

THE LINEAR AND NON-LINEAR FLEXURAL VIBRATIONS OF ROTATING
IMPERFECT DISCS.

by

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Tables and Figures.

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T A B L E S .

Formulae for resonant speeds.

	Nodal diameters				
	1	2	3	4	5
3d Minor Resonance Ω_{s3}	$\frac{\lambda_s}{\sqrt{(16-B)}}$	$\frac{\lambda_s}{\sqrt{(25-B)}}$	$\frac{\lambda_s}{\sqrt{(36-B)}}$	$\frac{\lambda_s}{\sqrt{(49-B)}}$	$\frac{\lambda_s}{\sqrt{(64-B)}}$
2nd Minor Resonance Ω_{s2}	$\frac{\lambda_s}{\sqrt{[(2+s)^2 - B]}}$	$\frac{\lambda_s}{\sqrt{(16-B)}}$	$\frac{\lambda_s}{\sqrt{(25-B)}}$	$\frac{\lambda_s}{\sqrt{(36-B)}}$	$\frac{\lambda_s}{\sqrt{(49-B)}}$
1st Minor Resonance Ω_{s1}	$\frac{\lambda_s}{\sqrt{[(1+s)^2 - B]}}$	$\frac{\lambda_s}{\sqrt{(9-B)}}$	$\frac{\lambda_s}{\sqrt{(16-B)}}$	$\frac{\lambda_s}{\sqrt{(25-B)}}$	$\frac{\lambda_s}{\sqrt{(36-B)}}$
Critical Speed Ω_s	$\frac{\lambda_s}{\sqrt{[s^2 - B]}}$	$\frac{\lambda_s}{\sqrt{(4-B)}}$	$\frac{\lambda_s}{\sqrt{(9-B)}}$	$\frac{\lambda_s}{\sqrt{(16-B)}}$	$\frac{\lambda_s}{\sqrt{(25-B)}}$
1st Sub-Minor Resonance Ω_{sA}	$\frac{\lambda_s}{\sqrt{[(s^2 - 1)^2 - B]}}$	$\frac{\lambda_s}{\sqrt{(4-B)}}$	$\frac{\lambda_s}{\sqrt{(9-B)}}$	$\frac{\lambda_s}{\sqrt{(16-B)}}$	$\frac{\lambda_s}{\sqrt{(25-B)}}$
2nd Sub-Minor Resonance Ω_{sB}	$\frac{\lambda_s}{\sqrt{[(s^2 - 2)^2 - B]}}$	$\frac{\lambda_s}{\sqrt{(4-B)}}$	$\frac{\lambda_s}{\sqrt{(9-B)}}$	$\frac{\lambda_s}{\sqrt{(16-B)}}$	$\frac{\lambda_s}{\sqrt{(25-B)}}$
3d Sub-Minor Resonance Ω_{sC}	$\frac{\lambda_s}{\sqrt{[(s^2 - 3)^2 - B]}}$	$\frac{\lambda_s}{\sqrt{(4-B)}}$	$\frac{\lambda_s}{\sqrt{(9-B)}}$	$\frac{\lambda_s}{\sqrt{(16-B)}}$	$\frac{\lambda_s}{\sqrt{(25-B)}}$

Table I.

Kinetic Energy

 α_s

$$\iiint_V \rho U^2(r) \cos^2 s \theta \, r \, d\theta \, dr \, dz$$

 β_s

$$\iiint_V \rho U^2(r) \sin^2 s \theta \, r \, d\theta \, dr \, dz$$

 γ_s

$$\frac{1}{2} \iiint_V \rho U^2(r) \sin 2s \theta \, r \, d\theta \, dr \, dz$$

Strain Energy

 α_s

$$\iiint_V \frac{E}{1-\nu^2} z^2 (\bar{A} \cos^2 s \theta + \bar{B} \sin^2 s \theta) \, r \, d\theta \, dr \, dz$$

 β_s

$$\iiint_V \frac{E}{1-\nu^2} z^2 (\bar{A} \sin^2 s \theta + \bar{B} \cos^2 s \theta) \, r \, d\theta \, dr \, dz$$

 γ_s

$$\frac{1}{2} \iiint_V \frac{E}{1-\nu^2} z^2 (\bar{A} - \bar{B}) \sin 2s \theta \, r \, d\theta \, dr \, dz$$

where

$$\bar{A} = \left(\frac{dU}{dr}\right)^2 + \frac{2\nu}{r} \frac{dU}{dr} \frac{d^2U}{dr^2} - \frac{2\nu s^2}{r^2} U \frac{d^2U}{dr^2} + \frac{1}{r^2} \left(\frac{dU}{d\theta}\right)^2 - \frac{2s^2}{r^3} U \frac{dU}{d\theta} + \frac{s^4}{r^4} U^2$$

$$\bar{B} = \frac{2(1-\nu)}{r^2} \left[\frac{U^2}{r^2} - 2 \frac{U}{r} \frac{dU}{dr} + \left(\frac{dU}{dr}\right)^2 \right]$$

Table II.

Nodal diameters S	Preferential frequencies c/s		Logarithmic decrement	
	λ^P	λ^Q	δ^P	δ^Q
1	68.90	71.65	$770 \cdot 10^{-4}$	$680 \cdot 10^{-4}$
2	143.62	144.15	$7.16 \cdot 10^{-4}$	$7.22 \cdot 10^{-4}$
3	325.48	325.90	$2.37 \cdot 10^{-4}$	$2.50 \cdot 10^{-4}$
4	574.80	575.70	-	-

Table III.

Part I. Rotating Observer					Part II. Stationary Observer					
Effective frequencies	$u(\omega + s\Omega)$		$u(\omega - s\Omega)$		Frequencies recorded by pick-up	$u(\omega + s\Omega)$		$u(\omega - s\Omega)$		Frequencies rec by pick-up
	Travelling Wave Component	Fixed Vibration Component	Travelling Wave Component	Fixed Vibration Component		Travelling Wave Component	Fixed Vibration Component	Travelling Wave Component	Fixed Vibration Component	
	I. $\frac{\omega + s\Omega}{\omega - s\Omega} < \lambda_d^p$	$\frac{Q_M \cos[s\theta - (\omega + s\Omega)t]}{2 R^p(\omega + s\Omega)}$	$\frac{1}{2} \left[\frac{Q_M}{R^p(\omega + s\Omega)} - \frac{Q_N}{R^q(\omega + s\Omega)} \right] \cos s\theta \cos(\omega + s\Omega)t$	$\frac{Q_M \cos[s\theta + (\omega - s\Omega)t]}{2 R^q(\omega - s\Omega)}$		$\frac{1}{2} \left[\frac{Q_M}{R^p(\omega - s\Omega)} - \frac{Q_N}{R^q(\omega - s\Omega)} \right] \cos s\theta \cos(\omega - s\Omega)t$	Frequencies due to $u(\omega - s\Omega)$; $(\omega + s\Omega)$ due to $u(\omega + s\Omega)$;	$\frac{Q_M \cos[s\theta - \omega t]}{2 R^q(\omega + s\Omega)}$	$\frac{1}{4} \left[\frac{Q_M}{R^p(\omega + s\Omega)} - \frac{Q_N}{R^q(\omega + s\Omega)} \right] \{ \cos[s\theta + (\omega + 2s\Omega)t] + \cos[s\theta - \omega t] \}$	
II. $\frac{\omega + s\Omega}{\omega - s\Omega} = \lambda_d^p$	—	$\frac{1}{2} \left[\frac{Q_M}{R^p(\omega + s\Omega)} \cos s\theta + \frac{Q_N}{R^q(\omega + s\Omega)} \sin s\theta \right] \sin(\omega + s\Omega)t$	—	$\frac{1}{2} \left[\frac{Q_M}{R^p(\omega - s\Omega)} \cos s\theta - \frac{Q_N}{R^q(\omega - s\Omega)} \sin s\theta \right] \sin(\omega - s\Omega)t$	—	$\frac{1}{4} \frac{Q_M}{R^p(\omega + s\Omega)} \{ \sin[s\theta + (\omega + 2s\Omega)t] - \sin[s\theta - \omega t] \}$ $-\frac{1}{4} \frac{Q_N}{R^q(\omega + s\Omega)} \{ \cos[s\theta + (\omega + 2s\Omega)t] - \cos[s\theta - \omega t] \}$		—	$\frac{1}{4} \frac{Q_M}{R^p(\omega - s\Omega)} \{ \sin[s\theta + \omega t] - \sin[s\theta - (\omega - 2s\Omega)t] \}$ $+\frac{1}{4} \frac{Q_N}{R^q(\omega - s\Omega)} \{ \cos[s\theta + \omega t] - \cos[s\theta - (\omega - 2s\Omega)t] \}$	
III. $\lambda^p < \frac{\omega + s\Omega}{\omega - s\Omega} < \lambda_d^q$	$\frac{Q_M \cos[s\theta + (\omega + s\Omega)t]}{2 R^p(\omega + s\Omega)}$	$-\frac{1}{2} \left[\frac{Q_M}{R^p(\omega + s\Omega)} - \frac{Q_N}{R^q(\omega + s\Omega)} \right] \cos s\theta \cos(\omega + s\Omega)t$	$\frac{Q_M \cos[s\theta - (\omega - s\Omega)t]}{2 R^q(\omega - s\Omega)}$	$-\frac{1}{2} \left[\frac{Q_M}{R^p(\omega - s\Omega)} - \frac{Q_N}{R^q(\omega - s\Omega)} \right] \cos s\theta \cos(\omega - s\Omega)t$	$-\frac{Q_M \cos[s\theta + (\omega + 2s\Omega)t]}{2 R^p(\omega + s\Omega)}$	$-\frac{1}{4} \left[\frac{Q_M}{R^p(\omega + s\Omega)} - \frac{Q_N}{R^q(\omega + s\Omega)} \right] \{ \cos[s\theta + (\omega + 2s\Omega)t] + \cos[s\theta - \omega t] \}$		$-\frac{Q_M \cos[s\theta - (\omega - 2s\Omega)t]}{2 R^q(\omega - s\Omega)}$	$-\frac{1}{4} \left[\frac{Q_M}{R^p(\omega - s\Omega)} - \frac{Q_N}{R^q(\omega - s\Omega)} \right] \{ \cos[s\theta + \omega t] + \cos[s\theta - (\omega - 2s\Omega)t] \}$	
IV. $\frac{\omega + s\Omega}{\omega - s\Omega} = \lambda_d^q$	$\frac{Q_M \cos[s\theta + (\omega + s\Omega)t]}{2 R^q(\omega + s\Omega)}$	—	$\frac{Q_M \cos[s\theta - (\omega - s\Omega)t]}{2 R^p(\omega - s\Omega)}$	—	$\frac{Q_M \cos[s\theta + (\omega + 2s\Omega)t]}{2 R^q(\omega + s\Omega)}$	—		$-\frac{Q_M \cos[s\theta - (\omega - 2s\Omega)t]}{2 R^p(\omega - s\Omega)}$	—	
V. $\lambda^q < \frac{\omega + s\Omega}{\omega - s\Omega} < \lambda_d^p$	$\frac{Q_M \cos[s\theta + (\omega + s\Omega)t]}{2 R^p(\omega + s\Omega)}$	$-\frac{1}{2} \left[\frac{Q_M}{R^p(\omega + s\Omega)} - \frac{Q_N}{R^q(\omega + s\Omega)} \right] \cos s\theta \cos(\omega + s\Omega)t$	$\frac{Q_M \cos[s\theta - (\omega - s\Omega)t]}{2 R^q(\omega - s\Omega)}$	$-\frac{1}{2} \left[\frac{Q_M}{R^p(\omega - s\Omega)} - \frac{Q_N}{R^q(\omega - s\Omega)} \right] \cos s\theta \cos(\omega - s\Omega)t$	$\frac{Q_M \cos[s\theta + (\omega + 2s\Omega)t]}{2 R^p(\omega + s\Omega)}$	$-\frac{1}{4} \left[\frac{Q_M}{R^p(\omega + s\Omega)} - \frac{Q_N}{R^q(\omega + s\Omega)} \right] \{ \cos[s\theta + (\omega + 2s\Omega)t] + \cos[s\theta - \omega t] \}$		$-\frac{Q_M \cos[s\theta - (\omega - 2s\Omega)t]}{2 R^q(\omega - s\Omega)}$	$-\frac{1}{4} \left[\frac{Q_M}{R^p(\omega - s\Omega)} - \frac{Q_N}{R^q(\omega - s\Omega)} \right] \{ \cos[s\theta + \omega t] + \cos[s\theta - (\omega - 2s\Omega)t] \}$	
VI. $\frac{\omega + s\Omega}{\omega - s\Omega} = \lambda_d^p$	—	$-\frac{1}{2} \left[\frac{Q_M}{R^p(\omega + s\Omega)} \cos s\theta + \frac{Q_N}{R^q(\omega + s\Omega)} \sin s\theta \right] \cos(\omega + s\Omega)t$	—	$-\frac{1}{2} \left[\frac{Q_M}{R^p(\omega - s\Omega)} \cos s\theta - \frac{Q_N}{R^q(\omega - s\Omega)} \sin s\theta \right] \cos(\omega - s\Omega)t$	—	$-\frac{1}{4} \frac{Q_M}{R^p(\omega + s\Omega)} \{ \cos[s\theta + (\omega + 2s\Omega)t] + \cos[s\theta - \omega t] \}$ $-\frac{1}{4} \frac{Q_N}{R^q(\omega + s\Omega)} \{ \sin[s\theta + (\omega + 2s\Omega)t] + \sin[s\theta - \omega t] \}$		—	$-\frac{1}{4} \frac{Q_M}{R^p(\omega - s\Omega)} \{ \cos[s\theta + \omega t] + \cos[s\theta - (\omega - 2s\Omega)t] \}$ $+\frac{1}{4} \frac{Q_N}{R^q(\omega - s\Omega)} \{ \sin[s\theta + \omega t] + \sin[s\theta - (\omega - 2s\Omega)t] \}$	
VII. $\frac{\omega + s\Omega}{\omega - s\Omega} > \lambda_d^q$	$\frac{Q_M \cos[s\theta - (\omega + s\Omega)t]}{2 R^p(\omega + s\Omega)}$	$-\frac{1}{2} \left[\frac{Q_M}{R^p(\omega + s\Omega)} - \frac{Q_N}{R^q(\omega + s\Omega)} \right] \sin s\theta \sin(\omega + s\Omega)t$	$\frac{Q_M \cos[s\theta + (\omega - s\Omega)t]}{2 R^q(\omega - s\Omega)}$	$-\frac{1}{2} \left[\frac{Q_M}{R^p(\omega - s\Omega)} - \frac{Q_N}{R^q(\omega - s\Omega)} \right] \sin s\theta \sin(\omega - s\Omega)t$	$\frac{Q_M \cos[s\theta - \omega t]}{2 R^p(\omega + s\Omega)}$	$\frac{1}{4} \left[\frac{Q_M}{R^p(\omega + s\Omega)} - \frac{Q_N}{R^q(\omega + s\Omega)} \right] \{ \cos[s\theta + (\omega + 2s\Omega)t] - \cos[s\theta - \omega t] \}$		$-\frac{Q_M \cos[s\theta + \omega t]}{2 R^q(\omega - s\Omega)}$	$+\frac{1}{4} \left[\frac{Q_M}{R^p(\omega - s\Omega)} - \frac{Q_N}{R^q(\omega - s\Omega)} \right] \{ \sin[s\theta + \omega t] - \sin[s\theta - (\omega - 2s\Omega)t] \}$	
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.

Table IV.

F I G U R E S .

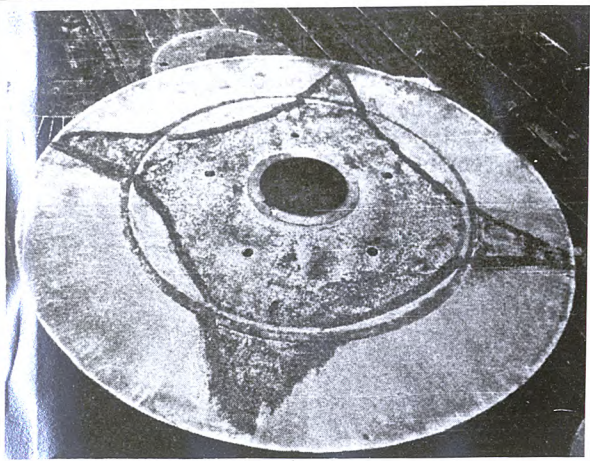


Fig. 1

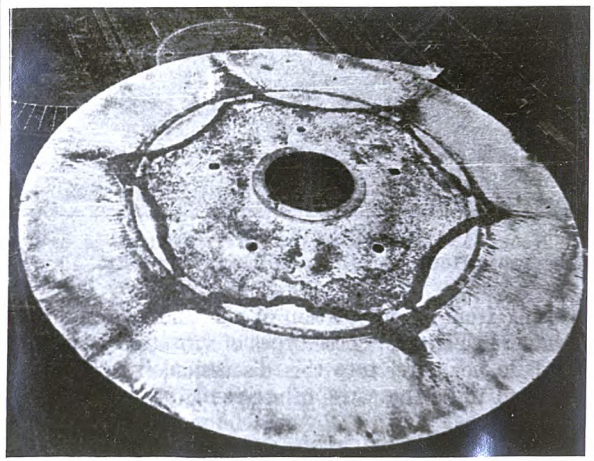


Fig. 2

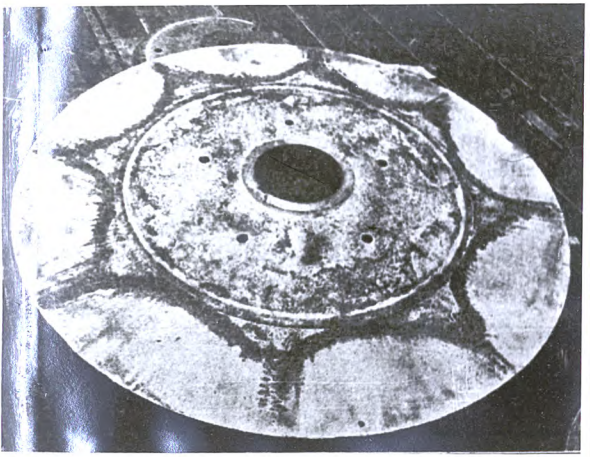


Fig. 3

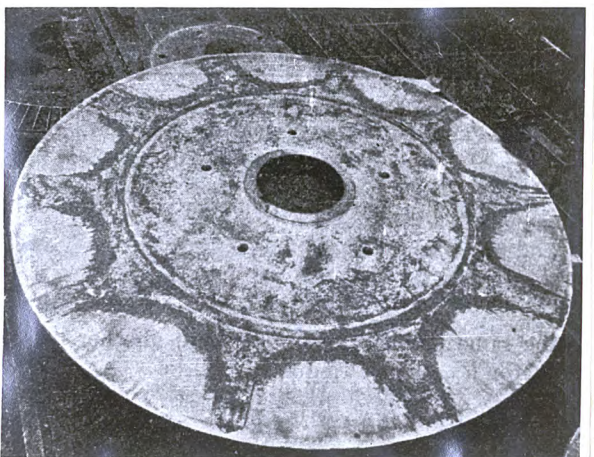


Fig. 4

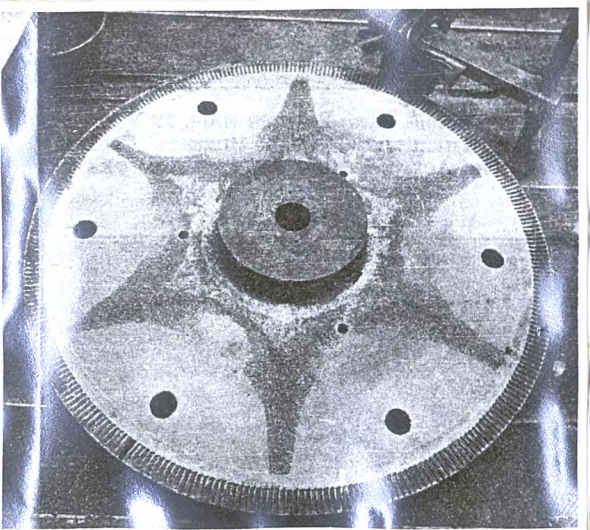


Fig. 5

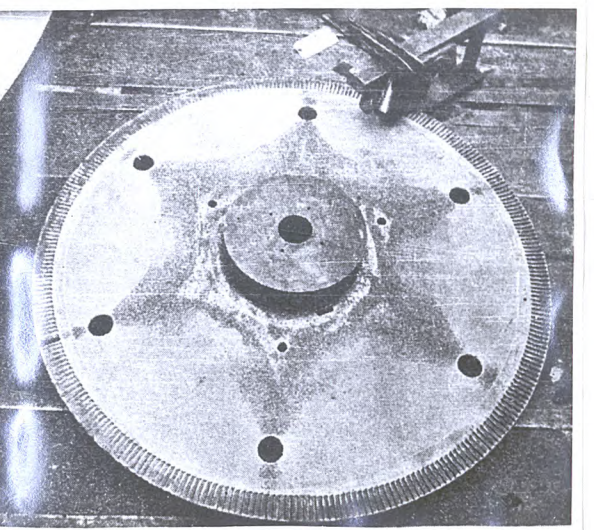


Fig. 6

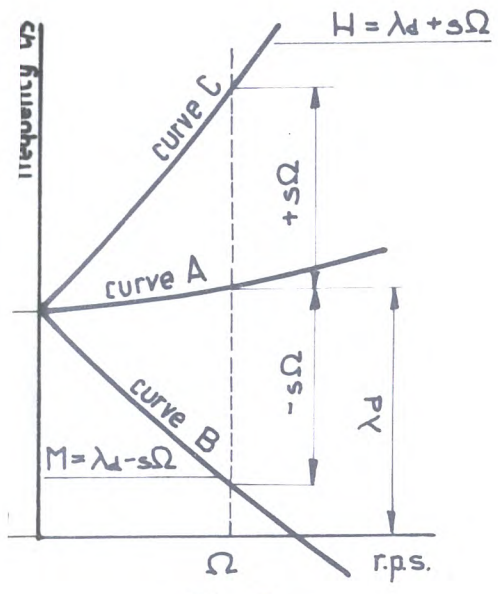


Fig. 7

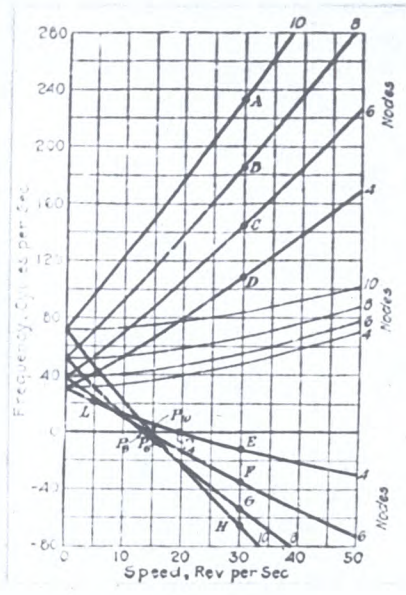


Fig. 8

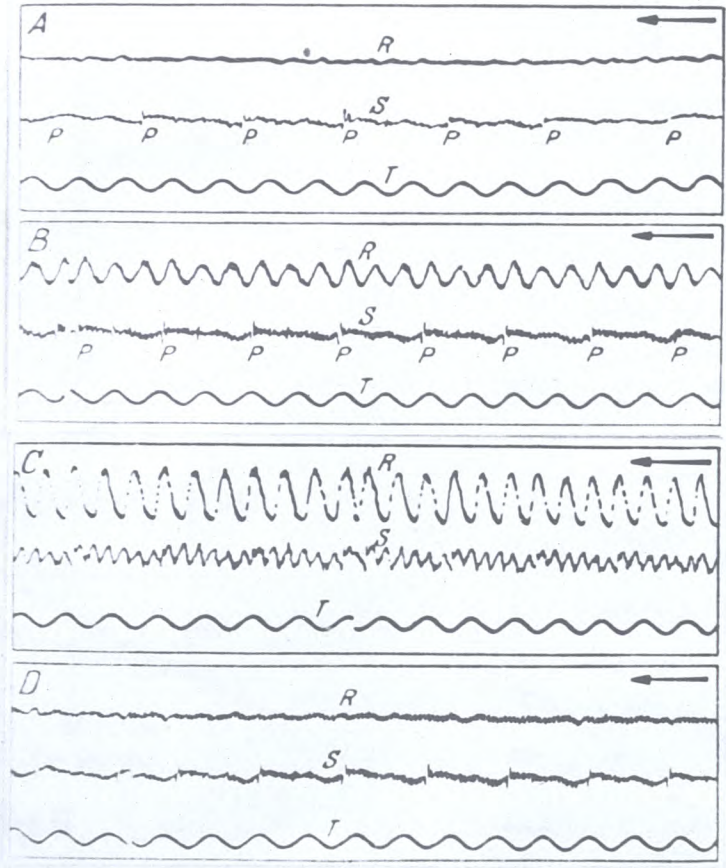


Fig. 9

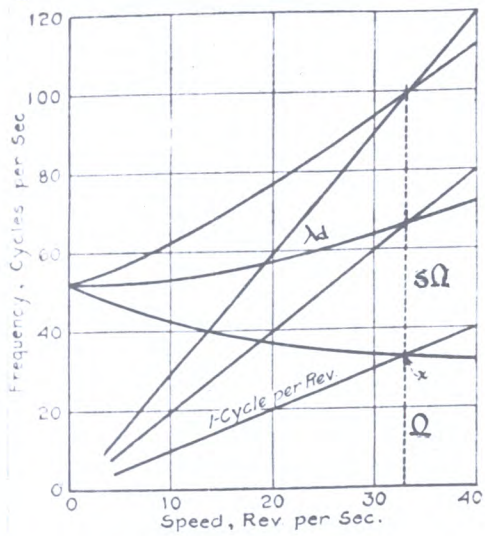


Fig. 10

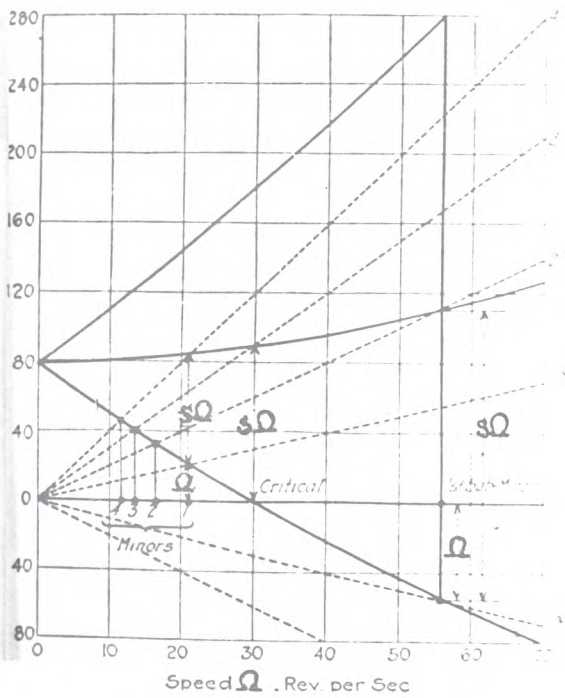


Fig. 11

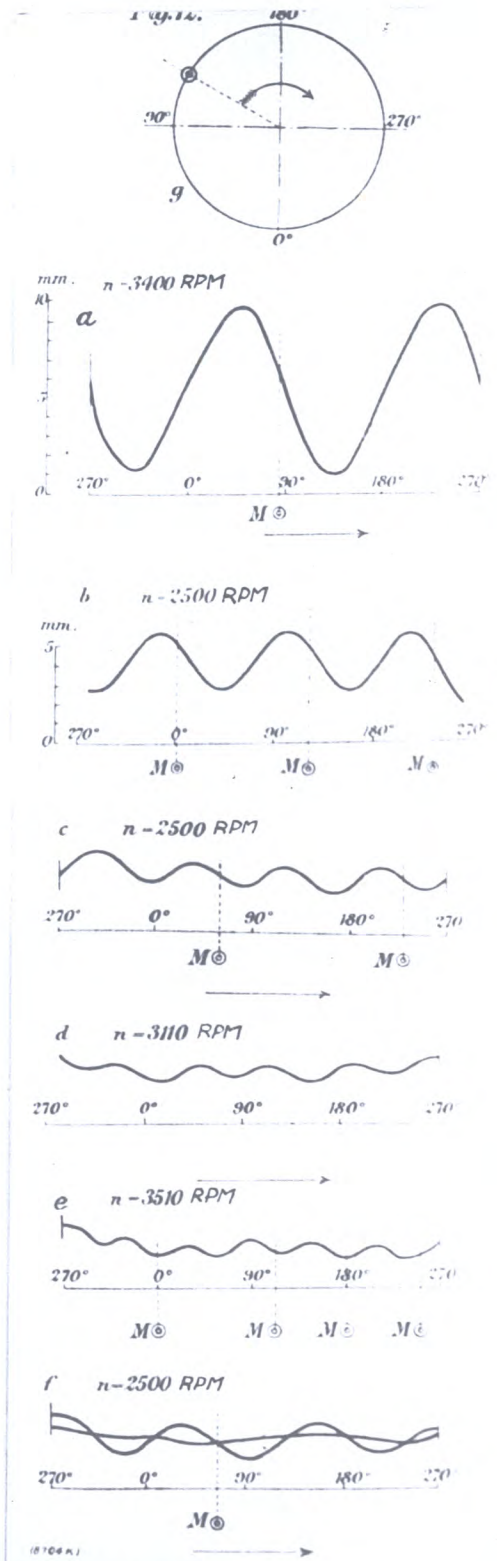


Fig. 12

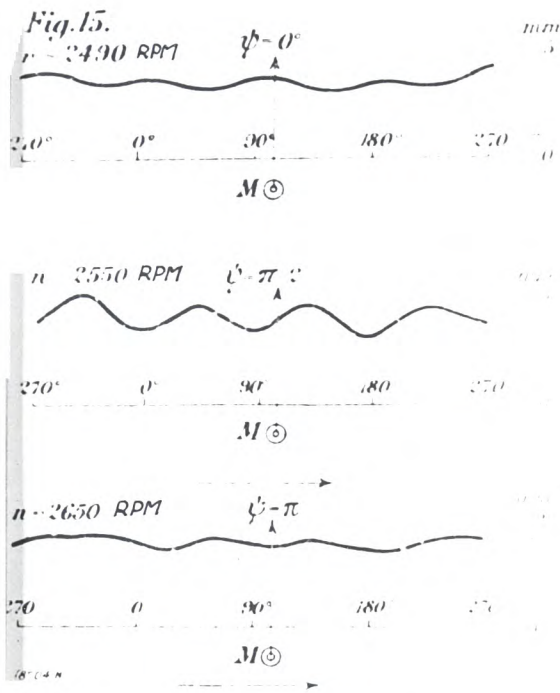


Fig. 13

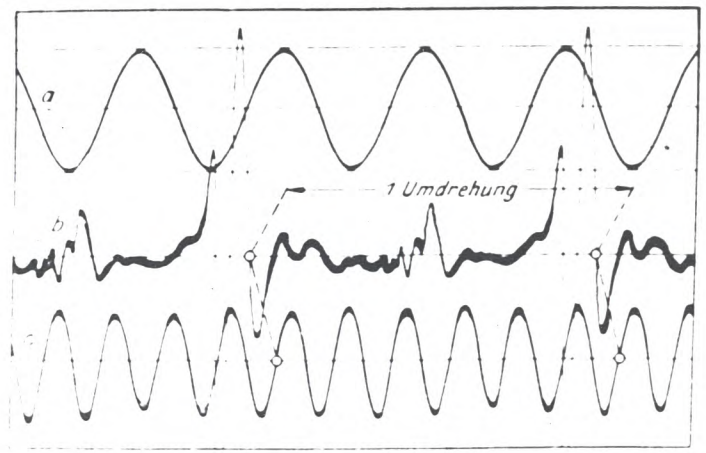


Fig. 14

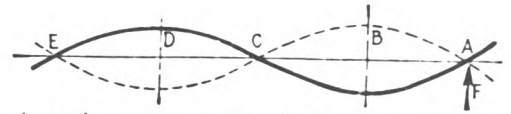


FIG. 202.—A steady, stationary force F can put work into a rotating and vibrating disk.

Fig. 15

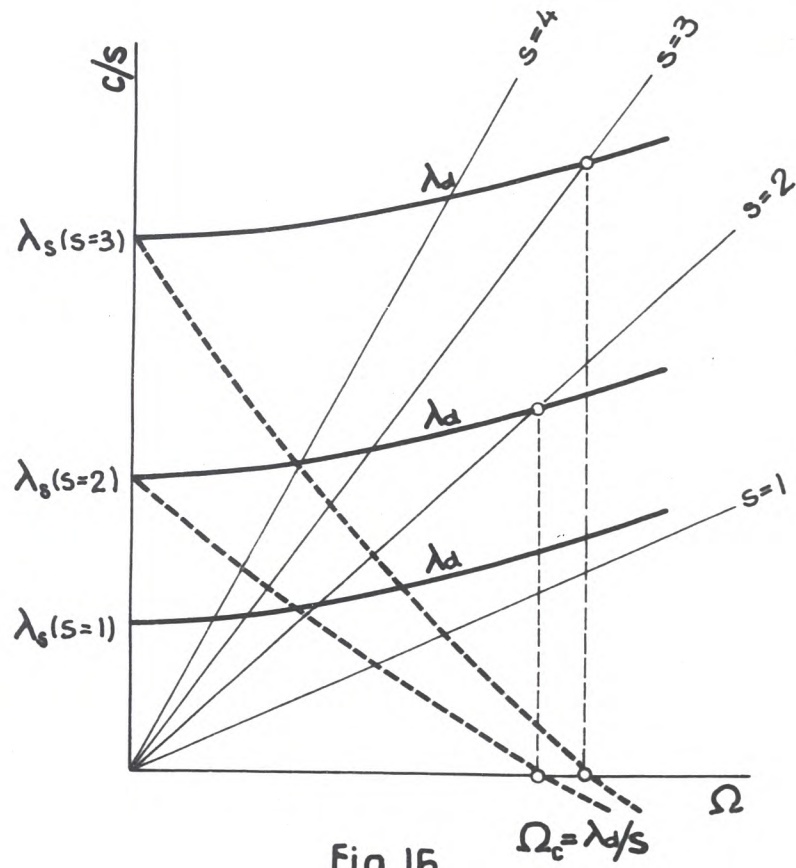


Fig. 16

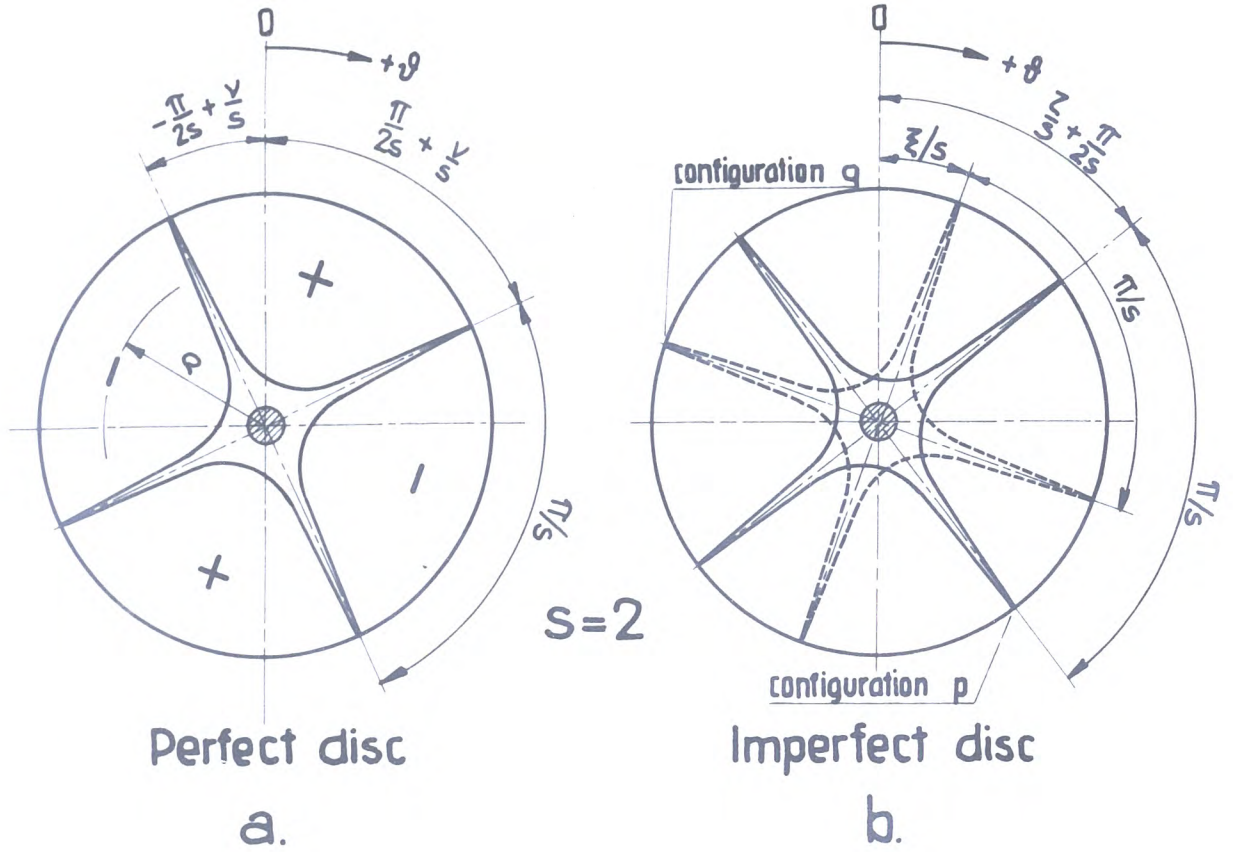


Fig. 17

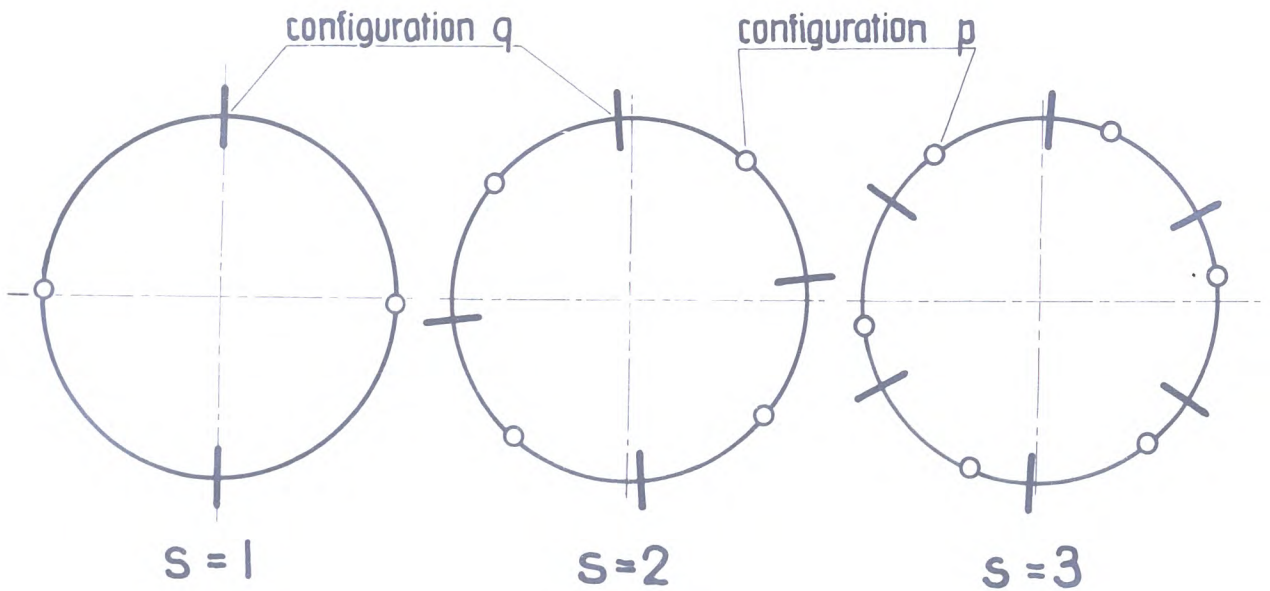


Fig. 18

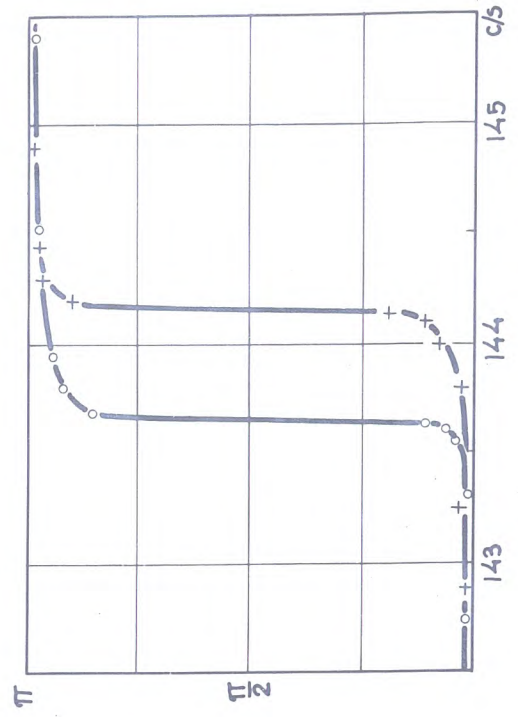
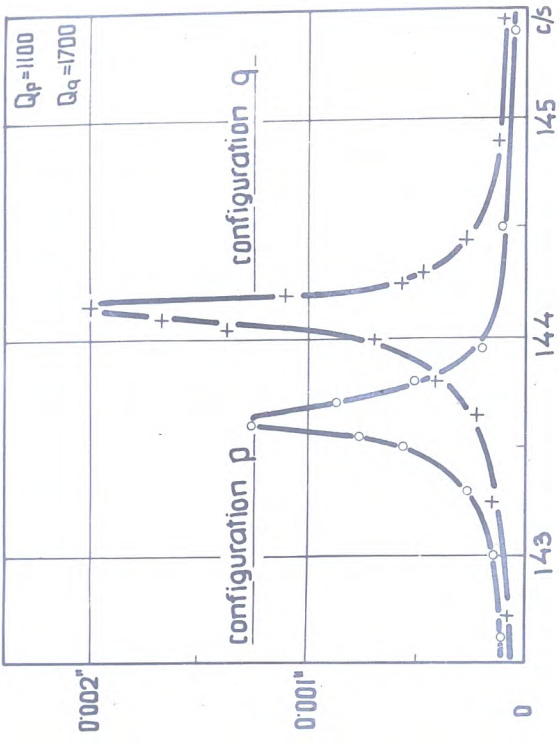


Fig. 20

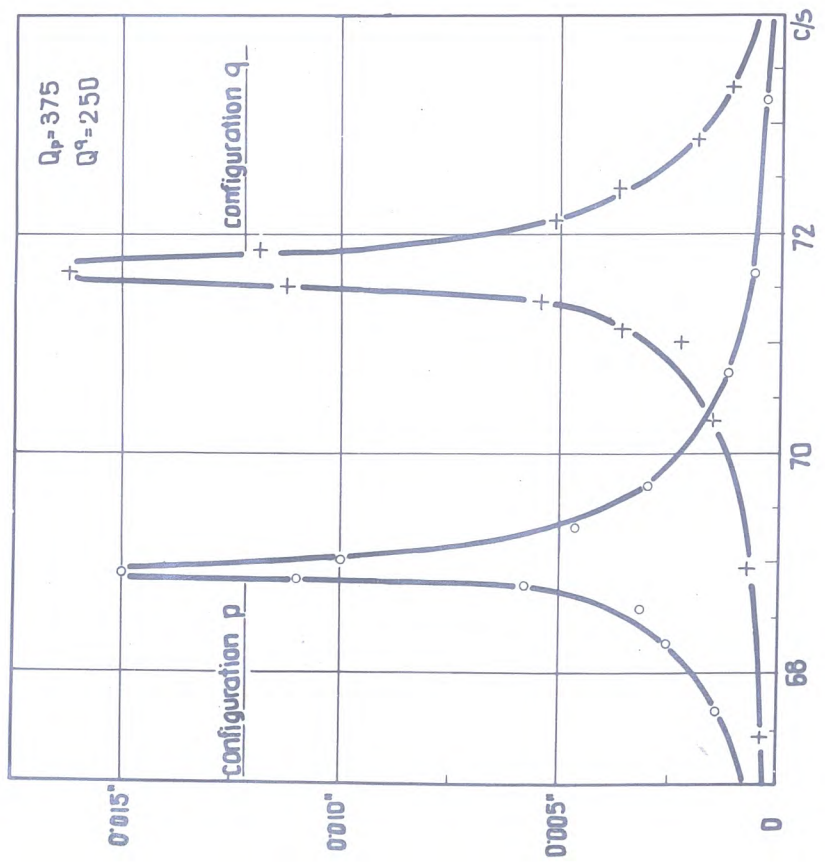


Fig. 19

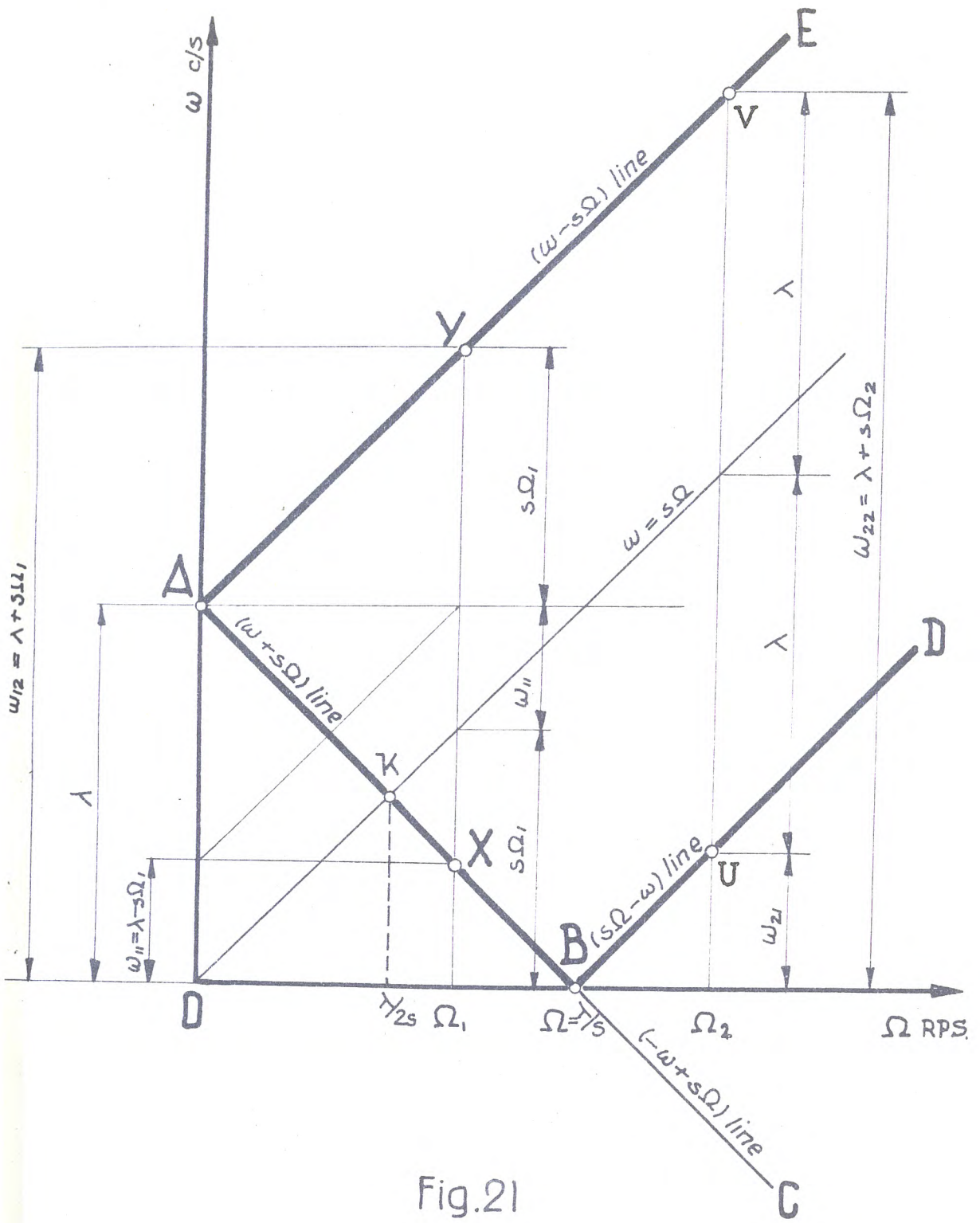


Fig.21

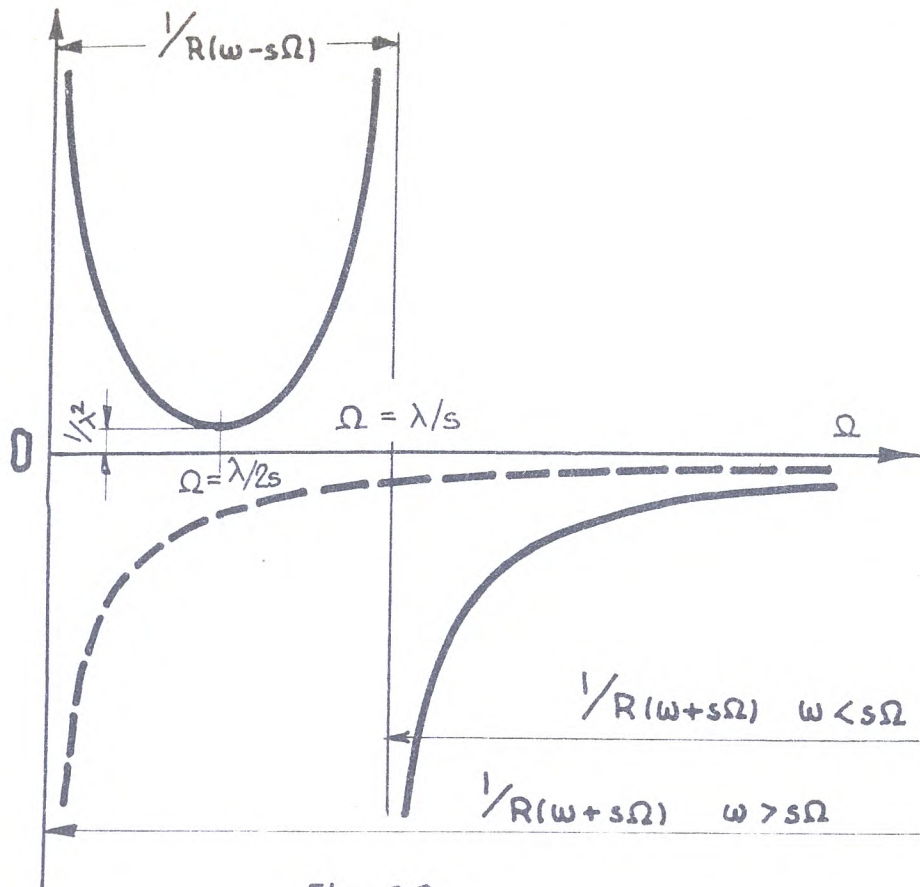


Fig. 22

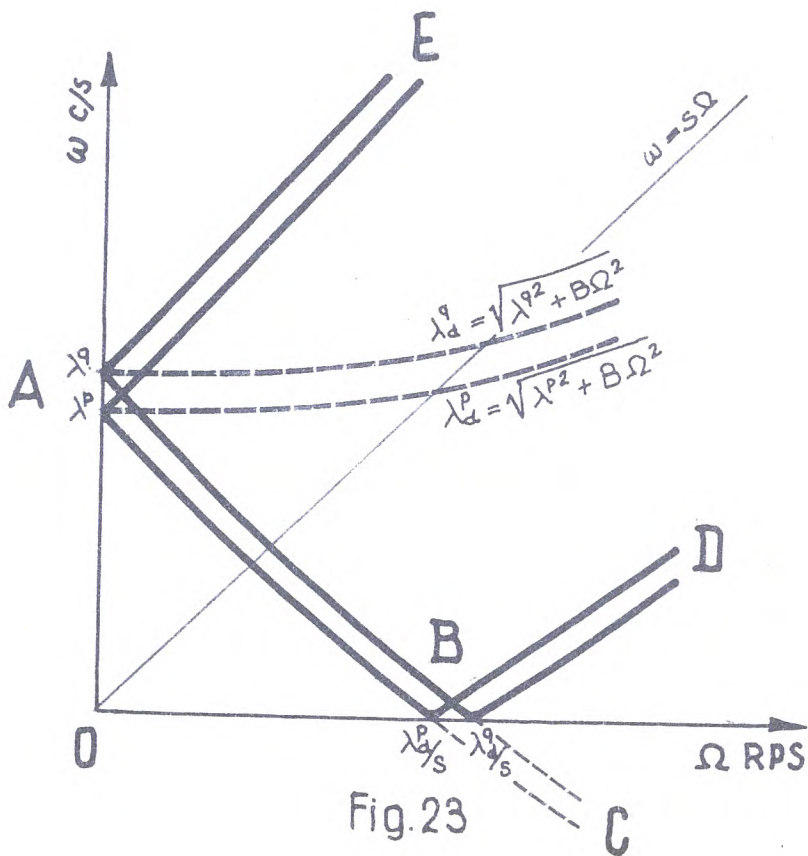
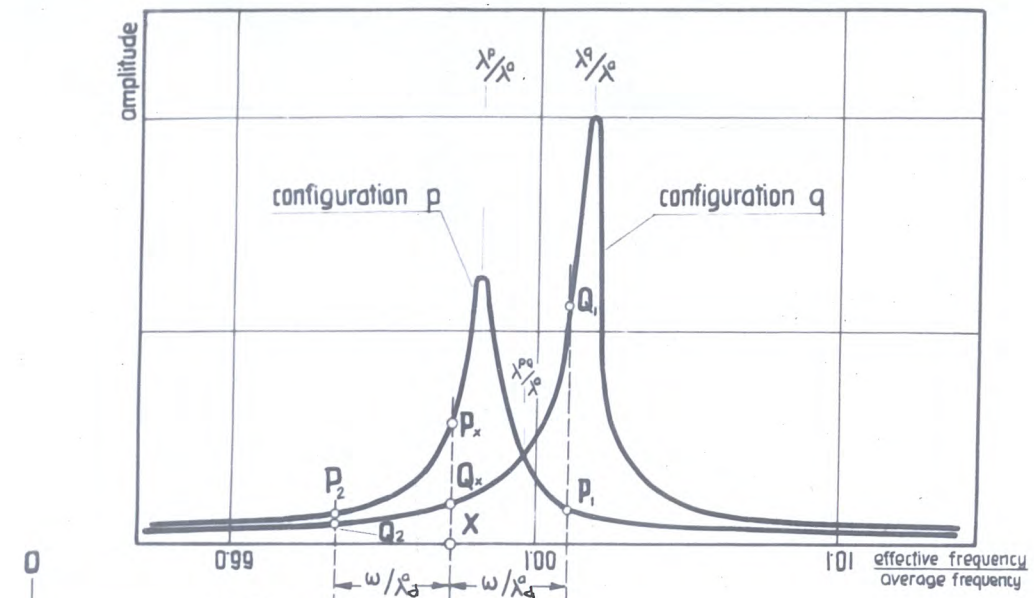
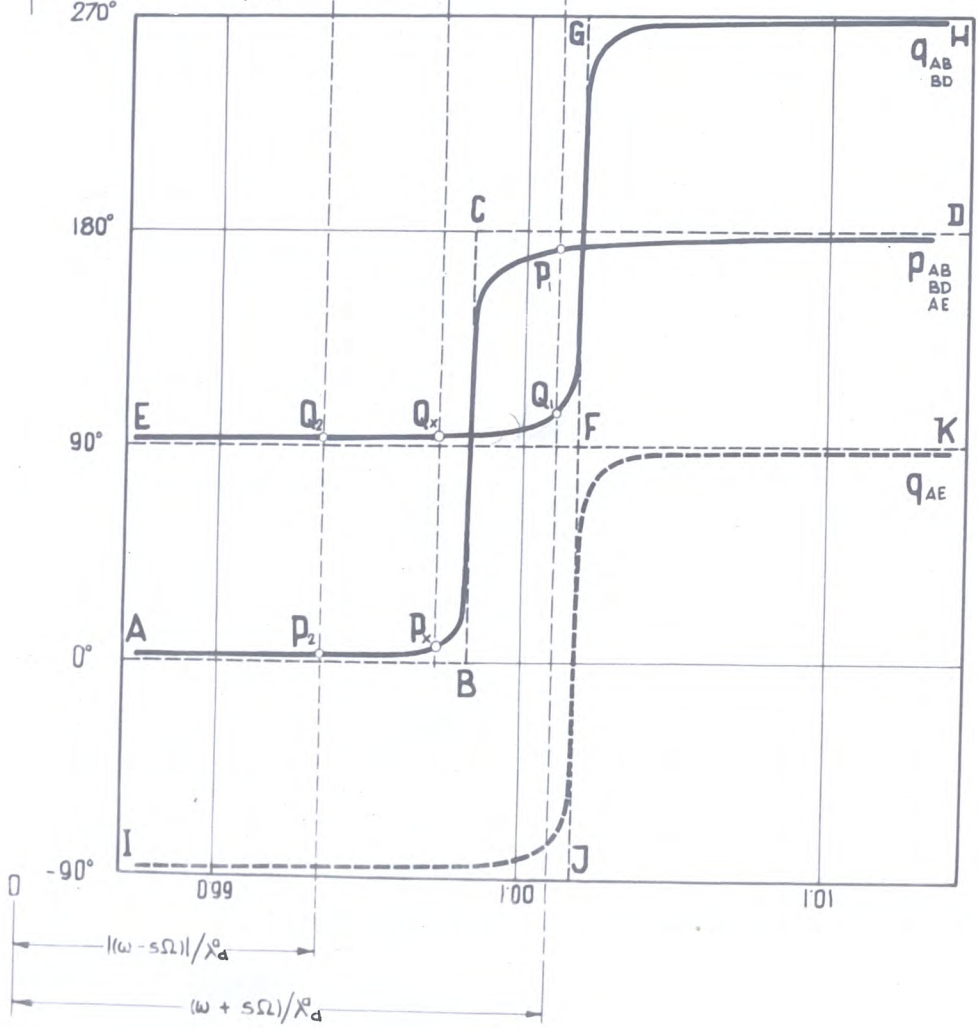


Fig. 23



a.



b.

