ABSTRACT OF THESIS

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Title of Thesis PHOTOPRODUCTION AND ELECTROPRODUCTION IN THE QUARK MODEL

The simple non-relativistic model of quarks in an harmonic oscillator potential as developed by Faiman and Hendry and Copley, Karl and Obryk is used in resonance photo and electroproduction to calculate transverse and longitudinal cross-sections and investigate their frame dependence.

Due to conflicting experimental data on the helicity structure of electroproduction amplitudes the ratio of transverse helicity amplitudes is also calculated and compared in various frames.

The results are compared with similar ones using a Coulomb-like 1/r potential which improves the behaviour of the ratio but decreases the overall normalisation of the wavefunctions. Returning to the harmonic oscillator, modifications due to spin orbit terms are found to give better fits to the data.

Now we go on to the problems of SU(6) breaking and of relativistic quark models. They are dealt with together and the two main approaches investigated. The Melosh approach is to classify the constituent quarks and the current quarks according to distinct SU(6) groups and have a unitary transformation between them to account for configuration mixing. The angular momentum structure imposed on the electromagnetic interaction by the Melosh transformation is shown to be that actually present in the current used. Also a modification to first order to account for electromagnetic interactions is introduced.

The other approach is to attribute the symmetry breaking to the interaction and calculate it directly. For this, we use the solution of the Bethe-Salpeter equation developed by Kellett.

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Electroproduction matrix elements are calculated and compared with previous predictions. In the conclusion the possibility of unifying the two approaches is mentioned briefly.

PHOTOPRODUCTION AND ELECTROPRODUCTION IN THE QUARK MODEL

Thesis

Submitted by

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INTRODUCTION

Quarks as constituents of hadrons were proposed by Gell Mann[1] and independently by Zweig in 1964 as a physical realisation of SU(3). The fundamental non-trivial multiplet in SU(3) is a triplet, so they suggested that there exists a fundamental triplet of "quarks" out of which all mesons and baryons are constituted as bound states. In this way the mesons and baryons would form SU(3) multiplets.

This was just a classification scheme, but when quarks as either real or fictitious particles found acceptance it was naturally asked how the bound states would form. Baryons would consist of three quarks and mesons of a quark anti-quark pair which is a stronger requirement than just SU(3) symmetry as only singlets, octets and decuplets are allowed. The question is, what are the interquark forces? In this thesis we will be entirely concerned with baryons.

Dalitz had been suggesting[2] that the known N^* resonances, which can be grouped into reasonably well defined bands, should be interpreted as excitations of a basic three quark system. This would be an SU(6) classification scheme where multiplets of the SU(3) scheme with different spins but the same parity would be grouped in SU(6) supermultiplets. The resonances fit this scheme very well and the bands of positive and negative parity respectively seem to be approximately evenly spaced. This suggested to Faiman and Hendry[3] that the quarks should have spring forces between them and they set up a model of quarks in an harmonic oscillator potential. They gave the detailed wavefunctions describing the non-spurious states left after centre of mass separation. Copley, Karl and Obryk (CKO)[4] then used this model to calculate resonance photoproduction off nucleons. The model will be referred to as the CKO model since they established the helicity formalism which we will use. For photoproduction, helicity amplitudes are very convenient to calculate, the possible helicities being $\frac{1}{2}$ and $\frac{3}{2}$. For electroproduction the photon is virtual and has an additional degree of freedom, zero helicity. This case has been considered by Thornber[5] who calculated matrix elements in the same model all in terms of electric, magnetic and longitudinal multipoles. Taking linear combinations of the electric and magnetic multipoles also gives a useful check on the consistency of the two sets of calculations.

Abdullah and Close[6] pointed out that since the model is non-relativistic, amplitudes and cross-sections will be frame dependent. The calculations of CKO and Thornber were all in the isobar rest frame so we have calculated transverse and longitudinal cross-sections and the ratio of transverse amplitudes in the isobar frame as before, and then in the Breit frame, in order to compare them and see the degree of frame dependence. The Breit frame was chosen because it has been argued by Abdullah and Close and others that this is the most physical frame. Amplitudes were calculated for the $F_{15}(1688)$ in order to establish the value of the oscillator spring constant, the one free parameter, from the assumption of zero helicity $\frac{1}{2}$ excitation in photoproduction. This aasumption is made because photoproduction experiments indicate zero

F_{15} excitation in the forward and backward directions. The helicity $\frac{3}{2}$ amplitude vanishes here by angular momentum conservation, so the helicity $\frac{1}{2}$ amplitude must be zero. Then cross-sections were derived for the two main resonances in the second resonance region, the $D_{13}(1520)$ and $S_{11}(1535)$. The ratio of transverse helicity amplitudes was calculated for the D_{13}.

The ratio of transverse amplitudes was the original motivation for this work. Close and Gilman[7] pointed out that according to the harmonic oscillator model the ratio changes very rapidly with photon four-momentum squared (q^2), while the data existing at that time indicated no change. Here was a very direct contradiction. Since the ratio had been calculated for the isobar frame we calculated it in the Breit frame but there is little difference.

The next step was to try another potential with better behaved form factors to see what effect this would have on the ratio. The Coulomb-like $\frac{1}{r}$ potential was chosen, as its wavefunctions reproduce the observed dipole elastic form factors. Thornber[8] had calculated the multipole moments in the isobar frame and found that cross-sections were far too small, but again we wanted to see the effect of frame dependence. Also Cho[9] had compared the ratios in the two models but only at two values of q^2 and only in the isobar frame. So we compared the ratios in the Breit frame all the way from $q^2 = 0$ to -1. The ratio indeed changes much less but, because the cross-sections are still far too small, this is not really a suitable potential.

Returning to the harmonic oscillator model, second order (spin-orbit) terms were added to the interaction Hamiltonian. This was first done by Bowler[10] but he used heavy quarks with an anomalous magnetic moment, while the CKO model which we use has light quarks with a g factor of 1. Spin orbit terms add only small contributions to the amplitudes but by changing the value of the spring constant, the ratio change becomes much flatter. It was found that the ratio is very sensitive to the value of the spring constant. Fixing it by the F_{15} cancellation as before, this second order model still gives cross-sections in good agreement since the new terms are small, in fact better agreement as there is experimental evidence for a spin orbit term, as will be discussed in Chapter 3. The only inconcistency is that we need a large quark mass without changing the g factor in order to maintain mutual cancellation for the D_{13} and F_{15} helicity amplitudes. However it may be argued that this is less of an inconsistency than the light quark mass. The ratio is also flatter and therefore in better agreement with the data, especially as more recent experiments do indicate some change in the ratio. So finally this is the model with the best overall agreement of all the variants we have been investigating. For more details of this type of model and its successes and failures see conference reviews such as References 11 and 12.

Now we come to two problems. First of all the model is SU(6) invariant with respect to particle classification only, while the real world has also symmetry with respect to interactions (vertex symmetry) which is not the same SU(6) invariance. The vertex symmetry, current algebra, is derived from the quark model too, so while not equal, there may be some relationship between them. Note that we are not concerned here with the SU(6) symmetry breaking which creates mass splittings within SU(3) multiplets. Secondly, the model is non-relativistic and a complete description of interactions would require a relativistic extension. The two problems are linked since a Lorentz boost affects spins and SU(6) is a non-relativistic group only

The first attempts to create a relativistic model were very simple[13, 14], just reproducing the CKO model with four vectors essentially and patching up difficulties in an <u>ad hoc</u> way. Since then, two distinct lines of progress have developed. In one, two SU(6)'s are postulated ("current" and "constituent" quarks) and a transformation between them constructed. This was developed by Melosh[15]. It has its greatest power in predicting the symmetries involved in interactions. Hey and Weyers[16] found the spin and angular momentum content to be expected for the electromagnetic current and we have compared this with the content of the current derived by Foldy-Wouthuysen transforming the relativistic current. We found that the terms required are there and no extra ones, even though in practice one term with $\Delta L_Z = 2$ is so small that it was ignored in theory and not distinguished by experiment (L = orbital angular momentum). The transformation was explicitly constructed by Melosh only for the free quark model. Explicit predictions for cross-sections etc. can be obtained using it, but the transformation is not exact when interactions are involved. On the one hand, there are the interactions binding the quarks to form hadrons and then there are external forces like the electromagnetic interaction for electroproduction. Melosh gave a prescription for including interactions to first order, and using that, a first order transformation fulfilling the requirements of the Melosh transformation in the presence of an external electromagnetic field was found.

The other approach is just to have classification SU(6) symmetry. The dynamics is obtained by solving the Bethe-Salpeter equation and the SU(6) symmetry is dynamically broken. This approach was used by Böhm, Joos and Krammer(17) with heavy quarks and a smooth ladder-type kernel mainly for mesons. The only really detailed and realistic description of baryons along the same lines is that of Kellett(18), generalising the work of Brodsky and Primack(19) on relativistic composite systems, to three quarks and strong binding. We will therefore refer to it as Kellett's model, even though the ideas are much older. We use the model to calculate electroproduction matrix elements both as a first approximation to the non-relativistic ones and the complete solutions.

These two approaches are different and independent. There has been no attempt to unify them but the possibility is discussed at the end.

The plan of this thesis is as follows. In Chapter 1 the non-relativistic harmonic oscillator quark model applied to resonance photo- and electroproduction off nucleons is discussed. In Chapter 2 the $\frac{1}{r}$ potential is tried and in

Chapter 3 we return to the harmonic oscillator including higher order terms. Chapter 4 is a short intermediate chapter, reviewing the problems of relativistic quark models and the forms of symmetry breaking. The algebraic approach of Melosh and its implications are set out in Chapter 5 and Kellett's explicit dynamical symmetry breaking in Chapter 6. In the conclusion the future unification of the two approaches is briefly considered.

CHAPTER 1

THE HARMONIC OSCILLATOR POTENTIAL

Only a brief review is needed as this model has been thoroughly discussed in the literature: for instance References 11 and 12.

The potential is $V(r) = \frac{1}{2} m\omega^2 r^2$.

There is one free parameter, the spring constant $\alpha^2 = m\omega$.

a. The Quark States

The quarks can be in the ground state representing the nucleon N or the $\Delta(1236)$, or have orbital angular momentum L and represent resonances. The states and particles are denoted by spectroscopic notation. We are specifically interested in the proton (p) in the $[56, 0^+]$, the $D_{13}(1520)$ and $S_{11}(1535)$ in the $[70, 1^-]$ and the $F_{15}(1688)$ in the $[56, 2^+]$.

The resonance classification scheme and state wave-functions are discussed further in the Appendix.

b. The Electromagnetic Interaction

Following CKO[4] the total interaction, taken as three times the interaction with the third quark (because there are three quarks each of which has the same chance of interacting and has the same Hamiltonian for doing so) is

$$H = 6\left(\frac{\pi}{k}\right)^{\frac{1}{2}} \mu\, q_3\, e^{-ikz_3} \left[k(s_x + is_y)_3 - \frac{1}{g}(p_x + ip_y)_3\right]$$

where the photon has helicity +1 which is right-hand circular polarisation $\underline{\varepsilon} = \frac{-1}{\sqrt{2}}$ (1, i, 0). The photon 3 momentum $\underline{k}$ is taken as the quantisation axis along the Z_3 direction with $k = |\underline{k}|$. Also q_3 is the average charge on the third quark as a fraction of the electron charge e, $\underline{s} = \frac{1}{2}\underline{\sigma}$ where σ is a Pauli spin matrix, and $\underline{\mu} = q \frac{eg}{m_q} \underline{s}$ is the quark magnetic moment with $\mu = \mu_p = \frac{eg}{2m_q}$ the scale magnetic moment.

Matrix elements of H between the wavefunctions of the appendix yield transverse amplitudes. There are two independent ones, (the photon always has helicity +1):

	initial nucleon	final resonance
$A_{1/2}$	$-\frac{1}{2}$	$+\frac{1}{2}$
$A_{3/2}$	$+\frac{1}{2}$	$+\frac{3}{2}$

These helicity amplitudes are obtained by use of Clebsch-Gordan coefficients as linear combinations of the spin flip and orbital flip amplitudes which are actually calculated.

$$\text{Spin flip} \qquad R_{L0} = \left\langle \psi_{L0}^{s \text{ or } \lambda} \middle| e^{-ikz_3} \middle| \psi_{00}^{s} \right\rangle$$

$$\text{Orbital flip} \qquad R_{L1} = \left\langle \psi_{L1}^{s \text{ or } \lambda} \middle| e^{-ikz_3} \, (p_x + ip_y)_3 \middle| \psi_{00}^{s} \right\rangle$$

where s = symmetric representation and λ is the even mixed symmetry representation.

All the wavefunctions have a common exponential $\exp(\frac{-k^2}{4\alpha^2})$ which becomes $\exp(\frac{-k^2}{6\alpha^2})$ upon elimination of the motion of the centre of mass.

Doing the calculations CKO found for instance for the D_{13} that

$$R^{\lambda}_{10} = \frac{ik}{\sqrt{3}\alpha} e^{-k^2/6\alpha^2}$$

$$R^{\lambda}_{11} = i\sqrt{\frac{2}{3}}\,\alpha\; e^{-k^2/6\alpha^2}$$

and $$A_{1/2} = \left(\frac{2}{3}\frac{\pi}{k}\right)^{\frac{1}{2}} \mu\left[\sqrt{2}\,k\,R^{\lambda}_{10} - \frac{1}{g}R^{\lambda}_{11}\right]$$

$$A_{3/2} = -\left(\frac{2\pi}{k}\right)^{\frac{1}{2}} \frac{\mu}{g} R^{\lambda}_{11} \quad .$$

There is a summary of helicity amplitudes for the three resonances we are interested in at the end of the next section.

Assuming that the magnetic moment of a hadron is the sum of the magnetic moments of its quarks leads to the magnetic moment of the proton.

$$\mu_p = \mu = 2.79\,\frac{e}{2m_p} \quad .$$

Writing the quark magnetic moment as $g\,\mu_q\,\underline{\sigma}_q$ with $\mu_q = \frac{e_q}{2m_q}$ we have $g = 2.79\,\frac{m_q}{m_p}$. A consistent choice of gyromagnetic ratio is $g = 1$ which, combined with μ from experiment, leads to $m_q = \frac{1}{3}M_N$ (about 0.33 GeV). Any larger g would imply an anomalous magnetic moment which would require an additional term in the electromagnetic current.

The spring constant α is fixed so that the helicity $\frac{1}{2}$ amplitude for the F_{15} is zero for photoproduction. A well known success of this model is that the D_{13} helicity $\frac{1}{2}$ amplitude is also almost zero as required by experiment. In this way $\alpha^2 = 0.17\ \text{GeV}^2$. Now with no other parameters the model has made accurate predictions for the amplitudes for other resonances and other processes. For details there are many reviews such as References 11 and 12. These successes have made it worthwhile to continue using the model and see just how far one can take it.

c. Multipole Moments

A useful check on the amplitudes of the previous section can be made by calculating the electric and magnetic multipole moments following Thornber[5]. Operators now act on eigenstates of total angular momentum rather than helicity but appropriate linear combinations of the amplitudes give $A_{1/2}$ and $A_{3/2}$. Checking that both methods give the same result and finding the relative normalisation between them, the longitudinal multipole for virtual photons easily follows and thus the longitudinal amplitude A_o in the CKO scheme.

Transverse multipoles

They are distinguished by parity, the magnetic having parity $(-1)^L$ and the electric $(-1)^{L+1}$.

Electric current $\quad \underline{j} = \underline{j}_1 + \underline{j}_2$

where $\quad \underline{j}_1 = \frac{1}{2im_q} (\psi_b^* \underline{\nabla} \psi_a - \psi_a \underline{\nabla} \psi_b^*)$

and $\quad \underline{j}_2 = \frac{g}{2m_q} \underline{\nabla} \times (\psi_b^* \underline{\sigma} \psi_a)$.

Magnetic multipole is $M_{LM} = \int d^3\underline{r}(j_L(kr)\ \underline{Y}_{LLM})\cdot\underline{j}$

Electric multipole is $E_{LM} = \frac{1}{k}\int d^3\underline{r}(\underline{\nabla}\times j_L(kr)\ \underline{Y}_{LLM})\cdot\underline{j}$

where $\underline{Y}_{LLM}$ is a vector spherical harmonic and $j_L(kr)$ is a spherical Bessel function.

For instance for the D_{13} we have a negative parity transition from an angular momentum state $\frac{1}{2}$ to $\frac{3}{2}$. So orbital angular momentum $L = 1$ or 2 and we have multipoles E1 and M2.

$$A_{1/2} = \tfrac{1}{2}E1 - \frac{\sqrt{3}}{2}M2$$

$$A_{3/2} = \tfrac{1}{2}M2 + \frac{\sqrt{3}}{2}E1 \quad .$$

Only the L value of the multipole matters. The M is accounted for by calculating reduced matrix elements using the Wigner Eckart theorem. So the E1 and M2 above are reduced matrix elements.

The relative normalisation between the two sets of amplitudes is $(-4i\frac{\pi}{\sqrt{k}})$. So we can calculate the longitudinal multipole

$$C_{LM} = \int j_L(kr)\ Y_{LM}(\theta\ \phi)\ \rho(r)\ d^3\underline{r}$$

where $\rho(r)$ is the charge density

$$\rho_i(r) = \psi_b^*\ q_i\ e\ \psi_a \ .$$

Then we put it in CKO normalisation.

Finally, the CKO amplitudes are

D_{13}: $$A_o = -i\,\frac{2}{3}\left(\frac{\pi}{3}\right)^{\frac{1}{2}} e\,\frac{\sqrt{k}}{\alpha}\, e^{-k^2/6\alpha^2}$$

$$A_{1/2} = i\,\frac{2}{3}\left(\frac{\pi}{k}\right)^{\frac{1}{2}} \mu\alpha \left(\frac{k^2}{\alpha^2} - \frac{1}{g}\right) e^{-k^2/6\alpha^2}$$

$$A_{3/2} = -i\,\frac{2}{\sqrt{3}}\,\frac{\mu\alpha}{g}\left(\frac{\pi}{k}\right)^{\frac{1}{2}} e^{-k^2/6\alpha^2}$$

S_{11}: $$A_o = i\,\frac{\sqrt{\pi}}{3}\left(\frac{2}{3}\right)^{\frac{1}{2}} e\,\frac{\sqrt{k}}{\alpha}\, e^{-k^2/6\alpha^2}$$

$$A_{1/2} = -i\,\frac{\sqrt{2}}{3}\left(\frac{\pi}{k}\right)^{\frac{1}{2}} \frac{\mu}{\alpha}\left(k^2 + \frac{2\alpha^2}{g}\right) e^{-k^2/6\alpha^2}$$

F_{15}: $$A_o = -i\,\frac{\sqrt{2}}{15}\left(\frac{\pi}{k}\right)^{\frac{1}{2}} \frac{ek^2}{\alpha^2}\, e^{-k^2/6\alpha^2}$$

$$A_{1/2} = 2\left(\frac{2\pi}{5k}\right)^{\frac{1}{2}} \frac{\mu}{6}\left(\frac{2k}{g} - \frac{k^3}{\alpha^2}\right) e^{-k^2/6\alpha^2}$$

$$A_{3/2} = \frac{4}{3\sqrt{5}}\left(\frac{\pi}{k}\right)^{\frac{1}{2}} \mu k\, e^{-k^2/6\alpha^2} .$$

d. Frame Dependence

Thornber's electroproduction calculations[5], although reasonable at low q^2, diverged rather strongly from experiment for $-q^2 > 1$ GeV^2 and were not a success for the quark model. But Abdullah and Close[6] showed that this was because she had calculated in the isobar frame, while the same calculations in the Breit frame would have given better results. For a process $eA \longrightarrow eB$ in the Breit frame $|\underline{p}_A| = |\underline{p}_B|$ and in the isobar frame $\underline{p}_B = 0$. Of course one can work in any frame one likes but these two are the most usual. Intuitively one would expect the Breit frame to be the most natural one to use physically because momenta are shared out equally between the particles, not one at rest and one carrying all the momenta. As this is a non-relativistic model, the slower the particles the better the approximation. Abdullah and Close[6] and Gutbrod[20] among others have put forward more quantitative reasons for preferring the Breit frame.

At low q^2 the frame dependence will not be so great but it is interesting to see the detailed behaviour. So let us calculate the transverse and longitudinal cross-sections σ_t, σ_ℓ for resonance electroproduction off protons. From Ravndal[21]

$$\sigma_\ell = -8\pi^3 \alpha \frac{q^2}{k^{*3}} |A_o|^2 \frac{\Gamma/2\pi}{(W-M)^2 + \Gamma^2/4}$$

$$\sigma_t = \frac{-8\pi^3 \alpha}{k^*} \cdot \tfrac{1}{2}(|A_{1/2}|^2 + |A_{3/2}|^2) \frac{\Gamma/2\pi}{(W-M)^2 + \Gamma^2/4}$$

where k^* is k in the isobar frame

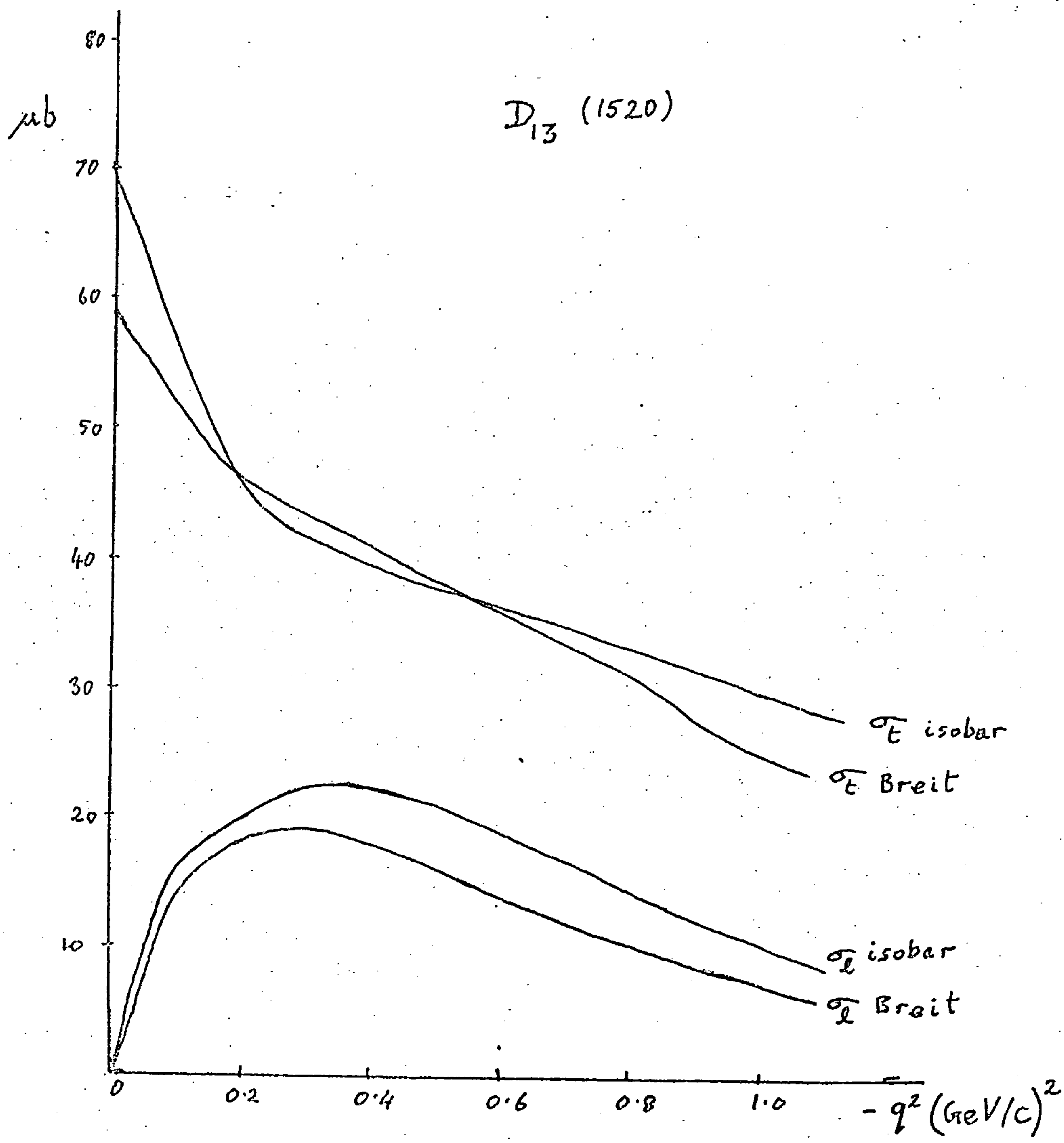
$\alpha = \frac{1}{137}$ = fine structure constant

and $\frac{\Gamma/2\pi}{(W-M)^2 + \Gamma^2/4}$ is the Breit-Wigner factor for a resonance with width Γ, mass M calculated at energy W. Assuming that all these resonances have the Breit-Wigner form is rather a crude approximation. It hopefully fits the level of approximation of the rest of the model to reality and works in a way, but for a more detailed and realistic model something better might be needed. Apart from distinct resonances there is also a background of less distinct resonances and non-resonant excitation which we do not take into account. This adds a large error to all predictions and also adds to the uncertainty when analysing data and separating out the contributions of several resonances. Clegg[12] has discussed this and estimated the background cross-section to be about the same size, or even larger, than the total peak cross-sections.

