

THE DUAL ABSORPTIVE MODEL AND ELASTIC HADRONIC SCATTERING
AT HIGH ENERGIES AND SMALL MOMENTUM TRANSFERS

Thesis

Submitted by

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To

my parents

and

my friends

DECLARATION

I declare that all the work done in this thesis is entirely my own except where otherwise indicated.

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"I feel that all theorists should do some phenomenology occasionally to anchor them on earth, in the same spirit as St. Paul used to sew tents between epistles."

C. Lovelace at the 1966 Berkeley Conference.

ABSTRACT OF THESIS

An attempt has been made to assess the status of Harari's Dual Absorptive Model (M) in regard to elastic hadronic scattering at high energies and small momentum transfers by parameterising the scattering amplitudes for several elastic reactions using the DAM "rules", and some standard phenomenological assumptions, and then trying to fit data using these amplitudes. The reactions considered are $\pi^\pm p \rightarrow \pi^\pm p$, $\pi^\pm p \rightarrow K^\pm p$, $pp \rightarrow pp$, $np \rightarrow np$ and $\bar{p}p \rightarrow \bar{p}p$. The data has been considered for each reaction from $p_{\text{lab}} \gtrsim 10$ GeV/c up to the highest energy at which data is available. The experimental data to be considered is surveyed in Chapter 1 while Chapter 2 is devoted to an exposition of several concepts and theoretical models which are widely employed in high-energy hadronic phenomenology. Chapter 3 describes the DAM. In Chapter 4, using the DAM "rules" $\pi^\pm p$ elastic data is fitted up to 280 GeV/c ($|t| \lesssim 1.2$ (GeV/c)²). We find that it is possible to explain the main features of $\pi^\pm p$ data using the DAM "rules". Chapter 5 is concerned with $K^\pm p$ elastic scattering. Provided a real part to the Pomeron contribution to the non-flip amplitude is introduced, the small $\frac{d\sigma}{dt}(K^\pm p)$ can be explained using the DAM "rules". We find that if one fits $\pi^\pm p$ using the DAM, a discrepancy of at least 30% exists in the numerical value of one parameter as obtained from $P(K^+ p)$ fits and $P(K^- p)$ fits. In Chapter 6 the reactions $pp \rightarrow pp$, $np \rightarrow np$ and $\bar{p}p \rightarrow \bar{p}p$ are considered. Fits to $\sigma_{\text{tot}}(pp, np)$ indicate a breaking of EXD (at both the strong and weak levels). $\frac{d\sigma}{dt}$ data for all these reactions can be fitted (up to $|t| \approx 1.3$ (GeV/c)²) by taking the non-Pomeron contributions to the non-flip amplitude as peripheral. $P(pp)$, on the other hand, can be fitted up to 300 GeV/c by parameterising the single-flip amplitude in accordance with the DAM, provided one introduces an imaginary Pomeron-flip contribution. $P(np)$ can be predicted very accurately using the flip amplitude from $P(pp)$, provided one assumes that both $I = 1$ and $I = 0$ exchanges contribute to the single flip amplitude with equal strength. In Chapter 7 we mention attempts to fit $\pi^\pm p$ data by including a real Pomeron contribution to the non-flip amplitude, and describe a fit to $P(K^\pm p)$ which restores the inconsistency in parameter values observed in Chapter 5, and then summarise the results of Chapter 4-6. In Chapter 8 we refit elastic data by a unified treatment of all reactions, treating trajectory parameters as free parameters, including various phases, and enforcing isospin constraints on the parameter values. Our final conclusions are that the $\rho - A_2$ trajectories /

e not EXD but that the f and ω trajectories approximately coincide. $\rho - A_2$
 well as $f - \omega$ EXD is broken at the level of residues in both KN and NN
 scattering, the EXD breaking being particularly severe in the NN case. $\rho - A_2$
 and $f - \omega$ EXD breaking in exotic channels contradicts the DAM expectations in this
 regard. The imaginary parts of the ρ , ω and P contributions are in accord with
 the DAM with the usual Regge pole phases. A Pomeron slope of 0.23 yields a sub-
 stantial Pomeron phase away from $t = 0$, contradicting the DAM approximation
 $\phi = 0$. For the leading $\tau = +1$ "ordinary" exchanges the DAM ansatz for imaginary
 parts has to be modified (" $J_{\Delta\lambda}$ " $\rightarrow$ " $J_{\Delta\lambda}$ " $\sin \frac{\pi\alpha}{2}$) if their phases $(-\cot \frac{\pi\alpha}{2} \text{Im } T_{\Delta\lambda})$
 are to be prevented from blowing up at $\alpha = 0$. This modification which is necessary
 in the $\Delta\lambda = 0$ case has been adapted for the $\Delta\lambda = 1$ case for uniformity. Fits
 to the data yield $R_f, R_{A_2} \sim 0.6 - 0.7$ fm in this modified ansatz, values which are
 all below 1 fm. There is evidence of a Pomeron flip term in πN and NN scattering
 though in the latter case there are problems that lead to interesting speculation.
 There is also evidence of the presence of low-lying exchanges in πN , KN and NN
 amplitudes.

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CHAPTER 1

EXPERIMENTAL SURVEY

1.1 Introduction

We shall, in this chapter, describe the main features of experimental results pertaining to measurements of quantities whose explanation remains the unending goal of theoretical models. We emphasise that we are going to deal with elastic hadronic scattering in this work and this will therefore dictate the quantities to be dealt with. The various physical quantities actually measured in a particular reaction depend not only on the nature of the particles involved but also on the feasibility of their experimental determination, this latter being a factor which changes with time since more sophisticated instruments and techniques are continually being developed. The main quantities that can be measured in elastic scattering are:

- i) Total cross-section σ_{tot} ;
- ii) The ratio of the real and imaginary parts of the elastic scattering amplitude in the forward direction, ρ ;
- iii) The elastic differential cross-section $\frac{d\sigma}{dt}$;
- iv) The polarisation parameter P for elastic scattering;
- v) The spin rotation parameters R and A for elastic scattering;
- vi) Various spin correlation parameters for elastic nucleon-nucleon scattering.

Of the various quantities mentioned above, all except the first two can be measured in suitable two-body inelastic reactions as well. Furthermore, all except the first two quantities depend on two independent variables, σ_{tot} and ρ depending on only one variable. The choice of the variables employed in describing various quantities depends on various factors such as the angular region, energy region etc. and this

will become evident as we go along. Usually (but not always) it is convenient to employ relativistically invariant variables. Some of these variables will be defined in Chapter 2. Kinematical relations can be found in Byckling & Kajantie (1973). We shall not be concerned with quantities of the type (vi) above and shall therefore discuss only the first five quantities. The reason for neglecting these is simply the fact that these spin correlation parameters have generally been measured at rather low energies ($p_{\text{lab}} < 10 \text{ GeV/c}$), which fall outside the energy regime explored in this thesis. σ_{tot} , $\frac{d\sigma}{dt}$ and P are defined in most standard texts (e.g. Martin and Spearman (1970), Williams (1971), Bransden and Moorhouse (1974)). Definitions of spin rotation parameters R and A can be found in Hamilton and Peterson (1975).

1.2 Total Cross-Sections

The total cross-section in the collision of two particles is a function of a single variable which is chosen variously as the momentum (p_{lab}) or the energy (E_{lab}) of the projectile with respect to the laboratory frame, or the square of the total energy (s) of the colliding particles in their centre of mass (c.m.) frame. The following features of the total cross-sections for various colliding particles may be noted:

a) At energies* corresponding to $p_{\text{lab}} \lesssim 4 \text{ GeV/c}$ the total cross-sections for $\pi^{\pm}N$, $K^{\pm}N$ (N denotes a proton or a neutron), $\bar{p}p$ collisions show very rapid variations with energy and are characterised by several bumps which are usually associated with the formation of unstable particles called resonances. [At these bumps σ_{tot} can rise as high as 80-100 mb or more.] The K^+N and NN total cross-sections however do not display resonances and vary smoothly with energy.

* Energy in the following means $\sqrt{s}$ the total energy in the c.m. frame.

b) Beyond $p_{lab} \sim 4$ GeV/c however a striking regularity emerges in the energy dependence of the various total cross-sections. Cross-sections for all pairs of colliding particles (including $\sigma_{tot}(\pi^{\pm}N)$, $\sigma_{tot}(K^{\pm}N)$, $\sigma_{tot}(\bar{p}p)$) are slowly varying functions of energy up to the highest energies at which measurements exist (Fig. 1.1). The highest p_{lab} at which measurements exist (Carrol et al. (1976), Amaldi et al. (1977)) are as follows: 280 GeV/c for π^-p , K^+n , pn ; 240 GeV/c for π^+p , K^+p , K^-n ; 200 GeV/c for $\bar{p}n$, $\bar{p}p$; 2092 GeV/c ($\sqrt{s} = 62$ GeV) for pp . Over the entire range of energies above $p_{lab} \sim 4$ GeV/c meson-nucleon total cross-sections vary between 20-30 mb and nucleon (anti-nucleon)-nucleon total cross-sections range between 38-60 mb. The data in the above range can be parameterised by a variety of forms, some of which have a theoretical basis while others are mere empirical fits to data. Some parameterisations are displayed in Table 1.1.

c) The $\pi^{\pm}N$, $K^{\pm}N$ and NN cross-sections fall slowly beyond $p_{lab} \sim 4$ GeV/c to a shallow minimum in the range $30 \lesssim p_{lab} \lesssim 80$ GeV/c, beyond which they start rising. $\sigma_{tot}(\bar{p}p)$ on the other hand is falling up to 200 GeV/c (the highest p_{lab} at which measurements exist) although the rate of fall seems to decrease at higher laboratory momenta. Whether this foreshadows an eventually rising $\sigma_{tot}(\bar{p}p)$ remains to be seen. $\sigma_{tot}(K^+N)$ does not display any fall in value over the entire p_{lab} range and is rising at the highest p_{lab} at which measurements exist. The rise is most spectacular in $\sigma_{tot}(pp)$ and a fit by Amaldi et al. (1977) seems to indicate that its rise saturates the Froissart bound* at ISR energies ($\sqrt{s} \approx 23-62$ GeV).

d) Anti-particle-target cross-sections approach the corresponding particle-target cross-sections from above as p_{lab} increases. The difference $\Delta\sigma(XN) \equiv \sigma_{tot}(\bar{X}N) - \sigma_{tot}(XN)$ can be accurately parameterised as (Giacomelli (1976))

$$\Delta\sigma(XN) = A(p_{lab})^{-n} \quad (1.1)$$

*The Froissart bound, first derived by Froissart using unitarity, Mandelstam representation (and the Optical theorem) states (c.f. Eden(1971))
 $\sigma_{tot}(s) \leq \text{const.}(\ln s)^2$

TABLE 1.1 - Some Parameterisations of Hadron-Proton Total Cross-Sections.

Parameterisation	Reference
$\sigma_{\text{tot}}(\text{Hp}) = G_P s^{0.07} + \frac{G_R}{\sqrt{2}} [1 - (\pm\beta)] s^{-0.5}$	H denotes $\pi^\pm$, $K^\pm$, p and $\bar{p}$. G_P, G_R, β are reaction dependent parameters. + refers to cases when H is π^+, K^+ or p and to cases when H is π^-, K^-, $\bar{p}$. Collins et al. (1974a)
$\sigma_{\text{tot}}(\text{Hp}) = 13 N_q^H (p_{\text{lab}}/20)^{1.13} + 4.4 N_q^H N_{\text{ns}}^H (p_{\text{lab}}/20)^{0.8} + 1.75 (N_n^H + 2N_p^H) (p_{\text{lab}}/20)^{-0.5}$	N_q^H denotes number of quarks in H, the quark type indicated by subscript. N_q^H is the total number of quarks in H. $H \equiv \pi^\pm, K^\pm, \bar{p}, p$, N_{ns}^H is the number of non-strange quarks in H. Lipkin (1975)
$\sigma_{\text{tot}}(\text{Hp}) = \left[\begin{array}{l} \text{Diffractive Term} \\ C_i \ln\left(\frac{p_{\text{lab}} + m_i}{b_i}\right) \\ \text{OR} \\ a_i + b_i \ln p_{\text{lab}} + C_i \ln^2 p_{\text{lab}} \\ \text{OR} \\ a_i + b_i (\ln p_{\text{lab}})^{C_i} \\ \text{OR} \\ a_i + b_i \left(\ln\left(\frac{p_{\text{lab}}}{C_i}\right)\right)^{d_i} \end{array} \right] +$	$\left[\begin{array}{l} \text{Non-Diffractive Term} \\ C(p_{\text{lab}})^{-n} \text{ for } \pi^\pm, K^\pm \\ \\ 2\beta_{\text{pp}}^p (p_{\text{lab}} - p_0)^{\alpha p - 1} + 2\beta_{\text{pp}}^{\omega} (p_{\text{lab}} - p_0)^{\alpha \omega - 1} \\ \text{for } p, \bar{p} \end{array} \right]$ Hendrick et al.⁷⁵ (1975)
$\sigma_{\text{tot}}(\text{pp}) = 38.5 + 0.5 \ln^2(s/120) \text{ mb}$	E. Leader and U. Maor (1973)

where $n > 0$ and A and n are different for different cases. A compilation of n values is given in Giacomelli (1976). Very roughly $n \approx 0.5$ in most cases. The asymptotic approach to equality, of $\sigma_{\text{tot}}(\bar{x}p)$ and $\sigma_{\text{tot}}(xp)$,

is in accord with the Pomeranchuk theorem. (The Pomeranchuk theorem for total cross-sections states (c.f. Eden(1971)) that asymptotically the particle-tgt total cross-section approaches the corresponding anti-particle total cross-section (i.e. $\sigma_{\text{tot}}(ab) \xrightarrow{s \rightarrow \infty} \sigma_{\text{tot}}(\bar{a}b)$).

1.3A Elastic Differential Cross-Sections

The two-body differential cross-section is a function of two independent variables whose choice is determined by the momentum transfer (t) or centre-of-mass scattering angular region being explored. The phenomenological description of elastic differential cross-sections at any (high) energy is divided into four regions given below (c.f. Giacomelli, 1976):

- i) Coulomb-Nuclear Interference Region: This region is, roughly, $.001 \lesssim |t| \lesssim .02 \text{ (GeV/c)}^2$ and, in this work, is occasionally referred to as the very small $|t|$ region.
- ii) Small $|t|$ Region: This is characteristically $.02 \lesssim |t| \lesssim 1.5 \text{ (GeV/c)}^2$ and is also called the near-forward or small-angle region*.
- iii) Large $|t|$, Large $|u|$ Region: It is also known as the large angle region.
- iv) Small $|u|$ Region: It is also known as the near-backward region and is roughly $u_{\text{min}} \lesssim |u| \lesssim 1.5 \text{ (GeV/c)}^2$ where u_{min} is determined by the masses of the particles involved.

In this work we shall be concerned only with the small momentum-transfer region, i.e. region (ii) above.

In the following the main features of elastic differential cross-sections

*

This terminology in which the phrases small $|t|$ and small angle are used equivalently has become standard although it is imprecise if the energies are not very high. For instance, in $pp \rightarrow pp$ $|t| = 1 \text{ (GeV/c)}^2$ corresponds to $\theta = 41.8^\circ$ at $p_{\text{lab}} = 5 \text{ GeV/c}$ which is not a small angle. At $p_{\text{lab}} = 200 \text{ GeV/c}$ however it corresponds to $\theta = 5.9^\circ$. θ is the centre of mass scattering angle. In the following we shall follow the standard convention.

are described.* The emphasis will be on the small $|t|$ region. The reactions considered are $\pi^\pm p \rightarrow \pi^\pm p$, $K^\pm p \rightarrow K^\pm p$, $pp \rightarrow pp$, $\bar{p}p \rightarrow \bar{p}p$ and $np \rightarrow np$.

1.3B Coulomb-Nuclear Interference and the Real Part of the Elastic Forward Amplitude

Measurements of elastic differential cross-sections in the very small $|t|$ region (roughly $0.001 \lesssim |t| \lesssim 0.02$ (GeV/c)²) enable us to deduce the real part of the spin-nonflip amplitude for elastic meson-nucleon scattering in the forward direction ($t = 0$), under the assumption that the spin-flip amplitude (which vanishes at $t = 0$ owing to conservation of angular momentum) can be neglected in this region (c.f. e.g. Eden, 1967). For nucleon-nucleon elastic scattering one introduces an additional assumption of spin-independence of the very small $|t|$ scattering since, in this case, there exist two amplitudes which do not vanish at $t = 0$. One can then speak unambiguously of a spin-nonflip amplitude and elicit its value in the forward direction from differential cross-section measurements. Many times, as a theoretical alternative, instead of the real part of the forward elastic amplitude one employs its ratio with the corresponding imaginary part, the latter being obtained from total cross-sections via the optical theorem. Throughout this section we shall denote the nonflip amplitude by f_N and the ratio of its real and imaginary parts by $\rho(s,t)$ or simply ρ i.e. $\rho \equiv \frac{\text{Re} f_N}{\text{Im} f_N}$. In both, the meson-nucleon and nucleon-nucleon cases, two additional simplifying assumptions are implicit in the procedure whereby one extracts $\rho(s,0)$ from the very small $|t|$ elastic differential cross-sections. These are:

- i) In the very small $|t|$ region the ratio of the real and imaginary parts of the non-flip amplitude is independent of t .

*Regions ii, iii, and iv mentioned above are common to both elastic and two-body inelastic reactions. Region i however is characteristic only of elastic scattering.

- ii) The imaginary part of f_N falls exponentially in t as one moves away from $t = 0$.

With the aforementioned points in mind one proceeds as follows. From a theoretical point of view the case when both particles are charged is slightly more involved than that when one or both the particles are uncharged (since the Coulomb amplitude vanishes for the latter case), although the inability to have free neutron targets can make the latter case somewhat tedious (by requiring Glauber corrections). When two charged hadrons collide both "nuclear" and Coulomb interactions contribute to the scattering amplitude. The Coulomb amplitude (f_C) is known exactly in the entire $|t|$ range and is dominant at $|t|$ values close to zero. As one moves away from $t = 0$ one comes across a range (called the Coulomb-nuclear interference region) where Coulomb and nuclear contributions are comparable. This region is, roughly, $0.001 \lesssim |t| \lesssim 0.02(\text{GeV}/c)^2$. Beyond this region the Coulomb contribution is negligible. One can then fit the differential cross-section measured in this region by the equation

$$\frac{d\sigma}{d\Omega} = |f_N + f_C|^2 = |(\rho+i)\text{Im}f_N|^2 + |f_C|^2 + 2f_C\rho\text{Im}f_N \quad (1.2)$$

where $\text{Im}f_N = \frac{q}{4\pi} \sigma_{\text{tot}} e^{bt/2}$, using the optical theorem and assumption (ii) above (q is the magnitude of the 3-momentum of any one of the colliding particles in the centre-of-mass frame). This fit, at any energy, yields the values of ρ and b at that energy.

Measurements of ρ indicate that, in general, $\rho \neq 0$, contrary to the Optical Model expectation (Chapter 2). The detailed behaviour of ρ as a function of energy or laboratory momentum varies with the reactions under consideration (Fig. 1.2). For the $\pi^\pm p$ case (Baillon et al. (1974, 1976), Foley et al. (1965, 1969), Apokin et al. (1975), Lach (1977)) ρ is negative above $p_{\text{lab}} \approx 1 \text{ GeV}/c$ and approaches zero at the highest p_{lab} at which measurements exist (Lach (1977)). Whether it becomes positive beyond this point or remains

at zero is a matter to be decided by further measurements. $\rho(K^-p)$ approaches zero from above at about 10 GeV/c and, at most p_{lab} values beyond this, is compatible with zero. $\rho(K^+p)$ on the other hand is negative above 1 GeV/c and up to 10 GeV/c has a very large magnitude (about 0.5 approx.) compared with all the preceding cases. However it vanishes around $p_{lab} \approx 70$ GeV/c, becomes positive and falls to zero again at about 150 GeV/c (Lach, 1977). Data for $\rho(K^\pm p)$ can be found in Chinowsky et al. (1965), Dansyz et al. (1969), Baillon et al. (1970a,b), Foley et al. (1963), Campbell et al. (1973), Baillon et al. (1974, 1976a,b), Carnegie et al. (1975), Lach (1977). Very few measurements of $\rho(\bar{p}p)$ exist but the few measurements that do exist (Foley et al. (1967), Jenni et al. (1975), Lach (1977)) indicate that beyond $p_{lab} \approx 2$ GeV/c it is consistent with zero. $\rho(pp)$ has large negative values in the range $3 \leq p_{lab} \leq 20$ GeV/c but it is an increasing function of energy and its magnitude decreases with energy up to $p_{lab} \approx 200$ GeV/c, where it is zero and beyond which ρ_{pp} becomes positive (Foley et al. (1967), Taylor et al. (1964), Beznogikh et al. (1972, 1974), Bellentini et al. (1965), Kirillova et al. (1966), Bartenev et al. (1973), Amaldi et al. (1973, 1977), Gross et al. (1978)). Isospin invariance leads us to expect $\rho_{pp} \xrightarrow{s \rightarrow \infty} \rho_{pn}$. This result seems to hold, within errors, in the region $50 \leq E_{lab} \leq 400$ GeV, at which measurements have been made by Gross et al. (1978). They find that in this range one can fit ρ_{pn} by:

$$\rho_{pn} = (-.49 \pm .034) + (.076 \pm .006) \ln s \quad (1.3)$$

Measurements of ρ_{pn} and ρ_{pp} have been made by Beznogikh et al. (1974) in the range $10 \leq p_{lab} \leq 70$ GeV/c, who find that in this range ρ_{pn} is close to ρ_{pp} .

The real part of a scattering amplitude is related to its imaginary part by a dispersion relation (Chapter 2). At $t = 0$, the imaginary part

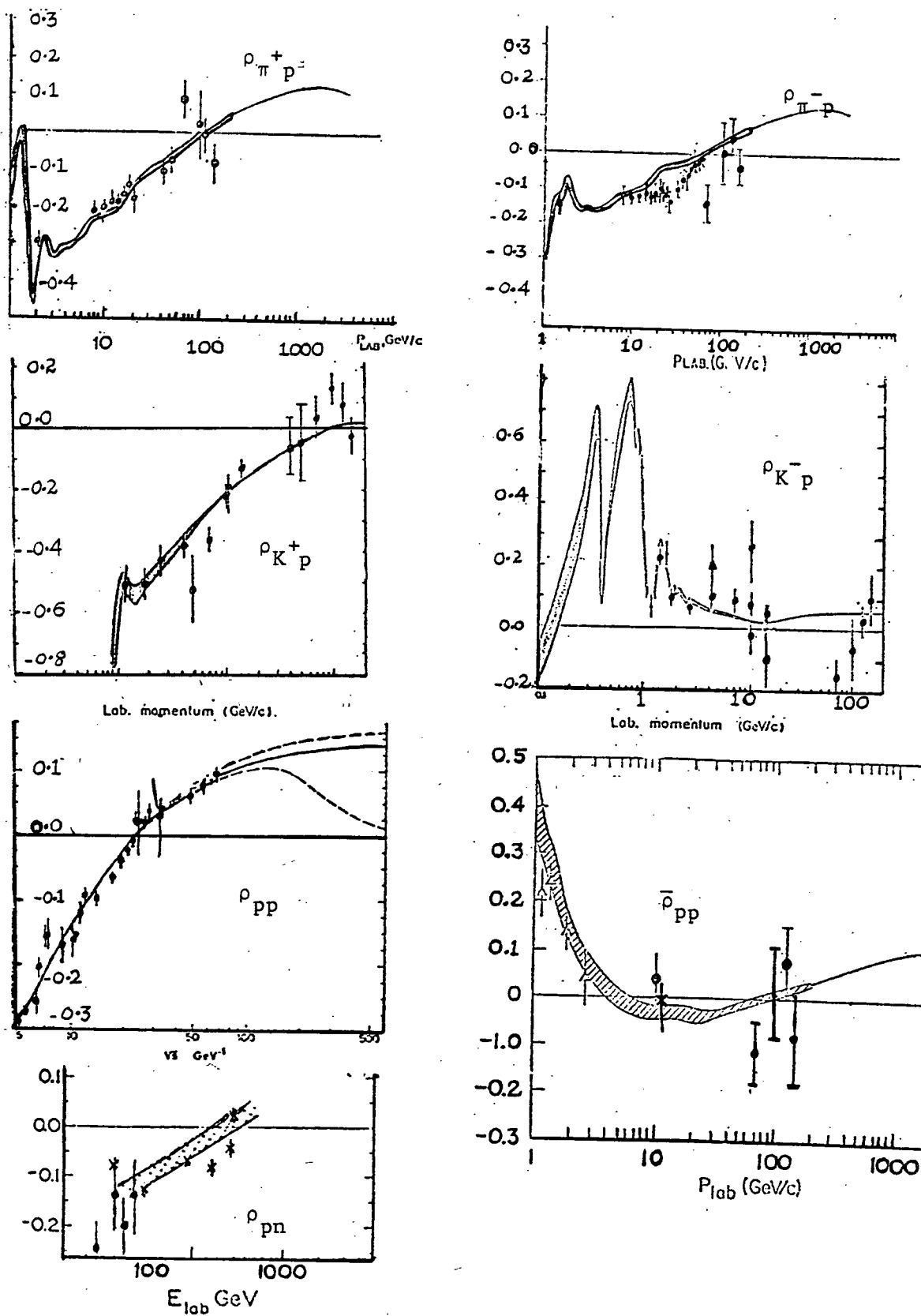


Fig. 1.2. ρ measurements for various elastic reactions. The various curves depict dispersion relation predictions except for the pn case where the shaded area represents the expression (1.3) described in the text.

of the non-flip amplitude is related to the total cross-section via the optical theorem (Chapter 2). Thus one can determine ρ using dispersion relations, using σ_{tot} as input. Hendrick and Lautrup (1975) and Engelmann and Hendrick (1977) have dealt with $\rho_{\pi^{\pm}p}$ from 1-240 GeV/c and $\rho_{K^{\pm}p}$, $\rho_{p^{\pm}p}$ from 1-200 GeV/c and have found good agreement with data. $\rho_{\pi^{-}p}$ however seems to disagree with dispersion-relation calculations in $20 \leq p_{\text{lab}} \leq 40$ GeV/c range.

1.3C The Near-forward or Small $|t|$ Region:

This region is $0.02 \lesssim |t| \lesssim 1.5$ (GeV/c)² and is further subdivided into (i) the diffraction region ($0.02 \lesssim |t| \lesssim 0.5$), (ii) the region $0.5 \lesssim |t| \lesssim 1.5$ (GeV/c)².

1.3Ca Diffraction Region: The $|t|$ region up to $|t| \lesssim 0.5$ (GeV/c)² is known as the diffraction region. It is characterised by two features. The first of these, known as the diffraction peak (on account of an analogy with optical diffraction - Chapter 2) characterises all individual elastic reactions and accounts for the name diffraction region. The other is known as the cross-over effect and refers to a feature of antiparticle-target and corresponding particle-target differential cross-section differences (i.e. $\frac{d\sigma}{dt}(\bar{A}B) - \frac{d\sigma}{dt}(AB)$) in this $|t|$ region. In the following we explain and summarise the main aspects of these features of the diffraction region.

1.3Cb The Diffraction Peak: It is observed (cf. e.g. Bertocchi and Ferrari (1967), Barger and Cline (1967), Perl (1974), Giacomelli (1976)) that as one moves away from $t = 0$ all elastic differential cross-sections fall very rapidly from maximum values in the forward direction and that up to $|t| \lesssim 0.5$ (GeV/c)² this fall is exponential in $|t|$. This implies

that on a $\frac{d\sigma}{dt}$ vs $|t|$ graph this feature appears as a sharp peak. This phenomenon itself does not depend on the nature of the interacting hadrons (although its details do) and is akin to the diffraction of light waves by obstacles where the intensity of the diffracted beam falls rapidly away from a maximum in the forward direction - hence the name diffraction peak. The exponential nature of the drop implies that a $\ln \frac{d\sigma}{dt}$ vs t graph would be a straight line up to $|t| \approx 0.5 (\text{GeV}/c)^2$ as depicted in Fig. 1.3a (see also Fig. 1.4a). The slope of this line is a parameter whose value is a measure of the rapidity or steepness of the fall in $\frac{d\sigma}{dt}$. The slope parameter b is obtained by fitting $\frac{d\sigma}{dt}$, in the diffraction region, by

$$\frac{d\sigma}{dt} = Ae^{bt} \quad (1.4)$$

{We have in the foregoing taken $|t| \approx 0.5 (\text{GeV}/c)^2$ as the upper limit of the diffraction region. This is somewhat arbitrary, though conventional, because it is somewhere between $|t| \approx 0.5 - 1.0 (\text{GeV}/c)^2$ that the drop in $\frac{d\sigma}{dt}$ no longer remains exponential in $|t|$, there being no sharp dividing line between the regions of exponential and non-exponential fall.}

Empirically the value of b is found to depend on the following factors (cf. e.g. Giacomelli (1976))

- i) The Reaction.
- ii) The Energy.
- iii) The $|t|$ range over which the fit is carried out.

In order to study the energy dependence of b for a particular reaction it is therefore necessary to fix the $|t|$ range considered in fitting $\frac{d\sigma}{dt}$ according to equation (1.4). Once this is done b can be regarded as t independent. In Fig. 1.3b b values for several elastic reactions are shown. If b increases with energy b is said to shrink, whereas b decreasing with energy is said to show anti-shrinkage. A compilation

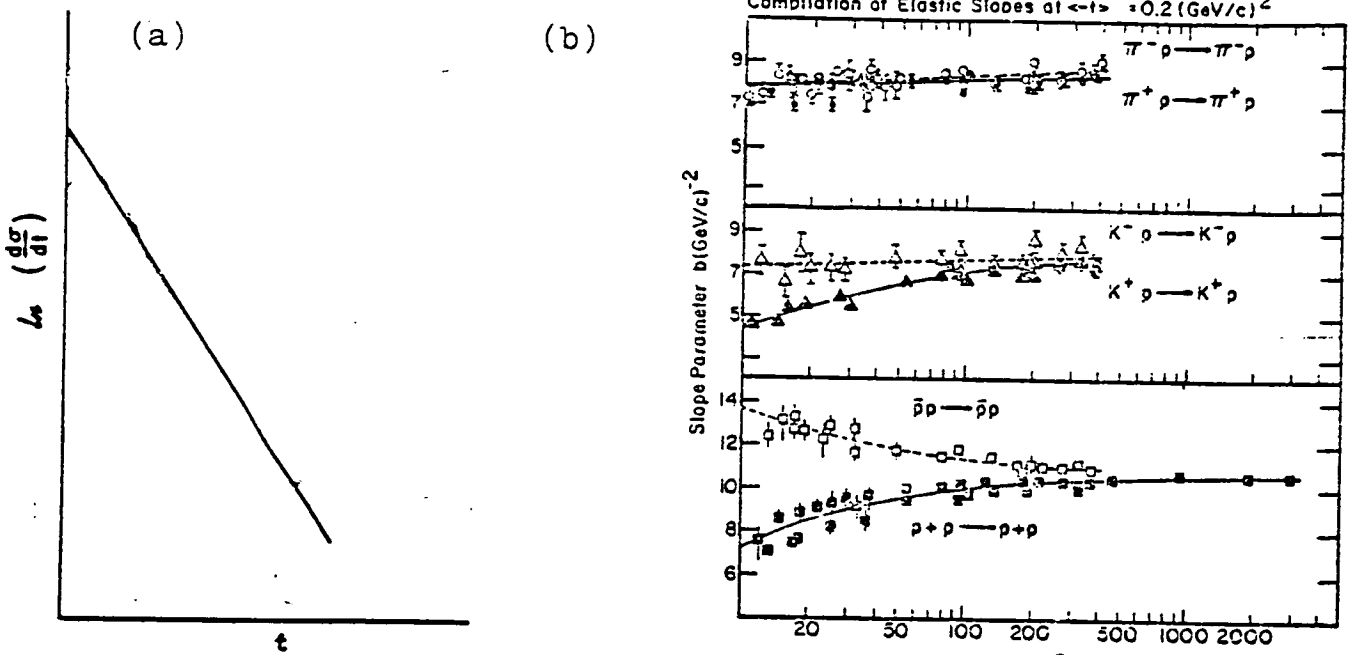


Figure 1.3 a) b is defined as the slope of a $\ln \frac{d\sigma}{dt}$ vs. t plot (which is a straight line for $|t| \lesssim 0.5 \text{ (GeV/c)}^2$).

b) Compilation of several elastic slopes (from Giacomelli (1976))

of $\pi^\pm p$, $K^\pm p$, $p^\pm p$ slopes up to $p_{\text{lab}} = 70 \text{ GeV/c}$ has been given by Lasinski et al. (1972). Values of b are almost invariably given in papers reporting $\frac{d\sigma}{dt}$ measurements in the small $|t|$ region.

The following points may be noted:

- (i) b_{pp} , b_{pn} , b_{K^+p} and b_{pp}^- show strong energy-dependence, the first three exhibiting shrinkage and the last one anti-shrinkage.
- ii) $b_{\pi^\pm p}$ and b_{K^-p} are weakly energy dependent.
- iii) Asymptotically the following trends are visible:

$$b_{\pi^+p} \rightarrow b_{\pi^-p}, \quad b_{pp} \rightarrow b_{pp}^-, \quad b_{K^+p} \rightarrow b_{K^-p}$$

$b_{pp} \approx b_{pn}$ over a wide p_{lab} range (De Haven et al. (1978), Beznogikh et al. (1974)).

- iv) The energy dependence of $b_{\pi^\pm p}$, b_{K^+p} , b_{pn} can be fitted with the form $b = b_0 + 2\alpha \ln s$ for $s > 100 \text{ GeV}^2$ as suggested by the Regge-pole model (Fig. 1.3b). This however does not rule out alternative parameterisations because of the large errors and because the range in s is not sufficiently large to differentiate between $\ln s$ and other forms varying slowly with s (Giacomelli

- v) Measurements for pp elastic scattering indicate that, at any given energy, the value of b obtained by fitting $\frac{d\sigma}{dt}$ in the range $0.02 < |t| < 0.15 \text{ (GeV/c)}^2$ differs very sharply from the value obtained if we have $.15 < |t| < .3$ (Barbiellini et al. (1972), Holder et al. (1971a,b), Amaldi et al. (1971), Carrigan et al. (1970)). This is known as the break in the pp slope parameter and the difference in the two values is $\Delta b \approx 1.5 \text{ GeV}^{-2}$ (Barger et al. (1972)). Data for other reactions are not precise enough to enable one to detect similar breaks in the slope parameters.
- vi) It has been noted (Buras & Dias de Deus (1974), Barger & Phillips (1976)) that for $p_{\text{lab}} \geq 50 \text{ GeV/c}$, the ratio $\frac{b}{\sigma_{\text{tot}}} = \text{constant}$ for a given reaction. This property is consistent with the hypothesis of geometrical scaling (Barger (1974), Barger & Phillips (1976), Buras & Dias de Deus (1974)).

1.3C^c The Cross-over Effect: It is empirically observed (cf.e.g. Barger & Cline (1969), Bertocchi and Ferrari (1967), Giacomelli (1976)) that if, at some energy, one plots elastic differential cross-sections for the processes $\bar{A} + B \rightarrow \bar{A} + B$ and $A + B \rightarrow A + B$ as functions of t , the two curves cross at a value of t (called the cross-over point) which, although different for different pairs of reactions, generally lies in the $|t| \approx 0.1 - 0.2 \text{ (GeV/c)}^2$ range. This intersection of the two curves known as the cross-over effect, is illustrated in Fig. 1.4a for the πp , Kp and pp systems. At lower energies the cross-over point can be localised accurately because the difference between the antiparticle-target and corresponding particle-target differential cross-sections is sufficiently large compared to the errors in the data. At higher energies, where this difference narrows down, it is difficult to isolate the cross-over point due to relatively larger errors. However, neglecting spin-flip amplitudes, using ^{the} optical theorem, measurements of ρ and the slope parameters for

the reactions one can obtain the cross-over point t_c indirectly according to ^{the} relation (Giacomelli (1976))

$$-t_c = \left[\ln \left(\frac{\sigma_{\text{tot}}^2(\bar{A}B)(1 + \rho^2(\bar{A}B))}{\sigma_{\text{tot}}^2(AB)(1 + \rho^2(AB))} \right) \right] / (b(\bar{A}B) - b(AB)) \quad (1.5)$$

where b denotes the slope parameter, ρ the ratio of real to imaginary part at $t = 0$ and σ_{tot} the total cross-section. For any pair of anti-particle-target and corresponding particle-target differential cross-section differences the important question is whether or not t_c is energy dependent and, if it is energy dependent, what is the nature of this dependence. The measured values of t_c are not accurate enough to enable us to answer this question, although theoretical models have differing predictions or implications.

In the Regge pole model the cross-over points are correlated to wrong-signature nonsense zeroes (WSNZ's) in the residue functions of the appropriate Regge-pole contributions to the amplitudes, and therefore have energy-independent locations. These WSNZ's however, owing to the factorisation of residues, lead to predictions for other reactions which are not always fulfilled, in which case one has to resort to conspiracies etc. Thus a consistent explanation of cross-overs in the Regge pole model is complicated.

For a review of the cross-over phenomenology in Regge theory one could consult Collins (1977), Irving and Worden (1977), Bransden and Moorhouse (1974), Phillips (1970), etc.

In a simple optical model (Chapter 2), where the scattering is represented by an opaque disc, one obtains (Barger and Cline (1969))

$$\frac{d\sigma}{dt} = \frac{\sigma_{\text{tot}}^2}{16\pi} e^{t\sigma_{\text{tot}}/8\pi} \quad (1.6)$$

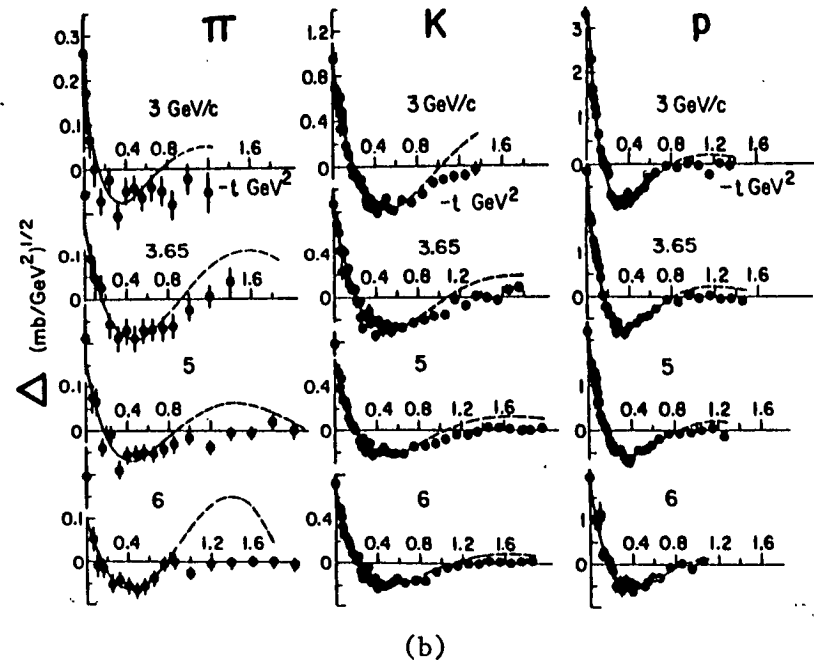
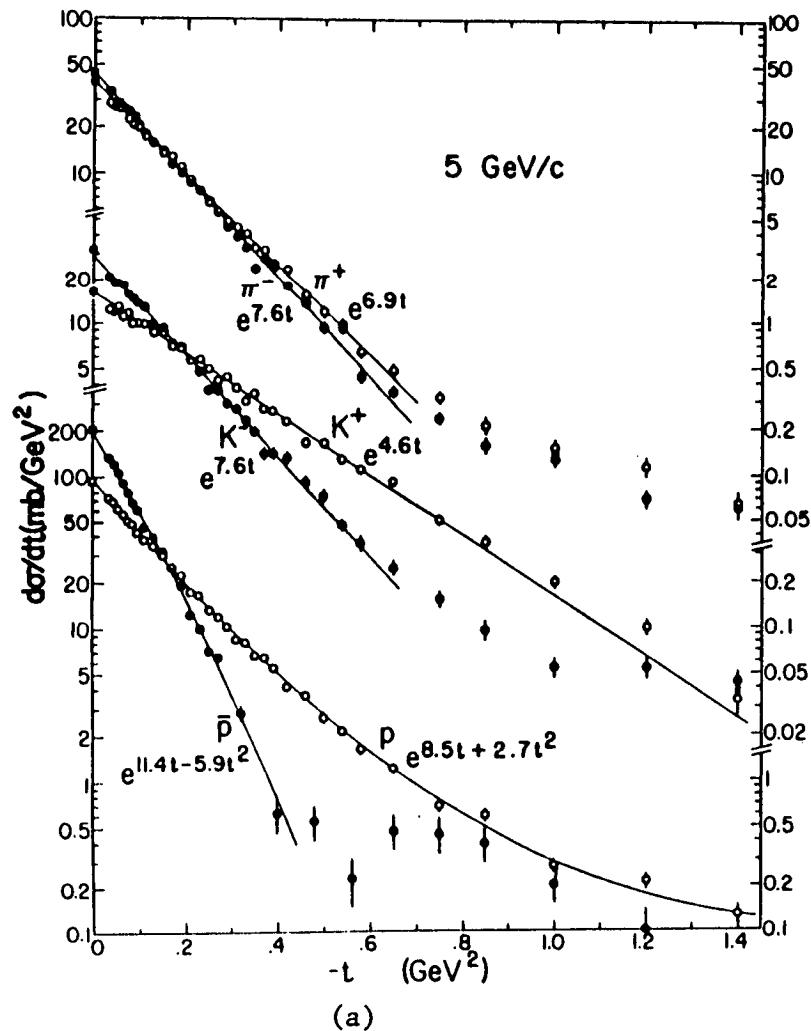


Fig.1.4: a) The elastic differential cross-section $\frac{d\sigma}{dt}(\bar{X}p)$ vs $\frac{d}{dt}(Xp)$ at 5 GeV/c as a function of t . The curves cross indicating the cross-over phenomenon (see text).

b) $\frac{\frac{d\sigma}{dt}(\bar{X}p) - \frac{d\sigma}{dt}(Xp)}{8[\frac{d\sigma}{dt}(\bar{X}p) + \frac{d\sigma}{dt}(Xp)]^{1/2}}$ plotted as a function of t in-

dicating a " J_0 " type behaviour for $\frac{d\sigma}{dt}(\bar{X}p) - \frac{d\sigma}{dt}(Xp)$ (also see Fig. 3.4a for the profile of the " J_0 " - " J_0 " denotes the Bessel function J_0 multiplied by a smooth function of s and t).

(Fig. from Ambats et al. (1974)).

This leads to a qualitative explanation of the cross-over phenomenon if one observes that, at energies where measurements exist, $\sigma_{\text{tot}}(\bar{A}B) > \sigma_{\text{tot}}(AB)$. Thus $\frac{d\sigma}{dt}(\bar{A}B) > \frac{d\sigma}{dt}(AB)$ at $t = 0$ but the former falls more rapidly and hence the cross-over results.

There is strong evidence (Ambats et al. (1974), Davier and Harari (1971)) that the difference $\Delta(Xp) \equiv \frac{d\sigma}{dt}(\bar{X}p) - \frac{d\sigma}{dt}(Xp)$ behaves as " $J_0(R\sqrt{-t})$ ", R being a radius of interaction ($R \approx 1 \text{ fermi} \approx 5.07 \text{ GeV}^{-1}$) where, at any energy, " J_0 " is the Bessel function $J_0(R\sqrt{-t})$ multiplied by a smooth function of t . On this interpretation, t_c coincides with the zero of the Bessel function $J_0(R\sqrt{-t})$. This type of behaviour can be interpreted in the framework of Harari's Dual Absorptive Model (Chapter 3). Depending on whether R is energy dependent or not, this approach can accommodate ~~the~~ energy dependent as well as fixed $-t$ cross-overs. A priori, however, no prediction can be made. Fig. 1.4b shows the " J_0 " type behaviour of $\Delta(Xp)$ for several cases.

I.3D The Region $0.5 \lesssim |t| \lesssim 1.5 \text{ (GeV/c)}^2$:

This large $|t|$ region within the small angle regime reveals interesting structure in most elastic differential cross-sections, which, depending on the reaction, appears or disappears as one moves from lower to higher energies. The details of the structure however may or may not be energy dependent. In $\pi^{\pm}p$, K^-p and $\bar{p}p$ elastic scattering one finds a dip-bump structure at lower energies which fades out as one proceeds to higher energies. In pp and pn elastic scattering however the lower energy data is structureless but, at higher energies, a distinct dip-bump structure develops. For $K^+p \rightarrow K^+p$, no structure is seen at any energy in this $|t|$ range. For most elastic reactions the laboratory momentum range $5 \lesssim p_{\text{lab}} \lesssim 20 \text{ GeV/c}$ appears to be a sort of transition regime in the sense that somewhere in this region either the already existing structure begins to fade out ($\pi^{\pm}p$, K^-p , pp) or else new structure manifests itself (pn , pp). Some representative data are

shown in Fig. 1.5 . We shall now mention a few details.

At lower energies, differential cross-sections for $\pi^\pm p \rightarrow \pi^\pm p$ have a dip at $|t| \approx 0.6 \text{ (GeV/c)}^2$ followed by a bump at $|t| \approx 1.5 \text{ (GeV/c)}^2$. This dip-bump structure however fades out by $p_{\text{lab}} \sim 10 \text{ GeV/c}$ (Foley et al. (1963a,b), Harting et al. (1965), Brandt et al. (1963)) and measurements up to 200 GeV/c (Antipov et al. (1974), Akerloff et al. (1976), Ayres et al. (1977), Bruneton et al. (1977)) laboratory momentum reveal a smooth structureless $\frac{d\sigma}{dt}$ for both reactions. (The location of the dip is regarded as energy independent in both cases. If, at all, there is any energy dependence in the dip location it must be slight and is masked by errors in data.)

The data for $K^+ p \rightarrow K^+ p$ do not reveal any structure in this $|t|$ region at any energy. ($\frac{d\sigma}{dt}$ measurements extend up to 200 GeV/c). This behaviour of $\frac{d\sigma}{dt} (K^+ p \rightarrow K^+ p)$ contrasts sharply with that of other elastic reactions which exhibit structure either at lower or higher energies. Data can be found in Foley et al. (1963b), Akerloff et al. (1976), Bruneton et al. (1977), Ayres et al. (1977).

The differential cross-section for $K^- p \rightarrow K^- p$ has a dip-bump structure at lower energies (Duam et al. (1968), Owen et al. (1968)) with the dip occurring around $|t| \approx 0.8 \text{ (GeV/c)}^2$. This structure however is reduced to a faint inflection by 10.1 GeV (Bartsch et al. (1971)). Measurements at higher energies (Foley et al. (1963b), Akerloff et al. (1976), Bruneton et al. (1977), Antipov et al. (1974), Ayres et al. (1977)) extending up to $p_{\text{lab}} = 200 \text{ GeV/c}$ reveal a structureless $\frac{d\sigma}{dt}$. The location of the dip seen at lower energies can be regarded as energy-independent, as in the πp case.

Differential cross-sections for $pp \rightarrow pp$ span the widest energy range, (up $\sqrt{s} = 62 \text{ GeV}$ or equivalently $p_{\text{lab}} \approx 2092 \text{ GeV/c}$) as well as the widest $|t|$ range at higher energies (Amaldi et al., (1977), Kwak et al. (1975), Böhm et al. (1974), Ayres et al. (1977), Akerloff et al. (1976), Foley et al. (1963b), Allaby et al. (1973)). In the $|t|$ region being considered here

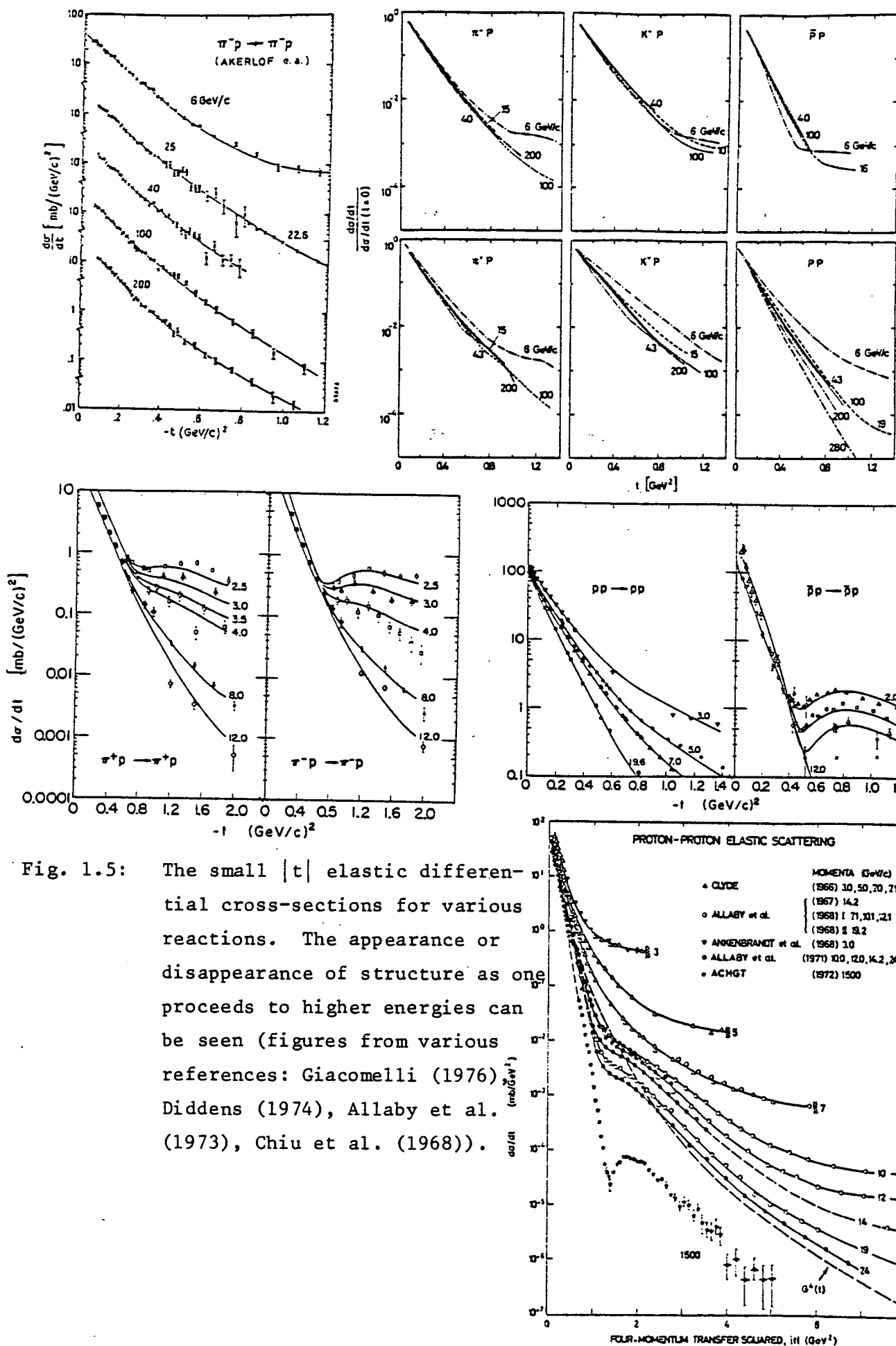


Fig. 1.5: The small $|t|$ elastic differential cross-sections for various reactions. The appearance or disappearance of structure as one proceeds to higher energies can be seen (figures from various references: Giacomelli (1976) Diddens (1974), Allaby et al. (1973), Chiu et al. (1968)).

the lower energy $\frac{d\sigma}{dt}$ is structureless. However a break in the $\frac{d\sigma}{dt}$ vs t graph is first seen (Allaby et al., (1968, 1973)) at $p_{lab} = 8 \text{ GeV/c}$ around $|t| \approx 1.5 \text{ GeV/c}^2$ which becomes sharper as the laboratory momentum increases. This structure develops into a dip at $|t| \approx 1.4 \text{ (GeV/c)}$ somewhere between $p_{lab} = 100 - 200 \text{ GeV/c}$ (Akerloff et al., (1976)) and persists throughout the ISR range up to the highest energy ($\sqrt{s} = 62 \text{ GeV}$) at which measurements exist (Kwak et al., (1975)). The dip is followed by a bump at $|t| \approx 1.8 \text{ (GeV/c)}^2$. The location of the dip is found to be energy dependent in the ISR range (Kwak et al., (1975)) where it moves from $|t| = 1.45$ at 290 GeV/c ($\sqrt{s} = 23 \text{ GeV}$) to $|t| = 1.25$ at 1480 GeV/c ($\sqrt{s} = 53 \text{ GeV}$). The height of the bump increases by a factor of two over this range. $\frac{d\sigma}{dt}$ falls by 5 orders of magnitude from $|t| \approx 0.075$ to $|t| \approx 2 \text{ (GeV/c)}^2$.

$\frac{d\sigma}{dt} (\bar{p}p \rightarrow \bar{p}p)$ has a dip-bump structure at lower energies (Parker et al. (1971), Ambats et al. (1974)). The dip seems to drift from $|t| \approx 0.4 \text{ (GeV/c)}^2$ at $p_{lab} = 2.45 \text{ GeV/c}$ (Parker et al, 1974) to $|t| \approx 0.7 \text{ (GeV/c)}^2$ at 8 GeV/c (Birnbaum et al, (1969)) where the bump becomes attenuated. At 16 GeV/c (Birnbaum et al., (1969)) it is difficult to speak of a dip-bump structure since all that remains is a faint inflection. Measurements (Akerloff et al, (1976), Ayres et al. (1977)) at NAL energies (up to $p_{lab} = 200 \text{ GeV/c}$) out to $|t| \approx 1.5 \text{ GeV/c}^2$ reveal a structureless $\frac{d\sigma}{dt}$.

The $np \rightarrow np$ differential cross-section measurements have recently been extended to 380 GeV/c laboratory momentum (De Haven et al., 1978). The measurements from $2 - 7 \text{ GeV/c}$ (Perl et al., (1970)) do not reveal any structure in the $|t|$ range being considered here. However data at and above 10 GeV/c (Engler et al. (1973), Böhmer et al. (1975), De Haven et al. (1978)) show structure around $|t| \approx 1.2 \text{ (GeV/c)}$ in the form of a break in the slope which tends to become prominent as the energy increases and

TABLE 1.2. Summary of Elastic $\frac{d\sigma}{dt}$ in the region $0.5 \leq |t| \leq 1.5$ (GeV/c)² at "lower" and "higher" energies in the high-energy region.

Reaction	Structure at lower energies? Comments. {t in (GeV/c) ² }	Structure at higher energies? Comments. {t in (GeV/c) ² }	Highest p _{lab} at which measurements exist (GeV/c)	Does σ_{tot} show resonances?
$\pi^{\pm}p \rightarrow \pi^{\pm}p$	Yes. Dip at $ t \approx 0.6$. Bump at $ t \approx 1.5$.	No	200	Yes
$K^+p \rightarrow K^+p$	No	No	200	No
$K^-p \rightarrow K^-p$	Yes. Dip at $ t \approx 0.6$	No	200	Yes
$pp \rightarrow pp$	No	Yes. Break around $ t \approx 1.5$ at 8 GeV/c develops into a dip-bump structure above 100 GeV/c. The dip occurs at $ t \approx 1.4$ and moves inward with increasing energy. The bump is at $ t \approx 1.8$.	2092 ($\sqrt{s} = 62$ GeV)	No
$\bar{p}p \rightarrow \bar{p}p$	Yes. Dip at $ t \approx 0.5$ drifts to $ t \approx 0.8$ with increasing p _{lab} .	No	200	Yes
$np \rightarrow np$	No	Yes. Break at $ t \approx 1.2$ and above 10 GeV/c develops into a dip at $ t \approx 1.4$ above 200 GeV/c.	380	No

develops into a dip at $|t| \approx 1.4 \text{ (GeV/c)}^2$ above 200 GeV/c (De Haven et al., 1978). In the $|t|$ range considered here the pp and np elastic differential cross-sections are, generally, rather similar and although it is not uncommon to compare the two cross-sections, a systematic comparison does not exist below 100 GeV/c. For instance, comparison at 19 GeV/c (Engler et al., 1973) indicates that the two cross-sections are, within errors, equal out to $|t| \approx 3 \text{ GeV/c}^2$. Amaldi et al. (1972) find a similar conclusion for their 24 GeV/c data up to $|t| \approx 4 \text{ (GeV/c)}^2$. At 100 GeV/c the two cross-sections are comparable out to $|t| \approx 0.8 \text{ (GeV/c)}^2$; at 200 GeV/c out to $|t| \approx 0.95 \text{ (GeV/c)}^2$, and at 280 GeV/c out to $|t| \approx 1.2 \text{ (GeV/c)}^2$ beyond which, in each case, they diverge. At 100 and 200 GeV/c, at $|t| \approx 1.25 \text{ (GeV/c)}^2$, $\frac{d\sigma}{dt}(\text{pn}) \sim 3 \frac{d\sigma}{dt}(\text{pp})$ (De Haven et al (1978)).

1.3E Large Momentum Transfer Region:

In a crude way one may regard small momentum-transfer collisions as "glancing blows" and large momentum-transfer collisions as "violent blows". One is thus led to expect that large momentum-transfer data might provide clues to the sub-structure, if any, of the colliding hadrons (Perl (1974), Blankenbecler (1974), Sivers (1975), Sivers et al. (1976)). This is an important motivation for studying large momentum transfer collisions, although, for large $|t|$, $\frac{d\sigma_{el}}{dt}$ is very small since the colliding hadrons tend to "break up" (i.e. give rise to inelastic reactions) under "violent blows". It may be useful to note that as a crude measure of the various two-body differential cross-sections it is usual to consider their values at $\theta = 90^\circ$ because it is located equally far from both the t and u channel boundaries. Several models have been proposed to explain the large angle scattering data and are reviewed in Sivers et al. (1976). We shall however content ourselves with a survey of the main features

of the data. The following points are worth noting:

i) Like the small $|t|$ case, the large $|t|$ $\frac{d\sigma_{el}}{dt}$ have different features for different reactions. Some of these features are mentioned in table 1.3 at the end of this section while others are mentioned below. The overall shapes of the various cross-sections are however broadly similar as may be seen in Fig. 1.6

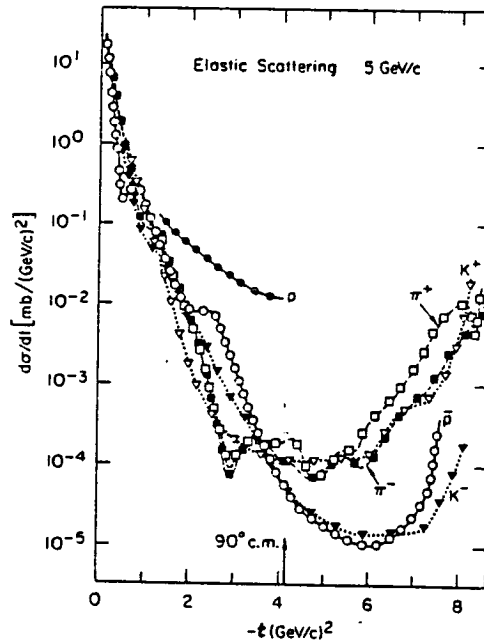


Fig. 1.6: Elastic Scattering at 5 GeV/c. Note the size of pp relative to meson-proton scattering. (Figure from Blankenbecler (1974)).

ii) At $\theta = 90^\circ$ the differential cross-sections for various two-body meson-baryon scattering processes (elastic and inelastic) are roughly equal. This led to the suggestion that hadrons have composite structure and that large angle scattering basically results from a rearrangement of the hadronic constituents (Constituent Interchange Model, CIM). The CIM predicts that at large angles

$$\frac{d\sigma}{dt} = s^{-N} f(\theta) \quad (1.7)$$

with $N \approx 8$ for meson-baryon data and 10 for pp data. The meson-baryon and, at least, the non-ISR $pp \rightarrow pp$ data, seem to agree with this prediction. For instance Landshoff and Polkinghorne (1973) have fitted the non-ISR pp data ($\theta > 30^\circ$) by the expression

$$\frac{d\sigma}{dt} \propto s^{-9.7 \pm .5} (\sin\theta)^{-14} \quad (1.8)$$

However Hojvat and Orear (1976) have recently shown that the ISR pp data goes as $s^{-6.6}$ rather than s^{-10} and have suggested that the agreement of the non-ISR data with the CIM prediction may be a numerical accident.

iii) The large $|t|$ differential cross-sections can be fitted very well by the introduction of the transverse momentum variable $q_\perp \equiv q \sin\theta$ (q being the magnitude of the C.M. 3-momentum)*. This was first discovered by Orear (1964) on empirical grounds. Hojvat and Orear (1976) have recently fitted the pp data over a wide energy range by

$$\frac{d\sigma}{dt} = A e^{-7q_\perp} \quad (1.9)$$

For $s > 300 \text{ GeV}^2$ A is energy independent while at lower energies it varies slowly with energy (see also Conetti et al., (1978)). It was also noticed sometime ago by Schrempp and Schrempp (1975a), that, when extrapolated to large angles (for the pp data upto 25 GeV/c), their DualPeripheral Model (Schrempp & Schrempp, (1973a,b,c, 1974, 1975a,b)), suggested an exponential behaviour in $q_\perp$ (modulated by a regular oscillation in $q_\perp$ of period $2\pi R$, $R \approx 1$ fermi) more naturally than a power law s^{-N} . This led them to suggest that the large $|t|$ behaviour is a manifestation of the same geometrical length scale $R \approx 1$ fermi which also controls the small

* In a very recent preprint (DAMTP 79/3) Donnachie and Landshoff have shown that the large $|t|$ pp differential cross-section behaves as t^{-8} .

$|t|$ data. This suggestion coupled with that of Hojvat and Orear mentioned above could lead one to conclude that the large-angle elastic scattering data do not provide conclusive evidence of the composite structure of hadrons (Schrempp and Schrempp, 1975a).

I.3F Near-Backward or Small $|u|$ Region:

The near-backward (small $|u|$) differential cross-section measurements are limited in energy range. These measurements cannot be made for $pp \rightarrow pp$ due to the indistinguishability of the protons. For the pn case, backward elastic scattering is identical with charge-exchange forward scattering ($np \rightarrow pn$). The following main features are worth noting in the backward data:

- i) As with the small $|t|$ case, all elastic differential cross-sections exhibit peaks in the backward direction. However the backward differential cross-sections are much smaller than the corresponding forward cross-sections (about 2 orders of magnitude for $\pi^\pm p$, $K^\pm p$ cases and 3 orders of magnitude for $K^\pm p$, $\bar{p}p$ cases). All peaks show shrinkage.
- ii) The $\pi^+ p$ system shows a beautiful dip at $|u| \approx 0.15 \text{ (GeV/c)}^2$ (Baker et al. (1971, 1969), Ashmore et al. (1967), Chabaud et al. (1972a), Orear et al. (1966, 1968), Owen et al. (1969)). Measurements for the $\pi^- p$ system (same ref. as for $\pi^+ p$) do not give any evidence of structure in the small $|u|$ region.
- iii) The $K^- p$ data does not evince any evidence of structure in the near-backward region (Baker et al. (1969, 1971), Chabaud et al. (1971b)). The situation for $K^+ p \rightarrow pK^+$ however is not so clear. The data of Baker et al. (1969, 1971) and Baglin et al (1971b) show no structure ($p_{\text{lab}} = 2.85 - 10.0 \text{ GeV/c}$) but the measurements of Chabaud et al. (1971b) ($p_{\text{lab}} = 5 \text{ GeV/c}$) indicate a mild dip around $|u| \approx 1 \text{ (GeV/c)}^2$.

TABLE 1.3*

Elastic Reaction	p_{lab} (GeV/c)	Position of 1st dip, if any, in the large $ t $ region (t in GeV/c) ²	Position of 2nd dip, if any, in the large $ t $ region (t in GeV/c) ²
$\pi^- p$	5	2.8 approx.	4.8 and also at 6
	10	2.8 approx.	-
$\pi^+ p$	5	2.8 approx.	4.8
	10	2.8 approx.	-
pp	3 - 2092	-	-
np	5 - 380	<u>Probable</u> inflection around $ t \approx 2.5$ (GeV/c) ² but data does not extend much beyond that to enable one to deduce anything definite.	-
$\bar{p}p$	5	2 approx.	-
$K^+ p$	5	2.2 approx.	-
$K^- p$	5	2.5 approx.	-

*
Table borrowed from Giacomelli (1976).

Berger and Fox (1971) have carried out an exhaustive analysis of πN backward data in the light of various theoretical models.

I.4 Polarisations for Elastic Reactions in the Small $|t|$ Region:

The measurement of the polarisation parameter complements differential cross-section measurements in enabling one to understand the structure of scattering amplitudes in a two-body scattering process. Whereas the differential cross-sections are determined merely by the gross magnitudes of the various amplitudes, the polarisation parameter is sensitive to the structure of even those amplitudes whose magnitude is small enough to be neglected in fitting differential cross-section data. Hence the importance of measuring this parameter. Unlike most elastic differential cross-sections, elastic polarisations exhibit structure, which, although it might change, does not die out with increasing energy. However the overall magnitude of all elastic polarisations decreases with increasing energy. The main features of polarisations for several elastic reactions are reviewed below.

The polarisation parameter for $\pi^{\pm}p \rightarrow \pi^{\pm}p$ has been measured up to 100 GeV/c laboratory momentum (Auer et al. (1977), Borghini et al. (1966, 1971a), Gaidot et al. (1975, 1976)). The $|t|$ range usually extends up to $|t| \approx 1$ or 1.5 (GeV/c)². It exhibits, in both reactions, a double zero around $|t| \approx 0.6$ (GeV/c)². For these reactions the polarisation is roughly equal in magnitude and opposite in sign, being positive for π^+p and negative for π^-p . This phenomenon is known as the mirror symmetry of πp polarisation and persists up to the highest energies at which measurements exist. There are departures from exact mirror symmetry however which are conspicuous at very low energies ($p_{\text{lab}} = 2.73$ GeV/c e.g.) but by 14 GeV/c these departures become insignificant.

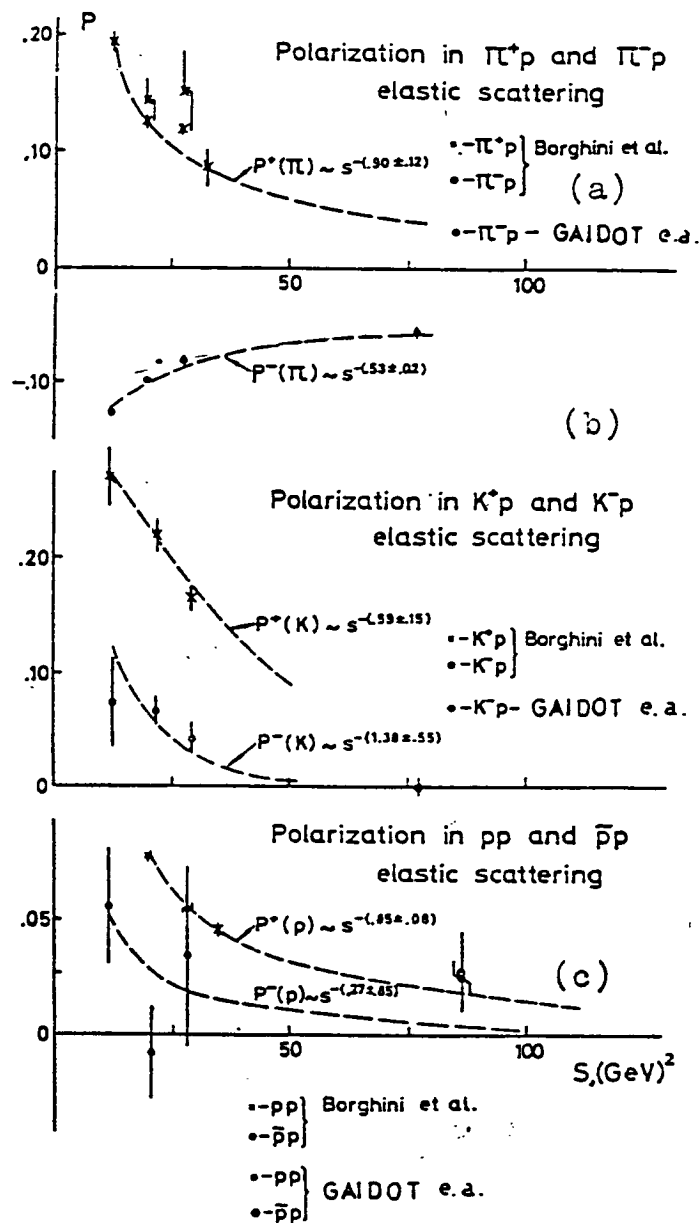


Fig. 1.7 Energy dependence of polarisation in several elastic reactions (fig. from Diddens (1974)).

Martin and Navelet (1977) have recently proposed that these departures are due to the contribution of the Pomeron, and of a new low-lying σ -trajectory to the spin-flip amplitude. The S -dependence of $P(\pi^\pm p)$ follows an $\alpha_R - \alpha_P$ behaviour, α_P and α_R being the Pomeron and Regge trajectories respectively (Gaidot et al. (1976), Diddens (1974), Fig. 1.7a).

Polarisation measurements for $K^+p \rightarrow K^+p$ exist up to $p_{\text{lab}} = 45 \text{ GeV}/c$ (Borghini et al. (1971b), Gaidot et al. (1976)) and usually span the range $|t| \approx 1 (\text{GeV}/c)^2$. The polarisation for this reaction is large and

positive at smaller $|t|$ values and falls gradually to zero at $|t| \lesssim 1$ (GeV/c)². Since data beyond $|t| \approx 1$ (GeV/c)² does not exist in most cases, it is difficult to say whether the polarisation changes sign beyond this zero (in which case we have a single zero) or remains positive (in which case we have a double zero or two closely spaced zeroes). At 14 GeV/c, where data exist up to $|t| \approx 1.5$ (GeV/c)², measurements do not favour a single zero although large errors on data make such a conclusion doubtful.

K^-p elastic polarisations are positive for $|t| \lesssim 1$ (GeV/c)², although unlike the K^+p case, they are very small at smaller $|t|$ values (i.e. in the diffraction region), attaining a peak around $|t| \approx 0.6$ (GeV/c)² (Borghini et al. (1971b), Gaidot et al. (1975)). The polarisation then falls to zero around $|t| \approx 1$ (GeV/c)², beyond which it becomes negative as indicated by data up to 14 GeV/c. Beyond this data is available only at 40 GeV/c laboratory momentum. The 40 GeV/c data extends up to $|t| \approx 1.5$ (GeV/c)² and is consistent with zero at most points so that it is not possible to draw any inference about the structure of 40 GeV/c K^-p polarisations. The energy dependence of $P(K^-p)$ is shown in Fig. 1.7b along with those of other reactions.