Fig.24

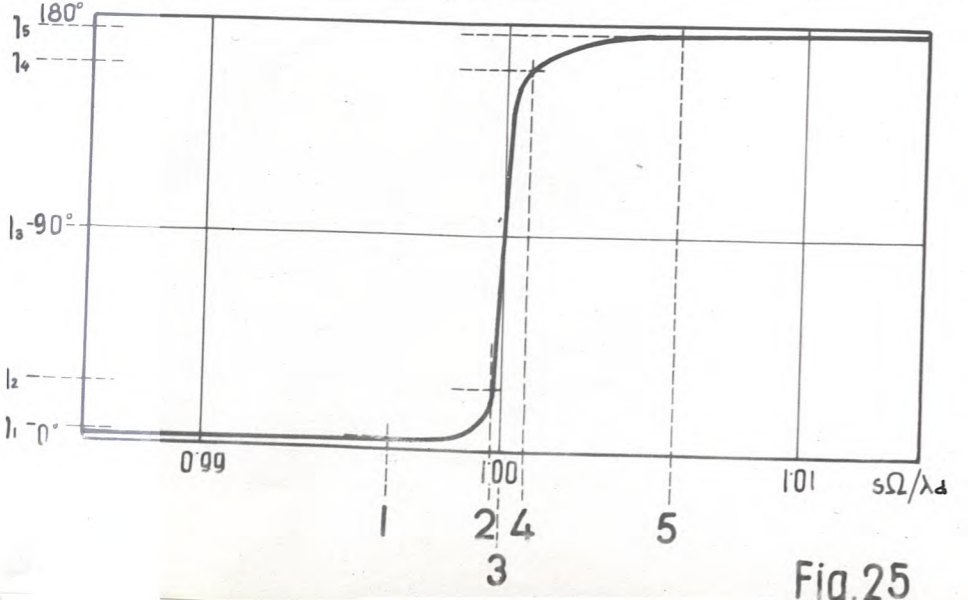
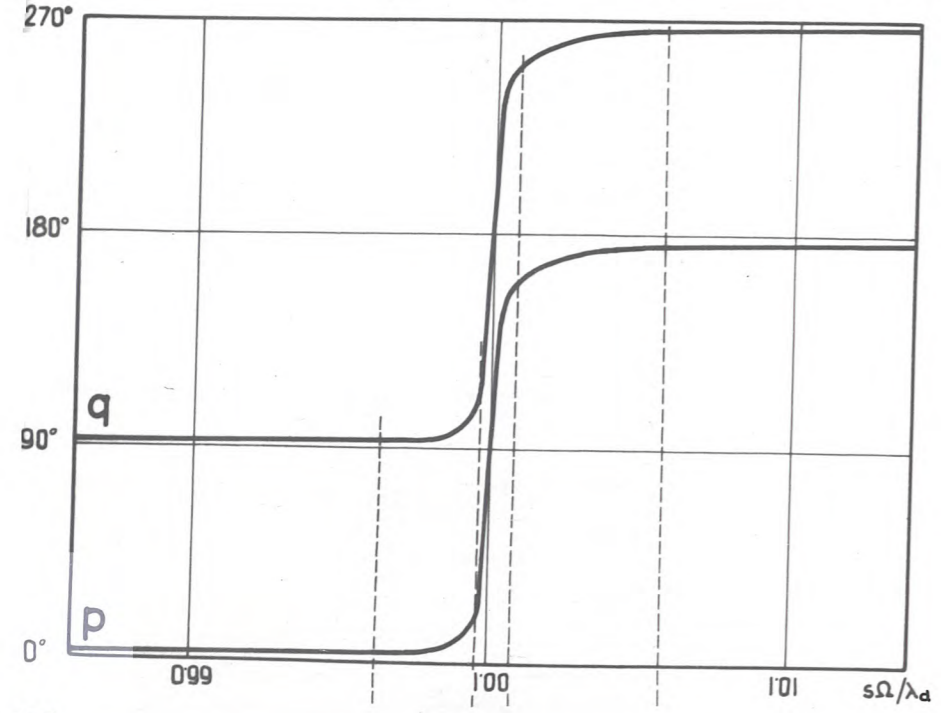
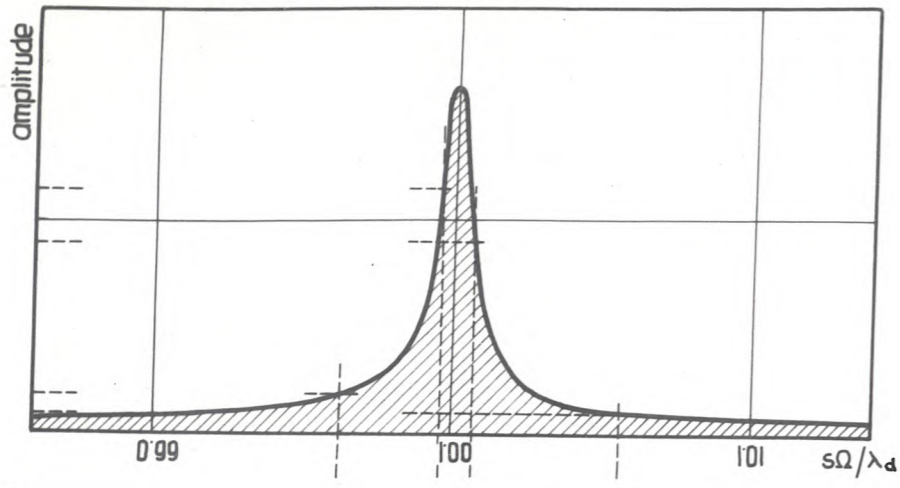
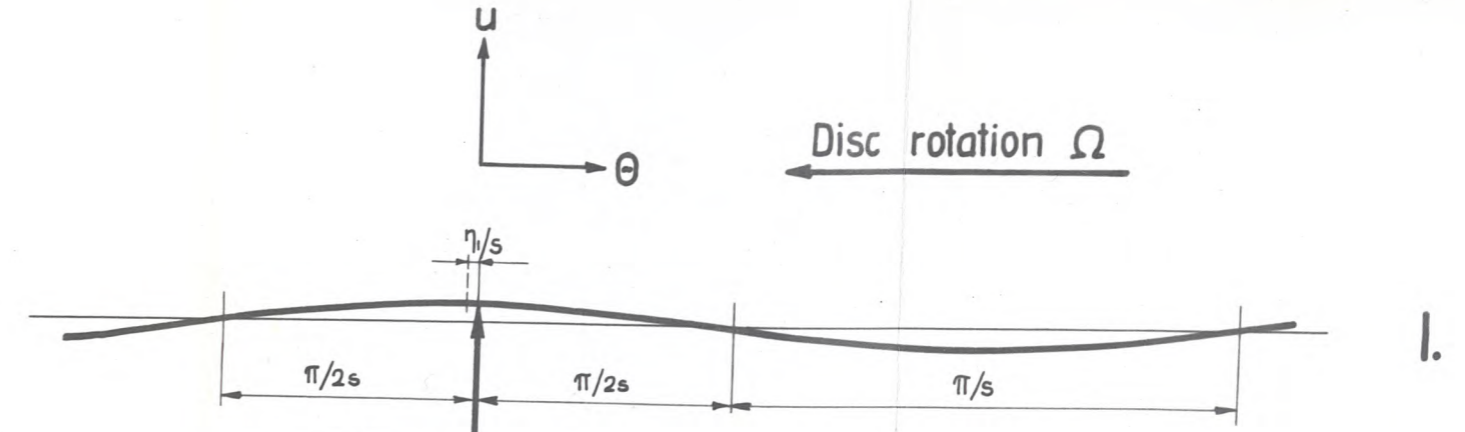
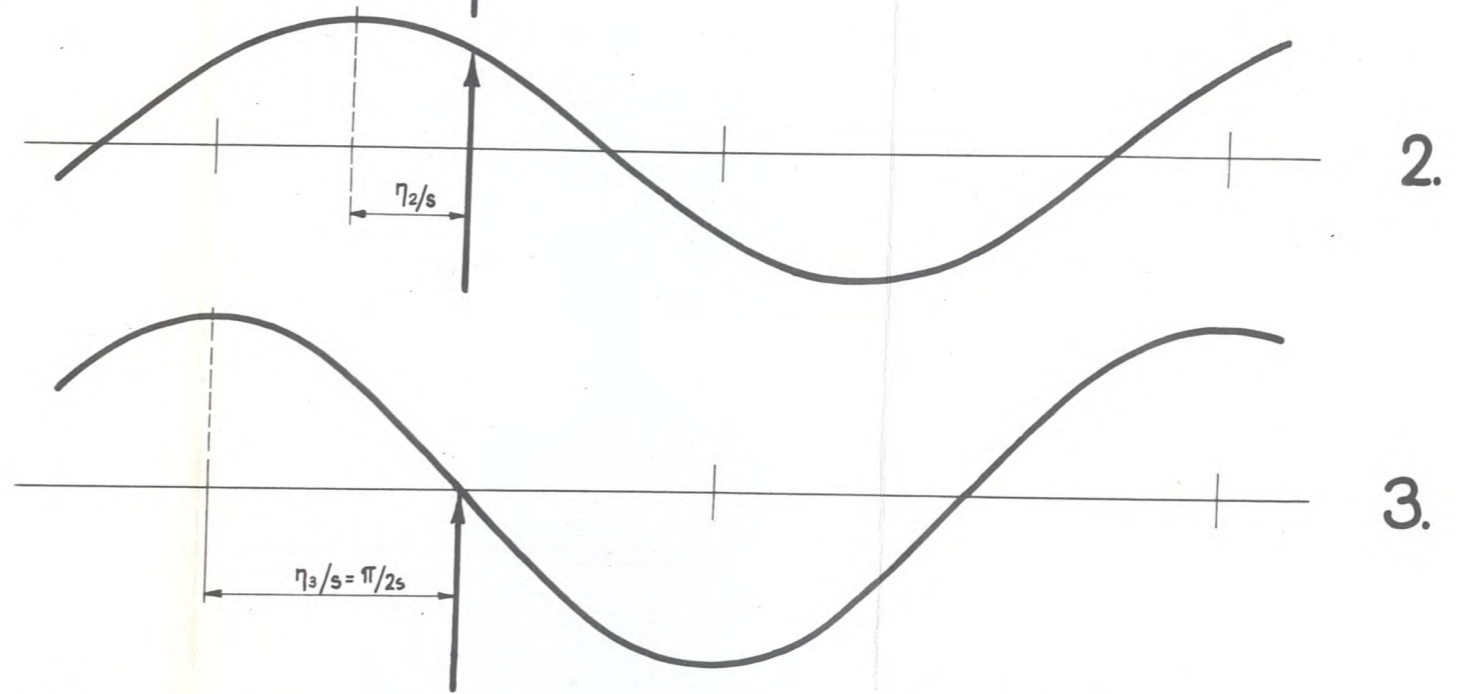


Fig.25

a.



b.



c.

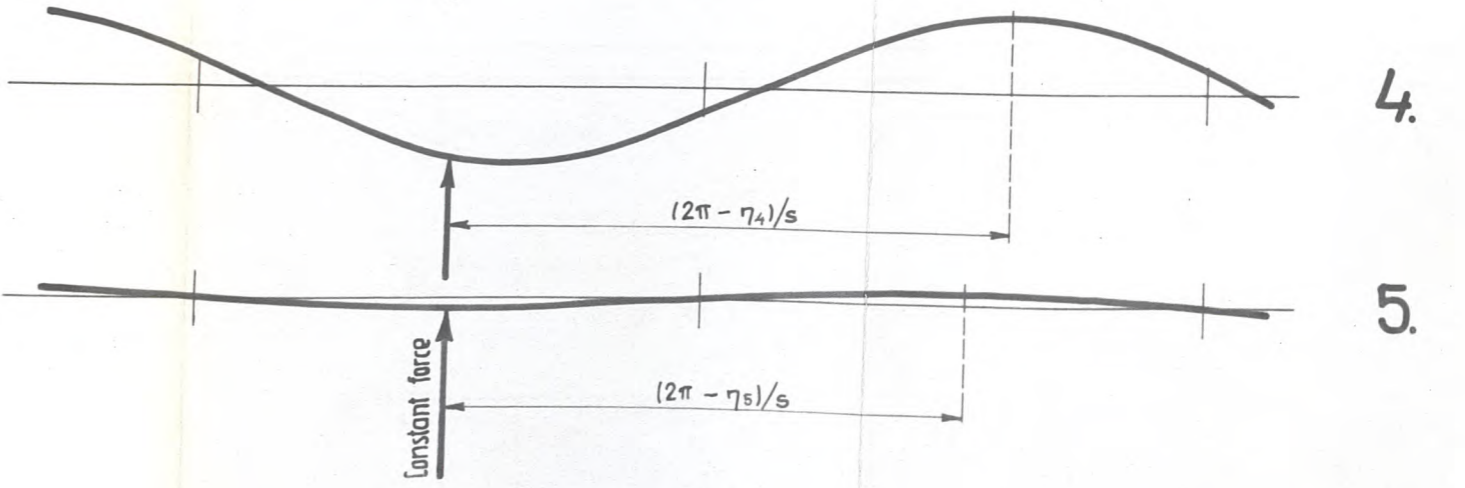


Fig.26

Force rotation \rightarrow

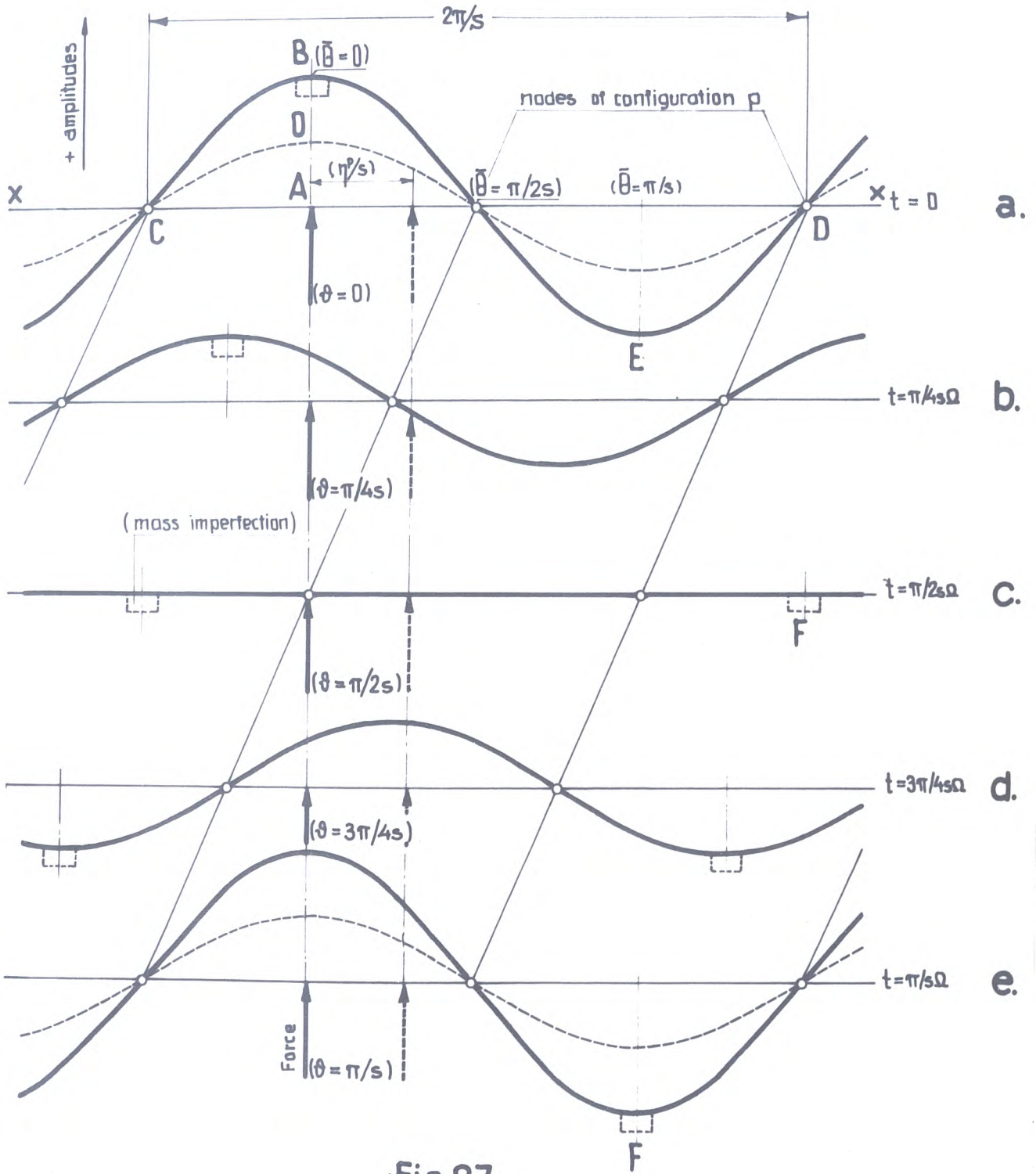
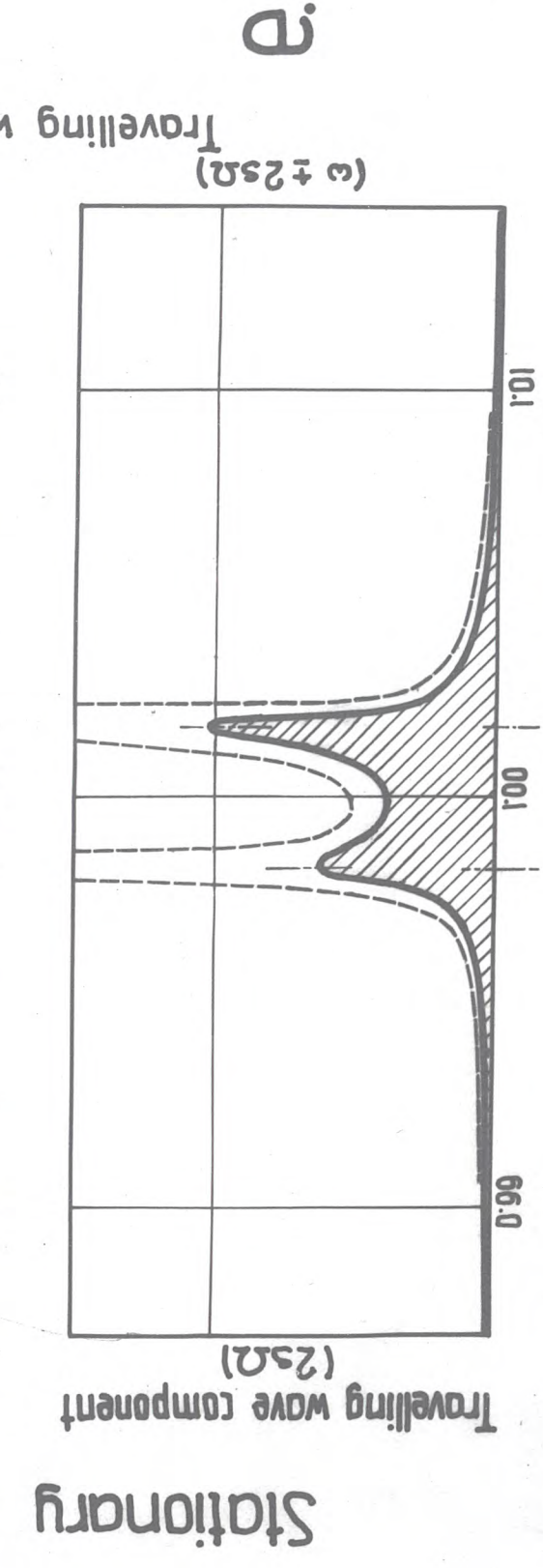
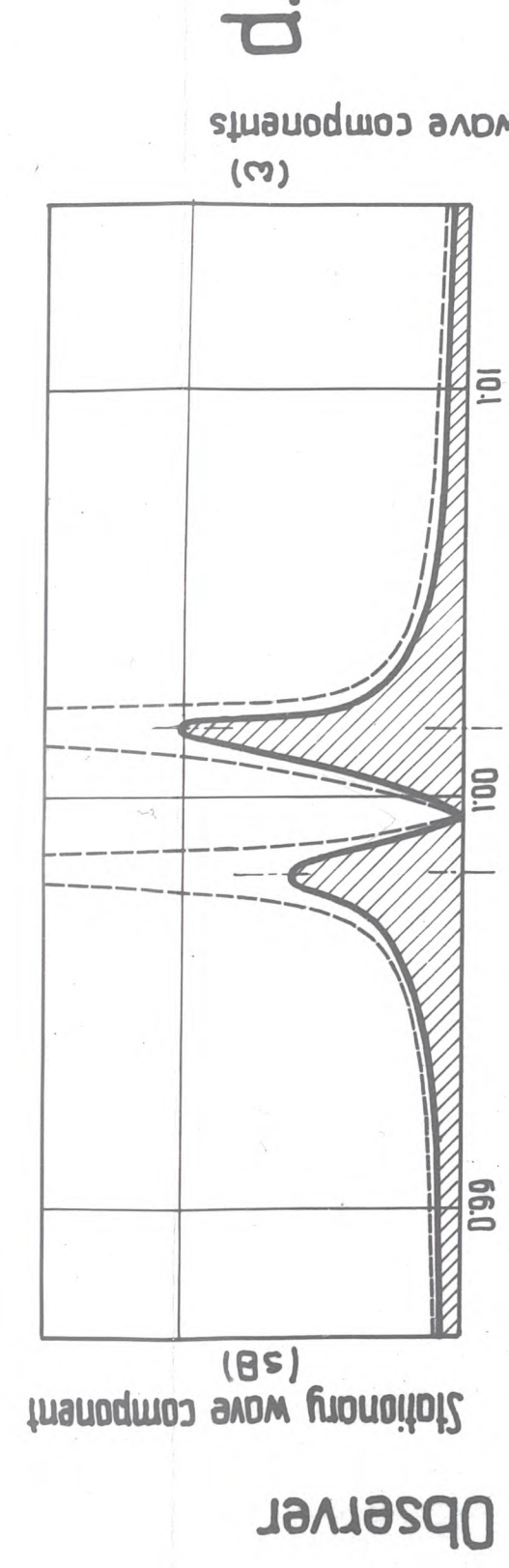
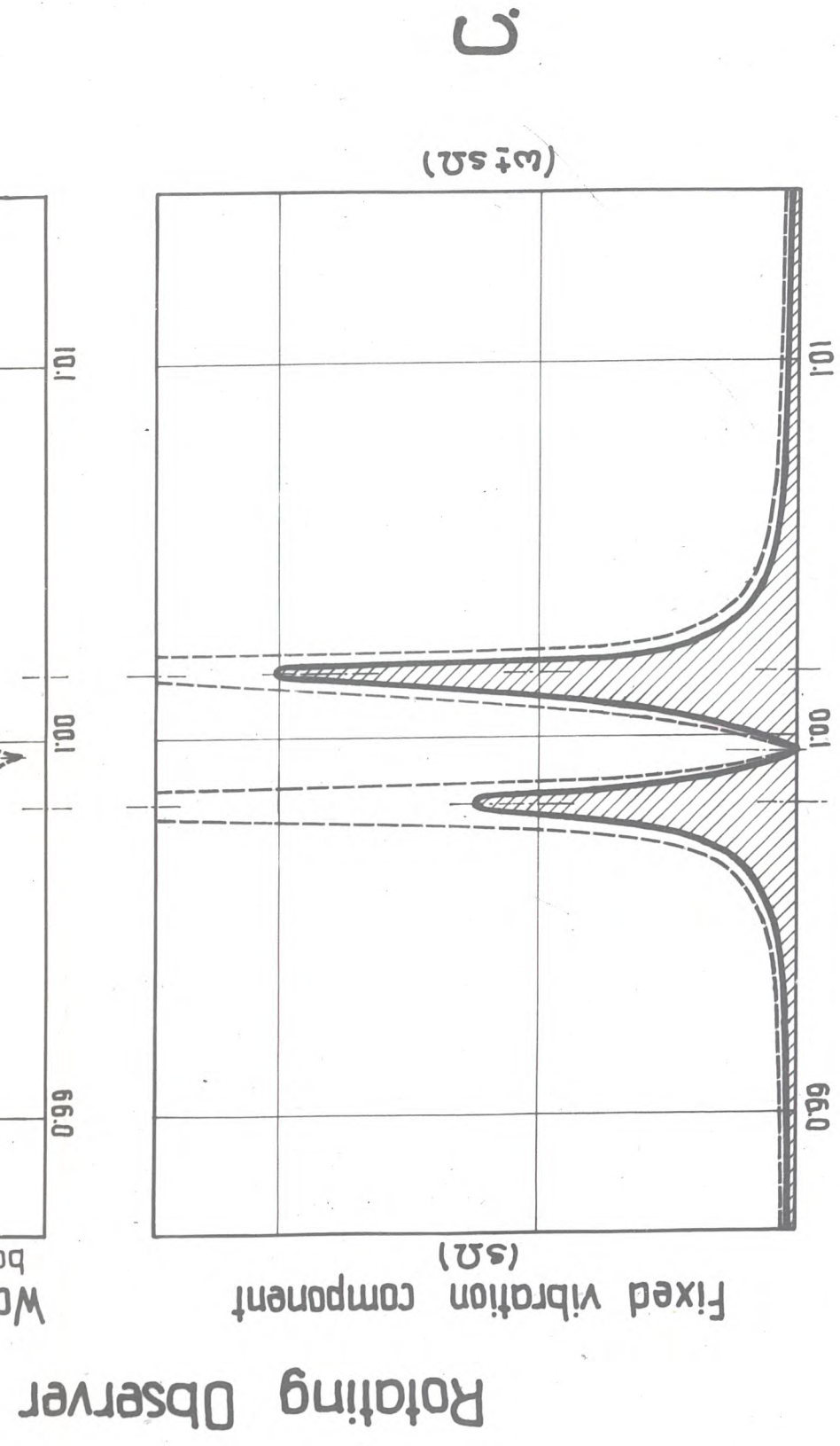
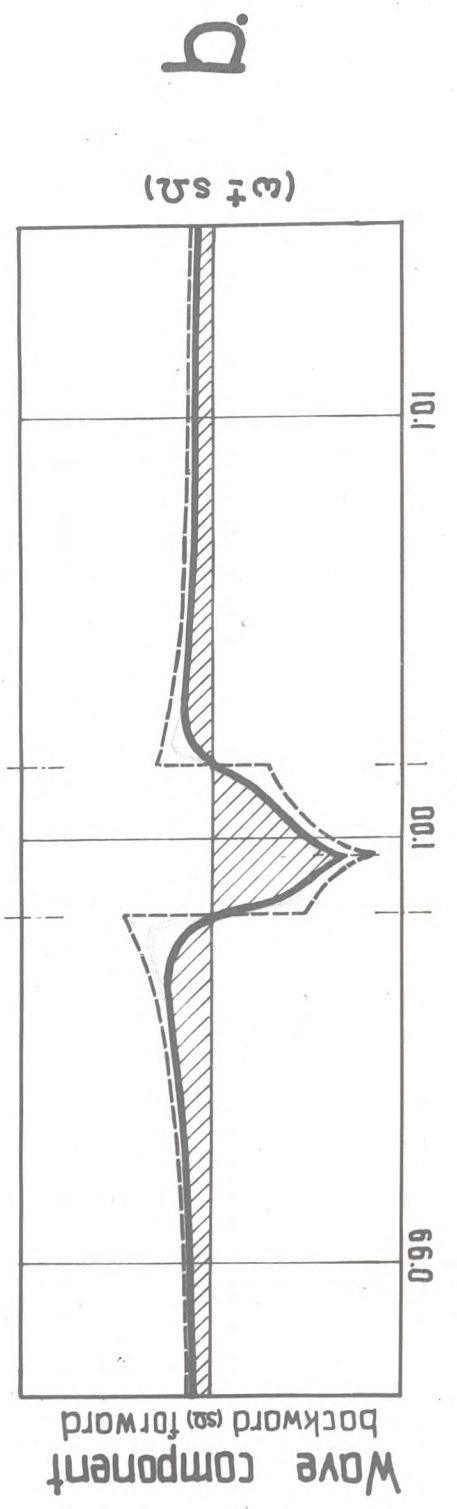
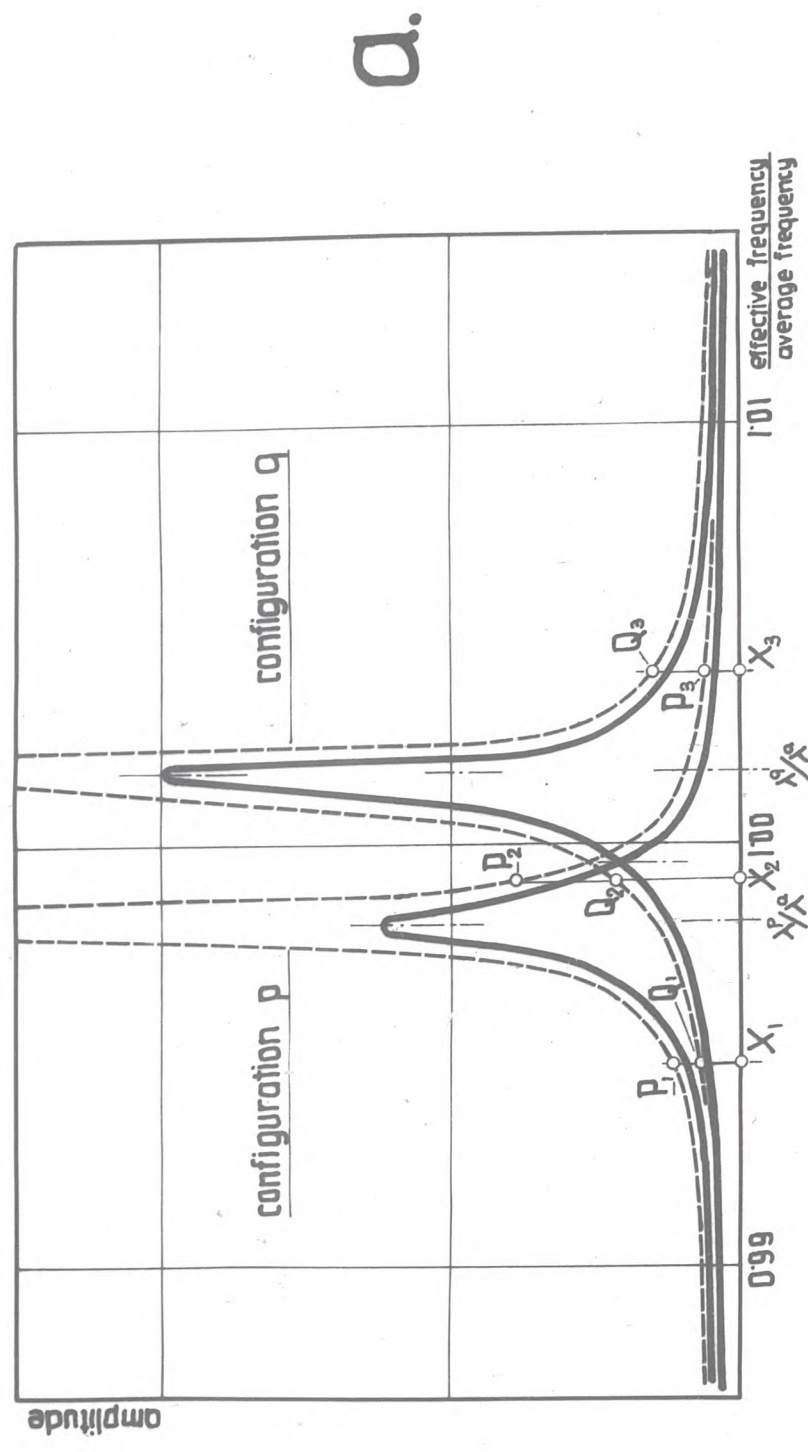


Fig.27



Stationary wave $\omega=0$

Forced vibration $\omega \neq 0$

Fig.28

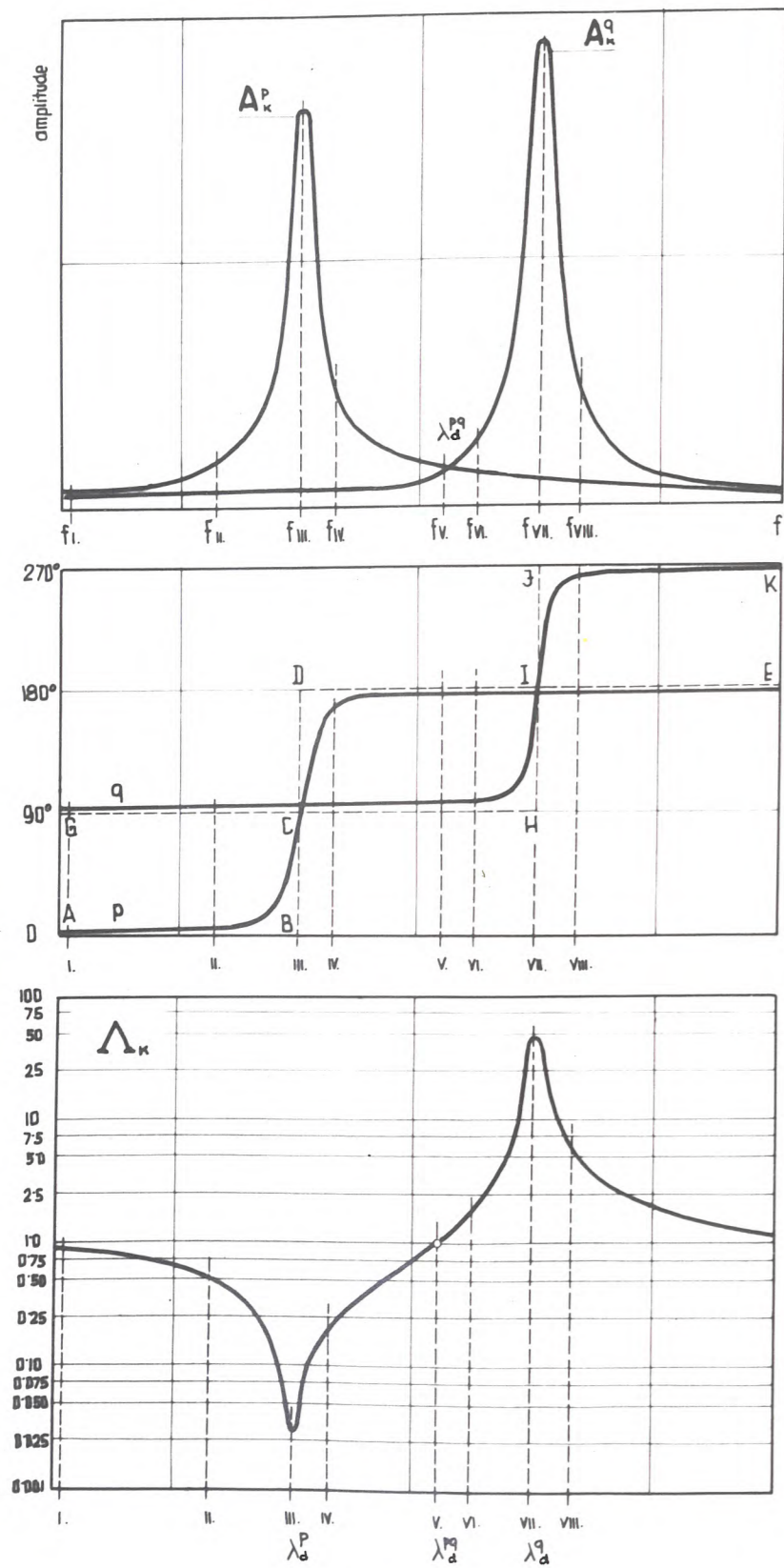


Fig.29

a.

b.

c.

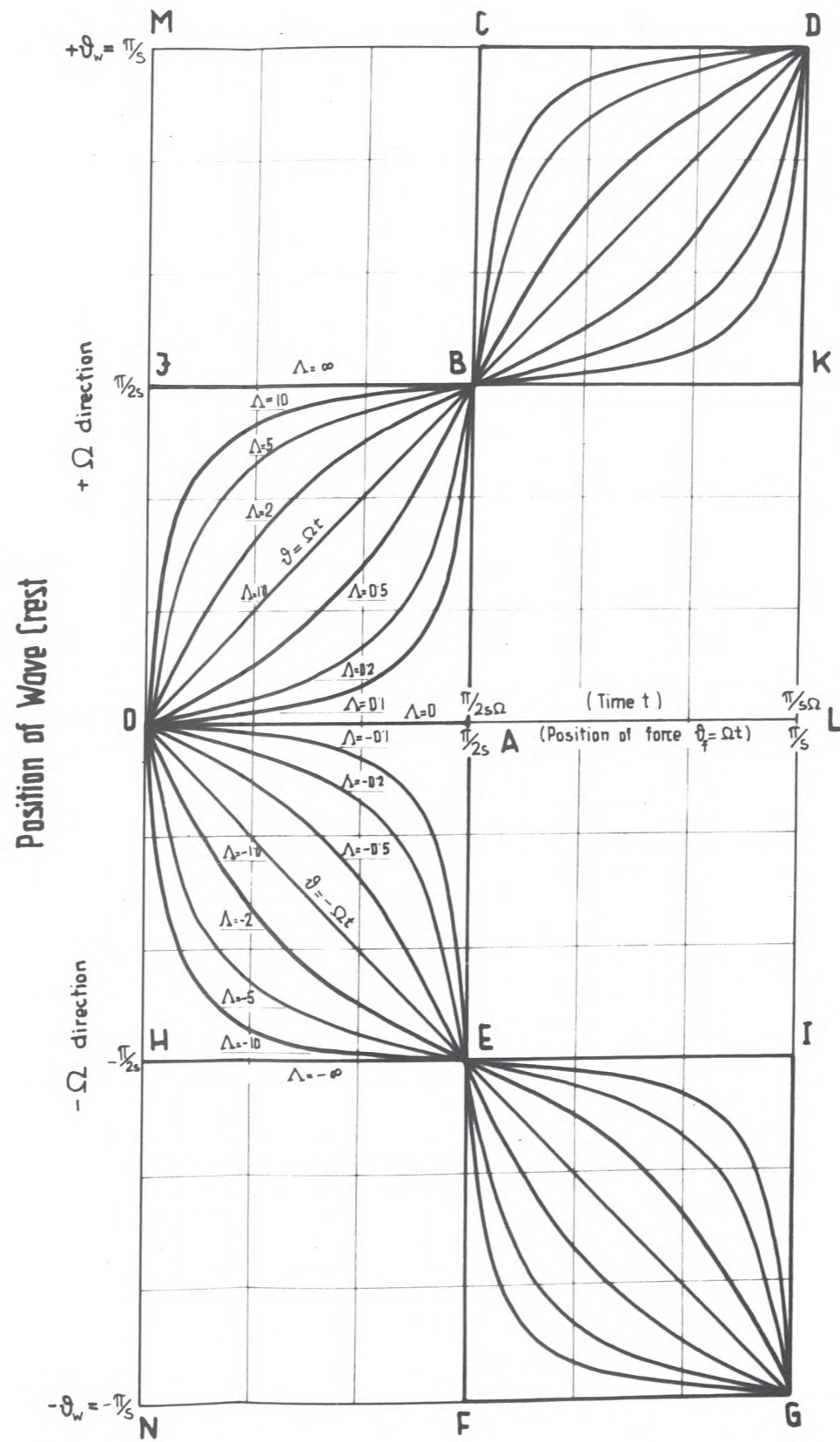
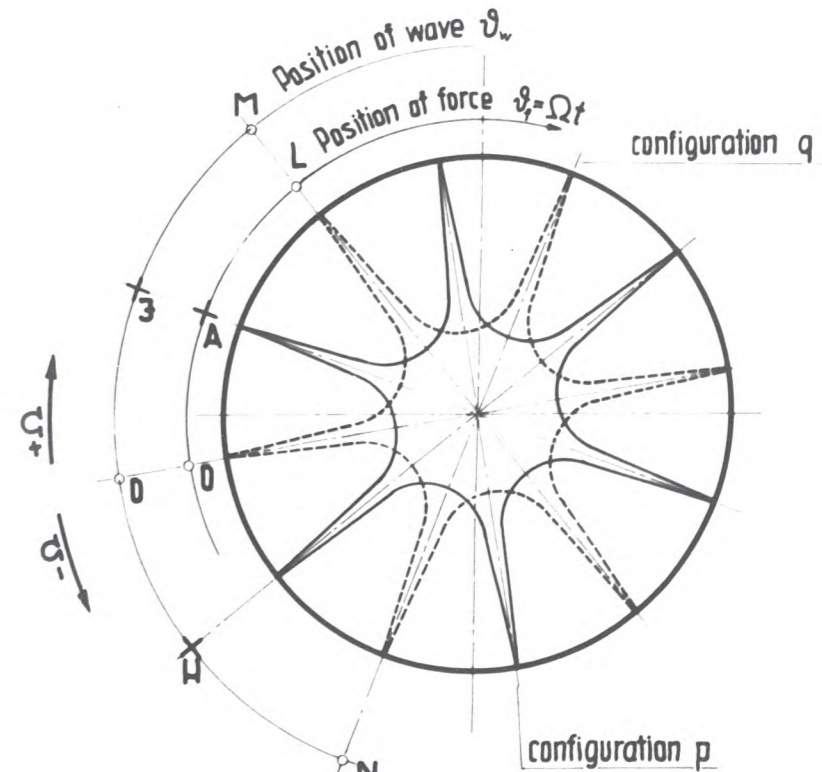


Fig.30



$s=3$

Fig.31

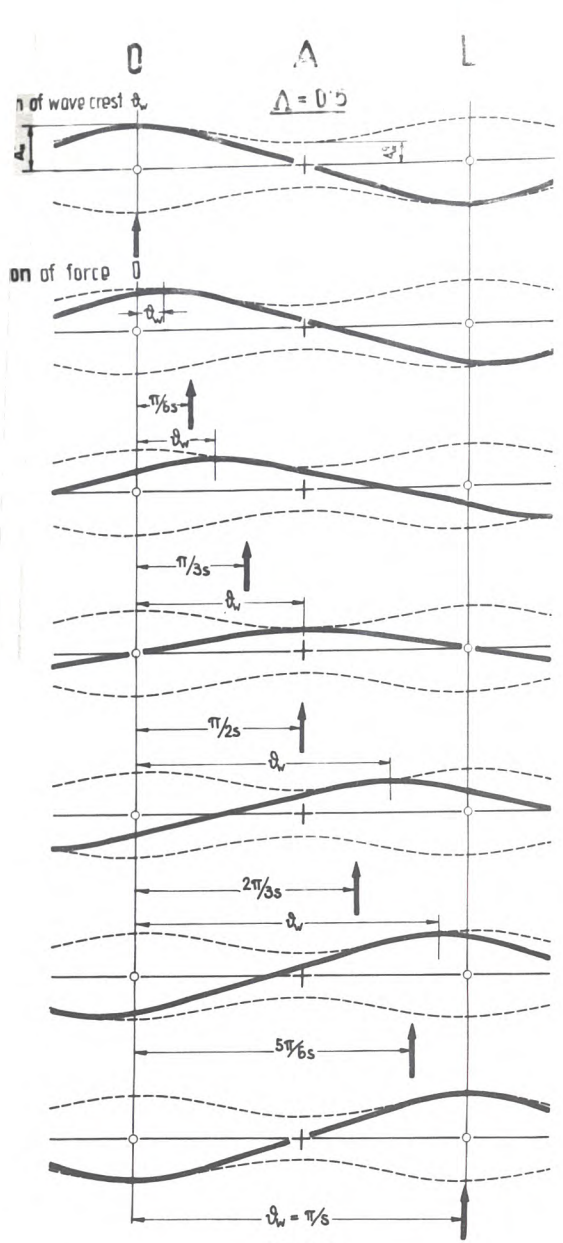


Fig.34a

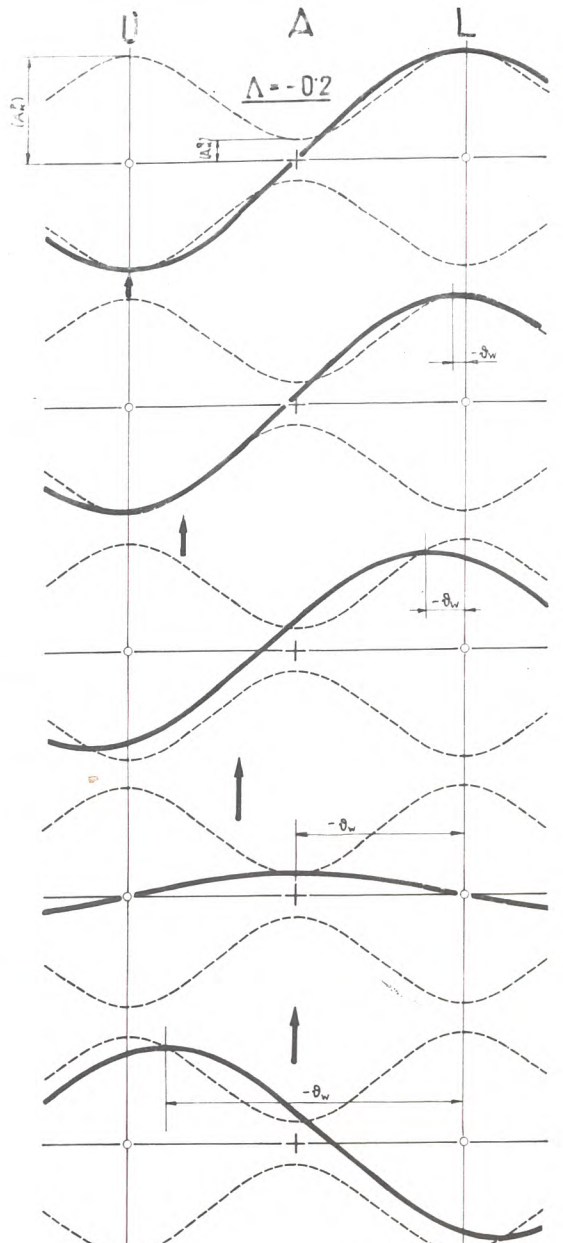


Fig.34c

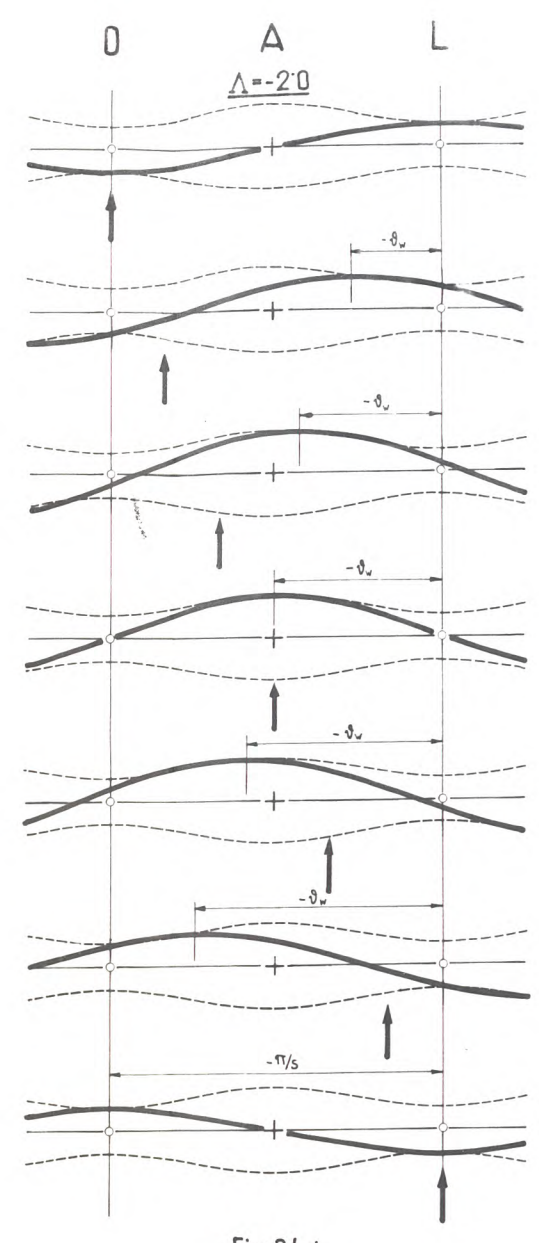


Fig.34d

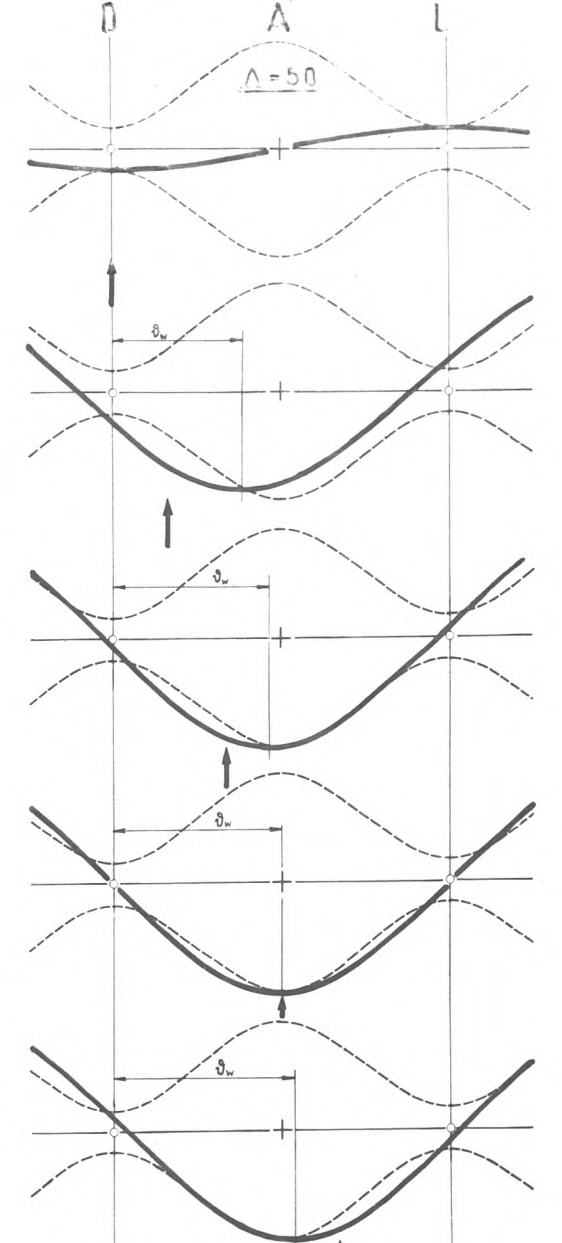


Fig.34f

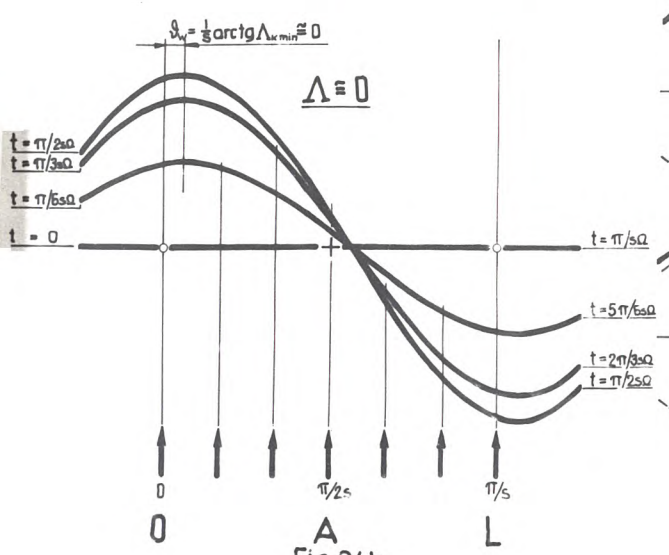


Fig.34b

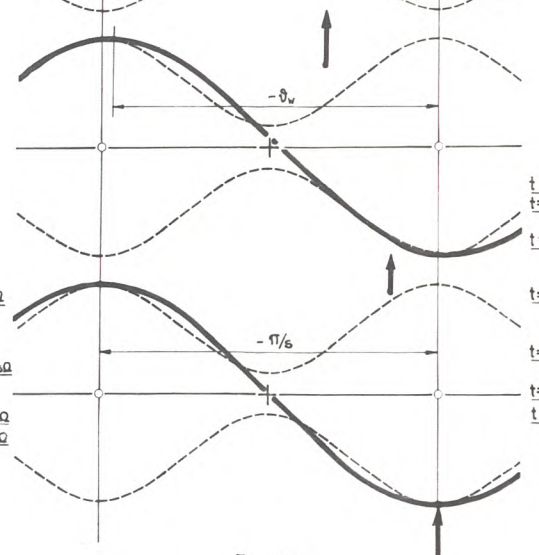


Fig.34e

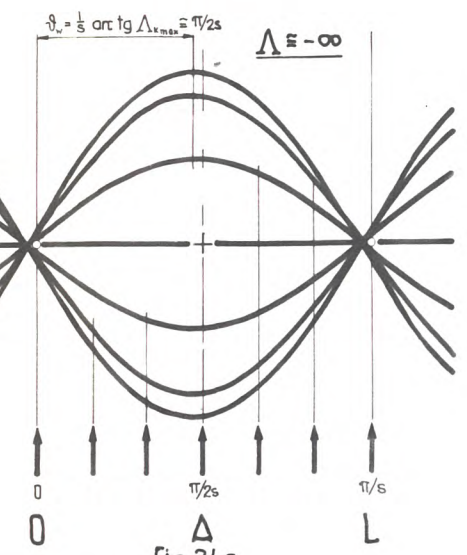


Fig.34e

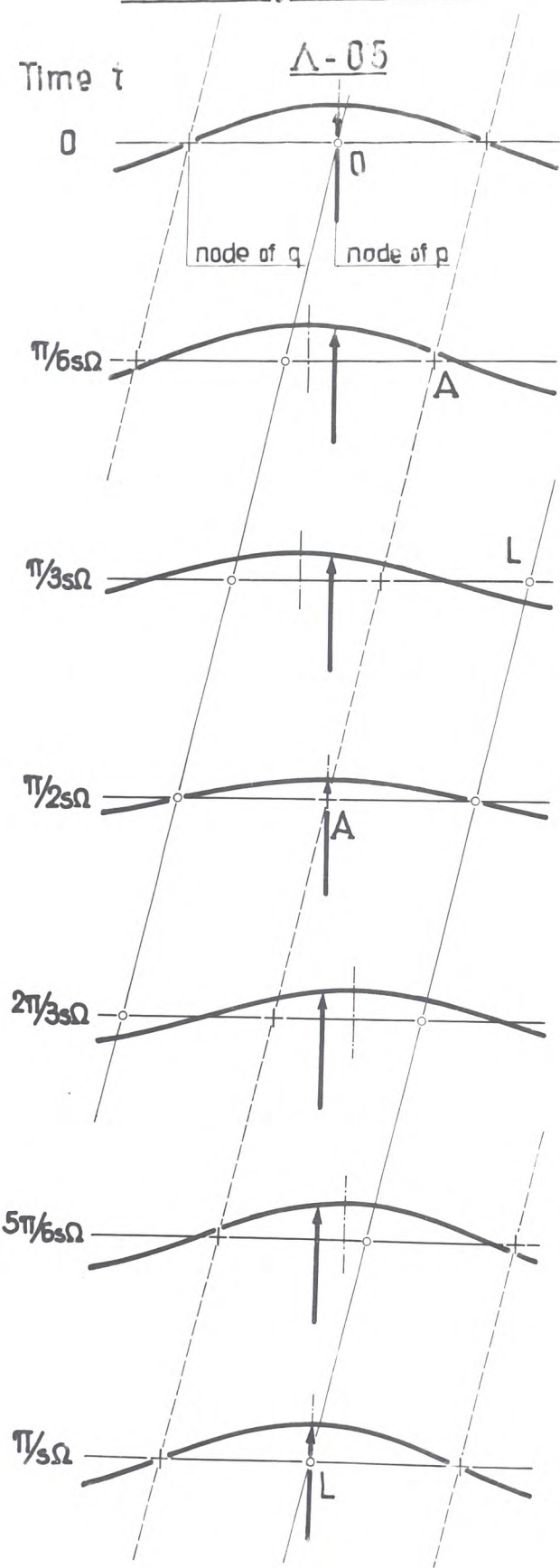


Fig.35a
(Oscillating stationary wave)

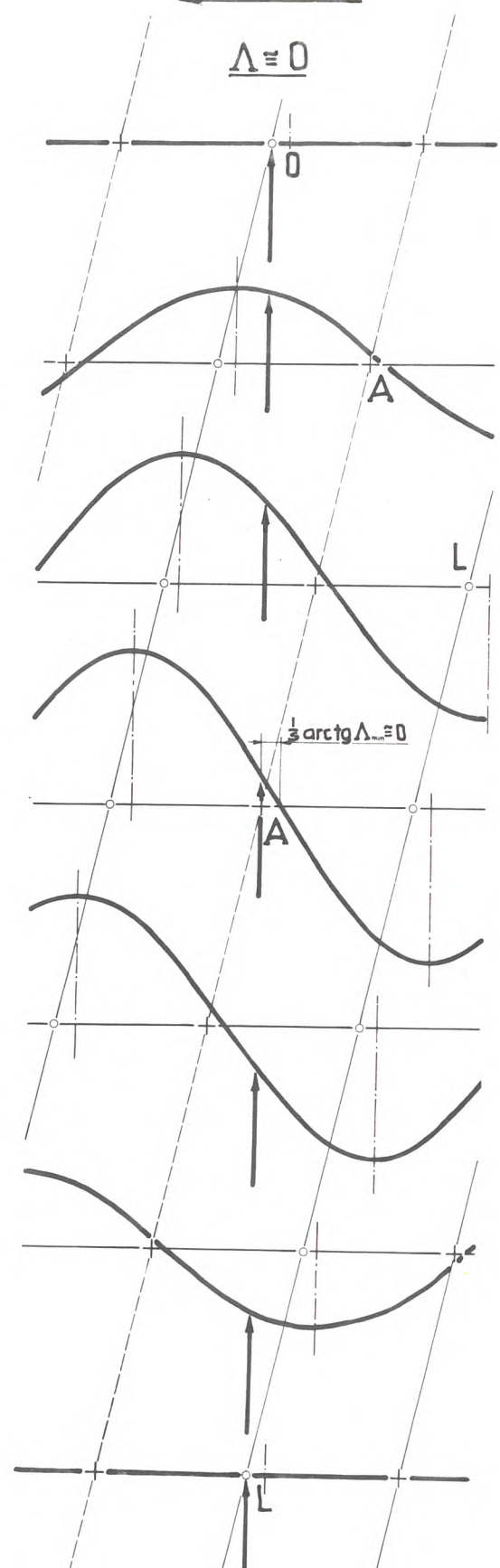


Fig.35b
(Fixed vibration p)

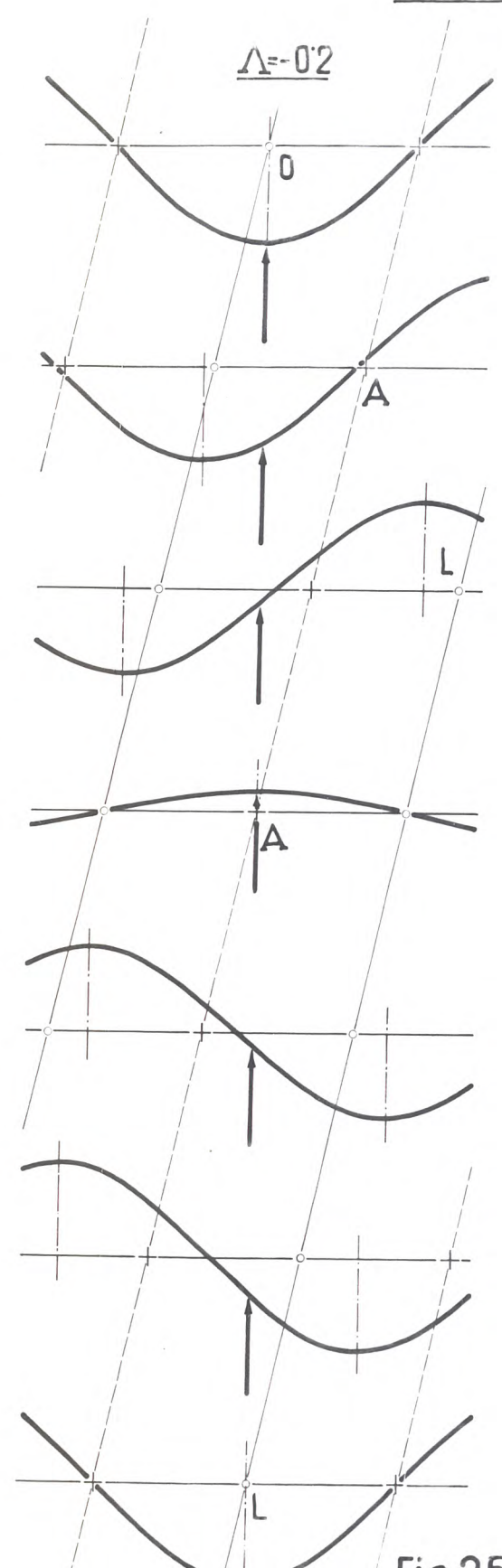


Fig.35c
(Beating forward wave)

Stationary Observer.

