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Rock physics theory and analysis for fractured carbonate reservoirs

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“Experience arises together with theoretical assumptions not before them, and an experience without theory is just as incomprehensible as is (allegedly) a theory without experience.”

(Paul K. Feyerabend, *Against Method*, 1975)

Declaration

I declare that this thesis has been composed solely by myself and that it has not been submitted, either in whole or in part, in any previous application for a degree. Except where otherwise acknowledged, the work presented is entirely my own.

Phillip A. Cilli

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Abstract

The analysis of a geological reservoir’s elastic and electrical properties forms the cornerstone of geophysical reservoir characterisation. These properties are significantly influenced by a reservoir’s pore and fracture geometry. A multitude of pore shapes occurs in carbonate rocks, making their electrical and elastic properties particularly challenging to model, especially by first principles physics. In addition to this, many of the world’s carbonate reservoirs are located beneath salt diapirs, which can alter the path of incidental seismic waves and impede seismic reservoir characterisation. Thus, developing new rock physics theory and methods for fractured carbonate reservoirs may serve to advance reservoir characterisation efforts.

Many rock physics models assume pore and fracture shape is constant with varying pore volume fraction, however carbonate porosity is highly susceptible to diagenesis and other physical changes in reality. In this thesis, I show a systematic change in pore shape with porosity exists in many carbonate rocks, even when measured at constant effective pressure. By accounting for this changing pore shape, electrical and elastic rock physics modelling errors are reduced on seven public domain, laboratory data sets, and two well logs from a carbonate reservoir, while new calcite $V_p - V_s$ and $V_p/V_s - \phi$ relationships are derived from first principles. This proposed porosity-pore shape relationship leads to alternative models for Archie’s first law, the Humble equation, the Castagna and Pickett

relations, and the Focke and Munn relations, while being correct in the porosity asymptotes. Unlike these empirical relations, the proposed model may extrapolate to other rock and fluid types as it is derived from first principles.

Reservoir characterisation uncertainty can be reduced using analysing both electrical and elastic measurements. Transforming from one physical property to the other, however, typically involves estimating reservoir porosity as an intermediate, bridging parameter. By integrating existing electrical and elastic differential effective medium models, I obtain new cross-property differential effective medium (DEM) expressions which have no explicit mathematical porosity dependence, allowing elastic moduli to be estimated directly from electrical measurements without estimating porosity intermediately. This new DEM method models public domain, electrical-elastic laboratory measurements from wet, mixed sandstones, and predicts synthetic velocity from resistivity well logs with lower error than the typical method, which estimates porosity logs as an intermediate step.

Fracture characterisation in Pre-salt reservoirs can be challenging by walkaround vertical seismic profile (VSP) analysis, as algorithms often assume a simple overburden or separable S-waves. I extend the spectral ratio method to estimate interval relative azimuthal attenuation in a reservoir beneath arbitrarily complex overburden given VSP receivers in an arbitrarily deviated well. Characterising a Brazilian Pre-salt reservoir's fracturing using a pre-existing, multifluid, fractured, viscoelastic model, it seems there are two non-orthogonal, vertical fracture sets present, with a depth-dependent fracture density and orientation.

Estimating tilted transverse isotropy (TTI) parameters using the slowness-polarisation method given walkaway VSP measurements shows the Brazilian Pre-salt reservoir interval has negative Thomsen's parameters, ϵ and δ . These findings are not contradictory to those of the fracture characterisation study, as a negative ϵ in a vertical or TTI system could be linked to (sub)vertical fracturing.

Lay Summary

Fluids, such as oil, gas, and ground water, can be found underground within the small holes in rocks, called pores, as well as the longer, flatter cavities, often referred to as fractures. Just like some shapes are better at resisting deformation than others (e.g., an equilateral triangle is especially good at this, and favoured in engineering designs for this reason), the ability of a rock's pore to resist deformation depends upon its shape. When a rock contains many pores, the rock's overall ability to resist deformation therefore depends upon its pores' shapes. Another interesting (yet non-obvious) fact is that the rock's electrical conductivity also depends upon its pores' shapes in almost the same way.

Pore shape, however, is not the only thing that affects a rock's resistance to deformation, or electrical conductivity. Gasses are less electrically conductive and more compressible than oil or water, for example. This effect leads to a rock's resistance to deformation, or electrical conductivity, also depending on the type of fluid contained within the pores. In fact, the rock's properties also depend on what minerals the solid rock is made of to some extent: for example, whether it is a sandstone or a limestone.

Rock physics aims to predict the resistance to deformation, or electrical conductivity, of rocks, among other things, to improve our understanding of underground reservoirs. A common way to predict these physical properties is to assume the

rock contains a known, unchanging fluid in its pores, as well as known, unchanging minerals and pore shapes. In this case, the rock's physical properties of interest only depend on how much of the rock is solid material and how much is pore space.

In reality, however, a rock's pores' shapes can change through natural processes, such as when minerals dissolve or the rock is compacted. In this thesis, I account for this change in a rock's pore shapes when predicting both its resistance to deformation, and its electrical conductivity. By accounting for this change in pore shape due to physical processes, predicting these physical properties becomes more accurate, which I show using many laboratory and field-measured data sets.

I also uncover an interesting mathematical result which lets us predict a rock's resistance to deformation using its measured electrical conductivity, without ever knowing how much pore space is actually present in the rock, as is usually required. Avoiding the need for this extra information, I predict the resistance to deformation of real rocks from their measured electrical conductivities using public, laboratory measurements, and reduce the prediction error by over 20% compared to typical methods on synthetic data sets which mimic those typically acquired inside reservoirs.

Understanding the nature of fracturing in an underground reservoir is useful as it helps us predict how fluid will flow through the reservoir during production; fluid flow can either be enhanced or hindered by fractures. When a reservoir is situated beneath extremely complex geology, for example beneath underground "mountains" of rock salt, existing methods for understanding fractures can be insufficient. I therefore extend a pre-existing fracture analysis method, which analyses how a rock attenuates sound depending on the sound's direction of travel, to be suitable for reservoirs located beneath complex geology. Applying this method to a Brazilian oil reservoir situated beneath complicated underground salt

structures, I find there seems to be two sets of vertical fractures in the reservoir, changing their orientation with depth.

Typically, sound will travel slower through a rock when it has to pass through fractures, than when it is moving in line with the fractures and not crossing them. By analysing how the speed of sound in this Brazilian reservoir changes with the direction in which it travels, I find the reservoir's speed of sound is slower when travelling in the horizontal direction than in the vertical direction. This could be caused by the presence of vertical fractures in the reservoir, which would be consistent with results from the fracture analysis performed in this thesis.

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List of Acronyms

<i>AIC</i>	Akaike information criterion
<i>AIC_C</i>	Corrected Akaike information criterion
CSEM	Controlled source electromagnetic
DEM	Differential effective medium
DTC	Compressional wave slowness log
DTS	Shear wave slowness log
EGAR	Effective grain aspect ratio
EPAR	Effective pore aspect ratio
FM	Focke and Munn
HTI	Horizontal transverse isotropic/isotropy
LMO	Linear moveout
M&C	Mendelson and Cohen
MD	Measured depth
NPHI	Neutron porosity log
RHOB	Bulk density log
RMSE	Root mean square error
SCA	Self-consistent approximation
S.D.	Standard deviation
SSR	Sum of squared residuals
TI	Transverse isotropy/isotropic
TVDSS	True vertical depth sub-sea
VSP	Vertical seismic profile
VTI	Vertical transverse isotropic/isotropy

Chapter 1

Introduction

Fractures, cracks, and pores are prevalent throughout the Earth's crust, forming the storage space for fluids in subsurface reservoirs. The assessment of these features is therefore an integral part of reservoir characterisation workflows in hydrocarbon, groundwater, and carbon sequestration studies. Elastic and electrical geophysical measurements are used to estimate the physical properties of subsurface reservoirs (Rider, 2002; Avseth *et al.*, 2005; Dvorkin *et al.*, 2014). These measured elastic and electrical responses depend on the reservoir's matrix and pore fluid compositions, as well as its pore and fracture geometries (Aguilera, 1974; Herrick and Kennedy, 1994; Kumar and Han, 2005; Schoenberg and Sayers, 1995). It is thus desirable to develop geophysical models for reservoir characterisation which consider both the contribution of a reservoir's pore and fracture architecture, as well as its matrix and fluid composition, to its overall elastic or electrical properties.

Carbonate reservoirs are estimated to make up the majority of the world's hydrocarbons in place (Schlumberger, 2007) and so understanding their fracture and flow properties is becoming increasingly relevant. Carbonate reservoirs can

have a notoriously diverse range of pore types (Choquette and Pray, 1970; Tucker and Wright, 1990; Braithwaite, 2005), which can lead to complex elastic and electrical properties (Focke and Munn, 1987; Xu and Payne, 2009). Anselmetti and Eberli (1993, 1997) showed the velocities of carbonates can vary with pore type, while Xu and Payne (2009) and Kittridge (2014) showed the elastic moduli can vary with pore geometry. Somewhat analogously, Focke and Munn (1987) showed the electrical properties of carbonates can vary with pore type.

One popular way to account for a rock's pore or grain geometry when modelling its electrical or elastic properties is through inclusion modelling (Eshelby, 1957; Kuster and Toksöz, 1974; Mori and Tanaka, 1973; Berryman, 1980b; Sen *et al.*, 1981; Mendelson and Cohen, 1982; Norris, 1985; Berryman, 1995). Inclusion models are a popular choice when estimating the physical properties of carbonates due to their ability to account for pore or grain shape (Xu and Payne, 2009; Kittridge, 2014; Fournier *et al.*, 2011, 2014, 2018), however, the assumption of an idealised pore geometry underpinning these models is a typical source of criticism (Vernik and Kachanov, 2012). Furthermore, typical implementation of an inclusion model assumes pore or grain geometry is constant with porosity. This may not be appropriate in the case of carbonates, which are highly susceptible to physical processes which alter pore or grain geometry (Tucker and Wright, 1990; Braithwaite, 2005). Indeed, Fournier *et al.* (2011) showed the inclusion model's pore geometry parameter, effective pore aspect ratio (EPAR), can change due to diagenesis in carbonates, while Fournier *et al.* (2014) showed EPAR increases when a carbonate undergoes vug-forming dissolution. In a different line of study, Vanorio and Mavko (2011) highlighted the complexity of carbonate rock physics modelling by finding the acoustic and transport properties of carbonates could be characterised effectively using the content of a rock's microcrystalline matrix (micrite), as opposed to its pore or rock type.

Reservoir characterisation uncertainty can be lowered by rock physics modelling

which uses both controlled source electromagnetic and seismic measurements (Alcocer *et al.*, 2013). However, the challenge still remains to produce a simple, physics-based, and accurate model to link a porous rock’s electrical and elastic properties while accounting for its pore or grain shape. The majority of published electrical-elastic multiphysics workflows and models require porosity estimation as an intermediate step when transforming from a rock’s electrical to elastic properties, and vice versa (Werthmüller *et al.*, 2013), which can be poorly constrained away from well control. Electrical-elastic multiphysics modelling has been attempted with mixed success (Han *et al.*, 2011a; Wang and Gelius, 2010; Jensen *et al.*, 2013). Regardless, Han *et al.* (2016) concluded a new model is required to relate the electrical and elastic properties of a porous rock using a single set of model parameters. Furthermore, Werthmüller *et al.* (2013) claimed there is no known direct link between a rock’s velocity and resistivity. Modelling a rock’s shear modulus from its electrical properties using first principles physics is another unresolved issue in multiphysics modelling.

Narr *et al.* (2006) proposed that all reservoirs should be considered fractured unless proven otherwise. The presence of fractures is understood to influence the low-frequency anisotropic elastic (Hudson, 1980, 1981; Thomsen, 1995), and viscoelastic (Chapman, 2003) properties of a rock, as well as its permeability anisotropy (Snow, 1969). Understanding the seismic anisotropy of a subsurface reservoir can improve seismic imaging (Alkhalifah *et al.*, 1996) and assist in building realistic geological and stress models. The inversion of multicomponent walkaway and walkaround VSP data for seismic anisotropy and fracture characterisation in the subsurface is becoming increasingly common (MacBeth, 2002; Liu and Martinez, 2013), however it remains challenging beneath complex, asymmetric overburden. Fracture characterisation by azimuthal attenuation modelling (Maultzsch *et al.*, 2003, 2005), for example, typically assumes a simple overburden. Shear wave splitting analysis (Winterstein and De, 2001; MacBeth, 2002; Mueller, 1991) can

be challenging in the case of complex overburdens, where multiple “first arrival” wave-fronts may be coincident on a VSP receiver from numerous azimuths and polar angles, and fast and slow shear waves may be difficult to separate in the seismic record.

The rock physics challenges presented in carbonates are due predominantly to their commonly complex pore structures (Xu and Payne, 2009). Figure 1.1 shows a popular classification scheme for carbonate porosity types, as presented by Choquette and Pray (1970). The difficulty in predicting the elastic properties of carbonate rocks, for example, is now evident: Some pores, such as vugs, are generally quite stiff simply due to their geometry, while others, such as fractures, are quite compliant. It follows that incorporating the shape of carbonate pores into elastic or electrical rock physics models may lead to more accurate modelling. However, carbonates rocks are more often interpreted and categorised in terms of their sedimentological and diagenetic features than their pore structures in practice, meaning investigating the dependence of carbonate rock physics modelling on rock type is more likely to be feasible than investigating its dependence on pore type.

The most renowned classification schemes for carbonate rocks are those proposed by Folk (1962) and Dunham (1962). The Folk classification differentiates between four different grain types (allochems), as well as muddy matrix, and cement (the orthochemical component). The allochems defined in the Folk classification system are intraclasts, ooids, fossils, and pellets. The muddy matrix is made of microcrystalline calcite, or micrite, and is deposited concurrently with the allochems. The crystal cement, or spar, is introduced after deposition through chemical processes. By contracting these names, Folk created the classification scheme seen in Figure 1.2.

While the Folk classification system focusses on a rock’s texture, the Dunham

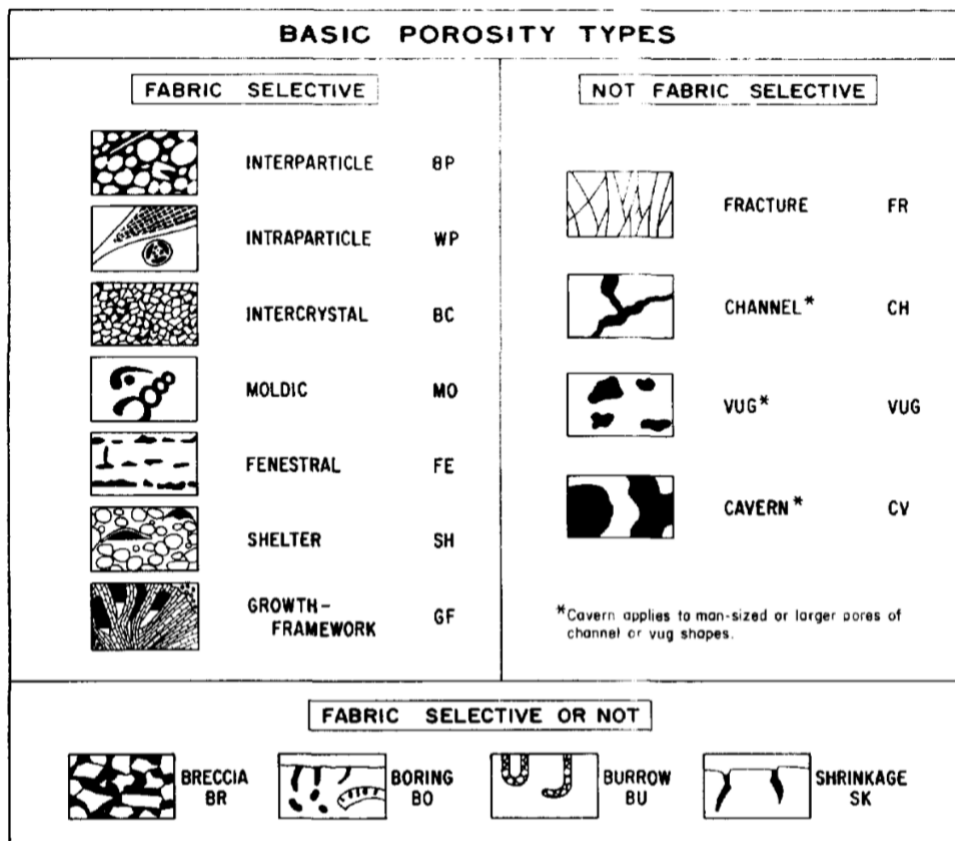


Figure 1.1: Typical carbonate porosity classification from Choquette and Pray (1970).

