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MONEY DEMAND AND MONEY SUPPLY IN AN EXTENDED IS-LM FRAMEWORK: AN ECONOMETRIC STUDY

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Ph.D. University of Edinburgh 1976

I declare that the work in this thesis has been composed by myself.

A C K N O W L E D G E M E N T S

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S.N.B.

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ABSTRACT

This study represents the first empirical piece of work on the IS-LM model which takes into account simultaneously the main criticisms levelled against the model and estimates its equations as a unified set of hypotheses. Specifically (a) by extending the model to include a price determination equation, expressing real sector variables in real terms and financial variables in nominal terms and introducing the price level in expenditure equations as well as in the money demand function, we have been able to determine the proportions in which each time nominal income changes are divided between output changes and price changes in a model in which the IS and LM parts ceased to be two independently shifting sets of relationships; and (b) by specifying price expectations as an endogenous element in the system and examining the relationship between the returns on bonds and equities, we have been able to distinguish between the nominal interest rate, which is appropriate for the demand for and supply of money, and the real interest rate which is relevant to investment decisions. Additionally we have admitted government activity, international trade and an endogenous money supply in the model, thus increasing its usefulness for the study of policy problems.

The model's equations were estimated by two stage least squares and first order autogressive errors (where necessary) and some of its dynamic properties as expressed in the appropriate impact and interim multipliers were analysed in relation to the set of empirical propositions known in the literature as monetarism. The validity of the propositions concerning the effect of monetary policy on interest rates and on real income and the price level in the short-run has been ascertained whereas propositions which refer to the long-run and imply an independence of real from monetary variables were not found to hold true.

A number of other issues dealt with in this study are the possibility of the accommodation of the short-run and the long-run within the same structure, the existence of money illusion in expenditure equations both as a short-run and a long-run phenomenon, the relaxation of the assumption that the money demand function is homogeneous of the first degree in prices and nominal income, the effect of interest rate and special deposits on money supply, the introduction of a lag distribution characterised by weights alternating in sign and declining geometrically in absolute value and the estimation of a realistic series of full-employment output for use in the price equation.

Finally, we have examined the multiplier effects of exogenous on endogenous variables in linear dynamic econometric models of higher than first order in which the lags of exogenous variables are extended to more than one period. A mistake in the theory of the calculation of multipliers in such models has been pointed out and the analysis and correct formulae have been provided. Furthermore, the asymptotic distribution of impact and interim multipliers has been derived for the case of a generalised linear dynamic model.

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CHAPTER 1

CHAPTER 1

INTRODUCTION

In this thesis, a small scale linear structural model of the British Economy is constructed and estimated for the period 1955 to 1972, and its dynamic properties are explored through multiplier analysis. The model is a version of the well known IS-LM model expanded to take into account recent theoretical developments. The work presented here aims at filling a gap which exists between one-equation studies (demand for money function, investment function, etc.) and large scale models (Klein *et al.* Model, Southampton Econometric Model, London Business School Model, etc.) of the British Economy which contain at least one equation for the financial sector.[1] The need for such a work has been stressed by various researchers in the field of macroeconomics.[2] Another secondary aim of the study is the building up of the money supply function (which represents an important part of the model) by taking into account the idiosyncracies of the British banking system and the functional dependence of the elements of the monetary multiplier on other variables. Again such a formal specification of the function is something that was missing.

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[1] Hendry recently estimated an eight equation model which however contains demand equations for the product market only. See D. F. Hendry: "Stochastic Specification in an Aggregate Demand Model of the United Kingdom." Econometrica, 1974, pp.559-78.

[2] For example Hines and Catephores at the concluding remarks in a study of investment in U.K. Manufacturing Industry call for the construction and estimation of an *"adequately specified IS-LM model for the U.K. along lines attempted by Chow for the United States."* See A. G. Hines and G. Catephores: "Investment in U.K. Manufacturing Industry, 1956-1967." in The Econometric Study of the United Kingdom. Proceedings of the 1969 Southampton Conference on Short-Run Econometric Models of the U.K. Economy. Edit. by K. Hilton and D. F. Heathfield, MacMillan, 1970, p.222. The model to which the authors refer is contained in: G. C. Chow "Multiplier, Accelerator and Liquidity Preference in the Determination of National Income in the United States." Review of Economics and Statistics, 1967, pp.1-15.

Alongside with meeting these needs specifically for the British Economy, it is hoped that the present research will contribute to our knowledge in economics by

(a) presenting the first empirical study using the IS-LM model, in which price expectations effects are an endogenous element in the system and the price level interacts with the level of real income. Although theoretical work has sometimes encompassed these aspects by enlarging the IS-LM model to include a price determination equation,[3] there has been no attempt (with one exception)[4] to pursue the issues empirically. This is partly accounted by the fact that theoretical research has in many cases resorted to drastic simplifications or has produced models which are not estimable.

(b) introducing a dynamic adjustment specification (used in the estimation of the price equation) leading, in the context of a single equation model, to a new lag distribution.

(c) pointing out a mistake in the theory of the calculation of dynamic multipliers for econometric models of order higher than one and providing the analysis and correct formulas which apply in such models.

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[3] See among others: T. J. Sargent: "The Optimum Monetary Instrument Variable in a Linear Economic Model." Canadian Journal of Economics, 1971, pp.50-60. T. J. Sargent: "Anticipated Inflation and the Nominal Rate of Interest." Quarterly Journal of Economics, 1972, pp.212-225. D. Laidler: "Simultaneous Fluctuations in Prices and Output." Economica, 1973, pp.60-72. S. T. Turnovsky: "Optimal Choice of Monetary Instrument in a Linear Economic Model with Stochastic Coefficients." Journal of Money, Credit and Banking, 1975, pp.51-80.

[4] The exception is a study by McCallum which will be discussed in some detail in chapter 2, see B. T. McCallum: "Friedman's Missing Equation: Another Approach." Manchester School of Economic and Social Studies, 1973, pp.311-28.

(d) giving answers, on the basis of the model's properties, to some disputed issues in monetary economics, such as the effects of monetary policy on interest rates and the lag in the effect of monetary policy on real income and the price level, insofar as the answer to these questions is independent of the particular (developed) economy examined.

(e) deriving the asymptotic distribution of impact and interim multipliers of econometric models of higher than first order and applying the results to the multipliers of Klein's Model I.

The Model: The model consists of four demand equations explaining gross domestic product components (consumption, private investment, inventories and imports), one price equation, one equation relating the return on capital to the return on bonds, two equations explaining the demand for and supply of money and three identities. The latter are one identity connecting gross domestic product to its components, the second relating stock and flow of inventories, and the third showing how the government borrowing requirement is met. Moreoever eight identities are appended to the model, which are useful for imposing certain restrictions on estimated coefficients. There are accordingly nineteen endogenous variables in the model and the number of basic exogenous variables is ten. A second version of the model is presented firstly which excludes the identity showing the financing of the government borrowing requirement since it is found that its inclusion in a model of the IS-LM type leads to unacceptable multiplier results. In this version the monetary base is considered as exogenous and the number of exogenous variables is reduced to nine.

Real sector variables are expressed in real terms (all deflated by the implicit GDP deflator)[5] and financial variables such as the money stock, the monetary base and special deposits in nominal terms. The price level (and sometimes its change) are contained as explanatory variables in the expenditure equations. While a significant effect of the price level on real expenditure decisions might be called money illusion, the effect of price changes is rationalised on other grounds. Also, the price level itself is determined to some extent by real GDP in the price equation although not simultaneously. The money demand function is not assumed to be homogeneous of the first degree in prices and nominal income and this leads to a formulation of the function in which the stock of nominal money demanded depends on real GDP, the price level and the nominal interest rate.

With the above interactions of the price level and real GDP, the link in the model between the product market and the money market ceases to be the interest rate alone, but includes also the price level. The appropriate interest rate for the demand for and supply of money is the nominal rate whereas the real rate is relevant in real investment decisions. Accordingly this distinction is made in the model where price expectations are an endogenous element. The wealth constraint is not included in either the demand for money function or in the consumption function because of the lack of data on it. All

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[5] Strictly speaking a variable X (in current prices) deflated by the implicit GDP deflator is not the real variable but the product of the real variable and the ratio of its deflator to the implicit GDP deflator $(\frac{X}{P} = \frac{X}{P*} \cdot \frac{P*}{P})$. As our model is not intended to explain relative prices we shall try to explain $\frac{X}{P}$ which will be referred to as the real variable.

data employed are not adjusted for seasonal variation which is picked up by seasonal dummies. The use of unadjusted data poses a problem in the estimation of full-employment expenditure and certain accommodating assumptions have to be made.

The model is constructed in a way preserving linearity in its variables. In defending the use of linearity in econometric models Chow and Moore argue:[6]

> *"Nonlinearities have been introduced in some business-cycle theories partly because the theories are nonstochastic. Once we allow a stochastic model, a linear one can capture many aspects of business fluctuations. Experience with large, nonlinear econometric models has indicated that the nonlinearities introduced often do not produce results much different from the linear versions."*

However, to be able to retain linearity throughout a model, certain approximations sometimes have to be made and these in our model concern the money supply function, nominal GDP, the identity showing the finance of the government borrowing requirement, and the rate of inflation.

Finally, the model is dynamic and makes use in several equations of the partial adjustment mechanism which in the context of a singe equation model leads to a distributed lag relationship with geometrically declining weights. An account of the most commonly used dynamic adjustment mechanisms together with the associated distributed lags will be given in chapter 3. Furthermore a new lag distribution will be introduced and the adjustment mechanism which generates it will be used in the price equation.

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[6] See discussion following G. C. Chow and G. H. Moore: "An Econometric Model of Business Cycles." in Econometric Models of Cyclical Behavior. Edit. B. Hickman, National Bureau of Economic Research, 1972, p.807.

An Overview of the following Chapters

In chapter 2 we present the basic framework of our study, namely the IS-LM model, and the assumptions on which it is based. A simplified version of the model is examined first and it is shown how inferences about the efficacy of monetary and fiscal policy are drawn on the basis of its properties. A number of criticisms are then examined regarding the slope of the IS curve, the independence of the two curves, the lack of interaction of real income and the price level in the IS-LM model and the neglect of price expectations effects. At the end a critical review is made of the existing empirical studies on the IS-LM model showing the need to estimate an expanded model on the lines suggested by theoretical research.

Chapter 3 presents the most commonly used dynamic adjustment mechanisms which, if combined with behavioural equations, imply that the sum of weights in the resulting lag distribution is unity. Furthermore a new lag distribution is introduced with weights alternating in sign and declining geometrically in absolute value, and the adjustment mechanism which generates it, is stated. Some propositions are put forward at the end of the chapter concerning the properties of a simultaneous equations model including equations to which the examined dynamic adjustment mechanisms have been applied. The next two chapters give priority to the money market which is examined in greater detail.

Chapter 4 has two sections. The first deals with the problem of the definition of money. The theoretical and empirical approaches to it are reviewed and our choice is made. The same section also touches upon the issue of money being used as an indicator of the thrust of monetary policy. The second section is devoted to the money demand function. It deals with the concept of demand for money and elucidates the motives for holding money balances. The developments of the function and the arguments in it are examined before specifying the form to be estimated.

Chapter 5 contains the theory on the money supply function. The role of the monetary authorities, the banking system and the public in the process of money creation is examined first in general terms and then with reference to the British institutional framework. The function is built formally and has the identity connecting the money stock and the monetary base as the starting point. In this context the operation of special deposits is analysed and incorporated in the function.

In chapter 6 the price equation is specified and analysed. Cost-push and demand-pull inflation are examined in the context of this equation, and price expectations are related to it. A review of empirical work on this equation is also given. Since no satisfactory series of full-employment GDP, a variable pertinent to the equation, exists, a method to calculate such a series is used which takes into account the cyclical response of labour force participation rate to changes in economic activity and the observed rise in the minimum rate of unemployment.

In chapter 7 the equations of the rest of the model are formulated, namely the equation relating the return on capital to the return on bonds, the consumption function, the investment function, the inventory function and the import function, and the identity showing the financing of the government borrowing requirement is analysed. Where necessary, a review of theoretical and empirical work is given.

Chapter 8 presents the estimation results for the one version of the model and discusses the findings. The estimation method and the identifaction problem are accounted for at the beginning of the chapter.

In chapter 9 we develop the theory of multiplier effects of exogenous on endogenous variables in a dynamic model of order higher than one, after examining the accepted theory for systems of order one and pointing out a mistake in the theory for systems of higher order. The estimated model is brought to a form appropriate for multiplier analysis.

In chapter 10 our complete model is put into the right perspective after a detailed examination of certain aspects of the debate between Monetarists and Neo-Keynesians. The anlysis of multipliers is then carried out, which offers some evidence on monetarist propositions and on the effect of Special Deposits on market interest rates.

In chapter 11 we summarise the findings of the study.

Appendix A contains the data used for this study.

In Appendix B we present the estimates of the other version of our model, and graphs of actual and fitted values of the dependent variables from the preferred equations of the first model.

Appendix C contains the input matrices for the calculation of multipliers, the computer programs for these calculations and the tables of impact and eighty interim multipliers of the model.

Finally in Appendix D we derive the asymptotic distribution of impact and interim multipliers of an econometric model of order higher than one and examine the significance of multipliers in Klein's model I.

CHAPTER 2

CHAPTER 2

THE IS-LM MODEL: RELATED THEORETICAL DISCUSSION AND EMPIRICAL RESEARCH

The framework of analysis which was given the name IS-LM model originated in J. R. Hicks' work in the late thirties.[1] The model has been proved a most useful tool of analysis and is the fundamental reference basis for the discussion of almost every macroeconomic proposition at a highly aggregative level.[2] It is very adaptable and can incorporate assorted considerations at the cost of increasing comlexity. It includes demand and supply equations and equilibrium conditions which pertain only to two markets, namely the output market and the money market. The variables which are determined from the interaction of the equations for the two markets are income and the interest rate. Regarding the latter, the one used in the model is assumed to be the only interest rate. Given the existence of a host of return -yielding real and financial assets this assumption can be justified by the following considerations:

(a) Financial assets - debts and real capital-equities are assumed to be perfect substitutes so that in equilibrium the returns on both types of assets are equal after allowance is made for

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[1] J. R. Hicks: "Mr. Keynes and the Classics: A Suggested Interpretation." Econometrica, 1937, pp.147-59.

[2] As a few indicative examples: J. M. Holmes and D. J. Smyth in "The Specification of the Demand for Money and the Tax Multiplier" Journal of Political Economy, 1972, pp.179-185, examine the effects of the inclusion of income taxes in the money demand function in the context of the IS-LM model. In the same context, W. Poole in "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macromodel" Quarterly Journal of Economics, 1970, pp.197-216, presents a solution to the problem of choice of the optimum monetary policy instrument.

a risk premium required on real capital

$$R^D + \rho = R^C \tag{2.1}$$

where R^D and R^C are the returns on debts and real capital and ρ is the risk premium. The last is taken constant and empirical studies have shown it to be as high as two-and-a-half percentage points.[3] The assumption of perfect substitutability between debts and real capital has been questioned but there is also some evidence that it may be quite justified. For instance Hamburger in a study of the demand of financial assets by households found that marketable bonds are close substitutes for equities in household portfolios and that a rise in market rates induces a large shift from the latter to the former.[4] His results generally show that when a market rate changes, it triggers a process of substitution involving a wide range of financial and real assets.

(b) Further within the class of debts itself there is a predictable relation between yields of instruments with differing maturities. It is commonly assumed that the yield on long term debts of n period life ($n > 1$) will generally exceed the short term rate by the amount of expected capital gains which in turn are inversely affected by the change in yield. This leads to the formulation of the

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[3] See P. H. Hendershott and J. C. Van Horne: "Expected Inflation Implied by Capital Market Rates." Journal of Finance, Papers and Proceedings, 1973, p. 309.

[4] M. J. Hamburger: "Household Demand for Financial Assets." Econometrica, 1968, pp.97-118.

expectations hypothesis used to explain the term structure of interest rates, i.e. the spread between long and short term rates. The available econometric evidence shows that the term structure is quite unaffected by changes in the maturity structure of the debt.[5]

The IS-LM model is too wellknown to require detailed examination.[6] So it will suffice to give a short delineation of it with the emphasis placed on particular assumptions and characteristics, criticisms and unsolved issues surrounding it and on the question of how far has empirical research gone in juxtaposing the theoretical model with real world data.

The IS and LM schedules are summary equations which, cast in static form, represent the equilibrium solution of the output and money markets. They determine simultaneously a unique level of income and interest rate. The IS curve is the locus of all combinations of income and interest rate which bring equilibrium to the output market by equating planned investment to planned saving. Keeping the initial formulation which employed money variables (with the underlying assumption of a rigid price level) we can derive the IS equation with

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[5] See W. D. Nordhaus and H. C. Wallich, "Alternatives for Debt Management, p. 15, and C. A. E. Goodhart, Discussion p. 28 in Issues in Federal Debt Management. Proceedings of a Conference held in June 1973, sponsored by the Federal Reserve Bank of Boston.

[6] For more detailed descriptions see among others, D. Laidler: The Demand for Money: Theories and Evidence. International Textbook Company, Scranton, 1969, Chapt. 1, and J. T. Boorman - T. M. Havrilesky: Money Supply, Money Demand and Macroeconomic Models. Ally and Bacon Inc., Boston 1972, Chapt. 6.

the use of simple linear functions. Let consumption C be a function of disposable income Y-T

$$C = a + b\,(Y-T) \qquad 0 < b < 1 \tag{2.2}$$

and investment I decreasing function of the interest rate i

$$I = c - di \qquad d > 0 \tag{2.3}$$

Then the equality of planned investment and saving is read as

$$Y - a - b\,(Y-T) - G = c - di \tag{2.4}$$

where G and T refer to government expenditures and taxes. The IS curve is then

$$i = \frac{a + c + G - bT}{d} - \frac{(i-b)}{d}\,Y \tag{2.5}$$

On the basis of the above equation and without consideration of the money market, income determination theory would proceed on the assumption of a constant interest rate. Clearly this is an unrealistic assumption which is removed once the money market enters the stage. With the simplification that the monetary authorities supply the economy exogenously with money, the monetary sector is summarised by the money demand equation

$$M = a' + b'Y - c'i \qquad b', c' > 0 \tag{2.6}$$

which can be solved to give the LM curve, i.e. all combinations of income and interest rate which are consistent with equilibrium in the money market

$$i = \frac{a' - M}{c'} + \frac{b'}{c'} \cdot Y \tag{2.7}$$

The system of equations (2.5) and (2.7) can be solved for the equilibrium level of income

$$Y^E = \frac{a + c + G - bT - \frac{a'}{c'} + \frac{M}{c'}}{\frac{1 - b}{d} + \frac{b'}{c'}} \tag{2.8}$$

If the model is to be insulated from price level movements real sector variables and the money stock are deflated by the price level and equations (2.5) and (2.7) are replaced by

$$i = \frac{a + c + \frac{G}{P} - b\frac{T}{P}}{d} - \frac{i - b}{d} \cdot \frac{Y}{P} \tag{2.9}$$

and

$$i = \frac{a' - \frac{M}{P}}{c'} + \frac{b}{c'} \cdot \frac{Y}{P} \tag{2.10}$$

The above results are the basis for carrying out comparative static analysis. Simple multipliers are easily obtained with respect to government expenditures, taxes and money. Their strength depends on the value of the denominator

$$\frac{1 - b}{d} + \frac{b'}{c'} .$$

For a given IS curve the greater the interest sensitivity (absolutely) or the smaller the income sensitivity of money demand, the greater the relevant multiplier is. For a given LM curve, the greater the marginal propensity to consume or the interest sensitivity of investment (absolutely) the greater the multiplier is.

We observe that insofar as the theory of the static model is concerned, policy action in it is depicted by parallel shifts of the two curves. Typically each term in the equations other than income and interest rate represents a shift factor whose effect can be studied in a comparative static sense. On the other hand the answer to the question of how effective policy measures are in influencing aggregate demand necessitates the examination of (a) the slope properties (interest and income elasticities) and (b) the independence of the IS and LM curves. When we leave static analysis and move on to dynamic there is a systematic dependence of the position of the two curves and lagged values of income and interest rate. Thus, under the influence of past and current policies a continuous shift of the IS-LM curves is taking place and the graphical comparative static study is a simplification to which it may be no longer possible to resort.

The use of the IS-LM analysis was uninterrupted over the first decades of its existence. Only during the last ten years there have been some criticisms and a number of questions revolving around the model. Practically all of them are closely linked to the debate between Neo-Keynesians and Monetarists.[7] The framework as expressed in the second version (equations (2.9) and (2.10) is accepted by both sides. But it is not complete and as such it has not been tested to

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[7] For a recent statement of the state of the debate see L. C. Anderson, "The State of the Monetarist Debate." Federal Reserve Bank of St. Louis Review, September 1973, pp.2-8. Commentaries follow by L. R. Klein and K. Brunner, people standing on opposite sides of it. Klein strongly recommends that for settlement of the disputable issues "the debate should shift from speculative theorising, casual empirical referencing and unsupported asserting to serious work in applied econometrics".

the satisfaction of the participants of the debate. Therefore many assertions are cast in the form of questions about properties and implications of it.

Is the IS curve positively or negatively sloped? How are changes in nominal income divided between changes in real output and changes in the price level? Are the IS and LM curves independent of each other? How are price expectations effects taken into account in the model? How far has empirical research progressed with the model?

We shall give our attention to the meaning of the above questions and attempt a review of the recent developments both at a theoretical and empirical level.

The slope of the IS curve: It has been of remarkable concern to monetarists whether the slope of the IS curve is negative or positive. Consider the case of a positively sloped curve as in diagram 2.A. The two schedules are drawn so that the slope of the LM curve is greater than the slope of the IS curve. This complies with the stability condition derived from a dynamic form of the static framework on the assumption that money market adjustments are much more rapid relative to income adjustments.[8]

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[8] For details of the derivation see T. Denburg and J. D. Denburg: Macroeconomic Analysis: An Introduction to Comparative Statics and Dynamics. Addison-Wesley, 1969, Chapt. 13.

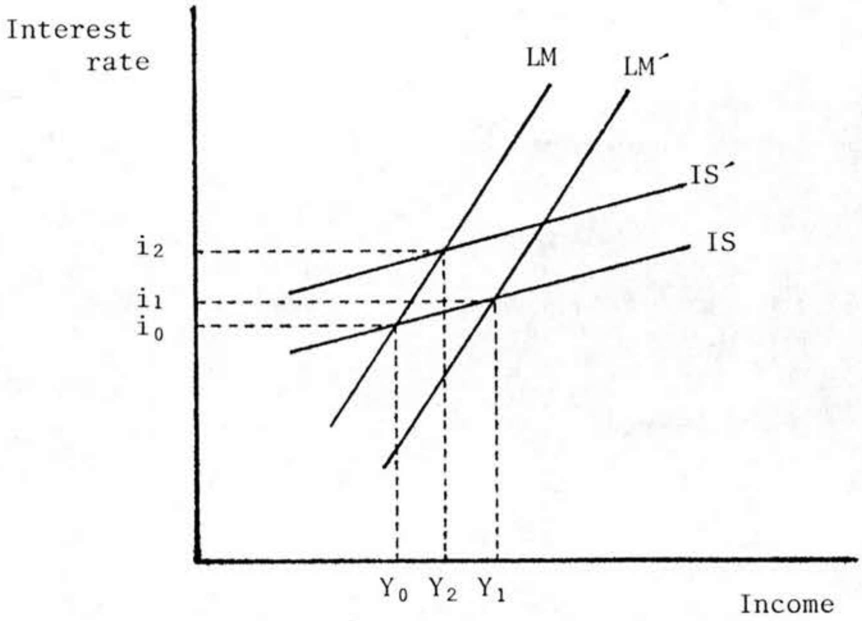

Diagram 2.A

The condition is that $\left(\frac{di}{dY}\right)_{IS} - \left(\frac{di}{dY}\right)_{LM} < 0$

i.e. the slope of the IS curve must be smaller than the slope of the LM curve. With a positively sloped IS curve, it is met if the latter is the steeper between the two. It satisfies Samuelson's correspondence principle which states that the model should be checked for dynamic stability before the comparative-static analysis is carried out and results are accepted. In terms of the diagram an increased stock of money causes the LM curve to shift rightwards to the position LM' and a higher level of government expenditures makes the IS curve shift upwards to the position IS'. In both instances interest rate and income move in the same direction and it becomes difficult to identify the disturbance with the implementation of monetary or fiscal policy.[9] It is held in the Keynesian system that

[9] D. Meiselman in the Discussion in Controlling Monetary Aggregates Proceedings of a Conference held in June 1969, sponsored by the Federal Reserve Bank of Boston, pp. 147, 149 has stressed that in the case of a positively sloped IS curve it is very difficult to find out whether the rise in both interest rate and income variables is real in the sense that it is initiated by a shift in the IS curve or monetary in the sense that it is initiated by a shift of the LM curve.

an expansive monetary policy lowers the interest rate and raises output while monetariat analysis is associated with movements of money and interest rate in the same direction. One possible way of obtaining this result while retaining the IS-LM framework is through the slope of the IS curve, and the concern about it is easily understandable. Another way is through price expectation effects on interest rates and this second possibility will be examined later. As regards the parallel movement of money and interest rates the discussion may well refer to the dynamics of the system and it may be the case that the conflicting views can be accommodated in a dynamic framework (e.g. a negative impact multiplier with positive interim and final multipliers).

Returning to the static IS-LM model we observe that equation (2.5) or (2.9) as it stands can in no way have a positive slope since the marginal propensity to consume will never exceed unity. But in the simple model examined it is only consumption spending which is related to income. However, other expenditure functions, e.g. investment, might well have income as an argument and the conclusions can differ substantially. Assume that the investment function is

$$I = c - di + eY \qquad d, e > 0 \tag{2.11}$$

then the IS curve (2.5) is replaced by

$$i = \frac{a + c + G - bT}{d} - \frac{1 - b - e}{d} \cdot Y \tag{2.12}$$

which has a positive slope if $b + e > 1$ i.e. the sum of the marginal propensities to consume and invest with respect to income sum to more than unity. With the additional restriction that $\frac{b + e - 1}{d} > \frac{b'}{c'}$ the curves are the ones shown in diagram 2.A.

Now as Silber pointed out, a positively sloped IS curve reverses the conclusions as to the potency of monetary policy. For a given IS curve the steeper the LM curve the less potent monetary policy is as it can be seen from the following diagram 2.B.

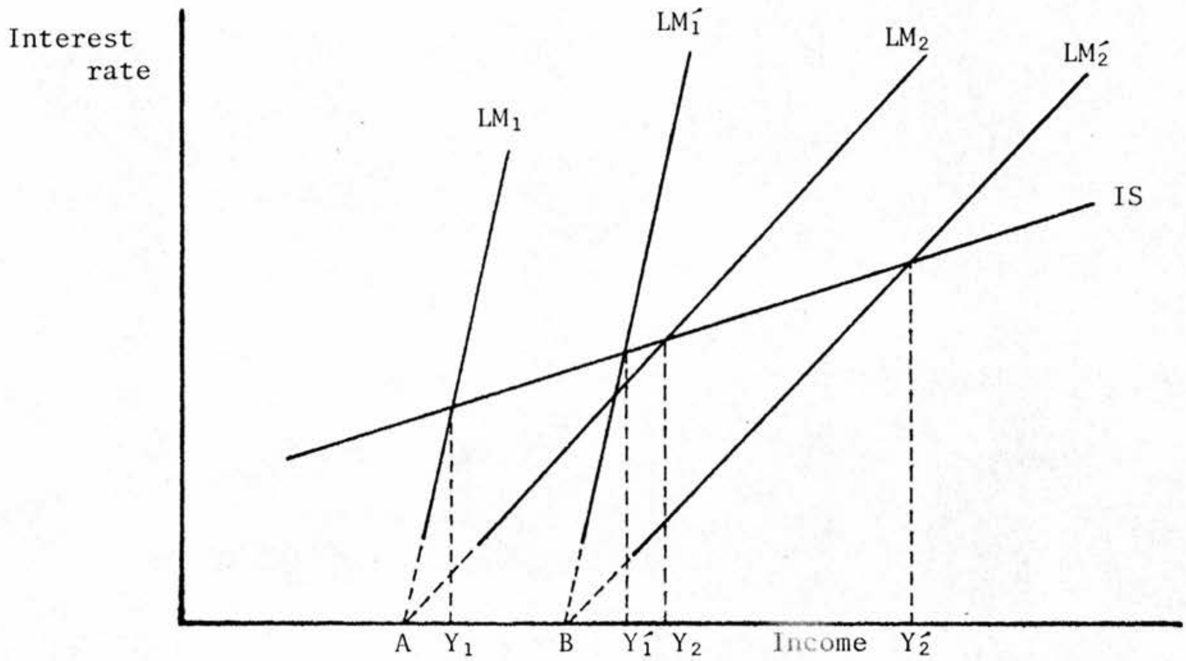

Diagram 2.B

An increase in the money stock by an equal amount AB is more expansionary ($Y_2Y_2' > Y_1Y_1'$) in the case of the LM_2 curve than in the case of LM_1. Since the tangent of the LM curve is b'/c' the relation can be expressed in terms of the relevant sensitivities of money demand to changes in income and interest rate. The greater the

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[10] W. L. Silber, "Monetary Policy Effectiveness: The Case of a Positively Sloped IS Curve." Journal of Finance, 1970, pp.1077-1082.

interest sensitivity and the smaller the income sensitivity the more powerful is monetary policy.

Division of changes in nominal income to output changes and price changes: One of the major weaknesses of the IS-LM model written in either form is that it fails to determine the proportions in which nominal income changes are divided between output changes and price changes. A simple model set forth by M. Friedman as a common one accepted by both monetarists and Keynesians shows explicitly this deficiency.[11] The model is given by six equations with seven unknowns

$$\frac{C}{P} = f\left(\frac{Y}{P}, i\right) \qquad \text{consumption function}$$

$$\frac{I}{P} = g(i) \qquad \text{investment function}$$

$$\frac{Y}{P} = \frac{C}{P} + \frac{I}{P} \qquad \text{equilibrium condition for the product market}$$

$$\frac{M^D}{P} = \ell\left(\frac{Y}{P}, i\right) \qquad \text{money demand function}$$

$$M^S = h(i) \qquad \text{money supply function}$$

$$M^D = M^S \qquad \text{equilibrium condition for the money market}$$

The first three equations describe the product market and correspond to the notion of the IS curve. The remaining three equations refer to the money market with demand for and supply of money functions and give the LM curve. Since the number of unknowns exceeds the number

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[11] M. Friedman, A Theoretical Framework for Monetary Analysis. National Bureau of Economic Research Occasional Paper 12, New York, 1971, pp.29-31.

of equations one of them should be set exogenously or the system should be extended to include an additional equation. While all of the empirical studies on the IS-LM model followed the first alternative, the second possibility was pursued theoretically.
B. T. McCallum proposed the completion of the system by the addition of an equation explaining price changes

$$\frac{dP}{dt} = \phi\left(\frac{Y}{P} - \frac{Y^*}{P}\right) \quad \text{with} \quad \phi(0) = 0$$
$$\text{and} \quad \phi' > 0$$

where $\frac{Y}{P} - \frac{Y^*}{P}$ is the discrepancy between actual output and its full employment level, a measure of excess demand.[12]

Some other contributions to the problem will be examined subsequently because they incorporate simultaneously price expectations effects on interest rates.

Independence of the IS and LM Curves: The question arose in relation to the interpretation of the transmission process of monetary policy. As already noted the interpretation attributed to Keynesian analysis considers the relation of money and debt instruments as the starting point. People respond to an increased supply of money by buying bonds and there is no direct substitution of money and real capital and goods. The effect on aggregate demand comes indirectly since the change in bond prices and the interest rate induce changes in investment spending. In this interpretation the LM curve is thought to move independently, the IS curve being fixed. But monetarists have stressed that money can be spent on financial assets as well as directly on goods and real capital. A 'real balance' effect is

[12] B. T. McCallum, "Friedman's Missing Equation: Another Approach." Manchester School of Economic and Social Studies, 1973, pp.311-323.

introduced in the expenditure function and the IS and LM curves cease to be independent of each other. A rightward shift in the LM curve is followed by an upward shift of the IS curve. The effect on the interest rate is not clear. Explanations are put forward suggesting that the new equilibrium rate will be higher than the initial.

A different type of interdependence is obtained from a modified second version of the IS-LM model and a supply function for nominal money. To illustrate consider the following model as presented by Bailey.[13]

$$\frac{L}{P} = \frac{a}{P} + b\,\frac{Y}{P}$$

$$\frac{I}{P} = \frac{g_0}{P} + g\,\frac{Y}{P}$$

$$\frac{Y}{P} = \frac{C}{P} + \frac{I}{P}$$

$$\frac{M}{P} = L\left(i, \frac{Y}{P}\right)$$

$$\frac{M}{P} = \frac{1}{P}, h(i)$$

It is a five equation model with six variables. When the price level is taken as the exogenous variable both the IS and LM schedules considered on the $i, \frac{Y}{P}$ plane are dependent on it. We note that the influence of the price level is not restricted to plain shifts of the curves but to altering their slopes as well.

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[13] M. J. Bailey, National Income and the Price Level. McGraw Hill, 1962, pp.28-33.

Price expectations effects: The analysis based on the IS-LM model has generally ignored price expectations effects. By this it is meant that there are anticipations of a changing price level, and these in turn affect other macroeconomic variables. The reason for neglecting them is simply that price expectations are of dynamic nature and as such are not tenable in a static framework. Moreover, the Keynesian model is abstracted from examining changes in the price level and price expectations in these models would be completely unbacked. There has been an attempt by Teigen to demonstrate the possible differentiation of the comparative static results. obtained from the IS-LM model when price expectations effects on interest rates are allowed.[14]

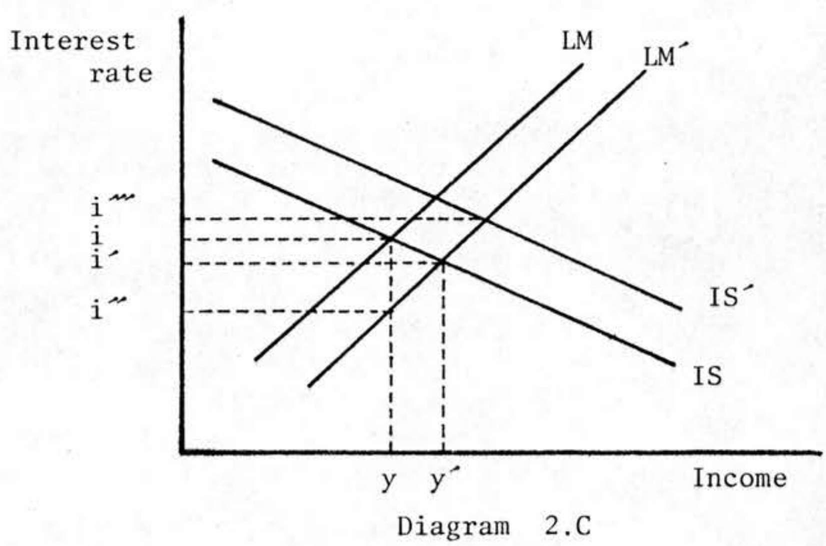

Diagram 2.C

In diagram 2.C, where the IS curve has the usual negative slope and the initial equilibrium values of interest rate and real income are i and y respectively, a monetary expansion will induce a shift of the LM curve to the position LM' . This involves the so-called 'liquidity effect', that is a fall of the interest rate to i" and

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[14] R. L. Teigen, "A Critical Look at Monetarist Economics". Federal Reserve Bank of St. Louis Review, January 1972, p.20.

the 'income effect' that is the subsequent rise of both real income and interest rate to y' and i'. The presence of price expectations will result in a rightward movement of the IS curve and the final equilibrium value of the interest rate might be higher than the initial. As is evident Teigen's discussion is heuristic and does not give the exact conditions under which equilibrium implies a higher value of the interest rate.

It was pointed out that the dynamic nature of price expectations does not permit their formal inclusion in a static IS-LM model. For this reason it suffices to emphasise their importance in shifting the IS curve and altering equilibrium positions. But a dynamic theory should contain explicitly the mechanism by which expectations are formed and the way they are supposed to influence other variables. Notwithstanding a review of a number of empirical studies which deal with the IS-LM model and most of which estimate the parameters of dynamic forms of the system, reveals that none of them has incorporated price expectations effects. This is not unrelated to the observation that empirical work has kept in line with the Keynesian System and the variables are either expressed all in nominal terms or they are deflated by the price level to represent quantities in constant prices. On the other hand, as already noted, there is a growing literature on the IS-LM model extended to include a price determination equation. The attempt on these lines has been confined so far to dynamic theoretical models simple enough to permit derivation of their implications.[15] In

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[15] For an empirical test on a part of the model consisting of the money demand and price equations see D. Laidler, "The Influence of Money on Real Income and Inflation: A Simple Model with Some Empirical Tests for the United States, 1953-72". Manchester School of Economic and Social Studies, 1973, pp.267-83.

all of them the expected rate of price change is an important constituent determining crucially the system's properties. We can distinguish the case where (a) the expected rate of inflation is an exogenous parameter determining the real rate of interest which is the appropriate rate influencing investment decisions. No relation is written or even alleged between expected and actual price changes. (b) the expected rate of inflation is again exogenous but its connection with changes in the price level is recognised but not put in an explicit form. The implications of the latter are distinctly different from those of the former. In equilibrium, although both cases accept that output is equal to full employment output in (a) the expected rate of inflation is zero, and in (b) is equal to the actual rate. Finally, (c) there is a clear link between expected and actual rate of inflation. Price developments are influenced by the variables of the model and feedback through endogenous expectations. An example of expectations falling under each category will be given below.

Sargent considered price expectations effects within the expanded IS-LM framework.[16] His main purpose was to assess the validity of Fisher's doctrine which links nominal with real rates through the anticipated rate of inflation. The model has a fairly standard formulation on the equations of the output market but innovates on the money market relationship where nominal money balances are connected with the price level, real output and nominal interest rate.

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[16] T. J. Sargent, "Anticipated Inflation and the Nominal Rate of Interest". Quarterly Journal of Economics, 1972, pp.212-225.

The anticipated rate of inflation is allowed in the model solely through the investment function which is

$$I_t = a_0 + a_1 \Delta Y_t + a_2 (i_t - \pi_t) + a_3 I_{t-1}$$

where I, Y and i are respectively real investment, real output and nominal rate of interest. In contrast the price equation does not contain any relation between expected rate of inflation and the price level. It is simply

$$P_t = P_{t-1} + \gamma (Y_{t-1} - Y^F) \qquad \gamma > 0$$

where P is the price level and Y^F the full employment level of output taken as constant over time in the absence of technological change and growth in the supplies of productive inputs.

The workings of the complete model show that a sustained increase in π has as a long run effect an increase in the rate of interest by an equal amount.

Brunner and Meltzer introduced expectations in an 'amended' form of the Hicksian model but their relations are not expressed in a way amenable to econometric estimation.[17] Nevertheless, they are important in pointing out that in equilibrium the actual and anticipated rate of inflation should be equal. Their price equation is written as

$$\frac{dP}{P} = h (y - y_0 , \pi) \qquad h_1 \; h_2 > 0$$

With $\pi = \frac{dP}{P}$ in equilibrium, where $\frac{dP}{P}$ is the actual rate of

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[17] K. Brunner and A. H. Meltzer, "Mr. Hicks and the Monetarists". Economica, 1973, pp.44-59.

inflation, π the anticipated rate of inflation, h denotes a general functional dependence, y is real output and y_0 full employment output. The last is the outcome of a production function $y = f(K,L)$ relating output to capital and labour inputs, when

> *"capital stock, productivity and tastes are given, anticipations are equal to actual values and the constant population supplies the man-hours of labour $L = L_0$ consistent with their lifetime consumption plans and the current and anticipated prices".*[18]

The influence of anticipated price changes on the IS curve comes from the replacement of the nominal interest rate i by the real rate $i - \pi$ in the expenditure function. In the discussion which follows the set-up of equations, anticipated rates of inflation can rise as a result of increases in the rate of price change which influences them tacitly. With no explicit feedback mechanism written between them the relation is presented as a leftward shift of the curve on the $\frac{dP}{P}$, y plane.

Laidler has adopted the 'error learning' model of the formation of expectations.[19]

$$\Delta P_t^E = d\,\Delta P_t + (1 - d)\,\Delta P_{t-1}^E \qquad 0 < d \leqslant 1$$

where ΔP^E denotes expected rate of inflation, and has thus made expectations an endogenous element in the system. Together with the adjustment mechanism, the price equation appears as

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[18] K. Brunner and A. H. Meltzer, *op.cit.*, p.46

[19] D. Laidler, "Simultaneous Fluctuations in Prices and Output: A Business Cycles Approach". Economica, 1973, pp.60-72.

$$\Delta P_t = \Delta P_{t-1}^E + c\,(Y_t - Y_t^*) \qquad \text{when} \quad Y_t > Y^*$$

$$\text{and} \quad \Delta P_t = \Delta P_{t-1}^E + g\,(Y_t - Y_t^*) \qquad \text{when} \quad Y_t \leqslant Y^*$$

where Y is real output (or natural log of it) and Y^* the full employment output which is again taken as constant. The real rate of interest $R - \Delta P^E$ is the one used in the investment function. A weakness of the model recognised by the author is that it can generate a continuously increasing level of excess demand so that the form chosen for the price equation can lead to an increasing under-prediction of the rate of inflation. On the other hand, the condition taken for equilibrium is that $Y_t = Y_{t-1} = \ldots Y^*$ and not a situation of no error with $d = 1$. Although the model yields interesting results (e.g. that an increasing output can create inflation before full employment output is reached), it has the serious drawback that the path of the real output variable is given by a difference equation which includes unobservable quantities, namely lagged expected rates of inflation.

Empirical research on the IS-LM model: The model has not remained a theoretical apparatus useful merely for expository purposes. There have been some attempts to estimate IS-LM models with interest either focused genuinely on the model and its slope properties or on some aspects of macrotheory within the context of this model. Below we present a review of a number of studies that came to our attention with a description of their principal features and specification of the equations. They are ordered chronologically so that one can follow the pace of the headway made in empirical applications.

1. ESTIMATES OF HICKSIAN IS AND LM CURVES FOR THE UNITED STATES (R. H. Scott, Journal of Finance, 1966, pp.479-487)

Scott estimated linear reduced forms corresponding to the notion of the IS and LM curves. They were derived from the structural equations

$$C_t = a_1 + b_1 Y_t$$

$$I_t = a_2 + b_2 Y_t + c_2 R_t$$

$$G_t = \overline{G}_t$$

$$Y_t = C_t + I_t + G_t$$

$$M_t = M_{1t} + M_{2t}$$

$$M_{1t} = a_3 Y_t$$

$$M_{2t} = a_4 R_t$$

$$M_t = \overline{M}_t$$

where

- Y : G.N.P. seasonally adjusted current prices
- C : consumption
- I : investment
- G : government spending plus net exports
- M_1 : transaction balances of money
- M_2 : liquidity balances of money
- M : money supply including time deposits, seasonally adjusted
- R : yield on Aaa corporate bonds, seasonally unadjusted

The findings, based on quarterly data covering 1951 to 1964 and two stage least squares estimating technique, were

$$Y_t = 2019.65 - 1545.33\ R_t + 43.94\ G_t \qquad \text{(IS)}$$

$$R_t = 2.441 + 0.0233\ Y_t - 0.046\ M_t \qquad \text{(LM)}$$

The reduced form equations satisfy the order condition for identifiability. However, not all of the structural parameters can be exactly identified from the reduced forms parameters. The latter are statistically significant and the sign of the coefficient of the interest rate in the IS equation indicates a sum of propensities to consume and invest less than one. The model is highly simplified, static and does not include any price effects. The specification of the demand for money seems to take no account of the criticisms related at least to the separation and independence of money held for transactions from money held for speculative purposes. A discriminatory device was to test the system separately for the periods where income was high and the periods where income was low. The IS curve turned out to be more elastic in the former case, a result in line with the Keynesian contention of an inelastic investment schedule during periods of low activity.

2. MONEY SUPPLY AND DEMAND; A COBWEB? (P. E. Smith, International Economic Review, 1967, pp.1-12)

It is admittedly true that predictions derived from short term models of income determination depend crucially on how money demand and supply interact. Smith explored the effect of including a money supply function in a simple IS-LM model, an aspect generally overlooked in empirical work. He estimated with U.S. data two versions of the model, the second being slightly more disaggregated. Below we present the less disaggregated version of it which has exhibited better forecasting properties.

$$C_t = a_1 + b_1 Y_{t-1}$$

$$I_t = a_2 + b_2 Y_{t-1} + c_2 R_{t-1}$$

$$Y_t = C_t + I_t$$

$$M_t^D = a_3 + b_3 Y_t + c_3 R_t$$

$$M_t^S = a_4 + b_4 R_{t-1} + c_4 (Y_{t-1} - Y_{t-2})$$

where

- C : private consumption, government current expenditures and net balance of trade
- I : private investment including inventories
- Y : G.N.P. current prices seasonally adjusted
- M^D, M^S : money demand and supply (currency and demand deposits)
- R : yield on corporate bonds

Banks are assumed to maintain some desired ratio of money supplied to expenditures. This is an awkward attempt alien to the theory describing money supply process. Since the inclusion of the level of output and current interest rate would create an identification problem Smith introduces an acceleration relationship into the money supply equation. This is again questionable because money is a stock variable.

The model is of limited use since all of the predetermined variables are lagged dependent variables and no policy instrument appears explicitly. Thus Smith discusses monetary policy in terms of the parameter C_4 being adjusted by the monetary authorities when Y_{t-2} is replaced by desired output. The inclusion of the money supply is found to lead to fluctuations. The solution to the system of difference equations provides a dominant root equal to 1.01, whereas in the enlarged version this is reduced to 0.997. The IS-LM curves are not obtained but as it can be seen the model is recursive.

3. MULTIPLIER, ACCELERATOR AND LIQUIDITY PREFERENCE IN THE DETERMINATION OF NATIONAL INCOME IN THE UNITED STATES (G. C. Chow, Review of Economics and Statistics, 1967, pp.1-15)

The model built by Chow is an attempt to scrutinise statistically the relevance of the multiplier, accelerator and liquidity preference in the determination of national income of the United States. The model set out in a dynamic form which includes simple distributed lags, has a peculiar mixture of differences and levels of the variables. The equations estimated from annual data for the periods 1931 to 1940 and from 1948 to 1963 are

$$\Delta C_t = a_1 \Delta Y_t^* + b_1 \Delta G_t + c_1 \Delta M_t + d_1 P_{t-1} + e_1 \Delta C_{t-1} \quad (1)$$

$$\Delta I_t^N = a_2 \Delta Y_t^* + b_2 \Delta G_t + c_2 Y_{t-1} + d_2 P_{t-1} + $$
$$+ e_2 \Delta R_t + f_2 R_{t-1} + g_2 I_{t-1}^N \quad (2)$$

$$\Delta I_t^C = a_3 \Delta Y_t^* + b_3 \Delta G_t + c_3 Y_{t-1} + d_3 P_{t-1} + $$
$$+ e_3 \Delta R_t + f_3 R_{t-1} + g_3 I_{t-1}^C \quad (3)$$

$$\Delta M_t = a_4 \Delta Y_t^* + b_4 \Delta G_t + c_4 \Delta R_t + d_4 \Delta M_{t-1} \quad (4)$$

$$\Delta Y_t^* = \Delta C_t + \Delta I_t^N + \Delta I_t^C \qquad (5)$$

$$\Delta Y_t = (1 - a_5)\ (\Delta Y_t^* + \Delta G_t) \qquad (6)$$

where

C : personal consumption expenditures

I^N : gross private investment in durable equipment plus inventories

I^C : new construction

Y^* : private expenditures

G : government purchases of goods and services

Y : private expenditures plus government deficit, current prices seasonally unadjusted

M : money stock - currency and demand deposits

R : yield of 20 year corporate bonds

P : G.N.P. deflator

The income variable is neither Y^* nor Y , but is $Y + G$. Net exports are excluded from it. Disposable income which is relevant for the consumption function is defined as $Y^* + G - T = Y$ (7) with T representing total taxes minus total transfers. Thus there is a neglect of depreciation and undistributed profits. The last equation is arrived at by assuming that net taxes are a linear function of domestic expenditure.

$$T_t = a_5\ (Y_t^* + G_t) + b_5 \qquad (8)$$

Substituting in (7) and differencing gives (6). The coefficient a_5 is estimated extraneously and (6) is treated as an identity. The reason for this was the presence of a major change in the tax rate which occurred in 1943, so that leaving a_5 unchanged over the whole period of estimation would affect all of the parameters in the system. Two values of a_5 were estimated; one for 1931 to 1943 and the other for 1943 to 1963. The method was free-hand drawing to find the relevant slopes in the scattergrams of T and (Y* + G) and then adjustment of the data according to the finding. Obviously the desire to keep the system as simple as possible averted the author from using some other more objective method to handle this change, e.g. a slope dummy.

Income was separated to the two components Y* and G in order to discriminate their multiplier effect. The results support the suggestion that G may have less importance than Y* as a component of aggregate demand. However, complete consistency is not observed in the specification of the equations since in the consumption function the change in disposable income ΔY is split into ΔY^* and ΔG but not the level Y_{t-1} of disposable income. The reason is that splitting Y_{t-1} would simply increase the number of predetermined variables. The same remark applies to the investment functions where the outside estimation of the tax rate permits scaling from disposable income appearing therein to simple income which is pertinent to the investment function.

Equations (1) to (3) include the stock of money to test the effects of liquid assets on consumption and investment expenditures. The coefficients of interest rate were found insignificant in the equation for I^N and the interest rate was suppressed from it. Similarly b_3 was insignificant. The acceleration principle was formulated in such a way that gross rather than net investment appeared as the dependent variable. The resulting equation was over-identified but the empirical results conformed quite well to expectations about equality of the derived depreciation rate from two different reduced form coefficients.

Chow contrived a linear deflation device in order to replace the real values in which the expenditure relationships are formalised by their nominal counterparts. The transformed, say, consumption equation is

$$\Delta \overset{*}{C}_t{}^N = \beta \, \Delta Y_t{}^N + cP_{t-1} \tag{9}$$

where the variables are in nominal terms and * denotes equilibrium values. The rationalisation stems from the approximation

$$\Delta C_t{}^N = \Delta (C_t{}^R P_t) \simeq P_t \Delta C_t{}^R + C_t{}^R \Delta P_t \simeq P_t \Delta C_t{}^R \simeq$$

$$\simeq P_{t-1} \Delta C_t{}^R \tag{10}$$

similarly

$$\Delta Y_t{}^N \simeq P_{t-1} \Delta Y_t{}^R \tag{11}$$

Presumably (9) expressed in real terms would be

$$\Delta C_t^{*R} = \beta \, \Delta Y_t{}^R + c \tag{12}$$

But the inclusion in (12) of a constant term allowing P_{t-1} to appear linearly would require a linear trend in the consumption function

$$C_t^{*R} = a + \beta \, Y_t{}^R + ct \tag{13}$$

and such an assumption is clumsy. An insignificant coefficient of the price level would indicate the possibility of applying unchangeably the model to nominal values of the variables. The ratio of the real consumption to real income would not change when the price level changed. The opposite situation could be defined as money illusion and was found to exist in the model.

The study does not cover the determination of the price level and the mechanism of money creation.

The IS-LM curves are cited and on the basis of their slopes the multiplier and accelerator are found to be more important than the liquidity preference in the determination of private aggregate demand. The two curves are shown below.

$$
\begin{aligned}
Y_t^* = {} & Y_{t-1}^* - 0.7171\ (R_t - R_{t-1}) + 0.6323\ C_{t-1} - \\
& - 2.161\ I_{t-1}^N - 1.664\ I_{t-1}^C + 0.1631\ Y_{t-1} + \\
& + 1.331\ \Delta M_t + 0.8046\ \Delta G_t + 1106\ P_{t-1} - \\
& - 55195 \qquad \text{(IS)}
\end{aligned}
$$

$$
\begin{aligned}
Y_t^* = {} & Y_{t-1}^* + 9.017\ (R_t - R_{t-1}) + 6.663\ \Delta M_t - \\
& - 2.866\ \Delta M_{t-1} + 1.688\ \Delta G_t + 8770 \qquad \text{(LM)}
\end{aligned}
$$

The salient characteristic of the two schedules is their dynamic nature and in such a system shifts can be caused by changes in exogenous and past endogenous variables. It has been ascertained that for a dynamic system, under an appropriate specification of the relationships, there are in each period parallel shifts of the one or the two curves (in both disequilibrium or equilibrium situations).

The dynamic properties of the model were explored by Arzac.[20] He started with a deterministic simulation and proceeded to stochastic runs with random shocks imposed on the endogenous variables. The exogenous variables were either kept constant or were also disturbed. In the deterministic run the 1952 initial conditions were used and the exogenous variables were kept constant. All endogenous variables followed a growth trend without any fluctuations.

Next the simulation experiment was continued with random variables added to the equations. They were drawn from a multivariate normal distribution defined by the covariance matrix of residuals in Chow's model. The paths followed by the endogenous variables oscillated about the corresponding paths in the deterministic run. This however, does not tell anything about how successful is the interaction of variables in the model's structure. Arzac did not compare the fluctuations produced from the shocked model with fluctuations of some kind of naive model with no economic interaction of the variables. The example was given by Adelman[21] who simulated

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[20] E. R. Arzac, "The Dynamic Characteristics of Chow's Model: A Simulation Study." Journal of Financial and Quantitative Analysis, 1967, pp.383-397.

[21] I. Adelman, "Business Cycles Endogenous or Stochastic?" Economic Journal, 1960, pp.783-96

the Klein-Goldberger model and found it superior to a naive model in terms of a stronger tendency towards a clustering of turning points of produced cycles at reference peaks and troughs. One cannot, therefore, argue whether the specification of the Chow model has led to improvement of the description of dynamic behaviour over the purely stochastic interpretation of the cycles. Finally, Arzac repeated the stochastic simulation with the exogenous variables being stochastic rather than deterministic. The shocks for the exogenous variables were drawn from the distribution described by the covariance matrix which was obtained from the linear trend of the exogenous variables for the period 1948-63. The pattern of the new fluctuations generated in this last run was similar to the previous one without any substantial change in the amplitude of fluctuations. It indicated the minor contribution of the stochastic behaviour of exogenous variables to the observed cycles.

4. MONEY SUPPLY, CYCLICAL FLUCTUATIONS AND INCOME DETERMINATION (V. K. L. Gupta, Jahbücher für NationalöKonomie und Statistik, 1968-69, pp.465-78)

In a similar vein to Smith's paper, Gupta's exercise assesses the implications of introducing an endogenous money supply to a system free from oscillations. He concludes, with empirical reference to Canada for the period 1958-1965, that money supply can bring about fluctuations and instability. The IS-LM model is

$$C_t^N = a_1 + b_1Y_{t-1} + c_1C_{t-1}^N$$

$$C_t^D = a_2 + b_2Y_{t-1} + c_2R_{t-1} + d_2C_{t-1}^D$$

$$C_t^S = a_3 + b_3Y_{t-1} + c_3C_{t-1}^S$$

$$I_t = a_4 + b_4 Y_{t-1} + c_4 R_{t-1} + d_4 I_{t-1}$$

$$H_t = a_5 + b_5 (Y_{t-1} - Y_{t-2})$$

$$L_t = a_6 + b_6 Y_{t-1}$$

$$G_t = \overline{G}_t \quad \text{and} \quad X_t = \overline{X}_t$$

$$Y_t = C_t^N + C_t^D + C_t^S + I_t + H_t + G_t + X_t - L_t$$

$$M_t^D = a_7 + b_7 Y_t + c_7 R_t$$

$$M_t^S = a_8 + b_8 (Y_{t-1} - Y_{t-2}) + c_8 R_{t-1}$$

with C^N, C^D, C^S, I, H, G, X, L and Y being consumption of nondurables, durables and services, gross fixed investment, investment in inventories, government expenditure, exports, imports and gross national expenditure respectively all in current prices seasonally adjusted.

M^D, M^S : money demand and supply - currency, adjusted demand deposits and saving deposits

R : interest rate on long term government bonds.

Again the lack of exogenous variables is noticeable in the model. The author reports the predictive performance of the equations six quarters ahead. Although the percentage error between actual and forecasted values is generally increasing as we get further beyond the sample period, the three cited components of Theil's coefficient of inequality give the impression of a remarkably good prediction.

The IS-LM curves finally obtained from the estimated structure are

$$Y_t = -4484.79 + 1.08\, Y_{t-1} - 0.26\, Y_{t-2} - 0.28\, C^D_{t-1} + 0.60\, C^S_{t-1} - 0.85\, I_{t-1} - 573.65\, R_{t-1} \qquad (IS)$$

$$Y_t = 46846 + 3.98\, M_t + 12048.23\, R_t \qquad (LM)$$

The LM curve is taken by omitting the money supply function which was shown to affect the stability of the model. The IS curve looks unorthodox, giving no combinations of current income and interest rate, since the last appears lagged one period as a result of the particular lags chosen for the expenditure functions. Essentially there is no IS curve and the interest rate is determined not simultaneously but from the LM schedule alone.

5. LAGS IN THE EFFECTS OF MONETARY POLICY: A STATISTICAL INVESTIGATION (J. E. Tanner, American Economic Review, 1969, pp.794-805)

The study aims at finding the timing of the effects of monetary policy. The impetus for Tanner's enquiry was given by Tucker who has shown that lags in the adjustment to desired money balances can offset long lags that were found to exist in the response of investment to changes in interest rates.[22] The results depend crucially upon the size of the coefficients which characterise speeds of adjustment. Tanner has extended the partial adjustment mechanism

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[22] D. P. Tucker, "Dynamic Income Adjustment to Money Supply Changes". American Economic Review, 1966, pp.433-449.

assumed by Tucker in the demand for money and expenditure equations to a combined with adaptive expectations form. From

$$X_t = X_{t-1} + (1 - \lambda)\ (X_t{}^* - X_{t-1})$$

$$X_t{}^P = a + b\ Z_{t+1}^E$$

$$Z_{t+1}^E = Z_t{}^E + (1 - \beta)\ (Z_t - Z_t{}^E)$$

the reduced form for estimation is

$$X_t = (1 - \lambda)\ (1 - \beta)\ (a + b\ Z_t) + (\lambda + \beta)\ X_{t-1} -$$

$$- \lambda\beta\ X_{t-1}$$

and this has been applied to both money demand and aggregate private expenditure equations. The equations of the model retaining the basic properties of the IS-LM framework are

$$X_t = a_1 + b_1\ (Y_t - T_t) + c_1 R_t + d_1 X_{t-1} + e_1 X_{t-2}$$

$$G_t = \overline{G}_t$$

$$T_t = \overline{T}_t$$

$$E_t = \overline{E}_t$$

$$Y_t = X_t + G_t + E_t$$

$$M_t{}^D = a_2 + b_2 Y_t + c_2 R_t + d_2 M_{t-1} + e_2 M_{t-2}$$

$$M_t{}^S = \overline{M}_t{}^S$$

$$M_t{}^D = M_t{}^S$$

with

X	:	private consumption and investment expenditures
G	:	government expenditures
T	:	government taxes net of transfers
Y	:	gross national expenditure, current prices
R	:	short term interest rate
M^D, M^S	:	money demand and supply

The parameters were estimated from a sample of U.S. quarterly data for the period 1947 to 1967 using augmented two stage least squares. Since in 2SLS in order to obtain consistent parameter estimates when lagged dependent variables are treated as predetermined in the first stage, the assumption is required that the error term is non-autocorrelated, Tanner has used augmented two stage least squares which yield consistent estimates in the presence of autocorrelated disturbances. Nevertheless, an error in the treatment of the residuals admitted later by the author has let inconsistency creep in.[23] The marginal propensity to spend is less than one in the short run and greater than one in equilibrium. Though this is an interesting feature giving a positively sloped IS curve, it might well be the case of a classical negative slope if imports were made endogenous.

The issue on which the estimated IS-LM curves attempted to shed light was the path that money supply would have to follow over time if a desired change in aggregate private demand was to be occasioned. To that end, the system's equations were solved by elimination of income and the interest rate to give money in terms of

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[23] J. E. Tanner, "Lags in the Effects of Monetary Policy: Reply and Some Further Thoughts". American Economic Review, 1972, p.235 footnote 1.

private expenditures and its own past values. The solution of the obtained difference equation showed that to effect a given permanent change in private expenditures money should change in the first period at about 95 per cent of the total equilibrium change required and the implied lag in monetary policy is accordingly very short. The variable which bears the impact of change in monetary policy is the interest rate which exhibits two very brisk, and in opposite directions, overshootings of its own equilibrium until it is settled to it. The situation might be undesirable to monetary authorities who, however, can use money for short run stabilisation policy.

The work of Tanner was extended by P. E. Smith who removed the inadmissible movement of the interest rate found in the other study by a more sophisticated specification of the relationships.[24] Specifically private expenditures were disaggregated into consumption and investment, the former being subject to a more complex lag distribution than the geometric posited for the investment and the money demand functions. Liquid assets were included in the equation for consumption and a linear accelerator in the investment demand.

The IS-LM model consisting of three structural equations is

$$C_t = (1 - a)\,[\beta_0 + (\beta_1 - \beta_2\eta)\,Y_t] + \beta_2 L_t - $$

$$- \beta_2 a L_{t-1} + a\,C_{t-1}$$

$$I_t = (1 - b)\,[\lambda_0 + \lambda_1(Y_t - Y_{t-1}) + \lambda_2 R_t + $$

$$+ \lambda_3 t] + b\,I_{t-1}$$

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[24] P. E. Smith, "Lags in the Effects of Monetary Policy: Comment American Economic Review, 1972, pp.230-233.

$$Y_t = C_t + I_t + G_t$$

$$M_t^D = (1 - g)\,[\delta_0 + \delta_1 Y_t + \delta_2 R_t] + g\,M_{t-1}$$

$$M_t^D = M_t^S$$

where

C : private consumption

I : gross private domestic investment

G : government expenditure plus net foreign investment

Y : GNE deflated and seasonally adjusted

R : yield on all corporate bonds

M : currency plus demand deposits, in real terms

L : money plus time deposits in commercial banks

t : time

The interesting finding implied by the estimated system is that the existence mainly of the accelerator and to a smaller extent of the liquid assets variable mitigates the required intitial increase in the money supply and the resulting downward movement of the interest rate is of acceptable size.

6. THE CHOICE OF OPTIMAL INTERMEDIATE ECONOMIC TARGETS (R. Holbrook - H. Shapiro, American Economic Review, Papers and Proceedings, 1970, pp.40-46)

The authors are actually concerned with the selection of an optimal intermediate target for guiding monetary policy in the presence of uncertainty about exogenous influences, and the IS-LM model is a 'vehicle' for their analysis. All variables with the exception of the interest rate are expressed in real terms and the model runs as follows

$$C_t = a_1 + b_1 Y_t$$

$$I_t = a_2 + b_2 R_t$$

$$Y_t = C_t + I_t + X_t$$

$$M_t^D = a_3 + b_3 Y_t + c_3 R_t$$

$$M_t^S = a_4 + b_4 R_t + c_4 B_t$$

$$M_t^D = M_t^S$$

where

C : personal consumption expenditure

I : gross private domestic investment

X : net balance of trade plus government expenditure

Y : GNP seasonally adjusted

R : yield of long term government bonds

M^D, M^S : demanded and supplied money balances (currency plus demand deposits)

B : monetary base

The constant terms stand as functions of lagged endogenous and other exogenous variables. The model was fitted to U.S. quarterly data covering 1952(1) to 1966(4). While the monetary sector posed no problems in the estimation of its parameters, the existence of lags in the investment expenditures compelled the authors to experiment on the IS curve. The preferred form on the basis of uncited criteria led the IS-LM model to be recursive. The estimated equations were

$$Y_t = 1.26 - 1.25\, X_{t-2} - 0.16\, C_{t-2} - 0.82\, I_{t-2} +$$

$$+ 1.32\, Y_{t-1} - 6.28\, R_{t-1} + X_t \qquad (\text{ IS })$$

$$R_t = 3.94 + 0.0073\, Y_t - 0.0791\, B_t \qquad (\text{ LM })$$

7. AUTONOMOUS EXPENDITURES VERSUS MONEY SUPPLY: AN APPLICATION OF DYNAMIC MULTIPLIERS (J. Kmenta - P. E. Smith, Review of Economics and Statistics, 1973, pp.229-307)

Kmenta and Smith's study represents an approach to ascertaining the relative effectiveness of autonomous expenditures and money through a structural equations system as opposed to single equation tests largely in use by Monetarists.[25] The model is of the IS-LM type and its equations are

$$C_t = a_1 + b_1 Y_t + c_1 L_t + d_1 L_{t-1} + e_1 C_{t-1}$$

$$I_t^D = a_2 + b_2 R_t + c_2 (S_{t-1} - S_{t-2}) + d_2 t + e_2 I_{t-1}^D$$

$$I_t^R = a_3 + b_3 R_t + c_3 (S_{t-1} - S_{t-2}) + d_3 t + e_3 I_{t-1}^R$$

$$I_t^I = a_4 + b_4 R_t + c_4 (S_{t-1} - S_{t-2}) + d_4 t + e_4 I_{t-1}^I$$

$$R_t = a_5 + b_5\, Y_t + c_5\, M_t + d_5\, M_{t-1}$$

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[25] One equation or 'reduced form' tests that received wide publicity and strong criticism as well are M. Friedman and D. Meiselman. "The Relative Stability of Monetary Velocity and the Investment Multiplier in the United States 1897-1958", in Stabilization Policies, Commission of Money and Credit. Englewood Cliffs: Prentice Hall, 1963. Also L. C. Andersen and J. C. Jordan, "Monetary and Fiscal Actions: A Test of their Relative Importance in Economic Stabilization". Federal Reserve Bank of St. Louis Review, November 1968.

$$Y_t = C_t + I_t^D + I_t^R + I_t^I + G_t$$

$$S_t = Y_t - I_t^I$$

$$L_t = M_t + D_t$$

where

C : consumption expenditures

I^D : investment on plant and equipment

I^R : residential investment

I^I : investment in inventories

G : government expenditure on goods and services plus net foreign investment

S : final sales

Y : gross national expenditure, seasonally adjusted, 1958 prices

t : time

R : yield on corporate bonds

M : currency and demand deposits

D : time deposits

L : liquid assets, constant prices

The forms of the assumed adjustment mechanisms are similar to the ones used by Smith to his study just examined.[26] What discriminates the present model is the breakdown of investment to its components, the definition of and lag in the accelerator variable and the estimation of an inverted money demand function.

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[26] P. E. Smith, *op.cit.*, 1972.

The predictive performance of the estimated model was examined eight quarters ahead and it was found reasonably satisfactory. In 40 out of 48 cases there was no divergence between actual and predicted endogenous variables greater than two standard deviations. Additionally, the model was subjected to a number of specification error tests all of which accepted the hypothesis of no specification error. The dynamic stability of the model was established by examination of the roots of the auxiliary part of the 'fundamental dynamic equation' relating income to its past values and current and lagged values of the exogenous variables. It should, however, be accepted with caution since the modulus of a pair of conjugate complex roots was 0.8513 with a standard error of 0.2275. In order to examine the relative potency of fiscal and monetary policies the expression

$$Y_t = a_0 + b_0 G_t + b_1 G_{t-1} + \dots + b_t G_0 +$$

$$+ c_0 M_t + c_1 M_{t-1} + \dots + c_t M_0$$

was used, derived from the fundamental dynamic equation by successive substitution of the lagged values of income. The picture which emerged from the calculations showed monetary and fiscal policies to be equi-potent, during the period of the study. A choice between them for stabilisation purposes should be done by establishing some arbitrary evaluation criterion. Smith and Kmenta suggested that the criterion should be a target quarterly increase of GNP of 1.5 billion dollars and missing it is a deflationary situation whilst exceeding it puts inflationary pressure in the economy. Then the operation of monetary policy is inflationary if

$$\hat{Y}_t - \hat{Y}_{t-1} - c_0^* (M_t^* - M_{t-1}^*) \geq 1.5 \quad \text{for}$$

$$\text{for} \quad c_0^* (M_t^* - M_{t-1}^*) > 0$$

where hats denote estimated values, $M_t^* = M_t + L_t$, c_0^* is the corresponding impact multiplier and the difference presents influences on the increase of GNP other than monetary policy. With analogous definitions for fiscal policy and deflationary operation, Smith and Kmenta found that the effects of changes in monetary and fiscal variables were destablising in many actual cases.

The general picture which emerges from all of the empirical work reviewed is far from being entirely satisfactory. It is a common feature of them that movements in the price level have either been neglected or confounded with movements in real variables. This accounts for the lack of distinction between nominal and real rate of interest and the independence of the responses of the IS and LM curves, to fiscal and monetary policy measures.

Moreover, the system's dynamic properties have not been sufficiently explored and competing assertions are not answered unequivocally. Still the IS-LM model as developed recently on a theoretical level, represents a promising approach which meets the criticisms against 'pseudo-reduced' forms of equations relating GNP directly to monetary and fiscal variables.[27] Without entering in the details of the large systems whose upsurge was buttressed by the progress made in econometrics and the advent of electronic computers,

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[27] For the nature of the objections see J. Tobin's "The Role of Money in National Economic Policy" in Controlling Monetary Aggregates. Proceedings of the Monetary Conference held on Nantucket Island, 1969. Sponsored by the Federal Reserve Bank of Boston, pp.21-24.

the IS-LM model, appropriately expanded, remains a paradigm of a small scale system of structural equations which are worth investigating. The latter have the additional attraction of being more easily comprehended by policy makers, and tend to be preferred especially by those who are concerned with the aggregate effects of policy.[28]

As regards extended forms of the IS-LM model which have been subjected to empirical testing we have been able to trace the following two pieces of work in the literature:

1. THE DYNAMIC IMPACTS OF AUTONOMOUS EXPENDITURES AND THE MONETARY BASE ON AGGREGATE INCOME (J. R. Moroney and J. M. Mason, Journal of Money, Credit and Banking, 1971, pp.793-814)

The object of the study was to investigate the response of income to changes in alternative policy instruments. What makes Moroney and Mason's model to be an extended IS-LM model is the fact that the authors have used the short term interest rate in their money demand function, the long term interest rate in the investment function and that they have appended a term structure equation to the plain IS-LM model in order to close it. However this extension of the model does not lessen the criticisms raised above regarding the lack of interaction between the price level and real income and the independence of the IS and LM curves.

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[28] See for example D. R. Francis, "The Usefulness of Applied Econometrics to the Policy Maker." Federal Reserve Bank of St. Louis Review, May 1973, p.9.

The equations of the model were as follows:-

$$Y_t = C_t + I_t + G_t + E_t - O_t$$

$$C_t = a_1 + b_1 Y_t^d + c_1 C_{t-1} + d_1 M_t + e_1 M_{t-1}$$

$$I_t + a_2 + b_2 (C_t - C_{t-1}) + c_2 Y_t + d_2 R_{t-2}^L + e_2 I_{t-1}$$

$$Y_t^d = a_3 Y_t$$

$$O_t = a_4 + b_4 Y_t$$

$$R_t^S = a_5 + b_5 Y_t + c_5 M_t^D$$

$$M_t^S = a_6 + b_6 R_t^S + c_6 R_t^D + d_6 B_t$$

$$M_t^D = M_t^S$$

$$R_t^L = a_7 + b_7 R_t^S + c_7 Y_t$$

where

- C : consumption expenditures
- I : gross private domestic investment
- G : government purchases of goods and services
- E : exports
- O : imports
- Y : gross national product, current prices, seasonally adjusted
- Y^d : disposable personal income
- M : money stock, currency plus adjusted demand deposits, seasonally adjusted
- R^S : short term interest rate, seasonally adjusted
- R^L : long term interest rate, seasonally adjusted
- R^D : rediscount rate
- B : adjusted monetary base, seasonally adjusted

Three stage least squares were applied in order to estimate the parameters of the model. Since in preliminary investigations autocorrelation had been tracked, Moroney and Mason estimated auto-regressive schemes separately for each equation and then the original observations were transformed according to the estimated coefficient of autocorrelation. Even after that, autocorrelation was not eliminated in the money supply equation. From the estimated equations, the final form for income was obtained and impact and twenty interim multipliers were calculated for unit changes in the monetary base amd government expenditure. The results showed changes in the monetary base to be more powerful than changes in government expenditure in influencing income. However, a serious mistake in the procedure followed by Moroney and Mason for the calculation of multipliers is that they treated the variables of the model as if they were absolute levels, whereas in fact they were transformations of the original variables according to the autocorrelation coefficient found in each equation. The sound procedure would be to write the behavioural equations explicitly in terms of the tranformed variables and carry out multiplier analysis to this system.

2. FRIEDMAN'S MISSING EQUATION : ANOTHER APPROACH
(B. T. McCallum, Manchester School of Economic and Social Studies, 1973, pp.311-28)

McCallum's study is the first empirical step towards extending the IS-LM model by adding to it a price equation and expressing all its variables in real terms. There are, however, drastic simplifications in it which reduce significantly its usefulness. The model has already been presented above in general terms (pp.20-1). For estimation purposes McCallum dynamised his price equation, introduced government expenditure in the IS and LM parts so that the basic system he set forth appeared as follows (the variables were expressed logarithmically):

$$\log y_t = a_1 + b_1 \log M_t + c_1 \log G_t + d_1 \log P_t$$

$$\Delta \log P_t = a_2 + b_2 (\log y_t - \log q_t) + b_3 \Delta \log P_{t-1}$$

where

y : real output

q : full-employment output

M : nominal money

G : nominal government expenditure

and P : the price level

As it is seen, the IS-LM part of the model is essentially a reduced form, too simple to stand any serious criticism. The price equation incorporates a demand-pull inflation term only and neglects cost-push inflation. The distinction between nominal rate of interest and real rate of interest is not made and price expectations are not mentioned in the price equation. The dynamic properties of the system are not examined in detail but a remark is only made about impact fiscal and monetary multipliers.

We conclude that much more detail is needed in order to extend the IS-LM framework in a satisfactory and more realistic way. We have pointed out above what are the deficiencies of the standard IS-LM model and the specification of the equations in our model will try to remove them.

CHAPTER 3

CHAPTER 3

DYNAMIC ADJUSTMENT HYPOTHESES

It has been very often found necessary by economists to introduce hypotheses of dynamic adjustment of variables to equilibrium values in order to reconcile economic theory which in most cases is static, with empirical findings. Lag effects in the adjustment to equilibrium were rationalised by appealing to various reasons such as institutional and technological constraints, psychological inertia of economic subjects, uncertainties about the market, frictions, etc. Generally the lag in the adjustment of a variable to equilibrium will have either the length of a single lag or the length of a distributed lag when the adjustment is spread over a period of time. In macro-economics which deal with aggregates it seems reasonable to assume that the total reaction will be distributed over a longer period of time since it is the sum of individual reactions which may be distributed or have the length of a single lag which however is not identical for all individuals.

The distributed lag relationship between a variable Y and the explanatory variable X will be written as

$$Y_t = c_0X_t + c_1X_{t-1} + c_2X_{t-2} + \ldots + c_mX_{t-m} + \varepsilon_t \qquad (3.1)$$

where Y and X are the values of the dependent and independent variables (nonstochastic), ε a random variable with zero mean and the c's are the reaction coefficients giving the response of variable Y to a shock in variable X over m periods earlier. The impact or short-run response is given by c_0 and the total or long-run response by $\sum_{i=0}^{m} c_i$. To eliminate the possibility of explosive values for $E(Y_t)$ we assume that the c's have a finite sum, i.e. $\sum_{i=0}^{m} c_i < \infty$. Usually the c's are normalised to sum to one. c_i can be regarded as the weight given to the lag of length i and the sequence of the c_i's describes the lag distribution.

Since in (3.1) the number of parameters to be estimated may be large, very often restrictions are placed on the coefficients $c_0, c_1, \ldots, c_m$, so that this number can be sustantially reduced. These restrictions result in a sequence of weights which either decline monotonically in a specified way or first rise and then decline. Then simple tranformations can be applied and (3.1) will be reduced to manageable forms.

The most popular lag distribution is the geometric characterised by

$$Y_t = a(X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots) + \varepsilon_t \qquad (3.2)$$

$$0 \leqslant \lambda < 1$$

This distribution was derived by Cagan[1] and Nerlove[2] by applying hypotheses of dynamic adjustment to economic relationships.

According to Nerlove's hypotheses - the partial adjustment hypothesis - there is an equilibrium level of Y , say Y* (other names for it being desired or planned level of Y), which depends on the current value of an exogenous variable X .

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[1] P. Cagan: "The Monetary Dynamics of Hyperinflation." in Studies in the Quantity Theory of Money. Edit. M. Friedman, University of Chicago Press, Chicago, 1956.

[2] M. Nerlove: Distributed Lags and Demand Analysis. USDA, Agriculture Handbook No.141, Washington, 1958.

$$Y_t^* = a X_t + u_t \qquad (3.3)$$

but only a fixed fraction of the required adjustment is accomplished within a particular period of time, i.e.

$$Y_t - Y_{t-1} = \lambda(Y_t^* - Y_{t-1}) \qquad (3.4)$$

$$0 < \lambda \leqslant 1$$

(3.3) and (3.4) reduce to

$$Y_t = a\lambda X_t + (1 - \lambda) Y_{t-1} + u_t' \qquad (3.5)$$

$$u_t' = \lambda u_t \qquad (3.6)$$

For the range of values assumed for λ (3.5) is stable because the stability condition requires that $0 < \lambda < 2$. Reduced form (3.5) after repeated substitutions of lagged values of Y gives as final form the geometric lag model

$$Y_t = a\lambda X_t + a\lambda(1 - \lambda) X_{t-1} + a\lambda(1 - \lambda)^2 X_{t-2} + \ldots \qquad (3.7)$$

If (3.3) includes more than one independent variable the same lag distribution applies to all of them. The partial adjustment model has been used widely in practice. To quote an example, Chow and Moore have estimated the reduced form of the partial adjustment model for all but two of the twenty three equations of a model of business cycles which they have built.[3]

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[3] G. C. Chow and G. H. Moore: <u>op.cit.</u>

The same reduced form was obtained by Cagan's hypothesis - the adaptive expectations hypothesis - according to which there is an expected level X^* of a variable X (another name for it being a permanent level of X), which is revised in proportion to the error associated with the previous level of expectations, i.e.

$$X_t^* - X_{t-1}^* = \mu(X_t - X_{t-1}^*) \qquad (3.8)$$

$$0 < \mu \leqslant 1$$

This hypothesis together with a relationship of the type

$$Y_t = a X_t^* + u_t \qquad (3.9)$$

yields after a simple transformation

$$Y_t = a\mu X_t + (1 - \mu) Y_{t-1} + u_t' \qquad (3.10)$$

$$u_t' = u_t - \mu u_{t-1} \qquad (3.11)$$

the same reduced form as (3.5) except that the errors are a first order moving average and are not generally independent of each other unless it is assumed that the errors in (3.9) follow a first order auto-regressive process with parameter μ .

The partial adjustment and the adaptive expectations models were combined into one compound geometric lag model as follows:

$$Y_t^* = a X_t^* + u_t \qquad (3.12)$$

$$Y_t - Y_{t-1} = \lambda(Y_t^* - Y_{t-1}) \qquad (3.13)$$

$$0 < \lambda \leqslant 1$$

$$X^*_t - X^*_{t-1} = \mu(X_t - X^*_{t-1}) \tag{3.14}$$

$$0 < \mu \leqslant 1$$

From (3.13) we have

$$Y^*_t = \frac{1}{\lambda} Y_t + \frac{\lambda - 1}{\lambda} Y_{t-1} \tag{3.15}$$

(3.12) gives

$$X^*_t = \frac{1}{a} Y^*_t - \frac{1}{a} u_t \tag{3.16}$$

and

$$X^*_{t-1} = \frac{1}{a} Y^*_{t-1} - \frac{1}{a} u_{t-1} \tag{3.17}$$

so that (3.14) becomes

$$Y^*_t - Y^*_{t-1} - u_t + u_{t-1} = \mu(a X_t - Y^*_{t-1} - u_{t-1}) \tag{3.18}$$

and substituting for Y^* and Y^*_{t-1} we get finally

$$Y_t = a \lambda\mu X_t + [(1 - \lambda) + (1 - \mu)]Y_{t-1} -$$

$$- (1 - \lambda)(1 - \mu) Y_{t-2} + u'_t \tag{3.19}$$

where

$$u'_t = \lambda u_t - \lambda(1 - \mu) u_{t-1} \tag{3.20}$$

The present model contains the additional variable Y_{t-2} and its errors are serially correlated. The model is stable since the condition for stability is that $0 < \lambda, \mu < 2$.

Despite the fact that the reduced form (3.14) has been used in practice,[4] the implied lag distribution has not been examined. The weights of lagged X's are obtained formally by substituting $\left[\lambda\mu X_{t-1} + [(1-\lambda) + (1-\mu)]Y_{t-2} - (1-\lambda)(1-\mu)Y_{t-3}\right]$ for Y_{t-1} in (3.19), then $\left[\lambda\mu X_{t-2} + [(1-\lambda) + (1-\mu)]Y_{t-3} - (1-\lambda)(1-\mu)Y_{t-4}\right]$ for Y_{t-2} in the resulting expression, and the procedure is repeated until all lagged values of the endogenous variable have been substituted.

We have carried out the above substitution six times with the following results:

Weight of X_t : $\lambda\mu$

" X_{t-1} : $\lambda\mu[(1-\lambda) + (1-\mu)]$

" X_{t-2} : $\lambda\mu\left[[(1-\lambda) + (1-\mu)]^2 - (1-\lambda)(1-\mu)\right]$

" X_{t-3} : $\lambda\mu\left[[(1-\lambda) + (1-\mu)]^3 - 2(1-\lambda)(1-\mu)\,[[(1-\lambda) + (1-\mu)]\right]$

" X_{t-4} : $\lambda\mu\left[[(1-\lambda) + (1-\mu)]^4 - 3(1-\lambda)(1-\mu)\,[[(1-\lambda) + (1-\mu)]^2 + (1-\lambda)^2(1-\mu)^2\right]$

" X_{t-5} : $\lambda\mu\left[[(1-\lambda) + (1-\mu)]^5 - 4(1-\lambda)(1-\mu)\,[[(1-\lambda) + (1-\mu)]^3 + 3(1-\lambda)^2(1-\mu)^2[[(1-\lambda) + (1-\mu)]\right]$

" X_{t-6} : $\lambda\mu\left[[(1-\lambda) + (1-\mu)]^6 - 5(1-\lambda)(1-\mu)\,[[(1-\lambda) + (1-\mu)]^4 + 6(1-\lambda)^2(1-\mu)^2\,[(1-\lambda) + + (1-\mu)]^2 - (1-\lambda)^3(1-\mu)^3\right]$

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[4] See for example J. E. Tanner: "Lags in the Effects of Monetary Policy: A Statistical Investigation." American Economic Review, 1969, pp.794-805 and E. L. Feige: "Expectations and Adjustments in the Monetary Sector." American Economic Review, Papers and Proceedings, 1967, pp.461-73.

A generalisation for these terms has been provided by Griliches.[5] Write (3.18) as

$$Y_t = a_0 X_t + t_1 Y_{t-1} + t_2 Y_{t-2} \qquad (3.21)$$

and the final form as

$$Y_t = z_0 X_t + z_1 X_{t-1} + z_2 X_{t-2} + \ldots \qquad (3.22)$$

Then the z_i's are given by the recursion

$$z_j = t_1 z_{j-1} + t_2 z_{j-2} \qquad (3.23)$$

with $z_0 = a_0$ and $z_1 = t_1 a_0$

The sum of the z_i's is

$$\sum_{i=o}^{\infty} z_i = \frac{a_0}{1-t_1-t_2} = \frac{a\lambda\mu}{1 - (1 - \lambda) - (1 - \mu) + (1 - \lambda)(1 - \mu)} = a \qquad (3.24)$$

We see that the compound geometric lag model implies a sequence of weights which sum up to unity as in the case of the partial adjustment and adaptive expectations models.

We have calculated the weights of the X's for three different pairs of values for λ and μ and for lags up to six. Table 3.1 below contains the findings. We note that in column 1 the weights decline rapidly, in column 2 they remain constant for one period and then decline, and in column 3 they first rise and after reaching a peak they decline.

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[5] Z. Griliches: "Distributed Lags : A Survey." Econometrica, 1967, p.23.

TABLE 3.1

Weight of	1 λ = 0.9 μ = 0.9	2 λ = 0.9 μ = 0.1	3 λ = 0.3 μ = 0.4
X_t	0.81	0.09	0.12
X_{t-1}	0.162	0.09	0.156
X_{t-2}	0.0243	0.0819	0.1524
X_{t-3}	0.00324	0.0738	0.1326
X_{t-4}	0.000405	0.066429	0.108372
X_{t-5}	0.0000486	0.059787	0.0851916
X_{t-6}	0.00000567	0.0538039	0.0652328

Concerning the distribution of weights exhibited in the last column, it was found after some experimentation that the lower the values of both λ and μ are, the later the peak in this distribution occurs.

The general conclusion is that lag distributions of different types can be obtained from the above model depending on the relative sizes of the parameters λ and μ . These findings increase the usefulness of the compound geometric distribution in certain applications since it is seen to be more general: it conveys to the data the task of determining (a) whether the weights will decline monotonically or they will first remain constant and then decline or they will initially increase, reach a peak and then decline, and (b) the speed of these movements.

Introduction of a new lag distribution

A common characteristic of the lag distributions mentioned in the previous section as well as of the ones not examined here (e.g. the Pascal lag model, the polynomial lag model, etc.) is that the weights applied to the values of X are assumed to be positive numbers.[6] Moreover, in the case where they are normalised to sum to one, they are all lying in the range (0,1) . This means that the variable tending to reach the equilibrium value (as this is determined by economic theory), falls always short of it until finally it approaches it asymptotically. Or to put it differently the cumulative sum of the weights is always positive and less than one in value. However, there exists another plausible description of the adjustment process to equilibrium: the variable in question instead of falling short of the equilibrium value in the first period, overshoots it so that in the next period it is necessary to reverse its movement in order to correct the error. With a new overshooting of the equilibrium value the process will be repeated until the variable is settled to it. This characterisation of the adjustment implies a sequence of weights alternating in sign but finally converging to zero.

We can see some ways in which this lag distribution can be obtained formally. Consider the equilibrium relationship of the partial adjustment model and the dynamic adjustment mechanism of the adaptive expectations model

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[6] Griliches noted that the assumption of positive weights is not a necessary one and it is not implied by either optimal prediction theory or optimal control theory when these are used to provide a theoretical underpinning to distributed lag models, *op.cit.*, p.44.

$$Y^*_t = a X_t + u_t \tag{3.25}$$

$$Y^*_t - Y^*_{t-1} = \lambda(Y_t - Y^*_{t-1}) \tag{3.26}$$

$$0 < \lambda \leqslant 1$$

(3.25) and (3.26) together imply that

$$Y_t = \frac{a}{\lambda} X_t + a(1 - \frac{1}{\lambda}) X_{t-1} + u'_t \tag{3.27}$$

$$u'_t = \frac{1}{\lambda} u_t + (1 - \frac{1}{\lambda}) u_{t-1} \tag{3.28}$$

In (3.27) Y overshoots equilibrium in the first period $(\frac{1}{\lambda} > 1)$, but the situation is exactly corrected in the second period $[(1 - \frac{1}{\lambda}) < 0$ and $\frac{1}{\lambda} + (1 - \frac{1}{\lambda}) = 1]$.

Now, if in (3.26) Y^*_{t-1} is replaced by its actual value Y_{t-1} , we get

$$Y^*_t - Y_{t-1} = \lambda(Y_t - Y_{t-1}) \tag{3.29}$$

or

$$Y^*_t = \lambda Y_t + (1 - \lambda)Y_{t-1} \tag{3.30}$$

and (3.29) is the <u>inverse adjustment mechanism</u> of the one used in the partial adjustment model.

The above mechanism has been used by Lovell[7] for the estimation of the function for inventories of finished goods in the

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[7] See M. Lovell: "Manufacturers' Inventories, Sales Expectations and the Acceleration Principle." <u>Econometrica</u>, 1961, pp.293-314.

U.S. economy. However because it was combined with another adjustment mechanism and a different notion of the equilibrium relationship there is no connection whatsoever with the lag distribution discussed below.[8]

(3.29) together with (3.25) give the following reduced form

$$Y_t = \frac{a}{\lambda} X_t + (1 - \frac{1}{\lambda}) Y_{t-1} + u_t' \qquad (3.31)$$

$$u_t' = \frac{1}{\lambda} u_t \qquad (3.32)$$

The lag distribution can be obtained by repeated substitution of the lagged Y's in (3.31). It is characterised by

$$Y_t = \frac{a}{\lambda} X_t + \frac{a}{\lambda} (1 - \frac{1}{\lambda}) X_{t-1} + \frac{a}{\lambda} (1 - \frac{1}{\lambda})^2 X_{t-2} + + \frac{a}{\lambda} (1 - \frac{1}{\lambda})^3 X_{t-3} + \ldots \qquad (3.33)$$

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[8] Lovell's relationships were

$$H_t^P - H_{t-1} = \delta(H_t^D - H_{t-1}) \qquad 0 < \delta \leqslant 1 \qquad (a)$$

$$H_t^D = a + b \hat{S}_t \qquad (b)$$

$$H_t^P = H_t + S_t - \hat{S}_t \qquad (c)$$

$$\hat{S} = \rho S_{t-1} + (1 - \rho) S_t \qquad 0 < \rho \leqslant 1 \qquad (d)$$

where H is the actual level of inventories at the end of period t ,
H^P is the planned level of inventories at the end of period t ,
H^D is the desired level of inventories at the end of period t ,
S is the actual level of sales in period t and
$\hat{S}$ is the expected level of sales in period t .

(d) is of course the inverse adjustment mechanism. The combination of (a), (b), (c) and (d) produced the following equation for estimation:

$$H_t = \delta a + \delta b S_t + (\delta b + 1)\rho (S_{t-1} - S_t) + (1 - \delta) H_{t-1}$$

with weights $\frac{1}{\lambda}$, $\frac{1}{\lambda}(1 - \frac{1}{\lambda})$, $\frac{1}{\lambda}(1 - \frac{1}{\lambda})^2$, $\frac{1}{\lambda}(1 - \frac{1}{\lambda})^3$, . . . which are positive and negative alternately and decline geometrically in absolute value. For this distribution to be acceptable the stability condition $\left|1 - \frac{1}{\lambda}\right| < 1$ must be satisfied, which implies that $\lambda > 0.5$. This means that the initial overshooting cannot exceed the double of the equilibrium value. We shall term the above distribution "hyperacontistic"[9] lag distribution. Table 3.2 below shows the weights for two different values of λ .

TABLE 3.2

Weight of	1 $\lambda = 0.6$	2 $\lambda = 0.9$
X	1.6667	1.1111
X_{t-1}	- 1.11118889	- 0.123443
X_{t-2}	0.74082963	0.0137145
X_{t-3}	- 0.49391111	- 0.00152368
X_{t-4}	0.32929053	0.000128028
X_{t-5}	- 0.21953799	- 0.0000142239
X_{t-6}	0.14636598	0.00000158027

It is observed that the higher the value of λ is, the quicker the equilibrium value is approached.

While the stability condition $\lambda > 0.5$ must hold in the context of a single equation model, this does not necessarily apply in

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[9] From the Greek ὑπερακοντίζειν = overshoot.

the case where reduced form (1.24) is a part of a simultaneous equations model. In such models, the stability condition imposes restrictions which might or might not imply the $\lambda > 0.5$ one. Consider for illustrative purposes the following two models and their stability conditions:

<u>Model A</u>

$$\left.\begin{aligned} Y_t &= \lambda b Z_t + (1 - \lambda)\, Y_{t-1} \\ Z_t &= \frac{a}{\mu} X_t + (1 - \frac{1}{\mu})\, Z_{t-1} \end{aligned}\right\} \tag{3.34}$$

Y and Z are the endogenous variables and X is the exogenous, and the geometric and hyperacontistic lag distributions are imposed to the two equations respectively.

The system written in matrix form is

$$\begin{bmatrix} 1 & -\lambda b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = \begin{bmatrix} 1 - \lambda & 0 \\ 0 & 1 - \frac{1}{\mu} \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{a}{\mu} \end{bmatrix} X_t \tag{3.35}$$

The reduced form is

$$\begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = \begin{bmatrix} 1 - \lambda & \lambda b(1 - \frac{1}{\mu}) \\ 0 & 1 - \frac{1}{\mu} \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix} + \begin{bmatrix} \lambda b \frac{a}{\mu} \\ \frac{a}{\mu} \end{bmatrix} X_t \tag{3.36}$$

For stability the characteristic roots of the coefficient matrix of lagged endogenous variables must be less than one in absolute value. The characteristic equation of this matrix is

$$\begin{vmatrix} K - (1 - \lambda) & -\lambda b(1 - \frac{1}{\mu}) \\ 0 & K - (1 - \frac{1}{\mu}) \end{vmatrix} = 0 \tag{3.37}$$

or $$K^2 - [(1 - \lambda) + (1 - \frac{1}{\mu})]K + (1 - \lambda)(1 - \frac{1}{\mu}) = 0 \qquad (3.38)$$

The two roots are $K_1 = 1 - \lambda$ and $K_2 = 1 - \frac{1}{\mu}$ and for this model the restriction $\mu > 0.5$ is necessary.

Model B
$$\left.\begin{aligned} Z_t &= \frac{a}{\mu} X_t + \frac{b}{\mu} Y_t + (1 - \frac{1}{\mu}) Z_{t-1} \\ Y_t &= W_t + Z_t \end{aligned}\right\} \qquad (3.39)$$

Z , Y are the endogenous variables and X , W the exogenous and the hyperacontistic lag distribution is imposed to the first equation whereas the second is an identity. The reduced form corresponding to this model is

$$\begin{bmatrix} Z_t \\ Y_t \end{bmatrix} = \frac{1}{1 - \frac{b}{\mu}} \begin{bmatrix} 1 - \frac{1}{\mu} & 0 \\ 1 - \frac{1}{\mu} & 0 \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ Y_{t-1} \end{bmatrix} + \frac{1}{1 - \frac{b}{\mu}} \begin{bmatrix} \frac{a}{\mu} & \frac{b}{\mu} \\ \frac{a}{\mu} & 1 \end{bmatrix} \begin{bmatrix} X_t \\ W_t \end{bmatrix} \qquad (3.40)$$

The characteristic equation of the coefficient matrix of the lagged endogenous variables is

$$\frac{1}{1 - \frac{b}{\mu}} \begin{vmatrix} K(1 - \frac{b}{\mu}) - (1 - \frac{1}{\mu}) & 0 \\ - (1 - \frac{1}{\mu}) & K(1 - \frac{b}{\mu}) \end{vmatrix} = 0 \qquad (3.41)$$

or $$K[K(1 - \frac{b}{\mu} - (1 - \frac{1}{\mu})] = 0 \qquad (3.42)$$

with roots $K_1 = 0$ and $K_2 = (1 - \frac{1}{\mu})/(1 - \frac{b}{\mu})$ and the restriction $\mu > 0.5$ need not necessarily hold.

Having examined the partial adjustment model, the adaptive expectations model, the compound geometric lag model and the inverse adjustment model, the following remarks are in order: (a) If equilibrium is defined as the situation in which adjustment is completed within one period, then (3.3), (3.9) and (3.12) collapse into a single relationship

$$Y_t = a X_t + u_t \tag{3.43}$$

which is the equilibrium relationship. (b) Several authors use the terms equilibrium relationship and long-run relationship without any discrimination. Thus Chow in a study of the demand for money function writes:

> *"One of the major weaknesses in the available theoretical formulations of demand functions for money seems to be the failure to distinguish between long run or equilibrium demand and short run demand by introducing a mechanism for the adjustment of the actual stock of money to its equilibrium level."*[10]

In terms of our notation $a X_t$ is the equilibrium value for Y (neglecting the error term), given the values of X and a, and it is simultaneously the long run (or total) value of Y. This will be made clear if we recall that the actual effect of X on Y is one distributed in time. With the assumption that the weights follow a geometric progression

$$Y_t = \lambda a X_t + \lambda a(1 - \lambda) X_{t-1} + \lambda a(1 - \lambda)^2 X_{t-2} + \ldots$$

and $\lambda < 1$, if X changes by one unit the impact effect on Y will be λa, the first period effect $\lambda a(1 - \lambda)$, the second period effect $\lambda a(1 - \lambda)^2$ and so on. The <u>total or long run</u> effect will be the sum *ad infinitum* of the partial effects

$$\lambda a + \lambda a(1 - \lambda) + \lambda a(1 - \lambda)^2 + \ldots = \frac{\lambda a}{1 - (1 - \lambda)} = a$$

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[10] G. C. Chow: "On the Long-Run and Short-Run Demand For Money." <u>Journal of Political Economy</u>, 1966, pp.111-31.

the same as the one obtained from the equilibrium relationship (3.43). The above equivalence is legitimate only if the equilibrium relationship is static, i.e. all variables are written without time subscripts. As before, equilibrium is defined as the situation in which adjustment is completed within a single period so that the variable for which the dynamic adjustment mechanism applies, attains immediately its equilibrium (or planned or desired or permanent or expected as the case may be) value. Technically this happens when the coefficients of adjustment in (3.4), (3.8) and (3.29) are equal to one. Thus

$$Y_t - Y_{t-1} = 1.(Y_t^* - Y_{t-1}) \quad \text{and} \quad Y_t = Y_t^* = a\,X_t$$

for the partial adjustment hypothesis,

$$X_t^* - X_{t-1}^* = 1.(X_t - X_{t-1}^*) \quad \text{and} \quad Y_t = a\,X_t^* = a\,X_t$$

for the adaptive expectations hypothesis,

$$Y_t^* - Y_{t-1} = 1.(Y_t - Y_{t-1}) \quad \text{and} \quad Y_t = Y_t^* = a\,X_t$$

for the inverse adjustment hypothesis.

However, the equilibrium relationship, so defined, can be dynamic, whence the notions of equilibrium and long run relationship diverge. We can of course get the long-run relations from the dynamic equilibrium ones (and this will be shown later) but this need not mean that the two coincide. Consider for example the following form for the investment function

$$I_t^* = a(Y_t - Y_{t-1}) + b\,R_t \tag{3.44}$$

$$I_t - I_{t-1} = \lambda(I_t^* - I_{t-1}) \tag{3.45}$$

where I , Y and R are investment, income and interest rate respectively. (3.44) can be thought of as the equilibrium relationship if I* is replaced by its actual value I . The static form corresponding to (3.44) is simply $I = b R$, since the first term in (3.44) vanishes when the variables are not subscripted. (3.44) and (3.45) together imply that

$$\begin{aligned} I_t &= \lambda a(Y_t - Y_{t-1}) + \lambda a(1 - \lambda)(Y_{t-1} - Y_{t-2}) + \\ &\quad + \lambda a(1 - \lambda)^2(Y_{t-2} - Y_{t-3}) + \ldots + \lambda b R_t + \\ &\quad + \lambda b(1 - \lambda)R_{t-1} + \lambda b(1 - \lambda)^2 R_{t-2} + \ldots \end{aligned} \tag{3.46}$$

The long run effect of a change in Y on I is given by

$$\lambda a + [\lambda a(1 - \lambda) - \lambda a] + [\lambda a(1 - \lambda)^2 - \lambda a(1 - \lambda)] + \ldots =$$

$$= \frac{\lambda a}{1 - (1 - \lambda)} - \frac{\lambda a}{1 - (1 - \lambda)} = 0$$

and the long run effect of a change in R on I is given by

$$\lambda b + \lambda b(1 - \lambda) + \lambda b(1 - \lambda)^2 + \ldots = \frac{\lambda b}{1 - (1 - \lambda)} = b$$

so that it is seen that these are the effects predicted by performing comparative static analysis on the static form of (3.44). As a matter of fact we would get the same long-run effects from (3.44), as these effects are $a - a = 0$ for income and b for the interest rate. This observation has important implications and it will be explored later in the context of a simultaneous equations model with interacting lags.

We can sum up the discussion held so far: In dynamic models of one behavioural equation and an adjustment mechanism of one of the types (3.4), (3.8), both (3.4) and (3.8), and (3.29), three forms can be distinguished

(a) the disequilibrium model in which the adjustment mechanism(s) is assumed at work

(b) the equilibrium model in which it is assumed that adjustment is completed within one period and which can be either dynamic or static, and

(c) the static model in which the variables are not subscripted.

The latter will obviously coincide with (b) if the equilibrium model is static.

When we move from single equation models to simultaneous equation models and in some equation of the latter the reduced form (3.5) or (3.10) or (3.31) is set for estimation, the distinction of three types of models (disequilibrium, equilibrium, static) can be still valid. Thus for example the system

$$\left.\begin{aligned} Y_t &= \lambda a(X_t - X_{t-1}) + \lambda b\, Z_t + (1-\lambda)\, Y_{t-1} \\ Z_t &= c\, X_t + d\, W_t + e\, Y_t \end{aligned}\right\} \qquad (3.47)$$

in which the reduced form of the partial adjustment model is the one shown in the first equation, can be termed the disequilibrium model. Y and Z are the endogenous variables and X and W the exogenous. The equilibrium model would be read as

$$\left.\begin{aligned} Y_t &= a(X_t - X_{t-1}) + b\, Z_t \\ Z_t &= c\, X_t + d\, W_t + e\, Y_t \end{aligned}\right\} \qquad (3.48)$$

and the static model as

$$\left.\begin{aligned} Y &= b\,Z \\ Z &= a\,X + d\,W + e\,Y \end{aligned}\right\} \qquad (3.49)$$

For such models certain useful conjectures[11] can be put forward. These are:

Conjecture 1 : "Although in a single equation model (of one of the types examined in the previous sections) with more than one independent variables, the distributed lag relationship is the same and has identical weights for all of the independent variables, in simultaneous equations models having the above equation as a part, this is not generally true."

Thus $$Y_t = \lambda a\, X_t + \lambda b\, W_t + (1 - \lambda)\, Y_{t-1} \qquad (3.50)$$

implies that the distributed lag relationship between Y and X and between Y and W is the same (geometric) and has identical weights λ , $\lambda(1 - \lambda)$, $\lambda(1 - \lambda)^2$ . . . However in a simultaneous equation models this will not generally happen because some of the independent variables of the equation in question may be determined endogenously in the model or others may appear in other equations as well, and this will differentiate the distributed lag relationships between these variables and the dependent variable.

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[11] The conjectures were verified in small models which were easy to handle analytically but, since a formal proof of their validity is not provided, the term conjecture is considered as the most appropriate.

Conjecture 2 : "The total or long run effect on endogenous variables of changes in exogenous variables in both the disequilibrium and equilibrium model is the one we get by performing comparative static analysis to the static form of the model. A corollary of this is that this effect is independent of the parameters indicating the speed of adjustment in the disequilibrium model."

The total effect on endogenous variables of changes in exogenous variables in the model

$$y_t = A\, y_{t-1} + B\, x_t + C\, x_{t-1} \qquad (3.51)$$

where y and x are the vectors of endogenous and exogenous variables, is given by the elements of the matrix

$$(I - A)^{-1}\,(B + C)$$

provided that the model is stable.[12] It can then be easily verified that this matrix is independent of the parameters indicating the speed of dynamic adjustment. For example in model (3.47) the above matrix does not contain the parameter λ and coincides with the matrix connecting endogenous and exogenous variables in the static model (3.49).

Conjecture 3 : "The stability conditions for the disequilibrium and equilibrium models will in general differ. The former will depend on the parameters of the adjustment mechanisms."

This is true because the stability condition is derived by examining the characteristic roots of the matrix A , and in the disequilibrium model, A will contain the above parameters.

[12] See H. Theil: Principles of Econometrics. John Wiley, 1971, p.465.

The above discussion was held in order to put in the right perspective the implications of using dynamic adjustment mechanisms in a simultaneous equations model. Since we make use of some of them in certain equations in our model, it seems necessary to be aware of their properties.

CHAPTER 4

CHAPTER 4

THE DEFINITION OF MONEY - THE MONEY DEMAND FUNCTION

A. The Definition of Money

The first task one faces in the specification of money demand and money supply relationships is the explicit determination of what is the definitional content of 'money'. This is necessary in order to develop an analytical framework within which behavioural assumptions will be formulated and tested. The issue to be elucidated next is the position of money, however it is defined, in a dynamic model of income and price determination. The anwer to this query can be partly located beyond the importance it has because of its operational characteristics.

There is a remarkable discord in the economic literature regarding the appropriate definition of money. The appearance in the fifties of some publications on the demand for money or the velocity of money function[1] prompted a parallel search to theoretically establish and empirically substantiate the composition of the money stock concept in that function.

Should money be defined in the traditional way as currency plus demand deposits or should its coverage be enlarged to include

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[1] See for instance M. Friedman, "The Demand for Money : Some Theoretical and Empirical Results." Journal of Political Economy, 1959, pp.327-51, and R. Selden, "Monetary Velocity in the United States" in M. Friedman ed. Studies in the Quantity Theory of Money, University of Chicago Press, 1956, pp.179-257.

financial assets that possess a high degree of liquidity and are regarded as potentially useful in settling market commitments?

We can examine briefly some proposed measures of liquidity and the possibility of the money variable stretching according to liquidity considerations.

Financial assets are said in general to have a closer affinity to the perfect liquidity situation than do real assets. The price obtained from immediately liquidating one unit of the asset is much nearer to the maximum expected price in the case say of a loan share than is the case of a unit of a productive physical asset. This view of liquidity considers the time available between the decision to sell an asset and the actual sale of it.[2] A cursory disposal of the asset can in most of the cases be realized at a price which is less than P_t^* the highest price obtainable from the sale. However as time lapses the ratio of the actual price P_{t+i} to P_t^* will tend to unity. $P_t/P_t^* = 1$ characterises perfect liquidity and the liquidity curve, that is the plotting of P_{t+i}/P_t^* against time describes the liquidity of the asset.

Another suggested measure of liquidity[3] stresses the subjective element in the individual owning the asset and takes as a basis of computation the cost of acquisition. Perfect liquidity then means the certainty that its holder can obtain one hundred per cent of its cost

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[2] J. L. Pierce, "Commercial Bank Liquidity." Federal Reserve Bulletin, 1966, p.1094.

[3] M. Bronfenbrenner, "Some Fundamentals in Liquidity Theory." Quarterly Journal of Economics, 1945, pp.407-8.

immediately upon deciding to dispose of it and it entails a utility of the form U (1.00, 100, 0), where the three numbers indicate the certainty of a price covering the full cost at zero time. The utility derived from selling a liquid asset can be represented as U (n, k, t) with n , k , t being a probability, a percentage and time respectively. The particular constellation of n , k and t relevant to this view is the one which maximizes the ratio of the marginal utilities ${}_{x}U_{a}$ (n, k, t) / ${}_{x}U_{a}$ (1.00, 100, 0) of the asset a in use x . The latter is taken as the desired measure of liquidity.

Apart from the fact that liquidity curves might intersect thus providing a non unique ranking of assets and utility indices might not be constructed easily, it appears that considerations of liquidity cannot be of practical aid in drawing the line between money and non-money since they do not put forward any objective criterion for selecting a borderline degree of liquidity and virtually all assets can be liquidated provided the allowance of the appropriate time is made.

Proponents of the liquidity position recognize that "*the boundary for the class of assets which replace money has neither sharpness, nor certainty nor permanence*".[4] It leaves us with those marketable assets which individuals consider each time to be readily usable to settle outstanding obligations. The range is wide enough to include not only liabilities of commercial banks and non-bank financial intermediaries but also of firms engaged in manufacturing and trading

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[4] R. S. Sayers, "Monetary Thought and Monetary Policy in England." Economic Journal, 1960, p.711.

which grant trade credit to their customers. Thus a magnitude comes up which in short is unmeasurable. In this view the 'traditional' money supply is not an unimportant quantity but forms a part of the wider liquidity of the economy.[5] Acceptance of this thesis implies that the attention of policy makers should turn to the control of liquidity with the view to affect aggregate demand. Nevertheless their supporters seem unable to offer an explanation of the operational utility of their suggested concept through relationships with other key variables as national income and interest rates.

This shortcoming due partly to measurement restraint led another school of thought to reject any *a priori* line between money and near-moneys. Instead they pursue a distinction which emerges from statistical investigations. The problem of the proper definition of money is thus considered as fundamentally empirical.

Two lines of approach can be distinguished among those who seek the definition in empirical findings. The first tries to assess the degree of substitutability among alternative financial assets.[6] The method commonly used is to estimate separate demand functions for each potential component of the money variable, the equations containing as arguments the assets' in question own rate of return just as the other assets' rates and an income and/or wealth constraint. This

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[5] Committee on the Working of Monetary System, Report, London : Her Majesty's Stationary Office, 1959, p.232.

[6] See E. Feige: The Demand for Liquid Assets : A Temporal Cross Section Analysis. Englewood Cliffs, N.J. : Prentice Hall, Inc. 1964 and M. J. Hamburger, "Household Demand for Financial Assets." Econometrica, 1968, pp.97-118.

procedure permits an explicit evaluation of the substitution relationships after the parameters of various rates of return have been estimated. Usually time series of cross section data provide the basis for such enquiries.

Some more rather arbitrary theoretical assumptions contributed to the impetus for undertaking these studies. One was advanced by Friedman, who argues that time deposits at commercial banks are nearly perfect substitutes for currency and demand deposits and as such they should be included in the definition of money.[7] The other originated from Gurley and Shaw whose principal contention is that liabilities of non bank financial intermediaries are sufficiently close substitutes for commercial banks liabilities and the latter are weak substitutes for primary securities and other assets. If the financial structure is taken into consideration with the influence of the emergence and growth of numerous non-bank financial intermediaries the demand for currency and demand deposits has declined relative to other assets. A correct definition should not then ignore liabilities of these institutions.[8]

It can be easily seen that the above studies do not come to coincide in their results on all issues involved. For instance the evidence offered by Hamburger indicates that time deposits at commercial

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[7] M. Friedman and D. Meiselman, "The Relative Stability of Monetary Velocity and the Investment Multiplier in the United States, 1897-1958." In Commission on Money and Credit, Stabilization Policies, Englewood Cliffs, N.J. : Prentice Hall, Inc. 1963, pp.165-268.

[8] J. Gurley and E. Shaw : Money in a Theory of Finance, Washington : Brookings Institution, 1960.

banks can not be substantially discriminated from savings deposits at non-bank financial institutions as regards their substitutability.[9] This diverges from Feige's results; indeed in Feige's study most of his individual regressions were found to be independent in demand. There is however a widespread unanimity as far as the following finding is concerned : currency and demand deposits are differentiated from other financial assets to a degree allowing their treatment as a separate asset.

The empirical research examined so far was concerned with making explicit the substitution relationship among assets and on the basis of relevant information draw inference at least as to which definitions of money are inadmissible.

A second group of economists considers as the appropriate composition of the money variable the one providing the most stable demand function. But what is meant by a stable function? From what can be gathered from the existing evidence there is no connection whatever with stability in the dynamic sense. A stable function is one which having few variables as its arguments renders it possible to predict the demand for money with sufficient accuracy. Moreover its parameters do not shift under gradually or swiftly changing economic conditions or differing institutional arrangements. The point is made that one can obtain a desirable degree of explanatory power by including more and more arguments in the function and the real success lies in designating a stable function with the features just described. Criteria that were used to evaluate stability include the

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[9] M. J. Hamburger, *op.cit.* p.104-5.

coefficient of determination, some measure of predictive performance, e.g. the root mean square error of predicition, and the Chow test. With a few exceptions,[10] the general conclusion reached from these studies is that the narrow definition of money gives an at least equally stable function compared to the inclusive of time deposits definition and in consequence there is no need to extend it to cover liabilities of non-bank financial intermediaries.

The above empirical studies are important for predicating the existing relationships between money and other variables either at the aggregate level or with a more detailed breakdown in groups, of individuals demanding it. Yet they focus their attention on that facet of money which interprets it as a store of wealth, whose capital value does not vary with the rate of interest. It is the 'money' in the money - bonds Kenesian distinction. When money is looked upon only as a riskless store of wealth the dividing line between assets becomes debatable and inquiry of what determines more stably its demand and the substitution relationships among candidate assets for inclusion in it seems worth undertaking. Nevertheless, whatever differences are spelt out from these inquiries, are differences in degree and refer to the asset status from the standpoint of the owner. If one is prepared to look at the other characteristic of money, that is its exchangeability for goods and services, a distinct subset of assets emerges which functions directly as a medium of exchange. It is on this account that we must concentrate to trace *a priori* a

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[10] See D. Laidler, "The Rate of Interest and the Demand for Money : Some Empirical Evidence." Journal of Political Economy, 1966, pp.552-3.

discernible money concept. The only constraint acting to blur such a choice could be the availability of reliable statistical information on the assets that will form the empirical counterpart of the theoretical setting. When unhampered by this limitation we can define narrow money as a measurable quantity and try to investigate the supply of and demand for it. In the process of doing so the existence and attractiveness of other income-earning assets will be taken account of. Indeed, considerations of this kind are the roots of an argument put forward to justify the use of a narrow definition of money, namely that more inclusive definitions encompass substitution effects between money functioning as a medium of exchange and other assets. These effects are partly hidden within this broader composition when seeking to establish the interest responsiveness in the demand for money function. Time deposits when included in the money variable will tend to obscure the role of the interest rate as the opportunity cost of retaining non-interest earning cash holdings. A rise in the interest rate affects in the same direction the opportunity cost of keeping currency and demand deposits but at the same time increases the return from the other component. However the assertion that the inclusion of time deposits 'stacks the cards' against finding any significant interest elasticity of demand for money[11] can not be accepted in the light of several studies on the demand function which produced a significant interest rate coefficient when employing the

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[11] R. L. Teigen, "The Demand for and Supply of Money" in Readings in Money, National Income and Stabilization Policy. Edit. by W. L. Smith and R. L. Teigen, R. D. Irwin, Inc, Homewood Ill., 1970, p.88.

broad definition. True enough more inclusive definitions are expected to lower the value of the above coefficient[12] but they are not likely to nullify it.

Finally we shall give a sketchy description of two criteria proposed to deal with the difficulty of unequivocally defining money.

Pesek and Saving[13] view the debate on the definition of money as a result of the inability to distinguish between money and debt. The argument that money is a debt resting on the balance sheets of the banking system is believed to be wrong and the balance sheets to be constructed with the prejudice that money is a debt. Their analysis suggests that what separates money from debt is the income stream money yields to the owner in its use as a medium of exchange, which is not matched by a negative income to its producer. Because of this positive flow of services - the saving of time in barter transactions - money should be included to the net wealth of a society. Debts on the other hand yield income to the owner only because they yield negative income of equal size to their creator. Consequently they are not part of the net wealth. The latter statement represents Pesek and Saving's

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[12] For a confirmation see A. H. Meltzer, "The Demand for Money: The Evidence from the Time Series". Journal of Political Economy, 1963, p.225. The author reports on elasticity equal to -.949 when he uses the narrow definition as compared to -.50 pertinent to the broader one where time deposits are included. For a similar reasoning on this matter we quote from D. M. Jones, "The Demand for Money: A Review of the Empirical Literature." Staff Economic Studies, Board of Governors of the Federal Reserve System, 1965, p.4: The substitution effect of these rate changes is partly hidden by changes within the composition of money. This of course tends to lower the observable responsiveness of money balances to interest rate movements.

[13] B. P. Pesek, and T. R. Saving, Money, Wealth and Economic Theory. Macmillan, New York, 1967, pp.139,163,249.

vulnerable argument. It may be the case that debt creators do not get any services from the last units of debts supplied; this need not imply lack of services from all units, in addition to the income they yield. This makes the distinction of money and debt as conceived by Pesek and Saving an ambiguous proposition.