Polarisation measurements for $pp \rightarrow pp$ have now been extended up to 300 GeV/c (Snyder et al., (1978)) laboratory momentum. At lower energies the data spans a wider $|t|$ range than that at higher energies. The data at 10, 14 and 17.5 GeV/c (Borghini et al., 1971c) indicate double (or possibly two closely spaced) zeroes at $|t| \approx 0.9$ and 2 (GeV/c)². The 11.8 and 24 GeV/c data (Kramer et al. (1978), Crabb et al. (1977)) is restricted to $|t| \lesssim 1$ (GeV/c)² and shows a double zero around $|t| \approx 0.8$ (GeV/c)². The 45 GeV/c data (Gaidot et al. (1976)) spans a similar $|t|$ range and reveals a single zero at $|t| \approx 0.5$ (GeV/c)² and a zero at $|t| \approx 1$ (GeV/c)². The 100 and 300 GeV/c (Snyder et al. (1978)) data shows single zeroes at $|t| \approx .325$ and .23 (GeV/c)² although large errors make

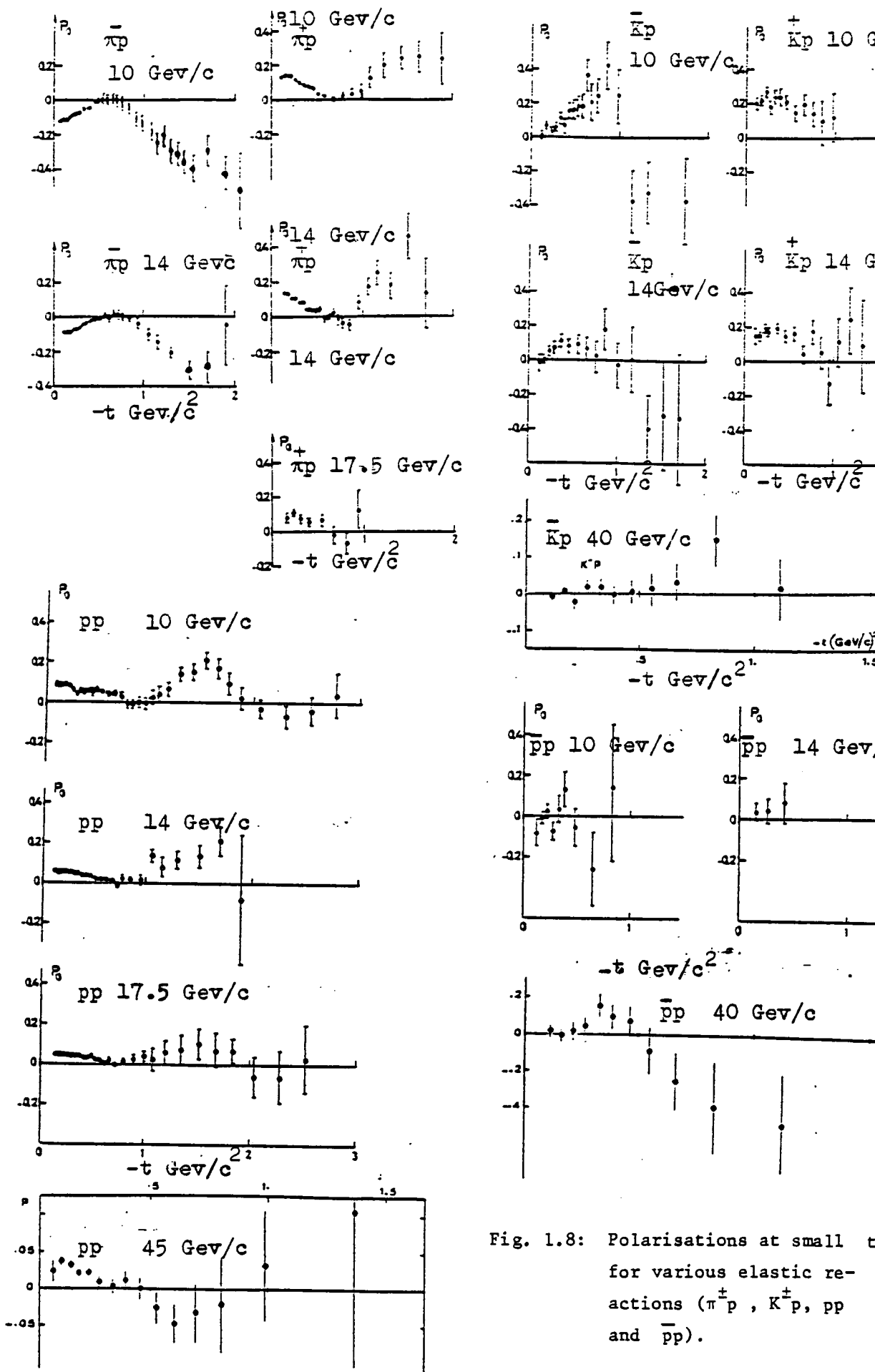


Fig. 1.8: Polarisation at small t for various elastic reactions ($\pi^\pm p$, $K^\pm p$, pp and $\bar{p}p$).

make it difficult to say anything certain about the nature of this zero^{*}. The 150 GeV/c data (Fidecaro et al., (1978)), while showing a single zero at $|t| \approx .475$ is more negative than either the 100 or 300 GeV/c data beyond $|t| \approx .8$ and shows another zero at $|t| \approx 1.30$ (GeV/c)² instead of the usual $|t| \approx 1$ (GeV/c)². $pp \rightarrow pp$ polarisations thus undergo interesting changes as one goes from the lower to the higher energies. It appears that somewhere between 17.5 and 45 GeV/c a single zero appears around $|t| \approx 0.5$ and moves inwards with increasing energy. Whether the zero structure around $|t| \approx 1$ (GeV/c)² remains a double zero or is converted into a single zero or follows some other pattern is unclear on account of large errors. The polarisation in pp elastic scattering shows an anomalously rapid energy dependence in the small $|t|$ region falling as s^{-1} approximately (Fig. 1.7c).

Data on $\overline{pp}$ elastic polarisations are relatively sparse, limited in the $|t|$ range, and have large errors. The 10 and 14 GeV/c data are not clear cut, having large errors and a limited $|t|$ range but the 40 GeV/c data (Gaidot et al. (1975a)) is comparatively more accurate and seems to indicate a single zero near $|t| \approx 0.5$ (GeV/c)² with a (positive) peak between $|t| \approx 0$ and $|t| \approx 0.5$ (GeV/c)². The polarisation beyond this zero is negative up to $|t| \approx 1$ (GeV/c)², the highest $|t|$ value at which measurements extend.

The $np \rightarrow np$ polarisation at 6 GeV/c shows an oscillatory behaviour, with zeroes at 0.5 and 1.3 (GeV/c)² approximately, being positive between $|t| \approx 0 - 0.5$ and negative between $|t| \approx 0.5 - 1.3$ (GeV/c)². The very recent 11.8 GeV/c measurements up to $|t| \approx 1$ (GeV/c)² (Kramer et al., (1978)) reveal very small polarisation and do not show any discernible structure. Averaging $P(pn)$ from $|t| \approx .18$ to 0.5 gives a value $P(pn)_{\text{average}} = (-0.9 \pm 0.5)$.

* 300 GeV/c data seems to show another zero between $|t| \approx .30$ to .35 (GeV/c)².

1.5 Spin-Rotation Parameters

In two-body meson-nucleon scattering one has two complex amplitudes $f_{++}(s,t)$ and $f_{+-}(s,t)$ and the determination of these amplitudes, at any (s,t) , requires the measurement of four quantities. However we can only measure real bilinear combinations of these amplitudes, which leaves a common phase factor $e^{i\psi(s,t)}$ (ψ real) undetermined and implies that we can, in principle, measure only three observables at any (s,t) . A similar argument for $pp \rightarrow pp$ implies that, at any (s,t) , nine observables are needed for a determination (up to an overall undetermined phase) of the five amplitudes. Of the various quantities needed for a determination of the various 2-body amplitudes we have so far described only two, the differential cross-section $\frac{d\sigma}{dt}$ and the polarisation parameter P . We now describe results pertaining to two more parameters, the so-called spin-rotation parameters R and A . Together with P these constitute a set of three parameters of which only two are independent, the third being determined through $P^2 + R^2 + A^2 = 1$. Usually R is measured and A inferred from the above relation or else a suitable combination of the R and A parameters is measured. In the laboratory frame the quantities R and A are the polarisations of the recoiling nucleon when the target nucleon is polarised perpendicularly to and along the beam direction respectively. There exist very few measurements for these parameters and these too are only for $\pi^\pm p \rightarrow \pi^\pm p$ (Lesquen et al. (1972), Pierrard et al. (1975, 1976), $K^- p \rightarrow K^- p$ (Pierrard et al. (1975), and $pp \rightarrow pp$ (Deregel et al. (1973), Pierrard et al. (1976)). For the reaction $pp \rightarrow pp$ a parameter $C \equiv R \sin\alpha - |A| \cos\alpha$ (α being the angle between the incident moment and the target polarisation in the lab. frame) is usually measured (Deregel et al. (1973), Pierrard et al. (1976)). For all reactions measurements of R (or C) exist over a very limited $|t|$ range, roughly $0.2 < |t| < 0.5$ (GeV/c) and the errors are

rather large. Below we give the average values of these parameters on this
t range as an indicator of their overall magnitude and sign. The average
value of $R(\pi^- p)$ and $C(pp)$ are equal (within errors) at all corres-
ponding lab. momenta.

TABLE 1.4.

Laboratory Momentum (GeV/c)	<u>Average value of</u>			
	$R(\pi^- p)$	$R(\pi^+ p)$	$R(K^- p)$	$C(pp)$ or $R(pp)$
3.83				-0.28 ± 0.06
6	-0.23 ± 0.05	-0.08 ± 0.04		-0.29 ± 0.07
16	-0.23 ± 0.05			-0.29 ± 0.04
40	-0.20 ± 0.02		-0.16 ± 0.16	
45		-0.22 ± 0.16		-0.22 ± 0.04

CHAPTER 2THEORETICAL BACKGROUND2.1 Introduction

Our objective in this work is the application and assessment of the status of a certain model (Harari's Dual Absorptive Model - DAM) of two-body hadronic reactions with respect to elastic scattering data at high energies and small momentum transfers. In order to state this model we outline some theoretical concepts and models which constitute the background from which the model gradually emerged. The introduction of these concepts and models is the purpose of this chapter. As the name 'Dual Absorptive Model' suggests, this model (apart from other things) combines the concept of duality with ideas taken from the so-called optical analogy (absorption). The optical analogy, which preceded duality historically, is essentially an analogy between the diffraction of light waves as described by Huygen's principle and the scattering of hadronic waves in non-relativistic physics. In the following section (Sec. 2) therefore we describe the optical analogy, also called the Optical Model. It is also convenient at this point to introduce the eikonal idea, so widely used in high-energy physics. The concept of duality, however, cannot be stated without the notion of Regge exchanges which, in turn, is introduced into relativistic scattering in the S-matrix framework. Crucial to the concept of Regge exchanges is the idea of crossed channels and the use of Mandelstam variables. We therefore introduce the notion of crossed channels and define Mandelstam variables in Section 3 of this chapter. This is followed by a definition of the S-matrix and a very brief mention of some of its properties (unitarity, crossing, analyticity) used in high-energy physics (Sec. 4). After this we state Regge theory (Sec. 5). Sec. 6 deals with helicity amplitudes (which are employed widely in phenomenology) and their partial-wave expansion. In Sec. 7 we introduce the impact-

parameter representation of amplitudes which enables one to incorporate geometrical and optical ideas into a relativistic amplitude. Finally we state the finite energy sum rules (FESR's) and duality (Sec. 8A, B). The chapter is concluded by assessing the status of two-component duality, a concept vital to the application of DAM to elastic scattering (Sec. 8C).

2.2 The Born Approximation, The Optical Analogy, and the Eikonal Expansion:

(Some references: Collins (1977), Amaldi et al. (1976), Jochain and Quigg (1974), Perl (1974), Gottfried (1972, 1965), Ommes (1970), Roman (1965), Ommes and Froissart (1963), Glauber (1959)).

2.2 A The Born Approximation

The Born approximation is an approximation to the solution of the problem of scattering of a projectile by a target in non-relativistic quantum mechanics. In the centre of mass frame of two particles of rest masses m_1 and m_2 and with position vectors $\vec{r}_1$ and $\vec{r}_2$ the Schrödinger wave equation assumes the form*

$$[\nabla^2 + k^2] \psi(\vec{r}) = U(\vec{r})\psi(\vec{r}) \quad (2.1)$$

In equation (2.1) $\vec{r} \equiv \vec{r}_1 - \vec{r}_2$, $k^2 = 2mE/\hbar^2$ and $U(\vec{r}) = 2mV(\vec{r})$ with $m \equiv \frac{m_1 m_2}{m_1 + m_2}$. E is the total energy of the system and $V(\vec{r})$ the potential due to the forces between the particles. It is well known that one can transform this differential equation into an integral equation (called the Lipmann-Schwinger equation) by using the Green's function, subject to a boundary condition stated in (2.2) below, for the operator $[\nabla^2 + k^2]$, and solve the integral equation by an iterative technique. The solution must however satisfy the boundary condition

$$\lim_{r \rightarrow \infty} \psi(\vec{r}) \sim e^{ikz} + f(\theta, \phi) \frac{e^{ikr}}{r} \quad (2.2)$$

where $\vec{r} = (r, \theta, \phi)$ and, for convenience, the incident beam is taken to

*Strictly speaking, form (2.1) of the Schrödinger equation is valid only when the potential $V(r)$ has a finite range (Cf. e.g. Roman (1965)).

travel along the z -axis. This asymptotic boundary condition merely reflects the observation that in a scattering experiment, one ends up, away from the scattering region, with a scattered wave travelling outwards from the scattering region in all directions $(f(\theta, \phi) \frac{e^{ikr}}{r})$ and an incident beam travelling along the z -axis (e^{ikz}) . $f(\theta, \phi)$ is called the scattering amplitude and can be shown to satisfy

$$\frac{d\sigma}{d\Omega}(\theta, \phi) = |f(\theta, \phi)|^2 \quad (2.3)$$

$\frac{d\sigma}{d\Omega}(\theta, \phi)$ being the differential cross-section for scattering into a unit solid angle around θ, ϕ .

The infinite series which represents the iterative solution, subject to the asymptotic boundary condition (2.2), of the Lipmann-Schwinger equation, is called the Born series. It is a series whose successive terms contain increasing powers of $U(\vec{r})$. One interprets this series as a multiple scattering series (cf. e.g. Jochain and Quigg (1974), Omnes (1970), Omnes and Froissart (1963), Roman (1965)) i.e. a series in which individual terms correspond to repeated scatterings of the particle inside the region of influence of the potential, the number of scatterings increasing as one goes to higher terms. The zeroth term thus corresponds to no scattering, the first term corresponds to a single scattering, the second to two scatterings and so on. Between successive scatterings the particle is assumed to travel freely. On this interpretation then the Born series is expected to converge only if terms representing greater numbers of scatterings can be neglected. In other words if the potential is "weak" in the sense that it cannot "hold back" the projectile for a sufficient period of time so that it can undergo several scatterings before emerging out into the free region. If V_0 characterises the strength of the potential and a its range and if $\vec{v}$ is the velocity of the particle, the above condition can be written as $\frac{V_0 a}{\hbar v} \ll 1$. The truncation of the Born series after the term

representing j scatterings of the particle yields the j^{th} Born approximation. In the first Born approximation, using plane-wave states, the solution can be shown to be

$$\psi(\vec{r}) = e^{ikz} - \frac{1}{4\pi} \int d^3r' U(\vec{r}') e^{ikz'} \frac{e^{i\vec{k} \cdot (\vec{r} - \vec{r}')}}{|\vec{r} - \vec{r}'|} \quad (2.4)$$

Taking the asymptotic limit of the second term in (2.4) and comparing it with the corresponding term in (2.2) yields

$$f(\theta, \phi) = -\frac{1}{4\pi} \int d^3r' e^{i(\vec{k}_i - \vec{k}_f) \cdot \vec{r}'} U(\vec{r}') \quad (2.5)$$

where $\hbar\vec{k}_i$ and $\hbar\vec{k}_f$ are the initial and final 3-momentum vectors with $k \equiv |\vec{k}_i| = |\vec{k}_f|$. Equations (2.4) and (2.5) provide us with a basis for formalising the optical analogy as described briefly below.

2.2B The Optical Analogy:

The phenomenon of diffraction arises when light waves impinge on a target whose dimensions are small enough to be comparable with the wavelength of the incident waves. The phrase optical analogy refers to an analogy between the description of diffraction of light waves, using Huygen's principle, and the description of small angle elastic scattering of high energy hadronic projectiles (hadronic waves) by hadronic targets in the first Born approximation (cf. e.g. Perl (1974), Gottfried (1972), Amaldi et al. (1976)). Most hadronic targets have dimensions of the order of a fermi* or so and the wavefunction of a beam of monoenergetic particles is characterised by a wavelength λ such that $\lambda = \frac{2\pi}{k}$ which, for k

*

A few examples are:

r.m.s. radius of a proton's charge distribution	$\approx 0.8 \text{ fm}$
Radius of a nucleus of mass number A	$\approx 1.1 A^{1/3} \text{ fm}$
Radius of a deuteron	$\approx 2.7 \text{ fm}$

($\hbar k$ is the magnitude of the 3-momentum) ... of the order of a few GeV/c ... is comparable to the dimensions of a hadronic target. Hence it is meaningful to speak of an optical analogy. This intuitive approach can be formalised by comparing the scattered light-wave-function at any point behind the obstacle (at small angles of scattering), constructed according to Huygen's principle with the scattered wavefunction of equation (2.4). By the scattered light-wave-function at some point one means the wavefunction obtained when the obstacle is present minus the wavefunction obtained when there is no obstacle. If one considers a plane disc of radius R and small thickness, characterised by a coefficient of absorption $\Gamma(\vec{b}, z)$ where $\vec{b}$ is any vector lying in its plane, then, assuming the monochromatic incident wave to travel along the z -axis, as shown, one obtains the scattered wave-

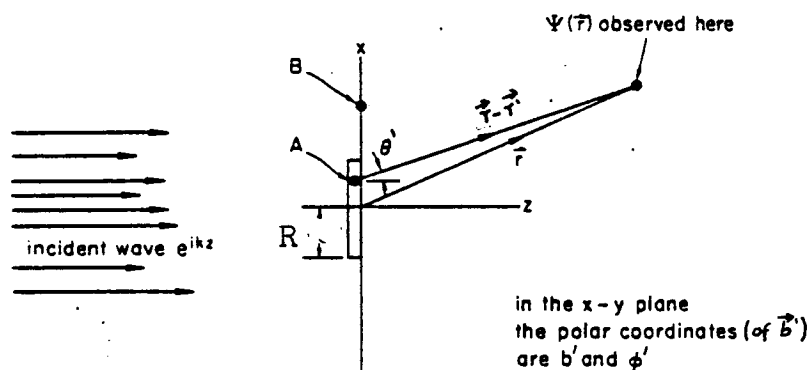


Fig. 2-1 Scattering of a plane wave by an absorbing disk.

function defined above, using Huygen's principle, as (cf. e.g. Perl (1974), Amaldi et al. (1976), Gottfried (1972))

$$\psi_{\text{scatt.}}(\vec{r}) = \frac{-ik}{4\pi} \int_{\Sigma} d^3r' (1 + \cos \theta') (-\Gamma(b')) e^{ikz'} \frac{e^{ik|\vec{r} - \vec{r}'|}}{|\vec{r} - \vec{r}'|} \quad (2.6)$$

where $\vec{r}' = (b', z')$, k being the wave number. Σ is the surface of the disc. For small θ' the inclination factor $1 + \cos \theta' \sim 2$. Then comparing equation (2.6) with the scattered wavefunction in equation (2.4) yields the relations:

* θ' is the angle between the normal to the disc and the direction of $\vec{r} - \vec{r}'$.

$$U(\vec{r}) = -2ik \Gamma(\vec{r}) \quad (2.7a)$$

$$f(\theta, \phi) = \frac{ik}{2\pi} \int d^3r e^{i(\vec{k}_i - \vec{k}_f) \cdot \vec{r}} \Gamma(\vec{r}) \quad (2.7b)$$

The identification (2.7a, b) constitutes the essence of the optical analogy (Cf. e.g. Perl (1974), Amaldi et al. (1976)). Since Γ is a coefficient of absorption it is real with $0 \leq \Gamma \leq 1$, $\Gamma = 1$ representing complete absorption. On the above identification U becomes imaginary and is called an absorptive potential. The optical term absorption is thus smuggled into scattering theory. Absorption is essentially the removal of energy from the incident beam and its reappearance in a mode different from the initial one. In optics the removed energy may be retained as the thermal energy of the absorbing material, etc. - in scattering of particles it can only reappear in the form of particles different from the incident ones if it is to be in a mode different from the original one. The absorbed part of the incident wave is thus, in this context, associated with the occurrence of inelastic reactions. The function $\Gamma(\vec{r})$ is also called the profile function. Many different forms of $\Gamma(\vec{r})$ are possible, subject to the condition $0 \leq \Gamma(\vec{r}) \leq 1$ (Cf. e.g. Perl (1974), Feld (1969) for some forms).

One form which we mention is:

$$\begin{aligned} \Gamma(\vec{r}) &= \delta(z)\beta \quad \text{for } r \leq R \quad \beta \text{ is a const. } \beta \leq 1 \\ &= 0 \quad \text{for } r > R \end{aligned} \quad (2.8)$$

(2.8) is the so-called grey disc model ($\beta = 1$ gives the black disc model) which when put into equation (2.7b) leads to the following results:

$$i) \quad \frac{d\sigma}{dt} = \pi\beta^2 a^4 \left[\frac{J_1(R-t)}{R-t} \right]^2 \quad (2.9a)$$

$$ii) \quad f(\theta) \text{ is imaginary}$$

$$\text{iii)} \quad \text{The slope parameter } b = \frac{R^2}{4} \quad (2.9b)$$

$$\text{iv)} \quad \sigma_{\text{tot}} = 2\pi\beta R^2 \quad (2.9c)$$

If R is taken to increase with energy then this model gives a qualitative explanation of shrinkage and rising σ_{tot} . The dip structure observed in some reactions can also be explained by (2.9a). But the model incorrectly predicts $f(\theta)$ to be imaginary at all energies. Models of this kind however are unsatisfactory because they are non-relativistic. One noteworthy feature, however, is that this model, regardless of the value of β , predicts a constant ratio σ_{tot}/b .

2.2C Eikonal Expansion: (Collins (1977), Perl (1974), Jochain and Quigg (1974), Glauber (1959))

The condition $\frac{V_0 a}{\hbar v} \ll 1$, so vital for the physical justification of the Born approximation, is ^{*}not always fulfilled in hadronic scattering, as indicated by the capture of the projectile by the target and the consequent formation of resonances. Furthermore the treatment of the incident wave as a plane wave, even when the particle is inside the potential, is physically unsatisfactory. Also the first Born approximation gives results with large errors. Since higher order terms of the Born series are difficult to calculate, an alternative approach was introduced by Glauber (Glauber (1959)). It assumes that the energy is high enough so that the potential energy is much smaller than the total energy and that the wavelength of the incident wave is much smaller than the range a of the potential V , i.e.

$\frac{|V_0|}{E} \ll 1$ and $ka \gg 1$, k being the wavenumber. Furthermore one replaces $\psi(r) = e^{ikz}$ by $\phi(\vec{r})e^{ikz}$ with $\phi(\vec{r})$ varying slowly with r and satisfying $\phi(\vec{r}) \rightarrow 1$ as $z \rightarrow -\infty$. This takes into account the distortion in the incident wave caused by the potential. The resulting amplitude is (Jochain and Quigg (1974)),

* Only when one goes to sufficiently high energies where resonance formation ceases to be a feature of hadronic scattering is this condition fulfilled.

Perl (1974))

$$f(\theta, \phi) = -\frac{ik}{2\pi} \int e^{i\vec{\Delta} \cdot \vec{b}} [e^{i\chi(\vec{b})} - 1] d^2\vec{b} \quad (2.10)$$

where $\vec{\Delta} \equiv \vec{k}_i - \vec{k}_f$, $\vec{r} = (\vec{b}, z)$ and $\chi(b) \equiv -\frac{1}{2k} \int_{-\infty}^{\infty} V(\vec{b}, z) dz$ is called the eikonal. The expansion of $e^{i\chi(b)}$ gives us the eikonal series or expansion

$$f(\theta, \phi) = -\frac{ik}{2} \sum_{n=1}^{\infty} [e^{i\vec{\Delta} \cdot \vec{b}} \frac{[i\chi(\vec{b})]^n}{n!} d^2\vec{b}] \quad (2.11)$$

The eikonal amplitude satisfies the optical theorem and is equally valid for real and complex potentials. This makes it a rather powerful tool and the eikonal method is widely used in hadronic phenomenology. For potentials with azimuthal symmetry one has

$$f(\theta) = -ik \sum_{n=1}^{\infty} \left[\int_0^{\infty} J_0(\Delta b) \frac{[i\chi(b)]^n}{n!} b db \right] \quad (2.12)$$

This form is the basis of many calculations based on the eikonal idea. One can not that the essential feature in the eikonal amplitude is the parameterisation of χ . This parameterisation however is subject to broad constraints. For instance one could expect χ to vanish when b is larger than the range of interaction. At high energies the first term of the eikonal expansion is identical with the first Born approximated amplitude (Jochain and Quigg (1974)). The eikonal idea is resurrected in relativistic scattering in a variety of situations (cf. section 2.7 of this chapter for a few examples).

2.3 Crossed Channels and Mandelstam Variables:

For the sake of convenience in stating certain principles we introduce the notion of crossed channels and define the Mandelstam variables.

Consider the following set of related reactions:

$$1 + 2 \rightarrow 3 + 4 \quad \text{I}$$

$$1 + \bar{3} \rightarrow \bar{2} + 4 \quad \text{II}$$

$$1 + \bar{4} \rightarrow 3 + \bar{2} \quad \text{III}$$

One then says that for reaction I, reactions II and III are the crossed reactions or crossed channels, for reaction II reactions I and III are the crossed channels and so on.

We now associate with each of the particles involved in the three mutually crossed channels a 4-momentum which will be denoted as follows: in any channel the 4-momentum of particle i ($i = 1, 2, 3, 4$) will be p_i , while that of particle $\bar{i}$ ($\bar{i} = \bar{1}, \bar{2}, \bar{3}, \bar{4}$) will be $-p_i$. Thus, in channels I and III, p_3 will be the 4-momentum of particle 3, while in channel II the 4-momentum of $\bar{3}$ shall be denoted as $-p_3$. One then introduces three invariant variables, the Mandelstam variables, which can be associated with the three channels, and their meaning with respect to any one channel is to be interpreted with the aforementioned notation in mind.

$$s \equiv (p_1 + p_2)^2 \quad (2.13a)$$

$$t \equiv (p_1 - p_3)^2 \quad (2.13b)$$

$$u \equiv (p_1 - p_4)^2 \quad (2.13c)$$

The meaning of these variables with respect to the various channels can be seen from the table below. We note that in any channel conservation of 4-momentum and $p_i^2 = m_i^2$ yield

$$s + t + u = \sum_i m_i^2 \quad (2.14)$$

Only two of the three Mandelstam variables are thus independent.

TABLE 2.1

Meaning of the Mandelstam Variables in different channels.

Reaction	Meaning of the variable		
	s	t	u
I	c.m. energy squared; $s \geq (m_1 + m_2)^2$	4-momentum transfer squared; t spacelike	4-momentum transfer squared; u spacelike
II	4-momentum transfer squared; s spacelike	c.m. energy squared $t \geq (m_1 + m_3)^2$	Do.
III	Do.	4-momentum transfer squared; t spacelike	c.m. energy squared; $u \geq (m_1 + m_4)^2$

The variables s , t , u thus acquire a certain symmetry with respect to the three channels. Because of the meaning of the various variables in the three channels it is usual to call a channel by the name of that variable which represents its centre-of-mass energy squared. Thus the channels I, II and III are called, respectively, s , t , and u channels.

If one introduces ordered sets of numbers (s, t, u) and associates with each of these a point in a two-dimensional space, one obtains for each channel a domain in this space which will be the collection of points representing the various possible (s, t, u) values for that channel. One finds that the domains of the three channels are non-overlapping (Kibble (1964)).

For certain purposes one interprets (s, t, u) as three complex variables, related through equation (2.14), and which, for appropriate real values can be interpreted as in equation (2.13a, b, c).

2.4 The S-matrix, Unitarity, Crossing and Analyticity

2.4A Definition and Some Relations:

The concept of the S-matrix was introduced for the purpose of having a theory of strong interactions of elementary particles directly in terms of observable quantities, rather than the usual fields (which themselves are unobservable), and objects and equations involving these fields. This line of thinking was necessitated by the inapplicability of the usual field-theoretic approach to strong interactions because the field theoretical approach involves an expansion of the scattering amplitude in terms of the coupling constant and such an expansion does not converge for strong interactions because the strong interaction coupling constant is large. The philosophy behind the S-matrix approach and its properties are discussed in detail in Chew (1961, 1966), Eden et al. (1966), Barut (1967), Martin and Spearman (1970). Summaries of this approach can be found in Bransden and Moorhouse (1973), Collins (1977) and Collins and Squires (1968). We shall however be content with the definition of the S-matrix and a brief mention of its properties that lead us to Regge theory and to certain aspects of high-energy phenomenology. These properties are (i) Unitarity, ii) Crossing, iii) Analyticity.

The definition of the S-matrix and these properties are discussed below.

We introduce an operator S , called the scattering operator, whose matrix element $\langle f|S|i\rangle$, between two free particle states $|i\rangle$ and $|f\rangle$ is, by definition, interpreted as the transition probability amplitude for the process $|i\rangle \rightarrow |f\rangle$. We denote $\langle f|S|i\rangle$ as:

$$S_{fi} \equiv \langle f|S|i\rangle, \quad (2.15)$$

where $|i\rangle$ is a state a long time before the particles interact and $|f\rangle$

is a state a long time after the interaction has occurred. (If the interaction occurs around time $t \sim 0$ the phrase "a long time before(after)" means $t \rightarrow -\infty(+\infty)$). It then follows from the definition of the operator S that the probability P_{fi} for the transition $|i\rangle \rightarrow |f\rangle$ is

$$P_{fi} = |S_{fi}|^2 \quad (2.16)$$

The array of objects $\{S_{fi}\}$ for all conceivable states $|i\rangle$ and $|f\rangle$ of a system is then called the S -matrix and, like the scattering operator, is denoted by S as well. Before we go on to describe a few properties of the S -matrix several points should be mentioned:

- a) In defining the S -matrix elements the time interval over which the transition $|i\rangle \rightarrow |f\rangle$ occurs is infinite, i.e. P_{fi} is the transition probability "sub-specie aeternitatis". Since experimental measurements are performed over a finite time interval, one, in connecting P_{fi} with observed quantities, uses transition probabilities/unit time/unit volume.
- b) In both the initial and final states the particles are assumed to be free. This assumption holds only if the range of the interaction between them is small compared to the distances between them initially and finally. This is indeed the case with strong interactions. The fact of the finite range of strong interactions is thus incorporated into the definition of S -matrix by using free states asymptotically in time.
- c) S_{fi} is the probability amplitude for transition from a state $|i\rangle$ to any state $|f\rangle$ of the system. It is quite possible that sometimes $|f\rangle$ is identical with $|i\rangle$ i.e. no "transition" occurs. To separate out this possibility from the remaining possibilities it is usual to break up S into two parts. Usually* one writes:

$$S = I + 2iT \quad (2.17)$$

* This way of writing S , though common, is not unique. Many authors (e.g. Eden et al. (1967)) write

$$S = I + iR \quad (2.17a)$$

I is the unit matrix and T is called the transition matrix. The factor $2i$ is for convenience. The matrix element S_{fi} is then usually written as* :

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4(P_f - P_i) T_{fi}. \quad (2.18)$$

It may be noted that the object $T_{fi} \neq \langle f|T|i \rangle$ but $\langle f|T|i \rangle$ divided by some factors. The relations between the objects T_{fi} and the measured quantities depend upon the normalisation chosen for the free particle states**. In two-body high-energy phenomenology one usually employs two-particle helicity states in the c.m. frame, in which case T_{fi} is called a helicity amplitude.

2.4B Unitarity

The probability that starting from a state $|i\rangle$ the system ends in any final state $|f\rangle$ is unity, i.e.

$$\sum_f P_{fi} = 1. \quad (2.19)$$

If the states $\{|j\rangle\}$ constitute a complete orthonormal set then this coupled with the above equation yields the result

$$\sum_f S_{jf}^* S_{fi} = \delta_{ji} \quad (2.20a)$$

$$\text{or} \quad S^\dagger S = S S^\dagger = 1. \quad (2.20b)$$

i.e. the S-matrix is unitary. This is known as the unitarity equation or simply unitarity. One consequence of the unitarity relation is the optical theorem, which connects the total cross-section with the imaginary part of

* Here again there are different expressions used by different authors. Equation (2.18) used above, however, is employed in most standard texts (e.g. Collins (1977), Perl (1974), Williams (1971), Eden (1967), Eden et al. (1964), Muirhead (1965)).

** In the above-mentioned references the normalisation is

$$\langle p'|p \rangle = (2\pi)^3 2p_0 \delta^3(\vec{p}' - \vec{p}). \quad (2.18a)$$

the elastic forward scattering amplitude* and is extensively used in calculations. The unitarity relation is usually referred to as a statement of conservation of probability.

2.4C Crossing Relations

We now state the crossing relations or the principle of crossing symmetry. We will state it for two-body reactions when all particles involved are spinless. A two-body reaction amplitude is a function of two independent variables, which may be taken as any two of the three Mandelstam variables. Let T be an invariant amplitude associated with the s -channel. We write $T = T(s,t,u)$ to exhibit the equal status of s, t, u although only two of these are independent. The crossing principle then states that the same analytic function $T(s,t,u)$ is to be regarded as the scattering amplitude for all three channels. In other words

For s -channel	$T(s, t)$ $12 \rightarrow 34$	$\equiv$	$T(s,t,u)$	s,t,u values in s -channel physical region
For t -channel	$T(t, s)$ $1\bar{3} \rightarrow 2\bar{4}$	$\equiv$	$T(s,t,u)$	s,t,u values in t -channel physical region
For u -channel	$T(u, t)$ $1\bar{4} \rightarrow 3\bar{2}$	$\equiv$	$T(s,t,u)$	s,t,u values in u -channel physical region.

* When T_{fi} is a helicity amplitude then, if the helicities of the corresponding initial and final state particles in an elastic reaction are the same, the optical theorem becomes

$$\sigma_{\text{tot}} = \frac{1}{2q\sqrt{s}} \text{Im } T(s,t=0) \quad (2.20c)$$

q being the magnitude of the c.m. 3-momentum and $\sqrt{s}$ the total c.m. energy. (Equations (2.18) and (2.18a) are assumed here.)

Since the (s,t,u) regions for the three reactions are non-overlapping, we are looking upon the three L.H.S. amplitudes as the same analytic function $T(s,t,u)$ evaluated in different domains^{*}.

2.4D Analyticity:

In potential scattering one can, by regarding one or other of the scattering amplitude variables as complex, arrive at analyticity properties of the amplitude regarded as a function of a complex variable (Cf. e.g. De Alfaro and Regge (1965), Omnes and Froissart (1963), Newton (1969)). Thus one discovers that resonances as well as bound states are poles in partial-wave amplitudes in the complex-energy plane. By interpreting the angular momentum variable ℓ as a complex parameter in the (radial) Schrödinger wave equation one arrives at the heuristic concept of Regge poles, which are poles of the scattering amplitude in the complex ℓ plane and which can correlate resonances and bound states for appropriate physical values of ℓ . In relativistic scattering of hadrons where the perturbation expansion of a field-theoretic scattering amplitude does not converge, one arrives at some analyticity properties of amplitudes by assuming that

* One, however, must have a procedure for connecting up $T(s,t,u)$ over the three regions because otherwise one would have three different functions defined over three different regions rather than a single analytic function in three disconnected regions. Analyticity enables us to achieve this end.

the individual terms in the expansion, though giving wrong numerical results, give the correct analytic structure of the amplitude (Cf. e.g. Eden et al. (1966)). Other properties might be suggested by potential scattering. In the framework of the S-matrix one then goes beyond this by asserting analyticity properties which, though suggested to some extent by the kind of analyses mentioned, are regarded as postulates whose ultimate justification lies in the correctness of conclusions drawn from them. These postulates are the following (Cf. e.g. Collins (1977), Collins and Squires (1968)):

- I) Maximal Analyticity of the first kind: The scattering amplitudes are the real boundary values of analytic functions of the invariants s, t, u , whose only singularities are poles corresponding to stable and unstable particles and additional singularities demanded by the unitary relation.
- II) Maximal Analyticity of the second kind: The S-matrix should be analytically continuable in the complex angular momentum plane with only isolated singularities.

For a two-body amplitude, statement (I) enables us to write single- and double-variable dispersion relations. A dispersion relation is a Cauchy formula expressing an analytic function with pole and branch point singularities only, in terms of the residues of its poles and discontinuities across the cuts. For instance, considering t to be fixed and real and regarding the two-body amplitude $T(s, t, u)$ for the scattering of spinless hadrons as an analytic function of s satisfying the condition:

$$\lim_{|s| \rightarrow \infty} s^\alpha T(s, t) = 0 \quad \alpha < N \quad (N \text{ positive integer}) \quad (2.22)$$

one can apply Cauchy's theorem to $T(s, t)/(s-s_1)^N$

$$T(s,t) = \left[\begin{array}{l} \text{terms} \\ \text{due to} \\ \text{residues} \\ \text{of poles} \end{array} \right] + \sum_{n=0}^{N-1} \frac{(s-s_1)^n}{n!} T^{(n)}(s,t) + \frac{(s-s_1)^N}{\pi} \int_{s_0}^{\infty} \frac{\text{Im}T(s',t) ds'}{(s'-s)(s'-s_1)^N} \\ + \frac{(s-s_1)^N}{\pi} \int_{u_0}^{\infty} \frac{\text{Im}T(u',t) du'}{(u'-u)(s'-s_1)^N} \quad (2.23)$$

This sort of dispersion relation can be used to evaluate the real part of the forward elastic scattering amplitude from total cross-section measurements. Further, by regarding t as complex and analytically continuing $\text{Im}T(s,t)$ in the complex t -plane one can, on the right hand side, replace $\text{Im}T$ by a dispersion relation in t to obtain the double variable dispersion relation.

Statement (II), in conjunction with crossing symmetry, leads us to Regge theory which is briefly discussed below.

2.5 Regge Theory

(Some references on Regge theory and Regge phenomenology are:

Irving and Worden (1977), Collins (1977, 1971a,b), Azimov et al. (1974), Perl (1974), Bransden and Moorhouse (1973), Fox and Quigg (1973), Chiu (1972), Fayyazuddin and Riazuddin (1971), Phillips (1970, 1971), Jackson (1970), Barger and Chene (1969), Bertocchi (1968), Bertocchi and Ferrari (1967), Collins and Squires (1968)).

In potential scattering the scattering amplitude when continued analytically into the complex angular momentum plane is found to possess poles whose location depends upon energy, and each of which, as the energy is varied, can, in suitable cases, correlate bound states and resonances for appropriate physical values of ℓ . These poles are known as Regge poles and their discovery, though interesting, does not lead to anything else in potential scattering. In relativistic scattering, where the

principles of crossing symmetry and of the maximal analyticity of the first and second kind hold, one wishes to be able to continue an amplitude of a channel to the complex J plane in a manner that enables one to interpret not only the resonances in that channel as J -plane poles but also at the same time to continue it to the crossed channel physical region and explore its behaviour there in the hope that it might be convergent. The former possibility is obviously suggested by the example of potential scattering, while the latter is motivated by the absence of a theory which can give non-diverging and numerically correct results in hadronic scattering, and is made possible by the principle of crossing symmetry -- it is therefore characteristic only of relativistic scattering. Investigations indicate that both the aforementioned purposes can be realised approximately. There is evidence for the existence of families of resonances, each family comprising objects with the same I , B , Y , P etc., but with spins differing by units of two, which are correlated by the same J -plane pole of a suitably constructed amplitude* (although, empirically, all known resonances cannot be correlated in this manner). The behaviour of the amplitude (which has been continued to the complex J -plane), in the high-energy region of the crossed channel, under certain simplifying assumptions, can be written down simply and does explain a great variety of data. It is found that the Regge poles encountered in correlating resonances in one channel also exert a dominant influence on the expression of the crossed channel amplitude at high energies. In potential scattering the only complex angular momentum singularities of the

* The manner in which a Regge pole $\alpha(t)$ is related to resonance parameters is as follows: At $t = m_R^2$ (m_R is the mass of a resonance of spin J_R , width Γ_R)

$$\left. \begin{aligned} \operatorname{Re} \alpha(t) \\ t = m_R^2 \end{aligned} \right\} = J_R \quad (2.24a)$$

$$\left. \begin{aligned} \operatorname{Im} \alpha(t) \\ t = m_R^2 \end{aligned} \right\} = (m_R \Gamma_R) \left(\frac{d}{dt} \operatorname{Re} \alpha(t) \right) \Big|_{t = m_R^2} \quad (2.24b)$$

amplitude are poles but in relativistic scattering the amplitudes, in addition to poles, seem to require branch point singularities as well. However, it is generally assumed that in most (if not all) instances, the contribution of a branch point singularity can be neglected in the crossed channel high-energy region. In the high-energy region of the s-channel the contribution $R^{(j)}$ of a t-channel (mesonic) Regge pole $\alpha_j(t)$ is given below in equation (2.23). We have, for convenience, taken the amplitude in (2.25) to be a helicity amplitude, although the amplitude could be a spin-independent invariant amplitude, etc.

$$R^{(j)}_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(s, t) = \beta^{(j)}_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(t) \left[\frac{1 + \tau e^{-i\pi \alpha_j(t)}}{s^{i\pi \alpha_j(t)}} \right] \left(\frac{v}{v_0} \right)^{\alpha(t)} \quad (2.25)$$

where $v \equiv \frac{s-u}{2}$; (as $s \rightarrow \infty$ $u \rightarrow s$); $\tau = +1$ if $\alpha(t)$ correlates mesons of even spin and -1 if the mesons lying on it have odd spin. The term in the square bracket is called the signature factor:

for $\tau = +1$ it has poles when $\alpha(t)$ is even, while
for $\tau = -1$ it has poles when $\alpha(t)$ is odd.

This leads to a very simple energy-dependence of total and differential cross-sections and of other observables. For total cross-sections one obtains, via the optical theorem (equation (2.20c))

$$\sigma_{\text{tot}}(s) = \frac{1}{2q\sqrt{s}} \sum_j \beta_j(0)(s) s^{\alpha_j(0)} \quad (2.26)$$

The functions $\alpha_j(t)$ can be determined from the curve correlating the appropriate t-channel resonances. The above form explains most total cross-section data. For differential cross-sections the energy dependence is given by

$$\frac{d\sigma}{dt} \sim (s)^{2\alpha(t)-2} \quad (2.27)$$

When a single Regge pole is exchanged $\alpha(t)$ is its trajectory. When more than one pole of comparable $\alpha(t)$ or a single leading pole is exchanged $\alpha(t)$ is called the effective trajectory. Where a single Regge pole exchange occurs one can use equation (2.27) to determine the function $\alpha(t)$ and compare it with $\alpha(t)$ determined by correlating the corresponding t-channel resonances. In any case $\frac{d\alpha}{dt} > 0$ implies that the rate at which a differential cross-section falls as one moves away from $t = 0$ increases with increasing energy. This is the phenomenon

of shrinkage (Sec. I.3 C) and is observed in most, though not all, reactions.

ii) Phase: The ratio of the real and imaginary parts of a Regge pole amplitude is s independent and is:

$$\begin{aligned} \text{Re } R_{\lambda_3 \lambda_4, \lambda_1 \lambda_2} / \text{Im } R_{\lambda_3 \lambda_4, \lambda_1 \lambda_2} &= \cot \pi \alpha(t) / 2 \quad \text{for } \tau = +1 \\ &= \tan \pi \alpha(t) / 2 \quad \text{for } \tau = -1 \end{aligned} \quad (2.28)$$

Not only is the ratio s -independent, it is also independent of the helicity structure of the amplitude which implies zero polarisation for those reactions where a single Regge pole exchange is allowed or is dominant. In general such a relation is difficult to test experimentally because of the difficulty in isolating individual Regge pole exchanges (c.f., e.g. Collins (1977), Phillips (1970)).

iii) Factorisation: This means that the residue of a Regge pole contribution to an amplitude for the reaction $1+2 \rightarrow 3+4$ can be written as a product of two factors, one depending on particles 1 and 3 and the other on particles 2 and 4:

$$\beta_{\lambda_3 \lambda_4, \lambda_1 \lambda_2} = g_{\lambda_3 \lambda_1}(t) g_{\lambda_4 \lambda_2}(t) . \quad (2.29)$$

This implies numerical relations between the contributions of the same pole in different reactions. Cut contributions do not factorise. Factorisation cannot be widely tested as all hadronic processes studied require nucleon targets. Its success is mixed - there are successes and failures (c.f., e.g. Collins (1977), Irving and Worden (1977) for the latest situation). One could attribute failure to the presence of cuts however.

iv) Exchange Degeneracy (EXD): EXD is not a necessary part of the Regge theory framework but leads to interesting consequences. The term EXD is used in two senses:

- a) Weak EXD: is the coincidence of two Regge trajectories carrying the same quantum numbers but having opposite signatures;
- b) Strong EXD: The numerical equality of the residues of two weakly EXD trajectories in some reaction. Strong EXD is thus with respect to specific reactions.

There is strong evidence of weak EXD of several pairs of trajectories in the t -channel region but in the s -channel region these same pairs do not seem exactly coincident. Nonetheless for many phenomenological purposes they are

taken as coincident. The evidence for strong EXD is not very clear. This is partly because failure of strong EXD predictions might be caused by cuts, secondary poles etc. (C.f., e.g. Collins (1977), Irving and Worden(1977) and references cited therein.

v) Dips and Wrong Signature Nonsense Zeroes (WSNZ's):

Poles $\alpha(t)$ of the amplitude in the complex-J plane can only be interpreted as resonances if they satisfy the criterion of equation (2.24a,b). If this criterion is not fulfilled the pole is physically meaningless and therefore must not be present in the amplitude. Poles $\alpha(t)$ in the s-channel region are meaningless or nonsense since they imply the existence of resonances for which the square of mass is negative. Furthermore, for $\tau = 1$ even values of $\alpha(t)$ are right signature points (RSP) and odd values of $\alpha(t)$ are wrong signature points (WSP) and vice versa for $\tau = -1$. The poles, in the signature factor, at RSP's (there are no poles at WSP's), correspond to negative mass squared values if they occur in the s-channel physical region, and since negative mass squared values are physically meaningless ("nonsense") one eliminates ("kills") such poles by requiring $\beta(t)$ to vanish at such points (nonsense zeroes). If, however, we have strong EXD of two trajectories then both will have the same residue associated with them. Thus, for any one of these, $\beta(t)$ will vanish not only at RSP's but also at WSP's (what is an RSP for one trajectory is a WSP for the other and vice versa). Such zeroes are wrong signature nonsense zeroes (WSNZ's) and can be associated with dips in $\frac{d\sigma}{dt}$. The dip systematics from this point of view has been discussed in Bertocchi (1968). The systematic explanation of dips in Regge pole models leads to complications and in many instances one has to invoke cuts. $pp \rightarrow pp$ is an important example where the dip in $\frac{d\sigma}{dt}$ is explained by invoking destructive pole-cut interference (e.g. Collins et al. (1974a)). Models which rely on pole-cut interference to explain dips in differential cross-section can differ radically on the relative importance of the cut terms and on the broad form of the Regge pole terms - this has given rise to the well-known Argonne-Michigan controversy (C.f. Chapter 3 and references cited therein).

2.6 Helicity Amplitudes and their Partial-Wave Expansion:

In two-body hadronic phenomenology the helicity amplitudes of Jacob and Wick (Jacob and Wick (1959)) are widely used when one cannot or does not wish to ignore spin effects. These are defined with respect to binary reactions and are the matrix elements T_{fi} (or Γ_{fi} multiplied by suitable factors) introduced in equation (2.18) when the initial and final states are labelled by individual 3-momenta and individual helicities (2-particle plane-wave helicity states). Great simplification occurs when one is considering these states and amplitudes in the c.m. frame and it is the c.m. helicity amplitudes that are usually employed in two-body hadronic phase meson energy. All further statements in this section will therefore refer to the c.m. mass frame. For the reaction $1 + 2 \rightarrow 3 + 4$, if one takes the 3 momenta of 1 and 3 to be $\vec{p}_i$ ($p_i = p_i, 0, 0$) and $\vec{p}_f$ ($p_f = (p_f, \theta, \phi)$) respectively, and if one denotes the helicity of the j th particle by λ_j , the initial and final plane wave helicity states are $|p_i, 0, 0, \lambda_1, \lambda_2\rangle$ and $|p_f, \theta, \phi, \lambda_3, \lambda_4\rangle$. The corresponding amplitude T_{fi} is then denoted as $T_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(s, \theta, \phi)$. Rotational invariance of $T_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(s, \theta, \phi)$ then yields a relationship between the helicity amplitude introduced above and the amplitude defined with respect to angular momentum helicity states $|J, J_z, \lambda_1 \lambda_2\rangle$ and $|J, J_z, \lambda_3, \lambda_4\rangle$. This relationship is known as the partial-wave expansion. We will state this expansion, for compactness, in terms of an amplitude $f_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(s, \theta, \phi)$ defined by

$$f_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(s, \theta, \phi) = \frac{1}{8\pi\sqrt{s}} \left(\frac{p_f}{p_i}\right)^{\frac{1}{2}} T_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(s, \theta, \phi). \quad (2.30)$$

For the elastic scattering of spinless particles this amplitude becomes identical with $f(\theta, \phi)$ introduced in equation (2.2). The partial-wave expansion of $f_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}$ is:

$$f_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(s, \theta, \phi) = \left(\frac{1}{p_i}\right) e^{i(\mu_i - \mu_f)\phi} \sum_J (2J+1) \langle \lambda_3 \lambda_4 | T_{(s)}^J | \lambda_1 \lambda_2 \rangle_{\mu_i \mu_f} d^J(\cos \theta) \quad (2.31)$$

where $\mu_i \equiv \lambda_1 - \lambda_2$, $\mu_f \equiv \lambda_3 - \lambda_4$ and $d^J_{\mu_i \mu_f}$ are the usual rotation functions. $\langle \lambda_3 \lambda_4 | T_{(s)}^J | \lambda_1 \lambda_2 \rangle$ is called a partial-wave amplitude. The partial-wave expansion serves as the basis for the impact-parameter representation of helicity amplitudes.

2.7 Impact-Parameter Representation of Helicity Amplitudes

In the partial-wave expansion (2.31) the summation runs over all possible values of the total angular momentum quantum number J . According to the principles of quantum mechanics this variable can only have discrete values. However it is a heuristic practice, in at least strong interaction physics, that, if need be, one can invoke some classical and semiclassical ideas to proceed further. With this in mind and remembering that in classical physics angular momentum is a real, non-negative and continuous variable, we now introduce a real non-negative, continuous variable (called the impact-parameter) to convert the partial-wave expansion into an integral representation of helicity amplitudes. The impact-parameter b is defined with respect to the c.m. mass frame of a system of two interacting particles and is given by the relation

$$J + \frac{1}{2} \equiv p_i b \quad (2.32)$$

where J is real non-negative and continuous and, for appropriate integral values, is interpreted as an angular momentum quantum number whereas p_i is the magnitude of the (initial state) c.m. 3-momentum.

We now assume the existence of an energy- and helicity-dependent function $\tilde{f}_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(b, s)$ which depends also on the impact-parameter b but in a manner such that, for appropriate J values, it coincides with the



corresponding partial-wave amplitude. We also note that the rotation function $d_{\mu_i \mu_f}^J(\cos\theta)$ is defined even when J is not discrete. At high energies where large angular momenta are involved one can then make the replacement

$$\sum_J \xrightarrow{p_i \rightarrow \infty} \int p_i db \quad . \quad (2.33)$$

At high energies and small scattering angles one has the simple relation (Durand and Chiu (1965))

$$\begin{aligned} d_{\mu_i \mu_f}^J(\cos\theta) &\rightarrow (-1)^n J_n \{ (J + \frac{1}{2}) \sqrt{2(1 - \cos\theta)} \} \\ &= (-1)^n J_n \{ b \sqrt{t_{\min} - t} \} \end{aligned} \quad (2.34)$$

$n \equiv |\mu_i - \mu_f|$ and $t_{\min} = m_1^2 + m_3^2 - 2E_1 E_3 + 2|\vec{p}_1||\vec{p}_3|$
being 0 for elastic scattering. J_n is a Bessel function of order n (n integer).

One thus obtains the relation, the so-called impact parameter representation of helicity amplitudes,

$$f_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(s, \theta, \phi) = 2p_i (-1)^n e^{in\phi} \int_0^\infty b db \frac{\gamma_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(b, s) J_n \{ b \sqrt{t_{\min} - t} \}}{\lambda_3 \lambda_4, \lambda_1 \lambda_2} \quad (2.35)$$

This result holds for all two-body reactions, elastic as well as inelastic.

All the known helicity dependence is contained in the Bessel function, while

the function $\gamma_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(b, s)$ contains the unknown helicity dependence.

By choosing different forms for $\gamma_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(b, s)$ one can arrive at

different forms for the amplitude. It is usual to set $\phi = 0$ in phenomenological calculations. Henceforth we will set $\phi = 0$ and also ignore the inessential factor $(-1)^n$. We quote some standard forms for

$\gamma_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(b, s)$ for later reference in Table 2.2 below. Frequently one

expresses $\tilde{f}_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(b, s)$ as a product of two factors, one depending only on b , the other only on s . For clarity in writing we drop the helicity subscripts.

TABLE 2.2

Some Standard Forms for the Impact-Parameter Partial Wave Amplitude

$\tilde{f}(b, s)$

Expression	Name of form	Equation No.
$\tilde{f}(b, s) = \beta(s) \quad 0 \leq b \leq R \quad 0 \leq \beta \leq 1$ $= 0 \quad b > R$	Step function or disc	2.36a
$\tilde{f}(b, s) = A(s) e^{-b^2/\beta(s)}$	Gaussian	2.36b
$\tilde{f}(b, s) = A(s) b^n e^{-b^2/\beta(s)} \quad n \neq 0$	Strongly Absorbed Gaussian	2.36c
$\tilde{f}(b, s) = A(s) \delta(b-R)$ or $\tilde{f}(b, s) = A(s) \quad R-\delta R \leq b \leq R+\delta R, \quad \delta R \ll R$ $= 0 \quad \text{elsewhere}$	Peripheral	2.36d
		2.36e
$\tilde{f}(b, s) = A(s) [1 - \exp(-(E-b)/b)]$	Wood Saxon	2.36f
$\tilde{f}(b, s) = \frac{A(s)}{\{1 + \exp((b-B_0(s))C(s))\}}$	Fermi distribution	2.36g

The Gaussian form leads to an amplitude which has an exponential t -dependence and is thus structureless. The peripheral form on the other hand gives rise to an amplitude which has essentially the same structure in t (maxim, zeroes, etc.) as the Bessel function $J_n(R\sqrt{t_{\min} - t})$. The peripheral form (2.36e) gives an amplitude $\tilde{f}_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(s, t)$ as:

$$\tilde{f}_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(s, t) = 2p_i AR e^{Bt} J_n(R\sqrt{t_{\min} - t}) \quad (2.37)$$

For the form (2.36d) $B = 0$.

The process of representing $f_{\lambda_1 \lambda_2, \lambda_3 \lambda_4}^{\gamma}(b, s)$ by simple forms of the type mentioned above to obtain different expressions for $f_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}^{\gamma}(s, t)$ goes by the name of "geometry" in high-energy hadronic phenomenology.

Apart from enabling one to employ "geometrical" ideas in analysing high-energy data, the impact-parameter representation permits the introduction of the eikonal idea into phenomenological calculations in a relativistic fashion. Thus, if for $n = 0$, one sets

$$f_o^{\gamma} = \frac{e^{i\chi_o} - 1}{2i} \quad (2.38a)$$

one recovers the old eikonal formula (2.10). However not only is our approach in this section relativistic, the "eikonalisation" process can be extended naturally to helicity-flip amplitudes as well. The eikonal idea is employed very widely in high-energy hadronic phenomenology and a variety of widely differing forms are used. To give an idea of the diversity of its application some of the forms (to which we shall allude in subsequent chapters) are quoted below*. For $n \neq 0$ the usual eikonal form of $f_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}^{\gamma}(b, s)$ is:

$$f_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}^{\gamma}(b, s) = \chi_{\lambda_3 \lambda_4, \lambda_1 \lambda_2} e^{i\chi_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}^{\gamma}(b, s)} \quad (2.38b)$$

The expressions (2.38a,b) which "eikonalise" an amplitude, though widely used, are not the only ones. Thus for instance Collins (1977) and Collins et al. (1974b) essentially replace χ_o in (2.38a) by $\lambda\chi_o$ where λ is an enhancement factor. Similarly Arnold and Blackmon (1968) use expressions which differ from (2.38a,b). We however do not concern ourselves with such differences here. In the Chou-Yang (Chou and Yang (1968)

* We do not concern ourselves with different normalisations or with the meanings of various parameters occurring in the expressions since we are not interested in numerical details.

and (Durand and Lipes (1968)) model for elastic pp scattering the $n \neq 0$ amplitudes are neglected and the eikonal χ_0 in (2.38a) is taken to be imaginary and s-independent, the eikonal being given as a Fourier-Bessel transform of the product $G_E^2(t)$ of the form-factors of the colliding particles:

$$\chi_0 = ik \int_0^\infty \sqrt{-t} d\sqrt{-t} G_E^2(t) J_0(b\sqrt{-t}) \quad (2.39)$$

In contrast the basis of the Reggeised Eikonal Model is the identification of the first term in the eikonal series (i.e. the Born approximation) with a single Regge pole exchange term or a sum of such terms, $A_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}^R(s, t)$ (say). This gives χ as the Fourier-Bessel transform of $A_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}^R(s, t)$, i.e.

$$\chi_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}(b, s) = \frac{1}{8\pi s} \int_0^\infty \sqrt{-t} d\sqrt{-t} J_n(b\sqrt{-t}) A_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}^R(s, t) \quad (2.40)$$

Frautschi and Margolis (1968) replaced $A_{\lambda_3 \lambda_4, \lambda_1 \lambda_2}^R(s, t)$ by a Pomeron exchange term to explain various features of elastic cross-section. (c.f. p 63)

Some authors do not take χ to be a mere Fourier-Bessel transform of Regge contributions but introduce additional considerations as well. Thus, for instance, in explaining pp data, Majerotto (1976) separates the eikonal into two parts, a diffractive part and a Regge part. While the Regge part is the usual Fourier-Bessel transform of appropriate "ordinary" Regge contributions, the diffractive part is constructed in a way as to accommodate the geometrical scaling. He writes the diffractive part (which contributes only to the non-flip amplitude) as:

$$\chi_0^D(b, s) = -i \ln(1 + i\tilde{\phi}_1(b, s)) \quad (2.41a)$$

$\tilde{\phi}_1(b, s)$ is an amplitude with

$$\text{Im}\tilde{\phi}_1(b, s) = 1 - \sqrt{1 - (0.94e^{-\beta} + 0.54\beta e^{-2.6\beta})} \quad (2.41b)$$

and where $\beta \equiv \frac{b^2}{R^2}$ with $R^2 = R_0 + R_1^2 \ln s$ ($R_0 \sim 0.84 \text{ fm}$, $R_1 \sim 0.22 \text{ fm}$).

$\text{Re} \phi_1(b,s)$ is then given by

$$\text{Re} \phi(b,s) \sim \frac{-\pi R_1^2}{2R^2} \frac{d}{d\beta} \text{Im} \phi(b,s) \quad (2.41c)$$

In the Cheng-Walker-Wu model (Cheng et al. (1973)) on the other hand, one has

$$\chi_0 = f e^{-\lambda\{b^2 + b_0^2\}^{\frac{1}{2}}} (s)^{0.08} \quad (2.41d)$$

As a final example we mention the parameterisation of Daboul and Rek (1977a, b). Assuming the additivity of eikonals in a quark model approach they assume the eikonal χ_0 in equation (2.38a) to be imaginary and parameterise it as

$$\chi_0(b,s) = i\chi(b,s) = i9 \left(g_c e^{-b^2/\overline{R}_c^2} + g_p e^{-b^2/\overline{R}_p^2} + g_l e^{-b^2/\overline{R}_l^2} \right) \quad (2.41e)$$

The various parameters are described in Daboul and Rek (1977a,b).

Because of the geometrical nature of the impact-parameter variable it is intuitively satisfying to study the impact-parameter properties of amplitudes. Thus the impact-parameter representation of a Regge-pole amplitude is Gaussian provided the residue function is taken as an exponential in t . The same is true for a cut generated by two such Regge poles - only its range is shorter than that of a single Regge pole (Cf. e.g. Collins (1977)). Harari and Schwimmer (1972) have carried out a study of the impact parameter structure of several classes of models. An interesting conclusion emerging from their investigations is the incompatibility of the following assumptions:

- i) Peripherality of the imaginary part $\text{Im} f(s,t)$ of some amplitude;
- ii) Indefinite shrinkage of $\text{Im} f(s,t)$ as $s \rightarrow \infty$;
- iii) Approach of R (the radius of the periphery), to a constant value as $s \rightarrow \infty$.

2.8 Finite Energy Sum Rules and Duality

2.8A Finite Energy Sum Rules (FESR's)

A finite energy sum rule is an integral relation following from the application of Cauchy's theorem, using the closed contour shown, to an odd function $[v^n A(v,t)]$ of the variable v (where $A(v,t)$ is an amplitude which itself is an even or odd function of v and n is an appropriate non-negative integer ($v \propto (s-u)$) under the assumption that the function is analytic everywhere on and inside that contour and that for large v (say $v > N$ where N is unspecified), it is a sum of a few Regge-pole/cut exchanges. The number n is called the moment

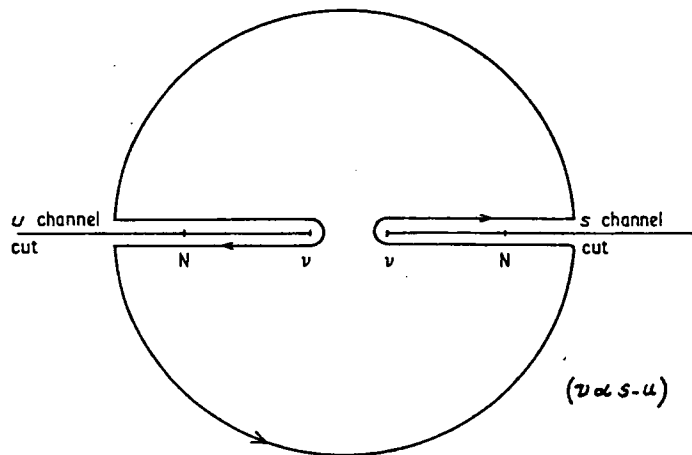


Figure 2.2 The integration contour.

of the sum rule and is odd(even) if $A(v,t)$ is an even(odd) function of v . A FESR of moment n looks like

$$S_n \equiv \int_{v_0}^N v^n \text{Im}T(v,t) dv = \sum_j \beta_j(t) \frac{N^{j(t)+n+1}}{\alpha_j(t) + n + 1} \quad (2.42)$$

One can note that it is $\text{Im}T$ and not T that occurs on the left-hand side. Also the value of N , though stated to be "high" has been left unspecified. In practice N is the upper limit of partial-wave analysis. FESR's were first derived in Logunov et al. (1967), Igi and Matsuda (1967), Dolen et al. (1967). It is clear that the FESR's connect the high-energy regime (i.e. the Regge regime) with the low-energy regime (i.e. the resonance regime), enabling one to determine the various Regge parameters after using the partial-wave amplitudes to calculate the left-hand side. Clearly the results are expected to be better in a reaction where there exists an accurate partial-wave (or phase-shift) analysis and where a small number of trajectories are exchanged. Consequently πN CEX, where only the ρ is exchanged, has been the subject of extensive applications. Worden (1973, 1974) has reviewed the subject. Recent reviews that include FESR's are Fukugita and Igi (1977) and Irving and Worden (1977).

2.8B Duality

Duality is the hypothesis that hadronic amplitudes are dual. An amplitude is said to be dual if its high-energy approximation when extrapolated down to low and intermediate energies gives a local average of the actual amplitude there (Cf. e.g. Worden (1973, 1974)). The above statement is, by standard usage, understood to apply only to the imaginary part of the amplitude. Mathematically, denoting the high energy approximation by T_{Regge} , and the actual amplitude as T duality may be expressed as

$$\int_{v_0}^N [\text{Im}(T(v,t) - T_{\text{Regge}}(v,t))] dv = 0 \quad (2.43)$$

(This relationship can be "derived" from an FESR of moment zero ($n=0$))

but duality gives it physical meaning). In the above definition of a dual amplitude the averaging usually covers a range $\Delta p_{\text{lab}} \sim 1 \text{ GeV/c}$ although in many cases this interval can be somewhat smaller on account of a lack of availability of partial-wave (or phase-shift) analyses over a wider range. The phrase 'high-energy' refers to the energy region where a Regge exchange behaviour begins to show up in data and in most reactions corresponds to $p_{\text{lab}} \geq 4 \text{ GeV/c}$, although occasionally the Regge regime can extend as low as $p_{\text{lab}} \sim 1 \text{ GeV/c}$, as in $K^+p \rightarrow K^+p$. The phrase 'low and intermediate energies' usually covers the range $p_{\text{lab}} > 0.5 \text{ GeV/c}$ up to the point beyond which partial-wave or phase-shift analyses are not available and this point corresponds roughly to $p_{\text{lab}} \sim 1.5 - 2 \text{ GeV/c}$. The cut off N in the duality relation corresponds to this point. It is obvious from the above remarks that sometimes there is an overlap between the regions corresponding to the phrases "high-energy" and "low and intermediate energies". [According to the duality hypothesis the actual amplitude in this region of overlap (and elsewhere in the intermediate and low energies) can be represented either by the high energy approximation or by the partial wave series (which is traditionally employed at lower energies) in its approximate (or exact) form but not by their sum. This latter possibility is known as the interference model and has been shown to be inconsistent with data (Dolen et al. (1967)).] We describe below briefly how two descriptions which originally refer to different energy regimes and have apparently different origins (s-channel for partial waves and t-channel for high energy approx.) can be used outside their respective regimes as well. For detailed discussion of the topic see Harari (1968), Kugler (1970), Schmid (1970), Phillips (1974), Collins (1977), Fukugita and Igi (1977) and the original papers Igi and Matsuda (1967), Dolen et al. (1967), Schmid (1968, 1969), Collins et al. (1968), Allesandrini et al. (1968a,b,c).

We note the following points:

(i) The partial-wave expansion of an amplitude is an exact expression which is valid for all s, t values in the s -channel region (and a little beyond) and can therefore in principle be used to describe both high- and low-energy data in the s -channel.

(ii) In practice the partial-wave expansion is used in a truncated form and this description is economical only in the low- and intermediate-energy regions. It is usual to break up the (truncated) partial-wave expansion into two parts: (a) a sum of appropriate Breit-Wigner terms, each term corresponding to some resonance observed in that reaction - this is the resonance part; (b) a smooth function of energy called the background term. In most reactions (b) can be neglected and what is left is an approximation called the resonance saturated amplitude.

(iii) Resonance saturation does not work for elastic scattering since it is found that the background is non-negligible for such reactions.

(iv) The higher mass resonances in a given channel are found to be closely spaced in energy and have larger widths. Consequently, their overlap can lead to a rather smooth energy dependence of the amplitude in this region, even when it is resonance saturated, provided a sufficiently large number of Breit-Wigner terms (i.e. resonances) are present. Thus, under suitable conditions, even a resonance saturated amplitude, which is normally "wiggly", can lead to a smooth energy dependence. However such a description at high energies would be uneconomical.

(v) One can also obtain an exact expression for the s -channel amplitude by starting from the t -channel partial wave expansion via the complex J -plane. This is the Regge representation which generally involves three types of terms:- a) Regge poles, b) Regge cuts and c) the background integral. It is a matter of experience that at high energies the amplitude can be approximated by a few Regge pole terms

and occasionally a cut or two as well. Such an amplitude is a smooth function of energy and is simple in appearance.

(vi) Elastic scattering involves a large contribution from the Pomeron pole. The Pomeron cannot be exchanged in inelastic scattering, has a larger intercept and smaller slope compared to all known Regge poles, and no known resonances lie on it.

(vii) It was shown by Schmid (Schmid (1968)) that a Regge-pole type amplitude, on being partial wave analysed, can give rise to resonance-like loops on an Argand diagram. (the Pomeron^{*} is an exception to this). He showed that one can obtain such loops for the leading resonances in πN scattering in this way. If one considers loops generated by such a mechanism as representing resonances, and such an interpretation appears reasonable (Cf. references cited above for this debate), then one can use either the Regge approximation or the usual low-energy amplitude to describe the amplitude at lower energies. For reactions whose amplitudes can be resonance saturated this means

$$\int_{v_0}^N \text{Im} [T_{\text{res}} - T_{\text{Regge}}] dv \sim 0 \quad (2.44)$$

where in this expression T_{Regge} cannot contain the Pomeron and T_{res} is the amplitude minus the background. This relation is of great significance since it implies a relationship between s- and t-channel resonances or equivalently between s- and t-channel Regge trajectories, since resonances are correlated by trajectories.

(viii) The Pomeron^{*} is an exception to the above scheme and this coupled with (iii) suggests that a relationship might exist between the Pomeron and the background. This will lead us to two-component duality, discussed in the next section.

(ix) An interesting consequence of equation (2.44) occurs if no resonances exist in a channel. Then since $\text{Im} T \sim \text{Im} T_{\text{res}} \sim 0$ the FESR

*c.f. footnote on p 63 about the Pomeron.

gives

$$\sum \beta_i(t) (v)^{\alpha_i(t)} = 0. \quad (2.45)$$

This implies strong EXD of the various trajectories exchanged in that reaction. In channels with exotic quantum numbers (e.g. K^+p , pp), duality implies strong EXD for various Regge poles. It may however be noted that the Pomeron is not included in this. Similarly, if the t -channel has exotic quantum numbers then, since no Regge poles exist, one concludes via duality that the s -channel amplitudes will be predominantly real.

