

Modelling of Spreader Hoist Systems in Mobile Gantry Cranes

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Dedicated to my mother

Abstract

This thesis presents an analysis of the kinematics and dynamics of a mobile rubber tyred gantry crane (RTG Crane), as typically used for container handling operations, for the ultimate purposes of research into automation.

Dynamical models are proposed for the crane based on a simplification of the structure into the gantry frame, the overhead trolley and the cable mounted spreader slung below the trolley. The underlying trolley/spreader cabling geometry for commonly encountered designs has been analysed and novel mathematical relationships have been established for the first time. Full three dimensional motion is encompassed in the dynamical equations of motion and various movement scenarios are examined.

As an adjunct to the geometrical modelling, a new phenomenon, described herein as geometrical extension, is defined in its role as an inherent anti-sway property in multi-cabled spreader suspension systems. The phenomenon is explained and investigated using pure geometry and a numerical method to assess its effect in practice.

The final section of the thesis uses the single and multi-cable dynamics to evolve an accurate, and generic, fuzzy logic control simulation for the RTG machine. Various performance scenarios are highlighted for typical freight manoeuvres. This work is now ready for experimental testing and evaluation in an allied project, thus the remit for the theoretical work of this thesis has been completed.

Declaration

This thesis is submitted in part fulfillment of the requirements for the degree of Doctor of Philosophy at the University of Edinburgh. Unless otherwise stated, the work described is original and has not been previously submitted in whole, or in part, for any degree at this, or at any other, University.

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Chapter 1

A Background to the Mechanics of Gantry Cranes

1.1 Introduction

Gantry cranes are used in many industries where there is a need to handle very large and heavy loads, a common example being the overhead factory crane. Cranes of this type consist of a gantry beam running on rails attached to factory walls, a motorised trolley which is free to move across the beam and a hook or electromagnetic clamp used as lifting gear. The latter is connected to the crane by means of cables or ropes and can be lowered and raised by the hoist drum attached to the trolley. Such cranes provide a facility for lifting and lowering of the payload across the lateral span of the gantry. Many factory cranes also allow a longitudinal movement of the gantry beam by means of a pair of parallel rails fitted onto the walls to guide the gantry translation. References Abeid et.al [1], Al-Garni et.al [4], Fliess et.al [36], Marttinen et.al [54], Moustafa and Abeid [56], Moustafa [57], Swevers et.al [73], and many others, deal with this type of crane.

Another common example is the mobile gantry crane as used widely in ports and container terminals to handle heavy ISO freight containers. In the case of mobile cranes, the trolley moves along the top beam of a rail guided, or wheeled, n-shaped, portal gantry frame (Figure 1.1). This type of gantry crane allows for the lifting and lowering of the payload (container), moving it laterally and longitudinally, and in the latter case (the tyre mounted gantry) a rotation of the crane is also possible.

Of particular theoretical and practical interest to researchers and operators alike is the capability and potential for automation of the mobile Rubber Tyred Gantry Crane (RTG Crane) as used world-wide in the containerisation industry. The study of these cranes is presented in the thesis.

1.2 Rubber Tyred Gantry (RTG) Cranes

1.2.1 RTG crane structure

Figure 1.1 shows a typical RTG crane deployed in a container terminal application.

As has been briefly mentioned in section 1.1, the structure of the crane consists of the following parts:

- The portal *gantry* frame.

The portal frame is really two portal frames tied together, each comprising a top beam and two side beams. There are engine bays fitted onto the bottom end of the side beam pairs; the wheels are fitted to transmission equipment underneath the engine bays. The wheels are arranged so that they can be separately motored, thus providing a capability for drive in fore-aft, lateral, and even steered modes.

Two pairs of side beams and top beams form a rigid, doubled, n-shaped structure capable of carrying containers of up to 40 tonnes.

- The overhead *trolley*.

The trolley is a fabricated structure which runs on wheels guided by rails on the top beams of the gantry. The trolley contains hoist hydraulics, motors, and a winching drum. The drivers cab is frequently attached to the trolley (slung underneath and offset to the side).



a.



b.

Figure 1.1
A typical Rubber Tyred Gantry (RTG)
Crane as used in the container terminals
with laden (a.) and unladen (b.) spreader

- The *spreader* (or lifting frame).

This is a rectangular fabricated structure, often with a telescopic subassembly fitted, to provide for containers of different lengths.

- Hoist ropes (or *cables*).

The cables are used to attach the spreader to the trolley. Figure 1.2 shows a typical cable suspension arrangement. Four pairs of parallel, closely running, cables connect four corresponding corners of the trolley and the spreader, running over the set of headblock and intermediate sheaves and the hoist drum to allow the spreader to be lowered or hoisted. Thus, due to this configuration, the spreader is potentially free to swing below the trolley and also to rotate about its own local central axis.

1.2.2 The RTG crane at work

RTG cranes are generally used for one reason: to move a payload (container) in order to get it into a more desirable physical location. Currently this is achieved by a highly skilled human operator (driver).

The driver sits in a cabin attached to the trolley (the cabin can be clearly seen in Figure 1.1) so that he can see the spreader and the container below. The driver controls the crane motors so that the combination of gantry and trolley movements and hoisting and lowering of the spreader bring the spreader on top of the container. There are four *twistlocks* (hardened, contoured, locking pins) fitted in each corner of the spreader which drop into four 'corner boxes' on the container topside. The twistlocks are then remotely rotated through ninety degrees to provide a positive lock-on of the spreader onto the container.

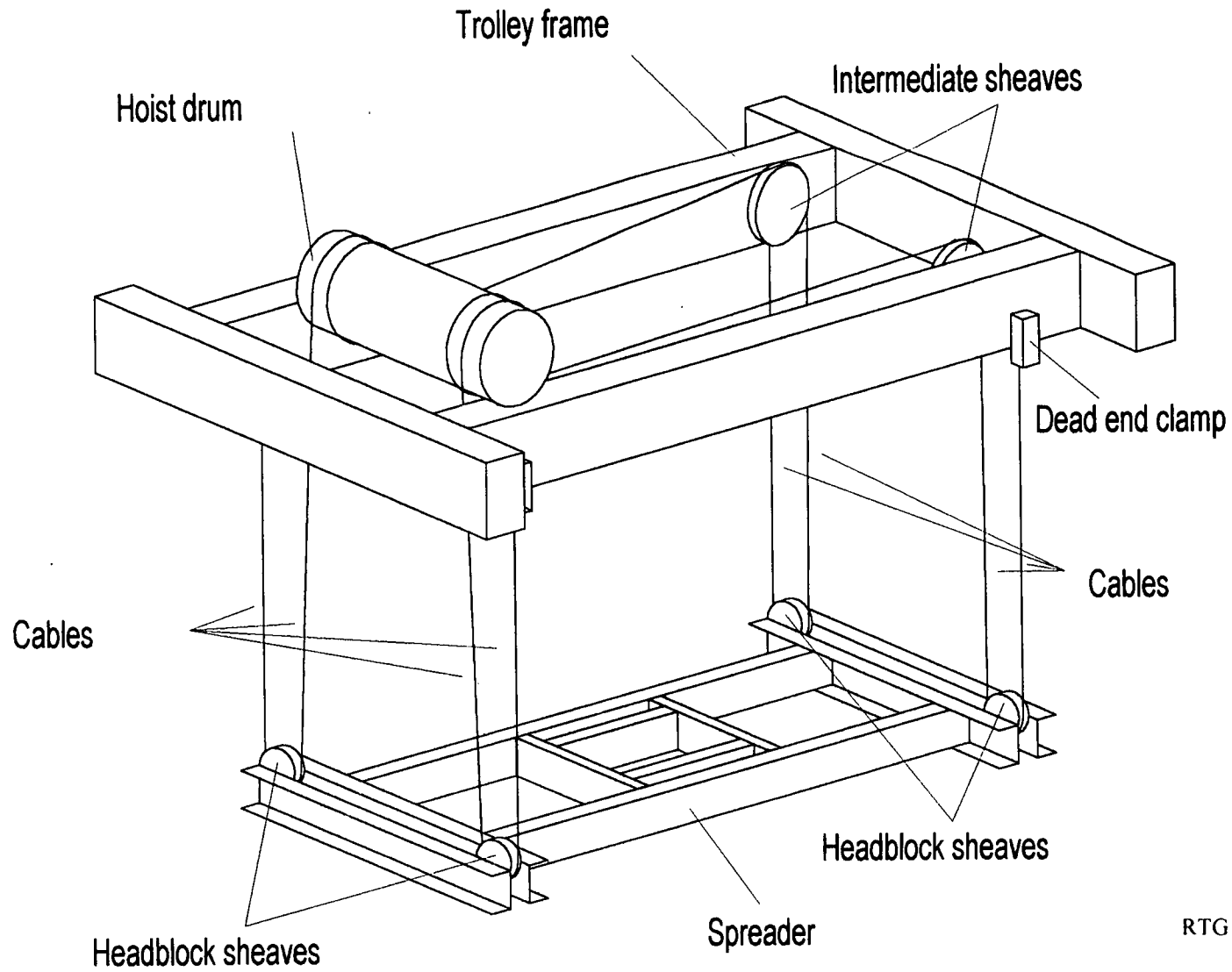


Figure 1.2
Reeving of a typical
RTG trolley and spreader system

1.3 The Port/Container Terminal

A typical port consists of a storage and stacking area, and a shipside area. The first is used for keeping containers in stacks while awaiting their turn to be loaded onto the ship.

The containers are brought from the stacking area to the shipside area by means of flat-backed lorries, and then so-called Quay Cranes¹⁾ are used to pick the containers up from the lorry and bring them onto the ship (or vice versa in case of unloading the ship).

RTG cranes are used for handling the containers in the storage area: for placing them onto the lorries, for stacking them, and shuffling them if necessary (in order to get a container off the stack to be loaded onto the ship). The work of RTG cranes is organised so that they can only (currently) hoist/lower and trolley simultaneously. Translation of the gantry (gantry modes) and its rotation (steer modes) have to be actuated before or after hoist/lower and trolley.

A typical container stack is three or four containers high, and six or seven wide. The width of the stack is constrained by two factors: the dimensions of the crane and the ease of access of the lorries that move the containers.

1.4 Problems encountered in the manual control of Gantry Cranes, and a discussion of the needs for RTG automation

RTG cranes have a common problem due to the flexibility of the cable suspension system used on them in that the potential for the swing of the spreader makes its positional control especially difficult. Only very skilled drivers can cope with large spreader oscillations and then position the spreader accurately to within some allowed minor misalignment at each twistlock and it is relatively common for drivers to engage in the practice of crash-location of the spreader onto the top of the container. This can

save operation time (if it works), but often leads to expensive twistlock failures, and then the whole vehicle has to be taken out of service whilst necessary repairs are carried out.

There are several types of spreader sway described in detail by Casper [20], Oostwouder [60], Rudolf [68]. The main one that the driver has to cope with is the so-called 'primary' (or 'pendulum') sway induced by the trolley accelerations 'where the container moves in an arc about the trolley and parallel to the trolley boom²⁾', Rudolf [68]. Any experienced driver can stop pendulum swing relatively easily by applying an acceleration to the trolley in the direction of the swing. Various anti-sway systems have been constructed to assist the driver. They introduce additional damping in the system by using wide cable spacing or by employing various cable reeving designs. Anti-sway systems are frequently fitted, or retrofitted when not supplied as original equipment, although some drivers do not use them due to their side effects. For example, Casper [20] mentions that 'operators are understandably intolerant of yaw motion induced by anti-sway devices. A good operator can control primary sway but he cannot control yaw. That is the main reason most anti-sway devices end up unused and disliked by good operators'.

Most drivers appear to cope with oscillations of the spreader by accelerating the trolley so that the spreader does not swing too much, and therefore it is then relatively easy to stop its oscillations before placing it onto the container. These are described as 'non-swing' drivers. There are also drivers, referred to as '*swing drivers*', who not only do not try to stop the swing, but actually *use* the swing in their work. These drivers accelerate the trolley maximally first with the spreader lagging behind, and when the trolley is near enough to the final destination it is decelerated and the spreader is lowered at the same time (then the spreader moves forward due to its inertia and remains in front of the trolley). This is effected in such a way that the spreader finishes its movement exactly on the container topside. The method of swing drivers is very attractive because the operation time is much lower than that in the case of non-swing

drivers (which reduces the cycle time for handling one container, and therefore increases the potential for improved throughput for the whole port).

Although a lot of research is going on world wide to assist the driver in his work (a more detailed review of this is given in section 1.5), to date little has been practically implemented and a control system which is able to position the spreader on the container fast and accurately is highly desirable. The commercial need to reduce the average cycle time, the very high costs of training good drivers and down-time for repairs are some very good reasons for doing this work.

Thus, there is a recognised industrial need for fast, reliable control systems to improve the all round performance of RTG machines.

1.5 Review of existing research on the automation of Gantry Cranes

1.5.1 Existing mathematical models of the Gantry Cranes

As has been mentioned above, active research on RTG automation is world-wide, but most researchers and operators believe that the key for building a successful control system is a good mathematical model of the gantry crane. A number of papers (e.g. Abeid et.al [1], Al-Garni et.al [4], DeSantis and Krau [30], Marttinen et.al [54], Moustafa and Abeid [56], Swevers et.al [73] and others) can be referred to for one, two and three dimensional pendulum models of gantry cranes. The Lagrangian approach is commonly used for deriving the equations of motion either by assuming a full vehicle model excited by generalised forces which are resident in a right hand side torque vector, or by regarding the system inputs as motion perturbations of directly actuated coordinates.

All the existing models have one thing in common: they do not take in consideration the multi-cable reeving system arrangement. In the case of the cranes with a hook or

magnetic clamp used for lifting gear this is acceptable, but, in the case of the RTG cranes used for container handling, the cables in the hoisting system (Figure 1.2) are widely spaced both laterally and longitudinally and this leads to a considerable spreader lift due to its rotation (e.g. in the case of so-called *secondary spreader sway* described by Casper [20] and Rudolf [68]).

One of the novel and major achievements of the research presented in this thesis is a new, so-called, *Multi-Cable Model* (MCM) of the RTG crane spreader suspension configuration. This is *an absolutely new version* of the three dimensional pendulum Single-Cable RTG Model taking into consideration trolley and gantry movements, hoisting and lowering of the spreader, and also the changes in the system dynamics due to the spreader rotation about its central axis. The Single-Cable Model is presented in Chapter 2 of this thesis, the additional hoist component due to spreader rotation is explored in detail in Chapter 3 and Chapter 5 presents the Multi-Cable version of the RTG model.

1.5.2 Control of Gantry Cranes

There are many ways of reducing the time needed for one container operation, e.g. (as suggested by Bryfors in [13]):

- division of the cycle into several independent parts,
- increasing speeds and accelerations,
- providing automated operation,
- combinations of the three above.

Bryfors comments that the first way is possible by using two or three trolleys, but this is quite complicated and expensive; the second way also is currently not very feasible as very skilled drivers would be needed, meaning expensive additional training as well as a whole new generation of very high powered RTG machines. Current on-board power supply requirements are already demanding. Thus, it is concluded that automation looks the most practically realisable.

As has been mentioned above, one of the features of the gantry crane is the presence of flexible hoist cables leading to compliance in the system. Some interesting achievements during the last decade in the control of compliant robot arms have been presented in the literature and discussed in detail by Albus et.al [2], Arocena [5], Cannon and Schmitz [15], Cashmore et.al [19], and Daniel et.al [28]. There are some parallels to be drawn between these pieces of work and the problems inherent in the gantry crane control problem in that both sorts of system possess complicated kinematic and constraint relationships and both are essentially noncollocated manipulator systems.

A number of authors investigate classical approaches for the control of gantry cranes, such as linear and non-linear feedback control (Moustafa and Abeid [56], Moustafa [57], Ridout [66], Yu et.al [85]), adaptive control (d'Andréa-Novel and Boustany [27], Virkkunen et.al [78]), model reference control (Butler et.al [14], Caron et.al [17]), control based on Lyapunov theory (Joshy and Rahn [44]), optimal control (Al-Garny et.al [4]), and many others (DeSantis and Krau [30], Takahashi and Inooka [74], Yoon et.al [84] to name some of the most well known authors). There are also a few papers, e.g. Benhidjeb and Gissinger [8], Cansever et.al [16], Itoh et.al [42], Itoh et.al [43], Suzuki et.al [72], Yamada et.al [82], Yasunobu and Hasegawa [83], which investigate the application of fuzzy control theory to this problem domain.

There are several obvious advantages to using a fuzzy rather than classical controller, and these are underlined in all the papers on fuzzy control mentioned above:

- the gantry cranes dynamics are highly non-linear and can be more easily and efficiently handled by use of a fuzzy approach rather than the classical methods;
- fuzzy controllers demonstrate much higher robustness to disturbances often present in the system which cannot easily be modelled such as wind, friction and 'time lags of the trolley drive and hoist motors' (Yasunobu and Hasegawa [83]);
- fuzzy controllers are simple to implement, and can be readily used to reflect the experience of a good driver.

The possibilities of introducing the velocity and displacement-proportional active terms into the modelled system by linearised feed-back control have been investigated by Cartmell and Alberts and reported in [18]. The Single-Cable RTG Model has been used for this research because the MCM proved to be too computationally slow to be used for simulation of real-time control. As far as the work presented in this thesis is concerned the novelty of application of fuzzy logic lies in the considerable extensions to existing strategies that are required to encode the highly complex multi-cable dynamics presented herein.

1.5.3 Computer packages for mathematical modelling of Multi-Body Mechanical Systems

It is appropriate to mention that, in addition to analytical models, various computer software packages are being developed for studying multi-body mechanical systems (Crolla et.al [22], Dharwadkar et.al [31], Holt and Cornish [40], Kerr and Goodchild [46], and others). The potential usage of such packages is two-fold: firstly in the development of the mathematical model and secondly in the graphical representation or animation of the results. This can be very helpful in studying the kinematics and dynamics of a multi-body mechanical system and was found to be of particular value for validation of the analytical models of the RTG crane described in the thesis. The

ADAMS package (presented by Puttré [63] in more detail), has been used for this specific purpose.

1.6 'The Container Terminal of the Future'

In parallel with research into the automation of RTG crane operation, the first steps are being made in introducing more automation into the organisation of ports themselves. The papers by Eller [35], Macleod [51], Verschoof [77] describe such ports (Rotterdam and Thames port) showing that these ports have a storage and stacking area, a shipside area with Quay Cranes in it and Automatically Guided Vehicles (AGVs) used to transport containers between the areas, which potentially replace manually driven lorries. Fully Automated Stacking Cranes (ASC), Verschoof [77], or Rail Mounted Gantries (RMG), Macleod [51], are used to handle containers in storage areas. Another paper by Bohlman et.al [9], describes an overhead rail system for container handling called 'GRAIL' which is checked on a scaled model. The expert system used there helps to make it fully automated.

The significant part of terminal automation is 'information and control', Macleod [51], and all the papers reviewed describe how control is organised at the locations discussed.

1.7 The objectives of research presented in the thesis and chapters contents

It has already been underlined that the importance of mathematical modelling of the RTG crane is paramount in order that a controller can be built for it, and it has been also emphasised that no known model takes the spreader rotation about its axis into consideration.

The objectives of this thesis are to:-

- Show how a three dimensional pendulum model of an RTG crane using Lagrangian dynamics with seven degrees of freedom can be constructed. This work is discussed in Chapter 2.
- Propose a geometrical model of the multi-cable spreader suspension in detail, and suggest an explicit formula for the spreader hoist due to rotation as a function of the angle of its rotation - Chapter 3.
- Investigate the phenomenon of asymmetrical cable stretch in the system which generally takes place when the spreader is translated from its equilibrium position under the trolley and then rotated, and to propose strategies for its future integration in MCM dynamics - Chapter 4.
- Present a novel Multi-Cable dynamical Model (MCM) based on the detailed geometry of the spreader suspension system investigated in Chapter 3 - Chapter 5.
- Describe the numerical RTG model constructed by means of the ADAMS package - Chapter 6.
- Give a comparative analysis of the numerical results of both the MCM and the ADAMS models - Chapter 6.
- Describe the basic concepts required for building a self-organised time-optimal fuzzy controller for spreader positioning for the RTG crane - Chapter 7.
- Give a résumé and plans for future research - Chapter 8 (conclusions).

Note to Chapter 1

- 1) The Quay Crane is another type of gantry cantilever crane which differs from the RTG cranes by the shape of the portal frame: the top beam of such cranes spreads beyond one of the sides beams. This allows the trolley to get into position above the ship when the crane is placed on the quay near the sealine.
- 2) The trolley boom is the top part of the RTG crane on which the trolley runs.

Chapter 2

Single-Cable Model of the RTG Crane

As has been stated in section 1.5.1 there is a recognised need to derive an accurate and complete mathematical model of the RTG crane in order to explore the controllability of the machine.

Systems of coordinates used for mathematical modelling are presented in this chapter and the geometry of the RTG is idealised to produce a single-cable model of the spreader suspension. In this case the spreader is considered as a simple lumped mass, for which rotation does not affect the dynamics of the system.

A system of generalised coordinates is proposed from which the equations of motion have been derived using Lagrange's equations. The equations of motion are derived in section 2.4.3.

The geometry of the multi-cable spreader suspension system, which is actually more representative of the RTG crane, is investigated in detail in Chapter 3 and the equations of motion for this case are given in Chapter 5.

2.1 Introduction

Recalling section 1.2.1, an RTG Crane can be regarded as comprising three rigid bodies, namely the n-shaped vehicle, the trolley, and the spreader. In theory the vehicle can move horizontally in any direction and also rotate about a vertical axis symmetrically placed through its mass centre. The trolley's movement is restricted to translation along the top beam of the vehicle.

In this chapter, the spreader is considered to be a lumped mass attached to the trolley by means of *one* flexible cable as shown in Figure 2.1. It can swing below the trolley as a pendulum in any direction. It is clear that in this particular case the rotation of the spreader does not affect the dynamics of the system since rotation cannot cross-couple with other coordinates.

The model presented here neglects the tyre and structure deformation and flexure effects. Incorporation of these phenomena is recommended for a future research programme.

2.2 Notations and systems of coordinates used for mathematical modelling

2.2.1 Global system of coordinates

Figure 2.1 depicts the RTG crane in the global system of coordinates XYZ .

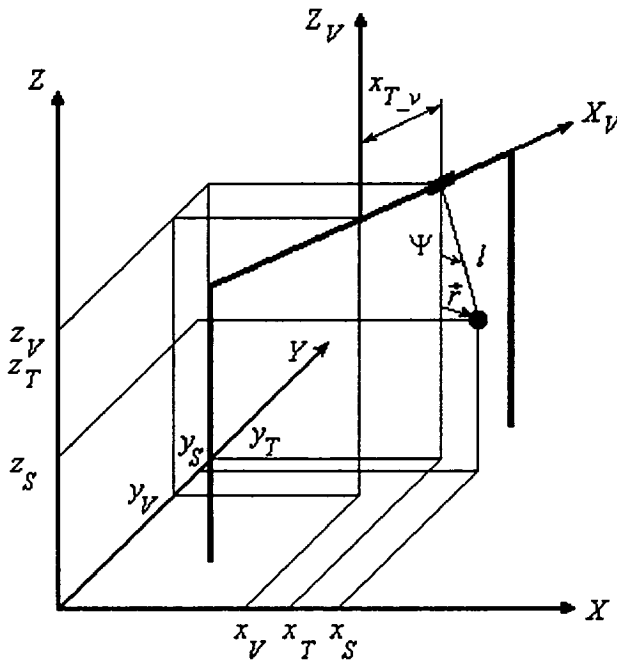


Figure 2.1

The RTG Crane in the global system of coordinates, single-cable case

The vehicle is shown in Figure 2.1 by means of three bold lines representing the two side beams and the top beam. The trolley attaches to the top beam, and the spreader is suspended by a cable of length l . The spreader is shown swung away from its equilibrium position under the trolley where the swing angle is denoted by Ψ .

The global system of coordinates XYZ is chosen with the X and Y axes in the horizontal plane and the Z axis pointing upwards. The vehicle global coordinates are denoted by x_V , y_V and z_V . These are the coordinates of the top beam centre. The trolley and spreader global coordinates are denoted by x_T, y_T, z_T and x_S, y_S, z_S respectively.

2.2.2 Vehicle system of coordinates

It is also convenient to use a vehicle system of coordinates, with its datum at the top beam centre, the X_V axis parallel to the top beam, and the Z_V axis pointing upwards. The Y_V axis is chosen to complete the right handed system but is not shown in the picture.

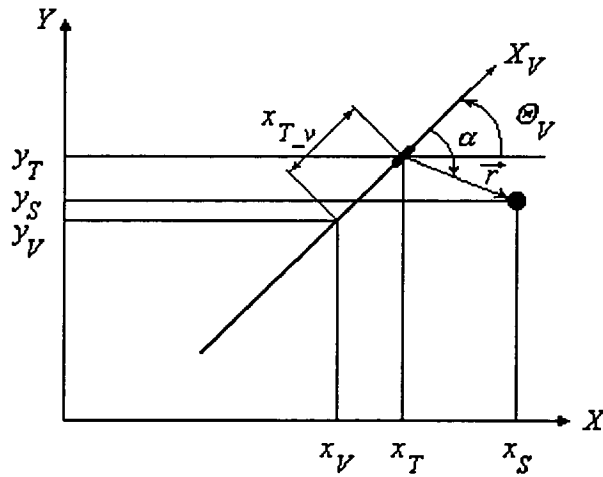


Figure 2.2

RTG in global system coordinates, plan view

(here $\alpha < 0$, $\Theta_V > 0$)

The x -coordinate of the trolley in the vehicle system of coordinates is denoted by x_{T_v} and is, in fact, the distance between the top beam centre and the trolley's geometrical centre, measured along the top beam.

Figure 2.2 shows the plan view of the RTG crane, where the angle of vehicle rotation is clearly seen. This is the angle between the global and vehicle X -axes, denoted by Θ_v , and it is defined as positive in the case presented in Figure 2.2.

2.3 The relationship between vehicle, trolley and spreader coordinates

2.3.1 Trolley coordinates

The position of the vehicle is defined by its global coordinates x_v and y_v and angle of rotation Θ_v . Given the distance x_{T_v} , the trolley global coordinates can be found from Figure 2.2:

$$x_T = x_v + x_{T_v} \cos \theta_v \quad (2.1)$$

$$y_T = y_v + x_{T_v} \sin \theta_v \quad (2.2)$$

2.3.2 Spreader coordinates

2.3.2.1 Vector of spreader translation

When the spreader swings away from its equilibrium position under the trolley, it can be said to be translated through a horizontal vector of translation, $\vec{r}(r, \alpha)$, (in the case presented in Figure 2.2 the angle of spreader translation α is negative) and that accompanying this will be a spreader hoist defined as h_r . The latter will be referred to

as 'hoist due to spreader translation', and for a known magnitude r of the vector of translation can be found from Figure 2.1:

$$h_t = l - \sqrt{l^2 - r^2} \quad (2.3)$$

The swing angle, Ψ , can be defined from the Figure as:

$$\Psi = \sin^{-1} \frac{r}{l} \quad (2.4)$$

2.3.2.2 The coordinates of the translated spreader

The coordinates of the spreader after translation from the equilibrium position can be found from Figures 2.1 and 2.2:

$$x_s = x_T + r \cos(\theta_V + \alpha) \quad (2.5)$$

$$y_s = y_T + r \sin(\theta_V + \alpha) \quad (2.6)$$

$$z_s = z_T - l + h_t \quad (2.7)$$

Substitution of equations (2.1)-(2.4) into equations (2.5)-(2.7) gives:

$$x_s = x_V + x_{T_v} \cos \theta_V + r \cos(\theta_V + \alpha) \quad (2.8)$$

$$y_s = y_V + x_{T_v} \sin \theta_V + r \sin(\theta_V + \alpha) \quad (2.9)$$

$$z_s = z_T - \sqrt{l^2 - r^2} \quad (2.10)$$

2.4 Obtaining the Equations of Motion for the Single-Cable model of the RTG crane

2.4.1 Lagrange's equations and the set of generalised coordinates for the RTG crane

The Lagrange's equations for a conservative system are given by equation (2.11):

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = 0 \quad (2.11)$$

where T and U are kinetic and potential energies of the system, and q_i , $i = 1-7$, denote seven generalised coordinates (this system having seven degrees of freedom: gantry translation in the horizontal plane and rotation about its central axis, trolley translation along the top beam, spreader hoist/lowering, and two dimensional pendulum swing):

$$q_1 = x_V \quad (2.12)$$

$$q_2 = y_V \quad (2.13)$$

$$q_3 = \Theta_V \quad (2.14)$$

$$q_4 = x_{T_v} \quad (2.15)$$

$$q_5 = l \quad (2.16)$$

$$q_6 = \alpha \quad (2.17)$$

$$q_7 = r \quad (2.18)$$

They completely determine the system at this level of modelling: q_1 - q_3 define the position of the vehicle, q_4 defines the position of the trolley, and q_5 , q_6 and q_7 define the spreader position. Other coordinates used in the previous chapters are dependent on q_1 - q_7 .

Note that dissipative terms are added in later, and that generalised force terms are not introduced because the equations are directly excited by motions of the motor actuated coordinates, i.e. q_1 - q_5 .

2.4.2 Kinetic and potential energy of the system

The kinetic energy function can be written as follows:

$$T = \frac{1}{2}I_V \dot{\theta}_V^2 + \frac{1}{2}(I_T + m_T x_{T-V}^2) \dot{\theta}_V^2 + \frac{I_S}{2}(\dot{\theta}_S + \dot{\theta}_V)^2 + \frac{1}{2}m_V(\dot{x}_V^2 + \dot{y}_V^2) + \frac{1}{2}m_S(\dot{x}_T^2 + \dot{y}_T^2) + \frac{1}{2}m_S(\dot{x}_S^2 + \dot{y}_S^2 + \dot{z}_S^2) \quad (2.19)$$

where m_V , m_T and m_S and I_V , I_T and I_S are the masses and mass moments of inertia of the vehicle, trolley and spreader respectively. The moments of inertia of the various RTG components are taken to be acting about their own respective centres of rotation.

The second term in equation (2.19), $\frac{1}{2}(I_T + m_T x_{T-V}^2) \dot{\theta}_V^2$, is written using the parallel axes theorem to refer the moment of inertia of the moving trolley to the assumed vehicle centre, i.e. the axis passing vertically through its centre, about which θ_V is taken to act.

The potential energy can be written as:

$$U = m_S g z_S \quad (2.20)$$

It only has a gravitational component because it was decided that torsional strain energy could be neglected for the sort of cable length-to-diameter ratios of the typical cable geometries used on RTG's, and that the energy due to the cables wrapping and unwrapping about the headblocks and hoist drum was also seen to have a minor

contribution to the overall potential energy, although it was carefully modelled in an earlier version of the system potential energy model.

2.4.3 Derivation of the Equations of Motion using Lagrange's equations

The trolley velocity terms, $\dot{x}_T$ and $\dot{y}_T$, used in the kinetic energy expression (2.19) are given in the global system of coordinates. They can be found by differentiating equations (2.1) and (2.2) to give:

$$\dot{x}_T = \dot{x}_v + \dot{x}_{T_v} \cos \theta_v - x_{T_v} \dot{\theta}_v \sin \theta_v \quad (2.21)$$

$$\dot{y}_T = \dot{y}_v + \dot{x}_{T_v} \sin \theta_v + x_{T_v} \dot{\theta}_v \cos \theta_v \quad (2.22)$$

By differentiating equations (2.8)-(2.10), the spreader Cartesian velocities can be found:

$$\begin{aligned} \dot{x}_s = & \dot{x}_v + \dot{x}_{T_v} \cos \theta_v - x_{T_v} \sin \theta_v \dot{\theta}_v \\ & + \dot{r} \cos(\theta_v + \alpha) - r \sin(\theta_v + \alpha) (\dot{\theta}_v + \dot{\alpha}) \end{aligned} \quad (2.23)$$

$$\begin{aligned} \dot{y}_s = & \dot{y}_v + \dot{x}_{T_v} \sin \theta_v + x_{T_v} \cos \theta_v \dot{\theta}_v \\ & + \dot{r} \sin(\theta_v + \alpha) + r \cos(\theta_v + \alpha) (\dot{\theta}_v + \dot{\alpha}) \end{aligned} \quad (2.24)$$

$$\dot{z}_s = \frac{r \dot{r} - l \dot{l}}{\sqrt{l^2 - r^2}} \quad (2.25)$$

Then substituting equations (2.21)-(2.25) into equation (2.10), and substituting equation (5.5) directly into equation (2.20) the expressions for kinetic and potential energies are obtained as functions of the generalised coordinates (defined by equations (2.12)-(2.18)). Finally, by differentiating those expressions as necessary for Lagrange's equations (2.11) one gets the equations of motion in local coordinates r and α :

$$\begin{aligned}
& \ddot{r} + C_r \frac{\dot{r}}{m_s} - r \frac{\ddot{l}}{l} + r \frac{(r\dot{l} - \dot{r}l)^2}{l^4 - l^2 r^2} + rg \frac{\sqrt{l^2 - r^2}}{l^2} \\
& - r \frac{l^2 - r^2}{l^2} (\dot{\theta}_v + \dot{\alpha})^2 \\
& - \frac{l^2 - r^2}{l^2} \left(x_{T-v} \dot{\theta}_v^2 - \ddot{x}_{T-v} \right) \cos \alpha \\
& + \frac{l^2 - r^2}{l^2} \left(2 \dot{x}_{T-v} \dot{\theta}_v + x_{T-v} \ddot{\theta}_v \right) \sin \alpha \\
& + \ddot{x}_v \frac{l^2 - r^2}{l^2} \cos(\theta_v + \alpha) + \ddot{y}_v \frac{l^2 - r^2}{l^2} \sin(\theta_v + \alpha) = 0
\end{aligned} \tag{2.26}$$

$$\begin{aligned}
& \ddot{\alpha} + \frac{C_\alpha \dot{\alpha}}{I_s r} + \ddot{\theta}_v \\
& + 2 \frac{\dot{r}}{r} (\dot{\theta}_v + \dot{\alpha}) \\
& + \frac{\ddot{y}_v}{r} \cos(\theta_v + \alpha) - \frac{\ddot{x}_v}{r} \sin(\theta_v + \alpha) \\
& + \left(2 \frac{\dot{x}_{T-v}}{r} \dot{\theta}_v + \frac{x_{T-v}}{r} \ddot{\theta}_v \right) \cos \alpha \\
& + \left(\frac{x_{T-v}}{r} \dot{\theta}_v^2 - \frac{\ddot{x}_{T-v}}{r} \right) \sin \alpha = 0
\end{aligned} \tag{2.27}$$

$C_r \frac{\dot{r}}{m_s}$ and $C_\alpha \frac{\dot{\alpha}}{I_s r}$, in equations (2.26) and (2.27) respectively, are dissipation terms with damping coefficients C_r and C_α taking from experimental observations showing the dominance of linear viscous damping characteristics.

Thus, equations (2.26) and (2.27) are the equations of motion for the single-cable system in r and α . They define the position of the spreader for given accelerations $\ddot{x}_v$,

$\ddot{y}_v$, $\ddot{\theta}_v$, $\ddot{x}_{T_v}$ and $\ddot{l}$ applied to the RTG crane (the coordinates x_v , y_v , θ_v , x_{T_v} , and l are directly 'motored' coordinates in the system).

2.5 Conclusions

In this chapter:-

- A system of coordinates and notations has been introduced;
- Equations of Motion of the RTG with simplified single-cable spreader suspension system have been derived using Lagrange's equations.

The next chapter presents the geometrical analysis of a multi-cable spreader suspension system and gives a formula which is to be used to develop the Multi-Cable Model for an RTG crane. This model is derived in Chapter 5 with references back to this chapter.

Chapter 3

Geometry of the Multi-Cable Spreader Suspension on the RTG Crane

In the previous chapter the Lagrangian equations of motion were derived for an RTG crane with simplified single-cable spreader suspension system for the spreader considered as a lumped mass. This chapter considers the geometry of a more realistic multi-cable suspension system for the spreader. In this case, the rotation of the spreader leads to an associated hoist which significantly changes the dynamics of the system.

This chapter shows that combined translations and rotations of the spreader from the equilibrium position (i.e. directly underneath the trolley) are geometrically complicated, and that the exact relationship (in the form of an algorithm), and also an approximate explicit formula relating the hoist of the spreader due to rotation and the angle of spreader rotation, can both be proposed, and that they are intrinsically non-linear.

The explicit approximate formula relating spreader hoist to rotation is found to be highly accurate over the practical range of rotation of the spreader, and is used for the derivation of the Equations of Motion for the RTG on the basis of a multi-cable spreader suspension system (in Chapter 5).

3.1 Geometry of the multi-cable spreader suspension used on RTG cranes

Reeving (i.e. suspension geometry) designs on RTG cranes vary between manufacturers. Figure 1.2 shows the reeving geometry arrangement for a typical spreader hoist system,

but, strictly speaking, there cannot be a truly generalised analysis for all configurations. Despite this, it appears that the concept of four pairs of essentially parallel cables attaching the spreader to the trolley above (Figure 3.1) can be taken to represent the majority of designs where cabling is parallel.

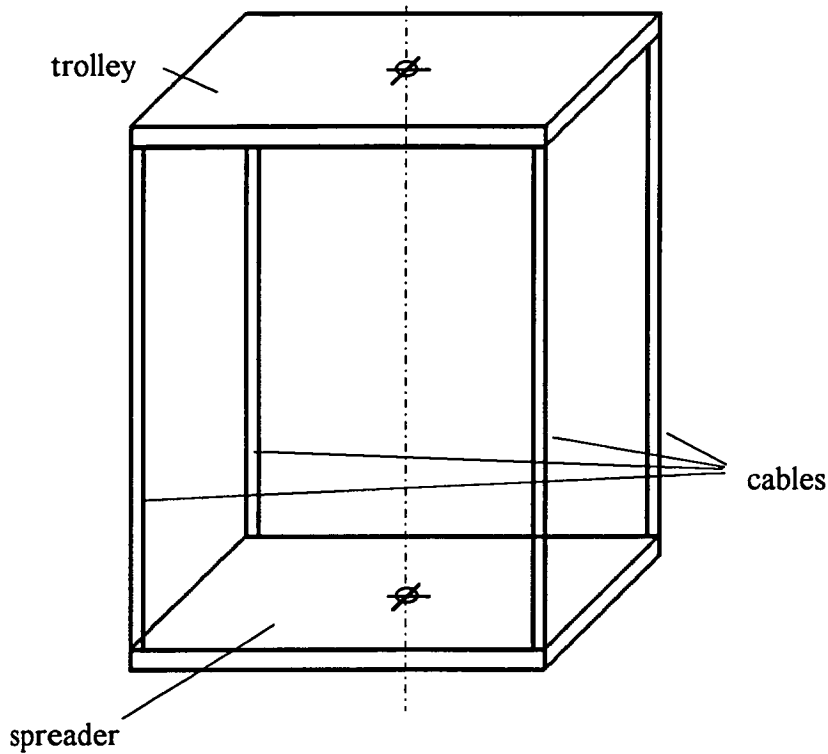


Figure 3.1

Simplified geometry of the spreader suspension

3.2 Notations and the local system of coordinates

The trolley and the spreader are considered, schematically (Figure 3.2), as rectangles $ABCD$ and $EFGH$ where points P and Q are their respective geometrical centres. The datum of the local system of coordinates is chosen to be in the geometrical centre of the spreader Q , with the X and Y axes being parallel to the spreader and trolley sides. The Z axis is chosen to point upwards.

The coordinates of the trolley and spreader corners are denoted by:

$$\begin{aligned}
A(A_x, A_y, A_z) & \quad E(E_x, E_y, E_z) \\
B(B_x, B_y, B_z) & \quad F(F_x, F_y, F_z) \\
C(C_x, C_y, C_z) & \quad G(G_x, G_y, G_z) \\
D(D_x, D_y, D_z) & \quad H(H_x, H_y, H_z)
\end{aligned} \tag{3.1}$$

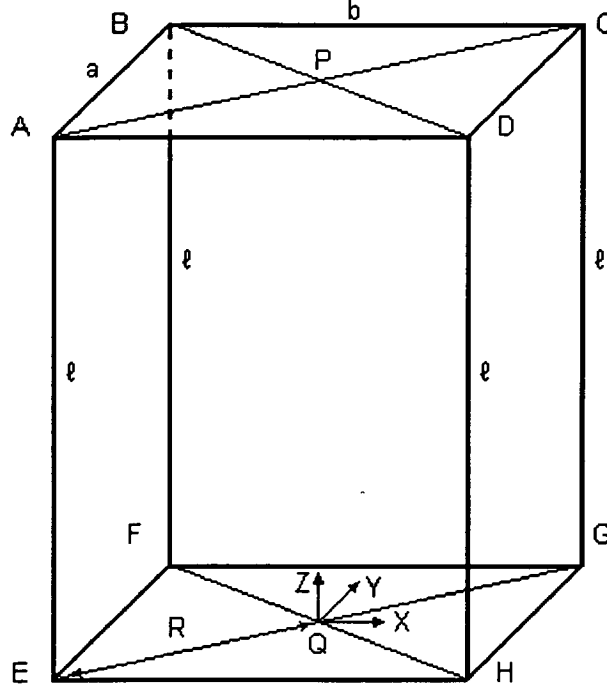


Figure 3.2

Trolley, spreader and cables schematic
used in the geometrical analysis

with their centres:

$$P(P_x, P_y, P_z) \quad Q(Q_x, Q_y, Q_z) \tag{3.2}$$

where l is the cable length. The side dimensions of the trolley/spreader are denoted by a , b :

$$a = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2} \tag{3.3}$$

$$b = \sqrt{(B_x - C_x)^2 + (B_y - C_y)^2} \quad (3.4)$$

and R is a half diagonal dimension of the trolley and of the spreader:

$$R = \frac{\sqrt{a^2 + b^2}}{2} \quad (3.5)$$

3.3 Pure translation of the spreader

Translation of the spreader along the vector $\vec{r}(r, \alpha)$ is shown on Figure 3.3. The angle of translation α is specified as relative to the X -axis.

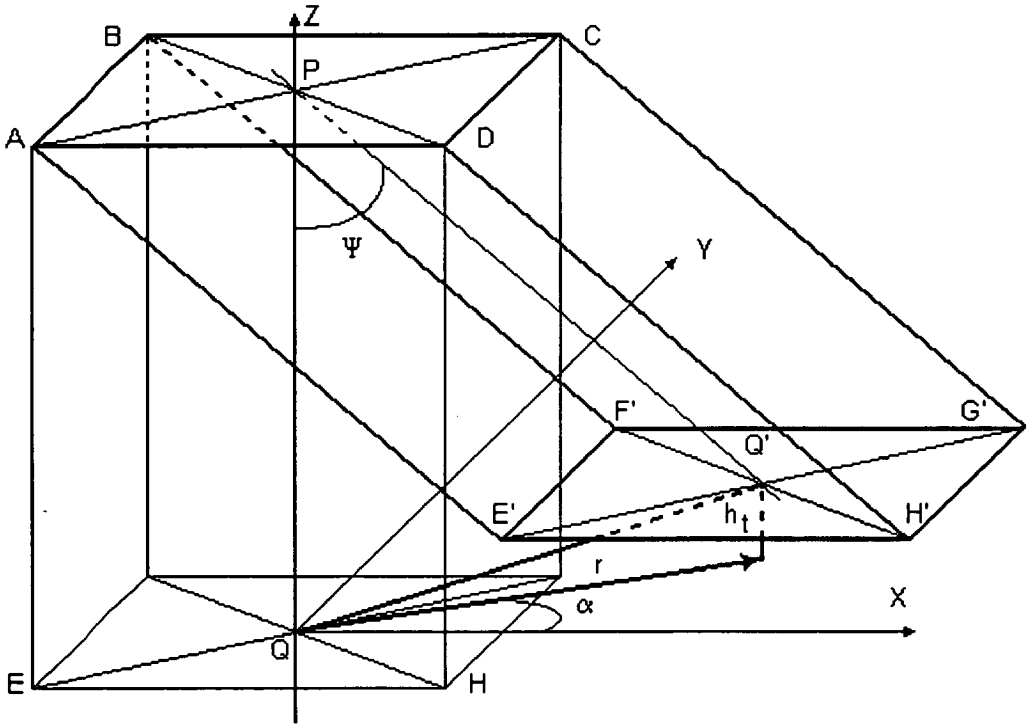


Figure 3.3
The translated spreader

The horizontal movement of the spreader along the vector $\vec{r}(r, \alpha)$ is accompanied by a hoist, or vertical translation, denoted by h_t . The centre of the spreader has therefore moved from Q to Q' :

$$Q'(Q'_x, Q'_y, Q'_z) = (Q_x + r_x, Q_y + r_y, Q_z + h_t) \quad (3.6)$$

and the new spreader position is $E'F'G'H'$, with coordinates:

$$\begin{aligned} E'(E'_x, E'_y, E'_z) &= (E_x + r_x, E_y + r_y, E_z + h_t) \\ F'(F'_x, F'_y, F'_z) &= (F_x + r_x, F_y + r_y, F_z + h_t) \\ G'(G'_x, G'_y, G'_z) &= (G_x + r_x, G_y + r_y, G_z + h_t) \\ H'(H'_x, H'_y, H'_z) &= (H_x + r_x, H_y + r_y, H_z + h_t) \end{aligned} \quad (3.7)$$

Here r_x and r_y are the components of the vector $\vec{r}(r, \alpha)$ (Figure 3.4):

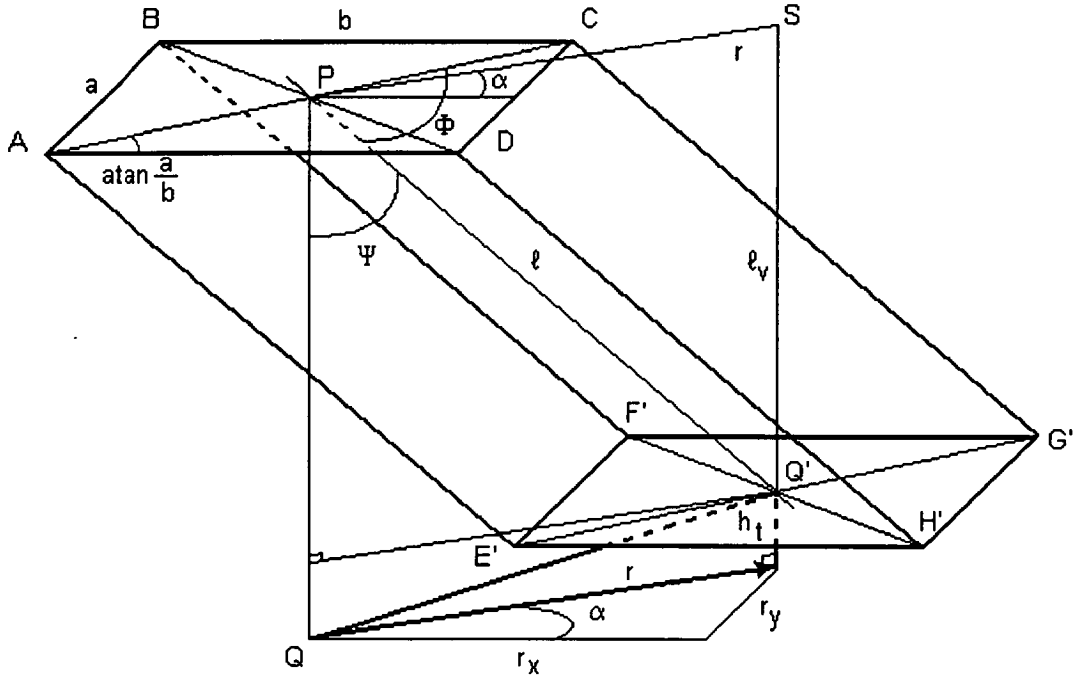


Figure 3.4

The translated spreader - new notations

$$r_x = r \cos \alpha, \quad r_y = r \sin \alpha \quad (3.8)$$

Figure 3.4 shows useful new geometrical quantities. The additional line PS of length r is constructed from the point P so that it is parallel to vector $\vec{r}(r, \alpha)$. Now the line SQ' is of length l_v , which is the vertical distance between the trolley and the spreader after translation:

$$l_v = l - h_t = \sqrt{l^2 - r^2} \quad (3.9)$$

By introducing the 'swing angle' Ψ , i.e. the angle between the central axis PQ' of the trolley-spreader system and the vertical axis PQ (Z axis of the local system of coordinates), equation (3.9) can be rewritten as:

$$l_v = l \cos \Psi \quad (3.10)$$

Also the angle lying between PQ' and PC is denoted by Φ , and it can be shown that:

$$\cos \Phi = \sin \Psi \cos \hat{CPS} = \sin \Psi \cos \left(\tan^{-1} \frac{a}{b} - \alpha \right) \quad (3.11)$$

for which a proof is given in Appendix I (proof I).

3.4 Combined translation and rotation of the spreader

3.4.1 Coordinate system transformation

Figure 3.5 shows a new coordinate system $X'Y'Z'$ which will be used for a more convenient understanding of the spreader rotation.

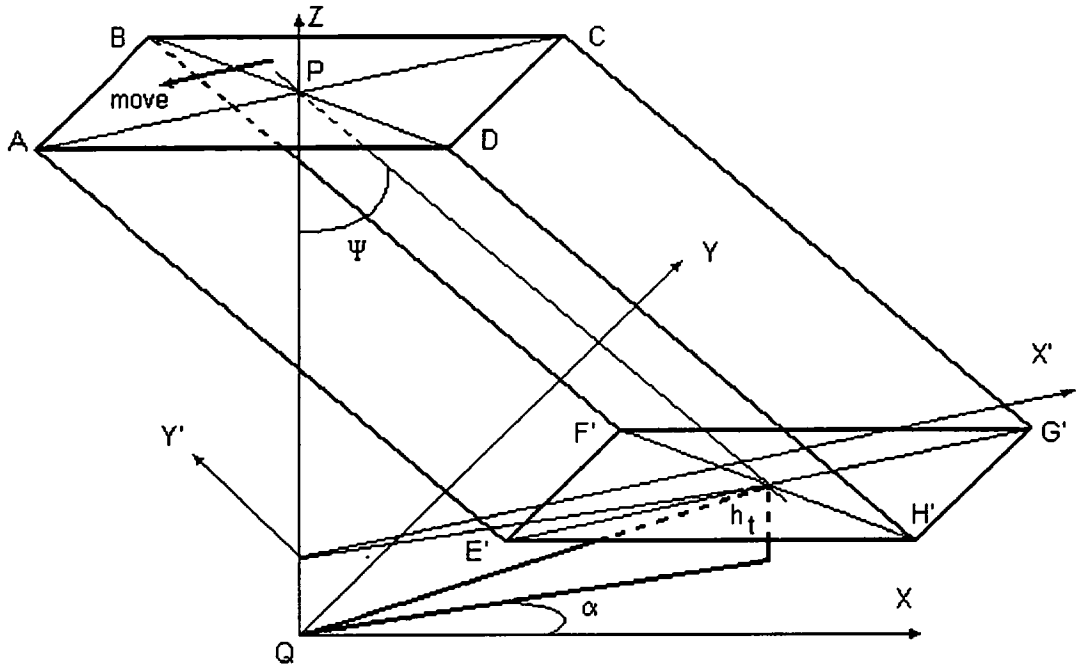


Figure 3.5
New coordinate system

The transformation is performed in three steps:

- The datum Q of the old system is lifted by h_t so that the X' and Y' axes are now lying on the spreader plane $E'F'G'H'$.
- The system is rotated about the Z axis so that the X' axis becomes parallel to the spreader diagonal $E'G'$.
- The axis set is moved along the new X' axis, through distance R , so that the Z' axis now passes through point A rather than point P .

Figure 3.6 shows the diagonals of the trolley and the spreader (AC and $E'G'$ respectively) in the new coordinate system.

3.4.2 Applying a rotation to the spreader

If a rotation is applied to the spreader after it has been translated through $\vec{r}(r, \alpha)$ then the diagonal corners E' and G' of the spreader move along the base of the cones with

vertices A and C respectively, as shown on Figure 3.7, and the centre point Q' translates along the $Q'P$ line up to the point Q'' . The distance $Q'Q''$ represents the rotational hoist of the spreader and its vertical component is denoted by h_r :

$$h_r = Q'Q'' \cos \Psi \quad (3.12)$$

The aim of the remaining investigation in this section is to find an analytical representation for the rotational hoist h_r for any angle of rotation of the spreader, Θ_s .

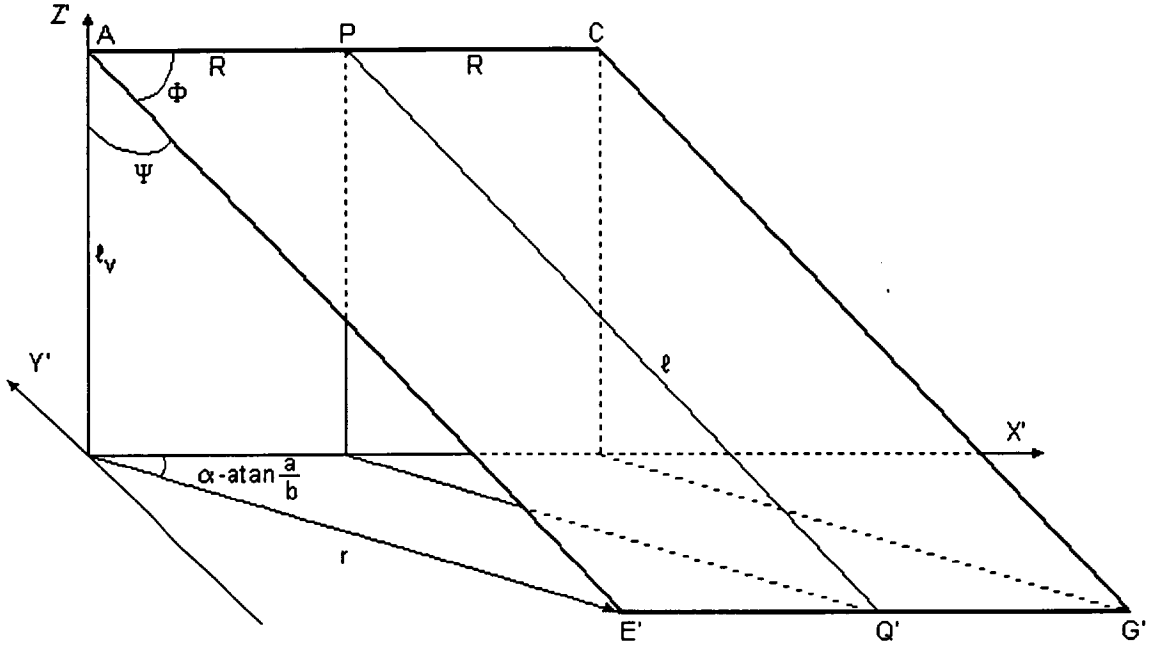


Figure 3.6

The trolley and spreader diagonals in the new coordinate system
(after the spreader has been translated)

It should be noted that the maximum rotational hoist is achieved when the points E' and G' have moved through half a cone base, and take the positions E''' and G''' respectively. All the points A , C , E''' and G''' are lying on one plane, that being the plane $ACE'G'$ (the proof of this statement is given in Appendix II, proof II). This means physically that two cables AE' and CG' have come to a position (i.e. AE''' and

CG''^* respectively) where they touch each other at some point, T_{cables} , and so further rotation would lead to the cables wrapping around each other. This situation is far too extreme to ever happen in practice, and therefore will not be dealt with further (note that in practice, $0 \leq |\theta_s| \leq 20^\circ$, represents the normal range for spreader rotations, and that even the upper bound here is rarely reached).

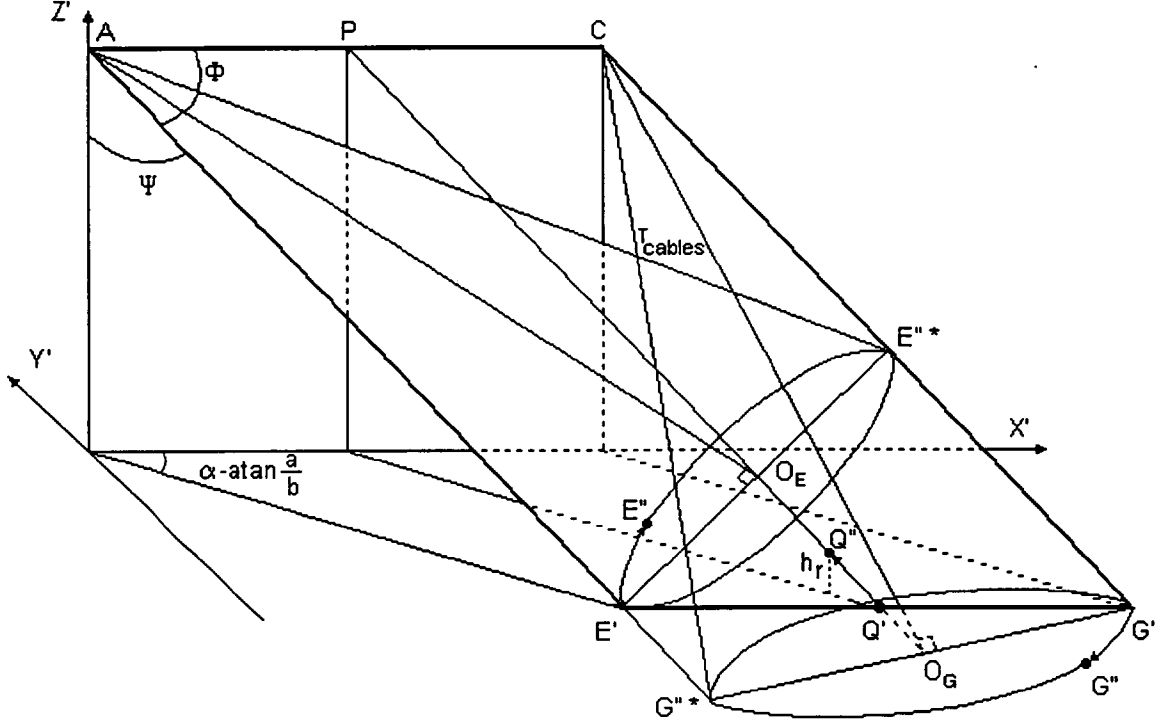


Figure 3.7

The loci of two spreader corners when it is under rotation

3.4.3 The angle of spreader rotation θ_s

Definition.

It seems reasonable to define the angle of the spreader rotation, θ_s , as the angle which lies between the projection of the diagonal of the spreader $E''G''$ onto the horizontal plane and the X' axis.

The extreme position of the spreader diagonal,
the case where $-90^\circ < \alpha - \tan^{-1} \frac{a}{b} < 0$

Figure 3.9 shows the similar case, i.e. the spreader's diagonal in an extreme position $E''G''$. The difference between the Figures 3.8 and 3.9 is only in the angle of the spreader translation. In the former case it is such that $-90^\circ < \alpha - \tan^{-1} \frac{a}{b} < 0$, but in

the latter case the relationship $-180^\circ < \alpha - \tan^{-1} \frac{a}{b} < -90^\circ$ holds. The angle of rotation and the rotational hoist can both be clearly seen in the two Figures, and $\Theta_s^* > 180^\circ$ in the first case, and $\Theta_s^* < 180^\circ$ in the second (due to the plane $ACG'E'$ not being vertical).

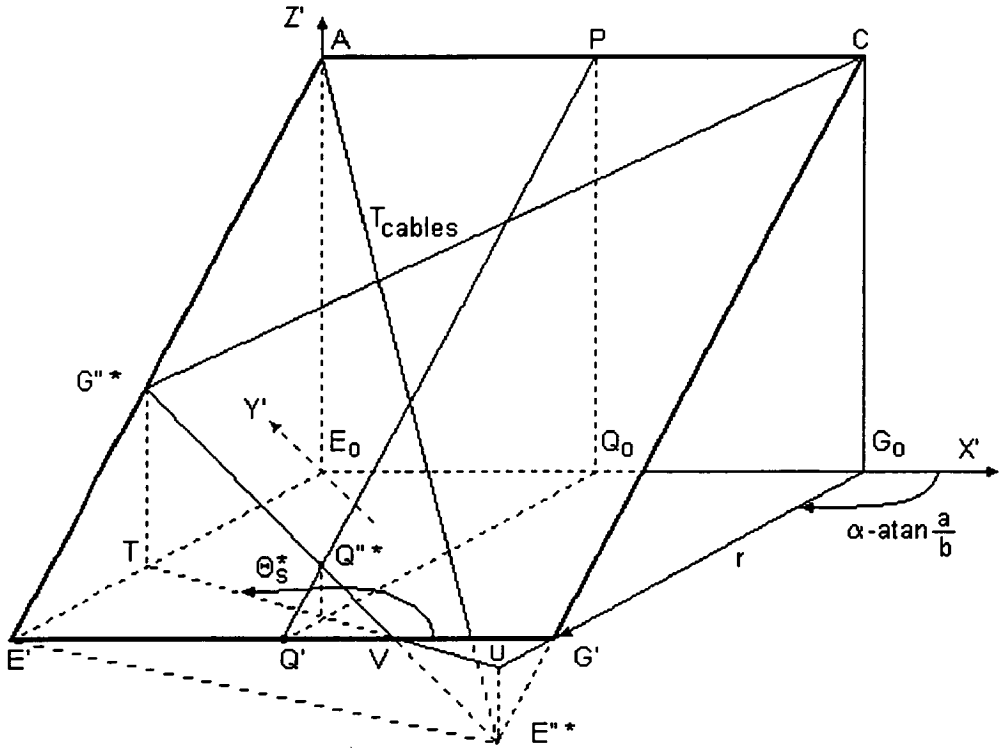


Figure 3.9

The extreme position of the spreader diagonal,

the case where $-180^\circ < \alpha - \tan^{-1} \frac{a}{b} < -90^\circ$

3.4.4 Relationship between the rotational hoist and spreader angular position

3.4.4.1 The angle Φ

The relationship between h_r and Θ_s cannot easily be derived unless another important relationship is established involving the angle Φ which denotes the angle

$\hat{P\hat{A}E'} = P\hat{Q'E'}$ (see, say, Figure 3.6). It is not intuitively obvious, but can easily be proved (see Appendix III, proof III), that this angle remains unchanged for any position of the line $E''G''$, i.e.

$$\Phi = P\hat{Q''*E''*} \quad (3.13)$$

for the extreme case (Figures 3.8 and 3.9) and for

$$\Phi = P\hat{Q''}E'' \quad (3.14)$$

in general (Figure 3.10). Now, with equation (3.14), the relationship $h_r = f(\Theta_S)$ can be established. In order to do so some more definitions and statements are needed. They are given in the next section.

3.4.4.2 More notations and statements

- Referring to Figure 3.10, the spreader diagonal is in the position $E''F''$. This line is not lying in the plane $ACG'E'$.
- The angle Θ_S is labelled on the Figure 3.10. According to the chosen definition (section 3.4.3) this is the angle between lines $E''_vG''_v$ and $E'G'$ ($E''_vG''_v$ is the projection of $E''G''$ onto the horizontal plane).
- The projection of the point E'' onto the plane $ACG'E'$ is the point E''_p .
- The angle $E''\hat{Q''}E''_p$ is denoted by β . This angle is equal to zero in the case of a pure spreader translation, i.e. when $E' = E''$; ($\Theta_S = 0$). When the spreader starts rotating β increases up to its maximal value of $\beta = \Phi$ (henceforth to be defined as the *first part of rotation*) and then decreases (defined as the *second part of rotation*). β is equal to zero again when $\Theta_S = \Theta_S^*$.

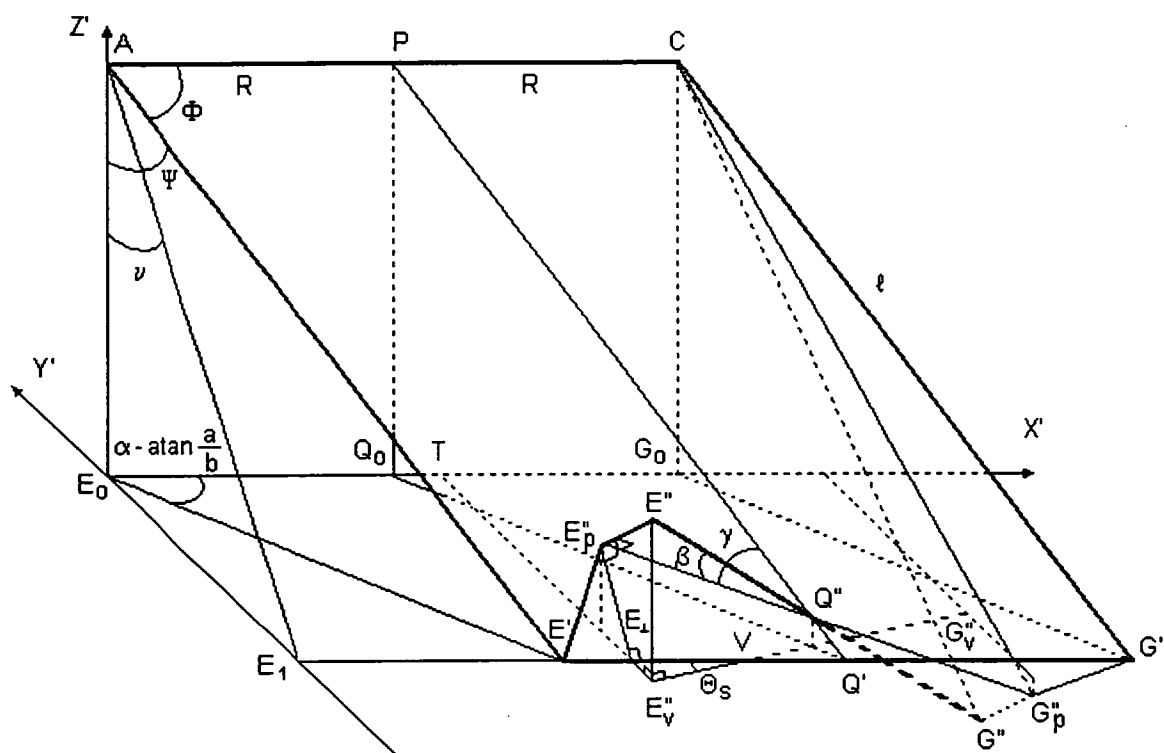


Figure 3.10

The general position of the spreader diagonal,

the case: $-90^\circ < \alpha - \tan^{-1} \frac{a}{b} < 0$

first part of rotation,

$$\theta_s > 0$$

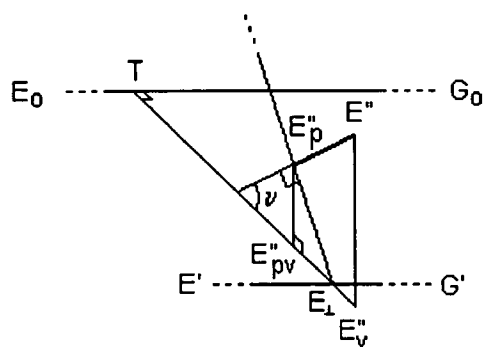


Figure 3.11

Fragment of Figure 3.10

Both h_r and Θ_S can be found as functions of β after some more geometrical constructs are defined, as summarised in the following sequence of points:

- The line $E''_p E_\perp$ is perpendicular to $E'G'$ lying on the plane $ACG'E'$ (Figures 3.10, 3.11). The line $E'E''_p$ is also on the plane $ACG'E'$
- The line $E_\perp E''_v$ is on the horizontal plane and is perpendicular to $E'G'$. On extension it intersects with the X' axis at the point T .
- The projection of E''_p onto the horizontal plane (point E''_{pv} , Figure 3.11) is lying on $E''_v T$.
- The angle between the $ACG'E'$ and $X'Z'$ planes is denoted by ϑ and can be calculated by applying similar ideas to those described in Appendix AI to the right-angled wedge $AE'E_1E_0$ (Figure 3.10):

$$\tan \vartheta = \tan \Psi \times \sin \left(\alpha - \tan^{-1} \frac{a}{b} \right) \quad (3.15)$$

- Applying the relationship (A1.8) established in Appendix I to the wedge $Q''E''E''_pP$:

$$\cos \gamma = \frac{\cos \Phi}{\cos \beta} \quad (3.16)$$

- Both ϑ and γ only depend on the vector of spreader translation $\vec{r}(r, \alpha)$ and the angle β (the independent variable in this analysis). This means that for a given β the projection of the point E'' onto the plane $ACG'E'$ is the same point E''_p for both negative and positive Θ_S as on Figures 3.12 and 3.13.

- Figure 3.14 shows the plane $ACG'E'$. The E''_p point is in its left half which means that this is the first part of rotation for both $\Theta_S > 0$ and $\Theta_S < 0$. Figure 3.15 shows the plane $ACG'E'$ with the E''_p point in its right half, which means this is the second part of rotation, again for both $\Theta_S > 0$ and $\Theta_S < 0$.