Newlyn develops a neutrality criterion permitting the distinction between money and near moneys to be couched in terms of their operational characteristics. According to this criterion an asset is classified as money if the effect of paying with this neither alters its total quantity nor disturbs the market for loans.[14] Application of the criterion to the United Kingdom paradigm leads to the inclusion of time deposits at commercial banks in the definition of money, since demand deposits and time deposits are not subject to different reserve requirements. Thus financing a payment by drawing down a commercial bank time deposit does not change the total quantity of money; moreover the respective increase in the payee's demand deposit account leaves the reserve position of the banking system unaffected and no impact on the loan market is felt.

Newlyn's criterion is subject to a twofold criticism. First a shift from demand to time deposits would represent a higher cost of acquiring funds for the banks with possible repercussions in the loan market from changes in the earning assets portfolio. A more serious defect, pointed out by Crouch,[15] is present in case an owner of a demand deposit cashes it to effect a payment. Then the banking system as a

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[14] W. T. Newlyn, "The Supply of Money and Its Control." Economic Journal, 1964, pp.335-6.

[15] R. L. Crouch, "A Model of the United Kingdom's Monetary Sector." Econometrica, 1967, p.404.

whole is faced with a loss of reserves and the corresponding shrinking in loans or securities held by banks, clearly invalidates the neutrality criterion.

In conclusion, examination at the aggregate level of the spending flows in the economy suggests as a natural choice the set of assets that are actually being spent; those that are directly exchanged for goods and services. The differences in degree of either substitutability or liquidity refer to the asset status of possible candidates for participation in the money concept formation and cannot provide any *a priori* clear-cut distinction; this is the reason why they have recourse to empirical support. On the other hand differences in functioning seem to point to an empirically identifiable item namely currency in the hands of the public and demand deposits with the banking system. We shall adopt this definition bearing in mind that the theoretical discussion of money demand and money supply will be dependent upon it.

Having defined money so far we can go on and specify demand and supply relationships for it. What would then be the role that money takes in a model of aggregate demand determination? The answer clearly shows two important merits of it.

(1) Money provides a direct link connecting monetary instruments outright with goal economic variables as real output and prices. Even if the supply of money is regarded as being influenced by factors which do not affect the demand for money, lagged monetary balances appearing in the latter function are channelling the impact of monetary action.

The case is more evident when the money supply is interest elastic. The interest rate appears to be one of the key arguments in the money demand function. Indeed there are a few studies which have recognized the simultaneous equations bias arising from neglect of the interest elasticity of the money supply.[16] By establishing an interest responsive supply function and the appropriate transmission mechanism for monetary policy, it is quite a straightforward matter to ally monetary instruments to target endogenous variables.

(2) Money is a potential candidate to be used as an indicator of monetary policy. What is meant by a monetary indicator and by what criteria do we determine how successful monetary indicators are? Because indicators can be of significant importance in guiding monetary authorities we shall deal peripherically with them and give a concise overview of their role and usefulness.

Brunner and Meltzer[17] posit that the monetary indicator is an index which summarizes the changes in social utility brought about by monetary policy operations. If the indicator is represented by I then

$$I = \frac{du}{dy_1}\frac{dy_1}{dx_1}dx_1 + \frac{du}{dy_2}\frac{dy_2}{dx_1}dx_1$$

where u is the social utility function, y_1 and y_2 are endogenous variables and x_1 a policy controlled monetary instrument. It can be

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[16] See R. L. Teigen, "Demand and Supply Functions for Money in the United States : Some Structural Estimates." Econometrica, 1964, p.476.

[17] K. Brunner and A. H. Meltzer, "The Meaning of Monetary Indicators" in Monetary Process and Policy : A Symposium. Edit. by E. Horwich. R. D. Irwin, Inc., Homewood Illinois, 1967, p.189.

easily realised that I is the total differential of the social utility function. All that would then be required to use this indicator is knowledge of structural parameters and their stability with changing institutional arrangements; also a choice is needed concerning the utility function. The authors exemplify their measure by taking a utility function containing as argument a single goal variable: real output. Then a rising index is identified with easy monetary policy whereas a falling one indicates a tighter policy. The monetary indicator in that form could be used more profitably in the case of endogenous variables which are themselves conflicting goals and some sort of trade off is needed between them. An obvious example is given by the price level which is traded off against unemployment. Some kind of utility function taking account of the desirability of opposing situations might be a preferable solution.[18]

However a careful glance at the literature shows that no indicator used has any connection with the indicator function introduced by Brunner and Meltrer. Usually money market variables are being watched and the primary concern is to evaluate the monetary authorities' actions by current money market conditions. For reasons that will be mentioned later, a variable is required which will tell us whether the stance of current monetary policy has been restrictive or expansive. Those who focus on interest rates feel that a rise in interest rates as a result of, say, a change in monetary base, forecasts a slowdown in real output. Their assumption implies that all other influences (past monetary actions, current fiscal policy measures

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[18] Cf. P. A. Samuelson, "Reflections on Recent Federal Reserve Policy." Journal of Money, Credit and Banking, 1970, p.42.

and exogenous influences) are less important on their indicator candidate than current monetary changes. Similarly, money quantity watchers assign a dominant change to the money supply as a result of current monetary action. They associate an expansion in economic activity with this increase in the money supply. To state the above example briefly: for $\frac{\partial Y}{\partial B} > 0$ where Y is real aggregate demand and B the monetary base, in order to have correct indicator information by the interest rate R or the money supply M : the sign of ΔR should be the opposite of ΔB for the interest rate watchers and the sign of ΔM the same as ΔB for the quantity of money watchers. Both parts assume that movements in their indicators are dominated by current monetary policy. However the two indicators do not always provide true information. Both the elements just mentioned (past monetary policy, current fiscal policy and exogenous shocks) and other factors (for example price expectations, effects on interest rates that shift the IS and LM curves can make one or both indicators yield false information. For instance there could be rising real output accompanied by rising interest rate because of those factors and one would assign false information to the interest rate indicator. It would then be worth investigating the conditions under which true indicator information is provided. Zecher has shown money to be a superior indicator in four econometric models containing explicit interest rate transmission mechanism.[19] His results are essentially an exercise in the analysis of dynamic multipliers for the models examined, and include the assumption of a sustained change of a monetary policy instrument and the examination of its effect simultaneously on the dependent variables income, money and interest rate.

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[19] R. Zecher, "Implications of Four Econometric Models for the Indicators Issue." American Economic Review Papers and Proceedings, 1970, pp.47-54.

The reasons for the search of a monetary indicator can be placed on (a) the lack of complete information about the structure of the economy. Even in the case a macromodel is set up, this lack is depicted in the error term of the model. Specific assumptions about the stochastic elements tend to replenish perfect quantitative knowledge. (b) the information lag involved in the availability of statistical figures. It can be present independently of how well the true structure has been approximated. The data on real output are a typical example often mentioned; they are brought into appearance after a considerable lapse of time. (c) the error in the forecasts of the current exogenous influences.

For these reasons the study of the indicator performance from ex post information might be rewarding regarding the future guiding of policy.

B. The Demand for Money

The problem of the definition of the money variable was examined with the opening of this chapter because it was deemed necessary to assign an indubitable content to the money concept, would this be possible on an *a priori* basis. As we have already noted many empirical investigations incorporated it in their list of queries to be answered by the data. Having in mind the qualifications they brought to the results we can focus our attention to how the demand and supplied quantities of money - the definition accepted above - are determined starting from the demand side.

The literature exhibits a rich variety of tests on the demand for money, perhaps the richest among every other highly aggregative macroeconomic function. The rationale for holding cash balances as well as the actual procedure by which individuals dispose of them consist the cornerstones in the theoretical underpinnings of the demand for money. They are nevertheless open to several interpretations. Theories with a similar kind of interpretation despite the different possible analytic derivations can be grouped for convenience under four broad headings: the Quantity Theory of Money, the Liquidity Preference Theory, the Inventory Approach to the demand for transaction balances and the Modern Quantity Theory of the demand for money.

The Quantity Theory of Money: The Quantity Theory in its original rigid form appears in two different versions of the same equation. This equation is essentially a tautology but it was converted to a theory of demand by particular assumptions about its behavioural parameter and variables. The one form of the equation is the famous exchange equation closely associated with the work of Irving Fisher[20]

$$MV = PT \tag{4.1}$$

where M is the money supply in circulation for a given time period. Clearly M is a stock, not a flow variable.

V is the velocity of circulation defined as the number of times each unit of money is transferred on the average in exchange for items transacted on a current or capital account.

P is the average price level for the same period,

and T is a suitably selected aggregate of quantities that changed hands during the examined period. T can contain items counted more than once if transactions on them occurred a multiplicity of times.

[20] I. Fisher, The Purchasing Power of Money, New York, MacMillan, 2nd ed.1913.

The above identity depicts both sides of total transactions taking place in a given time period. The value of sales should be equal to the value of receipts. Now assume as Fisher did that (a) the volume of transactions bears a constant relation to the level of full employment output and can thus be regarded as given, (b) the money is determined exogenously to the other variables of the equation, and (c) the velocity of circulation, a parameter depending on institutional and behavioural factors changes very slowly through time and can be considered for practical purposes as constant in the short run. Then the above equation leads to the determination of the final variable, the price level P. With the further assumption of constancy of V and T the proportionality of the Quantity Theory of Money between P and M is established. This approach was subsequently modified because of the difficulties connected with the distinction of current and capital transactions in relation to the price level concept. The development of national accounts has replaced total transactions by income transactions and the exchange equation is stated in the form

$$MV = Py \qquad (4.2)$$

where P is the price level implicit in the deflation of nominal income to give income in constant prices y. What is remarkable in this modification is that it leaves out transactions on existing assets as well as transactions on intermediate goods. Consequently with a given level of income a change in the factors affecting the excluded categories, e.g. vertical integration of firms and shortening of the productive process would lead to a different demand for money. To get around this situation of a changing demand because of the non constant ratio of income to total transactions, it can again be assumed that in the short run these factors are not likely to bring about any significant change in the ratio.

Perhaps the most notable feature of the income approach to Fisher's equation is its view of money as a medium of exchange which is held for transactions, not actually transferred. This makes it "a way station"[21] between the Fisher and the Cambridge versions.

The Cambridge cash-balance approach to the demand for money shifted the emphasis from the mechanical aspects of transactions to the reasons why individuals choose to hold money which yields no utility to the holder. It was upheld that money is a convenient asset to have because it is the universally acceptable means of payment. This fact together with the disparity between receipts and expenditures in a dynamic economy and the possibility of unforeseen events causes people to hold money rather than to convert all cash balances into income yielding assets. The Cambridge approach recognizes that the income foregone by holding a sterile asset might be an important determinant of the demand for money. However they assume for simplification on a *ceteris paribus* basis, that the demand for money in nominal terms is proportional to the nominal income. The latter is thought to be in a constant relation to the potential purchases of individuals. Stated in symbols

$$M = k.P.y \qquad (4.3)$$

and this is Fisher's equation in a rearranged form.

The omission of the interest rate from the formal statement of the Quantity Theory equation and the assunption of a constant velocity of circulation or income velocity, as the case may be, have important implications as to the separation of monetary and real variables and the

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[21] H. Friedman: A Theoretical Framework for Monetary Analysis. National Bureau of Economic Research, New York, 1971, p.8.

inability of fiscal policy to influence the level of real activity in the economy. As it has already been noted because of the overwhelming evidence on the role of the interest rate in the demand for money recent work in the Quantity Theory spirit has shown that a zero interest elasticity of the demand for money, as implied by this rigid form of the equation, is not a necessary condition for the separation of real and monetary phenomena. What is required is the existence of a stable velocity, i.e. with zero elasticity of the velocity with respect to monetary changes.

The Liquidity Preference Theory. In the Classical tradition the focus is on money as a medium of exchange which is necessary to bridge the discrepancies in the timing of receipts and expenditures. In Keynes' Liquidity Preference Theory[22] the other aspect of money, namely the store of value property, is also examined. The path-breaking contribution of Keynes to the Classical scheme is the analysis of the motives that lead people to keep their wealth in the form of money and through them the explicit appearance of the interest rate as a price equilibrating desired money holdings with the existing quantity of money.

Keynes did not keep away completely from the classical ideas. The transactions demand for money coupled with a precautionary component, which includes money to meet unforeseen emergencies or take advantage of favourable opportunities, is what the Quantity theorists stressed in their writings. This is but only one part of money demanded which Keynes made dependent on the level of income. The interest rate may

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[22] J. M. Keynes: The General Theory of Employment, Interest and Money. Macmillan and Co., London, 1936.

admittedly have an influence on these motives acting as an effective cost of holding cash. But Keynes suppresses it from this component with the assumption that *"the liquidity preference due to the transactions and the precautionary motives absorbs a quantity of money not very sensitive to changes in the interest rate as such."*[23]

The possibility of money being held as a store of value, with no risk of capital loss arising from changes in interest rates, leads to a consideration of a final motive, the speculative which is of crucial importance in the Keynesian system. The indivual faces the problem of what specific form his saving - command over future consumption - will take. He can hold it in the form of immediate liquid command or part with immediate command and leave to the future market conditions the determination of the terms in which he can convert deferred command into immediate command.[24] The rate of interest on bonds is the reward for dispensing with liquidity for a specified period of time. However, because of the uncertainty as to the future rate of interest and consequently the bond price, individuals have a norm expected interest rate which they regard as safe regarding their capital losses from fluctuations in bond prices. Then the amount of money held for speculative purposes can be thought as varying inversely with the difference of the actual rate and the norm rate, which measures the "risk of illiquidity". With a given state of expectations, the same inverse relation can be written between money and the current interest rate.[25]

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[23] J. M. Keynes, *op.cit.*, p.171.

[24] *Ibid*, p.166.

[25] *Ibid*, p.171.

The relationship emerging from Keynes' analysis is

$$M = M_1(Y) + M_2(r) \tag{4.4}$$

where the two components of money demand appear in additive form. This does not mean that they are independent. A change in M changes r and this leads to a new equilibrium implying a different Y and therefore M_1 .

There are three points in the Keynesian analysis that should be emphasised. (1) Keynes used the term money for all liquid assets which provide safety from capital losses induced by interest rate movements. Phrases like *"money or its equivalent"* or *"in a form which yields little or no interest to holding it"* are clear indications of this.[26] He did not, however, consider the composition of money, namely the question why people should keep their balances in the form of currency and demand deposits and should not move into income earning highly liquid assets. The reason is that at his time the interest rates on those assets were extremely low, so he could treat them as yielding not considerably high income. (2) The money demand function specified is interest elastic and the velocity of circulation is no longer constant. With an interest rate approaching a minimum rate, the interest elasticity tends to infinity with the well known implications of the liquidity trap. (3) The determination of the interest rate by the money demand for speculative purposes destroys the independence between the real and monetary variables.

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[26] J. M. Keynes, *op.cit.* pp.166,168.

The Inventory Approach. The centre of interest in this approach is the transactions component of money. The contributors to it, while showing that this part of cash is interest elastic, have divergent views about money held for speculative purposes. They either accept or prove the interest elasticity of the latter or they assert that such a demand is nonexistent under present institutional arrangements.

A systematic theory showing interest elasticity of demanded cash for transactions was worked out by Baumol and Tobin.[27] Their hypothesis states that there is an opportunity cost for holding idle cash. It is the return on liquid income-earning assets in which cash could be invested temporarily. But cost incurred from switching into and out of these assets can make it unprofitable to individuals. The costs involved, Baumol's broker's fees, are not only material costs associated with transactions but include also "psychic" costs, i.e. the trouble of frequent transactions. Under fairly restrictive assumptions and by different methods, both writers obtain the same result, namely that cash for transactions varies proportionally to the square root of income and in inverse proportion to the square root of the interest rate. The formula is analogous to the well known "square root law" of inventory management. Their results, which concern one individual, imply an income elasticity less than unity, that is the demand for transactions cash exhibits "economies of scale".

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[27] W. Baumol, "The Transactions Demand for Cash: An Inventory Theoretic Approach." Quarterly Journal of Economics, 1952, pp.545-56.

J. Tobin, "The Interest Elasticity of Transactions Demand for Cash." Review of Economics and Statistics, 1956, pp.241-7.

As regards the demand for money satisfying the speculative motives Baumol simply admits its existence.[28] Tobin goes further to provide explanation to it.[29] It was discussed above that liquidity preference theory as advanced by Keynes analyzes the interest elasticity of money held for speculative purposes by assuming uncertainty as to the future rate of interest leading to capital gains or losses, and stickiness of the expected norm rate. This permits the direct association of the third component of money to the rate of interest in the place of the difference between the two rates. Tobin seeks the rationalization of the inverse relationship between cash which is part of the speculative or investment balances as he calls them and the interest rates, on a different set of assumptions. The latter do not involve the norm rate of Keynes but take account of the individual's probability distribution of capital gains or losses and of the risk stemming from uncertainty about their movement. Assume that the most probable event for investors is zero capital loss or gain and decisions are based on this expectation. Then Tobin's analysis shows that the optimum portfolio allocation with a desired combination of yield and risk results in a negative relation between speculative cash and the interest rate. The size of the portfolio, allocated between cash and assets with a variable market yield, is considered to be independent of the allocation itself.

What is confusing in Tobin's analysis, is the composition of assets, fixed in money value, which stand as alternatives to transactions

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[28] W. Baumol, *op.cit.* pp.555-6.

[29] J. Tobin, "Liquidity Preference as Behavior Towards Risk." Review of Economic Studies, 1958, pp.65-86.

cash and because of this the non explicit enumeration of those assets. We state the following passages.

> *"By cash I mean generally acceptable media of payment . . . Let bonds represent the alternative asset in which transaction balances might be held. Bonds and cash are the same except in two respects. One difference is that bonds are not a medium of payment. The other is that bonds bear an interest rate. There is no risk of default on bonds nor any risk of a change in the rate of interest."*[30]
>
> *"In contrast to transaction balances, the investment balances are balances which will not have to be turned into cash within the year. If cash is to have any part in the composition of investment balances it must be because of expectations of loss on other assets. The alternative to cash, in examining the speculative motive for holding cash, are assets that differ from cash only in having a variable market yield."*[31]

The above postulated property of bonds, namely the absence of risk in relation to changes in interest rate, is true if individuals wait for the maturity of the title. In this case one could infer that all monetary assets that are fixed in money value will be entitled to form transactions balances. However this is not correct since transaction balances have to be disposed within the year as from their task of covering seasonal discrepancies between expenditures and receipts. This is equivalent to saying that the part of transactions balances which has been invested in income earning assets will have to be converted in cash within the year. But an attempt to monetise these assets at their face money value is not always feasible. It might be easy for a saving deposit but not realisable for a government bond. This is the situation of differing degrees of liquidity that has been

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[30] J. Tobin, op.cit. 1956, pp.241-2.

[31] J. Tobin, op.cit. 1958, p.66.

already discussed. From these considerations the conclusion is reached that transactions balances as viewed by Tobin, i.e. a class of assets wider then cash, cannot include bonds and other monetary assets whose prices fluctuate with the rate of interest if their maturity date is beyond the unit time period. The only assets permissible are the riskless ones. Not surprisingly this is identical with Keynes' transaction component which consisted of money interpreted to be inclusive of riskless assets. On the other hand, while Keynes examined through the speculative motives the relation between this inclusive of riskless assets money and the rate on risky bonds, Tobin is limited to the relation of the means of payment and the interest rate on risky consols within the class of investment balances of the individual.

A more satisfactory view was taken by Teigen[32] who questioned the existence of money for speculative purposes. Holding money in one's portfolio of assets can be considered as irrational action, when other riskless and almost perfectly liquid assets with a positive return exist. These alternatives are seen to be superior to the means of payment with respect to the store of value property. Therefore it appears that money is dominated by other assets and should not rationally have a place in the investment balances.

The Modern Quantity Theory. The Modern Quantity Theory is associated with the name of Friedman[33] who emphasised that the Quantity Theory of Money is a theory of the demand for money and not a theory of output, money income or the price level, and reformulated the classical Quantity

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[32] R. L. Teigen, *op.cit.*, 1964, pp.482-3.

[33] M. Friedman: The Quantity Theory of Money - A Restatement. In Studies in the Quantity Theory of Money, edit. by M. Friedman, the University of Chicago Press, Chicago, 1956, pp.3-21.

Theory in a more sophisticated way. Friedman is not concerned with the motives that lead people to hold money in the way that the Liquidity Preference Theory is and consequently he does not separate the existing stock of money into two different components, the active and idle balances, featuring some versions of that theory. Instead, Friedman distinguishes two types of money demanders: (a) wealth-owning units who demand money as a means to hold part of their wealth. By treating money as such as asset, Friedman fosters the view that money is a temporary abode of purchasing power and that it provides certain services which can be considered as consumption services. (b) business enterprises to whom money is a capital input combined with other inputs to produce an output. The reason why money can be considered as a factor of production and the nature of its productive services are not stated explicitly.[34]

The analysis of the demand for money by ultimate wealth owners is made analogously to the analysis of the demand for consumption goods by consumers. Thus wealth is the analogue of the budget constraint that the consumer faces. Similarly, as the demand for goods depends on their own prices as well as the prices of other goods, the demand for money depends on its own price and return and the price and return of alternative forms of wealth. Finally, the tastes and preferences of the wealth-owing units are taken into account.

Concerning the concept of wealth, the constraint to the demand for money balances, Friedman accepts that wealth includes all sources of income and consumable services. Thus wealth, apart from consisting of

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[34] Friedman discussed this issue in a later article, see M. Friedman: The Optimum Quantity of Money and Other Essays. Macmillan, 1969, p.14.

the forms of non-human wealth that will be mentioned later, includes the productive capacity of human beings termed human wealth. The individual's holdings of human wealth can not however be bought and sold in markets and are not easily adjustable. Because of this limited ability of wealth-owning units to substitute human wealth for other forms of wealth or to convert the whole of their human wealth into money when they wish, Friedman added as a variable in the money demand function the ratio w of human to non-human wealth, while retaining at the same time the definition of wealth inclusive of the human component of it. It is not clear at this point why wealth, since it is defined inclusive of human wealth, should not encompass other goods which are free goods but yield utility or consumable services, e.g. climatic conditions, etc. Obviously, all these other components of wealth as well as human wealth are unmeasurable quantities and empirical research on the money demand function, which employed a wealth variable as a determinant of money demand has restricted it to non-human wealth, while assuming that the ratio of human to non-human wealth is constant.[35]

Assets in which non-human wealth can be held according to Friedman are: (a) money, (b) bonds, (c) equities, and (d) physical non-human goods. It is thus seen that Friedman does not include only assets which yield (or potentially can yield) a pecuniary nominal return, but also consumption goods and money which yield a return in kind, the former in the form of the satisfaction from consuming them and the latter in the form of convenience, security, etc. There is, of course, a fundamental difference between the services emanating from consumption

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[35] See for example A. H. Meltzer: "The Demand for Money: The Evidence from the Time Series." Journal of Political Economy, 1963, pp.219-46.

goods and money if this is defined as the medium of exchange, i.e. the sum of currency and demand deposits. While the utility derived from the use of consumption goods is dependent upon the quantity consumed, the utility effect of the medium of exchange is largely unrelated to the quantity of money in use.[36] This is so because the saving of time and effort and the greater efficiency in transactions resulting from the use of money as a medium of exchange appear to have occurred at the beginning, when money has been firstly introduced; thereafter the contribution of money has been very little and it is unrelated to its quantity. It must be pointed out that the above mentioned utility is the only non pecuniary service of money. Friedman views also money as rendering another service - the feeling of security and pride of possession.[37] He admits that bonds can also yield this service but *"money dominates bonds in the provision of nonpecuniary services"* because there is no default risk associated with it. On the same token one could argue that other assets which are free from default risk and yield a positive pecuniary return, e.g. deposit accounts, will dominate money in the provision of the above non-pecuniary services so that the only service left to money is that of facilitating transactions by acting as the medium of exchange.

Wealth, thus conceived, i.e. human and non-human wealth, is a stock magnitude and the income and consumable services provided by it are a flow magnitude. The connection between the stock, wealth and the flow, income is according to Friedman 'the' interest rate, which makes

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[36] This second point was made by G. Pierson in "The Role of Money in Economic Growth." Quarterly Journal of Economics, 1972, p.393.

[37] M. Friedman, *op.cit.*, 1969, pp.24-6.

the one the capitalised value of the other, that is

$$W = \frac{Y}{r} \tag{4.5}$$

where W is wealth, Y is income and r is the interest rate. $\frac{Y}{r}$ is substituted for W in the money demand function. Friedman recognises that, since in practice income from national accounts does not include any imputed return from the stock of money, $\frac{Y}{r}$ refers to total wealth exclusive of money. A further step in his analysis is to delete from his equation the variable r without any explanation. This would be a valid step if the interest rate is constant or if it varies in some systematic way with the other rates of return which are included in the equation and will be examined later.

It has already been noted that empirical studies which used a wealth variable confined it to non-human wealth. Even so the measurement of it poses certain difficulties. To quote Meltzer:[38] "*Do we consider each economic entity (individual or firm) to be constrained by its gross wealth and the aggregate constrained by the unconsolidated sum? How do we treat the assets and liabilities of the government?*" If the answer to the first question is that the appropriate wealth constraint is the unconsolidated sum of gross wealth of households and business this will involve substantial double counting. For example if households own firms, the value of the stocks they hold will be included as a part of their assets while at the same time plant and equipment will be included as a part of their assets of business. The answer to this question can not be given *a priori*. The simplest choice which is now

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[38] A. H. Meltzer, *op.cit.*, p.224.

the prevalent practice in countries which have data on national wealth (e.g. U.S.) is to consider the consolidated net worth of the private sector including the sector's ownership of monetary and non-monetary (interest bearing and non-interest bearing) government debt. The last are liabilities of the government which appear as assets in the public's balance sheets.

Alternative definitions of the wealth variable were considered by Meltzer[39] who compared the empirical findings for the U.S. from four different definitions. These were

(a) W : total consolidated wealth of the country (from Goldsmith's tables) minus government structures, inventories, public land and the monetary gold and silver stock plus monetary and non-monetary debt of the government. This estimate is referred to as consolidated net non-human wealth of the public.

(b) G : total consolidated wealth of the country as before without any adjustment for the government sector.

(c) N : aggregate net worth of households and business.

(d) A : unconsolidated total assets of households business and government. This measure includes the group of all financial assets which are absent from measures of net worth and is closer to Friedman's definition of non-human wealth.

Measure A seems to treat government as owned by the public because it includes government assets to the assets of the public. On the other hand, measures W , N and A have the common characteristic that they all include government liabilities as a part of the assets of the public. The three of them were found to have roughly the same effect on the demand for money whereas measure G which has no government debt as a component of wealth appeared to have no effect on the demand for money.

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[39] A. H. Meltzer, *op.cit.*

A way around the problem of using a wealth variable where satisfactory data on wealth do not exist or human wealth is not disregarded, is to use an income variable or, as Friedman and several other researchers have done, a permanent income variable.[40] The last variable will be seen in more detail in the section for the consumption function (chapter 7). It will be enough to say here that it is a weighted sum of present and past values of income the weights declining geometrically and has been used as a better proxy, compared to actual income, for the sum of human and non-human wealth. The effect of including it in the money demand function is to give a dynamic character to it as it will be argued later.

The second group of variables affecting the demand for money are the price and return of money and the other assets. Money does not yield any nominal return (where charges are levied on demand deposits there is a negative nominal return but this is not the case for the U.K.). Also a price of unity is associated with a unit of money because it is the numéraire for other goods. Friedman includes the price level P as a variable affecting the real return on money, the latter being in the form of convenience, security, etc. However we have seen that the return in kind from money (the medium of exchange) can be viewed as largely independent of its quantity so that the inclusion of P can be questioned on these grounds given that it is not meant to denote the effect of the depreciation of the monetary unit on rates of

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[40] For an example of the former case see N. J. Kavanagh and A. A. Walters: "Demand for Money in the U.K., 1877-1961." Bulletin of the Oxford University Institute of Economics and Statistics, 1966, pp.93-116. For the use of permanent income see among others M. Friedman, *op.cit.*, 1959, and D. Laidler: "Some Evidence on the Demand for Money." Journal of Political Economy, 1966, pp.55-68.

return on assets and the demand for money. For the last the term $\frac{1}{P}\frac{dP}{dt}$, the expected rate of price changes in Friedman's notation, is appropriate and is included as a separate argument. The usefulness of P will be understood if we consider that Friedman uses it to deflate nominal money balances and nominal income.

The expected nominal return on bonds, which for the purpose of convenience are consols, i.e. claims to a perpetual income stream of constant nominal amount, is approximated by

$$r_b - \frac{1}{r_b}\frac{dr_b}{dt} \tag{4.6}$$

where r_b is the market rate of interest on bonds at time zero and $\frac{dr_b}{dt}$ denotes the derivative with respect to time. (4.6) can be obtained in the following way. The annual fixed interest income $y_b(0)$ from a bond at time zero is

$$y_b(0) = B(0).r_b(0) \tag{4.7}$$

with $B(0)$ being the market value of the bond at time period zero and $r_b(0)$ the market rate of interest at that period. The expected market value $B(t)$ of the bond over time will be

$$B(t) = \frac{y_b(0)}{r_b(t)} \tag{4.8}$$

where $r_b(t)$ is the expected rate of return on bonds. Combining (4.7) and (4.8) we get

$$B(t) = B(0)\frac{r_b(0)}{r_b(t)} \tag{4.9}$$

The expected capital gains or losses will be

$$\frac{dB(t)}{dt} = B(0)\ r_b(0)\ \frac{d\ \frac{1}{r_b(t)}}{dt} = -\ B(0)\ r_b(0)\ \frac{1}{r_b^2(t)}\ \frac{dr_b(t)}{dt} \qquad (4.10)$$

and the return purchased at time zero is

$$B(0)\ r_b(0) - B(0)\ r_b(0)\ \frac{1}{r_b^2(t)}\ \frac{dr_b(t)}{dt} \qquad (4.11)$$

Setting $B(0) = 1.0$ and approximating (4.11) by its value at time zero we get (4.6).

The expected nominal return on equities is also approximated by the functional

$$r + \frac{1}{P}\frac{dP}{dt} - \frac{1}{r_e}\frac{dr_e}{dt} \qquad (4.12)$$

where r_e is the market interest rate on equities. (4.12) is obtained if we consider equities as claims to a perpetual income stream of constant real amount. The expected nominal income from them is then the product of this constant real income multiplied by the ratio of the expected price index over time to the price index at time zero, that is

$$y_e(t) = y_e(0).\ \frac{P(t)}{P(0)} = E(0).r_e(0)\ \frac{P(t)}{P(0)} \qquad (4.13)$$

where $y_e(t)$ is the expected nominal income flow over time, consisting of the constant coupon payment $y_e(0) = E(0).r_e(0)$ [B(0) is the market value of an equity at time zero and $r_e(0)$ is the market rate of interest on equities at time zero] times the purchasing power escalator clause or the price level ratio $\frac{P(t)}{P(0)}$. The expected market value of the equity is

$$E(t) = \frac{y_e(t)}{r_e(t)} = \frac{E(0).r_e(0)}{P(0)} \cdot \frac{P(t)}{r_e(t)} \tag{4.14}$$

with $r_e(t)$ being the expected rate of interest on equities. The expected capital gains or losses are found from (4.14) by differentiating it with respect to time.

$$\frac{dE(t)}{dt} = \frac{E(0).r_e(0)}{P(0)} \frac{d\frac{P(t)}{r_e(t)}}{dt} = \frac{E(0).r_e(0)}{r_e(t)} \frac{1}{P(0)} \frac{dP(t)}{dt} -$$

$$- \frac{P(t)}{P(0)} \frac{E(0).r_e(0)}{r_e^2(t)} \frac{dr_e(t)}{dt} \tag{4.15}$$

and the expected nominal return over time will be

$$E(0).r_e(0) \frac{P(t)}{P(0)} + \frac{dE(t)}{dt} \tag{4.16}$$

which is approximated by its value at time zero and $E(0) = 1.0$, giving (4.12).

Friedman goes on to simplify (4.6) and (4.12) by *"restricting it to the case in which r_b and r_e are taken to be stable over time"*. This results in considering only r_b and r_e as explanatory variables in the function. However if r_b and r_e are stable over time so that $\frac{dr_b}{dt} = 0$ and $\frac{dr_e}{dt} = 0$, there will be no variability at all in r_b and r_e and one wonders why they are considered as explanatory variables and they are not absorbed in the constant term of the money demand function. The proper assessment of what Friedman has done is to say that he omitted parts of the variables representing the returns from bonds and equities rather than saying that interest rates are not changing over time. Also Friedman split the sum of r_e and $(\frac{1}{P}\frac{dP}{dt})$ to its components which appear as separate arguments of the function.

Concerning the return on physical goods Friedman repeats that these goods yield a return in kind and an additional return due to the depreciation or appreciation of the monetary unit. The last is equal to

$$\frac{1}{P}\frac{dP}{dt} \tag{4.17}$$

for \$1.00 value of physical goods. This is so because this return can be found by differentiating PQ , the nominal value of physical goods with respect to time

$$\frac{d(PQ)}{dt} = Q\frac{dP}{dt} = PQ\frac{1}{P}\frac{dP}{dt} \tag{4.18}$$

from which we get (4.17) if $PQ = 1.00$.

Finally other factors that affect the demand for money are included in the term tastes and preferences and should be considered as constant if empirical testing of the theory is attempted. Tastes and preferences are formed by such things as the degree of economic uncertainty, geographical movement of wealth-owners, etc., and are denoted by u .

Gathering all the influences examined above in one functional form, we get the money demand function as specified by Friedman.

$$M = f(P, r_b, r_e, \frac{1}{P}\frac{dP}{dt}, w, Y, u) \qquad (4.19)$$

Since (4.19) is the analogous of a demand function for goods which according to accepted theory is defined in terms of real magnitudes, it must be independent of the unit in which the money variables are measured. Thus Friedman makes use of the homogeneity assumption, i.e. if the price level changes, the quantity of money demanded should change proportionately, or

$$f(\lambda P, r_b, r_e, \frac{1}{P}\frac{dP}{dt}, w, \lambda Y, u) = \lambda f(P, r_b, r_e, \frac{1}{P}\frac{dP}{dt}, w, Y, u) = \lambda M \qquad (4.20)$$

This enables him to write the function in the following way by setting $\lambda = \frac{1}{P}$

$$\frac{M}{P} = f(r_b, r_e, \frac{1}{P}\frac{dP}{dt}, w, \frac{Y}{P}, u) \qquad (4.21)$$

Thus the precondition for being able to express the demand for real money balances as a function of real income is the absence of money illusion from the money demand function. The term money illusion in the context of demand equations means that when there is a rise in nominal income (or other nominal variables influencing spending decisions) due to a rise in the general price level, there follows a corresponding rise in the quantity of the goods demanded. That is, there is no perfect conversion of nominal magnitudes to real ones but a certain degree of illusion exists which shows up as a significant coefficient of the price level if that is included in regressions with real variables. The assumption, however, of the non-existence of money illusion to demand equations has been criticised by various economists and it will be removed from our model. Given that money illusion is an empirical issue and should not be excluded *a priori*, Friedman's homogeneity assumption in his demand function for money should also be relaxed, since it is based on the analogy in specification to demand functions for other commodities.

The demand for money on the part of business firms can be explained, as suggested by Friedman, by the same variables and the same kind of function as those used to explain the demand by wealth-owning units and therefore that equation can be thought of as representing the aggregate demand for money by the whole economy.

The theoretical developments traced in the previous sections have been accompanied by a considerable amount of empirical work most of which has been concentrated on versions of the Keynesian or the Modern Quantity Theory of Money.[41] Despite the explicit recognition of the influence of more than one rates of return on money demand in the latter theory, empirical research has generally used one representative interest rate thus admitting some common ground with the Keynesian theory. Moreover, it has been recognised unanimously that the interest rate which enters the money demand function is the nominal interest rate. This is the appropriate rate to measure the differential between

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[41] For work based on the Inventory Theoretic Approach, see for example R. L. Teigen, *op.cit.*, 1964.

the rate of return on income earning assets and that on money. In nominal terms this differential is measured by the nominal rate minus the nominal rate on money which is zero. In real terms it is measured by the real rate minus the negative of the expected rate of inflation which is the real rate on money. In either case the differential is equal to the nominal interest rate.

On the question whether this interest rate should be a long term or a short term one the answer is that it does not make much difference on theoretical grounds which one we choose, if we accept that a stable term structure of interest rates exists. In empirical investigations using systems of structural equations - the example is given by the IS-LM model - the question becomes relevant because if we use a short term rate in the money demand function and a long term rate in the investment function, then an equation describing the term structure of interest rates needs to be involved, whereas if a long term rate is used in money demand as well, the term structure equation does not appear necessary.

If the expected rate of inflation is included additionally to the interest rate variable in the money demand equation, this amounts to splitting interest rate into two components, the real rate and the expected rate of inflation and assume a different response of money demand to the two of them.

Of the main differences between the Keynesian money demand function and the Modern Quantity Theory one the following need to be stressed as relevant to empirical research: (a) in the first, income is used as a measure of the 'work' to be done by money and in the second, as a surrogate for wealth.[42] This difference is reflected in empirical work in the inclusion in the latter of permanent income as opposed to current income, as a proxy for wealth; (b) the money demand function which stems from Keynesian analysis expresses the demand for nominal money balances as a function of nominal income whereas in the Modern Quantity Theory nominal magnitudes are replaced by real ones.

Having examined the theoretical underpinnings of the money demand function and some problems associated with the specification of the explanatory variables in it, we can now survey the empirical work on the function which is specific to the U.K.

The study by Kavanagh and Walters[43] represents the first attempt to estimate money demand functions for the U.K. The authors started from a version of the Modern Quantity Theory, which, after successive simplifications (elimination of variables) reduced to

$$\frac{M}{P} = f(R, \frac{Y}{P}) \qquad (4.22)$$

where R is the interest rate on bonds. If the variables are expressed in logarithmic form and (4.22) is linear we get

$$\log M = a \log R + b \log Y + (1 - b) \log P \qquad (4.23)$$

and (4.23) was one of the forms estimated by K - W. Other versions estimated were an inverted money demand with the interest rate as the

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[42] See M. Friedman: "A Theoretical Framework for Monetary Analysis." Journal of Political Economy, 1970, p.203.

[43] N. J. Kavanagh and A. A. Walters: "Demand for Money in the U.K., 1877-1961. Some Preliminary Findings." Bulletin of the Oxford University Institute of Economics and Statistics, 1966, pp.93-116.

dependent variable and forms including *ad hoc* lags in some or all of the independent variables. The money stock series employed was the broad one. In all estimated equations the DW statistic indicated the existence of positive autocorrelation in the residuals, which however was not taken into account. The interest elasticity of money demand was found to range between -0.28 and -0.83.

Fisher's study[44] is the first one which used quarterly data in order to investigate the demand for money in postwar Britain. Both definitions of money, i.e. narrow and broad, were employed in the regressions. In the spirit of Chow's[45] work for the U.S. Fisher distinguished a long run demand function for money and a short run one. The latter was simply the long run function coupled with an extended partial adjustment mechanism assumed to describe the adjustment of actual money demand to long run demand. The income variable was personal disposable income at current prices; also a permanent income series was tried. The weights used to find permanent income were those which are appropriate for the U.S. economy resulting from annual data, i.e. those used by Friedman[46] in his consumption function. Obviously it is not known whether they are also appropriate for the U.K. and for quarterly data and Fisher did not test this. As far as the interest rate variable is concerned Fisher used a short-term and a long-term rate. The results for the long run demand function in which the variables were expressed in a first difference form showed an insignificant interest

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[44] D. Fisher: "The Demand for Money in Britain: Quarterly Results 1951 to 1967." Manchester School of Economic and Social Studies, 1968, pp.329-44.

[45] G. C. Chow: "On the Long Run and Short Run Demand for Money." Journal of Political Economy, 1966, pp.111-31.

[46] M. Friedman: A Theory of the Consumption Function. National Bureau of Economic Research, Princeton University Press, Princeton, 1957, p.147.

rate coefficient for both interest rates and no appreciably different performance of income as compared to permenent income. The long run function expressed in terms of levels of the variables and transformed to eliminate autocorrelation of the residuals showed again interest rates to be an insignificant argument in the equation.

On the side of the short run function the picture changed as regards the interest rate which was significant in the demand for narrow money. The interest elasticity was -0.11 for the short-term rate and -0.3 for the long-term one.

Laidler and Parkin[47] approached the specification and estimation of the money demand function from a more sophisticated point of view. Their model was

$$M_t^* = a + b\,Y_t^E + c\,R_t \qquad (4.24)$$

$$Y_t^E = \lambda Y_t + (1 - \lambda)\,Y_{t-1}^E \qquad (4.25)$$

$$M_t = \theta M_t^* + (1 - \theta)\,M_{t-1} + u_t \qquad (4.26)$$

i.e. the partial adjustment mechanism was applied to money demand and the adaptive expectations mechanism to income. Here M refers to real money balances - the broad definition - and Y to real income (both expressed logarithmically and in *per capita* terms). (4.24), (4.25) and (4.26) gave the following form for estimation

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[47] D. Laidler and M. Parkin: "The Demand for Money in the United Kingdom 1956-1967: Preliminary Estimates." Manchester School of Economic and Social Studies, 1970, pp.187-208.

$$M_t = b_0 + b_1 Y_t + b_2 R_t + b_3 R_{t-1} + b_4 M_{t-1} +$$

$$+ b_5 M_{t-2} + v_t \qquad (4.27)$$

where the paremeters are non linear functions of the original parameters. In view of the fact that the adaptive expectations model, if applied alone to money demand, implies a geometrically declining income series as does (4.25), the simultaneous assumption of (4.25) and (4.26) can be explained by L - P's desire to obtain different patterns of dynamic response of money demand to changes in real income, price level, population and interest rate.[48] Equation (4.27) was estimated by a constrained least squares technique allowing for the accounting of restrictions connecting coefficients b_0 to b_5. The preferred equation gave the following speed of adjustment parameters: $\lambda = 0.948$ and $\theta = 0.052$. It is perhaps noteworthy that with the exception of λ all estimated coefficients were highly insignificant and no particular confidence can be placed to any inference based on them.

Goodhart and Crockett's article[49] on the importance of money is the first study on the demand for money function conducted in the Bank of England. G - C tried several formulations of the function starting from the simplest, the one without lags

$$M_t = a_0 + a_1 Y_t + a_2 R_t \qquad (4.28)$$

where M is nominal money stock

Y is nominal income

R is some interest rate

[48] Cf D. Laidler and M. Parkin: "The Demand for Money in the United Kingdom 1956-1967: Preliminary Estimates." Manchester School of Economic and Social Studies, 1970, p.194.

[49] C. A. E. Goodhart and A. D. Crockett: "The Importance of Money." Bank of England Quarterly Bulletin, 1970, pp.159-98.

Three definitions of the money stock (the M_1, M_2 and M_3 definitions) and two interest rates (the long-term and short-term rates expressed as the ratio of 100 plus the relevant rate to 100) were considered and the variables were expressed in logarithmic form. The results from estimation in which quarterly data were used, showed that, despite the very high coefficient of determination, autocorrelation was present in the residuals and the interest rate variable had in some cases a positive although not significant coefficient. G - C went on to correct these deficiencies by assuming that it takes some time for money holders to be aware of changes in the variables that determine their demand for money, and also that the adjustment of actual money holdings to equilibrium ones is given by the partial adjustment model. Thus

$$M_t^* = a_0 + a_1 Y_t + a_2 R_t + u_t \qquad (4.29)$$

$$M_t - M_{t-1} = b\,(M_t^* - M_{t-1}) + v_t \qquad (4.30)$$

gave the following reduced form

$$M_t = ba_0 + ba_1 Y_t + ba_2 R_t + (1 - b)\,M_{t-1} +$$

$$+ b_t u + v_t \qquad (4.31)$$

The estimated equation showed better properties although autocorrelation was still present. The interest elasticities for M_1 reported on the basis of the coefficient estimates and the mean values of the interest rate were -1.05 and -0.80 for the short-term and long-term rates respectively. Paradoxically the elasticity for the short-term rate is larger in absolute value than the one for the long-term rate and G - C's study is the first which reverses the relative size of elasticities. The available studies for the U.S. and the U.K. which use both a short-term and a long-term rate, consistently find the elasticity for the short-term rate to be absolutely smaller.[50] Because of the particular way in which variables are measured (the interest rate is the ratio of 100 plus the short-term or long-term rate to 100 and both this variable and the money stock are expressed logarithmically) the interest elasticity should be computed by the formula

$$\frac{\partial \ln M}{\partial \ln R} = \frac{\bar{R}}{100 + \bar{R}} \cdot \frac{\partial \ln M}{\partial \ln(100 + R)} = \frac{\bar{R}}{100 + \bar{R}} a_2 \qquad (4.32)$$

where a_2 is the estimated coefficient in (4.31) and $\bar{R}$ is the mean value of the interest rate. However there is no explicit reference in the study that G - C have used this formula. We note that in the most recent work done in the Bank of England which will be examined later, use has been made of the above formula in the computation of elasticities and the findings conform with the previous ones as to the relative ranking of elasticities.

Further versions of the money demand function tried by G - C were the following

$$\left(\frac{M}{NP}\right)_t = b\, a_0 + ba_1\left(\frac{Y}{NP}\right)_t + ba_2 R_t + (1 - b)\left(\frac{M}{NP}\right)_{t-1} \qquad (4.33)$$

and

$$\left(\frac{M}{N}\right)_t = ba_0 + ba_1\left(\frac{Y}{N}\right)_t + ba_2 R_t + ba_3 P_t + (1 - b)\left(\frac{M}{N}\right)_{t-1} \qquad (4.34)$$

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[50] The example from the U.K. is Fisher's study reviewed above, Fisher, *op.cit*.

where N is the adult population of the U.K. and P is the price level. In (4.33) both the money stock and nominal income variables were deflated by the population and the price level and were thus cast in real *per capita* form, because it was thought that there was *"no sound reason for expecting a change in the price level or a change in population size to have an effect on the equilibrium money/income ratio"*. In (4.34) *per capita* nominal money was regressed on the price level and *per capita* nominal income thus allowing the last two variables to have different effects on the demand for money. The estimated equations (4.33) and (4.34) were worse than (4.31) above as the standard error of the regression was greater. Moreover one of the forms of (4.34) - the one with the M_1 money and the short-term interest rate - gave a coefficient of the lagged money term greater than one and this implies that the equation was unstable. The explanation of this deterioration could only be sought in either the separation of real income and price as explanatory variables and the expression of the variables in *per capita* terms. Since the first change would improve if anything the standard error if price and real income had a different effect on money demand, Goodhart and Crockett concluded that the *per capita* adjustment caused the deterioration and that the demand for money is not homogeneous in population.

The work of G - C was extended by Price[51] who ran a regression similar to (4.34) except that money and income data were not deflated by population. Generally the results were not very satisfactory. The

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[51] L. D. D. Price: "The Demand for Money in the United Kingdom: A Further Investigation." Bank of England Quarterly Bulletin, 1972, pp.43-55.

real income elasticity in the equation with a M_1 money variable and a short-term interest rate turned out to be negative. The interest elasticity for M_1 was again greater (in absolute value) for the short-term rate than for the long-term rate. The former was reported to be -9.52, an unusually high value. Price recognised the fact that the reduced form of the money demand function which he estimated will be the same if the adaptive expectations model rather than the partial adjustment one is postulated (neglecting the error terms), and the speed of adjustment is identical for every explanatory variable. By assuming a different speed of adjustment for these variables a more complicated reduced form is obtained. Thus

$$M_t = a_0 + a_1 Q_t^* + a_2 P_t^* + a_3 R_t^* + v_t \tag{4.35}$$

$$Q_t^* - Q_{t-1}^* = \lambda_1 (Q_t - Q_{t-1}^*) \quad \text{or} \quad Q_t^* = \frac{\lambda_1}{1 - (1 - \lambda_1) L} Q_t \tag{4.36}$$

$$P_t^* - P_{t-1}^* = \lambda_2 (P_t - P_{t-1}^*) \quad \text{or} \quad P_t^* = \frac{\lambda_2}{1 - (1 - \lambda_2) L} P_t \tag{4.37}$$

$$R_t^* - R_{t-1}^* = \lambda_3 (R_t - R_{t-1}^*) \quad \text{or} \quad R_t^* = \frac{\lambda_3}{1 - (1 - \lambda_3) L} R_t \tag{4.38}$$

where Q is real income and L is the lag operator defined by $L x_t = x_{t-1}$, reduce to

$$\begin{aligned} M_t = {} & c_0 + c_1 Q_t + c_2 Q_{t-1} + c_3 Q_{t-2} + c_4 P_t + c_5 P_{t-1} + \\ & + c_6 P_{t-2} + c_7 R_t + c_8 R_{t-1} + c_9 R_{t-2} + c_{10} M_{t-1} + \\ & + c_{11} M_{t-2} + c_{12} M_{t-3} + w_t \end{aligned} \tag{4.39}$$

with $$w_t = [1 - (1 - \lambda_1) L] [1 - (1 - \lambda_2) L] [1 - (1 - \lambda_3) L] v_t \tag{4.40}$$

Coefficients c_0 to c_{12} are nonlinear functions of the initial parameters and an iterative technique was used to obtain the latter. On the basis of these estimates long-run elasticities were calculated for the three independent variables. The income elasticity was found to be at around unity but the price elasticity was only 0.2 . Interest elasticities were very small although rightly ranked with the elasticity of long-term rate being absolutely higher. As an alternative means of computing elasticities Price used equation (4.39) without the constraints implied by (4.35) to (4.38). Specifically, ordinary least squares were applied as before to (4.39) and the distributed lag relationships corresponding to this equation (which is a rational lag equation) were studied. The new findings again showed a price elasticity considerably varying around unity. Moreover the interest rate elasticities were as expected, absolutely higher for the long-term rate and for narrower definitions of money. The estimates of the equation from which these elasticities were derived are not presented by Price, who nevertheless recognises that the rational lag model is in general an equation with autocorrelated errors. To correct for autocorrelation a first order autoregressive scheme was assumed to apply to the residuals of (4.39). Of the new estimates only the parameters of the autoregressive scheme and the standard error of the equation were presented whereas the dynamic properties of the equation were not examined.

Hacche's article[52] on the demand for money function presents the most recent work done in the Bank of England. The specification of the

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[52] G. Hacche: "The Demand for Money in the United Kingdom: Experience since 1971." Bank of England Quarterly Bulletin, 1974, pp.284-305.

$$\ln M_t^* = a_0 + a_1 \ln Y_t + a_2 \ln P_t + a_3 \ln R_t + u_t \qquad (4.41)$$

$$\ln M_t - \ln M_{t-1} = (1 - \gamma)(\ln M_t^* - \ln M_{t-1}) \qquad (4.42)$$

or, after a transformation

$$\ln M_t = a_0(1 - \gamma) + a_1(1 - \gamma)\ln Y_t + a_2(1 - \gamma)\ln P_t +$$

$$+ a_3(1 - \gamma)\ln R_t + \gamma M_{t-1} + (1 - \gamma)u_t \qquad (4.43)$$

where M^* is desired money stock

M is the actual money stock

Y is real income

P is the price level

and R is a representative interest rate.

The above specification uses logarithms of the variables rather than absolute levels so that the coefficient estimates can be interpreted as elasticities $[\frac{\partial \ln Z}{\partial \ln X} = \frac{\partial Z}{\partial X}\frac{X}{Z}]$. Some modifications were applied to reduced form (4.43): (a) the interest rate variable R was replaced by $1 + R$ because it was thought that a doubling of the interest rate from say 1% to 2% can not have the same effect on money demanded as a doubling from 10% to 20%. Also the long-term and short-term interest rate were used simultaneously in the regression; (b) the long run price elasticity was constrained to be one; (c) the equation was estimated after a first difference transformation has been applied to the variables. This introduced autocorrelation in the residuals which was eliminated by use of the Cochrane-Orcutt transformation. As a result, the disturbances in (4.43) were assumed to follow a second order autoregressive process.

The equation estimated was thus of the form

$$\ln(\frac{M_t}{P_t}) - \ln(\frac{M_{t-1}}{P_{t-1}}) = b_1(\ln Y_t - \ln Y_{t-1}) + b_3[\ln(1 + R^S_t) -$$

$$- \ln(1 + R^S_{t-1})] + b_4[\ln(1 + R^L_t) - \ln(1 + R^L_{t-1})] +$$

$$+ \gamma[\ln(\frac{M_{t-1}}{P_t}) - \ln(\frac{M_{t-2}}{P_{t-1}})] + v_t \qquad (4.44)$$

with $$v_t = \rho v_{t-1} + e_t \qquad (4.45)$$

Equation (4.44-4.45) was fitted for the period from 1963.4 to 1971.3 and its predictive performance was examined from 1971.4 to 1974.1, namely after the implementation of the September 1971 measures of monetary policy.[53] Both the M_1 and M_3 definitions of money as well as the two separate components of M_3 (the personal sector and the company sector holdings of M_3) were used, as also were the long-term and short-term interest rates. Specifically for the M_1 variable which is of interest to us, the long-run interest elasticity for the long term rate was -0.184, whereas the long run elasticity for the short term rate was -0.081 . Both of them were significant. The adjustment for M_1 was found to be faster than for M_3 , 95% of the desired adjustment being completed after one year. The most important result of the study seems to be that the equation for M_1 predicted fairly satisfactorily the demand for money for the period beyond 1971.3, in contrast to the equation for M_3 which uniformly under-predicted the demand for M_3 . The explanation given by Hacche was that the reforms of 1971 have led to an increasing attractiveness of interest-earning money balances and this implied that the interest rate

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[53] Bank of England Quarterly Bulletin, 1974, pp.284-305. These measures will be discussed in some detail in the following chapter.

on time deposits had become a more significant determinant of the demand for M_3 since 1971.4 . However, regression results of the demand function for M_3 balances in which the clearing banks' seven-day deposit rate replaced the market rate, showed that the relevant coefficient was insignificant or had the wrong sign both when the estimation period ended in 1971.3 and when it ended in 1972.4 . The conclusion therefore should be modified to say that the 1971 reforms had as an effect an increasing competitiveness of banks relative to other financial intermediaries at a given market rate, and this means that more time deposits were attracted to banks after 1971.3 , so that it could be expected that actual time deposits (and consequently the M_3 money) would be greater than that predicted by an equation describing the pre-1971.3 behaviour.[54]

Having examined the most important pieces of work on the demand for money in the U.K., we can summarise some conclusions which emerge from them and criticise certain aspects of them.

(1) All studies reviewed are single equation studies and ignore simultaneity in the determination of their independent variables. Therefore coefficient estimates are subject to simultaneous equations bias.

(2) The results from these studies are not in general comparable because of differences in the underlying theory or in the variables used or in the length of the sample period of each study. Thus some are more closely connected to the modern quantity theory of money and

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[54] For a different interpretation of the underprediction of the demand equation for money, based on supply considerations, see M. J. Artis and M. K. Lewis: "The Demand for Money: Stable or Unstable?" The Banker, 1974, pp.239-47.

relate real money balances to real income and some in the spirit of the Keynesian demand function express the demand for nominal money as a function of nominal income. As regards the variables used, personal disposable income, gross national product, permanent income, the average of the three official estimates of gross domestic product at factor cost or total final expenditure are the variants of the income variable; the Treasury bill rate, the interest rate on 3-month local authority debt, the interest rate on 3-month local authority deposits, the interest rate on 2½% Consolidated Stock and the interest rate on 4% Consolidated Stock are the variants of 'the' interest rate variable; finally currency plus demand deposits of London Clearing Banks or of the whole banking system, currency plus total deposits, currency plus deposits of the banking system and other financial intermediaries are the alternative definitions of money employed.

(3) A common characteristic shared by all studies is that the interest rate variable has not been adjusted to account for seasonality whereas all other variables have been seasonally adjusted. This can be a potential source of bias in coefficient estimates unless money demand responds similarly to seasonal and cyclical interest rate movements.[55]

(4) The key to getting better results (better in terms of explanatory power, significant coefficients and non autocorrelated errors) for the function when using quarterly data, is to assume that some kind of dynamic adjustment mechanism applies to the demand for money. Some researchers wishing to differentiate the pattern of

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[55] This point has been made by R. Lombra and H. M. Kaufman in "Interest Rate Seasonality and the Specification of Money Demand Functions." Review of Economics and Statistics, 1975, pp.252-5. L - K have tested this proposition for the U.S. and found that it holds true.

dynamic behaviour of money demand in response to changes in different independent variables, applied more complicated lag structures to the function with the worsening of the significance of estimated coefficients as a result of this. While these sophisticated assumptions might be deemed necessary in a single equation context, it seems that in simultaneous equations models (having money demand as a part) we can get differentiated patterns of dynamic behavior without having recourse to them. Indeed, the assumption of a partial adjustment to one of the equations need not mean the same lag structure for all independent variables (Cf conjecture 2 in chapter 3).

(5) Goodhart's study on money demand has shown that *per capita* deflation of money and income caused a marked deterioration of regression results and should not be adopted when specifying the form of the equation for estimation.

(6) The most recent work on the money demand revealed that the demand function for M_1 has not been affected by the reforms of 1971 as did the function for broad money which underpredicted the demand for the years 1972-3.

(7) The postulate of the homogeneity (of the first degree) of money demand with respect to nominal income and the price level is not generally supported by the evidence.

(8) Most studies contain inferences about monetary policy from examination of short run and long run elasticities of money demand with respect to interest rate, real income and the price level. However, this is a poor substitute for dynamic analysis and if we recognise that the money demand function is a part of a larger system of structural

equations the above variables are endogenous as well and the analysis of dynamic multipliers becomes the appropriate tool for studying dynamic properties of the system. If in such models we wish to calculate elasticities, we need to do this for every exogenous variable. Thus if we want to obtain the interest elasticity of money demand we must calculate

$$\frac{\partial M}{\partial R}\frac{R}{M} = \frac{\frac{\partial M}{\partial X_i}}{\frac{\partial R}{\partial X_i}}\frac{R}{M}$$

for every exogenous variable X_i , and these elasticities will be, in principle, different for different exogenous influences.

Regarding the specification of the money demand function for our model, we shall assume that in equilibrium the demand for nominal money depends on nominal income Y^* and on nominal interest rate R , i.e. it is the standard Keynesian demand function in linear form

$$R_t^* = a_1' Y_t^* + b_1' B_t + w_{1_t} \tag{4.46}$$

$$a_1' > 0 \qquad b_1' < 0$$

where M^* is equilibrium money demand and R the long term rate. A linear approximation will be used for nominal income because it is real income and the price level which will appear as the variables to be explained in the model. Thus

$$Y_t^* = c_1' Y_t + d_1' P_t + v_{1_t} \tag{4.47}$$

$$c_1', d_1' > 0$$

where Y is real income and P is the price level. Substituting in (4.46) we get

$$M_t^* = a_1'c_1' Y_t + a_1'd_1' P_t + b_1' R_t + u_{1t}^* \quad (4.48)$$

$$u_{1t}^* = w_{1t} + a_1' v_{1t} \quad (4.49)$$

However adjustment to equilibrium does not take place instantaneously but it may take time for individuals to become aware of changes in the variables which influence their demand for money. The partial adjustment mechanism will be assumed to apply for the adjustment of actual money holdings to equilibrium ones.

$$M_t - M_{t-1} = \lambda_1 (M_t^* - M_{t-1}) \quad (4.50)$$

(4.48) and (4.50) combined produce for estimation

$$M_t = \lambda_1 a_1'c_1' Y_t + \lambda_1 a_1'd_1' P_t + \lambda_1 b_1' R_t +$$

$$+ (1 - \lambda_1) M_{t-1} + \lambda_1 u_{1t}^* \quad (4.51)$$

Following G. Chow,[56] this equation is normalised as to the interest rate coefficient for the purpose of estimation. The reason for doing so is that the money stock variable will also be the dependent variable in the money supply function which will be specified in the following chapter. The equation with the interest rate as the dependent variable is

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[56] G. Chow: *op.cit.*,1967, p.7.

$$R_t = \frac{1}{\lambda_1 b_1'} M_t - \frac{a_1' c_1'}{b_1'} Y_t - \frac{a_1' d_1'}{b_1'} P_t - \frac{(1 - \lambda_1)}{\lambda_1 b_1'} M_{t-1} +$$

$$+ \frac{1}{b_1'} u^*_{1t} = \frac{1}{\lambda_1 b_1'} (M_t - M_{t-1}) - \frac{a_1' c_1'}{b_1'} Y_t -$$

$$- \frac{a_1' d_1'}{b_1'} P_t + \frac{1}{b_1'} M_{t-1} + \frac{1}{b_1'} u^*_{1t} = a_1 (M_t - M_{t-1}) +$$

$$+ b_1 Y_t + c_1 P_t + d_1 M_{t-1} + u_{1t} \qquad (4.52)$$

$$a_1,\ d_1 < 0 \qquad b_1,\ c_1 > 0$$

and (4.52) is the equation to be estimated.

We note that if the error term in (4.51) is non autocorrelated, so will be the error term in (4.52). The normalisation applied above has been widely used in empirical work on the IS-LM model.[57] It is known that the normalisation choice may affect the explanatory power of the equation and parameter estimates for small samples but this effect is likely to be minor for larger samples.[58] Also, large sample properties of the estimators such as consistency are not impaired by normalisation.

We shall end this chapter by summarising some propositions which we discussed and accepted and the conclusions which emerge from them.

(a) The demand for money (the definition accepted in the first section) by rational money holders is a demand for transactions balances and as such it can be interest elastic.

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[57] See J. Kmenta and P. E. Smith, *op.cit.*, and J. R. Moroney and J. M. Mason, *op.cit*.

[58] Cf. Chow's remarks, *op.cit.*, 1967, p.10. Chow had used a small sample of only 20 observations and noticed that the explanatory power substantially lower in the inverted money demand function. Subsequent studies on the IS-LM model which have employed larger samples showed very little sensitivity to normalisation.

(b) There is no *a priori* reason to make the demand for money homogeneous of the first degree in prices and nominal income. This proposition is based on an analogy to the demand function for commodities in which absence of money illusion is assumed to exist. The latter however is a testable assumption and, if relaxed, leaves us with a demand for nominal money as a function of nominal income (the product of the price level and real income) or of the price level and real income separately which are assumed to approximate their product.

(c) We need not worry about the fact that the partial adjustment model assumed to hold, implies in a single equation context the same speeds of adjustment for each explanatory variable, since the interaction of the variables in the model will differentiate this pattern.

(d) The so commonly used practice in single equations studies of computing different elasticities of the demand for money is a poor substitute of dynamic analysis and should be replaced by the analysis of dynamic multipliers in a simultaneous equations model.

CHAPTER 5

CHAPTER 5

THE MONEY SUPPLY FUNCTION

An assumption made frequently in theoretical macroeconomic analyses is that the money stock can be fixed at any desired level or can be increased at any desired rate by monetary authorities. This assumption attributes to the Central Bank precision of control upon the money creation process so that money can be safely viewed as a monetary policy instrument. On the other hand it is widely accepted that money creation is a result of a complex interaction of Central Bank's monetary management associated with the implementation of Government policy and portfolio decisions of the banking system and the public. Thus an expansionary government policy might imply an increase in base money[1] which in turn disturbs the conditions under which the banking system and the public were in equilibrium regarding their holdings of wealth in the form of either real or financial assets. The adjustment of the latter to a new equilibrium results in an allocation of the base money between them and the money supply emerges from these adjustments. The transition to a new equilibrium can be influenced by policy actions of the Central Bank which may aim at the rate at which money is growing but may also pursue other objectives such as the management of government debt, the preservation of favourable conditions for government borrowing in financial markets, the correction of a balance of payments deficit, etc.

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[1] Base money (or monetary base or high-powered money) is defined as the net monetary liabilities of the government held by the banking system and the public. A more comprehensive definition of it will be given later.

The outcome of the joint behaviour of the monetary authorities and the banking system and public is money and it can only be said that money can be determined by the monetary authorities if the latter (a) are capable and/or willing to control base money and (b) are able to predict and offset influences on the money supply from the side of the banking system and the public. It is a practice frequently met in textbooks of monetary economics to call money supply exogenous in this case while labelling it endogenous when it is not actually controlled.[2]

To avoid any confusion that might possibly arise because of the exogenous-endogenous distinction of variables in econometric models, we shall be content with the terms controllable and non-controlled money supply and keep the established definition of an exogenous variable, which implies that money supply is exogenous when it is regarded as being influenced by variables that are not determined anywhere else in the model that will be constructed; an endogenous money supply would mean that at least another endogenous variable is included among its determinants.

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[2] For a recent example see D. G. Pierce and D. M. Shaw: Monetary Economics: Theories, Evidence and Policy. Butterworth, 1974. We quote the following passage: *"Those who argue that the money supply is in fact exogenous claim that it is determined by the monetary authorities and is therefore autonomous of the macro-economic variables whose relationship to money is being analysed. However difficulties arise, if the monetary authorities, though capable of determining the money supply, do not in fact do so, but instead, because of their pursuit of other objectives, simply act passively and allow the money supply to increase or decrease in response to changes in the demand for it. When this happens the money supply has effectively ceased to be exogenous and has become endogenous."* pp.141-2.

There are several theoretical or empirical studies in which reference has been made to money supply functions in the same vein regarding the definition of an endogenous money supply. Some empirical research was reviewed in chapter 2 (nos. 2, 4 and 6). Money supply was made endogenous through dependence on the interest rate or the income variable both of which were endogenous in the models considered there. At a theoretical level the role of the interest rate was examined and the existence of a positive relationship between money supplied and the interest rate was concluded. The basis of such a link is the behaviour of excess reserves. In the case where commercial banks are holding excess cash reserves the latter tend to be sensitive to interest rate changes in the sense that when market rates rise banks are induced to minimise their holdings of reserves by expanding loans and investments with a consequent increase in the money supply. Neglect of the interest elasticity of the money supply will lead to biased estimates of the coefficients in the demand for money function[3] (as well as in other equations in a model).

The situation has a certain similarity with the case of a good traded in the market whose price and quantity sold that are observed do not correspond to either a demand curve or a supply curve alone but are the intersection of the two schedules. Concerning the income variable the rationale for using this as a determinant of money supplied can be found in the presumption that the volume of transactions may influence the desired allocation of money between currency and demand deposits and in this way it may impinge upon the ability of the

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[3] For elaboration of the point see R. L. Teigen "The Demand for and Supply of Money" in Readings in Money, National Income and Stabilization Policy, edit. by W. L. Smith and R. L. Teigen, Irwin 1970, pp.83-4.

banking system to expand money supply further. Closely connected to this is the suggestion that the above allocation is determined by real wealth considerations of the public and real income represents a proxy for wealth.[4] By contrast, the inclusion of an accelerator type income variable as an argument in the function (studies 2 and 4 reviewed) is lacking sufficient theoretical justification.

Money supply functions are constructed from a basic core identity and assumptions regarding parameters of this identity. The method of developing a behavioural relationship with an identity as the starting point is not unknown in economics. The demand for money function has its origin in the identity $M V = P Y$ where M is the money stock, V is its velocity of circulation, P is the price level and Y the volume of transactions. Assumptions about the behaviour of V, P and Y result in a theory of the demand for money. Similarly by a set of assumptions on a number of identities, Kaldor formulated his theory of income distribution.[5]

For the money supply the relevant identity is derived from simpler identities:

(5.1)	$M_t = C^P_t + D_t$	the money supply is the sum of currency held by the public and demand deposits.
(5.2)	$B_t = R^B_t + C^P_t$	the monetary base consists of cash reserves of the banking system plus currency circulation.

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[4] See K. Brunner and R. Crouch: "Money Supply Theory and British Monetary Experience." Operations Research Verfahren, 1968, pp.77-112.

[5] N. Kaldor: "Alternative Theories of Distribution." Review of Economic Studies, 1955, pp.83-100.

$$(5.3) \qquad R_t^B/D_t = b$$

and — are the ratios to demand deposits of banks' reserves and currency respectively, with $b < 1$.

$$(5.4) \qquad C_t^P/D_t = a$$

Combining (5.1), (5.2), (5.3) and (5.4) we obtain easily

$$M_t = \frac{1 + a}{a + b} B_t \tag{5.5}$$

and this is the money supply identity which holds true for every set of values for B , a and b . Nevertheless as it stands it is by no means a behavioural relation. An obvious way to assign a behavioural content to it is to express explicitly the dependence of parameters a and b in the expression $(1 + a)/(a + b)$ called the money multiplier on other variables of the economic system and thus make the money supply indirectly determined by them. Then the identity can be appropriately written as:

$$M_t = \frac{1 + a_t}{a_t + b_t} \cdot B_t \tag{5.6}$$

with

$$\frac{\partial M_t}{\partial a_t}\Big|_{B_t, b_t} = \frac{b_t - 1}{(b_t + a_t)^2} \cdot B_t < 0 \tag{5.7}$$

$$\frac{\partial M_t}{\partial b_t}\Big|_{B_t, a_t} = - \frac{(1 + a_t)}{(b_t + a_t)^2} \cdot B_t < 0 \tag{5.8}$$

$$\frac{\partial M_t}{\partial B_t}\Big|_{a_t, b_t} = \frac{1 + a_t}{a_t + b_t} > 0 \tag{5.9}$$

i.e. the partial derivatives of the money supply with respect to the ratio of currency to demand deposits and the ratio of banks' reserves to demand deposits are negative whereas the derivative with respect to the monetary base is positive.

By concentrating our attention to (5.1) we can follow seriatim the steps of money creation. The primary source of money generation is the Government and to a lesser extent the Central Bank and the External Sector. When for example the Government's expenditure on current and capital account is in excess of its total revenue, then this deficit appears in accounting statements as an increase of the monetary liabilities of the Central Bank which acts as an agent of the Government. What is recorded in this example, is an increase in government securities held by the Central Bank and a corresponding increase of liabilities of the Central Bank held by the banking system or the public. Similarly an inflow of foreign currency or a granting of credit by the Central Bank to the commercial banks causes an increase of the Central Bank's liabilities. All these influences are captured in the monetary base (5.2) which measures the monetary liabilities of the Government and the Central Bank. The monetary base which has been proved a very useful concept in the analysis of money supply is derived from consolidation of the balance sheets of the Government and the Central Bank. In (5.2) only one side of it is presented and this is the uses side which shows how the liabilities in question are allocated between the public (currency held by the public C^P) and the banking system (cash reserves of the banking system R^B). The other side of it (sources side) depicts the items causing variation of these liabilities. As an example consider the items of the source base for the U.S.[6]

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[6] See L. C. Andersen and J. L. Jordan: "The Monetary Base-Explanation and Analytical Use." Federal Reserve Bank of St. Louis Monthly Review, August 1968, p.7.

	Holdings of Securities	+
Federal Reserve Credit	Discounts and Advances	+
	Float	+
Gold Stock		+
Treasury Currency Outstanding		+
Treasury Deposits at Federal Reserve		-
Treasury Cash Holdings		-
Other Deposits and Other Federal Reserve Accounts		-

where the plus or minus sign indicates whether the item is added or subtracted.

The monetary base provides the basis for a secondary expansion of the money supply originating in the banking system (commercial banks). Not all of the monetary base goes to the public in the form of currency. Some is deposited with the banks which are then in a position to expand money supply multiplely by creating demand deposits. The last are a type of bank liability which is generally accepted as a means of payment and thus constitutes a part of the money supply. But let us have a closer look at the way the banking system is functioning.

Commercial banks are financial institutions which issue deposit liabilities (demand deposits and time deposits) and acquire income earning assets with the funds made available to them by the public. The banks' costly item on the liabilities side of their balance sheets are time deposits which bear interest while demand deposits do not. The assets side of balance sheets includes cash and other financial assets (commercial bills, government securities, loans etc.) with liquidity ranging from very high to very low levels. Two basic considerations govern the allocation of these assets to classes of assets with different time to run to maturity. First, the banks should have enough cash and liquid assets which are readily convertible to cash to meet any withdrawal of funds by the public. Second the banks want to maximise the return from these assets. Since these two objectives are in conflict - an asset with lower liquidity yields higher return - banks have to trade-off at any time liquidity and return. Their decisions will be influenced by the total of their liabilities as well as by the distribution of these liabilities between demand and time deposits.

The characteristic of commercial banks which distinguishes them from other institutions called financial intermediaries, is their ability to create money, i.e. issue demand deposits that can be transferred to another person by cheque. On the other hand what they have in common with financial intermediaries is that both types of institution provide the services of financial intermediation, i.e. of channelling funds from sectors which are in surplus (their expenditure falls short of their income) to sectors which are in deficit. The latter are frequently those which engage in productive investment and through borrowing from intermediaries they are able to absorb existing savings.

Demand deposits come into existence and the money supply is increased when commercial banks extend credit, that is when they make loans or buy securities. For example when a loan is made by the bank, typically a demand deposit is created for the borrower's use. Because of this unique power within the financial system to expand the money supply commercial banks are subject to supervision by the Central Bank, which sets limits to this expansion. Thus banks may be required to hold some fraction of their assets in the form of cash (or equivalently deposits with the Central Bank) or to hold some other fraction of their assets in an approved by the Central Bank asset form (e.g. liquid asset ratio). The basis of reference for these fractions may be total deposits or demand and time deposits separately. This fractional reserve requirement system places a constraint on the infinite expansion of the money supply. If for example commercial banks are asked to hold 10% of their demand deposit liabilities in the form of cash reserves then it is not possible for them to extend credit beyond the limit imposed by this requirement. In a case of a uniform reserve requirement ratio referring to total deposit liabilities the maximum possible volume of deposits that can be supported by a given amount of the monetary base is B/r, where r is the fraction of total deposits which are required to be held as reserves. In order to find this it is assumed that the public does not want to keep any money in the form of currency so that the whole of the monetary base is held as cash reserves by the banking system.

Finally the public is a very important factor influencing the supply of money by the Central Bank and the banking system by allocating its funds between cash and other income earning assets, the latter part providing finance to the Government to meet its borrowing requirement and reserves to the commercial banks to expand their income earning assets.

All movements sketched above are interconnected and result in a final supply of money in the economy. Since they involve income earning assets as well, they are influenced by the rates of return on them so that the amount of money that is wanted by the public and is supplied by the commercial banks crucially depends on these rates of return. The above discussion has not referred at all to the particular framework of institutions and markets in the U.K. However, for a proper understanding of the behaviour of different economic agents in the process of money supply creation, a brief description of the Bank of England and the institutions comprising the banking system seems necessary at this point.

The Bank of England (sometimes called simply the Bank) is the Central Bank of the U.K. and as such it has certain functions:

(i) it acts as a banker to the Government and manages its account (public deposits) and the National Debt;

(ii) it has the exclusive right in England to issue bank notes;

(iii) it is the banker to the banks constituting the banking system. In this function it holds most of their cash reserves which it can influence by open market operations;

(iv) it manages foreign exchange reserves;

(v) it acts as the 'lender of last resort' to the discount houses by rediscounting first class bills at the Bank Rate; and

(vi) it has the responsibility for monetary policy. In this function it acts as the Government's agent.

For accounting purposes the Bank is separated into two departments, the Issue Department and the Banking Department. The former is classified traditionally in the National Accounts as part of the public sector, while the latter as part of the banking sector. The position of the Issue and Banking Departments on January 1969 was

£ millions

Issue Dept.	Government Securities	3199	Notes in circulation		3165
	Other Assets	1	Notes in Banking Dept.		35
		3200			3200
Banking Dept.	Government Securities	530	Deposits	Public	14
	Other Securities	38		Special	227
	Discounts-Advances	65		Bankers	267
	Notes and Coin	36		Other Accounts	143
		669			651

Source: Bank of England Quarterly Bulletin. (The discrepancy between assets and liabilities in the Banking Department's balance sheet as well as other balance sheets which will be presented later on, are due to other items, e.g. capital, which do not appear in the statistics reported in the Bulletin.)

The Issue Department holds mainly government securities in exchange for the notes it issues. Notes in circulation are with the public and the banking system and the item notes in the Banking Department would not be present in a consolidated balance sheet for the two departments. The assets of the Banking Department include government and other securities, a part of the Bank's portfolio of government debt; discounts and advances which are mainly bills of exchange obtained from the discount houses (lender of last resort function) and from other private customers of the Bank; and notes and coin. The liabilities of the Banking Department are mainly deposits: public deposits which are the government's account at the Bank of England; bankers' deposits which are the commercial banks' accounts at the Bank and serve as a reserve to meet a sudden demand for cash by their customers and to settle indebtedness to other banks resulting from the clearing of cheques; special deposits, a special type of deposit by commercial banks which will be discussed later in greater detail; and other deposits by foreign banks, Commonwealth and private customers.

One of the most important functions of the Bank of England is, as we have seen, its responsibility for monetary policy. The Bank carries out this function with the use of reserve requirement ratios and a number of policy instruments. As far as reserve requirements are concerned, before September 1971 Clearing Banks had agreed with the Bank to maintain a minimum cash ratio of 8% of their total deposits and a minimum liquidity ratio of 28%. The last included the banks' more liquid assets - cash, money at call and short notice and bills discounted. In September 1971 these rules were abandoned in favour of a single obligatory minimum reserve ratio of 12½%. Eligible reserve assets for this ratio are:[7] (a) balances with Bank of England (other than special deposits), (b) British government and Northern Ireland government Treasury bills, (c) money at call with the London money market, (d) British government securities with a year or less to run to maturity, (e) local authority bills eligible for rediscount at the Bank of England, and (f) commercial bills eligible for rediscount at the Bank of England up to a maximum of 2% of total eligible liabilities. The amount of reserves will be calculated by reference to eligible liabilities, i.e. sterling deposits obtained outside the banking system, including sterling resources acquired by switching foreign currencies into sterling. The reserve asset ratio of 12½% applies to all banks in the banking system.

In addition to reserve requirements the Bank of England can use certain other instruments. These are:

1. The Bank Rate. Over the years the Bank Rate developed three major functions, (a) the minimum lending rate (penal rate) to the

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[7] See Bank of England: "Competition and Credit Control" Bank of England Quarterly Bulletin, 1971, p.190.

discount houses. Since borrowing from the Bank at a penal rate would imply losses for the discount houses, a rise in the Rate induced the latter to revise upwards their discount rates and this leads to a rise in many other rates in the market for loanable funds; (b) the prime rate. It was the rate to which certain rates (deposit rates, rates for overdrafts) were linked; (c) an economic signal. The announcement effect meant that a rise in the Rate internally was the herald of disinflationary measures. On the other hand a rise in the Rate could be signalling an attempt to attract foreign funds when serious balance of payments problems existed. Because these functions often conflicted to each other (e.g. stimulating measures and a low Rate were needed internally, whereas a high Rate was necessary for external equilibrium) the importance of Bank Rate as an active policy instrument declined and movements in it were an adjustment to market conditions rather than a decisive action by the Bank. Economic signals were given by other means such as directives by the Bank, special deposit calls, etc. After 1971 deposit rates were freely determined and the method of influencing discount rates by the Bank weakened further the importance of the Bank Rate. Finally at the last quarter of 1972 an automatic rule was set which formalised the passive adjustment of the Bank Rate and this was that the minimum lending rate is calculated by taking the last Treasury bill rate, adding half a percentage point and rounding up to the next quarter point.

2. Open market operations. By buying and selling securities in the market the Bank of England is able to influence banks' cash reserves and also the level of interest rates. For example, a sale of long term securities by the Bank will directly influence the long term market rate and also it will reduce banks' reserves because customers will pay by

cheques drawn on a commercial bank so that after the clearing of the cheque bankers' deposits at the Bank will suffer a reduction.

3. Funding. It is the process whereby the Bank of England converts short term (floating) government debt to long term (funded) debt. By funding operations the liquidity of the banks is reduced and their capacity for expansion of credit is affected.

4. Special deposits. The Bank of England can ask the commercial banks to deposit with it a specified percentage of their deposit liabilities in the form of SDs and these deposits are completely illiquid for the banks in the sense that they have no determinate day of release. SDs bear interest and are not counted as part of banks' cash reserves or even liquid assets (or reserve assets after 1971).

5. Direct controls. They take the form of official directives to the banks aiming at bank lending both in its distribution and overall level. Qualitative requests give priority to lending for specific purposes (e.g. financing exports) while they discourage lending for other purposes (e.g. personal consumption). Quantitative restrictions concern the size of increase of advances or the rate of growth of bank lending.

We are now going to examine briefly the institutions in the banking system. The banking system comprises the Discount Market, the Deposit Banks, the National Giro and the Accepting Houses, Overseas Banks and Other Banks.

The Discount Market. The discount market consists of eleven discount houses and a number of other firms having some interest in discounting operations. Discount houses stand between the banks and the Bank of England, because the banks have not direct access to the Bank when they need funds and the discount houses are the only financial institutions which do have this access. The term 'lender of last resort' for the Bank is used in connection with this lending and the rate charged is equal to or greater than the Bank Rate (minimum lending rate). The sources and uses of funds of the discount houses are shown below (March 1969).

		£ millions
Assets	British government stocks	254
	" " Treasury bills	261
	Other sterling bills	520
	Local authorities securities	140
	Negotiable certificates of deposit	84
	Other	99
		1358
Borrowed funds	Bank of England	-
	London Clearing banks	913
	Scottish banks	52
	Other deposit funds	15
	Accepting houses, overseas & other banks	166
	Other sources	125
		1271

Source: BEQB

Most of their funds come from the banking system and are at call or short notice, i.e. they can be called immediately or after a very short notice. The proceeds are used to discount Treasury bills, commercial bills, short term government securities, local authorities securities and certificates of deposit. Discount houses have a special position in the Treasury bill market: they guarantee that the market is cleared at the weekly issues of bills. They tender, as a syndicate a single bid for the whole issue, usually lower than the outside bids and if the market is not cleared they take up the residual amount on offer at the syndicated rate. The Bank is willing to deal at these prices in its day-to-day operations but it can regulate the offer of bills to the market in a way which will influence interest rates (e.g. by offering a greater supply of bills than it knows the market can absorb). With the September 1971 changes the syndicated tender is abandoned and the Bank can choose the price at which it will discount bills in its day-to-day operations.

Deposit Banks. These are the London and Scottish clearing banks and the Northern Ireland banks. They undertake all kinds of banking business including the granting of credit. The balance sheet of London clearing banks is shown below (December 1968).

		£ millions
Liabilities	Current accounts	5487
	Deposit accounts	4273
	Other accounts	976
		10736
Assets	Coin, notes and balances with Bank of England	865
	Money at call and short notice	1487
	Bills discounted	1034
	Special deposits	214
	Investments	1432
	Advances and other accounts	5039
		10071

Source: BEQB

The first item on the assets side contains the banks' cash balances with the Bank of England, encountered as bankers' deposits at the Bank's accounts and their vault cash. Money at call and short notice represents loans to the discount houses, to members of the stock exchange and to money markets in other centres and can be recalled, as we have seen when we examined the discount houses, immediately or after a very short notice. Bills discounted are Treasury bills or commercial and other bills which are sold to the banks at a discount. Less liquid assets are (a) investments most of which are government stocks, (b) special deposits which are deposits with the Bank of England deposited at the Bank's request and (c) advances which are the most profitable asset of banks and include personal loans, loans to manufacturing industry, commercial undertakings, etc. The liabilities side is dominated by deposits: current accounts (demand deposits) at which no interest is paid and deposit accounts (time deposits) bearing interest to the holders.

The National Giro. The national giro which was opened in 1968 and is operated by the Post Office, makes a new kind of current account banking facility available to the public.[8] Giro cheques can be cleared through deposit banks and money from a giro account can be easily transferred to a bank account. No interest is paid on giro account balances and until very recently there were no overdraft facilities.

Accepting Houses. Acceptance credit is the basic business of accepting houses.[9] A bill of exchange (commercial or financial) is drawn on an

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[8] See Central Statistical Office: British Financial Institutions. H.M.S.O., London 1969, p.17.

[9] *Ibid*. p.25

accepting house by the seller or purchaser of goods and after it has been accepted by that house against documents, the drawer can discount it immediately with a discount house (prime bill). Accepting houses also provide normal banking facilities for customers including foreign banks as it can be seen from the following balance sheet (December 1967).

		£ millions
Liabilities	Current and deposit accounts U.K. banks	320
	" " other U.K. residents	593
	" " overseas residents	552
		1465
Assets	Coin, notes and balances with Bank of England	1
	Balances with other U.K. banks	299
	Money at call and short notice	80
	Sterling bills discounted	32
	British government stocks	78
	Loans to U.K. local authorities	272
	Advances	667
	Other assets	194
	Acceptances	291
		1914

Source: BEQB

Overseas and other banks. These banks carry their banking activities overseas but at the same time they pursue banking business in London and as such they are included in the banking system.

Operation of monetary policy. We have seen in the preceding sections what instruments are available to monetary policy in the context of the British institutional framework. We are now going to examine how monetary policy was actually operated on the basis of monetary authorities' stated objectives. The relevant periods are two, the second being characterised by some changes in the functioning and control of the banking system and in the degree of the Bank's intervention in the market for government debt.

The main objective of monetary policy during the fifties and sixties was the proper management of national debt. The meaning of proper management was placed in the Bank's contribution to the existence of a stable market for government debt, i.e. a market in which large amounts could be invested in government securities without too much of an effect on prices-interest rates. The authorities believed that the market was unstable and the demand for gilts not well specified.[10] Investors expectations were thought to be volatile so that in certain circumstances a rapid downward movement in the prices, instead of inducing a higher demand for government stocks could demoralise the market, having as effect a net sale of stocks by the public;[11] or sharp movement of prices in either direction which are abruptly halted by official intervention could be considered as unpredictable by investors thus discouraging them from investing their funds in this market.[12] There is some evidence supporting the authorities' view of the public's behaviour. Norton has found that for the period 1955 to 1966 a sale of Government stocks by the banks which results in a reduction of their prices did not stimulate the

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[10] C. Goodhart: Discussion in Monetary Theory and Monetary Policy in the 1970s. Proceedings of the 1970 Sheffield Money Seminar, Oxford University Press, 1971, p.228.

[11] Bank of England: "The Aims of Debt Management." Bank of England Quarterly Bulletin, 1966, p.146, and Bank of England: "The Operation of Monetary Policy since the Radcliffe Report." Bank of England Quarterly Bulletin, 1969, pp.455-6.

[12] Bank of England: *op.cit.*, 1966, p.147.

public to increase their demand.[13] Instead, in the same quarter, the public followed the same pattern of behaviour by selling Government stocks in the market. However, the instability implied by this behaviour seems not to extend beyond three months. In the next quarter the pattern is reversed and the demand for government debt by the public shows a positive response to further sales of debt by the banks.

A secondary element relevant to debt management was the authorities' wish to alleviate the cost of servicing the national debt given the relatively (compared to other countries) higher burden from Government's past borrowings.

The above considerations were at the roots of monetary policy during the fifties and sixties and explain the authorities' concern for the stability of interest rates both short term and long term and their intervention in the Treasury bills and gilt-edged markets. The Bank of England was always ready to exchange cash for Treasury bills or gilt-edged securities, thus allowing injection of new base money in the system. This rendered the control of the expansion of bank lending, by means of dealings in the market loose enough and not always possible to be effected through the conventional cash and liquidity ratios. Since the cash ratio was more or less kept at the minimum level by the banks following their agreement with the Bank of England the most serious leakages were coming from the liquidity ratio and the authorities mainly concentrated on this ratio. Control of the liquid assets ratio was sought by (a) the use of the Bank Rate to influence short term rates, and open market

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[13] W. E. Norton: "Debt Management and Monetary Policy in the United Kingdom." Economic Journal, 1969, pp.481-2.

operations that helped in making Bank Rate effective, (b) funding operations, i.e. converting floating debt to longer term debt, (c) ceilings and qualitative restrictions to bank lending, and (d) the use of special deposits affecting bank liquidity directly.

Nevertheless the liquid assets ratio could not be a reliable fulcrum for controlling bank advances mainly because the Bank was prepared to buy long term government securities and the banks would resort to massive sales of investments when there was pressure on their liquidity position. Moreover, imposed quantitative and qualitative restrictions had the drawback of inhibiting competition and innovation in the banking system and creating distortions in the allocation of credit.

The period after 1971 is featured by a change in the conception of how debt management should be carried out. The Bank now allows sharper movements of prices in the gilt-edged market and is no longer prepared to buy stocks of more than one year to maturity. The argument about the instability of the market retreats to give place to an explanation of the new tactics based on monetarist suggestions. In the words of the Bank:

> *"Rising nominal rates can often be illusory when seen in real terms; and to hold rates artificially low can only create a consistently weak market and lead to steady monetization of the debt. It is this last consideration which has perhaps become more important in our minds recently as inflation has accelerated. Consequently unprecedentedly high nominal rates have seemed appropriate and our tactics in market management have become more flexible so that the market has been allowed to make sharper adjustments than in the past."*[14]

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[14] Bank of England: "Monetary Management in the United Kingdom." Bank of England Quarterly Bulletin, 1971, p.44.

On the other hand monetary authorities retained the power to influence the Treasury bill rate and other short term rates, with their own choosing of the prices at which exchange of liquid assets with cash will take place. It has been argued that this indicates that the Bank is not entirely convinced about the stability of the market for government debt and wants to prevent volatile short term rates from unsettling the gilt-edged market;[15] and that control of short term rates implies ability to influence flows of short term funds across foreign exchanges.[16] The importance of the former objective can be concluded from the fact that the Bank has never used the cash ratio as a means of controlling bank lending and the money supply. Before 1971 the minimum cash ratio was agreed to be 8% of total deposits of the Clearing Banks. With the new measures no attention at all is paid to the cash ratio because of the Bank's intention not to employ it. The impact of a change in the cash ratio would be felt on short term rates, because with varying needs for cash banks would turn first to assets in the lower end of the liquidity spectrum. The Bank is prepared to accommodate any sales of government stock of less than a year to maturity with prices of its own choosing and this implies ability of neutralisation of the cash ratio fluctuations. We quote the pronouncement made by the Bank:[17]

> *"The Bank has not used the cash ratio because it would have been likely to produce large fluctuations in short term interest rates with unwelcomed consequences not merely for the money markets but for many areas (as the housing mortgage markets) of wider economic significance. These considerations have led the authorities to*

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[15] A. B. Cramp: "Implications for Monetary Policy." Bankers' Magazine, July 1971, p.4.

[16] *Ibid.*, p.4.

[17] Bank of England: *op.cit.*, 1971, p.38.

> *continue with the present system under which Treasury bills are always interchangeable with cash through the mechanism of the discount market. The ready marketability of Treasury bills as the residual source of government finance is ensured. This argument coupled with the use of the Bank Rate has given the authorities control over most domestic short term interest rates."*

Concerning long term rates, although much more freedom was allowed in the adjustment of prices of gilt-edged securities, the Bank has still at its disposal most of the traditional instruments which it can use for control if things appear to slip out of hand and the resulting levels of interest rates are thought to be unacceptable. Control now will be concentrated on the reserve asset ratio which replaced the liquidity ratio. If the focus of the monetary authorities is on the liquid assets of the banks rather than their cash balances, the impact of any action by the Bank is felt on longer term rates as the banks move to the lower ranges in the liquidity spectrum. The instruments of control are again (a) open market operations, (b) funding, (c) qualitative "guidance", and (d) special deposits.

The counterpart for the banks to the authorities' measures was the abandonment of their agreement on interest rates and the free operation of price competition in attracting funds and allocating credit.

We now turn to a review of applied work for the U.K. on money supply theory.

There are a few studies which produced empirical results regarding the money supply process. The first attempt to estimate a money supply function relevant for the U.K. experience was done by R. L. Crouch, who derived from an analysis of a homogeneous multi-bank system the form of the function corresponding to the following relations:

$$M = C^P + D \qquad (1)$$

$$B = C^P + R \qquad (2)$$

$$C^P = C_0 + cM \qquad (3)$$

$$R = (r_1 + r_2)\, D \qquad (4)$$

where D represents total deposits of the banking system, R includes special deposits, C_0 is the exogenous demand for currency by the public and r_1 and r_2 the cash and special deposits ratios. The money supply function can be obtained as

$$M = \frac{1}{(r_1 + r_2)(1 - c) + c} B - \frac{1 - (r_1 + r_2)}{(r_1 + r_2)(1 - c) + c} C_0 \qquad (5)$$

but this was not what Crouch presented as the final relationship because of a slip in the derivation pointed out by Parkin.[19] The C_0 series was computed from $C^P - cM$ where c is arbitrary and was chosen so that M when regressed on B and C_0 yields a unit elasticity $\partial M/\partial B.\ B/M$. The latter was justified by the consideration that the function should intersect the origin on the M,B plane: a zero level of monetary base logically should imply a zero money supply. The parameter estimates showed little sensitivity when annual and quarterly data were used or when levels of variables were replaced by first differences.

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[18] R. L. Crouch: "The Genesis of Bank Deposits: New English Version." Bulletin of the Oxford University Institute of Economics and Statistics, 1965, pp.185-99.

[19] J. M. Parkin: "The Genesis of Bank Deposits: New English Version - A Comment." Bulletin of the Oxford University Institute of Economics and Statistics, 1967, pp.79-84.

In a later study, Crouch estimated a model of the U.K.'s monetary sector in which supply equations for total deposits and demand deposits were presented.[20] The argument in the equations was not the monetary base but banks' reserves. The latter according to Crouch are not interest responsive since the banks are not holding excess reserves over and above the 8% cash ratio. Their liquid assets are almost perfectly liquid and transactions costs in these assets are extremely low. The same assumption, namely lack of interest elasticity, was implicitly and without further discussion carried over to the supply of demand deposits equation where banks' reserves and time deposits were the determinants of the amount of demand deposits supplied. We shall argue later that there is some interest elasticity of the ratio of reserves to demand deposits and indicate the reasons for this.

Finally, Brunner and Crouch formulated and tested a more comprehensive money supply equation for the U.K. by adjusting the theoretical and empirical work of the former regarding the money creation process in the U.S., to take account of the institutional characteristics of the British monetary system.[21] The authors obtained formally the monetary base (both uses and sources side) through consolidation of the balance sheets of the Bank of England, Issue and Banking Department, the Exchange Equalisation Account and the Royal Mint. Details will not be given here because the derivation will be followed when the discussion of the monetary base for the U.K. will be held.

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[20] R. L. Crouch: "A Model of the United Kingdom's Monetary Sector." Econometrica, 1967, pp.398-418.

[21] K. Brunner and R. L. Crouch: *op.cit.*

The money supply function resulted from a number of equilibrium conditions assumed for the banks and the public. These conditions together with the accompanying identities were:

$M = C^P + D + T$	(1)	the definition of money, with C^P, D and T being currency, demand deposits and time deposits respectively.
$B = B^1 + K = R + C^P$	(2)	the definition of the monetary base - sources and uses side of it - with B^1 discounts and advances of Bank of England, K the rest of items on the sources side and R the banks' reserves.
$A = M + F^{1P} + F^{2P}$	(3)	the public's liquid assets, where F^{1P} and F^{2P} are Treasury bills and long term government securities holdings of the public.
$R + F^{1B} + F^{2B} + L^D = D + T$	(4)	the banks' balance sheet where F^{1B} and F^{2B} are banks' holdings of Treasury bills and long term government securities and L^D loans.
$R = b_1 (D + T)$	(5)	the banks' desired (equilibrium) allocation of assets, with $b_1 = b_{11} + b_{12}$, b_{11} the cash ratio and b_{12} the special deposit requirement ratio.
$F^{1B} = b_2 (D + T)$	(6)	
$F^{2B} = b_3 (D + T)$	(7)	
$L^D = b_4 (D + T+$	(8)	
$b_i = f(i^1, i^2, i^3)$	(9)	the dependence of the ratios b_i on the Treasury bill rate i^1, the bond yield i^2, and the loan rate i^3.

$$C^P = \pi^1 A \tag{10}$$

$$D = \pi^2 A \tag{11}$$

$$T = \pi^3 A \tag{12}$$

$$F^{1P} = \pi^4 A \tag{13}$$

$$F^{2P} = \pi^5 A \tag{14}$$

$$L^S = \pi^6 A \tag{15}$$

the public's desired allocation of assets where L^S refers to loans.

$$\pi^i = h\,(a,\ r - c,\ i^1,\ i^2,\ i^3,\ W,\ \sigma_w,\ p) \tag{16}$$

the dependence of the ratios on banks' service charges on checking accounts s , the rate paid on deposits $r - c$, where r is the Bank Rate and c a constant, the three rates i^1, i^2, i^3, mentioned before, real wealth W and its distribution with a standard deviation σ_W and the ratio of prices of goods purchased with currency to prices of goods purchased through a transfer of deposits.

$$B^1 = \pi^7 A \tag{17}$$

the discount houses' desired indebtedness to the Bank of England.

$$F^{1B} + F^{1P} = F^1 - B^1 \tag{18}$$

$$F^{2B} + F^{2P} = F^2 \tag{19}$$

$$L^D = L^S \tag{20}$$

the equilibrium conditions in the market for Treasury bills, long term government securities and loans, with F^1 and F^2 the amounts of Treasury bills and long term government securities outstanding.

$$G^a = K + F^1 + F^2 \tag{21}$$

the public's and the banks' total financial claims against the government sector.

The authors combined relations (1) to (21) to derive a reduced form for the money supply, namely

$$M = \frac{\pi^1 + \pi^2 + \pi^3}{1 - b_4 (\pi^2 + \pi^3) - \pi^7} G^a \qquad (22)$$

and three more relations. Without obtaining an explicit solution they claimed that the latter can be solved for the three interest rates i^1, i^2 and i^3 so that from (22) money supply is determined by G^a , r , W , σ_w , s , p and the special deposits requirement ratio. For simplification or because of lack of statistical information they omitted the last four factors and estimated a linear form of the function having K , F^1 , F^2 , r and W as arguments. Real income was used as a proxy for wealth and the authors introduced without any reference to their formal system of relations another explanatory variable, i.e. the ratio of demand and time deposits to total money supply. The empirical results showed that the monetary base was the major determinant of money supply while floating debt F^1 was found insignificant. No DW statistics were presented for the regressions which employed quarterly as well as yearly data and levels and first differences of the variables.

As far as our analysis of the money supply process is concerned we shall concentrate on (5.6) and try to establish a money supply equation after examination of the behaviour of the monetary authorities, the banking system and the public. This necessitates an analysis of the monetary base, the ratio of currency to current accounts and the ratio of banks' reserves to current accounts.

The monetary base. The monetary base for the U.K. was obtained by Brunner and Crouch in their study mentioned above.[22] The derivation will be followed here with a slight modification and the inappropriateness of the definition will be pointed out.

In order to derive the monetary base the balance sheets of the Bank of England, the Royal Mint and the Exchange Equalisation Account are consolidated.

The function of the Bank of England has been discussed previously.

The Royal Mint is the place where coins are manufactured according to exact dimensional and composition standards. It is a government department and the cost of producing coins (metals, labour, etc.) is met by drawing on the Public Deposits account at the Bank of England. This adds to the reserves of the banking system which on the other hand suffer a reduction when the public wants more coin.

The Exchange Equalisation Account is a fund established in 1932 and operated by the Bank of England under the control of the Treasury.[23] Its function consists of buying and selling gold and foreign currencies to prevent excessive fluctuations in the exchange rate for sterling. The Account was initially provided with a capital of £150 million of sterling most of which was lent to the Exchequer on "tap" Treasury bills.[24] When the Account purchases foreign currency it reduces its loan to the Exchequer by drawing on its sterling capital to finance the purchase. The Exchequer in turn has to raise

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[22] K. Brunner - R. L. Crouch: *op.cit.*, pp.109-112.

[23] Bank of England: "The Exchange Equalisation Account: its origins and developments." Bank of England Quarterly Bulletin, 1968, pp.377-87.

[24] Tap bills are Treasury bills which do not ordinarily enter the market but are issued directly to Government agencies such as the Issue Department of the Bank of England, the National Debt Commissioners and the Exchange Equalisation Account. See Committee on the Working of the Monetary System: Report, H.M.S.O. 1959, paragraph 98.

more money from outside sources, that is from the public or if the latter's demand for government debt is weak, from the banking system. In either case there is a counter effect on banks' cash reserves which at the outset had risen because of the inflow of foreign currency. But while the effect on reserves is likely to be small there can be an increase in banks' liquid assets depending on whether banks take up or not the additional government debt and on whether the latter takes the form of Treasury bills or Government securities with more than one year to run to maturity. Also the effect of these operations on the money supply depends on the particular form these operations take. For instance sale of the additional government debt to the public reduces the money supply; sales of long term government debt to banks leaves the money supply unaffected; and sale of Treasury bills to banks is a source of a potential increase in the money supply. Finally, if neither the public nor the banks take up the additional government debt there is a corresponding increase in the monetary base reflected in an increased portfolio of government securities held by the Bank of England.

The balance sheets of these three institutions appear as follows although the last two are the authors' versions since the Royal Mint and the Exchange Equalisation Account do not publish accounts.

Bank of England

(a) Issue Department

Government Securities	S_1	Notes issued and held by:	
Other Securities	S_2	the public	N^O
Coin	C^I	the banks	N^C
Other assets	A	the Bank, Banking Dept.	N^B

(b) Banking Department

Government Securities	S_3	Capital and rest		K^B
Other Securities	S_4	Deposits:	Public	P^D
Discounts and Advances	E		Bankers'	B^D
Notes	N^B		Special	S^D
Coin	C^B		Others	O^D

Royal Mint

Minted coin	C^T	Coin held by:		
		Bank of England	Issue Dept.	C^I
			Banking Dept.	C^B
		Public		C^O
		Banks		C^C

Exchange Equalisation Account

Gold and foreign exchange reserves	GF	Total borrowing rights	TB
Tap bills	S_5		

Consolidation of the three balance sheets yields:

$$S_1 + S_2 + S_3 + S_4 + S_5 + C^I + C^B + C^T + N^B + E + GF + A =$$

$$= N^O + N^C + N^B + P^D + B^D + S^D + K + O^D + C^I + C^B + C^O + C^C + TB$$

The terms C^I , C^B , and N^B are cancelled out and are not shown below. Setting

(a) $S_1 + S_2 + S_3 + S_4 + S_5 = S^T$, the Bank's total securities portfolio,

(b) $N^O + C^O = C^P$, currency circulation with the public,

(c) $C^C + N^C + B^D = R^B$, banks' cash reserves,

(d) $A - K^B = W^C$, a constant (difference between two approximately constant quantities),

and rearranging gives

$$E + S + C^T + GF - P^D = O^D - TB + W^C =$$

$$= R^B + C^P + S^D \qquad (5.10)$$

The monetary base is defined as

$$B = C^P + R^B + S^D \quad \text{or} \qquad (5.11)$$

$$B = E + S^T + C^T + GF - P^D - O^D + W^C - TB \qquad (5.12)$$

Definition (5.11) represents the sources side of the monetary base and reflects the particular institutional setting of the U.K. The uses side, on the other hand, does not fully agree with the typical definition which shows how the system's "high powered" assets are used by allocating them between the public and the banks. The public's component consists of currency in circulation with the public. The banks' component consists of the banks' vault cash as well as their balances with the Bank of England on which the banks have free access at any time. Their sum forms the basis for multiple expansion of bank advances and investments and therefore of the money supply. But Brunner and Crouch's definition of the monetary base contains an additional item not to be found in the corresponding grouping of variables for the U.S., namely special deposits.

Thus $$B = C^P + R^B + S^D$$

where S^D is the amount of cash called from the banks by the Bank of England in the form of special deposits. Given that SDs are a completely illiquid asset for the banks in the sense that they have no determinate future date of release, the question immediately arises: is it legitimate to assign SDs to the sum of C^P and R^B, a magnitude whose changes are followed by multiple changes in the same direction of the money supply? Clarification of the issue can be achieved only after examination of the nature of SDs, a relatively new instrument of monetary policy introduced in 1958 and not employed until 1960.

Special Deposits are a type of deposit at the Bank of England to which London Clearing and Scottish banks transferred funds when asked to do so. The banks concerned were required to make the transfers in cash and could not draw cheques on SDs as they could do with their free balances with the Bank. Funds called on SDs account were completely illiquid for the banks and so they were not eligible for inclusion in the 30 percent (28 percent after 1963) liquidity ratio, i.e. the ratio of liquid assets to deposits. The basis for computation of the amount to be called was the total of banks' deposits and the calls were expressed as a percentage of them. This percentage was twice as high for the London Clearing banks as it was for the Scottish banks. SDs carried from the day of deposit an interest which was close to the rate at which Treasury bills were allotted at the preceding tender. The reason was that the funds were not kept idle at the Bank but as we shall see were used for short term financing of the Exchequer while being at the same time a useful instrument for credit control.

What was the expected reaction of banks to a call for SDs? Naturally the immediate response would be to call short money or sell bills to acquire the necessary cash for effecting the payment and subsequently to sell investments or cut down advances in order to restore the disturbed equilibrium in which the banks had well defined liquidity ratios. Since this movement, if not supported by official intervention, could put pressure in the market for debts and could presumably lead to a rise in interest rates to unduly high levels, there has actually been official intervention in the market explained by the authorities' concern about the level of interest rates. Before giving our attention to the Bank's intervention in the market, we shall examine more closely the likely adjustment of banks' balance sheets to a call for SDs and understand the rationale for the use of this instrument.

The operation of SDs as conceived by the authorities was aimed at supplementing measures to reduce credit given by banks and consequently the available money supply. We can recall at this instance that the money supply we are referring to is the narrowly defined one. This will avoid the imputation that SDs were aimed at reducing total deposits and the broadly defined money supply. It seems that many economists have tended to concentrate on total deposits rather than bank advances and investments with the result that they assessed the ineffectiveness of SDs as an instrument of monetary policy. As a few examples, N. J. Gibson found that the reduction of banks' liquid assets *"has not inhibited the expansion of total deposits"* and the observed fall in banks' advances is due to other factors.[25] R. L. Crouch examining longer series of data reached the same conclusion, namely that SDs have had a perverse effect being associated with an increased level of deposits.[26] Nevertheless one should not regard total deposits as the magnitude upon which the impact of SDs would be felt through a reduction of their total. Instead the intention of the authorities was to affect the size of the banks' advances so that recourse to lending ceilings could be abandoned.

The expected sequence of events following a call for SDs was firstly a reduction of the cash balances of the banks. Since, as it has been already mentioned, the banks actually had a minimum cash ratio of 8 percent which they maintained relatively stable at around this level, the pressure was immediately transmitted to their liquidity position. They would sell bills to the Bank which was prepared to accommodate such sales and would not acquire others in the place of the maturing ones. The cash position corrected the banks were then in the need of rebuilding their liquid assets to restore the altered liquidity position. They were thus forced to sell some investments and apply rationing to their advances. To the extent that the Bank did not fully accommodate investment sales the market rate was likely to rise. On the other hand advances would follow the reduction of investments holdings by the banks, because they were related to investments and this relationship was determined in terms

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[25] N. J. Gibson: "Special Deposits as an Instrument of Monetary Policy." Manchester School of Economic and Social Studies, 1964, p.256.

[26] R. L. Crouch: "Special Deposits and the British Monetary Mechanism." Economic Studies, 1970, p.15.

of risk and return. It might not, however, have been constant for different levels of return and certainly it was not independent of developments in the costly element of the banks' liabilities side, namely deposit accounts, in relation to current accounts.

These adjustments to a new equilibrium position can be illustrated by considering the balance sheet of a bank in hypothetical examples. Let Balance Sheet A represent the initial equilibrium position of the bank in question.

Assets		A	Liabilities
Cash	800	Current Accounts	6800
Liquid Assets	2200	Deposit Accounts	3200
Advances	4000		10000
Investments	3000		
	10000		

It is assumed that the bank maintains the minimum cash (800/10000). After the call of SDs to the amount of 100 and the ensuing adjustment of the bank, the new equilbrium might be as in Balance Sheet B.

Assets		B	Liabilities
Cash	795	Current Accounts	6745
Liquid Assets	2188	Deposit Accounts	3200
SDs	100		9945
Advances	3947		
Investments	2915		
	9945		

The bank reduced its investments and advances to replenish the initial loss of liquid assets. Related to this reduction is the fall in current accounts in this bank. For the moment we assume that deposit accounts were left unaltered, while the bank managed to restore the minimum cash and liquid assets ratios. The resulting total of deposits is smaller than the one with which the bank started. This situation, however, is not likely to remain unchanged. The sales by the banks of investments results in a rise in the market rate to a greater or smaller extent depending on the degree of intervention of the Bank of England. Sooner or later there will be a rise in the Bank Rate, the rate offered on deposit accounts, and other rates. People will be induced to economise on money and an inflow of money in deposit accounts could be confidently expected to happen. The final equilibrium could be as in Balance Sheet C .

Assets		C	Liabilities	
Cash	802		Current Accounts	6720
Liquid Assets	2205		Deposit Accounts	3300
SDs	100			10020
Advances	3975			
Investments	2938			
	10020			

The fact that in stage C liquid assets are greater than at the start need not mean that short-term rates have fallen. It is simply associated with a higher supply of liquid assets by the bank (e.g. money at call and short notice) made possible by the inflow of deposits in deposit accounts and the cash which will be available to the bank after any intervention of the Bank of England. Also the increase in deposit accounts does not represent the inflow of the public's money only, but is partly created by the bank and reflects its secondary expansion of advances and investments and the public's preference for deposit accounts relative to current accounts.

We note that in the new equilibrium position an increase in special deposits is associated with an increase in deposit accounts and total deposits and this is precisely what has been observed in practice and made some economists conclude that the SDs instrument was ineffective. But advances, investments, current accounts and the money supply were reduced and in this respect SDs were successful. Indeed there is some empirical evidence showing that advances were negatively affected by special deposits. R. T. Coghlan, using quarterly data from 1955.3 to 1970.4 and first differences of the variables, performed a regression of advances on a number of variables including special deposits.[27] The results were

$$\Delta A_t = \underset{(8.0)}{120.3} - \underset{(2.2)}{0.62}\, \Delta S_{t-2} - \underset{(1.6)}{171}\, \Delta B_t - \underset{(5.4)}{99.5}\, \Delta P_t + \underset{(1.4)}{6.9}\, \Delta I_t$$

$$R^2 = 0.70 \qquad DW = 2.17$$

where
- A_t: advances seasonally adjusted
- B_t: Bank Rate
- S_t: Special Deposits
- I_t: Index of Industrial Production
- P_t: Price expectations (difference between equity and bond rate)

and indeed show a negative and significant impact of SDs on advances with a lag of two quarters.

Having demonstrated how SDs could act as a brake to the expansion of credit and the money supply, we now turn to examine in

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[27] R. T. Coghlan: "Special Deposits and Bank Advances." Bankers' Magazine, September 1973, pp.107-8.

brief the role of the Bank of England in this process. The adjustment of the banks to the new equilibrium could be eased by the Bank's intervention to accommodate the pressure that was likely to be put on the market for debts. The intervention could take the form (a) of the Bank buying through its agent in the market bills from the banks or the discount houses, or it could take the form of a reduction of the issue of Treasury bills to be allotted at the Treasury bills tender. This provided the market with the additional cash required by the banks and prevented the latter from pushing short term interest rates up too much by calling in money at call and short notice or trying to sell bills. At the same time the Bank would take up internal 'tap' Treasury bills to the extent of underallotment. In this way SDs were operating towards substituting the possibly higher rise in rates and provided finance to the Exchequer in a way that circumvented the market. (b) of the Bank being also prepared to buy to a smaller or greater extent Government stocks of more than a year to maturity. The second step of the process, as we have seen, was the sale of investments by the banks which tried to build up their liquid assets portfolio and raise money from outside sources. This movement was the one desired by the authorities who in this way passed to the banks the task of disposing to the public of medium and long term Government stocks and thus incurring the losses this entailed. Nevertheless in actual cases the banks were not left unhelped in this function because the official view prior to the introduction of competition and credit control measures was that the main purpose of debt management was the maintenance of such market conditions as would maximise present and future demand for government debt.[28] Thus there was a certain degree

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[28] See Bank of England: _op.cit.,_ 1966, pp.146-7.

of intervention when prices were falling rapidly as the banks were seeking to dispose of government securities, because it was thought that much lower prices would result in a decline rather than an increase in future demand and sharp movements in the prices would undermine confidence in the market.

The new system of control instituted in September 1971 has changed the scope of the Bank's intervention by limiting it to the stocks with one year or less to run to maturity. The Bank is now concerned with monetary aggregates rather than interest rate stabilisation, thus permitting free functioning of the price mechanism in the allocation of credit.[29] On the other hand the situation has not changed much regarding the Bank's dealings in Treasury bills; it will be prepared to meet sales of Treasury bills but free to determine the price at which this will be done. With these modifications established, SDs were not abolished but are still in existence and apply to the whole of the banking system. One would, of course, have thought that control through reserve requirements would be adequate as long as any high level of interest rates was accepted, and the behaviour of the public could be predicted. But it might be the case that very high rates are not socially acceptable so that SDs can be potentially useful. Implicit in this is the view that the operation of the SDs scheme implies smaller increase in interest rates than control exerted through the monetary base. After the introduction of the new credit control measures SDs were not used until the last quarter of 1972 and their future usefulness requires understanding and the testing of their effectiveness during the period of our study.

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[29] See Bank of England: "Key Issues in Monetary and Credit Policy." Bank of England Quarterly Bulletin, 1971, p.196.

The preceding analysis has demonstrated that it is not appropriate to blend SDs together with the sum of currency circulation with the public and banks' reserves (defined for our purposes as the monetary base) since they are functioning in a different way and the direction of influence on the money supply is the opposite of that of the monetary base. Consequently we include SDs and the monetary base as separate arguments of the function with the expected signs of the money supply derivatives with respect to them being negative for SDs and positive for the monetary base, i.e.

$$\frac{\partial M_t}{\partial S_t^D} < 0 \qquad\qquad \frac{\partial M_t}{\partial B_t} > 0$$

The usual textbook exposition of money supply theory assumes that monetary authorities can control in a fairly precise way the monetary base so that they are able to offset predicted movements in the individual elements of the monetary multiplier and thus exercise control on the growth of money. This makes possible an active role on the part of monetary authorities and the term control of money emphasises this aspect. However, there has been a marked divergence between what theory suggests and how monetary policy is actually implemented. In reality the monetary base emerges as a result of other actions, especially the management of national debt and the financing of government deficits so that it is considered as a passive outcome of debt management policy rather than a policy instrument.

We shall now proceed to examine more closely the factors which determine the changes of the monetary base and see why some of them are outside the control of the monetary authorities. This can be achieved by considering how the borrowing requirement of the public sector is met. The public sector is defined as[30] (i) Central Government, including the National Insurance Funds, the Exchange Equalisation Account, the National Savings Bank Fund and the Issue Department of the Bank of England; (ii) Local Authorities including such local bodies as water boards, harbour boards and passenger transport authorities; (iii) Nationalised industries and other public corporations.

The borrowing requirement can arise because of (a) an excess of public sector spending over receipts F^1, (b) a pay off requirement for government stocks reaching maturity F^2, and (c) a net inflow of foreign currency in the Exchange Equalisation Account GF. The funds required for this purpose come from (a) sales of marketable debt (government stocks of different maturities) to the public or overseas holders MP, (b) sales of non-marketable debt (National Saving Certificates, Defence Bonds, Deposits with the National Savings Bank, etc.) to the public NP, (c) sales of marketable debt to the banking system MB, and (d) changes in the total of the monetary base plus SDs, $\Delta B + \Delta S^D$. There is an accounting identity between the borrowing requirement of the public sector and the way it is financed.[31]

$$F^1 + F^2 + GF = MP + NP + MB + \Delta S^D + \Delta B \qquad (5.13)$$

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[30] For definition see notes and definitions following flow of funds tables in Bank of England Quarterly Bulletin.

[31] Cf. C. Goodhart: "Analysis of the Determination of the Stock of Money." in Essays in Modern Economics. The Proceedings of University Teachers of Economics: Aberystwyth, 1972, pp.243-261.

