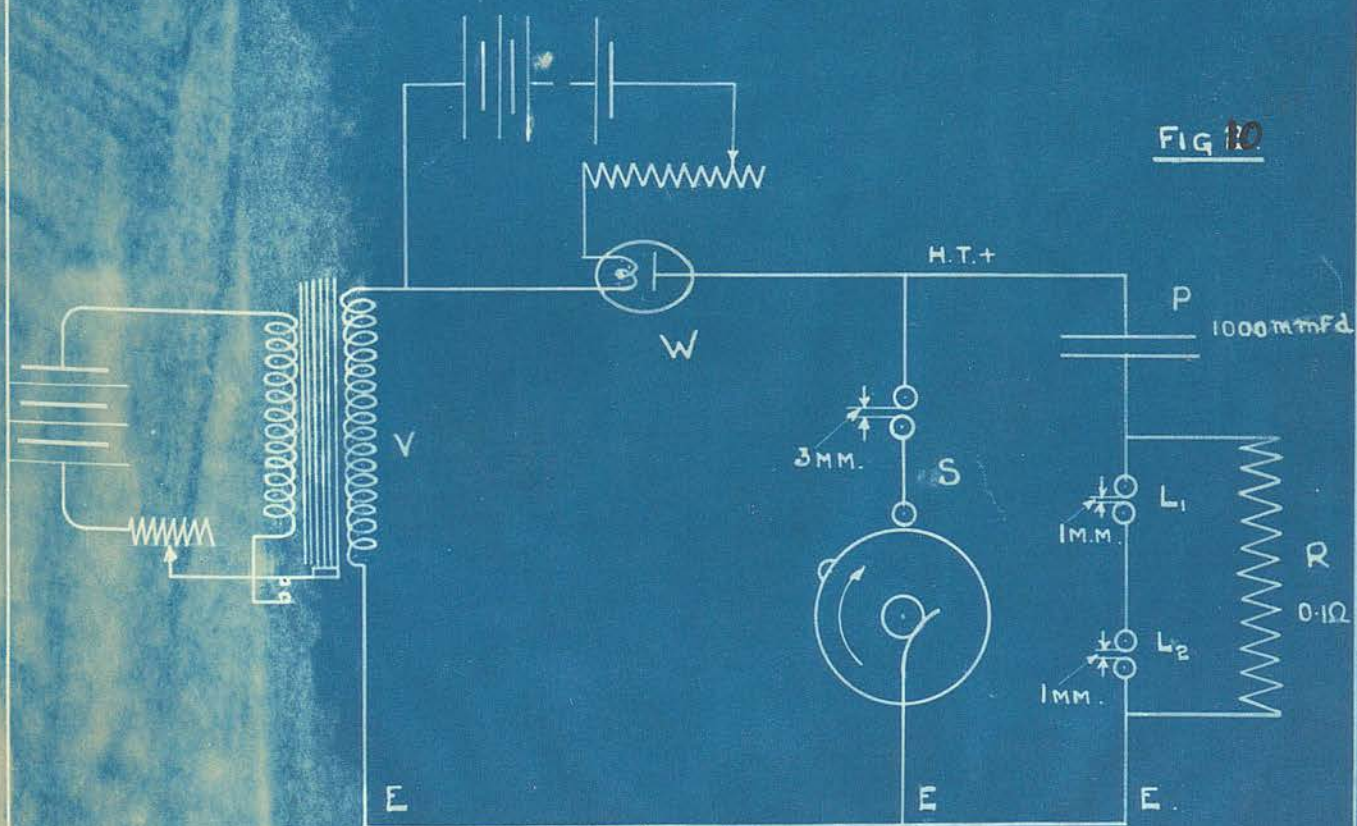


FIG 10



9. ILLUMINATING APPARATUS.

The electrical circuit was suggested by Dr. G.E. Bairsto of the Electrical Research Section, Royal Aircraft Establishment, who designed the electrical details to meet the author's requirements. Figs. 9 and 10 show the details in the channel and the electric circuit arrangement. The current from the secondary of the spark coil V is rectified by the valve W and charges the condenser P through a resistance R to a given voltage depending on the current in the primary winding of the spark coil. This will also be the voltage between the sparking points of the rotating gap S. As the moving point rotates towards the fixed point, they eventually come sufficiently close to allow a spark to pass between them, this distance being nearly constant for a given voltage difference. The illuminating gaps L_1 and L_2 are in series with the rotating gap and as the instantaneous rush of current prefers to jump the gaps rather than pass through the high resistance R, we get a simultaneous spark there. In order that the spark shall always occur at the same point of the revolution it is necessary to adjust the voltage so that a spark can just pass between the rotating gap points when they are at their minimum distance apart. To make this minimum distance as sensitive to position as possible, the rotating gap was cut down to a few thousandths of an inch and an auxiliary gap introduced in series with it. When the turbine was running at 10,000 r.p.m. and the voltage was properly adjusted, the illuminating spark did not vary more than 0.1° of a revolution.

Fig. 9 shows the details of the channel fittings. The illuminating arrangement consists simply
of

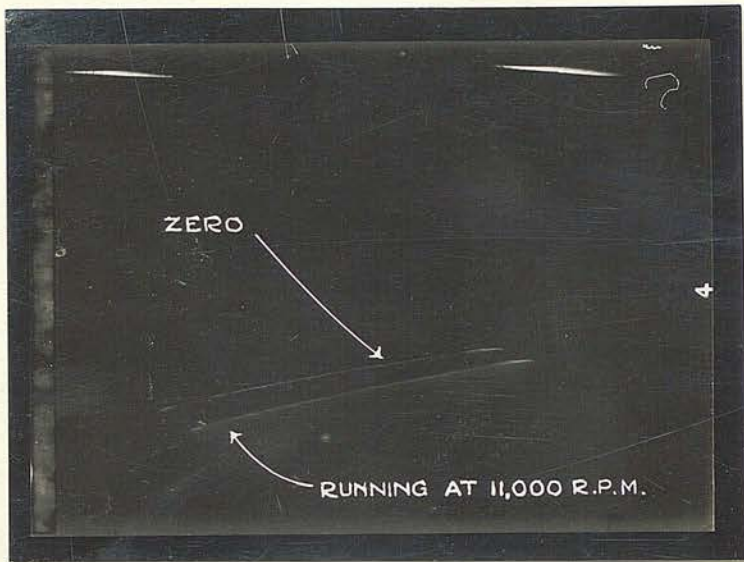


Photo. 2.

of two plain spark gaps in streamline ebonite holders, which were painted white inside to reflect as much light as possible. The two spark gaps were used to ensure adequate illumination at the ends of the chord line. This illuminating arrangement is very simple and the light, although not very bright, is of high actinic value.