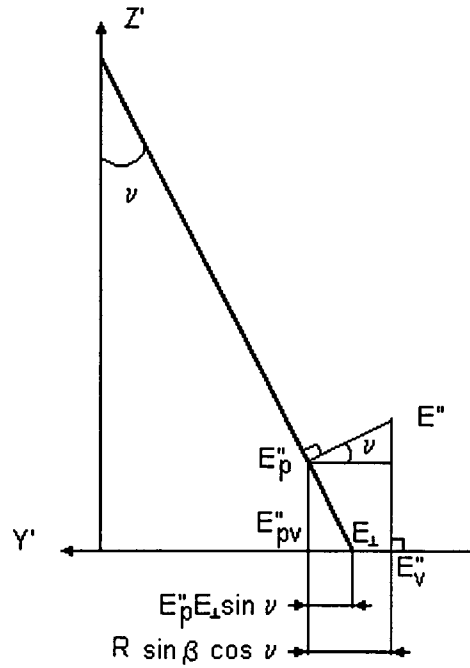


Figure 3.12

The view from $(-X)$ direction

the case: $-90^\circ < \alpha - \tan^{-1} \frac{a}{b} < 0$,

first part of rotation,

$\Theta_S > 0$

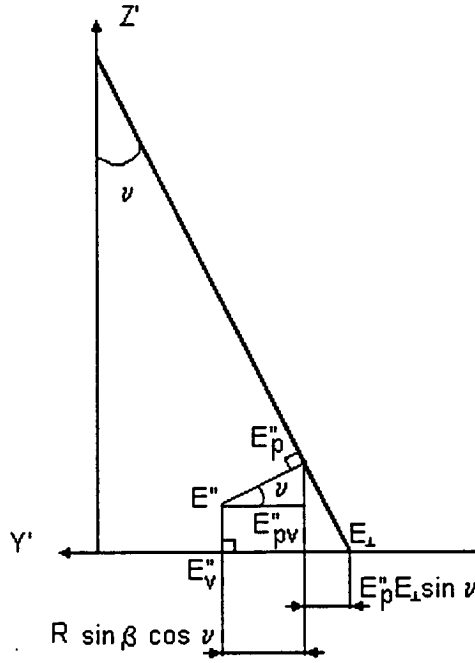


Figure 3.13

The view from $(-X)$ direction,

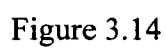
the case: $-90^\circ < \alpha - \tan^{-1} \frac{a}{b} < 0$,

first part of rotation,

$$\Theta_S < 0$$

- Finally, Figure 3.16 shows all possible principal positions of the projection $E''_v G''_v$ of the spreader diagonal onto the horizontal plane. It can be seen on this Figure that the intersection between $E''_v G''_v$ and $E'G'$ is the point V . Q''_{pv} denotes the projection of Q'' onto the horizontal plane, and $Q''_{pv} W \perp E'G'$.

Having established the necessary geometrical constructs for the system in general an algorithm for calculating $Q'Q''$ and Θ_S as functions of β can be proposed.



the case: $-90^\circ < \alpha - \tan^{-1} \frac{a}{b} < 0$,

both $\Theta_S > 0$ and $\Theta_S < 0$

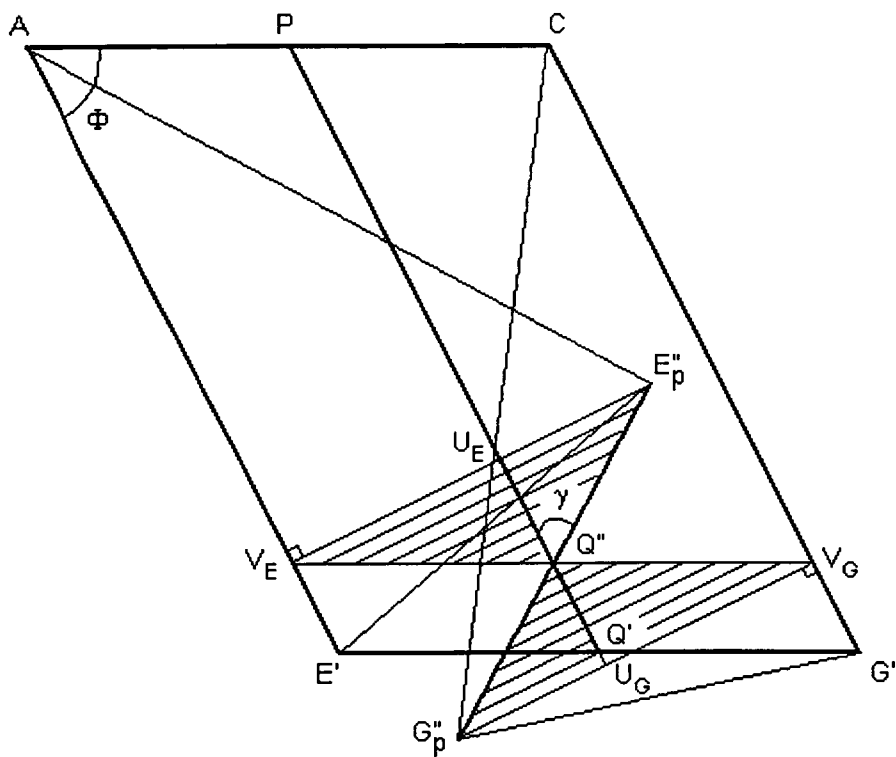


Figure 3.15

The plane $ACG'E'$,

the case: $-90^\circ < \alpha - \tan^{-1} \frac{a}{b} < 0$,

second part of rotation,

both $\Theta_S > 0$ and $\Theta_S < 0$

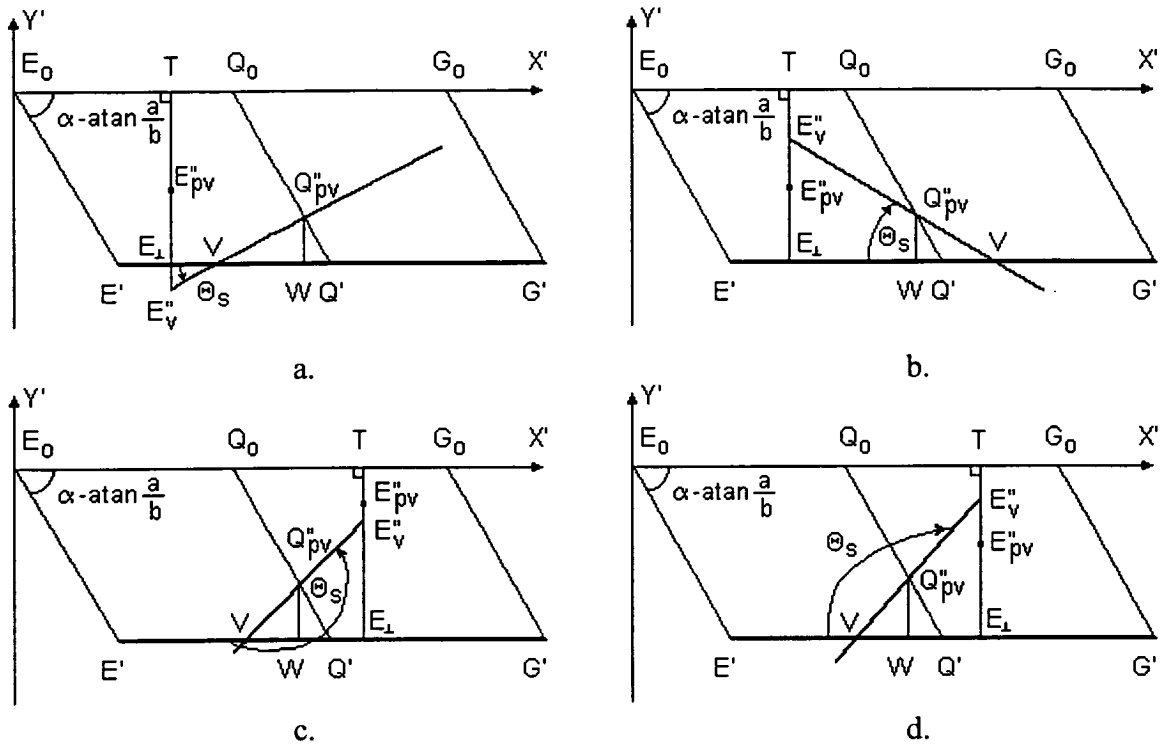


Figure 3.16

The horizontal plane,

$$-90^\circ < \alpha - \tan^{-1} \frac{a}{b} < 0$$

case a.	first part of rotation,	$\Theta_S > 0$
case b.	first part of rotation,	$\Theta_S < 0$
case c.	second part of rotation,	$\Theta_S > 0$
case d.	second part of rotation,	$\Theta_S < 0$

3.4.4.3 An algorithm for calculating the distance $Q'Q''$ and the angle of rotation Θ_S as functions of β

The distance $Q'Q''$ can be calculated using the following equation (Figures 3.14 and 3.15):

$$Q'Q'' = l - \sqrt{l^2 - 2R^2 \sin \Phi (\sin \Phi \mu \sqrt{\cos^2 \beta - \cos^2 \Phi}} \quad (3.17)$$

The proof of equation (3.17) is given in Appendix IV (proof IV). The '-' sign should be chosen for the first, and '+' for the second, parts of rotation. From equation (3.17) the vertical rotational hoist h_r can be calculated using equation (3.12).

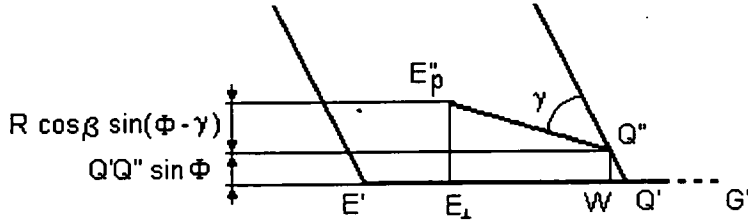


Figure 3.17

Fragment of Figure 3.14

Finally, an algorithm can be constructed for calculating Θ_S as a function of β . It starts with the equations (3.9)-(3.11), (3.15) and (3.16) for l_v , Ψ , Φ , ϑ and γ . Then the algorithm splits into four parts for all four cases (two parts of rotation and two signs of Θ_S). Equation (3.17) is written for each part separately, with the corresponding sign before the square root. All subsequent formulae are derived separately for each part using Figures 3.11-3.17 by applying simple trigonometrical equations, Pythagoras' theorem, and the geometry of triangles. The formulae numbers in the algorithm have subscript 'I' for the first or 'II' for the second part of rotation, and '>0' or '<0' for the cases $\Theta_S > 0$ or $\Theta_S < 0$.

1. $l_v = \sqrt{l^2 - r^2}$
2. $\Psi = \cos^{-1} \frac{l_v}{l}$
3. $\Phi = \cos^{-1} \left[\sin \Psi \cos \left(\tan^{-1} \frac{a}{b} - \alpha \right) \right]$
4. $\vartheta = \tan^{-1} \left[\tan \Psi \times \sin \left(\alpha - \tan^{-1} \frac{a}{b} \right) \right]$

$$5. \quad \gamma = \cos^{-1} \left(\frac{\cos \Phi}{\cos \beta} \right)$$

The first part of rotation for $\Theta_s > 0$:

$$6_{I>0}. \quad Q' Q'' = l - \sqrt{l^2 - 2R^2 \sin \Phi (\sin \Phi - \sqrt{\cos^2 \beta - \cos^2 \Phi})}$$

$$7_{I>0}. \quad W Q''_{pv} = -Q' Q'' \sin \Psi \sin \left(\alpha - \tan^{-1} \frac{a}{b} \right)$$

$$8_{I>0}. \quad E''_p E_{\perp} = R \cos \beta \sin(\Phi - \gamma) + Q' Q'' \sin \Phi$$

$$9_{I>0}. \quad E_{\perp} E''_{pv} = -R \sin \beta \cos \vartheta - E''_p E_{\perp} \sin \vartheta$$

$$10_{I>0}. \quad E_{\perp} W = R \cos \beta \cos(\Phi - \gamma)$$

$$11_{I>0}. \quad \theta_s = \tan^{-1} \frac{W Q''_{pv} - E_{\perp} E''_{pv}}{E_{\perp} W}$$

The first part of rotation for $\Theta_s < 0$:

$$6_{I<0}. \quad Q' Q'' = l - \sqrt{l^2 - 2R^2 \sin \Phi (\sin \Phi - \sqrt{\cos^2 \beta - \cos^2 \Phi})}$$

$$7_{I<0}. \quad W Q''_{pv} = -Q' Q'' \sin \Psi \sin \left(\alpha - \tan^{-1} \frac{a}{b} \right)$$

$$8_{I<0}. \quad E''_p E_{\perp} = R \cos \beta \sin(\Phi - \gamma) + Q' Q'' \sin \Phi$$

$$9_{I<0}. \quad E_{\perp} E''_{pv} = R \sin \beta \cos(-\vartheta) + E''_p E_{\perp} \sin(-\vartheta)$$

$$10_{I<0}. \quad E_{\perp} W = R \cos \beta \cos(\Phi - \gamma)$$

$$11_{I<0}. \quad \theta_s = \tan^{-1} \frac{W Q''_{pv} - E_{\perp} E''_{pv}}{E_{\perp} W}$$

The second part of rotation for $\Theta_s > 0$:

$$\begin{aligned}
6_{\Pi, >0}. \quad Q'Q'' &= l - \sqrt{l^2 - 2R^2 \sin \Phi (\sin \Phi + \sqrt{\cos^2 \beta - \cos^2 \Phi})} \\
7_{\Pi, >0}. \quad WQ''_{pv} &= -Q'Q'' \sin \Psi \sin \left(\alpha - \tan^{-1} \frac{a}{b} \right) \\
8_{\Pi, >0}. \quad E''_p E_{\perp} &= R \cos \beta \sin(\Phi + \gamma) + Q'Q'' \sin \Phi \\
9_{\Pi, >0}. \quad E_{\perp} E''_v &= -R \sin \beta \cos \vartheta - E''_p E_{\perp} \sin \vartheta \\
10_{\Pi, >0}. \quad E_{\perp} W &= R \cos \beta \cos(\Phi + \gamma) \\
11_{\Pi, >0}. \quad \theta_s &= \begin{cases} \pi + \tan^{-1} \frac{WQ''_{pv} - E_{\perp} E''_{pv}}{E_{\perp} W} & \text{if } \beta < \beta_{90^\circ} = \cos^{-1} \frac{1}{\tan \Phi} \\ \theta_s = \tan^{-1} \frac{WQ''_{pv} - E_{\perp} E''_{pv}}{E_{\perp} W} & \text{otherwise} \end{cases}
\end{aligned}$$

The second part of rotation for $\Theta_s < 0$:

$$\begin{aligned}
6_{\Pi, <0}. \quad Q'Q'' &= l - \sqrt{l^2 - 2R^2 \sin \Phi (\sin \Phi + \sqrt{\cos^2 \beta - \cos^2 \Phi})} \\
7_{\Pi, <0}. \quad WQ''_{pv} &= -Q'Q'' \sin \Psi \sin \left(\alpha - \tan^{-1} \frac{a}{b} \right) \\
8_{\Pi, <0}. \quad E''_p E_{\perp} &= R \cos \beta \sin(\Phi + \gamma) + Q'Q'' \sin \Phi \\
9_{\Pi, <0}. \quad E_{\perp} E''_v &= R \sin \beta \cos \vartheta - E''_p E_{\perp} \sin \vartheta \\
10_{\Pi, <0}. \quad E_{\perp} W &= R \cos \beta \cos(\Phi + \gamma) \\
11_{\Pi, <0}. \quad \theta_s &= \begin{cases} -\pi + \tan^{-1} \frac{WQ''_{pv} - E_{\perp} E''_{pv}}{E_{\perp} W} & \text{if } \beta < \beta_{90^\circ} = \cos^{-1} \frac{1}{\tan \Phi} \\ \theta_s = \tan^{-1} \frac{WQ''_{pv} - E_{\perp} E''_{pv}}{E_{\perp} W} & \text{otherwise} \end{cases}
\end{aligned}$$

β_{90° used in the algorithm is the threshold value of β corresponding to both $\Theta_s = +90^\circ$ and $\Theta_s = -90^\circ$. It means that $E_{\perp} W = 0$ when $\beta = \beta_{90^\circ}$, and:

$$\Phi + \gamma = 90^\circ \quad (3.18)$$

Equation (3.18) allows the expression for β_{90° to be found as used in formulae 11_{II,<0} and 11_{II,>0} of the algorithm. Note, using (3.16) and (3.18):

$$\beta_{90^\circ} = \cos^{-1} \frac{\cos \Phi}{\cos \gamma} = \cos^{-1} \frac{\cos \Phi}{\cos(90^\circ - \Phi)} = \cos^{-1} \frac{1}{\tan \Phi} \quad (3.19)$$

3.4.4.4 $Q'Q''$ as an exact function of Θ_S ($Q'Q'' = f(\Theta_S)$)

Using the algorithm given in section 3.4.4.3 provides a means for calculating the distance $Q'Q''$ and the angle of rotation Θ_S for given β (formulae 6 and 11 of the algorithm). This means that $Q'Q''$ (and the value of vertical rotational hoist h_r using equation (3.12)) can be found as a function of the angle of rotation Θ_S . Figure 3.18 shows one example of the $Q'Q'' = f(\Theta_S)$ function (both the exact relationship and a quadratic approximation).

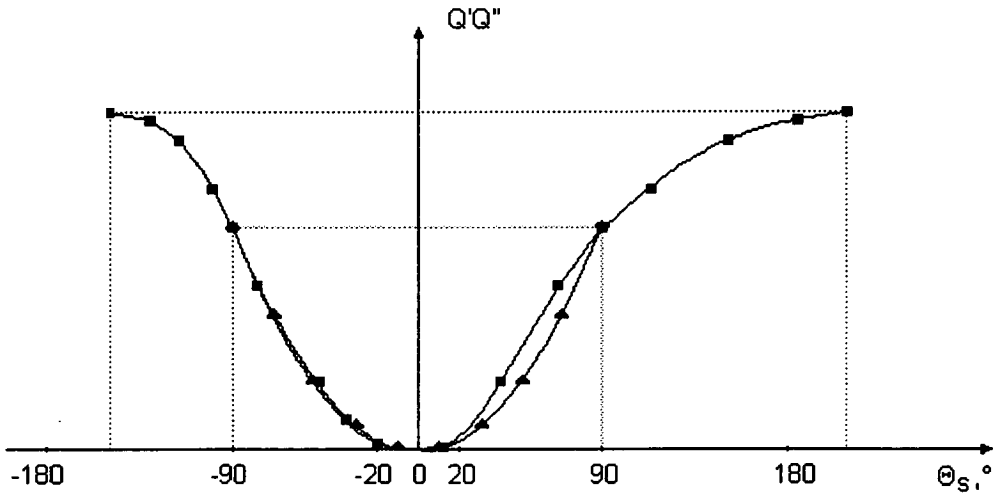


Figure 3.18

- exact relationship
- ▲ quadratic approximation

Thus the algorithm gives the exact value of $Q'Q''$ (and therefore h_r) for values of Θ_S , and it is therefore theoretically possible to find an explicit formula for $Q'Q'' = f(\Theta_S)$ (or

$h_r = f(\Theta_S)$)). But it is clear that this will lead to very tedious algebraic manipulations to produce a 'one-line' formula which will be exact, and which will be useless practically, due to its length and complexity. The next section suggests an approximation for $Q'Q'' = f(\Theta_S)$ which is accurate enough over the domain, and which can readily be used in further analytical investigations.

3.4.4.5 Vertical rotational hoist as an approximate explicit function $h_r = f(\Theta_S)$

Using equations (3.17) and (3.19) the value of $Q'Q''$ for $\Theta_S = \pm 90^\circ$ can be calculated:

$$Q'Q''_{\pm 90^\circ} = l - \sqrt{l^2 - 2R^2 \sin \Phi (\sin \Phi \pm \sqrt{\cos^2 \beta_{90^\circ} - \cos^2 \Phi})} =$$

$$l - \sqrt{l^2 - 2R^2 \sin \Phi (\sin \Phi \pm \sqrt{\frac{1}{\tan^2 \Phi} - \cos^2 \Phi})}$$

which leads to:

$$Q'Q''_{\pm 90^\circ} = l - \sqrt{l^2 - 2R^2} \quad (3.20)$$

Equation (3.20) shows that for $\Theta_S = \pm 90^\circ$ the value of $Q'Q''$ does *not* depend on the vector of translation $\vec{r}(r, \alpha)$.

This suggests the potential for using a quadratic approximation (see Figure 3.18) for the function $Q'Q'' = f(\Theta_S)$. For three known values of $Q'Q''$:

$$Q'Q''_{+90^\circ} = Q'Q''_{-90^\circ} = l - \sqrt{l^2 - 2R^2}$$

$$Q'Q''_{0^\circ} = 0$$

a quadratic relationship can be proposed in the following form:

$$Q'Q'' = \frac{l - \sqrt{l^2 - 2R^2}}{\left(\frac{\pi}{2}\right)^2} \theta_s^2 \quad (3.21)$$

and finally, using equation (3.12):

$$h_r = \frac{l - \sqrt{l^2 - 2R^2}}{\left(\frac{\pi}{2}\right)^2} \theta_s^2 \cos \Psi \quad (3.22)$$

Thus, equation (3.22) is an explicit formula from which one can calculate the value of the hoist due to the spreader rotation h_r for a given value of the rotation angle θ_s . This formula is approximate, but as the calculations show it is highly accurate within the domain of realistic values of θ_s ($|\theta_s| < 20^\circ$).

3.5 Coordinates of the spreader's centre of mass after undergoing combined translation and rotation

Back in the original local $QXYZ$ frame (Figure 3.2), equations (3.6), (3.8), and (3.9) can be used to establish the coordinates of the centre of mass of the spreader after translation:

$$Q'_x = Q_x + r \cos \alpha \quad (3.23)$$

$$Q'_y = Q_y + r \sin \alpha \quad (3.24)$$

$$Q'_z = Q_z + l - \sqrt{l^2 - r^2} \quad (3.25)$$

and after its rotation,

$$Q''_x = Q'_x - Q'Q'' \sin \Psi \cos \alpha \quad (3.26)$$

$$Q''_y = Q'_y - Q'Q'' \sin \Psi \sin \alpha \quad (3.27)$$

$$Q''_z = Q'_z + Q'Q'' \cos \Psi \quad (3.28)$$

The positive sign in equation (3.28) is explained by the fact that the spreader *lifts* while rotating, i.e. its z-coordinate *increases*, while the minus sign in (3.26) and (3.27) together with the value of α guarantee the fact that, while rotating, the spreader's mass centre moves *towards* that of the trolley.

Using equation (3.21) the coordinates of the spreader mass centre, Q'' , after the spreader has been rotated through Θ_s can be written:

$$Q''_x = Q'_x - \frac{l - \sqrt{l^2 - 2R^2}}{\left(\frac{\pi}{2}\right)^2} \theta_s^2 \sin \Psi \cos \alpha \quad (3.29)$$

$$Q''_y = Q'_y - \frac{l - \sqrt{l^2 - 2R^2}}{\left(\frac{\pi}{2}\right)^2} \theta_s^2 \sin \Psi \sin \alpha \quad (3.30)$$

$$Q''_z = Q'_z + \frac{l - \sqrt{l^2 - 2R^2}}{\left(\frac{\pi}{2}\right)^2} \theta_s^2 \cos \Psi \quad (3.31)$$

Equations (3.29)-(3.31) are given in the local system of coordinates (as in Figure 3.2). They should be rewritten in the global system of coordinates for further use when deriving the equations of motion (in chapter 5). Returning to the notations introduced in section 2.2.1, the global coordinates of the spreader after translation are given in section 2.3.2.2 as functions of the trolley and vehicle global coordinates. These are the equations (2.8)-(2.10) restated here:

$$x_s = x_v + x_{T-v} \cos \theta_v + r \cos(\theta_v + \alpha) \quad (3.32)$$

$$y_s = y_v + x_{T_v} \sin \theta_v + r \sin(\theta_v + \alpha) \quad (3.33)$$

$$z_s = z_T - \sqrt{l^2 - r^2} = z_T - l \cos \Psi \quad (3.34)$$

Comparison of (3.32)-(3.34) with (3.23)-(3.25) shows that they are the same. Clearly, the relationships:

$$x_v = Q_x$$

$$y_v = Q_y$$

$$x_T = Q_z + l$$

$$l \cos \Psi = \sqrt{l^2 - r^2}$$

are true for both systems of coordinates as their Z axes are collocated. The angle between their X axes is Θ_v (also between their Y axes). This explains the additional (second) terms in (3.32) and (3.33) (see Figure 2.2) and the substitution of α for $(\alpha + \Theta_v)$.

Now (3.29)-(3.31) can be rewritten in the global coordinate system:

$$x_s = x_v + x_{T_v} \cos \theta_v + r \cos(\theta_v + \alpha) - \frac{l - \sqrt{l^2 - 2R^2}}{\left(\frac{\pi}{2}\right)^2} \theta_s^2 \sin \Psi \cos(\theta_v + \alpha) \quad (3.35)$$

$$y_s = y_v + x_{T_v} \sin \theta_v + r \sin(\theta_v + \alpha) - \frac{l - \sqrt{l^2 - 2R^2}}{\left(\frac{\pi}{2}\right)^2} \theta_s^2 \sin \Psi \sin(\theta_v + \alpha) \quad (3.36)$$

$$z_s = z_T - l \cos \Psi + \frac{l - \sqrt{l^2 - 2R^2}}{\left(\frac{\pi}{2}\right)^2} \theta_s^2 \cos \Psi \quad (3.37)$$



3.6 Conclusions

The geometry of combined translations and rotations of the spreader has been thoroughly analysed and it has been shown that:

- The exact relationship between the hoist of the spreader due to rotation, and an angle of spreader rotation, can be derived in the form of an algorithm.
- An explicit formula has also been discussed. It is approximate, but highly accurate over the realistic range of angles of rotation of the spreader, and is used for constructing the Multi-Cable Model of the RTG crane in later sections of the thesis.
- Formulae (3.35)-(3.37) have been established for the global coordinates of the spreader mass centre after both translation and rotation. These are used in chapter 5 for deriving the equations of motion for the multi-cable model.

The results of this chapter are used for the proof of a asymmetrical cable stretch phenomenon, hitherto unknown, which takes place in a typical spreader suspension systems as used on RTG cranes. This is described in the next chapter in detail.

Chapter 4

Geometrical Extension and Asymmetrical Cable Stretch Phenomenon

The previous chapter discussed the geometry of a typical multi-cable spreader suspension system in order to investigate how the hoist of the spreader due to its rotation, h_r , depends on various parameters of the system (cables length, l , polar coordinates of vector of the spreader translation, r and α , and angle of the spreader rotation, Θ_s). The chapter proposed an accurate algorithm and an approximate formula for calculation of this part of the hoist as a function of the above-mentioned parameters, $h_r = f(l, r, \alpha, \Theta_s)$. The latter is used in Chapter 5 where the complete Multi-Cable model of the RTG is derived.

By means of the algorithm proposed in Chapter 3, a proof of the existence of a hitherto unknown phenomenon which takes place to some extent or another in all RTG cranes is offered. This phenomenon is shown to exist where a rectangular trolley/spreader suspension system is fitted, namely that:-

- the combined translation and rotation of the spreader is generally accompanied by a cable stretch or compression (henceforth called *geometrical extension*) due *only* to the geometry of the system. This is on top of extensions/compressions due to inertial and gravitational loads in the cables;
- the geometrical extension is unevenly allocated across the cables, causing an *asymmetrical cable stretch* effect in the system. In other words, when the spreader swings away from its position under the trolley and then rotates each cable will experience *different* tensile/compressive load due entirely to the instantaneous geometry of the system.

This phenomenon is thoroughly investigated in this chapter, and two methods are suggested for calculation of the numerical values of the geometrical extension for given parameters. The first method is exact and can be easily implemented, but only gives a theoretical upper bound for the geometrical extension in the system. The second one, being more sophisticated, offers a relatively practical approach to the problem and produces a numerical algorithm for calculating more realistic values for geometrical extension shared out across the four cables.

It is shown that the geometrical extension (and therefore the stretch asymmetry in the system) increases both with translation and rotation of the spreader, and inversely with the length of the cables.

Although the investigations propounded in this chapter are purely theoretical in character, it is clear that this phenomenon has the potential for significant practical applications whereby additional stiffening in the dynamical modelling will be apparent through modifications to the strain energy terms in the system potential energy formulation. This has a practical analogue in the form of natural enhancement to anti-sway properties in the system, but, as far as the author is aware, it is the first documented attempt to define and analyse this naturally occurring phenomenon in multi-cable spreader suspension systems.

4.1 Geometrical extension

4.1.1 Theoretical proof of the existence of the phenomenon in the four-cable suspension system

4.1.1.1 Fundamental development of theory: triangular laminas

The generalised system for a triangular lamina suspended from another triangular lamina by means of three cables is shown in Figure 4.1. The sides of the upper lamina are

denoted by a , b , c , and the sides of the lower lamina are denoted e , f , g . Three cables connect three corresponding corners of each lamina and their lengths are l_a , l_b , l_c (the cables are supposed to be straight and would intuitively be considered to be in equal tension).

The coordinates of the lamina corners are referred to a global reference frame and are denoted by:

$$\begin{array}{ll} A(A_x, A_y, A_z) & E'(E'_x, E'_y, E'_z) \\ B(B_x, B_y, B_z) & \text{and } F'(F'_x, F'_y, F'_z) \\ C(C_x, C_y, C_z) & G'(G'_x, G'_y, G'_z) \end{array} \quad (4.1)$$

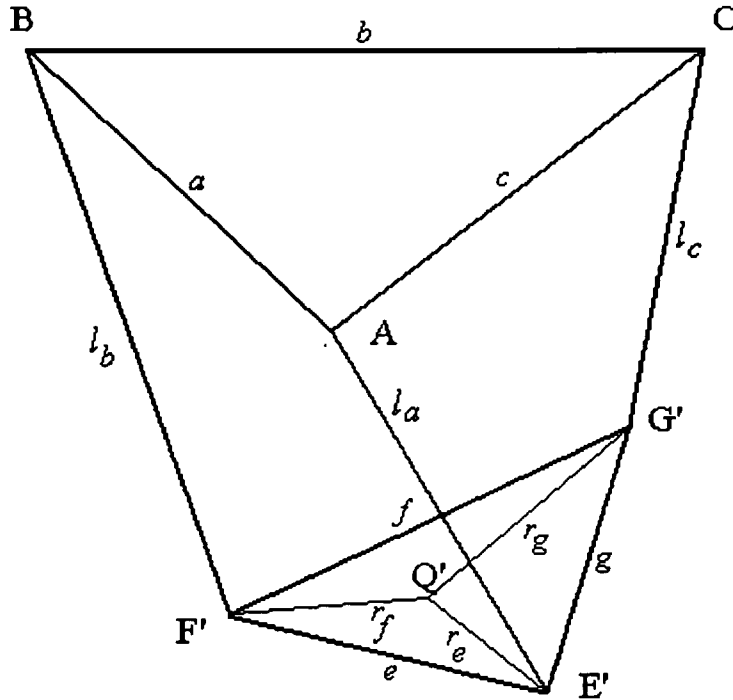


Figure 4.1

General three cable triangular lamina suspension system

The objectives are to show that it *is possible* to move the lower lamina away from this position to any other without any changes in the shape of either lamina or in any cable

lengths (provided the new position of the lower lamina is feasible, i.e. neither too far from nor too near to the lamina ABC). The upper lamina is supposed to stay fixed in space with the coordinates of its corners A, B, C being as in statements (4.1).

The coordinates of one (any) point on the surface of the lower lamina, say Q' , after this lamina is translated away from its initial position, are:

$$Q'(Q'_x, Q'_y, Q'_z) \quad (4.2)$$

It will be shown here that the coordinates of the three corners E', F', G' of the lower lamina can be identified without violation of the conditions described above (i.e. with $e, f, g, r_e, r_f, r_g, l_a, l_b, l_c$ remaining unchanged).

It is clear that four points E', F', G' and Q' lie on one plane. Three points on the plane, say E', F' and Q' , can be used to write the equation of this plane as follows.

The general equation of a plane is:

$$Ax + By + Cz = D \quad (4.3)$$

and the constants A, B, C and D in equation (4.3) can be written as functions of the coordinates of the chosen points E', F' and Q' . These are the coordinates of the vector $\vec{n}$ representing the normal to the plane. The vector product of $\vec{E'Q'}$ and $\vec{F'Q'}$ can be used as the normal to the plane $E'F'Q'$ giving:

$$\vec{n} = \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} [(E'_y - Q'_y)(F'_z - Q'_z) - (E'_z - Q'_z)(F'_y - Q'_y)] \\ [(E'_z - Q'_z)(F'_x - Q'_x) - (E'_x - Q'_x)(F'_z - Q'_z)] \\ [(E'_x - Q'_x)(F'_y - Q'_y) - (E'_y - Q'_y)(F'_x - Q'_x)] \end{Bmatrix} \quad (4.4)$$

The equation of the plane (4.3) in terms of the chosen points E' , F' , and Q' can now be rewritten:

$$\begin{aligned} & \left[(E'_y - Q'_y)(F'_z - Q'_z) - (E'_z - Q'_z)(F'_y - Q'_y) \right] x + \\ & \left[(E'_z - Q'_z)(F'_x - Q'_x) - (E'_x - Q'_x)(F'_z - Q'_z) \right] y + \\ & \left[(E'_x - Q'_x)(F'_y - Q'_y) - (E'_y - Q'_y)(F'_x - Q'_x) \right] z = D \end{aligned} \quad (4.5)$$

The constant D can be found by substituting the coordinates of Q' , say, into equation (4.5):

$$\begin{aligned} D = & \left[(E'_y - Q'_y)(F'_z - Q'_z) - (E'_z - Q'_z)(F'_y - Q'_y) \right] Q'_x + \\ & \left[(E'_z - Q'_z)(F'_x - Q'_x) - (E'_x - Q'_x)(F'_z - Q'_z) \right] Q'_y + \\ & \left[(E'_x - Q'_x)(F'_y - Q'_y) - (E'_y - Q'_y)(F'_x - Q'_x) \right] Q'_z \end{aligned} \quad (4.6)$$

Thus all four constants in equation (4.3) are identified, and the first equation for identifying the coordinates of the corners of the lower lamina is:

$$\begin{aligned} & \left[(E'_y - Q'_y)(F'_z - Q'_z) - (E'_z - Q'_z)(F'_y - Q'_y) \right] G'_x + \\ & \left[(E'_z - Q'_z)(F'_x - Q'_x) - (E'_x - Q'_x)(F'_z - Q'_z) \right] G'_y + \\ & \left[(E'_x - Q'_x)(F'_y - Q'_y) - (E'_y - Q'_y)(F'_x - Q'_x) \right] G'_z = D \end{aligned} \quad (4.7)$$

meaning that the fourth point G' also lies on the plane $E'F'Q'$. Note that D in equation (4.7) can be found from equation (4.6).

On the plane $E'F'G'Q'$, the lines $Q'E'$, $Q'F'$ and $Q'G'$ of given lengths r_e , r_f and r_g are constructed:

$$Q'E' = r_e = \left[(E'_x - Q'_x)^2 + (E'_y - Q'_y)^2 + (E'_z - Q'_z)^2 \right]^{1/2} \quad (4.8)$$

$$Q'F' = r_f = \left[(F'_x - Q'_x)^2 + (F'_y - Q'_y)^2 + (F'_z - Q'_z)^2 \right]^{1/2} \quad (4.9)$$

$$Q'G' = r_g = \left[(G'_x - Q'_x)^2 + (G'_y - Q'_y)^2 + (G'_z - Q'_z)^2 \right]^{1/2} \quad (4.10)$$

so that the distances:

$$E'F' = e = \left[(E'_x - F'_x)^2 + (E'_y - F'_y)^2 + (E'_z - F'_z)^2 \right]^{1/2} \quad (4.11)$$

$$F'G' = f = \left[(F'_x - G'_x)^2 + (F'_y - G'_y)^2 + (F'_z - G'_z)^2 \right]^{1/2} \quad (4.12)$$

Equations (4.8) to (4.12) inclusive, fully describe the triangular shape of the lower lamina $E'F'G'Q'$. Length g could be calculated, but it is not required for the triangular description.

Finally, the triangle $E'F'G'$ can be positioned so that the cables are of given lengths:

$$\ell_a = \left[(E'_x - A_x)^2 + (E'_y - A_y)^2 + (E'_z - A_z)^2 \right]^{1/2} \quad (4.13)$$

$$\ell_b = \left[(F'_x - B_x)^2 + (F'_y - B_y)^2 + (F'_z - B_z)^2 \right]^{1/2} \quad (4.14)$$

$$\ell_c = \left[(G'_x - C_x)^2 + (G'_y - C_y)^2 + (G'_z - C_z)^2 \right]^{1/2} \quad (4.15)$$

The set of nine independent equations, (4.7)-(4.15) inclusive, entirely describes the system, and without any violations of constraints. There are nine unknowns in this

system $E'_x, E'_y, E'_z, F'_x, F'_y, F'_z, G'_x, G'_y, G'_z$, i.e. the coordinates of the three corners of the lower lamina, and these can be determined from this system of equations.

4.1.1.2 Fundamental development of theory: quadrilateral laminas

Figure 4.2 shows a similar system but where the two laminas are now quadrilaterals in shape. The objective remain the same as in the previous case. Denoting the coordinates of the fourth corners of the upper and lower laminas, D and H' respectively:

$$D(D_x , D_y , D_z) \qquad H'(H'_x , H'_y , H'_z) \qquad (4.16)$$

the set of equations for identifying the position of the lower lamina with the given coordinates of the point Q' (statement (4.2) is to be determined next.

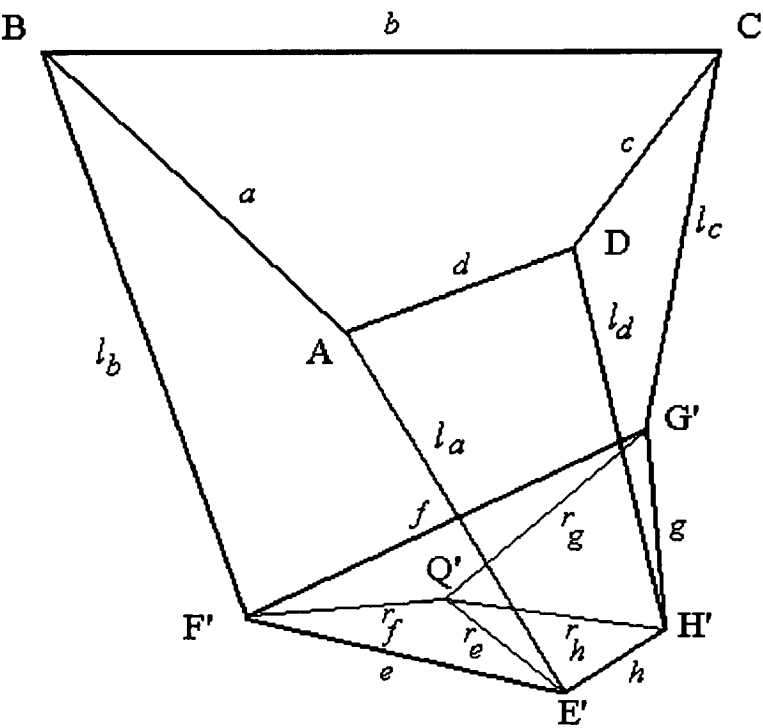


Figure 4.2
General four cable quadrilateral lamina suspension system

The four corners E' , F' , G' , H' , and the point Q' on the lower lamina lie on the same plane. The equation of the plane $E'F'Q'$ has been derived already (this is given by equation (4.5) with (4.6)), and two remaining points G' and H' give the first two equations of the set. One of these equations is exactly the same as equation (4.7), and the second one is:

$$\begin{aligned} & \left[(E'_y - Q'_y)(F'_z - Q'_z) - (E'_z - Q'_z)(F'_y - Q'_y) \right] H'_x + \\ & \left[(E'_z - Q'_z)(F'_x - Q'_x) - (E'_x - Q'_x)(F'_z - Q'_z) \right] H'_y + \\ & \left[(E'_x - Q'_x)(F'_y - Q'_y) - (E'_y - Q'_y)(F'_x - Q'_x) \right] H'_z = D \end{aligned} \quad (4.17)$$

On the plane $E'F'G'H'Q'$, four lines have to be constructed, three of given lengths r_e , r_f and r_g for which equations (4.8)-(4.10) can be re-stated here, and the fourth line of length r_h gives the following equation:

$$Q'H' = r_h = \left[(H'_x - Q'_x)^2 + (H'_y - Q'_y)^2 + (H'_z - Q'_z)^2 \right]^{1/2} \quad (4.18)$$

The conditions (4.11)-(4.12) have to be satisfied together with, say:

$$G'H' = g = \left[(G'_x - H'_x)^2 + (G'_y - H'_y)^2 + (G'_z - H'_z)^2 \right]^{1/2} \quad (4.19)$$

The fourth condition, $H'E' = h$ in this case, is not needed.

And finally, the system can be prescribed so that the cables are of given lengths. This gives equations (4.13)-(4.15) together with:

$$\ell_d = \left[(H'_x - D_x)^2 + (H'_y - D_y)^2 + (H'_z - D_z)^2 \right]^{1/2} \quad (4.20)$$

Thus, the set of thirteen equations (4.7), (4.17), (4.8)-(4.10), (4.18), (4.11), (4.12), (4.19), (4.13)-(4.15) and (4.20) is constructed, however only twelve unknowns E'_x , E'_y , E'_z , F'_x , F'_y , F'_z , G'_x , G'_y , G'_z , H'_x , H'_y , H'_z are identifiable. All thirteen equations are independent (they have been derived to satisfy independent geometrical constraints). This means that generally there is no solution to this system unless one of the constraints is violated.

4.1.2 Geometrical extension - the definition

4.1.2.1 The case of a parallel cable spreader suspension system

The theory in section 4.1.1.2 can be applied to a real RTG spreader suspension system with a trolley and a spreader of rectangular shape. In order to do this, the location of the mass centre of the spreader, Q'' , (this is the location of the mass centre of the spreader after its translation and rotation, and has been defined in Chapter 3), should be used as the point Q' in the general case described in section 4.1.1.2, and the general distances should be changed for the actual ones (also see chapter 3 for a definition of these quantities):

$$l_a = l_b = l_c = l_d = l \quad (4.21)$$

$$e = g = a \quad (4.22)$$

$$f = h = b \quad (4.23)$$

$$r_e = r_f = r_g = r_h = R \quad (4.24)$$

Figure 4.3 shows the spreader after having undergone combined translation and rotation¹⁾. By applying the generalised theory of 4.1.1.2 to the specific problem of the spreader and trolley it can be seen that, for a given Q'' , no solution to the problem can

be obtained without relaxing one of the thirteen constraints, recalling that there are thirteen equations for identifying only twelve unknowns in this type of problem²⁾.

Since the shape of either the trolley or the spreader cannot change (in practical systems they both are rigid bodies), and the cables are relatively flexible then the conclusion is that when the spreader is translated away from its equilibrium position from under the trolley and then rotated, the cables then undergo some sort of deformation, i.e. either extension or compression. The change in cables lengths is defined here as the *geometrical extension*, e , which can be either positive or negative.

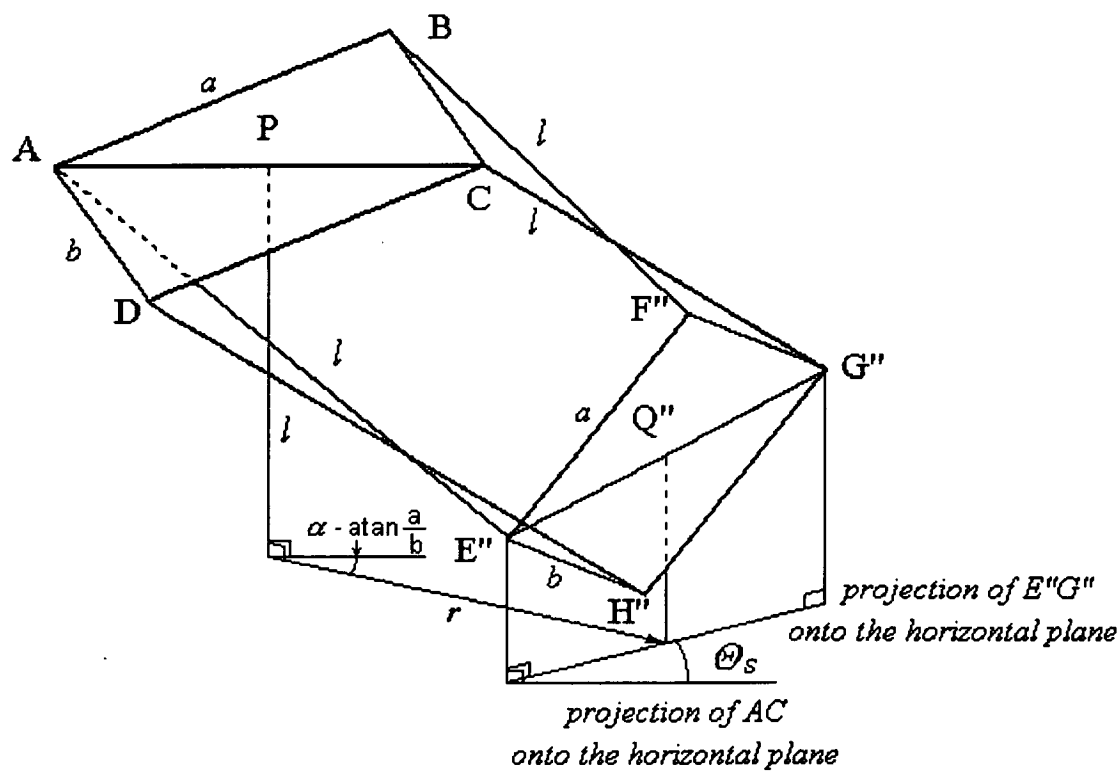


Figure 4.3

Four parallel cable suspension system as employed on RTG.
The spreader after being translated and rotated (shown schematically)

4.1.2.2 The case of non-parallel cable spreader suspension system

It is clear that the generalised theory is also applicable to a non-parallel suspension system, as shown in Figure 4.4., where both the spreader and the trolley are of rectangular shape, but the spreader's dimensions are smaller ($c < a$, $d < b$).

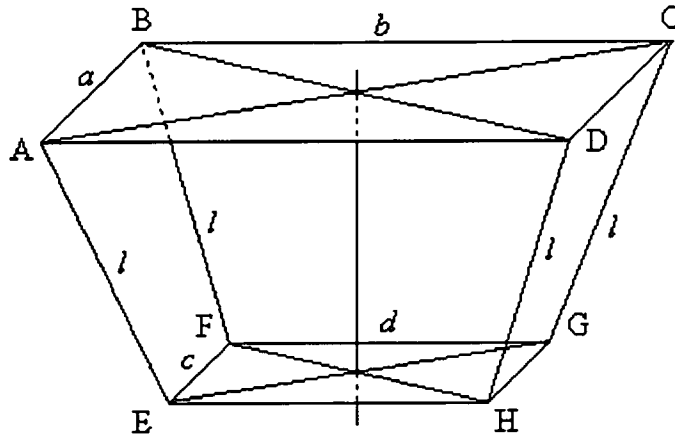


Figure 4.4

Four *non-parallel* cable suspension system
(equilibrium position)

When the spreader is moved away from its equilibrium position under the trolley (Figure 4.5) a geometrical extension can be shown to take place. There is no obvious way of solving analytically the set of equations given in section 4.1.1.2 (with one or more relaxed constraint(s)) for the non-parallel case, but it has been solved numerically to prove the above claim. The non-parallel system is not discussed in detail in this thesis.

4.2 Calculation of the numerical values for the geometrical extension

Two methods for the calculation of numerical values for geometrical extension for a typical system are proposed in this chapter.

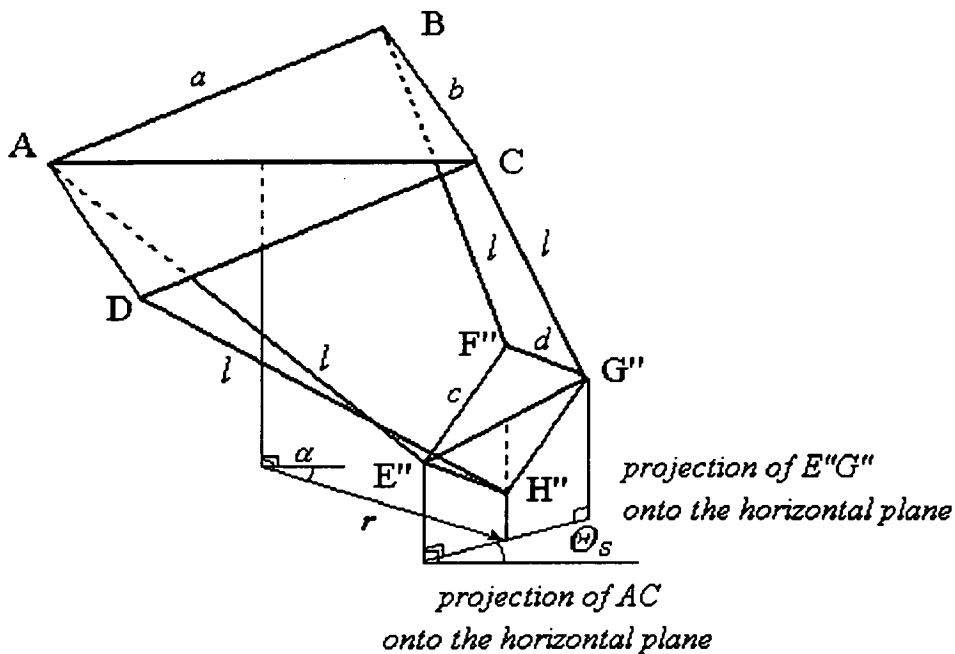


Figure 4.5

Four non-parallel cable suspension system.

The spreader after being translated and rotated (shown schematically)

The first one (called method [a]) is a strict algorithm for the solution of the set of thirteen equations described in section 4.1.1.2. One of these equations has to be omitted and that can be only one of the equations (4.13)-(4.15) or (4.20). This means that for given dimensions of the spreader suspension system, and the position of its mass centre, method [a] allows the determination of twelve coordinates for the corners of the spreader, thus identifying the position of the spreader with three undeformed cables and *one* cable being stretched or compressed. This method is strict, and therefore very simple in realisation, but can only give an *upper bound* for the geometrical extension in the four-cable system.

The second method (method [b]) is more complicated, but more realistic in the sense that it shares out the value of geometrical extension across all *four* cables to satisfy a least squares minimisation criterion. Importantly, it provides a numerical basis for the calculation of the geometrical extension distributed in a *non-uniform* manner across the four cables.

4.2.1 Method [a]

The coordinates of the spreader mass centre Q'' are completely identified by the vector $\vec{r}(r, \alpha)$ of the spreader translation and the angle Θ_s of the spreader rotation. An algorithm was proposed in Chapter 3 for calculating the coordinates of Q'' (section 3.4.4.2 and 3.5) and this algorithmic solution is conveniently used here. Thus, by referring to Figure 3.10 one can determine the coordinates of corner E'' :

$$E''_x = l \sin \Psi \cos\left(\alpha - \tan^{-1} \frac{a}{b}\right) + R - Q' E_{\perp} \quad (4.25)$$

$$E''_y = l \sin \Psi \sin\left(\alpha - \tan^{-1} \frac{a}{b}\right) + E_{\perp} E_v'' \quad (4.26)$$

$$E''_z = E'' E_v'' \quad (4.27)$$

and corner G'' , as the other end of the diagonal $E''G''$ of the spreader with Q'' at its centre:

$$G''_x = 2Q''_x - E''_x \quad (4.28)$$

$$G''_y = 2Q''_y - E''_y \quad (4.29)$$

$$G''_z = 2Q''_z - E''_z \quad (4.30)$$

Thus the points E'' and G'' are identified without any violation of the system, i.e. the following are correct:

$$AE'' \equiv l_{AE''} = l \quad (4.31)$$

$$CG'' \equiv l_{CG''} = l \quad (4.32)$$

$$E''Q'' = Q''G'' = R \quad (4.33)$$

The third corner, say F'' , can be determined by using the conditions that:

$$BF'' \equiv l_{BF''} = l \quad (4.34)$$

$$E''F'' = a \quad (4.35)$$

$$F''G'' = b \quad (4.36)$$

Then the set of equations for the coordinates of F'' is:

$$\begin{cases} (B_x'' - F_x'')^2 + (B_y'' - F_y'')^2 + (B_z'' - F_z'')^2 = \ell^2 \\ (E_x'' - F_x'')^2 + (E_y'' - F_y'')^2 + (E_z'' - F_z'')^2 = a^2 \\ (G_x'' - F_x'')^2 + (G_y'' - F_y'')^2 + (G_z'' - F_z'')^2 = b^2 \end{cases} \quad (4.37)$$

which after some manipulation leads to a quadratic equation in F_z'' :

$$\begin{aligned} & (P_x^2 + P_y^2 + 1)F_z''^2 + (P_x Q_x + P_y Q_y - 2B_x P_x - 2B_y P_y - 2B_z)F_z'' \\ & + \frac{Q_x^2}{4} + \frac{Q_y^2}{4} - B_x Q_x - B_y Q_y + B_x^2 + B_y^2 + B_z^2 - \ell^2 = 0 \end{aligned} \quad (4.38)$$

where:

$$P_x = \frac{(E_z'' - B_z)(E_y'' - G_y'') - (E_z'' - G_z'')(E_y'' - B_y)}{(E_x'' - G_x'')(E_y'' - B_y) - (E_x'' - B_x)(E_y'' - G_y'')} \quad (4.39)$$

$$P_y = \frac{(E_z'' - B_z)(E_x'' - G_x'') - (E_z'' - G_z'')(E_x'' - B_x)}{(E_y'' - G_y'')(E_x'' - B_x) - (E_y'' - B_y)(E_x'' - G_x'')} \quad (4.40)$$

$$\begin{aligned} Q_x = & \left[(B_x^2 - E_x''^2 + B_y^2 - E_y''^2 + B_z^2 - E_z''^2 + a^2 - \ell^2)(E_y'' - G_y'') - \right. \\ & \left. (G_x''^2 - E_x''^2 + G_y''^2 - E_y''^2 + G_z''^2 - E_z''^2 + a^2 - b^2)(E_y'' - B_y) \right] \times \end{aligned}$$

$$\left[1 / \left((E_x'' - G_x'')(E_y'' - B_y) - (E_x'' - B_x)(E_y'' - G_y'') \right) \right] \quad (4.41)$$

$$\begin{aligned} Q_y = & \left[(B_x^2 - E_x''^2 + B_y^2 - E_y''^2 + B_z^2 - E_z''^2 + a^2 - \ell^2)(E_x'' - G_x'') - \right. \\ & \left. (G_x''^2 - E_x''^2 + G_y''^2 - E_y''^2 + G_z''^2 - E_z''^2 + a^2 - b^2)(E_x'' - B_x) \right] \times \\ & \left[1 / \left((E_y'' - G_y'')(E_x'' - B_x) - (E_y'' - B_y)(E_x'' - G_x'') \right) \right] \end{aligned} \quad (4.42)$$

After obtaining the two roots for F''_z , the remaining two coordinates are:

$$F_x'' = P_x F_z'' + \frac{Q_x}{2} \quad (4.43)$$

$$F_y'' = P_y F_z'' + \frac{Q_y}{2} \quad (4.44)$$

Now the coordinates of the fourth corner H'' can be calculated as the other end of the diagonal $F''H''$ with Q'' at its centre:

$$H_x'' = 2Q_x'' - F_x'' \quad (4.45)$$

$$H_y'' = 2Q_y'' - F_y'' \quad (4.46)$$

$$H_z'' = 2Q_z'' - F_z'' \quad (4.47)$$

and finally the length of the cable DH'' can be now calculated:

$$DH'' = \left[(H_x'' - D_x)^2 + (H_y'' - D_y)^2 + (H_z'' - D_z)^2 \right]^{1/2} \quad (4.48)$$

Thus all four corners are found without violation of any constraint apart from imposing the condition $DH'' = l$. The geometrical extension in the fourth (DH'') cable can be calculated from:

$$e_{DH''} = l - DH'' \quad (4.49)$$

and it is not equal to zero. This is examined numerically in section 4.3.

4.2.2 Method [b]

4.2.2.1 General idea of the method [b]

The idea of this method is as follows. For given initial conditions (translational vector $\vec{r}(r, \alpha)$ and rotational angle Θ_s) the position of the spreader is calculated by means of the method [a]. This position is used as a datum and then several different discrete positions of the spreader around the datum are constructed so that:

- the mass centre of the spreader remains in the same point, Q'' ;
- the shape of the spreader, a and b , is unchanged;
- all four cables slightly vary their lengths:

$$l_{AE''} = l + e_{AE''}; \quad l_{BF''} = l + e_{BF''}; \quad l_{CG''} = l + e_{CG''}; \quad l_{DH''} = l + e_{DH''}.$$

For each constructed position the sum of the squares of the geometrical extension values in each cable is calculated. That position which gives the minimal value of the sum is considered to be the one which the spreader will actually take. Thus, this method gives approximate, but realistic, values of geometrical extension in the system.

4.2.2.2 Geometrical illustration of the method [b]

Figure 4.6 shows the point Q'' , the datum E'' calculated by the method [a] (using (4.25)-(4.27)), and several (m) discrete points E''_i around the datum such that:

$$E''_i Q'' = R \quad i = 1, 2, \dots, m \quad (4.50)$$

For each E''_i the corresponding G''_i can be found using modified (4.28)-(4.30) (i.e. preserving the distance $E''_i G''_i = 2R$):

$$G''_{ix} = 2Q''_x - E''_{ix} \quad (4.51)$$

$$G''_{iy} = 2Q''_y - E''_{iy} \quad (4.52)$$

$$G''_{iz} = 2Q''_z - E''_{iz} \quad (4.53)$$

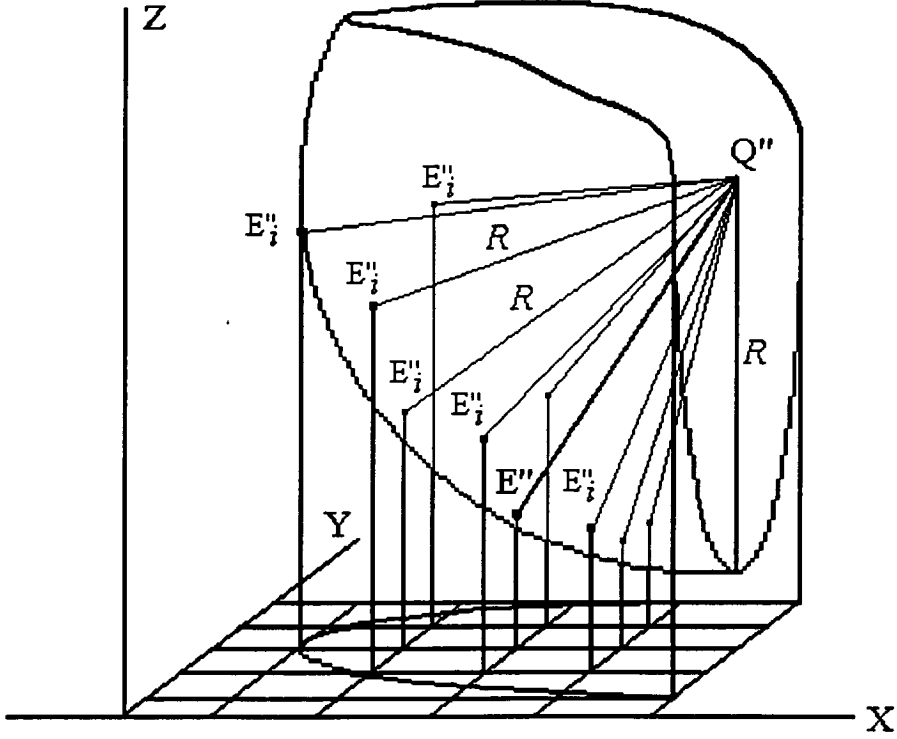


Figure 4.6

First discretisation scheme about the datum E''

For each pair of E''_i, G''_i , the point F''_i can be calculated from the conditions:

$$BF''_i = l \quad (4.54)$$

$$E''_i F''_i = a \quad (4.55)$$

$$F''_i G''_i = b \quad (4.56)$$

and by using correspondingly modified equations (4.38), (4.43) and (4.44) from method [a]. This point is used as a new datum of another discretisation scheme (Figure 4.7).

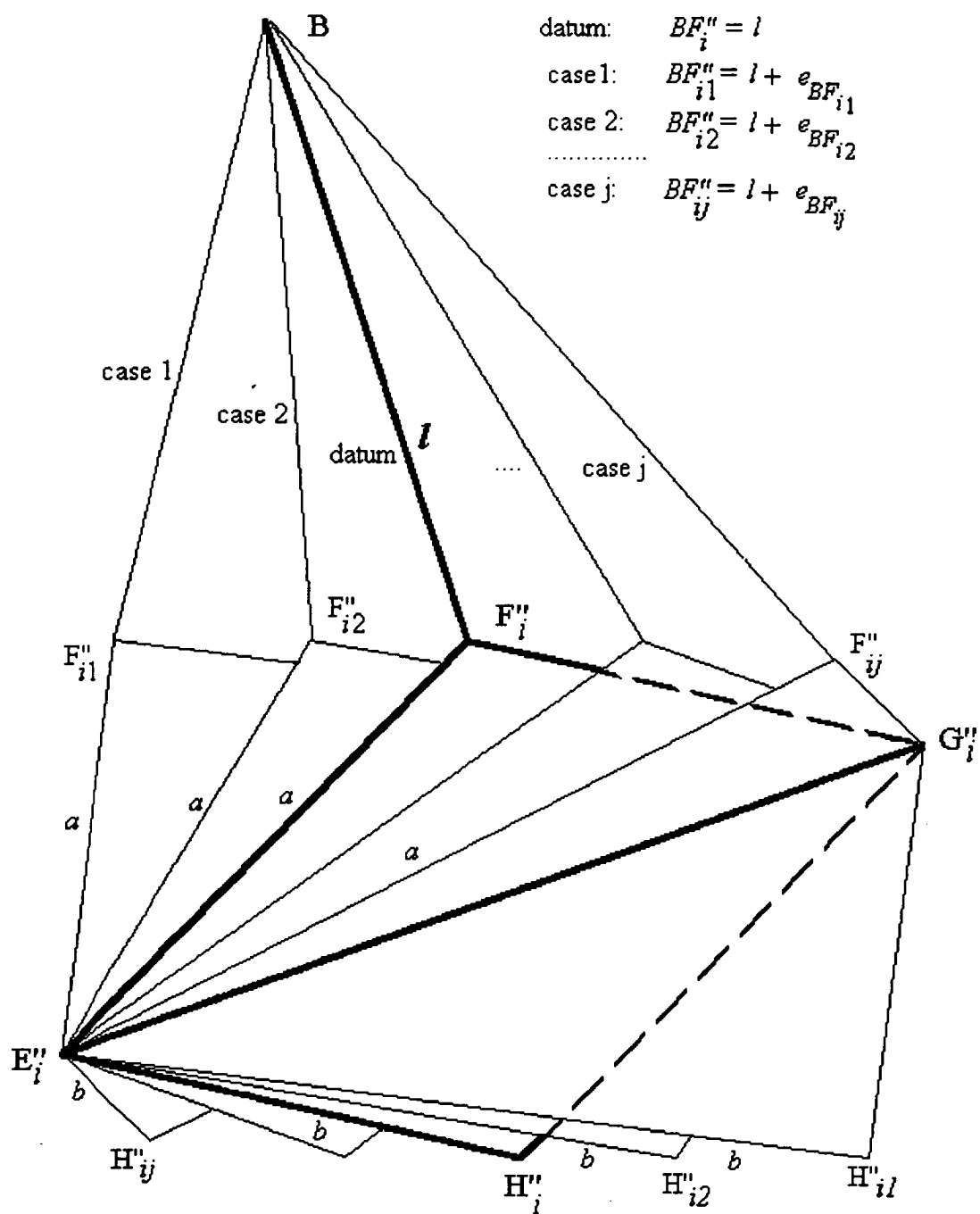


Figure 4.7

Second discretisation scheme with diagonal $E''_i G''_i$ fixed
 and F''_i used as a datum

By incrementing $l_{BF''}$ by $e_{BF''_{ij}}$ (positive and negative) several (n) discrete points F''_{ij} can be found satisfying equations (4.55) and (4.56) (only now it is with F''_{ij} instead of F''_i), and then the corresponding H''_{ij} can be calculated as before:

$$H''_{ijx} = 2Q''_x - F''_{ijx} \quad (4.57)$$

$$H''_{ijy} = 2Q''_y - F''_{ijy} \quad (4.58)$$

$$H''_{ijz} = 2Q''_z - F''_{ijz} \quad (4.59)$$

Thus, $n \times m$ positions of the spreader with its mass centre at Q'' have been found. For each position the lengths of all four cables can be calculated:

$$AE''_i = \left[(A_x - E''_{ix})^2 + (A_y - E''_{iy})^2 + (A_z - E''_{iz})^2 \right]^{1/2} \quad (4.60)$$

$$BF''_{ij} = \left[(B_x - F''_{ijx})^2 + (B_y - F''_{ijy})^2 + (B_z - F''_{ijz})^2 \right]^{1/2} \quad (4.61)$$

$$CG''_i = \left[(C_x - G''_{ix})^2 + (C_y - G''_{iy})^2 + (C_z - G''_{iz})^2 \right]^{1/2} \quad (4.62)$$

$$DH''_{ij} = \left[(D_x - H''_{ijx})^2 + (D_y - H''_{ijy})^2 + (D_z - H''_{ijz})^2 \right]^{1/2} \quad (4.63)$$

leading to the geometrical extension for each cable:

$$e_{AE_i} = \ell - AE''_i \quad (4.64)$$

$$e_{BF_{ij}} = \ell - BF''_{ij} \quad (4.65)$$

$$e_{CG_i} = \ell - CG''_i \quad (4.66)$$

$$e_{DH_{ij}} = \ell - DH''_{ij} \quad (4.67)$$

and from this a minimisation criterion δ'_{ij} can be calculated:

$$\delta_{ij} = \left[e_{AE_i}^2 + e_{BF_{ij}}^2 + e_{CG_i}^2 + e_{DH_{ij}}^2 \right]^{1/2} \quad (4.68)$$

$$\delta'_{i'j'} = \min_{ij}(\delta_{ij}) \quad (4.69)$$

The position which satisfies criterion (4.69) is taken to be one nearest to the actual spreader position. The value $\delta'_{i'j'}$ is taken to be an estimate of the actual value of the geometrical extension in the four cable spreader suspension system:

$$\delta'_{i'j'} \sim \delta_{\text{actual}} \quad (4.70)$$

4.2.2.3 Numerical procedure for the method [b]

A summary of the numerical procedure for the calculation of $\delta'_{i'j'}$ follows:

1. For given initial conditions, the coordinates of the datum E'' are calculated using method [a], equations (4.25)-(4.27). Then the coordinates of corresponding G'' are also calculated (4.28)-(4.30).

2. The spacing for both discretisation schemes is chosen to be $s = s_1$.

Loop Start:

3. For the spacing s , the coordinates of m points E''_i around the datum are calculated satisfying equation (4.50) with the coordinates of m corresponding G''_i , equations (4.51)-(4.53).
4. For each pair of E''_i and G''_i the coordinates of F''_i are calculated satisfying equations (4.54)-(4.56).
5. Coordinates for n points F''_{ij} are calculated, also the coordinates of the corresponding H''_{ij} , equations (4.57)-(4.59).
6. For each set of coordinates of the four spreader corners E''_i , F''_{ij} , G''_i , H''_{ij} , the value of the averaged geometrical extension δ_{ij} is calculated using equations (4.64)-

(4.68). There are $n \times m$ such sets, and Figure 4.8 shows schematically the surface on which all the $n \times m$ calculated values of δ_{ij} lie.