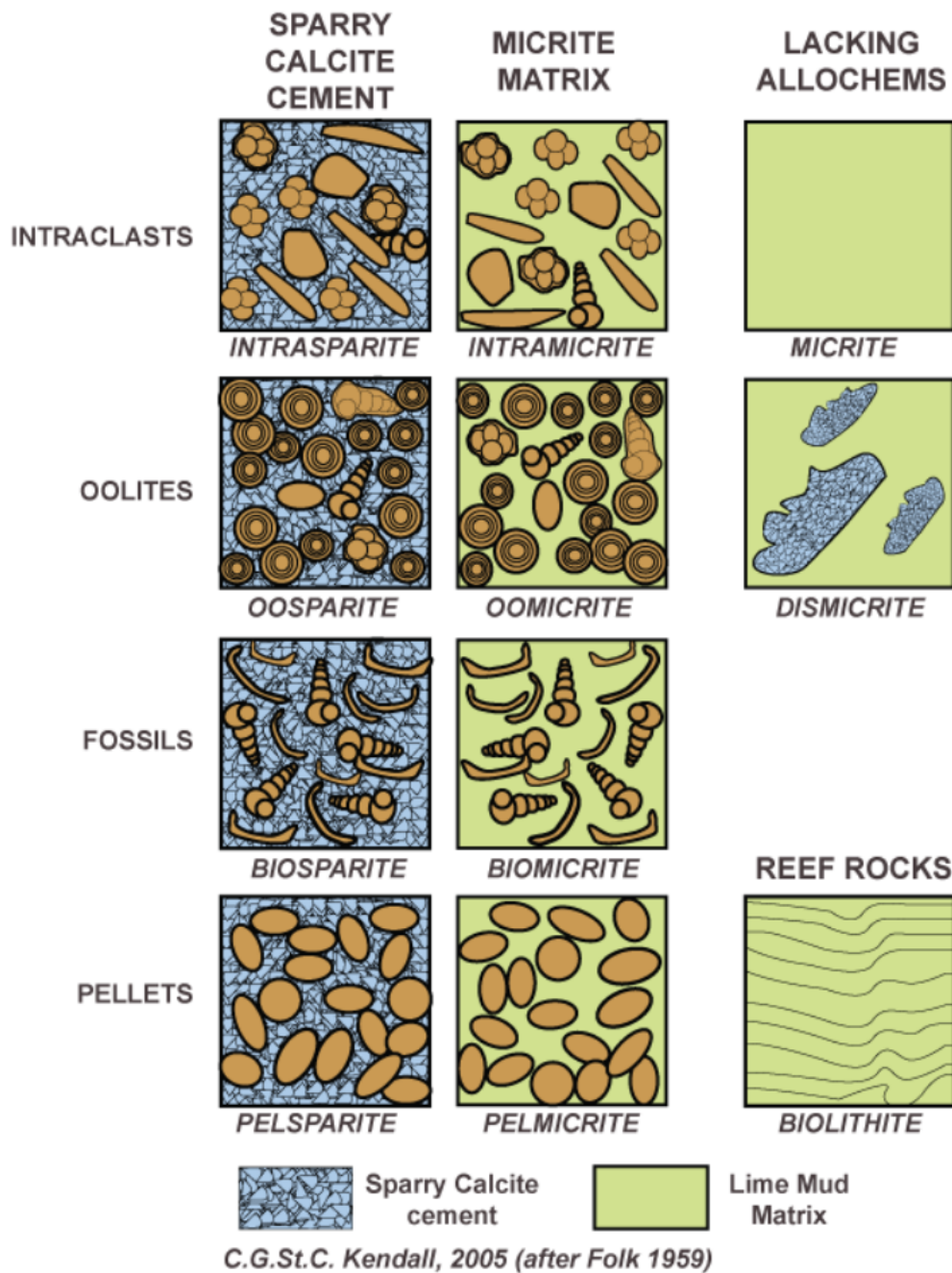


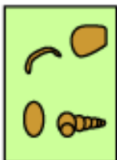
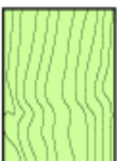
Figure 1.2: Folk classification. From Kendall and Flood (2005) after Folk (1962).

model categorises carbonate rocks by their proportion of mud and grains. There are five key rock types in the Dunham classification. Four of which have their original components unbound at deposition, while the fifth, a boundstone, has its original components bound together. The two mud-supported rock types are mudstones, which are less than 10% grains, and wackestones, which are more than 10% grains. The two grain-supported rocks are packstones, which contain some mud, and grainstones, which lack mud entirely. Figure 1.3 shows the original Dunham classification.

Important modifications to the Dunham classification scheme were provided by Embry and Klovan (1972), where more types were distinguished from those previously defined through considering grain size. Mud-supported carbonates with large grains are called floatstones, while grain-supported carbonates with large grains are rudstones. The updated Dunham classification system of Embry and Klovan (1972) also contains three distinct types of boundstones.

The Brazilian Pre-salt reservoirs are a collection of carbonate petroleum reservoirs which are particularly challenging to model, due to the presence of variable pore types, fracturing, and highly complex overburden. These reservoirs are situated in ultra-deep water offshore of Brazil, with water depths typically greater than 2000 m. These reservoirs often reside beneath one to two vertical kilometres of salt structures (Rosa *et al.*, 2018), as schematically demonstrated in Figure 1.4. The reservoirs' depositional basin was formed in a lacustrine environment during the Cretaceous breakup of Gondwana, when the South Atlantic began to open, separating South America from Africa.

It is not uncommon for Brazilian Pre-salt carbonate reservoirs to have multiple carbonate facies present within the reservoir interval (Al-Maghlouth *et al.*, 2017). Geologically, a *facies* can be defined as *the sum of lithologic and paleontologic characteristics of a sedimentary rock from which its origin and the environment*

Original components not bound together at deposition				Original components bound together at deposition. Intergrown skeletal material, lamination contrary to gravity, or cavities floored by sediment, roofed over by organic material but too large to be interstices
Contains mud (particles of clay and fine silt size)		Lacks Mud		
Mud-supported		Grain-supported		
Less than 10% Grains	More than 10% Grains			
Mudstone	Wackestone	Packstone	Grainstone	
				Boundstone
				

C. G. St. C. Kendall, 2005 (after Dunham, 1962, AAPG Memoir 1)

C. G. St. C. Kendall, 2005 (after Dunham, 1962, AAPG Memoir 1)

Figure 1.3: Dunham classification. From Kendall and Flood (2005) after Dunham (1962).

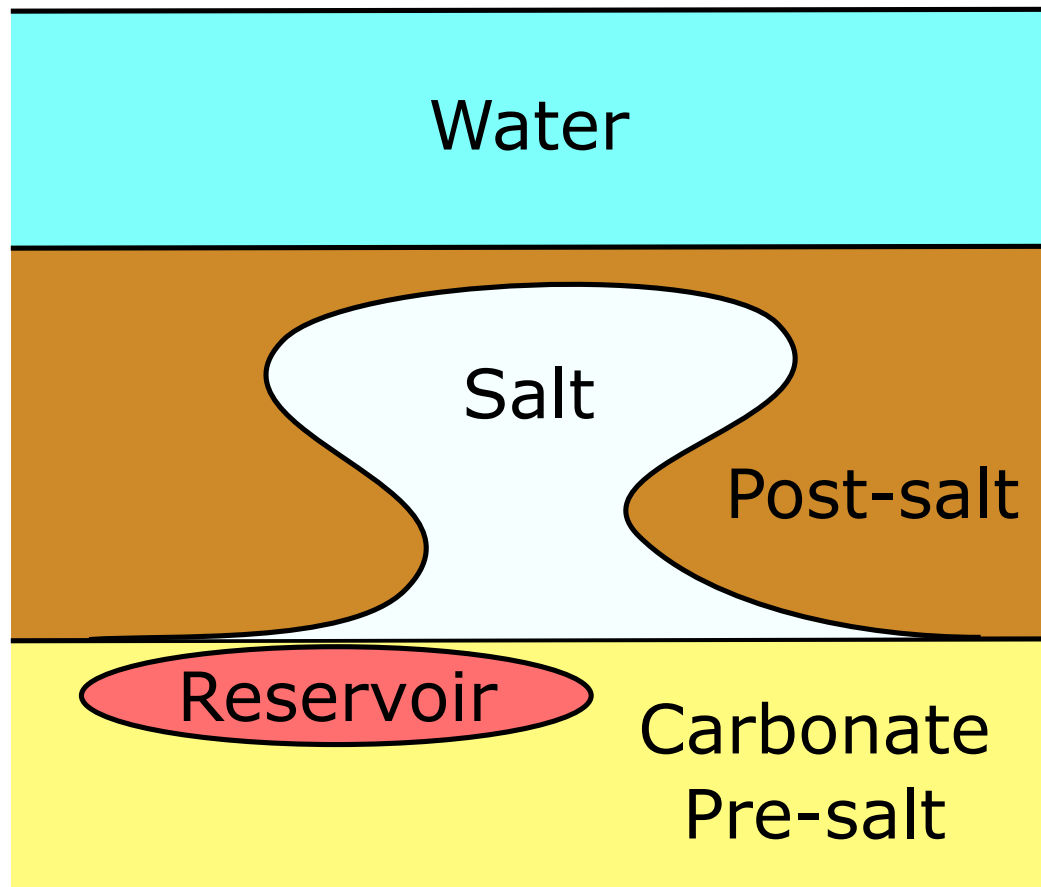


Figure 1.4: A simplified schematic of an offshore, carbonate Pre-salt reservoir and its surrounding geology. The asymmetric salt overburden can pose a challenge when studying the reservoir using seismic methods.

of its formation may be inferred (Teichert, 1958). In a reservoir characterisation setting, however, the term facies tends not only to refer to the depositional environment of rocks but also other distinguishing characteristics, such as the modern-day grain, mud, and matrix proportions. The Brazilian Pre-salt's primary reservoir formation is predominantly made of three carbonate facies: Carbonates with shrub-like texture, reworked carbonates, and low energy carbonates (Rosa *et al.*, 2018). Detailed reviews of the Brazilian Pre-salt reservoirs are presented by da Costa Fraga *et al.* (2015), Bruhn *et al.* (2017), and Rosa *et al.* (2018).

Figure 1.5 shows two perpendicular cross-sections through a real seismic image of a Brazilian Pre-salt reservoir, which was provided by this thesis' industrial sponsors. The image highlights some of the complexities which occur in the seismic analysis of Pre-salt carbonate reservoirs, obviating the asymmetric salt overburden, the deviated well orientation, and the dipping reservoir interval, among other features of interest.

At least three outstanding questions stand between rock physicists and the effective rock physics modelling of Pre-salt carbonate reservoirs containing multiple facies. Firstly,

How do we perform VSP analysis for reservoir properties, such as anisotropy and fracture characteristics, effectively beneath an asymmetric, salt overburden?

Secondly,

Should we, and if so, how would we account for pore shape in our carbonate rock physics models?

Finally, we ask,

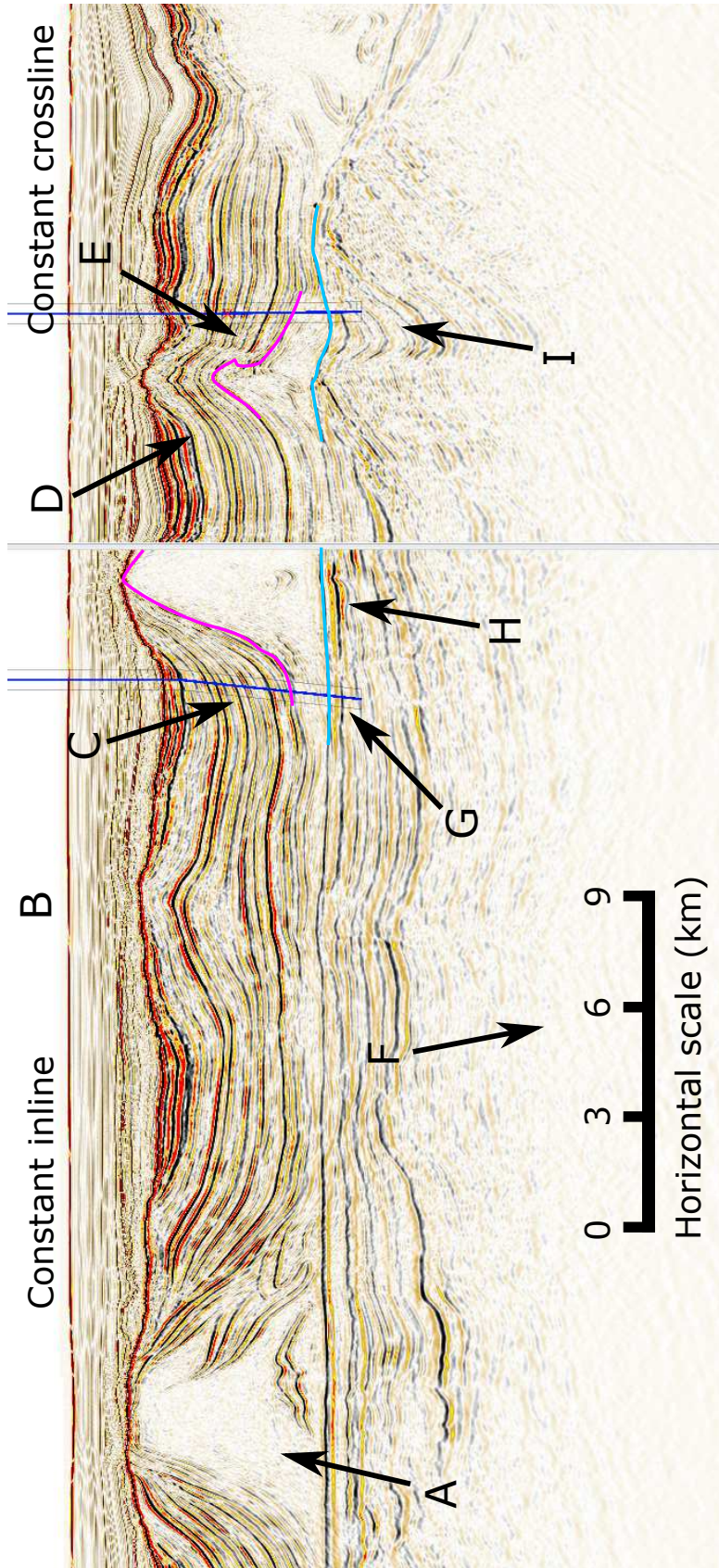


Figure 1.5: Two mutually perpendicular cross-sections through the Brazilian Pre-salt reservoir studied in this thesis. The cross-sections intersect at the blue vertical line, which represents the trajectory of Well A, studied herein. The top of salt (magenta) and base of salt (light blue) are interpreted in the vicinity of the well track. Features of interest are annotated, including A) Complex salt structures, B) the water bottom, C) the deviated well trajectory, D) post-salt geology, E) asymmetry of the Pre-salt well's overburden, F) basement, G) the Pre-salt carbonate interval where VSP receivers and well-log measurements are co-located, H) the Pre-salt reservoir "bright spot" (direct hydrocarbon indicator), and I) dipping geology at and beneath the reservoir. The horizontal scale is shown, however the vertical to horizontal scale ratio is not 1:1.

Is there a relationship between carbonate facies and rock physics properties, and should we incorporate any such relationship into our rock physics modelling?

With these questions in mind, I declare the aim of this thesis is:

To develop and extend rock physics modelling theory and methods for fractured carbonate reservoirs, with a focus on those possessing complex pore geometries, containing multiple carbonate facies, and overlain by asymmetric, salt overburdens.

I begin by reformulating the electrical DEM model of Mendelson and Cohen (1982) to embed ellipsoidal pores into a background of rock matrix material, as opposed to the original formulation which embeds insulating grains into a background of water. Through this reformulation, the DEM model of Mendelson and Cohen (1982) can be expressed in terms of the pre-existing electrical inclusion model function $R^{(*2)}$ proposed by Berryman (1995).

