REML AND THE ANALYSIS OF SERIES OF VARIETY TRIALS

by

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DECLARATION

This thesis is a record of research submitted for the degree of Doctor of Philosophy of the University of Edinburgh and has not been submitted for any other degree. Except where acknowledgement is made, the work is original.

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Dedicated to my parents:

Gedion & Kefuziba Nabugoomu

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ABSTRACT

In the UK, official series of trials are grown annually at several centres with the objective of predicting future variety performance under the growing conditions sampled by the trials. For this purpose, the centres are chosen to be representative of the growing conditions in the region to which results will be applied. Analysis involves combination of trial results over centres and years. The analysis for individual years is also important as it predicts performance under conditions of a particular year and is also required for monitoring the trials. Varieties × environments tables are inevitably incomplete and the use of interactions as error makes the REML algorithm suitable for analysis.

The models for analysis are determined solely by the objectives of analysis and the data structure. To predict variety performance for a range of conditions sampled by the trials, only variety effects should contribute to the systematic part of the model, all other effects and interactions are error. In this thesis we use REML to analyse the varieties × centres × years table, varieties × years/centres table and the varieties × regions/centres × years table.

Simple methods based on least-squares analysis of two-way tables have been used to provide a combined analysis. We show that these methods give the same means as a full analysis if the within years tables are complete. Moreover, if centres are nested within years, the use of REML in a two-stage analysis also gives correct standard errors. If some or all within-years tables are incomplete, simple methods can be inefficient.

Analysis of series of trials is often complicated by heterogeneous interactions. Sometimes this heterogeneity can be explained by differences in response to centres by groups of varieties. We show how the REML algorithm is used to deal with this type of heterogeneity. Another form of heterogeneity is when varieties have different sensitivities to centre differences. This leads to a mixed multiplicative model and we extend the REML algorithm to fit such models. This analysis

adjusts means for both incompleteness and heterogeneity and provides appropriate standard errors.

Heterogeneity of varieties $\times$ years interaction is more sensitive to departures from randomness. Successive year effects may lead to systematic effects either because the environment has changed or varieties have changed. If there are systematic effects, the objective of analysis shifts to finding a model which best describes variety performance under the conditions experienced in the trials. We give an example in which variety yields declined and associate this decline in yields to a change in type of seed used. This example demonstrates how other factors can lead to complications not only in the analysis but also on the long-term performance of the trial system.

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Chapter 1

INTRODUCTION

1.1 The UK variety testing scheme

Many developed countries have set up official schemes for testing and identifying new crop varieties to improve food production. Silvey (1978a) and Kempton & Talbot (1988) observed that new crop varieties will become increasingly important as a means of improving production. A new variety takes 10 – 15 years to develop and release for marketing.

Official variety testing in the UK, for purposes of identifying varieties suitable for commercial farming, started as early as 1920 (Bell 1976). Breeders submit new varieties for testing and only those varieties judged to be as good as the best currently available for a particular purpose are published in the Recommended List (RL). These trials provide information on the 'expected relative merit of varieties when grown on commercial scale' (Silvey 1978b).

Since entering the European Economic Community in 1973, sale of seed for specified agricultural crops in the UK is restricted to varieties in the National List (NL). To be included in the National List, a variety must be shown to be distinct from other listed varieties, uniform and stable. Consequently a new system was set up to provide information for both NL and RL (Silvey 1978b and Patterson & Silvey 1980). Bell (1976) records:

> It was considered that modification of the current Recommended List (RL) system would meet the needs of both the NL and RL by introducing a common trial system for the initial years of trials which would provide data for both purposes ... Of course a major change was to be the inclusion, in the initial years, of *all* varieties submitted on the UK basis, rather than the limited number of only the most promising varieties for England and Wales to meet the RL requirements.

NL and RL trials are organised separately for the three parts of the UK. For England and Wales, trials are grown by the National Institute of Agricultural Botany (NIAB), Cambridge; in Scotland, trials are grown by the Agricultural Colleges and, in Northern Ireland, by the Department of Agriculture. Series of trials for the main agricultural crops are grown annually at several centres. For some crops the same sample of centres is used in each year of the trials and for others a new sample is taken every year. The centres are not strictly a random sample but are selected to reflect a wide range of growing conditions in the UK.

NIAB trials are managed on a regional basis with one main centre in each region, assisted in some regions by commercial farms hired from year to year. England is divided into six regions and Wales is considered a region. Bell (1976) noted:

> The evolution of the seven regions was based on a number of features — the cropping characteristics, geographical and climatological features, the influence of latitude and longitude, ease of communication and political considerations.

The main centres are located either at university farms or agricultural college farms. For example, the main centre for the North of England is located at Cockle Park Experimental Farm of the University of Newcastle; that of East-Central region is located at the Field Station of the University of Leeds (Table 1.1.1).

Table 1.1.1:

Centres used in NIAB trials

centre	region
Cambridge	East England
Cockle Park	North England
Harper Adams	West Central England
Headley Hall	East Central England
Seale-Hayne	South West England
Bridgets	South England
Trawscoed	Wales

Testing authorities are obliged to accept for testing any variety submitted. A candidate variety is in trial for 2 or 3 years before it can be considered for entry on the National List. Existing widely grown varieties (controls) are included in the trials to provide a suitable standard of comparison. A control variety should therefore be stable over conditions sampled in the trials (Silvey 1978b). Once a NL decision has been taken, a variety does not continue in the trials unless used as a control. A variety may leave the trial system if it is withdrawn by the breeder after its first year in trial.

To be included in the RL trials, varieties in the National List must be at least as good as RL control varieties. The standards in the RL trials are higher than those in NL trials and only the best varieties in the NL trials enter RL trials. RL trials are grown for at least three years. Varieties that fall below the RL standard at the end of each year drop out of the system. Provisional recommendation is sometimes made at the end of the second year of RL trials, and only confirmed to full recommendation after a further year in trial.

In making recommendations, variety mean yields provide a criterion for selection. The yields in individual trials are first analysed and the resulting variety means are then assembled in a varieties $\times$ centres table for each year. Recommen-

Table 1.1.2:

Number of trials for winter wheat data 1974 – 1978

	year				
variety	1974	1975	1976	1977	1978
Huntsman	16	16	14	10	13
Atou	16	1	5	10	13
Armada	4	4	14	9	12
Mardler	*	4	4	9	12
Sentry	*	*	4	4	12
Stuart	*	*	4	4	12

dations are made each year using trial results for the previous five years for most crops. For some crops, such as perennial ryegrass, ten years of trial data are used.

In this thesis we describe methods for estimating variety means and standard errors of variety mean differences for recommended list trials. Our aim is to predict variety performance for the range of conditions sampled by the trials. We take examples mainly from winter wheat trials grown in Scotland during 1974 to 1978; NIAB perennial ryegrass trials sown during 1979 to 1988 and NIAB sugar beet trials grown during 1987 to 1991. A brief description of these trials follows.

Wheat trials

The data are part of winter wheat trials considered for the 1979 Scottish winter wheat recommended list. Sixteen varieties were considered and recommendation was based on the yields over the five years, 1974–1978. We consider trial data for only six varieties. This data were analysed by Patterson & Silvey (1980).

In these trials new centres were chosen in each year within the three regions of Scotland: East, North and West. Within each year, some varieties were tested at subsets of centres (Table 1.1.2). In 1977, for example, 4 of the ten centres were in the East, 5 in the North and 1 in the West. Huntsman and Atou were controls and so they were grown in all trials in that year; Armada and Mardler were in the second and first year of RL trials respectively, and Sentry and Stuart were in the second year of NL trials. The data are given in Appendix *A*.1.

Perennial ryegrass trials

The NIAB perennial ryegrass trials system is described by Silvey (1978b) and Talbot (1983, 1984). Data are available for two types of management: conservation and frequent cutting. The conservation cutting represents a system in which grass is cut for feeding to the animals whereas the frequent cutting system simulates pasture conditions grazed by animals. Harvest starts a year after sowing and continues for two years.

Thirty varieties in late cutting trials were considered for the 1990 recommended list. Trial~~s~~ designs for the conservation trials grown during 1978 to 1988 ~~is~~ are given in Table 1.1.3. The same seven centres listed in Table 1.1.1 are used each year. Each variety is initially tested in all trials for two consecutive years and is then considered for recommendation. In later years it may be re-tested in two years out of ten for as long as it stays on the recommended list. Variety 28 was a control throught the period and variety 1 became a control in the third year. Variety 6 was initially a control but was discontinued in the seventh year.

We consider data for 7 of the 30 varieties in late conservation trials and only for 5 years. Total dry matter yield is the response to be analysed. The data are given in Appendix *A*.2.

Sugar beet trials

NIAB considered 29 varieties for recommendation early in 1993. For these trials new centres are chosen each year. The number of centres each year varied from 11 to 16. A new variety is grown in seven of the trials in its first year; thereafter it is grown in all trials (Table 1.1.4). Of the 29 varieties considered, 10 were new varieties.

Table 1.1.3:

Design for conservation trials: number of centres used

	year									
variety	1	2	3	4	5	6	7	8	9	10
1	0	0	7	7	7	7	7	7	7	7
2	0	0	0	0	0	7	7	0	0	0
3	0	0	0	0	0	0	0	0	7	7
4	0	0	0	0	0	0	0	7	7	0
5	0	0	0	0	0	0	0	0	7	7
6	7	7	7	7	7	7	7	0	0	0
7	0	0	0	0	0	0	0	0	7	7
8	0	0	0	0	0	0	0	0	7	7
9	0	0	0	0	7	7	0	0	0	7
10	0	0	0	0	0	0	0	0	7	7
11	0	0	0	0	7	0	0	0	0	7
12	0	0	0	0	0	0	0	0	0	7
13	0	0	0	0	0	7	7	0	0	0
14	0	0	0	0	0	7	7	0	0	0
15	0	7	0	0	0	0	7	0	0	0
16	0	0	0	0	0	0	0	0	7	7
17	7	7	0	0	0	7	7	7	0	0
18	0	0	0	0	0	0	0	0	7	7
19	0	0	0	0	0	0	0	7	7	0
20	0	7	7	0	0	0	0	7	0	0
21	0	0	0	0	0	0	0	0	7	7
22	0	0	0	0	0	0	0	0	7	7
23	0	0	0	0	7	7	0	0	0	7
24	0	0	0	0	0	7	7	0	0	0
25	0	0	0	0	0	0	0	0	7	7
26	0	0	0	0	0	0	7	7	0	0
27	0	0	7	0	0	7	0	0	0	0
28	7	7	7	7	7	7	7	7	7	7
29	7	0	0	0	7	0	0	0	0	7
30	0	0	0	0	0	7	7	0	0	0

Table 1.1.4:

Number of trials for sugar beet 1987 – 1991

V1 – V16	16	16	16	11	13
V17	7	16	16	11	13
V18, V19		7	16	11	13
V20 – V29			7	11	13

All recommended varieties continue in the trials and the recommended list is revised annually. Variety recommendations are based on five years of performance. Several field characteristics of varieties are measured, including root yield, sugar yield, grower's income and sugar content, but only root yield will be analysed here. The data are given in Appendix *A*.3.

1.2 Linear modelling: Fixed, random and mixed models

Linear modelling has a long history as a method of analysing variety trials and will be our main tool of analysis. The sources of variation in a response variate, say yield, are modelled as components of a linear model. A full analysis of series of trials is based on multi-classified tables in which the environments are classified by centres and years. For example, a full analysis of series of trials in which the same sample of centres is used in each year is based on a varieties × centres × years table.

Yates (1933, 1934) described how least squares methods used in regression could be adopted to analyse multi-classified data with only one error term. He referred to this use of least squares as *fitting constants.* The acronym FITCON is also used. In the analysis of series of trials FITCON refers to a least squares analysis of a varieties × environments table.

Eisenhart (1947) was the first to categorize effects as either fixed or random. Effects of a factor are regarded as fixed if inference is restricted to the observed levels of the factor. If, however, the observed levels are considered to be a random sample from a population on which inference is sought, then the effects are random. If a model has only fixed effects apart from the residual term, it is a fixed model; and if a model has all its effects random it is a random model. Mixed models have both fixed and random effects.

This classification of effects raises controversies in rules for calculating expected mean squares especially in a mixed two-way classification (Wilks & Kempthorne 1955; Placket 1960 and Kempthorne 1975). Yates (1967) and Hocking (1973, 1985 page 303) observed that the controversy in calculation of expected mean squares is because of a difference in definition of components of variance. They affirmed that the F-test for the random factor should use the interaction mean square as denominator and not the residual mean square. Some statisticians hold the view that the interaction of a fixed effect and a random effect should have levels corresponding to the fixed effect constrained to a zero sum. Yates (1967), Nelder (1977) and Hocking (1973) maintained that the constraints are of no practical use.

In recommended list trials, varieties are distinct entities and not a random sample. They are therefore treated as fixed (Patterson & Silvey 1980). The environment can be categorized by centres, years and combinations of centres and years. For some crops, centres are classified by regions. In general we regard environmental factors as random because our objective is to predict variety means for a range of conditions. Consequently varieties $\times$ environments interactions are random, but there are exceptions, for example, when there is complete sampling with respect to a stratum of the environments.

The REML algorithm

The estimation of means of fixed effects in a mixed model depends on the variances of random effects that need to be estimated. When an effect or interaction is random we shall assume that it is normally distributed and use the REML[1] algorithm for analysis (Patterson & Thompson 1971). REML is preferred because, among other reasons, it gives the same results as regression methods when the model has only a residual component of variance; and when the data are complete REML gives the same results as ANOVA.

The REML algorithm estimates components of variance from the residual likelihood. The effects of the systematic part of the model are estimated from a generalised least squares analysis conditional on REML estimates of components of variance. Tables of means are obtained as margins to a table of expectation. Several statistical packages have facilities for REML. In this thesis we use the Genstat[2] REML algorithm (Supplement to Genstat 5.2 1990).

The principle of parsimony

One concept that needs comment is that of parsimony (Aitkin 1978 and Aitkin, Anderson, Francis & Hinde 1992 page 68). Models are chosen for which there is empirical evidence in the data that each of the factors is significant. If an interaction is not significant it is then dropped from the model. Aitkin (1978) wrote:

> In the *smoothing* stage, the complex full model is reduced to a parsimonious one, by setting unnecessary interactions to zero ... The persimonious model may then be used in a *prediction* stage ...

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[1]REML stands for REsidual Maximum Likelihood; see Chapter 3

[2]Genstat is a registered trade mark of Numerical Algorithms Group Ltd.

But as Cox (1978) remarked:

> There must be many situations in which all main effects should be included in the model regardless of significance and quite a number where some or all two-factor interactions should be included also.

The analysis of series of trials is one such situation where models are determined by objectives of analysis and the principle of parsimony does not apply. There is considerable empirical evidence that varieties $\times$ environments interactions exist (Patterson, Silvey, Talbot & Weatherup 1977 and Talbot 1984) and model building using the principle of parsimony can lead to inadequate variety means and standard errors.

Prediction

The word *predict* is usually used if we require estimates of future realisation of an observed variable. In analysing series of trials we estimate future performance of a variety under conditions sampled by the trials. That is, we project results to a population from which conditions sampled are regarded as a random sample. Recently, Nelder (1994) has argued that prediction is one of the four important general notions in statistical science. He identified two components of prediction. The first is the formation of summary statistics following an analysis. On the second component Nelder wrote:

> The second component of prediction is the combination of information from current experiments with relevant past experiments ... where no trial is sufficient on its own. Yates & Cochran were writing about this in agriculture in 1937, yet this vital process has been ignored by statisticians ...

We are concerned with Nelder's second component.

Bross (1953, page 33) made the following analogy in explaining prediction:

> Even in the mysterious and erratic world in which we live, there are some threads of continuity. There is chaos and confusion all about, but also some system and stability. Our progress in the real world is like driving along a road that is shrouded in a heavy fog; there are no sharp clear details, but only vague outlines. By looking very hard through the swirling, random fog shadows we can distinguish enough of the more permanent road shadows to enable us go ahead successfully if we go slowly and use caution.
>
> Similary the first step toward prediction is the search for stable characteristics — those characteristics which persist over a period of time *and space* (our emphasis).

Thus in predicting we estimate variety means that are valid for future years and over the regions sampled by the trials. As noted previously (Section 1.1), centres are not random but to the extent that they reflect a wide range of growing conditions in the UK we regard them as effectively random.

Year effects can include weather effects, trends and other systematic changes related to time. In making years random, it is weather effects that are under consideration. Immer, Hayes & Powers (1934) wrote:

> In so far as these six *centres* consititute a random sample of conditions to be found in the entire state and that these 2 years are a random sample of weather conditions to be encountered in future years, general recommendations may be drawn up for the entire state with reasonable assurance that the variety or varieties recommended will prove to be consistently superior in most places of the state and in most years.

And according to Fisher (1935):

> There seems, in fact, in no part of the world to be any such similarity between successive seasons as would make the experience of a sequence of trials unreliable for future application in the absence of genuine secular *(long-term)* changes of the climate.

Yates & Cochran (1938) advise that years could be considered random if the years in the trials provide a wide range of weather conditions. But if there are trends or systematic effects in variety yields, the concept of a population of years is not valid. The objective of the analysis is then not to predict but to describe the performance of varieties under the actual conditions in the trials.

1.3 Objectives

The analysis of series of trials is by no means a new area but has always raised fresh questions for statisticians (Yates & Cochran 1938; Kempthorne 1952, chapter 28 and Cochran & Cox 1957, chapter 14). To quote Yates (1967):

> Further work for which I believe computers will be particularly useful is the combined analysis of sets of experiments, and the analysis of accumulated results of long-term experiments. The techniques required for the analysis require much development ...

If all varieties are tested in each trial, all varieties $\times$ centres tables and the varieties $\times$ years table are complete, the analysis is easy and ANOVA is used. Yates & Cochran (1938) discussed the use of ANOVA when tables are complete. Simple means are adequate in estimating variety performance and standard errors of variety mean comparisons are functions of varieties $\times$ environments variances estimated from the ANOVA table. Fisher (1935, section 65) gives general advice on the use of interactions as error.

Incomplete tables

However, varieties × centres and varieties × years tables are often incomplete. Incompleteness arises because of the trials design and the number of years used in the analysis.

For some crops, for example wheat, not all varieties are tested at all centres in a given year because either the number of varieties is too large or some varieties have insufficient seed. Varieties × centers tables and the varieties × years table are then incomplete. For others crops, for example ryegrass, each variety is grown in every trial during the testing period. Varieties × centres tables are complete but the varieties × years table is incomplete. Sugar beet trials provide yet another form of incompleteness. If recommendation is based on three years of trials, the varieties × centres table are complete for two years and the varieties × years table is also complete. However, recommendation for these varieties is sometimes based on five years of trials. The varieties × centres tables are then incomplete for the first two years and the varieties × years table is also incomplete. See pages 4 and 5.

Incompleteness introduces complications in the analysis not considered by Yates & Cochran (1938). Estimation of variety means and standard errors depend on varieties × environments variances. Long-term estimates of varieties × environments variances show that varieties × environments interactions exist (Patterson *et al.*'s 1977 and Talbot 1984). Ignoring these interactions will lead to biased estimation of variety performance. Patterson & Silvey (1980) assert that assessment of variety performance is incomplete without knowledge of varieties × environments variances.

The problem of incompleteness is, in principle, solved by the use of REML provided every variety experiences a random sample of environments. Thus subsets of environments should be random samples of environments. Patterson & Silvey (1980) gave a general modelling framework for analysis; we explore some of these methods with examples. Patterson & Nabugoomu (1992) discussed the application of REML in analysing series of trials. We provide more examples.

Simple methods

Before suitable software was available for REML, simple methods were used to analyse series of trials. These include: adjustment by control varieties, the percentage method and least squares analysis of two-way varieties × environments tables (Patterson 1978; Patterson & Silvey 1980 and Silvey 1978b).

Least squares analysis can be done in a single stage or in two stages. A single-stage analysis estimates variety means from varieties × trials table ignoring the years classification. The first stage of a two-stage analysis is the analysis of trials for the individual years i.e, varieties × centres table. This analysis is important in its own right as it provides means for varieties under conditions of a particular year. In the second stage variety means from the first stage are analysed in a varieties × years table. We compare single-stage and two-stage analysis with a full analysis based on multi-way classification models.

Heterogeneity of varieties × environments variances

Analysis of series of trials is often complicated by heterogeneity of varieties × environments interactions and a wide range of plot error variances (Yates & Cochran 1938). In the UK trials, the range of plot error variances is not large enough to raise problems in the analysis (Patterson & Silvey 1980). We deal with complications due to varieties × environments interactions.

Yates & Cochran (1938) observed that varieties × centres variance and varieties × years variance may be heterogeneous. In their example they found that one variety varied more from centre to centre than others. Yates & Cochran showed how partitioning degrees of freedom in the ANOVA table provides a satisfactory analysis. In some cases groups of varieties may be identifed that respond similary within groups but with a groups × environments interaction. We extend this principle to incomplete tables and show how REML is used to analyse data with heterogeneous varieties × environments interaction.

Mixed multiplicative models

To identify the variety reponsible for heterogeneity, Yates & Cochran used regressions of each variety yields on centre means averaged over the two years. The method of regressing variety yields on environment means was used by Finlay & Wilkinson (1963) and Perkins & Jinks (1968) and is commonly referred to as *joint regression.* We refer to the estimates of regression slopes as *sensitivities.* The joint regression model is multiplicative and can be used to allow for a more general form of heterogeneity.

When varieties × environments table is incomplete a joint regression analysis gives a new set of variety and environment means. Digby (1979) described a least squares method in which the new environment means provide improved estimates of sensitivities. These are in turn used to improve estimates of variety and environment means. Oman (1991) used maximum likelihood to fit mixed two-way classification models with a multiplicative interactions. Shortcomings of maximum likelihood in mixed model situations are wellknown, for example, Harville (1977). We extend the REML algorithm to this class of models.

1.3.1 Notation

We use the notation of Wilkinson & Rogers (1973), also used by Nelder (1977) and McCullagh & Nelder (1983, section 3.4). The factors in a model are denoted by the capital of the first letter of the factors: i.e varieties (V), environments (E), centres (C) and years (Y). The interaction between two factors is denoted by letters denoting the factors separated by a dot. For example, $V.E$ denotes varieties

$\times$ environments interaction. When we refer to a varieties $\times$ environments table of means the notation $V \times E$ is used.

A model formula is divided into two parts separated by a full colon. The terms before the colon enter the model as fixed effects, whereas the terms after the colon enter the model as random effects. The residual part of the model is not included in the model formula. For example,

$$V : C$$

specifies a model in which variety effects are fixed, centres effects and varieties $\times$ centres interaction are random.

In specifying models with interaction, an asterisk is used to shorten formulae. For example, $V * C$ has the expansion: $V * C = V + C + V.C$ Thus the data structure for a $V \times C \times Y$ table can be written as $V * C * Y$.

A '/' is used for nested effects. The data structure in which trials are grown at new centres each year has centres nested within years and is denoted by

$$V * Y/C = V + Y + V.Y + V.C.Y$$

The term $V.C.Y$ is then the varieties $\times$ centres within years interaction.

When an effect is random, its component of variance is denoted by σ^2 subscripted by first letter(s) of the factor(s). For example σ^2_C is the centres component of variance and σ^2_{VY} is the varieties $\times$ years component of variance. The residual variance in any model will be referred to as *units variance.*

1.3.2 Overview

We review previous work in the analysis of series of variety trials in Chapter 2. In Chapter 3 we describe the REML method and its historical development.

We discuss the analysis of a varieties $\times$ centres table in Chapter 4. We give examples on REML and FITCON analysis and show how REML is used to analyse data with groups (of varieties) × centres interaction. In Chapter 5 we describe the method of Digby (1979) for fitting a two-way multiplicative model. We extend the REML algorithm to fit a mixed multiplicative model to give results that apply to a population of centres.

In Chapter 6, we describe the use of REML to combine trials over centres and years. We compare simple methods with the analysis based on full models and give examples of series of trials in which the data structure is $V * C * Y$, $V * Y/C$ and $V * (R/C) * Y$.

Methods for modelling heterogeneity of varieties × environments interaction are desribed in Chapter 7. We extend methods of Chapter 4 and 5 to a combined analysis over centres and years. This chapter also describes a complex form of varieties × environments heterogeneity in which recommended varieties in sugar beet trials declined in their performance over time. We investigate differential seed quality as a possible explanation of heterogeneity.

Chapter 8 contains suggestions for further work.

Chapter 2

ANALYSIS OF SERIES OF TRIALS — literature review

2.1 Analysis of variance methods

2.1.1 Early studies

Student (1923), one of the first statisticians to be involved in design and analysis of variety trials, observed that the main objective of testing varieties is to find out which variety will increase yield; and that considerable improvement in yield was a result of replacing native varieties with improved varieties. Trials grown at different places and seasons experience weather variation that could lead to results inconsistent with experimental error determined at either place.

Student (1923) analysed part of a series of trials carried out by the Department of Agriculture in Ireland to find the best variety to grow in that country. The experiment lasted six years, during which time only two of the seven varieties of barley tested completed the series. The two varieties tested throughout were analysed using a t-test on differences.

Student (1931) observed that only by growing trials at several centres and in several years could information be obtained for identifying good varieties. After an excellent exposition of principles underlying the analysis of variance, Student wrote:

> While it is obvious that there is a lot to be gained by planning field trials in such a way as both to reduce experimental error and to obtain an accurate estimate of it, it is important to remember that conclusions *can only be drawn applicable to the particular conditions in which the trials were carried out.* For this reason trials should be repeated from season to season, and in so many places to cover probable variation ...in which practical application is made.

Neyman (1935) also considered the analysis of a series of experiments and noted that conditions of analysis are different from those of a single experiment. In series of experiments large differences between varieties at different environmental conditions indicate the presence of varieties × environments interactions with a possibility that the interaction is unequal. Neyman suggested that such experiments be analysed by a series of regressions of yields of one variety against those of another. Neyman gave two advantages of this method of analysis: to avoid inconsistencies arising out of different environments in the experiment and to provide a better standard of comparison as often experimental stations gave better yields than ordinary farms.

One of the earliest data sets on series of variety trials to be published is that of Immer *et al.* (1934). The trials were grown to identify varieties which would give the best yields if grown throught the state of Minnesota. For this purpose the trial conditions were considered to be a satisfactory sample of practical growing conditions. Fisher (1935) included part of this data in his book, *The Design of Experiments*, as an example for the reader to practise analysis of variance technique. The attempts by Immer *et al.* (1934) to project results to other centres and future years required the use of interactions as error.

If the objective of the analysis is to select the best variety over the the region from which centres have been selected, Fisher (1935) advised that the appropriate error for variety comparisons includes a varieties × centres interaction. Fisher argued that the test employed is equivalent to considering only the average yield from individual trials, and the estimation of error in individual trials only provides assurance on the reliability of individual trials.

The within-year analysis indicates what varieties performed best in that year but significant variety differences based on varieties × years interaction shows varieties most successful in conditions of which those experienced may be taken as a random sample.

The use of interactions as error has been explored by Kempthorne (1952) who noted that although the estimates of variety differences are independent of the nature of recommendations when the data are complete, the standard error associated with the estimate reflects the use to which the estimate is put. Cochran & Cox (1957) and Kempthorne (1952) devote a complete chapter in their books on complications that arise in analysing series of trials and on the use of ANOVA. The principles of analysis are however given by Yates & Cochran (1938).

2.1.2 Yates & Cochran (1938)

Yates & Cochran (1938) uncovered endless complications in Immer *et al.*'s (1934) data. They analysed the data published by Fisher (1935). The estimation of variety means is of considerable importance even when there is a varieties × centres and varieties × years interaction and unless the causes are known, future variety performance can only be based on these means. They pointed out that the error mean square in a varieties × centres table is constituted by varieties × centres interaction and plot error variance.

In making recommendations to cover a wide range of environmental conditions, estimation of variety differences can be complicated by a heterogeneous varieties × environments interaction. Yates & Cochran wrote:

> There is, however, no reason to suppose that the variation of variety differences from place to place is the same for each pair of varieties.

In their example, one variety Trebi accounted for most of varieties × centres interaction. In large series of trials, however, it is rare for heterogeneity to be explained by a single contrast.

To identify Trebi, Yates & Cochran (1938) used the regression of each variety yields against total yields at each centre. The degrees of freedom in ANOVA table were accordingly partitioned for Trebi versus the rest. The mean squares from the partitioned ANOVA table provides a basis for appropriate standard errors of variety comparisons.

Yates & Cochran (1938) demonstrated that in making recommendations for future years but the same set of centres, standard errors include a varieties × years variance and units variance. Recommendations for the whole state of Minnesota but years similar to those in the trials depended on the extent to which the centres were representative. Standard errors of comparisons with Trebi would include a varieties × centres variance. Recommendations for future years could be made if the two years provided a wide range of weather conditions.

2.1.3 Weighted variety means

One other problem identified by Yates & Cochran (1938) is that individual experiments can be of a wide range of precision so that the estimate of units variance though unbiased is no longer efficient. This problem was also considered by Cochran (1937, 1954) and Cochran & Cox (1957) who recommended a weighted or semi-weighted analysis and suggested appropriate tests of significance. In the weighted analysis the weights are functions of error variances and replication whereas in the semi-weighted analysis the weights also depend on varieties × environments variances which need to be estimated.

Little has been reported in the literature on weighted analysis. Patterson & Silvey (1980) noted that in the UK the range of plot error variance is typically only about ten-fold and does not warrant use of a weighted analysis. This contrasts with a much larger range in Yates & Cochran (1938).

When within-years variety means have a wide range of precision, Patterson & Silvey (1980) recommended a weighted analysis in which weights are inversely proportional to the number of trials and varieties × environments variances. Inaccuracies in varieties × centres variance affect overall standard errors of variety comparisons far more than heterogenous units variance. Even when the variances of within-year variety means are within a ten-fold range a weighted analysis is worthwhile (Patterson & Silvey 1980 and Patterson & Nabugoomu 1992).

2.2 Least squares analysis

In variety trials, the varieties × centres table and varieties × years table are often incomplete (Section 1.1). Use of simple averages is only suitable for comparing varieties that are tested in all trials. Methods of analysis are required that account for differences in environments for those varieties that are tested in subsamples of environments. Least squares methods provide these adjustments (Silvey 1978b; Patterson 1978, 1982; Finney 1980; Patterson & Silvey 1980).

2.2.1 Percentage method

Prior to the use of least squares methods in analysis of UK variety trials, comparisons were based on complete subsets of data resulting in inconsistencies and inefficient use of information (Patterson 1978, 1982). The main method of analysis was the percentage method in which each candidate variety is considered in turn relative to the appropriate control grown in the same trial. Variety mean yield per trial is expressed as a percentage of the control and the relative percentage figures so obtained are averaged over all available trials, equal weight being given to each trial (Silvey 1978b). The standard errors of the means are derived from the percentage data.

The method depends upon finding stable control varieties on which to base the calculations and permits only comparisons between candidate varieties and the controls. The resulting variety comparisons based on different sets of trials can be misleading if the control yield is abnormally low. Silvey (1978b) gave an example of inconsistent comparisons. Silvey (1978b) and Finney (1980) pointed out that the method of percentages does not overcome the difficulty of incompleteness, and averaging of ratios even when the data is complete tends to exaggerate the variance of estimates and gives too much weight to varieties tested in trials where controls performed poorly.

The method works well when a single control variety can be identified which is also the market leader for the crop, and the varieties × trials is complete (Silvey 1978b). Rapid breakdown of disease resistance pathogens in some varieties makes stability over years untenable and the identification of a single suitable control difficult. NIAB trials consequently involve more than one control and the method of analysis shifted to least squares analysis.

2.2.2 Fitting constants (FITCON)

Least squares analysis (FITCON) was introduced in analysis of non-orthogonal designs and multi-classified data by Yates (1933, 1934) who also advocated its use in analysis of sample surveys (Yates 1960). Variety means are estimated from the model

$$y_{ij} = \tau_i + \beta_j + \varepsilon_{ij} \tag{2.1}$$

where τ_i's are variety means and β_j's are constants for environment effects. The units variance in model (2.1), σ^2, is a combination of varieties × environments variance and plot error variance.

In analysing a varieties × environments table, environments in the trials are treated as a random sample. Variety means are estimated from the model

$$y_{ij} = \tau_i + \epsilon_{ij} \tag{2.2}$$

where y_{ij} is the yield for variety i in environment j, τ_i the mean for variety i in the population of environments and ϵ_{ij} is an error term with variance $\sigma_\epsilon^2 = \sigma^2(1+\gamma)$. γ the ratio of environments variance to units variance also measures the between-environment covariance.

Models (2.1) and (2.2) are the same if β_j's are random variables with a zero mean and a homogeneous variance. Efficient estimates of variety means from model (2.2) are obtained from a generalised least squares analysis with weights proportional to components of variance. In practice, differences in the environments are large and provide little information on variety comparisons (Talbot 1984). Under these circumstances variety means are estimated by setting γ^{-1} to zero. This is equivalent to fitting model (2.1) with β_j's fixed (Patterson & Silvey 1980).

When FITCON was introduced in the analysis of variety trials there was criticism that the method was not appropriate when varieties × environments interaction exists. Finney (1980), Patterson & Silvey (1980) and Patterson (1982) have replied to this criticism: FITCON gives unbiased estimates of future performance of varieties regardless of how they vary from environment to environment. However, in the presence of a heterogeneous interaction, FITCON estimates are not in general efficient. FITCON adjustments in a varieties × environments table are valid if the sample of environmental conditions in the trials are random and the subsamples are effectively random.

2.2.3 Joint regression

When variety performance is predicted for a range of environments, varieties × environments interactions measure to some extent the reproducibility of experimental results. Yates & Cochran's (1938) method of using regression to identify the more variable varieties was ignored by statisticians until its rediscovery by Finlay & Wilkinson (1963) and Perkins & Jinks (1968). Finlay & Wilkinson (1963) observed that homogeneous components of variance though useful fail to give an adequate account of differential variety response to the environment. The method has been effectively used in selecting varieties well adapted to specific environments. Freeman (1973), Hills (1975), and Wescott (1987) provide comprehensive reviews.

The regressions can be incorporated in ANOVA table to assess significance of their contribution to explaining variation in the model. Silvey (1982) found that joint regression accounted for about 20% of varieties $\times$ environments variance for NIAB winter wheat trials grown during 1979 - 1981. Some authors refer to the regression coefficients as *stability coefficients*; we prefer *sensitivities* as the coefficients measure the differential response of a variety to environmental differences.

Finlay & Wilkinson (1963) considered sensitivities and variety means as two important indices for adapting varieties to specific environments. Knight (1970) observed that joint regression transforms the data to a scale of unit average sensitivity. For this reason, it is appropriate to scale sensitivities to unit mean.

Joint regresion has been criticised on two counts. Firstly, that both the regressor and the regressand are measured with error. Hills (1975) maintained that provided a large number of varieties are included in the experiment and the between-environments mean square is significantly greater than the error mean square, the bias is not serious.

Secondly, that the variety means contribute to, and are not therefore independent of environment means. Yates and Cochran (1938) justified the method as a partitioning of the varieties $\times$ environments sum of squares. Freeman (1973) noted that the validity holds only for those varieties and environments in the data. Despite these criticisms, the method is very popular, especially in variety adaptability studies.

In analysis of recommended list trials we use sensitivities as a diagnostic tool to identify varieties that are responsible for heterogeneity. Partitioning the degrees of freedom in the ANOVA table, as in Yates & Cochran (1938), forms the basis of appropriate standard errors for variety comparisons. The RL trials are, however, far from complete and we extend this method to incomplete tables.

2.2.4 Modified FITCON

FITCON adjustments are not appropriate if some varieties in the trials vary more than others to differences in environments. This is likely to be the case when varieties of different genetic or botanical grouping are tested together. Variety mean adjustment by FITCON can be improved by fitting the non-linear model

$$y_{ij} = \tau_i + \theta_i \beta_j + \varepsilon_{ij} \tag{2.3}$$

where θ_i's are variety sensitivities and the other terms are as in model (2.1).

Digby (1979) described an algorithm for fitting model (2.3) by successive least square approximations. The sensitivities are first set to 1.0 and FITCON used. ~~Each~~ Variety yields are then regressed against environment means to give sensitivities. The sensitivities after scaling to unit mean are used in (2.3) to estimate new variety means and environment means. The process is repeated iteratively until convergence. In Patterson (1978, 1982) and Patterson & Silvey (1980) the method is called *augmented FITCON.* We refer to this method as *modified FITCON.*

Modified FITCON provides a new set of adjusted means quite different from FITCON means for those varieties that are sensitive or insensitive to environmental differences when they are not tested in all environments. Patterson (1982) showed that one variety in the 1975 NIAB potato trials had its own pattern of variability and FITCON underestimated its variety mean. Modified FITCON detected this variety and also adjusted its FITCON mean accordingly.

Patterson & Silvey (1980) used modified FITCON to analyse the 1977 winter wheat data (Section 1.1, page 4). Two of the new varieties were more variable than the rest and their means were accordingly adjusted to an extent proportional to their sensitivities. Control varieties which were grown at all centres were not adjusted.

Patterson & Silvey (1980) pointed out that the standard errors from modified FITCON are not appropriate for a population of environments. Modified FITCON models varieties × environments interaction as linear in environmental variance. If environments are a random sample, it follows that this portion of variance contributes to standard errors of variety comparisons. We discuss this problem in Chapter 5.

2.2.5 Varieties × environments interaction

Varieties × environments variances have been estimated for complete data by Immer *et al.* (1934), Yates & Cochran (1938) and Sprague & Federer (1951). Estimates have been compiled for the UK trials by Patterson *et al* (1977), Patterson & Silvey (1980) and Talbot (1983, 1984). These figures indicate that varieties × environments interactions exist.

All crops in the UK showed large environmental variances. The centres × years component contributed substantially to total environmental variation. The varieties × centres component results from relative performance of some varieties changing from centre to centre in a manner that is consistent from year to year. The varieties × years component arises from differences between years in the performance of varieties that are apparent at all centres. The varieties × centres × years variance measures differences in between-centre variety performance which are not consistent from year to year.

Thus efficient estimation of variety performance should take into account varieties × environments interactions. Lack of appropriate software, however, led to use of simple methods using FITCON for combining data over centres and years.

2.2.6 Simple methods for analysing series of trials

Patterson (1978) observed that a single-stage analysis (Section 1.3, page 14) gives equal weight to each trial and ignores the classification of environments into centres and years. Consequently the method ignores consistent varieties × centres and varieties × years variation. Standard errors of variety comparisons are, therefore, underestimated, often severely. Patterson (1978) gave an example in which standard error were underestimated by 35% on average. A single-stage analysis often gives means similar to those from efficient methods and is still used routinely.

A two-stage analysis (Section 1.3, page 14) gives equal weight to each year and underestimates standard errors of variety comparisons because it ignores varieties × centres variance. In a simulation study for a system of spring wheat trials, Patterson (1978) found that a two-stage had definite advantage over a single-stage when varieties × years variance is not small and each variety appears in about average number of trials. Variety contrasts based on a smaller number of trials were poorly estimated by a two-stage analysis.

A two-stage analysis can be seriously affected by inaccuracies in entries of the second stage. When within-years means are based on a wide range of number of trials and variances of entries in the second stage are larger than expected from varieties × years variance, a weighted analysis should be done. Patterson & Silvey (1980) suggested that weights should be calculated using estimates of components of variance based on past trials data.

Efficient methods of estimating variety performance in general employ a generalised least squares analysis with weights that depend on variance components or their estimates. Now that suitable software is available there is less need for simple methods. The REML algorithm is suitable for combining trials over centres and years and can be modified to deal with some of the complications of heterogeneity.

Chapter 3

RESIDUAL MAXIMUM LIKELIHOOD (REML)

3.1 Introduction

The REML algorithm for analysing incomplete classified data was first described by Patterson & Thompson (1971). In this chapter we review the development of maximum likelihood (ML) methods for analysing classified data and summarise main theoretical results for REML. Details of that derivation are well documented in the literature, for example, by Patterson & Thompson (1971), Harville (1977), Hocking (1985, chapter 8), and Engel (1990).

Before maximum likelihood methods were developed, analysis of variance methods were in general use and the best known of these methods is Henderson's (1953) method III. Section 3.2 is devoted to this method. Section 3.3 is a review on ML methods for analysing mixed linear models based mainly on Hartley & Rao (1967). Section 3.4 deals with the development of REML.

3.2 ANOVA methods

When the data are complete, the ANOVA table provides a basis for estimating components of variance. The main feature of ANOVA methods is to estimate components of variance by equating mean squares to their expectation — essentially a method of moment estimators (Anderson & Bancroft 1952). It is known that, for complete data, ANOVA estimates of components of variance are unbiased; and if normality is assumed they are efficient in the sense of having minimum variance in the class of unbiased estimators, regardless of whether the model is mixed or random (Graybill & Hultquist 1961 and Albert 1976).

However, incomplete data present problems because there is no unique way of identifying mean squares that are unbiased for linear combinations of components of variance. Some of the methods developed are model specific and not much about their statistical properties is known. An example of this is the classical problem of recovery of inter-block information in balanced incomplete block designs.