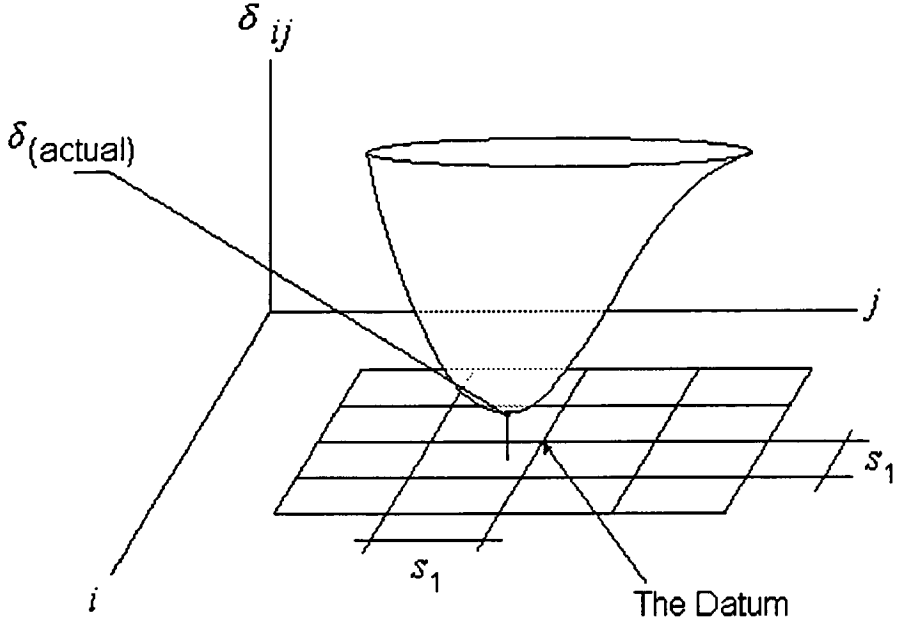


Figure 4.8

The surface showing the values of δ_{ij}
calculated for all i and j using formula (4.68)
and that the actual minimum of the surface can be identified as $\delta_{(actual)}$

7. The position of the spreader $E''_i F''_{ij} G''_i H''_{ij}$ which corresponds to the minimal δ_{ij} value is now taken as a new datum (see Figure 4.9.), and the loop 3.-7. is repeated with the smaller spacing $s = s_2 (< s_1)$.

Loop End.

8. After the procedure is repeated several times, the calculated coordinates $E''_i F''_{ij} G''_i H''_{ij}$ can be taken for the estimated actual spreader position, and δ_{ij} for the estimated actual geometrical extension $\delta_{(actual)}$, equations (4.69) and (4.70).

It is clear that decreasing the spacing s and increasing the number of the loops in the procedure increases the accuracy of the $\delta_{(actual)}$ estimation but leads to some

computational difficulties. Furthermore, there are several other difficulties in the practical use of this procedure:

- Each time the value $\delta'_{i'j'}$ is found it should be checked that it is not lying on the edge of the domain, meaning that the actual minimum is somewhere outside the domain. In such cases the datum should be shifted correspondingly.
- It is not necessarily true that there is only one minimum on the surface δ_{ij} . If there is more than one local minimum (and the numerical examples have shown that in some cases this is true) then the computational procedure becomes prohibitively time consuming.

Some numerical results are given in the next section.

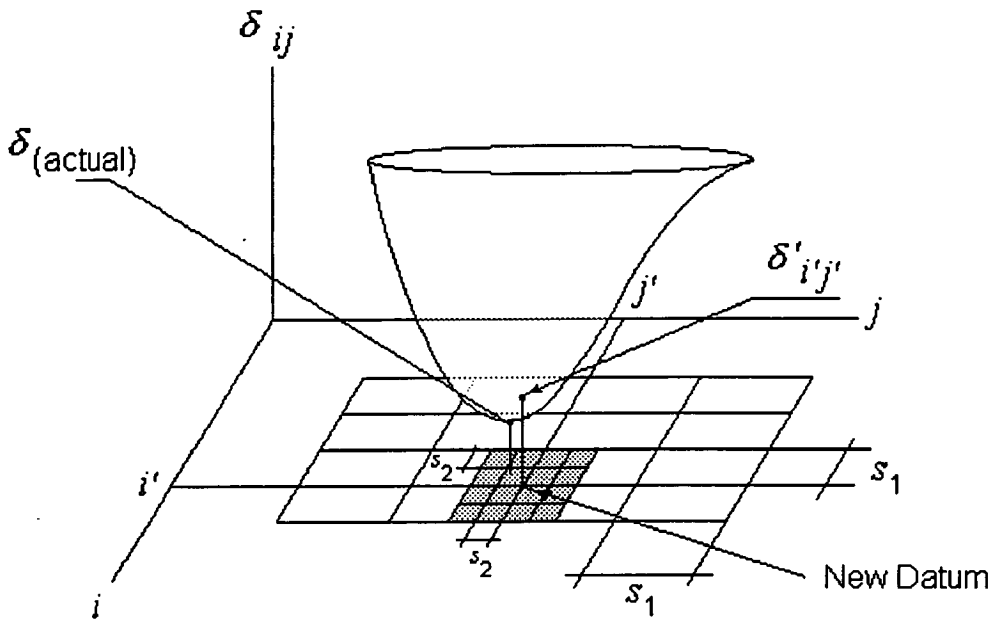


Figure 4.9

The surface showing the values of δ_{ij} for all i and j
 with the actual minimum of the surface $\delta_{(actual)}$
 and calculated minimum $\delta'_{i'j'}$

4.3 Numerical results

4.3.1 The data for numerical experiments

The data used for the calculations is as follows.

The width and the length of the spreader³⁾ are taken to be $a = 1.765\text{m}$ and $b = 4.938\text{m}$; these dimensions are constant for all the numerical experiments.

The cable length l , the magnitude of the translational vector r , the angle of translation α and the rotational angle Θ_S vary in the following ranges:

$$6\text{m} \leq l \leq 10\text{m}$$

$$0.5\text{m} \leq r \leq 2.5\text{m}$$

$$1^\circ \leq \alpha \leq 89^\circ$$

$$-30^\circ \leq \Theta_S \leq 30^\circ$$

4.3.2 Graphical presentation of the results

Figures 4.10 - 4.13 show the geometrical extension as a function of rotational angle Θ_S . The thick lines are for method [a] (the value of geometrical extension concentrated in one (DH'') cable, i.e. $e_{DH''}$ as in equation (4.49)), and the thin ones for method [b] (the averaged geometrical extension between all four cables $\delta'_{i,j'}$, equation (4.69)). As expected, method [a] gives the upper bound for geometrical extension in the system as all the values for method [b] are smaller (all the curves for method [b] lie lower than those for method [a]).

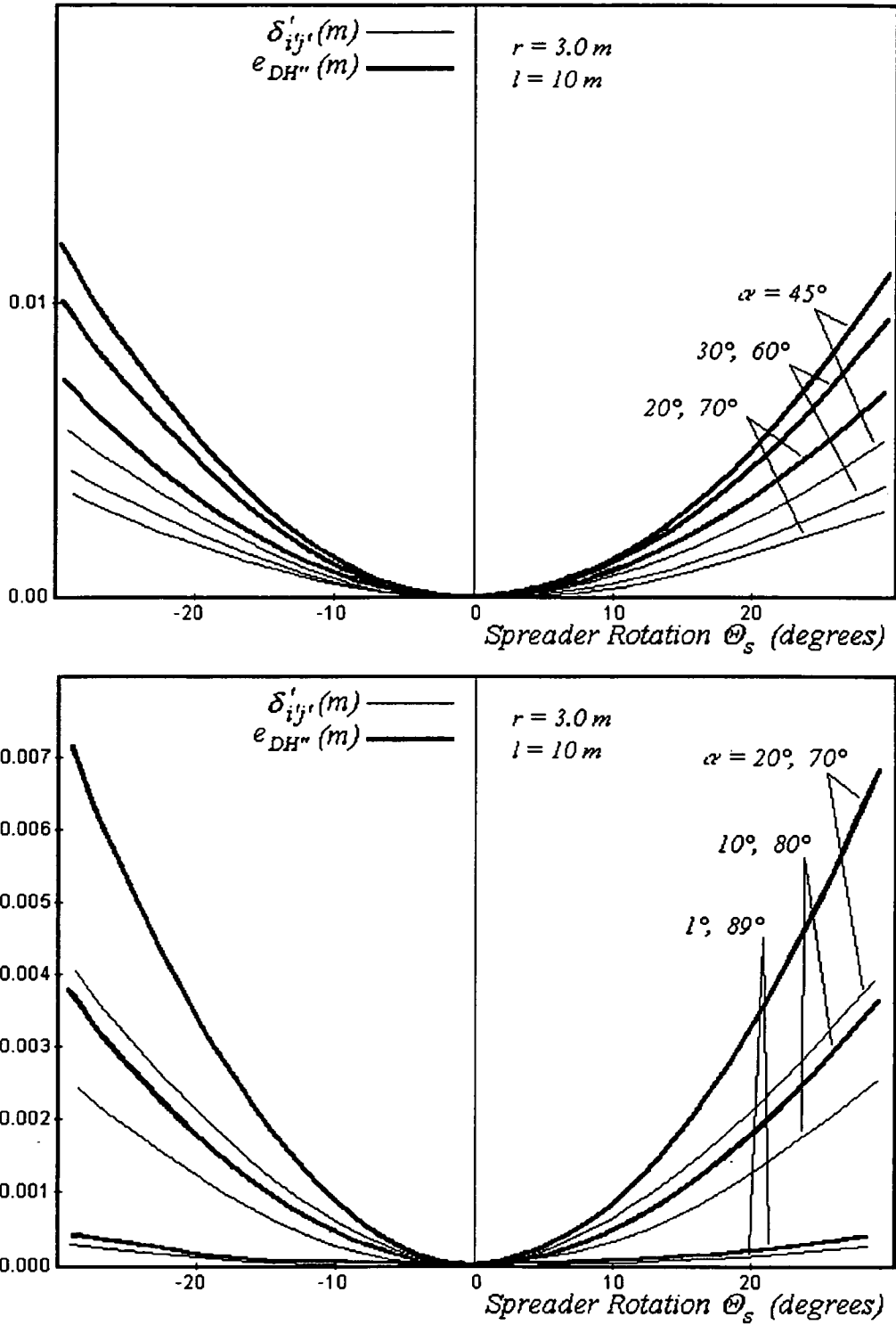


Figure 4.10 a, b

Geometrical extension $e_{DH''}$ (method [a])

and a shared out value of geometrical extension $\delta'_{ij'}$ (method [b])

as functions of the angle of the spreader rotation Θ_s and the angle of spreader translation α

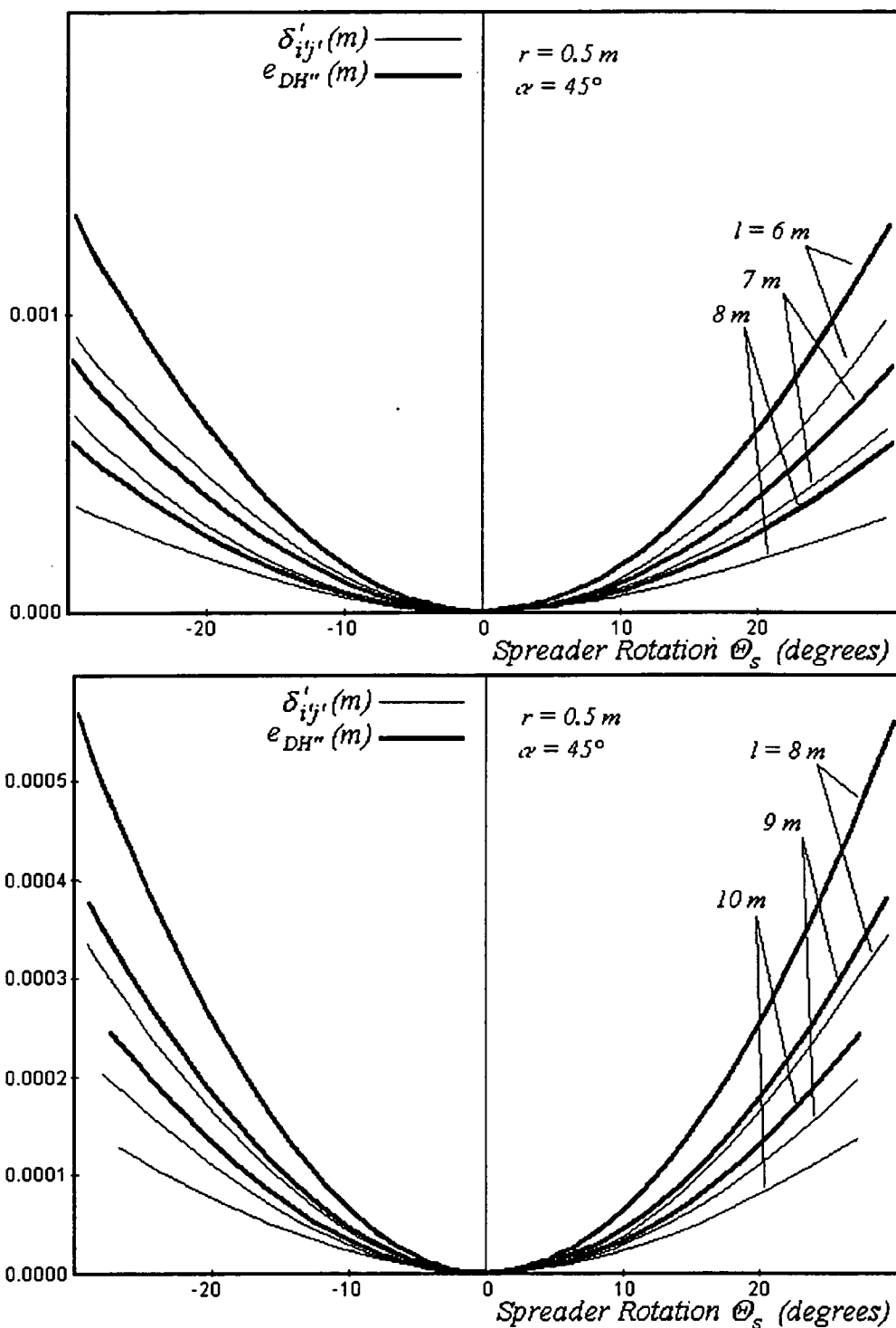


Figure 4.11 a, b

Geometrical extension $e_{DH''}$ (method [a])

and a shared out value of geometrical extension $\delta'_{ij'}$ (method [b])

as functions of the angle of the spreader rotation Θ_s and the cables length l

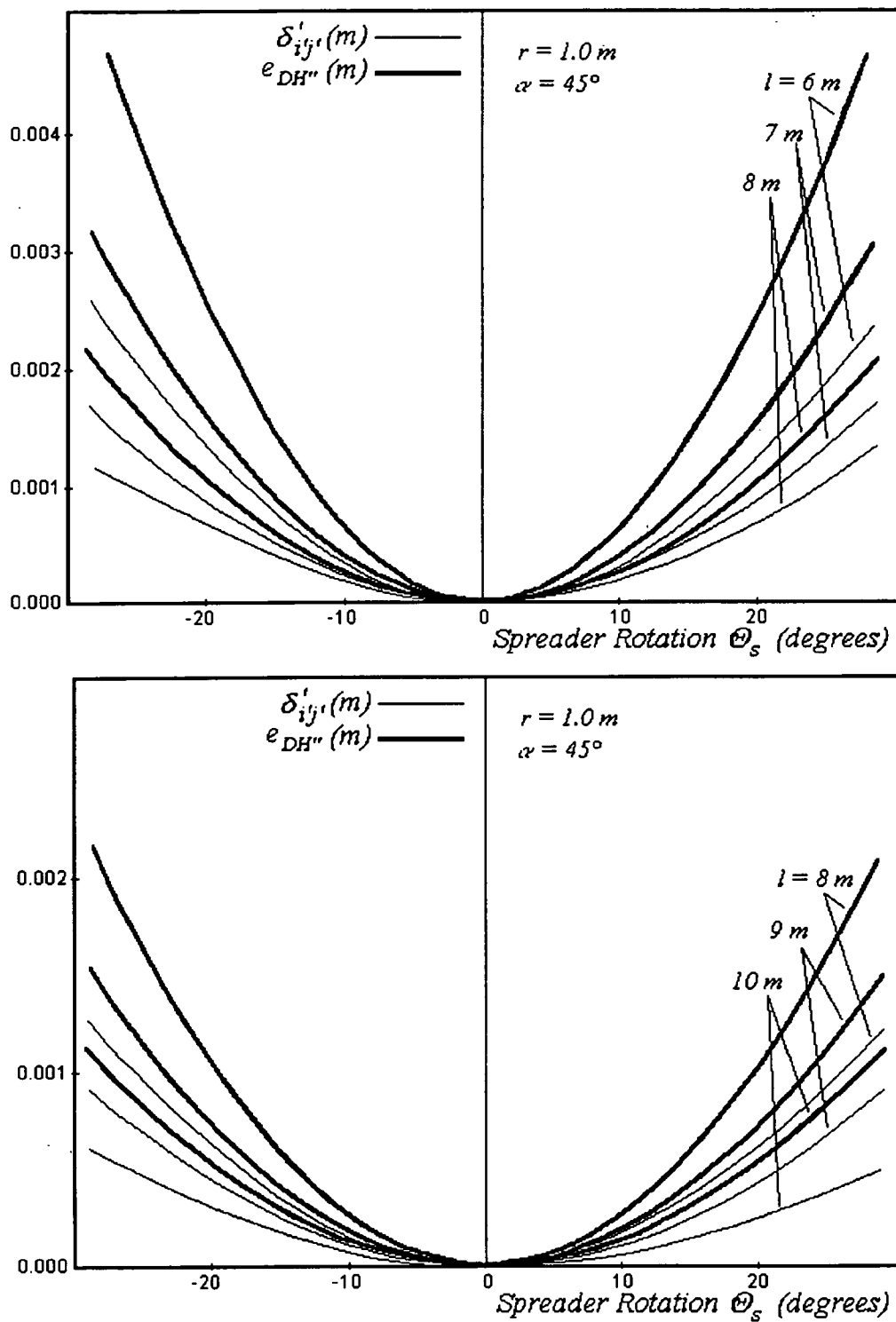


Figure 4.12 a, b

Geometrical extension $e_{DH''}$ (method [a])

and a shared out value of geometrical extension $\delta'_{ij'}$ (method [b])

as functions of the angle of the spreader rotation Θ_s and the cables length l

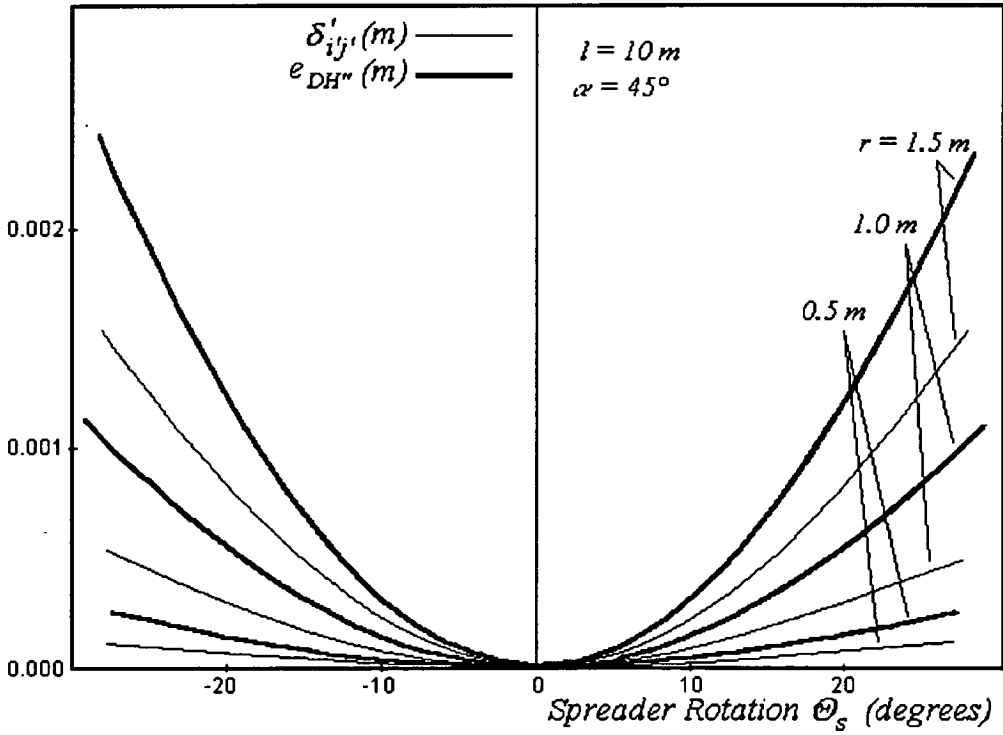
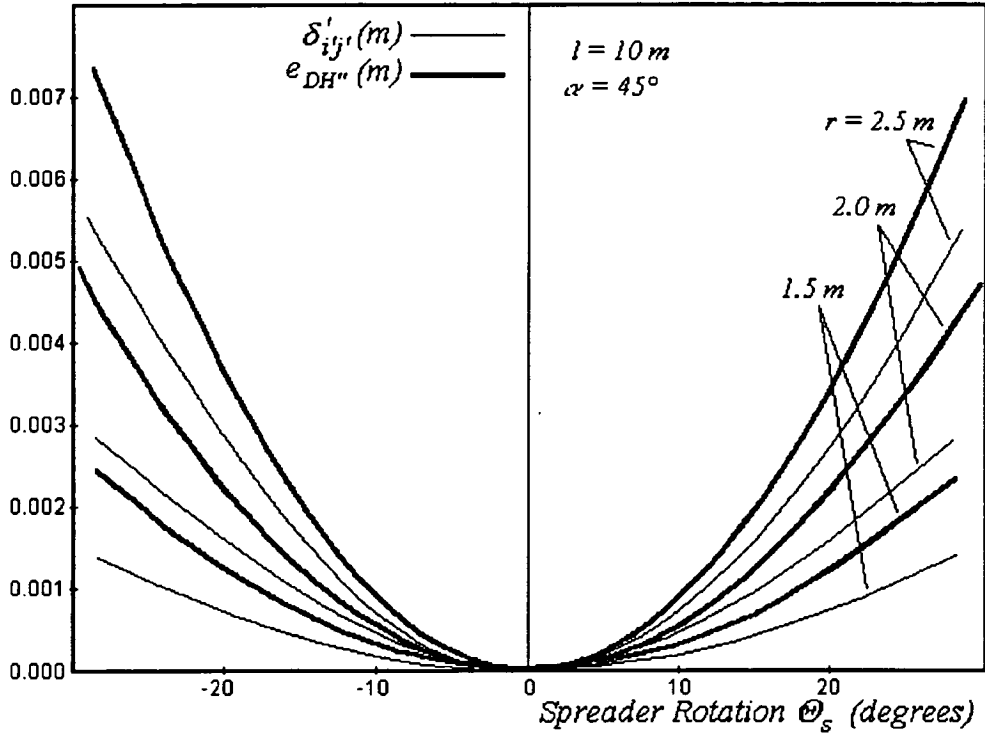


Figure 4.13 a, b

Geometrical extension $e_{DH''}$ (method [a])

and a shared out value of geometrical extension $\delta'_{ij'}$ (method [b])

as functions of the angle of the spreader rotation Θ_s and the magnitude of spreader translation r

4.3.3 Analysis of numerical results

Figures 4.10a, b show several pairs of curves ($e_{DH''}$ and $\delta'_{i,j'}$) for the case of fixed $r = 3\text{m}$ and $l = 10\text{m}$, and different α . This set of numerical experiments shows that the geometrical extension phenomenon does not take place when $\alpha = 0^\circ$ but with increasing α both $e_{DH''}$ and $\delta'_{i,j'}$ increase, reaching their maximums at $\alpha = 45^\circ$ and then decreasing again down to zero when $\alpha = 90^\circ$. This reveals the fact that if the spreader's translation is parallel to the trolley length/width dimensions (i.e. when $\alpha = 0^\circ, 90^\circ, 180^\circ, 270^\circ$) there is no geometrical extension effect in the system.

All further calculations were made for $\alpha = 45^\circ$.

Figures 4.11 - 4.13 demonstrate that $e_{DH''}$ and $\delta'_{i,j'}$ increase with decreasing l and increasing r , as one would intuitively expect.

4.3.4 Asymmetrical stretch in four cables

Figure 4.14 shows the four components, equations (4.64)-(4.67), of the averaged value of geometrical extension $\delta'_{i,j'}$ calculated by method [b] (equations (4.68) and (4.69)) against θ_s . This data is given only for one case: $r = 3\text{m}$, $l = 10\text{m}$, $\alpha = 45^\circ$.

As one can see, the stretch in all four cables is *different* for the same spreader position. This shows that the stretch phenomenon in the system is asymmetrical across the cables when the spreader is simultaneously translated and rotated.

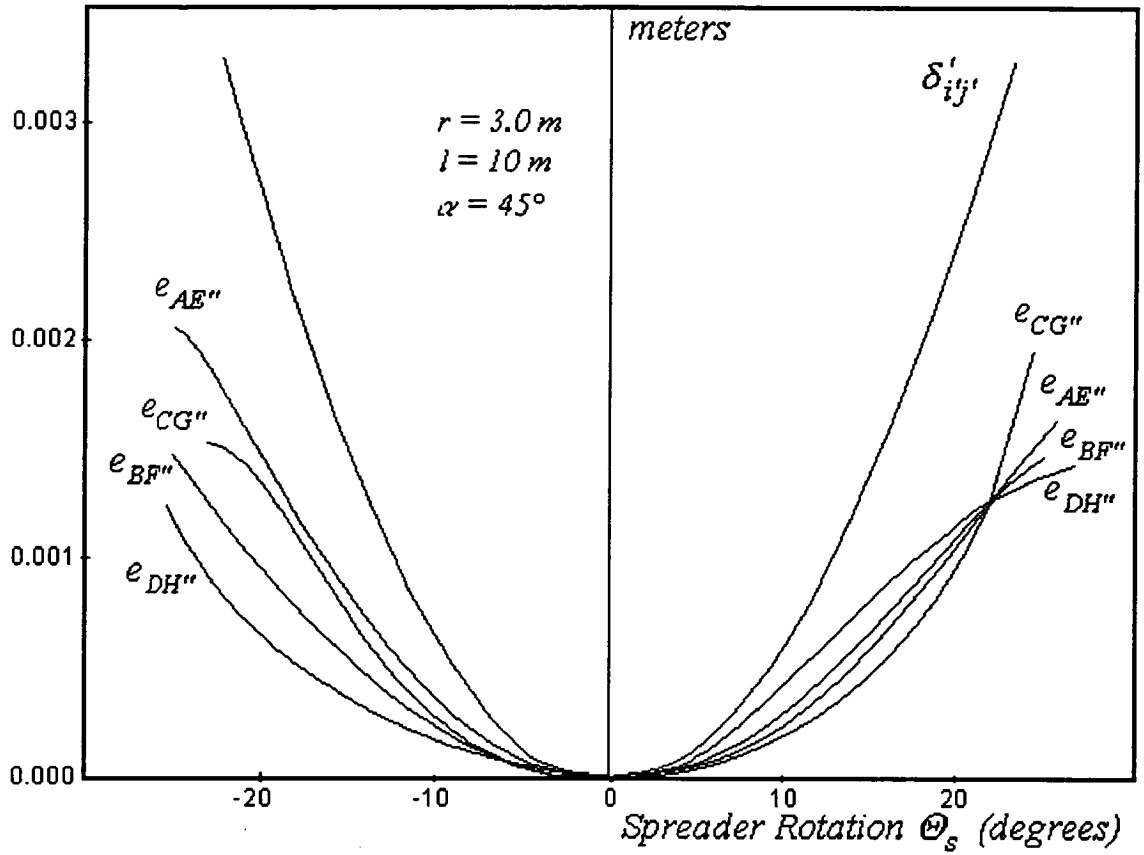


Figure 4.14
Geometrical extension components
(for method [b] showing $e_{AE''}$, $e_{BF''}$, $e_{CG''}$, $e_{DH''}$)

4.3.5 The sensitivity of the method [a] to the choice of the relaxed cable

For the same set of initial data as for Figure 4.14, $r = 3\text{m}$, $l = 10\text{m}$, $\alpha = 45^\circ$, Figure 4.15 shows the geometrical extension e against Θ_s calculated by method [a]. One of the curves is $e_{DH''}$ from equation (4.49). Three others are calculated similarly, but with different choice of the relaxed cable. It can be seen that for realistic values of Θ_s , i.e. within about $\pm 20^\circ$, the curves are very close indeed. For example, at $\Theta_s = 10^\circ$ the percentage difference is about 2.6%, and at $\Theta_s = 28^\circ$ the difference is 7.8%. Neither is considered significant, meaning that method [a] is not noticeably sensitive to the choice of the relaxed cable.

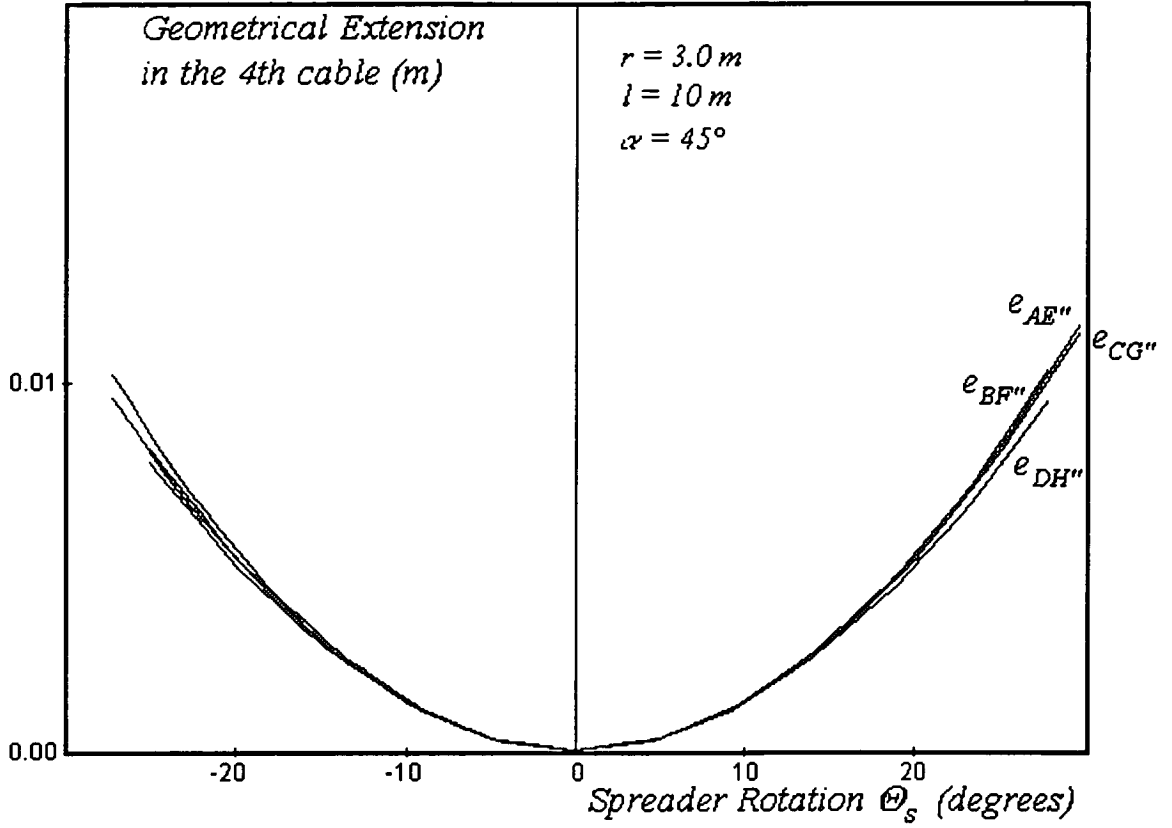


Figure 4.15

Geometrical extension calculated by method [a]
 with different choice of the cable where it is concentrated

4.3.6 Tensile load in the cables due to geometrical extension effect

Finally, assuming simple linear stress/strain behavior the load in the cables due to geometrical extension effect can be stated as:

$$F_{geom} = EA \frac{e}{l} \quad (4.71)$$

This approximate relationship is regarded as acceptable for the application here and serves to provide a means of obtaining numerical values relatively straight-forwardly. In equation (4.71), E is Young's modulus, A is the cross-sectional area of the cable, e is

the geometrical extension, and l is the cable length. Taking $E = 72 \text{ GN/m}^2$ (for commercial stranded cable of nominal diameter³⁾ 0.024m) one can calculate the cable load for a given value of e , where e is analogous to δ'_{ij} , here, of course. For example, in the case where $r = 3.0\text{m}$, $\alpha = 45^\circ$, $l = 10\text{m}$, $\Theta_s = -23^\circ$ (Figure 4.10a) the geometrical extension $\delta'_{ij} = 0.0034\text{m}$, and it can therefore be found using equation (4.71) that $F_{geom} = 11.05\text{KN}$.

Table 4.1 gives a set of data for various realistic Θ_s , r and l ($\alpha = 45^\circ$). They are used to obtain the value of combined translational and rotational spreader hoist ($h_s = h_{tr} + h_{rot}$), the load F_{geom} and corresponding energy U_{ext} , all due to the geometrical extension phenomenon, which can then be compared with the basic gravitational potential energy, U_{grav} , also given in the table. A comparison shows that although the latter is much greater than the former in all cases investigated the ratio of strain energy due to the geometrical extension phenomenon to gravitational potential energy reaches a maximum of 1/5000. Intuitive experiments on a small scale model imply that the extra stiffening generated by geometrical extension may be practically significant, and a new programme of research is planned to investigate this.

4.4 Conclusions

The existence of the geometrical extension phenomenon in the trolley/spreader suspension system is demonstrated, and two methods for the calculation of numerical values are suggested. This phenomenon leads to an additional tensile load in the cables due purely to the geometry of the system, and the load is generally different in all four cables.

The phenomenon takes place in the system when the spreader has undergone both a translation from the equilibrium position below the trolley and a rotation, with

exception of the cases where the translation of the spreader is parallel to the trolley length/width dimensions.

$\mathcal{Q}_s$	r	l	$e_{DH''}$	$\delta'_{i'j'}$	h_{tr}	h_{rot}	h_S	$\frac{\delta'_{rj'}}{h_S}$	$\frac{\delta'_{rj'}}{l}$	F_{geom}	U_{ext}	U_{grav}	$\frac{U_{ext}}{U_{grav}}$
[o]	[m]	[m]	[m]	[m]	[m]	[m]	[m]	%	%	[kN]	[N m]	[kN m]	%
			method[a]	method[b]									
7	0.5	10.0	1.7e-05	1.7e-05	0.01	0.01	0.02	9.4e-02	1.7e-04	0.05	0.000 ⁴⁾	1.26	3.6e-10
10	0.5	10.0	3.2e-05	3.2e-05	0.01	0.01	0.02	1.4e-01	3.2e-04	0.10	0.002	1.60	1.0e-09
13	0.5	10.0	5.7e-05	5.7e-05	0.01	0.02	0.03	1.9e-01	5.7e-04	0.18	0.005	2.15	2.4e-09
7	0.5	8.0	3.2e-05	3.2e-05	0.02	0.01	0.02	1.5e-01	4.0e-04	0.13	0.002	1.56	1.3e-09
10	0.5	8.0	6.2e-05	6.2e-05	0.02	0.01	0.03	2.2e-01	7.7e-04	0.25	0.008	1.98	3.9e-09
13	0.5	8.0	1.1e-04	1.1e-04	0.02	0.02	0.04	2.9e-01	1.4e-03	0.44	0.024	2.66	9.1e-09
7	0.5	6.0	7.4e-05	7.4e-05	0.02	0.01	0.03	2.5e-01	1.2e-03	0.40	0.015	2.07	7.3e-09
10	0.5	6.0	1.6e-04	1.6e-04	0.02	0.02	0.04	4.1e-01	2.6e-03	0.86	0.068	2.74	2.5e-08
13	0.5	6.0	2.5e-04	2.5e-04	0.02	0.03	0.05	5.1e-01	4.2e-03	1.37	0.173	3.50	4.9e-08
7	1.0	10.0	6.3e-05	6.3e-05	0.05	0.00	0.06	1.1e-01	6.3e-04	0.21	0.007	3.89	1.7e-09
10	1.0	10.0	1.3e-04	1.3e-04	0.05	0.01	0.06	2.2e-01	1.3e-03	0.44	0.030	4.29	6.9e-09
13	1.0	10.0	2.3e-04	2.3e-04	0.05	0.02	0.07	3.4e-01	2.3e-03	0.76	0.088	4.83	1.8e-08
7	1.0	8.0	1.2e-04	1.2e-04	0.06	0.01	0.07	1.7e-01	1.5e-03	0.49	0.029	4.85	6.1e-09
10	1.0	8.0	2.6e-04	2.6e-04	0.06	0.01	0.08	3.4e-01	3.2e-03	1.04	0.133	5.33	2.5e-08
13	1.0	8.0	4.4e-04	4.4e-04	0.06	0.02	0.08	5.2e-01	5.5e-03	1.80	0.397	5.99	6.6e-08
7	1.0	6.0	3.2e-04	3.2e-04	0.08	0.01	0.09	3.4e-01	5.3e-03	1.71	0.270	6.54	4.1e-08
10	1.0	6.0	6.4e-04	5.2e-04	0.08	0.02	0.10	5.1e-01	8.7e-03	2.84	0.742	7.18	1.0e-07
13	1.0	6.0	1.1e-03	7.8e-04	0.08	0.03	0.11	6.8e-01	1.3e-02	4.21	1.631	8.05	2.0e-07

Table 4.1

Notes to Chapter 4

- 1) Figure 4.3 is only a schematic picture as it does not in fact show the rotational hoist. This problem was dealt with in detail in Chapter 3.
- 2) Strictly speaking, it should be *proved* that these thirteen equations remain independent in the case of the parallel cable suspension (with conditions (4.21)-

(4.24)) and that the solution generally does not exist. Numerical calculations show that the effect of geometrical extension generally does take place. A number of numerical examples are given in section 4.3.

- 3) All the data used here, and later in the thesis, are taken from typical RTG cranes.
- 4) Rounding off has been employed in this table.

Chapter 5

Multi-Cable Model of the RTG Crane

The equations of motion for the RTG crane with simplified single-cable spreader suspension system are derived in Chapter 2 using Lagrange's equations.

This chapter describes the Multi-Cable dynamical Model (MCM) of the RTG crane. It has been derived using the same approach as in Chapter 2, and, like the single-cable model, it takes into consideration the gantry and trolley movements, hoisting and lowering of the spreader. The assumptions for the MCM are as before: it neglects the tyre and the gantry structure deformation and flexure effects. However, the principal difference between the single-cable model and MCM is that the latter introduces lift of the spreader due to its rotation into the equations of motion. This cannot be neglected in the multi-cable reeving systems as used on the RTG cranes for container handling (refer to section 1.5.1). In order to do this, the results of the Chapter 3 are used. Chapter 3 gives an explicit formula which relates spreader hoist to parameters of its rotation and spreader/trolley size, and henceforth allows the introduction of additional terms into the equations for spreader coordinates corresponding to spreader hoist due to its rotation. The latter (equations (3.35)-(3.37)) are used in this present chapter for deriving the equations of motion for the RTG crane.

5.1 Spreader Cartesian coordinates for a given spreader translation vector and angle of rotation

Figure 5.1 shows the RTG in the global system of coordinates. This is similar to Figure 2.1 but presents the multi-cable spreader suspension. As in Figure 2.1, the spreader is shown swung away from the equilibrium position through swing angle Ψ . However in this case, it has also undergone some rotation.

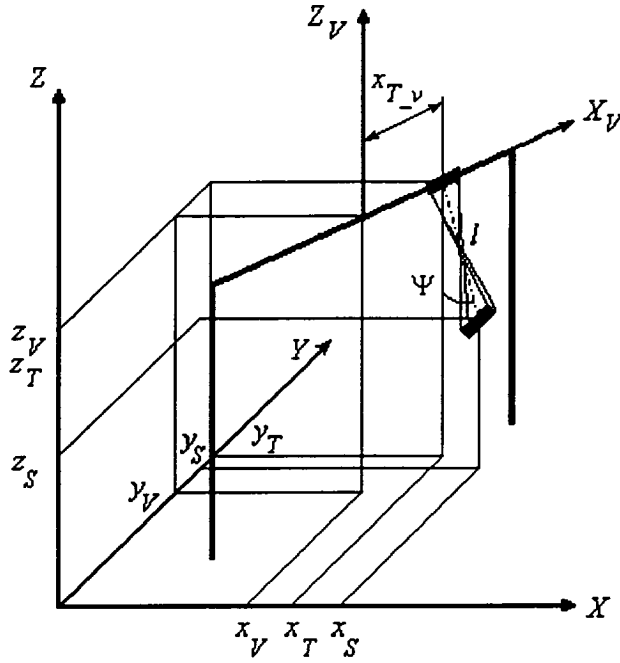


Figure 5.1

The RTG Crane in the global system of coordinates,
multi-cable case

The global coordinates of the spreader mass centre after being translated and rotated are given by equations (3.35)-(3.37) (please refer to Chapter 3 where the geometry of multi-cable suspension was considered in detail). Substituting for 'sinΨ' and 'cosΨ' in these equations by the following expressions:

$$\sin \Psi = \frac{r}{l} \quad (5.1)$$

$$\cos \Psi = \frac{\sqrt{l^2 - r^2}}{l} \quad (5.2)$$

(these expressions follow straight from Figures 2.1, 5.1 or 3.4) and so equations (3.35)-(3.37) can be modified to:

$$x_s = x_v + x_{T_v} \cos \theta_v + r \cos(\theta_v + \alpha) - \frac{l - \sqrt{l^2 - 2R^2}}{l\left(\frac{\pi}{2}\right)^2} \theta_s^2 r \cos(\theta_v + \alpha) \quad (5.3)$$

$$y_s = y_v + x_{T_v} \sin \theta_v + r \sin(\theta_v + \alpha) - \frac{l - \sqrt{l^2 - 2R^2}}{l\left(\frac{\pi}{2}\right)^2} \theta_s^2 r \sin(\theta_v + \alpha) \quad (5.4)$$

$$z_s = z_T - \sqrt{l^2 - r^2} + \frac{l - \sqrt{l^2 - 2R^2}}{l\left(\frac{\pi}{2}\right)^2} \theta_s^2 \sqrt{l^2 - r^2} \quad (5.5)$$

Thus, equations (5.3)-(5.5) take into account the change in the spreader coordinates due *both* to its translation and rotation. They present the spreader coordinates as functions of R (half diagonal dimension of both the trolley and the spreader), l (cable length), r and α (coordinates of the vector of spreader translation $\vec{r}(r, \alpha)$), θ_s (angle of spreader rotation), vehicle global coordinates, and x -coordinate of the trolley in the vehicle coordinate system, x_{T_v} .

5.2 Lagrangian Multi-Cable Model of the RTG crane

5.2.1 The set of generalised coordinates for the RTG crane with Multi-Cable spreader suspension

In the case of the single-cable model, there were seven generalised coordinates given by statements (2.12)-(2.18), but in the MCM another coordinate is required:

$$q_8 = \theta_s \quad (5.6)$$

Thus, the system now has eight degrees of freedom.

5.2.2 Derivation of the Equations of Motion for the Multi-Cable Model using Lagrange's equations

The process of derivation of the equations of motion is exactly the same as in the single-cable case (section 2.4). The difference is only that instead of equations (2.8)-(2.10) for the spreader coordinates equations (5.3)-(5.5) are used. After performing the necessary differentiations for substitution into Lagrange's equations, the following equations of motion for α , r and θ_s are obtained:

$$\begin{aligned}
 & \ddot{\alpha} + \frac{C_\alpha \dot{\alpha}}{I_s r} + \ddot{\theta}_v - \frac{1}{\left(1 - \frac{4}{\pi^2} \left(1 - \left(1 - 2 \frac{R^2}{l^2}\right)^{1/2}\right) \theta_s^2\right)^2} \left\{ -2 \dot{r} (\dot{\theta}_v + \dot{\alpha}) \right. \\
 & + \frac{32}{\pi^4} \left(1 - \left(1 - 2 \frac{R^2}{l^2}\right)^{1/2}\right) (\dot{\theta}_v + \dot{\alpha}) \theta_s^3 \left[\frac{2 R^2 \dot{r} \theta_s}{l^3 \left(1 - 2 \frac{R^2}{l^2}\right)^{1/2}} - \left(1 - \left(1 - 2 \frac{R^2}{l^2}\right)^{1/2}\right) (\dot{r} \theta_s + 2 r \dot{\theta}_s) \right] \\
 & + \left(\frac{4}{\pi^2} \left(1 - \left(1 - 2 \frac{R^2}{l^2}\right)^{1/2}\right) \theta_s^2 - 1 \right) \left[\begin{aligned} & \ddot{y}_v \cos(\theta_v + \alpha) - \ddot{x}_v \sin(\theta_v + \alpha) \\ & - \ddot{x}_{T_v} \sin \alpha + 2 \dot{x}_{T_v} \dot{\theta}_v \cos \alpha \\ & + x_{T_v} \ddot{\theta}_v \cos \alpha + x_{T_v} \dot{\theta}_v^2 \sin \alpha \end{aligned} \right] \\
 & \left. - \frac{8}{\pi^2} (\dot{\theta}_v + \dot{\alpha}) \theta_s \left[\frac{2 R^2 \dot{r} \theta_s}{l^3 \left(1 - 2 \frac{R^2}{l^2}\right)^{1/2}} - 2 \left(1 - \left(1 - 2 \frac{R^2}{l^2}\right)^{1/2}\right) (\dot{r} \theta_s + r \dot{\theta}_s) \right] \right\} = 0 \quad (5.7)
 \end{aligned}$$

$$\begin{aligned}
& \ddot{r} + C_r \frac{\dot{r}}{m_s} - r \frac{\ddot{l}}{l} + r \frac{(r\dot{l} - \dot{r}l)^2}{l^4 - l^2 r^2} - \left\{ r \left(\dot{\theta}_v + \dot{\alpha} \right)^2 + x_{T_v} \dot{\theta}_v^2 \cos \alpha \right. \\
& - \left[1 - \frac{4}{\pi^2} \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right) \theta_s^2 \right] \left[\ddot{x}_v \cos(\theta_v + \alpha) + \ddot{y}_v \sin(\theta_v + \alpha) + \ddot{x}_{T_v} \cos \alpha \right. \\
& \quad \left. \left. + 2 \dot{x}_{T_v} \dot{\theta}_v \sin \alpha + x_{T_v} \ddot{\theta}_v \sin \alpha \right] \right. \\
& + \frac{16R^2 \dot{l} \theta_s}{\pi^2 l^3 \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2}} \left[\frac{4 \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right) \theta_s^3 (\dot{r} l^2 - r l \dot{l})}{\pi^2 (l^2 - r^2)} - \frac{\theta_s (\dot{r} l^2 - r l \dot{l})}{(l^2 - r^2)} + r \dot{\theta}_s \right] \\
& + \frac{16 \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right)^2}{\pi^4} \theta_s^3 \left[4 \dot{\theta}_s \frac{r l \dot{l} - \dot{r} l^2}{l^2 - r^2} + r \left(\dot{\theta}_v + \dot{\alpha} \right)^2 \theta_s \right] \\
& \frac{4 \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right)}{\pi^2} \left[2 r \dot{\theta}_s \dot{\theta}_s - 2 r \theta_s \ddot{\theta}_s - 2 r \dot{\theta}_s^2 - x_{T_v} \dot{\theta}_v^2 \theta_s^2 \cos \alpha \right. \\
& \quad \left. - 2 r \left(\dot{\theta}_v + \dot{\alpha} \right)^2 \theta_s^2 - 4 r \frac{(l \dot{l} - r \dot{r})}{l^2 - r^2} \theta_s \dot{\theta}_s \right] \\
& - \left. \left[\frac{1 - \frac{4}{\pi^2} \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right) \theta_s^2}{(l^2 - r^2)^{1/2}} \right] r g \right\} \frac{l^2 - r^2}{l^2 \left(1 - \frac{4}{\pi^2} \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right) \theta_s^2 \right)^2} = 0 \quad (5.8)
\end{aligned}$$

$$\begin{aligned}
& \ddot{\theta}_s + \frac{C_{\theta_s} \dot{\theta}_s}{m_s} - \frac{1}{\left(I_s + \frac{64}{\pi^4} m_s \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right)^2 l^2 \theta_s^2 \right)} \left\{ -I_s \ddot{\theta}_v \right. \\
& - m_s \left(- \frac{128 R^2 \dot{l} \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right)}{\pi^4 l^2 \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2}} \theta_s^2 \left[2 l \dot{\theta}_s + \dot{l} \theta_s \right] \right. \\
& \left. - \frac{8}{\pi^2} \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right) r \theta_s \left[\ddot{x}_v \cos(\theta_v + \alpha) + \ddot{y}_v \sin(\theta_v + \alpha) + \ddot{x}_{T_v} \cos \alpha + x_{T_v} \ddot{\theta}_v \sin \alpha \right. \right. \\
& \left. \left. + 2 \dot{x}_{T_v} \dot{\theta}_v \sin \alpha - x_{T_v} \dot{\theta}_v^2 \cos \alpha - r \left(\dot{\theta}_v + \dot{\alpha} \right)^2 \right] \right\} \\
& + \left(\frac{32}{\pi^4} \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right)^2 \theta_s^3 - \frac{8}{\pi^2} \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right) \theta_s \right) \left[\frac{2 r \dot{r} \dot{l} \dot{l} + l^3 \ddot{l} - r^2 \dot{l}^2}{-l^2 \dot{r}^2 - r^2 \ddot{l}} \right] \frac{1}{l^2 - r^2} \\
& + \frac{32}{\pi^4} \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right)^2 \theta_s \left[4 l \dot{l} \theta_s \dot{\theta}_s + 2 l^2 \dot{\theta}_s^2 - r^2 \left(\dot{\theta}_v + \dot{\alpha} \right)^2 \theta_s^2 \right] \\
& - \frac{64 R^2 \left(\ddot{l} l^3 - 2 R^2 \ddot{l} l - 3 l^2 \dot{l}^2 + 4 R^2 \dot{l}^2 \right)}{\pi^4 l^4 \left(1 - 2 \frac{R^2}{l^2} \right)^{3/2}} \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right) \theta_s^3 \\
& + \frac{8}{\pi^2} \left(1 - \left(1 - 2 \frac{R^2}{l^2} \right)^{1/2} \right) (l^2 - r^2)^{1/2} \theta_s g \} = 0
\end{aligned} \tag{5.9}$$

As in the case of the single-cable model, $C_r \frac{\dot{r}}{m_s}$, $C_\alpha \frac{\dot{\alpha}}{I_s r}$ and $\frac{C_{\theta_s} \dot{\theta}_s}{m_s}$ (in equations (5.7)-(5.9) respectively) are dissipation terms with damping coefficients C_r , C_α and C_{θ_s} obtained from experimental observations.

Equations (5.7)-(5.9) are the equations of motion for the multi-cable system in coordinates α , r and θ_s . They define the position of the spreader for accelerations of

given 'motored' coordinates, $\ddot{x}_v$, $\ddot{y}_v$, $\ddot{\theta}_v$, $\ddot{x}_{T_v}$ and $\ddot{l}$, applied in practice by direct torquing of the motors driving these coordinates.

There is a restriction to the model which follows from the fact that the model takes into consideration the rotation of the spreader. The restriction is made to the cable length which has to lie within certain limits determined by the spreader dimensions. The requirement comes from the terms $\sqrt{1 - 2\frac{R^2}{l^2}}$ in the equations of motion meaning that the cable length must be longer than:

$$l > R\sqrt{2} \quad (5.10)$$

For the standard spreader dimensions used later in the thesis, $a = 1.765\text{m}$, $b = 4.938\text{m}$, giving $R = 2.622\text{ m}$ (equation (3.5)) and, from equation (5.10):

$$l > 3.708\text{ m} \quad (5.11)$$

5.3 Conclusions

In this Chapter, Equations of Motion for the RTG crane with multi-cable spreader suspension system have been derived using Lagrange's equations taking into consideration the hoist of the spreader due to its rotation. Highly intricate nonlinear cross-coupling is evident in many of the terms in equations (5.7)-(5.9), many of which are of relatively high order. Indiscriminate linearisation will lead to significant coordinate decoupling, and so the MCM is retained in its complete form for subsequent applications.

The numerical integration of the equations of motion was utilised, and the results are discussed in next chapter.

Chapter 6

Multi-Cable Model Simulations, and Analysis by Means of ADAMS

The equations of motion for the RTG crane with Multi-Cable spreader suspension have been derived in Chapter 5. This chapter describes how the equations are solved numerically, and gives several examples of MCM simulations for various data cases. In addition to this the MCM has been verified against an RTG model based on ADAMS (section 6.2) The chapter discusses also an experiment made on the Paceco Transtainer RTG crane at Felixstowe port.

6.1 Simulations using the Multi-Cable Model

6.1.1 Numerical solution of the Equations of Motion

The equations of motion for the RTG crane (5.7)-(5.9) are highly non-linear, second order, ordinary differential equations and cannot be solved analytically due to their complexity. Numerical integration by means of a standard Rungë-Kutta procedure was utilised using a series of time-steps sizes, $\Delta t = 0.1, 0.05, 0.02, 0.01s$. The time-step $\Delta t = 0.01s$ was found to be small enough to ensure the stability of the calculations. This time-step was used for all runs presented in the thesis.

The numerical integration scheme and the MCM have been implemented within a commercial graphical interface shell which offers the user a convenient interactive regime for running the simulations, and presents the results numerically and graphically.

6.1.2 The data used in the simulations

Several simulations have been carried out in order to test the model against an intuitive understanding of the RTG behaviour with different combinations of the $\ddot{x}_v$, $\ddot{y}_v$, $\ddot{\theta}_v$, $\ddot{x}_{T_v}$ and $\ddot{l}$ acceleration-time profiles applied to the system.

6.1.2.1 The system of coordinates

The position of the vehicle, the trolley and the spreader is given in the global system of coordinates (section 2.2.1). The velocity and acceleration of the vehicle, and the velocity and acceleration of the spreader lifting/lowering are also given in the global system.

The trolley velocity and acceleration are given in the vehicle system of coordinates (as described in section 2.2.2).

6.1.2.2 The RTG data

The following RTG data¹⁾ are used and are unchanged through all the simulations:

- The effective trolley and spreader width:- $a = 1.765\text{m}$
- The effective trolley and spreader length:- $b = 4.938\text{m}$
- The spreader mass:- $m_s = 10\,000\text{kg}$
- Mass moment of inertia of the spreader²⁾:- $I_s = 50\,000\text{ kg m}^2$
- Dissipation in the system³⁾:-
 $D_a = 0.02\text{ sec}^{-1}$ (6.1)
 $D_r = 0.1\text{ sec}^{-1}$ (6.2)
 $D_{\theta} = 0.09\text{ sec}^{-1}$ (6.3)

where:

$$D_{\alpha} = \frac{C_{\alpha}}{I_s} \quad (6.4)$$

$$D_r = \frac{C_r}{m_s} \quad (6.5)$$

$$D_{\phi} = \frac{C_{\phi}}{m_s} \quad (6.6)$$

- industry specified restrictions for accelerations and velocities applied to the RTG crane are:

$$|\ddot{x}_v| \leq 0.25\text{m/s}^2 \quad |\dot{x}_v| \leq 2.23\text{m/s} \quad (6.7)$$

$$|\ddot{y}_v| \leq 0.25\text{m/s}^2 \quad |\dot{y}_v| \leq 2.23\text{m/s} \quad (6.8)$$

$$|\ddot{\theta}_v| \leq 1.00^\circ/\text{s}^2 \quad |\dot{\theta}_v| \leq 15.1^\circ/\text{s} \quad (6.9)$$

$$|\ddot{x}_{T_v}| \leq 0.25\text{m/s}^2 \quad |\dot{x}_{T_v}| \leq 0.87\text{m/s} \quad (6.10)$$

$$|\ddot{l}| \leq 0.1\text{m/s}^2 \quad |\dot{l}| \leq 0.452\text{m/s (unladen spreader)} \quad (6.11)$$

- Working restriction to the cable length⁴⁾ is:

$$4\text{m} \leq l \leq 10\text{m} \quad (6.12)$$

The software was organised so that all these restrictions were universally implemented within the model.

6.1.2.3 The initial position of the RTG crane

In all the simulations presented in this chapter the RTG vehicle is initially arbitrarily positioned, with its top beam parallel to the global X axis (Figure 6.1), i.e.:

$$\vartheta_s(0) = 0 \quad (6.16)$$

In all except one of the simulations the displacement of the spreader is made to the right, i.e. in the direction of the global X axis, so that:

$$\alpha(0) = 0 \quad (6.17)$$

There is one run presented, though, when the spreader is initially translated forward, i.e. in the direction of the global Y axes:

$$\alpha(0) = 90^\circ \quad (6.18)$$

Thus, chosen initial parameters of the RTG (6.13)-(6.16) are universally applied, and the following initial data varies between the experiments and are given for each particular simulation:

- the coordinates of the vehicle, x_V and y_V ;
- the trolley displacement along the top beam of the gantry, x_{T_v} ;
- the angle of initial spreader translation, α .

6.1.2.4 MCM inputs: accelerations and velocities applied to the RTG crane

There are five inputs to the MCM:

- $\ddot{x}_V$ - lateral vehicle (gantry) acceleration;
- $\ddot{y}_V$ - longitudinal vehicle acceleration;
- $\ddot{\theta}_V$ - rotational acceleration of the vehicle about its central axis;

- $\ddot{x}_{T_v}$ - trolley acceleration along the top beam;
- $\ddot{l}$ - acceleration of the spreader lowering/hoisting.

In the experiments presented, combinations of the following accelerations are applied:

$\ddot{y}_v$, $\ddot{\theta}_v$, $\ddot{x}_{T_v}$, and are described for each experiment in its corresponding section.

In some experiments lowering of the spreader also takes place. It is organised in the following way: $\dot{l} = 0.452\text{m/s}$ ($\ddot{l} = 0$) for about the first 10 seconds, i.e. until the cables reach their maximum length limit, after which the lowering stops, and $l(t > 10\text{s}) = l_{\max} = 10\text{m}$ till the end of experiment.

6.1.2.5 Graphical presentation of the numerical results

The results of the numerical integration are presented graphically. There are two sets of graphs given for each experiment:

- velocities profiles ($\dot{x}_v$, $\dot{y}_v$, $\dot{\theta}_v$, $\dot{x}_{T_v}$, $\dot{l}$) corresponding to the inputs applied to the MCM, and profiles of the coordinates (r, α, θ_s) showing the response of the spreader;
- system responses in the form of loci of the gantry, trolley and spreader movements mapped onto the horizontal XY plane (and in some cases also onto the vertical plane XZ) with the time spacing between the plotted points equal to 1.5s.

6.1.3 The numerical results

6.1.3.1 Experiment No 1:- gantry translation only

The inputs: $\ddot{y}_v \neq 0$ $\ddot{\theta}_v = 0$ $\ddot{x}_{T_v} = 0$ $\dot{l} = 0$

In this set of experiments only the gantry acceleration in the Y direction is applied. The gantry is accelerated first at its maximum rate of 0.25m/s^2 for about 9sec (i.e. till the velocity reaches its maximum value of 2.23m/s). Then the gantry is left to move with constant velocity for some time, and then it decelerates at the same rate until it stops.

Run 1.

Initial data:

- the position of the vehicle: $x_v(0) = -2\text{m}, y_v(0) = -30\text{m}$
- the trolley displacement: $x_{T_v} = 2\text{m};$
- the spreader position: $r(0) = 0.1\text{m}, \alpha(0) = 0^\circ$

Inputs and outputs presented on: Figure 6.2

Hard gantry acceleration during the first stage results in the spreader swinging back and staying behind the gantry, as expected, but there is also an oscillatory movement of the spreader (this can be seen from the horizontal plane plot). During the constant velocity stage, which lasts for about 7sec, the magnitude of the spreader's oscillations in r decreases, and the spreader catches up with the trolley. The oscillations do not stop though, and the spreader swings around all the time in such a manner that the absolute value of its positional angle $|\alpha|$ monotonically increases. This does not mean that the spreader rotates (this sort of RTG movement does not induce spreader rotation, and $\dot{\theta}_s \equiv 0$ during all the experiment), it means that the spreader swings in small circles around

the trolley ($r = 10\text{-}12\text{cm}$ at this stage) staying for some time behind, for some time to the left, then to the right, and in front of the trolley, but maintaining its orientation throughout at $\theta_s = 0$.

During the deceleration stage the spreader behaviour is similar to that during the acceleration stage, but the spreader now stays in front of the trolley, as one would expect.

Run 2.

Initial data:

- the position of the vehicle: $x_v(0) = -2\text{m}$, $y_v(0) = -30\text{m}$
- the trolley displacement: $x_{T_v} = 2\text{m}$;
- the spreader position: $r(0) = 0.1\text{m}$, $\alpha(0) = 90^\circ$

Inputs and outputs presented on: Figure 6.3

This is a similar experiment to the previous one. The gantry is accelerated in the same manner (only the constant velocity stage is slightly longer), but the spreader is initially displaced by 0.1m forward ($\alpha(0) = 90^\circ$) rather than sideways, i.e. in the direction of the Y axis.

As can be seen from the horizontal plane plot, the spreader oscillates during the constant velocity stage as before, but in contrast with the case when $\alpha(0) = 0$, the amplitude r is now very small. The spreader naturally follows the trolley on the acceleration stage, and then swings forward when the vehicle decelerates.

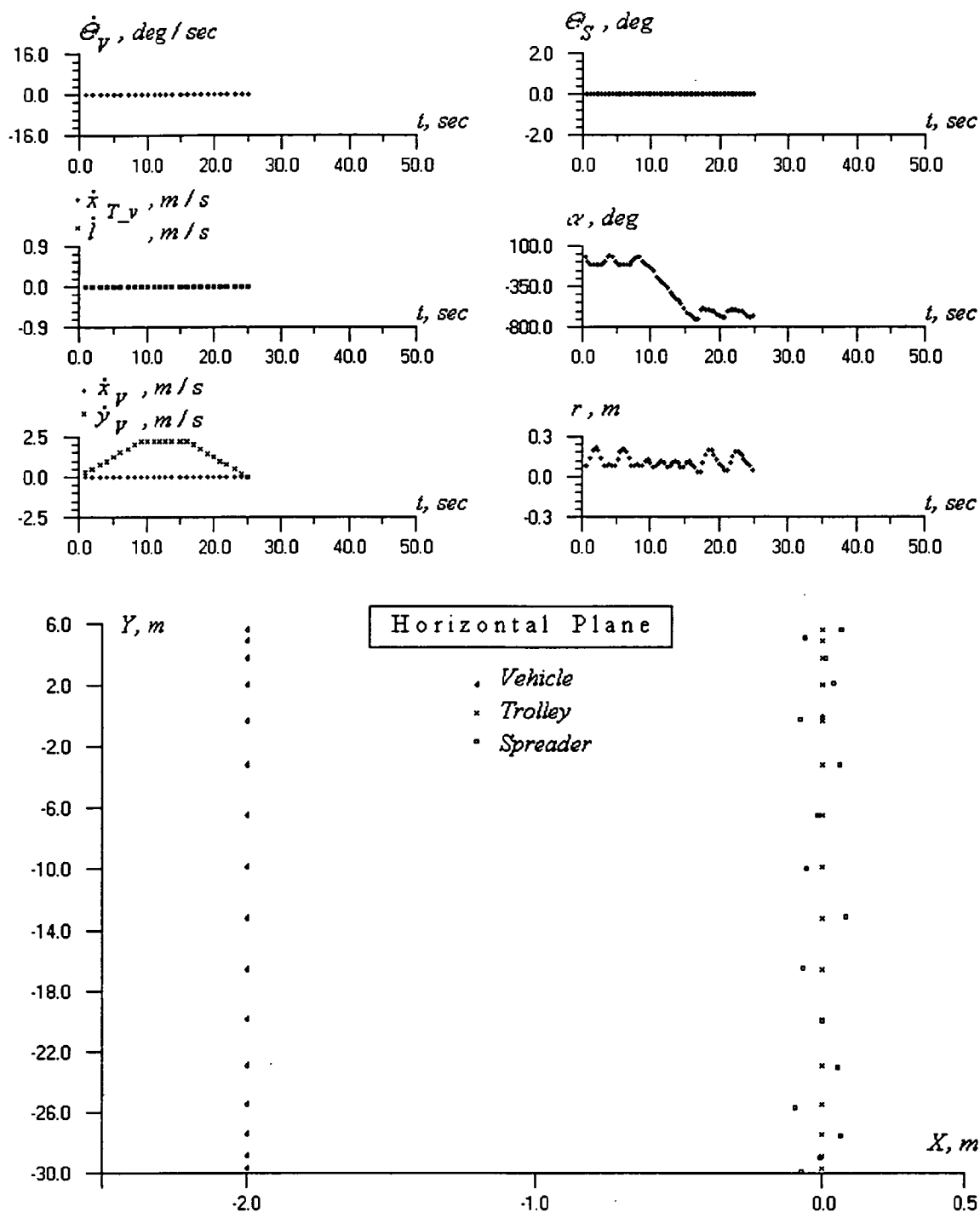


Figure 6.2

Experiment 1 (run 1): gantry translation only, $\alpha(0) = 0$

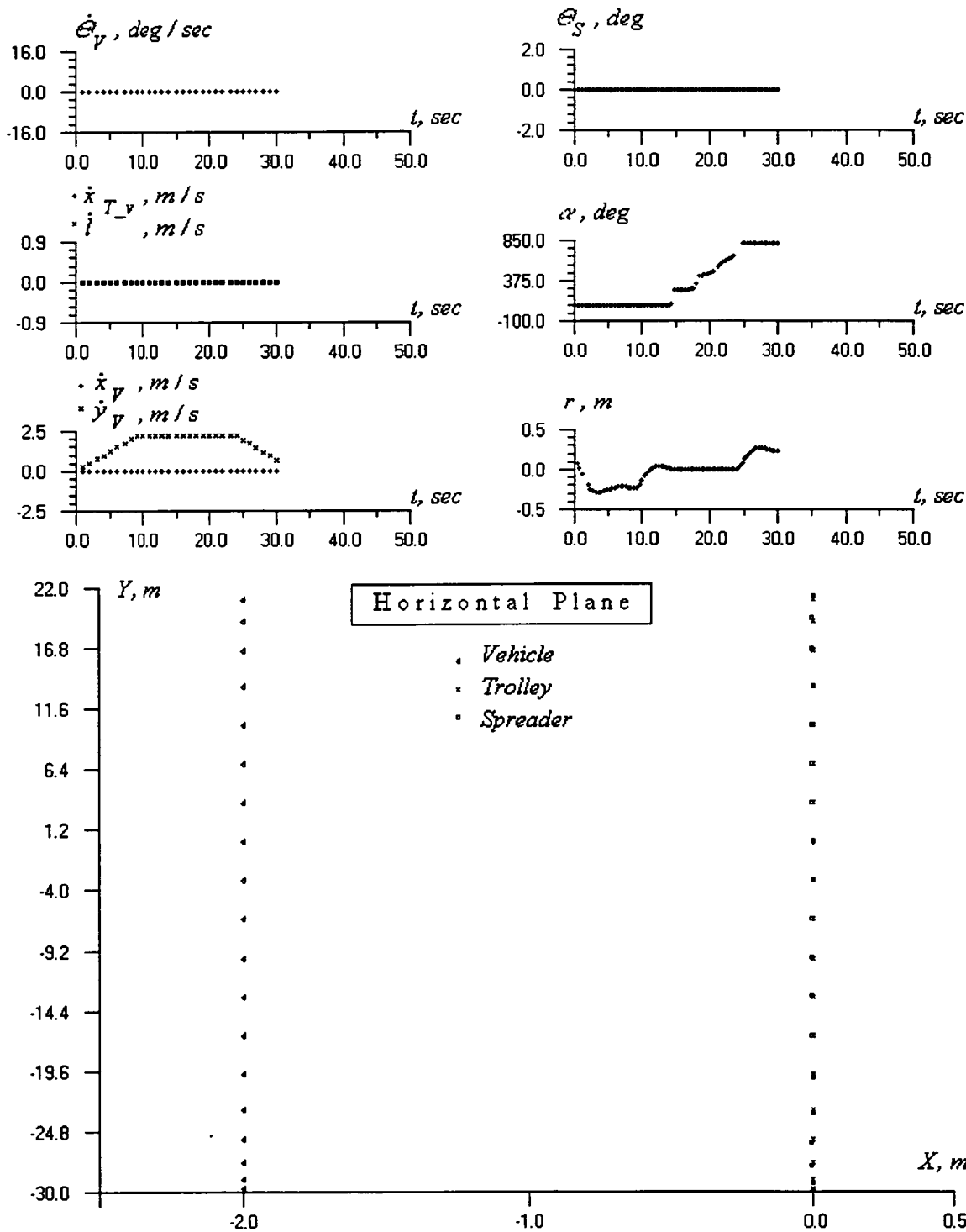


Figure 6.3

Experiment 1(run 2): gantry translation only, $\alpha(0) = 90^\circ$

6.1.3.2 Experiment No 2:- gantry translation and spreader lowering

Initial data:

- the position of the vehicle: $x_V(0) = -2\text{m}, y_V(0) = -30\text{m}$
- the trolley displacement: $x_{T_v} = 2\text{m};$
- the spreader position: $r(0) = 0.1\text{m}, \alpha(0) = 0^\circ$

The inputs: $\ddot{y}_V \neq 0 \quad \ddot{\theta}_V = 0 \quad \ddot{x}_{T_v} = 0 \quad \dot{l} \neq 0$

Inputs and outputs presented on: Figure 6.4

The similar three phase acceleration is applied to the gantry, but in this case lowering of the spreader (as described in section 6.1.2.4) takes place.