Cross-sections were calculated for the $D_{13}(1520)$ at $W = 1520$ MeV and also for the $S_{11}(1535)$ at $W = 1520$ MeV. These are the two main resonances dominating the second resonance region. Their sum should bear some relation to the numbers measured experimentally although comparison is difficult because the data is for $\sigma_t + \varepsilon\sigma_\ell$ (taken from Ref. 12) where $\varepsilon^{-1} = 1 + 2(1 + \frac{\nu^2}{q^2}) \tan^2(\frac{1}{2}\theta)$, with θ the electron scattering angle and ν the virtual photon energy.

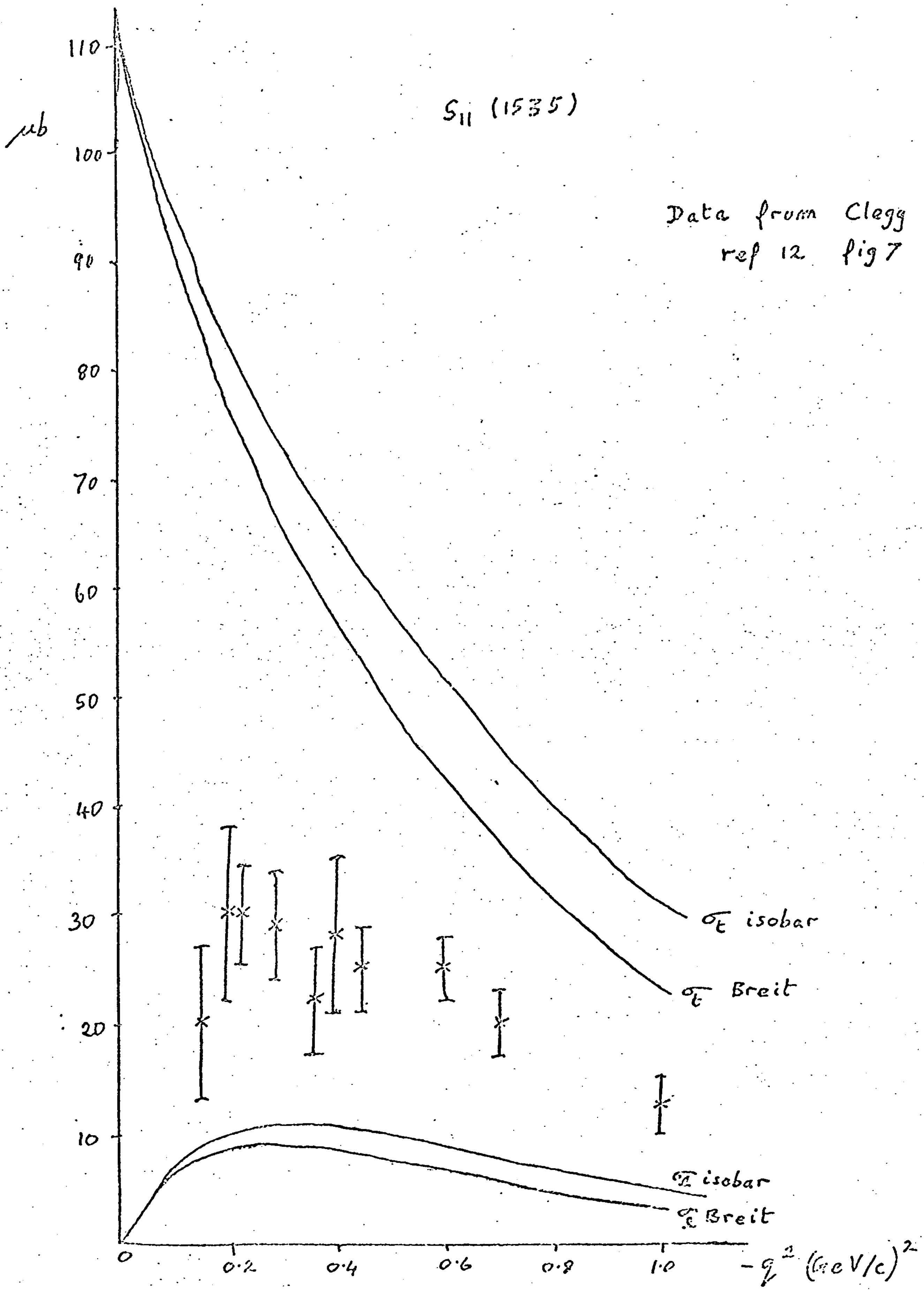
The results are shown in Figures 1, 2 and 3. There is a definite but not very great frame dependence as expected. The sum $D_{13} + S_{11}$ is compatible with the data points providing that ε is small, which it is in this kind of experiment. The S_{11} prediction is not in very good agreement with the data. The cross-sections are of course sensitive to the assumed resonance mass through the Breit-Wigner factor. For instance, if the mass of the S_{11} were taken as 1520 instead of 1535 MeV, one would multiply the cross-sections by 0.87. There is at least this much uncertainty in the phase shift analyses by which the resonances are identified.

It is also noticeable that σ_ℓ is much smaller than σ_t in all cases. This is in agreement with most data



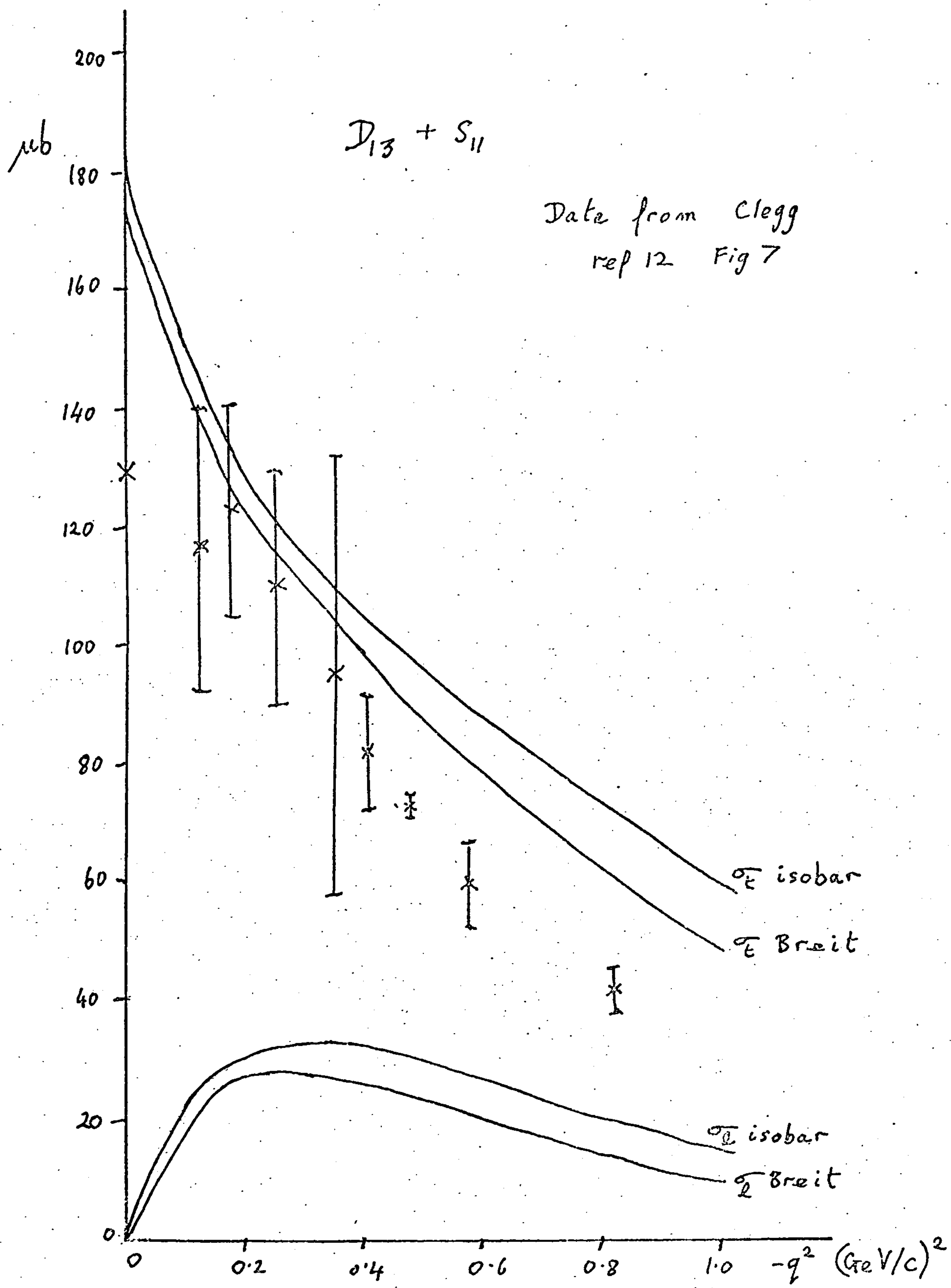
μb
D_{13} (1520)
80
70
60
50
40
30
20
10
0
0·2
0·4
0·6
0·8
1·0
$-q^2\ (\mathrm{GeV}/c)^2$
σ_t isobar
σ_t Breit
σ_ℓ isobar
σ_ℓ Breit

FIG 1



S_{11} (1535)
μb
110
100
90
80
70
60
50
40
30
20
10
Data from Clegg
ref 12 fig 7
σ_t isobar
σ_t Breit
σ_ℓ isobar
σ_ℓ Breit
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ (GeV/c)^2

FIG 2



$D_{13} + S_{11}$
μb
Data from Clegg
ref 12 Fig 7
σ_t isobar
σ_t Breit
σ_ℓ isobar
σ_ℓ Breit
200
180
160
140
120
100
80
60
40
20
0
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ $(GeV/c)^2$

FIG 3

analyses which seem to suggest that σ_ℓ never rises above 20°/o of σ_t [22]. However some interpretations of the data would require large, even dominant σ_ℓ so the situation is not at all clear[23,24].

e. The Ratio $R = A_{1/2}/A_{3/2}$

For the resonances D_{13} and F_{15} the helicity 1/2 amplitudes are zero or nearly zero experimentally[11]. This is accounted for in the CKO model by fixing the value of the spring constant α so that the helicity amplitude of the F_{15} is zero and then it is non-trivial that the helicity amplitude of the D_{13} is very small. However this is a contrived coincidence and as q^2 departs from zero the $A_{1/2}$ rapidly becomes dominant as was noted by Close and Gilman[3]. This should be seen as a marked change in the angular distribution of the pions (emitted as the resonance decays) as $-q^2$ changes from 0 to 0.6 GeV^2. The data at that time[25] did not show any change at all out to $-q^2 = 0.5\ GeV^2$, the exact opposite of what was predicted[7,26].

Close and Gilman did their analysis in the isobar frame so the first thing we looked for was the amount of frame dependence since the Breit frame is probably more suitable physically.

For the D_{13} (with $\alpha^2 = 0.17$, $g = 1$)

$$R = \frac{-1}{\sqrt{3}} \left(\frac{k^2}{\alpha^2} - 1\right)$$

	$-q^2$	$-R$
Isobar frame	0 0.6	0.17 1.70
Breit frame	0 0.6	0.496 2.45

There is considerable frame dependence here with even greater change in the Breit frame.

As the Breit frame is the most physical, let us calculate more points in it even though the slope is greater and more unsatisfactory than before. This is drawn in Figure 4.

For the F_{15}

$$R = \frac{\sqrt{2}}{4}\left(2 - \frac{k^2}{\alpha^2}\right) .$$

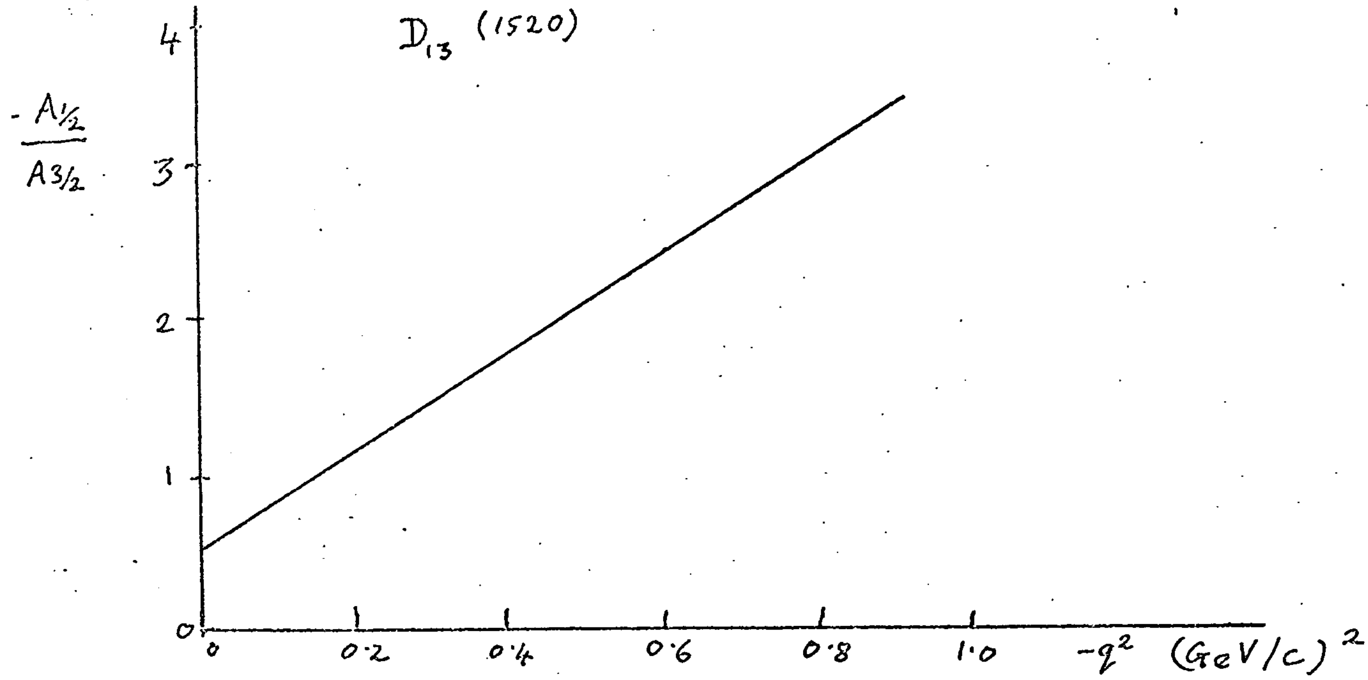
It is plotted in the Breit frame in Figure 5. We see that the slope is slightly less steep than for the D_{13}. R is not zero at photoproduction because the cancellation was fixed in the isobar frame and these calculations are in the Breit frame. There is little enough data for the D_{13} ratio but even less for the F_{15} so we will not consider the F_{15} ratio any further.

The fairly steep slopes of both ratios are due to the Gaussian wavefunctions. Other potentials would not have quite such a steep change. A peculiarity of the oscillator dynamics leads to

$$R = \frac{k^2 - \text{constant}}{\text{constant}}$$

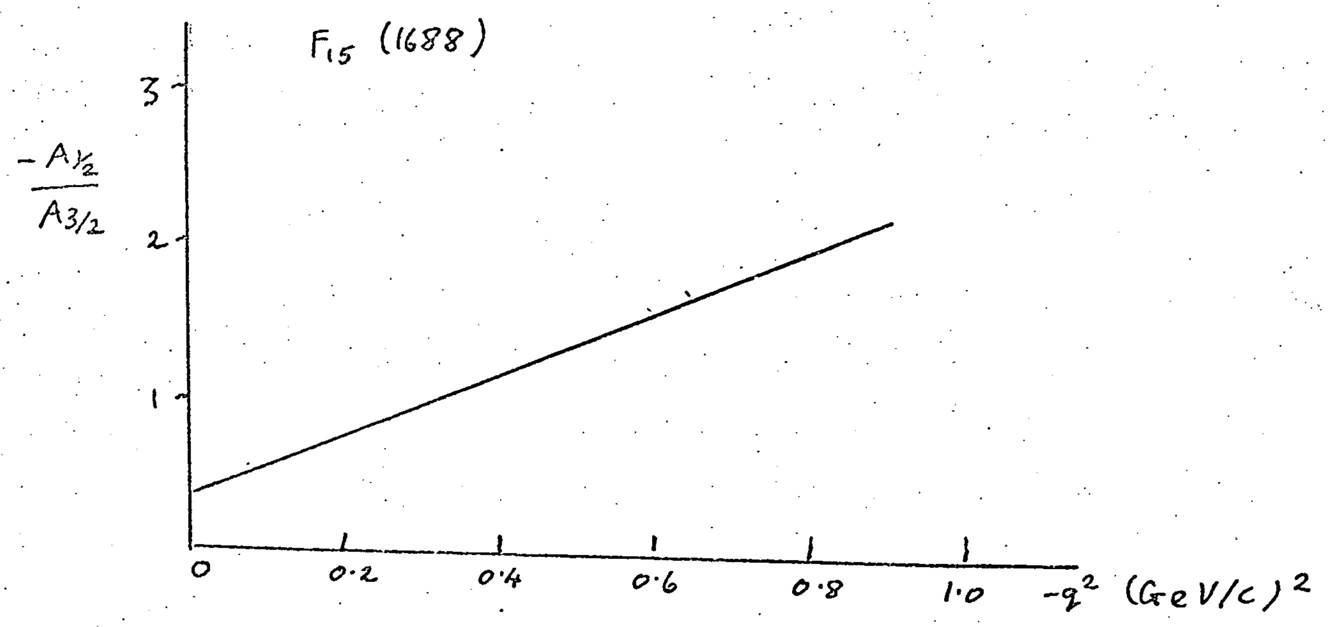
while the general form is

$$R = \frac{k^2 - \text{constant}}{k^2 - \text{constant}} .$$



D_{13} (1520)
$\frac{-A_{1/2}}{A_{3/2}}$
4
3
2
1
0
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ $(GeV/c)^2$

FIG 4



F_{15} (1688)
$\frac{-A_{1/2}}{A_{3/2}}$
3
2
1
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ $(GeV/c)^2$

FIG 5

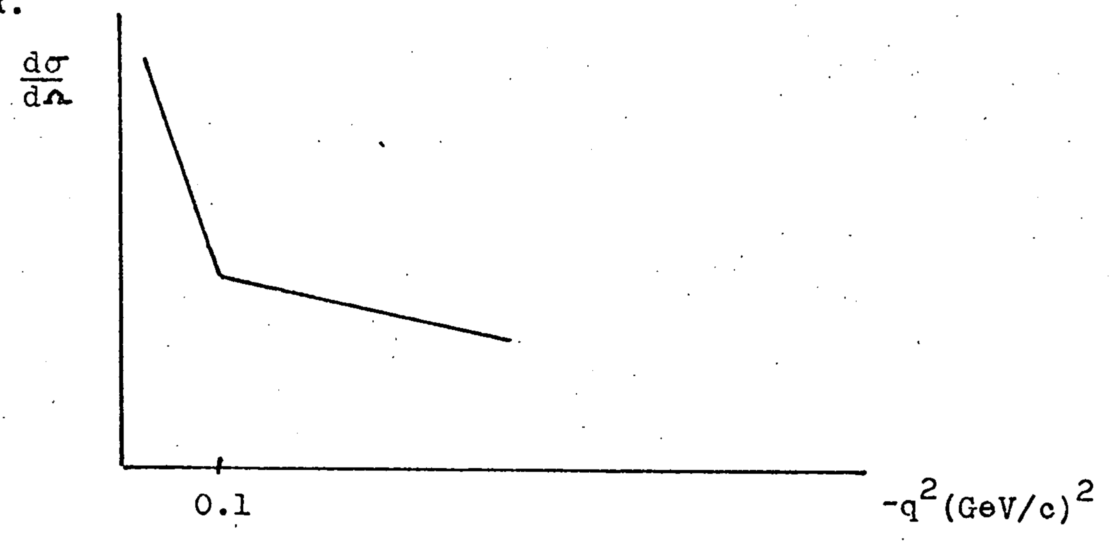
The extra k^2 dependence in the denominator will slow down the rate of change of $R^{(27,9)}$. Maybe a different potential will be the most effective solution of the difficulty.

CHAPTER 2

THE COULOMB-LIKE POTENTIAL

We would like a potential with a more realistic form factor. The Coulomb-like $\frac{1}{r}$ potential gives the electromagnetic form factor $(1 + q^2/b^2)^{-2}$, the Fourier transform of e^{-br}. Fitting to the experimental dipole form factor for elastic proton proton scattering gives $b^2 = 0.71$.

One objection may be that the higher excited states rapidly get closer together and approach a limit, after which there are only unbound states. But as Chu and Hendry(28) pointed out, the ground and first excited levels fit well. They suggested the possibility of a Coulomb force to explain the break in the slope of the proton proton elastic scattering cross-section at very forward angles at the ISR.



If it is assumed that the smaller slope is that due to diffraction, then the increase at small q^2 would come from a long range component of the strong interaction which could be a Coulomb force between the quarks.

This suggestion was not received with much enthusiasm because Thornber[8] did some work on the Coulomb potential, using the multipole formalism as for the harmonic oscillator, but found that the cross-sections were too small by about an order of magnitude. However, as before, she had worked in the isobar frame and it was worth re-investigating the cross-sections to see the effect of frame dependence which hopefully could be stronger than in the oscillator case. So let us set up wavefunctions and repeat the calculations of Chapter 1.

a. The Wavefunctions

$$V(r) = \frac{-\alpha}{r} \qquad \text{and} \qquad a_o = \frac{1}{m\alpha}$$

$$\psi(\underline{r}) = |n\ \ell\ m\rangle = R_{n\ell}(r)\ Y_{\ell m}(\theta\ \phi)\ .$$

The radial wavefunctions $R_{n\ell}(r)$ are well known and tabulated in elementary quantum mechanics text books.

$$R_{10} = \left(\frac{1}{a_o}\right)^{3/2} 2\ e^{-r/a_o}$$

$$R_{21} = \left(\frac{1}{2a_o}\right)^{3/2} \frac{r}{\sqrt{3}\ a_o}\ e^{-r/2a_o}$$

$$R_{32} = \frac{1}{(3a_o)^{3/2}} \frac{2\sqrt{2}}{27\sqrt{5}} \left(\frac{r}{a_o}\right)^2 e^{-\frac{1}{3}\frac{r}{a_o}}\ .$$

They only need to be multiplied by spherical harmonics to give the one particle wavefunctions. Then the required three particle wavefunctions are

$(1S)^3$ ground state, symmetric $L = 0^+$

$$\psi = \frac{1}{(4\pi)^{3/2}} \frac{8}{a_o^{9/2}} \exp \frac{-1}{a_o} (r_1 + r_2 + r_3)$$

$(1S)^2(2p)$ $L = 1^-$

$$\psi = \frac{1}{4\pi} \sqrt{\frac{2}{3}} \frac{1}{a_o^{11/2}} r_k Y_{1m}(\Omega_k) \exp \frac{-1}{2a_o} \left[2(r_i + r_j) + r_k \right] ,$$

the even mixed symmetric representation being taken.

$(1S)^2(3d)$, $L = 2^+$, symmetric

$$\psi = \frac{1}{4\pi} \frac{4}{\sqrt{5}} (\frac{2}{27})^{3/2} \frac{1}{a_o^{13/2}} r_k^2 Y_{2m}(\Omega_k) \exp \frac{-1}{3a_o} (3r_i + 3r_j + r_k$$

where the states are denoted by spectroscopic notation. The i,j,k refer to the three quarks and we must sum over all permutations.

Total wavefunctions are products of these with the SU(6) wavefunctions in the Appendix.

It has not been possible to factor out the centre of mass behaviour as was done for the harmonic oscillator. That is a very complex problem but in view of later results it must be said that this component would be expected to make the amplitudes larger than they should be [8]. This expectation is based on comparison with the harmonic oscillator where the centre of mass motion is accounted for. One cannot say exactly what would happen because of the algebraic complexity involved.

b. The Amplitudes

These are obtained, as in Chapter 1, by CKO's method and then Thornber's method as a check.