Henderson (1953) developed general methods for estimating components of variance using ANOVA. He suggested three methods but the best and most popular is his method III. These estimators are unbiased in estimating components of variance. Henderson's methods are also described by Henderson (1990), Hocking (1985, chapter 10) and Searle (1968, 1971, chapter 10).

Henderson's method III is a methodological contradiction because it assumes a fixed effects model but calculates expectations on a mixed model. A generalised least squares estimate of fixed effects using estimated components of variance would in general be different from fixed effects estimated from the fixed model. Cunningham & Henderson (1968), with a correction by Thompson (1969), improved Henderson's method III to make it iterative and to include the estimation of fixed effects. However this improved Henderson's method still suffered from the shortcomings of the original method and was overtaken by maximum likelihood estimators, especially after the publication of Hartley & Rao's (1967) paper.

3.3 Maximum likelihood (ML)

Maximum likelihood analysis of mixed linear models was given a general framework by Hartley & Rao (1967). Before Hartley & Rao (1967), maximum likelihood methods had been used in estimating components of variance. But much of this was for complete data and in simple models where estimates can be obtained analytically (Crump 1951; Ehernberg 1950; Anderson & Bancroft 1952; Russell & Bradley 1958 and Graybill 1961). In other cases computational difficulties hindered further development.

In addition to unifying the theory of estimation, Hartley & Rao (1967) hoped that the development of appropriate numerical techniques and computer technology would lead to increased use of maximum likelihood. The existence of large sample properties and other known properties of ML estimates were a strong incentive.

The numerical aspect received attention from Hemmerle & Hartley (1973) who described the W-transformation technique that considerably reduces computations and provides an efficient way of obtaining ML estimates. Further work in this area was by Jenrich & Sampson (1976) who adapted the technique for a modified Newton-Raphson method, with further refinement by Thompson (1975), Hermmerle & Lorens (1976), and Goodnight & Hemmerle (1979). Hemmerle & Downs (1978) showed how the the ML algorithms can be used to fit mixed linear models with heterogeneous error.

The major set back in using ML estimates of components of variance is that they tend to be downward biased and do not reduce to ANOVA results when the data are complete. The bias, which sometimes is considerable, arises largely because the method does not take account of the loss in degrees of freedom from estimation of fixed effects. Moreover, as when estimating variance for one sample, ML estimators can be inconsistent (Eherberg 1950). These deficiencies in ML are overcome by the use of REML (Patterson & Thompson 1971).

3.4 REML

3.4.1 Historical introduction

The REML method automatically adjusts for the degrees of freedom lost in estimating fixed effects by estimating components of variance from the joint likelihood of all contrasts with zero expectation. The method, originally called *modified maximum likelihood*, was first described by Patterson & Thompson (1971) in the context of recovery of inter-block information.

The likelihood is partitioned into two parts: the joint likelihood of all error contrasts and a portion that depends on both fixed and random effects. Components of variance are estimated by maximising the joint likelihood of all error contrasts. The resulting estimates of components of variance are then used in maximising the other part of the likelihood to estimate fixed effects. This is equivalent to a generalised least squares analysis. The method is now known as Residual Maximum Likelihood (REML).

REML can be justified on two grounds: firstly, it gives the same results as Nelder's (1968) method for balanced incomplete block designs whereas ML does not, and secondly, in the absence of any knowledge about the fixed effects, the error contrasts contain all the information for estimating components of variance (Patterson & Thompson 1971 and Harville 1977).

Patterson & Thompson (1971) used a Fisher's scoring scheme to solve the likelihood equations. They showed how Henderson's mixed model equations reduce the computations required. Relationships between REML, ML and the modified Henderson's method were identified. They pointed out that REML and ML use the same quadratic forms but differ in the distribution from which expectations were calculated. The REML expectations account for the estimation of fixed effects whereas those for ML do not.

The residual likelihood had been used for estimation of components of variance before Patterson & Thompson (1971) but only in special cases. It can be inferred from Anderson & Bancroft's (1952) treatment of random models. The likelihood used by Anderson & Bancroft (1952), though a residual likelihood, is arrived at by a different principle. That is, the joint likelihood of sufficient statistics other than the mean, in particular the mean squares from the ANOVA table.

This principle was given a general treatment by Thompson (1962) for some, not all, complete random models. Thompson (1962) wrote:

> For estimating scale parameters in general, the restricted maximum likelihood method, used in this paper, consists of maximising the joint likelihood of that portion of a set of sufficient statistics which is location invariant.

Thus although the likelihood considered by Thompson (1962) in his models is the residual likelihood, his starting point is the ANOVA table. Thompson (1962), however, dealt with how to obtain non-negative estimates of components of variance from the joint likelihood of mean squares. Patterson & Thompson (1971) extend the work of Thompson (1962) to provide a general method that yields a portion of the likelihood that is independent of fixed effects. The principle of REML is no doubt imbedded in the work of Thompson (1962) and therefore Anderson & Bancroft (1952), but obscurely and only for cases where answers can be obtained from ANOVA.

The most clear statement and application of REML prior to Patterson & Thompson (1971) is in Russell & Bradley (1958). While dealing with the problem of estimating means in the presence of heterogeneous error variance and referring to the analysis of variance estimator in a two-way cross classification, they wrote:

> That estimator of σ^2 depends on $(n-1)(r-1)$ contrasts formed from the original observations. The contrasts have zero means on .... The estimator of σ^2 is not the maximum likelihood estimator from (3.1) .... But it is the maximum likelihood estimator with reference to the likelihood function of any set of $(n-1)(r-1)$ linear and linearly independent error contrasts.

Russell & Bradley (1958) then defined a specific error contrasts matrix which they used to obtain REML estimates of variance. Earlier work on REML is also in Patterson (1964) where REML equations are given for estimating components of variance for crop rotation experiments. But neither the principle nor detailed derivation is given. Patterson & Thompson (1971) give a general framework for the work of Russell & Bradley (1958) and put in place the beginning of REML as a method for analysing incomplete mixed linear models.

3.4.2 Developments since 1971

Detailed account of how REML could be generalised to complex linear models is given by Patterson & Thompson (1975). The main thrust of that paper was the relationship between REML and other methods used in estimating variance components including Rao's (1971) MINQUE (minimum norm quadratic unbiased estimation) method. Patterson & Thompson (1975) showed that MINQUE is equivalent to a single iteration of REML when normal errors are assumed. Differences between REML and ML in the general mixed linear model were given and the equivalence of REML with Nelder's (1968) method established for generally balanced designs. Patterson & Thompson (1975) showed the equivalence of REML with ANOVA in orthogonal designs.

Corbeil & Searle (1976) derived REML estimates for the general mixed linear model and adapted the W-transformation to Newton-Raphson methods for REML. They also gave numeric examples to illustrate the method and compared REML with ML and ANOVA. ML estimation was shown to be biased in all examples. Corbeil & Searle (1976) computed asymptotic standard errors for components of variance and observed that these standard errors are dependent on the magnitude of the estimate of component of variance and conclusions from them would be misleading.

Corbeil & Searle (1976) were the first to link the work of Patterson & Thompson (1971) to that of Thompson (1962). The acronym REML was first used here to stand for restricted maximum likelihood. This link was useful in pointing out that ML had deficiencies long known to statisticians and that attempts to modify the likelihood method to give acceptable results did not begin in 1971.

Unfortunately, however, the link misdirected the philosophy of REML to that of estimating non-negative components of variance. Many understood 'restriction' as referring not only to the residual likelihood but also to the constraining of components of variance to be non-negative. Consequently there was too much emphasis on the first part of REML i.e estimation of variance components, to the neglect of estimation of fixed effects.

Harville (1977) extensively reviewed ML methods in general, including REML. He detailed the theoretical and computational aspects of likelihood-based methods and the relationships between various methods. This review gave REML a firm basis in relation to other methods and was probably more influential than any other paper in tilting the balance in favour of REML versus its competitors.

Thompson (1977) showed how a linear model can be split into two so that REML estimates are obtained by absorbing the equations of one group into the other. In the process only small matrices are inverted. He suggested an improvement in the computations for REML using an algorithm akin to the scoring method. A stand-alone program for general use was written by Robinson, Thompson & Digby (1982) largely an implementation of computational modifications of Thompson (1977). The stand-alone program has recently been included within the Genstat statistical package.

3.4.3 Estimation theory

Model formulation

Consider the linear model

$$y \sim N(X\alpha, V) \tag{3.1}$$

where y is an $n \times 1$ response vector, α a p-vector of fixed effects and V a positive definite variance matrix of y. The design matrix X is assumed to be of rank p. For most of this thesis the variance covariance matrix is linear in a finite number of parameters $\theta = (\theta_0, \theta_1, \ldots, \theta_m)$, i.e

$$V = \theta_0 I + \sum_{j=1}^{m} \theta_j V_j \tag{3.2}$$

where the matrices V_j's are known and linearly independent. We refer to elements of θ as variance parameters.

An alternative formulation is to write y as the linear model

$$y = X\alpha + \sum_{j=1}^{m} B_j \beta_j + \varepsilon \tag{3.3}$$

where β_j is a $q_j \times 1$ vector with $\beta_j \sim N(0, \theta_j I_{q_j})$ and ε an n-vector with $\varepsilon \sim N(0, \theta_0 I)$. The B_j's are design matrices associated with random terms β_j's. The variance covariance matrix can now be written as $V = \theta_0 H$ where $H = I + B\Gamma B'$, B is an $n \times q$ matrix $B = (B_1|B_2|\cdots|B_m)$ with $q = \sum q_j$, and Γ is a block diagonal matrix with elements $\gamma_j I_{q_j}$ where $\gamma_j = \theta_j/\theta_0$. The variance parameters are then components of variance.

In the formulation (3.1) and (3.2) the variance parameters could be interpreted as covariances and hence can be negative provided V is positive definite (see Yates

(1967) and Hocking (1985, chapter 8) for examples). In the alternative formulation (3.3), the parameters modelled as variances must be positive. However this is not a requirement of the REML algorithm. Moreover, positive parameters can have negative estimates. For most of our applications we use the latter formulation and denote the variances with σ_j^2's.

Estimation of components of variance

The residual likelihood is the likelihood of the transformation Sy where S is any $(n-p) \times n$ matrix of rank $n-p$ satisfying the condition $SX = 0$. In particular we choose $S = I - X(X'X)^{-1}X$. The likelihood of Sy is then the likelihood of all error contrasts.

The log-likelihood of Sy is

$$L = -(n-p)\ln|SVS'| - y'S'(SVS')^{-}Sy \qquad (3.4)$$

where $(SVS)^{-}$ is a generalised inverse of SVS. Since S is idempotent and symmetric, a matrix A exists such that $S = AA'$ and $A'A = I$. The residual likelihood can then be written as

$$L = -[(n-p)\ln\sigma^2 - \ln|A'HA| + y'A(A'HA)^{-1}A'y/\sigma^2]/2 \qquad (3.5)$$

Now, let $P = A(A'HA)^{-1}A$. Then the score vector s is given by

$$\begin{aligned} s(\gamma_j) &= \partial L/\partial\gamma_j = -trace\ (B_j'PB_j)/2 + y'PB_jB_j'Py/2\sigma^2 \\ s(\sigma^2) &= \partial L/\partial\sigma^2 = (n-p)/2\sigma^2 + y'Py/2\sigma^2. \end{aligned}$$

REML estimates of parameters of the variance matrix V are obtained by equating the score vector to zero. These equations are

$$\left.\begin{array}{rcl} trace\ (B_j'PB_j) & = & y'PB_jB_j'Py \\ \sigma^2 & = & y'Py/(n-p) \end{array}\right\} \qquad (3.6)$$

Estimation of means

Given REML estimates of components of variance, fixed effects are estimated by generalised least squares, i.e

$$\hat{\alpha} = (X'\hat{V}^{-1}X)^{-1}X'\hat{V}^{-1}y$$

with variance matrix,

$$var(\hat{\alpha}) = (X'\hat{V}^{-1}X)^{-1}$$

More generally we are interested in predictions of the form $K\hat{\alpha} + G\hat{\beta}$ with estimated variance matrix $(K|G)C^{*}(K'|G')'$ where C^{*} is the inverse of the matrix of coefficients in the mixed model equations (Henderson 1975 and Thompson 1979).

3.4.4 Computational aspects

REML equations (3.6) are solved numerically. Computational schemes for REML include Newton-Raphson methods (Corbeil & Searle 1976; Jennrich & Sampson 1976), the EM algorithm of Dempster, Laird & Rubin (1977) and Thompson & Meyer (1986), and Fisher's scoring scheme (Patterson & Thompson 1971, 1975 and Thompson 1977). We describe Fisher's scoring scheme.

A Fisher's scoring scheme updates estimates at the $(k+1)$-th iteration by

$$\{\zeta\}^{k+1} = \{\zeta\}^{k} + \{F^{-1}\}^{k}\{s\}^{k}$$

where F is Fisher's information matrix with elements

$$\begin{aligned} f_{i,j} &= trace\ (B_i'PB_jB_j'PB_i) \\ f_{i,m+1} &= trace\ (B_i'PB_i)/\sigma^2 \\ f_{m+1,m+1} &= (n-p)/\sigma^4 \end{aligned}$$

and ζ is a vector of all variance paremeters. The units variance is taken to be the last element of ζ.

The mixed model equations for (3.6) are

$$\begin{pmatrix} X'X & X'B \\ & B'B+\Gamma^{-1} \end{pmatrix} \begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix} = \begin{pmatrix} X'y \\ Z'y \end{pmatrix} \tag{3.7}$$

Let C^* be the inverse of the matrix of coefficients in (3.7) then

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix} = C^* \begin{pmatrix} X'y \\ B'y \end{pmatrix}$$

It can be shown that $P = S - SBCB'S$, $B'PB = \Gamma^{-1} - \Gamma^{-1}C\Gamma^{-1}$ and $C = (B'SB + \Gamma^{-1})^{-1}$ is a submatrix of C^* corresponding to random effects. Let U_{ij} be a submatrix of $B'PB$ corresponding to the i-th and j-th elements of ζ other than the units variance. Then

$$\begin{aligned} 2f_{i,j} &= trace\ \{U_{ij}\}^2 \\ 2f_{i,m+1} &= trace\ U_{ii}/\sigma^2. \end{aligned}$$

Moreover,

$$y'PB_iB_i'Py = \hat{\beta}_i'\hat{\beta}_i/\gamma_i^2$$

and

$$y'Py = y'(y - X\hat{\alpha} - B\hat{\beta})/\sigma^2$$

Thus all elements required for Fisher's scheme are products of the mixed model equations (Patterson & Thompson 1971, Harville 1977 and Engel 1990).

3.4.5 Tests of hypotheses

A consequence of incomplete data is that construction of exact tests of hypotheses regarding parameters or their linear combinations is often intractable. In practice one has to resort to either approximate or asymptotic methods.

Fixed effects

For a general hypothesis regarding fixed effects, the Wald test is the most popular. Hocking (1985, chapter 8) proposed a scaling of the Wald test to an F-type statistic, an approach also advocated by Berk (1987). Tests based on the normal distribution are the most common for single contrasts. In many studies, it is already known that the factors have an effect, the objective of the analysis is to estimate these effects and test specific linear contrasts. Asymptotic normal distribution theory leads to the conclusion that under the assumptions of (3.3), $\hat{\alpha}$ is asymptotically distributed as normal with mean α and variance matrix $(X'\hat{V}^{-1}X)^{-1}$.

Kackar & Harville (1984) noted that $(X'\hat{V}^{-1}X)^{-1}$ is only a lower bound for the variance of $\hat{\alpha}$ and suggested an improvement by adding a quantity proportional to the mean squared error matrix of $\hat{\alpha}$. Studies by Nabugoomu (1988) for selected block designs show that the benefit is small and only affects small designs. Giesbrecht & Burns (1985) suggested a t-statistic for a single contrast with the error degrees of freedom computed by a method based on the Satterthwaite (1946) approximation. The method was recommended worthwhile by Giesbreicht & Burns (1985) and Engel (1990) especially for small designs.

Components of variance

Tests on variance components are far from attaining a consensus. Likelihood ratio tests using the residual likelihood have been suggested (Hocking 1985) and in some packages, for example Genstat, they are scaled to deviances (McCullaugh & Nelder 1983). Experience however shows that these tests can be misleading.

In the first place only one degree of freedom is assigned to a component of variance and this is inconsistent with the practice for complete data or when the effect is fixed. Sometimes the tests have suggested a non-significant component of variance, especially interactions, when the effect on standard errors is substantial if the interactions are included in the model. Tests based on asymptotic standard errors of components of variance ought not be made as these standard errors are irrelevant to the hypothesis of a zero component of variance. In the absence of proper tests of significance on components of variance, a test treating random effects as fixed is likely to be more sensitive than likelihood ratio tests based on the REML likelihood function. Further research is required in this area.

3.4.6 Applications of REML to variety testing

In variety testing, REML is used in analysis of individual experiments and also offers a flexible framework for combining series of trials (Robinson 1987a, 1987b and Patterson & Nabugoomu 1992). Other uses of REML in this area include estimation of components of variation for design and evaluation of systems of trials (Patterson *et al.* 1977 and Talbot 1983, 1984). The use of REML in analysing series of variety trials will be explored further in subsequent chapters with emphasis on estimation of variety means.

Chapter 4

ANALYSIS OF VARIETIES $\times$ CENTRES TABLE

4.1 Introduction

This chapter describes methods for analysing trials in individual years. In a within-year analysis environments are defined by centres. If the $V \times C$ table is complete unadjusted means adequately predict variety performance. Sometimes, however, the resources available do not allow all varieties to be tested in each trial or yields of some of the varieties may be discarded before the analysis, for example, if some of the plots were affected by pests. The $V \times C$ table is then incomplete and the means of varieties with incomplete results need to be adjusted.

REML and FITCON are used to provide these adjustments. Section 4.2 describes FITCON and REML analysis when variances are regarded as homogeneous. We refer to the analysis with homogeneous variances as *basic*. The basic analysis applies to any other definition of environments, for example years.

If the varieties $\times$ centres variance is heterogeneous, the basic analysis requires modification. We show in Section 4.3 how REML is used to allow for groups (of varieties) $\times$ centres variance.

Section 4.4 gives a summary.

4.2 Basic analysis of a $V \times C$ table

The analysis of a $V \times C$ table for each year estimates variety performance in environmental conditions of that year. This is of interest to both breeders and testing authorities. Estimates of variety means and their standard errors relate to a population of centres from which those in the trials are considered a random sample.

4.2.1 FITCON analysis

In this analysis the centre effects are regarded as fixed. The FITCON model is

$$V + C : \tag{4.1}$$

Let y be an n-vector of yields. Then $y \sim N(X\tau + B\beta, \sigma^2 I)$ where X and B are design matrices for varieties and centres respectively, σ^2 is the units variance, τ is a v-vector of variety means and β is a c-vector of centre effects (Section 2.2.2). The units variance in FITCON analysis is regarded as homogeneous and includes varieties × centres variance and plot error variance. It is not possible to separate varieties × centres variance and plot error variance unless there is an independent estimate of one of them.

The FITCON model equation can be written as

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \tag{4.2}$$

where μ is the general mean, α_i the effect of variety i, β_j the effect of centre j and ε_{ij} is an error term. Model (4.2) is overparameterized and a solution to the normal equations can be obtained by putting restrictions on the effects so that a regular inverse is used or a generalized[1] inverse may be used (Searle 1971 and Hocking 1985).

Whatever the method of estimating parameters in model (4.2) variety means are calculated as marginal means to a $V \times C$ table of expectations. Each cell in the table of expectations is $\hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j$ and the means are possible because the varieties × centres interaction is random. If varieties × centres are regarded as fixed, the model cannot be fit because the systematic part of the model uses all the degrees of freedom and we have none for the error.

By defining an appropriate matrix of coefficients K, variety means can be calculated as $K\hat{\alpha}^*$ where $\hat{\alpha}^*$ is a vector of all effects estimated from the model. There are many ways of defining dummy variables for the parameters in the model and this may result in a complex structure for the matrix K. Given the variance matrix of the estimates of effects, the variance matrix of variety means is obtained in a straightforward manner. Centre means are similary calculated.

4.2.2 REML analysis

In this analysis the centre effects are regarded as random, and normally distributed with a zero mean and a common variance σ_C^2 i.e, $y \sim N(X\tau, \sigma^2 I + \sigma_C^2 BB')$ where σ^2 is the units variance and τ a v-vector of variety means (Section 4.2.1).

The REML model can be written as

$$V : C \tag{4.3}$$

Note that model (4.3) implicitly regards the varieties × centres interaction as random and homogeneous. The units variance is a combination of the plot error variance and the varieties × centres variance.

[1]Any generalised inverse may be used provided it takes into account the model structure. A Moore-Penrose inverse, for example, can lead to wrong results.

For the REML[2] model, variety means are calculated as marginal means to the table of expectations using estimates of effects obtained from a generalised least squares analysis (Section 3.4.3). If the general mean is omitted in the model, variety means are obtained directly from the generalised least squares analysis.

The centre means are calculated by setting up an appropriate K matrix. Moreover, centre means can be calculated from the REML model even though centre effects are regarded as random. The variance matrix of centre means is obtained by using the variance matrix of the fixed effects and that of centre effects (Section 3.4.3).

The trial means are assumed to be equally accurate and the only correlations in the data are those specified in the FITCON or REML model. Adjustments by REML and FITCON are valid if the subsamples of centres are effectively random samples. Thus every pair of varieties are highly correlated. This is often the case because varieties in trials usually come from a similar genetic background.

As discussed in Section 2.2.2, if σ_C^2/σ^2 is large, FITCON and REML results are similar. The use of FITCON in UK trials is justified on grounds of large centre variances (Patterson *et al.* 1977; Patterson 1978, 1982 and Talbot 1984).

4.2.3 L-pattern $V \times C$ table

An L-table is a $V \times C$ table which is complete except that a subset of varieties is tested only at a subset of centres. After some re-arrangement of columns and rows, the missing patterns are positioned in the top right hand corner — thus the name L-table.

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[2]Genstat REML gives means and standard errors of difference as part of its output. Note that FITCON analysis can be done using Genstat REML.

FITCON adjustments in an L-table are easy to follow and examples can be found in Patterson (1978, 1982). The variety means for varieties tested at all centres are not adjusted. The means of varieties tested at a subsample of centres are adjusted by an amount

$$d = \bar{X}_A - \bar{X}'_A$$

where $\bar{X}_A$ and $\bar{X}'_A$ are the means for varieties tested at all centres over all centres and over a subsample of centres respectively. FITCON assumes that the subsample varieties would have responded the same way as varieties tested in all centres. Thus the mean for variety i (m_i) can be written as

$$m_i = \begin{cases} \bar{y}_i + d & \text{if variety } i \text{ is tested at a subsample of centres} \\ \bar{y}_i & \text{if variety } i \text{ is tested at all centres} \end{cases}$$

REML shrinks the FITCON adjustment d by a factor dependent on the ratio σ_c^2/σ^2. The smaller the ratio the smaller the adjustment. If σ_C^2 is zero no adjustment is made. In contrast, when centre differences are large REML uses the same adjustment as that of FITCON. Thus when effects are random one is protected to some extent from making adjustments in circumstances which are not justified.

4.2.4 Examples

Example 4.2.4.1: L-pattern $V \times C$ table (Sugar beet data, 1989)

We analyse the 1989 sugar beet data[3] in which varieties 1–19 were tested in 16 centres and new varieties 20–29 were tested in a subsample of 7 centres. The yields in t/ha are given in Appendix A.3.

FITCON and REML do not adjust means of varieties 1–19 because they were tested at all centres. The average yield of varieties 1–19 over all centres is 55.393

[3]Sugar beet trials are described in Section 1.1 and are analysed in Sections 6.3.2 and 7.4.

Table 4.2.1:

FITCON and REML estimation of variety means for sugar beet data, 1989

Variety	unadjusted mean	FITCON mean	REML mean
1	53.429	53.429	53.429
2	57.096	57.096	57.096
3	58.042	58.042	58.042
4	57.455	57.455	57.455
5	56.312	56.312	56.312
6	54.877	54.877	54.877
7	56.436	56.436	56.436
8	52.515	52.515	52.515
9	53.946	53.946	53.946
10	53.242	53.242	53.242
11	57.086	57.086	57.086
12	53.856	53.856	53.856
13	56.187	56.187	56.187
14	56.604	56.604	56.604
15	54.321	54.321	54.321
16	56.192	56.192	56.192
17	55.335	55.335	55.335
18	54.516	54.516	54.516
19	55.021	55.021	55.021
20	53.851	55.416	55.412
21	53.156	54.720	54.716
22	53.206	54.770	54.766
23	56.473	58.038	58.033
24	56.061	57.626	57.622
25	53.741	55.306	55.302
26	50.239	51.803	51.799
27	55.513	57.078	57.073
28	54.211	55.776	55.772
29	53.553	55.118	55.113

Table 4.2.2:

Correlation matrix for new varieties in sugar beet trials 1989

V21	0.928	1.000							
V22	0.978	0.961	1.000						
V23	0.984	0.925	0.975	1.000					
V24	0.995	0.942	0.974	0.988	1.000				
V25	0.987	0.948	0.978	0.984	0.995	1.000			
V26	0.974	0.927	0.978	0.987	0.980	0.983	1.000		
V27	0.968	0.921	0.967	0.993	0.972	0.963	0.983	1.000	
V28	0.989	0.955	0.973	0.978	0.997	0.995	0.978	0.962	1.000
V29	0.956	0.960	0.978	0.973	0.962	0.955	0.975	0.986	0.961
	V20	V21	V22	V23	V24	V25	V26	V27	V28

t/ha compared to 53.828 t/ha for the subsample of centres. FITCON adjusts means of new varieties by 1.565 t/ha, an amount by which varieties 1 – 19 yielded more on average over all centres than in the subsample. REML shrinks this adjustment to 1.560 t/ha. Adjustments by FITCON and REML are valid if the new varieties experienced a random sample of environmental conditions. FITCON and REML means are displayed in Table 4.2.1.

The centres and units components of variance are 118.953 and 6.481 repectively. REML and FITCON standard errors of variety comparisons are identical up to the fourth significant place. The standard error for a comparison between any two varieties with complete results is 0.90 t/ha. A difference between any two new varieties has a standard error of 1.361 t/ha. A comparisons between a new variety and any of the varieties 1 – 19 has a standard error of 1.165 t/ha.

Correlation between pairs of varieties from the seven centres with complete results have a range 0.884 to 0.997 with only two pairs of varieties having a correlation below 0.90. Under these conditions a basic analysis is valid provided the subsample of centres is effectively a random sample. The correlation matrix for new varieties is given in Table 4.2.2.

Table 4.2.3:

Yields (t/ha) of 6 varieties for wheat data, 1977

	centre									
variety	1	2	3	4	5	6	7	8	9	10
Huntsman	5.79	6.12	5.12	4.50	5.49	5.86	6.55	7.33	6.37	4.21
Atou	5.96	6.64	4.65	5.07	5.59	6.53	6.91	7.31	6.99	4.62
Armada	5.97	6.92	5.04	4.99	5.59	6.57	7.60	7.75	7.19	*
Mardler	6.56	7.55	5.13	4.60	5.83	6.14	7.91	8.93	8.33	*
Sentry	*	*	*	*	*	*	7.34	8.68	7.91	3.99
Stuart	*	*	*	*	*	*	7.17	8.72	8.04	4.70

Example 4.2.4.2: An incomplete V × C table (wheat data, 1977)

FITCON and REML are used to analyse wheat data[4] displayed in Table 4.2.3. The data were analysed by Patterson & Silvey (1980) using FITCON. Estimates of variety means and their adjustments by REML and FITCON are displayed in Table 4.2.4. The centre component of variance is 1.421 and the units variance is 0.157. There are small differences between FITCON and REML variety means because of large centres variation. The plot variance for a mean of 3 plots for these trials is 0.036. Thus the variety × centre variance is $0.121 = 0.157 - 0.036$.

Control varieties had complete results and so FITCON and REML do not adjust their means. Armada and Mardler have their means adjusted downwards by 0.202 t/ha and 0.197 t/ha by FITCON and REML respectively. The only centre where these varieties were not grown was low yielding. FITCON and REML assume that Armada and Mardler would have given low yields if they had been grown at this centre. The centres at which Sentry and Stuart were tested yielded better on average than other centres. The variety means of Sentry and Stuart

[4]Wheat trials are described in Section 1.1 and are analysed in Sections 6.4, 6.5 and 7.2

Table 4.2.4:

Variety means (t/ha) and adjustment by FITCON and REML
(wheat data, 1977)

variety	unadjusted mean (1)	FITCON mean (2)	FITCON adjustment (3) = (2) − (1)	REML mean (4)	REML adjustment (5) = (4) − (1)
Huntsman	5.734	5.734	0.0	5.734	0.0
Atou	6.027	6.027	0.0	6.027	0.0
Armada	6.402	6.201	-0.201	6.206	-0.196
Mardler	6.776	6.574	-0.202	6.579	-0.197
Sentry	6.980	6.417	-0.563	6.429	-0.551
Stuart	7.158	6.595	-0.563	6.606	-0.552

Table 4.2.5:

Standard error of variety mean differences (t/ha) by REML
(wheat data, 1977)

Atou	0.177	*			
Armada	0.184	0.184	*		
Mardler	0.184	0.184	0.187	*	
Sentry	0.248	0.248	0.257	0.257	*
Stuart	0.248	0.248	0.257	0.258	0.280
	Huntsman	Atou	Armada	Mardler	Sentry

are therefore adjusted downwards by 0.563 t/ha and 0.552 t/ha by FITCON and REML respectively. The adjustments to Sentry and Stuart are valid if centres 1–6 and 7–10 are random samples from the same population. The REML adjustments are smaller than those of FITCON because random centre effects cause a shrinkage in the adjustments toward the origin. FITCON, as well as REML, gives equal adjustment to varieties that experience the same environmental conditions.

The standard error of variety mean differences from REML and FITCON analysis are very similar. REML standard errors are displayed in Table 4.2.5.

4.3 Heterogeneity in a $V \times C$ table

4.3.1 Identification of homogeneous groups of varieties

For the basic analysis to be valid every variety must experience a random sample of conditions sampled by the trials. Thus all subsamples of centres in an incomplete $V \times C$ table should be random samples of the centres in the trials. The requirement that subsamples of centres be random samples is only implied in the REML model and ignored in the FITCON model. If conditions at some of the centres favour some varieties and these varieties are tested only at these centres, varieties × centres interaction may no longer be homogeneous (Patterson 1982).

In this section we deal with heterogeneity that is characterised by differential variation to centre differences by groups of varieties. The basic analysis assumes that any two variety means have the same variation to centre differences. Sometimes, however, groups of varieties can be identified that vary similary within-groups but differently between groups.

Groups of varieties may be identified on the basis of a field characteristic, for example, stage of maturity, size of leaf, status i.e control or not control or on a botanical characteristic such as ploidy. Sensitivities can also be used to identify varieties causing heterogeneity (Section 2.2.3). But like any other method, the actual forming of groups may involve~~s~~ a degree of subjectivity.

If the data are complete, entries in the ANOVA table can be partitioned in a manner similar to that of Yates & Cochran (1938) from which standard errors which account for heterogeneity are calculated. Variety means require no adjustments. For incomplete data we use REML to adjust the means and provide standard errors of variety comparisons that account for heterogeneity.

4.3.2 REML analysis with groups (of varieties) × centres variance

Once groups of varieties have been identified, we fit the model[5]

$$V : C + GV.C \tag{4.4}$$

where GV is a factor for groups of varieties. If groups × centres variance is large, model (4.4) is used for the analysis. This model assumes equal within-groups variance. Centre effects may be regarded as fixed and this leads to the model

$$V + C : GV.C \tag{4.5}$$

Hemmerle & Downs (1978) showed how standard algorithms for analysing mixed linear models could be used to fit models with heterogeneous error by fitting supplementary components of variance. Although they used maximum likelihood the method is also suitable for REML (Robinson 1987a, Example 4) and can be used to analyse data with groups (of varieties) × centres interaction.

The method enables estimation of an extra component of variance for the difference in variation between groups of varieties. In the case of two groups, we define a new factor D similar to a centres factor except that the factor levels are specified only for one group of varieties. We then fit[6] the model

$$V : C + D \tag{4.6}$$

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[5]The data structure suggests the model $GV/V : C + GV.C$. This model, however, leads to unnecessary complications especially when Genstat REML is used. See Example 4.3.3.2.

[6]The REML algorithm used should have facilities to allow missing levels to be treated as zeros. In Genstat this is achieved by the MVINCLUDE option of the REML directive.

We may regard centre effects as fixed and use the model

$$V + C : D \tag{4.7}$$

We give two examples of heterogeneity associated with groups of varieties. In the first example we partition the degrees of freedom in the ANOVA table and show how the same analysis can be done using REML. The second example is an incomplete $V \times C$ table.

4.3.3 Examples

Example 4.3.3.1: A complete $V \times C$ table (Potato data)

The potato data consists of yields of 7 varieties of potato tested by NIAB in 1975. The trials were grown at 16 centres in England. Two subsets of the data were analysed by Patterson (1982) using FITCON. Simple sensitivities show that Cara was less sensitive than average (Tables 4.3.1 and 4.3.2). These sensitivities are regresion coeffients of variety yields against centre means (Section 2.2.3).

Analysis of variance

Since the $V \times C$ table is complete, we follow Yates & Cochran (1938) and partition the degrees of freedom for Cara versus the rest. We define a contrast for the comparison Cara versus the rest and use the Genstat ANOVA algorithm. The ANOVA table provides all the information for calculation of standard errors (Table 4.3.3).

The standard error of variety mean difference not involving Cara is $\sqrt{(}2 \times 20.46/16) = 1.599$ t/ha. The standard error of a difference involving Cara includes a varieties × centres variance estimated as $7 \times (110.20 - 20.46)/6 = 104.5$. Thus the standard error of a difference involving Cara is $\sqrt{(}2 \times 20.46 + 104.5)/16 = 3.015$ t/ha. Without allowing for heterogeneity, any variety mean difference is estimated

Table 4.3.1:

Yield (t/ha) of seven varieties of potato data

				variety			
centre	Cara	Desiree	Drayton	K.Edward	Estima	Majestic	P.Crown
1	36.6	39.2	38.2	37.4	45.5	39.5	44.0
2	51.5	43.6	43.0	46.8	46.4	49.0	48.4
3	40.1	20.1	31.0	29.1	28.1	38.4	28.9
4	39.3	40.5	38.5	43.1	45.2	46.1	43.9
5	37.2	38.2	30.5	25.6	28.3	31.1	47.1
6	42.7	46.1	41.4	46.8	45.5	49.3	48.2
7	49.4	52.3	55.7	51.4	55.0	53.8	59.1
8	42.1	42.4	34.4	41.1	32.1	40.1	48.0
9	35.3	34.2	31.4	32.7	27.9	32.5	37.3
10	53.3	79.1	69.4	82.4	73.5	71.7	87.8
11	44.2	54.3	32.2	30.1	50.3	37.6	50.0
12	61.0	55.8	53.5	54.0	62.0	60.8	62.3
13	48.4	40.9	36.0	32.4	39.0	34.4	52.9
14	58.4	27.5	29.7	28.3	24.9	28.0	37.9
15	58.2	39.7	44.4	42.3	44.5	38.1	44.6
16	31.3	33.6	35.5	37.5	30.6	36.5	38.8

Table 4.3.2:

Variety means (t/ha) and sensitivities for potato data

		simple sensitivity	
variety	mean	estimate	s.e
Cara	45.56	0.450	0.182
Desiree	42.97	1.123	0.118
Drayton	40.30	0.954	0.071
KingEdward	41.31	1.183	0.110
Estima	42.43	1.173	0.101
Majestic	42.93	0.986	0.097
PentlandCrown	48.70	1.131	0.106

Table 4.3.3:

ANOVA table for potato data

source	df	MS
V	6	128.18
Cara vs rest	1	82.74
Deviation	5	137.27
C	15	859.49
$V \times C$	90	35.41
(Cara vs rest)×C	15	110.20
Deviations	75	20.46

with a standard error of $\sqrt{(2 \times 35.41/16)} = 2.104$ t/ha. The standard error of comparisons not involving Cara is overestimated by 32% and the standard error of comparisons involving Cara is underestimated by 30%. Estimates of contrasts are not affected by heterogeneity.

REML analysis

We define a factor GV with two levels one for Cara and the other for the rest, and use models (4.4) and (4.5). Components of variance and standard errors of difference are given in Table 4.3.4.

Comparisons involving Cara are estimated with a standard error of 3.096 t/ha and those not involving Cara have a standard error of 1.590 t/ha if model (4.4) is used. Model (4.5) gives the same results as ANOVA. Unlike ANOVA in which extra calculations are required for standard errors, Genstat REML gives the appropriate standard errors automatically.

An alternative analysis is given by models (4.6) and (4.7). We define a dummy factor D for centres but specify levels for only Cara. Variety means are not adjusted. Components of variance and standard error of differences are given in Table 4.3.5.

Table 4.3.4:

Components of variance and standard errors of difference from models $V : C$, $V : C + GV.C$ and $V + C : GV.C$ (Potato data)

			component		standard error	
	model	centres	groups × centres	units	minimum	maximum
(4·3)	$V : C$	117.72		35.41	2.104	2.104
(4·4)	$V : C + GV.C$	53.82	56.42	20.22	1.590	3.096
(4·5)	$V + C : GV.C$		52.35	20.46	1.599	3.017

Table 4.3.5:

Components of variance and standard errors of difference from models $V : C$, $V : C + D$, $V + C : D$, $V : C + E$ and $V + C : E$ (Potato data)

			component		standard error	
	model	centres	dummy factor	units	minimum	maximum
(4·6)	$V : C + D$	57.98	99.10	20.64	1.605	2.967
(4·7)	$V + C : D$		104.70	20.46	1.599	3.017
(4·8)	$V : C + E$	139.31	100.01	20.61	1.606	2.962
(4·9)	$V + C : E$		104.70	20.46	1.599	3.017

The dummy component of variance is an estimate of (Cara vs the rest) × centres variance. The REML estimate from model (4.6) compares well with the ANOVA estimate but they are not identical unless centre effects are regarded as fixed. Note that the component of variance from model (4.7) is twice the groups × centres variance from model (4.5). The standard error for Cara versus any other variety is 2.9~~70~~67 t/ha and that of any difference not involving Cara is 1.605 t/ha if model (4.6) is used. The range of standard errors is smaller than that given by model (4.4). Standard errors from model (4.7) are ~~the same as~~ similar to those from ANOVA.

Instead of defining levels of the dumy factor for Cara we could define levels for the other varieties. We denote this factor by E. Model

$$V + C : E \tag{4.8}$$

gives the same results as model (4.7) but there is a small difference in standard errors of difference from model

$$V : C + E \tag{4.9}$$

compared to those from model (4.6) (Table 4.3.5).

Example 4.3.3.2: An incomplete $V \times C$ table (wheat data, 1977)

Simple sensitivities, for wheat data, are shown in Table 4.3.6. These sensitivities are regressions of variety yields on REML centre means. It is clear that some varieties such as Mardler and Sentry varied more than others to centre differences. Thus the varieties × centres variance is heterogeneous. Ignoring this heterogenity results in overestimation of some variety comparisons and underestimation of others.

If varieties × centres variance is homogeneous all variety sensitivities are 1.0. Thus tests of hypothesis on sensitivities should assess departures from 1.0 and not from zero. For example, the test for Sentry is based on a t-statistic (.225/.054) = 4.17 on 26 degree of freedom. Though approximate, there is strong evidence that Mardler and Sentry had more than average sensitivity, and the control varieties were less sensitive to centre differences.

From the sensitivities we can group controls versus the rest or Mardler and Sentry, the more sensitive varieties, versus the rest.

Controls versus the rest

Components of variance from model (4.4) are displayed in Table 4.3.7. The groups × centres component of variance is positive but smaller than the units variance. In addition, the change in units variance from 0.157 to 0.113 suggests that this grouping does not account for significantly more variation than the basic analysis.

Table 4.3.6:

Variety means (t/ha) and simple sensitivities by basic REML (wheat data, 1977)

		simple sensitivity	
variety	mean	estimate	s.e
Huntsman	5.734	0.744	0.053
Atou	6.027	0.780	0.064
Armada	6.~~402~~ 206	0.915	0.078
Mardler	6.579	1.308	0.081
Sentry	6.429	1.225	0.054
Stuart	6.606	1.027	0.130

Table 4.3.7:

Components of variance from model (4.4) for controls versus the rest (wheat data, 1977)

component	estimate
centres	1.349
groups × centres	0.071
units	0.113

If the varieties were coded 1 and 2 for the controls and 1, 2, 3, 4 for the rest, and model

$$GV/V : C + GV.C \tag{4.10}$$

is used for the analysis, Genstat REML gives means only for the control varieties. Even then, the means are adjusted and yet control varieties had complete results. The means are obtained as margins to a $GV \times V$ table of expectations, but because the table is incomplete means for varieties which were not controls are not estimable (Table 4.3.8). For example, the mean of Huntsman is given as the average of 5.734 and 6.189 i.e 5.961 t/ha. This is because Genstat REML treats $GV.V$ not as within group variety effects but as an interaction. In fact, if

Table 4.3.8:

$G \times V$ table of means from model (4.10) with varieties coded 1 and 2 for the controls and 1,2,3 and 4 for the rest (wheat data, 1972)

group	variety			
controls	5.734	6.207	*	*
other varieties	6.189	6.562	6.335	6.533

Table 4.3.9:

Components of variance from models $V : C$ and $V : C + GV.C$ (wheat data, 1977)

model	centres	groups × centres	units
$V : C$	1.421		0.157
$V : C + GV.C$	1.581	0.138	0.082

we code varieties as 1, 2, ..., 6 and use model (4.10) no variety means are given by Genstat REML.