10. PHOTOGRAPHIC DETAILS.

The camera was clamped to the air intake pipe with its axis vertical. As there is considerable vibration at high speeds both of the channel and of the turbine, a very rigid camera mounting, as shown, was used to ensure that there would be no relative movement between the camera and turbine. The camera was designed to give approximately a full size image of the chord line. Kodak film packs were used for convenience in changing.

A white line was fixed on the blade at each section where a measurement of twist was required. In order that the line should be clearly visible when looking along the blade, it was necessary that it should project slightly from the surface, so the procedure adopted was to twist together a few elementary fibres of silk and to tie this tightly round the blade. The silk was then given a thin coat of clear shellac varnish to hold it in position, and when the varnish had thoroughly set, the line was given a thin coat of nitro-cellulose white paint. A very fine sharply defined line was thus obtained as shown in photo 2. The method of attachment proved very satisfactory; when run up to 12,000 r.p.m. the thread at the blade tip showed a tendency to break away from the shellac in the middle, but the ends remained fast and so there was no uncertainty as to the angle.

The procedure in using the apparatus was as follows:-

A "zero" photograph was first taken with the airscrew at rest (photo 2.) the same apparatus being used to illuminate the line as when running. The channel and turbine were then started up and when conditions had become steady at the required speeds, the camera shutter was opened and an exposure of about 30 sec. given. The revolution counter was under the observation of the camera operator all the time so that the shutter could be closed at once if the speed became at all unsteady. The "crescents" at the top of the photograph are the edges of the illuminating spark gap reflectors. Any relative movement of the camera or film and the turbine is shown by a double image of these. To measure up a record, a second "zero" film attached to a long straight edge was superposed upon it, and moved about until parallel in turn to each of the chord lines on the film, the change in angle of the straight edge giving the twist of the blade.

The blade angles of the airscrew were measured up on the marking out table before and after each series of tests.

The apparatus worked very satisfactorily throughout the tests and twelve to twenty photographs could be obtained in an hour. The only trouble experienced was the pitting of the platinum points of the sparking coil contact breaker causing variations in the H.T. supply voltage. The use of a mercury contact breaker would eliminate this trouble. The voltage adjustment

adjustment was rather critical and when in the channel it was rather difficult to observe when the adjustment was correct. It is estimated that in general the error in angle does not exceed 0.1° . An asterisk after any angle given in the tables indicates that the record was not sharply defined and was liable to an error about 0.25° .

11. RANGE OF TESTS.

The blade twist was measured at three radii on each blade at one value of $\frac{V}{nD}$ for a range of tip speed from 630 to 1200 ft./sec/. The value of $\frac{V}{nD}$ was 0.34 as in the previous tests, and the values of r were 0.32, 0.40 and 0.46.

Blade twist at one section ($r = 0.40$) was investigated over the same range of tip speeds for values of $\frac{V}{nD}$ from 0.26 to 0.40.

The blade twist results at the three radii are plotted in fig. 11. The heavy lines show the variation of twist of each blade with velocity. The twist increases steadily with velocity until the blade tip has reached 90 to 100% of the velocity of sound after which it decreases rapidly. The difference in the twist angles of the two blades will be noticed; at the largest radius this difference varies from 0.4° below the velocity of sound to 1.5° above that velocity.

To see to what extent the measured blade twist accounted for the observed variation with speed of the thrust loading coefficients the twist required to account for the whole variation has been calculated by the method given in § 13.

The

VARIATION OF TWIST WITH VELOCITY.

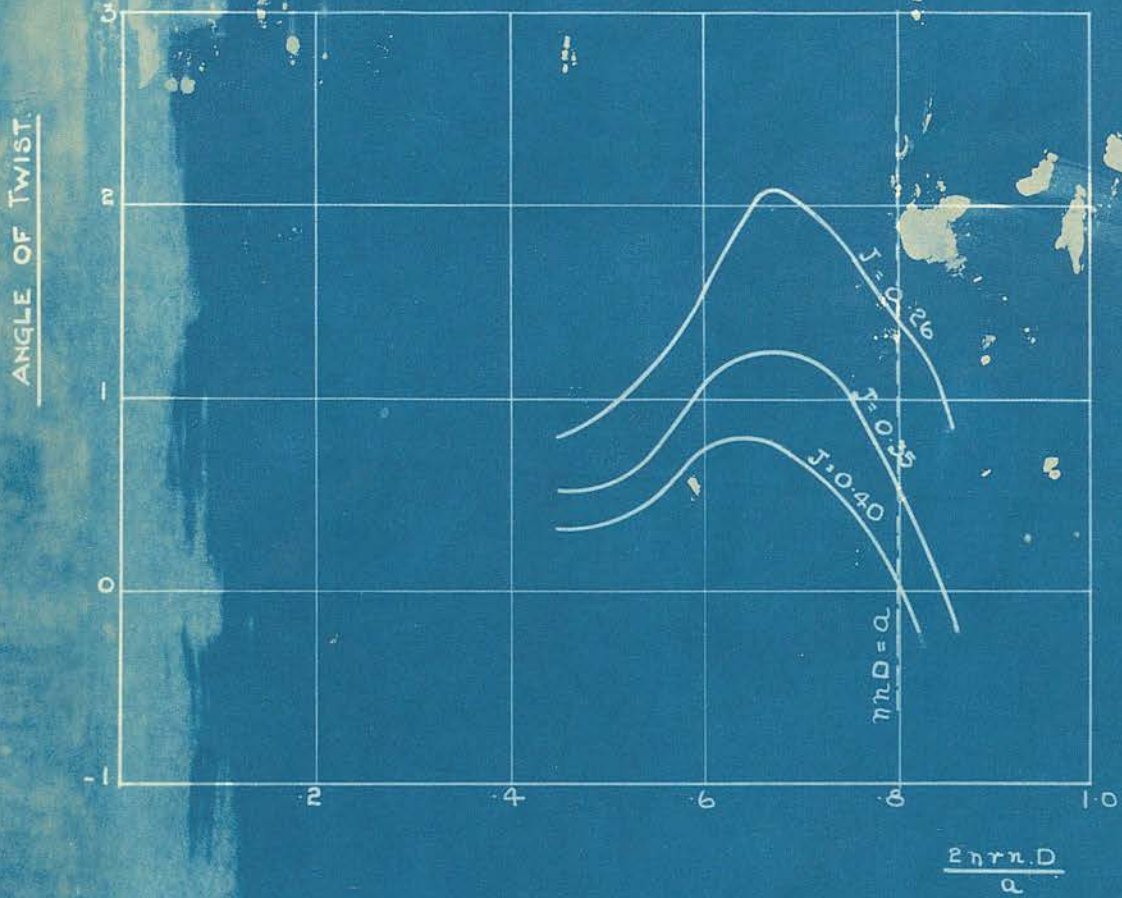
$J = 0.34$



- BLADE 1 (MEASURED)
- BLADE 2 (MEASURED)
- x TWIST REQUIRED IF VARIATION OF THRUST WERE ENTIRELY DUE TO TWIST.