By integrating this electrical DEM model with the elastic DEM model of Berryman (1992), I propose new electrical-elastic DEM modelling expressions which relate the effective conductivity and elastic moduli of a two phase, linearly elastic, electrically conductive composite with randomly oriented spheroidal inclusions embedded in an isotropic background. Porosity is rendered a dummy variable in this multiphysics DEM formulation and is not present in the model's final mathematical form. This multiphysics DEM formulation is derived from first principles physics and can be expressed in terms of the pre-existing inclusion model functions $P^{(*2)}$, $Q^{(*2)}$ (Berryman, 1992), and $R^{(*2)}$ (Berryman, 1995), offering a new link between electrical and elastic DEM models. The electrical-elastic model is correct in the high and low porosity limit and reduces errors when estimating elastic moduli from electrical measurements, compared with the more

typical method where porosity is inverted for as an intermediate step, bridging the gap between electrical and elastic moduli.

The model's single parameter, effective pore aspect ratio (EPAR), seems weakly sensitive to clay pore volume fraction when modelling measurements made by Han *et al.* (2011b) on wet, clay-bearing sandstones from mixed localities. The uncertainty in shear modulus modelling also seems weakly sensitive to clay volume fraction in modelling this data set. The model obeys the electrical-elastic Hashin-Shtrikman bounds (Carcione *et al.*, 2007) for this data set. Interestingly, the new method prefers to approximate pore shapes with prolate spheroids, as opposed to the more commonly used oblate spheroids. This is significant as a rock containing oblate spheroidal inclusions can have a stiffness as low as the Reuss bound, at least in elastic modelling, while rocks with prolate spheroidal inclusions cannot.

I observe that the EPAR parameter of the elastic DEM model of Berryman (1992) changes by a power law with porosity on 3 public domain carbonate laboratory data sets. Further evidence for this was provided by Singh *et al.* (submitted) through digital rock physics analysis, which is reproduced in Appendix I. Similarly, the effective grain aspect ratio (EGAR) parameter of the electrical DEM model of Mendelson and Cohen (1982) also changes by a power law with porosity on 4 public domain electrical carbonate laboratory data sets.

Establishing this power law relation between porosity and EPAR or EGAR, I propose alternative models based on first principles for Archie's first law (Archie, 1942), the Humble equation (Winsauer *et al.*, 1952), and the empirical relations of Pickett (1963), Focke and Munn (1987), and Castagna *et al.* (1993) for wet limestones. The proposed power law DEM models are correct in the low and high porosity limits, and provide comparable modelling accuracy to the empirical models in the range of data. This study allows us to infer it is the interplay between the changing pore shape and the changing proportion of resistive material

with porosity which can lead to the non-monotonic porosity-resistivity trends observed by Focke and Munn (1987) in carbonates.

Analysing Brazilian Pre-salt reservoir well log data, I find the elastic power law DEM parameters vary with carbonate facies type, finding their analysis may be useful for in situ carbonate facies classification using elastic well log measurements. Pre-salt velocity well log prediction is improved when using a facies-dependent power law DEM model compared to a facies-dependent single aspect ratio DEM model, or a DEM model with a single aspect ratio for all facies.

To address the challenge of fracture characterisation beneath complex overburdens, I extend the fracture characterisation method of azimuth-dependent relative attenuation (Maultzsch, 2005), making it suitable for the analysis of VSP receivers in an arbitrarily deviated well beneath complex overburden. Applying this method to VSP data from a Brazilian Pre-salt reservoir, I find the apparent presence of two vertical fracture sets with depth-dependent azimuths and fracture densities. Sensitivity testing on the anisotropic, two fluid, viscoelastic model of Jin *et al.* (2018) shows the modelled azimuthal attenuation has low sensitivity to fluid patchiness and relative permeability model at a water saturation of 15%.

Applying the slowness-polarisation inversion method of Grechka and Mateeva (2007) to walkaway VSP data, I estimate the reservoir interval has a dip between 10 and 15 degrees, and negative Thomsen parameters, ϵ and δ . This result seems consistent with the preceding fracture characterisation study, as vertical fractures may lead to negative ϵ values if the reservoir's horizontal P-wave velocity is slower than its vertical velocity accordingly.

After a review of elasticity theory in Chapter 2 and poroelasticity theory in Chapter 3, I present the derived electrical-elastic DEM modelling method in Chapter 4. I then present the power law DEM model in Chapter 5, which I

apply in Chapter 6 to well log data from a Brazilian Pre-salt reservoir. I then invert walkaround VSP data from the Brazilian Pre-salt reservoir for fracture characteristics in Chapter 7, followed by the inversion of walkaway VSP data for the reservoir's tilted transverse isotropic parameters in Chapter 8. I integrate and analyse the Pre-salt well log, walkaround VSP, and walkaway VSP analyses in Chapter 9 before the Discussion and Conclusion of this thesis.

Appendices G, H, I, and J are publications and publication manuscripts which I have produced or contributed to during my PhD and which are not chapters in the body of this thesis.

Chapter 2

Elasticity and Effective Medium Theory Review

By analysing seismic data, rock physics allows us to infer (with some assumptions) conclusions about a reservoir's physical properties which are often of economic interest. Seismic wave propagation depends on the elastic or anelastic properties of the host medium. Hence, to build models which help us understanding how seismic waves propagate through media, and ultimately gain understanding of the rock being modelled, we first need to understand the fundamentals of elasticity or anelasticity theory. In this chapter I review the fundamentals of elasticity theory, and in the following chapter I review anelasticity theory, specifically considering poroelasticity. In this chapter, I first review Hooke's law and the stiffness tensor, followed by the Christoffel equation, which is used in Chapter 8 to model plane wave propagation in an arbitrarily anisotropic elastic medium. Finally, I review the historical development of effective medium theory, considering the key models which have been used by the rock physics community over the decades to model the elastic properties of a porous rock. The novel findings throughout this are

primarily extensions of the traditional effective medium models reviewed in this chapter.

2.1 Elasticity Fundamentals

2.1.1 Hooke's law and the stiffness tensor

A material which dissipates no energy under external stress is termed an elastic material (Christensen, 1979). An elastic material obeys Hooke's law, of which the linearised format in one dimension is:

$$F = -k\Delta x; \quad (2.1)$$

where F is an applied force, k is the material-dependent spring constant, and Δx is the change in displacement of a particle in the medium.

Now suppose we have some three-dimensional elastic material. This material's stress is related to its strain through the general linearised Hooke's law:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \quad i, j, k, l = 1, 2, 3. \quad (2.2)$$

where σ_{ij} is the 3×3 stress tensor and ε_{kl} is the 3×3 strain tensor.

The strain tensor is defined in terms of the displacement field u_i with $i \in \{1, 2, 3\}$ as:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) ; \quad (2.3)$$

where the indicial comma represents the usual partial derivative with respect to its trailing indices.

The strain energy W in an elastic medium can be defined as:

$$W = \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl} ; \quad (2.4)$$

with which we can define the stress in a medium as:

$$\sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}} . \quad (2.5)$$

The strain tensor ε_{ij} is symmetrical by its definition in equation 2.3. A typical, additional assumption we make is that stress tensor is also symmetrical, which requires symmetries in the tensor C_{ijkl} by equation 2.2. The components of ε_{ij} can thus be interpreted as the change in displacement along the i^{th} axis with respect to a change in displacement along the j^{th} axis.

The inverse of C_{ijkl} is the compliance tensor s_{ijkl} . A rank-4, 3-dimensional tensor can have up to 81 independent components. However, C_{ijkl} and s_{ijkl} have fewer independent components due to additional assumptions imposed.

Under the assumption that the stress tensor σ_{ij} is symmetrical, i.e., $\sigma_{ij} = \sigma_{ji}$, it follows that:

$$C_{ijkl} = C_{jikl} . \quad (2.6)$$

This condition lowers the number of independent components in C_{ijkl} from 81 to 54. Similarly, from the assumptions of a symmetrical strain tensor, i.e., $\varepsilon_{kl} = \varepsilon_{lk}$, we see:

$$C_{ijkl} = C_{ijlk} . \quad (2.7)$$

This condition reduces the number of independent stiffness components to 36. A third symmetry of the stiffness tensor must also hold:

$$C_{ijkl} = C_{klij} . \quad (2.8)$$

To prove equation 2.8, we consider the derivative of equation 2.4 with respect to strain:

$$\frac{\partial W}{\partial \varepsilon_{ij}} = \frac{1}{2} C_{ijkl} \epsilon_{kl} . \quad (2.9)$$

Then:

$$\frac{\partial^2 W}{\partial \varepsilon_{mn} \partial \varepsilon_{ij}} = \frac{\partial}{\partial \varepsilon_{mn}} \left[\frac{1}{2} C_{ijkl} \epsilon_{kl} \right] \quad (2.10)$$

$$= \frac{1}{2} C_{ijkl} \delta_{km} \delta_{ln} \quad (2.11)$$

$$= \frac{1}{2} C_{ijmn} . \quad (2.12)$$

By the symmetry of second derivatives, equation 2.8 therefore holds as:

$$C_{ijkl} = 2 \frac{\partial^2 W}{\partial \varepsilon_{kl} \partial \varepsilon_{ij}} = 2 \frac{\partial^2 W}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}} = C_{klij} \quad (2.13)$$

The number of independent components in C_{ijkl} is reduced to 21 by equation 2.8, and can only be reduced further by imposing extra, specific symmetries on the medium.

With only 21 independent stiffness tensor components, we can express C_{ijkl} in Voigt notation (Auld, 1990), which reduces Equation 2.2 to a vector-matrix expression by the following mappings:

$$\sigma_1 = \sigma_{11} ; \quad (2.14)$$

$$\sigma_2 = \sigma_{22} ; \quad (2.15)$$

$$\sigma_3 = \sigma_{33} ; \quad (2.16)$$

$$\sigma_4 = \sigma_{23} ; \quad (2.17)$$

$$\sigma_5 = \sigma_{13} ; \quad (2.18)$$

$$\sigma_6 = \sigma_{12} ; \quad (2.19)$$

and:

$$\varepsilon_1 = \varepsilon_{11} ; \quad (2.20)$$

$$\varepsilon_2 = \varepsilon_{22} ; \quad (2.21)$$

$$\varepsilon_3 = \varepsilon_{33} ; \quad (2.22)$$

$$\varepsilon_4 = 2\varepsilon_{23} ; \quad (2.23)$$

$$\varepsilon_5 = 2\varepsilon_{31} ; \quad (2.24)$$

$$\varepsilon_6 = 2\varepsilon_{12} . \quad (2.25)$$

The 21 independent components of the stiffness tensor can thus be represented in a symmetric 6×6 matrix, yielding the following representation of Hooke's law:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}; \quad (2.26)$$

or compactly:

$$\sigma_I = C_{IJ}\varepsilon_J, \quad I, J = 1, \dots, 6. \quad (2.27)$$

2.1.2 Symmetries of the stiffness tensor

Many linearly elastic materials exhibit a certain level of symmetry in their elastic properties. An isotropic material is the simplest case, having only two independent stiffness parameters and being completely described by the stiffness tensor (e.g., Christensen (1979)):

$$\begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(C_{11} - C_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(C_{11} - C_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(C_{11} - C_{12}) \end{bmatrix}; \quad (2.28)$$

which can also be written in the form:

$$\begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu \end{bmatrix}, \quad (2.29)$$

where λ and μ are the Lamé parameters.

The isotropic Hooke's law can accordingly be expressed in Einstein notation as:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}. \quad (2.30)$$

The next most simple form of symmetry is called cubic symmetry. In the case that the material's symmetry axes and the coordinate system's axes are aligned, the stiffness tensor of a material with cubic symmetry can be written in terms of three independent constants:

$$\begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix}. \quad (2.31)$$

Cubic symmetry, with three independent elastic constants, is often relevant to

fields such as crystallography and petrology but is much less commonly observed in geophysical data (Thomsen, 2014).

By lowering the symmetry in medium's stiffness tensor further, we obtain hexagonal or transverse isotropic symmetry (TI). The transverse isotropic stiffness tensor has five independent elastic components and is represented in Voigt notation as:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(C_{11} - C_{12}) \end{bmatrix}; \quad (2.32)$$

when the medium's symmetry axis aligns with an axis of the coordinate system.

A medium with hexagonal symmetry has a single symmetry plane with constant elastic properties in any direction parallel to this plane. Rocks with parallel layering or fractures often exhibit TI symmetry (Tsvankin, 2001). Shales can also exhibit TI symmetry (Sayers, 1994).

A material with vertical transverse isotropy (VTI) has its symmetry axis pointing vertical. Similarly, a horizontal transverse isotropy (HTI) medium has its symmetry axis pointing horizontal, and a tilted transverse isotropic medium has its symmetry axis pointing in any other direction.

Lowering the symmetry of the stiffness tensor further, we consider the case where a material possesses 3 orthogonal axes of symmetry with unique lengths, called orthotropic or orthorhombic symmetry (Schoenberg and Sayers, 1995).

Orthorhombic symmetry can occur in rocks with orthogonally oriented parallel fractures as well as parallel layering, or in rocks with multiple fracture sets which are orthogonally oriented. The stiffness tensor for orthorhombic symmetry has 9 independent components:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}. \quad (2.33)$$

A material exhibiting monoclinic symmetry possesses three symmetry axes of unique length, where two of three angles between axes are right angles. The monoclinic stiffness tensor has 13 independent stiffness tensor components and can be associated geophysically with multiply fractured, layered rocks where fracture sets are not orthogonal to one another but perpendicular to layering. The lowest level of elastic symmetry, triclinic symmetry, has 21 independent stiffness tensor components and three non-orthogonal symmetry axes.

2.1.3 The Christoffel equation

The Christoffel equation, derived by substituting a trial plane wave solution into the generalised linear elastic wave equation, provides the relationship between a plane wave's polarisation direction, propagation direction, the host medium's density, and its anisotropic slowness (Musgrave, 1970). The Christoffel equation can be used to estimate a rock's seismic anisotropy, as is done in Chapter 8.

To derive the Christoffel equation, we consider the generalised anisotropic wave equation:

$$C_{ijkl}u_{k,lj} = \rho\ddot{u}_i; \quad (2.34)$$

where $u_{k,lj}$ is the second spatial derivative of displacement u_k , ρ is material density, and $\ddot{u}_i$ is displacement's second time derivative. The plane wave solution to equation 2.34 can be written in the form:

$$u_k = Ap_k e^{i\omega(s_r x_r - t)}; \quad (2.35)$$

where A is the wave amplitude, p_k is the polarisation vector, ω is the angular frequency, x_r is the element's position vector in a 3-dimensional Cartesian coordinate system, s_r is the wave's phase slowness vector, and t is time.

The values A and ω are scalars in equation 2.35 for a homogeneous medium. Also, we assume p_k and s_k are independent of time and space. Evaluating second time and space derivatives, we find:

$$\ddot{u}_i = -\omega^2 Ap_i e^{i\omega(s_r x_r - t)}; \quad (2.36)$$

and:

$$u_{k,lj} = -\omega^2 s_l s_j Ap_k e^{i\omega(s_r x_r - t)}. \quad (2.37)$$

Substituting these derivatives into equation 2.34 and recognising $p_i = \delta_{ik}p_k$, we arrive at the Christoffel equation:

$$[C_{ijkl}s_l s_j - \rho\delta_{ik}]p_k = 0. \quad (2.38)$$

Equation 2.38 is an eigenvector-eigenvalue problem with three eigenvalues, found by solving:

$$|C_{ijkl}s_l s_j - \rho\delta_{ik}| = 0. \quad (2.39)$$

If we define phase velocity V as the magnitude of the reciprocal of phase slowness s_r . Then:

$$s_i s_j = \frac{n_i n_j}{V^2}; \quad (2.40)$$

where n_r is the unit vector in the direction of phase velocity propagation. With equation 2.40, we can rewrite equation 2.38 as:

$$|C_{ijkl}n_l n_j - \rho V^2 \delta_{ik}| = 0. \quad (2.41)$$

The characteristic equation of this matrix is cubic in ρV^2 and possesses up to three unique eigenvalues. The matrix $C_{ijkl}n_l n_j$ is a symmetric and non-negative matrix, and thus possesses real, orthogonal eigenvectors.