The response of the spreader is generally the same: it lags behind the trolley at the acceleration stage, catches it up when the gantry moves with constant velocity, and leads at the deceleration stage. The difference is only in the phase of the spreader oscillations, and the amplitude r is higher (as expected for the longer cable length).

6.1.3.3 Experiment No 3:- trolley and gantry translation with spreader lowering

Initial data:

- the position of the vehicle: $x_V(0) = -25\text{m}, y_V(0) = -25\text{m}$
- the trolley displacement: $x_{T_v} = 0;$
- the spreader position: $r(0) = 0.1\text{m}, \alpha(0) = 0^\circ$

The inputs: $\ddot{y}_V \neq 0 \quad \ddot{\theta}_V = 0 \quad \ddot{x}_{T_v} \neq 0 \quad \dot{l} \neq 0$

Inputs and outputs presented on: Figure 6.5

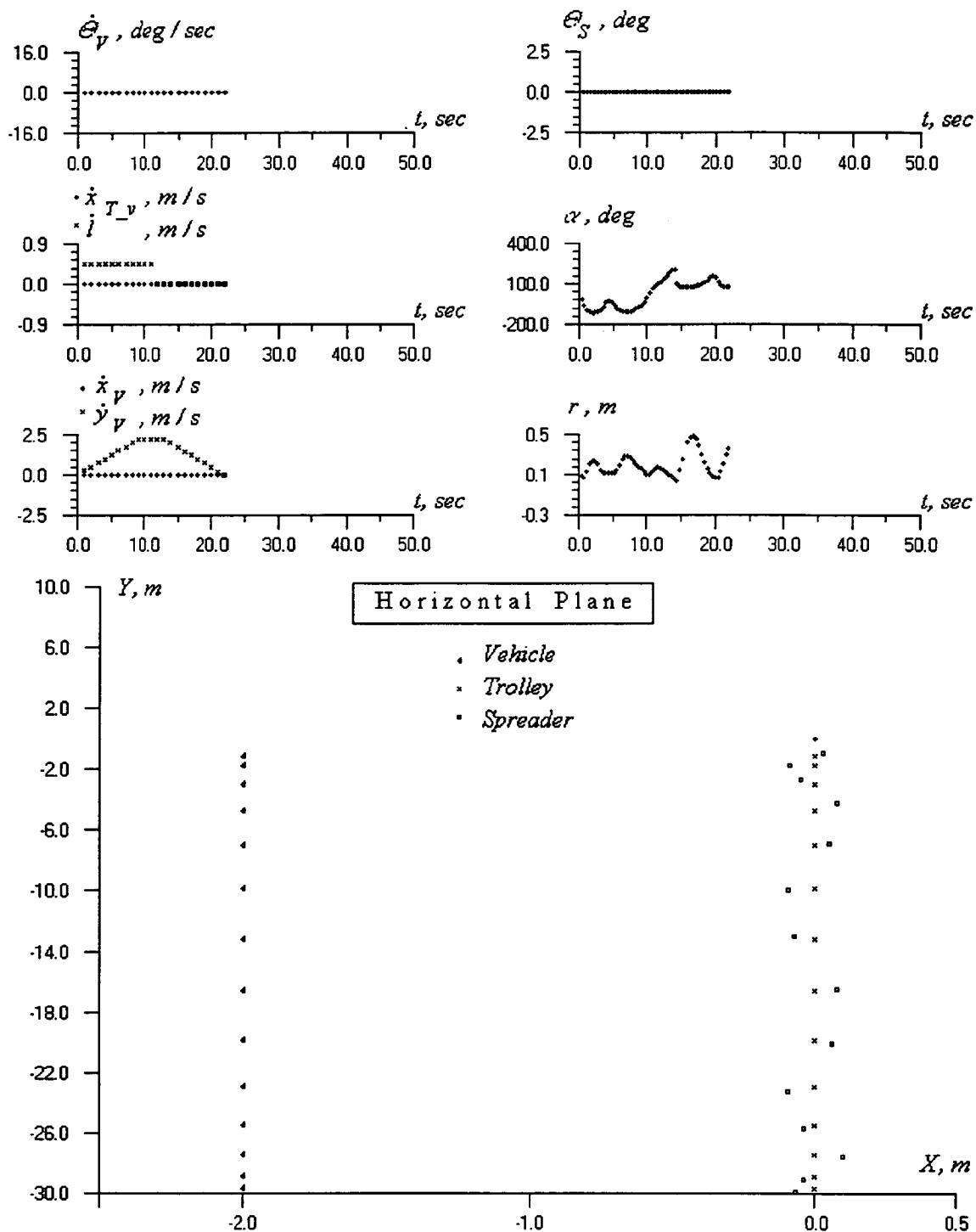


Figure 6.4

Experiment 2: gantry translation and spreader lowering

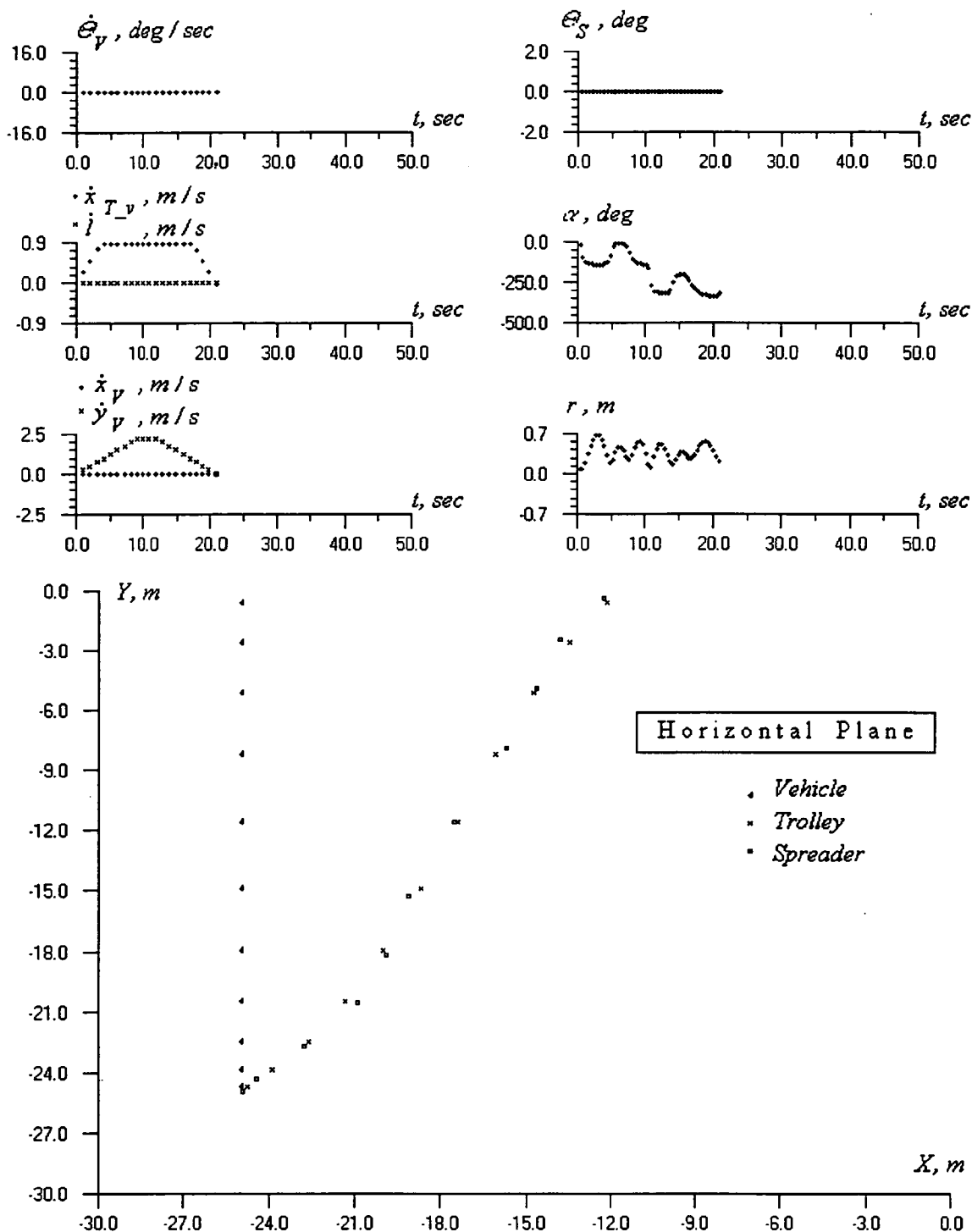


Figure 6.5

Experiment 3: trolley and gantry translation with spreader lowering

Three phase accelerations are applied to the gantry and the trolley. They both accelerate and decelerate at their maximal rate, $\ddot{x}_v = \ddot{y}_v = 0.25\text{m/s}^2$ without exceeding the velocity limits, $|\dot{y}_v| \leq 2.23\text{m/s}$ and $|\dot{x}_{T-v}| \leq 0.87\text{m/s}$. Thus, the length of the constant velocity stage is longer for the trolley than for the vehicle. These lengths are chosen to be such that both trolley and vehicle stop at the same time, t_{end} about 22sec.

The spreader, as expected, lags behind both the trolley and gantry on the acceleration stages, and leads at the deceleration stages.

There is only one point on the plot corresponding to the vehicle constant velocity stage (because it is very short), but there are five points on the plot corresponding to that of the trolley. The spreader has had enough time to catch up with the trolley, and it can be seen to follow the trolley's path closely, staying either behind or in front of it, depending on the gantry acceleration at a time.

6.1.3.4 Experiment No 4:- gantry pure rotation with the lowering of the spreader

Initial data:

- the position of the vehicle: $x_v(0) = 0\text{m}, \quad y_v(0) = 0\text{m}$
- the trolley displacement: $x_{T-v} = 2\text{m};$
- the spreader position: $r(0) = 0.1\text{m}, \quad \alpha(0) = 0^\circ$

The inputs: $\ddot{y}_v = 0 \quad \ddot{\theta}_v \neq 0 \quad \ddot{x}_{T-v} = 0 \quad \dot{l} \neq 0$

Inputs and outputs presented on: Figure 6.6

The trolley stays offset by 2m during all the experiment.

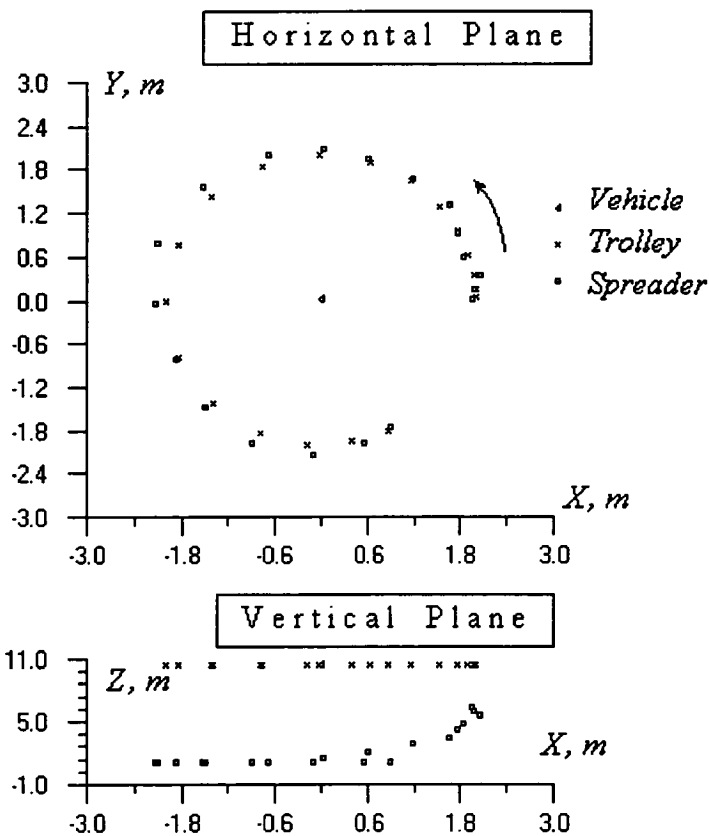
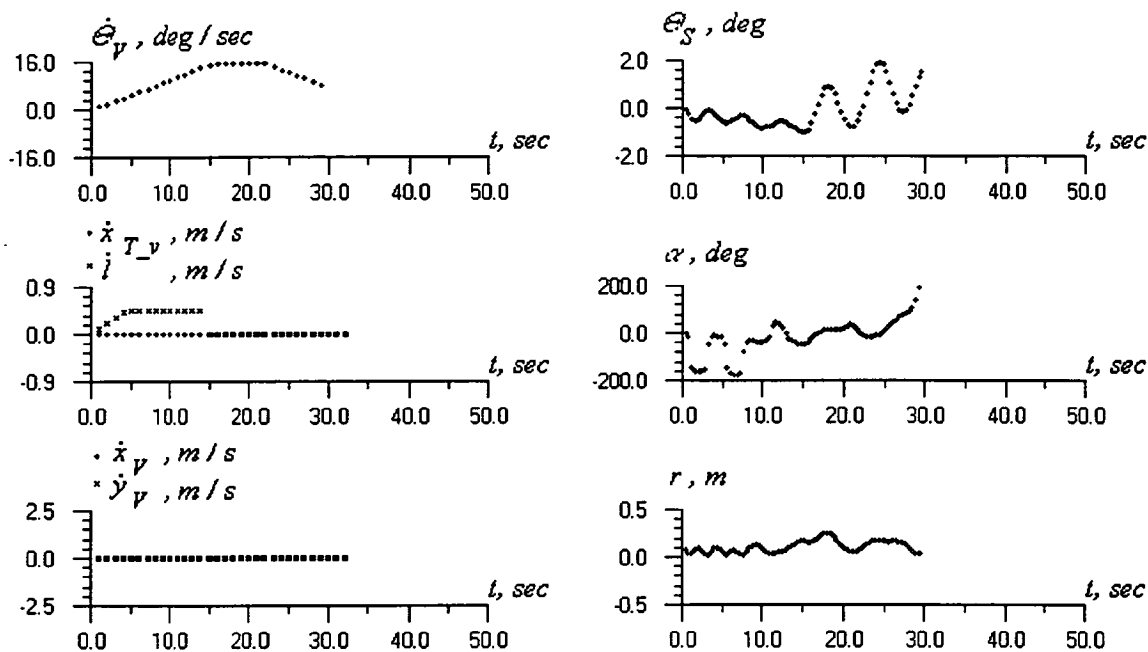


Figure 6.6

Experiment 4: gantry translation and rotation with spreader lowering

In this case, the gantry undergoes pure rotation about the vertical axis through the vehicle's geometrical centre, with the angular velocity $\dot{\theta}_v$ profile of trapezoidal shape. Lowering of the spreader also takes place in the manner described before.

The lag, catch up, and the ultimate tendency to lead of the spreader during last few seconds are evident, as usual, but in this case, the rotation of the vehicle induces the rotation of the spreader, thus $\dot{\theta}_s \neq 0$ meaning that the orientation of the spreader is different to that of the trolley.

Figure 6.6 shows the spreader and trolley loci not only in the horizontal, but also in the vertical plane. On the horizontal plane, the trolley moves anticlockwise with the starting point at 3 o'clock. On the vertical plane plot the lowering motion of the spreader can be seen to be spiral in characteristic.

Two more simulations with the MCM are presented here. Simulations 5 and 6 involve the simultaneous translation and rotation of the vehicle and lowering of the spreader, and in case of simulation 6 the trolley is also translated. These experiments are of theoretical rather than current practical interest, and Figures 6.7 and 6.8 demonstrate the response of the spreader corresponding to an intuitive understanding of RTG dynamics, gained from studying the results of earlier simulations.

6.1.3.5 Experiment No 5:- gantry translation and rotation with spreader lowering

Initial data:

- the position of the vehicle: $x_v(0) = -10\text{m}, y_v(0) = -10\text{m}$
- the trolley displacement: $x_{T_v} = 2\text{m};$
- the spreader position: $r(0) = 0.1\text{m}, \alpha(0) = 0^\circ$

The inputs: $\ddot{y}_v \neq 0$ $\ddot{\theta}_v \neq 0$ $\ddot{x}_{T_v} = 0$

Inputs and outputs presented on: Figure 6.7

6.1.3.6 Experiment No 6:- gantry translation and rotation, trolley translation, and spreader lowering

Initial data:

- the position of the vehicle: $x_v(0) = 0$, $y_v(0) = -25\text{m}$
- the trolley displacement: $x_{T_v} = 2\text{m}$;
- the spreader position: $r(0) = 0.1\text{m}$, $\alpha(0) = 0^\circ$

The inputs: $\ddot{y}_v \neq 0$ $\ddot{\theta}_v \neq 0$ $\ddot{x}_{T_v} \neq 0$

Inputs and outputs presented on: Figure 6.8

Note that the simulation 6 starts at the point A and finishes at the point B marked on the horizontal plane plot, Figure 6.8.

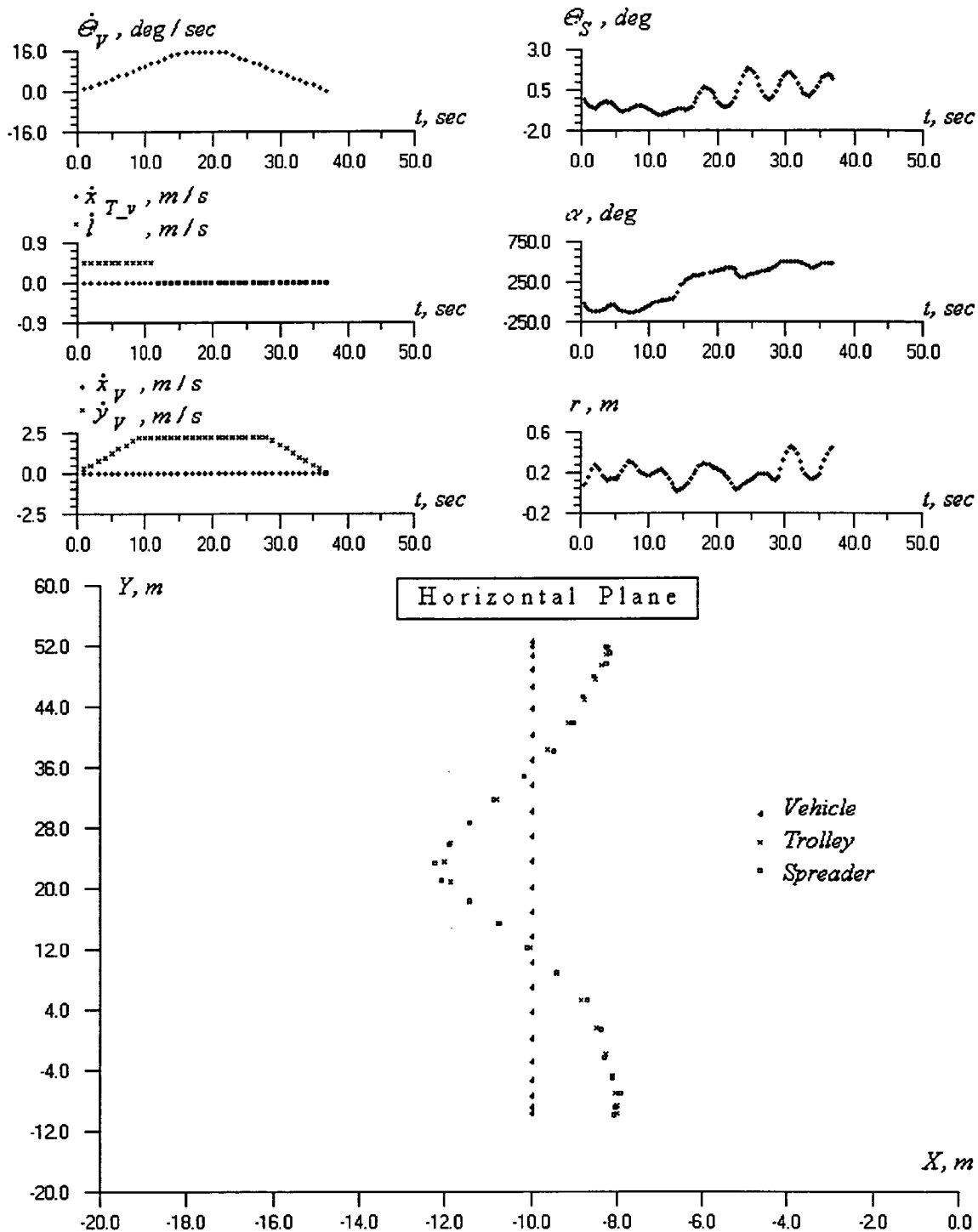


Figure 6.7
Experiment 5: gantry translation and rotation with spreader lowering

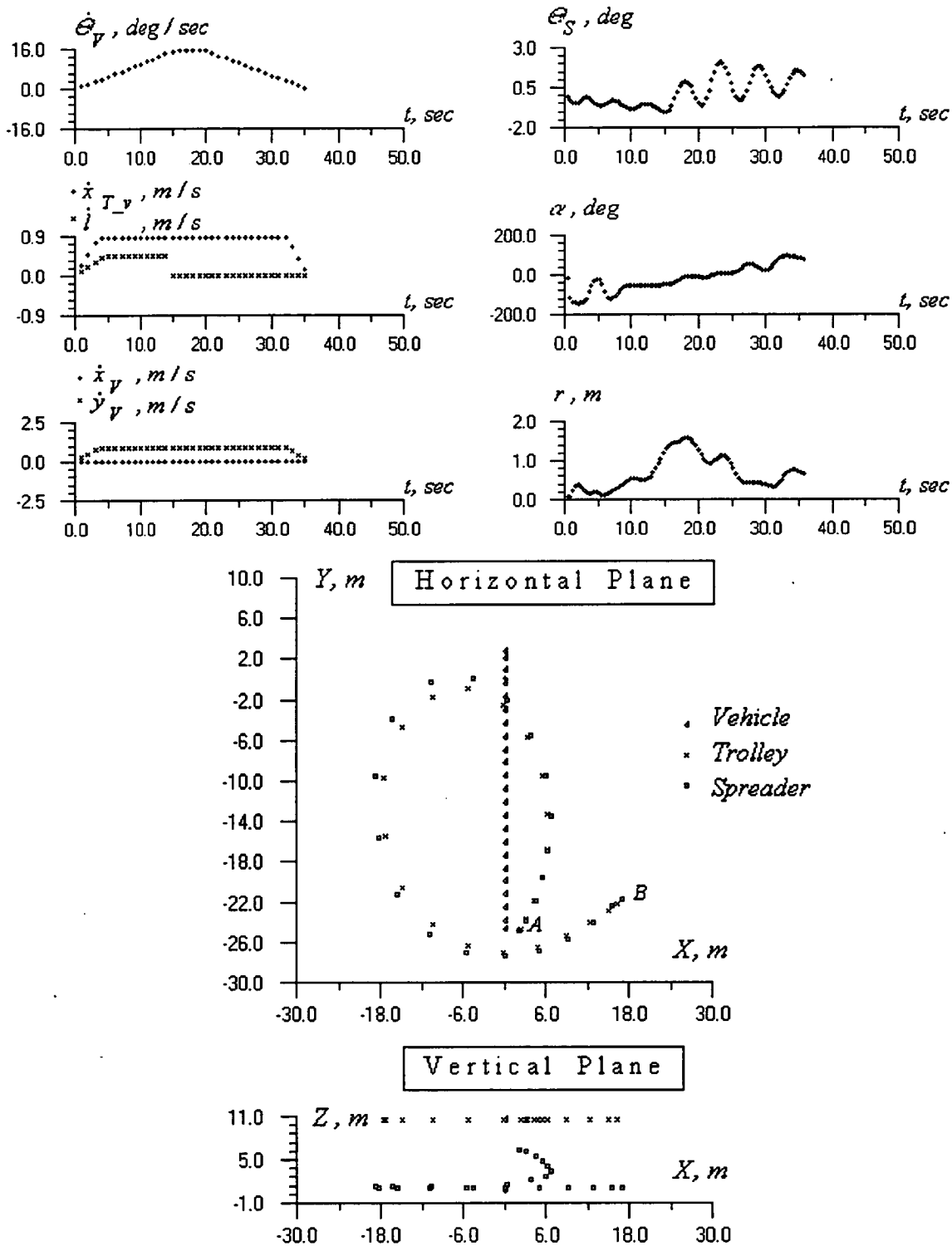


Figure 6.8

Experiment 6: gantry translation and rotation, trolley translation, and spreader lowering

6.2 The ADAMS Model of the RTG Crane

In order to validate the Multi-Cable Model (MCM), an ADAMS model of the RTG crane has been built. ADAMS (Analysis, Design, Animation of Mechanical Systems) is a well-known software package produced by Mechanical Dynamics, Inc., USA, which offers the user a very powerful tool for multi-body mechanical systems design in order to carry out kinematic and dynamic analysis. The package has advanced facilities for presentation of the results numerically, graphically, and in a form of animation.

This section describes how an ADAMS model of the RTG crane has been set up and presents two simulations using it. A comparison of the ADAMS simulations with the results of two analogous simulations with the MCM is made.

Some terminology used in ADAMS is adopted in this section, and all the specific ADAMS words are typed in *italics* with necessary explanations. The figures demonstrating the ADAMS model are composed in and printed directly from the package.

6.2.1 Building the ADAMS Model of the RTG Crane

Figure 6.9 shows three views of the ADAMS model of an idealised RTG crane schematic, the top view, the front view, and an isometric view. The global system of coordinate $OXYZ$ is chosen to be the same as in the MCM: the OZ axis pointing upwards, and the OX axis parallel to the upper beam of the portal frame.

In order to construct an ADAMS *model* of any mechanical system, it has to be split into *parts*. As has been already defined, the parts of the RTG crane are: the n-shaped gantry frame consisting of one top and two sides beams, the trolley, the spreader, and four cables.

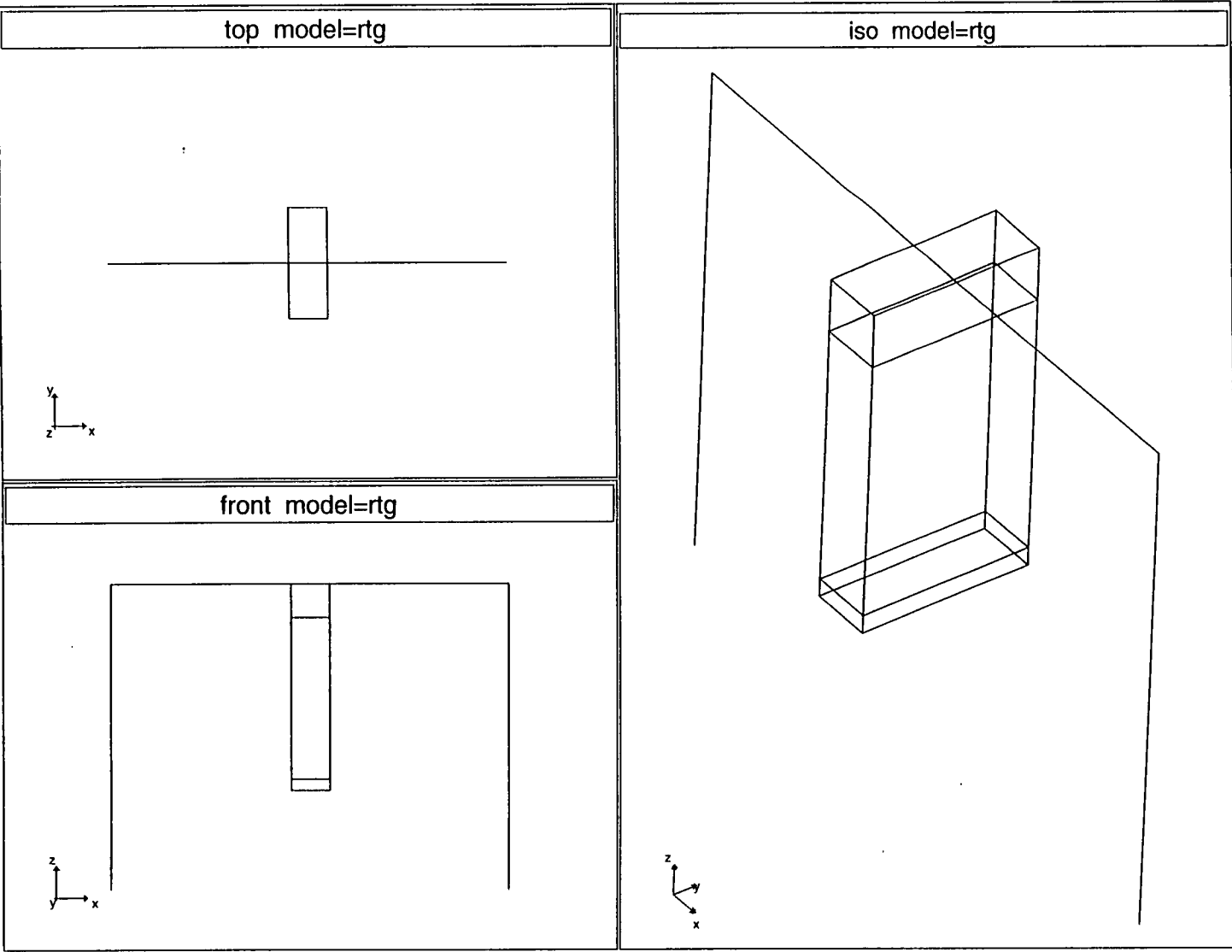


Figure 6.9
Three views of the RTG crane in
ADAMS model

The first *part* of ADAMS model is always *ground*. It is configured by ADAMS automatically to provide the global system of coordinates. All other *parts* are defined by the user by entering their *names* and then their translational and rotational coordinates.

Thus, there are nine *parts* in the RTG *model* (Figure 6.10). They are called:

- | | | |
|------------------------|------------------|-------------|
| 1. the <i>ground</i> , | 4. right beam, | 7. cable A, |
| 2. left beam, | 5. the trolley, | 8. cable B, |
| 3. upper beam, | 6. the spreader, | 9. cable C. |

Only *three* cables are used in the ADAMS model. This is made in order to avoid the effect of geometrical extension in the four cable system of spreader suspension⁵⁾ (refer to the Chapter 4).

Figure 6.10 shows the RTG structure with local *XYZ* triads for each *part* with its *name* typed next to it.

The global *z*-coordinate of the trolley is chosen: $z_T = 12.2\text{m}$. This is the clear height under the spreader for the Paceco RTG crane when the spreader is in its highest position. The spreader *z*-coordinate is arbitrarily chosen to be: $z_S = 5\text{m}$, thus the cable length is $l = 7.2\text{m}$.

All *parts* of the *model* are connected by means of *joints* (Figures 6.11 and 6.12). Three beams are joined together by means of *fixed joints* imitating a rigid structure of the portal frame (these joints are not shown on the Figures). Figure 6.11 shows the *translational joint* (named P1) between the trolley and the upper beam which allows the trolley's lateral movement along the beam. Three cables are joined to the trolley by means of *spherical joints* (Figure 6.12), and three *universal joints* join their other ends to the spreader. This allows the spreader to move in any direction and to be rotated.

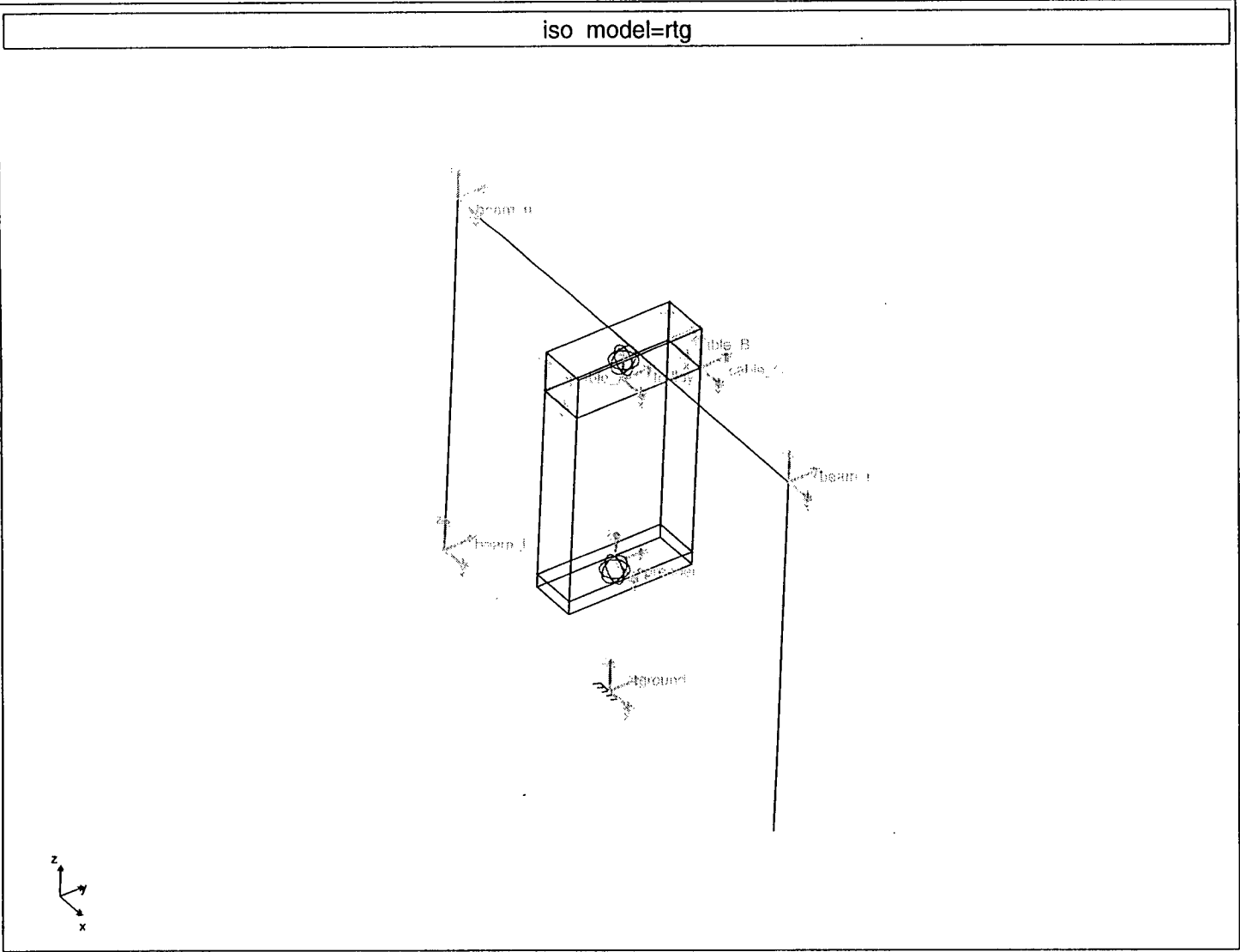


Figure 6.10

Parts of the RTG cranes as used in
ADAMS model

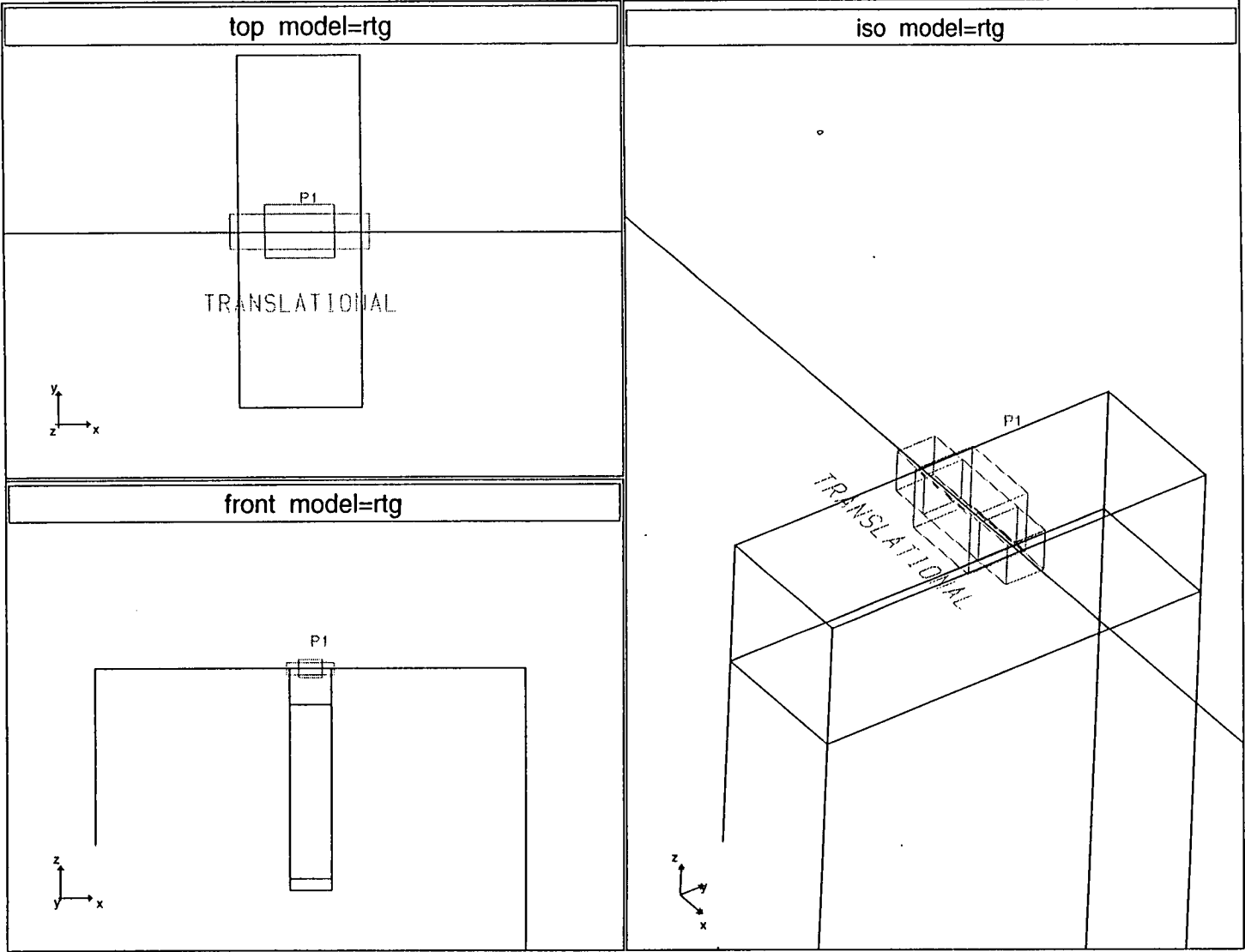


Figure 6.11

*Translational joint connecting the
trolley and the gantry top beam*

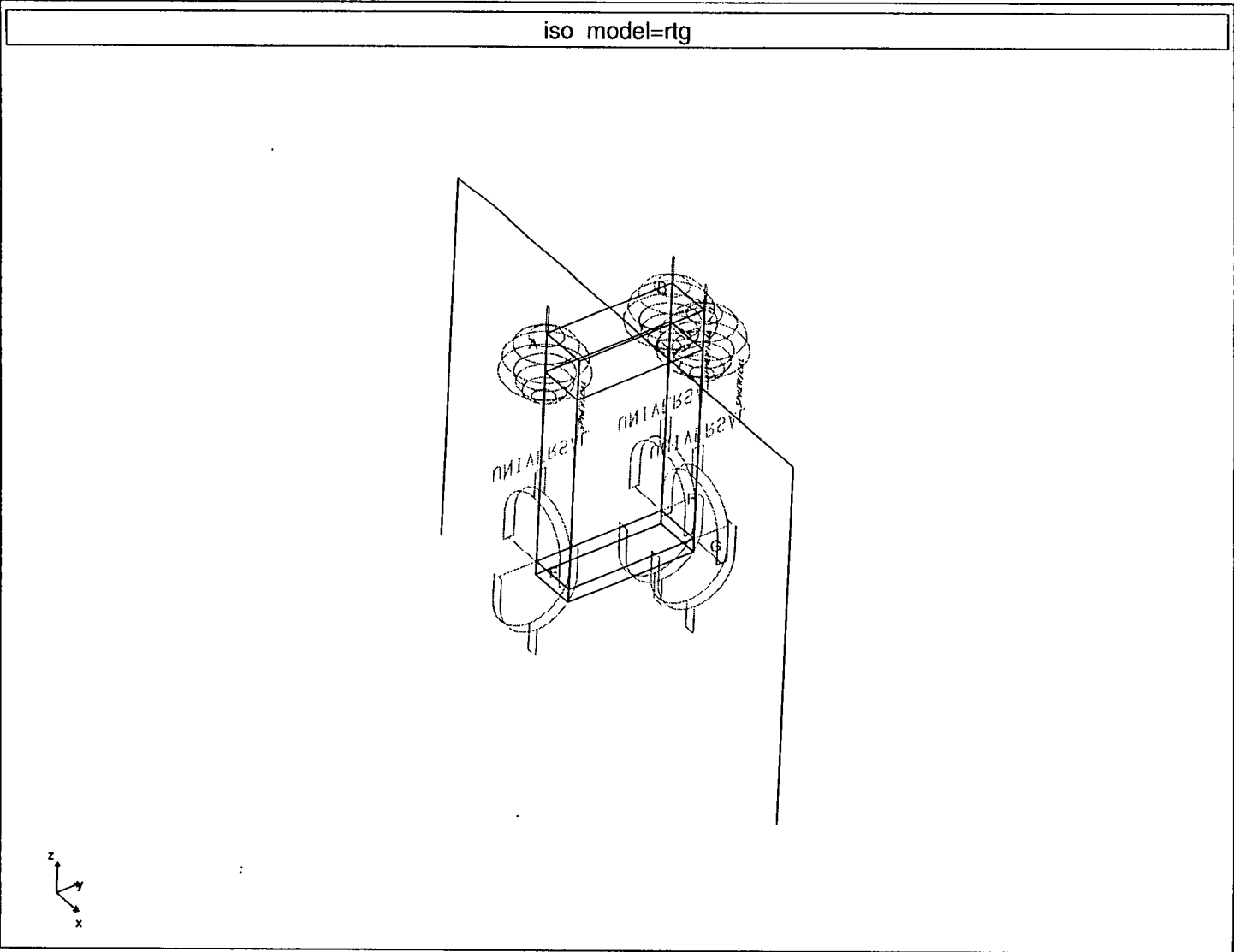


Figure 6.12
*Spherical and universal joints used for
the spreader suspension*

The ADAMS model supplies *tire-like-joints* which could be used to complete the RTG Crane's model. However, the choice was made to simplify the imitation of the gantry's translation and rotation in the following way. Two additional *parts* called 'tire' and 'gantry' were organised. The lower ends of the left and right beams were rigidly connected to the 'gantry' *part*, the latter was connected to the 'tire' *part* through the *revolute joint* (see 'tire_rot' *joint* on Figure 6.13) imitating the rotation of the gantry about its vertical centre axis. Finally, the 'tire' was connected to the *ground* by means of *translational joint* (see 'tire_tr' on Figure 6.13) imitating the translation of the gantry in the *Y* direction. Thus, the gantry movement in the ADAMS Model is restricted to a fore-aft translation, and to a rotation about its vertical symmetry axis. It was decided to concentrate the efforts in the development of the ADAMS model only on the trolley and gantry motion, and the spreader lowering was not investigated here.

This completes the kinematic model of the RTG crane.

For dynamic analysis of the model, *motions* are applied to the joints used in the ADAMS model. It can be done by writing the *acceleration function* for each joint. This function shows how the translational/angular acceleration of one part connected by the joint changes with time, with respect to another part.

In the case of RTG model as it has been set up, there are three possible *motions* (Figure 6.14) called: 'trolley_tr', 'gantry_tr', 'gantry_rot'. They are applied to the *joints* P1, tire_tr, and tire_rot respectively. Any combination of these motions can be applied in the same experiment.

The *mass* and *inertia properties* of the spreader are chosen as in the MCM (section 6.1.2.2):

$$m_s = 10\,000 \text{ kg}, \quad I_s = 50\,000 \text{ kg m}^2$$

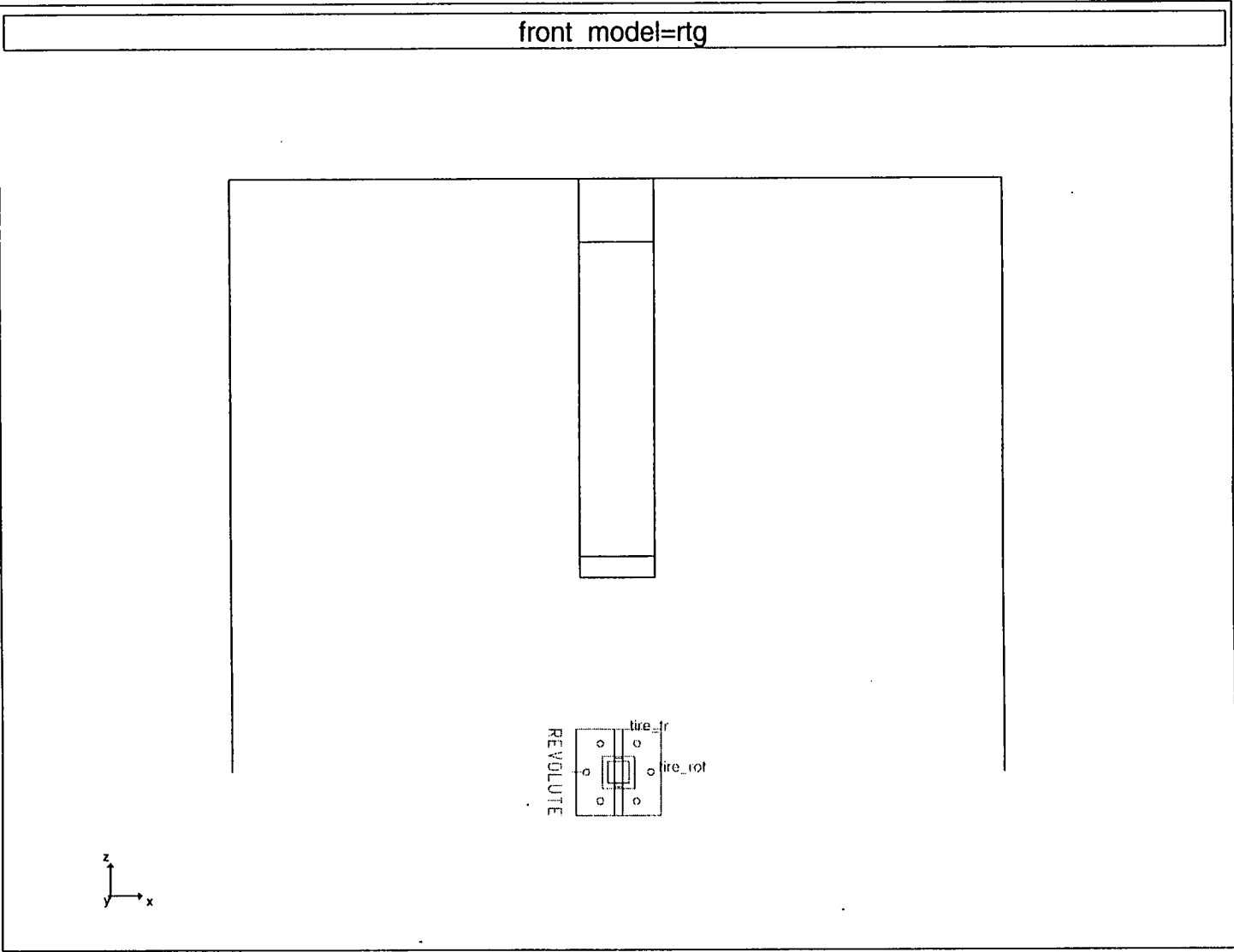


Figure 6.13

Imitation of the gantry movements
by means of *translational* and
rotational joints

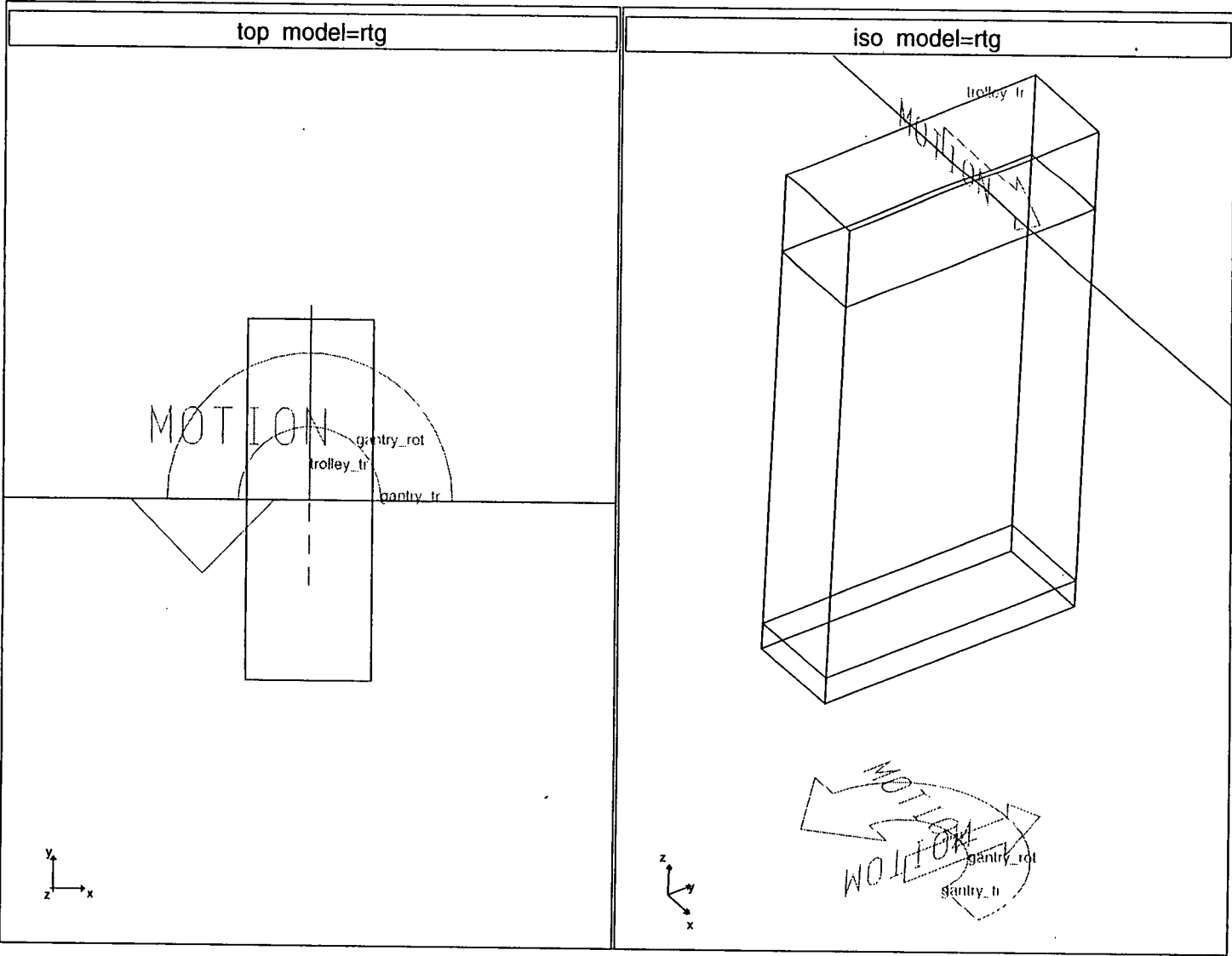


Figure 6.14

*Motions applied to the RTG crane in
ADAMS model*

The inertia term I_s represents the spreader moment of inertia about the OZ axis when it is fully extended. The moments of spreader inertia about the remaining OX and OY axes are calculated approximately and are taken to be

$$I_{sx} = 50\,000 \text{ kg m}^2, \quad I_{sy} = 5\,000 \text{ kg m}^2$$

This data has been derived on the assumption that a fully retracted spreader has the form of a parallelepiped of size¹⁾ $1.765 \times 4.938 \times 0.5 \text{ m}^3$.

6.2.2 Animation of the ADAMS Model of the RTG Crane

Two experiments with ADAMS model have been undertaken and compared to analogous experiments with the MCM. The first experiment involves the translation of the trolley along the upper beam (in the positive X direction) and the simultaneous translation forward of the gantry (in the positive Y direction), and the second one involves the trolley translation along the upper beam while the gantry is rotated about its vertical symmetry axis.

It should be noted that the initial spreader position in the ADAMS model is where the spreader is situated exactly below the trolley, while in the case of the Multi-Cable Model the spreader has to be initially translated away by a small amount due to numerical integration requirements previously discussed (see section 6.1.2.3).

6.2.2.1 Experiment No 1:- trolley and gantry translation

This experiment is designed in the same way as experiment 3 discussed in section 6.1.3.3 (only without lowering of the spreader): the trolley and the vehicle are accelerated to full extend first ($\ddot{x}_{T_v} = \ddot{y}_v = 0.25 \text{ m/s}^2$) and then the constant velocity stage takes place once the velocities reach their maximum values. Finally, the trolley and the gantry

decelerate fully and stop at the same time. Figure 6.15 shows the profile of the vehicle and trolley velocities.

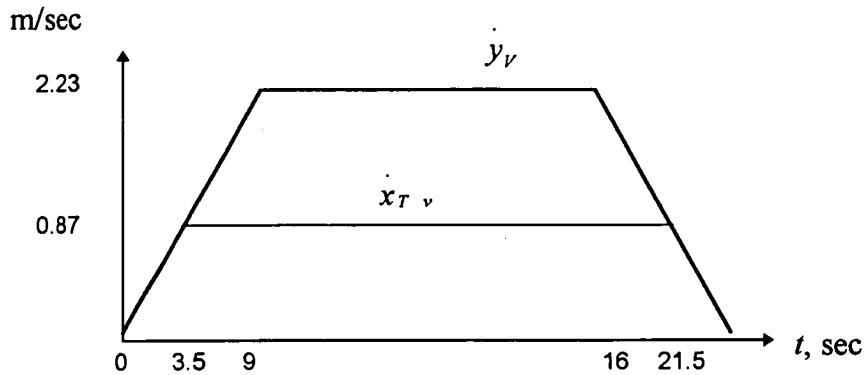


Figure 6.15

Experiment 1

the profiles of the vehicle and trolley velocities

The numerical data for the ADAMS model are presented graphically in Figures 6.16 - 6.17. Figures 6.16 and 6.17 show the X and Y Cartesian coordinates of the gantry, the trolley, and the spreader changing with time, Figure 6.18 shows the spreader hoist.

It can be clearly seen that on the first stage the spreader lags behind the trolley in both the X and Y directions, as expected. After 3.5 seconds, when the trolley does not accelerate any more, the spreader swings forward and keeps oscillating during the trolley constant velocity stage, still remaining mostly behind the trolley in the Y direction as the gantry continues to accelerate forward. After $t = 16$ sec the gantry decelerates, and the spreader moves forward catching the trolley up and then remaining most of the time in front of the trolley in the Y direction first, and then for the last 3.5 seconds in both directions.

Figures 6.19-6.21 show the data taken from the identical experiment with the Multi-Cable Model. One can clearly see that they are very similar, both qualitatively and quantitatively, to corresponding data on Figures 6.16-6.18.

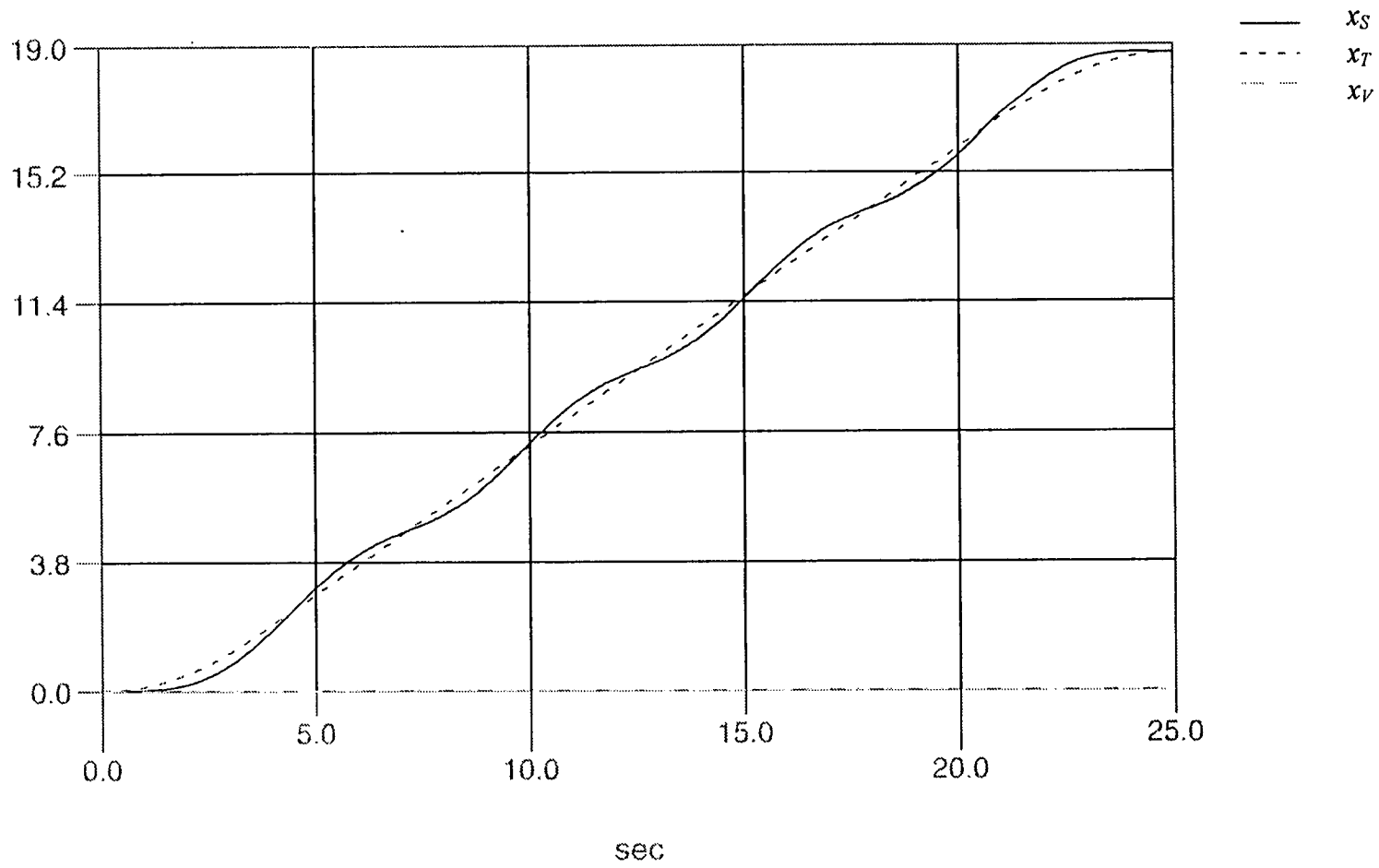
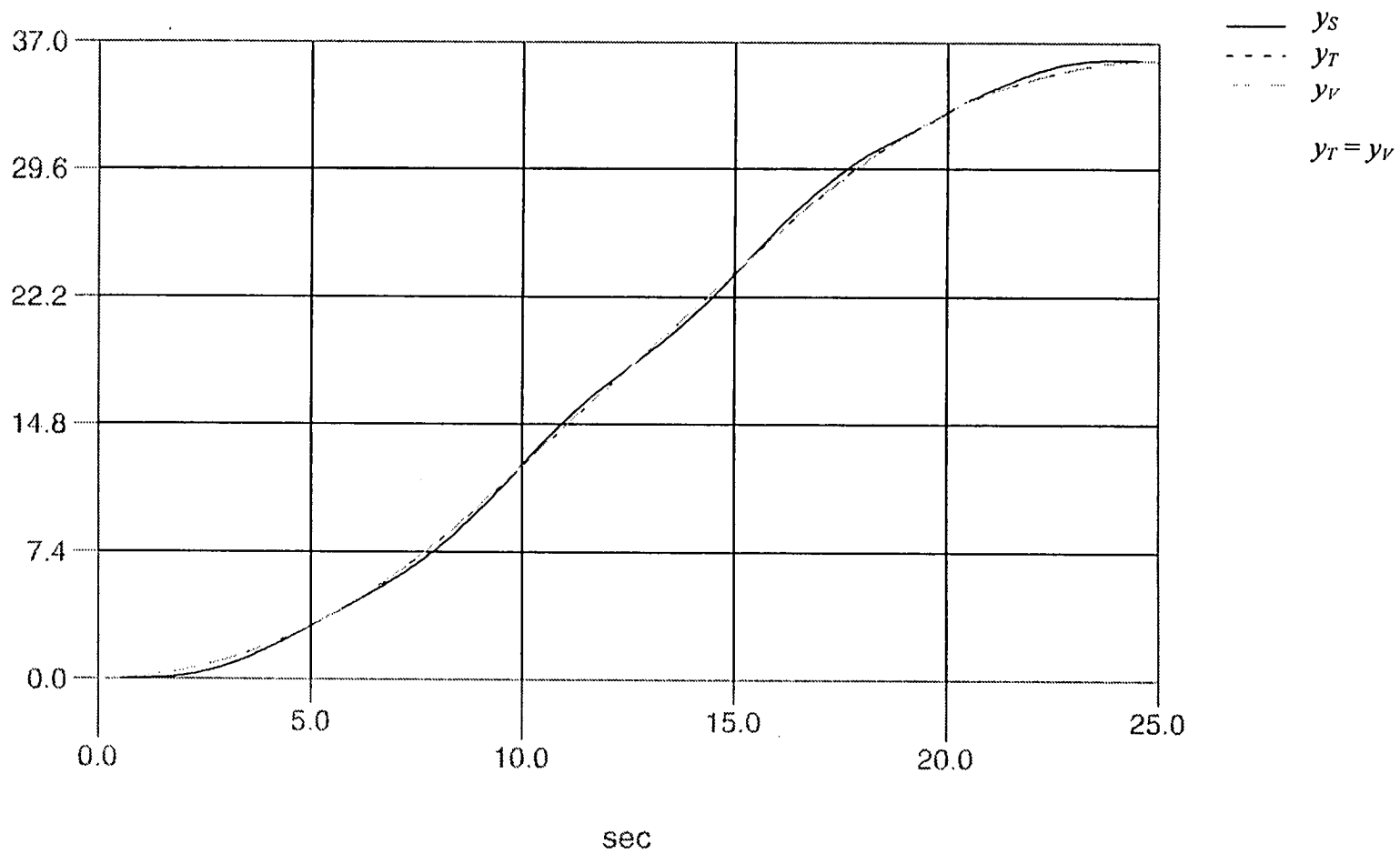


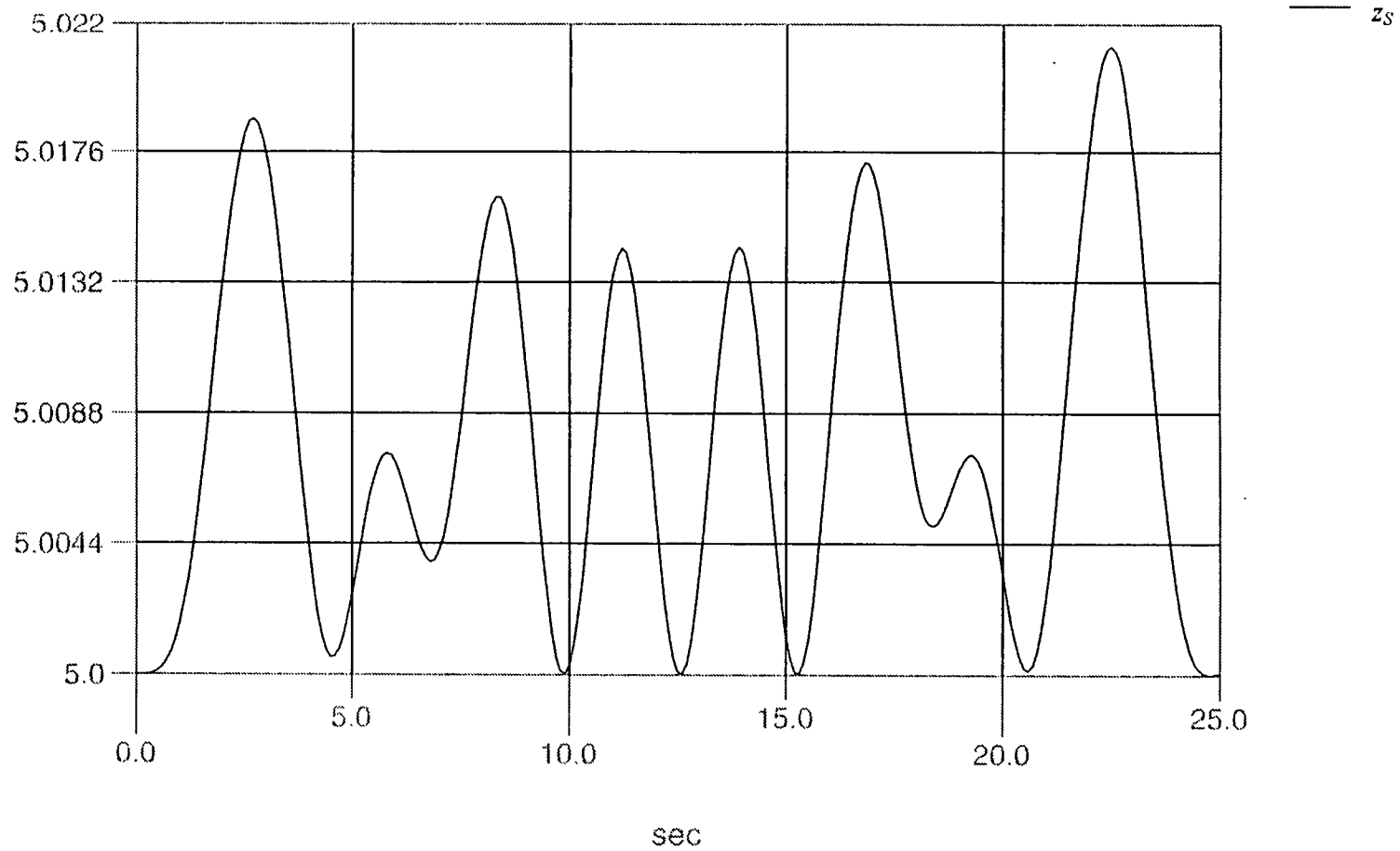
Figure 6.16
Experiment 1, ADAMS model



jejeu

Figure 6.17

Experiment 1, ADAMS model



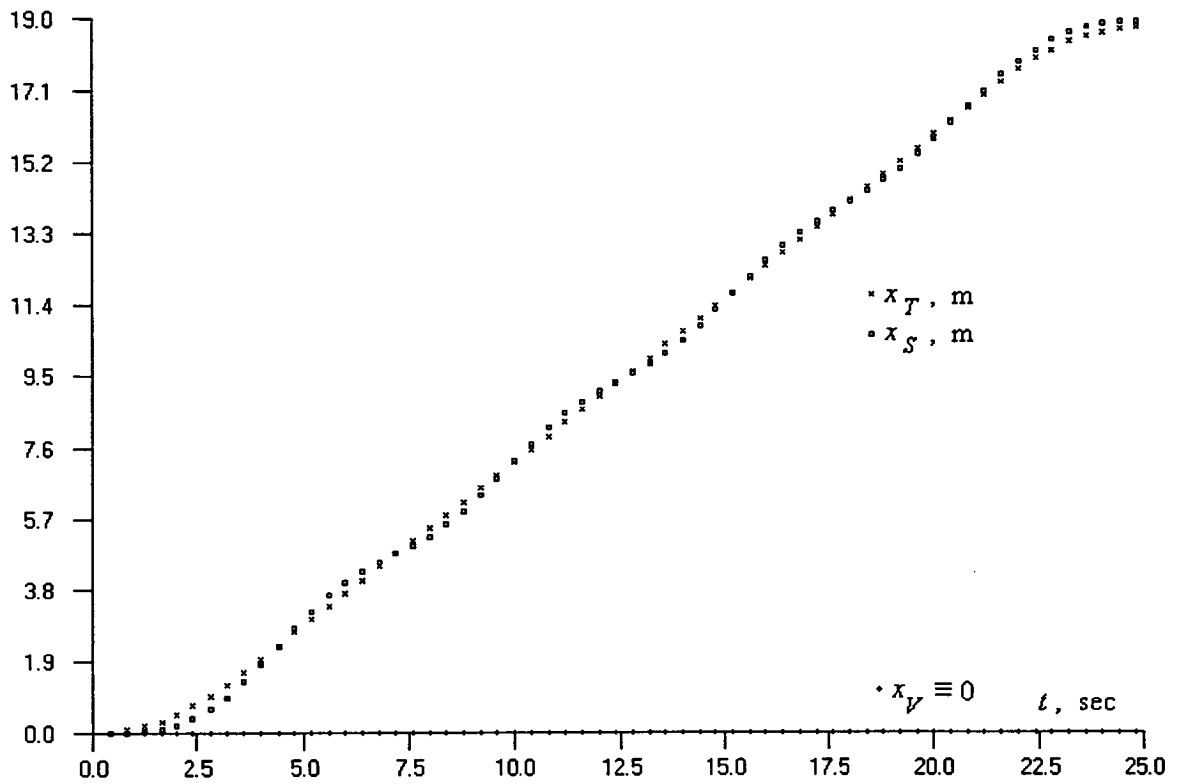


Figure 6.19

Experiment 1, MCM

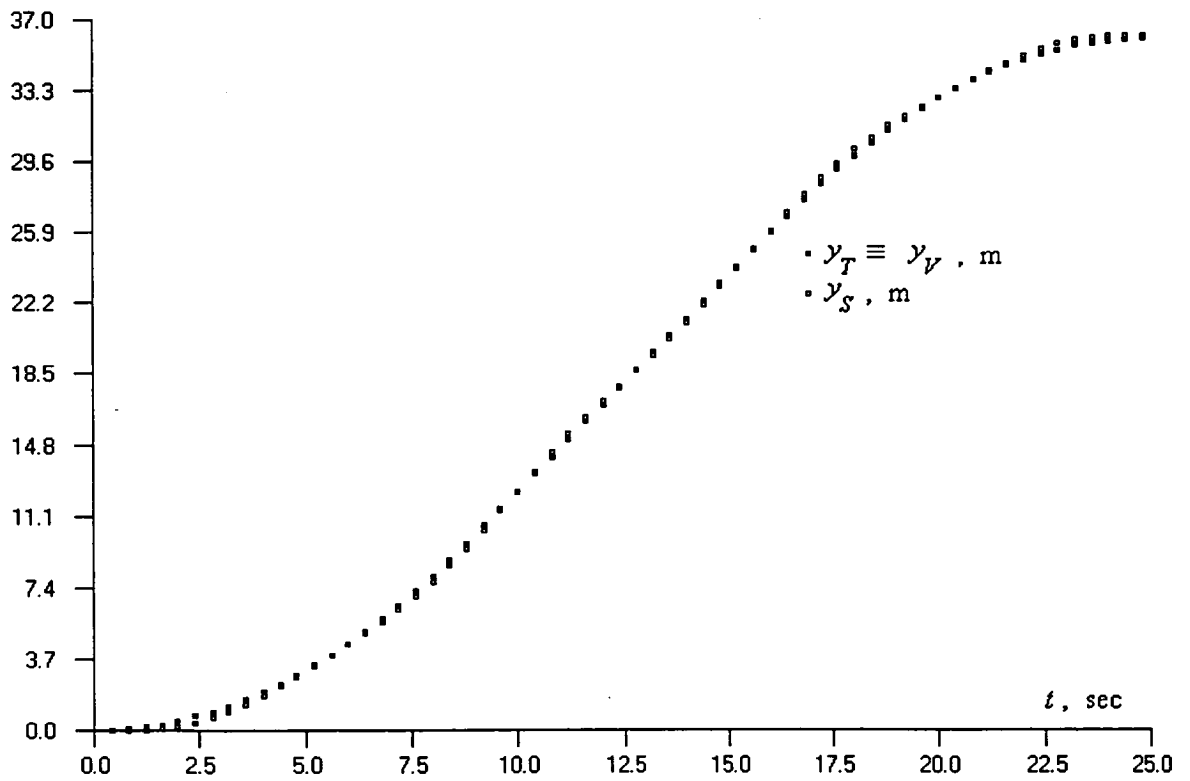
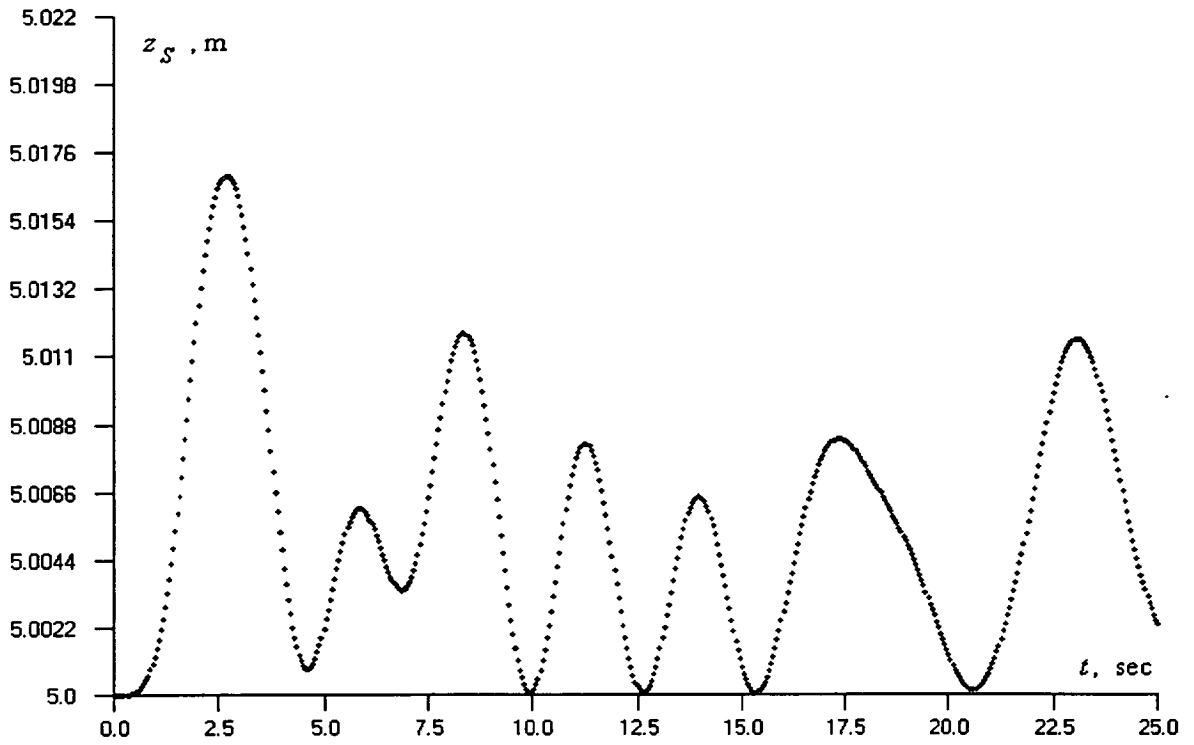


Figure 6.20

Experiment 1, MCM



Experiment 1, MCM

6.2.2.2 Experiment No 2:- trolley translation and gantry rotation

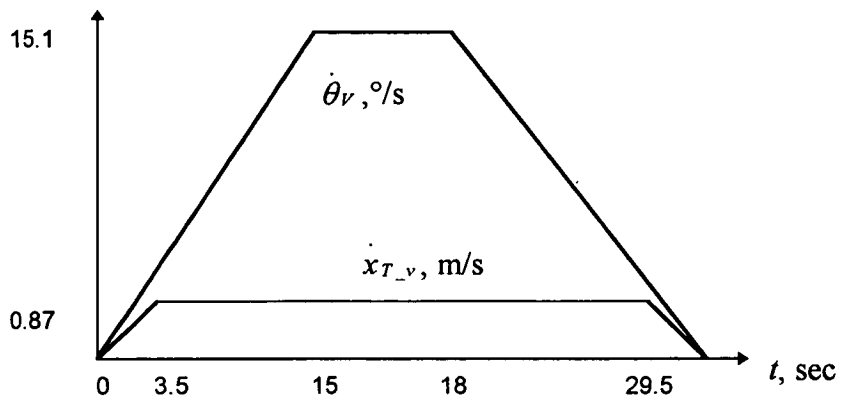


Figure 6.22

Experiment 2

the profiles of the vehicle angular velocity and trolley velocity

In this experiment the trolley accelerates fully for the first 3.5 seconds and decelerates fully for the last 3.5 seconds, Figure 6.22, with a second (constant velocity) stage of 26

seconds. The gantry rotates simultaneously about its vertical symmetry axis with angular acceleration of $\ddot{\theta}_v = 1^\circ/\text{s}^2$. Then its acceleration drops to zero when the gantry reaches the limit of its angular velocity, $\dot{\theta}_v = 15.1$ degree/s, and after another 3 seconds it decelerates. The gantry angular velocity profile is also shown on Figure 6.22.