For both forms of coding, however, the $GV \times V$ table of means from Genstat REML is correct except that the format is a bit awkward especially when standard errors of variety comparisons are required. For these reasons we use model (4.4) with the second form of coding.

Mardler and Sentry versus the rest

Components of variance from model (4.4) are displayed in Table 4.3.9. The groups × centres variance is large, almost twice the units variance.

Except for the control varieties, the means from (4.4) are different from those given by a basic analysis (Table 4.3.10). The control varieties are not adjusted because they were tested at all centres. Although Armada and Mardler were tested in the same environments, they are adjusted differently because they are in different groups. Armada and Mardler are adjusted downwards by a further

Table 4.3.10:

Variety means (t/ha) and their adjustments from basic REML and REML model (4.4) (wheat data, 1977)

variety (group)	unadjusted mean (1)	basic REML mean (2)	basic REML adjustment (3) = (2) − (1)	REML with *GV.C* mean (4)	REML with *GV.C* adjustment (5) = (4) − (1)
Huntsman (1)	5.734	5.734	0.0	5.734	0.0
Atou (1)	6.027	6.027	0.0	6.027	0.0
Armada (1)	6.402	6.206	-0.196	6.218	-0.184
Mardler (2)	6.776	6.579	-0.197	6.547	-0.229
Sentry (2)	6.980	6.429	-0.551	6.213	-0.767
Stuart (1)	7.158	6.606	-0.552	6.678	-0.480

Table 4.3.11:

Standard error of variety mean differences (t/ha) from the model $V : C + GV.C$ (wheat data, 1977)

Atou	0.128	*			
Armada	0.133	0.133	*		
Mardler	0.214	0.214	0.216	*	
Sentry	0.261	0.261	0.265	0.216	*
Stuart	0.181	0.181	0.187	0.250	0.287
	Huntsman	Atou	Armada	Mardler	Sentry

0.012 and 0.032 t/ha respectively compared to their means from the basic REML analysis. Similary, Sentry and Stuart are adjusted differently because they are in different groups. Compared to the basic REML analysis, Sentry is adjusted downwards by a further 0.22 t/ha but Stuart is adjusted upwards by 0.07 t/ha. Sentry is the more affected by heterogeneity.

Standard errors of mean differences from model (4.4) are displayed in Table 4.3.11. Standard errors for within-group comparisons are smaller than in the basic analysis. For example, the standard error for Mardler versus Sentry is reduced

Table 4.3.12:

Levels for dummy factor D defined for Mardler and Sentry

variety	centre									
Huntsman	*	*	*	*	*	*	*	*	*	*
Atou	*	*	*	*	*	*	*	*	*	*
Armada	*	*	*	*	*	*	*	*	*	*
Mardler	1	2	3	4	5	6	7	8	9	10
Sentry	1	2	3	4	5	6	7	8	9	10
Stuart	*	*	*	*	*	*	*	*	*	*

Table 4.3.13:

Components of variance and standard errors of difference from models $V : C$, $V : C + D$, $V + C : D$, $V : C + E$ and $V + C : E$ (wheat data, 1977)

			component		standard error	
	model	centres	dummy factor	units	minimum	maximum
(4.6)	$V : C + D$	1.161	0.318	0.079	0.126	0.292
(4.7)	$V + C : D$		0.294	0.080	0.127	0.289
(4.8)	$V : C + E$	2.163	0.238	0.085	0.131	0.282
(4.9)	$V + C : E$		0.294	0.080	0.127	0.289

from 0.257 t/ha to 0.216 t/ha. The reduction in standard errors is more evident in the less variable group. Standard errors of between-group comparison are increased from 3% to 16%.

We can also use models (4.6) and (4.7) to account for the difference in variation between the two groups. We define the dummy factor D for the more variable varieties Mardler and Sentry (Table 4.3.12). Components of variance and average standard error of differences are displayed in Table 4.3.13.

Variety means from model (4.6) and model (4.7) are displayed in Table 4.3.14. The means of control varieties are not adjusted and the means of other varieties are similar to those given by model (4.4).

Table 4.3.14:

Variety means (t/ha) from basic REML, and REML models (4.4) ~~and~~ (4.6) and (4.7) (wheat data, 1977)

Variety	model (43)	model (4.4)	model (4.6)	model (4.7)
Huntsman	5.734	5.734	5.734	5.734
Atou	6.027	6.027	6.027	6.027
Armada	6.206	6.218	6.220	6.216
Mardler	6.579	6.547	6.543	6.543
Sentry	6.429	6.213	6.195	6.200
Stuart	6.606	6.678	6.685	6.675

Table 4.3.15:

Standard error of variety mean differences (t/ha) from REML model $V : C + D$ (wheat data, 1977)

Atou	0.126	*			
Armada	0.130	0.130	*		
Mardler	0.222	0.222	0.224	*	
Sentry	0.268	0.268	0.271	0.215	*
Stuart	0.177	0.177	0.184	0.256	0.292
	Huntsman	Atou	Armada	Mardler	Sentry

Standard errors for variety comparisons from model (4.6) (Table 4.3.15) are also similar to those from model (4.4). ~~(Table 4.3.15)~~. The standard error of difference between Mardler and Huntsman is increased to 0.222 t/ha from 0.184 t/ha. Standard errors for within-group comparisons are decreased. The standard error for the differences Atou versus Huntsman and Mardler versus Sentry are decreased to 0.126 t/ha and 0.215 t/ha from 0.177 t/ha and 0.257 t/ha respectively.

Instead of defining levels of the dummy factor for Mardler and Sentry, levels could be defined for the other varieties resulting in factor E say (Table 4.3.16). REML models (4.8) and (4.9) give similar variety means and standard errors as

Table 4.3.16:

Levels for dummy factor E defined for other varieties except Mardler and Sentry

variety	centre									
Huntsman	1	2	3	4	5	6	7	8	9	10
Atou	1	2	3	4	5	6	7	8	9	10
Armada	1	2	3	4	5	6	7	8	9	10
Mardler	*	*	*	*	*	*	*	*	*	*
Sentry	*	*	*	*	*	*	*	*	*	*
Stuart	1	2	3	4	5	6	7	8	9	10

Table 4.3.17:

Variety means (t/ha) by basic REML, and REML model (4.4) and models (4.8) and (4.9)

(wheat data, 1977)

variety	model (4.3)	model (4.4)	model (4.8)	model (4.9)
Huntsman	5.734	5.734	5.734	5.734
Atou	6.027	6.027	6.027	6.027
Armada	6.206	6.218	6.216	6.216
Mardler	6.579	6.547	6.550	6.543
Sentry	6.429	6.213	6.232	6.200
Stuart	6.606	6.678	6.671	6.675

model (4.6) (Tables 4.3.13 and 4.3.17). Model (4.9) gives the same variety means and standard errors as model (4.7).

The slight differences between results from models (4.6) and (4.8) raises the question of which means and standard errors to use. Firstly, we would fit centres as random. Secondly, because Mardler and Sentry are the more variable varieties, it is more justifiable to declare dummy levels for these varieties.

If there are more than two groups of varieties, then more than one dummy factor need to be defined. For m groups, $(m-1)$ dummy factors are required. In these circumstances it is possible to obtain negative components of variance. The components of variance for the dummy factors estimate extra variance relative to the dummy factor not in the model. Thus negative components of variance would be valid.

A disadvantage of using the method of Hemmerle & Downs (1978) is that it slows the rate of convergence of the REML algorithm.

4.4 Summary

REML and FITCON give similar results when units variance is homogeneous. FITCON and REML do not adjust means of varieties that are tested at all centres but will adjust means upwards for varieties from a subsample of centres that are on average low-yielding, and downwards for varieties from a subsample of centres that are on average high-yielding. In this way FITCON and REML adjust variety means in an incomplete V × C table to a common standard. In general REML adjustments are smaller than those by FITCON.

When varieties × centres interaction is heterogeneous, we have shown how REML analysis can be modified if heterogeneity can be associated with groups of varieties. Groups of varieties can be identified by a field characteristic of the varieties or using sensitivities. REML is used to check whether the groups × centres variance is large. If so, a REML analysis takes account of this interaction in the adjustments to variety means when the data are incomplete.

In the next chapter we describe a method of analysis that provides separate adjustments for each variety. The method is based on a multiplicative interaction model in which sensitivities are used to adjust variety means.

Chapter 5

MULTIPLICATIVE INTERACTION IN $V \times C$ TABLE

5.1 Introduction

A basic FITCON analysis equally adjusts means of varieties with incomplete results that are grown in the same trials. In so doing FITCON makes no allowance for differential variety sensitivity to centre differences. If a variety has more than average sensitivity, then its adjustment should be larger than that given by FITCON and if a variety has less than average sensitivity then its adjustment should be smaller than that of FITCON. FITCON adjustment is, however, adequate if a variety has unit sensitivity. In the extreme case if a variety is insensitive to centre differences then its mean should not be adjusted.

FITCON, for example, makes the adjustment $\sum_j \hat{\beta}_j / n_i$ to the mean of a variety tested at n_i centres where the summation is over the centres at which the variety was not tested and provided the centre effects are constrained to a zero sum (Section 4.2.1 and Digby 1979). An analysis which allows for differences in variety sensitivity adjusts the mean of variety i by $\theta_i \sum_j \hat{\beta}_j / n_i$ where θ_i is the sensitivity relative to other varieties in the trials.

For example, using simple sensitivities for the 1977 wheat data (Table 4.3.5) adjustments to basic FITCON analysis are displayed in Table 5.1.1. Huntsman and Atou have less than average sensitivity but because they had complete results

Table 5.1.1:

Adjustments by sensitivities for wheat data, 1977

variety	unadjusted mean (1)	mean (2)	FITCON adjustment (3) = (2) − (1)	sensitivity (4)	readjusted mean (5) = (1) + (4) × (3)
Huntsman	5.734	5.734	0.0	0.744	5.734
Atou	6.027	6.027	0.0	0.780	6.027
Armada	6.402	6.201	-0.201	0.915	6.218
Mardler	6.776	6.574	-0.202	1.308	6.511
Sentry	6.980	6.417	-0.563	1.225	6.290
Stuart	7.158	6.595	-0.563	1.027	6.580

their means are not adjusted. But the other varieties are adjusted in proportion to their sensitivities.

5.1.1 Multiplicative models

More formally, the use of sensitivities to adjust variety means leads to the non-linear model

$$y_{ij} = \mu + \alpha_i + \theta_i \beta_j + \varepsilon_{ij} \tag{5.1}$$

(Sections 2.2.4 and 4.2.1).

Model (5.1) belongs to a family of models used in analysing two-way cross classified data in which the interaction effects are multiplicative. These models have been explored by a number of authors, for example, by Mandel 1971; Freeman 1973, 1975; Gabriel 1978 and Kempton 1984. A general multiplicative model~~s~~ has several multiplicative interaction terms in addition to additive main effects. Gauch (1988) has advocated their use in analysing yield trials. These models have been used almost exclusively for describing the interaction in complete tables.

If the data are complete the multiplicative part of the model is fit ted using principal components. Gauch (1988) found one or two principal axes enough to explain the interaction. Complication in analysis arise if the data are incomplete. Freeman (1975) extended this analysis to incomplete tables by replacing missing values with their estimates from the additive model.

The disadvantage in estimating missing values is that the analysis is based on different assumptions from those used in estimating missing values. Digby (1979) pointed out that the means are not adjusted in a manner exemplified in Table 5.1.1 and estimates of sensitivities tend to be biased towards 1.0. A method of analysis that handles incompleteness without estimating missing data is therefore preferrable.

If only one axis is fitted, a multiplicative model can be written as

$$E(y_{ij}) = \alpha_i + \beta_j + \delta_i \varsigma_j \tag{5.2}$$

where μ is the general mean, α_i's are variety means, β_j's are centre effects and $\delta_i \varsigma_j$ are multiplicative terms for the interaction. Model (5.1) is a special case of model (5.2). In model (5.1) the β_j's estimate effects of the environments and θ_j's measure variety sensitivities to those environments.

The objective of our analysis is to predict variety performance that take into account differential sensitivities to centre differences. The most useful varieties are high yielding with small variation (Patterson & Silvey 1980). Although model (5.2) has been used by some authors to describe the varieties $\times$ environments interaction (Hills 1975), we prefer model (5.1) because we can easily interpret parameters of the model.

If the $V \times C$ table is incomplete, including sensitivities in the model leads to a new set of estimates for variety means and centre means. Digby (1979) used this fact to provide a method of fitting model (5.1), i.e modified FITCON (Section 2.2.4). Modified FITCON treats environments as fixed and uses information from one principal component. Patterson & Silvey (1980) found the method useful in analysing UK trials. We discuss modified FITCON in Section 5.2.

Oman (1991) used maximum likelihood to fit models similar to (5.2) with the β_j's random but in view of shortcomings of maximum likelihood methods we use REML (Section 3.2). Section 5.3 describes a REML analysis that fits model (5.1) with random centre effects by iterating between two REML models: one conditional on sensitivities and the other conditional on centre means. A full REML analysis using a Fisher's scoring scheme is discussed in Section 5.4. Patterson & Silvey (1980) referred to model (5.1) with random centre effects as model B.

5.2 Modified FITCON analysis

Define an $n \times c$ matrix W with a typical element $w(h, j) = \theta_i$ if $y(h)$ is the yield of variety i at centre j and zero otherwise, where θ_i is the sensitivity for variety i. Modified FITCON iteratively estimates τ, β and θ from the model (5.1) with centre effects regarded as fixed, i.e $y \sim N(X\tau + W\beta, \sigma^2 I)$ (Section 4.2.1). The estimates of parameters are obtained by minimising residual sum of squares.

Digby (1979) noted, firstly, that conditional on $\hat{\theta}$, variety means and centre means are estimated using ordinary least squares. We write this model as

$$V + C.T : \tag{5.3}$$

where T is a variate such that $T(h) = \theta_i$ if $y(h)$ is the yield for variety i. Secondly, conditional on centre means, variety means are estimated from a regression of yields of each variety on the centre means i.e a joint regression analysis. Iterating between the two regressions, leads to estimates of variety means, centre means and sensitivities that minimise residual sum of squares. A variety mean will differ from its estimate by basic FITCON unless it is grown at all centres or has unit sensitivity.

5.2.1 Algorithm for modified FITCON

Algorithm 5.2.1

(a) Estimate variety means and centre means from the model $V + C$:

(b) For each variety, regress yield on the centre means to give the slope as the sensitivity for that variety.

(c) Scale sensitivities to unit mean.

(d) Use model (5.3) to estimate new variety means and centre means.

(e) Repeat steps (b) to (d) to convergence.

(f) Estimate variety means and standard errors from model (5.3).

Williams& Matheson (1993) in their algorithm for modified FITCON replaced step (b) with one regression model

$$V + V.M : \tag{5.4}$$

where $M(h)$ = mean of centre j if $y(h)$ is yield for a variety tested at that centre. The slopes in model (5.4) are variety sensitivities. Use of model (5.4) in place of step (b) shortens the time to convergence considerably.

We use Genstat REML algorithm with extra programming to estimate variety means and sensitivities by modified FITCON (Algorithm 5.2.1). Our algorithm differs from that of Williams & Matheson (1993) in that sensitivities are scaled to unit mean and standard errors of variety mean differences are given. A Genstat code is given in Appendix B.1.

5.2.2 Example 5.2.1: wheat data, 1977

Sensitivities

Variety sensitivities and their standard errors from Algorithm 5.2.1 are shown in Table 5.2.1. Standard errors of sensitivities are obtained from the regression step.

Though approximate, simple t-tests indicate that Mardler and Sentry varied more than other varieties to centre differences. These tests assess departures of sensitivities from one and not from zero. Such tests are useful because adjusted variety means are valid if there are genuine departures from a uniform sensitivity (Example 4.3.2.2). The standard errors for sensitivities are from separate regressions of individual varieties.

Variety means

Table 5.2.1 displays variety means from Algorithm 5.2.1. Given estimates of variety sensitivities any linear model algorithm can be used to estimate $\hat{\tau}$ and $\hat{\beta}$ in model (5.1) but care has to be taken in predicting variety means from these estimates if estimates of centre effects are not constrained to a zero sum. From model (5.1), variety means are estimated[1] as $\hat{\tau}_i + \hat{\theta}_i\bar{\hat{\beta}}$.

Let $\hat{\alpha}^* = (\hat{\mu}, \hat{\alpha}', \hat{\beta}')$ be a vector of estimates of all effects in (5.3) given $\hat{\theta}$. We construct a $v \times (1 + v + c)$ matrix of coefficients K such that $K\hat{\alpha}^*$ is a vector of variety means (Section 4.2.1). K takes the form:

$$K = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0.0741 & 0.0741 & \dots & 0.0741 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0.0777 & 0.0777 & \dots & 0.0777 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0902 & 0.0902 & \dots & 0.0902 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0.1297 & 0.1297 & \dots & 0.1297 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0.1241 & 0.1241 & \dots & 0.1241 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0.1042 & 0.1042 & \dots & 0.1042 \end{pmatrix}$$

The means of control varieties are not adjusted but other varieties are adjusted in proportion to their sensitivities. For example, basic FITCON adjusts Sentry and Stuart downwards by 0.563 t/ha but modified FITCON adjusts Sentry and Stuart downwards by 0.74 t/ha and 0.62 t/ha respectively.

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[1] Variety means from Genstat REML ignore variety sensitivities and are estimated as $\hat{\tau} + \bar{\hat{\beta}}$.

Table 5.2.1:

Variety means (t/ha) and sensitivities from Algorithm 5.2.1

(wheat data, 1977)

	basic FITCON	modified FITCON		
	mean	mean	sensitivity	
variety			estimate	s.e
Huntsman	5.734	5.734	0.741	0.055
Atou	6.027	6.027	0.777	0.064
Armada	6.201	6.218	0.902	0.078
Mardler	6.574	6.511	1.297	0.066
Sentry	6.417	6.240	1.241	0.047
Stuart	6.595	6.536	1.042	0.122

Table 5.2.2:

Standard error of variety mean differences (t/ha) from Algorithm 5.2.1

(wheat data, 1977)

Atou	0.117	*			
Armada	0.121	0.121	*		
Mardler	0.121	0.121	0.123	*	
Sentry	0.170	0.170	0.177	0.179	*
Stuart	0.166	0.166	0.172	0.174	0.185
	Huntsman	Atou	Armada	Mardler	Sentry

Standard errors of difference

Given the variance matrix of $\hat{\alpha}^*$, $V_{\hat{\alpha}^*}$ say, for example from Genstat REML, the variance matrix of variety means is given by $KV_{\hat{\alpha}^*}K'$. Standard errors of differences of variety means are given in Table 5.2.2. These standard errors account for loss in degrees of freedom in estimating slopes (Section 5.2.3). Standard errors from modified FITCON are much smaller than those given by basic FITCON (Table 4.~~3.9~~2.5). For example, the standard error for Sentry versus Atou is decreased from 0.248 t/ha to 0.170 t/ha.

5.2.3 Inadequacy of modified FITCON

From model (5.1) the variance for the difference between means of varieties i and i' is proportional to $((\theta_i - \theta_{i'})^2\sigma_c^2 + \sigma^2)$. Since centre effects are regarded as fixed, standard errors from (5.3) ignore the portion of variance that depends on centres component of variance and are therefore not valid for a population of centres (Patterson & Silvey 1980).

A FITCON estimation of variety sensitivities gives an estimate of residual variance that ignores loss in degrees of freedom from the regressions. The error degrees of freedom may be reduced by $v-1$ to account for slopes and a constraint of unit mean. But as we have shown adjustment of degrees of freedom is not enough. We need to treat centre effects in model (5.1) as random and use REML to estimate components of variance and sensitivities.

5.3 Conditional approximations using REML

The principle in modified FITCON algorithm can be extended to REML. Firstly, conditional on sensitivities, variety means and centre means are estimated from the model,

$$V : C.T \tag{5.5}$$

Secondly, conditional on variety means from (5.5) sensitivities are estimated from the REML model $V : V.M$

Thus we replace steps (a), (b) and (d) in Algorithm 5.2.1 by REML models.

Algorithm 5.3.1

(a) Estimate variety means and centre means from the model $V : C$

(b) Estimate sensitivities from the model $V : V.M$

(c) Scale sensitivities to a unit mean.

(d) Use model (5.5) to estimate new variety means and centre means.

(e) Repeat steps (b) to (d) to convergence.

(f) Estimate variety means and standard errors from model (5.5).

Genstat code for Algorithm 5.3.1 is given in Appendix B.2.

5.3.1 Example 5.3.1 : wheat data, 1977

Variety means need no extra calculations since model terms associated with sensitivities are now treated as random. Variety means and standard errors of differences are displayed in Tables 5.3.1 and 5.3.2 respectively.

Variety means and sensitivities from Algorithm 5.3.1 are similar to those from modified FITCON. For example, variety means for Sentry and Stuart differ by not more than 0.0~~8~~1 t/ha compared to means from modified FITCON. Variety means from Algorithm 5.3.1 include information from centre differences but computed as the variance of the columns of W.

Standard errors of mean differences reflect the fact that Mardler and Sentry are more variable. The standard error of Huntsman versus ~~Atou~~ Sentry, for example, is increased from 0.170 t/ha to 0.252 t/ha. These standard errors include the portion of variance that depends on centres variance (Section 5.2.3). However, Algorithm 5.3.1 does not use all the information to estimate variety sensitivities and consequently variety means and their standard errors.

We consider a REML algorithm that jointly estimates all variance parameters using a Fisher's scoring scheme.

Table 5.3.1:

Variety means (t/ha) and sensitivities from basic REML and Algorithm 5.3.1 (wheat data, 1977)

	basic REML	Algorithm 5.3.1		
	mean	mean	sensitivity	
variety			estimate	s.e
Huntsman	5.734	5.734	0.742	0.062
Atou	6.027	6.027	0.778	0.062
Armada	6.206	6.220	0.903	0.073
Mardler	6.579	6.513	1.298	0.073
Sentry	6.429	6.248	1.239	0.081
Stuart	6.606	6.543	1.040	0.081

Table 5.3.2:

Standard error of variety mean differences (t/ha) by Algorithm 5.3.1 (wheat data, 1977)

Atou	0.110	*			
Armada	0.129	0.123	*		
Mardler	0.247	0.234	0.194	*	
Sentry	0.252	0.241	0.212	0.168	*
Stuart	0.194	0.186	0.169	0.191	0.190
	Huntsman	Atou	Armada	Mardler	Sentry

5.4 REML with sensitivities (SREML)

The REML model (5.5) can be written as $y \sim N(X\tau, \sigma^2 I + \sigma_c^2 WW')$ where $\tau_i = \mu + \alpha_i$ (Sections 4.2.2 and 5.2). The residual likelihood is of the same form as in the basic REML except that B is replaced by W a function of θ (Section 3.4.3). To estimate $\theta, \sigma_c^2, \sigma^2$ we equate the score vector from the residual likelihood to zero and solve the resulting equations. We will work with the ratio γ instead of centres component of variance.

5.4.1 Fisher's scoring scheme

A Fisher's scoring scheme iteratively solves the system of equations

$$\zeta^{l+1} = \zeta^{l} + F^{-1}(\zeta^{l})s(\zeta^{l}) \tag{5.6}$$

at the $(l+1)$-th iteration where ζ is a $(2+v)$-vector of all variance parameters and F is Fisher's information matrix (Section 3.4.3).

The score vector elements are

$$\begin{aligned} s(\gamma) &= -\text{trace}(W'PW)/2 + y'PWW'Py/2\sigma^2 \\ s(\sigma^2) &= -(n-v)/2\sigma^2 + y'Py/2\sigma^4 \\ s(\theta_k) &= -\gamma(\text{trace}P\Delta_k - y'P\Delta_k Py/\sigma^2)/2 \\ & k = 1, 2, \ldots, v \end{aligned}$$

where $\Delta_k = \partial/\partial\theta_k WW'$, $P = H^{-1}(I - X(X'H^{-1}X)^{-1}X'H^{-1})$ and $H = I + \gamma WW'$.

The elements of F are

$$\left.\begin{array}{lcl} f_{\gamma,\gamma} &=& trace(W'PW)^2/2 \\ f_{\gamma,\sigma^2} &=& trace(W'PW)/2\sigma^2 \\ f_{\sigma^2,\sigma^2} &=& (n-v)/\sigma^4 \\ f_{\theta_k,\gamma} &=& \gamma\ trace(W'P\Delta_k PW)/2 \\ f_{\theta_k,\sigma^2} &=& \gamma\ trace(P\Delta_k)/2\sigma^2 \\ f_{\theta_k,\theta_{k'}} &=& \gamma^2\ trace(P\Delta_k P\Delta_{k'})/2 \\ && k, k' = 1, 2, \ldots, v \end{array}\right\} \tag{5.7}$$

The first two equations are of the same form as those in the basic REML algorithm except B is replaced by W. The extended REML algorithm to estimate variance parameters including variety sensitivities is given the acronym SREML. Details of implementing this algorithm are discussed in Section 5.4.3.

Table 5.4.1:

Variety means and sensitivities for potato data by SREML

Variety	mean	sensitivity ($\pm$0.115)
Cara	45.56	0.405
Desiree	42.97	1.139
Drayton	40.30	0.954
KingEdward	41.31	1.193
Estima	42.43	1.178
Majestic	42.93	0.993
PentlandCrown	48.70	1.139

5.4.2 Examples

Example 5.4.2.1: potato data

When the data are complete, modified FITCON (Algorithms 5.2.1), Algorithm 5.3.1 and SREML give the same estimates of sensitivities. Variety means are not adjusted. Standard errors of differences from modified FITCON analysis are different from those given by Algorithm 5.3.1 and SREML. However, standard errors from Algorithm 5.3.1 and SREML are the same.

Any variety comparison is estimated with a standard error of 1.877 t/ha in the modified FITCON analysis. The comparison Cara versus Estima has a standard error 2.785 t/ha where as KingEdward versus Estima has a standard error of 1.823 t/ha. The units variance from the REML models is 26.585 which is greater than the units variance of 20.46 using methods of Section 4.3. This suggests a preference of methods of Section 4.3 (Examples 4.3.3.1).

Table 5.4.1 shows variety means and sensitivities. Standard errors of difference are shown in Table 5.4.2.

Table 5.4.2:

Standard error of variety mean differences (t/ha) by SREML (potato data)

Desiree	2.707	*				
Drayton	2.358	1.892	*			
KingEdward	2.816	1.829	1.935	*		
Estima	2.785	1.826	1.922	1.823	*	
Majestic	2.427	1.866	1.826	1.902	1.891	*
Pent. Crown	2.705	1.823	1.891	1.829	1.826	1.866
	Cara	Desiree	Drayton	KingEdward	Estima	Majestic

Example 5.4.2.2: wheat data, 1977

Variety means and sensitivities from SREML are displayed in Table 5.4.3. Variety means and standard errors of difference are similar to those from Algorithm 5.3.1 (Table 5.3.1).

SREML standard errors of some comparisons are slightly different from Algorithm 5.3.1 standard errors. For example the comparison Armada versus Sentry has a standard error of 0.217 t/ha compared to 0.212 t/ha using Algorithm 5.3.1.

The centre component of variance is 26.16 and a units variance of 0.0595. The units variance is the same as that from Algorithm 5.3.1. There are slight differences in sensitivities. Sensitivities for Armada and Mardler are slightly decreased whereas those of Sentry and Stuart are slightly increased. From the inverse of the information matrix sensitivities are independent of the units variance but are dependent on centres and between themselves (Appendix B.3).

Thus when centres are random conditional methods of estimation such as Algorithm 5.3.1 ignore some information but this information is very small. It is immediate from the inverse of information matrix that asymptotic standard errors of sensitivities are too high (Section 3.4.5).

Table 5.4.3:

Variety means (t/ha) and sensitivities from SREML

(wheat data, 1977)

	basic REML	Algorithm 5.3.1	SREML	
variety	mean	mean	mean	sensitivity
Huntsman	5.734	5.734	5.734	0.739
Atou	6.027	6.027	6.027	0.775
Armada	6.206	6.220	6.223	0.895
Mardler	6.579	6.513	6.518	1.287
Sentry	6.429	6.248	6.230	1.252
Stuart	6.606	6.543	6.527	1.052

Table 5.4.4:

Standard error of variety mean differences (t/ha) from SREML

(wheat data, 1977)

Atou	0.110	*			
Armada	0.129	0.123	*		
Mardler	0.245	0.232	0.193	*	
Sentry	0.258	0.247	0.217	0.168	*
Stuart	0.198	0.190	0.172	0.187	0.190
	Huntsman	Atou	Armada	Mardler	Sentry

Example 5.4.2.3: spring wheat data

The data, copied from Silvey (1978b), are NIAB trials grown in 1975 in which 10 varieties of spring wheat were tested at 11 centres (Table 5.4.5). Varieties 1, 2, 5 and 10 were controls and so had complete results.

Sensitivities from Algorithms 5.2.1, 5.3.1 and SREML are shown in Table 5.4.6. Some varieties varied more than others to centre differences. Variety 8 was insensitive to centre differences. Variety means and standard errors of differences from the controls by the three methods are displayed in Table 5.4.7 and Table 5.4.8 respectively. Except for the control varieties which were not adjusted, there are

Table 5.4.5:

Yield (t/ha) of ten varieties of spring wheat data, 1975

	centre										
variety	1	2	3	4	5	6	7	8	9	10	11
V1	2.93	3.59	4.22	4.01	5.38	4.67	5.29	2.88	2.97	4.61	2.47
V2	3.20	3.73	4.06	3.88	5.01	4.48	4.40	3.78	2.70	4.05	2.33
V3	4.05	4.56	5.09	4.54	*	4.97	*	*	3.32	*	2.95
V4	3.49	4.57	4.13	4.17	*	4.99	*	*	2.74	*	2.67
V5	3.38	4.02	4.67	4.50	4.97	5.24	5.41	3.71	3.15	4.72	3.45
V6	*	3.83	3.85	4.00	5.49	*	4.63	4.24	*	3.90	*
V7	*	4.02	3.88	4.13	5.41	*	4.87	4.03	*	4.13	*
V8	*	3.78	4.10	3.76	4.88	*	4.23	4.66	*	3.63	*
V9	3.65	4.38	5.54	4.36	5.68	4.68	5.45	*	3.49	*	2.89
V10	2.69	4.05	3.27	3.73	4.91	4.61	4.32	3.74	2.35	3.75	1.99

Table 5.4.6:

Sensitivities from Algorithms 5.2.1, 5.3.1 and SREML

(spring wheat data, 1975)

	Algorithm 5.2.1		Algorithm 5.3.1		SREML
Variety	sensitivity	s.e	sensitivity	s.e	sensitivity
V1	1.166	0.122	1.193	0.126	1.180
V2	0.958	0.122	0.976	0.126	0.967
V3	1.045	0.171	1.053	0.175	1.056
V4	1.216	0.171	1.225	0.175	1.228
V5	0.883	0.122	0.903	0.126	0.893
V6	1.063	0.252	1.015	0.250	1.030
V7	1.072	0.252	1.028	0.250	1.041
V8	0.492	0.252	0.463	0.250	0.474
V9	1.013	0.123	1.034	0.126	1.028
V10	1.092	0.122	1.111	0.126	1.101

Table 5.4.7:

Variety means (t/ha) for spring wheat data, 1975
from Algorithms 5.2.1, 5.3.1 and SREML

Variety	Algorithm 5.2.1	Algorithm 5.3.1	SREML
V1	3.911	3.911	3.911
V2	3.784	3.784	3.784
V3	4.552	4.541	4.545
V4	4.297	4.283	4.287
V5	4.293	4.293	4.293
V6	3.884	3.917	3.909
V7	3.956	3.989	3.981
V8	3.967	3.984	3.978
V9	4.473	4.470	4.471
V10	3.583	3.583	3.583

Table 5.4.8:

Variety mean differences from controls
from Algorithms 5.2.1, 5.3.1 and SREML
(spring wheat data, 1975)

	Algorithm 5.2.1		Algorithm 5.3.1		SREML	
variety	difference	s.e	difference	s.e	difference	s.e
V3	0.660	0.139	0.649	0.131	0.652	0.131
V4	0.404	0.141	0.391	0.139	0.395	0.140
V6	-0.008	0.139	0.025	0.131	0.016	0.131
V7	0.064	0.139	0.096	0.131	0.088	0.131
V8	0.074	0.139	0.092	0.189	0.086	0.186
V9	0.581	0.124	0.577	0.116	0.578	0.116

very slight differences in the means. Standard error of differences by Algorithm 5.3.1 and SREML are similar.

5.4.3 Computational aspects

In the basic REML algorithm, Fisher's information matrix, F, and the score vector, s, may be calculated from solutions to the mixed model equations and the matrix P is not computed (Section 3.4.3). This simplification still holds for calculating elements of F and s not associated with sensitivities. The elements of F and s associated with sensitivities may also be calculated without direct computation of P.

Let $W_k^* = \partial/\partial\theta_k W$, i.e the matrix W with all elements zero except those associated with variety k. Then,

$$\left.\begin{array}{lcl} trace(P\Delta_k) & = & 2\ trace(W'PW_k^*) \\ trace(W'P\Delta_k PW) & = & 2\ trace(W'PW_k^* W'PW) \\ trace(P\Delta_k P\Delta_{k'}) & = & 2\ trace(W_{k'}^{*\prime} PW_k^* W'PW) \\ & & +\ 2\ trace(W_{k'}^{*\prime} PWW_k^{*\prime} PW) \\ y'P\Delta_k Py & = & 2\ y'PWW_k^{*\prime} Py. \end{array}\right\} \qquad (5.8)$$

The matrix $P = S - SWCW'S$ where $S = I - X(X'X)^{-1}X'$ and C is a submatrix of the inverse of the matrix of coefficients in the mixed model equations corresponding to centre effects (Section 3.4.4). Thus (5.7) is obtained from (5.8).

Let $Q = I - W'SWC$, $Q_1 = A_1'SW$, $Q_2 = W'SA_2$ and $Q_3 = A_1'SA_2$ for any conformable matrice A_1 and A_2, then

$$A_1'PA_2 = \begin{cases} Q_1 Q & \text{if } A_2 = W \\ Q_3 - Q_1 C Q_2 & \text{if } A_1 \neq W \text{ and } A_2 \neq W \end{cases} \qquad (5.9)$$

Thus most of the elements for computing scores and Fisher's information matrix for sensitivities are already available. Only a careful computing organization is required and use of (5.9).

5.4.4 The SREML Algorithm

STEP 0

Use basic REML to obtain initial estimates $\hat{\gamma}$ and $\hat{\sigma}^2$.

STEP 1

(a) Calculate

$$\begin{aligned} M_1 &= (X'X)^{-1}X'W \\ M_2 &= (X'X)^{-1}X'y \\ Q_0 &= W'SW = W'W - W'XM_1 \\ V_0 &= W'Sy = W'y - W'XM_2 \end{aligned}$$

(b) Calculate the matrix $C = (Q_0 + \gamma^{-1}I)^{-1}$

(c) Estimate centre effects, variety means and residuals:

$$b = CV_0$$

$$a = (M_2 - M_1 b)$$

$$r = y - Xa - Wb$$

STEP 2

Calculate elements of information matrix for γ and σ^2.

(a) Calculate $U = \gamma^{-1}I - \gamma^{-2}C$ and $Q = I - W'SWC$.

(b) Calculate $f_{\gamma,\gamma}, f_{\gamma,\sigma^2}$ and f_{σ^2,σ^2} :

$$f_{\gamma,\gamma} = trace(U^2)/2$$

$$f_{\gamma,\sigma^2} = trace(U)/(2\hat{\sigma}^2)$$

$$f_{\sigma^2,\sigma^2} = (n - v)/(2\hat{\sigma}^4)$$

(c) Calculate the scores for γ and σ^2 are:

$$s(\gamma) = trace(U)/2 + b'b/2\hat{\sigma}^2/\hat{\gamma}^2$$
$$s(\sigma^2) = -(n-v)/2\hat{\sigma^2} + y'(y - Xa - Wb)/2\hat{\sigma}^4$$

STEP 3

Calculate information and scores associated with variety sensitivities:
For each variety $k = 1, 2, \ldots, v$

(a) Set up $W_v^{*'}$ and calculate

$$\begin{array}{lllll} M_3 & = & (X'X)^{-1}X'W_v^{*'} & & \\ Q_1 & = & W_v^{*'}SW & = & W_v^{*'}W - W_v^{*'}XM_1 \\ Q_2 & = & W_v^{*'}SW_v^{*'} & = & W_v^{*'}W_v^{*'} - W_v^{*'}XM_3 \\ V_1 & = & W_v^{*'}Sy & = & W_v^{*'}y - W_v^{*'}XM_2 \end{array}$$

(b) Calculate $U_1 = W_v^{*'}PW$, $U_2 = W_v^{*'}PW_v$ and $V_2 = W_v{*}'Py$ as

$$U_1 = Q_1CQ$$
$$U_2 = Q_2 - Q_1CQ_1$$
$$V_2 = V_1 - V_1CV_0$$

(c) The score for the sensitivity is

$$s(\theta_k) = \gamma\{trace(W'PW_k^{*'}) - y'PWW_k^{*}Py/\sigma^2\}$$

Thus calculate:

$$s(\theta_k) = \hat{\gamma}\{trace(U_1) - b'V_2/(\hat{\gamma}\hat{\sigma}^2)\}$$

(d) Calculate f_{γ,θ_k}, f_{σ^2,θ_k} and f_{θ_k,θ_k} using (5.7), (5.8) and (5.9):

$$f_{\gamma,\theta_k} = \hat{\gamma}\ trace(U_1U)$$
$$f_{\sigma^2,\theta_k} = \hat{\gamma}\ trace(U_1)/\hat{\sigma}^2$$
$$f_{\theta_k,\theta_k} = \hat{\gamma}^2\{trace(U_2U) + trace(U_1^2)\}$$

(e) For each variety $l = 2, \ldots, k$,

i. set up $W_l^{*\prime}$ and calculate

$$\begin{aligned} Q_3 &= W_l^{*\prime} S W &= W_l^{*\prime} W - W_l^{*\prime} X M_1 \\ Q_4 &= W_l^{*\prime} S W_v^{*\prime} &= W_l^{*\prime} W_v^{*\prime} - W_l^{*\prime} X M_3 \end{aligned}$$

ii. calculate $U_3 = W_v^{*\prime} P W = Q_3 C Q$ and $U_4 = W_l^{*\prime} P W_v = Q_4 - Q_3 C Q_2$.

iii. calculate

$$f_{\theta_l, \theta_k} = \hat{\gamma}^2 \{trace(U_3 U) + trace(U_1 U_4)\}$$

STEP 4

(a) Perform a scoring step

(b) Scale sensitivities and construct W'

STEP 5

Repeat STEP 1 to STEP 4 until convergence

STEP 6

(a) Estimate final means and standard errors of differences from a generalised least squares analysis.

5.5 Summary

In this chapter we discussed three methods of fitting a mixed multiplicative model in which sensitivities are used to adjust variety means. Modified FITCON (Algorithm 5.2.1) underestimates standard errors of variety comparisons even though its means are not very different from those of Algorithm 5.3.1 or SREML.

Algorithm 5.3.1 gives results very similar to those of SREML. Algorithm 5.3.1 and SREML provide standard errors that are valid for a population of centres. Algorithms 5.2.1 and 5.3.1 have the advantage of providing approximate standard errors for sensitivities.

The means given by the three methods are valid if the sensitivities are genuine. Approximate standard errors can be used to test hypotheses regarding sensitivities. A lot remains to be known about the power of these tests. In the meantime regression tests may be combined with tests based on change in units variance to decide whether a multiplicative model should be used.

In addition to genuine sensitivities the subsamples must be effectively random samples and varieties $\times$ centres variance should contribute substantially to units variance.

We now turn to the problem of combining trials over centres and years.

Chapter 6

COMBINING TRIALS OVER CENTRES AND YEARS

6.1 Introduction

A combined analysis of trials over centres and years is based on a $V \times C \times Y$ table if the same sample of centres is used in each year, or on a $V \times Y/C$ table if a new sample of centres is chosen in each year, or on a $V \times R/C \times Y$ table if centres are classified by regions. We describe models for analysing a $V \times C \times Y$ table and a $V \times Y/C$ table in Sections 6.2 and 6.3. Methods for analysing series of trials in which centres are grouped by regions are discussed in Section 6.5. In this chapter we regard variances as homogeneous.

Simple methods are also used to provide estimates of means over all trials (Section 2.2.4). Trials data are often bulky and REML analysis based on full models may not be feasible. Examples of simple methods of analysis are given in Section 6.2.3 and 6.3.2. Sometimes the entries in the second stage of a two-stage analysis have a wide range of inaccuracy. In Section 6.4 we describe an algorithm for a weighted two-stage analysis in which more weight is given to trials that have more accurate information.