VARIATION OF TWIST WITH $\frac{v}{\pi D}$

$\gamma = 0.4$



The results are shown by the dotted curves in fig. 11. Comparing these with the measured values it appears that very roughly one third of the variation is due to blade twist.

Fig. 12 shows the mean twist of the two blades for three values of $\frac{V}{nD}$. The twist increases steadily as $\frac{V}{nD}$ is decreased i.e. as the thrust increases. The maximum twist recorded is just over 2° .

12. THRUST GRADING DETERMINATION.

For a full analysis of the tests thrust distribution and blade twist are both required. To reduce uncertainty due to possible warping of the blades since the previous test, the measurement of thrust grading was repeated at the same time as the above blade twist measurements.

The method of obtaining the thrust grading coefficient $\frac{dk}{dr} T$ has already been described. For the present experiment the bank of pitots (C, Photo 1) was fixed in the same position relative to the turbine as before, except that the pitots were brought up closer to the back of the blade to reduce the uncertainty of position of the thrust grading points. By locking the floating thrust gear this distance could be safely reduced to 1/10 inch. The thrust grading coefficients were deduced from the difference in pitot pressure at the bank of pitots and that at a forward pitot set up about 12 feet in front of the turbine. When the channel was running with the airscrew removed, a difference of pressure was observed between the forward pitot reading and that at the bank of pitots. The correction was measured at 100 ft/sec. and assumed to vary as V^2 . The correction to $\frac{dk}{dr} T$ was $0.25 r \left(\frac{V}{nD}\right)^2$

The twist of the airscrew blades was measured at a different portion of the revolution from the thrust grading. The analysis in the appendix shows that no serious error would be expected on this account.

13.

ANALYSIS OF RESULTS.

The author has analysed the above results to show the variation with speed of the lift coefficient of the airscrew blade section. Practically the whole thrust of a blade element and 80% of its torque ^{at low speeds.} depend on its lift coefficient so that, apart from distortion, the knowledge of the variation of the lift coefficient gives the designer the greater part of the information he requires for the design of high speed airscrews.

The equation to calculate the lift coefficient from the observations is derived as follows -

- $\varphi =$ angle of resultant wind to the plane of rotation.
- $c =$ blade width as a fraction of the diameter.
- $r =$ radius as a fraction of the diameter
- $k_L =$ lift coefficient = $\frac{\text{(Force normal to resultant velocity)}}{\rho (\text{resultant velocity})^2 \times \text{area.}}$
- $k_D =$ drag coefficient.

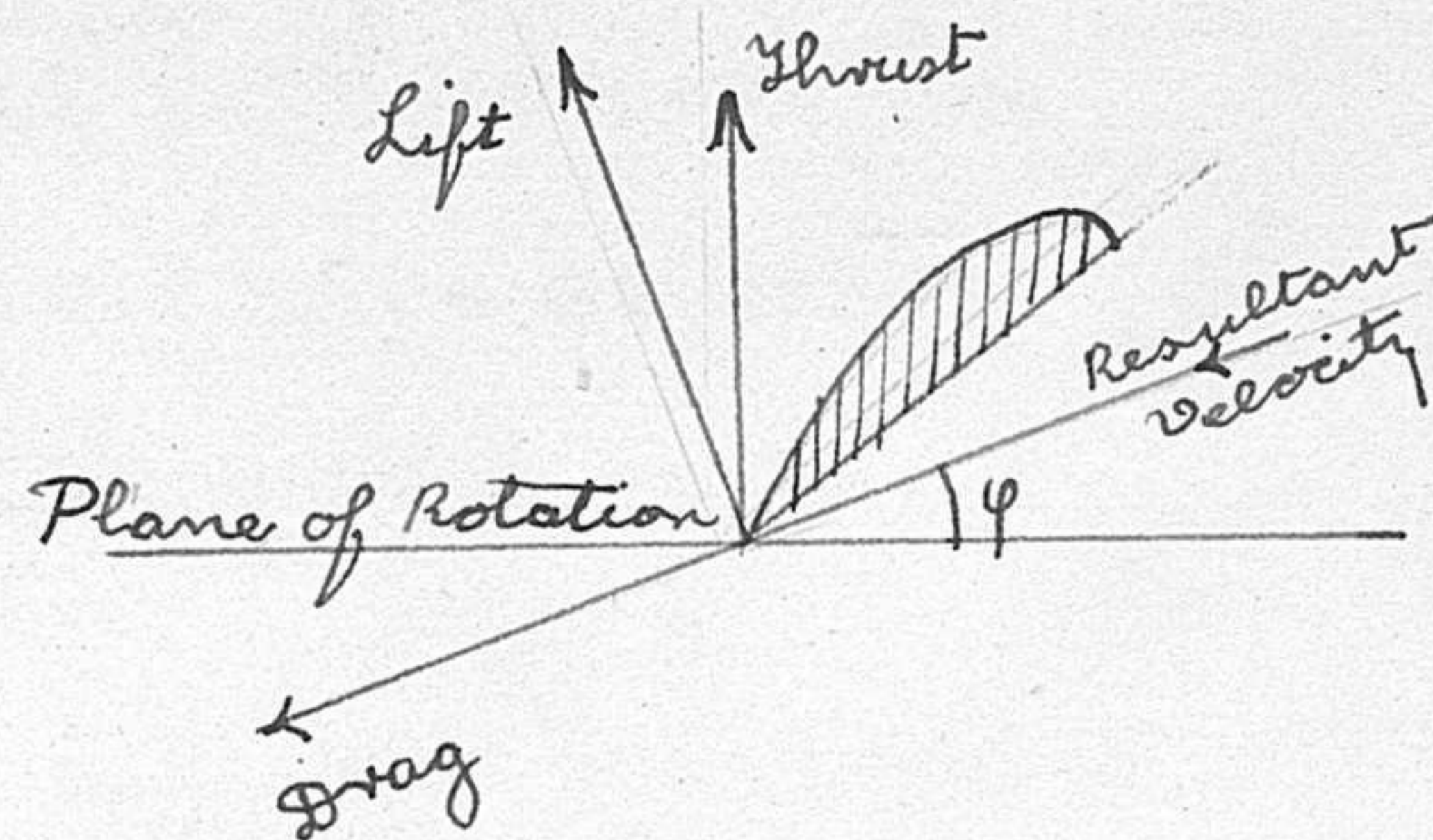
The resultant velocity will

$$\text{be } \frac{2\pi n r D}{\cos \varphi}$$

hence for an element of blade

$$\begin{aligned} dT &= \rho n^2 D^4 c (k_T) \\ &= \rho \left(\frac{2\pi n r D}{\cos \varphi} \right)^2 c dr 2 D^2 (k_L \cos \varphi - k_D \sin \varphi) \end{aligned}$$