The results are

$$D_{13}: \quad A_o = i \frac{8}{\sqrt{3}} \left(\frac{4}{9}\right)^3 e \left(\frac{\pi}{k}\right)^{\frac{1}{2}} \frac{a_o k}{Z_1^3}$$

$$A_{1/2} = \frac{i}{3a_o} \frac{64}{81} \left(\frac{\pi}{k}\right)^{\frac{1}{2}} \frac{\mu}{Z_1^2} \left(\frac{8}{3} \frac{a_o^2 k^2}{Z_1} - \frac{1}{g}\right)$$

$$A_{3/2} = -i \frac{64}{81\sqrt{3}} \left(\frac{\pi}{k}\right)^{\frac{1}{2}} \frac{\mu}{g a_o} \frac{1}{Z_1^2}$$

$$S_{11}: \quad A_o = -i \frac{8}{\sqrt{6}} \left(\frac{4}{9}\right)^3 e \left(\frac{\pi}{k}\right)^{\frac{1}{2}} \frac{a_o k}{Z_1^3}$$

$$A_{1/2} = -i \frac{8}{3\sqrt{2}} \left(\frac{4}{9}\right)^2 \left(\frac{\pi}{k}\right)^{\frac{1}{2}} \frac{\mu}{a_o} \frac{1}{Z_1^2} \left(\frac{4a_o^2 k^2}{3Z_1} + \frac{1}{g}\right)$$

$$F_{15}: \quad A_o = -4i \frac{128 a_o^2 k^2}{135\sqrt{27}} \left(\frac{\pi}{k}\right)^{\frac{1}{2}} F \left(\frac{9}{16}\right)^4 \frac{1}{Z_2^4}$$

$$A_{1/2} = -\left(\frac{2\pi}{5k}\right)^{\frac{1}{2}} \mu F \frac{9\sqrt{3}}{4} \frac{k}{Z_2^3} \left(\frac{9}{8} \frac{a_o^2 k^2}{Z_2} - \frac{1}{g}\right)$$

$$A_{3/2} = -\frac{9}{8\sqrt{15}} \left(\frac{\pi}{k}\right)^{\frac{1}{2}} \frac{\mu}{g} F \frac{k}{Z_2^3}$$

where $Z_1 = 1 + \dfrac{4a_o^2 k^2}{9}$

$$Z_2 = 1 + \frac{9a_o^2 k^2}{16}$$

and F is the fraction of the $(1S)^2(3d)$ wavefunction which would be present after centre of mass elimination. For the harmonic oscillator $F = \sqrt{\frac{2}{3}}$.

As before there is one free parameter a_o which will be fixed by demanding zero or very small helicity $\frac{1}{2}$ amplitudes for the D_{13} and F_{15} resonances.

F_{15}: Taking $g = 1$ and $k^2 = 0.34\ GeV^2$

$A_{1/2} = 0$ leads to $a_o^2 = 5.24\ GeV^{-2}$

or $a_o = 0.46$ fm.

D_{13}: $A_{1/2} = 0$ leads to $a_o^2 = 2.04\ GeV^{-2}$

or $a_o = 0.286$ fm.

There is rather a large difference between these two values of a_o. One could perhaps take the average as a compromise: $a_o^2 = 3.64\ GeV^{-2}$.

Because the Coulomb potential leads to the correct electromagnetic form factors we have as an independent check, the value of a_o required to fit the elastic form factor from experiment: $a_o^2 = 5.65\ GeV^{-2}$.

We now have a choice of values. The cancellations in the helicity $\frac{1}{2}$ amplitude for the D_{13} and F_{15} are measured through the backward or forward differential cross-section. How sensitive is it to changes in a_o ? From CKO(4)

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=\pi} = \frac{1}{3}\ \frac{2J+1}{4\pi}\ \frac{\varkappa}{\Gamma}\ \frac{m}{M}\ \left|A_{1/2}\right|^2$$

where J = resonance spin, $\mathcal{X}$ = elasticity and m, M are the masses of the proton and resonance respectively.

For the D_{13}:

$$J = \frac{3}{2}, \quad \mathcal{X} = 0.5, \quad \Gamma = 0.115 \text{ GeV}$$

for photoproduction k^2 = 0.22 GeV, so substituting three values of a_o^2 gives

	a_o^2 GeV^{-2}	$\left.\frac{d\sigma}{d\Omega}\right\|_{\theta=\pi}$ μb sr^{-1} × 10^{-2}
average	3.64	2.34
F_{15} canc.	5.24	3.38
El. scatt.	5.65	3.43

Walker's analysis gives 8×10^{-2} μb sr^{-1} experimentally and the harmonic oscillator model[4] gives 5×10^{-2} μb sr^{-1} so these results are a little small but they are fairly constant and do not place any restriction on the value used.

However calculating the backward differential cross-section for the F_{15} gives us

	a_o^2 GeV^{-2}	$\left.\frac{d\sigma}{d\Omega}\right\|_{\theta=\pi}$ μb sr^{-1}
average	3.64	0.035
El. scatt.	5.65	0.0011

These are upper limits taken with F = 1. F will be somewhere between 0 and 1 so $\frac{d\sigma}{d\Omega}$ will be some fraction of the numbers just given (e.g. $\frac{2}{3}$ for the harmonic oscillator) but we have the order of magnitude. For the elastic scattering value this is acceptable, the average value gives a rather large differential cross-section, larger than that for the D_{13} and even when reduced by some fraction will probably be too large to agree with experiment. The analysis of Moorhouse and Oberlack[30] gives 1.1×10^{-2} μb as the experimental backward differential cross-section.

We will use the average value of a_o for the ratio behaviour just to see what the magnitudes are but we need not take the numerical values very seriously. The F_{15} cancellation and elastic scattering values of a_o are both consistent with the cancellations of the F_{15} and D_{13} resonances.

c. <u>The Ratio of Transverse Amplitudes</u>

For the Coulomb potential with $g = 1$, for the D_{13}

$$R = \frac{-1}{\sqrt{3}} \left(\frac{8a_o^2 k^2}{3(1 + \frac{4}{9} a_o^2 k^2)} - 1 \right) .$$

Frame dependence:

	$-q^2$	$-R(a_o^2 = 5.24\ \text{GeV}^{-2})$
Isobar	0	0.59
	0.6	1.54
Breit	0	0.89
	0.6	1.76

The change is less in the Breit frame, unlike the harmonic oscillator potential although the absolute value in the Breit frame is still greater.

We now calculate the variation of R with q^2 in the Breit frame, using both the average and the F_{15} cancellation values of a_o. These are plotted in Fig. 6 and are certainly less steep than the oscillator's ratio in Fig. 4. So a modification of the potential can solve this contradiction without losing one of the oscillator's major successes, the D_{13} and F_{15} cancellations. A more sophisticated potential may do things still better.

d. Cross-sections

Let us in this section use $a_o^2 = 5.65$, the elastic scattering value, as it may be a more significant number, coming from experiment rather than from the model.

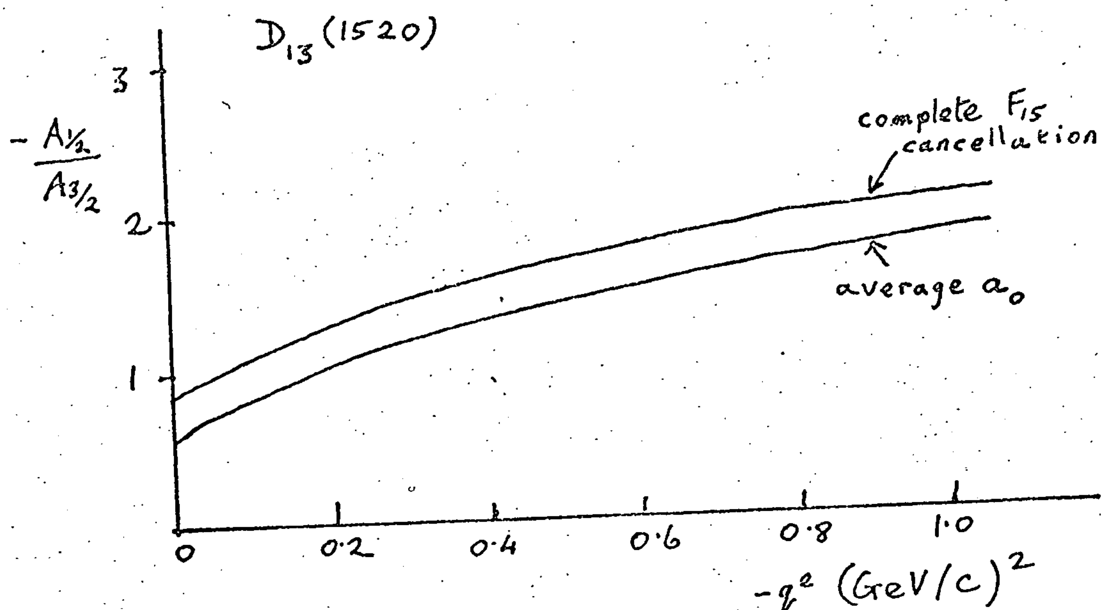
We calculate σ_ℓ and σ_t just as in Chapter 1, for the isobar and Breit frames and for the D_{13} and S_{11} resonances.

We present the Breit frame results only. There is frame dependence and the isobar frame numbers are slightly larger but the qualitative picture is unchanged.

The results are presented in Figures 7, 8, 9.

It is very disappointing to see that the cross-sections are so small in absolute magnitude as to make the Coulomb potential quite untenable. The reason is that for higher resonances the exponential does not damp as strongly as the ground state and to normalise the wavefunctions one must divide by higher numbers the more excited the state, and so the less they will agree with experiment.

However, this was not quite a waste of time. The cross-section shapes are reasonable and the ratio change is moderate



D13 (1520)
3
2
1
- A½ / A3/2
complete F15 cancellation
average a0
0
0·2
0·4
0·6
0·8
1·0
-q² (GeV/c)²

FIG 6

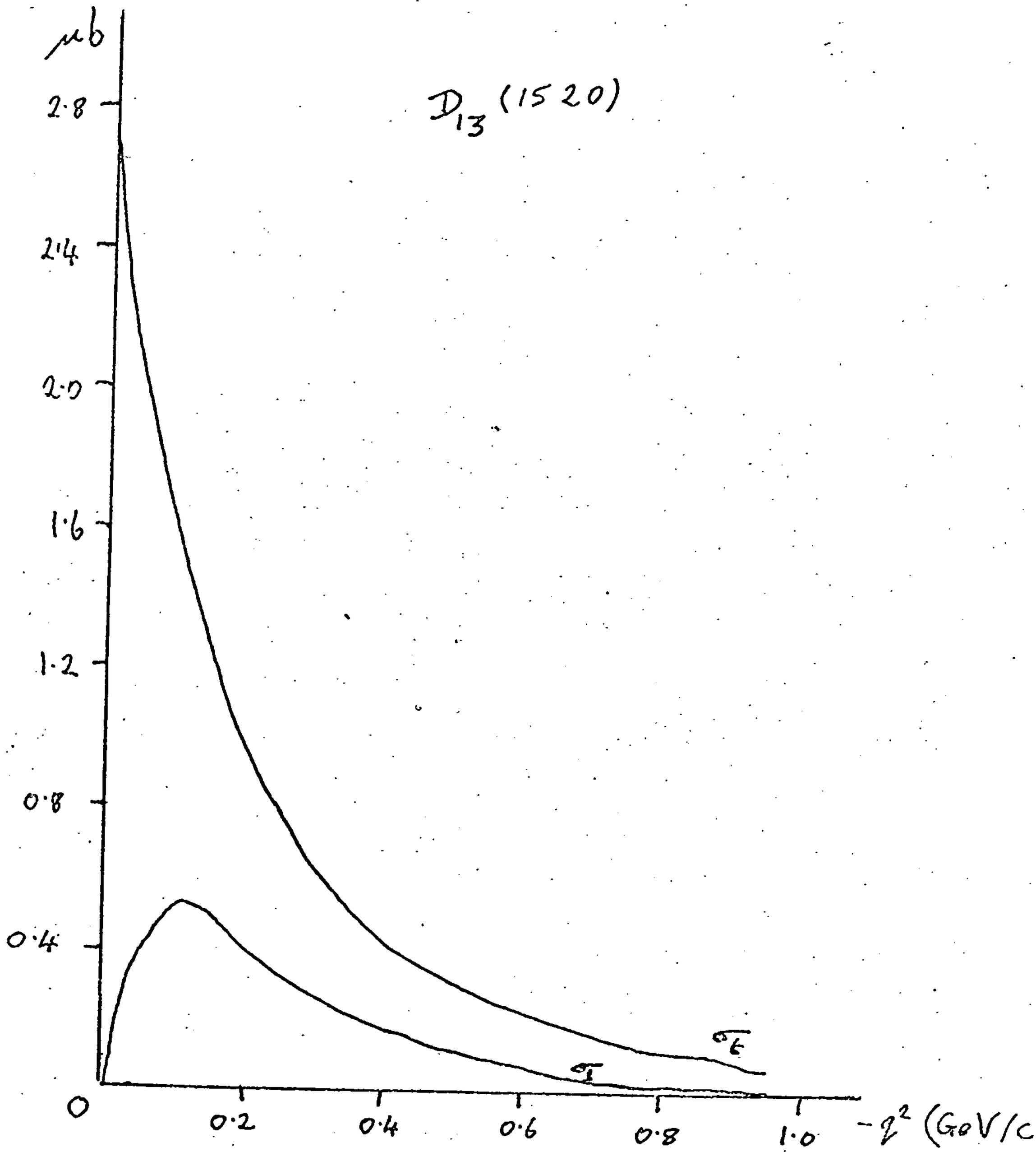
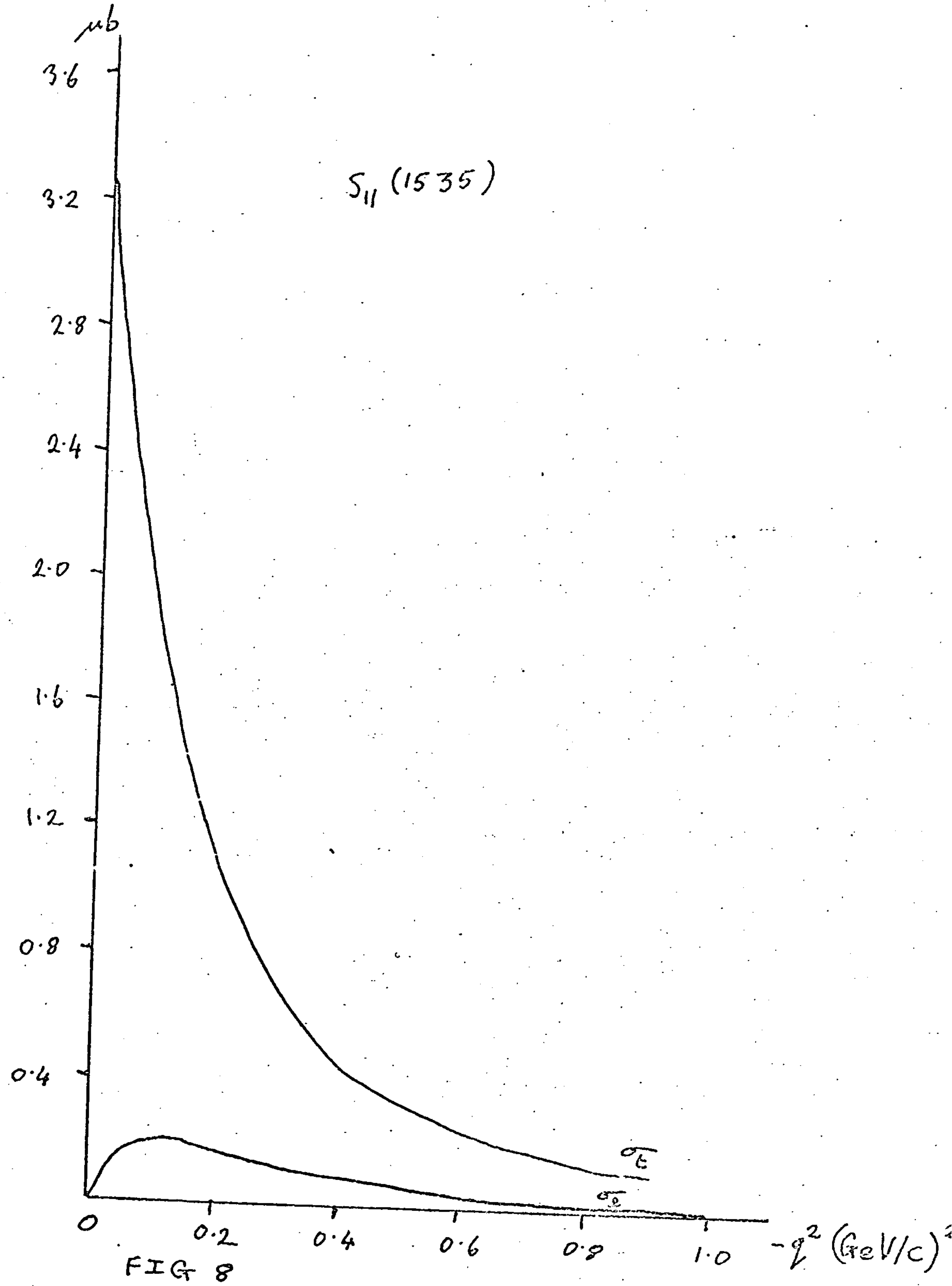

μb
2·8
2·4
2·0
1·6
1·2
0·8
0·4
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ (GeV/c
D_{13} (1520)
σ_t
σ_s

FIG 7



μb
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
0.4
S_{11} (1535)
σ_t
σ_e
0
0.2
0.4
0.6
0.8
1.0
$-q^2$ (GeV/c)

FIG 8

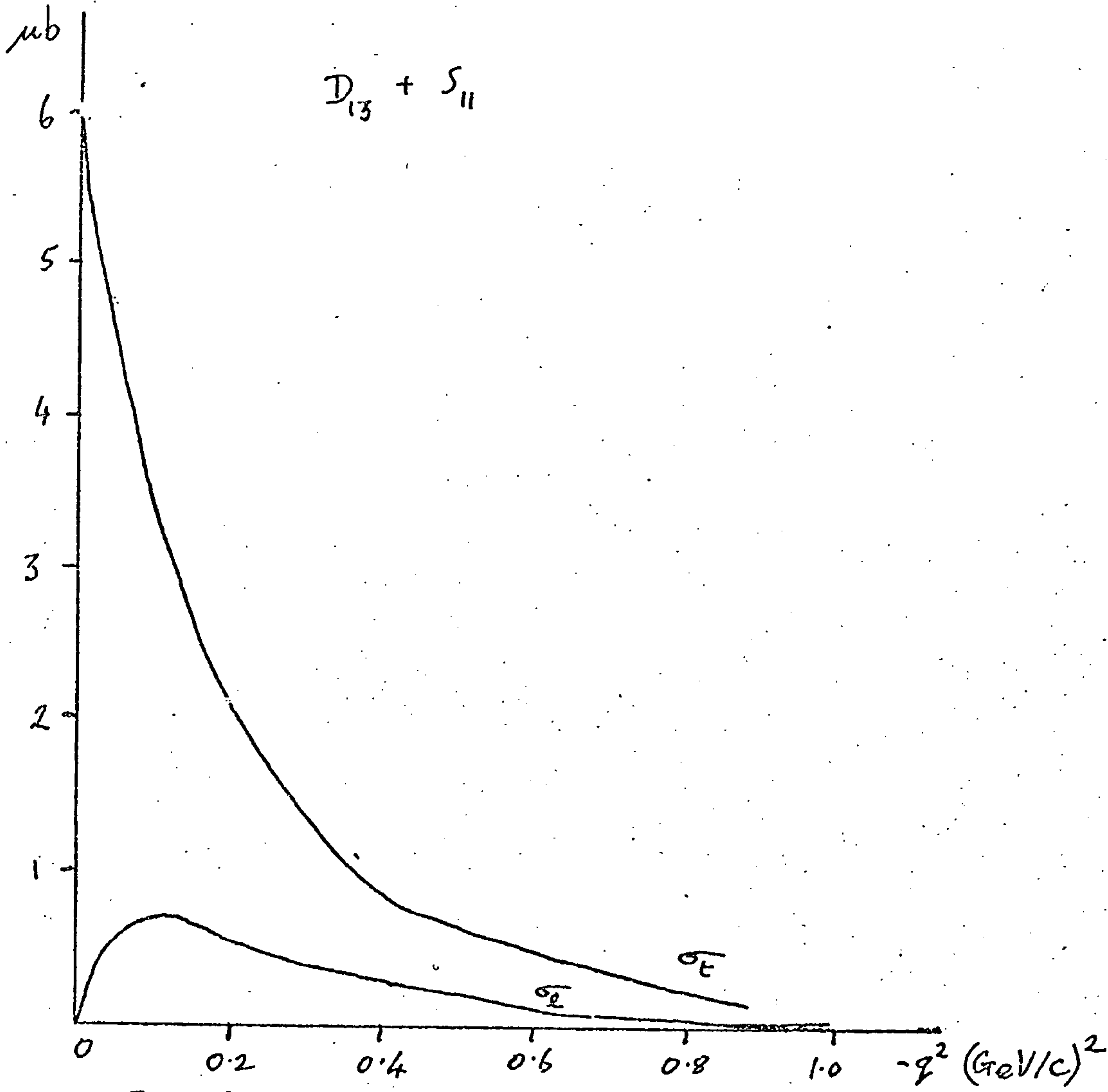


FIG 9

and these things depend on the form factor. Since the Coulomb potential form factor has the correct q^2 dependence, the general shape of the cross-sections and the moderate change in the ratio could be expected to hold even if the potential (or other mechanism) which gives rise to this dependence is changed. For instance, Lipes[14] has a relativistic harmonic oscillator model in which the form factors have correct asymptotic behaviour because of the effect of the Lorentz transformation on the spins of the three quarks. He does it in a very ad hoc way but we can see that Coulomb potential results may still be relevant as an indication of the behaviour of a more complete model.

Let us now return to the harmonic oscillator.

CHAPTER 3

HIGHER ORDER TERMS

The electromagnetic interaction used by CKO and Thornber is the lowest order non-relativistic interaction. This can be derived rigorously from the relativistic current by applying the Foldy-Wouthuysen (FW) transformation to it[29].

The Dirac Hamiltonian is $H = \beta m + \underline{\alpha}.(\underline{p} - e\underline{A})$ and the FW transformation removes odd operators O (for which $\beta O = -O\beta$) order by order to any order in m. It is a unitary transformation $\exp i S$

$$\text{with} \quad S_{n+1} = \frac{-i\beta}{2m} \left(\begin{array}{l}\text{odd operators in } H_n \text{ of} \\ \text{lowest order in } 1/m\end{array}\right) .$$

In this way it is found that the next order term in the series is spin orbit coupling.

$$H_{SO} = -6\sqrt{\pi k}\,\mu \left[q e^{-i\underline{k}.\underline{r}} \frac{1}{2m_q}\left(2 - \frac{1}{g}\right) \left[S_z(p_x + ip_y) - S_+ p_z\right]\right] .$$

Apart from the desire to improve the level of approximation, there is experimental evidence that this model needs spin-orbit coupling. CKO discovered a selection rule that the F_{15} could not be photoproduced from a neutron in an helicity $\frac{3}{2}$ state. This rule is violated by the spin orbit term and experimentally a value of this amplitude, compatible with the spin orbit term in magnitude and sign, seems to have been found in partial wave analysis[30].