Figures 6.23-6.25 (for the ADAMS Model) and Figures 6.26-6.28 (for the MCM) show how the Cartesian coordinates of the gantry, trolley, and the spreader depend on time. The similarity is obvious again.

In order to analyze the spreader behaviour more easily in this case, the plot of the horizontal plane is also created (Figure 6.29 for the MCM and Figure 6.30 for the ADAMS Model). It can be seen that the spreader lags behind the trolley at the first stage of small gantry angular acceleration, but then it catches the trolley up rather soon and remains farther from the gantry's centre than the trolley till the gantry rotation decelerates. At the end of the experiment the spreader is ahead of the trolley as the latter is also decelerating.

Figure 6.31 shows the angle Θ_s of spreader rotation in the Multi-Cable Model. It is oscillating with time, but it is clear that the spreader has a tendency to rotate in the direction opposite to that of the gantry angular acceleration.

There is no angle analogous to Θ_s in the ADAMS model. The angular position of each *part* in ADAMS Model is given by three Euler angles. But it is possible to calculate the Θ_s angle by using one of Euler angles, ψ (angle of rotation about a local Z axis), for both the trolley and the spreader, and assuming that the tilt of the spreader is very small then the following is correct:

$$\Theta_s \approx \psi_s - \psi_T \quad (6.19)$$

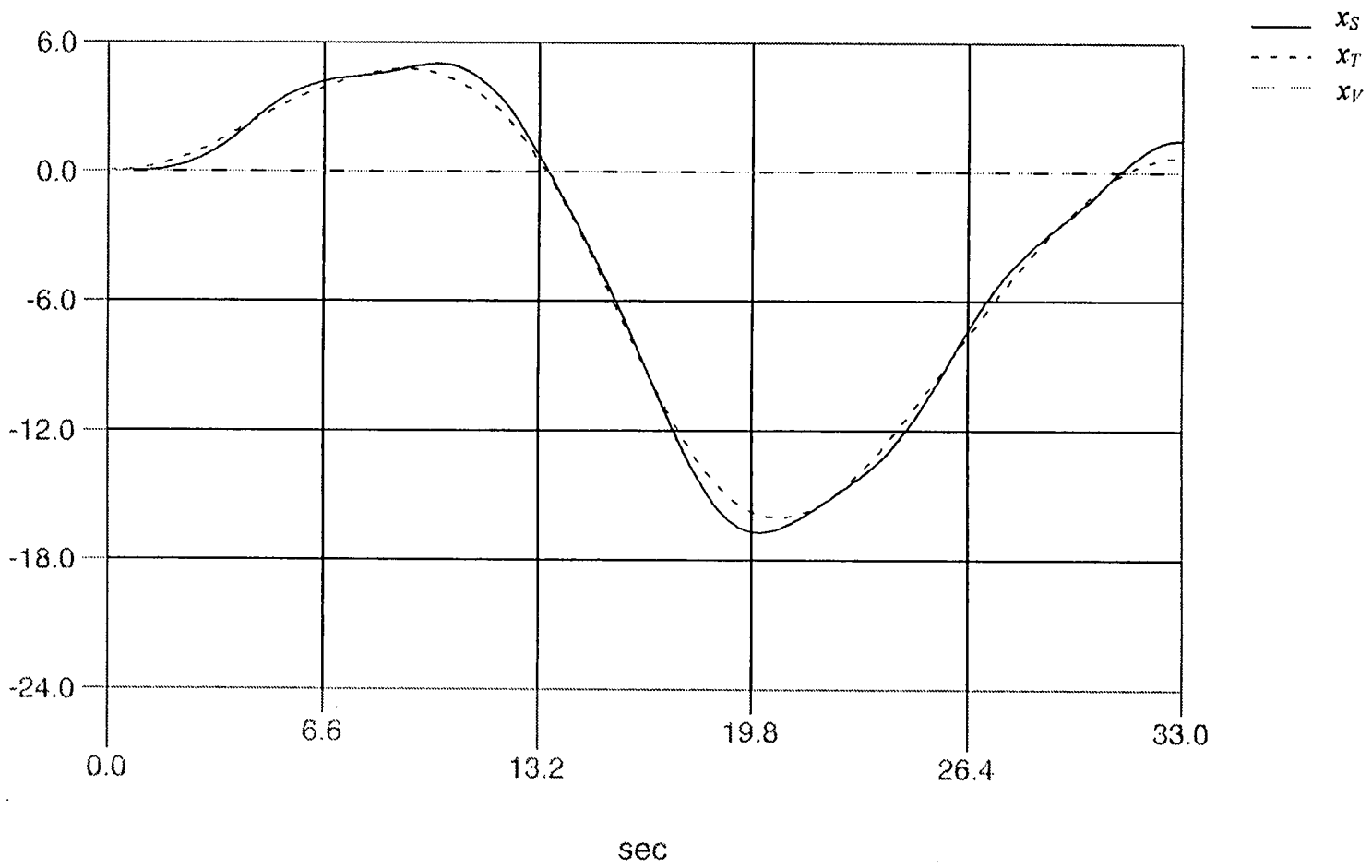


Figure 6.23
Experiment 2, ADAMS model

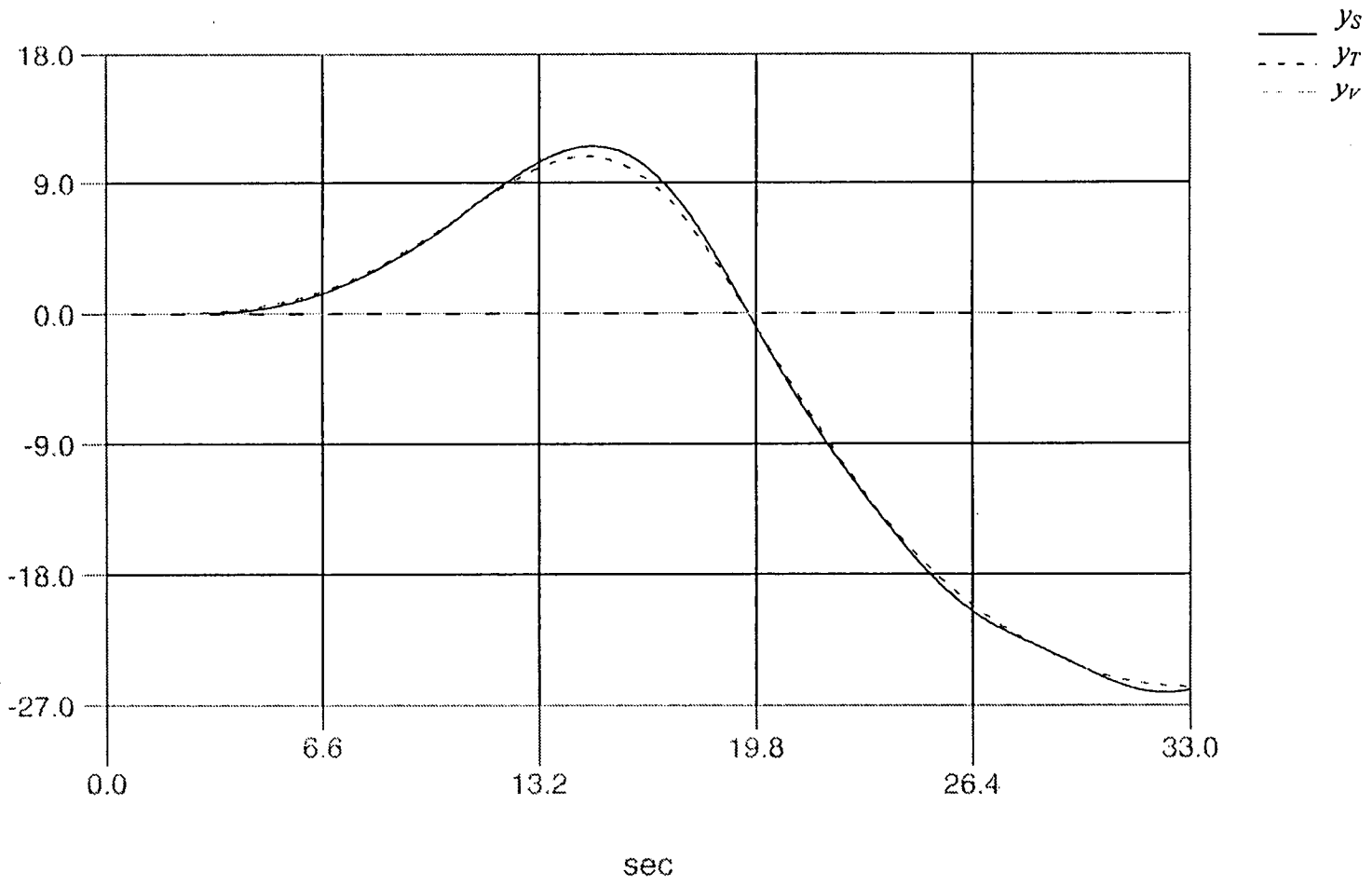


Figure 6.24

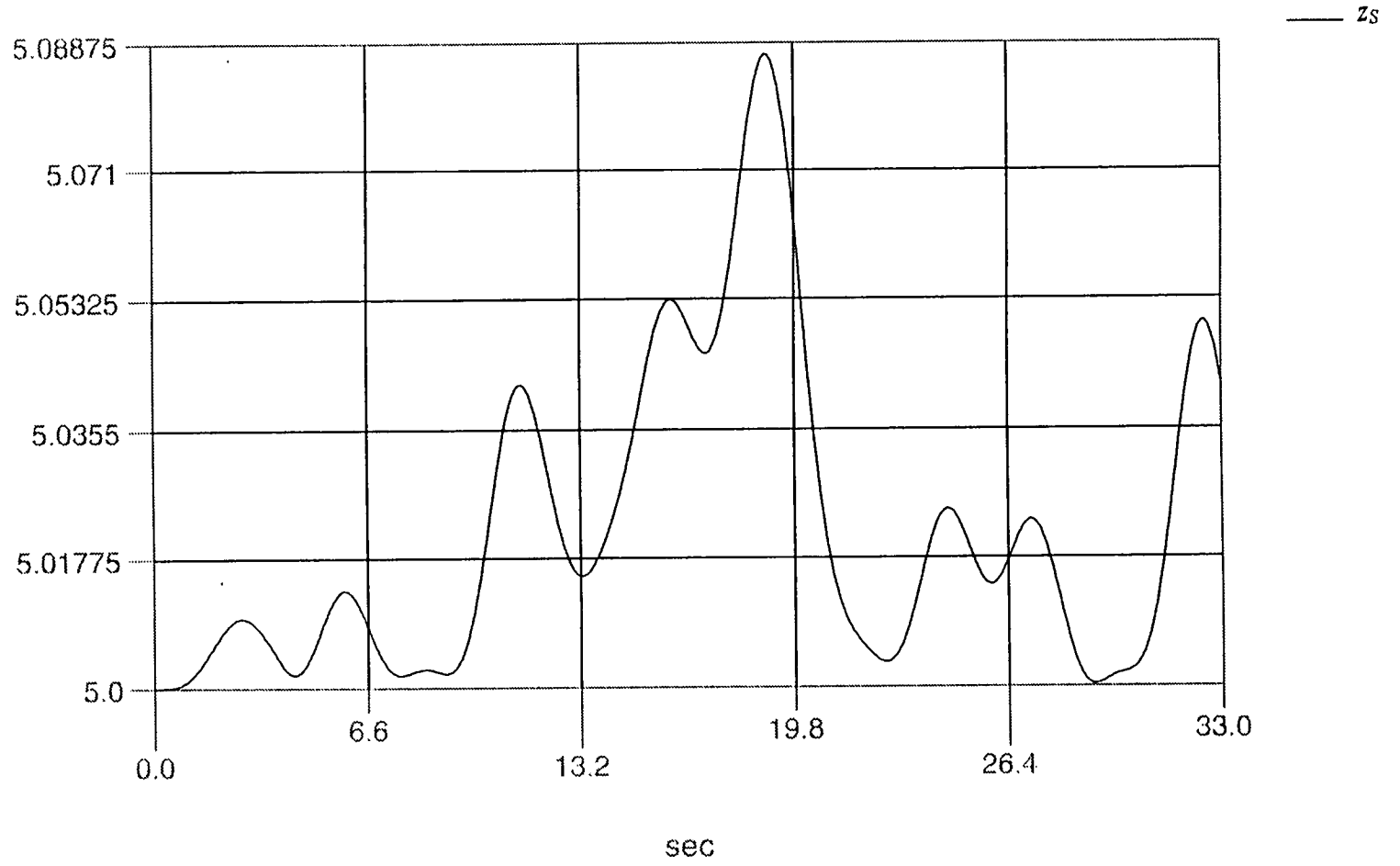
jetem

Experiment 2, ADAMS model

meter

Experiment 2, ADAMS model

Figure 6.25



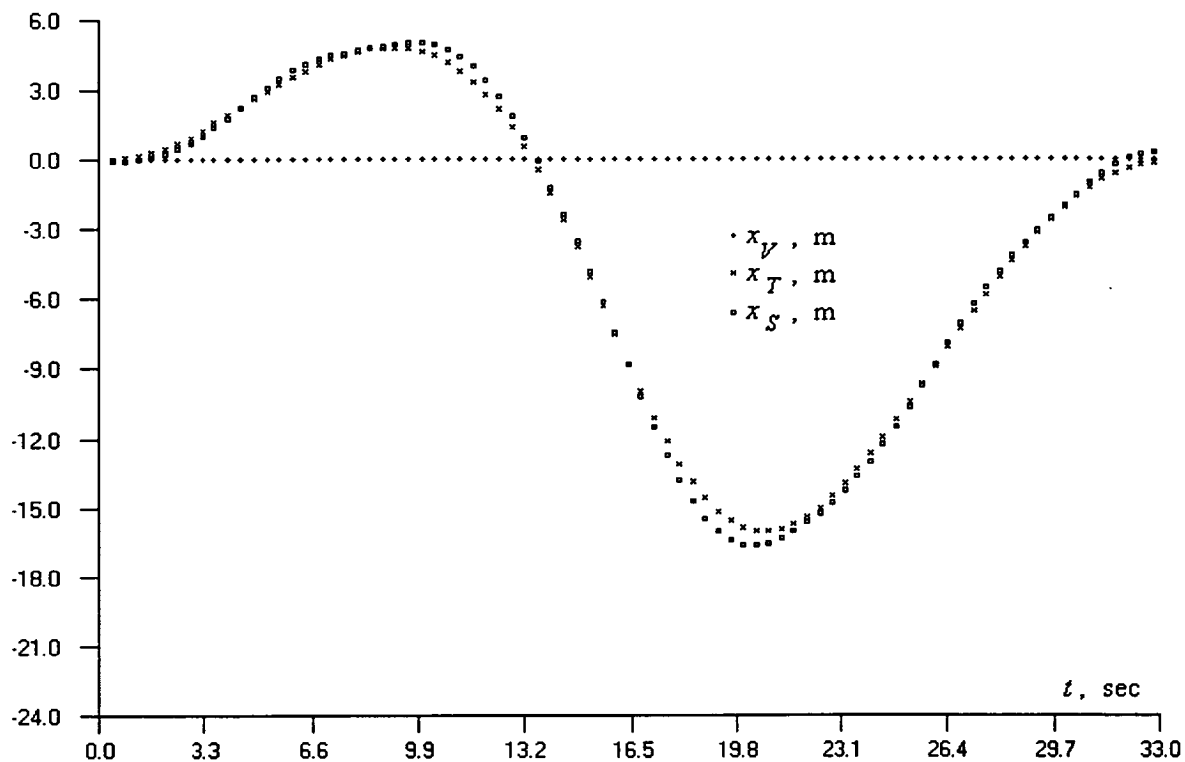


Figure 6.26

Experiment 2, MCM

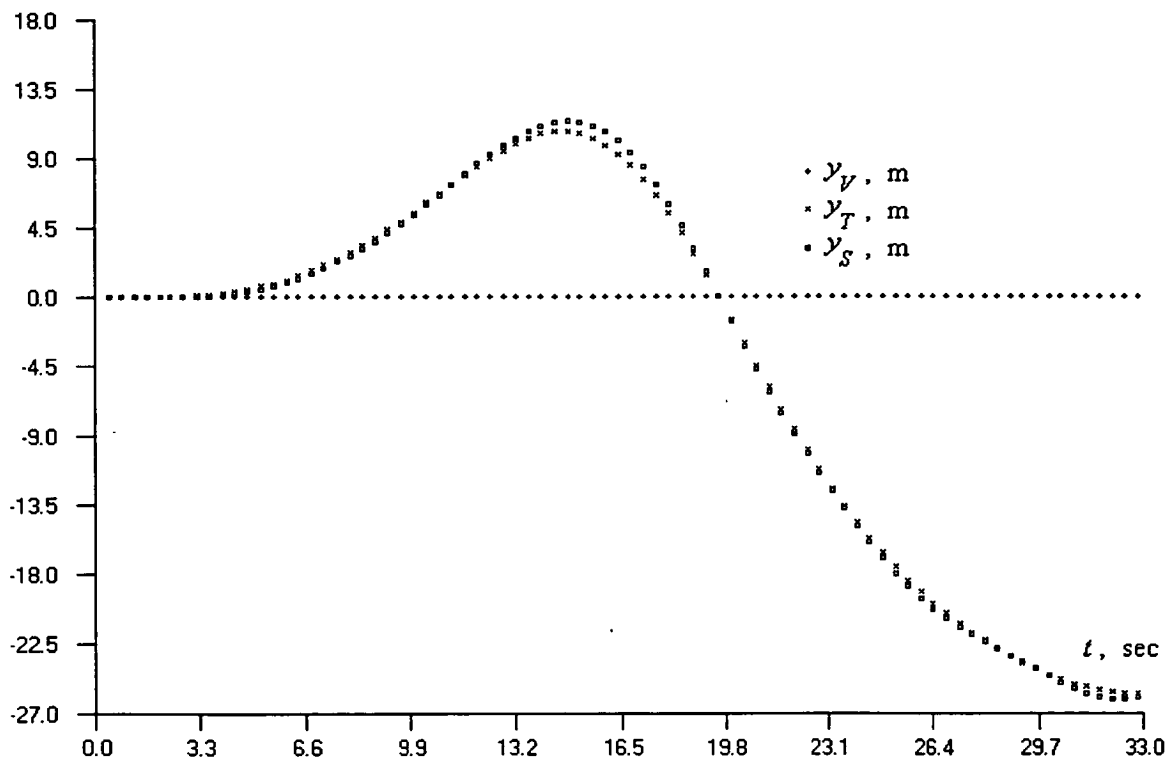
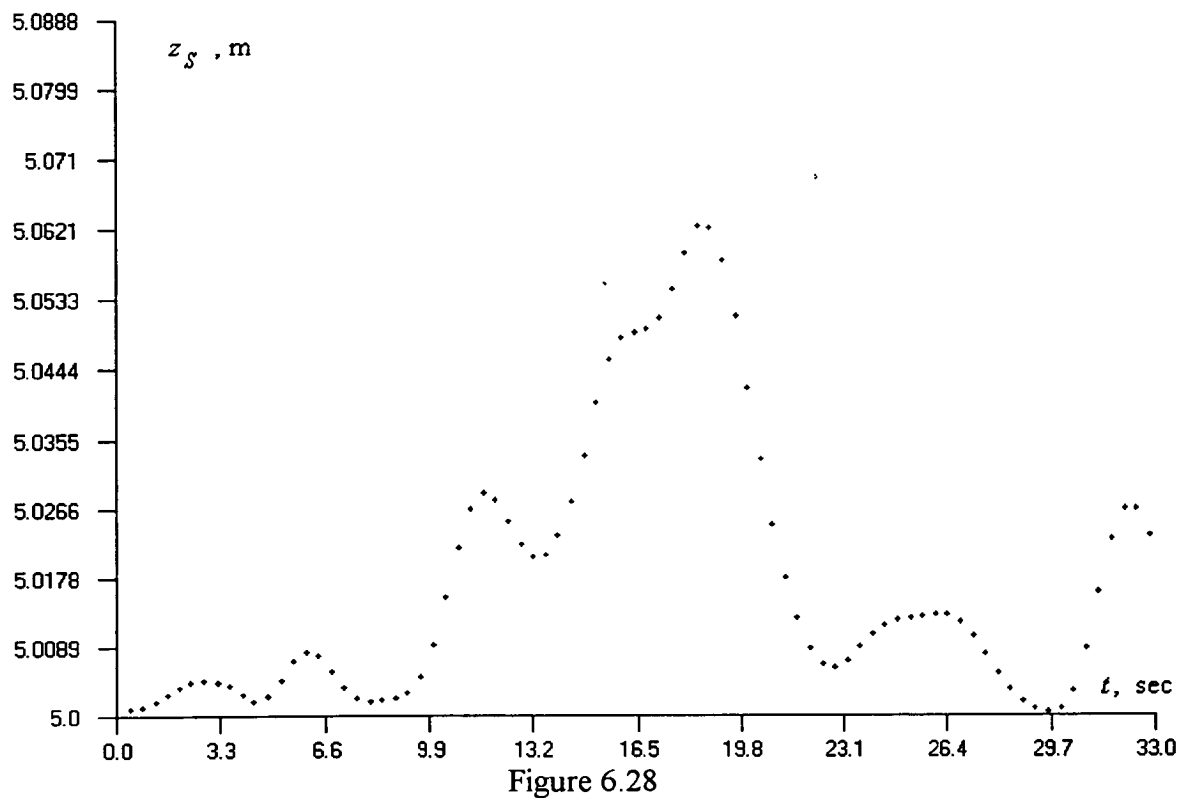
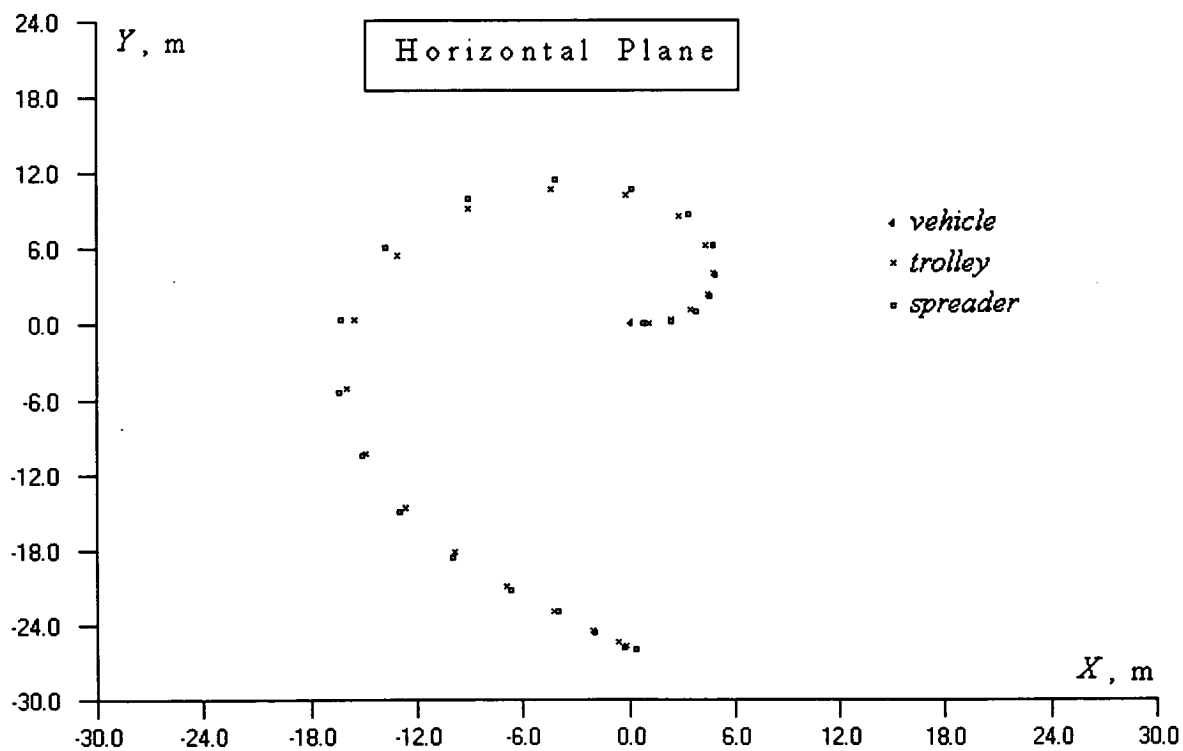


Figure 6.27

Experiment 2, MCM



Experiment 2, MCM



Experiment 2, MCM

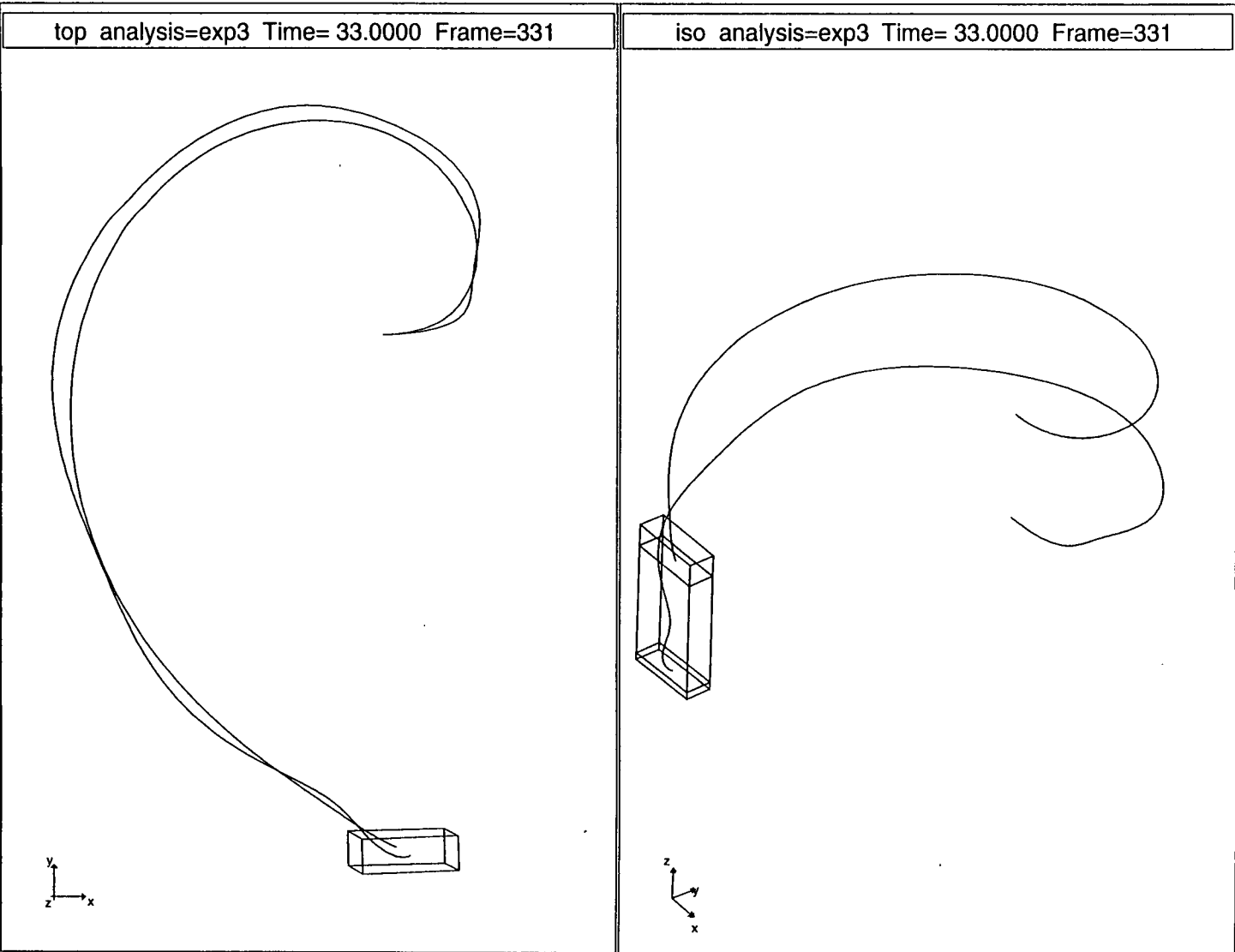


Figure 6.30

Experiment 2. ADAMS model

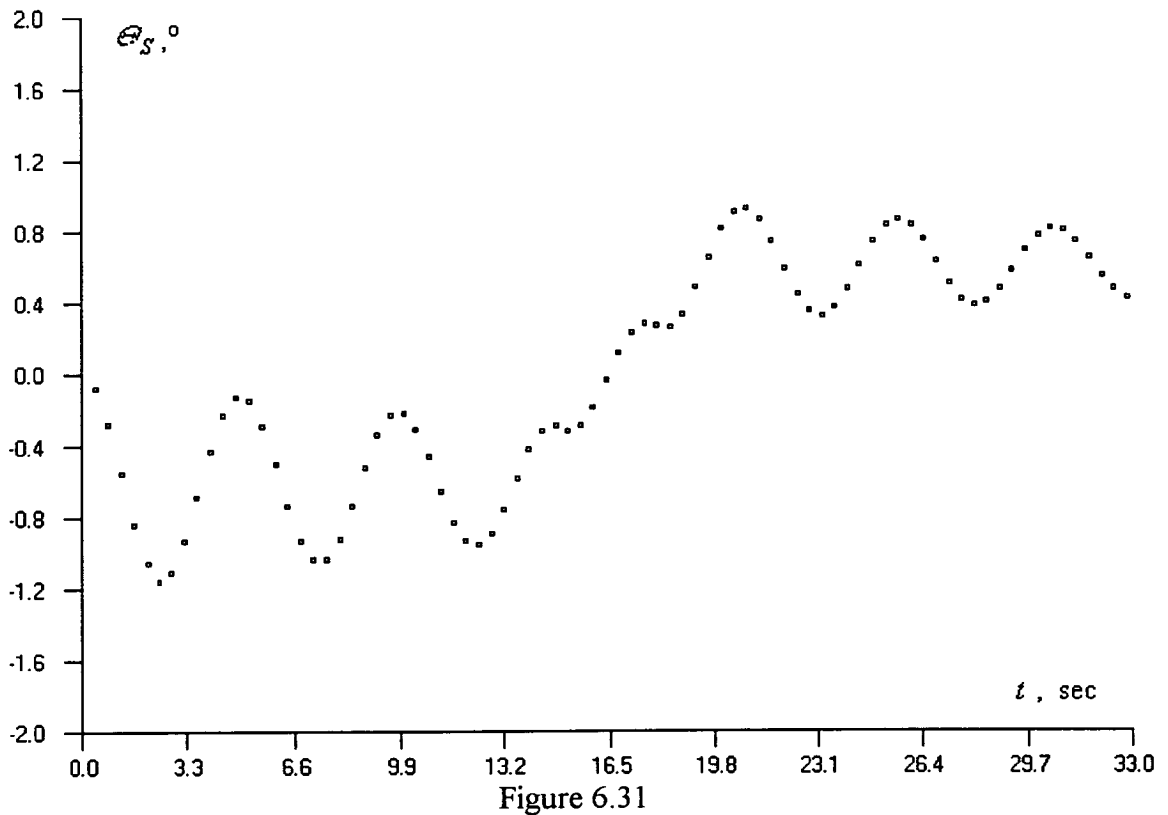


Figure 6.31
Experiment 2, MCM

Figure 6.32 shows the ψ_T and ψ_S angles in the ADAMS model. Most of the time ψ_S is very close to ψ_T , but when the spreader tilts ψ_S changes dramatically. It can be explained the following way. When the spreader tilts two other Euler angles are not equal to zero any more, and as all three Euler angles can be only positive in ADAMS the ψ_S angle takes the value between 0° and 360° which describes correctly the angular spreader position together with two other angles. So, the large deviations of ψ_S on the Figure 6.32 are not directly informative.

In order to get ADAMS data for θ_s equivalent to the MCM, the following calculations have been made. The values of ψ_S were taken equal to those given by ADAMS if the spreader is not tilted, otherwise a linear interpolation was made for ψ_S . Then the formula for θ_s (6.19) was used, and the Figure 6.33 shows the resulting ADAMS angle of the spreader rotation. The similarity in spreader rotation for both models can be clearly seen by comparing the Figures 6.31 and 6.33.

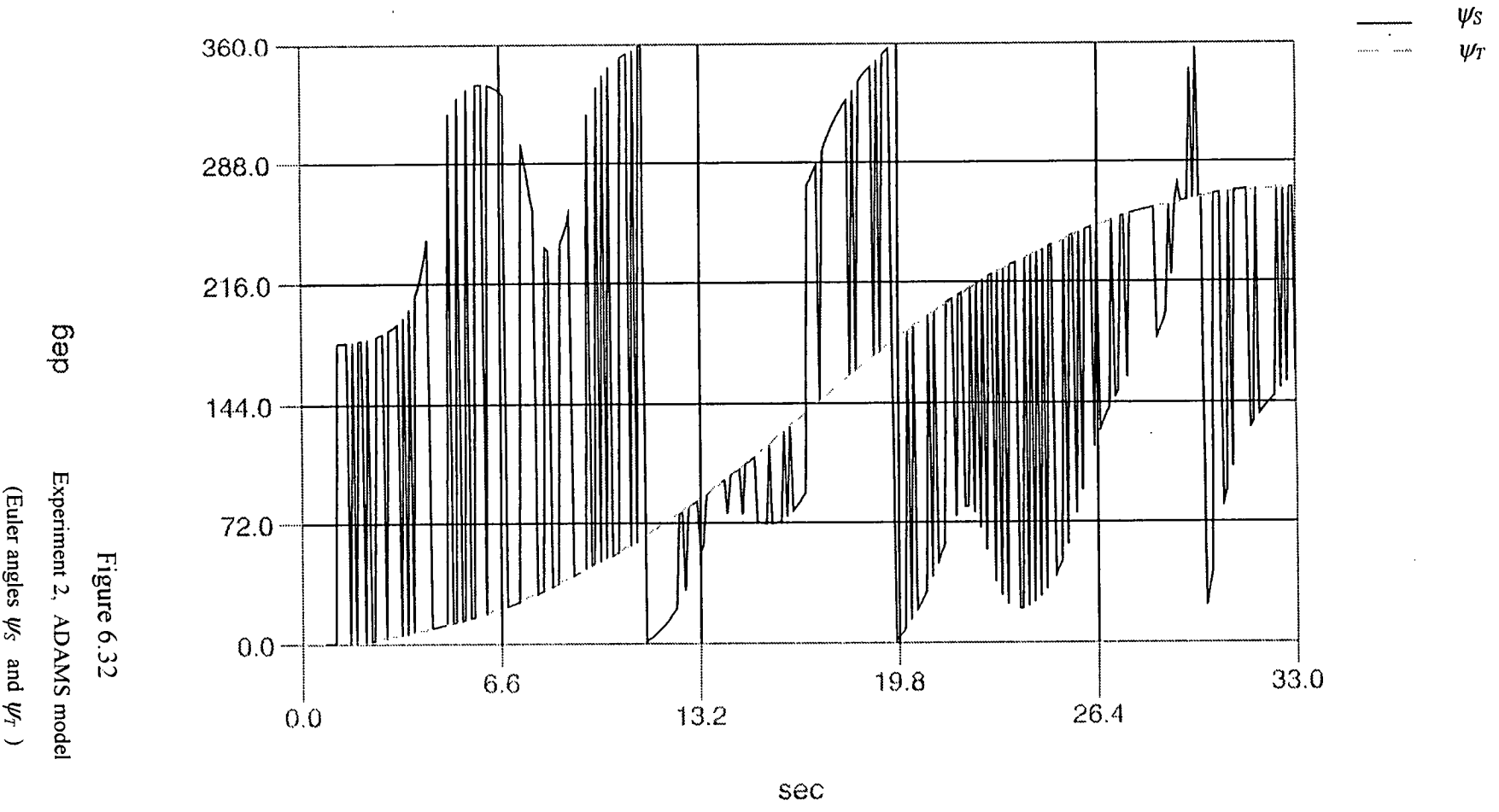


Figure 6.32
 Experiment 2, ADAMS model
 (Euler angles ψ_S and ψ_T)

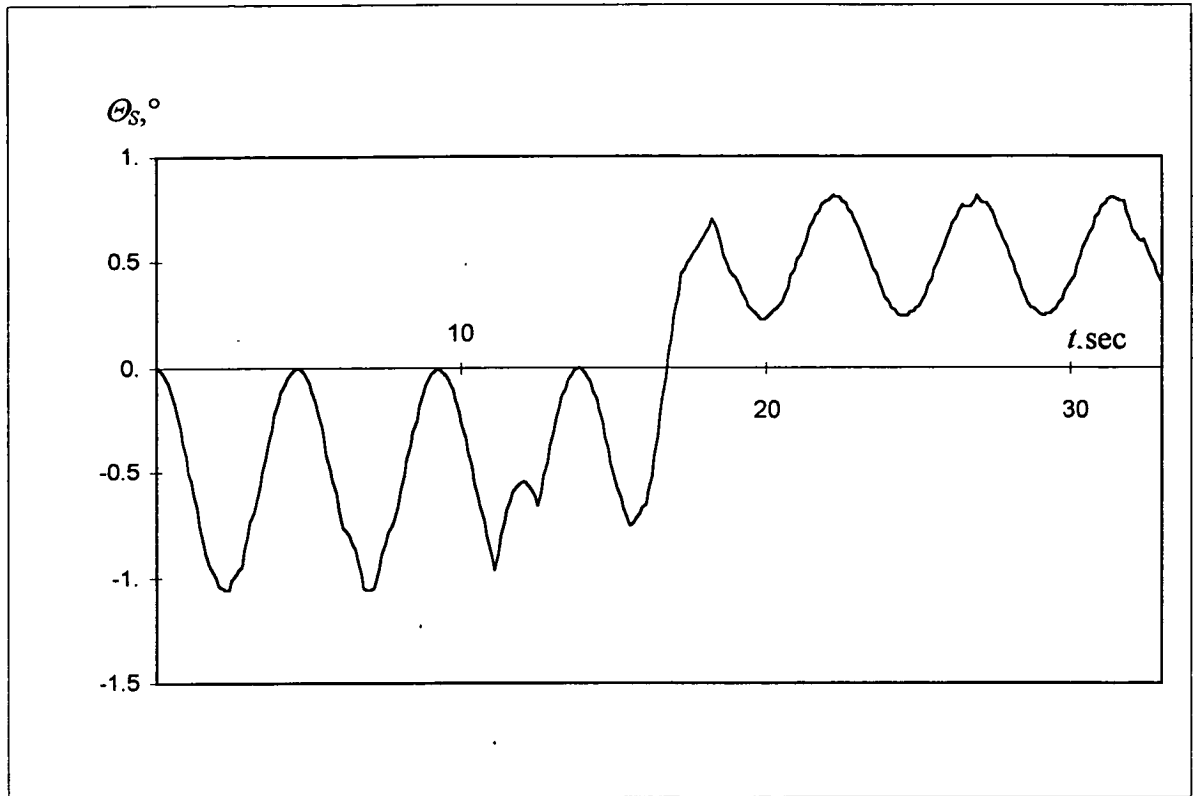


Figure 6.33

Experiment 2

θ_s calculated using ADAMS data

6.3 Performance test conducted on a Paceco Transtainer RTG crane at Felixstowe port

6.3.1 Description of the experiment and experimental data

The experiment described here was made on the Paceco Transtainer RTG crane. The trolley is placed initially somewhere near the gantry side beam (with the unladen spreader), then it moves to another side of the top beam, accelerating fully first, then with constant velocity, and at the end, it decelerates and stops. At the same time, the spreader is lowered.

The experimental data is given in the Table 6.1.

t, s	x_{T_v}, m	x_S, m	z_S, m	l, m
0.00	0.0000	-0.0319	9.8054	3.8947
0.16	0.0000	0.0201	9.8474	3.8527
0.52	0.0000	0.0015	9.9732	3.7268
0.80	0.0000	-0.0338	10.0152	3.6850
1.12	0.0000	-0.1564	10.0571	3.6463
1.44	0.0000	-0.2437	10.0571	3.6510
1.88	0.0000	-0.4100	10.0991	3.6242
2.20	0.0000	-0.4453	10.1410	3.5867
2.68	0.0000	-0.4017	10.1410	3.5816
3.20	0.0000	-0.3144	10.1410	3.5729
3.68	0.0000	-0.1398	10.1410	3.5617
4.20	0.0873	-0.0608	10.0991	3.6039
4.68	0.2619	0.0223	10.0781	3.6298
5.20	0.5675	0.1491	10.0571	3.6668
5.68	0.9603	0.3674	10.0571	3.6908
6.20	1.3532	0.8475	10.0571	3.6778
6.68	1.7897	1.4586	10.0571	3.6579
7.20	2.2698	2.1756	9.9313	3.7699
7.68	2.7500	2.8490	9.8054	3.8959
8.20	3.3611	3.5141	9.6376	4.0653
8.68	3.8849	4.0129	9.5118	4.1902
9.20	4.4523	4.4680	9.3859	4.3141
9.68	4.9761	4.9148	9.2181	4.4823
10.20	5.4999	5.3180	9.0503	4.6533
10.68	5.9801	5.7648	8.8825	4.8223
11.20	6.4602	6.3072	8.7567	4.9457
11.68	6.9840	6.8933	8.6308	5.0700
12.20	7.5078	7.5667	8.5050	5.1953
12.68	7.9443	8.1361	8.2952	5.4082
13.20	8.5554	8.7221	8.1694	5.5331
13.68	8.9919	9.2043	7.9596	5.7443
14.20	9.4721	9.6075	7.7918	5.9098
14.68	9.9959	10.0709	7.7079	5.9926
15.20	10.3451	10.5177	7.5401	6.1623
15.68	10.5633	10.8419	7.4143	6.2919
16.20	10.8689	11.1931	7.2045	6.5036
16.68	11.0435	11.4736	7.0787	6.6353
17.20	11.2181	11.6585	6.9109	6.8034
17.68	11.3927	11.7201	6.7850	6.9227
18.20	11.5236	11.7311	6.6172	7.0858
18.68	11.5673	11.6541	6.4494	7.2511
19.20	11.6109	11.6124	6.2396	7.4604
19.68	11.6109	11.5958	6.1558	7.5443
20.20	11.6109	11.5541	5.9460	7.7542
20.68	11.6109	11.5207	5.7782	7.9223

Table 6.1
The experimental data with Paceco Transtainer RTG Crane

The data x_{T_v} , x_S and z_S was measured using the computer system called 'Target' ('High Resolution Video Digitising Software' developed by Sport Science Department of Loughborough University) in the following way. There were bright marks placed as necessary on crane parts when making the experiment, and it was recorded on the video. While playing the video, 'Target' allows the experimenter to place the cursor on the marks on the video screen and stores all data on a computer. Then, by applying the appropriate scaling factors, the real distances between marked points are calculated.

The cable length also given in Table 6.1 is calculated using:

$$l = \sqrt{(x_T - x_S)^2 + (y_T - y_S)^2 + (z_T - z_S)^2} \quad (6.20)$$

All the data in Table 6.1 is given in the global system of coordinates as described in Chapter 2., with the Z axis coming through the centre mass of trolley in its initial position, and it is chosen that:

$$\Theta_V = 0 \quad (6.21)$$

meaning that the global X axis coincides with the gantry top beam. The experiment is two dimensional, in the vertical plane (global XZ plane, see Figures 2.1, 5.1, 6.1), as only accelerations $\ddot{x}_{T_v}$ and $\ddot{l}$ are applied. Thus, for this experiment the global y -coordinates are:

$$y_V = y_T = y_S \equiv 0. \quad (6.22)$$

The vertical distance between the ground and the underside of the trolley for the Paceco RTG crane is 13.7m, therefore z -coordinates for the vehicle and the trolley in the MCM are taken to be:

$$z_V = z_T = 13.7 \quad (6.23)$$

The spreader's initial z -coordinate is:

$$z_S = 9.8 \quad (6.24)$$

It can be seen from the experimental data that some oscillations of the spreader took place at the beginning leading to changes in both x_S and z_S , thus the experiment really started not at $t = 0$, but later, at about $t = 4\text{sec}$, when the driver started applying the trolley acceleration.

However, the MCM starts working at $t = 8.2\text{sec}$ when cable length just reaches 4m ($l(8.2\text{s}) = 4.06\text{m}$). This is a necessary act due to existing restriction of the model given by statement (6.12) and discussed in section 5.2.2.

6.3.2 MCM inputs:- accelerations $\ddot{x}_{T_v}$ and $\ddot{l}$

In order to run the MCM, it is necessary to work out the accelerations $\ddot{x}_{T_v}$ and $\ddot{l}$ which the driver applies. They were not measured during the experiment, but they can be approximately calculated using the data available.

6.3.2.1 Trolley acceleration, $\ddot{x}_{T_v}$

Figure 6.34 shows the trolley velocity profile derived from numerical differentiation of the experimental x_{T_v} data. The profile has strong noise which is due to inaccuracy of the measurements. The noise is neglected when the $\ddot{x}_{T_v}$ input for MCM is being derived.

Figure 6.35 shows the $\ddot{x}_{T_v}$ profile as it is used in the MCM. The $\ddot{x}_{T_v}$ profile has been chosen so that, after numerical integration in the MCM, the $\dot{x}_{T_v}$ profile is similar to that of the experimental data ($\dot{x}_{T_v}$ in Figures 6.34 and 6.35).

Figure 6.36 can be used to compare x_T profiles. The *line* in Figure 6.36 corresponds to x_T in MCM (after numerical integration of $\dot{x}_{T_v}$). The experimental x_T data is shown in Figure 6.37 by *crosses*. This is due to the fact that experimental data is given with a time-step about 0.5sec while the integrating in MCM has been performed with the step of 0.1sec⁶⁾. The Figure shows that actual values of experimental x_T data are very close to the values of x_T data calculated in the MCM.

6.3.2.2 The spreader lowering acceleration, $\ddot{l}$

Using the data for cable length l , the profile for $\ddot{l}$ has been built using exactly the same procedure as for $\ddot{x}_{T_v}$ in previous section. Figures 6.37 and 6.38 show the $\dot{l}$ profile for the experimental data (calculated using numerical differentiating), and $\dot{l}$, $\ddot{l}$ used in the MCM. Figure 6.39 allows to compare the cable length calculated by the MCM (by integration the $\ddot{l}$ from Figure 6.38 twice) with its actual values.

6.3.3 The results of the experiment with the MCM

$\ddot{x}_{T_v}$ and $\ddot{l}$ as in Figures 6.35 and 6.38 were applied to the MCM. Figures 6.40 and 6.41 show the horizontal (x_S) and vertical (z_S) position of the spreader both calculated by the MCM and measured in the experiment with a real RTG crane. The comparison shows that results of the MCM correlate well with the experimental data. Further experimental work in order to validate the model is planned for future research.

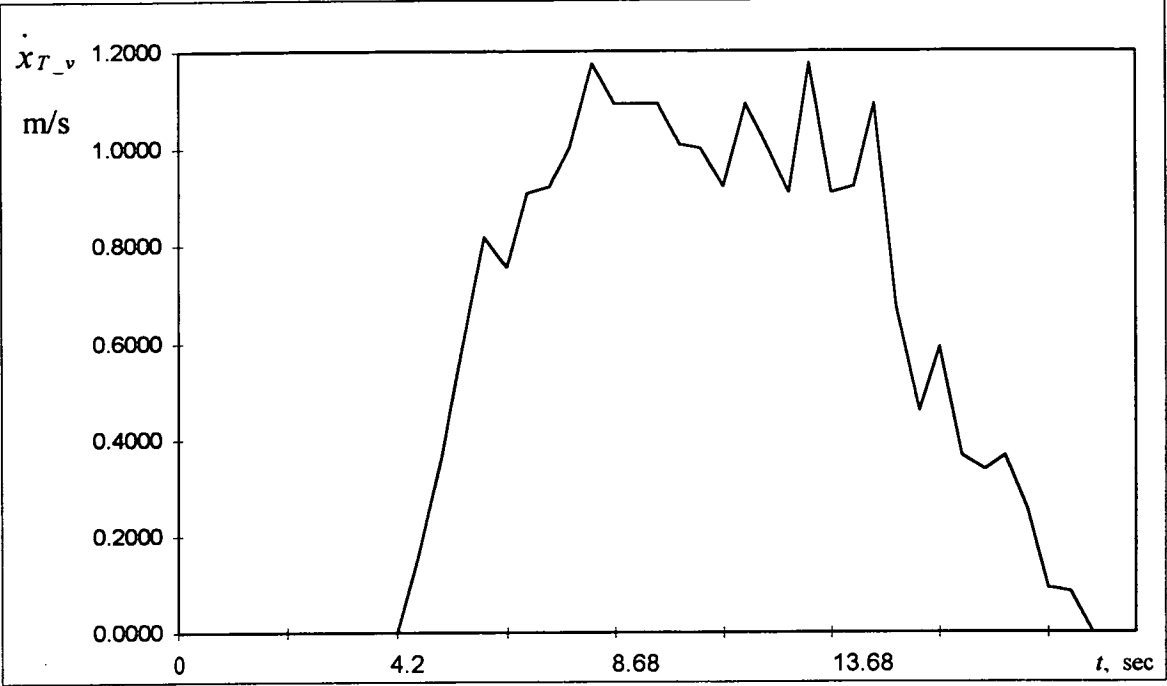


Figure 6.34

$\dot{x}_{T_v}$ profile calculated from experimental data

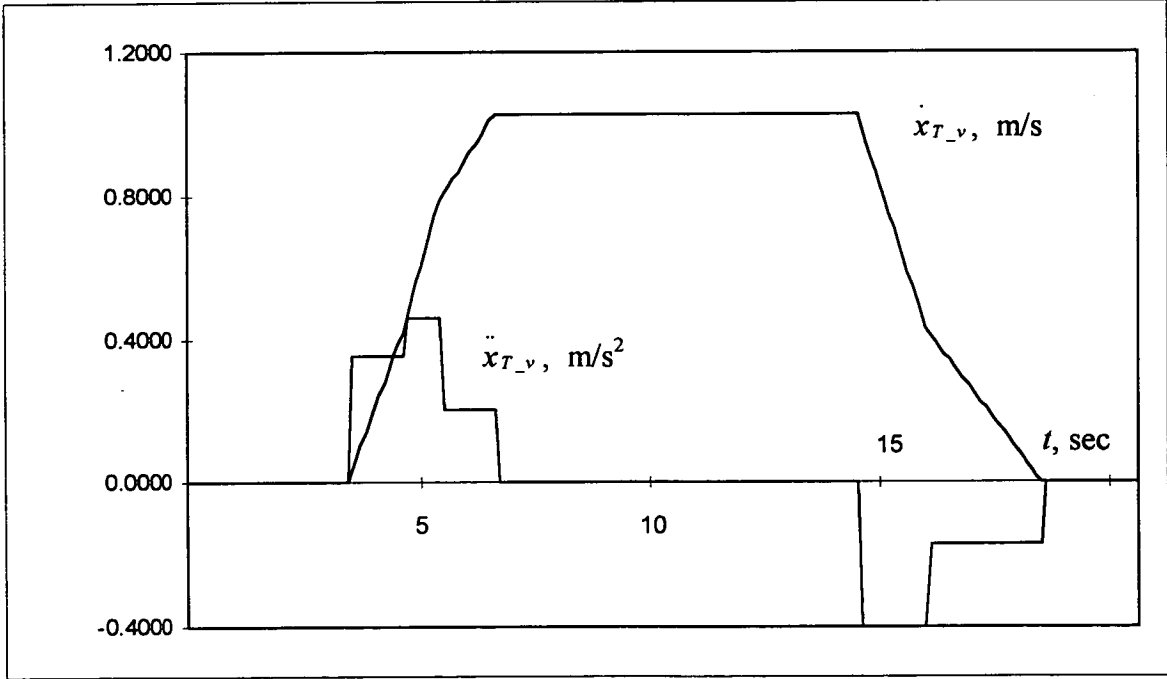


Figure 6.35

$\dot{x}_{T_v}$ and $\ddot{x}_{T_v}$ profiles as used in MCM

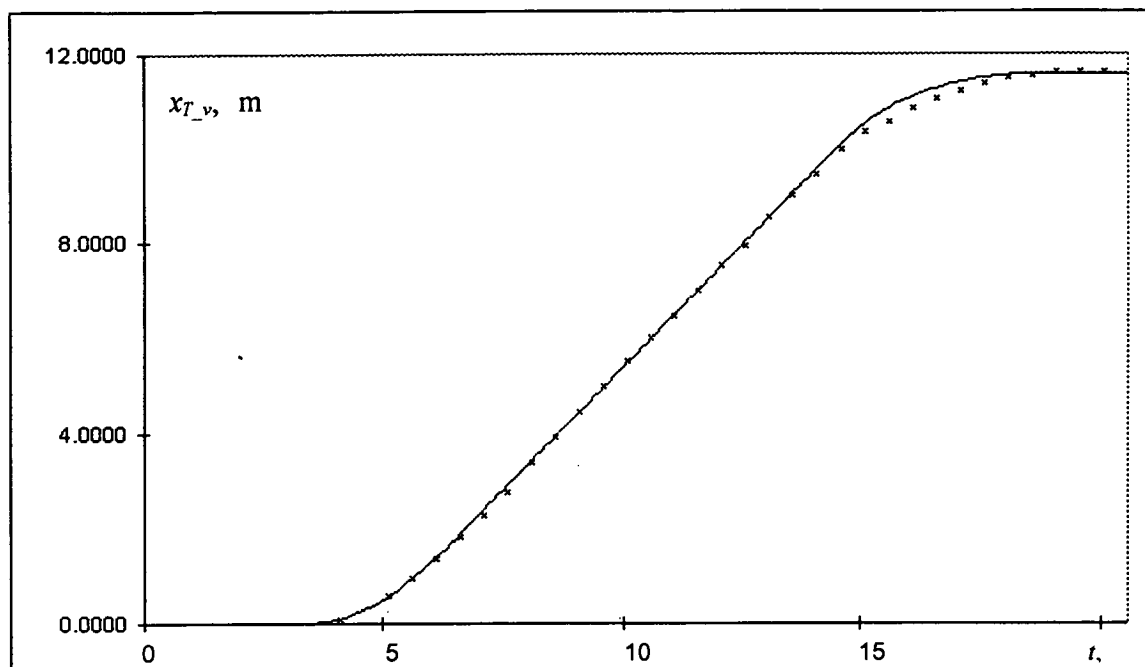


Figure 6.36

x_T coordinate changing with time:
experimental data (x) and MCM (—)

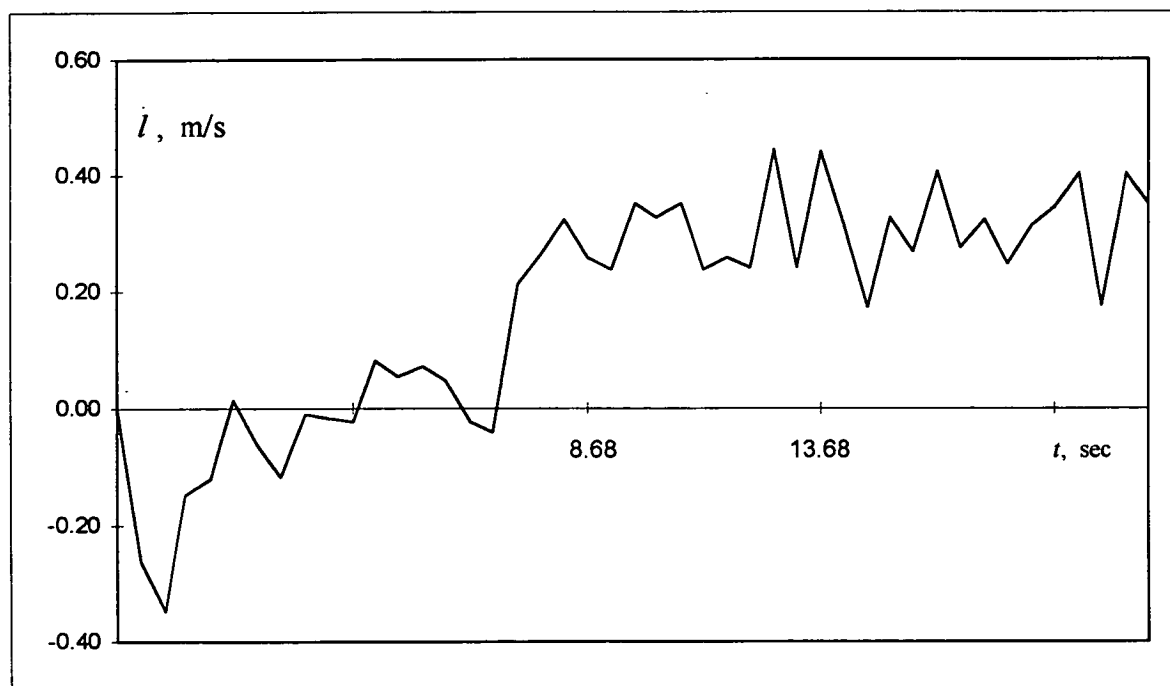


Figure 6.37

l profile calculated from experimental data

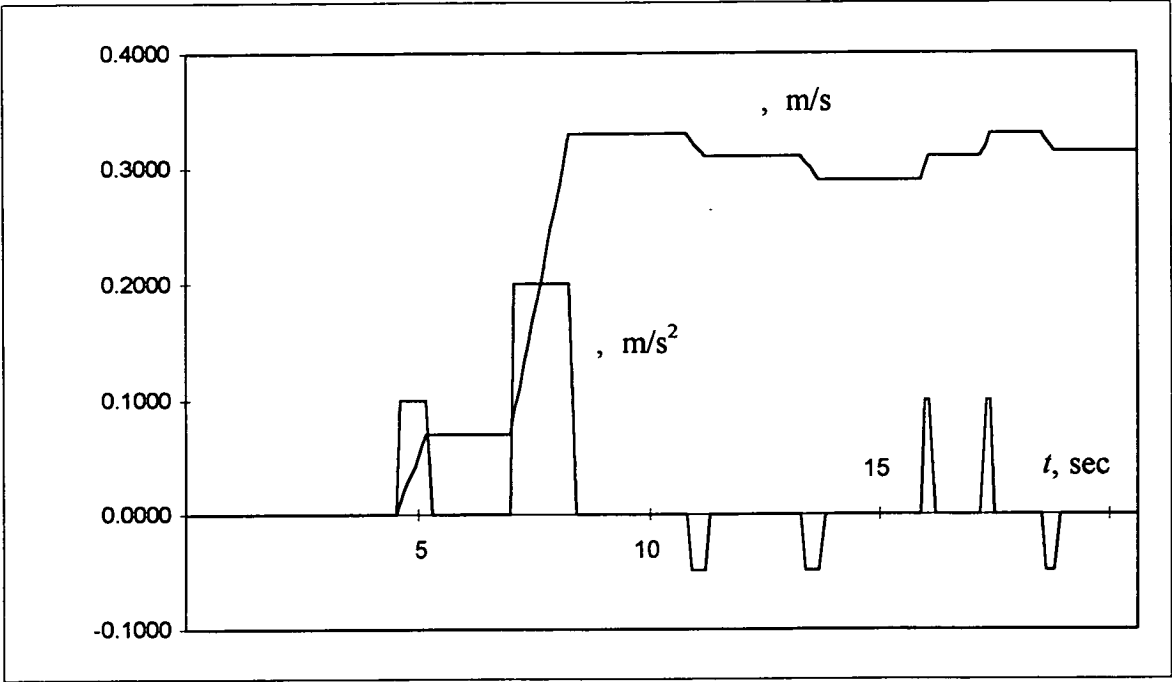


Figure 6.38
 $\dot{l}$ and $\ddot{l}$ profiles as used in MCM

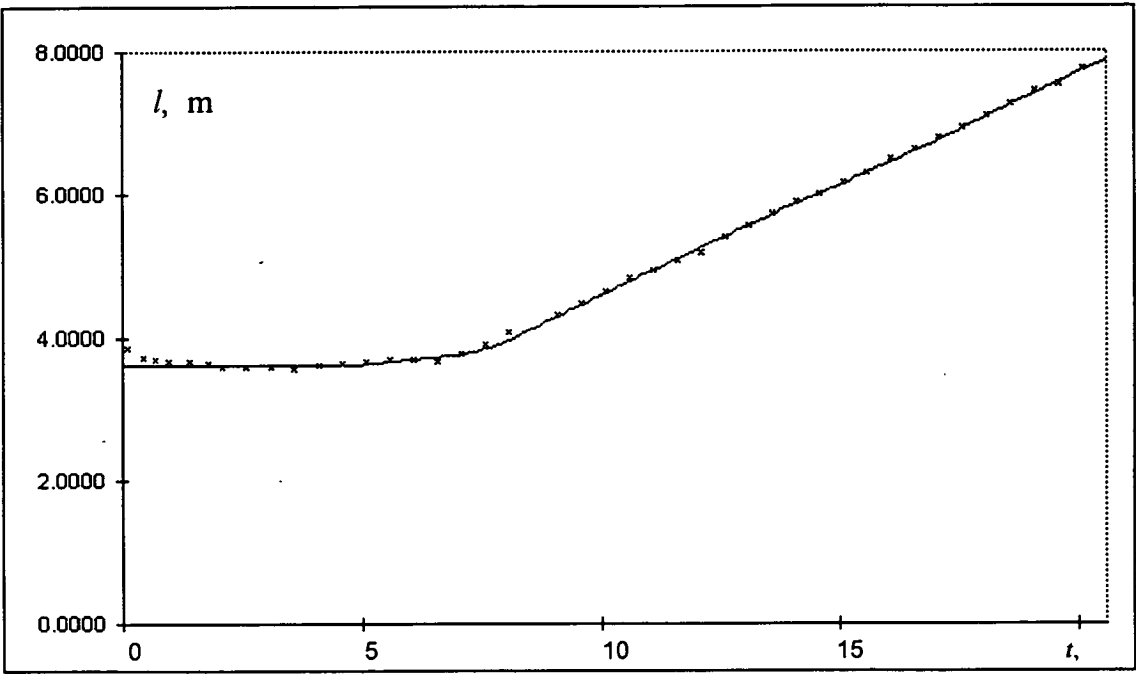


Figure 6.39
 l changing with time:
 experimental data (x) and MCM (—)

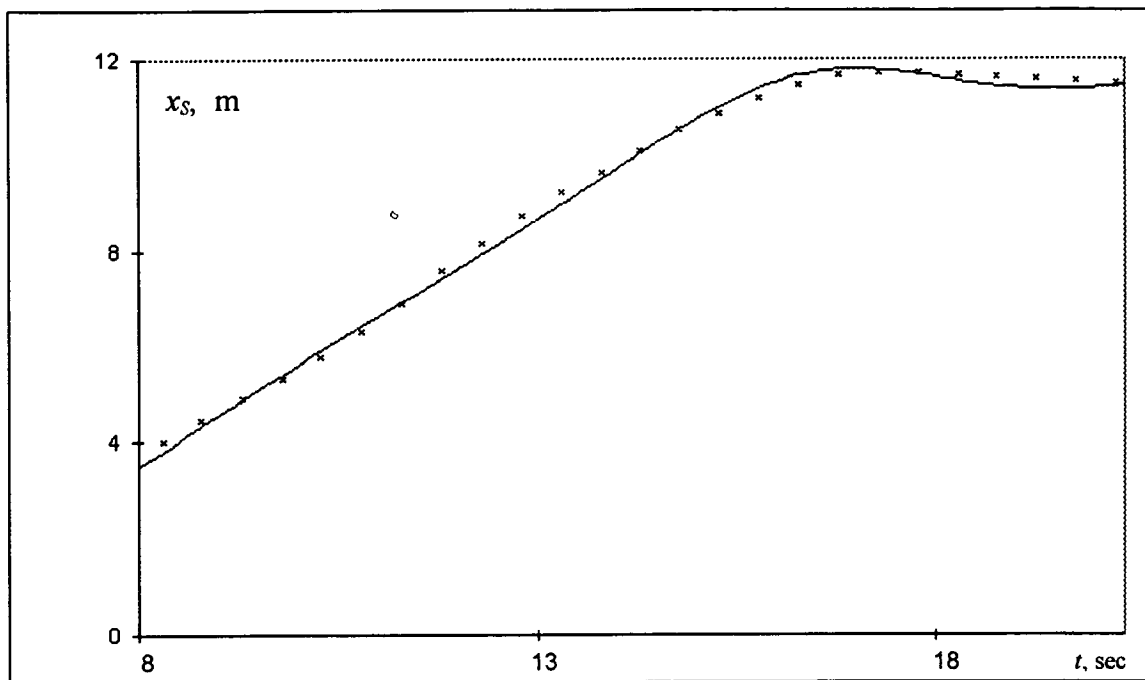


Figure 6.40

x_S coordinate changing with time:
experimental data (x) and MCM (—)

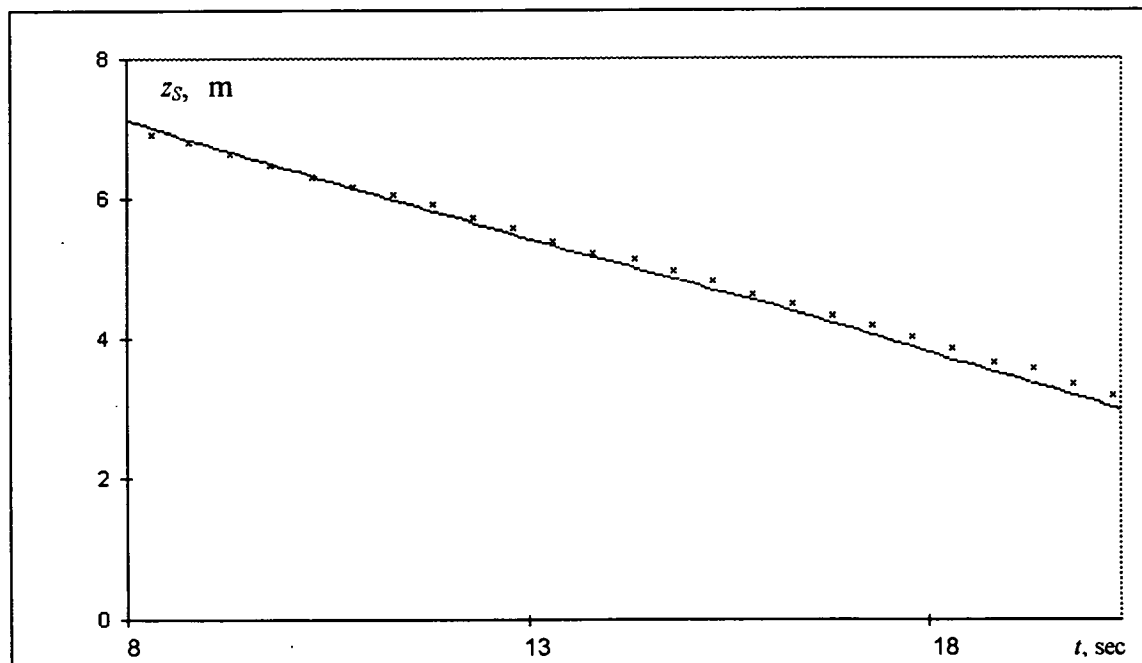


Figure 6.41

z_S coordinate changing with time:
experimental data (x) and MCM (—)

6.4 Conclusions

Six runs with the MCM have been made with different combinations of acceleration inputs, $\ddot{y}_v$, $\ddot{\theta}_v$, $\ddot{x}_{T_v}$, with and without lowering of the spreader. The integration step has been kept small, at $\Delta t = 0.01s$, to guarantee the stability of the model.