2.8C Two-Component Duality

While the duality hypothesis connects the approximation of a hadronic amplitude at high energies to the actual amplitude in the low and intermediate energies, two-component duality (Harari (1968), Freund (1968)) splits both, the Regge approximation at high energies and the actual amplitude in the low-intermediate energy regime, into two parts each and establishes a dual relation between these parts. It may be stated as follows: "The Pomeron is dual to the non-resonating background while the "ordinary" trajectories are dual to the resonance part." Thus one has

$$\int_{s_0}^N v^n \text{Im } T_{\text{b.g.}}(s, t) dv = \beta_P(t) \frac{N^{\alpha_P(t)+n+1}}{\alpha_P(t) + n + 1} \quad (2.46a)$$

$$\int_{s_0}^N v^n \text{Im } T_{\text{res}}(v, t) dv = \sum_{i \neq P} \beta_i(t) \frac{N^{\alpha_i(t)+n+1}}{\alpha_i(t) + n + 1} \quad (2.46b)$$

It may be noted that this two-component version of duality also applies only to the imaginary part. Several interesting consequences emerge from this conjecture. One of these, by requiring strong EXD between $f-\omega$ and $\rho-A_2$ trajectories, explains the constancy of K^+p , pp , K^+n , pn total cross-

sections even at low energies, as mentioned in Sec. 1.2. Other consequences are listed in Harari (1968).

This hypothesis can be tested more directly in the following manner (Harari and Zarmi (1969)): if one were to consider all partial-wave amplitudes of an elastic reaction with definite isospin I_t in the t-channel then (a) the $I_t \neq 0$ amplitudes should, according to two-component duality, be accounted for completely by the s-channel resonances (since the Pomeron has $I_t = 0$) and (b) the $I_t = 0$ amplitudes should have a dominant, predominantly imaginary, background term which varies smoothly with energy and on which some of the s-channel resonances are superposed. Harari and Zarmi (1968) have verified this for πN elastic scattering. Fukugita and Inami (1972) have confirmed this hypothesis for K^+p elastic scattering. They have also considered the energy dependence of the forward imaginary part of the background for the overall non-flip amplitude and found it to increase linearly with s , thus making the connection of the background with the Pomeron more credible. Quigg (1974) has considered $\pi\pi$ scattering ($I_t = 0, 1, 2$) and found the results to be consistent with two-component duality. Field (1974) has reported a very interesting test of this conjecture by Quigg in the reaction $\pi N \rightarrow \rho N$ which has $I_t = 0$ but no Pomeron exchange. In this case no significant background is found in the $I_t = 0$ amplitude. One could thus conclude that the evidence for 2-component duality is good and that the concept should, wherever possible, be tested.

footnote to p 55, 61 concerning the Pomeron:

The Pomeron is a Regge pole of intercept ≈ 1 which is needed to explain total and elastic-differential cross-section data. The quantum numbers associated with the Pomeron pole can be inferred to be those of vacuum (i.e. $B=S=Q=I=0$; $P=G=\tau=1$). However there are two surprising features of this (phenomenologically necessary) pole. Firstly, no known particles are correlated by the Pomeron (c.f. equation 2.24 on p 46 for a relationship between the intercept (and slope) of a Regge pole and various particles). Secondly the value of the slope of the Pomeron is inferred from data fitting is much less than that of other known Regge poles ($\alpha'_P \approx 0.25$ against $\alpha' \approx 1$ for other trajectories).

CHAPTER 3

THE DUAL ABSORPTIVE MODEL

3.1 Introduction:

In this chapter we shall describe the Dual Absorptive Model (DAM). In order to be able to introduce this model we shall, after making some general remarks on two-body phenomenology (Sec. 3.2), delineate the major problem confronting the various standard phenomenological models of the late sixties, i.e. a systematic explanation of the occurrence or absence of $|t| \approx 0.6 \text{ (GeV/c)}^2$ dips in many two-body inelastic reactions (Sec. 3.3). After this we shall note some empirical features of elastic data and, in particular, point out the peripherality of prominent elastic resonances in impact-parameter space and its consequences in a two-component dual framework (Sec. 3.3B). We shall then state the "rules" of DAM (Sec. 3.4) and briefly describe how these permit a qualitative understanding of the dip problem and of regularities in elastic data (Sec. 3.5).

3.2 Phenomenology. Some General Remarks

In Chapter 1 we reviewed the major features of elastic scattering data at high energies. In addition to the elastic data there exists a great amount of data on two-body inelastic reactions which has accumulated steadily over the past two decades. By far the bulk of this data exists in the form of differential cross-section measurements over a wide energy range. The dominant feature of these differential cross-sections is the absence or occurrence of dips in the small $|t|$ (or small $|u|$) region, particularly around $|t| \approx 0.6 \text{ (GeV/c)}^2$. The systematic explanation of the occurrence or absence of these dips is the first major challenge a theoretical model must face. Even when a model can explain differential

cross-section data there remain other quantities, such as the polarisation and spin rotation parameters for instance, to be explained. Furthermore, there are several models which, while explaining the same $\frac{d\sigma}{dt}$ data, differ widely in their interpretation of the zeroes in the amplitudes and the location of these zeroes. One could hope that measurements of other quantities, when confronted with these models, might enable one to choose one model over the others. However experience over the years has shown the inability of any single model to explain all features of the data satisfactorily. This had led to what Fox and Quigg (1973) have called the "demise" of explicit models and an emphasis on model independent amplitude extraction from data using general theoretical principles instead. This latter process ("amplitude analysis"), however, requires detailed measurements on a variety of quantities which are generally not available, except for a few reactions, and these only at relatively low energies. Therefore at present the phenomenologists, over and above the explicit models, possess some general ideas which are employed in elucidating amplitude structure. These are the notions of Regge exchanges, duality, and geometry (i.e. construction of amplitudes by considering profiles of partial wave amplitudes in impact parameter space as described in Chapter 2). It is well known (cf. e.g. Barger and Cline (1969), Fox and Quigg (1973), Irving and Worden (1977), Collins (1977)) that the over-all energy dependence of the various s-channel observables is unambiguously predicted and successfully correlated with the t-channel quantum numbers by Regge theory. Although finer measurements do reveal some problems in this regard (cf. e.g. Leader (1978) and references cited therein), the predictions are correct in an overwhelming majority of cases. In regard to the t-dependence the original hope that everything might be explicable in terms of a few Regge poles turned out to be simpler than reality, in spite of a fair amount of freedom on the choice of the residues. Consequently by the end of the

last decade models for cut corrections to basic Regge poles were devised. In the next section we will dilate on the mechanism of dip production in some of these models with the object of comparing their predictions with some commonly observed reactions. The purpose behind this comparison is to expose the pattern in the success and failure of these models in explaining $|t| \approx 0.6 \text{ (GeV/c)}^2$ dips in inelastic $\frac{d\sigma}{dt}$. It was the recognition of this pattern which, combined with the contemporaneous development of duality and the empirical impact-parameter peripherality of prominent elastic resonances (Sec. 3.3B), provided the basic motivation for the Dual Absorptive Model.

3.3A The "Dip Puzzle"

We will now discuss the difficulty faced by several ambitious models in systematically explaining the occurrence or non-occurrence of $|t| \approx 0.6 \text{ (GeV/c)}^2$ dips in two-body inelastic differential cross-sections. For this purpose we will briefly explain the mechanism by which dips are generated in some of these models. In Reggeised Absorption Models one modifies the original Regge pole exchange term by adding to it an absorbed pole-Pomeron cut term which is supposed to represent elastic rescattering corrections. (The absorption is carried out by using the Sopkovich prescription). This term interferes destructively with the original pole term, giving rise to dips, and its magnitude depends on whether one assumes the presence of WSNZ's (due to EXD) in the pole term or whether one regards the pole term as structureless. If one assumes the existence of WSNZ's, then the resulting change of sign in the Regge pole term in the integrand of the cut integral leads to a suppression of the magnitude of the cut term (weak cut). As a consequence one obtains dips, not exactly at WSNZ's as in the unmodified Regge pole case, but displaced slightly to smaller $|t|$. This is the Argonne version of the Reggeised Absorption

model (Arnold and Blackmon (1968), Blackmon (1969), Blackmon and Goldstein (1969)). Two things are to be noted in the Argonne Model:

i) It preserves, in a sense, the association of dips with WSNZ's in that dips, though generated by pole-cut interference, occur near the WSNZ's. Consequently reactions which do not exhibit dips around WSNZ's mean trouble for this Argonne model.

ii) The zero structure of the Regge-pole term is the same for all helicity amplitudes and is determined by the trajectory function and the signature of the exchanged Reggeon. The imaginary parts thus have single zeroes while the real parts have double zeroes.

In the Michigan version of the Reggeised Absorption Model the Regge pole terms have no WSNZ's and are constructed in a manner such as to have no zeroes other than those required by general analyticity arguments (Heyney et al. 1968, 1969, 1970). In addition the cut term is multiplied by an empirical factor λ which is interpreted as representing the effect of the diffractive contributions. λ is found to be somewhat larger than unity ($\lambda \approx 2$). Both, the absence of WSNZ's in cut integrand, and the multiplication by λ , serve to enhance the cut contribution - hence the name Strong Cut Reggeised Absorption Model (SCRAM). In SCRAM pole-cut interference tends to generate dips around $|t| \approx 0.2(\text{GeV}/c)^2$ if $\Delta\lambda = 0$ dominates and around $|t| \approx 0.6 (\text{GeV}/c)^2$ if $\Delta\lambda = 1$ dominates (as a mnemonic these are the same as the zeroes of $J_0(R\sqrt{-t})$ and $J_1(R\sqrt{-t})$ respectively* with $R \approx 1 \text{ fm}$). The dip positions are actually sensitive

* The first and second zeroes of the Bessel functions $J_0(x)$, $J_1(x)$, $J_2(x)$ ($x \equiv R\sqrt{-t}$, $R = 1 \text{ fm}$) are as follows (t in $(\text{GeV}/c)^2$).

Bessel Function	First Zero	Second Zero
J_0	$x = 2.4$ $-t = 0.22$	$x = 5.5$ $-t = 1.18$
J_1	$x = 0.0$ $-t = 0.0$	$x = 2.83$ $-t = 0.57$
J_2	$x = 0.0$ $-t = 0.0$	$x = 5.15$ $-t = 1.03$

to the value of parameter λ and to those of other parameters and tend toward $t = 0$ as $s \rightarrow \infty$. Both real and imaginary parts have single zeroes and the location of the zeroes depends not on the exchanged object as such but on which amplitude dominates. For comparable contributions from $\Delta\lambda = 0$ and $\Delta\lambda = 1$ no dip is expected. In this the location of SCRAM zeroes is similar to that of the Dar-Weisskopf model (Dar (1964), Dar et al. (1964, 1969)) which uses absorbed particle-exchange amplitudes in the Born approximation rather than Reggeon exchanges. In this model the central partial waves are completely absorbed, while in a peripheral ring of small thickness, the partial-wave amplitude (in impact-parameter space) is approximated by a Wood-Saxon expression multiplied by a factor $C_s J_e^{-1} e^{-m_e b} / \sqrt{m_e b}$ (m_e, J_e being the mass and spin of the exchanged particle). All these models belong to what has often been called the first generation of phenomenological models which attempt to make a unified treatment of the dip systematics. Another model belonging to this generation is the weak cut model of Capella and Tran Thanh Van (Capella and Tran Thanh Van (1969), Tran Thanh Van (1971)). The weak cut and Regge pole models predict more or less similar patterns of dips and could be lumped together as one family (say Class I) from this point of view. The strong cut and Dar-Weisskopf models could be grouped together as another family (say Class II) since their dip predictions are also similar (cf. e.g. Harari, 1971). Harari (1968) first reviewed the problem of dips and, in a table constructed on the basis of Harari (1971) and Table 6.7 of Collins (1977), we compare these predictions with the actual data in regard to the $|t| \approx 0.6 \text{ (GeV/c)}^2$ dips in several two-body inelastic reactions. In drawing this comparison several phenomenological assumptions have been made (cf. Harari (1971), Collins (1977))

TABLE 3.1

 $|t| \approx 0.6 \text{ (GeV/c)}^2$ Dips and Some Models

(Processes dominated by odd exchanges but exhibiting no $|t| \approx 0.6$ dips do not agree with Class I models regardless of what $\Delta\lambda$ dominates. Reactions dominated by $\Delta\lambda = 1$ but showing no $|t| \approx 0.6$ dips do not agree with Class II models no matter what exchanges dominate.)

Reaction	Dominant Exchange	Crossing Property of Dominant Exchange	Does $\Delta\lambda = 1$ dominate?	Dip seen at $t \approx 0.6$ (GeV/c) ?	Does Data agree with Class I Models?	Class II Models?
$\pi^- p \rightarrow \pi^0 n$	ρ	Odd	Yes	Yes	Yes	Yes
$\pi^+ p \rightarrow \pi^0 \Delta^{++}$	ρ	Odd	Yes	Yes	Yes	Yes
$\gamma p \rightarrow \pi^0 p$	ω	Odd	Yes	Yes	Yes	Yes
$\pi^\pm p \rightarrow \rho^\pm p$	ω		Yes	Yes	Yes	Yes
$\pi^- p \rightarrow \eta n$	A_2	Even	Yes	No	Yes	No
$\pi^+ p \rightarrow \eta \Delta^{++}$	A_2	Even	Yes	No	Yes	No
$K^- p \rightarrow \bar{K}^0 n$	$\rho + A_2$	Odd + Even	Yes	No	Yes	No
$K^- n \rightarrow \bar{K}^0 \Delta$	$\rho + A_2$	Odd + Even	Yes	No	Yes	No
$K^+ n \rightarrow K^0 p$	$\rho - A_2$	Odd - Even	Yes	No	Yes	No
$K^+ p \rightarrow K^0 \Delta^{++}$	$\rho - A_2$	Odd-Even	Yes	No	Yes	No
$\gamma p \rightarrow \eta p$	ρ	Odd	No	No	No	Yes
$\gamma p \rightarrow \pi^+ n$	ρ	Odd	No	No	No	Yes
$\gamma p \rightarrow \pi^- \Delta^{++}$	ρ	Odd	No	No	No	Yes
$\pi^- p \rightarrow \omega n$	ρ	Odd	No	No	No	Yes
$\pi^+ n \rightarrow \omega p$	ρ	Odd	No	No	No	Yes
$\pi^+ p \rightarrow \omega \Delta^{++}$	ρ	Odd	No	No?	No	Yes

and references cited therein)*.

From the table we know that there are several reactions dominated by $\tau = -1$ exchanges but which do not show $|t| \sim 0.6 \text{ (GeV/c)}^2$ dips and hence can not be explained by Class I models. On the other hand, there do not exist predominantly $\Delta\lambda = 1$ reactions which do not show $|t| \sim 0.6 \text{ (GeV/c)}^2$ dips as expected in Class II models.

3.3B Some Empirical Regularities in Elastic Data

We have, in the preceding pages, briefly delineated the "dip problem" in two-body inelastic reactions. We now collect together some empirical regularities in elastic scattering data in the $|t| \sim 1 \text{ (GeV/c)}^2$ region and if any of these have not been mentioned in Chapter I, we amplify them a little if necessary.

- i) Apparent constancy of total cross-sections over a wide energy range in exotic channels such as π^+p , K^+p , pp etc. (Chapter I, Table 1.2 and Fig. 1.1).
- ii) The structureless $\frac{d\sigma}{dt}$ in elastic exotic channels over a wide energy range starting at the lower end of the Regge energy regime (Cf. Chapter I, Table 1.2 and Fig. 1.5).

*The assumptions are:

- i) π exchange has been ignored in all these reactions where it is allowed since its influence, though strong near the forward direction, dies out by $|t| \sim 0.6 \text{ (GeV/c)}^2$.
- ii) ρ and A_2 are assumed to have dominantly flip NN and $N\bar{A}$ couplings, while ω is dominantly non-flip.
- iii) From SU(3) the following coupling relations have been assumed

$$\gamma_{\omega\pi\gamma} > \gamma_{\rho\pi\gamma} \quad \text{and} \quad \gamma_{\rho\eta\gamma} > \gamma_{\omega\eta\gamma}$$
- iv) The ρ, ω couplings to $\pi\gamma$ and πV are flip.

- iii) The disappearance of structure in non-exotic elastic differential cross-sections with increasing energy (Cf. Chapter I, Table 1.2, Fig. 1.5).
- iv) Cross-overs in elastic anti-particle-target and corresponding particle-target differential cross-section differences around $|t| \approx 0.2 \text{ (GeV/c)}^2$. (Cf. Chapter 1, Sec. 1, Fig. 1.5).
- v) The qualitative similarity in the appearance of the elastic polarisation sum $P_{K^+p}(s,t) + P_{K^-p}(s,t)$ and $P_{\pi^+p}(s,t) + P_{\pi^-p}(s,t)$ and the difference $P_{K^+p}(s,t) - P_{K^-p}(s,t)$ and $P_{\pi^+p}(s,t) - P_{\pi^-p}(s,t)$ as shown in Fig. 3.1. This was first noted by Harari (1970, 1971).

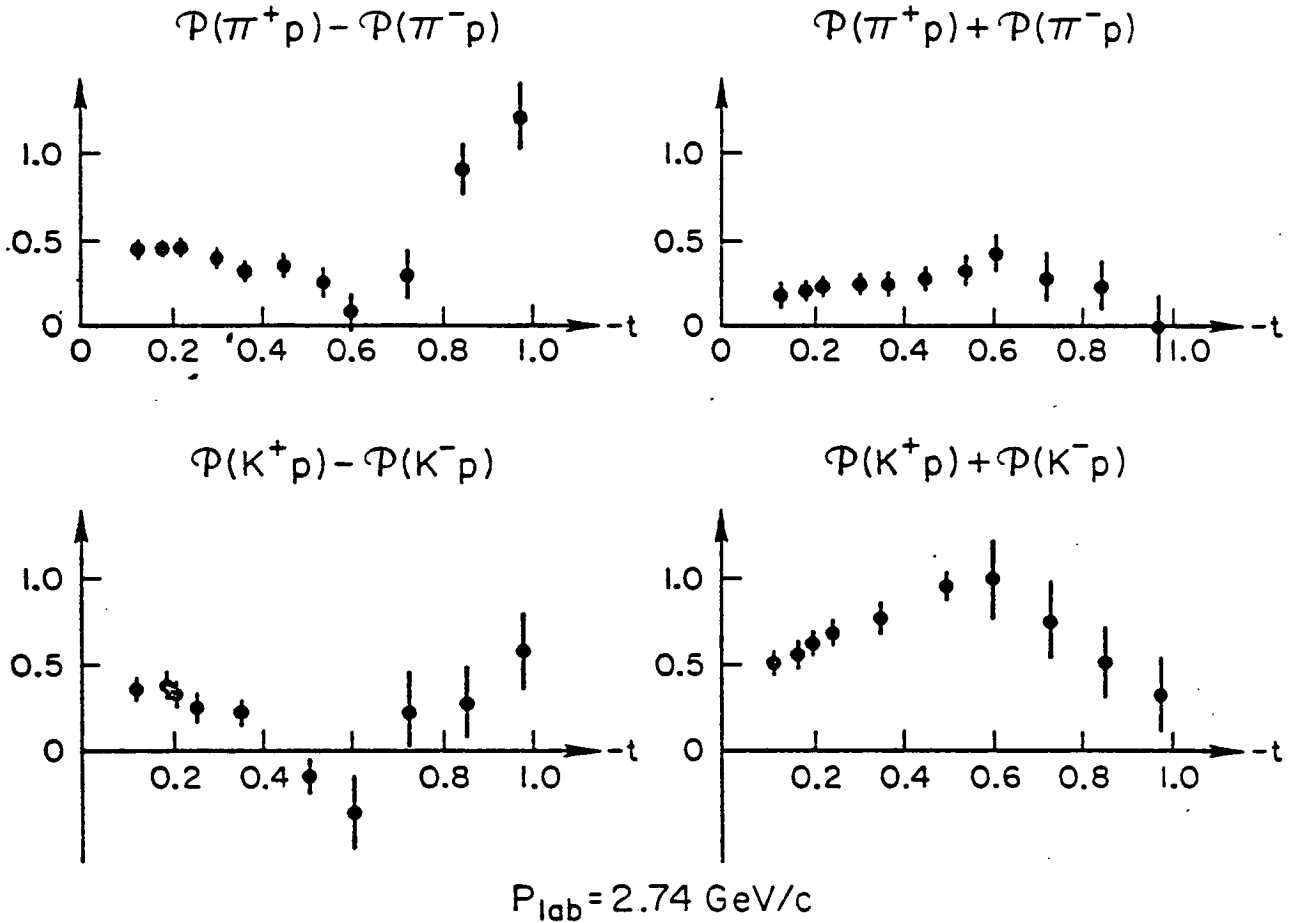


Fig. 3.1. The sum $P_{\pi^+p} + P_{\pi^-p}$ is qualitatively similar to the sum $P_{K^+p} + P_{K^-p}$ and the difference $P_{\pi^+p} - P_{\pi^-p}$ is qualitatively similar to $P_{K^+p} - P_{K^-p}$ (fig from Harari (1970a)).

vi) Peripherality of leading elastic resonances:

It was noted by Dolen et al. (1968) that the leading πN resonances occur in a "peripheral" band in impact-parameter space with the periphery having a radius (radius of interaction) of about 1 fm . By this one means that a leading πN resonance of spin J_i with a mass corresponding to a πN pair carrying a c.m. 3-momentum of magnitude q_i , has J_i and q_i related in a manner such that the impact-parameter b has a value b_i

$$b_i = \frac{J_i + \frac{1}{2}}{q_i} \sim 1 \text{ fm} \quad (3.1)$$

This is shown in Fig. 3.2, borrowed from Harari (1970). Since the scattering amplitude has a large value at those values of the variables which correspond to the formation of a resonance, one can conclude that a helicity amplitude in πN scattering will contain dominant contributions from partial waves which correspond to $b \sim 1 \text{ fm}$. Thus the profile of such an amplitude will be "peaked" in impact-parameter space around $b \sim 1 \text{ fm}$. As a consequence the amplitude will have a " $J_{\Delta\lambda}(b\sqrt{-t})$ " ($b \sim 1 \text{ fm}$) behaviour for an

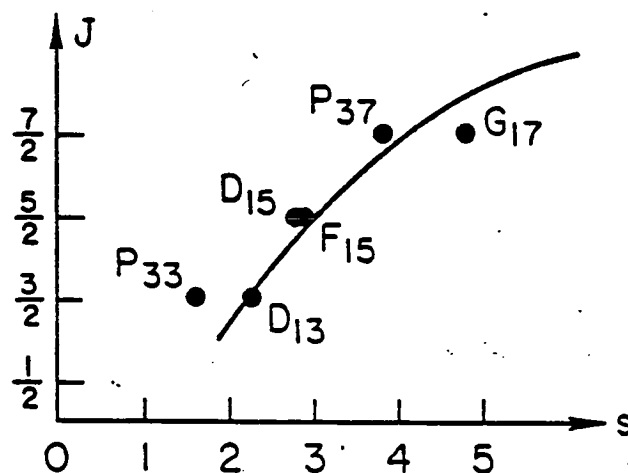
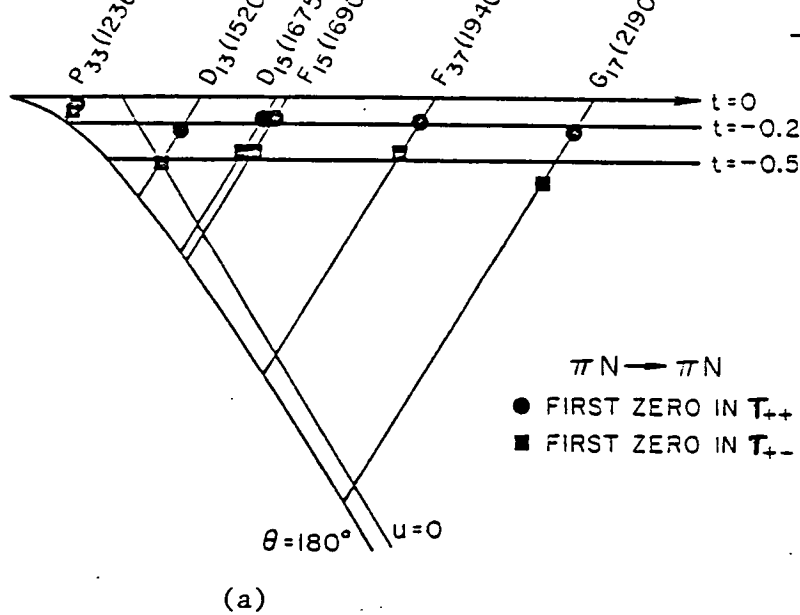
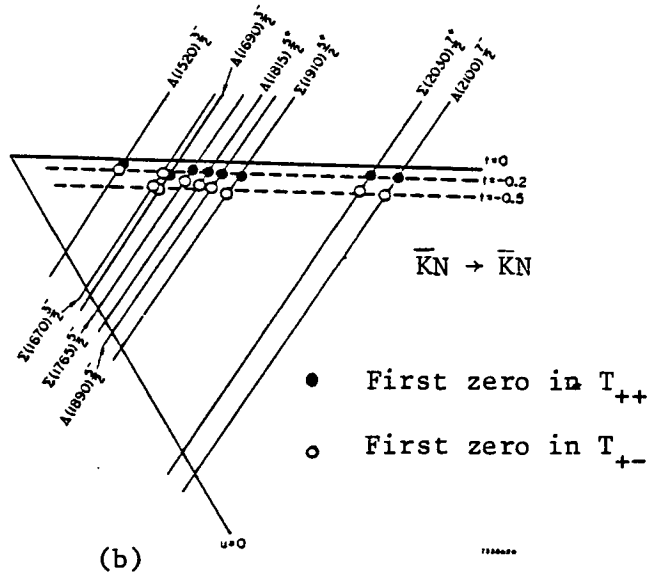


Fig. 3.2 . The spin-energy plot for prominent πN . elastic resonances, each resonance being represented as a dot. The curve represents the relation $J_i + \frac{1}{2} = q_i R$ with $R \sim 1 \text{ fm}$



(a)



(b)

Fig. 3.3a,b. The first zeroes in the contributions of prominent resonances to s-channel helicity amplitudes T_{++} and T_{+-} in πN and $\bar{K}N$ elastic scattering occur at fixed- t values ((a) from Harari (1970a), (b) from Davier (1974)).

s-channel helicity amplitude with s-channel helicity flip $\Delta\lambda$. This can be inferred by considering the partial-wave expansion of a helicity amplitude, making the small angle approximation $d_{\lambda_f \lambda_i}^J(z_c) \approx J_{\Delta\lambda}(b\sqrt{-t})$, ($\Delta\lambda \equiv |\lambda_f - \lambda_i|$), and then noting that the dominant contribution comes from $b \approx 1$ fermi on account of (3.1). Thus a $\Delta\lambda = 1$ amplitude will have a $|t| \approx 0.6 \text{ (GeV/c)}^2$ zero. For the πN case this was confirmed by Dolen et al. (1968)(Fig. 3.3a). A similar argument may be assumed to hold for

$\overline{K}N$ (Fig. 3.3b) and $\overline{N}N$ resonances. If then one uses the duality relation (2.45) whereby the imaginary parts of high-energy amplitudes are built up by these peripheral resonances, one arrives at the conclusion that these high energy amplitudes will also have peripheral structure. Going one step further and invoking two-component duality (relations 2.46a, b) yields the result that this peripherality is implied only for the imaginary parts of "ordinary" Regge exchanges in the amplitude but not for the Pomeron, which is dual to the non-resonating background. The status of peripherality as a general feature of resonances is still a subject of discussion (Cf. e.g. Sections 3 and 4B or Irving and Worden (1977)).

3.4 The Dual Absorptive Model

Having outlined the dip problem confronting the first generation of phenomenological models, and the various empirical features of elastic data, in particular the peripherality of various elastic resonances and its implications in a two-component dual framework, we can now state the "rules" of DAM as formulated by Harari (Harari, (1971)). After stating these "rules" we shall briefly comment on them.

- i) The imaginary part of a two-body hadronic amplitude is given by a sum of two parts - a Pomeron exchange term $P(s,t)$ which is dual to the s-channel background and an "ordinary" (i.e. non-Pomeron) exchange term $R(s,t)$ which is dual to s-channel resonances.
- ii) The P component approximately conserves s-channel helicity and it involves significant contributions from all partial waves within an impact-parameter radius of about 1 fm (also called radius of interaction). [This component is expected to have an $s^{\alpha_P(t)}$ type energy dependence.]
- iii) The R component is dominated by partial waves corresponding to the periphery of the interaction region in impact-parameter space, the

periphery having a radius ≈ 1 fm. [This component is expected to have $s^{\alpha_R(t)}$ behaviour, $\alpha_R(t)$ being an "ordinary" Regge trajectory. This behaviour also holds for the case when weak cut terms are present. However for strong cuts $\ln s$ terms as well as a modified "effective" $\alpha_R(t)$ function will appear.]*

iv) The real part of the P term is negligible.

v) The real part of the R term is unknown, although in principle a knowledge of the imaginary part might enable one to deduce it, using dispersion relations. {At infinite energies one might therefore expect it to have a form $g(t)J_{\Delta\lambda}(R\sqrt{-t}) \cot \frac{\pi\alpha}{2}$ or $g(t)J_{\Delta\lambda}(R\sqrt{-t}) \tan \frac{\pi\alpha}{2}$ depending on its crossing properties where $g(t)$ is a smooth function such as e^{at} . Empirically this form appears to be attained by $\Delta\lambda = 1$ (but not $\Delta\lambda = 0$) amplitudes at energies of a few GeV.}

In Table 3.2 borrowed from Harari (1970) we express these "rules" in a simple way. Here " J_0 " and " J_1 " denote functions possessing the

TABLE 3.2

Amplitude of net helicity flip $\Delta\lambda$	Non Pomeron Term		Pomeron Term	
	Im	Re	Im	Re
$\Delta\lambda = 0$	" J_0 "	?	e^{at}	0
$\Delta\lambda = 1$	" J_1 "	$\tan \frac{\pi\alpha}{2} "$ J_1 "	0	0
		$\cot \frac{\pi\alpha}{2} "$ J_1 "		

general characteristics (i.e. zeroes, maxima etc.) of the Bessel functions J_0 and J_1 respectively (e.g. $e^{bt}J_0$ or $e^{ct}J_1$) and are shown in Fig. 3.4 below.

* The treatment of poles and cuts on an equal footing in the DAM is a theoretically unsatisfactory feature of the model as pointed out in Collins and Fittner (1974)

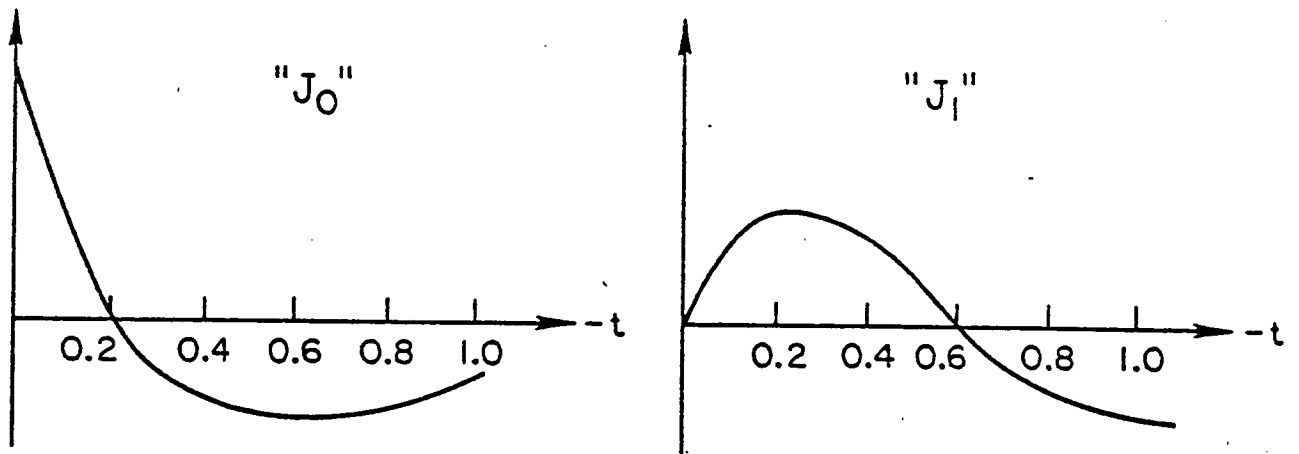


Fig. 3.4. The t -dependence of $T_{\Delta\lambda}$ is qualitatively given by $J_{\Delta\lambda}(R\sqrt{-t})$ with $R \approx 1$ fm. For $\Delta\lambda = 0, 1$ we have functions of the form " J_0 " and " J_1 " whose dips, bumps and zeroes are the same as those of the Bessel functions J_0 and J_1 (fig. from Harari (1970a)).

The basic feature of this model is a two-component picture of imaginary parts of two-body hadronic amplitudes coupled with empirically motivated assumptions on the impact-parameter structure of these components. As has been noticed by Schrempp and Schrempp (1973a), the $s^{\alpha(t)}$ behaviour is then inserted "by hand" in these components on empirical grounds. The reason for using the phrase "by hand" is that all one can infer from equation (2.46) and peripherality (or nonperipherality) is that one should have a " $J_{\Delta\lambda}(R\sqrt{-t})$ (or e^{at})" behaviour for amplitudes, where the different parameters could be energy dependent - it does not imply that one has an $s^{\alpha(t)}$ factor in it. It would perhaps be more rigorous if, instead of inserting an a priori energy dependence $s^{\alpha(t)}$, one decides this by fitting data over a wide energy range. Insertion of such an energy

dependence then implies an asymptotic expression for the real parts which is assumed to hold for $\Delta\lambda = 1$ amplitudes on empirical grounds but not for $\Delta\lambda = 0$. Thus all we have is a loose qualitative prescription for writing down amplitudes, which requires further empirical assumptions on dominance or non-dominance of various helicity amplitudes before it can be used to fit data and on the contribution to these amplitudes by various trajectories. Nonetheless by making some plausible assumptions of a general nature one can still make some qualitative predictions and confront them with data. A quantitative comparison can only be made after detailed data fitting and it is the purpose of this thesis to do so in the case of elastic scattering. In the following section we present a qualitative comparison of the model with notable features of the data as discussed in detail in Harari (1971).

3.5 Experimental Tests

3.5A $|t| \approx 0.6 \text{ (GeV/c)}^2$ Dips in Inelastic Reactions:

If $\Delta\lambda = 0, 2, 3$ etc. amplitudes dominate a two-body inelastic reaction, no dip is expected in the differential cross-section at $|t| \approx 0.6 \text{ (GeV/c)}^2$. If the $\Delta\lambda = 1$ amplitude dominates the reaction then the dip may or may not occur depending upon the crossing properties of the dominant exchange. If a crossing-odd amplitude dominates then a dip is expected, if a crossing-even amplitude dominates no dip is expected. Also, if a predominantly $\Delta\lambda = 1$ reaction contains comparable contributions from crossing-even and crossing-odd exchanges, no dip is expected. This state of affairs is summarised in the table below and can be seen to solve the dip puzzle qualitatively for the inelastic reactions mentioned in Table 3.1. It may be recalled that SCRAM predicts $|t| \approx 0.6 \text{ (GeV/c)}^2$ dips in all predominantly $\Delta\lambda = 1$ reactions, while

the weak cut model predicts $|t| \approx 0.6 \text{ (GeV/c)}^2$ dips in all reactions dominated exchanges which have a WSNZ at $|t| \approx 0.6 \text{ (GeV/c)}^2$. This is not the case with the DAM as can be seen from Table 3.3 below.

TABLE 3.3

Does $\Delta\lambda = 1$ amplitude domi- nate the Reaction?	Crossing Property of dominant ex- change	$\frac{d\sigma}{dt}$	Dip Predicted at $ t \approx 0.6$ (GeV/c)^2 ?
Yes	Odd	${}''J_1''^2 (1 + \tan^2 \frac{\pi\alpha}{2}) =$ $= {}''J_1''^2 / \cos^2 \frac{\pi\alpha}{2}$	Yes
Yes	Even	${}''J_1''^2 (1 + \cot^2 \frac{\pi\alpha}{2})$ $= \frac{{}''J_1''^2}{\sin^2 \frac{\pi\alpha}{2}}$	No
Yes	equal amounts of even and odd	$[{}''J_1'' (\cot \frac{\pi\alpha}{2}) \pm$ $\pm \tan \frac{\pi\alpha}{2}]^2$ $= \frac{4{}''J_1''^2}{\sin^2 \pi\alpha} \quad \text{OR}$ $\frac{4{}''J_1''^2 \cos^2 \pi\alpha}{\sin \pi^2 \alpha}$	No

3.5B Total Cross-sections and Elastic Differential Cross-Sections:

The "rules" of the DAM yield the following relations for the imaginary parts of the non-flip ($\Delta\lambda = 0$) and flip ($\Delta\lambda = 1$) elastic amplitudes

$$\text{Im } T_{\Delta\lambda=0} = {}''P'' + {}''J_0'' \quad (3.2a)$$

$$\text{Im } T_{\Delta\lambda=1} = {}''J_1'' \quad (3.2b)$$

If one makes the usual phenomenological assumption that the non-flip amplitude dominates elastic scattering at high energies and that $\text{Re } T_{\Delta\lambda=0} = 0$ at these energies one can write

$$\frac{d\sigma}{dt} \sim "P" + 2"P" "J_0" \quad (3.3)$$

In equation (3.3) the $"J_0"$ term has been neglected since it differs from $"P"$ by a factor of order s and from $"P" "J_0"$ by a factor of order $\sqrt{s}$. One can then draw the following inferences from equations (3.2a, b, 3.3) (Cf. Harari (1971)).

(i) In exotic channels the $"J_0"$ (and of course $"J_1"$) term will be absent if one assumes that exotic resonances do not exist. In the framework of the DAM the non-Pomeron terms are dual to s-channel resonances and if the s-channel has exotic quantum numbers, the contribution of these terms must vanish. This implies strong EXD of various Regge-exchanges with respect to exotic channels and yields

$$\text{Im } T_{\Delta\lambda=0} = "P" \quad (3.4a)$$

$$\frac{d\sigma}{dt} \sim "P" \quad (3.4b)$$

for such channels. Equation (3.4a) implies, in agreement with experimental data, a constant total cross-section for exotic channels (K^+p , K^-n , pp etc.) if one assumes a unit intercept for the Pomeron. Equation (3.4b) predicts a structureless differential cross-section for all exotic reactions in the $|t| \lesssim 1 \text{ (GeV/c)}^2$ region throughout the Regge energy regime. This is in agreement with experimental data (Cf. Table 1.2 of Chapter 1).

(ii) For non-exotic elastic channels the term $2"P"J_0"$ will introduce structure in differential cross-sections which will, however, fade out as one goes to high enough energies where $"P" \gg 2"P" "J_0"$. Let us examine the nature of this structure for $|t| < 1 \text{ (GeV/c)}^2$. The Bessel

function $J_0(R\sqrt{-t})$ ($R \sim 1$ fm) has a maximum (positive) value at $|t| = 0$, a zero at $|t| \sim 0.2$ (GeV/c)² and a minimum (negative) value at $|t| \sim 0.6$ (GeV/c)². Since " J_0 " corresponds to a sum of s-channel resonances it must be positive at $t = 0$; the above features of the Bessel function J_0 will therefore be reproduced by " J_0 ", without a sign change (Cf. Fig. 3.4). We can thus infer that

a) Separating the $C = +1$ and $C = -1$ terms in " J_0 " and writing it as

$$"J_0" = \sum_{C=+1} "J_0" + \sum_{C=-1} "J_0" \quad (3.5)$$

one finds that

$$\Delta(xp) \equiv \frac{d\sigma}{dt}(\bar{xp}) - \frac{d\sigma}{dt}(xp) = 4"P" \sum_{C=-1} "J_0" \quad (3.6)$$

since $C = -1$ exchanges contribute with opposite signs to the two reactions. Since " P " is structureless $\Delta(xp)$ is predicted to have a " J_0 " type behaviour. This prediction has been verified by several authors (e.g. Davier and Harari (1971), Ambats et al. (1974)) for various elastic channels. Davier and Harari (1971) found that (Fig. 3.5a) $\frac{\Delta(Kp)}{2\sqrt{K^+p}} = Ae^{Bt} J_0(R\sqrt{-t})$ with $A = 1.6 \text{ mb}^{\frac{1}{2}} \text{ GeV}^{-1}$, $B = 1.3 \text{ GeV}^{-2}$, $R = 4.8 \text{ GeV}^{-1} \sim 0.95 \text{ fm}$ (they used $\frac{d\sigma}{dt}(K^+p) = P^2$ or $P = \sqrt{\frac{d\sigma}{dt}}$). The projection, in impact parameter space, of $Ae^{Bt} J_0(R\sqrt{-t})$, showed a strong peripheral form sharply peaked around $b \sim 0.9 \text{ fm}$ (Fig. 3.5b) As a corollary to equation (3.6) one predicts the cross-over points to lie around $|t| \sim 0.2$ (GeV/c)² where " J_0 ", and hence $\Delta(xp)$, vanishes. This agrees with experimental data.

b) At $|t| \sim 0.6$ (GeV/c)² where " J_0 " has a (negative) minimum " $P^2 + 2"P" "J_0"$ will have a smaller value compared to its value at other $|t|$ values in its vicinity. One thus expects a dip in non-exotic elastic differential cross-sections at $|t| \sim 0.6$ (GeV/c)² at lower energies in the Regge region where the $2"P" "J_0"$ term is non-

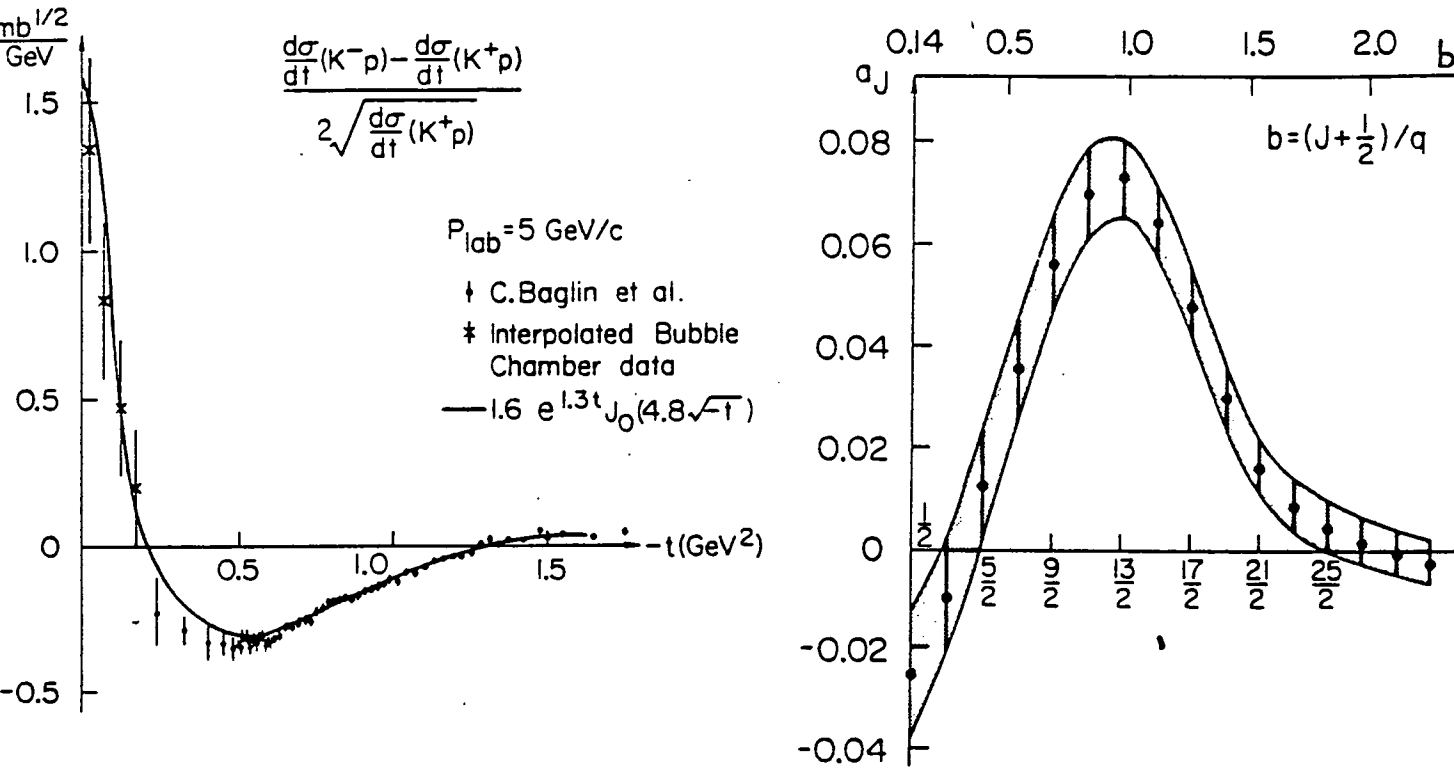


Fig. 3.5a,b: a) $C = -1$ non-flip amplitude in $K^\pm p$ elastic scattering at 5 GeV/c
b) Impact-parameter distribution of the amplitude shown in Fig. (5.6a) (fig. from Davier (1974)).

negligible. This expectation is in agreement with experimental data (Table 1. Chapter 1).

c) At sufficiently high energies where $s \gg 2m^2$ all non-exotic elastic differential cross-sections are predicted to become structureless for $|t| \lesssim 1$ (GeV/c)² in agreement with data (Table 1.2 Chapter 1).

3.5C Elastic Polarisation: (Harari (1970, 1971c, d))

The assumption that $\text{Re } T_{\Delta\lambda=0} \sim 0$ for elastic reactions implies that elastic polarisation are given by

$$P \frac{d\sigma}{dt} \sim \text{Re } T_{\Delta\lambda=1} \times \text{Im } T_{\Delta\lambda=0} \quad (3.7)$$

where $\text{Im } T_{\Delta\lambda=0} = "P" + "J_0"$ as given by equation (3.2a). Since $\text{Im } T_{\Delta\lambda=0}$ is dominated by "P" and since the DAM requires "P" to be structureless one can conclude that $\text{Im } T_{\Delta\lambda=0}$ will be structureless too. This observation when combined with equation (3.7) enables us to infer that the structure of polarisation is essentially the same as that of $\text{Re } T_{\Delta\lambda=1}$. The DAM gives definite predictions about $\text{Re } T_{\Delta\lambda=1}$ as discussed previously and shown in Table 3.3. Thus if $\tau = -1$ exchange dominates a $T_{\Delta\lambda=1}$ amplitude one expects

$$\text{Re } T_{\Delta\lambda} = \tan \frac{\pi\alpha}{2} "J_1" \quad (3.8)$$

Such a term will show a double zero (or two closely spaced zeroes) around $|t| \approx 0.6 \text{ (GeV/c)}^2$. For the case when $\tau = +1$ dominates the flip amplitude the DAM yields

$$\text{Re } T_{\Delta\lambda=1} = \cot \frac{\pi\alpha}{2} "J_1" \quad (3.9)$$

$\cot \frac{\pi\alpha}{2}$ has a pole at $\alpha = 0$ ($|t| \approx 0.6 \text{ (GeV/c)}^2$) which can be cancelled if one chooses the radius of interaction in " J_1 " in such a way that " J_1 " has a zero at $|t| \approx 0.6 \text{ (GeV/c)}^2$. Alternatively a logarithmic term in the energy dependence of $\text{Im } T_{\Delta\lambda=1}$ can give rise to a phase which is finite at $|t| \approx 0.6 \text{ (GeV/c)}^2$.

If $\text{Re } T_{\Delta\lambda=1}$ contains contributions from both $\tau = \pm 1$ (with $C = \pm 1$) exchanges then the $\tau = -1$ term will not be present in the sums of particle-target and corresponding anti-particle-target $P \frac{d\sigma}{dt}$ while their differences will not have $\tau = +1$ terms. Thus one could expect

$$(P \frac{d\sigma}{dt})_+ + (P \frac{d\sigma}{dt})_- \sim \cot \frac{\pi\alpha}{2} "J_1" \text{Im } T_{\Delta\lambda=0} \quad (3.10a)$$

$$(P \frac{d\sigma}{dt})_+ - (P \frac{d\sigma}{dt})_- \sim \tan \frac{\pi\alpha}{2} "J_1" \text{Im } T_{\Delta\lambda=0} \quad (3.10b)$$

where $(P \frac{d\sigma}{dt})_{\pm}$ refer to $\pi^{\pm}p$ or $K^{\pm}p$. The sum thus has no zeroes around $|t| \approx 0.6 \text{ (GeV/c)}^2$ while the difference has a double zero around $|t| \approx 0.6 \text{ (GeV/c)}^2$ as can be seen clearly in Fig. 3.1 [In Fig. 3.1 $P_{+} \pm P_{-}$ have been shown instead of $(P \frac{d\sigma}{dt})_{+} \pm (P \frac{d\sigma}{dt})_{-}$ but this does not affect the qualitative features discussed above.] We thus have a qualitative explanation of the similar behaviour of $(P_{\pi^{+}p}^{+} \pm P_{\pi^{-}p}^{-})$ and $(P_{K^{+}p}^{+} \pm P_{K^{-}p}^{-})$.

CHAPTER 4

THE DUAL ABSORPTIVE MODEL AND $\pi^\pm p$ ELASTIC SCATTERING UP TO 280 GeV/c

4.1 Introduction and Motivation

The πN system, because of the central and historic importance of the πN interaction, is one of the most basic systems in strong interaction physics (c.f. e.g. Cence (1969), Bransden and Moorhouse (1973)). Added to this is its great theoretical simplicity - it is the simplest strongly interacting system involving spin. The pion, which is the lightest hadron, is nonstrange and spinless, while the nucleon, which is the lightest baryon, has zero strangeness, and spin one half. The importance of the πN system is greatly enhanced by the fact that the copious availability of pion projectiles and proton targets has enabled experimentalists to amass an enormous amount of data on high-energy πN scattering over the past two decades. This data has sometimes led to the discovery of entirely new features of strong interactions and at others, provided a field of applicability of theoretical models. Thus for instance, resonances, now a fact of life in hadronic physics, were first observed in πN scattering. Similarly the first application of the Argonne model was carried out on $\pi^\pm p$ elastic, and $\pi^\mp p$ charge exchange data (Arnold and Blackman (1968)). When Frautschi and Margolis (1968) introduced their Reggeised Eikonal Model, $\pi^\pm p$ elastic scattering was among the first set of reactions to which it was applied. Although SCRAM is, in the view of its authors (Heyney et al. (1968)), inapplicable to elastic scattering on account of the unclear status of the Pomeron in this model, Phillips and Ringland (1971) employed a phenomenological Pomeron in conjunction with SCRAM non-Pomeron Regge exchanges to $\pi^\pm p$ elastic scattering in an effort to determine whether the data favoured either SCRAM or Argonne. They found that the data favoured Argonne. Earlier, on $\pi^\pm p$ elastic data was a favourite ground for the application of Regge pole and other

models. Some of the more well known Regge pole fits to the $\pi^\pm p$ elastic data are those of Phillips and Rarita (1965a,b), Chiu et al. (1967) and Rarita et al. (1968), Cohen-Tannoudji et al. (1967) also applied their phenomenological model to $\pi^\pm p$ elastic data. Chiu and Finkelstein (1969) applied their hybrid model (Chiu and Finkelstein (1968)) to several reactions, including backward $\pi^+ p$ elastic data. From the aforementioned it is obvious that $\pi^\pm p$ elastic data is a field to which most phenomenological models are applied. The same should therefore be the case with DAM. The first application of the DAM to $\pi^\pm p$ elastic scattering was carried out by Davier (1972). He considered sums of $\pi^\pm p$ differential and total cross-sections in the range $2.5 \lesssim p_{lab} \lesssim 17 \text{ GeV/c}$ and $|t| \lesssim 0.9 (\text{GeV/c})^2$. We shall discuss the quantitative aspects of his fits at the end of this chapter. Minami (1972) carried out a study of the properties of the $\pi^\pm p$ and pp slope parameters $(b \equiv \frac{d}{dt} \ln \frac{d\sigma}{dt} \Big|_{t=0})$ employing the DAM to separate Pomeron exchange from other Regge exchanges. Both Davier (1972) and Minami (1972) neglected the helicity flip amplitude and took the nonflip amplitude to be imaginary in the region considered. Minami and Terada (1974) refined the analysis of Minami (1972) by considering a parameterisation of σ_{tot} over a wide energy range and incorporating it into the expression for the slope parameter through the optical theorem. They took an energy independent radius of interaction for the non-Pomeron Regge term, included a term of the type $(D \ln^2(s/s_0))$ in the total cross-section fit, and in addition to $\pi^\pm p$ scattering considered $K^\pm p$ data. Savoy-Navarro (1974) applied the DAM to 5 and 10 GeV/c small $|t|$ $\pi^\pm p$ (and $K^\pm p$) elastic differential cross-section sums on the same lines as Davier (1972) and found the model to be in satisfactory agreement with the data. Earlier Barger and Halzen (1972) carried out a systematic investigation of the successes and failures of the DAM in regard to elastic scattering, including $\pi^\pm p \rightarrow \pi^\pm p$ data. They confined themselves to the then available range

2.5 $\lesssim p_{\text{lab}} \lesssim 16 \text{ GeV/c}$ for near-forward $\pi^\pm p$ differential cross-section sums. They also considered combinations of $K^\pm p$, (and $p^\pm p$) polarisations and differential cross-sections but did not consider $\pi^\pm p$ polarisations. Their conclusions, though based on a unified treatment of the then available elastic data, are marred by the fact that they assume an energy independent Pomeron term (i.e. corresponding to a flat Pomeron of intercept unity) which is in contradiction with the current state of knowledge about the Pomeron pole trajectory (c.f. e.g. Irving and Worden (1977), Collins (1977), Collins et al. (1974a,b), Lasinski et al. (1972)). This flaw in their analysis was first pointed out by Davier (1972). Gordon et al. (1972) applied the DAM to various elastic differential cross-sections at 8 and 16 GeV/c and found the non Pomeron Regge exchanges to have a " $J_0(R\sqrt{-t})$ " behaviour in the nonflip amplitude in the region $|t| \lesssim 0.8 (\text{GeV/c})^2$, in agreement with the model. The model was extended to the backward direction by Aye (1972) who showed it to be consistent with the qualitative features of backward πN data. Gonczi (1973) then attempted a quantitative fit to the 6 GeV/c near-backward $\pi^\pm p$ elastic polarisation, using the DAM, and found that, with a slight modification a fit could be obtained. Saleem et al. (1976) have applied this model successfully to the 8 and 16 GeV/c near-backward differential cross-sections for $\pi^\pm p \rightarrow \pi^\pm p$.

In the light of the aforementioned applications of the DAM to $\pi^\pm p$ elastic data, two general inferences may be drawn.

- i) In fitting near-forward data, polarisations have been ignored completely by all authors from a quantitative angle.
- ii) Although the emphasis has been on fitting near-forward differential cross-sections the energy range in which these are considered is limited (up to $p_{\text{lab}} \lesssim 17 \text{ GeV/c}$) and hence one cannot draw any safe inference about the energy dependence of the various parameters.

Over the past few years measurements of $\pi^\pm p$ elastic data at energies extending up to the FNAL range have become available. The total cross-section measurements extend up to 280 GeV/c, while differential cross-section measurements up to 200 GeV/c and polarisations up to 100 GeV/c in the small $|t|$ region are now available. These measurements coupled with the previous neglect of $\pi^\pm p$ polarisations necessitate the application of the DAM to $\pi^\pm p$ elastic scattering over the entire high-energy range at which data is available. This would hopefully enable us to assess the status of the DAM in a better way and also permit us to probe the energy dependence of the various parameters involved. In the following, therefore, we discuss $\pi^\pm p$ elastic data in the range $6.8 \lesssim p_{\text{lab}} \lesssim 280$ GeV/c, $|t| \lesssim 1$ (GeV/c)², in the light of this model. We discuss our parameterisation and fits below. However, contrary to usual practice in applying the DAM to data, we do not consider sums or differences of corresponding differential cross-sections, polarisations, etc., but instead concentrate on individual quantities. The reason for this is that the evaluation of combinations of quantities pertaining to two different (though related) reactions should be done when the quantities have been measured with the same apparatus, not only at the same beam momentum but also with the same t values for both particle and antiparticle data. These conditions are generally not met by existing data.

4.2 Assumptions and Parameterisation:

The "rules" of the DAM have already been stated in Chapter 3. We therefore parameterise our data in the light of these "rules" and some standard phenomenological assumptions. We recall that the trajectories exchanged in $\pi^\pm p$ elastic scattering are the P , f and ρ . The specific assumptions invoked and the procedure adopted in parameterising

and fitting data is discussed below.

i) The Pomeron contributes only to the helicity non-flip amplitude ($\Delta\lambda = 0$), is imaginary, and receives substantial contributions from all partial waves within the radius of interaction. We therefore write the Pomeron amplitude as

$$P(s,t) = i A_P e^{B_P t} (s)^{\alpha_P(0)} \quad (4.1)$$

$\alpha_P(0)$ being the Pomeron intercept. We find that πp total cross-section data requires $\alpha_P(0) = 1.06$, which is consistent with the results of Collins et al. (1974b) but somewhat smaller than that of Collins et al. (1974a) and of Joynson et al. (1977).

ii) Following Davier (1972) we regard f -exchange as imaginary and s -channel helicity conserving. The DAM requires this term to be peripheral. It is also well known that there is a small ρ contribution to the non-flip amplitude, as indicated by the small non-zero difference between $\pi^- p$ and $\pi^+ p$ total cross-sections. This contribution too is peripheral according to the "rules" of the DAM. The analysis of Gordon et al. (1972) (at 8 and 16 GeV/c) however indicates that it is not very fruitful to try to separate the ρ non-flip contribution from the f on account of its small value. For economy of parameterisation therefore, we lump the f and ρ together in the imaginary part of the non-flip amplitude. Assuming that the ρ and f trajectories have equal intercepts we can write the "R" term of the DAM in the non-flip amplitude as

$$R = i A_R e^{B_R t} (s)^{\alpha_R(0)} J_0(R_R \sqrt{-t}) \quad (4.2)$$

Since R has a small ρ component we expect A_R to have a slightly larger magnitude in the $\pi^- p$ case compared to the $\pi^+ p$ case, as the ρ coupling has opposite signs in the two cases, being positive for $\pi^- p$.

As will be shown below our fits to total cross-section data yield

$\alpha_R(0) = 0.5$, which is in accord with the standard intercept value for the EXD ρ -f-w- A_2 trajectory (c.f. e.g. Irving and Worden (1977)).

iii) In keeping with the standard phenomenological practice we assume that the real part of the non-flip amplitude can be neglected in fitting data.

iv) We assume that the flip amplitude receives contributions only from the ρ -exchange. Since the DAM "rules" specify the flip phase for $\tau = -1$ exchange as $\tan^{\pi\alpha}/2$ and since this in turn depends on an $s^{\alpha(t)}$ behaviour of the imaginary part we assume that the energy dependence of the flip amplitude is given by $s^{\alpha_\rho(t)}$. Since the DAM requires ρ exchange to be peripheral, like any other non-Pomeron Regge exchange, we have:

$$\text{Im } T_{+-} = A_\rho e^{B_\rho t} J_1(R_\rho \sqrt{-t}) s^{\alpha_\rho(t)} \quad (4.3a)$$

$$\text{Re } T_{+-} = \tan \frac{\pi\alpha_\rho}{2} \text{Im } T_{+-} \quad (4.3b)$$

We take $\alpha_\rho(t) = 0.5 + 0.9t$ which is the standard expression for the EXD ρ -f-w- A_2 trajectory (e.g. Irving and Worden (1977)).

v) We assume, as is usual, that the flip amplitude can be neglected in fitting differential cross-sections (i.e. $|T_{+-}|^2 \ll |T_{++}|^2$). This practice, for instance, has been followed in most papers mentioned in the previous section.

In the light of the above statements we arrive at the following expressions:

$$T_{++}(s,t) = i[A_P e^{B_P t} s^{1.06} + A_R e^{B_R t} \sqrt{s} J_0(R_R \sqrt{-t})] \quad (4.4)$$

$$T_{+-}(s,t) = [\tan \frac{\pi\alpha_\rho}{2} + i] A_\rho e^{B_\rho t} J_1(R_\rho \sqrt{-t}) s^{\alpha_\rho(t)} \quad (4.5)$$

$$\sigma_{\text{tot}}(s) = \frac{0.389}{2q \sqrt{s}} [A_P s^{1.06} + A_f \sqrt{s}] \quad (4.6)$$

$$\frac{d\sigma}{dt} \approx \frac{0.389}{64\pi s q^2} |T_{++}|^2 \quad (4.7)$$

$$P(s,t) \approx \frac{2\text{Re } T_{+-} - \text{Im } T_{++}}{|T_{++}|^2 + |T_{+-}|^2} \quad (4.8)$$

The factor 0.389 gives σ_{tot} directly in mb and $\frac{d\sigma}{dt}$ in $\frac{\text{mb}}{(\text{GeV}/c)^2}$. We recall that the ρ contributes with opposite signs to the two reactions which will automatically be reflected in the opposite signs of the polarisation P for these reactions. We thus have, for each reaction, a total of 8 parameters of which A_P and A_R are obtained by fitting σ_{tot} according to equation (4.6) and are expected to be energy independent. The values of A_P and A_R thus obtained are utilised in obtaining values for B_P , B_R and R_R at each energy by fitting $\frac{d\sigma}{dt}$ at that energy according to equation (4.7). The remaining parameters A_P , B_ρ and R_ρ are then determined by fitting polarisations according to equation (4.8). If the energy dependence of the flip amplitude is given entirely by $s^{\alpha_\rho(t)}$, as is generally believed (c.f. e.g. Gaidot et al (1975)), then one might expect A_ρ , B_ρ and R_ρ to be energy independent as well. We aim at obtaining good χ^2 values rather than visually good fits to data (visually good fits on a logarithmic scale for $\frac{d\sigma}{dt}$ can be quite deceptive as far as χ^2 values go) and this occasionally entails slight modifications in the values of energy independent parameters. Thus for instance we find that the A_P , A_R values obtained from fitting σ_{tot} have occasionally to be modified in fitting $\frac{d\sigma}{dt}$ if good χ^2 values are to be ensured. These modifications seem to be required mostly in fits to π^+p data. Finally we see whether we can discover any simple energy dependence for parameters that vary

with energy. We, however, point out that there are broad constraints on the various parameters. In particular the radius of interaction is expected to have values around 5 GeV^{-1} (1 fm approximately) in the intermediate energy region.

4.3 Fits and Discussion

We take up the two systems (π^+p and π^-p) separately.

4.3A Fits to π^+p data:

i) Total Cross-Sections: The fit to $\sigma_{\text{tot}}(\pi^+p)$ in accordance with equation (4.6) is shown in Fig. 4.1a. The laboratory momentum range covered is $7.73 \leq p_{\text{lab}} \leq 240 \text{ GeV}/c$ and the data is taken from Carroll et al. (1976) and Foley et al. (1963). The $\chi^2/\text{degree of freedom}$ is $\frac{32.14}{18} = 1.78$. {The numerical values of the parameters are shown in Table 4.1. The fit can be improved slightly to $\frac{25.56}{18} = 1.42$ if we let $\alpha_p(0) = 1.067$. This value however worsens the fit for $\sigma_{\text{tot}}(\pi^-p)$ so that we retain $\alpha_p = 1.06$ }. Fits are shown in Figs. 4.1a, b.

TABLE 4.1

A_P and A_R values from fits to $\sigma_{\text{tot}}(\pi^\pm p)$ using equation (4.6)*

Quantity	A_P	A_R	χ^2/n^{**}
$\sigma_{\text{tot}}(\pi^+p)$	$40.72 \pm 0.063\%$	54.524 ± 0.439	$\frac{32.14}{18} = 1.78$
$\sigma_{\text{tot}}(\pi^-p)$	41.03 ± 0.060	74.413 ± 0.520	$\frac{64.57}{26} = 2.48$

* Although, in the above table, we have quoted numerical values for various parameters up to 3 decimal places, they must not, in general, be taken seriously beyond the first or second decimal places. The errors on parameter values correspond to a change of 1.0 in the χ^2 value obtained by using mean values of these parameters. The quoted χ^2 values correspond to the mean values of the various parameters. It will be evident from the errors on the parameter values as to after what number of decimal places should a particular parameter value be rounded off. The above remarks hold for all tables of numerical values in this work which are based on our calculations.

** n denotes number of data points.

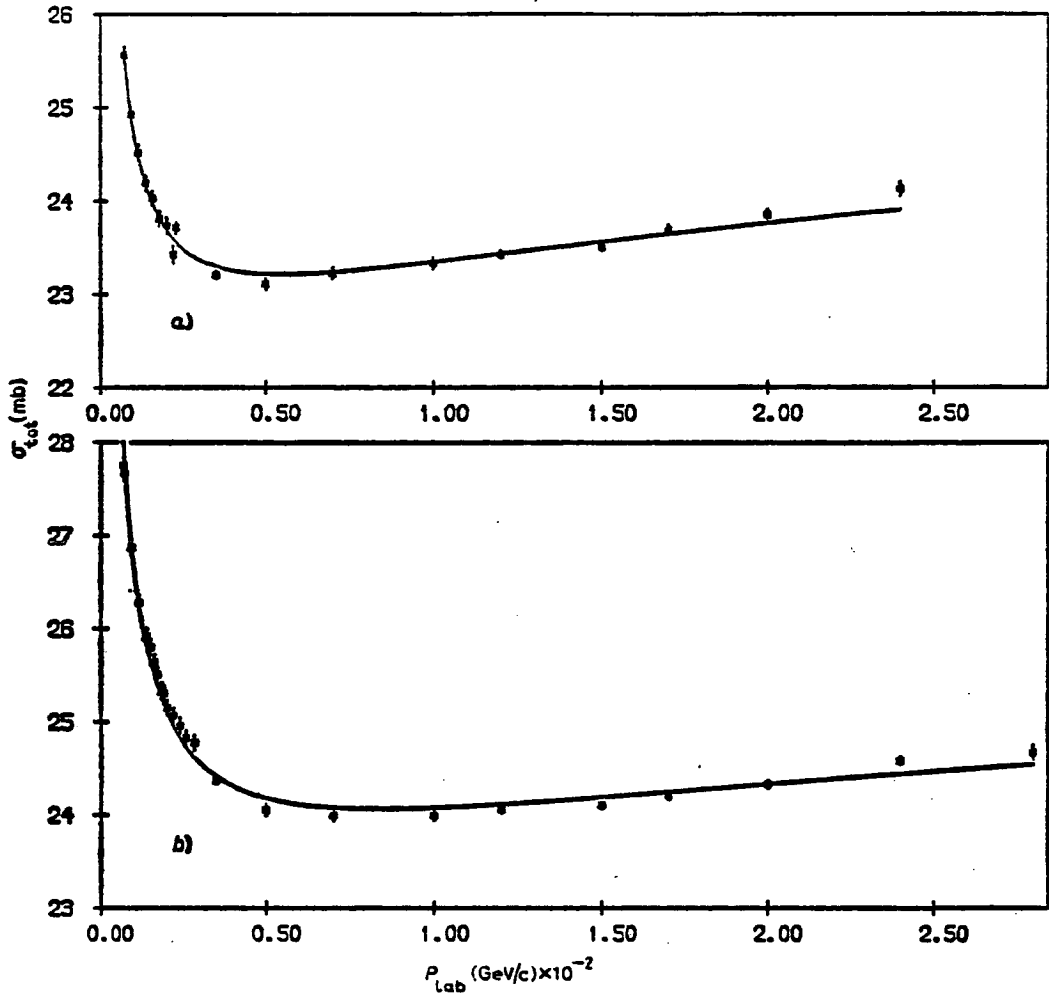


Fig.4-1 - Fits to $\sigma_{tot}(\pi^{\pm}p)$ (a) π^+p , (b) π^-p . The curves represent the theoretical fits.

ii) Differential Cross-Sections: Using the A_p and A_f values obtained above, π^+p elastic differential cross-sections are fitted in the range $6.8 \lesssim p_{lab} \lesssim 200 \text{ GeV}/c$, $|t| \lesssim 1.2 (\text{GeV}/c)^2$ in accordance with equation (4.7). Numerical results are given in Table 4.2 and some typical fits are shown in Fig. 4.2a-e. Although the χ^2 values are good (overall $\chi^2/n = \frac{407.01}{355} = 1.145$) we have had to allow for several

TABLE 4.2

Parameter Values for Fits to π^+p Elastic Differential Cross Sections

P_{Lab} (GeV/c)	A_P	A_R	B_P (GeV^{-2})	B_R (GeV^{-2})	R_R (GeV^{-1})	χ^2/n *
6.8	40.72	54.524	$2.249 \pm .093$	$0.398 \pm .285$	5.057 ± 0.98	22.2/14
8.8	48.594	44.141	$3.124 \pm .095$	$1.631 \pm .508$	$5.7888 \pm .271$	3.8/14
10.8	50.375	38.825	$3.471 \pm .068$	2.800 ± 1.26	$6.351 \pm .176$	8.22/14
12.8	40.72	54.524	$2.693 \pm .065$	$0.651 \pm .284$	$5.246 \pm .131$	23.4/14
14.8	40.72	54.524	$3.014 \pm .01$	$1.929 \pm .266$	$4.783 \pm .173$	21.98/13
16.7	40.72	54.524	$3.03 \pm .031$	$2.045 \pm .11$	$5.539 \pm .362$	16.01/13
44.5	40.72	72.524	$3.725 \pm .033$	$2.096 \pm .521$	$6.467 \pm .206$	26.79/16
50.0 a)	40.72	54.524	$3.441 \pm .014$	$1.78 \pm .239$	$6.166 \pm .119$	42.39/64
50.0 b)	41.379	44.203	$3.816 \pm .063$	$1.343 \pm .06$	$8.05 \pm .551$	15.23/19
70.0	40.72	54.524	$3.699 \pm .018$	9.006 ± 2.45	$4.93 \pm .053$	26.4/17
100.0 a)	40.72	54.524	$3.754 \pm .013$	$.797 \pm .279$	$7.053 \pm .084$	62.9/54
100.0 b)	41.112	49.134	$3.767 \pm .123$	$1.546 \pm .652$	$7.621 \pm .484$	14.4/19
140.0	39.839	85.506	$3.633 \pm .104$	$1.406 \pm .047$	$6.845 \pm .358$	11.01/19
175.0	40.72	61.549	$3.838 \pm .029$	$0.862 \pm .686$	$7.823 \pm .195$	12.32/19
200.0	37.752	55.904	$3.915 \pm .045$	2.289 ± 1.01	$17.56 \pm .543$	99.96/46

$$\text{Overall } \chi^2/n = \frac{407.01}{355} = 1.146$$

* n denotes number of data points.

a) Data from Akerloff et al. (1976).

b) Data from Ayres et al. (1977).