Note that it is an assumption to use the current acting on one quark for the entire interaction. Brodsky and Primack[19,31] and others have demonstrated the need for non-additive terms involving all the quarks to satisfy certain sum rules like the

Drell-Hearn-Gerasimov sum rule. We will not worry about such terms here, they are very small. But in relativistic models they become important and later we will discuss a model which is explicitly non-additive.

a. D_{13} Matrix Elements

Spatial matrix elements are

$$R_{11} = \left\langle D_{13} \right| (p_x + ip_y) e^{-i\underline{k}\cdot\underline{r}} \left| p \right\rangle = i\left(\frac{2}{3}\right)^{\frac{1}{2}} \alpha \quad e^{-k^2/6\alpha^2}$$

$$R_{1Z} = \left\langle D_{13} \right| p_z \, e^{-i\underline{k}\cdot\underline{r}} \left| p \right\rangle = -i \frac{\alpha}{\sqrt{3}} \left(1 - \frac{k^2}{2\alpha^2}\right) e^{-k^2/6\alpha^2} \quad .$$

These contribute to the helicity amplitudes as

$$A^{SO}_{1/2} = \frac{1}{\sqrt{3}} R_{11} + \left(\frac{2}{3}\right)^{\frac{1}{2}} R_{1Z}$$

$$A^{0}_{3/2} = R_{11} \quad .$$

The extra terms in $A_{1/2}$ affect the cancellation.

$$A^{SO}_{1/2} = i \, \frac{1}{3}\left(2 - \frac{1}{g}\right) \frac{\mu}{m_q} (\pi k)^{\frac{1}{2}} \quad \alpha \quad \left(\frac{k^2}{2\alpha^2} - \frac{1}{2}\right) e^{-k^2/6\alpha^2} \quad .$$

But adding this to the previous $A_{1/2}$ we find that for $A_{1/2} = 0$ $\quad \frac{k^2}{\alpha^2} = 1$ as before. The spin orbit interaction has n effect here but this may just be a fluke and we must also check the F_{15} matrix elements to see if mutual cancellation is still possible.

b. F_{15} Matrix Elements

Here

$$A^{SO}_{1/2} = -\frac{\sqrt{\pi k}\,\mu}{m_q}\left(2 - \frac{1}{g}\right)\frac{\sqrt{2}}{6\sqrt{5}}\left(2k + \frac{k^3}{2\alpha^2}\right) e^{-k^2/6\alpha^2} .$$

Setting $g = 1$, the new condition for cancellation is

$$\alpha^2 = \frac{\frac{k^2}{2}\left(2 + \frac{k}{2m_q}\right)}{\left(2 - {}^{k}/m_q\right)} .$$

$\alpha^2 = \frac{(k^o_{F_{15}})^2}{2}$ was the condition previously and is compatible with the D_{13} condition $\alpha^2 = (k^o_{D_{13}})^2$ where k^o resonance name is the value of k for that resonance at photoproduction. As the D_{13} condition has remained the same, clearly for mutual cancellation the F_{15} condition must also remain the same approximately. So $\frac{2 + k/2m_q}{2 - k/m_q}$, the rest of the fraction, should be approximately 1. It will approach one, clearly, as m_q gets larger and larger compared to $k^o_{F_{15}} = 0.59$. So we really want a heavy quark here. For consistency, with $g = 1$, $m_q = \frac{1}{3}M_N$ but this very light quark mass has always been a mystery for this type of model. A heavy quark would fit in better with the fact that quarks have not been discovered, but m_q would have to vary independently of g and μ. Choosing $g = 1$ and $\mu = \mu_p$ we have m_q fixed by numerical consistency and it is not at all reasonable to try to change m_q. However, the inconsistency is rather forced on us and some people[10] claim that a light quark mass is less bearable than numerical inconsistency.

If we have $g \neq 1$ then both the D_{13} and F_{15} cancellations change and the only way to make them compatible is to have a large m_q so that the k/m_q terms are negligible whatever the q value we choose. Varying g as well as m_q starts getting very complicated; there are too many free parameters and the model loses its predictive power and simplicity.

Would it not be possible to leave the parameters as they are? With $\alpha^2 = 0.17$ and $m_q = 0.33$ GeV one finds for photoproduction of the F_{15}:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=\pi} = 0.874\ \mu b.$$

Moorhouse and Oberlack's analysis[30] and that of Metcalf and Walker[32] yields 1.1×10^{-2} μb as the experimental value. So obviously the heavy m_q is necessary here.

c. <u>The Ratio R for the D_{13}</u>

Including the spin orbit terms

$$R = -\frac{k^3 + 4m_q k^2 - (k + 4m_q)\alpha^2}{(\sqrt{3}\,k + 4\sqrt{3}\,m_q)\alpha^2} .$$

From the D_{13} cancellation $\alpha^2 = (k^o_{D_{13}})^2 = 0.22.$ Previously we took $\alpha^2 = 0.17$ to fit the F_{15} cancellation. For a heavy enough quark this would still hold, otherwise α^2

could be a little bigger and still give zero F_{15} and small D_{13} helicity $\frac{1}{2}$ amplitudes with a less heavy quark. So to be arbitrary we use two values of α^2, 0.17 and 0.25 and plot R against q^2 in Figures 10 and 11.

The $\alpha^2 = 0.17$ graph with spin orbit coupling turns out to be exactly the same as without. So we checked by plotting R for $\alpha^2 = 0.25$ without spin orbit coupling (Fig. 12). This graph is again the same as with spin orbit coupling.

The obvious conclusion is that R is sensitive to α but not to the spin orbit coupling. The effect of the spin orbit coupling is only to change the value of α required for cancellation. It could then be left out as far as R is concerned.

So the important parameter is α. Without spin orbit coupling we plot R at two values of q^2 versus a range of α^2 (Fig. 13). It is seen that the magnitude of R and the amount that R changes both decrease with increasing α^2. This variation in α^2 is quite fair because as R is independent of spin orbit coupling there is a whole range of possible α^2 values leading to the F_{15} cancellation even though in one model there is only one specific value.

It can also be noted that as Clegg recently pointed out[12] the photoproduction values of k for the D_{13} and F_{15} used to fix the cancellation are from the isobar frame and it might be more consistent to take them from the Breit frame since that is the frame we used for electroproduction. The values of k are slightly larger and we would have $\alpha^2 = 0.24$ instead of 0.17. The cross-section values will change but probably only slightly - we have seen how little

$D_{13}(1520)$

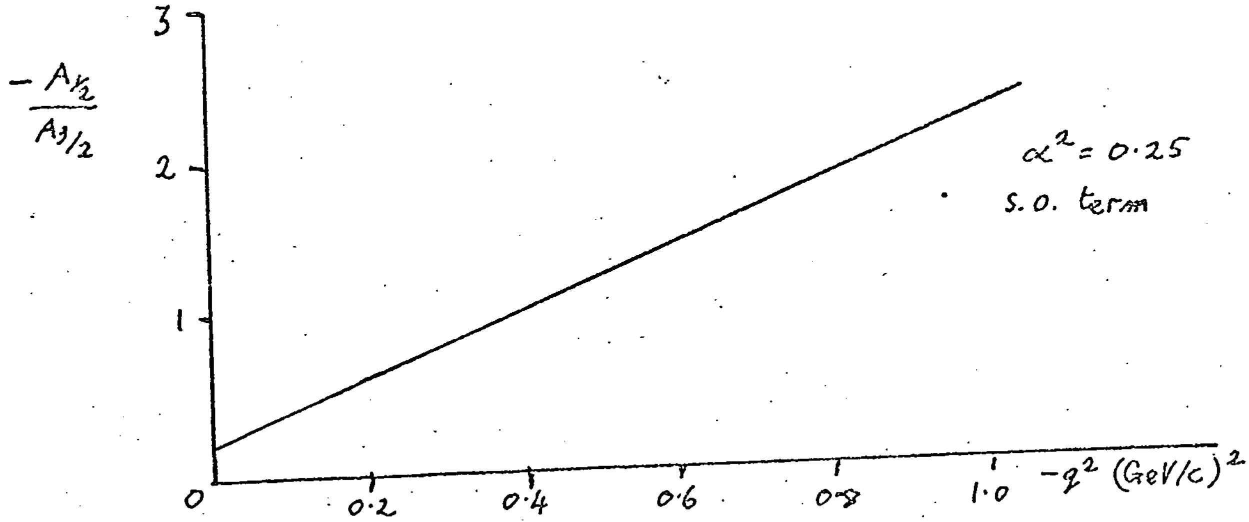


FIG 10

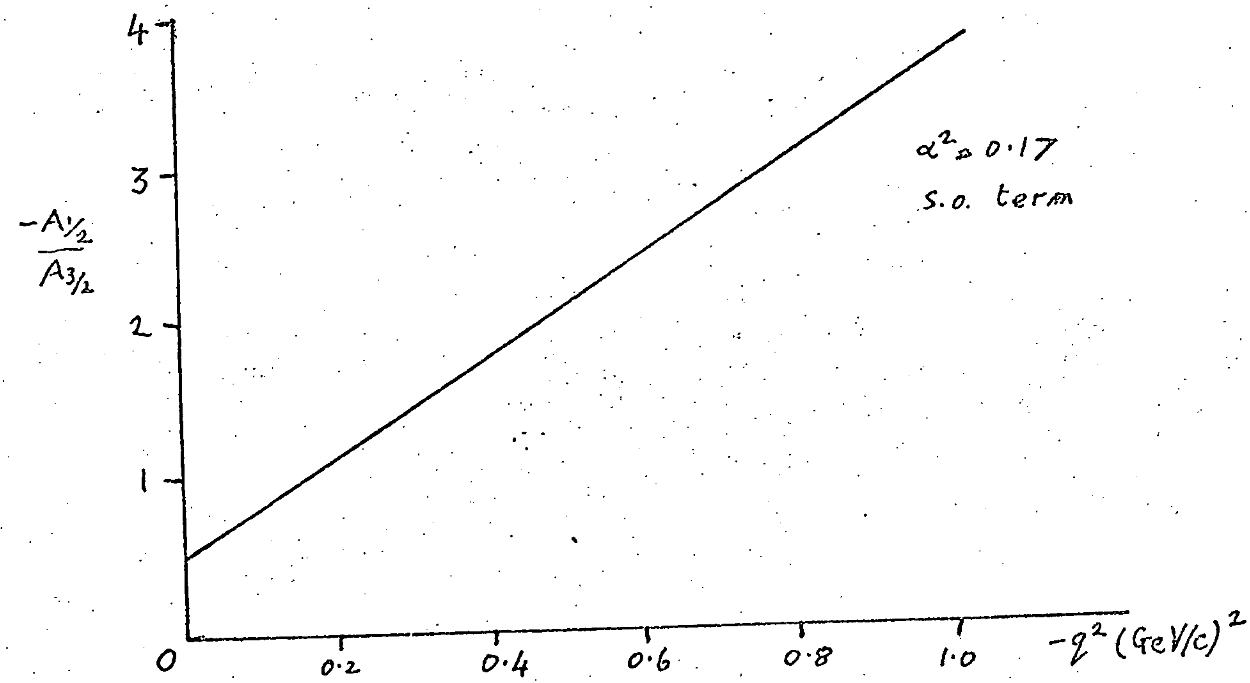


FIG 11

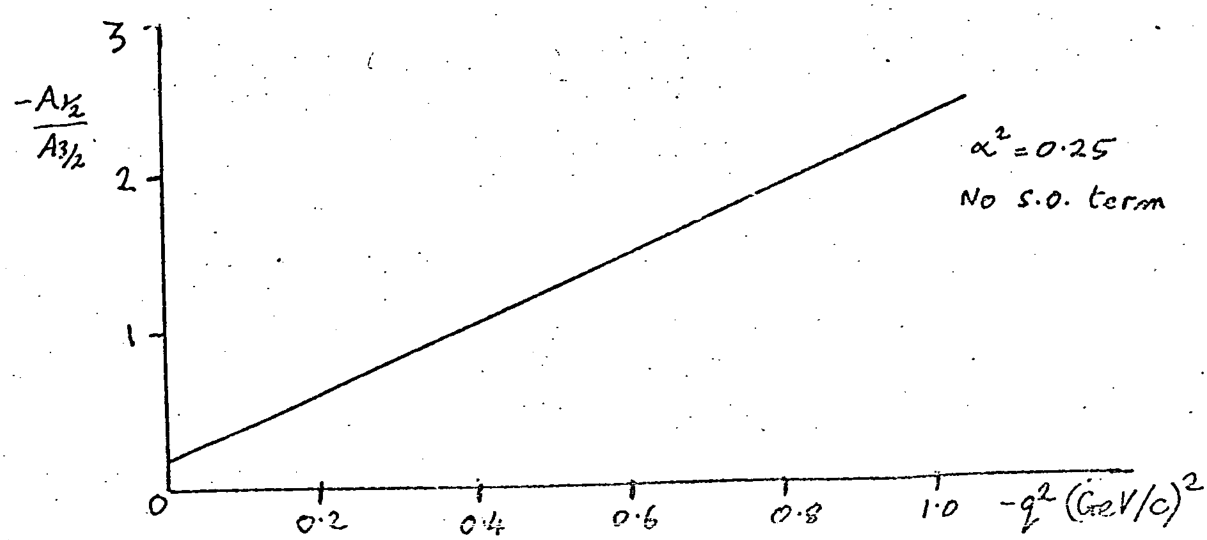
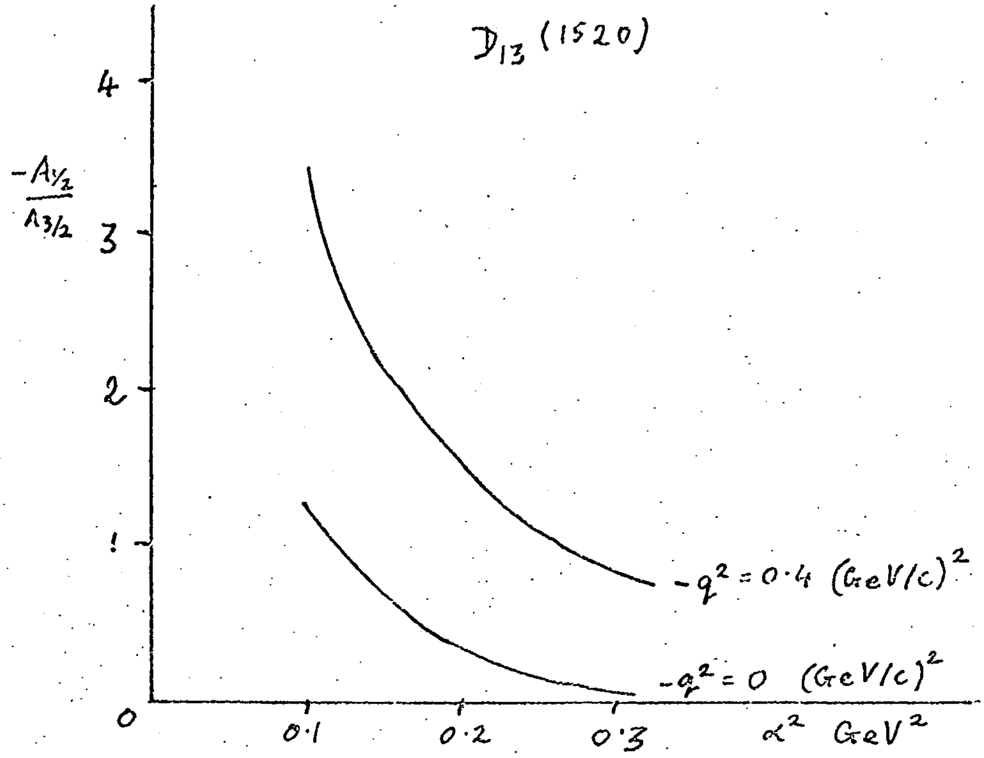


FIG 12



D_{13} (1520)
$-A_{1/2}/A_{3/2}$
4
3
2
1
0
0·1
0·2
0·3
α^2 GeV^2
$-q^2 = 0{\cdot}4$ $(GeV/c)^2$
$-q^2 = 0$ $(GeV/c)^2$

FIG 13

effect frame dependence has on cross-sections. From Fig. 13 we can see that R will be smaller and decreasing less, almost as flat as for the Coulomb potential. When CKO used the isobar frame values(4) they said that these values of k were taken from experiment. However even at $q^2 = 0$ the different frames give different k:

isobar : $$k^2 = \frac{(M^2 - m^2)^2}{4M^2}$$

Breit : $$k^2 = \frac{(M^2 - m^2)^2}{2(M^2+m^2)}$$

where M = resonance mass, m = proton mass.

So this is another factor to be taken into account when trying to get the best fit to all the data. One can vary α by fixing it in different frames or by using the spin orbit interaction (or both).

d. Summary

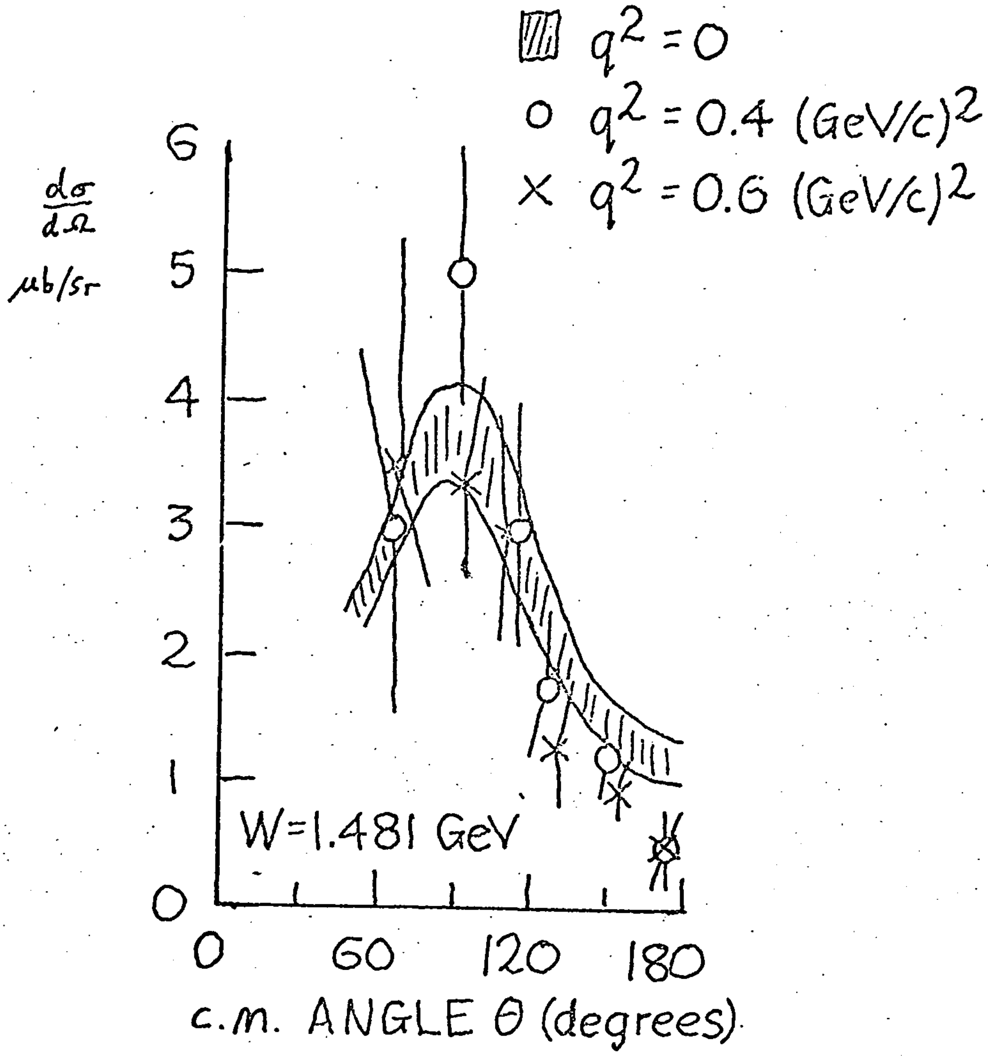
We have investigated several versions of the quark model mainly with an eye on the behaviour of R. The preliminary data on π^0 production(25) seemed to show no change at all but later data on π^+ production(33) is compatible with a change in R although probably not as great as the harmonic oscillator gives but perhaps like that of the Coulomb potential.

This information is inferred from the angular distributions of the pions into which the resonance decays in the process $\gamma N \longrightarrow N^* \longrightarrow \pi N$. For the D_{13} the distribution should go from being nearly $\sin^2\theta$ at $q^2 = 0$ where R = 0 to isotropic when $R = 1$ to approximately $1 + 3\cos^2\theta$ when $R^{-1} = 0$. So the change should be very distinct.

A sample of the data for π^0 production at $W = 1.481$ GeV [26] is shown on Fig. 14. We see that there is little or no variation from photoproduction. In Fig. 15 we show data from the same experiment but for π^+ production at $q^2 = -0.45$ GeV^2 where there is a great deal of structure different from photoproduction[33]. This structure has been interpreted as interference between resonances and background in a dispersion relation model by Devenish and Lyth[34]. The two curves are their predictions. The dotted line is under the assumption that the ratios $A_{1/2}/A_{3/2}$ retain their photoproduction values throughout the region: 0.03 for the D_{13} and 0 for the F_{15}. The solid line is for large R's: 3 for the D_{13} and 0.3 for the F_{15}. The solid line is in much better agreement with the data, implying substantial helicity 1/2 amplitudes for both the D_{13} and the F_{15}. The π^0 data which did not seem to allow noticeable helicity 1/2 excitations can be manipulated to agree with the π^+ data. At the moment the situation is very uncertain because the data can be manipulated to fit a wide range of predictions.

The main thing is that some change does take place and it is possible to fit this change using the standard harmonic oscillator model and changing α or changing the potential or modifying the Gaussian form factors by the Lorentz transformation as in Lipes.

So the quark model is by no means dead yet and we can go on to discuss other aspects of it.



$q^2 = 0$
o $q^2 = 0.4$ (GeV/c)^2
× $q^2 = 0.6$ (GeV/c)^2
$\frac{d\sigma}{d\Omega}$
μb/sr
6
5
4
3
2
1
0
W=1.481 GeV
0
60
120
180
c.m. ANGLE θ (degrees)

FIG 14

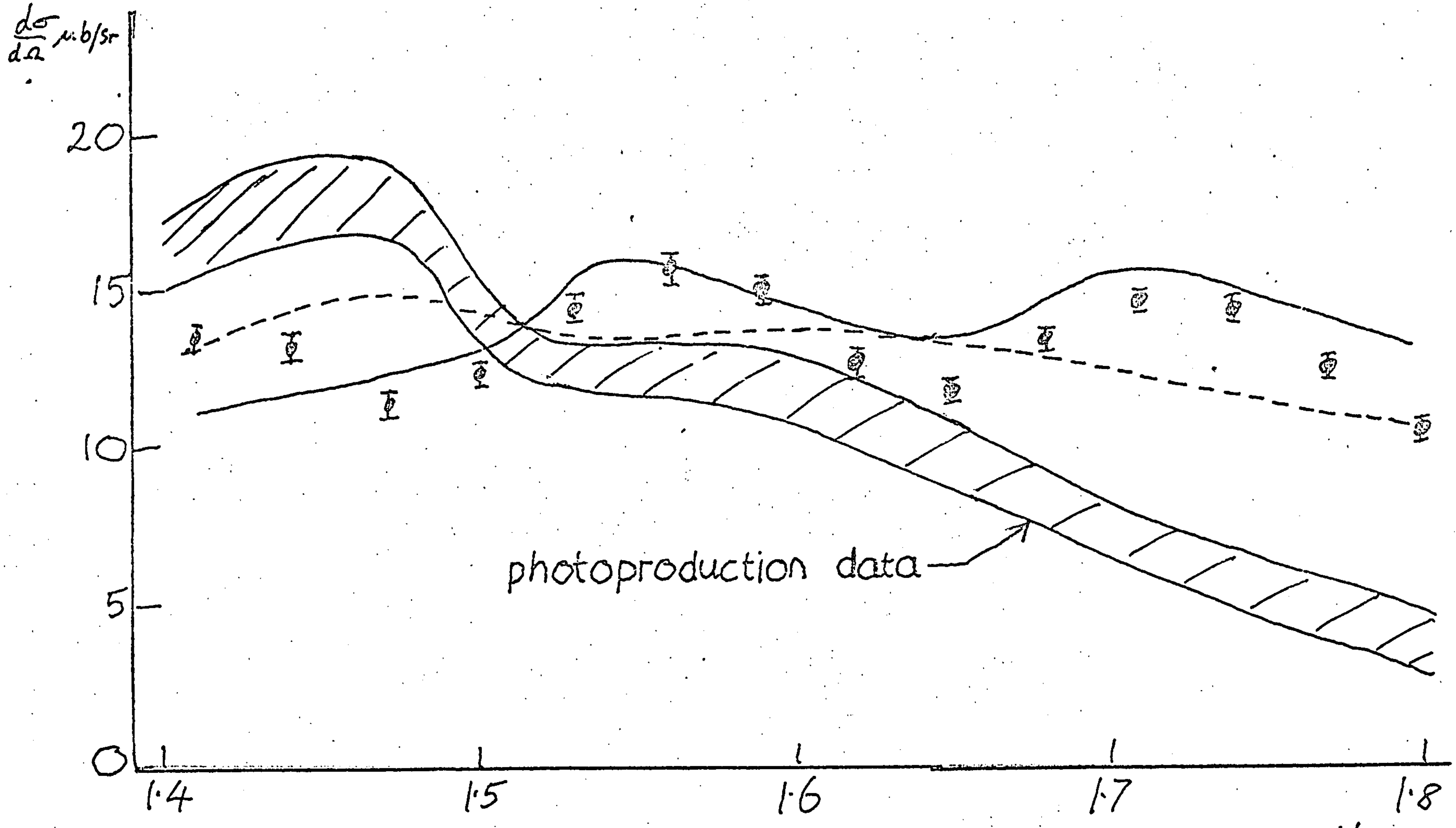


FIG 15

CHAPTER 4

EXTENSIONS OF THE MODEL

a. SU(6) Breaking

SU(6) is an exact symmetry of the non-relativistic quark model discussed in previous chapters. It is not an exact symmetry of nature since it makes predictions which are not fulfilled. Some predictions are good: the ratio of proton and neutron magnetic moments is predicted to be -3/2 which agrees with observation to within about 2°/o. In addition, the SU(6) resonance classification is very successful. But the ratio of the axial and vector form factors in neutron beta decay is given by SU(6) as $g_A/g_V = -5/3$, while in reality it is around 25°/o smaller. Also since SU(2) spin is a subgroup of SU(6), spin and orbital angular momentum are separately conserved in any interaction invariant under SU(6). This leads to such predictions as forbidding the decays of the D_{13} and F_{15} resonances to πN in the helicity 3/2 state. These results seem to indicate a broken SU(6) symmetry. Note that this symmetry breaking has nothing to do with lack of mass degeneracy in multiplets. Mass splitting interactions break the resonance classification symmetry but even if it were exact, the symmetry of current algebra dealing with interactions would be distinctly different.