2.2 Elastic Effective Medium Theory

2.2.1 Eshelby's ellipsoid problem

The effect on a medium's gross elastic field due to the presence of inhomogeneities is of interest in many physical sciences, including engineering (Aggelis and Shiotani, 2008), materials science (Aboudi, 1991), and geosciences (Mavko *et al.*, 2009). General ellipsoids, with up to three unique principal axis lengths, are commonly used in elastic modelling to approximate the presence of inhomogeneities in a medium (Aboudi, 1991; Mura, 2013). This approximation of a material's inhomogeneities by ellipsoidal inclusions when calculating a medium's average elastic properties was made possible by the results of Eshelby (1957), who demonstrated the effective elastic field of an ellipsoidal inhomogeneity embedded in an infinite background material possessing different elastic properties was analytically solvable. In this section, we outline Eshelby's method and review the key results which underpin much of this thesis.

The ultimate aim of the work of Eshelby (1957) is to calculate the effective elastic field of a homogeneous background material dilutely embedded with one or more inhomogeneities which possess a different stress state under a constant, applied strain field around the material's boundary at infinity. This abstract scenario is, in practice, similar to many mechanical laboratory experiments, obviating the wide-reaching applications and employment of the Eshelby model.

To solve this inhomogeneity problem, Eshelby first addresses a similar problem known as the inclusion problem. The problem, as proposed by Eshelby, is:

“A region (the ‘inclusion’) in an infinite homogeneous isotropic elastic medium undergoes a change of shape and size which, but for

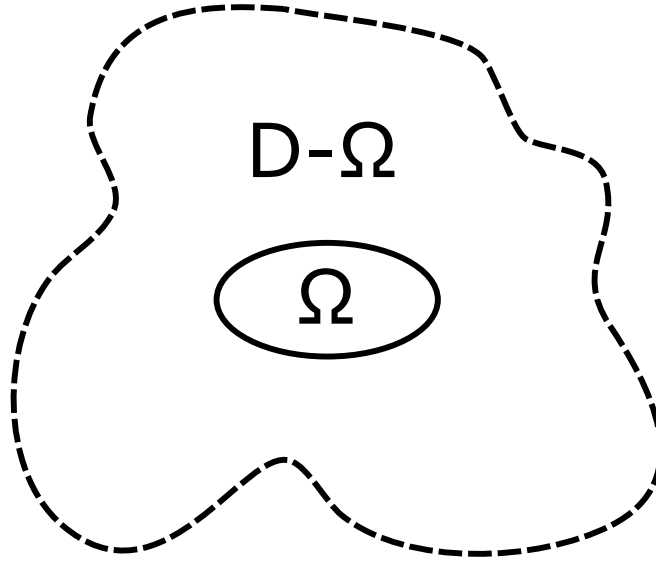


Figure 2.1: The Eshelby inclusion model considers an infinite elastic material with a single ellipsoidal inclusion embedded within it. The entire material is represented here by domain D , while the inclusion is subdomain Ω and the matrix material is the complimentary subdomain $D - \Omega$.

the constraint imposed by its surroundings (the ‘matrix’), would be an arbitrary homogeneous strain. What is the elastic state of inclusion and matrix?” (J. D. Eshelby, 1957)

Following the notation of Mura (2013), we define an inclusion as a subdomain Ω in D . The complementary subdomain $D - \Omega$ is then referred to as the matrix. This partition of an infinite medium is schematically demonstrated in Figure 2.1. The elastic moduli in the inclusion and matrix are the same. In this inclusion problem, as opposed to the subsequent inhomogeneity problem to be addressed, the matrix is stress-free at infinity. The eigenstrain or ‘stress-free-strain’, $\epsilon_{ij}^*(x)$, is the transformation strain an inclusion under stress would undergo in the absence of its constraining matrix.

To solve the inclusion problem, Eshelby considers an infinite body of homogeneous matrix material with zero stress throughout. Inclusion Ω is then arbitrarily

defined and cut out, removing its contents from the matrix and leaving a void. An unconstrained transformation is then applied to the inclusion. That is, the inclusion undergoes a stress-free strain $\epsilon_{ij}^*(x)$. After this transformation, the necessary surface tractions are applied which perfectly reshape the inclusion to fit into its previously created void. The surface tractions $-\sigma_{ij}^* n_j$, where n_j is the outward normal vector on the surface of Ω , are defined through the eigenstress σ_{ij}^* , related to the eigenstrain in an isotropic, homogeneous medium by:

$$\sigma_{ij}^* = \lambda \epsilon_{ij}^* \delta_{ij} + 2\mu \epsilon_{ij}^* ; \quad (2.42)$$

where λ and μ are the material's Lamé parameters.

The stressed inclusion is placed into its void and re-joined along the cut. At this point, the matrix has zero stress while the inclusion has a known, constant stress. Finally, the inclusion is allowed to relax to equilibrium within the matrix, which in practice is performed by applying an equal and opposite layer of body forces $+\sigma_{ij}^* n_j$ over the boundary of Ω to cancel out the previously applied surface tractions.

Eshelby uncovers four key results for an arbitrarily shaped inclusion in the above described thought experiment. Eshelby determines: the elastic field far from the inclusion; the stresses and strains at a point immediately outside the inclusion; the matrix and inclusion's total strain energy; and the interaction energy of the inclusion's elastic field with another elastic field. Of these results, the matrix and inclusion's total strain energy is undoubtedly the most valuable in the field of rock physics, as we will see it can be used to calculate a composite material's effective elastic moduli. Eshelby finds the total strain energy in matrix and inclusion for an arbitrary inclusion shape to be:

$$E = -\frac{1}{2} \int_V \sigma_{ij}^I \epsilon_{ij}^* dv; \quad (2.43)$$

where V is the volume of inclusion Ω , ϵ_{ij}^T is the strain of the inclusion in its stress-free state after expansion outwith the medium, and σ_{ij}^I is the inclusion's stress which is calculated as the difference in its constrained stress and eigenstress states.

We now consider the case when the inclusion is an ellipsoid and Ω is defined by:

$$\mathbf{x} \in \Omega \quad \text{if} \quad \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \frac{x_3^2}{a_3^2} \leq 1; \quad (2.44)$$

where a_i , $i \in \{1, 2, 3\}$, are the ellipsoid's semi-major axes, and a_i/a_j are the ellipsoid's aspect ratios for $i \neq j$ and $j \in \{1, 2, 3\}$.

For an ellipsoidal inclusion embedded in a homogeneous, infinite matrix, as prescribed above, Eshelby found:

$$\epsilon_{ij} = S_{ijkl} \epsilon_{kl}^*, \quad \text{for } x \in \Omega \quad (2.45)$$

where ϵ_{ij} is the inclusion's constrained strain, and S_{ijkl} is known as the Eshelby tensor. As the theory is derived with constant eigenstrain ϵ_{kl}^* , we see the constrained strain is also constant throughout the ellipsoid. The Eshelby tensor S_{ijkl} is a function of the medium's Poisson's ratio and coefficients I_a , I_b , and I_c , which form the expression for the Newtonian potential Ψ within an ellipsoid of unit density:

$$\Psi = \frac{1}{2} (a_1^2 - x_1^2) I_a + \frac{1}{2} (a_2^2 - x_2^2) I_b + \frac{1}{2} (a_3^2 - x_3^2) I_c. \quad (2.46)$$

Coefficients I_a , I_b , and I_c are dependent on the ellipsoid's aspect ratios, and are defined explicitly by Kellogg and Courant (2013), Osborn (1945), and Eshelby (1957).

In the case of ellipsoidal inclusions, equation 2.43 becomes:

$$E = -\frac{1}{2} V \sigma_{ij}^I \epsilon_{ij}^*, \quad V = \frac{4}{3} \pi a_1 a_2 a_3; \quad (2.47)$$

thus, finding the total elastic energy in an ellipsoidal inclusion and matrix. After analysis of the ellipsoidal inclusion problem, Eshelby then approaches the ellipsoidal inhomogeneity problem, posing the following question:

“An ellipsoidal region in a solid has elastic constants differing from those of the remainder (if, in particular, the constants are zero within the ellipsoid we have the case of a cavity). How is an applied stress, uniform at large distances, disturbed by this inhomogeneity?” (J. D. Eshelby, 1957)

In addressing this problem, Eshelby obtains multiple distinctly important results: the elastic field far from an inhomogeneity; the stresses and strains at a point immediately outside the inhomogeneity; and the interaction of the inhomogeneity's elastic field with another elastic field.

To derive these inhomogeneity-related results, we replace the contents of ellipsoidal inclusion Ω with another material possessing different elastic moduli. We then say the composite material possesses stiffness tensors C_{ijkl}^* in Ω and C_{ijkl}

in $D - \Omega$. We also apply a homogeneous stress σ_{ij}^A upon the matrix at infinity. Eshelby reasoned that the perturbation in the originally homogeneous stress field due to the presence of an ellipsoidal inhomogeneity can be found by solving the ellipsoidal inclusion problem with a specially chosen eigenstrain ϵ_{ij}^* , which is found as the solution to the equation (Mura, 2013):

$$(\Delta C_{ijkl} S_{klmn} - C_{ijmn}) \epsilon_{mn}^* = -\Delta C_{ijkl} \epsilon_{kl}^A - C_{ijkl}^* \epsilon_{kl}^P; \quad (2.48)$$

where ϵ_{kl}^A is defined through $\sigma_{kl}^A = C_{ijkl} \epsilon_{kl}^A$, ϵ_{kl}^P is some arbitrary, initial eigenstrain, and $\Delta C_{ijkl} = C_{ijkl} - C_{ijkl}^*$.

Mura (2013), after Eshelby, then shows the stress and strain fields within the inhomogeneity Ω are:

$$\epsilon_{ij} = \epsilon_{ij}^A + S_{ijkl} \epsilon_{mn}^*; \quad (2.49)$$

$$\sigma_{ij} = \sigma_{ij}^A + C_{ijkl} (S_{klmn} \epsilon_{mn}^* - \epsilon_{kl}^*). \quad (2.50)$$

Similarly, the stress and strain fields at point $\mathbf{x}$ in the matrix $D - \Omega$, given a coordinate system centred in the ellipsoid's centre, is:

$$\epsilon_{ij}(\mathbf{x}) = \epsilon_{ij}^A + D_{ijkl}(\mathbf{x}) \epsilon_{mn}^*; \quad (2.51)$$

$$\sigma_{ij}(\mathbf{x}) = \sigma_{ij}^A + C_{ijkl} D_{klmn}(\mathbf{x}) \epsilon_{mn}^*; \quad (2.52)$$

where Mura (2013) defines D_{ijkl} analogously to S_{ijkl} , such that:

$$\epsilon_{ij}(\mathbf{x}) = D_{ijkl}(\mathbf{x})\epsilon_{kl}^* . \quad (2.53)$$

The tensor $D_{ijkl}(\mathbf{x})$ therefore relates the strain at any point $\mathbf{x}$ in the matrix to the embedded inclusion's eigenstrain. We note $D_{ijkl}(\mathbf{x})$ is dependent on position as $\epsilon_{ij}(\mathbf{x}) \rightarrow 0$ when $|\mathbf{x}| \rightarrow \infty$ to satisfy the problem's stress and strain boundary conditions.

Once stresses and strains inside and outside an inhomogeneity are formulated in terms of an equivalent ellipsoidal inclusion problem, the material's total elastic energy is calculated by equation 2.47.

The final result of Eshelby, which is arguably of the most practical importance in rock physics, is an expression of:

“... The change in the gross elastic constants of a material when a dilute dispersion of ellipsoidal inhomogeneities is introduced into it.”
(J. D. Eshelby, 1957)

To find this, Eshelby defines ΔE_{int} as the interaction energy of the applied stress and the inhomogeneity, and in the case of a single ellipsoidal inhomogeneity, finds it to be:

$$\Delta E_{int} = -\frac{1}{2}V\sigma_{ij}^A\epsilon_{ij}^* ; \quad (2.54)$$

where V is the volume of the ellipsoidal inhomogeneity.

Consider the total elastic energy E_0 of a medium which is initially homogeneous under surface tractions that produce a uniform stress σ_{mn}^A throughout.

Eshelby argues using prior results (Eshelby, 1951, 1956) this energy augments by $-\Delta E_{int}(\sigma_{mn}^A)$ when a dispersion of inhomogeneities is added. That is, Eshelby shows a material's total elastic energy E , given a dilute, uniformly dispersed collection of inhomogeneities, is:

$$E = E_0 - \Delta E_{int}(\sigma_{mn}^A). \quad (2.55)$$

Finally, the effective elastic moduli of the composite material are calculated by recognising:

$$E = \frac{1}{2} s_{ijkl} \sigma_{ij} \sigma_{kl}; \quad (2.56)$$

for compliance tensor $s_{ijkl} = C_{ijkl}^{-1}$. It thus follows that:

$$\frac{1}{2} s_{ijkl} \sigma_{ij} \sigma_{kl} = E_0 - \Delta E_{int}(\sigma_{mn}^A). \quad (2.57)$$

Therefore, given a known dispersion of arbitrarily oriented ellipsoidal inhomogeneities with a given aspect ratio, in a homogeneous background material under a prescribed, constant strain at infinity, we have the means to calculate the material's effective compliance and hence stiffness tensor through consideration of interaction energies resultant from the embedded ellipsoidal inclusions. Alternatively, given measurements of stiffness components, and knowledge of matrix and inhomogeneity composition, as well as stress states, one can solve for the ellipsoidal aspect ratio parameters, although assumptions may need to be made about orientation and the number of unique ellipsoidal semi-major axes to ensure a unique solution.

A body of literature concerning inclusion-based effective medium modelling has built upon Eshelby's fundamental results in the modelling of elastic composites. Although varied in their implementation, the methods and theories within this collection of literature all fundamentally aim to do one thing: calculate the effective properties of a composite material consisting of ellipsoidal inhomogeneities embedded in matrix material.

We now chronologically address the development of inclusion modelling up to the present day, at which point Chapters 4, 5, and 6 extend the field by presenting new theoretical and applied developments.

2.2.2 Wu's T tensor and self-consistent model

It was the work of Wu (1966) which made the Eshelby inclusion model more accessible by explicitly relating the strain field ϵ_{ij} within an ellipsoidal inclusion to the uniform strain field ϵ_{kl}^A applied at infinity through the linear relationship:

$$\epsilon_{ij} = T_{ijkl} \epsilon_{kl}^A; \quad (2.58)$$

where T_{ijkl} is sometimes referred to as the “Wu T tensor”. The usefulness of this relationship is that it relates the strain in the inclusion to an experimentally measurable quantity; the prescribed, low-frequency strain field at a medium's boundary.

To arrive at equation 2.58, Wu (1966) substitutes the expression of Eshelby (1957) for the strain field in a composite containing an ellipsoidal inclusion into a previously derived expression (Wu, 1965) for the wavefield in a composite material with a small volume of arbitrarily shaped inclusions, obtaining the expression:

$$\epsilon_{ij} = \epsilon_{ij}^A + \frac{1}{2\mu_1} (A\epsilon_{lk} + B\delta_{lk}\epsilon_{qq}) \left(S_{ijkl} - \frac{\nu_1}{1 + \nu_1} \delta_{lk} S_{ijpp} \right); \quad (2.59)$$

where A is a function of matrix and inclusion shear modulus; B is a function of matrix and inclusion shear modulus and Poisson's ratio; μ_1 is the matrix shear modulus; and ν_1 is the matrix Poisson's ratio. Solving equation 2.59 yields the resultant equation 2.58. The components of the Wu T tensor are transcribed by Wu (1966) and Berryman (1980b), and omitted here.

In the same paper, Wu (1966) proposed a method of calculating the effective elastic moduli of a material comprised of a dispersion of Eshelby's ellipsoids throughout a homogeneous elastic background. Wu calculates the total strain energy in the material as a function of strain ϵ_{ij} throughout the volume of material as:

$$E = \frac{1}{2} \int_V \sigma_{ij} \epsilon_{ij} dV; \quad (2.60)$$

where σ_{ij} and ϵ_{ij} are the stress and strain respectively at any location in the volume V .

However, the total strain energy in the material of volume V is also given by:

$$E = \frac{1}{2} \sigma_{ij}^0 \bar{L}_0 \left(\sigma_{ij}^0 \right) V; \quad (2.61)$$

where $\bar{L}_0$ is the composite material's "linear Hookean operator" (i.e. the composite's effective compliance tensor).