Table 6.2.1:

Basic REML models for analysing $V \times C \times Y$ table

model	formula
$\mathcal{A}$	$V : C + Y + V.C + V.Y + C.Y$
$\mathcal{B}$	$V + C : Y + V.C + V.Y + C.Y$
$\mathcal{C}$	$V + Y : C + V.C + V.Y + C.Y$
$\mathcal{D}$	$V + C + Y : V.C + V.Y + C.Y$
$\mathcal{E}$	$V + C + V.C : Y + V.Y + C.Y$
$\mathcal{F}$	$V + Y + V.Y : C + V.C + C.Y$
$\mathcal{G}$	$: V * C * Y$

6.2 Analysis of $V \times C \times Y$ table

The model structure $V * C * Y$ for a $V \times C \times Y$ table has the effects: C, Y, $C.Y$, V, $V.C$, $V.Y$ and $V.C.Y$ The units variance is made up of varieties $\times$ centres $\times$ years variance and plot error variance. It is not possible to separate varieties $\times$ centres $\times$ years variance and plot error variance unless there is an independent estimate of one of them.

6.2.1 REML models

Basic models for analysing a $V \times C \times Y$ table are displayed in Table 6.2.1. In specifying models for analysis it is important to bear in mind marginality relations on parameters of the model (Nelder 1977). Except for nested interactions, an interaction in the model has all its main effects in the model. If an interaction is regarded as fixed then all its main effects are fixed.

Model $\mathcal{A}$ predicts variety performance for a range of environmental conditions sampled by the trials. In this model variety effects are regarded as fixed and all other effects and interactions are error. The components of variance for centres, years and their interaction contribute to the error for the general mean. This model gives full weight to all information in the data for estimating variety means. Effects of centres and years may be regarded as fixed because environmental variation is known to be large (Talbot 1984). This leads to models $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$. If the $C \times Y$ table is complete, i.e every centre is used at least once each year, centres $\times$ years interaction can be treated as fixed in the analysis. If this condition is not met, some of the variety means cannot be estimated and so, we treat centres $\times$ years interaction as random.

It is possible for a random effect or an interaction to have a negative component of variance (Section 3.4.3, page 36). A random interaction should only enter the analysis if its component of variance is positive. Otherwise it is deleted from the model. A main effect with a negative component of variance should be made fixed.

Model $\mathcal{E}$ estimates future variety performance for the centres in the trials. The model also gives a $V \times C$ table of predicted means which are used to identify varieties well-adapted to specific regions or centres. In this analysis the varieties $\times$ centres interaction is regarded as fixed. If variety means are required for a population of centres but for the years in which the trials were grown, then varieties $\times$ years interaction does not contribute to error and model $\mathcal{F}$ is used.

Incompleteness imposes conditions under which a $V \times C$ or a $V \times Y$ table of means is given by model $\mathcal{E}$ or $\mathcal{F}$ respectively. A $V \times C$ or a $V \times Y$ table of means is obtained from margins of a $V \times C \times Y$ table of expectations using REML estimates. Thus varieties $\times$ centres effects can be regarded as fixed if every variety is tested at every centre in at least one year. Similarly to make varieties $\times$ years effects fixed, the $V \times Y$ table should be complete. Failing these conditions only means of varieties tested in all centres or all years are estimable. Often we have no choice but to make varieties $\times$ centres and varieties $\times$ years interaction random otherwise we have no analysis.

Model $\mathcal{G}$ is used mainly to estimate components of variance (Patterson *et al.* 1977 and Talbot 1984). Components of variance are required to estimate genetic gains for the varieties in trials and are also used in design and monitoring of trial systems (Patterson & Silvey 1980 and Talbot 1983, 1984).

The choice of the model to use is determined solely by objectives of the analysis and not on any model-building criteria. And as discussed in Section 1.2.2 the aim is not to find a model which best fits the data but to predict variety means for a specified population of environmental conditions. In particular, we prefer model $\mathcal{A}$ for analysing recommended list trials.

Rules for model specification

(a) All main effects and interaction are included in the model.

(b) If an interaction is fixed then all its main effects are fixed.

(c) Model $\mathcal{E}$ is used only if every variety is tested at every centre in at least one year. Similarly, model $\mathcal{F}$ is used only when $V \times Y$ table is complete.

(d) If an interaction has a negative component of variance, it should be removed from the model.

(e) A main effect with a negative component of variance should be made fixed if it is associated with an interaction with a positive component of variance.

6.2.2 The complete $V \times C \times Y$ table

When the data are complete, estimates of variety means are independent of components of variance. However, estimates of components of variance are needed to estimate standard errors of variety comparisons. Varieties $\times$ environments interactions that are random in the model contribute to error.

For model $\mathcal{A}$, the variance of a difference between any two means is

$$var(m_i - m_j) = 2(\sigma^2 + n_y\sigma^2_{VC} + n_c\sigma^2_{VY})/n_cn_y$$

Table 6.2.2:

Yields (bushels/acre) of five varieties of barley at six centres in Minnesota State (Yates & Cochran 1938)

		centre					
year	variety	1	2	3	4	5	6
1931	Manchuria	27.00	48.87	27.43	39.93	32.97	28.97
	Svansota	35.13	47.33	25.77	40.47	29.67	25.70
	Velvet	39.90	50.23	26.13	41.33	23.03	26.30
	Trebi	36.57	63.83	43.77	46.93	29.77	33.93
	Peatland	32.77	48.57	29.87	41.60	34.70	32.00
1932	Manchuria	26.90	33.47	34.37	32.97	22.13	22.57
	Svansota	27.43	38.50	35.03	20.63	16.63	22.23
	Velvet	26.80	37.40	38.83	32.07	32.23	22.47
	Trebi	29.07	49.23	46.63	41.83	20.63	30.60
	Peatland	28.07	36.03	43.20	25.23	26.77	31.37

where n_y and n_c are the number of years and centres respectively. If the varieties $\times$ centres or the varieties $\times$ years interaction is fixed then the corresponding component of variance does not contribute to error. Thus standard errors of variety comparisons will be smaller than those given by model $\mathcal{A}$ because the population of inference is reduced. For example the variance of a difference of means under model $\mathcal{F}$ is

$$var(m_i - m_j) = 2(\sigma^2 + n_y \sigma_{VC}^2)/n_c n_y$$

Efficient estimates of components of variance can be obtained from the ANOVA table.

Yates & Cochrans's (1938) example

The data analysed by Yates & Cochran (1938) are displayed in Table 6.2.2. Since the data are complete, the Genstat ANOVA algorithm can be used. The ANOVA

Table 6.2.3:

ANOVA table for data in Table 6.2.2

source	df	MS
centres	5	471.58
years	1	422.06
centres × years	5	153.20
varieties	4	147.50
varieties × centres	20	24.63
varieties × years	4	8.11
units	20	15.47

Table 6.2.4:

Variety means for Yates & Cochran's (1938) example

	year		
variety	1931	1932	mean
Manchuria	34.19	28.73	31.46
Svansota	34.01	26.74	30.38
Velvet	34.49	31.63	33.06
Trebi	42.47	36.33	39.40
Peatland	36.58	31.78	34.18

table is displayed in Table 6.2.3. Variety means are efficiently estimated by unadjusted means (Table 6.2.4). However extra calculations are required to obtain standard errors of difference.

There is no evidence for a significant varieties × years interaction. We therefore pool the varieties × years mean square and the units variance to obtain a pooled units variance[1] of $14.24 = (32.424 + 309.359)/24$.

[1]The pooled units variance can also be obtained by excluding the varieties × years term from the TREATMENTSTRUCTURE directive of Genstat.

Table 6.2.5:

Components of variance for Yates & Cochran's (1938) example using REML

	model	
component	V:C*Y+V.C	V:C*Y + V.C + V.Y
centres	30.80	30.92
years	8.96	9.31
centres × years	27.79	7.55
varieties × centres	5.19	4.58
varieties × years		-1.23
units	14.24	15.47

To calculate the standard error of difference we need estimates of components of variance. From the preceeding discussion, $\hat{\sigma}^2_{VY} = 0$ and $\hat{\sigma}^2 = 14.241$. The estimate of the varieties × centres component of variance is obtained by equating the mean sum of squares to its expectation i.e $\sigma^2 + n_y\sigma^2_{VC}$ from which we get $\hat{\sigma}^2_{VC} = (24.628 - 14.241)/2 = 5.19$. Thus the standard error of difference between any two means from model $\mathcal{A}$ is $\sqrt{(14.241/6 + 5.19/3)} = 2.026$. If model $\mathcal{F}$ is used the standard error of difference is obtained as $\sqrt{(14.241/6)} = 1.541$.

The analysis using ANOVA can be reproduced by REML using the model $V : Y + C + V.C + C.Y$ The term for varieties × years interaction is excluded from the model because varieties × years component of variance is negative (Table 6.2.5).

6.2.3 Example 6.2.3: Ryegrass data

Models $\mathcal{A}$ - $\mathcal{E}$ (Table 6.2.1) are used to analyse ryegrass data from Appendix $A.2$ (Section 1.1, page 5). Model $\mathcal{F}$ is excluded from the analysis because the varieties × years table is incomplete. The within-years means for 7 varieties is displayed in Table 6.2.6. Variety 1 was a control and so has complete results; varieties 12 and

Table 6.2.6:

Within-years yields of seven varieties of ryegrass trials

	year				
Variety	1984	1985	1986	1987	1988
1	10.84	14.46	13.37	10.83	10.03
4	*	*	13.42	11.38	*
12	*	*	*	9.91	8.78
14	11.14	13.90	*	*	*
17	11.61	14.66	13.99	*	*
18	*	*	*	10.75	9.60
19	*	*	13.94	10.92	*

18 were new varieties and varieties 4, 17 and 19 were already recommended but were in trial for further testing.

Components of variance

Components of variance from models $\mathcal{A} - \mathcal{E}$ and $\mathcal{G}$ are shown in Table 6.2.7. The centres × years component of variance is large compared to other environmental components of variance. The varieties × centres and the varieties × years components of variance are small but nevertheless positive. The centres and years component of variance are large, and so only slight differences are expected in estimation of variety means and their standard errors if centre effects and year effects are treated as fixed.

Variety means

Table 6.2.8 shows estimates of variety means from models $\mathcal{A} - \mathcal{E}$. Average standard errors of difference are displayed in Table 6.2.9.

Differences in centres supply no information on variety means because within-years tables are complete. Thus treating centre effects as fixed or random makes

Table 6.2.7:

Components of variance for ryegrass data (Table 6.2.6)

	model					
component	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{C}$	$\mathcal{D}$	$\mathcal{E}$	$\mathcal{G}$
centre	3.174		3.174			3.174
year	2.786	2.786			2.810	2.969
centre × year	3.446	3.445	3.446	3.445	3.279	3.446
variety						0.2095
variety × centre	0.0393	0.0398	0.0393	0.0398		0.0393
variety × year	0.0376	0.0377	0.0376	0.0376	0.0339	0.0379
units	0.2144	0.2140	0.2143	0.2140	0.2405	0.2413

no difference to estimation of variety means. On the other hand, because the varieties × years table is incomplete, REML estimates of variety means include weighted information on indirect differences for varieties not tested in all years. This weighting, however, is inversely proportional to the years component of variance. When there is large years variation, as is typical of variety trials in the UK, a random year term in the models makes only a slight difference in estimation of variety means and their standard errors (Talbot 1984). Consequently models $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{E}$ give the same variety means. Variety means from model $\mathcal{C}$ are the same as those from model $\mathcal{D}$ but slightly different from variety means given by model $\mathcal{A}$ for varieties with incomplete results.

There is little to choose between standard errors from models $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ because of large centre and years variances (Table 6.2.9). In general, treating varieties × environments interactions as random increases standard errors of variety mean differences, but treating centres or year effects as random makes the adjustments to means smaller and standard errors smaller.

Model $\mathcal{E}$ gives the same variety means as model $\mathcal{A}$ because within-years tables are complete. Standard errors from model $\mathcal{E}$ are, however, smaller than those from models $\mathcal{A}$ – $\mathcal{D}$ by about 7% on average. The standard errors are functions of σ^2_{VY} and σ^2 and reflects the fact that the population of inference has been reduce

Table 6.2.8:

Variety means (t/ha) for ryegrass data

using models in Table 6.2.1

variety	model $\mathcal{A}, \mathcal{B}, \mathcal{E}$	$\mathcal{C}, \mathcal{D}$
1	11.91	11.91
4	12.13	12.13
12	10.81	10.83
14	11.81	11.80
17	12.44	12.43
18	11.64	11.66
19	12.16	12.16

Table 6.2.9:

Standard error of variety mean difference

(ryegrass data)

model	minimum	maximum	average
$\mathcal{A}$	0.2310	0.3581	0.2982
$\mathcal{B}$	0.2313	0.3583	0.2984
$\mathcal{C}$	0.2311	0.3590	0.2986
$\mathcal{D}$	0.2314	0.3592	0.2988
$\mathcal{E}$	0.2053	0.3421	0.2785

to that of future years only. Although varieties $\times$ environments components of variance are small their effect on standard errors is not negligible.

Single-stage and two-stage analysis

A single-stage analysis can be obtained from the model

$$V + C.Y :$$

(Sections 1.3 and 2.2.6) The first stage of a two-stage analysis is easy because $V \times C$ tables are complete; variety means are not adjusted and Table 6.2.6 is used in the second stage.

Components of variance from simple methods are displayed in Table 6.2.11. The units variance from a two-stage analysis is related to the units variance from model $\mathcal{A}$ by:

$$\sigma_s^2 \simeq \sigma_{VY}^2 + \sigma_f^2/n_c$$

where σ_f^2 and σ_s^2 are the units variance from model A and two-stage analysis respectively. For example, the varieties $\times$ years variance can be estimated as $0.0682 - (0.2144)/7 = 0.0376$.

Variety means from single-stage and two-stage analysis are the same as those from full models in which variety and year effects are fixed but all other effects and interactions are random i.e models $\mathcal{C}$ and $\mathcal{D}$ (Table 6.2.8). In general if within-years tables are complete simple methods provide the same estimates of means as full models in which year effects are regarded as fixed. If REML is used in the second stage of a two-stage analysis variety means obtained are the same as those given by model $\mathcal{A}$.

Standard errors from simple methods are however smaller than those obtained from full models. Standard errors of differences from single-stage, two-stage and models $\mathcal{A}$ and $\mathcal{D}$ are given in Table 6.2.10. Standard errors from single-stage analysis ignore consistent varieties $\times$ centres and varieties $\times$ years variation. Consequently, standard errors of variety mean comparisons are underestimated by 29% on average. A two-stage analysis ignores consistent varieties $\times$ centres variation; standard errors are underestimated but by 6% on average in this example. A correction can be made to the two-stage standard errors of difference by adding $2\sigma_{VC}^2/n_c$ using an estimate from previous trials or long-term averages, for example Talbot (1984).

Table 6.2.10:

Standard error of variety mean difference for ryegrass data from single-stage, two-stage and models $\mathcal{A}$ and $\mathcal{D}$ (Table 6.2.10)

model	minimum	maximum	average
$\mathcal{A}$	0.2310	0.3581	0.2982
$\mathcal{D}$	0.2314	0.3592	0.2988
Single-stage	0.1557	0.2601	0.2114
Two-stage (FITCON)	0.2055	0.3431	0.2789
Two-stage (REML)	0.2055	0.3421	0.2785

Table 6.2.11:

Components of variance from single-stage and two-stage analysis (ryegrass data)

		two-stage	
component	single-stage	FITCON	REML
years	-	-	3.2782
units	0.2742	0.0682	0.0683

A single-stage analysis gives equal weight to each trial so that varieties tested in a small number of trials are over-valued. A two-stage analysis gives equal weight to each year and varieties tested in years with small number of trials are over-valued. A full analysis gives an optimum weight to each trial that takes into account centres and years variances. If both $V \times C$ and $V \times Y$ tables are incomplete the three methods can give different means. In this example a two-stage analysis is better than a single-stage and should be used especially if a full analysis is not feasible.

Table 6.3.1:

Basic REML models for analysing V × Y/C table

model	formula
$\mathcal{H}$	V : Y/C + V.Y
$\mathcal{I}$	V + Y : Y.C + V.Y
$\mathcal{J}$	: V + Y/C + V.Y

6.3 Analysis of $V \times Y/C$ table

6.3.1 REML models

The model structure for a $V \times Y/C$ table has the terms: V, Y/C, $V.Y$ and $Y/V.C$. There is no centres × years interaction and Y/C denotes year effects plus within-years centre effects. The within-years varieties × centres variance is part of units variance. Basic models for analysing a $V \times Y/C$ table are displayed in Table 6.3.1.

Model $\mathcal{H}$ predicts means for a range of conditions in a $V \times Y/C$ table and has all effects except variety effects random and all interactions random. Model $\mathcal{I}$ also predicts means over a range of conditions since typically variation from year to year is large. Model $\mathcal{J}$ is used mainly to estimate components of variance.

6.3.2 Example 6.3.2: Sugar beet data

Complete within-years tables

We use a subset of the sugar beet data for only $V13$ to $V21$ at seven centres in each year with the design (Section 1.1, page 5 and Appendix A.3):

Table 6.3.2:

Components of variance from complete, single-stage and two-stage analysis
(subset of sugar beet data)

source	model $\mathcal{H}$	model $\mathcal{I}$	single-stage	two-stage	
years	-11.102			5.002	
centres × years	112.727	112.727			
varieties × years	3.298	3.297			
units	5.721	5.721	8.330	4.115	4.114

	year				
variety	1	2	3	4	5
$V13$	7	7	7	7	7
$V14$	7	7	7	7	7
$V15$	7	7	7	7	7
$V16$	7	7	7	7	7
$V17$	7	7	7	7	7
$V18$	0	7	7	7	7
$V19$	0	7	7	7	7
$V20$	0	0	7	7	7
$V21$	0	0	7	7	7

Components of variance ~~and variety means~~ from model $\mathcal{H}$ and $\mathcal{I}$, single-stage and two-stage analysis are displayed in Table 6.3.2. The years component of variance is negative and so we treat year effects as fixed.

Variety means from model $\mathcal{I}$, single-stage and two-stage (FITCON) analysis are the same (Table 6.3.3). Standard errors from two-stage analysis are the same as those from the model $\mathcal{I}$ but standard errors from single-stage analysis are smaller because they ignore consistent varieties × years variation. In general a two-stage REML analysis gives the same means and standard errors of variety contrasts as model $\mathcal{H}$.

Table 6.3.3:

Variety means (t/ha) for (a subset) sugar beet data
from single-stage, two-stage and model $\mathcal{I}$ (Table 6.2.10)

variety	model $\mathcal{I}$	single-stage	two-stage	
			FITCON	REML
$V13$	55.51	55.51	55.51	55.51
$V14$	55.82	55.82	55.82	55.82
$V15$	56.64	56.64	56.64	56.64
$V16$	56.29	56.29	56.29	56.29
$V17$	58.36	58.36	58.36	58.36
$V18$	58.21	58.21	58.21	58.15
$V19$	58.52	58.52	58.52	58.46
$V20$	58.11	58.11	58.11	57.94
$V21$	58.23	58.23	58.23	58.06

Table 6.3.4:

Standard error of variety mean difference for (a subset) sugar beet data
from single-stage, two-stage and model $\mathcal{I}$

model	minimum	maximum	average
$\mathcal{I}$ $(V + Y : Y.C + V.Y)$	1.283	1.656	1.418
Single-stage	0.690	0.891	0.763
Two-stage (FITCON)	1.283	1.656	1.418
Two-stage (REML)	1.283	1.656	1.416

Table 6.3.5:

Components of variance from models $\mathcal{H}$ & $\mathcal{I}$, single-stage and two-stage analysis
(sugar beet data)

	analysis				
source	model $\mathcal{H}$	model $\mathcal{I}$	single-stage	two-stage	
years	-1.453			5.436	
centres $\times$ years	80.97	80.40			
varieties $\times$ years	1.877	1.876			
units	5.022	5.022	6.506	2.255	2.255

Table 6.3.6:

Within-years variety means (number of trials) (sugar beet data)

	year				
variety	1	2	3	4	5
1	57.52 (16)	58.29 (16)	57.10 (16)	51.88 (11)	59.94 (13)
2	57.66 (16)	56.51 (16)	54.88 (16)	52.04 (11)	59.52 (13)
3	57.24 (16)	57.87 (16)	56.19 (16)	53.10 (11)	57.21 (13)
4	56.40 (16)	56.98 (16)	57.46 (16)	55.02 (11)	58.20 (13)
5	55.30 (16)	55.40 (16)	53.24 (16)	50.00 (11)	55.32 (13)
6	59.51 (16)	57.17 (16)	56.44 (16)	53.79 (11)	57.86 (13)
7	58.57 (15)	57.95 (16)	57.09 (16)	52.78 (11)	53.24 (13)
8	57.38 (16)	58.09 (16)	56.60 (16)	53.43 (11)	56.33 (13)
9	56.76 (16)	60.38 (16)	53.43 (16)	49.36 (11)	54.20 (13)
10	57.51 (16)	58.38 (16)	52.51 (16)	51.13 (11)	51.41 (13)
11	57.49 (16)	57.86 (16)	53.95 (16)	52.25 (11)	57.68 (13)
12	57.02 (16)	59.53 (16)	53.86 (16)	50.54 (11)	56.09 (13)
13	58.27 (16)	58.66 (16)	54.32 (16)	50.21 (11)	53.79 (13)
14	55.26 (16)	55.82 (16)	56.19 (16)	52.70 (11)	57.81 (13)
15	59.97 (16)	59.75 (16)	55.33 (16)	51.28 (11)	55.76 (12)
16	57.90 (16)	59.63 (16)	54.52 (16)	50.71 (11)	54.33 (13)
17	58.42 (7)	57.98 (16)	58.04 (16)	55.91 (11)	57.09 (13)
18	*	59.46 (7)	56.31 (16)	53.71 (11)	59.46 (13)
19	*	59.21 (7)	55.02 (16)	54.96 (11)	60.63 (13)
20	*	*	54.72 (7)	51.96 (11)	59.57 (13)
21	*	*	54.77 (7)	54.43 (11)	59.99 (13)
22	*	*	57.63 (7)	55.63 (11)	61.11 (13)
23	*	*	58.04 (7)	54.13 (11)	58.22 (13)
24	*	*	55.31 (7)	53.72 (11)	58.14 (13)
25	*	*	51.80 (7)	53.26 (11)	56.72 (13)
26	*	*	55.42 (7)	51.47 (11)	56.70 (13)
27	*	*	57.08 (7)	55.48 (11)	59.36 (13)
28	*	*	55.78 (7)	51.89 (11)	56.96 (13)
29	*	*	55.12 (7)	54.74 (11)	58.65 (13)

Incomplete within-years tables

We analyse all the sugar beet data (Section 1.1 and Appendix A.3). The years component of variance is negative and so we use model $\mathcal{I}$ (Table 6.3.5). There were 72 trials and a single-stage is based on a 29×72 two-way table. This analysis is given by the model $V + C.Y$:

In each of the first three years, new varieties were tested at a subset of centres. For these years, the within-year tables are of L-pattern, except for the first year in which the yield for variety 7 at centre 5 is missing. The fourth year had complete results, so no adjustment is required. The fifth year had complete results except for a missing value for variety 15. We use FITCON to adjust means of varieties with incomplete results (Section 4.2.2). The within-years variety means are displayed in Table 6.3.6 and are used in the second stage of a two-stage analysis.

Variety means from model $\mathcal{I}$, single-stage and two-stage analysis are shown in Table 6.3.7. Single-stage variety means differ from those given by model $\mathcal{I}$ by as much as 0.4 t/ha in some cases. For example variety 20 single-stage mean exceeds the mean from model $\mathcal{I}$ by 0.48 t/ha. Two-stage means are similar to those from model $\mathcal{I}$ for varieties tested in all the years. Slight differences are however observed for new varieties. The maximum difference is about 0.07 t/ha. Two-stage FITCON and two-stage REML give the same means for varieties tested in all the years; new variety means from REML are however slightly different because REML makes use of information on varieties from differences in years.

Average standard errors of difference are displayed in Table 6.3.8. Standard errors of differences are underestimated by single-stage analysis by 52% on average compared to standard errors from model $\mathcal{I}$. FITCON two-stage standard errors are very similar to those from two-stage REML because of large years variation (Table 6.3.5). Two-stage standard errors differ from those from model $\mathcal{I}$ by only about 2% on the average. There is therefore not much to loose in using a two-stage analysis for this data (see Section 7.4 for further analysis of sugar beet data).

Table 6.3.7:

Variety means from model $\mathcal{I}$, single-stage and two-stage analysis (sugar beet data)

Variety	model $\mathcal{I}$	single-stage	two-stage FITCON	two-stage REML
1	56.95	57.17	56.95	56.95
2	56.11	56.27	56.12	56.12
3	56.32	56.51	56.32	56.32
4	56.78	56.88	56.81	56.81
5	53.85	54.06	53.85	53.85
6	56.95	57.14	56.95	56.95
7	55.94	56.24	55.93	55.93
8	56.36	56.57	56.37	56.37
9	54.86	55.23	54.83	54.83
10	54.21	54.52	54.19	54.19
11	55.84	56.02	55.85	55.85
12	55.42	55.72	55.41	55.41
13	55.08	55.44	55.05	55.05
14	55.54	55.66	55.56	55.56
15	56.45	56.83	56.42	56.42
16	55.44	55.79	55.42	55.42
17	57.50	57.71	57.49	57.49
18	57.67	57.90	57.66	57.65
19	57.89	58.05	57.88	57.87
20	56.76	57.24	56.68	56.65
21	57.76	58.30	57.66	57.63
22	59.44	59.84	59.39	59.36
23	58.03	58.19	58.06	58.03
24	57.03	57.40	56.99	56.96
25	55.30	55.85	55.19	55.16
26	55.79	56.02	55.79	55.76
27	58.61	58.93	58.57	58.54
28	56.13	56.36	56.14	56.11
29	57.50	57.93	57.43	57.40

Table 6.3.8:

Standard error of variety mean difference from single-stage, two-stage and models $\mathcal{I}$ (sugar beet data)

model	minimum	maximum	average
$\mathcal{I}$ $(V+Y:Y/C+V.Y)$	0.945	1.262	1.073
Single-stage	0.425	0.648	0.518
Two-stage (FITCON)	0.950	1.226	1.060
Two-stage (REML)	0.950	1.226	1.060

6.4 Analysis of $V \times Y$ table with varying precision

A two-stage analysis can be greatly affected by inaccuracies in entries in the second stage. This is much more likely if the single entries in the $V \times Y$ table are based on a wide range of trials. A weighted analysis is worthwhile if variances of single entries are wider than expected using long-term averages such as Talbot (1984). More weight is given to means that have been more accurately estimated. Efficient weights depend on the varieties $\times$ years variance and a within-year variance of each entry i.e

$$w_{ij} = (\sigma_{VY}^2 + \sigma^2/n_{ij})^{-1} \tag{6.1}$$

where σ^2 is a within-year units variance and n_{ij} the number of trials on which variety i mean is based in the j-th year of trials. Weighting by trial numbers (n_{ij}) is efficient if there is no varieties $\times$ years interaction.

Since varieties $\times$ years variance (σ_{VY}^2) is unknown, the method of analysis involves an iterative search for the unknown component of variance with weights given by (6.1). The value of σ_{VY}^2 which gives a unit variance is the required estimate. This analysis also gives the weighted variety means. Better weights are obtained by using a pooled estimate of within-years unit variances.

6.4.1 Algorithm for analysis of $V \times Y$ table weighted by components of variance

The unit variance of a weighted REML[2] analysis is taken as a function $F(X)$ of X, where X is the varieties × years component of variance. The required analysis is given by the value of X for which $F(X) = 1$.

(a) Guess two value for σ^2_{vy}, say x and z.

(b) Set up a variate $W = 1/(x + U)$ where U is a variate of within-year variances for each entry. The values of U are approximately $0.204/n_i$.

(c) Do a weighted REML with weights W. Save the units variance as f_1.

(d) Repeat steps (b) and (c) with z in place of x and unit variance saved in f_2.

(e) Update z using the secant root-finding method i.e, at the i-th iteration

$$z^i = x^{i-1} - (x^{i-1} - z^{i-1})(f_1^{i-1} - 1)/(f_1^{i-1} - f_2^{i-1})$$

(f) Repeat steps (d) and (e) until $f_2 = 1$.

6.4.2 Example 6.4.2: wheat data

The distribution of trials in the wheat data is shown in Table 6.4.1. $V \times C$ tables and the $V \times Y$ table are incomplete. We use models $\mathcal{H}$ and $\mathcal{I}$ (Table 6.3.1), single-stage and two-stage to analyse the data (Appendix A.1). Within-years variety means are displayed in Table 6.4.1. Variety means and average standard errors of difference are displayed in Tables 6.4.2 and 6.4.3 respectively.

Variety means from models $\mathcal{H}$ and $\mathcal{I}$ differ slightly by 0.01t/ha at the most. Differences in years contributes only a small amount of information to estimation

[2] Genstat REML has facilities for a weighted analysis.

of variety means because of large years variance. If 0.04 t/ha is subtracted from single-stage means, the resulting means are very similar to those from full models. Also if 0.05 t/ha is added to two-stage means the resulting means are similar to those from full models.

Components of variance are shown in Table 6.4.4. The varieties × years component is small but positive. The units variance from two-stage is much smaller than expected. Average standard errors of difference are displayed in Table 6.4.3. Standard errors from single and two-stage analysis underestimate standard errors of difference by about 24% on the average.

The within-year variance for each entry range from 0.008 to 0.358 (Table 6.4.5); the number of trials range from 1 to 16 and the units variance from 0.082 to 0.483. Talbot's (1984) long-term average for varieties × years variance for wheat is 0.022 which is much smaller than σ^2/n_i for most entries. Thus a weighted analysis is appropriate.

Variety means from a weighted two-stage analysis with weights proportional to trial numbers and weighted two-stage with weights given by (6.1) are displayed in Tables 6.4.6 and 6.4.7 respectively. The estimate of varieties × years variance is 0.052.

Even though this method has produced an answer it is far from being satisfactory. The weights allow for unequal variances but take no account of ~~an~~ unequal covariances.

Table 6.4.1:

Within-years mean yields (t/ha) (wheat data, 1974 - 1978)

	year				
variety	1974	1975	1976	1977	1978
Huntsman	6.30 [16]	6.46 [16]	6.17 [14]	5.73 [10]	4.98 [13]
Atou	6.02 [16]	6.36 [1]	5.96 [5]	6.03 [10]	5.69 [13]
Armada	6.12 [4]	6.61 [4]	6.20 [14]	6.20 [9]	5.81 [12]
Mardler	*	6.82 [4]	6.60 [4]	6.57 [9]	5.75 [12]
Sentry	*	*	6.39 [4]	6.42 [4]	5.84 [12]
Stuart	*	*	6.54 [4]	6.60 [4]	5.95 [12]

[n] number of trials

Table 6.4.2:

Variety means (t/ha) from models $\mathcal{H}$, $\mathcal{I}$, single-stage and two-stage analysis (wheat data, 1974 - 1978)

variety	model $\mathcal{H}$	model $\mathcal{I}$	single-stage	two-stage FITCON	two-stage REML
Huntsman	5.976	5.958	6.001	5.928	5.928
Atou	6.075	6.077	6.110	6.012	6.012
Armada	6.259	6.256	6.291	6.188	6.188
Mardler	6.506	6.508	6.534	6.459	6.461
Sentry	6.441	6.458	6.495	6.378	6.390
Stuart	6.558	6.579	6.602	6.525	6.536

Table 6.4.3:

Average standard error of variety mean differences from models $\mathcal{H}$ and $\mathcal{I}$, single-stage and two-stage analysis (wheat data, 1974 - 1978)

model	minimum	maximum	average
$\mathcal{H}$ $(V : Y/C + V.Y)$	0.1426	0.2057	0.1762
$\mathcal{I}$ $(V + Y : Y.C + V.Y)$	0.1411	0.2035	0.1747
single-stage	0.1018	0.1609	0.1341
two-stage (REML)	0.1166	0.1505	0.1331
two-stage (FITCON)	0.1166	0.1506	0.1334

Table 6.4.4:

Components of variance from models $\mathcal{H}$ and $\mathcal{I}$, single-stage and two-stage analysis (wheat data, 1974 – 1978)

			analysis		
source	model $\mathcal{H}$	model $\mathcal{I}$	single-stage	two-stage	
years	0.034			0.125	
centre × years	1.145	1.143			
varieties × years	0.025	0.023			
units	0.227	0.227	0.247	0.034	0.034

Table 6.4.5:

Within-years variance of variety means (wheat data, 1974 – 1978)

			year		
	1974	1975	1976	1977	1978
variety	(0.483)[1]	(0.230)	(0.082)	(0.157)	(0.159)
Huntsman	0.040	0.200	0.008	0.023	0.016
Atou	0.154	0.358	0.020	0.023	0.016
Armada	0.040	0.083	0.008	0.025	0.017
Mardler	*	0.083	0.025	0.025	0.017
Sentry	*	*	0.025	0.047	0.017
Stuart	*	*	0.025	0.047	0.017

[1]within-year units variance

Table 6.4.6:

Variety means and standard error of differences (t/ha) from a two-stage analysis weighted by trial numbers (wheat data 1974 – 1978)

variety	mean	standard error of difference				
Huntsman	5.95	*				
Atou	6.04	0.124	*			
Armada	6.22	0.124	0.138	*		
Mardler	6.45	0.143	0.154	0.152	*	
Sentry	6.40	0.166	0.174	0.172	0.183	*
Stuart	6.54	0.166	0.174	0.172	0.183	0.197
		Huntsman	Atou	Armada	Mardler	Sentry

Table 6.4.7:

Variety means and standard error of differences (t/ha) from a two-stage analysis weighted by (6.1) (wheat data 1974 – 1978)

variety	mean	standard error of difference				
Huntsman	5.94	*				
Atou	6.04	0.123	*			
Armada	6.21	0.128	0.131	*		
Mardler	6.45	0.135	0.146	0.142	*	
Sentry	6.40	0.156	0.162	0.159	0.168	*
Stuart	6.53	0.156	0.162	0.159	0.168	0.181
		Huntsman	Atou	Armada	Mardler	Sentry

6.5 Analysis of $V \times R/C \times Y$ table

6.5.1 Within-year analysis

Consider a subset of wheat data (1977) only for Huntsman, Atou, Armada and Mardler (Section 1.1 and Example 4.2.4.2). For the moment, we exclude Sentry and Stuart and trials grown at centre 10 from the analysis. The $V \times C$ table is then complete and the overall variety performance is estimated by unadjusted means (Table 6.5.1).

The centres were selected within the three region of Scotland; East (1,2,8,9), North (3,4,5,6,10) and West (7). A complete analysis of the data should take into account the regions classification. Moreover, a varieties $\times$ region table of means may be required for purposes of identifying varieties well-adapted to specific regions. If it is known that regions contribute disproportionately to overall yield appropriate weights may be applied to each region. These weights should not depend on trials results.

The model $V + R + V.R$:

The data structure formula is $V * R/C$. Since sampling is complete with respect to regions, varieties $\times$ regions interaction contributes to expectation and not to error. One model to use is

$$V + R + V.R : \tag{6.2}$$

The within-regions varieties $\times$ centres interaction is part of error and is not specified in the model. If the regions had been sampled by a fraction of n/N, then varieties $\times$ regions interaction would contribute to error but with a correction factor $(1 - n/N)$ applied to the component of variance. For the present, there are no facilities in the REML algorithm to analyse data from samples of finite populations.

Table 6.5.1:

A subset of wheat data, 1977

	centre									
	East				North				West	
variety	1	2	8	9	3	4	5	6	7	mean
Huntsman	5.79	6.12	7.33	6.37	5.12	4.50	5.49	5.86	6.55	5.90
Atou	5.96	6.64	7.31	6.99	4.65	5.07	5.59	6.53	6.91	6.18
Armada	5.97	6.92	7.75	7.19	5.04	4.99	5.59	6.57	7.60	6.40
Mardler	6.56	7.55	8.93	8.33	5.13	4.60	5.83	6.14	7.91	6.78

Table 6.5.2:

Varieties × regions means from model (6.2) using Genstat REML

	region			
variety	1	2	3	mean
Huntsman	6.402	5.242	6.550	6.06
Atou	6.725	5.457	6.910	6.36
Armada	6.957	5.548	7.590	6.70
Mardler	7.843	5.425	7.910	7.06

The varieties × regions table of means and overall variety means from Genstat REML are given in Table 6.5.2. The overall variety means have been adjusted. Classifying of centres into regions should not make any adjustment to variety means when the data are complete. Examination of Table 6.5.2 shows that the model gives correct $V \times R$ cell means but estimates overall means as simple averages ignoring the distribution of centres over regions. For example the mean of Huntsman is computed as $(6.40 + 5.24 + 6.55)/3 = 6.06$ t/ha instead of $(4 \times (6.40 + 5.24) + 6.55)/9 = 5.90$ t/ha. We conclude that model (6.2) gives appropriate $V \times R$ table but is not suitable for overall prediction.

An alternative model for prediction

An alternative model for analysing a $V \times R/C$ table is to start with a $V \times C$ table and introduce regions as groups of centres. The regions enter the analysis as a varieties $\times$ groups of centres interaction. This leads to the model

$$V + C + V.R : \tag{6.3}$$

Model (6.3) gives predictions that take into account the disproportionate number of trials within each region. We prefer model (6.3) to model (6.2) because the model gives predictions suitable for combining trials over years. However, Genstat REML does not have adequate facilities for prediction when the systematic part of the model includes nested terms. We describe how to perform supplementary computations.

A V $\times$ R table of means

A set of estimates of effects from model (6.3) for the data in Table 6.5.1 is:

$$\begin{aligned} \text{general mean} &= 8.08 \\ \text{variety effects} &= (-1.360, -1.000, -0.320, 0.000) \\ \text{centre effects} &= (-1.150, -0.412, -3.089, -3.284 \\ &\quad -2.449, -1.801, -0.171, 0.610, 0.000) \end{aligned}$$

and varieties $\times$ region effects are:

	region		
variety	1	2	3
Huntsman	-0.080	1.178	0.000
Atou	-0.118	1.033	0.000
Armada	-0.565	0.443	0.000
Mardler	0.000	0.000	0.000

From model (6.3) a typical element of a $V \times R$ table is estimated[3] as

$$\hat{\mu} + \hat{\alpha}_i + \bar{\hat{\beta}}_{(k)} + \hat{\gamma}_{ik}$$

where $\bar{\hat{\beta}}_{(k)}$ is the mean of centre effects for only those centres in region k. For example, means[4] for Huntsman are calculated as

$$\begin{pmatrix} 8.08 \\ 8.08 \\ 8.08 \end{pmatrix} + \begin{pmatrix} -1.36 \\ -1.36 \\ -1.36 \end{pmatrix} + \begin{pmatrix} -0.08 \\ 1.18 \\ 0.00 \end{pmatrix} + \begin{pmatrix} 0.24 \\ -2.66 \\ -0.17 \end{pmatrix} = \begin{pmatrix} 6.40 \\ 5.24 \\ 6.55 \end{pmatrix}$$

It is satisfying that these means are the same as those in Table 6.5.2.

To estimate overall variety means we need the $V \times C$ table of means. Appropriate margins from this table give the $V \times R$ table of means and overall variety means. A typical cell mean in the $V \times C$ table is estimated as

$$\hat{\mu} + \hat{\alpha}_i + \bar{\hat{\beta}}_j + \bar{\hat{\gamma}}_{i.} \tag{6.4}$$

From (6.4) we observe that for a cell mean in a $V \times C$ table to be estimable, the variety must be tested in at least one trial in each region, i.e $\hat{\gamma}_{ik}$ must be estimable for all k. It is easy to verify that the margins of the $V \times C$ table of means give adequate means[5] for varieties and varieties $\times$ regions table (Table

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[3]Genstat REML estimates a typical element of a $V \times R$ table as

$$\hat{\mu} + \hat{\alpha}_i + \bar{\hat{\beta}} + \hat{\gamma}_{ik}$$

where $\hat{\mu}$ is an estimate of the general mean, $\hat{\alpha}_i$ the effect of variety i, $\bar{\hat{\beta}}$ is the mean of centre effects and $\hat{\gamma}_{ik}$ is the varieties $i \times$ region k effect.

[4]Genstat REML calculates Huntsman means as

$$\begin{pmatrix} 8.08 \\ 8.08 \\ 8.08 \end{pmatrix} + \begin{pmatrix} -1.36 \\ -1.36 \\ -1.36 \end{pmatrix} + \begin{pmatrix} -1.31 \\ -1.31 \\ -1.31 \end{pmatrix} + \begin{pmatrix} 0.24 \\ -2.66 \\ -0.17 \end{pmatrix} = \begin{pmatrix} 5.33 \\ 6.59 \\ 5.41 \end{pmatrix}$$

[5]Genstat REML means are again not adequate. Moreover, if we include yields at centre 10, Genstat REML gives no variety means and yet all means are estimable.