The input accelerations have been chosen such that the corresponding velocity profiles are of trapezoidal shape to imitate the common RTG crane operations. There have been also some tests undertaken which involve complicated simultaneous trolley and vehicle movements but it is recognised that these are only of theoretical interest.

The response of the spreader has been examined for each simulation, and it corresponds with intuitive understanding of the RTG crane dynamics: a lag, a catch up, and a lead phase of the spreader are evident in all simulations, depending specifically on the changes in velocities of the vehicle and the trolley.

The spreader moves in an oscillatory manner in all the simulations that have been examined. The magnitude of these oscillations, r , is higher at the beginning of each experiment due to the presence of the initial displacement ($r(0) = 0.1$, this is needed to satisfy the MCM computational requirements), then it decays slowly with time due to the passive damping operative in the system.

In order to verify the MCM, an ADAMS Model of the RTG crane has been devised, and two pairs of simulations were carried out with both models. They reveal a real similarity between the models. The data for an experiment with a Paceco Transtainer RTG crane at Felixstowe port has been also presented in this chapter. An analogous test has been conducted using the MCM and results have been discussed.

The MCM presented in this thesis has been derived to provide a facility for testing a Fuzzy control system for the RTG crane. In the next chapter a few ideas for the Fuzzy

controller for basic two dimensional operation are discussed and outlines for future research in developing a Fuzzy controller for more complicated RTG tasks are presented.

Notes to Chapter 6

- 1) As before, all the data used here are taken from typical RTG cranes.
- 2) The inertial quantity was calculated for a fully retracted telescopic spreader.
- 3) The dissipation coefficients have been obtained from experimental data based on transient tests undertaken on a Paceco Transtainer RTG crane.
- 4) The restriction to the cable length $l \geq 4\text{m}$ is to provide the necessary conditions for convergence of the MCM discussed in section 5.2.2. The upper boundary $l \leq 10\text{m}$ is determined by RTG cranes dimensions.
- 5) Only three cables are used for the following reason: the cables in the ADAMS model are represented as rigid bodies of fixed lengths, and when the spreader is translated from its equilibrium position under the trolley and simultaneously rotated, the ADAMS model *locks up*. Note here that the Multi-Cable Model does not take into consideration the effect of geometrical extension, either.
- 6) The smaller step is used here for convenience. It is satisfactory for this simplified RTG operation.

Chapter 7

A Self-Organised Time-Optimal Fuzzy Controller for Two Dimensional Control of the RTG Crane

Chapter 1 includes a brief review of international research on RTG cranes control and highlights the advantages of using a fuzzy rather than classical controller to this particular problem. This chapter presents a *time-optimal* fuzzy controller for spreader positioning for the automation a common two-dimensional operation for the typical RTG crane (as described in the first section of this chapter). A number of experiments have been carried out to verify the performance of the controller on the Multi-Cable Model, and they are discussed in the specific conclusions to this chapter.

The concept of time-optimal control as presented here is very new, and as this appears to be a first attempt to implement it in an actual fuzzy controller, there is obviously still plenty of room for improvement.

There is little (if any) doubt, though, about the general suitability of the fuzzy approach to the RTG problem or the perspectives of the time-optimal fuzzy control. Further development of the two-dimensional controller presented here in order to improve its performance is possible, and research on the development of a three-dimensional positional controller is planned for the future.

7.1 The objectives

7.1.1 The task description

The fuzzy controller presented in this thesis has been devised for the simplified two dimensional RTG control problem (Fig. 7.1) in which the gantry does not move ($\ddot{x}_v =$

$\ddot{y}_v = \ddot{\theta}_v = \dot{x}_v = \dot{y}_v = \dot{\theta}_v = 0$), but the trolley is free to move along the top beam, and the spreader can be lowered.

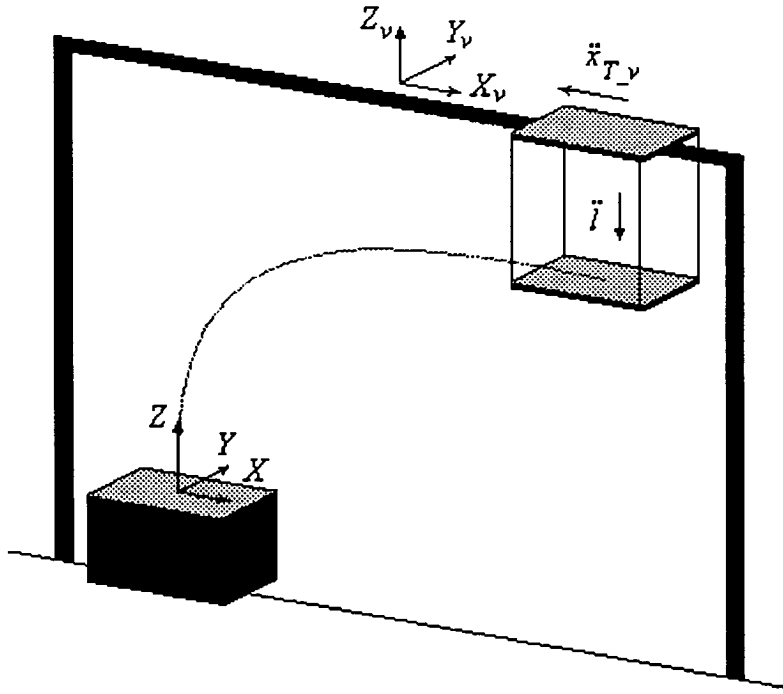


Figure 7.1

A typical spreader trajectory

It is assumed that the spreader is initially hoisted to its highest possible position. This reduces its swing during the traversing motion of the trolley, and the spreader is left to be lowered as late as possible so that its lowering finishes at approximately the same time when the trolley is positioned above the target. The effectiveness of this strategy is also mentioned in the paper by Ren et.al [64] which deals with gantry crane dynamics in detail. The paper says: 'to reduce the dynamic effect of changes of rope tension, it is more effective to perform hoisting motion prior to traversing motion'. Thus, this strategy is to be adopted here, and will be discussed more when the fuzzy rules are considered.

Initially, the container is placed exactly below the gantry top beam, and the trolley is placed somewhere on the top beam with the spreader in the extreme hoisted position ($l(0) = 4\text{m}$ due to the restriction of the MCM (6.12)).

The required task is to bring the trolley to some point above the container with the spreader being lowered at the same time, so that the simultaneous trolley and spreader movements result in the spreader covering the top of the container. Each of the four spreader corners should ultimately get to positions within 25mm¹⁾ of the corresponding corner of the container. This misalignment quantity is henceforth defined by ε .

A reasonable path for the spreader trajectory is shown in Figure 7.1. This can be achieved by the controlled application of two accelerations to the system, namely $\ddot{x}_{T_v}$ (trolley acceleration) and $\ddot{l}$ (the acceleration of the spreader when lowering).

7.1.2 The systems of coordinates and initial data

As usual, two systems of coordinates are used here (Figure 7.1), a global system XYZ , and a vehicle system, $X_v Y_v Z_v$. The global X axis is chosen to be parallel to the gantry top beam pointing to the right, and the Z axis is pointing upwards. The Y axis is chosen appropriately to complete the right handed system. The datum of the vehicle system of coordinates has been chosen to coincide with the geometrical centre of the gantry along the top beam, and that its axes are parallel to the corresponding axes of the global system (Figure 7.1).

Y -coordinates are not used in this particular problem due to the fact that it resides purely in the two dimensional X - Z plane, thus:

$$y_v = y_T = y_S = 0 \quad (7.1)$$

In this chapter, the origin of the global system is at the geometrical centre of the top of the container, thus the container coordinates in this system are:

$$x_C = y_C = z_C = 0 \quad (7.2)$$

The displacement of the vehicle along the X -axis is arbitrarily chosen to be 8m, which means that the horizontal distance between the geometrical centre of the vehicle and that of the container is eight metres. The height of the top beam is 12.59m from the ground²⁾. Thus, the coordinates of the geometrical centre of the vehicle are (taking the height of the container to be 2.59m):

$$x_V = 8; \quad (7.3)$$

$$y_V = 0; \quad (7.4)$$

$$z_V = 10; \quad (7.5)$$

Generally (in the three dimensional case), it will be more convenient to use the vehicle (rather than the global) system of coordinates to identify the position of the trolley because the position of the trolley in the vehicle system (as usual, denoted by subscript ' $_v$ ') is completely identified by the x_{T_v} coordinate (which is in fact the distance between the geometrical centres of the trolley and the gantry measured along the top beam of the gantry). The first and second derivatives, $\dot{x}_{T_v}$ and $\ddot{x}_{T_v}$, are, respectively, the velocity and acceleration of the trolley. In the case of the two dimensional task the trolley velocity and acceleration are respectively the same in each coordinate system:

$$\dot{x}_{T_v} = \dot{x}_T \quad (7.6)$$

$$\ddot{x}_{T_v} = \ddot{x}_T \quad (7.7)$$

but the vehicle system will continue to be used for consistency and clarity.

7.1.3 Parameters of the RTG crane used in the Multi-Cable Model for the Fuzzy Controller design

The MCM described in Chapter 5 is used here to simulate the actual crane. The outputs of the MCM are the spreader coordinates, x_S and z_S , for the specific accelerations $\ddot{x}_{T_v}$ and $\ddot{l}$ 'applied' to the MCM. This allows the calculation of all the necessary data used for the fuzzy controller inputs.

The data for the experiments with the Fuzzy Controller has been chosen to be the same as in the experiments on the Multi-Cable Model described in section 6.1.2.2, and the operating width for the trolley is taken to be 18.2m^2 .

For this aspect of the work the coefficient D_r is taken to be $0.5[1/\text{s}]$, (i.e. five times greater than that calculated from experimental data with a real RTG crane, statement (6.2)). The introduction of higher passive damping into the system in order to reduce the spreader oscillations in the control scenario is in order to make effective progress with controller design, and damping modelling criteria are to be re-considered in a future research programme.

Finally it should be noted that for the two dimensional task considered here the horizontal misalignment ε is simply the difference between the x -coordinates of the spreader and the container calculated at that moment when their z -coordinates are the same, i.e. when the spreader has covered the top of the container:

$$\varepsilon = x_C - x_S \quad (\text{when } z_C = z_S) \quad (7.8)$$

Thus, the criterion for the controller to be successful is:

$$|\varepsilon| = |x_C - x_S| \leq 25\text{mm} \quad (\text{when } z_C = z_S) \quad (7.9)$$

7.1.4 The Fuzzy Controller implementation

The commercially written CubiCalc package has been used to implement the controller described here. It has a convenient shell allowing quick changes of rules and adjectives of the fuzzy variables for tuning the fuzzy controller. It also offers the user a useful facility for designing the numerical and graphical presentation of the results.

The MCM has been implemented into the CubiCalc shell to represent the RTG crane in the simulation of the controller and crane on the computer.

7.1.5 Terminology used to define Fuzzy control concepts

Fuzzy control is a relatively new field, but since the first papers on fuzzy control (Mamdani and Assilian [52], Mamdani [53], Tong [75], Zadeh [86]) were published, a lot of research has successfully been carried out (Baxter and Bumby [7], Daley and Gill [25], Daley and Gill [26], Linkens and Nie [47], Linkens and Abbod [48], Linkens and Abbod [49], Linkens and Abbod [50], and others). The basic fuzzy concepts can also be found in the textbook by Driankov et.al [33].

A large number of different people have been working on this subject in different parts of the world which has led to some differences in the terminology currently in use. Therefore some definitions and explanations are given here to avoid any confusion in the thesis.

- *Fuzzy rules* - linguistic expressions of a form:

if < the state of the process > *then* < control input >

- *Input and output fuzzy variables* - the variables used as fuzzy controller inputs and outputs.

- *Fuzzy set, or adjective* - the *membership* function on a fuzzy variable *domain* which takes any value between 0 and 1.

Example. The variable $\ddot{l}$ is a *fuzzy output* in the RTG problem presented here. Its *domain* is given by statement (6.11), and there are five *adjectives* (fuzzy sets) for $\ddot{l}$ used in the fuzzy controller which are shown on Figure 7.10³). They are called, in the standard way, PB, PS, ZE, NS, NB (the abbreviations are explained in the caption to Figure 7.10).

- *Rules weights* - numbers from 0 to 1 which are assigned to each rule. Weights can be used for disabling the rule: if the weight is equal to 0 the rule will never *fire* even when the '*if*' part of the rule is satisfied. If the weight is equal to 1, then the rule fires in the usual manner. The weight values between 0 and 1 can be used to change the effectiveness of a particular rule with respect to others (the latter is not used in the controller presented in the thesis).
- *Simple and self-organised fuzzy controllers.* According to Driankov et.al [33], these fuzzy logic terms are completely analogous to 'non-adaptive' and 'adaptive' terms used in classical control. The controller described in the thesis is self-organised because the set of rules and the fuzzy sets are altered on-line in order to enlarge its operating range (in order to 'increase the nonlinearity the fuzzy controller can cope with', Driankov et.al, [33]).

7.2 Building and tuning the Fuzzy Controller

7.2.1 The block-diagram of the two-dimensional Fuzzy Controller

The block diagram of the Fuzzy Controller to perform the two-dimensional task described in section 7.1.1 is shown in Figure 7.2.

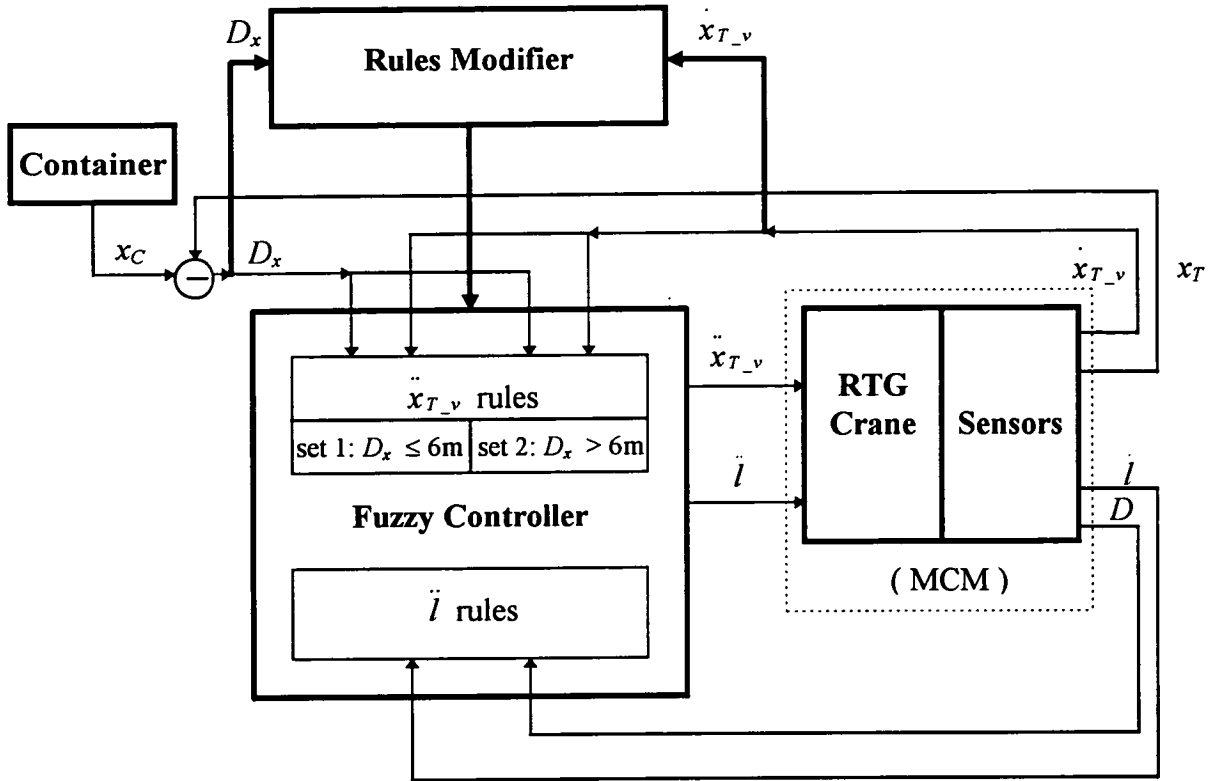


Figure 7.2

Block diagram of the self-organised time-optimal Fuzzy Controller

The controller has four fuzzy inputs (D , D_x , $\dot{x}_{T_v}$, $\dot{l}$) and two fuzzy outputs ($\ddot{x}_{T_v}$ and $\ddot{l}$). Here D_x is a horizontal distance between the trolley and the container:

$$D_x = x_C - x_T \quad (7.10)$$

All the rules implemented in the controller can be divided into two groups: the rules for $\ddot{x}_{T_v}$ and the rules for $\ddot{l}$. These groups are derived independently using both the driver's experience and practical understanding of the problem, but the simultaneous applications of those rules have to be organised in a way which always results in an optimal spreader trajectory. This is described in the sections 7.2.1.1, 7.2.1.2 and 7.2.1.3.

7.2.1.1 Rules for $\ddot{x}_{T_v}$

The trolley is initially placed somewhere on the top beam and the horizontal distance from it to the container, at $t = 0$, is denoted by $D_x(0) = D_0$. The group of rules to be described is intended to bring the trolley to a point on the top beam which is exactly above the container, i.e. for which $D_x = 0$, or as near to it as possible.

The value of the trolley acceleration $\ddot{x}_{T_v}$ depends on the current horizontal distance between the trolley and the container, D_x , and on the current velocity $\dot{x}_{T_v}$ of the trolley. Both D_x and $\dot{x}_{T_v}$ are used as fuzzy inputs.

The rules are required to fulfil two objectives:

- the absolute value of the trolley acceleration has to be greater for greater $|D_x|$ and smaller for smaller $|D_x|$, and directed so that D_x is reduced;
- the absolute value of the trolley acceleration has to get smaller as the trolley velocity approaches its maximum allowed value, independently of the current value of $|D_x|$.

The practical limits on the domain for each fuzzy variable are given by the following:

$$0 \leq |D_x| \leq 18.2\text{m} \quad (7.11)$$

$$0 \leq |\dot{x}_{T_v}| \leq 0.87\text{m/s} \quad (7.12)$$

$$0 \leq |\ddot{x}_{T_v}| \leq 0.25\text{m/s}^2 \quad (7.13)$$

Limit (7.11) is determined by the maximum operating width of the trolley for a typical RTG Crane, and limits (7.12) and (7.13) follow from speed and acceleration restrictions for the trolley, (6.10).

In order to 'cover' such a wide range of D_x and to make the changes in trolley velocity as smooth as possible, the domain of each fuzzy variable comprises seven adjectives (rather than the conventional three or five), namely:

$$\begin{aligned} \text{NB} &= \text{negative and big;} & \text{PB} &= \text{positive and big;} \\ \text{NM} &= \text{negative and medium;} & \text{PM} &= \text{positive and medium;} \\ \text{NS} &= \text{negative and small;} & \text{PS} &= \text{positive and small;} \\ & & \text{ZE} &= \text{zero} \end{aligned}$$

For tuning the adjectives, the following are regarded as reasonable ideas:

- when the trolley is far from the container, its acceleration should be as near to a maximum as possible so that it reaches the maximum allowed speed (0.87m/s) as soon as possible, in order to reduce the performance time of the controller.
- the trolley should start decelerating when it is near to the container, and it should decelerate in such a manner that the spreader oscillations are minimised⁴⁾.
- the fuzzy set for D_x and $\dot{x}_{T_v}$ should be constructed so that the NS/PS adjectives are narrower than the NM/PM adjectives, and so that the latter are narrower than the NB/PB adjectives (i.e. somewhat like the adjectives for D_x and $\dot{x}_{T_v}$ shown in Figure 7.3), this being intended to reduce the error in the final alignment ('to increase the sensitivity of the controller to small values of the input', Driankov et.al [33]).

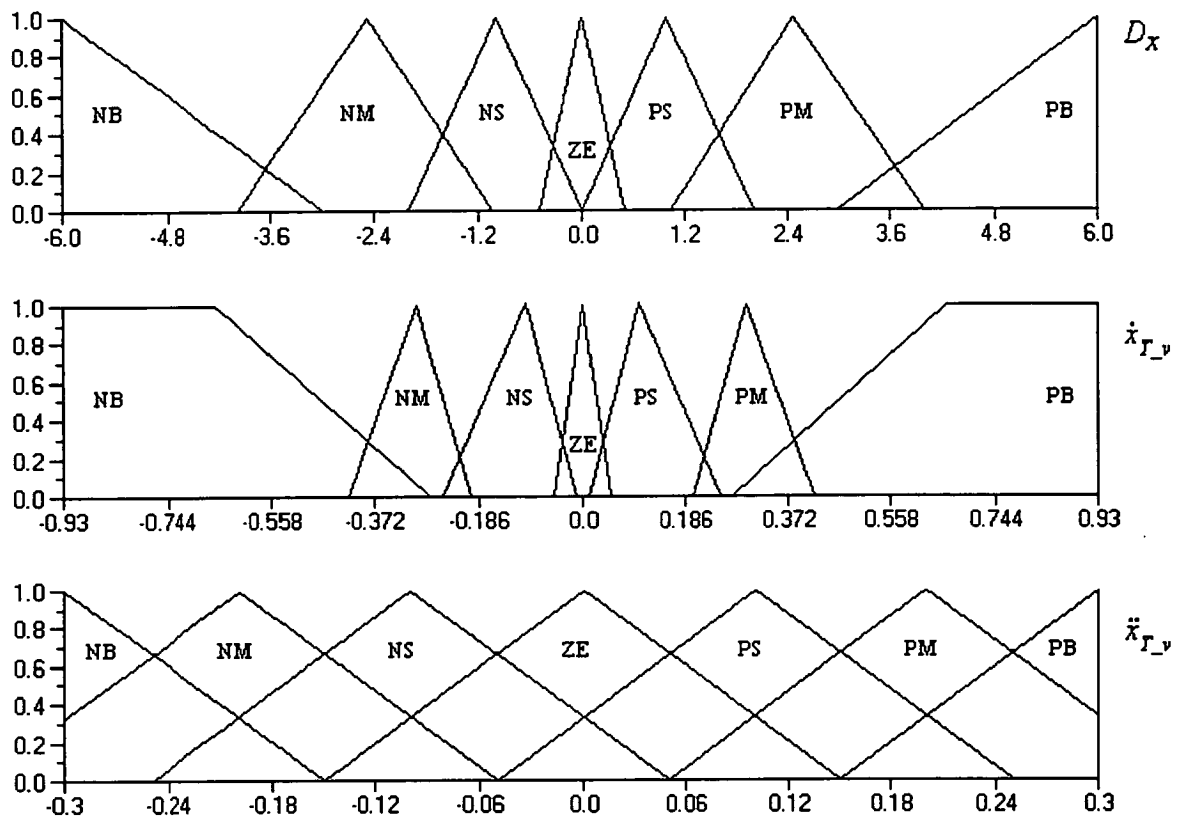


Figure 7.3

Adjectives for D_x , $\dot{x}_{T_v}$, $\ddot{x}_{T_v}$
(for $D_x \leq 6m$)

NB = negative and big;	PB = positive and big;
NM = negative and medium;	PM = positive and medium;
NS = negative and small;	PS = positive and small;
ZE = zero	

Figure 7.4a shows five desirable curves (1-5) for $D_x(t)$ as functions of time for five different initial values of horizontal distance D_0 . Figure 7.4b shows the corresponding trolley velocities and accelerations profiles.

Curves 1, 2 and 3 correspond to D_0 greater than about 6m and it can be seen that the performance of the controller corresponding to these curves would be near to ideal: the trolley accelerates at a maximum when it is far from the container (up to its velocity limit), and then it decelerates when it is near to the container in such a way that the

horizontal distance between the trolley and the container reduces down to zero (as do the velocity and acceleration of the trolley).

For the case of D_0 smaller than about 6m (curves 4 and 5) the procedure does not work the same way. It is clear that the trolley cannot accelerate fully, as then it will have no time to stop directly above the container. Thus, in case 4, the trolley starts to accelerate at its maximum value, but its speed never actually reaches its maximum value of 0.87m/s, and in case 5 the acceleration is below maximum all the time.

This means that the algorithm cannot be the same for such a wide range of D_x values (refer to limit (7.11)), i.e. a simple⁵⁾ fuzzy controller is not suitable for the task and some adaptation of the rules (or, in other words, the self-organised⁵⁾ fuzzy controller) is required.

It has been decided that two sets of rules are necessary, one for $D_0 \leq 6\text{m}$, and the other for $D_0 > 6\text{m}$. The rules are shown in the form of matrices in Figures 7.5 ($D_0 \leq 6\text{m}$) and 7.6 ($D_0 > 6\text{m}$). The adjectives are shown in Figures 7.3 ($D_0 \leq 6\text{m}$) and 7.7 ($D_0 > 6\text{m}$).

The first set of rules ($D_x \leq 6\text{m}$) is a standard set of rules described widely in the literature (the reader can refer to the papers by Daley et.al [24], Linkens et.al [49], Procyk and Mamdani [62], and many others). In order to illustrate how the rules work in practice, three typical examples are discussed below.

1. Rule 35:- *if the velocity of the trolley, $\dot{x}_{T_v}$, is NS (negative and small) and the horizontal distance D_x is NS, then the trolley acceleration is ZE (zero).*

In practice this means that if the trolley is moving slowly in the negative X direction and the container is not far from the trolley to the left on X ($D_x < 0$ means $x_C < x_T$ (7.10)), then there is no change needed in the speed of the trolley.

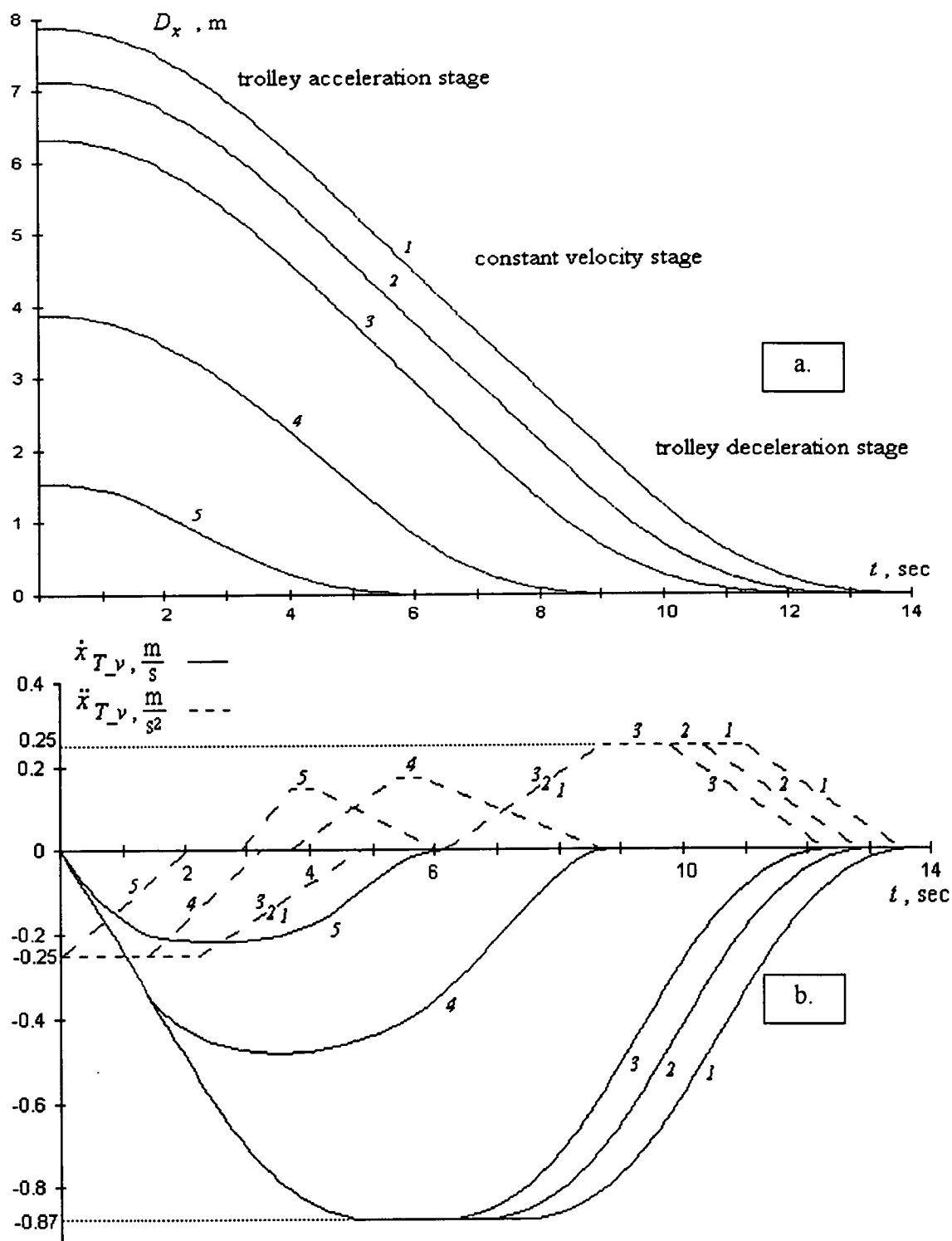


Figure 7.4

The desirable curves showing possible changes with time of the horizontal distance from the trolley to the container (a) and corresponding velocity and acceleration of the trolley (b) for several D_0

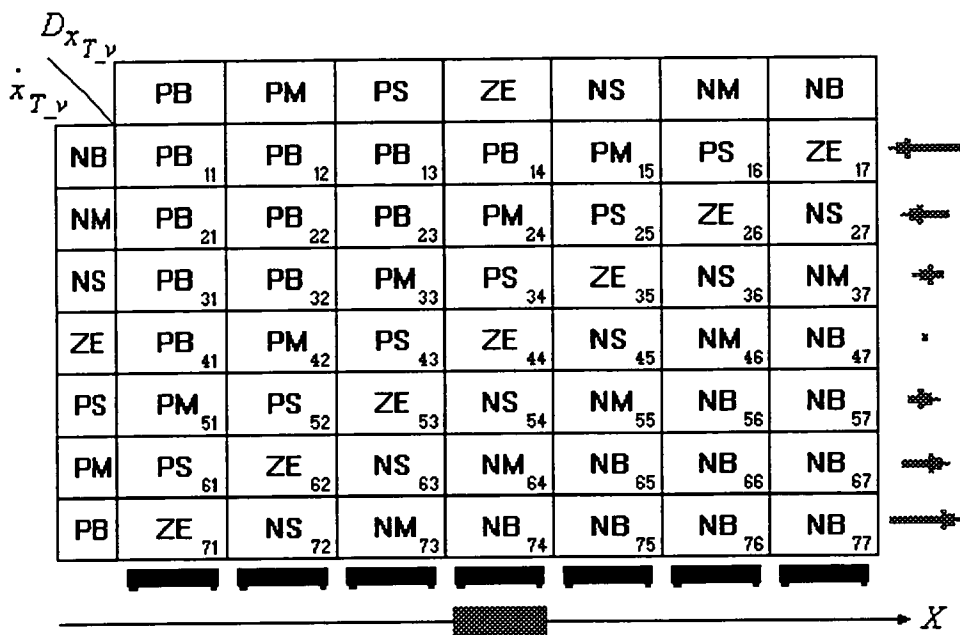


Figure 7.5

$\dot{x}_{Ty}$ rules,
set 1: $D_0 \leq 6m$

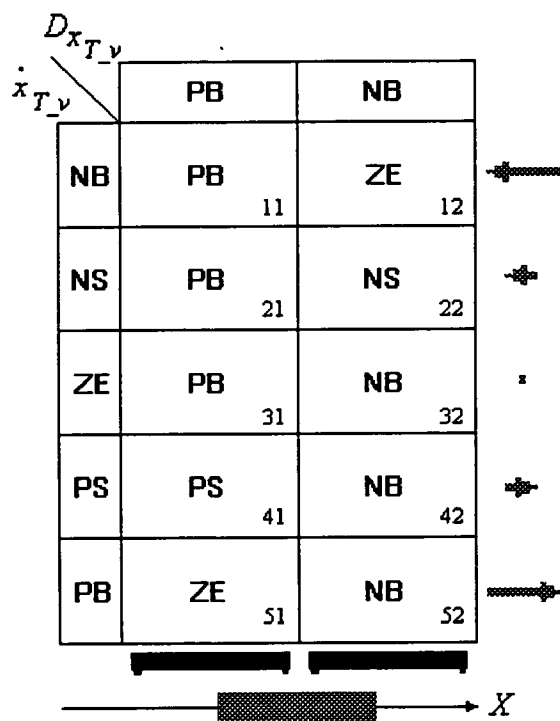


Figure 7.6

$\dot{x}_{Ty}$ rules,
set 2: $D_0 > 6m$

2. Rule 51:- if the velocity of the trolley, $\dot{x}_{T_v}$, is PS (positive and small) and the horizontal distance D_x is PB (positive and big), then the trolley acceleration is PM.

This means that if the trolley is moving slowly in the positive X direction, but the container is still very far from the trolley to the right on X , then the speed of the trolley should be increased in the same direction.

3. Rule 17:- if the velocity of the trolley, $\dot{x}_{T_v}$, is NB (negative and big) and the horizontal distance D_x is NB (negative and big), then the trolley acceleration is ZE.

In this case: the trolley moves in the correct direction. Although the container is still very far from the trolley the acceleration is assigned to ZE because the speed of the trolley cannot be increased any more (the trolley is already moving with maximum allowed velocity NB).

This set of rules (together with the adjectives shown in Figure 7.3) works very well for trajectories below 6m. For trajectories longer than 6m the second set of rules (Figure 7.6) is needed firstly to accelerate the trolley at its maximum rate (see the adjectives in Figure 7.7), and then when D_x becomes smaller than 6m the rules in Figure 7.4 come into action. The logical procedure of this 'switch' is implemented in the 'Rules modifier' block (Figure 7.2) by means of changing the rule weights⁵⁾.

Each rule in the fuzzy controller has a weight which is either equal to zero (the rule cannot fire at all) or to one (the rule can fire). The weights of the $\ddot{x}_{T_v}$ rules are dependent on whether D_x is greater or smaller than 6m.

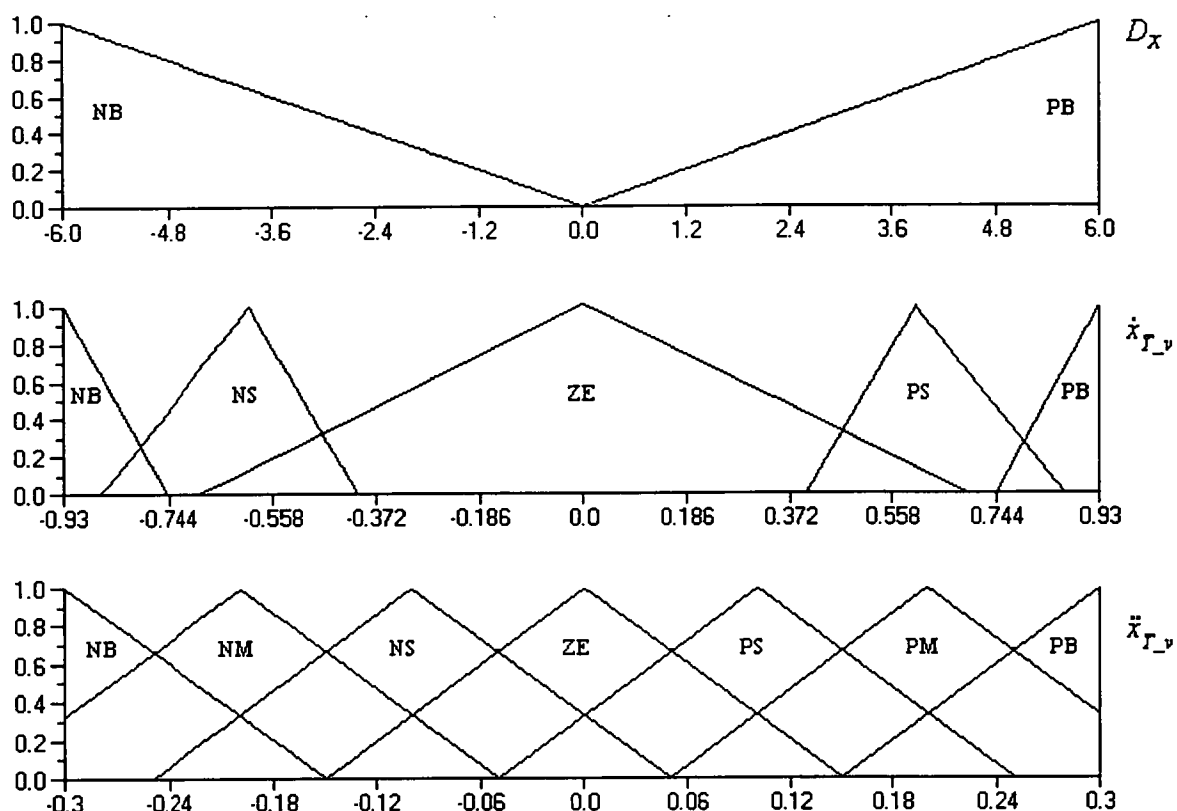


Figure 7.7

Adjectives for D_x , $\dot{x}_{T_v}$, $\ddot{x}_{T_v}$
(for $D_x > 6m$)

NB	= negative and big;	PB	= positive and big;
NM	= negative and medium;	PM	= positive and medium;
NS	= negative and small;	PS	= positive and small;
	ZE	= zero	

7.2.1.2 Rules for $\ddot{l}$

Intuitively, the decision as to when the spreader should start to be lowered depends upon the angle α_v (Figure 7.8) through which the driver can see the container from the trolley, or, simply on the distance between the spreader and the container D (the choice of the fuzzy input will not change the conceptual structure of the rules, but only their tuning, as a one-to-one mapping between α_v and D takes place for the optimal spreader trajectory).

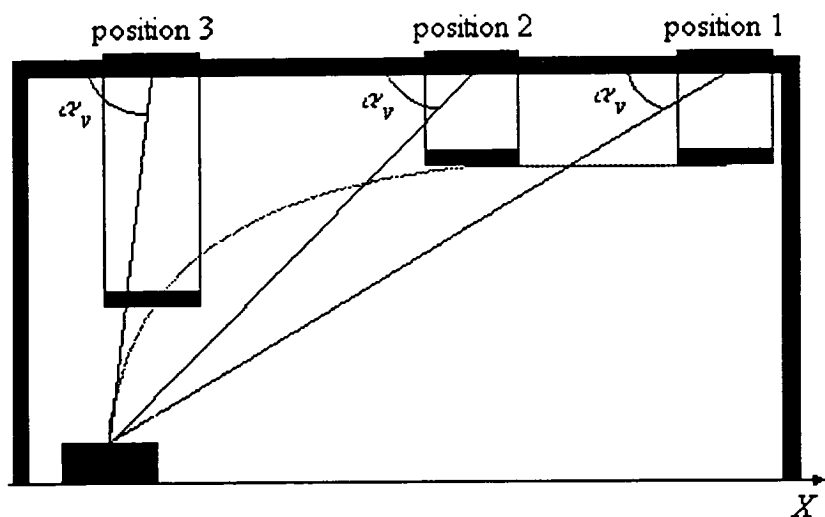


Figure 7.8

Definition of the angle α_v for the spreader at different XZ locations

$i \backslash D$	ZE	VS	S	L	
NB	PB 11	PB 12	PB 13	PB 14	↑
NS	PS 21	PS 22	PB 23	PB 24	↕
ZE	ZE 31	PS 32	PS 33	PB 34	•
PS	NS 41	NS 42	ZE 43	PS 44	↕
PB	NB 51	NB 52	NS 53	ZE 54	↓

Figure 7.9

$\ddot{l}$ rules

If the driver can see the container at a very small angle, α_v , (position 1 on Figure 7.8) there is no need to lower the spreader yet. The spreader should start being lowered

when the trolley is nearer to the point above the container (in position 2 on the same Figure), fully accelerating ($\ddot{l} = 0.1 \text{ m/s}^2$) till $\dot{l}$ reaches its maximum (which is 0.452m/s for an unladen spreader). When the spreader is nearer to the container (position 3) its rate of lowering has to be reduced in order to avoid a subsequent crash between the underside of the spreader and the container top-side.

The corresponding set of rules is shown in Figure 7.9 with the adjectives in Figure 7.10.

7.2.1.3 Simultaneous application of the $\ddot{x}_{T_v}$ and the $\ddot{l}$ rules

The $\ddot{x}_{T_v}$ and the $\ddot{l}$ rules have been designed independently, and it is now necessary to organize them so that they can be applied simultaneously so as to ensure that the trajectory of the spreader is as near to the optimal condition (Figures 7.1, 7.8) as possible. Some points have to be made so that this can be done:

- the $\ddot{l}$ rules are designed so that in order to lower the spreader from $l = 4\text{m}$, initially, down to $l = 10\text{m}$ (this is how the task has been designed, please refer to the section 7.1.1) the $\ddot{l}$ rules fire in a certain order for all different possible trajectories (i.e. for all different initial D_0): starting from rule 34, then rule 44 fires next, after which the group of 44-54-53-43 fire, and finally the combination of rules 43,42-52,41,41-31 fire finishing at rules 41-31⁶⁾. Thus, for this simplified task, it is possible to work out the time (denoted by t') needed for the cable length to change from 4m to 10m, in other words, for the spreader to be fully lowered down. Numerical experiments have shown that this time is approximately $t' = 20\text{sec}$;
- the longer the spreader remains in the upper position the better, because in the case of smaller l its oscillations are smaller. This leads to the general concept of lowering the spreader not earlier than about 20sec before the trolley reaches the position exactly above the container;

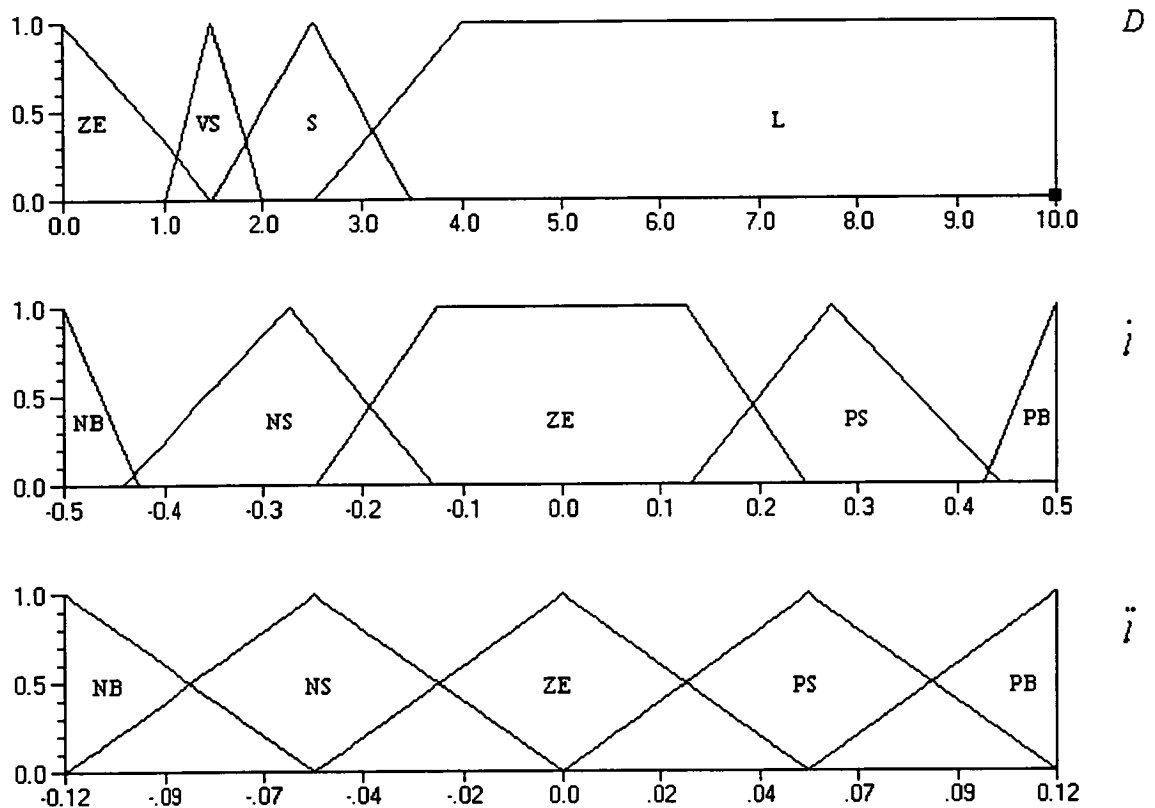


Figure 7.10

Adjectives for D, i, j

ZE = zero;	VS = very small;	S = small;	L = large
NB = negative and big;	PB = positive and big;		
NS = negative and small;	PS = positive and small;		

- it is also clear that the distance D'_x (the horizontal distance between the trolley and the container when the spreader should start lowering) has to be different for different trajectories. This is due to the fact that the trolley velocity profiles are different for different trajectories, and for the same D_x a different time will be needed for different trajectories to accomplish the trolley movement.

Thus, some on-line algorithm is needed to establish the value of D'_x at which to start firing the j rules. This can be done in the following way.

The last 20 second phase corresponds to the deceleration stage⁷⁾ of the trolley movement⁸⁾ with the velocity showing some sort of parabolic shape with time, as shown in Figure 7.11. The equation of this parabola can be easily written:

$$\dot{x}_{T-v}(t) = \dot{x}_{T-v}^l \left(\frac{t^2}{400} - \frac{t}{10} + 1 \right) \quad (7.14)$$

Using simple integration, the horizontal distance D_x^l can be found:

$$D_x^l = \int_0^{20} \dot{x}_{T-v}^l \left(\frac{t^2}{400} - \frac{t}{10} + 1 \right) dt \approx 7 \dot{x}_{T-v}^l \quad (7.15)$$

which means that the value $t^l = \frac{D_x}{\dot{x}_{T-v}}$, [sec], has to be calculated on-line, and the $\ddot{l}$

rules should start to fire when the following is correct:

$$t^l = \frac{D_x}{\dot{x}_{T-v}} < 7 \quad (7.16)$$

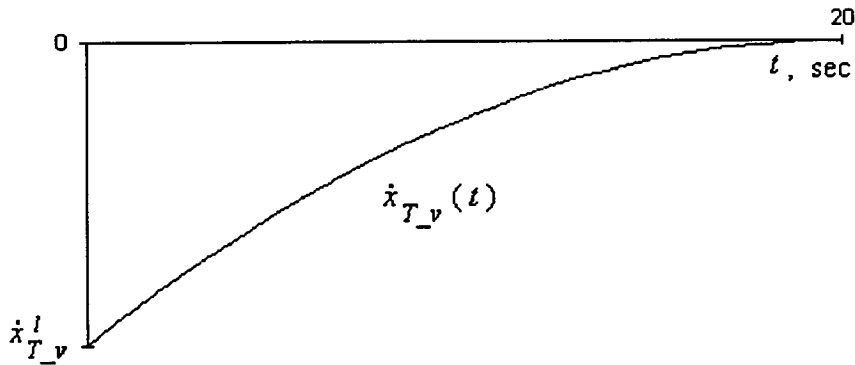


Figure 7.11

Generalised change of trolley velocity with time
during the final part of the trajectory - a parabolic profile

This requirement is activated by means of the weights applied to the $\ddot{l}$ rules in the 'Rules Modifier' block. Diagrams at the end of this chapter show that the algorithm works very effectively.

7.3 The results

The Fuzzy controller has been tuned for the RTG crane with parameters given in section 7.1.3. Its performance was checked by several numerical experiments using the Multi-Cable Model of the RTG Crane.

Different values for initial trolley horizontal displacement were used:

$$D_0 = 2, 4, 6, 8, \text{ and } 10\text{m} \quad (7.17)$$

and five corresponding sets of plots are given in Figures 7.12-7.41. They show:

- a. The path of the spreader in the vertical plane (XZ);
- b. $\ddot{x}_{T_v}$ and $\dot{x}_{T_v}$ (against time);
- c. x_{T_v} (against time);
- d. $\ddot{l}$ and $\dot{l}$ (against time);
- e. l (against time);
- f. r (against time).

The performance of the fuzzy controller for the initial displacement values mentioned in (7.11) is excellent with the error in final alignment, ε , varying from fractions of a millimetre to about 8mm, which well satisfies the stipulation in equation (7.9). The trolley acceleration and velocity profiles are very near to the desired profiles (shown on Figure 7.4b), and the spreader path is near to the optimal path. Clearly this performance will be reviewed when definitive damping values are obtained during the

future stages of this research, but this work clearly demonstrates the viability of this approach to RTG control.

It is obvious that the controller will work well for longer trajectories, $D_0 > 10\text{m}$, too, with the same value of final misalignment ε as that for $D_0 = 10\text{m}$. This can be explained by the fact that there is no difference between trajectories longer than 10m and those for which $D_0 = 10\text{m}$, apart from the length of the constant velocity stage. Therefore when the set of rules for $D_x < 6\text{m}$ starts to fire the trolley velocity is constant and equal to the maximum value attainable (0.87m/s), in both cases. The spreader path during the deceleration stage will be identical for both trajectories with only small differences in the oscillation of r , which can be neglected at this stage of the research given the presence here of relatively high passive damping. The controller fails only for $D_0 < 2\text{m}$.

7.4 Conclusions

This chapter has explained how a Fuzzy Controller for the RTG Crane problem has been devised, along with an analysis of the controller performance. The task of the controller has been simplified to a lateral trolley movement and lowering of the spreader, but it clearly proves that the Fuzzy approach for the RTG control works very well for a wide range of initial trolley displacements, $2\text{m} < D_0 < \infty$ (where the performance of the controller is really excellent, with the alignment error well within the implementation demands of the real RTG crane). The results presented here suggest that the Fuzzy Controller as it has been developed so far can be used as a real-time controller for the described RTG task.

It is clear that better tuning of the suggested controller is needed to produce smoother changes in trolley and lowering accelerations in line with operational levels of passive damping in different machines. This will reduce both operational time and spreader oscillations.

The controller suggested is only a first attempt of applying the ideas described, and this work opens the field for further research on fuzzy control of RTG cranes:

- the possibility of a 'swing-fuzzy controller' can be investigated;
- a more complicated, self-organised, fuzzy controller for the tasks involving spreader hoist, gantry translations and gantry rotation could be investigated.

Notes to Chapter 7

- ¹⁾ The design of the twistlock pins allows some limited float, thus a small, finite, misalignment in positioning of the spreader is allowed. It is limited to about 25mm (this value was suggested by the principal industrial collaborator on the project and was defined from a practical understanding of the problem).
- ²⁾ As before, all the data used in the model and experiments was taken from the typical RTG crane performance data.
- ³⁾ The domain of $\ddot{l}$ on Figure 7.10 is slightly larger than that given by statement (6.11). This is due to the defuzzification procedure adopted in the fuzzy controller which uses the centroid method. The centroids of the PB and NB sets, as they are in Figure 7.10, correspond to $\ddot{l}$ exactly equal to $\pm 0.1 \text{ m/s}^2$.
- ⁴⁾ As it has been mentioned in section 1.4, there are two driver methods: the so-called 'swing' and 'non-swing' drivers. A 'swing-based' Fuzzy Controller is an attractive idea, but likely to be rather complicated and time consuming to be build properly. The controller suggested here is a 'non-swing controller', and, as mentioned in section 7.1.3, the higher passive damping is used to reduce the spreader oscillations. Also, it is expected that smoother changes in the trolley acceleration which can be achieved by thorough tuning of the described controller also will reduce the spreader oscillations.
- ⁵⁾ An explanation of this term can be found in section 7.1.5.

- 6) Other rules do not fire at all for the primitive task (as in section 7.1.1) considered in the thesis. They are left in the table 7.9 for the 'full picture' only as they will be brought into play in future research where the task will be extended to the more general 'Super RTG' task in which all motored degrees of freedom can be actuated simultaneously.
- 7) Figure 7.4a,b explains what 'the deceleration stage' is.
- 8) It can be easily checked by comparing the adjectives for $\ddot{x}_{T_v}$ and $\ddot{l}$ rules that this actually takes place. It also can be worked out from the plots of $\ddot{x}_{T_v}$ and $\ddot{l}$ against time for actual experiments analysed in section 7.4.

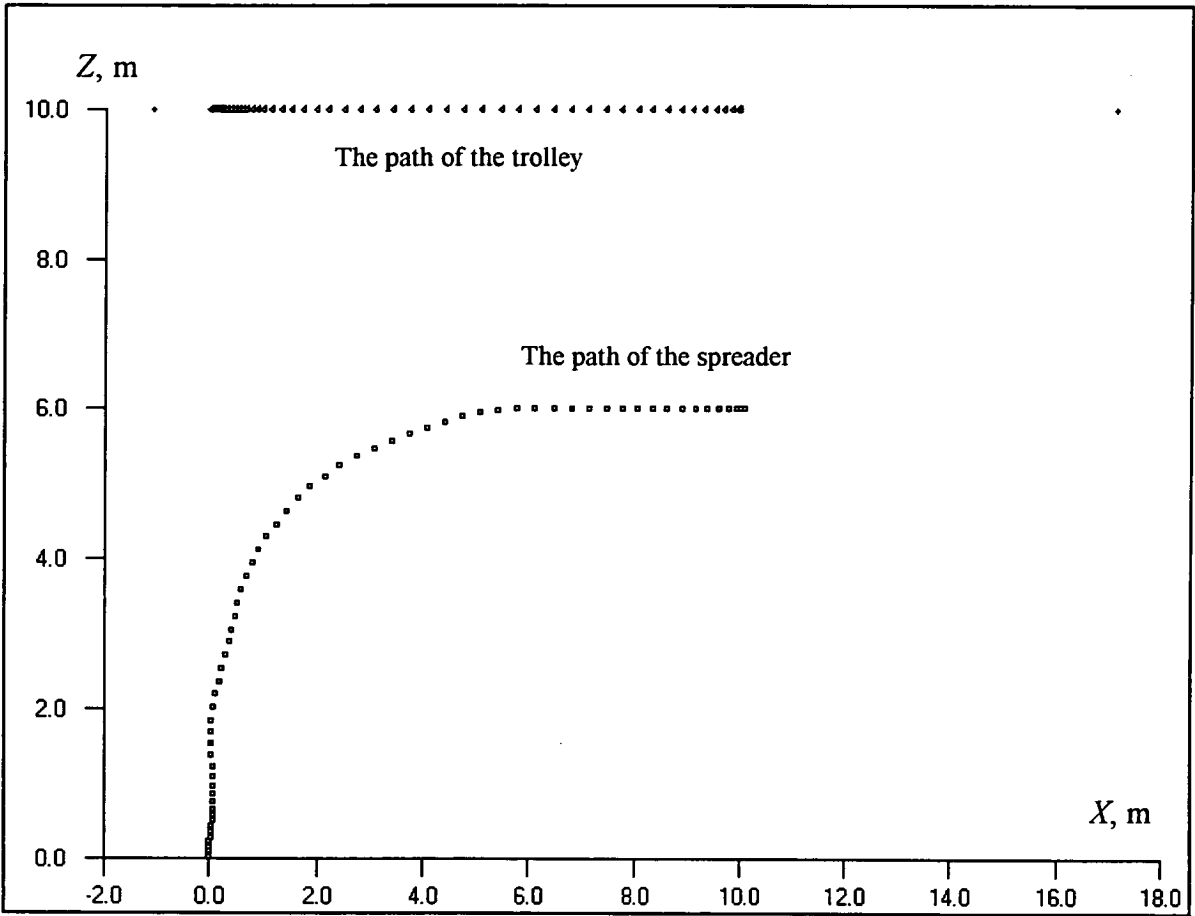


Figure 7.12
Performance of the Fuzzy Controller
 $D_0 = 10$ m
Vertical plane

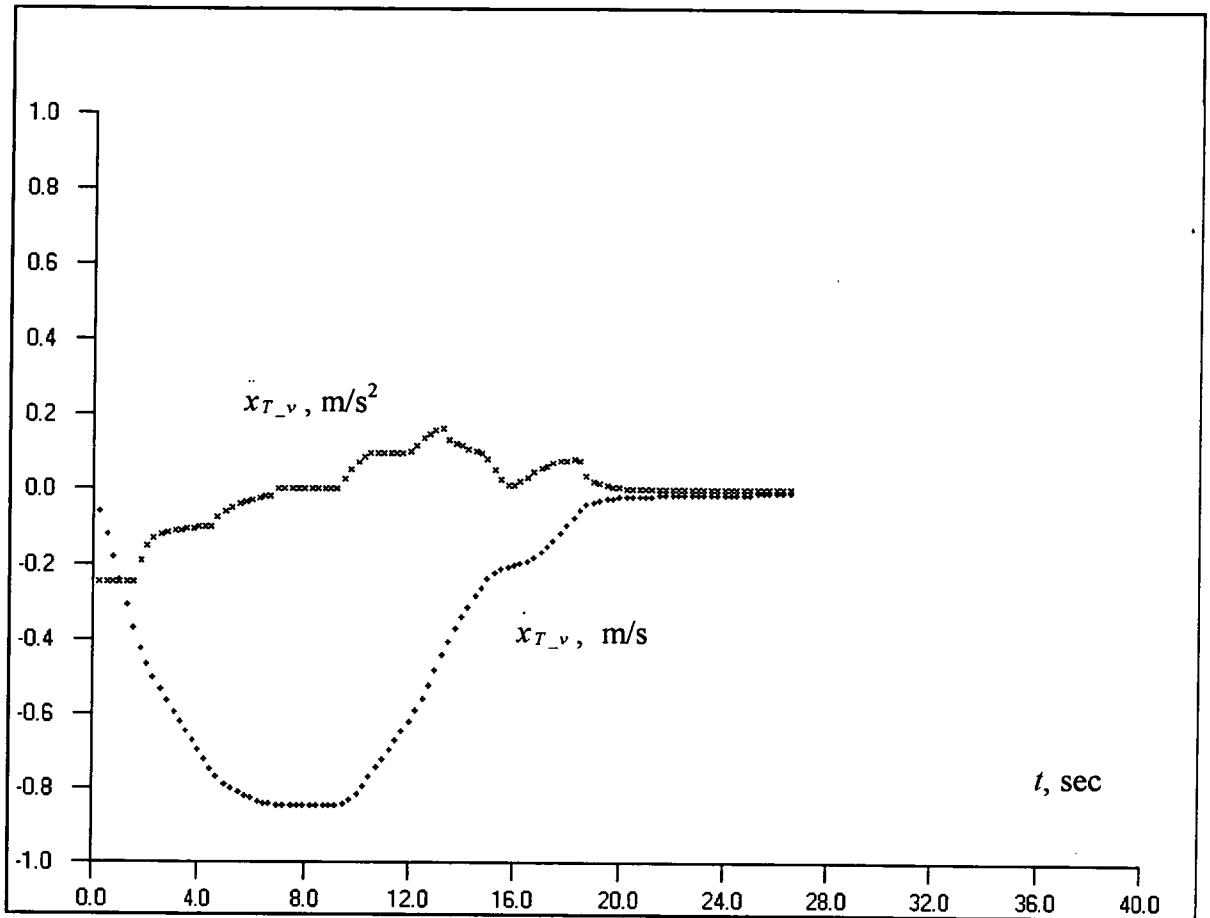


Figure 7.13
Performance of the Fuzzy Controller
 $D_0 = 10$ m
Velocity and acceleration of the trolley

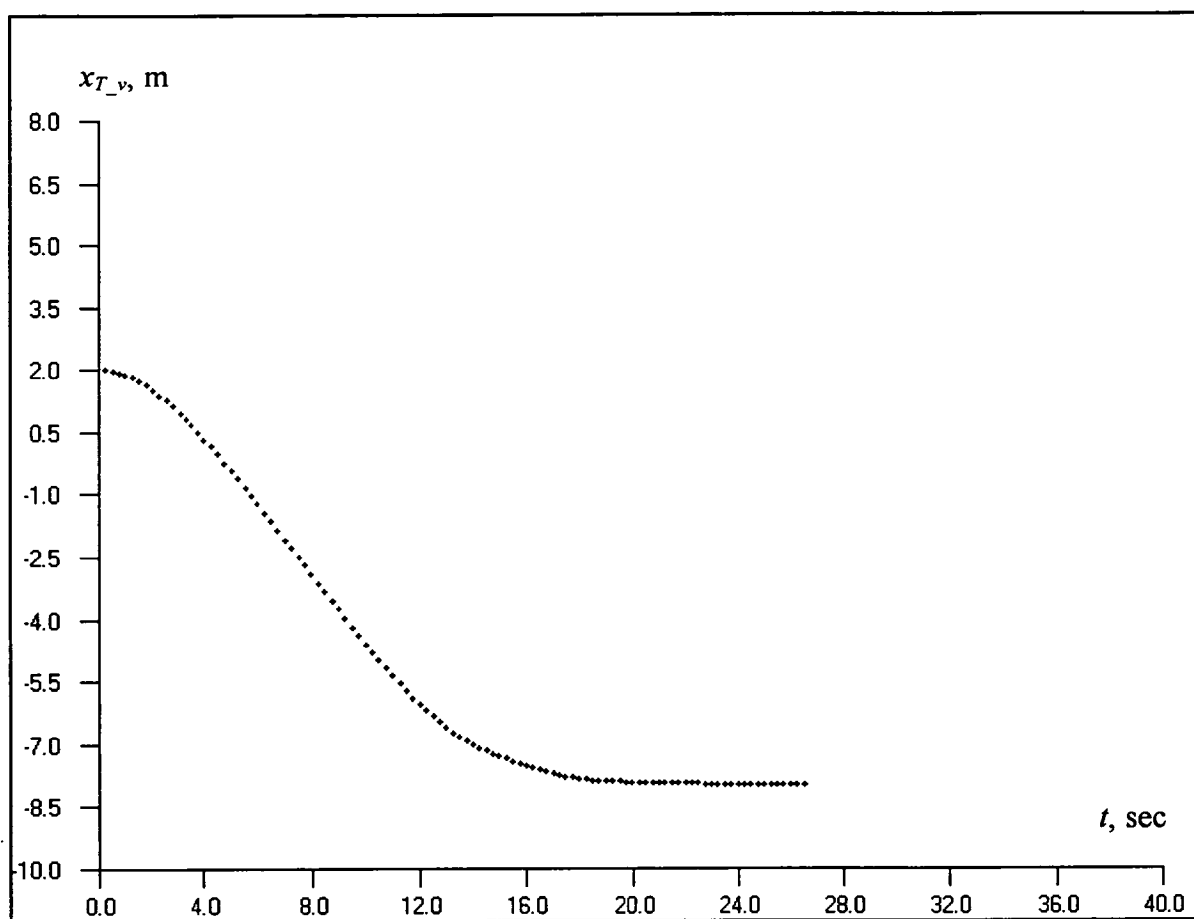


Figure 7.14
Performance of the Fuzzy Controller
 $D_0 = 10$ m
Displacement of the trolley