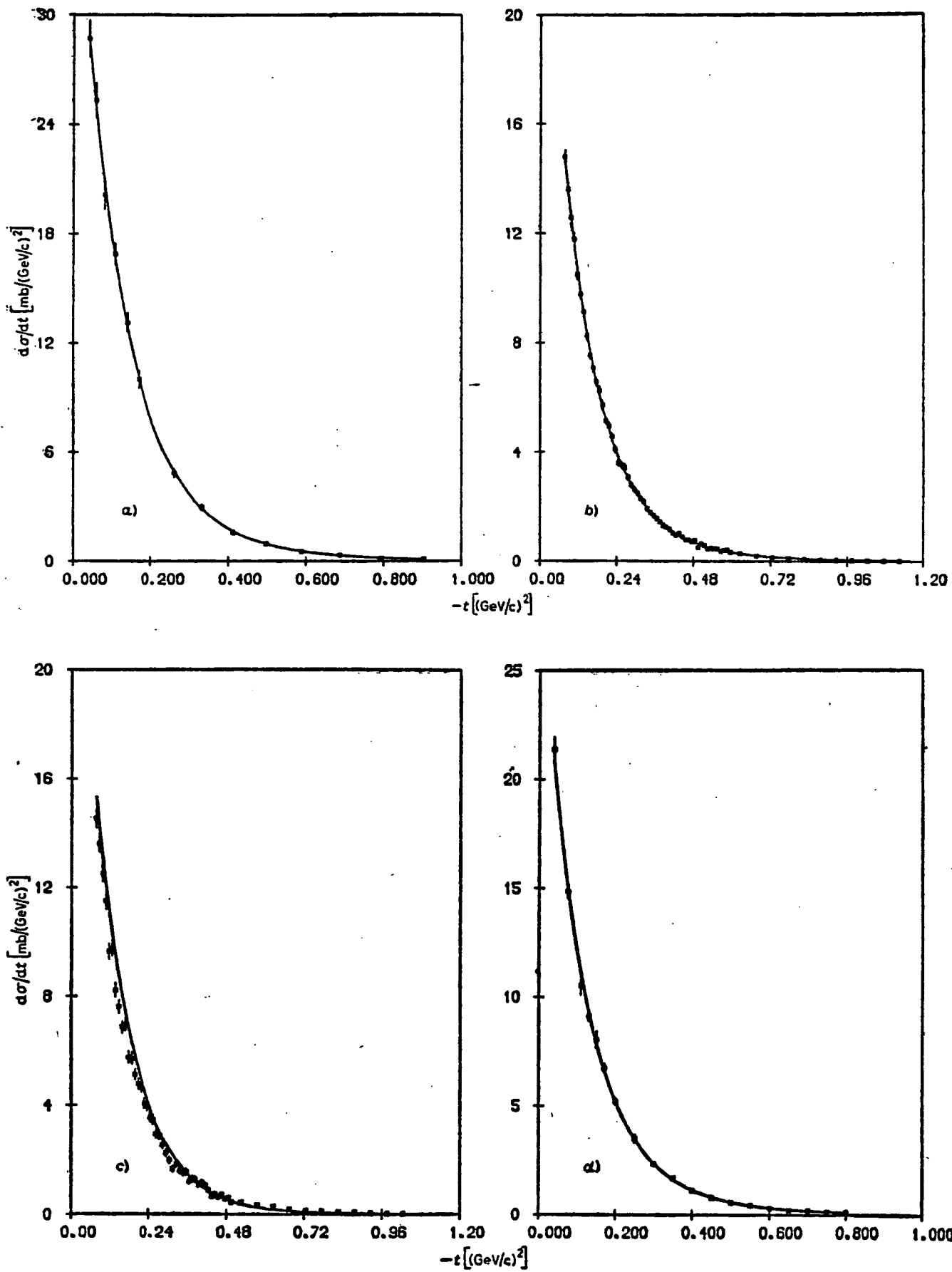


Fig.4.2 - Fits to $(d\sigma/dt)$ ($\pi^+p \rightarrow \pi^+p$) at a) $8.8 \text{ GeV}/c$, b) $50.0 \text{ GeV}/c$, c) $100.0 \text{ GeV}/c$, d) $140.0 \text{ GeV}/c$.

alterations in the values of A_P and A_R as determined by fitting $\sigma_{\text{tot}}(\pi^+p)$. This, in part, seems to have been caused by the fact that data from several different experiments was used (Akerloff et al. (1976), Ayres et al. (1977), Bruneton et al. (1977), Harting et al. (1965), Foley et al. (1963)) but it is nonetheless not very satisfactory from an aesthetic angle. The parameters B_P and R_R show a small overall increase with energy when considered over the entire laboratory momentum range, whereas B_R does not seem to vary in any definite manner, which is rather disappointing. We have tried to fit the parameters B_P and R_R with several alternative forms shown in Table 4.3 (see next page)

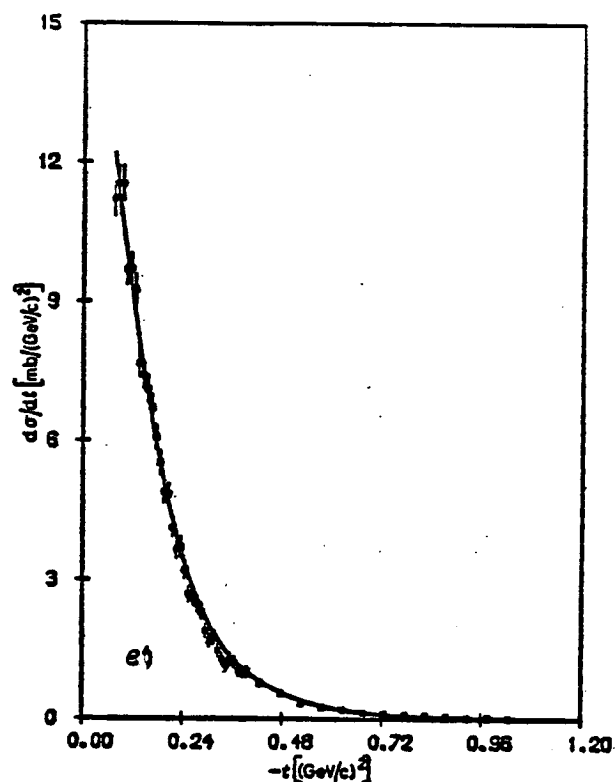


Fig. 4.2- Fits to $(d\sigma/dt)(\pi^+p \rightarrow \pi^+p)$ at e) 200 GeV/c.

Although, on account of very small errors on B_P values, the χ^2 values are large in all three parameterisations the various fits have comparable χ^2 values. If one translates form (1)* for B_P into Regge pole language one obtains $e^{B_P t} \rightarrow e^{b_0 t} e^{b_1 \ln s} = e^{b_0 t} e^{b_1 \ln s}$

* See Table 4.3.

TABLE 4.3

Various Parameterisations of B_P and R_R for the π^+p case.

No	$B_P =$	$R_R =$
1	$b_0 + b_1 \ln s = 1.7418 + 0.3827 \ln s$ $\pm 0.0516 \quad \pm 0.0131$	$R_0 + R_1 \ln s = 4.101 + 0.3461 \ln s$ $\pm 0.264 \quad \pm 0.0523$
2	$b_2 (\ln s)^{\beta_2} = 1.6919 (\ln s)^{0.4795 \pm .0108}$ ± 0.0240	$R_2 (\ln s)^{\gamma_2} = 4.025 (\ln s)^{.229 \pm .025}$ $\pm .229$
3	$b_3 (\ln \ln s)^{\beta_3} = 2.6568 (\ln \ln s)^{0.6934 \pm .0123}$ ± 0.0131	$R_3 (\ln \ln s)^{\gamma_3} = 5.075 (\ln \ln s)^{.2827 \pm .047}$ $\pm .074$

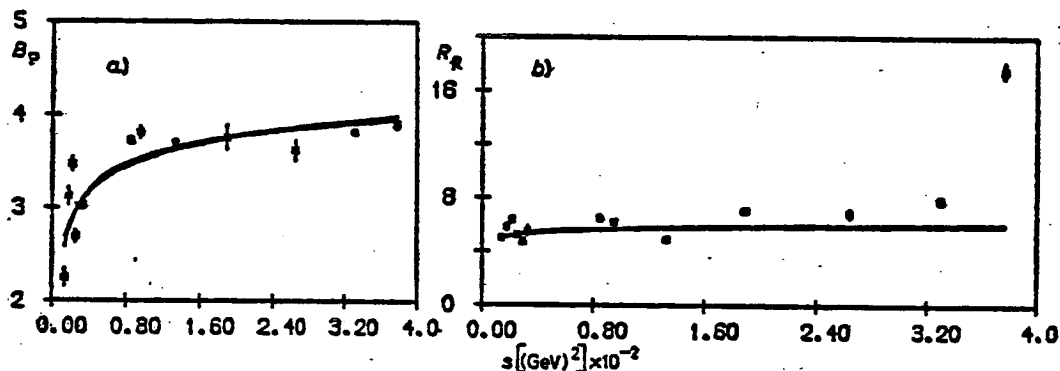


Fig. 4.3. Fits to B_P and R_R according to parameterisations of Table 4.3

which gives a slope of about 0.38 for the Pomeron trajectory. In Fig. 4.3a, we show the curves corresponding to parameterisations 1, 2, 3 of Table 4.3 along with the values of B_P from Table 4.2. As can be seen, it is difficult to distinguish the various curves. The same is true of R_R which can be fitted comparably with the three different parameterisations (Fig. 4.3b).

iii) Polarisations: The π^+p elastic polarisations (data from Borghini et al. (1971), Gaidot et al. (1975), Auer et al. (1977) are fitted in the range $10 \lesssim p_{\text{lab}} \lesssim 100 \text{ GeV/c}$ for $|t| \lesssim 1.3 (\text{GeV/c})^2$ according to equation (6.8). The A_P, A_R, B_P, B_R, R_R values obtained from fits to $\sigma_{\text{tot}}(\pi^+p)$ and $\frac{d\sigma}{dt}(\pi^+p)$ above are used in fits to polarisations. Where P and $\frac{d\sigma}{dt}$ measurements are not available at the same laboratory momentum we

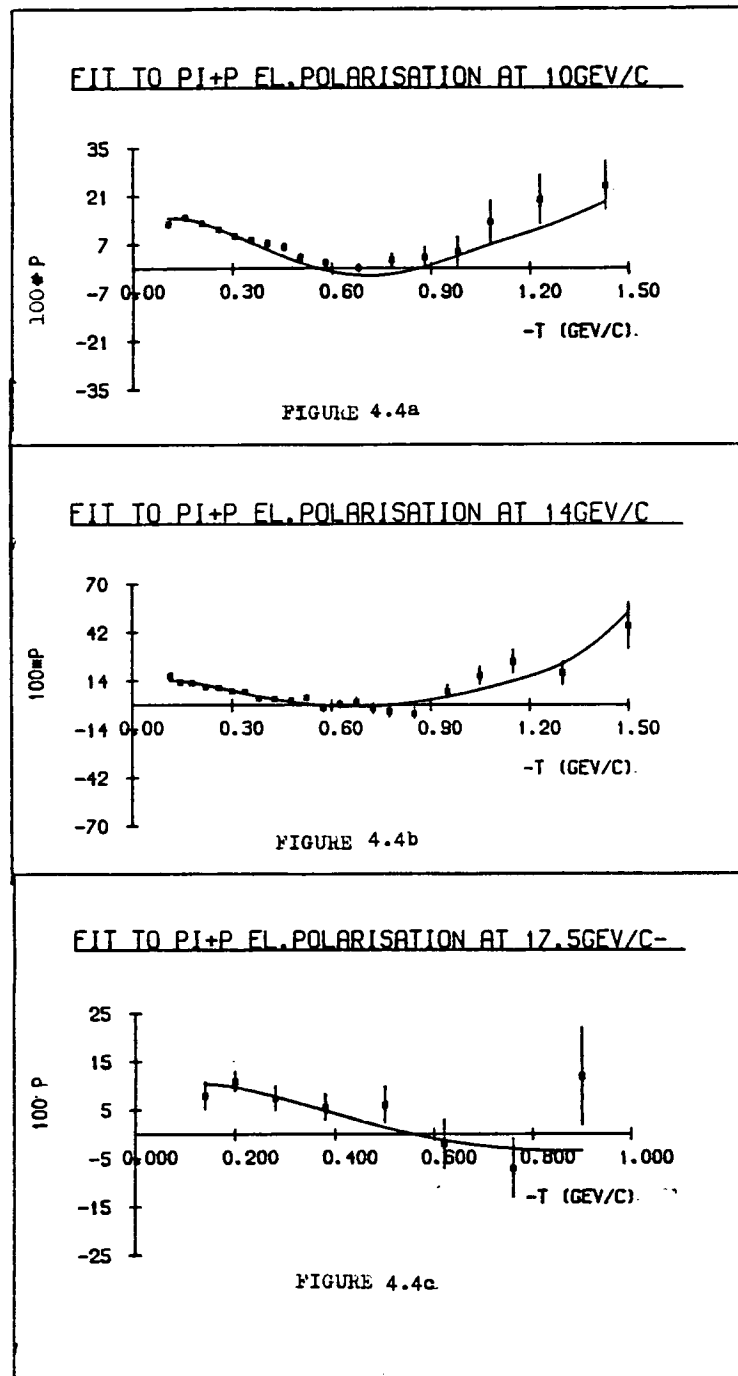
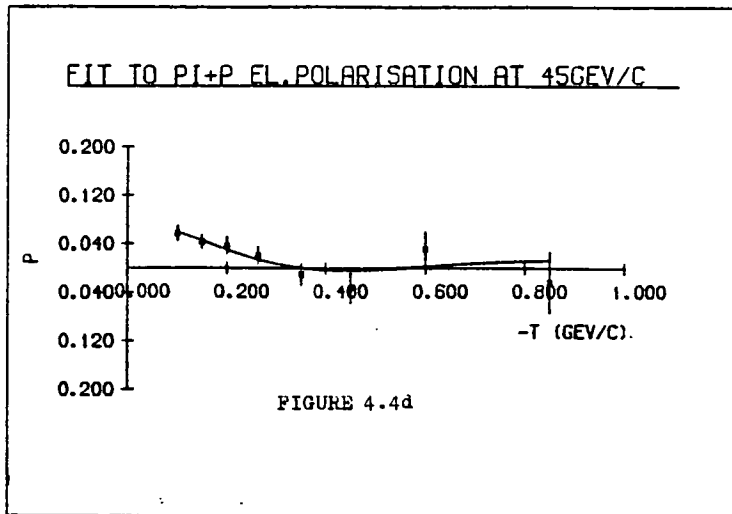


Fig. 4.4a-e. Fits to $\pi^+ p$ elastic polarisations according to equations 4.4, 4.5 and 4.8 (Fig. 4.4d, c on following page).

use parameter values from fits to $\frac{dg}{dt}$ at a nearby laboratory momentum. Our fits indicate that B_ρ is consistent with zero so that we drop it as a parameter. The numerical results of our fits are shown in Table 4.4 and some fits are depicted in Fig. 4.4 a-e. The A_ρ and R_ρ values, though slightly different at different momenta in general, are close in all cases. The R_ρ values are close to about 1 fm, as expected, the average value of R_ρ being



approximately 4.72 ($\approx .93$ fm). The average value of A_p is 45.83. The overall χ^2 is 95.78 for 68 degrees of freedom (approx. 1.408/degree of freedom). If one refits the data using these average values of A_p and R_p one obtains an overall χ^2 per degree of freedom of 2.43 which is perhaps still acceptable. (Most of the contribution to this value comes from the 10 GeV/c data, for which the use of average A_p and R_p values yields a χ^2 /degree of freedom of 5.39. Excluding the 10 GeV/c data yields a χ^2 /degree of freedom of 1.69).

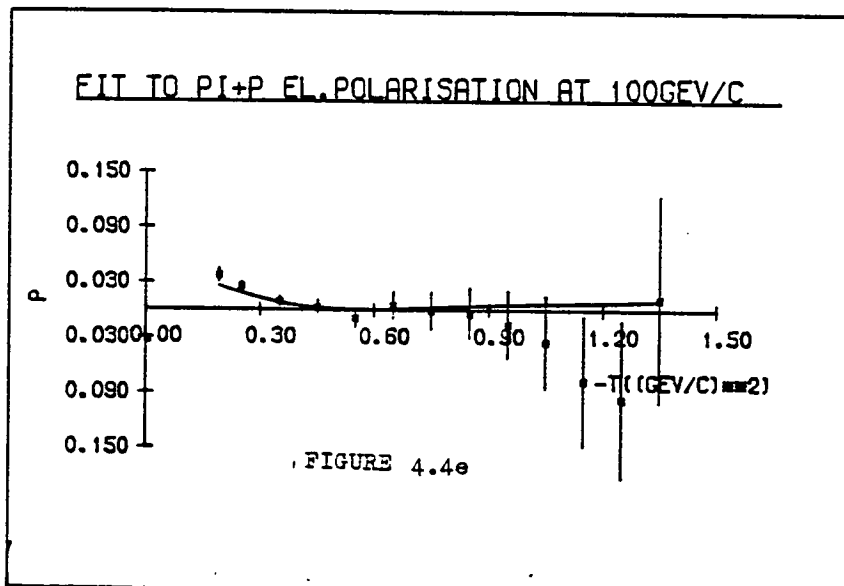


TABLE 4.4

Fits to π^+p Polarisations according to Equations (4.4,4.5,4.8)*

p_{lab} (GeV/c)	A_ρ	R_ρ (GeV^{-1})	χ^2/n **
10	45.295 ± 1.143	$4.179 \pm .079$	47.68/17
14	50.805 ± 2.105	$4.357 \pm .158$	32.72/22
17.5	46.449 ± 9.691	3.471 ± 1.028	5.77/8
45	35.873 ± 8.045	6.497 ± 1.181	3.98/8
100	50.735 ± 12.251	$5.092 \pm .877$	5.63/13

Overall $\chi^2/\text{degree of freedom} = \frac{95.78}{68} = 1.408.$

* B_ρ in equations 4.3a,b;4.5 is consistent with zero and has been dropped.

**
n denotes number of degrees of freedom.

4.3B Fits to π^-p Data:

i) Total Cross-Sections: The fit to π^-p total cross-sections is shown in Fig. 4.1b and the numerical results are given in Table 4.1. The laboratory momentum range covered is $10 \leq p_{lab} \leq 280$ GeV/c and the data is from Carroll et al. (1976), Foley et al. (1963). The $\chi^2/\text{degrees of freedom}$ has a value $\frac{64.57}{26} = 2.48$. The A_ρ value is slightly larger than the corresponding value in the π^+p case, the discrepancy being about 0.75%. The A_R value is, as expected, larger in the π^-p case, indicating a ρ component of about 15% of the contribution of the f

$$\text{at } t = 0 \text{ (i.e. at } t = 0, \frac{(A_R^{\pi^-p} - A_R^{\pi^+p})}{2} \cdot \frac{(A_R^{\pi^-p} + A_R^{\pi^+p}/2)}{2} \approx \frac{9.9}{64.47} \approx .154$$

TABLE 4.5

Parameter Values from Fit to $\pi^- p$ Elastic Differential Cross-Sections

p_{lab} (GeV/c)	A_P	A_R	B_P (GeV $^{-2}$)	B_R (GeV $^{-2}$)	R_R (GeV $^{-1}$)	χ^2/n
10.0	41.03	74.413	3.059 $\pm$.031	3.547 $\pm$.572	4.945 $\pm$.383	23.77/21
12.4	41.03	74.413	3.047 $\pm$.078	2.584 $\pm$.536	4.835 $\pm$.180	19.83/20
13.0	41.03	74.413	2.961 $\pm$.006	1.986 $\pm$.164	4.913 $\pm$.093	23.85/13
15.0	41.03	74.413	2.8223 $\pm$.092	1.011 $\pm$.242	5.363 $\pm$.107	13.56/13
17.0	41.03	74.413	3.122 $\pm$.047	2.065 $\pm$.064	5.099 $\pm$.138	5.53/12
18.4	41.03	74.413	3.003 $\pm$.038	1.637 $\pm$.29	5.177 $\pm$.044	23.03/18
25.2	41.03	74.413	3.664 $\pm$.038	7.34 $\pm$.121	4.902 $\pm$.224	32.26/38
40.0	41.03	74.413	3.537 $\pm$.053	1.684 $\pm$.631	6.685 $\pm$.145	52.91/38
50.0 a)	43.899	50.516	4.062 $\pm$.107	8.58 $\pm$ 4.096	6.367 $\pm$.458	36.06/57
50.0 b)	41.03	74.413	3.490 $\pm$.029	1.305 $\pm$.213	6.472 $\pm$.052	25.49/18
70.0	41.03	74.413	3.814 $\pm$.023	8.374 $\pm$ 1.69	5.075 $\pm$.147	4.79/13
100.0 a)	41.03	74.413	3.671 $\pm$.018	1.647 $\pm$.278	6.6313 $\pm$.063	58.08/55
100.0 b)	40.857	72.692	3.781 $\pm$.091	1.968 $\pm$.472	7.55 $\pm$.596	26.92/19
140.0	41.03	74.413	3.868 $\pm$.010	7.706 $\pm$.897	5.032 $\pm$.403	14.52/18
175.0	41.03	74.413	3.939 $\pm$.007	1.000 $\pm$.054	7.867 $\pm$.100	33.56/19
200.0	41.03	74.413	4.111 $\pm$.018	.052 $\pm$.054	7.695 $\pm$.127	111.69/50

$$\text{Overall } \chi^2 = \frac{506.15}{422} = 1.199$$

a) Data from ref. 24.

b) Data from ref. 25.

gives a measure of the ratio of the ρ contribution to the f contribution in the non-flip amplitude).

ii) Differential Cross-Sections: We have fitted the near-forward differential cross-sections in the range $10 \leq p_{\text{lab}} \leq 200$ GeV/c and $|t| \lesssim 1.2$ (GeV/c)². Data was taken from Akerloff et al. (1976), Ayres et al. (1977), Bruneton et al. (1977), Harting et al. (1965), Foley et al. (1963). The numerical results are shown in Table 4.4 and some typical fits in Fig. 4. a-f. The overall $\chi^2/n \sim \frac{500.32}{422} \sim 1.2$ per point is comparable with the π^+p case. The π^-p fits have the satisfactory feature that, for good χ^2 values, A_p and A_R do not require any change from the values obtained from $\sigma_{\text{tot}}(\pi^-p)$ fits, unlike the π^+p case. As in the π^+p case, both B_p and R_R seem to increase slowly with energy whereas B_R does not seem to vary in any definite manner. We have fitted these parameters with the three alternative forms used in the π^+p case. The parameter values are as shown in Table 4.6 below. The parameter values for $B_p^{\pi^-p}$ and $B_p^{\pi^+p}$ are in close agreement for all three parameterisations.

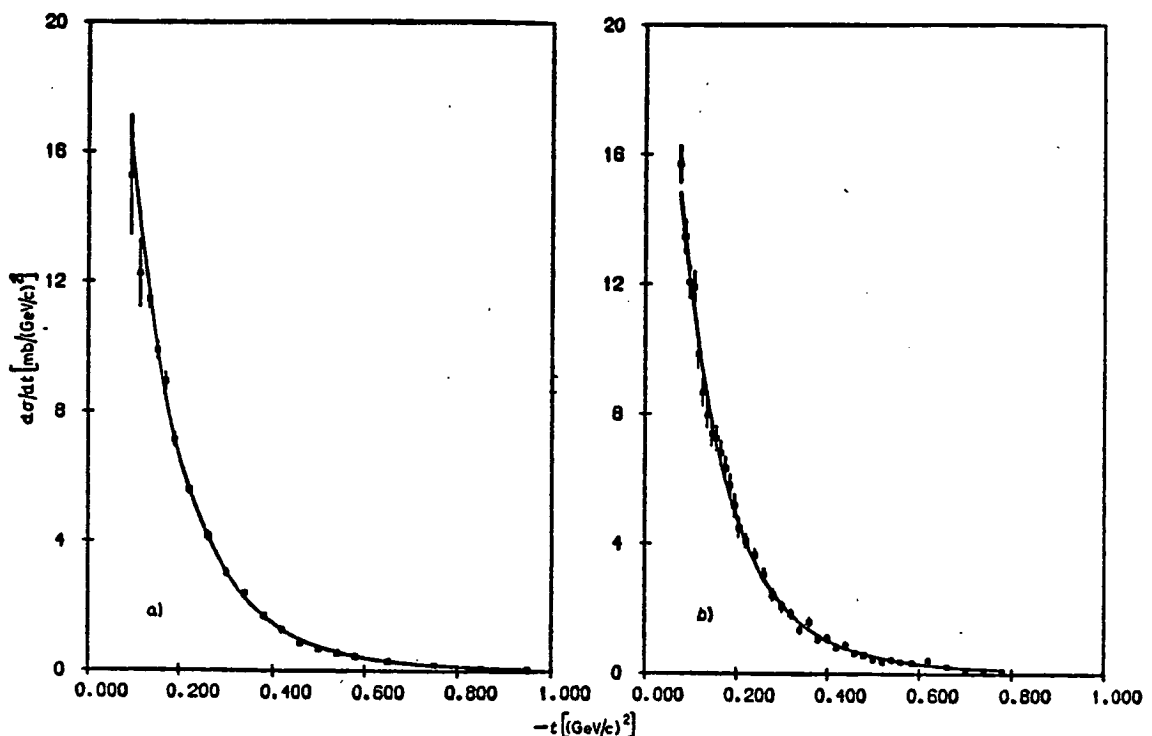


Fig. 4.5 - Fits to $(d\sigma/dt)$ ($\pi^-p \rightarrow \pi^-p$) at a) 12.4 GeV/c, b) 40.0 GeV/c.

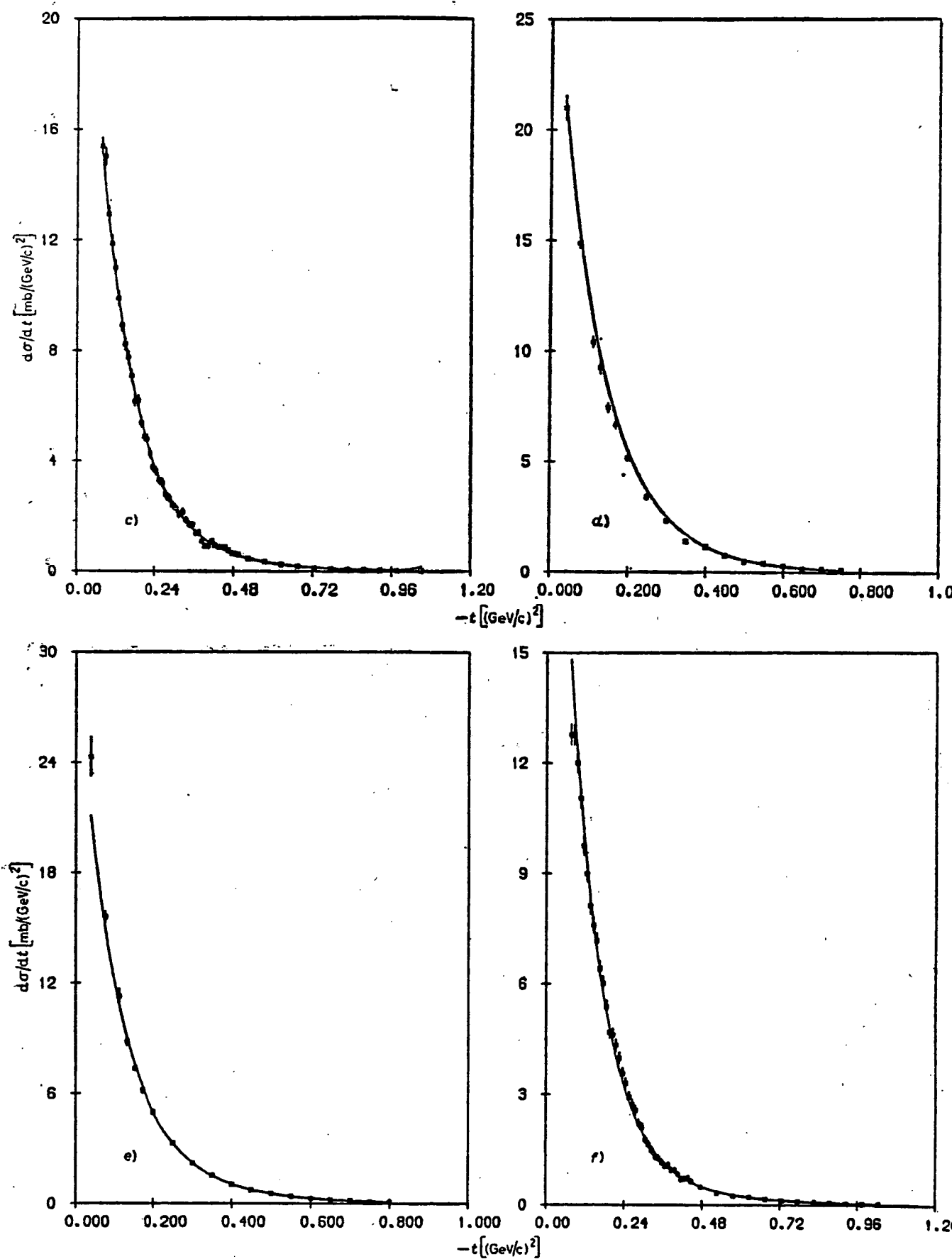


Fig.4.5 - Fits to $(d\sigma/dt)$ ($\pi^-p \rightarrow \pi^-p$) at c) 100.0 GeV/c, d) 140.0 GeV/c, e) 175 GeV/c, f) 200.0 GeV/c.

The first parameterisation in Table 4.5 for $B_P^{\pi^- p}$ implies a Pomeron slope of 0.365 compared to a value of 0.382, as implied by the corresponding parameterisation for $B_P^{\pi^+ p}$.

TABLE 4.6

Various Parameterisations for B_P and R_R for the $\pi^- p$ Case

Parameterisation No	$B_P =$	$R_R =$
1	$b_0 + b_1 \ln s = 1.797 + 0.365 \ln s$ $\pm .011 \quad \pm .002$	$R_0 + R_1 \ln s = 1.830 + 0.946 \ln s$ $\pm .13 \quad \pm .034$
2	$b_2 (\ln s)^{\beta_2} = 1.723 (\ln s)^{0.466 \pm .006}$ $\pm .018$	$R_2 (\ln s)^{\gamma_2} = 2.153 (\ln s)^{0.694 \pm .048}$ $\pm .143$
3	$b_3 (\ln \ln s)^{\beta_3} = 2.796 (\ln \ln s)^{0.611 \pm .025}$ $\pm .038$	$R_3 (\ln \ln s)^{\gamma_3} = 4.095 (\ln \ln s)^{1.010 \pm .050}$ $\pm$

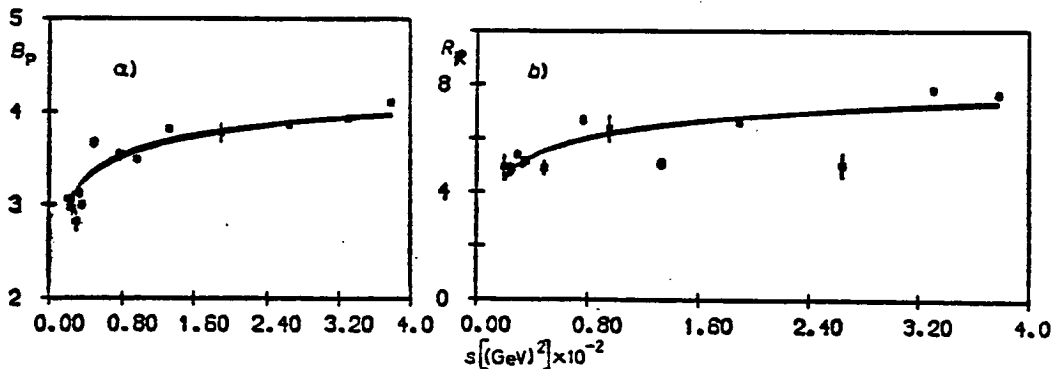


Fig. 4.6a,b. Fits to $\pi^- p$ parameter B_P and R_F according to parameterisations in Table 4.6. As may be noticed it is virtually impossible to distinguish between the various parameterisations.

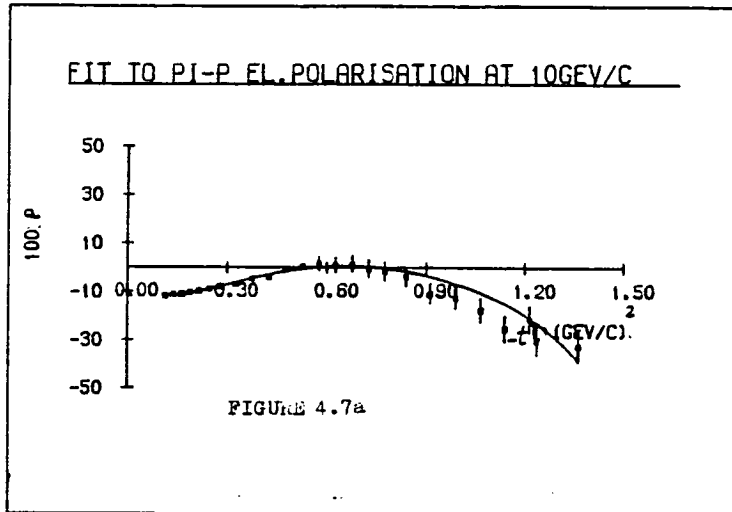


Fig. 4.7a-d (b-d on following page). Fits to π^-p elastic polarisation according to equations 4.4, 4.5, 4.8. The curve represents results of our calculations while points with error bars show experimental values.

iii) Polarisations: The procedure adopted in fitting π^-p polarisations is the same as that for π^+p polarisations discussed previously. The numerical results are shown in Table 4.7 and Figs. 4.7a-d depict the fits. B_p is, as with the π^+p case, consistent with zero and can be dropped as a parameter. The values of A_p and R_p , though slightly different at different momenta, vary little. The average value of A_p is 39.46 and that of R_p is 4.23 GeV^{-1} (approximately 0.83 fm). The overall $\chi^2/\text{degrees of freedom}$ is $\frac{81.79}{73} = 1.12$. If one employs the average values of A_p and R_p given above to fit the polarisation data, one ends up with a $\chi^2/\text{degree of freedom}$ of 3.43, which is bad. However, as in the π^+p case, this large value is due to the 10 GeV/c

FIT TO PI-P EL. POLARISATION AT 14GEV/C

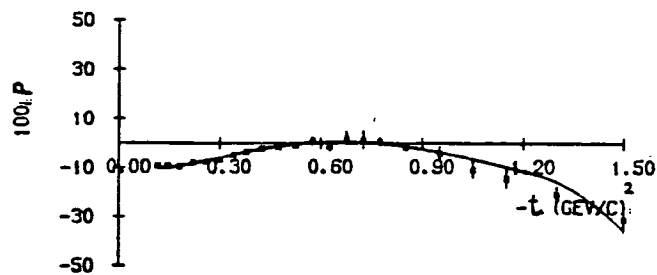


FIGURE 4.7b

FIT TO PI-P EL. POLARISATION AT 40GEV/C

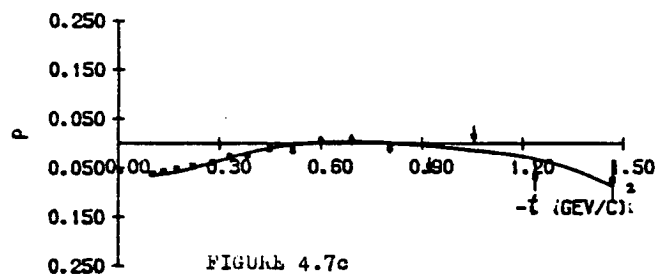


FIGURE 4.7c

FIT TO PI-P EL. POLARISATION AT 100GEV/C

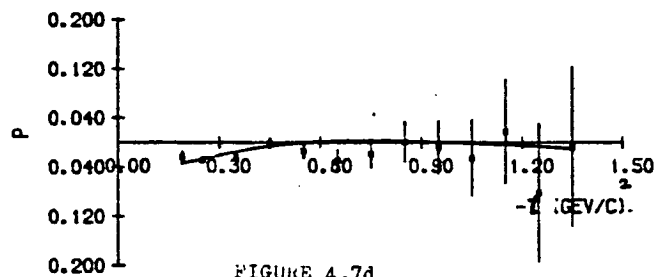


FIGURE 4.7d

data, which yield a $\chi^2/\text{degree of freedom}$ of 8.54 with the average A_ρ and R_ρ values. Excluding the 10 GeV/c data improves the overall χ^2 with average R_ρ , A_ρ values to 1.73 per degree of freedom.

TABLE 4.7

Fits to π^-p Polarisation

p_{lab} (GeV/c)	$-A_\rho$	R_ρ (GeV^{-1})	χ^2/n^*
10	33.92 ± 0.689	4.439 ± 0.075	29.44/24
14	37.148 ± 1.199	4.3286 ± 1.1437	17.47/20
40	39.74 ± 0.655	4.270 ± 0.176	23.23/16
100	47.026 ± 9.511	3.895 ± 0.537	11.65/13

Overall $\chi^2/\text{degree of freedom} = 81.79/73 = 1.12$.

* n denotes number of data points.

4.3C Discussion

As mentioned earlier, several authors have previously applied the DAM to various pieces of πp elastic data. We shall now discuss our results and, wherever appropriate, compare them with some of the earlier applications of the DAM to elastic πp scattering.

After explaining sums of $\pi^\pm p$ elastic differential cross-sections by using the DAM to separate P exchange from f exchange, Davier (1972) draws the following conclusions about the various parameters:

- The slope in t of the Pomeron amplitude increases with energy at a rate similar to that of K^+p elastic scattering. (He refers to

the compilation of Lasinski et al. (1972) in this context, who give $\alpha_p' \approx 0.4$.)

- b) The f amplitude has an s -dependence which, at $t = 0$, is compatible with standard Regge behaviour.
- c) The f radius of interaction has no striking energy dependence and has a value of ~ 1 fm.

Although we have dealt with individual differential cross-sections rather than their sums, our results for the Pomeron slopes in both π^+p and π^-p cases agree with a) as shown in Tables 4.3 and 4.6.

We also agree with conclusion b) of Davier (1972) as may be noted by the parameterisation of the Regge amplitude in equation (2) and by recalling that the amplitude is dominated by f -exchange. However, unlike Davier (1972), we find that the radius of interaction in the non-Pomeron term does have a definite energy dependence, compatible with several different forms, as shown in Tables 4.3 and 4.6. In the energy range explored in the above reference ($2.5 \leq p_{lab} \leq 17$ GeV/c) our radius of interaction is also ~ 1 fm.

Minami (1972) and Minami and Terada (1974) have employed the DAM to analyse the slope parameters for various elastic reactions. We shall concern ourselves with the πp case in this chapter. The two papers are close in spirit though the latter involves total-cross-section parameterisations which are incorporated into the slope parameter expressions via the Optical theorem ($b \equiv \frac{d}{dt} \ln \frac{d\sigma}{dt} \Big|_{t=0}$). Both papers employ an energy-independent radius of interaction for the non-Pomeron Regge (or f) exchange term. The values and expressions for various parameters are as follows: (both papers employ the notation $b_P^{\pi p}$ and $b_R^{\pi p}$ where in our notation $b_P^{\pi p} = 2B_P$, $b_R^{\pi p} = 2B_R$)

Reference	B_P	B_R	$R_R(\text{GeV}^{-1})$
Minami (1972)	$2.08 + 0.27\ln s$	$-2.2 + 2\ln s$	5.09
Minami & Terada (1974)	$2.25 + 0.39\ln s$	$0.3 + 0.85\ln s$	4.58

If we compare the parameterisations of Minami (1972) and Minami and Terada (1974) for B_P with our corresponding expressions in Tables 4.3 and 4.6 ($B_P^{\pi^- p} = 1.797 + .365\ln s$ and $B_P^{\pi^+ p} = 1.7418 + .3827\ln s$) we find that our coefficients of $\ln s$ agree closely with that of Minami and Terada (1974). The other term in our expressions however is closer to the corresponding term of Minami (1972) rather than of Minami and Terada (1974). Unfortunately our B_R values do not show any pattern in their energy dependence. Nonetheless we show our B_R values against those of Minami (1972) and Minami and Terada (1974) in Fig. 4.8 .

TABLE 4.8

Values of Various Parameters Predicted by Different Parameterisations in Tables 4.3 and 4.6 at $p_{\text{lab}} = 2000 \text{ GeV}/c$

Parameterisation No.	$B_P^{\pi^+ p}(\text{GeV}/c^{-2})$	$B_P^{\pi^- p}(\text{GeV}/c^{-2})$	$R_R^{\pi^+ p}(\text{GeV}/c^{-1})$	$R_R^{\pi^- p}(\text{GeV}^{-1})$
1	4.89	4.80	6.95	9.62
2	4.65	4.60	6.522	9.297
3	4.455	4.41	6.266	8.696

The radius of interaction in our fits shows a definite energy dependence (compatible with several alternative forms) unlike that of Minami (1972) and Minami and Terada (1974). We would like to

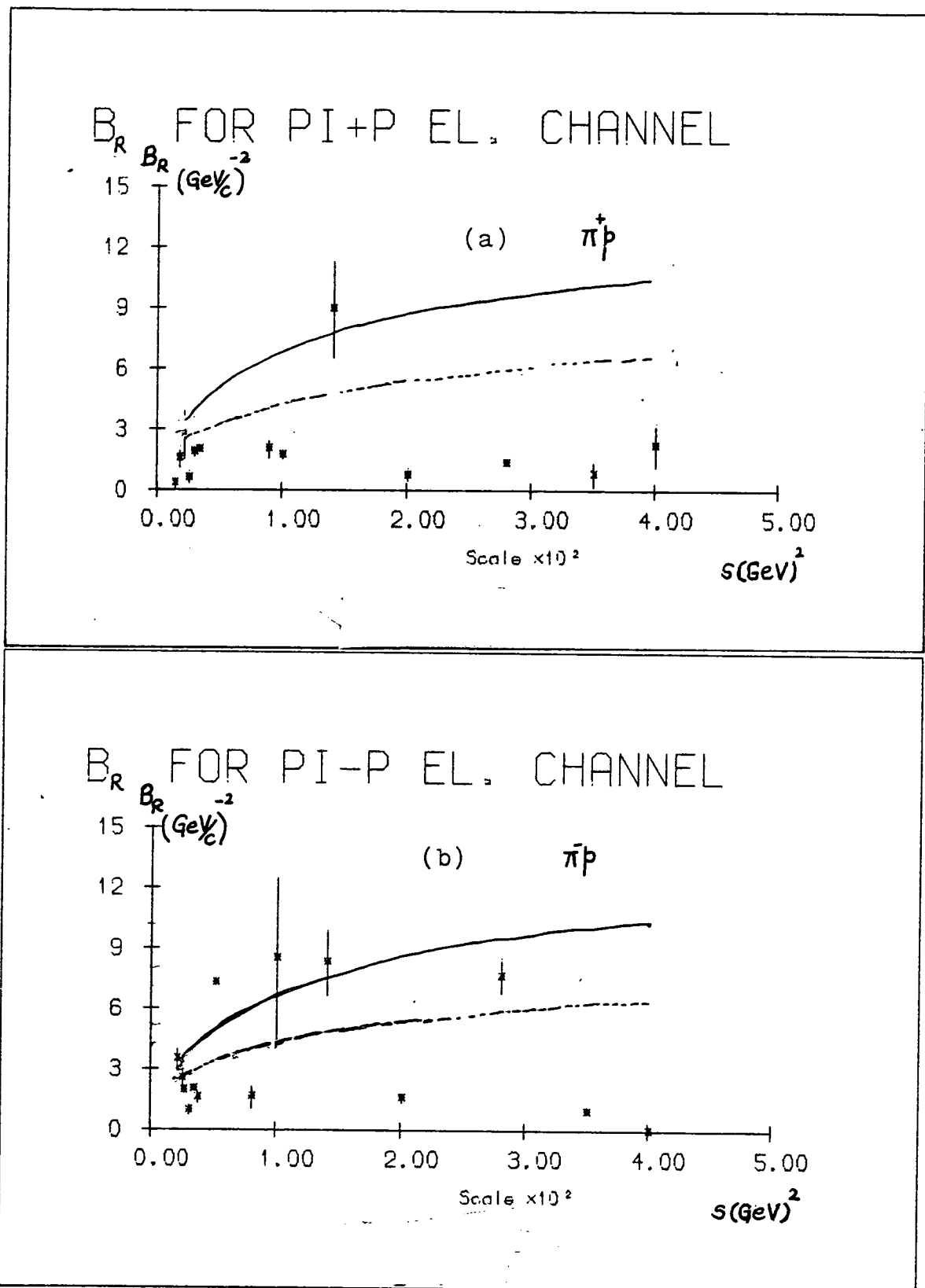


Fig 4.8 a,b): Comparison of B_R values for a) π^+p b) π^-p cases with the expressions of Minami(1972)(solid line) and of Minami and Terada(1974)(dashed line). B_R values have been taken from tables 4.2 and 4.5 for π^+p and π^-p cases respectively.

emphasise that we treat the radius of interaction as a free parameter whose energy dependence is to be determined by fitting data over a wide energy range while, in the aforementioned references, the radius of interaction is a priori assumed to be energy independent. In the energy range considered we cannot choose any particular form of energy dependence over the other but at high energies this might perhaps be possible. As a speculative exercise we quote the values of various parameters at 2000 GeV/c.

Let us now consider the elastic slope parameter b ($b \equiv \frac{d}{dt} \ln \frac{d\sigma}{dt} \Big|_{t=0}$). Starting from the definition of b and employing equations 4.4 and 4.7 we arrive at the following expression

$$b = 2 \left[\frac{B_P s^{2\alpha_P(0)} + \frac{A_R}{A_P} (B_P + B_R + \frac{R_R^2}{4}) s^{\alpha_P(0) + R(0) (\frac{A_R}{A_P})^2 (B_R + \frac{R_R^2}{4}) s^{2\alpha_R(0)}}}{\left| s^{\alpha_P(0)} + \frac{A_R}{A_P} s^{\alpha_R(0)} \right|^2} \right] \quad (4.9)$$

where $J_0(R_R \sqrt{-t}) \Big|_{t=0} = 1$ and $\frac{d}{dt} J_0(R_R \sqrt{-t}) \Big|_{t=0} = \frac{R_R^2}{4}$ have been used and where $\alpha_P(0) = 1.06$, $\alpha_R(0) = 0.5$ while the remaining parameters can be read off from Tables 4.2 and 4.5 for the π^+p and π^-p cases. Using the values of the various parameters we calculate b values at the various laboratory momenta and find that our results agree closely with the experimental measurements of b for both π^+p and π^-p elastic scattering as shown in Fig. 4. a, b.

Our fits to polarisation data indicate that the two parameters A_ρ and R_ρ can be regarded as energy independent except for the 10 GeV/c data which cannot be fitted satisfactorily if one employs fixed A_ρ and R_ρ values. Furthermore there is, in general, a slight discrepancy between the A_ρ values for π^+p and π^-p cases and also the R_ρ values in the two cases. The aforementioned discrepancies could have been caused

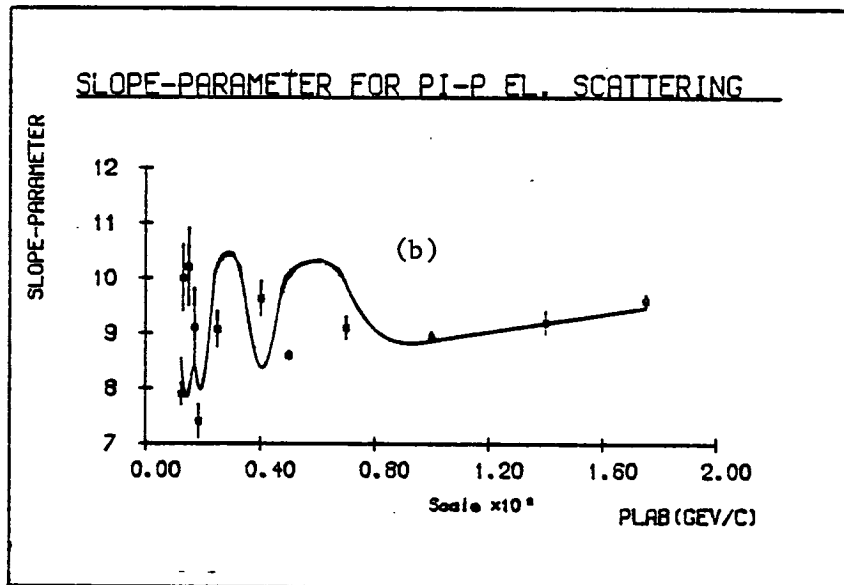
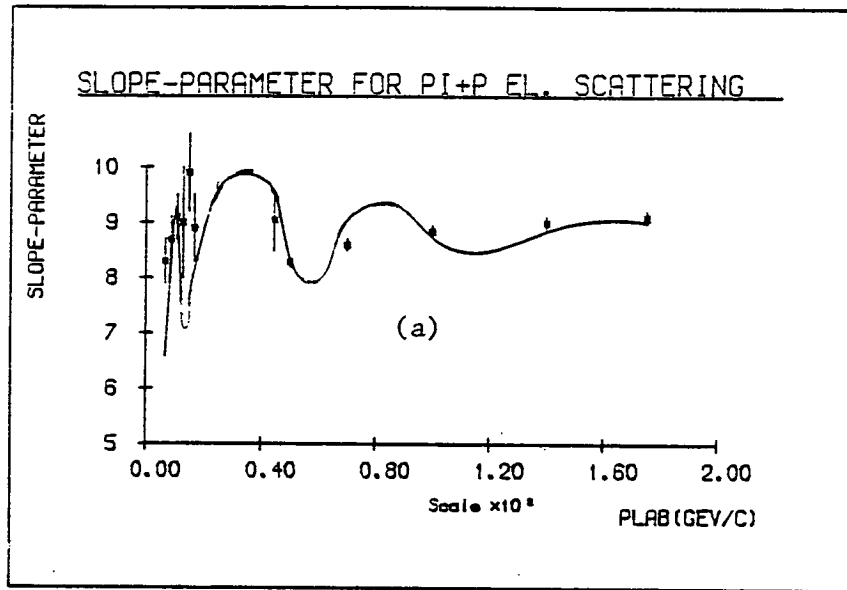


Fig. 4.9a, b. Slope parameter values for $\pi^\pm$ -p elastic scattering calculated according to equation (4.9), shown with the experimental values. The curve represents theoretical results while the points with error bars represent experimental values.

by either or both of the following factors:

- a) Failure to take into account the real part of the non-flip amplitude.
- b) A possible contribution to the flip amplitude by the P or f exchange or by both.

The fact that the effect is small and does not seem to be present in the data above 10 GeV/c, coupled with the fact that we do not have any definite procedure for calculating the above effects without introducing new parameters, has prevented us from pursuing this point further. Martin and Navelet (1977) have argued that the departure of $\pi^{\pm}p$ polarisations from mirror symmetry can be explained by introducing a small Pomeron flip contribution and invoking a new low-lying trajectory. They have considered polarisations in the range $2.73 \leq p_{\text{lab}} \leq 40$ GeV/c. However since these departures from mirror symmetry are pronounced only at and below 10 GeV/c, we have not pursued the matter. The fact that there is a real part of the isovector contribution to the non-flip amplitude has been established by model-independent amplitude analysis at 6 GeV/c (c.f. Halzen and Michael (1972)). Our neglect of the real part completely, therefore, is somewhat unsatisfactory although it does not affect our overall explanation of σ_{tot} , $\frac{d\sigma}{dt}$, b and P in the light of the DAM. As a last point we mention that our parameterisation is in general agreement with the measurements of ^{the} spin rotation parameter R. For π^+p only one measurement of R exists (Table 1., Chapter 1) at $p_{\text{lab}} = 45$ GeV/c $|t| = 0.315$. Our calculated value of -0.2794 agrees closely with the experimental value of -0.22 ± 0.16 . For the π^-p case measurements exist at 16 and 40 GeV/c. Our calculations for the 16 GeV/c data are shown in Fig. 4.8 and can be seen to agree with the data. Our 40 GeV/c calculations are slightly more negative than the experimental data. Our average value is -0.338 as against the experimental value of -0.193 ± 0.08 . The formula for $R(\pi p)$ is as follows (c.f., e.g. Pierrard et al. (1976), Kane and Seidl (1976):

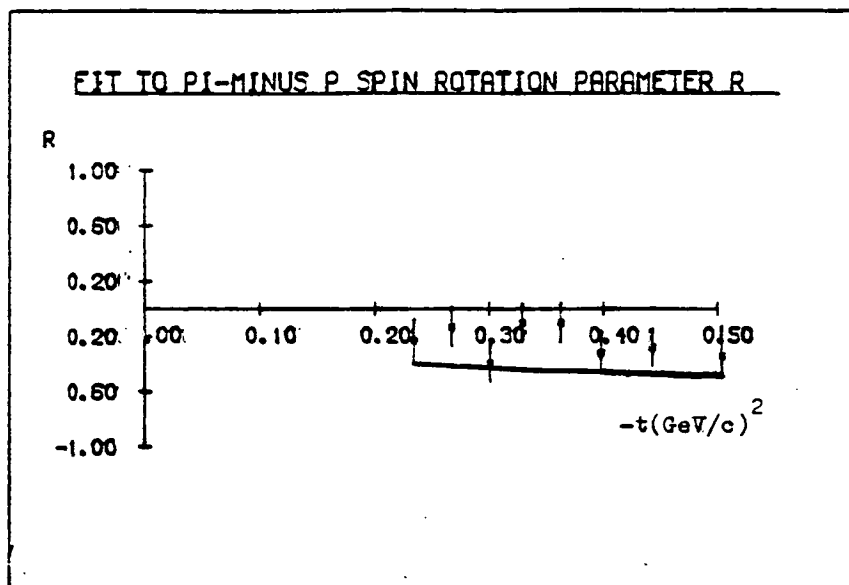


Fig. 4.10. Prediction for $\pi^- p$ spin rotation parameter at 16 GeV/c (curve) shown against experimental values (points with error bars).

$$R = (-r \sin \theta_R + a \cos \theta_R) / (|T_{++}|^2 + |T_{+-}|^2) \quad (4.10a)$$

where

$$r = |T_{++}|^2 - |T_{+-}|^2 \quad (4.10b)$$

$$a = 2 \operatorname{Re}(T_{++} T_{+-}^*) \quad (4.10c)$$

$$\theta_R = \text{recoil angle in the laboratory frame.}$$

CHAPTER 5

THE DUAL ABSORPTIVE MODEL AND $K^{\pm}_p$ ELASTIC SCATTERING UP TO 280 GeV/c

5.1 Introduction and Motivation:

In the previous chapter we applied the DAM to the high-energy, small $|t|$ $\pi^{\pm}_p$ elastic scattering data. In this chapter we consider the $K^{\pm}_p$ elastic data at high energies ($6 \leq p_{lab} \leq 280$ GeV/c) and small momentum transfers ($|t| \lesssim 1$ (GeV/c)²) in the light of the DAM. The kaon-nucleon system is the simplest hadronic system with non-zero strangeness for which extensive data is available. Its simplicity resides in the fact that the kaon, which is the lightest hadron with non-zero strangeness, is spinless. As a consequence only two independent amplitudes are needed in a description of elastic kaon-nucleon scattering and of most two-body kaon-nucleon reactions involving the exchange of non-vacuum quantum numbers (charge-exchange, hypercharge exchange, etc.). In this the kaon-nucleon system is similar to the pion-nucleon system. However, unlike the π^+_p and π^-_p elastic scattering data, which exhibit common features, the K^+_p and K^-_p elastic data reveal many mutually contrasting features which have made the kaon-nucleon system a source of great fascination to the phenomenologist. For instance, whereas both the π^+_p and π^-_p total cross-sections display resonances, this is not the case with the $K^{\pm}_p$ total cross-sections - while K^-_p total cross-section has a rich resonance spectrum the K^+_p total cross-section shows none. Furthermore while the K^-_p total cross-section behaves like other hadron-nucleon total cross-sections in the Regge region - in that it falls to a shallow minimum before rising again, the K^+_p total cross-section does not fall anywhere in the Regge region. Similarly unlike the $\pi^{\pm}_p$ case, while $\frac{d\sigma}{dt}(K^-_p)$ shows a dip for $|t| \lesssim 1$ (GeV/c)² at lower energies in the Regge region, $\frac{d\sigma}{dt}(K^+_p)$ is structureless at all energies in this region. A consistent explanation of these contrasting

features of K^+p and K^-p elastic data therefore constitutes an interesting challenge to any model. It may be noted that the K^+p system has exotic quantum numbers. The distinctive behaviour of K^+p data is connected with this fact.

There exist numerous studies of $K^\pm p$ elastic data based on different approaches and models. Phillips and Rarita (1965) considered Regge pole fits to the then available $K^\pm p$ (and $\pi^\pm p$) elastic data. Cohen-Tannoudji et al. (1967) also applied their hybrid model to kaon-nucleon elastic data. However their fits predict $K^\pm p$ elastic polarisations to be roughly mirror-symmetric up to $|t| \sim 0.5 \text{ (GeV/c)}^2$ - a feature contradicted by subsequent measurements. Berger and Fox (1969) attempted a Veneziano ansatz in a comprehensive study of the low- and high-energy features of KN (and πN) data. Such an ansatz is simpler for $K^\pm p$ elastic data on account of the (empirical) absence of exotic resonances. They concluded that their failure in fitting $K^\pm p$ elastic data (after including a Pomeron term) might be indicative of effects outside the framework of the model. Blackmon and Goldstein (1969) however successfully applied the Argonne version of the Reggeised Absorption Model to $K^\pm p$ elastic data including polarisations. A phenomenological investigation of πN and KN scattering employing Regge cuts was carried out by Hong Tuan et al. (1971) who found that in order to be able to fit $K^\pm p$ elastic polarisations satisfactorily strong EXD between the ρ and A_2 exchanges in the flip amplitude was necessary. Henzi et al. (1972) also obtained a satisfactory fit to $K^\pm p$ elastic data including polarisations on the basis of the Reggeised Eikonal Model of Frautschi and Margolis (1968). $K^\pm p$ elastic data has also been looked at in the context of the impact-parameter model of Cheng-Walker and Wu (Cheng and Wu (1970), Walker (1973), Cheng et al. (1973). Collins et al. (1974b) have carried out a detailed

phenomenological investigation of various meson-baryon elastic reactions including $K^{\pm}p$ data. They predict, on the basis of their analysis, a dip in the $K^{\pm}p$ differential cross-sections somewhere in the region $1.5 \leq |t| \leq 2.5$ (GeV/c)² at high energies. Barger and Phillips (1976) found $K^{\pm}p$ elastic data to be consistent with the hypothesis of geometrical scaling. Recently considerable effort has been devoted to phenomenological investigations of kaon-nucleon scattering by Joynson and Martin (1977), Joynson (1977) and Joynson et al. (1977). Joynson et al. (1977) consider 5-pole (P, f, ρ , ω , A_2) Regge pole model fits to data as well as fits which are characterised by a mixture of Regge pole and (empirical) non-Regge pole terms. They find that the Regge pole model is in good agreement with data except for certain discrepancies at and near $t = 0$. Joynson (1977) had earlier found a breaking of the ρ - A_2 EXD from an FESR analysis of kaon-nucleon data. Subsequently Joynson and Martin (1977) extracted Regge pole residues and trajectories at fixed t values by simultaneously fitting CMSR's and available data in the region $0 \leq |t| \leq 1.0$ (GeV/c)² in the range $1.5 \leq p_{lab} \leq 280$ GeV/c. They included several secondary trajectories (f' , A_2' , ρ' , ω'), in addition to the usual leading trajectories, in their analysis and suggested that the previously suspected ρ - A_2 EXD breaking observed in Joynson (1977) might be attributed to the f' .

From the preceding description it is evident that phenomenological models are extensively applied to $K^{\pm}p$ elastic data. On account of the exotic quantum numbers of the $K^{\pm}p$ channel the $K^{\pm}p$ elastic data has a special relevance to the DAM. The assumption of non-exoticity of the resonances then implies that at least the imaginary part of the $K^{\pm}p$ non-flip amplitude is pure Pomeron on account of the consequent strong EXD of the various Regge trajectories with respect to this channel. At high energies this offers an opportunity to determine the structure of

the Pomeron accurately from $K^{\pm}p$ data. However the DAM-based analyses of data have not been carried out from this point of view - they have generally aimed at establishing the peripheral character of the $C = -1$ contributions to the non-flip amplitude. The first quantitative application of the DAM was carried out by Davier and Harari (1972) who established that the 5 GeV/c $K^{\pm}p$ data indicated a " J_0 " type behaviour for $C = -1$ exchange in the non-flip amplitude as described earlier (Chapter 3, Section 3.4). They assumed that $\text{Re}T_{\Delta\lambda=0} = 0$ (empirically $\frac{\text{Re}T(s,0)}{\text{Im}T(s,0)} \sim -0.4$ at 5 GeV/c; see Fig. 1.2, Chapter 1.) in their analysis. Gordon et al. (1972) carried out a similar analysis and considered the $K^{\pm}p$ elastic differential cross-section data at 8 and 16 GeV/c. Barger and Halzen (1972) included the $K^{\pm}p$ elastic data in a comprehensive DAM based study of various elastic reactions. As pointed out earlier, however, their analysis assumes a flat Pomeron trajectory, in contradiction to the current state of our knowledge of the Pomeron. They also considered $K^{\pm}p$ polarisations qualitatively. Ambats et al. (1974) also confirmed the peripheral character of the $C = -1$ exchange in the non-flip amplitude in a comprehensive analysis of various elastic cross-sections, including $K^{\pm}p \rightarrow K^{\pm}p$, in the range $3 \leq p_{\text{lab}} \leq 6$ GeV/c. Savay-Navarro (1974) repeated the analysis of Davier and Harari in a phenomenological study of $K^{\pm}p$ (and $\pi^{\pm}p$) elastic cross-sections at 5 and 10 GeV/c. Minami and Terada (1974) studied the $K^{\pm}p$ slope parameters by using the DAM to separate the P exchange from the other exchanges and by including fits to total cross-sections over a wide energy range (the same paper includes an analysis of $\pi^{\pm}p$ as mentioned in Chapter 4).

It is therefore evident that a comprehensive DAM based study of $K^{\pm}p$ elastic scattering over the entire laboratory momentum range over which data is available is lacking. Furthermore $K^{\pm}p$ elastic polarisations have never been quantitatively investigated in the light of the DAM. Therefore we analyse the $K^{\pm}p$ elastic data up to 280 GeV/c using the DAM.

For the same reasons as mentioned in connection with $\pi^\pm p$ data we consider the corresponding quantities for these reactions separately rather than considering appropriate combinations of such quantities. The procedure adopted and assumptions invoked in fitting data are given below, along with the results.

5.2 Assumptions and Parameterisation

The K^+p channel has exotic quantum numbers. The assumption of absence of exotic resonances then implies, on account of duality, the strong EXD of the contributions of the appropriate non-Pomeron trajectories to the imaginary parts of the amplitudes in this channel. This strong EXD is then automatically implied for the K^-p channel. Although this strong EXD is an attractive possibility, its validity must ultimately rest on its ability to yield results that agree with the data. We start by assuming strong EXD of the appropriate non-Pomeron trajectories with respect to the $K^\pm p$ channels (see iii below) and then see whether the data implies any results to the contrary. The contributions of the various leading trajectories to any helicity amplitude $T_{\lambda\lambda'}$ in the $K^\pm p$ channels can be written as:

$$T_{\lambda\lambda'}(K^+p) = P_{\lambda\lambda'} + f_{\lambda\lambda'} - \omega_{\lambda\lambda'} + A_{2\lambda\lambda'} - \rho_{\lambda\lambda'} \quad (5.1a)$$

$$T_{\lambda\lambda'}(K^-p) = P_{\lambda\lambda'} + f_{\lambda\lambda'} + \omega_{\lambda\lambda'} + A_{2\lambda\lambda'} + \rho_{\lambda\lambda'} \quad (5.1b)$$

where each letter on the right-hand side of (5.1a,b) denotes the contribution of the corresponding trajectory to the amplitude.

(i) We assume that the P and f conserve s -channel helicity. In conformity with general practice we also assume that the ω contributes only to the non-flip amplitude while the A_2 and ρ contributions dominate the flip amplitude. Thus we can write (denoting $\lambda = \pm \frac{1}{2}$ as $\pm$)

$$\begin{array}{ll}
 P_{++} = P & P_{+-} = 0 \\
 f_{++} = f & f_{+-} = 0 \\
 \omega_{++} = \omega & \omega_{+-} = 0 \\
 \rho_{++} = 0 & \rho_{+-} = \rho \text{ (say)} \\
 A_{2++} = 0 & A_{2+-} = A_2 \text{ (say)}
 \end{array} \tag{5.2}$$

Breaking the individual terms into real and imaginary parts we thus obtain

$$T_{++}(K^+p) = \text{Re } T_{++} + i \text{Im}[P + f - \omega] \tag{5.3a}$$

$$T_{+-}(K^+p) = \text{Re } T_{+-} + i \text{Im}[A_2 - \rho] \tag{5.3b}$$

$$T_{++}(K^-p) = \text{Re } T_{++} + i \text{Im}[P + f + \omega] \tag{5.3c}$$

$$T_{+-}(K^-p) = \text{Re } T_{+-} + i \text{Im}[A_2 + \rho] \tag{5.3d}$$

where we have, for the moment, left the real parts unspecified.

(ii) Leaving $\text{Re } T_{++}$ unspecified and assuming that $\text{Re } T_{+-}$ satisfies the usual phase rule, as given in the "rules" of the DAM, gives:

$$\text{Re } T_{+-}(K^+p) = \cot \frac{\pi\alpha}{2} \text{Im } A_2 + \tan \frac{\pi\alpha}{2} \text{Im } \rho \tag{5.4a}$$

$$\text{Re } T_{+-}(K^-p) = \cot \frac{\pi\alpha}{2} \text{Im } A_2 - \tan \frac{\pi\alpha}{2} \text{Im } \rho \tag{5.4b}$$

where $\alpha = 0.5 + 0.9t$ is the usual EXD ρ -f- ω - A_2 trajectory.

iii) Assuming strong EXD for f- ω and ρ - A_2 (i.e. $\text{Im } f = \text{Im } \omega$, $\text{Im } A_2 = \text{Im } \rho$) then yields

$$T_{++}(K^+p) = \text{Re } T_{++} + i \text{Im } P \tag{5.5a}$$

$$T_{+-}(K^+p) = \frac{2}{\sin\pi\alpha} \text{Im } A_2 \tag{5.5b}$$

$$T_{++}(K^-p) = \text{Re } T_{++} + i \text{Im}[P + 2f] \tag{5.5c}$$

$$T_{+-}(K^-p) = \frac{2}{\sin\pi\alpha} \cos\pi\alpha \text{Im } A_2 + i 2\text{Im } A_2 \tag{5.5d}$$

It may be noted that $\text{Re } T_{++}(K^-p) = \cos\pi\alpha \text{Re } T_{++}(K^+p)$ according to equation (5.5b,d). The structure of polarisations is given by $\text{Re } T_{+-}$ for values of $|t|$ close to zero (Chapter 3, Section 3.5C). Since $\alpha(t) \approx 0.5$ close to $t = 0$ the above relation predicts that K^-p polarisations will be suppressed relative to K^+p polarisations in this $|t|$ region.

iv) The various "rules" of the DAM (peripheral imaginary parts for Regge exchanges, non-peripheral $\text{Im } P$) then give the following parameterisation:

$$T_{++}(K^+p) = \text{Re } T_{++} + i A_P e^{B_P t} (s)^{\alpha_P(0)} \quad (5.6a)$$

$$T_{+-}(K^+p) = \frac{2}{\sin\pi\alpha} A_1 e^{B_1 t} J_1(R_1 \sqrt{-t}) (s)^{\alpha(t)} \quad (5.6b)$$

$$T_{++}(K^-p) = \text{Re } T_{++} + i [A_P e^{B_P t} (s)^{\alpha_P(0)} + A_R e^{B_R t} J_0(R \sqrt{-t}) \sqrt{s}] \quad (5.6c)$$

$$T_{+-}(K^-p) = 2 \left[\frac{\cos\pi\alpha}{\sin\pi\alpha} + i \right] A_1 e^{B_1 t} J_1(R_1 \sqrt{-t}) (s)^{\alpha(t)} \quad (5.6d)$$

(The factor 2 in 2f (5.5c) has been absorbed in A_R (5.6c)).

Before we go on to discuss $\text{Re } T_{++}$ (in (v) below) several remarks on the above parameterisation are in order:

- a) The parameters A_P , A_R and $\alpha_P(0)$ in (5.6a,b) can be determined by fitting total cross-sections. We start by letting $\alpha_P(0) \approx 1.06$ on the basis of the fits to $\pi^\pm p$ data in Chapter 4 but, if the $K^\pm p$ data does not yield satisfactory fits, alter it accordingly.
- b) $\alpha(t)$ is known while B_P , B_R and R (the latter two occur only in $T_{++}(K^-p)$) are obtained from fits to $\frac{d\sigma}{dt}$ data using A_P , A_R values obtained from fitting total cross-sections.
- c) The parameter R_1 in the flip amplitudes is predetermined on account of the factor $\frac{1}{\sin\pi\alpha(t)}$. This factor has a pole at $\alpha(t) = 0$ (i.e. $-t \approx 0.55 \text{ (GeV/c)}^2$) which can only be removed by taking R_1

to be such that the second zero of $J_1(R_1 \sqrt{-t})$ occurs at $-t \approx 0.55$ $(\text{GeV}/c)^2$ (the first zero of $J_1(R_1 \sqrt{-t})$ occurs at $t = 0$). The second zero of $J_1(R_1 \sqrt{-t})$ is at $R_1 \sqrt{-t} = 3.83$ which, for $-t \approx 0.55$ $(\text{GeV}/c)^2$ yields $R_1 = 5.164$ $(\text{GeV}/c)^{-1}$. We are thus left with two parameters A_1 and B_1 to fit $K^\pm p$ polarisations. Our experience with $\pi^\pm p$ scattering leads us to expect $B_1 = 0$ in which case we are left with only one parameter which is to be used in both $K^+ p$ and $K^- p$ polarisations. Substantial disagreement between those parameter values as obtained from fits to $K^+ p$ polarisation and as obtained from fits to $K^- p$ polarisations could be regarded as indicative of EXD breaking or of other contributions or both.

- d) It may be pointed out that the parameterisation of $\text{Im } T_{++}(K^- p)$ (5.6c) is such that contributions from ρ and A_2 to the non-flip amplitude are difficult to disentangle if they are peripheral as expected on the basis of the DAM. Hence one can also look upon the " J_0 " term in (5.6c) as an "effective" parameterisation in which various Regge contributions have been lumped together. The same can be said of the $f-\omega$ trajectories in relation to the flip amplitudes.

We now take up $\text{Re } T_{++}$.