$$\therefore \frac{dk_T}{dr} = 8\pi^2 r^2 c (k_L - k_D \tan \varphi) \sec \varphi \quad (1)$$



to which $\frac{d k_T}{dr} = 8\pi^2 r^2 c k_L$ is a good approximation for an airscrew of fine pitch.

To calculate the angle of incidence an application of the Froude momentum theory due to Mr. McKinnon Wood has been used.

It was shown in § 7 that $\frac{d k_T}{dr} = \frac{P}{\rho n^2 D^4} \times 2\pi r$ where P is the pressure over an elementary annulus. Now if the air velocity through the annulus is $V(1+a)$ and its final increase of velocity is $2Va$

$$\begin{aligned} P &= \rho V(1+a) \times 2Va \\ &= 2\rho a(1+a) \left(\frac{V}{nD}\right)^2 n^2 D^2 \\ \frac{d k_T}{dr} &= 4\pi r \left(\frac{V}{nD}\right)^2 a(1+a) \end{aligned}$$

or using J the usual symbol for $\frac{V}{nD}$

$$\frac{d k_T}{dr} = 4\pi r J^2 a(1+a) \quad (2)$$

from the definition of ψ

$$\tan \psi = \frac{V(1+a)}{2\pi n D r} = \frac{J}{2\pi r} (1+a) \quad (3)$$

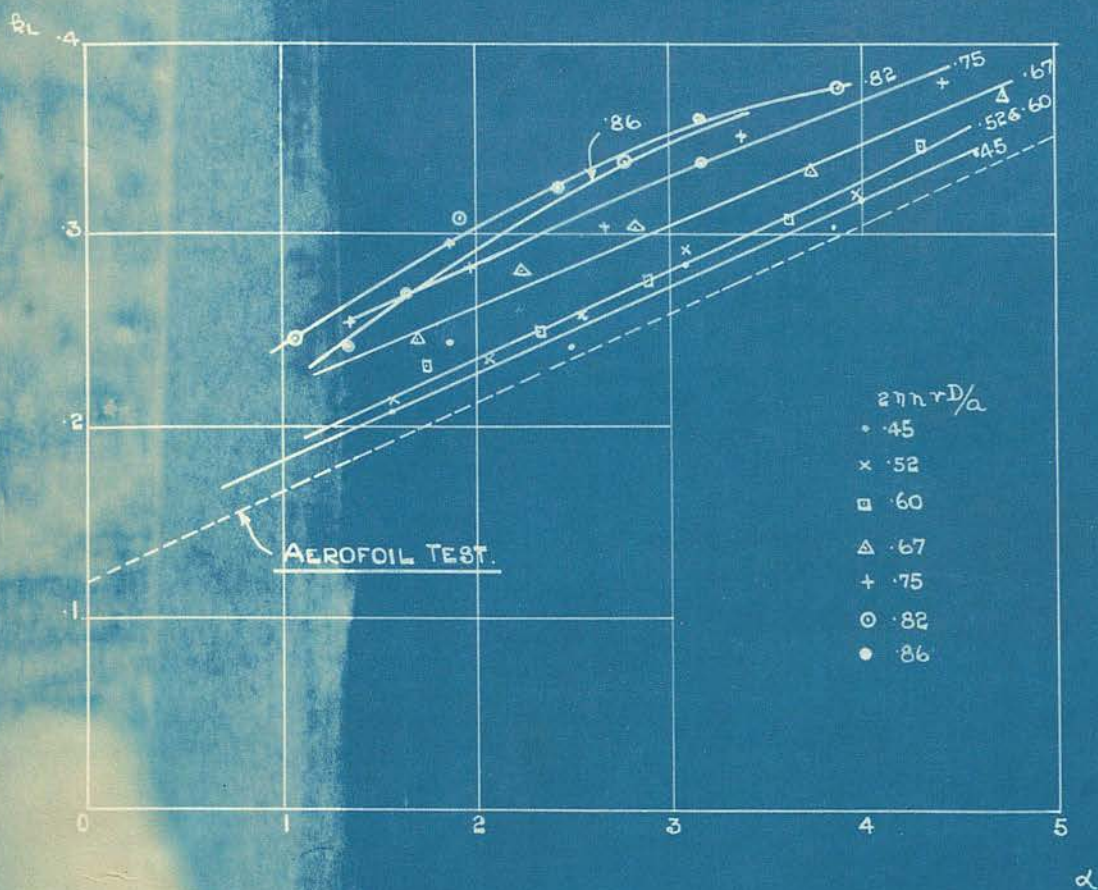
$$\text{and } \alpha = \theta - \psi + \text{twist.} \quad (4)$$

where α is the angle of incidence and θ is the angle of the blade element to the plane of rotation.