Table 6.5.3:

Varieties × centres means from model (6.3) using (6.4)

	centre									
	East				North				West	
variety	1	2	8	9	3	4	5	6	7	mean
Huntsman	5.49	6.23	7.25	6.64	4.81	4.61	5.45	6.10	6.55	5.90
Atou	5.81	6.55	7.57	6.96	5.02	4.83	5.66	6.31	6.91	6.18
Armada	6.05	6.78	7.81	7.20	5.11	4.92	5.75	6.40	7.59	6.40
Mardler	6.93	7.67	8.69	8.08	4.99	4.80	5.63	6.28	7.91	6.78

6.5.3). The process of estimating variety means from the output of model (6.3) is more conveniently expressed in matrix notation. This enables estimation of standard errors.

6.5.2 Estimation of variety means — a matrix approach

Let $\hat{\varphi}$ be an n-vector of all estimates of effects in model (6.3). Partition $\hat{\varphi}$ to correspond to the estimates of the general mean, variety effects, year effects and varieties × regions effects i.e,

$$\hat{\varphi} = (\hat{\mu}, \hat{\alpha}', \hat{\beta}', \hat{\gamma}')'$$

We define a $v \times n$ matrix K of constants corresponding to $\hat{\varphi}$ such that $K\hat{\varphi}$ is a vector of variety means equivalent to the marginal means for varieties in Table 6.5.3. Let r the number of regions and n_l the number of centres in region l. We partition the matrix K correspondingly to the vector $\hat{\varphi}$, i.e,

$$K = (J_v | K_v | K_c | K_\gamma)$$

where J_v is a v-vector with every element equal to 1 and

$$K_v = I_v$$

$$K_\gamma = r^{-1}(J'_r \otimes I_v)$$
$$K_c = (k_{ij})$$

where

$$k_{jl} = \begin{cases} n_l^{-1} & \text{if centre } j \text{ is in region } l \\ 0 & \text{otherwise} \end{cases}$$

In our example the submatrices K_c and K_γ are

$$K_c = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1 \\ 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1 \\ & & & & \vdots & & & & \\ 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1 \end{pmatrix}$$

and

$$K_\gamma = \begin{pmatrix} 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \end{pmatrix}$$

If weights w_1, w_2, w_3 such that $\sum w_m = 1$ are given to the three regions, then weighted means are obtained by modifying K_γ as follows:

$$K_\gamma = \begin{pmatrix} w_1 & w_2 & w_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_1 & w_2 & w_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & w_1 & w_2 & w_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_1 & w_2 & w_3 \end{pmatrix}$$

Similar matrices can be defined for estimating a $V \times C$ and a $V \times R$ table of means. Given the variance matrix of $\hat{\varphi}$, say $V_{\hat{\varphi}}$, the variance matrix of variety means is given by $KV_{\hat{\varphi}}K'$. This method can be applied to any model of the form:

$$A + B + GA.B : \tag{6.5}$$

where GA is factor for groups of A.

Algorithm 6.5.1: Means from $A + B + GA.B$:

(a) Fit the model $A + B + GA.B$: using REML. Save estimates of effects in $ALLEFF$ and their variance matrix in $VCOV$.

(b) Set up coefficient matrix K for varieties (factor B).

(c) Estimate variety means as the product of K and $ALLEFF$.

(d) Estimate variance matrix of variety means and obtain standard errors of difference.

(e) Set up another coefficient matrix K for centres (factor A).

(f) Estimate centre means as the product of K and $ALLEFF$.

(g) Estimate variance matrix of centre means and obtain standard errors of difference.

Although model (6.5) is a fixed effects model, we use the REML algorithm because a full specification would include centre terms which are treated as random. A Genstat code for Algorithm 6.5.1 is given in Appendix C.

6.5.3 Example 6.5.3: wheat data, 1977

Variety means and standard erors of differences for winter wheat 1977 trials using Algorithm 6.5.1 are displayed in Table 6.5.4; see Appendix C for output from Algorithm 6.5.1 on this data. Although variety means are not much different from those given by methods of Chapter 4, standard errors of variety comparisons more than double when varieties $\times$ regions interaction is fitted in the model. This analysis assumes an equal within-regions variance.

Table 6.5.4:

Variety means and standard error of difference by Algorithm 6.5.1

(wheat data, 1977)

variety	mean	standard error of difference				
Huntsman	5.73	*				
Atou	6.03	0.489	*			
Armada	6.22	0.495	0.583	*		
Mardler	6.55	0.268	0.520	0.399	*	
Sentry	6.20	0.492	0.481	0.510	0.471	*
Stuart	6.58	0.271	0.410	0.338	0.237	0.396
	Huntsman	Atou	Armada	Mardler	Sentry	

6.5.4 Over-years analysis

The distribution of trials in each region over years is shown in Table 6.5.5 (Appendix A.1). A full analysis is based on the model

$$V + R + V.R : Y + R.Y + V.Y$$

Although this is a small data set, it is too bulky for a full analysis using Genstat REML and because of the comments made in Section 6.5.1, we use a two-stage analysis.

Inspection of Table 6.5.5 shows that variety means are not estimable for 1974 and 1975 if model (6.3) is used for analysis. We therefore exclude Armada from the 1974 analysis and Atou from the 1975 analysis. Within-years means from model (6.3) are displayed in Table 6.5.6. Over-years variety means are obtained from a REML analysis of that table. Variety means from the model $V : Y$ and standard errors of difference are displayed in Table 6.5.7.

Table 6.5.5:

The distribution of trials for each region (wheat data, 1978 – 1979)

year	region	Huntsman	Atou	Armada	Mardler	Sentry	Stuart
	E	7	7	3	0	0	0
1974	N	5	5	0	0	0	0
	W	4	4	1	0	0	0
	E	7	0	2	2	0	0
1975	N	6	0	1	1	0	0
	W	3	1	1	1	0	0
	E	7	3	7	2	2	2
1976	N	6	1	6	1	1	1
	W	1	1	1	1	1	1
	E	4	4	4	4	2	2
1977	N	5	5	4	4	1	1
	W	1	1	1	1	1	1
	E	5	5	5	5	5	4
1978	N	5	5	4	4	1	1
	W	1	1	1	1	1	1

Table 6.5.6:

Within-years variety means (t/ha) (wheat data 1974 – 1978)

			year		
variety	1974	1975	1976	1977	1978
Huntsman	6.30	6.46	6.17	5.73	4.98
Atou	6.02	6.49	5.93	6.03	5.69
Armada	*	6.55	6.20	6.22	5.84
Mardler	*	*	6.63	6.55	5.76
Sentry	*	*	6.34	6.20	5.83
Stuart	*	*	6.61	6.58	5.99

Table 6.5.7:

Variety means and standard error of differences (t/ha) from Algorithm 6.5.1 (wheat data 1974–1978)

variety	mean	standard error of difference				
Huntsman	5.93	*				
Atou	6.03	0.121	*			
Armada	6.24	0.131	0.131	*		
Mardler	6.50	0.146	0.146	0.149	*	
Sentry	6.31	0.146	0.146	0.149	0.156	*
Stuart	6.58	0.146	0.146	0.149	0.156	0.156
	Huntsman	Atou	Armada	Mardler	Sentry	

6.6 Summary

The model to use for combining trials over centres and years is determined by the objectives of the analysis and the structure of the data. For recommended list trials, we use the model in which only variety effects contribute to expectation and all other effects and interactions are random. A good procedure for the combined analysis is always to start with the full model and if any component of variance is negative, the random term with a negative component should be excluded from the model.

Simple methods provide estimates of variety means not very different from those from a full analysis. In general, however, simple methods underestimate standard errors of variety comparisons.

If within-years tables are complete a two-stage analysis using REML gives the same means as a full analysis in which only variety effects are treated as fixed. In a $V \times C \times Y$ table, standard errors will be underestimated, but in a $V \times Y/C$ a two-stage REML gives the same standard errors as a full analysis.

A two-stage analysis can be very inefficient if entries in a $V \times Y$ table have a wide range of inaccuracies. REML can be used to provide a weighted analysis so that trials with more accurate information are given more weight. This method however ignores unequal covariances.

When centres are classified by regions and all regions have been sampled, varieties $\times$ regions interaction does not contribute to error. Predictions given by Genstat REML are not appropriate. We described an algorithm to perform supplemetary calculations.

In all these models we have regarded varieties $\times$ environments interactions as homogeneous. In practice several factors may lead to a serious departure from homogeneity. This causes further complications in the analysis which are described in Chapter 7.

Chapter 7

HETEROGENEOUS INTERACTIONS

7.1 Introduction

Complications arise in the combined analysis if the varieties $\times$ centres or the varieties $\times$ years variances, or both, are heterogeneous. Heterogeneous varieties $\times$ centres variance may result from a single variety or groups of varieties responding differently to centre differences. We extend methods of Chapters 4 and 5 to a combined analysis in Section 7.2. Similar methods can be applied to heterogeneous varieties $\times$ years interaction, providing there are no systematic year effects. In dealing with heterogeneous varieties $\times$ years interaction it is more convenient to analyse a $V \times Y$ table of means, i.e the second stage of a two-stage analysis.

If there are systematic effects the objective of analysis is no longer to predict variety performance for a range of conditions but to describe variety performance under the actual conditions in the sample. Anscombe (1981, page 278) re-analysed Immer *et al.*'s data and declared his objective to be modelling varieties $\times$ environments interaction. He used a simple multiplicative model and claimed that it was 'just as inteligible as an additive model'.

Systematic year effects may be caused by changes in the environment, for example drought. If the dry years provide significantly different conditions, we estimate variety means that take into account a fixed varieties $\times$ (dry versus wet years) interaction. Changes in years may affect some varieties more than others.

Groups of varieties can sometimes be identified with a systematic groups $\times$ years interaction. In either case, the analysis is obtained using Algorithm 6.5.1 with the factor G as either groups of varieties or groups of years (Section 6.5.2).

Patterson & Silvey (1980) observed:

> Varieties $\times$ years tables are more affected than varieties $\times$ centres within years tables by departures from the random environments model. Differential trends over the years are particularly troublesome. A change in environment can have permanent effects on variety performance. For example, a variety may lose its resistance to disease through the appearance of new races of pathogens. Future yields of a variety affected in this way will be smaller than predicted by the analysis.

We give two examples of series of trials with systematic year effects. In the first example, for winter wheat, yields of the control Huntsman, declined over time. Since the yields of candidate varieties are compared with the controls, they are more likely to be recommended in later years. We show how to correct for this trend in Section 7.3. The second example, for sugar beet, is more complex. Yields of recommended varieties in trials declined. The systematic effects are related to age of varieties and we investigate change in types of seed as a possible explanation in Section 7.4.

Table 7.2.1:

Variety sensitivities for barley trials (Yates & Cochran 1938)

variety	mean	sensitivity	
		1931	1932
		±0.194[1]	±.263
Manchuria	31.46	0.889	0.755
Svansota	30.38	0.935	1.153
Velvet	33.06	1.139	0.693
Trebi	39.40	1.280	1.606
Peatland	34.18	0.757	0.793

[1]average standard error

7.2 Heterogeneous varieties × centres variance

7.2.1 Groups of varieties: Yates & Cochran's (1938) example

In Yates & Cochran's (1938) example (Section 6.2.2), they observed that the variety Trebi had more than average sensitivity to centre differences averaged over two years. Table 7.2.1 shows the sensitivities estimated for individual years. Correspondingly, variation in differences between Trebi and the other four varieties accounted for a large part of the varieties × centres interaction; the varieties × centres interaction is therefore heterogeneous. Yates & Cochran (1938) then partioned the degrees of freedom into Trebi versus the rest and deviation.

Analysis of variance

The Genstat ANOVA algorithm can be used to give a complete partition of degrees of freedom (Appendix *D*.1). The ANOVA table is given in Table 7.2.2. The

Table 7.2.2:

ANOVA table for Yates & Cochran's (1938) example

(Trebi versus the rest)

source	df	MS
years	1	422.057
centres	5	471.576
centres × years	5	153.198
varieties	4	147.499
Trebi vs rest	1	487.920
deviations	3	34.026
varieties × years	4	8.106
Trebi vs rest	1	2.576
deviations	3	9.949
varieties × centres	20	24.628
Trebi vs rest	5	62.539
deviations	15	11.991
varieties × centres × years	20	15.468
Trebi vs rest	5	10.808
deviations	15	17.021

varieties × years interaction and the varieties × centres interaction between other varieties are not significant. Thus we pool their degrees of freedom with that of varieties × centres × years interaction to give an estimate of the units variance, i.e $(4 \times 8.106 + 15 \times 11.991 + 20 \times 15.468)/39 = 13.38$.

The standard errors for future performance of varieties in the growing region sampled by the trials depend on whether or not the comparison involves Trebi. The varieties × years variance is zero and the varieties × centres variance is zero for comparisons not involving Trebi. The standard error of difference for these comparisons is $\sqrt{(13.38/6)} = 1.493$.

The standard error for the difference between Trebi and any other variety includes a large varieties × centres variance. An estimate of the varieties × centres component of variance is obtained by equating the mean sum of squares for Trebi versus the rest to its expectation, i.e $\sigma^2 + (4/5)n_y\sigma^2_{VC}$ from which we get $\hat{\sigma}^2_{VC} = 5 \times (62.54 - 13.38)/8 = 30.73$. The standard error for the difference between Trebi and one other variety is $\sqrt{(13.38/6 + 30.73/6)} = 2.711$. Thus if varieties $\times$ centres heterogeneity is taken into account, the standard error for Trebi versus any other variety is increased by 36% whereas the standard error for a difference not involving Trebi is decreased by 36%.

REML analysis

The analysis can be done using Genstat REML with the advantage that standard errors are automatically calculated. We define a factor GV for groups of varieties with two levels, one for Trebi and the other level for other varieties. We fit the model

$$V : Y + C + C.Y + GV.C + V.C + V.Y \tag{7.1}$$

The varieties $\times$ centres and the varieties $\times$ years component of variance are negative and so we delete the corresponding terms from the model and fit the models:

$$V : Y + C + C.Y + GV.C \tag{7.2}$$

$$V + Y + C : C.Y + GV.C \tag{7.3}$$

Components of variance are displayed in Table 7.2.3.

Standard errors of difference are displayed in Table 7.2.4. Model (7.2) gives the same standard errors as ANOVA. If centres and years effects are regarded as random the range of standard errors is reduced.

The method of Hemmerle & Downs (1978) can also be used to partition varieties $\times$ centres variance (Section 4.3.2). To do this we define a factor D similar to that of centres but with levels specified for only Trebi. We then fit the model

$$V : C + Y + C.Y + D + V.C + V.Y \tag{7.4}$$

Table 7.2.3:

Components variance for Yates & Cochran (1938) example

	model		
component	(7.1)	(7.2)	(7.3)
Y	9.221	8.96	
C	43.62	42.91	
$C \times Y$	27.55	27.93	27.96
$GV \times C$	14.11	13.57	15.36
$V \times C$	-1.56		
$V \times Y$	-1.23		
units	15.47	13.55	13.38

Table 7.2.4:

Average standard error of difference for Yates & Cochran (1938) example

	model	
	(7.2)	(7.3)
minimum	1.503	1.493
maximum	2.602	2.711
average	1.942	1.980

The residual varieties × centres and the varieties × years variances from model (7.4) are negative (Table 7.2.5) and so we use the models

$$V : C + Y + C.Y + D \tag{7.5}$$

$$V + Y + C : C.Y + D \tag{7.6}$$

Model (7.5) gives the same standard errors as model (7.3). Note that the component of variance for D is twice that of groups × centres in model (7.2). If year effects are treated as fixed (model 7.6) the results are the same as those given by ANOVA. A disadvantage of fitting supplementary components of variance is the decrease in the rate of convergence of the REML algorithm.

Varieties may be grouped by ploidy, stage of maturity or status i.e control or not control. The use of REML extends the analysis to incomplete tables.

Table 7.2.5:

Components variance using Hemmerle & Downs' (1978) method
(Yates & Cochran's (1938) example)

	model		
component	(7.4)	(7.5)	(7.6)
Y	9.221	8.96	
C	21.22	21.07	
$C \times Y$	27.55	27.99	27.96
D	34.11	33.47	30.73
$V \times C$	-1.85		
$V \times Y$	-1.23		
units	13.27	13.55	13.38

Often groups of varieties are not consistent from year to year. Different group of varieties may be responsible for heterogeneity in different years and in some years the varieties $\times$ centres variance may be homogeneous. In these circumstances a two-stage analysis should be done using methods of Chapters 4 and 5.

7.2.2 Multiplicative models

In section 7.2.1 we used sensitivities as a diagnostic to identify varieties responsible for heterogeneity. As in Chapter 5, multiplicative models in which sensitivities are variance parameters can be used to model heterogeneous varieties $\times$ centres variance. These models account for heterogeneity as differences in regressions of varieties against centre means.

The $V \times C \times Y$ table

A multiplicative model for a $V \times C \times Y$ table can be written as

$$y_{ijk} = \mu + \alpha_i + \theta_{ik}\beta_j + \phi_k + \delta_{ijk} \tag{7.7}$$

where μ is the general mean, α_i the effects for variety i, β_j the effect for centre j, ϕ_k the effect of year k, θ_{ik} the sensitivity for variety i in year k and δ_{ijk} a general error term which includes centres $\times$ years and varieties $\times$ environments interactions. Although the same sample of centres is used each year, environmental conditions at these centres differ from year to year. It is therefore possible for a variety to have different sensitivities to centre differences from year to year.

In equation (7.7) variety effects are regarded as fixed. Year effects may be regarded as fixed or random but centre effects are always random. In principle the varieties $\times$ centres variance is modelled as linear in centres variance. Treating centres as fixed would exclude that part of varieties $\times$ centres interaction which depends on centres variance (Section 5.3.2). A Fisher's scoring scheme can be employed to estimate parameters in (7.7) using REML, thus extending SREML algorithm to $V \times C \times Y$ tables (Section 5.4.2).

But in view of the results in Chapter 5 we can ignore correlations between sensitivities and use conditional approximations to fit model (7.7). Firstly, if variety sensitivities were known, then (7.7) is fitted by a basic REML model $\mathcal{A}$ or $\mathcal{D}$ according as years are regarded as random or fixed (Section 6.2.1). Let T be a variate defined as

$$T(h) = \begin{cases} \hat{\theta}_{ik} & \text{if } y(h) \text{ is the yield of variety } i \text{ at centre } j \text{ in year } k \\ 0 & \text{otherwise} \end{cases}$$

Given $\hat{\theta}$, we write (7.7) as

$$V : Y + C.T + C.Y + V.C + V.Y \tag{7.8}$$

Secondly, given estimates of centre effects variety sensitivities are estimated from the regression model

$$E[y_{ijk}] = \mu + \alpha_i + \theta_{ik}\beta_j \tag{7.9}$$

We write this model as

$$V + V.Y.M : \tag{7.10}$$

where M is a variate defined as

$$M(h) = \begin{cases} \hat{\beta}_j & \text{if } y(h) \text{ is the yield of variety } i \text{ at centre } j \\ 0 & \text{otherwise} \end{cases}$$

Then (7.7) can be fitted by conditional approximations using Algorithm 7.2.1 — an extension of Algorithm 5.3.1 to $V \times C \times Y$ tables.

Algorithm 7.2.1: REML for a $V \times C \times Y$ table with multiplicative varieties $\times$ centres interaction

(a) Estimate variety means and centre means from the model $V : Y + C + C.Y + V.C + V.Y$

(b) Estimate sensitivities from model (7.10).

(c) For each year scale sensitivities to unit mean.

(d) Use model (7.8) to estimate new variety means and centre means.

(e) Repeat steps (b) to (d) to convergence.

(f) Estimate variety means and standard errors from model (7.8).

Analysis of Yates & Cochran's (1938) example using Algorithm 7.2.1 leads to a negative component of variance for the residual variaties $\times$ centres variance providing further evidence that heterogeneity of variety $\times$ centres variance is largely due to the abnormal behaviour of Trebi.

Yates & Cochran (1938) identified Trebi by regressing variety yields against centre means i.e the mean of all variety yields at each centre. This can be generalised to the model

$$y_{ijk} = \mu + \alpha_i + \theta_i \beta_j + \phi_k + \delta_{ijk} \tag{7.11}$$

(See model (7.7)). Model (7.11) can be fitted by a slight modification of Algorithm 7.2.1. If sensitivities from model (7.11) show that varieties $\times$ centres variance is fairly homogeneous, i.e θ_i are all equal, there is little point in using model (7.7) for analysis.

The $V \times Y/C$ table

A multiplicative model for a $V \times Y/C$ table can be used to account for heterogeneity of within-years varieties $\times$ centres variance. Since new centres are chosen each year, sensitivities are defined in the centres within years stratum. This model can be written as

$$y_{ijk} = \mu + \alpha_i + \theta_{ik}\beta_{j(k)} + \phi_k + \delta_{ijk} \tag{7.12}$$

where μ is the general mean, α_i the effects for variety i, $\beta_{j(k)}$ the effect for centre j in year k, ϕ_k the effect of year k, θ_{ik} the sensitivity for variety i in year k and δ_{ijk} a general error term which includes varieties $\times$ years interaction.

Model (7.12) can be fitted by conditional approximations using Algorithm 7.2.2.

Algorithm 7.2.2: REML for a $V \times Y/C$ table with multiplicative within-years varieties $\times$ centres interaction

(a) Estimate variety means and centre means from the model $V : Y + C.Y + V.Y$

(b) Estimate sensitivities from model $V + Y + V.Y.M$: where M is a variate defined in (7.10).

(c) For each year scale sensitivities to unit mean.

(d) Estimate new variety means and centre means from the model $V : Y + C.Y.T + V.Y$ where T is a variate defined in (7.8).

(e) Repeat steps (b) to (d) to convergence.

(f) Estimate variety means and standard errors from the model $V : Y + C.Y.T + V.Y$

Table 7.2.6:

Components variance for wheat trials from Algorithm 7.2.2 and basic model

V+Y:C.Y+V.Y

component	Algorithm 7.2.2	basic model
C × Y	1.154	1.146
V × Y	0.0147	0.0235
units	0.1024	0.1375

Example 7.2.2: wheat data

We analyse the wheat data for 1976 – 1978 data (ignoring regions) using Algorithm 7.2.2 (Appendix *D*.2). We treat year effects as fixed because of a negative years component of variance. Tables 7.2.6 and 7.2.7 show components of variance and sensitivities respectively.

Variety means from Algorithm 7.2.2 and standard errors of difference are displayed in Table 7.2.8. Variety means from the model $V + Y : C.Y + V.Y$ and standard errors of difference are displayed in Table 7.2.9. Although variety means are similar, standard errors of difference are decreased by about 15% on the average compared to a basic analysis (Table 7.2.10). This may seem surprising, but can be explained by the fact that no variety had more than average sensitivity consistently over the three years.

A general multiplicative model

Patterson & Silvey (1980) referred to multiplicative models (7.7), (7.11) and (7.12) as extended model B. Multiplicative models can be extended to model differences in varieties to year means, but we make years fixed so that the varieties × years interaction which depends on sensitivities does not contribute to error.

A general multiplicative model can be used to allow for different sensitivities in the years and centres strata. For a $V \times C \times Y$ table, such a model can be written

Table 7.2.7:

Variety sensitivities from Algorithm 7.2.2

(wheat data 1974–1978)

variety	1976 sensitivity (s.e)	1977 sensitivity (s.e)	1978 sensitivity (s.e)
		(first iteration)	
Huntsman	1.100 (0.089)	0.743 (0.093)	1.097 (0.095)
Atou	0.826 (0.141)	0.775 (0.093)	0.827 (0.095)
Armada	1.046 (0.089)	0.912 (0.108)	0.869 (0.097)
Mardler	1.102 (0.190)	1.326 (0.108)	1.046 (0.097)
Stuart	1.027 (0.190)	1.204 (0.112)	1.087 (0.097)
Sentry	0.899 (0.190)	1.040 (0.112)	1.074 (0.099)
		(final iteration)	
Huntsman	1.184 (0.090)	0.741 (0.086)	1.173 (0.089)
Atou	0.820 (0.131)	0.769 (0.086)	0.793 (0.090)
Armada	1.049 (0.090)	0.925 (0.099)	0.821 (0.092)
Mardler	1.094 (0.178)	1.340 (0.099)	1.075 (0.092)
Stuart	0.987 (0.178)	1.190 (0.102)	1.068 (0.092)
Sentry	0.866 (0.178)	1.034 (0.102)	1.070 (0.093)

Table 7.2.8:

Variety means and standard error of differences (t/ha) from Algorithm 7.2.2

(wheat data 1974–1978)

variety	mean	standard error of difference				
Huntsman	5.65	*				
Atou	5.94	0.138	*			
Armada	6.06	0.130	0.133	*		
Mardler	6.27	0.145	0.151	0.143	*	
Sentry	6.22	0.145	0.151	0.145	0.147	*
Stuart	6.39	0.144	0.147	0.144	0.149	0.148
		Huntsman	Atou	Armada	Mardler	Sentry

Table 7.2.9:

Variety means and standard error of differences (t/ha) from model $\mathcal{I}$ (Table 6.5.9) (wheat data 1976–1978)

variety	mean	standard error of difference				
Huntsman	5.63	*				
Atou	5.92	0.160	*			
Armada	6.07	0.154	0.161	*		
Mardler	6.31	0.163	0.166	0.164	*	
Sentry	6.24	0.170	0.173	0.171	0.175	*
Stuart	6.38	0.170	0.173	0.172	0.175	0.179
		Huntsman	Atou	Armada	Mardler	Sentry

Table 7.2.10:

Average standard error of difference for wheat trials 1976–1978 from Algorithm 7.2.2 and basic model V+Y:C.Y+V.Y

	Algorithm 7.2.2	basic model
minimum	0.130	0.154
maximum	0.151	0.179
average	0.144	0.168

as

$$y_{ijk} = \mu + \alpha_i + \theta_{ik}\beta_j + \psi_i\phi_k + \delta_{ijk} \tag{7.13}$$

(see model (7.7)) where ψ_i is the sensitivity to year differences for variety i. In fitting this model centres are treated as random but years are fixed. The analysis is valid if the sensitivities are genuine and subsamples of centres are effectively random samples.

If varieties $\times$ centres variance is heterogeneous only for some of the years, then a two-stage analysis should be done using methods described in Chapters 4 and 5. Complications due to years may then be handled using Modified FITCON or Algorithm 6.5.1 (Sections 5.2 and 6.5.2).

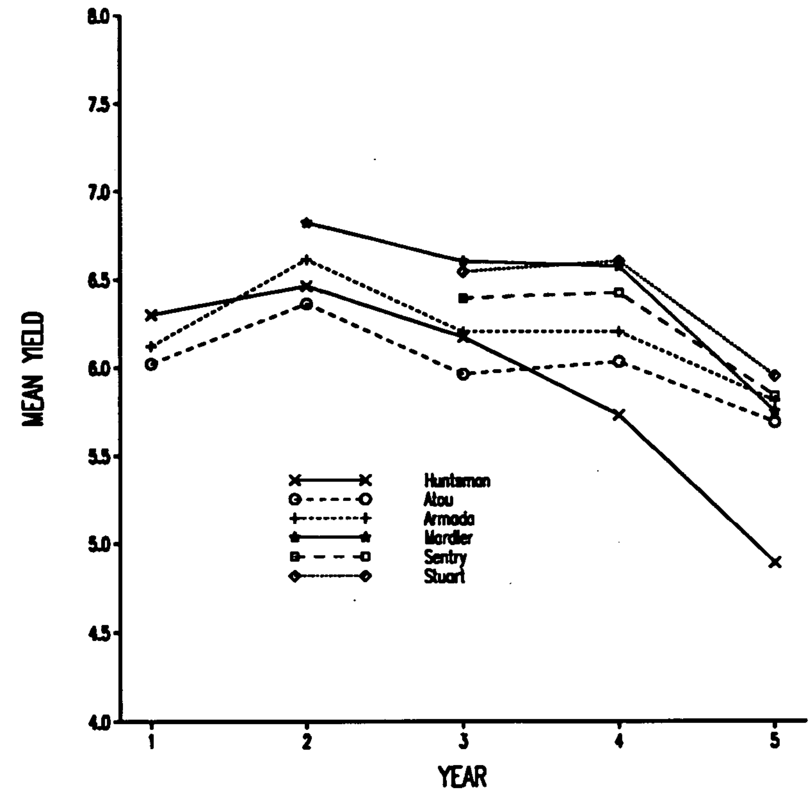

Figure 7.3.1:

Within-years mean yields of wheat data 1974 – 1978

7.3 Example of heterogeneity due to trend in the control variety

Figure 7.3.1 shows a decline in the performance of all varieties in the wheat data, but Huntsman declined much more than the rest. REML can be used to provide adjustment for the differential trend in Huntsman yields (Table 6.4.5). We define

Table 7.3.1:

Dummy covariate for trend adjustment for Huntsman (wheat trials)

variety	1974	1975	year 1976	1977	1978
Huntsman	-2	-1	0	1	2
Atou	*	*	*	*	*
Armada	*	*	*	*	*
Mardler	*	*	*	*	*
Sentry	*	*	*	*	*
Stuart	*	*	*	*	*

a time variate centred at 1976 and taking on unit values of time in either direction (Table 7.3.1). The covariate Z has values defined only for Huntsman.

We fit[1] the model

$$V + Y + Z:$$

Variety means and standard error of differences are displayed in Table 7.3.2. The mean of Huntsman is not adjusted because the variety had complete results. Standard errors of variety comparisons are reduced by almost 50% compared to a basic analysis (Table 7.3.3). It is impossible to separate how much of this decline is due to permanent effects in the latter years on Huntsman and that due to general changes in the environment (Patterson & Silvey 1980).

[1]Genstat REML specifications for fitting this model are similar to methods described in Chapter 4, Section 4.3.

Table 7.3.2:

Variety means and standard errors of difference with trend adjustment for Huntsman (wheat data)

variety	mean	standard error of difference				
Huntsman	5.93	*				
Atou	6.01	0.068	*			
Armada	6.19	0.068	0.068	*		
Mardler	6.43	0.074	0.074	0.074	*	
Sentry	6.33	0.082	0.082	0.082	0.084	*
Stuart	6.47	0.082	0.082	0.082	0.084	0.088
		Huntsman	Atou	Armada	Mardler	Sentry

Table 7.3.3:

Variety means and standard errors of difference from the model $V + Y$: (wheat data)

variety	mean	standard error of difference				
Huntsman	5.93	*				
Atou	6.01	0.117	*			
Armada	6.19	0.117	0.117	*		
Mardler	6.46	0.126	0.126	0.126	*	
Sentry	6.39	0.139	0.139	0.139	0.143	*
Stuart	6.54	0.139	0.139	0.139	0.143	151
		Huntsman	Atou	Armada	Mardler	Sentry

Table 7.4.1:

Seed classes in sugar beet data 1987–1991

(b — breeder seed; p — pre-commercial seed; c — commercial seed)

	number of	year				
age in year 5	varieties	1	2	3	4	5
≥ 9	3	c	c	c	c	c
8	2	p	c	c	c	c
7	3	b	p	c	c	c
6	8	b	b	p	c	c
5	1	b	b	b	p	c
4	2		b	b	b	p
3	10			b	b	b

7.4 Seed effects in sugar beet trials

For many years the sugar beet trial system worked well. Recommended varieties performed as well on farms as in the trials. Recently, however, breeders and trial officers have noted that the performance of some recommended varieties has proved disappointing. Several hypothesis were put forward which might explain this behaviour including a run of unfavourable seasons. In this section we suggest another possible explanation: a relationship between the changing yields and seed classes.

In the first three years that a variety in trial, only small quantities of seed produced by breeders are used. In the fifth and later years, seed is produced commercially in greater bulk. The pre-commercial seed used in the fourth year is intermediate. We refer to types of seed as *seed classes* and the breeder or consortium of breeders that produce seed as the *seed house* (Tables 7.4.1 and 7.4.2).

Since commercial seed is the one used by farmers, means predicted from com-

Table 7.4.2:

Variety codes, age in year 5 and seed-house code

(sugar beet data, 1987–1991)

variety	age	seed house	variety	age	seed house
1	9	1	16	6	7
2	9	2	17	5	2
3	9	2	18	4	8
4	8	3	19	4	2
5	8	1	20	3	2
6	7	4	21	3	4
7	7	5	22	3	3
8	7	2	23	3	9
9	6	6	24	3	9
10	6	5	25	3	4
11	6	1	26	3	1
12	6	4	27	3	8
13	6	3	28	3	2
14	6	2	29	3	2
15	6	7			

ercial seed are appropriate for decision making. Methods are sought that take account of seed class effects and predict yields from commercial seed. The following quote from *NIAB Farmers leaflet No 5* emphasises the importance of seed quality control in sugar beet trials:

> Seed production in sugar beet is more complex than for most crops and the breeder has to maintain a close involvement in selection and multiplication. The biennial nature of the plant complicates the work and a breeder may have variable success with those processes under his control, while the effects of the environment on the seed crop can also influence the quality and performance of particular seed lots.

The complications of analysing incomplete $V \times Y$ tables for sugar beet trials have led Silvey (1978b) to suggest growing all recommended varieties each year. Silvey noted:

> In the cross-fertilised root and fodder crops like sugar beet, the possibility of a variety showing shifts in some characters over a period of years makes it desirable to grow all the RL varieties in trial each year to monitor their performance.

Kempton (1980) observed that heterogeneity of varieties $\times$ years interaction in sugar beet trials could arise when a variety is 'gradually improved each year by reselection within its own gene pool'.

Our analysis in Section 7.4.2 found large differences in variety means which could be attributed to seed classes. Further examination revealed that the effect of seed classes varied from variety to variety. This type of heterogeneity of varieties $\times$ years interaction, raises complications in analysis not considered by Yates & Cochran (1938).

Variety sensitivities for each year are displayed in Table 7.4.3. This provided no evidence of unusually low or high sensitivity in any variety, except for variety 19 which had a sensitivity of 1.19 in the fourth year and variety 26 with a sensitivity of 0.80 in the fifth year. However adjustment for these sensitivities would not have much effect on the analysis. The $V \times Y$ table of means from a two-stage analysis is therefore a good starting point (Table 6.3.7); see Section 6.3 for a basic analysis of sugar beet trials.

7.4.1 Adjustment for seed effects

To include the effects of seed classes in the model, we introduce a new factor S with levels b, p, c corresponding to breeder, pre-commercial and commercial seed (Table 7.4.1). Since there are only three possible classes of seed we regard seed effects as fixed and because years variation is large we regard year effects as fixed.

Table 7.4.3:

Variety sensitivities (sugar beet data, 1987–1991)

variety	year 1 (±0.082)[1]	2 (±0.074)	3 (±0.057)	4 (±0.052)	5 (±0.095)
1	0.902	1.106	1.106	0.925	1.093
2	1.094	0.978	1.025	0.963	1.002
3	1.047	0.850	1.117	1.016	1.013
4	1.009	0.888	0.970	1.082	0.983
5	0.888	0.954	1.015	0.977	1.035
6	1.035	0.987	1.072	0.947	1.060
7	1.007	0.990	1.075	1.074	0.976
8	1.014	0.969	1.034	0.927	0.878
9	1.056	1.166	1.162	1.051	0.889
10	0.878	1.004	1.052	1.101	1.033
11	1.005	1.086	0.974	1.003	1.000
12	0.962	1.005	0.916	0.915	0.955
13	0.872	1.008	0.860	0.930	0.930
14	0.993	0.936	0.916	0.950	0.878
15	0.994	0.934	0.957	0.901	1.056
16	1.144	1.183	1.068	0.963	1.089
17	1.098	1.033	1.085	1.087	1.014
18	*	0.881	0.904	0.934	1.134
19	*	1.040	0.991	1.190	1.178
20	*	*	0.921	0.982	0.912
21	*	*	0.956	1.089	0.986
22	*	*	1.092	0.993	1.144
23	*	*	1.104	0.988	1.066
24	*	*	0.856	1.017	1.020
25	*	*	0.912	1.017	1.049
26	*	*	1.055	0.933	0.800
27	*	*	1.003	1.053	0.996
28	*	*	0.911	1.039	0.892
29	*	*	0.892	0.954	0.940

[1] average standard error

We fit the model

$$V + Y + S : \tag{7.14}$$

Variety means predicted for breeder, pre-commercial and commercial seed[2] for selected varieties are displayed in Table 7.4.4. Model (7.14) makes a uniform variety adjustment of 1.49 t/ha for change from breeder to pre-commercial seed; and a uniform adjustment of 2.74 t/ha from breeder to commercial seed. The average standard error of difference for the change from breeder to commercial seed is 0.673 t/ha and that for the change to pre-commercial seed is 0.542 t/ha. Differences in variety means attributed to change in seed are large. The average variety means over seed classes are of little practical value.

Modelling varieties × seed classes interaction

Equal adjustments to each variety are not justified if there is a significant varieties × seed classes interaction. We therefore consider the model

$$V + Y + S + V.S : \tag{7.15}$$

Model (7.15) is a better model as it takes into account a significant varieties × interaction (Table 7.4.5). This rules out the use of model (7.14) for analysis.

Partitioning the seed class degrees of freedom (Table 7.4.6) shows that there is no significant difference between pre-commercial and commercial means so we combine pre-commercial and commercial seed into one class. We define a new factor S^* with two levels, b for the first three years of a variety in trials and c from the fourth year onwards and use the model

$$V + Y + S^* + V.S^* : \tag{7.16}$$

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[2]We refer to these means as *breeder means, pre-commercial means* and *commercial means* respectively.

Table 7.4.4:

Breeder, pre-commercial and commercial means for selected varieties from the model $V + Y + S$:

(sugar beet data, 1987–1991)

	seed class		
variety	breeder	pre-commercial	commercial
6	58.90	57.41	56.15
7	57.87	56.38	55.13
8	58.31	56.82	55.57
9	56.22	54.73	53.48
10	55.58	54.09	52.84
11	57.24	55.75	54.50
12	56.80	55.31	54.06
13	56.44	54.96	53.70
14	56.95	55.46	54.21
15	57.81	56.32	55.07
16	56.81	55.32	54.07
17	58.33	56.85	55.59

Table 7.4.5:

Mean squares for models fitting seed effects

(sugar beet data, 1987–1991)

		model
effects	df	$V + Y + S + V.S$:
V	28	5.035
Y	4	128.759
S	2	16.116
$V \times S$	26	3.002
residual	88	1.445

Table 7.4.6:

Partitioning of seed class degrees of freedom

(sugar beet data, 1987–1991)

	df	MS	F-ratio	probability
Seed classes (S)				
b vs. pooled p,c (S^*)	1	20.786	14.12	< 0.001
p vs. c	1	0.853	0.53	0.473
Varieties × Seed ($V \times S$)				
$V \times S^*$	13	5.403	3.67	< 0.001
$V\times$ (p vs. c)	13	1.133	0.70	0.745

The $V \times S^*$ table of means from model (7.16) is incomplete. Only 14 varieties had completed exactly 4, 5, 6 or 7 trial years at the end of the series. The five oldest varieties were not grown from breeder seed in any year of the series and the ten youngest were not grown from commercial seed. Consequently only varieties 6 – 19 have means predicted for both classes (Table 7.4.7).

Figure 7.4.1 shows breeder and commercial means for these varieties. Seed effects are large and variable. Most varieties show a loss of about 10% in changing from breeder to commercial seed. Figure 7.4.1 also shows the lack of any apparent relationship between the two figures. The rank orders are entirely different and the correlation almost zero.

In these circumstances it is hardly surprising that FITCON variety means are poor predictors of commercial means (Figure 7.4.2). Discrepancies depend on age of variety as well as size of seed effect. The older varieties are least affected. Model (7.16) is however of limited value. It cannot provide predictions for the ten varieties grown with only breeder seed.

Table 7.4.7:

Variety means from the model $V + Y + S + V.S$: and adjustment from breeder mean to commercial mean

(sugar beet data, 1987–1991)

		mean		
Seed house	variety	breeders	commercial	difference
1	11	56.64	55.32	1.32
2	8	56.79	56.26	0.53
	14	54.50	56.26	-1.76
	17	57.59	57.34	-1.25
	19	57.04	59.29	-2.25
3	13	57.43	53.46	3.97
4	12	57.24	54.19	3.05
	6	58.92	56.46	2.27
5	10	56.91	52.37	2.46
	7	57.98	55.41	2.57
6	9	57.53	53.02	4.51
7	16	57.73	53.88	3.85
	15	58.82	54.81	4.01
8	18	57.14	58.12	-0.98

Allowing for seed class × seed houses interaction

Further inspection of Table 7.4.7 provides small but indicative evidence that varieties varied more between seed houses than within seed house. For example four varieties with smallest seed effects are from seed house 2 and there is not much difference between seed effects of seed house 7. Differences between seed house 2 and seed house 7 are much larger than differences within seed houses. This

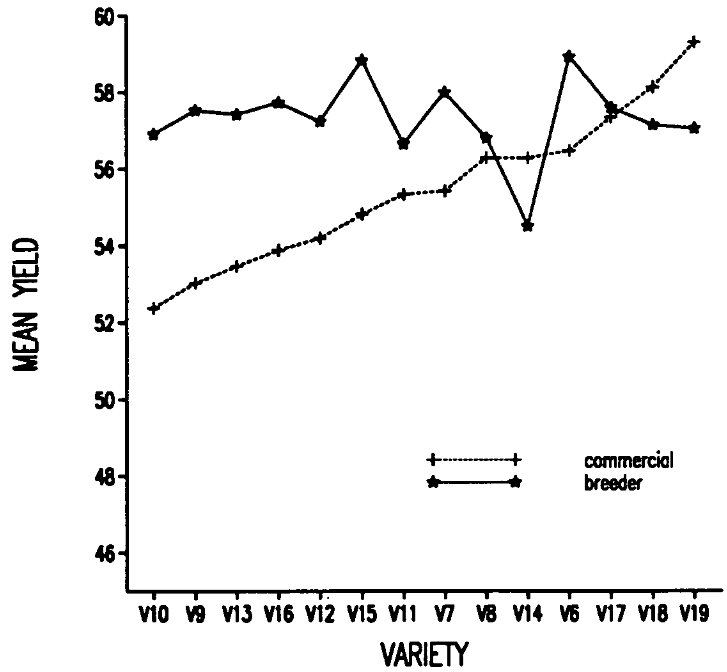

Figure 7.4.1:

Breeder and commercial means from the model $V + Y + S^* + V.S^*$: for 14 varieties

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suggests a model which allows for variety differences between seed houses, i.e

$$V + Y + S^* + S^*.H : \tag{7.17}$$

where H is a factor for seed houses (Table 7.4.2). Table 7.4.8 shows that seed classes $\times$ seed houses interaction is highly significant and the analysis of variance confirms that most of the varieties $\times$ seed classes interaction is explained by differences between seed houses.