Current algebra is successful where SU(6) symmetry fails. g_A/g_V is given correctly by the Adler-Weisberger sum-rule. This indicates that the symmetry is broken because the physical nucleon states are not members of representations of the group generated by the charges of current algebra.

This problem exists for non-relativistic quark models but it is dealt with in the relativistic case where it is more acute. One can then include the further difficulty that SU(6) is a static group and must therefore be modified to include the spin Lorentz transformation properties which will be induced during reactions.

b. <u>Relativistic Quark Models</u>

We have already mentioned one problem. Ths spins of the spectator quarks are affected by being included in the centre of mass acceleration during a collision. They are Wigner rotated. In Lipes's model(14) this was used in an *ad hoc* way to get the good behaviour of the electromagnetic form factors despite his use of Gaussian wavefunctions.

Another problem of relativistic quark models is unitarity. Having four dimensional excitations instead of three leads to states of imaginary mass which can only be eradicated by banning time-like excitations. This destroys unitarity because all excitations are needed to form a complete set of states. So they must be compensated for in an *ad hoc* fashion. Lorentz covariance is also lost as one space-time direction is singled out to be banned. Nearly all relativistic quark models seem to suffer from this as they are constructed covariantly. If one starts with a non-covariant model such as that of Le Yaouanc et al.(35) the problem need not arise.

The underlying unitary symmetry of relativistic models is not SU(6) as this is a static group. As soon as the quarks are allowed to move, the SU(2) spin subgroup classification is distorted by Wigner rotations in the plane perpendicular to the motion. The group proposed by Lipkin and Meshkov(36) and commonly used instead of SU(6) is $SU(6)_W$. SU(6) has as a subgroup the spin group SU(2) but by using other representations of SU(2) one can construct different representations of SU(6). This is analogous to the use of U spin and V spin subgroups of SU(3). $SU(6)_W$ has an SU(2) subgroup generated by the operators of W spin as shown in the table. They satisfy SU(2) commutation rules and represent a conserved spin similar to ordinary spin.

Spin all particles	W spin quarks	W spin anti-quarks
$S_x = \frac{1}{2}\sigma_x$	$W_x = \beta \frac{\sigma_x}{2}$	$-\beta \frac{\sigma_x}{2}$
$S_y = \frac{1}{2}\sigma_y$	$W_y = \beta \frac{\sigma_y}{2}$	$-\beta \frac{\sigma_y}{2}$
$S_z = \frac{1}{2}\sigma_z$	$W_z = \frac{\sigma_z}{2}$	$\frac{\sigma_z}{2}$

W spin operators commute with generators of Lorentz boosts in the z direction so that this is a relativistic replacement for SU(6) only for collinear processes in the z direction, which two body resonance formation is.

c. Possible Solutions

We see that the relativistic quark model is a non-trivial extension of the non-relativistic case. It is linked with SU(6) breaking through the spin Wigner rotations, adding that problem to the specifically relativistic ones.

Two possible solutions are to have separate vertex and classification symmetries or to have one classification symmetry and break it dynamically.

The next two chapters describe first steps in these directions and in the conclusion a possible unification is mentioned.

CHAPTER 5

CURRENT AND CONSTITUENT QUARKS

a. $SU(6)_W$ and its Two Representations

As we saw in the last chapter, one method of simultaneously constructing a relativistic quark model and dealing with SU(6) breaking is to have separate vertex and classification symmetries. They will be distinct representations of $SU(6)_W$.

Classification $SU(6)_W$ will have as subgroup the SU(3) representation used to arrange hadrons in multiplets. Its associated quarks are called constituent quarks. Vertex symmetry involves the SU(3) of current algebra which, as remarked in Chapter 4, is successful where classification symmetry fails. So the two representations are distinct and the quarks of the vertex symmetry are called current quarks. The concept of two representations was introduced by Melosh(15) and discussed at great length in the reviews by Weyers(37) and Gilman(38).

$SU(6)_W$ strong is the group introduced by Lipkin and Meshkov(36). It is the algebra of classification $SU(6)_W$. The strong interaction Hamiltonian H_{st} approximately commutes with 35 operators W^i_α ($i = 0...8$ except for $W^o_{\alpha=0}$ which is left out, and $\alpha = 0, x, y, z$).

$$W^i \sim \int d^3x\, q^+ \frac{\lambda_i}{2} q$$

$$W^i_x \sim \int d^3x\, q^+ \beta\, \sigma_x \frac{\lambda_i}{2} q$$

$$W^i_y \sim \int d^3x\, q^+ \beta\, \sigma_y \frac{\lambda_i}{2} q$$

$$W^i_z \sim \int d^3x\, q^+ \sigma_z \frac{\lambda_i}{2} q$$

where $\sim$ means "transforms as".

The classification for baryons under W spin is the same as for spin. For mesons the only differences are

$$|s = 0, \; s_z = 0\rangle \longrightarrow -|W = 1, \; W_z = 0\rangle$$

$$|s = 1, \; s_z = 0\rangle \longrightarrow -|W = 0, \; W_z = 0\rangle \; .$$

This symmetry is only approximate, as is seen from the fact that the hadron mass spectrum badly breaks it. $SU(6)_W$ currents is an extension of the chiral $SU(3) \times SU(3)$ algebra of vector and axial charges from current algebra[39]. The charges F^i_α and F^{i5}_α and associated currents have the same commutation relations, charge conjugation and parity as the W^i_α. These are well defined operators with known Lorentz transformation properties and their algebraic relations could be exact even if the symmetry of the particle mass spectrum is broken. Calculations are usually done in the infinite momentum frame where one need only consider "good" operators whose matrix elements do not vanish between finite mass states with infinite p_z[15]. All the F^i's are good.

Particle classification

The $SU(6)_W$ classification scheme assigns a value of W spin to each particle from which ordinary spin is obtained. But the relevant quantity for a particle is its total angular momentum, not the spin of its quarks. So an additional quantum number to those given by the $SU(6)_W$ multiplet is needed to complete the classification. We take orbital angular momentum L. $\frac{1}{2}W_z^o$ and $\frac{1}{2}F_z^o$ act like quark spin operators for constituent and current quarks, respectively. The total angular momentum component J_z is the same for both classifications, so

$$L_z(W) = J_z - \tfrac{1}{2}W_z^o$$

$$L_z(F) = J_z - \tfrac{1}{2}F_z^o \quad .$$

The particles are classified[38] as

$$\left[(A,\ B)_{s_z}\ ,\quad L_z\right]$$

where A and B are the two SU(3) representations corresponding to right and left handed helicity currents. The "ordinary" SU(3) representation is $A \otimes B$.

One can see that the two algebras are not identical. Suppose the physical nucleon were a $\left|\, (6,3)_{\frac{1}{2}} \quad 0 \right\rangle$ in $SU(6)_W$ currents. Different states decaying to N will be described by matrix elements of Q and Q_5, the vector and axial charges. But Q and Q_5 are generators and only connect states within a representation. So only $N^* \longrightarrow N$ or Δ would be allowed. From this and other arguments it is clear that all hadrons must be complicated mixtures of many irreducible representations under currents. They are irreducible representations under constituents by definition.

But $W^i = F^i$ is a statement of CVC and the two algebras are very similar, so Melosh suggested[15, 37] that there exists a unitary transformation V between them. The transformation need not be unitary, that is an assumption because the SU(6) algebra does not include a complete set of quantum numbers. V must possess certain general properties.

1. V is an SU(3) singlet so that $W^i = F^i$.
2. V has parity and charge conjugation +1 and $[V, J_z] = 0$.
3. $[V, \partial^\mu F^{i5}_\mu] = 0$ between states with nearly the same momenta because PCAC says that the F^{i5} in the 35 of currents and the π in the 35 of constituents are equal.
4. V contains only good operators so that in the infinite momentum limit good operators only transform to good and bad to bad.

Using these properties Melosh(15) constructed an explicit form of V in the free quark model and investigated its algebraic properties. Abstracting the algebraic properties and saying that these hold in the real world as well as in the quark model, enables connections to be made between many different types of decays. These have been mostly in good agreement with experiment, for instance Refs. 40, 41.

As we saw above, the hadronic states which take part in interactions are complicated combinations of many irreducible representations of the constituent quarks. That same hadron in a classification scheme consists of just three quarks. Letting V act on the interacting hadron will transform it to an effective three-quark state where the three effective or current quarks form an irreducible representation (IR) of $SU(6)_W$ currents. These three are, in field theoretic terms, dressed quarks while the constituent representation involves the same three quarks bare.

$$|\text{IR constituents}\rangle = |\text{hadron}\rangle = V\,|\text{IR currents}\rangle$$

$$\langle \text{IR' constituents}|\ \text{op}\ |\text{IR constituents}\rangle$$

$$= \langle \text{IR' currents}\ |\ V^{-1}\,\text{op}\ V\ |\ \text{IR currents}\rangle \quad .$$

We know the $|\text{IR currents}\rangle$, they are the states $\left[(A,B)_{s_z}\ \ L_z\right]$ already mentioned. So, knowing the transformation, V allows one to use both simple operators and simple states in matrix elements.

b. Algebraic Structure of the Electromagnetic Current

Following Melosh, Hey and Weyers[16] investigated the photoproduction matrix element. Under $SU(6)_W$ currents, in the free quark model, there are four components. The operator is the first moment of the vector current, a dipole operator

$$D_+ = \int d^3x\ \frac{-(x + iy)}{\sqrt{2}}\ V_o(\underline{x}, t)$$

and when transformed, is assumed in general to have the free quark model properties.

$$\begin{aligned} V^{-1} D_+ V \sim\ & A\left[\underline{35},\ W = 0,\ W_z = 0,\ \Delta L_z = \pm 1\right] \\ & + B\left[\underline{35},\ W = 1,\ W_z = \pm 1,\ \Delta L_z = 0\right] \\ & + C\left[\underline{35},\ W = 1,\ W_z = 0,\ \Delta L_z = \pm 1\right] \\ & + D\left[\underline{35},\ W = 1,\ W_z = \mp 1,\ \Delta L_z = \pm 2\right] \end{aligned}$$

where A, B, C, D are some coefficients.

In the simple CKO model of Chapters 1 and 2, $C = D = 0$. C is non-zero when spin orbit coupling is included but D is still zero.

We would like to show that the FW and Melosh approaches are mutually consistent. The Melosh approach suggests a term D which is apparently not present in the FW decomposition of the current even with spin orbit coupling. However, since the FW transformation in the presence of an electromagnetic field is not exact, but is carried out order by order in a $^1/m$ power series expansion, there is always the possibility that the term we require may be found at a higher order. If the D term were definitely not present at all, or if terms not predicted by Melosh's algebra were found, then this would suggest that either the algebra or the transformation did not fit. So let us investigate the FW transformation more carefully to see if a $\Delta L_z = 2$ term is present anywhere and also if it generates any more terms at higher orders not predicted by the algebra.

The FW transformation of order n is e^{iS_n} where

$$S_{n+1} = \frac{-i\beta}{2m} \times \left(\begin{array}{l} \text{odd operators in } H_n \text{ of lowest order} \\ \text{in } 1/m \end{array} \right) .$$

The transformation acts on the Hamiltonian H_o and removes odd operators from it to order n. Odd operators 0 are such that $0\beta = - \beta 0$ where $\beta = \gamma_o$ is a Dirac matrix. Odd operators mix large and small components in the Dirac equation. The Hamiltonian should be left with just even operators to the desired order. Even operators ε do not mix the large and small components of the Dirac equation and $\beta\varepsilon = \varepsilon\beta$.

The FW transformation is a non-relativistic reduction because, if the Dirac equation can be separated into a large component and a small component equation, then the small component equation can be neglected and we are left with the non-relativistic two component Pauli equation.

$$H_n = e^{iS_n} H_{n-1} e^{-iS_n}$$

$$= H_{n-1} + i\left[S_n, H_{n-1}\right] - \tfrac{1}{2}\left[S_n, \left[S_n, H_{n-1}\right]\right] + \ldots - \dot{S}_n - \tfrac{i}{2}\left[S_n, \dot{S}_n\right] + \tfrac{i}{6}\left[S_n, \left[S_n, \dot{S}_n\right]\right] + \ldots$$

We start with the Dirac Hamiltonian including an electromagnetic field.

$$H_o = \beta m + \underline{\alpha}.(\underline{p} - e\underline{A}) = \beta m + O$$

$$S_1 = \frac{-i\beta}{2m} O \quad .$$

Let us calculate keeping terms to order $1/m^5$ in kinetic energy and $1/m^4$ in kinetic energy × field energy. This is two orders of magnitude greater than the transformation which led to the spin orbit terms, but two extra orders are needed to eliminate odd operators of the intermediate one.

Calculating the various commutators for H_o, S_1, $\dot{S}_1$ gives

$$H_1 = \beta\left(m + \frac{O^2}{2m} - \frac{O^4}{8m^3} + \frac{O^6}{144m^5}\right) - \frac{O^3}{3m^2} + \frac{O^5}{30m^4} + \frac{i\beta}{2m}\dot{O}$$

$$- \frac{i}{8m^2}\left[O, \dot{O}\right] - \frac{i\beta}{48m^3}\left(O^2\dot{O} - O\,\dot{O}\,O\right) + \frac{i}{384m^4}\left(O^3\dot{O} - O^2\dot{O}\,O\right)$$

Now $S_2 = \frac{-i\beta}{2m}\left(\frac{i\beta\dot{O}}{2m}\right) = \frac{1}{4m^2}\,\dot{O}$.

Calculating commutators for H_1 and S_2 leads to H_2 and the process is repeated as often as required. In this case, as we are working to fifth order, we need H_5 before all terms are even.

$$H_5 = \beta\left(m + \frac{O^2}{2m} - \frac{O^4}{8m^3} + \frac{O^6}{16m^5}\right) - \frac{i}{8m^2}\left[O, \dot{O}\right]$$

$$- \frac{\beta}{8m^3}\dot{O}^2 - \frac{i}{384m^4}\left(O^3\dot{O} - O^2\dot{O}\,O\right) - \frac{i}{12m^4}\left[\dot{O}, O^3\right]$$

$$- \frac{i}{32m^4}\left[\dot{O}, \ddot{O}\right]$$

with $O = \underline{\alpha}.(\underline{p} - e\underline{A})$

and $\frac{O^2}{2m} = \frac{(p - e\underline{A})^2}{2m} - \frac{e}{2m}\,\underline{\sigma}\cdot\underline{B}$.

The terms up to third order are well known. All we are interested in here is finding a $\Delta L_z = 2$ term. The lowest order such term is

$$-\frac{\beta\, O^4}{8m^3}$$

$O^4 = \left[(\underline{p} - e\underline{A})^2 - e\,\underline{\sigma}.\underline{B}\right]^2$ which includes $p^2(\underline{\sigma}.\underline{B})$

$p^2 = p_z^{\,2} + (p_x \pm ip_y)^2 \mp i\,2p_y(p_x \pm i\,p_y)$.

$(\underline{\sigma}.\underline{B})$ contains $\sigma_+\,e^{-ikz}$ or $\sigma_- e^{ikz}$

so that O^4 can yield $p_+^{\,2}\,\sigma_-$ or $p_-^2\,\sigma_+$ which would be the fourth term, one order of magnitude smaller than the spin orbit terms and therefore negligible both theoretically and experimentally so far. But the term is there in principle as required by the algebra.

$$H_5 = \beta\Big[m + \frac{(\underline{p} - e\underline{A})^2}{2m} - \frac{e}{2m}\,\underline{\sigma}.\underline{B} - \frac{(\underline{p} - e\underline{A})^4}{8m^3} - \frac{e^2B^2}{8m^3}$$

$$+ \frac{e}{4m^3}(\underline{p} - e\underline{A})^2\,\underline{\sigma}.\underline{B} + \frac{(\underline{p} - e\underline{A})^6}{16m^5} - \frac{3e}{16m^5}(\underline{p} - e\underline{A})^4\,\underline{\sigma}.\underline{B}$$

$$+ \frac{3e^2}{16m^5}(\underline{p} - e\underline{A})^2 B^2 - \frac{e^3B^2}{16m^5}\,\underline{\sigma}.\underline{B}\Big]$$

$$+ (-ie\,\mathrm{div}\underline{E} + e\,\underline{\sigma}.\mathrm{curl}\,\underline{E} - 2ie\,\underline{\sigma}.\underline{E}\times\underline{p}) \times$$

$$\times\left(\frac{-i}{8m^2} + \frac{95i}{384m^4}\left((\underline{p} - e\underline{A})^2 - e\,\underline{\sigma}.\underline{B}\right)\right) - \frac{\beta}{8m^3}e^2E^2 .$$

One might ask whether more terms might turn up at still higher orders. The answer is clearly no. In order to have $|\Delta L_z| \geqslant 3$ it would have to be matched by $|\Delta\sigma_z| \geqslant 2$ in order that $\Delta J_z = 1$. But spin can only be changed from $+\frac{1}{2}$ to $-\frac{1}{2}$ and vice versa, no further. Mathematically $\sigma_i^2 = 1$ and as this means that one does not have $\Delta\sigma_z$ greater than 1, one cannot have ΔL_z greater than 2.

So now we see that the FW transformation does match the algebra. It gives the correct number of independent terms in the current: no more and no less.

c. The Melosh transformation

Now that we have explored the algebra let us explore the specific transformation,

The transformation given by Melosh for the free quark model is $\quad V = e^{iy}$

where $\quad Y = \frac{1}{2}\int d^3x\ q^+ \arctan(\frac{\gamma_\perp \cdot \partial_\perp}{m}) q$

where the symbol "$\perp$" means the two-vector in the plane perpendicular to the motion, in this case the x and y directions. For the rest of this work we can leave out the q and q^+ integrated over all space, which are necessary in a general field theory but not for us because we are in a one quark subspace of the total Hilbert space.

Expressions such as arctan are defined by their infinite power series expansions:

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} \ldots\ldots .$$

Using this definition of arctan one can rewrite Y as

$$Y = \frac{\gamma_\perp \cdot \partial_\perp}{|\partial_\perp|} \cdot \frac{1}{2} \arctan(\frac{\partial_\perp}{m}) = \frac{\gamma_\perp \cdot \partial_\perp}{|\partial_\perp|} \theta$$

and $\quad e^{iY} = \cos\theta + i \frac{\gamma_\perp \cdot \partial_\perp}{|\partial_\perp|} \sin\theta \quad .$

We are now in the relativistic region and the metric is such that $a_\mu b_\mu = a_o b_o - \underline{a}.\underline{b}$. We will be very careful in signifying the scalar product of 3 vectors with a dot, and no dot when the quantities are scalars or just the magnitudes of vectors.

With this V, Melosh showed that the free Dirac Hamiltonian

$$H = -i\ \underline{\alpha}.\underline{\partial} + m\beta$$

transforms to

$$H_W = V^{-1}HV = -i\ \alpha_z\ \partial_z + \beta(m^2 + (\gamma_\perp \cdot \partial_\perp)^2)^{\frac{1}{2}} .$$

The z component is not affected. In the x and y directions V is just an FW transformation. Looking at the definition of V we see that if $p_\perp = 0$ then V = 1. So it is the transverse momenta of the quarks which breaks the identity of the current and constituent algebras. V transforms away the tranverse momenta in the current states in such a way that when the resulting state of $SU(6)_W$ strong is boosted in the z direction, the spin Wigner rotation is counteracted and the classification algebra is independent of boosts in the z direction. This is purely a matter of kinematics (the spin rotation being a consequence of the momentum change).

The FW transformation also transforms momenta to enable one to boost from a relativistic to a non-relativistic system so that the identification of V with the transverse FW transformation could have been expected.

However, if the quarks are interacting, which they will be, the FW transformation alters and the identity between Melosh and FW is broken. Since the two transformations were of the same form in the free case and have the same function of affecting momenta and spins, V now needs to be altered for consistency to become the appropriate interaction FW transformation in its transverse components. Such a transformation would need to be constructed by a power series approach similar to that used for the FW transformation in the presence of interactions.

From the general properties of V cited in Section a), it is not possible to construct an interaction V beyond first order in $1/m$ without violating one of these rules, but Melosh suggested a prescription to first order. He found that to first order all models give the same structure

$$V_{model} = 1 + \frac{i}{2m} \int d^3x \; q^+(x)(\gamma_\perp \cdot \partial_\perp) q(x) + \dots$$

where $\partial_\perp$ is an ordinary derivative for scalar and pseudoscalar gluon models but the gauge-invariant derivative $\nabla_\perp = \partial_\perp - ig\, B_\perp$ for vector gluons with $B_\mu(x)$ the gluon field operator.

We see that V_{model} is just V_{free} to first order from expanding the arctan in the definition of V_{free}. So as to satisfy the constraints on the general properties of V exactly, Melosh then assumed that $V_{model} = V_{free}$ to <u>all</u> orders for scalar and pseudoscalar gluon models. For vector gluon models $V_{model} = V_{free}$ with all ∂_i replaced by ∇_i. Because of non-commutativity of the ∇_i one can only replace ∂_i^n by ∇_i^n for all integers n when $[B_\mu(x), B_\nu(y)] = 0$ for all x, y and when curl $\underline{B} = 0$. In, for instance, electrodynamics, these conditions correspond to a potential representing a static electric field and no magnetic field. Otherwise, without these conditions, we only have a first order prescription for the vector gluon model.

Melosh evaluated the free Melosh transformation of the free Dirac Hamiltonian. Let us now find the interacting Melosh transformation of the Dirac Hamiltonian, including an electromagnetic field.