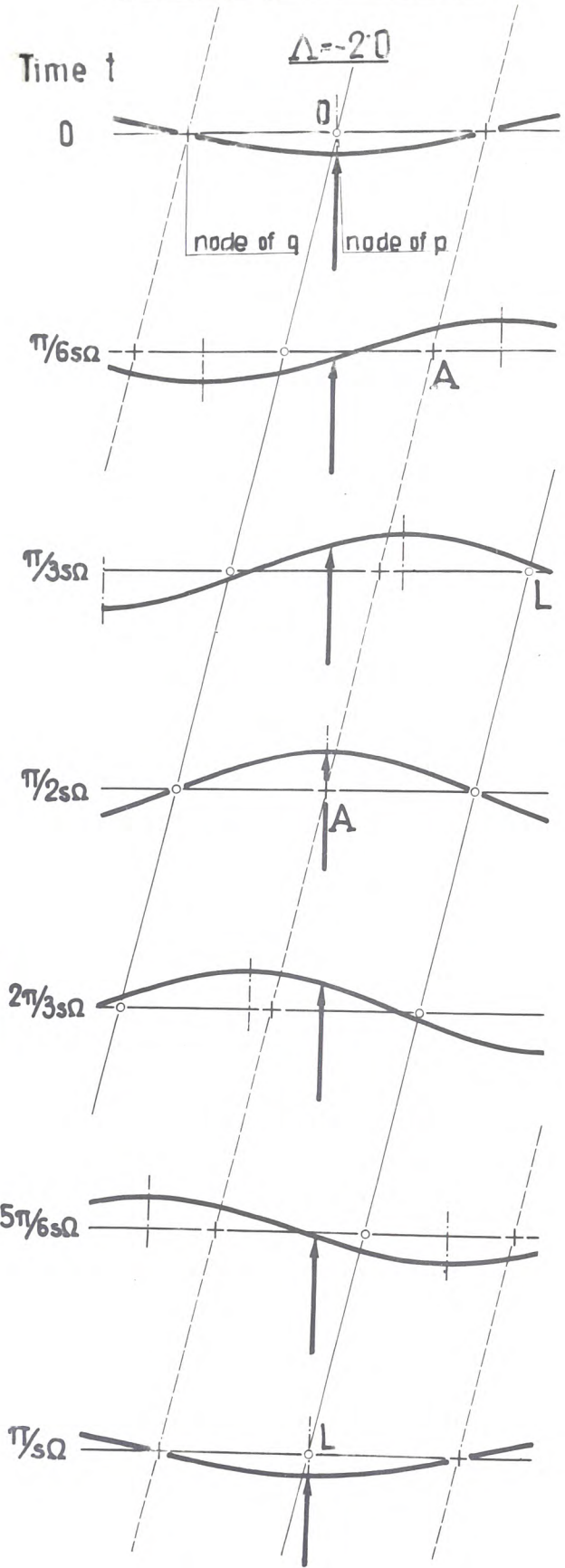


Fig. 35 d
(Beating forward wave)

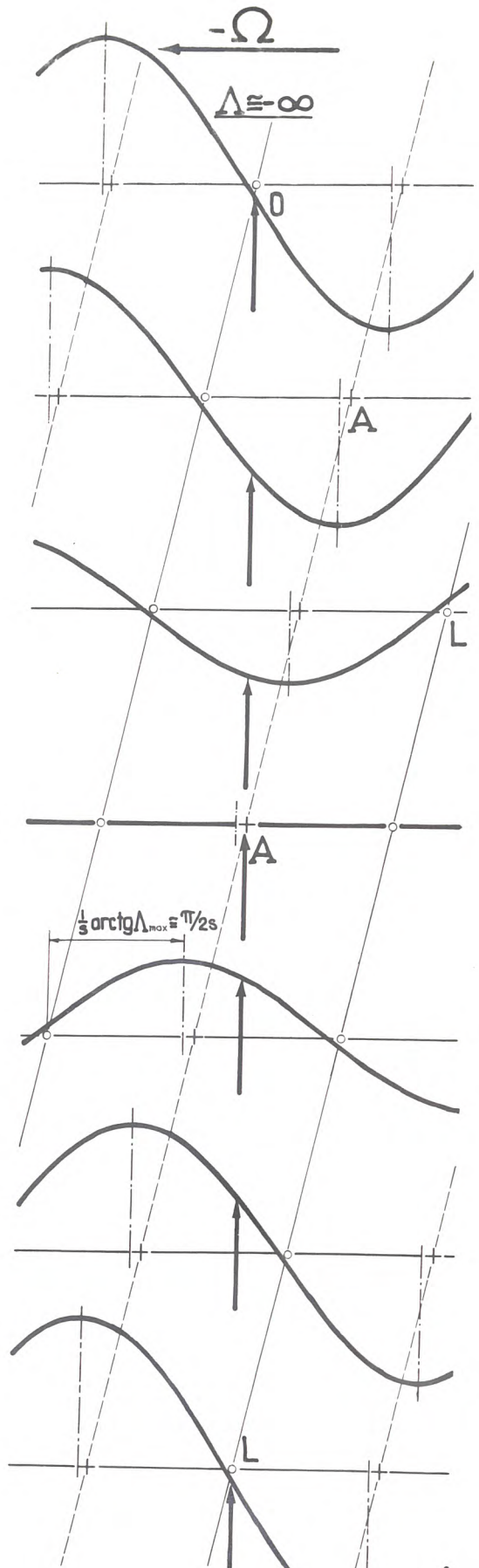


Fig. 35 e
(Fixed vibration q)

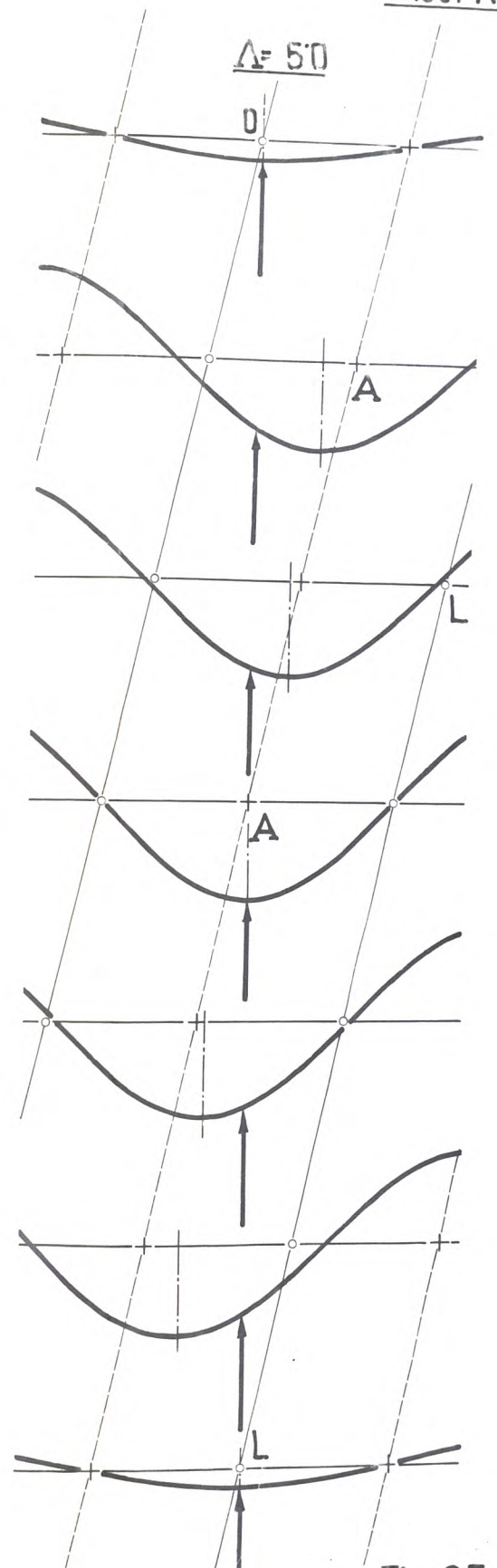


Fig. 35 f
(Oscillating stationary wave)

Fig. 35

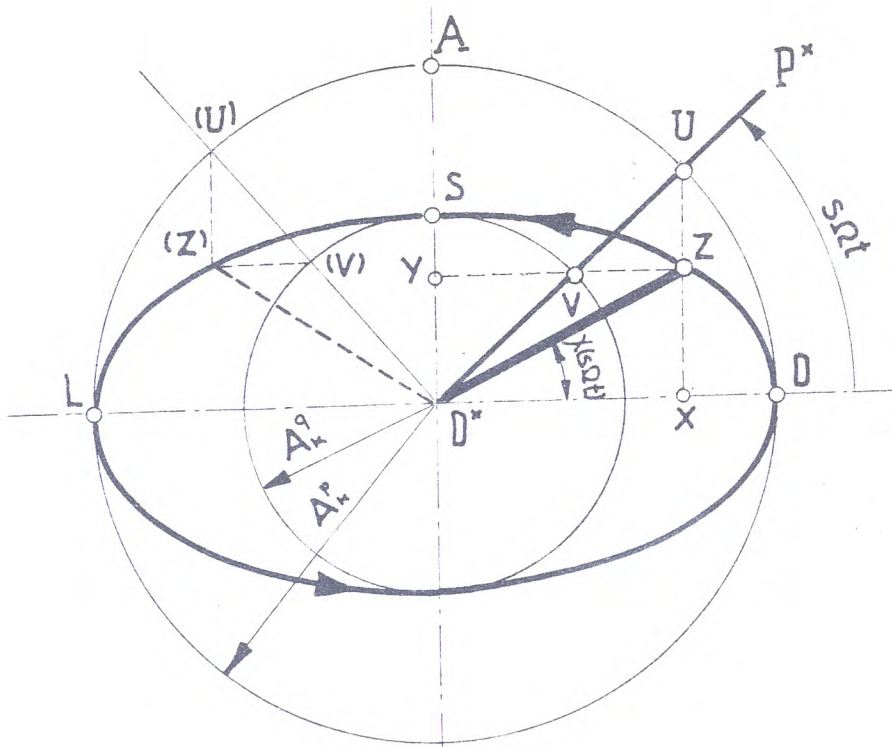


Fig. 36

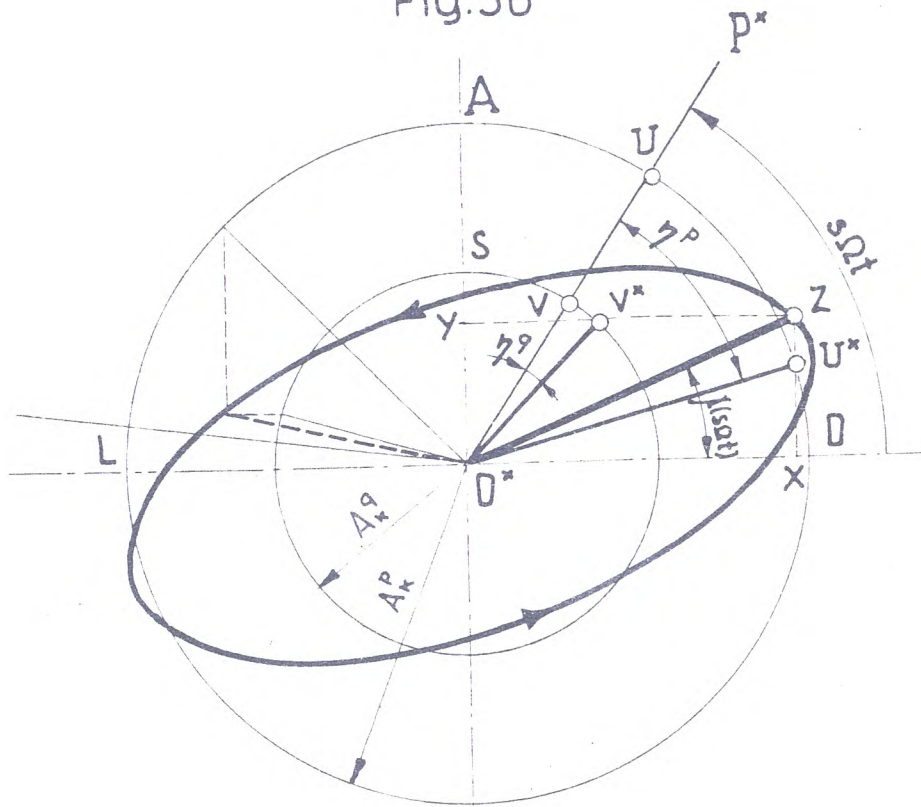


Fig. 37

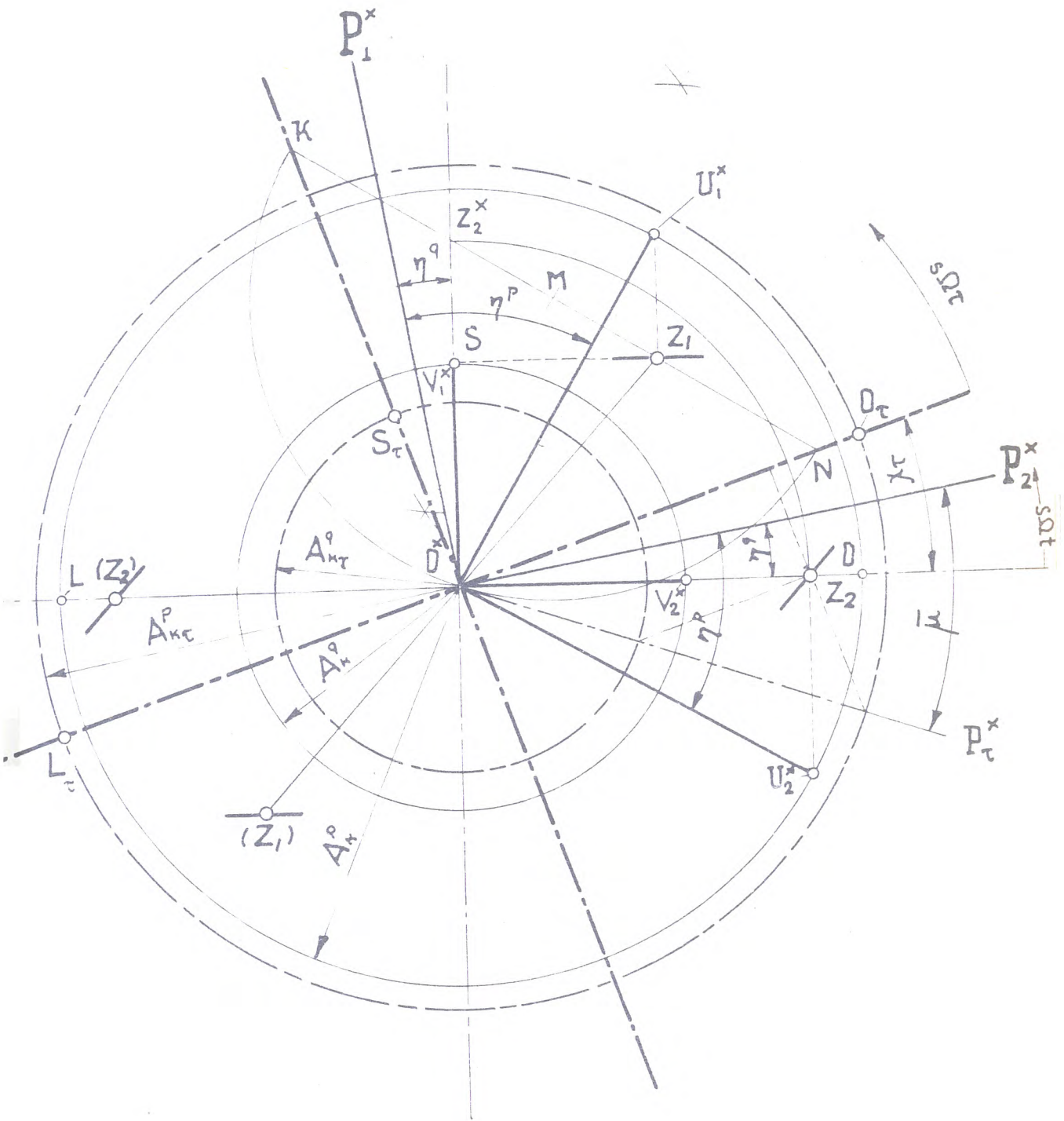


Fig. 38

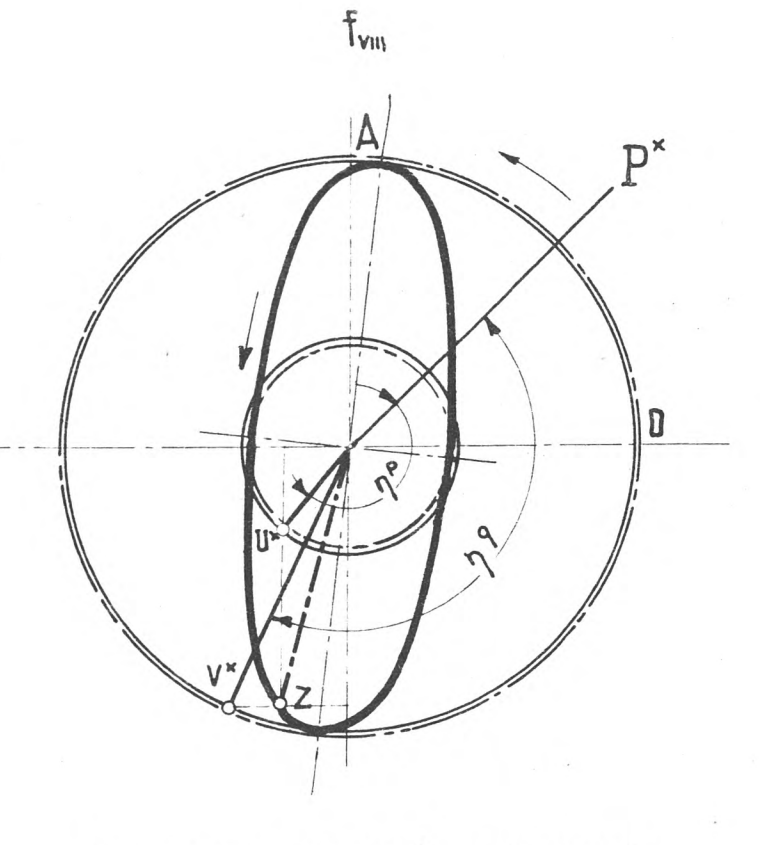
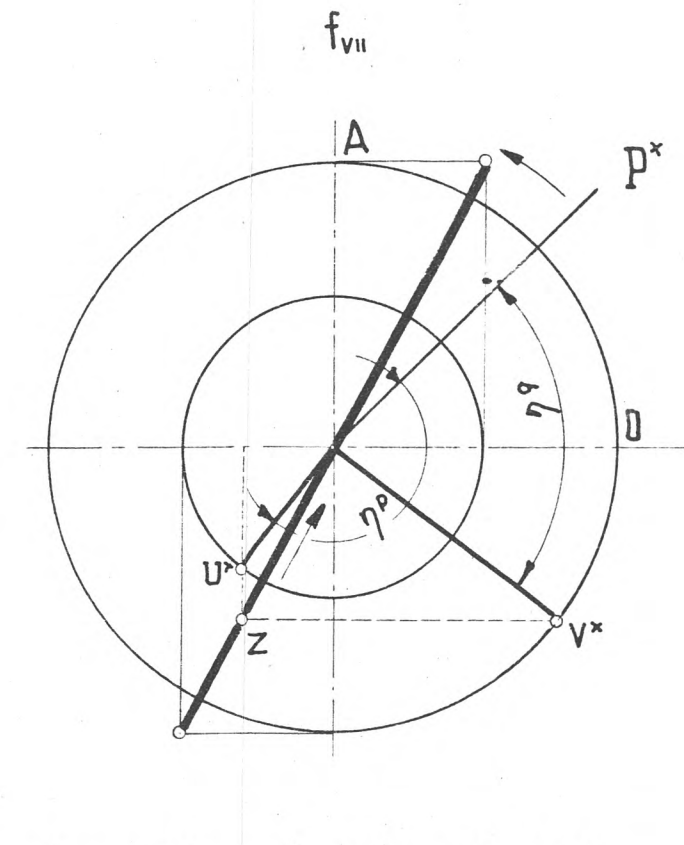
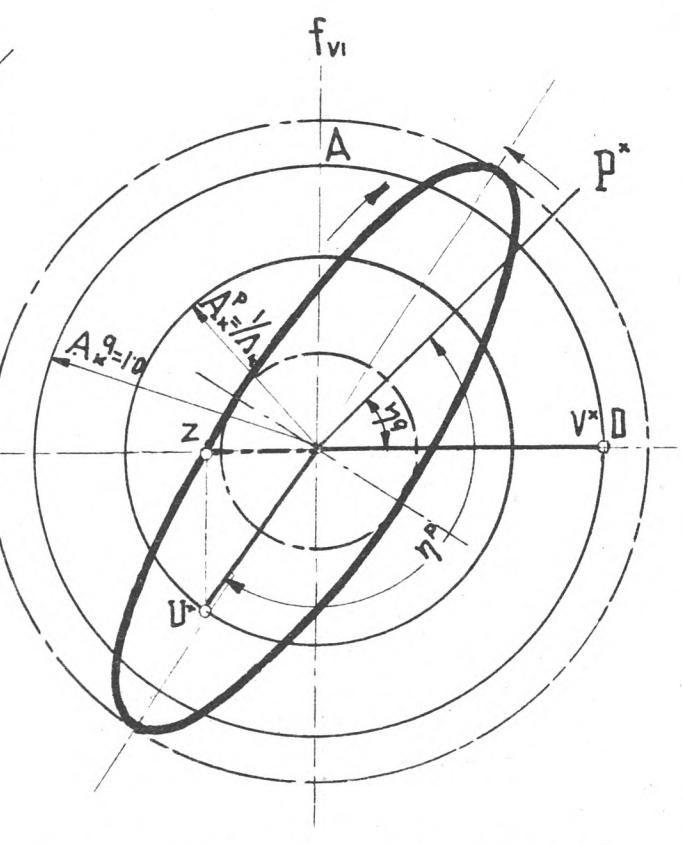
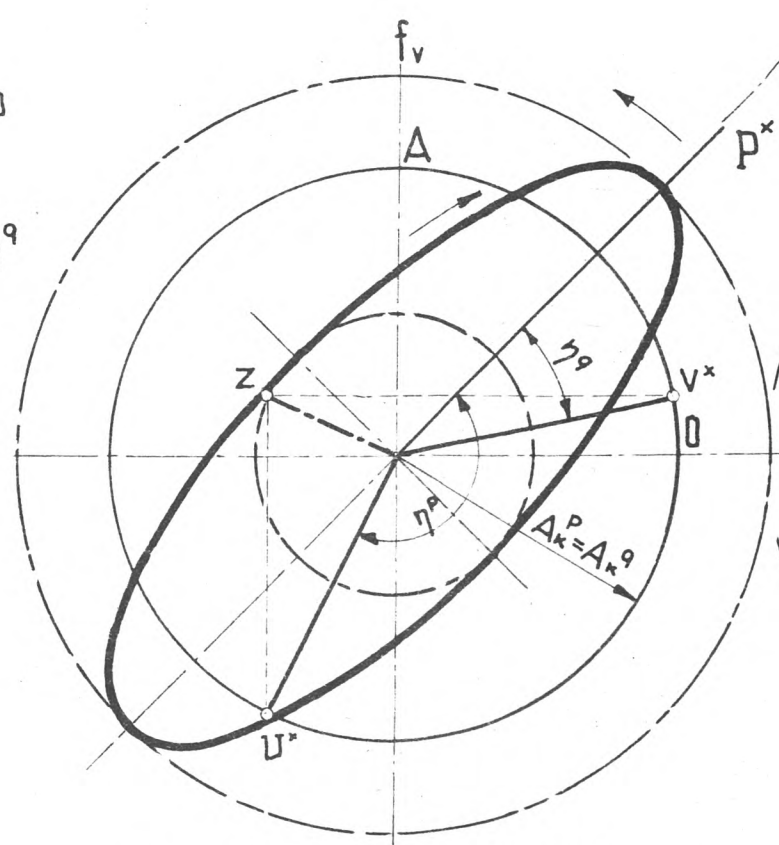
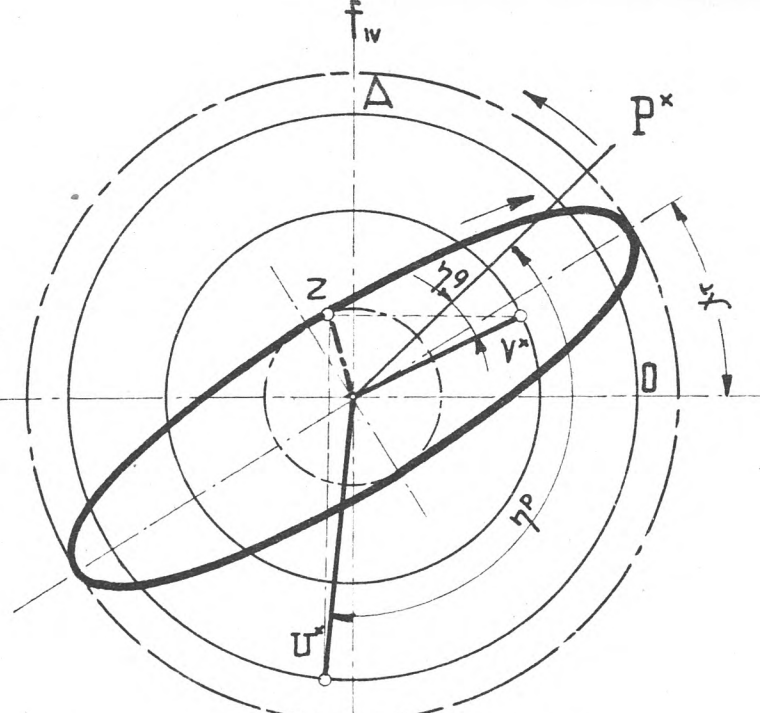
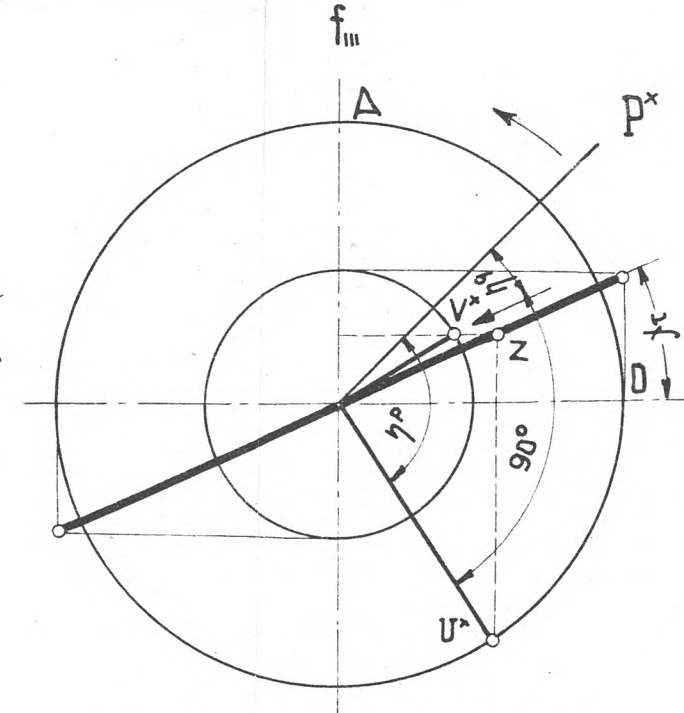
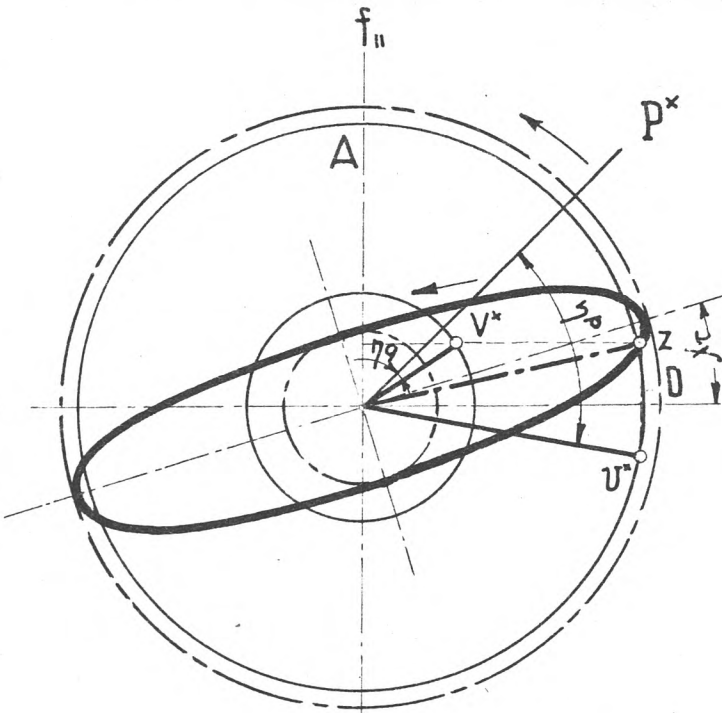
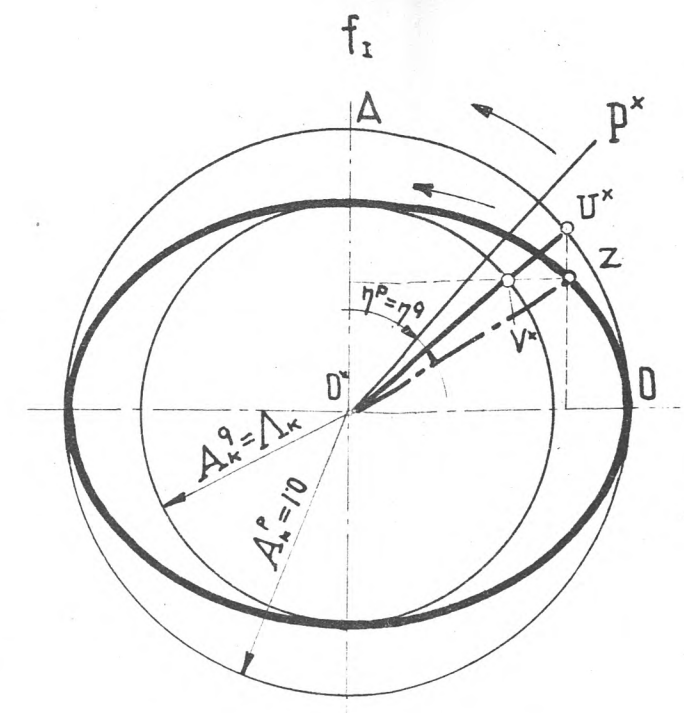
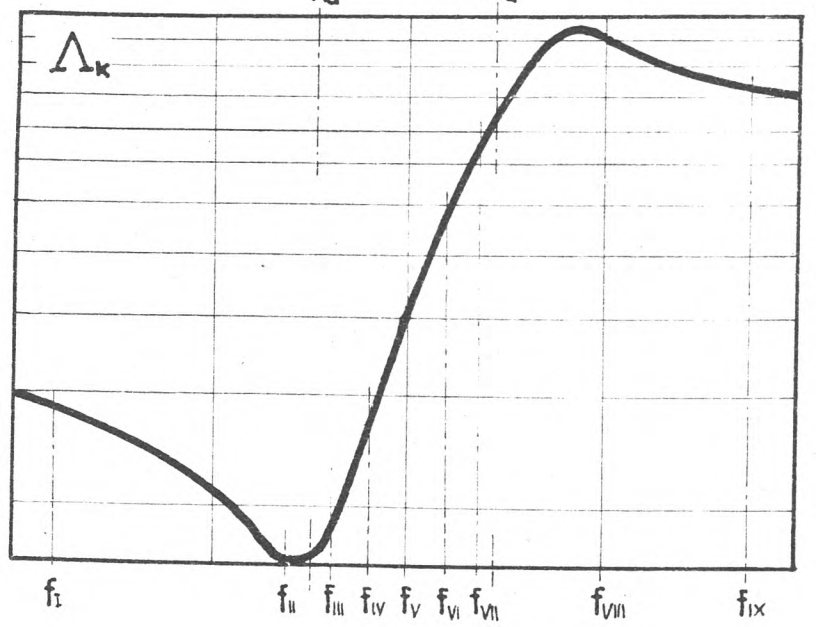
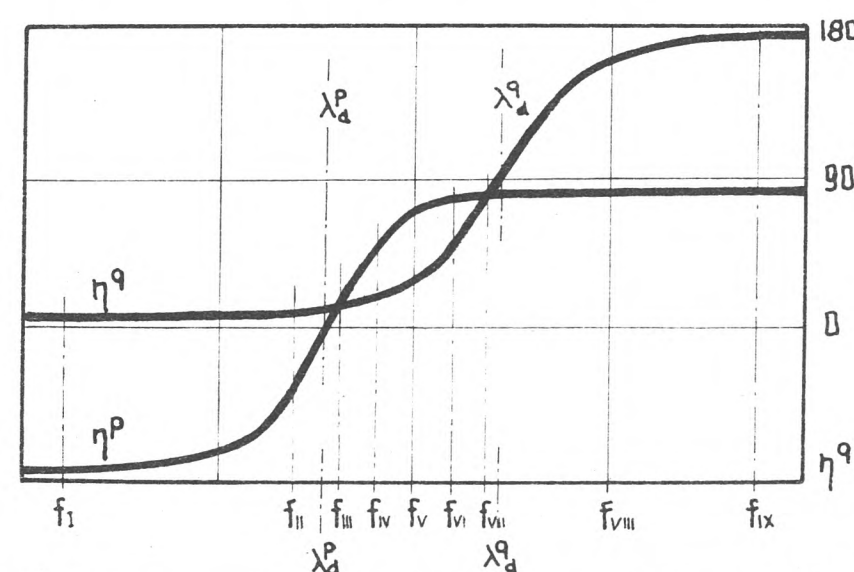
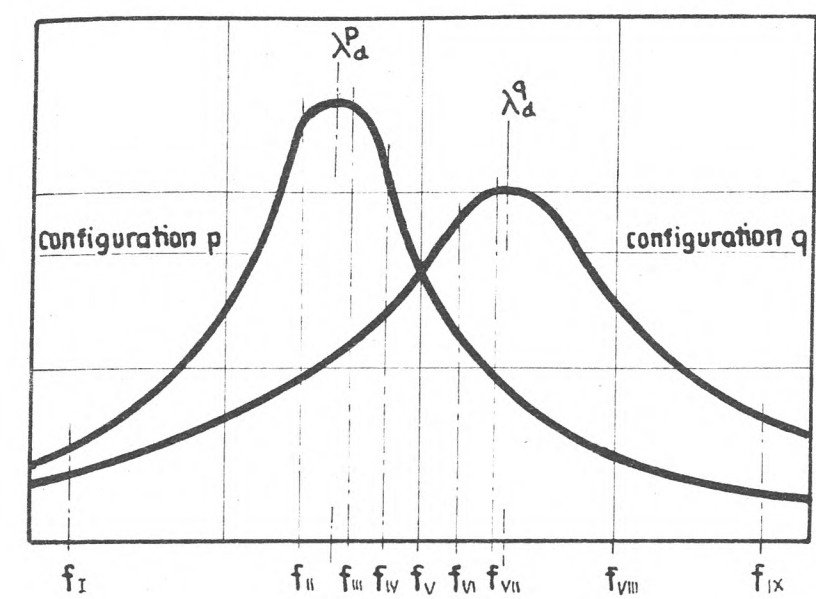


Fig.39

Fig.40

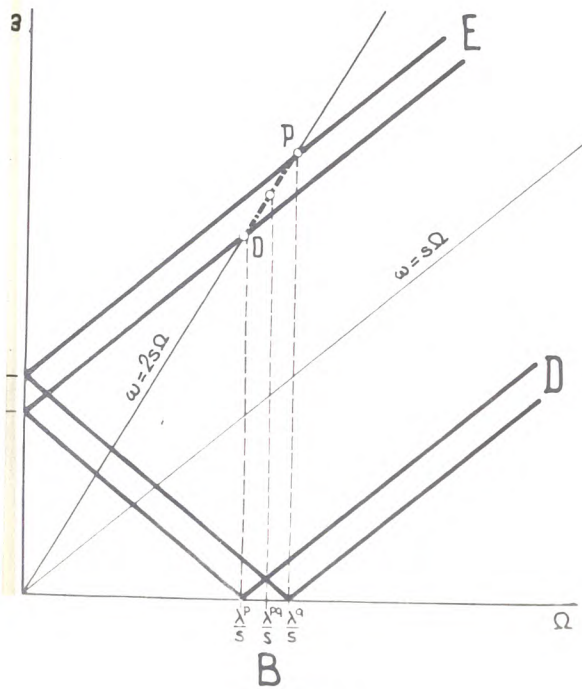
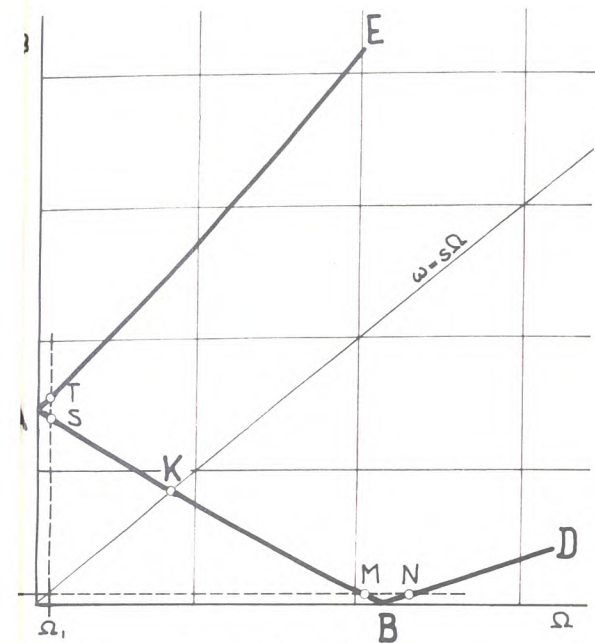


Fig. 43

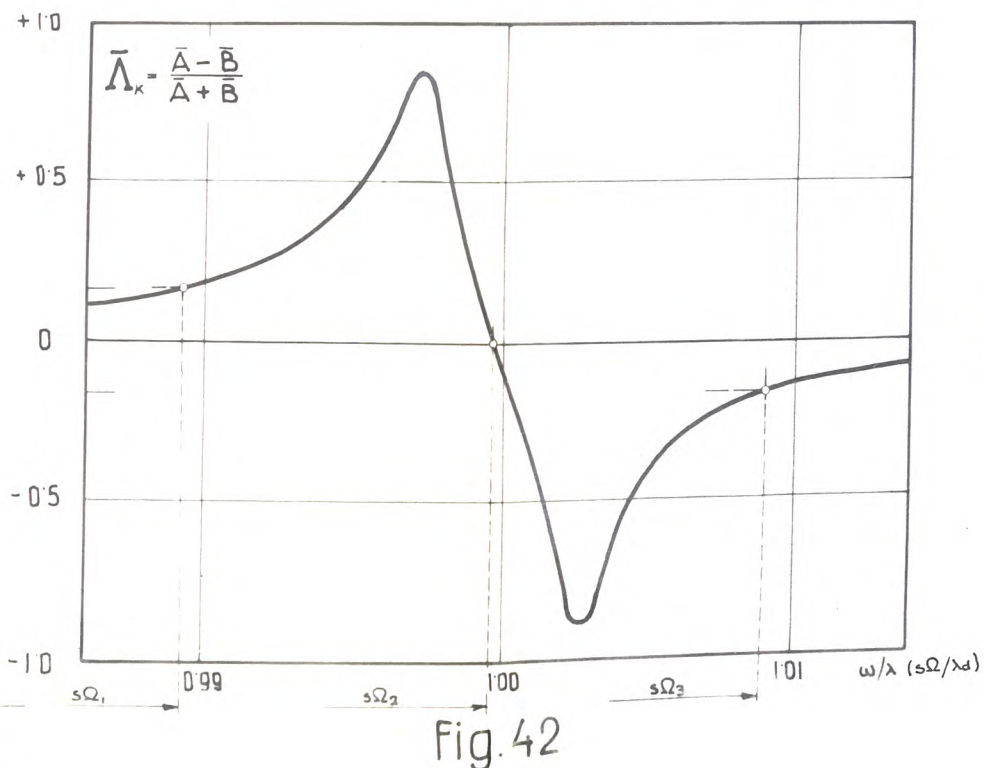
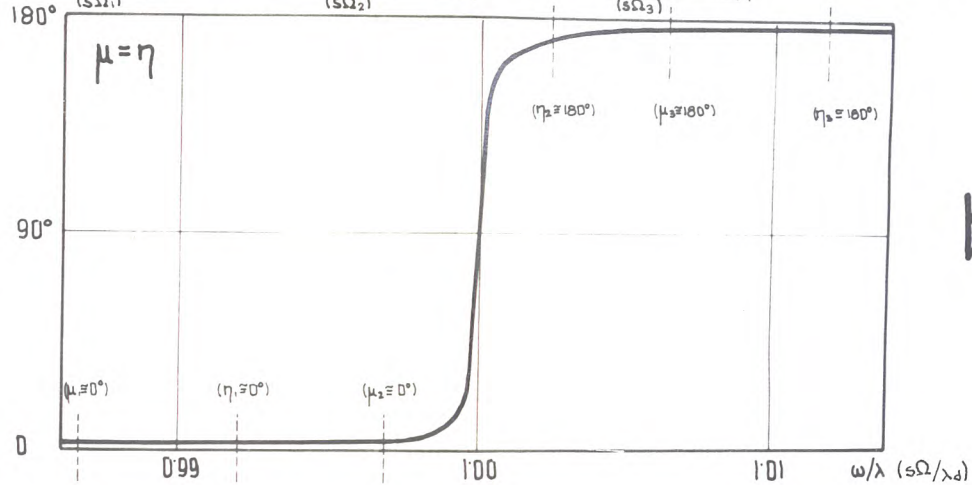
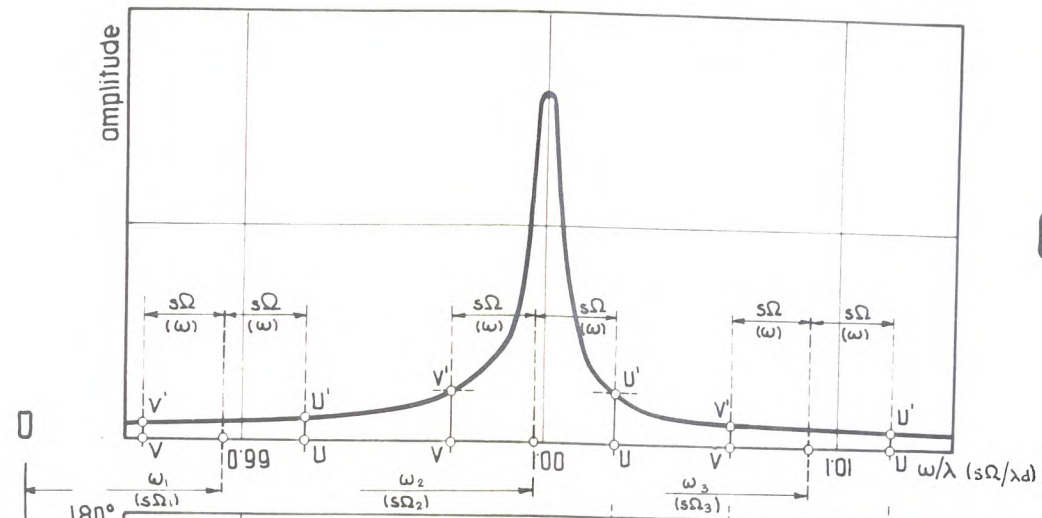
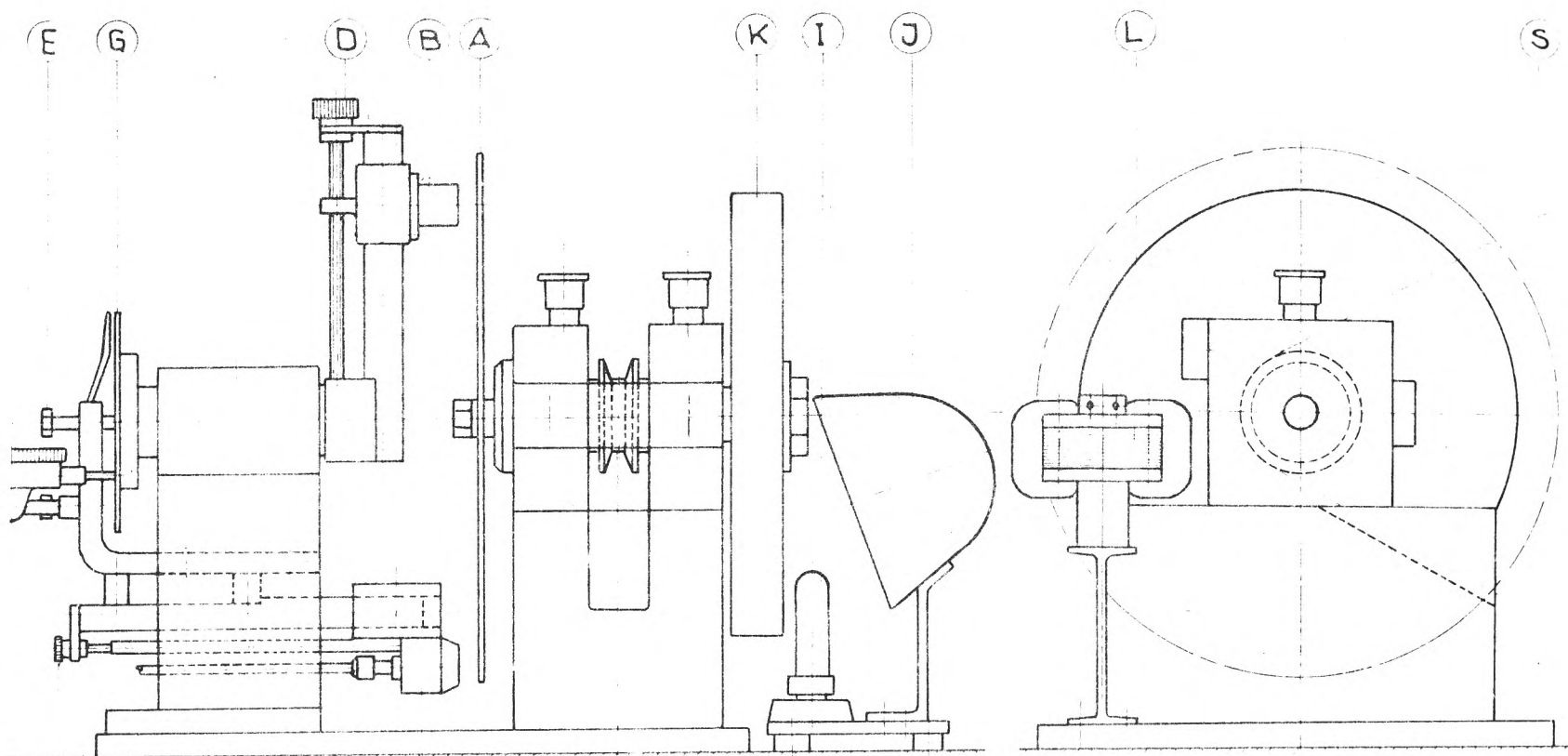


Fig. 42

a.

b.

c.



Experimental disc

J Reflector

Pick-up No.1

K Flywheel

Pick-up No.2

L Magnet

Radial adjustment P.U.No.1

M DC -AC current wires

Axial adjustment P.U.No.1

N Air-jet

Axial adjustment P.U.No.2

D Pressure gauge

Protractor

P Air-tap

Calibration dial gauge

Q Compressed air supply

Photo-electric cell

R V-belt pulley

S V-belt

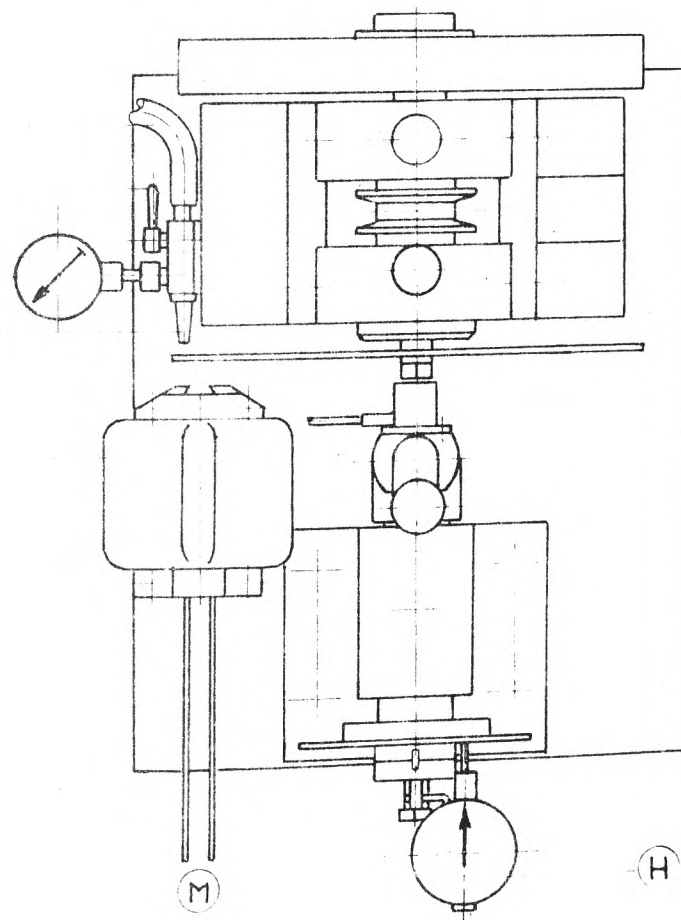


Fig. 44

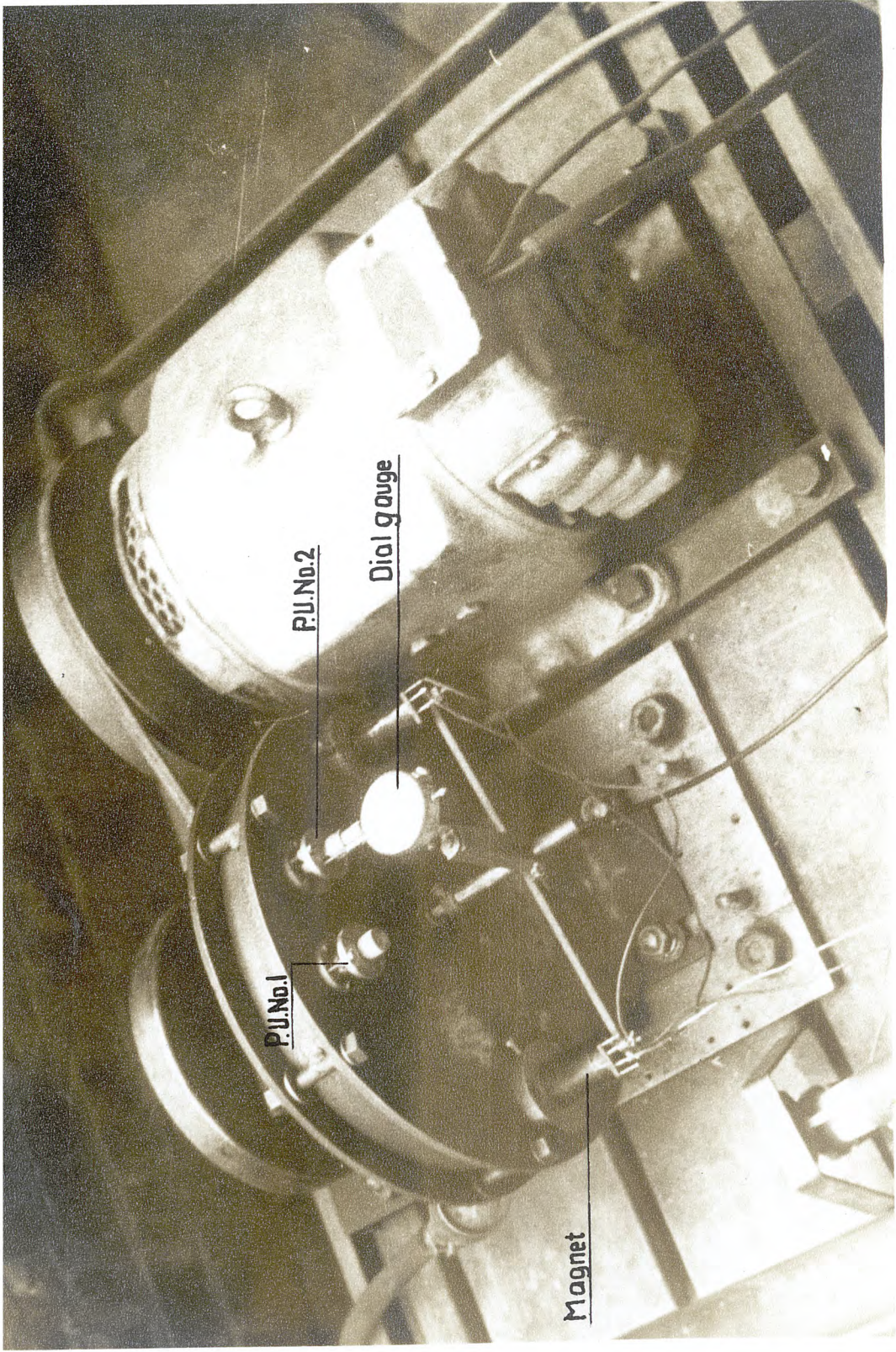
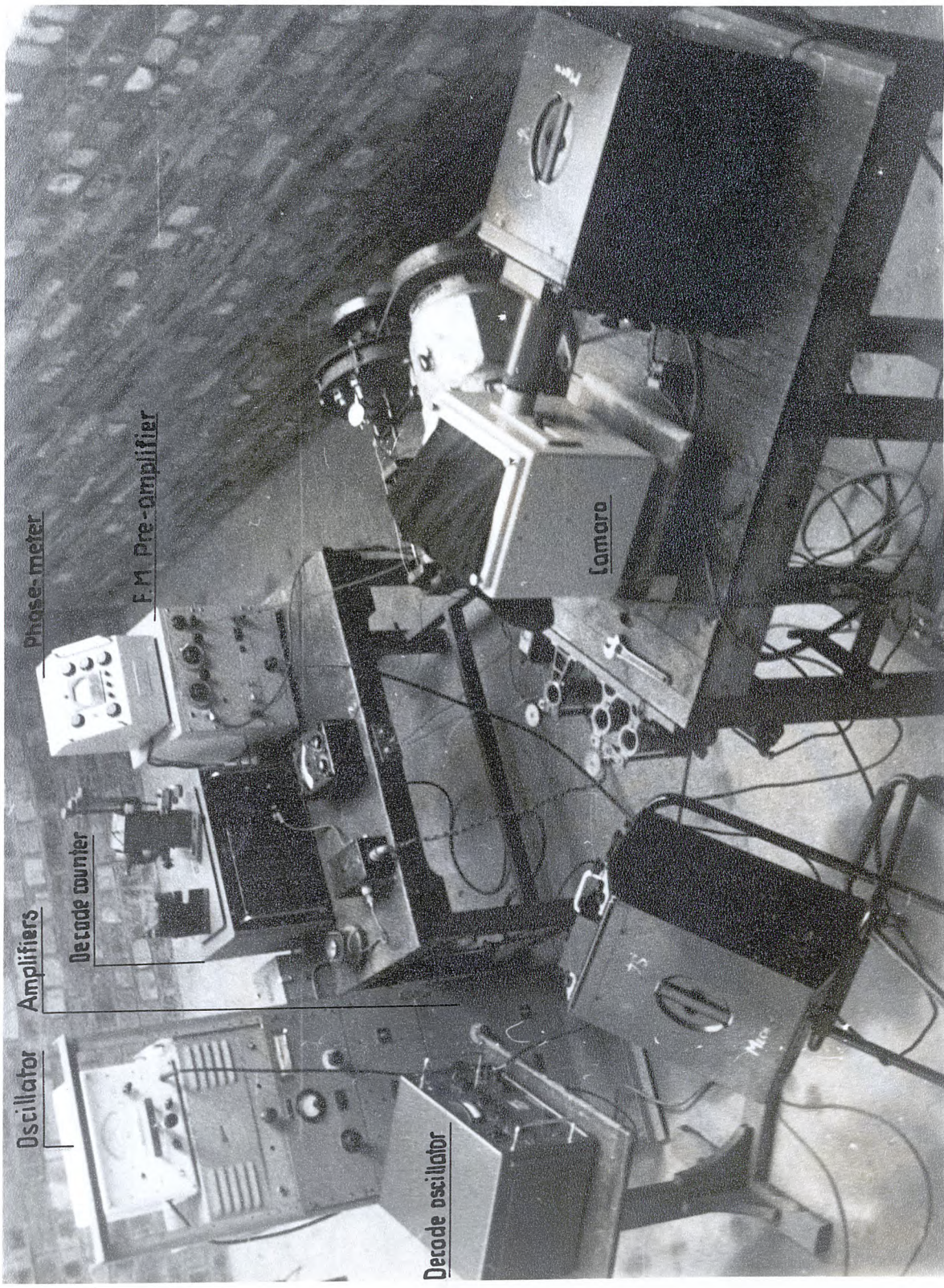


Fig. 45a



Oscillator

Amplifiers

Decode counter

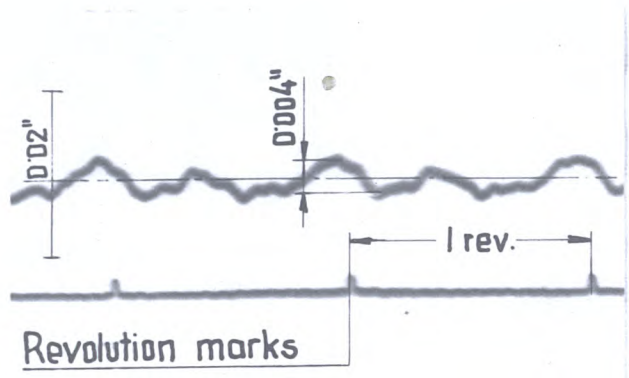
Phase-meter

F.M. Pre-amplifier

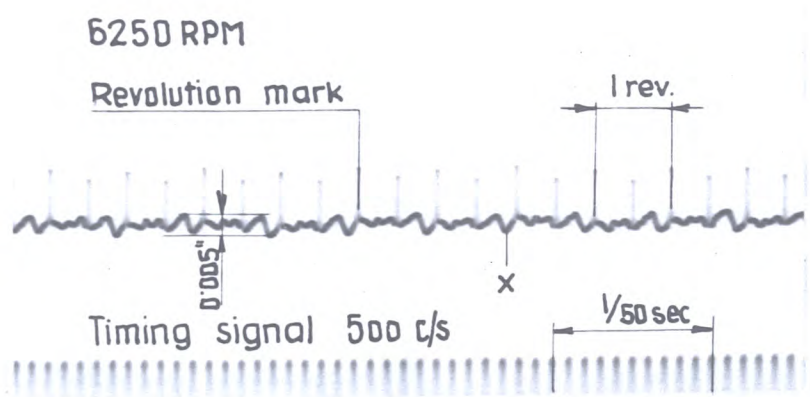
Decode oscillator

Tamara

Fig.45b



a.



b.