It can be seen from the above identity that ΔB is the residual finance requirement after the public, overseas sector and the banking system have taken up interest bearing government debt according to the balance they wished to maintain or the banks have been forced to finance a part of the borrowing requirement through SDs calls. The increase is allocated between currency circulation with the public and banks' cash reserves since the public deposits part of its higher cash holdings with the banks. A subsequent step of the process will be a new take-up of government debt by banks as well as an expansion of advances and the money supply because of the increased cash reserves which are now available to the banks. To the extent that Government is not successful in financing its borrowing requirement through sales of government debt to the public, overseas sector or the banking system and there is an increase in the monetary base, there should be most probably a corresponding increase of Government securities held by the Bank of England, Issue Department in exchange of the notes issued and an increase of coins in circulation.

Concerning the allocation of Government debt between the public and the banking system, it is evident from the above discussion how important the role of the latter is in providing finance to the public sector when the public seems reluctant to take up large quantities of debt. On the other hand since this is associated with increased bank assets and money supply it can be a threat to official objectives. As the Bank of England states:

> *"One of the difficulties that has confronted the authorities in their efforts to further national economic policy through control of the banking sector is that restraint of bank lending has often seemed appropriate at times when it has been difficult to sell large quantities of government debt to the private*

sector. The consequent reliance on the banking sector for residual borrowing generates the liquidity with which the banks could frustrate official objectives. Particularly in some of the earlier years of this period the banks had such large holdings of liquid assets that, even when conditions allowed for large sales of debt to the public, it was hardly possible to put severe pressure on their liquidity position. Such excess liquidity can be sterilised by requesting the banks to place Special Deposits with the Bank of England, which are not counted as liquid assets for the purpose of meeting the required ratio."[32]

At this point the role of Special Deposits comes naturally into consideration and it is easily realised that SDs are an additional source of financing of the borrowing requirement but unlike the other contribution made by the banking system, they are a compulsory form of financing.

Changes in the monetary base are thus determined by the elements of identity and in order to find out whether the monetary base can be controlled at will by the monetary authorities attention should be focused on those elements. This has been done by C. Goodhart[33] who suggested that because certain flows in (5.13) might require a large amount of financing and are outside the control of the authorities, it is impossible to control the monetary base and consequently the money supply without putting high pressure on interest rates, a result of the Bank's attempt to counteract these flows by operations in marketable debt. The above flows are (a) the public sector deficit F^1 which is inflexible in the short run and its

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[32] Bank of England: "The U.K. Banking Sector 1952-67." Bank of England Quarterly Bulletin, 1969, p.179.

[33] C. Goodhart: *op.cit.*, 1972.

implications for monetary expansion are sometimes banished by other considerations, (b) Government stocks reaching maturity F^2 which need to be refinanced and reflect past fiscal policy decisions. In addition two other flows which respond perversely to interest rate changes tend to frustrate any attempt at controlling the base. These are (a) flow of funds NP in non-marketable debt; the rates offered on this have been in general less competitive than the rates offered in other financial institutions, so that the funds flowing in it have been observed to move inversely with interest rates, (b) capital inflows from abroad which add to the net inflow of foreign currency GF which has to be financed.

The above discussion does not preclude the monetary base as a determining factor in the money supply equation. It rather indicates that control of the money stock by the monetary authorities might be unachievable without the co-operation of the Government.

<u>The ratio of currency to current accounts</u>. The ratio describes the behaviour of the public, which has some bearing on the creation of money. While currency circulation is determined only by the demand for it (always within the limit imposed by the supply of the monetary base), current accounts are determined both by demand and supply considerations and the ratio of currency to current accounts impinges upon the supply of current accounts by the banking system. To state this explicitly, the demand for currency by the public relative to current accounts influences banks' reserves and their ability to create new current accounts and increase the amount of money supplied. The ratio, which is a part of the multiplier connecting the monetary base and the

money supply, has not been stable over time but has fluctuated considerably in the short-run as can be seen from chart 5.A where the graph of the ratio is presented. Thus the assumption stating that it can be considered as a constant is not very realistic. It simplifies, however, the picture of the money supply process and justifies the assertion that control of the monetary base is a necessary and sufficient condition for the control of the money supply - the so called 'orthodox' position.[34] A changing ratio cannot be reconciled with this proposition unless one finds a relationship to predict movements of the ratio and the arguments of this relation are exogenous to the process studied.[35] A more direct alternative that can be followed easily is to try to predict the ratio from its own history. The method was applied by N. J. Gibson and D. R. Thom[36] who estimated the relationship of the ratio of currency to the broadly defined money supply to its own past values. They used monthly data from 1955(7) to 1969(12), the lag being extended six months back. Of the set of lagged values of the ratio only the first two were found significant in explaining the current value. Doubt was expressed on the utility of the equation when greater interest rate variability will be allowed after the implementation of the September 1971 measures.

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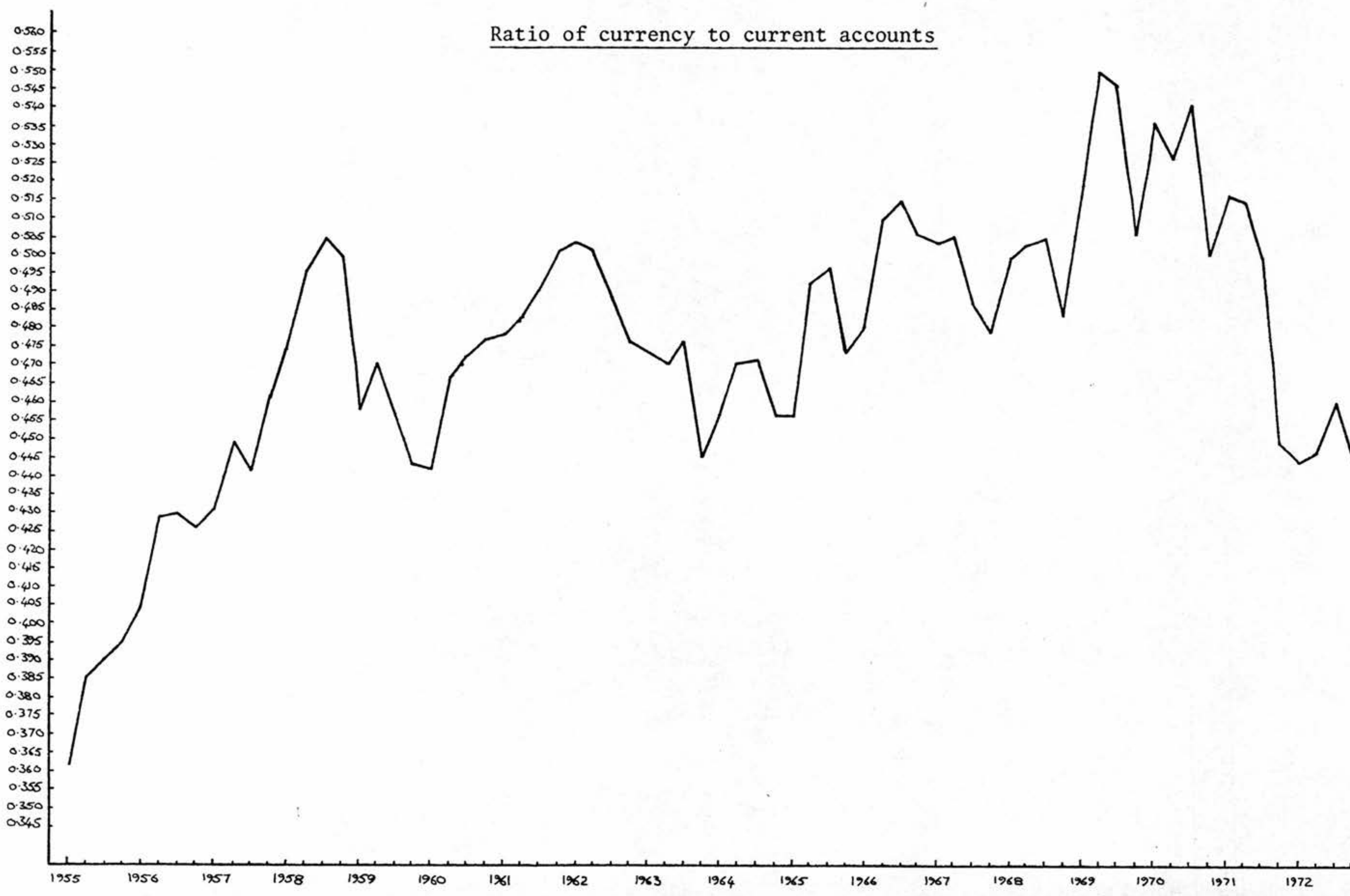
[34] See for instance W. T. Newlyn: Theory of Money, Oxford 1971, Chapt.II pp.19-41 where the author recognises only a trend and seasonal fluctuations to the ratio although the latter refers to the ratio of currency to total deposits and the author admits that *"since it is the result of rough adjustments to an imprecise concept of mere convenience, it is not so rigid as the banks' cash ratio."* R. L. Crouch has given some examples where changes in this ratio pertaining to the broad definition of money, dominated the monetary process. See R. L. Crouch "Money Supply Theory and the United Kingdom's Monetary Contraction, 1954-56." Bulletin of the Oxford University Institute of Economics and Statistics, 1968, pp.149-50.

[35] Cf. Crouch's concern about the exogeneity to the monetary mechanism of the arguments in the relative demand function for currency, Crouch op. cit., 1968, p.151. Crouch is one of the proponents of the 'orthodox' position.

[36] N. J. Gibson and D. R. Thom: "Can the Money Stock Be Controlled?" Bankers' Magazine, 1971, p.208.

CHART 5.A

Ratio of currency to current accounts



Unrelated to the method used to predict the behaviour of the public, the effect of policy actions by the monetary authorities crucially depends on this ratio. For example if the value of the ratio falls between two points in time then the increase in the money supply resulting from an increase in the monetary base will be greater than it otherwise would be. And a higher ratio, say at a downturn in economic activity, would mean that to expand money supply the monetary base has to be increased at a comparatively higher rate.

The orthodox approach to study the behaviour of the public is to specify the factors that determine the ratio of currency to current accounts and examine the direction of their influence. These are: (a) interest rates. They might have an impact on the ratio, if higher rates, which induce switching from money to financial or real assets are accompanied by payments by cheque to a greater extent than payment by currency. Then a positive relationship can be expected between the interest rate and the ratio and this simply means that current accounts are reduced relatively more than currency holdings. Such a result was found by R. L. Teigen[37] for the U.S. with the yield on treasury bills and another composite yield (weighted average of yields on deposit accounts, savings and loan association shares and mutual savings banks deposits) used as the interest rates variables. Similarly a positive although not significant coefficient was reported for Germany.[38] In both studies the ratio of currency to demand deposits was the dependent variable. The conclusions differ when the ratio of currency to total deposits or to the broad money supply is

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[37] See R. L. Teigen: "An Aggregated Quarterly Model of the U.S. Monetary Sector 1953-1964" in Targets and Indicators of Monetary Policy ed. by K. Brunner, San Francisco, Chandler Publishing Company, 1969, pp.175-218.

[38] See V. H. Mattfeldt: "Zum Verhaltnis verschiedener finanzieller Aktiva im Portefeuille inlandischer Nichtbanken der BRD." Jahrbücher für Nationalökonomie und Statistik, 1972/73, pp.237-244.

used instead. The expected increase in deposit accounts is likely to outweigh the positive effect on the ratio of a change in interest rates and the coefficient of the interest rate variable will be negative.[39] (b) real national product or the ratio of this to real permanent national product. Permanent national product is defined as a weighted average of past levels of national product (*ad infinitum*) with the weights declining geometrically and summing to unity. In this instance it is used as a deflator so that the ratio of national product to it may be thought as being a better indicator of short run income movements. The effects on the ratio of an increase in income can be examined in the same way as the interest rate effect, namely it will be negative if current accounts are used to a greater extent than currency to finance the additional transactions associated with the higher level of expenditure. This means that although the demand for both currency and current accounts rises, the demand for the latter rises proportionally more than the demand for currency so that their ratio falls. Again the empirical findings in the studies mentioned previously are in line with this conclusion.[40] (c) other factors such as: the ratio of the price level of goods normally purchased with currency (e.g. food) to the price level of goods purchased with cheques. In a developed economy currency is held primarily to facilitate transactions of a relatively small value whereas current accounts to effect larger transfers; the cost of current accounts - the banks' service charges on them - and the level of financial sophistication of the general public are mentioned as well.

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[39] For empirical results confirming this conclusion see V. H. Mattfeldt, *op.cit.*, p.239 and G. Macesich, "Demand for Currency and Taxation in Canada." Southern Economic Journal, 1962-63, p.35.

[40] Teigen, *op.cit.*, p.181, and Mattfeldt, *op.cit.*, p.239.

Cagan and Macesich considered the proportion of income taxed as relevant in accounting for the variation in the ratio of currency to total money supply during wartime.[41] The reason is that it was easier to evade income tax by making more transactions with currency and not reporting the receipts. Their empirical findings confirmed this. Also Crouch[42] has suggested that for the years 1954-56 in which he studied the behaviour of the currency ratio for the U.K. the ratio of armed service personnel to the total population might be an important factor because *"military service disrupts established banking connections"* so that a rise of the ratio might be expected.

We denote all possible influences mentioned under (c) by O but we do not include them explicitly in the macroeconomic model we are building. Instead possible autocorrelation will be allowed in the residuals of the money supply function, which might partly be due to these factors. Since there are no results for the U.K. indicating the importance of the first two factors examined we performed some exploratory regressions (OLS) to find the significance of the effects discussed above to the ratio of currency to current accounts. The results are:

$$(C_t^P/D_t) = \underset{(13.72)}{0.420077} - \underset{(-\ 0.93)}{0.00000758615}\ Y_t + \underset{(3.02)}{0.0175358}\ R_t + \underset{(1.13)}{0.18305}\ S_t^2 +$$

$$+ \underset{(0.54)}{0.00505017}\ S_t - \underset{(-\ 0.53)}{0.00543451}\ S_t^2 \qquad (5.14)$$

$$R^2 = 0.3938 \qquad DW = 0.2756$$

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[41] P. Cagan: "The Demand for Currency Relative to Total Money Supply." Journal of Political Economy, 1958, pp.303-328, and Macesich, *op.cit.*

[42] Crouch, *op.cit.*, 1968.

$$(C^P/D)_t = \underset{(9.75)}{0.628946} - \underset{(-\ 4.07)}{0.00003398}\, Y_t + \underset{(3.20)}{0.0180752}\, R_t + \underset{(5.14)}{0.0182043}\, S_t^2 +$$

$$+ \underset{(4.01)}{0.0144222}\, S_t^3 + \underset{(2.31)}{0.0120168}\, S_t^4 + u_t \qquad (5.15)$$

$$u_t = 0.89638\, u_{t-1} + e_t$$

(t values in parenthesis)

$R^2 = 0.8598 \qquad DW = 2.2124$

where C^P/D is the ratio of currency to current accounts, Y real gross domestic product, R the yield on consols and S^2, S^3, S^4 seasonal dummies. The coefficients of the income and interest rate variable have the expected sign although the significance of income is strengthened when autocorrelation is allowed in the residuals. Seasonal factors are important as well. The evidence thus supports the assumption that income and interest rate movements influence the ratio of currency to current accounts in the direction suggested above.

The ratio of banks' cash reserves to current accounts. It was argued by Crouch that the ratio of cash reserves to total deposits was not interest sensitive because banks did not hold excess reserves over and above the 8% minimum cash ratio.[43] The same assumption was made for the ratio of banks' reserves to current accounts without further discussion. While the first assumption is true and the conventional cash ratio was always kept close to 8% by Clearing Banks,[44] the second assumption can be questioned. The distribution of the total of deposits

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[43] See R. L. Crouch, *op.cit.*, 1967, p.402.

[44] We note that only Clearing Banks were subject to this minimum ratio. Newlyn has estimated that between 1959 and 1969, the ratio for the whole of the banking system has varied between 0.094 and 0.105, see Newlyn, *op.cit.*, p.214.

between current accounts and deposit accounts was influenced by the rate of return of investments and advances and the rate paid to deposit accounts. Generally in period of high interest rates banks were induced to expand their investments and advances and consequently current accounts while they observed an increase in their deposit accounts. Then any elasticity of the ratio of their cash reserves to current accounts with respect to the market rate could be justified, (a) by the existence of a margin in their liquidity position which could be converted very easily and with low transaction costs into cash reserves by the banks. The minimum liquidity ratio was not observed so closely as the cash ratio so that any existing margin could play the role of excess reserves given the ease with which banks could convert liquid assets into cash. Then a proportionally greater expansion of current accounts would lead to a negative relationship between the market rate and the ratio of banks' reserves to current accounts. There have been examples in which a decline in the banks' liquid assets in a particular period has been associated with an increase in the volume of total deposits and this in a different context has been used to question the possibility of using liquid assets for the control of deposits. (b) by the lag that might intervene between changes in the interest rate receivable (on investments) and changes in interest rate payable (on deposits).[45] (c) by the fact that the rate charged by banks on advances exhibited larger (compared to the rate offered to deposit accounts) dispersion around the norm dictated by the Bank Rate. Of course there were cases where practically no variation at all was observed, e.g. lending on a short term basis to

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[45] Cf. The Banker, 1958, p.111 and The Banker, 1959, p.124.

regular commercial customers financing inventories. Also lower variation was present when the demand for credit was low. In this case the banks would accept a lower range of variation in order to expand advances to more reluctant customers. But generally the rate on new advances varied considerably depending on a lot of factors such as the credit worthiness of customers, the asset structure of companies, the period, regularity and type of loan, government directives for investment, etc. These variations could not be unlimited because by overcharging customers the banks would face the danger of financial intermediaries taking away the market whereas undercharging could lead to the appearance of the so called disintermediation, i.e. companies borrowed money from the banks and lent it to other customers at higher rates.

The aforementioned factors suggest that the ratio of banks' cash reserves to current accounts was likely to show a negative response to changes in the market rate. As far as the influence of the deposit accounts rate is concerned, a rise in this rate would be followed on the part of the public by a switching from current accounts to deposit accounts which became more attractive with the rise of their yield; and on the part of the banks by an attempt to increase their cash reserves and an enhancement of their willingness to incur higher risk and higher return implied by less liquid assets, because of the increase in their costly element, namely deposit accounts. These reactions lead us to expect that a rise in the rate for deposit accounts will have a positive effect on the ratio *ceteris paribus*.

To see whether these hypotheses come true in reality we again perform some exploratory regressions. The Bank Rate replaces the rate offered on deposit accounts because for most part of the period of our study the former exceeded the latter by a constant. This is not true for the last five quarters after the September 1971 measures. One of the authorities' intentions was to strengthen competition between the banks in order to achieve more efficient allocation of credit and the rate offered on deposit accounts is no longer in a fixed relation to the Bank Rate but it is allowed to be determined competitively. How good a proxy Bank Rate will be in subsequent years given the variation in rates for deposit accounts, remains to be examined. The regression results are presented below:

$$(R^B/D_t) = \underset{(15.66)}{0.143853} - \underset{(-\ 2.47)}{0.00349697}\ R_t + \underset{(4.46)}{0.00830097}\ R^D_t + \underset{(1.03)}{0.0050177}\ S^2_t$$

$$+ \underset{(0.46)}{0.00220858}\ S^3_t - \underset{(-\ 0.47)}{0.00226392}\ S^4_t \tag{5.16}$$

$$R^2 = 0.2506 \qquad DW = 0.3040$$

$$(R^B/D)_t = \underset{(2.10)}{0.104849} + \underset{(0.78)}{0.00296941}\ R_t + \underset{(1.82)}{0.00284568}\ R^D_t + \underset{(2.00)}{0.00325739}\ S^2_t +$$

$$+ \underset{(1.31)}{0.00236882}\ S^3_t - \underset{(-\ 0.35)}{0.000527032}\ S^4_t + u_t \tag{5.17}$$

$$u_t = 0.98149\ u_{t-1} + e_t$$

$$R^2 = 0.8133 \qquad DW = 2.2301$$

where R^B/D is the ratio of banks' cash reserves to current accounts and R^D is the Bank Rate. The negative impact of the market rate and positive of the Bank Rate is evidenced from these results. However, when autocorrelation is allowed in the residual, the coefficient of the interest rate changes sign and becomes insignificant.

The money supply function. Having discussed so far the behaviour of different economic agents in the process of money supply creation and having examined the factors affecting the elements in the money multiplier we can write a general money supply function

$$M_t = f(B_t, S_t^D, Y_t, R_t, O_t, R_t^D) \quad (5.18)$$

In the empirical investigation factors included in O_t will be omitted and the linear form

$$M_t = a_2 B_t + b_2 S_t^D + c_2 Y_t + d_2 B_t + e_2 B_t^D \quad (5.19)$$

assumed to approximate (5.18) will be tested. Autocorrelation will be allowed in the residuals to account for other factors not appearing explicitly or possible nonlinearities in the function.

Regarding the signs of the coefficients, a_2 should be positive and b negative as already noted. For the signs of the others we must look at the regression results (5.14), (5.15), (5.16) and (5.17) in conjunction with the sign of the derivatives (5.7) and (5.8). Thus for example real income has a negative sign in (5.14) and the derivative of the money supply with respect to the ratio of currency to current accounts is negative so that we can expect c_2 to be positive. The sign of d_2 is ambiguous because from (5.16) and (5.8) we get a positive influence and from (5.14) and (5.7) a negative influence so that the direction of the total influence can not be determined *a priori* and d_2 can be positive, negative or zero. This is a conclusion which markedly contrasts with what is found in the literature where the money supply was assumed to be positively, if at all, related to the interest rate. In fact the presence of autocorrelation in the residuals of the equations explaining the ratios of currency to current accounts and of banks' cash reserves to current accounts can give us a hint about the prevalence of the negative sign, since the interest rate coefficient turns out to be insignificant in (5.17).

We shall end this chapter by emphasising two major conclusions which emerged from our analysis of the determinants of money supply.

(a) Specials deposits are expected to have a contractive influence on the narrow money supply and should not be blended together with the sum of currency circulation with the public and cash reserves of the banking system which form the basis of a multiple expansion of the money supply.

(b) The effect that the interest rate has on the ratio of currency to current accounts and the money supply may outweigh its effect on the ratio of banks' cash reserves to current accounts and the money supply so that the combined effect on the money supply may be positive, negative or zero and not necessarily positive or zero as accepted theory suggests.

CHAPTER 6

CHAPTER 6

THE PRICE EQUATION*

We have seen in previous sections that recent theoretical work within the IS-LM model indicated the significance of price expectations for sorting the real from the nominal rate of interest and suggested the extension of the model towards including a price equation, while real sector variables are cast in real terms. Price expectations were either not connected with the price equation or indirectly or directly connected with it. Thus, for example, Sargent[1] utilised the following price and price expectations equations

$$\frac{P_t - P_{t-1}}{P_{t-1}} = \gamma \, (Y_{t-1} - Y^F) \tag{6.1}$$

and

$$\pi_t = e\left(\frac{P_t - P_{t-1}}{P_{t-1}}\right) \tag{6.2}$$

where P is the price level, Y is real income, Y^F its full-employment level and π are price expectations.

Unfortunately in some cases the particular way of connecting price expectations with the price equation in the context of the IS-LM model, led to forms of equations not offering themselves to estimation.[2]

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* An article containing material from this chapter and entitled "Estimating Full-Employment Output for the United Kingdom" has been accepted for publication in Applied Economics.

[1] T. J. Sargent, *op.cit.*, 1971.

[2] D. Laidler, *op.cit.*, 1973a.

In others estimable forms could be obtained in principle but certain simplifying assumptions (made in order to render the model's manipulation manageable), such as the one of a constant over time full-employment output, would detract much from the realism of the results.

Here, we shall attempt to develop a price determination equation in which some simplifications present in the literature of the extended IS-LM model, will be removed and price expectations will be incorporated in the equation in a way which yields consistent results (where consistency refers to coefficient estimates which are admissible in terms of sign and size) with the ones obtained from their use in another part of the model. Our discussion will be organised as follows: first we shall examine the price equation and its arguments and review some empirical work for the U.K. and second we shall consider price expectations and see how they were linked to the price equation and how they were estimated. Price expectations are an unobservable variable and they were related to actual price changes through a dynamic adjustment mechanism (the adaptive expectations scheme). This should be emphasised because the price equation as it appeared in the literature before and even after the Monetarists called our attention to price expectations, did not lack dynamic characteristics. On the contrary both single period lags and distributed lags were applied to the explanatory variables on different occasions. But the fact that in general, price expectations and the distinction between the real and the nominal rate of interest have been ignored in empirical work, made the dynamic specification of the price equation quite independent from these considerations.

The price equation purports to describe the inflationary process in an economy, where inflation is taken to mean a rise in the price level or, to state it differently, a depreciation of the monetary unit. The term inflation has been accompanied by a variety of characterisations (hyperinflation, suppressed inflation, repressed inflation, creeping inflation) which, however, need not concern us here.

Generally prices can be distinguished between those which are sensitive to conditions of demand for and available supply of goods and services and those which are set on the basis of costs and change when some cost element changes. The first situation is described as demand inflation and the second as cost inflation and each of them has been used by different authors and at different times as the sole explanation of price changes. In the first case it is assumed that prices are fixed in response to the state of demand relative to supply that is, if at the existing price demand relative to supply is pressing prices will be raised more than if demand is comparatively slacker. The degree of excess demand can of course be altered when either demand or supply change or when both of them change. In the second case prices move in response to changes in average costs of production, namely the costs of the factors of production which take into account their average productivity. Usually it is posited that prices are the result of adding a profit mark-up to the average costs of production or sometimes the result of taking a profit and other fixed costs mark-up as a proportion of prime costs plus an additional fixed absolute charge per unit of output.

In distinguishing between the above two types of inflation is is customary to refer to particular market situations in which each type applies. Thus in conditions of perfect competition or of some forms of imperfect competition[3] prices are likely to be responsive to excess demand. On the other hand in oligopolistic or monopolistic markets it is likely that prices are set on a prime cost plus constant percentage mark-up basis. Similarly the buyer's market sometimes determines that prices will rise only with cost increases (e.g. some government purchases). Since all market situations subsist together in the economy the general price level can be taken to be subject both to the influence of excess demand and to the pressure of cost changes. Indeed, a lot of work on the price equation has examined the relative importance of demand and cost influences by estimated 'mixed inflation' equations.

Concerning the influence of excess demand most of the empirical work has used the unemployment rate as a proxy for excess demand in the labour market. In some studies however, it has been recognised that the excess demand variable should relate to the product market, the excess demand for labour being a derived demand. These studies have used a distributed lag relationship between excess demand in the labour market and excess demand in the product market in order to derive expressions for the latter in terms of the former. An alternative proxy for excess demand employed in theoretical studies

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[3] See A. C. Enthoven: "Monetary Disequilibria and the Dynamics of Inflation." Economic Journal, 1956, pp.256-70. The forms of imperfect competition referred to, are *"profit-maximising monopoly and monopolistic competition where it exists on only one side of the market, with perfect competition on the other so that sellers or buyers are confronted with determinate and differentiable demand and supply functions."* p.267.

is the degree to which actual output differs from its full-employment level.[4] According to the prevailing view it is the rate of change of prices (or the absolute price change) which is related to the excess demand variable and not the price level itself because the latter, as McCallum has noted *"would imply that a non-zero level of excess demand which remained constant period after period would not induce, ceteris paribus, a change in the price level. The 'Law of Supply and Demand' would not be in force"*.[5] In mixed inflation models the qualification should be added to the above remark that cost elements remained constant as well.

Concerning the influence of cost on price inflation some studies use the unit prime costs (consisting of unit labour costs and unit material costs) multiplied by one plus the profit and fixed costs mark-up, and some use variables that are either part of prime costs or are in some way related to prime costs. For the mark-up of prices over prime costs, most studies seem to assume that although it can vary as an absolute size (being a multiple of variable prime costs), as a percentage it is fixed. However, it may be that, even as a percentage, it is sensitive to changes in demand, being larger when demand pressure is high and smaller when demand pressure slackens. This is related to the interaction of demand with cost inflation of which mention will be made later when we discuss the form of our equation.

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[4] See T. J. Sargent, *op.cit.*, 1971. T. J. Sargent, *op.cit.*, 1972. D. Laidler, *op.cit.*, 1973a. D. Laidler, *op.cit.*, 1973b. B. T. McCallum, *op.cit.*,1973. D. W. Peterson - E. M. Lerner and E. J. Lusk: "The Response of Prices and Income to Monetary Policy: An Analysis Based upon a Differential Phillips Curve." Journal of Political Economy, 1971, pp.57-66.

[5] B. T. McCallum: "The Effect of Demand on Prices in British Manufacturing: Another View." Review of Economic Studies, 1970, p.149.

In what follows a brief overview will be offered of some pieces of empirical work and particular aspects of it will be emphasised. The studies at which we look are not by any means the whole of the work done for the United Kingdom regarding the price equation, but they cover all forms of the price equation, i.e. having as explanatory variables cost variables only, excess demand variables only and both cost and excess demand variables. Moreover, the dynamic specification of some of them will be indicated.

Dow[6] examined the effect of changes in cost elements on final prices as expressed by the price index of total final expenditure at factor cost. He did not estimate explicitly any price equation and the basis of his study was the identity showing that the price level is obtained by multiplying unit variable costs of production by one plus the profit mark-up. The variables that he found important regarding variable costs, were money wages per head, the cost of imported materials and output per head. As far as the influence of demand is concerned Dow noted that *"examination of the actual behaviour of labour costs and of profits suggests that demand is capable of modifying the responses of the system at each stage; but it seems unlikely that such modifications would completely offset strong cost impulses."*[7]

Klein and Ball estimated a price equation as a part of a larger model of the United Kingdom by using an estimation method taking

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[6] J. C. R. Dow: "A Study of Cost and Price Changes in the United Kingdom, 1946-54." Oxford Economic Papers, 1956, pp.252-301.

[7] J. C. R. Dow: *op.cit.*, p.252.

into account the interdependence of the variables in the model.[8] The equation is of the mark-up of prices over costs type and one of its main characteristics is that it relates the level of the price variable (consumer price index) to the level of the relevant cost variables. The latter are wage earnings, import prices and the ratio of indirect taxes less subsidies to consumer expenditures. Since it was thought that the constant of the equation, representing a measure of the mark-up over costs, varies cyclically, the authors experimented with a productivity variable which may moderate this variation. The coefficient of the new term turned out to be insignificant and with the wrong sign.

Dicks-Mireaux[9] was concerned with changes in both the level of prices and wages by estimating price and wage equations for the U.K. and for the period 1946-59. The price equation included only cost elements as arguments. The dependent variable was the annual percentage change between twelve-month averages of an index of final prices and the independent variables were also annual percentage changes in wages and salaries per person employed, import prices and output per man. Though the pressure of demand was not included explicitly in the equation, it was assumed to determine wages and thus simultaneously prices.

Solow[10] tried to combine a mixed inflation equation with price expectations. The latter will have our attention later on so it suffices to examine the cost and demand elements of the equation.

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[8] L. R. Klein and R. J. Ball: "Some Econometrics of the Determination of Absolute Prices and Wages." Economic Journal, 1959, pp.465-82. The complete model was developed in L. R. Klein - R. J. Ball - A. Hazlewood and P. Vandome: An Econometric Model of the United Kingdom. Basil Blackwell, Oxford, 1961.

[9] L. A. Dicks-Mireaux: "The Interrelationship between Cost and Price Changes, 1946-1959." Oxford Economic Papers, 1961, pp.267-92.

[10] R. M. Solow: Price Expectations and the Behaviour of the Price Level. Manchester University Press, 1969, pp.18-32.

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[9] L. A. Dicks-Mireaux: "The Interrelationship between Cost and Price Changes, 1946-1959." Oxford Economic Papers, 1961, pp.267-92.

[10] R. M. Solow: Price Expectations and the Behaviour of the Price Level. Manchester University Press, 1969, pp.18-32.

As regards cost variables Solow made an attempt to split unit labour costs into wage rate changes and changes in unit labour requirements. But to do so, he had to assume that hours worked per man in the economy moved like hours worked per man in manufacturing industry and the results were not satisfactory. The excess demand side was represented by a measure of capacity utilisation constructed by Paish. The index was a measure of unemployment, i.e. of pressure in the labour and not the product market and its inclusion in the price equation led to an insignificant and of the wrong sign coefficient when annual data were employed and a significant and of the right sign coefficient when the price variable was a series of overlapping four-quarts changes in the price of final product.

Neild[11] was concerned with pricing behaviour in British manufacturing industry. His equation which was of a mark-up type included a variable to test the effect of excess demand on prices. His cost variables consisted of indices of unit labour costs (rather than wages and productivity per head separately) and material costs (rather than measures of parts of material costs such as import prices). His equation will be spelt out in more detail since it incorporated distributed lag functions of some variables in contrast to previous studies in which single period delays were applied to the independent variables. Price is obtained as a function of prime costs c

$$p_t = a_0 + a_1 c_t \qquad (6.3)$$

$$a_1 > 1$$

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[11] R. R. Neild: Pricing and Employment in the Trade Cycle - A Study of British Manufacturing Industry, 1950-1961. National Institute of Economic and Social Research, Occasional Paper 21, 1963.

that is, other costs and profits are calculated as a proportion of prime costs but also a part of them might be better represented by a fixed absolute charge a_0 . Prime costs are given by

$$c_t = b_1 w_t + b_2 m_t \tag{6.4}$$

where w and m are unit costs of labour and materials and b_1 and b_2 the requirements of labour and materials per unit of output. w_t and m_t were replaced by distributed geometric lags of their previous values, w_t by

$$(1 - \lambda) w_t + (1 - \lambda) w_{t-1} + (1 - \lambda)^2 w_{t-2} + \ldots$$

and m_t by

$$\mu_0 m_t + \mu_1 m_{t-1} + \mu_2(1 - \lambda) m_{t-2} + \mu_3(1 - \lambda) m_{t-3} + \ldots$$

After the appropriate transformation the equation for estimation was

$$p_t = a_0' + a_1' w_t + a_2' m_t + a_3' m_{t-1} + a_4' m_{t-2} + $$

$$+ a_5' p_{t-1} \tag{6.5}$$

with the a_i' 's being functions of the initial parameters. To this form Neild added an excess demand variable d which was the cumulative sum of an index of excess demand for labour. The reason why the variable appeared in cumulative form is that *"it seemed most reasonable to assume that a given level of excess demand would be associated with a change in price."*[12] Consider for example the simple function

$$p_t - p_{t-1} = a e_t \tag{6.6}$$

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[12] R. R. Neild, *op.cit.*, p.20.

where e is a measure of excess demand. (6.6) by repeated substitution of lagged price levels gives

$$p_t = a(e_t + e_{t-1} + e_{t-2} + \ldots) = a\, d_t \qquad (6.7)$$

i.e. the price level is a function of the cumulative sum of excess demand. The estimated price equation gave an insignificant coefficient for the demand variable and Neild argued that the pressure of demand is not important in accounting for price changes. However he added that *"it remains possible that a more complex formulation or a different indicator (of demand pressure) might lead to a different conclusion."*[13]

Neild's negative results concerning the effect of the demand variable were critisised by Rushdy and Lund,[14] who noted that the inclusion of d in the reduced form (6.5) implies a distributed lag for e with increasing weights, and attributed the unsatisfactory results to this reversal of the acceptable distributed lag form. Therefore Rushdy and Lund used changes in the price level as the dependent variable in the equation and e as the independent variable together with changes in the same cost variables as in Neild's study. They also experimented with single period delays in excess demand which was generally found an important explanatory variable so that R - L concluded that *"the level of demand cannot be dismissed as being an insignificant factor in the explanation of the price changes of manufactured goods, even after its effects on costs (wages and materials) have been accounted for."*[15]

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[13] R. R. Neild, op.cit., p.20.

[14] F. Rushdy and P. J. Lund: "The Effect of Demand on Prices in British Manufacturing Industry." Review of Economic Studies, 1967, pp.361-73.

[15] B. T. McCallum, op.cit., 1970.

Unlike Neild and Rushdy and Lund who examined the existence of both change of costs and excess demand effects on prices, McCallum[16] argued that in competitive markets price changes are brought about by excess demand alone and formulated the following equation

$$\Delta P_t = (1 - \lambda)\, a\, k_t + \lambda \Delta P_{t-1} \tag{6.8}$$

where now k is a measure of excess demand for the product market, and the partial adjustment mechanism has been applied to price changes. In order to relate the excess demand variable in the product market to the corresponding variable in the labour market McCallum assumed that the relationship between them is one which is distributed in time, with the response of the labour market beginning one period after a change occurs in the product market, i.e.

$$e_t = (1 - \mu)\, [k_{t-1} + \mu k_{t-2} + \mu^2 k_{t-3} + \ldots] \tag{6.9}$$

$$0 < \mu < 1$$

implying that

$$k_t = \frac{1}{1 - \mu} (e_{t+1} - \mu e_t) \tag{6.10}$$

so that the price equation he estimated was

$$\Delta P_t = \frac{(1 - \lambda)}{(1 - \mu)} e_{t+1} - \mu \frac{(1 - \lambda)}{(1 - \mu)} e_t + \lambda \Delta P_{t-1} + u_t$$

and the results showed the significance of excess demand in accounting for price changes.

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[16] B. T. McCallum: "The Effect of Demand on Prices in British Manufacturing: Another View.' Review of Economic Studies, 1970, pp.147-56.

A further point that merits examination in McCallum's article is his defence of Neild's dynamic specification of the price equation which included the price level lagged one period and the cumulative sum of the excess demand variable. McCallum departed from the specification

$$P_t^* = a_0 + a_1 b_1 w_t + a_1 b_2 m_t + \delta d_t \tag{6.11}$$

where the symbols are the same as above and the * denotes equilibrium value. As already noted McCallum argued that it is simple excess demand which should be related to price changes and its cumulative sum to the price level. (6.11) was combined with the partial adjustment hypothesis applied to the price level

$$P_t - P_{t-1} = (1 - \lambda)(P_t^* - P_{t-1}) \tag{6.12}$$

to give a form of equation for estimation similar to Neild's.

However there is a basic fallacy in McCallum's and Neild's assumptions. We have seen above (equation 6.6) that when excess demand is the only determinant of price changes, the price level can be obtained as a function of cumulative excess demand by expanding the distributed lag relationship implicit in (6.6). But if cost influences are at work as well (6.6) must be written as

$$P_t - P_{t-1} = a\, e_t + b(x_t - x_{t-1}) \tag{6.13}$$

or

$$P_t = \sum_{i=0}^{\infty} a\, e_{t-i} + \sum_{i-0}^{\infty} b(x_{t-i} - x_{t-1-i}) \tag{6.14}$$

where x contains cost influences, so that the price level is a function of the cumulative sum of both excess demand and cost changes and not of the level of cost. If a partial adjustment hypothesis is assumed to this formulation, i.e.

$$P^*_t - P_{t-1} = a\, e_t + b(x_t - x_{t-1}) \qquad (6.15)$$

and

$$P_t - P_{t-1} = \lambda(P^*_t - P_{t-1}) \qquad (6.16)$$

it will yield for estimation

$$P_t - P_{t-1} = \lambda a\, e_t + \lambda b(x_t - x_{t-1}) \qquad (6.17)$$

or

$$P_t = \sum_{i=0}^{\infty} a\, e_{t-i} + \sum_{i=0}^{\infty} \lambda b(x_{t-i} - x_{t-1-i}) \qquad (6.18)$$

the same as (6.14) above with the exception that its parameters are not identified. But the important point is that the lagged price level does not appear in (6.18) and this avoids imposing a distributed lag to a relationship where variables are already distributed in time *ad infinitum*.

The above remarks put an end to our cursory review of the empirical work and we now turn to examine price expectations.

Price expectations were linked to the price equation after they have been extensively used in the augmented form of the traditional Phillips Curve. The Phillips Curve is the relationship between wage inflation and unemployment and was initiated in the work of Phillips in the late fifties.[17] The general form of the augmented Phillips Curve is

$$\dot{W}_t = f(X) + a\, \dot{P}^E_t \qquad (6.19)$$

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[17] See A. W. Phillips: "The Relationship between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957." Economica, 1958, pp.283-99.

where $\dot{W}$ is the rate of change of money wage rates, $f(X)$ a function of real variables of which predominant is the unemployment rate (a proxy for excess demand for labour) and $\dot{P}^E$ is the expected (or anticipated) rate of price changes.

Since the formulation of the 'expectations hypothesis' (also called accelerationist position in the literature) by its major proponents Phelps and Friedman,[18] a similar form to (6.19) was written and tested for price inflation, i.e.

$$\dot{P}_t = f(X) + a\, \dot{P}^E_t \tag{6.20}$$

where now $\dot{P}$ is the rate of price level changes and $f(X)$ a function of the real variables which are relevant for price inflation. The expectations hypothesis states that inflation responds fully to anticipated inflation and the observed short-run trade-off between inflation and unemployment is due to unanticipated inflation. The hypothesis in terms of equation (6.20) means that the a coefficient is unity and as in the short-run the expected rate of inflation does not coincide with the actual rate of inflation we have $f(X) \neq 0$ and a non-zero short-run relationship between the inflation rate and the unemployment rate. However, in the long-run (or equilibrium, cf our remark in page 68 , chapter 3) the expected rate of inflation is equal to the actual rate so that $f(X) = 0$ and real variables are not determined by the particular rate that results. Friedman has hypothesised (for the U.S. economy) that *"the initial effects of a higher and unanticipated rate of inflation last for something like two to five years; that this initial effect then begins to be reversed; and that a full adjustment to the new rate of*

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[18] E. S. Phelps: "Phillips Curves, Expectations of Inflation and Optimal Unemployment over Time." Economica, 1967, pp.254-81 and M. Friedman: "Monetary Theory and Policy." American Economic Review, 1968, pp.1-17.

inflation takes about as long for employment as for interest rates, say, a couple of decades."[19]

An attempt has been made by Solow to test the proposition that a is unity by estimating a price equation in which f(X) was a function of excess demand and cost variables.[20] A review of this has been made above, but what we are concerned with here is the testing of the expectations hypothesis and, more important, the way in which price expectations have been measured. Solow used the adaptive expectations scheme

$$\dot{P}^E_t - \dot{P}^E_{t-1} = \theta(\dot{P}_t - \dot{P}^E_{t-1})$$

or

$$\dot{P}^E_t = \theta\dot{P}_t + (1 - \theta)\dot{P}^E_{t-1}$$

or

$$\dot{P}^E_t = \theta\dot{P}_t + (1 - \theta)\theta\,\dot{P}_{t-1} + (1 - \theta)^2\theta\,\dot{P}_{t-2} + + (1 - \theta)^3\theta\,P_{t-3} + \ldots \quad (6.21)$$

to generate expectations series for different values of the parameter θ . He then substituted these series in equation (6.20) and chose that value of θ which gave the best fitting equation. His results indicated that a is in all cases significantly below unity.

Several criticisms were directed against this approach:
(a) It has been argued that a is bound to be less than one since one of the explanatory variables in Solow's price equation is the rate of

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[19] M. Friedman, *op.cit.*, 1968, p.11.
[20] R. M. Solow, *op.cit.*

change of money wage rates which by virtue of equation (6.19) respond to expected rate of inflation, so that the expected rate of inflation term, when included in the price equation, will pick up the effects on actual inflation that were not transmitted through the labour market.[21] (b) The value of a is contingent upon the adjustment mechanism posited for price expectations. It has been shown by Saunders and Nobay in the context of equation (6.19) that by varying this mechanism one can get values of a closer to unity.[22] (c) Price expectations cannot be determined solely by the historical record as represented by the weighted past observed values of price changes, because in this way there is no separate role to be played by current economic events and this point has been strongly emphasised by Walters.[23] This last criticism could be overcome if the price equation (6.20) is tranformed to take into account the price expectations mechanism. The approach was never pursued for the price equation although it has been frequently followed in equation (6.19).[24]

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[21] See Discussion Paper by D. Laidler in Money in Britain 1959-1969. Edit. by D. R. Croome and H. G. Johnson, Oxford University Press, 1970, p.120.

[22] P. G. Saunders and A. R. Nobay: "Price Expectations, the Phillips Curve and Incomes Policy." In Incomes Policy and Inflation. Edit. by M. Parkin and M. T. Sumner, Manchester University Press, 1972, pp.237-49.

[23] A. A. Walters: "Consistent Expectations, Distributed Lags and the Quantity Theory." Economic Journal, 1971, pp.273-81.

[24] See for example M. T. Sumner: "Aggregate Demand, Price Expectations and the Phillips Curve." In Incomes Policy and Inflation, edit. by M. Parkin and M. T. Sumner, M.U.P., 1972, pp.163-81, and M. Parkin: "Some Further Results on the Rate of Change of Money Wages." In Incomes Policy and Inflation, edit. by M. Parkin and M. T. Sumner, M.U.P., 1972, pp.112-29.

Let us see the implications of transforming equation (6.20).

$$\dot{P}_t = b\, X_t + a\, \dot{P}^E_t + u_t \qquad (6.22)$$

$$a > 0$$

where for simplicity a single variable X appears in the place of f(X) ,

and
$$\dot{P}^E_t = \theta\, \dot{P}_t + (1 - \theta)\, \dot{P}^E_{t-1} \qquad (6.23)$$

$$0 < \theta \leqslant 1$$

reduce to
$$\dot{P}_t = c_1\, X_t + c_2\, X_{t-1} + c_3\, \dot{P}_{t-1} + e_t \qquad (6.24)$$

where

$$c_1 = \frac{b}{1 - a\theta}$$

$$c_2 = \frac{-\, b(1 - \theta)}{1 - a\theta}$$

$$c_3 = \frac{1 - \theta}{1 - a\theta}$$

and
$$e_t = \frac{1}{1 - a\theta}\, u_t - \frac{(1 - \theta)}{1 - a\theta}\, u_{t-1}$$

If we assume the classical properties for the error term in (6.24), i.e.

$$E(e_t) = 0 \qquad \text{and} \qquad E(e_t e_t') = \sigma^2$$

which is equivalent to assuming that the disturbance term in (6.22) follows a first order autoregressive process

$$u_t = (1 - \theta)\, u_{t-1} + \varepsilon_t$$

then (6.24), in order to be an acceptable equation, must satisfy the stability condition

$$\left|\frac{1 - \theta}{1 - a\theta}\right| < 1 \qquad \text{or} \qquad 1 < a < \frac{1}{\theta}$$

and this contradicts the expectations hypothesis. The important conclusion can be drawn that (6.22) and (6.23) are not the appropriate framework to account for the interaction between price expectations and the short-run determinants of price changes since (6.24) excludes *a priori* any test of the proposition on which its construction was based.

The conclusion is the same if we recognise that successive error terms e_t are a first order moving average of u and posit, as for example Parkin did for the wage equation,[25] that

$$e_t = \rho \, e_{t-1} + z_t \tag{6.25}$$

where $E(z_t) = 0$ and $E(z_t z_t) = \sigma_z^2$

A further transformation of equations (6.24) and (6.25) gives

$$\dot{P}_t = d_1 X_t + d_2 X_{t-1} + d_3 X_{t-2} + d_4 \dot{P}_{t-1} +$$

$$+ d_5 \dot{P}_{t-2} + z_t \tag{6.26}$$

where

$$d_1 = c_1$$

$$d_2 = c_2 - \rho \, c_1$$

$$d_3 = - \rho \, c_2$$

$$d_4 = \rho + c_3$$

$$d_5 = - \rho \, c_3$$

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[25] M. Parkin, op.cit., 1972, p.116. We note that this assumption is completely arbitrary and an autoregressive process of higher order might equally well be assumed to approximate e_t.

For stability we must examine the roots of the homogeneous part of the difference equation (6.26). They are

$$k = \rho \quad \text{and} \quad k_2 = c_3$$

so that again the restriction $|c_3| < 1$ must be imposed which results in the same condition as above.

The following variant of equation (6.22)

$$\dot{P}_t = b\,X_{t-1} + a\,\dot{P}^E_{t-1} + u_t \tag{6.27}$$

also to be found in the literature, combined with (6.23) similarly necessitates a restriction which involves the parameter a. (6.27) and (6.22) are transformed to

$$\dot{P}_t = e_1\,X_{t-1} + e_2\,X_{t-2} + e_3\,\dot{P}_{t-1} + u'_t \tag{6.28}$$

where $e_1 = b$

$e_2 = -\,b(1 - d)$

$e_3 = ad + 1 - d$

and $u'_t = u_t - (1 - d)\,u_{t-1}$

and the condition for stability is $e_3 < 1$ or $ad + 1 - d < 1$ or $a < 1$. Consequently studies which force a to be one must be viewed with scepticism as to their dynamic stability properties.

Having been through the theory and some of the empirical work on the price equation and having indicated the role of price expectations and the attempts made so far to relate them to the price equation, we can now write the form of the equation that will be used in our model. We shall assume that the expected rate of inflation is a function of previous period's excess demand and the previous period's pressure of the unit cost of production on profit margins, i.e.

$$\left(\frac{\Delta P}{P}\right)^E = a_3'(Y^*_{t-1} - Y_{t-1}) + b_3'(P_{t-1} - c^U_{t-1}) \qquad (6.29)$$

where $\left(\frac{\Delta P}{P}\right)^E$ is the expected rate of inflation,

$Y^* - Y$ is the difference between the full-employment and the actual level of gross domestic product and is taken as a measure of excess demand in the economy,

and $P - C^U$ is the difference between the price level and the unit prime cost of production and is taken as the variable signalling the cost pressure.

The coefficients a_3 and b_3 determine the relative importance of excess demand and cost influences in the formation of expectations of price changes. This formulation makes the expected rate of inflation dependent on other economic variables and avoids its determination from its own past values alone. Thus individuals observe last period's economic events which are relevant to pricing and on the basis of this information they form their expectations about future price changes. We shall examine each form in (6.29) in turn and we shall also specify how the expected rate of inflation which is an unobservable quantity is related to the actual rate of inflation.

The dependent variable $\left(\frac{\Delta P}{P}\right)^E$ written in discrete time form would be

$$\frac{P^E_t - P^E_{t-1}}{P^E_{t-1}} 100 \qquad \text{or} \qquad \frac{P^E_t - P_{t-1}}{P_{t-1}} 100$$

since the price level of the previous period is known when expectations of price changes are formed. Similarly the actual rate of inflation is $\frac{P_t - P_{t-1}}{P_{t-1}} 100$. This is unfortunately a non-linear expression and we should approximate it by a linear one in order to preserve linearity in the model. A formal way of obtaining an approximation is to consider the Taylor formula for functions of several variables

$$f(X,Y) = f(a,b) + \frac{1}{1!}\left\{f'_X(a,b)(X - a) + f'_Y(a,b)(Y - b)\right\} + \frac{1}{2!}\left\{f''_{XX}(a,b)(X - a)^2 + 2\, f''_{XY}(a,b)(X - a)(Y - b) + f''_{YY}(a,b)(Y - b)^2\right\} + \ldots \tag{6.30}$$

where a and b are particular points where the value of the function f and its partial derivatives are calculated,[26] and approximate it by taking only the linear terms.

For $\frac{P_t - P_{t-1}}{P_{t-1}} 100 = f(P_t, P_{t-1})$ we have

$$\frac{P_t - P_{t-1}}{P_{t-1}} 100 \simeq \frac{P^*_t - P^*_{t-1}}{P^*_{t-1}} 100 + \frac{100}{P^*_{t-1}} P_t - \frac{100\ P^*_t}{P^{*2}_{t-1}} P_{t-1} \tag{6.31}$$

with P^*_t , P^*_{t-1} the price levels at which the first derivatives are calculated. Taking as such the price levels at the second and third quarter of 1963, where they were both very close to 100, the constant term above vanishes and the coefficients of P_t and P_{t-1} are approximately equal to one so that

$$\frac{P_t - P_{t-1}}{P_{t-1}} 100 \simeq P_t - P_{t-1}$$

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[26] See E. Hille: <u>Analysis</u>, Vol. II, Blaisdell Publishing Company, 1966, p.309.

This approximation has been used in both theoretical and empirical work[27] but it must be pointed out that it should not necessarily be introduced if price expectations were not used for distinguishing the real from the nominal rate of interest. We have seen pieces of empirical work where the dependent variable in the price equation was the absolute change in the price level rather than its rate of change. However, since we are going to make the distinction between real and nominal rates of interest in our model the above approximation is deemed necessary.

The first term in (6.29) represents the influence of excess demand conditions in the product market in changing the price level. Coefficient a can be expected to be negative, i.e. the higher the pressure of demand or equivalently the smaller the difference between Y^*_{t-1} and Y_{t-1} is, the higher the expected change in the price level will be. We have seen that various authors have used this term as a measure of excess demand instead of proxies for excess demand for labour (the unemployment rate) or proxies for excess demand for product derived from the above measure (distributed lags of the unemployment rate). But their work has remained on a theoretical level and the accommodating assumption has been made that full-employment product was constant over time. This assumption will be removed and a series of data of full-employment product growing in time will be used. We shall therefore examine in some detail the concept of full-employment output and the way it can be estimated.

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[27] For an example of the former case see T. J. Sargent, *op.cit.*, 1971, p.54 and for an example of the latter see R. S. Pindyck: Optimal Planning for Economic Stabilization. North Holland Publishing Company, Amsterdam, 1973, p.49.

The concept and measurement of full-employment output

Full-employment output is a concept related to the more general concept of potential output. Their difference lies in that in the latter a greater number of assumptions about changes in the quantity and quality of the inputs which determine it over time, have to be made explicitly. Potential output has been defined as a measure of the optimum output which the economy can attain over a prolonged period of time *"without running into serious instabilities of employment, output or prices"*.[28] It is thus the full equilibrium output which is viewed from the supply side, when all inputs determining it are free from cyclical and random fluctuations. Specifically potential output estimation aims at measuring the volume of goods and services that would be produced with the use of available technologies and the efficient utilisation of both capital and labour inputs at the norms of full employment prevailing in the economy. Taking into consideration all the relevant factors implies the need of a complete model accounting separately for each input. Or, as an easier alternative, a production relationship can be set up and some kind of assumption is required to relate actual and potential levels of labour and capital utilised. Klein and Preston, for instance, assume proportionality between them and a production function of the Cobb-Douglas type.[29] A similar method was adopted by Briscoe, O'Brien and Smyth for the U.K.[30] As the authors

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[28] J. W. Knowles: "The Potential Economic Growth in the United States." Study Paper No.20 in Study of Employment, Growth and Price Levels, Congress of the United States, Joint Economic Committee, U.S. Government Printing Office, Washington, 1960, p.6. The same emphasis however should not be put on price stability in conditions in which inflation is being caused by other factors in addition to excess demand.

[29] L. R. Klein and R. S. Preston: "Some New Results in the Measurement of Capacity Utilisation." American Economic Review, 1967, p.37.

[30] G. Briscoe, P. O'Brien and D. J. Smyth: "The Measurement of Capacity Utilization in the United Kingdom." Manchester School of Economic and Social Studies, 1970, pp.95-98.

admit their approach is cruder than the other because no provision is made for correcting unemployment to take account of a minimum unemployment rate and the full-employment supply of labour is independent of the rate of utilisation of labour. Kuh's production relationship has a linear dynamic form and the writer avoids complications by dropping out the capital variable, invoking collinearity with trend and poor quality of data on capital stock.[31]

At the other extreme, as regards the extent to which various supply factors are calculated explicitly, the linked-peaks method offers no other assumption than maintaining that potential output grows at a constant rate between peak levels of economic activity, which are believed to represent full resource utilisation. The method has been used by many economists, e.g. Denison,[32] at Wharton School in U.S., because of its simplicity and is described in Klein and Summers.[33] The basic criticisms of the method are that peaks may in fact correspond to disparate degrees of resource utilisation and output may not follow a constant growth rate path between peaks, with capital formation not being spread evenly through the cycle.[34]

Falling between these extremes, namely no accounting at all and complete accounting for both labour and capital inputs, is a study using British data which makes a calculation of potential labour supply and applies it to a smoothed trend of productivity.[35] Our method

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[31] E. Kuh: "Measurement of Potential Output." American Economic Review, 1966, p.759.

[32] E. F. Denison, The Sources of Economic Growth in the United States and the Alternatives Before Us. Supplementary Paper No.13, Committee for Economic Development, 1962, pp.16-21.

[33] R. L. Klein and R. Summers: The Wharton Index of Capacity Utilization. Studies in Quantitative Economics No.1, University of Pennsylvanie, 1966.

[34] See A. Phillips: "An Appraisal of Measures of Capacity." American Economic Review, Papers and Proceedings, 1963, pp.290-91.

[35] W. H. Godley and J. R. Shepherd: "Long-Term Growth and Short-Term Policy - The Productive Potential of the British Economy and Fluctuations in the Pressure of Demand for Labour 1951-1962." National Institute Economic Review, August 1964, pp.26-38.

resembles the one applied by Godley and Shepherd in that capital input and individual factors affecting the growth rate of productivity are left implicit, a time trend taking account of them. This does not examine whether or not they are unaffected by the level of demand but simply it is posited implicitly that the required adjustments in those factors at full-employment levels of expenditure are attainable so that the overall level of productivity remains unaffected. However their method lacks sufficient theoretical justification despite the detailed system of relations they present. The approach which will be used here follows the work of Tella and Simler and Tella[36] on relations for labour force participation rates and the way they can yield predictions of full employment labour force. The latter is necessary since actual output, which is formed from the interaction of labour force and average labour productivity, varies in response to changing business conditions while it is desirable to eliminate cyclical and random fluctuations of employment in order to arrive at an equilibrium estimate. We now turn to examine labour force and productivity separately.

Labour Force. The size of the labour force is determined by population growth and the labour force participation rate.

(a) The rate of growth of population is determined by the birth rate, the death rate and the net emigration rate. As the last two were of limited variability in Great Britain the birth rate remained the main factor which contributed to the acceleration or deceleration in the growth rate of the population. Indeed for the period 1954 to 1962

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[36] A. Tella: "The Relation of Labor Force to Employment." Industrial and Labor Relations Review, 1964, pp.454-69, and

N. J. Simler and A. Tella: "Labor reserves and the Phillips Curve." Review of Economics and Statistics, 1968, pp.32-49.

for which the birth rate was, with one exception, rising the average annual compound growth rate of home population was 0.61 per cent. After 1962, when attitudes towards birth controls seem to have changed effectively and the birth rate has started to decline steadily, the average growth rate was cut down to 0.45 per cent. The birth rate, while contributing to the current growth of population makes its impact on the population of labour force age to be felt after at least 15 years, since the latter is defined to include all persons 15 years and over. This is particularly important because this total is used as a deflator in the computation of participation rates. A faster than average rate of growth in population of labour force age is reflected in a lower response of participation to labour force than would have been observed otherwise.

(b) It has been observed that variations in unemployment do not match variations in employment in a one-to-one relationship. The phenomenon is basically due to the cyclical response of participation to labour force in different phases of the business cycle. Participation rates defined as the percentage of labour force to population of labour force age are sensitive to changing economic activity in the short run, because reserve labour which is not registered as either employed or unemployed tends to respond to fluctuations in the demand for labour. At any time there are two forces acting to alter the proportion of labour force to total population. Since they work in opposite directions their combined effect weakens the individual impact of either. The one known as the 'discouraged worker' effect maintains that decreased likelihood of employment in the downswing of the cycle discourages a number of workers and makes them withdraw from the labour force. On the other hand the 'added worker' effect suggests that the loss of income by primary workers at low levels of activity induces secondary workers to enter the labour force to supplement this loss. The reverse movements are true for the upswing of the cycle.

The applicability of these hypotheses is tested by use of the basic formulation:[37]

$$\left(\frac{L}{P}\right)^{i}_{t} = a + b \left(\frac{E}{P}\right)_{t} + c\, t \qquad (6.32)$$

where $\left(\frac{L}{P}\right)^{i}_{t}$ = the ith group in labour force as a percentage of the respective population of labour force age, in period t .

$\left(\frac{E}{P}\right)_{t}$ = the aggregate employment population ratio in period t .

t = time.

Evidence of a net discouraged worker effect will be offered if the coefficient of $\frac{E}{P}$ turns out to be significantly positive. On the contrary a negative coefficient will be evidence of a net additional worker effect.

In studies using quarterly data it was found that the equation is performing better with the inclusion of the employment-population ratio lagged one period. Both current and lagged ratio were tried and indeed the results improved with the latter. Another

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[37] See Tella, *op.cit.*, p.458.

suggested modification is the replacement of the linear trend term by a logarithmic one on the grounds that the trend in participation rates is likely to diminish gradually and reach finally an asymptotic limit. However, inspection of Charts 6A, 6B and 6C where participation rates and employment population ratios of all workers, males and females are plotted, reveals that this may not be the case with British participation rates at least within the period 1954-72. What is immediately evident is a break in trend for the aggregate participation rate at the end of 1966. While the participation rate of all workers does not have any trend until that period, it is markedly falling afterwards. The reasons for this diversion of trend were discussed in the June 1970 issue of the Employment and Productivity Gazette,[38] where it was shown that for most part this was a result of an increased number of persons in full-time education and of a tendency to earlier retirement. Some other apparently inexplicable features were examined and explained later in the light of new information after the 1971 Census.[39] For instance it has been found that there was no fall in participation rates of males aged 25-64 as it had been reported in the first article.

In view of these characteristics of participation rates a linear trend term will be included in the regressions together with a slope dummy to take up the change in trend after 1966.3 . The form for estimation is:

$$\left(\frac{L}{P}\right)^{i}_{t} = a + b\left(\frac{E}{P}\right)_{j} + c\,t + d\,t' + e\,S^2_t + f\,S^3_t + g\,S^4_t \qquad (6.33)$$

[38] U.K. Department of Employment: "The Fall in the Working Population since 1966." Employment and Productivity Gazette, 1970, pp.492-95.

[39] U.K. Department of Employment: "The Fall in the Labour Force between 1966 and 1971." Department of Employment Gazette, 1973, pp.1083-87.

CHART 6.A

All workers

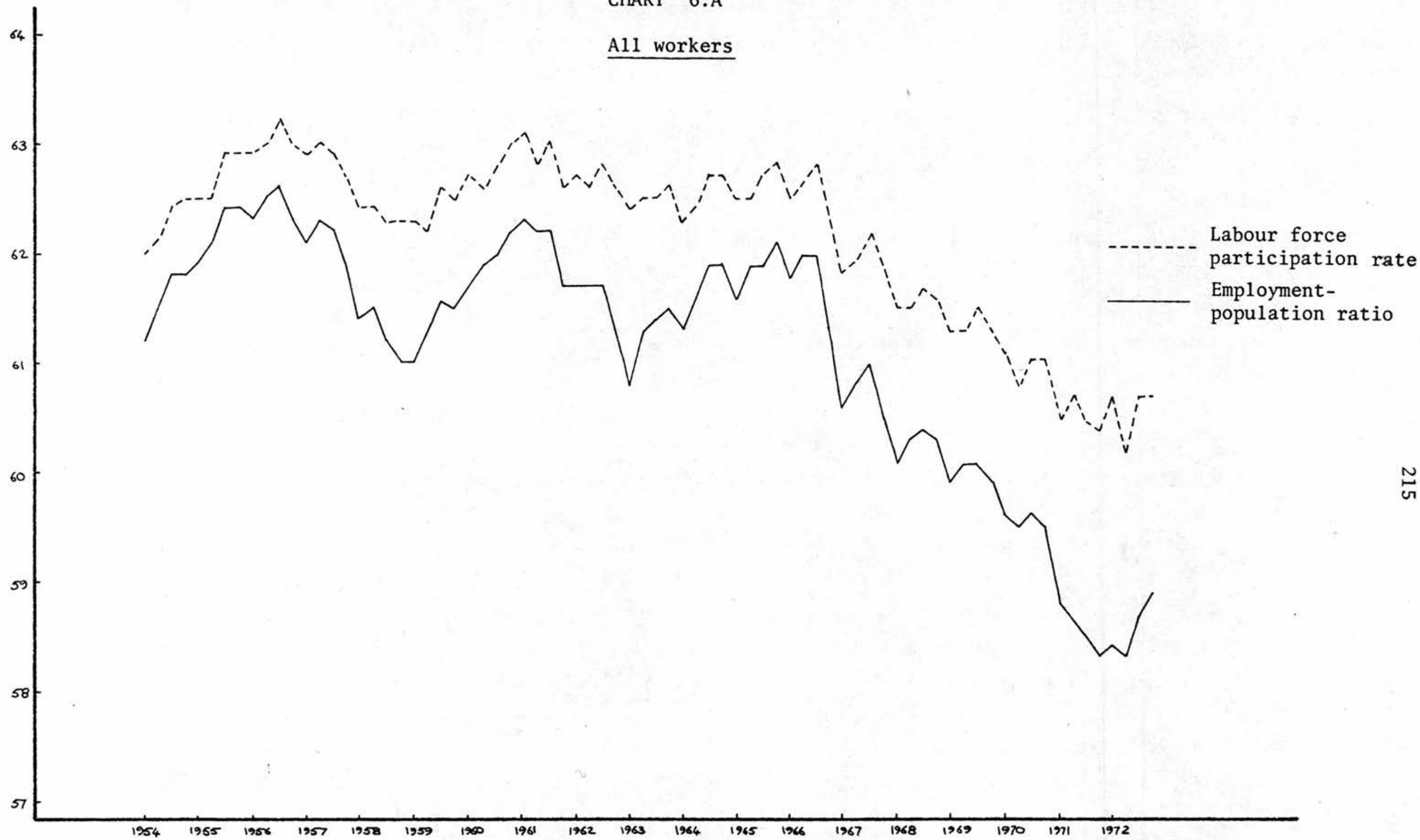
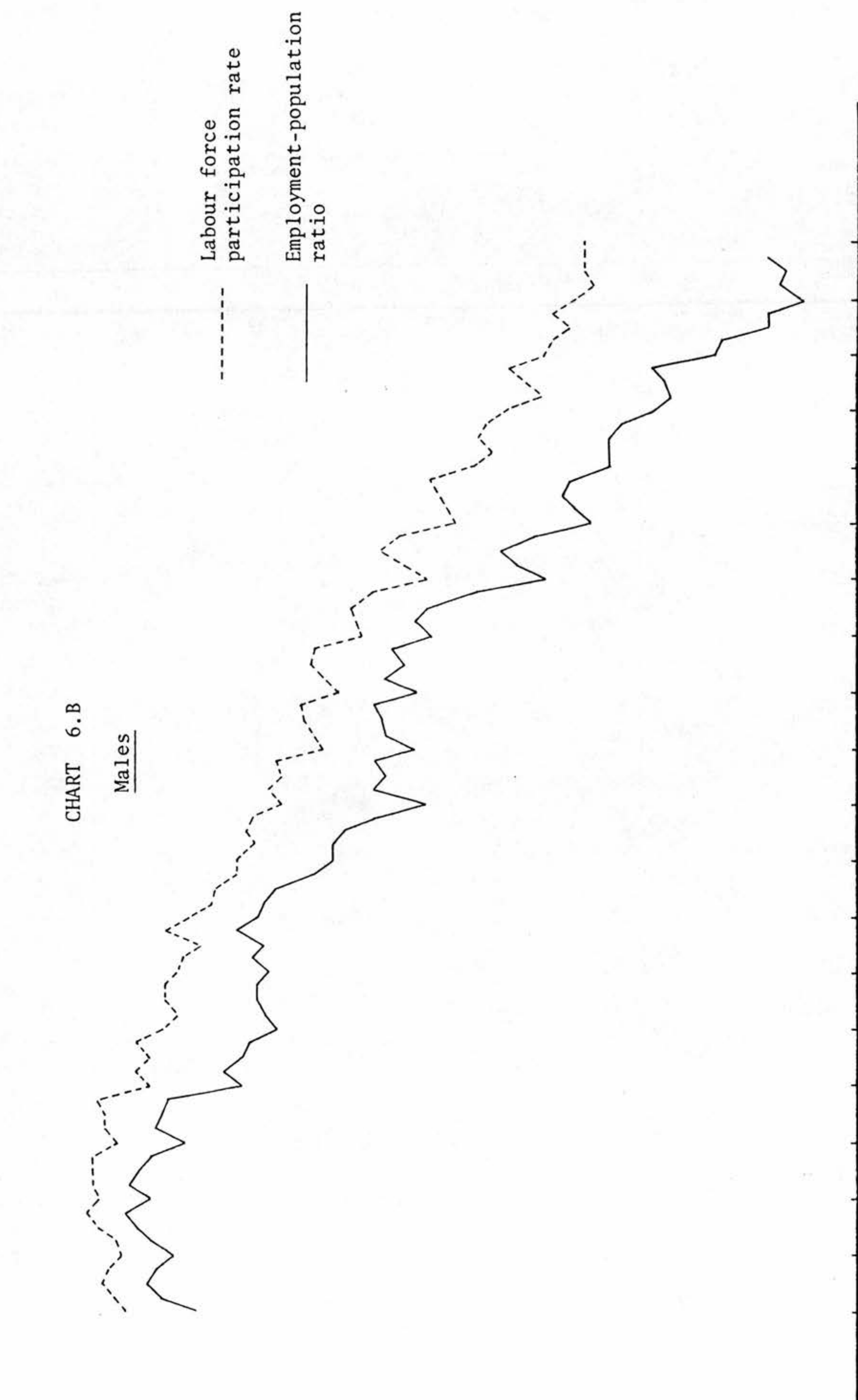


CHART 6.B

Males



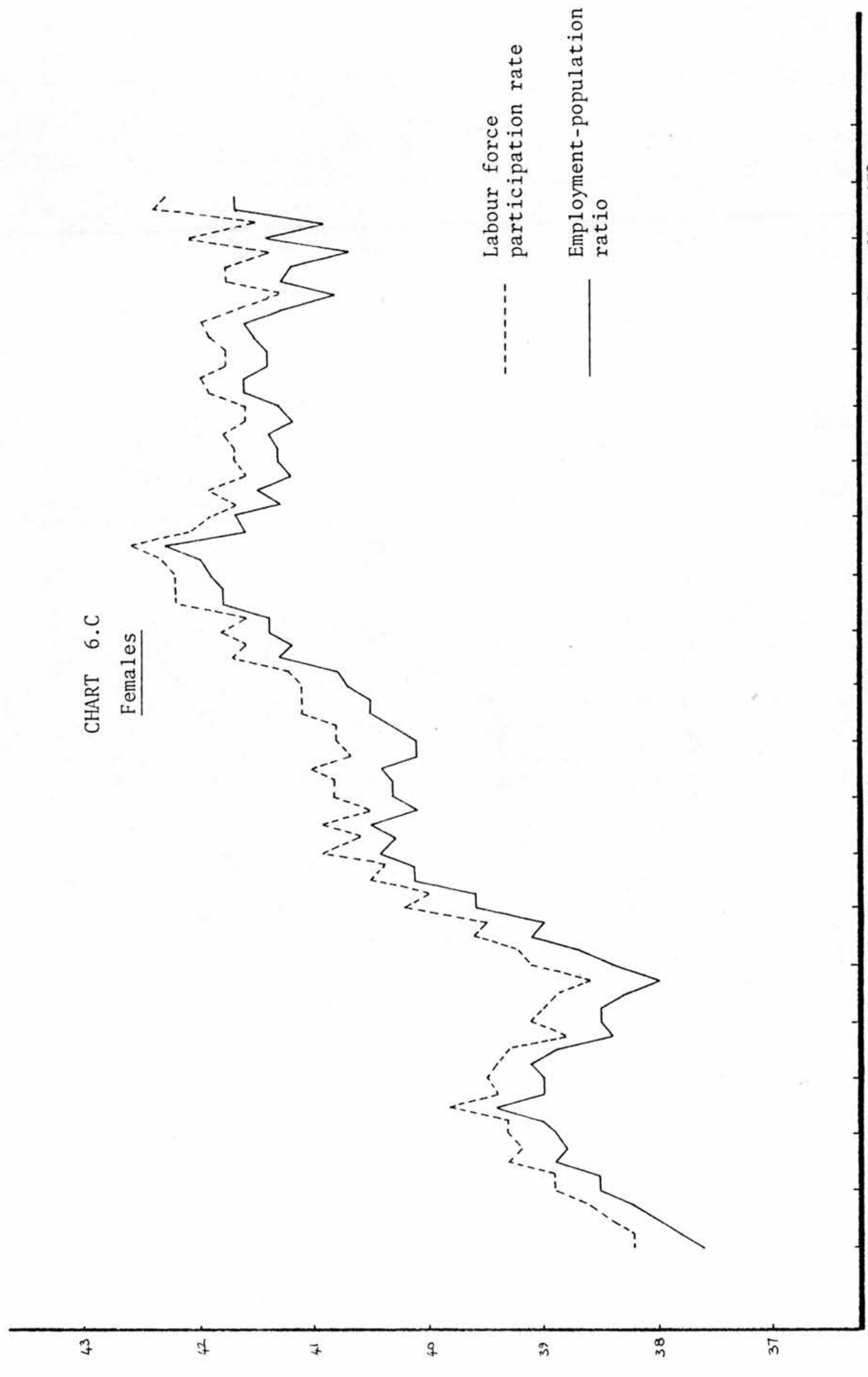


CHART 6.C
Females
Labour force participation rate
Employment-population ratio
43
42
41
40
39
38
37
1954
1955
1956
1957
1958
1959
1960
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973

where i refers to total (T), male (M) or female (F) participation rate, j is t or t_{-1} , t' is a slope dummy taking the value 0 up to 1966.3 and the values 51 , 52 , . . . afterwards, and S^2 , S^3 , S^4 are seasonal factors. The results using the data of Tables A.9, A.10 are presented in Table 6.1 . Two comments on these results are in order. (1) The discouraged worker effect is the prevalent effect for both male and female workers. As expected female participation rates are more sensitive to employment changes. On average an increase of employment by one thousand persons implies 595 new registrations in the labour force and a decrease of unemploymnet by 405 persons. These estimates are approximately the same with the estimates reported by Shepherd for low levels of unemployment.[40] The difference could to a certain extent be justified by the fact that equation 6.1a is not the most appropriate for such conclusions since the DW statistic indicates serial correlation in the residuals. If we focus on equation 6.1d the relation between unemployed and new entrants to the labour force becomes roughly one to one and this lies in the middle of Shepherd's upper and lower estimates. (2) The evidence conforms with Strant and Denburg's[41] finding for the U.S. that the added worker effect is operating subsequently to the discouraged worker effect to mitigate its dominance. The coefficient of the lagged employment-population ratio is consistently lower than that of the current ratio in all fitted equations. This can be interpreted as being a result of the offsetting - although to a small extent - influence of the secondary workers' movement in the opposite direction to that of the discouraged workers. In the same quarter there is a certain net discouraged

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[40] J. R. Shepherd: "Productive Potential and the Demand for Labour." Economic Trends, August 1968, pp.xxv-xxvi.

[41] K. Strand and T. Denburg: "Cyclical Variation in Civilian Labor Force Participation." Review of Economics and Statistics, 1964, pp.278-391.

TABLE 6.1: Regression results for participation rates

Equation	Dependent variable	Constant	Coefficient of							Statistics	
			$(\frac{E}{P})_t$	$(\frac{E}{P})_{t-1}$	t	t´	S_t^2	S_t^3	S_t^4	$\overline{R}^2$	DW
6.1a	$(\frac{L}{P})_t^T$	25.81	0.595 (20.13)		0.00256 (2.11)	-0.00454 (3.80)	0.15 (3.61)	0.13 10^{-3} (0.003)	0.13 10^{-1} (0.31)	0.976	0.73
6.1b	$(\frac{L}{P})_t^M$	67.93	0.356 (4.54)		-0.09949 (3.09)	-0.09491 (3.00)	0.023 (0.22)	0.13 (1.18)	0.34 (3.20)	0.987	0.46
6.1c	$(\frac{L}{P})_t^F$	-7.25	0.736 (15.48)		0.08238 (42.12)	0.00109 (0.57)	-0.21 (3.36)	-0.069 (1.06)	0.26 (3.99)	0.975	0.87
6.1d	$(\frac{L}{P})_t^T$	31.86		0.496 (12.98)	0.00142 (0.83)	-0.00821 (5.21)	0.069 (1.22)	0.17 (3.09)	0.071 (1.26)	0.953	1.82
6.1e	$(\frac{L}{P})_t^M$	70.62		0.312 (4.30)	-0.10002 (30.85)	-0.01128 (3.77)	0.11 (0.99)	0.23 (2.16)	0.37 (3.49)	0.986	0.73
6.1f	$(\frac{L}{P})_t^F$	1.85		0.587 (9.98)	0.08068 (30.65)	-0.00415 (1.71)	0.041 (0.47)	0.14 (1.67)	-0.18 (2.10)	0.955	1.12

(t values are entered in parenthesis)

worker effect as expressed by the positive coefficients in the regressions. During the next period this effect is reduced because of the increasing inflow or outflow of additional workers.

Generally the results show that changes in labour force participation have acted to a remarkable extent towards moderating pressures of excess demand for labour or not aggravating excess supply of labour. Full-employment labour force will be obtained from equation 6.1d as follows. The equation can be rewritten as

$$\left(\frac{L}{P}\right)_t = a + b \left(\frac{E}{L}\right)_{t-1} \left(\frac{L}{P}\right)_{t-1} + c\ t + d\ t' + $$

$$+ e\ S_t^2 + f\ S_t^3 + g\ S_t^4 \qquad (6.34)$$

Starting from an initial value for $\left(\frac{L}{P}\right)$ and substituting a full-employment ratio for $\frac{E}{L}$, converts it to a predicting equation for full-employment labour force. Full employment does not mean that every one who is in the labour force has a job, the last implying a value of a hundred per cent for $\frac{E}{L}$. Due to various factors there is always an unavoidable minimum of unemployment that cannot be attributed to demand deficiency. Important factors responsible for the above-zero minimum level of unemployment are among others: the composition of demand resulting in particular patterns of employment distribution, the degree of labour mobility and retraining facilities, technological change and institutional setting especially legislation ruling labour market relations. Substantial changes in them can be reflected in varying degrees of the full employment norm. It seems that during the period 1954-72 two shifts have taken place of the U.K.'s minimum unemployment rate. The first at the beginning of 1957 was of rather small size compared to the one after 1966.3 which was by far more serious and of a different origin. In the years from 1954 to 1956 unemployment among males and females was approximately of equal magnitude, with the minimum rate estimated at 0.75%. From 1957 onwards the pattern changes with male unemployment rising above female unemployment.[42] The evidence adduced by Gillion and Black[43] suggests that the reason for this change lies in a shift of the pattern of demand towards industries with a higher female-male employment ratio. Their explanation is corroborated by the fact that vacancies for women show a significant rise after 1956 relatively to vacancies for men. For the quarters 1957.1 to 1966.3 the overall minimum rate was put to 1%, a level approximately reached at 1961.2 and 1966.2 . With the end of 1966 unemployment figues have exhibited a sudden rise and in subsequent years they have not given any sign of retreating. Finally in the last years they have reached levels not previously experienced. We note that the particular movement observed in the total unemployment rate was a reflection of a similar but more pronounced movement of the male unemployment rate; female unemployment followed the fluctuations in business activity on a trendless route. The phenomenon has received a good deal of attention because unemployment ceased to provide reliable information as an indicator of the pressure of demand and moreover unemployment itself is undesirable. In general two views about the explanation of this change can be distinguished. In one of them the change is attributed to a process of dishoarding labour in overmanned

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[42] See Charts 6D, 6E and 6F where total unemployment rate and the rates for males and females separately, are shown. The basis for deflating the data is the total of the labour force and this gives slightly lower percentages than those reported in the Department of Employment Gazette.

[43] C. Gillion and I. Black: "Some Characteristics of Unemployment." National Institute Economic Review, August 1966, p.33.

CHART 6.D

Unemployment rate : all workers

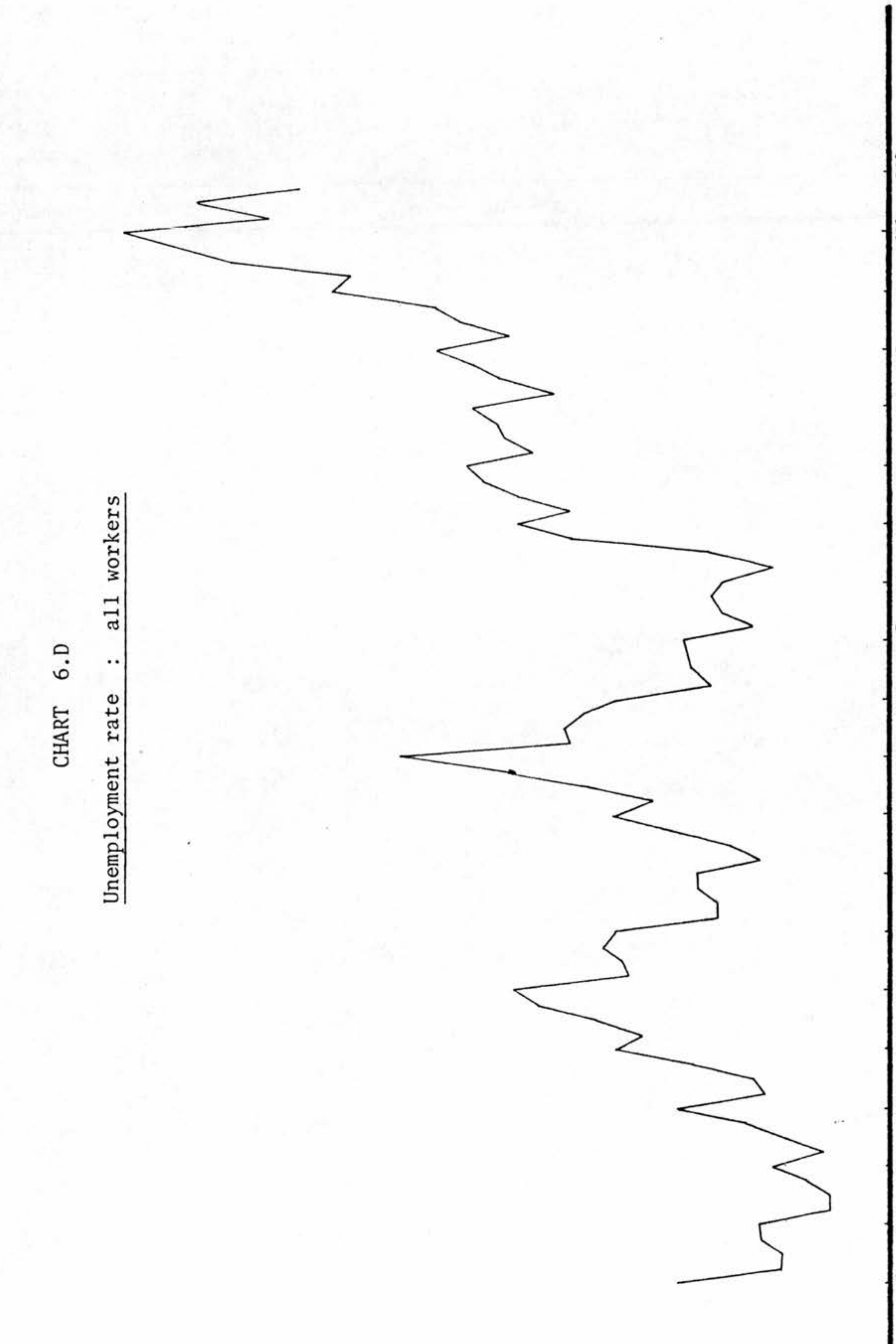


CHART 6.E

Unemployment rate : Males

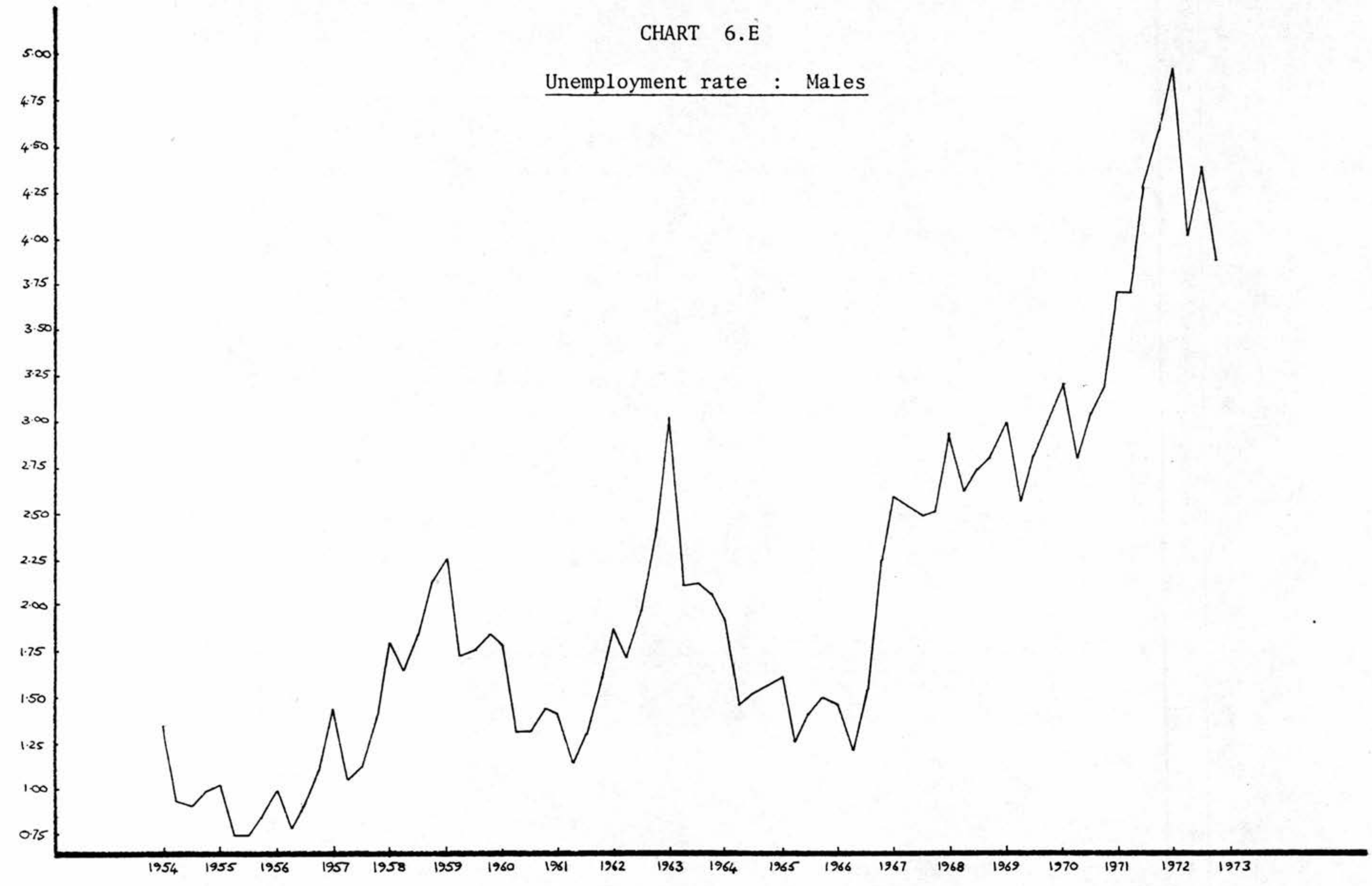
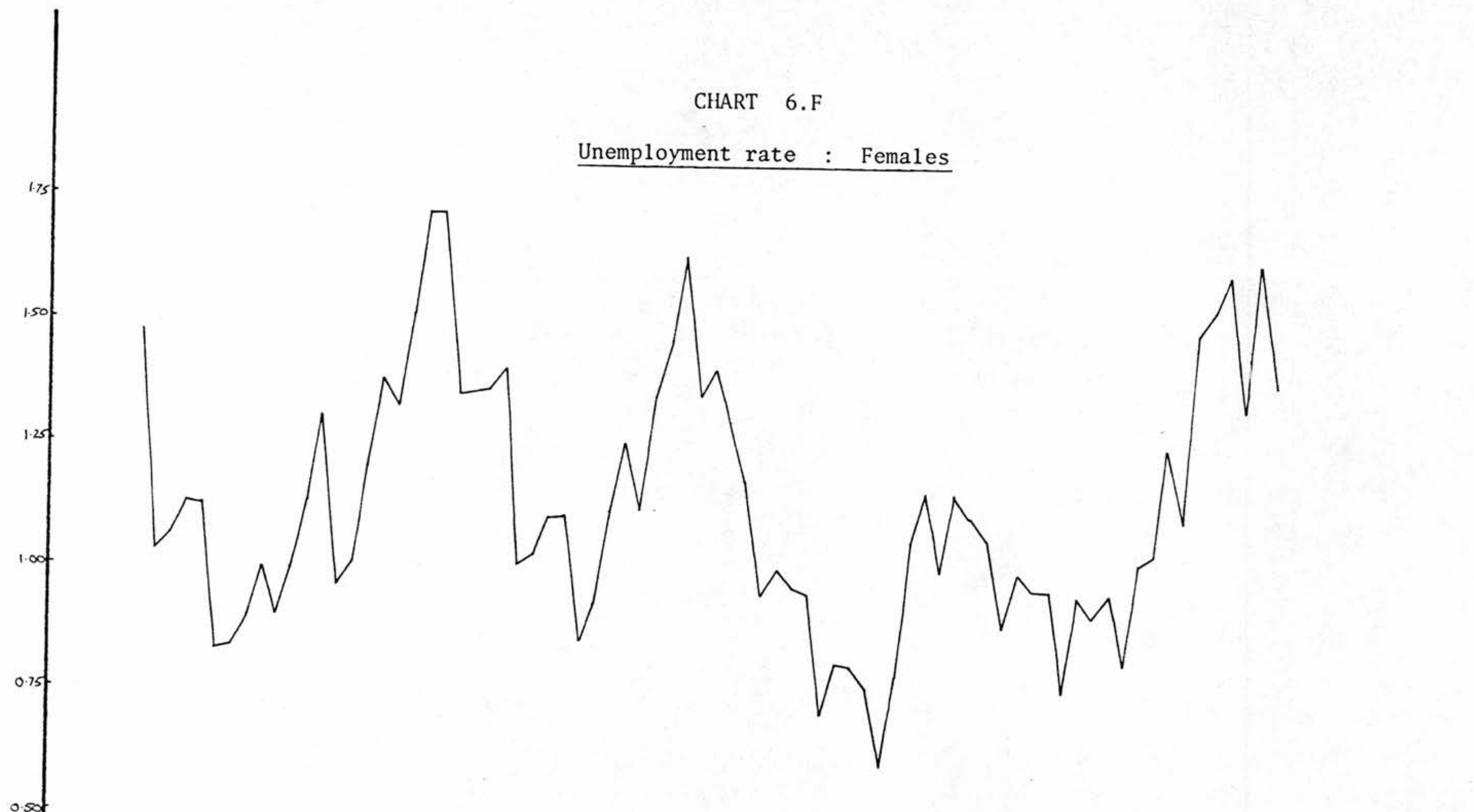


CHART 6.F

Unemployment rate : Females



industries. Employees found it easier to increase redundancies after the introduction of the redundancy payments scheme in 1965 and of the earnings related unemployment benefits in 1966. It is thought that these measures have eased the way to redundancies which were formerly deterred by a strong resistance to redundancy by employees leading to industrial disputes and deterioration in employers-employees relations.[44] On the other hand unemployed probably have used their eased situation to be more choosy in search of new jobs, no matter whether they were unemployed through redundancy or voluntarily.

The impact of higher redundancies or voluntary quits that will concern us here, is the upward shift of the minimum unemployment rate brought about by the increase in the average duration of unemployment. This has featured male unemployment and indeed it has

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[44] The scheme for redundancy payments was introduced with the objective of tackling problems of overmanning. It entitles workers to lump-sum payments if their dismissal is due to redundancy and is graded according to age, length of service and pay. It might be reasonable to assume that there are volunteers to be dismissed when a reduction of employment is impending, especially older people who have a longer period of service. For the extreme assumption, which is not grounded on any evidence, that increased unemployment is all voluntary see D. Gujarati: "The Behaviour of Unemployment and Unfilled Vacancies: Great Britain, 1958-1971." Economic Journal, 1972, p.195. The earnings related benefit scheme was designed in the same spirit as the redundancy payment scheme. It alleviates the consequences of unemployment by granting earnings related benefits in addition to the basic flat-rate unemployment and sickness benefit. It is thus a highly supplementary measure, which takes into account earnings usually reflecting higher skills. For a discussion of some other factors that might explain the unemployment upsurge, like selective employment tax, devaluation and incomes policy see J. K. Bowers - P. C. Cheshire and A. E. Webb: "The Change in the Relationship Between Unemployment and Earnings Increases: A Review of Some Possible Explanations." National Institute Economic Review, November 1970, pp.58-60.

been suggested by Mackay, who examined samples of redundant employees, that long run adjustment costs have been higher for males declared redundant and females have shown greater industrial mobility than males.[45] The second view which can be thought as complementary to the first and emphasises the monetary aspects of the phenomenon, is gaining much support after the recent experience with high unemployment and high rates of inflation. According to this view the rise of the minimum unemployment rate reflects a rise in the natural rate of unemployment. The latter has been defined by Friedman as *"the rate which has the property that it is consistent with equilibrium in the structure of real wages"*, and at which *"real wage rates are tending on the average to rise at a 'normal' secular rate, i.e. at a rate that can be indefinitely maintained as long as capital formation, technological improvements, etc. remain on their long-run trends."*[46]

The rise is explained by the distortions brought about by inflation in which *"more and more workers are drawn into kinds of jobs which depend on continuing or even accelerating inflation."*[47] Thus the labour unions' action which creates unemployment is met by government's expansionary action because the latter has always been committed to a full-employment target. The result is the above mentioned distortions which make more and more employment depend on further inflation and monetary expansion and any attempt to curb inflation will show sooner or later in an increasing minimum number of unemployed. This view will be further elaborated when we discuss the dynamic adjustment

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[45] D. I. Mackay: "After the Shake-Out." Oxford Economic Papers, 1972, pp.89-110.

[46] M. Friedman: "The Role of Monetary Policy." American Economic Review, 1968, p.8.

[47] F. A. Hayek: "Inflation: the Path to Unemployment." in Inflation: Causes, Consequences, Cures. Institute of Economic Affairs, 1975, p.119.

mechanism which will be applied to our price equation.

To obtain an estimate of the shift, which did not occur as a once and for all jump, a mechanical procedure is used. The scattergram of vacancies-unemployment points[48] suggested a three step movement to a shifted relationship between unemployment and vacancies from 1966.4 to 1967.4, from 1968.1 to 1970.4 and from 1971.1 to 1972.4 . The total unemployment rate is then regressed on female unemployment rate and three shift dummy variables for the respective intervals, which are hoped to pick up the shift. Female unemployment rate is included as a regressor because it has not been subjected to any remarkable shift over the whole period, while following the movements of the overall rate. The result is

$$U_t^T = \underset{(13.20)}{1.27\, U_t^F} + \underset{(7.69)}{0.67\, K_t^1} + \underset{(17.00)}{1.05\, K_t^2} + \underset{(18.97)}{1.45\, K_t^3} + 0.05 \qquad (6.35)$$

$$\overline{R}^2 = 0.927 \qquad D.W. = 0.38$$

Allowing for the shift of the rate after 1956 by 0.25 percentage points the minimum rate is raised to 1.40%, 1.80%, 2.20% respectively for the three spans. These estimates are not inconsistent with considerations of degree of capital utilisation. For instance examination of the capital utilisation index constructed by the Bank of England[49] and the unemployment figures shows that comparable degrees of capital utilisation before and after 1966.3 are associated with unemployment differentials including in most of the cases the estimated numbers.

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[48] Unfilled vacancies are an alternative proxy for excess demand and have not exhibited any irregular movement. See J. K. Bowers - P. C. Cheshire and A.E. Webb, *op.cit.*, p.44.

[49] Bank of England. "Capital Utilisation in Manufacturing Industries." Bank of England Quarterly Bulletin, 1971, pp.490-6.

Substituting the full-employment ratio in (6.34) and starting from an initial value for $(\frac{L}{P})$ we can easily get the full-employment participation rates $(\frac{L^F}{F})$ and the full-employment labour force (L^F) . It is straightforward to obtain full-employment figures from the latter by subtracting minimum numbers of unemployed.

Productivity. Average labour productivity, that is output produced per person employed follows a rising trend on which short run fluctuations are imposed. There are a multitude of factors affecting productivity trend growth most of which are quantitatively unmeasurable. A list of them, by no means exhaustive, includes management skill, quality of effort of workers, scale of operation, plant size, education, age of mechanical equipment, state of industrial relations, capital intensity, etc. As a result of their influence productivity has been moving on a rising trend. Since the end of 1966 there has been an acceleration in the growth rate of productivity both in manufacturing and distribution. Whether productivity gains were caused by a common factor or not is an issue which is not definitely settled.[50]

The second notable feature of productivity is its variation with the trade cycle. Labour usage is not adjusted fully to changes in output so that productivity level is depressed on the downswing of the cycle and rising rapidly on the upswing. It has been argued that there are two distinct reasons for this. First, the existence of

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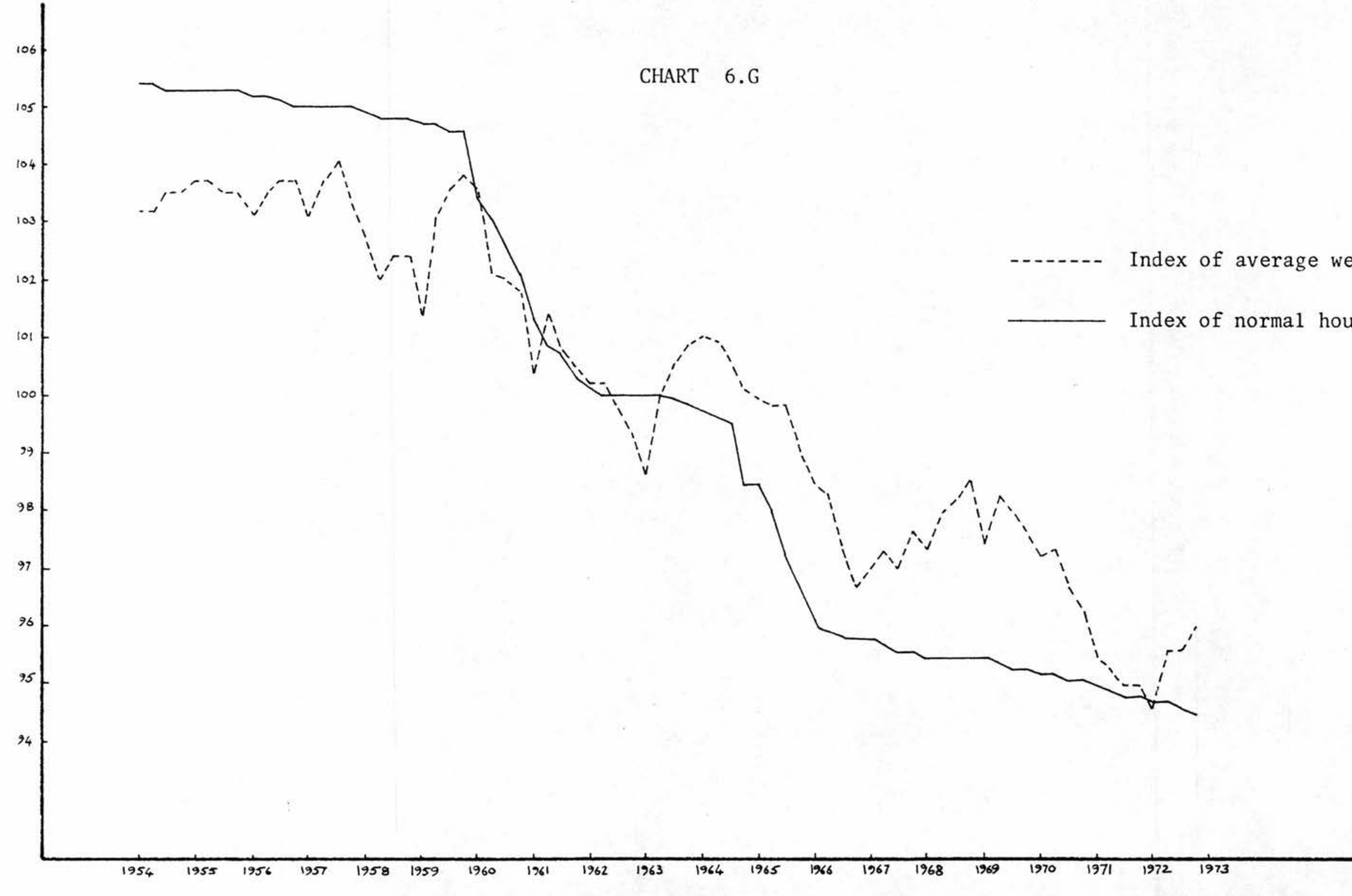
50 For opposing views see J. D. Whitley, and G. D. N. Worswick: "The Productivity Effects of Selective Employment Tax." National Institute Economic Review, May 1971, pp.36-40. The authors, without denying that S.E.T. might have an effect, argue that the shake-out of labour, already discussed in the other section, is the common factor responsible for this behaviour. They reached this conclusion after examination and rejection of the possibility that productivity increases were due to a higher rate of investment in both sectors. But R. D. Sleeper in "S.E.T. and the Shake-Out: A Note on the Productivity Effects of the Selective Employment Tax." Oxford Economic Papers, 1971, pp.197-211, has questioned this and suggested a new plausible explanation, namely that the shake-out vindicates productivity increases only in manufacturing whereas Selective Employment Tax and the abolition of Resale Price Maintenance account for the relevant rises in distribution.

overhead labour means that there is a proportion of the labour force aligned with the size and capital equipment of the firm, which employers are unable to adjust to short-run variations in demand and output. Secondly, there is a belated adjustment of employment because employers are generally reluctant to dismiss trained personel unless marketing possibilities are considered unfavourable over the long run.

Besides all of the before-mentioned factors which shape productivity there is another one which should be borne in mind, namely changes in the average worked hours in the economy. Its influence is singled out when productivity is expressed on a man-hour basis, but productivity measured as output per person employed incorporates this effect. Our preference has been for the latter measure since indices of average hours worked are of partial coverage; they include average hours worked by all operatives in manufacturing industries only. Moreover they are not available for the years 1954-55, from 1956 to 1960 they are given for six months, and in 1961 for nine months. Chart 6G shows the movement of the index during the period of the study. Estimates for 1954-55 were computed from data on average hours worked in April and October according to the 1948 Standard Industrial Classification, but because of their limitations they are of indicative usefulness. It is observed that average hours worked, which are inclusive of hours worked overtime, respond to changes in output, declining in recessions and expanding in booms. Their variations are imposed on a downward trend which characterises the last two decades. A visual impression of the trend can be gained by observing the index of normal hours worked, plotted beside the other, although the two indices are not directly comparable. The latter refers to normal hours worked by

CHART 6.G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1973
Index of average weekly hours
Index of normal hours

manual workers in all industries and services, does not take into account the effects of overtime and is based on changes in normal conditions of employment as laid down in collective agreements, statutory orders, etc.,[51] thus being a good indicator of the trend in hours worked. One can discern two major movements towards a shorter working time. The one, from 1959.4 to 1961.4 established a reduction of the normal week by two hours after a number of long term settlements that affected most industries. The other from 1964.2 to 1966.1 brought into operation the 40 hours normal working week in a number of sectors covering approximately half of all manual workers. Similar sharply falling segments can be seen in the index of average hours worked despite its short-run fluctuations. Normally the expected effect of these reductions is in the way of improving efficiency[52] by pulling up produced output per man-hour. However the net effect or output per person employed is not clear on *a priori* grounds and such a quantitative enquiry will not be undertaken here.

The normal procedure to follow in the estimation of full-employment output is to apply the employment figures deduced above to a smoothed trend of productivity. This is essentially application of the same level of productivity to those unemployed who would have been at work at full-employment level. A polynomial trend of second degree is probably the best choice because it can capture satisfactorily the acceleration in the growth rate of productivity. The fitted equation is

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[51] See for example technical note in <u>Department of Employment Gazette</u>, 1974, p.58.

[52] <u>Ministry of Labour Gazette</u>, 1961, p.2.

$$Z_t = \underset{}{79.19} + \underset{(6.61)}{0.42085}\ t + \underset{(4.57)}{0.00371}\ t^2 \qquad (6.36)$$

$$\overline{R}^2 = 0.965 \qquad D.W. = 2.63$$

with the trend line being a smooth line with mild curvature. However our decision to use untransformed data including seasonal variation requires a corresponding provision for this influence. No such allowance is made in any of the studies concerned with the estimation of full-employment output; seasonally adjusted figures underlie the calculation and the interpretation is similar to the one outlined at the beginning. In the following computation of output at full-employment level it will be assumed that seasonal factors that have led productivity to reach specific levels in different quarters will retain their relative strength at different levels of output and employment so that the actual within the year fluctuations of productivity will be reproduced to the full-employment level of labour supply. To this end the trend was re-estimated with the inclusion of seasonal dummies. The result is

$$Z_t = 75.80 + \underset{(12.17)}{0.43582}\ t + \underset{(7.65)}{0.00348}\ t^2 + \underset{(5.42)}{2.95}\ S_t^2 +$$

$$+ \underset{(5.60)}{3.05}\ S_t^3\ )\ \underset{(12.55)}{6.83}\ S_t^4 \qquad (6.37)$$

$$\overline{R}^2 = 0.989 \qquad D.W. = 2.06$$

where S^2 a dummy taking the value 1 in the second quarter and 0 in the others and S^3 , S^4 the respective dummies for the third and fourth quarters. Equation (6.37) will be the basis of computation of the productivity data that will be applied to the estimates of full employment. The estimated series is shown in Table A.11, Appendix A

Combining the findings of this and the preceding section gives us the figures of full-employment GDP presented in Table A.12, Appendix A.

The last term of the price equation. The second term in the right hand side of (6.29) represents the influence of cost elements in raising expectations of price changes. It recognises the fact that the mark-up over costs can vary not only in absolute size but as a percentage as well.[53] When the difference between the previous period's price level and the unit prime cost of production (that is the profit per unit of output) becomes smaller because of cost inflation, people come to expect that the price level will be higher in the next period so that the expected price change will be positive and the coefficient b_3' will be negative. The unit prime cost of production C^U consists of labour costs C^L and material costs C^M.

$$C^U = C^L + C^M \qquad (6.38)$$

Unit labour costs are defined by

$$C^L = \frac{\text{Wage bill}}{\text{G.D.P.}} \qquad \frac{\text{(W)(Hours worked)}}{\text{G.D.P.}} \qquad (6.39)$$

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[53] An alternative interpretation for this term was given by J. D. Pitchford in "Cost and Demand Elements in the Inflationary Process", Review of Economic Studies, 1956-7, pp.139-48. Pitchford specified the price equation for cost determined prices in a closed economy as

$$P_t - P_{t-1} = k_2 \,(hW_{t-1} - P_{t-1}) \qquad k_2 > 0$$

where P is the general price level and W the general wage level, and interpreted h to be a constant (greater than one) which determines a target price. The gap between actual and target price due to an increase in the money wage level induces a change in prices according to the above equation.

where W is the wage rate. This is a non-linear expression (the product of the wage rate and the inverse of average productivity expressed on a man-hour basis) and can be approximated by[54]

$$C^L = c_3' W + d_3' \left(\frac{\text{G.D.P.}}{\text{Hours worked}}\right) \qquad (6.40)$$

$$c_3 > 0 \quad \text{and} \quad d_3 < 0$$

and the unit prime cost is

$$C^L = c_3' W + d_3' \left(\frac{\text{G.D.P.}}{\text{Hours worked}}\right) + C^M \qquad (6.41)$$

From this expression the influence of the last two terms will not be considered explicitly in the price equation, because data on them are not available for non-manufacturing activities, and this lack of data has already been discussed as regards the productivity term. Instead, we shall allow autocorrelation in the residuals to pick up their influence. Consequently (6.29) can be written as

$$P_t^E - P_{t-1} = a_3' (Y_{t-1}^* - Y_{t-1}) + b_3' (P_{t-1} - c_3' W_{t-1}) \qquad (6.42)$$

We note that cost and demand inflation can interact or the one can be the cause of the other with some time lag involved. Thus if prices are demand determined an increase in the pressure of demand in a competetive sector will increase prices and this may in turn raise the wage level in this sector and may also lead to wage claims in other sectors with a further impact on prices. Moreover, even if wages to non-competitive sectors are not affected there might be a tendency for prices to rise if some of the products of competitive sectors are inputs to the former. If prices are cost determined variations in

[54] For a similar approximation in applied work see R. S. Pindyck, *op.cit.*, pp.50-51.

demand may affect the profit mark-up or conversely the widening of the gap between the previous period's price and unit costs might lead to a fall of aggregate demand thus adding to the slowing down of the rate of price increases. This depends on how the general price level interacts with the different components of demand. To the extent that there is no overall effect on the growth of demand such changes in the gap can be thought as autonomous.

The above interactions are partly accounted by equation (6.42) since both real expenditure and the price level are endogenous variables and they appear as determinants of expected price changes with the same time lag (Y_{t-1} , P_{t-1}) . On the other hand the nexus between wages and prices, not examined here, will not affect the unbiasedness of the coefficients if the induced part of wage changes is assumed to depend on prices lagged at least one period. Apart from this induced part, there will be an autonomous part whose explanation lies on wage determination theory and this can be considered as exogenous. For example, there might be disparity of demand between different markets (in some markets there is excess demand and in some others deficient demand so that on the aggregate excess demand is not affected) and the wage increases in the sectors where demand has risen may spill over into the sectors where it is slack.

Equation (6.42) contains price expectations which are an unobservable quantity, so we must link them to actual price changes. We shall posit that price expectations are falling short of actual price changes by a constant percentage, i.e.

$$P_t^E - P_{t-1} = \lambda_2 (P_t - P_{t-1}) \qquad 0 < \lambda_2 \leq 1 \qquad (6.43)$$

and this is equivalent to assuming the inverse adjustment mechanism discussed in chapter 3, for the price level. Thus if $\lambda_2 = 0.8$, the initial price level 100 and the price change expected by people, on the basis of last period's information as this is contained in equation (6.43), is 5%, then the actual value of the rate of inflation will be 6%. The lower the value of λ_2 the higher is unanticipated inflation. A possible rationalisation for this hypothesis might be sought in the influence of monetary conditions and Government's commitment to maintain full-employment. This can be understood if we consider that a higher price level caused by excessive wage claims will cause some unemployment. The latter is an undesirable situation and the government wanting to preserve full employment will draw people in newly created jobs whose existence depends on increasing budget deficits and continuing or perhaps accelerating monetary expansion. This is the view put forward by Hayek who convincingly remarked that we have allowed *"a long inflationary boom to bring about a misdirection of labour and other resources into employments in which they can be maintained only so long as inflation exceeds expectations."*[55] This effect would be captured by the adjustment mechanism (6.43). The above view is more or less accepted by Monetarists as well, who point out that the initiating force can not be wage claims alone but they have to be accompanied by government deficits financed by money creation.

According to Monetarists if a union bids up its wages this will result in a changed structure of wages, with higher real wages of the one particular group of workers and some unemployment, which however will be absorbed in other sectors of the economy where real wages are relatively lower. This will only happen if the government does not

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[55] F. A. Hayek, op.cit., p.120.

intervene to eliminate the unemployment which has been created. In the opposite situation where expansionary policy is undertaken there will be a once and for all effect on inflation, so long as these unemployed are absorbed later on by the private sector. This is the monetarist thesis. But in the case where the newly created jobs or the benefits for the unemployed are for some reason perpetuated there will be a continuing although not accelerating process of inflation. Finally if the movement of one union is followed by others and the government again intercedes to maintain full employment, inflation can be proceeding at an accelerating rate.

As far as relationship (6.43) is concerned we note that it is not reasonable to assume that it is fully symmetric, holding equally for price decreases as for price increases. As however in our sample there are only six cases of decrease in the price level of a rather small size, it is hoped that it will not make much difference to the results if we consider (6.43) on a uniform basis.

(6.42) and (6.43) can be reduced to

$$\lambda_2 \ (P_t - P_{t-1}) = a_3' \ (Y^*_{t-1} - Y_{t-1}) + b_3' \ (P_{t-1} - c_3' \ W_{t-1})$$

$$\text{or} \quad P_t - P_{t-1} = \frac{a_3'}{\lambda_2} (Y^*_{t-1} - Y_{t-1}) + \frac{b_3'}{\lambda_2} P_{t-1} - \frac{b_3' \ c_3'}{\lambda_2} W_{t-1}$$

$$\text{or} \quad P_t = \frac{a_3'}{\lambda_2} (Y^*_{t-1} - Y_{t-1}) - \frac{b_3' \ c_3'}{\lambda_2} W_{t-1} + (1 - \frac{b_3'}{\lambda_2}) P_{t-1} \quad (6.43)$$

$$\text{or} \quad P_t = a_3 \ (Y^*_{t-1} - Y_{t-1}) + b_3 \ W_{t-1} + c_3 \ P_{t-1} \quad (6.44)$$

where the expected signs of the coefficients are $a_3 > 0$, $b_3 > 0$ and $0 < c_3 < 1$ if $|b_3'| < \lambda_2$. Of the parameters of (6.43) only c_3 can be identified. Autocorrelation will be allowed in the residuals of (6.44) to take account of the possible influence of the omitted variables.

CHAPTER 7

CHAPTER 7

THE OTHER EQUATIONS OF THE MODEL

The equation relating returns on bonds and equities. One of the assumptions underlying the IS-LM model is, as we have seen, that real capital and debts are perfect substitutes. This permits the exclusion of both markets for real capital and debts from explicit consideration and on the additional assumption of a rigid price level it allows the presence of one interest rate, the market rate, in the model. This rate is simultaneously the rate of return on bonds (opportunity cost of holding money) and the marginal efficiency of capital (after allowing for the appropriate risk premium), and it will be determined endogenously when the money supply is endogenous as well. However, the return on capital is not an observable variable and the risk premium is unknown and if we wish to estimate them, an appropriate measure has to be selected (including most probably proxy variables) which will represent the return on capital and another equation has to be introduced relating the market rate to this return and the risk premium. This is a necessary step even if we retain the assumption of perfect substitutability and price level rigidity.

There are further complications if (a) we consider the price level as variable and not fixed, whence we must make the distinction between nominal and real rates of return, and (b) we accept that the returns on debts and capital are interdependent but there is not a one to one relationship between them, i.e. there is not perfect substitutability.

In view of the interesting questions which are posed and our wish to examine in a dynamic context the possibly different timing of the effect of monetary and fiscal policies on the markets for capital and debts, we shall include in our model a relationship between rates of return appropriate for the two markets. Regarding the market for capital (or under a simplifying assumption the equities market) an appeal has been made recently by Walters to study this in relation to the gilt market (government bonds) and monetary policy.[1] Walters attributes the growth of the price of equities to real, monetary and institutional factors. A real factor is basically the increase in productivity of real capital. The monetary factor is the increase in the quantity of money which generates (at a later stage) inflation and further inflationary expectations and depresses the real return on gilts. This causes people to substitute equities for gilts and the price of the former rise. Finally, the institutional factors are changes in the law, in the organisation of financial institutions, etc. Walters concentrates further on monetary policy and tries to assess its impact effect on equity prices by observing the coincidence of turning points in the money stock and equity price index series. He suggests however that *"one would need to do more research on the nature of the series in order to determine precisely the relationship between the series (it seems to be a useful task for a scholar seeking a subject)."*[2]

Concerning the causation between changes in the money stock and changes in the price index for equities, it will be the case that either changes in the former cause the turn down (or up) in the equity

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[1] A. A. Walters: "Monetary Policy, Gilts and Equities." Investment Analyst, December 1970, pp.3-6.

[2] *Ibid.* p.6.

market or both changes are the consequences of some third force. Walters, being a monetarist, favours the first: *"My predilection is to believe that movements in the money stock are the cause of the oscillations in the equity market. But it must be admitted that sufficient properly documented evidence has not yet been produced to establish such a claim. One needs to specify how the monetary oscillations affect the stock market and to trace the linkages. Such a research program seems worth exploring."*[3] It is exactly the presentation of this evidence regarding the equity market that we are interested in, by giving our attention to this market. Since in our model money will be an endogenous variable, it follows that both the money stock and the variable pertaining to the equity market will respond to third forces - the exogenous influences in the model.