- (v) The DAM does not give any prescription for $\text{Re } T_{++}$. In all DAM-based fits to elastic data it is always assumed that $\text{Re } T_{++} = 0$. In fact this practice extends to many other fits as well. The $\frac{d\sigma}{dt}(K^+ p)$ data however show a curvature that persists up to the highest energies at which measurements exist. If one assumes strong EXD and takes $\text{Re } T_{++} = 0$ as suggested by the "rules" of the DAM then, since the Pomeron has a behaviour $A_p e^{\frac{B_p t}{2}} \alpha_p(0)$ (s) one obtains

a differential cross-section without curvature in contradiction with the data. The assumption $\text{Re } T_{++} = 0$ has therefore to be abandoned if, as is desirable from the point of view of DAM, one does not break strong EXD in $\text{Im } T_{++}$. If one assumes that $\text{Re } T_{++}$ is due to "ordinary" Regge exchanges and that (as assumed in the DAM) the Pomeron is imaginary one has two problems. Firstly a real part due to non-Pomeron exchanges will, presumably, behave as $\sqrt{s}$ in energy. Since now $\text{Im } T_{++}$ is pure Pomeron and $\text{Re } T_{++}$ is pure non-Pomeron and, since the energy-dependence is different in the two cases, one will again end up with a differential cross-section $((\text{Im } T_{++})^2 + (\text{Re } T_{++})^2)$ without curvature at high energies where the Pomeron dominates. Secondly the only asymptotic $\text{Re } T_{++}$ one can reasonably think of is the usual Regge phase rule which for $\tau = +1$ exchanges (i.e. f in this case) has a $\cot \frac{\pi\alpha}{2}$ "J₀" behaviour - there is a pole in $\cot \frac{\pi\alpha}{2}$ at $t = 0.55 \text{ (GeV/c)}^2$ which can not be eliminated if "J₀" has a radius of interaction of approximately 1 fm as required in the DAM and as generally accepted. The only reasonable alternative that we are left with, without making a major break with the framework of the DAM is to relax the "rule" that the Pomeron is imaginary and introduce a $\text{Re } T_{++}$ due to the Pomeron. Here we note that at lab. momenta around 10 GeV/c the experimental value of $\frac{\text{Re } T_{++}(s,0)}{\text{Im } T_{++}(s,0)}$ is -0.2 to -0.3 for K^+_p but is consistent with zero for K^-_p . We therefore suggest the ansatz

$$\text{Re } T_{++} = \left(\cot \frac{\pi\alpha_p}{2} + \rho e^{\gamma t} \right) \text{Im } P \quad (5.7a)$$

where $\rho = 0$ ($\rho \equiv \frac{\text{Re } T_{++}(s,0)}{\text{Im } T_{++}(s,0)}$ here) for the K^-_p case and can be neglected for the K^+_p case above 20 GeV/c. $\alpha_p(t)$ is taken as a linear Pomeron trajectory

$$\alpha_p(t) = \alpha_p(0) + 0.3t \quad (5.7b)$$

where $\alpha_P(0)$ is obtained from fits to total cross-sections. The parameter γ is determined by data fitting and is not needed for the K^-p

case. Although a value of ρ of the order 0.2 does not affect differential cross-sections appreciably it could presumably affect polarisations more perceptibly.

With the aforementioned points in mind the different observables can be expressed as follows:

$$\sigma_{\text{tot}}(K^+p) = \frac{0.389}{2q \sqrt{s}} A_P(s)^{\alpha_P(0)} \quad (5.8a)$$

$$\sigma_{\text{tot}}(K^-p) = \frac{0.389}{2q \sqrt{s}} [A_P(s)^{\alpha_P(0)} + A_R \sqrt{s}] \quad (5.8b)$$

$$\frac{d\sigma}{dt}(K^+p) = \frac{0.389}{64\pi s q^2} \left| \left\{ \left(\cot \frac{\pi\alpha}{2} + \rho e^{\gamma t} \right)^2 + 1 \right\} A_P^2 s^{2\alpha_P(0)} \right| \quad (5.9a)$$

$$\begin{aligned} \frac{d\sigma}{dt}(K^-p) = \frac{0.389}{64\pi s q^2} & \left| \cot \frac{\pi\alpha_P}{2} A_P s^{\alpha_P(0)} + \right. \\ & \left. + i \{ A_P(s)^{\alpha_P(0)} + A_R e^{B_R t} J_0(R\sqrt{-t}) \sqrt{s} \} \right|^2 \quad (5.9b) \end{aligned}$$

$$P(K^\pm p) = 2 \operatorname{Im}[T_{++}(K^\pm p) T_{+-}^*(K^\pm p)] / [|T_{++}(K^\pm p)|^2 + |T_{+-}(K^\pm p)|^2] \quad (5.10)$$

where $T_{++}(K^\pm p)$ and $T_{+-}(K^\pm p)$ are given in equations (5.6a-d).

5.3A Fits to K^+p Data

i) $\sigma_{\text{tot}}(K^+p)$: (Data from Carroll et al. (1976), Galbraith et al. (1965)).

Fixing $\alpha_p(0) = 1.06$ on the basis of results obtained in the previous chapter and fitting $\sigma_{\text{tot}}(K^+p)$ (in the range $6.8 \leq p_{\text{lab}} \leq 280$ GeV/c) according to equation (5.8a) yields an unsatisfactory fit ($\chi^2/n = \frac{113.4}{21}$). The fit can be improved to $\chi^2/n = \frac{35.53}{21}$ if $\alpha_p(0)$ is treated as a free parameter along with A_p . The resulting values are shown below.

TABLE 5.1

Parameter values from fits to $\sigma_{\text{tot}}(K^+p)$ according to equation (5.8a)

$\alpha_p(0)$	A_p	χ^2/n
1.06	$35.279 \pm .079$	113.4/21
$1.069 \pm .0004$	$33.806 \pm .039$	35.53/21

We choose to work with $\alpha_p(0) = 1.069$ (fit shown in Fig. 5.1) instead if $\alpha_p(0) = 1.06$ since we are interested in good χ^2 values although the slight discrepancy with the $\pi^\pm p$ case is aesthetically undesirable. (We have checked up that while $\sigma_{\text{tot}}(\pi^+p)$ can be fitted well with $\alpha_p(0) = 1.069$, $\sigma_{\text{tot}}(\pi^-p)$ cannot be fitted satisfactorily with this value.) However such minor discrepancies are encountered in several well known fits. For instance Collins et al. (1974b) find $\alpha_p(0) \sim 1.06$ but in Collins et al. (1974a) this is revised to $\alpha_p(0) \sim 1.07$. Joynson et al. (1977) find $\alpha_p(0) = 1.08$ on the other hand.

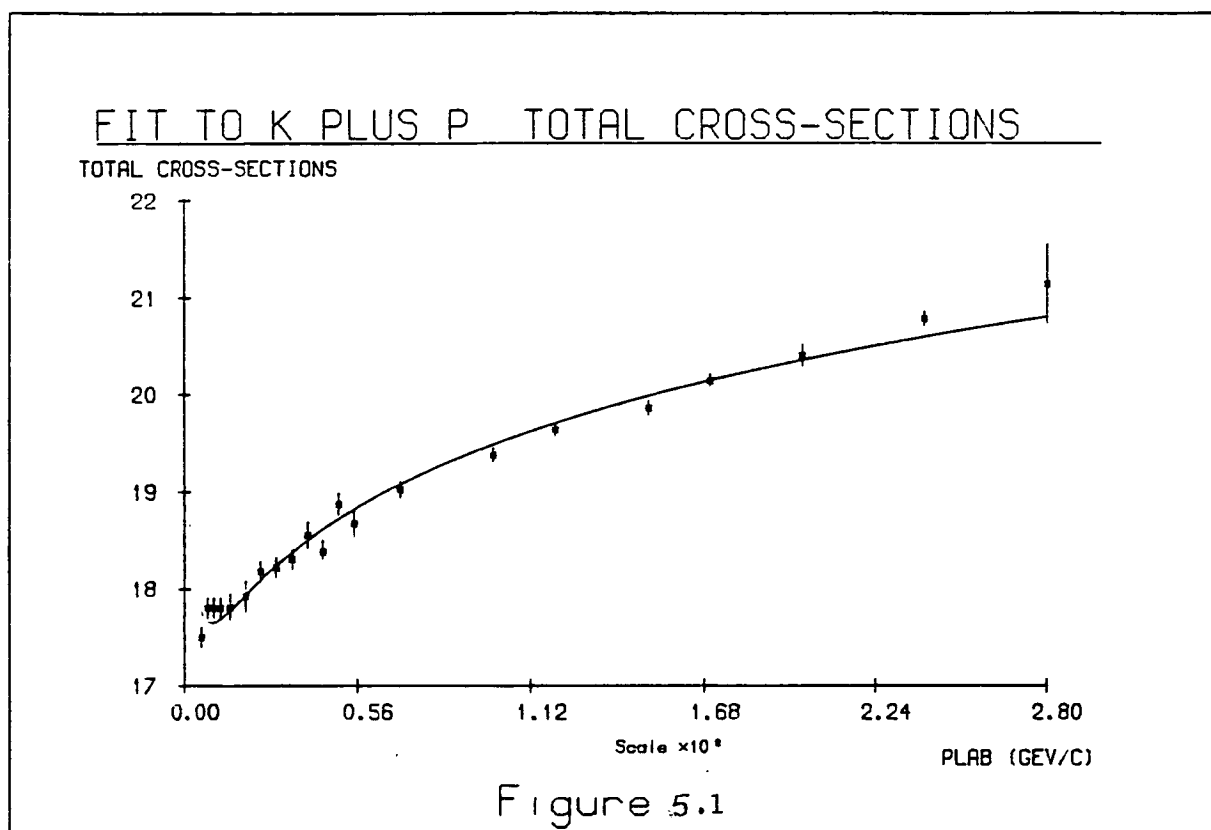


Fig. 5.1. Fit to $\sigma_{\text{tot}}(K^+p)$ according to eqn. (5.8a) and Table 1. Curve shows the theoretical calculation and points with error bars the experimental values.

Differential Cross-Sections: The differential cross-sections have been fitted in the range $6.8 \leq p_{\text{lab}} \leq 200$ GeV/c using equations (5.6a, 5.7a,b and 5.9). Although the inclusion of ρ ($\rho \equiv \frac{\text{Re } T_{++}(s,0)}{\text{Im } T_{++}(s,0)}$) is unnecessary above 9.8 GeV/c or so in fitting $\frac{d\sigma}{dt}(K^+p)$ data we have continued using it in our fits above this laboratory momentum so as to see whether γ might show some pattern in its energy dependence. The results of our fits to $\frac{d\sigma}{dt}(K^+p)$ are shown in Table 5.2. Some typical fits are shown in Fig. 5.2a-e. In our fits we used the A_p and $\alpha_p(0)$ values obtained from our fits to $\sigma_{\text{tot}}(K^+p)$. We find the 140 GeV/c and 175 GeV/c data require minor alterations in the A_p value if good χ^2 values are to be obtained. The overall $\chi^2/n = \frac{295.2}{185} \approx 1.6$. The parameter of interest is B_p which shows a steady increase with energy and in Table 5.3 we show three different fits to this parameter in an attempt to ascertain its energy-dependence. It is clear from Table 5.3

TABLE 5.2

Fits to $\frac{d\sigma}{dt}$ for $K^+p \rightarrow K^+p$ (equations 5.6a,b; 9a)

P_{Lab}	$\rho \equiv \frac{Re T_{++}(s,0)}{Im T_{++}(s,0)}$	γ	B_P	χ^2/n
6.8	-0.28	2.827 ± 5.37	2.429 ± 0.085	$\frac{5.0}{11}$
9.8	-0.21	0.941 ± 0.191	2.584 ± 0.039	$\frac{14.25}{15}$
12.8	-0.185	0.505 ± 0.042	2.724 ± 0.044	$\frac{31.31}{15}$
14.8	-0.15	0.921 ± 0.229	2.667 ± 0.056	$\frac{21.07}{15}$
50.0	-0.046	7.42 ± 6.97	3.407 ± 0.088	$\frac{16.65}{21}$
70.0	+0.04	0.952 ± 0.224	3.703 ± 0.194	$\frac{33.61}{16}$
100.0	+0.13	7.99 ± 7.89	3.692 ± 0.058	$\frac{62.65}{35}$
140.0*	+0.04	1.011 ± 0.017	3.538 ± 0.077	$\frac{21.65}{16}$
175.0**	0.0	-	3.66 ± 0.054	$\frac{46.4}{17}$
200.0	0.0	-	3.954 ± 0.087	$\frac{42.8}{24}$

*Here $A_P = 31.3$ instead of 33.805

**Here $A_P = 32.3$ instead of 33.805 Overall $\chi^2/n = \frac{295.21}{185} = 1.59$

TABLE 5.3

Energy Dependence of $B_P(K^+p)$

Parameterisation No.	Parameterisation	χ^2/n
1.	$B_P = b_0 + b_1 \ln s = 1.344 + 0.420 \ln s$ $\pm 0.118 \pm 0.029$	$\frac{25.3}{10}$
2.	$B_P = b_2 (\ln s)^{\beta_2} = 1.390 (\ln s)^{0.56 \pm 0.40}$	$\frac{23}{10}$
3.	$B_P = b_3 [\ln(\ln s)]^{\beta_3} = 2.401 [\ln(\ln s)]^{0.790 \pm 0.046}$ ± 0.053	$\frac{22.7}{10}$

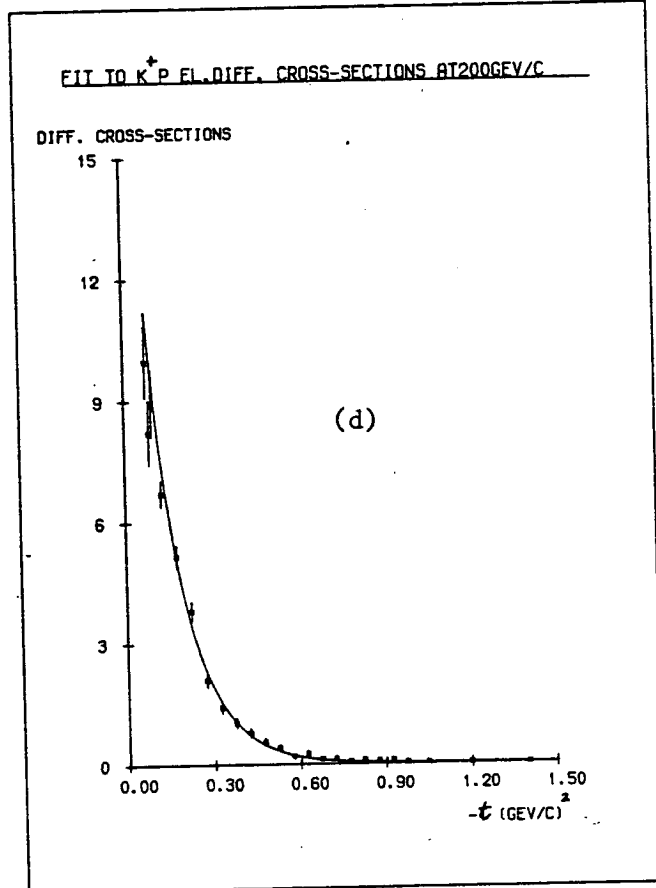
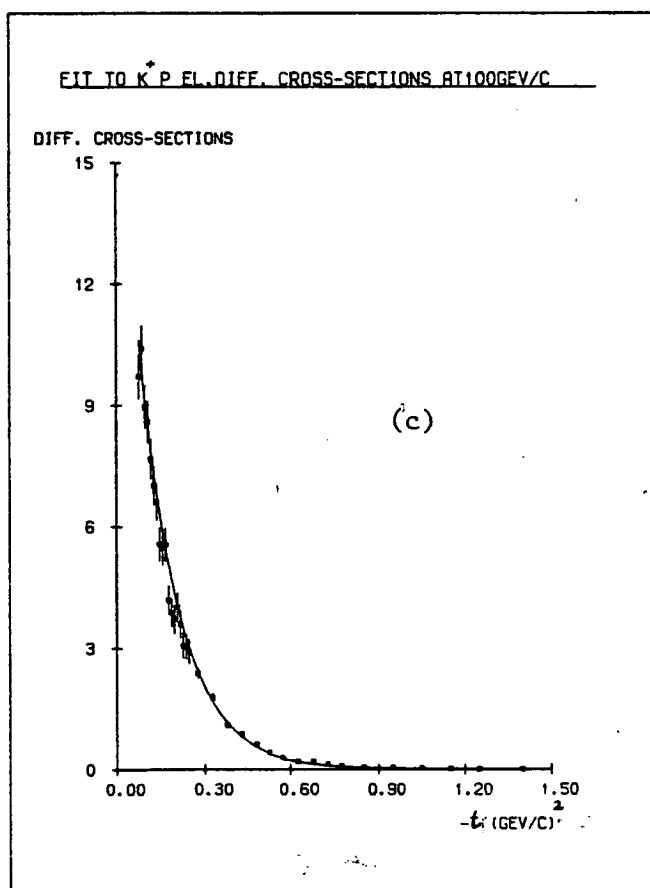
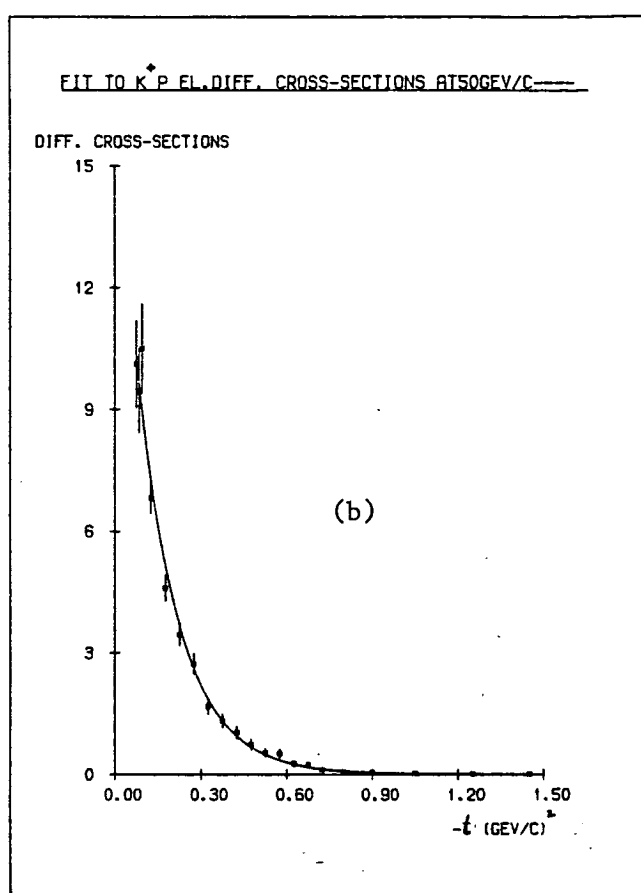
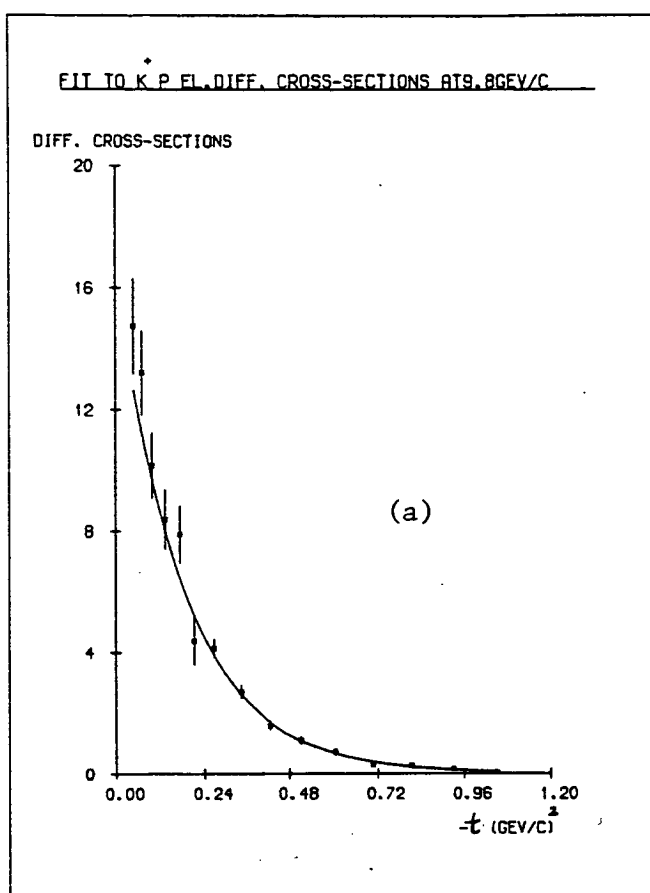


Fig. 5.2a-d. Some typical fits to $\frac{d\sigma}{dt}(K^+p)$ according to equation 5.9a (see also Table 6.2).

that the different parameterisations describe the energy dependence of B_p equally well so that one cannot prefer one form over the other except perhaps marginally. The virtual indistinguishability of these parameterisations can also be seen from Fig. 5.3. The parameter γ (equation 5.7a) can be represented quite well as $\gamma = 0.18 \ln s$ or $\gamma = 0.42 (\ln \ln s)^{1.5}$ $\{\chi^2/n \sim 1.47$ for the first case and 1.33 for the second case}.

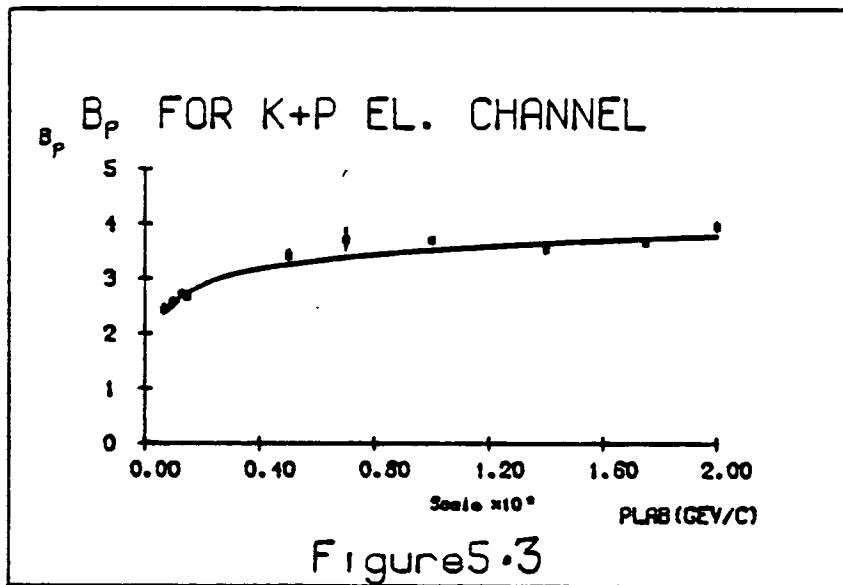


Fig. 5.3 Fits to B_p according to parameterisations of Table 5.3. As may be seen, the different fits are virtually indistinguishable.

Polarisations: Having determined the various parameters occurring in $T_{++}(K^+p)$ by fitting total and differential cross-section data we consider K^+p polarisations at 10, 14 and 45 GeV/c (Borghini et al. (1971b), Gaidot et al. (197)). The parameterisation of the amplitudes is given in equations (5.6a,b) and the formula for polarisation is given in equation (5.10). The two parameters of T_{+-} there are A_1 and B_1 . Our "best fits" to the data give A_1 and B_1 values shown in Table 5.4 below. The overall χ^2/n is $\frac{42.94}{34} = 1.26$.

TABLE 5.4

Fits to K^+p Polarisations

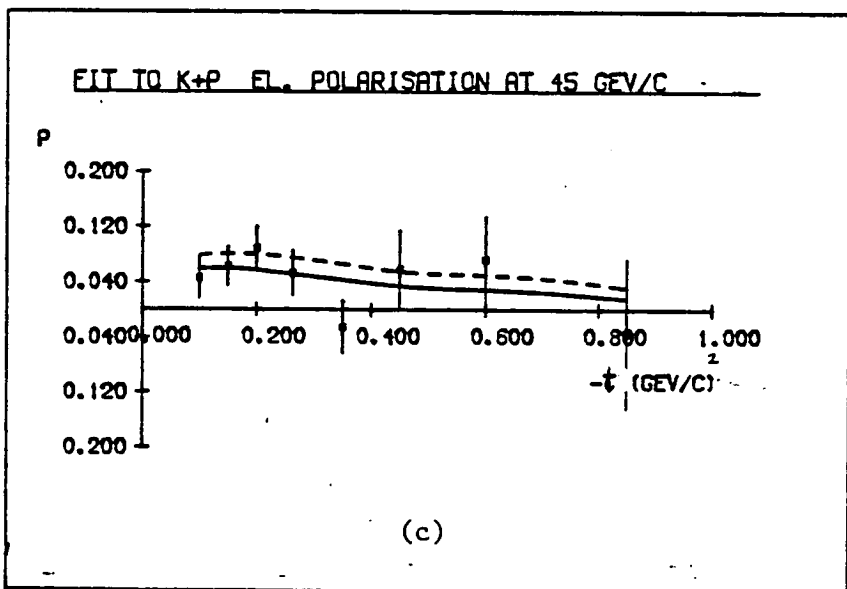
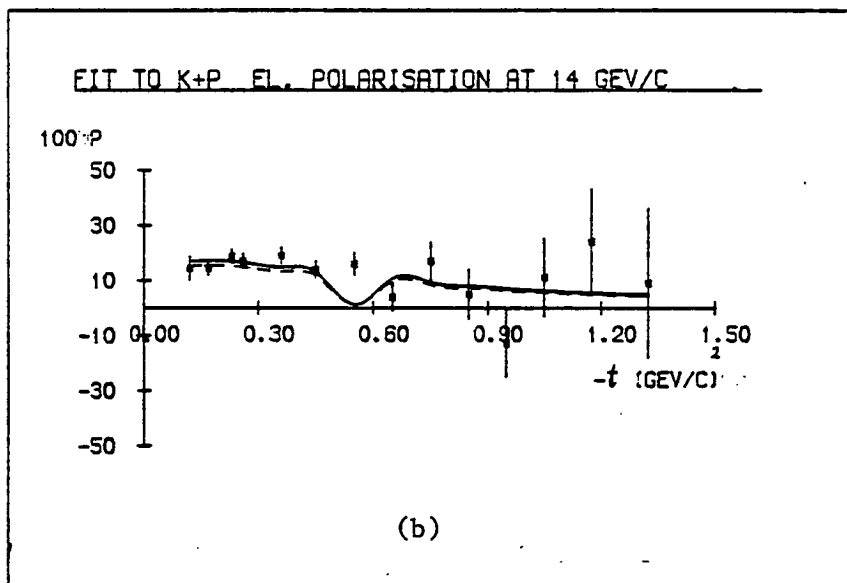
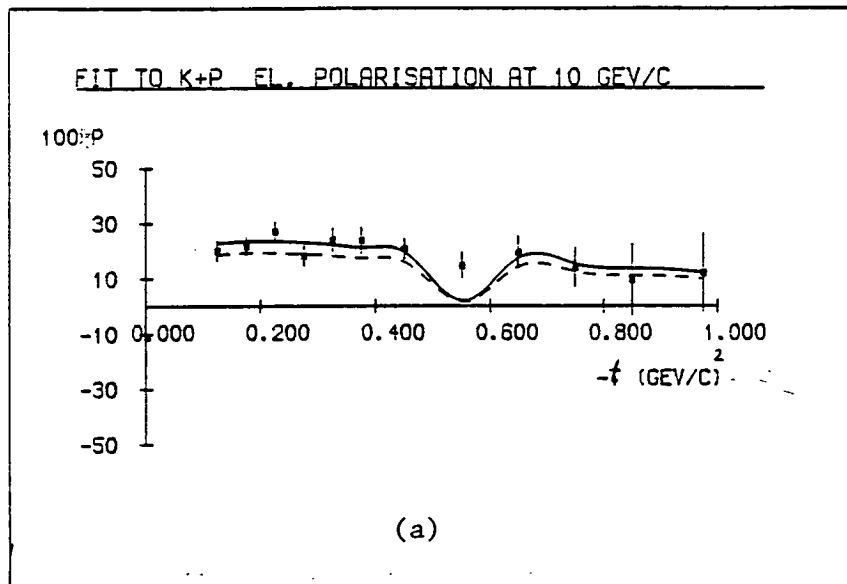
$p_{\text{Lab}} \text{ (GeV/c)}$	A_1	$B_1 \left(\frac{\text{GeV}}{c}\right)^{-2}$	χ^2/n
10	18.408 ± 1.843	0.000 ± 0.269	$\frac{12.69}{12}$
14	16.65 ± 2.074	0.002 ± 0.426	$\frac{24.8}{14}$
45	11.987 ± 5.65	0.568 ± 0.279	$\frac{5.45}{8}$

$$\text{Overall } \chi^2/n = \frac{42.94}{34} = 1.26$$

The parameter A_1 has the same value at all three momenta, within errors. B_1 is consistent with zero at 10 and 14 GeV/c. We have explored the possibility whether one can take $B_1 = 0$ throughout and obtain acceptable χ^2 values since, in that case, only one (energy independent) parameter determines both K^+p and K^-p polarisations. We will take up this point in Sec. 5.3B (iii) below. The fits obtained by using A_1 values quoted in Table 5.4 (and with $B_1 = 0$) are shown as solid curves in Fig. 5.4a-c.

5.3B Fits to K^-p Data

(i) Total Cross-sections: Using $\sigma_{\text{tot}}(K^-p)$ from the same references as cited in connection with $\sigma_{\text{tot}}(K^+p)$ we have fitted $\sigma_{\text{tot}}(K^-p)$ in the range $6 \leq p_{\text{lab}} \leq 240$ GeV/c. The parameterisation is given in equation (5.8b). Since K^-p is not an exotic channel a non-negligible Regge contribution to $\sigma_{\text{tot}}(K^-p)$ is expected. The numerical results are given in Table 5.5 below for both the $\alpha_p(0) = 1.06$ and $\alpha_p(0) = 1.069$ cases.



Figs. 5.4a-c. Fits to $P(K^+p)$ according to equations (5.6a,b) and (5.10). The solid curves are fits with the values of A_1 given in Table 5.4, while dashed lines represent fits with $A_1 = 15$.

TABLE 5.5

Parameter Values from fits to $\sigma_{\text{tot}}(K^-p)$ according to equation (5.8b)

$\alpha_p(0)$	A_p	A_R	χ^2/n
1.06	35.350 ± 0.087	52.503 ± 1.357	$\frac{27.85}{29}$
1.069	33.058 ± 1.026	60.680 ± 3.973	$\frac{21.80}{29}$

It is clear that, from the χ^2 point of view, the fit with $\alpha_p(0) = 1.069$ is marginally better. However its real advantage over the fit with $\alpha_p(0) = 1.06$ lies in the fact that it is aesthetically more satisfying since $\sigma_{\text{tot}}(K^+p)$ can be fitted satisfactorily only with $\alpha_p(0) = 1.069$. Furthermore the values of $A_p(K^-p)$ (with $\alpha_p(0) = 1.069$) is equal (within errors) to that of $A_p(K^+p)$ (with $\alpha_p(0) = 1.069$). If we treat $\alpha_p(0)$ as a free parameter fitting $\sigma_{\text{tot}}(K^-p)$ we end up with $\alpha_p(0) = 1.070 \pm .004$ as giving the best fit with the same A_p , A_R and χ^2/n values as displayed in Table 5.5 for the $\alpha_p(0) = 1.069$ case. We therefore work with $\alpha_p(0) = 1.069$ in the following calculations using the A_p and A_R values quoted against $\alpha_p(0) = 1.069$ in Table 5.5. The fit is shown in Fig. 5.5.

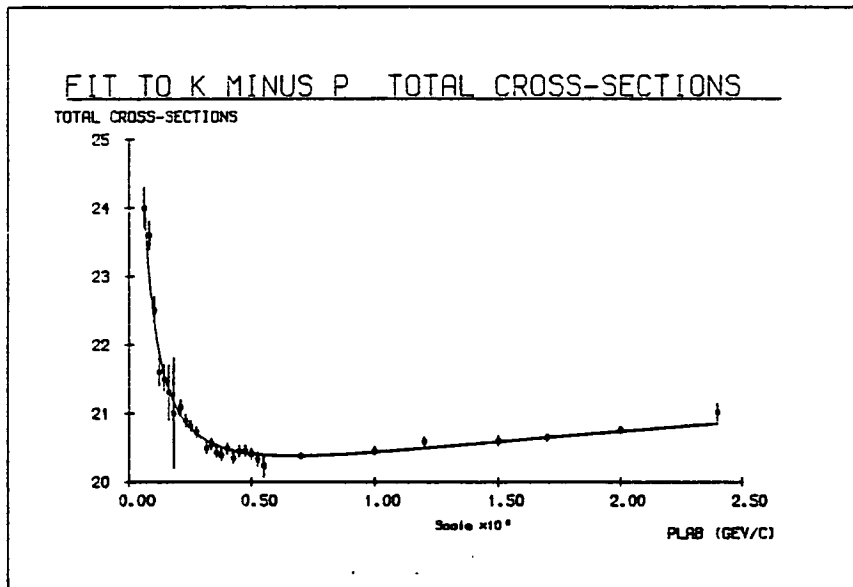


Fig. 5.5. Fit to $\sigma_{\text{tot}}(K^-p)$ according to equation (5.8b).

(ii) Differential Cross-sections:

We have fitted the $\frac{d\sigma}{dt}(K^-p)$ data in the range $7.2 \leq p_{\text{lab}} \leq 200$ GeV/c up to $|t| \sim 1$ (GeV/c)². The A_P and A_R values used in the fitting $\frac{d\sigma}{dt}(K^-p)$ were taken from fits to $\sigma_{\text{tot}}(K^-p)$ [$A_P = 33.058$, $A_R = 60.680$]. The values of the three parameters B_P , B_R , and R (equations 5.6b,d) are shown below in Table 5.6. (Fig. 5.6a-d show some typical fits). We find that we can obtain satisfactory fits to data without altering A_P and/or A_R values at any energy. It can be seen from the table that, as in the π^+p and K^+p cases, B_P increases slowly with energy. However, unlike the $\pi^\pm p$ case, the parameters B_R and R associated with the non-Pomeron contribution appear to be energy independent. To quantitatively ascertain these impressions we have fitted each of the three parameters B_P , B_R , R with three different parameterisations. The results are shown in Table (5.7) below.

TABLE 5.6

Fits to K^-p differential cross-sections (equations 5.6c,d, 5.9b)

p_{lab} (GeV/c)	B_P	B_R	R	χ^2/n
7.2	2.7456 ± 0.110	2.0916 ± 0.861	4.5248 ± 0.002	$\frac{11.16}{10}$
9.0	2.9711 ± 0.106	3.149 ± 0.368	4.5133 ± 0.2016	$\frac{7.51}{10}$
10.1	2.9246 ± 0.037	3.404 ± 0.409	4.5199 ± 0.0315	$\frac{26.28}{23}$
11.88	2.8511 ± 0.074	2.6062 ± 0.776	4.4578 ± 0.0332	$\frac{3.74}{10}$
14.3	2.8989 ± 0.038	2.1367 ± 0.287	5.1558 ± 0.082	$\frac{84.71}{59}$
15.91	3.1381 ± 0.1045	8.2132 ± 9.0799	4.7445 ± 0.5788	$\frac{8.85}{8}$
25.2	3.5115 ± 0.0312	2.8996 ± 0.2889	4.471 ± 0.1468	$\frac{35.5}{38}$
40.1	3.5108 ± 0.0201	3.732 ± 0.0976	7.0390 ± 0.3382	$\frac{37.67}{38}$
50.0	3.1976 ± 0.0882	2.934 ± 0.3144	4.4326 ± 0.6325	$\frac{28.05}{25}$
70.0	3.7622 ± 0.1023	2.862 ± 0.0220	4.4670 ± 0.0204	$\frac{20.83}{12}$
100.0	3.553 ± 0.0337	6.5138 ± 1.532	4.4938 ± 0.9988	$\frac{66.85}{39}$
140.0	3.947 ± 0.0826	2.8614 ± 0.009	4.4697 ± 0.0081	$\frac{38.7}{13}$
175.0	3.9368 ± 0.1222	2.8613 ± 0.0135	4.4684 ± 0.0164	$\frac{28.27}{15}$
200.0	3.6945 ± 0.1103	2.897 ± 0.0923	4.5198 ± 0.1497	$\frac{13.65}{17}$

$$\text{Overall } \chi^2/n = \frac{411.77}{317} = 1.298$$

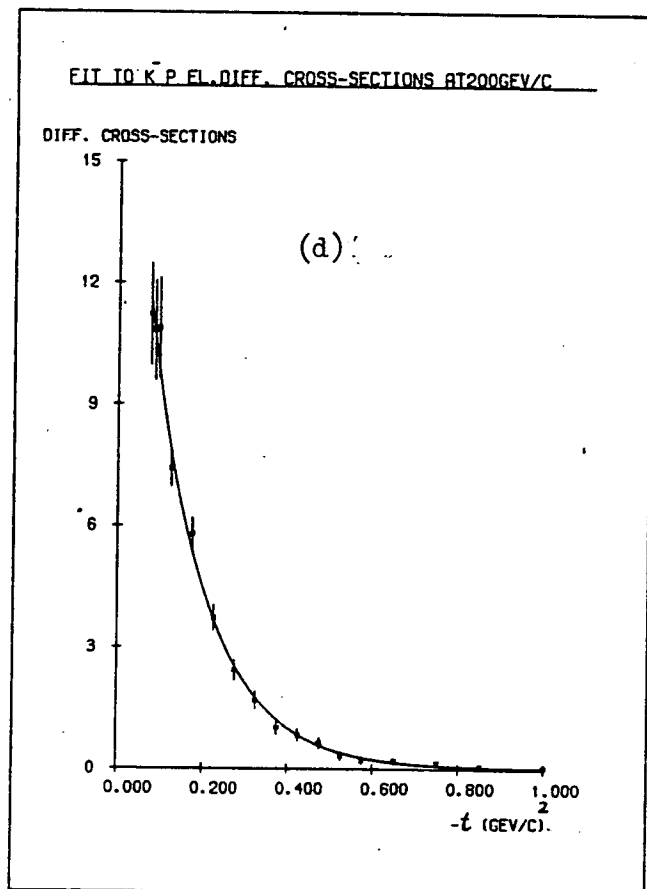
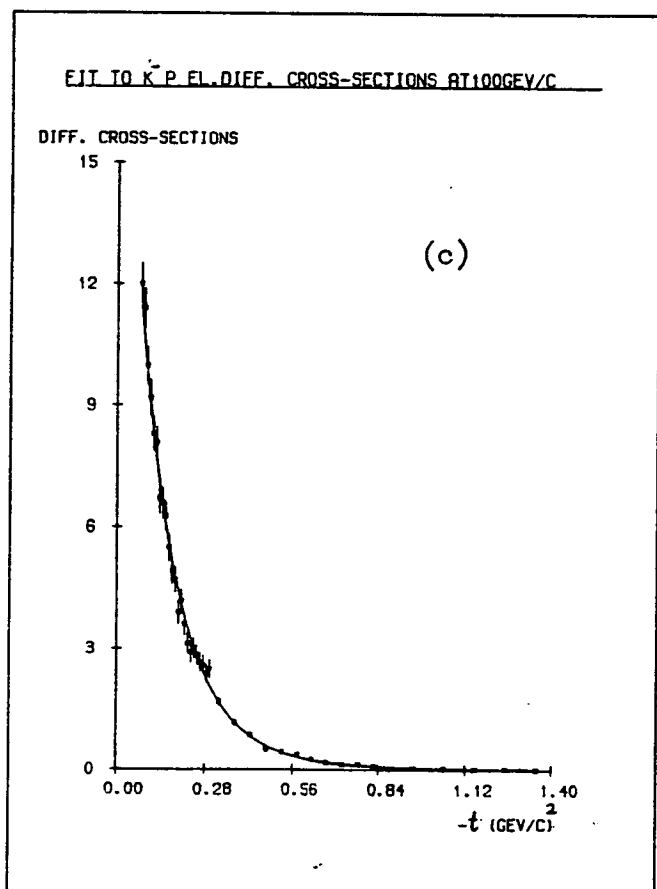
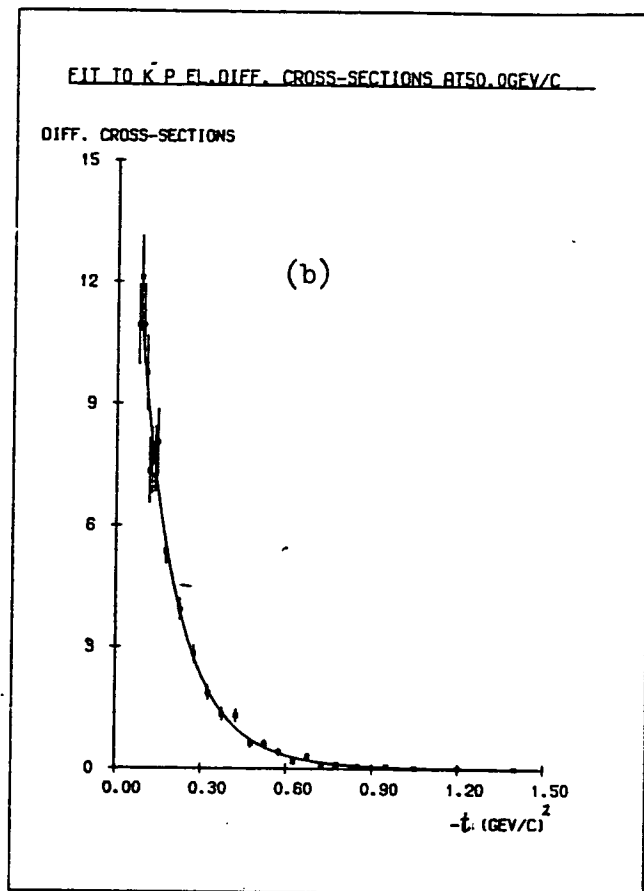
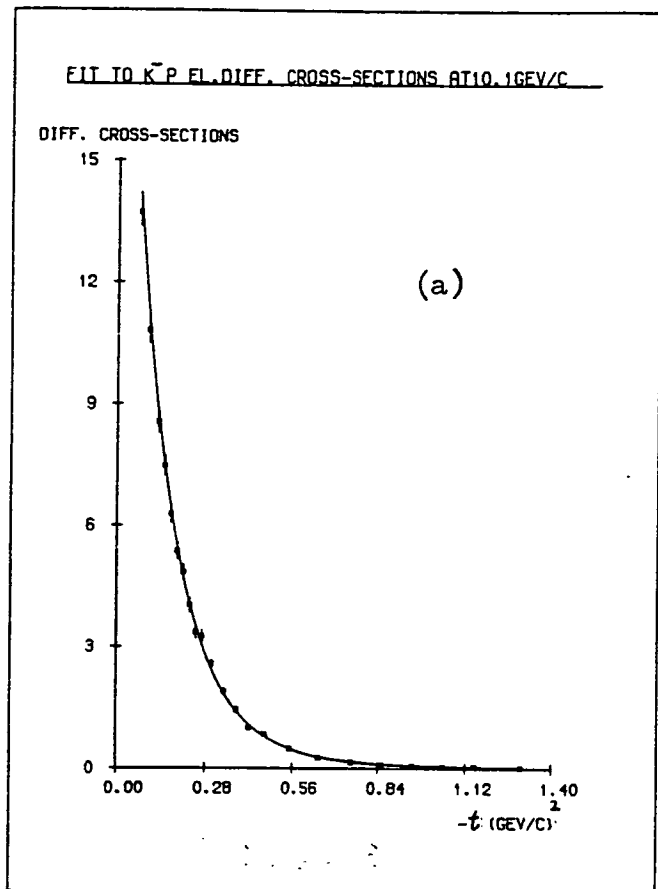


Fig. 5.6.a-d Typical fits to $\frac{d\sigma}{dt}(\bar{K}^0 p)$ according to equation (5.9b).

TABLE 5.7

Energy Dependence of K^-p Parameters B_P , B_R and R

Parameterisation for energy dependence		(1)	(2)	(3)
		$x + y \ln s$	$x (\ln s)^y$	$x (\ln \ln s)^y$
Parameter				
B_P	$x =$	1.9988 ± 0.0416	1.9022 ± 0.0389	2.7869 ± 0.0292
	$y =$	0.3309 ± 0.0141	0.4053 ± 0.0155	0.5672 ± 0.0239
B_R	$x =$	2.8665 ± 0.0166	2.8626 ± 0.0201	2.8681 ± 0.0066
	$y =$	0.0000 ± 0.0005	0.0012 ± 0.0014	0.0000 ± 0.0025
R	$x =$	4.5205 ± 0.0023	4.5201 ± 0.0031	4.5206 ± 0.0032
	$y =$	0.0000 ± 0.0004	0.0000 ± 0.0005	0.0000 ± 0.0002

Parameterisation No. 1 for B_P , when interpreted as a Regge pole type dependence, yields a Pomeron slope of $\alpha_P' = 0.3309$ close to that obtained for π^+p and K^+p cases. One can also infer that both B_R and R are energy independent with $B_R = 2.86 (\text{GeV})^{-2}$ and $R = 4.52 (\text{GeV})^{-1}$.

(iii) Polarisations: The parameterisation for the helicity flip amplitude for $K^-p \rightarrow K^-p$ is given in equation (5.6d). The parameterisation is based on strong EXD of ρ - A_2 exchanges and on their peripherality in impact-parameter space. The two parameters whose values are to be obtained by fitting polarisation data are A_1 and B_1 . If the assumptions on which our parameterisation is based are correct then one should obtain A_1 and B_1 values close to or equal to those

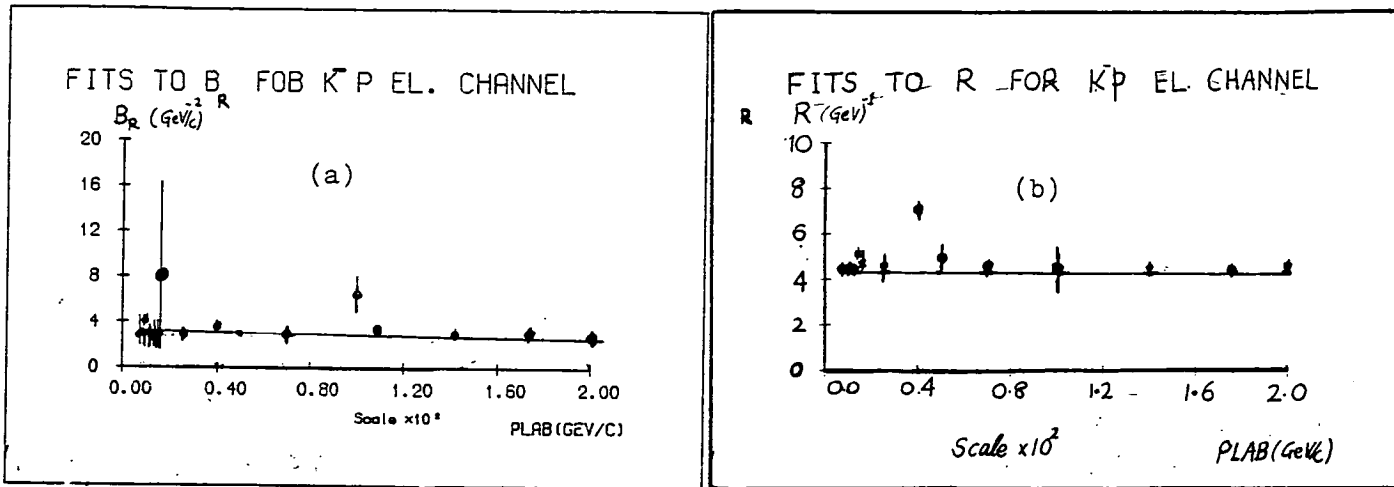
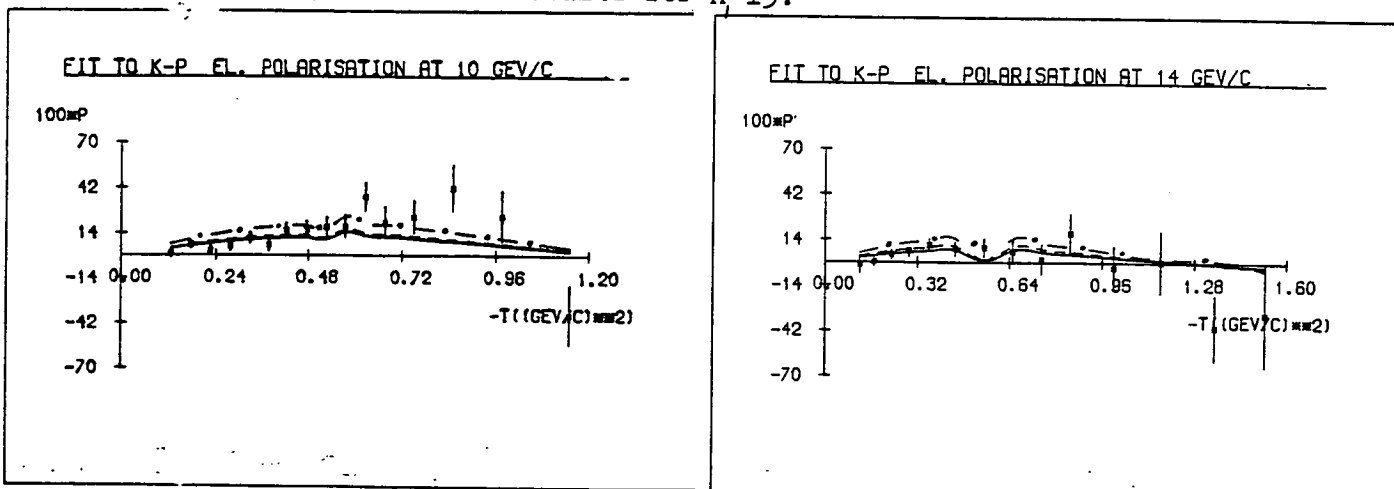


Fig. 5.7 a,b): Values of parameters B_R and R for Kp from table 5.6. The straight lines correspond to $B=2.86$ and $R=4.52$.

Fig. 5.8 a,b): Fits to $P(Kp)$ according to equations 5.6 and 5.10. The solid curve depicts results obtained by using parameter values from table 5.8 while the dashed curve (---) corresponds to $A_1=10$. The curve with dashes and dots (-.-.-) shows the results for $A_1=15$.



obtained by fitting K^+p polarisations. Treating A_1 and B_1 as free parameters yields the values given in Table 5.8 below for the "best fits" (data from Borghini et al. (1971b), Gaidot et al. (1975)). As can be seen there is a serious discrepancy between the A_1 values

TABLE 5.8

Fits to K^-p Polarisations

p_{lab} (GeV/c)	A_1	B_1	χ^2/n
10	8.96 ± 1.51	0.000 ± 0.217	$\frac{33.6}{16}$
14	7.681 ± 1.60	0.000 ± 1.835	$\frac{12.6}{14}$
40	0.203 ± 4.48	0.185 ± 5.087	$\frac{8.00}{11}$

$$\text{Overall } \chi^2/n = \frac{54.2}{41} = 1.32$$

obtained by fitting K^+p polarisations (Table 5.4) and the A_1 values obtained by fitting K^-p polarisations (Table 5.8). However before concluding that the assumptions on which our parameterisation is based are wrong, we have tried to explore the possibility whereby one could obtain fits, using A_1 and B_1 values common to $K^\pm p$ data, which, although inferior to those described in Tables 5.4 and 5.8 above, still have reasonable χ^2 values. Since B_1 is generally consistent with zero in both cases it is natural to set $B_1 = 0$. Then we have tried different A_1 values to fit data. For instance $A_1 = 15$ gives a reasonable fit to K^+p data with an overall $\chi^2/n = 1.6$ (the break-up is as follows: $\chi^2/n = 1.8$ for 10 GeV/c, 1.99 for 14 GeV/c, 1.05 for 45 GeV/c data). However the corresponding χ^2/n for K^-p is

4 which is unacceptable. We have found that the discrepancy cannot be removed but that for $A_1 = 10$ we can fit K^-p polarisations reasonably with an overall χ^2/n of 2.2 - (the break up is as follows: $\chi^2/n = 2.14$ for 10 GeV/c, 2.06 for 14 GeV/c and 2.48 for 40 GeV/c data). This still leaves a discrepancy of about 30% in the A_1 values required for the K^+p and K^-p cases.

5.4 Discussion:

As remarked earlier, most DAM-based fits to $K^\pm p$ data have aimed at establishing the " J_0 " character of the $C = -1$ exchange in the non-flip amplitude and all of them assume an imaginary Pomeron term. Our fits to $\frac{d\sigma}{dt}(K^+p)$ indicate that, as discussed in Section 5.2, the K^+p data force on us a real Pomeron part ($\text{Re } P \sim \cot \frac{\pi\alpha_P}{2} \text{Im } P$) if we wish to retain the assumption on the impact-parameter structure of the Pomeron made in the "rules" of the DAM. One could perhaps introduce structure in a pure imaginary Pomeron term by introducing an ad hoc peripheral component. However since this assumption is ad hoc and since it introduces additional parameters it is, from our point of view, not very satisfactory. The total cross-section fit for K^+p collisions determines $\alpha_P(0)$ very accurately and confirms the strong EXD of the $f-\omega$ exchanges in the non-flip amplitude. We assumed $\alpha_P' \sim 0.3$. Our fits to $B_P(K^+p)$ yield $\alpha_P' \sim 0.4$ (Table 5.4). Our fits to $B_P(K^-p)$ yield $\alpha_P' \sim 0.33$, however (Table 5.7). These values for α_P' obtained from $K^\pm p$ data agree closely with those obtained from $\pi^\pm p$ data in the previous chapter. It is interesting to note that both B_R and R are energy independent, contrary to the $\pi^\pm p$ case.

Before we discuss polarisations let us consider the slope parameters $b_{K^\pm p}$. For the K^+p case

$$b \equiv \frac{d}{dt} \ln \left. \frac{d}{dt} \right|_{t=0} = 2B_P + \frac{2\gamma\rho^2}{1 + \rho^2} \quad (5.11)$$

It is clear that at higher energies the second term vanishes. The values of b calculated according to this relation, using B_p and γ values from Table 5.2, are shown against the experimental values in table 5.9a. The theoretical results, although slightly lower, are in general agreement with the experimental values. For the K^-p case the expression for b is the same as that in the $\pi^\pm p$ case (equation 4.9 of Chapter 4). Using that equation (i.e. 4.9) and B_p , B_R and R values from Table 5.6 we have calculated b_{K^-p} values. The results are shown in table 5.9b and can be seen to agree with the experimental values displayed there. The only DAM-based analysis of $K^\pm p$ slope parameters of which we are aware is that of Minami and Terada (1974). They take B_p , (B_R) (their B_p (B_R) equals twice our B_p (B_R) - we have therefore divided their expressions by two for comparison) as $B_p = 1.95 + 0.39 \ln s$ and $B_R = 0.05 + 0.9 \ln s$. They also assume an energy independent $R = 4.58$. Their expressions and values for B_p are the same for both $K^\pm p$ cases, while B_R and R are related only to the K^-p case, there being no 'ordinary' Regge term for $K^+p \rightarrow K^+p$. However a small additional term of the type $D \ln^2 s/s_0 e^{(B_t \ln^2 s/s_0)}$ is included in their analysis, which derives from their fits to $\sigma_{tot}(K^\pm p)$. Our analysis indicates that $B_p(K^\pm p) \approx 1.344 + 0.42 \ln s$ and $B_p(K^\pm p) \approx 1.99 + 0.33 \ln s$. A comparison with the expression of Minami and Terada (1974) indicates that our expression for $B_R^{K^-p}$ is quite close to theirs but for the K^+p case there is a difference of about 30% in the first term, while the coefficients of $\ln(s)$ are very close. Our analysis disagrees with their expression for B_R since it indicates that B_R is energy dependent. Our R value of 4.52 agrees closely with their value of 4.582.

Our fits to polarisation indicate that there is at least a 30% discrepancy in the values of the parameter A_1 (equations 5.6b,d)

TABLE 5.9a

Slope parameter for K_p^+ elastic scattering according to equation 5.11

p_{lab} (GeV/c)	Theoretical value (GeV/c) ⁻²	Experimental value (GeV/c) ⁻²
6.8	5.269	4.9 ± 0.3
9.8	5.247	5.6 ± 0.2
12.8	5.462	6.1 ± 0.2
14.8	5.374	5.9 ± 0.2
50.0	7.279	7.5 ± 0.45
70.0	7.409	8.1 ± 0.3
100.0	7.649	8.02 ± 0.2
140.0	7.079	8.3 ± 0.2
175.0	7.32	8.4 ± 0.2
200.0	7.808	8.13 ± 0.37

TABLE 5.9b

Slope parameter for K_p^- elastic scattering according to equation 4.9

7.2	8.238	6.2 ± 0.9
9.0	8.942	8.1 ± 0.9
10.1	8.467	7.8 ± 0.2
11.88	7.849	6.0 ± 0.3
14.3	8.549	8.3 ± 0.1
15.91	10.135	6.9 ± 0.8
25.2	9.675	8.71 ± 0.25
40.1	10.306	8.9 ± 0.23
50.0	7.880	7.83 ± 0.37
70.0	9.478	8.8 ± 0.4
100.0	8.652	8.53 ± 0.15
140.0	9.230	9.1 ± 0.3
175.0	9.061	9.7 ± 0.3
200.0	8.465	8.79 ± 0.53

obtained from fitting data. The parameter A_2 (equations 5.6b,d) can be taken as zero. (For each reaction however one can fit data by using the same A_1 value.) Any one (or a combination) of the following could give rise to this discrepancy.

- i) There is a breaking of ρ - A_2 (strong) EXD in the flip amplitude.
- ii) Either or both exchanges are non-peripheral.
- iii) The phase rule for the flip amplitudes is incorrect.
- iv) There are other contributions such as cuts or a small Pomeron flip term.

It is extremely difficult to disentangle these factors quantitatively on account of lack of data at more energies (there is no polarisation data between 14 and 40 GeV/c) and on account of the low accuracy of 40 and 45 GeV/c data. A breaking of strong ρ - A_2 EXD has been noted by Joynson (1977) and Joynson and Martin (1977) using FESR's and CMSR's. As for the non-peripherality of the various exchanges it seems unlikely, in view of our experience with $\pi^\pm p$ polarisations (Chapter 4), that the ρ is nonperipheral. The analysis of Joynson (1977) confirms that the ρ is peripheral. The peripherality of A_2 has been a source of controversy (c.f. e.g. Minami and Terada (1973)). However the analysis of Joynson (1977) indicates that $\text{Im}(A_{2+-})$ has a zero at $-t \sim 0.5 \text{ (GeV/c)}^2$ and that the real part is given correctly by Regge theory. This agrees with the DAM picture qualitatively whereby A_2 is expected to be peripheral and the flip amplitudes have Regge phases. However Joynson (1977) also finds that there is an A_2 contribution to the non-flip amplitude and that $\text{Im}A_{2++}$ is non-peripheral. Joynson (1977) and Joynson and Martin (1977) also find that in their analysis of kaon-nucleon scattering additional trajectories are also required (f' , ω' , ρ' , A_2'). It is quite possible that some of these trajectories might be a source of the discrepancy observed in our analysis. However

Martin and Navelet (1977) have argued for a Pomeron flip contribution to all elastic reactions. In view of the relative scarcity and inaccuracy of data it is difficult to ascertain the causes of the discrepancy we observe in our parameter value. We also recall that earlier Barger and Halzen (1972) carried out a DAM-based analysis of data and considered $K^\pm p$ polarisations qualitatively. They tested the relation

$$\left(P \frac{d\sigma}{dt}\right)_{K^- p} \approx \cos \pi\alpha \left(P \frac{d\sigma}{dt}\right)_{K^+ p} \quad (5.12)$$

up to $p_{lab} = 14 \text{ GeV/c}$ and found it to be in qualitative agreement with data. We however point out that this is not a test of the DAM but rather of strong ρ - A_2 EXD in the flip amplitude (the non-flip amplitude is imaginary in their analysis) since it does not depend upon the peripherality of these exchanges. Our parameterisation (equation 5.6b,d) however indicates that, although there is a discrepancy, the DAM parameterisation is not far off the mark. It might even be regarded as correct if one blames the discrepancy on other exchanges. However we shall have to await further measurements to decide this point.

As a final point we mention the spin-rotation parameter R . Only one measurement of this parameter for $K^\pm p$ elastic scattering exists in the energy range considered here. The measurement by Pierrard et al. (1975) gives the value -0.16 ± 0.16 for this parameter for $K^- p$ elastic scattering at 40 GeV/c laboratory momentum and $|t| = 0.331 \text{ (GeV/c)}^2$ (c.f. Table 1.4). Our calculation (using $B_R = 2.86 \text{ GeV}^{-2}$, radius of interaction $R = 4.52 \text{ GeV}^{-1}$ and A_p, A_R, B_p and A_1 values at 40 GeV/c from Tables 5.5, 5.6 and 5.8) yields the value -0.236 , in agreement with the experimental value. (The spin rotation parameter is given by formula (4.10) in Chapter 4).

In conclusion therefore we can say that the DAM can be used successfully to explain the main features of $K^{\pm}p$ elastic data by introducing a real part for the Pomeron contribution. There is however a discrepancy of at least 30% in the value of the parameter A_1 (equations 5.6b,d) as obtained from fitting $K^{\pm}p$ polarisations whose origin cannot be accurately pinpointed on account of the relative paucity of accurate data.

CHAPTER 6

THE DUAL ABSORPTIVE MODEL AND NUCLEON-NUCLEON ELASTIC SCATTERING

UP TO ISR ENERGIES

6.1 Introduction and Motivation

The importance of the nucleon-nucleon forces cannot be overemphasised. The nucleons constitute the building blocks of all atomic nuclei and it is therefore not a coincidence that the nucleon is the hadron with which we have the oldest acquaintance. A great amount of data exists on nucleon-nucleon scattering, in particular for $pp \rightarrow pp$. The experimental results have already been summarised in Chapter 1. A great amount of theoretical effort has been devoted to model building for pp scattering as extensive and accurate data is available. We shall very briefly describe some of these efforts in the following. Reviews of some of the models for pp scattering are contained in Coconni (1973), Phillips (1973) and Barger (1974). Sukhatme (1977) gives a quick and useful review of most models. A lucid review of the various features of the ISR data is Winter (1977).

One can subdivide the models for pp scattering into three categories although there can be an overlap between these categories in some cases:

- i) s-channel models
 - ii) t-channel models ("Reggeology")
 - iii) hybrid models (i.e. involving s and t notions) and model independent analyses.
-
- i) s-channel models: One could further subdivide these as
 - a) Geometrical models, b) Eikonal models, c) Inelastic Overlap Function models.

a) Geometrical Models: We mention a few examples. The grey disc approximation (c.f. e.g. Cocconi (1973)) has already been mentioned in Chapter 2 as an optical model. It predicts $\frac{d\sigma}{dt} \propto \left[\frac{J_1(R\sqrt{-t})^2}{R\sqrt{-t}} \right]$ (equation 2.9a) which, for $R \approx 0.66\text{fm}$ gives a dip at $|t| \approx 1.3 (\text{GeV}/c)^2$ (and another at $|t| \approx 4.9 (\text{GeV}/c)^2$). However the dips are predicted at all energies in contradiction with data and spin effects are ignored. The Chu-Hendry model (Chu and Hendry (1970, 1973)) is a sophisticated version of the grey disc model and includes spin effects. The (imaginary) non-flip amplitude is parameterised as a sum of peripheral and diffractive pieces (i.e. " J_0 " + $\frac{J_1 R \sqrt{-t}}{R \sqrt{-t}}$) while the flip amplitude is real and peripheral. This model describes the polarisation and $d\sigma/dt$ data quite well up to 24 GeV/c (c.f. e.g. Cheng et al. (1973) for a comparative study). Gotsman and Maor (1976b) have refined this model in the light of the ISR data by including an empirical " J_2 " type term in the non-flip amplitude and taking the various radii in the Bessel function arguments to be energy dependent. Their parameterisation yields a qualitative fit to ISR data. Kane (1972) has introduced a parameterisation of the non-flip amplitude as a sum of two terms - ~~one~~ corresponding to a black core of radius $\sim \frac{1}{2}$ fm and a smoothed edge of radius ~ 1 fm - and has fitted the ISR data quantitatively for $|t| \leq 0.4 (\text{GeV}/c)^2$. Torgerson and Kamal (1975, 1976) have parameterised the profile function $\Gamma(b)$ [$\Gamma(b) = 1 - \tilde{f}(b,s)$, $\tilde{f}(b,s)$ being described in Chapter 2] as a sum of Fermi and Gaussian distributions on empirical grounds. This yields an excellent fit to the ISR data. The hypothesis of geometrical scaling (GS) (Dias de Deus (1973), Buras and Dias de Deus (1974); also see Barger et al. (1975), Barger (1974)) asserts that the energy dependence of the elastic amplitude is contained entirely in a radial parameter R and that the elastic amplitude scales in impact-parameter as $T(b,s) = T(b/R(s))$. This hypothesis can correlate the changes observed

in the high-energy total, differential, and inelastic cross-sections, the slope parameter, and the inward movement of the dip in the ISR region.

b) Eikonal Models: There are numerous eikonal model studies of pp elastic scattering. We shall briefly mention the well known ones. We have already mentioned the Chou-Yang model (Chou and Yang (1968), Durand and Lipen (1968), the Frautschi-Margolis model (Frautschi and Margolis (1968)) and the Cheng-Walker-Wu model (Cheng et al. (1973), Walker (1973)) in Chapter 2. The Chou-Yang model, in its pristine form, is incompatible with the rising total cross-section. Hayot and Sukhatme (1974) have considered a modified version of the Chou-Yang model by considering a factorisable eikonal with an energy dependent factor. Leader et al. (1976) have carried out a critical assessment of both the modified and the unmodified, Chou-Yang models. The Frautschi-Margolis model consists in identifying the Pomeron exchange with the "Born term" of the eikonal expansion. The differential cross-section dips are then generated by destructive interference between the various terms of the series. The Cheng-Walker-Wu model is motivated by studies in QED and they assume an eikonal (equation 2.41d) which is designed to approach a Yukawa potential at large impact parameters. This model is another example of a factorisable eikonal model. It has been pointed out (Barger (1974)) that models with factorisable eikonals do not give the correct energy dependence of slope parameter at ISR energies and also violate geometrical scaling. The above-mentioned models ignore spin-flip amplitudes. The generalisation of the Chou-Yang model to include spin effects has been carried out by Bourrely et al. (1975, 1976, 1977) who take the Born amplitude to be proportional to the product of the currents corresponding to the colliding particles. The resulting eikonals are then given in terms of the Gamma function and the modified Bessel functions K_n ($n = 2, 1$). This model has been used to fit pp data in the range $5.5 \leq p_{lab} \leq 2000$

GeV/c including polarisations.

Gerhold and Majerotto

(1978) have taken the eikonal to be a Fourier-Bessel transform of Regge pole amplitudes to obtain a good fit to pp polarisations.

c) Inelastic Overlap Function Model: This model consists of expressing the imaginary part of the elastic amplitude in terms of inelastic amplitudes through unitarity (Van Hove (1964)). The relationship between the elastic amplitude and terms involving inelastic amplitudes is simple in impact-parameter space. To obtain the elastic amplitude one then parameterises the inelastic term in b -space (inelastic overlap function). The inelastic overlap function is usually taken as a Gaussian of the impact parameter (e.g. Henzi and Heckman (1972)). Henzi and Valin (1972) have also considered a parameterisation of the overlap function as a sum of a Gaussian and a strongly absorbed Gaussian term.

ii) t -channel Models: There exist numerous studies of pp scattering using the Regge pole/cut model. It is generally believed that the P , f and ω exchanges dominate the non-flip amplitude while the ρ , A_2 exchanges dominate the $\Delta\lambda = 1$ amplitude. Instead of referring to individual cases we refer to some general features. Martin (1974), Collins et al. (1973, 1974a,b) have carried out a detailed phenomenological investigation of pp and $\bar{p}p$ elastic and total cross-sections up to the ISR energies in the light of a variety of Regge pole and cut models. They find that only two types of models are able to explain the data over the whole energy range satisfactorily. The simplest of these has no cuts but a Pomeron pole with trajectory $\alpha_P(t) = 1.06 + 0.25t$ and a non-shrinking "core" term at large $|t|$. The interference of the two terms leads to the observed dip in pp scattering. An effective-trajectory approach clearly brings out the dominance of such a Pomeron trajectory for $|t| \lesssim 1.3 \text{ (GeV)}^2$ beyond which the data appears

energy independent (Martin, 1974; Collins et al. 1973). The Regge approach predicts that polarisation varies as $s^{-\frac{1}{2}}$ with energy if one assumes that "ordinary" Regge exchange contributions (usually taken as ρ and A_2) dominate the flip amplitude. This is however in contrast with a rapid fall with increasing energy. Irving (1975) has however been able to explain NN polarisation up to 45 GeV/c and $|t| \approx 1(\text{GeV}/c)^2$ by including a Pomeron flip contribution. . . . It has been pointed out (Bouquet and Diu (1975), Bouquet et al. (1975, 1976)) that there are difficulties in a Regge-pole based phenomenological analysis of NN and $\bar{\text{N}}\text{N}$ total cross-sections. They conclude that the problem lies with the A_2 contribution to the non-flip amplitude which seems to contain a growing asymptotic component. Analysis of pp data based on Gribov's Reggeon Calculushave also been considered, e.g. Craigie and Preparata (1973).

iii) Hybrid Models and Model Independent Analyses: There exists a wide range of models employing s- and t-channel ideas simultaneously, e.g. the dual model of Coon et al. (1973), the hybrid model of Gotsman and Maor (1975a), the DAM, etc. Coon et al. use a dual model with logarithmic trajectories and an eikonalised P exchange to fit $\frac{d\sigma}{dt}$ data in the range $20 \lesssim s \lesssim 2600 (\text{GeV})^2$ (for a critical assessment of the model c.f. Barger (1974)). Gotsman and Maor (1975a) have attempted to explain the entire pp data in the range $21 \lesssim s \lesssim 2860 (\text{GeV})^2$ by a hybrid model in which the Pomeron is represented as a sum of Bessel functions supplemented by Regge exchange contributions to single and double flip amplitudes parameterised in terms of the appropriate Bessel functions. Their fits are in qualitative agreement with $\frac{d\sigma}{dt}$ up to the ISR energies but do not agree with polarisation data above 24 GeV/c. The DAM has been applied by Barger and Halzen (1972) to pp and $\bar{\text{p}}\text{p}$

data but their analysis, apart from being confined to a limited s range, suffers from defects mentioned in Chapters 4 and 5. Saleem et al. (1975) have used the DAM to parameterise the ISR data by assuming a peripheral Pomeron. While their parameterisation reproduces the ISR dip it yields a constant total cross-section and has a very small Pomeron slope $\alpha'_p = 0.05$ which is unsatisfactory, as borne out by the analysis of Collins et al. (1973, 1974b, Martin (1974) who find $\alpha'_p \approx 0.2$ in the ISR range. Also this parameterisation yields a dip at all energies in contradiction with data and is theoretically unpalatable since it is inconsistent with the general belief that the Pomeron amplitude is predominantly central in impact-parameter space. Barger et al. (1972) have considered a fit to pp and $\bar{p}p \frac{d\sigma}{dt}$ and σ_{tot} data above 20 GeV by taking a Pomeron with a central and a peripheral part along with suitably parameterised f and ω contributions. They consider data in the range $|t| \leq 0.4 \text{ (GeV/c)}^2$. As a last case we mention an excellent model independent parameterisation of $\frac{d\sigma}{dt}(pp)$ in the range 12-1496 GeV/c and up to $|t| \approx 5.0 \text{ (GeV/c)}^2$ by Phillips and Barger (1973) who employ two interfering exponential amplitudes to reproduce the data.