This suggests that model (7.17) is adequate for describing the data but how well does it predict? Model (7.17) gives equal adjustment within seed houses but

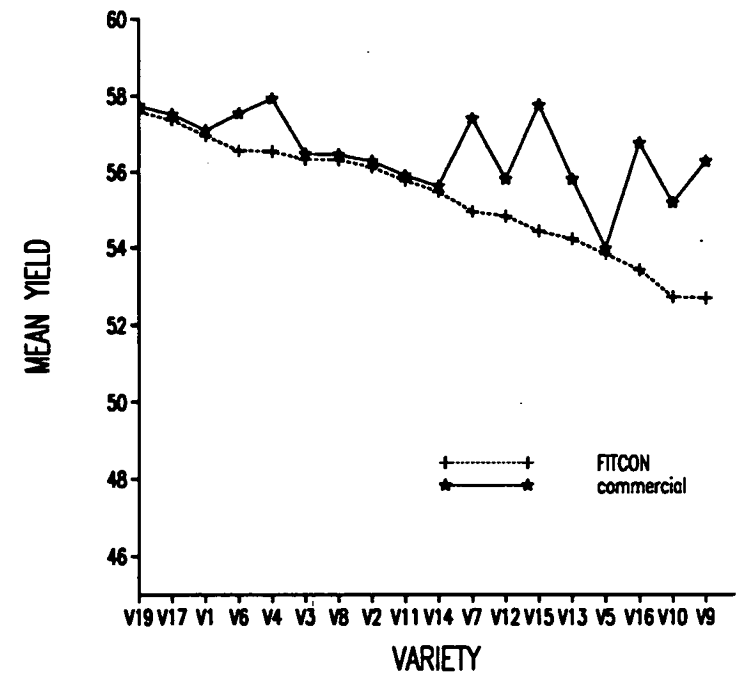

Figure 7.4.2:
FITCON means and commercial mean from the model $V + Y + S^{*} + V.S^{*}$:

Table 7.4.8:

ANOVA table for the model $(V/H) * S^{*} + Y$:
(sugar beet data, 1987–1991)

class	df	MS	F-ratio
Y	4	75.93	50.82
H	8	5.04	3.38
V within H	20	10.52	7.04
S^{*}	2	40.12	26.85
$H \times S^{*}$	13	7.69	5.14
$V \times S^{*}$ within H	13	2.44	1.63
residual	62	1.49	

not between seed houses. As variety effects are fitted in the model, the main effect of H is completely aliased with V. Variety means obtained from model (7.17) using Genstat REML estimate variety performance as if a variety is produced by all breeders. Since each variety is uniquely produced by a breeder, variety means should be obtained from interaction terms of its seed house.

We discuss results of analysis from model (7.17) in Section 7.4.3. But first, we describe an algorithm for estimating variety means from model (7.17) using output from Genstat REML. We use a subset of the data to show how variety means are calculated.

7.4.2 Computational aspects for the model $V+Y+S+S.H$:

Example 7.4.1: Model $V+Y+S+S.H$: (subset of sugar beet data)

The subset of data is given in Table 7.4.9. We fit the model

$$V + Y + S + S.H : \tag{7.18}$$

Estimates of effects from model (7.18) using Genstat REML are:

$$\begin{aligned} \text{constant} &= 57.89 \\ \text{variety effects} &= (0.0, -1.625, -0.511, -3.921, -1.823) \\ \text{year effects} &= (0.0, 1.833, -0.482, -3.994, -1.342) \\ \text{class effects} &= (0.0, -1.123, 0.013) \end{aligned}$$

and effects for $S \times H$ interaction are:

	seed house		
seed class	2	4	5
breeder	0.000	1.400	2.807
pre-commercial	0.000	-1.017	0.559
commercial	0.000	0.000	0.000

Table 7.4.9:

Subset sugar beet data

Variety (V)	seed class (S)	seed house (H)	year (Y)	yield
6	b	4	1	59.51
6	p	4	2	57.17
6	c	4	3	56.44
6	c	4	4	53.79
6	c	4	5	57.86
7	b	5	1	58.57
7	p	5	2	57.95
7	c	5	3	57.09
7	c	5	4	52.78
7	c	5	5	53.24
8	b	2	1	57.38
8	p	2	2	58.09
8	c	2	3	56.60
8	b	2	4	53.43
8	c	2	5	56.33
10	b	5	1	57.51
10	p	5	2	58.38
10	c	5	3	52.51
10	c	5	4	51.13
10	c	5	5	51.14
12	b	4	1	57.02
12	p	4	2	59.53
12	c	4	3	53.86
12	c	4	4	50.54
12	c	4	5	56.09

The required means[3] are estimated as

constant + variety effect + mean of year effects + mean of class effects
+ mean of $S \times H$ effects for the seed house to which a variety belongs.

See Table 7.4.10.

The mean for seed class j of seed house k is estimated[4] as:

constant + mean of variety effects for breeder k + mean of year effects
+ effect of class j + effect of seed class $j \times$ seed house k.

With reference to Table 7.4.10, the S $\times$ H table of means is obtained as

$S \times H2$: effects of row $4 +$ effects of row 2 + [A] + [C] + [H]

$S \times H4$: effects of row $5 +$ effects of row 2 + [A] + [C] + [I]

$S \times H5$: effects of row $6 +$ effects of row 2 + [A] + [C] + [J]

---

[3]The variety means from model (7.18) using Genstat REML are estimated as:

constant + variety effect + mean of year effects + mean of class effects +
mean of $S \times H$ effects.

The last term is averaged over all levels of seed class and seed house. These means are not appropriate because they measure average performance of a variety as if its seed is produced by all seed houses.

[4]The $S \times H$ table of means obtained from Genstat REML is also inappropriate because it ignores grouping of varieties within seed houses. The cell means of $S \times H$ table involve the mean of all varieties instead of the mean of only those varieties produced by a particular seed house.

Table 7.4.10:

Variety mean computations for subset data

row	effects	estimates					average
1	constant	57.89					57.89 [A]
2	S	0.00	-1.12	0.01 [B]			
3	Y	0.00	1.83	-0.48	-3.99	-1.34	-0.80 [C]
4	$S \times H$ 2	0.00	0.00	0.00			0.00 [D]
5	$S \times H$ 4	1.40	-1.02	0.00			0.13 [E]
6	$S \times H$ 5	2.81	0.56	0.00			1.12 [F]
7		all seed $S.H$ effects					0.42 [G]
8	variety 8	-0.51					-0.51 [H]
9	variety 6,12	0.00	-1.82				-1.82 [I]
10	variety 7,10	-1.63	-3.92				-2.77 [J]
		commercial means					
11	variety 8	56.60		(row 8 + [A] + [B] + [C] + [D])			
12	variety 6,12	57.11	55.28	(row 9 + [A] + [B]+ [C] + [E])			
13	variety 7,10	55.48	53.19	(row 10 + [A] + [B] + [C] + [F])			

Thus the $S \times H$ table of means is:

	seed house		
seed class	2	4	5
breeder	56.58	57.58	57.13
pre-commercial	55.46	55.16	54.88
commercial	56.60	56.18	54.32

Calculations of this kind are involving and mistakes are easy to make. It is therefore desirable to have an automated procedure to calculate variety means and standard errors of difference from Genstat REML output. To do this we use matrices.

Matrix computation scheme

Let $\hat{\varphi}$ be a vector of estimates of effects from model (7.17), partitioned as

$$\hat{\varphi} = \begin{pmatrix} \hat{\varphi}_v \\ \hat{\varphi}_y \\ \hat{\varphi}_s \\ \hat{\varphi}_{sh} \end{pmatrix}$$

where $\hat{\varphi}_v$ is a vector of estimates for the general mean and variety effects, $\hat{\varphi}_y$ is a vector of estimates for year effects; other vectors are similarly defined. In our example,

$$\begin{aligned} \hat{\varphi}_v &= (57.89, 0.0, -1.63, -0.51, -3.92, -1.82)' \\ \hat{\varphi}_y &= (0.0, 1.83, -0.48, -3.99, -1.34)' \\ \hat{\varphi}_s &= (0.0, -1.12, 0.01)' \\ \hat{\varphi}_{sh} &= (0.0, 0.0, 0.0, 1.40, -1.02, 0.0, 2.81, 0.56, 0.0)' \end{aligned}$$

Conformable with vector $\hat{\varphi}$, define a 5×18 matrix of coefficients K, partioned as $K = (K_v|K_y|K_s|K_{sh})$, i.e

$$K_v = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K_y = 1/5 \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$K_s = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$K_{sh} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The breeder means are given by $K\hat{\varphi}$. Pre-commercial means are obtained by shifting to the right the positions of 1's in the K_s and K_{sh} matrices by one and a further shift in the position of 1's yields commercial means. The variance matrix of the variety means can be calculated given the variance matrix of $\hat{\varphi}$.

Algorithm 7.4.1: Estimating variety means from the model $V + Y + S + S.H$:

(a) Let $\hat{\varphi}$ be an n-vector of all effects from the model $V + Y + S + S.H$: For each seed class, define a $v \times n$ matrix $K_l = (k_{ij})$ where

$$k_{ij} = \begin{cases} 1 & j = 1, i+1, n_l \\ 1/y & j = m_1, m_1 + 1, m_1 + 2, \ldots, m_2 \\ 0 & \text{elsewhere} \end{cases}$$

where i indexes the varieties, j the elements of $\hat{\varphi}$; n_l is the column corresponding to the l-th seed class, and m_1 and m_2 index the columns of K corresponding to the effect of the first and last year of the trials respectively.

(b) Let n_v and n_s be the number of years, varieties and seed classes respectively. We define an $n_s n_v \times n$ matrix

$$K = \begin{pmatrix} K_1 \\ K_2 \\ \vdots \\ K_s \end{pmatrix}$$

from which we obtain the vector of variety means as $K\hat{\varphi}$.

Table 7.4.11:

Variety means and standard errors relative to variety 6 for a subset of sugar beet data (Table 7.4.9)

	class		
variety	breeder	pre-commercial	commercial
6	58.49 (-)	54.95 (-)	57.11 (-)
7	58.28 (1.311)	54.91 (1.414)	55.48 (0.940)
8	56.58 (1.620)	55.46 (1.725)	56.60 (1.007)
10	55.98 (1.292)	52.61 (1.476)	53.19 (1.097)
12	56.67 (0.937)	53.13 (0.937)	55.28 (0.937)

(c) Given the variance of $\hat{\varphi}$ we compute the variance matrix of variety means as $Kvar(\hat{\varphi})K'$. From which we calculate standard errors of variety comparisons.

A genstat code for Algorithm 7.4.1 is given in Appendix D.3. Variety means for Example 7.4.1 using Algorithm 7.4.1 are displayed in Table 7.4.11.

7.4.3 Predictions from the model $V + Y + S^* + S^*.H$:

Seed classes × seed houses means

Unlike Example 7.4.1 (Section 7.4.2), for the full data set, not every seed house produced seed in all classes. Seed house 9 had no varieties with commercial seed. Thus commercial means are not estimable for varieties of seed house 9 and the $S^* \times H$ table is incomplete (Table 7.4.12).

Except for seed house 9 for which we have no information on its commercial seed, most seed houses show a decline from breeder to commercial means (Figure 7.4.3). The ~~other~~ only exceptions are seed house 2 and 8 which had better commercial seed than breeder seed. Note that seed house 2 produced seed for 30% of the

Table 7.4.12:

Seed house × class means (sugar beet data 1987 – 1991)

seed house	class	
	breeder	commercial
1	56.30	55.02
2	56.09	56.89
3	58.99	55.06
4	57.00	54.22
5	57.65	53.94
6	57.51	53.04
7	58.25	54.36
8	57.58	58.55
9	56.97	*

varieties in the trials (Table 7.4.2). Thus except for seed house 9, i.e varieties 23 and 24, commercial means can be predicted for all varieties. Certainly, an improvement over model (7.17).

Breeder means versus commercial means

Variety means for recommended varieties in the trials are displayed in Table 7.4.13. Varieties 1 – 5 were grown with commercial seed throughout the five years and so their commercial means are not adjusted. Breeder means are, however adjusted. Model (7.17) adjust equally varieties in the same seed house. For example, seed house 2 breeder means are adjusted downwards by 0.8 t/ha because commercial seed yielded better than breeder seed for seed house 2, and breeder means for varieties in seed house 1 are adjusted upwards by 1.28 t/ha because breeder seed yielded better than commercial seed (Table 7.4.12).

Variety 9 is the only variety in seed house 6 and so its means are the same as the means of seed house 6 in Table 7.4.12. Variety 8 had only one year of breeder seed, its commercial mean is adjusted upwards by 0.16 t/ha whereas variety 19

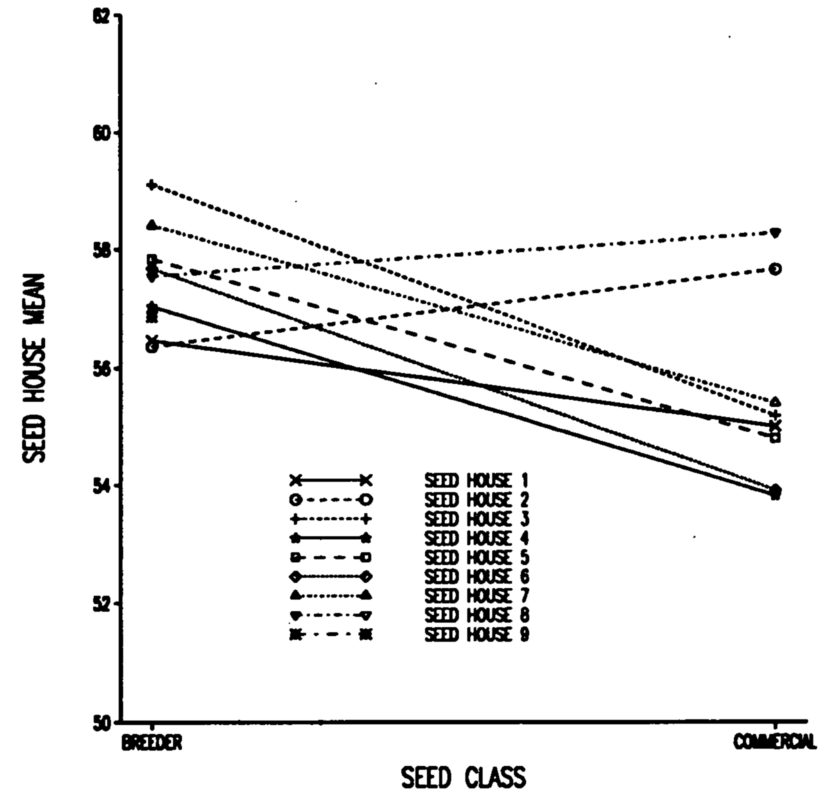

Figure 7.4.3: Seed house means

with three years of breeder seed is adjusted upwards by 0.76 t/ha; both varieties are from seed house 2 and their breeder means are smaller than commercial means by 0.8 t/ha. Adjustments to variety means depend on the age of the variety and its seed house.

Although we have made a case for using commercial means as a basis for recommendation, this leads to complications for new varieties. In the first place new varieties have not been grown with commercial seed: their commercial potential is based on commercial performance of other varieties. Their means will therefore be estimated with less precision. Secondly, if a seed house has not produced commer-

Table 7.4.13:

Breeder and commercial means for recommended varieties from model (7.4)
(sugar beet data, 1987–1991)

		class	
variety (seed house)	unadjusted mean	breeder	commercial
1 (1)	56.95	58.23	56.95
2 (2)	56.12	55.32	56.12
3 (2)	56.32	55.52	56.32
4 (3)	56.81	60.74	56.81
5 (1)	53.85	55.14	53.85
6 (4)	56.95	59.17	56.40
7 (5)	55.93	58.89	55.18
8 (2)	56.37	55.72	56.53
9 (6)	54.83	57.51	53.04
10 (5)	54.19	56.41	52.70
11 (1)	55.85	56.62	55.33
12 (4)	55.41	57.07	54.30
13 (3)	55.05	57.41	53.48
14 (2)	55.56	55.07	55.88
15 (7)	56.42	58.75	54.86
16 (7)	55.42	57.75	53.86
17 (2)	57.49	57.17	57.97
18 (8)	57.24	57.15	58.11
19 (2)	57.45	57.41	58.21

cial seed for any variety, commercial means for all varieties in the seed house are not estimable. Thirdly, if recommended varieties in the trials are declining in their performance, failure to adjust for seed effects gives new varieties unfair advantage over old varieties. Table 7.4.14 shows breeder and commercial means for the ten third year varieties. Evidently varieties by seed house 2 are least affected in their prediction of commercial means (Table 7.4.14).

Table 7.4.14:

Breeder and commercial means for new varieties from model (7.4)

(sugar beet data, 1987–1991)

	~~class~~	class	
variety (seed house)	unadjusted mean	breeder	commercial
20 (2)	55.42	56.12	56.93
21 (4)	56.40	57.10	54.33
22 (3)	58.12	58.83	54.90
23 (9)	56.80	57.50	*
24 (9)	55.72	56.43	*
25 (4)	53.93	54.63	51.86
26 (1)	54.53	55.24	53.95
27 (8)	57.31	58.02	58.98
28 (2)	54.88	55.58	56.39
29 (2)	56.17	56.88	57.68

Standard errors of variety comparisons

Standard errors for contrasts of variety means depend on the age of the variety and the number of varieties at each seed class within a seed house. The more the varieties in a seed house at a given seed class the smaller the standard errors. Also the older the variety in the trial the smaller the standard errors. Standard errors for new varieties for contrasts based on commercial or pre-commercial means will be larger than for the old varieties. Similary standard errors for contrasts of older varieties based on breeder means will in general be large.

To assess information loss due to use of different seed classes in the trials, standard errors of selected contrasts are compared with what they would be if a single seed class was used. The units variance from the model $V + Y$: is inflated by seed class effects; standard errors from this analysis are therefore scaled by a factor $\sqrt{s_1/s_0}$ where s_1 and s_0 are units variances from models $V+Y+S^*+S^*.H$: and $V + Y$: respectively. Standard errors of selected contrasts and percentage loss of information are given in Table 7.4.15.

There is no loss of information when differences between varieties using only

Table 7.4.15:

Standard errors and percentage loss in information for selected contrasts (Sugar beet data)

contrast	class breeder	%	commercial	%	FITCON[1]
$V3 - V1$	1.476	48	0.763	0	0.763
$V6 - V7$	1.230	38	0.800	5	0.763
$V27 - V4$	1.507	41	1.692	48	0.887
$V27 - V13$	1.165	24	1.730	48	0.887
$V25 - V21$	0.985	0	0.985	0	0.985
$V26 - V22$	0.985	0	1.842	47	0.985

[1](FITCON s.e) $\times\sqrt{(1.455/2.225)}$

commercial seed in trials are estimated on the basis of commercial means. No adjustment is made on the FITCON means of these varieties. However these comparisons are innaccurate when breeder means are used. For example, the loss of information in the comparison $V3 - V1$ is $(1 - 0.763/1.476)100\% = 48\%$. Similarly, comparisons on new varieties based on commercial seed are very poorly estimated whereas no information is lost if breeder means are used. No single set of means, whether breeder or commercial is free from loss of information for every variety. In particular the fact that old varieties use mostly commercial seed in the trials, new varieties use only breeder seed and that some breeders have either one variety or only new varieties would lead to large standard errors for some comparisons regardless of the set of means used.

7.5 Summary

We have shown how REML can be used to deal with heterogenous varieties $\times$ environments interactions. If groups of varieties can be identified with a consistent groups $\times$ centres variance then extra components of variance are required. Multiplicative models can also be used to model heterogeneity. Heterogeneity of varieties $\times$ centres variance is often present in some of the years. In this case a two-stage analysis should be done. Extension of these methods to varieties $\times$ years interaction are straight forward, except that if there are trends or systematic effects we do not project results to a population of years.

An example of systematic effects due to years is that of seed effects in sugar beet trials. The heterogeneity associated with seed effects introduces problems of modelling and prediction of commercial means. The seed effects were large with a significant varieties $\times$ seed interaction. This ruled out use of basic FITCON model and model (7.14). Commercial means for new varieties cannot be predicted from model (7.15) because $V \times S^*$ table is incomplete. This model provides some evidence of heterogeneity of varieties × seed classes interaction and that improvement in model fitting could be achieved by grouping varieties by their seed houses.

The model (7.17) allows equal adjustment for varieties within seed houses and predicts commercial means for all varieties, except those in seed house 9. The adjustment to varieties depend on both the age of the variety and its seed house. This led to different ranking depending on the means used. The estimation of commercial means is, however, imprecise for new varieties.

We must also bear in mind the possibility of varieties × seasons interaction in the data. The two types of interaction are highly correlated. More information will be needed either from more trials or from designed experiments with seed classes as a factor to enable more efficient adjustment of seed effects and to separate varieties × seed classes interaction from varieties × seasons interaction. The data demonstrates that control of seed processes in seed houses offers a practical solution to the problem. Seed class may not be the only factor responsible for the observed heterogeneity. Other factors, such as ploidy, may be confounded with seed effects and require further investigation.

This analysis was possible because we knew the seed classes and were able to include their effects in the model. Without this information other analysis could have been done, for example, seed house 2 versus the rest, varieties or groups of varieties × dry versus wet, or indeed a covariate associated with the age of varieties. A good starting point is the examination of residuals.

Chapter 8

FURTHER WORK

Multiplicative models

FITCON or REML can be used to analyse varieties $\times$ centres tables. If the units variance is heterogeneous, basic FITCON analysis can be inefficient. REML analysis can be modified to account for heterogeneity associated with groups of varieties. Knowledge of varieties in trials, for example grouping by ploidy or sensitivity to centre differences, may be used to identify homogeneous groups of varieties. It is possible for varieties grouped by the first criterion to have different sensitivities to centre differences.

A weakness of the REML analysis with groups $\times$ centres variance is that it assumes equal within-groups variances. If varieties are grouped by sensitivities, the more variable group should have a higher within-groups variance. This problem may be tackled by fitting a multiplicative model with sensitivities in the groups stratum. This extension of the multiplicative model applies to a varieties $\times$ years table and to a $V \times R/C$ table.

We made the point in Section 7.2 that a general multiplicative model can be used to model heterogeneity of varities $\times$ years and varieties $\times$ centres interactions. Further study is required here, particularly the tests of significance regarding sensitivities.

Combined analysis over centres and years

In our analysis we have treated series of trials as having a $V * C * Y$ or $V * Y/C$ or $V * R/C * Y$ data structure. Often series of trials are a mixture of these sampling schemes. We need a REML algorithm that permits mixtures of sampling schemes in series of trials. Moreover, if regions have been sampled, then the algorithm should allow for sampling of finite populations.

Shrunken means

In variety trials, several factors introduce bias in predicting variety performance. Commercial farms have generally a lower standard of management than sites often chosen for variety trials. Thus variety performance for commercial farming is overestimated by trials results. Standardization of non-experimental treatments such as seed rates and seed treatments may favour certain varieties. Selection bias also arises as a variety is more likely to be recommended if its trial mean exceeds its true mean (Patterson & Silvey 1980).

One way of dealing with this bias is to use estimation methods that shrink variety effects towards the origin. Plant breeders are cautious about a new variety that has exceptionally high or low mean. Shrinkage makes adjustments that are consistent with this caution. Another justification for shrunken means is to consider a case in which there are no real differences between varieties. Trial results may then indicate differences in varieties when no differences exist. Shrinkage with variety component of variance equal to zero would give the correct results.

Finney (1964) used regression analysis to provide an estimate of this bias. The regression coefficients can be estimated indirectly from variety components of variance (Patterson & Silvey 1980). A more general method involves estimating variety effects from a model in which all effects are random. Shrunken means are a by-product of the REML algorithm if model $\mathcal{G}$ or model $\mathcal{J}$ is used (Sections 6.2 and 6.3).

If a $V \times C$ table is complete and REML is used to fit the model : $V + C$, variety means are shrunk by a factor

$$n_c\sigma_V^2/\{n_c\sigma_V^2 + \sigma^2\},$$

which is the same as that given by regression analysis.

For a complete $V \times C \times Y$ table, REML model $\mathcal{G}$ shrinks variety means by a shrinkage factor,

$$\lambda/\{\lambda + \sigma^2/(n_c n_y)\},$$

where $\lambda = \sigma_V^2 + \sigma_{VC}^2/n_c + \sigma_{VY}^2/n_y$. However, regression analysis would shrink variety means by a factor

$$\sigma_V^2/\{\lambda + \sigma^2/(n_c n_y)\}$$

Similary, for a complete $V \times Y/C$ table, REML model $\mathcal{J}$ shrinks variety means by a shrinkage factor

$$\lambda/\{\lambda + \sigma^2/(n_c n_y)\}$$

where $\lambda = \sigma_V^2 + \sigma_{VY}^2/n_y$ However, regresion analysis would shrink variety means by a factor

$$\sigma_V^2/\{\lambda + \sigma^2/(n_c n_y)\}$$

Much more needs to be known about shrunken means and the method of shrinkage should take into account the intensity of selection.

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Appendix A

Data sets

A.1 Wheat data, 1974 – 1978

year	region	variety	trial 1	2	3	4	5	6	7
74	ES	HUNTSMAN	6.51	8.18	6.39	8.49	5.57	4.92	7.26
		ATOU	6.27	6.59	8.06	6.20	5.74	5.74	6.95
		ARMADA	6.42	8.38	5.92	*	*	*	*
	NS	HUNTSMAN	6.12	6.31	6.54	6.02	6.46	*	*
		ATOU	4.97	5.98	7.36	5.30	5.68	*	*
	WS	HUNTSMAN	7.19	3.62	6.61	4.61	*	*	*
		ATOU	7.12	3.75	6.01	4.63	*	*	*
		ARMADA	7.28	*	*	*	*	*	*
75	ES	HUNTSMAN	6.43	7.94	5.36	6.61	8.74	5.66	6.40
		ARMADA	6.17	7.66	*	*	*	*	*
		MARDLER	6.77	8.47	*	*	*	*	*
	NS	HUNTSMAN	5.72	6.81	4.59	4.43	8.28	7.74	*
		ARMADA	5.48	*	*	*	*	*	*
		MARDLER	5.13	*	*	*	*	*	*
	WS	HUNTSMAN	6.75	6.08	5.86	*	*	*	*
		ATOU	6.77	*	*	*	*	*	*
		ARMADA	*	*	7.10	*	*	*	*
		MARDLER	7.36	*	*	*	*	*	*
76	ES	HUNTSMAN	7.74	6.70	6.25	5.82	7.92	7.33	5.54
		ATOU	6.99	6.60	*	*	7.59	*	*
		ARMADA	7.32	7.08	6.51	6.21	7.94	6.81	4.75
		MARDLER	7.73	7.59	*	*	*	*	*
		SENTRY	7.73	6.95	*	*	*	*	*
		STUART	7.50	7.38	*	*	*	*	*
	NS	HUNTSMAN	4.96	6.26	7.43	4.15	4.74	6.04	*
		ATOU	5.10	*	*	*	*	*	*
		ARMADA	5.62	6.48	7.48	4.11	5.64	5.52	*
		MARDLER	5.73	*	*	*	*	*	*
		SENTRY	5.41	*	*	*	*	*	*
		STUART	5.94	*	*	*	*	*	*

	WS	HUNTSMAN	5.56	*	*	*	*	*	*
		ATOU	5.68	*	*	*	*	*	*
		ARMADA	5.30	*	*	*	*	*	*
		MARDLER	5.77	*	*	*	*	*	*
		SENTRY	5.89	*	*	*	*	*	*
		STUART	5.76	*	*	*	*	*	*
77	ES	HUNTSMAN	7.33	6.37	5.79	*	6.12	*	*
		ATOU	7.31	6.99	5.96	*	6.64	*	*
		ARMADA	7.75	7.19	5.97	*	6.92	*	*
		MARDLER	8.93	8.33	6.56	*	7.55	*	*
		SENTRY	8.68	7.91	*	*	*	*	*
		STUART	8.72	8.04	*	*	*	*	*
	NS	HUNTSMAN	4.21	5.12	4.50	5.49	5.86	*	*
		ATOU	4.62	4.65	5.07	5.59	6.52	*	*
		ARMADA	*	5.04	4.99	5.59	6.57	*	*
		MARDLER	*	5.13	4.60	5.83	6.14	*	*
		SENTRY	3.99	*	*	*	*	*	*
		STUART	4.70	*	*	*	*	*	*
	WS	HUNTSMAN	6.55	*	*	*	*	*	*
		ATOU	6.91	*	*	*	*	*	*
		ARMADA	7.59	*	*	*	*	*	*
		MARDLER	7.91	*	*	*	*	*	*
		SENTRY	7.34	*	*	*	*	*	*
		STUART	7.17	*	*	*	*	*	*
78	ES	HUNTSMAN	5.31	6.39	6.56	4.08	5.09	*	*
		ATOU	6.35	6.48	6.61	5.38	6.96	*	*
		ARMADA	6.57	6.71	6.94	5.33	6.24	*	*
		MARDLER	7.18	6.49	6.86	4.77	5.95	*	*
		SENTRY	6.58	6.57	7.01	5.52	7.01	*	*
		STUART	7.25	*	7.55	5.55	7.32	*	*
	NS	HUNTSMAN	4.56	3.55	6.05	2.68	3.96	*	*
		ATOU	4.95	4.17	5.93	4.47	5.29	*	*
		ARMADA	*	4.53	6.61	4.87	4.88	*	*
		MARDLER	*	4.17	7.11	4.86	3.97	*	*
		SENTRY	*	4.20	6.61	4.07	4.48	*	*
		STUART	5.09	4.19	6.19	4.57	4.94	*	*
	WS	HUNTSMAN	6.01	5.87	4.58	*	*	*	*
		ATOU	6.69	5.71	4.91	*	*	*	*
		ARMADA	6.47	6.53	4.76	*	*	*	*
		MARDLER	6.41	6.85	4.99	*	*	*	*
		SENTRY	7.12	6.43	5.16	*	*	*	*
		STUART	6.28	6.59	4.98	*	*	*	*

A.2 Ryegrass data 1985 – 1989

		centre						
year	variety	1	2	3	4	5	6	7
84	1	9.28	12.68	11.06	10.91	9.98	8.63	13.31
	14	10.15	12.38	10.68	10.78	9.91	10.16	13.90
	17	10.30	14.24	11.10	11.13	11.06	9.51	13.90
85	1	17.61	13.17	13.29	15.77	13.95	10.75	16.67
	14	15.95	11.76	13.59	14.79	13.90	10.15	17.16
	17	17.61	12.38	14.20	15.32	14.52	10.83	17.76
86	1	14.01	11.99	14.01	14.87	14.74	9.51	14.47
	4	14.18	12.54	14.78	14.96	13.22	9.05	15.21
	17	14.08	12.92	15.11	14.69	16.27	9.51	15.36
	19	14.44	13.25	15.36	14.37	15.41	9.26	15.49
87	1	7.55	13.09	8.73	13.49	11.89	6.62	14.44
	4	7.92	12.40	9.31	13.81	13.68	6.96	15.59
	12	5.47	12.09	8.48	12.09	12.04	5.64	13.53
	18	7.20	13.02	8.57	12.64	12.23	6.52	15.05
	19	8.20	12.69	9.05	12.72	12.51	6.68	14.60
88	1	6.12	14.67	11.96	10.54	8.15	6.50	12.28
	12	4.52	12.74	11.07	7.81	7.82	5.05	12.42
	18	5.82	13.50	12.23	8.79	8.57	5.64	12.65

A.3 Sugar beet data 1987 — 1991

1987

centre / variety	1	2	3	4	5	6	7	8
1	62.08	57.69	64.13	60.68	57.21	51.03	52.16	67.92
2	61.33	53.50	63.71	58.83	65.86	47.14	52.67	64.61
3	61.84	56.13	65.07	57.11	59.47	48.63	48.70	63.78
4	61.68	52.45	62.62	57.13	64.90	48.33	50.83	60.13
5	61.69	54.98	59.02	58.32	57.55	49.42	46.82	65.66
6	63.77	56.91	66.49	58.69	61.84	51.06	55.95	63.63
7	62.62	56.98	68.25	61.90	*	51.09	54.82	67.17
8	62.11	54.01	66.47	59.11	58.71	49.74	52.51	64.68
9	60.33	53.58	66.02	52.77	63.38	49.21	51.86	64.74
10	63.24	55.17	63.66	57.48	61.89	50.69	55.86	66.33
11	62.69	55.10	64.50	59.55	59.06	51.44	50.07	65.14
12	62.26	54.58	64.61	54.79	57.69	48.59	54.11	63.49
13	63.88	56.55	69.57	57.16	62.98	51.15	51.36	61.31
14	59.68	53.55	66.41	57.28	56.62	46.77	46.03	62.67
15	63.87	57.67	69.30	63.99	64.56	52.20	55.87	67.91
16	61.91	56.84	65.70	57.20	62.34	49.34	51.10	66.72
17	63.14	55.85	67.87	56.87	63.39	50.41	52.83	*

centre / variety	9	10	11	12	13	14	15	16
1	53.95	52.04	69.29	56.70	54.12	55.19	54.24	51.84
2	52.11	49.17	72.94	56.49	57.64	58.27	56.74	51.58
3	53.09	52.74	72.95	56.49	57.27	56.11	56.71	49.78
4	53.20	47.24	70.03	58.55	56.20	54.10	57.36	47.65
5	51.61	51.89	66.48	52.61	54.53	51.54	54.08	48.58
6	55.59	52.20	76.78	58.75	56.23	63.51	59.53	51.29
7	53.93	52.29	70.78	55.30	54.59	56.13	58.10	51.00
8	52.06	49.51	71.01	55.26	58.23	57.91	55.58	51.19
9	55.19	50.18	72.68	52.91	54.96	53.01	55.09	52.25
10	52.44	51.61	68.25	52.27	55.69	56.56	56.65	52.34
11	51.74	51.25	72.03	60.50	53.12	56.73	56.09	50.81
12	54.43	50.06	71.54	53.54	57.10	57.21	57.84	50.46
13	54.60	54.63	68.42	55.95	60.53	54.46	57.50	52.21
14	53.11	50.61	66.12	56.45	51.84	56.35	54.30	46.41
15	55.62	52.51	71.20	52.75	58.51	62.55	58.26	52.79
16	52.38	50.45	75.65	55.07	54.42	58.29	55.22	53.78

1988

centre	1	2	3	4	5	6	7	8
variety								
1	66.73	62.73	61.72	68.77	56.12	58.23	41.61	56.30
2	61.79	62.20	61.15	57.77	51.31	57.98	40.48	54.79
3	66.44	62.66	61.94	60.55	52.48	57.41	45.96	55.74
4	63.80	62.21	59.68	62.10	54.06	57.13	43.48	57.34
5	61.92	62.33	61.38	59.85	52.68	55.15	39.98	54.80
6	61.09	64.29	63.06	69.06	56.34	53.89	39.63	57.11
7	63.19	63.82	64.44	63.16	54.00	54.77	43.40	57.98
8	66.86	64.83	64.32	66.82	54.27	53.42	44.07	56.88
9	67.74	69.33	67.69	65.44	57.48	57.75	42.65	60.55
10	61.49	65.92	65.94	66.93	57.50	52.87	45.17	58.46
11	63.09	65.40	64.75	63.04	52.42	55.00	39.42	57.18
12	63.63	68.60	64.07	66.51	57.75	58.05	45.59	61.03
13	66.48	65.88	63.77	65.31	59.36	54.00	41.32	57.03
14	61.20	62.93	60.11	57.39	52.14	54.19	40.84	56.12
15	64.18	65.26	64.57	67.18	56.80	59.49	46.21	59.11
16	69.09	68.19	66.43	70.62	58.13	57.41	46.14	60.72
17	66.63	64.05	62.24	66.51	55.99	57.78	43.07	54.94
18	66.62	63.70	65.12	63.17	54.48	62.86	45.48	*
19	63.17	64.47	67.20	67.78	56.95	56.81	43.33	*

centre	9	10	11	12	13	14	15	16
variety								
1	66.23	63.67	61.46	48.90	49.99	57.70	59.18	53.36
2	65.69	65.00	60.85	52.77	48.45	55.08	57.08	51.84
3	64.74	63.13	61.41	52.31	50.77	57.26	58.32	54.73
4	63.66	62.85	59.21	53.47	47.44	54.80	56.80	53.66
5	63.31	56.37	59.93	47.81	48.13	55.29	53.92	53.58
6	62.86	59.00	57.88	53.40	51.72	54.73	57.76	52.88
7	68.29	61.88	60.54	55.12	48.18	56.62	58.27	53.54
8	63.53	60.53	59.17	53.68	49.35	58.04	58.35	55.30
9	68.26	66.14	63.17	54.57	50.33	58.16	63.50	53.26
10	67.20	63.90	59.48	51.05	51.60	55.88	59.38	51.36
11	65.99	62.46	62.37	49.46	52.35	57.77	60.25	54.78
12	69.62	63.73	60.77	54.32	50.63	56.39	58.68	53.19
13	63.59	64.23	60.07	53.49	51.52	57.46	59.80	55.20
14	62.64	62.87	60.00	53.13	45.58	55.71	56.27	51.95
15	66.43	65.82	62.66	53.08	52.96	60.77	58.88	52.65
16	67.76	63.74	63.40	48.78	50.11	54.86	57.96	50.72
17	63.70	61.59	64.23	52.62	48.10	56.22	57.72	52.27

1989

centre	1	2	3	4	5	6	7	8
variety								
1	43.90	41.52	55.10	64.30	69.19	78.66	39.41	50.94
2	44.43	40.76	51.65	62.23	64.73	70.19	39.43	50.99
3	44.79	40.67	51.53	62.83	61.00	77.00	38.33	51.30
4	44.79	43.43	53.98	62.91	67.28	78.89	44.47	54.87
5	43.11	39.37	50.63	58.89	64.07	69.73	38.05	46.52
6	45.93	42.81	53.61	63.79	70.12	71.83	42.03	54.60
7	43.00	43.09	50.33	62.19	70.14	80.55	44.38	51.14
8	45.68	39.76	52.53	61.93	61.41	71.89	39.86	53.01
9	41.77	37.56	50.04	60.53	65.86	73.08	36.65	43.82
10	41.75	36.59	49.02	58.37	61.69	71.72	35.00	48.50
11	40.27	38.57	49.97	58.16	62.81	70.38	37.48	48.29
12	42.04	41.78	51.34	62.91	64.74	70.48	40.21	51.93
13	42.93	41.56	50.52	62.79	68.80	62.81	43.67	52.63
14	44.31	42.63	52.34	64.74	65.12	71.26	39.79	57.30
15	40.63	45.07	49.03	58.69	65.37	68.22	39.68	49.40
16	40.00	38.04	53.54	61.46	64.59	76.42	35.85	49.99
17	46.61	44.76	55.24	64.50	68.06	82.64	40.20	52.55
18	47.24	39.81	57.56	63.98	62.79	70.51	40.49	52.30
19	43.86	41.00	54.31	61.75	66.26	72.04	36.79	53.07
20	42.89	36.94	55.89	64.19	66.17	64.92	41.09	*
21	40.51	42.67	52.62	63.50	64.37	69.31	39.46	*
22	45.20	41.93	53.25	62.47	71.35	76.52	41.71	*
23	46.46	42.57	52.24	65.60	67.79	79.02	41.63	*
24	46.00	43.26	51.07	60.20	66.32	68.42	40.92	*
25	42.18	40.37	49.56	57.21	59.73	67.98	34.64	*
26	40.86	41.96	50.73	60.05	68.37	74.21	40.78	*
27	46.28	41.93	54.02	63.99	63.13	77.10	42.14	*
28	45.50	41.91	53.24	59.45	67.81	70.16	41.41	*
29	43.70	40.46	55.74	62.34	60.81	70.43	41.39	*

centre	9	10	11	12	13	14	15	16
variety								
1	45.43	53.33	73.42	53.44	53.43	63.79	51.99	75.68
2	44.62	54.72	76.54	48.23	50.07	64.71	47.56	67.17
3	48.47	52.07	83.52	50.57	54.24	65.02	52.13	65.52
4	48.33	54.32	74.97	52.86	54.67	63.84	53.03	66.64
5	47.41	46.74	75.97	48.70	48.70	60.48	48.03	65.48
6	45.52	53.63	79.44	47.83	50.04	66.73	46.03	69.03
7	46.44	52.90	79.44	48.97	57.67	63.24	56.19	63.70
8	49.63	54.90	84.14	51.80	54.70	63.14	54.50	66.78
9	46.97	50.87	81.72	46.71	49.39	63.69	46.37	59.83
10	45.13	49.00	73.51	49.57	48.57	60.67	46.35	64.80
11	48.47	54.69	73.60	47.44	54.83	64.68	52.02	61.47