The key assumption which underpins Wu's "self-consistent" model is that the

composite's stress σ_{ij}^0 and strain ϵ_{ij}^0 at infinity are related by Hooke's law through the matrix material's linear Hookean operator $\bar{L}_1$, as opposed to the composites operator $\bar{L}_0$. That is, Wu makes the assumption:

$$\epsilon_{ij}^0 = \bar{L}_1 \left(\sigma_{ij}^0 \right), \quad (2.62)$$

which is demonstrated schematically in Figure 2.2.

With the self-consistent assumption (equation 2.62), Wu equates equations 2.60 and 2.61 and finds the implicit expressions for the composite's effective bulk and shear moduli:

$$\frac{1}{K_0} = \frac{1}{K_1} \left[1 + \frac{c}{3} T_{ijj} \left(\frac{K_1 - K_2}{K_0} \right) \right]; \quad (2.63)$$

and

$$\frac{1}{G_0} = \frac{1}{G_1} \left[1 + \frac{c}{15} (3T_{ijij} - T_{ijj}) \left(\frac{G_1 - G_2}{G_0} \right) \right]; \quad (2.64)$$

where K and G are bulk and shear moduli respectively, c is the concentration of inclusions, and subscripts 1, 2, and 0, represent the matrix, inclusion, and composite materials respectively. Equations 2.63 and 2.64 are coupled through T_{ijij} and T_{ijj} .

Wu (1966) then evaluates the T tensor and hence equations 2.63 and 2.64 for specific ellipsoidal forms. For simplicity, only spheroids are considered, having at least two equal primary axis lengths and thus a single aspect ratio α which

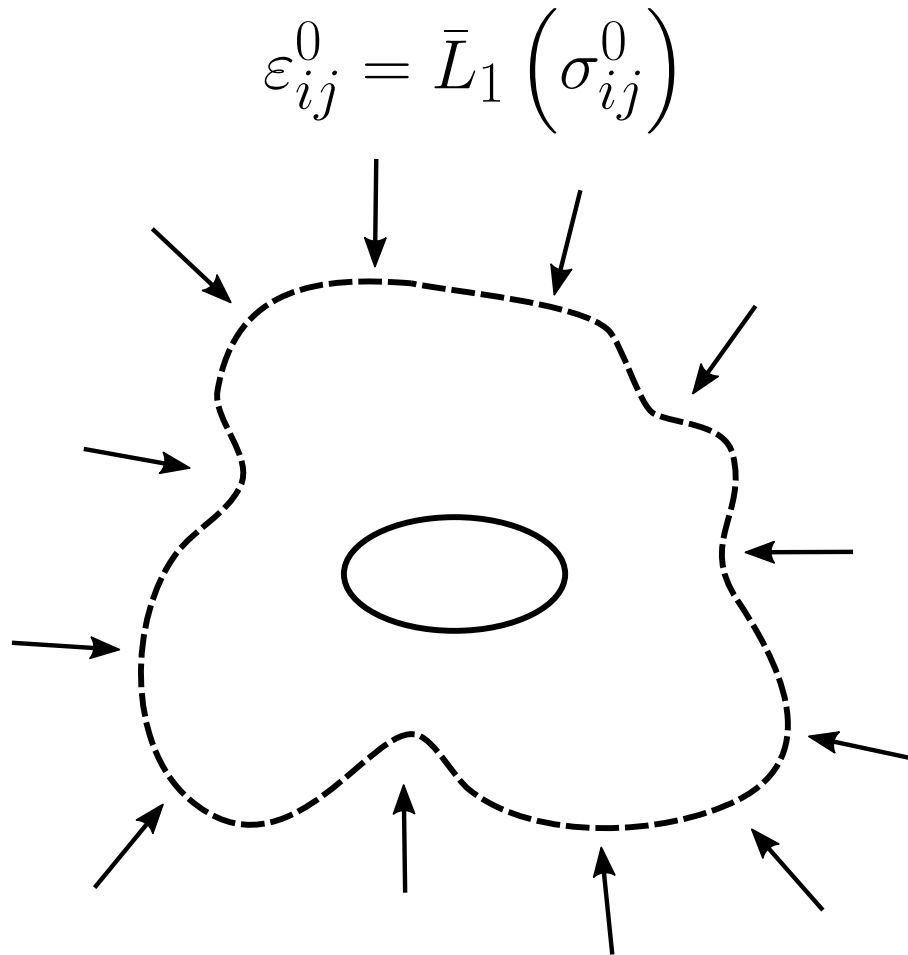


Figure 2.2: The Wu self-consistent assumption is that the composite's stress and strain at infinity are related by the linear Hookean operator (compliance tensor) of the matrix material. This is an approximation to the true relationship, which would require the linear Hookean operator of the composite material.

completely describes the inclusion's geometry. Wu calculates the T tensor for spherical ($\alpha = 1$), needle shaped ($\alpha \gg 1$) and disk shaped ($0 < \alpha \ll 1$) inclusions.

Other popular self-consistent models contemporary with that of Wu are the models of Budiansky (1965) and Hill (1965). The similarity between these models is that the effective elastic moduli are calculated in some sense through the replacement of the matrix material in Eshelby's thought experiment with the yet-to-be-determined, effective composite material itself.

2.2.3 Mori-Tanaka model

The most popular inclusion models published in the 1950s and 1960s modelled the elastic properties of a composite by arguments centred on its total or average elastic energy. The model of Mori and Tanaka (1973), however, was derived with a different approach.

Mori and Tanaka (1973) constructed an effective medium with effective elastic stiffness tensor $\mathbf{L}^*$ by embedding disperse, arbitrarily aligned, ellipsoidal inclusions of n phases into a background of matrix material. They consider a scenario where, for every phase $i = 0, 1, 2, \dots, n$, the anisotropic Hooke's law is obeyed. That is:

$$\sigma_i = \mathbf{L}_i \epsilon_i ; \quad (2.65)$$

where index $i = 0$ represents matrix material, and $\mathbf{L}_i$, σ_i , and ϵ_i , are the stiffness, anisotropic stress, and anisotropic strain tensors within the i^{th} phase, respectively. The average strain $\bar{\epsilon}_i$ in phase i is then related to that in the matrix $\bar{\epsilon}_0$ by the result of Wu (1966):

$$\bar{\epsilon}_i = \bar{\mathbf{T}}_{i0} \bar{\epsilon}_0 ; \quad (2.66)$$

where $\bar{\mathbf{T}}_{i0}$ is some average of the Wu T tensor for all inclusions of the i^{th} phase embedded in matrix material. Invoking the approximation of dilute inclusions, Mori and Tanaka (1973) let:

$$\bar{\mathbf{T}}_{i0} = \mathbf{T}_{i0} ; \quad (2.67)$$

where $\mathbf{T}_{i0}$ is the stiffness tensor defined in equation 2.58. It has been shown (E.g., see Mura (2013)) for aligned, ellipsoidal inclusions, that:

$$\mathbf{T}_{i0} = [\mathbf{I} + \mathbf{S}_{i0} \mathbf{L}_0^{-1} (\mathbf{L}_i - \mathbf{L}_0)]^{-1} ; \quad (2.68)$$

where $\mathbf{S}_{i0}$ is the Eshelby tensor for inclusions of phase i given matrix stiffness tensor $\mathbf{L}_0$, inclusion stiffness tensor $\mathbf{L}_i$, and phase i 's ellipsoid aspect ratios, while $\mathbf{I}$ is the rank-4 identity tensor.

Mori and Tanaka then formalise the following concept: Under a constant strain ϵ^A applied at infinity, as per the Eshelby thought experiment, introducing an ellipsoidal inclusion into a homogeneous matrix with average strain $\bar{\epsilon}_0$ leads to the strain within the inclusion, ϵ_{I_1} , being defined by Wu's result (equation 2.58):

$$\epsilon_{I_1} = \mathbf{T} \bar{\epsilon}_0 . \quad (2.69)$$

The matrix is then deformed by this process and has the average strain $\bar{\epsilon}_{I_1}$. Thus,

introducing a new inclusion into this now-deformed matrix will result in this new inclusion's strain $\bar{\epsilon}_{I_2}$ being given by:

$$\epsilon_{I_2} = \mathbf{T} \bar{\epsilon}_{I_1}. \quad (2.70)$$

Generalising equation 2.70 so ϵ_{I_2} is the strain in any given inclusion, and $\bar{\epsilon}_{I_1}$ is the average strain due to all other inclusions embedded in the background material, Mori and Tanaka (1973) estimate the elastic state in the composite. Although the dilute inclusion approximation is utilised in the Mori-Tanaka model, this concept shows its advantage on pre-existing models, where inclusions are elastically uninteracting with one-another. By formalising this idea, Mori and Tanaka derive the effective stiffness tensor $\mathbf{L}^*$, which we show here in the rederived form of Benveniste (1987), as:

$$\mathbf{L}^* = \mathbf{L}_1 + c \{ (\mathbf{L}_2 - \mathbf{L}_1) \mathbf{T} \} [(1 - c) \mathbf{I} + c \{ \mathbf{T} \}]^{-1}; \quad (2.71)$$

where curly brackets denote averaging over all inclusion orientations. Benveniste (1987) also presented an explicit form of the Mori-Tanaka model's effective compliance tensor and proved it to be the inverse of the Mori-Tanaka effective stiffness tensor.

2.2.4 Kuster-Toksöz model

We have now reviewed ellipsoidal inclusions models which consider the average energy and average strain of a composite to calculate its effective moduli. A markedly different approach to estimating the effective moduli of a composite

with ellipsoidal inclusions was presented by Kuster and Toksöz (1974), which was derived using scattering theory.

The model of Kuster and Toksöz (1974) is built on the foundational work of Mal and Knopoff (1967), who derived integral equations for the displacement field caused by low-frequency incident waves scattered by an isolated inclusion of arbitrary shape in an infinite, homogeneous matrix. Combining this result with that of Eshelby (1957), Kuster and Toksöz (1974) calculated the first-order scattered displacement field in an infinite matrix due to the presence of an ellipsoidal inclusion.

With this scattered field calculated, the thought experiment is posed: Consider a finite, representative sphere V_0 embedded within an infinite matrix. Within V_0 , N inclusions are randomly embedded dilutely such that multiple scattering effects are negligible under very long incident wavelengths, as seen in Figure 2.3. Let $\mathbf{x}_0$ denote the centre of V_0 , $\mathbf{x}_s$ represent the centre of the s^{th} scattering inclusion, and $\mathbf{x}$ represent an arbitrary position in the infinite medium. It follows to first order scattering that the total observed displacement $\mathbf{u}(\mathbf{x})$ at position $\mathbf{x}$ is a superposition of the displacement $\mathbf{u}^0(\mathbf{x})$ due to the incident wavefield, and all the displacements $\mathbf{u}^s(\mathbf{x}, \mathbf{x}_s)$ caused by all scatterers centred at locations $\mathbf{x}_s$ for $s \in 1, \dots, N$. That is:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}^0(\mathbf{x}) + \sum_{s=1}^N \mathbf{u}^s(\mathbf{x}, \mathbf{x}_s). \quad (2.72)$$

Given the scale of the incidence wavelength is significantly larger than the representative sphere V_0 , the sphere V_0 can itself be seen as a single spherical inhomogeneity embedded within an infinite body of matrix material. As the region V_0 behaves as a single homogeneous scatterer, its effective, homogeneous properties must be some average of its constituent embedded ellipsoidal inclusions

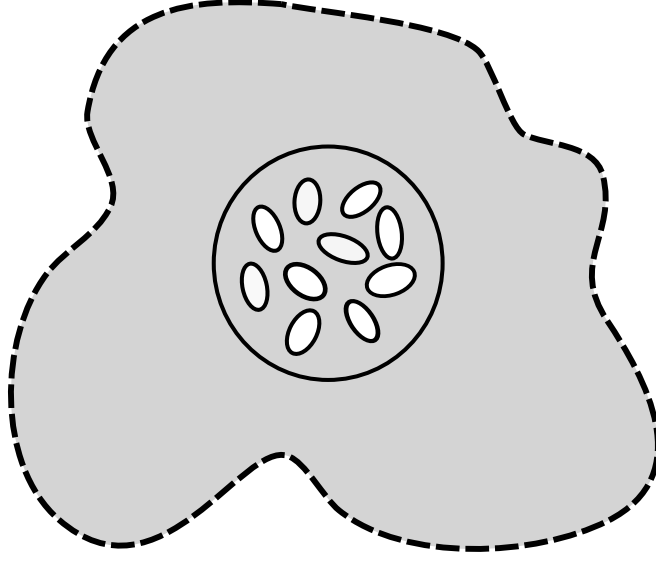


Figure 2.3: The Kuster-Toksöz model considers an infinite linearly elastic material containing a spherical region which itself contains many dilutely dispersed, ellipsoidal inclusions. In the low frequency limit, the sphere scatters the incident wavefield to first order as if it were a single inclusion with effective elastic properties determined by its constituents. Similarly, each constituent ellipsoid within the spherical region also scatters the incident wavefield individually, and the sum of their scattered wavefields indicates the total scattered wavefield in the medium. Equating these effective and total scattered wavefields leads to estimation of the material's effective elastic moduli.

and matrix materials, and would depend upon inclusion concentration and aspect ratios, following the results of Eshelby (1957).

If we then say the scattered displacement field measured at location $\mathbf{x}$ due to the effective inhomogeneity V_0 centred at $\mathbf{x}_0$ is $\mathbf{u}^*(\mathbf{x}, \mathbf{x}_0)$, the total displacement field throughout the medium can be written as:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}^0(\mathbf{x}) + \mathbf{u}^*(\mathbf{x}, \mathbf{x}_0). \quad (2.73)$$

Equating equations 2.72 and 2.73, Kuster and Toksöz (1974) obtain the fundamental equation which defines their effective medium:

$$\mathbf{u}^*(\mathbf{x}, \mathbf{x}_0) = \sum_{s=1}^N \mathbf{u}^s(\mathbf{x}, \mathbf{x}_s). \quad (2.74)$$

By assuming that query point $\mathbf{x}$ is so far from V_0 that $\mathbf{x}_s \approx \mathbf{x}_0$ for all $s \in 1 \dots N$, the need to understand the spatial distribution of inclusions is removed. The resultant expression for effective bulk modulus K^* in terms of matrix modulus K_1 and ellipsoidal inclusion modulus K_2 is then solved as:

$$\frac{K^* - K_1}{3K^* + 4\mu_1} = c \frac{K_2 - K_1}{3K_1 + 4\mu_1} \frac{1}{3} T_{ijjj}. \quad (2.75)$$

Similarly, the Kuster-Toksöz expression for effective shear modulus μ^* is:

$$\frac{\mu^* - \mu}{6\mu^* (K_1 + 2\mu_1) + \mu_1 (9K_1 + 8\mu_1)} = \frac{c (\mu_2 - \mu_1)}{25\mu_1 (3K_1 + 4\mu_1)} [T_{ijjj} - T_{iijj}]. \quad (2.76)$$

2.2.5 Berryman's self-consistent model

Following the work of Budiansky (1965), Hill (1965), and Wu (1966), Berryman (1980a,b) proposed a self-consistent approximation (SCA) model for spherical and spheroidal inclusions in an infinite matrix background. The key distinction between Berryman's model and the preceding SCA models is that Berryman, in the style of Kuster and Toksöz (1974), derived a self-consistent model by considering low-frequency scattering theory.