We have $\partial_\perp \longrightarrow \partial_\perp - ie\, A_\perp$

and $$Y = \tfrac{1}{2} \arctan\left(\frac{\gamma_\perp . \partial_\perp - ie\, \gamma_\perp . A_\perp}{m}\right) .$$

Taking just the first term of the arctan power series expansion

$$Y = \frac{1}{2m} (\gamma_\perp . \partial_\perp - ie\, \gamma_\perp . A_\perp) .$$

The Dirac Hamiltonian including an electromagnetic field is

$$H = \beta m + \underline{\alpha} . (\underline{p} - e\underline{A}) .$$

Since this is a first order calculation we cannot use $e^{iY} = \cos Y + i \sin Y$ as one would in the free case to all orders, but instead we have .

$$e^{-iY} H e^{iY} = H_W = H - i[Y,H] - \tfrac{1}{2} [Y [Y,H]] + \ldots -\dot{Y} +$$

For the first order we need few commutators

$$\dot{Y} = \frac{-e\beta}{2m} k \quad \alpha_\perp . A_\perp$$

$$[Y,H] = \frac{-i\beta}{m} \partial_\perp^2 + \alpha_\perp \partial_\perp - 2 \frac{e\beta}{m} \partial_\perp A_\perp + \frac{iek}{2m} \beta \alpha_z (\alpha_\perp A_\perp)$$
$$- ie\, \alpha_\perp A_\perp + i \frac{e^2 \beta}{m} A_\perp^2$$

$$[Y, [Y,H]] = \frac{-\beta}{m} \partial_\perp^2 + \frac{e^2 \beta}{m} A_\perp^2 + i \frac{2e}{m} \beta\, \partial_\perp A_\perp .$$

Finally then

$$H_W(\text{em}) = \beta m + \underline{\alpha}\cdot(\underline{p}-e\underline{A}) + \frac{1}{m}\Big[\tfrac{1}{2}\beta p_\perp^2 + \tfrac{1}{2}e^2\beta A_\perp^2$$

$$+ \frac{1}{\sqrt{2}}\, e\beta(p_x + ip_y)e^{-ikz} - \frac{e\beta}{2\sqrt{2}}\, k(\sigma_x + i\sigma_y)e^{-ikz}\Big]$$

$$- i\,\alpha_\perp \partial_\perp - e\,\alpha_\perp A_\perp + \frac{ek\beta}{2m}\,\alpha_\perp A_\perp \;.$$

Since it is only the transverse part which has been transformed, let us leave out the z components

$H_W(\text{em})$, transverse

$$= \beta m + \frac{\beta}{2m}\, p_\perp^2 + \frac{e^2\beta}{2m}\, A_\perp^2 + \frac{ek\beta}{2m}\,\alpha_\perp A_\perp$$

$$+ \frac{e\beta}{\sqrt{2}m}\,(p_x + ip_y)e^{-ikz} - \frac{e\beta k}{2\sqrt{2}m}\,(\sigma_x + i\sigma_y)e^{-ikz} \;.$$

We must now compare this with the Hamiltonian which has undergone the first order FW transformation:

$$H_1 = \beta m + \frac{\beta p^2}{2m} + \frac{e^2A^2\beta}{2m} + \frac{\beta}{2m}\, ek\,\underline{\alpha}\cdot\underline{A}$$

$$+ \frac{e\beta}{\sqrt{2}m}\,(p_x + ip_y)e^{-ikz} - \frac{e\beta}{\sqrt{2}m}\,\frac{k}{2}\,(\sigma_x + i\sigma_y)e^{-ikz} \;.$$

We see that the transverse parts of $H_W(\text{em})$ and of H_1 are equal. The FW transformation can of course be carried out to any order but not the Melosh transformation unfortunately. This is only a first order result because $(\underline{\partial} - ie\underline{A})$ raised to higher powers no longer equals $\underline{\partial}$ to the same power when curl $\underline{A} \neq 0$. As soon as the necessary extra terms are introduced we would be violating one or more of the general constraints on the Melosh transformation.

We could also attempt to include an interaction binding the quarks in the Melosh transformation. In our case this would be the harmonic oscillator force. When one evaluates the FW transformation in the presence of an electromagnetic field, the quarks are assumed to be free until the electromagnetic interaction is introduced. It is reasonable that the Melosh transformation should be compared under the same assumption and we saw that they do agree. Since the FW transformation has its desired effect, one can infer that the free quark assumption is reasonable and therefore no further terms are necessary.

Ref. page 40

The charges F_α^i and F_α^{i5} and associated currents form an $SU(6)_W$ algebra which is an extension of the chiral $SU(3) \times SU(3)$ current algebra.

In terms of the free quark model the F^i's are defined as

$$F^i = \int d^3x \; q^+ \frac{\lambda_i}{2} q$$

$$F_x^i = \int d^3x \; q^+ \beta\sigma_x \frac{\lambda_i}{2} q$$

$$F_y^i = \int d^3x \; q^+ \beta\sigma_y \frac{\lambda_i}{2} q$$

$$F_z^i = \int d^3x \; q^+ \sigma_z \frac{\lambda_i}{2} q$$

and $$F^{i5} = \int d^3x \; q^+ \frac{\lambda_i}{2} \gamma_5 q \quad \text{etc.}$$

The charges F^i are space integrals of the 0-th components of vector, axial vector and tensor current densities which are either the directly observable weak and electromagnetic currents or appear on the right hand side of commutators of these currents.

Using the canonical equal-time anticommutation relations for the quark fields one can derive the commutators for the F^i's:

$$[F^i, F^j] = i f_{ijk} F^k$$

$$[F^i, F^{j5}] = i f_{ijk} F^{k5}$$

and $$[F^{i5}, F^{j5}] = i f_{ijk} F^k .$$

The big assumption is to abstract these algebraic relations from the model and assume that they hold in the real world even though the operators may no longer be expressed in simple bilinear forms.

The electromagnetic current $\underline{j}$ consists of two parts $j_\perp = j_x \pm i j_y$ which raises or lowers the orbital angular momentum, and j_z which flips spin instead.

j_z is a good operator and can be used as it is. The CKO current $j_\perp$ is

$$\langle N^* | j_x + i j_y | N \rangle = e \int d^3x \, \langle N^* | (p_x + i p_y) e^{-ikz} | N \rangle$$

$$= e \int d^3x \, \langle N^* | r \sin\theta \, e^{i\phi} \, e^{-ikr\cos\theta} | N \rangle$$

The dipole operator:

$$D_+ = \int d^3x (x + iy) j_o(\underline{x}\ t) = \int d^3x (x + iy) j_o(0\ t) e^{-ikz} .$$

Taking matrix elements (j_o is the charge density. Its matrix element between quark SU(3) wavefunctions gives the quark charge e)

$$\langle N^* | D_+ | N \rangle = e \int d^3x \, \langle N^* | (x + iy) e^{-ikz} | N \rangle$$

$$= e \int d^3x \, \langle N^* | r \sin\theta \, e^{i\phi} \, e^{-ikr\cos\theta} | N \rangle$$

which is the same as above.

The spatial matrix elements for spin flip transitions are

$$\langle N^* | e^{-ikz} | N \rangle$$

which contains only good operators (z components of vectors) and therefore need not be changed. This gives the rest of the interaction.

The Melosh transformed D_+ contains the full angular momentum structure as shown by Hey and Weyers[41] because the Melosh transformation mixes spin and orbital angular momentum representations. The untransformed D_+ has only part of the structure.

Ref. page 45

The FW transformation in the presence of interactions is a series of transformations taking the initial Hamiltonian H_o to H_1, then H_1 to H_2 and so on, removing odd operators to any required order.

Following Bjorken and Drell[29] let us consider the initial full Hamiltonian H_o and the transformation e^{iS_1}, which transforms H_o to H_1 and also the initial wavefunction ψ_o to ψ_1.

$$\psi_1 = e^{iS_1} \psi_o$$

$$i \frac{\partial}{\partial t} e^{-iS_1} \psi_1 = H_o \psi_o = H_o e^{-iS_1} \psi_1$$

$$= e^{-iS_1} (i \frac{\partial \psi_1}{\partial t}) + (i \frac{\partial}{\partial t} e^{-iS_1}) \psi_1 \quad .$$

So $$i \frac{\partial \psi_1}{\partial t} = \left[e^{iS_1} (H_o - i \frac{\partial}{\partial t}) e^{-iS_1} \right] \psi_1 = H_1 \psi_1$$

and $$H_1 = e^{iS_1} (H_o - i \frac{\partial}{\partial t}) e^{-iS_1} \quad .$$

Expanding the exponentials: $e^x = 1 + x + \frac{x^2}{2} \; . \; . \; . \; .$

and multiplying out gives the series of commutators written for a general H_n on page 45.

Ref. page 46

We are interested in finding the lowest order $\Delta L_z = 2$ term. It would also be of interest to calculate its matrix element. We want to consider all $\frac{1}{m^3}$ terms in order to be consistent. From H_5 the relevant terms are

$$\frac{-e}{2m^3} p_+^2 p_- e^{-ikz} + \frac{ek}{4m^3} p_z^2 \sigma_+ e^{-ikz} + \frac{ek}{4m^3} p_-^2 \sigma_+ e^{-ikz}$$

$$= R_+ + R_z + R_- .$$

They yield matrix elements with $\Delta L_z = +1, 0, -2$ respectively.

The $\Delta L_z = 2$ term, R_-, clearly cannot contribute to the excitation of $L = 1$ resonances such as the D_{13} and S_{11} from the proton, but it will contribute when $L \geqslant 2$. We are interested in the contribution to the F_{15}. How will the new type of term, of which R_- is the lowest order representative, affect the matrix elements already calculated from the CKO model?

R_- does not contribute to $A_{\frac{1}{2}}$ but only to $A_{3/2}$ for any resonance (like the F_{15}) where the total quark spin is $\frac{1}{2}$ and not $\frac{3}{2}$: $R_- \approx p_-^2 \sigma_+$.

Starting with the state with $J_z = -\frac{1}{2}$ which we will denote $|-\frac{1}{2}\rangle$ we have $\sigma_+ |-\frac{1}{2}\rangle = |+\frac{1}{2}\rangle$ and

$$p_-^2 \sigma_+ |-\tfrac{1}{2}\rangle = p_-^2 |+\tfrac{1}{2}\rangle = |-\tfrac{3}{2}\rangle .$$

So the helicity $\frac{1}{2}$ amplitude of the F_{15} is unaffected by this term. The $A_{3/2}$ will have two new terms R_+ and R_-.

Ref. page 46 (contd.)

$$R_+^{F_{15}} = -\frac{10}{3} \alpha^2 k e^{-k^2/6\alpha^2}$$

$$R_-^{F_{15}} = \frac{2}{3} \alpha^2 e^{-k^2/6\alpha^2}$$

$$A_{3/2} = \sqrt{\frac{4}{5}} (\Delta L_z = 1) + \sqrt{\frac{1}{5}} (\Delta L_z = 2) .$$

So the new terms in $A_{3/2}$ are

$$\left(\frac{e}{2m^3} \sqrt{\frac{4}{5}} \frac{10}{3} \alpha^2 k + \frac{ek}{4m^3} \sqrt{\frac{1}{5}} \frac{4\alpha^2}{3}\right) e^{-k^2/6\alpha^2} .$$

Because of uncertainty over the magnitude of the quark mass m it is difficult to compare the relative magnitudes of these terms with the lower order ones. If the FW expansion is to have any validity they should be small and m quite large.

A less uncertain comparison can be made between the two terms themselves:

$$\frac{\Delta L_z = 2}{\Delta L_z = 1} = \frac{\sqrt{\frac{1}{5}} \frac{4\alpha^2}{3} \frac{ek}{4m^3}}{\sqrt{\frac{4}{5}} \frac{10}{3} \alpha^2 k \frac{e}{2m_3}} = \frac{1}{10} .$$

So R_- is ten times smaller than R_+ and the resulting cross-section would be 100 times smaller. This is truly negligible.

There will of course be higher order $\Delta L_z = 2$ terms. We can see some in H_5. As with all other terms the higher orders will be even smaller and less important.

CHAPTER 6

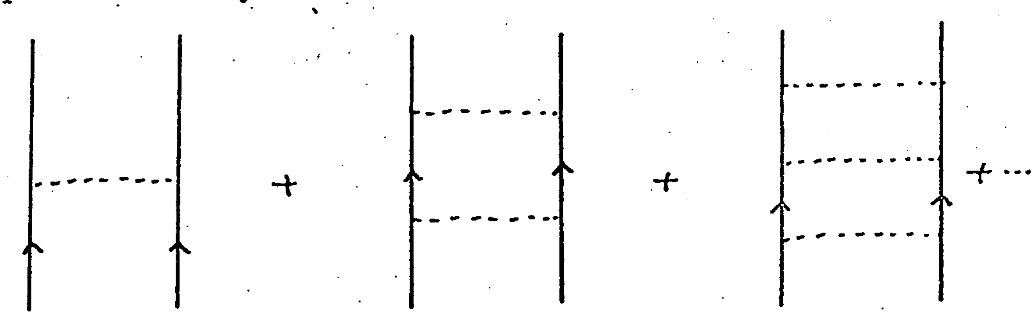
THE BETHE SALPETER EQUATION

This is a dynamical approach to symmetry breaking. A realistic equation is set up and solved for constituent quarks. Phenomena that in the previous chapter were treated as manifestations of $SU(6)_W$ currents are now dealt with as consequences of the dynamics.

a. Review

The Bethe-Salpeter (BS) equation is a covariant generalisation of the two body Schrödinger equation for dealing with bound state problems in relativistic quantum mechanics(42).

In practice the BS equation can only be treated in the ladder approximation. Here the two body propagator is approximated by



The BS equation in this approximation is

$$\chi_k(x_1 x_2) = -G_o^2 \int d^4x_3 \, d^4x_4 \; S_F^A(x_1-x_3) \; S_F^B(x_2-x_4) \times \Delta_F(x_3-x_4) \, \chi_k(x_3 \, x_4)$$

where $\mathcal{X}_k(x_1\ x_2)$ is the two body BS wavefunction,

$S_F(x_1-x_2)$ is the propagator function - the bare value since there are no diagrams with loops

G_o is the coupling constant for the interaction

$\Delta_F(x_1-x_2)$ is a delta function .

As a differential equation, the above BS equation becomes

$$(\gamma^A.\partial + m)(\gamma^B.\partial + m)\mathcal{X}_k(x_1\ x_2) = G_o^2\Delta_F(x_1-x_2)\mathcal{X}_k(x_1, x_2).$$

In momentum space one has

$$S_F^A(\tfrac{1}{2}k+p)^{-1}\ S_F^B(\tfrac{1}{2}k-p)^{-1}\mathcal{X}_k(p) = \frac{-G_o^2}{(2\pi)^4}\int d^4p'\,\Delta_F(p-p')\mathcal{X}_k(p')$$

Salpeter[43] rewrote the equation for the case of instantaneous interactions in the ladder approximation, and Brodsky and Primack[19] generalised it to include the effect of an external field. Brodsky and Primack found exact solutions for weakly bound two fermion systems in the ladder approximation with instantaneous interactions.

First attempts at using the BS equation as the basis of a relativistic quark model were directed at mesons describing them as quark anti-quark systems. Among others Böhm, Joos and Krammer[17] developed a model with heavy strongly bound scalar quarks in a smooth potential approximating an harmonic oscillator at short distances. Their solutions represented linear Regge trajectories with the correct slope and acceptable electromagnetic meson form factors (within 10°/o). This model has been refined since then to include spin ½ quarks[44]. Recently Meyer has tried to construct a three quark baryon model along the same lines[45]. However he treats scalar quarks as a first approximation which is rather pointless since the spin behaviour is so crucial as shown in even primitive models such as Lipes's modification of FKR[14]. So we will use the model of Kellett[18] which appears to be the first quark model based on the BS equation to deal with spin in a realistic fashion. He generalises the Brodsky and Primack solution to three quarks and strong binding. Although their method is independent of the strength of binding, Kellett found that only for a particularly simple class of potentials can the simple explicit solution be derived for strong binding. However the form of the radial wavefunctions remains open. One is at liberty to use harmonic oscillator, Coulomb or other more complicated potentials. We will continue to use the harmonic oscillator.

The BS equation for three particles in momentum space in the ladder approximation and with an instantaneous interaction is

$$\left[(\tfrac{1}{3}P-p)\cdot\gamma^{a} - m_q\right]\left[(\tfrac{1}{3}P+\tfrac{1}{2}p-q)\cdot\gamma^{b} - m_q\right]$$

$$\times\left[(\tfrac{1}{3}P+\tfrac{1}{2}p+q)\cdot\gamma^{c} - m_q\right]\psi(p,q,P)$$

$$= \frac{1}{(2\pi)^2}\int d^4p'\,d^4q'\; g(\underline{p}-\underline{p}', \underline{q}-\underline{q}')\,\psi(p', q', P)$$

where P, p, q are centre of mass and relative coordinates

$$P = p_a + p_b + p_c$$
$$p = \frac{1}{3}(p_b + p_c - 2p_a)$$
$$q = \tfrac{1}{2}(p_c - p_b)$$

and the quarks are all assumed to have equal mass m_q which is a reasonable assumption since we shall only consider non-strange baryons.

Kellett's choice of an instantaneous interaction in all frames means that although the model is nominally relativistic it is no longer covariant. Suppressing the time-like excitations saves unitarity but it is just as ad hoc as the methods used by FKR(13) and Lipes(14). Still the formal simplification is so great that Kellett thought it worth trying. The neglect of these retardation effects in the instantaneous approximation implies a static potential. This may not be unreasonable.

To obtain Salpeter's equation(43) from the momentum space BS equation one must first multiply by $\gamma_o^a \gamma_o^b \gamma_o^c$ which gives

$$F(p,q,P)\psi(p,q,P) = \frac{1}{(2\pi)^2}\int d^4p' \, d^4q' \, \tilde{g}\,(\underline{p}-\underline{p}', \underline{q}-\underline{q}') \times \psi(p',q',P)$$

with $\tilde{g}(\underline{p}\ \underline{q}) = \gamma_o^a \gamma_o^b \gamma_o^c \, g(\underline{p}\ \underline{q})$

and

$$F(p,q;P) = \left[\tfrac{1}{3}P_o - H_a(p_a) - p_o\right]\left[\tfrac{1}{3}P_o - H_b(p_b) + \tfrac{1}{2}p_o - q_o\right]$$

$$\times \left[\tfrac{1}{3}P_o - H_o(p_c) + \tfrac{1}{2}p_o + q_o\right]$$

$$H_a(p_a) = \underline{\alpha}^a.\underline{p}_a + \beta^\beta m_q \quad \text{etc.}$$

m_q is the physical free quark mass and H_a etc are free quark Hamiltonians. Salpeter's next step was the introduction of projection operators

$$\Lambda_{\pm}^{a}(p_a) = \frac{E_a \pm H_a(p_a)}{2E_a}$$

and similarly for quarks b and c. They will be free particle projection operators in this case.

Integrating with respect to p_o and q_o gives

$$\Phi_{P_o}(p,q,P) = \int dp_o \, dq_o \, \psi(p,q,P)$$

and the equation becomes

$$\left[P_o - H_a(p_a) - H_b(p_b) - H_c(p_c)\right] \Phi_{P_o}(\underline{p}, \underline{q}, \underline{P})$$

$$= -(\Lambda_+^a\Lambda_+^b\Lambda_+^c + \Lambda_-^a\Lambda_-^b\Lambda_-^c)\int d^3\underline{p}' \, d^3\underline{q}' \, \tilde{g}\,(\underline{p} - \underline{p}', \underline{q} - \underline{q}')$$

$$\times \Phi_{P_o}(\underline{p}', \underline{q}', \underline{P})$$

which is the Salpeter equation(43) and also the three particle generalisation of the equation considered by Brodsky and Primack(19).

For weak binding the projection operators can be dropped and Brodsky and Primack found an exact solution to the resulting equation. In the strong binding case one can only drop the projection operators if the rest frame wavefunction is a product of positive energy free quark spinors. This is not in general the case.

Kellett looked for a wavefunction of the form

$$\Phi_M(\underline{p},\underline{q}) = \binom{1}{\omega_a} \otimes \binom{1}{\omega_b} \otimes \binom{1}{\omega_c} \phi_M(\underline{p}, \underline{q})\chi_s$$

where χ_s is a constant spinor, ϕ_M is the radial wavefunction and depends on the potential used. He found that the wavefunction would be a product of free quark spinors if

$$\omega_a = -\frac{1}{2m_q + k_a}\, \underline{\sigma}^a \cdot \underline{p}$$

with $k_a = E_a - m_q$

and $E_a = (\underline{p}^2 + m_q^2)^{\frac{1}{2}}$.

Similarly for the b and c quarks with momenta $(\frac{1}{2}\underline{p} - \underline{q})$ and $(\frac{1}{2}\underline{p} + \underline{q})$ respectively.

For later ease of calculation the spinor products are finally absorbed into the Hamiltonian so that only the standard non-relativistic wavefunctions are needed to evaluate matrix elements. The trial wavefunction is that in the rest frame. The general wavefunction is obtained by Lorentz boosting the rest frame solution. The normalisation is fixed by

$$\int d^3\underline{x}_a . d^3\underline{x}_b \; d^3\underline{x}_c \quad \Phi^+(\underline{x}_a \; \underline{x}_b \; \underline{x}_c)(\Lambda_+^a\Lambda_+^b\Lambda_+^c \; + \Lambda_-^a\Lambda_-^b\Lambda_-^c) \times$$

$$\times \; \Phi \; (\underline{x}_a \; \underline{x}_b \; \underline{x}_c) \quad = \quad 1$$

together with

$$\int d^3\underline{p} \; d^3\underline{q} \; \left| \phi_M(\underline{p} \; \underline{q}) \right|^2 \quad = \quad 1 \; .$$

Finally then Kellett's solution of the equation produces a general matrix element

$$3 \int d^3\underline{p} \; d^3\underline{q} \qquad \left\langle B \left| H^a(\underline{p} \; \underline{q}) \right| A \right\rangle \quad .$$

$H^a(\underline{p}\;\underline{q})$ is the effective interaction on quark a and the whole expression is multiplied by 3 to account for all the quarks. $|A\rangle$ and $|B\rangle$ are standard non-relativistic SU(6) and spatial wavefunctions. All relativistic effects are in H^a which is a product of spinors, projection operators and the appropriate part of the quark current.

Exact SU(6) symmetry is assumed for the three quarks so that they have the same mass, magnetic moment etc.

$$H^a(pq) \;\; = \;\; \text{normalisation factors}$$

$$\times \; (1 + \rho_a \xi_a, \; \xi_a \; + \rho_a)\Lambda_+^a \; (\tfrac{1}{3}k - \tilde{\underline{p}})\gamma_o^a j_\mu^a \varepsilon^\mu \; (\tfrac{1}{\eta_a})$$

$$\times \; (1 + \rho_b \xi_b, \; \xi_b \; + \rho_b)(\tfrac{1}{\eta_b}) \; \times \; (1 + \rho_c \xi_c, \; \xi_c \; + \rho_c)(\tfrac{1}{\eta_c})$$

where λ_i = SU(3) matrices

k = $p' - p$

K = anomalous magnetic moment

$$\varepsilon_\mu = \text{photon polarisation 4 vector}$$

$$j_\mu = \text{quark current:}$$

$$j^v_\mu = \bar{\Psi}(p')\left[\gamma_\mu + \frac{K}{2m_q}\, i\, \sigma_{\mu\nu}\, k^\nu\right]\frac{\lambda_i}{2}\,\psi(p)$$

$$\xi_i = \frac{\underline{\sigma}^i \cdot \underline{P}}{E_B + M_B}$$

$$\rho_a = -\frac{\underline{\sigma}^a \cdot \underline{p}}{E_a + m_q},$$

$$\rho_b = \frac{\underline{\sigma}^b \cdot (\frac{1}{2}\underline{p} - \underline{q})}{E_b + m_q},$$

$$\rho_c = \frac{\underline{\sigma}^c \cdot (\frac{1}{2}\underline{p} + \underline{q})}{E_c + m_q}$$

with the η_i obtained from the ρ_i by $\underline{p} \to \tilde{\underline{p}} + \frac{2}{3}\underline{k}$ and $\underline{q} \to \tilde{\underline{q}}$. $\underline{P}$ is the centre of mass momentum and $\tilde{p}$, $\tilde{q}$ are quark relative momenta for the Lorentz contracted wavefunction. For more details see Ref. (18).

Now let us apply this model to resonance photo- and electroproduction and compare with our other results.

b. Second Order Amplitudes

Kellett expanded H^a as a series in $^1/m$ where m was any of the masses involved. To first order this gives the CKO Hamiltonian, so the second order Hamiltonian is the first order in corrections to the non-relativistic model. D_{13} electroproduction matrix elements were accordingly calculated. To order $^1/m^2$ for the transverse modes

$$H_I = 3e_a \left[F_a + \frac{2\underline{p}.\underline{\varepsilon}}{\alpha^2} G_a \right] \qquad e_a = \text{charge on quark a} = \tfrac{1}{3}e$$

$$F_a = i \frac{\underline{\sigma}^a.\underline{\varepsilon}\times\underline{k}}{2m_q} + \frac{i\nu}{24m_q^2} \Big[(4\underline{\sigma}^a.\underline{\varepsilon}\times\underline{p} + (\underline{\sigma}^b+\underline{\sigma}^c).\underline{\varepsilon}\times\underline{p} - 2(\underline{\sigma}^b-\underline{\sigma}^c).\underline{\varepsilon}\times\underline{q}) \Big]$$

$$G_a = \nu+\underline{k}^2 \left(\frac{1}{6m_q} + \frac{\nu}{4M_f m_q} - \frac{\nu}{6m_q^2} + \frac{3\nu}{8M_f^2}\right) + \frac{\underline{p}.\underline{k}}{m_q} + \frac{i\nu}{24m_q^2}\Big[4\, \underline{\sigma}^a.\underline{p}\times\underline{k} + (\underline{\sigma}^b+\underline{\sigma}^c).\underline{p}\times\underline{k} - 2(\underline{\sigma}^b-\underline{\sigma}^c).\underline{q}\times\underline{k} \Big] .$$

The interaction can be written as three times the interaction on quark a, where the one quark terms are of the form $\underline{\sigma}^a.\underline{\varepsilon} \times \underline{p}_a$ and $(\underline{\sigma}_b + \underline{\sigma}_c).\underline{\varepsilon} \times \underline{p}_a$, i.e. the spins of the other two quarks still contribute.