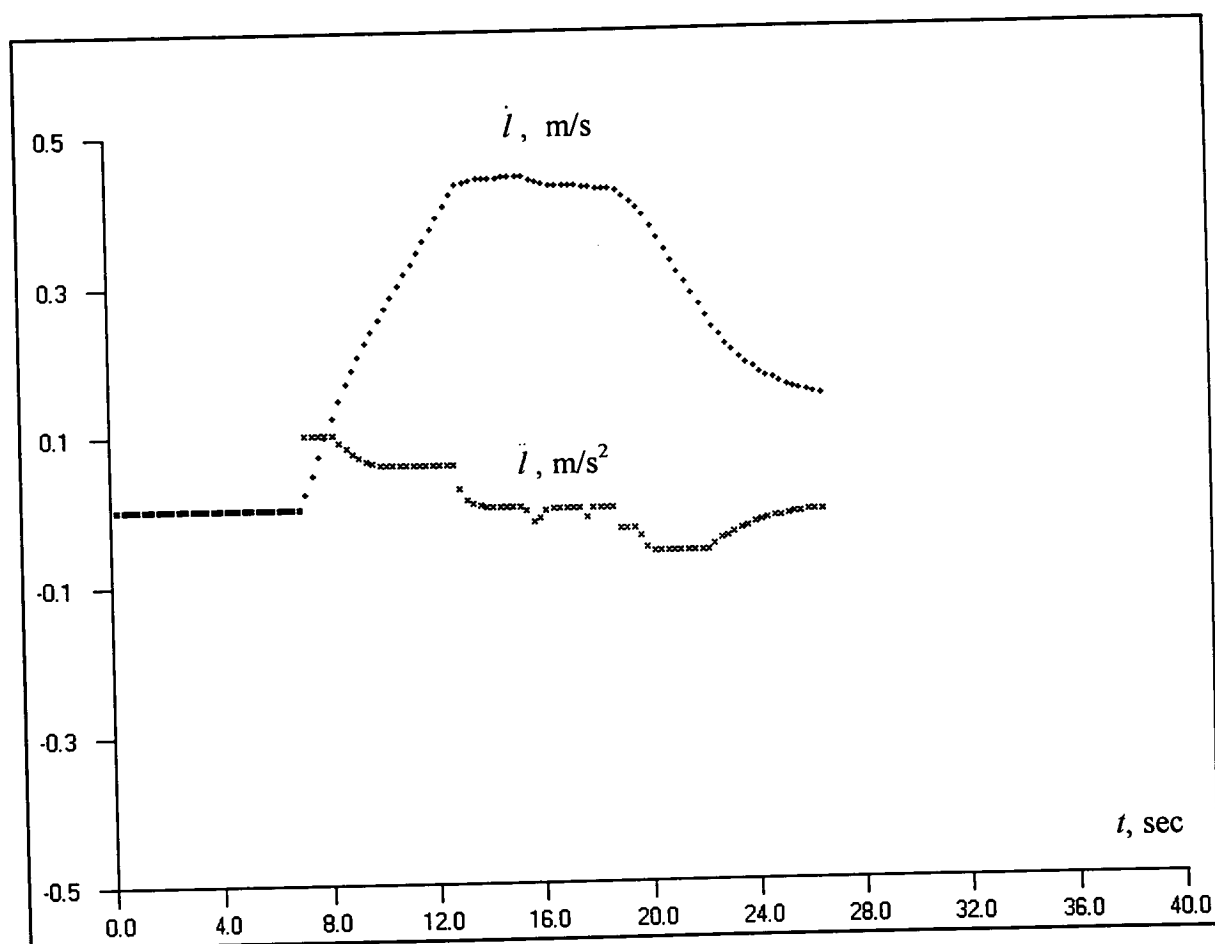


Figure 7.15
 Performance of the Fuzzy Controller
 $D_0 = 10$ m
 Velocity and acceleration of the spreader lowering

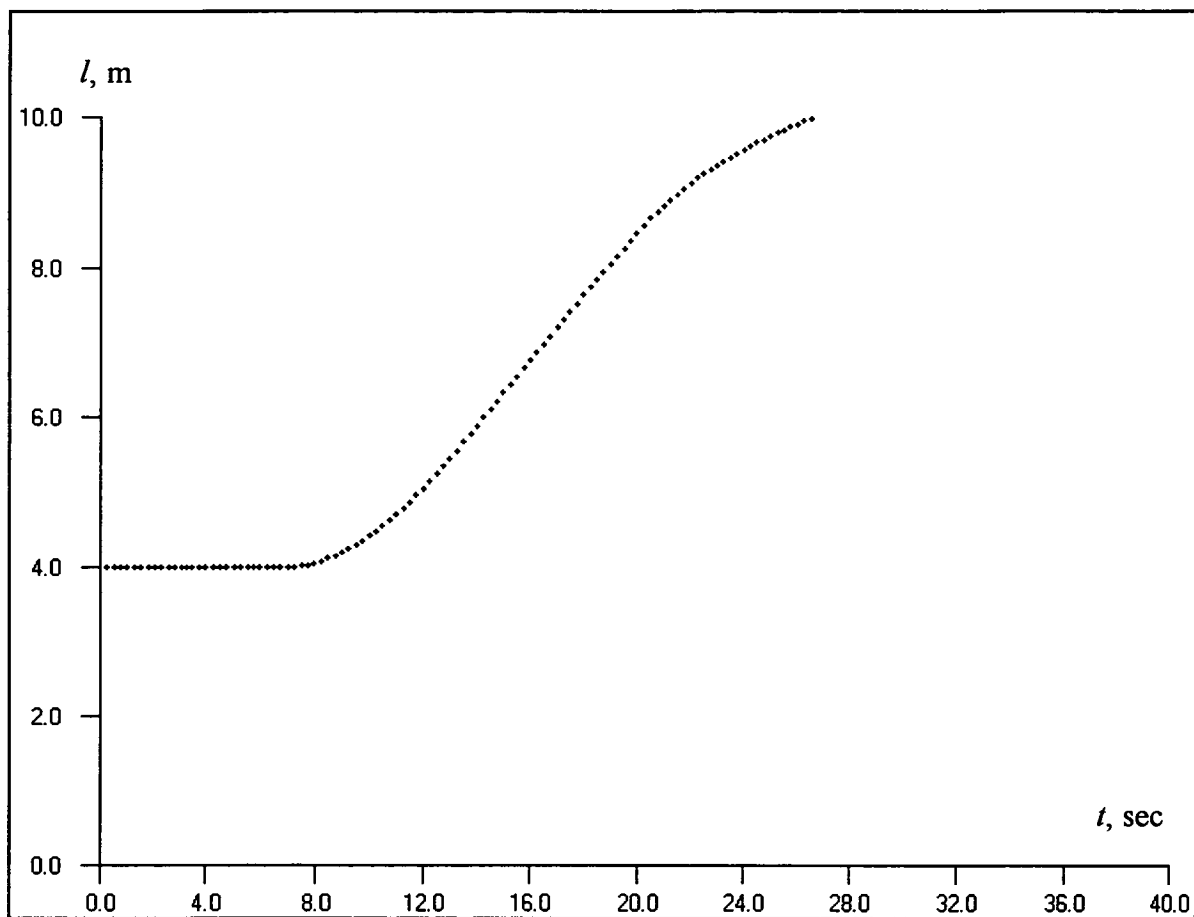


Figure 7.16
Performance of the Fuzzy Controller
 $D_0 = 10$ m
Cable length changes with time

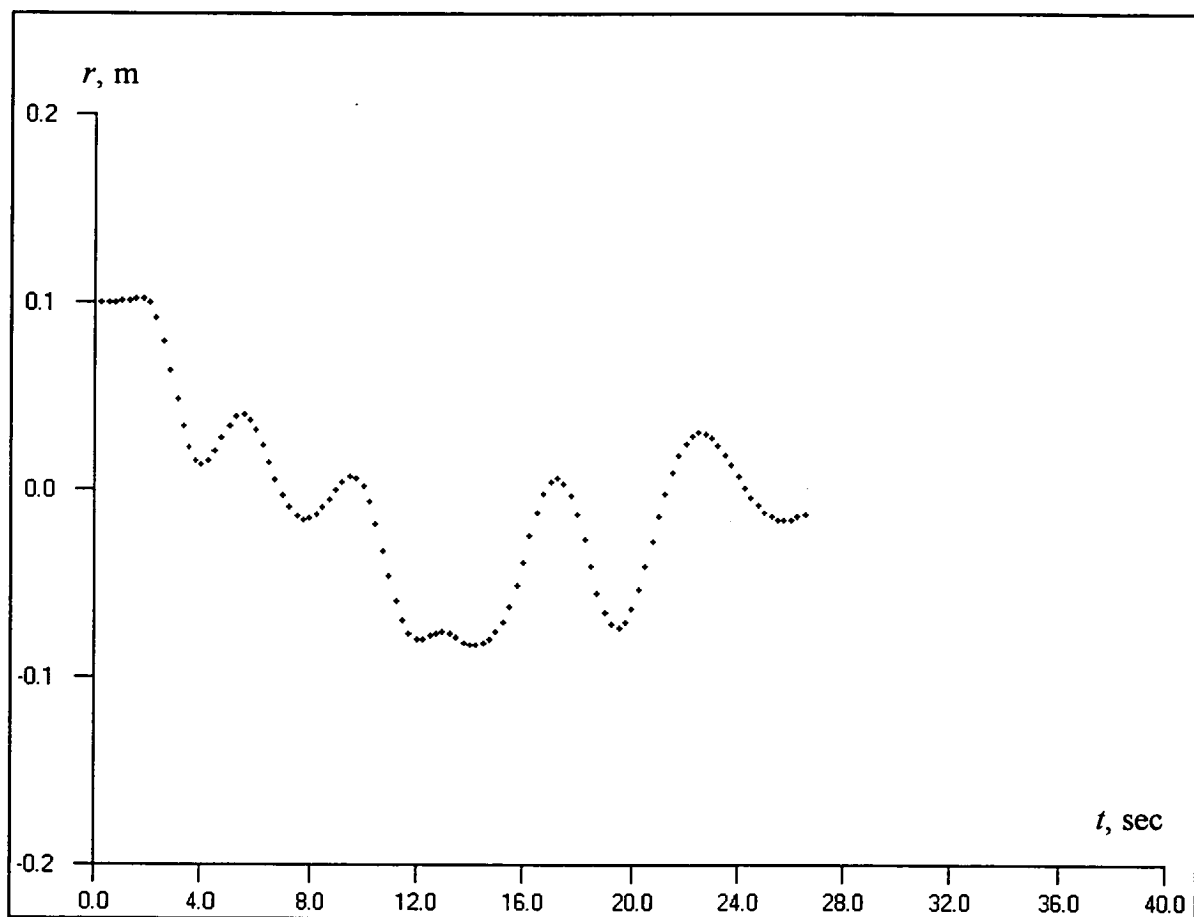


Figure 7.17
Performance of the Fuzzy Controller
 $D_0 = 10$ m
Magnitude of the spreader translation

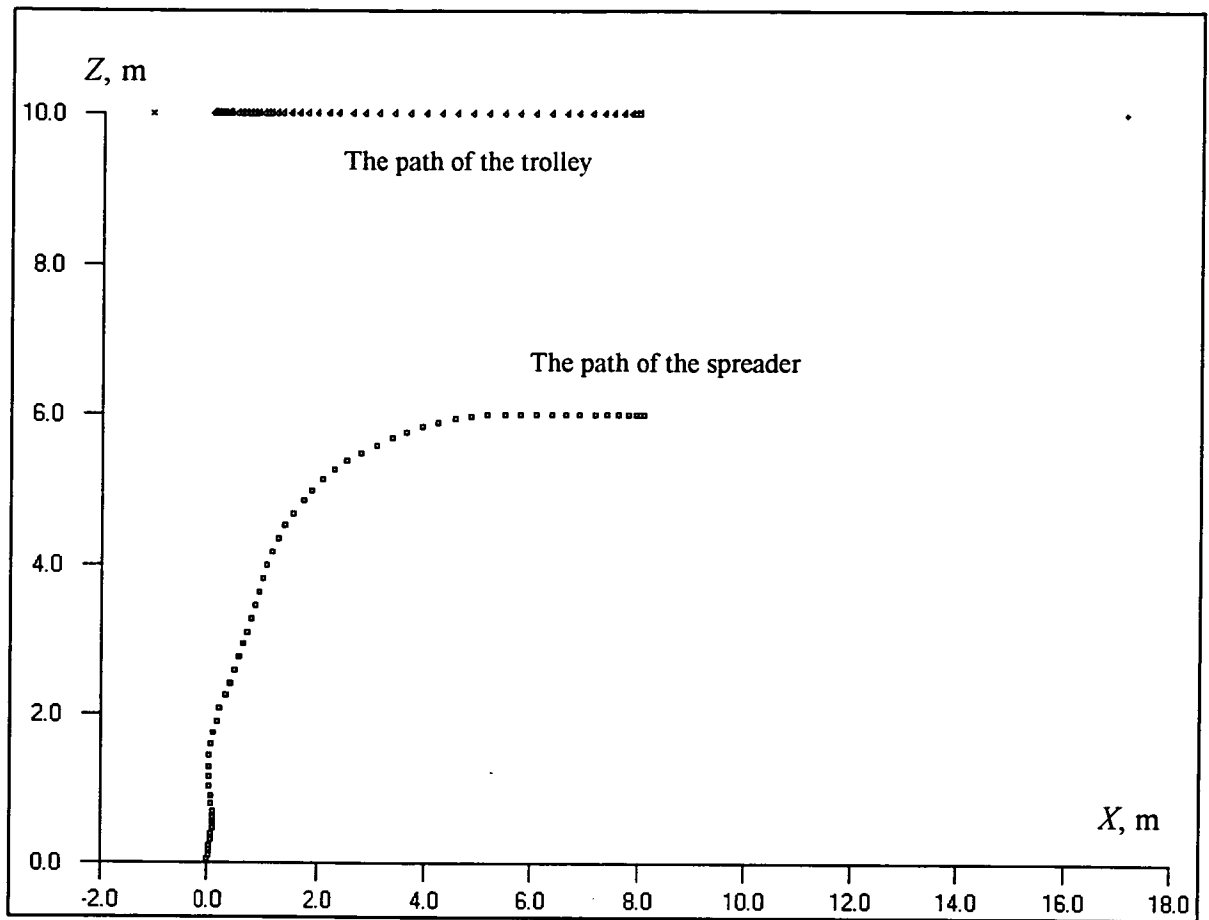


Figure 7.18
 Performance of the Fuzzy Controller
 $D_0 = 8 \text{ m}$
 Vertical plane

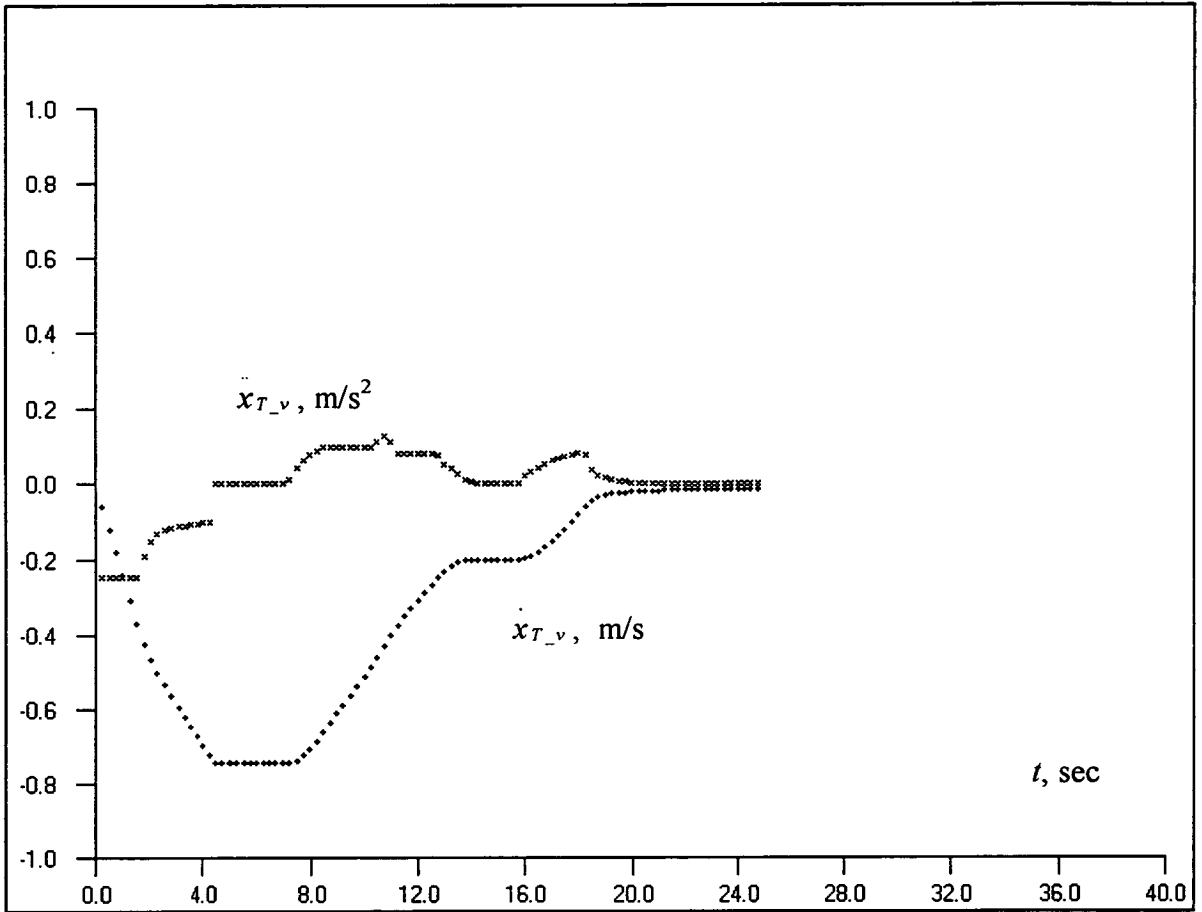


Figure 7.19
Performance of the Fuzzy Controller
 $D_0 = 8$ m
Velocity and acceleration of the trolley

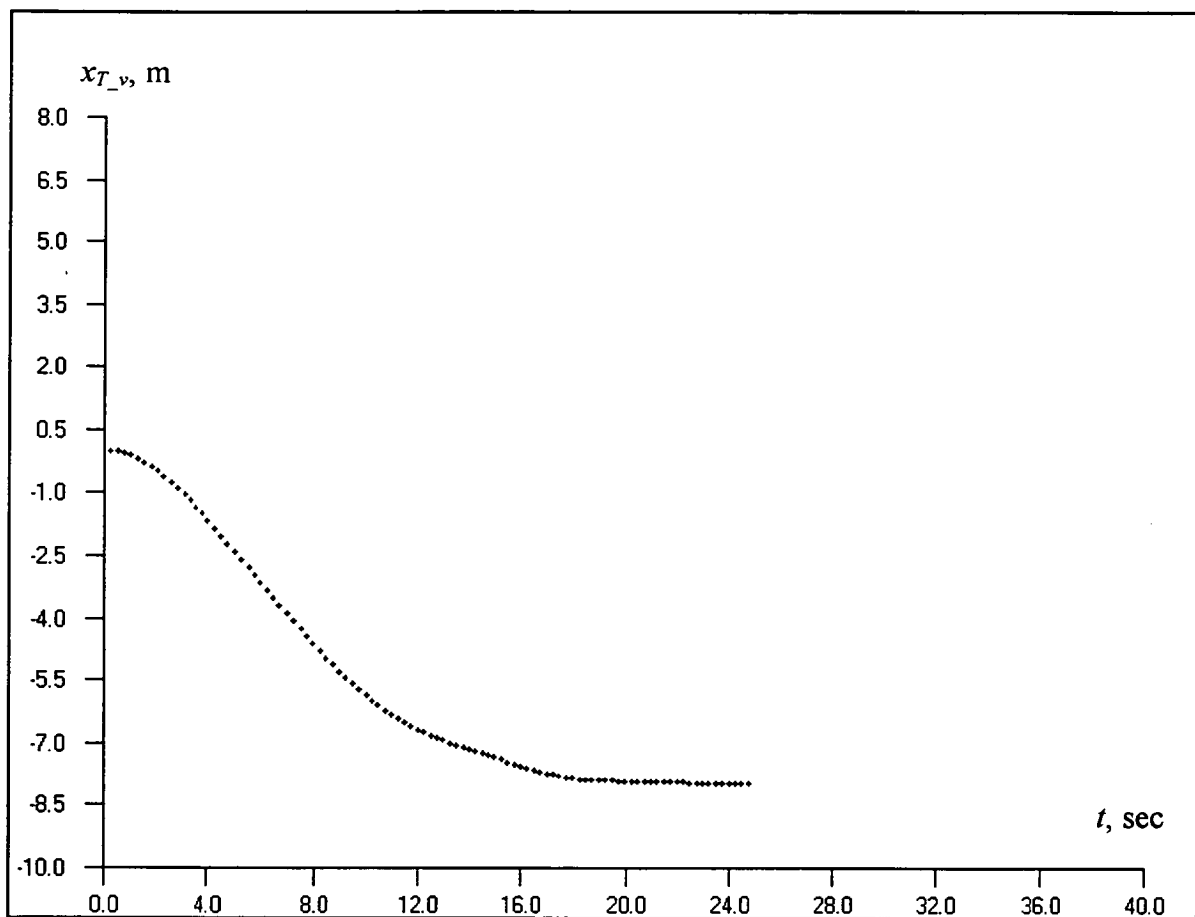


Figure 7.20
Performance of the Fuzzy Controller
 $D_0 = 8$ m
Displacement of the trolley

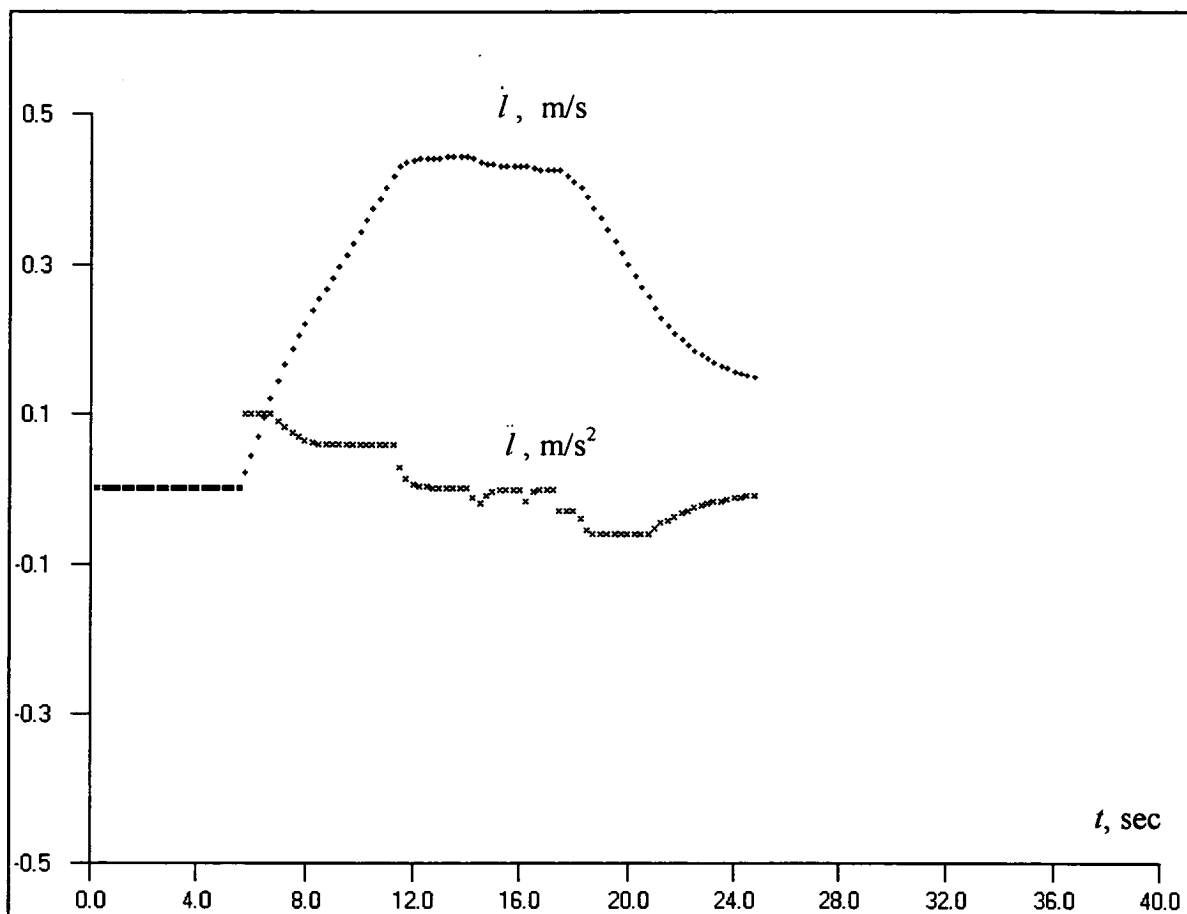


Figure 7.21
Performance of the Fuzzy Controller
 $D_0 = 8$ m
Velocity and acceleration of the spreader lowering

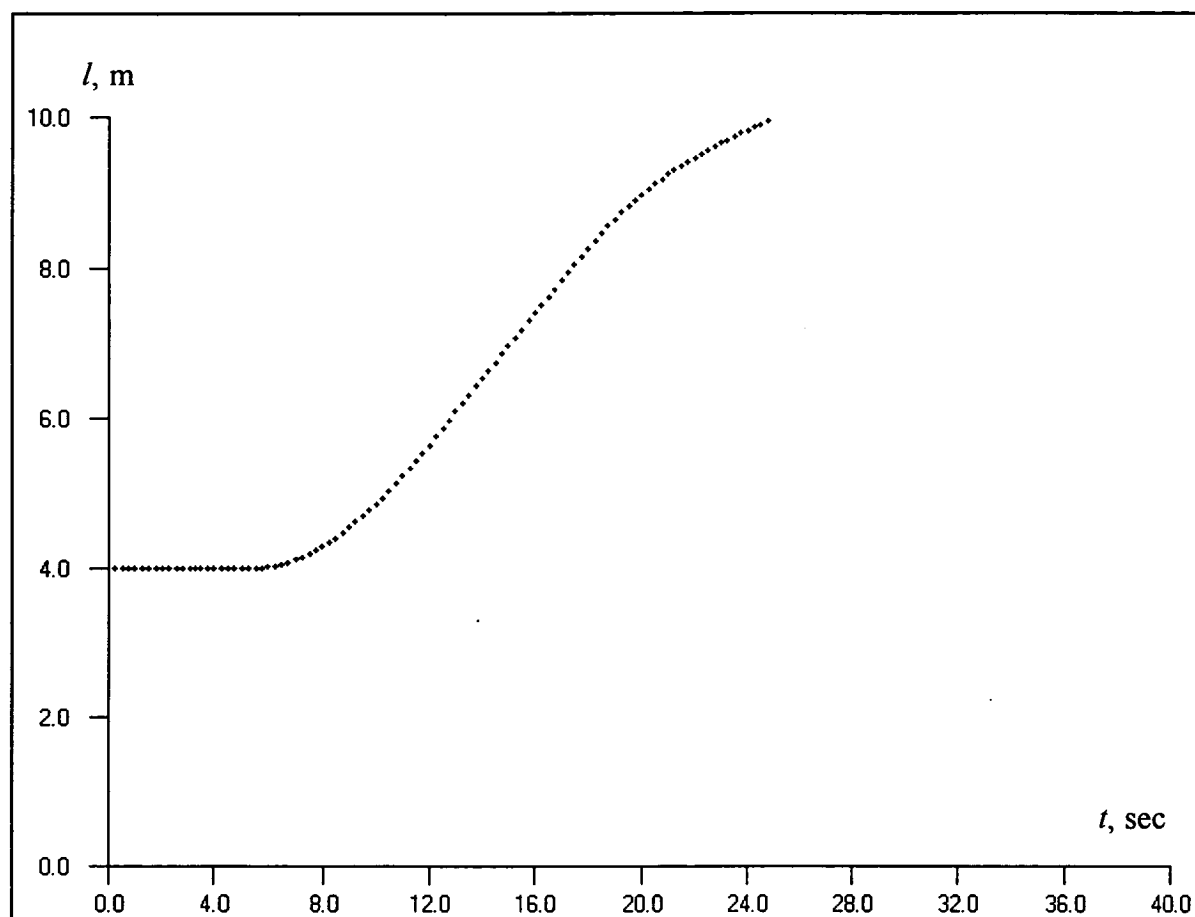


Figure 7.22
Performance of the Fuzzy Controller
 $D_0 = 8$ m
Cable length changes with time

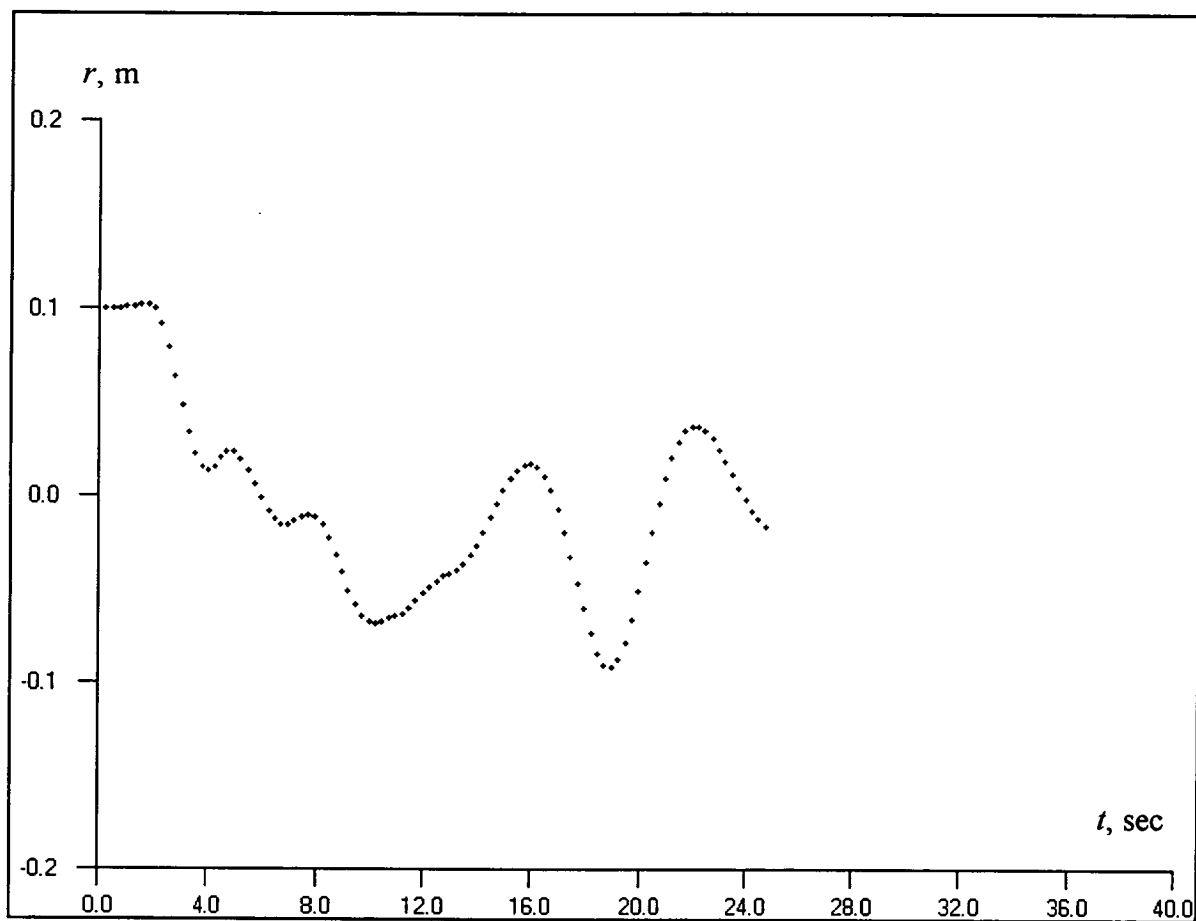


Figure 7.23
 Performance of the Fuzzy Controller
 $D_0 = 8$ m
 Magnitude of the spreader translation

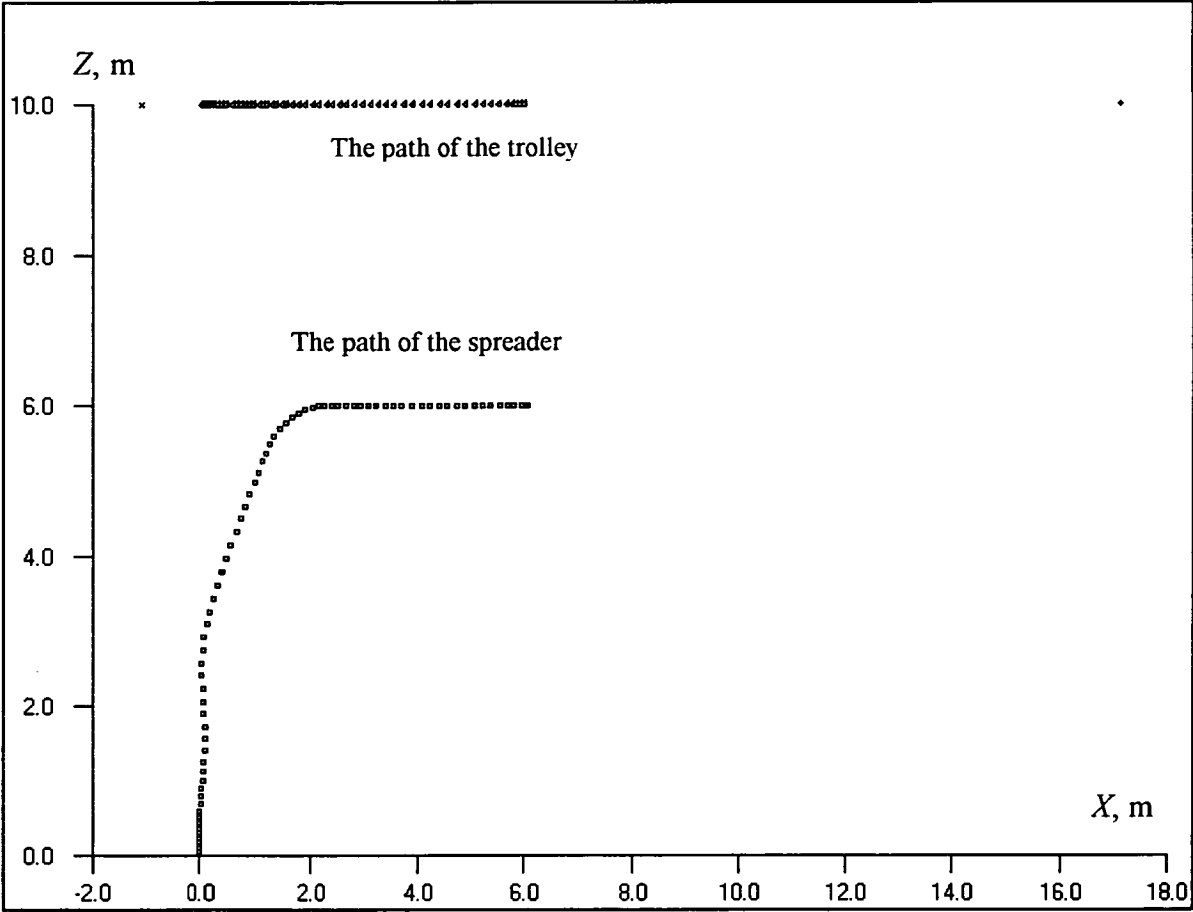


Figure 7.24
Performance of the Fuzzy Controller
 $D_0 = 6\text{ m}$
Vertical plane

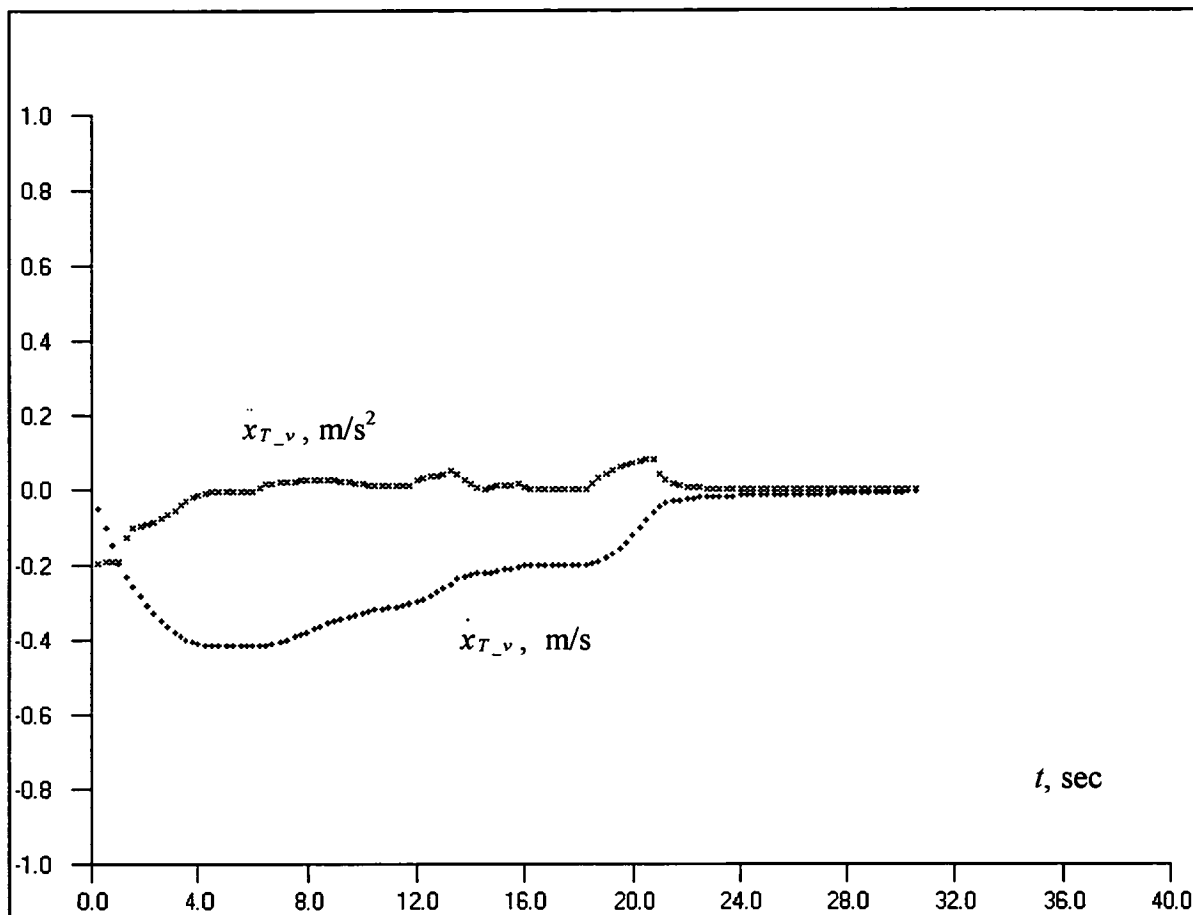


Figure 7.25
 Performance of the Fuzzy Controller
 $D_0 = 6 \text{ m}$
 Velocity and acceleration of the trolley

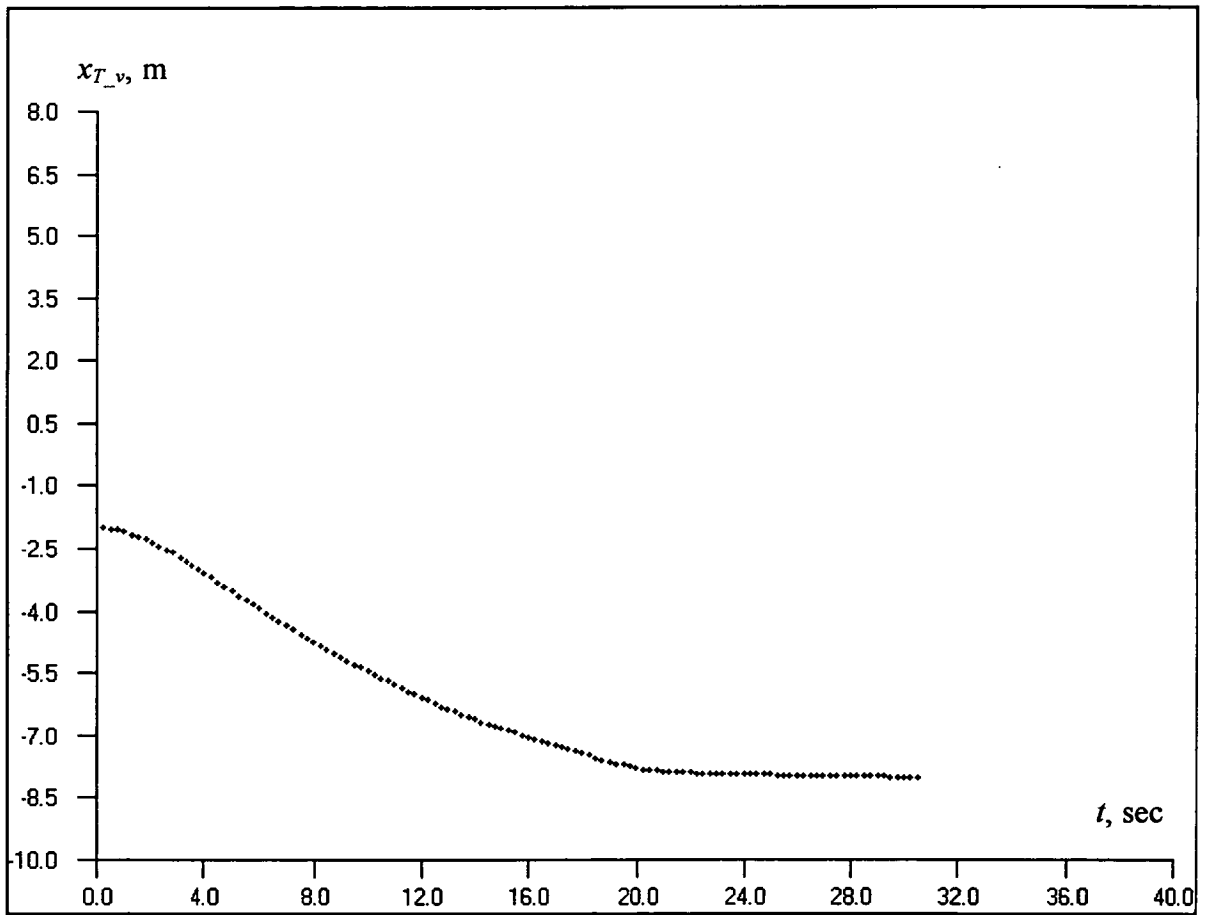


Figure 7.26
Performance of the Fuzzy Controller
 $D_0 = 6$ m
Displacement of the trolley

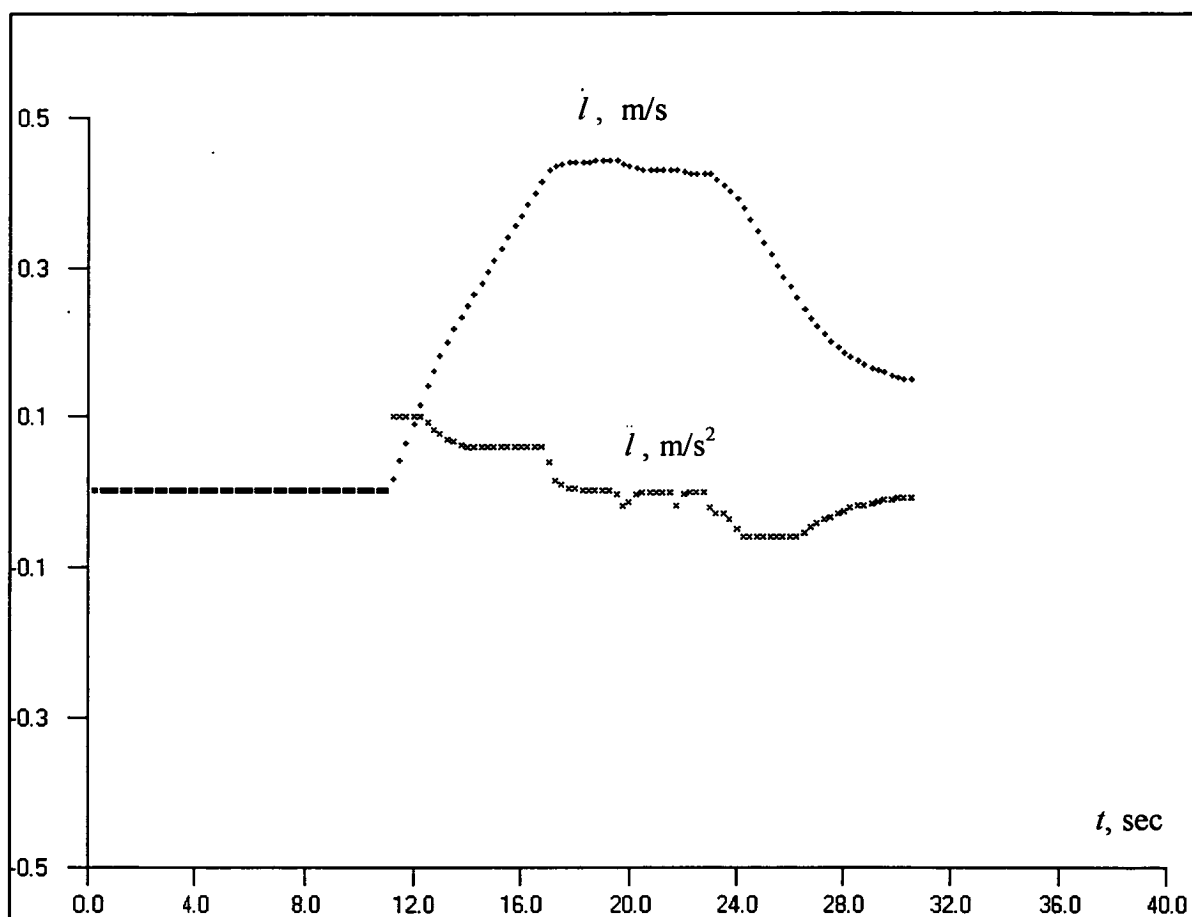


Figure 7.27
 Performance of the Fuzzy Controller
 $D_0 = 6$ m
 Velocity and acceleration of the spreader lowering

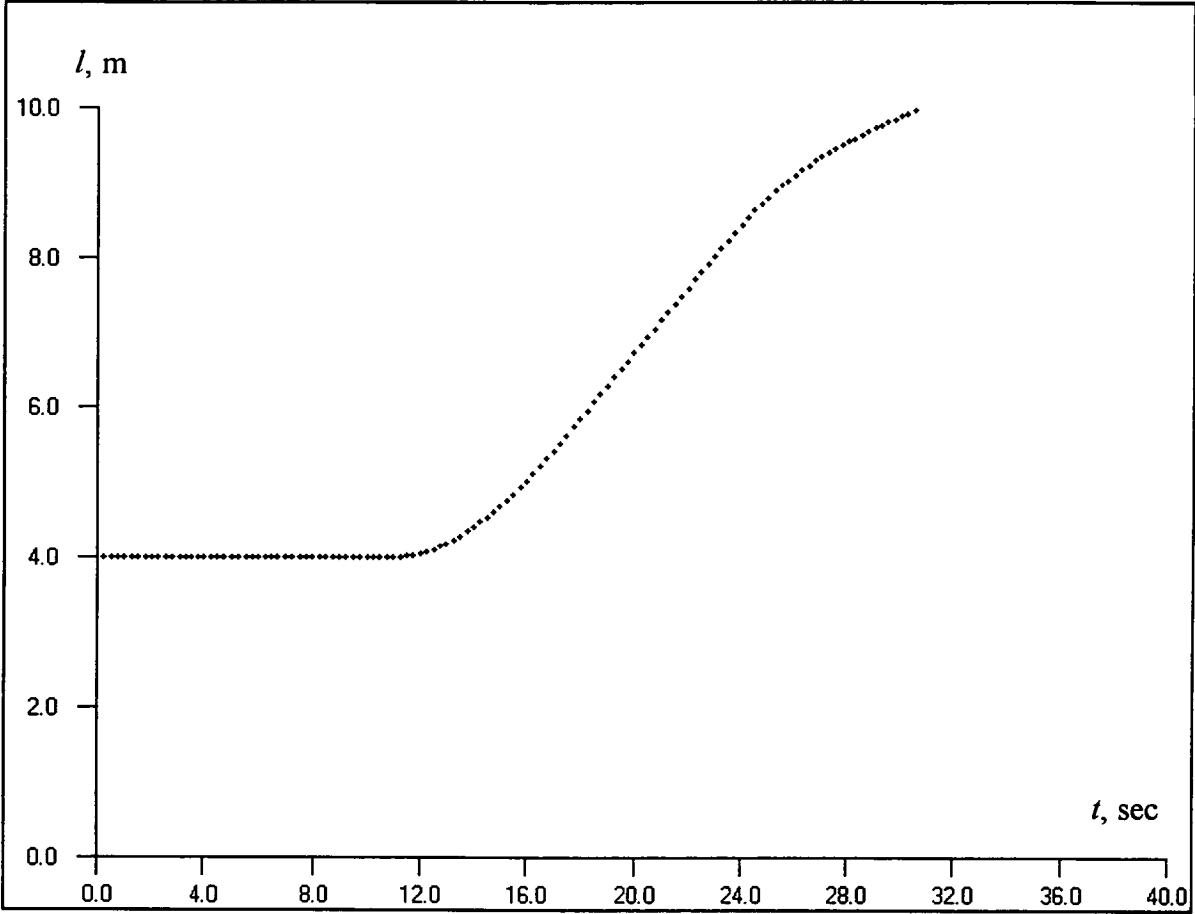


Figure 7.28
Performance of the Fuzzy Controller
 $D_0 = 6$ m
Cable length changes with time

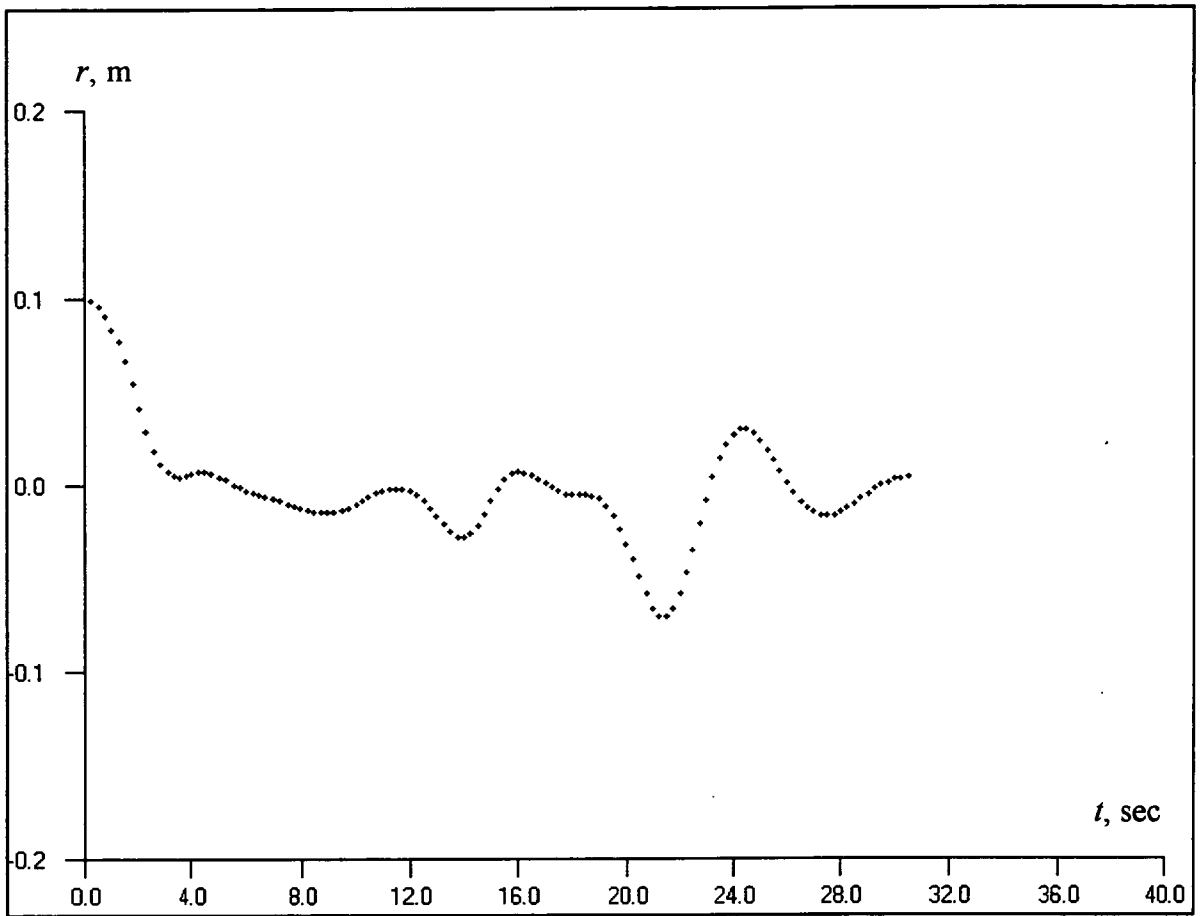


Figure 7.29
Performance of the Fuzzy Controller
 $D_0 = 6$ m
Magnitude of the spreader translation

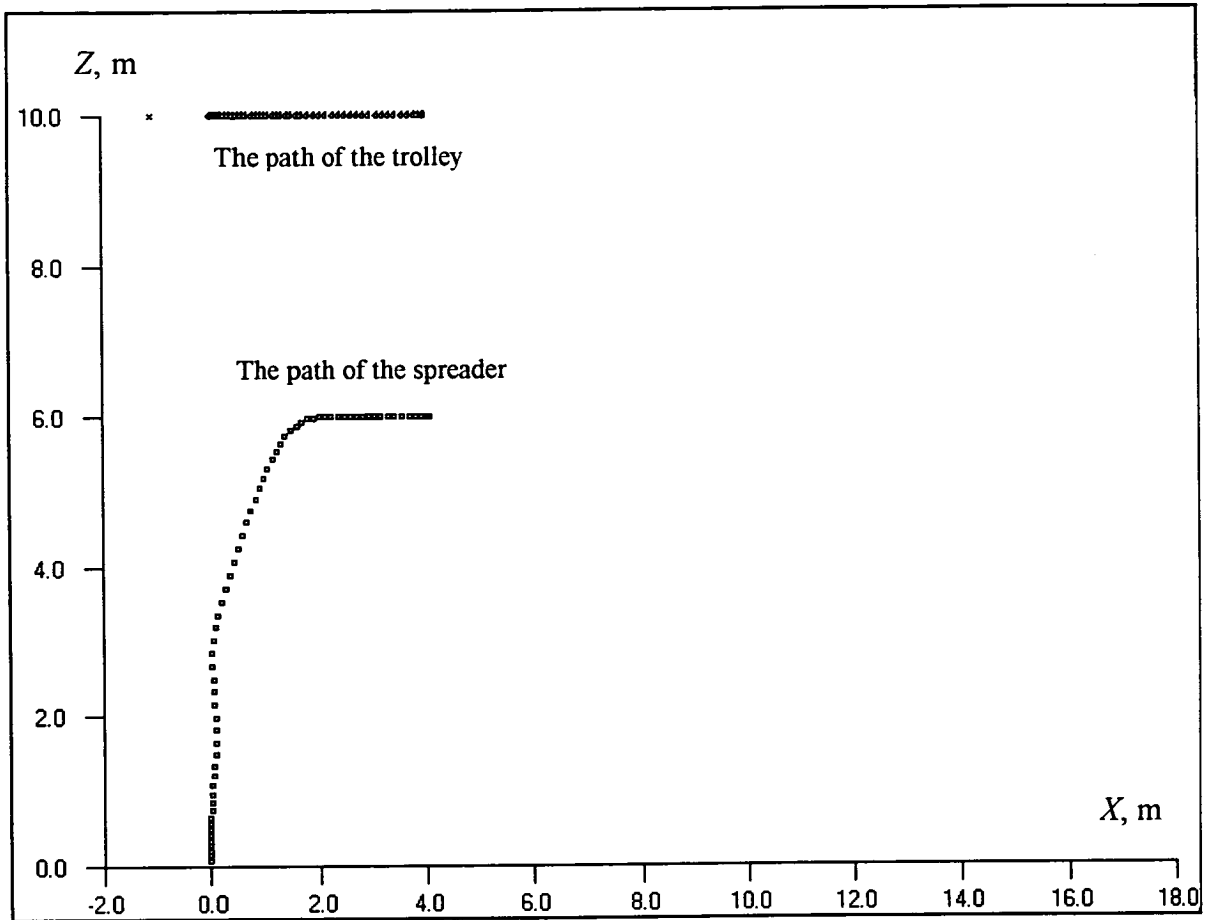


Figure 7.30
Performance of the Fuzzy Controller
 $D_0 = 4$ m
Vertical plane

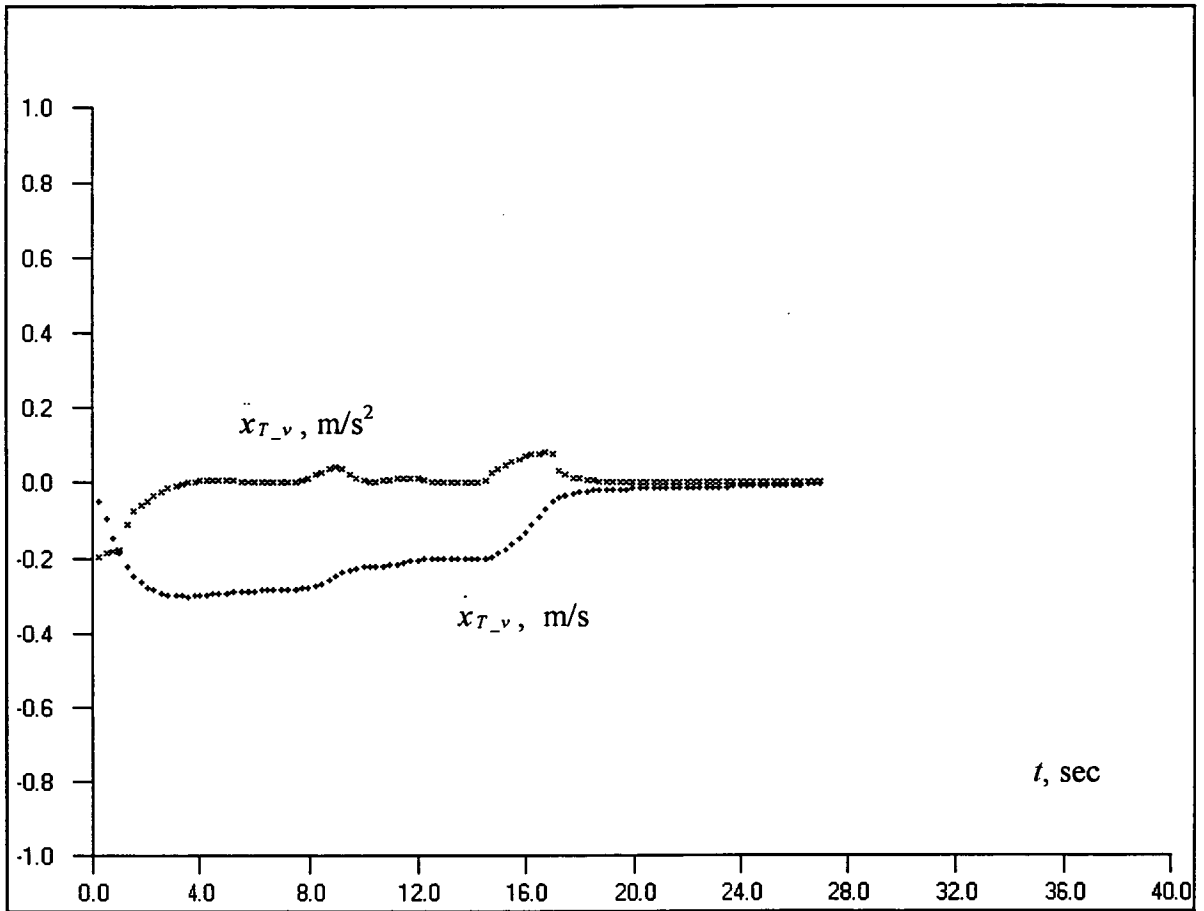


Figure 7.31
 Performance of the Fuzzy Controller
 $D_0 = 4$ m
 Velocity and acceleration of the trolley

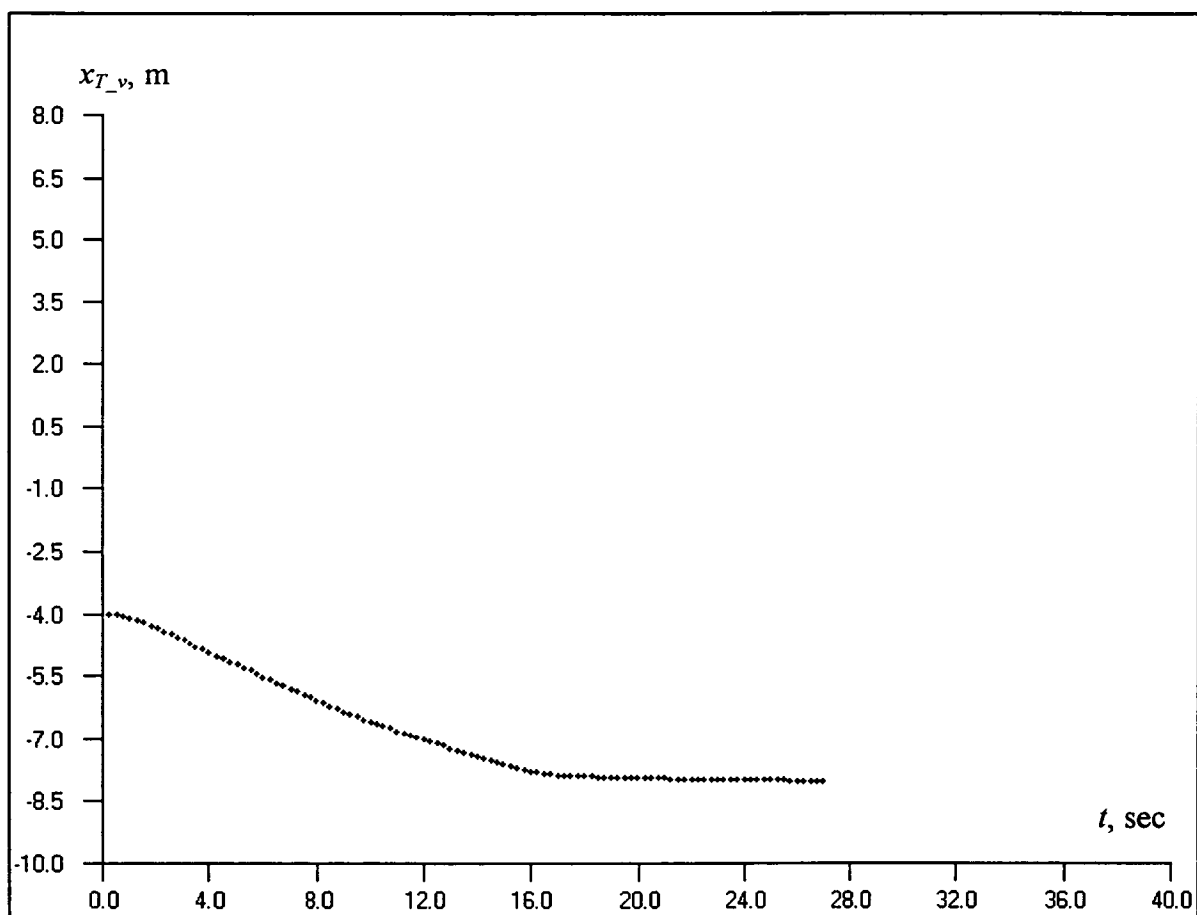


Figure 7.32
Performance of the Fuzzy Controller
 $D_0 = 4$ m
Displacement of the trolley

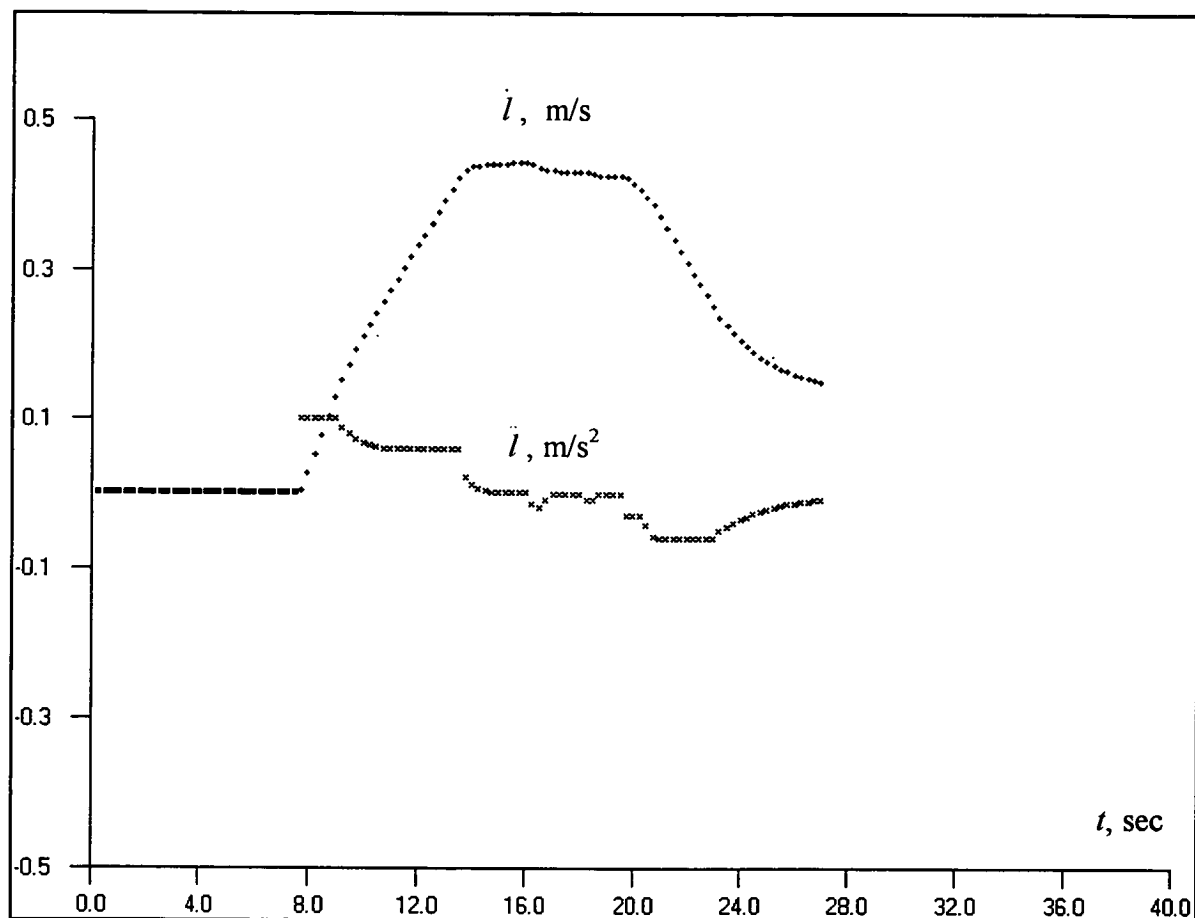


Figure 7.33
 Performance of the Fuzzy Controller
 $D_0 = 4$ m
 Velocity and acceleration of the spreader lowering