It is clear from the aforementioned survey that a careful and comprehensive DAM based study of pp (as well as $\bar{p}p$ and np) data over a wide energy range and including spin effects (polarisations and spin rotation parameters) is lacking. In the following we undertake such a study. But before doing that we discuss some simplifications of NN amplitudes below.

6.2 Amplitudes

The total number of independent amplitudes for nucleon-nucleon elastic scattering is five. Useful reviews of the formalism of elastic nucleon-nucleon scattering are given in Moravcsik (1963), Abshire et al.

(1974), Kane and Sukhatme (1974), Bystricky et al. (1978). The standard notation for the five amplitudes in helicity space is: (denoting individual helicities $\lambda_i = \pm \frac{1}{2}$ as $\pm$ and the net s-channel helicity flip by $\Delta\lambda$)

			$\Delta\lambda$	
ϕ_1	$\equiv$	$\langle ++ T ++ \rangle$	0	
ϕ_2	$\equiv$	$\langle -- T ++ \rangle$	0	
ϕ_3	$\equiv$	$\langle +- T +- \rangle$	0	(6.1)
ϕ_4	$\equiv$	$\langle -+ T +- \rangle$	2	
ϕ_5	$\equiv$	$\langle ++ T +- \rangle$	1	

Of these the last two amplitudes vanish in the forward direction. In our analysis we make the usual simplifying phenomenological assumptions for the high energy region

$$\phi_1 = \phi_3 \quad (6.2a)$$

$$\phi_2 = 0 \quad (6.2b)$$

$$\phi_4 = 0 \quad (6.2c)$$

and write

$$\phi_1 = \phi_3 = \phi \quad (6.3)$$

The aforementioned assumptions can be loosely justified on empirical and theoretical grounds as follows. The quantities $\Delta\sigma_L$ and $\Delta\sigma_T$ $\{\Delta\sigma_L \equiv \sigma_{\text{tot}}(\vec{+}) - \sigma_{\text{tot}}(\vec{-}), \Delta\sigma_T \equiv \sigma_{\text{tot}}(\uparrow\uparrow) - \sigma_{\text{tot}}(\uparrow\downarrow)^*\}$ are related to the imaginary parts of ϕ_1 , ϕ_2 and ϕ_3 at $t = 0$ by

* Arrows refer to beam and target polarisations relative to the beam direction, horizontal arrows ($\vec{+}$) referring to longitudinal polarisations and vertical arrows ($\uparrow\uparrow$) referring to transverse polarisations.

$$\Delta\sigma_L = \frac{1}{4q\sqrt{s}} \operatorname{Im}(\phi_1 - \phi_3) \Big|_{t=0} \quad (6.4a)$$

$$\Delta\sigma_T = \frac{-1}{4q\sqrt{s}} \operatorname{Im} \phi_2 \Big|_{t=0} \quad (6.4b)$$

In Fig. 6.1 adapted from Spinka (1977) we show these quantities. It is

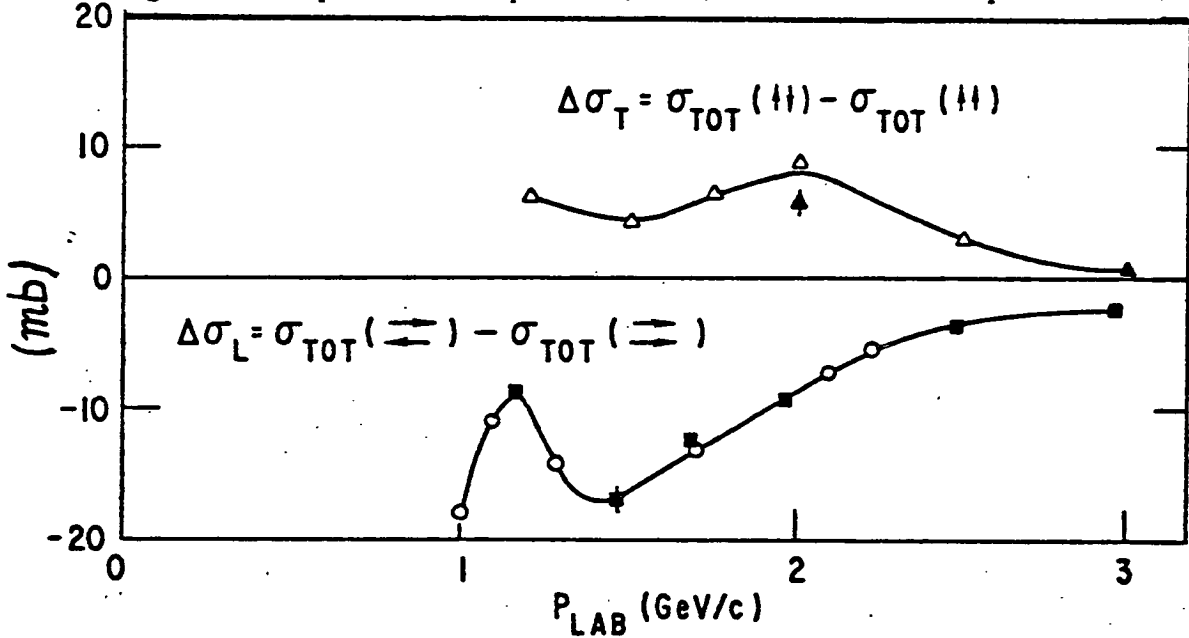


Fig. 6.1.

clear that between $p_{lab} = 1.5$ and 3 GeV/c $\Delta\sigma_L$ falls by a factor of about 7-8, becoming very small by 3 GeV/c. It would then be reasonable to assume that by 10 GeV/c $\Delta\sigma_L \approx 0$ so that $\operatorname{Im}\phi_1(s,0) = \operatorname{Im}\phi_3(s,0)$. One can further assume this to hold even for $t \neq 0$ and if the imaginary parts are equal it is reasonable to expect that the real parts also coincide. It is also clear from Fig. 1 that by 3 GeV/c $\Delta\sigma_T \approx 0$ so that $\operatorname{Im}\phi_2(s,0) \approx 0$ by 3 GeV/c. One can then assume $\phi_2(s,t) = 0$ at small $|t|$ and large s on lines similar to those for $(\phi_1 - \phi_3)$ above. The assumption $\phi_4 = 0$ can be justified by invoking the vector coupling hypothesis (VCH) of Collins and Gault (1976a,b). According to this hypothesis the leading natural parity exchanges (P, f, ρ, ω, A_2) couple to high-spin vertices like vector particles, so that in the t -channel the net helicity change $\Delta\lambda_t = 0, \pm 1$ only. This then implies that for small

approximately

t in the s-channel the net helicity flip $\Delta\lambda = |\Delta\lambda_s| = 0, 1$. Since ϕ_4 corresponds to $\Delta\lambda = 2$ one expects $\phi_4 \sim 0$ on the basis of VCH. We are therefore left with only two independent amplitudes ϕ ($\phi \equiv \phi_1 = \phi_3$ and ϕ_5 corresponding to $\Delta\lambda = 0$ and 1 respectively. We will determine ϕ from fits to total and differential cross-sections and ϕ_5 from polarisation data. With the above assumptions the relevant formulae are

$$\sigma_{\text{tot}} = \frac{0.389}{2q\sqrt{s}} \text{Im}\phi(s,0) \quad (6.5a)$$

$$\frac{d\sigma}{dt} \sim \frac{0.389}{64\pi s q^2} |\phi|^2 \quad (6.5b)$$

$$P \sim \frac{2\text{Im}(\phi \phi_5^*)}{|\phi|^2 + 2|\phi_5|^2} \quad (6.6a)$$

$$R \approx \frac{-\cos \theta_R |\phi|^2 + \sin \theta_R \text{Re}(\phi \phi_5^*)}{|\phi|^2 + 2|\phi_5|^2} \quad (6.6b)$$

6.3 Parameterisation of Amplitudes and Fits to Data for $pp \rightarrow pp$ and $np \rightarrow np$

Total Cross-Sections

The pp channel has exotic quantum numbers. Consequently one would expect, on the basis of 2-component duality, strong EXD between the various non-Pomeron exchanges in this channel. This strong EXD is then automatically implied for the np case in which the same trajectories contribute as in $pp \rightarrow pp$ except that the $I = 1$ contributions change sign. This implies that the imaginary parts of the various amplitudes in both reactions contain only Pomeron exchange. In particular the imaginary part of the non-flip amplitude is expected to contain no "ordinary" Regge exchanges. Consequently 2-component duality predicts $\sigma_{\text{tot}}(pp, pn)$ to be flat or slowly rising with energy depending on whether the Pomeron intercept is taken as equal to or greater than unity. Experimentally both total cross-sections display shallow minima in contradiction to 2-component duality and hence the DAM. The presence of minima in

* θ_R is the laboratory angle of the slow or recoil proton.

$\sigma_{\text{tot}}(\text{pp}, \text{np})$ is indicative of "ordinary" Regge contributions to the non-flip amplitude ϕ . We therefore attempt to parameterise the total cross-section data by including "ordinary" Regge exchange terms. The simplest and most natural possibility is to include a single "ordinary" Regge term with an intercept $\alpha_R(0) = 0.5$ in addition to a Pomeron term with intercept $\alpha_P(0) = 1.06$ or 1.069 . Surprisingly enough we find that $\sigma_{\text{tot}}(\text{pp})$ cannot be fitted satisfactorily at all if one writes

$$\text{Im}\phi(s, 0) = A_P(s)^{\alpha_P(0)} + A_R(s)^{\alpha_R(0)} \quad (6.7)$$

with $\alpha_R(0) = 0.5^*$ and uses equation (6.5a). We have varied $\alpha_P(0)$ within reasonable limits, keeping $\alpha_R(0)$ fixed at 0.5 , to see if good fits can be obtained. We find that as long as $\alpha_R(0) = 0.5$ the expressions (6.5a, 6.7) do not give good fits. Some illustrative cases are shown in Table 6.1. We have then fixed $\alpha_P(0)$ and allowed $\alpha_R(0)$ to vary. We find that the nearest one can get to an acceptable fit is with $\alpha_P(0) = 1.06$ and $\alpha_R(0) \sim 0.7$. The latter intercept however is not physically appealing, being much higher than the generally accepted value $\alpha_R(0) \sim 0.37 - 0.57$ (see below). We find however that a third Regge term with an intercept of 0.4 yields satisfactory fits if subtracted from (6.7), i.e. if one has

$$\text{Im}\phi(s, 0) = A_P(s)^{\alpha_P(0)} + A_1(s)^{0.5} - A_2(s)^{0.4} \quad (6.8)$$

with $\alpha_P(0) = 1.06$ as shown in Table 6.1. As a matter of interest we also find that an excellent fit to data can be obtained with a Pomeron of

* Collins et al. (1974b) have fitted $\sigma_{\text{tot}}(\text{pp})$ with an amplitude of the form (6.7) with $\alpha_P(0) = 1.06$ and $\alpha_R(0) = 0.5$. They however make the approximation $s \sim 2q\sqrt{s}$. We have found that if one makes this approximation a very good fit ($\chi^2/n = \frac{27.95}{28}$) to data can be obtained ($A_P = 66.22 \pm 0.108$, $A_R = 111.51 \pm 2.06$) but if one uses the exact expression the fits obtained are not satisfactory. This point has to some extent already been noted in Gotsman and Maor (1975). The lower limit of the energy range considered in fitting $\sigma_{\text{tot}}(\text{pp}, \text{np})$ above corresponds to $p_{\text{lab}} = 6 \text{ GeV}/c$. We have tried to explore whether one could fit these total cross-sections with reasonable intercept values ($\alpha_P(0) \sim 1.06 - 1.07$ and $0.3 < \alpha_R(0) < 0.6$) by moving the lower limit of the energy range upwards. We find that it is possible to do this for $\sigma_{\text{tot}}(\text{pp})$ only when the lower end of the lab. momentum range has been moved up to $20 \text{ GeV}/c$ ($A_P = 66.875 \pm 0.062$, $\alpha_P(0) = 1.06$, $A_R = 84.479 \pm 1.22$, $\alpha_R(0) = 0.5$, $\chi^2/n = 42/22$; taking $\alpha_P(0) = 1.069$ gives a bad fit ($\chi^2/n = 88.4/22$). However the same intercept values still do not give an acceptable fit to $\sigma_{\text{tot}}(\text{np})$ [$A_P = 67.774 \pm 0.077$, $A_R = 80.032 \pm 0.14$, $\chi^2/n = 88/18$; $\alpha_P(0) = 1.069$ gives a worse fit.] It/

intercept 1.069 and an $A[\log(s/s_0)]^B$ type term. This type of fit is also shown in Table 6.1a. Although parameterisations 6 and 7 in Table 6.1a give a better fit than parameterisation 8, we have chosen to work with the last one because the intercept values are physically more acceptable. The determination of intercepts of the leading Regge trajectories ρ , f , ω , A_2 in the literature seem to vary between 0.37 to 0.57 approximately. For instance Roberts et al. (1977) find a ρ intercept of 0.42 from KN charge exchange data with ρ - A_2 EXD. Hendrick et al. (1975) find $\alpha_\rho(0) = 0.57$, while Bouquet and Diu (1975) find $\alpha_\rho = 0.51$. Similarly Bock et al. (1979) find $\alpha_\omega(0) = 0.41 - 0.44$, Rohring et al. (1977) find $\alpha_\omega(0) \sim 0.39 - 0.41$, Joynson and Martin (1977) find $\alpha_\omega(0) = 0.5$. A_2 intercept determinations also yield values varying from $\alpha_{A_2}(0) = 0.37$ (Bouquet and Diu (1975)) to $\alpha_{A_2}(0) = 0.52 \pm 0.04$ (Bolotov et al. (1974a,b)). Very recently Nakata (1978) has carried out a detailed analysis of FNAL CEX data and concludes that ρ - A_2 EXD is broken by 20%. It is for these reasons that we have preferred to work with the parameterisation (8) of Table 6.1a rather than parameterisations (4) or (6). Similar remarks are valid for the np case. (Fits are shown in Fig. 6.1a,b). Some of our parameterisations of $\sigma_{\text{tot}}(\text{np})$ are displayed in Table 6.1b. Data for $\sigma_{\text{tot}}(\text{pp}, \text{np})$ has been taken from Galbraith et al. (1965), Denisov et al. (1971b), Carroll et al. (1976), the pp total cross-sections in the ISR region being taken from Amendolia et al. (1973) and Amaldi et al. (1976).

It is probable that by pushing the lower end of the energy region further up it might be possible to obtain satisfactory fits to $\sigma_{\text{tot}}(\text{pp}, \text{np})$ with reasonable intercept values. We have however not explored this point further since in our view even 20 GeV/c is unconventionally high as a lower limit for such phenomenological fits (the Durham group take $s \sim 12 (\text{GeV}/c)^2$ or $p_{\text{lab}} \sim 7 \text{ GeV}/c$ as the lower end of their fits to $\sigma_{\text{tot}}(\text{pp}, \text{pn})$ and a great deal of polarisation data exists below 20 GeV/c ($P(\text{pp})$ at 10, 12, 14, 17.5 GeV/c, $P(\text{np})$ at 11.8 GeV/c). We therefore conclude that if one attempts to fit $\sigma_{\text{tot}}(\text{pp}, \text{np})$ data in the conventional energy range $p_{\text{lab}} \geq 6 \text{ GeV}/c$, it is not possible to do so by preserving weak EXD with reasonable intercept values ($\alpha_\rho(0) = 1.06 - 1.07$, $0.3 < \alpha_R(0) < 0.6$).

TABLE 6.1a

Some Parameterisations of $\text{Im}\phi(s,0)$ for $pp \rightarrow pp$

No.	Parameterisation of $\text{Im}\phi(s,0)$; $\sigma_{\text{tot}} = \frac{0.389}{2q\sqrt{s}}$	$\text{Im}\phi(s,0)$	χ^2/n
1.	$A_P(s)^{1.069} + A_R(s)^{0.5}$	$A_P = 63.219 \pm 0.059$ $A_R = 92.013 \pm 1.617$	324.4/28
2.	$A_P(s)^{1.06} + A_R(s)^{0.5}$	$A_P = 67.246 \pm 0.1036$ $A_R = 75.793 \pm 1.595$	204.2/28
3.	$A_P(s)^{1.069} + A_R(s)^{\alpha_R(0)}$	$A_P = 62.633 \pm 0.043$ $A_R = 85.117 \pm 0.002$ $\alpha_R(0) = 0.5396 \pm 0.0051$	276.4/28
4.	$A_P(s)^{1.06} + A_R(s)^{\alpha_R(0)}$	$A_P = 64.51 \pm 0.137$ $A_R = 45.869 \pm 0.895$ $\alpha_R(0) = 0.6999 \pm 0.002$	79.7/28
5.	$A_P(s)^{1.05} + A_R(s)^{\alpha_R(0)}$	$A_P = 72.87 \pm 0.146$ $A_R = 45.869 \pm 0.895$ $\alpha_R(0) = 0.376 \pm 0.011$	185.9/28
6.	$A_P(s)^{1.069} + A_1(s)^{\alpha_1(0)} - A_2(s)^{\alpha_2(0)}$	$A_P = 58.826 \pm 0.674$ $A_R = 85.161 \pm 0.0005$ $A_2 = 116.35 \pm 3.1587$ $\alpha_1(0) = 0.6712 \pm 0.0012$ $\alpha_2(0) = 0.1849 \pm 0.0013$	27.207/28
7.	$62.52(s)^{1.069} + 98.63[\ln s/3.25]^{1.96}$		7.5/28
8.	$A_P(s)^{1.06} + A_1(s)^{0.5} - A_2(s)^{0.4}$	$A_P = 65.2 \pm 0.09$ $A_1 = 297. \pm 2.14$ $A_2 = 300. \pm 2.2$	40.7/28

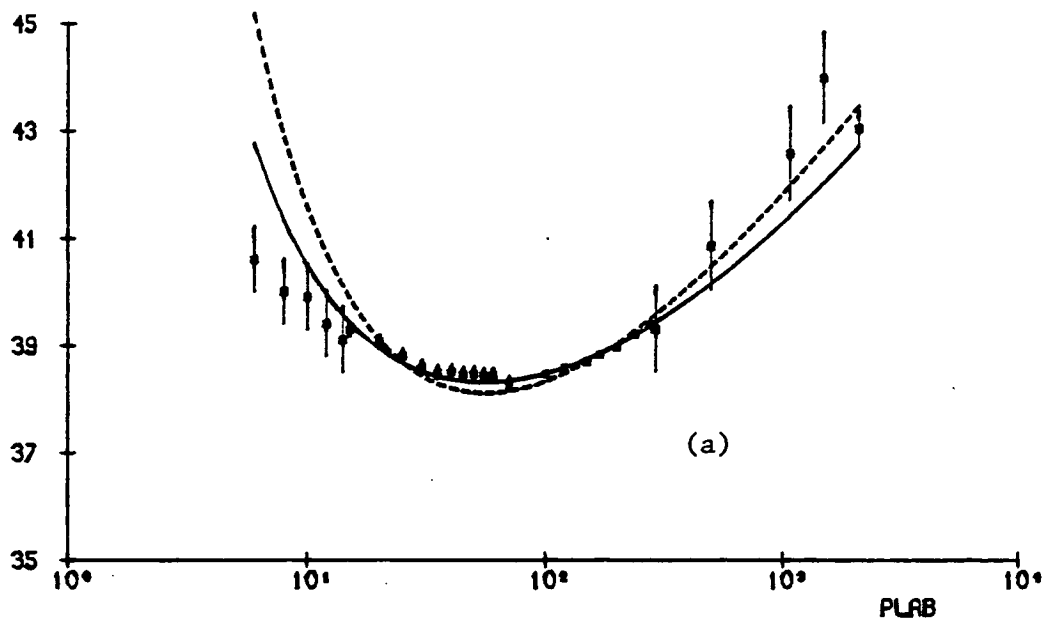
TABLE 6.1b

Some Parameterisations of $\text{Im}\phi(s,0)$ for $np \rightarrow np$

No.	Parameterisation of $\text{Im}(s,0)$; $\sigma_{\text{tot}} = \frac{0.389}{2q\sqrt{s}} \text{Im}\phi(s,0)$;	χ^2/n
1.	$A_P(s)^{1.069} + A_R\sqrt{s}$ $A_P = 63.414 \pm 0.1548$ $A_R = 102.81 \pm 3.548$	123.6/25
2.	$A_P(s)^{1.06} + A_R\sqrt{s}$ $A_P = 67.534 \pm 0.111$ $A_R = 84.393 \pm 3.174$	90.05/25
3.	$A_P(s)^{1.069} + A_R(s)^{\alpha_R(0)}$ $A_P = 62.148 \pm 0.0722$ $A_R = 80.180 \pm 0.0049$ $\alpha_R(0) = 0.5914 \pm 0.0025$	93.4/25
4.	$A_P(s)^{1.06} + A_R(s)^{\alpha_R(0)}$ $A_P = 64.867 \pm 0.3069$ $A_R = 49.575 \pm 0.749$ $\alpha_R(0) = 0.6939 \pm 0.0056$	51.2/25
5.	$A_P(s)^{1.05} + A_R(s)^{\alpha_R(0)}$ $A_P = 72.752 \pm 0.1542$ $A_R = 78.535 \pm 0.4407$ $\alpha_R(0) = 0.440 \pm 0.008$	70.3/25
6.	$A_P(s)^{1.069} + A_1(s)^{\alpha_1(0)} - A_2(s)^{\alpha_2(0)}$ $A_P = 58.356 \pm 0.601$ $A_1 = 80.306 \pm 0.0184$ $A_2 = 185.65 \pm 9.945$ $\alpha_1(0) = 0.6994 \pm 0.0008$ $\alpha_2(0) = 0.0698 \pm 0.0134$	33.7/25
7.	$65.2(s)^{1.06} + A_1(s)^{0.5} - A_2(s)^{0.4}$ $A_1 = 362.4 \pm 9.338$ $A_2 = 388.6 \pm 13.652$	36.5/25

FIT TO PP TOTAL CROSS-SECTIONS

TOTAL CROSS-SECTIONS



FIT TO NP TOTAL CROSS-SECTIONS

TOTAL CROSS-SECTIONS

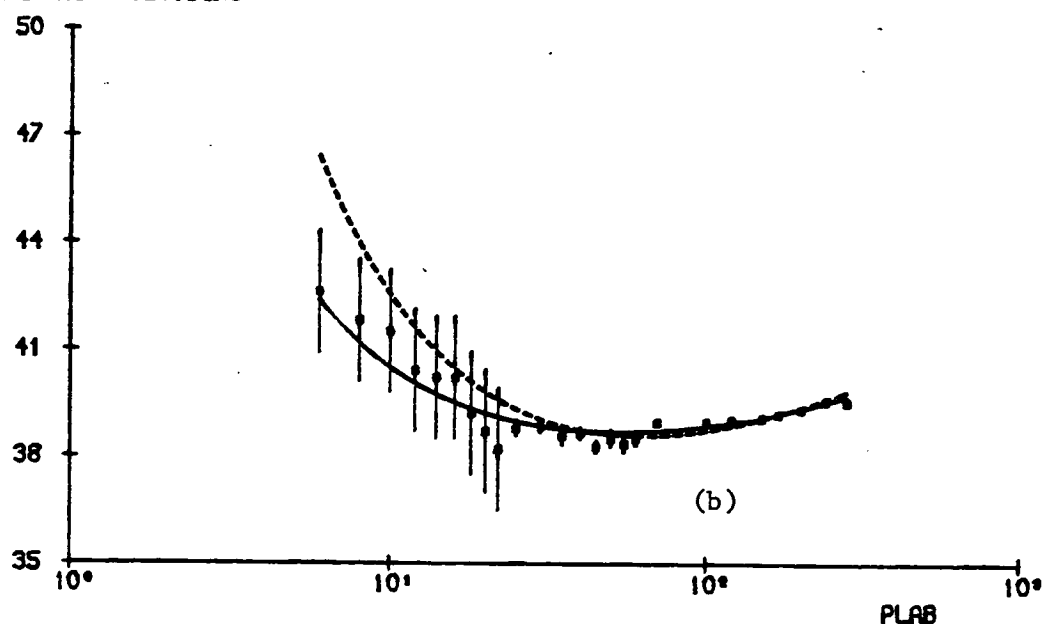


Fig. 6.2a,b. Fits to $\sigma_{\text{tot}}(\text{pp}, \text{np})$. The dashed curves correspond to parameterisation No. (2) in Table 6.1a,b (i.e. with $\alpha_p(0) = 1.06$, $\alpha_R(0) = 0.5$). The solid curve is the fit No. 8 in Table 6.1a for pp and fit No. 7 in Table 6.2b for np.

Differential Cross Sections: In fixing the $|t|$ range for fitting differential cross-section data we have been guided by the work of Collins et al. (1973, 1974a,b) and Martin (1974). They have carried out a detailed phenomenological analysis of various elastic reactions with particular emphasis on $pp \rightarrow pp$. For the pp case they have considered data from $p_{lab} = 8 \text{ GeV/c}$ up to the ISR region over a wide $|t|$ range (up to $|t| \approx 5 \text{ (GeV/c)}^2$ wherever data at large $|t|$ is available). After testing a variety of cases they conclude that the simplest model that explains the data consists of a shrinking Pomeron pole (with f and ω contributions at lower energies) that dominates the amplitude for $|t| < 1.4 \text{ (GeV/c)}^2$, and a non-shrinking "core" term interfering destructively with it to give rise to the high-energy differential cross-section dip. The "core" term is essentially empirical in character. It is therefore not possible to understand and parameterise the "core" term from the point of view of the DAM. We have consequently confined ourselves to $|t| < 1.4 \text{ (GeV/c)}^2$ in fitting differential cross-sections. Since $np \rightarrow np$ is in many ways similar to $pp \rightarrow pp$ we consider it in the same $|t|$ range as the pp data, i.e. $|t| < 1.4 \text{ (GeV/c)}^2$. We start from the imaginary part of the non-flip amplitude in the forward direction, as determined from fits to total cross-sections, and parameterise the Pomeron term, as in the previous cases, according to the DAM prescription. The "ordinary" Regge terms are assumed to be peripheral.

We thus write

$$\text{Im}\phi(s,t) = P + R \quad (6.9a)$$

$$P = 65.2 e^{B_P t} (s)^{1.06} \quad (6.9b)$$

$$R = [A_1 e^{B_1 t} (s)^{0.5} - A_2 e^{B_2 t} (s)^{0.4}] J_0(R\sqrt{t}) \quad (6.9c)$$

where A_1 and A_2 are known for the pp and np case from fits to total cross-sections. The two Regge terms have been assigned the same radius of interaction R in order to reduce the number of parameters. The parameters B_P, B_1, B_2 and R are determined by fitting data. The plausibility of the parameterisation depends on its ability to fit data well with reasonable parameter values, e.g. R should be close to $5 (\text{GeV}/c)^{-1}$ in general the the parameters B_P, B_1, B_2 should not have large values. The real part of the amplitude is assumed to be

$$\text{Re}\phi(s,t) = \rho e^{\gamma t} \text{Im}\phi(s,t) + \frac{\cot \frac{\pi \alpha_P}{2}}{2} P \quad (6.10a)$$

$$\alpha_P(t) = 1.06 + 0.3t \quad (6.10b)$$

where ρ and γ are free parameters.

Asymptotically this yields $\text{Re}\phi(s,t) = \left(\cot \frac{\pi \alpha_P}{2}\right) P$.

Values of $|\rho|$ are non-negligible up to 40 GeV/c or so (e.g. $\rho(\text{pp}) \approx -0.3$ around 20 GeV/c, $\rho(\text{np}) \approx -0.35$ around 25 GeV/c) and hence we retain the first term in equation (6.10a) up to 50 GeV/c beyond which we drop it. γ is expected to be large so that away from $t = 0$ the first term in (6.10a) is negligible, its only role being to ensure that its (quite significant - approximately 10%) contribution to $\frac{d\sigma}{dt}$ at and around $t = 0$ is included. We find

$$\gamma = 10.0 (\text{GeV}/c)^{-2} \quad \text{for} \quad \text{pp} \rightarrow \text{pp} \quad (6.11a)$$

$$\gamma = 14.5 (\text{GeV}/c)^{-2} \quad \text{for} \quad \text{np} \rightarrow \text{np} \quad (6.11b)$$

Using equations (6.9a,b,c, 6.10a,b, 6.5b) we have fitted data in the range $10 \leq p_{lab} \leq 2092$ GeV/c for pp and $11 \leq p_{lab} \leq 65$ GeV/c for np, up to $|t| \approx 1.3$ (GeV/c)². The pp differential cross-section data is taken from Harting et al. (1965), Allaby et al. (1973), Akerloff et al. (1976), Ayres et al. (1977), Bruneton et al. (1977), Kwak et al. (1975)*. ρ values for both cases were taken from Foley et al. (1967) and Beznogikh et al. (1972). The np differential cross-sections were taken from Engler et al. (1973) and Böhmer et al. (1975). The measurements of Böhmer et al. have a large uncertainty in laboratory momentum values and we have used the mean p_{lab} values to fit data. For instance the $\frac{d\sigma}{dt}$ values quoted for $p_{lab} = 20 - 30$ GeV/c have been fitted with $p_{lab} = 25$ GeV/c and so on. The results of our fits are shown in Tables 6.2a, b. (Some fits are depicted in Fig. 6.3a-h). The 45 GeV/c data of Bruneton et al. for pp elastic scattering require $\gamma = 7.4$ instead of $\gamma = 10$ as for other cases. The 10, 12, 14.2 and 24 GeV/c pp data is from Allaby et al. (1973) who do not quote errors on their cross-sections. Consequently χ^2 values cannot be calculated. The parameter values shown against these momenta seem to reproduce the data reasonably and have been quoted mainly because they are used in fitting 10, 12, 14 and 24 GeV/c polarisation data later (see following section). The pp parameter B_p shows a slow increase with energy and can be roughly parameterised by several forms

$$B_p = 2.05 + 0.51 \ln s \quad (6.12a)$$

$$B_p = 1.92 (\ln s)^{0.55} \quad (6.12b)$$

$$B_p = 3.1 (\ln \ln s)^{0.88} \quad (6.12c)$$

* We are grateful to the Hamburg group of the CHOV collaboration for supplying us with tables of $\frac{d\sigma}{dt}$ values which were used to fit data.

TABLE 6.2a

Fits to pp elastic differential cross-sections according to equations

6. and 6. (p_{lab} in GeV/c, B_P, B_1, B_2 in $(GeV/c)^2$ and R in (GeV^{-1})).

p_{lab}	B_P	B_1	B_2	R	χ^2/n
10*	3.55	1.7	1.7	5.2	
12*	3.43	2.15	2.41	4.8	
14.2*	3.72	2.52	3.0	4.9	
18.4	3.897 ± 0.133	2.42 ± 0.134	2.43 ± 0.62	5.00 ± 0.249	$\frac{15.4}{17}$
24*	4.10	5.5	5.0	5.2	
44,5	4.495 ± 0.014	3.79 ± 0.086	3.722 ± 0.173	2.67 ± 4.775	$\frac{58.8}{23}$
50	4.436 ± 0.097	2.369 ± 0.075	2.131 ± 0.202	4.177 ± 0.421	$\frac{66.6}{50}$
70	4.662 ± 0.016	3.512 ± 0.166	2.486 ± 0.006	4.829 ± 0.262	$\frac{47.8}{17}$
100	4.709 ± 0.089	2.908 ± 0.118	2.496 ± 0.026	4.94 ± 0.094	$\frac{61.3}{54}$
140	5.029 ± 0.034	5.758 ± 0.399	2.531 ± 0.013	4.778 ± 0.104	$\frac{15.7}{19}$
175	5.072 ± 0.027	4.628 ± 1.093	2.206 ± 0.607	5.06 ± 0.025	$\frac{20.4}{15}$
200	5.173 ± 0.008	3.62 ± 0.683	2.053 ± 0.487	5.185 ± 0.171	$\frac{127.3}{45}$
291	5.416 ± 0.028	2.542 ± 0.092	3.146 ± 0.065	7.441 ± 0.072	$\frac{29.6}{40}$
1480	5.634 ± 0.020	2.905 ± 0.168	7.541 ± 4.340	8.48 ± 0.46	$\frac{19.2}{11}$
2091.776	5.704 ± 0.037	8.487 ± 5.653	2.486 ± 0.029	6.218 ± 0.211	$\frac{25.9}{26}$

* 10, 12, 14.2, 24 GeV/c data is taken from Allaby et al. (1973) who do not quote any errors on their data. Consequently χ^2 cannot be evaluated for their measurements. The quoted parameter values reproduce their data reasonably and are used to fit polarisations.

$$\text{Overall } \chi^2/n = \frac{487.8}{317}$$

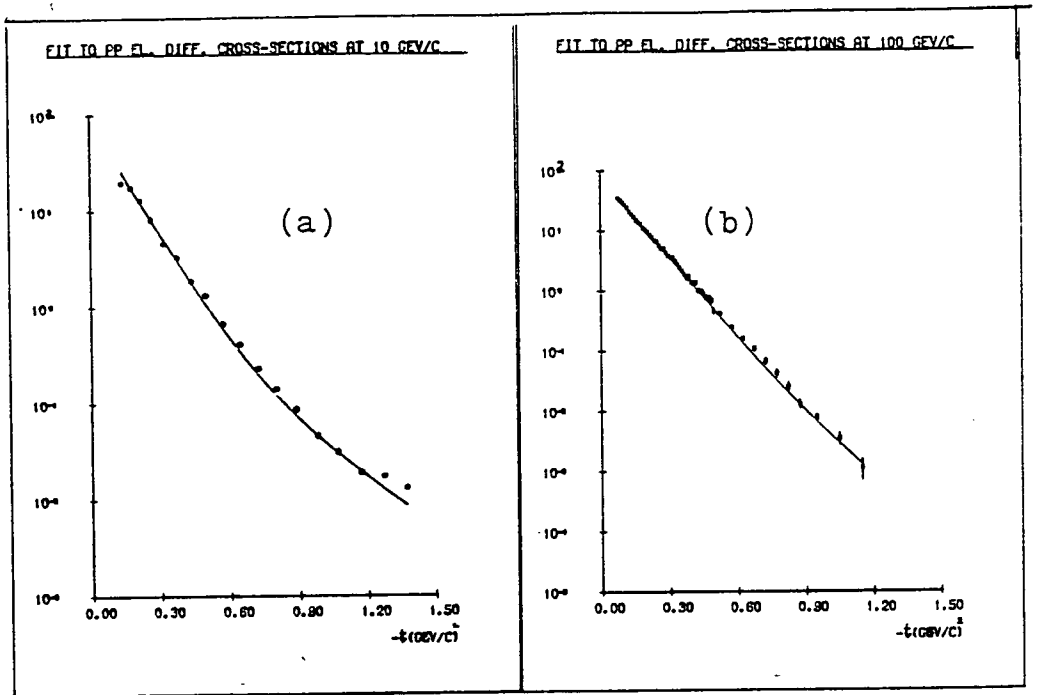
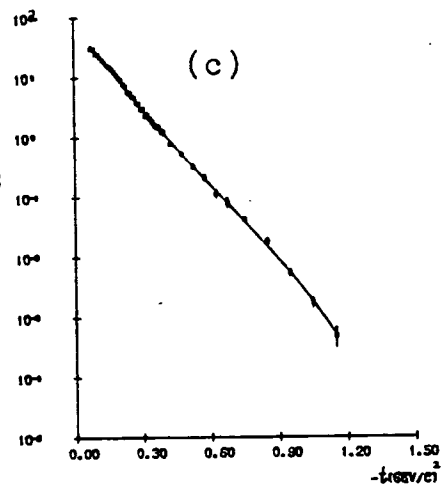
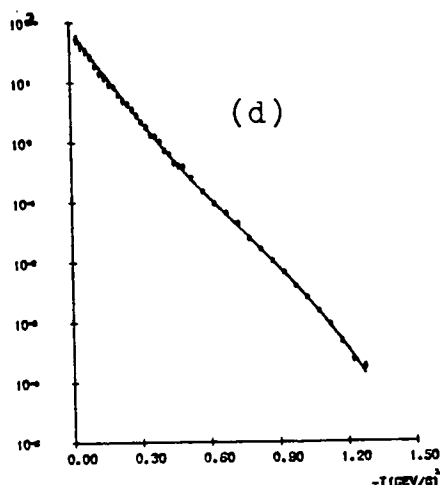


Fig6.3 a-e: Fits to pp elastic differential cross-sections at several energies (from $p_{\text{lab}} = 10 \text{ GeV}/c$ upto $\sqrt{s} \approx 62 \text{ GeV}$) according to equations 6.5b, 6.9 and 6.10. (Parameter values from table 6.2a).

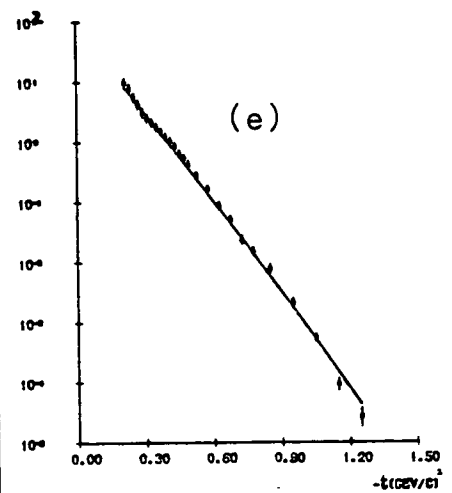
FIT TO PP EL. DIFF. CROSS-SECTIONS AT 200 GEV/C



FIT TO PP EL. DIFF. CROSS-SECTIONS AT 23 GEV (ISR)



FIT TO PP EL. DIFF. CROSS-SECTIONS AT 62 GEV (ISR)



The first of these parameterisations indicates a Pomeron slope of about 0.5. The last parameterisation however reproduces the actual values somewhat better compared to the other two. The parameters B_1 and B_2 can similarly be (roughly) parameterised comparably with several forms

$$B_1 = 1.65 + 0.20 \ln s \quad (6.13a)$$

$$B_2 = 1.64 + 0.17 \ln s \quad (6.13b)$$

$$B_1 = 1.58 (\ln s)^{0.35} \quad (6.13c)$$

$$B_2 = 1.42 (\ln s)^{0.35} \quad (6.13d)$$

$$B_1 = 2.18 (\ln(\ln s))^{0.55} \quad (6.13e)$$

$$B_2 = 1.92 (\ln(\ln s))^{0.55} \quad (6.13f)$$

R can also be parameterised. (The rough energy dependence of the various parameters becomes plausible if one looks at fits for $p_{lab} \geq 50$ GeV/c. The 10, 12, 14.2 and 24 GeV/c values are obtained by guessing and hence should not be taken seriously as data fitting is not possible as no errors are quoted on data at these momenta. The 45 GeV/c data are from Bruneton et al. (1977) and even in πp and Kp cases the parameters required to fit their data are generally somewhat outside the trends indicated by fits to data from other experiments.)

For the np case a careful look at Table 6.2b reveals that B_p does not increase with energy for the 11-21 GeV/c data of Engler et al. (1973) but for 25-65 GeV/c data it shows an increase with energy. We have not therefore attempted to parameterise it. Both B_1 and B_2 show rough and slow increase with energy and can be (roughly) parameterised comparably as follows:

$$B_1 = 1.41 + 0.20 \ln s \quad (6.14a)$$

$$B_2 = 0.39 + 0.47 \ln s \quad (6.14b)$$

$$B_1 = 1.34 (\ln s)^{0.35} \quad (6.14c)$$

$$B_2 = 0.74 (\ln s)^{0.82} \quad (6.14d)$$

$$B_1 = 1.88 (\ln(\ln s))^{0.46} \quad (6.14e)$$

$$B_2 = 1.61 (\ln(\ln s))^{1.12} \quad (6.14f)$$

TABLE 6.2b Fits to np elastic differential cross-sections according to equations (6.5b,6.9,6.10,6.11)
(p_{lab} in (GeV/c), B_p , B_1 , B_2 in (GeV/c)², R in (GeV/c)⁻¹).

p_{lab}	B_p	B_1	B_2	R	χ^2/n
11	3.755 ± 0.0027	1.9382 ± 0.0092	1.7018 ± 0.0100	3.5191 ± 0.0035	$\frac{38.7}{38}$
13	3.5805 ± 0.0008	2.1883 ± 0.0031	2.2965 ± 0.0041	4.5747 ± 0.0023	$\frac{23.7}{38}$
15	3.8769 ± 0.0024	2.1569 ± 0.0076	1.9813 ± 0.009	3.9755 ± 0.0265	$\frac{46.4}{38}$
17	3.7237 ± 0.0018	2.4655 ± 0.0021	2.5704 ± 0.0036	4.0659 ± 0.0046	$\frac{25.5}{38}$
19	3.4836 ± 0.0008	2.0257 ± 0.0013	2.2353 ± 0.0032	4.1619 ± 0.0028	$\frac{46.4}{38}$
21	3.6689 ± 0.0018	2.1226 ± 0.0124	2.0195 ± 0.0189	3.8473 ± 0.01676	$\frac{83.7}{38}$
25	4.0230 ± 0.0029	2.5473 ± 0.0130	2.3397 ± 0.0195	5.7201 ± 0.0244	$\frac{66.4}{30}$
35	4.0711 ± 0.0057	2.1370 ± 0.0009	2.4707 ± 0.0055	5.1972 ± 0.0019	$\frac{25.4}{30}$
45	4.2442 ± 0.0009	2.2673 ± 0.0021	2.5045 ± 0.0004	5.3582 ± 0.0046	$\frac{22.6}{25}$
65	4.3814 ± 0.0116	2.4148 ± 0.0079	2.5389 ± 0.0057	5.1890 ± 0.0017	$\frac{28.8}{22}$

$$\text{Overall } \chi^2/n = \frac{407.6}{335} = 1.21 .$$

As mentioned on p 149 the data of Bohmer et al (1975) has large uncertainties in p_{lab} values. The 50-60 GeV/c data (average $p_{lab} = 55$ GeV/c) cannot be fitted at all by our parameterisation probably on account of these uncertainties.

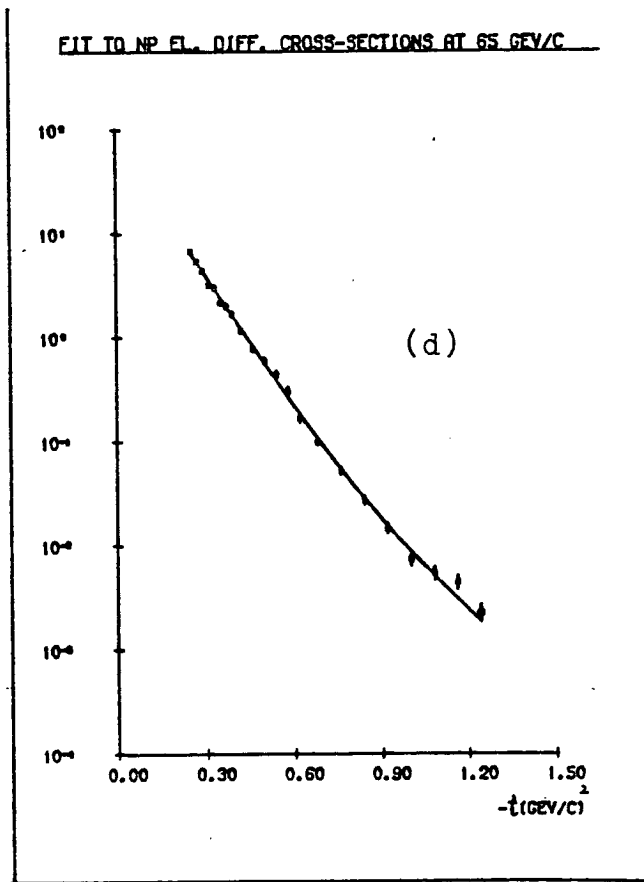
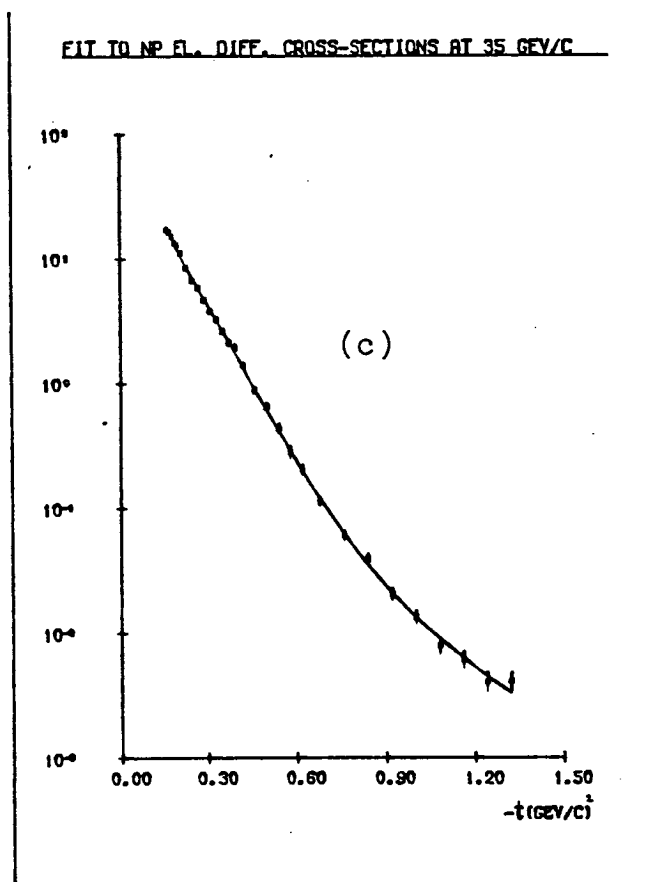
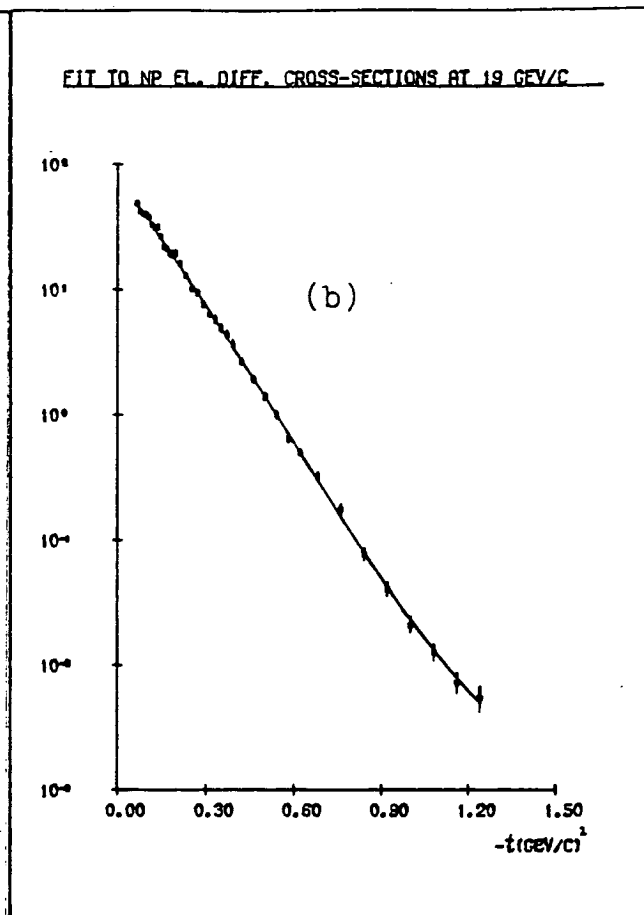
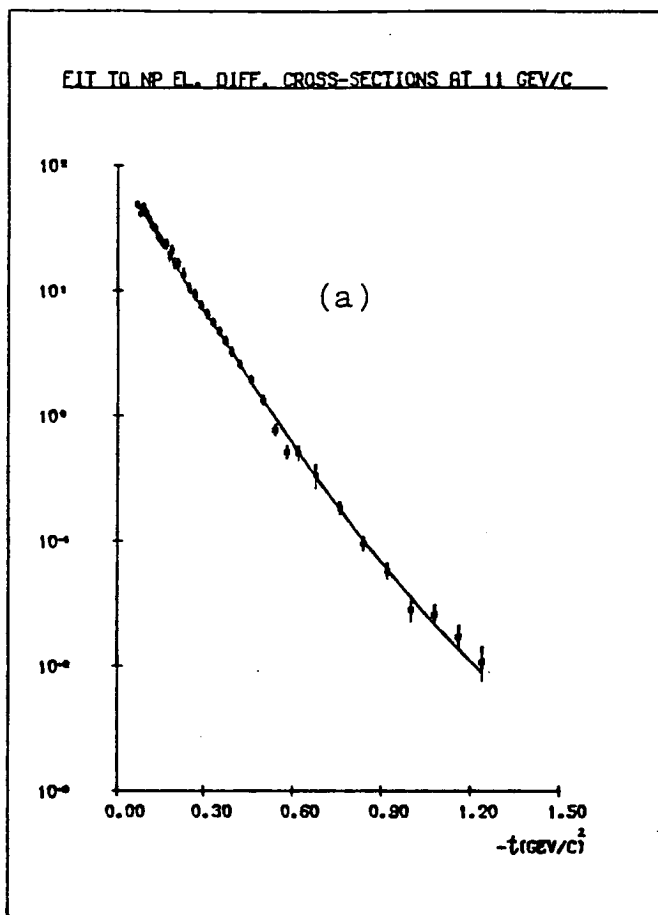


Fig 6.4 a-d): Fits to np elastic differential cross-sections at several momenta according to equations(6.5b,6.9,6.10).

The parameter R can be similarly parameterised. The aforementioned parameterisations of the various pp and np parameters should be taken more as indicators of trends in their values rather than serious numerical fits.

Polarisations: pp elastic polarisations have two interesting features which depart from conventional expectations.

- (a) A very rapid fall with increasing energy ($P(pp)$ falls as s^{-1} approximately, in contrast with the expected $s^{-\frac{1}{2}}$ behaviour shown by πp and Kp polarisations (Gaidot et al. (1976)).
- (b) The presence of a minimum (indicative of a double zero or two closely spaced zeroes) in the region $0.7 \lesssim |t| \lesssim 1$ (GeV/c)² up to 24 GeV/c and the startling appearance of a single zero at $|t| \approx 0.5$ (GeV/c)² in the 45 GeV/c data (Gaidot et al. (1976)) which then seems to move inward with increasing energy (Snyder et al. (1978)). (The data at and above 45 GeV/c is consistent with a zero around $|t| \sim 1$ (GeV/c)² but on account of larger errors it is difficult to ascertain the nature of this zero).

The conventional Regge models (e.g. Arnold and Logan (1969)) predict a structureless t -behaviour for $P(pp)$ and an $s^{-\frac{1}{2}}$ energy dependence. The $s^{-\frac{1}{2}}$ energy dependence is caused by the fact that the polarisation results from an interference of the, predominantly real, flip amplitude (containing "ordinary" Regge contributions only) with a predominantly imaginary, Pomeron dominated non-flip amplitude. However the departure of $P(pp)$ from the conventional expectations has led to the view that its unexpected behaviour might be due to a Pomeron contribution to the flip amplitude (Irving (1975), Martin and Navelet (1977)). Irving (1975) has studied pp , pn and $\bar{p}p$ polarisations up to 45 GeV/c ($|t| \lesssim 1$ (GeV/c)²), by introducing a structureless Pomeron flip contribution

We have found that if one parameterises the "ordinary" Regge exchanges to the flip amplitude according to the DAM "rules" (strong EXD, peripherality, Regge pole phases) then one can explain $P(pp)$ in the $10 \lesssim p_{lab} \lesssim 300$ GeV/c region reasonably well for $|t| \lesssim 1.2$ (GeV/c)² provided one introduces an imaginary, peripheral Pomeron flip contribution with a radius of interaction of 3.36 (GeV/c)⁻¹. Our parameterisation of the amplitude ϕ_5 thus is (also see Chapter 5, where a similar parameterisation of "ordinary" Regge exchanges has been carried out for the $K^\pm p$ case):

$$\phi_5 = 2C e^{ft} \frac{J_1(5.164\sqrt{-t})}{\sin\pi\alpha(t)} s^{\alpha(t)} + i D e^{gt} J_1(3.36\sqrt{-t}) s^{\alpha_P(t)} \quad (6.15)$$

where $\alpha(t) = 0.5 + 0.9t$ is the usual EXD ρ, f, ω, A_2 trajectory and $\alpha_P(t) = 1.06 + 0.3t$ is the Pomeron trajectory. We find that a good overall description can be obtained with the following values for the parameters (f and g in (GeV/c)⁻²):

$$\begin{aligned} C &= 11 ; \\ f &= 1.5 ; \\ D &= 8 ; \\ g &= 3.5 , \end{aligned}$$

the data has been fitted at 10, 12, 14, 17.5, 24, 45, 100 and 300 GeV/c^{*} (the overall χ^2 per point ~ 2). (The 45 GeV/c data can be explained better if one uses $f = 2$ instead of 1.5).^{**} Some fits are shown in Fig. 6.4a-e.

* Data from Borghini et al. (1971c, 1966), Crabb et al. (1977), Gaidot et al. (1976), Snyder et al. (1978).

** The χ^2 per point break up is as follows: χ^2 per point = 3.9, 1.9, 1.3, 1.2, 3.6, 1.9, 1.1 for 10, 12, 14, 17.5, 24, 100 and 300 GeV/c data respectively. For 45 GeV/c data $f = 1.5$ gives a χ^2 per point of 3.6 while $f = 2$ gives a χ^2 per point of 2.1.

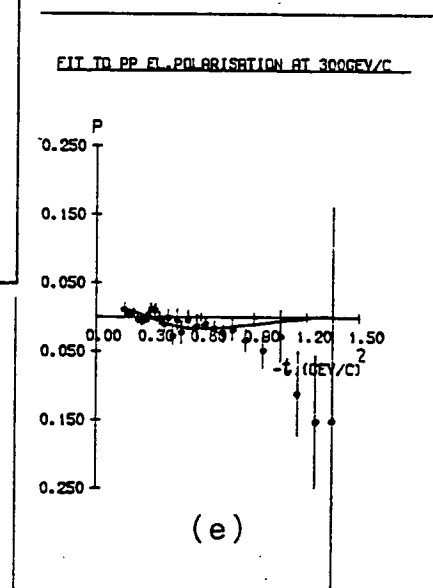
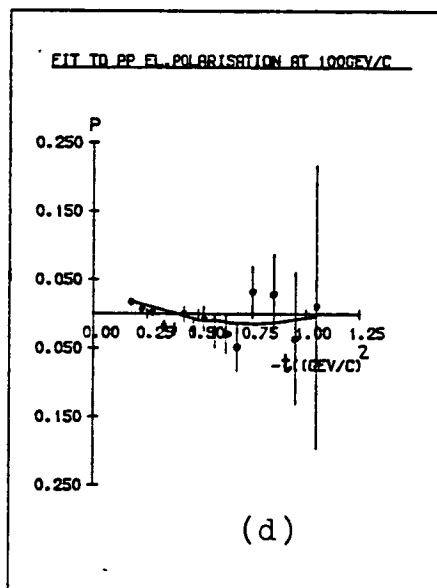
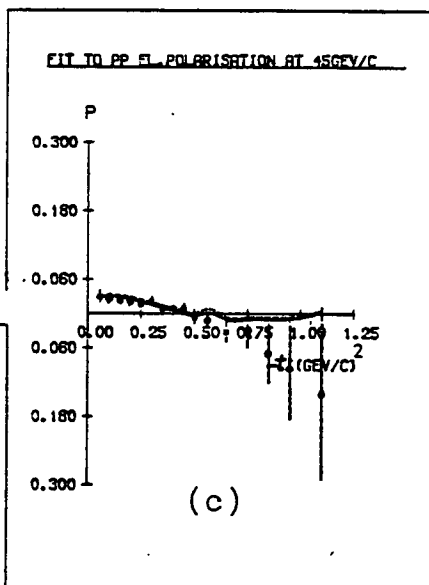
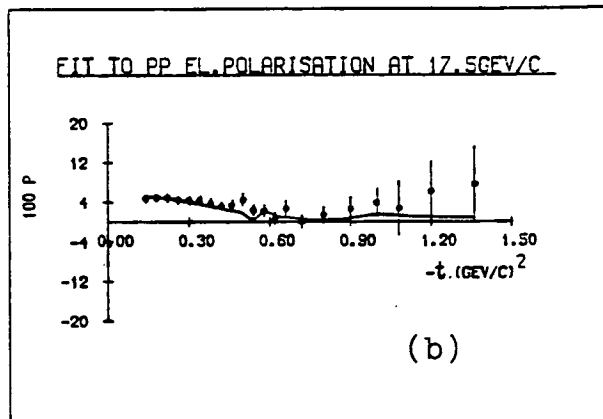
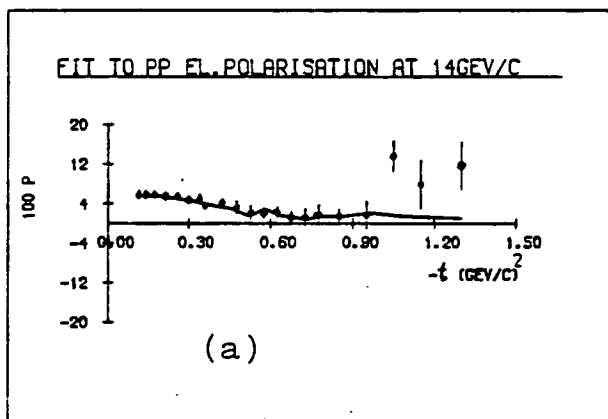


Fig 6.6 a-e): Fits to pp elastic polarisations at 14,17.5 45,100 and 300 GeV/c (solid curve) according to equations 6.6a and 6.15. For the 45 GeV/c case the solid line represents the fit corresponding to $f=2$ while the dashed line shows the fit for $f=1.5$ (cf footnote on p 154).

We now take up np polarisations. It may be noted that in going over from the pp amplitudes to the corresponding np amplitudes the $I = 1$ terms will change sign while the $I = 0$ contributions remain unaltered. Thus if $\text{Re}\phi_5$ contains only $I = 1$ contributions we will have $\text{Re}\phi_5(\text{np}) = -\text{Re}\phi_5(\text{pp})$. This implies $P(\text{np}) \sim -P(\text{pp})$ but polarisation measurements at 11.8 GeV/c (Kramer et al. (1978)) and lower momenta, contradict this. If, on the other hand, $\text{Re}\phi_5$ contains both $I = 1$ and $I = 0$ contributions then $|\text{Re}\phi_5(\text{np})|$ will be suppressed relative to $|\text{Re}\phi_5(\text{pp})|$, the extent of suppression depending on the relative strength of the two contributions. Thus if one assumes that both $I = 1$ and $I = 0$ terms contribute equally to $\text{Re}\phi_5(\text{pp})$ we will have $\text{Re}\phi_5(\text{np}) = 0$. (For some time evidence for non-negligible $I = 0$ contributions to flip amplitudes has been building up - c.f. e.g. Irving and Vanryckeghem (1975)). We find that one can obtain an excellent prediction for $P(\text{np})$ (data at 11.8 GeV/c is available - Kramer et al. (1978)) from $\phi_5(\text{pp})$ (i.e. ϕ_5 as given in equation (6.15)) provided one assumes that $I = 0$ and 1. Regge exchanges contribute equally to the flip amplitude. This assumption gives $\text{Re}\phi_5(\text{np}) = 0$ and thus one has

$$\phi_5(\text{np}) = i D e^{gt} J_1(3.36\sqrt{-t}) s^{\alpha_p(t)} \quad (6.16)$$

where the different parameters have the same values as for pp, i.e. $D = 8$, $g = 3.5 (\text{GeV}/c)^{-2}$ and $\alpha_p(t) = 1.06 + 0.3t$. Using (6.16) for ϕ_5 and constructing ϕ from 11 GeV/c parameter values in Table 6.2b and equation (6.9 and 6.10), we obtain a prediction for 11.8 GeV/c np polarisation which is in close agreement with data (χ^2 per point ≈ 0.8). The comparison between theory and experiment is shown in Fig. 6.6.

Fig.6.6 : Fit to $P(np)$ at 11.8 GeV/c according to equation 6.16.

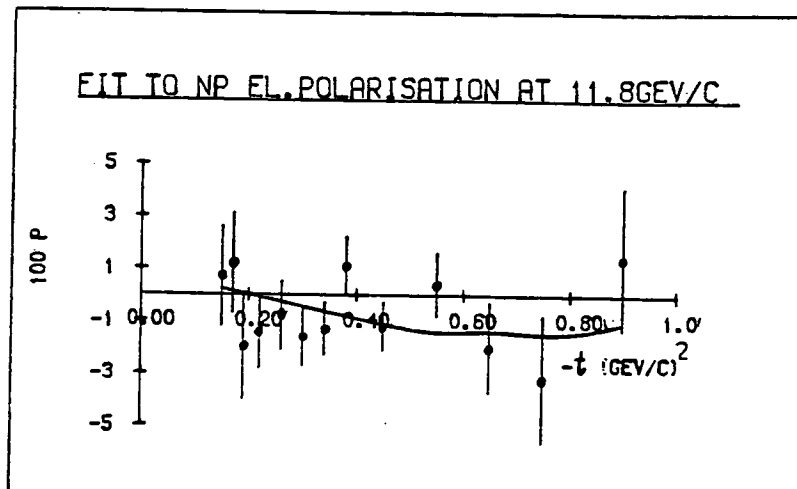


Fig.6.7 : Prediction for the pp spin rotation parameter R at 45 GeV/c according to equations 6.6b 6.9 and 6.15(parameter values from table 6.2a).

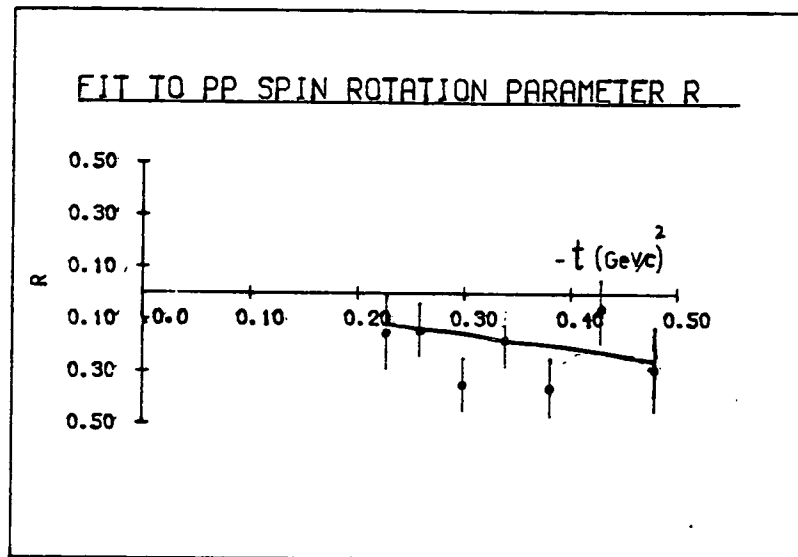
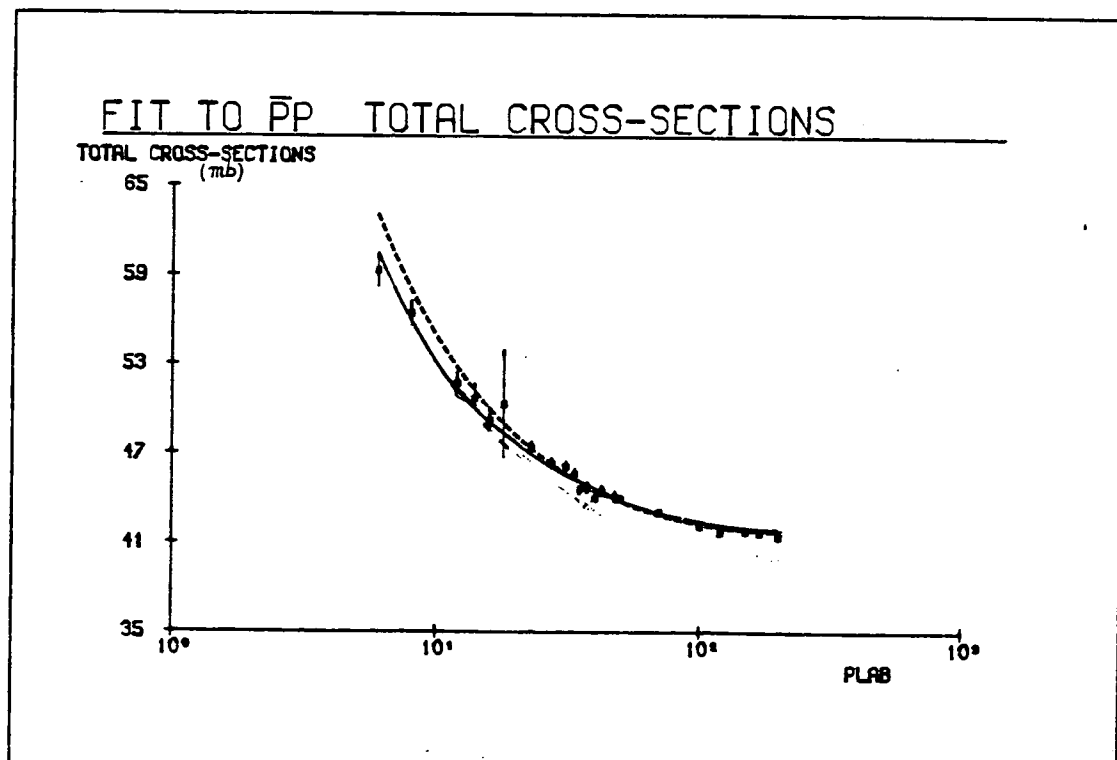


Fig 6.8 : Fits to $\sigma_{tot}(\bar{p}p)$. The solid curve corresponds to parameterisation 2 of table 6.3 while the dashed curve corresponds to parameterisation 3 of the same table.



Spin Rotation Parameter R : This parameter has been measured for pp scattering at 45 GeV/c by Pierrard et al. (no measurement for np scattering exists). Using (6.15) for ϕ_5 and construction ϕ from Table 6.2a and equations (6.9, 6.10) gives a prediction for R(pp) in close agreement with data ($\chi^2 = \frac{8.07}{8}$ for $f = 2.0$ (GeV/c)⁻² and $\frac{8.08}{8}$ for $f = 1.5$ (GeV/c)⁻²).

6.4 Fits to $\bar{p}p \rightarrow \bar{p}p$

Total Cross-Sections: Unlike the pp and np cases it is possible to fit $\bar{p}p$ total cross-sections with an expression of the form (6.7) with $\alpha_p(0) = 1.06$ and $\alpha_R(0) = 0.5$.^{*} It is also possible to fit $\sigma_{tot}(\bar{p}p)$ with an expression of the form (6.8). The two fits are comparable and are given below in Table 6.3. The range covered is $6 \leq p_{lab} \leq 200$ GeV/c. Data from Galbraith et al. (1965), Dennis et al. (1971b), Carroll et al. (1976).

TABLE 6.3

Parameterisations of $\text{Im}\phi(s,0)$ for $\bar{p}p$. $\sigma_{tot} = \frac{0.389}{2q\sqrt{s}} \text{Im}\phi(s,0)$

No.	Parameterisation		χ^2/n
1.	$65.2(s)^{1.06} + A_R(s)^{0.5}$	$A_R = 244.02 \pm 1.759$	$\frac{109.1}{22}$
2.	$65.2(s)^{1.06} + 362.4(s)^{0.5} - A_2(s)^{0.4}$	$A_2 = 187.6 \pm 2.93$	$\frac{31.08}{22}$
3.	$A_P(s)^{1.06} + A_R(s)^{0.5}$	$A_P = 66.91 \pm 0.22$ $A_R = 219.54 \pm 3.04$	$\frac{51.4}{22}$

In fitting data with parameterisation (2) in Table 6.3 the coefficients of $(s)^{0.5}$ and $(s)^{0.4}$ were, to begin with, treated as free parameters (the Pomeron parameters being fixed from pp data). In this way

^{*} This however entails a slight alteration (~3%) in the Pomeron residue at $t=0$ if a satisfactory fit is to be obtained (compare A_p values for 1 and 3 in table 6.3)

it was found that the coefficient of $(s)^{0.5}$ was close to that for the np case for the corresponding parameterisation (as expected from Regge considerations), but that the coefficient of $(s)^{0.4}$ was different from what one might expect from simple Regge-pole considerations. The coefficient of $(s)^{0.5}$ was then set equal to that for the np case (i.e. 362.4) and treating the coefficient of $(s)^{0.4}$ as a free parameter the data was fitted. The resulting fit has been shown in Table 6.3.

Differential Cross-Sections: The two fits for σ_{tot} suggest two parameterisations for the non-flip amplitude for $-t \geq 0$ according to the DAM "rules" if one identifies the non-Pomeron terms as "ordinary" Regge exchanges. The first of these is

$$\text{Im}\phi(s,t) = 66.91 e^{B_P t} (s)^{1.06} + 219.54 e^{B_R t} (s)^{0.5} J_0(R\sqrt{-t}) \quad (6.17a)$$

since $\frac{\text{Re}\phi(s,0)}{\text{Im}\phi(s,0)} \sim 0$ for $\bar{p}p$ one has

$$\text{Re}\phi(s,t) = \frac{\cot \frac{\pi\alpha_P}{2}}{2} \times (\text{imaginary part of Pomeron contribution}) \quad (6.17b)$$

where $\alpha_P(t)$ is the usual Pomeron trajectory mentioned in previous cases. The other parameterisation is similar to that for the pp and np cases, i.e.

$$\begin{aligned} \text{Im}\phi(s,t) = & 55.2 e^{B_P t} (s)^{1.06} + [362.4 e^{B_1 t} (s)^{0.5} \\ & - 187.6 e^{B_2 t} (s)^{0.4}] J_0(R\sqrt{-t}) \end{aligned} \quad (6.18)$$

with $\text{Re}\phi(s,t)$ given by (6.17b). We have used (6.17a,b) as well as (6.18), (6.17b) to fit $\bar{p}p$ differential cross-sections up to $|t| \approx 1 \text{ GeV/c}^2$ in the range $25.2 \leq p_{\text{lab}} \leq 200 \text{ GeV/c}$ [$\frac{d\sigma}{dt}(\bar{p}p)$ data below 25.2 GeV/c is too sparse to make any attempt at data fitting a useful exercise.] Data has been taken from Ayres et al. (1977), Brumet et al. Akerloff et al. (1976) and

The results of these fits are given in Tables (6.4a,b). Table (6.4a) gives the results for parameterisation (6.17a,b), while Table 6.4b) gives the results for parameterisation (6.18), 6.17b). (Some fits are shown in Fig. 6.9a-d).