A study made by Hendershott and Van Horne[4] will be the basis of our version of the equation. Hendershott and Van Horne departed from the equality of real returns on bonds and equities the latter adjusted for the risk premium, i.e.

$$R^D = R^C - \rho \tag{7.1}$$

We shall discuss each term in (7.1) separately.

(1) The real return on bonds. Here, bonds must be interpreted to be debts both government and private. Monetarists have pointed out that nominal yields on debts do not reflect real productivity because they are influenced by expected price changes and therefore they can not

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[3] A. A. Walters, *op.cit.*, 1970, p.6.

[4] P. H. Hendershott and J. C. Van Horne, *op.cit.*

be safely used as a proxy for the real rate of interest. On this point Fand writes:[5] *"In a high-pressure economy with rising prices such as the US since 1965 interest rates may be misleading indicators of the monetary posture and do not provide an appropriate measure of the cost of capital. The interest rate used in the econometric models is a nominal rate - and is therefore affected by rising prices and inflationary expectations."*

Hence nominal rates must be corrected for the expected rate of price changes to give the real rate, the Fisherian real rate of interest. In symbols

$$R^D = R - 4\left(\frac{\Delta P}{P} 100\right)^E \qquad (7.2)$$

Recalling our discussion in the previous chapter (pp.206-207) in which we approximated the expected rate of inflation by

$$\left(\frac{\Delta P}{P} 100\right)^E \simeq P_t^E - P_{t-1} = \lambda_2 \left(P_t - P_{t-1}\right) \qquad (7.3)$$

we shall write the real rate on debts as

$$R_t - 4\, a_4' \,\lambda_2 \left(P_t - P_{t-1}\right) \qquad (7.4)$$

where a_4' is expected to lie in the vicinity of one but as a result of the approximation may not be exactly one.

(2) The real return on equities. The starting notion is the real return on capital but, as it will be explained later, for reasons

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[5] D. I. Fand: "The Monetary Theory of Nine Recent Quarterly Econometric Models of the United States: A Comment." Journal of Money, Credit and Banking, 1971, p.455.

of quantification of this notion one needs to switch to equities. In dynamic equilibrium the real return on capital will be the one at which every moment in time all existing capital will be held by the community. This was defined by Tobin as the supply price of capital. As Tobin puts it:[6] *"The strategic variable is the rate of return that the community of wealth-owners require in order to absorb the existing capital stock (valued at current prices), no more, no less, into their portfolios and balance sheets. This rate may be termed the supply price of capital."* It is seen that the supply price of capital despite its name is not a monetary valuation but it is rather an equilibrium rate of return.

If the supply price of capital is defined in this way then it must be that rate of return which equalises the present value of all expected future income flows from a unit of capital to its market price, i.e.

$$P_0 = \frac{X_1}{1 + k} + \frac{X_2}{(1 + k)^2} + \frac{X_3}{(1 + k)^3} + \ . \ . \ . \qquad (7.5)$$

where P_0 is the market price of this unit of capital, X_1 , X_2 , X_3 . . . is the sequence of expected income flowing from it and k is the supply price of capital. k bears the alternative labels of marginal efficiency of investment and of cost of capital and it is not a market rate but an internal rate of return. We shall avoid the use of the term supply price of capital because, as we shall see later on when we examine the investment function, the term supply price of capital has been used by Keynes to denote the price P_0 in (7.5), which is equilibrium is of course equal to the market price. We shall use instead

[6] J. Tobin: "Monetary Theory: New and Old Looks. Money, Capital and Other Stores of Value." American Economic Review, Papers and Proceedings, 1961, p.35

the term cost of capital.

At this point a switch must be made in order to be able to quantify the cost of capital. The latter refers to physical capital and is the return at which wealth-owners are willing to hold all existing capital. As however data on the value of physical units of capital and expected future income from them are not easily available, we should switch from physical capital to the other form of capital, namely equity capital. This is a valid step if we assume that all of physical capital is owned by corporations. The assumption is not unreasonable since divergences from it are rather insignificant in practice. What exactly this switch means is that instead of examining physical capital as such we examine its ownership, because corporations are owned by the holders of equity capital (or common stock shares). Consequently in (7.5) we should value equities rather than capital with the understanding that if wealthowners require a certain rate of return on their capital, the corporation also requires this return since the latter represents its shareholders and acts in their interests. This is a very important point which will help to clarify the nature of the incone variables which appear in the valuation formula (7.5). It has been sometimes wrongly assumed that this variable is represented by the earnings of the corporation, namely the sum of its dividends and its retained earnings. But what the investor (shareholder) buys is dividends and it can be shown that dividends alone should enter the present value calculations.[7]

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[7] The following demonstation appears in J. F. Weston and E. F. Brigham: Managerial Finance. Holt, Rinehart and Winston Inc., New York, 1966, pp.297-8.

Suppose that an investor buys a stock which he is going to sell in a year's time. The value of the stock to him will be

$$P_0 = \frac{D_1}{1 + k} + \frac{P_1}{1 + k} \tag{7.6}$$

i.e. discounted dividends from the stock plus the discounted price of the stock at the end of the year. The value of the stock is set equal to its price today P_0, because by definition the cost of capital k is the discount rate at which all capital is absorbed, i.e. the value obtained from the present value calculations is equal to the market price. Some other investor will buy the stock at the end of the year at the price P_1 equal to the value

$$P_1 = \frac{D_2}{1 + k} + \frac{P_2}{1 + k} \tag{7.7}$$

By substituting (7.7) into (7.6) we get

$$P_0 = \frac{D_1}{1 + k} + \frac{D_2}{(1 + k)^2} + \frac{P_2}{(1 + k)^2} \tag{7.8}$$

If we continue this *ad infinitum* we get

$$P_0 = \frac{D_1}{1 + k} + \frac{D_2}{(1 + k)^2} + \frac{D_3}{(1 + k)^3} + \ldots \tag{7.9}$$

So it is seen that dividends are the appropriate income variable in the valuation formula (7.5). The fact that dividends are considered for an infinite period of time answers at a microeconomic level the criticisms concerning the application of this formula to corporations which at the present time do not pay dividends at all. The value of these corporations is not zero as it would be implied if we considered only the first term in (7.9) but it is positive, since by the higher retention of earnings the corporations increase their earning potentiality so that higher dividends can be expected to accrue to shareholders in the future.

Now assume that dividends are expected to grow at an average compound rate g . Then (7.9) can be written as

$$P_0 = \frac{D_0 \ (1 + g)}{1 + k} + \frac{D_0 \ (1 + g)^2}{(1 + k)^2} + \frac{D_0 \ (1 + g)^3}{(1 + k)^3} + \ . \ . \ . \qquad (7.10)$$

By using the formulas for annuities (7.10) is reduced to

$$P_0 = \frac{D_0}{k - g} \qquad (7.11)$$

Solving (7.11) for the cost of capital we obtain

$$k = \frac{D_0}{P_0} + g = R^C \qquad (7.12)$$

so that the cost of capital consists of the current dividend yield plus the average rate of growth of dividends expected in perpetuity. Clearly (7.12) refers to one firm and there will be a range of cost of capital depending on the degree of risk associated with the operation of this particular firm. Nevertheless it will make sense to construct an average series for all corporations. The expected rate of growth of dividends is not observable and it will be replaced by proxy variables. Homa and Jaffee[8] have used the money stock and its rate of growth in estimating the level of stock prices and indirectly the growth in expected future dividends. As they put it[9]

> *"Given the demand for money, a decrease in the supply of money will raise interest rates and reduce interest sensitive expenditures such as capital investment. The*

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[8] K. E. Homa and D. M. Jaffee: "The Supply of Money and Common Stock Prices." Journal of Finance, 1971, pp.1045-66.

[9] *Ibid*. p.1047.

> *decrease in expenditures together with the standard multiplier, will then cause a reduction in the firm's sales and thus a decrease in its earnings. The timing of the effect of the decreased earnings on dividends may depend on the firm's cash flow and liquidity position, but ultimately the full effect must be a decrease in dividends. Although the current price of the common stock share will fall if current dividends are reduced, the main point of leverage for the effect of the money supply is on the expcted growth rate of dividends."*

The growth rate of the money supply is also included because *"it may be particularly useful in accounting for short-run variations in expectations."*

Following Homa and Jaffee we shall include, as a proxy for g, the level and the change in the money supply rather than its growth rate because our model is linear. Thus

$$R_t^c = R_t' + b_4' M_t + c_4' (M_t - M_{t-1}) \tag{7.13}$$

where R' is the dividend yield.

(3) The risk premium. If the return on equity capital was known with certainty, it would be equal to the real return on debts in the case of perfect substitutability. But this return is not known with certainty. Equity capital participates in earnings after all suppliers of funds have received their interest payments so that there is a risk associated with this residual participation as the level of earnings may vary depending on the state of the economy. Equity funds will then be forthcoming only if a risk premium is added on top of the real return on debts. This premium is denoted by ρ and is assumed constant.

We have so far examined the three terms in (7.1). Acceptance of the perfect substitutability hypothesis would imply that

$$R_t - 4a_4'\lambda_2 (P_t - P_{t-1}) = R_t' + b_4' M_t + c_4' (M_t - M_{t-1}) - \rho \qquad (7.14)$$

We are however going to relax this assumption and let the substitution characteristic be freely determined by the data. Thus we have

$$R_t' + b_4' M_t + c_4' (M_t - M_{t-1}) = d_4' [R_t - 4a_4'\lambda_2 (P_t - P_{t-1})] + \rho + u_{4t}' \qquad (7.15)$$

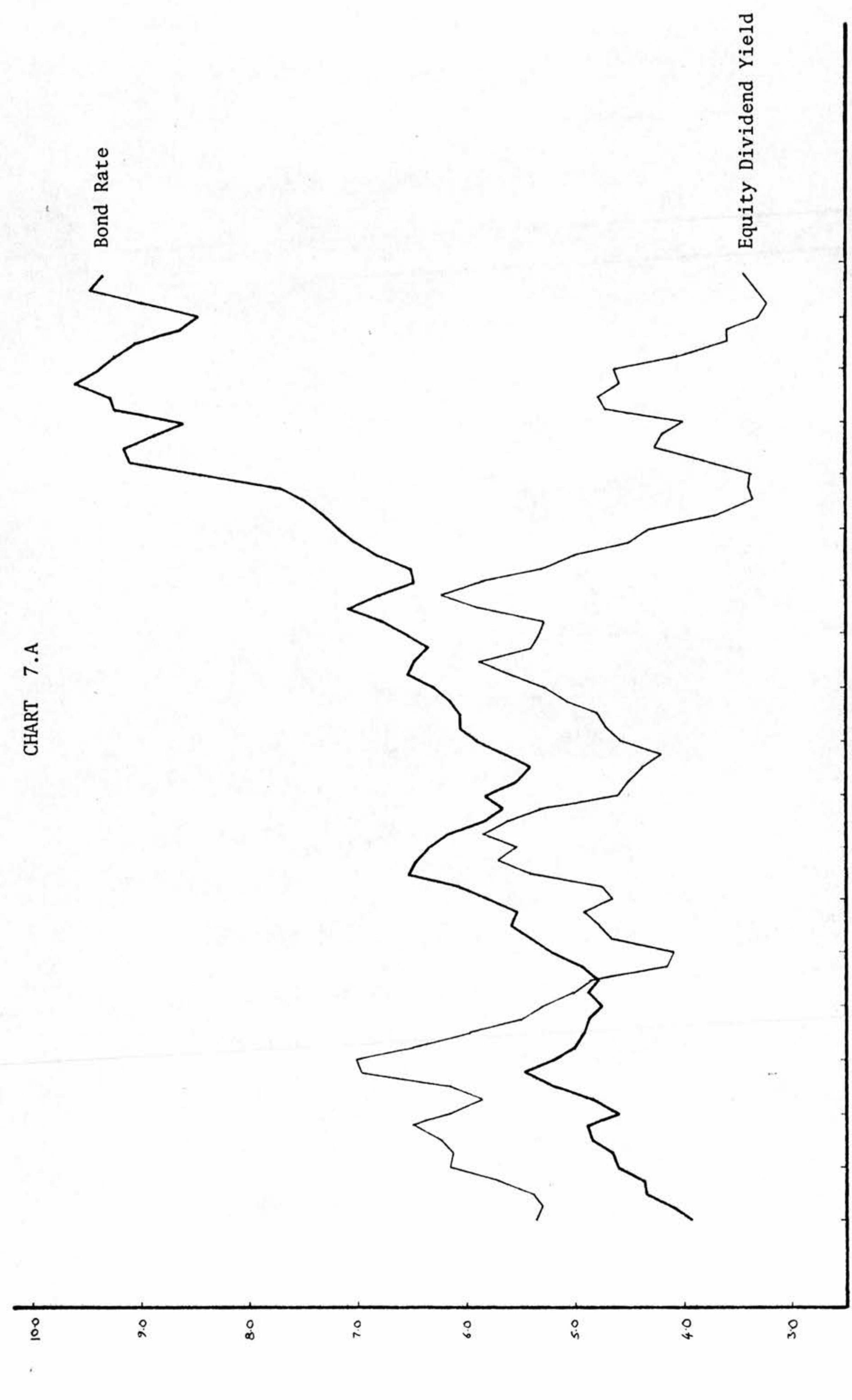
A value of d_4' not different from one would confirm that bonds and equities are perfect substitutes, whereas a value of d_4' less than one, and a higher value of ρ , would indicate the preponderance of the risk element against the substitutability characteristic. In other words in the last case the effect of risk will be to make smaller the size of the effect of a change in the real bond yield on the cost of capital. Hendershott and Van Horne forced d_4' to be one by estimating an equation in which the difference between the nominal bond yield and the dividend yield of equities was the dependent variable. The two yields are shown in Chart 7.A where it is observed that similar fluctuations are imposed on two different trends in the two series.

Further we assume that the partial adjustment mechanism applies to the dividend yield component of the cost of capital rather than to the whole of the latter, i.e.

$$R_t' - R_{t-1}' = \lambda_3 (R_t^{*\prime} - R_{t-1}') \qquad (7.16)$$

where $R^{*\prime}$ is the equilibrium value of R' , which appears in (7.15). The reason for the application of the adjustment mechanism to the dividend yield is that the cost of capital is an internal rate of return

CHART 7.A


Bond Rate
Equity Dividend Yield
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
1955
1956
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1972
1973

used in present value calculations whereas the dividend yield is a market magnitude which moreover correlates stronger with the price of equities; this happens because the second component of the cost of capital, namely the expected rate of dividend growth is negatively related to dividend yield[10] and positively to equity prices so that if the latter change as a result of some external force the dividend yield exhibits these movements better than the cost of capital. Also we note incidentally that the dividend yield is a better indicator of the movement in equity prices than the other market yield pertaining to equities, i.e. the ratio (in percentage terms) of earnings to the price of equities. This is true because earnings generally fluctuate more than dividends as the companies usually follow a policy of more or less smooth payment of dividends. Consequently there is a better correlation of the dividend yield with the index of market prices. For the U.K. and for the years 1962.2 to 1972.4 the correlation between the price index of common stocks and their dividend yield was -0.88 and the corresponding correlation with the earnings yield was -0.84 .

Combining (7.15) and (7.16) we obtain the equation to be estimated as

$$\begin{aligned} R'_t &= \lambda_3 d'_4 R_t - 4\lambda_3 d'_4 a'_4 \lambda_2 (P_t - P_{t-1}) - \lambda_3 b'_4 M_t - \\ &\quad - \lambda_3 c'_4 (M_t - M_{t-1}) - \lambda_3 \rho + (1 - \lambda_3) R'_{t-1} + \lambda_3 u'_{4t} = \\ &= a_4 R_t + b_4 (P_t - P_{t-1}) + c_4 M_t + d_4 (M_t - M_{t-1}) + \\ &\quad + e_4 + f_4 R'_{t-1} + u_{4t} \end{aligned} \tag{7.17}$$

[10] M.J. Gordon: <u>The Investment, Financing and Valuation of the Corporation</u>. Irwin, Homewood Illinois, 1962, pp.51-2.

Again we shall examine the possible existence of autocorrelation in the residuals due to the approximation of the expected rate of inflation by a linear term. Since λ_3 can be identified, an approximation to the cost of capital series can be taken from the estimates of the parameters of this equation and it will be

$$\tilde{R}_t^c = R_t' + \tilde{b}_4' M_t + \tilde{c}_4' (M_t - M_{t-1}) \qquad (7.18)$$

This variable will be the one to use in the investment function as it will be explained there. The parameter λ_2 of the price equation can not be identified from (7.17) unless we assume that $a_4' = 1$. Unlike $\dot{\lambda_2}$, the parameter of substitutability d_4' and the risk premium ρ are identifiable.

<u>The consumption function</u>. The aim of specifying a consumption function is to explain aggregate consumption expenditure of the economy as this is defined in the national accounts. Thus the distinction of different types of consumption expenditure according to the durability of the goods will not be made here but the act of purchase and its relationship to total income and production will be examined in the aggregate.[11] Consumption expenditures refer to expenditures made by households but not all expenditures by households are classified as consumption expenditure, e.g. expenditure on new houses.

On what variables does consumption depend? The first and more important variable is real income. According to Keyne's 'fundamental psychological law' *"men are disposed, as a rule and on the average, to*

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[11] Where data on depreciation and the stock of the so called consumer durable goods exist it may be profitable to distinguish between expenditure functions for durables and consumption functions for durables. But the lack of satisfactory data for this country (and the purpose for which we build our model) will restrict our consumption variable to total private consumer expenditure.

increase their consumption as their income increases, but not by as much as the increase in their income."[12] The appropriate income variable suggested by Keynes is 'net income' which *"a man has in mind when he is deciding his scale of consumption."*[13] The generally accepted and currently used measure of net income is personal disposable income.

Keynes expected the average propensity to consume to fall as income increased so that a redistribution of income in favour of the poor would presumable raise consumption. Tests of this proposition have usually been conducted by dividing income in the consumption function into labour income and property income and testing whether the relevant coefficients are unequal. The evidence in general did not offer support to the above inequality.[14] However these tests rested on the assumption that 'functional shares vary by income bracket'[15] so that distributive shares can be considered as a proxy for distribution of incomes by size. But this was questioned by Blinder[16] who noticed that labour's share in the U.S. was increased during the past two decades whereas conventional measures of inequality in the size distribution of

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[12] J. M. Keynes: The General Theory of Employment, Interest and Money. Harcourt Brace, 1936, p.36.

[13] *Ibid*, p.42.

[14] For a recent study see L. D. Taylor: "Saving Out of Different Types of Income." Brookings Papers on Economic Activity, 1971, pp.383-407.

[15] See D. B. Suits: "The Determinants of Consumer Expenditure: A Review of Present Knowledge." In The Impacts of Monetary Policy. Commission on Money and Credit. Prentice Hall, 1963, p.15.

[16] A. S. Blinder: "Distribution Effects and the Aggregate Consumption Function." Journal of Political Economy, 1975, pp.447-75.

incomes have been rather constant. Blinder used measures of income inequality in the function and found that they as well do not influence consumption.

Further propositions about consumption behaviour associated with Keynes are: (a) the short-run marginal propensity to consume is less than the long-run marginal propensity to consume and the response to a change in current income is small because *"a man's habitual standard of life usually has the first claim on his income and he is apt to save the difference which discovers itself between his actual income and the expense of his habitual standard"*[17] while over a longer period of time his consumption habits are adjusting more flexibly. Estimated consumption functions in the interwar period did not take into account this proposition and were of a static form. But postwar studies incorporated it in the analysis by assuming the partial adjustment model to hold for consumption expenditures.[18] (b) *"changes in the money value of wealth should be classified amongst the major factors capable of causing short period changes in the propensity to consume."*[19] This proposition although ignored in empirical research is of interest in view of the concern of economists with the Pigou effect of which it is in some respects the opposite.[20] The Pigou effect refers to the effect of real wealth on consumption and represents a post Keynesian development. It

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[17] J. M. Keynes, *op.cit.*, p.97.

[18] See for example L. R. Klein and A. S. Goldberger: An Econometric Model of the United States, 1929-52. North-Holland, 1955; and R. Stone and D. A. Rowe: "Aggregate Consumption and Investment Functions for the Household Sector Considered in the Light of British Experience." Nationalökonomisk Tidskrift, 1956, pp.1-32.

[19] J. M. Keynes, *op.cit.*, p.93.

[20] D. B. Suits, *op.cit.*, p.11.

has been introduced by Pigou[21] as a means of showing that with flexible prices and wages the long-run equilibrium position of the economy would be one in which full employment would prevail. The reason is that consumption is an increasing function of real wealth and a change in the price level would change the real value of wealth shift the consumption function and warrant that full-employment output is obtained. The effect of wealth on consumption has been analysed and tested extensively either as a Pigon effect or as an effect specified by other hypotheses on consumer behaviour. Such hypotheses are: (1) the life-cycle hypothesis according to which the consumption of a household *"reflects a more or less conscious attempt at achieving the preferred distribution of consumption over the life cycle subject to the constraint imposed by the size of resources accruing to the household over its lifetime."*[22] This hypothesis gives the consumption function of a household and, by aggeegating over households, the aggegate consumption function as a linear function of expected income and of aggregate net worth.[23] (2) the permanent income hypothesis also suggests that the consumer takes account of the resources he has but only until a relatively near 'horizon'.[24] While this assumption may be important for some purposes the reduced form corresponding to this hypothesis may be translated into life-cycle terms.[25] According to the permanent income hypothesis[26]

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[21] A. C. Pigou: "Economic Progress in A Stable Environment." Economica, 1947, pp.180-8.

[22] F. Modigliani: "The Life Cycle Hypothesis of Saving, the Demand for Wealth and the Supply of Capital." Social Research, 1966, p.167.

[23] See A. Ando and F. Modigliani: "The Life Cycle Hypothesis of Saving: Aggregate Implications and Tests." American Economic Review, 1963, pp.55-87 and 1964, pp.111-3.

[24] M. Friedman: "The Concept of Horizon in the Permanent Income Hypothesis." In C. F. Christ *et.al.*, Measurement in Economics, Stanford University Press, 1963.

[25] See K. Hilton and D. H. Crossfield: "Short-Run Consumption Functions for the U.K., 1955-66." In The Econometric Study of the United Kingdom. Edited by K. Hilton and D. F. Heathfield. Macmillan, 1970, p.60.

[26] M. Friedman: A Theory of the Consumption Function. Princeton University Press, 1957.

planned or permanent consumption is a proportion of permanent or expected income, i.e.

$$C_t^P = k\, Y_t^P \tag{7.19}$$

Permanent income is used again as a surrogate for wealth (cf our discussion on the money demand function) by assuming either that the discount rate is constant or that the coefficient k in (7.19) depends on this discount rate. There remains the problem of estimating permanent income. Friedman employed the adaptive expectations hypothesis slightly extended to include a growth effect. In continuous formulation

$$Y_t^P = \beta \int_0^{\infty} e^{(\beta - a)(T - t)}\, Y(T)\, dT \tag{7.20}$$

where a is the growth effect and if $a = o$, (7.20) is the solution to the continuous adaptive expectations model

$$\frac{d\, Y_t^P}{dt} = \beta\, (Y_t - Y_t^P) \tag{7.21}$$

A discrete approximation of (7.20) is given by

$$Y_t^P = a \sum_{i=o}^{\infty} \lambda^i\, Y_{t-i} + u_t \tag{7.22}$$

i.e. permanent income is a distributed lag function of current and post incomes with geometrically declining weights. A further approximation is to truncate (7.22) after a certain number of terms. Friedman truncated it after seventeen terms and constructed a series of permanent income (annual U.S. data) with a value of λ which produced the highest

R^2 in the consumption function. But it is not at all necessary to truncate the above distributed lag function and carry out the laborious process of finding the appropriate λ . Klein[27] pointed out that by applying the Koyck transformation the consumption function one gets for estimation has the following form

$$C_t = a\, Y_t + \lambda\, C_{t-1} + u_t - \lambda\, u_{t-1} \tag{7.23}$$

which has been used in various Keynesian models and which is identical to Brown's[28] consumption function based on his work on lags in consumer behaviour.

It will be remembered that the concept of wealth used by Friedman in the money demand function included both human and non-human wealth and that empirical research which used a wealth variable restricted it to measurable non-human wealth. In the same way, where estimates of the net worth of individuals are available the wealth variable can be used as an alternative to permanent income and there is no *a priori* reason for preferring the one or the other. The marginal propensity to consume out of a change in wealth is presumably smaller than the marginal propensity to consume out of a change in permanent income because the change in wealth which is an exhaustible stock as opposed to a recurring flow (income), will be spread over a longer period of time. Spiro[29] and Ball and Drake[30] have shown that in the case where consumption

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[27] L. R. Klein: "The Friedman-Becker Illusion." Journal of Political Economy, 1958, p.541.

[28] T. M. Brown: "Habit Persistence and Lags in Consumer Behaviour." Econometrica, 1952, pp.355-71.

[29] A. Spiro: "Wealth and the Consumption Function." Journal of Political Economy, 1962, pp.339-54.

[30] R. J. Ball and P. S. Drake: "The Relationship Between Aggregate Consumption and Wealth." International Economic Review, 1964, pp.63-81.

is proportional to wealth the long-run marginal propensity to consume out of income in stationary equilibrium with no growth and even when wealth is not included in the consumption function, is unity. Their final equation is

$$C_t = \frac{k}{k+1} Y_t + \frac{1}{k+1} C_{t-1} \tag{7.24}$$

where

$$k = \frac{C_t}{W_t} \tag{7.25}$$

(7.24) provides a test of the above hypothesis since the coefficients of income and lagged consumption must sum to unity.

The wealth effects on consumption stem from changes in real wealth following a revaluation of the existing stock of assets held by the private sector. A change in the real value of these assets may be induced by changes in output prices (price induced wealth effect) or by changes in the rate of interest (interest induced wealth effect). The first effect is the Pigou effect mentioned above. The second effect will operate if changes in the market rate of interest affect the market value of equities and government bonds and may be an important channel of transmission of policy changes on aggregate demand.[31] Another

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[31] Modigliani recently reported simulations with the FMP model which showed the importance of the interest induced wealth effect. Net worth in that model is endogenous and includes among other items changes in the market value of corporate equity which in turn depends on the real rate of interest. See F. Modigliani: "Monetary Policy and Consumption: Linkages via Interest Rate and Wealth Effects in the FMP Model." In Consumer Spending and Monetary Policy: The Linkages. Proceedings of a Monetary Conference held on Nantucket Island, Massachusetts, June 1971. Federal Reserve Bank of Boston, 1971, pp.3-84.

possible determinant of consumption expenditures is liquid assets, a variable which is part of wealth and which is considered either as a proxy for it or as providing a direct incentive to the consumer to spend.[32]

As far as our specification of the consumption function is concerned we shall concentrate on the personal disposable income variable rather than wealth since there are no quarterly estimates of the latter for the United Kingdom. The notion of permanent income is not absolutely necessary in the specification because the partial adjustment model provides the same lag distribution for income and has been successfully used in consumption functions which are part of an IS-LM model.[33] The difference between using permanent income and the partial adjustment mechanism for consumption lies in the properties of the error term in each case. While the former implies an error term which is a first order moving average of the error term of the equilibrium consumption function, the latter gives us an error term which is a constant multiple of the error term in the equilibrium consumption function.

The use of the reduced form corresponding to the partial adjustment model in the consumption function can be attributed to Brown.[34] who stressed the gradual adjustment of the consumer to changes in his

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[32] For empirical studies on the importance of liquid assets on consumption expenditures see among others K. Hilton and D. H. Crossfield, *op.cit.*; A. Zellner: "The Short Run Consumption Function." Econometrica, 1957, pp.552-66; and A. Zellner, D. S. Huang and L. C. Chau: "Further Analysis of the Short Run Consumption Function with Emphasis on the Role of Liquid Assets." Econometrica, 1965, pp.570-81.

[33] See for example G. C. Chow, *op.cit.*, 1967.

[34] T. M. Brown, *op.cit.*

income. His theory is based on consideration of psychological characteristics of consumers (inertia, habit persistence) and assumes that the decline of the effects of past habits is continuous rather than discontinuous as Duesenberry's[35] theory on consumer behaviour had suggested. Duesenberry had also appealed to psychological characteristics but the adjustment of consumers he invoked was discontinuous. The consumption function under his hypothesis can be approximately written as

$$C_t = a + b\,Y_t + c\,Y^0 \tag{7.26}$$

where Y^0 is the peak previous income of consumers. Brown's argument was to include previous consumption rather than income as the relevant lagged variable so that the consumption function is

$$C_t = a + b\,Y_t + c\,C_{t-1} \tag{7.27}$$

which is exactly the reduced form of the partial adjustment model.

So far the discussion of the determinants of consumer expenditure has been done under the implicit assumption that real consumption is not affected by the price level, i.e. consumers are able to convert their nominal income (or wealth) into real income (or wealth) which determines their real consumption expenditures, i.e. consumers do not suffer from price-level illusion (or money illusion as it is commonly named). This assumption however may not be true and money illusion may exist in consumers. Then instead of considering say $\frac{Y}{P}$ (Y is

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[35] J. S. Duesenberry: Income, Saving and the Theory of Consumer Behavior. Cambridge, Harvard University Press, 1949.

nominal income and P the price level) as a determinant of $\frac{C}{P}$ (C is nominal consumption), one should consider $\frac{Y}{P^a}$ instead, where a is a positive parameter.[36] When $a = 1$ consumers are free from money illusion. When $a > 1$ real income is underestimated and prices are thought to be higher than they really are. Conversely, when $a < 1$ consumers overestimate their purchasing power. A linear approximation of $\frac{Y}{P^a}$ would be $b \frac{Y}{P} + c P$, where c is positive when $a < 1$, zero when $a = 1$ and negative when $a > 1$.

The existence of money illusion in the consumption function has been tested for the U.S. by Branson and Klevorick[37] who used a log linear form of the function having both real income and real wealth as arguments. To this formulation they have added the price level as an additional variable to indicate the existence or non existence of money illusion. The results showed that money illusion was indeed present during their sample period.[38] Branson and Klevorick argued that money illusion is a short-run phenomenon and the consumption function which displays it is a short-run consumption function because *"this sensitivity of real consumption to the price level will lead to the conclusion that real consumption will exceed Gross National Product if prices rise relative to real income and real wealth for a long period of time."*[39] This statement however is not necessarily true because

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[36] See E. J. Kane and A. K. Klevorick: "Absence of Money Illusion: A Sine Qua Non for Neutral Money?" Journal of Finance, 1967, p.420.

[37] W. H. Branson and A. K. Klevorick: "Money Illusion and the Aggregate Consumption Function." American Economic Review, 1969, pp.832-49.

[38] Further tests about money illusion were carried out by A. Cukierman: "Money Illusion and the Aggregate Consumption Function: Comment." American Economic Review, 1972, pp.198-206. Cukierman introduced in the function five individual additional price levels in order to allow for substitution effects in response to changes in relative prices and found that the existence of money illusion is weakened but not altered.

[39] W. H. Branson and A. K. Klevorick, *op.cit.*, p.841.

(a) the effect on consumption of a rise in the price level depends on the size of the estimated coefficients, and (b) more importantly, real income even in the simplest models of income determination is endogenous through the identity connecting income with consumption and therefore an increase in consumption itself represents an increase in real income which, if not accompanied by an analogous expansion of investment, will be realised at the expense of real wealth of the economy. We conclude that money illusion can exist even as a long-run phenomenon and need not be restricted to the short-run.

Another effect of the price level on consumption mentioned by Branson and Kleverick was the effect that expected price changes might have on the allocation of consumption in time. That is "*if consumers expect prices to rise in the future, they will restructure the time pattern of their consumption by moving consumption from the future toward the present. Then, if their expectations are realized they will reduce their consumption in the future.*"[40] Such a pattern of behaviour would mean that if the current rate of change of prices was only included in the function one would get a positive coefficient attached to it. However it is conceivable that expectations of price changes might have the opposite effect, namely if consumers expect a rise in the rate of change of prices in relation to the expected rate of change of their incomes they might reduce their consumption expenditure hoping that in the future the rate of increase of prices will slow down relative to the rate of increase in their incomes, so that their real position (real income, real wealth) will be improved.

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[40] W. H. Branson and A. K. Klevorick, op.cit., p.835.

Accordingly the expected rate of price changes will have a negative sign if included as an argument in the function.

Following the discussion above we shall specify the consumption function for our model as follows. Assume that equilibrium consumption is a function of disposable income, the price level and expected price changes, i.e.

$$C_t^* = a_5' Y_t^d + b_5' P_t + c_5' \lambda_2 (P_t - P_{t-1}) + u_{5t}' \qquad (7.28)$$

where Y^d is disposable personal income and $\lambda_2 (P_t - P_{t-1})$ stands as before for the expected rate of inflation (approximately). Disposable income is given by

$$Y_t^d = Y_t - T_t \qquad (7.29)$$

where T includes U.K. taxes on income, national insurance and health contributions, net transfers abroad and taxes paid abroad, business undistributed profits, business taxes and statistical discrepancies. The price level is included to test for the existence of money illusion which, if it exists, will be as we have argued above, both a short-run and a long-run phenomenon. In this case b_5' can be positive or negative depending on whether consumers overestimate or underestimate their purchasing power. On the other hand the effect of price changes on consumption will be a short-run phenomenon because in long-run stationary equilibrium $P_t = P_{t-1}$ and this term will vanish. Again c_5', if significant, can be positive or negative depending on the interpretation which is found to be true for consumers behaviour.

the relation

$$C = \frac{R_1}{1 + k} + \frac{R_2}{1 + k} + \; . \; . \; . \; . \; . \qquad (7.32)$$

In equilibrium the marginal efficiency of capital equals the market rate of interest and the supply price of the asset equals its market price. Then the above formula becomes precisely the same as the one which we have used to derive the equation relating rates of return on bonds and equities. We have noted there that Tobin has defined k as the supply price of capital, a definition which we shall avoid so that confusion is not created, because for Keynes it is C in (7.32) which is the supply price of capital.

Equation (7.32) in equiblibrium shows the equality between the demand price and supply price of capital goods. Demand arises from those who use capital goods for the production of consumer goods (or conceivably of other capital goods) and is determined by the prospective yield of investment to investors (wealthowners). On the other hand the supply of capital goods will be continued if the price offered for capital goods is greater than or equal to the supply price of capital, i.e. *"that price which would just induce a manufacturer newly to produce an additional unit of such assets."*[43] Thus in (7.32) both supply and demand considerations are present. This has been noted by Haavelmo who emphasised that *"Keynes has often been misinterpreted on this point due to an unfortunate slip on page 136 of the General Theory, where he talks about 'the demand for investment' instead of what he actually means,*

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[43] J. M. Keynes, *op.cit.*, p.135.

namely, an equation resulting from a conjunction of the supply of investment goods and the marginal efficiency of capital."[44]

Having defined the marginal efficiency of capital Keynes goes on to specify the investment function by noting that if more investment takes place during any period of time this will reduce the marginal efficiency of capital for two reasons, (a) the prospective yield on investment falls due to the existing higher supply of capital (long-run), and (b) the supply price of capital rises because of the pressure which is put on resources for the production of more capital goods (short-run). The above assumption means that in equilibrium the interest rate will also be reduced. Conversely, if there is a fall in the marginal efficiency of capital and the interest rate (in equilibrium) there will be an increased volume of investment precisely for the same reasons described above.

The relation between aggregate real investment and the marginal efficiency of capital (or interest rate in equilibrium) is the schedule of marginal efficiency of capital. It is also called by Keynes the investment demand schedule although it is derived from a condition of equality of supply and demand price of capital.

The next important issue to clarify is the equality between the marginal efficiency of capital and the interest rate in equilibrium. Is this interest rate the nominal interest rate or the real interest

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[44] T. Haavelmo: A Study in the Theory of Investment. The University of Chicago Press, 1960, p.196.

rate? Keynes gives clear indications in his General Theory that it is the real interest rate which is equal to the marginal efficiency of capital. We quote the following passages:

> *"Professor Fisher uses his rate of return over cost in the same sense and for precisely the same purpose as I employ the marginal efficiency of capital."*[45]

> *"Professor Fisher's theory could be best rewritten in terms of a real rate of interest."*[46]

> *"The prices of existing assets will always adjust themselves to changes in expectation concerning the prospective value of money. The significance of such changes in expectations lies in their effect on the readiness to produce new assets through their reaction on the marginal efficiency of capital. The stimulating effect of the expectation of higher prices is due not to its raising the rate of interest but to its raising the marginal efficiency of capital."*[47]

Thus it is seen that the marginal efficiency of capital is equal to the real rate of interest which is given by the nominal rate minus the expected rate of inflation, and the investment schedule is simply

$$I = f\,[R - (\frac{\Delta P}{P})^{E}] \qquad (7.33)$$

or in linear form

$$I = a\,[R - (\frac{\Delta P}{P})^{E}] \qquad a < o \qquad (7.34)$$

where I is real investment.

If (7.34) is plotted on a nominal rate, real investment plane, the expected rate of inflation becomes a shift factor raising

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[45] J. M. Keynes, op.cit., p.141.

[46] Ibid, p.143.

[47] Ibid. p.142.

the marginal efficiency schedule when there are expectations of a higher rate of inflation in the future. In this way the marginal efficiency of capital is a factor through which "*(much more than through the interest rate) the expectation of the future influences the present.*"[48]

The (nominal) rate of interest is a "*current phenomenon; and if we reduce the marginal efficiency of capital to the same status, we cut ourselves off from taking any direct account of the influence of the future in our analysis of the existing equilibrium.*"[49]

The above rather extensive discussion of Keynesian theory has been given in order to establish that the assumption that real investment is a function of real interest rate has a Keynesian origin. This has not been recognised in the literature where Keynesian theory has been sometimes associated with rigid prices. It is monetarists who are supposed to emphasise the dependence of real investment on the real rate of interest without however attributing this to Keynes. It is noteworthy that empirical work so far has either not used a real rate of interest in the investment function or used an inappropriate proxy for the cost of capital such as the earnings yield on ordinary shares.[50]

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[48] J. M. Keynes, *op.cit.*, p.145.

[49] *Ibid.*, p.146.

[50] For an example where the earnings yield is considered as the cost of capital determining the volume of private investment see D. R. Hodgman: "Credit Controls in Western Europe: An Evaluative Review." In Credit Allocation Techniques and Monetary Policy. Federal Reserve Bank of Boston, Conference Series No.11, 1973, p.145.

There are other factors which affect investment besides the interest rate. We shall mention here the accelerator and flexible accelerator approaches to investment demand because they have both been found important in empirical applications. According to the acceleration principle[51] there is a fixed capital - output ratio a, i.e.

$$K_t = a\, Y_t \tag{7.35}$$

where K is the capital stock at the end of the period and Y is the output of this period. Then by differencing (7.35) we get that net investment is a function of the change in output

$$\Delta K_t = I_t = a\, \Delta Y_t \tag{7.36}$$

However, this simple form of the function gave very poor results when estimated from actual data and coefficient a did not correspond well to capital - output ratio estimates. The main criticism against its use was that it is not appropriate for situations of underutilisation of the capital stock. This specification has been improved in two alternative ways which both involved dynamising (7.35).

According to the one of them known as the flexible accelerator approach desired capital (K^*_t) is proportional to output (Y_t)

$$K^* = a\, Y_t \tag{7.37}$$

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[51] The best known early study based on this principle is Clark's study, see J. M. Clark: "Business Acceleration and the Law of Demand." Journal of Political Economy." 1917, pp.217-35.

and the partial adjustment model applies to the adjustment of actual capital to desired one

$$K_t - K_{t-1} = \lambda (K^*_t - K_{t-1})$$

or

$$I_t = \lambda a Y_t - \lambda K_{t-1} \qquad (7.38)$$

This equation implies that actual capital stock is a distributed lag function of output with the weights declining geometrically. This was the original formulation of the hypothesis by Koyck.[52]

Because the quality of data on the capital stock is not particularly good an alternative hypothesis was sought. According to it

$$K^*_t = a Y_t \qquad (7.39)$$

$$I^*_t = K^*_t - (1 - d) K^*_{t-1} \qquad (7.40)$$

$$I_t - I_{t-1} = \lambda (I^*_t - I_{t-1})$$

or

$$I_t = \lambda a [Y_t - (1 - d) Y_{t-1}] + (1 - \lambda) I_{t-1} \qquad (7.41)$$

where now I is gross investment, the starred I indicates desired gross investment and d is the depreciation rate. The above model is based on the standard assumption that depreciation is a constant proportion d of the capital stock. Equation (7.41) may be preferable to (7.38) because the latter involves the capital stock variable for

[52] L. M. Koyck: Distributed Lags and Investment Analysis. North-Holland, Amsterdam, 1954.

which an initial estimate is required whereas (7.41) involves only the knowledge of the depreciation rate which may be credible even if the initial estimate of the capital stock is not so precise. Alternatively d can be estimated freely from the data.[53]

Most of the recent empirical work on the investment function emanates from Jorgenson's work,[54] and estimates equations which are detailed investment functions rather appropriate for large scale models of the economy. We shall however examine briefly Jorgenson's work because it has influenced the specification of the investment functions (to be used in small models) as regards the interest rate variable.

Jorgenson developed a theory of investment on the basis of the neoclassical theory of optimal capital accumulation. The theory considers a firm producing an output by using labour and capital inputs, both the output and inputs being traded in perfectly competitive markets. The combination of the inputs in order to produce the output is done in such a way as to maximise the net receipts of the firm but is subject to the constraints of the production function and the identity relating capital investment and depreciation. Thus the problem is to find the maximum of

$$\int_0^\infty e^{-rt} \Big[p(t)\, X(t) - s(t)\, L(t) - q(t)\, I(t) - u(t)[p(t)\, X(t) - s(t)\, L(t) - q(t)\, \{v(t)\, \delta + w(t)\, r - x(t)\, \frac{\dot{q}}{q}\}\, K(t)]\Big]\, dt \qquad (7.42)$$

[53] See G. C. Chow, *op.cit.*, 1967, for such an approach.

[54] D. W. Jorgenson: "Capital Theory and Investment Behavior." American Economic Review, 1963, pp.247-59; D. W. Jorgenson: "Anticipations and Investment Behavior." In The Brookings Quarterly Model of the United States, Edit. by J. S. Duesenberry *et.al.*, North Holland, 1965, pp.35-92 and D. W. Jorgenson: "The Theory of Investment Behavior." In Determinants of Investment Behavior. Edit. by R. Ferber, NBER, 1967, pp.129-55.

subject to

$$X(t) = A\,[K(t)]^{\alpha}\,[L(t)]^{\beta} \qquad (7.43)$$

and

$$\dot{K}(t) = I(t) - \delta\,K(t) \qquad (7.44)$$

where

r	=	the cost of capital	u	=	tax rate of net income
p	=	price of a unit of output	v	=	tax allowance on depreciation
X	=	output	δ	=	depreciation rate
s	=	wage rate	w	=	tax allowance for debt finance
L	=	labour	x	=	tax allowance for capital loss
q	=	price of a capital good	K	=	capital stock
I	=	investment	A	=	constant

and the dot on the variable denotes its rate of change. The above maximisation problem was solved to give that the optimum capital stock is

$$K = \alpha\,\frac{pX}{c} \qquad (7.45)$$

where

$$c = q\left[\left(\frac{1 - uv}{1 - u}\right)\delta + \left(\frac{1 - uw}{1 - u}\right)r - \left(\frac{1 - ux}{1 - u}\right)\frac{\dot{q}}{q}\right] \qquad (7.46)$$

The variable c can be interpreted as the cost of using a unit of capital (implicit rental value of capital services supplied by the firm to itself) and has a similarity with Keynes' user cost of capital, i.e. the reduction in value of equipment due to using it compared with not using it.

Several criticisms were raised against Jorgenson's approach. Firstly Klein noted that it is unfortunate to confound a general assumption such as a profit maximisation assumption with a particular production function (the Cobb-Douglas) and with the assumption of market perfection.[55] Klein would not *"find fault with those who would use theory simply to generate a list of variables involved in the equation for desired capital stock and then use some tractable statistical approximation for the estimation of the implied relationships."* Second, (7.45) gives the equilibrium capital stock which will be demanded for given relative prices. But defining equilibrium as the stationary solution $\frac{dK}{dt} = 0$, *"there is no positive net investment unless something (e.g. prices) changes and disrupts the previous equilibrium. In this sense, net investment is viewed as a disequilibrium phenomenon."*[56] Indeed the investment function is obtained by considering net desired investment as a difference between desired capital stock at the end of the period and desired capital stock at the beginning of the period and assuming that replacement investment is proportional to desired capital stock at the beginning of the period, i.e.

$$I_t = \Delta K_t + \delta K_{t-1} \tag{7.47}$$

where I is gross investment and δ is the depreciation rate. To the above formulation a dynamic adjustment mechanism was added by Jorgenson to account for time lags involved in the completion of investment projects and the non instantaneous achievement of desired net investment, i.e.

$$I = \frac{\gamma(L)}{w(L)} \Delta \left(\alpha\frac{pX}{c}\right)_t \tag{7.48}$$

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[55] L. R. Klein: "Issues in Econometric Studies of Investment Behavior." Journal of Economic Literature, 1974, p.44.

[56] Z. Griliches: "On Crockett-Friend and Jorgenson" In Determinants of Investment Behavior. Edit. by R. Ferber, NBER, 1967, p.160. For exactly the same criticism see also J. Tobin "On Crockett-Friend and Jorgenson.", pp.156-60.

where $\frac{\gamma(L)}{w(L)}$ is the rational lag function and L the lag operator defined by $Lx_t = x_{t-1}$. The dynamisation of the function does not alter the basic postulate of it, namely that in static equilibrium net investment will be zero.

The result of such an assumption is that the cost of capital (interest rate) when used as a determinant of desired capital stock enters the investment function in first difference form. Consider for example the simplified function

$$K_t = a\, c_t \qquad a < 0 \tag{7.49}$$

where

$$c_t = q_t\, (d_t + r_t) \tag{7.50}$$

(7.50) is simply (7.46) with $\frac{\dot{q}}{q} = 0$ and $u = w = 1$. By ignoring the effects of relative prices and assuming constant rates of depreciation,[57] we can write that net investment is related to the change in the interest rate

$$\Delta K_t = a\, (r_t - r_{t-1}) \tag{7.51}$$

If we want to consider gross investment instead of net investment, (7.51) becomes

$$I_t = a\, [r_t - (1 - d)\, r_{t-1}] \tag{7.52}$$

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[57] See for example G. C. Chow, *op.cit.*, 1967, p.2.

and this is the specification introduced by Chow and used for the study of investment in U.K. manufacturing industry by Hines and Cataphores.[58] On the other hand there are studies which, apart from the accelerator variable have employed the level of the interest rate rather than its difference following the Keynesian specification.[59]

In specifying therefore our investment function two alternatives can be examined as regards the cost of capital variable. The one which uses the level of the cost of capital and the other which uses its change. Thus desired gross private investment is given by

$$I^*_t = a'_6 \, [Y_t - (1 - d) \, Y_{t-1}] + b'_6 \, R^c_t \tag{7.53}$$

or

$$I^*_t = a'_6 \, [Y_t - (1 - d) \, Y_{t-1}] + b'_6 \, [R^c_t - (1 - d) \, R^c_{t-1}] \tag{7.54}$$

where d is the depreciation rate of privately owned capital stock estimated at 0.0095 (or 0.038 at annual rate) and R^c is the cost of capital, an estimated proxy for which will be obtained from (7.18). Furthermore, the price level will be included in both versions of the equations to test for the existence of money illusion in investment decisions. Also the effect of expected price changes will be examined. The prospect of continued price increases may make firms decide to invest more in fixed capital now while prices are relatively low, whereas if price increases are expected to be temporary the effect on investment

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[58] A. G. Hines and G. Catephores, *op.cit.*

[59] See for example the studies by J. Kmenta - P. E. Smith, *op.cit.*, and J. R. Moroney - J. M. Mason, *op.cit.*, both of which present empirical work on the IS-LM model.

will be the opposite. Desired investment taking account of the above possible influences will be

$$I^*_t = a'_6 [Y_t - (1 - d) Y_{t-1}] + b'_6 R^c_t + c'_6 P_t + $$
$$+ d'_6\lambda_2 (P_t - P_{t-1}) + u'_{6_t} \qquad (7.55)$$

or

$$I^*_t = a'_6 [Y_t - (1 - d) Y_{t-1}] + b'_6 [R^c_t - (1 - d) R^c_{t-1}] + $$
$$+ c'_6 P_t + d'_6\lambda_2 (P_t - P_{t-1}) + u'_{6_t} \qquad (7.56)$$

We next assume that a fraction λ_2 of the discrepancy between desired investment and last period's actual investment is realised, i.e.

$$I_t - I_{t-1} = \lambda_5 (I^*_t - I_{t-1}) \qquad (7.57)$$

Combining (7.55) or (7.56) and (7.57) we get for estimation

$$I_t = \lambda_5 a'_6 [Y_t - (1 - d) Y_{t-1}] + \lambda_5 b'_6 R^c_t + \lambda_5 c'_6 P_t + $$
$$+ \lambda_5 d'_6\lambda_2 (P_t - P_{t-1}) + (1 - \lambda_5) I_{t-1} + \lambda_5 u'_{6_t} = $$
$$= a_6 [Y_t - (1 - d) Y_{t-1}] + b_6 R^c_t + c_6 P_t + $$
$$+ d_6 (P_t - P_{t-1}) + e_6 I_{t-1} + u_{6_t} \qquad (7.58)$$

or

$$I_t = \lambda_2 a'_6 [Y_t - (1 - d) Y_{t-1}] + \lambda_5 b'_6 [R^c_t - (1 - d) R^c_{t-1}] + $$
$$+ \lambda_5 c'_6 P_t + \lambda_5 d'_6\lambda_2 (P_t - P_{t-1}) + (1 - \lambda_5) I_{t-1} + \lambda_5 u'_{6_t} = $$
$$= a_6 [Y_t - (1 - d) Y_{t-1}] + b_6 [R^c_t - (1 - d) R^c_{t-1}] + $$
$$+ c_6 P_t + d_6 (P_t - P_{t-1}) + e_6 I_{t-1} + u_{6_t} \qquad (7.59)$$

In (7.58) and (7.59) the parameter d_6' is not uniquely identified. All other parameters can be obtained from estimates of coefficients a_6 , b_6 , c_6 and e_6 .

The Inventory Function. Inventory investment is the most volatile component of GDP and is thought to contribute to the strength of expansions in economic activity and to the severity of recessions because it tends to rise in the former and fall in the latter. Inventory investment is the difference between a period's production and that period's final sales of goods and services and can be positive, zero or negative. The latter will be the case when businesses run down their inventories because their sales exceed what they are producing. The stock of inventories consists of three types of goods: goods in progress, finished goods and raw materials. The function of the first is to support the time needed for the production and distribution processes to be carried out. There are always some goods which are either undergoing some physical transformation or are in transit through the network of distribution. Finished goods are stocked in order to act as a buffer against unexpected variations in demand or in order to meet regularly known fluctuations in sales thus avoiding the cost of production varying in line with demand. Finally businesses hold stock of raw materials because of fear of unexpected changes in supply or because costs may be significantly lower when they order raw materials in large lots and in excess of the current needs of production.

For the estimation of inventory functions the flexible accelerator approach mentioned in the investment function has been proved quite fruitful.[60] For the case of inventories the method is a little more complicated since the distinction is made between desired or equilibrium stock of inventories and planned stocks of inventories. Thus the desired stock of inventories S_t^* is a function of expected sales $\hat{Y}_t$

$$S_t^* = a_7 \hat{Y}_t \tag{7.60}$$

This desired level of inventories will be generally different from last period's actual level and the firm will plan to change its inventory according to this discrepancy. But it is more realistic to assume a partial adjustment rather than a complete one:[61]

$$H_t^p = \lambda_6 (S_t^* - S_{t-1}) \tag{7.61}$$

where H^p is planned investment in inventories and λ_6 the coefficient of adjustment. This equation is the one corresponding to the flexible accelerator equation in the investment function. (7.61) is one possible formulation of the adjustment process. An alternative has been suggested by Johnston[62] namely

$$H_t^p = S_t^* - S_{t-1}^* + \mu (S_{t-1}^* - S_{t-1}) \tag{7.62}$$

The firm may plan to accumulate inventories but the planned stock will differ from the actual stock if expected sales differ from actual ones. The following relation describes this situation

$$S_t = S_t^* + \hat{Y}_t - Y_t \tag{7.63}$$

[60]One of the most wellknown studies based on the flexible accelerator approach is M. Lovell's "Manufacturers' Inventories, Sales Expectations and the Acceleration Principle." Econometrica, 1961, pp.293-314.

[61]This was first tried by R. L. Klein in Economic Fluctuations in the United States, 1921-1941. Cowles Commission Monograph 11, Wiley, New York, 1950. Klein however did not make the distinction between desired and planned inventories.

[62]J. Johnston, "An Econometric Study of the Production Decision." Quarterly Journal of Economics, 1961, pp.234-61.

It is therefore important to know how expectations of sales are formed. Ball and Drake[63] in an empirical study of inventory functions for the U.K. considered the following two forecasting devices for expected sales

$$\hat{Y}_t = Y_t - \theta\ (Y_t - Y_{t-1}) \tag{7.64}$$

and

$$\hat{Y}_t = \sum_{i=0}^{\infty} \tau^{i+1}\ Y_{t-1} \tag{7.65}$$

Ball and Drake combined each of the equations (7.61) and (7.62) with (7.64) or (7.65) and together with (7.60) and (7.63) they obtained four equations for estimation. Of these only one permits the identification of the initial parameters and it will be for this reason the one which we shall use in our model. This consists of equations (7.60), (7.61), (7.63) and (7.64). We shall however expand equation (7.60) to include a price level variable which will test the existence of money illusion, and an expected price change variable to account for possible accumulation of inventories by businesses for speculative reasons. The rate of inflation has been included in an inventory equation by Burrows[64] who found that it is not significant. However our preliminary work indicated that price changes can not be discarded from the equation. Therefore we assume that

$$S_t^* = a_7' \hat{Y}_t + b_7' P_t + c_7'\lambda_2\ (P_t - P_{t-1}) \tag{7.66}$$

and (7.66) replaces (7.60). Eliminating H^p , $\hat{Y}$ and S^* we get for estimation

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[63] R. J. Ball and P. S. Drake: "Stock Adjustment Inventory Models of the United Kingdom Economy." Manchester School of Economic and Social Studies, 1963, pp.87-103.

[64] P. Burrows: "Explanatory and Forecasting Models of Inventory Investment in Britain." Applied Economics, 1971, pp.275-89.

$$H_t = \lambda_6 a_7' Y_t - \theta (1 + \lambda_6 a_7') (Y_t - Y_{t-1}) + \lambda_6 b_7' P_t +$$

$$+ \lambda_6 c_7' \lambda_2 (P_t - P_{t-1}) - \lambda_6 S_{t-1} + u_{7t}$$

$$= a_7 Y_t + b_7 (Y_t - Y_{t-1}) + c_7 P_t + d_7 (P_t - P_{t-1}) +$$

$$+ e_7 S_{t-1} + u_{7t} \qquad (7.67)$$

With the exception of c_7' which is not uniquely identified, all other parameters are identifiable. The variable S will be obtained as in Ball and Drake's article, namely by using the identity $S_t = S_{t-1} + H_t$ and assuming that the initial unknown stock of inventories S_0 is absorbed in the constant term of the equation.

The Import Function. The effect of including an import function (expressing imports as a function of income) in simple IS-LM models is to show explicitly the leakage from the expenditure that imports represent and to reduce the size of fiscal and monetary policy multipliers. A realistic analysis therefore of government policies in the context of the IS-LM apparatus should take account of the import function. For this reason we shall specify an equation for import demand and incorporate it in our model.

The basic argument in an import function is the level of home demand. It might be argued that personal disposable income determines import demand if imports consisted entirely of consumption goods. But since a large part of British imports are raw materials we shall include real GDP as the relevant variable.[65]

Another variable which is assumed to influence the demand for imports is the ratio of U.K. prices of goods and services to prices of goods and services in the rest of the world. This variable is in turn influenced by the rate of exchange between the U.K. and the rest of the world. Empirical work which included such a variable in the function showed however that it was not significant.[65] This may be partly due to the existing inadequacies of import and export price indices. Instead of this ratio we shall include in the equation the exchange rate for which also there is some evidence that it has had a perverse effect on imports.[67]

Two more variables will be included in the equation. One is a variable to pick up the effect of the import surcharge imposed at the last quarter of 1964 and lifted at the last quarter of 1966. The

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[65] Many researchers analysed import demand in terms of the individual components of aggregate demand and in particular inventories which were found to have a higher marginal import content, see for example W. A. H. Godley and J. R. Shepherd: "Forecasting Imports." National Institute Economic Review, August 1965, pp.35-42, and I. G. Black, J. E. Kidgell and G. F. Roy: "Forecasting Imports: A Re-examination." National Institute Economic Review, November 1967, pp.52-7. Such a detailed analysis however is beyond the scope of our work and we shall be content to find the marginal propensity to consume on the average by including total GDP as an explanatory variable.

[66] See for example H. S. Houthakker and S. P. Magee. "Income and Price Elasticities in World Trade." Review of Economics and Statistics, 1969, pp.111-25, and National Institute of Economic and Social Research: "The Effects of the Devaluation of 1967 on the Current Balance of Payments." Economic Journal, 1972, pp.442-64.

[67] See National Institute of Economic and Social Research, *op.cit.*, and M. C. Deppler: "Some Evidence on the Effects of Exchange Rate Changes on Trade." International Monetary Fund Staff Papers, 1974, pp.605-36.

variable takes on the value 15 in 1964.4 and 1965.1 and 10 in 1965.2 to 1960.4 and it has been found significant in previous empirical work. The second variable is a dummy variable to pick up the effect of the import deposit scheme. Import deposits were introduced in the last quarter of 1968 and abolished in the last quarter of 1970 after being gradually wound down. The variable takes on the value 1 in 1968.4 to 1970.4 and 0 elsewhere. Finally the price level will be again included in the equation to test for money illusion. Therefore we specify equilibrium import demand as

$$X_t^* = a_8' Y_t + b_8' E_t^R + c_8' P_t + d_8' D_t^1 + e_8' D_t^2 + u_{8t}' \qquad (7.68)$$

where E^R is the exchange rate, D^1 the import surcharge variable and D^2 the import deposit variable. Assuming that the partial adjustment mechanism applies to the adjustment of actual imports to equilibrium imports we get for estimation

$$\begin{aligned} X_t &= \lambda_7 a_8' Y_t + \lambda_7 b_8' E_t^R + \lambda_7 c_8' P_t + \lambda_7 d_8' D_t^1 + \\ &\quad + \lambda_7 e_8' D_t^2 + (1 - \lambda_7) X_{t-1} + \lambda_7 u_{8t}' \\ &= a_8 Y_t + b_8 E_t^R + c_8 P_t + d_8 D_t^1 + e_8 D_t^2 + f_8 X_{t-1} + u_{8t} \end{aligned} \qquad (7.69)$$

where λ_7 is the coefficient of adjustment. All parameters in (7.68) as well as λ_7 are identifiable.

<u>Identities</u>. The following identities complete the model.

(A) $$Y_t = C_t + I_t + H_t + G_t + I_t^P + E_t - X_t \qquad (7.70)$$

where G is government current expenditure on goods and services

I^P is public investment expenditure, and

E are exports.

(B) $$S_t = S_{t-1} + H_t \qquad (7.71)$$

(C) The identity of finance of the public sector's borrowing requirement.

An additional identity in the second of the two versions of our model will be the identity describing how the net borrowing requirement of the public sector is met. Christ has repeatedly called for the government budget constraint to be taken into account explicitly into econometric models.[68] His review of nine major U.S. econometric models, including detailed financial submodels, revealed that none of them has incorporated the government budget restraint.[69] The latter is written by Christ as[70]

$$G + r_f B = T + \Delta B + \Delta H - \Delta D - \Delta F + \Delta Z \qquad (7.72)$$

where

G : the sum of federal purchases of goods and services plus transfers payments other than interest

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[68] See C. F. Christ: "A Short-Run Aggregate Demand Model of the Interdependence and Effects of Monetary and Fiscal Policies with Keynesian and Classical Interest Elasticities." American Economic Review, 1967, pp.434-43. C. F. Christ: "A Simple Macroeconomic Model with a Government Budget Restraint." Journal of Political Economy, 1968, pp.53-67. C. F. Christ: "Econometric Models of the Financial Sector." Journal of Money, Credit and Banking, 1971, pp.419-49 and C. F. Christ: "Monetary and Fiscal Influences on U.S. Money Income, 1891-1970." Journal of Money, Credit and Banking, 1973, pp.279-300.

[69] C. F. Christ, *op.cit.*, 1971, p.421. The models considered were the Wharton model, the OBE model, the 1968 Michigan Model, the original 1965 Brookings Model, the 1968-69 condensed Brookings Model, the December 1967 FRB-MIT model, the January 1968 FRB-MIT model, the December 1968 FRB-MIT model and the FRB-Chicago Model.

[70] C. F. Christ, *op.cit.*, 1973, pp.280-1.

$B, \Delta B$: the level and increase in privately held federal debt

r_f : interest rate on federal debt

$r_f B$: the federal debt interest payments to private sector

T : sum of federal tax receipts

ΔH : the increase in unborrowed high powered money

ΔD : the increase in federal government deposits at commercial banks

ΔF : the increase in federal government and Federal Reserve holdings of gold and foreign exchange

ΔZ : the net increase in other liabilities of the consolidated federal government and Federal Reserve.

A striking deficiency of this budget restraint is that it ignores the capital account of the federal government and considers only the current account. Thus public investment expenditures are excluded and so is the amount paid to finance maturing debt. But clearly changes in, say, ΔB or ΔH are intended to finance the borrowing requirement on capital account which includes the balance on current account. We shall remove this shortcoming by examining in detail the capital and current accounts of the Public Sector.

The alleged importance of the government budget constraint consists in (a) making one of the variables included in it endogenous. If for example the government has decided upon its expenditure and taxation policies, amount of borrowing and other means of financing a deficit except printing money, then it has no choice about how much money to issue: this will be given by the above identity;[71]

[71] See C. F. Christ, *op.cit.*, 1968, p.53.

(b) affecting the analysis of macroeconomic policies: *"the multiplier effect of a change in government purchases can not be defined until it is decided how to finance the purchases, and the value of the multiplier given by the generally accepted analysis (which ignores the government budget restraint) is in general incorrect. The one year impact multiplier effect of government purchases may be greater or less than the value obtained by ignoring the budget restraint, depending on whether the method of financing is mainly by printing money or mainly by taxation."*[72]

While the first point is clear and may have implications for estimation, the second one blends two distinctly different issues. The first is that the multiplier for government expenditure cannot be considered independently of other multipliers showing how government spending is financed, i.e. the tax multiplier or the multiplier with respect to a change in high powered money. A multiplier needs to be examined which shows the effect on income resulting from a change in government expenditure compensated by an equal change, say, in taxes or high powered money a mix of these two. This is a point stressed by Monetarists who, moreoever, argue that government spending is expansionary only when it is financed by an increase in high powered money.[73] The second is that multipliers corresponding to compensated changes are different when the government budget constraint is taken into account explicitly from those where it is not. We shall try to make clear these two issues by a simple example. Consider the simplified static IS-LM model

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[72] C. F. Christ, op.cit., 1968, pp.53-4.

[73] See for example D. I. Fand: "Some Issues in Monetary Economics." Federal Reserve Bank of St. Louis Review, January 1970, pp.10-27.

$$Y_t = E_t + G_t \tag{7.73}$$

$$E_t = a_1 + b_1 (Y_t - T_t) + c_1 R_t \tag{7.74}$$

$$b_1 > 0 \qquad c_1 < 0$$

$$R_t = a_2 + b_2 B_t + c_2 Y_t \tag{7.75}$$

$$b_2 < 0 \qquad c_2 > 0$$

where E is the sum of private consumption and investment expenditures

G and T are government expenditure and taxes respectively

R is the interest rate, and

B is high powered money.

The price level is assumed rigid. Equation (7.75) can be thought of as a reduced form corresponding to a money demand, a money supply function and the equilibrium condition.

$$M_t^D = d_1 + d_2 Y_t + d_3 R_t \tag{7.76}$$

$$M_t^S = d_4 B_t \tag{7.77}$$

$$M_t^D = M_t^S \tag{7.78}$$

The above system does not include the government budget constraint but multipliers on income of compensated changes in exogenous variables G , T and B can be taken. We write the system in matrix form

$$\begin{bmatrix} Y_t \\ E_t \\ R_t \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ b_1 & 0 & c_1 \\ c_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y_t \\ E_t \\ R_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & -b_1 & 0 \\ 0 & 0 & b_2 \end{bmatrix} \begin{bmatrix} G_t \\ T_t \\ B_t \end{bmatrix} \quad \text{or}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ -b_1 & 1 & -c_1 \\ -c_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y_t \\ E_t \\ R_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -b_1 & 0 \\ 0 & 0 & b_2 \end{bmatrix} \begin{bmatrix} G_t \\ T_t \\ B_t \end{bmatrix} \quad \text{or}$$

$$\begin{bmatrix} Y_t \\ E_t \\ R_t \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -b_1 & 1 & -c_1 \\ -c_2 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -b_1 & 0 \\ 0 & 0 & b_2 \end{bmatrix} \begin{bmatrix} G_t \\ T_t \\ B_t \end{bmatrix} =$$

$$= \frac{1}{1 - b_1 - c_1 c_2} \begin{bmatrix} 1 & -b_1 & c_1 b_2 \\ b_1 + c_1 c_2 & -b_1 & c_1 b_2 \\ c_2 & -b_1 c_2 & b_2(1-b_1) \end{bmatrix} \begin{bmatrix} G_t \\ T_t \\ B_t \end{bmatrix} \qquad (7.79)$$

From (7.79)

$$\frac{\partial Y}{\partial G} = \frac{1}{1 - b_1 - c_1 c_2} > 0 \quad \frac{\partial Y}{\partial T} = \frac{-b_1}{1 - b_1 - c_1 c_2} < 0 \quad \text{and}$$

$$\frac{\partial Y}{\partial B} = \frac{c_1 b_2}{1 - b_1 - c_1 c_2} > 0$$ and the effect corresponding to

$\Delta G = \Delta T$ is

$$\frac{\partial Y}{\partial G} + \frac{\partial Y}{\partial T} = \frac{1 - b_1}{1 - b_1 - c_1 c_2} \quad \text{with} \quad 0 < \frac{1 - b_1}{1 - b_1 - c_1 c_2} < 1$$

whereas the effect corresponding to $\Delta G = \Delta B$ is

$$\frac{\partial Y}{\partial G} + \frac{\partial Y}{\partial T} = \frac{1 + c_1 b_2}{1 - b_1 - c_1 c_2} > 0$$ which is always greater

than the previous one. Now according to Christ the inclusion of the government budget constraint leads to different multiplier results. With the identity

$$G_t = T_t + X_t - X_{t-1} + B_t - B_{t-1} \tag{7.80}$$

added (X is the amount of government debt in the hands of the public), the static system of equations (7.73) to (7.75) becomes dynamic as Christ notes.[74] By considering B as the endogenous variable and assuming that the values of the parameters are the true values, the enlarged system is written as

$$\begin{bmatrix} Y_t \\ E_t \\ R_t \\ B_t \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ b_1 & 0 & c_1 & 0 \\ c_2 & 0 & 0 & b_2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y_t \\ E_t \\ R_t \\ B_t \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ E_{t-1} \\ R_{t-1} \\ B_{t-1} \end{bmatrix} +$$

$$+ \begin{bmatrix} 1 & 0 & 0 \\ 0 & -b_1 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} G_t \\ T_t \\ X_t \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} G_{t-1} \\ T_{t-1} \\ X_{t-1} \end{bmatrix} \tag{7.81}$$

or $$y_t = A\, y_{t-1} + B_0\, x_t + B_1\, x_{t-1} \quad \text{(in matrix notation)} \tag{7.82}$$

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[74] C. F. Christ, *op.cit.*, 1968, p.55.

with

$$A = \frac{1}{1 - b_1 - c_1c_2} \begin{bmatrix} 0 & 0 & 0 & c_1b_2 \\ 0 & 0 & 0 & c_1b_2 \\ 0 & 0 & 0 & b_2(1-b_1) \\ 0 & 0 & 0 & 1-b_1-c_1c_2 \end{bmatrix} \qquad (7.83)$$

From standard theory of multiplier analysis the matrix of total multipliers is given by

$$(I - A)^{-1}\ (B_0 + B_1) \qquad (7.84)$$

$(I - A)^{-1}$ is not however obtainable since $I - A$ is singular. Thus we see that the static model of equations (7.73) to (7.75) is not appropriate to accompany identity (7.80). A more realistic dynamic model is required which is not easy to handle analytically. It may be the case that consideration of the identity in a dynamic model differentiates multiplier results and this remains to be examined. Similarly the values of the parameter estimates in the other equations may be different when a previously exogenous variable is converted into endogenous by considering the identity; this is a statistical matter which cannot be answered *a priori*.

While the last considerations are generally recognised there has been a defense of the omission of the identity from large models based on theoretical grounds.[75] The identity represents the equilibrium condition for the government securities market and this market is excluded from consideration by virtue of Walras law.

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[75] E. M. Gramlich: "Comments on the Discussions of Carl Christ Paper." Journal of Money, Credit and Banking, 1971, p.466.

For the reasons explained above we shall include the identity in one of the two versions of our model. We have already briefly examined it in the form it was presented by Goodhart (p.172). The identity was expressed as

$$F^1 + F^2 + GF = MP + NP + MB + \Delta S^D + \Delta B \qquad (7.85)$$

where the variables are as defined in page 172. We shall now examine more carefully the net borrowing requirement of the public sector. To this end we need to consider the current and capital accounts of the Central Government, the Local Authorities and the Public Corporations. The accounts are shown below.

I Central Government (including National Insurance Funds)

A : Current Account

Receipts	Payments
1. Gross trading surplus	7. Current expenditure on goods and services
2. Rent, dividends and interest	8. Subsidies
3. Taxes on income	9. National Insurance Benefits
4. Taxes on expenditure	10. Other Current Grants to Personal Sector
5. National Insurance, National Health and Redundancy Fund Contributions	11. Debt Interest
6. Balance: Current Surplus (Saving)	12. Current grants to Local Authorities
	13. Current grants paid abroad

B : Capital Account

Receipts	Payments
1. Surplus from current account	9. Gross domestic fixed capital formation
2. Taxes on capital	10. Increase in value of stocks and work in progress
3. Proceeds of iron and steel disposals	11. Capital transfers to { Local authorities; Public corporations; Private sector }
4. Receipts from certain pension funds	

(cont.)

B : Capital Account
(cont.)

Receipts	Payments
5. Adjustments for purchase tax and subsidies	12. Net lending to Private Sector Local Authorities Public Corporations Overseas Governments Private Industry, etc. abroad International Organisations
6. Miscellaneous capital receipts	
7. Net Borrowing by Northern Ireland Central Government	
8. Balance: Net Borrowing Requirement	

II Local Authorities

A : Current Account

Receipts	Payments
1. Gross trading surplus	6. Current Expenditure on Goods and Services
2. Rent, Dividends and Interest	7. Housing Subsidies
3. Rates Cash Receipts Accruals Adjustment	8. Current Grants to Personal Sector
4. Current Grants from Central Government	9. Debt Interest
5. Balance: Current Surplus (Saving)	10. Taxes on Income

B : Capital Account

Receipts	Payments
1. Surplus from current account	6. Gross domestic fixed capital formation
2. Capital Grants from Central Government	7. Capital Grants to Persons
3. Net Borrowing Central Government Other	8. Net lending for House Purchase
4. Accruals Adjustment for Rates	
5. Unidentified Items	

III Public Corporations

A : Current Account

Receipts	Payments
1. Current Revenue	3. Current Expenditure
2. Balance: Current Surplus (Saving)	

B : Capital Account

Receipts	Payments
1. Surplus from current account	8. Gross domestic fixed capital formation
2. Capital transfers (net receipts)	9. Increase in value of stocks and work in progress
3. Loans from Central Government	10. Miscellaneous financial assets
4. Stock issued less stock redeemed	11. Unidentified Items
5. Bank lending	
6. Trade Credit (net)	
7. Borrowing from own Superannuation Fund	

Some remarks on these accounts are in order.

(a) The balances on current account of the Central Government, Local Authorities and Public Corporations constitute their savings and appear on the receipt side of the respective capital accounts.

(b) The sum of items 7 in I A and 6 in II A gives the total of government expenditure on goods and services which appears in the tables for the expenditure on gross domestic product. In the symbols of our model this is variable G in identity (7.70) multiplied by the price level P .

(c) Similarly the sum of items 9 in I B, 6 in II B and 8 in III B forms the total of public investment in fixed capital - variable I^P in identity (7.70) multiplied by the price level P .

(d) With the exception of variables G and I^P discussed above, statistical information regarding the variables in the capital account of the Central Government and in both accounts of the Local Authorities and Public Corporations is non existent for the years 1955 to 1959 (or 1961 in some cases) and this prevents explicit treatment of these variables in the identity showing the financing of the public sector borrowing requirement.

(e) The Public Sector deficit appears as the balance of the Capital Account of the Central Government only. The capital account of the Local Authorities and Public Corporations is always balanced and this is achieved through loans and current or capital transfers from the Central Government.

(f) The amount needed to finance maturing debt in the hands of the public is included in item 11 of I B, i.e. capital transfers to the private sector.

From the above accounts the net borrowing requirement can be written as

$$\begin{aligned} NBR = & \Big[(\text{items 7 to 13 in I A}) + (\text{items 9 to 12 in I B}) + \\ & + (\text{items 6 to 10 in II A}) + (\text{items 6 to 8 in II B}) + \\ & + (\text{item 3 in III A}) + (\text{items 8 to 11 in III B})\Big] - \\ & - \Big[(\text{items 1 to 5 in I A}) + (\text{items 2 to 7 in I B}) + \\ & + (\text{items 1 to 4 in II A}) + (\text{items 2 to 5 in II B}) + \\ & + (\text{item 1 in III A}) + (\text{items 2 to 7 in III B})\Big] \end{aligned} \qquad (7.86)$$

This is obtained from the capital accounts of the Central Government, Local Authorities and Public Corporations, in which the surpluses from current account were replaced by the differences between receipts and expenditure on current account. In the right hand side of (7.86) the items which balance the capital accounts of the Local Authorities and Public Corporations, namely loans and current or capital transfers from the Central Government, cancel out because they appear as receipts for the former and expenditure for the latter. From the host of variables in the above definition we can detach some which either are present elsewhere in the model or can be accounted for separately because of the existence of statistical data for them. These are (a) government expenditure on goods and services: since this variable is expressed in real terms in identity (7.70) this part of NBR will be written as

$\frac{PG}{100}$ where P is the GDP implicit deflator, G is real expenditure and their product is divided by 100 because of the specific scale considered for the price variable (see data appendix); (b) taxes on income and national insurance and health contributions. We have already included in the consumption function a variable which exceeds this by the amount of undistributed corporate profits, net transfers abroad, taxes paid abroad and possible statistical discrepancies. We shall therefore consider here the term $\frac{-PT}{100}$ where T is as defined in the consumption function section, and adjust the remaining items for the above variables; (c) items in the Central Government's current account other than taxes on income, National Insurance and National Health Contributions and current expenditure on goods and services. The statistical series on expenditure items minus receipts is pretty erratic and since most of the variables included are of no particular concern to us, we shall separate taxes on expenditure (net of subsidies) so that we can study the effects of changes in these taxes on other variables of the system. So, the term included separately is $\frac{-PT^i}{100}$ where T^i are taxes on expenditure net of subsidies adjusted to the total of the tables of expenditure on GDP . With the qualifications made above the net borrowing requirement is written as

$$NBR = \frac{PG}{100} - \frac{PT}{100} - \frac{PT^i}{100} + Z^A \qquad (7.87)$$

where Z^A includes all other variables not taken into account explicitly. The first three terms in (7.87) are products of variables which appear linearly in our model and the addition of the identity for the finance of the borrowing requirement would render it nonlinear.

To avoid nonlinearity a linear approximation will be introduced by using the Taylor expansion of a function of two variables.

The Taylor expansion of a function $f(x,y)$ of two variables x and y is given by

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[f'_x(a,b)(x-a) + f'_y(a,b)(y-b)\right] + \\ + \frac{1}{2!}\left[f''_{xx}(a,b)(x-a)^2 + 2f''_{xy}(a,b)(x-a)(y-b) + \\ + f'_{yy}(a,b)(y-b)^2\right] + \ldots \quad (7.88)$$

where the function and its partial derivatives are calculated for $x = a$ and $y = b$. The approximation occurs if we consider only the first two terms which contain x and y linearly, and neglect all others. Concerning the values of a and b around which the function is approximated, an obvious choice for them are the means of the variables.

For P, G, T and T^i we have

$$\overline{P} = 107.52 \qquad \overline{G} = 1947.375 \qquad \overline{T} = 2314.278$$
$$\overline{T}^i = 995.014$$

Then the approximations will be as follows

(a) $$\frac{PG}{100} \simeq \frac{\overline{P}\overline{G}}{100} + \frac{1}{1!}\left\{\frac{\overline{G}}{100}(P - \overline{P}) + \frac{\overline{P}}{100}(G - \overline{G})\right\} =$$

$$= \frac{-\overline{P}\overline{G}}{100} + \left(\frac{\overline{G}}{100}\right)P + \left(\frac{\overline{P}}{100}\right)G = -2093.8 + 19.47375\,P + 1.0752\,G$$

(b) $\frac{-PT}{100} \simeq \frac{-\bar{P}\bar{T}}{100} - \frac{1}{1!}\left\{\frac{\bar{T}}{100}(P - \bar{P}) + \frac{\bar{P}}{100}(T - \bar{T})\right\} =$

$$= \frac{\bar{P}\bar{T}}{100} - (\frac{\bar{T}}{100}) P - \frac{\bar{P}}{100} T = 2488.3 - 23.14278 P - 1.0752 T$$

(c) Similarly

$$\frac{-PT^i}{100} \simeq \frac{\bar{P}\bar{T}^i}{100} - (\frac{\bar{T}^i}{100}) P - (\frac{\bar{P}}{100}) T^i = 1069.8 - 9.95014 P - 1.0752 T^i$$

The detailed calculations are shown in Tables A.13, A.14, A.15, of Appendix A. The approximation is generally satisfactory, the average error of approximation being 4.3% in (a), 4.0% in (b) and 4.6% in (c). Consequently we can write as

$$NBR = 1464 - 13.61917 P + 1.0752 (G - T - T^i) + Z^A + Z^B \qquad (7.89)$$

where Z^B includes the error of the approximation.

We are now in a position to consider identity (7.85) showing the borrowing requirement of the public sector and the way it is financed. Again lack of statistical information for MP , NP and MB for the earlier years of the study compels us to lump them together with Z^B so that the final form of the identity which will be used in our model is

$$1464 + 1.0752 (G_t - T_t - T_t^i) - 13.61917 P_t + Z_t = \Delta B_t + \Delta S_t^D \qquad (7.90)$$

where $Z = Z^A + Z^B$ is the lump sum variable which includes (with a negative sign) the fundamental instrument of monetary policy namely the sales of marketable and nonmarketable debt by the authorities.

Taking account of this identity means that not all variables therein (except of course P which is determined by the price equation) can be determined independently. One of them, given the values of the others will be endogenous; this variable is for the purpose of our work the monetary base. Making the monetary base an endogenous variable is a practice commonly used in theoretical analyses which incorporate the government budget constraint.[76] Also the criticisms about the controllability of it[77] can be better expressed in terms of an endogenous variable. Thus identity (7.90) is written as

$$B_t = B_{t-1} - S_t^D + S_{t-1}^D + 1464 + 1.0752\ (G_t - T_t - T_t^i) -$$

$$- 13.61917\ P_t + Z_t \qquad (7.91)$$

(D) Identities - Linear restrictions

The model is completed by the following identities which are useful for imposing simple linear restrictions to certain coefficients when we estimate the model. These are

$$Z_t^1 = M_t - M_{t-1} \qquad (7.92)$$

$$Z_t^2 = Y_t^* - Y_{t-1} \qquad (7.93)$$

$$Z_t^3 = P_t - P_{t-1} \qquad (7.94)$$

$$Z_t^4 = Y_t - T_t \qquad (7.95)$$

$$Z_t^5 = Y_t - (1 - d)\ Y_{t-1} \qquad d = 0.0095 \qquad (7.96)$$

$$Z^6 = R_t' + \tilde{b}_4' M_t + \tilde{c}_4'\ (M_t - M_{t-1}) \qquad (7.97)$$

$$Z^7 = Z_t^6 - (1 - d)\ Z_{t-1}^6 \qquad d = 0.0095 \qquad (7.98)$$

$$Z^8 = Y_t - Y_{t-1} \qquad (7.99)$$

[76] See for example S. J. Turnovsky: "Optimal Choice of Monetary Instrument in a Linear Economic Model with Stochastic Coefficients." Journal of Money, Credit and Banking, 1975, p.55 and F. G. Steindl: "A Simple Macroeconomic Model with a Government Budget Restraint." Journal of Political Economy, 1971, p.676.

[77] C. Goodhart, *op.cit.*, pp.250-3.

CHAPTER 8

CHAPTER 8

ESTIMATION OF THE MODEL

In this chapter we shall estimate the structural equations of our model as specified in the previous four chapters. For this purpose an estimation method must be selected which meets the following requirements:

(a) it must eliminate the problem of the simultaneous equations bias[1] present in estimates of the parameters of a single equation when the last is a part of a larger system and some of its variables are simultaneously determined within the system. The simultaneous equations bias arises when simultaneity is ignored by using the ordinary least squares estimation technique; (b) it must give consistent parameter estimates in the presence of autocorrelation in the residuals and lagged dependent variables as regressors in the equations; and (c) it must produce estimates of each equation separately, because if we recall our discussion in chapter 7 we see that, in order to estimate the parameters of the investment function, we need an estimate of the cost of capital in terms of the variables and the attached parameters employed in equation (7.17) to represent it. This consideration would preclude the use of estimation techniques such as three stage least squares and full information maximum likelihood method unless we were prepared to split the above variable in its two parts and estimate freely the relevant parameters in the investment function.

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[1] We note that the term simultaneous equations bias is reproduced here because it is commonly used in monetary economics. However, it is a most unfortunate term for it does not refer to the bias properties of the estimator but to its lack of consistency. For this point see P. J. Dhrymes: Econometrics, Statistical Foundations and Applications. Harper & Row, New York, 1970, p.176.

The only methods that satisfy the above requisites and have been used in practice are two-stage least squares plus first-order serial correlation of the residuals (TSLSAUTO1) and two-stage least squares plus first and second-order serial correlation of the residuals (TSLSAUTO2).[2] However, computational facilities restricted our choice only to the first estimator, in the cases where autocorrelation was found and/or expected to exist. Fortunately, in all but one equations in which autocorrelation has been accounted, the first order autoregressive process for the residuals has been proved enough to eliminate serial correlation. In one equation (the money demand function) the DW statistic was in the indeterminate range and we would have liked to experiment with the second order case. In the absence of autocorrelation the two-stage least squares method was used.[3] Strictly speaking in some cases (a form of the money supply function and most equations of model B) the estimates obtained were instrumental estimates rather than two-stage least squares estimates (without or with autocorrelation), as the latter require the set of <u>all</u> predetermined variables in the first stage regression and it was found by trial that the available program would work only with a number of instruments not exceeding 29,[4] whereas the two-stage least square estimate would require in some cases more instruments than this number allows. The difference between instrumental estimates other than two-stage least squares and two-stage least squares concerns only asymptotic efficiency and will be discussed later on.

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[2] For a comparison of these two and several other estimators see R. C. Fair: "A Comparison of Alternative Estimators of Macroeconomic Models." International Economic Review, 1973, pp.261-77.

[3] For the source of the programs used see Appendix B.

[4] This was true despite the fact that there are not any restrictions as to dimensions, explicitly stated in the users manual.

We shall now state briefly the estimation methods employed and their properties.

Let

$$Y\,A + X\,B = U \qquad (8.1)$$

be a linear system of equations with

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2m} \\ \vdots & & & \\ y_{T1} & y_{T2} & \cdots & y_{Tm} \end{bmatrix}$$ a matrix of T observations on m endogenous variables

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1G} \\ x_{21} & x_{22} & \cdots & x_{2G} \\ \vdots & & & \\ x_{T1} & x_{T2} & \cdots & x_{TG} \end{bmatrix}$$ a matrix of T observations on G predetermined variables

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1m} \\ a_{21} & 1 & \cdots & a_{2m} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & 1 \end{bmatrix}$$ a non singular m x m matrix of parameters with 1's on the main diagonals. The 1's represent the normalised coefficients corresponding to one endogenous variable in each equation.

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & & & \\ b_{G1} & b_{G2} & \cdots & b_{Gm} \end{bmatrix}$$ a G x m matrix of parameters

and

$$U = \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1m} \\ u_{21} & u_{22} & \dots & u_{2m} \\ \vdots & & & \\ u_{T1} & u_{T2} & \dots & u_{Tm} \end{bmatrix}$$ a T x m matrix of disturbances

Suppose that the following assumptions are true[5]

(i) $E(u_{ti}) = 0 \qquad t = 1, 2, \dots T \qquad i = 1, 2, \dots m$

(ii) $E(u_{ti}\, u_{t'j}) = \delta_{tt'}\, \sigma_{ij}$ $\qquad \delta_{tt'}$ is the Kronecker delta

$t, t' = 1, 2, \dots T \quad i, j = 1, 2, \dots m$

i.e. the elements of the rows of U are independently and identically distributed random variables with mean 0 and finite covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1m} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2m} \\ \vdots & & & \\ \sigma_{m1} & \sigma_{m2} & \dots & \sigma_{mm} \end{bmatrix}$$

Σ has constant elements, in symmetric, positive semidefinite and it can be singular [when (8.1) contains identities]. Non contemporaneous covariances are equal to zero.

(iii) $E(x_{ti}\, u_{tj}) = 0 \qquad t = 1, 2, \dots T \qquad i = 1, 2, \dots G$

$j = 1, 2, \dots m$

i.e. predetermined variables (which may include lagged endogenous variables) are uncorrelated with disturbances.

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[5] See P. J. Dhrymes, *op.cit.*, p.172.

The first equation of the complete equation system can be written as

$$y_1 = Y_1 a_1 + X_1 b_1 + u_1 \qquad (8.2)$$

where

- y_1 is a $T \times 1$ vector of observations on the first dependent variable
- y_1 the $T \times m_1$ matrix of observations on the m_1 endogenous variables which are regressors in the first equation
- a_1 the $m_1 \times 1$ vector of parameters corresponding to the above matrix
- X_1 the $T \times G_1$ matrix of observations on the G_1 predetermined variables appearing as regressors in the equation
- b_1 the $G_1 \times 1$ vector of the parameters attached to X_1
- and u_1 the $T \times 1$ vector of disturbances.

(8.2) can be easily seen as a part of (8.1) if we consider the following partition of (8.1)

$$\begin{bmatrix} y_1 & Y_1 & Y_2 \end{bmatrix} \left[\begin{array}{c|c} 1 & \\ -a_1 & A_2 \\ 0 & \end{array}\right] + \begin{bmatrix} X_1 & X_2 \end{bmatrix} \left[\begin{array}{c|c} -b_1 & B_2 \\ 0 & \end{array}\right] = \begin{bmatrix} u_1 & U_1 \end{bmatrix} \qquad (8.3)$$

with the corresponding dimensions

$$\begin{bmatrix} T \times 1 & T \times m_1 & T \times m_2 \end{bmatrix} \begin{bmatrix} 1 \times 1 & \\ m_1 \times 1 & m \times m\text{-}1 \\ m_2 \times 1 & \end{bmatrix} +$$

$$+ \begin{bmatrix} T \times G_1 & T \times G_2 \end{bmatrix} \begin{bmatrix} G_1 \times 1 & G \times m\text{-}1 \\ G_2 \times 1 & \end{bmatrix} = \begin{bmatrix} T \times 1 & T \times m\text{-}1 \end{bmatrix}$$

with $m_1 + m_2 + 1 = m$ and $G_1 + G_2 = G$

<u>The two-stage least squares estimator</u>: The two-stage least squares estimator of the parameters a_1 and b_1 in (8.2) was proposed by Theil[6] and is derived by a two step procedure.

At the first step we consider the reduced form corresponding to the right hand jointly dependent variables in (8.2)

$$Y_1 = X \Pi_1 + V_1 \tag{8.4}$$

and apply ordinary least squares to obtain the estimator of Π_1

$$\tilde{\Pi}_1 = (X' X)^{-1} X' Y_1 \tag{8.5}$$

and the estimator $\tilde{V}_1$ of residuals

$$\tilde{V}_1 = Y_1 - X (X' X)^{-1} X' Y_1 \tag{8.6}$$

At the second step we replace Y_1 in (8.2) by $Y_1 - \tilde{V}_1$ and apply least squares to it. The two-stage least squares estimates of a_1 and b_1 are thus given by

$$\tilde{c}_1 = \begin{bmatrix} \tilde{a}_1 \\ \tilde{b}_1 \end{bmatrix} = \left[\begin{pmatrix} Y_1' - \tilde{V}_1' \\ X_1' \end{pmatrix} (Y_1 - \tilde{V}_1 \quad X_1) \right]^{-1} \begin{pmatrix} Y_1' - \tilde{V}_1' \\ X_1' \end{pmatrix} y_1 \tag{8.7}$$

which simplifies to

$$\tilde{c}_1 = \begin{bmatrix} Y_1' Y_1 - \tilde{V}_1' \tilde{V}_1 & Y_1' X_1 \\ X_1' Y_1 & X_1' X_1 \end{bmatrix}^{-1} \begin{bmatrix} Y_1' - \tilde{V}_1' \\ X_1' \end{bmatrix} y_1 \tag{8.8}$$

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[6] H. Theil: <u>Economic Forecasts and Policy</u>. North Holland Publishing Co., Amsterdam, 1961.

Estimator (8.8) has the following properties[7]

(i) it is consistent, i.e. $\operatorname*{plim}_{T\to\infty} \tilde{c}_1 = c_1$

(ii) it is generally biased since

$$E(\tilde{c}_1) = c_1 + E\left[\begin{bmatrix} Y_1' Y_1 - \tilde{V}_1' \tilde{V}_1 & Y_1' X_1 \\ X_1' Y_1 & X_1' X_1 \end{bmatrix}^{-1} \begin{bmatrix} Y_1' - \tilde{V}_1' \\ X_1' \end{bmatrix} u_1\right]$$

and Y_1 is correlated with u_1. Moreover the right hand side expectation above may not exist.

(iii) it is the most efficient - in an asymptotic sense - estimator within the class of instrumental variables estimators in which it belongs.

A variant of the method of two-stage least squares which is computationally easier consists of regressing $Z_1 = [Y_1 \; X_1]$ on X rather than Y_1 alone at the first stage,[8] i.e.

$$Z_1 = X \Pi_1 + V_1 \tag{8.9}$$

where now Π_1 is of dimension $G \times (G_1 + m_1)$. The OLS estimator of Π_1 is

$$\tilde{\Pi}_1 = (X' X)^{-1} X' Z_1 \tag{8.10}$$

and the estimator of the residuals is

$$\tilde{V}_1 = Z_1 - X (X' X)^{-1} X' Z_1 \tag{8.11}$$

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[7] For a proof see P. J. Dhrymes, *op.cit.*, pp.179-80 and 299-305.

[8] It is this variant of the method that the available program makes use of in estimating the parameters.

The deterministic component of Z_1 , namely $\tilde{Z}_1 = X \tilde{\Pi}_1 =$ $= X (X' X)^{-1} X' Z_1$ replaces Z_1 in (8.2) which is conveniently written as

$$y_1 = Z_1 c_1 + u_1 \quad (8.12)$$

or
$$y_1 = \tilde{Z}_1 c_1 + u_1 + \tilde{V}_1 c_1 \quad (8.13)$$

and the 2SLS estimator of c_1 is

$$\tilde{c}_1 = (\tilde{Z}_1 \tilde{Z}_1)^{-1} \tilde{Z}_1 y_1$$

or
$$\tilde{c}_1 = [Z_1' X (X' X)^{-1} X' X (X' X)^{-1} X' Z_1]^{-1} Z_1' X (X' X)^{-1} X' y_1$$

or
$$\tilde{c}_1 = [Z_1' X (X' X)^{-1} X Z_1]^{-1} Z_1' X (X' X)^{-1} X' y_1 \quad (8.14)$$

It is easy to show that (8.14) has exactly the same value as (8.8) above since

$$Y_1' Y_1 - \tilde{V}_1' \tilde{V}_1 = Y_1' X (X' X)^{-1} X' Y_1 \quad (8.15)$$

$$X (X' X)^{-1} X' X_1 = X_1 \quad \text{or} \quad X_1' X (X' X)^{-1} X' = X_1' \quad (8.16)$$

$$Y_1' X_1 = Y_1' X (X' X)^{-1} X' X_1 \quad (8.17)$$

$$X_1' Y_1 = X_1' X (X' X)^{-1} X' Y_1 \quad (8.18)$$

$$X_1' X_1 = X_1' X (X' X)^{-1} X' X_1 \quad (8.19)$$

and
$$Z_1 = [Y_1 \ X_1] \quad (8.20)$$

The two-stage least square estimator (8.14) can be interpreted as an instrumental variables estimator. Thus if we premultiply both sides of (8.12) by $P' = Z_1' X (X' X)^{-1} X'$ or $P' = Z_1' N$, and ignoring the error term solve for c_1 we get (8.14). Premultiplication of (8.12) by any other matrix of instruments $P_s' = Z_1' X_s (X_s' X_s)^{-1} X_s'$ gives the instrumental variable estimator

$$\tilde{c}_1 = [Z_1' X_s (X_s' X_s)^{-1} X_s' Z_1]^{-1} \quad Z_1' X_s (X_s' X_s)^{-1} X_s' y_1 \qquad (8.21)$$

which has the desirable property of consistency, is biased, and is asymptotically less efficient than (8.14) if the matrix X_s contains only variables which are among the predetermined variables of the model but are less in number than the whole set of predetermined variables. For the existence of the estimator (8.21) it is necessary that the inverse of the matrix $Z_1' N_s Z_1$ exists. This will happen if the rank of $Z_1' N_s$ is equal to the rank of Z_1'. A necessary and sufficient condition for the last is that the matrix X contains at least all of the predetermined variables of the equation, namely $X_s = [X_1 \; X_s^*]$ where X_s^* may be the null matrix or may include variables other than the predetermined variables of the model (assumed of course to be uncorrelated with the error term).[9]

Simultaneous estimation of equations with first order serially correlated errors.

One of the assumptions made in order to obtain the 2SLS estimators is that predetermined variables are uncorrelated with

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[9] A formal proof of this is contained in L. Gill: "The Existence of Instrumental Variables Estimators." Mimeographed, 1975. For a discussion of the same requisite but from the point of view of consistency of the estimator in large models see M. D. McCarthy: "Notes on the Selection of Instruments for Two-stage Least Squares and K Class Type Estimators of Large Models." Southern Economic Journal, 1970-71, pp.251-59 in which the author suggests that for consistency the same set of regressors should be used at the first stage for every dependent variable in Y_1 and this set should explicitly include X_1.

disturbances. When, however, the disturbances in some equations follow a first order autoregressive process, the 2SLS or the instrumental variables estimators of the parameters will be inconsistent and additional instruments must be added so that the resulting estimator is consistent. Consequently the above assumption should be modified accordingly. The instruments which are necessary to ensure consistency in this case were examined by Fair.[10] Let

$$Y A + X B = U$$

be the model in the same notation as before, where

$$U = U_{-1} R + E \qquad (8.21)$$

R is a diagonal $m \times m$ matrix of coefficients between one and minus one (for a stable model). The subscript -1 denotes the one period lagged values of the terms of U. The subscript t is avoided since U contains all of the sample values of the disturbances for each equation.

It is now assumed that

(i) $E(e_{ti}) = 0 \qquad t = 1,2, \ldots T \qquad i = 1,2, \ldots m$

(ii) $E(e_{ti}\, e_{t'j}) = \delta_{tt'}\, \sigma_{ij}$ is the Kronecker delta

$t, t' = 1,2, \ldots T \quad i, j = 1,2, \ldots m$

and these are the same assumptions as before except that the error term in them is e rather than u.

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[10] R. C. Fair: "The Estimation of Simultaneous Equations Models with Lagged Endogenous Variables and First Order Serially Correlated Errors." Econometrica, 1970, pp.507-16.

(iii)

$$E\,(x_{ti}\,e_{tj}) = 0$$

$$E\,(x_{t-1,i}\,e_{tj}) = 0 \qquad t = 1,2,\ldots T \qquad i = 1,2,\ldots G$$

$$E\,(y_{t-1,i}\,e_{tj}) = 0 \qquad j = 1,2,\ldots m$$

i.e. the disturbances are uncorrelated with the contemporaneous and one-period lagged values of the predetermined variables and with the one-period lagged values of the endogenous variables. This is the modified assumption corresponding to the third assumption of the previous section.

The first equation of the model can be written as

$$y_1 = Y_1\,a_1 + X_1\,b_1 + u_1 \tag{8.23}$$

where
$$u_1 = u_{1_{-1}}\,r_{11} + e_1 \tag{8.24}$$

with r_{11} the element in the first row and first column of R,

or $$y_1 - y_{1_{-1}}\,r_{11} = Y_1\,a_1 - Y_{1_{-1}}\,a_1\,r_{11} + X_1\,b_1 - X_{1_{-1}}\,b_1\,r_{11} + e_1 \tag{8.25}$$

To estimate a_1, b_1 and r_{11} consistently we use the following two-step procedure suggested by Fair.[11]

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[11] R. C. Fair, *op.cit.*, pp.508-9.

At the first stage we regress Y_1 on a set of instrumental variables which are uncorrelated with e_1 and include at least $y_{1_{-1}}$, $Y_{1_{-1}}$, X_1 , $X_{1_{-1}}$. Thus

$$Y_1 = X^* \hat{\Pi}_1 + \hat{V}_1 = \hat{Y}_1 + \hat{V}_1 \qquad (8.26)$$

where

$$X^* = [y_{1_{-1}} \; Y_{1_{-1}} \; X_1 \; X_{1_{-1}} \; X_2]$$

for the estimator corresponding to the 2SLS estimator in the case of uncorrelated errors and

$$X^* = [y_{1_{-1}} \; Y_{1_{-1}} \; X_1 \; X_{1_{-1}} \; X_s^*]$$

for the estimator corresponding to the instrumental variables estimator.

At the second stage, for a given value of r_{11} [12] we estimate (8.25) by ordinary least squares, using $\hat{Y}_1$ in place of Y_1 . Then an iterative procedure is used, which is the standard Cochrane Orcutt technique[13] adjusted to take into account simultaneity. Namely, the coefficient estimates $\tilde{a}_1{}^{(0)}$ and $\tilde{b}_1{}^{(0)}$ are fed into the OLS estimator of r_{11} from (8.24)

$$\tilde{r}_{11} = \frac{(u_{1_{-1}})'(u_1)}{(u_{1_{-1}})'(u_{1_{-1}})} \qquad (8.27)$$

to give a first estimate of it

$$\tilde{r}_{11}^{(1)} = \frac{(y_1 - Y_{1_{-1}} \tilde{a}_1^{(0)} - X_{1_{-1}} \tilde{b}_1^{(0)})'(y_1 - Y_1 \tilde{a}_1^{(0)} - X_1 \tilde{b}_1^{(0)})}{(y_1 - Y_{1_{-1}} \tilde{a}_1^{(0)} - X_{1_{-1}} \tilde{b}_1^{(0)})'(y_1 - Y_{1_{-1}} \tilde{a}_1^{(0)} - X_{1_{-1}} \tilde{b}_1^{(0)})} \qquad (8.28)$$

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[12] The available program uses an OLS regression to form an initial guess of r_{11} .

[13] D. Cochrane and G. H. Orcutt: "Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms." Journal of the American Statistical Association, 1949, pp.749-809.

This value is used in (8.25) to estimate $\tilde{a}_1^{(1)}$ and $\tilde{b}_1^{(1)}$, which give a new value $\tilde{r}_{11}^{(2)}$ for r_{11} and the procedure is repeated until the difference between two successive r_{11} 's is within a tolerance level. The tolerance level used by the available program was .005 and the maximum number of iterations was 20. After this number, iterations would terminate even if the difference between the two values of r_{11} was greater than .005 . For our model and for the equations estimated under the assumption of a first order autoregressive error the number of iterations never reached 20 so that iterations terminated after a difference of r_{11} 's within the tolerance level was reached.

Estimators $\tilde{a}_1^{(n)}$, $\tilde{b}_1^{(n)}$ and $\tilde{r}_{11}^{(n)}$ are consistent only when at least $y_{1_{-1}}$, $Y_{1_{-1}}$, X_1 and $X_{1_{-1}}$ are used as instruments in the first stage regression because in

$$y_1 - y_{1_{-1}} r_{11} = (Y_1 - Y_{1_{-1}} r_{11}) a_1 + (X_1 - X_{1_{-1}} r_{11}) b_1 + (e_1 + \hat{V}_1 a_1) \tag{8.29}$$

the error term $\hat{V}_1$ must be uncorrelated with $\hat{Y}_1$, $y_{1_{-1}}$, $Y_{1_{-1}}$, X_1 and $X_{1_{-1}}$ and it is, only when the above requisite is met. We have applied this rule to every equation of our model whose errors were autocorrelated. The increase of the number of instruments that the rule implies, made in some cases necessary the exclusion of some predetermined variables (variables in X_2) from the first stage regression, since as already noted, the program would not work with a number of instruments exceeding 29. For the two models estimated the endogenous and predetermined variables are given below. Model A is exclusive of identity (7.91) and the monetary base is exogenous, whereas Model B includes the above identity and an endogenous monetary base.

Model A

Endogenous Variables

1.	R_t	6.	I_t	11.	Z_t^1	16.	Z_t^6
2.	M_t	7.	H_t	12.	Z_t^2	17.	Z_t^7
3.	P_t	8.	X_t	13.	Z_t^3	18.	Z_t^8
4.	R_t'	9.	Y_t	14.	Z_t^4		
5.	C_t	10.	S_t	15.	Z_t^5		

We note again that variables Z^i $i = 1, \ldots 8$ correspond to identities (7.92) to (7.98) which are immediately substitutable. The latter's role is to place the appropriate restrictions on coefficients since the program does not provide for restrictions.

Predetermined variables

1.	Constant	11.	P_{t-1}	21.	D_t^1
2.	S_t^2	12.	Y_t^*	22.	D_t^2
3.	S_t^3	13.	R_{t-1}'	23.	X_{t-1}
4.	S_t^4	14.	T_t	24.	G_t
5.	M_{t-1}	15.	C_{t-1}	25.	I_t^P
6.	B_t	16.	Y_{t-1}	26.	E_t
7.	S_t^D	17.	I_{t-1}		
8.	R_t^D	18.	S_{t-1}		
9.	Z_{t-1}^2	19.	Z_{t-1}^3		
10.	W_{t-1}	20.	E_t^R		

Model B

Endogenous variables

1.	R_t	6.	I_t	11.	B_t	16.	Z_t^5
2.	M_t	7.	H_t	12.	Z_t^1	17.	Z_t^6
3.	P_t	8.	X_t	13.	Z_t^2	18.	Z_t^7
4.	R'_t	9.	Y_t	14.	Z_t^3	19.	Z_t^8
5.	C_t	10.	S_t	15.	Z_t^4		

Predetermined variables

1.	Constant	11.	Y_t^*	21.	D_t^2
2.	S_t^2	12.	R'_{t-1}	22.	X_{t-1}
3.	S_t^3	13.	T_t	23.	G_t
4.	S_t^4	14.	C_{t-1}	24.	I_t^P
5.	M_{t-1}	15.	Y_{t-1}	25.	E_t
6.	S_t^D	16.	I_{t-1}	26.	B_{t-1}
7.	R_t^D	17.	S_{t-1}	27.	Z_t
8.	Z_{t-1}^2	18.	Z_{t-1}^3	28.	S_{t-1}^D
9.	W_{t-1}	19.	E_t^R	29.	T_t^I
10.	P_{t-1}	20.	D_t^1		

Identification of the equations. Examining the identification of an equation in a structural equations model means examining whether the equation in question is sufficiently different so that it can be distinguished from any convex linear combination of the equations of the model. In general, there are two conditions for identifiability of an equation of a model, known as the rank and order conditions.[14]

The rank condition concerns the rank of certain matrices and can be stated in terms of either the reduced form coefficients or in terms of the structural form coefficients.[15] However, since it is impossible to check the rank condition prior to estimation we resort to a counting rule, known as the order condition for identifiability which is derived from the rank condition[16] and constitutes a necessary but not a sufficient condition. This is

$$G_2 \geqslant m_1 \tag{8.30}$$

i.e. the number of excluded predetermined variables from the equation must be at least as great as the number of endogenous variables included in the equation as regressors.

All the equations of both models A and B satisfy the order condition (8.30) and are therefore identifiable.

A different aspect of the identification problem concerns the identification of parameters. Typically the problem arises when the

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[14] See P. J. Dhrymes, *op.cit.*, pp.280-95.

[15] See for example J. L. Murphy: Introductory Econometrics. Richard D. Irwin Inc., Homewood, Illinois, 1973, p.429.

[16] See J. Johnston: Econometric Methods. McGraw-Hill, New York, 1972, p.348.

equation estimated is the reduced form[17] of a behavioural equation and one or several dynamic adjustment mechanisms. This type of system contains in principle unobservable variables which are removed by means of transformations and the resulting equation contains only observable variables but its parameters are functions of the original parameters. Identification in this context means to be able to get these parameters from the estimated coefficients. Thus, for example, in the partial adjustment model examined in chapter 3 we are in a position to obtain uniquely the speed of adjustment coefficient λ and the behavioural parameter a. In the compound geometric lag model, the parameters λ and μ pertaining to the partial adjustment and the adaptive expectations mechanisms, can be obtained but the symmetric form in which they appear in the reduced form (3.19) does not allow their allocation to the corresponding dynamic mechanisms. In most of the equations of our model the initial parameters can be obtained. The few cases where more than one parameter value is consistent with the estimated values are indicated in the text, e.g. in the price equation the ratio $\frac{b_3'}{\lambda_2}$ is compatible with more than one values of b_3' and λ_2.

The Results

The money demand function

(A) Exogenous monetary base. In estimating equation (4.52) the sample period taken is from 1955.3 to 1972.4, because two observations are lost by the dynamic specification of the overall model in which the maximum lag appearing to an endogenous variable is two quarters (see

[17] The term reduced form employed here should not be confused with the reduced form of a model. It is simply the result of removing by transformations unobservable variables from systems of equations.

results of equation for inventories). Given that the maximum lag for exogenous variables is one quarter, it follows that the degrees of freedom lost by the lag requirements are two. Furthermore, where autocorrelation in the residuals is accounted, an additional observation is taken up by the program so that the number of observations reduces to 69. The result for (4.52) is presented below.

$$R_t = \underset{(-4.24)}{-0.00160849}\,(M_t - M_{t-1}) + \underset{(1.80)}{0.000281549}\,Y_t +$$

$$+ \underset{(7.30)}{0.104944}\,P_t - \underset{(-4.07)}{0.000655507}\,M_{t-1} - \underset{(-4.49)}{2.1783} +$$

$$+ \underset{(0.79)}{0.133356}\,S_t^2 + \underset{(1.54)}{0.27053}\,S_t^3 + \underset{(1.35)}{0.284806}\,S_t^4 + u_{1_t}$$

(t values are shown in parenthesis) (8.31)

$R^2 = 0.9355 \quad D.W. = 0.8557$

We notice immediately that the existence of positive autocorrelation is indicated by the DW statistic. Since the partial adjustment model implies no autocorrelation, this finding may be attributed to either the omission of some important variable from the equation or the operation of the adaptive expectations mechanism to the regressors so that "permanent" levels of real income, price level and interest rate are relevant for the equilibrium relationship or finally to the effect of omitted nonlinear terms in the approximation of nominal income by a linear combination of real income and the price level. We emphasise that the second would imply the same speeds of adjustment for the above three variables only in the context of a single equation, irrespective of whether autocorrelation is or is not assumed away. The interaction of the variables in the model would most probably differentiate the pattern of dynamic behaviour pertaining to each variable (cf our discussion in chapter 3).

In view of the above traced autocorrelation, we try the change of the price level as an explanatory variable, because from equation (6.43) we see that a constant percentage of it approximates the expected rate of inflation $[\frac{\Delta P^E}{P} \simeq \Delta P^E = \lambda_2 (P_t - P_{t-1})]$. A significant coefficient on it would indicate a different response of money holdings to the real rate of interest and the expected rate of inflation. The result is

$$R_t = \underset{(-2.67)}{-\,0.00109108}\,(M_t - M_{t-1}) + \underset{(2.71)}{0.000436882}\,Y_t + \underset{(5.85)}{0.0883287}\,P_t + \underset{(2.96)}{0.27175}\,(P_t - P_{t-1}) - \underset{(-4.24)}{0.000665333}\,M_{t-1} - \underset{(-3.33)}{1.6763} - \underset{(-1.02)}{0.203824}\,S_t^2 - \underset{(-0.19)}{0.03724}\,S_t^3 + \underset{(0.09)}{0.02109}\,S_t^4 + u_{1t} \qquad (8.32)$$

$$R^2 = 0.9397 \qquad D.W. = 1.2925$$

The coefficient of ΔP turns out to be significant and with the opposite of the expected sign. However autocorrelation has not been eliminated and the next step is to estimate the equation assuming that the residuals follow a first order autoregressive process. The result is

$$R_t = \underset{(-1.95)}{-\,0.000497309}\,(M_t - M_{t-1}) + \underset{(2.40)}{0.000579969}\,Y_t + \underset{(2.89)}{0.0714868}\,P_t + \underset{(0.52)}{0.0260592}\,(P_t - P_{t-1}) -$$

$$- \underset{(-1.79)}{0.000458746}\, M_{t-1} - \underset{(-2.02)}{2.23824} - \underset{(-1.15)}{0.121014}\, S_t^2 -$$

$$- \underset{(-0.41)}{0.0399831}\, S_t^3 - \underset{(-1.54)}{0.206273}\, S_t^4 + u_{1_t}$$

$$u_{1_t} = 0.8258\, u_{1,t-1} + e_{1_t} \qquad (8.33)$$

$$R^2 = 0.9778 \qquad D.W. = 1.4874$$

In (8.33) the expected rate of inflation ceases to be a significant determinant of money holdings so that it is dropped from it and the equation is reestimated.

$$R_t = - \underset{(-2.60)}{0.000567239}\, (M_t - M_{t-1}) + \underset{(2.36)}{0.000542781}\, Y_t +$$

$$+ \underset{(3.62)}{0.0778209}\, P_t - \underset{(-2.08)}{0.00050263}\, M_{t-1} - \underset{(-2.11)}{2.29844} -$$

$$- \underset{(-1.05)}{0.0883988}\, S_t^2 - \underset{(-0.16)}{0.0131399}\, S_t^3 - \underset{(-1.48)}{0.169873}\, S_t^4 + u_{1_t}$$

$$u_{1_t} = 0.82072\, u_{1,t-1} + e_{1_t} \qquad (8.34)$$

$$R^2 = 0.9775 \qquad D.W. = 1.467$$

The DW statistic in (8.34) would prompt us to try a second order autoregressive process for the residuals, had we an available program for two-stage least squares and second order autoregressive errors. Also we might be induced to consider test statistics alternative to the DW one which are available, since it has been established that the latter is biased in the presence of the lagged dependent variable among the regressors.[18] These tests were suggested by Durbin[19] in the case where lagged dependent variables are among the predetermined variables, and their small sample bias was examined by Spencer.[20] Unfortunately these tests assume one equation models with no current endogenous variables as regressors and this is the situation we are likely to encounter in simultaneous equations models. For the last no similar test statistic is available as yet.

In (8.34), which is the final version of the money demand function, all variables are significant and have the expected sign. If autocorrelation is due to any other omitted factor, (8.34) will imply a coefficient of adjustment of actual money holdings to desired ones, equal to 0.886 which conforms with the *a priori* short lag expected to prevail.[21] We note that this estimate changes a lot as we go from (8.31), where we have not allowed autocorrelation in the residuals, to (8.34). The former equation implies an adjustment coefficient equal to 0.43, that is approximately half the size of the other. If equations (8.31) and (8.34) were considered alone and not as a part of the model and error terms were neglected, carrying out the exercise of computing the price elasticity of money holdings would give us the following values for elasticities calculated at the means $\overline{P} = 107.52$ and $\overline{M} = 7558.4$:

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[18] M. Nerlove and K. F. Wallis: "The Use of the Durbin-Watson Statistic in Inappropriate Situations." Econometrica, 1966, pp.235-8.

[19] J. Durbin: "Testing for Serial Correlation in Least-Squares Regression when Some of the Regressors are Lagged Dependent Variables." Econometrica, 1970, pp.410-21.

[20] B. G. Spencer: "The Small Sample Bias of Durbin's Test for Serial Correlation." Journal of Econometrics, 1975, pp.249-54.

[21] Cf D. E. W. Laidler: The Influence of Money on Economic Activity - A Survey of Some Current Problems. In Monetary Theory and Monetary Policy in the 1970s, edit. by G. Clayton, J. C. Gilber and R. Sedgwick, Oxford University Press, 1971, p.98. If, on the other hand, autocorrelation is introduced by the adaptive expectations scheme which is assumed for the explanatory variables, then we are implicitly assuming an instantaneous adjustment of actual money holdings to desired ones, the relevant coefficient being one, and the coefficient $\lambda_1 = 0.886$ found above attaches to the explanatory variables.

Table 8.1 : Price elasticity of money holdings

	Short-run	Long-run
Equation 8.31	0.93	2.28
Equation 8.34	1.95	2.20

In no case could we say that the long-run price elasticity is one. However, this type of exercise was criticised in chapter 4 and can not meaningfully substitute the analysis of dynamic multipliers in the context of a structural equations model. Correct analysis in a model, in which nominal money and the price level are both endogenous variables, would require that short-run and long-run elasticities be calculated for changes in every exogenous variable of the model.

(B) Endogenous monetary base. The equations corresponding to the ones presented above came up very close to them. This was also true for the other equations of Model B in so far as estimation is concerned. The small differences observed in parameter estimates are due to the use of different instruments in the first stage regressions. The equations of Model B are presented in Appendix B.

The money supply function

Equation (5.19) was estimated for the 1955.3 - 1972.4 period under the assumption of first order serial correlation of the error term. The result is shown below

$$M_t = \underset{(7.85)}{2.2253}\, B_t - \underset{(-2.93)}{1.30198}\, S_t^D + \underset{(4.15)}{0.510894}\, Y_t - \underset{(-1.22)}{136.93}\, R_t -$$

$$- \underset{(-1.17)}{42.6444}\, R_t^D - \underset{(-2.42)}{2430.36} - \underset{(-5.09)}{234.15}\, S_t^2 - \underset{(-3.94)}{180.13}\, S_t^3 -$$

$$- \underset{(-2.97)}{205.92}\, S_t^4 + u_{2_t}$$

$$u_{2_t} = 0.92372\, u_{2,_{t-1}} + e_{2_t} \qquad (8.35)$$

$$R^2 = 0.9923 \qquad D.W. = 2.3282$$

The coefficients of both interest rate and Bank Rate are insignificant. Since these two rates are highly correlated and a zero d_2 coefficient can be accepted theoretically, we dropped the interest rate variable and re-estimated the parameters of the function.

$$M_t = \underset{(8.20)}{2.06555}\, B_t - \underset{(-3.43)}{1.44907}\, S_t^D + \underset{(4.05)}{0.4768}\, Y_t - \underset{(-1.92)}{62.1773}\, R_t^D -$$

$$- \underset{(-2.49)}{2460.44} - \underset{(-5.20)}{235.90}\, S_t^2 - \underset{(-3.99)}{180.15}\, S_t^3 - \underset{(-2.80)}{184.10}\, S_t^4 + u_{2_t}$$

$$u_{2_t} = 0.92371\, u_{2,_{t-1}} + e_{2_t} \qquad (8.36)$$

$$R^2 = 0.9923 \qquad D.W. = 2.1926$$

The Bank Rate almost reaches the limit for significance (at 5% level). All other coefficients conform with expectations and are significant.