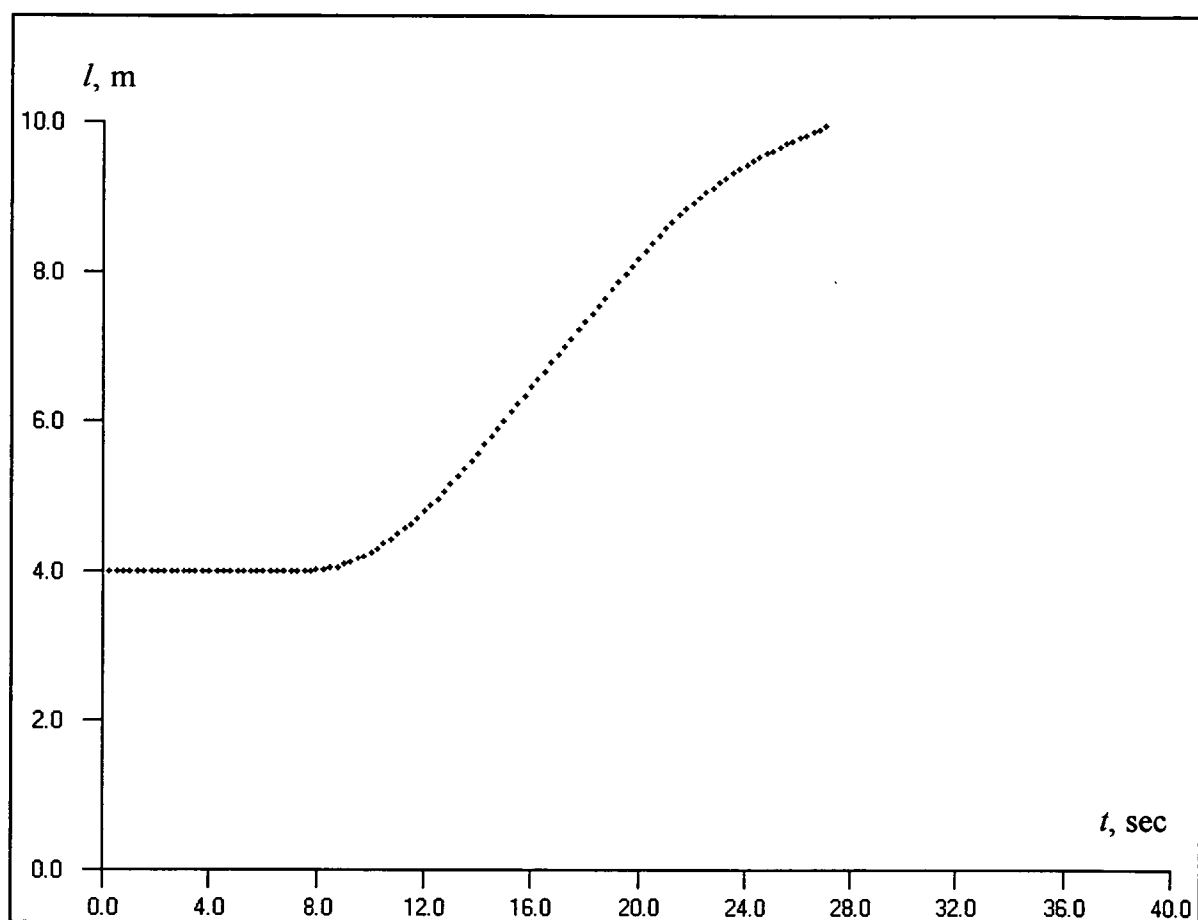


Figure 7.34
Performance of the Fuzzy Controller
 $D_0 = 4$ m
Cable length changes with time

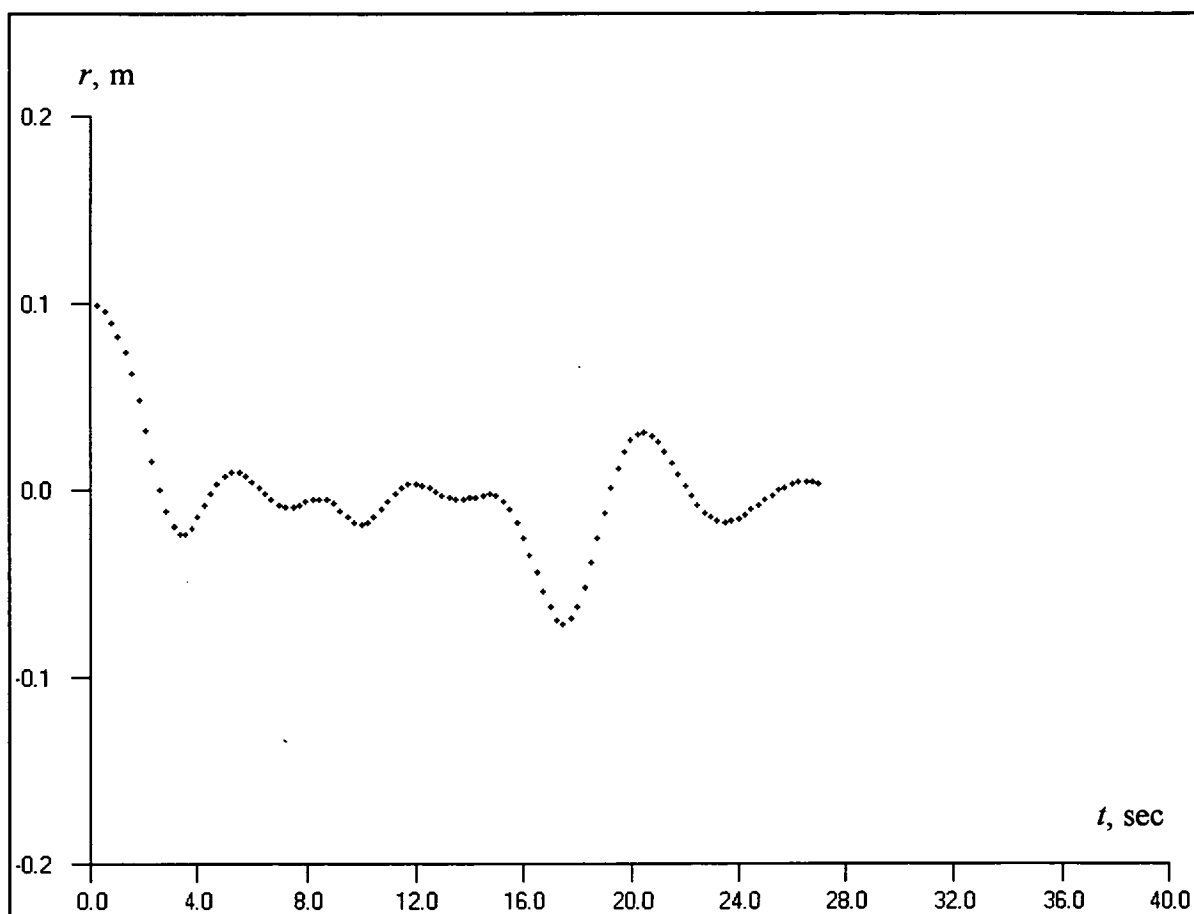


Figure 7.35
Performance of the Fuzzy Controller
 $D_0 = 4$ m
Magnitude of the spreader translation

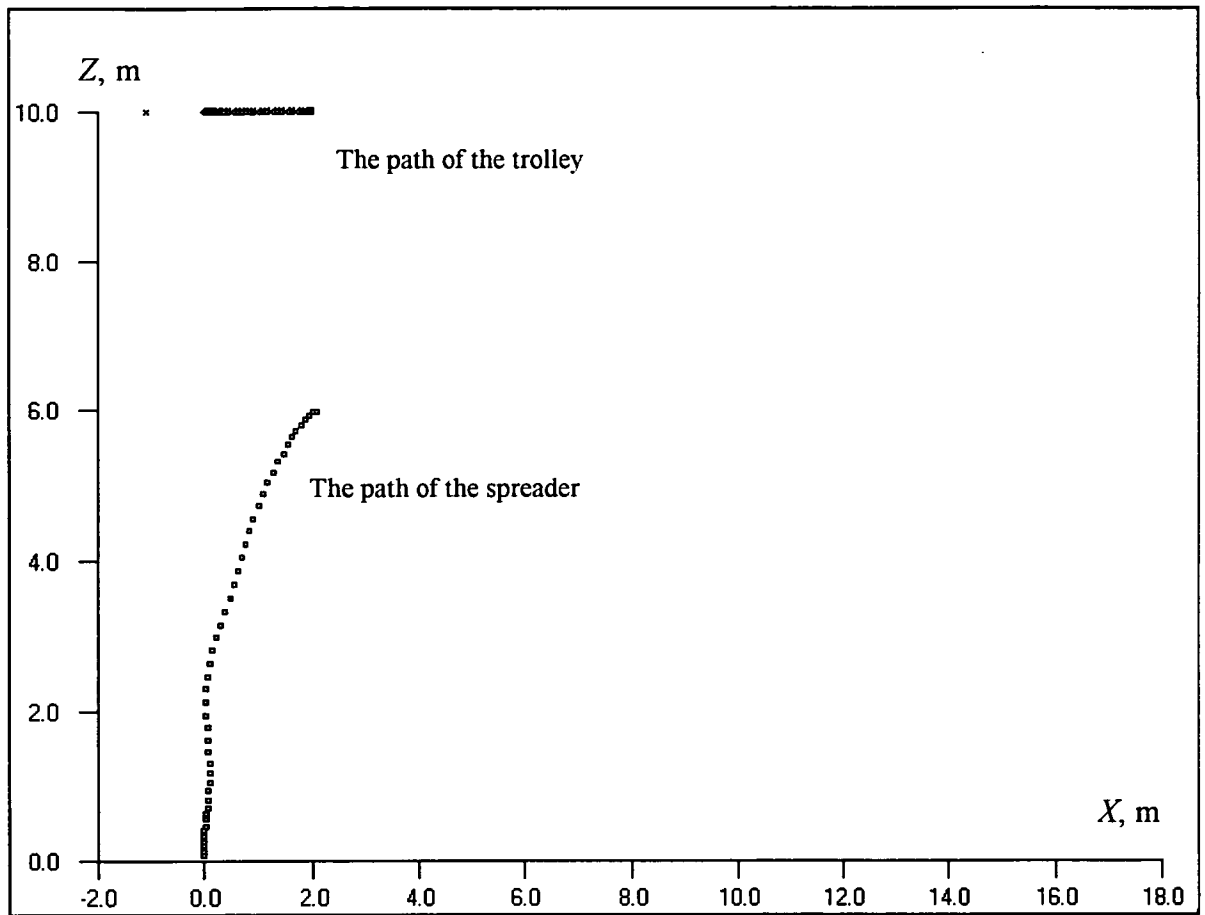


Figure 7.36
Performance of the Fuzzy Controller
 $D_0 = 2 \text{ m}$
Vertical plane

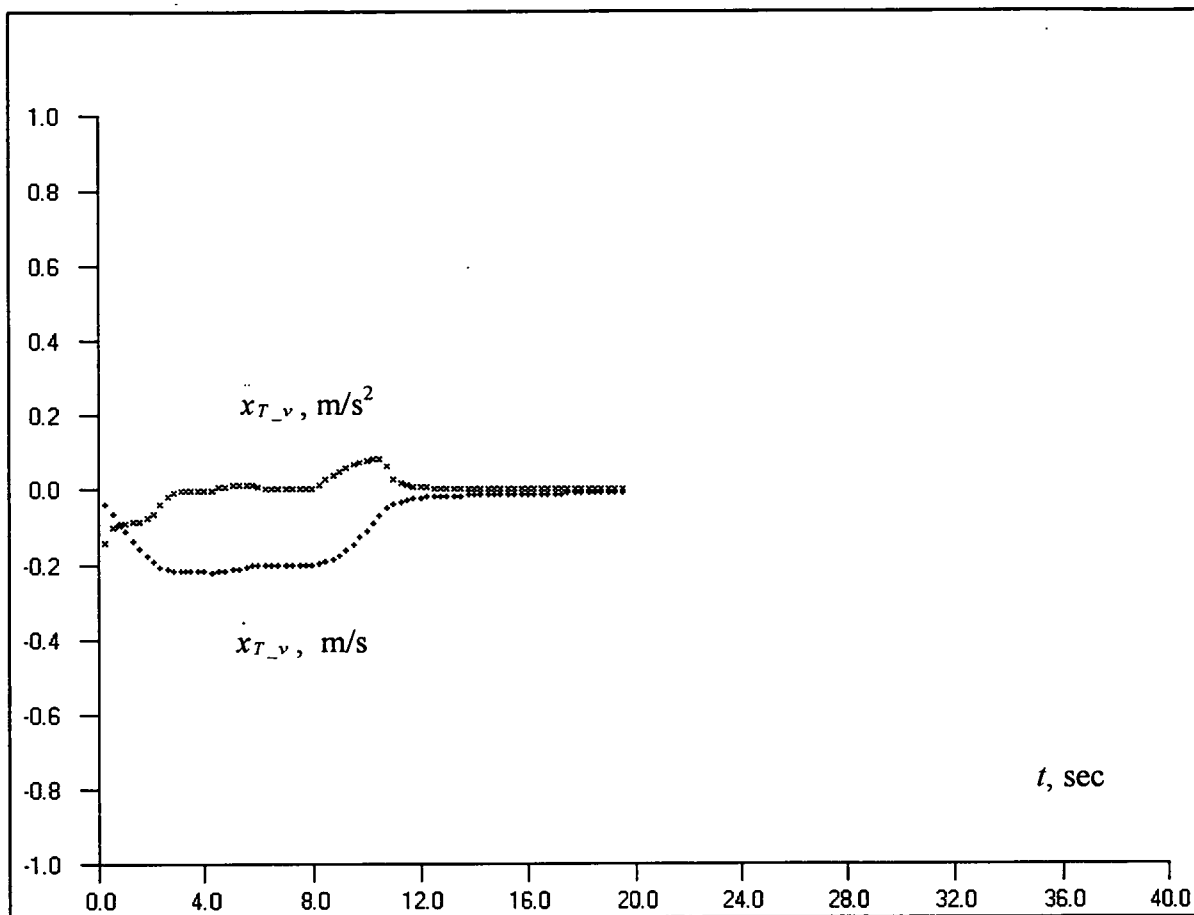


Figure 7.37
 Performance of the Fuzzy Controller
 $D_0 = 2$ m
 Velocity and acceleration of the trolley

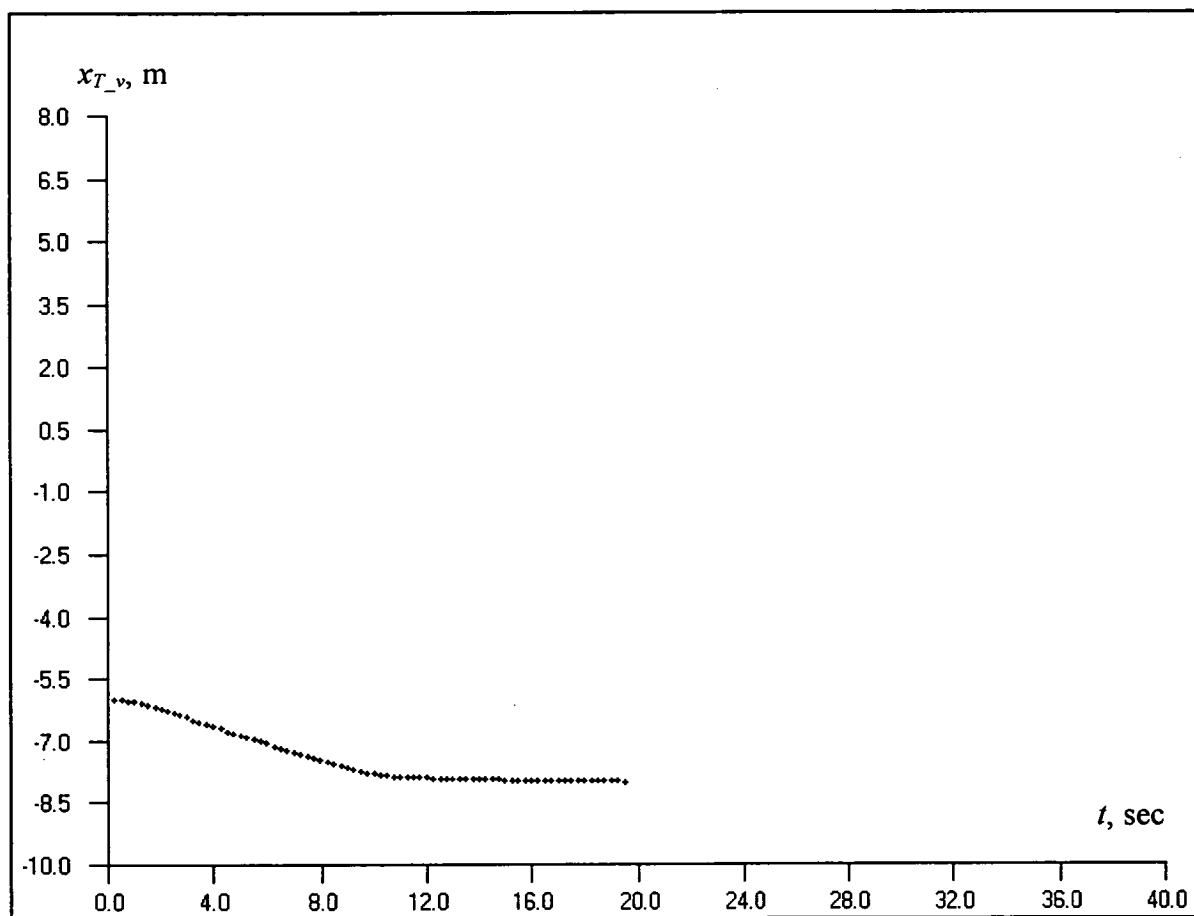


Figure 7.38
Performance of the Fuzzy Controller
 $D_0 = 2$ m
Displacement of the trolley

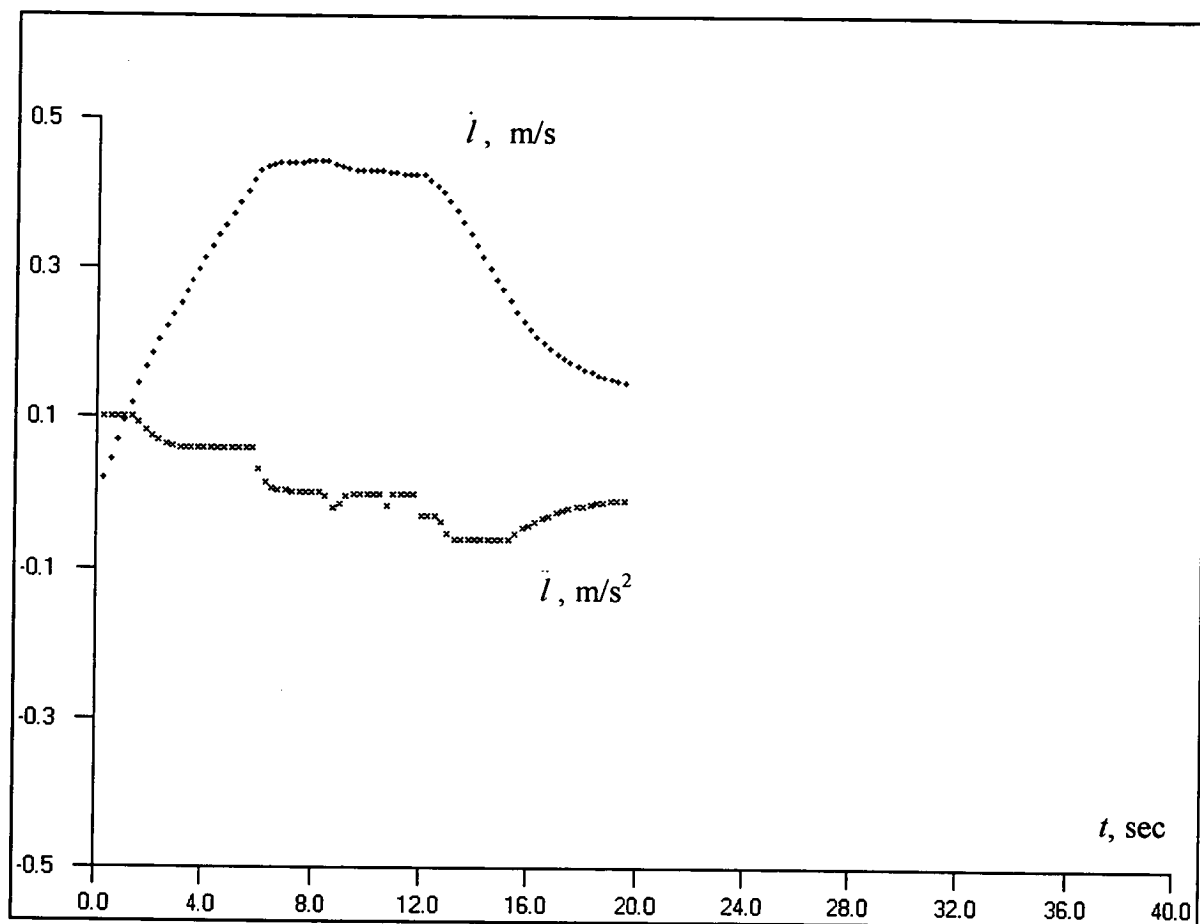


Figure 7.39
 Performance of the Fuzzy Controller
 $D_0 = 2$ m
 Velocity and acceleration of the spreader lowering

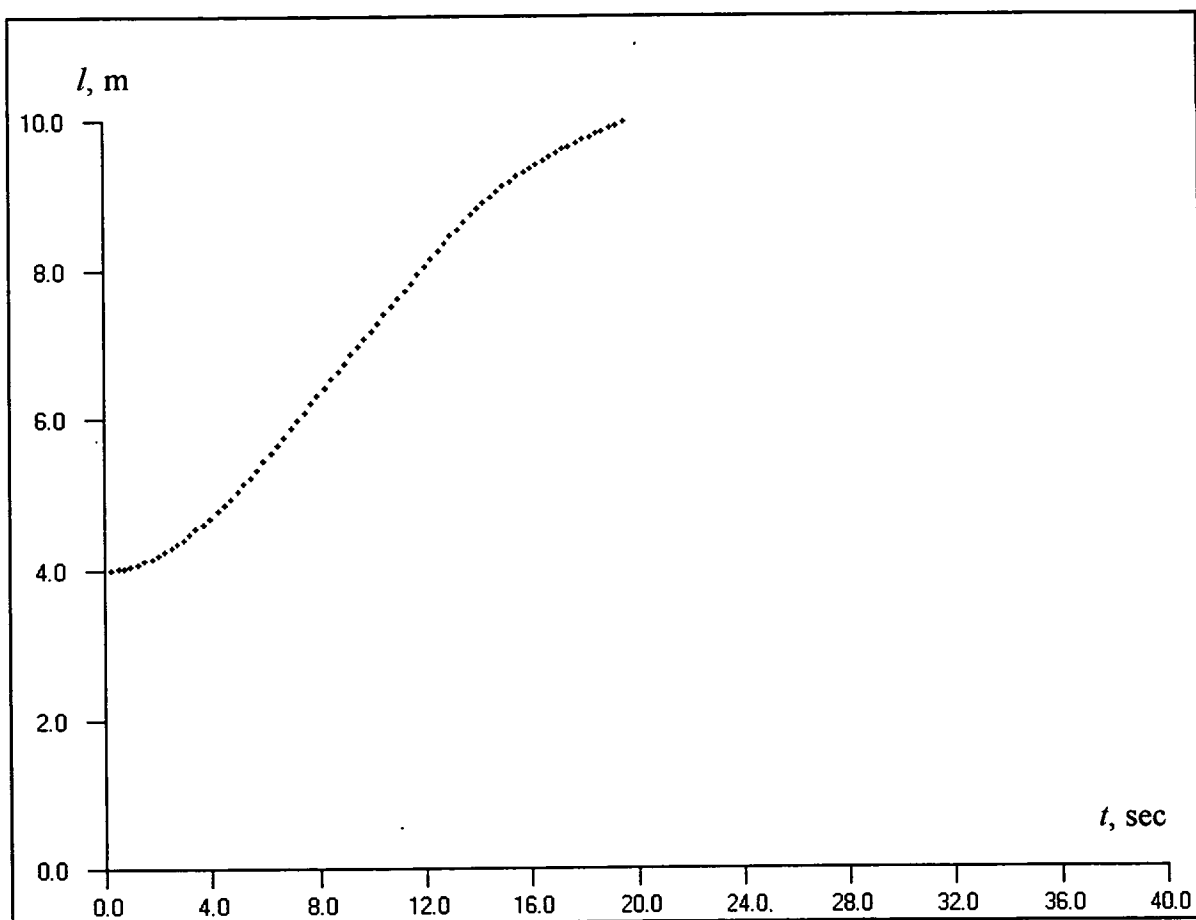


Figure 7.40
Performance of the Fuzzy Controller
 $D_0 = 2$ m
Cable length changes with time

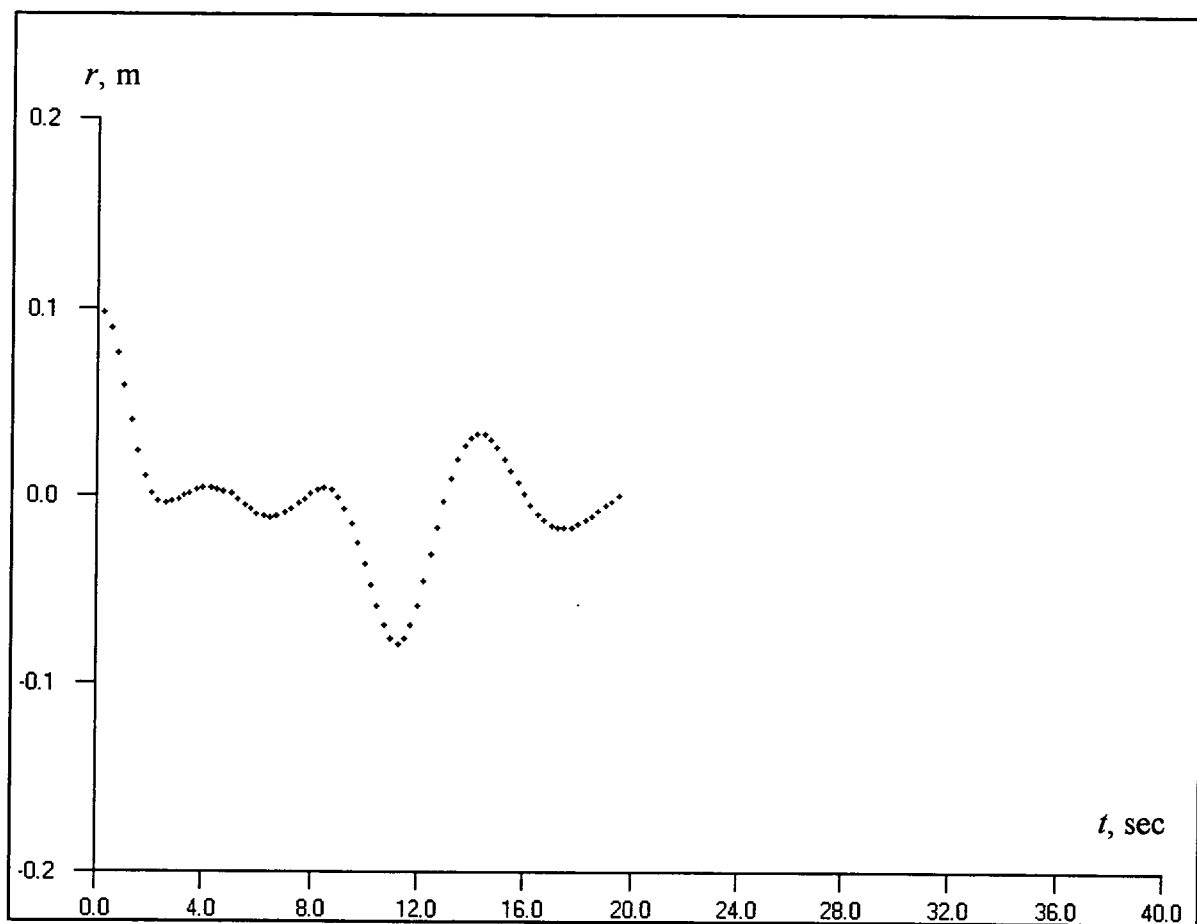


Figure 7.41
Performance of the Fuzzy Controller
 $D_0 = 2$ m
Magnitude of the spreader translation

Chapter 8

Conclusions and Plans for Future Research

8.1 Conclusions

This thesis studied the kinematics and dynamics of RTG cranes and proposed a complete three dimensional mathematical model for them which can be used as a basis for a simulation of a control system. The most important achievements were:

- The geometry of a typical multi-cable spreader suspension system was investigated and a novel exact relationship between the hoist of the spreader due to rotation and an angle of its rotation was established in a form of an algorithm.
- The unknown hitherto phenomenon called geometrical extension was discovered, and proved mathematically by means of the above algorithm. This means that an additional cable stretch or compression takes place in the system when the spreader is translated from its equilibrium and then rotated. This is due only to the geometry of multi-cable spreader suspension, and is on top of stretch due to static and dynamic effects.
- It was shown that generally the geometrical extension is different in all four cables, thus the tensile load due to system geometry is asymmetrically spread across the cables.
- The explicit approximate formula relating the spreader hoist due to parameters of its translation and rotation and trolley and spreader dimensions was derived. It proved to be very accurate over realistic values of angle of spreader rotation.
- The three-dimensional RTG crane model, which takes into consideration the gantry and the trolley movements and hoist/lowering of the spreader, was derived for both Single-Cable and Multi-Cable spreader suspension. The latter is based on use of the

explicit formula mentioned above and takes into account the spreader hoist due to rotation.

- The numerical experiments using the Multi-Cable Model were described and analysed, and the results seem to be intuitively reasonable.
- The RTG crane model built using the commercial ADAMS package was described and the results of the ADAMS and Multi-Cable models were compared and found to be almost identical.
- The data obtained from the site-experiment at Felixstowe port with the Paceco Transtainer RTG crane was given and the attempt was made to replicate this experiment using the Multi-Cable Model. The results of this preliminary work correlate well with the theory developed here.
- The concepts for two-dimensional time-optimal fuzzy controller for RTG cranes were discussed. The fuzzy controller for accurate positioning of the spreader was suggested and it was verified on the Multi-Cable Model simulation.

8.2 Future research

Further practical work is planned in order to validate the theoretical model described. Once fully validated, it can be used to design and test the controller for RTG cranes with ultimate purpose of building a practical controller.

First steps have already been made by building a time-optimal two-dimensional fuzzy controller. There are the following possible directions for its further development which are now under consideration:

- Better tuning for improving the controller performance.
- Reducing the spreader oscillations by applying the acceleration of the trolley in the direction opposite to this oscillations (this is the method used by 'non-swing' drivers).
- Developing a fuzzy 'swing-controller' which uses more progressive 'swing-driving' method.

- Considering the possibilities of 'Super-RTG Crane' control which involves not only the simultaneous trolley movements and spreader lowering, but gantry translations and rotations and spreader hoist.

In summary, the work so far as it stands provides a sound basis for building an actual working crane controller and opens the possibilities for further theoretical work. Current and future experimental work on a laboratory scale model (1/10) is intended to provide a demonstration facility so that technology based upon the theoretical advances explained in this thesis can be transferred to the appropriate industrial sector as quickly as possible.

References

- [1] **Abeid AM, Moustafa KAF, Emara-Shabaik HE.** Electromechanical Modelling of Overhead Cranes. *INT. J SYSTEMS SCI.*, Vol.23, No12, 1992, pp.2155-2169.
- [2] **Albus JS, Bostelman R, Dagdlakis N.** The NIST SPIDER. A Robot Crane. *Journal of Research of the National Institute of Standards and Technology*, Vol.97, Number 3, May-June 1992, pp.373-385.
- [3] **Albus JS, Goodwin K.** The NIST SPIDER - A Robot Crane. National Institute of Standards and Technology, U.S.A, Research Report, 25 March 1992.
- [4] **Al-Garni AZ, Moustafa KAF, Javeed Nizami SSAK.** Optimal Control of Overhead Cranes. *Control Engineering Practice*, Sept 1995, Vol.3, No.9, pp.1277-1284.
- [5] **Arocena JI.** Trip in Support of "Torque Ripple Control of Compliant Direct Drive Robots" GR/G 17516, 26 August 1991.
- [6] **Barrett D, Burns RS.** Fuzzy Control and its Application to a Servomotor. *Neural Networks and Fuzzy Logic in Measurement and Control*, colloquium proceedings, 10th March 1993.
- [7] **Baxter J W, Bumby J R.** Modifying the Fuzzy Fit Vector to Avoid Unwanted Outputs, With Particular Reference to Autonomous Vehicle Control. *Neural Networks and Fuzzy Logic in Measurement and Control*, colloquium proceedings, 10th March 1993.
- [8] **Benhidjeb A, Gissinger GL.** Fuzzy control of an overhead crane performance comparison with classic control. *Control Engineering Practice*, Dec 1995, Vol.3, No.12, pp.1687-1696.
- [9] **Bohlman MT, Dougherty EJ, Simmons III GR.** GRAIL - The Container Terminal of the Future. The Society of Naval Architects and Marine Engineers, Spring Meeting/STAR Symposium, New Arleans, Lousiana, April 12-15, 1989, S2-1.
- [10] **Bostelman R, Dagdlakis N, Albus JS.** A Robotic Crane System Utilizing the Stewart Platform Configuration. National Institute of Standards and Technology, U.S.A, Research Report, June 1992.
- [11] **Boustany F, D'Andréa-Novel B.** Adaptive Control of Non-Completely Controlled Mechanocal Systems Using Dynamic Feedback Linearization and Estimation Design. *International Journal of Adaptive Control and Signal Processing*, Vol.6, 1992, pp.589-610.
- [12] **Bradley RA.** Partial Periodic Trim Algorithm for Application to Advanced Mathematical Models of Rotocraft Vehicles. *Kinematics and dynamics of multi-body systems seminar*, IMechE HQ, 6 November 1992.
- [13] **Bryfors U.** Electronic Positioning of Containers Cranes. *ABB Crane Control. Freight International*, pp.41-44, 1992.
- [14] **Butler H, Honderd G, Van Amerongen J.** Model reference adaptive control of a gantry crane scale model. *IEEE Control Systems Magazine*, Jan 1991, Vol.11, No.1, pp.57-62.
- [15] **Cannon RH, Schmitz E.** Precise Control of Flexible Manipulators. "Robotics Research", the 1st International Symposium, MIT, 1984, pp.841-861.

- [16] **Cansever G, Engin SN, Ozguven OF, Uzam M.** Fuzzy logic controller in modern control systems: an industrial application. *Advances in Modelling & Analysis C: Systems Analysis, Control & Design*, 1994, Vol.43, No.1, pp.41-63.
- [17] **Caron B, Perrard P, Rozinoer J.** Reference model control of a variable length overhead crane. *IFAC Proceedings Series*, 1990, No.15., pp.475-480.
- [18] **Cartmell MP, Alberts TE.** Simultaneous Passive and Active Dissipation Mechanisms in a Gantry Crane and Spreader System. Working paper, Department of Mechanical Engineering of Edinburgh University, 1995.
- [19] **Cashmore RD, Halikias GD, Wilson DA.** Adaptive and Robust Control of a Flexible Manipulator System. 1993 SERC/INSTMC Postgraduate Research Symposium, 30-31 March 1993, proceedings.
- [20] **Casper B.** Electric Reactions - Cargo Handling. *Cargo Systems*, August 1989, pp.69-71.
- [21] **Champion V.** Swayed by the Arguments? - Cargo Handling, *Cargo Systems*, August 1989, pp.63-67.
- [22] **Crolla DA, Horton D, Firth G.** Applications of Multibody Systems Software to Vehicle Dynamics Problems. Kinematics and dynamics of multi-body systems seminar, IMechE HQ, 6 November 1992.
- [23] **Dagalakis NG, Albus JS, Goodwin KR, et al.** Final Report on Robot Crane Technology. NIST Technical Note 1267, National Institute of Standards and Technology, U.S.A, 1989.
- [24] **Daley S, Gill KF.** A Design Study of a Self-Organizing Fuzzy Logic Controller. *IMechE*, 1986, Vol.200, No C1, pp.59-69.
- [25] **Daley S, Gill KF.** A Justification for the Wider Use of Fuzzy Logic Control Algorithms. *IMechE* 1985, Vol.199 No C1, pp.43-49.
- [26] **Daley S, Gill KF.** Attitude Control of a Spacecraft using an Extended Self-Organizing Fuzzy Logic Controller. *IMechE* 1987, Vol.201 No C2, pp.97-106.
- [27] **D'Andr a-Novel B, Boustany F.** Adaptive control of a class of mechanical systems using linearization and Lyapunov methods: A comparative study on the overhead crane example. *Proceedings of the IEEE Conference on Decision and Control*, Jan 1992, No.Jan, pp.120-125.
- [28] **Daniel RW, Irving MA, Fraser AR, Lambert M.** The Control of Compliant Manipulator Arms. "Robotics Research", the 4th International Symposium, MIT, 1988, pp.119-126.
- [29] **Davidson P, Longmore DK, Burrows CR.** Efficient Introduction of distributed Elasticity in Multibody Dynamic Systems. Kinematics and dynamics of multi-body systems seminar, IMechE HQ, 6 November 1992.
- [30] **DeSantis RM, Krau S.** Bang-bang Motion Control of a Cartesian Crane. *Robotica*, Vol.12, 1994, pp.449-454.
- [31] **Dharwadkar PV, O'Connor JT, Catton TM.** 3D CAD Modeling and Graphical Simulation of Mobile Cranes. Congress on Computing in Civil Engineering, Washington DC, June 1994, pp.539-546.

- [32] **Dougherty EJ, Lee DE, Shively PD.** Automated All-Weather Cargo Transfer System. The Society of Naval Architects and Marine Engineers, Spring Meeting/STAR Symposium, New Orleans, Louisiana, April 12-15, 1989, S2-3.
- [33] **Driankov D, Hellendoorn H, Reinfrank M.** An Introduction to Fuzzy Control. Springer-Verlag, 1993.
- [34] **Eden JF, Homer P, Butler AJ.** The Dynamic Stability of Mobile Cranes. Proceedings of the Institution of Mechanical Engineers, Vol.199, NoD4, 1985, pp.283-293.
- [35] **Eller D.** Automation is key to new Sea-Land Hub. Cargoware International, January 1990, pp.28-30.
- [36] **Fliess M, Lévine J, Rouchon P.** A Simplified Approach of Crane Control Via a Generalized State-Space Model. Preceeding of the 30th IEEE conference on Decision and Control, Brighton, England, December 1991, pp.736-741.
- [37] **Hara K, Yamamoto T, Kobuyashi A, Okamoto M.** Jib Crane Control to Suppress Load Swing. The International Journal of System Science, Vol.20(5), 1989, pp.715-731.
- [38] **Hobbs RE, Sharp DM, Walton JM.** Cabling of Crane Ropes. Wire Industry, March 1993, 60, 193-196.
- [39] **Hollars MG, Rosenthal DE, Short K, Austin A.** Concurrent Design and Analysis of Mechanisms. Kinematics and dynamics of multi-body systems seminar, IMechE HQ, 6 November 1992.
- [40] **Holt M, Cornish R.** Simulation Tools for Vehicle Handling Dynamics. Kinematics and dynamics of multi-body systems seminar, IMechE HQ, 6 November 1992.
- [41] **Ito H, Senda Y, Fujimoto H.** Dynamic Behaviour of a Load Lifted by a Mobile Construction-Type Crane. Bulletin of the JSME, Vol.21(154), #154-9, 1978, pp.600-608.
- [42] **Itoh O, Migita H, Itoh J, Irie Y.** Application of Fuzzy Control to Automatic Crane Operation. IECON Proceedings (Industrial Electronics Conference), 1993, Vol.1, pp.161-164.
- [43] **Itoh O, Migita H, Itoh J, Irie Y.** KF dynamic fuzzy crane system. IEEE International Conference on Fuzzy Systems, 1995, Vol.5, pp.63-64.
- [44] **Joshi S, Rahn CD.** Position control of a flexible cable gantry crane: theory and experiment. Proceedings of the American Control Conference, 1995, Vol.4, pp.2820-2824.
- [45] **Karmakar R, Mukherjee A.** Dynamics of Electric Overhead Travelling Cranes. Mechanical Machines Theory, Vol.25(1), 1990, pp.29-39.
- [46] **Kerr D, Goodchild C.** The Automatic Generation of the Symbolic Dynamical Equations of Complex Mechanisms. Kinematics and dynamics of multi-body systems seminar, IMechE HQ, 6 November 1992.
- [47] **Linkens D A, Nie J.** A fuzzy-Neural Controller Based on Hybrid Neural Networks. Neural Networks and Fuzzy Logic in Measurement and Control, colloquium proceedings, 10th March 1993.

- [48] **Linkens DA, Abbod MF.** Self-Organizing Fuzzy Logic Control and the Selection of its Scaling Factors. Transactions of Institution of Measurements and Control, 1992, Vol.14, No3, pp.114-125.
- [49] **Linkens DA, Abbod MF.** Self-Organizing Fuzzy Logic Control for Real-Time Processes. International Conference 'Control 91', Edinburgh, 1991, Vol.2, No332, pp.971-976.
- [50] **Linkens DA, Abbod MF.** Supervisory Intelligent Control Using a Fuzzy Logic Hierarchy. Transactions of Institution of Measurements and Control, 1985, Vol.15, No3, pp.112-132.
- [51] **Macleod R.** Developing an Automated Container Terminal. Freight International, pp.37-38, 1992.
- [52] **Mamdani EH, Assilian S.** An Experiment in Linguistic Synthesis with a Fuzzy Logic Controller. International Journal of Man-Machine Studies, 1975, Vol.7, pp.1-13.
- [53] **Mamdani EH.** Advances in the Linguistic Synthesis of Fuzzy Controllers. Int.J.Man-Machine Studies, Vol.8, 1976, pp.669-678.
- [54] **Marttinen A, Virkkunen J, Salminen R T.** Control Study with a Pilot Crane. IEEE Transactions on Education, Vol.33, No3, 1990, pp.298-305.
- [55] **Medland AJ.** The Creation of a Constraint Modelling Approach to the Design and Optimisation of Complex Mechanisms. Kinematics and dynamics of multi-body systems seminar, IMechE HQ, 6 November 1992.
- [56] **Moustafa KAF, Abeid AM.** Nonlinear Modelling and Control of Overhead Crane Load Sway. Transactions of the ASME, Vol.110, September 1988, pp.266-271.
- [57] **Moustafa KAF.** Feedback control of overhead cranes swing with variable rope length. Proceedings of the American Control Conference, 1994, Vol.1, pp.691-695.
- [58] **Nicolodi S, Visentin R, Rotella A.** Supervisory Controlled Telemanipulation with Automatic Compensation of Manipulator Base Movement. "On The Shores of Tomorrow", Intervention '90 Conference and Exposition sponsored by The Marine Technology Society, June 25-27, 1990.
- [59] **Nyongesa HO.** Optimal Fuzzy Controller Design. 1993 SERC/INSTMC Postgraduate Research Symposium, 30-31 March 1993, proceedings.
- [60] **Oostwouder I WAF.** Controlling Sway the Hybrid Way - Cargo Handling, Cargo Systems, June 1992, pp.33-35.
- [61] **Peacock C, Bolouri H.** A Neural Network Controlled Mobile Robot. Neural Networks and Fuzzy Logic in Measurement and Control, colloquium proceedings, 10th March 1993.
- [62] **Procyk TJ, Mamdani EH.** A Linguistic Self-Organizing Process Controller. Automatica, 1979, Vol.15, pp.15-30.
- [63] **Puttré M.** Dynamic Simulations with Animation Software. Mechanical Engineering, March 1994, pp.108-111.
- [64] **Ren Yx, Abe M, Ito H.** Dynamic behavior of gantry cranes (3rd report, the hoisting-traversing combined motion). Nippon Kikai Gakkai Ronbunshu, C Hen/Transactions of the Japan Society of Mechanical Engineers, Part C, Apr 1990, Vol.56, No.524, pp.889-893 (in Japanese).

- [65] **Ren Yx, Abe M, Ito H.** Dynamic behavior of gantry cranes. (4th report, The hoisting dynamic coefficient). Nippon Kikai Gakkai Ronbunshu, C Hen/Transactions of the Japan Society of Mechanical Engineers, Part C, Apr 1990, Vol.56, No.524, pp.894-899 (in Japanese).
- [66] **Ridout AJ.** Anti-swing control of the overhead crane using linear feedback. Journal of Electrical and Electronics Engineering, Australia, Mar-Jun 1989, Vol.9, No.1-2, pp.17-26.
- [67] **Rooney GT.** Implementing Motions with Mechanisms. Kinematics and dynamics of multi-body systems seminar, IMechE HQ, 6 November 1992.
- [68] **Rudolf CD.** Swaying at gantries - Cargo Handling, Cargo Systems, June 1992, pp.27-31.
- [69] **Sakawa Y, Nakazumi A.** Modelling and Control of a Rotary Crane. Journal of Dynamic Systems, Measurement, and Control, Transactions of the ASME, Vol.107, 1985, pp.200-206.
- [70] **Sakawa Y, Shindo Y, Hashimoto Y.** Optimal Control of a Rotary Crane. Journal of Optimisation Theory and Applications, Vol.35(4), 1981, pp.535-557.
- [71] **Sillitoe IPW, Sumko S, Taylor A.** The Control of Tendon Transmission System in Manipulators. The 8th International Conference on System Engineering, 10-12 September 1991, Coventry Polytechnic, proceedings, pp.450-457.
- [72] **Suzuki Y, Yamada Si, Fujikawa H.** Anti-swing control of the container crane by fuzzy control. Proceedings of the 19th Annual International IEEE Conference on Industrial Electronics, Control, and Instrumentation, Maui, Hawaii, USA, November 1993, Vol.1 pp.230-235.
- [73] **Swevers J, Van Brussel H, Indrawanto, Torfs D.** Accurate Motion Control of an Overhead Crane. Proceedings of the IUTAM Symposium on the Active Control of Vibration, 1994, University of Bath, UK.
- [74] **Takahashi T, Inooka H.** Control of a pendulum under torque limitation (4th Report: Playback of a swinging-up motion). Nippon Kikai Gakkai Ronbunshu, C Hen/Transactions of the Japan Society of Mechanical Engineers, Part C, Jul 1989, Vol.55, No.515, pp.1703-1707 (in Japanese).
- [75] **Tong R M.** A Control Engineering Review of Fuzzy Systems. Automatica, Vol.13, Pergamon Press 1977, pp.559-569.
- [76] **Tong RM.** Analysis of Fuzzy Control Algorithms Using the Relation Matrix. Int.J.Man-Machine Studies, Vol.8, 1976, pp.679-686.
- [77] **Verschoof IJ.** Automated Stacking Cranes at ECT. Freight International, pp.31-33, 1992.
- [78] **Virkkunen J, Marttinen A, Rintanen K, Salminen R, Seitsonen J.** Computer control of overhead and gantry cranes. IFAC Symposia Series - Proceedings of a Triennial World Congress, 1991, Vol.4., pp.401-406.
- [79] **VonStempel A.** Price of comfort and safety. Cargo Systems International: The Journal of ICHCA, Aug 1989, Vol.16, No.8, p.73, 75.
- [80] **Wong CB, Moore PR, Weston RH.** A Solution to Servo Drive System Tuning Using an Integrated Rule-Base and Neural Network. Neural Networks and Fuzzy Logic in Measurement and Control, colloquium proceedings, 10th March 1993.
- [81] **Woods JD, Clements DJ.** Nonlinear control of overhead cranes. National Conference Publication - Institution of Engineers, Australia, 1992, No.92 pt 15, pp.29-40.

- [82] **Yamada Si, Fujikawa H, Takeuchi O Wakasugi Y.** Fuzzy control of the roof crane. Proceedings of 15th Annual Conference of IEEE Industrial Electronics Society - IECON'89, Philadelphia, Pennsylvania, November 1989, Vol.4, pp.709-714
- [83] **Yasunobu S, Hasegawa T.** Evaluation of an Automatic Container Crane Operation System Based on Predictive Fuzzy Control. Control - Theory and Advanced Technology, 1986, Vol.2, No.3, pp.419-432.
- [84] **Yoon JS, Park BS, Lee JS, Park HS.** Development of anti-swing control algorithm for the verhead crane. Proceedings of the Conference on Remote Systems Technology, 1991, No.1991., pp.89-94.
- [85] **Yu J, Lewis FL, Huang T.** Nonlinear feedback control of a gantry crane. Proceedings of the American Control Conference, 1995, Vol.6, pp.4310-4314.
- [86] **Zadeh LA.** Outline of a New Approach to the Analysis of Complex Systems and Decision Processes. IEEE Transactions on Systems Man and Cybernetics, 1973, SMC-3, pp.28-44

Appendix I

Proof I : to show that $\cos \Phi = \sin \Psi \cos \left(\tan^{-1} \frac{a}{b} - \alpha \right)$

Figure A1.1 shows the spreader translated through $\vec{r}(r, \alpha)$. $Q'S$ is perpendicular to $ABCD$ plane, so $PCSQ'$ is a wedge with the planes $PSC \perp PSQ'$ (the wedge $PCSQ'$ is also shown on Figure A1.2 at a different angle of view).

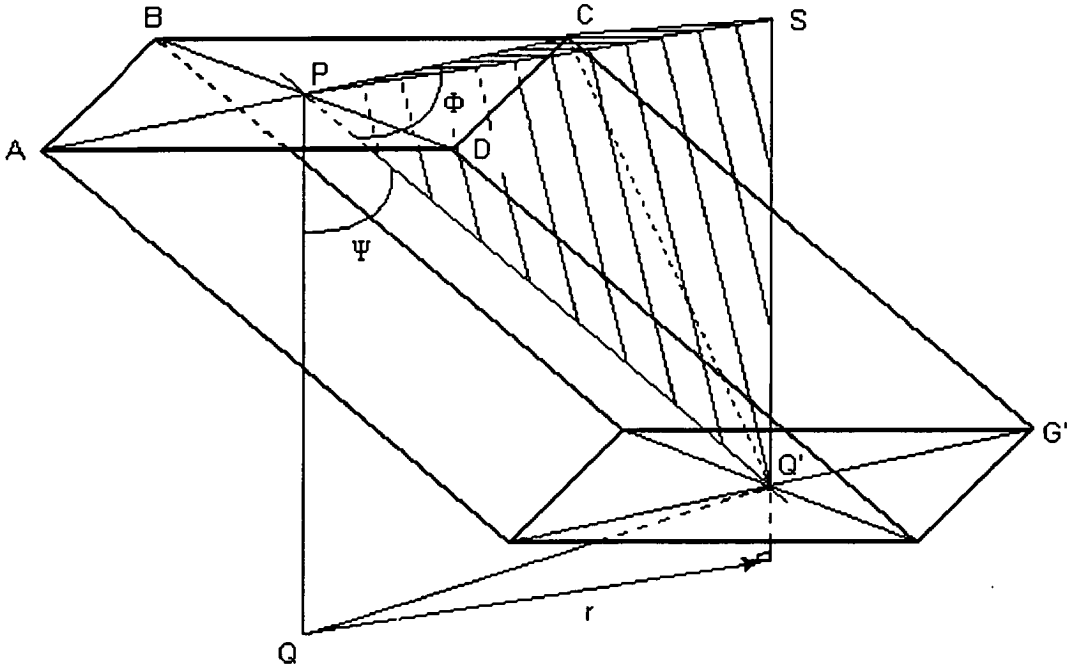


Figure A1.1

The spreader translated through $\vec{r}(r, \alpha)$

The point C' on PC (Figure A1.2) is obtained by constructing the line starting at S and continuing the perpendicular PC . From $\Delta C'PQ'$:

$$\cos \Phi = \frac{(PC')^2 + (PQ')^2 - (C'Q')^2}{2(PC')(PQ')} \quad (A1.1)$$

From the right-angled $\Delta SPC'$:

$$PC' = \frac{SP}{\cos \hat{SPC'}} \quad (\text{A1.2})$$

$$SC' = SP \tan \hat{SPC'} \quad (\text{A1.3})$$

and from the right-angled $\Delta SPQ'$:

$$PQ' = \frac{SP}{\cos \hat{SPQ'}} \quad (\text{A1.4})$$

$$SQ' = SP \tan \hat{SPQ'} \quad (\text{A1.5})$$

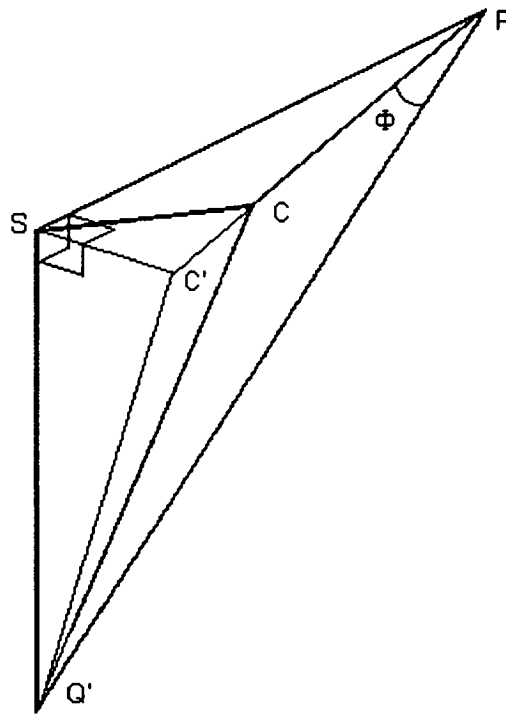


Figure A1.2

The wedge $PCSQ'$ from Figure A1.1

From the right-angled $\Delta SC'Q'$:

$$(C'Q')^2 = (SC')^2 + (SQ')^2 \quad (\text{A1.6})$$

Substituting equations (A1.2), (A1.4), and (A1.6) into equation (A.1.1) gives:

$$\cos \hat{CPQ'} = \frac{\left(\frac{SP}{\cos \hat{SPC'}}\right)^2 + \left(\frac{SP}{\cos \hat{SPQ'}}\right)^2 - ((SC')^2 + (SQ')^2)}{2\left(\frac{SP}{\cos \hat{SPC'}}\right)\left(\frac{SP}{\cos \hat{SPC'}}\right)}$$

Substituting SC' and SQ' from equations (A1.3) and (A1.5) means that this can be rationalised down to:

$$\cos \hat{CPQ'} = \cos \hat{SPC'} \cos \hat{SPQ'} \quad (\text{A1.7})$$

or

$$\cos \hat{CPQ'} = \cos \hat{SPC} \cos \hat{SPQ'} \quad (\text{A1.8})$$

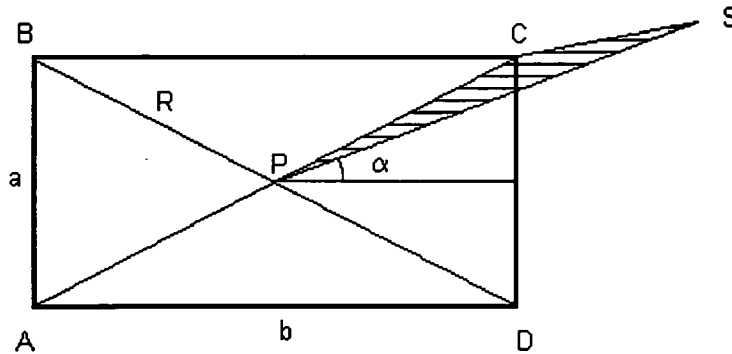


Figure A1.3

The top face of the wedge $PSCQ'$

It is clear that

$$\hat{CPQ'} = \Phi \quad (\text{A.1.9})$$

From the Figure A1.3 it can be clearly seen that:

$$\hat{DPC} = 2 \tan^{-1} \frac{a/2}{b/2} \quad (\text{A1.10})$$

and

$$\hat{CPS} = \tan^{-1} \frac{a}{b} - \alpha \quad (\text{A1.11})$$

From Figure A1.1:

$$\hat{SPQ'} = 90^\circ - \hat{QPQ'} = 90^\circ - \Psi \quad (\text{A1.12})$$

So, equations (A1.8), (A1.9), (A1.11) and (A1.12) give:

$$\cos \Phi = \sin \Psi \cos \left(\tan^{-1} \frac{a}{b} - \alpha \right) \quad (\text{A1.13})$$

End of proof I

Appendix II

Proof II : to show that the position of the spreader diagonal $E''^* G''^*$ is realistic, and that it corresponds to maximum rotational hoist

Figure A2.1 shows the spreader diagonal in two positions:

- $E'G'$, i.e. after the spreader is only translated, and
- $E''^*G''^*$, i.e. after the spreader is both translated and rotated. This is its extreme position when the cables touch each other at the point T_{cables} .

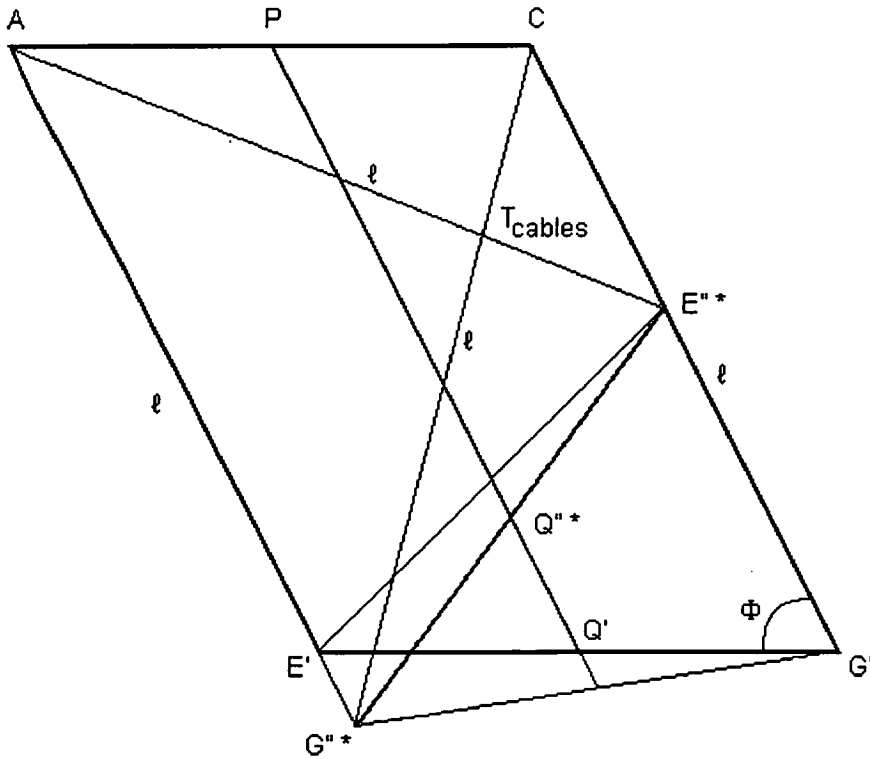


Figure A2.1

The extreme position of the spreader diagonal
after translation and rotation

It is necessary to prove that this position ($E''^*G''^*$) is physically realistic, i.e. that if:

$$AE''^* = AE' = l \quad (A2.1)$$

$$CG''^* = CG' = l \quad (A2.2)$$

then the following is satisfied:

$$E''G'' = E'G' = (2R) \quad (\text{A2.3})$$

It is deducible from Figure A2.1 that,

$$\Delta AE'E'' = \Delta CG'G'' \quad (\text{A2.4})$$

being two triangles with equal verticals (i.e. two mutually perpendicular lines lying between AE' and CG') and two equal pairs of sides as stipulated by equations (A2.1) and (A2.2). Therefore:

$$E'E'' = G'G'' \quad (\text{A.2.5})$$

So, the trapezium $E'E''G'G''$ has equal sides, via equation (A2.5), and therefore it has equal diagonals, i.e. equation (A2.3) is correct.

End of proof II

Appendix III

Proof III: to show that $\hat{PQ''E''} = \Phi$
for any position of the spreader diagonal

The projection of point E'' onto the plane $ACG'E'$ is E''_p , and the projection of the point G'' onto the same plane is G''_p . The following is defined on Figure A3.1:

$$E''\hat{Q''}E''_p = G''\hat{Q''}G''_p \equiv \beta \quad (\text{A3.1})$$

$$E''_p\hat{Q''}P = G''_p\hat{Q''}Q' \equiv \gamma \quad (\text{A3.2})$$

both pairs being opposite angles, see Figure A3.1.

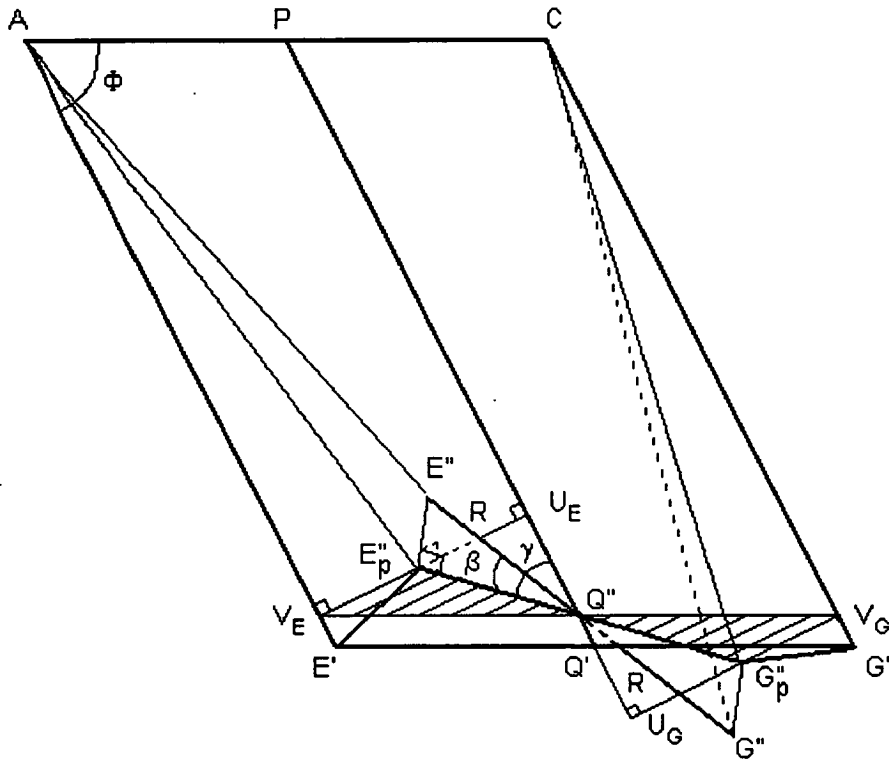


Figure A3.1

The plane $ACG'E'$ as in Figure 3.14
with additionally shown spreader diagonal $E''G''$

As the two halves of the spreader diagonal:

$$E''Q'' = Q''G'' = R \quad (\text{A3.3})$$

therefore:

$$E''_p Q'' = Q'' G''_p = R \cos \beta \quad (\text{A3.4})$$

Now, two lines $E''U_E$ and $G''U_G$ have to be constructed so that they are perpendicular to PQ' (they are not shown in Figure A3.1 in order to reduce its complexity). Then the lines on the plane $ACG'E'$ connecting the points U_E with E''_p and U_G with G''_p are also perpendicular to PQ' :

$$E''_p \hat{U}_E Q'' = G''_p \hat{U}_G Q'' = 90^\circ \quad (\text{A3.5})$$

The intersection of $U_E E''_p$ with AE' is denoted V_E , and the intersection of $U_G G''_p$ with CG' is denoted V_G .

Now the proof is straightforward:

$$\Delta U_E Q'' E''_p = \Delta U_G Q'' G''_p \quad (\text{A3.6})$$

as two triangles with one equal side (equation (A3.4)) and two pairs of equal angles (equations (A3.2) and (A3.5)), therefore:

$$E''_p V_E = G''_p V_G \quad (\text{A3.7})$$

Furthermore:

$$\Delta V_E Q'' E''_p = \Delta V_G Q'' G''_p \quad (\text{A3.8})$$

as two triangles with two equal sides (equations (A3.4) and (A3.7)) and the equal angles between them:

$$V_E \hat{E''}_p Q'' = V_G \hat{G''}_p Q'' = 90^\circ + \gamma \quad (\text{A3.9})$$

Therefore:

$$\Delta V_E Q'' U_E = \Delta V_G Q'' U_G \quad (\text{A3.10})$$

(N.B. Equation (A3.10) means that V_E , Q'' , and V_G are really lying on one line. This was not intuitively obvious at the stage when $V_E U_E$ and $V_G U_G$ were constructed).

These are two right-angled triangles. The hypotenuse of each triangle is equal to R , and the opposite side is given by:

$$U_E V_E = U_G V_G = R \sin \Phi \quad (\text{A3.11})$$

Therefore the adjacent side is:

$$U_E Q'' = U_G Q'' = R \cos \Phi \quad (\text{A3.12})$$

Finally, from the right-angled triangles $E'' Q'' U_E$ and $G'' Q'' U_G$:

$$Q'' E'' = Q'' G'' = R \quad (\text{A3.13})$$

(these are two halves of the spreader diagonal in the position after rotation) and applying equation (A3.12):

$$E''\hat{Q}''U_E = G''\hat{Q}''U_G = P\hat{Q}''E'' = \Phi \quad (\text{A3.14})$$

End of proof III

Appendix IV

Proof IV : to show that $Q'Q'' = l - \sqrt{l^2 - 2R^2 \sin \Phi (\sin \Phi \mp \sqrt{\cos^2 \beta - \cos^2 \Phi})}$

Figure A3.1 will be used in this proof.

From $\Delta U_E E''_p Q''$:

$$U_E E''_p = \sqrt{(Q'' E''_p)^2 - (Q'' U_E)^2} \quad (\text{A4.1})$$

From $\Delta E'' E''_p Q''$:

$$Q'' E''_p = E'' Q'' \cos \beta = R \cos \beta \quad (\text{A4.2})$$

and from $\Delta E'' U_E Q''$ using equation (A3.14):

$$Q'' U_E = E'' Q'' \cos \Phi = R \cos \Phi \quad (\text{A4.3})$$

Substituting equations (A4.2) and (A4.3) into equation (A4.1):

$$U_E E''_p = \sqrt{(R \cos \beta)^2 - (R \cos \Phi)^2} = R \sqrt{\cos^2 \beta - \cos^2 \Phi} \quad (\text{A4.4})$$

From $\Delta A E''_p V_E$:

$$A V_E = \sqrt{(A E''_p)^2 - (E''_p V_E)^2} \quad (\text{A4.5})$$

From $\Delta AE''_p E''$:

$$AE''_p = \sqrt{l^2 - (R \sin \beta)^2} \quad (A4.6)$$

Finally, the following is obvious:

$$V_E U_E = E' Q' \sin \Phi = R \sin \Phi \quad (A4.7)$$

therefore using equations (A4.4) and (A4.7):

$$E''_p V_E = V_E U_E \mp E''_p U_E = R \sin \Phi \mp \sqrt{(R \cos \beta)^2 - (R \cos \Phi)^2} \quad (A4.8)$$

Here the '-' sign corresponds to the first part of rotation, and the '+' sign for the second.

Substituting equations (A4.6) and (A4.8) into equation (A4.5):

$$\begin{aligned} AV_E &= \sqrt{\left(l^2 - (R \sin \beta)^2\right) - \left(R \sin \Phi \mp \sqrt{(R \cos \beta)^2 - (R \cos \Phi)^2}\right)^2} = \\ &= \sqrt{l^2 - R^2 \sin^2 \beta - R^2 \sin^2 \Phi - R^2 \cos^2 \beta + R^2 \cos^2 \Phi \pm 2R \sin \Phi \sqrt{R^2 \cos^2 \beta - R^2 \cos^2 \Phi}} = \\ &= \sqrt{l^2 - 2R^2 \sin^2 \Phi \pm 2R^2 \sin \Phi \sqrt{\cos^2 \beta - \cos^2 \Phi}} = \\ &= \sqrt{l^2 - 2R^2 \sin \Phi \left(\sin \Phi \mp \sqrt{\cos^2 \beta - \cos^2 \Phi}\right)} \end{aligned}$$

And finally:

$$Q' Q'' = l - AV_E = l - \sqrt{l^2 - 2R^2 \sin \Phi \left(\sin \Phi \mp \sqrt{\cos^2 \beta - \cos^2 \Phi}\right)} \quad (A4.9)$$

where the '-' sign corresponds to the first part of rotation, and the '+' sign for the second.

End of proof IV

Appendix V

Publications

Journal papers

Accepted for forthcoming publication:

- [1] **Morrish L, Cartmell MP, Taylor AJ.** Cable Stretch Asymmetries in Multi-Cable Spreader Suspension Systems Undergoing Combined Translations and Rotations. Accepted for publication in the Proc. Inst. Mech. Engineers Part C, Journal of Mechanical Engineering Sciences, 1996.

Accepted subject to modifications:

- [2] **Morrish L, Cartmell MP, Taylor AJ.** Geometry and Kinematics of Multi-Cable Spreader Lifting Gear. Submitted to Proc. Inst. Mech. Engineers Part C, Journal of Mechanical Engineering Sciences, 1996.

Under review:

- [3] **Cartmell MP, Morrish L, Taylor AJ.** Dynamics of Spreader Motion in a Gantry Crane. Submitted to Proc. Inst. Mech. Engineers Part C, Journal of Mechanical Engineering Sciences, 1996.
- [4] **Cartmell MP, Morrish L, Taylor AJ.** Controlling the Nonlinear Dynamics of Gantry Cranes. Submitted to Machine Vibration, 1996.

Conference papers

- [5] **Cartmell MP, Morrish L, Taylor AJ.** Control of a Mobile Gantry Crane incorporating Multiple Cable Suspension of the Lifting Gear. The Joint Hungarian-British Mechatronics Conference, Budapest, September 21-23, 1994, proceedings 'Mechatronics - the basis for new Industrial Development', edited by M.Acar, J.Makra, and E.Penney, pp.3-8.
- [6] **Cartmell MP, Morrish L, Taylor AJ.** The Dynamics of Spreader Motion in a Mobile Gantry Crane. The IUTAM Symposium at Bath University, September 5-8, 1994, proceedings 'The active control of vibration', edited by C.R.Burrows and P.S.Keogh, pp.39-44.