TABLE 6.4a

P_{lab} (GeV/c)	$B_p (GeV)^{-2}$	$B_R (GeV/c)^{-2}$	$R (GeV/c)^{-1}$	χ^2/n
25.2	4.994 ± 0.115	5.092 ± 0.107	4.757 ± 0.260	$\frac{26.7}{33}$
39.0	4.370 ± 0.056	2.415 ± 0.414	6.338 ± 0.105	$\frac{9.8}{11}$
40.1	5.248 ± 0.054	8.534 ± 1.516	4.974 ± 0.389	$\frac{32.4}{33}$
50.0	4.911 ± 0.243	2.663 ± 0.221	3.148 ± 0.440	$\frac{58.4}{31}$
70.0	5.265 ± 0.066	9.991 ± 1.304	5.021 ± 0.086	$\frac{9.3}{13}$
100.0	5.024 ± 0.037	5.000 ± 0.444	4.940 ± 0.252	$\frac{28}{29}$
140.0	5.145 ± 0.116	11.450 ± 10.17	5.147 ± 0.470	$\frac{4.6}{11}$
175.0	5.440 ± 0.218	11.98 ± 11.93	6.348 ± 2.066	$\frac{7.5}{13}$
200.0	5.519 ± 0.301	3.615 ± 2.256	4.191 ± 1.488	$\frac{4.5}{13}$

$$\text{Overall } \chi^2/n = \frac{181.2}{187} \approx 0.97$$

As can be seen both parameterisations yield more or less equivalent fits. The energy dependence of various parameters involved can be fitted quite well compared to the pp and $p\bar{n}$ cases (on account of larger errors on their values) with several parameterisations. Thus for instance parameters in Table 6.4a can be fitted as follows:

TABLE 6.4b. Fits to $\frac{d}{dt}(\bar{p}p)$ according to equations (6.18, 6.17b and 6.5b)

$p_{lab} \text{ (GeV/c)}$	$B_p \text{ (GeV/c)}^{-2}$	$B_1 \text{ (GeV/c)}^{-2}$	$B_2 \text{ (GeV/c)}^{-2}$	$R \text{ (GeV/c)}^{-1}$	χ^2/n
25.2	4.8474 ± 0.3124	3.7581 ± 0.4888	2.5755 ± 0.2436	4.9224 ± 0.6187	$\frac{31.6}{33}$
39.0	4.3631 ± 0.0959	2.731 ± 0.4401	2.3164 ± 0.2290	5.999 ± 0.0532	$\frac{12.1}{11}$
40.1	5.1257 ± 0.3935	4.5917 ± 0.4039	2.5660 ± 0.04308	5.3106 ± 1.6507	$\frac{38.5}{33}$
50	4.4338 ± 0.1064	2.1384 ± 0.0852	1.7084 ± 0.3821	4.0545 ± 0.1823	$\frac{60.22}{31}$
70	5.2103 ± 0.0792	5.7933 ± 0.5097	2.6424 ± 0.0338	5.0865 ± 0.0432	$\frac{16.1}{13}$
100	4.8979 ± 0.0105	3.9006 ± 0.4378	3.0447 ± 0.3989	5.058 ± 0.08608	$\frac{28.6}{29}$
140	5.0727 ± 0.1079	6.2061 ± 2.1363	2.7984 ± 0.4153	5.2382 ± 0.2447	$\frac{3.7}{11}$
175	5.3811 ± 0.1152	7.9897 ± 0.7168	2.5827 ± 0.0617	5.0036 ± 0.0582	$\frac{63}{13}$
200	5.111 ± 0.2411	2.6016 ± 0.1041	4.1944 ± 0.8021	5.4136 ± 0.5495	$\frac{4.6}{13}$

Overall $\chi^2/n = 201.72/187$

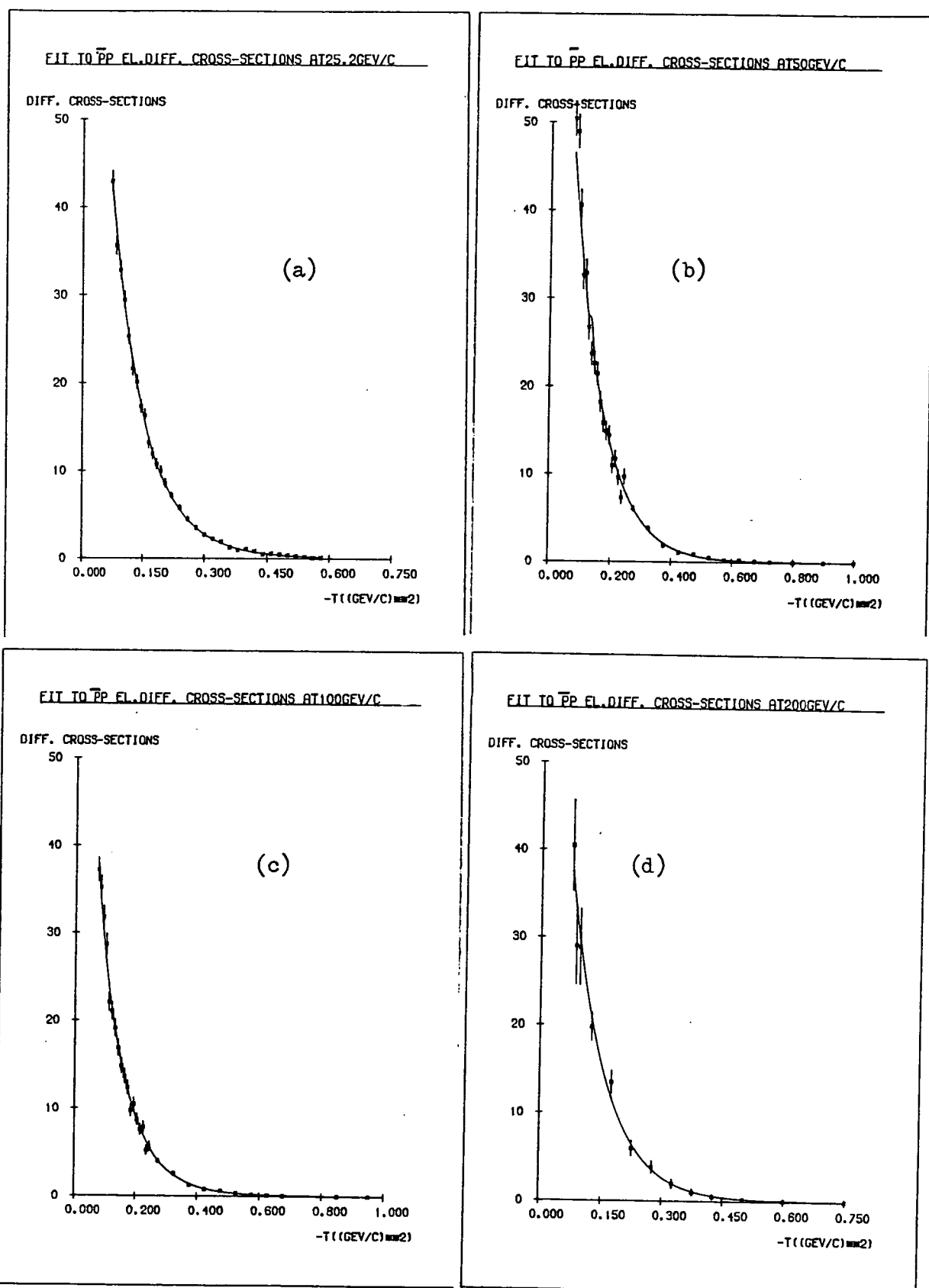


Fig. 6.9 a-d): Typical fits to $\bar{p}p$ differential cross-sections according to equations 6.17a,b and 6.5b. (Cross-sections in $\text{mb}/(\text{GeV}/c)^2$) .

$$B_P = 3.82 + 0.24 \ln s \quad (6.19a)$$

$$B_P = 3.45 (\ln s)^{0.23} \quad (6.19b)$$

$$B_P = 4.22 (\ln(\ln s))^{0.37} \quad (6.19c)$$

The parameters B_R and R are, as with the K^-p case, consistent with energy independence with $B_R = 4.58 (\text{GeV}/c)^{-2}$ and $R \sim 5.4 (\text{GeV})^{-1}$. Equation (6.19a) yields a Pomeron slope $\alpha'_P = 0.21$ which is closer to that determined from fits to pp data at ISR energies (Martin (1974)).

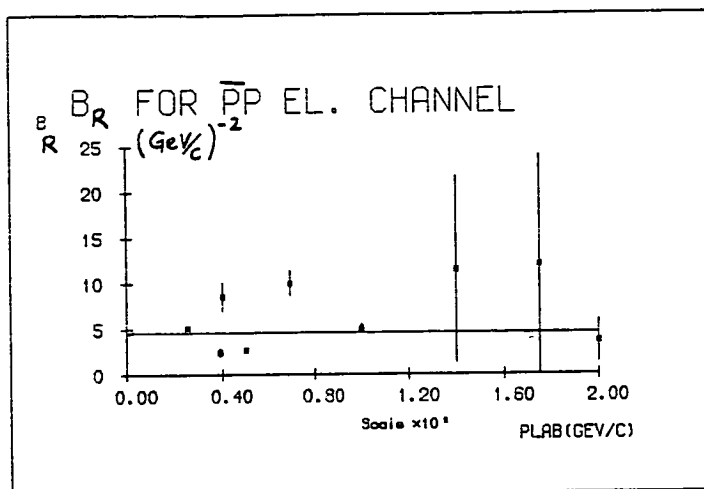
Polarisation: The only measurement of $\bar{p}p$ polarisations in the energy range considered is at 40 GeV/c by Gaidot et al. (1975). Since the $\bar{p}p$ channel is related to the pp channel through charge conjugation the spin-flip amplitude ϕ_5 for $\bar{p}p$ scattering is related to that for pp scattering as follows (provided the same assumptions are made as for the pp case, i.e. strong EXD in the imaginary part for "ordinary" exchanges; pure imaginary Pomeron (e.g. Chapter 5))

$$\phi_5(\bar{p}p) = \cos\pi\alpha(t) \text{Re}\phi_5(pp) + i[\text{Im}\phi_5(pp) + \text{"ordinary" Regge contributions}] \quad (6.20)$$

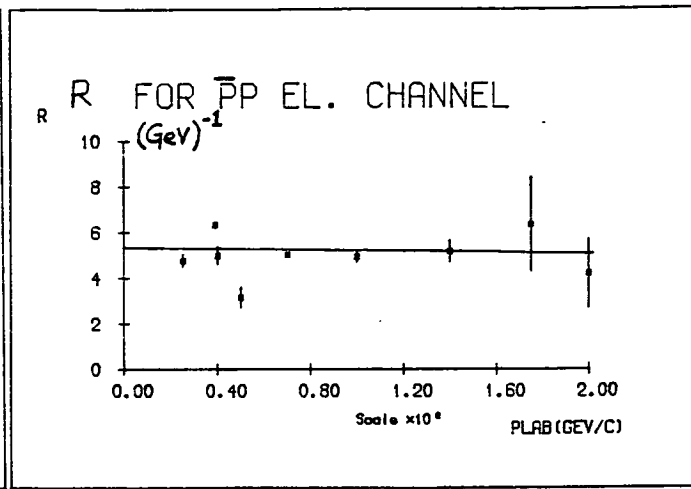
which, using equation (6.15) yields

$$\begin{aligned} \phi_5(\bar{p}p) = & 2C \cot\pi\alpha(t) e^{ft(s)} \alpha(t) J_1(5.164\sqrt{-t}) \\ & + i[De^{gt(s)} \alpha_P(t) J_1(3.36\sqrt{-t}) + 2Ce^{ft(s)} \alpha(t) J_1(5.164\sqrt{-t})] \end{aligned} \quad (6.21)$$

where the numerical values of the various parameters are the same as for the pp case. Using (6.22) for the flip amplitude and (6.17a,b) for the non-flip amplitude with 40 GeV/c parameter values from Table (6.4a) yields a prediction for the 40 GeV/c polarisation in agreement with data



(a)

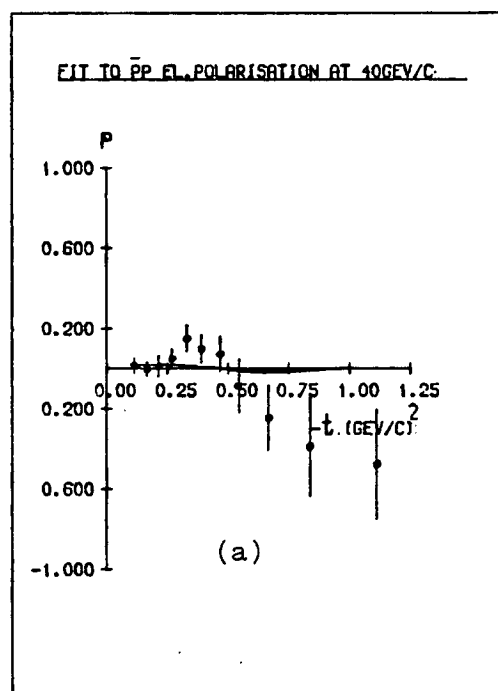


(b)

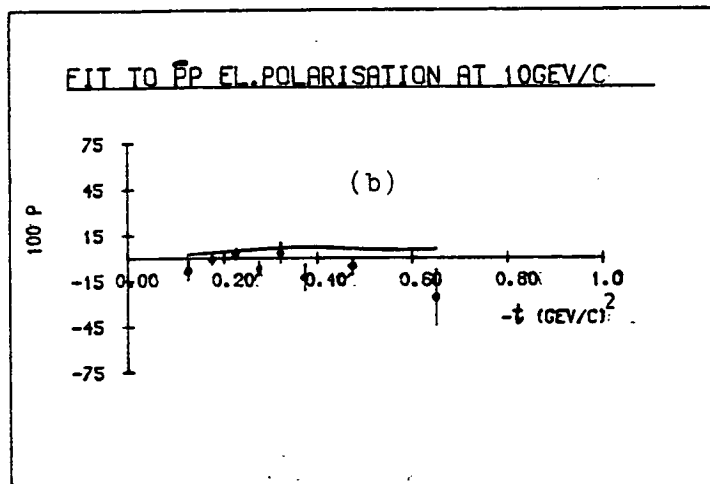
Fig 6.10 a,b): B_R and R values from table 6.4a for $\bar{p}p \rightarrow \bar{p}p$. Straight lines correspond to $B=4.58$ and $R=5.4$ in a) and b) respectively.

Fig 6.11 a): Fit to 40 GeV/c $\bar{p}p$ polarisation according to equations 6.6a, 6.17a and 6.22.

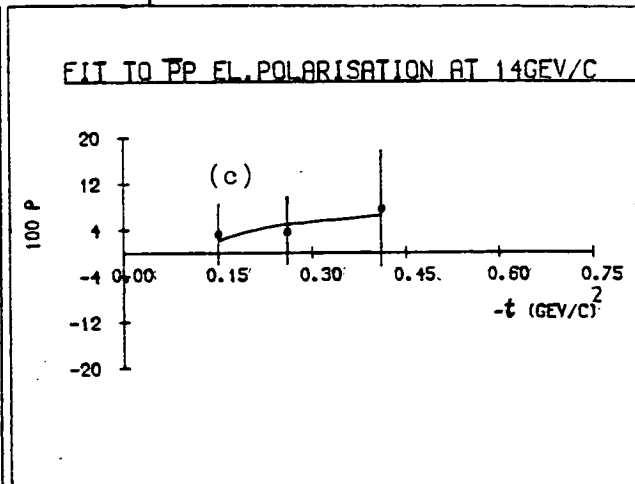
Fig 6.11 b,c): Predictions for 10 and 14 GeV/c $\bar{p}p$ polarisations using $B=3.82+0.24 \ln s$, $B=4.58$ and $R=5.4$ in conjunction with equations 6.6a, 6.17a, 6.22.



(a)



(b)



(c)

(χ^2 per point = 1.44). (If one uses the values $B_R = 4.58$ and $R = 5.4$ instead, a comparable χ^2 of 1.45 per point is obtained.) Using $B_R = 4.58 (\text{GeV})^{-2}$ and $R = 5.4 (\text{GeV})^{-1}$ along with $B_P = 3.82 + 0.24 \ln s$ enables us to determine ϕ at 10 and 14 GeV/c without fitting $\frac{d\sigma}{dt}(\bar{p}p)$ data at these momenta. One can consequently predict $\bar{p}p$ polarisations at these momenta. The results of our calculations are shown in Fig. 6.11a-c.

6.5 Conclusions and Discussion

We have, in this chapter, attempted to quantitatively assess the status of the DAM in regard to pp , np and $\bar{p}p$ scattering over a wide energy range in the region $|t| \leq 1.2 (\text{GeV}/c)^2$. We have, wherever required by data, attempted to parameterise the two amplitudes ϕ and ϕ_5 (that remain after simplifying phenomenological assumptions have been made) by going beyond the DAM "rules". As a result we fit total and differential cross-sections as well as polarisations and spin rotation parameters for the various reactions. We find that:

i) Fits to $\sigma_{\text{tot}}(pp, pn)$ in the range $6 \lesssim p_{\text{lab}} \lesssim 2092 \text{ GeV}/c$ suggest a breaking, not only of strong EXD between the appropriate trajectories but also of weak EXD. Contrary to general belief, we find that pp and pn total cross-sections cannot be fitted satisfactorily in the above-mentioned range by a Pomeron pole and a single Regge Pole term with reasonable intercept values ($0.3 < \alpha_R(0) \lesssim 0.6$) provided one does not make the approximation $s \approx 2q\sqrt{s}$ in the formula (equation 6.5a) for total cross sections (Table 6.1a,b). The data seem to require a Pomeron contribution (with $\alpha_P(0) = 1.06$) and two Regge pole like terms of opposite signs with intercepts of 0.5 and 0.4 (i.e. $A_1(s)^{0.5} - A_2(s)^{0.4}$).

Furthermore the residues (at $t = 0$) of these terms when considered for the pp and pn data together indicate that each of these Regge pole like terms contains $I = 0$ and $I = 1$ contributions. The breaking of EXD at both the strong as well as weak levels is inconsistent with two-component duality and hence with the DAM, provided one makes the generally accepted assumption that the pp and pn channels do not contain any resonances. $\sigma_{\text{tot}}(\bar{p}p)$ however is consistent with a single Regge pole term of intercept 0.5 and a Pomeron with intercept 1.06 (equation 6.6 and Table 6.3). It can also be fitted by a parameterisation of the same form as for the pp and np cases (i.e. equation 6.8) but the value of the residue of the $(s)^{0.4}$ term is not consistent with what one might expect on the basis of general Regge pole considerations. Bouquet and Diu (1975) have already referred to difficulties in Regge pole parameterisations of NN and $\bar{N}N$ total cross-sections.

ii) - We can fit pp and pn $\frac{d\sigma}{dt}$ data by taking the EXD breaking terms as peripheral and the imaginary part of the Pomeron as that required by DAM "rules" but with a real part given by the usual Regge pole phase. In this way one can reasonably fit pp, np

and $p\bar{p}$ differential cross-sections up to the ISR region ($\sqrt{s} \lesssim 62$ GeV) and for $|t| \lesssim 1.2$ (GeV/c)² (Tables 6.2a,b, 6.4a,b). The pp and $\bar{p}p$ fits are consistent with a shrinking Pomeron with slope $\alpha'_p \sim 0.25-0.5$.

The fits to np data below 21 GeV/c however do not indicate any shrinkage of the Pomeron term but the fits to 25 - 65 GeV/c data are consistent with shrinkage of the Pomeron term. The parameters B_R and R for $\bar{p}p \rightarrow \bar{p}p$ (eq. 6.17a) are very similar to the corresponding parameters for the K^-p case (Chapter 5) in that they are energy independent with $B_R \approx 4.58$ (GeV/c)² and $R \approx 5.4$ (GeV/c)⁻¹ (eqs. 6.20a,b, Table 6.4a).

The only other DAM parameterisations of pp and $\bar{p}p$ non-flip amplitudes that we are aware of are those of Barger and Halzen (1972) and Saleem et al. (1975). The former assume an imaginary Pomeron but with an undesirable non-shrinking Pomeron pole, as mentioned in previous chapters. They consider data in the range $2.5 \lesssim p_{lab} \lesssim 16$ GeV/c. The defects of the parameterisation of Saleem et al. (1975) have already been mentioned (Section 6.1).

iii) We can parameterise the flip amplitude by assuming the DAM prescription for the "ordinary" Regge exchanges (strong EXD, peripheralty, Regge phases) and can fit pp polarisations for $|t| \lesssim 1.2$ (GeV/c)² in the range $10 \lesssim p_{lab} \lesssim 300$ GeV/c provided a peripheral imaginary Pomeron flip term is introduced (eq. 6.15). As a byproduct we predict the spin rotation parameter R for pp scattering at 45 GeV/c as well as $\bar{p}p$ polarisations at 10, 14 and 40 GeV/c in agreement with data. Assuming that the $I = 0$ and 1 exchanges contribute with equal strength in the flip amplitude then yields an excellent fit to the 11.8 GeV/c np polarisations.

A Pomeron-flip contribution has previously been considered by Irving (1975) and by Martin and Navelet (1977). Irving (1975) writes the imaginary part of the contribution of the Pomeron to the flip amplitude as

$$\text{Im } P_{+-} = \sqrt{-t} r_p e^{b_3 t} (s)^{(\alpha' - \alpha'_p)t} \text{Im } P_{++}^{++}$$

where the term $\text{Im} P_{++}^{++}$ denotes the non-flip Pomeron contribution and is parameterised as (α'_p is the Pomeron slope, α' is the "ordinary" Regge trajectory slope)

$$\text{Im} P_{++}^{++} = g_p e^{b_1 t + b_2 t^2} s^{\alpha'_p t}.$$

In addition, the flip contribution of the Pomeron is given a real part. (The non-flip Pomeron contribution is taken as imaginary). The "ordinary" Regge contributions to the flip amplitude are taken as real and parameterised as $g_R e^{b_R t} s^{\alpha(t)}$. Such a parameterisation does not yield a zero in $P(pp)$ around $|t| \sim 1 \text{ (GeV/c)}^2$ at higher energies. However Snyder et al. (1978) have found that within the errors on data this parameterisation agrees with the 100 GeV/c pp polarisation up to $|t| \sim 1 \text{ (GeV/c)}^2$. In his parameterisation Irving (1975) has assumed that the $I = 0$ contributions to the flip amplitude dominate the $I = 1$ contributions - in fact he finds

$$\lambda \equiv \frac{I = 1 \text{ contribution to } \phi_5}{I = 0 \text{ contribution to } \phi_5} \sim \frac{1}{2}.$$

It has been noted by Kramer et al. (1978) that their measurement of $P(np)$ at 11.8 GeV/c is inconsistent with Irving's parameterisation. In particular at $|t| = 0.9$ the data is consistent with zero ($100 P(np) = 1.4 \pm 2.7$) while the parameterisation of Irving yields $100 P(np) \sim -11$ at $|t| = 0.9 \text{ (GeV/c)}^2$. Our parameterisation however reproduces this data very well ($\chi^2 = 0.8$ per point) with $\lambda = 1$ instead*. Also unlike Irving (1975) we have a real part for the non-flip Pomeron contribution. The parameterisation of the Pomeron-flip contribution by Martin and Navelet (1977) is structureless in t and has a real part given by the usual Regge pole phase.

*

The zero in $P(np)$ around $|t| \sim 1$ is a feature of the " J_1 " type parameterisation of the Pomeron flip contribution rather than of $\lambda = 1$.

In conclusion one can therefore say that:

- 1) The non-flip contribution of the Pomeron to elastic pp , np , $\bar{p}p$ scattering is given by the DAM "rules" but with a real part given by the usual Regge pole phase.
- 2) There is a breaking of EXD at both the strong and weak levels in the non-flip amplitude for pp and pn scattering which is inconsistent with the expectations of the DAM. Nonetheless the data can be parameterised by taking these EXD breaking terms as peripheral. The "ordinary" Regge contributions to $\text{Im}\phi(\bar{p}p)$ are however consistent with weak EXD although $\text{Im}\phi(\bar{p}p)$ can be parameterised equally well by breaking weak EXD and taking the EXD breaking terms as peripheral.
- 3) The "ordinary" Regge exchange contributions to $\phi_5(pp, \bar{p}p)$ are consistent with the DAM (peripherality, strong EXD, Regge phases) provided an imaginary peripheral Pomeron flip term is included. This leads to an explanation of $P(np)$ if one assumes that the $I = 1$ and $I = 0$ "ordinary" Regge exchanges contribute to $\phi_5(NN)$ with equal strength.

CHAPTER 7

TENTATIVE CONCLUSIONS

We have, in this work, attempted an assessment of the status of the DAM in regard to elastic scattering by trying to fit data on several elastic reactions in terms of amplitudes parameterised according to the DAM "rules". The reactions considered are $\pi^\pm p \rightarrow \pi^\pm p$, $K^\pm p \rightarrow K^\pm p$, $pp \rightarrow pp$, $np \rightarrow np$ and $\bar{p}p \rightarrow \bar{p}p$. For every reaction the data has been considered in the small $|t|$ region ($|t| \lesssim 1.3 \text{ (GeV/c)}^2$) while the energy region extends from $p_{lab} \sim 10 \text{ GeV/c}$ up to the highest energy at which data is available for the reaction under consideration. Before summarising our ^{tentative} conclusions we will briefly consider a few points of consistency.

We recall that the DAM "rules" have an imaginary s-channel helicity-conserving Pomeron contribution. The $\pi^\pm p$ data was fitted in Chapter 4 using such a Pomeron term. It may be recalled that the DAM does not specify the real part of the non-flip amplitude (c.f. e.g. Table 3.2 of Chapter 3). The common practice of phenomenologists, particularly in applying the DAM, is to neglect the real part of the non-flip amplitude in fitting elastic data. However in Chapter 5 we found it necessary to introduce a real part of the non-flip amplitude when fitting K^+p differential cross-sections. This real part is essentially given by the Pomeron phase asymptotically and accounts for the curvature present in $\frac{d\sigma}{dt}(K^+p)$. A real part of the Pomeron non-flip contribution was then assumed in all subsequent cases, primarily on grounds of consistency. We have therefore tried to refit $\pi^\pm p$ elastic data by including a real Pomeron contribution ($\text{Re}P = \cot \frac{\pi\alpha_P}{2} \text{Im}P$, $\alpha_P = 1.06 + 0.3t$). We find that $\frac{d\sigma}{dt}(\pi^\pm p)$ can be fitted comparably

to the fits in Chapter 4 except for the data of Ayres et al. (1977) which cannot be fitted satisfactorily if $\text{Re}P \neq 0$. However the introduction of a real Pomeron part worsens the polarisation fits, some of them being unacceptable.

In Chapter 5 we observed a discrepancy in the value of a parameter (parameter A_1 in equations 5.6b,d of Chapter 5) in the $K^\pm p$ flip amplitudes. We noted that, while the discrepancy could be reduced to a minimum of about 30%, it could not be entirely eliminated. Several causes of the discrepancy were suggested, but no quantitative effort was made to reparameterise the amplitudes on account of the paucity of data. However, partly prompted by the realisation of the presence of a Pomeron flip contribution in NN scattering, we have tried to reparameterise the $K^\pm p$ flip amplitude. We have found that the discrepancy observed in Chapter 5 can be removed provided one admits:

- i) an imaginary, peripheral Pomeron flip contribution as in the NN case;
- ii) an additional, EXD breaking, $C = -1$ exchange contribution (presumably ρ or ω) parameterised according to the DAM "rules".

With these modifications the $K^\pm p$ flip amplitudes become:

$$T_{+-}(K^+p) = \left[\frac{2A_1 J_1(5.164\sqrt{-t})}{\sin\pi\alpha(t)} + C \tan \frac{\pi\alpha}{2} J_1(R\sqrt{-t}) \right] s^{\alpha(t)} + i \left[D e^{gt} s^{\alpha_P(t)} J_1(3.36\sqrt{-t}) + C s^{\alpha(t)} J_1(R\sqrt{-t}) \right] \quad (7.1a)$$

$$T_{+-}(K^-p) = \left[2A_1 \cot\pi\alpha(t) J_1(5.164\sqrt{-t}) - C \tan \frac{\pi\alpha}{2} J_1(R\sqrt{-t}) \right] s^{\alpha(t)} + i \left[D e^{gt} s^{\alpha_P(t)} J_1(3.36\sqrt{-t}) + \{-C J_1(R\sqrt{-t}) + 2 A_1 J_1(5.164\sqrt{-t})\} s^{\alpha(t)} \right]. \quad (7.1b)$$

The above amplitudes when employed in conjunction with the non-flip amplitudes in Chapter 5 yield satisfactory fits to $K^{\pm}p$ polarisations, with $A_1 = 12$, $C = 6$, $D = 6$, $R = 4.5 \text{ (GeV)}^{-1}$ and $g = 2.5 \text{ (GeV)}^{-2}$. [Overall χ^2 per point ≈ 2.0 for K^+p and 1.9 for K^-p . The break up is as follows: χ^2 per point = 2.7, 2.3, 1.1 respectively for 10, 14 and 45 GeV/c K^+p polarisations and 2.07, 0.93, 2.7 respectively for 10, 14 and 40 GeV/c K^-p polarisations.]* Of course, both the introduction of a Pomeron flip term, and the presence of an EXD breaking "ordinary" Regge term in the flip amplitude, are inconsistent with the expectations of the DAM. The latter, in particular, is inconsistent with 2-component duality and therefore with the DAM.

Keeping the aforementioned comments in mind we can summarise the conclusions of our investigations in Chapters 4, 5 and 6 as follows:

(i) The imaginary part of non-flip amplitude in $\pi^{\pm}p$ elastic scattering is consistent with the DAM.

The flip amplitude for $\pi^{\pm}p$ elastic scattering is also consistent with the DAM.

The energy dependence of the relevant parameter in the Pomeron term implies a shrinking Pomeron pole with a slope $\alpha_p' \approx 0.35 - 0.40$. The radius of interaction of the non-flip "ordinary" Regge exchange contribution shows a slow increase with energy.

(ii) The imaginary part of the non-flip amplitude in $K^{\pm}p$ elastic scattering is consistent with the DAM - in particular the K^+p amplitude is consistent with strong EXD. The non-flip amplitude has a real part which is given asymptotically by the usual Regge-pole phase rule for the Pomeron contribution. The flip amplitude however cannot be parameterised adequately by assuming strong EXD for the "ordinary"

* The above fits could perhaps be improved by varying parameter values but we have not explored the possibility here on account of the relative paucity of data.

exchanges. In addition to a strongly EXD contribution, the flip amplitude contains a $C = -1$ ("ordinary" Regge) exchange term (presumably ρ or ω) and an imaginary peripheral Pomeron contribution (with a radius of interaction ~ 0.7 fm). Both of these terms are inconsistent with the expectations of the DAM - the former in particular conflicts with 2-component duality (and hence the DAM) if it is assumed that there are no resonances in the K^+p channel. All "ordinary" Regge exchanges present in the flip amplitude however can be parameterised according to the DAM "rules" (peripherality, Regge phases - see Table 3.2, e.g.).

The energy dependence of the relevant parameters in the K^+p non-flip amplitude implies a Pomeron slope of $\alpha'_p \approx 0.3 - 0.4$. The various parameters in the "ordinary" Regge-exchange term in the K^-p non-flip amplitude are energy independent - in particular the radius of interaction is consistent with a value $R \sim 4.52$ (GeV/c) $^{-1}$.

(iii) The imaginary parts of the pp and np elastic non-flip amplitudes contain terms which indicate a breaking of EXD (at both the strong and weak levels) which is inconsistent with the hypothesis of 2-component duality (and hence the DAM) if one assumes that these channels do not contain any resonances. The imaginary part of the $\bar{p}p$ non-flip amplitude however can be parameterised either by assuming weak EXD or by breaking weak EXD in accordance with the intercept values obtained in pp and np cases. Nonetheless the small $|t|$ ($|t| \lesssim 1.3$ (GeV/c) ^{2}) differential cross-section data for all three reactions can be fitted by taking the non-Pomeron terms in the non-flip amplitude as peripheral. The non-flip amplitude has a real part, given asymptotically, as in the K^+p case, by the Pomeron-pole phase in all three reactions. The "ordinary" Regge-exchanges in the single-flip amplitude (ϕ_5) for these reactions however appear to be

consistent with the DAM provided that an imaginary peripheral Pomeron flip term (radius of interaction ~ 0.7 fm) is included. Such a parameterisation of ϕ_5 yields adequate fits to $P(pp)$, $R'(pp)$, $P(\bar{p}p)$ and $P(pn)$, the np polarisation being obtained by the additional assumption that both $I = 0$ and 1 "ordinary" Regge exchanges contribute equally to this amplitude.

The relevant parameter in the Pomeron term in the pp and $\bar{p}p$ non-flip amplitude has an energy-dependence, which implies a shrinking Pomeron pole with slope $\alpha_p' \sim 0.25 - 0.5$. The parameters associated with the various "ordinary" Regge terms in the pp and np non-flip amplitude have a (rough) energy dependence, increasing very slowly with energy. The corresponding $\bar{p}p$ parameters are consistent with energy independence however (as in the K^-p case), the radius of interaction being consistent with a value $R \sim 5.4 \text{ (GeV)}^{-1}$ throughout.

However before drawing our final conclusions we will refit data after including the phases of all contributions, treating trajectory parameters as free parameters, and by enforcing isospin constraints on our parameter values. This is done in the following chapter.

CHAPTER 8

ELASTIC SCATTERING REVISITED - FINAL CONCLUSIONS

In Chapters 4, 5 and 6 we attempted a parameterisation of amplitudes for several elastic reactions in using the DAM and some additional assumptions. In Chapter 7 some tentative conclusions were drawn after attempting to remove some of the discrepancies noted earlier. However our approach remains open to several objections which must be met before any conclusions can safely be drawn. The following objections can be made against our previous analysis:

i) The assumption of weak EXD of the various "ordinary" trajectories used in Chapters 4 and 5 is now known to be violated. It is therefore desirable to determine the trajectory parameters by a careful and unified treatment of various reactions.

ii) We have neglected the phases of the various contributions to the non-flip amplitude. Also the real part of the Pomeron-flip contribution was set equal to zero in Chapter 5. Neglecting the phase of a contribution however violates analyticity. Furthermore it is quite possible that the neglect of the phases might, in conjunction with (i) above, be the source of some of the discrepancies observed in parameter values since there is an organic connection between the different parts of the amplitudes and various observables.

iii) In many instances the values of the parameters do not satisfy isospin constraints. Although some such variations in parameter values are probably due to normalisation discrepancies in the measurements from different experiments it is quite possible that one might, after taking (i) and (ii) above, into account, be able to fit data by using parameter values consistent with isospin invariance of strong interaction amplitudes.

In the following we undertake a re-analysis of the data by including the various phases and incorporating isospin constraints along with carefully determined trajectory parameters in our parameterisation.

TRAJECTORY INTERCEPTS AND TOTAL CROSS-SECTIONS

We shall, to begin with, consider the various trajectory intercepts as determined by fitting suitable total cross-section combinations. The P and f have the same quantum numbers and it is therefore not possible to form total cross-section combinations in which they can be separated. On the other hand the ρ , ω and A_2 trajectories correspond to different quantum numbers and their contributions to various total cross-sections can be isolated by forming suitable combinations of the appropriate total cross-sections. In addition the ρ and A_2 intercepts can also be obtained from the forward differential cross-sections for $\pi^- p \rightarrow \pi^0 n$ and $\pi^- p \rightarrow \eta n$ respectively. We shall first consider the ω , ρ and A_2 intercepts (and residues), since their contributions are easily isolated, and then we shall attempt a determination of the P and f contributions to the various systems.

ω Exchange: ω exchange can be isolated in several cases. The commonly used cases are

$$\Delta\sigma(Kp) + \Delta\sigma(Kn) = 4\omega^{KN} \quad (8.1a)$$

$$\Delta\sigma(pp) + \Delta\sigma(pn) = 4\omega^{NN} \quad (8.1b)$$

where

$$\Delta\sigma(xN) \equiv \sigma_{\text{tot}}(\bar{x}N) - \sigma_{\text{tot}}(xN) \quad (8.1c)$$

and ω^{ij} is explained in (8.1f) below. It is also well known that the

ω intercept can, to a good approximation, be determined from the following combinations (c.f. e.g. Bouquet and Diu (1975))

$$\Delta\sigma(pd) = 2\omega^{Nd} \quad (8.1d)$$

$$\Delta\sigma(Kd) = 2\omega^{Kd} \quad (8.1e)$$

In equations (8.1a,b,d,e),

$$\omega^{ij} \equiv \frac{0.3893}{2q\sqrt{s}} \beta_{\omega}^{ij}(0) s^{\alpha_{\omega}(0)} \quad (8.1f)$$

The factor 0.3893 arises because the cross-sections are in millibarns. $\beta_{\omega}^{ij}(0)$ is the residue, at $t = 0$, of the ω contribution to the system ij . For the $K(p)$ deuteron data however, $\beta_{\omega}^{id}(0)$ is, strictly speaking, not the residue but the residue and a correction term - this however does not matter because in (8.1d,e) we are interested only in the ω -intercept. The relevant cross-section combinations in (8.1a,b,d,e) are now available up to 310, 280, 280 and 310 GeV/c respectively (Carroll et al. (1979, 1976), Denisov et al. (1973), Galbraith et al. (1965)). We have considered data for $p_{lab} \geq 6$ GeV/c. In determining the ω -intercept (and residues) we have first fitted each of the above combinations separately and then taken these data sets simultaneously. Each data set is in excellent agreement with the Regge-pole model and yields almost the same intercept in each case. Our results are:-

			χ^2/n
$\alpha_{\omega}(0)$	$= 0.461 \pm 0.022$	$\beta_{\omega}^{KN}(0)$	$= 24.121 \pm 2.304 \quad (\frac{26}{25})$
$\alpha_{\omega}(0)$	$= 0.445 \pm 0.017$	$\beta_{\omega}^{NN}(0)$	$= 81.53 \pm 4.57 \quad (\frac{5.5}{25})$
$\alpha_{\omega}(0)$	$= 0.468 \pm 0.021$	$\beta_{\omega}^{Kd}(0)$	$= 63.652 \pm 6.233 \quad (\frac{42.6}{25})$
$\alpha_{\omega}(0)$	$= 0.450 \pm 0.011$	$\beta_{\omega}^{Nd}(0)$	$= 212.05 \pm 12.605 \quad (\frac{18.8}{25})$

When fitted simultaneously the data yield

$$\begin{aligned}\alpha_{\omega}(0) &= 0.449 \pm 0.001 & \beta_{\omega}^{KN}(0) &= 25.292 \pm 0.276 & \beta_{\omega}^{NN}(0) &= 81.106 \pm 0.131 \\ \beta_{\omega}^{Kd}(0) &= 69.833 \pm 8.287 & \beta_{\omega}^{Nd}(0) &= 212.05 \pm 12.605 \\ \chi^2/n &= \frac{97.3}{100} .\end{aligned}$$

From a somewhat similar analysis in a slightly smaller energy range

($p_{lab} \leq 240$ GeV/c) Bouquet and Diu (1975) obtained $\alpha_{\omega}(0) = 0.44 \pm 0.01$.

Our results are in close agreement with theirs.

ρ -Exchange: There are several ways of determining the ρ intercept, the most obvious and commonly used way being via the $\pi^{\pm}p$ total cross-section differences and the forward differential cross-sections for the CEX reaction $\pi^{-}p \rightarrow \pi^{0}n$. In addition to this it is also possible to determine the ρ intercept by isolating the ρ contribution to the various kaon-nucleon and nucleon-(anti-) nucleon total cross-sections. The relevant expressions for these cases are:

$$\Delta\sigma(\pi p) = 2\rho^{\pi N} \quad (8.2a)$$

$$\left. \frac{d\sigma}{dt} (\pi^{-}p \rightarrow \pi^{0}n) \right|_{t=0} = \frac{389.3}{64\pi s q^2} \left[\tan^2 \frac{\pi\alpha_{\rho}(0)}{2} + 1 \right] 2(\beta_{\rho}^{\pi N}(0))^2 (s)^{2\alpha_{\rho}(0)} \quad (8.2b)$$

$$\Delta\sigma(Kp) - \Delta\sigma(KN) = 4\rho^{KN} \quad (8.2c)$$

$$\Delta\sigma(pp) - \Delta\sigma(pn) = 4\rho^{NN} \quad (8.2d)$$

where, as in (8.1f)

$$\rho^{ij} = \frac{0.3893}{2q\sqrt{s}} \beta_{\rho}^{iN}(0) s^{\alpha_{\rho}(0)} \quad (8.2e)$$

We have attempted a determination of the ρ intercept by first treating

these data sets individually and then collectively, as in the ω case. Both approaches lead to difficulties which are now well known. Further investigation leads us to include an additional contribution, ρ' , to overcome these difficulties. We discuss our results below.

It is well known (Leader and Nicolescu (1973), Barger and Phillips (1974), Joynson et al. (1975), Bouquet and Diu (1975), Nakata (1977, 1978)) that problems arise when one considers the $\Delta\sigma(\pi p)$ and $\frac{d\sigma}{dt}(\pi^- p \rightarrow \pi^0 n)|_{t=0}$ data together for a determination of the ρ intercept. Taken separately, each of these quantities can be fitted reasonably with a ρ Regge pole but the intercept (and residue) values so obtained are quite different in the two cases (the difference $\Delta\alpha_\rho \approx 0.1$ - this is a discrepancy of approximately 20% !). It is also now known (Bouquet and Diu (1975), Nakata (1977)) that the Serpukhov data on $\Delta\sigma(\pi p)$ (Denisov et al. 1973)) is out of line with the $\Delta\sigma(\pi p)$ measurements of Foley et al. (1967) and the Fermilab data (Carroll et al. (1976)). The same holds true for the πN CEX data from Serpukhov (Bolotov et al. (1974a)) and from other experiments (Barnes et al. (1976), Apel et al. (1978), Stirling et al. (1965)). Initially we considered the various fits twice, firstly including the Serpukhov points and subsequently excluding them. This seemed to make practically no difference to the intercept values but the χ^2 values were inferior when the fits included the Serpukhov points. A similar conclusion was reached by Bouquet and Diu (1975) who then omitted the Serpukhov data in the final phases of their investigation. We shall do likewise. The results of our calculations are as follows:*

* The $\frac{d\sigma}{dt}(\pi^- p \rightarrow \pi^0 p)|_{t=0}$ data involves extrapolation to $t = 0$. Bolotov et al. (1974) have, in an appendix, quoted their values for this quantity with error bars) and also those of an earlier experiment (Stirling et al. (1965)). This is done by fitting the $\frac{d\sigma}{dt}$ data in a small $|t|$ range (say $|t| \lesssim 0.2$ (GeV/c)²) by the expression, due to Phillips and Rarita (1965),

Quantity	$\alpha_p(0)$	$\beta_p^{\pi N}(0)$	$\chi^2/\text{point} \approx$
$\Delta\sigma(\pi p)$ including Serpukhov	0.577 ± 0.010	8.722 ± 0.444	2.1
$\Delta\sigma(\pi p)$ excluding Serpukhov	0.574 ± 0.008	8.555 ± 0.488	1.4
$\frac{d\sigma}{dt}(\pi^- p \rightarrow \pi^0 n) \Big _{t=0}$ including Serpukhov	0.485 ± 0.004	12.123 ± 0.241	2.4
$\frac{d\sigma}{dt}(\pi^- p \rightarrow \pi^0 n) \Big _{t=0}$ excluding Serpukhov	0.485 ± 0.006	12.127 ± 0.424	1.9

For the kaon-nucleon and nucleon(anti-nucleon)-nucleon data one has:

$$\text{KN data: } \alpha_p(0) = 0.489 \pm 0.049 \quad \beta_p^{\text{KN}}(0) = 7.3111 \pm 1.447$$

$$(\chi^2/\text{pt} \approx 1.7)$$

$$\text{N(N)N data: } \alpha_p(0) = 0.501 \pm 0.005 \quad \beta_p^{\text{NN}}(0) = 4.111 \pm 1.001$$

$$(\chi^2/\text{pt} \approx 0.9)$$

These results merely confirm what has already been described and known for some time. The discrepancy in $\Delta\alpha_p(0)$ from the $\Delta\sigma$ and $\frac{d\sigma}{dt}$ data is much too large. As a first alternative, by carrying out a

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt} \Big|_{t=0} (1 - gct)e^{ct} . \quad (8.3)$$

The physical meaning of g and c need not concern us here. The FNAL group (Barnes et al. (1976)) do not give any error bars on their $\frac{d\sigma}{dt} \Big|_{t=0}$ values and in fact have used a Regge pole type fit over a wide t range to fit their data - as a by-product of which they get $\frac{d\sigma}{dt} \Big|_{t=0}$ values. Apel et al. (1979) have used (8.3) to give a $\frac{d\sigma}{dt} \Big|_{t=0}$ value (with error bars) for their 40 GeV/c measurements but have not quoted any values for their remaining measurements at 15, 20.2, 25 and 30 GeV/c. Therefore we have used (8.3) to determine the $\frac{d\sigma}{dt} \Big|_{t=0}$ values for the remaining measurements of Apel et al. (1979) and for all measurements of Barnes et al. (1976). As a cross-check we find that our $\frac{d\sigma}{dt} \Big|_{t=0}$ values for the data of Barnes et al. (1976) agree with theirs, within errors. For the sake of record we quote our (and their) numbers in Appendix I.

simultaneous fit to the various data sets one might hope to achieve a sort of "compromise" value for $\alpha_\rho(0)$ (and $\beta_\rho^{\pi N}(0)$) with which to work. This approach yields

	Including Serpukhov	Excluding Serpukhov
$\alpha_\rho(0)$	0.488 ± 0.003	0.491 ± 0.003
$\beta_\rho^{\pi N}(0)$	11.933 ± 0.211	11.753 ± 0.173
$\beta_\rho^{KN}(0)$	7.627 ± 0.427	7.526 ± 0.200
$\beta_\rho^{NN}(0)$	4.293 ± 0.141	4.302 ± 0.149
χ^2/pt	2.35	2

Using the values of $\alpha_\rho(0)$ and $\beta_\rho^{\pi N}(0)$ in the second column yields a $\chi^2/pt \approx 3.6$ for $\Delta\sigma(\pi p)$ (excluding Serpukhov). Despite giving an acceptable overall χ^2/pt of column 2 this is not a "compromise" value in the intended sense as the $\Delta\sigma(\pi p)$ data is not acceptably fitted by this value. Clearly an alternative approach is required. Henceforth we shall omit the Serpukhov points from our calculations following Nakata (1977) and Bouquet and Diu (1975) since their inclusion, while making practically no difference to the various parameter values, affects χ^2 quite adversely.

Difficulties with the ρ -exchange model in πN scattering have led people to add new terms to the non-flip amplitude. These terms are either non-Regge in character (e.g. Bialkowski et al. (1975), Joynson et al. (1975)) or are additional Regge singularities (Nakata (1978), Leader and Nicolescu (1973)). We would prefer to work with Regge singularities and hence, following Nakata (1978) and Leader and Nicolescu (1973), use a $\rho + \rho'$ amplitude instead of the ρ . The ρ' is parameterised in exactly the same form as the ρ in (8.2e). We first concentrate on the

πN data alone to obtain the ρ and ρ' parameters and then use the resulting ρ intercept to fit KN and $N(\bar{N})N$ data of equation (8.2c,d). No ρ' contribution will be assumed in these latter cases. The ρ' is expected to be a low-lying trajectory whose parameters will hopefully give us some clue to its nature. Using a $\rho + \rho'$ amplitude for the πN data ($\Delta\sigma(\pi p)$ and $\left. \frac{d\sigma}{dt}(\pi^- p \rightarrow \pi^0 n) \right|_{t=0}$) resolves the discrepancy observed previously and one obtains:

$$\begin{aligned}\alpha_\rho(0) &= 0.485 \pm 0.001 & \beta_\rho^{\pi N}(0) &= 12.184 \pm 0.063 \\ \alpha_{\rho'}(0) &= -1.745 \pm 0.047 & \beta_{\rho'}^{\pi N}(0) &= -459.28 \pm 3.93 \\ \chi^2/pt &= 2.2\end{aligned}$$

The χ^2/pt of 2.2, though not so good, is still acceptable for now we obtain comparable fits to both $\Delta\sigma$ and $\left. \frac{d\sigma}{dt} \right|_{t=0}$ CEX data and there is no longer a discrepancy in the $\alpha_\rho(0)$ values. Nakata (1978) obtains $\alpha_{\rho'}(0) = -1.80$ which is very close to our value of -1.745 ± 0.047 . It is quite tempting to identify our ρ' as a trajectory on which the first particle is $\rho'(1600)$ with $I^G(J)^P C$ of $1^+(1^-)-$ and the next one $T(2190)$ with quantum numbers $1^+(3^-)-$. Both these mesons are listed in the meson table of the Review of Particle Properties by the Particle Data Group (1978). Their masses are only known approximately and ρ' is quite wide ($\Gamma_\rho \approx 300$ MeV). Allowing for these uncertainties it is possible to put them on a trajectory of intercept -1.745 and having the universal slope $\alpha'_{\rho'} = 0.9$. Such a trajectory then yields $m_{\rho'} = 1.75$ and $m_T = 2.3$ which are quite close to the experimental values $m_{\rho'} \approx 1.6$ and $m_T = 2.19$ (the value m_T is, according to the data tables, an "educated guess").* Nakata (1978) did not make any physical identification of his

* Using the fact that at $t = m_R^2$ $\alpha(t=m_R^2) = J_R$ where m_R and J_R are the mass and spin respectively of a resonance on the trajectory $\alpha(t)$ we can find easily, knowing $\alpha(t)$ and J_R , what m_R is. Thus:

$$m_R = \sqrt{(J_R - \alpha_0)/\alpha'} \quad (8.4)$$

α' being the slope.

ρ' , probably because at the time of submission of his paper (June 1977) the 1978 Review of Particle Properties by the Particle Data Group had not appeared. The earlier reviews (e.g. Particle Data Group (1976)) did not regard these as established resonances. However the physical identification of the ρ' exchange that we have made, although providing additional credibility, is not the reason for invoking it - the need for such a contribution arises in order to restore consistency between the $\alpha_\rho(0)$ values as obtained from $\Delta\sigma(\pi p)$ and $\left. \frac{d\sigma}{dt}(\pi^- p \rightarrow \pi^0 n) \right|_{t=0}$ data.

Using $\alpha_\rho(0) = 0.485$ then gives the following values for $\beta_\rho^{NN}(0)$ and $\beta_\rho^{KN}(0)$ ($\chi^2/pt = 1.5$)

$$\beta_\rho^{NN}(0) = 5.41 \pm 1.0$$

$$\beta_\rho^{KN}(0) = 7.497 \pm 0.36.$$

A₂-Exchange: The A₂ intercept can be determined from $\left. \frac{d\sigma}{dt}(\pi^- p \rightarrow \eta n) \right|_{t=0}$ as well as from the following combinations: ($\sigma \equiv \sigma_{tot}$)

$$\sigma(K^+p) - \sigma(K^+n) + \sigma(K^-p) - \sigma(K^-n) = 4A_2^{KN} \quad (8.5a)$$

$$\sigma(pp) - \sigma(pn) + \sigma(\bar{p}p) - \sigma(\bar{p}n) = 4A_2^{NN} \quad (8.5b)$$

where we write

$$A_2^{ij} = \frac{0.389}{2q\sqrt{s}} \beta_{A_2}^{\pi N}(0) \sin \frac{\pi \alpha_{A_2}(0)}{2} (s)^{\alpha_{A_2}(0)} \quad (8.5c)$$

The factor $\sin \frac{\pi \alpha_{A_2}(0)}{2}$ has been factored out for future convenience.

Of the combinations 8.5a,b, the latter has been investigated in great detail in a series of papers by Bouquet et al. (1975, 1976) who find that the data on the L.H.S. of 8.5b leads to a contradiction between the FNAL and Serpukhov data. This in fact leads them to question the procedure

used at Serpukhov to extract $\sigma_{\text{tot}}(\bar{p}n, pn)$ data from $p(\bar{p})-d$ collisions. The upshot of their investigation is that the cross-sections have to be corrected and that their corrected values lead to a startling contradiction with the standard Regge pole model. They find that the (corrected) data indicate that the term A_2^{NN} on the R.H.S. of (7.5b) is not, as generally believed, simply the A_2 Regge pole contribution, but rather an ordinary A_2 Regge pole and another high lying $I = 1$ term which contributes oppositely to A_2 and eventually dominates it. In view of this controversy we have confined ourselves only to $\frac{d\sigma}{dt}(\pi^- p \rightarrow \eta n) \Big|_{t=0}$ data (available up to 199.3 GeV/c) and to the kaon-nucleon data (now available up to 280 GeV/c).^{*} The forward differential cross-section data involve, as in the $\pi^- p \rightarrow \eta n$ case discussed earlier, extrapolation to $t = 0$. Bolotov et al. (1974b) have, in their paper, quoted the extrapolated values (with error bars) not only for their measurements but also of some earlier experiments (Guisan et al. (1965)). The FNAL group (Dahl et al. (1976)) on the other hand do not quote any error bars on the forward differential cross-section values which they obtain as a by-product of a Regge pole type fit to the data over the $|t| \lesssim 1 \text{ (GeV/c)}^2$ range. We have used (8.3) to calculate the $\frac{d\sigma}{dt} \Big|_{t=0}$ values, as for the πN CEX case. The relevant numbers are, for the sake of completeness, quoted in Appendix 2. In determining the A_2 contribution for $\pi^- p \rightarrow \eta n$ at $t = 0$ we then fit the $\frac{d\sigma}{dt}(\pi^- p \rightarrow \eta n) \Big|_{t=0}$ data by

$$\frac{d\sigma}{dt}(\pi^- p \rightarrow \eta n) \Big|_{t=0} = \frac{389.3}{64\pi s q^2} \left[\cot^2 \frac{\pi \alpha_{A_2}(0)}{2} + 1 \right] \left[\beta_{A_2}^{\pi N}(0) \sin \frac{\pi \alpha_{A_2}(0)}{2} \right]^2 \times s^{2\alpha_{A_2}(0)} \quad (8.5d)$$

^{*}On account of this difficulty we have excluded np scattering from our subsequent analysis.

(The $\sin \frac{\pi \alpha_{A_2}(0)}{2}$ factor is for uniformity with 8.5c where it is factored out for future convenience; $\frac{d\sigma}{dt}$ is in μ -barns).

A simultaneous fit to the data involved in (8.5a,d) then yields two results of comparable χ^2 . These results are:

	Sol. 1	Sol. 2
$\alpha_{A_2}(0)$	0.3475 ± 0.002	0.3618 ± 0.007
$\beta_{A_2}(0)$	13.00 ± 1.82	11.732 ± 1.861
$\beta_{A_2}(0)$	10.95 ± 0.104	10.073 ± 2.625
χ^2/pt	1.4	1.3

Bouquet and Diu (1975) found $\alpha_{A_2}(0) = 0.36 \pm 0.1$, which agrees with solution 2. Later we find that a better fit is obtained for $S(Kp) \equiv \sigma_{tot}(K^+p) + \sigma_{tot}(K^-p)$ if we employ solution 1. Our results for $\alpha_{A_2}(0)$ and $\alpha_\rho(0)$ indicate that the ρ - A_2 EXD is badly broken (approx. 20%) so that it is unsafe to use the same intercept for ρ and A_2 as is widely done and as we did in the previous chapters. The breaking of ρ - A_2 EXD at any level (intercepts, trajectories, residues) is in contradiction with the expectations of the DAM.

The P and f Exchanges

Having determined the ω , ρ and A_2 intercepts we now turn to the P and f exchanges. As noted earlier, it is not possible to form total cross-section combinations where the P and f are separated. In order to determine their intercepts, we proceed as follows. According to standard ideas one has the following total cross-section combinations where the P and f appear together:

$$S(\pi p) \equiv \sigma_{\text{tot}}(\pi^- p) + \sigma_{\text{tot}}(\pi^+ p) = 2[P + f]^{\pi N} \quad (8.6a)$$

$$S(Kp) \equiv \sigma_{\text{tot}}(K^- p) + \sigma_{\text{tot}}(K^+ p) = 2[P + f + A_2]^{\text{KN}} \quad (8.6b)$$

$$S(pp) \equiv \sigma_{\text{tot}}(\bar{p}p) + \sigma_{\text{tot}}(pp) = 2[P + f + A_2]^{\text{NN}} \quad (8.6c)$$

One can also add $S(pn)$ data to the above list but on account of the controversy surrounding the A_2 contribution to the NN system, owing to the difficulties with $\sigma_{\text{tot}}(pn)$, $\sigma_{\text{tot}}(\bar{p}n)$ data referred to earlier (Bouquet and Diu (1975)), we have not included $S(pn)$ data in our calculations. This is a shortcoming about which we cannot at present do anything because Bouquet et al. (1975, 1976) have not published their corrected values for $\sigma_{\text{tot}}(pn)$ and $\sigma_{\text{tot}}(\bar{p}n)$. (In fact this has led us to exclude np data in subsequent investigations.) In equation 8.6a,b,c, P^{ij} , f^{ij} are written as:

$$P^{ij} = \frac{0.3893}{2q\sqrt{s}} \beta_P^{ij}(0)(s) \alpha_P^{(0)} \quad (8.7a)$$

$$f^{ij} = \frac{0.3893}{2q\sqrt{s}} \beta_f^{ij}(0) \sin \frac{\pi \alpha_f(0)}{2} (s) \alpha_f^{(0)} \quad (8.7b)$$

($\sin \frac{\pi \alpha_f(0)}{2}$ has been factored out for future convenience.)

A_2^{ij} is the same as in (8.5c). We then fit the data on the L.H.S. of equation (8.6a,b,c) using the parameterisation (8.5c), (8.7a,b) for the various terms on the R.H.S. of (8.6a,b,c). Here we note that while we know the A_2^{KN} contribution we do not know A_2^{NN} because of the previously mentioned difficulties. Hence we leave $\beta_{A_2}^{\text{NN}}(0)$ (c.f. equation (8.5c)) as a free parameter, but with $\alpha_{A_2}(0)$ fixed at a value obtained in the previous section ($\alpha_{A_2}(0) = 0.3618$ or 0.3475). The data ($p_{\text{lab}} \geq 6 \text{ GeV/c}$) has been taken from Foley et al. (1967), Galbraith

et al. (1965), Carroll et al. (1976, 1979) and Denisov et al. (1974) (The πN data from Denisov et al. (1974) has been excluded.) The $S(\pi p)$ data is available up to 340 GeV/c while the $S(Kp)$ and $S(pp)$ data extend up to 310 and 280 GeV/c respectively. We find, somewhat surprisingly, that it is impossible to fit the $S(hp)$ data satisfactorily using (8.6), (8.7) and (8.5c). Using $\alpha_{A_2}(0) = 0.3475$ and the corresponding $\beta_{A_2}^{KN}(0)$ value improves the result (very) slightly but the overall $\chi^2/\text{d.o.f}$ values are still large. The best we could manage was $\chi^2/\text{d.o.f.} = 4.9$ with the following values:

$$\begin{aligned} \alpha_P(0) &= 1.0738; & \beta_P^{\pi N}(0) &= 39.876; & \beta_P^{KN}(0) &= 33.043; & \beta_P^{NN}(0) &= 61.781 \\ \alpha_f(0) &= 0.434; & \beta_f^{\pi N}(0) &= 140.89; & \beta_f^{KN}(0) &= 48.492; & \beta_f^{NN} &= 345.83 \\ \alpha_{A_2}(0) &= 0.3475, & \beta_{A_2}^{KN} &= 11.27; & \beta_{A_2}^{NN} &= 0.171 \end{aligned}$$

On closer scrutiny the problem seems to lie with the πN and NN data. We have even attempted separate fits to $S(\pi p)$ and $S(pp)$ data allowing for differing intercepts in the two cases but satisfactory fits are still not possible. This convinced us that the equations (8.6) (particularly (8.6a,c)) are inadequate for describing $S(hp)$ data. Hence the need for a new term on the R.H.S. of (8.6a,c). In the past a failure of the Regge model has always been compensated for either by finding new Regge terms or else by invoking non-Regge terms which are then used either to replace or complement the Regge description. We will, as a first step, try to find a new Regge term whose inclusion might solve this difficulty. However before attempting to do so one could explore literature on the possibility of the presence of one or more additional Regge singularities in elastic amplitudes. Interestingly enough, Dash and Navelet (1976) proposed a new low-lying $I = 0$ crossing-even trajectory, which they called the σ ($\alpha_\sigma(0) = -0.4$), in order to explain the anomalous energy

dependence of pp polarisation. The contribution of such a term was invoked only in the (single-)flip amplitude ϕ_5 . Soon afterwards Girardi and Navelet (1976) used a similar term ($\alpha_\sigma(0) = -0.32$) in the πN flip amplitude to explain the departure of πp polarisations from mirror symmetry. Martin and Navelet (1977) have subsequently confirmed the need for such a trajectory (they use $\alpha_\sigma(0) = -0.5$) in the flip amplitudes in elastic scattering. Earlier, Field and Stevens (1975), while analysing nucleon-nucleon data, found that one needs a pair of $I = 0$ trajectories of opposite signature and opposite C values ($C = \pm 1$). They called these ϵ ($\tau = +1, C = +1$) and $\tilde{\omega}$ ($\tau = -1, C = -1$) trajectories ($\alpha_\epsilon(0) = \alpha_{\tilde{\omega}}(0) \approx -0.5$). Their investigations indicated that $\tilde{\omega}$ couples strongly to the non-flip amplitude while ϵ couples strongly to the single-flip amplitude. Berger, Irving and Sorensen (1978), while studying the implications of measurements of various spin observables at ANL, find that the ϵ and ω' (they denote $\tilde{\omega}$ as ω') contribute comparably to the nonflip amplitude. They point out however that there is some difficulty in reproducing the phase $\rho(pp)$ correctly at lower energies. Irving (1979) has made similar observations on the model of Berger, Irving and Sorensen (1978). It is clear therefore that a low-lying, crossing-even isoscalar trajectory is needed in flip amplitudes for various elastic reactions. In addition, there is some mention of a $C = -1$ lowlying exchange of odd signature in the nonflip $pp \rightarrow pp$ amplitude. However a trajectory (or trajectories) contributing to $S(hp)$ must have $C = +1$. We also note that no mention of such a low lying trajectory in connection with the nonflip πp amplitude appears to have been made. With these points in mind we add another pole (say) $\sigma^{\pi N}$ and σ^{NN} on the R.H.S. of (8.6a,c). We must bear in mind that the notation is misleading because we are, in using

the same symbol σ in both cases, implying that the same object is exchanged in the πN and NN reactions. However there is no strong reason to do so and the two terms could turn out to be entirely different in πN and NN scattering. (in which case different symbols would have to be used for the two cases). In πN scattering one could expect this to be the crossing-even object σ or ϵ mentioned previously but in the case of NN scattering one could expect it to be a more complicated object.

Writing σ^{ij} as

$$\sigma^{ij} = \frac{0.3893}{2q\sqrt{s}} \beta_{\sigma}^{ij}(0) s^{\alpha_{\sigma}(0)} \quad (8.7c)$$

and fitting $S(\pi p)$, $S(Kp)$, $S(pp)$ with (8.6a,b,c) and (8.7a,b,c) yields a drastic improvement in the results. We obtain an overall fit, with $\chi^2/pt \sim 1.5$ and the following values ($\alpha_f(0) \sim \alpha_{\omega}(0)$ according to these results)

$$\begin{array}{llll} \alpha_P(0) = 1.0697; & \beta_P^{\pi N}(0) = 38.652; & \beta_P^{KN}(0) = 33.707; & \beta_P^{NN}(0) = 62.428 \\ \alpha_f(0) = 0.4468; & \beta_f^{\pi N}(0) = 138.06; & \beta_f^{KN}(0) = 48.717; & \beta_f^{NN}(0) = 361.91 \\ \alpha_{A_2}(0) = 0.3475 & & \beta_{A_2}^{KN}(0) = 11.27; & \beta_{A_2}^{NN}(0) = 3.524 \\ \alpha_{\sigma}(0) = -0.5919 & \beta_{\sigma}^{\pi N}(0) = -148.89; & & \beta_{\sigma}^{NN}(0) = -1399. \end{array}$$

Since $\sigma_{tot}(pp)$ data extends far beyond the $S(pp)$ data ($\sigma_{tot}(pp)$ extends up to $p_{lab} \approx 2092$ GeV/c compared to $S(pp)$ which exists only up to 240 GeV/c) we have checked that our parameterisation extrapolates correctly to the higher-energy $\sigma_{tot}(pp)$ data. We find that our parameterisation reproduces $\sigma_{tot}(pp)$ quite well for $6 \leq p_{lab} \leq 2092$ GeV/c with $\chi^2/pt = 1.4$. However fits to the total cross-section data do not tell us much about the nature of the new term. In particular they do not tell us anything about its signature or the possibility that it might be made up of more than one

pole. It is also quite possible that such a term is an "effective" parameterisation of some low-energy effects. However one can hope that the various elastic phases $\rho(\text{hp})$ might enable us to deduce something about the crossing property of such a term.* With this in mind we turn to the measurements of various elastic phases at $t = 0$.

Phases of the Non-Flip Amplitudes (i.e. $\rho \equiv \frac{\text{Re } T(s,0)}{\text{Im } T(s,0)} \equiv \frac{\text{Re}}{\text{Im}}$)

The Regge-pole phase rule is:

$$\left. \begin{aligned} \text{Re } T_R(s,t) &= -\cot \frac{\pi\alpha}{2} \text{Im } T_R(s,t) \\ &\quad \tan \frac{\pi\alpha}{2} \text{Im } T_R(s,t) \end{aligned} \right\} \begin{array}{l} \tau = +1 \\ \tau = -1 \end{array} \quad \begin{array}{l} (8.8a) \\ (8.8b) \end{array}$$

where T_R is the contribution of a Regge pole $\alpha(t)$, with signature τ , to any amplitude T . Here we are concerned with the point $t = 0$ and the trajectories P , f , ρ , ρ' , A_2 , ω and σ . If we identify the σ as a crossing-even ($\tau = +1$) Regge pole then a satisfactory reproduction of $\rho(\pi^\pm p)$ for $p_{\text{lab}} \geq 10 \text{ GeV/c}$ can be made but the $\rho(pp)$ and $\rho(\bar{p}p)$ values are far too negative. Since the Kp system does not contain any contribution from the σ there is no ambiguity here and the $\rho(K^\pm p)$ data is reproduced quite well. If on the other hand one identifies the low lying contribution σ in $p(\bar{p})p$ scattering as a crossing-odd pole (while retaining it as a crossing-even object in πN scattering) a reasonable reproduction of the $\rho(pp)$ and $\rho(\bar{p}p)$ data is obtained although at lower energies our $\rho(pp)$ values are somewhat more negative than the data. We have however not attempted any further improvement here because a more careful analysis, by Berger et al. (1978), reveals the presence at lower energies of other contributions such as those due to the A_1 and Z exchanges. Since our original motive was to analyse

* As mentioned earlier there is no strong a priori reason to believe that the object σ is the same in πN and NN amplitudes in all respects.