The monetary base as defined here, that is net of special deposits, appears to be the most significant determinant of the money supply. The results support our contention that SDs should not be blended together with the sum of currency circulation with the public and cash reserves of the banking system, because they are shown to have a negative impact on the money supply, whereas the latter are the basis for multiple expansion of it. The present evidence seems to refute the monetary authorities' belief that there can not be *"a nicely calculated relationship between the size of calls for Special Deposits and the achievement of desired objectives."*[22] A significant link does exist between the size of calls for SDs and the money supply and in the complete structural model it is a straight forward exercise to relate SDs to desired objectives, e.g. real output.

The relative size of coefficients a_2 and b_2 and the finding of a negative impact relationship between the quantity of money demanded and the interest rate leads us to the conclusion that a call for SDs implies a smaller increase in the rate than an open market sale of equal size. Thus SDs can be viewed as substitutes for the increase in the market rate and present the Treasury with a successful means of financing itself through the banking system by bypassing the market. The conclusion cannot be extended to more than one period; obviously to be able to trace the future course of the interest rate following these two alternative policy actions, we need to examine the dynamic properties of the whole model.

Contrary to the widely held view that money supply is positively, if at all, related to the interest rate, the discussion in chapter 5 suggested that this is not necessarily true and that the sign of the interest rate coefficient can not be determined *a priori*. This is an

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[22] See Bank of England: "Key Issues in Monetary and Credit Policy." Bank of England Quarterly Bulletin, 1971, p.197.

important aspect of money supply theory not accounted for properly in previous studies. While the positive impact of the interest rate on the money supply (the intermediate steps being a negative impact of the interest rate on the ratio of banks' cash reserves to current accounts and a negative impact of the ratio on the money supply) has been examined in the context of money supply models, the negative impact of the interest rate on the money supply (the intermediate steps being a positive impact of the interest rate on the ratio of currency circulation to current accounts and a negative impact of this ratio on the money supply) has been neglected, despite the results from independent research indicating the importance of the interest rate to the ratio of currency to demand deposits (current accounts). A joint consideration of the two effects leads to the *a priori* indeterminancy of the coefficient attaching to interest rate. The findings here present an insignificant d_2 coefficient which is on the negative side. This is explained by the relative preponderance of the interest rate-ratio of currency to current accounts effect over the interest rate-ratio of banks' cash reserves to current accounts effect in the British institutional framework.

The preliminary findings in chapter 5 and the results presented here indicate that the ratio of currency to current accounts and consequently the money supply is dependent on income and this makes money supply an endogenous variable. This may have some bearing on the control of money issue, since it is found that the factors which determine money supply are not completely different from the factors which determine money demand.

The price equation

The estimates of the price equation under the assumption of a first order autoregressive process for the residuals are given below. They are the OLS estimates of equation (6.44) in which a Cochrane-Orcutt transformation has been applied. The reason for this is that there is no current endogenous variable among the regressors and consequently no first stage regression is necessary.

$$P_t = \underset{(-1.74)}{-\ 0.00138309} (Y^*_{t-1} - Y_{t-1}) + \underset{(1.98)}{0.0456771} W_{t-1} +$$

$$+ \underset{(18.30)}{0.930711} P_{t-1} + \underset{(0.16)}{0.222839} + \underset{(2.78)}{0.714097} S^2_t +$$

$$+ \underset{(1.64)}{0.440205} S^3_t + \underset{(0.17)}{0.0417283} S^4_t + u_{3_t}$$

$$u_{3_t} = 0.1058\, u_{1,t-1} + e_{3_t} \qquad (8.37)$$

$$R^2 = 0.9989 \qquad D.W. = 2.0107$$

The three coefficients have the expected sign and two of them are significant at a 5% level of significance while the coefficient of the excess demand variable is significant at a 10% level. The lagged dependent variable has the highest t ratio which however cannot indicate the significance of the lagged price level as a part of the cost inflation term in explaining price changes. To get an idea about the latter we perform a regression of the price change rather than the price level on the same variables with the following result

$$(P_t - P_{t-1}) = \underset{(-1.74)}{-\ 0.00138374}\ (Y^*_{t-1} - Y_{t-1}) + \underset{(1.98)}{0.0456171}\ W_{t-1} -$$

$$- \underset{(-1.36)}{0.0691286}\ P_{t-1} + \underset{(0.15)}{0.213528} + \underset{(2.80)}{0.717162}\ S^2_t +$$

$$+ \underset{(1.66)}{0.442553}\ S^3_t + \underset{(0.18)}{0.0446748}\ S^3_t + u'_{3_t}$$

$$u'_{3_t} = 0.10631\ u'_{3,t-1} + e'_{3_t} \qquad (8.38)$$

$$R^2 = 0.4912 \qquad D.W. = 2.0112$$

In (8.38) the coefficient of determination is reduced to .49, something that one usually gets when the first difference of the variable is the dependent variable in an equation. The price level has the correct sign but its significance is drastically reduced. All other coefficient estimates (including the autoregressive error parameter) are, as expected, equal in a statistical sense. The coefficient of the lagged price level in (8.38) is equal (also in a statistical sense) to the corresponding coefficient in (8.37) minus one which is what is expected as a result of the shifting of P_{t-1} in (8.37) from the right hand side to the left hand side. The coefficient λ_2 by which expectations of price changes fall short of actual price changes can not, as already noted in chapter 6, be identified from either (8.37) or (8.38).

The equation relating rates of return on bonds and equities

Equation (7.17) was estimated for the 1954.3 - 1972.4 period. The result is

$$R'_t = \underset{(0.52)}{0.034985}\, R_t + \underset{(0.72)}{0.0567197}\, (P_t - P_{t-1}) -$$

$$- \underset{(-1.26)}{0.0000953132}\, M_t - \underset{(-1.06)}{0.00036919}\, (M_t - M_{t-1}) +$$

$$+ \underset{(1.42)}{0.950729} + \underset{(0.60)}{0.0917584}\, S_t^2 + \underset{(1.27)}{0.198345}\, S_t^3 +$$

$$+ \underset{(1.54)}{0.238908}\, S_t^4 + \underset{(13.55)}{0.868078}\, R'_{t-1} + u_{4t} \qquad (8.39)$$

$$R^2 = 0.8878 \qquad D.W. = 1.4418$$

Equation (8.52) performs poorly in terms of significance and expected sign of some coefficients, but the DW statistic is in the indeterminate range and we shall postpone any discussion until we have estimated it assuming a first order autoregressive process for the residuals. The result in this case is

$$R'_t = \underset{(1.94)}{0.32162}\, R_t - \underset{(-1.09)}{0.0582486}\, (P_t - P_{t-1}) -$$

$$- \underset{(-2.86)}{0.000404827}\, M_t - \underset{(-0.12)}{0.0000267456}\, (M_t - M_{t-1}) +$$

$$+ \underset{(2.82)}{3.77661} + \underset{(0.33)}{0.0377291}\, S_t^2 + \underset{(1.17)}{0.109445}\, S_t^3 +$$

$$+ \underset{(2.08)}{0.162254}\, S_t^4 + \underset{(3.61)}{0.42281}\, R'_{t-1} + u_{4t} \qquad (8.40)$$

$$u_{4t} = 0.82604\, u_{4,t-1} + e_{4t}$$

$$R^2 = 0.908 \qquad D.W. = 2.0157$$

The picture changes as we go from (8.39) to (8.40). Clearly in the latter autocorrelation is eliminated. The nominal rate of interest on bonds becomes significant and its coefficient increases considerably. The expected rate of inflation assumes the right sign although it is significant only at a 20 per cent significance level. Moreover it is absolutely much smaller than the coefficient of the bond rate. Of the two proxies standing in the equation for the rate of growth of dividends only the level of the money supply is found important. The coefficient of the change in the money supply is highly insignificant. Therefore the equation is re-estimated without the money supply change as an explanatory variable. The result is

$$R'_t = \underset{(2.09)}{0.327681}\, R_t - \underset{(-1.26)}{0.0599152}\, (P_t - P_{t-1}) -$$

$$- \underset{(-2.95)}{0.000407314}\, M_t + \underset{(2.85)}{3.74734} + \underset{(0.59)}{0.046607}\, S_t^2 +$$

$$+ \underset{(1.33)}{0.112979}\, S_t^3 + \underset{(2.18)}{0.163998}\, S_t^4 + \underset{(3.66)}{0.423458}\, R'_{t-1} + u_{4t}$$

$$u_{4t} = 0.82794\, u_{4,t-1} + e_{4t} \qquad (8.41)$$

$$R^2 = 0.908 \qquad D.W. = 2.021$$

No coefficient changes significantly as a result of this. If, however, we discarded the change of the price level which is not significant (at a 5% level) but its t ratio exceeds a certain small value (say one or a value at the vicinity of one) we would get

$$R'_t = \underset{(1.38)}{0.17742}\, R_t - \underset{(-2.66)}{0.000313082}\, M_t + \underset{(3.23)}{3.52081} +$$

$$+ \underset{(0.27)}{0.0206177}\, S^2_t + \underset{(1.15)}{0.098813}\, S^3_t + \underset{(1.92)}{0.147452}\, S^4_t +$$

$$+ \underset{(4.67)}{0.515723}\, R'_{t-1} + u_{4t}$$

$$u_{4t} = 0.72769\, u_{4,t-1} + e_{4t} \qquad (8.42)$$

$$R^2 = 0.9071 \qquad D.W. = 1.9959$$

and now different coefficient estimates are obtained. Therefore it seems that the price change is not a superfluous variable in the equation.[23]

The explanation of the existence of autocorrelation in the residuals of the equation might be sought in one of the following reasons: (a) the adaptive expectations model rather than the partial adjustment model applies to equation (7.17); (b) in approximating the rate of inflation by a linear term, the nonlinear ones were omitted and this might have introduced autocorrelation if the latter were serially correlated; and (c) the real rate of interest may not be formed as the nominal rate minus the current expected rate of inflation but as the nominal rate minus a distributed lag of the current and past expected rates of inflation the weights of which sum to unity. Thus for a geometric lag pattern the real rate would be

$$R - \theta \left(\frac{\Delta P}{P}\right)^E - \theta\,(1-\theta)\left(\frac{\Delta P}{P}\right)^E_{-1} - \theta\,(1-\theta)^2 \left(\frac{\Delta P}{P}\right)^E_{-2} - \ldots .$$

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[23] For a discussion and some examples of the implications of discarding insignificant but non-superfluous variables see P. Rao and R. L. Miller: Applied Econometrics, Wadsworth Publishing Co., California, 1971, pp.35-40.

and in equilibrium, with $\theta = 1$ it would be $R - (\frac{\Delta P}{P})$ because the expected rate of inflation equals the actual rate of inflation. The above reasoning carried in the case of our model would give us as real rate

$$R_t - 4\theta k\lambda_2 (P_t - P_{t-1}) - 4\theta (1 - \theta)k\lambda_2 (P_{t-1} - P_{t-2}) - \ldots .$$

but what we did include in the equation for estimation were only the first two terms, with autocorrelation of the errors as a consequence, if the other terms were autocorrelated.

Coefficient b_4' - the coefficient of the money stock in the equilibrium relationship - is estimated (in absolute value) from equation (8.54) as $\frac{\tilde{c}_4}{(1 - \tilde{f}_4)} = \frac{0.000407314}{0.576542} = 0.000706477$. This enables us to form a proxy series for the cost of capital which will be used in the investment function. The variable in question is simply $Z_t = R_t' + 0.000706477 M_t$.

The adjustment in the market yields on equities (dividend yields) is found to occur at a moderate speed, 57 per cent of the equilibrium change being completed in the first quarter. The coefficient of the bond rate in (8.41) indicates that in the short-run the substitution occurring between bonds and equities does not result in a one to one relationship between changes in their yields and consequently equality between the yields is achieved through the addition of a higher risk-premium than the one which would have to be added if bonds and equities were perfect substitutes. In equilibrium the situation is improved and the coefficient of the bond rate rises to 0.568, which is still significantly different from unity.

The consumption function

The result of estimating equation (7.31) by 2SLS is

$$C_t = \underset{(4.34)}{0.344195}\,(Y_t - T_t) + \underset{(3.33)}{3.547}\,P_t - \underset{(-2.47)}{28.2428}\,(P_t - P_{t-1}) -$$

$$- \underset{(-0.09)}{11.5491} + \underset{(7.41)}{490.168}\,S_t^2 + \underset{(9.29)}{356.389}\,S_t^3 + \underset{(10.99)}{502.559}\,S_t^4 +$$

$$+ \underset{(4.99)}{0.497504}\,C_{t-1} + u_{5t} \qquad (8.43)$$

$$R^2 = 0.9926 \qquad D.W. = 2.147$$

It is seen that the partial adjustment model is likely to be operating, as the residuals of (8.60) do not show any serial correlation. Consumption exhibits a strong seasonal movement not accounted for by the other explanatory variables, the coefficients of the three seasonal dummies being the most significant in the equation. On the basis of the coefficient of the disposable income variable, we would say that the short-run marginal propensity to consume is (approximately) 0.34 and this is in agreement with the available evidence on the consumption function which generally reports a low income elasticity of consumption. However, if we remember that both consumption and income are endogenous variables in our model, we can see that the above inference about the short-run m.p.c. is not valid and we can get in principle as many short-run m.p.c. as there are exogenous variables in the model. Because, following a change in an exogenous variable, there is an associated impact (short-run) change in both consumption and income and the corresponding short-run m.p.c. can be computed. This can be repeated for every exogenous variable. Our remarks here are of exactly the same nature with the ones put forward when the interest elasticity of money demand has been examined.

The coefficient of adjustment of actual consumption to equilibrium consumption is 0.50 showing a moderately slow adjustment of consumption habits. On the other hand, consumers are found to be subject to a positive money illusion as the coefficient of the price level is significantly different from zero. Also the change of prices variable has a significant coefficient with a negative sign indicating that when prices are increasing faster consumers reduce their real consumption expecting a slower increase in the future and/or an improvement of their position in real terms (income or wealth). This hypothesis is seen to prevail over the hypothesis stating that the effect of changes in the rate of inflation is to restructure the time pattern of consumption by increasing present consumption and reducing it in the future. The last hypothesis would imply a positive coefficient of the current price change variable in the equilibrium consumption equation.

Despite the negative influence of price changes on real consumption in (8.43) this effect will vanish, in the static equilibrium model (Cf chapter 3, p.71) with the result that only the money illusion term will be among the crucial factors which determine, in the long run, the proportions in which a change in nominal income is divided between a change in real income and a change in the price level.

The investment function

Equations (7.58) and (7.59) were estimated for the period 1955.3 to 1972.4 with the following results

$$I_t = \underset{(6.64)}{0.132536}\,[Y_t - (1 - d)\,Y_{t-1}] - \underset{(-1.49)}{6.81359}\,[R'_t + \tilde{b}'_4 M_t] +$$

$$+ \underset{(1.72)}{0.665189}\,P_t + \underset{(0.31)}{1.57197}\,(P_t - P_{t-1}) + \underset{(1.58)}{69.9322} -$$

$$- \underset{(-0.54)}{10.1457}\,S^2_t - \underset{(-0.44)}{5.63826}\,S^3_t + \underset{(0.37)}{6.68334}\,S^4_t + \underset{(18.51)}{0.892272}\,I_{t-1} +$$

$$+ u_{6_t} \qquad (8.44)$$

$$R^2 = 0.9795 \qquad D.W. = 2.3602$$

$$I_t = \underset{(5.75)}{0.124065}\,[Y_t - (1 - d)\,Y_{t-1}] + \underset{(1.49)}{17.1316}\,[(R'_t + \tilde{b}'_4 M_t) -$$

$$- (1 - d)(R'_{t-1} + \tilde{b}'_4 M_{t-1})] + \underset{(1.63)}{0.63878}\,P_t -$$

$$- \underset{(-0.28)}{1.44964}\,(P_t - P_{t-1}) + \underset{(0.50)}{11.1371} - \underset{(-0.27)}{5.28975}\,S^2_t -$$

$$- 6.53794\,S^3_t + 6.22323\,S^4_t + 0.88334\,I_{t-1} + u_{6_t}$$

$$(8.45)$$

$$R^2 = 0.9784 \qquad D.W. = 2.3383$$

$$d = 0.0095 \qquad \tilde{b}'_4 = 0.000706477$$

The Durbin-Watson statistic is in both cases in the indeterminate range for negative serial correlation of the residuals. The lower and upper limits of this statistic are for our sample size: $d_L = 1.46$ and $d_U = 1.77$ and the indeterminate range for negative autocorrelation is from 2.23 to 2.54 which includes both values from (8.44) and (8.45). The equations were therefore re-estimated with the assumption of a first order autogregressive scheme for the residuals. The results were

$$I = \underset{(6.16)}{0.132414}\,[Y_t - (1 - d)\,Y_{t-1}] - \underset{(-1.80)}{6.63333}\,[R'_t + \tilde{b}'_4 M_t] +$$

$$+ \underset{(1.47)}{0.463001}\,P_t + \underset{(0.42)}{1.88332}\,(P_t - P_{t-1}) + \underset{(1.84)}{68.4497} -$$

$$- \underset{(-0.39)}{7.93443}\,S^2_t - \underset{(-0.21)}{2.73001}\,S^3_t + \underset{(0.40)}{8.14147}\,S^4_t +$$

$$+ \underset{(23.72)}{0.918564}\,I_{t-1} + u_{6t}$$

$$u_{6t} = - 0.23632\,u_{6,t-1} + e_{6t} \tag{8.46}$$

$$R^2 = 0.9801 \qquad D.W. = 2.0995$$

$$I = \underset{(5.75)}{0.12835}\,[Y_t - (1 - d)\,Y_{t-1}] + \underset{(1.60)}{15.8878}\,[R'_t + \tilde{b}'_4 M_t) -$$

$$- (1 - d)(R'_{t-1} + \tilde{b}'_4 M_{t-1})] + \underset{(1.45)}{0.482122}\,P_t -$$

$$- \underset{(-0.43)}{2.03628}\,(P_t - P_{t-1}) + \underset{(0.50)}{10.6324} - \underset{(-0.30)}{6.08744}\,S^2_t -$$

$$- \underset{(-0.41)}{5.36945}\,S^3_t + \underset{(0.21)}{4.28576}\,S^4_t + \underset{(21.64)}{0.907843}\,I_{t-1} + u_{6t}$$

$$u_{6t} = - 0.20896\,u_{6,t-1} + e_{6t} \tag{8.47}$$

$$R^2 = 0.979 \qquad D.W. = 2.0572$$

In both cases the coefficient pertaining to the autoregressive process is negative. We observe that between the two specifications of the investment function, one employing the level of the cost of capital variable and the other its difference, we are led to choose the former since it provides a more significant and of the right sign coefficient attaching to it. By concentrating our attention to (8.46) we see that the coefficient of the cost of capital is significant only at a 10% level. We shall seek an improvement of this result by splitting this variable to its components and re-estimating the equation. The result is

$$I_t = \underset{(5.86)}{0.12764}\,[Y_t - (1 - d)\,Y_{t-1}] - \underset{(-2.04)}{7.38475}\,R'_t +$$

$$+ \underset{(0.56)}{5.73736}\,(\tilde{b}'_4M_t) - \underset{(-0.46)}{0.292602}\,P_t + \underset{(0.65)}{2.86702}\,(P_t - P_{t-1}) +$$

$$+ \underset{(2.04)}{74.4219} - \underset{(-0.18)}{3.59744}\,S^2_t - \underset{(-0.05)}{0.680742}\,S^3_t + \underset{(0.52)}{10.7411}\,S^4_t +$$

$$+ \underset{(24.06)}{0.933048}\,I_{t-1} + u_{6t}$$

$$u_{6t} = - 0.2713\,u_{6,t-1} + e_{6t} \qquad (8.48)$$

$$R^2 = 0.9805 \qquad D.W. = 2.1624$$

In (8.48) the second component of the cost of capital is highly insignificant. Moreover the t value of the price variable drops well below unity and the conclusion emerges that real investment decisions are not subject to money illusion and are not influenced by changes of the general price level. The insignificant variables $\tilde{b}'_4M$, P and ΔP are removed from the equation which without them appears as follows

$$I_t = \underset{(6.17)}{0.126942}\,[Y_t - (1 - d)\,Y_{t-1}] - \underset{(-2.48)}{7.84702}\,R'_t +$$

$$+ \underset{(2.50)}{71.6251} - \underset{(-0.02)}{0.309837}\,S^2_t + \underset{(0.14)}{1.63257}\,S^3_t + \underset{(0.67)}{13.1478}\,S^4_t +$$

$$+ \underset{(48.04)}{0.940335}\,I_{t-1} + u_{6t}$$

$$u_{6t} = - 0.27151\,u_{6,t-1} + e_{6t} \qquad (8.49)$$

$$R^2 = 0.9806 \qquad D.W. = 2.1362$$

Equation (8.49) is the specification chosen for the investment function in our model. The accelerator variable is playing an important role in it. Also the findings confirm the long lags that were found to exist in the investment process since the coefficient of adjustment of actual investment to equilibrium one is only 0.06, signifying a very slow adjustment.

The inventory function

Equation (7.67) was estimated with the price change lagged one period because it had been found from preliminary exploratory regressions that the current price change was not significant at a 5% level and was affecting badly the significance of the ΔY term. The result is

$$H_t = \underset{(5.18)}{0.234623}\, Y_t - \underset{(-1.23)}{0.0656596}\, (Y_t - Y_{t-1}) - \underset{(-7.27)}{7.8684}\, P_t +$$

$$+ \underset{(2.90)}{26.0109}\, (P_{t-1} - P_{t-2}) - \underset{(-3.24)}{0.0928264}\, S_{t-1} -$$

$$- \underset{(-2.94)}{665.524} + \underset{(0.86)}{36.1017}\, S_t^2 - \underset{(-0.33)}{8.97039}\, S_t^3 - \underset{(-3.41)}{130.49}\, S_t^4 + u_{7t} \qquad (8.50)$$

$$R^2 = 0.6543 \qquad D.W. = 1.5712$$

The DW statistic lies in the range of indeterminacy concerning the existence of positive serial correlation in the residuals and the equation is re-estimated under the assumption of first order serial correlation of the error term

$$H_t = \underset{(4.13)}{0.227149}\, Y_t - \underset{(-1.60)}{0.0835261}\, (Y_t - Y_{t-1}) - \underset{(-6.15)}{7.86107}\, P_t +$$

$$+ \underset{(3.10)}{28.9116} (P_{t-1} - P_{t-2}) - \underset{(-2.51)}{0.088095} S_{t-1} - \underset{(-2.35)}{634.765} +$$

$$+ \underset{(1.35)}{53.4311} S_t^2 - \underset{(-0.05)}{1.27913} S_t^3 - \underset{(-2.97)}{112.48} S_t^4 + u_{7_t}$$

$$u_{7_t} = 0.19918\, u_{7,t-1} + e_{7_t} \qquad (8.51)$$

$$R^2 = 0.6529 \qquad D.W. = 2.0031$$

In estimating (8.51) care was taken not to include all of the following variables as instruments: inventory investment lagged one period (H_{t-1}), stock of inventories lagged one period (S_{t-1}) and stock of inventories lagged two periods (S_{t-2}). The reason is that by virtue of the identity $S_{t-1} = H_{t-1} + S_{t-2}$ they are not linearly independent and the simultaneous inclusion of them all, while necessary for consistency of the estimators, would introduce singularity in the matrix of instruments. Therefore the one period lagged stock of inventories was omitted.

In (8.51) no coefficient estimate changes sign as compared to (8.50). Autocorrelation has been eliminated although the fit of the equation has been slightly reduced. The coefficient of the price level is significant indicating the existence of money illusion in inventory investment decisions, but it is on the negative side and this means that decision makers are overacting when they convert nominal expected sales into real ones. The change of the price level is seen to affect positively the desired stock of inventories and this is related to a speculative motive for holding inventories rather than a precautionary motive. The estimated coefficients of sales expectations and of adjustment of planned inventories are est $\lambda_6 = 0.088$ and est $\theta = 0.068$ respectively.

<u>The import function</u>

The estimates of equation (7.69) are presented below

$$X_t = \underset{(5.09)}{0.131491}\, Y_t - \underset{(-1.52)}{2.04442}\, P_t + \underset{(4.68)}{366.965}\, E_t^R -$$

$$- \underset{(-1.52)}{3.33379}\, D_t^1 - \underset{(-2.04)}{55.1741}\, D_t^2 - \underset{(-4.44)}{773.013} - \underset{(-2.24)}{45.1033}\, S_t^2 -$$

$$- \underset{(-1.89)}{36.856}\, S_t^3 - \underset{(-5.23)}{115.448}\, S_t^4 + \underset{(3.55)}{0.366409}\, X_{t-1} + u_{8t} \qquad (8.52)$$

$$R^2 = 0.9589 \qquad D.W. = 1.7825$$

It is seen that gross domestic product is the most significant determinant of import demand. The exchange rate is found also to exert a significant influence on imports, which however is positive. This confirms the findings of previous studies according to which devaluation has had a perverse effect on imports. The money illusion term is significant only at a 15% level and has a negative sign, indicating that consumers of imported goods behave in a different way than when they are faced with consumption decisions of domestically produced goods. Namely, whereas they increase their consumption of the latter (when they observe a rise in their nominal income) to the extent that it is not justified in real terms, they are again overacting but to the opposite side when they are concerned with consumption of imported goods. Finally, of the two variables included in the equation to pick up the effect of the import surcharge and the import deposit scheme on imports, the latter seems to have been the most important.

The complete models

Model A consists of equations (8.34), (8.36), (8.37), (8.41), (8.43), (8.49), (8.51), (8.52) and identities (7.70) and (7.71) of the previous chapter. Graphs of actual values of the dependent variables and fitted values from the behavioural equations of the model are presented in Appendix B.

Model B includes equations (8.4), (8.6), (8.7), (8.11), (8.12), (8.19), (8.21), (8.22) and the same identities (7.70), (7.71) plus the additional one (7.91).

The discussion held above about the empirical findings from each equation separately, is of limited scope and it should be accompanied by an analysis of the properties of the whole set of structural equations which form the complete models. This will permit the interaction of the variables and will throw some light on the implied dynamic characteristics of the models constructed. The analysis required is the analysis of dynamic multipliers. The next chapter establishes the theory of dynamic multipliers for linear dynamic models of order higher than one and writes model A in a form amenable to multiplier analysis.

CHAPTER 9

CHAPTER 9

MULTIPLIER ANALYSIS OF DYNAMIC MODELS*

The subsequent discussion of the model's implications will be couched in terms of the impact and interim multipliers which describe the path over time of endogenous variables when an exogenous variable is shocked in a single period (a shock not sustained in following periods). Pioneering results in this area are due to Theil and Boot,[1] who obtained multiplier expressions for a dynamic econometric model from the so called final form of the system, i.e. the one in which the functional dependence is reduced to the endogenous variables being expressed only in terms of current and past values of the exogenous variables. Let

$$\acute{A}y_t = \acute{B}y_{t-1} + \acute{C}x_t + u_t' \qquad (9.1)$$

be the structural form of a linear econometric model, where y_t is the $G \times 1$ vector of endogenous variables, x_t the $K \times 1$ vector of exogenous variables, u_t' the $G \times 1$ vector of disturbances and A, B and C the $G \times G$, $G \times G$, $G \times K$ respectively matrices of estimated structural coefficients. Any system of higher order and/or any lags on the exogenous variables, can be reduced by means of a simple transformation to (9.1) as it will be shown later.

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* An article containing material from this chapter and entitled "Multiplier Effects for Higher than First Order Linear Dynamic Econometric Models" has been accepted for publication in Econometrica.

[1] H. Theil and J. C. G. Boot: "The Final Form of Econometric Equation Systems." Review of the International Statistical Institute, 1962, pp.136-152.

If A is nonsingular, (9.1) can be written as

$$y_t = A'^{-1}B'y_{t-1} + A'^{-1}C'x_t + A'^{-1}u_t = Ay_{t-1} + Bx_t + u_t \tag{9.2}$$

and (9.2) is the reduced form of the system.

The final form was taken by Theil and Boot by substituting $Ay_{t-2} + Bx_{t-1} + u_{t-1}$ for y_{t-1} in (9.2) and repeating the same procedure for y_{t-2} , y_{t-3} , ... , s times with the following result

$$y_t = A^{s+1}y_{t-s-1} + Bx_t + ABx_{t-1} + A^2Bx_{t-2} + \ldots + A^{s-1}Bx_{t-s} + u_t + Au_{t-1} + \ldots + A^s u_{t-s} \tag{9.3}$$

If for $s\to\infty$ the limit of A^s is a zero matrix (and the condition was shown to be that the characteristic roots of A are less than one in absolute value) (9.3) leads to

$$y_t = Bx_t + \sum_{j=1}^{\infty} A^j Bx_{t-j} + \sum_{j=0}^{\infty} A^j u_{t-j} \tag{9.4}$$

which was called the final form of the system. The coefficient matrices B , AB , A^2B etc. contain the impact and interim multipliers which describe the effect of changes in exogenous variables on the endogenous variables in the same period, the next period, etc. For instance the (i,j)th element of A^2B represents the effect of a change in the j-th exogenous variable on the i-th endogenous variable two periods after the current one.

Theil and Boot argued that the case of a higher than first order system with lags to the exogenous variables can be handled in a similar way after the system has been transformed to appear as in (9.1). Consider the simultaneous equations system (p-th order)

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + B_0 x_t +$$

$$+ B_1 x_{t-1} + \ldots + B_q x_{t-q} \qquad (9.5)$$

with y_t the G x 1 vector of endogenous variables, x_t the K x 1 vector of exogenous variables, A_1, A_2, . . . , A_p the G x G matrices and B_0, B_1, . . . , Bq the G x K matrices of estimated reduced form coefficients.

System (9.1), together with p-1 vector identities

$$\begin{aligned} y_{t-1} &= y_{t-1} \\ &\vdots \\ y_{t-p+1} &= y_{t-p+1} \end{aligned}$$

is transformed into

$$y_t^* = A^* y_{t-1}^* + B^* x_t^* \qquad (9.6)$$

where A* and B* are big matrices of order PG x PG and PG X K(Q+1) respectively. The matrices containing impact and interim multipliers are

$$B^* , \quad A^*B^* , \quad A^{*2}B^* , \quad A^{*3}B^* , \ldots \qquad (9.7)$$

but according to Theil and Boot the relevant matrices for the multiplier effects of the K exogenous variables on the G endogenous variables are the leading G x K submatrices of the above matrices. Thus the upper left G x K submatrix in B* gives the impact multipliers, the upper left G x K submatrix in A*B* the interim multipliers for the next period, etc.

It is our contention that this proposition is erroneous and that the multiplier effects for a given distance into the future are portioned out to more than one of the matrices that appear in sequence (9.6). The following section provides an illustration of a simple dynamic model with $G = 1$ and $K = 1$. Later we generalise the findings and obtain explicitly the matrices of multiplier effects of the K exogenous on the G endogenous variables.

Consider the simple case of only one equation with one exogenous variable

$$y_t = by_{t-1} + cy_{t-2} + dy_{t-3} + ex_t + fx_t + gx_{t-2} + hx_{t-3} \qquad (9.8)$$

The above equation can easily be written in the form (9.2) with

$$y^* = \begin{bmatrix} y_t \\ y_{t-1} \\ y_{t-2} \end{bmatrix} \quad x^* = \begin{bmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ x_{t-3} \end{bmatrix} \quad A^* = \begin{bmatrix} b & c & d \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } B^* = \begin{bmatrix} e & f & g & h \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The final form is obtained as

$$\begin{bmatrix} y_t \\ y_{t-1} \\ y_{t-2} \end{bmatrix} = \underbrace{\begin{bmatrix} e & f & g & h \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{B^*} \begin{bmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ x_{t-3} \end{bmatrix} + \underbrace{\begin{bmatrix} be & bf & bg & bh \\ e & f & g & h \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A^*B^*} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ x_{t-3} \\ x_{t-4} \end{bmatrix} +$$

$$\underbrace{\begin{bmatrix} b^2e + ce & b^2f + cf & b^2g + cg & b^2h + ch \\ be & bf & bg & bh \\ e & f & g & h \end{bmatrix}}_{A^{*2}B^*} \begin{bmatrix} x_{t-2} \\ x_{t-3} \\ x_{t-4} \\ x_{t-5} \end{bmatrix} + \ldots \tag{9.9}$$

Theil and Boot assert that $\frac{\partial y_t}{\partial x_t} = e$ (i)

$$\frac{\partial y_t}{\partial x_{t-1}} = be \quad \text{(ii)}$$

$$\frac{\partial y_t}{\partial x_{t-2}} = b^2e + ce \quad \text{(iii)}$$

However it is obvious that the second multiplier consits of another part which is to be found among the elements of matrix B^*, namely f, so that

$$\frac{\partial y_t}{\partial x_{t-1}} = be + f \quad \text{(iv)}$$

Similarly the third multiplier consists of two other parts, i.e. g in B^* and bf in A^*B^* so that

$$\frac{\partial y_t}{\partial x_{t-2}} = g + bf + b^2e + ce \quad \text{(v)}$$

All multipliers after the third period will have four parts portioned out to four consecutive matrices.

For a check of the validity of the above modifications we can examine the multipliers

$$\frac{\partial y_{t-1}}{\partial x_{t-1}} \text{ (vi)} \qquad \frac{\partial y_{t-2}}{\partial x_{t-2}} \text{ (vii)} \qquad \frac{\partial y_{t-1}}{\partial x_{t-2}} \text{ (viii)}$$

The first two must be equal to (i) whereas the last must be equal to (iv) because the value of the multiplier should be invariant with respect to an identical change in the time indices of both exogenous and endogenous variables. Indeed, from matrix A^* the relevant element for (vi) is zero and from A^*B^* is e and $\frac{\partial y_{t-1}}{\partial x_{t-1}} = e$. From matrices A^* and A^*B^* the elements that we are concerned with in (vii) are zeros and the one from $A^{*2}B^*$ is e , so that again we have $\frac{\partial y_{t-2}}{\partial x_{t-2}} = e$. Finally the part of the multiplier (viii) in B^* is zero, in A^*B^* is f and in $A^{*2}B^*$ is be , giving $\frac{\partial y_{t-1}}{\partial x_{t-2}} = f + be$.

It is thus seen that considering only the elements in the leading submatrices in (9.9) leads to an incomplete accounting of multiplier effects in all but one cases (impact multiplier).

We can now turn to the generalised reduced form (9.5) of a linear dynamic system and formalise the findings for the impact and interim multipliers. We have: non zero parts of the m-period multiplier

$$\frac{\partial y_t^i}{\partial x_{t-m}^j}$$

of the j-th exogenous on the i-th endogenous variable are portioned out to $m + 1$ consecutive matrices of the sequence (9.7) for $m = 0$, 1, 2, . . . , q and in $q + 1$ consecutive matrices for $m > q$. Correct multiplier analysis, then, necessitates the examination of generally more than one component parts of interim multipliers.

Expressions for these parts can be obtained explicitly after the matrices of sequence (9.7) have been written in terms of submatrices A_1, A_2, . . . , A_p and B_0, B_1, . . . , B_q. To this end we need to develop the powers of matrix A^*. It can be easily verified that, due to the special form of A^*, its powers have the following configuration:

$$A^* = \begin{bmatrix} C_1(1) & C_2(1) & C_3(1) & \dots & C_{p-1}(1) & C_p(1) \\ I & 0 & 0 & \dots & 0 & 0 \\ 0 & I & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & I & 0 \end{bmatrix}$$

$$A^{*2} = \begin{bmatrix} C_1(2) & C_2(2) & C_3(2) & \dots & C_{p-2}(2) & C_{p-1}(2) & C_p(2) \\ C_1(1) & C_2(1) & C_3(1) & \dots & C_{p-2}(1) & C_{p-1}(1) & C_p(1) \\ I & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & I & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & I & 0 & 0 \end{bmatrix}$$

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$$A^{*p} = \begin{bmatrix} C_1(p) & C_2(p) & C_3(p) & \dots & C_{p-1}(p) & C_p(p) \\ C_1(p-1) & C_2(p-1) & C_3(p-1) & \dots & C_{p-1}(p-1) & C_p(p-1) \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ C_1(1) & C_2(1) & C_3(1) & \dots & C_{p-1}(1) & C_p(1) \end{bmatrix}$$

$$A^{*p+1} = \begin{bmatrix} C_1(p+1) & C_2(p+1) & C_3(p+1) & \dots & C_{p-1}(p+1) & C_p(p+1) \\ C_1(p) & C_2(p) & C_3(p) & \dots & C_{p-1}(p) & C_p(p) \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ C_1(2) & C_2(2) & C_3(2) & \dots & C_{p-1}(2) & C_p(2) \end{bmatrix}$$

where the $C_j(t)$ ($j = 1, 2, \dots, p$ and $t = 2, 3, \dots,$) are matrices of order $G \times G$ connected for the same value of j by the recursive relations:

<u>j = 1</u> $C_1(t) = A_1C_1(t-1) + A_2C_1(t-2) + \dots + A_pC_1(t-p)$

<u>j = 2</u> For $t = 2, 3, \dots, p$

$C_2(t) = A_1C_2(t-1) + A_2C_2(t-2) + \dots + A_{p'-2}C_2(t-p'+2) +$

$+ A_{p'}C_2(t-p'+1)$ and the maximum subscript p' of matrices $A_1, A_2, \dots$ is $p' = t+1$.

For $t = p+1, p+2, \dots$

$C_2(t) = A_1C_2(t-1) + A_2C_2(t-2) + \dots + A_{p-1}C_2(t-p+1) +$

$+ A_pC_2(t-p)$

<u>j = 3</u> For $t = 2, 3, \dots, p$

$C_3(t) = A_1C_3(t-1) + A_2C_3(t-2) + \dots + A_{p'-3}C_3(t-p'+3) +$

$+ A_{p'}C_3(t-p'+2)$ and the maximum subscript p' of matrices $A_1, A_2, \dots$ is $p' = t+2$.

<u>j = 3</u> (cont.) For $t = p+1, \; p+2, \; . \; . \; .$

$$C_3(t) = A_1C_3(t-1) + A_2C_3(t-2) + \; . \; . \; . \; + A_{p-}\, C_3(t-p+1) + A_pC_3(t-p)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

<u>j = p-1</u> For $t = 2, 3, \; . \; . \; ., \; p$

$$C_{p-1}(t) = A_1C_{p-1}(t-1) + A_{p'}C_{p-1}(t-p'+p-2)$$

and the maximum subscript p' of matrices $A_1, \; A_2, \; . \; . \; .$ is

$p' = t+p-2$.

For $t = p+1, \; p+2, \; . \; . \; .$

$$C_{p-1} = A_1C_{p-1}(t-1) + A_2C_{p-1}(t-2) + \; . \; . \; . \; + A_{p-1}C_{p-1}(t-p+1) + $$
$$+ A_pC_{p-1}(t-p)$$

<u>j = p</u> $$C_p(t) = A_1C_p(t-1) + A_2C_p(t-2) + \; . \; . \; . \; + A_{p-1}C_p(t-p+1) + $$
$$+ A_pC_p(t-p)$$

where $A_{p'} = 0_G$, the null matrix when $p' \geq p$,

$C_j(0) = I_G$, the unit matrix, and $C_j(t) = 0_G$ for $t \leq 0$

$(j = 1, 2, \; . \; . \; ., \; p-1)$

and $C_p(t) = 0_G$ for $t \leq 0$.

Correspondingly the matrices of the sequence (9.7) appear as

$$B^* = \begin{bmatrix} B_0 & B_1 & B_2 & \cdots & B_q \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & & 0 \end{bmatrix}$$

$$A^*B^* = \begin{bmatrix} C_1(1)B_0 & C_1(1)B_1 & \cdots & C_1(1)B_q \\ B_0 & B_1 & \cdots & B_q \\ 0 & 0 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$A^{*2}B^* = \begin{bmatrix} C_1(2)B_0 & C_1(2)B_1 & \cdots & C_1(2)B_q \\ C_1(1)B_0 & C_1(1)B_1 & \cdots & C_1(1)B_q \\ B_0 & B_1 & \cdots & B_q \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & 0 \end{bmatrix}$$

$$\vdots$$

$$A^{*p}B^* = \begin{bmatrix} C_1(p)B_0 & C_1(p)B_1 & \cdots & C_1(p)B_q \\ C_1(p-1)B_0 & C_1(p-1)B_1 & \cdots & C_1(p-1)B_q \\ \vdots & \vdots & & \vdots \\ C_1(1)B_0 & C_1(1)B_1 & \cdots & C_1(1)B_q \end{bmatrix}$$

$$A^{*p+1}B^* = \begin{bmatrix} C_1(p+1)B & C_1(p+1)B & \cdots & C_1(p+1)B_q \\ C_1(p)B & C_1(p)B & \cdots & C_1(p)B_q \\ \vdots & \vdots & & \vdots \\ C_1(2)B & C_1(2)B & \cdots & C_1(2)B_q \end{bmatrix} \qquad (9.10)$$

From these matrices, the relevant submatrices for multiplier effects of the K exogenous on the G endogenous variables can be selected. Thus

For impact	multipliers:	B_0
" 1^{st} period	"	: $B_1 + C_1(1)B_0$
" 2^{nd} "	"	: $B_2 + C_1(1)B_1 + C_1(2)B_0$
" 3^{rd} "	"	: $B_3 + C_1(1)B_2 + C_1(2)B_1 + C_1(3)B_0$
.		. . .
.		. . .
.		. . .
" q^{th} "	"	: $B_q + C_1(1)B_{q-1} + C_1(2)B_{q-2} + \ldots + C_1(q)B_0$
" $q{+}1^{st}$ "	"	: $C_1(1)B_q + C_1(2)B_{q-1} + \ldots + C_1(q{+}1)B_0$
" $q{+}2^{nd}$ "	"	: $C_1(2)B_q + C_1(3)B_{q-1} + \ldots + C_1(q{+}1)B_0$

As already noted interim multipliers after the q-th period (inclusive) are portioned out to $q+1$ submatrices $C_1(t)B_k$ $(t = 1, 2, \ldots, k = 0, 1, \ldots, q)$ of the sequence (9.7) or (9.10).

It has been demonstrated that, with the exception of impact multipliers, Theil and Boot's analysis referring to higher than first order dynamic econometric models leads to an incomplete picture of multiplier effects because it does not consider all component parts of interim multipliers. The picture obtained if Theil and Boot's proposition is followed can be very misleading. We shall illustrate this within a simple model characterised by one equation in which (a) the exogenous variables specific to it appear lagged at least one period, and (b) these variables do not appear in any other equation. The example is particularly opportune since one of the equations in our model, namely the price equation, happens to be subject to these two restrictions. Let a two equation model, in which the above restrictions are embodied, be

$$\underbrace{\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}}_{A} \begin{bmatrix} y_{1_t} \\ y_{2_t} \end{bmatrix} = \underbrace{\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}}_{B} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}}_{C} \begin{bmatrix} y_{1,t-2} \\ y_{2,t-2} \end{bmatrix} +$$

$$\underbrace{\begin{bmatrix} 0 & 0 & d_{13} \\ 0 & 0 & 0 \end{bmatrix}}_{D} \begin{bmatrix} x_{1_t} \\ d_{2_t} \\ x_{3_t} \end{bmatrix} + \underbrace{\begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \end{bmatrix}}_{E} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ x_{3,t-1} \end{bmatrix} + \begin{bmatrix} u_{1_t} \\ u_{2_t} \end{bmatrix} \tag{9.11}$$

The reduced form of (9.11) is obtained as

$$y_t = A^{-1}By_{t-1} + A^{-1}Cy_{t-2} + A^{-1}Dx_t + A^{-1}Ex_{t-1} + A^{-1}u_t \tag{9.12}$$

or

$$y_t = A_1y_{t-1} + A_2y_{t-2} + B_0x_t + B_1x_{t-1} + u_t' \tag{9.13}$$

Now, because of the special form of D , B_0 will equal D . Multipliers according to Theil and Boot's suggestions will be given by the sequence of matrices B_0 , $C_1(1)B_0$, $C_1(2)B_0$, . . . It is easily observable that the left 2 x 2 block of elements in this sequence has zero entries so that impact and interim multipliers of all the endogenous variables with respect to exogenous variables x_1 and x_2 will be equal to zero. This is a striking result which however does not correspond to reality.

In practice no modelbuilder has followed Theil and Boot's formulas because all of the models which were explored as to their dynamic properties were nonlinear and in such models dynamic multipliers can not be derived analytically; one has to resort to simulations and multipliers in this case are not independent of the initial conditions and are not proportional to different changes in the exogenous variables. Linear models have the advantage that multipliers can be derived analytically and their size does not depend on initial conditions. While the wrong formulas were not applied in actual models, theoretical discussions of the general linear model seem to have followed a pattern yielding identical conclusions with Theil and Boot. For instance L. R. Klein[2] examines

$$\sum_{i=0}^{p} A_i y_{t-i} + Bx_t = e_t$$

as the general form of the linear model, noting that lagged independent variables can simply be defined as new independent variables so that there is no reason to deal with such lags explicitly. This may be true when multipliers are calculated by simulation of the model. When however analytical solutions are attempted the error is present and the results obtained are identical to Theil and Boot's.[3] For Fromm and Klein substitute for y_{t-1} in the system

$$y_t = \sum_{j=1}^{p} A_j y_{t-j} + B_0 x_t$$

and in the resulting equations

$$y_t = B_0 x_t + A_1 B_0 x_{t-1} + A_1 \sum_{j=1}^{p} A_j y_{t-j-1} + \sum_{j=2}^{p} A_j x_{t-j}$$

they assert that B_0 contains the impact effect on the endogenous variables of a change in the exogenous variables and $B_0 + A_1 B_0$ the one period multiplier effect of a sustained increase in x_t or

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[2] See R. L. Klein: <u>An Essay on the Theory of Economic Prediction</u>, Chicago, 1971, p.15.

[3] G. Fromm and R. L. Klein: Solutions of the Complete Model in <u>The Brookings Model</u>: Some Further Results. Chicago, 1969, pp.365-6.

equivalently A_1B_0 the interim effect for the first period of a non sustained increase in x_t. But the leading submatrix of A^*B^* is $C_1(1)B_0 = A_1B_0$ and thus it is seen that Fromm and Klein's implications coincide with Theil and Boot's erroneous formulas.

We can now write our model in a form amenable to multiplier analysis, bearing in mind that interim multipliers should be sought in more than one matrices of sequence (9.7). The model of which multipliers will be presented and discussed will be model A rather than model B because the latter has been found to lead to some unacceptable multiplier figures which are contained in Table C.1, Appendix C, and multipliers on some endogenous variables of a change in variable Z which is a composite variable, are not easily interpretable. It seems that the IS-LM model as traditionally used or as subsequently expanded does not offer itself to be associated with the identity showing how the government borrowing requirement is financed. This conforms with our inability to obtain theoretically multiplier values from simple IS-LM models to which the above identity has been added (see chapter 7). One possible explanation of this might be that the IS-LM model relates levels of stock variables (money stock, monetary base, special deposits) to levels of flow variables (domestic product and its components) whereas in the identity levels of flow variables (government expenditure, taxes) are related to changes in stock variables (monetary base, special deposits). The eight structural equations of Model A together with the two main identities can be written in matrix notation as follows

$$y_t = Ay_t + By_{t-1} + Cy_{t-2} + Dx_t + Ex_{t-1} + u_t \qquad (9.14)$$

$$u_t = Ru_{t-1} + E_t \qquad (9.15)$$

where the column vector y_t of endogenous variables and x_t of exogenous variables are

$$y_t = \begin{bmatrix} M_t \\ R_t \\ R'_t \\ P_t \\ Y_t \\ C_t \\ I_t \\ H_t \\ X_t \\ S_t \end{bmatrix} \qquad x_t = \begin{bmatrix} B_t \\ S^D_t \\ R^D_t \\ Y^*_t \\ W_t \\ G'_t \\ T_t \\ E^R_t \\ D^1_t \end{bmatrix}$$

where $G'_t = G_t + I^P_t + E_t$

A , B , C are the 10 x 10 coefficient matrices of endogenous variables , D , E the 10 x 9 coefficient matrices of exogenous variables, and R the 10 x 10 diagonal matrix of autogressive coefficients. Constant terms are omitted since they vanish with differentiation. Likewise seasonal dummies S^2_t , S^3_t and S^4_t and the dummy D^2_t from the import function are left out because their effect is on the constant term and differentiation with respect to them is meaningless.

The omission of the above variables will not alter multiplier results for the others. In general this is true for any exogenous variable which we do not wish to consider: its exclusion does not affect the values of the multipliers for the rest of the variables. This can be shown by partitioning the matrices B_0 , B_1 , . . . B_q in the following manner

$$B_0 = \begin{bmatrix} B_0^1 & B_0^2 \end{bmatrix} \quad B_1 = \begin{bmatrix} B_1^1 & B_1^2 \end{bmatrix} \quad . \; . \; . \quad B_q = \begin{bmatrix} B_q^1 & B_q^2 \end{bmatrix}$$

where the B_j^1 ,s $j = 0, 1, \ldots q$ are of dimension $G \times K_1$ $(K_1 < K)$ and contain the coefficients of the exogenous variables whose dynamic multipliers we are interested in and the B_j^2 's $j = 0, 1, \ldots q$ are of dimension $G \times K_2$ $(K_1 + K_2 = K)$ and contain the coefficients of the remaining exogenous variables with respect to which differentiation is of no interest or meaning.

The sequence of matrices $C_1(1)$, $C_1(2)$, . . . do not involve any coefficients of exogenous variables so that the matrices of multiplier effects can be partitioned conformably with the B_j 's and are written as

For impact multipliers	:	$\begin{bmatrix} B_0^1 & B_0^2 \end{bmatrix}$
" 1^{st} period "	:	$\begin{bmatrix} B_1^1 & B_1^2 \end{bmatrix} + \begin{bmatrix} C_1(1)B_0^1 & C_1(1)B_0^2 \end{bmatrix}$
" 2^{nd} " "	:	$\begin{bmatrix} B_2^1 & B_2^2 \end{bmatrix} + \begin{bmatrix} C_1(1)B_1^1 & C_1(1)B_1^2 \end{bmatrix} + \begin{bmatrix} C_1(2)B_0^1 & C_1(2)B_0^2 \end{bmatrix}$
.		. . . . .
.		. . . . .
.		. . . . .

Thus multiplier effects for the first K_1 exogenous variables do not depend on the coefficients of the other K_2 exogenous variables.

System (9.14) can be written as

$$(I - A)y_t = By_{t-1} + Cy_{t-2} + Dx_t + Ex_{t-1} + u_t$$

or

$$A'y_t = By_{t-1} + Cy_{t-2} + Dx_t + Ex_{t-1} + u_t \qquad (9.16)$$

Here A' is equal to $I - A$ and should not be confused with the transpose of A .

Premultiplying (9-14) by R and lagging once we get

$$RA'y_{t-1} = RBy_{t-2} + RCy_{t-3} + RDx_{t-1} + REx_{t-2} + Ru_{t-1} \qquad (9.17)$$

Subtracting (9.17) from (9.16) and collecting terms referring to the same vector we obtain

$$A'y_t = (RA' + B)y_{t-1} + (C - RB)y_{t-2} - RCy_{t-3} + Dx_t +$$

$$+ (E - RD)x_{t-1} - REx_{t-2} + u_t - Ru_{t-1} \qquad (9.18)$$

But from (9.15) $u_t - Ru_{t-1} = \varepsilon_t$ so that

$$A'y_t = (RA' + B)y_{t-1} + (C - RB)y_{t-2} - RCy_{t-3} + Dx_t +$$

$$+ (E - RD)x_{t-1} - REx_{t-2} + E_t \qquad (9.19)$$

and (9.19) is the structural form of the model in which the errors are uncorrelated. System (9.19) can be transformed to a first order one.

By adding the vector identities

$$y_{t-1} = y_{t-1} \qquad \text{and} \qquad y_{t-2} = y_{t-2}$$

and omitting the error term (9.19) is written as

$$\begin{bmatrix} A\acute{y}_t \\ y_{t-1} \\ y_{t-2} \end{bmatrix} = \begin{bmatrix} A' & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} y_t \\ y_{t-1} \\ y_{t-2} \end{bmatrix} = \begin{bmatrix} RA' + B & C - RB & -RC \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ y_{t-3} \end{bmatrix} +$$

$$\begin{bmatrix} D & E - RD & -RE \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_t \\ x_{t-1} \\ x_{t-2} \end{bmatrix} \tag{9.20}$$

or $$A''y^*_t = B'y^*_{t-1} + C'x^*_t$$

with A'' , B' being of order 30 x 30 and C' 30 x 27.

Submatrices A' , $RA' + B$, $C - RB$, $- RC$, D , $E - RD$ and $- RE$ are shown in Appendix C. A'' is a non singular matrix (det A'' = 1.9446) . It follows that we can write the reduced form corresponding to (9.20) as

$$y^*_t = A''^{-1}B'y^*_{t-1} + A''^{-1}C'x^*_t$$

or $$y^*_t = A^* y^*_{t-1} + B^* x^*_t \tag{9.21}$$

and this is exactly the form (9.6) in which (9.5) was transformed.

The necessary and sufficient condition for the derivation of dynamic multipliers is that the characteristic roots of A^* should be less than one in absolute value (for complex roots their modulus should be less than one). This is also known as the stability condition for deterministic systems. The non-zero roots of A^* are presented in Table 9.1 (the calculation of the roots is discussed in Appendix C).

Table 9.1

Non-zero characteristic roots of A*

Real Roots	Complex Roots
0.9553695	0.9235411 ± 0.0013863 i
0.8294939	0.8076310 ± 0.1174252 i
0.8133684	0.3160532 ± 0.0701858 i
0.4230511	
- 0.2629435	
- 0.2125368	
0.1995715	
0.1054255	
- 0.0002352	
0.0002271	
- 0.0000025	
0.0000003	

They are all less than one in absolute value so that our model is stable in the deterministic sense, although capable of generating damped oscillations because of the existence of complex roots.

It is a simple task to verify that the inverse of A* has the form

$$\begin{bmatrix} A'^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

and consequently A* can be written as

$$\begin{bmatrix} A_1 & A_2 & A_3 \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$$

and B* as

$$\begin{bmatrix} B_0 & B_1 & B_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with

$$A_1 = A'^{-1} (RA' + B) \tag{9.22}$$

$$A_2 = A'^{-1} (C - RB) \tag{9.23}$$

$$A_3 = A'^{-1} (- RC) \tag{9.24}$$

$$B_0 = A'^{-1} D \tag{9.25}$$

$$B_1 = A'^{-1} (E - RD) \tag{9.26}$$

$$B_2 = A'^{-1} (- RE) \tag{9.27}$$

Multiplier values of the ten endogenous variables of the model with respect to the nine exogenous variables are obtained according to the formulas on page 347, and all tables containing dynamic multipliers that will be presented in following sections are based on the analysis carried out above. Additionally multiplier values from Theil and Boot's formulas will be juxtaposed in some graphs to correct values for the sake of depicting the existing error. The calculations involved are performed by the programs presented in Appendix C.

CHAPTER 10

CHAPTER 10

ANALYSING SOME DYNAMIC PROPERTIES OF THE MODEL

In this chapter we shall present and discuss a number of results identified as certain dynamic properties of the model which we have constructed and estimated in the previous chapters. These results relate to propositions which are of genuine interest in monetary economics and which are at the centre of disputes among economists. It is natural to concentrate only on these issues since our work places a lot of emphasis on the money market (which so far has been examined in greater detail) and aspires to give some answers on empirical propositions involving monetary variables. However, the complete set of results about the dynamic properties of our model which are described by the impact and eighty interim multipliers is contained in Appendix D where the interested reader is referred to. Most of the propositions to be examined here, are known as being a part of a body of empirical propositions called monetarism. Additionally, evidence will be offered on the effect of Special Deposits on market interest rates as compared with that of open market operations.

Examining some monetarist propositions

In order to appraise monetarist propositions,[1] which are of an empirical nature, one has either to assume a structure which, after the estimation of its parameters, will offer an assessment of the truth

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[1] Propositions to be examined will be cited later.

of them or to use alternatives to model building such as one equation tests or examination of timing and turning point relationships between variables or rates of change of variables. Neither of the last two methods appears to be satisfactory and proponents of monetarism who have used them, were accused as testing a "black box" theory, i.e. one in which the underlying theoretical structure has not been made explicit.

In response to these criticisms Friedman has spelt out an aggregative framework of analysis which can be the common starting point of monetarist and income-expenditure theory. This common model is simply the IS-LM model in a static and highly simplified form. The model has already been presented in chapter 2 (p. 20) but it will be repeated here for the sake of a continuous presentation of the issues involved. It consists of the following equations

$$\frac{C}{P} = f\left(\frac{Y}{P}, r\right) \qquad (10.1)$$

$$\frac{I}{P} = g(r) \qquad (10.2)$$

$$\frac{Y}{P} = \frac{C}{P} + \frac{I}{P} \qquad (10.3)$$

$$\frac{M^D}{P} = \ell\left(\frac{Y}{P}, r\right) \qquad (10.4)$$

$$M^S = h(r) \qquad (10.5)$$

$$M^D = M^S \qquad (10.6)$$

These equations describe the product and money markets and can be identified as the IS and LM curves. It must be pointed out at this place that the IS-LM framework as presented in the above equations is not inconsistent with the introduction of a general portfolio adjustment mechanism emphasised by monetarists. The latter involves the supply schedules and rates of return of more than one asset and the dynamic mechanisms which shape the timing of changes in these returns and as such it gives a disaggregated picture of asset markets in contrast with the IS-LM model which is an aggregative tool of analysis. The aggregate model can however be appropriately expanded to include representative yields of more than one broader classes of assets yielding a pecuniary return. We have seen for example that Moroney and Mason[2] have expanded it by incorporating a term structure equation relating the yields on short-term and long-term debts. Our work here has also expanded it by including a relationship between the rates of return on long-term debts and equities. In these instances the model can potentially offer implications about the effect (size and timing) of policy actions on these representative rates of return, as our results will amply show, without at the same time losing its aggregative nature.

Equations (10.1) to (10.6) contain seven "unknown" variables: C , I , Y , r , P , M^D , M^S while there are only six of them. Consequently there is one missing equation. Friedman argued that the simple quantity theory adds to this system the equation

$$\frac{Y}{P} = y = y_0 \qquad (10.7)$$

i.e. real income is determined exogenously by a system of Walrasian

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[2] J. R. Moroney and J. M. Mason, *op.cit*.

equations of general equilibrium and the equilibrium solution for income in this case is, as it is well known, a full-employment level of it.

On the other hand, according to Friedman, the simple income-expenditure theory completes the system by appending the equation

$$P = P_0 \tag{10.8}$$

i.e. the price level is given exogenously.

To this challenge Keynesians have answered that prices, or both prices and wages, have been endogenously determined in large scale models constructed and estimated. And indeed Friedman has recognised this as a post-Keynesian development although not derivable from Keynes' theoretical system but rather from pre-Keynes classical system.[3] On the other hand, the Walrasian system of equations invoked by Friedman for the determination of real income, does this, in the short-term, in terms of the price level so that the assertion that y is exogenously supplied must be viewed in the context of a long-run equilibrium solution, where real income is determined by real magnitudes only and is independent of the price level. Consequently in the short-run the difference between the quantity theory of money and the post-Keynesian income-expenditure theory is narrowed down since both theories include an additional equation which makes both real income and the price level endogenously determined. Friedman has stressed the importance of introducing these interactions between real and monetary variables as a means of interpreting reality.[4]

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[3] See M. Friedman: A Theoretical Framework for Monetary Analysis. Occasional Paper 112. National Bureau of Economic Research, 1971, p.32, and M. Friedman: "Comments on the Critics." Journal of Political Economy, 1972, p.948, fn.11.

[4] M. Friedman: "Interest Rates and the Demand for Money." In The Optimum Quantity of Money and Other Essays. Macmillan, 1969, p.151.

But these interactions refer to short-run disequilibrium situations and not to the long-run equilibrium solution.

Thus the distinction between short-run disequilibrium and long-run equilibrium becomes a crucial one and the question arises whether these two situations can be accommodated within a single model. Some economists seem to believe that it is not possible to develop *"an empirically verifiable common framework applicable to both short-run and long-run processes and capable of generating testable implications for short-run and long-run positions of aggregate variables."* [5] This is not correct however and our discussion in chapter 3 has shown that the introduction of some dynamic adjustment mechanisms which are cited there and which have the property that in their reduced forms the weights attached to lagged values of the explanatory variables sum to unity, makes possible the distinction within the same framework of a disequilibrium model, an equilibrium model and a stationary model. The equilibrium model is the one in which equilibrium values of the variables (or anticipated or expected or any other name we might want to give to them) are equal to actual ones whereas in the disequilibrium model discrepancies between equilibrium values and actual ones describe the behaviour of some variables. Friedman is in complete agreement with this characterisation[6] and he himself has used one of the adjustment mechanisms discussed in chapter 3, namely the adaptive expectations mechanism.

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[5] See K. Brunner and A. H. Meltzer: "Friedman's Monetary Theory." Journal of Political Economy, 1972, p.839.

[6] See for example M. Friedman, *op.cit.*, p.152, and M. Friedman, *op.cit.*, 1971, pp.54-5.

However for the idea of the possible coexistence of a short-run and a long-run relation between variables we must give credit to Keynes who, although he concentrated on the 'first round' effects of changes of variables on other variables in his system, did not exclude the possibility of the existence of a long-run relationship. For example, he stated in his General Theory:[7]

> *"So far we have been primarily concerned with the way in which changes in the quantity of money affect prices in the short period. But in the long-run is there not some simpler relationship? This is a question for historical generalisation rather than for pure theory. If there is a tendency to a measure of long-run uniformity in the state of liquidity-preference, there may well be some sort of rough relationship between the national income and the quantity of money required to satisfy liquidity preference, taken as a mean over periods of pessimism and optimism together."*

In the above passage we note the following points raised by Keynes and relate them to our distinction of short-run disequilibrium and long-run equilibrium: (a) the long-run relationship, if it exists, will be a simpler one. Indeed the one we assume before applying any dynamic adjustment mechanism to it is simpler than the one resulting after the transformation which takes into account short-run dynamic adjustment; (b) to distinguish between long-run and short-run equations is not a matter of pure theory. Indeed this involves the specification of an adjustment mechanism which is rather *ad hoc*; (c) the long-run relationship can be considered as a mean over periods of pessimism and optimism together. Indeed discrepancies between equilibrium and actual values are the key elements in cyclical fluctuations which characterise the real world.

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[7] J. M. Keynes: The General Theory of Employment, Interest and Money. Macmillan, 1936, p.306.

Having made clear the distinction between long-run equilibrium and short-run disequilibrium, we take a further step and examine the form of the missing equation according to the specification offered by monetarists and NeoKeynesians for both the short-run and the long-run. To start with monetarists real income and the price level are assumed to interact in the short-run but in the long-run there is no such interaction and the solution is the one which is given by the Walrasian system of equilibrium equations, i.e. it is a solution corresponding to a full-employment output. On the other hand, as already noted, monetarists accept that in equilibrium anticipated values (or equilibrium values) are equal to actual ones so that the missing equation must be specified in such a way as to ensure simultaneously a full-employment output and equality of anticipated and actual values in equilibrium. To quote Friedman:[8]

> *"One subtle problem in this kind of a structure in which we have identified the absence of a discrepancy between actual and anticipated values as defining long period equilibrium is to assume that the feedback relations . . . . . are consistent with the expanded system of Walrasian equations which specify the long term equilibrium values. At least some values are implicitly determined in two ways: by a feedback relation such as equations . . . . . and by the system of long run equilibrium equations. The problem is to assure that at long-run eqiulibrium these two determinations do not conflict."*

This sheds light on the particular way in which the Phillips Curve has been specified by Monetarists. Three types of equation have appeared in the literature.[9] Two of them have already been mentioned in chapter 6.

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[8] M. Friedman, op.cit., 1971, p.55

[9] For empirical tests on them (one equations tests) see respectively, R. M. Solow, op.cit., M. Parkin: "The Short-Run and Long-Run Trade-Offs between Inflation and Unemployment in Australia." Australian Economic Papers, 1973, pp.117-44, and M. Artis: "Some Aspects of the Present Inflation and the National Institute Model." In The Current Inflation. Edit. by H. G. Johnson - A. R. Nobay. Macmillan, 1971, pp.3-37.

(a) $\dot{p} = g(u) + \dot{p}^e$ (10.9)

where $\dot{p}$ is the rate of change of prices

$\dot{p}^e$ is the expected rate of change of prices

and $g(u)$ a function of excess demand for product as a function of unemployment (excess demand for labour derived from excess demand from product).

In equilibrium when the expected rate of change of prices equals the actual rate, $g(u) = 0$ defines the natural rate of unemployment which, in a system with market imperfections, cost of information, etc., is the rate corresponding to a zero rate of unemployment in perfectly frictionless markets.

(b) $\dot{w} = f(u) + \dot{w}^e$ (10.10)

where $\dot{w}$ is the rate of change of wages

$\dot{w}^e$ is the expected rate of change of wages

and $f(u)$ a function of unemployment.

Again in equilibrium $f(u) = 0$ defines the natural rate of unemployment.

(c) $\dot{w} = h(u) + \dot{p}^e$ (10.11)

where $h(u)$ is a function of unemployment (excess demand for labour).

In equilibrium, if $\dot{w} - \dot{p} = \dot{q}$, the rate of productivity growth, the relationship $h(u) = \dot{q}$ defines the natural rate of unemployment.

The common feature of all the above specifications is that they restrict the coefficient of the expectational variable to be unity so that Friedman's postulate of full employment equilibrium and equality between expected and actual values is satisfied. At an empirical level attempts have been made to test whether that coefficient is indeed unity but all work has been done in the context of a single equation - a price or wage equation. A review and some criticisms on this approach was given in chapter 6. On the other hand, we have only one example where this type of specification has been used as a part of a full model and particularly as a part of the common framework spelt out by Friedman, and this example is McCallum's study examined in chapter 2.[10] Indeed McCallum estimated the following price equation appended to the IS-LM model

$$\Delta \log P_t = \lambda\gamma (\log y_t - \log q_t) + (1 - \lambda) \Delta \log P_{t-1} \qquad (10.12)$$

where P is the price level, y is real income, q is the full-employment level of it and the partial adjustment model rather than the adaptive expectations one has been applied to equilibrium values of $\Delta \log P$. His framework would be in principle acceptable by Monetarists because the stationary model (not simply the equilibrium one) implies that real output is equal to full-employment output. However the distinction between nominal and real rate of interest has not been made, and this distinction is relevant since in the short run the assumption of stability of the price level is not tenable. Also McCallum did not examine the model's dynamic properties in order to assess the validity of monetarist propositions.

Having sorted out monetarist assumptions we now turn our attention to the Keynesian system as it originally stemmed from Keynes'

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[10] B. T. McCallum, *op.cit.*, 1973.

analysis and as subsequently developed by NeoKeynesians. Keynes' challenge to the Quantity Theory has been his showing that there can be a long-run equilibrium position characterised by less than full-employment of resources. This proposition has been disputed by Friedman as shown by others to be false.[11] The argument he invoked was that it has been demonstrated by others (Pigou, Patinkin) that if the effect of real wealth on consumption is taken into account and all prices are flexible the equilibrium solution is necessarily a full-employment one.[12] However this statement of the argument by Friedman is very misleading since it does not mention explicitly Keynes' relaxation of the assumption of nominal wage flexibility. Indeed Pigou's and Patinkin's demonstration was a reply to the early Keynesian literature which disputed the classical view that price and wage flexibility are sufficient conditions for full employment equilibrium in a monetary economy.[13] Pigou and Patinkin have shown that this is the case but only on the condition of price and wage flexibility. But Keynes' challenge was essentially based on the assumption that the wage unit is sticky downwards and that it will tend to rise before full employment has been reached.

Various explanations have been put forward for the observed downward inflexibility of the money wage rate but the commonest is that workers suffer from money illusion, i.e. while they refuse to accept a reduction of their money wages as a means to reduce their real wages they accept price inflation in this role. But in this case in which workers

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[11] M. Friedman, *op.cit.*, 1971, p.16.

[12] The wealth effect alluded here is the price-induced wealth effect (cf. our discussion of wealth effects in chapter 7, p.256).

[13] See for example A. J. Hansen: Fiscal Policy and Business Cycles. W. W. Norton and Co., New York, 1941.

fail to perceive what price rises do to their wages, if real wages become flexible enough (and this in turn depends on liquidity preferences in conjunction with the available quantity of money) then the long run equilibrium solution will be one of full-employment but surely this is not what Keynes wanted to emphasise in his General Theory.

Another explanation has been suggested recently by Tobin[14] as interpreting Keynes who did not rely on the above type of money illusion.

> *"Instead, Keynes emphasised the institutional fact that wages are bargained and set in the monetary unit of account. Money wage rates are, to use an unKeynesian term, 'administered prices'. That is, they are not set and reset in daily auctions but posted and fixed for finite periods of time. This observation led Keynes to his central explanation: Workers, individually and in groups, are more concerned with relative than absolute real wages. They may withdraw labor if their wages fall relatively to wages elsewhere, even though they would not withdraw any if real wages fall uniformly everywhere."*

The resistance by workers of a reduction in their nominal wages violates one of the basic postulates of the Quantity Theory namely that in equilibrium the real wage rate is equal to the marginal disutility of labour (condition for maximisation of utility by labour). Friedman by using the Walrasian system of equations for obtaining equilibrium full-employment output has provided an alternative and has diverted attention from the Classical System which is summarised by the following equations:

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[14] J. Tobin: "Inflation and Unemployment. American Economic Review, 1972, p.3.

$$I\ (r) = S\ (y) \tag{10.13}$$

$$y\ P = \overline{M}\ k \tag{10.14}$$

$$y = \Phi\ (n) \tag{10.15}$$

$$\Phi'\ (n) = \frac{W}{P} \tag{10.16}$$

$$\Psi'\ (n) = \frac{W}{P} \tag{10.17}$$

where I , S and y are real investment, saving and income, r is the interest rate, n is employed labour, P is the price level $\overline{M}$ the quantity of money exogenously given, k the velocity of circulation of money, Φ' is the marginal product of labour and Ψ' the marginal disutility of labour. When (10.17) holds, (10.16) and (10.7) alone determine employment and the real wage rate, independently of the quantity of money. Similarly (10.15) and (10.13) determine y and i without the intervention of equation (10.14) which implies that money determines prices (and nominal wages) and does not affect the level of output and the interest rate.

Thus we see that the implications of the Classical System about the independence of real and monetary variables are identical with the implications derived from the 'common framework' if y is assumed to be determined exogenously by the system of Walrasian equations. Let us now see what changes are brought about by Keynes' innovations. First, equations (10.13) and (10.14) are replaced by the IS and LM curves respectively. And second, the assumption of an exogenous wage rate inflexible downwards violates (10.17) and makes it redundant. The solution of the system in this case makes the equilibrium values of y and r dependent upon the money supply and this equilibrium is generally a less than full employment equilibrium. Only for a single value of the money supply the resulting price level is such that the real wage rate corresponds to a full employment solution.[15]

Leaving aside the issue of full-employment equilibrium for the moment we can examine how empirical research in the Keynesian spirit has completed the common framework presented by Friedman. Has it produced an equation like (10.16) or has it followed alternative ways? The answer shows that alternatives have been used while the above framework provided the rationalisation of the assumption that there can be and there will generally be an equilibrium solution corresponding to less than full employment. The commonest procedure adopted by researchers was to add to this framework however detailed the latter was, price equations typically expressing prices as obtained by marking up prime costs per unit of output. These corresponded to the cost-push explanation of inflation and are accepted by Friedman as the only ones derivable from the Keynesian system. The advocates of cost-push inflation argued that the source of inflation is not excess demand but rather market power manifested either in strong labour unions raising the wages of their workers or in oligopolistic firms raising the prices of their products.

Subsequent developments recognised that different market situations are likely to coexist so that excess demand is appropriate for the determination of certain prices (competitive markets-demand pull

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[15] For a demonstration of this and excellent discussions of the fact that generally equilibrium will be at less than full employment see W. L. Smith: "A Graphical Exposition of the Complete Keynesian System." Southern Economic Journal, 1956, and R. A. Mundell: "An Exposition of Some Subtleties in the Keynesian System." Weltwirtschaftliches Archiv, 1964, both reprinted in Macroeconomics, Selected Readings. Edit. by W. L. Johnson and D. R. Komerschen, 1970, pp.24-6 and 36-7 respectively. The interdependence of real income and the price level in equilibrium could of course be elaborated within other frameworks but the above seems to be closer to Keynes' analysis.

inflation). The result of such developments was a price equation which merged cost-push and demand-pull inflation in one equation where the two components were not necessarily independent of each other but could interact. Certainly the excess demand component of them is not derivable from Keynes' system as Friedman correctly pointed out.

Also recent developments partly instigated by the monetarist invitation to pay more attention to the quantification of expected price changes and the distinction between nominal and real interest rate, studied inflation as a phenomenon of disequilibrium (shifting in time) in and between sectors, where the sectors are the private sector on the one hand and the public sector on the other.

Workers in competitive subsectors of the private sector, making real wage comparisons between themselves and other workers either in the private sector or in the public sector, are not prepared to allow a reduction of their nominal wages when there is a deficient demand. As a result unemployment is created and it can be taken up by the public sector if the latter is committed to a full-employment target. This in turn introduces more inflation if budget deficits are financed by monetary expansion. Also workers in non competitive subsectors of the private sector, making similar real wage comparisons do not accept nominal wage cuts when the state of demand dictates it. But in this case the market power of the firms which employ them sometimes permits the raising of the prices of their products, which the public sector accommodates later on by paying higher prices for the goods and services it buys or by granting higher wages or salaries to the persons already employed in it. This has the same consequences if it creates a budget deficit accompanied by an increase in the monetary base and the money supply.

The result of the above described events is likely to be an underestimation on the part of individuals of expected price changes in the short-run, as already noted in chapter 6. This element has been incorporated in the dynamic specification of the price equation.[16] The specification is such that in the long-run equilibrium, expected price changes are equal to actual ones but whether this equilibrium will be a full-employment one depends on the assumed determinants of expected price changes. If the latter is only excess demand, then the assumption of full-employment equilibrium can be maintained on the simultaneous acceptance of a rise in the natural rate of unemployment that the changing relationship between the private sector and public sector might imply. However this procedure ignores the pricing behavior of non competitive firms, which if taken into account makes the system a proper Keynesian one, i.e. one in which there will generally be a long-run equilibrium solution at less than full employment and in which money affects equilibrium real output and not only the price level. There is only one case in which long-run full-employment can be secured and this is only when the estimated coefficients of the cost-push terms in the price equation are like the ones obtained if, on the average, the rate of change of the wage rate minus the rate of change of the price level equals the rate of change of average labour productivity. The last however is not likely to be true in general.

The above rather extensive discussion of the two systems, the monetarist and the NeoKeynesian, aimed at putting in the right perspective our model by elucidating the differences between them, and at

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[16] See for example T. J. Sargent, *op.cit.*, 1971, and our specification of the equation.

providing a criterion to decide *a priori* on which monetarist propositions are likely not to be true and which ones may not be dependent on these differences. Hence the monetarist propositions must be cited at this place. They were recently listed by Friedman[17] and are given below in summary form.

(a) a change in monetary growth affects interest rates in one direction at first but in the opposite direction later on. Some monetarists have gone as far as saying that interest rates return to their initial level or even exceed it at long-run equilibrium.

(b) the long-run equilibrium effect of a monetary change is felt primarily on prices, while real output is influenced by real factors such as the enterprise, ingenuity and industry of people, etc. Again the extreme position of some monetarists is that the long-run elasticity of the price level with respect to money is unity and the long-run elasticity of output with respect to money is zero.

(c) in the short-run a monetary expansion exerts an impact on both real output and the price level. It shows up first in output, the (maximum) effect being felt two or three quarters later, and hardly at all in prices. The (maximum) effect on prices comes about two or three quarters after the (maximum) effect on real output.

(d) there is a consistent though not precise relation between the rate of growth of the quantity of money and the rate of growth of nominal income. This relation is not obvious to the naked eye largely because it takes time for changes in monetary growth to affect income and how long it takes is itself variable.

(e) government spending financed by taxes or by borrowing from the public will not be inflationary in the long-run but if it is financed by creating money it will be inflationary.

Clearly proposition (b) can be questioned *a priori* in a model of Keynesian persuasion because, as has been discussed above, there can be and there will most probably be a long-run equilibrium position at less than full-employment in which money affects in the long-run both the price level and real output and not only the former. An additional reason for believing this is that some components of real expenditure in our model

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[17] M. Friedman: The Counter-Revolution in Monetary Theory. Occasional Paper 33. Institute of Economic Affairs, London, 1970, pp.22-6.

were found to be subjected to money illusion or to be influenced by price changes and the homogeneity assumption has been relaxed from the money demand function.

Also proposition (e) seems to depend on (b) although in this case fiscal changes appear as well, and the monetary expansion is not stated alone. The remaining propositions can be examined without any prior diffidence as to their truth. It is to this task that we now turn.

Effects of monetary changes on interest rates

We have seen in chapter 2 how some criticisms on the IS-LM model, when put into its language, are translated into appreciably different implications regarding the course of the interest rate when an expansion of the money stock occurs. In the first place we noticed that monetarists were anxious about the slope of the IS curve, a positive slope implying that the equilibrium interest rate rises at a rightward shift of the LM curve due to a monetary expansion. In the simple static model the condition for the IS curve to be of a positive slope, was worked out to require that the marginal propensity to consume and invest out of income should exceed one. Also we have mentioned in a diagramatic representation of the IS-LM model, the existence of income and price expectations effects which work to revert the influence of the liquidity effect accompanying a monetary expansion. It is one of the extreme assertions of monetarists that the final level of the interest rate is the same or even higher than the initial so that growth in the supply of money is associated with higher and not lower levels of interest rates. This is a startling proposition and because of its importance in the choice of the best indicator of the thrust of monetary policy, we now turn our attention to the three effects mentioned above.

These effects together with their theoretical underpinnings are: (a) the liquidity effect. The only way in which the demand for money can adjust to an increased supply is through interest rates which are thus driven down at a monetary expansion. This represents a movement along the demand for money curve. Individuals find themselves holding a larger amount of money than they wish to hold so they attempt to exchange it and buy return yielding assets in the market. The prices of these assets (debts and real capital-equities) rise and the interest rates fall. Moreover the act itself of supplying more money to the economy may tend to depress interest rates on debts. For example an open market purchase will affect security prices and yields but the financing of maturing debt will probably not affect them directly. (b) the income effect. It refers to changes in interest rates which are caused by changes in nominal income. The fall in the rates of return of assets as a result of the sales of debts and subsequently of equities, will stimulate investment and income will rise. Regardless of whether this rise comes from a change in physical output or a change in the price level it implies a higher demand for money. With a constant supply of money, interest rates will be pushed up again by people trying now to sell assets in the market. (c) the price expectations effect. An increase in the money stock may also contribute to the upward movement of the interest rate through a price expectations effect. By that it is meant that interest rates on debts will rise when people expect further price rises, because lenders will seek to protect themselves from capital losses on debts through a higher nominal interest. The price expectations effect says nothing directly about the effect of a change in the stock of money on interest rates but it is a relationship between interest rates and price changes. But if a monetary expansion raises prices and creates expectations of further price increases then it triggers a price expectations effect.

The three effects of an increase in the money stock on interest rates were shown by Gibson[18] in the following diagram:

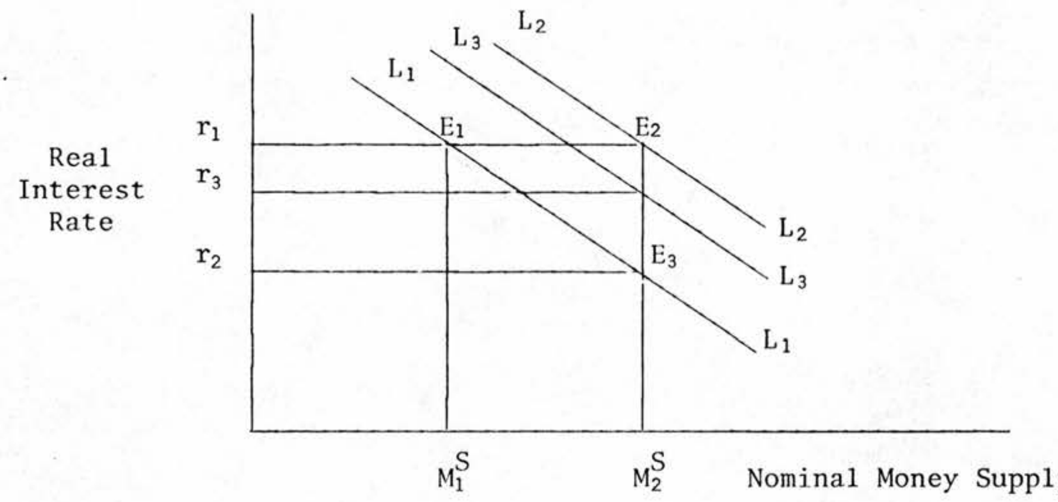

Diagram 10.A

The different curves represent the inverse relationship of money and interest rate as expressed in the money demand function. Let us assume a linear form of the function that Gibson quotes

$$M^d = -a\left[r + \left(\frac{\Delta P}{P}\right)^E\right] - b\left(\frac{\Delta P}{P}\right)^E + cY \qquad (10.18)$$

$$a, b, c, > 0$$

with M^d nominal money stock, r the real interest rate, $\left(\frac{\Delta P}{P}\right)^E$ the expected rate of inflation, $r + \left(\frac{\Delta P}{P}\right)^E$ the nominal rate and Y nominal

[18] W. E. Gibson: "Interest Rates and Monetary Policy." Journal of Political Economy, 1970, pp.431-455.

income. Now in terms of Gibson's diagram, the economy is initially at point E_1 where the money stock is M_1^S and the interest rate r_1. The liquidity effect means that an increase in M drives the interest rate down to r_2 and this movement takes place along the demand curve L_1. Subsequently income rises and because Y represents a shift factor in the function L_1 shifts upward to the position L_2 where the interest rate is restored to its initial level. This can be done only under the restrictive assumption that prices and real income have increased in the same proportion as the money stock. If now the expected rate of inflation increases, this results according to Gibson in a downward shift of the curve from the position L_2 to the position L_3 thus lowering real interest rate to r_3. But as Steindl rightly notices:[19] *"the rate of interest relevant for decisions regarding the allocation of wealth between money and other assets is the* nominal *rate because it is the appropriate measure of the opportunity cost of holding money* (Cf our discussion on the demand for money). *The appropriate interest rate on the ordinate in Diagram 10.A is therefore the nominal rate, and it is this rate - not the real rate - that is determined by the supply and demand for nominal money (those variables measured on the abscissa)."*

But Steindl goes on to argue that *"the interpretation of figure should be that the expectation of an increased rate of price change results in a fall in both the nominal and real rate of interest, with the real rate being less than the money rate by the expected rate of price increase."* This type of analysis leads Steindl to wonder on the apparent contradiction between the theoretical model implying that the nominal rate

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[19] F. G. Steindl: "Price Expectations and Interest Rates." Journal of Money, Credit and Banking, 1973, p.940.

falls with increased anticipated rate of inflation and the empirical results showing a rise of the nominal rate. Clearly such contradiction does not exist if we do not recognise a separate influence of the expected rate of inflation on money demand, since the former is supposed to represent the real rate of return on goods and services (the nominal pecuniary return being zero) and is equal to the real return on money.

The above example has presented a framework permitting a comparative static diagramatic representation of the liquidity, income and price expectations effects of a monetary change on nominal interest rates. It should be viewed only as an illustrative example showing the possibility that the final level of the interest rate can be higher than the initial because of the operation of the last two effects. Obviously the answer to the question of what happens to the interest rate is an empirical matter and it cannot be given from estimates of the money demand function alone. Account must be taken of the whole macroeconomic system because monetary phenomena are not independent of real. Thus alternative schemes might be used and the IS-LM model or any extension of it are possible candidates. We recall our discussion in chapters 2 and 7 in which it was pointed out that the real interest rate is the appropriate rate for investment decisions so that on the nominal rate-real income plane the expected rate of inflation represents a shift factor for the IS curve and price expectations effects lead to a higher nominal rate because of the outward shift of this curve. Consequently there is no need to force price expectations to appear in the money demand function.

The following step in the investigation of what happens to interest rates after a monetary change has occurred, concerns the yield on equities. There was no explicit treatment of this question by monetarists but it can be answered on similar lines if we concentrate on the portfolio adjustment of individuals which follows a monetary expansion. As we have seen individuals attempt to rid themselves of excessive money holdings by selling bonds to the market at the first stage. In this way capital goods-equities become more attractive because of their lower prices and higher yields and people will turn to them and substitute them for bonds in their portfolios. With a given stock of capital this will bid up their prices and lower their yields. Higher prices of capital goods will instigate the production of new capital goods by firms which engage in these activities. The latter will find it profitable to increase the supply since their labour costs were not yet affected by the conditions in the markets for debts and capital. The production of capital goods will raise income and will result in a chain of further adjustments in which more consumption goods will be produced and income will rise further. As a result of the increase in income the downward movement of the yield on equities will be reversed as people will try to meet the increased need for money for transactions purposes by selling bonds and equities. Additionally the supply of capital goods has increased and this might reinforce the upward tendancy of yields. Expectations of price increases work also in the same direction by raising the yield of equities further, only if they add to the higher demand for money. As we have seen the expectations effect pertains to the nominal rate on debts and as such is not applicable to the yield of equities.

We now turn to the empirical evidence on the effects of a monetary change on interest rates. To the best of our knowledge there is no evidence at all as to what happens to the yield of equities. The existing work focuses on the interest rate on debts, concerns the U.S. economy and comes from single equation estimates or from simulations of large scale econometric models.

In the first case the work of Gibson can be mentioned. Gibson estimated the following equations in two different articles which appeared at the same time, using three different interest rates and two definitions of the money stock.

(a)[20] $$i = c + a_0 \ln M_t + a \ln M_{t-1} + \ldots + a_{12} \ln M_{t-12}$$

(b)[21] $$i = c + a_0 \left(\frac{1}{M}\frac{dM}{dt}\right)_t + a_1 \left(\frac{1}{M}\frac{dM}{dt}\right)_{t-1} + \ldots + a_{12}\left(\frac{1}{M}\frac{dM}{dt}\right)_{t-12}$$

He interpreted equation (a) (for which a high R^2 was obtained) as one of the reduced forms of a simultaneous equations system with the exception that money was the only exogenous variable; and equation (b) (for which a very low R^2 was obtained) as not being advanced *"to explain all or even most of the observed variation in interest rates"* but *"as being addressed to rate movements due to monetary disturbances"*, on the assumption that other exogenous influences (that should appear in (b) but were omitted) are uncorrelated with money, a_0 measures the effect on

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[20] W. E. Gibson, *op.cit.*, 1970, pp.431-455.

[21] W. E. Gibson: "The Lag in the Effect of Monetary Policy on Income and Interest Rates." Quarterly Journal of Economics, 1970, pp.288-300.

interest rate of a monetary change or a change in the rate of increase of money supply during the current period, a_1 the same effect after one period and so on. Income and price expectations effects should show their presence with positive coefficients and the sum of all coefficients measures the total effect on interest rate.

The results from the estimation, in which monthly seasonally adjusted data were used for the same period of time, showed that for the narrow definition of money: (1) in (b) only the constant term c was significant and in (a) c and a_0 ; (2) there was marked autocorrelation in the residuals; (3) the pattern of signs of the coefficients was as follows

	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
(a)	< 0	> 0	> 0	< 0	> 0	> 0	> 0	> 0	> 0	< 0	< 0	< 0	< 0
(b)	< 0	< 0	< 0	> 0	> 0	> 0	> 0	> 0	> 0	> 0	> 0	> 0	> 0

In (a) Gibson explained the unexpected negative sign of a_4 , a_9 , a_{10} , a_{11} , a_{12} as a result of "*short lived price expectations effects or of cyclical movements in measured income involved in the adjustment of permanent income*"; and (4) the sum $\sum_{j=0}^{12} a_j$ was positive in both cases and Gibson concluded that income and price expectations effects outweigh the negative liquidity effect.

Apart from the fact that not much confidence can be placed on these estimates because of their statistical insignificance, the equations can not even be considered as a part of some reduced form. This is so because the existing literature shows that in no theoretical or empirical study of the demand and supply functions for money the rate of change of money or the logarithm of it were linked to the level of the interest rate. One notices that either levels or changes (or logarithms) are used for both variables. By adopting the former specification one would set in the final form the interest rate against current and past levels of the money stock and in such an environment all experiments with varying rates of growth of money can be carried out. Moreover if we consider that since $\frac{1}{M} \cdot \frac{dM}{dt}$ concerns discrete time intervals with $dt = 1$ and $\frac{1}{M} \cdot dM = d\ln M \simeq \Delta \ln M$ it is hard to reconcile the evidence as regards the sign of coefficients in (a) and (b).

As far as the evidence from econometric models is concerned, it must be pointed out that this evidence does not support the total positive effect reported by Gibson. Smith, discussing the possibility that nominal rates will be pushed above their initial level due to the last two effects which operate subsequently to the liquidity effect, quotes simulations with the FRB-MIT model which show that an increase in bank reserves causes a sharp fall of interest rates at a first stage, which is followed by a gradual rise of rates but only part of the way back to their original level.[22] He admits that the sign of the total effect can not be determined *a priori*; however he expresses doubt whether it is the easing of monetary policy alone which causes nominal rates to rise above their initial level and not this easing together with other forces operating simultaneously.

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[22] W. L. Smith: "A Neo-Keynesian View of Monetary Policy" in Controlling Monetary Aggregates. Proceedings of the Monetary Conference Held on Nantucket Island 1969. Sponsored by Federal Reserve Bank of Boston, pp.118-9.

Zecher provided values of the multipliers of a change in the open market instrument on the short-term interest rate for four major U.S. models, namely the Brookings, Ando-Goldfeld, FRS-MIT and FRS-CHI-MIT models.[23] Information is given only for the three first cumulative multipliers (corresponding to a sustained increase in the open market instrument). The values of the multipliers are shown below.

TABLE 10.1

Changes in the short term interest rate in basis points due to a billion dollar change in open market instrument (sustained)

Lag	Brookings	Ando-Goldfeld	FRS-MIT	FRS-CHI-MIT
0	-148.1	-83.6	-114.7	-210.9
1	- 57.6	-54.1	- 27.3	- 24.5
2	- 47.9	-43.0	- 43.9	- 55.9
3	- 41.5	-11.9	- 39.0	- 35.4

Evidently all models imply that income and price expectations effects start operating in the second quarter but in no model do we get a positive total effect nor is such an effect likely. The evidence is not uniform concerning the sign of the second interim multiplier and the speed at which multipliers converge. Since all four models are nonlinear, multipliers depend on the initial values of the variables. Consider the way dynamic multipliers are calculated in nonlinear models.[24] The nonlinear model

[23] R. Zecher: "Implications of Four Econometric Models for the Indicators Issue." American Economic Review, Papers and Proceedings, 1970, p.48.

[24] G. Fromm and R. L. Klein: "Solutions of the Complete System" in The Brookings Model: Some Further Results. Rand McNally, Chicago, 1969, pp.363-421.

$$F\,(Y_t,\ Y_{t-1},\ .\ .\ .\ .\ ,\ Y_{t-p},\ X_t,\ .\ .\ .\ .\ ,\ X_{t-q})$$

is solved for Y_t given initial values of $Y_{t-1}, \ldots, Y_{t-p}, X_t, \ldots, X_{t-q}$ and the values of its parameters. Using the computed value of Y_t and $Y_{t-1}, \ldots, Y_{t-p+1}, X_{t+1}, \ldots, X_{t-q+1}$, Y^c_{t+1} is computed and the procedure is repeated for as many periods ahead as it is desired. Then Y_t, $Y_{t+1}, \ldots$ are recomputed but with the elements of X_t, $X_{t+1}, \ldots$ changed by ε. Denote the new values by $Y^{c'}_t$, $Y^{c'}_{t+1}, \ldots$ Dynamic multipliers are given by

$$\frac{Y^{c'}_{it} - Y^c_{it}}{\varepsilon}\ ,\quad \frac{Y^{c'}_{i,t+1} - Y^c_{i,t+1}}{\varepsilon}\ ,\ .\ .\ .\ .\ .$$

and these multipliers vary with the initial values of the variables chosen for the calculations as well as with ε. For the four models examined by Zecher neither the initial conditions are reported nor the difference it would have made to the results had these conditions been altered.

Further evidence has been adduced by Fisher and Sheppard[25] in their review article of the interrelationships between real and monetary variables, and is contained in the following table.

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[25] G. R. Fisher and D. K. Sheppard: "Interrelationships between Real and Monetary Variables: Some Evidence from Recent U.S. Empirical Studies." In Issues in Monetary Economics. Proceedings of the 1972 Money Study Group Conference. Edit. by H. G. Johnson and A. R. Nobay, Oxford University Press, 1974, pp.226-7.

TABLE 10.2

Changes in the short-term and long-term rate due to a $1 billion increase (sustained) in open market instrument

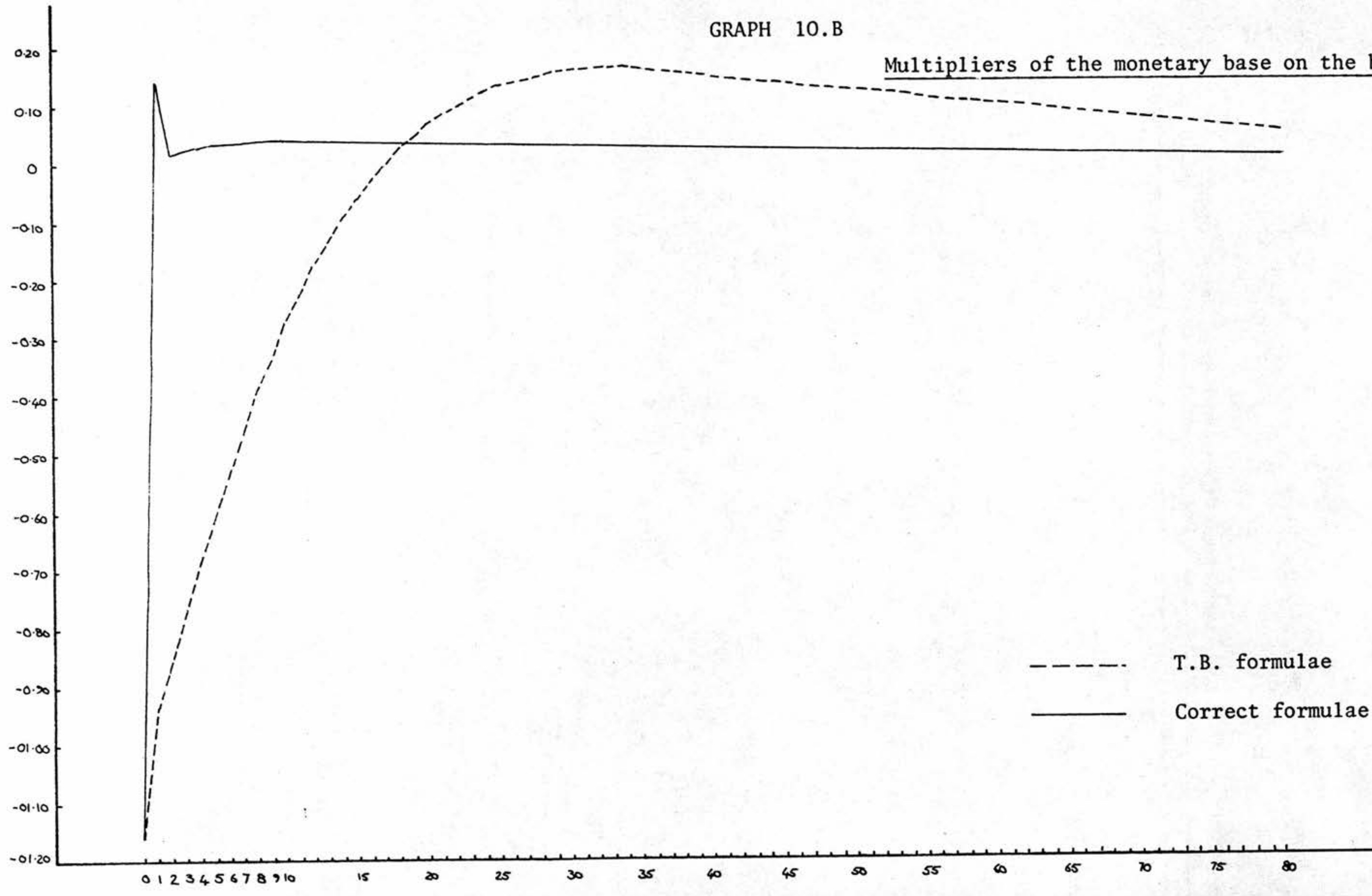
Lag	FRB-MIT January 1968 version	Condensed Brookings	Ando-Goldfeld
	Short-term rate Initial Conditions: 1963(1)	Long-term rate Initial Conditions: 1960(2)	Long-term rate Initial Conditions: (1965(1)
0	-1.2	-0.70	-0.305
1	-0.6	-0.11	-0.240
2	-0.5	-0.19	-0.233
3	-0.5	-0.23	-0.152
4	-0.45	-0.26	-
5	-0.4	-0.33	-
6	-0.38	-0.29	-
7	-0.35	-0.30	-
8	-	-0.31	-
9	-	-0.30	-

Again the assumption of a positive total effect is not supported from the above table.

The evidence from our model on the effects of a monetary change on interest rates is contained in Table 10.3 and Graphs 10.B and 10.C . Since money is an endogenous variable the monetary change is shown as a £1,000 millions increase in the monetary base which is the exogenous variable in the model. The dotted lines on the graphs show the course of the interest rates eighty quarters ahead following a monetary expansion if Theil and Boot's formulae are used in the calculation of multipliers. Although multipliers converge to zero in both cases the sizes of them are quite different and of the opposite sign for the first few interim multipliers.

GRAPH 10.B

Multipliers of the monetary base on the bond rate



GRAPH 10.C

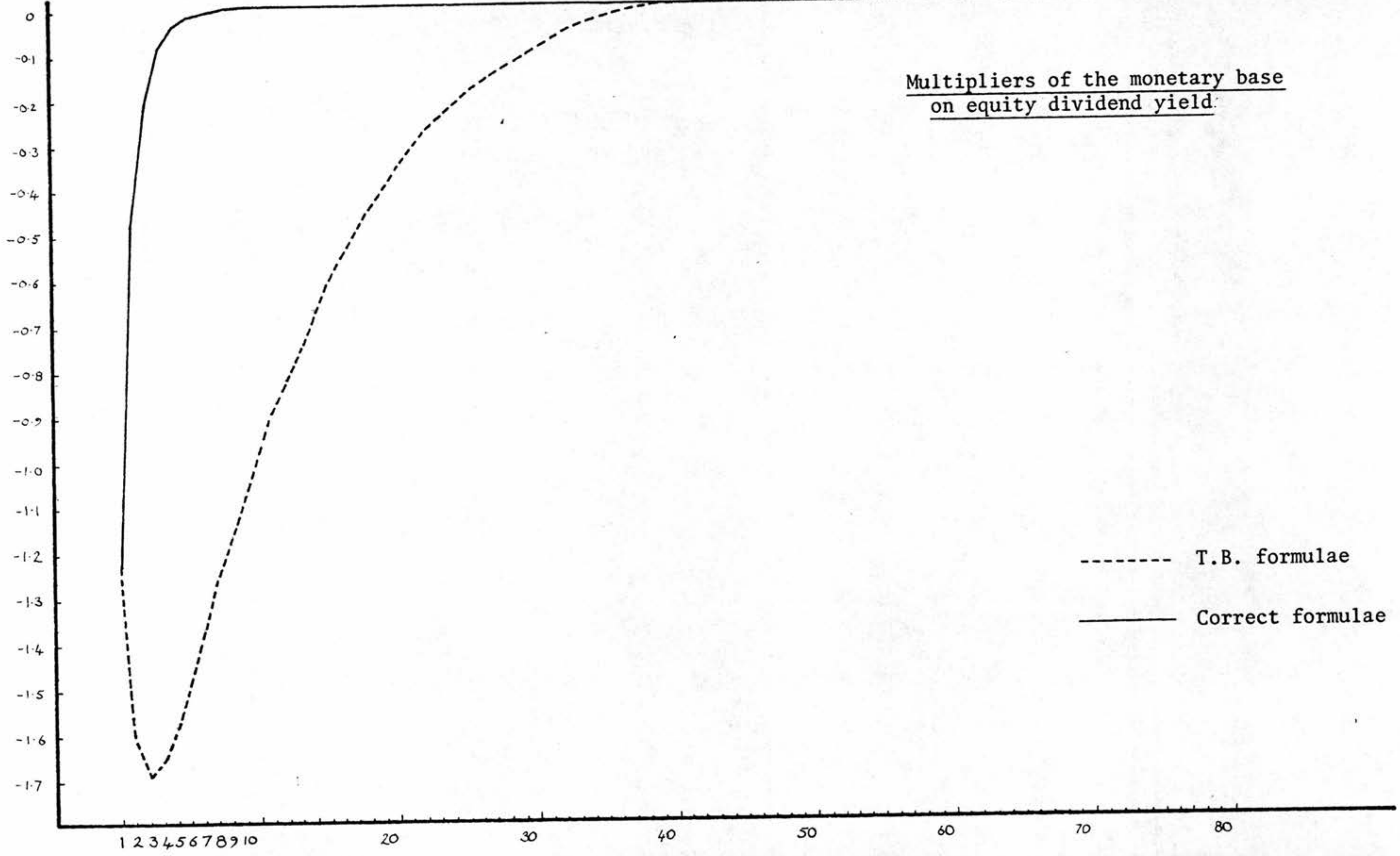

Multipliers of the monetary base
on equity dividend yield
------- T.B. formulae
——— Correct formulae
0
-0·1
-0·2
-0·3
-0·4
-0·5
-0·6
-0·7
-0·8
-0·9
-1·0
-1·1
-1·2
-1·3
-1·4
-1·5
-1·6
-1·7
1 2 3 4 5 6 7 8 9 10
20
30
40
50
60
70
80

TABLE 10.3

Changes in the bond rate and the equity dividend yield due to a £1,000 millions increase in the monetary base

Lag	Bond Rate	Equity dividend yield	Lag	Bond Rate	Equity dividend yield
0	-1.1666	-1.2272	11	0.0370	0.0097
1	0.1442	-0.4798	12	0.0369	0.0113
2	0.0162	-0.2075	13	0.0366	0.0125
3	0.0211	-0.0917	14	0.0360	0.0135
4	0.0252	-0.0416	15	0.0353	0.0142
5	0.0287	-0.0190	16	0.0345	0.0147
6	0.0345	-0.0079	17	0.0336	0.0150
7	0.0336	-0.0017	18	0.0326	0.0151
8	0.0352	0.0024	19	0.0316	0.0151
9	0.0362	0.0054	20	0.0306	0.0150
10	0.0368	0.0078			

Table 10.3 contains impact and interim multipliers for the bond rate and for the yield on equities, the interim multipliers covering twenty quarters ahead. It would not have made a great difference to the relative sizes of multipliers if we had tried to adjust multiplier values so that they were associated with a unit (£1,000 millions) increase in the money stock. This is so because interim multipliers of the monetary base on the money stock die out very fast, the impact multiplier being approximately 90% of the total multiplier, as is shown below.

Changes in the money stock following a 1,000 millions increase in the monetary base

Impact multiplier 2074	Total multiplier for 20 years 2323

Table 10.3 shows that the income and price expectations effects described by monetarists operate indeed in the U.K. economy and are seen to reverse the direction of the initial negative impact of an increase in the monetary base. Interim multipliers for all twenty quarters have a positive sign indicating the presence of these effects. The maximum value for a positive interim multiplier is recorded in the first quarter. After this, interim multipliers converge to zero although not in a steady manner but with a damped oscillation with a minor peak occurring at the eleventh quarter.

The present evidence, although it supports the presence of the income and price expectations effects, seems to reveal that the latter are not strong enough to outweigh the impact effect, if the monetary change is not accompanied by some other compensating change, e.g. in fiscal policy. The total multiplier for twenty years equals -0.0439,[26] i.e. the two effects account for a 96.24% coverage of the impact multiplier. If we consider the rate at which interim multipliers converge we see that it is quite unlikely that addition of more interim multipliers can lead to a positive total effect.

Regarding the response of the equity dividend yield to a monetary change some very important inferences are drawn from the contents of table 10.3 . First, the fact that the negative sign in the response of this yield to an increase in the monetary base persists for eight quarters indicates the strength of this delayed initial negative response. The evidence is offered to the monetarist assertion

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[26]See Appendix C for the remainder of the figures.

that money is spent directly on capital goods-equities thus affecting their prices and yields immediately. But also it is seen that the immediate effect in this direction is only a part of the total negative effect which is distributed in time more evenly in contrast with the negative effect on bond rate which has its main part concentrated in the current period. It is in this sense that we must accept the lagged effect of a monetary change on equity yields, the latter being widely accepted by Monetarists and NeoKeynesians.[27] Second, the factors which operate to revert the direction of the market prices and yields of equities are very weak. The dynamics of our model imply that after the lapse of certain time (2 years) a reversal of the movement of equity yields takes place due partly to the selling of equities by wealthowners in order to meet the increasing demand for money and partly to an increased supply of investment goods; these effects however account only for a minor part (19.42%) of the initial negative impact so that the total effect after 20 years is negative (-1.67).

To summarise the findings so far, a monetary change induces initially a negative movement in the yields of bonds and equities, earlier on for bonds and later on for equities, which is subsequently reversed for the reasons described in the previous sections. The forces acting in the opposite direction are not strong enough to outweight the initial effects.

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[27] M. Friedman and A. J. Schwartz (Monetarists) in: "Money and Business Cycles" Review of Economics and Statistics, 1963, Supplement p.61, discussing the adjustment following a monetary change conclude that the first impact should be expected *"on the financial markets, and there, first on bonds, and only later on equities, and only still later on actual flows of payments for real resources"*. A. M. Okun (Neo-Keynesian) in a comment to the above article is surprised to find that *"the transmission mechanism described by Friedman and Schwartz is so similar"* to the one he visualises, *op.cit.*, p.74.

The effects of a monetary change on interest rates were examined alone, that is without any compensating change in fiscal policy. This is the case when for example an open market purchase is conducted by the Bank of England or maturing debt is financed by increasing the amount of the monetary base. But it would be interesting to examine what happens to interest rates when the increase in the monetary base is intended to finance government expenditures. To this end we need to obtain the value of multipliers for an increase in government expenditure matched by an equal increase in the monetary base. A complication however arises because a unit increase in the monetary base should be accompanied by a unit increase in G* (nominal government expenditure) and our model gives the values of multipliers with respect to a change in G (real government expenditure). We solve this problem by dividing all values of multipliers $\frac{\partial R}{\partial G}$ and $\frac{\partial R'}{\partial G}$ by 1.613 the price level at 1972.4, the last quarter of our sample. The effect of this device is to give us the values of a £$\frac{1}{1.613}$ = 0.61966 millions change in real government expenditure or a £1 million change in nominal government expenditure. This procedure implies that in the future, when the price level will be higher, a smaller increase in real government expenditure will be required to obtain a unit increase in nominal government expenditure, with a consequent reduction of the multiplier effects on real output. The conclusion is that in a highly inflationary economy government expenditure will be becoming less and less effective in influencing real activity (always at a unit level of nominal expenditure G* and on the assumption that the parameters of the model do not vary in time). A necessary condition for the effectiveness of fiscal policy seems to be the containing of inflation or the increase of

G* at a rate equal or comparable to the rate of inflation. In the last case, if the rate of increase of real output in equilibrium is less than the rate of increase of inflation, the result of such a fiscal policy will be a larger share of the public sector in real output.

TABLE 10.4

Changes in the bond rate and the equity dividend yield due to a £1,000 millions increase in the exogenous variable(s)

Lag	$\frac{\partial R}{\partial G^*}$	$\frac{\partial R}{\partial G^*} + \frac{\partial R}{\partial B}$	$\frac{\partial R'}{\partial G^*}$	$\frac{\partial R'}{\partial G^*} + \frac{\partial R'}{\partial B}$
0	0.3272	-0.8394	-0.1261	-1.3533
1	0.2593	0.4035	-0.1343	-0.6141
2	0.2376	0.2538	-0.0503	-0.2578
3	0.2177	0.2388	0.0115	-0.0802
4	0.1989	0.2241	0.0550	0.0134
5	0.1796	0.2083	0.0814	0.0624
6	0.1604	0.1919	0.0958	0.0879
7	0.1422	0.1758	0.1017	0.1000
8	0.1251	0.1603	0.1019	0.1043
9	0.1097	0.1459	0.0981	0.1035
10	0.0959	0.1327	0.0920	0.0998
11	0.0839	0.1209	0.0845	0.0942
12	0.0735	0.1104	0.0764	0.0877
13	0.0645	0.1011	0.0683	0.0808
14	0.0570	0.0930	0.0605	0.0740
15	0.0505	0.0858	0.0533	0.0675
16	0.0451	0.0796	0.0467	0.0614
17	0.0406	0.0742	0.0409	0.0559
18	0.0368	0.0694	0.0357	0.0508
19	0.0335	0.0651	0.0313	0.0464
20	0.0308	0.0614	0.0275	0.0425

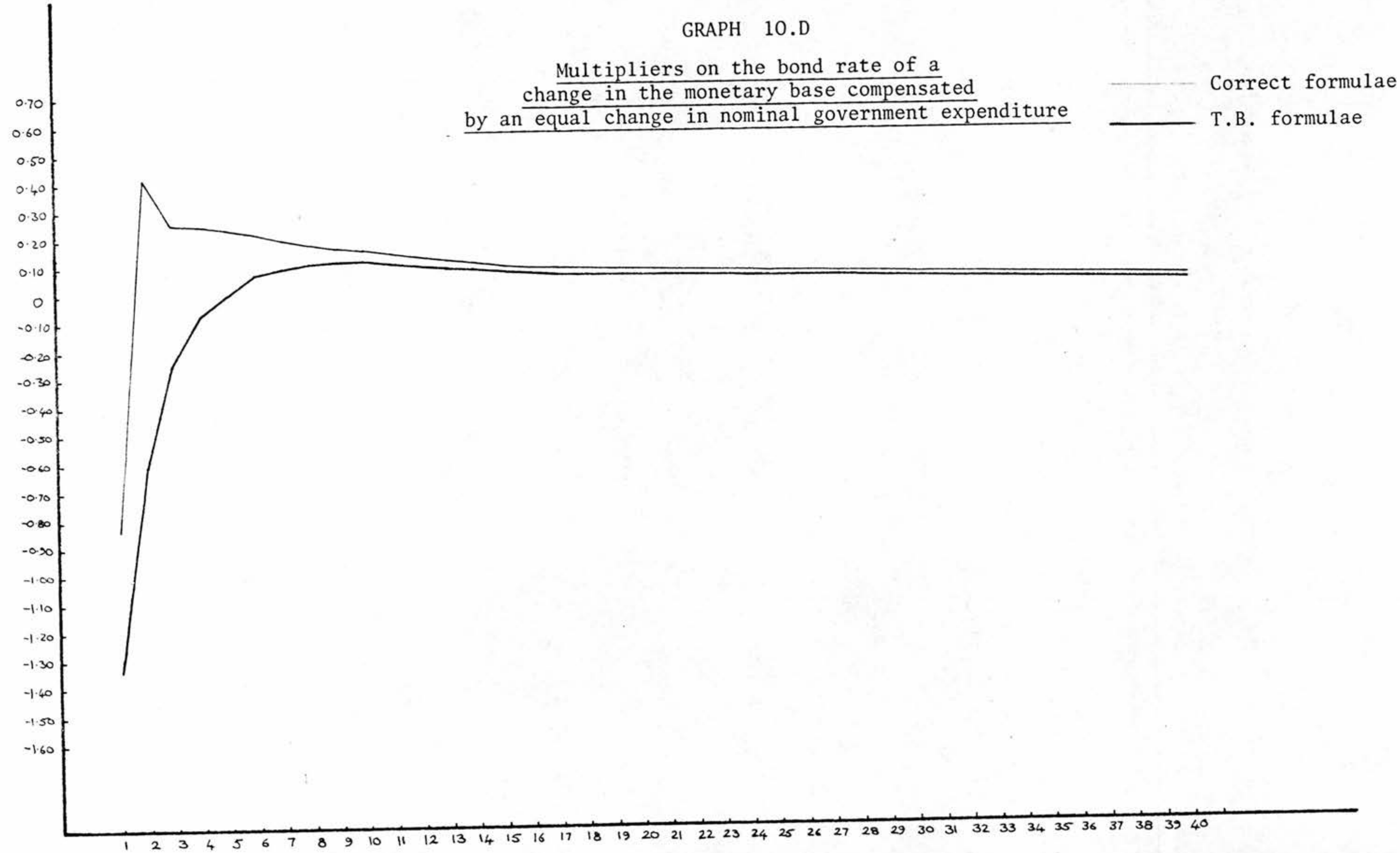
In table 10.4, column 2 shows the values of impact and interim multipliers of nominal government expenditure on the bond rate for twenty quarters. They are all positive in sign and this means that the increased demand for money is brought to equality with the supply of money by a higher interest rate. Clearly an increase in government expenditures raises real income, and the price level, so that a higher demand for money follows. But the supply of money is also affected; although no change in the monetary base occurs the money supply is increased through changes in the ratio of currency to demand deposits. Obviously the increase in supply is not enough to balance the increase in demand so that the interest rate is driven up by people wanting more cash and selling bonds to acquire it.

When this increase of nominal government expenditures is compensated by an equal increase in the monetary base the picture which emerges is consistent with the monetarist position that the resulting final level of the interest rate is higher than the initial (see table 10.4, column 3 and graph 10.D). For this to happen a monetary change should be accompanied by a fiscal change of equal size. In this case although the impact multiplier is negative by the end of the third quarter the positive influences will have achieved a total increase in the interest rate in excess of it. Finally in twenty years time the interest rate will be exceeding the initial level by 3.441 percentage points.

Now, as far as the yield on equities is concerned, we observe from table 10.4, column 4, where the multiplier values are presented for

GRAPH 10.D

Multipliers on the bond rate of a
change in the monetary base compensated
by an equal change in nominal government expenditure



a £1,000 millions change in nominal government expenditures, that the positive effect on this yield from the increased demand for money is not present for the first two quarters. The impact and first two interim multipliers are negative. This is explained if we view government expenditure as representing a bidding for existing capital for the production of consumer or investment goods demanded by the government. The price of existing capital is raised and the yield falls and this effect is stronger than the one which tends to raise equity yields. The further sequence of events which follow are similar to the ones described and observed above. But now there is no initial liquidity effect as it was the case with an increase in the monetary base so that the positive influences prevail and the total multiplier (20 years) is positive.

A combined monetary and fiscal policy at equal levels of the monetary base and nominal government expenditure gives a negative overall effect on equity yield as can be seen from column 5 in table 10.4 and the reason for this must be sought in the comparatively larger size of the liquidity effect which overshadows all others.

We conclude that consideration of the fiscal policy accompanying a monetary expansion remarkably alters the direction and magnitude of the effect on interest rates of the monetary change alone.