Fig.46

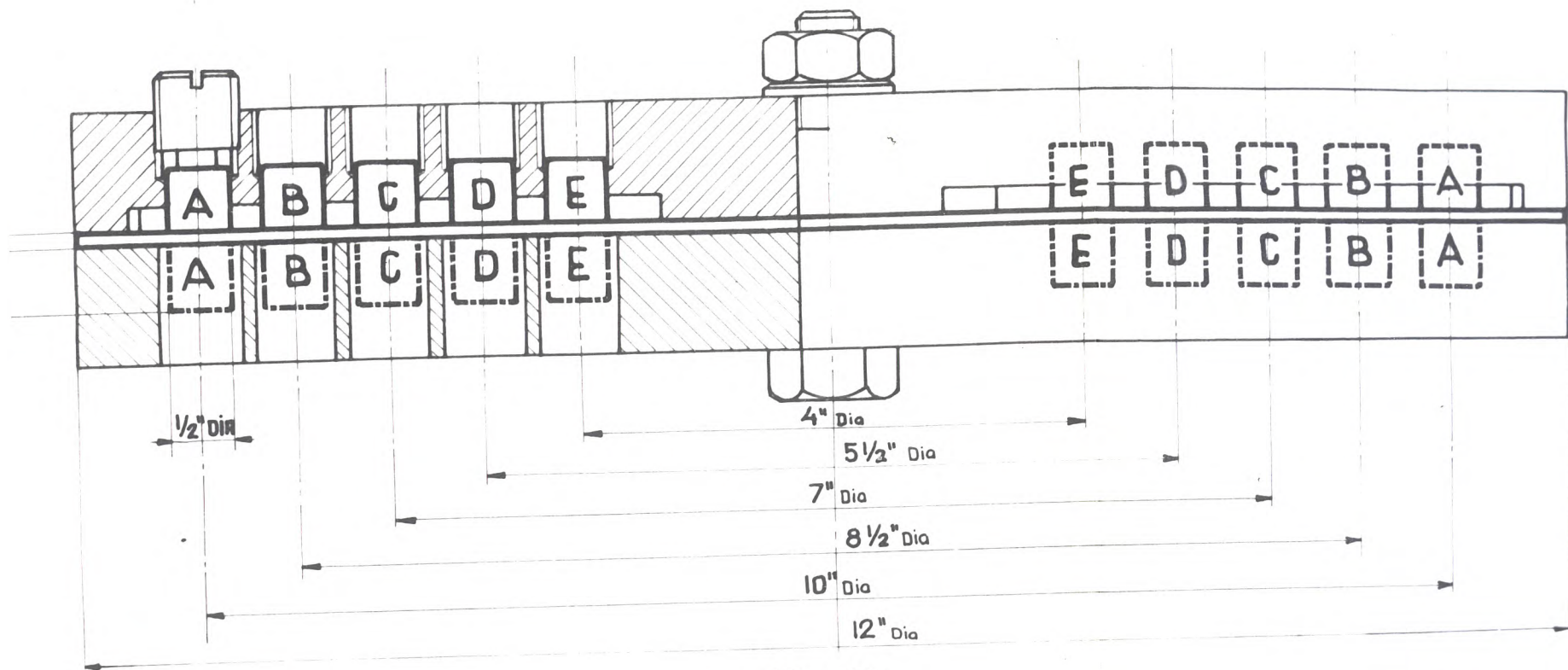
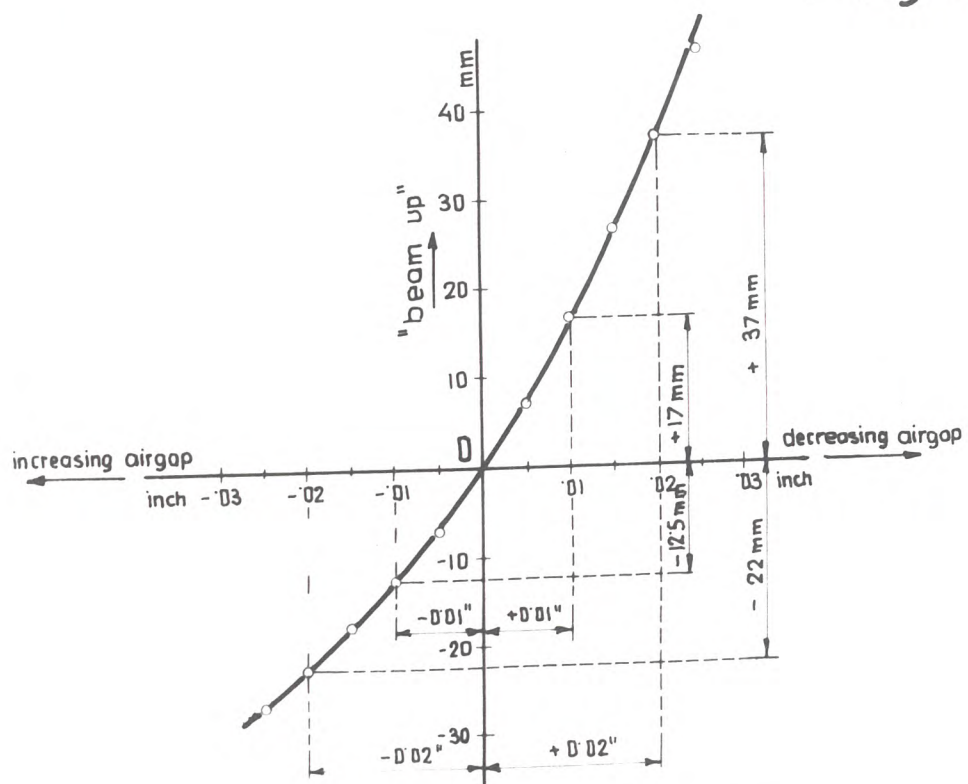
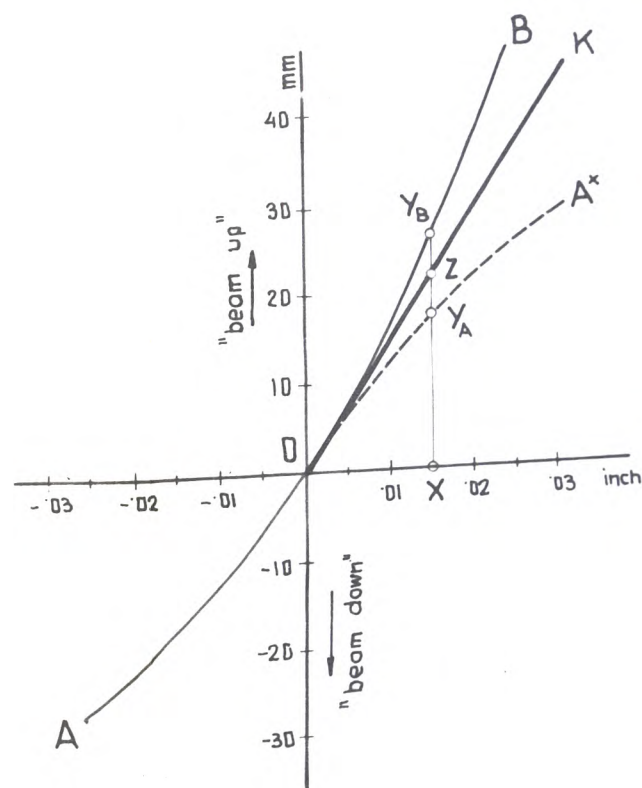


Fig.48



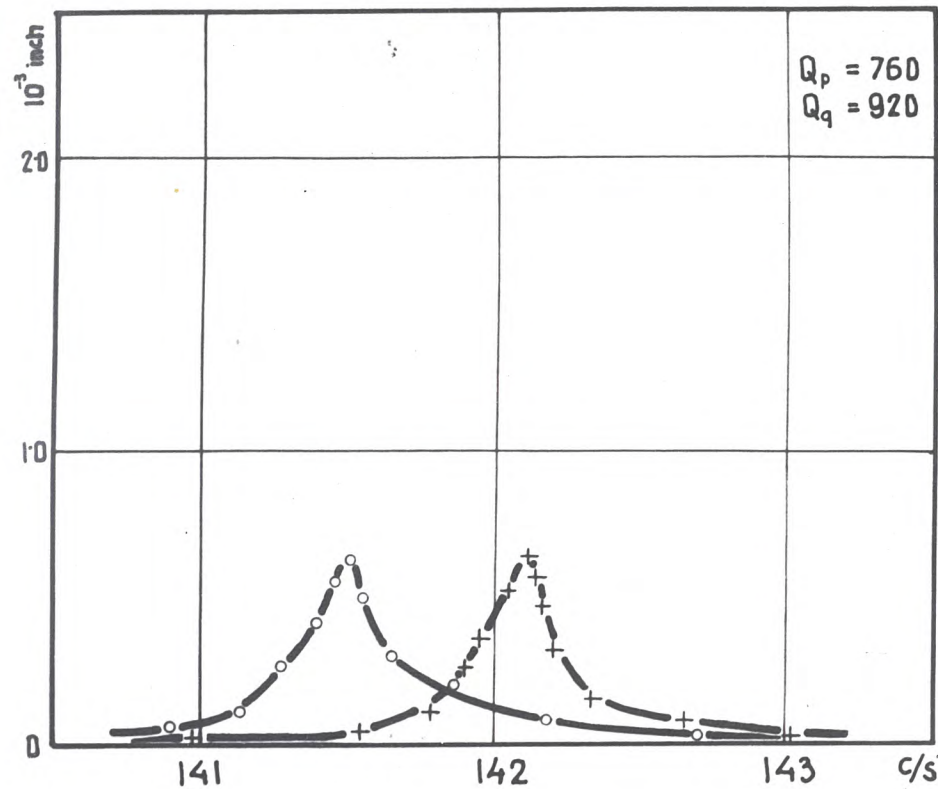
a.



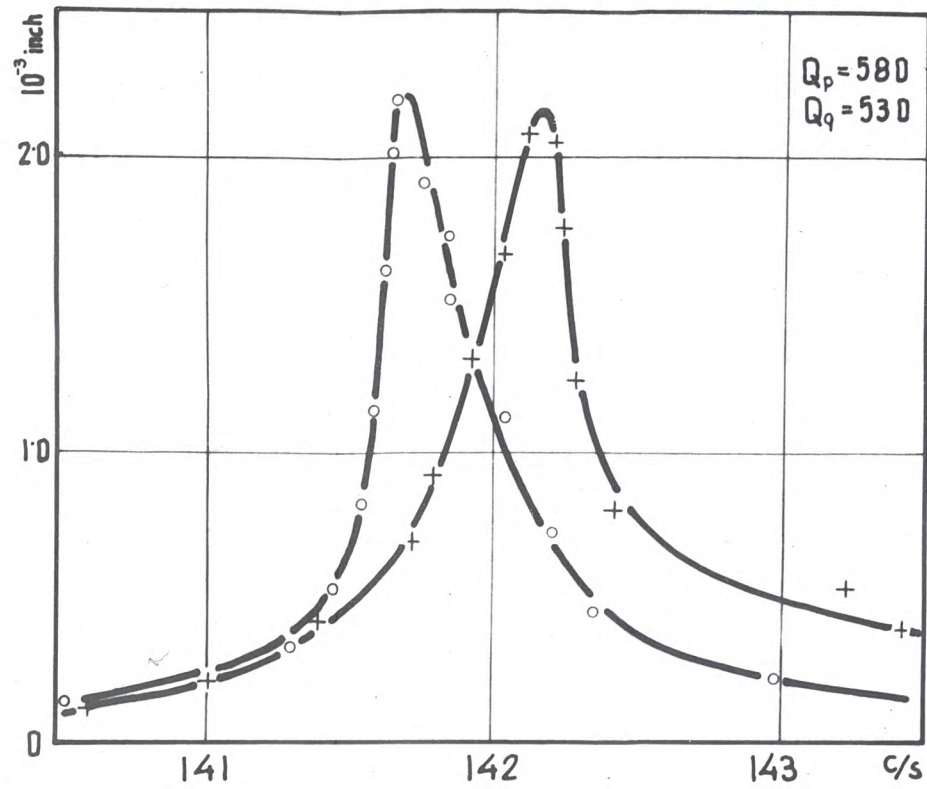
b.

Fig.47

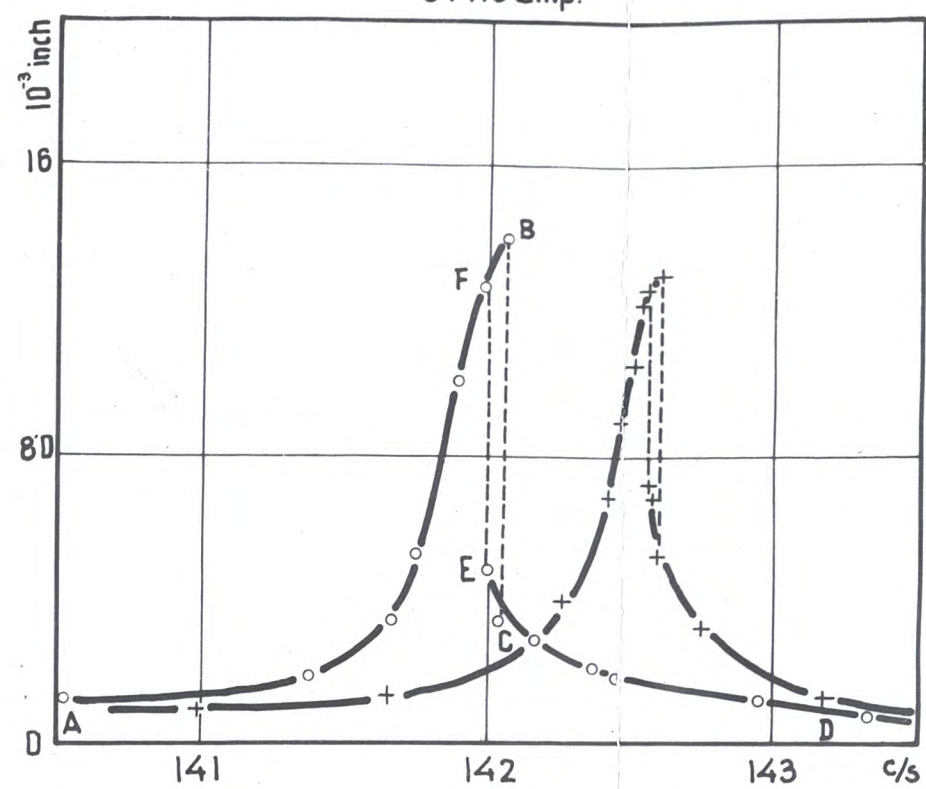
0.03 AC amp.



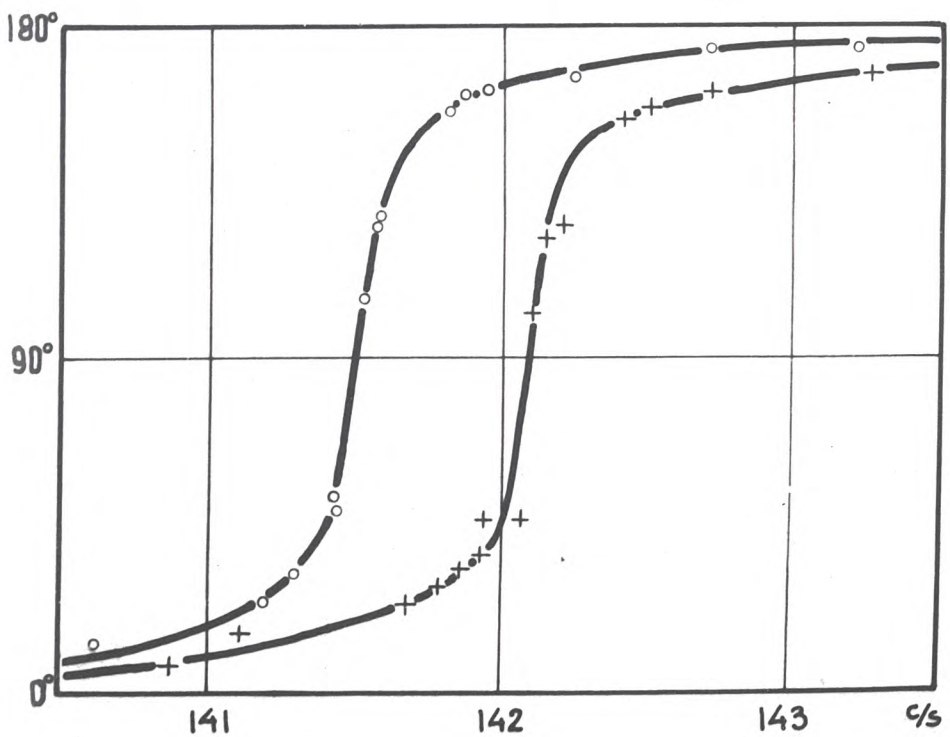
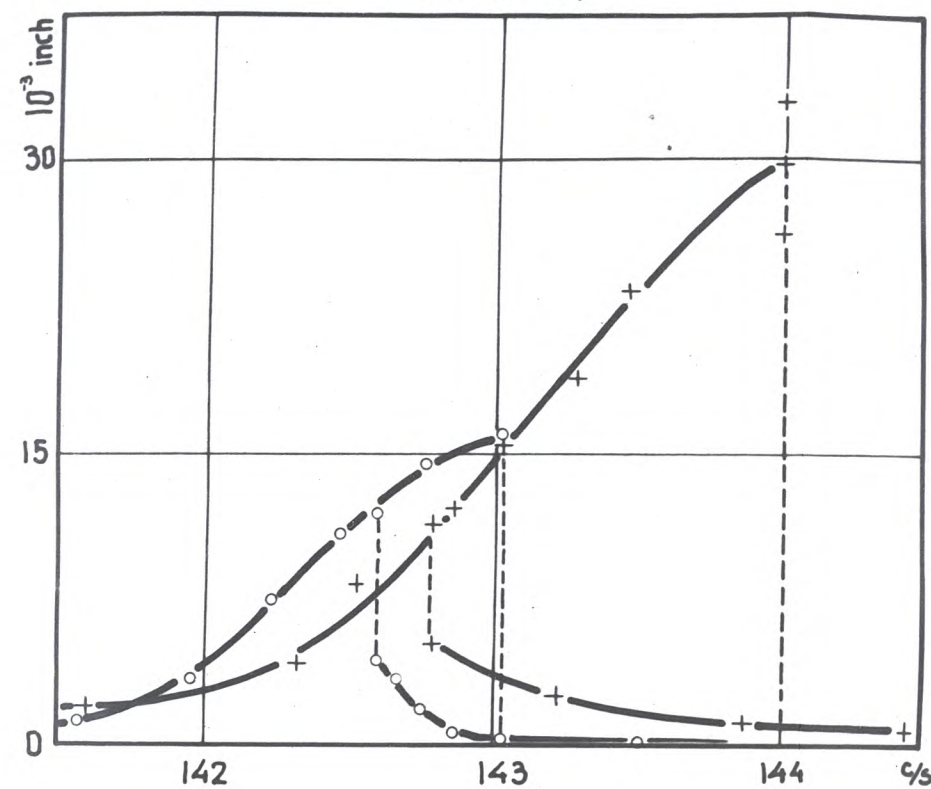
0.05 AC amp.



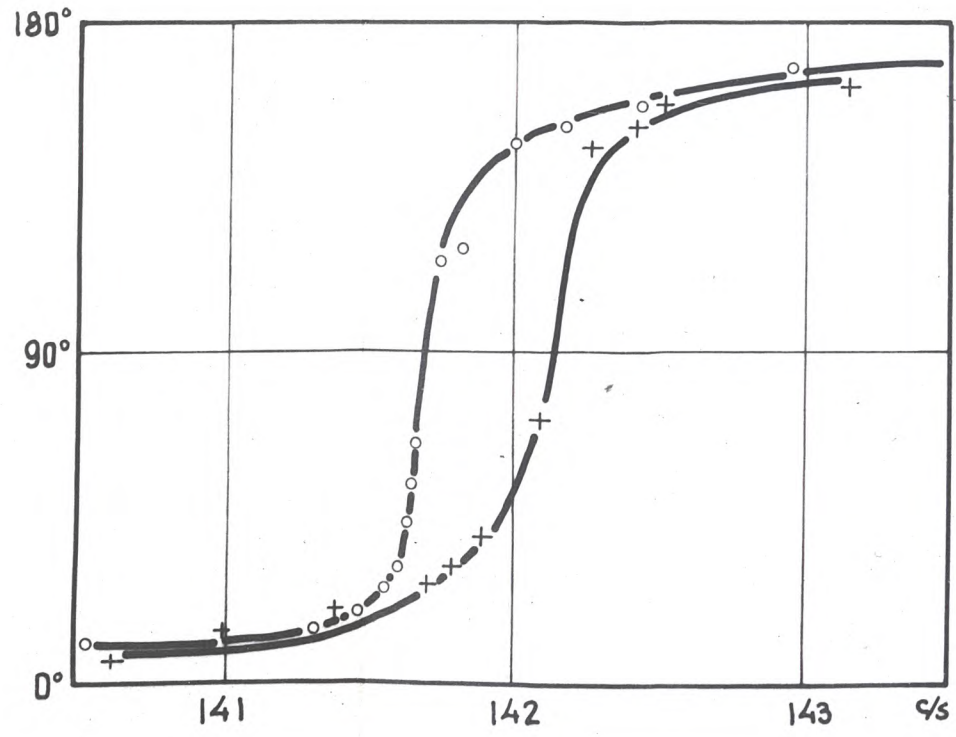
0.1 AC amp.



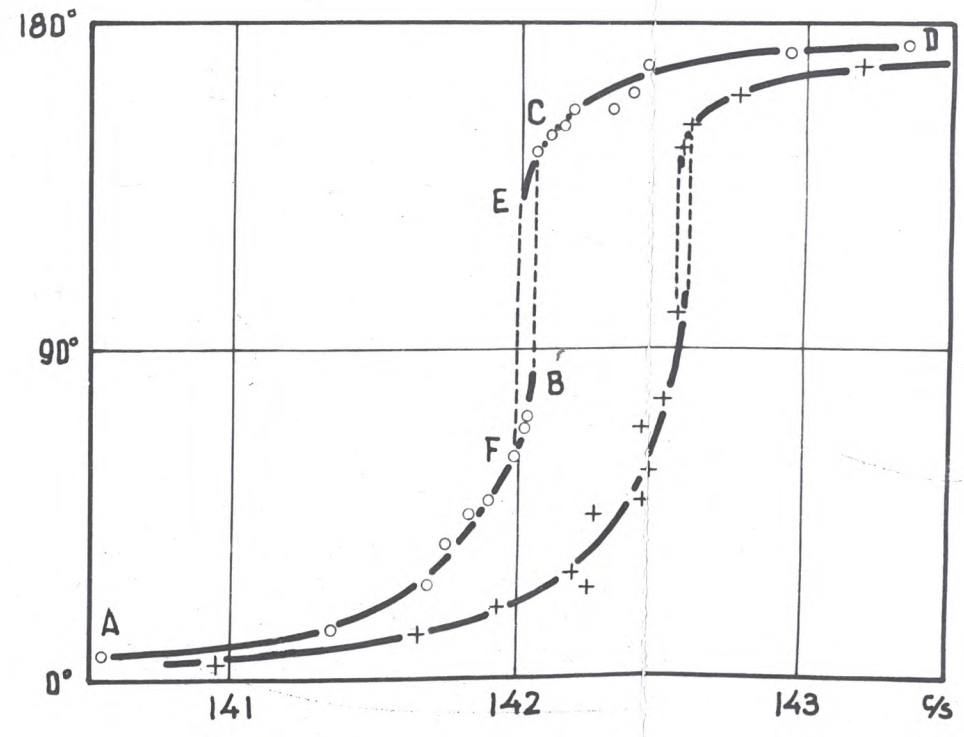
0.15 AC amp.



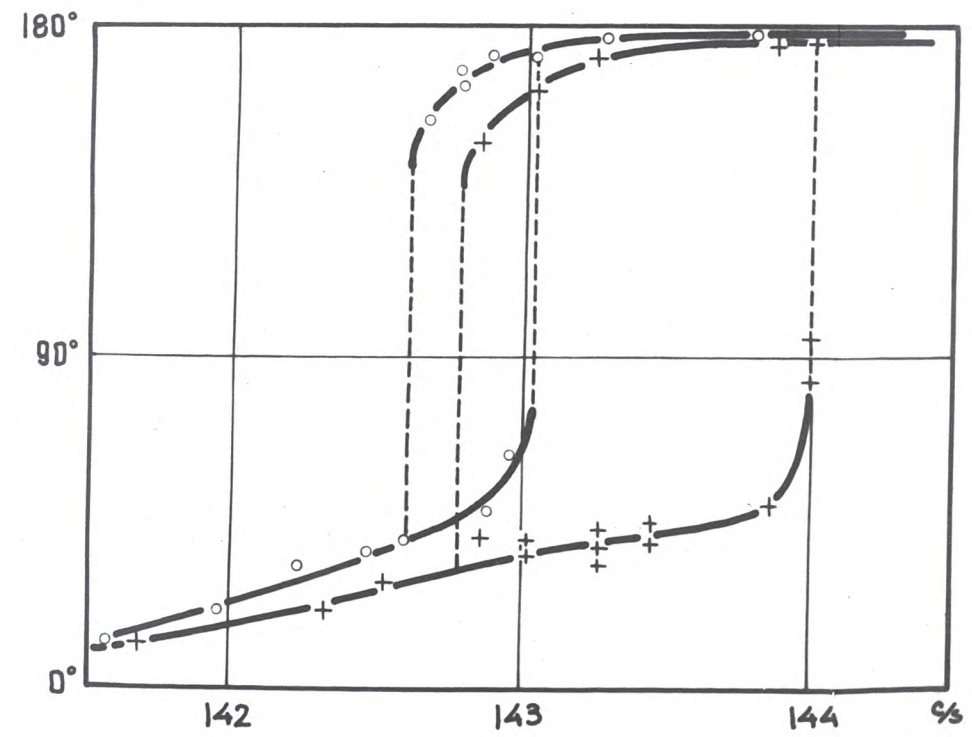
a.



b.



c.



d.

Fig. 49

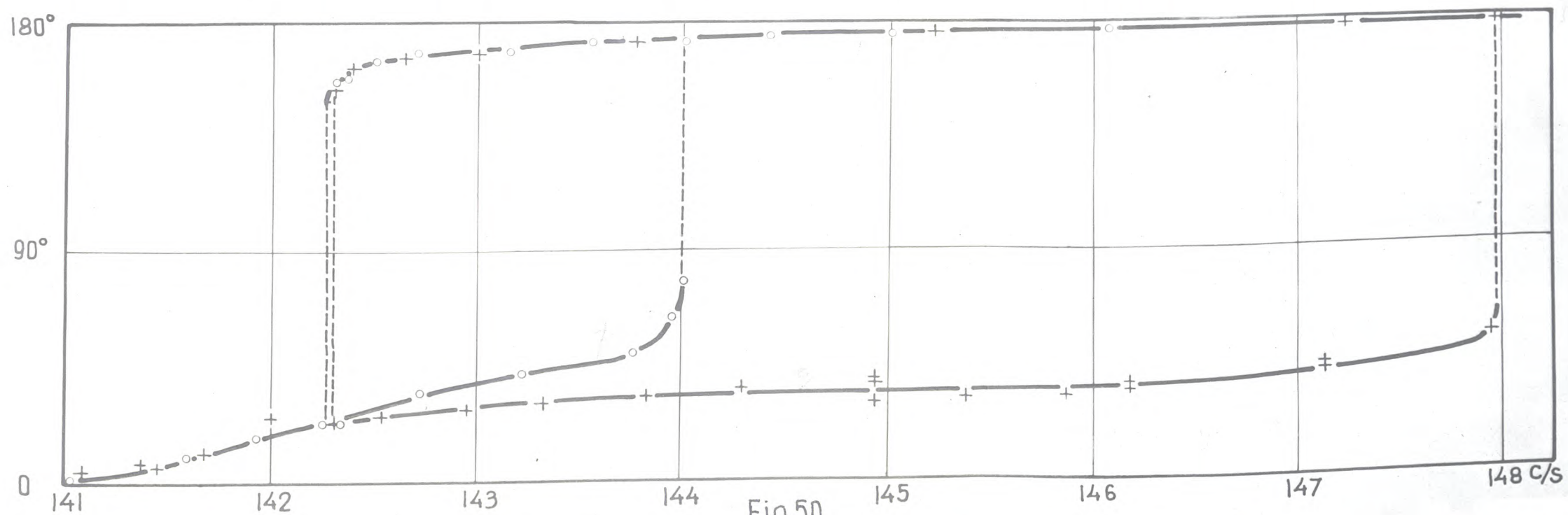
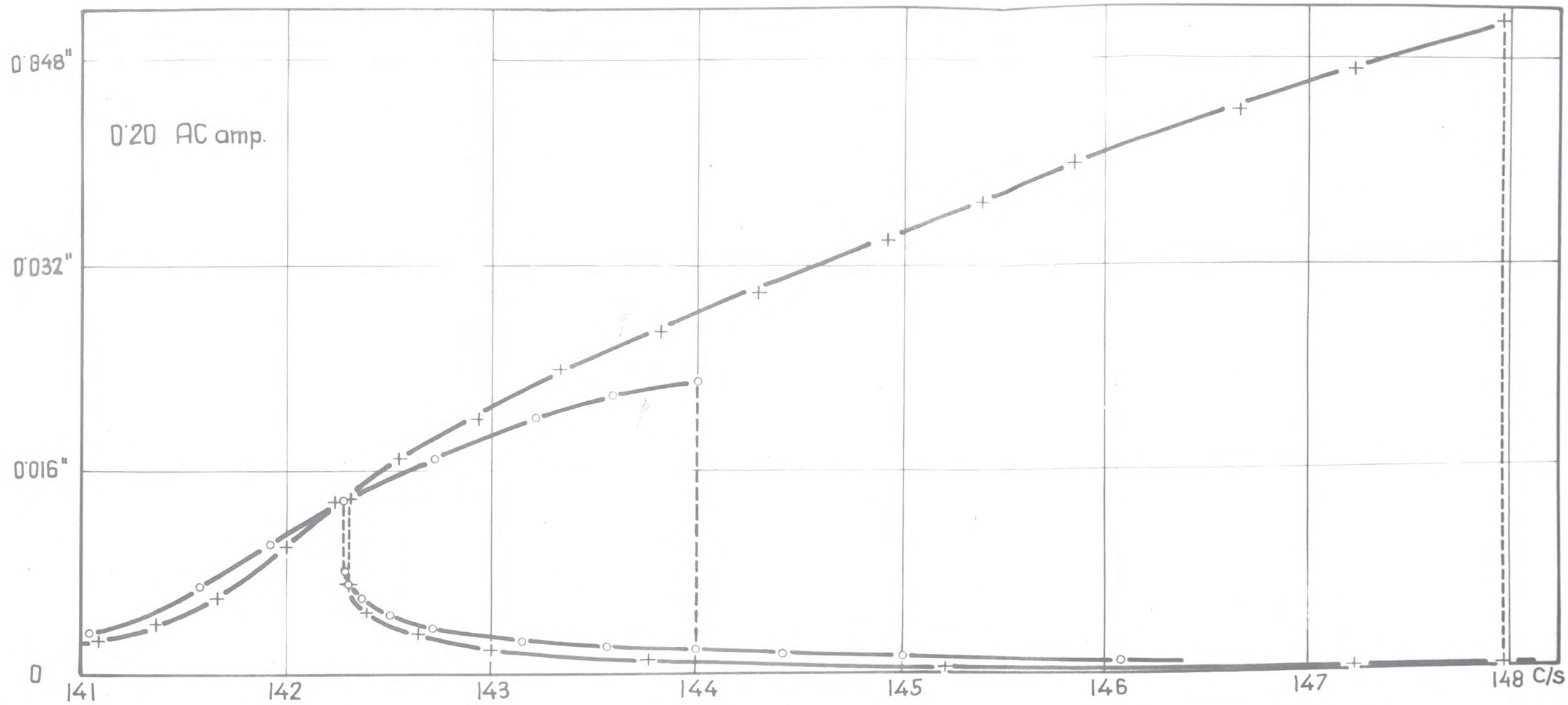


Fig. 50

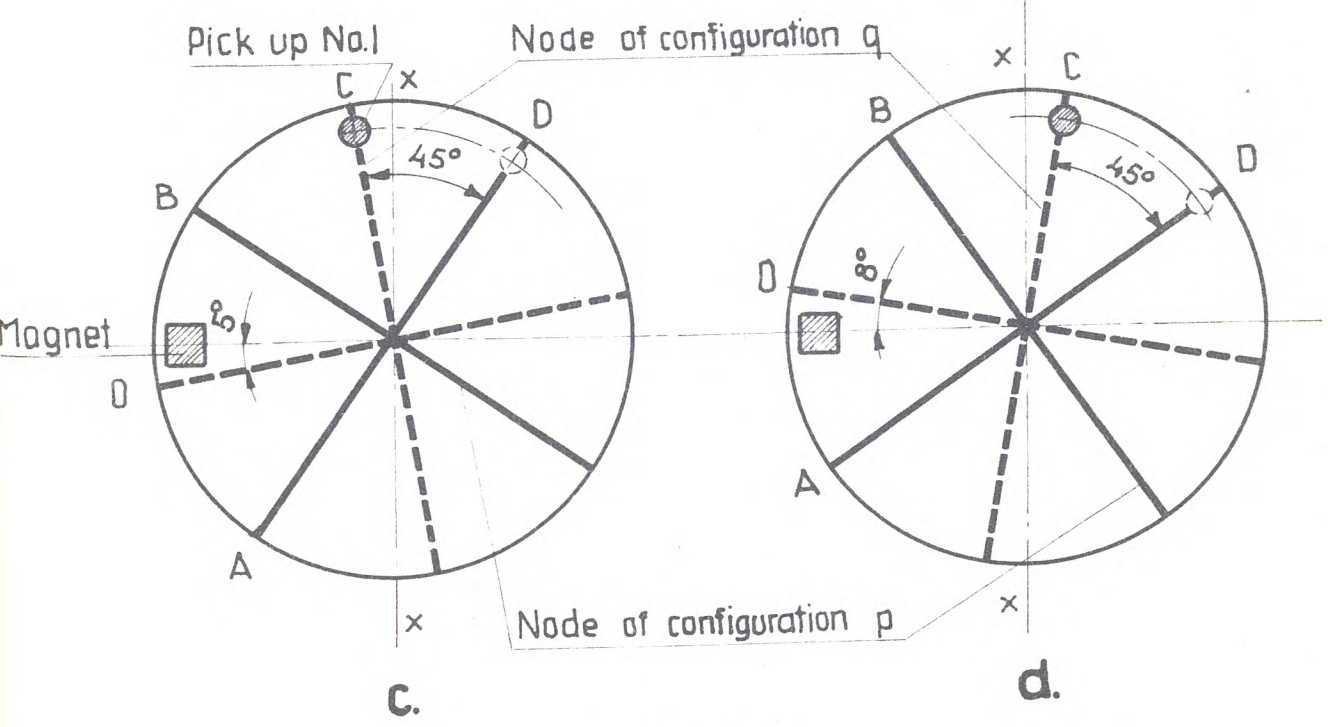
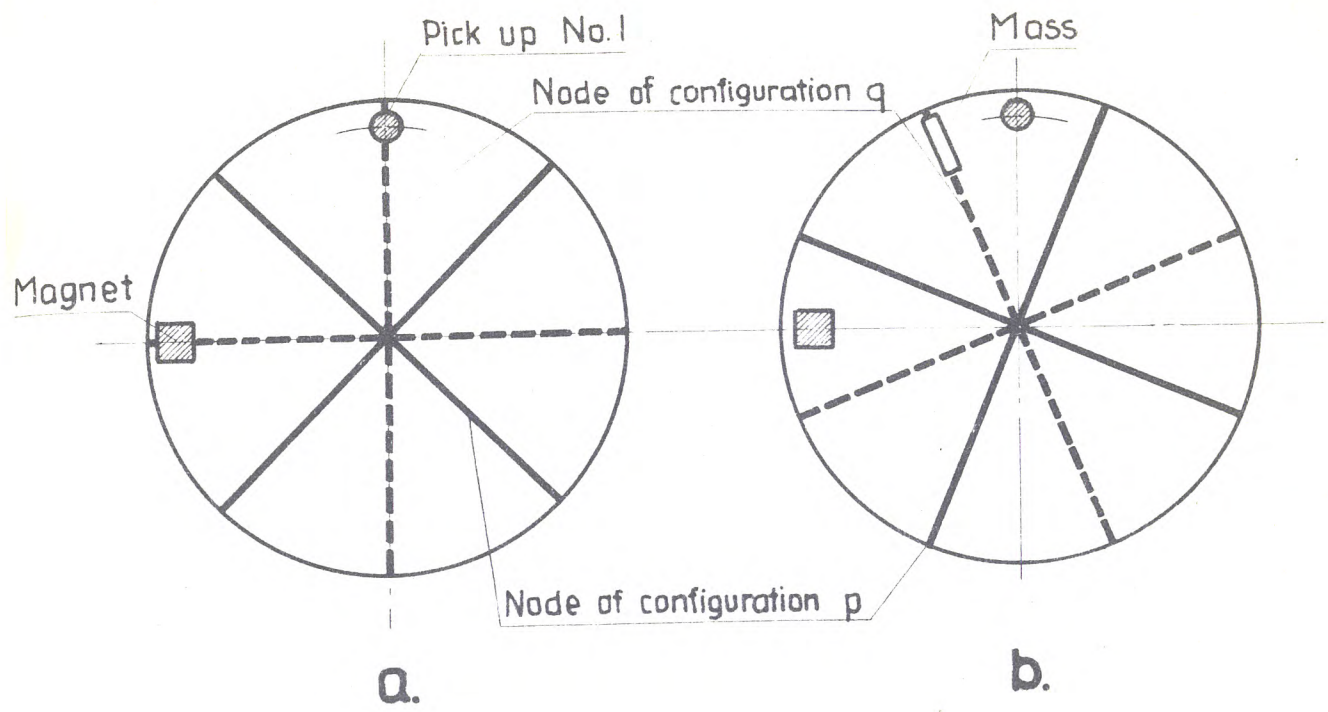


Fig.51

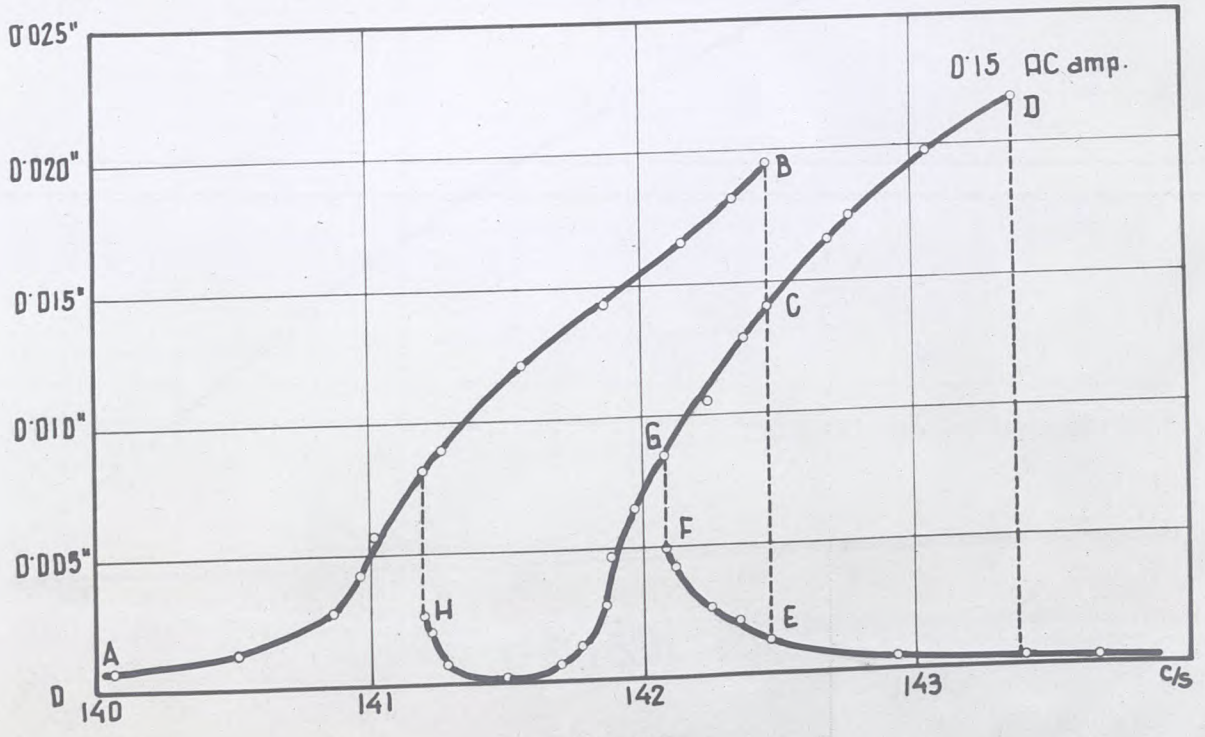
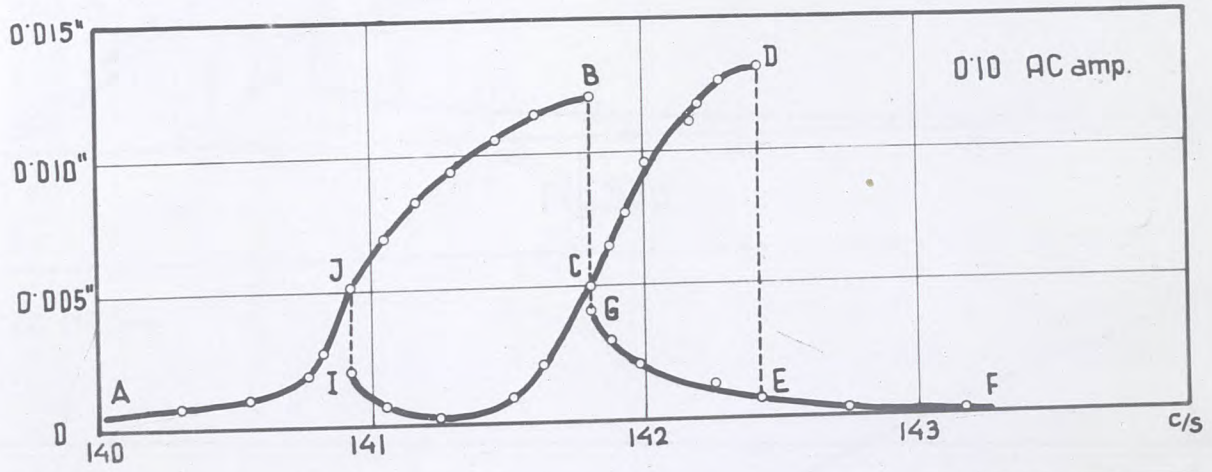
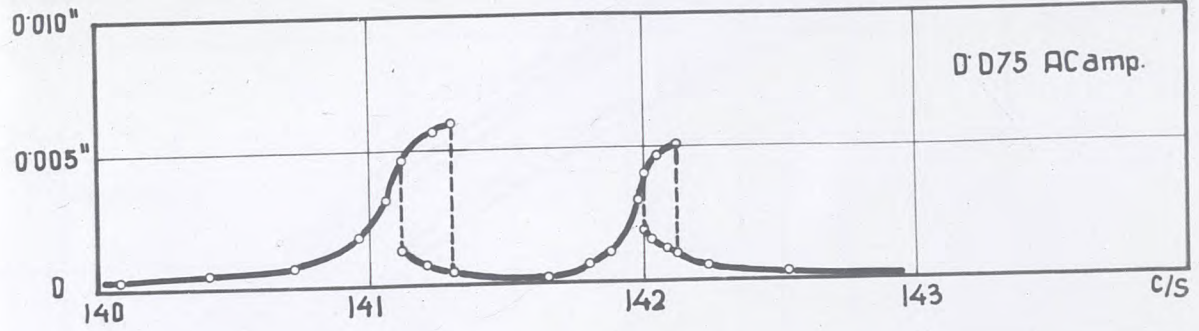
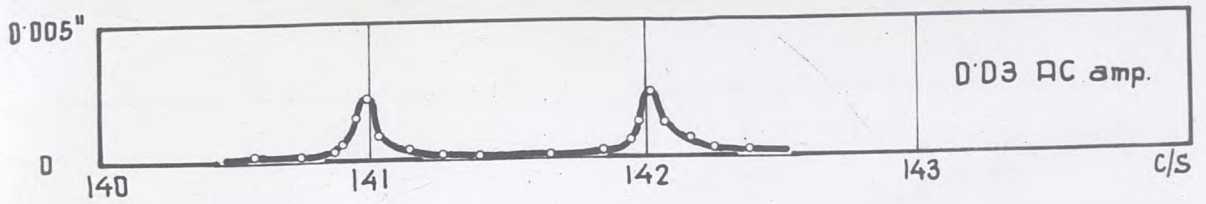


Fig.52

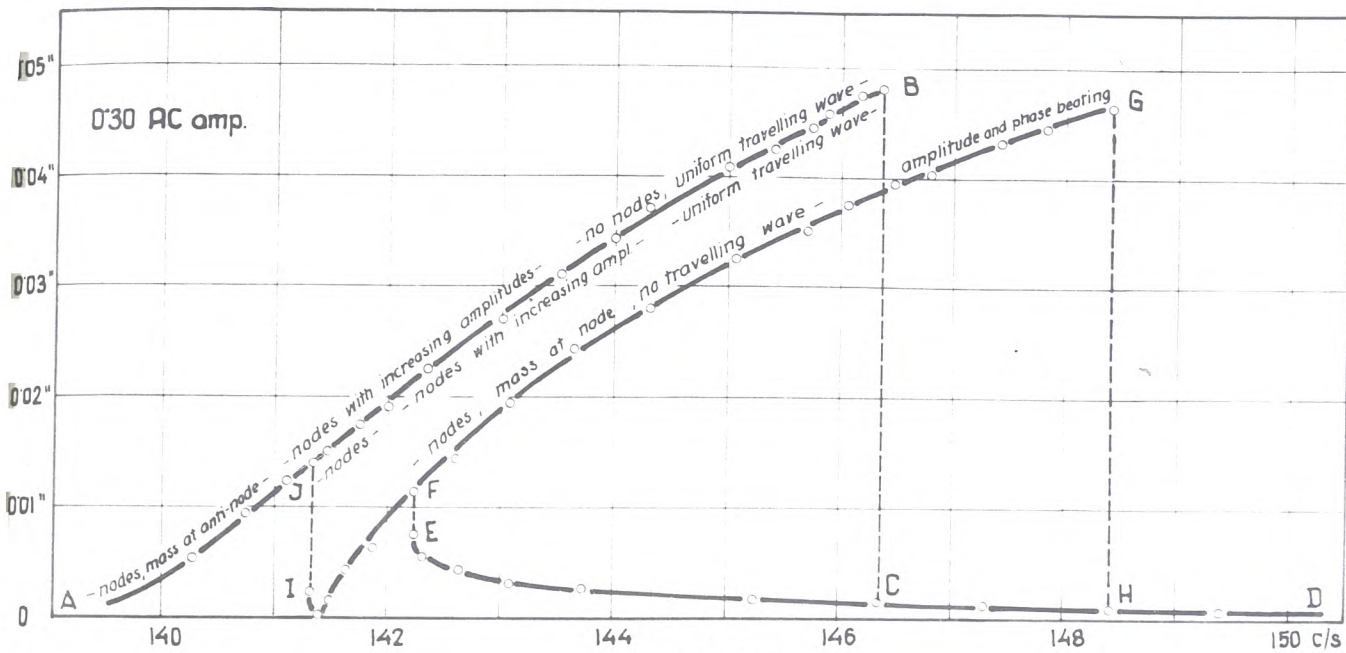


Fig. 53a

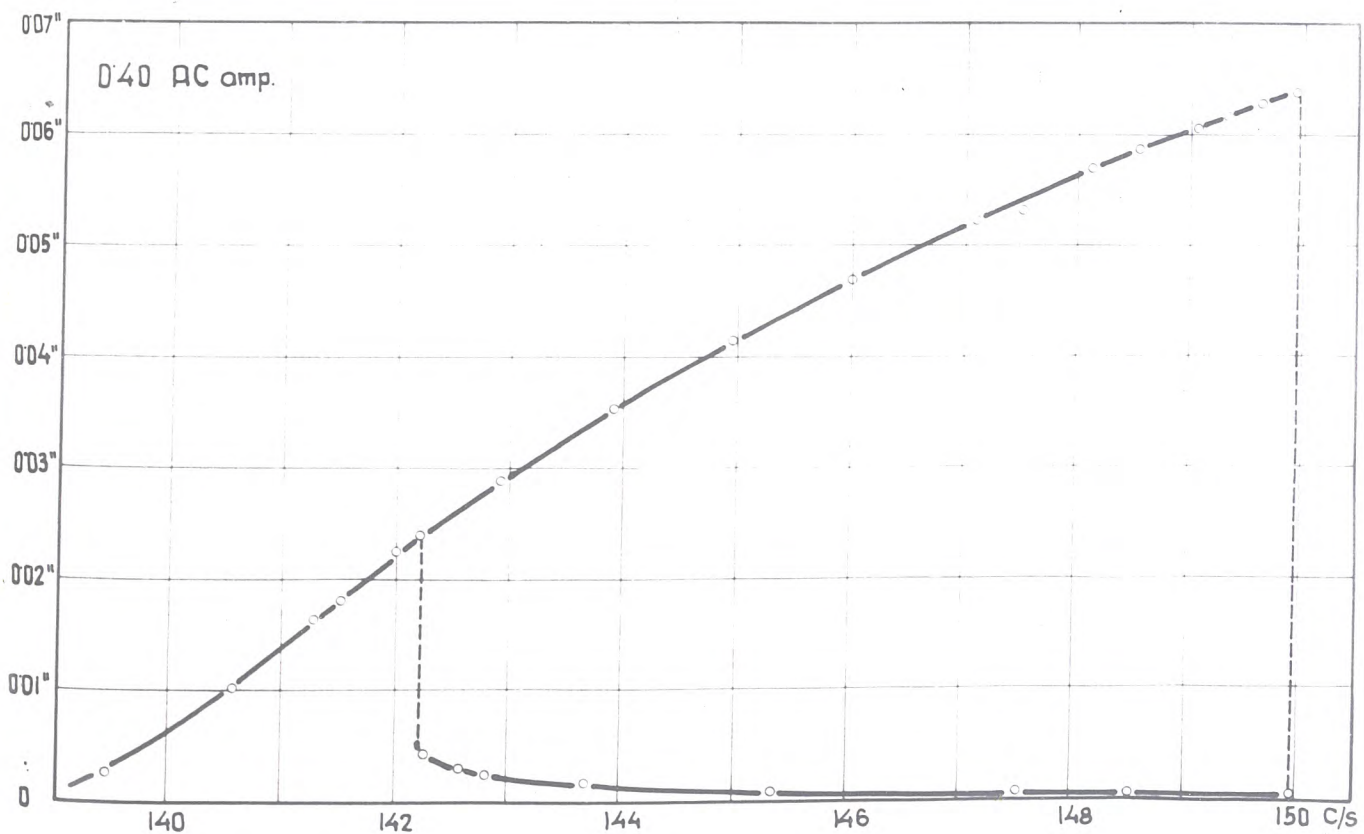
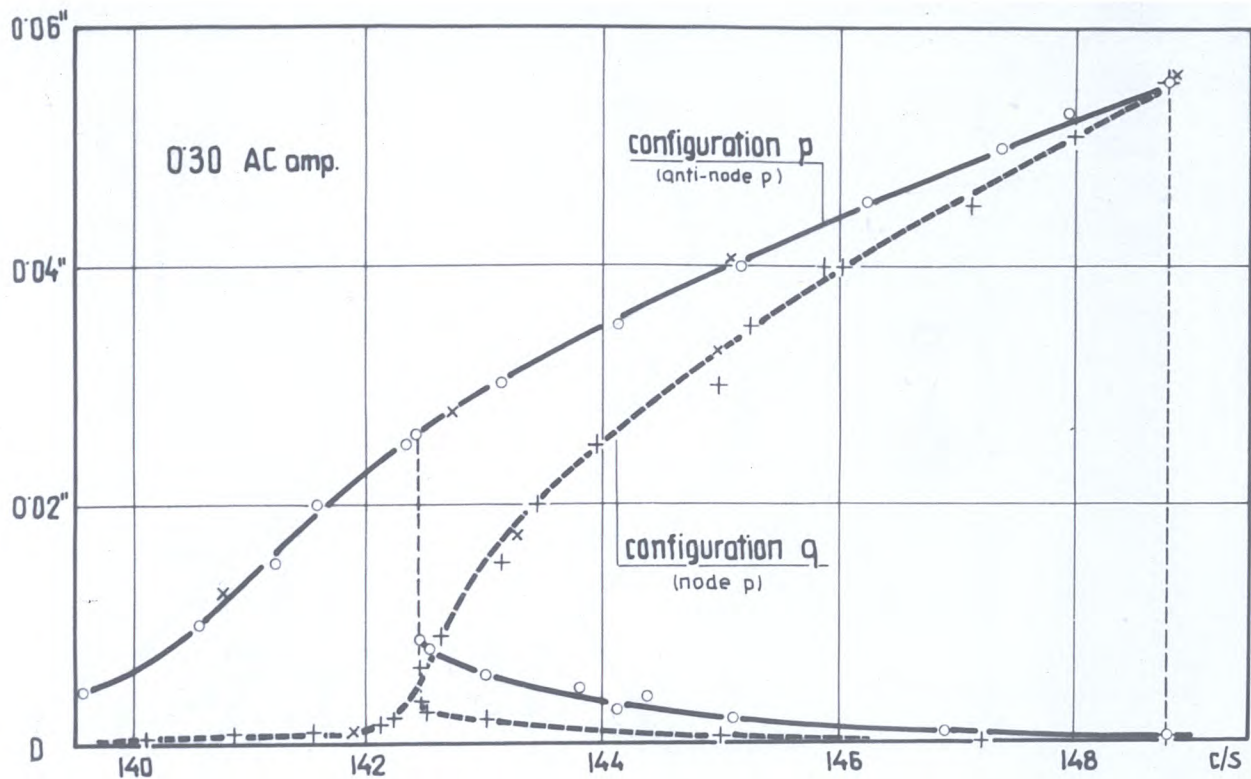
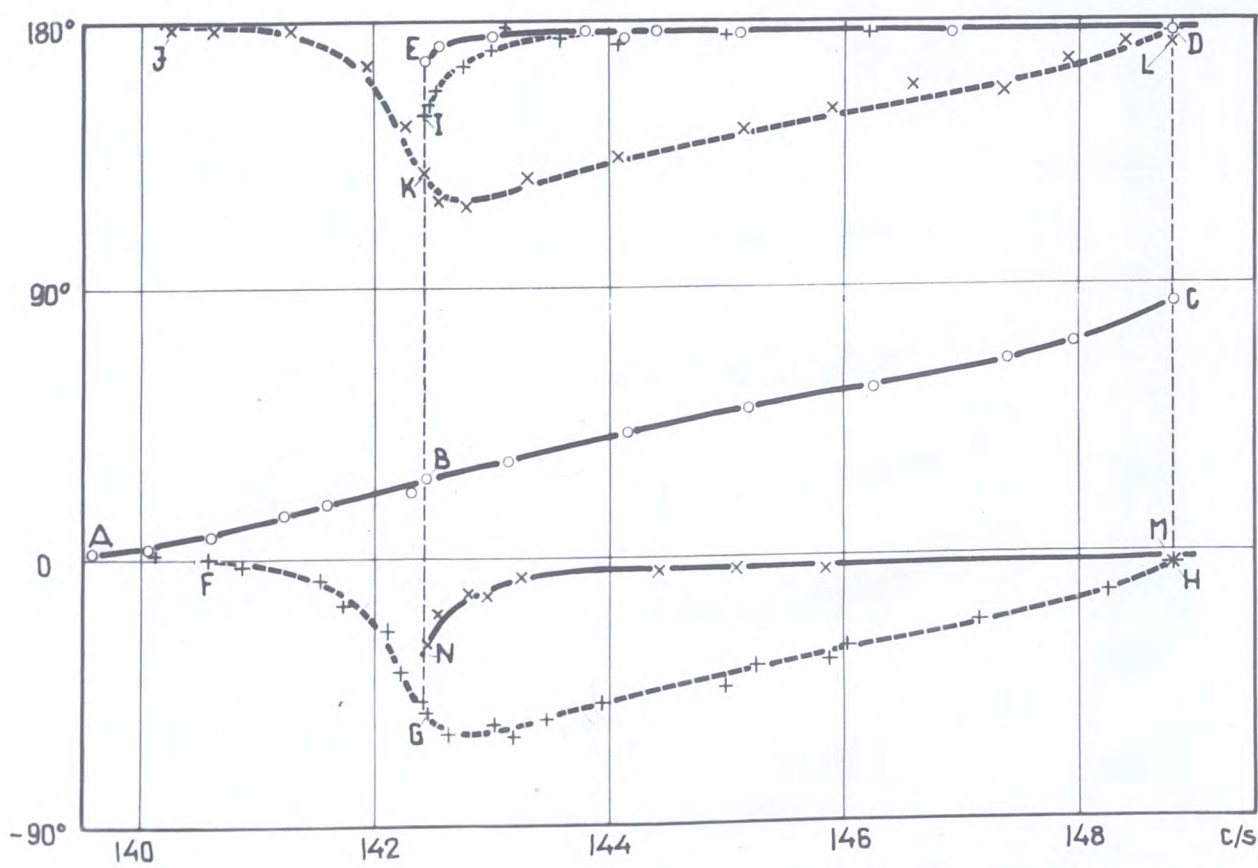


Fig. 53b



a.



b.

Fig.54

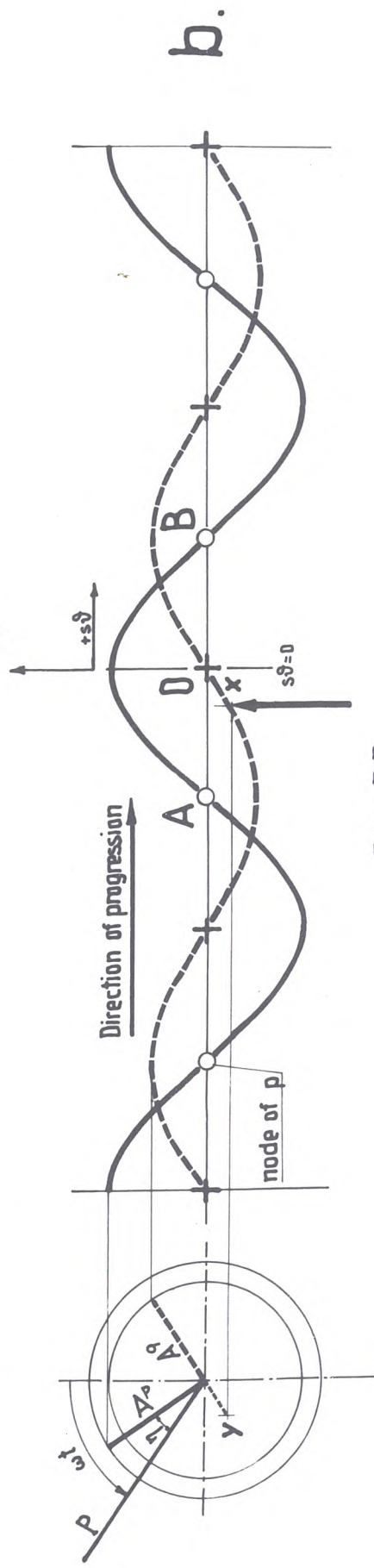
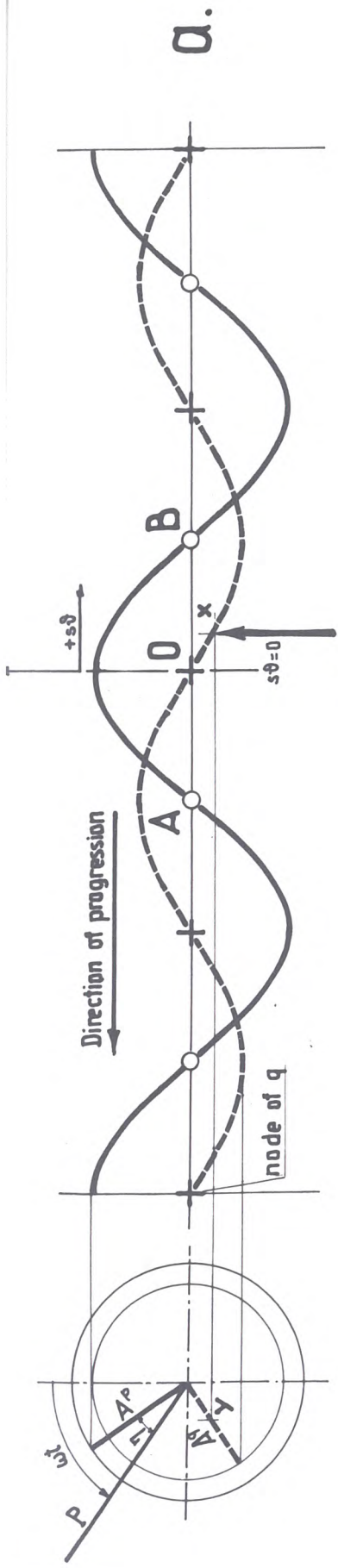


Fig. 55

2/0

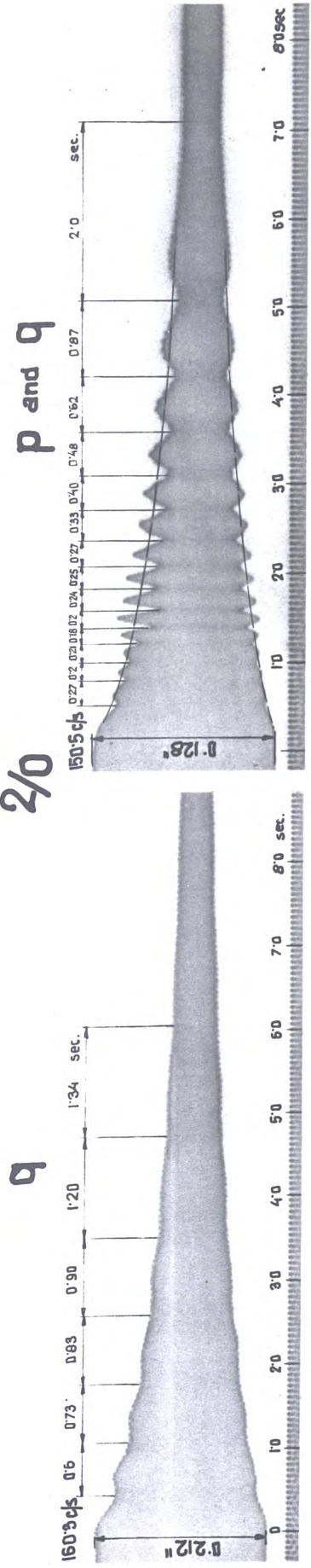


Fig. 56

a.

b.