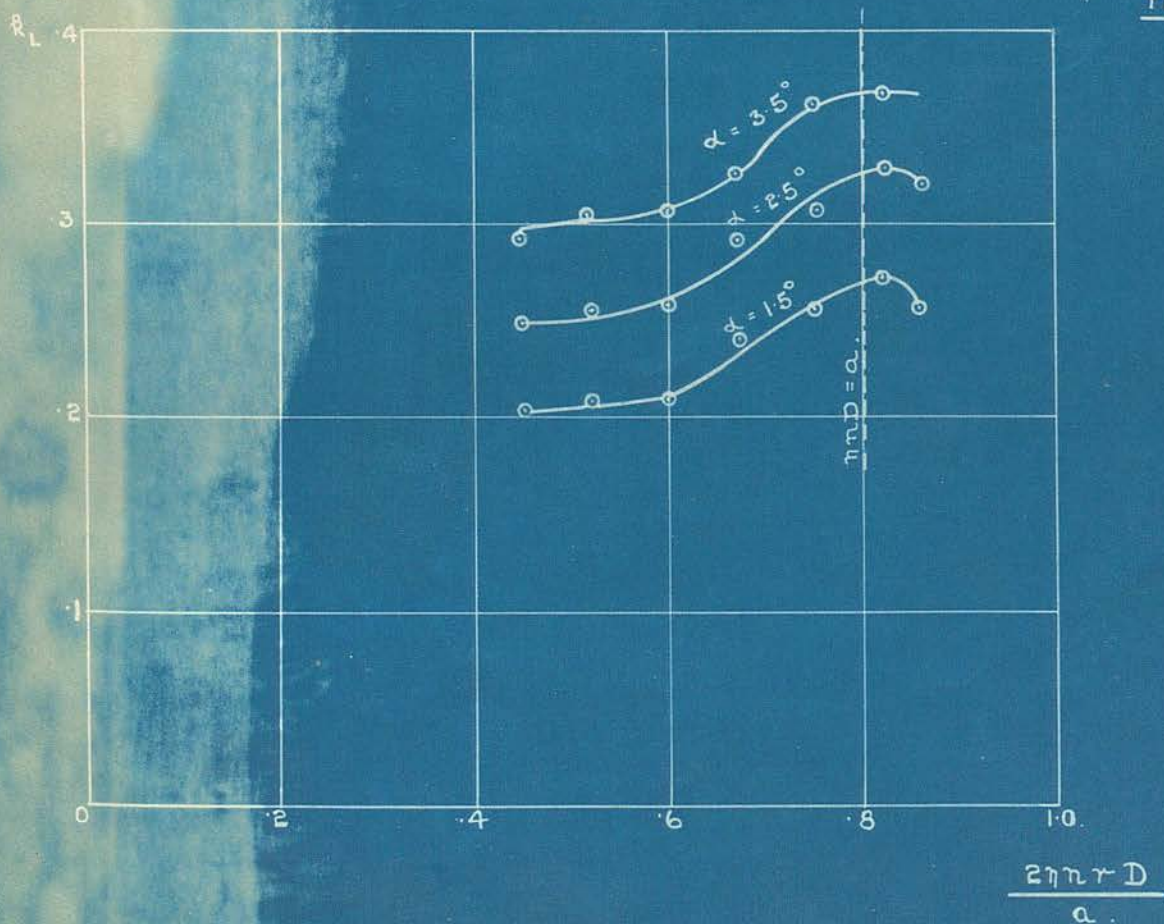
As shown by Glauert it follows from the Lanchester Prandtl theory that the incidence so calculated corresponds to an aerofoil of infinite aspect ratio.

In using the equations k_L was obtained by equation (1) from the $\frac{d k_T}{dr}$ values given in tables 8 + 9 .

SPEED EFFECT ON R_L . (ASPECT RATIO ∞)



SPEED EFFECT ON R_L .



To calculate incidence we first solve for " α " by equation (2) and then obtain ψ by equation (3). In equation (4) θ is the angle of the blade as given in fig. 2 corrected for error as given in table 10. The error was found by measuring up the blade on the marking out table before and after each test. The twist is the mean twist of the two blades.

The results of the analysis of the measurements given in tables 8, 9 are shown in figs. 13 and 14. They are all for the blade section at $r = 0.4$. In fig. 13, lift coefficient is plotted against incidence at different values of the circumferential velocity of the blade section. The dotted line shows the comparative values obtained from an ordinary wind channel test of this aerofoil section. All the results are for infinite aspect ratio. The curves are approximately parallel to that obtained from the aerofoil test and the lift coefficient increases with speed until we exceed 0.8 of the velocity of sound. In fig. 14 the results from fig. 13 have been cross plotted to show more clearly the speed effect on k_L at constant incidence. As the wind speed increases for 0.6 to 0.8 of the velocity of sound the lift coefficient rises rapidly; for a further 10% increase of speed there is no indication of a sudden falling off of the lift coefficient.

14. TRAVEL OF CENTRE OF PRESSURE.

The reversal in gradient of the twist curves as the tip speed approaches the velocity of sound must be due to a change in the resultant twisting moment of the

centrifugal and air forces. The centrifugal twisting moment is the product of the centrifugal force, the tilt of the blade and the distance from the centroid of the section to the flexural centre. The product of the centrifugal force and blade tilt (ψ) is proportional to $n^2 \psi$ where ψ is of the form $a + b n^2$; a is proportional to the initial tilt, and b is approximately proportional to k_L . Hence, if a and b are both positive, the centrifugal twisting moment must increase at an increasing rate with n . The blade was designed with an initial forward tilt so that the centrifugal bending moment should partially balance the air force bending moment. At the same J the forward tilt therefore increases with speed as is shown by the photographs. The twist depends on the moments due to the centrifugal and air forces and since the centrifugal twist is increasing more rapidly than n^2 it follows that the falling off of the twist curves from the parabolic curve indicates a backward travel of the C.P.

The interconnection of the blade elements make it difficult to see, from the twist curves, how the twisting moment at a particular section of the blade is varying with velocity. Some idea of the variation of twisting moment at one portion of the blade may be obtained from the difference between the twist angles shown in ^{table 7} ~~fig. 11~~ for the sections at $r = 0.40$ and $r = 0.46$. The differences are given in the table below. They will be proportional to the twisting moment of the centrifugal and air forces on the portion of the blade between $r = 0.4$ and $r = 0.5$.

$2\pi n r D$	@ $r = 0.46$	0.53	0.68	0.86	0.90	0.94	0.98
$(\delta\theta @ r = 0.46) - (\delta\theta @ r = 0.40)$		0.2°	0.6°	0.4°	0.15°	-0.35°	-0.75°

15.

CONCLUSION.

In the foregoing tests, a method has been established by which the lift incidence curves of aerofoils can be measured at speeds up to the velocity of sound and there is no reason why, with a more powerful turbine, the scheme should not be used up to much higher speeds. The only limit to the method is the ultimate strength of the airscrew. The wood airscrews stood up to the stresses remarkable well, but wood is liable to warp in storage and does not lend itself to mathematical analysis of distortion. Using a model airscrew, ^{of duralumin or} ground from "100 ton" steel we should get rid of these troubles and could extend the investigation to 20 to 50% higher speeds. Only the lift variation has so far been determined, but, until the flow is seriously modified by compressibility, analysis shows that this accounts for ^{practically the whole thrust and 80% of the torque} the airscrew tested. If we could measure the torque grading along the blade to correspond with the thrust grading curves of this report we could analyse the results to show the effects of high speeds on drag. It is hoped to obtain these from measurements of the rotational momentum in the slip stream. The remaining important factor is the travel of the centre of pressure; with a steel airscrew it should be possible to obtain this from an analysis of the blade twist.