Effects on monetary changes on real output and the price level

Propositions (b) and (c) which refer to the short-run and long-run effects of monetary changes on real output and the price level, will be examined together since the short-run and the long-run effects are obtained from the same model, the latter being simply the total multiplier effects. The rationalisation of these two propositions has been provided by Friedman who assumed that the monetary expansion occurs in order to reduce unemployment to a level below the natural level, corresponding to the notion of full-employment.[28] According to Friedman the result of an increased supply of money will be to reduce initially interest rates and thus stimulate spending and increase income. At the beginning the increase will be an increase in real output and employment rather than prices because people adjust slower as regards prices so that the expansion in demand will be met by a higher supply of output (at the initial prices) and a lower unemployment rate. But this is only the first part of the story. After that, prices start responding to an unanticipated higher demand. Prices of products lead this movement because they respond faster than wages to the state of higher demand. The result of this is that real wages received by employees go down and this permits the increased employment to continue for a while. But when employees become aware of rising prices they start demanding higher nominal wages, real wages rise and the downward movement in unemployment is reversed. Finally unemployment returns to its former 'natural' level and the effect of the increased monetary expansion is felt only by prices.

Friedman suggested the timing of the short-run effects to be as in proposition (c) by studying turning point relationships and correlations between rates of change of the relevant variables.[29]

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[28] M. Friedman, *op.cit.*, 1968, p.9

[29] See for example M. Friedman: "The Lag in Effect of Monetary Policy." Journal of Political Economy, 1961, pp.447-66, and M. Friedman: Money and Economic Development. The Horowitz Lectures of 1972, Praeger, New York, 1973.

However, even if one accepted his proposition for the short-run, there is no guarantee that the final position will be one of full employment with no permanent effect of money on real output. Our discussion at the beginning of this chapter has shown that this is not what one normally expects the long-run equilibrium position to be.

The evidence on the above propositions is very limited and comes from single equations or structural models of the U.S. economy. The first case includes a study from economists of the Federal Reserve Bank of St. Louis.[30] Andersen and Karnosky have run the following regressions:

$$\Delta \ln P = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^{n} m_i \Delta \ln M_{-i} + \sum_{i=0}^{4} e_i \Delta \ln E_{-i} \quad (10.19)$$

$$\Delta \ln Q = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^{n} m_i \Delta \ln M_{-i} + \sum_{i=0}^{4} e_i \Delta \ln E_{-i} \quad (10.20)$$

$$\Delta \ln Y = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^{n} m_i \Delta \ln M_{-i} + \sum_{i=0}^{4} e_i \Delta \ln E_{-i} \quad (10.21)$$

where P is the price level, Q is real output, $Y = PQ$, D_1 and D_2 are dummy variables for a major labour strike, M is the narrow money stock and E is high-employment government expenditure. The sum of the money coefficients for any lag on money represents the corresponding elasticity for this length of the lag. Andersen and Karnosky

[30] L. C. Andersen and D. S. Karnosky: "The Appropriate Time Frame for Controlling Monetary Aggregates: The St. Louis Evidence." In Controlling Monetary Aggregates II: The Implementation. Proceedings of a Conference Held at Melvin Village, New Hampshire, Sept. 1972. Federal Reserve Bank of Boston, 1973, pp.147-77.

have chosen the maximum value of n to be 28 , having as criterion that the value of $\overline{R}^2$ should attain a maximum. They also contended that their results were little affected by changes in the length of the lag on government expenditures, so they presented results for a lag of four quarters on that variable. The results were shown for values of n which are multiples of four so that no detailed view can be gained of the truth of proposition (c) concerning the short-run. As far as the long-run is concerned, for n = 28 the hypothesis $\Sigma m_i = 1$ for (10.19) and (10.21) and $\Sigma m_i = 0$ for (10.20) have been accepted.

Several doubts on these results were expressed by B. M. Friedman.[31] Friedman juxtaposed the implications of this study and other studies by economists in the St. Louis Bank, as regards the length of the lag of the impact of monetary growth on income, real output and prices and suggested that they do not coincide and because of this they can not offer any acceptable hypothesis about the length of the lag. Moreover, the reported value of $\overline{R}^2$ in the equations for the price level and for the value n = 28 is not probably a maximum; also the sum Σm_i in the same equations rises at an accelerating rate for each increment of the lag by four quarters so that Friedman wonders whether addition of more quarters would have led to a rejection of the hypothesis of unitary elasticity.

The evidence from structural equation models has been given by Fisher and Sheppard[32] and is contained in the following table.

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[31] B. M. Friedman: Discussion in Controlling Monetary Aggregates II: The Implementation. Proceedings of a Conference Held at Melvin Village, New Hampshire, Sept. 1972. Federal Reserve Bank of Boston, 1973, pp.183-4.

[32] G. R. Fisher and D. K. Shepperd, *op.cit.*, pp.226-7.

TABLE 10.5

Change in Real GNP and the GNP deflator due to a $1 billion increase in the open market instrument (non sustained)

	Condensed Brookings Model Initial conditions: 1960(2)		Ando-Goldfeld Model Initial conditions: (1965(1)	FRB-MIT Model December 1968 version Initial conditions: 1963(1)	
Lag	Real GNP	GNP deflator	GNP deflator	Real GNP	GNP deflator
0	-1.5	0.2	0.011	0.7	0
1	4.2	-0.6	0.002	1.3	0
2	-0.1	0.1	-0.002	1.6	0.1
3	1.9	-0.2	-0.002	1.8	0
4	0.6	-0.1	-	1.6	0.1
5	6.5	-0.6	-	1.3	0.1
6	-1.7	0.5	-	1.0	0.1
7	0	0	-	0.7	0.2
8	-1.0	0.1	-	-	-
9	0.7	0.1	-	-	-

It is obvious that the results obtained from the first two models are not credible since they imply that after an initial rise in prices the latter fall at an expansionary period. The reason for this is that prices in the above models are obtained *"by a mark-up equation on wage cost per unit of output. Wage rates are determined by a Phillips relationship. Thus an increase in the money supply generates an expansion in real output greater than the increase in employment, thus reducing the wage cost per unit of out-put - and so we have the paradox that prices go down in response to an expansion in the money* *supply. The change in unemployment takes a long time to affect wage rates and so to affect wage cost per unit of output."*[33]

On the other hand the latest version of the Federal Reserve-M.I.T. model offers a strong support to monetarism as regards proposition (b). As it can be observed from multipliers of a change in the open market instrument on real GNP and the GNP deflator, a peak is reached at a lag equal to three quarters for real GNP and at a lag equal to seven quarters for the GNP deflator. Multiplier values are not given for lags of more than seven quarters so that one can not judge on the validity of proposition (c) which concerns the long-run.

The results from our model are contained in the following table.

TABLE 10.6

Change in real GDP, GDP deflator and nominal GDP due to a £1,000 millions increase in the monetary base

Lag	Real GDP	GDP deflator	Nominal GDP
0	19	0	31
1	30	0.026	48
2	37	0.066	60
3	41	0.113	66
4	42	0.161	68
5	41	0.208	66
6	39	0.251	63
7	37	0.288	60
8	34	0.319	55
9	31	0.344	51
10	28	0.363	46
11	25	0.376	41
12	23	0.385	38
13	20	0.390	33
14	18	0.391	30
15	16	0.389	26
16	15	0.385	24
17	13	0.379	22
18	12	0.371	20
19	11	0.362	18
20	10	0.353	17

[33] A. A. Walters: Discussion Paper in Issues in Monetary Economics. Proceedings of the 1972 Money Study Group Conference. Edit. by H. G. Johnson and A. R. Nobay, Oxford University Press, 1974, p.485.

Our results confirm that real output peaks before the price level, the relevant peaks being reached at the fourth quarter for real GDP and at the fourteenth quarter for the GDP deflator. But it is also seen that there is greater delay as compared with the U.S. economy, before these peaks are attained, especially for the price level for which it takes as long as three years and a half for the maximum effect to occur. Despite the consistency of our results with monetarism as regards the short-run there is no indication at all from our model that in the long-run the total effect on real output will be zero and all the impact of the monetary expansion will be concentrated on the price level. Multipliers of monetary changes on real GDP converge steadily to zero without taking any negative values at all (see Appendix C). And indeed it has been argued above that this is what we must in general expect as a long-run solution, namely that money affects both real output and the price level.

The above findings can be combined to give us the effect of a monetary expansion on nominal income and thus examine proposition (d). The latter is not autonomous but depends on what happens to both real output and the price level after a monetary expansion. The way it is formulated is not precise and it lacks any concrete empirical content. The form that this proposition has taken is explained by the fact that Friedman has calculated the lead of the rate of change of money over the rate of change of nominal income as an average of all observed leads and found that the dispersion around this average is very large.[34]

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[34] M. Friedman, *op.cit.*, 1961.

The implications of our model for the effect of a monetary change on nominal GDP are contained in column 4 of table 10.6 . Changes in nominal GDP have been calculated by cumulating the effects of the monetary expansion on the price level at each lag and multiplying the resulting price level by multipliers on real GDP for every value of the lag. Nominal GDP reaches a peak at the same quarter as real GDP and the length of the lag for this maximum effect to happen is not so long as it is for the GDP deflator. It is worth noting that the size of multipliers of a monetary change on GDP are relatively small, the total effect after twenty quarters being only 88.3% of the change in the monetary base and exceeding this change by very little after eighty quarters. This is in agreement with the findings from one equation tests by Artis and Nobay[35] who reported low monetary multipliers and high fiscal multipliers for the United Kingdom.

Crowding out effects of fiscal policy operation on income

To the monetarists the impact of fiscal policy actions on income depends crucially on how government spending is financed. Expenditures financed either by taxation or borrowing from the public are said to be ineffective in raising income because they crowd out an equal amount of private expenditure. In the second case this happens because the rise in interest rates from the competition with private markets for the attraction of funds by the government induces an

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[35] See M. J. Artis and A. R. Nobay: "Two Aspects of the Monetary Debate." National Institute Economic Review, August 1969, p.42. Artis and Nobay run regressions relating quarterly changes in GDP (current prices) to changes in different measures of monetary and fiscal policy. After having selected the most important explanatory variables they applied the Almon technique for estimating the coefficients attached to current and lagged terms.

offsetting reduction in private expenditure through an interest rate effect or an interest-induced wealth effect. In the first case again funds are transferred to the public sector involuntarily by the private sector in the form of taxes and private consumption (and possibly investment) expenditures are reduced.

A definition of the crowding out effect has been given by Spencer and Yohe,[36] who distinguished between nominal crowding out and real crowding out. The former means that

$$\frac{dY^*}{dG^*} \Big| \, dM^* = 0 \quad \simeq \quad 0 \qquad (10.22)$$

while the latter means that

$$\frac{dY}{dG} \Big| \, dM = 0 \quad \simeq \quad 0 \qquad (10.23)$$

where the starred variables are nominal income, government expenditure and money stock and the plain ones are the corresponding real variables. In a later article this definition has been interpreted as referring to the steady state multipliers,[37] i.e. the long-run (total) multipliers. Thus it is monetary expansion which is important for monetarists and the one which exerts *"even unaccompanied by an increase in government spending a strong, stimulative influence on the economy."*[38]

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[36] R. W. Spencer and W. P. Yohe: "The 'Crowding Out' of Private Expenditures by Fiscal Policy Actions." Federal Reserve Bank of St. Louis Review, October 1970, p.14.

[37] K. M. Carlson and R. W. Spencer: "Crowding Out and Its Critics." Federal Reserve Bank of St. Louis Review, December 1975, p.3.

[38] R. W. Spencer and W. P. Yohe, *op.cit.*, p.14.

In order to examine how government expenditure will be financed one has to look at the identity showing public sector deficits and the sources of their finance. From this identity it is seen that government expenditure can be financed either by a cut in transfer payments or by a rise in tax receipts or by selling government debt to the public or by increasing the monetary base (or special deposits in the case of this country). The existence of this identity means that when we are dealing with the problem of finance of government expenditure, we should in principle consider compensating changes in some other variable in this identity so that it always holds true. This is the case when the identity is not explicitly included in a macroeconomic model. When it is formally included in the model, one of its variables is converted into endogenous and the problem of compensating changes becomes more complicated. The above definitions (10.22) and (10.23) of the crowding out seem to be appropriate only if this identity is incorporated in the model as a part of it and the tax variable or the variable 'government debt in the hands of public' is the endogenous variable. Otherwise the definitions are not consistent with the description of the phenomenon by monetarists and they should be as follows in the case of change, say, in taxes:

Nominal Crowding Out $$\left(\frac{dY^*}{dG^*} + \frac{dY^*}{dT^*}\right) \Big| \; dM^* = 0 \quad \simeq \quad 0 \qquad (10.24)$$

Real Crowding Out $$\left(\frac{dY}{dG} + \frac{dY}{dT}\right) \Big| \; dM = 0 \quad \simeq \quad 0 \qquad (10.25)$$

But let us concentrate on the first type of model, i.e. one which does not include this identity as one of its equations. If this is a model in which the money supply is an endogenous variable then the definition of the crowding out should be modified to read as follows:

Nominal Crowding Out $$\left(\frac{dY^*}{dG^*} + \frac{dY^*}{dT^*}\right) \bigg| \, dB^* = 0 \quad \simeq \quad 0 \tag{10.26}$$

and Real Crowding Out $$\left(\frac{dY}{dG} + \frac{dY}{dT}\right) \bigg| \, dB = 0 \quad \simeq \quad 0 \tag{10.27}$$

where now the monetary base has replaced the money stock.

On the other hand for the rise in nominal income due to an increased government expenditure financed by an increase in the monetary base we must examine the expressions

$$\frac{dY^*}{dG^*} \quad \frac{dY^*}{dB^*} \tag{10.28}$$

What is then assumed for these expressions? The monetarist answer is that in the long-run

$$\frac{dY^*}{dG^*} = 0 \tag{10.29}$$

and

$$\frac{dY^*}{dB^*} = Y_0 \frac{dP}{dB^*} \tag{10.30}$$

i.e. the long-run effect of fiscal action is zero and that of the monetary action will be felt only by the price level and not by real income, as proposition (b) suggests. This puts into the right perspective proposition (e) which states that the impact of government expenditure will be inflationary if the latter is financed by money creation and it will not be inflationary if it is financed by taxes or borrowing from the public. It will be reiterated here that this proposition is grounded on the assumption of the long-run independence between real and monetary variables and can be ruled out *a priori* if this independence is not accepted.

The available empirical results do not give any decisive answer on the above proposition. For example, the following table gives the value of the expressions $\frac{dY^*}{dG^*} + \frac{dY^*}{dT^*}$ and $\frac{dY}{dG} + \frac{dY}{dT}$ after forty quarters for some U.S. structural models.[39]

TABLE 10.7

Implications of Some U.S. Models on the Crowding Out Proposition (cumulative multipliers after forty quarters)

	BEA	Brookings	DHL III	DRI 71	Wharton III	Lin-Hwa Monthly
$\frac{dY^*}{dG^*} + \frac{dY^*}{dT^*}$	- 0.2	-0.9	-3.3	1.7	2.3	1.0
$\frac{dY}{dG} + \frac{dY}{dT}$	-22.2	-0.6	-0.9	2.6	-3.6	-

On nominal crowding out the first three models go beyond monetarists assertions implying that income is reduced after government spending financed by taxes, whereas the last three models give a positive and different from zero effect. Regarding real crowding four models give negative total effects and only one gives a positive effect. However not much confidence can be placed on these results given the reservations expressed above by Walters as regards the way models have explained price movements.

[39] For these results see: G. Fromm and L. R. Klein: "A Comparison of Eleven Econometric Models of the United States." American Economic Review, Papers and Proceedings, 1973, pp.391-2.

The evidence from our model is shown in the following table.

TABLE 10.8

Fiscal policy multipliers due to a non-sustained 1,000 millions increase in the exogenous variable

Lag	$\frac{\partial Y}{\partial G^*}$	$\frac{\partial P}{\partial G^*}$	$\frac{\partial Y^*}{\partial G^*}$	$\frac{\partial Y}{\partial T^*}$	$\frac{\partial P}{\partial T^*}$	$\frac{\partial Y^*}{\partial T^*}$
0	1,201	0	1,937	-414	0	-668
1	341	1.662	556	-323	-0.572	-519
2	257	2.019	424	-249	-0.980	-398
3	132	2.234	221	-169	-1.257	-268
4	69	2.262	117	-108	-1.404	-170
5	23	2.201	39	- 61	-1.456	- 95
6	- 7	2.080	- 12	- 28	-1.440	- 43
7	- 27	1.925	- 47	- 4	-1.379	- 6
8	- 38	1.754	- 67	11	-1.290	17
9	- 44	1.579	- 79	20	-1.185	30
10	- 45	1.409	- 81	26	-1.074	39
11	- 43	1.249	- 78	28	-0.964	42
12	- 40	1.102	- 73	28	-0.859	41
13	- 35	0.970	- 64	26	-0.761	38
14	- 31	0.854	- 57	24	-0.673	35
15	- 26	0.752	- 48	20	-0.593	29
16	- 21	0.664	- 39	17	-0.524	25
17	- 17	0.589	- 32	14	-0.463	20
18	- 13	0.525	- 24	12	-0.411	17
19	- 10	0.471	- 19	9	-0.366	13
20	- 7	0.424	- 13	7	-0.329	10

Columns 2, 3 and 4 show the effect on real GDP, the GDP deflator and nominal GDP of a unit change in nominal government expenditure. The figures for columns 2 and 3 are obtained by dividing multiplier values, as these are given in appendix C, by 1.613 so that they correspond to a unit increase in nominal, not in real, government expenditure. The figures for column 4 are obtained by cumulating the effects of the fiscal change on the price level at each lag and multiplying the resulting price level by the multipliers of column 2 for every value of the lag. It can be noted that multipliers on both real and nominal GDP take on negative values after the fifth quarter but are not large enough to warrant a zero total effect. Indeed the total effect after twenty quarters is implying a fiscal multiplier equal to 2.561 and again this finding conforms with the available evidence (as to the size of fiscal policy multipliers) from one equation tests for the United Kingdom.[40]

Similarly multiplier values are obtained for a unit change in the tax variable and are shown in columns 5 to 7 of table 10.8 . The total effect of a tax increase on nominal GDP implies after twenty quarters a negative multiplier equal to 1.811 . The combined multiplier effect of a unit increase in government expenditure financed by taxation is 0.802 . This figure is obtained by cumulating the effects on the price level of the above combined change at each lag, multiplying the resulting price level by the multipliers of columns 2 and 5 for each lag and adding the interim multipliers. The above value is a little less than unity implied by simple models of income determination for this case, but definitely it does not support the crowding out proposition, which is associated with a value equal to zero or near zero. We note that if we added more values of interim multipliers this would not change

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[40] M. J. Artis and A. R. Nobay, *op.cit.*

our main conclusion because interim multipliers of government expenditure are approximately equal (in absolute value) to multipliers of taxes after the twentieth quarter.

As far as government expenditure financed by money creation is concerned the findings in columns 2 and 4 indicate that the fiscal multiplier will not be zero in the long-run so that the effect of a combined change is felt on both prices and real output. The multiplier of a combined change on nominal GDP after twenty quarters will be equal to 3.44 but the multiplier on real GDP will be 2.161 and not zero (or approximately zero) as prescribed by monetarism.

Having completed our discussion of monetarist propositions we shall examine briefly another question which is of interest to the monetary authorities, namely the effect that the operation of Special Deposits has on market interest rates.

Special Deposits: a substitute for the higher rise in market rates implied by market operations?

When SDs were introduced in 1958, commentators directed their attention to the effect SDs would have on interest rates. For example we read in the Banker:

> *"If the scheme is brought into force it could in principle be worked in one of two opposite ways: (1) as a new device for causing a rise in interest rates, either in the money market or gilt-edged market or both; or (2) with the object of avoiding a rise in rates."*[41]

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[41] Banker: "The New Monetary Weapon: Full Description and Appraisal." The Banker, 1958, p.499.

The distinction as expressed in the Banker can be very misleading, so we must make its meaning absolutely clear. By (1) it is meant that the operation of the SDs scheme will lead to a rise in market rates (short term, long term or both). The discussion in chapter 5 pp. 166-7 showed that this will be the case because banks reduce their liquid assets and investments (stage B in the example cited there) with a consequent increase of both short term and long term market rates. But if SDs cause a rise in market rates so will any other measure of monetary policy which aims at reducing bank credit and the money supply, because deflationary pressure can be exerted only by forcing an impact rise in interest rates. So (1) must be interpreted to signify that SDs as well as any other deflationary monetary measure will lead to higher market interest rates.

Proposition (2) is more significant and it should be read as follows: with SDs the monetary authorities can avoid the higher rise in rates that would occur after an open market operation. Let us examine whether this proposition can be accepted theoretically. SDs are called instead of funding or open market sales in order to tighten excess liquidity in the private sector. The operation of funding is that the Bank tries to neutralise excess liquidity by buying short term stocks and selling longer term ones to the public. But suppose that at a moment the conditions in the gilt-edged market are such that sales are impossible despite the fact that rates can already be at a high level. This means that a contraction of liquidity that the authorities wished to achieve is impossible until they are again able to sell and this will be undoubtedly possible at still higher rates. Now if this task is conveyed to the banks and the latter are confronted with a declining appetite of the public for government securities, there is an alternative to them which does not exist for the monetary authorities. The alternative is bank advances; to the extent that the banks could not succeed in selling investments they could curtail their advances by calling in advances or refusing to grant new ones.

The distribution of the burden for the banks between investments and advances can also be thought in terms of risk and return and the relative position that investments and advances have in banks' balance sheets. Whatever the case might be a call for SDs will put less pressure in the gilt-edged market because a part of it will be transmitted to banks' advances. Consequently one can expect that the market rate will rise less than it would have risen if the authorities had forced an open market sale of equal size to the call for SDs. Regarding the rate charged by the banks on advances this will be initially unchanged, since it is linked to the Bank Rate by a more or less constant relationship. Of course the rise in the market rate will induce sooner or later a rise in the Bank Rate and all rates attached to it but the important observation is that due to the above distribution, the rise in the market rate can be expected to be smaller than that which would have resulted from market operations by the authorities.

It is interesting to note that although the question about the effect of a SDs call on interest rates has been posed the answer has never been pursued empirically. This is not unrelated to the view about the ineffectiveness of SDs which has wrongly prevailed for a long time among academic economists.

It has already been pointed out in chapter 8, p.320 that the relative sizes of the coefficients attached to the monetary base and special deposits in the money supply function in conjunction with a negative impact relation between money demand and the interest rate makes possible the inference that special deposits imply a smaller impact effect on the nominal market rate than an open market sale of equal size reflected in a change in the monetary base. We can now examine the course of the interest rate following a change in the above two variables. Table 10.9 contains the relevant results from our model.

TABLE 10.9

Changes in the bond rate and equity dividend yield due to a £1,000 millions change in the exogenous variable(s)

Lag	$\frac{\partial R}{\partial S^D}$	$\frac{\partial R}{\partial(-B)} - \frac{\partial R}{\partial S^D}$	$\frac{\partial R'}{\partial S^D}$	$\frac{\partial R'}{\partial(-B)} - \frac{\partial R'}{\partial S^D}$
0	0.8184	0.3482	0.8610	0.3662
1	-0.1012	-0.0480	0.3366	0.1432
2	-0.0114	-0.0048	0.1456	0.0619
3	-0.0148	-0.0063	0.0643	0.0274
4	-0.0177	-0.0075	0.0292	0.0124
5	-0.0201	-0.0086	0.0133	0.0057
6	-0.0221	-0.0094	0.0055	0.0024
7	-0.0236	-0.0100	0.0012	0.0005
8	-0.0247	-0.0105	-0.0017	-0.0007
9	-0.0254	-0.0108	-0.0038	-0.0016
10	-0.0258	-0.0110	-0.0054	-0.0024
11	-0.0260	-0.0110	-0.0068	-0.0029
12	-0.0259	-0.0110	-0.0079	-0.0034
13	-0.0257	-0.0109	-0.0088	-0.0037
14	-0.0253	-0.0107	-0.0095	-0.0040
15	-0.0248	-0.0105	-0.0100	-0.0042
16	-0.0242	-0.0103	-0.0103	-0.0044
17	-0.0236	-0.0100	-0.0105	-0.0045
18	-0.0229	-0.0097	-0.0106	-0.0045
19	-0.0222	-0.0094	-0.0106	-0.0045
20	-0.0215	-0.0091	-0.0105	-0.0045

Column 2 of the table indicates that a call for SDs causes an initial rise in the bond rate which is less by 0.3482 percentage points then the rise caused by an open market sale of equal size (column 3). The subsequent course of the interest rate is reversed following both policy changes because income decreases and expectations of price reductions are created to the public. However these effects are stronger when an open market sale is conducted so that the net change for all subsequent quarters has a negative sign (column 3). After twenty years the negative differences cover 96.32% of the initial positive difference so that in the long-run there is no overwhelming reason for preferring SDs calls to open market operations as regards their effect on the bond rate.

As regards the relative effect of the above policy actions on the equity dividend yield the relevant evidence is offered in columns 3 and 4 of table 10.9 . In the case of equities the timing of changes in their yield due to a call for SDs has similar characteristics to those we encountered when we examined the effects of monetary changes on interest rates. Consequently the initial positive difference stays on for eight quarters and then changes sign. After twenty years the negative differences cover only 19.38% of the positive differences and SDs are seen to be superior to open market sales as regards raising equity market yields and depressing their prices.

CHAPTER 11

CHAPTER 11

SUMMARY OF THE FINDINGS

We close by repeating in summary form what we believe to be the innovations introduced in this study and the main conclusions that can be drawn from the material presented in the previous chapters.

1. We have presented the first empirical study on the IS-LM model which took into account simultaneously the main criticisms levelled against the model and estimated its equations as a unified set of hypotheses. Specifically, (a) by extending the model to include a price determination equation, expressing real sector variables in real terms and financial variables in nominal terms and introducing the price level in demand for product equations as well as in the money demand function, we have been able to determine the proportions in which nominal income changes are divided between output changes and price changes in a model in which the IS and LM parts ceased to be two independent sets of relationships; and (b) by specifying price expectations as an endogenous element in the system and examining the relationship between the returns on bonds and equities we have been able to distinguish between the nominal interest rate, which is appropriate for the demand for and supply of money, and the real interest rate, which is relevant to real investment decisions. Additionally we have admitted government activity, international trade and an endogenous money supply in the model, thus increasing its usefulness for the study of policy problems.

2. The distinction between short-run disequilibrium and long-run equilibrium has been examined and although some economists had suggested that it is not possible to develop a common framework for short-run and long-run processes the discussion has shown that this is not necessarily the case. Through the use of certain distributed lag models like the partial adjustment one, we can distinguish three types of model within the same framework: the disequilibrium, the equilibrium and the stationary models. The above distributed lag models usually result from the application of a dynamic adjustment mechanism to a behavioural equation and have the common characteristic that their final form implies a lag distribution for the explanatory variables with weights which sum to unity. When we move from single equation models to simultaneous equations models and in some equations of the latter the reduced form of any of the above models are set for estimation, the distinction of three types of models can still be valid. Moreover, in this case, certain useful propositions have been conjectured as being true, namely that (a) total multipliers of the disequilibrium model are independent of the speed of adjustment coefficients and coincide with the multipliers obtained from the stationary model; (b) the dynamic stability of the disequilibrium model generally depends on the speed of adjustment coefficients; and (c) while single-equation distributed lag models of the above mentioned type imply a distributed lag relationship with equal weights for all explanatory variables, in simultaneous equations models, having the above equations as a part, there is in general a differentiation of the pattern of dynamic response of the relevant endogenous variable to changes in any of the explanatory variables in that equation.

3. Some of the model's dynamic properties as expressed in the appropriate impact and interim multipliers were analysed in relation to the set of empirical propositions known in the literature as monetarism. These propositions refer both to the short-run and the long-run and concern the effect of monetary policy on interest rates, the lag in the effect of monetary policy on real income and the price level, and the effect on real income and the price level of government spending financed by creating money, by taxes or by borrowing from the public. As regards the framework within which these propositions can be assessed it has been ascertained that the difference in theory between monetarists and Neo-Keynesians is that the former extend the standard IS-LM model in a way warranting the implication that the model's long-run equilibrium solution for output is one of full employment, whereas in the Keynesian system the possibility of this solution is not excluded but is only one out of a host of solutions all of which give a less than full employment level of output in equilibrium. More importantly, the solution suggested by monetarism for long-run equilibrium implies the independence of real and monetary variables so that the effect of a monetary expansion is felt only by prices and not by real output. For Neo-Keynesians there is in general short-run and long-run interaction between real output and the price level and the effect of a monetary expansion is felt by both.

Our findings are consistent with the Neo-Keynesian view as regards the long-run, namely a monetary change has been found to affect both prices and real output and a fiscal change has been shown to have a permanent effect on real and nominal income. An interesting characteristic of the examined multipliers on real and nominal income is the small size of monetary multipliers and the large size of fiscal multipliers, a finding which confirms the same evidence for the U.K. from other sources. On the other hand, the other monetarist propositions were supported by the results obtained from multiplier analysis. Specifically (a) it has been verified that the income and price expectations effects tend to revert the downward movement of the nominal interest rate following a monetary expansion and almost restore it to its initial level in the long-run. When the monetary expansion is accompanied by a fiscal change of equal size the interest rate increases outweight its initial fall by the third quarter. (b) it has been reported that the maximum effect of a monetary expansion on real output occurs three quarters later while the maximum effect on prices occurs after fourteen quarters and this conforms with the monetarist proposition that the maximum effect on real output precedes the one on prices, although the lags for the U.K. were found to be longer than those suggested by monetarists for the U.S.

4. It has been suggested by other economists that the adjustment of a variable to its equilibrium value need not be associated with positive weights only. A plausible adjustment would be one in which the variable in question overshoots its equilibrium value in the first period, reverses direction and overshoots it again in the second period and the process is continued until finally the equilibrium value is reached asymptotically. This description of the process implies a sequence of weights which alternate in sign and converge to zero in absolute value. In particular, the distribution of weights alternating in sign and declining geometrically in absolute value has been examined in this study and has been termed the hyperacontistic lag distribution. The adjustment mechanism which generates it has been stated and used in the price equation.

5. It has been argued that there is no *a priori* reason to make the demand for money function homogeneous of the first degree in prices and nominal income. This proposition is based on an analogy to the demand function for commodities in which absence of money illusion is assumed to exist. The term money illusion in the context of demand equations means that when there is a rise in nominal income (or other nominal variables influencing spending decisions) due to a rise in the general price level, there follows a corresponding rise or fall in the quantities of goods demanded. That is, there is no perfect conversion of nominal magnitudes to real ones but a certain degree of illusion exists which shows up as a significant coefficient of the price level if this is included in regressions with real variables. The assumption, however, of the non-existence of money illusion in demand for product equations has been criticised by various economists and has been removed from our model. Indeed the findings indicate that money illusion is present in the consumption and inventory functions and, less significantly, in the import function. Given that money illusion is an empirical issue and should not be excluded *a priori*, the homogeneity assumption in the money demand function should similarly be relaxed and has, in fact, been relaxed from our specification of the equation. This has left us with a demand for nominal money as a function of nominal income (the product of the price level and real income) or the price level and real income separately which are assumed to approximate their product.

A further point made here regarding money illusion has been that the latter can exist both as a short-run and long-run phenomenon and need not be restricted to the short-run. The main reason for this is that, even in the simplest macroeconomic model, real income is an endogenous variable and the price level effect on different types of expenditure can not be considered independently of movements in real income.

6. The results support our contention that Special Deposits should not be blended together with the sum of currency circulation with the public and cash reserves of the banking system, because they have been shown to have a negative impact on the narrow money supply whereas the latter are the basis of multiple expansion of it. The evidence offered in this study seems to refute the monetary authorities' belief in the non existence of a relationship between the size of calls for Special Deposits and desired objectives, since it has been found that a statistically significant link does exist between the size of calls for SDs and the money supply. We are thus able to relate SDs to desired objectives in the complete structural model.

7. Contrary to the widely held view that money supply is positively, if at all, related to the interest rate our analysis suggested that this is not necessarily true and that the sign of the interest rate coefficient can not be determined *a priori*. This is an important aspect of money supply theory not accounted properly in previous studies. While the positive impact of the interest rate on money supply has been examined in the context of the money supply function, the negative impact of the interest rate on the money supply has been neglected, despite the results from independent research indicating the importance of the interest rate to the ratio of currency to demand deposits (current accounts). A joint consideration of the effect of the interest rate on the two ratios

of the money multiplier leads to the *a priori* indeterminacy of the sign of the coefficient attached to interest rate. The findings here presented an insignificant coefficient which, however, was on the negative side. This is explained by the relative preponderance of the interest rate-ratio of currency to current accounts effect over the interest rate-ratio of banks' cash reserves to current accounts effect, in the British institutional context.

8. A series of estimates of the full-employment output of the United Kingdom economy have been made, using a method which took into account the cyclical response of labour force participation rate to expansions and contractions in economic activity. The findings revealed a high sensitivity of participation rates to employment opportunities - the relationship between new registrations in the labour force and the reduction in unemployment being approximately one to one. In the equation explaining variations in participation rates, which has been the basis for computation of the full-employment labour force, account has been taken of the break in trend for participation rates after 1966 and the increase in the minimum unemployment rate after 1956 and after 1966. In particular for the latter increase, which seems to have occurred by steps, a mechanical procedure was developed to provide as objective a measure as possible. The figures for full employment were applied to a trend of productivity to give the figures for full employment output. The computation took into account the acceleration in this trend after 1966. Also, in freeing the productivity data from cyclical fluctuations, provision was made for seasonal variations, so that the figures which have been obtained could profitably be used in our study employing data which have not been seasonally adjusted.

9. We have examined the multiplier effects of exogenous on endogenous variables in linear dynamic econometric models of higher than first order in which the lags of exogenous variables are extended to more than one period. A mistake in the theory of the calculation of multipliers in such models has been pointed out and the analysis and correct formulae have been provided. The crucial observation was that multipliers are not contained only in the leading submatrices of the relevant sequence of matrices for the system which has been transformed into a first order one but are portioned out to more than one matrix.

Furthermore the asymptotic distribution of impact and interim multipliers has been derived (in appendix D) for the case of a generalised linear dynamic model and the significance of multipliers in Klein's Model I has been re-examined. The evidence presented suggests that multipliers in that model are insignificant for those values for which the multiplier sequence changes sign.

APPENDIX A

TABLE

Year/Qtr		R	R^D	R'	Year/Qtr		R	R^D	R'
1955	1	3.93	3.67	5.35	1960	1	5.20	4.78	4.10
	2	4.08	4.50	5.31		2	5.41	5.09	4.65
	3	4.33	4.50	5.37		3	5.57	6.00	4.76
	4	4.36	4.50	5.71		4	5.53	5.51	4.90
1956	1	4.60	4.99	6.15	1961	1	5.82	5.00	4.64
	2	4.67	5.50	6.12		2	6.06	5.00	4.74
	3	4.82	5.50	6.22		3	6.52	6.46	5.41
	4	4.87	5.50	6.49		4	6.46	6.20	5.70
1957	1	4.60	5.21	6.13	1962	1	6.33	5.81	5.56
	2	4.82	5.00	5.86		2	6.18	4.09	5.83
	3	5.16	5.26	6.14		3	5.82	4.50	5.61
	4	5.44	7.00	6.96		4	5.66	4.50	5.31
1958	1	5.19	6.87	6.99	1963	1	5.80	4.01	4.58
	2	4.99	5.71	6.49		2	5.54	4.00	4.47
	3	4.90	4.74	5.96		3	5.39	4.00	4.36
	4	4.85	4.27	5.46		4	5.62	4.00	4.20
1959	1	4.76	4.00	5.30	1964	1	5.91	4.36	4.38
	2	4.84	4.00	5.00		2	6.04	5.00	4.51
	3	4.78	4.00	4.86		3	6.04	5.00	4.58
	4	4.92	4.00	4.13		4	6.13	5.83	5.03

R: Interest rate on 2½% Consolidated Stock, an undated government bond. Percentage per annum. Quarterly average of monthly figures based on the mean of the middle opening and middle closing prices each day.
Source: Monthly Digest of Statistics (M.D.S.)

A.1

Percentage per annum

Year/Qtr		R	R^D	R'	Year/Qtr		R	R^D	R'
1965	1	6.30	7.00	5.28	1970	1	8.60	7.85	4.02
	2	6.55	6.69	5.59		2	9.23	7.08	4.70
	3	6.48	6.00	5.86		3	9.28	7.00	4.77
	4	6.34	6.00	5.40		4	9.59	7.00	4.58
1966	1	6.53	6.00	5.32	1971	1	9.38	7.00	4.61
	2	6.78	6.00	5.27		2	9.24	6.00	4.05
	3	7.11	6.86	5.87		3	9.05	5.69	3.61
	4	6.81	7.00	6.21		4	8.59	5.00	3.61
1967	1	6.47	6.55	5.82	1972	1	8.45	5.00	3.30
	2	6.51	5.68	5.31		2	8.98	5.10	3.22
	3	6.82	5.50	5.01		3	9.46	6.00	3.30
	4	7.01	6.88	4.49		4	9.36	7.35	3.42
1968	1	7.13	7.94	4.33					
	2	7.30	7.50	3.71					
	3	7.47	7.44	3.35					
	4	7.69	7.00	3.38					
1969	1	8.42	7.37	3.36					
	2	9.10	8.00	3.82					
	3	9.15	8.00	4.23					
	4	8.87	8.00	4.18					

R^D: Bank Rate. Computed as weighted average of rates on specific dates, the weights being the days at which these rates applied. Source: M.D.S.

R': Dividend yield of industrial ordinary shares. Quarterly average of monthly, seasonally unadjusted figures. Source: M.D.S.

TABLE

Indices

Year/Qtr		P	W	E^R	Year/Qtr		P	W	E^R
1955	1	74.65	115.40	1.0431	1960	1	90.24	145.01	1.0028
	2	75.50	119.06	1.0411		2	91.27	146.48	1.0002
	3	77.36	119.84	1.0423		3	91.19	147.25	1.0048
	4	78.71	120.10	1.0361		4	92.31	148.40	0.9983
1956	1	79.56	124.22	1.0338	1961	1	93.06	151.28	1.0040
	2	80.89	128.66	1.0366		2	93.69	152.35	1.0098
	3	81.91	129.32	1.0453		3	95.01	153.12	1.0017
	4	83.40	129.93	1.0488		4	95.32	154.38	0.9986
1957	1	82.97	130.83	1.0441	1962	1	96.67	155.57	0.9971
	2	84.85	134.25	1.0442		2	97.96	157.77	0.9962
	3	85.58	136.17	1.0311		3	98.52	159.64	0.9987
	4	88.10	136.94	1.0171		4	98.64	160.82	0.9990
1958	1	88.29	137.59	1.0121	1963	1	98.65	162.05	0.9991
	2	89.21	138.16	1.0151		2	99.74	163.76	0.9998
	3	88.67	139.59	1.0179		3	100.40	164.49	1.0002
	4	89.81	141.92	1.0123		4	100.88	166.37	1.0010
1959	1	90.23	142.48	0.9987	1964	1	100.77	169.26	1.0005
	2	90.39	142.77	0.9987		2	102.79	171.18	1.0009
	3	89.47	143.21	1.0017		3	103.91	172.97	1.0055
	4	90.53	143.54	1.0038		4	104.36	174.11	1.0046

P: Implicit gross domestic product deflator derived from seasonally unadjusted totals of G.D.P. at current and constant 1963 prices. Source: Economic Trends (E.T.)

W: Wage rate, index referring to all industries and services and all workers. Quarterly average of monthly figures not seasonally adjusted, 1952 . 1 = 100. Source: M.D.S.

A.2

Indices

Year/Qtr		P	W	E^R	Year/Qtr		P	W	E^R
1965	1	105.59	176.35	1.0029	1970	1	131.49	231.05	1.1624
	2	107.61	177.98	1.0012		2	134.24	236.38	1.1684
	3	109.31	180.71	1.0004		3	138.15	242.46	1.1793
	4	109.68	182.34	0.9983		4	139.85	252.04	1.1794
1966	1	110.26	185.56	0.9991	1971	1	143.33	261.82	1.1670
	2	112.22	187.23	1.0018		2	147.03	267.20	1.1684
	3	113.84	189.03	1.0028		3	150.50	273.96	1.1641
	4	114.87	189.03	1.0019		4	151.88	283.17	1.1483
1967	1	114.73	190.70	1.0003	1972	1	153.50	292.75	1.1271
	2	115.79	192.25	1.0019		2	155.83	299.31	1.1539
	3	117.16	196.81	1.0047		3	159.68	315.43	1.2088
	4	117.35	199.50	1.1029		4	161.30	328.95	1.2526
1968	1	118.07	205.00	1.1619					
	2	120.16	206.14	1.1705					
	3	122.27	208.14	1.1705					
	4	122.67	211.73	1.1722					
1969	1	124.30	215.72	1.1677					
	2	126.02	216.66	1.1696					
	3	128.17	218.94	1.1614					
	4	129.12	223.10	1.1653					

E^R: Exchange rate, the sterling price of a unit of foreign exchange. Antilogarithms (to the base e) of the figures given by Llewellyn in "The Determinants of United Kingdom Import Prices" Economic Journal 1974, pp.20-31. The index is calculated as the weighted sum of the exchange rates of the U.K. with each of the six countries United States, Canada, France, Germany, Italy and Japan constituting the major part of the industrial world. The weights, i.e. the country's share in United Kingdom imports of goods in 1963, are United States 0.5489, Canada 0.0892, France 0.1102, Germany 0.1455, Italy 0.0526 and Japan 0.0537.

TABLE A.3

Money stock inclusive of current account deposits of all banks

£ millions

Year	Quarters			
	1	2	3	4
1963	6672	6855	6947	7322
1964	7146	7223	7376	7557
1965	7358	7461	7548	7848
1966	7764	7728	7806	7844
1967	7773	7899	8225	8442
1968	8210	8356	8461	8784
1969	8339	8188	8312	8812
1970	8507	8852	9032	9635
1971	9691	9831	10210	11088
1972	11225	11729	11930	12657

TABLE A.4

Money stock inclusive of current account deposits of London Clearing Banks

£ millions

Year	Quarters			
	1	2	3	4
1955	5285	5259	5344	5314
1956	5248	5200	5318	5345
1957	5244	5245	5415	5281
1958	5135	5104	5101	5132
1959	5367	5398	5646	5778
1960	5720	5703	5782	5744
1961	5689	5809	5856	5771
1962	5693	5702	5814	5909
1963	5894	5961	6178	6388
1964	6324	6366	6498	6610
1965	6496	6574	6691	6691
1966	6732	6919	6910	6817
1967	6749	6925	7112	7234
1968	7197	7265	7437	7513
1969	7363	7251	7281	7348
1970	7620	7713	8008	8215
1971	8572	8582	8797	9183

TABLE A.5

Data on Money Stock based on regression

£ millions

Year	Quarters			
	1	2	3	4
1955	5967	5936	6037	6001
1956	5923	5866	6006	6038
1957	5918	5919	6121	5962
1958	5788	5751	5748	5785
1959	6064	6101	6396	6553
1960	6484	6464	6558	6513
1961	6447	6590	6646	6545
1962	6452	6463	6596	6709

M: Money stock defined as currency plus net sterling current account deposits held by the private sector. Quarterly average of monthly figures, seasonally unadjusted. The money stock series employed is based on two sets of figures kindly supplied by C.A.E. Goodhart, Bank of England. The first spanning the years 1952 to 1971 (see Table A.4) includes currency and net current account deposits of London Clearing Banks. The second series which is more comprehensive and currently available goes back to 1963 and contains net current account deposits of all banks defined to constitute the banking system (see Table A.3). A regression of the new series on the old one was performed for the overlapping period 1963 to 1971. The fitted equation provided backward projection of the new series (see Table A.5). The regression results are presented below.

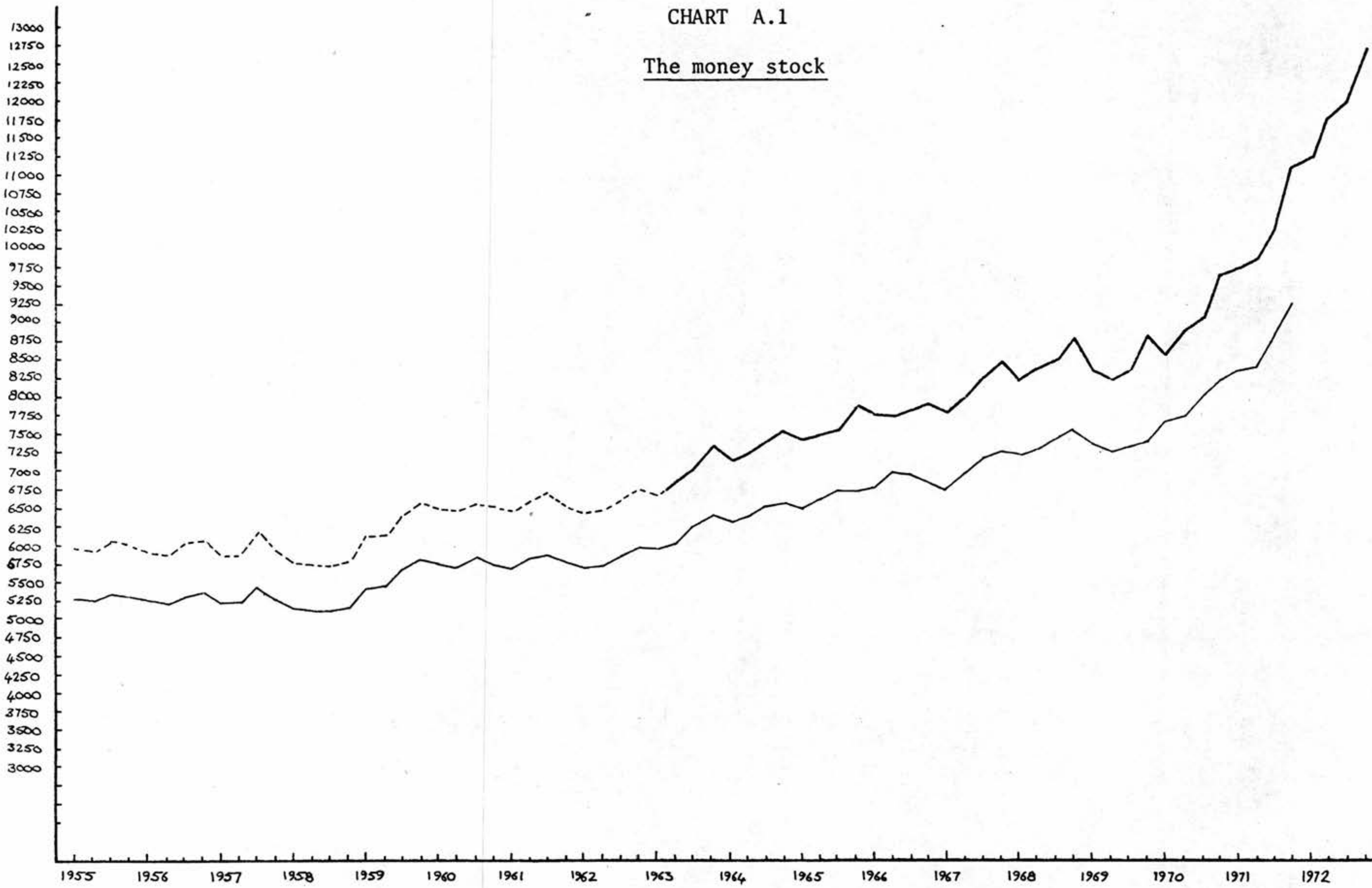
$$Y_t = -321.14 + 1.189761 X_t$$
$$(44.06)$$

$$\bar{R}^2 = 0.982279 \qquad D.W. = 2.63$$

The regression shows a remarkably good fit and produces figures for the years 1955 to 1962 which reproduce the fluctuations in the available series. This can be seen from the accompanying chart where the two series are plotted. The data of Tables A.5 and A.3 taken together form the money stock series actually used.

CHART A.1

The money stock



TABLE

Year/Qtr		C^P	B	S^D	Year/Qtr		C^P	B	S^D
1955	1	1587	2232	-	1960	1	1989	2715	-
	2	1649	2297	-		2	2059	2790	23
	3	1695	2347	-		3	2100	2842	132
	4	1696	2357	-		4	2103	2857	150
1956	1	1701	2340	-	1961	1	2084	2835	154
	2	1757	2389	-		2	2144	2900	152
	3	1802	2449	-		3	2186	2956	190
	4	1803	2474	-		4	2186	2958	232
1957	1	1779	2420	-	1962	1	2159	2920	237
	2	1832	2477	-		2	2158	2931	217
	3	1873	2552	-		3	2162	2946	159
	4	1885	2583	-		4	2163	2958	71
1958	1	1858	2524	-	1963	1	2147	2925	-
	2	1904	2589	-		2	2192	2986	-
	3	1926	2611	-		3	2241	3045	-
	4	1925	2631	-		4	2254	3089	-
1959	1	1903	2593	-	1964	1	2240	3065	-
	2	1951	2642	-		2	2308	3150	-
	3	2002	2722	-		3	2361	3232	-
	4	2012	2747	-		4	2372	3264	-

C^P: Currency holdings of the public. Quarterly average of monthly figures, seasonally unadjusted.
Source: Bank of England Statistical Abstract (B.E.S.A.) and Bank of England Quarterly Bulletin (B.E.Q.B.)

A.6

£ millions

Year/Qtr		C^P	B	S^D	Year/Qtr		C^P	B	S^D
1965	1	2371	3238	-	1970	1	2968	3980	221
	2	2459	3368	34		2	3055	4048	243
	3	2503	3419	94		3	3169	4228	267
	4	2520	3447	95		4	3213	4261	349
1966	1	2517	3438	98	1971	1	3298	4475	395
	2	2608	3549	98		2	3339	4462	406
	3	2646	3592	168		3	3401	4491	271
	4	2635	3586	198		4	3437	4487	-
1967	1	2600	3540	200	1972	1	3453	4418	-
	2	2652	3616	200		2	3615	4675	-
	3	2695	3678	206		3	3756	4808	-
	4	2731	3733	211		4	3892	4989	40
1968	1	2729	3734	217					
	2	2792	3813	217					
	3	2836	3882	220					
	4	2862	3927	223					
1969	1	2847	3883	229					
	2	2904	3988	225					
	3	2934	4008	224					
	4	2962	4038	223					

B: Monetary base, the sum of currency calculation with the public and notes, coin and balances with Bank of England of London, Scottish and Northern Ireland deposit banks. Quarterly average of monthly figures, seasonally unadjusted. Source: B.E.S.A. and B.E.Q.B.

S^D: Special deposits. Quarterly average of seasonally unadjusted monthly figures. Source: B.E.S.A. and B.E.Q.B.

TABLE

Year/Qtr		Y	C	G	I^P	I	H	E	X
1955	1	6055	4003	1074	445	430	224	1413	1534
	2	6056	4264	1034	406	486	-34	1289	1389
	3	6315	4301	1026	424	502	189	1366	1493
	4	6513	4539	1009	457	542	17	1387	1438
1956	1	6105	4001	1031	422	507	158	1424	1438
	2	6382	4256	1056	414	507	106	1461	1418
	3	6258	4244	1057	421	518	57	1345	1384
	4	6615	4469	1065	433	586	0	1416	1354
1957	1	6254	4006	1090	437	544	183	1473	1479
	2	6410	4251	1043	402	563	112	1456	1417
	3	6379	4318	1045	427	546	72	1381	1410
	4	6536	4490	1023	459	580	-81	1359	1294
1958	1	6236	4010	1054	436	544	112	1352	1272
	2	6303	4243	1048	396	552	17	1288	1241
	3	6406	4325	1047	405	563	74	1331	1339
	4	6637	4605	1064	430	599	-77	1318	1302
1959	1	6244	4115	1092	411	544	59	1285	1262
	2	6645	4435	1114	405	601	80	1351	1341
	3	6640	4511	1104	456	577	54	1332	1394
	4	7082	4815	1129	494	656	1	1409	1422
1960	1	6732	4381	1152	465	619	186	1438	1509
	2	6999	4657	1157	407	669	208	1418	1517
	3	6927	4656	1151	457	680	167	1374	1558
	4	7266	4856	1198	478	739	86	1410	1501
1961	1	6964	4440	1227	494	700	167	1447	1511
	2	7289	4752	1213	466	750	161	1448	1501
	3	7218	4758	1201	494	770	55	1387	1447
	4	7439	4962	1217	484	800	-43	1412	1393
1962	1	6961	4517	1258	520	688	14	1387	1423
	2	7390	4887	1245	470	736	46	1441	1435
	3	7302	4844	1239	506	721	74	1374	1456
	4	7516	5067	1270	508	742	-80	1415	1406
1963	1	7062	4641	1287	468	639	11	1425	1409
	2	7638	5076	1277	500	719	73	1467	1474
	3	7583	5115	1287	574	713	11	1430	1547
	4	8065	5298	1321	592	788	95	1488	1517

Y: Gross domestic product.

C: Consumer's expenditure.

G : Public authorities' current expenditure

I^P: Public authorities' gross fixed capital formation.

A.7

£ millions

Year/Qtr		Y	C	G	I^P	I	H	E	X
1964	1	7676	4915	1342	654	763	132	1491	1621
	2	8032	5231	1320	581	802	214	1520	1636
	3	7961	5256	1324	625	820	163	1428	1655
	4	8381	5485	1354	649	888	115	1506	1616
1965	1	7910	5012	1364	682	830	98	1465	1541
	2	8156	5297	1395	590	825	111	1538	1600
	3	8175	5344	1388	627	811	142	1505	1642
	4	8533	5522	1436	674	888	15	1563	1565
1966	1	8131	5139	1437	717	817	77	1571	1627
	2	8353	5459	1441	631	799	110	1512	1599
	3	8339	5392	1444	696	806	129	1531	1659
	4	8670	5499	1496	733	844	-79	1626	1449
1967	1	8307	5115	1573	825	776	51	1586	1619
	2	8583	5444	1527	717	838	117	1602	1662
	3	8562	5527	1534	769	793	36	1550	1647
	4	8794	5772	1612	805	825	-16	1494	1698
1968	1	8601	5483	1646	895	825	-115	1795	1928
	2	8714	5544	1567	729	841	125	1781	1873
	3	8845	5620	1577	746	875	119	1832	1924
	4	9278	5900	1600	773	954	44	1875	1868
1969	1	8654	5375	1615	798	885	84	1807	1910
	2	8963	5684	1562	659	885	110	1976	1913
	3	9013	5695	1571	716	905	63	1970	1907
	4	9459	5967	1635	759	950	38	2003	1893
1970	1	8752	5413	1621	780	878	-55	2001	1886
	2	9274	5757	1662	700	965	145	2103	2058
	3	9238	5786	1671	725	952	116	1981	1993
	4	9761	6033	1705	788	1023	65	2183	2036
1971	1	8827	5380	1730	866	832	40	1978	1999
	2	9351	5830	1723	674	951	1	2212	2040
	3	9487	5905	1747	709	946	-1	2157	1976
	4	9989	6200	1802	780	1004	5	2151	1953
1972	1	9020	5688	1837	813	918	182	2019	2083
	2	9646	6143	1792	666	1006	-28	2162	2095
	3	9528	6225	1847	715	952	-71	1900	2040
	4	10330	6617	1908	762	1091	-59	2310	2299

I: Private gross fixed capital formation.

H: Physical increase in stocks and work in progress.

E: Exports of goods and services.

X: Imports of goods and services.

All figures are derived by deflating totals at current prices by the implicit G.D.P. deflator and are not seasonally adjusted.
Source: Economic Trends (E.T.)

TABLE

Year/Qtr		T	T^i	Z	Year/Qtr		T	T^i	Z
1955	1	1637	715	432	1960	1	2046	783	1036
	2	1264	736	231		2	1931	784	1114
	3	1624	756	639		3	1889	783	1083
	4	1684	797	707		4	2062	831	1135
1956	1	1798	718	745	1961	1	2046	766	958
	2	1798	748	843		2	2057	812	1155
	3	1782	752	847		3	1941	775	1022
	4	2038	811	1149		4	2112	872	1258
1957	1	1916	738	823	1962	1	2005	780	903
	2	1829	745	963		2	2159	842	1243
	3	1902	736	1029		3	2033	843	1051
	4	1968	778	1126		4	2073	881	1067
1958	1	1918	720	913	1963	1	1935	787	818
	2	1787	730	970		2	2205	859	1331
	3	1900	730	1032		3	2034	885	1100
	4	2043	803	1233		4	2387	943	1554
1959	1	1843	748	897	1964	1	2220	879	1070
	2	1882	774	1038		2	2370	950	1546
	3	1888	768	1012		3	2262	944	1385
	4	2196	849	1323		4	2470	1039	1609

T: As defined in the section where the consumption function is discussed in chapter 6. Obtained as difference between G.D.P. and total personal disposable income. The latter is total personal disposable income at current prices deflated by the implicit G.D.P. deflator.
Source: E.T.

A.8

£ millions

Year/Qtr		T	T^i	Z	Year/Qtr		T	T^i	Z
1965	1	2271	989	1253	1970	1	2661	1297	1941
	2	2366	1004	1654		2	3021	1419	2689
	3	2406	1000	1631		3	2958	1425	2757
	4	2547	1077	1587		4	3281	1434	2945
1966	1	2131	1020	1104	1971	1	2609	1267	2124
	2	2506	1066	1788		2	3022	1391	2705
	3	2573	1087	1834		3	3076	1261	2502
	4	2729	1276	2033		4	3327	1354	2586
1967	1	2596	1047	1393	1972	1	2554	1174	1706
	2	2633	1110	1800		2	2823	1346	1754
	3	2548	1124	1672		3	2737	1278	2405
	4	2658	1162	1702		4	3247	1388	3024
1968	1	2538	1137	1370					
	2	2680	1194	1948					
	3	2790	1253	2122					
	4	3003	1387	2423					
1969	1	2543	1272	1699					
	2	2864	1352	2499					
	3	2911	1417	2495					
	4	3125	1451	2669					

T^i: Taxes on Expenditure net of subsidies. Totals at current prices deflated by the implicit G.D.P. deflator. Source: E.T.

Z: As defined in the last section of chapter 7 where the identity, showing how the Government borrowing requirement is financed, is discussed. Obtained as a residual from the terms in this identity.

TABLE A.9

Thousands

Year/Qtr		Civil Employment			H.M. Forces			Wholly Unemployed			Labour Force		
		Males	Females	Total	Males	Females	Total	Males	Females	Total	Males	Females	Total
1954	1	14927	7569	22496	823	23	846	214	113	327	15964	7705	23669
	2	15051	7603	22654	816	23	839	148	79	227	16015	7705	23720
	3	15109	7653	22762	814	22	836	145	82	222	16068	7757	23825
	4	15102	7694	22796	809	21	830	158	88	246	16069	7803	23872
1955	1	15083	7755	22838	803	21	824	163	88	251	16049	7864	23913
	2	15178	7770	22948	784	19	803	117	64	181	16079	7853	23932
	3	15243	7855	23098	772	18	790	117	66	183	16132	7939	24071
	4	15293	7835	23128	762	17	779	135	71	206	16190	7923	24113
1956	1	15246	7857	23103	756	17	773	159	79	238	16161	7953	24114
	2	15332	7874	23206	745	16	761	126	63	189	16203	7953	24156
	3	15307	7962	23269	764	16	780	145	80	225	16216	8058	24274
	4	15309	7886	23195	745	16	761	177	91	268	16231	7993	24224
1957	1	15248	7902	23150	704	15	719	231	104	335	16183	8021	24204
	2	15380	7917	23297	687	15	702	170	77	247	16237	8009	24246
	3	15402	7883	23285	660	14	674	180	80	260	16242	7977	24219
	4	15426	7787	23213	637	14	651	225	95	320	16288	7896	24184
1958	1	15226	7831	23057	612	14	626	285	110	395	16123	7955	24078
	2	15309	7826	23135	600	14	614	264	104	368	16173	7944	24117
	3	15296	7804	23100	585	14	599	296	120	416	16177	7938	24115
	4	15319	7743	23062	576	14	590	344	135	479	16239	7892	24131
1959	1	15264	7844	23108	564	15	579	364	137	501	16192	7996	24188
	2	15340	7907	23247	550	15	565	278	107	385	16168	8029	24197
	3	15408	8011	23419	536	15	551	285	109	394	16229	8135	24364
	4	15448	7998	23446	521	15	536	299	110	409	16268	8123	24391

1960	1	15454	8142	23596	511	15	526	287	115	402	16252	8272	24524
	2	15547	8163	23710	503	15	518	214	83	297	16264	8261	24525
	3	15550	8259	23809	498	15	513	213	85	298	16261	8359	24620
	4	15693	8243	23936	488	15	503	233	.90	323	16414	8348	24762
1961	1	15680	8348	24028	469	16	485	230	92	322	16379	8456	24835
	2	15722	8322	24044	459	15	474	185	70	255	16366	8407	24773
	3	15760	8402	24162	448	16	464	211	79	290	16419	8497	24916
	4	15718	8322	24040	438	16	454	262	93	355	16418	8431	24849
1962	1	15745	8399	24144	429	17	446	306	106	412	16480	8522	25002
	2	15804	8427	24231	425	17	442	278	95	373	16507	8539	25046
	3	15798	8460	24258	419	17	436	325	114	439	16542	8591	25133
	4	15740	8400	24140	415	18	433	399	125	524	16554	8543	25047
1963	1	15583	8410	23993	413	18	431	496	140	636	16492	8568	25060
	2	15792	8458	24250	410	17	427	346	115	461	16548	8590	25138
	3	15784	8531	24315	407	17	424	347	121	468	16538	8669	25207
	4	15860	8539	24399	406	17	423	340	112	452	16606	8668	25274
1964	1	15771	8578	24349	408	16	424	314	102	416	16493	8696	25189
	2	15898	8629	24527	408	16	424	240	77	317	16546	8722	25268
	3	15943	8738	24681	407	16	423	249	87	336	16599	8841	25440
	4	15982	8724	24706	409	16	425	255	85	340	16646	9825	25471
1965	1	15863	8780	24643	408	16	424	259	84	343	16530	8880	25410
	2	15990	8780	24770	407	16	423	207	63	270	16604	8859	25463
	3	15939	8889	24828	406	15	421	231	73	304	16576	8977	25553
	4	16002	8895	24897	405	15	420	247	72	319	16654	8982	25636
1966	1	15884	8923	24807	403	15	418	239	68	307	16526	9006	25532
	2	15956	8958	24914	402	15	417	199	54	253	16557	9027	25584
	3	15937	9022	24959	401	15	416	253	71	324	16591	9108	25699
	4	15788	8880	24668	403	16	419	373	94	467	16564	8990	25554

(cont.)

TABLE A.9
(cont.)

Thousands

Year/Qtr		Civil Employment			H.M. Forces			Wholly Unemployed			Labour Force		
		Males	Females	Total	Males	Females	Total	Males	Females	Total	Males	Females	Total
1967	1	15556	8843	24399	403	16	419	421	104	525	16380	8963	25343
	2	15688	8831	24519	401	16	417	378	88	466	16467	8935	25402
	3	15735	8864	24599	397	16	413	424	102	526	16556	8982	25538
	4	15622	8807	24429	396	16	412	461	98	559	16479	8921	25400
1968	1	15418	8841	24259	391	16	407	477	95	572	16286	8952	25238
	2	15491	8856	24347	385	15	400	429	77	506	16305	8948	25253
	3	15536	8884	24420	380	15	395	448	87	535	16364	8986	25350
	4	15545	8839	24384	376	14	390	457	83	540	16378	8936	25314
1969	1	15414	8856	24270	370	14	384	484	82	566	16268	8952	25220
	2	15439	8934	24373	366	14	380	415	68	483	16220	9016	25236
	3	15449	8945	24394	363	14	377	455	85	540	16267	9044	25311
	4	15404	8897	24301	362	14	376	483	82	565	16249	8993	25242
1970	1	15300	8905	24205	360	14	374	517	85	602	16177	9004	25181
	2	15253	8934	24187	358	14	372	450	73	523	16061	9021	25082
	3	15259	8933	24192	356	14	370	487	92	579	16102	9039	25141
	4	15249	8866	24115	356	15	371	513	92	605	16118	8973	25091
1971	1	15007	8752	23759	354	15	369	590	110	700	15951	8877	24828
	2	14972	8847	23819	353	15	368	589	98	687	15914	8960	24874
	3	14832	8828	23660	353	15	368	677	133	810	15862	8976	24838
	4	14854	8745	23599	357	15	372	732	136	868	15943	8896	24839
1972	1	14758	8903	23661	356	15	371	780	144	924	15894	9062	24956
	2	14848	8796	23644	356	15	371	648	119	767	15852	8930	24782
	3	14855	8979	23834	359	15	374	699	149	848	15913	9143	25056
	4	14945	8995	23940	357	15	372	620	125	745	15922	9135	25057

Source: Department of Employment Gazette (Employment and Productivity Gazette, Ministry of Labour Gazette).

TABLE

Population of

Year/Qtr		Males	Females	Total	Year/Qtr		Males	Females	Total
1954	1	17977	20168	38145	1959	1	18356	20458	38814
	2	17994	20176	38170		2	18386	20486	38872
	3	18011	20184	38295		3	18422	20521	38943
	4	18028	20192	38220		4	18459	20557	39016
1955	1	18045	20201	38246	1960	1	18495	20592	39087
	2	18062	20209	38271		2	18532	20628	39160
	3	18076	20216	38292		3	18580	20647	39227
	4	18090	20223	38313		4	18629	20666	39295
1956	1	18105	20231	38336	1961	1	18677	20685	39362
	2	18119	20238	38357		2	18726	20704	39430
	3	18138	20254	38392		3	18808	20760	39568
	4	18158	20270	38428		4	18890	20816	39706
1957	1	18177	20286	38463	1962	1	18973	20873	39846
	2	18197	20302	38499		2	19055	20929	39984
	3	18214	20320	38534		3	19085	20960	40045
	4	18231	20338	38569		4	19115	20991	40106
1958	1	18248	20355	38603	1963	1	19145	21023	40168
	2	18265	20373	38638		2	19175	21054	40229
	3	18295	20401	38696		3	19217	21082	40299
	4	18326	20429	38755		4	19260	21110	40370

Source: Monthly Digest of Statistics

A.10

Labour Force Age

Thousands

Year/Qtr		Males	Females	Total	Year/Qtr		Males	Females	Total
1964	1	19302	21139	40441	1969	1	19638	21499	41137
	2	19345	21167	40512		2	19652	21513	41165
	3	19371	21193	40564		3	19662	21523	41185
	4	19397	21219	40616		4	19672	21534	41206
1965	1	19424	21246	40670	1970	1	19682	21544	41226
	2	19450	21272	40722		2	19692	21555	41247
	3	19469	21289	40758		3	19654	21530	41084
	4	19488	21306	40794		4	19616	21505	41121
1966	1	19507	21322	40829	1971	1	19578	21481	41059
	2	19526	21339	40865		2	19540	21456	40996
	3	19548	21356	40904		3	19569	21477	41046
	4	19570	21373	40943		4	19598	21498	41096
1967	1	19592	21391	40983	1972	1	19626	21519	41145
	2	19614	21408	41022		2	19655	21540	41195
	3	19610	21421	41031		3	19681	21567	41248
	4	19605	21434	41039		4	19707	21594	41301
1968	1	19601	21446	41047					
	2	19596	21459	41055					
	3	19610	21472	41082					
	4	19624	21486	41110					

Quarterly data were obtained by interpolation.

TABLE

Productivity Index

Year/Qtr		Actual	Fitted	Year/Qtr		Actual	Fitted	
1954	1	77.82	75.80	1959	1	85.25	85.91	
	2	78.87	79.19		2	91.28	89.44	
	3	82.21	79.74		3	89.92	90.12	
	4	80.80	83.98		4	96.28	94.51	
1955	1	79.16	77.60	1960	1	90.67	88.27	
	2	78.71	81.02		2	94.36	91.82	
	3	82.67	81.59		3	92.82	92.54	
	4	84.60	85.86		4	97.04	96.95	
1956	1	80.11	79.51	1961	1	92.99	90.73	
	2	83.10	82.95		2	96.76	94.32	
	3	82.53	83.56		3	95.54	95.06	
	4	88.37	87.86		4	98.21	99.50	
1957	1	82.46	81.53	1962	1	91.92	93.31	
	2	85.38	85.00		2	96.52	96.93	
	3	84.72	85.63		3	96.22	97.69	
	4	88.63	89.96		4	99.76	102.16	
1958	1	85.11	83.66	1963	1	93.12	96.00	
	2	85.38	87.17		2	100.00	99.64	
	3	87.44	87.82		3	100.36	100.44	
	4	90.78	92.18		4	106.07	104.94	

A.11

(1963.2 = 100)

	Year/Qtr	Actual	Fitted	Year/Qtr	Actual	Fitted
	1964 1	100.18	98.81	1969 1	113.23	114.49
	2	104.28	102.47	2	116.53	118.30
	3	102.97	103.30	3	118.91	119.26
	4	107.87	107.82	4	124.60	123.92
	1965 1	102.32	101.72	1970 1	115.71	117.96
	2	104.32	105.42	2	122.30	121.80
	3	105.50	106.27	3	113.94	122.79
	4	109.52	110.82	4	130.30	127.48
	1966 1	104.55	104.74	1971 1	119.32	121.54
	2	106.82	108.47	2	124.88	125.41
	3	107.62	109.35	3	128.78	126.43
	4	113.33	113.93	4	135.42	131.14
	1967 1	108.99	107.88	1972 1	121.43	125.24
	2	112.15	111.63	2	129.48	129.13
	3	113.43	112.54	3	127.88	130.17
	4	116.14	117.15	4	136.17	134.92
	1968 1	113.49	111.13			
	2	113.93	114.91			
	3	116.70	115.85			
	4	121.55	110.48			

TABLE

Full employment Labour Force

Year/Qtr		Labour Force	G.D.P.	Year/Qtr		Labour Force	G.D.P.
1954	1	-	-	1959	1	24155	6471
	2	23691	6065		2	24202	6750
	3	23900	6092		3	24291	6763
	4	23880	6409		4	24320	7116
1955	1	23883	5949	1960	1	24329	6666
	2	23918	6207		2	24385	6950
	3	23982	6202		3	24473	6967
	4	23981	6528		4	24499	7325
1956	1	23962	6062	1961	1	24504	6873
	2	23986	6323		2	24557	7163
	3	24053	6319		3	24690	7197
	4	24060	6653		4	24759	7576
1957	1	23987	6177	1962	1	24810	7135
	2	23989	6431		2	24906	7444
	3	24041	6429		3	24992	7470
	4	24040	6762		4	25012	7841
1958	1	24022	6303	1963	1	25015	7384
	2	24053	6571		2	25064	7687
	3	24133	6581		3	25155	7718
	4	24153	6925		4	25182	8096

A.12

and Full Employment G.D.E.

Year/Qtr		Labour Force	G.D.P.	Year/Qtr		Labour Force	G.D.P.
1964	1	25190	7643	1969	1	24864	8757
	2	25244	7952		2	24882	9067
	3	25324	7982		3	24935	9109
	4	25340	8365		4	24922	9490
1965	1	25337	7908	1970	1	24891	9048
	2	25380	8218		2	24907	9363
	3	25450	8251		3	24911	9389
	4	25455	8634		4	24849	9756
1966	1	25440	8174	1971	1	24667	9260
	2	25474	8488		2	24585	9537
	3	25546	8525		3	24632	9584
	4	25280	8818		4	24627	9969
1967	1	25131	8321	1972	1	24607	9543
	2	25096	8608		2	24639	9868
	3	25111	8631		3	24711	9926
	4	25077	9000		4	24718	10325
1968	1	24929	8508				
	2	24885	8797				
	3	24917	8826				
	4	24898	9202				

TABLE A.13: Linear approximation of G* = G.P

Year/Qtr	G*	Linear terms	Residual	Percentage error (Absolute value)	Year/Qtr	G*	Linear terms	Residual	Percentage error (Absolute value)
1955 1	1134	993	141	12.4	1964 1	2011	2015	-4	0.2
2	1087	925	162	14.9	2	1954	1952	2	0.1
3	1122	972	150	13.4	3	2025	2025	0	0
4	1154	1015	139	12.0	4	2090	2092	-2	0.1
1956 1	1156	1018	138	11.9	1965 1	2160	2162	-2	0.1
2	1189	1062	127	12.0	2	2136	2136	0	0
3	1211	1090	121	11.1	3	2203	2201	2	0.1
4	1249	1141	108	9.5	4	2314	2311	3	0.1
1957 1	1267	1164	103	8.1	1966 1	2375	2369	6	0.3
2	1226	1112	114	9.3	2	2325	2319	6	0.3
3	1260	1156	104	8.3	3	2436	2424	12	0.5
4	1306	1215	91	7.0	4	2560	2540	20	0.8
1958 1	1316	1228	88	6.7	1967 1	2751	2719	32	1.2
2	1288	1196	92	7.1	2	2598	2574	24	0.9
3	1287	1194	93	7.2	3	2698	2664	34	1.3
4	1342	1261	81	6.0	4	2836	2790	46	1.6
1959 1	1356	1279	77	5.7	1968 1	3000	2938	62	2.1
2	1373	1300	73	5.3	2	2759	2715	44	1.6
3	1396	1326	70	5.0	3	2840	2785	55	1.9
4	1469	1414	55	3.7	4	2911	2846	65	2.2
1960 1	1459	1402	57	3.9	1969 1	2999	2921	78	2.6
2	1427	1365	62	4.3	2	2799	2748	51	1.8
3	1466	1411	55	3.8	3	2931	2861	70	2.4
4	1547	1505	42	2.7	4	3091	2995	96	3.1
1961 1	1602	1569	33	2.1	1970 1	3157	3048	109	3.5
2	1573	1536	37	2.4	2	3171	3060	111	3.5
3	1610	1579	31	1.9	3	3310	3173	137	4.1
4	1621	1591	30	1.9	4	3486	3310	176	5.0
1962 1	1719	1700	19	1.1	1971 1	3721	3489	232	6.2
2	1680	1658	22	1.3	2	3524	3346	178	5.1
3	1719	1701	18	1.0	3	3696	3478	218	5.9
4	1754	1739	15	0.9	4	3922	3640	282	7.2
1963 1	1735	1718	17	1.0	1972 1	4083	3755	328	8.0
2	1772	1759	13	0.7	2	3830	3584	246	6.4
3	1868	1862	6	0.3	3	4091	3771	320	7.8
4	1930	1851	79	4.1	4	4307	3918	389	9.0

TABLE A.14: Linear approximation of T* = T.P

Year/ Qtr	T*	Linear terms	Residual	Percentage error (Absolute value)	Year/ Qtr	T*	Linear terms	Residual	Percentage error (Absolute value)
1955 1	1222	999	223	18.2	1964 1	2237	2231	6	0.3
2	954	618	336	35.2	2	2436	2439	-3	0.1
3	1256	1048	208	16.6	3	2350	2349	1	0
4	1325	1144	181	13.7	4	2578	2583	-5	0.2
1956 1	1430	1286	144	10.1	1965 1	2398	2397	1	0
2	1454	1317	137	9.4	2	2546	2546	0	0
3	1460	1323	137	9.4	3	2630	2628	2	0.1
4	1700	1633	67	3.9	4	2794	2789	5	0.2
1957 1	1590	1492	98	6.2	1966 1	2350	2355	-5	0.2
2	1552	1442	110	7.1	2	2812	2803	9	0.3
3	1628	1537	91	5.6	3	2929	2913	16	0.5
4	1734	1667	67	3.9	4	3135	3104	31	1.0
1958 1	1693	1617	76	4.5	1967 1	2978	2958	20	0.7
2	1594	1498	96	6.0	2	3049	3022	27	0.9
3	1685	1607	78	4.6	3	2985	2963	22	0.7
4	1835	1787	48	2.6	4	3119	3085	34	1.1
1959 1	1663	1582	81	4.9	1968 1	2997	2973	24	0.8
2	1701	1627	74	4.4	2	3220	3174	46	1.4
3	1689	1612	77	4.6	3	3411	3341	70	2.1
4	1988	1968	20	1.0	4	3684	3579	105	2.9
1960 1	1846	1800	46	2.5	1969 1	3161	3123	38	1.2
2	1762	1700	62	3.5	2	3609	3508	101	2.8
3	1723	1653	70	4.1	3	3731	3608	123	3.3
4	1903	1865	38	2.0	4	4035	3860	175	4.3
1961 1	1904	1865	39	2.0	1970 1	3499	3416	83	2.4
2	1927	1892	35	1.8	2	4055	2867	188	4.6
3	1844	1798	46	2.5	3	4086	3889	197	4.8
4	2013	1989	24	1.2	4	4588	4276	312	6.8
1962 1	1938	1905	33	1.7	1971 1	3739	3634	105	2.8
2	2115	2100	15	0.7	2	4443	4164	279	6.3
3	2003	1978	25	1.2	3	4629	4302	327	7.1
4	2045	2023	22	1.1	4	5053	4604	449	8.9
1963 1	1913	1880	33	1.7	1972 1	3920	3810	110	2.8
2	2199	2191	8	0.4	2	4399	4153	246	5.6
3	2042	2022	20	1.0	3	4370	4150	220	5.0
4	2408	2413	-5	0.2	4	5173	4693	480	9.3

TABLE A.15: Linear approximation of $T^{i*} = T^{i}.P$

Year/ Qtr	T^{i*}	Linear terms	Residual	Percentage error (Absolute value)	Year/ Qtr	T^{i*}	Linear terms	Residual	Percentage error (Absolute value)
1955 1	534	442	92	17.2	1964 1	886	878	8	0.9
2	556	473	83	14.9	2	977	974	3	0.3
3	585	513	72	12.3	3	981	979	2	0.2
4	627	570	57	9.1	4	1084	1086	-2	0.2
1956 1	571	494	77	13.5	1965 1	1044	1044	0	0
2	605	539	66	10.9	2	1080	1080	0	0
3	616	554	62	10.1	3	1093	1093	0	0
4	676	632	44	6.5	4	1181	1081	1	0.1
1957 1	612	549	63	10.3	1966 1	1125	1124	1	0.1
2	632	576	56	8.9	2	1196	1193	3	0.3
3	630	573	57	9.0	3	1238	1232	6	0.5
4	685	643	42	6.1	4	1466	1445	21	1.4
1958 1	636	583	53	8.3	1967 1	1201	1198	3	0.2
2	651	603	48	7.4	2	1285	1276	9	0.7
3	647	597	50	7.7	3	1317	1305	12	0.9
4	721	687	34	4.7	4	1364	1347	17	1.2
1959 1	675	632	43	6.4	1968 1	1343	1328	15	1.1
2	700	662	38	5.4	2	1435	1410	25	1.7
3	687	646	41	6.0	3	1532	1494	38	2.5
4	769	744	25	3.3	4	1702	1642	60	3.5
1960 1	707	670	37	5.2	1969 1	1581	1535	46	2.9
2	716	681	35	4.9	2	1704	1638	66	3.9
3	714	680	34	4.8	3	1816	1729	87	4.8
4	767	742	25	3.3	4	1873	1775	98	5.2
1961 1	713	680	33	4.6	1970 1	1705	1633	72	4.2
2	761	736	25	3.3	2	1905	1792	113	5.9
3	736	709	27	3.7	3	1968	1837	131	6.7
4	831	816	15	1.8	4	2006	1864	142	7.1
1962 1	754	731	23	3.1	1971 1	1816	1719	97	5.3
2	825	810	15	1.8	2	2045	1889	156	7.6
3	831	817	14	1.7	3	1898	1784	114	6.0
4	869	859	10	1.2	4	2057	1897	160	7.8
1963 1	778	760	18	2.3	1972 1	1802	1720	82	4.6
2	849	838	11	1.3	2	2097	1928	169	8.1
3	889	881	8	0.9	3	2040	1894	147	7.2
4	951	948	3	0.3	4	2239	2028	211	9.4

APPENDIX B

CHART B.1

Interest rate (bonds)

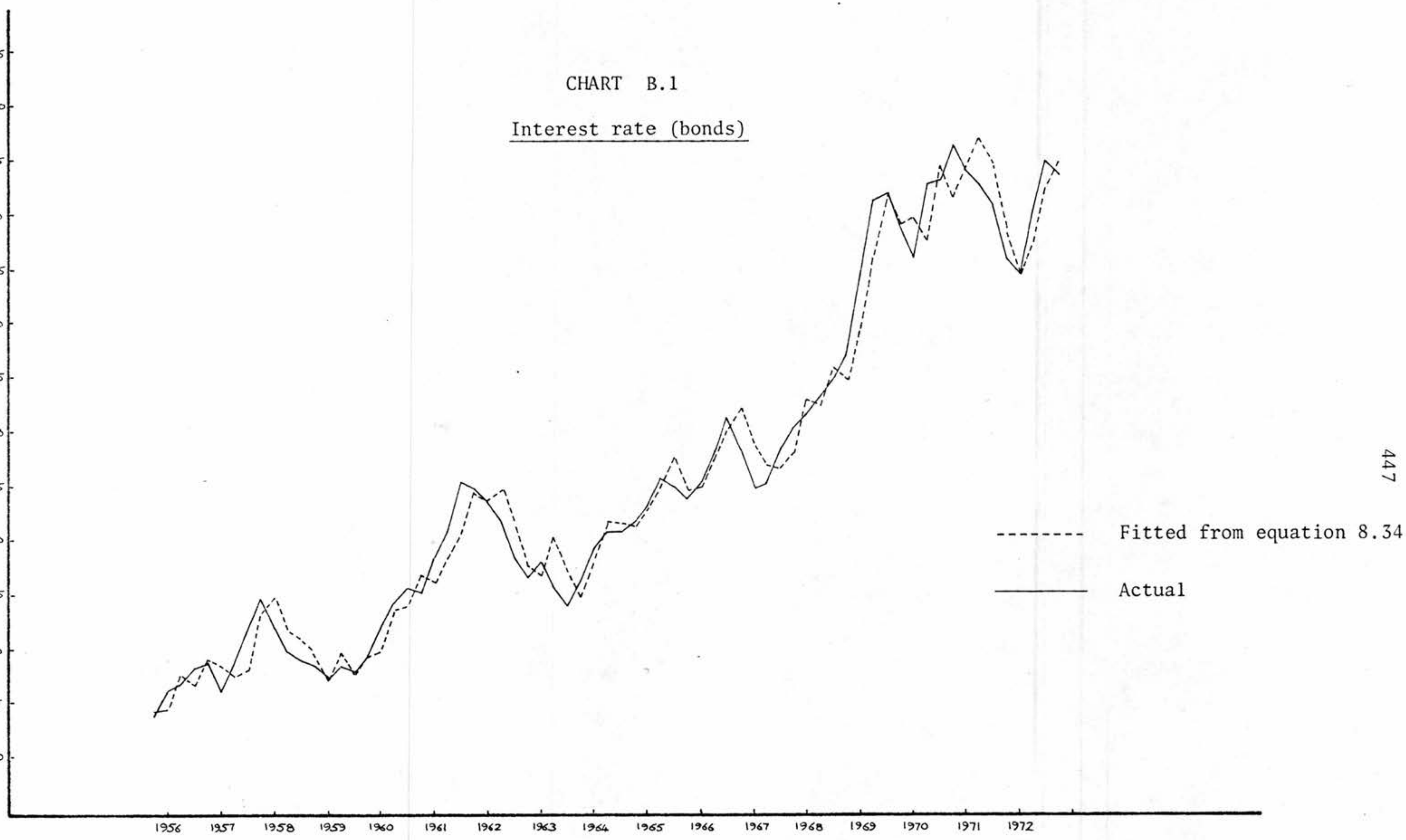


CHART B.2

Money stock

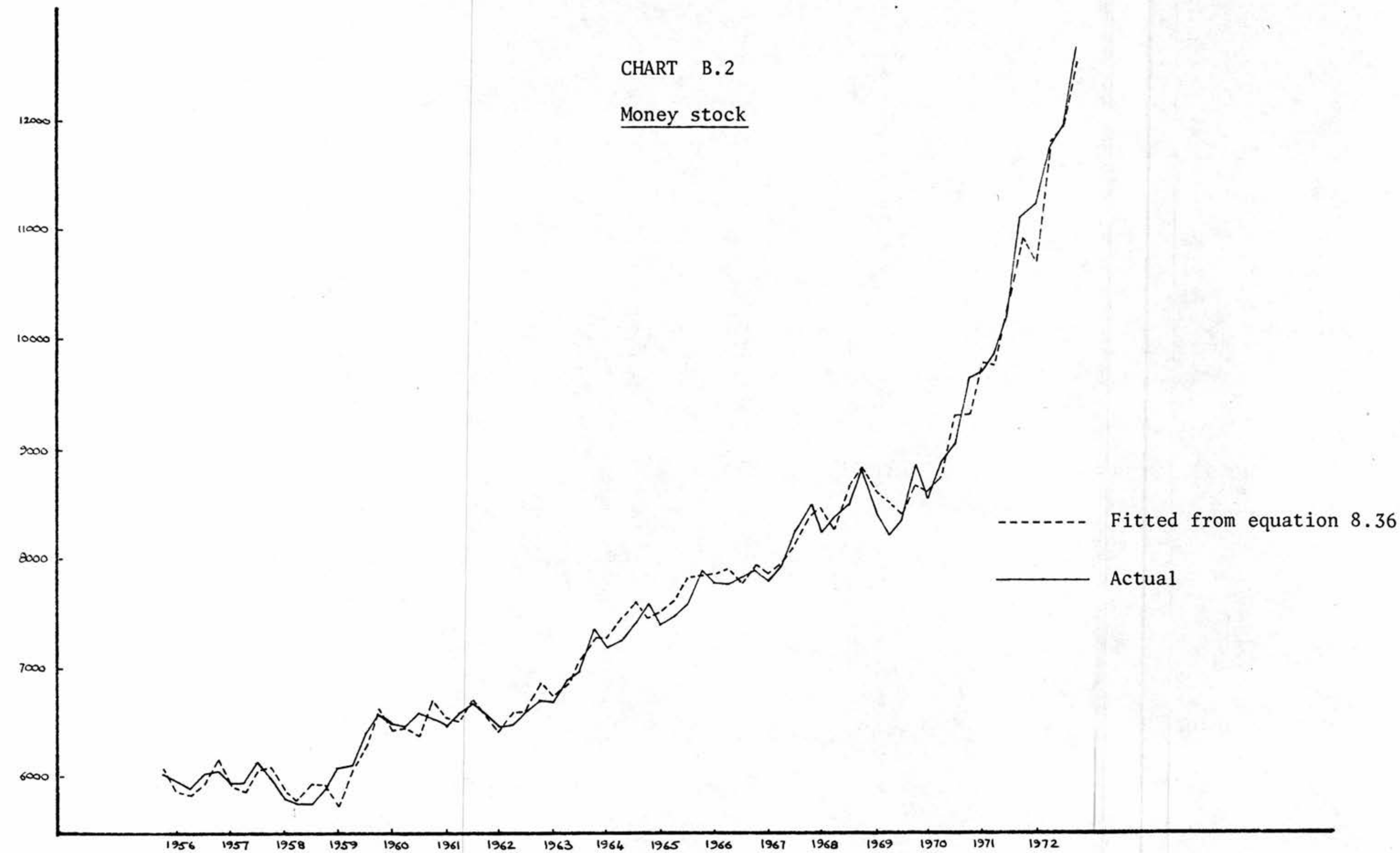


CHART B.3

Implicit G.D.P. deflator

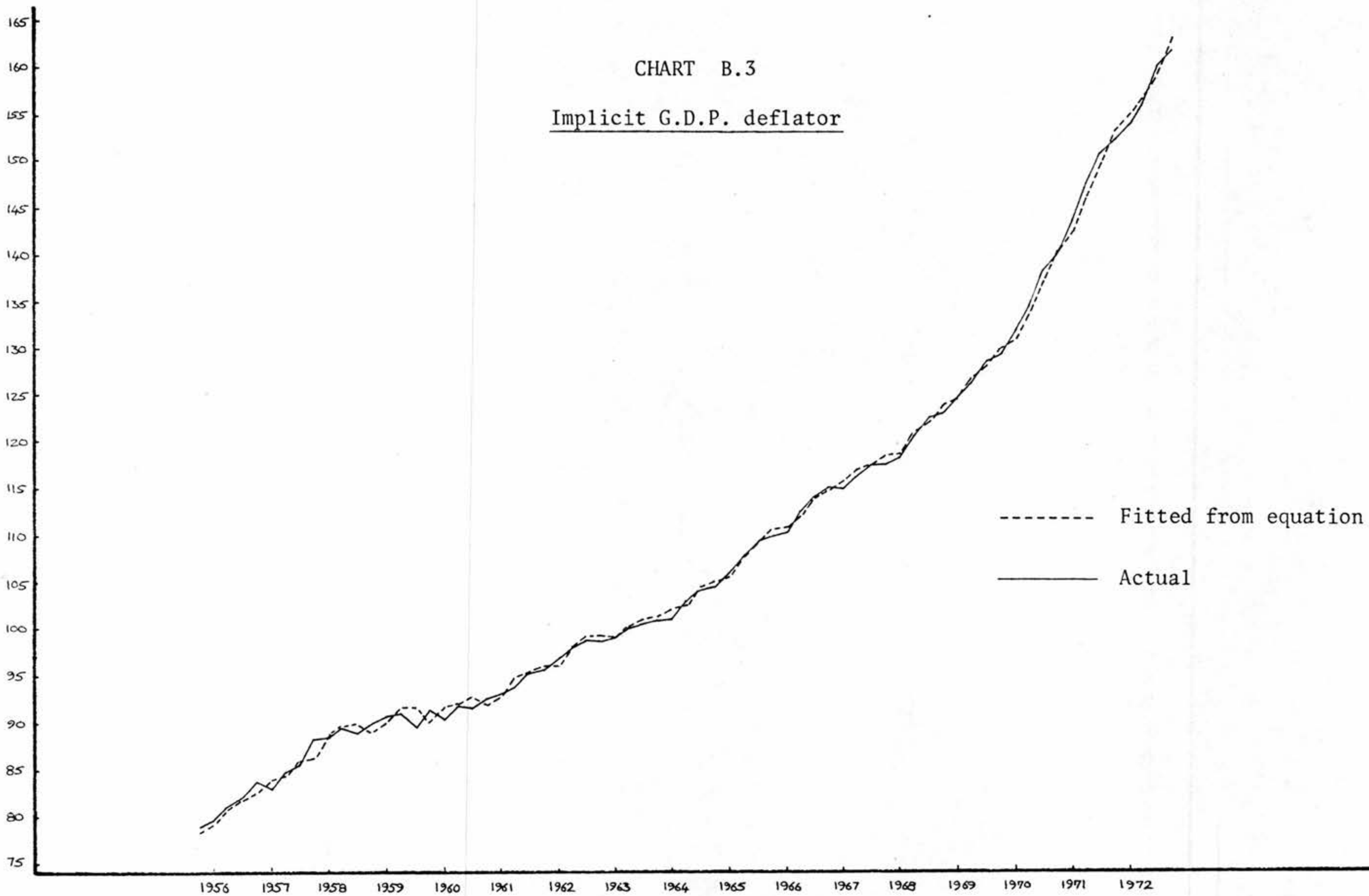


CHART B.4

Interest rate (equity dividend yield)

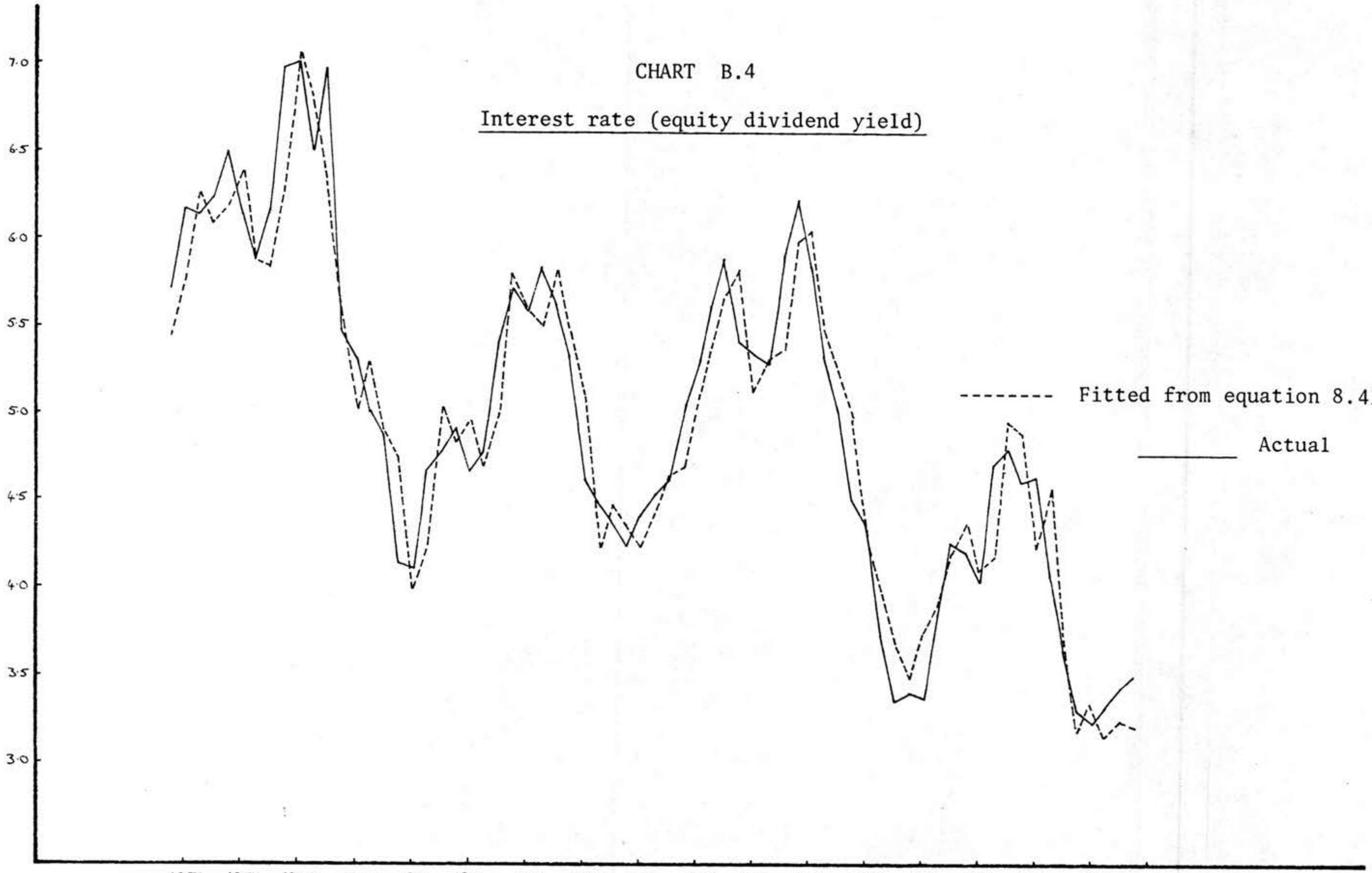


CHART B.5

Consumption

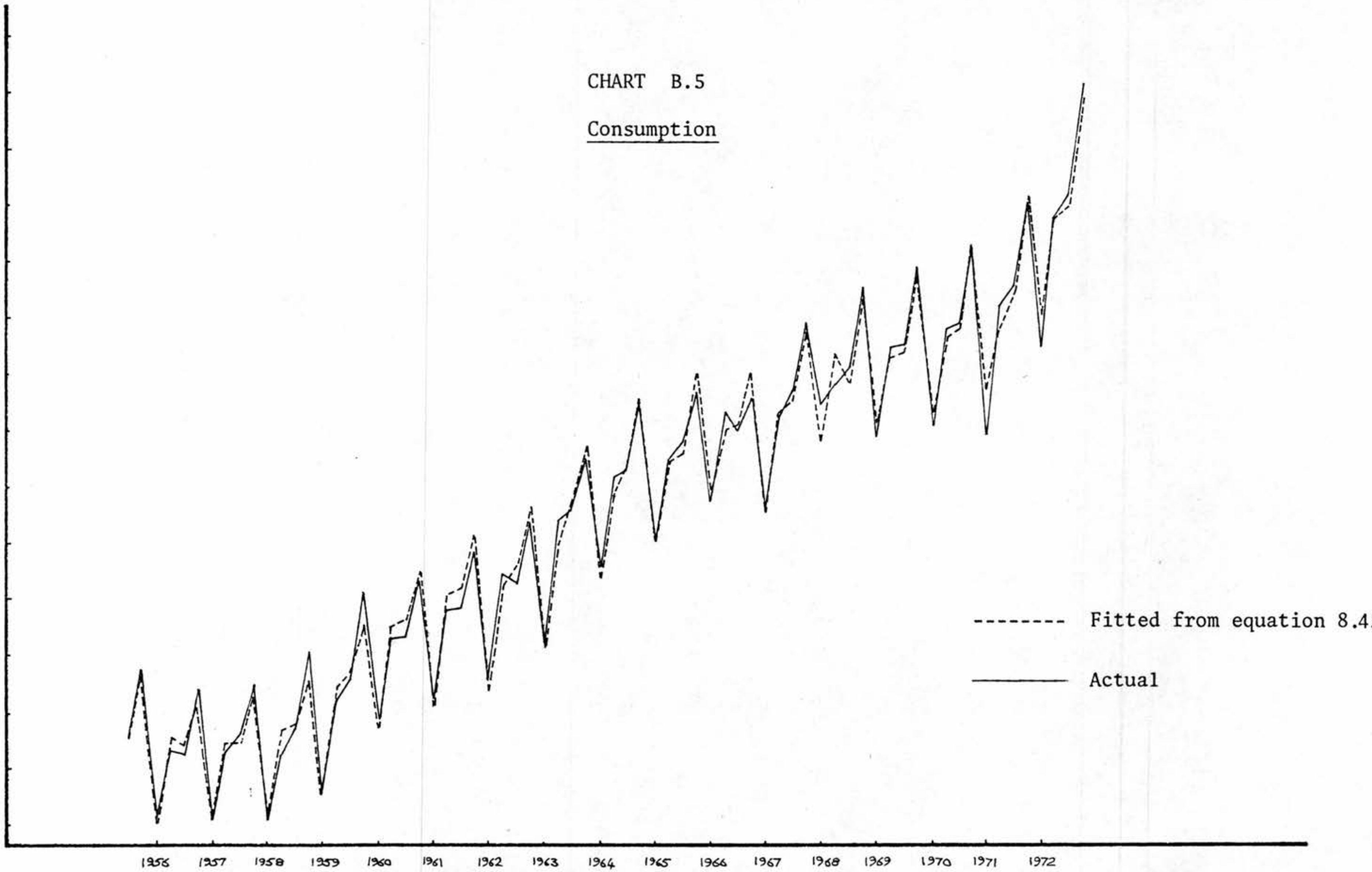
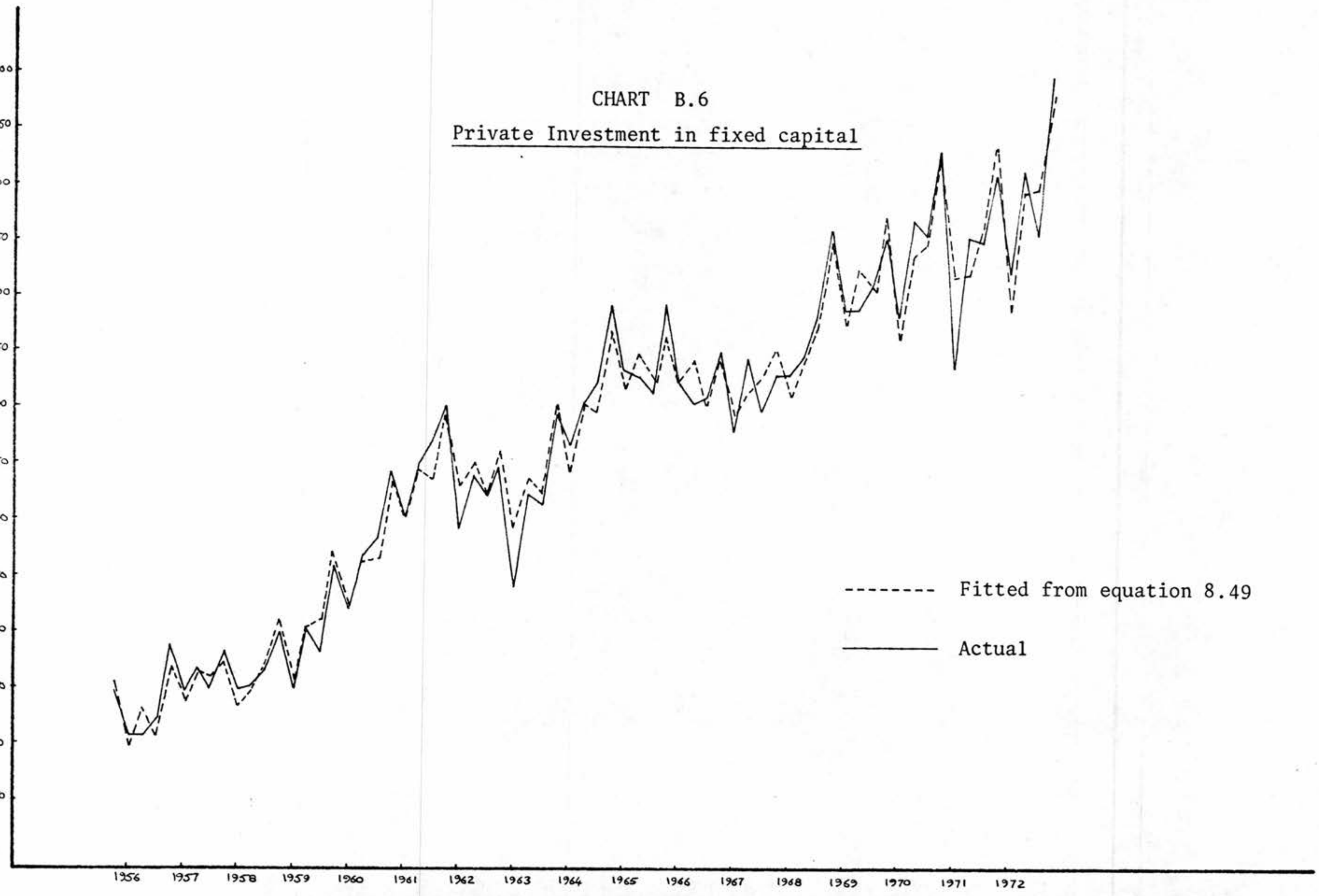


CHART B.6

Private Investment in fixed capital



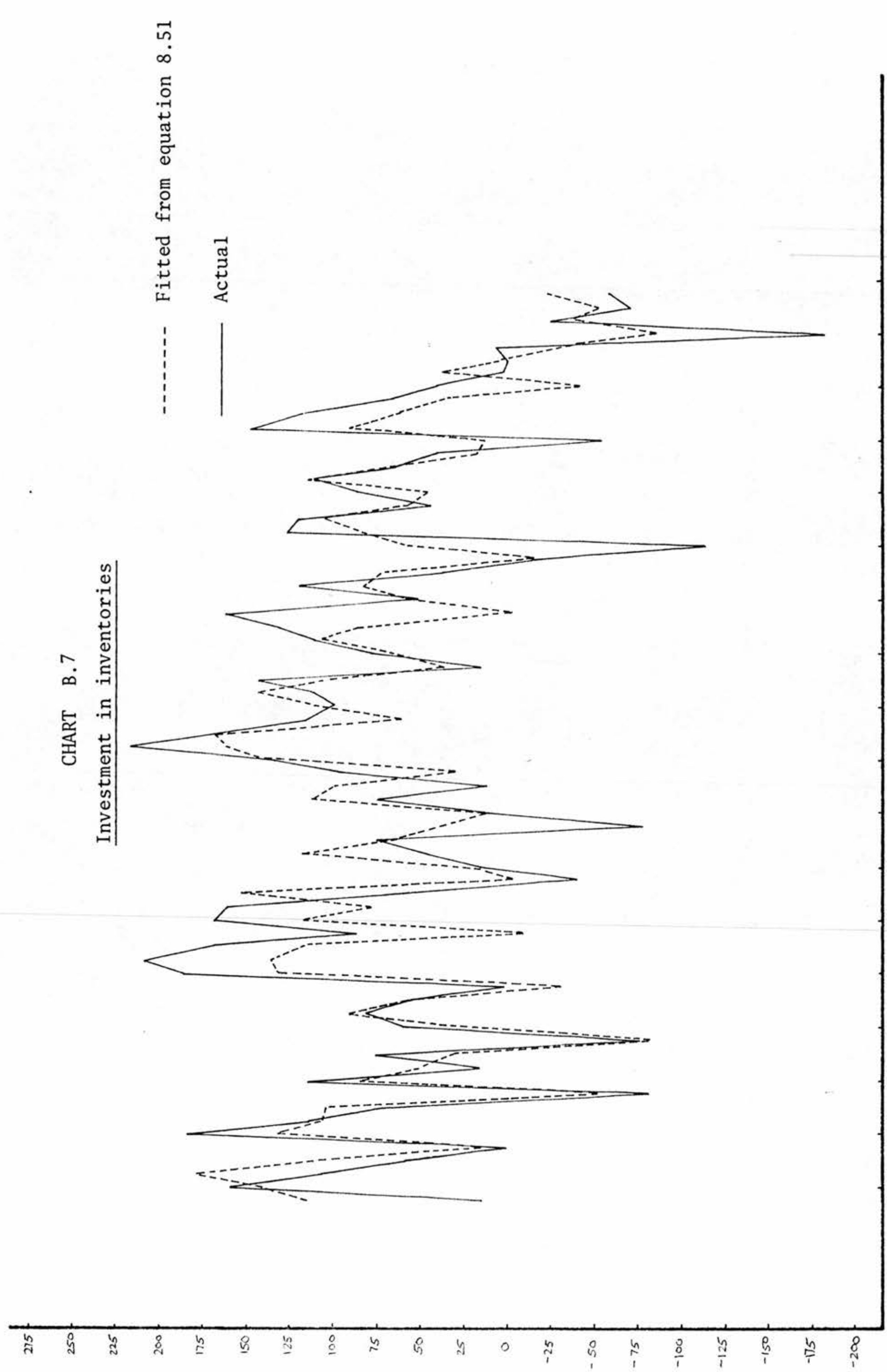
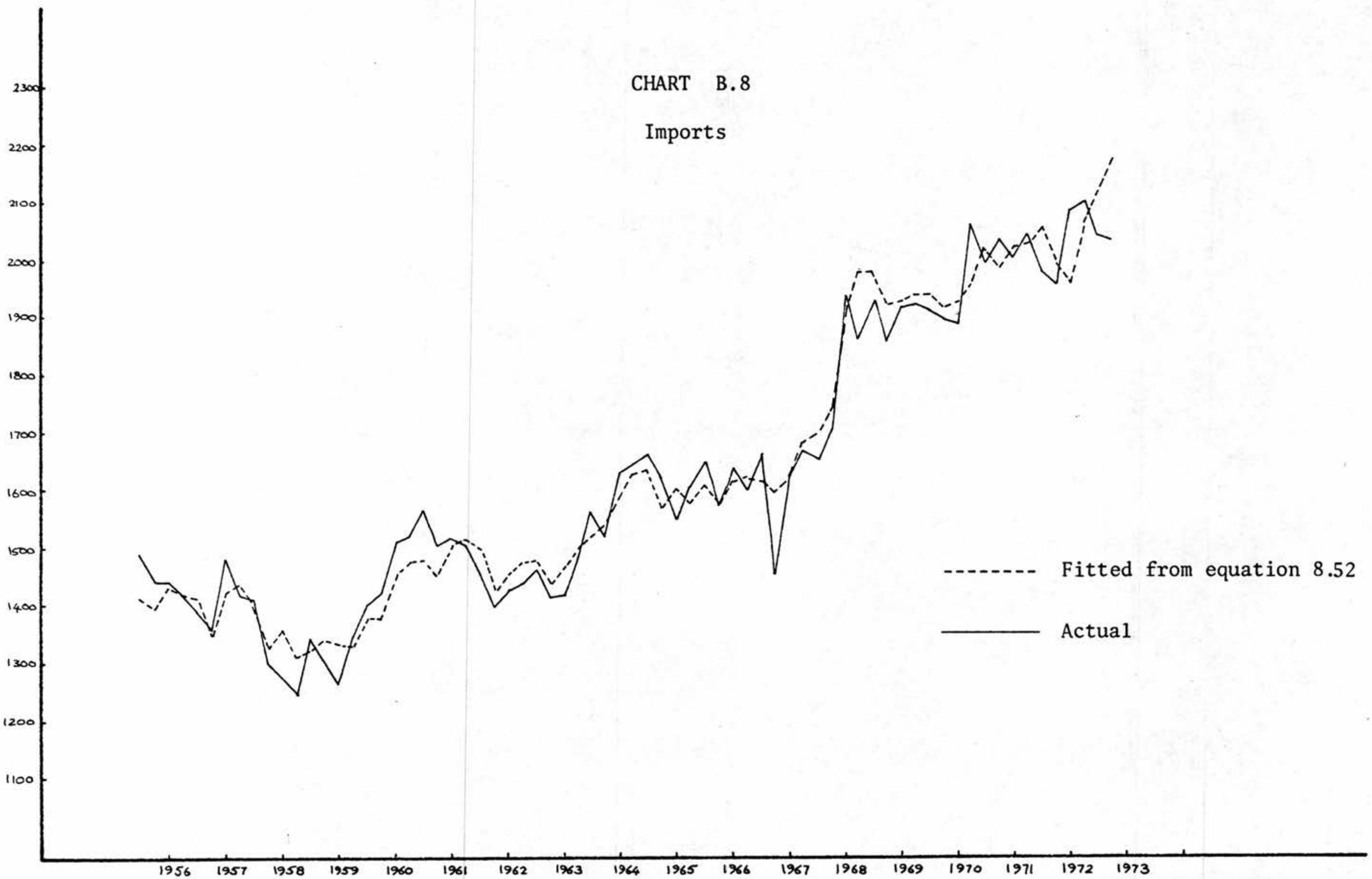


CHART B.7

Investment in inventories

CHART B.8

Imports



The equations of Model B*

The money demand function

$$R_t = \underset{(-4.23)}{-\,0.00163202}\,(M_t - M_{t-1}) + \underset{(1.72)}{0.000271395}\,Y_t +$$

$$+ \underset{(7.30)}{0.105425}\,P_t - \underset{(-4.06)}{0.000653872}\,M_{t-1} - \underset{(-4.46)}{2.16764} +$$

$$+ \underset{(0.82)}{0.140551}\,S_t^2 + \underset{(1.57)}{0.278367}\,S_t^3 + \underset{(1.40)}{0.297522}\,S_t^4 + u_{1t} \qquad (B.1)$$

$$R^2 = 0.9353 \qquad D.W. = 0.8758$$

$$R_t = \underset{(-2.62)}{-\,0.00109966}\,(M_t - M_{t-1}) + \underset{(2.61)}{0.000422191}\,Y_t +$$

$$+ \underset{(5.89)}{0.089227}\,P_t + \underset{(2.77)}{0.255921}\,(P_t - P_{t-1}) - \underset{(-4.24)}{0.000661264}\,M_{t-1} -$$

$$- \underset{(-3.34)}{1.67953} - \underset{(-0.92)}{0.185884}\,S_t^2 - \underset{(-0.11)}{0.023}\,S_t^3 + \underset{(0.14)}{0.03273}\,S_t^4 + u_{1t}$$

$$(B.2)$$

$$R^2 = 0.9404 \qquad D.W. = 1.2504$$

$$R_t = \underset{(-1.79)}{-\,0.000459324}\,(M_t - M_{t-1}) + \underset{(2.24)}{0.000536676}\,Y_t +$$

$$+ \underset{(2.85)}{0.0712482}\,P_t + \underset{(0.47)}{0.023686}\,(P_t - P_{t-1}) - \underset{(-1.69)}{0.000433074}\,M_{t-1} -$$

$$- \underset{(-1.87)}{2.0722} - \underset{(-1.03)}{0.108883}\,S_t^2 - \underset{(-0.34)}{0.033541}\,S_t^3 - \underset{(-1.42)}{0.191767}\,S_t^4 + u_{1t}$$

$$u_{1t} = 0.82542\,u_{1,t-1} + e_{1t} \qquad (B.3)$$

$$R^2 = 0.978 \qquad D.W. = 1.4561$$

*The programs used to estimate the parameters of the structural equations are the ones from the Econometric Software Package by J. P. Cooper, University of Chicago (1974 revised version, entitled ESP 74).

$$R_t = \underset{(-2.44)}{-\ 0.000526012}\ (M_t - M_{t-1}) + \underset{(2.21)}{0.000503035}\ Y_t +$$

$$+ \underset{(3.58)}{0.0772448}\ P_t - \underset{(-1.98)}{0.000475741}\ M_{t-1} - \underset{(-1.97)}{2.13347} -$$

$$- \underset{(-0.95)}{0.0793372}\ S_t^2 - \underset{(-0.11)}{0.0090958}\ S_t^3 - \underset{(-1.39)}{0.158249}\ S_t^4 + u_{1_t}$$

$$u_{1_t} = 0.82322\ u_{1,t-1} + e_{1_t} \qquad (B.4)$$

$$R^2 = 0.9777 \qquad D.W. = 1.4402$$

The money supply function

$$M_t = \underset{(7.84)}{2.37843}\ B_t - \underset{(-2.89)}{1.29548}\ S_t^D + \underset{(3.92)}{0.491526}\ Y_t - \underset{(-1.47)}{168.26}\ R_t -$$

$$- \underset{(-1.14)}{41.9218}\ R_t^D - \underset{(-2.59)}{2652.89} - \underset{(-5.01)}{232.24}\ S_t^2 - \underset{(-3.98)}{183.47}\ S_t^3 -$$

$$- \underset{(-2.92)}{204.63}\ S_t^4 + u_{2_t}$$

$$u_{2_t} = 0.92229\ u_{2,t-1} + e_{2_t} \qquad (B.5)$$

$$R^2 = 0.9921 \qquad D.W. = 2.4266$$

$$M_t = \underset{(8.26)}{2.1031}\ B_t - \underset{(-3.45)}{1.46013}\ S_t^D + \underset{(3.96)}{0.46631}\ Y_t - \underset{(-1.95)}{63.2641}\ R_t^D -$$

$$- \underset{(-2.54)}{2504.1} - \underset{(-5.17)}{234.7}\ S_t^2 - \underset{(-2.75)}{180.6}\ S_t^3 - \underset{(-3.45)}{180.9}\ S_t^4 + u_{2_t}$$

$$u_{2_t} = 0.92328\ u_{2,t-1} + e_{2_t} \qquad (B.6)$$

$$R^2 = 0.9923 \qquad D.W. = 2.214$$

The price equation

$$P_t = \underset{(-1.74)}{-\,0.00138316}\,(Y^*_{t-1} - Y_{t-1}) + \underset{(1.98)}{0.0456768}\,W_{t-1} +$$

$$+ \underset{(18.30)}{0.930711}\,P_{t-1} + \underset{(0.16)}{0.222745} + \underset{(2.78)}{0.714099}\,S^2_t +$$

$$+ \underset{(1.65)}{0.4402}\,S^3_t + \underset{(0.17)}{0.0417302}\,S^4_t + u_{3t}$$

$$u_{3t} = 0.10581\,u_{3,t-1} + e_{3t} \qquad \text{(B.7)}$$

$$R^2 = 0.9989 \qquad D.W. = 2.0107$$

$$(P_t - P_{t-1}) = \underset{(-1.74)}{-\,0.00138374}\,(Y^*_{t-1} - Y_{t-1}) + \underset{(1.98)}{0.045617}\,W_{t-1} -$$

$$- \underset{(-1.36)}{0.0691284}\,P_{t-1} + \underset{(0.15)}{0.213526} + \underset{(2.80)}{0.717162}\,S^2_t +$$

$$+ \underset{(1.66)}{0.442553}\,S^3_t + \underset{(0.18)}{0.0446748}\,S^4_t + u'_{3t}$$

$$u'_3 = 0.10631\,u'_{3,t-1} + e'_3 \qquad \text{(B.8)}$$

$$R^2 = 0.4912 \qquad D.W. = 2.0112$$

The equation relating rates of returns on bonds and equities

$$\acute{R}_t = \underset{(0.57)}{0.0383136}\,R_t + \underset{(0.64)}{0.050488}\,(P_t - P_{t-1}) - \underset{(-1.31)}{0.0000989714}\,M_t -$$

$$- \underset{(-0.89)}{0.0003109}\,(M_t - M_{t-1}) + \underset{(1.43)}{0.960987} + \underset{(0.56)}{0.085655}\,S^2_t +$$

$$+ \underset{(1.20)}{0.187717}\,S^3_t + \underset{(1.43)}{0.221717}\,S^4_t + \underset{(13.58)}{0.869344}\,\acute{R}_{t-1} + u_{4t} \qquad \text{(B.9)}$$

$$R^2 = 0.8879 \qquad D.W. = 1.4023$$

$$\acute{R}_t = \underset{(2.04)}{0.321396}\, R_t - \underset{(-0.95)}{0.052521}\, (P_t - P_{t-1}) - \underset{(-2.79)}{0.000414061}\, M_t$$

$$+ \underset{(0.39)}{0.0000957348}\, (M_t - M_{t-1}) + \underset{(2.84)}{3.82382} + \underset{(0.23)}{0.02444}\, S_t^2$$

$$+ \underset{(0.78)}{0.08751}\, S_t^3 + \underset{(1.33)}{0.135426}\, S_t^4 + \underset{(3.68)}{0.427307}\, \acute{R}_{t-1} + u_{4t}$$

$$u_{4t} = 0.82474\, u_{4,t-1} +_4 e \qquad \text{(B.10)}$$

$$R^2 = 0.9086 \qquad D.W. = 2.0014$$

$$\acute{R}_t = \underset{(1.99)}{0.311466}\, R_t - \underset{(-1.32)}{0.0633207}\, (P_t - P_{t-1}) - \underset{(-2.84)}{0.000391892}\, M_t +$$

$$+ \underset{(2.85)}{3.70887} + \underset{(0.65)}{0.05129}\, S_t^2 + \underset{(1.36)}{0.11626}\, S_t^3 + \underset{(2.14)}{0.16193}\, S_t^4$$

$$+ \underset{(3.71)}{0.429701}\, \acute{R}_{t-1} + u_{4t}$$

$$u_{4t} = 0.8216\, u_{4,t-1} + e_{4t} \qquad \text{(B.11)}$$

$$R^2 = 0.9076 \qquad D.W. = 2.0195$$

$$\acute{R}_t = \underset{(1.14)}{0.138085}\, R_t - \underset{(-2.43)}{0.000271952}\, M_t + \underset{(3.16)}{3.23048} + \underset{(0.36)}{0.028104}\, S_t^2 +$$

$$+ \underset{(1.20)}{0.10496}\, S_t^3 + \underset{(1.80)}{0.142044}\, S_t^4 + \underset{(5.24)}{0.562995}\, \acute{R}_{t-1} + u_{4t}$$

$$u_{4t} = 0.67914\, u_{4,t-1} + e_{4t} \qquad \text{(B.12)}$$

$$R^2 = 0.9065 \qquad D.W. = 1.9807$$

<u>The consumption function</u>

$$C_t = \underset{(4.16)}{0.332795}\, (Y_t - T_t) + \underset{(3.38)}{3.62037}\, P_t - \underset{(-2.60)}{30.1519}\, (P_t - P_{t-1}) -$$

$$- \underset{(-0.20)}{24.8937} + \underset{(7.51)}{499.789} S_t^2 + \underset{(9.39)}{361.724} S_t^3 + \underset{(11.05)}{508.43} S_t^4 +$$

$$+ \underset{(5.09)}{0.510421} C_{t-1} + u_{5_t} \qquad (B.13)$$

$$R^2 = 0.9926 \qquad D.W. = 2.1591$$

The investment function

$$I_t = \underset{(6.76)}{0.133682} [Y_t - (1 - d) Y_{t-1}] - \underset{(-1.51)}{6.80803} [\acute{R}_t + \tilde{b}_4' M_t] +$$

$$+ \underset{(1.82)}{0.698554} P_t + \underset{(0.04)}{0.182399} (P_t - P_{t-1}) + \underset{(1.54)}{67.6931} -$$

$$- \underset{(-0.54)}{10.008} S_t^2 - \underset{(-0.42)}{5.386} S_t^3 + \underset{(0.39)}{5.843} S_t^4 + \underset{(18.66)}{0.892575} I_{t-1} + u_{6_t}$$

$$(B.14)$$

$$R^2 = 0.9798 \qquad D.W. = 2.3580$$

$$I_t = \underset{(5.96)}{0.126644} [Y_t - (1 - d) Y_{t-1}] + \underset{(1.34)}{15.195} [(\acute{R}_t + \tilde{b}_4' M_t) -$$

$$- (1 - d)(\acute{R}_{t-1} + \tilde{b}_4' M_{t-1})] + \underset{(1.68)}{0.646082} P_t - \underset{(-0.29)}{2.38182} (P_t -$$

$$- P_{t-1}) + \underset{(0.44)}{9.67046} - \underset{(-0.32)}{6.04623} S_t^2 - \underset{(-0.50)}{6.48127} S_t^3 +$$

$$+ \underset{(0.27)}{4.92628} S_t^4 + \underset{(17.64)}{0.886316} I_{t-1} + u_{6_t} \qquad (B.15)$$

$$R^2 = 0.9807 \qquad D.W. = 2.3206$$

$$I_t = \underset{(6.86)}{0.146331} [Y_t - (1 - d) Y_{t-1}] - \underset{(-1.82)}{3.25152} [\acute{R}_t + \tilde{b}_4' M_t] +$$

$$+ \underset{(0.26)}{0.0875542} P_t + \underset{(0.40)}{1.86577} (P_t - P_{t-1}) + \underset{(1.73)}{54.4418} -$$

$$- \underset{(-0.85)}{17.1556} S_t^2 - \underset{(-0.52)}{6.51426} S_t^3 - \underset{(-0.09)}{1.69714} S_t^4 + \underset{(24.18)}{0.948805} I_{t-1} + u_{6_t}$$

$$u_{6_t} = - 0.2217 u_{6,t-1} + e_{6_t} \qquad (B.16)$$

$$R^2 = 0.9811 \qquad D.W. = 2.0698$$

$$I_t = \underset{(5.71)}{0.128186} [Y_t - (1 - d) Y_{t-1}] + \underset{(1.49)}{14.8712} [(\acute{R}_t + \tilde{b}_4^{\prime} M_t) -$$

$$- (1 - d)(R_{t-1} + \tilde{b}_4^{\prime} M_{t-1})] + \underset{(1.52)}{0.50694} P_t - \underset{(-0.64)}{3.08077} (P_t -$$

$$- P_{t-1}) + \underset{(0.37)}{7.89579} - \underset{(-0.24)}{4.88964} S_t^2 - \underset{(-0.34)}{4.42569} S_t^3 +$$

$$+ \underset{(0.24)}{4.90879} S_t^4 + \underset{(21.82)}{0.908871} I_{t-1} + u_{6_t}$$

$$u_{6_t} = - 0.21383 u_{6,t-1} + e_{6_t} \qquad \text{(B.17)}$$

$$R^2 = 0.9793 \qquad D.W. = 2.0568$$

$$I_t = \underset{(5.98)}{0.12986} [Y_t - (1 - d) Y_{t-1}] - \underset{(-2.02)}{7.29323} \acute{R}_t +$$

$$+ \underset{(0.69)}{7.06757} (\tilde{b}_4^{\prime} M_t) - \underset{(-0.52)}{0.328983} P_t + \underset{(0.36)}{1.61748} (P_t - P_{t-1}) +$$

$$+ \underset{(1.97)}{72.3318} - \underset{(-0.22)}{4.43487} S_t^2 - \underset{(-0.09)}{1.13344} S_t^3 + \underset{(0.43)}{8.75335} S_t^4 +$$

$$+ \underset{(24.12)}{0.93378} I_{t-1} + u_{6_t}$$

$$u_{6_t} = - 0.26521 u_{6,t-} + e_t \qquad \text{(B.18)}$$

$$R^2 = 0.9807 \qquad D.W. = 2.1579$$

$$I_t = \underset{(6.34)}{0.130193} [Y_t - (1 - d) Y_{t-1}] - \underset{(-2.55)}{8.06842} \acute{R}_t +$$

$$+ \underset{(2.62)}{75.1705} - \underset{(-0.16)}{3.07244} S_t^2 + \underset{(0.01)}{0.07796} S_t^3 + \underset{(0.53)}{10.3036} S_t^4 +$$

$$+ \underset{(47.84)}{0.938912} I_{t-1} + u_{6_t}$$

$$u_{6_t} = - 0.26544 u_{6,t-1} + e_{6_t} \qquad \text{(B.19)}$$

$$R^2 = 0.9807 \qquad D.W. = 2.1361$$

The inventory function

$$H_t = \underset{(5.11)}{0.231951}\, Y_t - \underset{(-1.32)}{0.0704302}\, (Y_t - Y_{t-1}) - \underset{(-7.22)}{7.8258}\, P_t +$$

$$+ \underset{(2.94)}{26.4053}\, (P_{t-1} - P_{t-2}) - \underset{(-3.19)}{0.0914977}\, S_{t-1} - \underset{(-2.90)}{656.263} +$$

$$+ \underset{(0.97)}{40.7546}\, S_t^2 - \underset{(-0.24)}{6.40007}\, S_t^3 - \underset{(-3.11)}{125.242}\, S_t^4 + u_{7_t} \qquad (B.20)$$

$$R^2 = 0.6536 \qquad D.W. = 1.5780$$

$$H_t = \underset{(4.30)}{0.235705}\, Y_t - \underset{(-1.38)}{0.0711378}\, (Y_t - Y_{t-1}) - \underset{(-6.10)}{7.80933}\, P_t +$$

$$+ \underset{(-2.87)}{26.1934}\, (P_{t-1} - P_{t-2}) - \underset{(-2.62)}{0.094789}\, S_{t-1} - \underset{(-2.52)}{677.873} +$$

$$+ \underset{(1.05)}{40.662}\, S_t^2 - \underset{(-0.28)}{6.81403}\, S_t^3 - \underset{(-3.45)}{126.106}\, S_t^4 + u_{7_t}$$

$$u_{7_t} = 0.21057\, u_{7,t-1} + e_{7_t} \qquad (8.21)$$

$$R^2 = 0.6571 \qquad D.W. = 2.0172$$

The import function

$$X_t = \underset{(5.00)}{0.129906}\, Y_t - \underset{(-1.46)}{1.96628}\, P_t + \underset{(4.65)}{365.188}\, E_t^R - \underset{(-1.50)}{3.29606}\, D_t^1 -$$

$$- \underset{(-2.01)}{54.4458}\, D_t^2 - \underset{(-4.38)}{766.059} - \underset{(-2.22)}{44.7197}\, S_t^2 - \underset{(-1.88)}{36.5844}\, S_t^3 -$$

$$- \underset{(-5.19)}{114.72}\, S_t^4 + \underset{(3.56)}{0.36743}\, X_{t-1} + u_{8_t} \qquad (B.22)$$

$$R^2 = 0.9589 \qquad D.W. = 1.7884$$

A P P E N D I X   C

Calculation of characteristic roots.