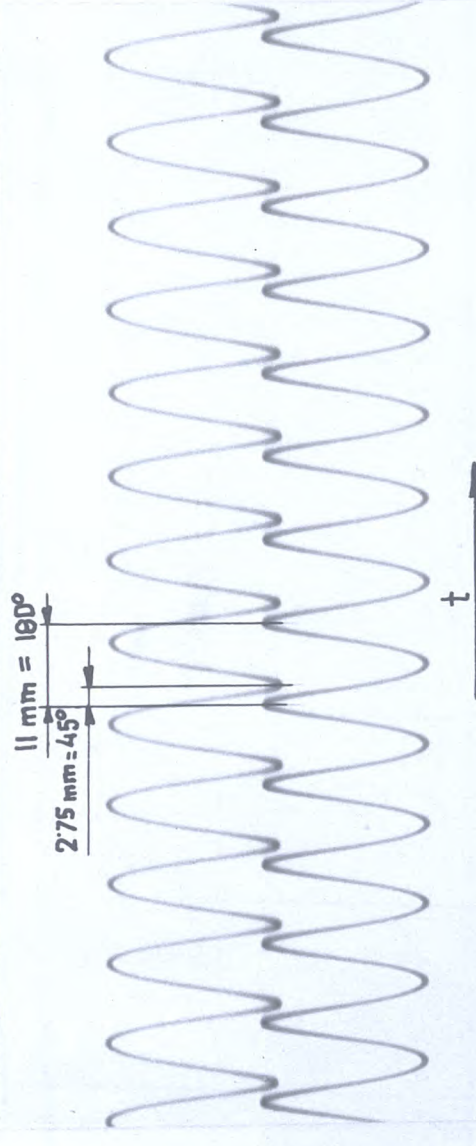
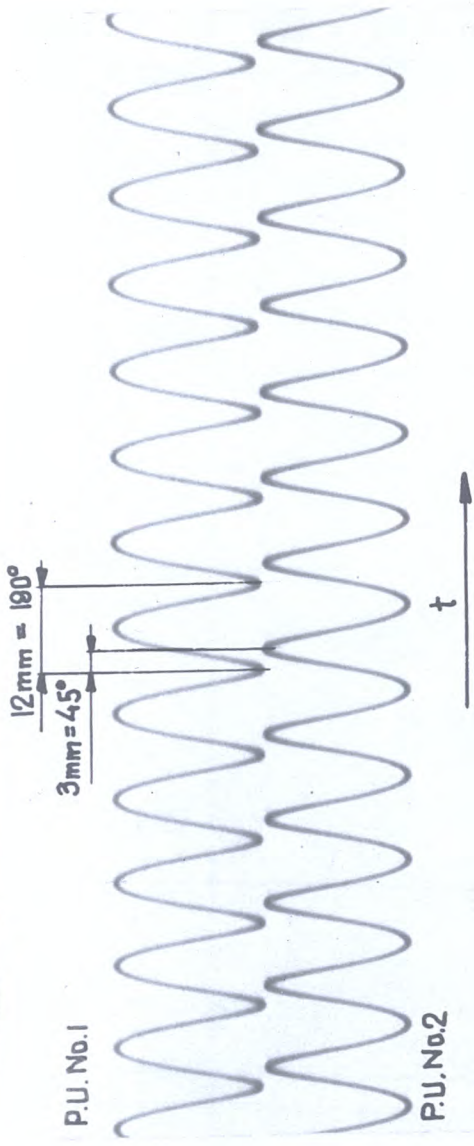
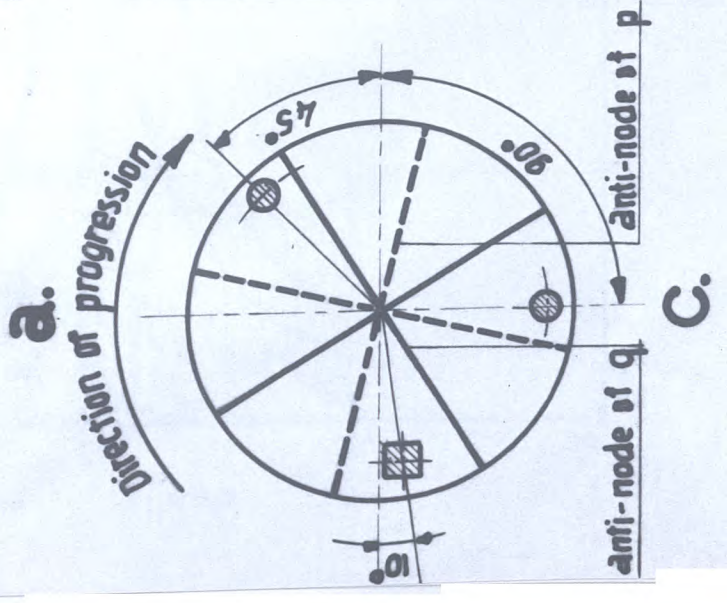
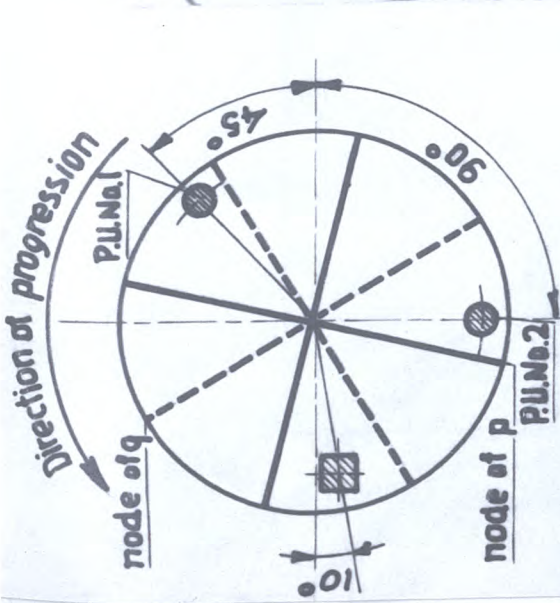


Fig. 57

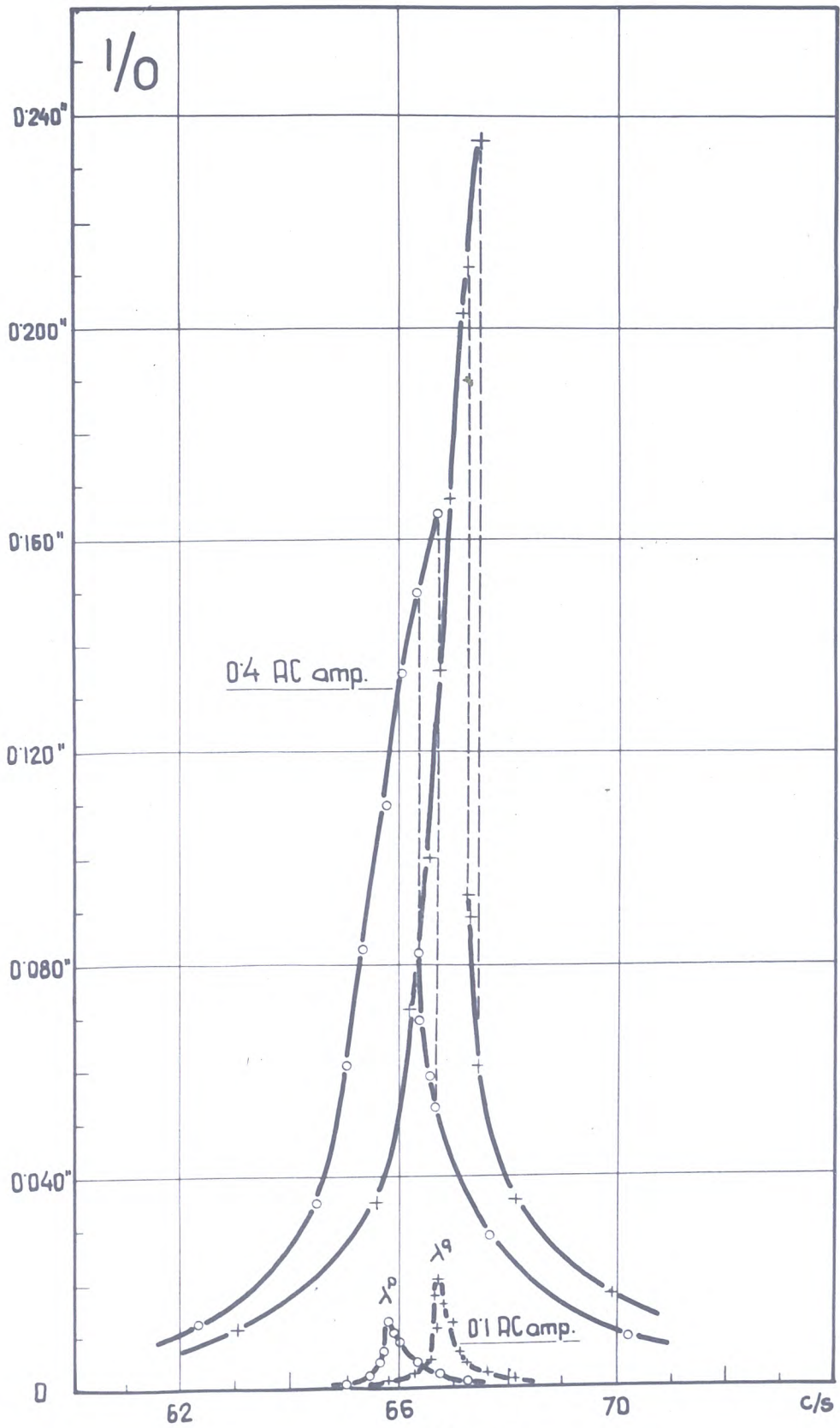


Fig.58

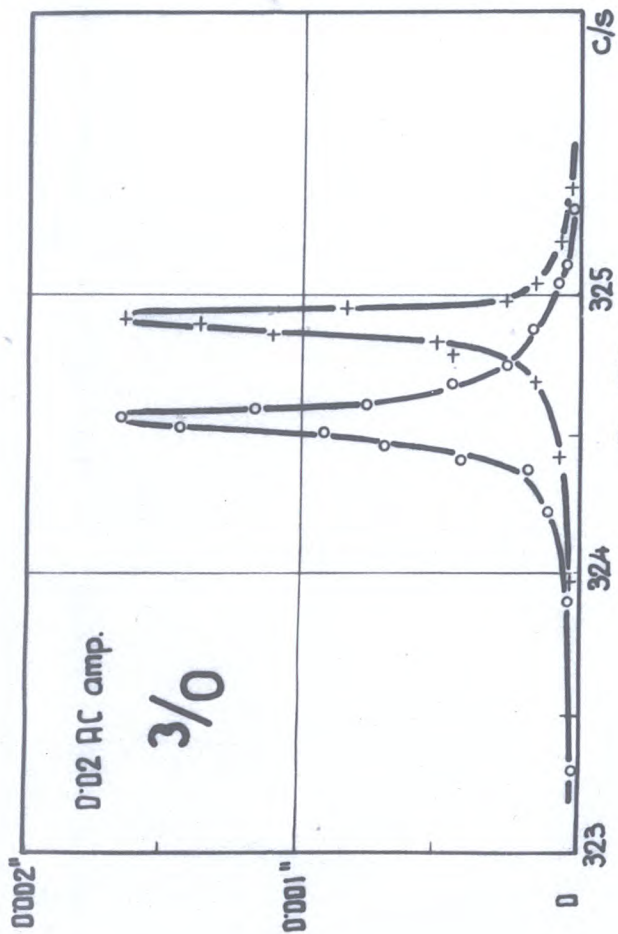


fig.59

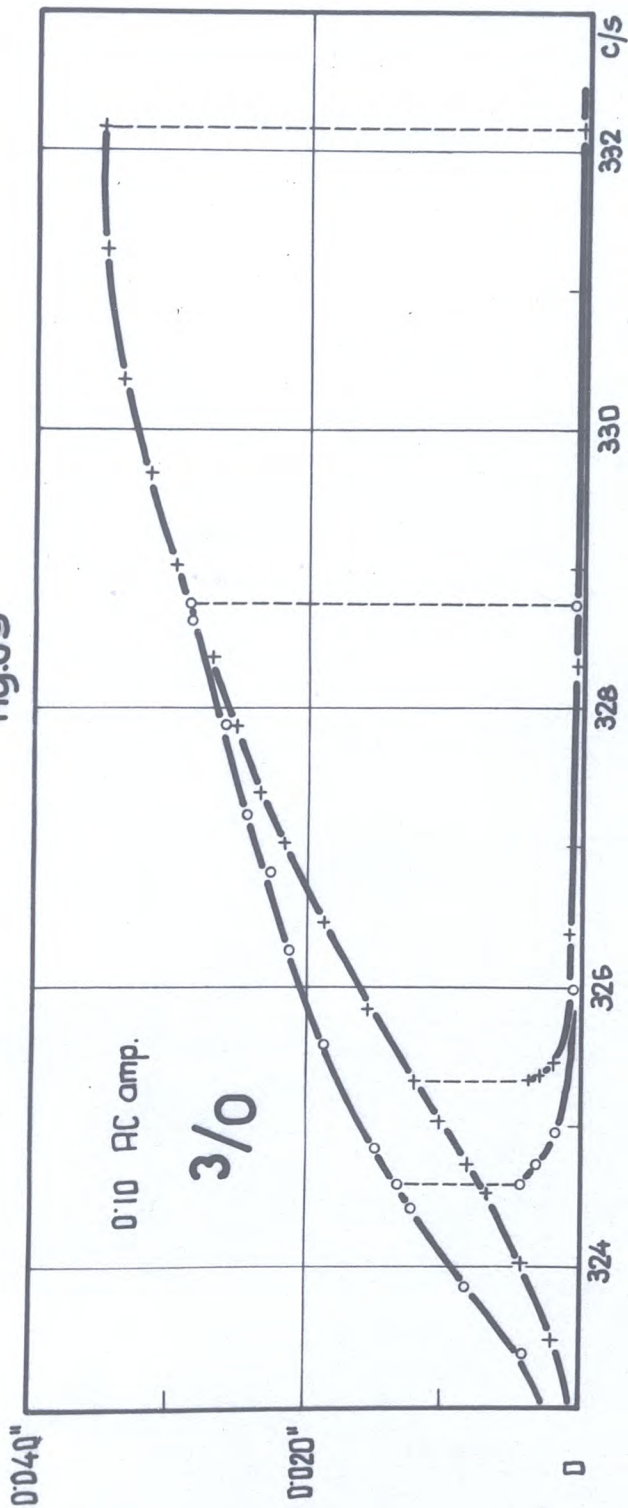


fig.60

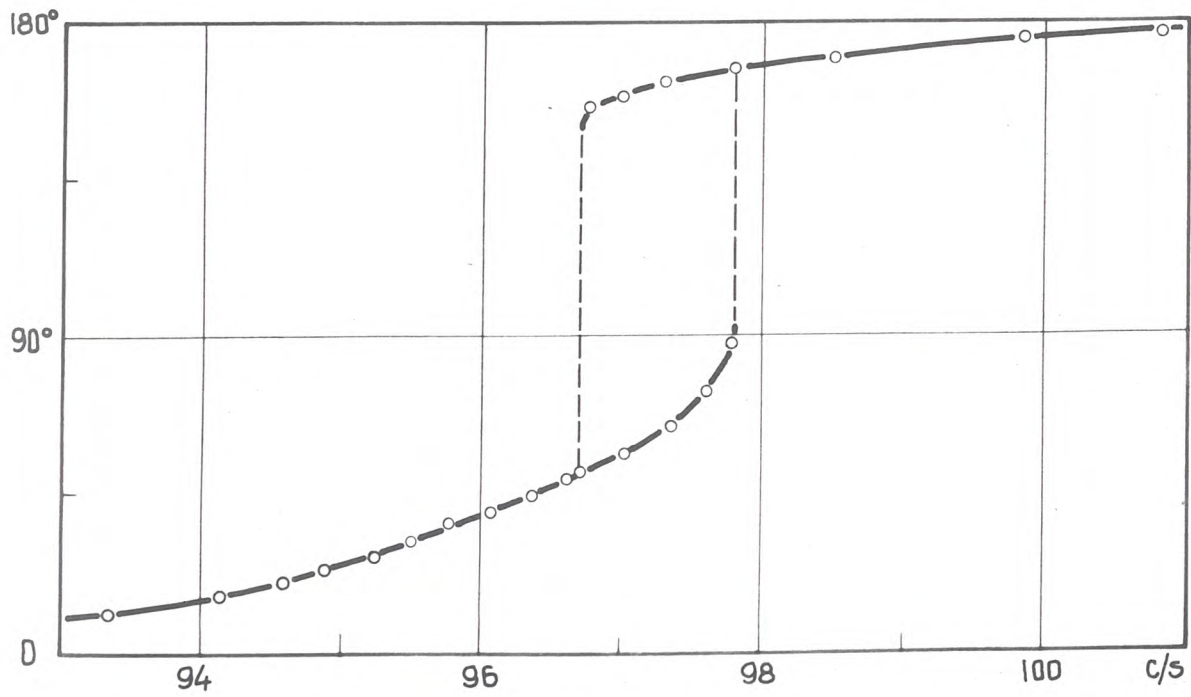
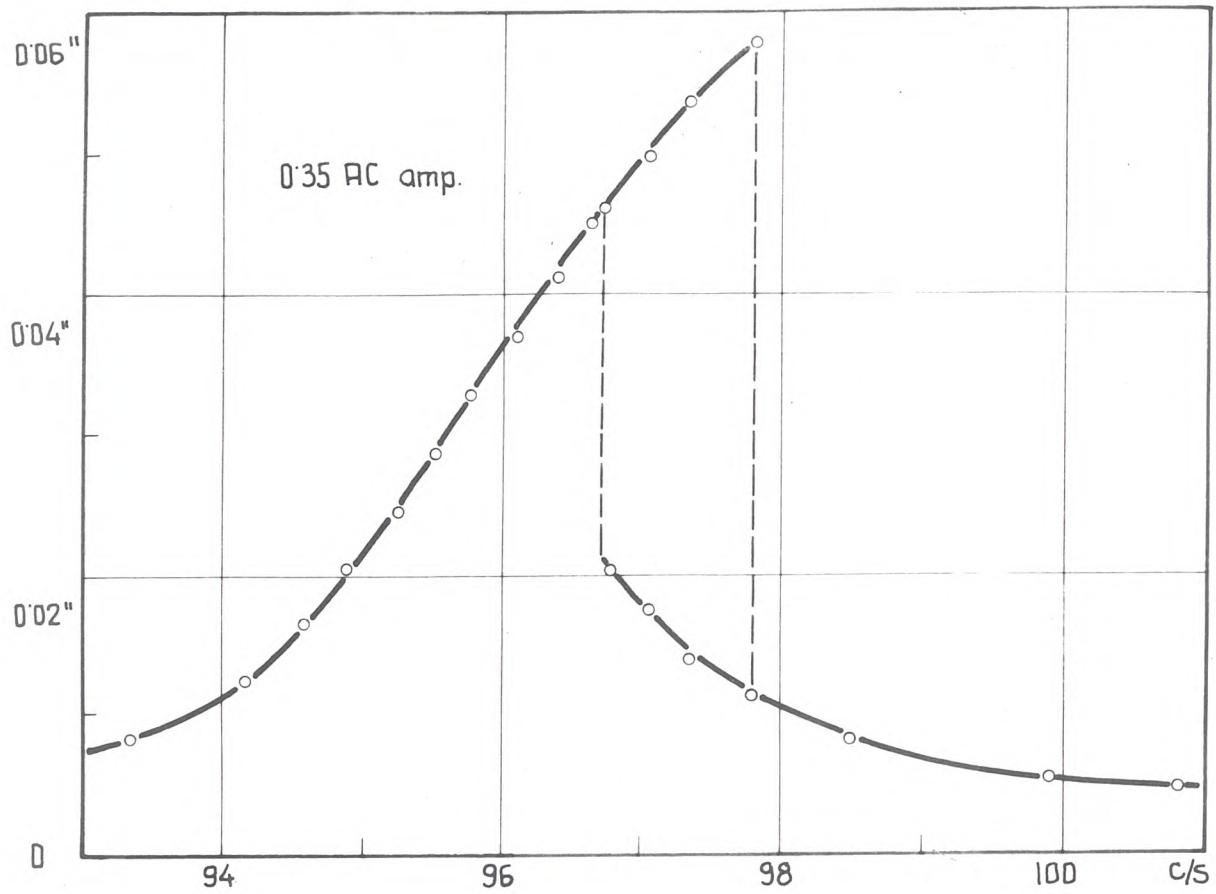


fig.61

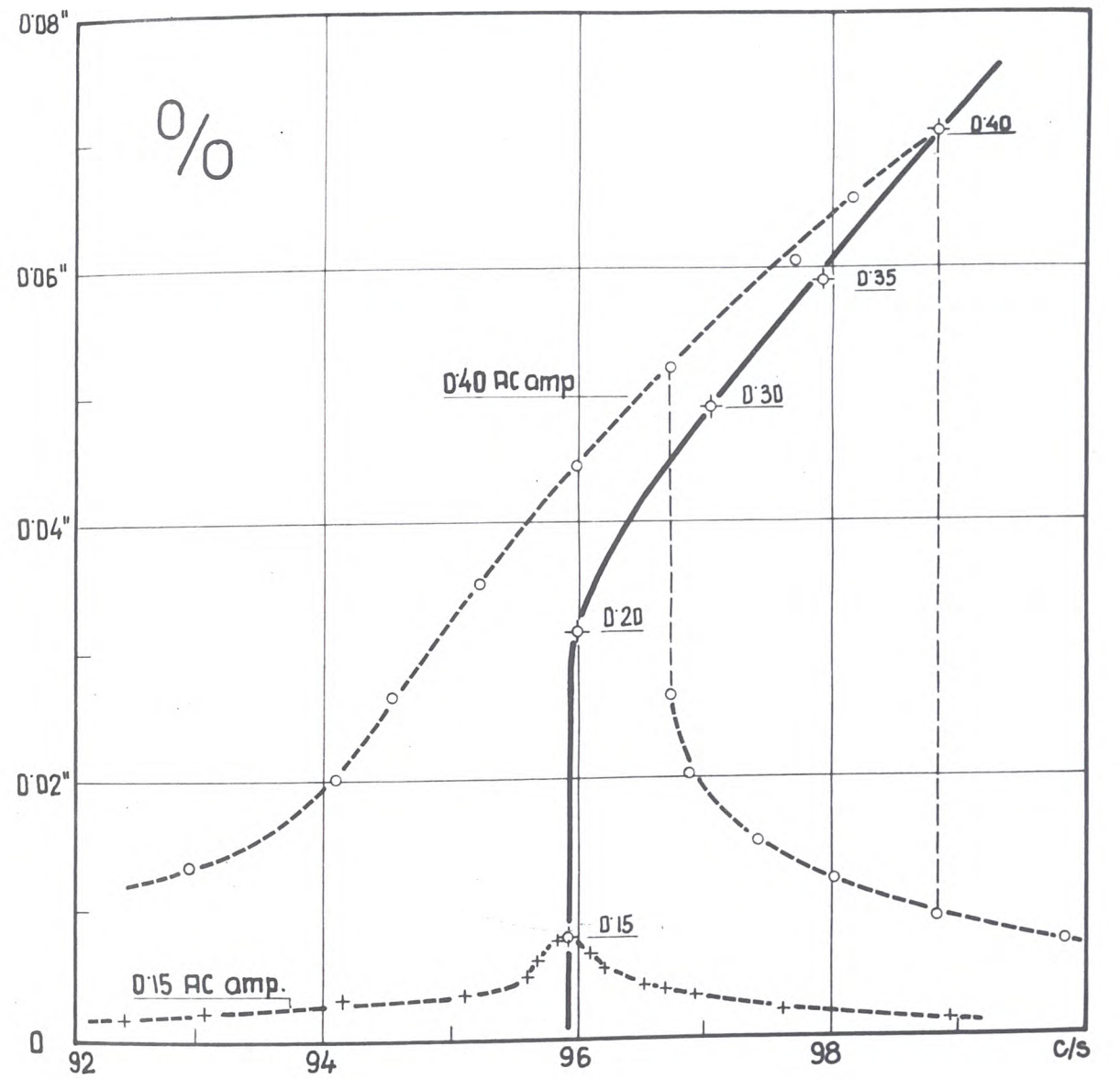


Fig.62

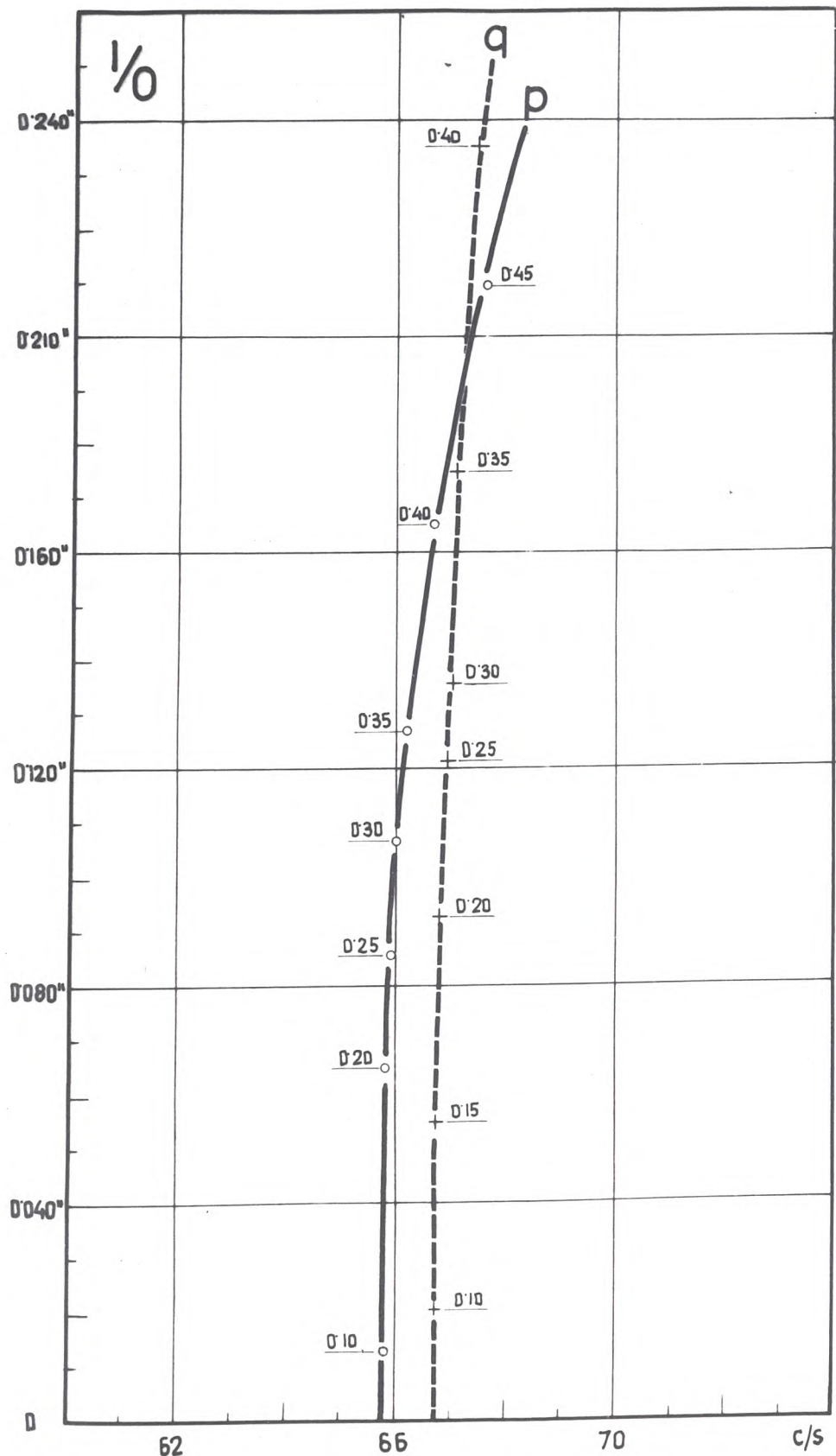


Fig. 63

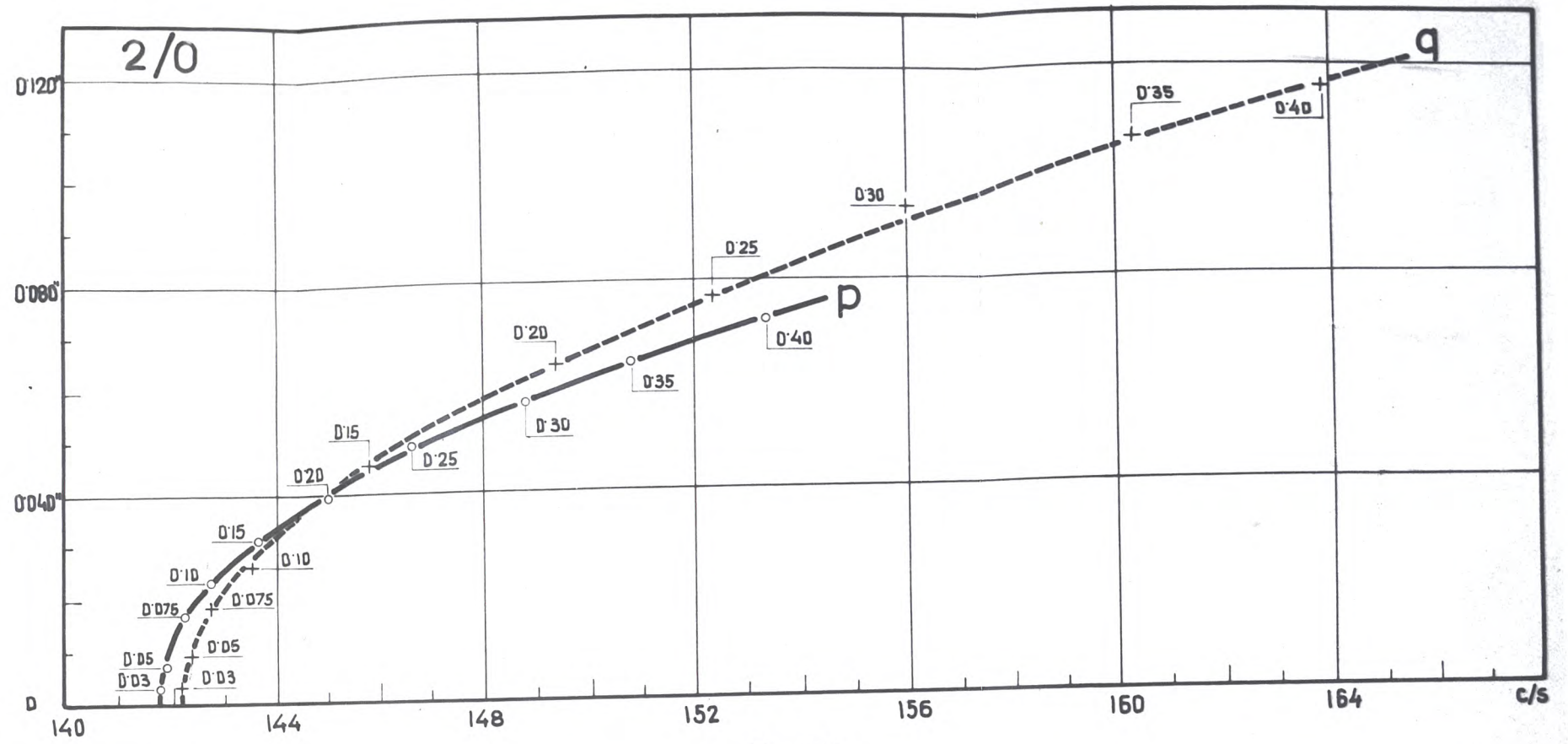


Fig. 64

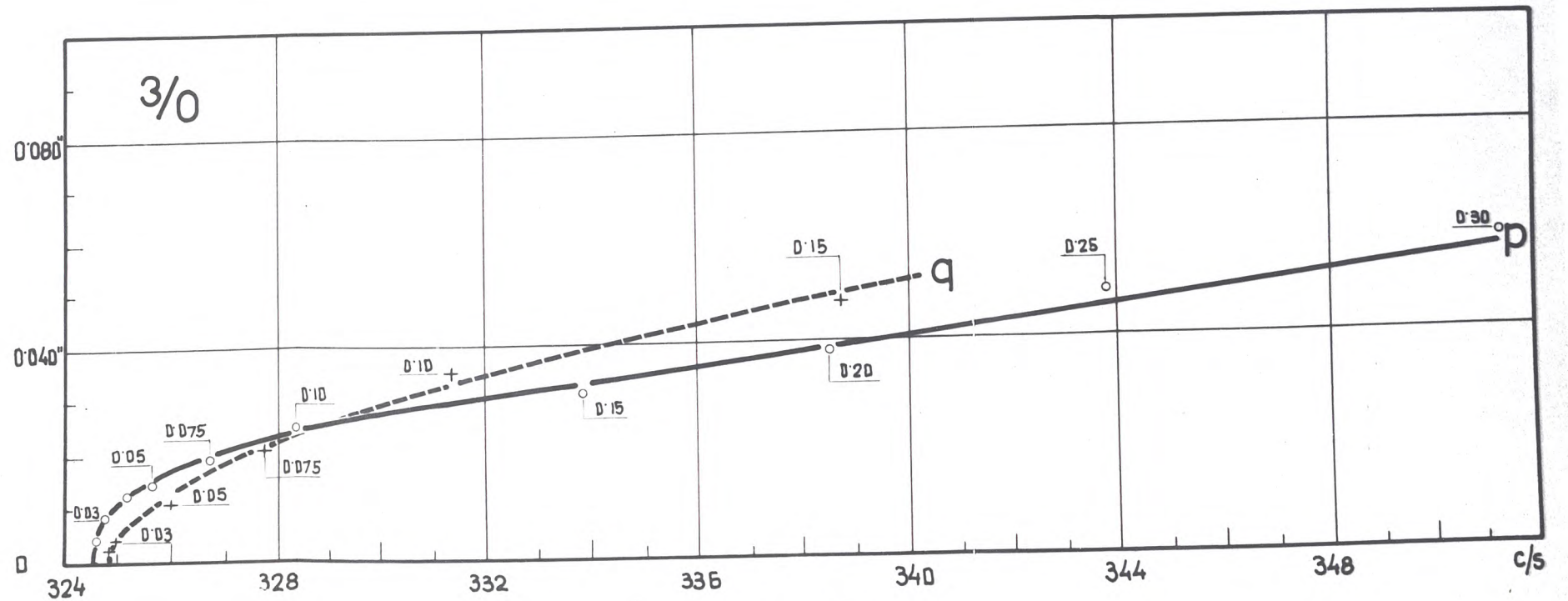
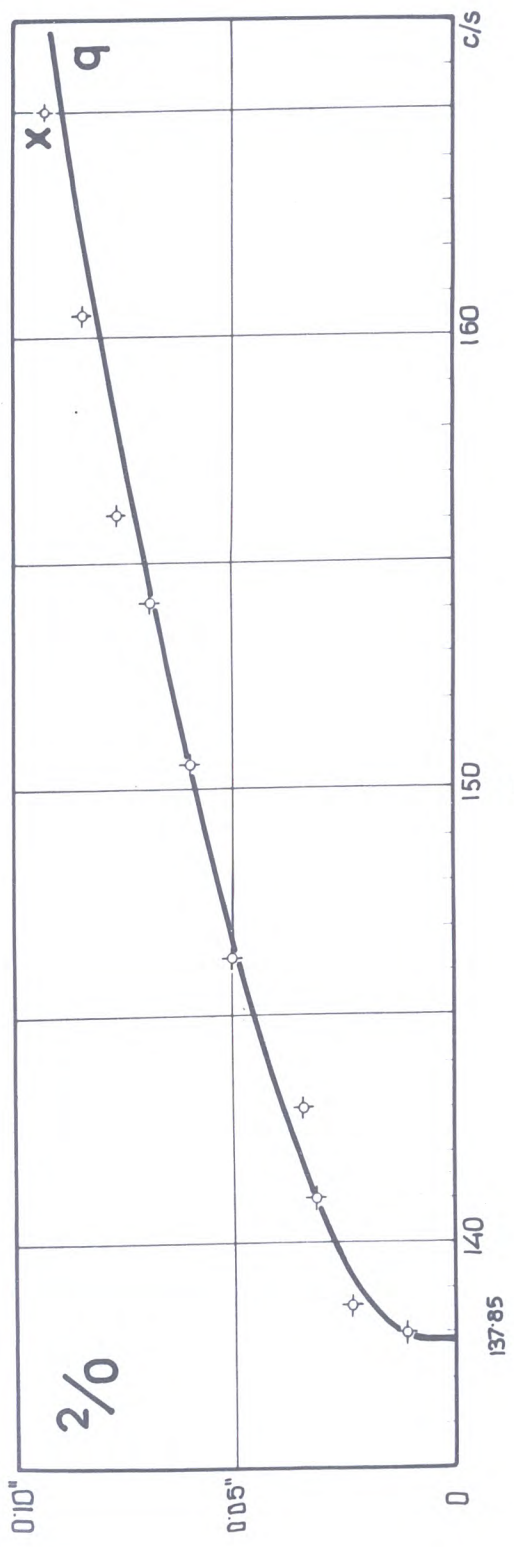
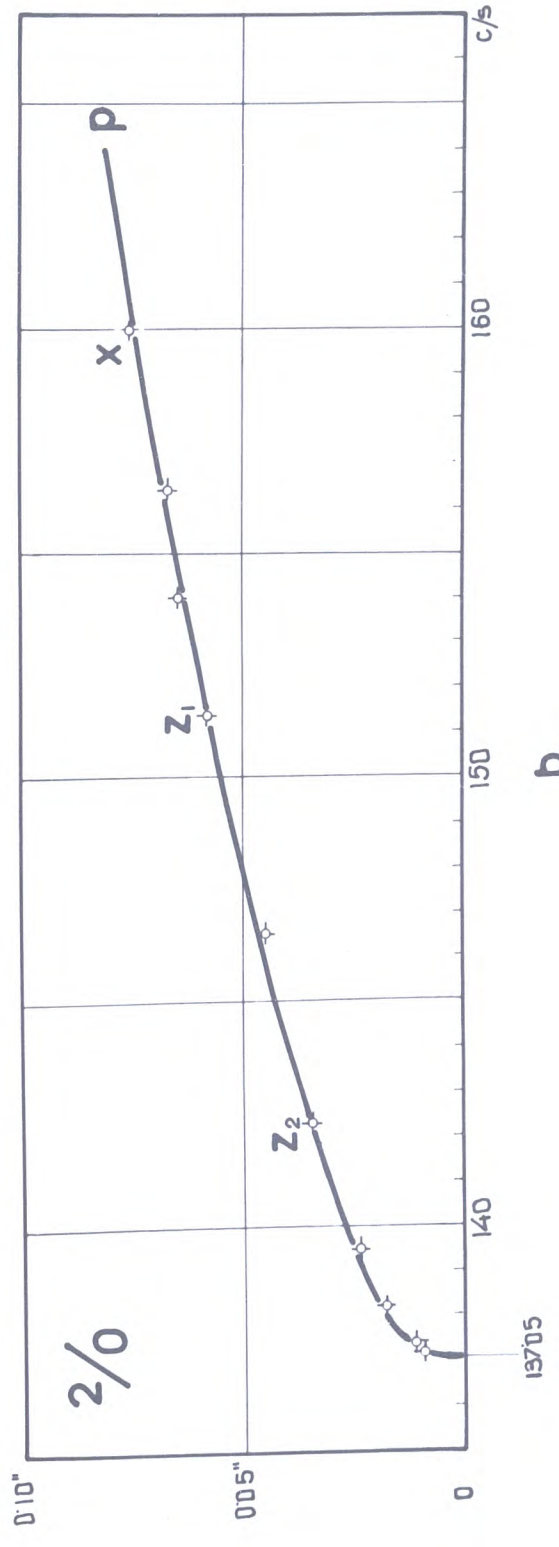


Fig. 65



a.



b.