Initially considering spherical inclusions, Berryman (1980a) devised a thought

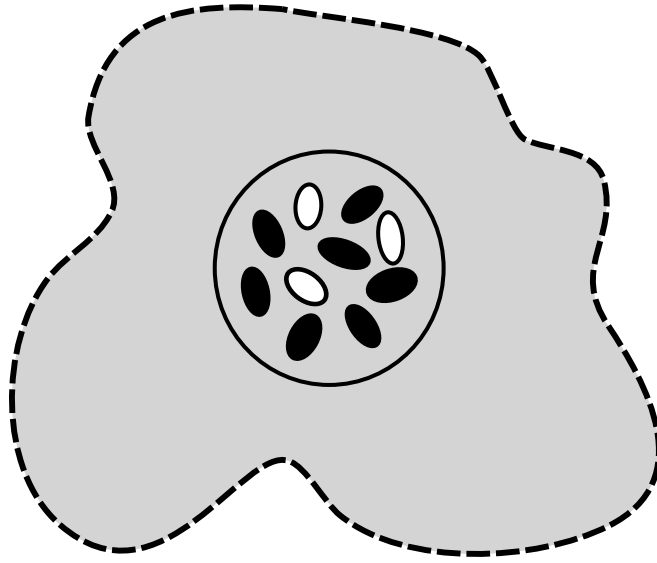


Figure 2.4: Berryman's self-consistent model embeds ellipsoids made of both pore and matrix material into the central spherical region using the same volume fraction of different inclusions as the desired volume fraction of constituents in the material being modelled. The elastic properties of the (third) background phase is then varied until there is no net scattering, at which point the background phase has the same elastic properties as the material's effective elastic properties.

experiment similar to that of Kuster and Toksöz (1974) where a sphere V_0 is embedded in a matrix of infinite, homogeneous material and V_0 itself is embedded with spherical inclusions. Berryman's model, however, lets the elastic moduli of the infinite embedding material within and outwith V_0 vary, while spherical inclusions embedded in V_0 are made of both the desired inclusion and matrix phases, distributed with the same volume fraction as that desired in the modelled composite. This is shown schematically in Figure 2.4. Using the method of Kuster and Toksöz, the first order scattered field due to each spherical inclusion within V_0 can be calculated. The yet-to-be-known effective embedding medium is then varied until the scattering from all embedded spheres vanishes. When this occurs, the spheres are transparent for the low-frequency incident wave and the elastic properties of the infinite matrix are therefore those of the composite.

Berryman (1980a) formulates an expression for the effective elastic moduli in the case of spherical inclusions, before calling upon the results of Eshelby (1957) and Wu (1966) to produce analogous results for ellipsoidal inclusions (Berryman, 1980b). The final, self-consistent expressions for effective elastic moduli, given ellipsoidal inclusions embedded in an infinite matrix, are:

$$\sum_{i=1}^N c_i (\rho_i - \rho^*) = 0, \quad (2.77)$$

$$\sum_{i=1}^N c_i (K_i - K^*) P^{(*i)} = 0, \quad (2.78)$$

$$\sum_{i=1}^N c_i (\mu_i - \mu^*) Q^{(*i)} = 0, \quad (2.79)$$

where subscript i denotes the i^{th} phase in a composite with N distinct phases, c_i is the volume fraction of the i^{th} phase, superscript $*$ indicates the effective medium, ρ signifies density, K signifies bulk modulus, and μ signifies shear modulus. Tensors $Q^{(*i)}$ and $P^{(*i)}$ are functions of the Wu T tensor components:

$$P = \frac{1}{3} T_{iijj}; \quad (2.80)$$

$$Q = \frac{1}{5} \left(T_{ijij} - \frac{1}{3} T_{iijj} \right). \quad (2.81)$$

Superscripts $(*i)$ on tensors $Q^{(*i)}$ and $P^{(*i)}$ in equations 2.78 and 2.79 indicate Wu's T tensor should be evaluated using the composite's effective elastic properties for the background phase and the i^{th} inclusion phase's elastic properties for the inclusion phase. Clearly, tensors $Q^{(*i)}$ and $P^{(*i)}$ are the SCA model's link to

Eshelby's S tensor and hence the calculated effective elastic moduli are dependent on inclusion aspect ratio.

2.2.6 Norris' generalised differential effective medium model

The differential effective medium (DEM) model (Bruggeman, 1935; McLaughlin, 1977; Cleary *et al.*, 1980; Walsh, 1980; Mendelson and Cohen, 1982; Zimmerman, 1991b; Berryman, 1992) is a popular inclusion-based effective medium model for composites which is generally stiffer than the SCA model at high porosities, and is more suitable for rocks with isolated pores in a mostly continuous background matrix (Berge *et al.*, 1993). The DEM model may also perform better than SCA at higher porosities (Roberts and Garboczi, 2000), although this conclusion remains contentious (Berge *et al.*, 1993). Norris (1985) presented a generalisation of pre-existing inclusion models which collapses to the DEM (McLaughlin, 1977) and SCA (Berryman, 1980a,b) models in special cases. Further, Norris (1989) showed the Mori-Tanaka model is also consistent with this generalised DEM model. The model of Norris (1985) is constructed as follows:

Consider an arbitrary, infinite background material, which we generally distinguish as different from the matrix and inclusion material of the composite. Let all phases be linearly elastic solids, and say their stiffness tensors are L_0 , L_1 , and L_2 , for the background, matrix, and inclusion phases respectively. We assume *a priori* that the final, modelled volume fractions of matrix and inclusion material in the composite are known, called c_1 and c_2 respectively. Also, let the volume fraction of the background material be c_0 such that $c_0 + c_1 + c_2 = 1$ at any point during the thought experiment.

To construct the composite, we start with an initial medium entirely composed of

background material and add a volume of ellipsoids composed of both matrix and inclusion material by displacing the same volume of background material. We then homogenise this rock using concepts from the results of Eshelby (1957) (to be discussed shortly), constructing an effective medium which we declare is a new background material. We replace a small amount of this new background material with a small volume of matrix and inclusion material once more. Importantly, the proportion of phases in the removed volume must be the same as the proportion of phases in the entire composite background rock at any stage. This is physically possible by allowing the added inclusions to increase in size by an order of magnitude every iteration of replacement and homogenisation (Norris, 1985). Allow this process to continue *ad infinitum* and the final composite is composed only of matrix and inclusion material with the desired proportion of phases and no background material remaining. A schematic of this process is shown in Figure 2.5.

The final composite's effective elastic properties are dependent on the material's construction path through the (c_1, c_2) phase-plane. Calling the iterative process' arc-length parameter t and referring to it as "time" for convenience, we arbitrarily choose the model boundary conditions $c_1(0) = c_2(0) = 0$.

Reviewing the model of Norris (1985) more closely, we let Δv be the volume of material removed at time t . This volume is replaced by volumes Δv_1 and Δv_2 of phases 1 and 2 such that $\Delta v_1 + \Delta v_2 = \Delta v$. If the invariant, total volume of the composite is V_0 , then the volume of material 1 in the composite after replacement at time t is:

$$V_0 (c_1 + \Delta c_1) = V_0 c_1 \left(1 - \frac{\Delta v}{V_0} \right) + \Delta v_1. \quad (2.82)$$

Hence we have related the added volume of material 1 to its change in volume

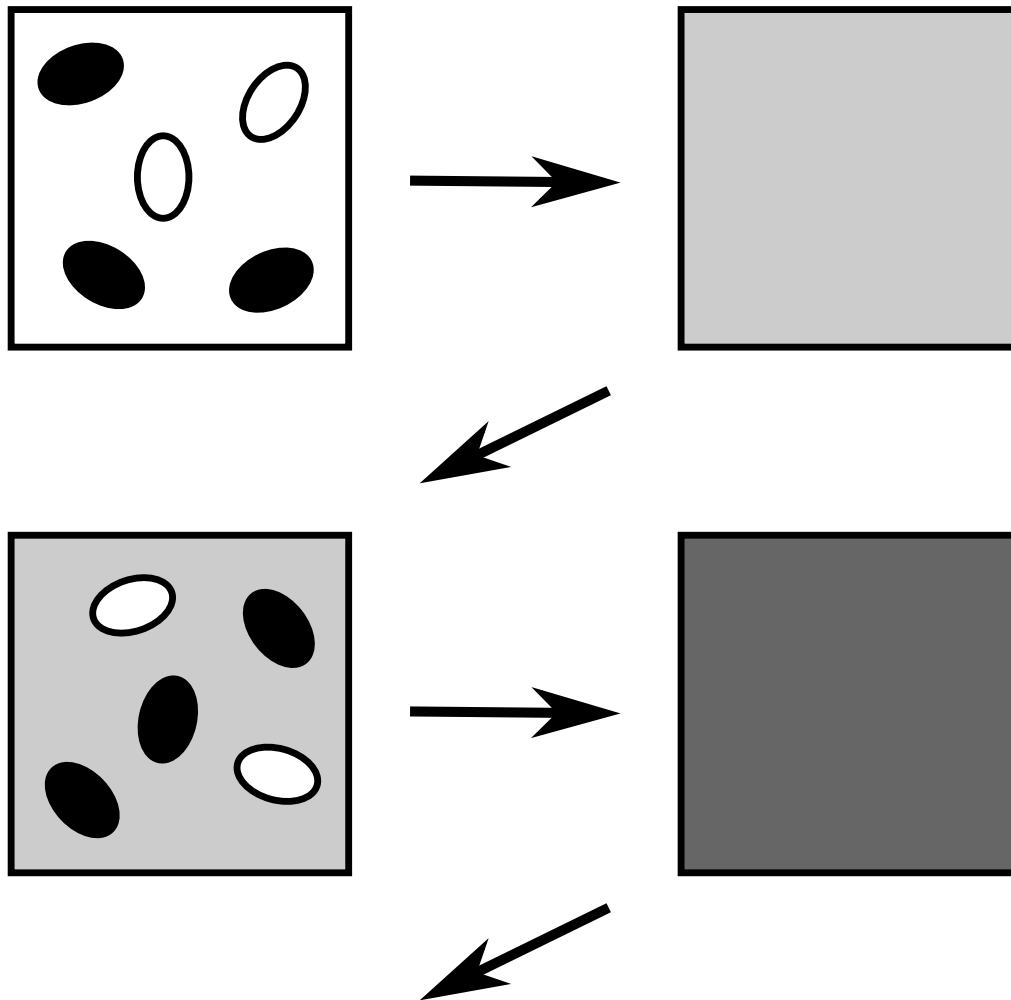


Figure 2.5: Norris' generalised differential effective medium model considers the addition of dilute ellipsoidal inclusions to a background material before setting this composite material's effective elastic properties as the background material for a subsequent iteration of the same steps. Inclusions of both phases can be added into a third background material until the background material is completely removed from the composite. Figure modified from Norris (1985).

fraction Δc_1 . The analogous relationship between v_2 and c_2 also holds. Norris then calculates time-derivatives $\dot{c}_1 = dc_1/dt$, $\dot{c}_2 = dc_2/dt$, $\dot{v}_1 = dv_1/dt$, and $\dot{v}_2 = dv_2/dt$, and shows:

$$\frac{\dot{v}_1}{V_0} = \frac{(1 - c_2) \dot{c}_1 + c_1 \dot{c}_2}{1 - \phi}; \quad (2.83)$$

$$\frac{\dot{v}_2}{V_0} = \frac{(1 - c_1) \dot{c}_2 + c_2 \dot{c}_1}{1 - \phi}; \quad (2.84)$$

for $\phi \equiv c_1 + c_2$. Equations 2.83 and 2.84 describe the composite-building process' movement through state space in terms of either volume fractions of materials or actual material volumes added.

Considering now the homogenisation step after each iteration of material replacement, let $\mathbf{L}^*$ be the effective stiffness tensor at time t . After replacement of material, the stiffness tensor of the composite becomes $\mathbf{L}^* + \Delta \mathbf{L}^*$, which we define by:

$$\bar{\sigma} = (\mathbf{L}^* + \Delta \mathbf{L}^*) \bar{\epsilon}; \quad (2.85)$$

where $\bar{\sigma}$ and $\bar{\epsilon}$ are the average stress and strain respectively over the composite after material replacement. Then:

$$V_0 \bar{\epsilon} = \int_{V_0 - \Delta v} \epsilon dv + \int_{\Delta v} \epsilon dv; \quad (2.86)$$

$$V_0 \bar{\sigma} = \mathbf{L}^* \int_{V_0 - \Delta v} \epsilon dv + \int_{\Delta v} \hat{\mathbf{L}} \epsilon dv; \quad (2.87)$$

where ϵ is the strain at any point in the medium, $\hat{\mathbf{L}} = \mathbf{L}_1$ in Δv_1 and $\mathbf{L}_2$ in Δv_s . Norris (1985) infers from equations 2.85 to 2.87 that:

$$\Delta \mathbf{L} \bar{\epsilon} = \frac{1}{V_0} \int_{\Delta v} (\hat{\mathbf{L}} - \mathbf{L}^*) \epsilon dv. \quad (2.88)$$

We now consider the situation where each individual inclusion added to the background is made completely of either matrix or inclusion material. Furthermore, we assume they are dilute and spheroidal. Under these assumptions, our thought experiment is similar to that of Eshelby's, and Wu's relationship (equation 2.58) holds for each phase, assuming inclusions are non-interacting. That is, under a constant strain ϵ^A applied at infinity, the strain ϵ in an added inclusion of material j , for $j = 1, 2$ is:

$$\epsilon = \mathbf{T}_j \epsilon^A \quad \text{in } \Delta v_j; \quad (2.89)$$

where $\mathbf{T}_j$ is the Wu T tensor corresponding to an inclusion of material j embedded into the background material. Assuming all inclusions of material j are randomly oriented, the average of equation 2.89 over all inclusions is then:

$$\frac{1}{\Delta v_j} \int_{\Delta v_j} \epsilon dv = \bar{\mathbf{T}}_j \epsilon^A, \quad j = 1, 2. \quad (2.90)$$

Tensor $\bar{\mathbf{T}}_j$ is the average Wu T tensor over all aspect ratios and orientations. With 2.90 established, equation 2.88 is expanded to:

$$\Delta \mathbf{L}^* \bar{\epsilon} = (\mathbf{L}_1 - \mathbf{L}^*) \bar{\mathbf{T}}_1 \epsilon \frac{\Delta v_1}{V_0} + (\mathbf{L}_2 - \mathbf{L}^*) \bar{\mathbf{T}}_2 \epsilon \frac{\Delta v_2}{V_0} \quad (2.91)$$

Finally, taking the limit where $\Delta v \rightarrow 0$ and hence $\Delta v_1 + \Delta v_2 \rightarrow 0$, and incorporating the system's state equations 2.83 and 2.84, Norris obtains the coupled differential equations which describe the generalised DEM model:

$$\dot{\mathbf{L}}(t) = (\mathbf{L}_1 - \mathbf{L}) \bar{\mathbf{T}}_1 \dot{c}_1 + (\mathbf{L}_2 - \mathbf{L}) \bar{\mathbf{T}}_2 \dot{c}_2 + [(\mathbf{L}_1 - \mathbf{L}) \bar{\mathbf{T}}_1 c_1 + (\mathbf{L}_2 - \mathbf{L}) \bar{\mathbf{T}}_2 c_2] \frac{\dot{c}}{1 - c}; \quad (2.92)$$

with $c(t) = c_1(t) + c_2(t)$ and boundary conditions $\mathbf{L}^*(0) = \mathbf{L}_0$.

The strength of this DEM scheme is that it can behave as a typical DEM or SCA inclusion model depending on the path taken through (c_1, c_2) state-space. By allowing the background material to be that of the matrix and adding only inclusion material into it iteratively, equation 2.92 takes the form:

$$\frac{d\mathbf{L}^*}{dc_1} = \frac{1}{1 - c_1} (\mathbf{L}_1 - \mathbf{L}^*) \bar{\mathbf{T}}_1; \quad (2.93)$$

with the initial conditions $\mathbf{L}^* = \mathbf{L}_2$ when $c_1 = 0$, which is the DEM model proposed by McLaughlin (1977). An expression for an effective medium approximation (EMA) can also be derived from equation 2.92 as:

$$c_1 (\mathbf{L}_1 - \mathbf{L}^*) \bar{\mathbf{T}}_1 + c_2 (\mathbf{L}_2 - \mathbf{L}^*) \bar{\mathbf{T}}_2 = 0. \quad (2.94)$$

Equation 2.94 is consistent with the self-consistent models of Hill (1965), Korringa (1973), Korringa *et al.* (1979), and Berryman (1980b). Other self-consistent models, such as those of Wu (1966), Walpole (1969), and Boucher (1974), are not consistent with equation 2.94 due to their asymmetry in treating a composite's phases as background and inclusion.

Chapter 3

Poroelasticity Theory

The water saturation of a hydrocarbon reservoir is a key factor in determining the economic viability of its development and production. Water saturation in a well is commonly calculated using electrical resistivity and porosity logs through Archie’s (Archie, 1942) law, exploiting the difference in the electrical conductivity of hydrocarbons and formation water. Away from well control, seismic inversion and joint seismic and electrical inversions have been used to estimate water saturation (Bachrach, 2006; Chen and Hoversten, 2012; Fattahi and Karimpouli, 2016).