From the present tests we can arrive at the following conclusions which will apply to similar airscrew sections of about 0.1 camber for incidences from 1° to 4° :-

- (1) As the relative wind speed increases from 0.6 to 0.8 of the velocity of sound the lift coefficient rises rapidly while the k_L incidence curves remain approximately parallel. For a further 10% increase of speed, there is no indication of a sudden falling off of the lift coefficients.

- (2) As the speed approaches the velocity of sound the centre of pressure moves back and, from the torque curves, it appears that the drag coefficients increase.

For the airscrew tested we can conclude:-

- (1) Of the total variation with speed of the thrust grading coefficients approximately one third is due to blade twist.
- (2) The thrust grading curves collapse towards the blade tip as the tip speed approaches the velocity of sound. This collapse appears to be due primarily to the reduction of incidence caused by the negative twist of the blade tip and not by a falling off of the lift coefficient.

Note B.
Since the stresses in an airscrew depend on the tip speed it would appear that, given good material and workmanship there should be no difficulty in constructing full scale airscrews of wood to run at tip speeds up to the velocity of sound.

APPENDIX I.CONSTANCY OF BLADE TWIST THROUGHOUT THE REVOLUTION.

As the twist of the airscrew was being measured at a different portion of the revolution from the thrust grading, it was necessary to verify that the mean twist of the blades did not change appreciably between the two positions. A control pitot was fixed at B (Photo 1) and the thrust grading was measured there simultaneously with that at the bank of pitots. The thrust grading coefficient at B was found to be 8 to 11% higher than at the other position.

We can show that the difference in thrust at the two points can be almost entirely accounted for by the slowing up of the air at the control pitot.

The change in thrust $\delta \frac{dk_T}{dr}$ at the two points corresponds to a change in incidence $\delta \alpha$ which we can calculate as below

$$\frac{dk_T}{dr} = 8\pi^2 r^2 c k_L$$

and from fig. 13 it appears that $\frac{dk_L}{d\alpha} = 0.047$

$$\therefore \delta \alpha = \delta \frac{dk_T}{dr} \frac{1}{0.047 \times 8\pi^2 r^2 c k_L} \quad (1)$$

The table below gives the velocity distribution at the airscrew disc. The measurements were made with the airscrew removed at 60 f.p.s. and are given as a percentage of this speed.

r	At bank of pitots.	At control pitot. Illuminating apparatus removed.	Mean velocity through disc.
.32	98.8	93.3	96.8
.40	100.4	93.5	97.8
.46	100.8	93.7	98.2

The presence of the illuminating apparatus slowed up the air by another 3.6%.

The velocity change in the above tables corresponds to a change in incidence $\delta\alpha$ which can be calculated from the equations

$$\tan \phi = \frac{J}{2\pi r} (1+a)$$

$$y = \frac{dk_T}{dr} = 4\pi r J^2 a (1+a)$$

$$\alpha = \theta - \phi + \text{twist.}$$

and assuming that the twist does not vary from the one position to the other these reduce to

$$\delta\alpha = \frac{\delta y + 8\pi^2 r^2 \tan \phi \delta J}{8\pi^2 r^2 (J - 4\pi r \tan \phi) \sec^2 \phi} \dots (2)$$

$$\text{For } r = .4 \text{ and } J = .34 \quad \delta\alpha \text{ (degrees)} = -8(\delta y + 2.3\delta J)$$

The table below shows the values of $\delta\alpha$ found by the two methods:

R.P.M.		6000	8000	10000	11000
$\delta \frac{dk_T}{dr}$.014	.022	.020	.030
δJ		-.035	-.035	-.035	-.035
$\delta\alpha$ from Eq ⁿ . (1)		0.35°	0.5°	0.45°	0.7°
$\delta\alpha$ from Eq ⁿ . (2)		0.55°	0.5°	0.5°	0.4°

The value of $\delta \frac{dk_T}{dr}$ at 11,000 is rather uncertain; so that agreement is as good as could be expected. It appears that to the accuracy of the experiment the slowing up of velocity accounts for the whole difference.

APPENDIX II.COMPARISON WITH OTHER WORK.

The only published comparatively high speed test on the lift of aerofoils is given in Report No. 83 of the American Advisory Committee for Aeronautics. Using a special high speed wind channel they measured the lift of aerofoils up to 650 ft./sec. i.e. $\frac{V}{a} = 0.57$. The tests confirm the result shown in fig. 13 that there is practically no variation of K_L with incidence up to this speed.

Professor G.P.Thompson in his book on Aerodynamics gives the curve below for the lift and moment on a shell. It will be noted that the curve of variation of lift coefficient with incidence is of similar shape to those of fig. 14. At the velocity of sound the lift coefficient is decreasing with speed while the moment coefficient is increasing. This would give a backward movement to the C.P. - rather an interesting result in view of the deductions as to C.P. travel given in Section 14.