Fig. 66

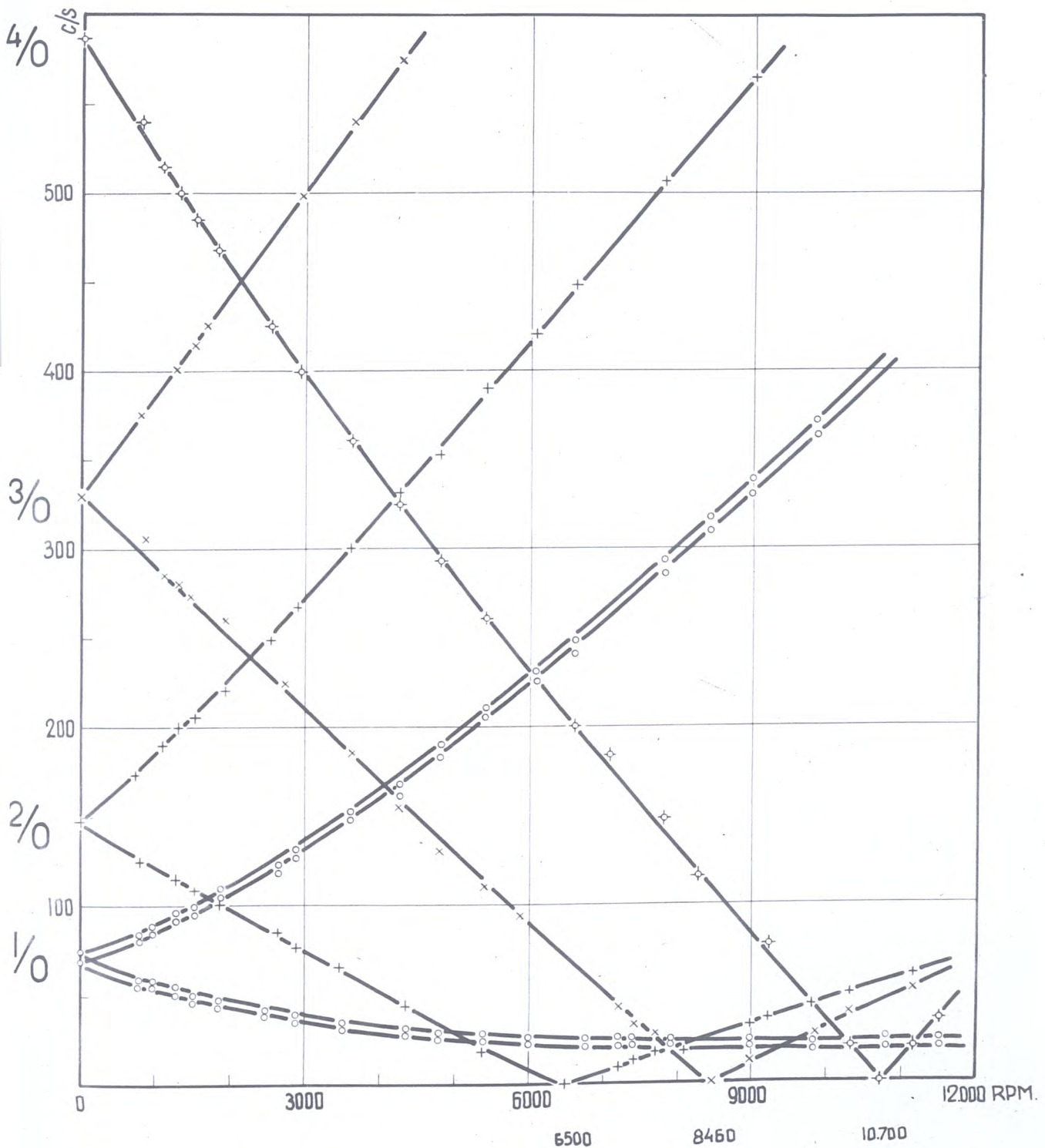


Fig.67

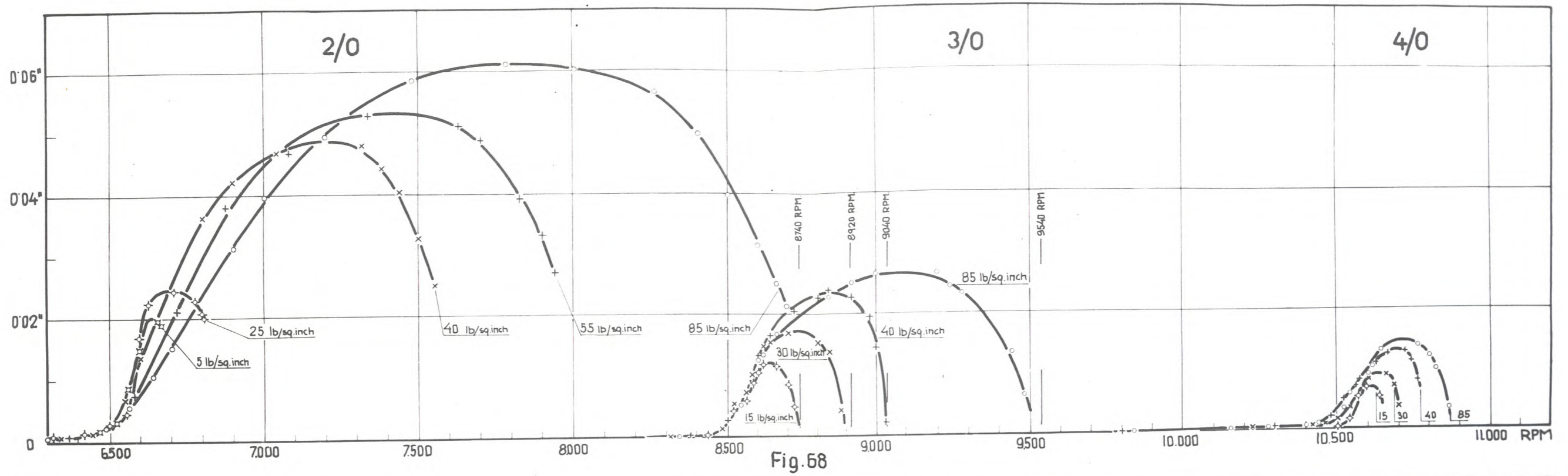


Fig. 68

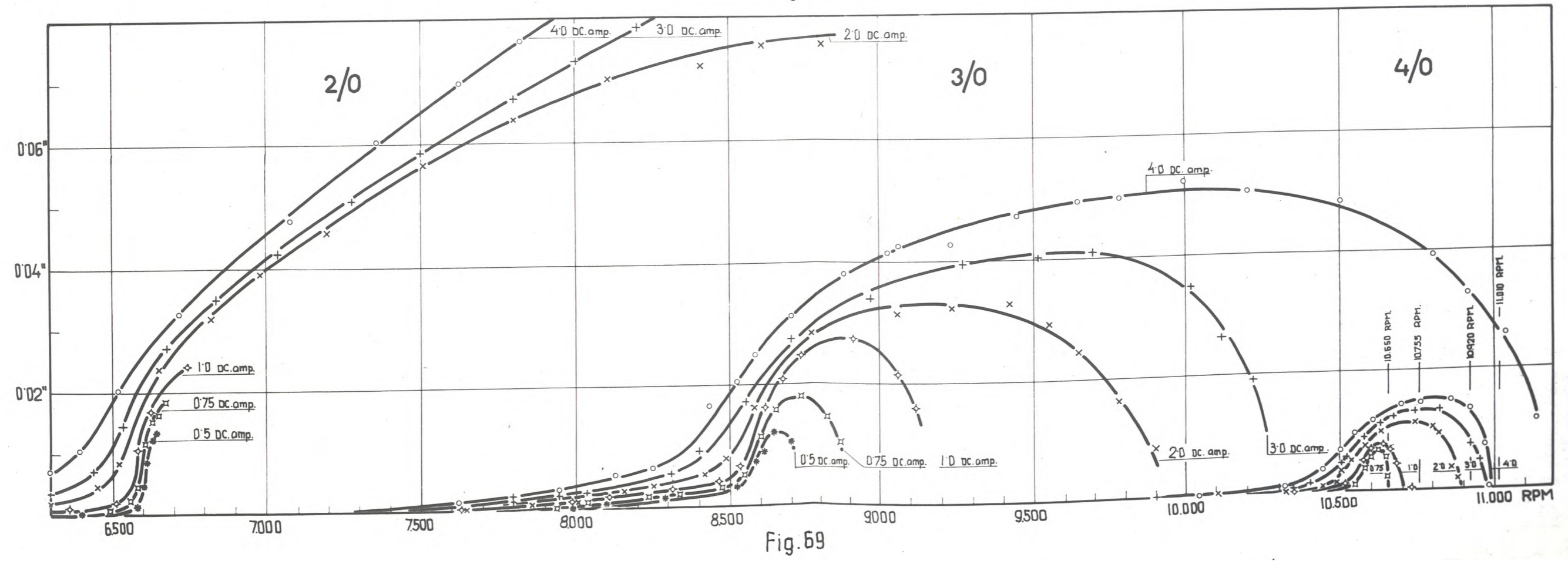


Fig. 69

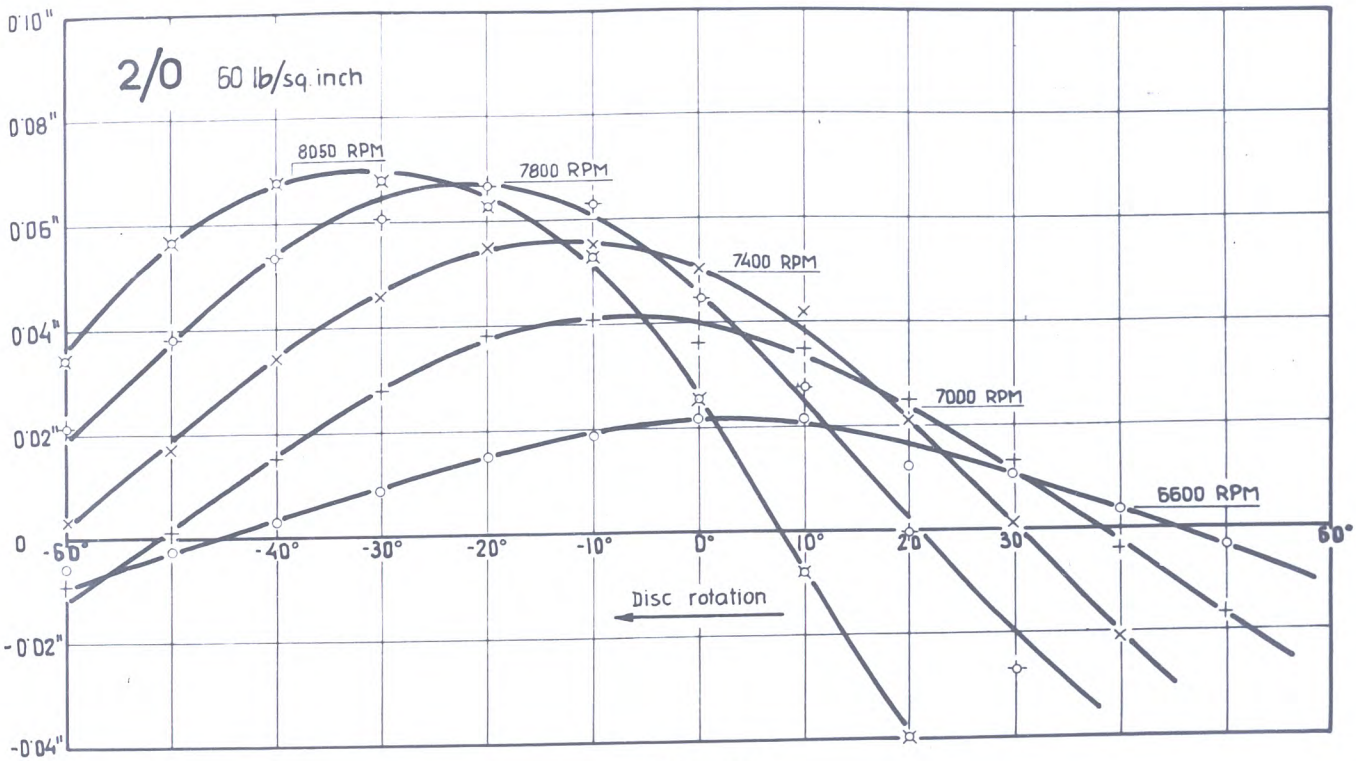


Fig.70

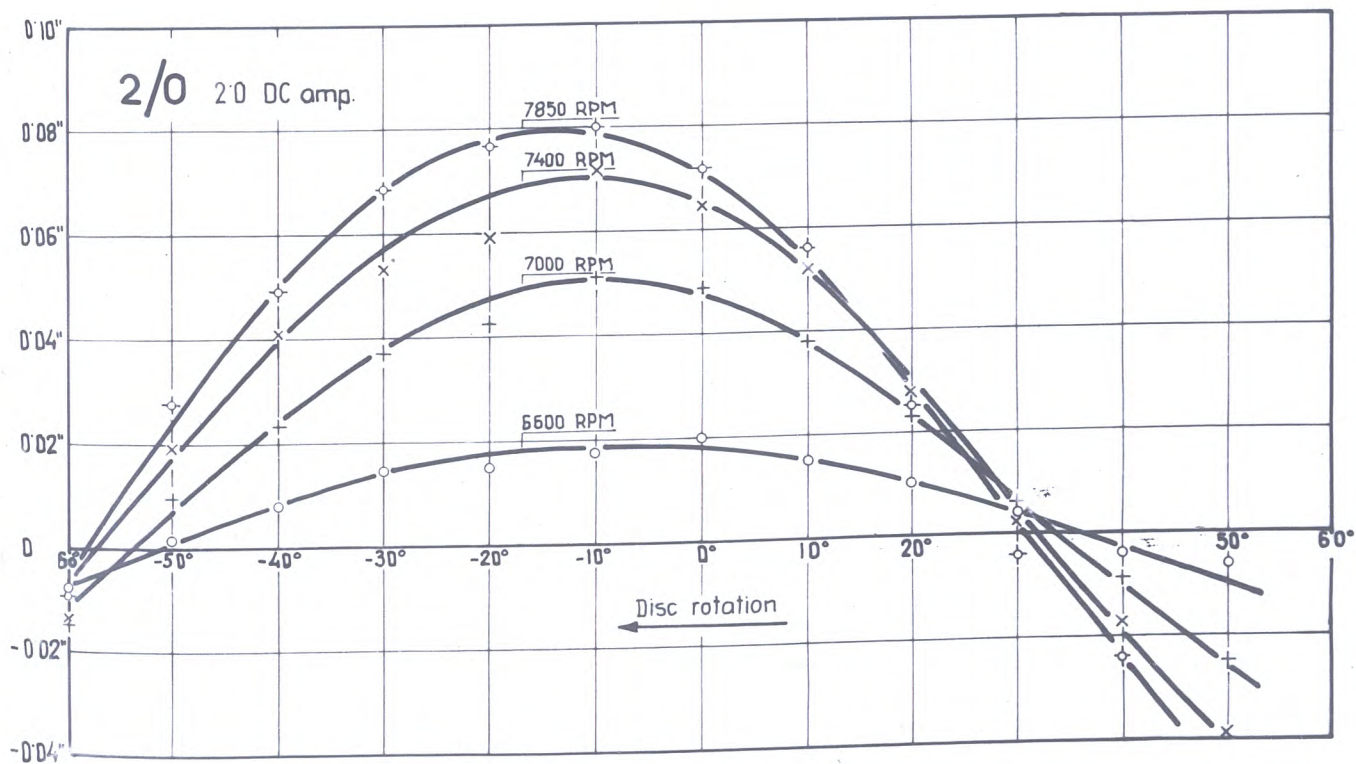


Fig.71

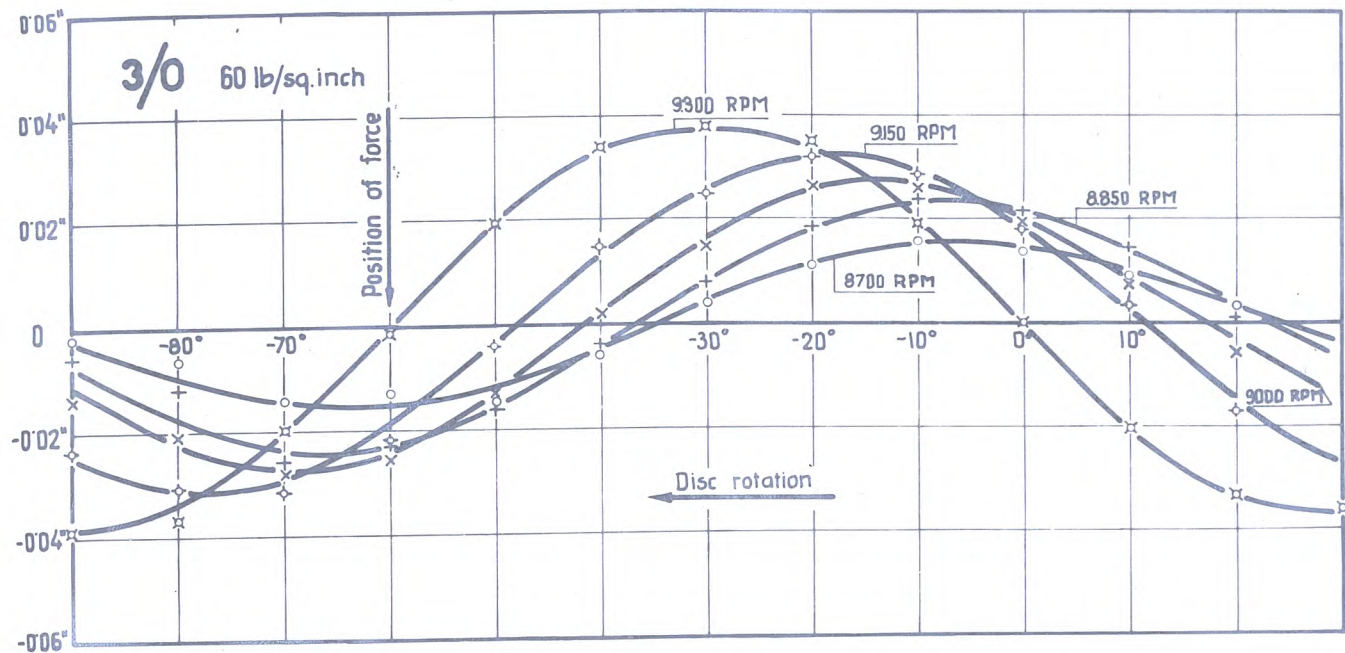


Fig. 72a

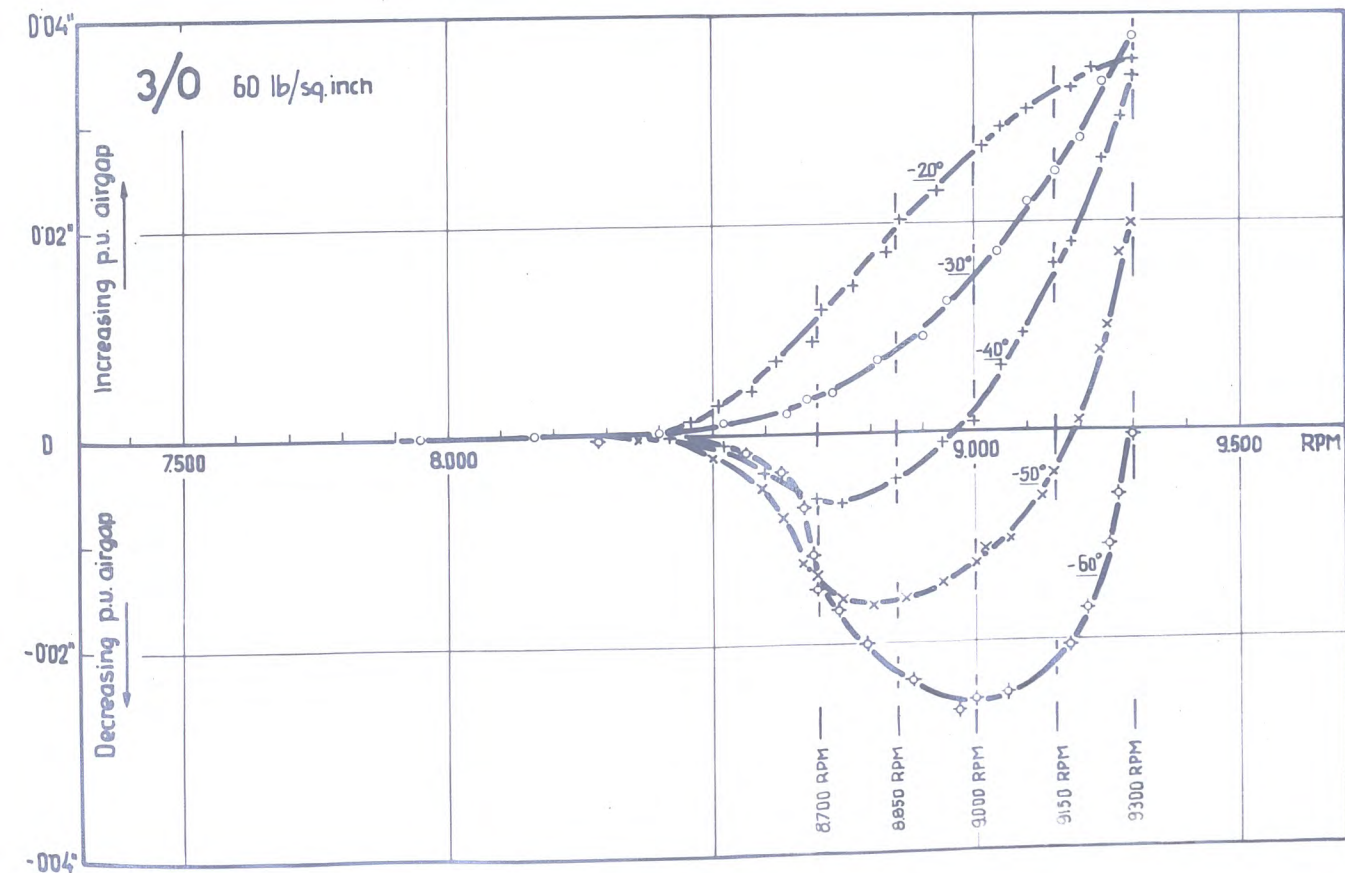


Fig. 72b

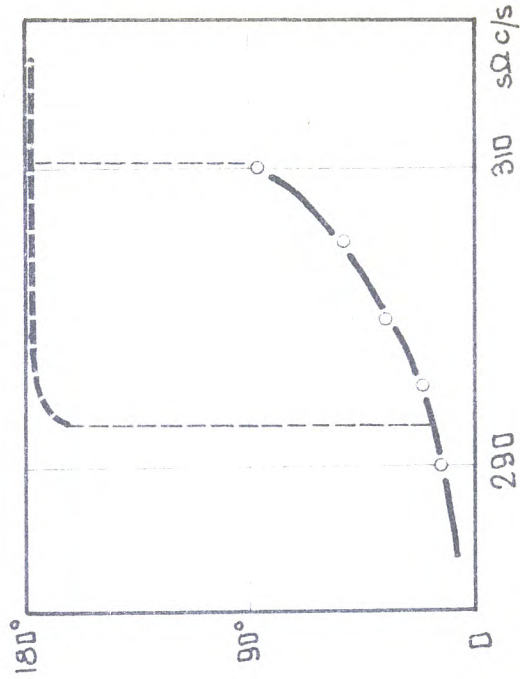
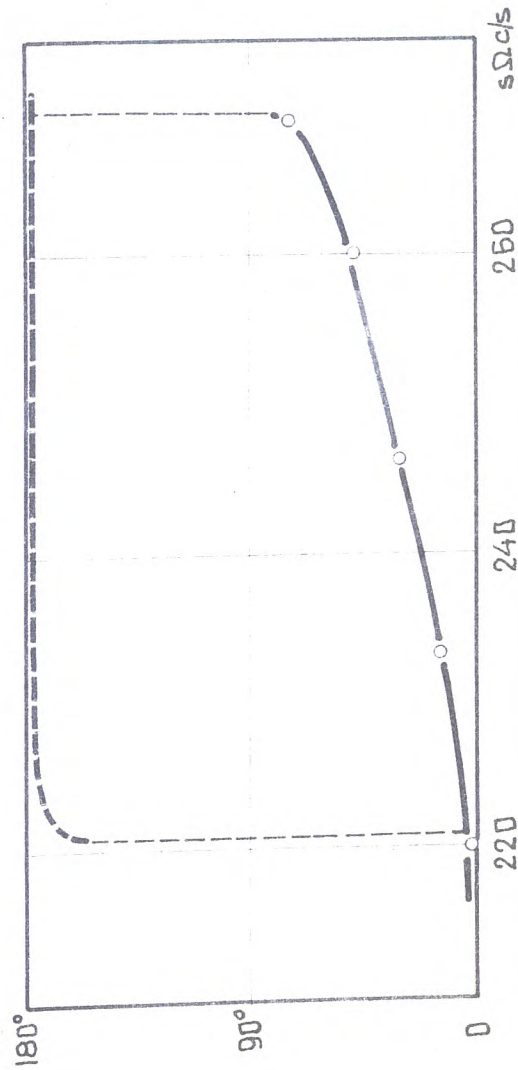
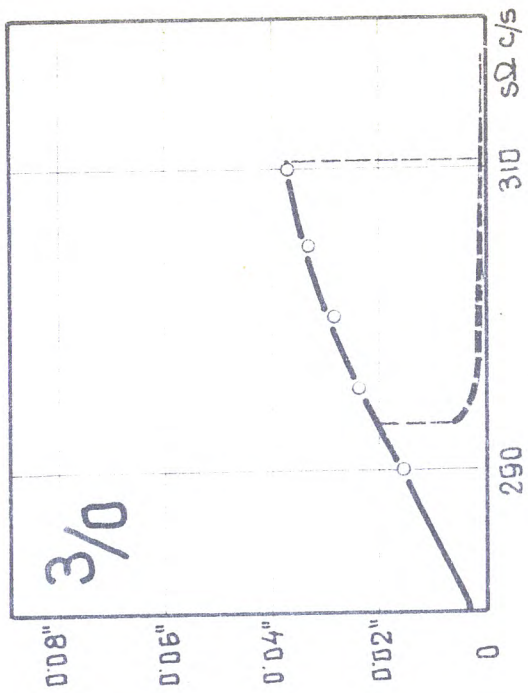
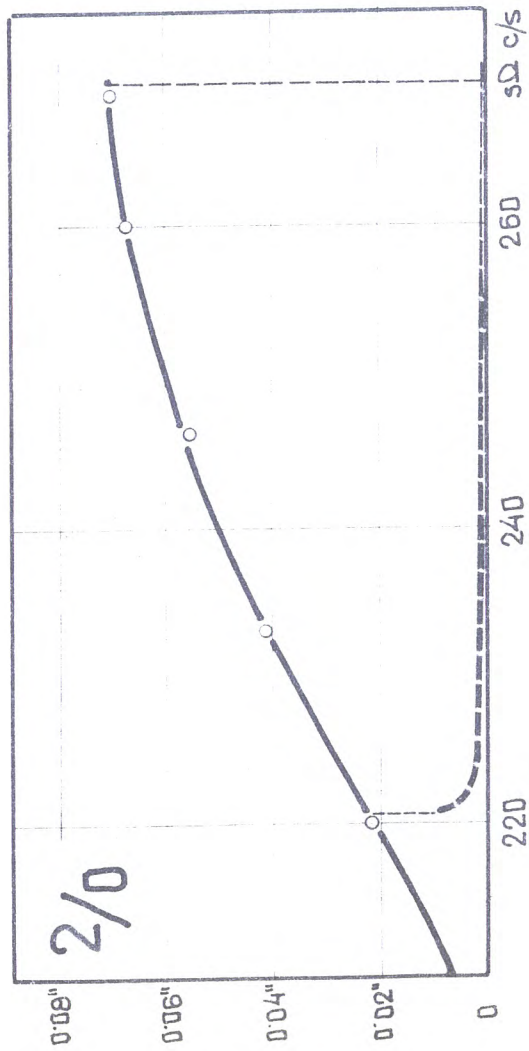
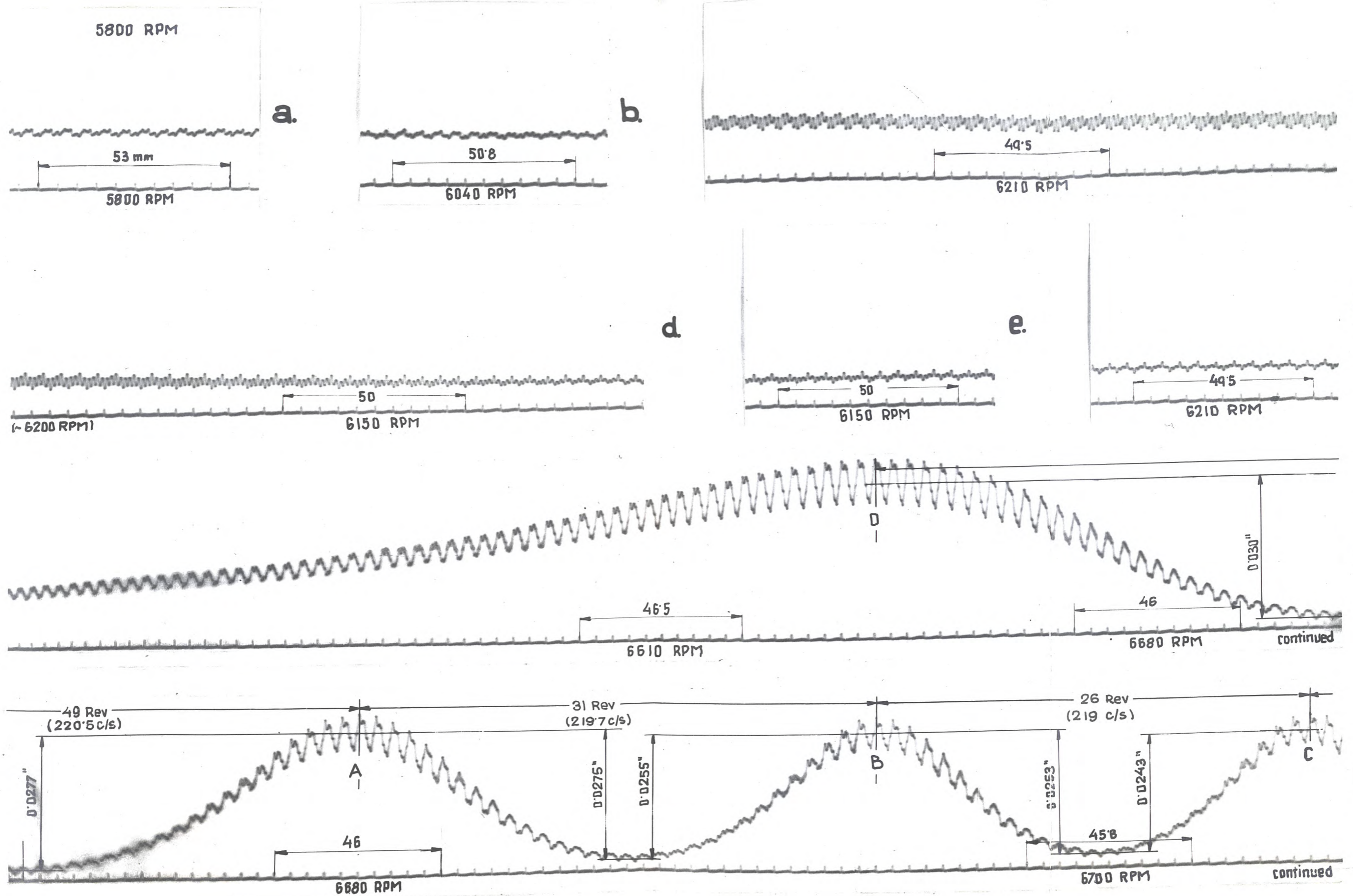


Fig. 73

Fig. 74



"Peak to peak" amplitude scale:
0 0.01" 0.02" 0.03"

fig.75

II.

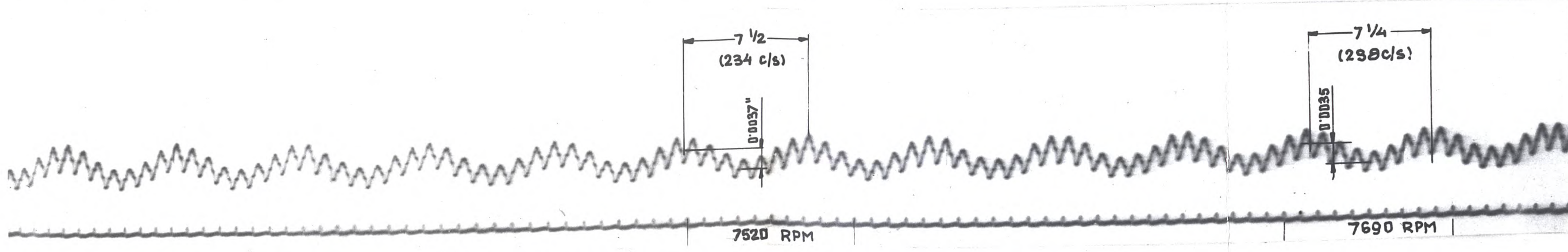
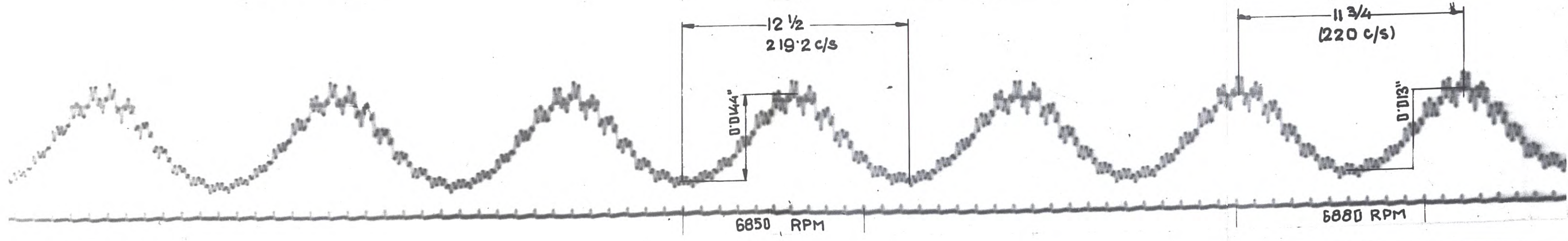
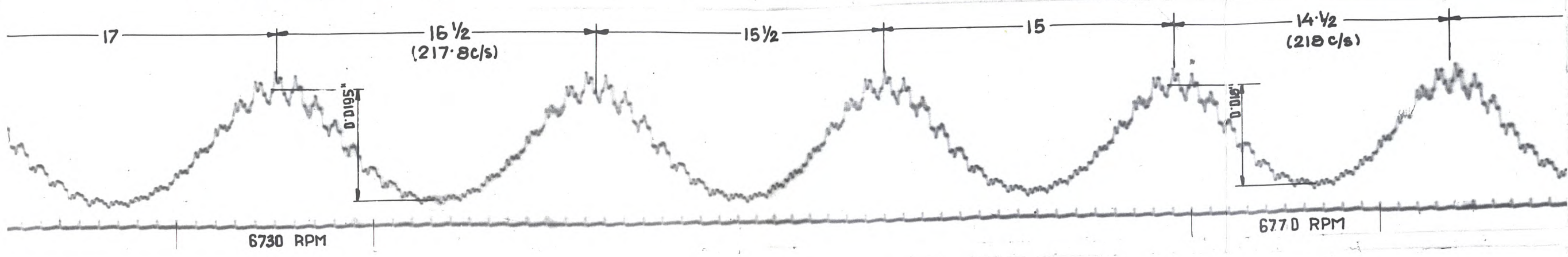
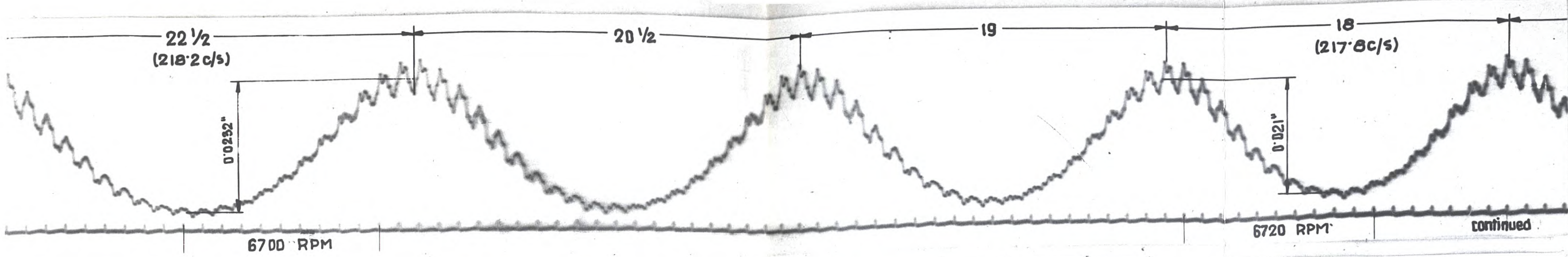
i.

j.

k.

l.

Fig. 75



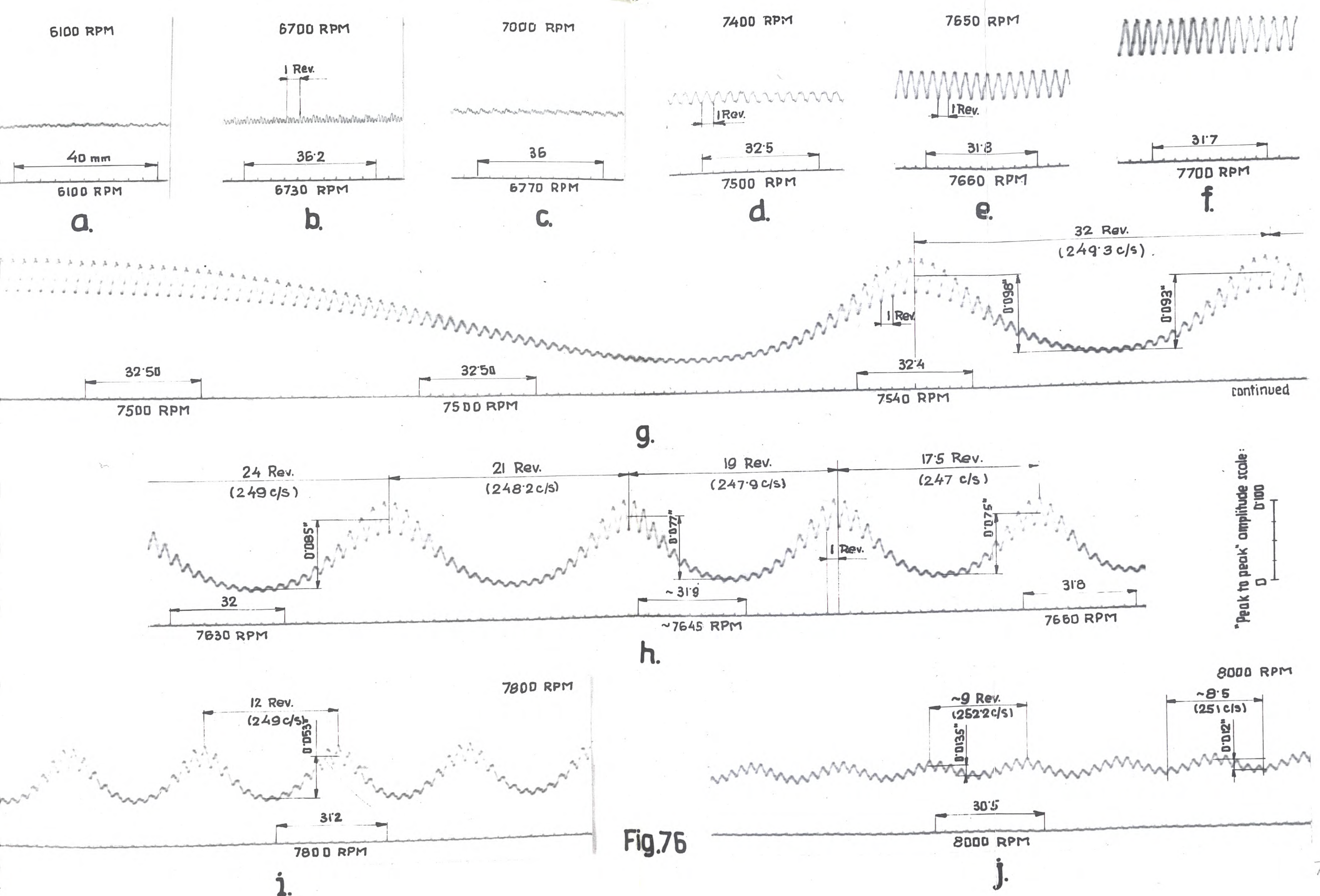


Fig.76

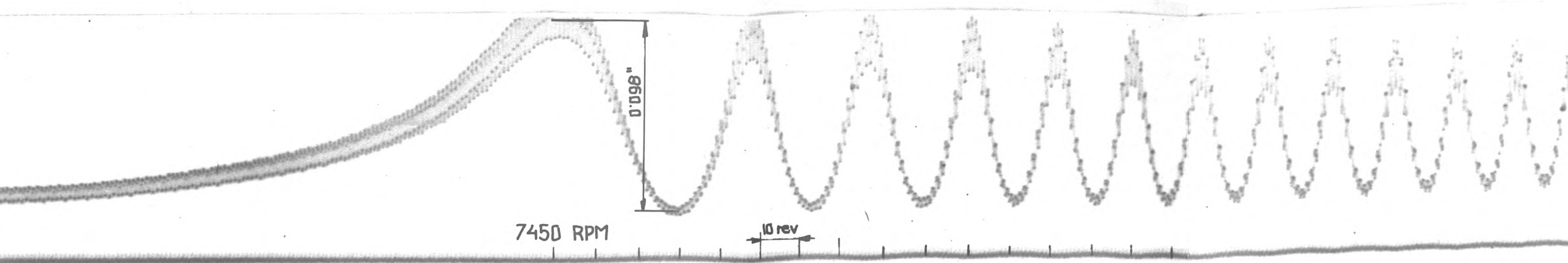


Fig. 77

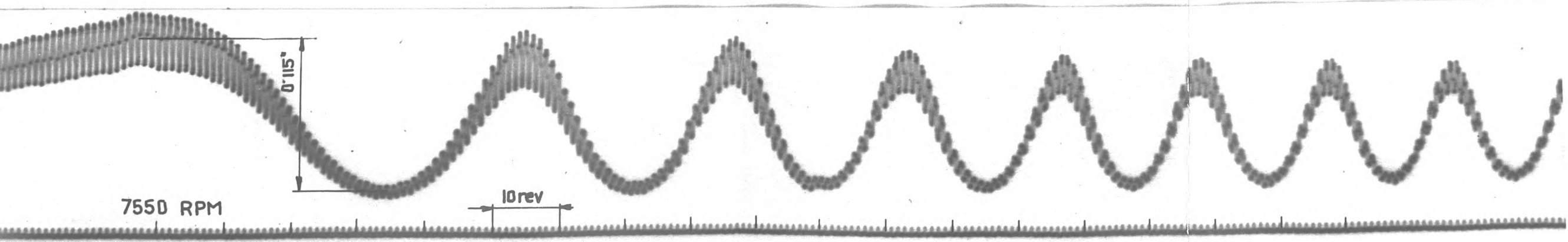


Fig. 78

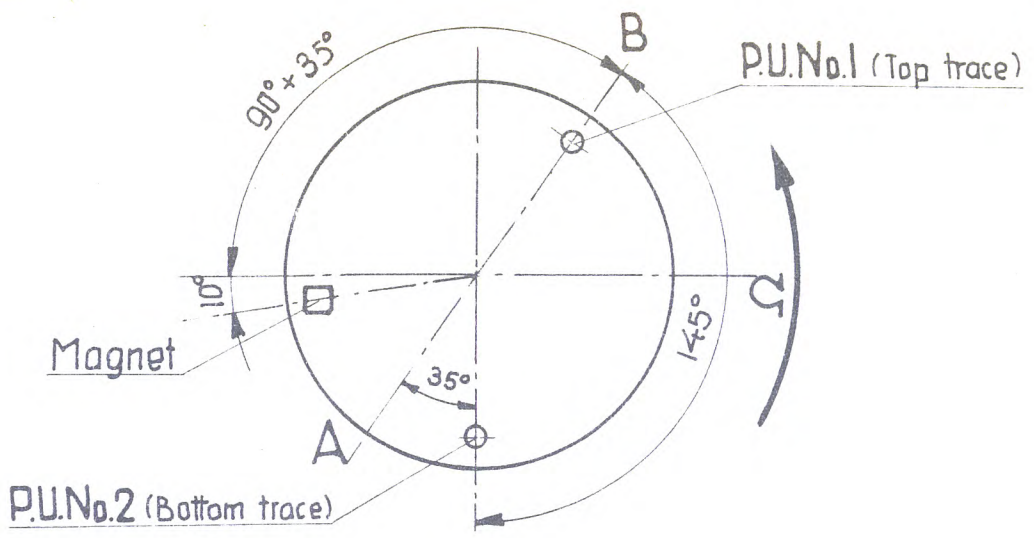


Fig.79a

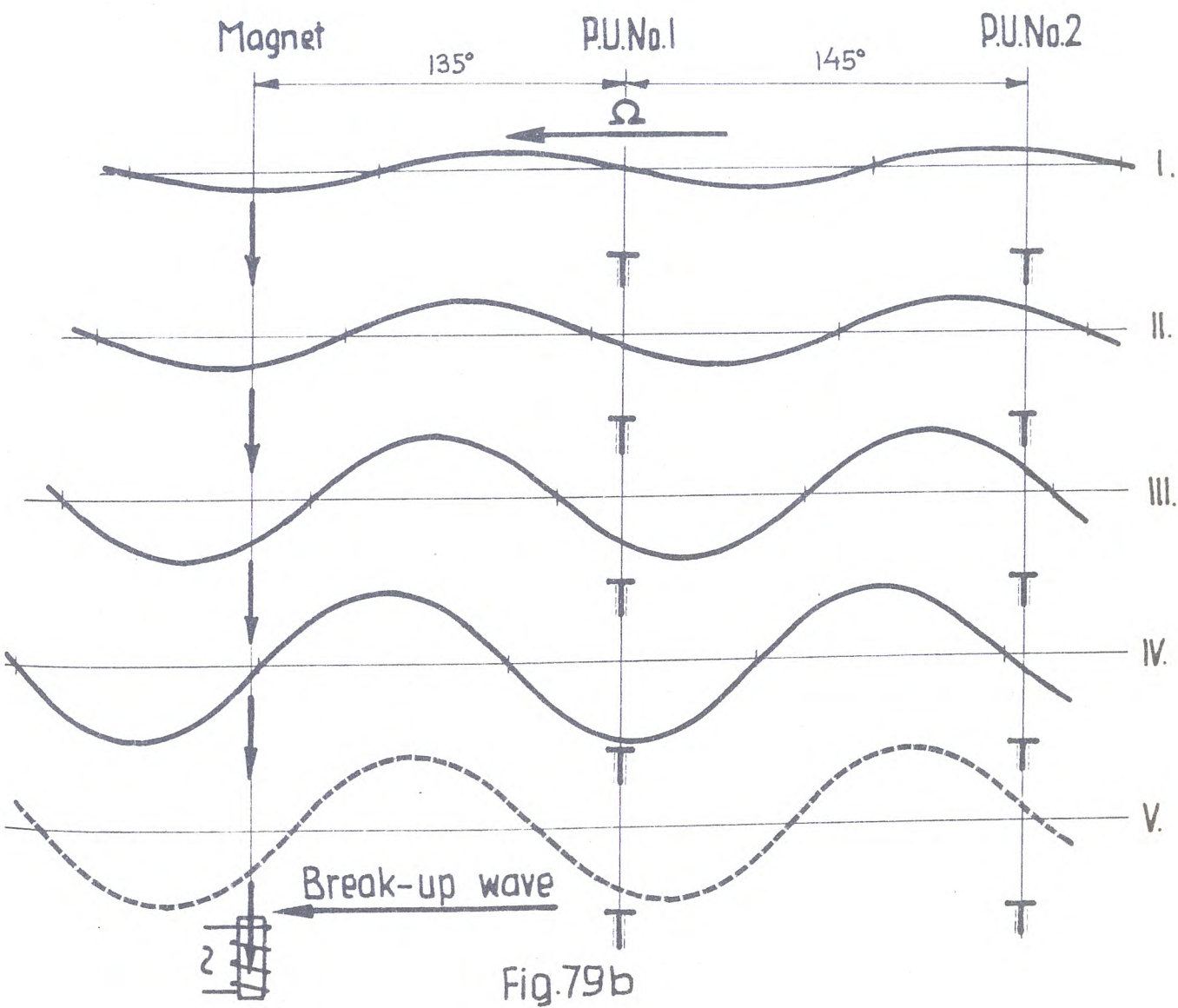


Fig.79b

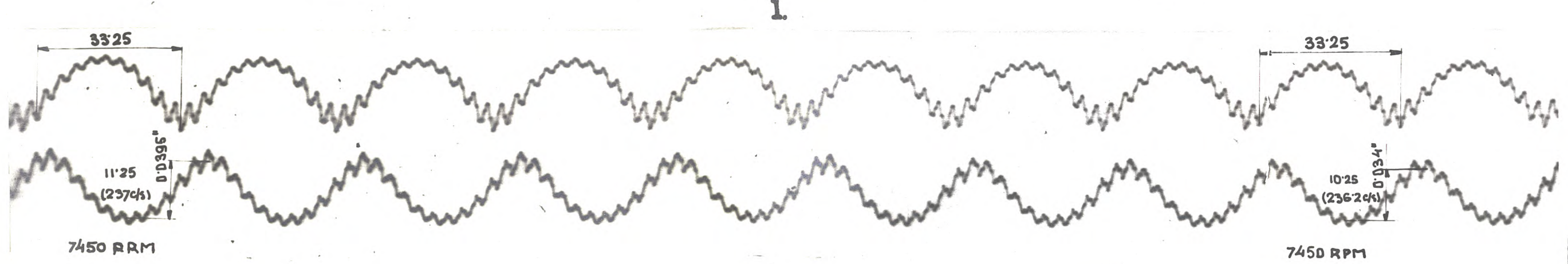
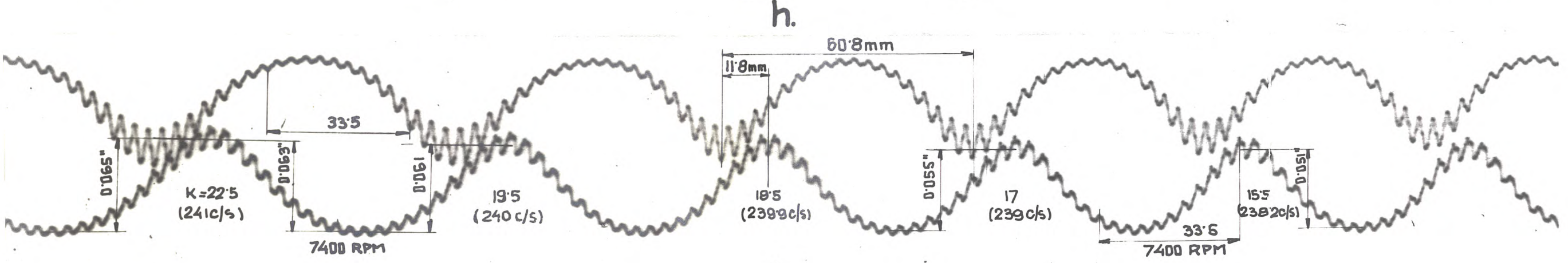
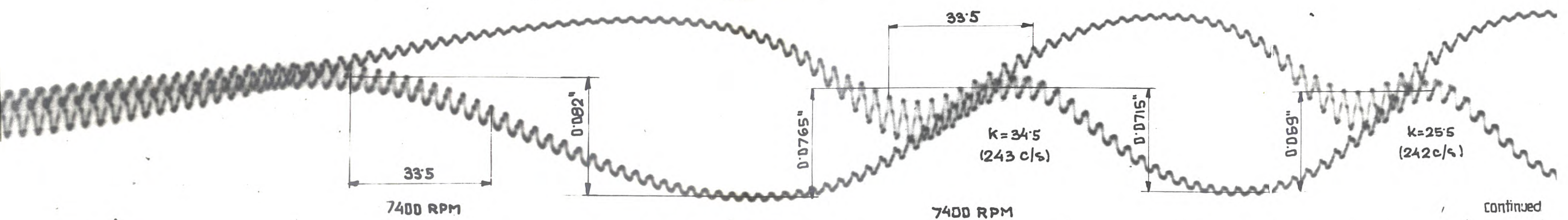
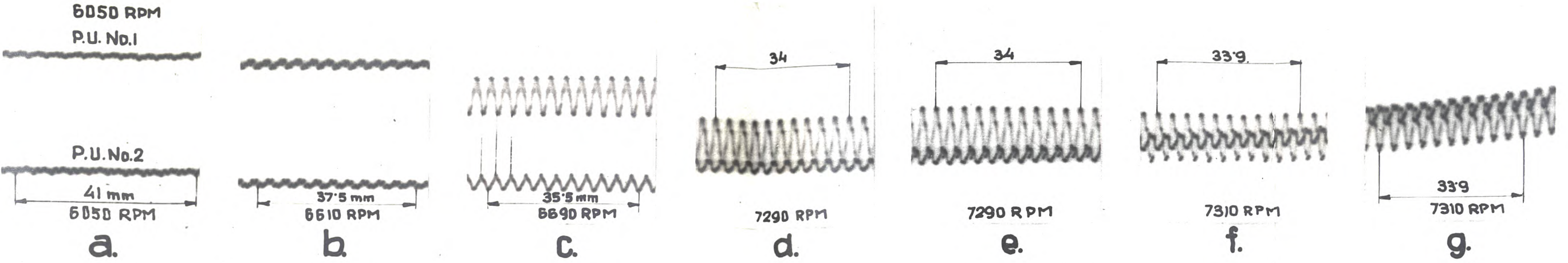


Fig. 80

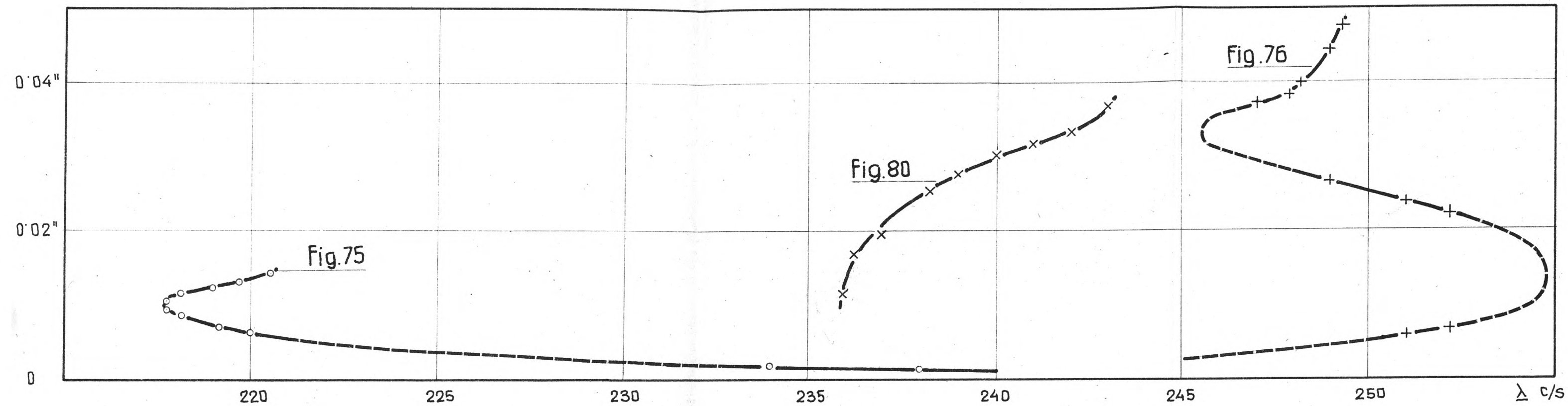


Fig. 81

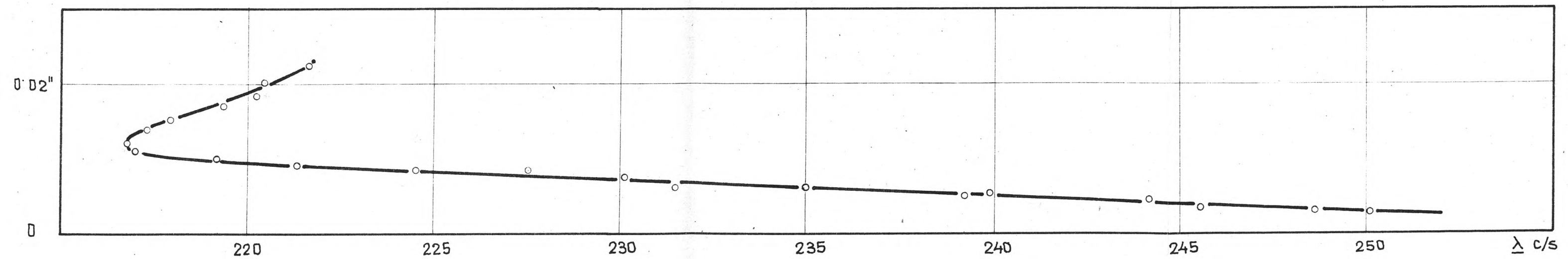


Fig. 82

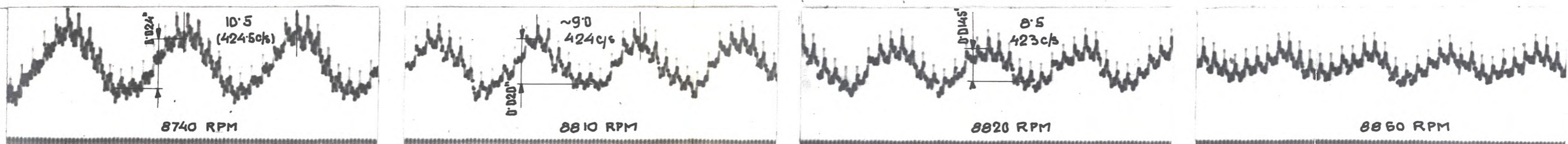
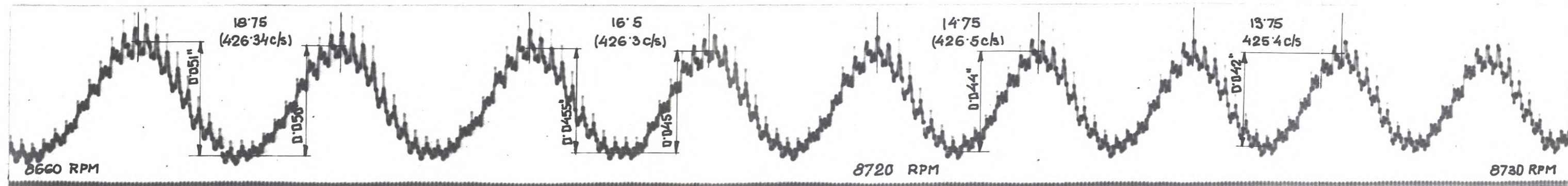
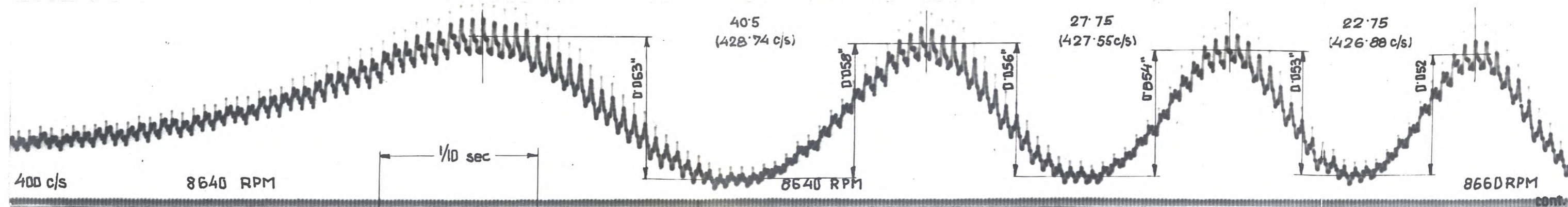
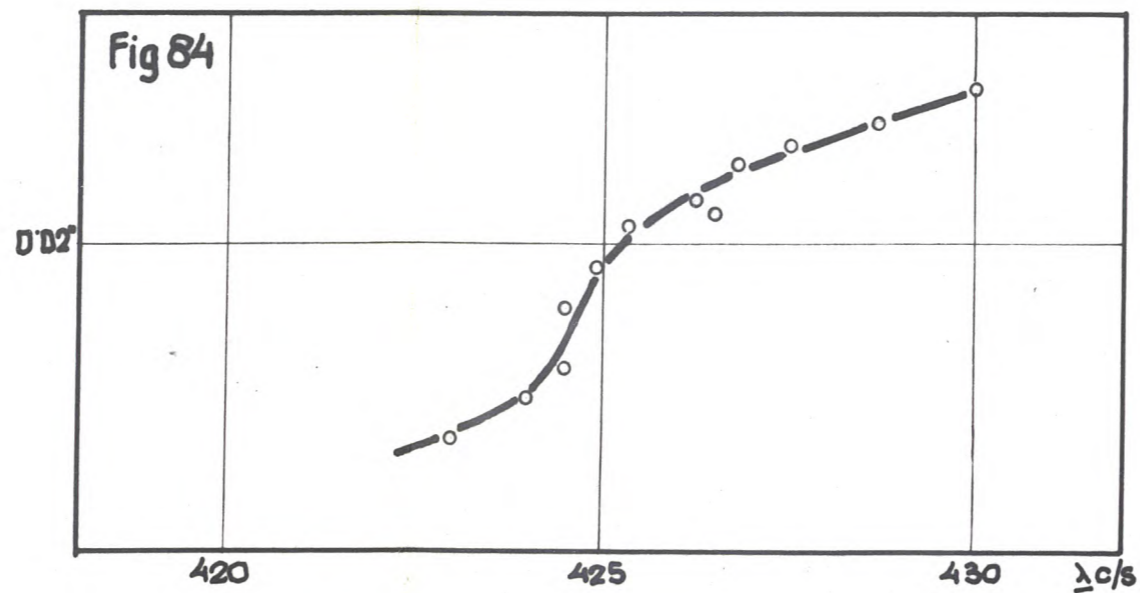
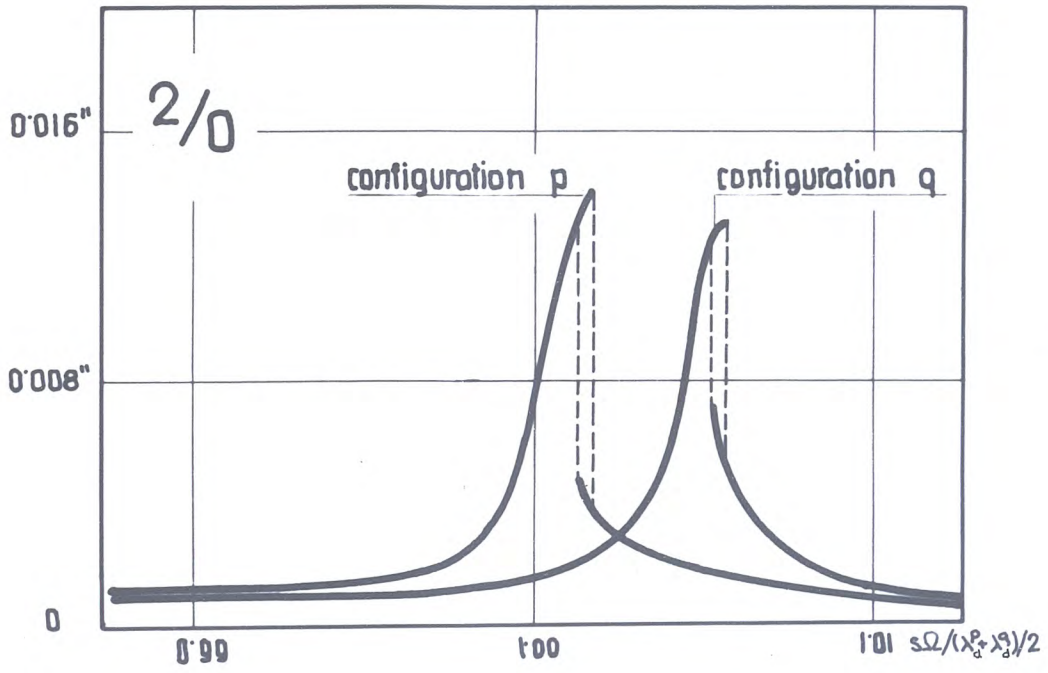


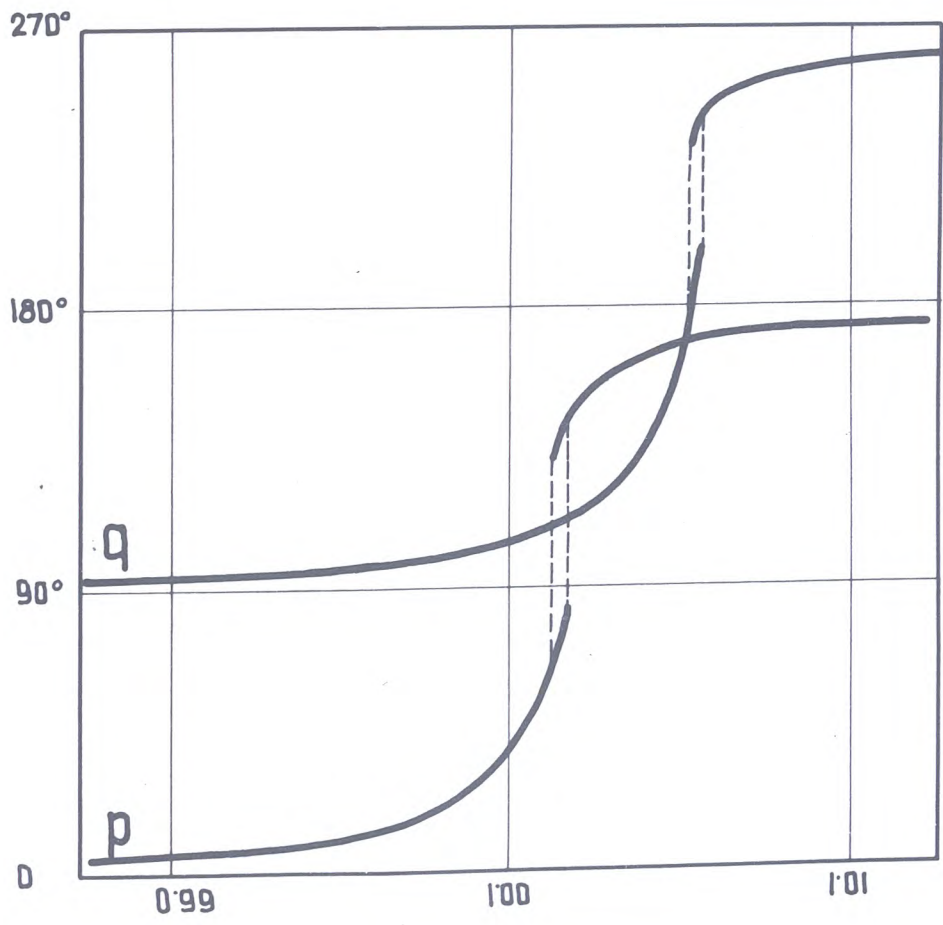
Fig. 83



"Peak to peak amplitude" scale



a.



b.

Fig.85

1/2 Lower Branch

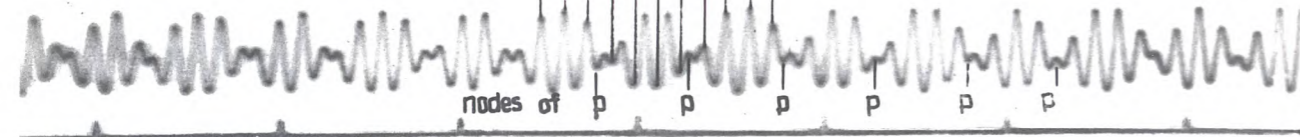
540 RPM

$\omega_0 = 60$ c/s

$\Omega = 9.0$ rev/s

1 2 3 4 5 6 7 8 9 10 11

1.



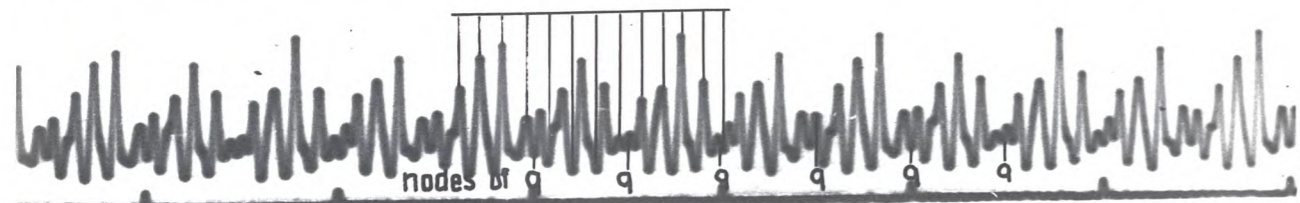
$\omega = 60$ c/s 100% , $\omega + 2s\Omega = 75.5$ c/s 100% , $\omega - 2s\Omega = -$ 0%
 (124 c/s 6.5% , 140 c/s 9% , 156 c/s 4%)

530 RPM

$\omega_0 = 61.9$ c/s

$\Omega = 8.83$ rev/s

2.



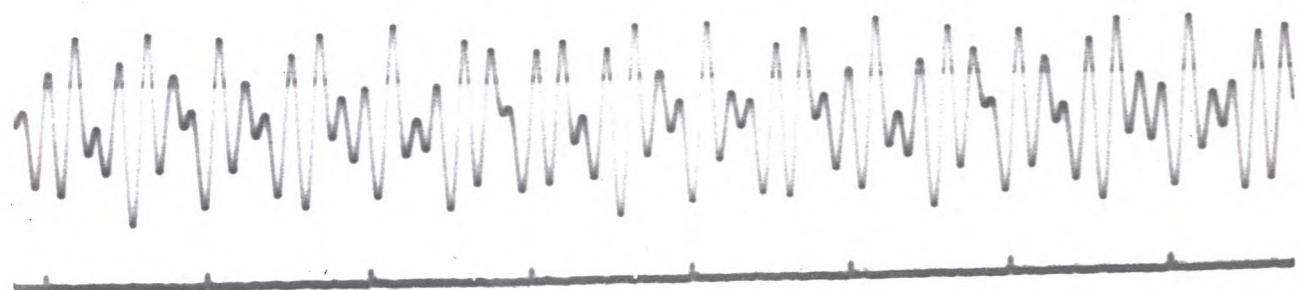
$\omega = 62.1$ c/s 100% , $\omega + 2s\Omega = 81$ c/s 100% , $\omega - 2s\Omega = -$ 0%
 (132 c/s 47% , 162 c/s 27%)

750 RPM

$\omega_0 = 56$ c/s

$\Omega = 12.5$ rev/s

5.



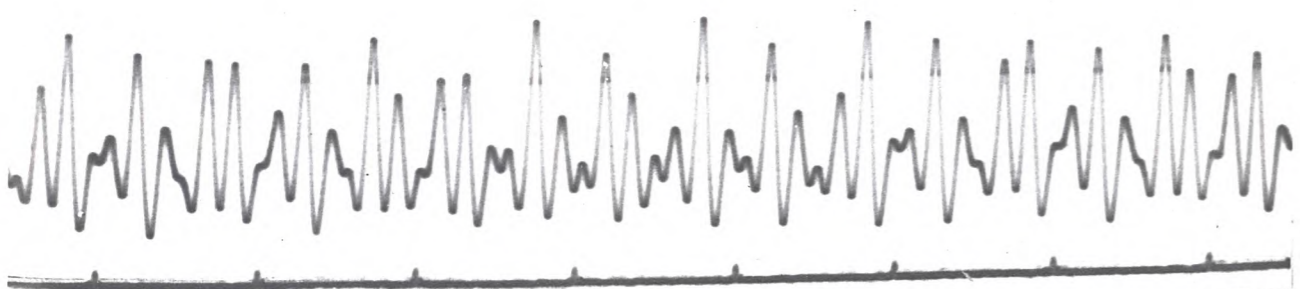
$\omega = 57.5$ c/s 100% , $\omega + 2s\Omega = 82.5$ c/s 50% , $\omega - 2s\Omega = -$ 0%

760 RPM

$\omega_0 = 59$ c/s

$\Omega = 12.6$ rev/s

6.



$\omega = 60$ c/s 100% , $\omega + 2s\Omega = 84.5$ c/s 82% , $\omega - 2s\Omega = -$ 0%

1/2 Higher Branch

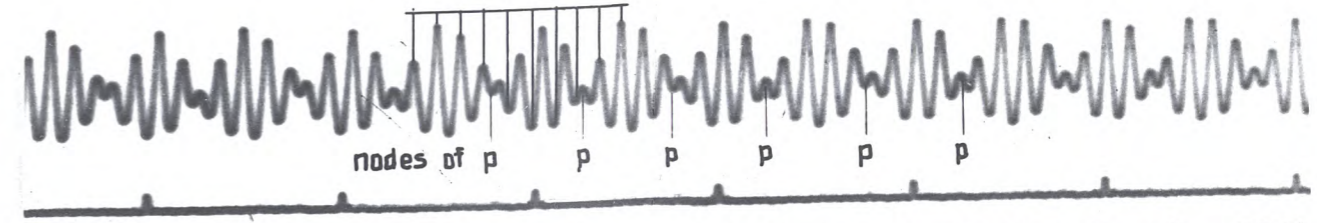
530 RPM

$\omega_0 = 78.5$ c/s

$\Omega = 8.83$ rev/s

p

3.



$\omega = 76$ c/s 100% , $\omega + 2s\Omega = -$ 0% , $\omega - 2s\Omega = 61$ c/s 93%
 (124 c/s 5% , 141 c/s 13% , 158 c/s 8%)

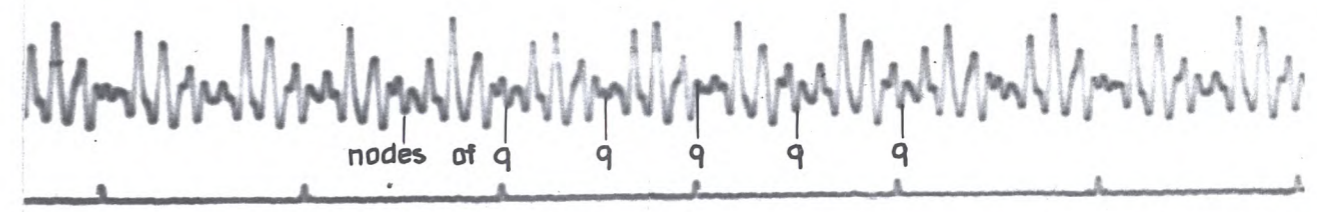
530 RPM

$\omega_0 = 80$ c/s

$\Omega = 8.83$ rev/s

q

4.



$\omega = 81$ c/s 100% , $\omega + 2s\Omega = -$ 0% , $\omega - 2s\Omega = 65$ c/s 95%
 (132 c/s 81% , 146 c/s 27% , 164 c/s 54%)

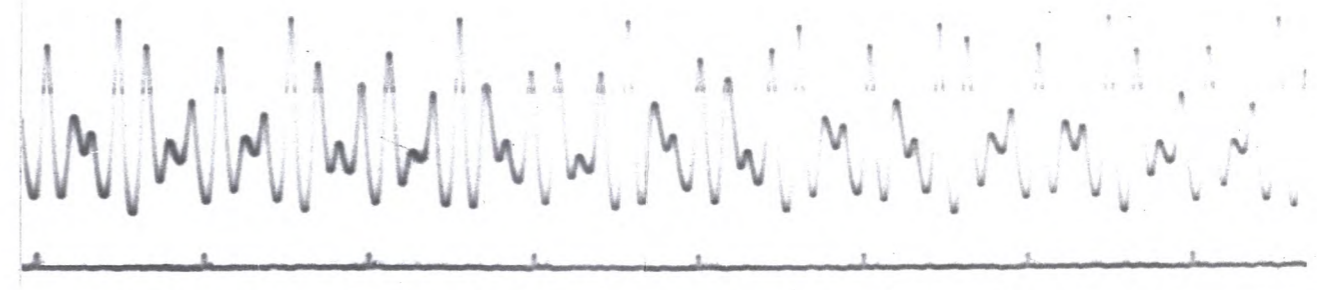
740 RPM

$\omega_0 = 80$ c/s

$\Omega = 12.3$ rev/s

p

7.



$\omega = 81$ c/s 100% , $\omega + 2s\Omega = -$ 0% , $\omega - 2s\Omega = 58$ c/s 93%
 (117 c/s 10%)

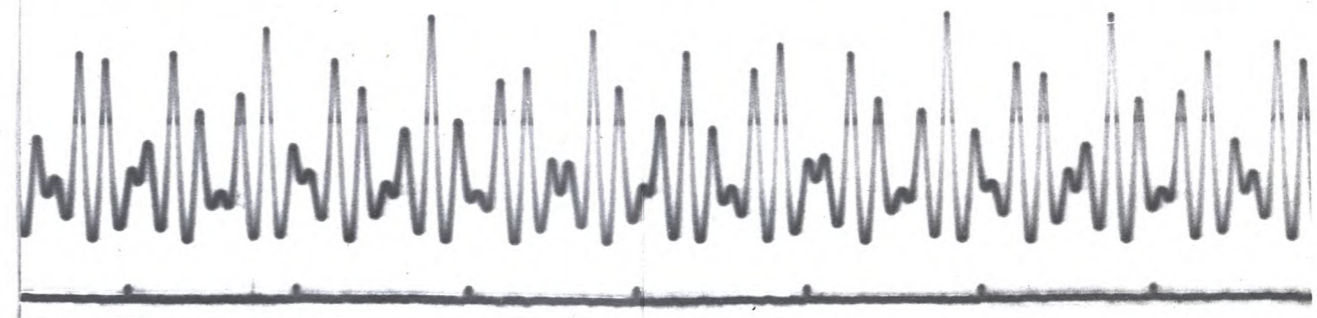
740 RPM

$\omega_0 = 84$ c/s

$\Omega = 12.3$ rev/s

q

8.