12	44.08	50.15	71.49	50.93	47.89	60.70	50.44	60.59
13	45.48	52.05	75.32	48.05	52.15	59.81	50.19	60.37
14	48.25	50.19	73.03	53.89	51.81	62.12	55.10	67.19
15	47.70	54.95	78.24	49.21	51.50	63.11	58.33	66.23
16	47.78	50.25	71.85	47.68	53.85	66.63	52.43	61.89
17	49.09	52.62	75.90	50.56	51.15	66.38	57.44	70.97
18	47.38	51.57	74.40	52.71	58.04	67.64	52.69	61.88
19	47.95	50.66	75.05	54.05	49.30	63.61	48.40	62.23

1990

centre variety	1	2	3	4	5	6	7	8
1	41.84	50.17	66.86	55.20	43.65	42.54	85.84	46.45
2	42.20	50.77	78.76	59.52	43.09	46.10	75.92	44.34
3	41.76	51.08	75.82	59.99	44.60	44.67	82.64	43.20
4	43.03	54.26	85.85	61.57	43.73	47.66	80.59	44.47
5	39.50	46.16	69.22	57.77	43.18	44.67	81.41	40.42
6	44.92	51.10	74.85	61.60	44.41	44.99	81.81	41.06
7	40.79	47.62	81.91	62.19	44.68	45.57	79.43	41.95
8	42.45	50.84	76.36	61.61	43.32	45.62	77.84	43.51
9	39.54	43.89	74.04	56.68	39.84	40.44	80.70	42.05
10	37.59	47.41	80.44	60.59	44.14	46.98	78.97	40.51
11	40.33	55.88	76.56	56.54	42.62	43.71	80.82	42.52
12	43.69	47.92	75.39	56.28	40.49	42.81	73.92	41.88
13	41.61	46.32	72.64	55.51	40.60	40.20	77.45	42.09
14	48.50	48.13	78.52	57.26	42.05	45.54	77.90	43.83
15	41.71	49.61	73.75	57.52	41.15	44.22	75.58	40.97
16	43.11	49.34	76.59	54.99	44.74	42.76	76.74	39.12
17	44.47	50.54	86.31	64.60	45.02	47.71	82.28	46.10
18	44.10	51.41	76.83	63.07	45.82	47.02	78.55	43.12
19	44.44	49.76	88.50	62.07	43.87	47.43	85.14	42.55
20	44.87	52.79	72.34	61.77	39.72	46.97	82.04	42.64
21	44.81	50.49	85.75	62.59	44.87	45.24	80.40	42.33
22	48.06	53.48	80.13	63.25	45.59	48.06	83.25	46.16
23	44.88	50.69	80.69	57.98	45.61	46.92	79.93	41.77
24	44.68	48.75	73.79	58.72	43.31	47.03	87.34	42.69
25	44.11	49.34	78.34	61.83	43.17	45.82	80.82	44.14
26	41.78	48.62	75.57	58.31	44.38	43.00	75.28	43.75
27	46.37	54.02	85.64	62.69	48.34	46.25	81.11	45.40
28	41.62	50.25	78.69	56.71	44.57	43.06	79.82	38.62
29	40.92	55.04	77.16	59.49	45.23	49.72	82.29	46.38

centre	9	10	11
variety			
1	41.08	52.44	44.56
2	39.01	48.04	44.74
3	42.61	55.16	42.61
4	42.68	55.77	45.63
5	40.55	48.27	38.82
6	47.00	52.25	47.72
7	46.15	51.87	38.44
8	45.59	54.38	46.25
9	41.61	45.80	38.40
10	40.06	48.40	37.38
11	41.93	49.40	44.44
12	41.58	50.32	41.66
13	46.43	48.87	40.64
14	42.29	52.35	43.32
15	45.34	51.41	42.80
16	42.58	47.14	40.65
17	45.56	55.87	46.56
18	43.08	50.61	47.19
19	44.38	51.77	44.67
20	42.96	45.01	40.50
21	43.77	52.96	45.55
22	44.23	52.07	47.66
23	46.44	56.90	43.61
24	43.68	54.71	46.21
25	40.12	52.69	45.45
26	40.80	54.63	40.07
27	43.13	51.81	45.63
28	44.63	53.10	39.69
29	45.23	54.01	46.72

1991

centre variety	1	2	3	4	5	6	7	8
1	57.86	60.12	46.90	57.97	53.06	67.72	65.96	60.32
2	54.88	62.34	48.08	58.51	55.42	64.44	64.30	58.32
3	55.65	55.46	44.45	55.70	53.82	62.91	66.69	54.75
4	51.85	59.66	46.58	57.82	57.03	60.31	68.43	60.05
5	49.74	53.90	43.13	54.97	53.54	55.30	62.15	59.03
6	55.71	56.76	43.05	58.23	55.44	64.89	62.13	58.51
7	51.39	51.40	39.11	56.01	47.84	57.37	59.29	53.97
8	53.38	55.81	45.12	56.93	55.77	55.04	59.36	55.98
9	51.26	48.45	43.33	56.07	50.95	65.73	57.59	48.42
10	52.76	48.60	38.26	52.50	45.94	57.07	57.68	46.93
11	52.56	56.31	46.44	58.18	51.32	63.01	65.11	56.23
12	53.50	52.70	46.07	57.62	51.57	62.40	64.93	54.56
13	50.42	52.23	42.11	54.20	51.15	57.04	59.67	57.80
14	55.20	55.97	48.65	58.54	54.75	64.25	62.98	57.16
15	52.23	56.50	44.73	52.92	52.52	60.22	61.90	54.80
16	50.07	48.74	40.43	54.50	53.05	63.13	61.46	54.24
17	55.74	57.09	44.00	57.70	52.00	63.05	63.58	55.57
18	56.31	58.22	44.73	58.31	57.76	66.46	64.97	58.79
19	56.44	57.05	44.66	65.82	54.86	68.20	61.98	62.19
20	58.17	59.65	47.74	60.31	56.49	66.38	68.24	57.22
21	56.14	52.18	49.33	63.45	54.31	67.16	63.09	62.35
22	54.61	57.89	49.00	58.56	58.31	68.04	67.37	58.66
23	55.80	53.14	45.62	58.64	55.16	67.21	65.88	55.27
24	54.47	55.64	46.04	59.70	55.30	63.05	67.03	60.26
25	50.56	56.71	43.59	58.06	51.14	65.35	66.29	61.82
26	55.92	57.84	46.77	56.32	50.79	57.80	61.52	56.49
27	54.27	57.21	48.57	58.56	56.91	62.69	63.54	59.73
28	53.84	60.60	48.03	52.33	52.32	60.08	65.52	56.35
29	56.50	55.85	47.71	60.58	59.43	63.01	64.87	59.86

centre	9	10	11	12	13
variety					
1	54.32	64.44	51.06	66.87	72.66
2	54.40	62.24	51.56	66.11	73.21
3	50.70	60.95	50.68	63.99	68.04
4	52.04	57.00	52.20	61.72	71.93
5	52.23	55.07	46.68	62.50	70.96
6	55.98	58.59	48.66	61.79	72.46
7	49.89	54.21	48.67	56.12	66.82
8	51.84	57.39	50.73	64.33	70.64
9	53.43	56.41	49.90	56.61	66.46
10	45.26	53.75	47.72	54.86	67.00
11	54.08	56.92	53.84	64.33	71.56
12	51.77	54.04	51.29	58.48	70.21
13	45.86	55.24	48.68	59.04	65.87
14	53.12	60.24	49.17	62.84	68.60
15	*	56.70	48.78	58.92	73.68
16	48.89	53.93	49.15	60.91	67.74
17	51.48	57.83	51.30	62.79	70.09
18	53.81	60.03	53.77	62.88	76.91
19	56.05	63.40	53.15	68.04	76.40
20	52.01	62.31	53.15	64.88	67.92
21	57.52	62.28	52.89	65.22	73.98
22	54.93	65.60	55.52	68.17	77.72
23	53.52	56.81	51.87	67.68	70.24
24	52.92	57.46	49.62	64.31	70.07
25	52.05	53.90	49.80	59.70	68.42
26	55.71	54.60	50.51	65.15	67.74
27	57.26	60.81	51.96	63.81	76.34
28	50.04	59.26	50.76	62.74	68.61
29	49.79	60.92	50.66	62.38	70.95

Appendix B

Genstat programs for Chapter 5

B.1 Algorithm 5.2.1: modified FITCON

B.1.1 Program description

The algorithm can be described compactly as:

SETUP
`FOR` *each iteration:*
do ENVIRONMENT
do SENSITIVITY
if `monitor` = 1, print iteration results
check for convergence
`ENDFOR`
if `monitor` = 0, print iteration results
do SED
`STOP`

SETUP:

(a) declares factors, scalars and variates. For example,
   i. `cons`: factor for the general mean
   ii. `variety`: factor for variety levels (1 to `v`)
   iii. `centre`: factor for centre levels (1 to `c`)
   iv. `yield`: variate for yield
   v. `kcoef`: matrix of coeffiecients to be used in calculating standard errors of differences.

(b) inputs data from a file

ENVIRONMENT:

(a) set up a variate `sens` i.e (T of (5.3)) from previous sensitivities `beta2` initially set to 1.

(b) fit the model `variety + centre.sens`:

(c) keep centre means in `cmns` and calculate variety means (`vmeans`).

SENSITIVITY:

(a) set up a variate **sensv** i.e (M of (5.4)) from **cmns**

(b) fit the model **variety + variety.sensv**:

(c) keep sensitivities in **beta** and standard errors of sensitivities in **ser**

(d) scale sensitivities to unit mean

SED:

(a) set up the matrix of coefficients K (**kcoef**)

(b) fit the model **const + variety + centre**: so as to obtain the value of the general mean

(c) keep estimates of effects in **alleff** and their variance matrix in **vcov**

(d) calculate the variance matrix for variety means **varmns**

(e) calculate standard errors of differences as **vse2**

(f) adjust degrees of freedom for units variance from $n - v$ to $n - 2v + 1$ (see Section 5.2.3).

B.1.2 Program listing

```
SET [inprint=*] "----------SETUP---------------------"
OUTPUT [width=65] 1
SCALAR nn,v,c,maxit,critical,monitor;48,8,6,25,0.0001,0
UNIT [nn]
OPEN   'cloverg.d'; CHAN = 2

FACTOR [LEVELS =c] centre
FACTOR [LEVELS=v] variety
&      [level=1] const;values=!(#nn(1))
VARIATE yield,sens,sensv
&   [nvalues=v] vmeans,vmns,beta,beta2,ser
&       [nvalues=c] cmns,envmeans,cmeans
SYMMETRICMATRIX [rows=v] vse, vse2

READ [CHAN = 2] variety,centre,yield

CALC beta2=1 & np1,np2,nv2=v+2,c,v & np,nv1 = np2,v+1
VARIATE [nvalues=np] alleff
MATRIX [rows=v;columns=np] kcoef;values=!(#v(0)#np)

FOR iter = 1 ... maxit
               "----------ENVIRONMENT---------------------"
"sets up a variate sens with sensitivities corresponding
 to appropriate positions of varieties in the factor centre"
```

```
  FOR i = 1 ... v
    RESTRICT sens,yield;cond=(variety==i);saveset=vv
    CALC ln = nobs(vv) & sens$[vv$[1...ln]] = #ln(beta2$[i])
  ENDFOR
  RESTRICT sens,yield

  VCOMP [fixed=variety+centre.sens]
  REML [print=*] yield
  VKEEP terms=variety+centre.sens;means=varmn,*;\
                     effects=*,cenmn
  EQUATE varmn;vmeans & cenmn;cmns
  CALC  vm= mean(cmns) & vmeans = vmeans + beta2*vm

            "----------SENSITIVITY---------------------"
"sets up a variate sensv with sensitivities corresponding
 to appropriate positions of varieties in the factor variety"

  FOR i = 1 ... c
    RESTRICT sensv,yield;cond=(centre==i);saveset=vv
    CALC ln = nobs(vv) & sensv$[vv$[1...ln]] = #ln(cmns$[i])
  ENDFOR
  RESTRICT sensv,yield


" regression of yields for each variety against centre means"
  MODEL yield
  FIT [print=*] variety + variety.sensv
  RKEEP estimate=beta1;se=se2
  CALC beta$[1...v] = beta1$[nv1...nv2] & ser$[1...v] = se2$[nv1...nv2]

" scale sensitivities to a unit mean"

  CALC cv = v/sum(beta)  & beta = beta*cv
  IF monitor==1
    PRINT 'iteration ...',iter;13,3;*,0
    & 'variety mean','sensitivity';15
    &     '             ','old    new'
    &  vmeans,beta2,beta,ser;12,7,7,7;4(3)
  ENDIF
  EXIT  MAX(ABS(beta-beta2)) .lt. critical

  EQUATE beta;beta2

 ENDFOR

 IF monitor==0
   PRINT 'variety mean','sensitivity';15
    &     '             ','old    new'
```

```
   &  vmeans,beta2,beta,ser;12,7,7,7;4(3)
 ENDIF

           "----------SED---------------------"
" final reml run to calculate sed"

 FOR i = 1 ... v
   RESTRICT sens,yield;cond=(variety==i);saveset=vv
   CALC ln = nobs(vv) & sens$[vv$[1...ln]] = #ln(beta$[i])
    & ii = i+1 & kcoef$[i;1,ii] = 1 & kcoef$[i;np1...np] = beta$[i]/c
 ENDFOR
 RESTRICT sens,yield

 VCOMP [fixed=const+variety+centre.sens;cons=o]
 REML [print=comp] yield
 VKEEP [fullcov=vcov] terms=const + variety+centre.sens;\
                      means=*,varmn,*;effects=ceff,veff,cenmn
 EQUATE veff;vmeans & cenmn;cmeans & ceff;alleff
 CALC  cmns = cmns + mean(vmeans) + alleff$[1]
 &  alleff$[2...np] = vmeans$[1...v],cmeans$[1...c]
 &    vmeans = kcoef*+alleff & vse = kcoef*+vcov*+trans(kcoef)

 FOR i = 2...v
  CALC ii = i-1 : FOR j = 1 ... ii
  CALC  vse2$[j;i] = SQRT(vse$[j;j] + vse$[i;i] - 2*vse$[i;j])
 ENDFOR  &

 CALC nn = NOBS(yield) & vse2 = vse2 *SQRT((nn - v-c+1)/(nn-v - v +2-c))
 PRINT  vse2;;4
STOP
```

B.1.3 Output from Genstat program for modified FIT-CON

```
Genstat 5  Release 2.2  (Sun/Unix)
                  Sat Jun 11 21:22:29 1994
Copyright 1990, Lawes Agricultural Trust
                  (Rothamsted Experimental Station)

     "SET [inprint=*]"
     OUTPUT [width=65] 1
     SCALAR nn,v,c,maxit,critical,monitor;46,6,10,25,0.0001,0
     UNIT [nn]
     OPEN   'ps80.dat'; CHAN = 2

     FACTOR [LEVELS =c] centre
     FACTOR [LEVELS=v] variety
     &      [level=1] const;values=!(#nn(1))
```

```
VARIATE yield,sens,sensv
&   [nvalues=v] vmeans,vmns,beta,beta2,ser
&         [nvalues=c] cmns,envmeans,cmeans
SYMMETRICMATRIX [rows=v] vse, vse2

READ [CHAN = 2] variety,centre,yield

  Identifier   Minimum      Mean   Maximum    Values   Missing
       yield     3.990     6.364     8.930        46         0

CALC beta2=1 & np1,np2,nv2=v+2,c,v & np,nv1 = np2,v+1
VARIATE [nvalues=np] alleff
MATRIX [rows=v;columns=np] kcoef;values=!(#v(0)#np)

FOR iter = 1 ... maxit

-------------SOME LINES DELETED ----------

 IF monitor==0
   PRINT 'variety mean','sensitivity';15

 variety mean    sensitivity

     &     '              ','old    new'

             old    new

     &  vmeans,beta2,beta,ser;12,7,7,7;4(3)

    vmeans  beta2   beta    ser
     5.734  0.741  0.741  0.062
     6.027  0.777  0.777  0.062
     6.219  0.902  0.901  0.072
     6.512  1.296  1.296  0.072
     6.238  1.242  1.242  0.081
     6.535  1.043  1.043  0.081

 ENDIF


" final reml run to calculate sed"

 FOR i = 1 ... v
   RESTRICT sens,yield;cond=(variety==i);saveset=vv
   CALC ln = nobs(vv) & sens$[vv$[1...ln]] = #ln(beta$[i])
    & ii = i+1 & kcoef$[i;1,ii] = 1
    & kcoef$[i;np1...np] = beta$[i]/c
 ENDFOR
 RESTRICT sens,yield
```

```
  86
  87   VCOMP [fixed=const+variety+centre.sens;cons=o]
  88   REML [print=comp] yield

88.................................................................

*** Estimated Components of Variance ***

                                                   s.e.
*units*                        0.05946      0.01510

  89   VKEEP [fullcov=vcov] terms=const + variety+centre.sens;\
  90                           means=*,varmn,*; effects=ceff,veff,cenmn


  91   EQUATE veff;vmeans & cenmn;cmeans & ceff;alleff
  92   CALC  cmns = cmns + mean(vmeans) + alleff$[1]
  93   &  alleff$[2...np] = vmeans$[1...v],cmeans$[1...c]
  94   &    vmeans = kcoef*+alleff & vse = kcoef*+vcov*+trans(kcoef)
  95
  96   FOR i = 2...v
  97    CALC ii = i-1 : FOR j = 1 ... ii
  98    CALC  vse2$[j;i] = SQRT(vse$[j;j] + vse$[i;i] - 2*vse$[i;j])
  99   ENDFOR  &
 100
 101   CALC nn = NOBS(yield)
        & vse2 = vse2 *SQRT((nn - v-c+1)/(nn-v - v +2-c))
 102   PRINT  vse2;;4

          vse2

   1          *
   2     0.1191          *
   3     0.1232     0.1232          *
   4     0.1241     0.1241     0.1257          *
   5     0.1739     0.1739     0.1804     0.1826          *
   6     0.1692     0.1692     0.1753     0.1773     0.1886

              1          2          3          4          5

   6          *

              6

 103  STOP

******** End of job.  Maximum of 20048 data units used at line 69
 (30556 left)
```

B.2 Algorithm 5.3.1: modified REML

B.2.1 Program description

The algorithm can be written as:

SETUP
FOR *each iteration:*
do ENVIRONMENT
do SENSITIVITY
if monitor = 1, print iteration results
check for convergence
ENDFOR
if monitor = 0, print iteration results
do ENVIRONMENT
STOP

The modules ENVIRONMENT and SENSITIVITY are the same as in Appendix B.1.1 except that:

(a) Step (b) of ENUMERATE is replaced by:
fit the model `variety : centre.sens`

(b) Step (b) of SENSITIVITY is replaced by:
fit the model `variety : variety.sensv`

(c) In the last execution of ENVIRONMENT variety means and standard errors of differences are printed.

B.2.2 Program listing

```
SET [inprint=*] "-----------------SETUP---------------"
OUTPUT [width=65] 1
SCALAR nn,v,c,maxit,critical,monitor;46,6,10,25,.0001,1
UNIT [nn]
OPEN   'ps80.dat'; CHAN = 2

FACTOR [LEVELS =c] centre
FACTOR [LEVELS=v] variety
VARIATE yield,sens,sensv
& [nvalues=v] vmeans,vmns,beta,beta2,ser
&        [nvalues=c] cmns,envmeans,cmeans
SYMMETRICMATRIX [rows=v] vse, vse2

READ [chann = 2] variety,centre,yield
```

```
CALC beta2=1 & np1,np2,nv2=v+2,c,v & np,nv1 = np2,v+1
& nv1,nv2=nv1,nv2+1

FOR iter = 1 ... maxit

                 "---------------ENVIRONMENT---------------"
"sets up a variate sens with sensistivities corresponding
 to appropriate positions of varieties in the factor centre"

FOR i = 1 ... v
RESTRICT sens,yield;cond=(variety==i);saveset=vv
CALC ln = nobs(vv) & sens$[vv$[1...ln]] = #ln(beta2$[i])
ENDFOR
RESTRICT sens,yield

VCOMP [fixed=variety] centre.sens
REML [print=*] yield
VKEEP terms=variety+centre.sens;means=varmn,*;\
                     effects=*,cenmn
EQUATE varmn;vmeans & cenmn;cmns

CALC  vm= mean(cmns) & vmeans = vmeans + beta2*vm

                  "-------------SENSITIVITY---------------"
"sets up a variate sensv with sensitivities corresponding
 to appropriate positions of varieties in the factor variety"

FOR i = 1 ... c
RESTRICT sensv,yield;cond=(centre==i);saveset=vv
CALC ln = nobs(vv) & sensv$[vv$[1...ln]] = #ln(cmns$[i])
ENDFOR
RESTRICT sensv,yield


" regression of yields for each variety against centre means"

VCOMP [variety + variety.sensv]
REML [print=*] yield
VKEEP [fullcov=vslope] terms=variety+variety.sensv;effects=*,beta1
EQUATE beta1;beta

FOR l = nv1 ... nv2 : calc l2 = l - v - 1
& ser$[l2] = sqrt(vslope$[l;l]) : endfor

" scale sensitivities to a unit mean"

CALC cv = v/sum(beta)  & beta = beta*cv
IF monitor==1
PRINT 'iteration ...',iter;13,3;*,0
```

```
PRINT 'variety mean','senstivity';15
&     '              ','old    new'
PRINT  vmeans,beta2,beta,ser;12,7,7;4(3)
ENDIF
EXIT MAX(ABS(beta-beta2)) .lt. critical

EQUATE beta;beta2
ENDFOR

IF monitor==0
PRINT 'variety mean','senstivity';15
&     '              ','old    new'
PRINT  vmeans,beta2,beta,ser;12,7,7;4(3)
ENDIF

                   "------------ENVIRONMENT---------------"
" final reml run to calculate sed "

FOR i = 1 ... v
RESTRICT sens,yield;cond=(variety==i);saveset=vv
CALC ln = nobs(vv) & sens$[vv$[1...ln]] = #ln(beta$[i])
ENDFOR
RESTRICT sens,yield

VCOMP [fixed=variety] centre.sens
REML [print=components,means;pterms=variety;pse=a] yield
VKEEP terms=variety;varmeans=vse
STOP
```

B.2.3 Output from Genstat program for modified REML

```
Genstat 5  Release 2.2  (Sun/Unix)
                  Sat Jun 11 21:22:29 1994
Copyright 1990, Lawes Agricultural Trust
                  (Rothamsted Experimental Station)

"SET [inprint=*]"
OUTPUT [width=65] 1
SCALAR nn,v,c,maxit,critical,monitor;46,6,10,25,.0001,0
UNIT [nn]
OPEN   'ps80.dat'; CHAN = 2

FACTOR [LEVELS =c] centre
FACTOR [LEVELS=v] variety
VARIATE yield,sens,sensv
& [nvalues=v] vmeans,vmns,beta,beta2,ser
```

```
&         [nvalues=c] cmns,envmeans,cmeans
SYMMETRICMATRIX [rows=v] vse, vse2

READ [chann = 2] variety,centre,yield

  Identifier   Minimum      Mean   Maximum    Values   Missing
       yield     3.990     6.364     8.930        46         0

 CALC beta2=1 & np1,np2,nv2=v+2,c,v & np,nv1 = np2,v+1
 &  nv1,nv2=nv1,nv2+1

 FOR iter = 1 ... maxit

-----------------SOME LINES ARE DELETED --------------------

 variety mean    sensitivity

 &     '            ','old    new'


           old    new

 PRINT  vmeans,beta2,beta,ser;12,7,7;4(3)

    vmeans  beta2   beta         ser
     5.734  0.741  0.741       0.063
     6.027  0.777  0.777       0.063
     6.220  0.902  0.902       0.073
     6.514  1.297  1.297       0.073
     6.245  1.241  1.241       0.082
     6.540  1.042  1.042       0.082

 ENDIF

" final reml run to calculate sed "

 FOR i = 1 ... v
 RESTRICT sens,yield;cond=(variety==i);saveset=vv
 CALC ln = nobs(vv) & sens$[vv$[1...ln]] = #ln(beta$[i])
 ENDFOR
 RESTRICT sens,yield

 VCOMP [fixed=variety] centre.sens
 REML [print=components,means;pterms=variety;pse=a] yield


88.................................................................
```

```
*** Estimated Components of Variance ***

                                          s.e.
centre.sens                  1.554      0.7397
*units*                    0.05944     0.01510


*** Table of mean effects for variety ***


    variety           1           2           3           4
                  5.734       6.027       6.220       6.514


    variety           5           6
                  6.245       6.540

Standard errors of differences between pairs


   variety  1          *
   variety  2     0.1100           *
   variety  3     0.1294      0.1231           *
   variety  4     0.2469      0.2344      0.1936           *
   variety  5     0.2534      0.2426      0.2124      0.1683
   variety  6     0.1951      0.1868      0.1696      0.1907

               variety  1  variety  2  variety  3  variety  4


   variety  5          *
   variety  6     0.1898           *

               variety  5  variety  6

  89
  90   stop

******** End of job.  Maximum of 15800 data units used at line 71
 (34804 left)
```

B.3 Algorithm 5.4.1: SREML

B.3.1 Program description

The algorithm can be written as:

```
SETUP
FOR each iteration:
do STEP 1 to sc step 4
if monitor = 1, print iteration results
check for convergence
ENDFOR
if monitor = 0, print iteration results
do STEP 6
STOP
```

The steps are described in detail in Section 5.4.4.

B.3.2 Program listing

```
SET [inprint=*]
OUTPUT [WIDTH=65] 1

"
  ...................................................................
  A REML PROGRAM FOR A FULL ANALYSIS OF A VARIETIES x CENTRES TABLE
                    WITH SENSISTIVITIES
            (using Fisher's scoring method)
      Written by F.Nabugoomu at Edinburgh, Jan 1994
  ...................................................................

"
 SCALAR nn,v,c,maxit,critical,monitor;46,6,10,50,0.0005,1

 UNITS      [nn]
 VARIATE     Yd
 OPEN 'ps80.dat';chann=2

 FACTOR      [levels=v] Variety & [levels=c] Centre
 READ        [print=*;chann=2] Variety,Centre,Yd

  CALC nv = v*(v > c) + c * (c>= v) & gma,ct=1 & np = v+2

 VARIATE [nvalues=v] beta2,vmns,ser,ymn
  & [nvalues=np] s,r,gold,gnew
  &     [nvalues=nn] B[1...nv],W[1...c],WV[1...c]
```

```
MATRIX [rows=v;colu=nn] XT  & [rows=c;colu=nn] WT,WVT,WLT
& [rows=c;columns=c] U,C,Q,Q0,Q1,Q2,Q3,U1,U2,U3,U4



DIAGONALMATRIX [rows=nn] I1  &  [rows=c] I2
SYMMETRICMATRIX [rows=np] f  &  [rows=v] VSE, IX
CALC I1,I2,gold = 1

FOR i = 1 ... v  "sets up X'"
RESTRICT Yd;cond=Variety==i;saveset=SV
CALC B[i]=expand(SV;nn) : ENDFOR : RESTRICT Yd
EQUATE !P(B[1 ... v]);XT

FOR i = 1 ... c "sets up columns for B"
RESTRICT Yd;cond=Centre==i;saveset=SV
CALC B[i],WV[i]=expand(SV;nn) : ENDFOR : RESTRICT Yd

VCOMP [fixed=Variety] rand=Centre "STEP 0"
REML [print=*] Yd
VKEEP [sigma2=s0] terms=Centre;components=gma
CALC gma = gma/s0 & gold$[1,2] = gma,s0
 & IX = inverse(rtproduct(XT;XT))

FOR iter=1 ... maxit "STEP 5 "
EQUATE !P(B[ ]);WT  "sets up W'"

CALC M1= IX *+ rtprod(XT;WT) & ymn = IX *+(XT*+Yd)
 & Q0 = rtprod(WT;WT) - (rtprod(WT;XT) *+ M1) "STEP 1(a)"
 &   V0 = WT*+Yd - rtprod(WT;XT) *+ ymn         "STEP 1(b)"
& C = Q0 + I2/gma & b1 = solution(C;V0) "STEP 1(c)"
&    a1 = IX*+(XT*+(Yd-(trans(WT)*+b1)))
&    C = inverse(C) & U=(I2-C/gma)/gma & Q = I2 -Q0*+C

"STEP 2 -- INFORMATION & SCORE FOR GAMMA & SO"
CALC f$[1;1] = trace(U *+ U) & f$[1;2] = trace(U)/s0
&    f$[2;2] = (nn-v)/(s0*s0)
& s$[1] = -s0*f$[1;2] + (trans(b1/gma)*+b1/gma)/s0
 & s$[2] = trans(Yd) *+ (Yd - trans(XT)*+a1 - trans(WT)*+b1)/s0/s0
 & s$[2] = - s0*f$[2;2] + s$[2]

FOR i = 1 ... v "STEP 3(a)"
CALC W□ = WV□ & k,l = i+ (1,2)
RESTRICT W□,Yd;condition=Variety .ne. i
CALC W□ = 0 : RESTRICT Yd,W□ : EQUATE !P(W□);WVT

CALC  M2 = IX *+ rtprod (XT;WVT) "STEP 3(b) & (c)"
 & Q1,Q2 = rtprod(WVT;WT,WVT) - rtprod(WVT;XT)*+(M1,M2)
 & V1 = WVT*+Yd - rtprod(WVT;XT)*+ymn
```

```
 &    Q3,U1 = Q1*+(C,Q) & V2,U2 = V1,Q2 - Q3*+(V0,Q1)

"STEP 3(d) & (e)"
CALC s$[1] = -gma* 2*(trace(U1) - trans(b1/gma)*+V2/s0)
&    f$[1;1] = 2*gma*trace(U1*+U) & f$[2;1] = 2*gma*trace(U1)/s0
&  f$[1;1] = 2*gma*gma*(trace(U2*+U) + trace(U1*+U1))

IF k .ge. 3  "STEP 3(f)i"
FOR j = 3...k :  calc W[] = WV[]
RESTRICT W[],Yd;cond=Variety .ne. (j-2)
CALC W[] = 0 : restrict Yd,W[]  : EQUATE !P(W[ ]);WLT

 "STEP 3(f)ii to iv"
CALC  Q4,Q3 = rtprod(WLT;WT,WVT) - rtprod(WLT;XT)*+(M1,M2)
 &  Q2,U3 = Q4*+(C,Q) & U4 = Q3 - Q2*+Q1
 & f$[j;1] = 2*gma*gma*(trace(U4*+U) + trace(U3*+U1))

ENDFOR : ENDIF : ENDFOR

CALC f = inv(f) &  gnew = gold + f*+s "STEP 4(a)"
& gma,s0= gnew$[1,2]  & ss = sum(gnew) - gma-s0

FOR i= 1 ...v : restrict B[1...c];cond=Variety==i "STEP 4(b)"
CALC l= i+2 & ct,gnew$[l] = gnew$[l]*v/ss
& ct = gnew$[l]/gold$[l] & B[ ] = B[ ]*ct
& ser$[i] =  SQRT(2*f$[l;l]) : ENDFOR : RESTRICT B[ ]

IF monitor==1
PRINT 'ITERATION ...',iter;*,3;*,0
& 'variety means';12 & [iprint=*] a1;12
& 'old ests', 'new ests';12 & [iprint=*] gold,gnew;12;5
ENDIF

CALC old = gold & new = gnew & old$[1],new$[1] = 0
&    cdif = sum(abs(old-new))/(v+1)
" EXIT cdif .lt. critical:"
CALC gold=gnew
ENDFOR "STEP 5"

IF monitor==0
PRINT 'variety means' & [iprint=*] a1;12
& 'old ests', 'new ests';12 & [iprint=*] gold,gnew;12;5
ENDIF

EQUATE !P(B[ ]);WT
"STEP 6  S.E.d"
CALC  Va = rtprod(XT;WT)*+((inverse(rtprod(WT;WT)+I2/gnew$[1])) \
                        **+rtprod(WT;XT))
&    Va = inverse(rtprod(XT;XT) - Va)
```

```
FOR j = 1...v & i = 1...j
IF (i==j) : CALC VSE$[i;i] = !(*)
ELSE
CALC VSE$[i;j] = SQRT(gnew$[2]*(Va$[i;i]+Va$[j;j]-2*Va$[i;j]))
ENDIF :  ENDFOR &

PRINT 'standard error of difference'
& VSE;12;4
STOP
```

B.3.3 Output from SREML

```
Genstat 5  Release 2.2  (Sun/Unix)
                  Sat Jun 11 21:22:29 1994
Copyright 1990, Lawes Agricultural Trust
                  (Rothamsted Experimental Station)

  1
  2    SET [inprint=*]

 ITERATION ...  1

 variety means

                         1

           1       5.734
           2       6.027
           3       6.206
           4       6.579
           5       6.429
           6       6.606

    old ests    new ests

     9.04924    13.51676
     0.15700     0.15703
     1.00000     0.59865
     1.00000     0.64231
     1.00000     0.80615
     1.00000     1.19757
     1.00000     1.59750
     1.00000     1.15783

 ITERATION ...  2

 variety means
```

```
                        1

          1         5.734
          2         6.027
          3         6.278
          4         6.590
          5         5.811
          6         6.310

   old ests     new ests

   13.51676     14.14560
    0.15703      0.09187
    0.59865      0.70181
    0.64231      0.74864
    0.80615      0.85395
    1.19757      1.23984
    1.59750      1.32862
    1.15783      1.12715

----------- SOME LINES DELETED -------------------

 ITERATION ... 11

 variety means
                        1

          1         5.734
          2         6.027
          3         6.223
          4         6.518
          5         6.230
          6         6.527

   old ests     new ests


   26.29680     26.08162
    0.05948      0.05948
    0.73879      0.73889
    0.77519      0.77530
    0.89498      0.89508
    1.28688      1.28702
    1.25163      1.25294
    1.05252      1.05076
```

```
ITERATION ... 12

variety means

                         1

           1        5.734
           2        6.027
           3        6.223
           4        6.518
           5        6.229
           6        6.528


   old ests     new ests


   26.08162     26.24027
    0.05948      0.05948
    0.73889      0.73883
    0.77530      0.77523
    0.89508      0.89502
    1.28702      1.28693
    1.25294      1.25195
    1.05076      1.05205


           f (inverse of information matrix)

  1   117.86851
  2    -0.05053     0.00011
  3    -0.28211     0.00000     0.00615
  4    -0.29601     0.00000     0.00419     0.00655
  5    -0.35190     0.00000     0.00498     0.00523     0.00871
  6    -0.50599     0.00000     0.00717     0.00752     0.00905
  7    -0.38103     0.00000     0.00540     0.00566     0.00654
  8     0.00000     0.00000     0.00000     0.00000     0.00000

                1           2           3           4           5

  6     0.01543
  7     0.00940     0.01559
  8     0.00000     0.00000     0.00000

                6           7           8
```

```
 standard error of difference


            VSE

  1           *
  2      0.1100          *
  3      0.1286     0.1223          *
  4      0.2445     0.2319     0.1929          *
  5      0.2579     0.2468     0.2173     0.1679          *
  6      0.1984     0.1898     0.1722     0.1871     0.1900

              1          2          3          4          5

  6           *

              6


******** End of job.  Maximum of 25678 data units used at line
 113 (24926 left)
```

Appendix C

Algorithm 6.5.1: Means from the model A + B + GA.B:

Program listing

```
output [wid=75] 1
units [60]
factor [levels=3] GA
factor [levels=6] B
factor [levels=10] A
variate YIELD

open 'wwr77.d';chann=2
read [chann=2] B,A,GA,YIELD
tabulate [class=A,B;margin=y] YIELD; means = mm
print mm;6;2
 set [inprint=*]
CALC NB = nlevels(B) & NA = nlevels(A)
&    NGRP = nlevels(GA) & NN    = nvalues(YIELD)
&    N2A,NY = NGRP*NA,NB & NEFF = 1+NA+NB + NY

VARIATE [nvalues=NA] ALEV
&       [nvalues=NB] BLEV

GETATTRIBUTE [levels] A, B;save=ALEV1, BLEV1
EQUATE !p(ALEV1[]);ALEV & !p(BLEV1[]); BLEV

FACTOR [nvalues=NN;level=1] GMEAN
VARIATE [nvalues=ALEV] AMNSE,AMEANS,AINDX,GINDX
&       [nvalues=BLEV] BMNSE,BMEANS,BINDX
&     [nvalues=NEFF] ALLEFF,AMISS
POINTER  PA; values= !P(1 ... NA)
SCALAR kk,GNO,GN2,N1,N2

SYMMETRICMATRIX [rows=ALEV] ASE,ASE3
```

```
MATRIX [rows=NB;COLUMNS=NGRP] HGRP
&      [rows=ALEV;columns=BLEV] ABMEANS
&      [rows=NA;COLUMNS=NEFF] K

CALC GMEAN = 1 &  K,AMISS = 0
& NB1 = 2 + NA & NB2 = NB1 + NB - 1

VCOMP [const=omit;fixed=GMEAN+A+B+GA.B]
REML [print=*] YIELD
VKEEP [full=VCOV] terms=GMEAN+A+B+GA.B;\
      effects=CON,AEF,BEF,GEF
EQUATE !p(CON,AEF,BEF);ALLEFF & GEF;HGRP

FOR g = 1 ... NGRP
  CALC    N1 = NB2 + 1 + (g-1)*NB
   &       N2 = NB2 + g*NB
   &   ALLEFF$[N1...N2] = HGRP$[1 ... NB;g]
ENDFOR

FOR k = 1 ... NB
  IF ALLEFF$[k] == !(*)
  CALC ALLEFF$[k],AMISS$[k] = 0,1
ENDIF : ENDFOR

FOR k = 1 ... NA
  CALC  kk = ALEV$[k] & k3 = k + 1
  RESTRICT GA;cond=A==kk;saveset=GRNO
  CALC  GN = GRNO$[1] &  GINDX$[k],GN2 = GA$[GN]
RESTRICT GA

CALC k2 = NB2 + 1 + NB*(GN2 - 1)
&    k4 = NB2 + NB + NB*(GN2 - 1)
&    K$[k;1,k3] = 1
&    K$[k;(NB1...NB2),(k2...k4)] = 1/NB
& ABMEANS$[k;1 ... NB] = ALLEFF$[1] + ALLEFF$[k3] \
                  + ALLEFF$[NB1...NB2] + ALLEFF$[k2 ...k4]
FOR j = 1 ... NB
  IF (HGRP$[j;GN2] .eq. !(*))
     CALC   ABMEANS$[k;j] = !(*)
  ENDIF
ENDFOR &

CALC   ASE = K*+VCOV*+trans(K)
 &    AMEANS,AINDX = K*+(ALLEFF,AMISS)
PRINT ABMEANS;7

FOR i = 1 ... NA
  IF (sum(AINDX) .gt. 0) .and. (AINDX$[i] .ne. 0)
    CALC AMEANS$[i] = !(*)
```

```
  ENDIF
ENDFOR

FOR i = 2 ... NA : CALC i2 = i-1
FOR j = 1 ... i2
  IF ((AMEANS$[i] .ne. !(*)) .and. (AMEANS$[j] .ne. !(*)))
     CALC ASE3$[j;i] = SQRT(ASE$[j;j]+ASE$[i;i] - 2*ASE$[j;i])
  ELSE : CALC ASE3$[j;i] = !(*)
  ENDIF
ENDFOR
CALC j2 = 1 + abs(round((i-1)/NA - .5))*NA
  &    AMNSE$[i]  = ASE3$[i;j2] : ENDFOR

PAGE
PRINT 'FACT A','MEAN','SED'
& [IPRINT=*] ALEV,AMEANS,AMNSE;;0,*,*
PAGE
PRINT '                         STANDARD ERROR OF DIFFERENCES (FACT A)'
&    [IPRINT=*] ASE3;7

delete [redefine=yes] K, ASE,ASE3
"=================================================================="
MATRIX  [rows=NB;columns=NEFF] K
SYMMETRICMATRIX [rows=BLEV] BSE, BSE3
VARIATE [nvalues=NGRP] NGS
FACTOR [nvalues=NA;levels=NGRP;values=GINDX$[1...#NA]] GFACT
TABULATE [class=GFACT] GINDX;nobs=NG1
EQUATE NG1;NGS : CALC K = 0


FOR k = 1 ... NB
  CALC  kk = BLEV$[k]
     &  k3 = k+1+NA & k5 = NA+1
     &  K$[k;1,k3] = 1 & K$[k;2...k5] = 1/NA
FOR g = 1 ... NGRP
  CALC  k2 = 1+NA+NB+(g - 1)* NB+k
     &    K$[k;k2] = NGS$[g]/NA
ENDFOR &

CALC   BSE = K*+VCOV*+trans(K)
&      BMEANS,BINDX = K*+(ALLEFF,AMISS)
&      BMEANS = MVINSERT(BMEANS;BINDX)

FOR i = 2 ... NB : CALC i2 = i-1
FOR j = 1 ... i2
  IF ((BINDX$[i] .gt. 0) .or. (BINDX$[j] .gt. 0))
     CALC BSE3$[j;i] = !(*)
  ELSE
     CALC BSE3$[j;i] = SQRT(BSE$[j;j]+BSE$[i;i] - 2*BSE$[j;i])
```

```
  ENDIF
ENDFOR
CALC j2 = 1 + abs(round((i-1)/NB - .5))*NB
  &    BMNSE$[i]  = BSE3$[i;j2]
ENDFOR

PAGE
PRINT  'FACT B','MEAN','SED'
& [IPRINT=*] BLEV,BMEANS,BMNSE;;0,*,*

PAGE
PRINT '           STANDARD ERROR OF DIFFERENCES (FACT B)'
&   [IPRINT=*] BSE3;7

STOP
```