However the spin matrix elements of the quarks are the same whichever quark interacts since one cannot say which quark had spin up or spin down to start with. So we can write $\underline{\sigma}.\underline{\varepsilon} \times \underline{p}_a$ for all three.

Now dropping the subscript from $\underline{p}_a$ we have

$$H_I = \frac{ek}{2\sqrt{2}\, m_q} \sigma_+ e^{-ikz} - \frac{\sqrt{2}e}{\alpha^2}\left(\nu + \frac{k^2}{6m_q} + \frac{\nu k^2}{4M_f\, m_q} - \frac{\nu k^2}{6m_q^{\ 2}} + \frac{3\nu k^2}{8M_f^{\ 2}}\right) p_+ e^{-ikz}$$

$$+ \frac{e\nu}{\sqrt{2}\, m_q^2} \frac{\sqrt{26}}{24} (\sigma_+ p_z - \sigma_z p_+) e^{-ikz} - \frac{2e}{m_q} \frac{k}{\sqrt{2}\, \alpha^2} p_+ p_z e^{-ikz}$$

$$- \frac{2iek\nu}{\sqrt{2}\, m_q^2 \alpha^2} \frac{\sqrt{26}}{24} p_+ (\sigma_x p_y - \sigma_y p_x) e^{-ikz} .$$

Setting the helicity $\frac{1}{2}$ amplitude of the F_{15} to zero determines α^2 just as in the CKO model. With $m_q = 0.33$ GeV as before, we have $\alpha^2 = 0.328\ \text{GeV}^2$. Kellett used $\alpha^2 = 0.35$ given by the slope of Regge trajectories. We will use 0.328 but it is worth noting that the values are so close. Kellett's argument, taken from Reference (13), does not apply in our model because it requires the interaction energy W to be small compared to the quark mass - in other words, heavy quarks. So the coincidence of values of α^2 may not have any significance.

Now the transverse D_{13} amplitudes can be calculated. We will work in the isobar frame. There will be frame dependence but Kellett has found it to be small[46] so we will not bother about it.

The corresponding longitudinal part of the Hamiltonian is

$$H^{long.} = \varepsilon_o \Big\{ 1 + (\frac{1}{12M_f m_q}) + \frac{3\nu}{8M_f^2} - \frac{\nu}{12m_q^2})\ \underline{k}^2$$

$$+ \frac{1}{4m_q^2}\Big[\underline{p}.\underline{k} + \frac{1}{3}\underline{k}^2 + \frac{i}{6}(4\underline{\sigma}^a + \underline{\sigma}^b + \underline{\sigma}^c).\underline{p}\times\underline{k}$$

$$- \frac{i}{3}(\underline{\sigma}^b - \underline{\sigma}^c).\underline{q}\times\underline{k}\Big] \Big\} \quad .$$

This is in the radiation gauge $\underline{\nabla}.\underline{A} = 0$.

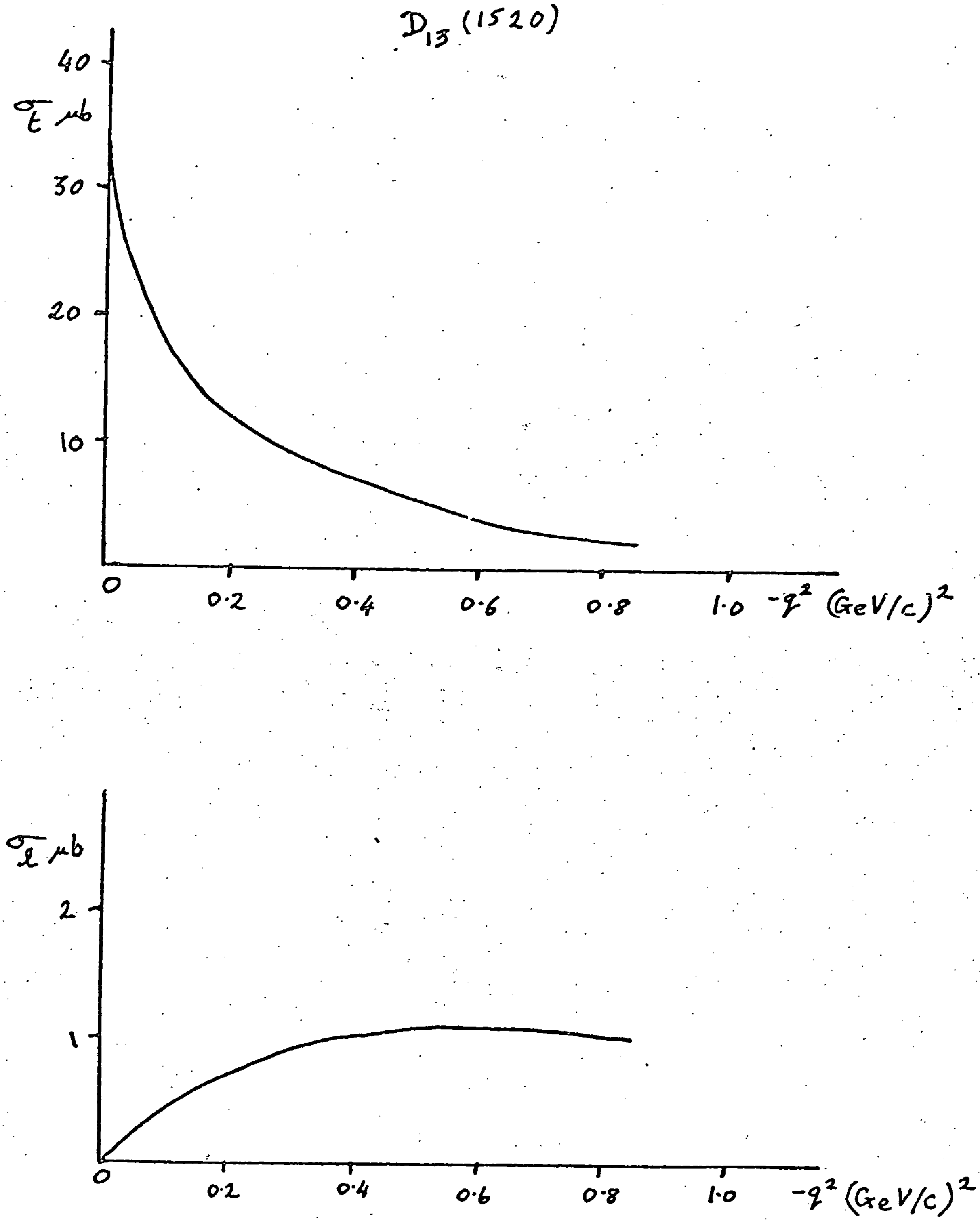
The longitudinal amplitude represents the actions of the charges of the quarks - the $^1/r$ Coulomb force felt even when no other interactions are occurring. So all the σ terms which change the quark spins give zero matrix elements and

$$H^{long} = \varepsilon_o \Big[1 + \frac{1}{12M_f m_q} + \frac{3\nu}{8M_f^2} - \frac{\nu}{12m_q^2} + \frac{1}{12m_q^2})\underline{k}^2\Big] e^{-ikz}$$

$$+ \frac{\varepsilon_o}{4m_q^2}\, k\, p_z\, e^{-ikz} \quad ,$$

The transverse and longitudinal cross-sections are drawn in Figure 16 and the ratio $R = \frac{A_{1/2}}{A_{3/2}}$ is plotted in Figure 17.

The ratio is very well behaved. As Lipes noted, including the Lorentz contraction of the wavefunction and the Wigner rotation of the spins strongly modifies the Gaussian behaviour of the form factors. As we saw earlier, this modification is bound to be flatter rather than steeper, in line with the behaviour



$D_{13}(1520)$
40
σ_t μb
30
20
10
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ (GeV/c)^2
σ_l μb
2
1
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ (GeV/c)^2

FIG 16

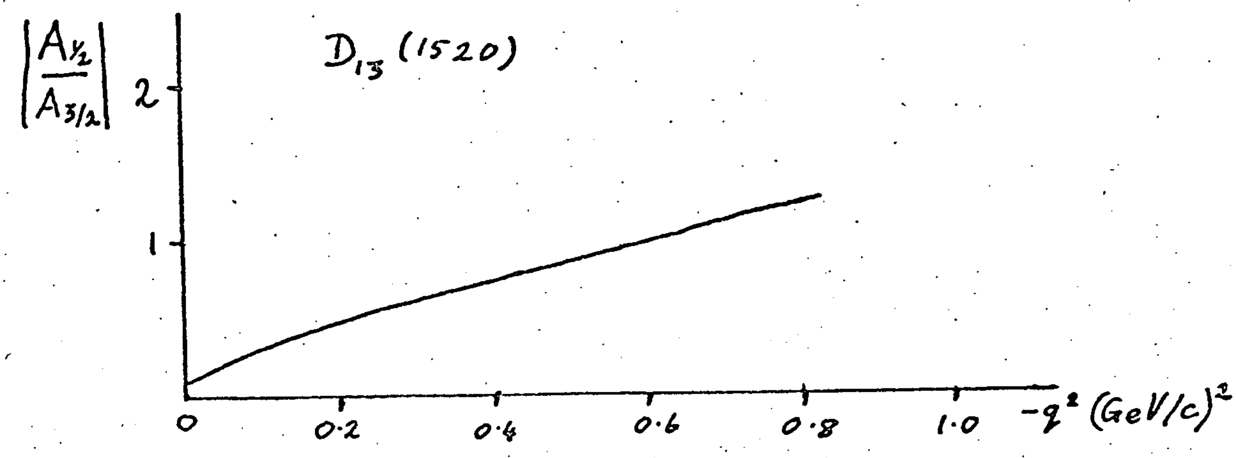


FIG 17

of the Coulomb-like potential.

It is difficult to compare the magnitudes of the cross-sections with experiment because of the presence of other resonances. σ_t is smaller than our previous results, σ_l is smaller in proportion to it. The photoproduction point is the clearest; there the D_{13} should be dominant. At $q^2 = 0$ experiment (11) gives $A_{1/2} = 0.03$, $A_{3/2} = 0.15$, while this model gives $A_{1/2} = 0.0319$, $A_{3/2} = 0.303$. Interestingly the shape of σ_t agrees well with Ravndal[21] whose model is naive relativistic. σ_l is very flat, unlike any other predictions.

c. The Full Integral

This $^1/m$ expansion is really only valid for large quark masses. But we are using $m_q = 0.33$ GeV which makes one wonder whether the expansion is still valid. So we now want to do the full integrals and compare them with the second order ones. The actual expressions are very complicated and the integration has to be done numerically by computer.

The general matrix element is written out in detail. There are three integrals: spin flip, orbital flip and longitudinal. The spin matrix elements were calculated by hand and then the spatial integrals were put on the computer. The integration method used was that of Gaussian quadratures and each of the integrands was calculated at five points. As a check on accuracy ten points were tried and the relative error turned out to be 2°/o. This is a small error, the integrands are smooth well behaved functions and were expected to converge rapidly because of being dominated by exponentials, so no further action was taken on errors and five cycles were used for all the integrals. Note that 2°/o on an amplitude means 4°/o for a cross-section.

The results for the D_{13} are shown in Figure 18.

These numbers are for the isobar frame, $m_q = 0.33$ GeV, $g = 1$ and $\alpha^2 = 0.35$. The formulae for σ_t and σ_ℓ are those used before. We need to check the effectiveness of the cancellation in the $A_{1/2}$ for photoproduction. To do this we calculate the backward differential cross-section which is given by CKO[4] as

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=\pi} = \frac{1}{3}\,\frac{2J+1}{4\pi}\,\frac{x}{\Gamma}\,\frac{m_N}{m_R}\left|A_{\frac{1}{2}}\right|^2$$

where J = resonance spin

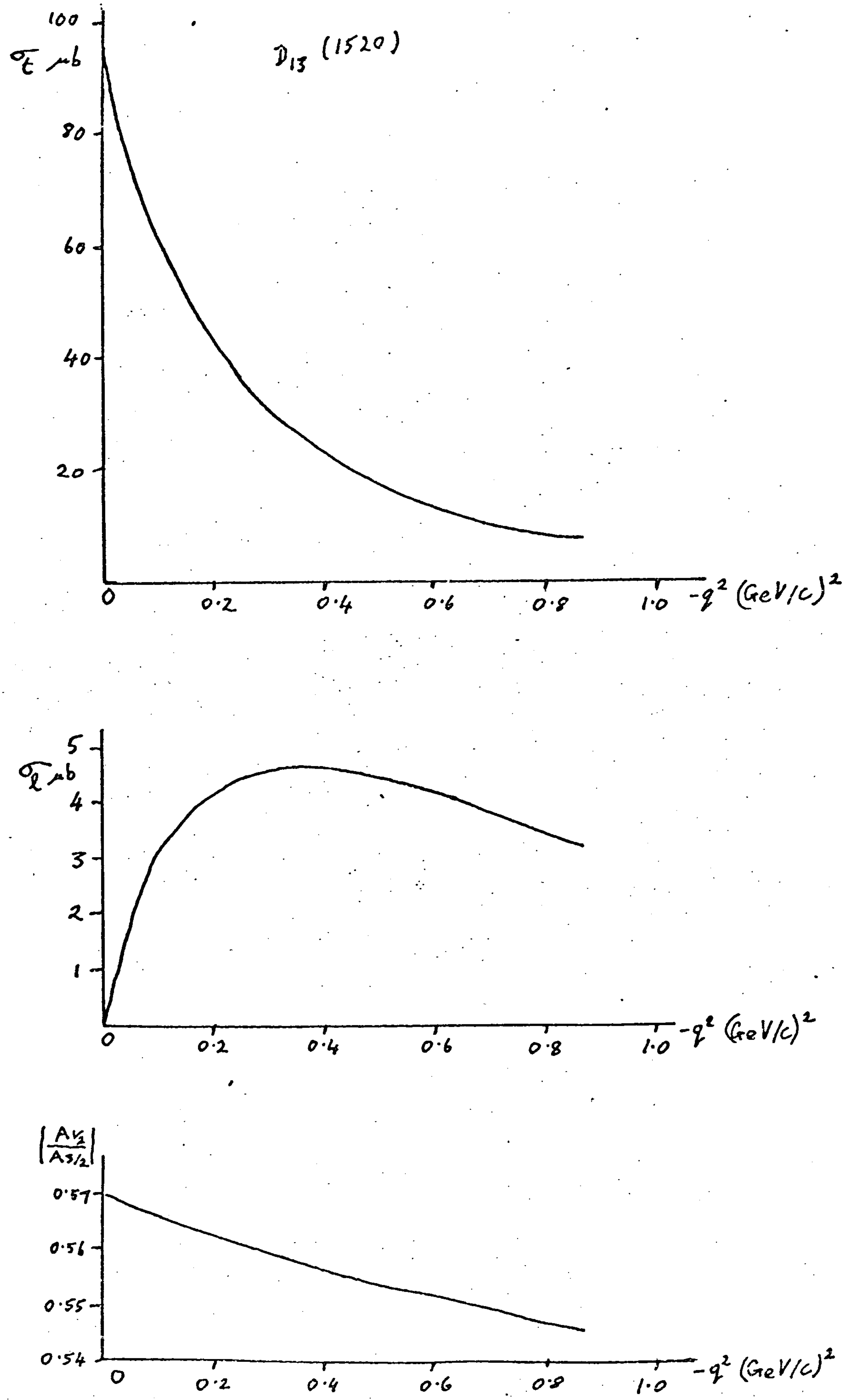
x = elasticity, Γ = width

m_N, m_R = masses of nucleon and resonance.

In this case $\left.\frac{d\sigma}{d\Omega}\right|_{\theta=\pi} = 0.62\ \mu b.sr^{-1}$

with $\alpha^2 = 0.35$.

So although the cross-sections are the correct shape and order of magnitude, we are far from cancellation. The experimental value[4] is about $0.08\ \mu b.sr^{-1}$.



D_{13} (1520)
σ_t μb
100
80
60
40
20
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ (GeV/c)^2
σ_ℓ μb
5
4
3
2
1
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ (GeV/c)^2
$\left|\frac{A_{1/2}}{A_{3/2}}\right|$
0·57
0·56
0·55
0·54
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ (GeV/c)^2

FIG. 1

The ratio is also rather strange: it falls rather than rises. But the numbers are almost constant. This behaviour is still consistent with the data. The only real discrepancy is the differential cross-section.

One could try to vary α^2. With $\alpha^2 = 1$ we have $\left.\frac{d\sigma}{d\Omega}\right|_{\theta=\pi} = 0.046\ \mu b.sr^{-1}$ smaller by a factor of 10. The point of doing this is not very clear. First of all α^2 needs to be greater than 0.35 to improve the cancellation. But this reduces the overall normalisation too so that we would lose the order of magnitude agreement for σ. The total transverse cross-section at $q^2 = 0$ and $\alpha^2 = 1$ is $\sigma_t = 0.17\mu b$ down from $\sigma_t = 95.5\ \mu b$ at $\alpha^2 = 0.35$. The main reduction comes from the factor of $\frac{1}{\alpha^{11/2}}$ outside the integrals. Also for an harmonic oscillator $\alpha^2 = m_q\omega$ where ω is the level spacing. If α^2 increases without changing m_q, ω will become unphysically large. A possible solution here is that, since the theory is basically a relativistic one, a literal interpretation of the potential may be inappropriate. In this case $\alpha^2 = m_q\omega$ would be a non-relativistic approximation.

Secondly, we could try an anomalous magnetic moment and a heavier quark. Kellett(18) has pointed out that an anomalous moment seems to be required in his model even for very light quarks. This appears to be the most suitable next step. Changing α on its own is blocked by the changes in overall normalisation since a small change, not enough to completely disrupt the order of magnitude agreement, will not affect the cancellation very much either.

It is highly probable that the situation can be improved by some sort of tinkering along these lines. The important thing is that the model is viable, the exact numbers are a secondary matter and we will not pursue it any further.

The fact that the second order and full integrals are not in very good agreement casts doubts upon the meaning and usefulness of the non-relativistic expansion and also on the full integral. Which is more reliable and trustworthy? Are either or neither of them a reasonable description of reality? The non-relativistic CKO quark model has been quite a good tool for exploring resonance decays. The electromagnetic current is the first or second order in a $\frac{1}{m}$ expansion. The Melosh theory of the previous chapter also relies on a $\frac{1}{m}$ series. These series seem to be useful despite the small mass. However the model is supposed to give a full description of events. There are no time-like excitations but these are known to be unwanted. The interaction is calculated in the ladder-approximation and neglected higher order graphs could perhaps be accounted for by the inclusion of an anomalous magnetic moment as discussed by Kellett. To sum up: it is difficult to assess the importance of the series expansion in $\frac{1}{m}$ but the full integral gives consistent numbers for the cross-sections and, by adjusting the parameters, can probably be improved.

One final test which should be made is for the mutual cancellation of the D_{13} and the F_{15}. The differential cross-section for the F_{15} should be about the same or a little smaller than that for the D_{13}. Then if, with adjustment, the F_{15} amplitudes were made vanishingly small, then the D_{13} amplitude would do likewise. Cancellation in one resonance requires only adjustment of the parameters. Cancellation in both is a non-trivial prediction of a successful model.

So, putting in the F_{15} wavefunction and repeating the previous calculation, we find for $\alpha^2 = 0.35$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=\pi} = 0.074\ \mu b.sr^{-1} \quad .$$

This is about ten times smaller than the D_{13} value. For $\alpha^2 = 1$ $\left.\frac{d\sigma}{d\Omega}\right|_{\theta=\pi} = 0.04 \times 10^{-2}\ \mu b.sr^{-1}$

which is a hundred times smaller.

So increase in α^2 has the same effect for both resonances. However the experimental value of $1.1 \times 10^{-2}\ \mu b.sr^{-1}$ (30,32) agrees better with the $\alpha^2 = 0.35$ value.

ADDENDUM TO CHAPTER 6

Ref. p. 68

We have tried to improve the fit of the model by including an anomalous magnetic moment term in the current. The quark mass was changed with the gyromagnetic ratio according to $\mu = \frac{eg}{2m_q}$ although this is arbitrary in a strong binding situation where the binding may also affect the mass.

Values of g from 2 to 10 were tried and the backward differential cross-sections for photoproduction of the D_{13} and F_{15} were calculated. The experimental values are 0.08 and 0.01 μb.sr^{-1} for the D_{13} and F_{15} respectively(4, 30, 32). Keeping $a^2 = 0.35$ GeV^2 the results were as follows:

	g	$\left.\frac{d\sigma}{d\Omega}\right\|_{\theta=\pi}$ μb.sr^{-1} (D_{13})	(F_{15})
	2	0.157	2.47
	4	0.514	3.43
	6	0.507	2.85
	8	0.481	2.20
	10	0.462	1.72
without anomalous moment	1	0.62	0.074

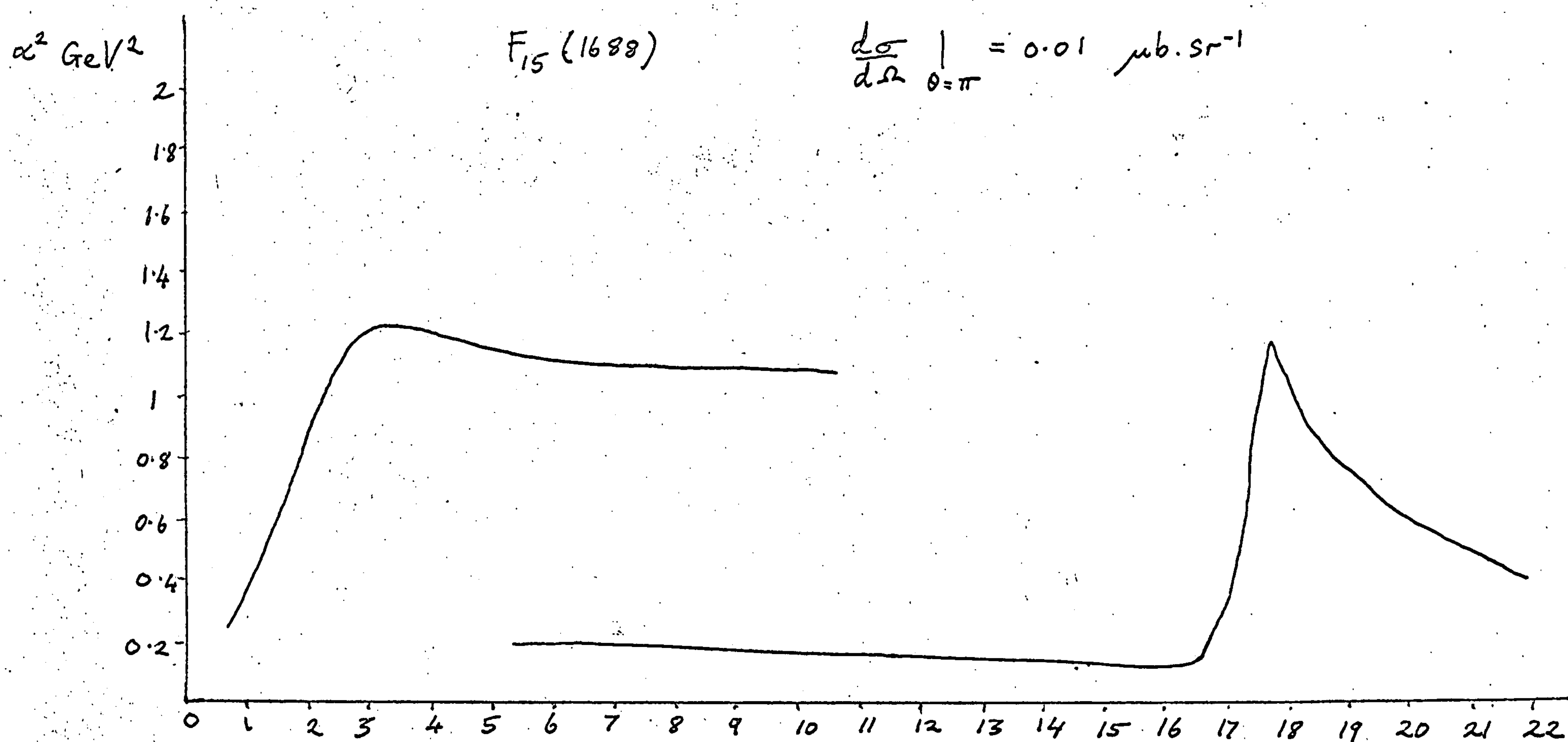
Values for the F_{15} are far from experiment with any anomalous moment although there was improvement for the D_{13}.