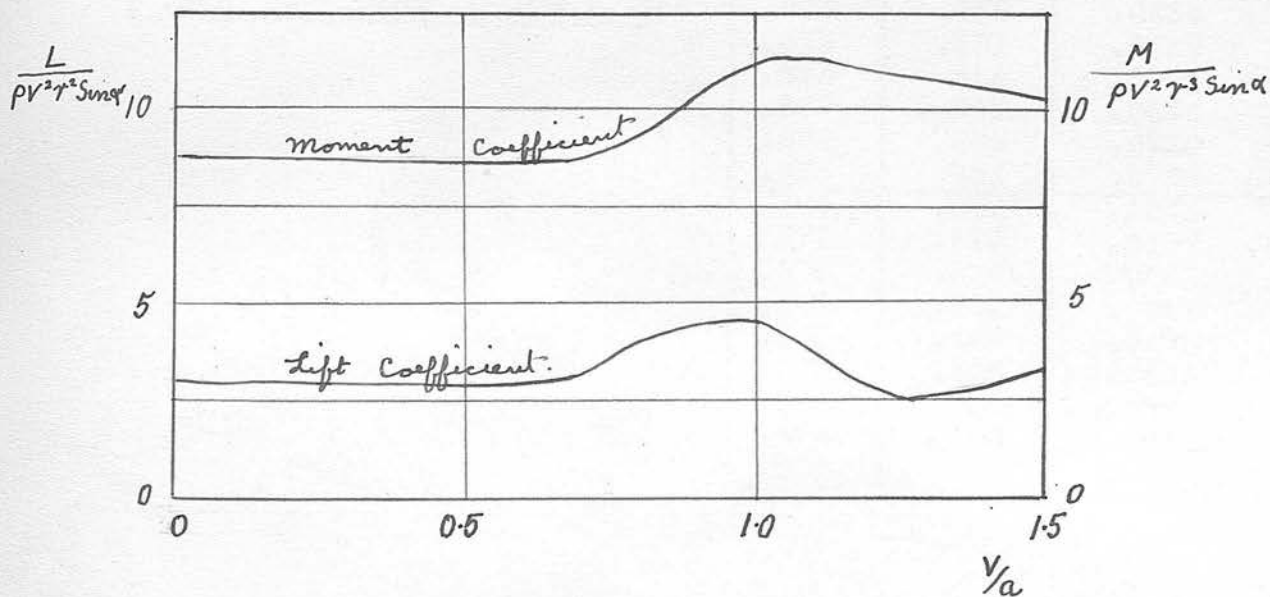


TABLE 1.

Model Airscrew. Thrust.

$\frac{\pi nD}{a}$	$\frac{V}{nD}$	k_T	$\frac{\pi nD}{a}$	$\frac{V}{nD}$	k_T
1.188	.346	.0466	.834	.485	.0392
1.198	.340	.0458	.849	.475	.0393
1.168	.328	.0524	.844	.441	.0478
			.844	.441	.0477
1.130	.360	.0480	.849	.406	.0553
1.120	.352	.0469	.844	.398	.0565
1.130	.338	.0530	.844	.381	.0626
1.131	.318	.0592	.844	.353	.0686
1.120	.295	.0652	.844	.353	.0654
			.849	.350	.0680
1.038	.396	.0504	.844	.323	.0741
1.037	.393	.0495	.849	.322	.0746
1.041	.361	.0564	.854	.317	.0745
1.041	.344	.0615	.858	.220	.0870
1.038	.318	.0674	.858	.194	.0906
1.062	.284	.0729			
1.037	.266	.0780	.765	.532	.0254
1.052	.262	.0780	.765	.488	.0344
1.037	.244	.0817	.765	.457	.0409
1.052	.239	.0822	.765	.420	.0483
1.028	.215	.0869	.760	.390	.0572
1.028	.189	.0904	.765	.348	.0614
1.018	.169	.0920	.765	.315	.0680
			.760	.278	.0755
.950	.431	.0462	.765	.240	.0802
.950	.426	.0472	.765	.212	.0844
.945	.400	.0536			
.945	.376	.0606	.669	.601	.0118
.945	.346	.0681	.669	.560	.0193
.964	.314	.0726	.669	.559	.0204
.945	.290	.0780	.669	.519	.0301
.945	.264	.0816	.677	.510	.0302
.945	.233	.0856	.668	.474	.0369
.945	.204	.0886	.673	.470	.0370
.945	.182	.0918	.673	.435	.0434
			.673	.391	.0519
			.673	.354	.0582
			.669	.313	.0644
			.668	.269	.0719
			.673	.233	.0762
			.576	.645	.0043
			.571	.610	.0132
			.576	.552	.0245
			.581	.501	.0341
			.581	.445	.0425
			.576	.406	.0535
			.576	.356	.0586
			.576	.305	.0664
			.576	.267	.0708

TABLE 2.

Model Airscrew - Torque.

$\frac{\pi n D}{a}$	$\frac{V}{n D}$	k_q	$\frac{\pi n D}{a}$	$\frac{V}{n D}$	k_q
1.171	.346	.00648	.756	.528	.00365
1.171	.330	.00684	.766	.521	.00399
1.171	.326	.00680	.756	.494	.00414
			.762	.490	.00410
1.106	.366	.00620	.756	.465	.00465
1.121	.366	.00625	.762	.462	.00457
1.123	.361	.00615	.756	.430	.00497
1.131	.361	.00640	.762	.389	.00540
1.131	.361	.00625	.762	.383	.00539
1.123	.340	.00700	.756	.356	.00558
1.131	.340	.00661	.756	.314	.00584
1.113	.320	.00716	.756	.282	.00582
1.123	.319	.00718	.762	.237	.00626
1.123	.319	.00694	.762	.214	.00638
1.125	.314	.00716			
1.113	.297	.00743	.575	.695	.00217
1.096	.277	.00792	.578	.645	.00280
			.578	.642	.00280
1.012	.397	.00626	.578	.602	.00326
1.031	.388	.00622	.583	.598	.00312
1.031	.369	.00674	.574	.556	.00380
1.031	.368	.00666	.578	.501	.00430
1.021	.348	.00715	.574	.500	.00436
1.031	.346	.00707	.578	.455	.00458
1.021	.324	.00755	.578	.404	.00495
1.068	.309	.00790	.583	.355	.00507
1.083	.305	.00767	.582	.299	.00537
1.012	.299	.00805	.582	.264	.00527
1.031	.262	.00825			
1.031	.239	.00856			
1.021	.219	.00886			
1.001	.171	.00925			
.992	.434	.00757			
.941	.424	.00572			
.855	.406	.00600			
.941	.402	.00580			
.936	.382	.00672			
.941	.376	.00675			
.936	.353	.00690			
.922	.326	.00730			
.941	.321	.00744			
.941	.276	.00757			
.941	.258	.00785			
.930	.237	.00806			
.936	.201	.00825			
.927	.182	.00825			

TABLE 3.

Low Speed Results. (No. 17 ft.).Model Airscrew $\frac{\pi n D}{a} = .177$ Thrust.Torque.

$\frac{V}{n D}$	k_T	$\frac{V}{n D}$	k_Q
0.039	0.0832.	0.039	0.00473
0.277	0.0622	0.263	0.00464
0.318	0.0576	0.316	0.00449
0.358	0.0530	0.359	0.00431
0.397	0.0471	0.398	0.00409
0.436	0.0400	0.437	0.00374
0.477	0.0337	0.478	0.00337
0.516	0.0255	0.518	0.00290
0.555	0.0166	0.559	0.00230
0.596	0.0073	0.597	0.00165
0.628	0.0001	0.637	0.00096

TABLE 4.
Pitot Readings $\left(\frac{dk_T}{dr}\right)$ (1st Test).