Experimental analysis has shown saturated, porous rocks exhibit a frequency-dependent seismic response, which is the result of fluid effects (Winkler and Nur, 1982; Jones, 1986; Batzle *et al.*, 2006). When the saturating fluid is a mixture of water and hydrocarbons, say, these fluid effects are sensitive to water saturation (Murphy III, 1982; Mochizuki, 1982; White, 1975). As such, it may be desirable for a reservoir rock physics model to account for saturation-dependent fluid effects when modelling frequency-dependent seismic responses. Furthermore, fluids can be distributed heterogeneously in a reservoir (i.e., patchy saturation) (Dvorkin

and Nur, 1998). Because of this, a useful reservoir rock physics model may also account for the effects of fluid patchiness when modelling its saturation- and frequency-dependent seismic response.

In this chapter, I review the low frequency poroelastic model of Gassmann (Gassmann, 1951), then its extension to higher frequencies by Biot (1956a,b). This is followed by a review of squirt flow effects; a grain-scale seismic attenuation phenomenon which is observed experimentally yet unaccounted for by Biot theory alone. I then review the poroelastic inclusion model of Chapman *et al.* (2002) which considers squirt flow in an isotropic rock, followed by the extension of this model to a rock with aligned fractures (Chapman, 2003). I discuss the isotropic inclusion model of Papageorgiou and Chapman (2017) where two immiscible fluids with patchy saturation are introduced to the isotropic squirt flow model of Chapman *et al.* (2002). Finally, I review the model of Jin *et al.* (2018) which introduces two immiscible fluids with patchy saturation into the fractured model of Chapman (2003).

3.1 Poroelastic Effective Medium Theory

3.1.1 Fluid effects on seismic waves

In a static loading experiment, pore pressures in a rock are allowed to equilibrate throughout the medium. In active source seismology, however, frequencies can range from decihertz in surface seismic surveys, to megahertz in laboratory experiments. Over this range of frequencies, pore fluid pressures are not necessarily able to equilibrate, which can in turn lead to viscoelastic behaviour (Cheng *et al.*, 1993).

The relations of Gassmann (1951) form a foundation for modern poroelasticity theory. At sufficiently low frequencies, Gassmann's relations (Gassmann, 1951; Berryman, 1999) relate the elastic moduli of a fluid saturated rock to those of the same rock in dry conditions, as well as the moduli of the saturating fluid, matrix material, and rock frame. The Gassmann relations are:

$$\frac{K_{sat}}{K_0 - K_{sat}} = \frac{K_{dry}}{K_0 - K_{dry}} + \frac{K_{fl}}{\phi (K_0 - K_{fl})}; \quad (3.1)$$

and:

$$\mu_{sat} = \mu_{dry}; \quad (3.2)$$

where K_{sat} and μ_{sat} are the saturated rock's bulk and shear moduli, K_{dry} and μ_{dry} are the dry rock's bulk and shear moduli, K_{fl} is the fluid bulk modulus, K_0 is the mineral bulk modulus, and ϕ is the rock's porosity.

The Gassmann relations assume the porous rock is homogeneous and all pores are in communication with one another (Wang, 2001). It also assumes the frequency is low enough such that pore pressure is always in equilibrium throughout the rock. The Gassmann relations are most commonly used for fluid substitution in rock physics modelling, where a porous rock's low-frequency elastic moduli are estimated given different saturating pore fluids.

Biot (1956a,b) presented theoretical relations which accounted for the viscous and inertial interactions between a rock's matrix and pore fluid to describe the frequency-dependent velocity of a porous rock in terms of its dry elastic properties. In the low-frequency limit, the Biot relations are consistent with the Gassmann relations.

The Biot model transitions from a relaxed (low-frequency) state to an unrelaxed (high-frequency) state near the characteristic frequency, ω_i , defined as:

$$\omega_i = \frac{1}{a - m_f} \frac{\phi \eta}{\rho_{fl} \kappa}; \quad (3.3)$$

where a is the rock's tortuosity, ϕ is the rock's porosity, η is the fluid viscosity, ρ_{fl} is the fluid density, and κ is the rock's permeability. Parameter m_f is the fluid mass fraction defined as $m_f = \phi \rho_{fl} / \rho_{bulk}$, where ρ_{bulk} is the rock's bulk density.

As frequency increases from the low-frequency (Gassmann) regime, a rock's pore fluid pressure becomes unable to equilibrate throughout the rock. This leads to the prediction of Biot theory that a porous rock's velocity increases with increasing frequency. A slower, diffusive P-wave is also predicted by the Biot model in the high-frequency regime, which has been observed experimentally by Plona (1980) and Kelder and Smeulders (1997).

Some predictions of Biot theory have been verified experimentally, such as the existence of a high and low frequency velocity regime, and the existence of a slow, diffusive P-wave. However, Biot theory has been shown experimentally to underestimate seismic attenuation (e.g., Winkler (1985); King *et al.* (2000)), suggesting another source of attenuation may also be in effect.

One cause of attenuation which Biot theory doesn't account for is a grain scale attenuative effect referred to as squirt flow (e.g., Mavko and Nur (1975), Dvorkin *et al.* (1995)). Squirt flow is a phenomenon in which dispersion results from fluid moving in and out of communicating pores with different compliances. This motion is caused by induced fluid pressure gradients due to passing seismic waves with wavelengths comparable to the grain scale. At low frequencies, the pressure gradient between pores is negligible and hence there is negligible squirt flow.

Importantly, squirt flow modelling has been used to fit laboratory attenuation measurements (Winkler, 1985; King and Marsden, 2002) where Biot theory has underestimated.

A common way to incorporate the effects of squirt flow into Biot theory is to calculate a rock's wet frame modulus using a squirt flow model (for example, that of Mavko and Jizba (1991)), then use this wet frame modulus to evaluate Biot's equations. Another approach to model the squirt-flow induced, poroelastic properties of rocks is by inclusion modelling, first proposed by Chapman *et al.* (2002), where the frequency-dependent exchange of fluids between cracks and stiff pores is modelled using a hydraulically connected network of oblate spheroids and spheres.

3.1.2 Chapman *et al.*'s isotropic squirt flow model

Chapman *et al.* (2002) modelled the poroelastic properties of a rock by devising a squirt-flow model based on Eshelby's inclusion model. To do this, they considered a medium comprised of ellipsoidal cracks at random orientations and spherical pores, all in hydraulic communication, as shown schematically in Figure 3.1. Due to the different compliances of oblate spheroids and spheres, fluid is squirted between inclusions as pressure waves pass with wavelengths comparable to the grain scale, causing squirt-flow dispersion. At low frequencies, all pore pressures equilibrate and the model is consistent with Gassmann's relations, unlike the contemporary squirt flow models of Dvorkin and Nur (1993) and Hudson *et al.* (1996). At high frequencies, the model behaves as if pores are hydraulically isolated from one another.

Chapman *et al.* (2002) calculated the effective stiffness tensor C_{ijkl} using the homogenisation method of Eshelby (1957). Hence, an underlying assumption

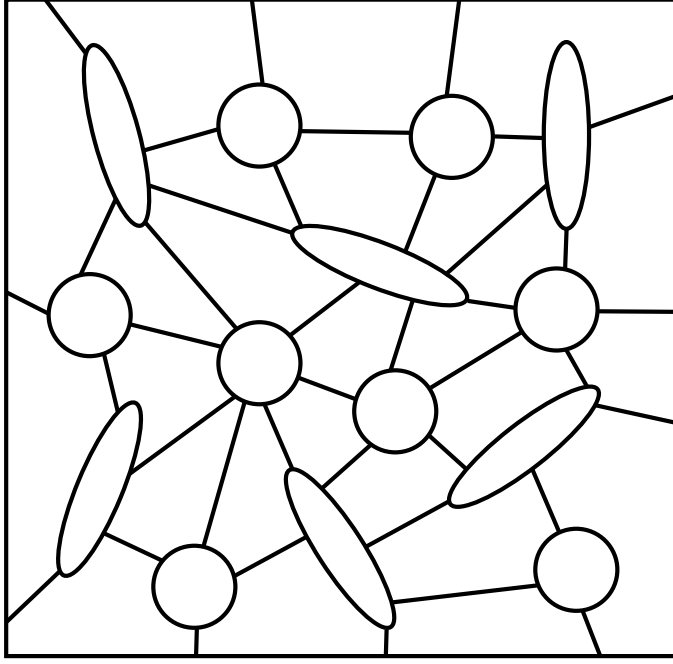


Figure 3.1: The hydraulically connected network of stiff and compliant ellipsoidal inclusions used to approximate a pore network in the model of Chapman *et al.* (2002).

of the Chapman model is that inclusions are in low concentration, meaning the model is most valid at low porosities and crack densities. The expression for the effective stiffness tensor is:

$$C_{ijkl} = C_{ijkl}^{(0)} - \phi_0 C_{ijkl}^{(1)} - \epsilon_c C_{ijkl}^{(2)}; \quad (3.4)$$

where $C_{ijkl}^{(0)}$ is the isotropic stiffness tensor of the matrix material, $C_{ijkl}^{(1)}$ is the contribution from stiff pores, and $C_{ijkl}^{(2)}$ is the contribution from microcracks. These inclusions' stiffness contributions are scaled by the stiff pore volume fraction ϕ_0 and microcrack density ϵ_c . Microcrack density is defined as:

$$\epsilon_c = \frac{3\phi_c}{4\pi\alpha_c}; \quad (3.5)$$

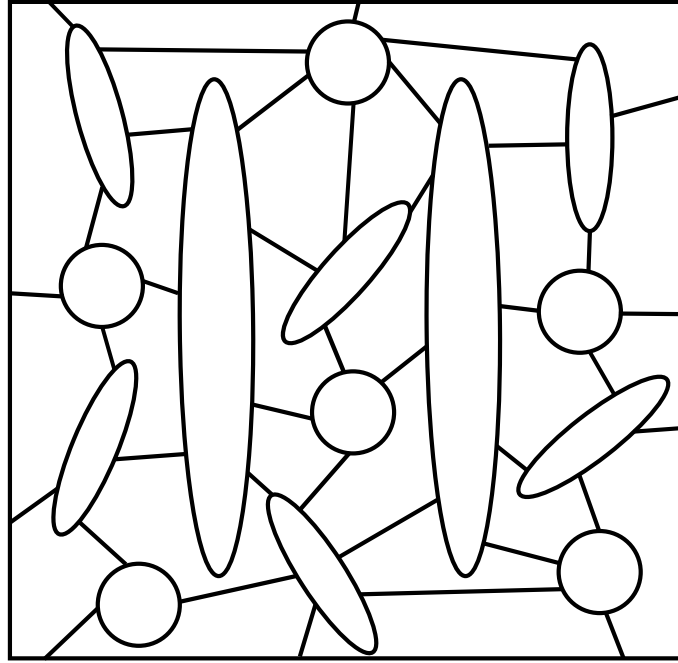


Figure 3.2: The hydraulically connected network of randomly oriented stiff and compliant ellipsoidal inclusions, and larger, aligned, compliant inclusions, used to approximate a fractured pore network in the model of Chapman (2003).

where ϕ_c is microcrack volume fraction and α_c is the microcrack aspect ratio.

3.1.3 Chapman's fractured squirt flow model

To model the frequency-dependent properties of fractured rocks, Chapman (2003) extended the squirt flow model of Chapman *et al.* (2002) to include aligned mesoscale fractures, which are bigger than pores and microcracks, yet much smaller than low frequency seismic wavelengths. In this model, the fractures and microcracks are both represented by oblate spheroids with equal aspect ratios for simplicity, as seen in Figure 3.2. The inclusion-based squirt-flow mechanism presented by Chapman *et al.* (2002) is then used to couple fluid flow at two scales, between the interconnected fractures, stiff pores, and microcracks.

The effective stiffness tensor of the fractured squirt-flow model takes the form:

$$C_{ijkl} = C_{ijkl}^{(0)} - \phi_0 C_{ijkl}^{(1)} - \epsilon_c C_{ijkl}^{(2)} - \epsilon_f C_{ijkl}^{(3)}. \quad (3.6)$$

Equation 3.6 is simply equation 3.4 with an added term, $\epsilon_f C_{ijkl}^{(3)}$, which represents the stiffness detracted from the solid matrix stiffness due to the presence of fractures. Analogous to the case of microcracks, ϵ_f is the fracture density, defined as:

$$\epsilon_f = \frac{3\phi_f}{4\pi\alpha_f}; \quad (3.7)$$

where ϕ_f is the fracture volume fraction, and α_f is the fracture aspect ratio.

Squirt flow is permitted to occur on two length-scales: between microcracks and pores, and between fractures and the surrounding small-scale inclusions. As such, two separate characteristic frequencies are expected from the model.

The frequencies over which dispersion occurs in the case where stiff and compliant inclusions are of the same scale is often discussed in terms of the time scale or relaxation parameter τ_c , which is (Liu and Martinez, 2013):

$$\tau_c = \frac{\eta c_v (1 - K_c)}{\sigma_c k a_g c_l}; \quad (3.8)$$

where a_g is the characteristic grain size, η is the fluid viscosity, κ is the rock's permeability, c_l is the number of connections to other pore space elements, c_v is an individual crack's volume, $\sigma_c = \pi\mu\alpha_c / [2(1 - \nu)]$ is the inverse of the crack space compressibility, $K_c = \sigma_c / k_f$, α_c is the crack aspect ratio, the fluid bulk

modulus is k_f , and ν and μ are the matrix material's Poisson's ratio and shear modulus respectively.

Having two scales of squirt-flow due to two scales of inclusions, the model of Chapman (2003) has two time scale parameters. The fracture squirt time scale parameter τ_f is related to the traditional squirt time scale parameter τ_c by:

$$\tau_f = \left(\frac{a_f}{a_g} \right) \tau_c; \quad (3.9)$$

where a_f is the fracture length scale. The fractured model of Chapman (2003) is consistent with the isotropic model of Chapman *et al.* (2002) as setting $\epsilon_f = 0$ in equation 3.6 yields equation 3.4.

With its vertical aligned fractures, the model of Chapman (2003) models azimuthal anisotropy in both seismic velocity and attenuation. Maultzsch *et al.* (2003) calibrated the Chapman model to the laboratory data of Rathore *et al.* (1995) with general agreement, modelling both measured azimuthal attenuation and velocity measurements. Rocks with aligned fractures of arbitrary orientation can be modelled by applying a rotation operator to the effective stiffness tensor C_{ijkl} as calculated by the model of Chapman (2003).

3.1.4 Papageorgiou and Chapman's multifluid squirt flow model

The relationship between velocity and porosity in a rock saturated with multiple fluids is often approximated by the Gassmann-Wood (Gassmann, 1951; Wood, 1955) equation, which assumes all liquid phases are homogeneously distributed.

In reality, rocks often exhibit patchy saturation (Dvorkin and Nur, 1998), a case not modelled by the Gassmann-Wood equation.

Papageorgiou and Chapman (2017) presented a poroelastic model which accounts for the partially saturated, patchy mixture of two immiscible fluids using the isotropic squirt flow framework of Chapman *et al.* (2002). This extends the work of Papageorgiou and Chapman (2015) who constructed a poroelastic model with a non-patchy (isostress) mixture of immiscible fluids. Papageorgiou and Chapman define “patchiness” in terms of the partial pressures of the two fluid phases, saying:

$$P_g = qP_w ; \quad (3.10)$$

where P_g is the partial pressure of fluid 1 and P_w is the partial pressure of fluid 2. For convenience, we use subscripts g and w to denote gas and water phases. In the case that the “patch” parameter q is unity, the patchy saturation squirt flow model of Papageorgiou and Chapman (2017) is equal to the isostress model of Papageorgiou and Chapman (2015). The bounds for patch parameter q are:

$$q_0 < q \leq 1 , \quad \text{where} \quad q_0 = \frac{K_g}{K_w} ; \quad (3.11)$$

for fluid bulk moduli K_g and K_w , assuming $K_g < K_w$. Considering multifluid Darcy’s law (Myron III *et al.*, 2013) with known fluid relative permeabilities and viscosities, Papageorgiou and Chapman (2017) obtain expressions for the dynamics of volume-averaged pore and crack pressures which they then use to calculate the q -dependent effective fluid compressibility and squirt-flow time scale parameter τ_c . Once these are calculated, the rock’s effective stiffness is calculated using the equations of Chapman *et al.* (2002). Figure 3.3 shows the model of Papageorgiou and Chapman (2017) modelling compressional velocities

and attenuations given a maximum patchiness of $q = q_0$ at different frequencies relative to a reference frequency.