TABLE 8.1

Values of Trajectories and Residues at $t = 0$ Trajectories:

$$\begin{array}{llll}
 \alpha_p(0) = 1.0697 & \alpha_f(0) = 0.4468 & \alpha_{A_2}(0) = 0.3475 & \alpha_\omega(0) = 0.449 \\
 \alpha_\rho(0) = 0.485 & \alpha_{\rho'}(0) = -1.745 & \alpha_\epsilon(0) = -0.5919 & \alpha_{\omega'}(0)
 \end{array}$$

Residues ^{*}:

Quantity	A_p	A_f	A_{A_2}	A_ω	A_ρ	$A_{\rho'}$	A_ϵ	$A_{\omega'}$ (+)	χ^2/pt
$S(\pi p)$	38.652	138.06	-	-	12.127	-459.28	185.77	-	1.9
$\Delta\sigma(\pi p)$									2.2
$S(Kp)$	33.707	48.717	11.27	25.292	7.497	-	-	-	1.8
$\Delta\sigma(Kp)$									1.2
$S(pp)$	62.428	361.91	3.524	81.106	5.41	-	-	-2340.68	0.74
$\Delta\sigma(pp)$									1.2

*Key to symbols is as follows:

$$A_i \equiv \beta_i(0) \quad \text{for } i = p, f, \rho, \omega, A_2, \rho'$$

$$A_\epsilon \equiv \beta_\epsilon(0) / \sin \frac{\pi\alpha_\epsilon(0)}{2}$$

$$A_{\omega'} \equiv \beta_{\omega'}(0) / \cos \frac{\pi\alpha_\epsilon(0)}{2}$$

(+)

where $\beta_i(0)$ occur on
previous pages (c.f. page 185)

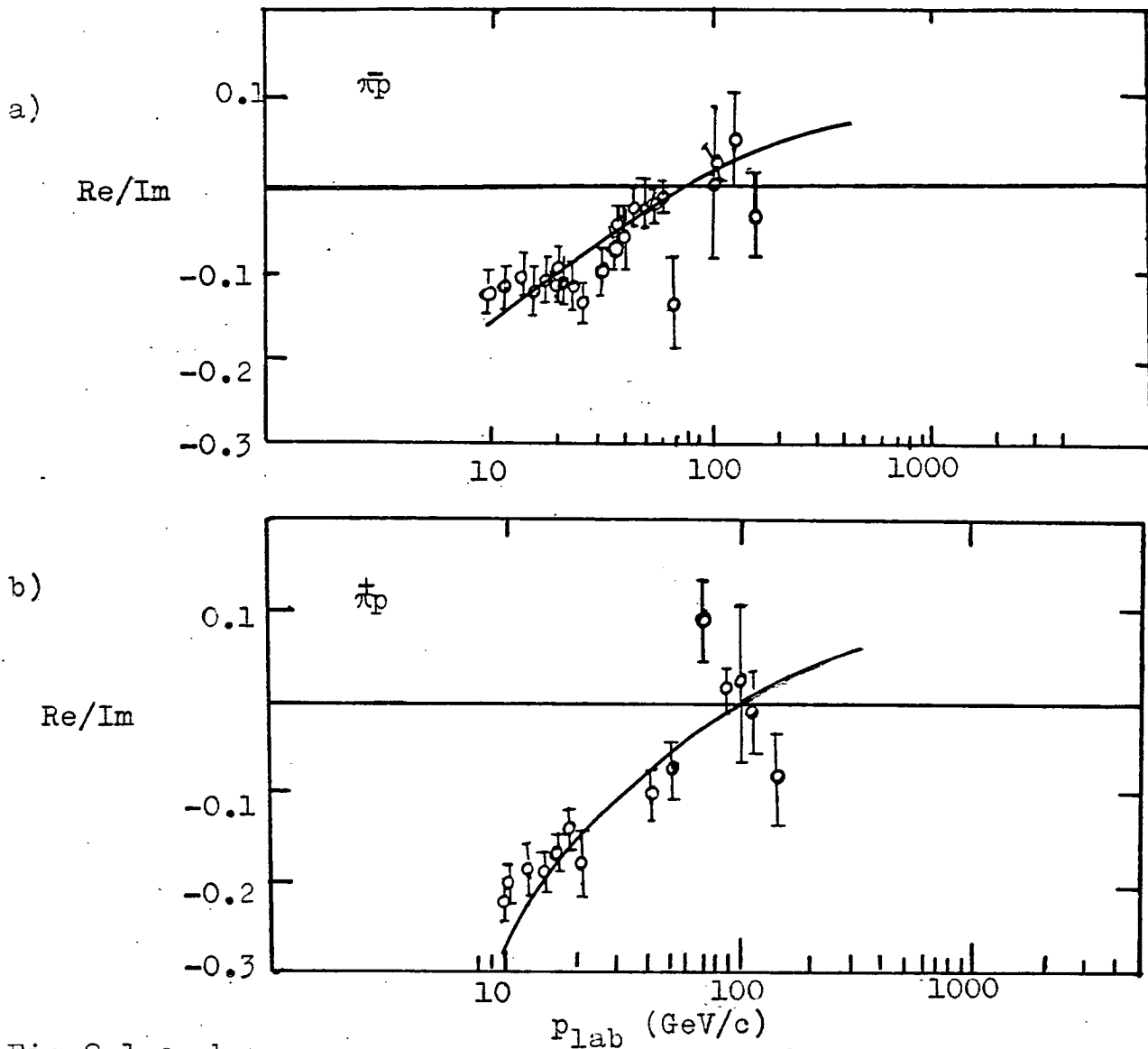
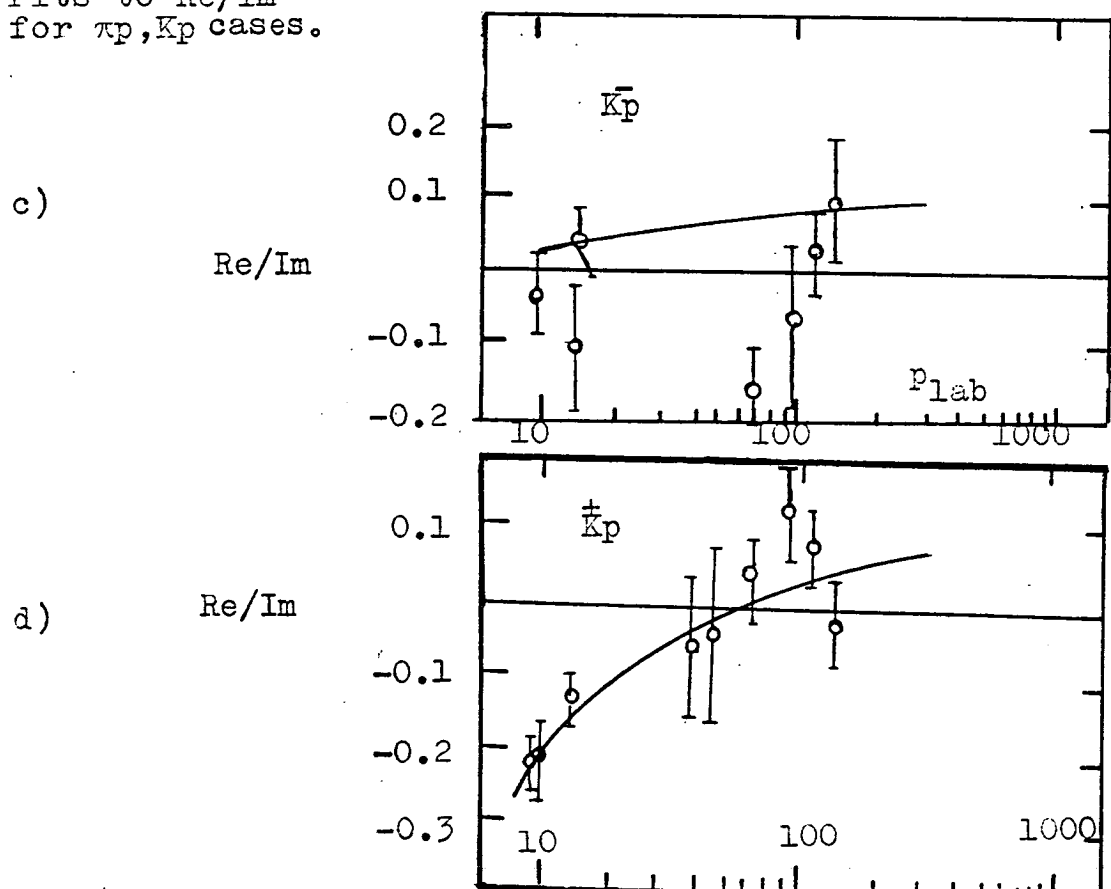
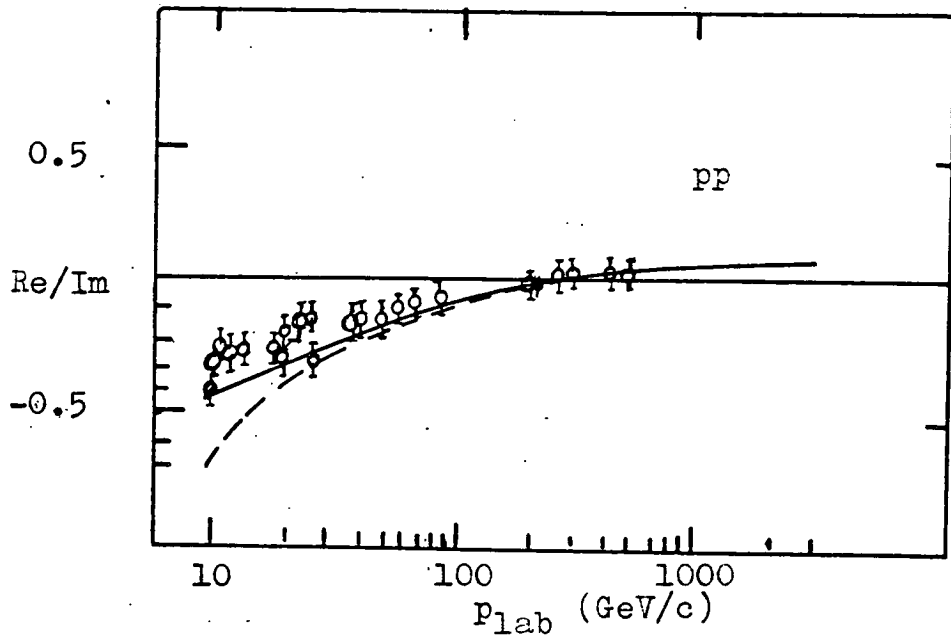


Fig 8.1 a-d :
Fits to Re/Im
for $\pi p, Kp$ cases.



e)



f)

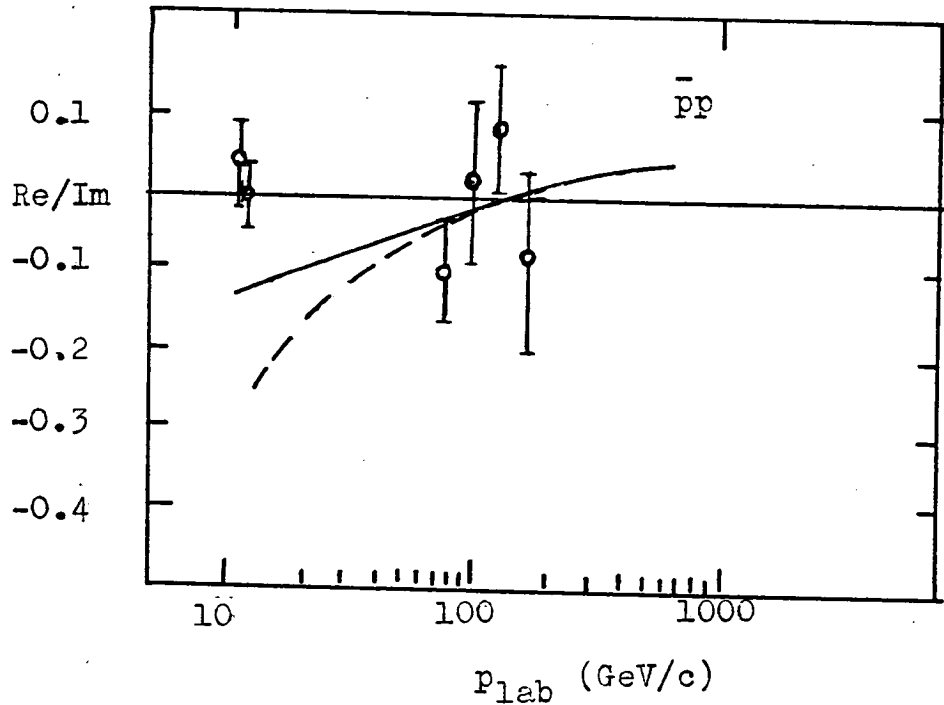


Fig 8.1e-f: Fits to Re/Im for pp and $\bar{p}p$ cases. Dashed curves correspond to the case when the low lying contribution (i.e. σ) is considered as a crossing-even object while solid curves correspond to the case when it is regarded as a crossing-odd Regge pole.

data for $p_{lab} \geq 10 \text{ GeV}/c$ we have let the matter rest. Thus from now on we use the nomenclature ϵ for the even signaturred pole (found in πp nonflip amplitude) and $\omega'_{(+)}$ for the odd-signaturred object found in $\phi(pp)$ ($\phi \equiv \frac{\phi_1 + \phi_3}{2}$). ($\omega'_{(+)}$ has $C = +1$, unlike the ω' contribution of Berger et al. which has $C = -1$.) We also point out that Berger et al. have used an f -dominated Pomeron and an EXD trajectory with intercept 0.5 for the ρ , ω , f , A_2 contributions. This latter feature of their analysis is certainly unsatisfactory. We will regard both ϵ and $\omega'_{(+)}$ as phenomenological objects which are "effective" parameterisations of effects which are not understood.

The Non-flip Amplitude and Differential Cross-sections

Parameterisation: Having determined the non-flip amplitude at $t = 0$, we now proceed to determine this amplitude in the small $|t|$ region by fitting differential-cross-section data. Our aim is to find out how well it is possible to fit this data by using the peripherality (in impact-parameter space) of the leading "ordinary" Regge exchange contributions as a prescription for parameterising their imaginary parts and, at the same time, by including the phases not adequately taken into account in previous chapters). Here of course we run into the well known difficulty that the phases of the leading crossing-even non-Pomeron contributions (i.e. f and A_2) have a pole at $\alpha(t) = 0$ (around $|t| \sim 0.5$) which is well inside the $|t|$ range considered. In Regge theory there are various ways of eliminating this pole (nonsense zeroes, signature zeroes etc. - c.f. Collins (1977, Ross et al. (1973), Bertocchi (1969)) all of which amount to the insertion of at least a single zero at the point where the phases have poles. Then, since the (crossing-even) phase is given by

$$\operatorname{Re} T_R^{(\tau=+1)} = -\cot \frac{\pi\alpha(t)}{2} \operatorname{Im} T_R^{(\tau=+1)} \quad (8.8a)$$

it is clear that, at the appropriate t value, the zero in the denominator of the phase cancels the zero in $\operatorname{Im} T_R$, yielding a finite value. In the DAM, the imaginary part of the non-flip amplitude is parameterised as " $J_0(R\sqrt{-t})$ " with $R \sim 1$ fm. Since " $J_0(R\sqrt{-t})$ " cannot have a zero around $|t| \approx 0.5$ if $R \sim 1$ fm we have to modify this prescription in order to obtain sensible results. We introduce the minimum additional structure required to prevent the phases from blowing up, i.e. we parameterise the imaginary parts in such a manner that in addition to the " $J_0(R\sqrt{-t})$ " structure these imaginary parts have a single zero at $\alpha(t) = 0$. There is no uniform practice in inserting such zeroes (c.f. e.g. Ross et al. (1973)) and various people use various factors. For instance one could insert a factor α or α^2 or $\sin \frac{\pi\alpha}{2}$ etc. (c.f. Collins (1977)) in the imaginary part of the relevant contribution - at $\alpha = 0$ all such factors vanish, which makes the phase in (8.8a) regular at $\alpha = 0$ in spite of the pole in $\cot \frac{\pi\alpha}{2}$. We therefore adopt the following ansatz for the leading crossing-even "ordinary" exchange contributions to the non-flip amplitude

$$\operatorname{Im} R_{\Delta\lambda=0} = "J_0(R\sqrt{-t})" \sin \frac{\pi\alpha}{2} \quad (8.9a)$$

$$\operatorname{Re} R_{\Delta\lambda=0} = -\cot \frac{\pi\alpha}{2} \operatorname{Im} R_{\Delta\lambda=0} \cdot \quad (f, A_2) \quad (8.9b)$$

There is no specific reason for choosing $\sin \frac{\pi\alpha}{2}$ rather than some other factor, e.g. α . One can in fact explore how the introduction of different factors affects the resulting fits but we have not pursued this point on account of lack of time.* For the leading crossing-odd

* The introduction of $\sin \frac{\pi\alpha}{2}$ in $\operatorname{Im} R$ enables us to write the resulting amplitude in a nice form (Collins et al. (1974))

ordinary exchanges (ρ, ω) we have not introduced any such factor and have retained

$$\text{Im } R_{\Delta\lambda=0} = "J_0(R\sqrt{-t})" \quad (\rho, \omega) \quad (8.10a)$$

$$\text{Re } R_{\Delta\lambda=0} = \tan \frac{\pi\alpha_R}{2} "J_0(R\sqrt{-t})" \quad (8.10b)$$

The Pomeron has been parameterised as a simple Regge pole with a structureless residue since the DAM prescription for a Pomeron becomes the same as that for any Regge pole if we choose to include the Pomeron phase (the DAM sets $\text{Re}P \sim 0$, c.f. Table 3.2, p. 75). We shall give the explicit parameterisation of the various contributions shortly.

A word about low-lying contributions. The contributions of the ρ' , ϵ and $\omega'_{(+)}$ to the relevant differential cross-sections are quite small for $p_{\text{lab}} > 10 \text{ GeV/c}$ but we have, for the sake of completeness, included them in our fits. (We will be considering $\frac{d\sigma}{dt}$ data for $p_{\text{lab}} \geq 10 \text{ GeV/c}$.) We assume $\alpha_\epsilon(t) = \alpha_{\omega'_{(+)}}(t)$. Also we fix the slopes of all low-lying exchanges as $\alpha_{\rho'} = \alpha'_\epsilon = \alpha_{\omega'_{(+)}} = 0.9$. The explicit parameterisation of the various exchanges is given below. The $\cos \frac{\pi\alpha_{\omega'_{(+)}}}{2}$

$$R_{\Delta\lambda=0}(s, t) = -\hat{\beta}_0(t) (e^{-i\pi/2})_s^{\alpha_R(0)} (e)^{(\ln s - i\pi/2)\alpha'_R t} \quad (8.11a)$$

with $\hat{\beta}_0(t) \approx "J_0"$. Explicitly the above expression looks like:

$$R_{\Delta\lambda=0} = -\hat{\beta}_0(t) \left[\cos \frac{\pi\alpha_R}{2} - i \sin \frac{\pi\alpha_R}{2} \right] s^{\alpha_R(t)} \quad (8.11b)$$

$$R_{\Delta\lambda=0} = \left[-\cot \frac{\pi\alpha_R}{2} + i \right] \hat{\beta}_0(t) \sin \frac{\pi\alpha_R}{2} s^{\alpha_R(t)} \quad (8.11c)$$

where

$$\alpha_R(t) = \alpha_R(0) + \alpha'_R t \quad (8.11d)$$

factor in the imaginary part of the $\omega'_{(+)}$ contribution is necessary to prevent $\tan \frac{\pi\alpha_{\omega'}(+)}{2}$ from blowing up the corresponding real part at $\alpha_{\omega'}(+)=1$ (which lies in the $|t|$ region considered, at $|t| \sim 0.4$). There is no significance in the presence of $\sin \frac{\pi\alpha_{\epsilon}}{2}$ in the ϵ contribution except that it makes all $\tau = +1$ "ordinary" exchange contributions look similar. The explicit expressions are as follows:

$$P(s,t) = \left[-\cot \frac{\pi\alpha_P}{2} + i \right] \{ A_P e^{b_P t + c_P t} (s)^{\alpha_P(t)} \} \quad (8.12a)$$

$$f(s,t) = \left[-\cot \frac{\pi\alpha_f}{2} + i \right] \{ A_f e^{b_f t + c_f t^2} J_0(R_f \sqrt{-t}) \sin \frac{\pi\alpha_f}{2} s^{\alpha_f(t)} \} \quad (8.12b)$$

$$A_2(s,t) = \left[-\cot \frac{\pi\alpha_{A_2}}{2} + i \right] \{ A_{A_2} e^{b_{A_2} + c_{A_2} t^2} J_0(R_{A_2} \sqrt{-t}) \sin \frac{\pi\alpha_{A_2}}{2} s^{\alpha_{A_2}(0)} \} \quad (8.12c)$$

$$\epsilon(s,t) = \left[-\cot \frac{\pi\alpha_{\epsilon}}{2} + i \right] \{ A_{\epsilon} e^{b_{\epsilon} t} \sin \frac{\pi\alpha_{\epsilon}}{2} s^{\alpha_{\epsilon}(t)} \} \quad (8.12d)$$

$$\rho(s,t) = \left[\tan \frac{\pi\alpha_{\rho}}{2} + i \right] \{ A_{\rho} e^{b_{\rho} t} J_0(R_{\rho} \sqrt{-t}) s^{\alpha_{\rho}(t)} \} \quad (8.12e)$$

$$\omega(s,t) = \left[\tan \frac{\pi\alpha_{\omega}}{2} + i \right] \{ A_{\omega} e^{b_{\omega} t + c_{\omega} t^2} J_0(R_{\omega} \sqrt{-t}) s^{\alpha_{\omega}(t)} \} \quad (8.12f)$$

$$\omega'_{(+)}(s,t) = \left[\tan \frac{\pi\alpha_{\epsilon}}{2} + i \right] \{ A_{\omega'_{(+)}} e^{b_{\omega'_{(+)}} t} \cos \frac{\pi\alpha_{\epsilon}}{2} s^{\alpha_{\epsilon}(t)} \} \quad (8.12g)$$

$$\rho'_{(s,t)} = \left[\tan \frac{\pi\alpha_{\rho'}}{2} + i \right] \{ A_{\rho'} e^{b_{\rho'} t} s^{\alpha_{\rho'}(t)} \} \quad (8.12h)$$

with $\alpha_R(t) = \alpha_R(0) + \alpha'_R t$ and $\alpha'_{\rho'} = \alpha'_{\epsilon} = \alpha'_{\omega'_{(+)}} = 0.9$ a priori.

We then have

$$T_{++}(\pi^{\pm} p) = P + f + \epsilon + \{\rho + \rho'\} \quad (8.13a)$$

$$T_{++}(K^{\pm} p) = P + f + \rho + A_2 + \omega \quad (8.13b)$$

$$\phi\left(\frac{p}{p}\right) = P + f + \rho + A_2 + \omega + \omega' \quad (8.13c)$$

The data is fitted by using

$$\frac{d\sigma}{dt}(hp) = \frac{0.3893}{64\pi s q^2} |T_{++}(hp)| \quad (8.13d)$$

$$\frac{d\sigma}{dt}\left(\frac{p}{p}\right) = \frac{0.3893}{64\pi s q^2} \left|\phi\left(\frac{p}{p}\right)\right|^2 \quad (8.13e)$$

where $h = \pi^\pm$ or $K^\pm$.

πp Differential Cross-Sections

The data has been fitted in the range $10 \lesssim p_{lab} \leq 200$ GeV/c. Our parameterisation reproduces the data up to $|t| \leq 0.8(\text{GeV}/c)^2$. The restriction on the $|t|$ range can be relaxed for data above 20 GeV/c in which case the $|t|$ range can be slightly extended ($|t| \leq 0.9 (\text{GeV}/c)^2$). At energies corresponding to $p_{lab} < 20$ GeV/c the data falls below our fit for $|t| > 0.8 (\text{GeV}/c)^2$. We have fitted the $\pi^+ p$ data at 12.8, 14.8, 16.7, 50, 70, 100, 140, 175 and 200 GeV/c. The $\pi^- p$ data has been considered at 13, 15, 17 (Foley et al. (1963)), 12.4, 18.4 (Harting et al. (1965)), 25.2, 40.1 (Antipov et al. (1974)), 32.82, 35.39, 42.02, 45.34, 48.61, 50, 50.96, 54.74 (Derevchekov et al. (1974a,b)), 50, 100, 200 (Akerloff et al. (1976)), 50, 70, 100, 140, 175 (Ayres et al. (1977)) GeV/c. We find that there exist data sets at several energies which cannot be fitted satisfactorily. This is almost invariably the case with $\pi^- p$ data. It seems rather odd that a parameterisation (parameter values in Table 8.2A) which fits data at most energies correctly fails badly at some energies. However before suggesting renormalisation of

TABLE 8.2A

PARAMETER VALUES FOR THE NON-FLIP AMPLITUDES AND TRAJECTORY SLOPES

Slopes of trajectories:

$$\alpha'_p = 0.23; \quad \alpha'_f = 0.85; \quad \alpha'_\rho = 0.85 \quad \alpha'_\omega = 0.9, \quad \alpha'_{A_2} = 0.9.$$

Other Parameters: *

Reaction Parameter	πp	Kp	pp
b_p	2.9207	2.8246	3.95
c_p	0.629	0.791	0.585
b_f	0.00	0.682	0.535
c_f	2.962	1.610	1.013
R_f	3.010	3.00	3.26
b_ρ	0.00	0.00	0.00
R_ρ	4.5	4.5	4.5
b_ω	-	0.5	0.0
c_ω	-	0.0	0.0
R_ω	-	3.875	5.784
b_{A_2}	-	0.0	0.0
R_{A_2}	-	4.5	4.5
b_ϵ	0.00	-	-
$b_{\omega'} (+)$	-	-	3.0
$b_{\rho'}$	0.00	-	-
Overall χ^2/pt	2.1	1.6	2.1

* b_i in $(\text{GeV}/c)^{-2}$
 c_i in $(\text{GeV}/c)^{-4}$
 R_i in $(\text{GeV}/c)^{-1}$

TABLE 8.2B

PARAMETER VALUES FOR NON-FLIP AMPLITUDES AND TRAJECTORY SLOPES

(Here $R_f = R_{A_2} = 0$)

Slopes of Trajectories:

$\alpha'_p = 0.23; \quad \alpha'_f = 0.85; \quad \alpha'_\rho = 0.85; \quad \alpha'_\omega = 0.9 \quad \alpha'_{A_2} = 0.9.$

Other Parameters:*

Parameter	Reaction		
	πp	Kp	pp
b_p	3.0588	2.894	3.97
c_p	0.86	0.906	0.572
b_f	0.806	3.00	2.88
c_f	-0.309	0.769	-3.575
b_ρ	0.00	0.00	0.00
R_ρ	4.5	4.5	4.5
b_ω	-	0.5	0.0
c_ω	-	-	2.37
R_ω	-	5.10	5.905
b_{A_2}	-	0.00	0.00
b_ϵ	0.00	-	-
$b_{\omega'} (+)$	-	-	3.0
$b_{\rho'}$	0.00	-	-
χ^2/pt	2.15	1.9	1.95

* b_i in $(\text{GeV}/c)^{-2}$
 c_i in $(\text{GeV}/c)^{-4}$
 R_i in $(\text{GeV}/c)^{-1}$

these data sets we also tried to fit data by using an alternative parameterisation in which the f contribution is parameterised without a J_0 factor (this is equivalent to setting $R_f = 0$). The resulting parameter values are quoted in Table 8.2B. Here again one finds the same difficulty as before and with the same data sets as in the previous case. One could therefore attempt a renormalisation of the relevant data sets. We find that a renormalisation of the $\pi^- p$ data at the following energies is needed: 13, 15 GeV/c (Foley et al. (1963)), 12.4, 18.4 GeV/c (Harting et al. 1965)), 35.39 GeV/c (Derevchekov et al. (1974a)), 50, 200 GeV/c (Akerloff et al. (1976)). The renormalisation factors are, respectively, 0.85, 0.85, 0.89, 0.91, 1.10 and 0.93, and 1.07. This yields a drastic improvement in the fits. Thus for instance the 50 GeV/c data of Akerloff et al. (1976) after being renormalised yields a $\chi^2/pt = \frac{72.8}{55}$ for solutions in Table 8.2A) as compared to the $\frac{264}{55}$ before the fit. There is no problem with renormalizing the data of Akerloff et al. (1976) since they do not have any accurate normalisation of their own (c.f. e.g. H hler et al. (1979b)). The original normalisation of Foley et al. (1963) yields $\left. \frac{d\sigma}{dt} \right|_{t=0}$ values which are higher by about 15% than those obtained from using $\sigma_{tot}(\pi^- p)$ measurements in conjunction with $\frac{Re}{Im}$ values. This can be checked as follows

$$\left. \frac{d\sigma}{dt} \right|_{t=0} = \frac{(1 + \rho^2) \sigma_{tot}^2}{0.3893 \times 16\pi} \quad (8.14)$$

where σ_{tot} is in millibarns and $\frac{d\sigma}{dt}$ in $mb/(GeV/c)^2$. Thus if we use $\rho \approx -0.2$ and $\sigma_{tot} = 26$ mb at 13 GeV/c we find that $\left. \frac{d\sigma}{dt} \right|_{t=0} \approx 34.6$ $mb/(GeV/c)^2$, well below the value 42.4 ± 1.6 quoted by Foley et al. (1963). We also find that while the 70, 100 and 140 GeV/c data of Ayres et al. (1977) are adequately described by both parameterisations this is not true of their 50 and 175 GeV/c data, where renormalisation does not help. It is curious that the 50 GeV/c data of Derevchekov et al. (1974b) and the

renormalised data of Akerloff et al. (1976) at 50 GeV/c are described well by this parameterisation while the 50 GeV/c data of Ayres et al. (1977) cannot be adequately described ($\chi^2/pt = \frac{64.6}{13}$). It was quite a relief when (after we had arrived at the possibility of discrepancies between data sets independently) Höhler et al. (1979a,b) drew attention to the discrepancies in the small $|t|$ data for $\frac{d\sigma}{dt}(\pi^- p)$. Renormalising the 50 and 175 GeV/c data of Ayres et al. (1977) does not improve the situation. Hence we have included the χ^2 values for their data as such in the overall χ^2/pt values quoted at the bottom of Tables 8.2A, B. As far as $\pi^+ p$ data is concerned only the 200 GeV/c data (Akerloff et al. (1976)) has to be renormalised (by a factor of 1.10). Here again there is a slight discrepancy in the 50 GeV/c data of Akerloff et al. (1976) and Ayres et al. (1977). Our fit describes the data of Akerloff et al. (1976) quite well ($\chi^2/pt = \frac{51.6}{55}$) but the data of Ayres et al. (1977) yields $\chi^2/pt = \frac{68.1}{19}$.

The dominant "ordinary" exchange is the f and our results indicate that, in order to obtain a satisfactory fit, the f -radius of interaction has to be reduced to $3.01 \text{ GeV}^{-1}/c$ ($\sim 0.6 \text{ fm}$). Such a value of R_f does not correspond to the sense in which peripherality is used in the DAM in that, although the f -contribution (apart from a $\sin \frac{\pi\alpha}{2}$ factor) can be parameterised in a form which indicates dominant contributions from a narrow band of partial waves (in impact-parameter space), such a band in this case does not occur at the periphery of the interaction region since the periphery is expected to occur at a radius of about 1 fm. The ρ has a very small contribution with a Regge pole phase and an imaginary part which can be parameterised as suggested by the DAM. The Pomeron dominates the amplitude and has a smooth structureless imaginary part as expected in the DAM. However there is a significant Pomeron phase ($\alpha_p' = 0.23$) which contradicts the DAM, for which one has the

approximation $\text{Re}P \sim 0$.

Kp Differential Cross-Sections:

We have fitted $\frac{d\sigma}{dt}(K^{\pm}p)$ data according to equations (8.12, 8.13) in the range $10 \lesssim p_{\text{lab}} \lesssim 200$ GeV/c and $|t| \lesssim 0.8$ (GeV/c)². The K^+p data has been considered at 9.8, 12.8, 14.8, 16.7, 50, 70, 100, 140, 175 and 200 GeV/c (Foley et al. (1963), Akerloff et al. (1976), Ayres et al. (1977)) while the K^-p data is considered at 9, 10.1, 11.88, 14.3, 25.2, 40.1, 50, 70, 100, 140, 175 and 200 GeV/c (Foley et al. (1963), Bartsch et al. (1971), Drevillon et al. (1975), Antipov et al. (1974), Akerloff et al. (1976), Ayres et al. (1977)). In the K^+p case the data of Foley et al. (1963) has to be renormalised* at 9.8, 12.8 and 14.8 GeV/c by factors of 0.81, 0.72 and 0.70 respectively. The values of the renormalisation factors are suggested by comparing the $\left. \frac{d\sigma}{dt} \right|_{t=0}$ values quoted by Foley et al. (1963) with those suggested by using equation (8.14). The K^-p data at 10.1 GeV/c (Bartsch et al. (1971)) and 14.3 GeV/c (Drevillon et al. (1975)) have to be renormalised by factors of 0.8 and 0.88 respectively. The 40.1 GeV/c data of Antipov et al. (1974) has also been renormalised by a factor of 1.05 (their normalisation uncertainty is $\pm 5\%$) to obtain a better χ^2/pt for their data.

Here the f and ω exchanges contribute comparably while the ρ contribution is comparable to the A_2 (see Table 8.1). All the leading "ordinary" exchanges are non-negligible, particularly at the lower p_{lab} values. The f radius of interaction ($R_f = 3.00$) implies the same sort of conclusion as in the πp case. The ρ , ω and A_2 have

* Before renormalising we have, as in the πp case, fitted $\frac{d\sigma}{dt}(K^{\pm}p)$ data by a parameterisation in which the f and A_2 contributions do not have any J_0 factors (i.e. we have set $R_f = R_{A_2} = 0$). Such a fit, like the previous one, does not reproduce data correctly at the same data sets as the earlier fit (see Table 8.2B).

radii of interaction close to 1 fm. The ρ and ω amplitudes are parameterised as in the DAM and the results agree with the DAM expectation ($R \sim 1$ fm and " J_0 " behaviour for imaginary parts).

pp, $\bar{p}p$ Differential Cross-Sections:

We have fitted $\frac{d\sigma}{dt}$ for both pp and $\bar{p}p$ data for $p_{lab} > 10$ GeV/c and $|t| \leq 0.8$ (GeV/c). The pp data was considered at 10.8, 12.8, 14.2, 14.8, 16.7, 18.4, 19.6, 50, 70, 100, 140, 175, 200, and 291 ($\sqrt{s} = 23$ GeV), 2091 ($\sqrt{s} = 62$ GeV) GeV/c. The data was taken from Foley et al. (1963), Harting et al. (1965), Akerloff et al. (1976), Ayres et al. (1977) and Kwak et al. (1975). Tables of $\frac{d\sigma}{dt}$ (pp) values supplied by the authors of Kwak et al. (1975) were used. The $\bar{p}p$ data was considered at 10, 12, 25.2, 40.1, 50, 70, 100, 140, 175 and 200 GeV/c. The data was taken from Foley et al. (1963), Antipov et al. (1974), Akerloff et al. (1976), Ayres et al. (1977). As in the earlier cases, we find that the $\frac{d\sigma}{dt}$ (pp) data of Foley et al. (1963) has to be renormalised if satisfactory fits are to be obtained. The values of the renormalisation factors can be deduced from the discrepancy between the $\frac{d\sigma}{dt}|_{t=0}$ values quoted in Foley et al. (1963) and the $\frac{d\sigma}{dt}|_{t=0}$ values deduced from using equation (8.14) in conjunction with $\sigma_{tot}(pp)$ and $\rho(pp)$ measurements at the relevant laboratory momenta. The renormalisation factors for the 12.8, 14.8, 16.7 and 19.6 GeV/c pp data are .802, .815, .882 and .878 respectively*. The 50 and 200 GeV/c data of Akerloff et al. (1976) have to be renormalised by factors 0.94 and 1.08 respectively. This does not

* We have, as in the πp and Kp cases, considered two parameterisations of $\phi(pp)$. In one of these the various contributions are expressed as in equations (8.12), while in the other we set the f and A_2 radii of interaction equal to zero with the result that in these contributions $J_0(R\sqrt{-t}) \approx 1$. Neither of these parameterisations can reproduce the 10.8 GeV/c data so that our parameterisation is valid for $p_{lab} > 10.8$ GeV/c. See Table 7.2B for this second parameterisation.

pose any difficulty for they do not have any accurate normalisation of their own. The ISR data of Kwak et al. (1975) at $\sqrt{s} = 23$ GeV has to be renormalised by a factor of 1.36. Our parameterisation, while reproducing the $\sqrt{s} = 62$ GeV data very well ($\chi^2/\text{pt} = \frac{35.4}{21}$) falls well above the $\sqrt{s} = 23$ GeV data ($\chi^2/\text{pt} = \frac{484.5}{31}$). Renormalising the $\sqrt{s} = 23$ GeV data by 1.36 yields $\chi^2/\text{pt} = \frac{56.8}{31}$.

The $\bar{p}p$ also has to be renormalised at several energies. The data sets are 12 GeV/c (Foley et al. (1963)), 40.1 GeV/c (Antipov et al.), 50 GeV/c (Akerloff et al. (1976)), 70, 175 GeV/c Ayres et al. (1977) and the corresponding renormalisation factors are, respectively. 0.8, 1.10, 0.75, 1.07 and 1.18. After renormalising, the resulting overall χ^2/pt is shown in Tables 8.2A and 8.2B. The general conclusions that can be drawn from the parameter values are the same as in the πp and Kp cases.

The Flip Amplitude and Polarisation

Having determined the non-flip amplitude in the various elastic reactions we now turn to the flip amplitude. In the context of $pp \rightarrow pp$, flip amplitude means the single-flip amplitude ϕ_5 . We shall determine the flip amplitude by using the previously determined non-flip amplitude in conjunction with polarisation data. Since our non-flip amplitude reproduces data up to $|t| \sim 0.8 \text{ (GeV/c)}^2$ we restrict ourselves to $|t| \leq 0.8 \text{ (GeV/c)}^2$ in fitting polarisation data. Let us first write down the various contributions to the flip amplitudes in the different reactions. According to standard ideas $T_{+-}(\pi p)$ is dominated by the ρ contribution (c.f. e.g. Irving and Worden (1977)). However definite evidence for the contribution of a low-lying even-signatured trajectory of zero iso-spin has been forthcoming (Girardi and Navelet (1976), Martin and Navelet (1977)). We have already discovered the need for a

low-lying contribution in evaluating $S(\pi p)$. In fitting polarisations such a contribution is needed in the flip amplitude to explain the departure of $P(\pi^\pm p)$ from mirror symmetry (mirror symmetry would be exact if only the ρ contributed to $T_{+-}(\pi p)$). In addition to the ρ and ϵ , Girardi and Navelet (1977) argue for the presence of a Pomeron flip as well as an f -flip term. We therefore write

$$T_{+-}(\pi^\pm p) = \pm \rho + \epsilon + f + P \quad (8.15)$$

but initially we set $P = f = 0$ and attempt to fit data without using P and f contributions. Before discussing the fits to πp polarisations we, in order to save space, discuss briefly the Kp and pp flip amplitudes and write explicitly all the contributions. The Kp flip amplitude is known to be dominated by the ρ and A_2 contributions. In addition Martin and Stevens (1977) have argued for a small Pomeron-flip contribution. The ϵ might also be expected to contribute. As a starting point we ignore the Pomeron flip term and only if the data can not be fitted shall we attempt to include the Pomeron flip contribution.

$$T_{+-}(K^\pm p) = A_2 \mp \rho + \epsilon \quad (8.16)$$

For the $pp, \bar{p}p$ case, in view of the rather unexpected energy dependence of $P(pp)$ at lower energies, we expect a low-lying contribution. This could be just ϵ or just $\omega'_{(+)}$ or both. Also the appearance of a zero in $P(pp)$, at $|t| \sim 0.5 \text{ (GeV/c)}^2$, which moves inwards with increasing energy, seems to demand a high lying flip contribution which grows with energy. Then we write

$$\phi_5(\frac{pp}{pp}) = A_2 \mp \rho + \epsilon + \omega'_{(+)} + \check{P} \quad (8.17)$$

For the (leading) crossing-odd "ordinary" exchanges (ρ, ω) we use the DAM prescription (Table 3.3) but for f and A_2 use (8.19) below. This

makes the prescription for f and A_2 uniform for $\Delta\lambda = 0$ and $\Delta\lambda = 1$ amplitudes. In each case we write

$$\rho = C_\rho e^{d_\rho t + g_\rho t^2} J_1(R_\rho \sqrt{-t}) \left[\tan \frac{\pi\alpha_\rho}{2} + i \right] s^{\alpha_\rho(t)} \quad (8.18a)$$

$$A_2 = C_{A_2} e^{d_{A_2} t + g_{A_2} t^2} \sin \frac{\pi\alpha_{A_2}}{2} J_1(R_{A_2} \sqrt{-t}) \left[-\cot \frac{\pi\alpha_{A_2}}{2} + i \right] s^{\alpha_{A_2}(t)} \quad (8.18b)$$

$$\epsilon = \sqrt{-t} C_\epsilon e^{d_\epsilon t + g_\epsilon t^2} \sin \frac{\pi\alpha_\epsilon}{2} \left[-\cot \frac{\pi\alpha_\epsilon}{2} + i \right] s^{\alpha_\epsilon(t)} \quad (8.18c)$$

$$\omega'_{(+)} = \sqrt{-t} C_{\omega'_{(+)}} e^{d_{\omega'_{(+)}} t + g_{\omega'_{(+)}} t^2} \cos \frac{\pi\alpha_\epsilon}{2} \left[\tan \frac{\pi\alpha_\epsilon}{2} + i \right] s^{\alpha_\epsilon(t)} \quad (8.18d)$$

$$P = C_P e^{d_P t + g_P t^2} J_1(3.5\sqrt{-t}) \left[-\cot \frac{\pi\alpha_P}{2} + i \right] s^{\alpha_P(t)} \quad * \quad (8.18e)$$

$$f = C_f e^{d_f t + g_f t^2} J_1(R_f \sqrt{-t}) \left[-\cot \frac{\pi\alpha_f}{2} + i \right] s^{\alpha_f(t)} \quad (8.18f)$$

Note that, for crossing-even leading exchanges (A_2 , f), we have written

$$"J_1((R\sqrt{-t}))" \rightarrow "J_1(R\sqrt{-t})" \sin \frac{\pi\alpha}{2} \quad (8.19)$$

This is for reasons of uniformity with the non-flip case.

* The P flip term, in spite of the $J_1(3.5\sqrt{-t})$ factor, is structureless in the $|t| \lesssim 0.8 \text{ (GeV/c)}^2$ region since $J_1(3.5\sqrt{-t})$, apart from a zero at $t=0$ (which is needed anyway in a flip amplitude on account of the constraint from angular momentum conservation which requires all $\Delta\lambda \neq 0$ amplitudes to vanish at $t=0$), is structureless in this $|t|$ range, its next zero lying outside this range. This parameterisation has been adopted for continuity with, and for reasons given in, Chapter 6. We have also fitted polarisation data by removing the $J_1(3.5\sqrt{-t})$ factor in (8.18e) and replacing it by the usual $\sqrt{-t}$ factor. Since this does not yield any qualitatively different results and conclusions we have not quoted the numbers here.

We use the above parameterisation in conjunction with solution 1 for the non-flip amplitudes (i.e. Table 8.2A which takes f and A_2 parameterised as " $J_0 \sin \frac{\pi\alpha}{2}$ "). Also we will, for the sake of completeness, attempt to fit the data by using solution (2) (i.e. Table 8.2B) for the non-flip amplitude and parameterising the f , A_2 flip contributions as (all other flip contributions remain the same) in (8.20) below:

$$A_2 = \sqrt{-t} \left(-\cot \frac{\pi\alpha_{A_2}}{2} + i \right) C_{A_2} e^{d_{A_2}t + g_{A_2}t^2} \sin \frac{\pi\alpha_{A_2}(t)}{2} s^{\alpha_{A_2}(t)} \quad (8.20)$$

$$f = \sqrt{-t} \left(-\cot \frac{\pi\alpha_f}{2} + i \right) C_f e^{d_ft + g_ft^2} \sin \frac{\pi\alpha_f}{2} s^{\alpha_f(t)}$$

We now discuss our results. The values of the various parameters are described in Tables 8.3A, B.

πp Polarisations: Using only the ρ and ϵ contributions in the flip amplitude and parameterising these as in equations (8.18) we first fit $P(\pi^\pm p)$ using $(T_{++}(\pi p))$ from Table 8.2A)

$$P \sim \frac{2\text{Im}(T_{++} T_{++}^*)}{|T_{++}|^2} \quad (8.21)$$

The resulting fit has a $\chi^2/\text{pt} = 2.57$ which needs some improvement. Allowing for a Pomeron flip and an f -flip contribution yields $\chi^2/\text{pt} = 2$. This is a substantial improvement so that there does appear to be some evidence for a P and f flip contribution. We considered $P(\pi^+ p)$ data at 10, 12, 14, 17.5, 45 and 100 GeV/c and $P(\pi^- p)$ at 10, 14, 40, 100 GeV/c. (Data from Borghini et al. (1971a), Gaidot et al. (1975), Auer et al. (1977), Fujisaki et al. (1979)). The results of our parameterisation are displayed in Table 8.3A. As can be seen from

inspecting the value of R_ρ it is indeed in agreement with the DAM expectations. The flip f radius of interaction is also ~ 1 fm. There is a substantial ϵ contribution as well which is stronger than its non-flip contribution

$$\left(\frac{\epsilon_{++}}{\epsilon_{+-}/\sqrt{-t}} \right) \Big|_{t=0} = \frac{185.77}{-345} \approx -0.54$$

Also using solution 2 for T_{++} (Table 8.2B) and parameterising f as in (8.20) we have attempted a fit to $P(\pi^\pm p)$. This yields a fit with $\frac{\chi^2}{n} = 2$ again. The values of the parameters are given in Table 8.3B.

Kp Polarisation: We find that an interplay of the ρ , A_2 and ϵ contributions yields good agreement with the $P(K^\pm p)$ data. Using equation (8.18) for ρ , A_2 , ϵ and solution 1 of T_{++} (i.e. T_{++} from Table 8.2A) yields a good fit ($\chi^2/pt = \frac{110.2}{76} = 1.45$). Also using an A_2 contribution given by equation (8.20) in conjunction with equations (8.18) for ρ and ϵ and solution 2 of T_{++} (T_{++} from Table 8.2B) yields a very good fit ($\chi^2/pt = \frac{73.3}{76} = .96$) to polarisation data. The first solution gives an A_2 flip interaction radius 3.2 (GeV/c)^{-1} and a ρ flip interaction radius of 3.6 (GeV/c)^{-1} . The values of the parameters are given in Table 8.3A. The second fit is quoted in Table 8.3B. (Data was taken from Borghini et al. (1971b), Gaidot et al. (1975) and Fujisaki et al. (1979)). The K^+p data was considered at 10, 12, 14 and 45 GeV/c, while $P(K^-p)$ was considered at 10, 14 and 40 GeV/c. The values of ρ and A_2 parameters indicate that $\rho - A_2$ EXD is also broken in the flip residues as well. This result holds for both parameterisations (either (8.18) or (8.20)) of the A_2 flip contribution.

TABLE 8.3A

PARAMETER VALUES FOR FLIP-AMPLITUDES

(Non-flip amplitudes involved in the polarisation fits quoted here correspond to those in Table 8.2A)

Parameter [*] /Reaction	πp	Kp	pp
C_p	34.41	-15.0	-26.16
d_p	0.918	1.10	2.045
g_p	0.855	0.70	0.0
R_p^-	4.75	3.60	3.85
C_ϵ	-345.05	-100.0	-91.0
d_ϵ	1.931	1.00	3.79
g_ϵ	0.244	2.00	0.00
C_f	5.02	-	-
d_f	4.047	-	-
g_f	-	-	-
R_f^-	4.5	-	-
C_P	-0.29	-	-1.57
d_P	1.016	-	2.24
g_P	0.437	-	0.00
C_{A_2}	-	-36.0	-19.66
d_{A_2}	-	0.70	1.45
g_{A_2}	-	1.20	0.00
$R_{A_2}^-$	-	3.20	3.44
$C_{\omega' (+)}$	-	-	-52.75
$d_{\omega' (+)}$	-	-	3.00
χ^2/pt	2	1.45	1.44

* d_i in $(\text{GeV}/c)^{-2}$
 g_i in $(\text{GeV}/c)^{-4}$
 R_i in $(\text{GeV}/c)^{-1}$

TABLE 8.3B

PARAMETER VALUES FOR FLIP-AMPLITUDES

(non-flip amplitudes involved in polarisation fits are those corresponding to Table 8.2B.)

* Parameter/Reaction			
	πp	Kp	pp
C_ρ	33.53	-21.0	-20.90
d_ρ	1.049	1.0	2.075
g_ρ	0.774	0.70	
R_ρ^-	4.669	4.5	3.67
C_ϵ	-352.75	-100.0	-271.8
d_ϵ	2.33	1.00	8.0
g_ϵ	0.583	2.00	0.0
C_f	4.486	-	-
d_f	4.478	-	-
g_f	0.00	-	-
C_P	-0.202	-	-2.274
d_P	3.439	-	2.457
g_P	4.58	-	-
C_{A_2}	-	-41.00	-18.47
d_{A_2}	-	1.50	2.65
g_{A_2}	-	1.20	-
$C_{\omega'}$	-	-	-219.6
$d_{\omega'(+)}$	-	-	3.00
$g_{\omega'(+)}$	-	-	0.00
χ^2/pt	2	0.96	1.28

*
 d_i in $(\text{GeV}/c)^{-2}$
 g_i in $(\text{GeV}/c)^{-4}$
 R_i in $(\text{GeV}/c)^{-1}$

pp and $\bar{p}p$ Polarisation

We now come to most interesting elastic polarisation data, that of $pp \rightarrow pp$. Here we recall that Dash and Navelet (1976), Martin and Navelet (1977) and Berger et al. (1978), have found evidence for the presence of a low lying even signature trajectory in $\phi_5(pp)$. Very recently Kroll et al. (1979) have also found the need for a low lying contribution of even signature in $\phi_5(pp)$. Earlier, Field and Stevens (1975) had found that one needs a pair of EXD exchanges of opposite signature ($\epsilon - \tilde{\omega}$) in both ϕ_1 and $\phi_5(pp)$. They also found that the $\tilde{\omega}$ contribution is dominantly non-flip and that of ϵ is dominantly single flip. There has also been speculation on a Pomeron-flip contribution for some time now. However there appear to be some difficulties. Irving (1975) set the real part of the Pomeron flip contribution equal to zero. This, though reproducing the polarisation data, violates analyticity. Subsequently Martin and Navelet (1977) have argued for for a Pomeron flip term in NN polarisations. Berger et al. (1978), while investigating the implications of spin-spin measurements at 6 GeV/c, concluded that "either the magnitude of the spin-flip Pomeron contribution or the Pomeron phase (or both) are poorly understood..." Irving (1979) concludes that a somewhat reduced Pomeron flip phase can reproduce the data using the model of Berger et al. (1978). Using solution 1 for ϕ (Table 8.2A) we have attempted to fit data employing (8.17) and (8.21) and with the replacement $T_{++} \rightarrow \phi$, $T_{+-} \rightarrow \phi_5$. This yields a good fit to the data ($\chi^2/pt \approx 1.45$). The parameters have been displayed in Table 8.3A. Similarly, using solution 2 for ϕ (Table 8.2B) and an A_2 contribution which does not involve any Bessel function J_1 (8.20), we obtain an equally good fit to the data. However in both cases the good χ^2 values are deceptive because neither of the parameterisations reproduces the inward moving zero observed in

$P(pp)$ at $|t| \lesssim 0.5 \text{ (GeV/c)}^2$ for $p_{lab} \geq 45 \text{ GeV/c}$. In both cases the $P(pp)$ values for $|t| \geq 0.5 \text{ GeV/c}$ and $p_{lab} \geq 45 \text{ GeV/c}$ are small and positive. The data in this (p_{lab}, t) range, though negative, however has large error bars and hence χ^2/pt is not bad. Another feature of the fits is that the $\omega'_{(+)}$ contribution, treated in terms of free parameters, turns out to be smaller than that of ϵ . Thus, of the low lying contributions the ϵ contributes more strongly to the helicity flip amplitude than to the non-flip amplitude. The $\bar{p}p$ polarisation is reproduced satisfactorily by the two parameterisations. The relevant numbers for the two fits to $P(pp)$ and $P(\bar{p}p)$ are quoted in Tables 8.3A,B. The $P(pp)$ data has been fitted at 10, 11.8, 12, 14, 17.5, 24, 45, 100 and 300 GeV/c, while $P(\bar{p}p)$ is fitted at 10, 14 and 40 GeV/c. Data was taken from Borghini et al. (1971b,c), Gaidot et al. (1975, 1976), Crabb et al. (1977), Snyder et al. (1978). Our parameterisation does not reproduce the measurements of $P(pp)$ at 150 GeV/c carried out by Fidecaro et al. (1978); their data give values which are far too negative beyond $|t| \gtrsim 0.5 \text{ (GeV/c)}^2$. However Kroll et al. (1979) have quoted a private communication of Fidecaro et al. according to which preliminary results from a repetition of the same experiment indicate less negative values than the ones published by Fidecaro et al. (1978). We shall therefore have to await their final results for the 150 GeV/c data. The values of C_ρ and C_{A_2} indicate clearly that, as in the KN case, the ρ and A_2 contributions to the flip amplitude break EXD. This is in contradiction with the expectations of the DAM in which $\rho - A_2$ trajectories and residues in exotic channels are expected to coincide.

Let us try to understand why a Pomeron flip contribution fails to reproduce the structure in $P(pp)$. The structure of polarisation is governed by the difference $\text{Re } \phi_5 \text{ Im } \phi - \text{Re } \phi \text{ Im } \phi_5$. Let us analyse

what happens to various parts of this difference as the energy increases. We are interested in making the second term $(\text{Re}\phi \text{ Im}\phi_5)$ much stronger than $\text{Re}\phi_5 \text{ Im}\phi$ with increasing energy, hoping that it will eventually dominate it, and produce a sign change. The first point to note is that $\text{Im}\phi$ is, at high energies, effectively the Pomeron imaginary part in the non-flip amplitude. It $(\text{Im}\phi)$ is positive and structureless. Re ϕ on the other hand $(-\cot \frac{\pi\alpha}{2} \text{Im}\phi)$ is negative and structureless in the $|t|$ region of interest. In order that $\text{Re}\phi \text{ Im}\phi_5$ should compete for dominance with $\text{Re}\phi_5 \text{ Im}\phi$ we must have a negative Pomeron flip term in $\text{Im}\phi_5$. This in conjunction with a negative $\text{Re}\phi$ makes $\text{Re}\phi \text{ Im}\phi_5$ positive. $\text{Re}\phi_5$ on the other hand is positive and at high energies, is essentially the Pomeron flip phase. Here lies the nub of the problem. No matter how strong the Pomeron contribution to $\text{Im}\phi_5$ is made, its phase is of opposite sign (i.e. +ve) and is sufficient to enable $\text{Re}\phi_5 \text{ Im}\phi$ to maintain an edge over $\text{Re}\phi \text{ Im}\phi_5$. That is why Irving (1975) set the Pomeron flip phase equal to zero (in contradiction with analyticity) and, following him, so did we in Chapter 7. This is probably also the source of difficulty encountered by Berger et al. (1978) and the reason for the remark by Irving (1979) that if one reduces the Pomeron flip phase the P(pp) data can be reproduced.

The question therefore arises as to what more can be done. Kroll et al. (1978) have used a parameterisation in which $\text{Re}\phi$ controls the behaviour of the polarisation. In our case, however, $\text{Re}\phi$ is structureless and negative at high energies and we cannot reproduce the important feature of a single zero at $|t| \sim 0.5$ at 45 GeV/c which moves inwards with increasing energy. The difficulty, as analysed above, is that the high-lying contribution to $\text{Im}\phi_5$ is that of a crossing-even pole, the Pomeron, which has a phase of opposite sign to its imaginary contribution in the $|t|$ region of interest. However if one were to use a high lying contribution of odd signature in $\text{Im}\phi_5$ the difficulty could be overcome

because the phase of such a contribution would have the same sign as its imaginary part in the $|t|$ region of interest. We would probably not have pursued this point were it not for Joynson et al. (1975) and Leader (.978) who, in a different context, have speculated on the existence of an odd-signatured partner of the Pomeron which they called the "Odderon". One could invoke the Odderon in the NN flip amplitude to reproduce the high energy features of the $P(pp)$. Following Joynson et al. (1975) we take the Odderon trajectory to be EXD with that of the Pomeron. This implies that one could write an Odderon contribution (say O) to the flip amplitude as:

$$O(s,t) = \sqrt{-t} \left[\tan \frac{\pi\alpha_P}{2} + i \right] C_o e^{d_o t + g_o t} \cos \frac{\pi\alpha_P}{2} s^{\alpha_P(t)} \quad (8.22)$$

where the $\cos \frac{\pi\alpha_P}{2}$ factor is necessitated by the need to keep $\text{Re}O$ regular at $\alpha_P = 1$ (where $\tan \frac{\pi\alpha_P}{2}$ has a pole). Here of course

$$\alpha_P(t) = 1.0697 + 0.23t \quad .$$

We then write

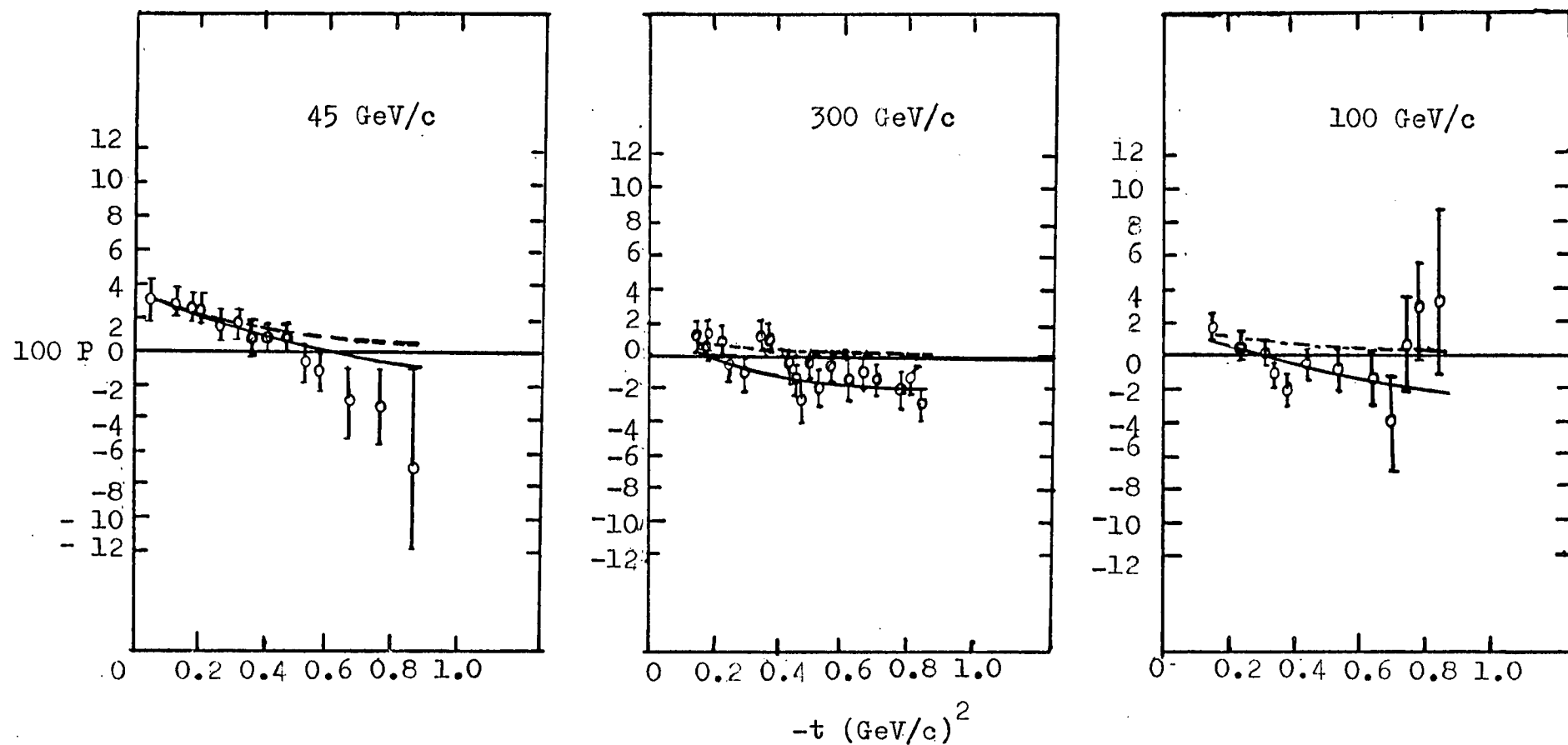
$$\phi_5\left(\frac{P}{P}\right) = A_2 + \epsilon + \omega_{(+)}^{+} \bar{\rho} + O \quad (8.23)$$

and fit polarisation data using (8.21). Using the parameters of Table 8.2A for the non-flip amplitude ϕ gives an extremely good fit ($\chi^2/pt = 1.1$) which reproduces all features of the $P(pp)$ data which eluded us when a Pomeron flip term was used. The parameter values are quoted in Table 8.4 as solution 1*. Using the parameters of Table 8.2B for the non-flip amplitude in conjunction with equation (8.20a) for the A_2 flip contribution and an Odderon flip term gives a comparable

*

Here the $\omega_{(+)}^{+}$ flip contribution turns out to be zero leaving ϵ as the only low-lying exchange in the flip amplitude.

Fig 8.2: Fits to pp polarisations. Solid curves correspond to an Odderon-flip contribution (table 8.4 sol.1) while dashed curves represent fits with a Pomeron-flip contribution (table 8.3A).



fit to $P(pp)$ and $P(\bar{p}p)$ ($\chi^2/pt = 1.15$) and such a parameterisation also reproduces the inward moving zero in $P(pp)$ observed for $|t| \leq 0.5$ and $p_{lab} \geq 45$ GeV/c. The numerical details are given in Table 8.4 as solution 2 (c.f. Figure 8.2 on the following page).

The question remains as to the utility of invoking a new trajectory as baffling as the Pomeron which remains inscrutable as ever. Furthermore what are its quantum numbers? Apart from having $C = +1$ and $\tau = -1$ what is its isospin? Does it have $I = 0$ or $I = 1$? If it has $I = 1$ then such a contribution will change sign as we go from pp to pn polarisations. This will imply, asymptotically, opposite polarisations for pp and pn polarisations. Joynson et al. (1975) discovered such a high-lying contribution in πN scattering at $t = 0$ and assigned it an isospin 1. Also, one might ask, does the Odderon couple dominantly to the flip amplitude? Here we discovered (after finishing our calculations and writing up most of this chapter) that some very interesting results have emerged recently from a study of the np CEX process $np \rightarrow pn$ at high energies. Bouquet and Diu ((1977), 1978), while investigating the energy-dependence of this process up to 300 GeV/c, discovered that the effective trajectory for this process has an intercept $\alpha_{eff}(0) \sim 0.74$. This is well above the ρ and A_2 intercepts which, according to conventional ideas, should dominate this process at high energies. In order to reproduce the high energy $\frac{d\sigma}{dt}$ data for np CEX Bouquet and Diu (1978) find it necessary to introduce a Pomeron-like term with a trajectory $\alpha(t) = 1 + 0.25t$. It may be noted that the Pomeron cannot be exchanged in $np \rightarrow pn$. This term was assumed to contribute only to the non-flip amplitude. Furthermore Diu and Ferraz De Camargo (1977) have found evidence for the presence of a similar term in $pp \rightarrow n\Delta^{++}$ at high energies. All this points to the existence of a high-lying trajectory with properties different from that of the

TABLE 8.4

Fit to $P(pp)$ and $P(\bar{p}p)$ involving an Odderon-flip term.

Parameter*	Solution 1	Solution 2
C_ρ	-21.09	-26.08
d_ρ	2.414	1.987
g_ρ	2.217	1.745
R_ρ	4.96	4.022
C_ϵ	-56.95	-49.935
d_ϵ	3.595	0.00
g_ϵ	0.00	0.00
C_{A_2}	-22.39	-16.86
d_{A_2}	1.262	2.023
g_{A_2}	1.523	1.545
R_{A_2}	3.58	-
$C_{\omega' (+)}$	0.00	0.00
$d_{\omega' (+)}$	-	
C_o	-0.33	-0.2933
d_o	2.542	2.358
g_o	0.00	0.00
χ^2/pt	1.1	1.15

* d_i in $(\text{GeV}/c)^{-2}$

g_i in $(\text{GeV}/c)^{-4}$

R_i in $(\text{GeV}/c)^{-1}$

Pomeron. Perhaps the introduction of an Odderon flip contribution may not be off the mark. Polarisation data for np scattering at high energies would shed significant, perhaps decisive, light on this point.

Conclusions

We now sum up our conclusions as follows:

- i) Contrary to the expectations of the DAM, $\rho - A_2$ trajectories are not EXD. Furthermore strong $\rho - A_2$ EXD appears to be broken by the corresponding residues in KN and NN scattering. In the context of our analysis this latter statement is definitely established for both amplitudes in KN scattering and for the flip amplitude in NN scattering. However since our determination of the A_2 non-flip contribution in NN scattering is unreliable the statement must be taken with caution for the NN non-flip amplitude. The $f - \omega$ trajectories are approximately degenerate but the corresponding (non-flip) residues in KN and NN scattering indicate a breaking of EXD at the level of residues. This is in contradiction with the DAM expectations where the $\rho - A_2$ as well as $f - \omega$ residues are expected to be EXD in NN and KN scattering. There is some evidence of an f-flip contribution in πN scattering but there appears to be no need for an f or ω flip contribution in KN and NN scattering.
- ii) The imaginary parts of the ρ contributions to both the flip and non-flip amplitudes, in πN , KN and NN scattering, are in accord with the DAM. Fits give a ρ radius of interaction in the range $R_\rho \sim 0.7 - 1$ fm which can be regarded as confirming the DAM expectation of $R_\rho \sim 1$ fm. The corresponding real parts for both amplitudes are then given by the usual Regge pole phase rule (which, in the DAM, is assumed valid only for the $\Delta\lambda = 1$ amplitude). A similar statement is valid

for the ω contribution to the non-flip amplitude in KN and NN scattering. There seems to be no need for an ω -flip contribution in these reactions. Since the ρ non-flip contributions are very much smaller than the P , f , ω contributions the precise numerical values of the non-flip ρ radius of interaction must not be taken too seriously.

iii) In order to incorporate the correct phases of the f and A_2 contributions to the $\Delta\lambda = 0$ and $\Delta\lambda = 1$ amplitudes we alter the DAM prescription

$$\text{Im } T_{\Delta\lambda}^{(f, A_2)} \approx "J_{\Delta\lambda}"$$

to

$$\text{Im } T_{\Delta\lambda}^{(f, A_2)} \approx "J_{\Delta\lambda}" \sin \frac{\pi\alpha}{2}$$

with the real parts given by

$$\text{Re } T_{\Delta\lambda} = - \cot \frac{\pi\alpha}{2} \text{Im } T_{\Delta\lambda} .$$

For the $\Delta\lambda = 0$ case this is necessitated by the need for preventing $\text{Re } T_{\Delta\lambda=0}$ from blowing up at $\alpha = 0$ but for the flip case this ansatz, though not necessary, has been taken over for reasons of uniformity.

With this change the f non-flip radius of interaction turns out to be in the range 0.60 - 0.65 fm for πN , KN and NN scattering, while the A_2 flip radius of interaction comes in the range 0.6 - 0.7 fm approximately. There is a small f -flip contribution in πN scattering and a small non-flip A_2 contribution in NN and KN scattering all of which have a radius of interaction of approximately 0.9 fm. However since these latter contributions are very much smaller than other contributions in the respective amplitudes the precise numerical values of their radii of interaction must not be taken too seriously. It is also possible to parameterise data by taking f and A_2 contributions

which do not involve any Bessel functions. Even when this is done the statements pertaining to $\rho - A_2$ and $f - \omega$ EXD breaking made in (i) are valid.

iv) The Pomeron dominates all non-flip amplitudes considered in this work with imaginary parts that can be parameterised as smooth structureless functions of t in agreement with the DAM prescription. However there is a substantial Pomeron phase as indicated by a Pomeron trajectory slope of 0.23 which contradicts the DAM approximation $\text{Re}P \sim 0$.

v) In contradiction to the DAM there is some evidence of a Pomeron flip contribution. Such a contribution seems necessary for a reasonable description of πp polarisations. There is no need for a Pomeron flip term in KN scattering. In NN scattering there is evidence of a high lying contribution to the flip amplitude. Using a Pomeron-flip term, while yielding good χ^2 values, does not reproduce the structure observed in pp polarisation in the region $p_{\text{lab}} \geq 45 \text{ GeV/c}$, $-t \leq 0.5 (\text{GeV/c})^2$. This difficulty is caused by a substantial Pomeron phase (determined by the Pomeron slope) and since it (phase) cannot be reduced we have, as a speculative exercise, invoked an Odderon-flip contribution which solves this difficulty. The Odderon is an odd-signatured partner of the Pomeron and its existence was suggested in another context by Joynson et al. (1975). There has also been some evidence of a high lying $I = 1$ trajectory ($\alpha(0) = 1$) in $np \rightarrow pn$ (Bouquet and Diu (1978)) so that an Odderon-flip contribution might well be present in NN scattering. np elastic polarisations at high energies can shed definite light on this point^{*}. The Pomeron flip

* As the last lines are being written Collins and Wright (1979) have published their work in which the Pomeron slope $\alpha'_P = 0.1$. This is very much smaller than our α'_P value of 0.23 and that of Collins et al. (1974a,b) ($\alpha'_P = 0.22$) and of Berger et al. (1978) ($\alpha'_P = 0.3$). Such

contribution is structureless in the $|t| \leq 0.8 \text{ (GeV/c)}^2$ region { where $J_1(3.5\sqrt{-t})$, while giving the zero at $t = 0$ required by angular momentum conservation, is structureless (its next zero occurs beyond $|t| = 0.8 \text{ (GeV/c)}^2$)}. The Odderon flip contribution is also structureless

$$(\beta_0(t) = C_0 e^{d_0 t + g_0 t^2}).$$

vi) There is strong evidence for low-lying contributions in the πN and $\bar{N}N$ non-flip amplitudes and in the πN , KN and NN flip amplitude. Apart from the ρ' needed in πN scattering, one needs an $I = 0$, $C = +1$ even-signatured trajectory in both the πN non-flip as well as the flip amplitudes. Such a contribution is also present in the KN flip amplitude. In pp , $\bar{p}p$ scattering however one needs two EXD trajectories of opposite signature ($C = +1$ for both of them), the even-signatured partner contributing only to the flip amplitude. The odd signatured term has a strong non-flip contribution and a comparatively weaker flip coupling.

The success of the DAM in describing elastic hadronic scattering is thus very mixed.

a small slope might solve the difficulty encountered by a Pomeron flip term in reproducing the pp polarisations at high energies since the resulting Pomeron phase structure will be much smaller compared to ours and shall increase very slowly (in magnitude) away from $t = 0$. However this can only form part of a new investigation because of lack of time here.

APPENDIX I

$\left. \frac{d\sigma}{dt} \right|_{t=0}$ Value for $\pi^- p \rightarrow \pi^0 n$ Using Equation (8.3)

P_{lab} (GeV/c)	$\left. \frac{d\sigma}{dt} \right _{t=0}$ ($\mu b / (GeV/c)^2$)	Comments
5.9	345 ± 30	Fits of Bolotov et al. (1974a) to the data of Stirling et al. (1966)
9.8	185 ± 17	
13.3	170 ± 17	
18.2	135 ± 20	
21.0	120 ± 18	Fits of Bolotov et al. (1974) to their own data.
25.0	107 ± 13	
40.0	74 ± 8	
48.0	62 ± 6	
15.0	151.55 ± 3.24	Our fits to data of Apel et al. (1979)
20.2	103.48 ± 1.72	
25.0	84.40 ± 0.42	
40.0	54.9 ± 1.5	Fit of Apel et al. (1979) to their own data
20.8	99.33 ± 3.36	Our fits to FNAL data (Barnes et al. (1976). Barnes et al do not quote any errors on their fits (c.f. comment on p.175) and their $\left. \frac{d\sigma}{dt} \right _{t=0}$ values are (in order of increasing momenta) 100.3, 49.8, 30.9, 19.51, 12.8 and 9.61 $\mu b / (GeV/c)^2$ respectively. Our fits give answers which agree with theirs within errors.
40.8	49.14 ± 1.60	
64.4	31.31 ± 0.81	
100.7	19.18 ± 0.68	
150.2	12.944 ± 0.260	
199.3	9.948 ± 0.332	

APPENDIX 2

$\left. \frac{d\sigma}{dt} \right|_{t=0}$ Values for $\pi^- p \rightarrow \eta n$ Using Equation (8.3)

p_{lab} (GeV/c)	$\left. \frac{d\sigma}{dt} \right _{t=0}$ ($\mu b / (GeV/c)^2$)	Comments
5.9	42.0 ± 10.0	Fits to data of Guisan et al. (1965) by Bolotov et al. (1974b).
9.8	23.0 ± 6.0	
13.3	15.0 ± 4.0	
18.2	8.0 ± 2.8	
21.0	9.4 ± 2.7	Fits of Bolotov et al. (1974b) to their own data.
25.0	9.0 ± 2.5	
32.5	7.1 ± 1.8	
40.0	6.7 ± 1.9	
48.0	4.5 ± 1.3	
20.8	7.754 ± 0.566	Our fits to FNAL data (Dahl et al. (1976). Dahl et al. do not quote errors on their $\left. \frac{d\sigma}{dt} \right _{t=0}$ values (c.f. previous page) and their corresponding values are (in order of increasing momentum) 6.71, 2.88, 1.62, 0.92, 0.558 and 0.391 $\mu b / (GeV)^2$) res- pectively.
40.8	3.269 ± 0.148	
64.4	1.574 ± 0.208	
100.7	0.9666 ± 0.1274	
150.2	0.5706 ± 0.0072	
199.3	0.4168 ± 0.0039	

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