However we can also vary α^2 and try to find a region of g and α^2 within which both cross-sections are small. A number of values of α^2 were tried (0.5, 0.75, 1) and run over the same g range as before. Plotting g vs. $\frac{d\sigma}{d\Omega}$ at fixed α^2 and α^2 vs. $\frac{d\sigma}{d\Omega}$ at fixed g and extrapolating, values of α^2 and g were found for $\frac{d\sigma}{d\Omega}$ fixed at the experimental numbers: 0.08 and 0.01 $\mu b.sr^{-1}$ for the D_{13} and F_{15} respectively. These are drawn in Figures 19, 20 and superimposed in Figure 21.

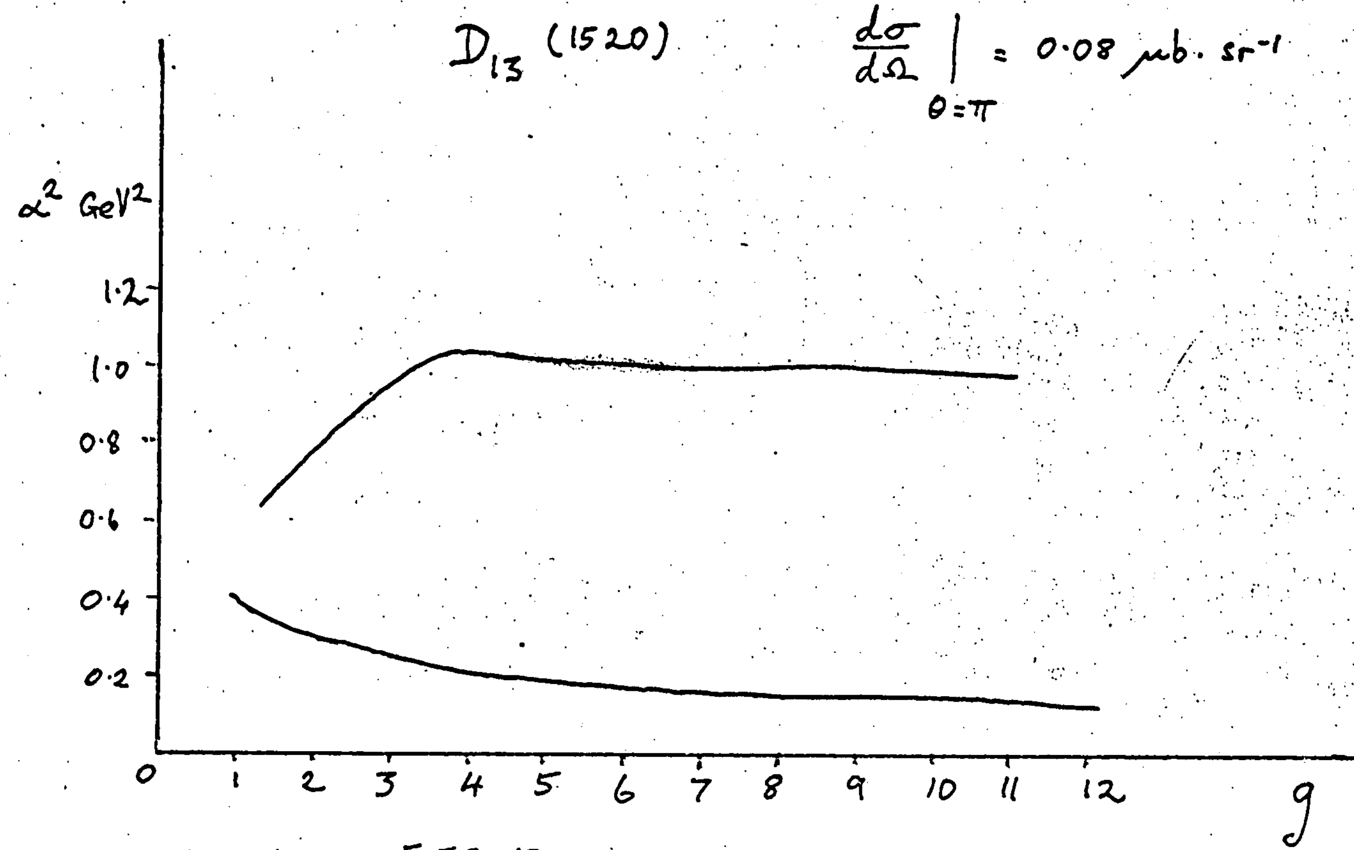
Note that the graphs are double valued. The upper one decreases and the lower one increases for a larger differential cross-section. As we see in Figure 21 there are points at which the D_{13} and F_{15} curves cross and a region where the curves lie almost on top of one another. This means that one can choose g and α^2 to satisfy experiment for both resonances together. This is a success for the theory.

The peak at around $g = 20$ for the F_{15} will occur for the D_{13} (if at all) at much larger g. We could not extrapolate far enough accurately enough. The extrapolations all have error bars of about ± 1 for g and ± 0.1 for α^2 but because some are drawn with known g and some with known α^2 and then smooth curves are drawn, this increases the accuracy.

There is one other resonance of particular interest, the $S_{11}(1535)$. According to new data presented at the Bonn Conference(12) the total cross-section for electroproduction of the S_{11} is constant with q^2. The CKO

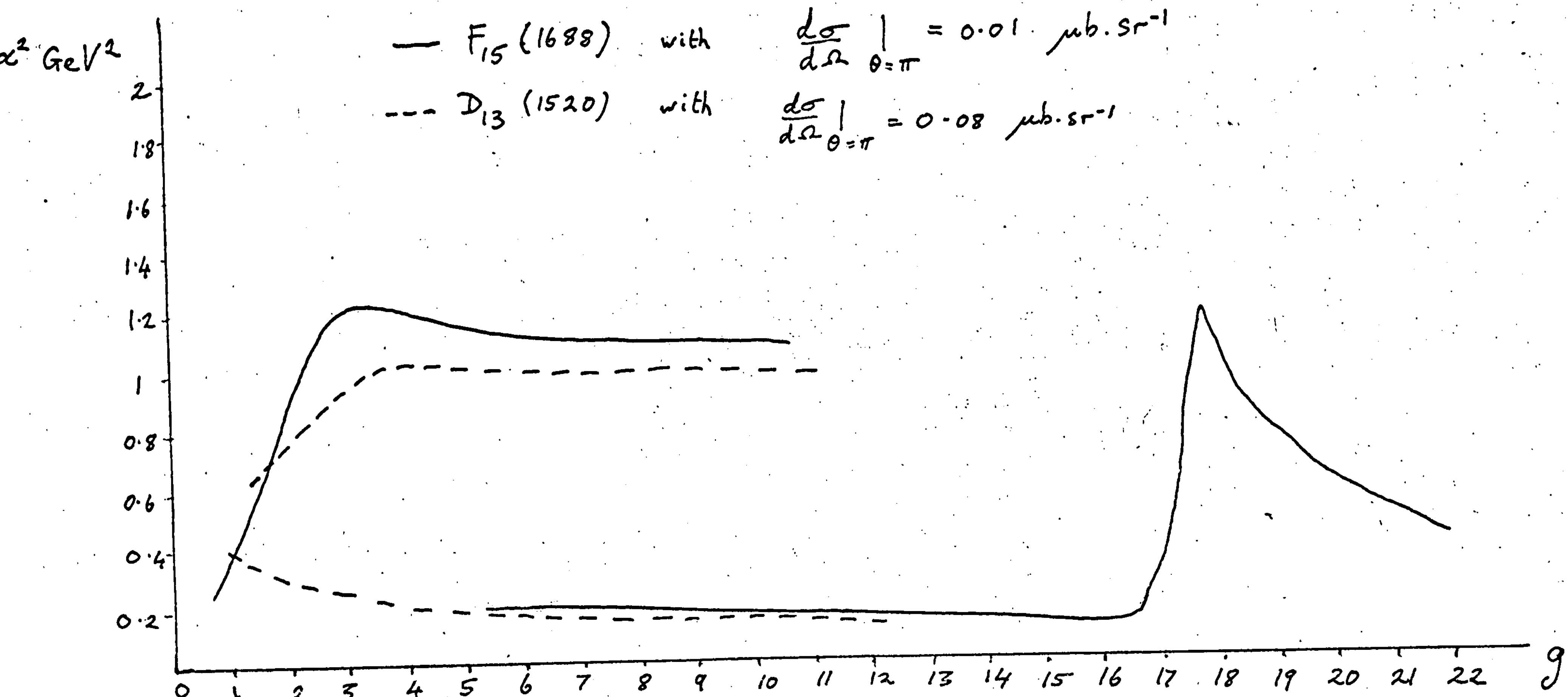


α² GeV²
F15 (1688)
dσ/dΩ |θ=π = 0·01 μb.sr⁻¹
2
1·8
1·6
1·4
1·2
1
0·8
0·6
0·4
0·2
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22



D_{13} (1520)
$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=\pi} = 0.08\ \mu b \cdot sr^{-1}$
α^2 GeV^2
1.2
1.0
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10 11 12
g

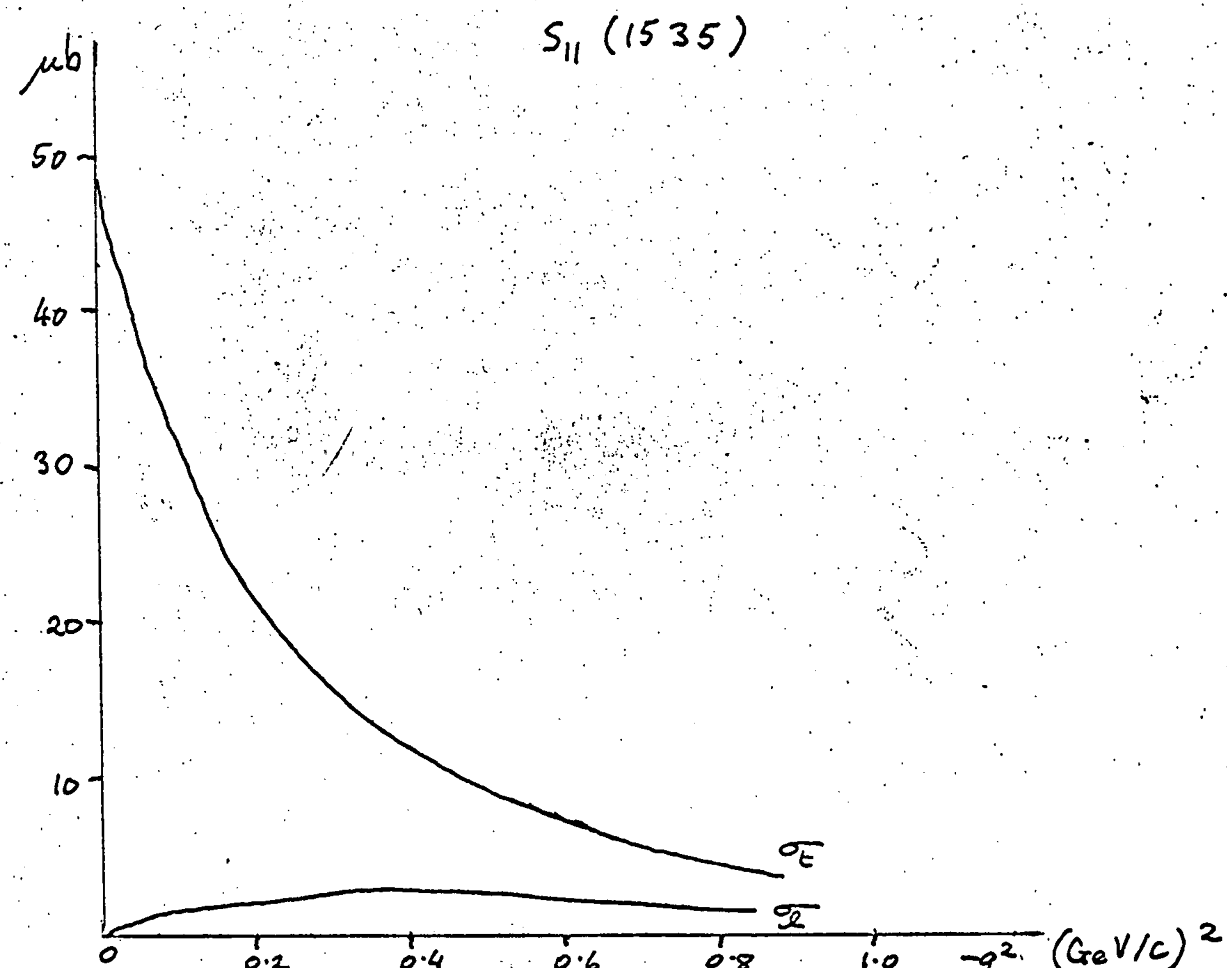
FIG 19



χ^2 GeV^2
— F_{15} (1688) with $\frac{d\sigma}{d\Omega}\Big|_{\theta=\pi} = 0.01\ \mu b.sr^{-1}$
--- D_{13} (1520) with $\frac{d\sigma}{d\Omega}\Big|_{\theta=\pi} = 0.08\ \mu b.sr^{-1}$
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
g

FIG 21

model (and others of similar type) predicts a fall as for the D_{13} (Figure 2). From the calculations discussed here, Kellett's model also predicts a sharp fall without an anomalous moment (Figure 22). Including the anomalous moment term makes no essential difference because, although two terms contribute to the amplitude, one is two orders of magnitude larger and causes the total transverse cross-section to fall in both the D_{13} and S_{11}. The dominance of the orbital-flip term is so great that over the whole range of g and a^2 investigated, there were no signs of flattening off. So even though the parameters can be chosen to fit the differential cross-sections for the D_{13} and F_{15}, none of the possible values will explain the observed behaviour of the S_{11} total cross-section. This is a definite failure of the model to agree with experiment even with two parameters to vary.



S_{11} (1535)
μb
50
40
30
20
10
0
0·2
0·4
0·6
0·8
1·0
$-q^2$ (GeV/c)^2
σ_t
σ_ℓ

FIG 22

CONCLUSION

Let us summarise our progress. The non-relativistic harmonic oscillator model is in good agreement with experiment for cross-sections but it predicted a very steep increase in the ratio of transverse helicity amplitudes, R. The Coulomb-like potential predicted a much gentler change in R but since the overall normalisation was far too small, we rejected it as unsuitable. But since R behaviour depends on the form factors, we still regard the R prediction as meaningful. Returning to the harmonic oscillator we found that with the addition of spin orbit terms R is made flatter without losing the good cross-section agreement. So the non-relativistic model can be made self-consistent and fairly accurate.

Next we had to face the problems of SU(6) breaking and of a relativistic extension to the previous model. We dealt with them together in two ways. We showed that the composition of terms in the Melosh theory was compatible with the angular momentum structure inherent in the electromagnetic current and extended the Melosh transformation in first order to include the electromagnetic interaction. Kellett developed a model of three quarks in an harmonic oscillator potential solved via the Bethe-Salpeter equation. We showed that when applied to resonance electroproduction, the results are consistent with those of other models and the data.

So what do we do next? The situation at present is that there are two independent models for extensions to the naive CKO quark model. Both work quite well. Our natural impulse is to try and unite them. It can be argued that this is possible as follows. In the Melosh picture the interaction is with one quark only, the other two are dragged along in the longitudinal direction and their $SU(6)_W$ classification is not affected. However, these quarks were already moving about in the stationary nucleon, at an angle to the longitudinal direction so that the boost will affect their spins contrary to the previous assumption.

This has been dealt with rigorously by Osborn[47]. He has shown that the one quark Melosh transformation discussed so far is only compatible with spin constraints (formulated as an angular condition) to first order in the arctan series, i.e. just $\frac{\gamma_\perp d_\perp}{m}$. Otherwise many-quark operators are required.

So the inclusion of the electromagnetic interaction in V to first order, discussed in Chapter 5, is still valid for very low energy interactions where the relativistic spin transformation is negligible. What is needed now is a specific model with a three quark Melosh transformation which affects spins in the same way as the dynamics of the Bethe-Salpeter equation to agree with Kellett's method. It is hoped to investigate this problem in the near future.

The Melosh transformation was a great step forward in providing a link between particle classification and interactions. A model with more or less realistic forces is also a step forward. To unite them would give the quark model more strength and certainty than it had previously to combat difficulties.

APPENDIX

a. SU(6) Resonance Classification

The supermultiplets are of the group SU(6): SU(3) combined with SU(2) quark spin. States are grouped in 56 or 70 plets, the symmetric and mixed symmetry configurations, and also labelled by orbital angular momentum and parity. The quarks are assumed to obey parastatistics and the wavefunctions will form representations of the permutation group on three objects with the ground state totally symmetric. The three lowest levels are shown on the next page.

b. Spatial wavefunctions

These are as in Faiman and Hendry[9]. The three quarks have radius vectors $\underline{r}_1$, $\underline{r}_2$, $\underline{r}_3$ and the complete spatial wavefunctions are the solutions of the Schrödinger equation with an harmonic oscillator potential for three particles.

The proton is the ground state $(1s)^3$, completely symmetric.

$$\psi = \frac{1}{(4\pi)^{3/2}} \left(\frac{4\alpha^3}{\sqrt{\pi}}\right)^{3/2} \exp\left[-\frac{\alpha^2}{2}\left(r_1^{\,2} + r_2^{\,2} + r_3^{\,2}\right)\right] .$$

The $\underline{D_{13}}$ and $\underline{S_{11}}$ are $(1s)^2(1p)$ in the $[70, 1^-]$ with mixed symmetry. There are two possible wavefunctions: ψ_a is even under permutation of quarks 1 and 2 and is the one we require. There is also ψ_b odd under permutations of quarks 1 and 2.

Level Number	Configurations	Supermultiplets	Resonances
0	$(1s)^3$	$56,\ 0^+$	$N\binom{p}{n}$, $\Delta(1236)$
1	$(1s)^2\ (1p)$	$70,\ 1^-$	$2S_{11}$, $2D_{13}$ etc.
2	$(1s)^2\ (2s)$ $(1s)\ (1p)^2$ $(1s)^2\ (1d)$	$56,2^+$ $56,0^+$ $20,1^+$ $70,0^+$ $70,2^+$	F_{15} P_{13} F_{17} etc.

$$\psi_a = \frac{1}{3}\left(\frac{4\alpha^3}{\sqrt{\pi}}\right)^{3/2} \frac{\alpha}{4\pi} (\underline{r}_1 + \underline{r}_2 - 2\underline{r}_3) \exp\left[-\frac{\alpha^2}{2}(r_1^2 + r_2^2 + r_3^2)\right]$$

The $\underline{F_{15}}$ is $(1s)^2(1d)$ in the $[56, 2^+]$ and completely symmetric.

$$\psi = \frac{2}{\sqrt{45}} \left(\frac{4\alpha^3}{\sqrt{\pi}}\right)^{3/2} \frac{\alpha^2}{4\pi}\left[r_1^2 Y_2(\Omega_1) + r_2^2 Y_2(\Omega_2) + r_3^2 Y_2(\Omega_3)\right] \times$$

$$\times \exp\left[-\frac{\alpha^2}{2}(r_1^2 + r_2^2 + r_3^2)\right] .$$

The Y's are spherical harmonics.

In the F_{15} case there is a second possibility:

$$\psi(56, 2^+) = \sqrt{\frac{2}{3}}\,(1s)^2(1d) - \sqrt{\frac{1}{3}}\,(1s)(1p)^2 .$$

The $(1s)(1p)^2$ does not contribute in this model because one quark excitation only is assumed to be of significance. But we must always remember to multiply by $\sqrt{\frac{2}{3}}$. The coefficients for the linear combination are chosen to eliminate dependence on the centre of mass radius vector $\underline{R} = \frac{1}{3}(\underline{r}_1 + \underline{r}_2 + \underline{r}_3)$. Any $\underline{R}$ dependence represents a spurious excitation as the three quarks remain in the same state and are just corporately moved along.

c. SU(6) Wavefunctions

SU(2) spin wavefunctions. Each quark has spin ½.

Defining α = spin ½ up β = spin ½ down, we have doublet

$$\left|\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle \begin{cases} \chi^1_{\frac{1}{2}+} = \frac{1}{\sqrt{6}}(2\alpha_1\,\alpha_2\,\beta_3 - \alpha_1\,\beta_2\,\alpha_3 - \beta_1\,\alpha_2\,\alpha_3) \\ \chi^2_{\frac{1}{2}+} = \frac{1}{\sqrt{2}}(\alpha_1\,\beta_2 - \beta_1\,\alpha_2)\alpha_3 \end{cases}$$

quartet

$$\left|\tfrac{3}{2}, +\tfrac{1}{2}\right\rangle \quad \chi_{3/2,+1/2} = \frac{1}{\sqrt{3}}(\alpha_1\,\alpha_2\,\beta_3 + \alpha_1\,\beta_2\,\alpha_3 + \beta_1\,\alpha_2\,\alpha_3)$$

$$\tfrac{3}{2}, +\tfrac{3}{2} \quad \chi_{3/2,+3/2} = \alpha_1\,\alpha_2\,\alpha_3 \quad .$$

SU(3) isospin wavefunctions

We are only interested in p and n quarks.

$p = \alpha$ and $n = \beta$ (isospin ½ up or down)

$$g^1_{8p} = \frac{1}{\sqrt{6}}(2p_1 p_2\, n_3 - p_1\, n_2\, p_3 - n_1\, p_2\, p_3)$$

$$g^2_{8p} = \frac{1}{\sqrt{2}}(p_1\, n_2 - n_1\, p_2)p_3$$

$$g_{10\ 3/2} = p_1\, p_2\, p_3$$

$$g_{10\ 1/2} = \frac{1}{\sqrt{3}}(p_1\, p_2\, n_3 + p_1\, n_2\, p_3 + n_1\, p_2\, p_3) \quad .$$

SU(6) wavefunctions are formed by combining the SU(2) and SU(3) wavefunctions. They must be representations of the permutation group. The relevant Clebsch-Gordan coefficients are

$$\begin{pmatrix} m & m & s \\ K & \lambda & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \delta_{K\lambda}$$

$$-\begin{pmatrix} m & m & m \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} m & m & m \\ 2 & 2 & 1 \end{pmatrix} = \begin{pmatrix} m & m & m \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} m & m & m \\ 2 & 1 & 2 \end{pmatrix} = \frac{1}{\sqrt{2}} \quad .$$

So SU(6) wavefunctions are

$$|p \quad \tfrac{1}{2} \quad +\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left[\chi^{1}_{\frac{1}{2}+}\, g^{1}_{8p} + \chi^{2}_{\frac{1}{2}+}\, g^{2}_{8p}\right]$$

$$|D_{13} \quad \tfrac{1}{2} \quad +\tfrac{1}{2}\rangle^{a} = \frac{1}{\sqrt{2}}\left[\chi^{2}_{\frac{1}{2}+}\, g^{2}_{8p} - \chi^{1}_{\frac{1}{2}+}\, g^{1}_{8p}\right] \quad \text{even}$$

$$|D_{13} \quad \tfrac{1}{2} \quad +\tfrac{1}{2}\rangle^{b} = \frac{1}{\sqrt{2}}\left[\chi^{1}_{\frac{1}{2}+}\, g^{2}_{8p} + \chi^{2}_{\frac{1}{2}+}\, g^{1}_{8p}\right] \quad \text{odd}$$

$$|F_{15} \quad \tfrac{1}{2} \quad +\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left[\chi^{1}_{\frac{1}{2}+}\, g^{1}_{8p} + \chi^{2}_{\frac{1}{2}+}\, g^{2}_{8p}\right] \quad .$$

Similarly for nett quark spin component $-\frac{1}{2}$.

The total wavefunctions are now products of SU(6) and spatial wavefunctions in the same way.

$$|p\rangle_{up} = \psi(56,\ 0^{+})\ |p_{\frac{1}{2}}\ {+\tfrac{1}{2}}\rangle$$

$$|D_{13}\rangle_{up} = \frac{1}{\sqrt{2}}\Big[\psi_a(70,\ 1^{-})\ |D_{13}\ \tfrac{1}{2}\ +\tfrac{1}{2}\rangle^{a} + \psi_b(70,\ 1^{-})\ |D_{13}\ \tfrac{1}{2}\ +\tfrac{1}{2}\rangle^{b}\Big]$$

$$|F_{15}\rangle_{up} = \psi(56,\ 2^{+})\ |F_{15}\ \tfrac{1}{2}\ +\tfrac{1}{2}\rangle \quad .$$

The S_{11} is not separately described here as it has the same wavefunctions as the D_{13}. The only difference between them is the way the quark spin and orbital angular momentum combine.

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