$\omega = 84$ c/s 100% , $\omega + 2s\Omega = -$ 0% , $\omega - 2s\Omega = 62$ c/s 90%

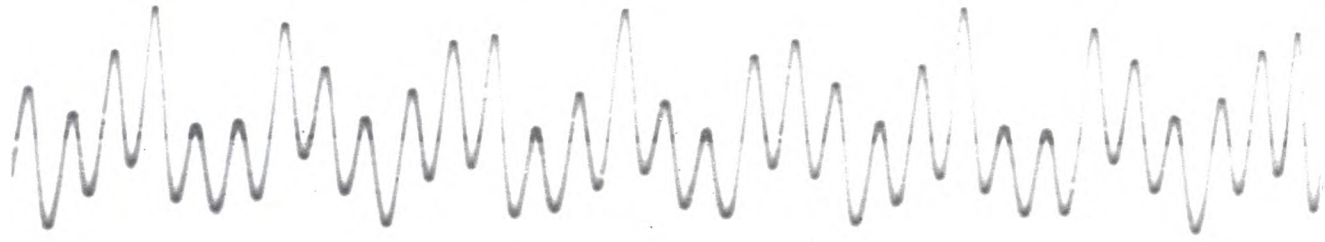
fig.86

1/0 Lower Branch

3000 RPM

$\omega_0 = 40 \text{ c/s}$

$\Omega = 50 \text{ rev/s}$



$\omega = 37 \text{ c/s } 100\%$, $\omega + 2s\Omega = 142 \text{ c/s } 120\%$, $\omega - 2s\Omega = -$ 0%

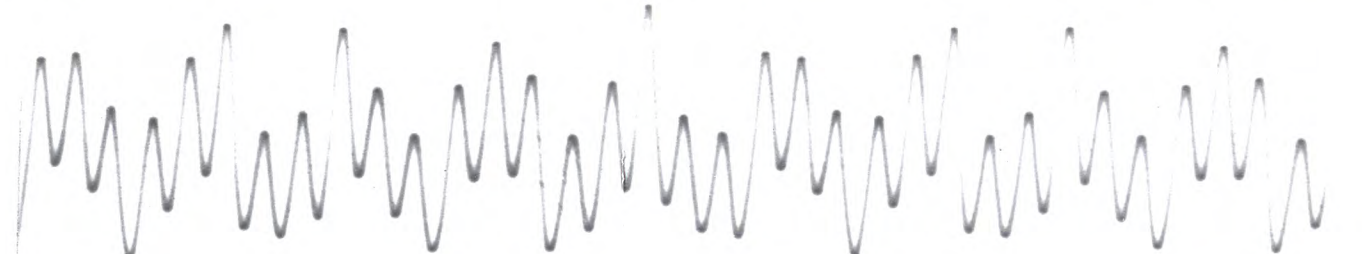
1/0 Higher Branch

3000 RPM

$\omega_0 = 137 \text{ c/s}$

$\Omega = 50 \text{ rev/s}$

p

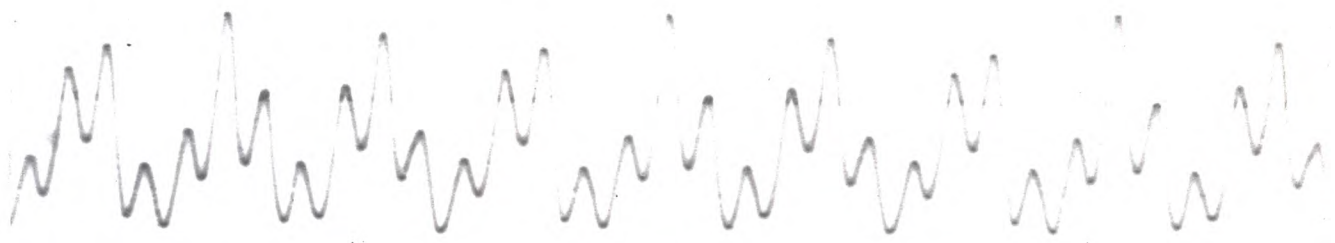


$\omega = 140 \text{ c/s } 100\%$, $\omega + 2s\Omega = -$ 0%, $\omega - 2s\Omega = 37 \text{ c/s } 90\%$

3000 RPM

$\omega_0 = 41.5 \text{ c/s}$

$\Omega = 50 \text{ rev/s}$



$\omega = 38.7 \text{ c/s } 100\%$, $\omega + 2s\Omega = 142 \text{ c/s } 90\%$, $\omega - 2s\Omega = -$ 0%

3000 RPM

$\omega_0 = 140 \text{ c/s}$

$\Omega = 50 \text{ rev/s}$

q



$\omega = 143 \text{ c/s } 100\%$, $\omega + 2s\Omega = -$ 0%, $\omega - 2s\Omega = 37.5 \text{ c/s } 50\%$

Fig.87

1/2 Lower Branch

p

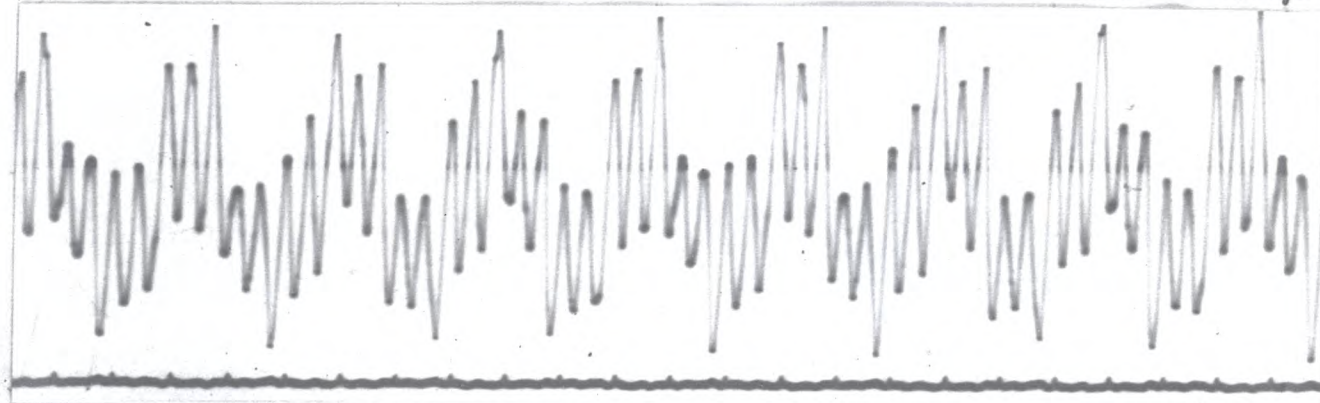
q

4600 RPM

$\omega_0 = 29.5 \text{ c/s}$

$\Omega = 76.8 \text{ rev/s}$

1.



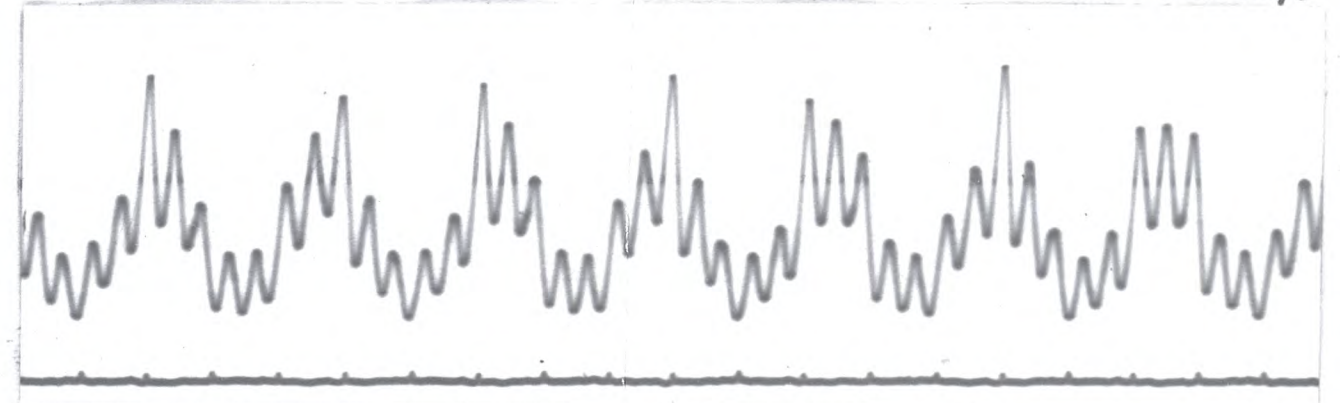
$\omega = 29 \text{ c/s } 100\%$, $\omega + 2s\Omega = 185 \text{ c/s } 130\%$, $\omega - 2s\Omega = -$ 0%.

4600 RPM

$\omega_0 = 30.5 \text{ c/s}$

$\Omega = 76.8 \text{ rev/s}$

2.



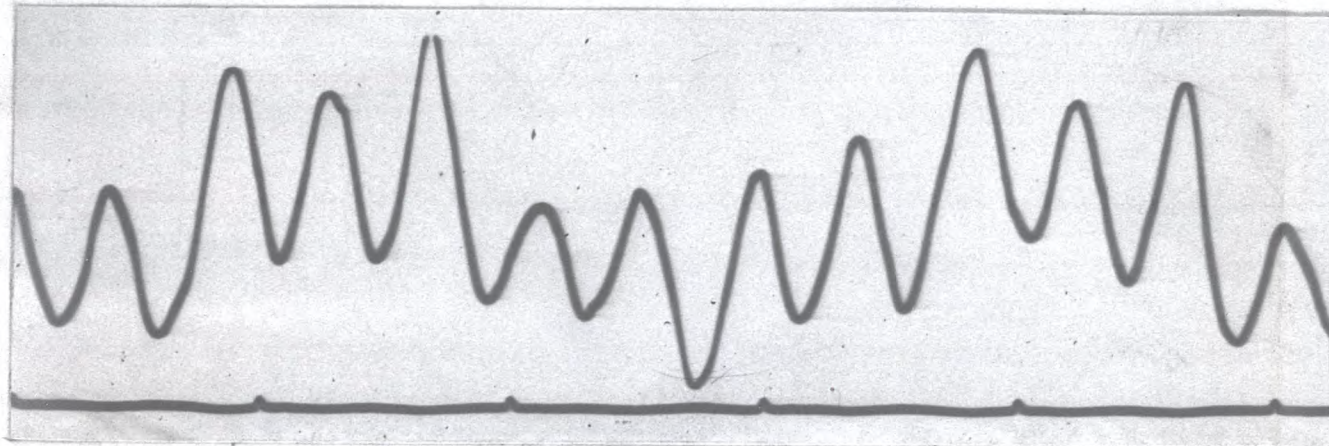
$\omega = 30.5 \text{ c/s } 100\%$, $\omega + 2s\Omega = 188 \text{ c/s } 70\%$, $\omega - 2s\Omega = -$ 0%.

4600 RPM

$\omega_0 = 29 \text{ c/s}$

$\Omega = 76.8 \text{ rev/s}$

3.



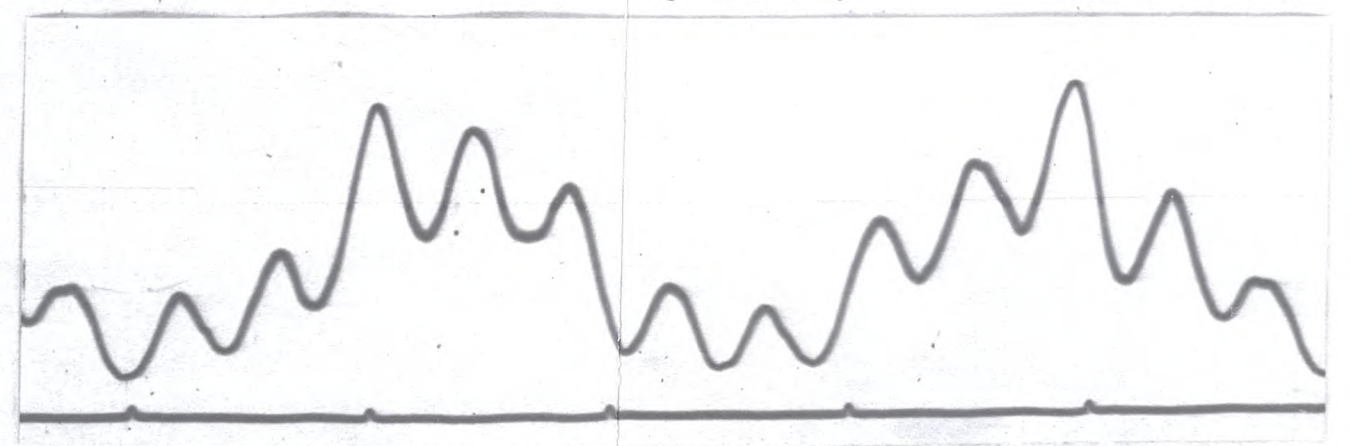
$\omega = 29 \text{ c/s } 100\%$, $\omega + 2s\Omega = 185 \text{ c/s } 120\%$, $\omega - 2s\Omega = -$ 0%.

4600 RPM

$\omega_0 = 30.5 \text{ c/s}$

$\Omega = 76.8 \text{ rev/s}$

4.



$\omega = 30.5 \text{ c/s } 100\%$, $\omega + 2s\Omega = 186 \text{ c/s } 40\%$, $\omega - 2s\Omega = -$ 0%.

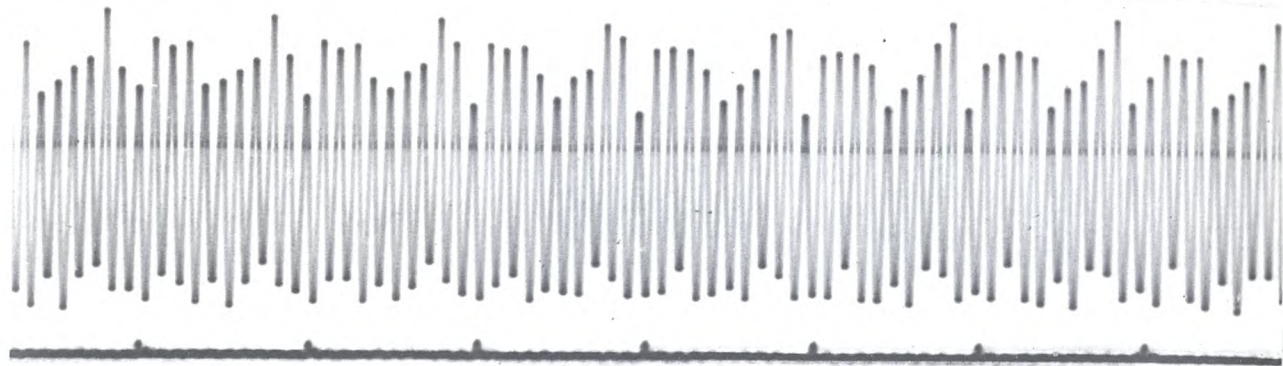
fig.88

2/0 Lower Branch

740 RPM

$$\omega_0 = 121 \text{ c/s}$$

$$\Omega = 12.3 \text{ rev/s}$$

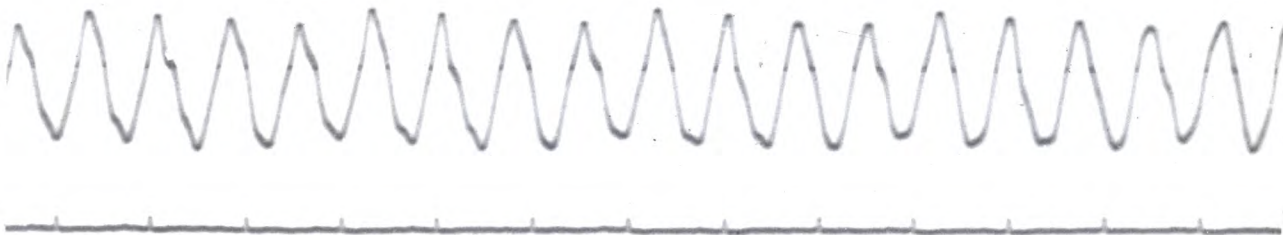


$$\omega = 121 \text{ c/s } 100\% , \quad \omega + 2s\Omega = 172 \text{ c/s } 7\% , \quad \omega - 2s\Omega = - \quad 0\%$$

3000 RPM

$$\omega_0 = 72 \text{ c/s}$$

$$\Omega = 50 \text{ rev/s}$$



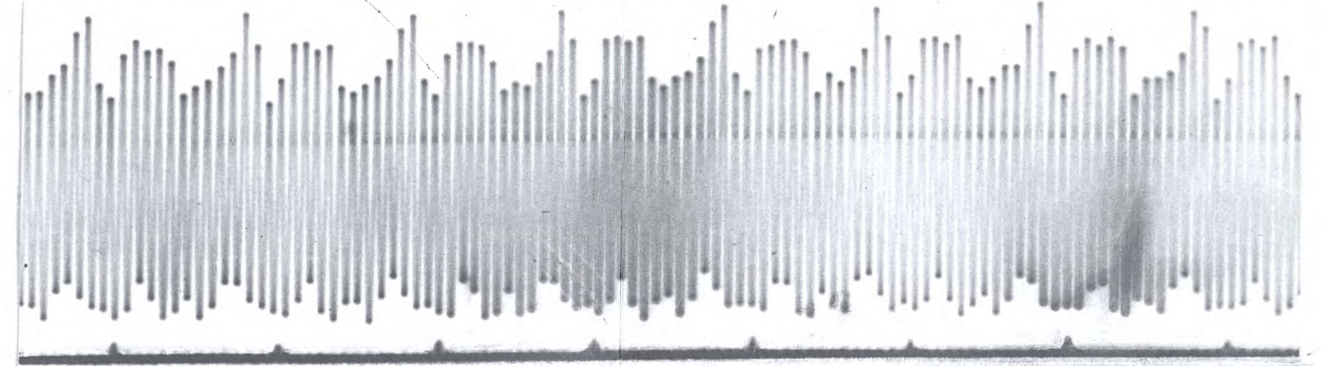
$$\omega = 72 \text{ c/s } 100\% , \quad \omega + 2s\Omega = - \quad 0\% , \quad \omega - 2s\Omega = - \quad 0\%$$

2/0 Higher Branch

765 RPM

$$\omega_0 = 172 \text{ c/s}$$

$$\Omega = 12.75 \text{ rev/s}$$

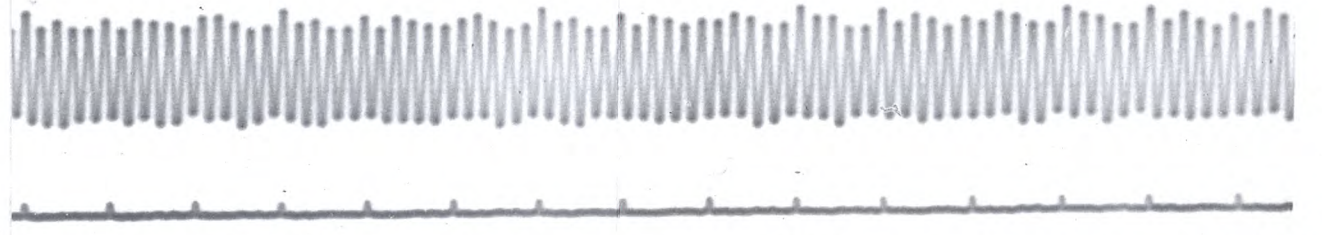


$$\omega = 174 \text{ c/s } 100\% , \quad \omega + 2s\Omega = - \quad 0\% , \quad \omega - 2s\Omega = 127 \text{ c/s } 6\%$$

3000 RPM

$$\omega_0 = 272 \text{ c/s}$$

$$\Omega = 50 \text{ rev/s}$$



$$\omega = 272 \text{ c/s } 100\% , \quad \omega + 2s\Omega = - \quad 0\% , \quad \omega - 2s\Omega = 70 \text{ c/s } 4\%$$

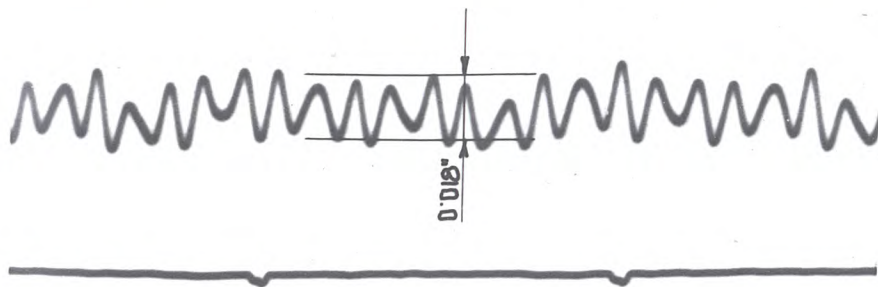
fig. 89

2/0 Lower Branch

724 RPM

$$\omega_0 = 118 \text{ c/s}$$

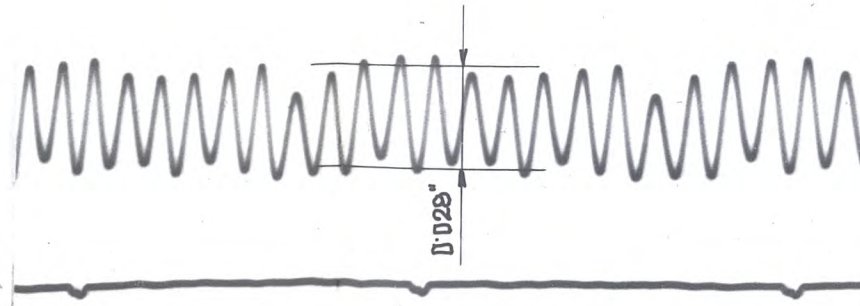
$$\Omega = 12.1 \text{ rev/s}$$



720 RPM

$$\omega_0 = 120 \text{ c/s}$$

$$\Omega = 12 \text{ rev/s}$$



745 RPM

$$\omega_0 = 128 \text{ c/s}$$

$$\Omega = 12.4 \text{ rev/s}$$

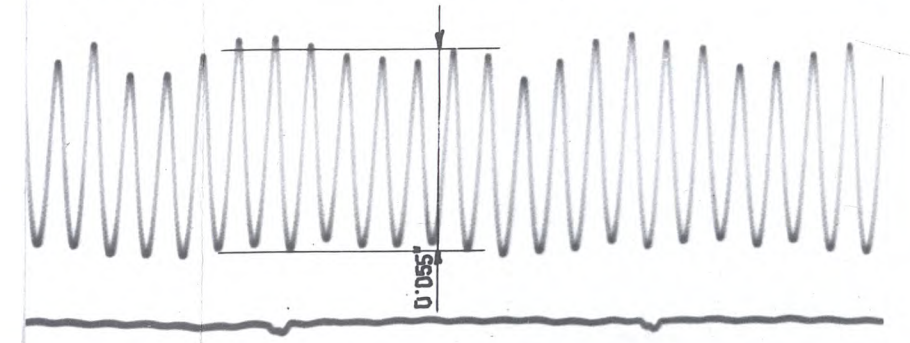


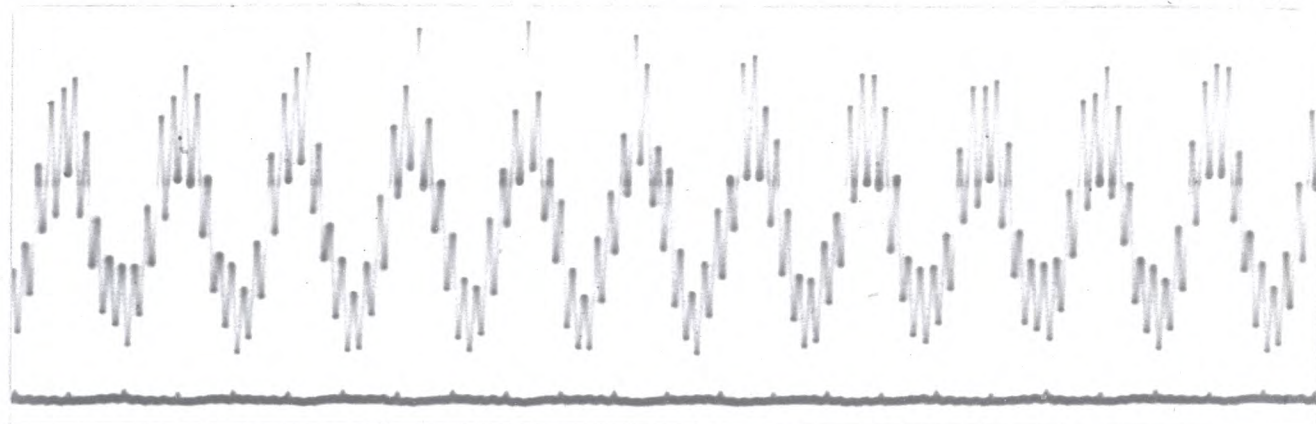
fig. 90

2/0 Lower Branch

4700 RPM

$\omega_0 = 38$ c/s

$\Omega = 78.3$ rev/s

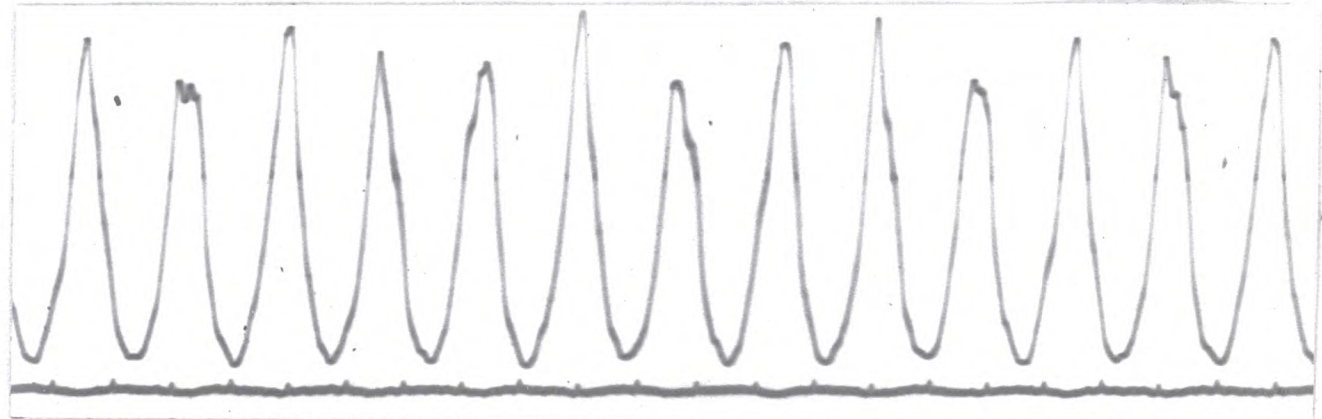


$\omega = 36.5$ c/s 100% , $\omega + 2s\Omega = 352$ c/s 70% , $\omega - 2s\Omega = -$ 0%

4700 RPM

$\omega_0 = 47$ c/s

$\Omega = 78.3$ rev/s

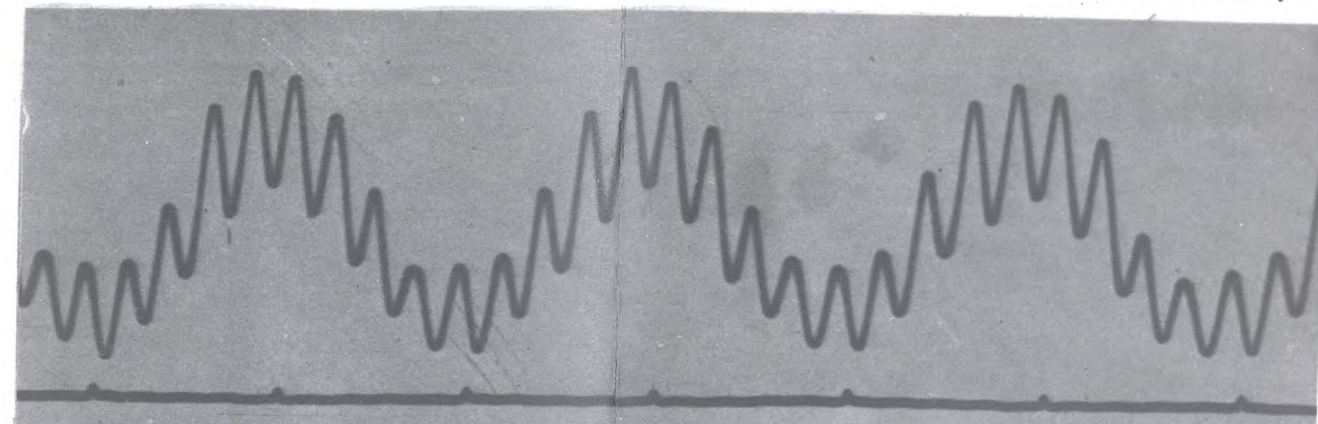


$\omega = 46$ c/s 100% , $\omega + 2s\Omega = -$ 0% , $\omega - 2s\Omega = -$ 0%

4600 RPM

$\omega_0 = 41$ c/s

$\Omega = 76.6$ rev/s

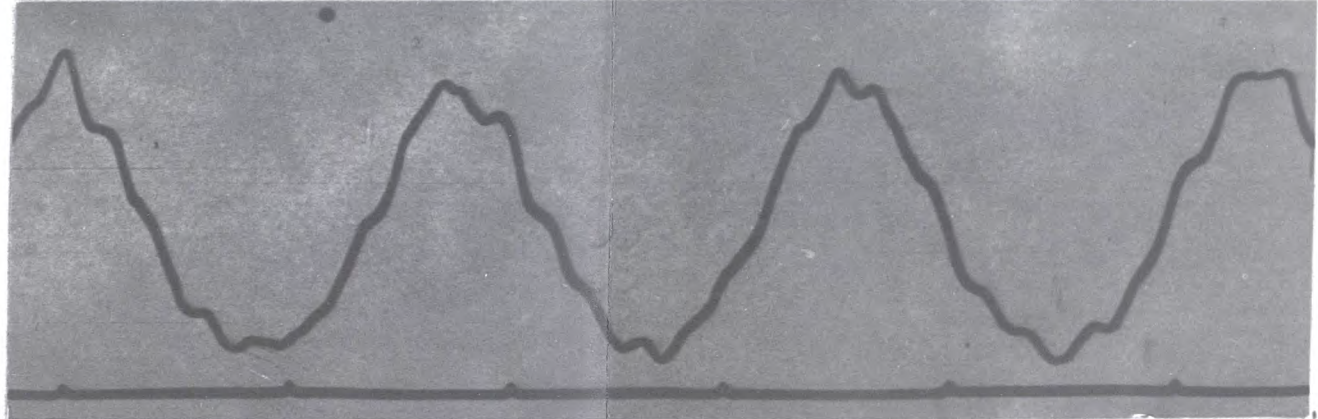


$\omega = 41$ c/s 100% , $\omega + 2s\Omega = 345$ c/s 60% , $\omega - 2s\Omega = -$ 0%

4600 RPM

$\omega_0 = 49$ c/s

$\Omega = 76.6$ rev/s



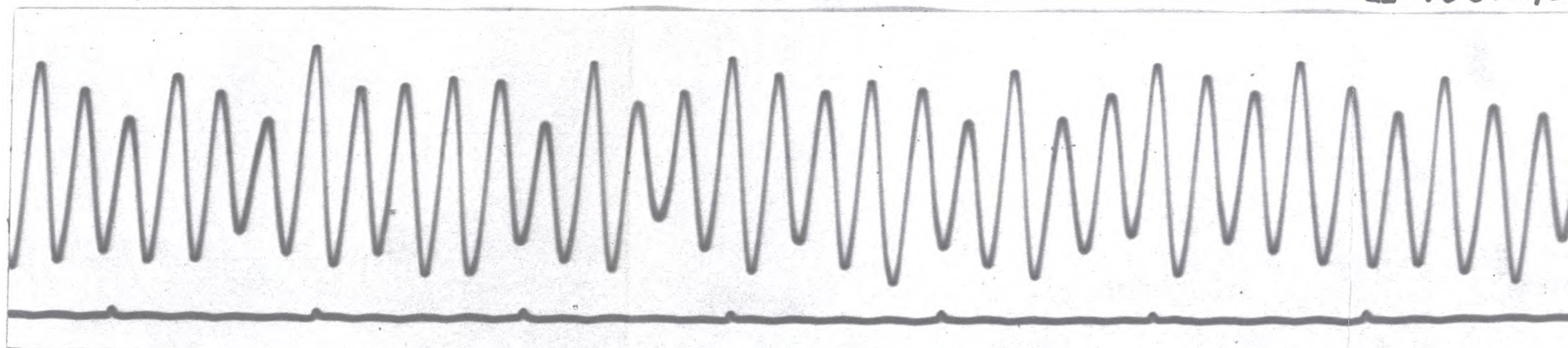
$\omega = 45$ c/s 100% , $\omega + 2s\Omega = 350$ c/s 3% , $\omega - 2s\Omega = -$ 0%

2/0 Higher Branch

4600 RPM

$\omega_0 = 345$ c/s

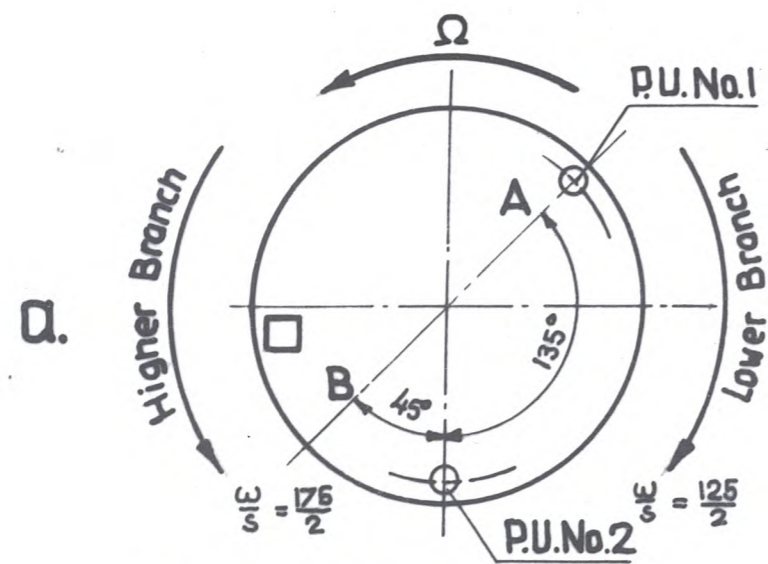
$\Omega = 76.6$ rev/s



$\omega = 349$ c/s 100% , $\omega + 2s\Omega = -$ 0% , $\omega - 2s\Omega = 39$ c/s 4%

5.

Fig. 91

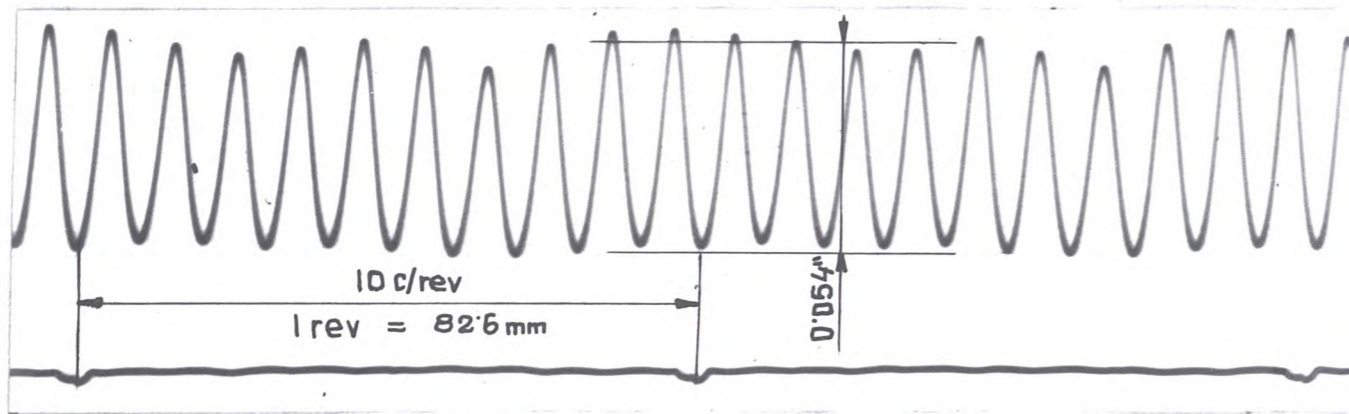


2/0 Lower Branch

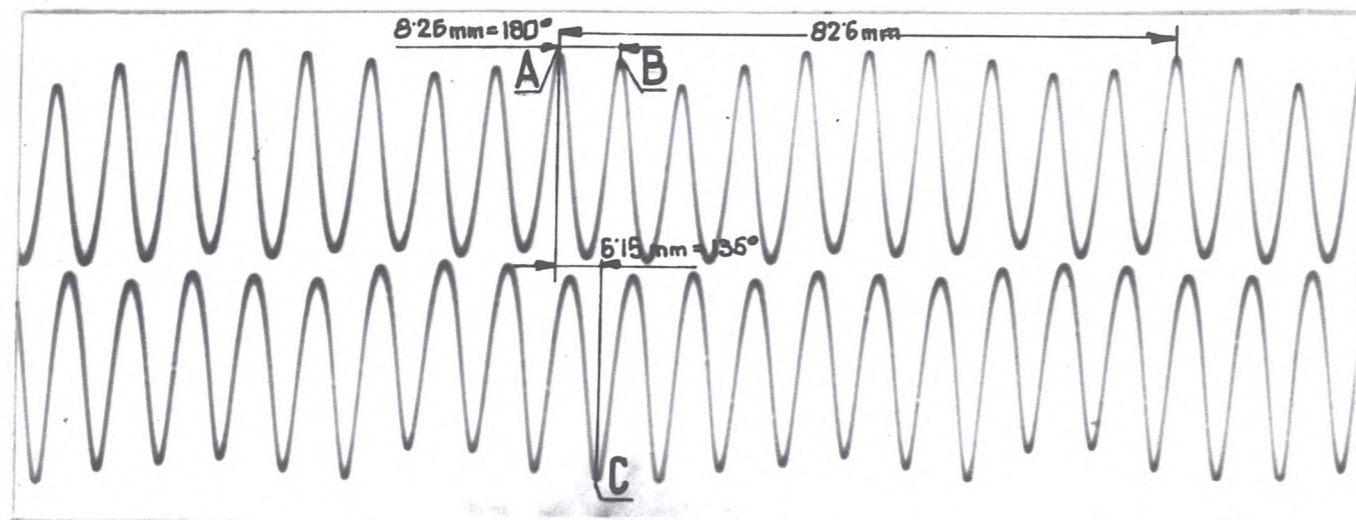
750 RPM

$\omega_c = 125 \text{ c/s}$

$\Omega = 12.5 \text{ rev/s}$

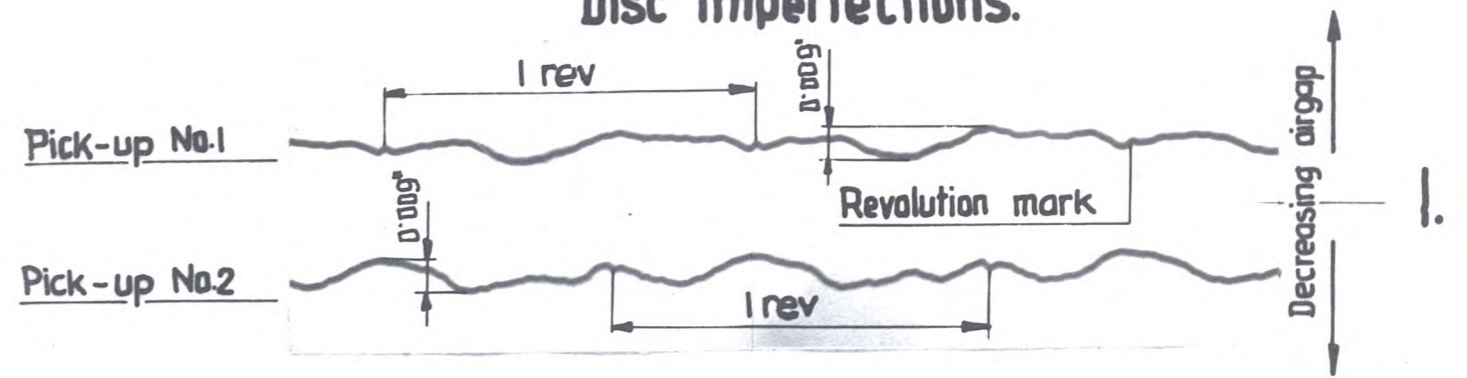


2.



3.

Disc imperfections.

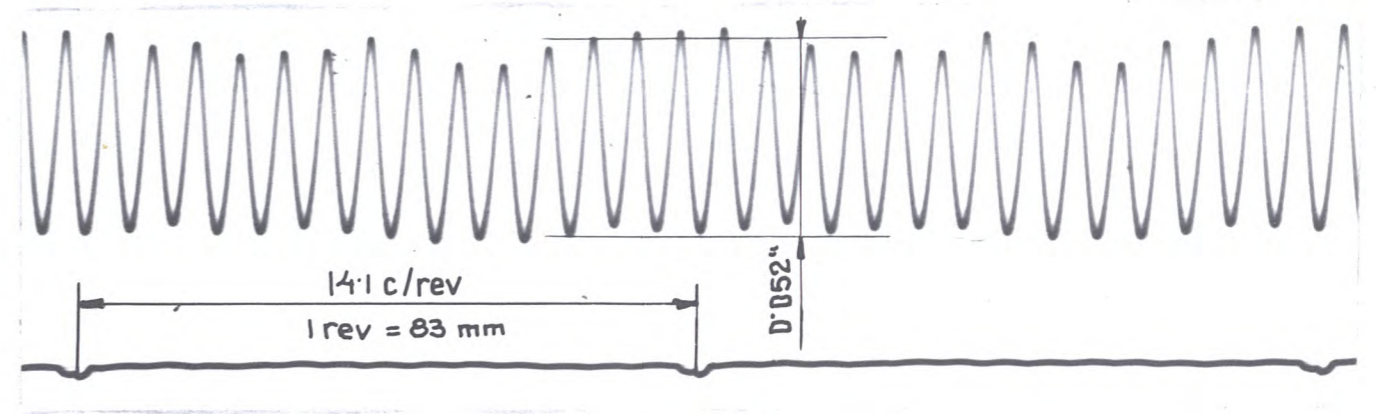


2/0 Higher Branch

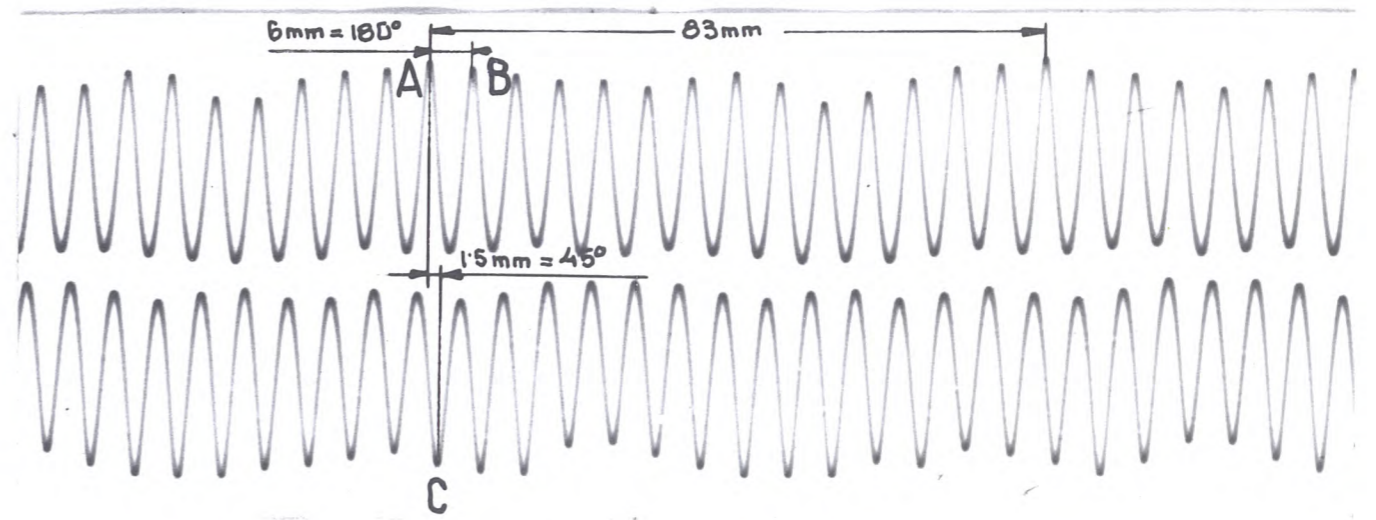
745 RPM

$\omega_c = 175 \text{ c/s}$

$\Omega = 12.4 \text{ rev/s}$



4.

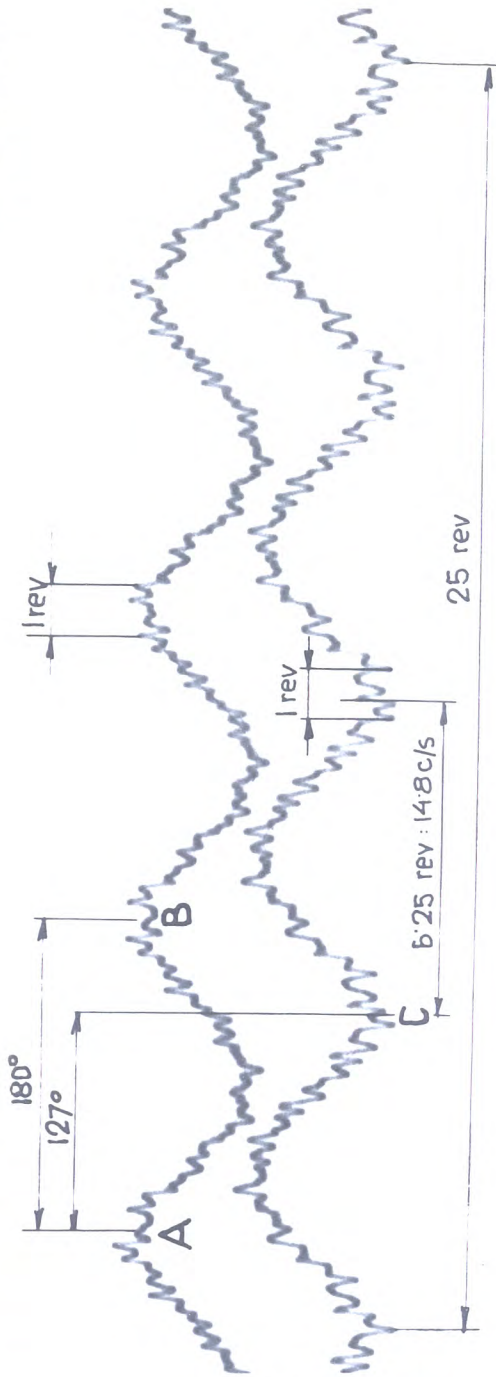


5.

fig. 92

5500 RPM

$$\omega = 14.8 \text{ c/s}$$



7500 RPM

$$\omega = 12.5 \text{ c/s}$$

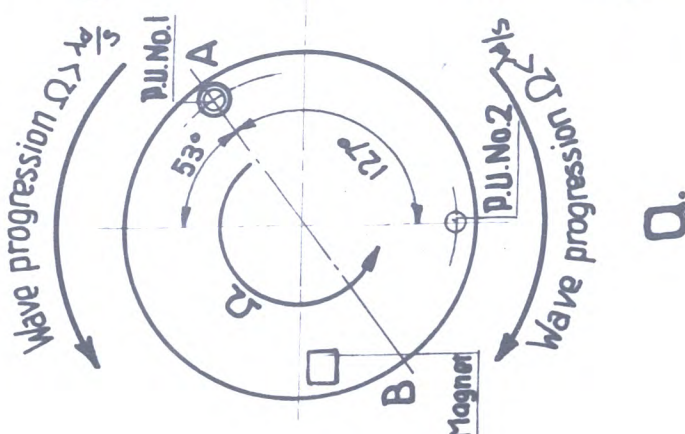
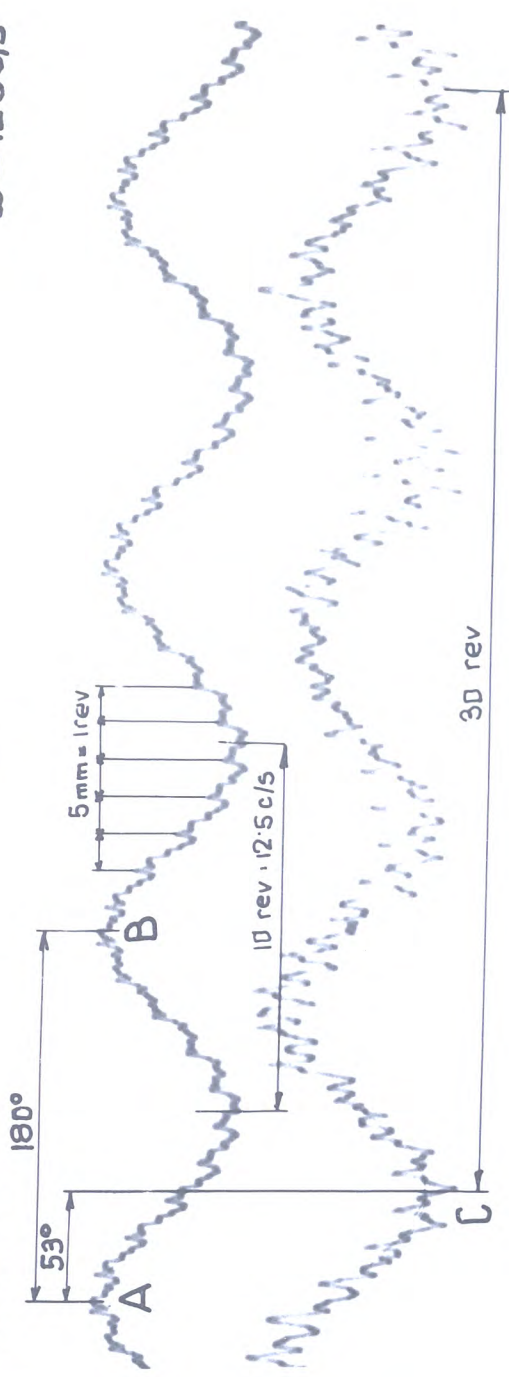


Fig. 93

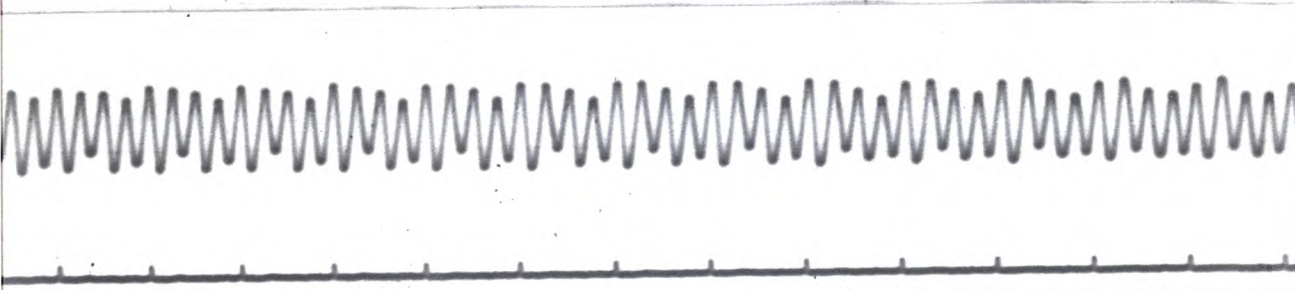
C.

3/0 Lower Branch

3000 RPM

$\omega_0 = 205 \text{ c/s}$

$\Omega = 50 \text{ rev/s}$



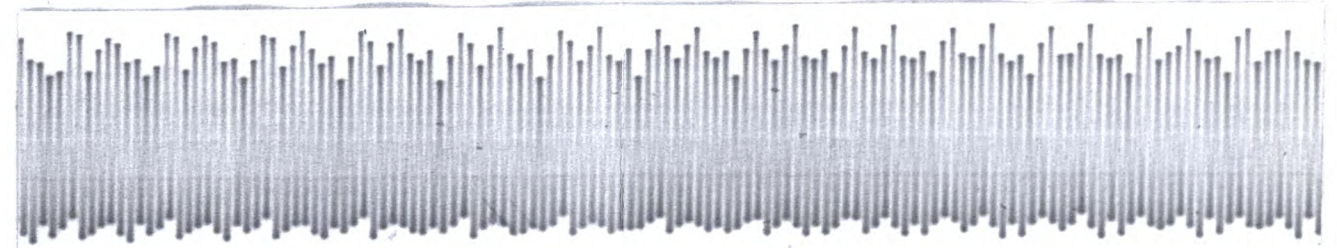
$\omega = 200 \text{ c/s } 100\%$, $\omega + 2s\Omega = - 0\%$, $\omega - 2s\Omega = - 0\%$

3/0 Higher Branch

3000 RPM

$\omega_0 = 502 \text{ c/s}$

$\Omega = 50 \text{ rev/s}$



$\omega = 500 \text{ c/s } 100\%$, $\omega + 2s\Omega = - 0\%$, $\omega - 2s\Omega = 200 \text{ c/s } 4\%$

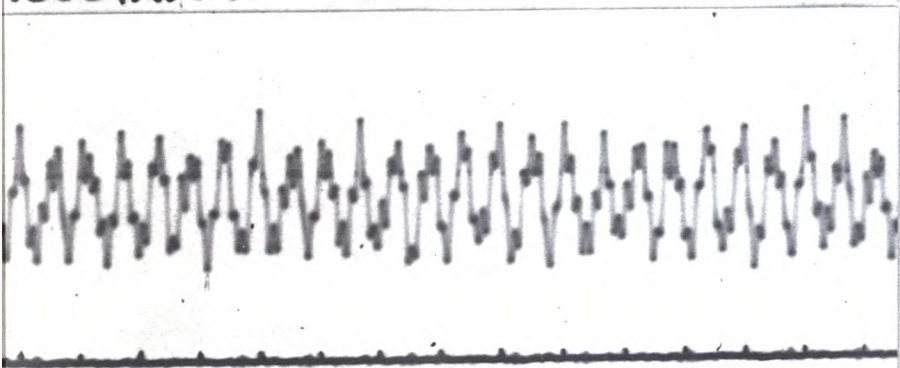
2.

3/0 Lower Branch

4650 RPM

$\omega_0 = 136 \text{ c/s}$

$\Omega = 775 \text{ rev/s}$

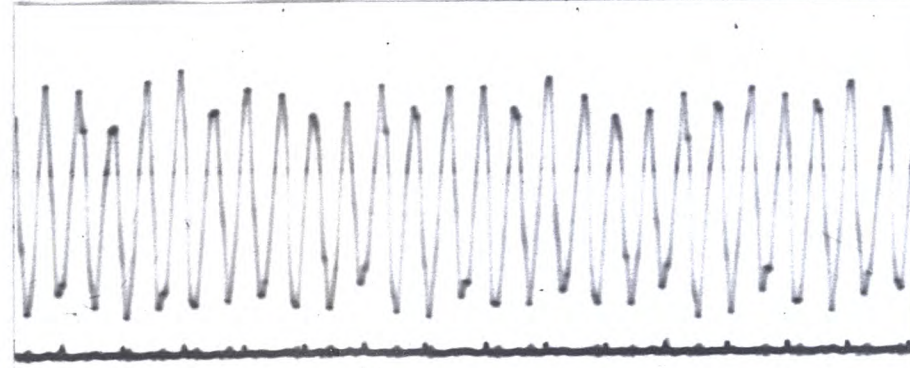


$\omega = 136 \text{ c/s } 100\%$, $\omega + 2s\Omega = 600 \text{ c/s } 20\%$, $\omega - 2s\Omega = - 0\%$

4660 RPM

$\omega_0 = 137 \text{ c/s}$

$\Omega = 778 \text{ rev/s}$

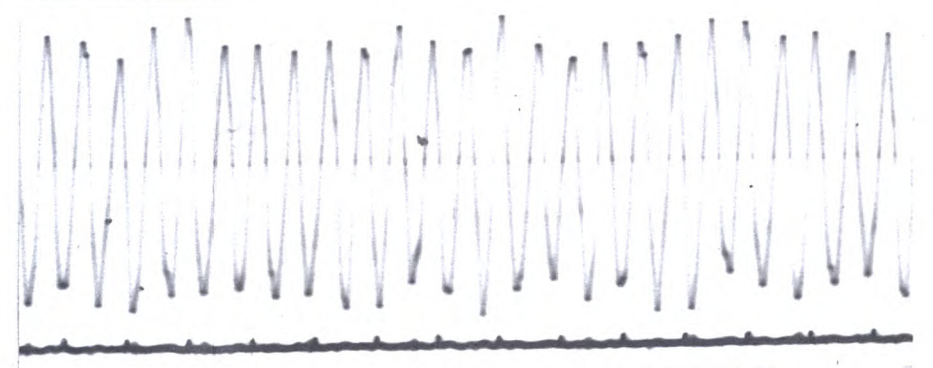


4.

4680 RPM

$\omega_0 = 139 \text{ c/s}$

$\Omega = 782 \text{ rev/s}$



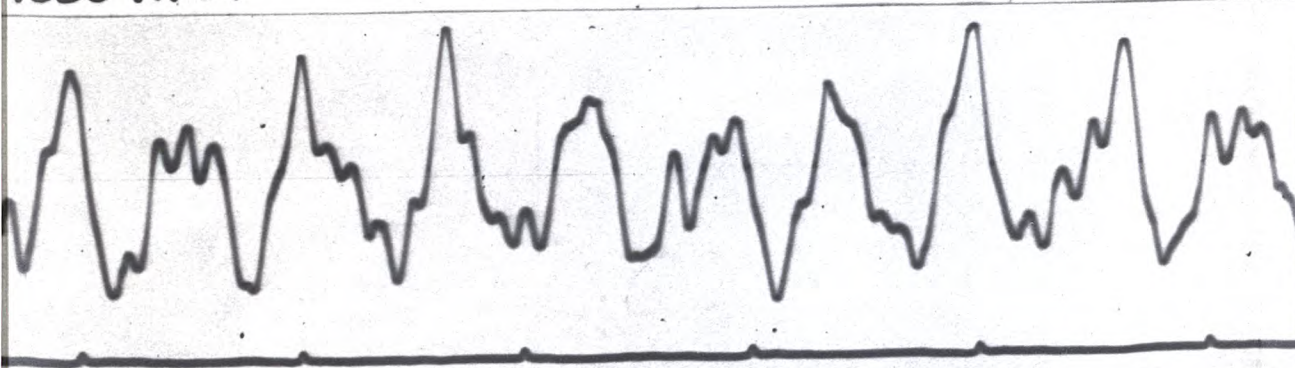
$\omega_0 = 139.5 \text{ c/s } 100\%$, $\omega + 2s\Omega = - 0\%$, $\omega - 2s\Omega = - 0\%$

5.

4600 RPM

$\omega_0 = 137 \text{ c/s}$

$\Omega = 767 \text{ rev/s}$

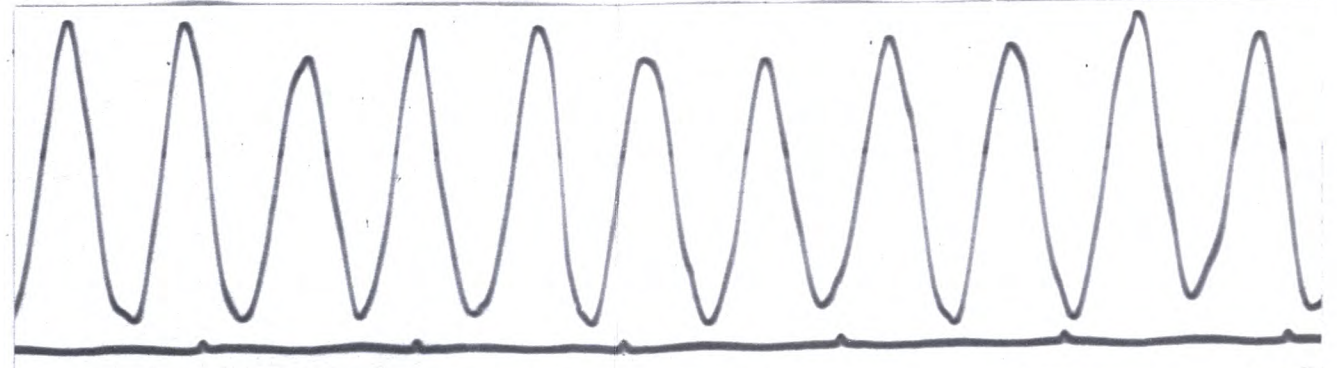


$\omega = 136 \text{ c/s } 100\%$, $\omega + 2s\Omega = 600 \text{ c/s } 10\%$, $\omega - 2s\Omega = - 0\%$

4600 RPM

$\omega_0 = 139.5 \text{ c/s}$

$\Omega = 767 \text{ rev/s}$



$\omega = 141.5 \text{ c/s } 100\%$, $\omega + 2s\Omega = - 0\%$, $\omega - 2s\Omega = - 0\%$

7.