Output for winter wheat data 1977

```
Genstat 5  Release 3.1  (Sun/Unix)    Tue Apr 12 21:47:12 1994
Copyright 1992, Lawes Agricultural Trust
 (Rothamsted Experimental Station)

   1
   2  output [wid=75] 1
   3  units [60]
   4  factor [levels=3] GA
   5  factor [levels=6] B
   6  factor [levels=10] A
   7  variate YIELD
   8
   9  open 'wwr77.d';chann=2
  10  read [chann=2] B,A,GA,YIELD

    Identifier   Minimum      Mean   Maximum    Values   Missing
         YIELD     3.990     6.364     8.930        60        14

    Identifier    Values   Missing    Levels
             B        60         0         6
             A        60         0        10
            GA        60         0         3

  11  tabulate [class=A,B;margin=y] YIELD; means = mm
  12  print mm;6;2

               mm
             B     1     2     3     4     5     6  Mean
             A
             1  5.79  5.96  5.97  6.56     *     *  6.07
             2  6.12  6.64  6.92  7.55     *     *  6.81
             3  5.12  4.65  5.04  5.13     *     *  4.99
             4  4.50  5.07  4.99  4.60     *     *  4.79
             5  5.49  5.59  5.59  5.83     *     *  5.62
             6  5.86  6.52  6.57  6.14     *     *  6.27
             7  6.55  6.91  7.59  7.91  7.34  7.17  7.25
             8  7.33  7.31  7.75  8.93  8.68  8.72  8.12
             9  6.37  6.99  7.19  8.33  7.91  8.04  7.47
            10  4.21  4.62     *     *  3.99  4.70  4.38

          Mean  5.73  6.03  6.40  6.78  6.98  7.16  6.36

  13   set [inprint=*]

               ABMEANS
          BLEV    1.00    2.00    3.00    4.00    5.00    6.00
          ALEV
```

1.00	5.491	5.813	6.046	6.931	6.840	6.925
2.00	6.228	6.551	6.783	7.668	7.578	7.663
3.00	4.790	5.044	5.114	4.992	4.492	5.202
4.00	4.595	4.849	4.919	4.797	4.297	5.007
5.00	5.430	5.684	5.754	5.632	5.132	5.842
6.00	6.077	6.331	6.402	6.279	5.779	6.489
7.00	6.550	6.910	7.590	7.910	7.340	7.170
8.00	7.270	7.592	7.825	8.710	8.619	8.704
9.00	6.621	6.944	7.176	8.061	7.971	8.056
10.00	4.288	4.542	4.613	4.490	3.990	4.700

FACT A	MEAN	SED
1	6.341	*
2	7.078	0.1691
3	4.939	0.4214
4	4.744	0.4214
5	5.579	0.4214
6	6.226	0.4214
7	7.245	0.2990
8	8.120	0.1619
9	7.472	0.1619
10	4.437	0.3686

STANDARD ERROR OF DIFFERENCES (FACT A)

1.00	*							
2.00	0.1691	*						
3.00	0.4214	0.4214	*					
4.00	0.4214	0.4214	0.1691	*				
5.00	0.4214	0.4214	0.1691	0.1691	*			
6.00	0.4214	0.4214	0.1691	0.1691	0.1691	*		
7.00	0.2990	0.2990	0.3406	0.3406	0.3406	0.3406	*	
8.00	0.1619	0.1619	0.4011	0.4011	0.4011	0.4011	0.2783	*
9.00	0.1619	0.1619	0.4011	0.4011	0.4011	0.4011	0.2783	0.1381
10.00	0.3686	0.3686	0.2156	0.2156	0.2156	0.2156	0.3055	0.3452
	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00

9.00	*	
10.00	0.3452	*

```
            9.00  10.00



    FACT B        MEAN          SED


         1        5.734           *
         2        6.026      0.4892
         3        6.222      0.4947
         4        6.547      0.2679
         5        6.204      0.4915
         6        6.576      0.2704


          STANDARD ERROR OF DIFFERENCES (FACT B)


       1.00      *
       2.00 0.4892      *
       3.00 0.4947 0.5832      *
       4.00 0.2679 0.5198 0.3987      *
       5.00 0.4915 0.4813 0.5099 0.4713      *
       6.00 0.2704 0.4104 0.3383 0.2366 0.3963      *

              1.00   2.00   3.00   4.00   5.00   6.00

******** End of job.  Maximum of 16380 data units used at line 72 (35404
left)
```

Appendix D

Genstat programs for Chapter 7

D.1 Yates & Cochran (1938) example

Program listing

```
output [width=65] 1
units [nvalues=60]
text vn; !T(MANCHURIA, SVANSOTA,VELVET,TREBI, PEATLAND)
text cn; !T(UNIVERSITY, WASECA, MORRIS, CROOKSTON,\
           'GRAND RAPIDS', DULUTH)
factor [levels=5] V
factor [levels=2] Y
factor [levels=6] C
factor [levels=2] GV
generate C,Y,V

read [serial=yes] yld
81.0  105.4 119.7 109.7 98.3
80.7   82.3  80.4  87.2 84.2
146.6 142.0 150.7 191.5 145.7
100.4 115.5 112.2 147.7 108.1
82.3   77.3  78.4 131.3  89.6
103.1 105.1 116.5 139.9 129.6
119.8 121.4 124.0 140.8 124.8
 98.9 61.9  96.2  125.5  75.7
 98.9 89.0  69.1  89.3 104.1
 66.4 49.9  96.7  61.9 80.3
 86.9 77.1  78.9  101.8 96.0
 67.7 66.7  67.4   91.8 94.1
 :

calc yld = yld/3
tabu[class=Y,V,C]yld;means=mm
print mm;6;2
TREATMENTSTRUCTURE V*C*Y - V.Y
ANOVA [print=aov] yld
MATRIX [rows=!t('Trebi vs rest');column=5;\
```

```
        values=-1,-1,-1,4,-1] contrasts
TREATMENTSTRUCTURE REG(V;1;contrasts)*Y*C
ANOVA [print=aov] yld

"REML Model A"
VCOMPONENTS [FIXED=V] Y + C + V.C + V.Y + C.Y
REML[PRINT=comp,means;pterms=V] yld

"V.Y term deleted from the model"
VCOMPONENTS [FIXED=V] Y + C + V.C + C.Y
REML[PRINT=comp,means;pterms=V] yld

"SET UP FACTOR GV WITH GV=1 for Trebi and GV=2 for the rest"
calc GV = 1
restrict yld;cond=V .ne. 4;save=vv
calc ln = nobs(vv)
for l = 1...ln
calc l2 = vv$[l] & GV$[l2] = 2
endfor
restrict yld

VCOMPONENTS [FIXED=V] Y+C + GV.C + C.Y
REML[PRINT=comp,means;pterms=V] yld

VCOMPONENTS [FIXED=V+Y+C]  GV.C + C.Y
REML[PRINT=comp,means;pterms=V] yld

stop
```

Output from Genstat REML

```
   1
   2
   3  output [width=65] 1
   4  units [nvalues=60]
   5  text vn; !T(MANCHURIA, SVANSOTA,VELVET,TREBI, PEATLAND)
   6  text cn; !T(UNIVERSITY, WASECA, MORRIS, CROOKSTON,\
   7              'GRAND RAPIDS', DULUTH)
   8  factor [levels=5] V
   9  factor [levels=2] Y
  10  factor [levels=6] C
  11  factor [levels=2] GV
  12  generate C,Y,V
  13
  14  read [serial=yes] yld

    Identifier   Minimum      Mean   Maximum     Values    Missing
           yld      49.9     101.1     191.5         60          0

  28
  29  calc yld = yld/3
  30  tabu[class=Y,V,C]yld;means=mm
  31  print mm;6;2

                                mm
                        C       1      2      3      4      5      6
           Y            V
           1            1  27.00  48.87  27.43  39.93  32.97  28.97
                        2  35.13  47.33  25.77  40.47  29.67  25.70
                        3  39.90  50.23  26.13  41.33  23.03  26.30
                        4  36.57  63.83  43.77  46.93  29.77  33.93
                        5  32.77  48.57  29.87  41.60  34.70  32.00
           2            1  26.90  33.47  34.37  32.97  22.13  22.57
                        2  27.43  38.50  35.03  20.63  16.63  22.23
                        3  26.80  37.40  38.83  32.07  32.23  22.47
                        4  29.07  49.23  46.63  41.83  20.63  30.60
                        5  28.07  36.03  43.20  25.23  26.77  31.37

  32  TREATMENTSTRUCTURE V*C*Y - V.Y
  33  ANOVA [print=aov] yld


33...............................................................

***** Analysis of variance *****

Variate: yld
```

```
Source of variation    d.f.       s.s.       m.s.     v.r.
V                         4    589.997    147.499
C                         5   2357.878    471.576
Y                         1    422.057    422.057
V.C                      20    492.558     24.628
C.Y                       5    765.989    153.198
V.C.Y                    24    341.783     14.241
Total                    59   4970.261

  34  MATRIX [rows=!t('Trebi vs rest');column=5;\
  35          values=-1,-1,-1,4,-1] contrasts
  36  TREATMENTSTRUCTURE REG(V;1;contrasts)*Y*C
  37  ANOVA [print=aov] yld


37.................................................................

***** Analysis of variance *****

Variate: yld

Source of variation    d.f.       s.s.       m.s.     v.r.
V                         4    589.997    147.499
  Trebi vs rest           1    487.920    487.920
  Deviations              3    102.077     34.026
Y                         1    422.057    422.057
C                         5   2357.878    471.576
V.Y                       4     32.424      8.106
  Trebi vs rest.Y         1      2.576      2.576
  Deviations              3     29.847      9.949
V.C                      20    492.558     24.628
  Trebi vs rest.C         5    312.696     62.539
  Deviations             15    179.862     11.991
Y.C                       5    765.989    153.198
V.Y.C                    20    309.359     15.468
  Trebi vs rest.Y.C       5     54.040     10.808
  Deviations             15    255.319     17.021
Total                    59   4970.261

  38
  39  "REML Model A"
  40  VCOMPONENTS [FIXED=V] Y + C + V.C + V.Y + C.Y
  41  REML[PRINT=comp,means;pterms=V] yld


41.................................................................
```

```
*** Estimated Variance Components ***

Random term                 Component        S.e.

Y                                9.21       20.16
C                               30.92       31.37
C.V                              4.58        4.60
Y.V                             -1.23        1.26
Y.C                             27.55       19.40
*units*                         15.47        4.89


*** Negative variance components present:

*   Fitting of fixed model terms is not sequential: effects and
means for
    any aliased fixed model terms may therefore be misleading.
Wald tests,
    likelihood tests and fitted values are unaffected.

See Genstat Noticeboard for more details.


*** Table of predicted means for V ***

          V          1            2            3            4
                 31.46        30.38        33.06        39.40


          V          5
                 34.18

Standard error of differences:       1.696


  42
  43  "V.Y term deleted from the model"
  44  VCOMPONENTS [FIXED=V] Y + C + V.C + C.Y
  45  REML[PRINT=comp,means;pterms=V] yld


45.............................................................


*** Estimated Variance Components ***

Random term                 Component        S.e.

Y                                8.96       20.16
```

```
C                             30.80        31.37
C.V                            5.19         4.40
Y.C                           27.79        19.40
*units*                       14.24         4.11


*** Table of predicted means for V ***

          V          1            2            3            4
                 31.46        30.38        33.06        39.40


          V          5
                 34.18

Standard error of differences:       2.026


  46
  47   "SET UP FACTOR GV WITH GV=1 for Trebi and GV=2 for the rest"
  48   calc GV = 1
  49   restrict yld;cond=V .ne. 4;save=vv
  50   calc ln = nobs(vv)
  51   for l = 1...ln
  52   calc l2 = vv$[l] & GV$[l2] = 2
  53   endfor
  54   restrict yld
  55
  56   VCOMPONENTS [FIXED=V] Y+C + GV.C + C.Y
  57   REML[PRINT=comp,means;pterms=V] yld


57..................................................................


*** Estimated Variance Components ***

Random term                Component         S.e.

Y                              8.96        20.15
C                             42.91        42.97
C.GV                          13.57        11.25
Y.C                           27.93        19.38
*units*                       13.55         3.07


*** Table of predicted means for V ***

          V          1            2            3            4
```

```
                31.46        30.38        33.06        39.40


        V           5
                34.18

Standard error of differences:    Average            1.942
                                  Maximum            2.602
                                  Minimum            1.503

Average variance of differences:                     4.063

  58
  59  VCOMPONENTS [FIXED=V+Y+C]  GV.C + C.Y
  60  REML[PRINT=comp,means;pterms=V] yld


60................................................................


*** Estimated Variance Components ***

Random term                 Component       S.e.

GV.C                            15.36      12.40
C.Y                             27.96      19.39
*units*                         13.38       3.03


*** Table of predicted means for V ***

        V           1            2            3            4
                31.46        30.38        33.06        39.40


        V           5
                34.18

Standard error of differences:    Average            1.980
                                  Maximum            2.711
                                  Minimum            1.493

Average variance of differences:                     4.278

  61
  62  stop
```

D.2 Algorithm 7.2.2: Multiplicative interaction in $V \times Y/C$ table

Program listing

```
"SET [inprint=*]"
OUTPUT [width=65] 1
SCALAR nn,v,c,y;222,6,37,3
& maxit,critical,monitor;25,.0001,1
UNIT [nn]
OPEN   'xreml2.d'; CHAN = 2

FACTOR [LEVELS =c] centre
FACTOR [LEVELS=v] variety
&      [levels=y] year
VARIATE yield,sens,sensv
& [nvalues=v] vmeans,vmns
MATRIX [rows=c;column=y] cmns,envmeans,cmeans
MATRIX [rows=v;column=y] beta,beta2,ser
SYMMETRICMATRIX [rows=v] vse, vse2

READ [chann = 2] variety,centre,year,yield

CALC vy = v*y
VARIATE [nvalues=vy] ser2,old,new
CALC beta2=1 & np1=v+2+y  & nv2 = v+vy+y+1


FOR iter = 1 ... maxit

"sets up a variate sens with sensistivities corresponding
 to appropriate positions of varieties in the factor centre"

FOR i = 1 ... v & j = 1...y
RESTR sens,yield;cond=((variety==i) .and. (year==j));save=vv
CALC ln = nobs(vv) & sens$[vv$[1...ln]] = #ln(beta2$[i;j])
ENDFOR &
RESTRICT sens,yield


VCOMP [fixed=variety + year]  centre.year.sens + variety.year
REML [print=*] yield
VKEEP terms=variety+centre.year.sens;means=varmn,*;\
                     effects=*,cenmn
EQUATE varmn;vmeans & cenmn;cmns

"sets up a variate sensv with sensitivities corresponding
```

```
to appropriate positions of varieties in the factor variety"

CALC kk = 0
FOR k = 1 ... y
RESTRICT yield;year==k
TABULATE [class=centre] yield;means = mm
RESTRICT yield
CALC k2 = nobs(mm) & k0,kk = kk + 1,k2

FOR i = k0 ... kk
RESTRICT sensv,yield;cond=(centre==i);saveset=vv
CALC ln = nobs(vv) & sensv$[vv$[1...ln]] = #ln(cmns$[i;k])
ENDFOR
RESTRICT sensv,yield
ENDFOR


" regression of yields for each variety against centre means"

VCOMP [variety + year + year.variety.sensv]
REML [print=*] yield
VKEEP [fullcov=vslope] terms=variety+year.variety.sensv;effects=*,beta1
EQUATE beta1;beta

FOR l = np1 ... nv2 : calc l2 = l - v - 1-y
& ser2$[l2] = sqrt(vslope$[l;l]): ENDFOR
EQUATE ser2;ser

" scale sensitivities to a unit mean"
FOR j = 1 ... y
CALC cv = 0 : FOR k = 1 ...v
CALC cv = cv + beta$[k;j]
ENDFOR : FOR k = 1 ... v
CALC beta$[k;j] = v/cv *beta$[k;j]
ENDFOR &

IF monitor==1
PRINT 'iteration ...',iter;13,3;*,0
PRINT 'variety means and sensitivities'
PRINT  vmeans & beta,ser;7;2(3)
ENDIF
EQUATE beta;new & beta2;old

EXIT MAX(ABS(new-old)) .lt. critical


EQUATE beta;beta2
ENDFOR
```

```
IF monitor==0
PRINT 'variety means and sensitivities'
PRINT  vmeans & beta2,beta,ser;12;3(3)
ENDIF

" final reml run to calculate sed "

FOR i = 1 ... v & j = 1...y
RESTRICT sens,yield;cond=((variety==i) .and. (year==j));saveset=vv
CALC ln = nobs(vv) & sens$[vv$[1...ln]] = #ln(beta$[i;j])
ENDFOR &
RESTRICT sens,yield

VCOMP [fixed=variety + year] centre.year.sens + variety.year
REML [print=compo,means;pterms=variety;pse=a;max=30] yield

VCOMP [fixed=variety+year] centre.year + variety.year
REML [print=compo,means;pterms=variety;pse=a;max=30] yield
STOP
```

Output for wheat data, 1976–78

```
"SET [inprint=*]"
OUTPUT [width=65] 1
SCALAR nn,v,c,y;222,6,37,3
& maxit,critical,monitor;25,.0001,1
UNIT [nn]
OPEN   'xreml2.d'; CHAN = 2

FACTOR [LEVELS =c] centre
FACTOR [LEVELS=v] variety
&      [levels=y] year
VARIATE yield,sens,sensv
& [nvalues=v] vmeans,vmns
MATRIX [rows=c;column=y] cmns,envmeans,cmeans
MATRIX [rows=v;column=y] beta,beta2,ser
SYMMETRICMATRIX [rows=v] vse, vse2

READ [chann = 2] variety,centre,year,yield

  Identifier   Minimum      Mean   Maximum    Values   Missing
       yield     2.680     6.042     8.930       222        57

CALC vy = v*y
VARIATE [nvalues=vy] ser2,old,new
CALC beta2=1 & np1=v+2+y  & nv2 = v+vy+y+1


FOR iter = 1 ... maxit

 --------------------LINES DELETED -------------------------

iteration ...  1


variety means and sensitivities


     vmeans
      5.632
      5.920
      6.072
      6.311
      6.237
      6.375
```

```
            1                 2                 3
         beta      ser     beta      ser     beta      ser

   1    1.100    0.089    0.743    0.093    1.097    0.095
   2    0.826    0.141    0.775    0.093    0.827    0.095
   3    1.046    0.089    0.912    0.108    0.869    0.097
   4    1.102    0.190    1.326    0.108    1.046    0.097
   5    1.027    0.190    1.204    0.112    1.087    0.097
   6    0.899    0.190    1.040    0.112    1.074    0.099
```

```
iteration ...  2


variety means and sensitivities


    vmeans
     5.636
     5.931
     6.062
     6.266
     6.193
     6.372
```

```
            1                 2                 3
         beta      ser     beta      ser     beta      ser

   1    1.150    0.089    0.741    0.087    1.149    0.091
   2    0.815    0.133    0.769    0.087    0.802    0.091
   3    1.047    0.089    0.918    0.100    0.837    0.094
   4    1.102    0.182    1.335    0.100    1.066    0.094
   5    1.005    0.182    1.197    0.104    1.076    0.094
   6    0.881    0.182    1.040    0.104    1.071    0.095
```

-------------------------LINES DELETED ----------------

```
iteration ...  9
```

```
variety means and sensitivities


     vmeans
      5.649
      5.938
      6.058
      6.274
      6.215
      6.393


                  1              2              3
               beta     ser   beta     ser   beta     ser

         1    1.184   0.090  0.741   0.086  1.173   0.089
         2    0.820   0.131  0.769   0.086  0.793   0.090
         3    1.049   0.090  0.925   0.099  0.821   0.092
         4    1.094   0.178  1.340   0.099  1.075   0.092
         5    0.987   0.178  1.190   0.102  1.068   0.092
         6    0.866   0.178  1.034   0.102  1.070   0.093
```

```

  IF monitor==0
  PRINT 'variety means and sensitivities'
  PRINT  vmeans & beta2,beta,ser;12;3(3)
  ENDIF

 " final reml run to calculate sed "

  FOR i = 1 ... v & j = 1...y
  RESTR sens,yield;cond=((variety==i) .and. (year==j));save=vv
  CALC ln = nobs(vv) & sens$[vv$[1...ln]] = #ln(beta$[i;j])
  ENDFOR &
  RESTRICT sens,yield

  VCOMP [fixed=variety + year] centre.year.sens + variety.year
  REML [print=compo,means;pterms=variety;pse=a;max=30] yield
```

```
107.............................................................


*** Estimated Components of Variance ***

                                                s.e.
```

```
centre.year.sens                    1.154      0.2851
year.variety                      0.01468     0.01388
*units*                            0.1024     0.01360


*** Table of mean effects for variety ***


    variety           1          2          3          4
                  5.649      5.938      6.058      6.274


    variety           5          6
                  6.215      6.393

Standard errors of differences between pairs


   variety  1         *
   variety  2    0.1382          *
   variety  3    0.1302     0.1327          *
   variety  4    0.1446     0.1511     0.1425          *
   variety  5    0.1452     0.1505     0.1450     0.1465
   variety  6    0.1435     0.1473     0.1436     0.1490

              variety  1 variety  2 variety  3 variety  4


   variety  5         *
   variety  6    0.1478          *

              variety  5 variety  6

 108
 109   VCOMP [fixed=variety+year] centre.year + variety.year
 110   REML [print=compo,means;pterms=variety;pse=a;max=30] yield


110.............................................................


*** Estimated Components of Variance ***

                                                s.e.
```

```
centre.year                     1.146      0.2873
year.variety                    0.02351    0.01898
*units*                         0.1375     0.01826


*** Table of mean effects for variety ***


    variety           1           2           3           4
                  5.632       5.920       6.072       6.311


    variety           5           6
                  6.237       6.375

Standard errors of differences between pairs



   variety  1         *
   variety  2    0.1601           *
   variety  3    0.1536      0.1612           *
   variety  4    0.1631      0.1657      0.1638           *
   variety  5    0.1702      0.1727      0.1714      0.1749
   variety  6    0.1702      0.1726      0.1717      0.1751

              variety  1  variety  2  variety  3  variety  4


   variety  5         *
   variety  6    0.1785           *

              variety  5  variety  6

 111   STOP
```

D.3 Algorithm 7.4.1: Means from the model $V+S+Y+S.H$:

Program listing

```
SET [inprint=*]
OUTPUT [wid=65] 1
UNITS [145]
FACTOR [levels=!(1,3)] seed  & [levels=9] house
& [levels=5] year & [levels=29] variety
VARIATE [nvalues=3;values=(1,1,1)] wt
VARIATE YIELD

OPEN 'sbeet8s.d';chann=2
READ [chann=2] variety,seed,house,year,yield

CALC NYEAR = nlevels(year)
&    NVAR = nlevels(variety)
&    NSOU = nlevels(seed)
&    NSH  = nlevels(house)
&    NN   = nvalues(yield)
&    N2VAR,N2SH = NSOU*(NVAR,NSH)
&    NEFF  = 1 + NVAR + NYEAR + NSOU + NSOU*NSH

VARIATE [nvalues=NVAR] VARLEV2
&       [nvalues=NSH] SHLEV2
&       [nvalues=NSOU] SLEV2

GETATTRIBUTE [levels] variety,seed,house;\
             save=VARLEV,SLEV,SHLEV
EQUATE !p(VARLEV[]);VARLEV2
&      !p(SHLEV[]);SHLEV2 & !p(SLEV[]);SLEV2


FACTOR [nvalues=NN;level=1] GMEAN
VARIATE [nvalues=VARLEV2] TST,TSH,VMISS2,VMNS[1 ... NSOU]
VARIATE [nvalues=N2VAR] VMNSE,VMEANS,VMISSING
VARIATE [nvalues=NEFF] ALLEFF,ZALLEFF
VARIATE [nvalues=SHLEV2] SHSM,SHIND
&       [nvalues=N2SH] SHMISS,SHMEANS
POINTER  PA; values= !P(1 ... N2VAR)
SCALAR KK,SNO,SN2,N1,N2

SYMMETRICMATRIX [rows=PA] VSE,VSE3
&                [rows=N2SH] SHVCOV,SHSE
MATRIX [rows=NSOU;columns=NSH] HSOU
&      [rows=N2VAR;columns=NEFF] H
```

```
&       [rows=N2SH;columns=NEFF] SHXSOU

CALC GMEAN,SHIND = 1

VCOMP [cons=o;fixed=GMEAN+variety+year+seed+seed.house]
REML [print=*] yield
VKEEP [full=VCOV] TERMS=GMEAN+variety+year+seed+seed.house;\
      effects=CON,VEF,YEF,SEF,SHEF
EQUATE !p(CON,VEF,YEF,SEF);ALLEFF
EQUATE SHEF;HSOU

CALC ALLMEANS,ZALLEFF,VMISSING,VMISS2,SHMISS,SHXSOU,H=0
&    NYR1 = 2 + NVAR &    NYR2 = NYR1 + NYEAR - 1

FOR K = 1 ... NVAR
CALC   K3 = K+1  &  KK = VARLEV2$[K]

RESTRICT house;COND=variety==KK;saveset=SDNO
CALC  SN = SDNO$[1] &  TSH$[K],SN2=house$[SN]
&    SNO = POSITION(SN2;SHLEV2)
RESTRICT house

IF SHIND$[SNO] == 1
RESTRICT variety;house==SN2;saveset=VRNO
TABU [CLASS=variety] yield;means=VVM
EQUATE VVM;TST
CALC SHSM$[SNO] = NOBS(TST)

RESTRICT variety
CALC SHIND$[SNO] = 0
ENDIF

FOR L = 1 ... NSOU
CALC K2 = K + (L-1)*NVAR
&    L2 = SNO + (L-1)*NSH
&    N3SH = SHSM$[SNO] &    K4 = NYR2 + L
&    K5 = NYR2 + NSOU + (L-1)*NSH + SNO
&    H$[K2;1,K3,K4,K5],SHXSOU$[L2;1,K4,K5] = 1
&    H$[K2;NYR1...NYR2],SHXSOU$[L2;NYR1...NYR2] = 1/NYEAR
&    SHXSOU$[L2;K3] = 1/N3SH
&    ALLEFF$[K5] = HSOU$[L;SNO]

IF HSOU$[L;SNO] ==!(*)
CALC VMISSING$[K2],VMISS2$[K],SHMISS$[L2] = 1
ENDIF

ENDFOR &

CALC ALLEFF=MVREPLACE(ALLEFF;ZALLEFF)
```

```
&      VSE = H*+VCOV*+trans(H)
&      VMEANS = H*+ALLEFF
&      VMEANS = MVINSERT(VMEANS;VMISSING)
&      SHMEANS  = MVINSERT((SHXSOU*+ALLEFF);SHMISS)
" computes variances for class x house means
&      SHVCOV   = SHXSOU*+VCOV*+TRANS(SHXSOU)
"
DELETE [REDEFINE=YES] H,VCOV,ALLEFF,SHXSOU

FOR I = 2 ... N2VAR
CALC I2 = I-1
FOR J = 1 ... I2
IF (VMEANS$[I] == !(*) .OR. (VMEANS$[J] == !(*)))
CALC VSE3$[J;I] = !(*)
ELSE
CALC VSE3$[J;I] = SQRT(VSE$[J;J]+VSE$[I;I] - 2*VSE$[J;I])
ENDIF
ENDFOR
CALC J2 = 1 + ABS(ROUND((I-1)/NVAR - .5))*NVAR
ENDFOR


"
computes standard error of differences for class x house means
FOR I = 2 ... N2SH
CALC I2 = I-1
FOR J = 1 ... I2
IF SHMISS$[I] == 1 .OR. SHMISS$[J] == 1
CALC SHSE$[J;I] = !(*)
ELSE
CALC SHSE$[J;I] = SQRT(SHVCOV$[J;J]+SHVCOV$[I;I] - 2*SHVCOV$[J;I])
ENDIF
ENDFOR
ENDFOR
"

EQUATE VMEANS;!p(VMNS[]) &   SHMEANS;HSOU


FOR K = 1 ... NSOU
CALC KS = SLEV2$[K]
PAGE
PRINT '                    Class of seed  =  ',KS;*,4;*,0

PRINT 'VARIETY','GROUP','MEAN';8
PRINT [IPRINT=*] VARLEV2,TSH,VMNS[K];8;0,0,2,3

CALC N1,N2 = 1,NVAR + (K - 1)*NVAR
```

```
POINTER PB;values=!P(PA[N1 ... N2])
SYMMETRICMATRIX [rows=PB] VSE5

CALC VSE5 = SUBMAT(VSE3)
&    VSE2 = VSE5
PAGE
PRINT '                        STANDARD ERROR OF DIFFERENCES'
&     '**********************************************'

PRINT [IPRINT=*] VSE2;7;3
DELETE [REDEFINE=YES] VSE5
ENDFOR

PAGE
PRINT '                        SEED CLASS BY SEED-HOUSE MEANS'
&     '***********************************************'
PRINT [IPRINT=*] HSOU
STOP
```

Output for sugar beet data

```
  1  SET [inprint=*]

  Identifier    Minimum       Mean    Maximum     Values    Missing
       yield      49.36      55.97      61.11        145         22

  Identifier     Values    Missing     Levels
     variety        145          0         29
        seed        145         22          2
       house        145          0          9
        year        145          0          5


                                          KS
                   Class of seed   =       1


VARIETY   GROUP    MEAN


      1       1   58.23
      2       2   55.32
      3       2   55.52
      4       3   60.74
      5       1   55.14
      6       4   59.17
      7       5   58.89
      8       2   55.72
      9       6   57.51
     10       5   56.41
     11       1   56.62
     12       4   57.07
     13       3   57.41
     14       2   55.07
     15       7   58.75
     16       7   57.75
     17       2   57.17
     18       8   57.15
     19       2   57.41
     20       2   56.12
     21       4   57.10
     22       3   58.83
     23       9   57.50
     24       9   56.43
     25       4   54.63
```

```
      26        1   55.24
      27        8   58.02
      28        2   55.58
      29        2   56.88


                     STANDARD ERROR OF DIFFERENCES

*****************************************************************


 1      *
 2  1.476      *
 3  1.476  0.763      *
 4  1.734  1.476  1.476      *
 5  0.763  1.476  1.476  1.734      *
 6  1.508  1.193  1.193  1.508  1.508      *
 7  1.508  1.193  1.193  1.508  1.508  1.230      *
 8  1.434  0.775  0.775  1.434  1.434  1.137  1.137      *
 9  1.501  1.183  1.183  1.501  1.501  1.222  1.222  1.122
10  1.444  1.106  1.106  1.444  1.444  1.147  0.785  1.042
11  0.894  1.183  1.183  1.501  0.894  1.222  1.222  1.122
12  1.444  1.106  1.106  1.444  1.444  0.785  1.147  1.042
13  1.501  1.183  1.183  0.894  1.501  1.222  1.222  1.122
14  1.404  0.810  0.810  1.404  1.404  1.095  1.095  0.775
15  1.427  1.086  1.086  1.427  1.427  1.130  1.130  1.020
16  1.427  1.086  1.086  1.427  1.427  1.130  1.130  1.020
17  1.387  0.865  0.865  1.387  1.387  1.069  1.069  0.810
18  1.473  1.123  1.123  1.473  1.473  1.172  1.172  1.046
19  1.426  0.976  0.976  1.426  1.426  1.114  1.114  0.908
20  1.507  1.160  1.160  1.507  1.507  1.202  1.202  1.079
21  1.507  1.160  1.160  1.507  1.507  1.202  1.202  1.079
22  1.507  1.160  1.160  1.507  1.507  1.202  1.202  1.079
23  1.507  1.160  1.160  1.507  1.507  1.202  1.202  1.079
24  1.507  1.160  1.160  1.507  1.507  1.202  1.202  1.079
25  1.507  1.160  1.160  1.507  1.507  1.202  1.202  1.079
26  1.507  1.160  1.160  1.507  1.507  1.202  1.202  1.079
27  1.507  1.160  1.160  1.507  1.507  1.202  1.202  1.079
28  1.507  1.160  1.160  1.507  1.507  1.202  1.202  1.079
29  1.507  1.160  1.160  1.507  1.507  1.202  1.202  1.079

        1      2      3      4      5      6      7      8

 9      *
10  1.133      *
11  1.206  1.133      *
12  1.133  1.051  1.133      *
13  1.206  1.133  1.206  1.133      *
```

```
14  1.076  0.991  1.076  0.991  1.076      *
15  1.112  1.032  1.112  1.032  1.112  0.969      *
16  1.112  1.032  1.112  1.032  1.112  0.969  0.763      *
17  1.045  0.957  1.045  0.957  1.045  0.775  0.934  0.934
18  1.136  1.060  1.136  1.060  1.136  0.980  1.035  1.035
19  1.080  0.998  1.080  0.998  1.080  0.856  0.974  0.974
20  1.165  1.086  1.165  1.086  1.165  1.009  1.067  1.067
21  1.165  1.086  1.165  1.086  1.165  1.009  1.067  1.067
22  1.165  1.086  1.165  1.086  1.165  1.009  1.067  1.067
23  1.165  1.086  1.165  1.086  1.165  1.009  1.067  1.067
24  1.165  1.086  1.165  1.086  1.165  1.009  1.067  1.067
25  1.165  1.086  1.165  1.086  1.165  1.009  1.067  1.067
26  1.165  1.086  1.165  1.086  1.165  1.009  1.067  1.067
27  1.165  1.086  1.165  1.086  1.165  1.009  1.067  1.067
28  1.165  1.086  1.165  1.086  1.165  1.009  1.067  1.067
29  1.165  1.086  1.165  1.086  1.165  1.009  1.067  1.067

        9     10     11     12     13     14     15     16

17      *
18  0.931      *
19  0.824  0.936      *
20  0.953  0.996  0.948      *
21  0.953  0.996  0.948  0.985      *
22  0.953  0.996  0.948  0.985  0.985      *
23  0.953  0.996  0.948  0.985  0.985  0.985      *
24  0.953  0.996  0.948  0.985  0.985  0.985  0.985      *
25  0.953  0.996  0.948  0.985  0.985  0.985  0.985  0.985
26  0.953  0.996  0.948  0.985  0.985  0.985  0.985  0.985
27  0.953  0.996  0.948  0.985  0.985  0.985  0.985  0.985
28  0.953  0.996  0.948  0.985  0.985  0.985  0.985  0.985
29  0.953  0.996  0.948  0.985  0.985  0.985  0.985  0.985

       17     18     19     20     21     22     23     24

25      *
26  0.985      *
27  0.985  0.985      *
28  0.985  0.985  0.985      *
29  0.985  0.985  0.985  0.985      *

       25     26     27     28     29


                                         KS
                   Class of seed  =       3
```

VARIETY	GROUP	MEAN
1	1	56.95
2	2	56.12
3	2	56.32
4	3	56.81
5	1	53.85
6	4	56.40
7	5	55.18
8	2	56.53
9	6	53.04
10	5	52.70
11	1	55.33
12	4	54.30
13	3	53.48
14	2	55.88
15	7	54.86
16	7	53.86
17	2	57.97
18	8	58.11
19	2	58.21
20	2	56.93
21	4	54.33
22	3	54.90
23	9	*
24	9	*
25	4	51.86
26	1	53.95
27	8	58.98
28	2	56.39
29	2	57.68

STANDARD ERROR OF DIFFERENCES

****

1	*						
2	0.763	*					
3	0.763	0.763	*				
4	0.763	0.763	0.763	*			
5	0.763	0.763	0.763	0.763	*		
6	0.785	0.785	0.785	0.785	0.785	*	
7	0.785	0.785	0.785	0.785	0.785	0.800	*

8	0.775	0.775	0.775	0.775	0.775	0.791	0.791	*
9	0.894	0.894	0.894	0.894	0.894	0.901	0.901	0.894
10	0.849	0.849	0.849	0.849	0.849	0.856	0.785	0.850
11	0.894	0.894	0.894	0.894	0.894	0.901	0.901	0.894
12	0.849	0.849	0.849	0.849	0.849	0.785	0.856	0.850
13	0.894	0.894	0.894	0.894	0.894	0.901	0.901	0.894
14	0.810	0.810	0.810	0.810	0.810	0.820	0.820	0.775
15	0.838	0.838	0.838	0.838	0.838	0.845	0.845	0.838
16	0.838	0.838	0.838	0.838	0.838	0.845	0.845	0.838
17	0.865	0.865	0.865	0.865	0.865	0.870	0.870	0.810
18	1.345	1.345	1.345	1.345	1.345	1.349	1.349	1.342
19	0.976	0.976	0.976	0.976	0.976	0.971	0.971	0.908
20	1.160	1.160	1.160	1.160	1.160	1.147	1.147	1.079
21	1.334	1.334	1.334	1.334	1.334	1.202	1.317	1.318
22	1.507	1.507	1.507	1.507	1.507	1.493	1.493	1.493
23	*	*	*	*	*	*	*	*
24	*	*	*	*	*	*	*	*
25	1.334	1.334	1.334	1.334	1.334	1.202	1.317	1.318
26	1.507	1.507	1.507	1.507	1.507	1.493	1.493	1.493
27	1.692	1.692	1.692	1.692	1.692	1.691	1.691	1.686
28	1.160	1.160	1.160	1.160	1.160	1.147	1.147	1.079
29	1.160	1.160	1.160	1.160	1.160	1.147	1.147	1.079
	1	2	3	4	5	6	7	8

9	*							
10	0.945	*						
11	0.985	0.945	*					
12	0.945	0.903	0.945	*				
13	0.985	0.945	0.985	0.945	*			
14	0.915	0.872	0.915	0.872	0.915	*		
15	0.934	0.893	0.934	0.893	0.934	0.860	*	
16	0.934	0.893	0.934	0.893	0.934	0.860	0.763	*
17	0.955	0.913	0.955	0.913	0.955	0.775	0.903	0.903
18	1.405	1.378	1.405	1.378	1.405	1.354	1.370	1.370
19	1.041	1.001	1.041	1.001	1.041	0.856	0.993	0.993
20	1.196	1.164	1.196	1.164	1.196	1.009	1.154	1.154
21	1.356	1.327	1.356	1.086	1.356	1.316	1.320	1.320
22	1.525	1.502	1.525	1.502	1.165	1.491	1.493	1.493
23	*	*	*	*	*	*	*	*
24	*	*	*	*	*	*	*	*
25	1.356	1.327	1.356	1.086	1.356	1.316	1.320	1.320
26	1.525	1.502	1.165	1.502	1.525	1.491	1.493	1.493
27	1.730	1.710	1.730	1.710	1.730	1.691	1.702	1.702
28	1.196	1.164	1.196	1.164	1.196	1.009	1.154	1.154
29	1.196	1.164	1.196	1.164	1.196	1.009	1.154	1.154
	9	10	11	12	13	14	15	16

17	*							
18	1.378	*						
19	0.824	1.437	*					
20	0.953	1.549	0.948	*				
21	1.327	1.684	1.357	1.443	*			
22	1.501	1.823	1.530	1.602	1.709	*		
23	*	*	*	*	*	*	*	
24	*	*	*	*	*	*	*	*
25	1.327	1.684	1.357	1.443	0.985	1.709	*	*
26	1.501	1.823	1.530	1.602	1.709	1.842	*	*
27	1.706	0.996	1.748	1.830	1.945	2.062	*	*
28	0.953	1.549	0.948	0.985	1.443	1.602	*	*
29	0.953	1.549	0.948	0.985	1.443	1.602	*	*
	17	18	19	20	21	22	23	24

25	*				
26	1.709	*			
27	1.945	2.062	*		
28	1.443	1.602	1.830	*	
29	1.443	1.602	1.830	0.985	*
	25	26	27	28	29

SEED CLASS BY SEED-HOUSE MEANS

****

	1	2	3	4
1	56.30	56.09	58.99	57.00
2	55.02	56.89	55.06	54.22

	5	6	7	8
1	57.65	57.51	58.25	57.58
2	53.94	53.04	54.36	58.55

	9
1	56.97
2	*