$r =$	0.205	0.277	0.323	0.363	0.394	0.427	0.454	0.484
$\frac{\pi nD}{a}$	dk_T/dr at $V/nD = 0.265$							
0.58	0.103	0.187	0.237	0.280	0.293	0.301	0.265	-0.069
0.67	0.100	0.184	0.238	0.281	0.298	0.306	0.271	-0.072
0.76	0.105	0.197	0.262	0.313	0.332	0.356	0.317	-0.079
0.85	0.101	0.196	0.266	0.322	0.345	0.360	0.330	-0.086
0.94	0.102	0.201	0.275	0.334	0.362	0.352	0.299	-0.081
0.99	0.103	0.203	0.283	0.339	0.362	0.336	0.301	-0.081
1.03	0.098	0.199	0.278	0.338	0.352	0.312	0.285	-0.065
1.06	0.102	0.200	0.276	0.338	0.350	0.311	0.263	-0.057
$\frac{\pi nD}{a}$	dk_T/dr at $V/nD = 0.34$							
0.58	0.092	0.167	0.204	0.240	0.251	0.254	0.225	0.019
0.67	0.081	0.161	0.202	0.238	0.250	0.257	0.229	0.022
0.76	0.091	0.173	0.225	0.268	0.285	0.294	0.270	0.019
0.85	0.087	0.171	0.230	0.277	0.300	0.317	0.297	0.015
0.94	0.084	0.168	0.230	0.279	0.302	0.311	0.264	-0.008
0.99	0.082	0.167	0.230	0.280	0.303	0.294	0.245	-0.018
1.03	0.080	0.166	0.225	0.277	0.298	0.267	0.216	-0.024
1.08	0.080	0.161	0.220	0.269	0.284	0.234	0.189	-0.017
1.12	0.075	0.152	0.207	0.247	0.250	0.195	0.139	-0.010
1.18	0.071	0.147	0.198	0.223	0.178	0.151	0.085	-0.007
r	.167	.229	.292	.333	.375	.417	.458	.479
$\frac{\pi nD}{a}$	dk_T/dr at $V/nD = 0.34$							
0.18	0.068	0.121	0.172	0.194	0.213	0.212	0.170	0.082
$\frac{\pi nD}{a}$	dk_T/dr at $V/nD = 0.5$							
0.18	0.043	0.074	0.093	0.098	0.102	0.100	0.073	0.047

TABLE 5.

Measured blade angles. Departure from drawing.1st Test.

Low speed tests.			Air turbine tests.	
r	Blade 1.	Blade 2.	Blade 1.	Blade 2.
0.24	-5'	-10'	- 10'	-10'
0.32	13'	- 8'	+ 3'	- 7'
0.40	20'	- 5'	+ 5'	0
0.46	11'	-11'	0	-11'

TABLE 6.

(2nd Test).

Values of $\frac{dk_T}{dr}$ $J = .34$ (approx:) (1st expt.)

r.		.32	.40	.46
J	$\frac{\pi n D}{a}$	$\frac{dk_T}{dr}$	$\frac{dk_T}{dr}$	$\frac{dk_T}{dr}$
0.332	0.57	0.202	0.248	0.229
0.342	0.74	0.216	0.274	0.260
0.342	0.93	0.222	0.304	0.270
0.343	0.98	0.221	0.304	0.243
0.344	1.02	0.217	0.292	0.212
0.345	1.07	0.208	0.264	0.181

TABLE 7.

Angles of twist (degrees) $J = .34$ (approx). (2nd Test).
(1st expt.)

r		.32		.40		.46	
J	$\frac{\pi n D}{a}$	Angle of twist.		Angle of twist.		Angle of twist.	
		Blade 1.	Blade 2.	Blade 1.	Blade 2.	Blade 1.	Blade 2.
0.332	0.57	0	0.10	0.55	0.40	0.60	0.80
0.342	0.74	0.35	0.25	0.85	0.65	1.20	1.55
0.342	0.93	0.45	0.50	1.00	1.00	1.20	1.65
0.343	0.98	0.40	0.45	0.50	1.20	0.70	1.35
0.344	1.02	0.10	0.50	0.10	0.70	-0.40	0.50
0.345	1.07	-0.05	0.05	-0.30	0.30	-1.55	0.05

TABLE 8.

Values of $\frac{dk_T}{dr}$ at $r = 0.40$ (2nd Test).
(2nd Expt).

$\frac{\pi nD}{a}$	Test. a		b		c		d		e	
	J.	$\frac{dk_T}{dr}$	J	$\frac{dk_T}{dr}$	J	$\frac{dk_T}{dr}$	J	$\frac{dk_T}{dr}$	J	$\frac{dk_T}{dr}$
.57	0.262	0.276	0.308	0.256	.352	0.218	0.379	.222	.405	0.187
.66	0.259	0.291	0.311	0.265	.345	0.235	0.373	.214	.406	0.194
.75	0.258	0.316	0.307	0.276	.346	0.250	0.375	.227	.406	0.209
.84	0.257	0.338	0.307	0.301	.347	0.276	0.376	.253	.407	0.224
.93	0.255	0.345	0.305	0.320	.346	0.276	0.376	(.267 (.256	.405	0.231
1.02	0.257	0.342	0.307	0.305	.347	0.278	0.377	-	.388	0.220
1.07	0.256	0.328	0.307	0.291	.347	0.245	0.371	.218	-	-

TABLE 9.

(2nd Test).
Angles of twist (degrees) at $r = 0.40$ (2nd expt).

Test		a		c			e		
$\frac{\pi n D}{a}$	J	Angle of twist		J	Angle of twist		J	Angle of twist	
		Blade 1	Blade 2		Blade 1	Blade 2		Blade 1	Blade 2
.57	0.262	0.7	0.85*	0.352	0.4	0.65*	0.405	0.2	0.65*
.66	0.259	1.0	0.85*	0.345	0.35	0.65*	0.406	0.35	0.4
.75	0.258	1.55	1.6	0.346	0.95	1.0*	0.406	0.75	0.7
.84	0.257	2.1	2.05	0.347	1.1	1.45	0.407	0.55	0.9
.93	0.255	1.55	1.85	0.346	0.8	1.3	0.405	0.4	0.65
1.02	0.257	1.05	1.65	0.347	-0.2	0.8	0.388	-0.75	0.2
1.07	0.256	0.45	1.15	0.347	-0.55	0.2	0.371	-0.8	-

* Photograph not sharply defined.

TABLE 10.Measured blade angles - Departure from drawing.2nd Test.

r	1st Expt.		2nd Expt.	
	Blade 1.	Blade 2.	Blade 1	Blade 2.
0.32	-4'	-10'	-	-
0.40	0	-10'	20'	-5'
0.46	-23'	-21'	-	-