To calculate the roots of A^* , we have used subroutines from the IBM 360 Scientific Subroutine Package. The non symmetric matrix A^* is reduced to an upper almost-triangular form by subroutine HSBG and the characteristic roots of the Hessenberg form (equal to the roots of the original matrix) are calculated by subroutine ATEIG.

Calculation of dynamic multipliers.

Calculations of dynamic multipliers are performed by the following program I which is based on the theoretical developments of chapter 9. Matrices A', $RA'+B$, $C-RB$, $-RC$, D, $E-RD$, $-RE$ are read in as input and the program computes the matrices A_1, A_2, A_3, B_0, B_1, B_2 . Then, given the recursive relationship $C_1(t) = A_1C_1(t-1) + A_2C_1(t-2) + A_3C_1(t-3)$ the sequence of matrices B_0, $B_1 + C_1(1)B_0$, $B_2C_1(1)B_1 + C_1(2)B_0$, $C_1(1) B_2 + C_1(2)B_1 + C_1(3)B_0$, ... $C_1(m)B_2 + C_1(m + 1)B_1 + C_1(m + 2)B_0$ (Cf p.) is computed up to the value $m = 78$.

The program makes use of subroutines MINV (calculation of the inverse of a given matrix) and MPRD (calculation of the product of two matrices) of the IBM 360 Scientific Subroutine Package. Also subroutine MATAB1, written and kindly supplied by R. Cressy, is employed, which prints out the resultant matrices in segmented form (5 column sections).

Verification of the results can be attained by computing directly the sequence of matrices B^*, A^*B^*, $A^{*2}B^*$, . . . and performing the relevant calculations manually. Program II below calculates and prints out this sequence.

PROGRAM I

```
      REAL E1(10,10),E2(10,10),E3(10,10)
      REAL E4(10,10),E5(10,9),E6(10,9)
      REAL E7(10,9),V1(10),V2(10)
      REAL A1(10,10),A2(10,10),A3(10,10)
      REAL B(10,9),B1(10,9),B2(10,9)
      REAL C(10,10,84),C17(10,10),C1(10,10)
      REAL C18(10,10),C19(10,10)
      REAL C2(10,10),C20(10,10),C21(10,10)
      REAL C22(10,10),O(10,10)
      REAL C23(10,9),C24(10,9),C25(10,9),D(10,9)
      REAL C26(10,9),C27(10,9),C28(10,9)
      REAL D1(10,9),D2(10,9)
      READ(5,1)((E1(I,J),J=1,10),I=1,10)
      READ(5,1)((E2(I,J),J=1,10),I=1,10)
      READ(5,1)((E3(I,J),J=1,10),I=1,10)
      READ(5,1)((E4(I,J),J=1,10),I=1,10)
      READ(5,1)((E5(I,J),J=1,9),I=1,10)
      READ(5,1)((E6(I,J),J=1,9),I=1,10)
      READ(5,1)((E7(I,J),J=1,9),I=1,10)
    1 FORMAT(5F15.0)
      CALL MATAB1(E1,10,10,6HD      ,10,2)
      CALL MATAB1(E2,10,10,6HD      ,10,2)
      CALL MATAB1(E3,10,10,6HD      ,10,2)
      CALL MATAB1(E4,10,10,6HD      ,10,2)
      CALL MATAB1(E5,10,9,6HD      ,10,2)
      CALL MATAB1(E6,10,9,6HD      ,10,2)
      CALL MATAB1(E7,10,9,6HD      ,10,2)
      CALL MINV(E1,10,DET,V1,V2)
      CALL MATAB1(E1,10,10,6HD      ,10,2)
      WRITE(6,28) DET
   28 FORMAT(E17.7)
      CALL MPRD(E1,E2,A1,10,10,0,0,10)
      CALL MATAB1(A1,10,10,6HD      ,10,2)
      CALL MPRD(E1,E3,A2,10,10,0,0,10)
      CALL MATAB1(A2,10,10,6HD      ,10,2)
      CALL MPRD(E1,E4,A3,10,10,0,0,10)
      CALL MATAB1(A3,10,10,6HD      ,10,2)
      CALL MPRD(E1,E5,B,10,10,0,0,9)
      CALL MATAB1(B,10,9,6HD      ,10,2)
      CALL MPRD(E1,E6,B1,10,10,0,0,9)
      CALL MATAB1(B1,10,9,6HD      ,10,2)
      CALL MPRD(E1,E7,B2,10,10,0,0,9)
      CALL MATAB1(B2,10,9,6HD      ,10,2)
      DO 29 I=1,10
      DO 29 J=1,9
      C26(I,J)=0.0
      DO 29 K=1,10
   29 C26(I,J)=C26(I,J)+A1(I,K)*B(K,J)
      DO 30 I=1,10
      DO 30 J=1,9
   30 D1(I,J)=C26(I,J)+B1(I,J)
      CALL MATAB1(D1,10,9,6HD      ,10,2)
      DO 10 I=1,10
      DO 10 J=1,10
   10 C(I,J,1)=A1(I,J)
      DO 11 I=1,10
      DO 11 J=1,10
      C17(I,J)=0.0
```

```
      DO 11 K=1,10
   11 C17(I,J)=C17(I,J)+C(I,K,1)*C(K,J,1)
      DO 12 I=1,10
      DO 12 J=1,10
   12 C1(I,J)=C17(I,J)+A2(I,J)
      DO 31 I=1,10
      DO 31 J=1,9
      C27(I,J)=0.0
      DO 31 K=1,10
   31 C27(I,J)=C27(I,J)+C1(I,K)*B(K,J)
      DO 32 I=1,10
      DO 32 J=1,9
      C28(I,J)=0.0
      DO 32 K=1,10
   32 C28(I,J)=C28(I,J)+C(I,K,1)*B1(K,J)
      DO 33 I=1,10
      DO 33 J=1,9
   33 D2(I,J)=B2(I,J)+C28(I,J)+C27(I,J)
      CALL MATAB1(D2,10,9,6HD     ,10,2)
      DO 13 I=1,10
      DO 13 J=1,10
   13 C(I,J,2)=C1(I,J)
      DO 14 I=1,10
      DO 14 J=1,10
      C18(I,J)=0.0
      DO 14 K=1,10
   14 C18(I,J)=C18(I,J)+C(I,K,1)*C(K,J,2)
      DO 15 I=1,10
      DO 15 J=1,10
      C19(I,J)=0.0
      DO 15 K=1,10
   15 C19(I,J)=C19(I,J)+A2(I,K)*C(K,J,1)
      DO 16 I=1,10
      DO 16 J=1,10
   16 C2(I,J)=C18(I,J)+C19(I,J)+A3(I,J)
      DO 17 I=1,10
      DO 17 J=1,10
   17 C(I,J,3)=C2(I,J)
      DO 18 L=4,81
      M1=L-1
      M2=L-2
      M3=L-3
      DO 19 I=1,10
      DO 19 J=1,10
      C20(I,J)=0.0
      DO 19 K=1,10
   19 C20(I,J)=C20(I,J)+C(I,K,1)*C(K,J,M1)
      DO 20 I=1,10
      DO 20 J=1,10
      C21(I,J)=0.0
      DO 20 K=1,10
   20 C21(I,J)=C21(I,J)+A2(I,K)*C(K,J,M2)
      DO 21 I=1,10
      DO 21 J=1,10
      C22(I,J)=0.0
      DO 21 K=1,10
   21 C22(I,J)=C22(I,J)+A3(I,K)*C(K,J,M3)
```

```
            DO 22 I=1,10
            DO 22 J=1,10
      22    D(I,J)=C20(I,J)+C21(I,J)+C22(I,J)
            DO 23 I=1,10
            DO 23 J=1,10
      23    C(I,J,L)=D(I,J)
            DO 24 I=1,10
            DO 24 J=1,9
            C23(I,J)=0.0
            DO 24 K=1,10
      24    C23(I,J)=C23(I,J)+C(I,K,M3)*B2(K,J)
            DO 25 I=1,10
            DO 25 J=1,9
            C24(I,J)=0.0
            DO 25 K=1,10
      25    C24(I,J)=C24(I,J)+C(I,K,M2)*B1(K,J)
            DO 26 I=1,10
            DO 26 J=1,9
            C25(I,J)=0.0
            DO 26 K=1,10
      26    C25(I,J)=C25(I,J)+ C(I,K,M1)*B(K,J)
            DO 27 I=1,10
            DO 27 J=1,9
      27    D(I,J)=C23(I,J)+C24(I,J)+C25(I,J)
            CALL MATAB1(D,10,9,6HD      ,10,2)
      18    CONTINUE
            STOP
            END
```

```
          SUBROUTINE MATAB1(X,N,M,KH,ND,IFO)
C IFO IS THE OUTPUT FORMAT DECISION
C VARIABLE.  VALUES 1 AND ANY OTHER NUMBER
C GIVE TWO POSSIBLE FORMATS .
          DIMENSION X(ND,M)
          IF (M.GT.1) GO TO 2Ø
          WRITE (6,1)
    1     FORMAT ('1',////)
          DO 1Ø I = 1,N,5
          J = MINØ (I + 4,N)
          GOTO(3,4),IFO
   3      WRITE(6,5)KH,(K,K=I,J)
          GOTO(8,9),IFO
   4      WRITE(6,6) KH,(K,K=I,J)
   7      GOTO(8,9),IFO
   8      WRITE(6,15)(X(K,1),K=I,J)
          GOTO10
   9      WRITE(6,16) (X(K,1),K=I,J)
   1Ø     CONTINUE
          RETURN
    2Ø    DO 45 K=1,M,5
          WRITE (6,25)
   25     FORMAT (1H1,1HØ,/)
          L= MINØ (K+4,M)
          GOTO(29,3Ø),IFO
   29     WRITE(6,5)KH,(J,J=K,L)
          GOTO39
   3Ø     WRITE(6,6)KH,(J,J=K,L)
    39    DO 45 I=1,N
          IF(I.EQ.26.OR.I.EQ.51.OR.I.EQ.76.OR.I.EQ.1Ø1)
         XGOTO(43,44),IFO
          GOTO(48,49),IFO
   43     WRITE(6,61)KH,(J,J=K,L)
   48     WRITE(6,5Ø)I,(X(I,J),J=K,L)
          GOTO45
   44     WRITE(6,62)KH,(J,J=K,L)
   49     WRITE(6,51)I,(X(I,J),J=K,L)
   45     CONTINUE
    5     FORMAT (/A8,5I11)
   6      FORMAT(/A8,5I16)
    15    FORMAT (1ØX,5F11.1)
   16     FORMAT(1ØX,5F16.6)
    5Ø    FORMAT (/I7,4X,5F11.7)
   51     FORMAT (/I7,4X,5F16.9)
   61     FORMAT (1H1,/////,A8,5I11)
   62     FORMAT (1H1,/////,A8,5I16)
          RETURN
          END
```

PROGRAM II

```
      REAL  A(30,30),B(30,30),Q(30,27),V1(30)
      REAL  V2(30),R1(30,30),R2(30,27)
      REAL  C(30,30,81),D(30,27)
      READ(5,1)((A(I,J),J=1,30),I=1,30)
      READ(5,1)((B(I,J),J=1,30),I=1,30)
      READ(5,1)((D(I,J),J=1,27),I=1,30)
    1 FORMAT(8F10.0)
      CALL MATAB1(A,30,30,6HD          ,30,1)
      CALL MATAB1(B,30,30,6HD          ,30,1)
      CALL MATAB1(D,30,27,6HD          ,30,1)
      CALL MINV(A,30,DET,V1,V2)
      CALL MATAB1(A,30,30,6HD          ,30,1)
      WRITE(6,21) DET
   21 FORMAT(E17.7)
      CALL MPRD(A,B,R1,30,30,0,0,30)
      CALL MATAB1(R1,30,30,6HD          ,30,1)
      CALL MPRD(A,D,R2,30,30,0,0,27)
      CALL MATAB1(R2,30,27,6HD          ,30,2)
      WRITE(7,26)((R1(I,J),I=1,30),J=1,30)
   26 FORMAT(5(F15.10))
      DO 10 I=1,30
      DO 10 J=1,30
   10 C(I,J,1)=R1(I,J)
      DO 20 L=2,81
      M=L-1
      DO 15 I=1,30
      DO 15 J=1,30
      C(I,J,L)=0.0
      DO 15 K=1,30
   15 C(I,J,L)=C(I,J,L)+C(I,K,M)*C(K,J,1)
      DO 5 I=1,30
      DO 5 J=1,27
      D(I,J)=0.0
      DO 5 K=1,30
    5 D(I,J)=D(I,J)+C(I,K,M)*R2(K,J)
      CALL MATAB1(D,30,27,6HD          ,30,2)
   20 CONTINUE
      STOP
      END
```

Input matrices for the calculation of multipliers.

The correspondence between numbers and variables is the one implied by the listing of variables in page 351, chapter 9.

$A' = E_1$

	1	2	3	4	5
1	1.0000000	0.0000000	0.0000000	0.0000000	-0.4768000
2	0.0005672	1.0000000	0.0000000	-0.0778209	-0.0005428
3	0.0004073	-0.3276809	1.0000000	0.0599152	0.0000000
4	0.0000000	0.0000000	0.0000000	1.0000000	0.0000000
5	0.0000000	0.0000000	0.0000000	0.0000000	1.0000000
6	0.0000000	0.0000000	0.0000000	24.6957855	-0.3441949
7	0.0000000	0.0000000	7.8470192	0.0000000	-0.1269420
8	0.0000000	0.0000000	0.0000000	7.8610697	-0.1436230
9	0.0000000	0.0000000	0.0000000	2.0444193	-0.1314909
10	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

	6	7	8	9	10
1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
5	-1.0000000	-1.0000000	-1.0000000	1.0000000	0.0000000
6	1.0000000	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	1.0000000	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	1.0000000	0.0000000	0.0000000
9	0.0000000	0.0000000	0.0000000	1.0000000	0.0000000
10	0.0000000	0.0000000	-1.0000000	0.0000000	1.0000000

$RA' + B = E_2$

	1	2	3	4	5
1	0.9237000	0.0000000	0.0000000	0.0000000	-0.4404200
2	0.0005301	0.8207000	0.0000000	-0.0638676	-0.0004455
3	0.0003372	-0.2712871	1.2513571	0.1095189	0.0000000
4	0.0000000	0.0000000	0.0000000	1.0365105	0.0013831
5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000	28.2427979	0.0000000
7	0.0000000	0.0000000	-2.1304998	0.0000000	-0.0912700
8	0.0000000	0.0000000	0.0000000	30.4775238	0.0546164
9	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

	6	7	8	9	10
1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
6	0.4975040	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.6688250	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	0.1992000	0.0000000	-0.0880949
9	0.0000000	0.0000000	0.0000000	0.3664089	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000	1.0000000

$-RB + C = E_3$

	1	2	3	4	5
1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
2	-0.0000530	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	-0.3505809	-0.0496038	0.0000000
4	0.0000000	0.0000000	0.0000000	-0.0984492	-0.0001463
5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000000	0.0000000	0.0000000	-0.0341386
8	0.0000000	0.0000000	0.0000000	-34.6707764	-0.0165786
9	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

	6	7	8	9	10
1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.2553090	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	0.0000000	0.0000000	0.0175485
9	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

$-RC = E_4$

	1	2	3	4	5
1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	0.0000000	5.7591906	0.0000000
9	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

	6	7	8	9	10
1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
9	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

$D = E_5$

	1	2	3	4	5
1	2.0655994	-1.4490995	-62.1772919	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
9	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

	6	7	8	9
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	1.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	-0.3441949	0.0000000	0.0000000
7	0.0000000	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	0.0000000	0.0000000
9	0.0000000	0.0000000	366.9648438	-3.3337898
10	0.0000000	0.0000000	0.0000000	0.0000000

$E - RD = E_6$

	1	2	3	4	5
1	-1.9079943	1.3385334	57.4331665	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	-0.0013831	0.0456771
5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
9	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

	6	7	8	9
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	0.0000000	0.0000000
9	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000

$-RE = E_7$

	1	2	3	4	5
1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0001463	-0.0048326
5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
9	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

	6	7	8	9
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000000	0.0000000	0.0000000	0.0000000
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000000	0.0000000	0.0000000
8	0.0000000	0.0000000	0.0000000	0.0000000
9	0.0000000	0.0000000	0.0000000	0.0000000
10	0.0000000	0.0000000	0.0000000	0.0000000

Matrices of impact and interim multipliers. Row numbers refer to endogenous variables and column numbers to exogenous variables. The correspondence between numbers and variables is the one implied by the listing of variables in page 351, chapter 9.

Lag : 0

	1	2	3	4	5
1	2,074482918	-1,455331802	-62,444717407	0,000000000	0,000000000
2	-0,001166612	0,000818424	0,035116583	0,000000000	0,000000000
3	-0,001227242	0,000860959	0,036941610	0,000000000	0,000000000
4	0,000000000	0,000000000	0,000000000	0,000000000	0,000000000
5	0,018636763	-0,013074424	-0,560991347	0,000000000	0,000000000
6	0,006414685	-0,004500154	-0,193090558	0,000000000	0,000000000
7	0,011995990	-0,008415662	-0,361095309	0,000000000	0,000000000
8	0,002676669	-0,001877789	-0,080571294	0,000000000	0,000000000
9	0,002450567	-0,001719169	-0,073765278	0,000000000	0,000000000
10	0,002676669	-0,001877789	-0,080571294	0,000000000	0,000000000

	6	7	8	9
1	0,924196601	-0,318103552	-339,147460938	3,081076622
2	0,000527850	-0,000181683	-0,193702340	0,001759741
3	-0,000203430	0,000070023	0,074653149	-0,000678207
4	0,000000000	0,000000000	0,000000000	0,000000000
5	1,938330650	-0,667163253	-711,299072266	6,461986542
6	0,667164207	-0,573829353	-244,825805664	2,224184990
7	0,247652352	-0,085240662	-90,879699707	0,825620890
8	0,278388917	-0,095820069	-102,158935547	0,928090096
9	0,254872978	-0,087725997	273,435302734	-2,484096527
10	0,278388917	-0,095820069	-102,158935547	0,928090096

Lag : 1

	1	2	3	4	5
1	0,014317513	-0,010044098	-0,431091309	0,038657229	-1,276670456
2	0,000144219	-0,000101176	-0,004341185	-0,000085554	0,002825469
3	-0,000479801	0,000336599	0,014442660	0,000039089	-0,001290944
4	0,000025776	-0,000018083	-0,000775901	-0,001383090	0,045677099
5	0,030042838	-0,021076236	-0,904328108	0,081076324	-2,677577972
6	0,012895368	-0,009046603	-0,388167918	0,062062591	-2,049641609
7	0,016515605	-0,011586342	-0,497141957	0,009985279	-0,329767883
8	0,005427461	-0,003807586	-0,163374245	0,022516996	-0,743632853
9	0,004795570	-0,003364281	-0,144353092	0,013488431	-0,445460796
10	0,008104131	-0,005685374	-0,243945599	0,022516996	-0,743632853

	6	7	8	9
1	0,262406051	-0,248576462	-220,560134888	2,003739357
2	0,000418212	-0,000234334	-0,224443257	0,002039016
3	-0,000216568	0,000109382	0,106827855	-0,000970508
4	0,002680885	-0,000922747	-0,983790576	0,008937508
5	0,550345957	-0,521342576	-462,583496094	4,202464104
6	0,455136657	-0,442137957	-256,725341797	2,332287788
7	0,060719837	-0,063307106	-55,581192017	0,504942060
8	0,194762468	-0,114706993	-108,902938843	0,989357293
9	0,160272300	-0,098808885	41,374771118	-0,375880480
10	0,473151505	-0,210527062	-211,061920166	1,917447090

Lag : 2

	1	2	3	4	5
1	0,017807007	-0,012492180	-0,536010742	-0,012110468	0,399953067
2	0,000016198	-0,000011364	-0,000487577	-0,000095868	0,003166086
3	-0,000207494	0,000145566	0,006245848	-0,000022389	0,000739392
4	0,000065542	-0,000045980	-0,001972908	-0,001175120	0,038808819
5	0,037348278	-0,026201345	-1,124234200	-0,025399577	0,838832021
6	0,018379979	-0,012894277	-0,553260267	0,012092158	-0,399347603
7	0,018121962	-0,012713272	-0,545493245	-0,003853296	0,127256691
8	0,007380497	-0,005177706	-0,222162783	-0,029633533	0,978657305
9	0,006534107	-0,004583936	-0,196685731	0,004004899	-0,132263422
10	0,015484646	-0,010863096	-0,466108799	-0,007116538	0,235024154

	6	7	8	9
1	0,197675049	-0,191706359	-153,354598999	1,393193245
2	0,000383262	-0,000248499	-0,222881973	0,002024832
3	-0,000081068	0,000082325	0,068893611	-0,000625882
4	0,003256305	-0,001579873	-1,555416107	0,014130604
5	0,414583623	-0,402067542	-321,632324219	2,921955109
6	0,364429235	-0,345399976	-227,799179077	2,069499969
7	0,041162901	-0,045663659	-35,470779419	0,322243929
8	0,115574837	-0,096847296	-82,314880371	0,747811854
9	0,106581748	-0,085842609	-23,951705933	0,217595816
10	0,588726580	-0,307374716	-293,376708984	2,665258408

Lag : 3

	1	2	3	4	5
1	0,019485474	-0,013669968	-0,586608887	0,009448998	-0,312056363
2	0,000021051	-0,000014768	-0,000633646	-0,000083234	0,002748849
3	-0,000091717	0,000064343	0,002760813	-0,000043374	0,001432452
4	0,000112657	-0,000079034	-0,003391122	-0,001128851	0,037280779
5	0,040875971	-0,028675966	-1,230421066	0,019817356	-0,654476345
6	0,022282373	-0,015631974	-0,670727313	0,007526278	-0,248558700
7	0,018253263	-0,012805387	-0,549444199	0,002426262	-0,080128133
8	0,007879052	-0,005527463	-0,237169921	0,016246021	-0,536531866
9	0,007538669	-0,005288679	-0,226924062	0,006381057	-0,210737050
10	0,023363695	-0,016390555	-0,703276277	0,009129483	-0,301505744

	6	7	8	9
1	0,101461887	-0,130297124	-93,423034668	0,848730803
2	0,000351197	-0,000244509	-0,210542977	0,001912734
3	0,000018621	0,000034551	0,018411428	-0,000167263
4	0,003604132	-0,002026518	-1,892507553	0,017193001
5	0,212794483	-0,273273289	-195,937210083	1,780040741
6	0,257507980	-0,260470867	-177,963943481	1,616759300
7	0,013445728	-0,027345832	-17,930892944	0,162898064
8	0,001506247	-0,048700381	-30,713592529	0,279025912
9	0,059664603	-0,063243270	-30,670928955	0,278638422
10	0,590233028	-0,356075287	-324,090332031	2,944286346

Lag : 4

	1	2	3	4	5
1	0,019963264	-0,014003754	-0,600936890	0,002879608	-0,095100403
2	0,000025223	-0,000017694	-0,000759255	-0,000077376	0,002555360
3	-0,000041612	0,000029193	0,001252595	-0,000051220	0,001691553
4	0,000161387	-0,000113220	-0,004857980	-0,001023249	0,033793263
5	0,041874290	-0,029376268	-1,260470390	0,006039247	-0,199447930
6	0,024694432	-0,017324246	-0,743340492	-0,000788915	0,026054118
7	0,017666601	-0,012393910	-0,531793594	0,000958330	-0,031649269
8	0,007451280	-0,005227376	-0,224293828	0,011093952	-0,366382420
9	0,007938366	-0,005569078	-0,238955379	0,005224124	-0,172528744
10	0,030814923	-0,021617927	-0,927571297	0,020223487	-0,667889059

	6	7	8	9
1	0,053213052	-0,083138764	-53,758346558	0,488381207
2	0,000320898	-0,000232095	-0,194903135	0,001770650
3	0,000088717	-0,000013344	-0,025808841	0,000234466
4	0,003648788	-0,002264096	-2,032409668	0,018463966
5	0,111600637	-0,174367130	-112,746612549	1,024275780
6	0,178204477	-0,190922320	-130,602264404	1,186490059
7	-0,000641551	-0,013383918	-6,334558487	0,057547808
8	-0,036885146	-0,011533037	2,281859398	-0,020730078
9	0,029076412	-0,041471776	-21,908126831	0,199030161
10	0,553347826	-0,367608428	-321,808349609	2,923555374

Lag : 5

	1	2	3	4	5
1	0,019616127	-0,013762474	-0,590438843	-0,000082935	0,002738595
2	0,000028692	-0,000020129	-0,000863675	-0,000073326	0,002421635
3	-0,000018994	0,000013326	0,000571765	-0,000050428	0,001665416
4	0,000208123	-0,000146006	-0,006264798	-0,000944019	0,031176653
5	0,041144252	-0,028864503	-1,238495827	-0,000174253	0,005754370
6	0,025865495	-0,018145580	-0,778589249	-0,006038513	0,199424207
7	0,016719460	-0,011729449	-0,503281593	0,000515401	-0,017021090
8	0,006452370	-0,004526597	-0,194225132	0,009170130	-0,302847624
9	0,007893246	-0,005537424	-0,237597406	0,003821209	-0,126197100
10	0,037267268	-0,026144415	-1,121798515	0,029393606	-0,970734715

	6	7	8	9
1	0,017432280	-0,047361739	-26,094345093	0,237059593
2	0,000289687	-0,000215177	-0,177719295	0,001614538
3	0,000131312	-0,000051834	-0,057642557	0,000523669
4	0,003550397	-0,002348426	-2,047564507	0,018601641
5	0,036556538	-0,099330962	-54,726257324	0,497175813
6	0,116612136	-0,135121942	-90,646316528	0,823500033
7	-0,011025202	-0,002863682	1,724884987	-0,015670221
8	-0,060827449	0,015198737	23,157669067	-0,210381925
9	0,008202218	-0,023455542	-11,037266731	0,100270629
10	0,492520213	-0,352409661	-298,650878906	2,713172913

Lag : 6

	1	2	3	4	5
1	0,018739700	-0,013146341	-0,564071655	-0,003402877	0,112380445
2	0,000031475	-0,000022081	-0,000947449	-0,000070344	0,002323140
3	-0,000007898	0,000005541	0,000237759	-0,000046920	0,001549555
4	0,000250612	-0,000175814	-0,007543780	-0,000878872	0,029025156
5	0,039305329	-0,027574241	-1,183141708	-0,007137239	0,235710382
6	0,026085615	-0,018300023	-0,785217285	-0,010418076	0,344060719
7	0,015600085	-0,010943960	-0,469584465	-0,000031271	0,001032796
8	0,005167458	-0,003625177	-0,155546129	0,005570538	-0,183969378
9	0,007548101	-0,005295292	-0,227205753	0,002258421	-0,074585140
10	0,042434871	-0,029769599	-1,277344704	0,034964159	-1,154704094

	6	7	8	9
1	-0,005953949	-0,021512918	-7,375895500	0,067008018
2	0,000258814	-0,000196134	-0,160093904	0,001454416
3	0,000154561	-0,000078985	-0,077838540	0,000707145
4	0,003355032	-0,002323139	-1,981425285	0,018000793
5	-0,012492400	-0,045117714	-15,467758179	0,140521526
6	0,071132779	-0,091707110	-59,316665649	0,538878620
7	-0,017762329	0,004689012	7,150255203	-0,064958572
8	-0,071358800	0,032122806	34,671371460	-0,314981580
9	-0,005496342	-0,009777408	-2,027149200	0,018416218
10	0,421160936	-0,320286691	-263,979492188	2,398190498

Lag : 7

	1	2	3	4	5
1	0,0175447	-0,0123084	-0,5281525	-0,0056824	0,1876622
2	0,0000336	-0,0000236	-0,0010119	-0,0000679	0,0022421
3	-0,0000017	0,0000012	0,0000508	-0,0000429	0,0014153
4	0,0002876	-0,0002018	-0,0086576	-0,0008279	0,0273407
5	0,0368007	-0,0258170	-1,1077471	-0,0119181	0,3936013
6	0,0256195	-0,0179730	-0,7711811	-0,0136622	0,4511978
7	0,0144119	-0,0101104	-0,4338188	-0,0003086	0,0101926
8	0,0037858	-0,0026559	-0,1139533	0,0030055	-0,0992578
9	0,0070166	-0,0049224	-0,2112074	0,0009529	-0,0314694
10	0,0462207	-0,0324255	-1,3913040	0,0379697	-1,2539606

	6	7	8	9
1	-0,021039594	-0,003460433	5,018445969	-0,045592874
2	0,000229258	-0,000176486	-0,142789483	0,001297210
3	0,000164112	-0,000095780	-0,088743448	0,000806214
4	0,003105368	-0,002224624	-1,865570068	0,016948294
5	-0,044130873	-0,007256627	10,527123451	-0,095636487
6	0,038264636	-0,058795147	-35,775863647	0,325015843
7	-0,022021394	0,009912524	10,701116562	-0,097217262
8	-0,074539602	0,041637510	40,057373047	-0,363912165
9	-0,014165428	0,000011368	4,455476761	-0,040476982
10	0,346621215	-0,278649390	-223,922073364	2,034277916

Lag : 8

	1	2	3	4	5
1	0,0161848	-0,0113555	-0,4872589	-0,0072527	0,2395223
2	0,0000352	-0,0000247	-0,0010588	-0,0000658	0,0021716
3	0,0000024	-0,0000017	-0,0000713	-0,0000392	0,0012941
4	0,0003186	-0,0002235	-0,0095900	-0,0007870	0,0259913
5	0,0339518	-0,0238186	-1,0219984	-0,0152115	0,5023663
6	0,0246870	-0,0173189	-0,7431135	-0,0159783	0,5276878
7	0,0132160	-0,0092716	-0,3978243	-0,0004151	0,0137107
8	0,0024326	-0,0017066	-0,0732211	0,0011398	-0,0376409
9	0,0063839	-0,0044786	-0,1921644	-0,0000421	0,0013889
10	0,0486531	-0,0341322	-1,4645271	0,0391094	-1,2916012

	6	7	8	9
1	-0,029774677	0,008527242	12,765498161	-0,115972617
2	0,000201805	-0,000157263	-0,126374960	0,001148087
3	0,000164298	-0,000104200	-0,092807531	0,000843135
4	0,002829238	-0,002080569	-1,721790314	0,015642077
5	-0,062451102	0,017885398	26,774871826	-0,243242741
6	0,015375394	-0,034542985	-18,750686646	0,170346618
7	-0,024375293	0,013321497	12,866029739	-0,116884768
8	-0,072636724	0,045716174	41,332534790	-0,375496864
9	-0,019186158	0,006609473	8,673218727	-0,078794241
10	0,273984253	-0,232933104	-182,589477539	1,658782959

Lag : 9

	1	2	3	4	5
1	0,0147715	-0,0103640	-0,4447021	-0,0082056	0,2709936
2	0,0000362	-0,0000254	-0,0010901	-0,0000638	0,0021069
3	0,0000054	-0,0000038	-0,0001616	-0,0000362	0,0011942
4	0,0003435	-0,0002410	-0,0103392	-0,0007535	0,0248858
5	0,0309868	-0,0217385	-0,9327459	-0,0172102	0,5683730
6	0,0234628	-0,0164601	-0,7062607	-0,0174911	0,5776510
7	0,0120500	-0,0084538	-0,3627224	-0,0003787	0,0125061
8	0,0011854	-0,0008316	-0,0356798	-0,0000782	0,0025822
9	0,0057114	-0,0040068	-0,1719227	-0,0007378	0,0243676
10	0,0498387	-0,0349638	-1,5002127	0,0390312	-1,2890205

	6	7	8	9
1	-0,033913430	0,015915558	17,122833252	-0,155556321
2	0,000176905	-0,000139128	-0,111223042	0,001010435
3	0,000158270	-0,000106315	-0,092085242	0,000836572
4	0,002546895	-0,001911720	-1,565498352	0,014222197
5	-0,071130574	0,033380799	35,913208008	-0,326262057
6	0,000174265	-0,017245267	-6,934294701	0,062996268
7	-0,025339734	0,015349470	14,013154030	-0,127306342
8	-0,067554712	0,045996014	39,934799194	-0,362798572
9	-0,021589920	0,010719426	11,100724220	-0,100847661
10	0,206429183	-0,186936975	-142,654479980	1,295982361

Lag : 10

	1	2	3	4	5
1	0,0133772	-0,0093858	-0,4027405	-0,0086898	0,2869849
2	0,0000368	-0,0000258	-0,0011081	-0,0000619	0,0020451
3	0,0000078	-0,0000054	-0,0002333	-0,0000338	0,0011151
4	0,0003625	-0,0002543	-0,0109131	-0,0007251	0,0239482
5	0,0280626	-0,0196872	-0,8447266	-0,0182257	0,6019090
6	0,0220793	-0,0154896	-0,6646194	-0,0183490	0,6059839
7	0,0109364	-0,0076723	-0,3292017	-0,0002408	0,0079523
8	0,0000884	-0,0000620	-0,0026608	-0,0008203	0,0270890
9	0,0050415	-0,0035368	-0,1517553	-0,0011844	0,0391141
10	0,0499271	-0,0350258	-1,5028715	0,0382110	-1,2619305

	6	7	8	9
1	-0,034799095	0,019896254	19,044357300	-0,173014104
2	0,000154747	-0,000122480	-0,097540140	0,000886129
3	0,000148362	-0,000103957	-0,088184893	0,000801139
4	0,002272103	-0,001733136	-1,407391548	0,012785845
5	-0,072988451	0,041729655	39,943298340	-0,362875581
6	-0,009215407	-0,005407736	0,841038704	-0,007640779
7	-0,025313389	0,016349413	14,423881531	-0,131037712
8	-0,060611948	0,043745574	36,874908447	-0,335000157
9	-0,022153143	0,012958016	12,196897507	-0,110805929
10	0,145816684	-0,143191278	-105,779434204	0,960981905

Lag : 11

	1	2	3	4	5
1	0,0120531	-0,0084559	-0,3628235	-0,0088203	0.2912930
2	0,0000370	-0,0000260	-0,0011146	-0,0000601	0.0019843
3	0,0000097	-0,0000068	-0,0002918	-0,0000319	0.0010532
4	0,0003762	-0,0002639	-0,0113254	-0,0007001	0.0231219
5	0,0252823	-0,0177366	-0,7610350	-0,0184993	0.6109457
6	0,0206344	-0,0144758	-0,6211195	-0,0186860	0.6171139
7	0,0098888	-0,0069374	-0,2976656	-0,0000329	0.0010860
8	-0,0008381	0,0005880	0,0252260	-0,0012154	0.0401405
9	0,0044024	-0,0030885	-0,1325188	-0,0014351	0.0473948
10	0,0490890	-0,0344380	-1,4776459	0,0369955	-1.2217884

	6	7	8	9
1	-0,0334956	0,0214280	19,2701263	-0,1750646
2	0,0001353	-0,0001075	-0,0854017	0,0007759
3	0,0001363	-0,0000986	-0,0823243	0,0007479
4	0,0020138	-0,0015554	-1,2546654	0,0113983
5	-0,0702539	0,0449420	40,4166107	-0,3671750
6	-0,0143271	0,0022411	5,5658770	-0,0505648
7	-0,0246130	0,0166058	14,3174114	-0,1300703
8	-0,0527851	0,0399321	32,8816223	-0,2987219
9	-0,0214719	0,0138373	12,3485441	-0,1121837
10	0,0930308	-0,1032589	-72,8975677	0,6622567

Lag : 12

	1	2	3	4	5
1	0,0108255	-0,0075951	-0,3259125	-0,0086937	0.2871140
2	0,0000369	-0,0000259	-0,0011118	-0,0000582	0.0019233
3	0,0000113	-0,0000079	-0,0003393	-0,0000304	0.0010045
4	0,0003852	-0,0002702	-0,0115934	-0,0006772	0.0223653
5	0,0227078	-0,0159304	-0,6835423	-0,0182340	0.6021842
6	0,0191963	-0,0134668	-0,5778303	-0,0186215	0.6149840
7	0,0089140	-0,0062536	-0,2683229	0,0002191	-0.0072372
8	-0,0015906	0,0011159	0,0478783	-0,0013706	0.0452637
9	0,0038116	-0,0026740	-0,1147337	-0,0015389	0.0508238
10	0,0474985	-0,0333220	-1,4297676	0,0356249	-1.1765232

	6	7	8	9
1	-0,0308171	0,0212681	18,3699951	-0,1668885
2	0,0001185	-0,0000943	-0,0747883	0,0006794
3	0,0001233	-0,0000915	-0,0754024	0,0006850
4	0,0017771	-0,0013855	-1,1118631	0,0101010
5	-0,0646363	0,0446067	38,5287170	-0,3500237
6	-0,0163884	0,0067560	8,0535784	-0,0731648
7	-0,0234833	0,0163446	13,8638620	-0,1259500
8	-0,0447646	0,0352743	28,4751892	-0,2586905
9	-0,0199999	0,0137680	11,8639364	-0,1077811
10	0,0482662	-0,0679846	-44,4223022	0,4035662

Lag : 13

	1	2	3	4	5
1	0,0097117	-0,0068135	-0,2923737	-0,0083879	0,2770143
2	0,0000366	-0,0000257	-0,0011014	-0,0000564	0,0018615
3	0,0000125	-0,0000088	-0,0003772	-0,0000292	0,0009654
4	0,0003899	-0,0002735	-0,0117357	-0,0006555	0,0216490
5	0,0203716	-0,0142915	-0,6132183	-0,0175926	0,5810014
6	0,0178114	-0,0124954	-0,5361500	-0,0182572	0,6029533
7	0,0080147	-0,0056226	-0,2412519	0,0004949	-0,0163438
8	-0,0021760	0,0015265	0,0654988	-0,0013672	0,0451519
9	0,0032782	-0,0022998	-0,0986795	-0,0015370	0,0507592
10	0,0453224	-0,0317955	-1,3642693	0,0342577	-1,1313715

	6	7	8	9
1	-0,0273743	0,0200037	16,7768402	-0,1524131
2	0,0001041	-0,0000827	-0,0656163	0,0005961
3	0,0001102	-0,0000834	-0,0680668	0,0006184
4	0,0015646	-0,0012278	-0,9815608	0,0089173
5	-0,0574154	0,0419547	35,1872864	-0,3196678
6	-0,0163640	0,0089938	8,9561949	-0,0813648
7	-0,0221078	0,0157413	13,1929903	-0,1198553
8	-0,0370193	0,0302910	24,0184631	-0,2182021
9	-0,0180765	0,0130716	10,9805803	-0,0997560
10	0,0112466	-0,0376935	-20,4035187	0,1853616

Lag : 14

	1	2	3	4	5
1	0,0087166	-0,0061152	-0,2624054	-0,0079640	0,2630134
2	0,0000360	-0,0000253	-0,0010848	-0,0000545	0,0017987
3	0,0000135	-0,0000095	-0,0004064	-0,0000282	0,0009327
4	0,0003910	-0,0002743	-0,0117710	-0,0006345	0,0209531
5	0,0182841	-0,0128270	-0,5503817	-0,0167033	0,5516344
6	0,0165086	-0,0115814	-0,4969301	-0,0176778	0,5838179
7	0,0071902	-0,0050442	-0,2164326	0,0007786	-0,0257152
8	-0,0026086	0,0018300	0,0785211	-0,0012665	0,0418281
9	0,0028059	-0,0019684	-0,0844612	-0,0014624	0,0482968
10	0,0427138	-0,0299655	-1,2857475	0,0329911	-1,0895414

	6	7	8	9
1	-0,0236085	0,0180784	14,8112154	-0,1345574
2	0,0000919	-0,0000728	-0,0577637	0,0005248
3	0,0000976	-0,0000751	-0,0607689	0,0005521
4	0,0013769	-0,0010848	-0,8649065	0,0078575
5	-0,0495182	0,0379171	31,0649109	-0,2822174
6	-0,0149979	0,0096370	8,7857437	-0,0798163
7	-0,0206216	0,0149296	12,4017878	-0,1126674
8	-0,0298482	0,0253436	19,7539215	-0,1794599
9	-0,0159495	0,0119930	9,8763981	-0,0897247
10	-0,0186020	-0,0123498	-0,6494627	0,0059014

Lag : 15

	1	2	3	4	5
1	0,0078375	-0,0054986	-0,2359467	-0,0074693	0,2466760
2	0,0000353	-0,0000248	-0,0010636	-0,0000525	0,0017349
3	0,0000142	-0,0000100	-0,0004279	-0,0000274	0,0009042
4	0,0003892	-0,0002731	-0,0117170	-0,0006136	0,0202647
5	0,0164407	-0,0115339	-0,4948893	-0,0156657	0,5173655
6	0,0153033	-0,0107360	-0,4606543	-0,0169520	0,5598469
7	0,0064374	-0,0045162	-0,1937780	0,0010586	-0,0349607
8	-0,0029060	0,0020387	0,0874751	-0,0011136	0,0367759
9	0,0023941	-0,0016796	-0,0720682	-0,0013413	0,0442959
10	0,0398077	-0,0279268	-1,1982660	0,0318775	-1,0527658

	6	7	8	9
1	-0,0198315	0,0158207	12,7050753	-0,1154231
2	0,0000815	-0,0000643	-0,0510881	0,0004641
3	0,0000860	-0,0000670	-0,0538072	0,0004888
4	0,0012130	-0,0009572	-0,7620352	0,0069229
5	-0,0415961	0,0331819	26,6475220	-0,2420865
6	-0,0128486	0,0092172	7,9346008	-0,0720839
7	-0,0191198	0,0140089	11,5607700	-0,1050270
8	-0,0234210	0,0206700	15,8328056	-0,1438373
9	-0,0137935	0,0107143	8,6806450	-0,0788615
10	-0,0420234	0,0083203	15,1834717	-0,1379380

Lag : 16

	1	2	3	4	5
1	0,0070686	-0,0049592	-0,2127533	-0,0069396	0,2291830
2	0,0000345	-0,0000242	-0,0010387	-0,0000506	0,0016703
3	0,0000147	-0,0000103	-0,0004427	-0,0000266	0,0008780
4	0,0003850	-0,0002701	-0,0115898	-0,0005928	0,0195766
5	0,0148277	-0,0104023	-0,4463348	-0,0145547	0,4806765
6	0,0142021	-0,0099634	-0,4275045	-0,0161344	0,5328455
7	0,0057531	-0,0040362	-0,1731787	0,0013262	-0,0437979
8	-0,0030876	0,0021661	0,0929413	-0,0009400	0,0310445
9	0,0020398	-0,0014310	-0,0614023	-0,0011934	0,0394125
10	0,0367200	-0,0257607	-1,1053257	0,0309375	-1,0217190

	6	7	8	9
1	-0,0162471	0,0134638	10,6180534	-0,0964618
2	0,0000728	-0,0000570	-0,0454406	0,0004128
3	0,0000753	-0,0000592	-0,0473657	0,0004303
4	0,0010714	-0,0008450	-0,6723969	0,0061086
5	-0,0340788	0,0282387	22,2704468	-0,2023217
6	-0,0103234	0,0081393	6,6961050	-0,0608326
7	-0,0176661	0,0130504	10,7191343	-0,0973809
8	-0,0178142	0,0164151	12,3386326	-0,1120935
9	-0,0117256	0,0093665	7,4836893	-0,0679874
10	-0,0598379	0,0247356	27,5222015	-0,2500325

Lag : 17

	1	2	3	4	5
1	0,0063999	-0,0044899	-0,1926575	-0,0064012	0.2114037
2	0,0000336	-0,0000236	-0,0010113	-0,0000486	0.0016052
3	0,0000150	-0,0000105	-0,0004516	-0,0000258	0.0008531
4	0,0003789	-0,0002658	-0,0114043	-0,0005718	0.0188854
5	0,0134252	-0,0094183	-0,4041176	-0,0134256	0.4433873
6	0,0132043	-0,0092634	-0,3974686	-0,0152674	0.5042107
7	0,0051319	-0,0036003	-0,1544790	0,0015755	-0.0520326
8	-0,0031730	0,0022260	0,0955119	-0,0007673	0.0253420
9	0,0017381	-0,0012194	-0,0523205	-0,0010335	0.0341328
10	0,0335468	-0,0235345	-1,0098104	0,0301701	-0.9963764

	6	7	8	9
1	-0,0129815	0,0111672	8,6549406	-0,0786272
2	0,0000655	-0,0000509	-0,0406753	0,0003695
3	0,0000659	-0,0000522	-0,0415425	0,0003774
4	0,0009501	-0,0007474	-0,5950221	0,0054056
5	-0,0272295	0,0234218	18,1531372	-0,1649169
6	-0,0077105	0,0067038	5,2835150	-0,0479997
7	-0,0163009	0,0121037	9,9097786	-0,0900282
8	-0,0130365	0,0126538	9,3050632	-0,0845343
9	-0,0098193	0,0080397	6,3455496	-0,0576478
10	-0,0728752	0,0373897	36,8275452	-0,3345695

Lag : 18

	1	2	3	4	5
1	0,0058208	-0,0040838	-0,1752319	-0,0058727	0.1939491
2	0,0000326	-0,0000229	-0,0009821	-0,0000466	0.0015400
3	0,0000151	-0,0000106	-0,0004556	-0,0000251	0.0008285
4	0,0003712	-0,0002604	-0,0111734	-0,0005508	0.0181905
5	0,0122101	-0,0085658	-0,3675442	-0,0123171	0.4067776
6	0,0123053	-0,0086327	-0,3704100	-0,0143830	0.4750049
7	0,0045689	-0,0032053	-0,1375341	0,0018029	-0.0595412
8	-0,0031804	0,0022312	0,0957344	-0,0006093	0.0201220
9	0,0014835	-0,0010408	-0,0446596	-0,0008722	0.0288054
10	0,0303667	-0,0213035	-0,9140778	0,0295607	-0.9762527

	6	7	8	9
1	-0,0101013	0,0090333	6,8787184	-0,0624905
2	0,0000593	-0,0000457	-0,0366556	0,0003330
3	0,0000576	-0,0000458	-0,0363737	0,0003304
4	0,0008466	-0,0006632	-0,5287000	0,0048031
5	-0,0211891	0,0189464	14,4279613	-0,1310744
6	-0,0052032	0,0051268	3,8459921	-0,0349399
7	-0,0150466	0,0112009	9,1529150	-0,0831522
8	-0,0090532	0,0094116	6,7318020	-0,0611570
9	-0,0081149	0,0067930	5,3030834	-0,0481772
10	-0,0819290	0,0468015	43,5596008	-0,3957275

Lag : 19

	1	2	3	4	5
1	0,0053198	-0,0037323	-0,1601410	-0,0053666	0,1772352
2	0,0000316	-0,0000222	-0,0009517	-0,0000447	0,0014751
3	0,0000151	-0,0000106	-0,0004555	-0,0000243	0,0008037
4	0,0003624	-0,0002542	-0,0109077	-0,0005297	0,0174931
5	0,0111599	-0,0078292	-0,3359280	-0,0112557	0,3717236
6	0,0114976	-0,0080660	-0,3460941	-0,0135049	0,4460062
7	0,0040590	-0,0028474	-0,1221800	0,0020062	-0,0662546
8	-0,0031266	0,0021934	0,0941153	-0,0004737	0,0156432
9	0,0012702	-0,0008911	-0,0382338	-0,0007167	0,0236695
10	0,0272399	-0,0191101	-0,8199587	0,0290870	-0,9606096

	6	7	8	9
1	-0,0076306	0,0071213	5,3212681	-0,0483420
2	0,0000540	-0,0000413	-0,0332571	0,0003021
3	0,0000565	-0,0000402	-0,0318531	0,0002894
4	0,0007587	-0,0005911	-0,4721254	0,0042891
5	-0,0160075	0,0149364	11,1614952	-0,1013995
6	-0,0029230	0,0035574	2,4825144	-0,0225531
7	-0,0139128	0,0103615	8,4595184	-0,0768529
8	-0,0058003	0,0066789	4,5952129	-0,0417465
9	-0,0066293	0,0056615	4,3759613	-0,0397545
10	-0,0877296	0,0534805	48,1549072	-0,4374754

Lag : 20

	1	2	3	4	5
1	0,0048861	-0,0034279	-0,1470528	-0,0048907	0,1615185
2	0,0000306	-0,0000215	-0,0009206	-0,0000427	0,0014108
3	0,0000150	-0,0000105	-0,0004521	-0,0000236	0,0007786
4	0,0003527	-0,0002474	-0,0106168	-0,0005086	0,0167956
5	0,0102507	-0,0071914	-0,3085632	-0,0102575	0,3387586
6	0,0107723	-0,0075572	-0,3242617	-0,0126498	0,4177642
7	0,0035969	-0,0025233	-0,1082754	0,0021846	-0,0721471
8	-0,0030264	0,0021232	0,0910994	-0,0003641	0,0120239
9	0,0010922	-0,0007662	-0,0328779	-0,0005717	0,0188796
10	0,0242136	-0,0169868	-0,7288675	0,0287229	-0,9485845

	6	7	8	9
1	-0,0055650	0,0054591	3,9926062	-0,0362715
2	0,0000496	-0,0000376	-0,0303705	0,0002759
3	0,0000444	-0,0000352	-0,0279463	0,0002539
4	0,0006840	-0,0005295	-0,4239857	0,0038518
5	-0,0116741	0,0114500	8,3745317	-0,0760805
6	-0,0009366	0,0020929	1,2539911	-0,0113923
7	-0,0129001	0,0095953	7,8337688	-0,0711681
8	-0,0032000	0,0044243	2,8581247	-0,0259654
9	-0,0053625	0,0046625	3,5713940	-0,0324453
10	-0,0909300	0,0579049	51,0131989	-0,4634412

Lag : 21

	1	2	3	4	5
1	0,0045103	-0,0031644	-0,1357746	-0,0044494	0,1469423
2	0,0000295	-0,0000207	-0,0008893	-0,0000408	0,0013473
3	0,0000148	-0,0000104	-0,0004461	-0,0000228	0,0007529
4	0,0003424	-0,0002402	-0,0103081	-0,0004875	0,0161007
5	0,0094615	-0,0066378	-0,2848072	-0,0093318	0,3081874
6	0,0101202	-0,0070996	-0,3046331	-0,0118287	0,3906494
7	0,0031782	-0,0022297	-0,0956707	0,0023383	-0,0772228
8	-0,0028925	0,0020292	0,0870680	-0,0002812	0,0092883
9	0,0009442	-0,0006624	-0,0284214	-0,0004398	0,0145249
10	0,0213210	-0,0149575	-0,6417913	0,0204416	-0,9392952

	6	7	8	9
1	-0,0038793	0,0040519	2,8871069	-0,0262284
2	0,0000457	-0,0000345	-0,0279009	0,0002535
3	0,0000391	-0,0000310	-0,0246017	0,0002235
4	0,0006205	-0,0004770	-0,3830349	0,0034798
5	-0,0081388	0,0084986	6,0559902	-0,0550170
6	0,0007280	0,0007914	0,1929588	-0,0017531
7	-0,0120028	0,0089052	7,2751932	-0,0660934
8	-0,0011669	0,0026029	1,4755402	-0,0134050
9	-0,0043036	0,0038010	2,8879824	-0,0262366
10	-0,0920972	0,0605079	52,4888611	-0,4768491

Lag : 22

	1	2	3	4	5
1	0,0041825	-0,0029344	-0,1259060	-0,0040442	0,1335602
2	0,0000285	-0,0000200	-0,0008580	-0,0000389	0,0012851
3	0,0000145	-0,0000102	-0,0004380	-0,0000220	0,0007268
4	0,0003318	-0,0002328	-0,0099881	-0,0004667	0,0154117
5	0,0087737	-0,0061553	-0,2641020	-0,0084820	0,2801213
6	0,0095319	-0,0066870	-0,2869272	-0,0110488	0,3648901
7	0,0027984	-0,0019633	-0,0842390	0,0024681	-0,0815094
8	-0,0027355	0,0019191	0,0823429	-0,0002237	0,0073890
9	0,0008213	-0,0005762	-0,0247207	-0,0003224	0,0106473
10	0,0185854	-0,0130386	-0,5594530	0,0282179	-0,9319063

	6	7	8	9
1	-0,0025369	0,0028897	1,9893503	-0,0180725
2	0,0000424	-0,0000317	-0,0257685	0,0002341
3	0,0000347	-0,0000274	-0,0217592	0,0001977
4	0,0005662	-0,0004322	-0,3481278	0,0031627
5	-0,0053240	0,0060613	4,1733370	-0,0379138
6	0,0020707	-0,0003183	-0,6885147	0,0062550
7	-0,0112117	0,0082896	6,7801600	-0,0615964
8	0,0003838	0,0011630	0,3998894	-0,0036329
9	-0,0034345	0,0030733	2,3186283	-0,0210642
10	-0,0917135	0,0616709	52,8888245	-0,4804811

Lag : 23

	1	2	3	4	5
1	0,0038944	-0,0027322	-0,1172304	-0,0036750	0.1213667
2	0,0000275	-0,0000193	-0,0008270	-0,0000371	0.0012244
3	0,0000142	-0,0000100	-0,0004283	-0,0000212	0.0007002
4	0,0003210	-0,0002252	-0,0096614	-0,0004461	0.0147317
5	0,0081697	-0,0057315	-0,2459211	-0,0077077	0.2545488
6	0,0089988	-0,0063130	-0,2708826	-0,0103135	0.3406085
7	0,0024537	-0,0017215	-0,0738630	0,0025753	-0.0850492
8	-0,0025643	0,0017990	0,0771891	-0,0001890	0.0062433
9	0,0007189	-0,0005044	-0,0216417	-0,0002197	0.0072543
10	0,0160213	-0,0112396	-0,4822655	0,0280288	-0.9256608

	6	7	8	9
1	-0,0014963	0,0319532	1,2785482	-0,0116150
2	0,0000394	-0,0000293	-0,0239067	0,0002172
3	0,0000310	-0,0000243	-0,0193553	0,0001758
4	0,0005197	-0,0003939	-0,3182426	0,0028912
5	-0,0031423	0,0040977	2,6828728	-0,0243731
6	0,0031081	-0,0012275	-1,3921871	0,0126477
7	-0,0105155	0,0077436	6,3432980	-0,0576275
8	0,0015323	0,0000512	-0,4157310	0,0037768
9	-0,0027339	0,0024701	1,8529282	-0,0168334
10	-0,0901812	0,0617221	52,4730530	-0,4767043

Lag : 24

	1	2	3	4	5
1	0,0036392	-0,0025532	-0,1095514	-0,0033403	0.1103160
2	0,0000265	-0,0000186	-0,0007964	-0,0000353	0.0011653
3	0,0000139	-0,0000097	-0,0004175	-0,0000204	0.0006734
4	0,0003100	-0,0002175	-0,0093324	-0,0004258	0.0140633
5	0,0076342	-0,0053558	-0,2298031	-0,0070059	0.2313733
6	0,0085132	-0,0059723	-0,2562590	-0,0096244	0.3178496
7	0,0021405	-0,0015016	-0,0644321	0,0026614	-0.0878941
8	-0,0023859	0,0016738	0,0718173	-0,0001740	0.0057475
9	0,0006334	-0,0004444	-0,0190672	-0,0001311	0.0043302
10	0,0136352	-0,0095657	-0,4104404	0,0278548	-0.9199139

	6	7	8	9
1	-0,0007142	0,0012182	0,7310871	-0,0066416
2	0,0000368	-0,0000273	-0,0222610	0,0002022
3	0,0000279	-0,0000217	-0,0173277	0,0001574
4	0,0004793	-0,0003609	-0,2924895	0,0026572
5	-0,0015027	0,0025564	1,5349989	-0,0139451
6	0,0038690	-0,0019417	-1,9292803	0,0175272
7	-0,0099026	0,0072611	5,9582863	-0,0541298
8	0,0023527	-0,0007842	-1,0156574	0,0092270
9	-0,0021792	0,0019791	1,4787149	-0,0134338
10	-0,0878286	0,0609378	51,4574738	-0,4674787

Lag : 25

	1	2	3	4	5
1	0,0034113	-0,0023931	-0,1026869	-0,0030382	0,1003366
2	0,0000255	-0,0000179	-0,0007664	-0,0000335	0,0011080
3	0,0000135	-0,0000095	-0,0004058	-0,0000196	0,0006465
4	0,0002991	-0,0002098	-0,0090039	-0,0004060	0,0134092
5	0,0071564	-0,0050207	-0,2154169	-0,0063722	0,2104446
6	0,0080677	-0,0056597	-0,2428493	-0,0089810	0,2966011
7	0,0018555	-0,0013017	-0,0558548	0,0027282	-0,0901006
8	-0,0022055	0,0015473	0,0663890	-0,0001752	0,0057875
9	0,0005616	-0,0003940	-0,0169039	-0,0000558	0,0018440
10	0,0114297	-0,0080185	-0,3440514	0,0276795	-0,9141256

	6	7	8	9
1	-0,0001482	0,0006577	0,3226131	-0,0029324
2	0,0000344	-0,0000254	-0,0207877	0,0001889
3	0,0000253	-0,0000195	-0,0156183	0,0001419
4	0,0004440	-0,0003324	-0,2701054	0,0024538
5	-0,0003159	0,0013811	0,6788347	-0,0061670
6	0,0043878	-0,0024756	-2,3164997	0,0210448
7	-0,0093612	0,0068346	5,6185265	-0,0510432
8	0,0029101	-0,0013918	-1,4400616	0,0130826
9	-0,0017478	0,0015863	1,1832895	-0,0107499
10	-0,0849186	0,0595460	50,0174408	-0,4543965

Lag : 26

	1	2	3	4	5
1	0,0032051	-0,0022487	-0,0964832	-0,0027658	0,0913416
2	0,0000245	-0,0000172	-0,0007371	-0,0000319	0,0010525
3	0,0000131	-0,0000092	-0,0003936	-0,0000188	0,0006196
4	0,0002883	-0,0002023	-0,0086781	-0,0003867	0,0127715
5	0,0067240	-0,0047172	-0,2024088	-0,0058010	0,1915808
6	0,0076565	-0,0053713	-0,2304707	-0,0083818	0,2768129
7	0,0015960	-0,0011197	-0,0480394	0,0027775	-0,0917277
8	-0,0020274	0,0014223	0,0610276	-0,0001893	0,0062515
9	0,0005005	-0,0003512	-0,0150678	0,0000074	-0,0002436
10	0,0094026	-0,0065963	-0,2830296	0,0274902	-0,9078730

	6	7	8	9
1	0,0002409	0,0002450	0,0304898	-0,0002768
2	0,0000323	-0,0000237	-0,0194519	0,0001767
3	0,0000230	-0,0000176	-0,0141741	0,0001288
4	0,0004128	-0,0003075	-0,2504579	0,0022753
5	0,0004996	0,0005157	0,0659208	-0,0005988
6	0,0047005	-0,0028489	-2,5731688	0,0233767
7	-0,0088802	0,0064569	5,3175068	-0,0483084
8	0,0032611	-0,0018148	-1,7243652	0,0156654
9	-0,0014187	0,0012776	0,9542513	-0,0086691
10	-0,0816574	0,0577311	48,2930450	-0,4387309

Lag : 27

	1	2	3	4	5
1	0,0030178	-0,0021171	-0,0908442	-0,0025205	0,0832391
2	0,0000235	-0,0000165	-0,0007085	-0,0000303	0,0009991
3	0,0000127	-0,0000089	-0,0003811	-0,0000180	0,0005930
4	0,0002776	-0,0001948	-0,0083569	-0,0003680	0,0121518
5	0,0063308	-0,0044413	-0,1905651	-0,0052865	0,1745886
6	0,0072741	-0,0051031	-0,2189665	-0,0078246	0,2584109
7	0,0013596	-0,0009538	-0,0409260	0,0028110	-0,0928336
8	-0,0018547	0,0013012	0,0558292	-0,0002130	0,0070347
9	0,0004483	-0,0003145	-0,0134946	0,0000598	-0,0019760
10	0,0075476	-0,0052950	-0,2271957	0,0272771	-0,9008372

	6	7	8	9
1	0,0004891	-0,0000457	-0,1677130	0,0015238
2	0,0000302	-0,0000222	-0,0182267	0,0001656
3	0,0000211	-0,0000160	-0,0129483	0,0001176
4	0,0003849	-0,0002854	-0,2330179	0,0021169
5	0,0010200	-0,0000941	-0,3497324	0,0031773
6	0,0048433	-0,0030839	-2,7196770	0,0247077
7	-0,0084495	0,0061207	5,0491533	-0,0458704
8	0,0034537	-0,0020917	-1,8992624	0,0172543
9	-0,0011726	0,0010393	0,7800257	-0,0070863
10	-0,0782036	0,0556394	46,3937378	-0,4214766

Lag : 28

	1	2	3	4	5
1	0,0028450	-0,0019961	-0,0856438	-0,0022993	0,0759358
2	0,0000226	-0,0000159	-0,0006806	-0,0000287	0,0009477
3	0,0000122	-0,0000086	-0,0003684	-0,0000172	0,0005666
4	0,0002672	-0,0001874	-0,0080417	-0,0003498	0,0115516
5	0,0059686	-0,0041874	-0,1796656	-0,0048227	0,1592721
6	0,0069166	-0,0048522	-0,2082024	-0,0073067	0,2413065
7	0,0011440	-0,0008026	-0,0344362	0,0028304	-0,0934747
8	-0,0016894	0,0011852	0,0508537	-0,0002435	0,0080425
9	0,0004028	-0,0002826	-0,0121267	0,0001029	-0,0033976
10	0,0058582	-0,0041098	-0,1763439	0,0270336	-0,8927952

	6	7	8	9
1	0,0006273	-0,0002380	-0,2910987	0,0026447
2	0,0000284	-0,0000208	-0,0170914	0,0001553
3	0,0000195	-0,0000147	-0,0119004	0,0001081
4	0,0003597	-0,0002658	-0,2173609	0,0019747
5	0,0013100	-0,0004973	-0,6084700	0,0055278
6	0,0048496	-0,0032030	-2,7757282	0,0252168
7	-0,0080603	0,0058195	4,8080072	-0,0436797
8	0,0035281	-0,0022550	-1,9906807	0,0180848
9	-0,0009927	0,0008588	0,6501713	-0,0059066
10	-0,0746754	0,0533844	44,4030914	-0,4033917

Lag : 29

	1	2	3	4	5
1	0,0026844	-0,0018833	-0,0808096	-0,0020997	0,0693429
2	0,0000217	-0,0000152	-0,0006535	-0,0000272	0,0008983
3	0,0000118	-0,0000083	-0,0003557	-0,0000164	0,0005408
4	0,0002569	-0,0001802	-0,0077332	-0,0003322	0,0109717
5	0,0056319	-0,0039510	-0,1695261	-0,0044040	0,1454448
6	0,0065804	-0,0046164	-0,1980791	-0,0068252	0,2254055
7	0,0009475	-0,0006647	-0,0285206	0,0028373	-0,0937037
8	-0,0015327	0,0010753	0,0461372	-0,0002784	0,0091927
9	0,0003630	-0,0002546	-0,0109259	0,0001378	-0,0045511
10	0,0043254	-0,0030345	-0,1302080	0,0267552	-0,8836010

	6	7	8	9
1	0,0006823	-0,0003526	-0,3564802	0,0032388
2	0,0000266	-0,0000195	-0,0160305	0,0001456
3	0,0000181	-0,0000135	-0,0109961	0,0000999
4	0,0003366	-0,0002481	-0,2031462	0,0018455
5	0,0014250	-0,0007375	-0,7455631	0,0067734
6	0,0047500	-0,0032280	-2,7597332	0,0250715
7	-0,0077051	0,0055473	4,5892801	-0,0416925
8	0,0035162	-0,0023322	-2,0197716	0,0183491
9	-0,0008644	0,0007249	0,5554969	-0,0050465
10	-0,0711591	0,0510521	42,3832855	-0,3850421

Lag : 30

	1	2	3	4	5
1	0,0025347	-0,0017782	-0,0763063	-0,0019191	0,0633789
2	0,0000208	-0,0000146	-0,0006272	-0,0000258	0,0008510
3	0,0000114	-0,0000080	-0,0003431	-0,0000156	0,0005155
4	0,0002469	-0,0001732	-0,0074320	-0,0003153	0,0104129
5	0,0053177	-0,0037305	-0,1600704	-0,0040253	0,1329370
6	0,0062622	-0,0043933	-0,1885052	-0,0063773	0,2106127
7	0,0007684	-0,0005391	-0,0231314	0,0028333	-0,0935702
8	-0,0013859	0,0009723	0,0417172	-0,0003155	0,0104183
9	0,0003274	-0,0002297	-0,0098568	0,0001658	-0,0054760
10	0,0029396	-0,0020623	-0,0884914	0,0264398	-0,8731828

	6	7	8	9
1	0,0006763	-0,0004075	-0,3782696	0,0034367
2	0,0000250	-0,0000183	-0,0150323	0,0001366
3	0,0000168	-0,0000125	-0,0102070	0,0000927
4	0,0003152	-0,0002319	-0,1901057	0,0017271
5	0,0014123	-0,0008527	-0,7911436	0,0071876
6	0,0045704	-0,0031786	-2,6879835	0,0244197
7	-0,0073774	0,0052992	4,3888655	-0,0398718
8	0,0034441	-0,0023458	-2,0039949	0,0182058
9	-0,0007755	0,0006276	0,4881533	-0,0044347
10	-0,0677150	0,0487064	40,3792572	-0,3668363

Lag : 31

	1	2	3	4	5
1	0,0023934	-0,0016791	-0,0720482	-0,0017552	0.0579674
2	0,0000200	-0,0000140	-0,0006016	-0,0000244	0.0008057
3	0,0000110	-0,0000077	-0,0003307	-0,0000149	0.0004909
4	0,0002371	-0,0001664	-0,0071386	-0,0002990	0.0098755
5	0,0050211	-0,0035225	-0,1511412	-0,0036816	0.1215873
6	0,0059602	-0,0041813	-0,1794128	-0,0059601	0.1968362
7	0,0006051	-0,0004246	-0,0182180	0,0028196	-0.0931192
8	-0,0012488	0,0008761	0,0375900	-0,0003531	0.0116622
9	0,0002954	-0,0002072	-0,0088913	0,0001880	-0.0062084
10	0,0016910	-0,0011861	-0,0508906	0,0260866	-0.8615203

	6	7	8	9
1	0,0006276	-0,0004181	-0,3683825	0,0033469
2	0,0000234	-0,0000172	-0,0140884	0,0001280
3	0,0000157	-0,0000116	-0,0095099	0,0000864
4	0,0002953	-0,0002170	-0,1780315	0,0016174
5	0,0013096	-0,0008747	-0,7702115	0,0069972
6	0,0043339	-0,0030727	-2,5749407	0,0233929
7	-0,0070719	0,0050705	4,2033253	-0,0381862
8	0,0033321	-0,0023140	-1,9571409	0,0177801
9	-0,0007158	0,0005586	0,4415639	-0,0040115
10	-0,0643827	0,0463923	38,4220886	-0,3490558

Lag : 32

	1	2	3	4	5
1	0,0022599	-0,0015855	-0,0680285	-0,0016061	0.0530423
2	0,0000192	-0,0000134	-0,0005768	-0,0000231	0.0007624
3	0,0000106	-0,0000074	-0,0003184	-0,0000141	0.0004669
4	0,0002277	-0,0001597	-0,0068532	-0,0002634	0.0093596
5	0,0047412	-0,0033261	-0,1427193	-0,0033689	0.1112585
6	0,0056724	-0,0039794	-0,1707487	-0,0055711	0.1839895
7	0,0004566	-0,0003203	-0,0137453	0,0027976	-0.0923917
8	-0,0011216	0,0007869	0,0337626	-0,0003900	0.0128809
9	0,0002662	-0,0001868	-0,0080134	0,0002053	-0.0067802
10	0,0005690	-0,0003992	-0,0171328	0,0256966	-0.8486387

	6	7	8	9
1	0,0005503	-0,0003967	-0,3363535	0,0030560
2	0,0000219	-0,0000161	-0,0131926	0,0001199
3	0,0000147	-0,0000109	-0,0088859	0,0000807
4	0,0002767	-0,0002032	-0,1667647	0,0015150
5	0,0011472	-0,0008298	-0,7029589	0,0063863
6	0,0040591	-0,0029255	-2,4327431	0,0221010
7	-0,0067845	0,0048578	4,0298548	-0,0366103
8	0,0031953	-0,0022511	-1,8897295	0,0171677
9	-0,0006771	0,0005110	0,4102965	-0,0037274
10	-0,0611874	0,0441412	36,5322876	-0,3318876

Lag : 33

	1	2	3	4	5
1	0,0021332	-0,0014966	-0,0642147	-0,0014700	0,0485459
2	0,0000184	-0,0000129	-0,0005528	-0,0000218	0,0007211
3	0,0000102	-0,0000071	-0,0003065	-0,0000134	0,0004438
4	0,0002185	-0,0001533	-0,0065759	-0,0002684	0,0088652
5	0,0044751	-0,0031396	-0,1347103	-0,0030833	0,1018279
6	0,0053974	-0,0037866	-0,1624727	-0,0052079	0,1719932
7	0,0003214	-0,0002255	-0,0096769	0,0027683	-0,0914247
8	-0,0010042	0,0007045	0,0302290	-0,0004252	0,0140415
9	0,0002394	-0,0001679	-0,0072056	0,0002186	-0,0072189
10	-0,0004352	0,0003054	0,0130997	0,0252714	-0,8345965

	6	7	8	9
1	0,0004554	-0,0003534	-0,2897816	0,0026328
2	0,0000205	-0,0000150	-0,0123407	0,0001121
3	0,0000138	-0,0000102	-0,0083203	0,0000756
4	0,0002591	-0,0001903	-0,1561860	0,0014189
5	0,0009486	-0,0007391	-0,6054425	0,0055003
6	0,0037619	-0,0027500	-2,2715988	0,0206368
7	-0,0065118	0,0046582	3,8662128	-0,0351236
8	0,0030459	-0,0021684	-1,8102417	0,0164456
9	-0,0006531	0,0004791	0,3900239	-0,0035433
10	-0,0581417	0,0419729	34,7220917	-0,3154422

Lag : 34

	1	2	3	4	5
1	0,0020122	-0,0014117	-0,0605755	-0,0013453	0,0444277
2	0,0000176	-0,0000123	-0,0005296	-0,0000206	0,0006816
3	0,0000098	-0,0000069	-0,0002948	-0,0000128	0,0004215
4	0,0002095	-0,0001470	-0,0063065	-0,0002541	0,0083920
5	0,0042220	-0,0029619	-0,1270924	-0,0028218	0,0931899
6	0,0051342	-0,0036019	-0,1545467	-0,0048682	0,1607741
7	0,0001986	-0,0001394	-0,0059795	0,0027328	-0,0902508
8	-0,0008961	0,0006287	0,0269753	-0,0004578	0,0151187
9	0,0002145	-0,0001505	-0,0064585	0,0002286	-0,0075481
10	-0,0013315	0,0009339	0,0400743	0,0248136	-0,8194768

	6	7	8	9
1	0,0003524	-0,0002965	-0,2349624	0,0021348
2	0,0000191	-0,0000141	-0,0115297	0,0001047
3	0,0000130	-0,0000095	-0,0078017	0,0000709
4	0,0002425	-0,0001781	-0,1462042	0,0013282
5	0,0007331	-0,0006199	-0,4907169	0,0044579
6	0,0034539	-0,0025567	-2,0995646	0,0190738
7	-0,0062511	0,0044691	3,7105875	-0,0337098
8	0,0028915	-0,0020741	-1,7244415	0,0156662
9	-0,0006386	0,0004582	0,3772833	-0,0034275
10	-0,0552501	0,0398988	32,9977112	-0,2997766

Lag : 35

	1	2	3	4	5
1	0,0018970	-0,0013310	-0,0571079	-0,0012307	0,0406432
2	0,0000168	-0,0000118	-0,0005070	-0,0000195	0,0006440
3	0,0000094	-0,0000066	-0,0002833	-0,0000121	0,0004000
4	0,0002008	-0,0001409	-0,0060456	-0,0002404	0,0079396
5	0,0039810	-0,0027928	-0,1198330	-0,0025814	0,0852515
6	0,0048817	-0,0034247	-0,1469469	-0,0045500	0,1502674
7	0,0000874	-0,0000613	-0,0026312	0,0026919	-0,0889000
8	-0,0007970	0,0005591	0,0239903	-0,0004874	0,0160973
9	0,0001915	-0,0001343	-0,0057632	0,0002358	-0,0077877
10	-0,0021281	0,0014929	0,0640583	0,0243261	-0,8033785

	6	7	8	9
1	0,0002471	-0,0002319	-0,1762811	0,0016015
2	0,0000178	-0,0000131	-0,0107573	0,0000977
3	0,0000122	-0,0000089	-0,0073210	0,0000665
4	0,0002267	-0,0001666	-0,1367558	0,0012424
5	0,0005130	-0,0004847	-0,3678485	0,0033418
6	0,0031450	-0,0023542	-1,9231892	0,0174714
7	-0,0060006	0,0042888	3,5616455	-0,0323567
8	0,0027386	-0,0019745	-1,6368876	0,0148707
9	-0,0006300	0,0004448	0,3694621	-0,0033565
10	-0,0525117	0,0379243	31,3608551	-0,2849061

Lag : 36

	1	2	3	4	5
1	0,0017871	-0,0012540	-0,0538015	-0,0011251	0,0371559
2	0,0000161	-0,0000113	-0,0004852	-0,0000184	0,0006081
3	0,0000090	-0,0000063	-0,0002722	-0,0000115	0,0003794
4	0,0001924	-0,0001350	-0,0057927	-0,0002273	0,0075076
5	0,0037501	-0,0026308	-0,1128798	-0,0023599	0,0779362
6	0,0046395	-0,0032548	-0,1396532	-0,0042517	0,1404149
7	-0,0000133	0,0000093	0,0003996	0,0026464	-0,0873981
8	-0,0007060	0,0004953	0,0212522	-0,0005137	0,0169661
9	0,0001699	-0,0001192	-0,0051133	0,0002408	-0,0079540
10	-0,0028341	0,0019882	0,0853109	0,0238123	-0,7864118

	6	7	8	9
1	0,0001443	-0,0001645	-0,1171041	0,0010641
2	0,0000166	-0,0000122	-0,0100222	0,0000910
3	0,0000114	-0,0000084	-0,0068718	0,0000624
4	0,0002117	-0,0001558	-0,1277917	0,0011610
5	0,0002982	-0,0003436	-0,2440298	0,0022169
6	0,0028419	-0,0021491	-1,7473078	0,0158736
7	-0,0057588	0,0041159	3,4183178	-0,0310546
8	0,0025906	-0,0018741	-1,5505219	0,0140861
9	-0,0006244	0,0004362	0,3645358	-0,0033117
10	-0,0499211	0,0360502	29,8103485	-0,2708203

Lag : 37

	1	2	3	4	5
1	0,0016825	-0,0011805	-0,0506535	-0,0010275	0,0339342
2	0,0000154	-0,0000108	-0,0004641	-0,0000174	0,0005740
3	0,0000087	-0,0000061	-0,0002614	-0,0000109	0,0003596
4	0,0001843	-0,0001293	-0,0055475	-0,0002148	0,0070953
5	0,0035304	-0,0024769	-0,1062737	-0,0021553	0,0711784
6	0,0044069	-0,0030916	-0,1326571	-0,0039717	0,1311657
7	-0,0001040	0,0000729	0,0031294	0,0025971	-0,0657685
8	-0,0006229	0,0004370	0,0187509	-0,0005366	0,0177213
9	0,0001497	-0,0001050	-0,0045056	0,0002441	-0,0080608
10	-0,0034571	0,0024253	0,1040583	0,0232757	-0,7686905

	6	7	8	9
1	0,0000476	-0,0000977	-0,0599879	0,0005451
2	0,0000154	-0,0000114	-0,0093231	0,0000847
3	0,0000107	-0,0000078	-0,0064489	0,0000586
4	0,0001975	-0,0001455	-0,1192772	0,0010836
5	0,0000956	-0,0002036	-0,1243155	0,0011292
6	0,0025498	-0,0019466	-1,5756836	0,0143147
7	-0,0055246	0,0039492	3,2798662	-0,0297968
8	0,0024503	-0,0017758	-1,4673681	0,0133307
9	-0,0006199	0,0004304	0,3610732	-0,0032803
10	-0,0474708	0,0342744	28,3429871	-0,2574894

Lag : 38

	1	2	3	4	5
1	0,0015824	-0,0011102	-0,0476379	-0,0009372	0,0309512
2	0,0000147	-0,0000103	-0,0004437	-0,0000164	0,0005415
3	0,0000083	-0,0000058	-0,0002509	-0,0000103	0,0003406
4	0,0001764	-0,0001238	-0,0053101	-0,0002029	0,0067023
5	0,0033206	-0,0023296	-0,0999546	-0,0019658	0,0649217
6	0,0041837	-0,0029351	-0,1259365	-0,0037085	0,1224741
7	-0,0001856	0,0001302	0,0055857	0,0025445	-0,0840318
8	-0,0005469	0,0003837	0,0164625	-0,0005560	0,0183612
9	0,0001308	-0,0000918	-0,0039373	0,0002458	-0,0081191
10	-0,0040040	0,0028088	0,1205225	0,0227197	-0,7503282

	6	7	8	9
1	-0,0000418	-0,0000338	-0,0062411	0,0000570
2	0,0000143	-0,0000106	-0,0086592	0,0000787
3	0,0000100	-0,0000074	-0,0060488	0,0000550
4	0,0001839	-0,0001357	-0,1111864	0,0010101
5	-0,0000911	-0,0000697	-0,0118998	0,0001080
6	0,0022723	-0,0017503	-1,4109478	0,0128181
7	-0,0052973	0,0037881	3,1457510	-0,0285784
8	0,0023186	-0,0016816	-1,3885756	0,0126149
9	-0,0006151	0,0004259	0,3580529	-0,0032528
10	-0,0451522	0,0325928	26,9544067	-0,2448745

Lag : 39

	1	2	3	4	5
1	0,0014873	-0,0010433	-0,0447683	-0,0008534	0,0281852
2	0,0000141	-0,0000099	-0,0004240	-0,0000155	0,0005105
3	0,0000080	-0,0000056	-0,0002407	-0,0000098	0,0003224
4	0,0001688	-0,0001184	-0,0050806	-0,0001916	0,0063279
5	0,0031195	-0,0021887	-0,0939102	-0,0017901	0,0591196
6	0,0039694	-0,0027846	-0,1194849	-0,0034610	0,1143003
7	-0,0002587	0,0001815	0,0077877	0,0024892	-0,0822061
8	-0,0004775	0,0003350	0,0143740	-0,0005719	0,0188806
9	0,0001131	-0,0000794	-0,0034051	0,0002464	-0,0081378
10	-0,0044815	0,0031439	0,1348953	0,0221478	-0,7314393

	6	7	8	9
1	-0,0001223	0,0000258	0,3429639	-0,0003902
2	0,0000132	-0,0000098	-0,0080295	0,0000729
3	0,0000094	-0,0000069	-0,0056689	0,0000515
4	0,0001710	-0,0001264	-0,1035008	0,0009403
5	-0,0002601	0,0000551	0,0913310	-0,0008298
6	0,0020111	-0,0015627	-1,2547398	0,0113990
7	-0,0050766	0,0036320	3,0156136	-0,0273961
8	0,0021962	-0,0015926	-1,3147850	0,0119445
9	-0,0006092	0,0004216	0,3547794	-0,0032231
10	-0,0429559	0,0310001	25,6395721	-0,2329297

Lag : 40

	1	2	3	4	5
1	0,0013964	-0,0009797	-0,0420351	-0,0007757	0,0256168
2	0,0000135	-0,0000094	-0,0004050	-0,0000146	0,0004812
3	0,0000077	-0,0000054	-0,0002308	-0,0000092	0,0003051
4	0,0001614	-0,0001132	-0,0048586	-0,0001808	0,0059713
5	0,0029291	-0,0020549	-0,0881739	-0,0016270	0,0537327
6	0,0037637	-0,0026404	-0,1132965	-0,0032281	0,1066093
7	-0,0003239	0,0002272	0,0097494	0,0024317	-0,0803078
8	-0,0004142	0,0002906	0,0124671	-0,0005846	0,0193075
9	0,0000966	-0,0000678	-0,0029082	0,0002460	-0,0081242
10	-0,0048956	0,0034345	0,1473637	0,0215631	-0,7121316

	6	7	8	9
1	-0,0001936	0,0000799	0,0871092	-0,0007914
2	0,0000122	-0,0000091	-0,0074332	0,0000675
3	0,0000088	-0,0000065	-0,0053074	0,0000482
4	0,0001588	-0,0001175	-0,0962049	0,0008740
5	-0,0004092	0,0001685	0,1838364	-0,0016702
6	0,0017678	-0,0013857	-1,1082373	0,0100679
7	-0,0048620	0,0034804	2,8891792	-0,0262475
8	0,0020831	-0,0015094	-1,2462149	0,0113216
9	-0,0006017	0,0004169	0,3508415	-0,0031873
10	-0,0408727	0,0294908	24,3933716	-0,2216083

Lag : 41

	1	2	3	4	5
1	0,0013096	-0,0009186	-0,0394239	-0,0007034	0,0232294
2	0,0000128	-0,0000090	-0,0003866	-0,0000137	0,0004532
3	0,0000073	-0,0000052	-0,0002212	-0,0000087	0,0002885
4	0,0001543	-0,0001082	-0,0046442	-0,0001705	0,0056320
5	0,0027471	-0,0019270	-0,0826921	-0,0014754	0,0487251
6	0,0035664	-0,0025019	-0,1073580	-0,0030089	0,0993690
7	-0,0003818	0,0002679	0,0114931	0,0023725	-0,0783511
8	-0,0003564	0,0002501	0,0107291	-0,0005942	0,0196238
9	0,0000812	-0,0000570	-0,0024441	0,0002448	-0,0080840
10	-0,0052521	0,0036846	0,1580963	0,0209689	-0,6925073

	6	7	8	9
1	-0,0002557	0,0001282	0,1260859	-0,0011455
2	0,0000113	-0,0000084	-0,0068698	0,0000624
3	0,0000082	-0,0000060	-0,0049630	0,0000451
4	0,0001473	-0,0001092	-0,0892866	0,0008112
5	-0,0005396	0,0002698	0,2656257	-0,0024133
6	0,0015427	-0,0012201	-0,9719893	0,0088302
7	-0,0046535	0,0033333	2,7663403	-0,0251316
8	0,0019785	-0,0014319	-1,1826477	0,0107441
9	-0,0005925	0,0004114	0,3460219	-0,0031435
10	-0,0388944	0,0280589	23,2107391	-0,2108643

Lag : 42

	1	2	3	4	5
1	0,0012271	-0,0008608	-0,0369377	-0,0006362	0,0210094
2	0,0000123	-0,0000086	-0,0003689	-0,0000129	0,0004267
3	0,0000070	-0,0000049	-0,0002119	-0,0000083	0,0002727
4	0,0001474	-0,0001034	-0,0044368	-0,0001608	0,0053093
5	0,0025740	-0,0018059	-0,0774860	-0,0013344	0,0440692
6	0,0033777	-0,0023696	-0,1016750	-0,0028024	0,0925515
7	-0,0004329	0,0003037	0,0130312	0,0023118	-0,0763483
8	-0,0003037	0,0002131	0,0091430	-0,0006009	0,0198450
9	0,0000669	-0,0000469	-0,0020140	0,0002429	-0,0080217
10	-0,0055559	0,0038977	0,1672392	0,0203680	-0,6726617

	6	7	8	9
1	-0,0003091	0,0001706	0,1599748	-0,0014532
2	0,0000104	-0,0000078	-0,0063379	0,0000576
3	0,0000077	-0,0000056	-0,0046348	0,0000421
4	0,0001363	-0,0001012	-0,0827345	0,0007516
5	-0,0006519	0,0003589	0,3367748	-0,0030596
6	0,0013358	-0,0010666	-0,8461547	0,0076870
7	-0,0044509	0,0031904	2,6469946	-0,0240473
8	0,0018818	-0,0013601	-1,1238794	0,0102102
9	-0,0005815	0,0004049	0,3402079	-0,0030907
10	-0,0370126	0,0266989	22,0868530	-0,2006543

Lag : 43

	1	2	3	4	5
1	0,0011486	-0,0008058	-0,0345745	-0,0005736	0,0189448
2	0,0000117	-0,0000082	-0,0003518	-0,0000122	0,0004015
3	0,0000067	-0,0000047	-0,0002029	-0,0000078	0,0002577
4	0,0001407	-0,0000987	-0,0042365	-0,0001515	0,0050025
5	0,0024098	-0,0016906	-0,0725403	-0,0012033	0,0397393
6	0,0031968	-0,0022428	-0,0962305	-0,0026081	0,0861318
7	-0,0004777	0,0003351	0,0143806	0,0022501	-0,0743114
8	-0,0002556	0,0001793	0,0076951	-0,0006049	0,0199782
9	0,0000537	-0,0000376	-0,0016151	0,0002405	-0,0079410
10	-0,0058114	0,0040770	0,1749325	0,0197630	-0,6526832

	6	7	8	9
1	-0,0003545	0,0002073	0,1890645	-0,0017174
2	0,0000096	-0,0000072	-0,0058370	0,0000530
3	0,0000072	-0,0000053	-0,0043223	0,0000393
4	0,0001260	-0,0000937	-0,0765373	0,0006953
5	-0,0007465	0,0004357	0,3975148	-0,0036114
6	0,0011465	-0,0009251	-0,7305938	0,0066372
7	-0,0042543	0,0030516	2,5310907	-0,0229944
8	0,0017924	-0,0012935	-1,0695248	0,0097164
9	-0,0005688	0,0003972	0,3334063	-0,0030289
10	-0,0352202	0,0254054	21,0173340	-0,1909379

Lag : 44

	1	2	3	4	5
1	0,0010742	-0,0007538	-0,0323410	-0,0005155	0,0170251
2	0,0000111	-0,0000078	-0,0003354	-0,0000114	0,0003776
3	0,0000065	-0,0000045	-0,0001942	-0,0000074	0,0002433
4	0,0001343	-0,0000942	-0,0040434	-0,0001426	0,0047110
5	0,0022537	-0,0015811	-0,0678415	-0,0010814	0,0357122
6	0,0030239	-0,0021213	-0,0910206	-0,0024250	0,0800869
7	-0,0005168	0,0003626	0,0155566	0,0021877	-0,0722501
8	-0,0002120	0,0001487	0,0063821	-0,0006066	0,0200316
9	0,0000414	-0,0000290	-0,0012458	0,0002375	-0,0078449
10	-0,0060235	0,0042257	0,1813107	0,0191565	-0,6326509

	6	7	8	9
1	-0,0003925	0,0002386	0,2135947	-0,0019405
2	0,0000088	-0,0000066	-0,0053658	0,0000487
3	0,0000067	-0,0000049	-0,0040248	0,0000366
4	0,0001162	-0,0000866	-0,0706857	0,0006422
5	-0,0008257	0,0005012	0,4488364	-0,0040776
6	0,0009740	-0,0007953	-0,6249575	0,0056775
7	-0,0040636	0,0029169	2,4186573	-0,0219729
8	0,0017094	-0,0012319	-1,0191498	0,0092587
9	-0,0005545	0,0003885	0,3256893	-0,0029588
10	-0,0335108	0,0241735	19,9981842	-0,1816792

Lag : 45

	1	2	3	4	5
1	0,0010037	-0,0007043	-0,0302191	-0,0004615	0,0152411
2	0,0000106	-0,0000074	-0,0003195	-0,0000107	0,0003549
3	0,0000062	-0,0000043	-0,0001858	-0,0000070	0,0002297
4	0,0001281	-0,0000899	-0,0038573	-0,0001343	0,0044340
5	0,0021055	-0,0014771	-0,0633783	-0,0009680	0,0319700
6	0,0028585	-0,0020052	-0,0860424	-0,0022527	0,0743959
7	-0,0005505	0,0003862	0,0165699	0,0021248	-0,0701733
8	-0,0001722	0,0001208	0,0051844	-0,0006060	0,0200122
9	0,0000301	-0,0000211	-0,0009050	0,0002342	-0,0077357
10	-0,0061957	0,0043465	0,1864977	0,0185505	-0,6126390

	6	7	8	9
1	-0,0004235	0,0002648	0,2339139	-0,0021252
2	0,0000081	-0,0000061	-0,0049233	0,0000447
3	0,0000062	-0,0000046	-0,0037421	0,0000340
4	0,0001070	-0,0000799	-0,0651687	0,0005920
5	-0,0008905	0,0005560	0,4914317	-0,0044646
6	0,0008173	-0,0006768	-0,5287477	0,0048035
7	-0,0038788	0,0027863	2,3096495	-0,0209826
8	0,0016321	-0,0011746	-0,9723383	0,0088335
9	-0,0005390	0,0003789	0,3171749	-0,0028814
10	-0,0318788	0,0229989	19,0258331	-0,1728457

Lag : 46

	1	2	3	4	5
1	0,0009372	-0,0006575	-0,0282125	-0,0004113	0,0135838
2	0,0000101	-0,0000071	-0,0003043	-0,0000101	0,0003333
3	0,0000059	-0,0000041	-0,0001776	-0,0000066	0,0002167
4	0,0001222	-0,0000857	-0,0036777	-0,0001263	0,0041711
5	0,0019653	-0,0013788	-0,0591612	-0,0008628	0,0284941
6	0,0027004	-0,0018944	-0,0812845	-0,0020905	0,0690400
7	-0,0005792	0,0004063	0,0174338	0,0020617	-0,0680893
8	-0,0001360	0,0000954	0,0040925	-0,0006034	0,0199281
9	0,0000197	-0,0000138	-0,0005923	0,0002306	-0,0076153
10	-0,0063316	0,0044419	0,1905872	0,0179471	-0,5927111

	6	7	8	9
1	-0,0004483	0,0002863	0,2504233	-0,0022752
2	0,0000074	-0,0000055	-0,0045084	0,0000410
3	0,0000057	-0,0000042	-0,0034739	0,0000316
4	0,0000984	-0,0000736	-0,0599749	0,0005449
5	-0,0009426	0,0006012	0,5260905	-0,0047795
6	0,0006753	-0,0005690	-0,4414281	0,0040102
7	-0,0037000	0,0026597	2,2040834	-0,0200236
8	0,0015599	-0,0011213	-0,9286799	0,0084368
9	-0,0005225	0,0003684	0,3079914	-0,0027980
10	-0,0303189	0,0218776	18,0971680	-0,1644088

Lag : 47

	1	2	3	4	5
1	0,0008741	-0,0006132	-0,0263128	-0,0003647	0,0120455
2	0,0000096	-0,0000067	-0,0002896	-0,0000095	0,0003129
3	0,0000056	-0,0000040	-0,0001698	-0,0000062	0,0002043
4	0,0001164	-0,0000817	-0,0035048	-0,0001187	0,0039216
5	0,0018331	-0,0012860	-0,0551796	-0,0007651	0,0252678
6	0,0025495	-0,0017886	-0,0767469	-0,0019380	0,0640013
7	-0,0006033	0,0004232	0,0181600	0,0019986	-0,0660049
8	-0,0001031	0,0000723	0,0031041	-0,0005991	0,0197861
9	0,0000102	-0,0000072	-0,0003070	0,0002267	-0,0074853
10	-0,0064347	0,0045142	0,1936933	0,0173480	-0,5729252

	6	7	8	9
1	-0,0004677	0,0003037	0,2635965	-0,0023948
2	0,0000067	-0,0000051	-0,0041199	0,0000374
3	0,0000053	-0,0000039	-0,0032198	0,0000293
4	0,0000903	-0,0000677	-0,0550928	0,0005005
5	-0,0009830	0,0006375	0,5535878	-0,0050294
6	0,0005470	-0,0004712	-0,3623439	0,0032917
7	-0,0035271	0,0025373	2,1019678	-0,0190959
8	0,0014919	-0,0010713	-0,8877320	0,0080648
9	-0,0005052	0,0003572	0,2982743	-0,0027098
10	-0,0288271	0,0208063	17,2094574	-0,1563441

Lag : 48

	1	2	3	4	5
1	0,0008142	-0,0005713	-0,0245132	-0,0003216	0,0106195
2	0,0000092	-0,0000064	-0,0002755	-0,0000089	0,0002936
3	0,0000054	-0,0000038	-0,0001621	-0,0000058	0,0001925
4	0,0001109	-0,0000778	-0,0033382	-0,0001116	0,0036849
5	0,0017076	-0,0011981	-0,0514078	-0,0006745	0,0222766
6	0,0024056	-0,0016877	-0,0724163	-0,0017945	0,0592638
7	-0,0006233	0,0004373	0,0187613	0,0019357	-0,0639268
8	-0,0000732	0,0000514	0,0022039	-0,0005933	0,0195931
9	0,0000015	-0,0000011	-0,0000465	0,0002225	-0,0073470
10	-0,0065080	0,0045656	0,1958979	0,0167547	-0,5533323

	6	7	8	9
1	-0,0004825	0,0003174	0,2738746	-0,0024882
2	0,0000061	-0,0000046	-0,0037566	0,0000341
3	0,0000049	-0,0000036	-0,0029793	0,0000271
4	0,0000826	-0,0000621	-0,0505108	0,0004589
5	-0,0010139	0,0006663	0,5750655	-0,0052245
6	0,0004312	-0,0003827	-0,2908845	0,0026425
7	-0,0033603	0,0024189	2,0033264	-0,0181998
8	0,0014277	-0,0010244	-0,8491845	0,0077146
9	-0,0004874	0,0003455	0,2881765	-0,0026180
10	-0,0273993	0,0197819	16,3602448	-0,1486293

Lag : 49

	1	2	3	4	5
1	0,0007579	-0,0005317	-0,0228128	-0,0002816	0,0092994
2	0,0000087	-0,0000061	-0,0002619	-0,0000083	0,0002753
3	0,0000051	-0,0000036	-0,0001548	-0,0000055	0,0001814
4	0,0001056	-0,0000741	-0,0031781	-0,0001048	0,0034605
5	0,0015894	-0,0011153	-0,0478430	-0,0005907	0,0195077
6	0,0022687	-0,0015916	-0,0682926	-0,0016597	0,0548114
7	-0,0006394	0,0004486	0,0192469	0,0018731	-0,0618604
8	-0,0000461	0,0000323	0,0013871	-0,0005861	0,0193553
9	-0,0000063	0,0000044	0,0001891	0,0002181	-0,0072016
10	-0,0065540	0,0045979	0,1972849	0,0161687	-0,5339770

	6	7	8	9
1	-0,0004933	0,0003279	0,2815861	-0,0025582
2	0,0000056	-0,0000042	-0,0034174	0,0000310
3	0,0000045	-0,0000034	-0,0027523	0,0000250
4	0,0000755	-0,0000569	-0,0462162	0,0004199
5	-0,0010364	0,0006883	0,5911741	-0,0053706
6	0,0003269	-0,0003028	-0,2264494	0,0020571
7	-0,0031994	0,0023047	1,9081259	-0,0173349
8	0,0013668	-0,0009801	-0,8127065	0,0073832
9	-0,0004692	0,0003334	0,2778044	-0,0025238
10	-0,0260325	0,0188019	15,5475674	-0,1412460

Lag : 50

	1	2	3	4	5
1	0,0007046	-0,0004944	-0,0212107	-0,0002446	0,0080787
2	0,0000083	-0,0000058	-0,0002489	-0,0000078	0,0002580
3	0,0000049	-0,0000034	-0,0001477	-0,0000052	0,0001708
4	0,0001005	-0,0000705	-0,0030242	-0,0000983	0,0032478
5	0,0014779	-0,0010368	-0,0444880	-0,0005132	0,0169474
6	0,0021382	-0,0015000	-0,0643616	-0,0015331	0,0506296
7	-0,0006520	0,0004574	0,0196254	0,0018111	-0,0598107
8	-0,0000217	0,0000152	0,0006525	-0,0005777	0,0190787
9	-0,0000134	0,0000094	0,0004024	0,0002135	-0,0070501
10	-0,0065757	0,0046131	0,1979368	0,0155910	-0,5148984

	6	7	8	9
1	-0,0005006	0,0003357	0,2870623	-0,0026079
2	0,0000050	-0,0000038	-0,0031010	0,0000282
3	0,0000042	-0,0000031	-0,0025382	0,0000231
4	0,0000689	-0,0000520	-0,0421969	0,0003834
5	-0,0010515	0,0007045	0,6026220	-0,0054747
6	0,0002331	-0,0002308	-0,1684147	0,0015300
7	-0,0030443	0,0021945	1,8163586	-0,0165012
8	0,0013087	-0,0009380	-0,7780317	0,0070682
9	-0,0004510	0,0003211	0,2672937	-0,0024283
10	-0,0247238	0,0178638	14,7695427	-0,1341779

Lag : 51

	1	2	3	4	5
1	0,0006545	-0,0004592	-0,0197020	-0,0002105	0,0069519
2	0,0000079	-0,0000055	-0,0002365	-0,0000073	0,0002416
3	0,0000047	-0,0000033	-0,0001409	-0,0000049	0,0001607
4	0,0000956	-0,0000670	-0,0028761	-0,0000922	0,0030463
5	0,0013728	-0,0009631	-0,0413246	-0,0004416	0,0145838
6	0,0020140	-0,0014129	-0,0606270	-0,0014142	0,0467047
7	-0,0006614	0,0004640	0,0199083	0,0017496	-0,0577825
8	0,0000004	-0,0000003	-0,0000128	-0,0005683	0,0187684
9	-0,0000197	0,0000138	0,0005943	0,0002087	-0,0068934
10	-0,0065753	0,0046128	0,1979241	0,0150227	-0,4961301

	6	7	8	9
1	-0,0005049	0,0003410	0,2906201	-0,0026402
2	0,0000046	-0,0000035	-0,0028062	0,0000255
3	0,0000038	-0,0000029	-0,0023368	0,0000212
4	0,0000626	-0,0000474	-0,0384404	0,0003492
5	-0,0010599	0,0007154	0,6098875	-0,0055408
6	0,0001490	-0,0001660	-0,1162648	0,0010561
7	-0,0020952	0,0020883	1,7279863	-0,0156983
8	0,0012532	-0,0008980	-0,7449698	0,0067679
9	-0,0004327	0,0003087	0,2567346	-0,0023324
10	-0,0234706	0,0169658	14,0245514	-0,1274099

Lag : 52

	1	2	3	4	5
1	0,0006073	-0,0004261	-0,0182819	-0,0001791	0,0059134
2	0,0000075	-0,0000052	-0,0002245	-0,0000068	0,0002260
3	0,0000045	-0,0000031	-0,0001343	-0,0000046	0,0001511
4	0,0000908	-0,0000637	-0,0027341	-0,0000865	0,0028554
5	0,0012738	-0,0008936	-0,0383444	-0,0003756	0,0124049
6	0,0018959	-0,0013301	-0,0570707	-0,0013027	0,0430234
7	-0,0006678	0,0004685	0,0201022	0,0016890	-0,0557793
8	0,0000203	-0,0000142	-0,0006099	-0,0005580	0,0184291
9	-0,0000254	0,0000178	0,0007653	0,0002039	-0,0067323
10	-0,0065550	0,0045986	0,1973143	0,0144646	-0,4777010

	6	7	8	9
1	-0,0005067	0,0003442	0,2925904	-0,0026581
2	0,0000041	-0,0000031	-0,0025321	0,0000230
3	0,0000035	-0,0000026	-0,0021475	0,0000195
4	0,0000568	-0,0000431	-0,0349341	0,0003174
5	-0,0010637	0,0007222	0,6139836	-0,0055779
6	0,0000734	-0,0001077	-0,0694243	0,0006307
7	-0,0027518	0,0019861	1,6429892	-0,0149262
8	0,0012000	-0,0008598	-0,7133271	0,0064804
9	-0,0004146	0,0002963	0,2462279	-0,0022369
10	-0,0222705	0,0161059	13,3112392	-0,1209295

Lag : 53

	1	2	3	4	5
1	0,0005630	-0,0003950	-0,0169468	-0,0001501	0,0049580
2	0,0000071	-0,0000050	-0,0002130	-0,0000064	0,0002114
3	0,0000042	-0,0000030	-0,0001279	-0,0000043	0,0001420
4	0,0000863	-0,0000605	-0,0025979	-0,0000810	0,0026748
5	0,0011808	-0,0008284	-0,0355444	-0,0003149	0,0104008
6	0,0017837	-0,0012513	-0,0536900	-0,0011983	0,0395734
7	-0,0006716	0,0004711	0,0202158	0,0016292	-0,0538051
8	0,0000382	-0,0000268	-0,0011494	-0,0005470	0,0180654
9	-0,0000305	0,0000214	0,0009177	0,0001989	-0,0065675
10	-0,0065168	0,0045718	0,1961641	0,0139176	-0,4596357

	6	7	8	9
1	-0,0005063	0,0003456	0,2931150	-0,0026629
2	0,0000037	-0,0000028	-0,0022772	0,0000207
3	0,0000032	-0,0000024	-0,0019699	0,0000179
4	0,0000514	-0,0000392	-0,0316648	0,0002877
5	-0,0010627	0,0007252	0,6150702	-0,0055878
6	0,0000058	-0,0000555	-0,0274656	0,0002495
7	-0,0026140	0,0018879	1,5612917	-0,0141840
8	0,0011488	-0,0008232	-0,6829223	0,0062042
9	-0,0003967	0,0002840	0,2358252	-0,0021424
10	-0,0211217	0,0152828	12,6283159	-0,1147251

Lag : 54

	1	2	3	4	5
1	0,0005212	-0,0003657	-0,0156908	-0,0001236	0,0040805
2	0,0000067	-0,0000047	-0,0002020	-0,0000060	0,0001975
3	0,0000040	-0,0000028	-0,0001218	-0,0000040	0,0001334
4	0,0000820	-0,0000575	-0,0024671	-0,0000758	0,0025039
5	0,0010932	-0,0007669	-0,0329056	-0,0002592	0,0085601
6	0,0016771	-0,0011765	-0,0504827	-0,0011004	0,0363421
7	-0,0006730	0,0004721	0,0202574	0,0015704	-0,0518627
8	0,0000542	-0,0000380	-0,0016303	-0,0005354	0,0176811
9	-0,0000350	0,0000245	0,0010533	0,0001938	-0,0063999
10	-0,0064627	0,0045338	0,1945342	0,0133823	-0,4419547

	6	7	8	9
1	-0,0005040	0,0003456	0,2924587	-0,0026569
2	0,0000033	-0,0000025	-0,0020407	0,0000185
3	0,0000029	-0,0000022	-0,0018035	0,0000164
4	0,0000464	-0,0000354	-0,0286204	0,0002600
5	-0,0010578	0,0007250	0,6136534	-0,0055749
6	-0,0000546	-0,0000087	0,0100715	-0,0000915
7	-0,0024818	0,0017935	1,4828463	-0,0134713
8	0,0010994	-0,0007879	-0,6536524	0,0059383
9	-0,0003793	0,0002718	0,2256112	-0,0020496
10	-0,0200224	0,0144949	11,9746513	-0,1087869

Lag : 55

	1	2	3	4	5
1	0,0004820	-0,0003382	-0,0145102	-0,0000992	0,0032761
2	0,0000064	-0,0000045	-0,0001915	-0,0000056	0,0001844
3	0,0000039	-0,0000027	-0,0001159	-0,0000038	0,0001252
4	0,0000778	-0,0000546	-0,0023417	-0,0000709	0,0023423
5	0,0010110	-0,0007093	-0,0304346	-0,0002081	0,0068730
6	0,0015759	-0,0011056	-0,0474377	-0,0010089	0,0333183
7	-0,0006722	0,0004715	0,0202331	0,0015126	-0,0499549
8	0,0000684	-0,0000480	-0,0020584	-0,0005232	0,0172800
9	-0,0000389	0,0000273	0,0011716	0,0001886	-0,0062299
10	-0,0063943	0,0044858	0,1924753	0,0128590	-0,4246749

	6	7	8	9
1	-0,0005001	0,0003441	0,2907634	-0,0026416
2	0,0000029	-0,0000023	-0,0018214	0,0000165
3	0,0000027	-0,0000020	-0,0016478	0,0000150
4	0,0000417	-0,0000320	-0,0257889	0,0002343
5	-0,0010496	0,0007220	0,6100855	-0,0055424
6	-0,0001084	0,0000331	0,0435590	-0,0003956
7	-0,0023551	0,0017029	1,4075918	-0,0127876
8	0,0010517	-0,0007540	-0,6254565	0,0056821
9	-0,0003622	0,0002599	0,2156073	-0,0019587
10	-0,0189707	0,0137409	11,3492069	-0,1031048