Fig. 94

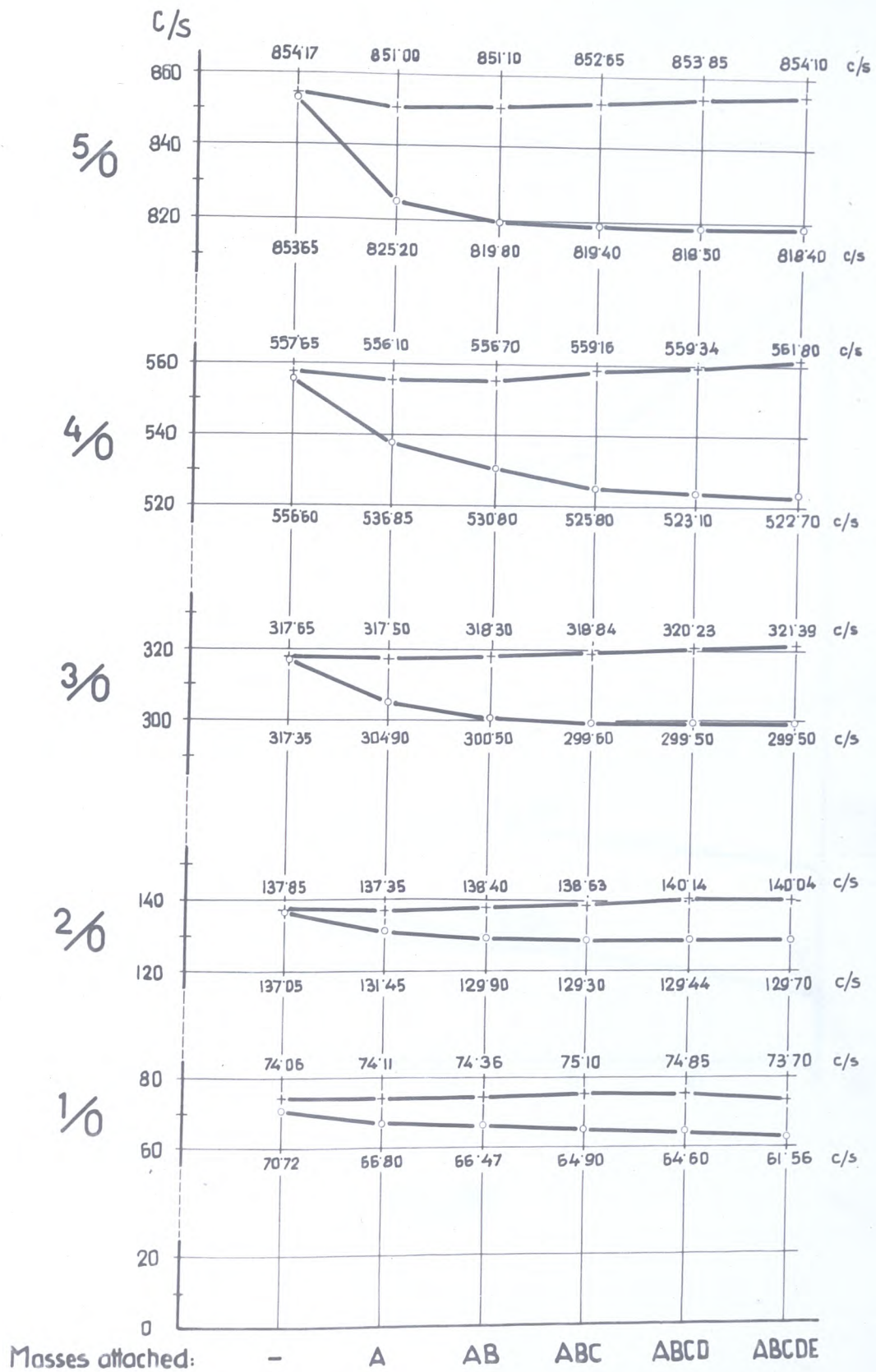


Fig. 95

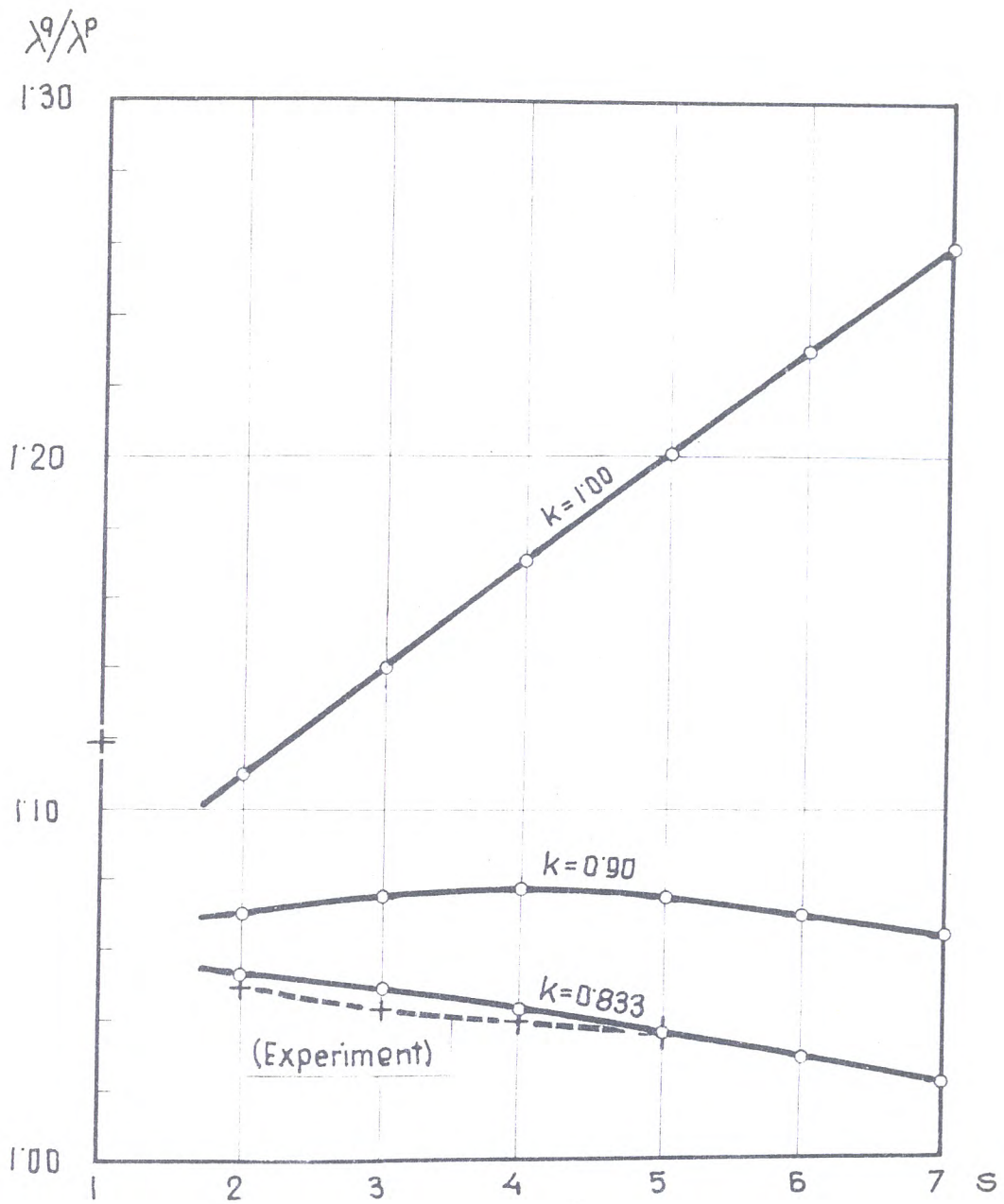


Fig.96

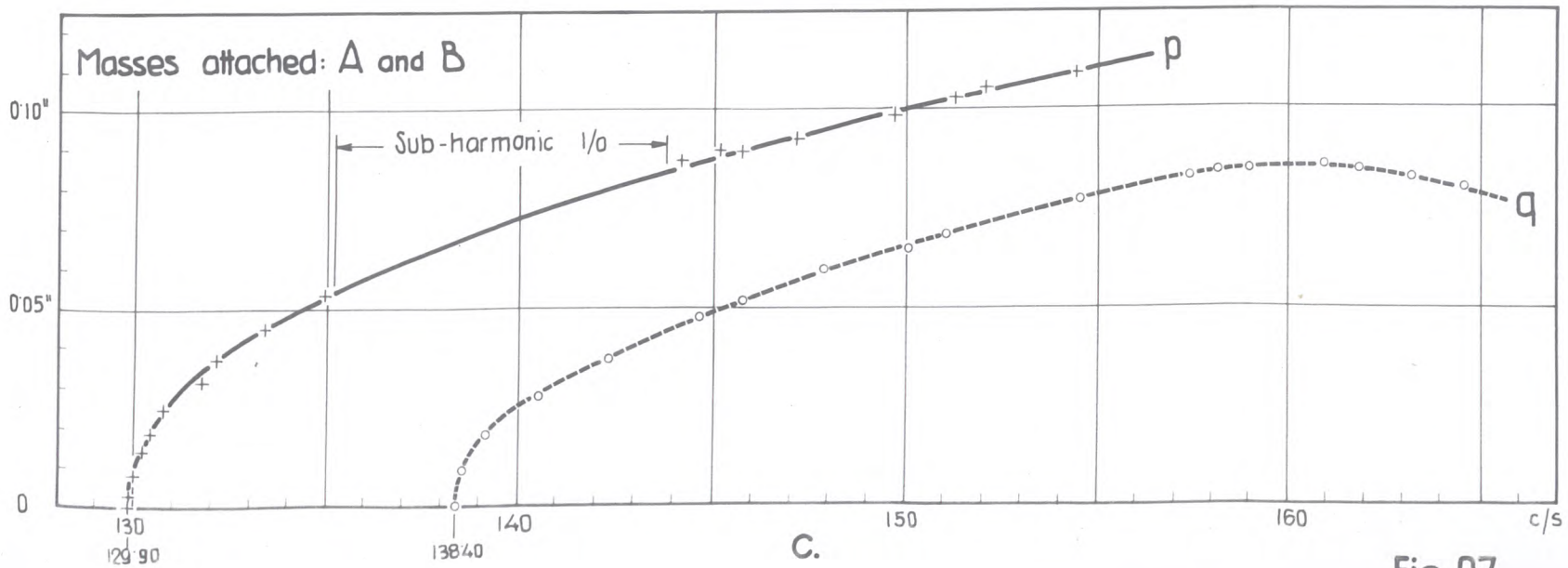
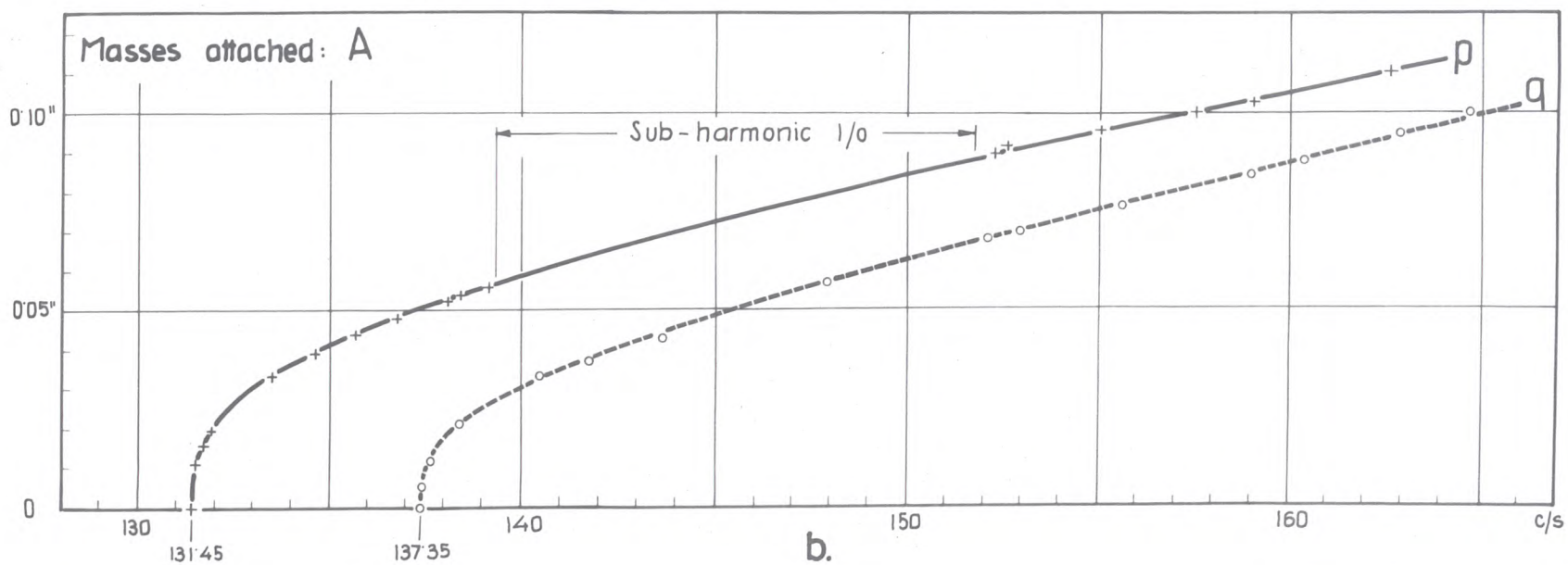
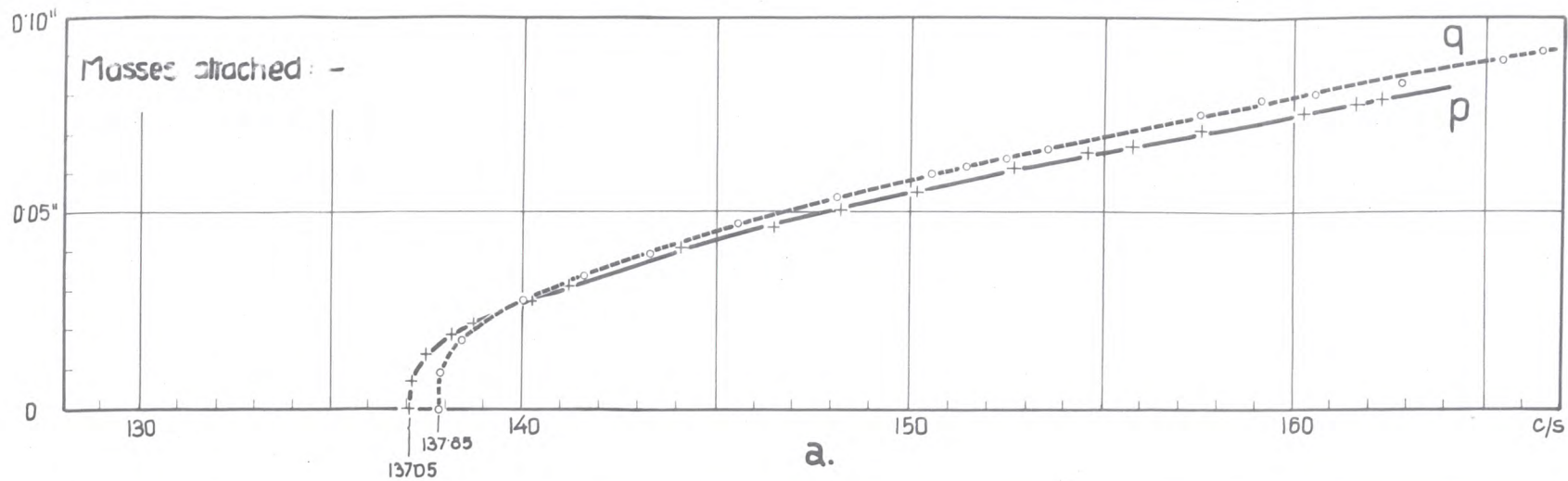
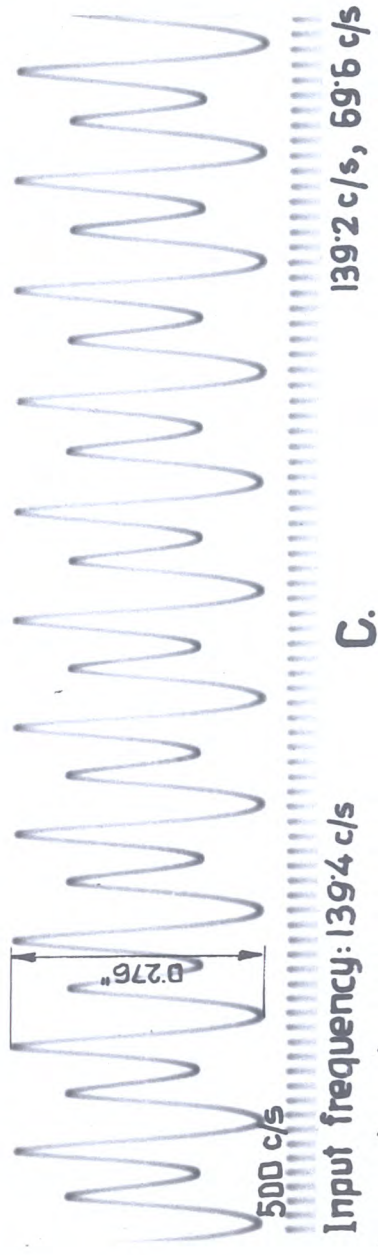


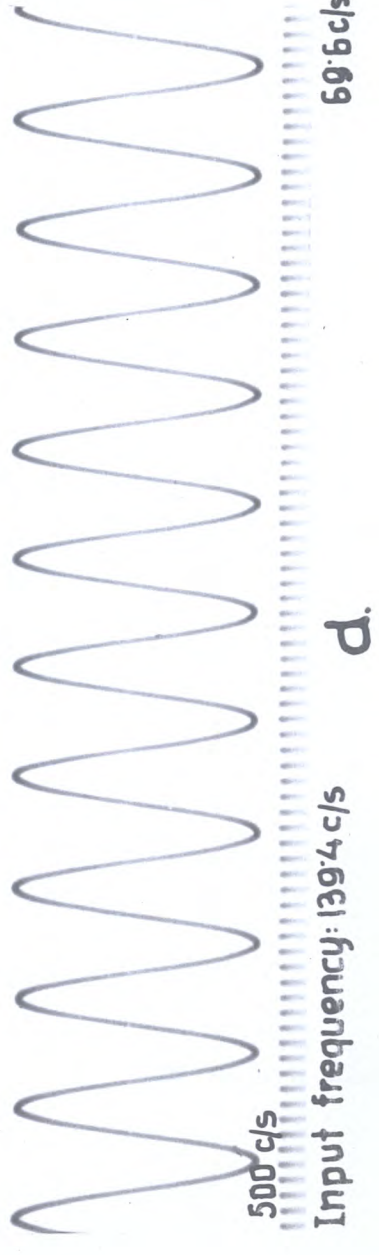
Fig. 97



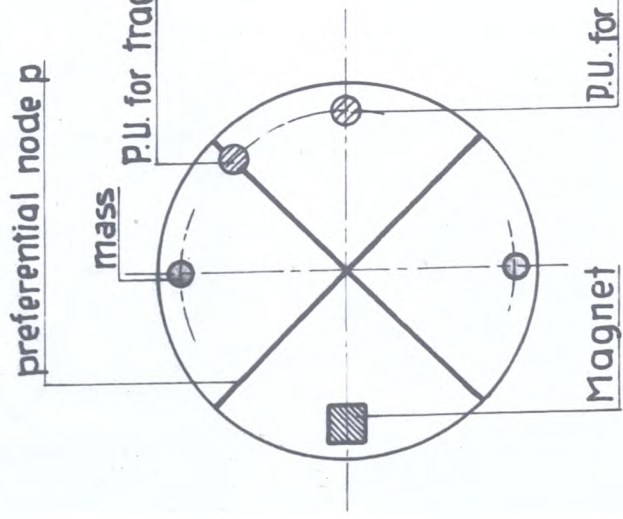
b.



c.



d.

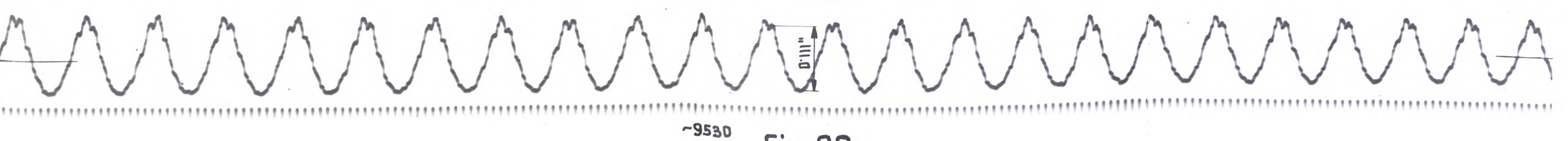
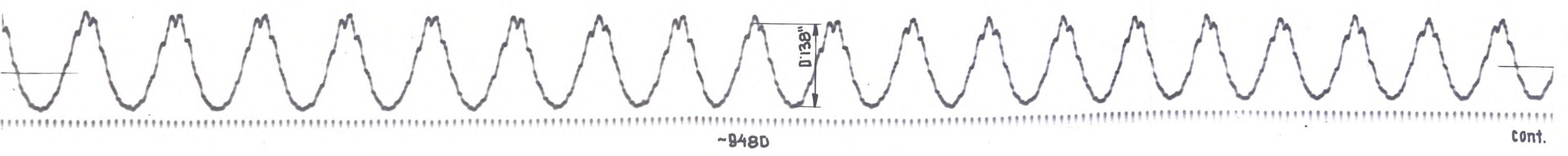
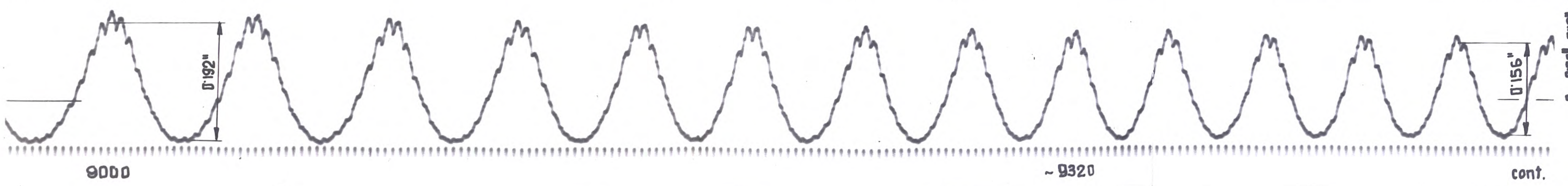
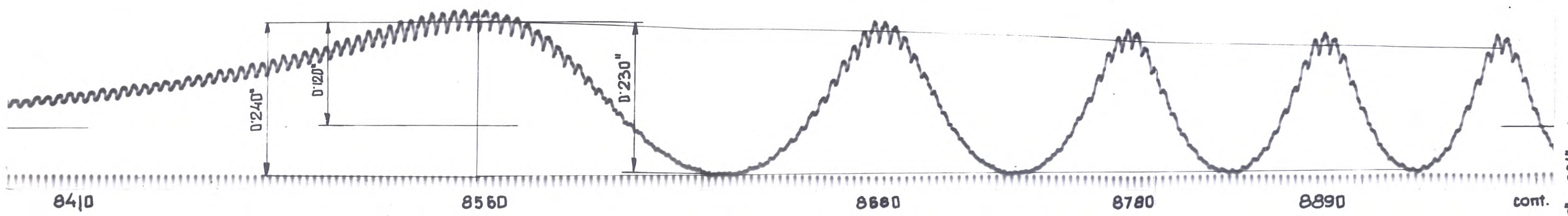
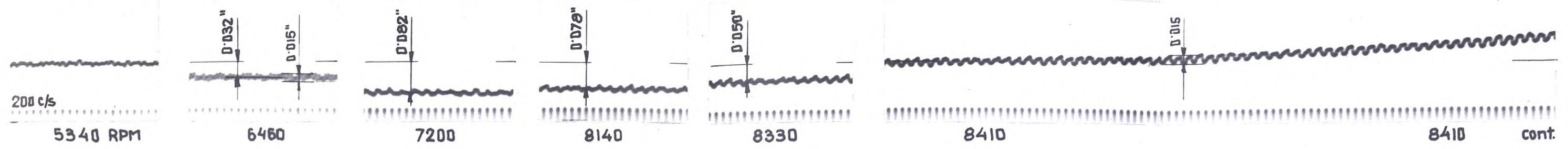


P.U. for traces b and c.

a.

Fig. 98

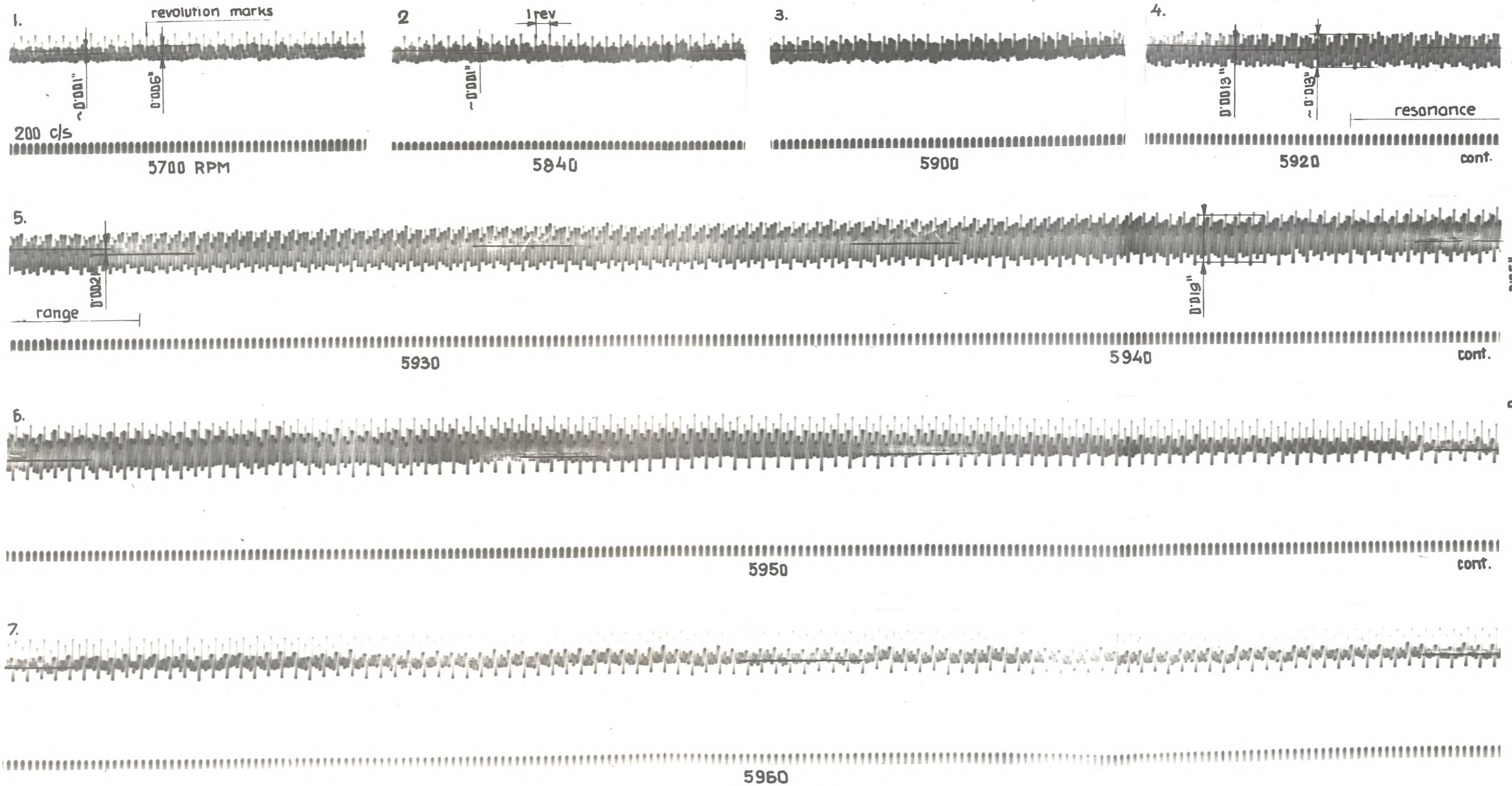
2/0 Non-linear Stationary Wave. (Masses attached: -)



"Peak to peak" amplitude scale: 0 0.06" 0.12" 0.18" 0.24"

Fig.99

2/0-p Non-linear Stationary Wave. (Masses attached: A)



5960

Fig. 100

2/0 - q Non-linear Stationary Wave. (Masses attached: A)

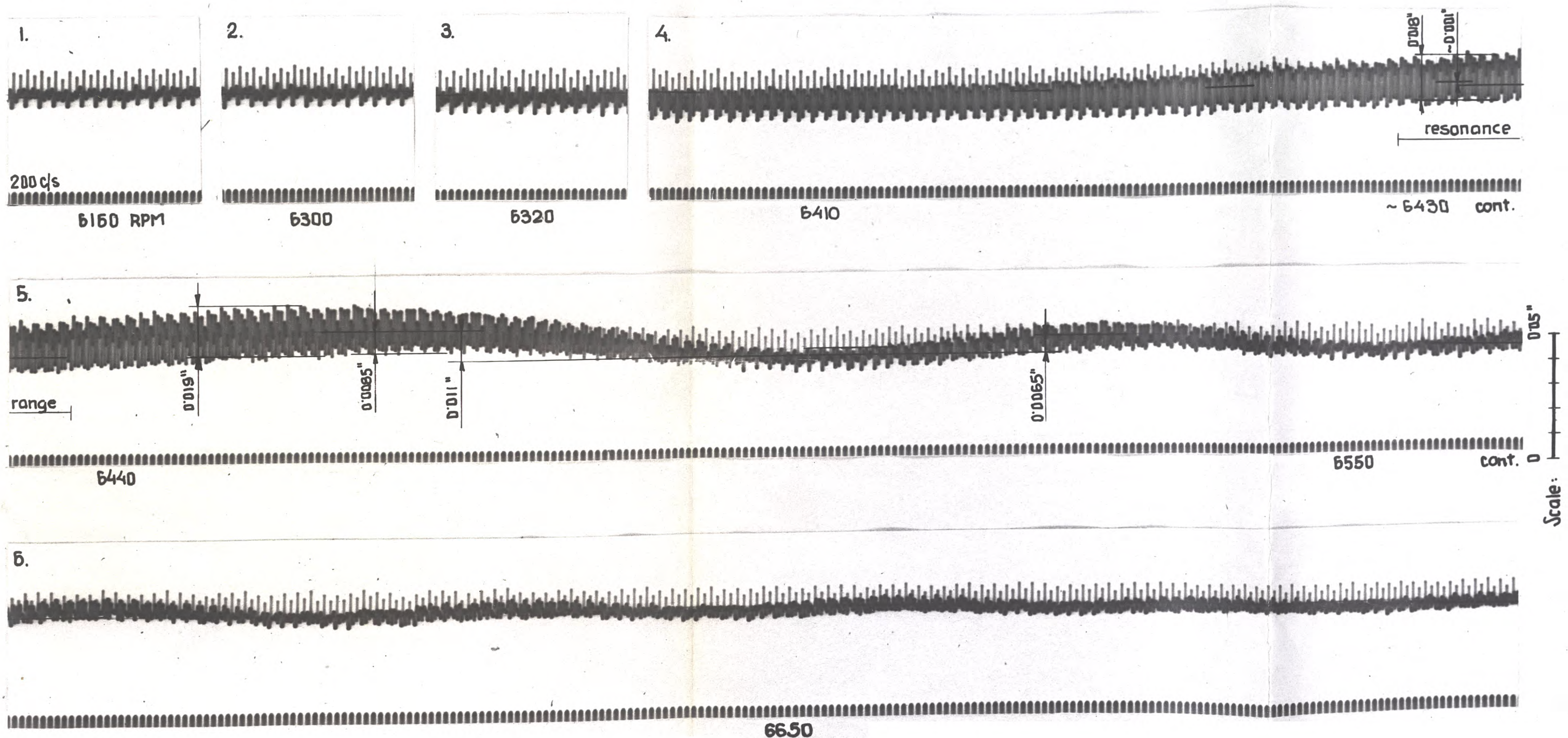
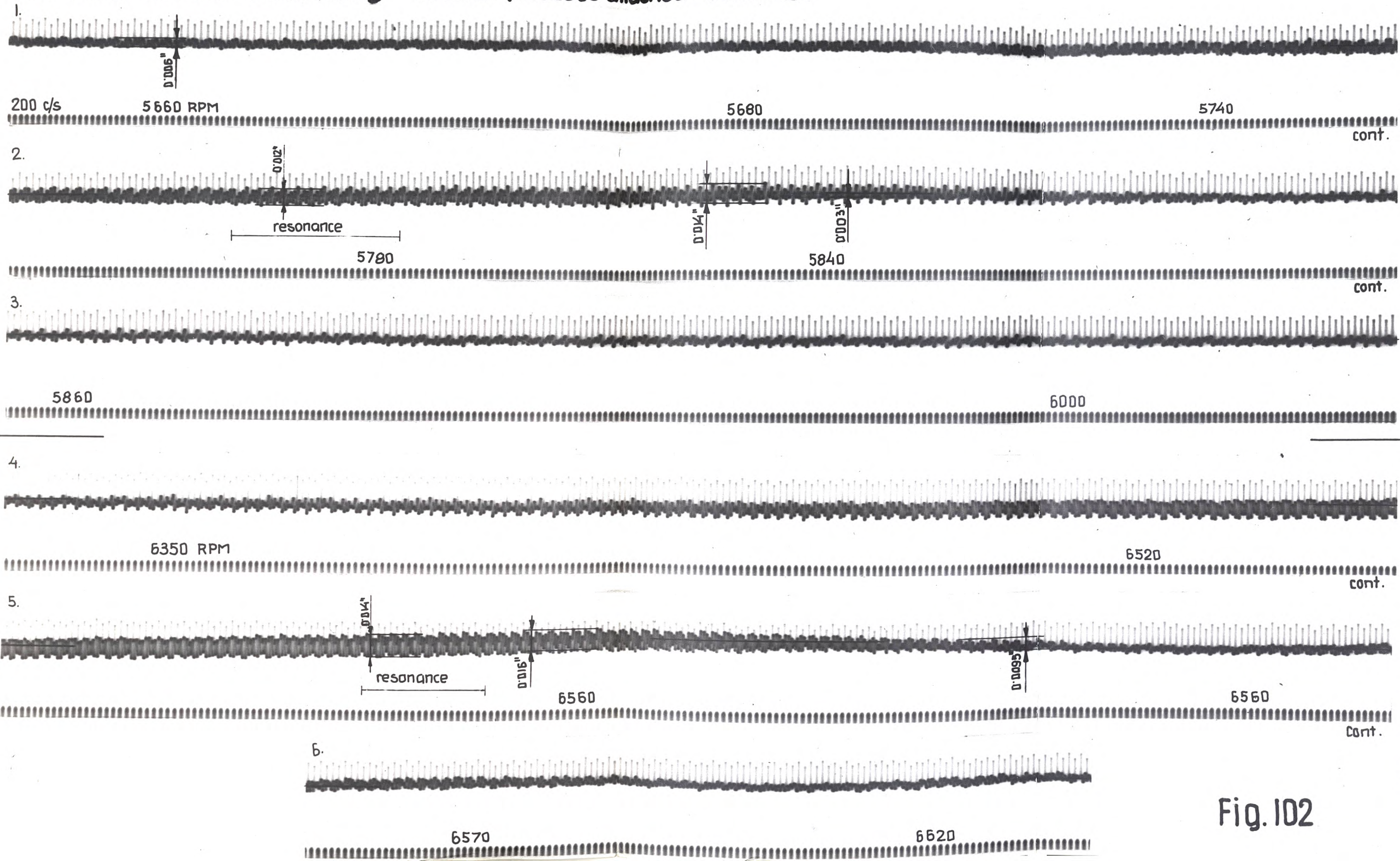


Fig. 101

2/0 Non-linear Stationary Wave. (Masses attached: A and B)



Configuration p

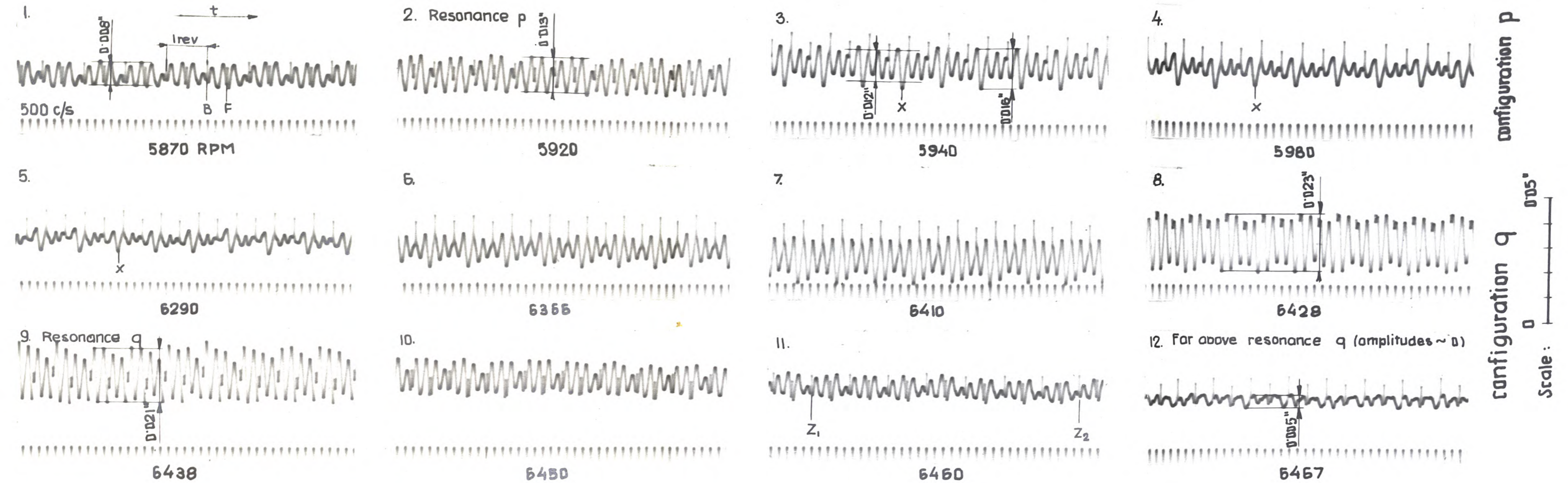
Scale: 0 0.015"

Configuration q

Fig. 102

2/0 Non-linear Stationary Wave.

(Masses attached: A)



(Masses attached: A and B)

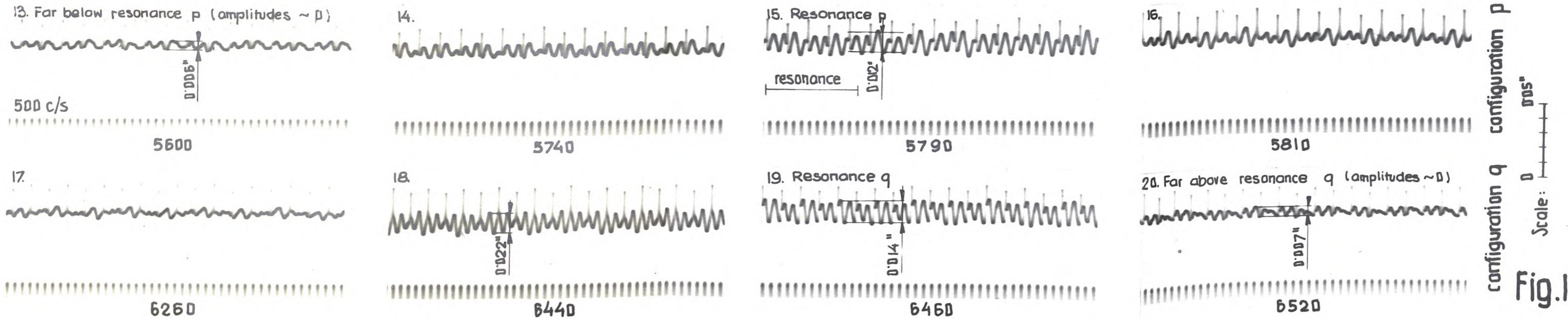


Fig. 103

displacement inch.

Force lb.

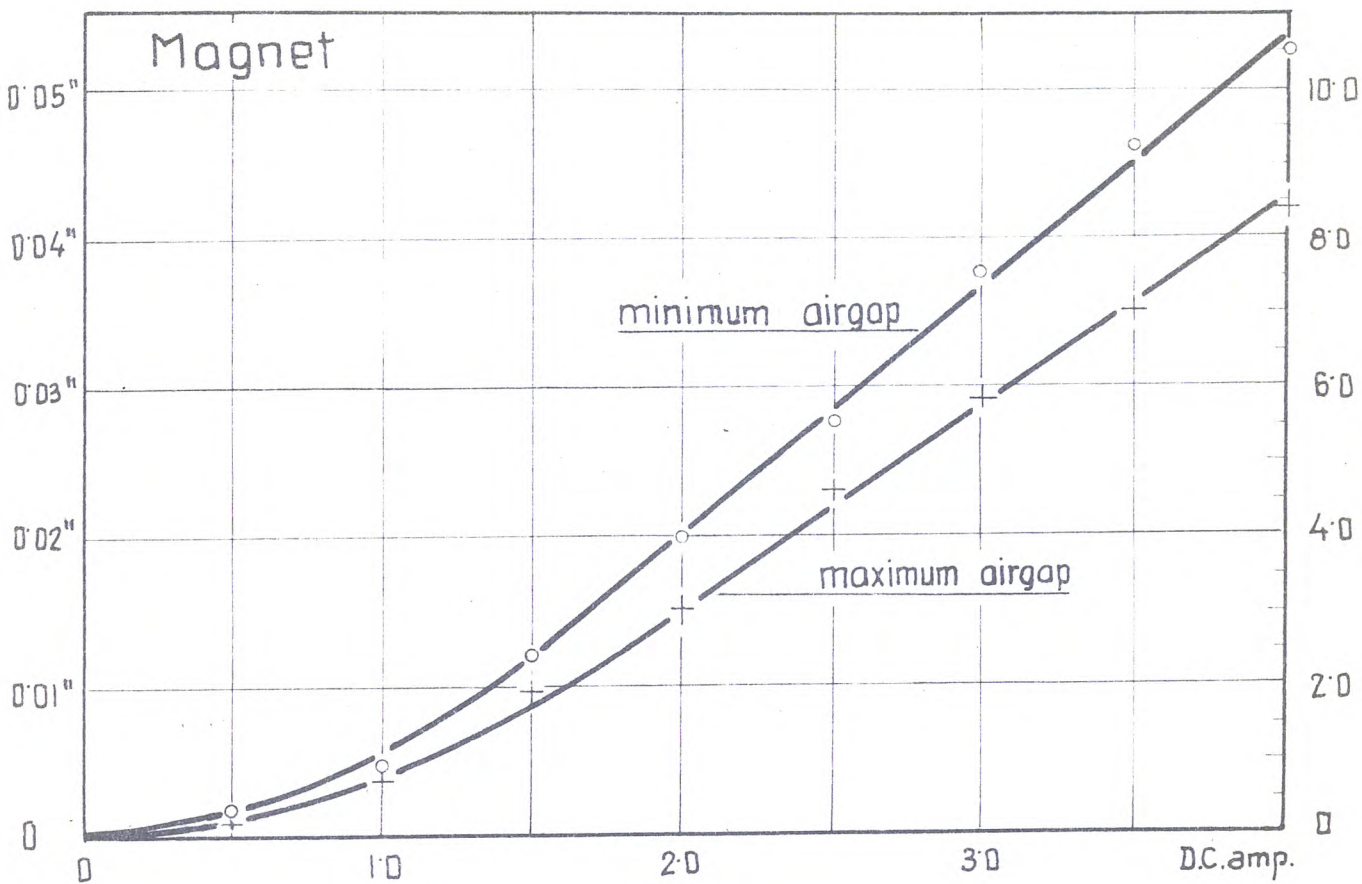


Fig. 104

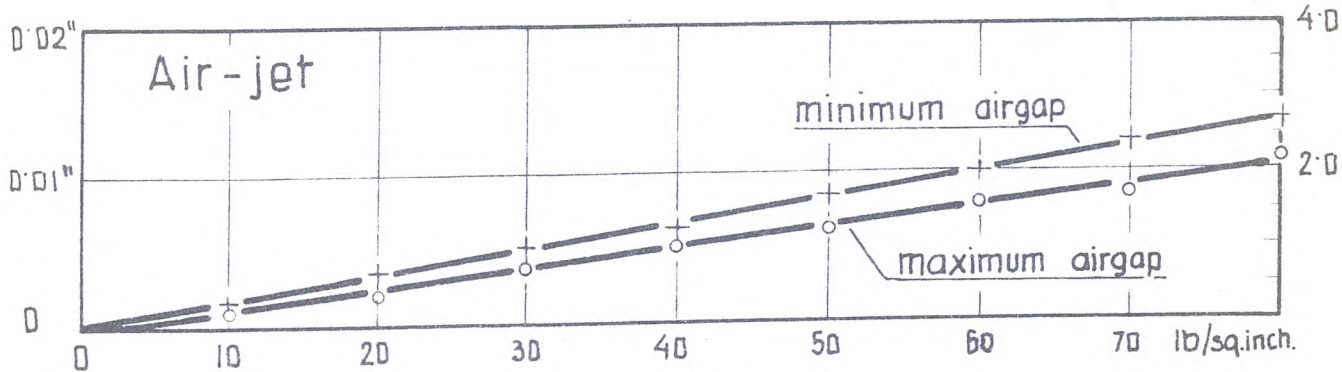


Fig. 105

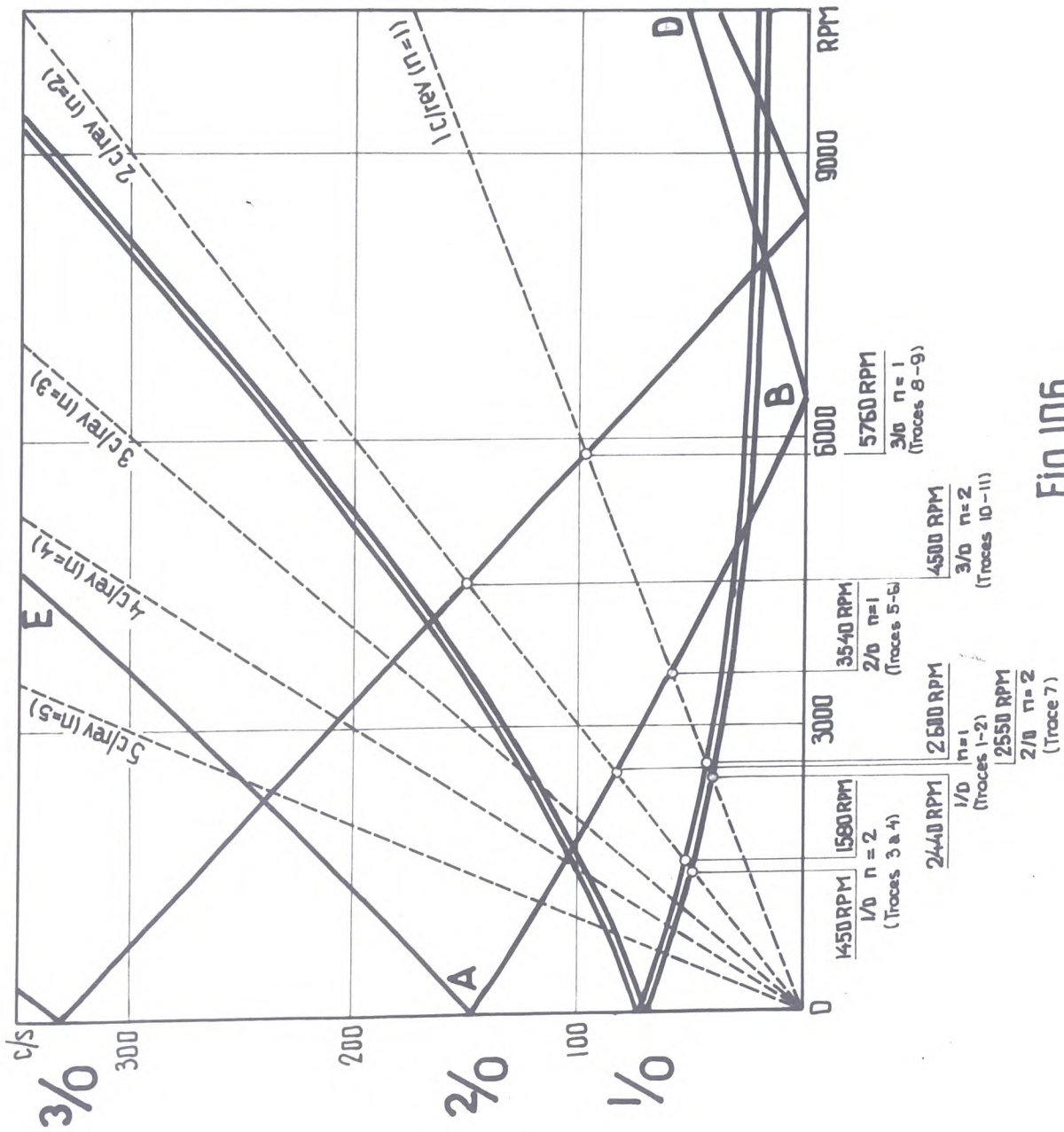


Fig.106

Minor Resonance.

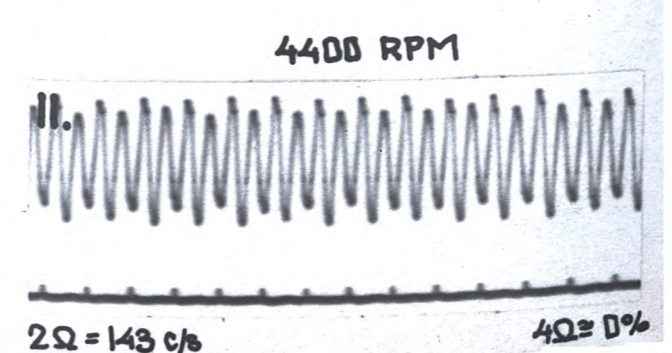
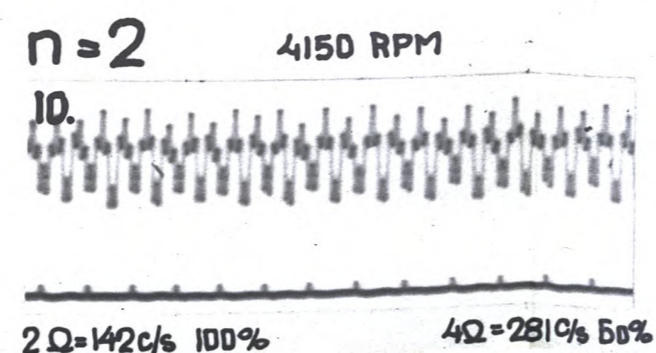
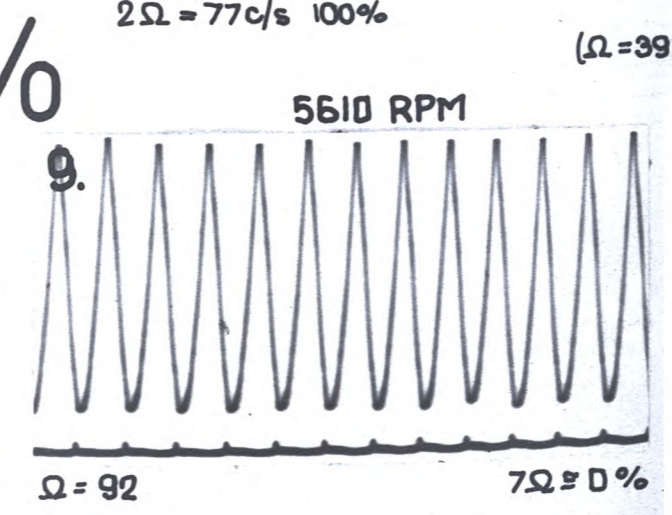
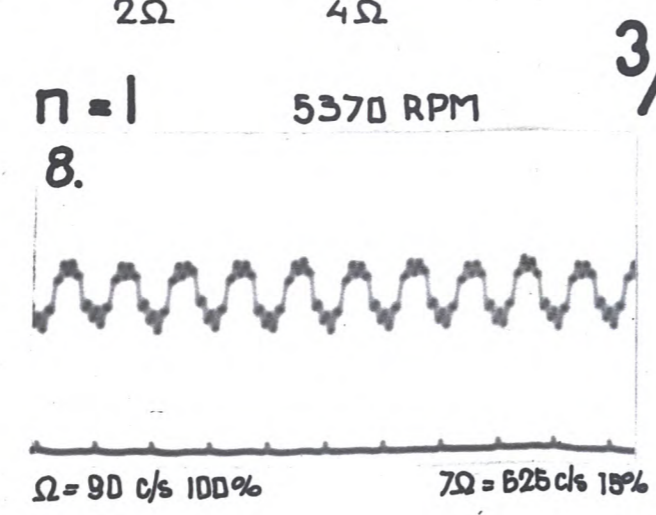
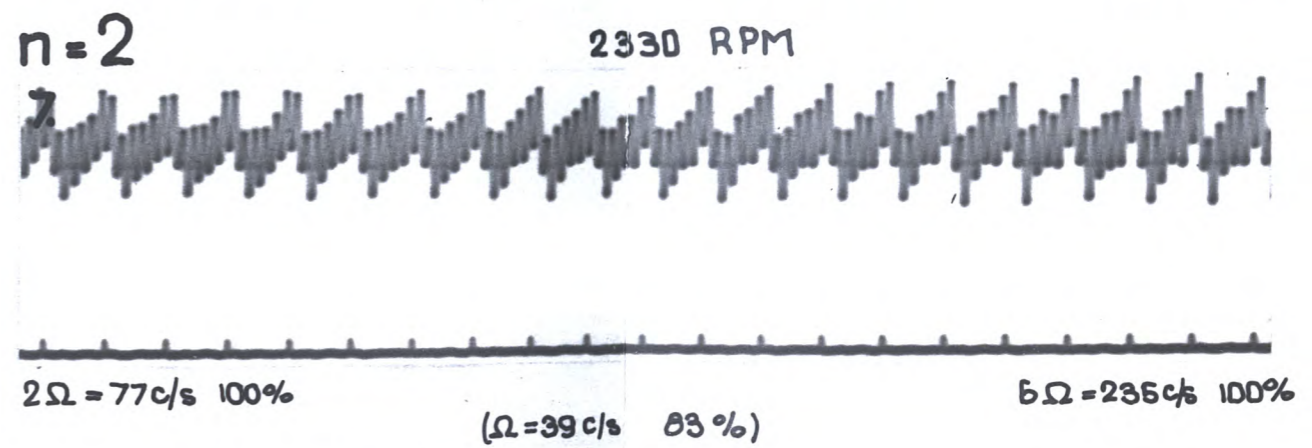
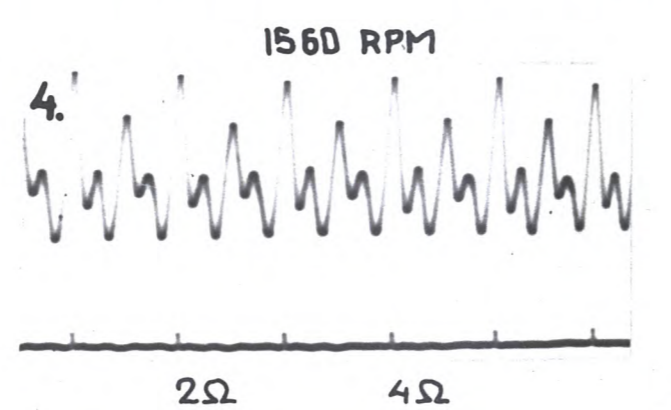
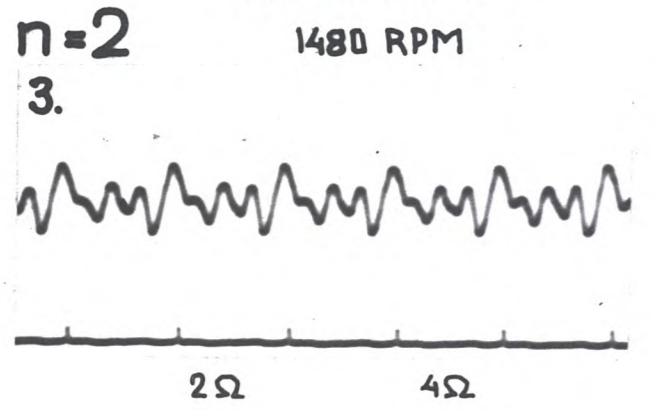
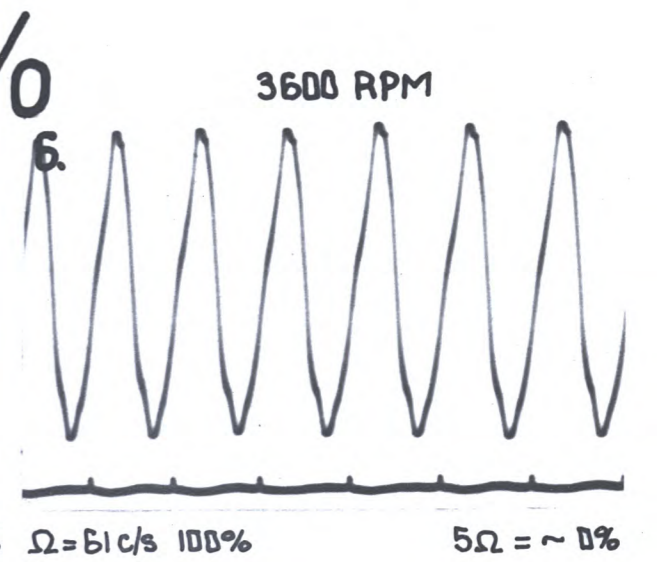
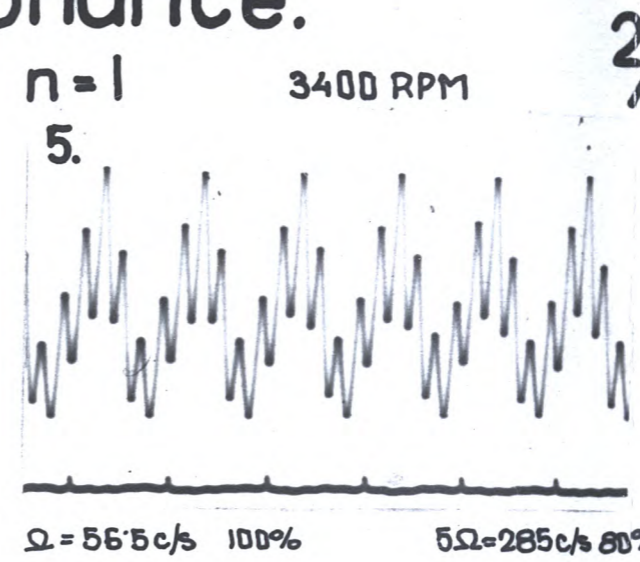
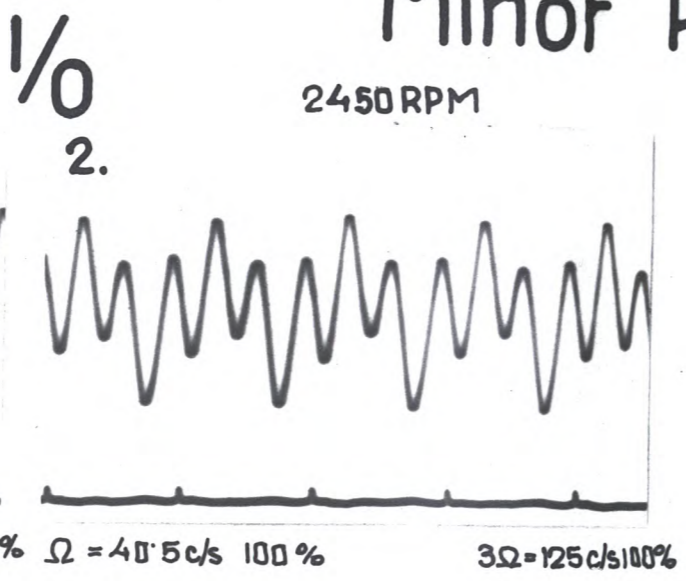
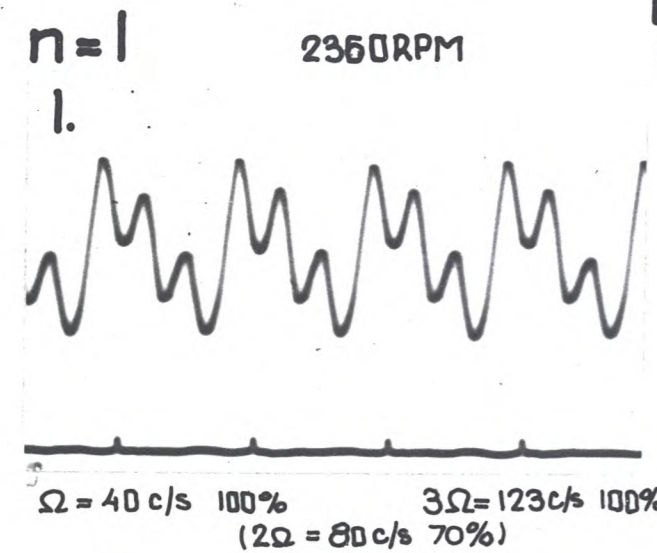


Fig.107