Lag : 56

	1	2	3	4	5
1	0,0004452	-0,0003123	-0,0134020	-0,0000769	0,0025404
2	0,0000060	-0,0000042	-0,0001815	-0,0000052	0,0001721
3	0,0000037	-0,0000026	-0,0001103	-0,0000036	0,0001175
4	0,0000738	-0,0000518	-0,0022216	-0,0000663	0,0021896
5	0,0009339	-0,0006552	-0,0281105	-0,0001614	0,0053297
6	0,0014799	-0,0010382	-0,0445490	-0,0009233	0,0304907
7	-0,0006694	0,0004696	0,0201488	0,0014560	-0,0480840
8	0,0000810	-0,0000568	-0,0024378	-0,0005107	0,0168649
9	-0,0000424	0,0000297	0,0012750	0,0001834	-0,0060583
10	-0,0063133	0,0044290	0,1900353	0,0123484	-0,4078100

	6	7	8	9
1	-0,0004948	0,0003416	0,2882150	-0,0026183
2	0,0000026	-0,0000020	-0,0016183	0,0000147
3	0,0000024	-0,0000019	-0,0015022	0,0000136
4	0,0000374	-0,0000288	-0,0231590	0,0002104
5	-0,0010385	0,0007167	0,6047081	-0,0054936
6	-0,0001563	0,0000703	0,0733886	-0,0006667
7	-0,0022337	0,0016160	1,3354454	-0,0121322
8	0,0010058	-0,0007213	-0,5982596	0,0054350
9	-0,0003457	0,0002483	0,2058596	-0,0018702
10	-0,0179649	0,0130196	10,7509336	-0,0976696

Lag : 57

	1	2	3	4	5
1	0,0004107	-0,0002882	-0,0123653	-0,0000566	0,0018691
2	0,0000057	-0,0000040	-0,0001719	-0,0000049	0,0001604
3	0,0000035	-0,0000024	-0,0001049	-0,0000033	0,0001102
4	0,0000700	-0,0000491	-0,0021066	-0,0000619	0,0020453
5	0,0008615	-0,0006044	-0,0259342	-0,0001187	0,0039215
6	0,0013890	-0,0009744	-0,0418081	-0,0008433	0,0278489
7	-0,0006648	0,0004664	0,0200122	0,0014005	-0,0462519
8	0,0000922	-0,0000646	-0,0027739	-0,0004978	0,0164390
9	-0,0000453	0,0000318	0,0013637	0,0001782	-0,0058856
10	-0,0062211	0,0043644	0,1872612	0,0118506	-0,3913712

	6	7	8	9
1	-0,0004884	0,0003382	0,2849239	-0,0025884
2	0,0000023	-0,0000018	-0,0014304	0,0000130
3	0,0000022	-0,0000017	-0,0013664	0,0000124
4	0,0000333	-0,0000258	-0,0207186	0,0001882
5	-0,0010250	0,0007095	0,5977920	-0,0054308
6	-0,0001986	0,0001334	0,0998518	-0,0009071
7	-0,0021174	0,0015328	1,2663364	-0,0115044
8	0,0009614	-0,0006898	-0,5720173	0,0051966
9	-0,0003296	0,0002370	0,1963880	-0,0017841
10	-0,0170034	0,0123298	10,1789007	-0,0924729

Lag : 58

	1	2	3	4	5
1	0,0003785	-0,0002655	-0,0113935	-0,0000381	0,0012579
2	0,0000054	-0,0000038	-0,0001627	-0,0000045	0,0001494
3	0,0000033	-0,0000023	-0,0000997	-0,0000031	0,0001032
4	0,0000663	-0,0000465	-0,0019965	-0,0000578	0,0019791
5	0,0007937	-0,0005568	-0,0238924	-0,0000799	0,0026391
6	0,0013027	-0,0009139	-0,0392141	-0,0007686	0,0253828
7	-0,0006587	0,0004621	0,0198278	0,0013462	-0,0444604
8	0,0001019	-0,0000715	-0,0030673	-0,0004846	0,0160045
9	-0,0000478	0,0000335	0,0014394	0,0001730	-0,0057124
10	-0,0061192	0,0042929	0,1841965	0,0113660	-0,3753667

	6	7	8	9
1	-0,0004811	0,0003339	0,2810427	-0,0025532
2	0,0000020	-0,0000016	-0,0012568	0,0000114
3	0,0000020	-0,0000015	-0,0012397	0,0000113
4	0,0000296	-0,0000230	-0,0184568	0,0001677
5	-0,0010096	0,0007005	0,5896167	-0,0053565
6	-0,0002360	0,0001328	0,1232800	-0,0011199
7	-0,0020061	0,0014531	1,2001877	-0,0109034
8	0,0009185	-0,0006593	-0,5466338	0,0049660
9	-0,0003141	0,0002260	0,1872182	-0,0017008
10	-0,0160849	0,0116705	9,6322603	-0,0875068

Lag : 59

	1	2	3	4	5
1	0,0003482	-0,0002443	-0,0104809	-0,0000213	0,0007029
2	0,0000051	-0,0000036	-0,0001539	-0,0000042	0,0001390
3	0,0000031	-0,0000022	-0,0000947	-0,0000029	0,0000967
4	0,0000628	-0,0000441	-0,0018913	-0,0000539	0,0017805
5	0,0007303	-0,0005123	-0,0219831	-0,0000447	0,0014750
6	0,0012211	-0,0008566	-0,0367584	-0,0006989	0,0230827
7	-0,0006512	0,0004568	0,0196014	0,0012933	-0,0427108
8	0,0001104	-0,0000775	-0,0033246	-0,0004713	0,0155642
9	-0,0000500	0,0000350	0,0015036	0,0001677	-0,0055391
10	-0,0060088	0,0042154	0,1808739	0,0108947	-0,3598028

	6	7	8	9
1	-0,0004730	0,0003290	0,2766373	-0,0025131
2	0,0000017	-0,0000014	-0,0010967	0,0000100
3	0,0000018	-0,0000014	-0,0011218	0,0000102
4	0,0000262	-0,0000205	-0,0163628	0,0001487
5	-0,0009926	0,0006903	0,5803698	-0,0052725
6	-0,0002689	0,0001587	0,1439136	-0,0013074
7	-0,0018997	0,0013768	1,1369209	-0,0103287
8	0,0008770	-0,0006298	-0,5221128	0,0047433
9	-0,0002991	0,0002154	0,1783628	-0,0016204
10	-0,0152079	0,0110406	9,1101465	-0,0827634

Lag : 60

	1	2	3	4	5
1	0,0003198	-0,0002244	-0,0096253	-0,0000061	0,0002003
2	0,0000048	-0,0000034	-0,0001455	-0,0000039	0,0001293
3	0,0000030	-0,0000021	-0,0000899	-0,0000027	0,0000905
4	0,0000595	-0,0000417	-0,0017907	-0,0000502	0,0016592
5	0,0006708	-0,0004706	-0,0201926	-0,0000127	0,0004208
6	0,0011438	-0,0008024	-0,0344305	-0,0006340	0,0209392
7	-0,0006424	0,0004507	0,0193377	0,0012416	-0,0410043
8	0,0001178	-0,0000826	-0,0035452	-0,0004578	0,0151198
9	-0,0000517	0,0000363	0,0015566	0,0001625	-0,0053663
10	-0,0058910	0,0041328	0,1773281	0,0104369	-0,3446829

	6	7	8	9
1	-0,0004642	0,0003236	0,2717892	-0,0024691
2	0,0000015	-0,0000012	-0,0009491	0,0000086
3	0,0000016	-0,0000013	-0,0010121	0,0000092
4	0,0000230	-0,0000181	-0,0144270	0,0001311
5	-0,0009741	0,0006788	0,5702043	-0,0051802
6	-0,0002976	0,0001814	0,1620139	-0,0014718
7	-0,0017981	0,0013039	1,0764370	-0,0097792
8	0,0008369	-0,0006014	-0,4984127	0,0045280
9	-0,0002847	0,0002052	0,1698243	-0,0015428
10	-0,0143710	0,0104392	8,6117258	-0,0782354

Lag : 61

	1	2	3	4	5
1	0,0002933	-0,0002058	-0,0088295	0,0000077	-0,0002533
2	0,0000046	-0,0000032	-0,0001375	-0,0000036	0,0001201
3	0,0000028	-0,0000020	-0,0000853	-0,0000026	0,0000846
4	0,0000563	-0,0000395	-0,0016946	-0,0000468	0,0015448
5	0,0006152	-0,0004316	-0,0185187	0,0000161	-0,0005307
6	0,0010706	-0,0007511	-0,0322266	-0,0005736	0,0189437
7	-0,0006326	0,0004438	0,0190411	0,0011913	-0,0393418
8	0,0001240	-0,0000870	-0,0037322	-0,0004443	0,0146732
9	-0,0000531	0,0000373	0,0015996	0,0001573	-0,0051943
10	-0,0057670	0,0040458	0,1735935	0,0099926	-0,3300098

	6	7	8	9
1	-0,0004549	0,0003177	0,2666066	-0,0024220
2	0,0000013	-0,0000010	-0,0008133	0,0000074
3	0,0000015	-0,0000011	-0,0009101	0,0000083
4	0,0000200	-0,0000159	-0,0126392	0,0001148
5	-0,0009546	0,0006663	0,5593054	-0,0050811
6	-0,0003225	0,0002014	0,1777917	-0,0016152
7	-0,0017010	0,0012342	1,0186567	-0,0092543
8	0,0007982	-0,0005739	-0,4755377	0,0043201
9	-0,0002708	0,0001953	0,1616040	-0,0014681
10	-0,0135728	0,0098653	8,1361799	-0,0739152

Lag : 62

	1	2	3	4	5
1	0,0002685	-0,0001884	-0,0080835	0,0000200	-0,0006615
2	0,0000043	-0,0000030	-0,0001299	-0,0000034	0,0001114
3	0,0000027	-0,0000019	-0,0000809	-0,0000024	0,0000790
4	0,0000532	-0,0000374	-0,0016028	-0,0000435	0,0014371
5	0,0005632	-0,0003951	-0,0169542	0,0000420	-0,0013869
6	0,0010015	-0,0007026	-0,0301447	-0,0005174	0,0170875
7	-0,0006217	0,0004362	0,0187154	0,0011423	-0,0377240
8	0,0001292	-0,0000906	-0,0038894	-0,0004308	0,0142262
9	-0,0000543	0,0000381	0,0016331	0,0001521	-0,0050237
10	-0,0056378	0,0039552	0,1697063	0,0095618	-0,3157837

	6	7	8	9
1	-0,0004452	0,0003113	0,2611157	-0,0023722
2	0,0000011	-0,0000009	-0,0006884	0,0000063
3	0,0000013	-0,0000010	-0,0008155	0,0000074
4	0,0000173	-0,0000139	-0,0109901	0,0000998
5	-0,0009341	0,0006531	0,5477722	-0,0049764
6	-0,0003440	0,0002186	0,1914422	-0,0017392
7	-0,0016084	0,0011677	0,9634871	-0,0087531
8	0,0007609	-0,0005474	-0,4534563	0,0041195
9	-0,0002575	0,0001858	0,1537082	-0,0013964
10	-0,0128119	0,0093179	7,6827145	-0,0697957

Lag : 63

	1	2	3	4	5
1	0,0002454	-0,0001722	-0,0073871	0,0000311	-0,0010274
2	0,0000041	-0,0000029	-0,0001227	-0,0000031	0,0001033
3	0,0000025	-0,0000018	-0,0000767	-0,0000022	0,0000738
4	0,0000503	-0,0000353	-0,0015152	-0,0000404	0,0013356
5	0,0005147	-0,0003611	-0,0154934	0,0000652	-0,0021545
6	0,0009361	-0,0006567	-0,0281760	-0,0004652	0,0153627
7	-0,0006101	0,0004280	0,0183657	0,0010947	-0,0361514
8	0,0001336	-0,0000937	-0,0040214	-0,0004173	0,0137799
9	-0,0000551	0,0000387	0,0016594	0,0001470	-0,0048546
10	-0,0055042	0,0038614	0,1656847	0,0091446	-0,3020039

	6	7	8	9
1	-0,0004350	0,0003047	0,2553731	-0,0023200
2	0,0000009	-0,0000007	-0,0005740	0,0000052
3	0,0000012	-0,0000009	-0,0007278	0,0000066
4	0,0000148	-0,0000120	-0,0094713	0,0000860
5	-0,0009127	0,0006391	0,5357121	-0,0048668
6	-0,0003623	0,0002335	0,2031510	-0,0018455
7	-0,0015200	0,0011041	0,9108413	-0,0062748
8	0,0007248	-0,0005218	-0,4321391	0,0039259
9	-0,0002447	0,0001767	0,1461293	-0,0013276
10	-0,0120871	0,0087960	7,2505636	-0,0658697

Lag : 64

	1	2	3	4	5
1	0,0002238	-0,0001570	-0,0067380	0,0000410	-0,0013540
2	0,0000038	-0,0000027	-0,0001157	-0,0000029	0,0000957
3	0,0000024	-0,0000017	-0,0000726	-0,0000021	0,0000689
4	0,0000476	-0,0000334	-0,0014317	-0,0000376	0,0012402
5	0,0004694	-0,0003293	-0,0141309	0,0000860	-0,0028395
6	0,0008743	-0,0006134	-0,0263196	-0,0004167	0,0137616
7	-0,0005978	0,0004194	0,0179945	0,0010484	-0,0346243
8	0,0001372	-0,0000962	-0,0041285	-0,0004038	0,0133358
9	-0,0000557	0,0000391	0,0016770	0,0001419	-0,0046875
10	-0,0053670	0,0037652	0,1615582	0,0087408	-0,2886682

	6	7	8	9
1	-0,0004245	0,0002978	0,2494042	-0,0022658
2	0,0000007	-0,0000006	-0,0004691	0,0000043
3	0,0000010	-0,0000008	-0,0006466	0,0000059
4	0,0000126	-0,0000103	-0,0080745	0,0000734
5	-0,0008908	0,0006246	0,5232309	-0,0047534
6	-0,0003776	0,0002462	0,2130784	-0,0019357
7	-0,0014358	0,0010435	0,8606342	-0,0078187
8	0,0006902	-0,0004972	-0,4116101	0,0037394
9	-0,0002325	0,0001679	0,1388551	-0,0012615
10	-0,0113969	0,0082988	6,8389521	-0,0621301

Lag : 65

	1	2	3	4	5
1	0,0002037	-0,0001429	-0,0061315	0,0000498	-0,0016443
2	0,0000036	-0,0000025	-0,0001092	-0,0000027	0,0000885
3	0,0000023	-0,0000016	-0,0000688	-0,0000019	0,0000642
4	0,0000449	-0,0000315	-0,0013521	-0,0000348	0,0011503
5	0,0004272	-0,0002997	-0,0128601	0,0001044	-0,0034482
6	0,0008161	-0,0005725	-0,0245654	-0,0003717	0,0122768
7	-0,0005849	0,0004103	0,0176049	0,0010036	-0,0331428
8	0,0001399	-0,0000982	-0,0042117	-0,0003905	0,0128951
9	-0,0000561	0,0000393	0,0016878	0,0001369	-0,0045227
10	-0,0052271	0,0036670	0,1573439	0,0083503	-0,2757730

	6	7	8	9
1	-0,0004138	0,0002906	0,2432806	-0,0022101
2	0,0000005	-0,0000005	-0,0003732	0,0000034
3	0,0000009	-0,0000007	-0,0005715	0,0000052
4	0,0000105	-0,0000087	-0,0067915	0,0000617
5	-0,0008683	0,0006097	0,5104108	-0,0046369
6	-0,0003903	0,0002569	0,2213739	-0,0020111
7	-0,0013555	0,0009857	0,8127757	-0,0073839
8	0,0006568	-0,0004734	-0,3918325	0,0035597
9	-0,0002207	0,0001595	0,1318812	-0,0011981
10	-0,0107401	0,0078254	6,4471035	-0,0585703

Lag : 66

	1	2	3	4	5
1	0,0001849	-0,0001297	-0,0055659	0,0000576	-0,0019009
2	0,0000034	-0,0000024	-0,0001029	-0,0000025	0,0000818
3	0,0000022	-0,0000015	-0,0000651	-0,0000018	0,0000598
4	0,0000424	-0,0000297	-0,0012762	-0,0000323	0,0010659
5	0,0003879	-0,0002721	-0,0116755	0,0001207	-0,0039863
6	0,0007611	-0,0005339	-0,0229093	-0,0003301	0,0109015
7	-0,0005714	0,0004009	0,0172002	0,0009601	-0,0317069
8	0,0001420	-0,0000996	-0,0042738	-0,0003773	0,0124589
9	-0,0000562	0,0000394	0,0016924	0,0001320	-0,0043604
10	-0,0050852	0,0035675	0,1530695	0,0079731	-0,2633142

	6	7	8	9
1	-0,0004029	0,0002833	0,2370449	-0,0021535
2	0,0000004	-0,0000004	-0,0002857	0,0000026
3	0,0000008	-0,0000006	-0,0005022	0,0000046
4	0,0000085	-0,0000073	-0,0056154	0,0000510
5	-0,0008454	0,0005944	0,4973347	-0,0045181
6	-0,0004006	0,0002658	0,2281761	-0,0020729
7	-0,0012790	0,0009307	0,7671792	-0,0069696
8	0,0006247	-0,0004505	-0,3728160	0,0033869
9	-0,0002095	0,0001515	0,1252021	-0,0011374
10	-0,0101154	0,0073749	6,0742731	-0,0551834

Lag : 67

	1	2	3	4	5
1	0,0001674	-0,0001175	-0,0050406	0,0000644	-0,0021263
2	0,0000032	-0,0000023	-0,0000969	-0,0000023	0,0000754
3	0,0000020	-0,0000014	-0,0000616	-0,0000017	0,0000556
4	0,0000400	-0,0000281	-0,0012040	-0,0000299	0,0009865
5	0,0003513	-0,0002464	-0,0105745	0,0001350	-0,0044589
6	0,0007092	-0,0004975	-0,0213480	-0,0002916	0,0096289
7	-0,0005576	0,0003912	0,0167834	0,0009180	-0,0303165
8	0,0001434	-0,0001006	-0,0043176	-0,0003642	0,0120278
9	-0,0000562	0,0000394	0,0016913	0,0001272	-0,0042008
10	-0,0049417	0,0034668	0,1487522	0,0076089	-0,2512863

	6	7	8	9
1	-0,0003918	0,0002759	0,2307100	-0,0020959
2	0,0000003	-0,0000003	-0,0002059	0,0000019
3	0,0000007	-0,0000006	-0,0004382	0,0000040
4	0,0000068	-0,0000059	-0,0045387	0,0000412
5	-0,0008223	0,0005788	0,4840460	-0,0043974
6	-0,0004086	0,0002729	0,2336191	-0,0021224
7	-0,0012062	0,0008782	0,7237579	-0,0065752
8	0,0005939	-0,0004286	-0,3545361	0,0032209
9	-0,0001987	0,0001438	0,1188031	-0,0010793
10	-0,0095215	0,0069463	5,7197380	-0,0519625

Lag : 68

	1	2	3	4	5
1	0,0001512	-0,0001061	-0,0045502	0,0000703	-0,0023229
2	0,0000030	-0,0000021	-0,0000913	-0,0000021	0,0000695
3	0,0000019	-0,0000014	-0,0000583	-0,0000016	0,0000517
4	0,0000377	-0,0000265	-0,0011352	-0,0000276	0,0009120
5	0,0003171	-0,0002225	-0,0095450	0,0001475	-0,0048712
6	0,0006603	-0,0004632	-0,0198752	-0,0002560	0,0084529
7	-0,0005434	0,0003812	0,0163569	0,0008772	-0,0289713
8	0,0001442	-0,0001012	-0,0043420	-0,0003513	0,0116029
9	-0,0000560	0,0000393	0,0016853	0,0001225	-0,0040443
10	-0,0047975	0,0033656	0,1444111	0,0072575	-0,2396834

	6	7	8	9
1	-0,0003808	0,0002684	0,2243199	-0,0020379
2	0,0000002	-0,0000002	-0,0001335	0,0000012
3	0,0000006	-0,0000005	-0,0003792	0,0000034
4	0,0000052	-0,0000047	-0,0035550	0,0000323
5	-0,0007991	0,0005631	0,4706551	-0,0042758
6	-0,0004147	0,0002786	0,2378299	-0,0021606
7	-0,0011370	0,0008283	0,6824347	-0,0061998
8	0,0005642	-0,0004074	-0,3369401	0,0030610
9	-0,0001884	0,0001364	0,1126840	-0,0010237
10	-0,0089573	0,0065389	5,3828030	-0,0489013

Lag : 69

	1	2	3	4	5
1	0,0001361	-0,0000955	-0,0040968	0,0000755	-0,0024929
2	0,0000029	-0,0000020	-0,0000859	-0,0000019	0,0000640
3	0,0000018	-0,0000013	-0,0030551	-0,0000015	0,0000481
4	0,0000355	-0,0000249	-0,0010698	-0,0000255	0,0008421
5	0,0002855	-0,0002003	-0,0085931	0,0001583	-0,0052276
6	0,0006142	-0,0004309	-0,0184802	-0,0002231	0,0073675
7	-0,0005290	0,0003711	0,0159222	0,0008379	-0,0276709
8	0,0001446	-0,0001014	-0,0043528	-0,0003387	0,0111848
9	-0,0000556	0,0000390	0,0016746	0,0001178	-0,0038909
10	-0,0046529	0,0032642	0,1400576	0,0069189	-0,2284986

	6	7	8	9
1	-0,0003696	0,0002608	0,2178807	-0,0019794
2	0,0000001	-0,0000031	-0,0000678	0,0000006
3	0,0000005	-0,0000004	-0,0003249	0,0000030
4	0,0000037	-0,0000036	-0,0026580	0,0000241
5	-0,0007757	0,0005472	0,4571640	-0,0041532
6	-0,0004189	0,0002828	0,2408993	-0,0021885
7	-0,0010711	0,0007808	0,6431189	-0,0058426
8	0,0005357	-0,0003871	-0,3200549	0,0029076
9	-0,0001786	0,0001293	0,1068331	-0,0009706
10	-0,0084216	0,0061518	5,0627203	-0,0459937

Lag : 70

	1	2	3	4	5
1	0,0001221	-0,0000856	-0,0036741	0,0000799	-0,0026385
2	0,0000027	-0,0000019	-0,0000808	-0,0000018	0,0000588
3	0,0000017	-0,0000012	-0,0000520	-0,0000014	0,0000446
4	0,0000335	-0,0000235	-0,0010075	-0,0000235	0,0007766
5	0,0002561	-0,0001796	-0,0077078	0,0001675	-0,0055330
6	0,0005708	-0,0004004	-0,0171817	-0,0001928	0,0063670
7	-0,0005143	0,0003608	0,0154822	0,0007998	-0,0264149
8	0,0001444	-0,0001013	-0,0043481	-0,0003262	0,0107740
9	-0,0000551	0,0000387	0,0016600	0,0001133	-0,0037408
10	-0,0045084	0,0031628	0,1357098	0,0065926	-0,2177246

	6	7	8	9
1	-0,0003584	0,0002532	0,2114248	-0,0019207
2	-0,0000000	-0,0000000	-0,0000784	0,0000001
3	0,0000004	-0,0000004	-0,0002750	0,0000025
4	0,0000024	-0,0000026	-0,0018418	0,0000167
5	-0,0007523	0,0005312	0,4436209	-0,0040302
6	-0,0004214	0,0002858	0,2429470	-0,0022071
7	-0,0010085	0,0007356	0,6057375	-0,0055030
8	0,0005084	-0,0003676	-0,3038469	0,0027604
9	-0,0001692	0,0001226	0,1012411	-0,0009198
10	-0,0079131	0,0057843	4,7588491	-0,0432331

Lag : 71

	1	2	3	4	5
1	0,0001091	-0,0000765	-0,0032835	0,0000836	-0,0027617
2	0,0000025	-0,0000018	-0,0000759	-0,0000016	0,0000539
3	0,0000016	-0,0000011	-0,0000491	-0,0000013	0,0000413
4	0,0000315	-0,0000221	-0,0009484	-0,0000217	0,0007151
5	0,0002288	-0,0001605	-0,0068864	0,0001754	-0,0057912
6	0,0005299	-0,0003718	-0,0159522	-0,0001649	0,0054461
7	-0,0004996	0,0003505	0,0150387	0,0007631	-0,0252027
8	0,0001439	-0,0001010	-0,0043318	-0,0003140	0,0103711
9	-0,0000545	0,0000383	0,0016416	0,0001088	-0,0035941
10	-0,0043645	0,0030619	0,1313782	0,0062786	-0,2073534

	6	7	8	9
1	-0,0003473	0,0002456	0,2049767	-0,0018621
2	-0,0000001	0,0000000	0,0000452	-0,0000004
3	0,0000004	-0,0000003	-0,0002293	0,0000021
4	0,0000012	-0,0000017	-0,0011007	0,0000100
5	-0,0007290	0,0005152	0,4300947	-0,0039073
6	-0,0004223	0,0002876	0,2440565	-0,0022172
7	-0,0009490	0,0006926	0,5702114	-0,0051802
8	0,0004823	-0,0003489	-0,2883031	0,0026192
9	-0,0001602	0,0001161	0,0958977	-0,0008712
10	-0,0074308	0,0054354	4,4705429	-0,0406139

Lag : 72

	1	2	3	4	5
1	0,0000970	-0,0000680	-0,0029198	0,0000867	-0,0028642
2	0,0000024	-0,0000017	-0,0000713	-0,0000015	0,0000494
3	0,0000015	-0,0000011	-0,0000463	-0,0000012	0,0000383
4	0,0000296	-0,0000208	-0,0008922	-0,0000199	0,0006576
5	0,0002035	-0,0001428	-0,0061256	0,0001819	-0,0060062
6	0,0004915	-0,0003448	-0,0147960	-0,0001393	0,0045998
7	-0,0004848	0,0003401	0,0145932	0,0007277	-0,0240336
8	0,0001430	-0,0001003	-0,0043031	-0,0003021	0,0099766
9	-0,0000538	0,0000378	0,0016201	0,0001045	-0,0034510
10	-0,0042216	0,0029616	0,1270742	0,0059765	-0,1973768

	6	7	8	9
1	-0,0003363	0,0002380	0,1985531	-0,0018038
2	-0,0000002	0,0000001	0,0000932	-0,0000008
3	0,0000003	-0,0000002	-0,0001873	0,0000017
4	0,0000001	-0,0000009	-0,0004295	0,0000339
5	-0,0007057	0,0004993	0,4166016	-0,0037847
6	-0,0004220	0,0002884	0,2443298	-0,0022197
7	-0,0008925	0,0006518	0,5364652	-0,0048737
8	0,0004572	-0,0003309	-0,2734288	0,0024840
9	-0,0001517	0,0001100	0,0907959	-0,0008249
10	-0,0069736	0,0051044	4,1971054	-0,0381298

Lag : 73

	1	2	3	4	5
1	0,0000859	-0,0000603	-0,0025852	0,0000893	-0,0029477
2	0,0000022	-0,0000016	-0,0000669	-0,0000014	0,0000451
3	0,0000015	-0,0000010	-0,0000437	-0,0000011	0,0000354
4	0,0000279	-0,0000196	-0,0008389	-0,0000183	0,0006037
5	0,0001801	-0,0001264	-0,0054227	0,0001872	-0,0061816
6	0,0004554	-0,0003195	-0,0137098	-0,0001158	0,0038232
7	-0,0004700	0,0003297	0,0141471	0,0006936	-0,0229070
8	0,0001417	-0,0000994	-0,0042646	-0,0002904	0,0095907
9	-0,0000530	0,0000372	0,0015957	0,0001003	-0,0033116
10	-0,0040799	0,0028622	0,1228075	0,0056861	-0,1877865

	6	7	8	9
1	-0,0003253	0,0002304	0,1921608	-0,0017457
2	-0,0000003	0,0000001	0,0001363	-0,0000012
3	0,0000002	-0,0000002	-0,0001489	0,0000014
4	-0,0000009	-0,0000001	0,0001765	-0,0000016
5	-0,0006826	0,0004834	0,4031900	-0,0036629
6	-0,0004204	0,0002882	0,2438385	-0,0022152
7	-0,0008389	0,0006130	0,5044261	-0,0045826
8	0,0004332	-0,0003138	-0,2591593	0,0023544
9	-0,0001435	0,0001041	0,0859237	-0,0007806
10	-0,0065403	0,0047907	3,9379349	-0,0357753

Lag : 74

	1	2	3	4	5
1	0,0000756	-0,0000530	-0,0022746	0,0000913	-0,0030141
2	0,0000021	-0,0000015	-0,0000628	-0,0000012	0,0000412
3	0,0000014	-0,0000010	-0,0000412	-0,0000010	0,0000327
4	0,0000262	-0,0000184	-0,0007883	-0,0000168	0,0005534
5	0,0001585	-0,0001112	-0,0047722	0,0001914	-0,0063207
6	0,0004215	-0,0002957	-0,0126886	-0,0000942	0,0031118
7	-0,0004552	0,0003193	0,0137021	0,0006608	-0,0218220
8	0,0001401	-0,0000983	-0,0042169	-0,0002790	0,0092138
9	-0,0000521	0,0000366	0,0015688	0,0000962	-0,0031758
10	-0,0039398	0,0027639	0,1185923	0,0054071	-0,1785724

	6	7	8	9
1	-0,0003144	0,0002229	0,1858274	-0,0016882
2	-0,0000003	0,0000002	0,0001747	-0,0000016
3	0,0000002	-0,0000002	-0,0001140	0,0000010
4	-0,0000018	0,0000006	0,0007218	-0,0000066
5	-0,0006598	0,0004676	0,3898939	-0,0035421
6	-0,0004177	0,0002872	0,2426584	-0,0022045
7	-0,0007881	0,0005762	0,4740199	-0,0043064
8	0,0004103	-0,0002973	-0,2455252	0,0022305
9	-0,0001357	0,0000985	0,0812753	-0,0007384
10	-0,0061300	0,0044933	3,6923943	-0,0335447

Lag : 75

	1	2	3	4	5
1	0,0000660	-0,0000463	-0,0019881	0,0000928	-0,0030646
2	0,0000020	-0,0000014	-0,0000589	-0,0000011	0,0000375
3	0,0000013	-0,0000009	-0,0000388	-0,0000009	0,0000302
4	0,0000246	-0,0000173	-0,0007403	-0,0000153	0,0005063
5	0,0001386	-0,0000972	-0,0041714	0,0001946	-0,0064268
6	0,0003897	-0,0002734	-0,0117299	-0,0000745	0,0024613
7	-0,0004405	0,0003090	0,0132595	0,0006291	-0,0207779
8	0,0001382	-0,0000970	-0,0041605	-0,0002679	0,0088462
9	-0,0000512	0,0000359	0,0015398	0,0000922	-0,0030437
10	-0,0038015	0,0026669	0,1144304	0,0051393	-0,1697263

	6	7	8	9
1	-0,0003036	0,0002155	0,1795549	-0,0016312
2	-0,0000004	0,0000002	0,0002088	-0,0000019
3	0,0000001	-0,0000001	-0,0000821	0,0000007
4	-0,0000026	0,0000012	0,0012109	-0,0000110
5	-0,0006372	0,0004520	0,3767281	-0,0034225
6	-0,0004140	0,0002854	0,2408650	-0,0021882
7	-0,0007398	0,0005413	0,4451797	-0,0040443
8	0,0003884	-0,0002816	-0,2324805	0,0021120
9	-0,0001282	0,0000931	0,0768407	-0,0006981
10	-0,0057416	0,0042117	3,4599152	-0,0314326

Lag : 76

	1	2	3	4	5
1	0,0000573	-0,0000402	-0,0017237	0,0000939	-0,0031008
2	0,0000018	-0,0000013	-0,0000552	-0,0000010	0,0000340
3	0,0000012	-0,0000009	-0,0000365	-0,0000008	0,0000278
4	0,0000231	-0,0000162	-0,0006948	-0,0000140	0,0004623
5	0,0001201	-0,0000843	-0,0036164	0,0001969	-0,0065027
6	0,0003598	-0,0002524	-0,0108301	-0,0000566	0,0018676
7	-0,0004259	0,0002988	0,0128204	0,0005987	-0,0197738
8	0,0001361	-0,0000955	-0,0040978	-0,0002570	0,0084880
9	-0,0000501	0,0000352	0,0015091	0,0000883	-0,0029155
10	-0,0036654	0,0025714	0,1103334	0,0048822	-0,1612383

	6	7	8	9
1	-0,0002930	0,0002081	0,1733650	-0,0015750
2	-0,0000004	0,0000003	0,0002389	-0,0000022
3	0,0000001	-0,0000001	-0,0000531	0,0000005
4	-0,0000033	0,0000017	0,0016481	-0,0000150
5	-0,0006149	0,0004366	0,3637161	-0,0033043
6	-0,0004094	0,0002829	0,2385235	-0,0021669
7	-0,0006941	0,0005082	0,4178366	-0,0037959
8	0,0003674	-0,0002666	-0,2200254	0,0019989
9	-0,0001211	0,0000880	0,0726119	-0,0006597
10	-0,0053741	0,0039452	3,2398710	-0,0294336

Lag : 77

	1	2	3	4	5
1	0,0000492	-0,0000345	-0,0014800	0,0000946	-0,0031239
2	0,0000017	-0,0000012	-0,0000517	-0,0000009	0,0000308
3	0,0000011	-0,0000008	-0,0000344	-0,0000008	0,0000256
4	0,0000216	-0,0000152	-0,0006517	-0,0000128	0,0004213
5	0,0001031	-0,0000724	-0,0031050	0,0001984	-0,0065514
6	0,0003317	-0,0002327	-0,0099861	-0,0000402	0,0013271
7	-0,0004115	0,0002887	0,0123857	0,0005695	-0,0188088
8	0,0001338	-0,0000939	-0,0040277	-0,0002465	0,0081394
9	-0,0000491	0,0000344	0,0014769	0,0000845	-0,0027910
10	-0,0035316	0,0024776	0,1063070	0,0046358	-0,1530989

	6	7	8	9
1	-0,0002826	0,0002008	0,1672688	-0,0015196
2	-0,0000005	0,0000003	0,0002652	-0,0000024
3	0,0000000	-0,0000000	-0,0000268	0,0000002
4	-0,0000039	0,0000022	0,0020368	-0,0000185
5	-0,0005930	0,0004213	0,3509063	-0,0031879
6	-0,0004040	0,0002799	0,2356895	-0,0021412
7	-0,0006508	0,0004769	0,3919291	-0,0035606
8	0,0003474	-0,0002522	-0,2081236	0,0018908
9	-0,0001144	0,0000832	0,0685838	-0,0006231
10	-0,0050266	0,0036929	3,0317450	-0,0275427

Lag : 78

	1	2	3	4	5
1	0,0000417	-0,0000293	-0,0012566	0,0000949	-0,0031353
2	0,0000016	-0,0000011	-0,0000483	-0,0000008	0,0000278
3	0,0000011	-0,0000008	-0,0000323	-0,0000007	0,0000235
4	0,0000203	-0,0000142	-0,0006108	-0,0000116	0,0003831
5	0,0000876	-0,0000615	-0,0026376	0,0001991	-0,0065752
6	0,0003055	-0,0002143	-0,0091963	-0,0000253	0,0008359
7	-0,0003972	0,0002786	0,0119560	0,0005415	-0,0178820
8	0,0001314	-0,0000922	-0,0039541	-0,0002362	0,0078005
9	-0,0000479	0,0000336	0,0014432	0,0000809	-0,0026704
10	-0,0034002	0,0023854	0,1023531	0,0043996	-0,1452985

	6	7	8	9
1	-0,0002723	0,0001937	0,1612567	-0,0014650
2	-0,0000005	0,0000003	0,0002882	-0,0000026
3	-0,0000000	-0,0000000	-0,0000330	0,0000000
4	-0,0000044	0,0000026	0,0023809	-0,0000216
5	-0,0005714	0,0004063	0,3382888	-0,0030733
6	-0,0003979	0,0002762	0,2324184	-0,0021115
7	-0,0006098	0,0004471	0,3673889	-0,0033376
8	0,0003284	-0,0002385	-0,1967652	0,0017876
9	-0,0001080	0,0000785	0,0647459	-0,0005882
10	-0,0046983	0,0034544	2,8349695	-0,0257551

Lag : 79

	1	2	3	4	5
1	0,0000349	-0,0000245	-0,0010516	0,0000950	-0,0031359
2	0,0000015	-0,0000011	-0,0000452	-0,0000008	0,0000250
3	0,0000010	-0,0000007	-0,0000304	-0,0000007	0,0000216
4	0,0000190	-0,0000133	-0,0005722	-0,0000105	0,0003474
5	0,0000733	-0,0000514	-0,0022054	0,0001991	-0,0065767
6	0,0002809	-0,0001971	-0,0084558	-0,0000118	0,0003908
7	-0,0003831	0,0002688	0,0115327	0,0005145	-0,0169924
8	0,0001287	-0,0000903	-0,0038743	-0,0002262	0,0074714
9	-0,0000468	0,0000328	0,0014084	0,0000773	-0,0025535
10	-0,0032716	0,0022951	0,0984793	0,0041734	-0,1378269

	6	7	8	9
1	-0,0002623	0,0001867	0,1553539	-0,0014113
2	-0,0000006	0,0000004	0,0003080	-0,0000028
3	-0,0000001	0,0000000	0,0000184	-0,0000002
4	-0,0000049	0,0000030	0,0026838	-0,0000244
5	-0,0005502	0,0003916	0,3258927	-0,0029606
6	-0,0003912	0,0002721	0,2287588	-0,0020782
7	-0,0005710	0,0004190	0,3441591	-0,0031266
8	0,0003102	-0,0002254	-0,1859339	0,0016892
9	-0,0001018	0,0000741	0,0610893	-0,0005550
10	-0,0043881	0,0032290	2,6490269	-0,0240658

Lag : 80

	1	2	3	4	5
1	0,0000287	-0,0000201	-0,0008636	0,0000947	-0,0031270
2	0,0000014	-0,0000010	-0,0000422	-0,0000007	0,0000225
3	0,0000009	-0,0000007	-0,0000285	-0,0000006	0,0000197
4	0,0000178	-0,0000125	-0,0005356	-0,0000095	0,0003143
5	0,0000602	-0,0000422	-0,0018113	0,0001986	-0,0065579
6	0,0002579	-0,0001809	-0,0077633	0,0000003	-0,0000114
7	-0,0003693	0,0002591	0,0111158	0,0004887	-0,0161390
8	0,0001260	-0,0000884	-0,0037914	-0,0002166	0,0071521
9	-0,0000456	0,0000320	0,0013728	0,0000739	-0,0024404
10	-0,0031456	0,0022068	0,0946875	0,0039568	-0,1306748

	6	7	8	9
1	-0,0002524	0,0001798	0,1495609	-0,0013587
2	-0,0000006	0,0000004	0,0003249	-0,0000030
3	-0,0000001	0,0000000	0,0000376	-0,0000003
4	-0,0000054	0,0000033	0,0029486	-0,0000268
5	-0,0005295	0,0003771	0,3137456	-0,0028503
6	-0,0003840	0,0002676	0,2247763	-0,0020420
7	-0,0005343	0,0003924	0,3221778	-0,0029269
8	0,0002929	-0,0002130	-0,1756029	0,0015953
9	-0,0000960	0,0000699	0,0576095	-0,0005234
10	-0,0040952	0,0030160	2,4734249	-0,0224704

TABLE C.1

Multipliers of special deposits on the money stock and the monetary base obtained from model B.

Lag	$\frac{\partial M}{\partial S^D}$	$\frac{\partial B}{\partial S^D}$
0	- 3.576985339	- 0.999999881
1	- 2.13156947	- 0.999448839
2	- 2.145062447	- 0.997728705
3	- 2.154821396	- 0.994240105
4	- 3.159033775	- 0.988408983
5	- 2.156230927	- 0.979730546
6	- 2.145270348	- 0.967746477
7	- 2.125286102	- 0.95219697
8	- 2.09564209	- 0.932734
9	- 2.055924416	- 0.909191012
10	- 2.005893707	- 0.88145715
11	- 1.94548893	- 0.849486172
12	- 1.874781609	- 0.813294947
13	- 1.793973923	- 0.772957146
14	- 1.703370094	- 0.728596568
15	- 1.603373528	- 0.680382907
16	- 1.494474411	- 0.628526092
17	- 1.377236366	- 0.573272347
18	- 1.252288818	- 0.5148983
19	- 1.120314598	- 0.453708291
20	- 0.98205775	- 0.390030384
Total	-39.89908008	-17.00803582

APPENDIX D

ON THE ASYMPTOTIC DISTRIBUTION OF IMPACT AND INTERIM MULTIPLIERS*

1. Introduction

The asymptotic distribution of interim multipliers of a first order linear dynamic model was recently derived by Schmidt.[1] Let the reduced form of such a model be

$$y_{t.} = y_{t-1,.} \Pi_0 + w_{t.} \Pi_1 + v_{t.} \tag{D.1}$$

where the notation is that of Dhrymes.[2] The matrices of impact and interim multipliers are given by

$$\Pi_1, \quad \Pi_1\Pi_0, \quad \Pi_1\Pi_0^2, \quad . \; . \; . \tag{D.2}$$

Schmidt has shown that $\sqrt{T}\, P\, (\tilde{\Pi}_1 \tilde{\Pi}_0^n - \Pi_1 \Pi_0^n)$, where P is the pack of a matrix, and $\tilde{\Pi}_1$, $\tilde{\Pi}_0$ consistent estimates of the matrices Π_1 and Π_0, is distributed asymptotically as $N\, [0,\, (\overline{A}_n,\; \overline{D}_n)\; \Psi\; (\overline{A}_n,\; \overline{D}_n)']$

with
$$\overline{A}_n = \sum_{j=0}^{n-1} (\Pi_0^j)' \otimes \Pi_1\, \Pi_0^{n-1-j} \tag{D.3}$$

$$\overline{D}_n = (\Pi_0^n)' \otimes I \tag{D.4}$$

and Ψ the variance-covariance matrix of $\sqrt{T} \begin{bmatrix} P(\tilde{\Pi}_0 - \Pi_0) \\ P(\tilde{\Pi}_1 - \Pi_1) \end{bmatrix}$ which is normally distributed with zero mean. Similar results were given by

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* This appendix represents the outcome of a joint research work with Mr. L. Gill.

[1] P. Schmidt: "The Asymptotic Distribution of Dynamic Multipliers." Econometrica, 1973, pp.161-4.

[2] P. J. Dhrymes: "Restricted and Unrestricted Reduced Forms: Asymptotic Distribution and Relative Efficiency." Econometrica, 1973, pp.119-34.

Schmidt for the matrices of cumulative multipliers. Remarks were added for the case where the lags of exogenous variables are extended to more than one period. As an example the impact and fifteen interim multipliers of the Klein's Model I as well as their standard errors were calculated.

In this appendix we give a generalisation of the results for the case of a higher than first order system and re-examine the significance of multipliers in Klein's Model I.

2. Generalisation to Higher Order Systems

Let

$$y_{t.} = y_{t-1,.} A_1 + y_{t-2,.} A_2 + \cdots + y_{t-p,.} A_p +$$

$$+ w_{t.} B_0 + w_{t-1,.} B_1 + \cdots + w_{t-q,.} B_q \qquad (D.5)$$

be the reduced form of an m-th order system of equations which together with the $m-1$ vector identities

$$\begin{aligned} y_{t-1,.} &= y_{t-1,.} \\ &\vdots \\ y_{t-p+1,.} &= y_{t-p+1,.} \end{aligned}$$

is transformed to

$$y^*_{t.} = y^*_{t-1,.} \Pi_0 + w^*_{t.} \Pi_1 \qquad (D.6)$$

where

$$y^*_{t.} = (y_{t.}, y_{t-1,.}, \ldots, y_{t-p+1,.})$$

$$w^*_{t.} = (w_{w.}, w_{t-1,.}, \ldots, w_{t-q,.})$$

$$\Pi_0 = \begin{bmatrix} A_1 & I_m & 0 & \ldots & 0 \\ A_2 & 0 & I_m & \ldots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ A_{p-1} & 0 & 0 & \ldots & I_m \\ A_p & 0 & 0 & \ldots & 0 \end{bmatrix} \qquad \Pi_1 = \begin{bmatrix} B_0 & 0 & \ldots & 0 \\ B_1 & 0 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ B_q & 0 & \ldots & 0 \end{bmatrix}$$

are row vectors of dimension $1 \times mp$ and $1 \times G\,(q+1)$ and matrices of dimension $mp \times mp$ and $G\,(q+1) \times mp$ respectively. Impact and interim multipliers are contained in the matrices

$$\Pi_1, \quad \Pi_1\Pi_0, \quad \Pi_1\Pi_0^2, \quad \ldots\ldots \qquad (D.7)$$

but are portioned out to more than one of them as shown in chapter 9. The part of the matrices in this sequence which is relevant for picking up multipliers is the first column of blocks of elements with dimensions $G \times m$ each. Thus the matrix of n-th period interim multipliers consists of the 1,1 block in $\Pi_1\Pi_0^n$ plus the 2,1 block in $\Pi_1\Pi_0^{n-1}$ plus . . . plus the $(q+1),1$ block in $\Pi_1\Pi_0^{n-q}$. The reason for selecting the first column of blocks is that this column corresponds to $y_{t.}$ in $y^*_{t.}$. The n-th power of Π_0 can be written as

$$\Pi_0^n = \begin{bmatrix} C_1(n) & C_1(n-1) & \cdots & C_1(n-p+1) \\ C_2(n) & C_2(n-1) & \cdots & C_2(n-p+1) \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ C_p(n) & C_p(n-1) & \cdots & C_p(n-p+1) \end{bmatrix} \tag{D.8}$$

Explicit expressions for the submatrices $C_i(n)$ $(i = 1, 2, \ldots p$ and $n = 1, 2, 3, \ldots)$ are given in chapter 9, p. .

Correspondingly, the term $\Pi_1\Pi_0^n$ is

$$\Pi_1\Pi_0^n = \begin{bmatrix} B_0C_1(n) & B_0C_1(n-1) & \cdots & B_0C_1(n-p+1) \\ B_1C_1(n) & B_1C_1(n-1) & \cdots & B_1C_1(n-p+1) \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ B_qC_1(n) & B_qC_1(n-1) & \cdots & B_qC_1(n-p+1) \end{bmatrix} \tag{D.9}$$

Since multipliers are to be sought in the first column of submatrices, $\begin{bmatrix} B_0C_1(n) \\ B_1C_1(n) \\ \vdots \\ B_qC_1(n) \end{bmatrix}$ we must look for the asymptotic distribution of

$$\sqrt{T}\ P \begin{bmatrix} \tilde{B}_0\tilde{C}_1(n) - B_0C_1(n) \\ \tilde{B}_1\tilde{C}_1(n) - B_1C_1(n) \\ \vdots \\ \tilde{B}_q\tilde{C}_1(n) - B_qC_1(n) \end{bmatrix}$$

The last can be derived from the distribution of $\sqrt{T}\ P\ (\tilde{\Pi}_1\tilde{\Pi}_0^n - \Pi_1\Pi_0^n)$. We have

THEOREM 1 : Let $\tilde{A}_1,\ \tilde{A}_2,\ \dots,\ \tilde{A}_p,\ \tilde{B}_0,\ \tilde{B}_1,\ \dots,\ \tilde{B}_q$ be consistent estimates of $A_1,\ A_2,\ \dots,\ A_p,\ B_0,\ B_1,\ \dots,\ B_q$ such that

$$\sqrt{T}\begin{bmatrix} P\begin{bmatrix}\tilde{A}_1 - A_1\\ \vdots \\ \tilde{A}_p - A_p\end{bmatrix} \\ P\begin{bmatrix}\tilde{B}_0 - B\\ \vdots \\ \tilde{B}_q - B_q\end{bmatrix}\end{bmatrix}$$

has asymptotic distribution $N\ (0,\ \Phi\ )$, with Φ being the $\left[m\ [mp + C\ (q+1)] \right]$ -th order variance covariance matrix of reduced form coefficients. Then the asymptotic distribution of $\sqrt{T}\ P\ (\tilde{\Pi}_1\tilde{\Pi}_0^n -$ $- \Pi_1\Pi_0^n)$ is $N\ [0,\ (\overline{A}_n,\ \overline{D}_n)\ \Omega\ (\overline{A}_n,\ \overline{D}_n)']$,

where

$$\overline{A}_n = \sum_{j=0}^{n-1} (\Pi_0^j)' \otimes \Pi_1\Pi_0^{n-1-j} \qquad (D.10)$$

$$\overline{D}_n = (\Pi_0^n)' \otimes I_{G(q+1)} \qquad (D.11)$$

and
$$\Omega = \begin{bmatrix} \Phi_{11} & 0 & \Phi_{12} & 0 \\ 0 & 0 & 0 & 0 \\ \Phi_{21} & 0 & \Phi_{22} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(of order } mp\,[mp + G(q+1)]) \tag{D.12}$$

Φ_{11} is the variance covariance matrix of the a's of order m^2p, Φ_{22} the variance covariance matrix of the b's of order mG $(q+1)$, $\Phi_{12} = \Phi_{21}'$ the covariance matrix of a's and b's of dimension $m^2p \times mG\,(q+1)$, and

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \tag{D.13}$$

PROOF : Schmidt has shown that

$$P\left[\tilde{\Pi}_1\tilde{\Pi}_0^n - \Pi_1\Pi_0^n\right] = (A_n, D_n)\begin{bmatrix} P\,(\tilde{\Pi}_0 - \Pi_0) \\ P\,(\tilde{\Pi}_1 - \Pi_1) \end{bmatrix} \tag{D.14}$$

But

$$P\left[\tilde{\Pi}_o - \Pi_o\right] = P\begin{bmatrix} \tilde{A}_1 - A_1 & 0 & 0 & \cdots & 0 \\ \tilde{A}_2 - A_2 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \tilde{A}_p - A_p & 0 & 0 & \cdots & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} P\begin{bmatrix} \tilde{A}_1 - A_1 \\ \tilde{A}_2 - A_2 \\ \vdots \\ \tilde{A}_p - A_p \end{bmatrix} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{D.15}$$

Similarly

$$P\,(\tilde{\Pi}_1 - \Pi_1) \;=\; P\begin{bmatrix} \tilde{B}_0 - B_0 & 0 & \cdots & 0 \\ \tilde{B}_1 - B_1 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \tilde{B}_q - B_q & 0 & & 0 \end{bmatrix} \;=\; \begin{bmatrix} P\begin{bmatrix} \tilde{B}_0 - B_0 \\ \tilde{B}_1 - B_1 \\ \vdots \\ \tilde{B}_q - B_q \end{bmatrix} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (D.16)$$

The theorem follows easily if we recognize that Ω is the variance covariance matrix of

$$\sqrt{T}\begin{bmatrix} P'\begin{bmatrix} \tilde{A}_1 - A_1 \\ \tilde{A}_2 - A_2 \\ \vdots \\ \tilde{A}_p - A_p \end{bmatrix} & 0 & 0 & \cdots & P'\begin{bmatrix} \tilde{B}_0 - B_0 \\ \tilde{B}_1 - B_1 \\ \vdots \\ \tilde{B}_q - B_q \end{bmatrix} & 0 & 0 & \cdots & 0 \end{bmatrix}'$$

THEOREM 2 : If the conditions of Theorem 1 hold, the asymptotic distribution of

$$\sqrt{T}\;P\begin{bmatrix} \tilde{B}_0\tilde{C}_1(n) - B_0C_1(n) \\ \tilde{B}_1\tilde{C}_1(n) - B_1C_1(n) \\ \vdots \\ \tilde{B}_q\tilde{C}_1(n) - B_qC_1(n) \end{bmatrix}$$

is $N[0, F_{n,n}]$ where

$$F_{n,n} = \left[\left[\sum_{j=0}^{n-1} C'(j) \otimes \overline{E}_n(j) \right] \Phi_{11} + \left[C_1'(n) \otimes I_{G(q+1)} \right] \Phi_{21} \right] \cdot$$

$$\cdot \left[\sum_{j=0}^{n-1} C_1(j) \otimes \overline{E}_n'(j) \right] + \left[\left[\sum_{j=0}^{n-1} C'(j) \otimes \overline{E}_n(j) \right] \Phi_{12} + \right.$$

$$\left. + \left[C_1(n) \otimes I_{G(q+1)} \right] \Phi_{22} \right] \left[C_1(n) \otimes I_{G(q+1)} \right] \qquad (D.17)$$

$$\overline{E}_n(j) = \text{plim}\, E_n(j) = \Pi_1 \Pi_0^{n-1-j} \qquad (D.18)$$

and the matrices $C_1(j)$ $(j = 1, 2, 3, \ldots)$ are given by

$$C_1(j) = C_1(j-1)A_1 + C_1(j-2)A_2 + \ldots + C_1(j-p)A_p \qquad (D.19)$$

with $C_1(0) = I_m$ and $C_1(j) = 0_m$, the zero matrix, for $j < 0$.

PROOF : The above variance covariance matrix $F_{n,n}$ is the upper left matrix of order $mG(q+1)$ of Ω and is obtained by writing Ω explicitly as in (D.12), $\overline{A}_n$ as

$$\overline{A}_n = \sum_{j=0}^{n-1} (\Pi_0^j)' \otimes \overline{E}_n(j) = \begin{bmatrix} \sum_{j=0}^{n-1} C_1'(j) \otimes \overline{E}_n(j) & \sum_{j=0}^{n-1} C_2'(j) \otimes \overline{E}_n(j) & \ldots & \sum_{j=0}^{n-1} C_p'(j) \otimes \overline{E}_n(j) \\ \sum_{j=0}^{n-1} C_1'(j-1) \otimes \overline{E}_n(j) & \sum_{j=0}^{n-1} C_2'(j-1) \otimes \overline{E}_n(j) & \ldots & \sum_{j=0}^{n-1} C_p'(j-1) \otimes \overline{E}_n(j) \\ \vdots & \vdots & & \vdots \\ \sum_{j=0}^{n-1} C_1'(j-p+1) \otimes \overline{E}_n(j) & \sum_{j=0}^{n-1} C_2'(j-p+1) \otimes \overline{E}_n(j) & \ldots & \sum_{j=0}^{n-1} C_p'(j-p+1) \otimes \overline{E}_n(j) \end{bmatrix}$$

(D.20)

$\bar{D}_n$ as

$$\bar{D}_n = (\Pi_0^n)' \otimes I_{G(q+1)} = \begin{bmatrix} C_1'(n) \otimes I_{G(q+1)} & C_2'(n) \otimes I_{G(q+1)} & \cdots & C_p'(n) \otimes I_{G(q+1)} \\ C_1'(n-1) \otimes I_{G(q+1)} & C_2'(n-1) \otimes I_{G(q+1)} & \cdots & C_p'(n-1) \otimes I_{G(q+1)} \\ \vdots & \vdots & & \vdots \\ C_1'(n-p+1) \otimes I_{G(q+1)} & C_2'(n-p+1) \otimes I_{G(q+1)} & \cdots & C_p'(n-p+1) \otimes I_{G(q+1)} \end{bmatrix} \tag{D.21}$$

performing the relevant multiplications of the conformable matrices and selecting from the result matrix the upper left block of dimension $mG(q+1) \times mG(q+1)$.

The estimated variances of the elements of the matrices $\tilde{B}_i\tilde{C}_1(n)$ $(i = 0, 1, \ldots, q)$ can be obtained as the diagonal elements of the variance covariance matrix Ω_{11} divided by the sample size.

REMARK 1 : Care should be taken regarding the selection of variances from the above matrix. For example, if we want the variance of the elements of $\tilde{B}_0\tilde{C}_1(n)$, which is the first component of the sum of matrices of n-th period interim multipliers, we need to select its diagonal elements which have the following positions: $1, 2, \ldots, G$, $[G(q+1) + 1]$, $[G(q+1) + 2]$, $\ldots$, $[G(q+1) + G]$, $[2G(q+1) + 1]$, $[2G(q+1) + 2]$, $\ldots$, $[2G(q+1) + G]$, $[3G(q+1) + 1]$, $\ldots$ $[(m-1)G(q+1) + 1]$, $[(m-1)G(q+1) + 2]$, $\ldots$, $[(m-1)G(q+1) + G]$. For this remark to be valid, the elements of Φ should have been laid out according to the definition of Φ in page 523.

The variances of the elements of the matrix of n-th period interim multipliers

$$B_0C_1(n) + B_1C_1(n-1) + \ldots + B_qC_1(n-q)$$

will be the variances of the elements of each component $B_iC_1(n-i)$, i.e. they will be included in $F_{n-i,n-i}$ $(i = 0, 1, \ldots, q)$, plus two times the covariances of the elements of all possible pairs of matrices $B_iC_1(n-i)$ (there are $q^2 - \frac{1}{2}q\,(q-1)$ such pairs). Covariances will be looked for in the matrices $F_{n-i,n-k}$, where

$$F_{n-i,n-k} = \left[\left[\sum_{j=0}^{n-i-1} C_1'(j) \otimes E_{n-i}(j)\right]\Phi_{11} + \left[C_1'(n-i) \otimes I_{G(q+1)}\right]\Phi_{21}\right]\left[\sum_{j=0}^{n-k-1} C_1(j) \otimes \overline{E}_{n-k}(j)\right] + \left[\left[\sum_{j=0}^{n-i-1} C_1'(j) \otimes \overline{E}_{n-i}(j)\right]\Phi_{12} + \left[C_1'(n-i) \otimes I_{G(q+1)}\right]\Phi_{22}\right]\left[C_1(n-k) \otimes I_{G(q+1)}\right] \qquad (D.22)$$

$$i, k = 0, 1, \ldots, q \qquad i \neq k$$

REMARK 2 : There is no meaning in adding the above matrices since the particular elements we are interested in lie in different positions. This increases the amount of manual computational work, which is greater, the higher the order of the econometric model is. On the other hand, formulae (D.17) and (D.22) reduce considerably the amount of computer calculations needed for such a model.

3. A Re-examination of Schmidt's Example

Schmidt has estimated the variances of the impact and fifteen interim multipliers of the Klein's Model I and found that interim multipliers are not significant for lags of more than one period. However the above model contains, apart from other predetermined variables, an exogenous variable lagged one period (T) and is not strictly of the type exhibited in (D.1). The rather heuristic discussion provided by Schmidt for this case did not reveal any detail on the course of computations followed by the author. We have repeated the calculations involved, by using the formulae developed in the previous section.

The data for Klein's Model I are

$$m = 6, \quad G = 5, \quad p = 1, \quad q = 1,$$

$$\Pi_0 = A_1, \quad \Pi_1 = \begin{bmatrix} B_0 \\ B_1 \end{bmatrix} \quad c_1(n) = C_1(n-1)\, A_1 \qquad (D.23)$$

and matrices of multipliers are given by the sequence

$$B_0, \quad B_0C_1(1)+B_1, \quad B_0C_1(2)+B_1C_1(1), \quad B_0C_1(3)+B_1C_1(2), \; . \; . \; . \qquad (D.24)$$

In computing the variances of the elements of each component $B_iC_1(n-i)$ $(i = 0,1)$ of the matrix of n-th period interim multipliers, we need to compute the matrices

$$\sum_{j=0}^{n-1} C_1'(j) \otimes \overline{E}_n(j) \qquad (D.25)$$

(or their transpose). Denote these matrices by G_n. Then it can be easily verified that the G's are related for different values of n, by the recursion

$$G_n = G_{n-1} (I_{mp} \times \Pi_0) + [C_1'(n-1) \times I_{G(q+1)}] G_1 \qquad (D.26)$$

where

$$G_1 = I_{mp} \times \Pi_1 \qquad (D.27)$$

Similarly these results are useful for the calculation of the covariances included in the matrix $F_{n,n-1}$. Also, we have calculated the variance-covariance matrix of reduced form coefficients by following an alternative formula to the one given in Goldberger, Nagar and Oden.[3] To this end we have used Dhrymes' identity 8[4] which implies that the distribution of $\sqrt{T}P(\tilde{\Pi} - \Pi)$, where $\Pi = [A_1' \; B_0' \; B_1']'$, and $\tilde{\Pi}$ is a consistent estimate of it, is $N(O, C\,H\,C')$ and the estimate of the variance-covariance matrix is $\tilde{C}\,\tilde{H}\,\tilde{C}'$

where $\tilde{H}$ is the estimated 132 x 132 variance-covariance matrix of structural equations coefficients, including the appropirate zeros in the places corresponding to *a priori* restricted coefficients,

$\tilde{C} = \tilde{D}' \times (\tilde{\Pi}, \; I_{16})$,

and $\tilde{D}$ is the 6 x 6 matrix defined in Dhrymes and obtained from coefficient estimates of structural equations.

The matrix which came out of the calculations was in general different from the one obtained by Goldberger, Nagar and Odeh and is presented in table D.2 in the same arrangement as theirs. For our calculations its elements had to be rearranged so that it conforms to our definition of Φ and its submatrices (pre and postmultiplication by the appropriate permutation matrix).

Table D.1 shows the same multipliers presented by Schmidt (with their standard errors in parenthesis) and the associated t ratio sequences.

[3] See A. S. Goldberger, A. L. Nagar and H. S. Odeh: "The Covariance Matrices of Reduced-Form Coefficients and Forecasts for a Structural Econometric Model." Econometrica, 1961, pp.556-73.
[4] See P. J. Dhrymes, *op.cit.*, 1973, p.121

TABLE D.1

Lag	$\frac{\partial Y}{\partial G}$	t ratio	$\frac{\partial Y}{\partial T}$	t ratio
0	1.8168 (0.421)	4.32	-1.3043 (0.483)	-2.70
1	1.8084 (0.393)	4.60	-1.7716 (0.475)	-3.73
2	1.1916 (0.333)	3.58	-1.4487 (0.408)	-3.55
3	0.4545 (0.245)	1.86	-0.6483 (0.308)	-2.10
4	-0.1782 (0.284)	-0.63	0.1314 (0.348)	0.38
5	-0.6072 (0.326)	-1.86	0.6940 (0.390)	1.78
6	-0.8101 (0.320)	-2.53	0.9832 (0.381)	2.58
7	-0.8141 (0.247)	-3.30	1.0205 (0.320)	3.19
8	-0.6748 (0.203)	-3.32	0.8693 (0.279)	3.12
9	-0.4571 (0.159)	-2.87	0.6094 (0.243)	2.51
10	-0.2215 (0.124)	-1.79	0.3169 (0.220)	1.44
11	-0.0143 (0.094)	-0.15	0.0523 (0.199)	0.26
12	0.1364 (0.083)	1.64	-0.1460 (0.185)	0.79
13	0.2204 (0.079)	2.79	-0.2623 (0.157)	1.67
14	0.2418 (0.074)	3.27	-0.2996 (0.134)	2.24
15	0.2148 (0.064)	3.36	-0.2738 (0.120)	2.28

The joint variation of the two sequences (i.e. the multiplier and t ratio sequences) exhibits in both cases the striking feature that as the multiplier sequence changes sign, the significance level and the t ratio fall sharply and recover. This feature is easily explained: so long as the standard error of a multiplier decreases or increases more slowly than the absolute value of the multiplier, then the t ratio will exhibit more erratic behaviour than either the multiplier or standard error sequence.

It is clear that Schmidt's assertion that all multipliers for lags of more than one period are insignificant is not confirmed by our calculations. Rather, the multipliers remain significant even though their absolute size is damping out, provided they are not close to a 'change of sign' point in the sequence. It is not possible to locate the source of these conflicting results since Schmidt used a different variance-covariance matrix and gave no details of his computational procedure.

TABLE D.2

Variance-Covariance Matrix of Reduced-Form Coefficients: Klein's Model I

A. DIAGONAL BLOCKS								
	1	P_{-1}	W_2	K_{-1}	T	$(W_1+T)_{-1}$	t	G
	Consumption							
1	68.72594	-0.09113	-0.41264	-0.30510	-1.76013	-0.13874	-0.07793	1.51083
P_{-1}		0.01394	-0.00065	-0.00023	0.00270	-0.00126	0.00056	-0.00087
W_2			0.00604	0.00165	0.01456	0.00140	0.00037	-0.00963
K_{-1}				0.00147	0.00832	0.00047	0.00047	-0.00730
T					0.08030	0.00124	0.00224	-0.06340
$(W_1+T)_{-1}$						0.00175	-0.00021	-0.00237
t							0.00093	-0.00189
G								0.05667
	Investment							
1	41.63878	-0.04556	-0.06683	-0.20001	-0.36917	-0.01615	-0.01522	0.31430
P_{-1}		0.00653	0.00045	-0.00033	0.00291	0.00011	0.00012	-0.00257
W_2			0.00151	0.00025	0.00918	0.00039	0.00036	-0.00799
K_{-1}				0.00101	0.00136	0.00006	0.00005	-0.00114
T					0.05679	0.00238	0.00218	-0.04957
$(W_1+T)_{-1}$						0.00011	0.00009	-0.00208
t							0.00009	-0.00190
T								0.04329
	Private Wage Bill							
1	48.80038	0.04329	-0.19037	-0.21484	-0.66568	-0.19878	-0.01696	0.81264
P_{-1}		0.00883	-0.00056	-0.00078	0.00174	-0.00104	-0.00014	0.00191
W_2			0.00210	0.00083	0.00710	0.00092	-0.00022	-0.00660
K_{-1}				0.00104	0.00310	0.00069	0.00030	-0.00379
T					0.04446	0.00107	0.00042	-0.03753
$(W_1+T)_{-1}$						0.00254	-0.00038	-0.00416
t							0.00146	-0.00085
G								0.03954
	Profits							
1	63.49063	-0.10930	-0.27797	-0.29033	-1.23785	-0.08913	-0.06141	1.04980
P_{-1}		0.01121	-0.00012	-0.00031	0.00202	0.00002	-0.00016	-0.00192
W_2			0.00338	0.00117	0.01245	0.00101	0.00099	-0.01010
K_{-1}				0.00142	0.00563	0.00036	0.00024	-0.00485
T					0.07420	0.00369	0.00325	-0.06542
$(W_1+T)_{-1}$						0.00041	0.00027	-0.00316
t							0.00036	-0.00254
G								0.05874
	National Income							
1	197.8424	-0.26334	-0.81351	-0.91080	-3.62079	-0.27204	-0.17884	3.11335
P_{-1}		0.03546	-0.00011	-0.00117	0.00879	-0.00090	0.00042	-0.00636
W_2			0.01018	0.00346	0.03781	0.00253	0.00113	0.02974
K_{-1}				0.00446	0.01663	0.00108	0.00099	-0.01453
T					0.23306	0.00781	0.00663	-0.19919
$(W_1+T)_{-1}$						0.00194	0.00002	-0.00790
t							0.00097	-0.00572
G								0.17734

(cont.)

TABLE D.2
(cont.)

B. OFF-DIAGONAL BLOCKS									
		1	P_{-1}	W_2	K_{-1}	T	$(W_1+T)_{-1}$	t	G
		Consumption							
Investment	1	43.73892	-0.04950	-0.21037	-0.19666	-0.80103	-0.08612	-0.05617	0.69763
	P_{-1}	-0.07714	0.00749	-0.00016	-0.00023	0.00137	0.00013	-0.00028	-0.00121
	W_2	-0.12368	0.00025	0.00131	0.00056	0.00817	0.00038	0.00005	-0.00724
	K_{-1}	-0.20905	-0.00037	0.00100	0.00099	0.00368	0.00041	0.00034	-0.00325
	T	-0.69047	0.00181	0.00590	0.00328	0.04798	0.00195	0.00027	-0.04439
	$(W_1+T)_{-1}$	-0.03103	0.00011	0.00035	0.00015	0.00225	0.00004	0.00003	-0.00190
	t	-0.02951	0.00003	0.00035	0.00013	0.00195	0.00011	-0.00003	-0.00170
	G	0.59060	-0.00171	-0.00488	-0.00283	-0.04183	-0.00155	-0.00024	0.03869
		Consumption							
Private Wage Bill	1	51.96127	0.02490	-0.26133	-0.22668	-0.94558	-0.18846	-0.03874	0.97970
	P_{-1}	-0.05109	0.01005	-0.00036	-0.00035	0.00329	-0.00081	-0.00010	-0.00027
	W_2	-0.24705	-0.00055	0.00319	0.00101	0.00910	0.00114	0.00017	-0.00753
	K_{-1}	-0.23535	-0.00057	0.00113	0.00111	0.00454	0.00068	0.00035	-0.00474
	T	-1.08586	0.00165	0.00895	0.00513	0.05553	0.00171	0.00109	-0.04742
	$(W_1+T)_{-1}$	-0.10285	-0.00149	0.00061	0.00031	-0.00080	0.00201	-0.00033	-0.00179
	t	-0.03513	0.00075	-0.00054	0.00035	0.00069	-0.00040	0.00109	-0.00132
	G	0.98374	0.00083	-0.00619	-0.00467	-0.04158	-0.00358	-0.00089	0.04252
		Consumption							
Profits	1	60.50354	-0.16553	-0.36168	-0.27506	-1.61558	-0.03639	-0.09536	1.22875
	P_{-1}	-0.11719	0.01138	-0.00045	-0.00012	0.00079	-0.00031	0.00037	-0.00181
	W_2	-0.28927	0.00015	0.00416	0.00120	0.01363	0.00065	0.00024	-0.00934
	K_{-1}	-0.27879	-0.00003	0.00151	0.00134	0.00746	0.00020	0.00046	-0.00582
	T	-1.36474	0.00286	0.01151	0.00647	0.07275	0.00148	0.00141	-0.06037
	$(W_1+T)_{-1}$	-0.06691	0.00035	0.00114	0.00030	0.00429	-0.00022	0.00015	-0.00248
	t	-0.07231	-0.00017	0.00126	0.00025	0.00349	0.00030	-0.00018	-0.00227
	G	1.11769	-0.00341	-0.00832	-0.00546	-0.06365	-0.00034	-0.00124	0.05284
		National Income							
National Income	1	112.4649	-0.14063	-0.62301	-0.50174	-2.56116	-0.22486	-0.13410	2.20846
	P_{-1}	-0.16827	0.02143	-0.00081	-0.00047	0.00407	-0.00112	0.00027	-0.00208
	W_2	-0.53632	-0.00040	0.00736	0.00221	0.02273	0.00178	0.00042	-0.01686
	K_{-1}	-0.51414	-0.00060	0.00265	0.00246	0.01200	0.00088	0.00081	-0.01056
	T	-2.45060	0.00451	0.02046	0.01160	0.12829	0.00319	0.00251	-0.10779
	$(W_1+T)_{-1}$	-0.16977	-0.00115	0.00175	0.00061	0.00349	0.00179	-0.00018	-0.00427
	t	-0.10744	0.00059	0.00072	0.00060	0.00419	-0.00010	0.00091	-0.00359
	G	2.10143	-0.00258	-0.01451	-0.01013	-0.10523	-0.00392	-0.00213	0.09536
		Investment							
Private Wage Bill	1	39.61487	-0.05732	-0.08618	-0.19112	-0.49503	-0.01743	-0.02096	0.41749
	P_{-1}	-0.02724	0.00649	0.00028	-0.00042	0.00174	0.00010	0.00006	-0.00166
	W_2	-0.13136	0.00008	0.00126	0.00060	0.00678	0.00032	0.00032	-0.00576
	K_{-1}	-0.18216	-0.00029	0.00037	0.00093	0.00217	0.00008	0.00008	-0.00183
	T	-0.52216	0.00184	0.00763	0.00226	0.04613	0.00202	0.00181	-0.04021
	$(W_1+T)_{-1}$	-0.07294	0.00024	0.00038	0.00034	0.00227	0.00002	0.00011	-0.00185
	t	-0.03956	-0.00029	-0.00003	0.00027	0.00021	0.00003	-0.00006	-0.00022
	G	0.49525	-0.00142	-0.00676	-0.00223	-0.04209	-0.00169	-0.00160	0.03655

(cont.)

TABLE D.2
(cont.)

B. OFF-DIAGONAL BLOCKS

		1	P_{-1}	W_2	K_{-1}	T	$(W_1+T)_{-1}$	t	G
		Investment							
Profits	1	45.76279	-0.06538	-0.10433	-0.21794	-0.56461	-0.02975	-0.02377	0.48741
	P_{-1}	-0.06782	0.00754	0.00043	-0.00028	0.00299	0.00012	0.00008	-0.00262
	W_2	-0.14583	0.00021	0.00157	0.00064	0.00830	0.00043	0.00040	-0.00711
	K_{-1}	-0.21450	-0.00028	0.00044	0.00108	0.00247	0.00012	0.00010	-0.00214
	T	-0.64804	0.00244	0.00972	0.00277	0.05865	0.00261	0.00231	-0.05119
	$(W_1+T)_{-1}$	-0.02933	0.000005	0.00040	0.00013	0.00206	0.00013	0.00010	-0.00178
	t	-0.03184	0.00012	0.00044	0.00012	0.00224	0.00012	0.00012	-0.00191
	G	0.51667	-0.00236	-0.00846	-0.00217	-0.05187	-0.00229	-0.00200	0.04543
		Investment							
National Income	1	85.37769	-0.12271	-0.19051	-0.40906	-1.05964	-0.04718	-0.04473	0.90490
	P_{-1}	-0.09507	0.01402	0.00070	-0.00070	0.00472	0.00022	0.00014	-0.00428
	W_2	-0.27719	0.00029	0.00283	0.00125	0.01508	0.00074	0.00071	-0.01287
	K_{-1}	-0.39667	-0.00057	0.00081	0.00200	0.00464	0.00020	0.00018	-0.00397
	T	-1.17020	0.00428	0.01735	0.00503	0.00477	0.00463	0.00413	-0.09139
	$(W_1+T)_{-1}$	-0.10227	0.00024	0.00078	0.00047	0.00433	0.00015	0.00021	-0.00363
	t	-0.07139	-0.00017	0.00041	0.00039	0.00245	0.00012	0.00006	-0.00213
	G	1.01192	-0.00378	-0.01523	-0.00440	-0.09396	-0.00398	-0.00359	0.08198
		Private Wage Bill							
Profits	1	42.77568	-0.12162	-0.18804	-0.20267	-0.94233	0.02298	-0.05772	0.66356
	P_{-1}	-0.07571	0.00771	0.00010	-0.00009	0.00175	-0.00021	0.00061	-0.00251
	W_2	-0.15714	0.00047	0.00235	0.00067	0.00948	0.00007	-0.00035	-0.00635
	K_{-1}	-0.20296	0.000001	0.00079	0.00100	0.00429	-0.00004	0.00032	-0.00311
	T	-0.77492	0.00328	0.00878	0.00361	0.05720	0.00039	0.00048	-0.04614
	$(W_1+T)_{-1}$	-0.00711	0.00033	0.00053	0.00007	0.00266	-0.00051	-0.00002	-0.00111
	t	-0.04274	0.00011	0.00071	0.00013	0.00248	0.00015	-0.00043	-0.00163
	G	0.58455	-0.00384	-0.00668	-0.00278	-0.05010	0.00053	-0.00069	0.03953
		Private Wage Bill							
National Income	1	91.57610	-0.07833	-0.37841	-0.41751	-1.60802	-0.17580	-0.07469	1.47899
	P_{-1}	-0.03242	0.01654	-0.00047	-0.00086	0.00349	-0.00125	0.00047	-0.00059
	W_2	-0.34750	-0.00009	0.00445	0.00150	0.01658	0.00099	-0.00057	-0.01296
	K_{-1}	-0.41781	-0.00077	0.00162	0.00204	0.00739	0.00065	0.00062	-0.00690
	T	-1.44061	0.00502	0.01588	0.00671	0.10166	0.00147	0.00090	-0.08367
	$(W_1+T)_{-1}$	-0.20589	-0.00071	0.00145	0.00076	0.00373	0.00203	-0.00040	-0.00527
	t	-0.59971	-0.00003	0.00049	0.00043	0.00291	-0.00023	0.00103	-0.00248
	G	1.39719	-0.00193	-0.01329	-0.00657	-0.08763	-0.00363	-0.00155	0.07907
		Profits							
National Income	1	106.26628	-0.18501	-0.43510	-0.49329	-2.01277	-0.09624	-0.10415	1.63436
	P_{-1}	-0.23092	0.01892	0.00036	-0.00031	0.00530	0.00035	-0.00005	-0.00577
	W_2	-0.46601	-0.00002	0.00573	0.00196	0.02123	0.00154	0.00170	-0.01678
	K_{-1}	-0.49300	-0.00040	0.00184	0.00242	0.00925	0.00043	0.00037	-0.00763
	T	-2.18019	0.00377	0.02193	0.00993	0.13140	0.00635	0.00573	-0.11552
	$(W_1+T)_{-1}$	-0.06614	-0.00019	0.00108	0.00032	0.00408	-0.00010	0.00042	-0.00263
	t	-0.11913	0.00045	0.00064	0.00056	0.00373	0.00025	-0.00006	-0.00324
	G	1.71616	-0.00443	-0.01645	-0.00796	-0.11156	-0.00427	-0.00418	0.09827

COLLECTED BIBLIOGRAPHY

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ADELMAN, I. (1960) 'Business Cycles Endogenous or Stochastic?' Economic Journal, pp.783-96.

ANDERSEN, L.C. (1973) 'The State of the Monetarist Debate.' Federal Reserve Bank of St. Louis Review, September, pp.2-8.

ANDERSEN, L.C. and J.L. JORDAN (1968a) 'The Monetary Base - Exploration and Analytical Use.' Federal Reserve Bank of St. Louis Review, August.

ANDERSEN, L.C. and J.C. JORDAN (1968b) 'Monetary and Fiscal Actions: A Test of their Relative Importance in Economic Stabilization.' Federal Reserve Bank of St. Louis Review, November, pp.11-24.

ANDERSEN, L.C. and D.S. KARNOSKY (1973) 'The Appropriate Time Frame for Controlling Monetary Aggregates: The St. Louis Evidence.' In Controlling Monetary Aggregates II: The Implementation. Conference Series No.9, Federal Reserve Bank of Boston, pp.147-77.

ANDO, A. and F. MODIGLIANI (1963) 'The Life-Cycle Hypothesis of Saving: Aggregate Implications and Tests.' American Economic Review, pp.55-87.

ARTIS, M. (1971) 'Some Aspects of the Present Inflation and the National Institute Model.' In The Current Inflation, edit. by H.G. Johnson and A.R. Nobay, Macmillan, pp.3-37.

ARTIS, M.J. and M.K. LEWIS (1974) 'The Demand for Money: Stable or Unstable?' The Banker, pp.239-47.

ARTIS, M.J. and A.R. NOBAY (1969) 'Two Aspects of the Monetary Debate.' National Institute Economic Review, August, pp.33-51.

ARZAC, E.R. (1967) 'The Dynamic Characteristics of Chow's Model: A Simulation Study.' Journal of Financial and Quantitative Analysis, pp.383-97.

BAILEY, M.J. (1962) National Income and the Price Level, McGraw Hill.

BALL, R.J. and P.S. DRAKE (1963) 'Stock Adjustment Inventory Models of the United Kingdom Economy.' Manchester School of Economic and Social Studies, pp.87-103.

BALL, R.J. and P.S. DRAKE (1964) 'The Relationship Between Aggregate Consumption and Wealth.' International Economic Review, pp.63-81.

BANK OF ENGLAND (1966) 'The Aims of Debt Management.' Bank of England Quarterly Bulletin, pp.145-7.

BANK OF ENGLAND (1968) 'The Exchange Equalisation Account: Its Origins and Developments.' Bank of England Quarterly Bulletin, pp.377-87.

BANK OF ENGLAND (1969a) 'The U.K. Banking Sector 1952-67.' Bank of England Quarterly Bulletin, pp.176-200.

BANK OF ENGLAND (1969b) 'The Operation of Monetary Policy since the Radcliffe Report.' Bank of England Quarterly Bulletin, pp.448-60.

BANK OF ENGLAND (1971a) 'Monetary Management in the United Kingdom.' Bank of England Quarterly Bulletin, pp.37-47.

BANK OF ENGLAND (1971b) 'Competition and Credit Control.' Bank of England Quarterly Bulletin, pp.189-93.

BANK OF ENGLAND (1971c) 'Key Issues in Monetary and Credit Policy.' Bank of England Quarterly Bulletin, pp.195-8.

BANK OF ENGLAND (1971d) 'Capital Utilisation in Manufacturing Industries.' Bank of England Quarterly Bulletin, pp.490-6.

BANKER (1958a) 'Credit Expansion at Seven per Cent.' Banker, pp.104-12.

BANKER (1958b) 'The New Monetary Weapon: Full Description and Appraisal.' The Banker, pp.493-506.

BANKER (1959) 'Credit after the Squeeze.' Banker, pp.113-21.

BAUMOL, W. (1952) 'The Transactions Demand for Cash: An Inventory Theoretic Approach.' Quarterly Journal of Economics, pp.545-56.

BLACK, I.G., J.E. KIDGELL and G.F. ROY (1967) 'Forecasting Imports: A Re-examination.' National Institute Economic Review, November, pp.52-7.

BLINDER, A.S. (1975) 'Distribution Effects and the Aggregate Consumption Function.' Journal of Political Economy, pp.447-75.

BOORMAN, J.T. and T.M. HAVRILESKY (1972) Money Supply, Money Demand and Macroeconomic Models. Ally and Bacon.

BOWERS, J.K., P.C. CHESHIRE and A.E. WEBB (1970) 'The Change in the Relationship between Unemployment and Earnings Increases: A Review of Some Possible Explanations.' National Institute Economic Review, No.54, pp.44-63.

BRANSON, W.H. and A.K. KLEVORICK (1969) 'Money Illusion and the Aggregate Consumption Function.' American Economic Review, pp.832-49.

BRISCOE, G., P. O'BRIEN and D.J. SMYTH (1970) 'The Measurement of Capacity Utilisation in the United Kingdom.' Manchester School of Economic and Social Studies, pp.91-117.

BRONFENBRENNER, M. (1945) 'Some Fundamentals in Liquidity Theory.' Quarterly Journal of Economics, pp.405-26.

BROWN, T.M. (1952) 'Habit Persistence and Lags in Consumer Behaviour.' Econometrica, pp.355-71.

BRUNNER, K. and R. CROUCH (1968) 'Money Supply Theory and British Monetary Experience.' Operations Research Verfahren, pp.77-112.

BRUNNER, K. and A.H. MELTZER (1967) 'The Meaning of Monetary Indicators.' In Monetary Process and Policy: A Symposium, edit. by G. Horwich. Richard Irwin, pp.187-217.

BRUNNER, K. and A.H. MELTZER (1972) 'Friedman's Monetary Theory.' Journal of Political Economy, pp.837-51.

BRUNNER, K. and A.H. MELTZER (1973) 'Mr. Hicks and the Monetarists.' Economica, pp.44-59.

BURROWS, P. (1971) 'Explanatory and Forecasting Models of Inventory Investment in Britain.' Applied Economics, pp.375-89.

CAGAN, P. (1956) 'The Monetary Dynamics of Hyperinflation.' In Studies in the Quantity Theory of Money, edit. by M. Friedman. University of Chicago Press, pp.25-117.

CAGAN, P. (1958) 'The Demand for Currency Relative to Total Money Supply.' Journal of Political Economy, pp.303-28.

CARLSON, K.M. and R.W. SPENCER (1975) 'Crowding-Out and Its Critics.' Federal Reserve Bank of St. Louis Review, December, pp.2-17.

CENTRAL STATISTICAL OFFICE (1969) British Financial Institutions. H.M.S.O.

CHOW, G.C. (1966) 'On the Long-Run and Short-Run Demand for Money.' Journal of Political Economy, pp.111-31.

CHOW, G.C. (1967) 'Multiplier, Accelerator and Liquidity Preference in the Determination of National Income in the United States.' Review of Economics and Statistics, pp.1-15.

CHOW, G.C. and G.H. MOORE (1972) 'An Econometric Model of Business Cycles.' In Econometric Models of Cyclical Behavior, edit. by B. Hickman. National Bureau of Economic Research, pp.739-81.

CHRIST, C.F. (1967) 'A Short-Run Aggregate Demand Model of the Interdependence and Effects of Monetary and Fiscal Policies with Keynesian and Classical Interest Elasticities.' American Economic Review, pp.434-43.

CHRIST, C.F. (1968) 'A Simple Macroeconomic Model with a Government Budget Restraint.' Journal of Political Economy, pp.53-67.

CHRIST, C.F. (1971) 'Econometric Models of the Financial Sector.' Journal of Money, Credit and Banking, pp.419-49.

CHRIST, C.F. (1973) 'Monetary and Fiscal Influences on U.S. Money Income, 1891-1970.' Journal of Money, Credit and Banking, pp.279-300.

CLARK, J.M. (1917) 'Business Acceleration and the Law of Demand.' Journal of Political Economy, pp.217-35.

COCHRANE, D. and G.H. ORCUTT (1949) 'Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms.' Journal of the American Statistical Association, pp.749-809.

COGHLAN, R.T. (1973) 'Special Deposits and Bank Advances.' Bankers' Magazine, 216, pp.103-8.

COMMITTEE ON THE WORKING OF THE MONETARY SYSTEM (1959) Report, H.M.S.O.

CRAMP, A.B. (1971) 'The Bank of England's Proposals: Implications for Monetary Policy.' Bankers' Magazine, 212, pp.1-6.

CROUCH, R.L. (1965) 'The Genesis of Bank Deposits: New English Version.' Bulletin of the Oxford University Institute of Economics and Statistics, pp.185-99.

CROUCH, R.L. (1967) 'A Model of the United Kingdom's Monetary Sector.' Econometrica, pp.398-418.

CROUCH, R.L. (1968) 'Money Supply Theory and the United Kingdom's Monetary Contraction, 1954-56.' Bulletin of the Oxford University Institute of Economics and Statistics, pp.143-52.

CROUCH, R.L. (1970) 'Special Deposits and the British Monetary Mechanism.' Economic Studies, pp.3-19.

CUKIERMAN, A. (1972) 'Money Illusion and the Aggregate Consumption Function: Comment.' American Economic Review, pp.198-206.

DENBURG, T. and J.D. DENBURG (1969) Macroeconomic Analysis: An Introduction to Comparative Statics and Dynamics. Addison-Wesley.

DENISON, E.F. (1962) The Sources of Economic Growth in the United States and the Alternatives before Us. Occasional Paper No.13. Committee for Economic Development.

DEPPLER, M.C. (1974) 'Some Evidence on the Effects of Exchange Rate Changes on Trade.' International Monetary Fund Staff Papers, pp.605-36.

DHRYMES, P.J. (1970) Econometrics, Statistical Foundations and Applications. Harper and Row.

DHRYMES, P.J. (1973) 'Restricted and Unrestricted Reduced Forms: Asymptotic Distribution and Relative Efficiency.' Econometrica, pp.119-34.

DICKS-MIREAUX, L.A. (1961) 'The Interrelationship between Cost and Price Changes, 1946-1959.' Oxford Economic Papers, pp.267-92.

DOW, J.C.R. (1956) 'A Study of Cost and Price Changes in the United Kingdom, 1946-54.' Oxford Economic Papers, pp.252-301.

DUESENBERRY, J.S. (1949) Income, Saving and the Theory of Consumer Behavior. Harvard University Press.

DURBIN, J. (1970) 'Testing for Serial Correlation in Least-Squares Regression when Some of the Regressors are Lagged Dependent Variables.' Econometrica, pp.410-21.

ENTHOVEN, A.C. (1956) 'Monetary Disequilibria and the Dynamics of Inflation.' Economic Journal, pp.256-70.

FAIR, R.C. (1970) 'The Estimation of Simultaneous Equations Models with Lagged Endogenous Variables and First Order Serially Correlated Errors.' Econometrica, pp.507-16.

FAIR, R.C. (1970) 'A Comparison of Alternative Estimators of Macroeconomic Models.' International Economic Review, pp.261-77.

FAND, D.I. (1970) 'Some Issues in Monetary Economics.' Federal Reserve Bank of St. Louis Review, pp.10-27.

FAND, D.I. (1971) 'The Monetary Theory of Nine Recent Quarterly Econometric Models of the United States: A Comment.' Journal of Money, Credit and Banking, pp.450-57.

FEIGE, E.L. (1964) The Demand for Liquid Assets: A Temporal Cross Section Analysis. Prentice Hall.

FEIGE, E.L. (1967) 'Expectations and Adjustments in the Monetary Sector.' American Economic Review, Papers and Proceedings, pp.461-73.

FISHER, D. (1968) 'The Demand for Money in Britain: Quarterly Results 1951 to 1967.' Manchester School of Economic and Social Studies, pp.329-44.

FISHER, G.R. and D.K. SHEPPERD (1974) 'Interrelationships between Real and Monetary Variables: Some Evidence from Recent U.S. Empirical Studies.' In Issues in Monetary Economics, edit. by H.G. Johnson and A.R. Nobay. Oxford University Press, pp.179-259.

FISHER, I. (1913) The Purchasing Power of Money. Macmillan.

FISHER, I. (1930) The Theory of Interest. Macmillan.

FRANCIS, D.R. (1972) 'The Usefulness of Applied Econometrics to the Policy Maker.' Federal Reserve Bank of St. Louis Review, May, pp.7-10.

FRIEDMAN, B.M. (1973) 'Discussion.' In Controlling Monetary Aggregates II: The Implementation. Conference Series No.9. Federal Reserve Bank of Boston.

FRIEDMAN, M. (1956) 'The Quantity Theory of Money - A Restatement.' In Studies in the Quantity Theory of Money, edit. by M. Friedman, University of Chicago Press, pp.3-21.

FRIEDMAN, M. (1957) A Theory of the Consumption Function. Princeton University Press.

FRIEDMAN, M. (1959) 'The Demand for Money: Some Theoretical and Empirical Results.' Journal of Political Economy, 1959, pp.327-51.

FRIEDMAN, M. (1961) 'The Lag in Effect of Monetary Policy.' Journal of Political Economy, pp.447-66.

FRIEDMAN, M. (1963) 'The Concept of Horizon in the Permanent Income Hypothesis.' In Measurement in Economics, edit. by C.F. Christ *et.al.* Stanford University Press.

FRIEDMAN, M. (1968) 'Monetary Theory and Policy.' American Economic Review, pp.1-17.

FRIEDMAN, M. (1969) The Optimum Quantity of Money and Other Essays. Macmillan.

FRIEDMAN, M. (1970) The Counter-Revolution in Monetary Theory. Occasional Paper 33. Institute of Economic Affairs.

FRIEDMAN, M. (1971) A Theoretical Framework for Monetary Analysis. Occasional Paper 112. National Bureau of Economic Research.

FRIEDMAN, M. (1972) 'Comments on the Critics.' Journal of Political Economy, pp.906-50.

FRIEDMAN, M. (1973) Money and Economic Development. The Horowitz Lectures of 1972. Praeger.

FRIEDMAN, M. and A.J. SCHWARTZ (1963) 'Money and Business Cycles.' Review of Economics and Statistics, Supplement, pp.32-64.

FRIEDMAN, M. and D. MEISELMAN (1963) 'The Relative Stability of Monetary Velocity and the Investment Multiplier in the United States 1897-1958.' In Commission of Money and Credit, Stabilization Policies. Prentice Hall, pp.165-268.

FROMM, G. and L.R. KLEIN (1969) 'Solutions of the Complete System.' In The Brookings Model: Some Further Results. North-Holland, pp.363-421.

FROMM, G. and L.R. KLEIN (1973) 'A Comparison of Eleven Econometric Models of the United States.' American Economic Review, Papers and Proceedings, pp.385-93.

GIBSON, N.J. (1964) 'Special Deposits as an Instrument of Monetary Policy.' Manchester School of Economic and Social Studies, pp.239-59.

GIBSON, N.J. and D.R. THOM (1971) 'Can the Money Stock Be Controlled?' Bankers' Magazine, 212, pp.206-11.

GIBSON, W.E. (1970a) 'The Lag in the Effect of Monetary Policy on Income and Interest Rate.' Quarterly Journal of Economics, pp.288-300.

GIBSON, W.E. (1970b) 'Interest Rates and Monetary Policy.' Journal of Political Economy, pp.431-55.

GILL, L. (1975) 'The Existence of Instrumental Variables Estimators.' Mimeographed.

GILLION, C. and I. BLACK (1966) 'Some Characteristics of Unemployment.' National Institute Economic Review, No.37, pp.33-8.

GODLEY, W.H. and J.R. SHEPHERD (1964) 'Long-Term Growth and Short-Term Policy - The Productive Potential of the British Economy and Fluctuations in the Pressure of Demand for Labour 1951-1962.' National Institute Economic Review, No.29, pp.26-38.

GODLEY, W.A.H. and J.R. SHEPHERD (1965) 'Forecasting Imports.' National Institute Economic Review, August, pp.35-42.

GOLDBERGER, A.S., A.L. NAGAR and H.S. ODEH (1961) 'The Covariance Matrices of Reduced-Form Coefficients and Forecasts for a Structural Econometric Model.' Econometrica, pp.556-73.

GOODHART, C. (1971) 'Discussion.' In Monetary Theory and Monetary Policy in the 1970s, edit. by G. Clayton, J.C. Gilbert and R. Sedgwick. Oxford University Press, pp.728-9.

GOODHART, C. (1972) 'Analysis of the Determination of the Stock of Money.' In Essays in Modern Economics. Longman, pp.243-61.

GOODHART, C.A.E. (1973) 'Discussion.' In Issues in Federal Debt Management, Monetary Conference Series No.10. Federal Reserve Bank of Boston.

GOODHART, C.A.E. and A.D. CROCKETT (1970) 'The Importance of Money.' Bank of England Quarterly Bulletin, pp.159-98.

GORDON, M.J. (1962) The Investment, Financing and Valuation of the Corporation. Irwin.

GRAMLICH, E.M. (1971) 'Comments on the Discussions of Carl Christ Paper.' Journal of Money, Credit and Banking, pp.458-67.

GRILICHES, Z. (1967a) 'Distributed Lags: A Survey.' Econometrica, pp.16-49.

GRILICHES, Z. (1967b) 'On Crockett-Friend and Jorgenson.' In Determinants of Investment Behavior, edit. by R. Ferber. National Bureau of Economic Research, pp.160-2.

GUJARATI, D. (1972) 'The Behaviour of Unemployment and Unfilled Vacancies.' Economic Journal, pp.195-204.

GUPTA, V.K.L. (1968-9) 'Money Supply, Cyclical Fluctuations and Income Determination.' Jahbücher für Nationalökonomie und Statistik, pp.465-78.

GURLEY, J. and E. SHAW (1960) Money in a Theory of Finance. Brookings Institution.

HAAVELMO, T. (1960) A Study in the Theory of Investment. The University of Chicago Press.

HACCHE, G. (1974) 'The Demand for Money in the United Kingdom: Experience since 1971.' Bank of England Quarterly Bulletin, pp.284-305.

HAMBURGER, M.J. (1968) 'Household Demand for Financial Assets.' Econometrica, pp.97-118.

HANSEN, A.J. (1941) Fiscal Policy and Business Cycles. Norton and Co.

HAYEK, F.A. (1975) 'Inflation: the Path to Unemployment.' In Inflation: Causes, Consequences, Cures. Institute of Economic Affairs, pp.115-20.

HENDERSHOTT, P.H. and J.C. VAN HORNE (1972) 'Expected Inflation Implied by Capital Market Rates.' Journal of Finance, Papers and Proceedings, pp.301-14.

HENDRY, D.F. (1974) 'Stochastic Specification in an Aggregate Demand Model of the United Kingdom.' Econometrica, pp.559-78.

HICKS, J.R. (1937) 'Mr. Keynes and the Classics: A Suggested Interpretation.' Econometrica, pp.147-59.

HILLE, E. (1966) Analysis, Vol.II. Blaisdell Publishing Company.

HILTON, K. and D.H. CROSSFIELD (1970) 'Short-Run Consumption Functions for the U.K., 1955-66.' In The Econometric Study of the United Kingdom, edit. by K. Hilton and D.F. Heathfield. Macmillan, pp.59-98.

HINES, A.G. and G. CATEPHORES (1970) 'Investment in U.K. Manufacturing Industry, 1956-1967.' In The Econometric Study of the United Kingdom, edit. by K. Hilton and D.F. Heathfield. Macmillan, pp.203-24.

HODGMAN, D.R. (1973) 'Credit Controls in Western Europe: An Evaluative Review.' In Credit Allocation Techniques and Monetary Policy. Conference Series No.11, Federal Reserve Bank of Boston, pp.137-61.

HOLBROOK, R. and H. SHAPIRO (1970) 'The Choice of Optimal Intermediate Economic Targets.' American Economic Review, Papers and Proceedings, pp.40-6.

HOLMES, J.M. and D.J. SMYTH (1972) 'The Specification of the Demand for Money and the Tax Multiplier.' Journal of Political Economy, pp.179-85.

HOMA, K.E. and D.M. JAFFEE (1971) 'The Supply of Money and Common Stock Prices.' Journal of Finance, pp.1045-66.

HOUTHAKKER, H.S. and S.P. MAGEE (1969) 'Income and Price Elasticities in World Trade.' Review of Economics and Statistics, pp.111-25.

JOHNSTON, J. (1961) 'An Econometric Study of the Production Decision.' Quarterly Journal of Economics, pp.234-61.

JOHNSTON, J. (1972) Econometric Methods. McGraw-Hill.

JONES, D.M. (1965) 'The Demand for Money: A Review of the Empirical Literature.' Staff Economic Studies, Board of Governors of the Federal Reserve System.

JORGENSON, D.W. (1963) 'Capital Theory and Investment Behavior.' American Economic Review, pp.247-59.

JORGENSON, D.W. (1965) 'Anticipations and Investment Behavior.' In The Brookings Quarterly Model of the United States, edit. by J.S. Duesenberry *et.al.*, North Holland, pp.35-92.

JORGENSON, D.W. (1967) 'The Theory of Investment Behavior.' In Determinants of Investment Behavior, edit. by R. Ferber. National Bueau of Economic Research, pp.129-55.

KALDOR, N. (1955) 'Alternative Theories of Distribution.' Review of Economic Studies, pp.83-100.

KANE, E.J. and A.K. KLEVORICK (1967) 'Absence of Money Illusion: A Sine Qua Non for Neutral Money?' Journal of Finance, pp.419-23.

KAVANAGH, N.J. and A.A. WALTERS (1966) 'Demand for Money in the U.K., 1877-1961.' Bulletin of the Oxford University Institute of Economics and Statistics, pp.93-116.

KEYNES, J.M. (1936) The General Theory of Employment, Interest and Money. Macmillan.

KLEIN, R.L. (1950) Economic Fluctuations in the United States, 1921-1941. Cowles Commission Monograph 11. John Wiley.

KLEIN, R.L. (1958) 'The Friedman-Becker Illusion.' Journal of Political Economy, pp.539-45.

KLEIN, R.L. (1971) An Essay on the Theory of Economic Prediction. Markham.

KLEIN, R.L. (1974) 'Issues in Econometric Studies of Investment Behavior.' Journal of Economic Literature, pp.43-9.

KLEIN, R.L. and R.J. BALL (1959) 'Some Econometrics of the Determination of Absolute Prices and Wages.' Economic Journal, pp.465-82.

KLEIN, R.L., R.J. BALL, A. HAZLEWOOD and P. VANDOME (1961) An Econometric Model of the United Kingdom. Basil Blackwell.

KLEIN, L.R. and A.S. GOLDBERGER (1955) An Econometric Model of the United States, 1929-52. North-Holland.

KLEIN, L.R. and R.S. PRESTON (1967) 'Some New Results in the Measurement of Capacity Utilization.' American Economic Review, pp.36-58.

KLEIN, R.L. and R. SUMMERS (1966) The Wharton Index of Capacity Utilization. Studies in Quantitative Economics No. 1, University of Pennsylvania.

KNOWLES, J.W. (1960) 'The Potential Economic Growth in the United States.' Study Paper 20 in Study of Employment, Growth and Price Levels, Congress of the United States, Joint Economic Committee. U.S. Government Printing Office.

KOYCK, L.M. (1954) Distributed Lags and Investment Analysis. North-Holland.

KUH, E. (1966) 'Measurement of Potential Output.' American Economic Review, pp.758-76.

LAIDLER, D. (1966a) 'Some Evidence on the Demand for Money.' Journal of Political Economy, pp.55-68.

LAIDLER, D. (1966b) 'The Rate of Interest and the Demand for Money: Some Empirical Evidence.' Journal of Political Economy, pp.543-55.

LAIDLER, D.E.W. (1969) The Demand for Money: Theories and Evidence. International Textbook Company.

LAIDLER, D. (1970) Discussion Paper in Money in Britain 1959-1969, edit. by D.R. Croome and H.G. Johnson. Oxford University Press.

LAIDLER, D. (1971) 'The Influence of Money on Economic Activity - A Survey of Some Current Problems.' In Monetary Theory and Monetary Policy in the 1970s, edit. by G. Clayton, J.C. Gilbert and R. Sedgwick. Oxford University Press, pp.75-135.

LAIDLER, D. (1973a) 'Simultaneous Fluctuations in Prices and Output - A Business Cycle Approach.' Economica, pp.60-72.

LAIDLER, D. (1973b) 'The Influence of Money on Real Income and Inflation: - A Simple Model with Some Empirical Tests for the United States 1953-72.' Manchester School of Economic and Social Studies, pp.367-95.

LAIDLER, D. and M. PARKIN (1970) 'The Demand for Money in the United Kingdom 1956-1967: Preliminary Estimates.' Manchester School of Economic and Social Studies, pp.187-208.

LLEWELLYN, G.E.J. (1974) 'The Determinants of United Kingdom Import Prices.' Economic Journal, pp.20-31.

LOVELL, M. (1961) 'Manufacturers' Inventories, Sales Expectations and the Acceleration Principle.' Econometrica, pp.293-314.

LOMBRA, R. and H.M. KAUFMAN (1975) 'Interest Rate Seasonality and the Specification of Money Demand Functions.' Review of Economics and Statistics, pp.252-5.

MACESICH, G. (1962-63) 'Demand for Currency and Taxation in Canada.' Southern Economic Journal, pp.33-8.

MACKAY, D.I. (1972) 'After the Shake-Out.' Oxford Economic Papers, pp.89-110.

MATTFELDT, V.H. (1972/73) 'Zum Verhältnis verschiedener finanzieller Aktiva im Portefeuille inländischer Nichtbanken der BRD.' Jahrbücher für Nationalökonomie und Statistik, pp.237-44.

McCALLUM, B.T. (1970) 'The Effect of Demand on Prices in British Manufacturing: Another View.' Review of Economic Studies, pp.147-56.

McCALLUM, B.T. (1973) 'Friedman's Missing Equation: Another Approach.' Manchester School of Economic and Social Studies, pp.311-23.

McCARTHY, M.D. (1970-71) 'Notes on the Selection of Instruments for Two-Stage Least Squares and k Class Type Estimators of Large Models.' Southern Economic Journal, pp.251-9.

MEISELMAN, D. (1969) 'Discussion.' In Controlling Monetary Aggregates, Monetary Conference Series No.1. Federal Reserve Bank of Boston, pp.145-51.

MELTZER, A.H. (1963) 'The Demand for Money: The Evidence from the Time Series.' Journal of Political Economy, pp.219-46.

MODIGLIANI, F. (1966) 'The Life-Cycle Hypothesis of Saving, the Demand for Wealth and the Supply of Capital.' Social Research, pp.160-217.

MORONEY, J.R. and J.M. MASON (1971) 'The Dynamic Impacts of Autonomous Expenditures and the Monetary Base on Aggregate Income.' Journal of Money, Credit and Banking, pp.793-814.

MUNDELL, R.A. (1964) 'An Exposition of Some Subtleties in the Keynesian System.' Weltwirtschaftliches Archiv. Reprinted in Macro-economics, Selected Readings, edit. by W.L. Johnson and D.R. Komerschen, 1970.

MURPHY, J.L. (1975) Introductory Econometrics. Richard D. Irwin.

NATIONAL INSTITUTE OF ECONOMIC AND SOCIAL RESEARCH (1972) 'The Effects of the Devaluation of 1967 on the Current Balance of Payments.' Economic Journal, pp.442-64.

NEILD, R.R. (1963) Pricing and Employment in the Trade Cycle - A Study of British Manufacturing Industry, 1950-1961. Occasional Paper 21. National Institute of Economic and Social Research.

NERLOVE, M. (1958) Distributed Lags and Demand Analysis. Agriculture Handbook No.141, United States Department of Agriculture.

NERLOVE, M. and K.F. WALLIS (1966) 'The Use of the Durbin-Watson Statistic in Inappropriate Situations.' Econometrica, pp.235-8.

NEWLYN, W.T. (1964) 'The Supply of Money and Its Control.' Economic Journal, pp.327-46.

NEWLYN, W.T. (1971) Theory of Money. Oxford University Press.

NORDHAUS, W.D. and H.C. WALLICH (1973) 'Alternatives for Debt Management.' In Issues in Federal Debt Management, Monetary Conference Series No.10. Federal Reserve Bank of Boston.

NORTON, W.E. (1969) 'Debt Management and Monetary Policy in the United Kingdom.' Economic Journal, pp.475-94.

OKUN, A.M. (1963) 'Comment.' Review of Economics and Statistics, Supplement, pp.72-7.

PARKIN, J.M. (1967) 'The Genesis of Bank Deposits: New English Version - A Comment.' Bulletin of the Oxford University Institute of Economics and Statistics, pp.79-84.

PARKIN, M. (1972) 'Some Further Results on the Rate of Change of Money Wages.' In Incomes Policy and Inflation, Edit. by M. Parkin and M.T. Sumner, Manchester University Press.

PARKIN, M. (1973) 'The Short-Run and Long-Run Trade-Offs between Inflation and Unemployment in Australia.' Australian Economic Papers, pp.117-44.

PESEK, B.P. and T.R. SAVING (1967) Money, Wealth and Economic Theory. Macmillan.

PETERSON, D.W., E.M. LERNER and E.J. LUSK (1971) 'The Response of Prices and Income to Monetary Policy: An Analysis Based upon a Differential Phillips Curve.' Journal of Political Economy, pp.857-66.

PHELPS, E.S. (1967) 'Phillips Curves, Expectations of Inflation and Optimal Unemployment over Time.' Economica, pp.254-81.

PHILLIPS, A. (1963) 'An Appraisal of Measures of Capacity.' American Economic Review, Papers and Proceedings, pp.275-92.

PHILLIPS, A.W. (1958) 'The Relationship between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957.' Economica, pp.283-99.

PIERCE, D.G. and D.M. SHAW (1974) Monetary Economics: Theories, Evidence and Policy. Butterworth.

PIERCE, J.L. (1966) 'Commercial Bank Liquidity.' Federal Reserve Bulletin, pp.1093-101.

PIERSON, G. (1972) 'The Role of Money in Economic Growth.' Quarterly Journal of Economics, pp.383-95.

PIGOU, A.C. (1947) 'Economic Progress in A Stable Environment.' Economica, pp.180-8.

PINDYCK, R.S. (1973) Optimal Planning for Economic Stabilization. North Holland Publishing Company.

POOLE, W. (1970) 'Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macromodel.' Quarterly Journal of Economics, pp.197-216.

PRICE, L.D.P. (1972) 'The Demand for Money in the United Kingdom: A Further Investigation.' Bank of England Quarterly Bulletin, pp.43-55.

RAO, P. and R.L. MILLER (1971) Applied Econometrics. Wadsworth Publishing Co.

RUSHDY, F. and P.J. LUND (1967) 'The Effect of Demand on Prices in British Manufacturing Industry.' Review of Economic Studies, pp.361-73.

SAMUELSON, P.A. (1970) 'Reflections on Recent Federal Reserve Policy.' Journal of Money, Credit and Banking, pp.33-44.

SARGENT, T.J. (1971) 'The Optimum Monetary Instrument Variable in a Linear Economic Model.' Canadian Journal of Economics, pp.50-60.

SARGENT, T.J. (1972) 'Anticipated Inflation and the Nominal Rate of Interest.' Quarterly Journal of Economics, pp.212-25.

SAUNDERS, P.G. and A.R. NOBAY (1972) 'Price Expectations, the Phillips Curve and Incomes Policy.' In Incomes Policy and Inflation, edit. by M. Parkin and M.T. Sumner. Manchester University Press.

SAYERS, R.S. (1960) 'Monetary Thought and Monetary Policy in England.' Economic Journal, pp.710-24.

SCHMIDT, P. (1973) 'The Asymptotic Distribution of Dynamic Multipliers.' Econometrica, pp.161-4.

SCOTT, R.H. (1966) 'Estimates of Hicksian IS and LM Curves for the United States.' Journal of Finance, pp.479-87.

SELDEN, R. (1956) 'Monetary Velocity in the United States.' In Studies in the Quantity Theory of Money, edit. by M. Friedman. University of Chicago Press, pp.179-257.

SHEPHERD, J.R. (1968) 'Productive Potential and the Demand for Labour.' Economic Trends, pp.xxv-xxvii.

SILBER, W.L. (1970) 'Monetary Policy Effectiveness: The Case of a Positively Sloped IS Curve.' Journal of Finance, pp.1077-82.

SIMLER, N.J. and A. TELLA (1968) 'Labor Reserves and the Phillips Curve.' Review of Economics and Statistics, pp.32-49.

SLEEPER, R.D. (1972) 'S.E.T. and the Shake-Out: A Note on the Productivity Effects of Selective Employment Tax.' Oxford Economic Papers, pp.197-211.

SMITH, P.E. (1967) 'Money Supply and Demand: A Cobweb?' International Economic Review, pp.1-12.

SMITH, P.E. (1972) 'Lags in the Effects of Monetary Policy: Comment.' American Economic Review, pp.230-3.

SMITH, W.L. (1956) 'A Graphical Exposition of the Complete Keynesian System.' Southern Economic Journal. Reprinted in Macroeconomics, Selected Readings, edit. by W.L. Johnson and D.R. Komerschen, 1970.

SMITH, W.L. (1969) 'A Neo-Keynesian View of Monetary Policy.' In Controlling Monetary Aggregates. Conference Series No.1. Federal Reserve Bank of Boston, pp.105-26.

SOLOW, R.M. (1969) Price Expectations and the Behavior of the Price Level. Manchester University Press.

SPENCER, B.G. (1975) 'The Small Sample Bias of Durbin's Test for Serial Correlation.' Journal of Econometrics, pp.249-54.

SPENCER, R.W. and W.P. YOHE (1970) 'The Crowding-Out of Private Expenditures by Fiscal Policy Actions.' Federal Reserve Bank of St. Louis Review, October, pp.12-24.

SPIRO, A. (1962) 'Wealth and the Consumption Function.' Journal of Political Economy, pp.339-54.

STEINDL, F.G. (1971) 'A Simple Macroeconomic Model with a Government Budget Restraint.' Journal of Political Economy, pp.675-9.

STEINDL, F.G. (1973) 'Price Expectations and Interest Rates.' Journal of Money, Credit and Banking, pp.939-49.

STONE, R. and D.A. ROWE (1956) 'Aggregate Consumption and Investment Functions for the Household Sector Considered in the Light of British Experience.' Nationalökonomisk Tidskrift, pp.1-32.

STRAND, K. and T. DENBURG (1964) 'Cyclical Variation in Civilian Labor Force Participation.' Review of Economics and Statistics, pp.378-89.

SUITS, D.B. (1963) 'The Determinants of Consumer Expenditure: A Review of Present Knowledge.' In The Impacts of Monetary Policy. Commission on Money and Credit. Prentice Hall, pp.7-57.

SUMNER, M.T. (1972) Aggregate Demand, Price Expectations and the Phillips Curve. In Incomes Policy and Inflation, edit. by M. Parkin and M.T. Sumner. Manchester University Press.

TANNER, J.E. (1972) 'Lags in the Effects of Monetary Policy. Reply and Some Further Thoughts.' American Economic Review, pp.234-7.

TANNER, J.E. (1969) 'Lags in the Effects of Monetary Policy: A Statistical Investigation.' American Economic Review, pp.794-805.

TAYLOR, L.D. (1971) 'Saving Out of Different Types of Income.' Brookings Papers on Economic Activity, pp.383-407.

TEIGEN, R.L. (1964) 'Demand and Supply Functions for Money in the United States: Some Structural Estimates.' Econometrica, pp.476-509.

TEIGEN, R.L. (1969) 'An Aggregated Quarterly Model of the U.S. Monetary Sector 1953-1964.' In Targets and Indicators of Monetary Policy, edit. by K. Brumner. Chandler Publishing Company, pp.175-218.

TEIGEN, R.L. (1970) 'The Demand for and Supply of Money.' In Readings in Money, National Income and Stabilization Policy, edit. by W.L. Smith and R.L. Teigen. Richard Irwin, pp.74-112.

TEIGEN, R.L. (1972) 'A Critical Look at Monetarist Economics.' Federal Reserve Bank of St. Louis Review, January, pp.10-25.

TELLA, A. (1964) 'The Relation of Labor Force to Employment.' Industrial and Labor Relations Review, pp.454-69.

THEIL, H. (1961) Economic Forecasts and Policy. North Holland.

THEIL, H. (1971) Principles of Econometrics. John Wiley.

THEIL, H. and J.C.G. BOOT (1962) 'The Final Form of Econometric Equation Systems.' Review of the International Statistical Institute, pp.136-52.

TOBIN, J. (1956) 'The Interest Elasticity of Transactions Demand for Cash.' Review of Economics and Statistics, pp.241-7.

TOBIN, J. (1958) 'Liquidity Preference as Behavior Towards Risk.' Review of Economic Studies, pp.65-86.

TOBIN, J. (1961) 'Monetary Theory: New and Old Looks. Money Capital and Other Stores of Value.' American Economic Review, Papers and Proceedings, pp.26-37.

TOBIN, J. (1967) 'On Crockett-Friend and Jorgenson.' In Determinants of Investment Behavior, edit. by R. Ferber. National Bureau of Economic Research, pp.156-60.

TOBIN, J. (1969) 'The Role of Money in National Economic Policy.' In Controlling Monetary Aggregates. Monetary Conference Series No.1, Federal Reserve Bank of Boston, pp.21-4.

TOBIN, J. (1972) 'Inflation and Unemployment.' American Economic Review, pp.1-18.

TUCKER, D.P. (1966) 'Dynamic Income Adjustment to Money Supply Changes.' American Economic Review, pp.433-49.

TURNOVSKY, S.T. (1975) 'Optimal Choice of Monetary Instrument in a Linear Economic Model with Stochastic Coefficients.' Journal of Money Credit and Banking, pp.51-80.

U.K. DEPARTMENT OF EMPLOYMENT (1970) 'The Fall in the Working Population since 1966.' Employment and Productivity Gazette, pp.492-5.

U.K. DEPARTMENT OF EMPLOYMENT (1973) 'The Fall in the Labour Force between 1966 and 1971.' Department of Employment Gazette, pp.1083-7.

WALTERS, A.A. (1970) 'Monetary Policy, Gilts and Equities.' Investment Analyst, December, pp.3-6.

WALTERS, A.A. (1971) 'Consistent Expectations, Distributed Lags and the Quantity Theory.' Economic Journal, pp.273-81.

WALTERS, A.A. (1974) 'Discussion Paper.' In Issues in Monetary Economics, edit. by H.G. Johnson and A.R. Nobay. Oxford University Press, pp.484-9.

WESTON, J.F. and E.F. BRIGHAM (1966) Managerial Finance. Holt, Rinehart and Winston.

WHITLEY, J.D. and G.D.N. WORSWICK (1971) 'The Productivity Effects of Selective Employment Tax.' National Institute Economic Review, No.56, pp.36-40.

ZECHER, R. (1970) 'Implications of Four Econometric Models for the Indicators Issue.' American Economic Review, Papers and Proceedings, pp.47-54.

ZELLNER, A. (1957) 'The Short Run Consumption Function.' Econometrica, pp.552-66.

ZELLNER, A., D.S. HUANG and L.C. CHAU (1965) 'Further Analysis of the Short Run Consumption Function with Emphasis on the Role of Liquid Assets.' Econometrica, pp.570-81.