3.1.5 Jin *et al.*'s fractured multifluid squirt flow model

Papageorgiou and Chapman (2017) showed the effect of having two patchy, viscous, immiscible fluids on a rock's effective stiffness and attenuative properties can be significant, depending on the fluids' mobilities and hence their abilities to squirt between inclusions. Chapman (2003) also showed the presence of aligned, mesoscale fractures can result in anisotropic elastic behaviour, with a frequency-dependence arising from squirt-flow effects between fractures, cracks, and stiff pores. In reality, many, if not all, reservoir rocks are fractured to some extent (Aguilera, 1998), and the fluid within these rocks is often a mixture, typically including oil, water, or gas. By incorporating the patchy, multifluid squirt-flow concepts of Papageorgiou and Chapman (2017) into the fractured rock squirt-flow model of Chapman (2003), Jin *et al.* (2018) developed a patchy multifluid, fractured, squirt flow model. The evolution of squirt flow inclusion models is shown in Figure 3.4.

The model of Jin *et al.* (2018) estimates the anisotropic, viscoelastic properties of a porous rock with aligned, meso-scale fractures by modelling squirt-flow effects, given two immiscible fluids of arbitrarily patchy saturation. A key finding in the development of this model is that the grain scale and fracture scale characteristic frequencies in partially saturated fractured rocks are different to those of a fully saturated rock, which were proposed by Chapman (2003). Calling the two immiscible fluids water and gas for convenience, Jin *et al.* (2018) find:

$$\omega_c = \frac{\omega_0 M_f}{M_w}; \quad \omega_f = \frac{\omega'_0 M_f}{M_w}; \quad (3.12)$$

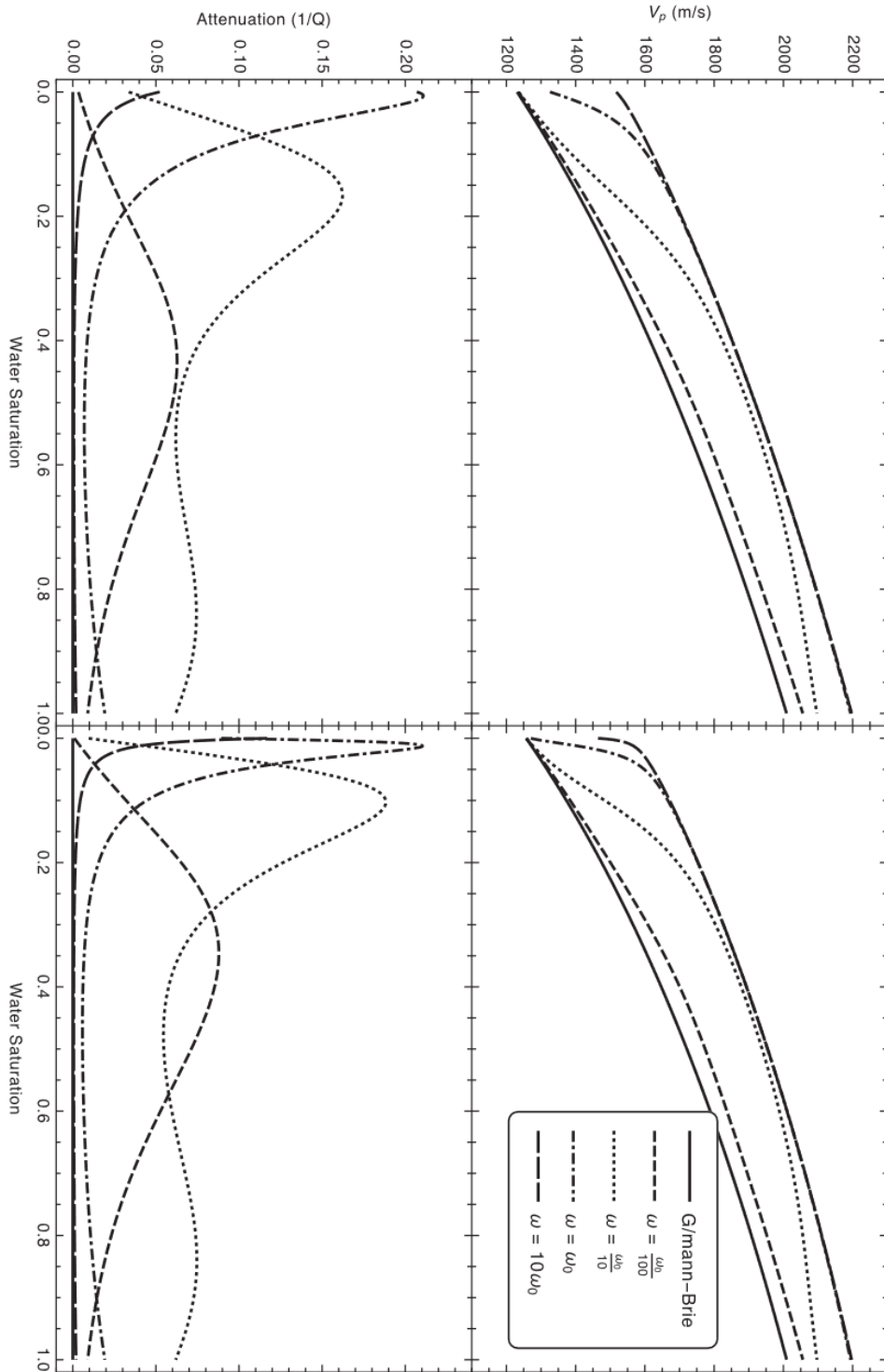


Figure 3.3: Modelled compressional velocities and attenuation using the model of Papageorgiou and Chapman (2017) with $q = q_0$ for a number of frequencies relative to a reference frequency ω_0 . Figure from Papageorgiou and Chapman (2017).

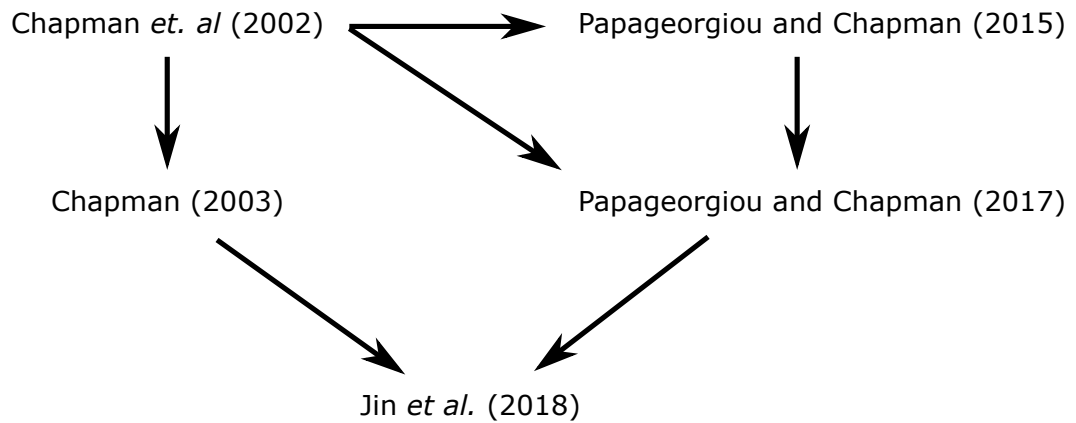


Figure 3.4: The progression of inclusion-based squirt flow models. Chapman *et al.* (2002) proposed a isotropic poroelastic model before aligned fractures were added to the model by Chapman (2003), creating an anisotropic, poroelastic inclusion model. Papageorgiou and Chapman (2015) extended the isotropic model of Chapman *et al.* (2002) to include two fluids with equal partial pressures, before being extended to two fluids with different partial pressures (i.e., patchy saturation) by Papageorgiou and Chapman (2017). Finally, Jin *et al.* (2018) followed the method of Papageorgiou and Chapman (2017) and extended the anisotropic poroelastic model of Chapman (2003) to include two pore fluids with patchy saturation.

where ω_c is the partially-saturated grain scale characteristic frequency, ω_f is the partially-saturated fracture scale characteristic frequency, and ω_0 and ω'_0 are ω_c and ω_f respectively at full saturation. Parameters M_w and M_f are the fluid mobility of water and the fluid mixture respectively, assuming water is the wetting fluid. The fluid mobility of water M_w and gas M_g are defined as:

$$M_w = \frac{\kappa}{\eta_w}; \quad M_g = \frac{\kappa}{\eta_g}; \quad (3.13)$$

where κ is the rock's total permeability, and η_w and η_g are the viscosities of water and gas respectively. Introducing parameter $\tilde{q}$, which is related to patch parameter q and water saturation S_w by $\tilde{q} = S_w + q(1 - S_w)$, the effective fluid mobility M_f is:

$$M_f = \frac{\kappa_{rw}}{\tilde{q}} M_w + \frac{q\kappa}{\tilde{q}} M_g; \quad (3.14)$$

where κ_{rw} is the relative permeability of water.

The effective fluid bulk modulus K_f is also dependent on the patch parameter, water saturation, and the bulk moduli of both fluids by:

$$\frac{1}{K_f} = \frac{S_w}{\tilde{q}K_w} + \frac{(1 - S_w)q}{\tilde{q}K_g}; \quad (3.15)$$

where K_w and K_g are the bulk moduli of water and gas respectively.

By incorporating these relationships in the fractured squirt-flow model of Chapman (2003), Jin *et al.* (2018) evaluated the rock's upscaled viscoelastic stiffness tensor. Figure 3.5 shows anisotropy as predicted by the model of Jin *et al.* (2018)

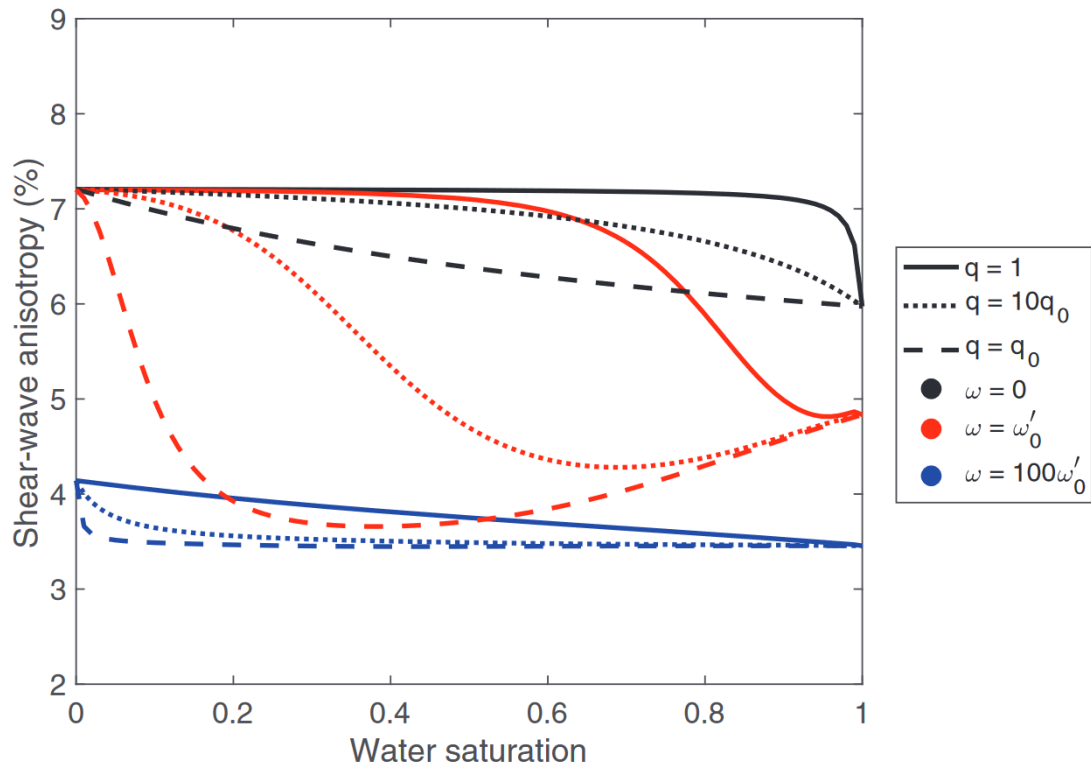


Figure 3.5: Shear wave splitting as a function of water saturation for different patch parameters and relative frequencies using the fractured, patchy saturation model of Jin *et al.* (2018). Figure from Jin *et al.* (2018).

in terms of shear wave splitting as a function of patch parameter, water saturation, and frequency relative to a reference frequency.

Chapter 4

Modelling the Electrical-Elastic Properties of Porous Rocks

4.1 Abstract

Joint electrical-elastic rock physics methods have been instrumental in lowering uncertainty in reservoir modelling. Typical multiphysics models, however, are empirical or require an intermediate step of porosity estimation to link a rock's electrical and elastic moduli, which can be error-prone away from well controls. Another prevailing challenge is modelling shear modulus from electrical measurements. By reformulating an existing electrical differential effective medium (DEM) theory to embed ellipsoidal pores into a background of matrix material, rather than the original method of embedding ellipsoidal grains into a background

of fluid, we express the model in terms of the geometrical parameter $R^{(*2)}$, popularly used in electrical ellipsoidal inclusion models. This reformulation is consistent with 2 other popular effective conductivity models and shares its geometrical parameter with the electrical self-consistent approximation (SCA) model, providing a new link between electrical DEM and SCA models. By taking the ratio of this reformulated electrical DEM model and a pre-existing elastic DEM model, we obtain expressions for a composite's effective elastic moduli with respect to effective conductivity. These expressions have no explicit porosity terms and are formulated in terms of the popular elastic DEM geometrical parameters $P^{(*2)}$, $Q^{(*2)}$, and the electrical parameter $R^{(*2)}$, linking electrical and elastic inclusion modelling in a novel way. Modelling the elastic moduli of clay-bearing sandstones using public domain laboratory measurements, there seems to be weak sensitivity of the model's single parameter, effective pore aspect ratio (EPAR), to clay volume fraction. Furthermore, the uncertainty in shear modulus modelling from conductivity measurements seems weakly sensitive to clay content. Predicting synthetic velocity well logs from resistivity well logs is also shown to have a lower error when using the proposed, direct electrical-elastic DEM approach, when compared to the more common, two-step method of inverting for porosity using electrical DEM modelling before predicting velocities using an elastic DEM model. The inverse problem of predicting resistivity from elastic measurements is significantly challenging, if at all possible, using the proposed, coupled differential equations and is not attempted here.

4.2 Introduction

Electrical-elastic multiphysics modelling, for example through the integration of controlled-source electromagnetic (CSEM) data into marine seismic reservoir

characterisation workflows, has led to reduced uncertainty in reservoir characterisation (Alcocer *et al.*, 2013). However, a simple, physics-based, and accurate multiphysics model which links a porous rock’s electrical and elastic properties remains elusive. Many electrical-elastic relationships proposed to date are at least partly empirical (Carcione *et al.*, 2007). In addition to this, the majority of workflows which relate a rock’s elastic and electrical properties require the estimation of porosity as an intermediate step (Werthmüller *et al.*, 2013), which can be uncertain away from well control.

Carcione *et al.* (2007) presented a set of electrical-elastic cross-property models with no explicit porosity terms by formulating pre-existing electrical models in terms of porosity and substituting them into pre-existing velocity-porosity relations. However, the individual electrical or elastic models involved were mostly empirical, or were physics-based models with sometimes physically incompatible derivations, making their physical interpretation complicated or self-contradictory. Chen and Dickens (2009) and Werthmüller *et al.* (2013) both assessed the intrinsic (theoretical) and extrinsic (parametrisation) uncertainties involved in these models, which were generally found to be significant. Indeed, Kwon and Snieder (2011) showed the uncertainty in these models had a larger contribution to overall uncertainty than that of the data. The electrical-elastic Hashin-Shtrikman bounds (Carcione *et al.*, 2007) have no explicit porosity terms and are physically meaningful, but can be too widely spaced to predict the electrical-elastic properties of any rock in particular.

Other challenges, such as accurately estimating a rock’s shear modulus from its electrical response, also remain largely unsolved (MacGregor, private communication, 2018). Joint electrical-elastic modelling with a single model parameter has been attempted with mixed success (Han *et al.*, 2011a; Wang and Gelius, 2010; Jensen *et al.*, 2013), however a scrutiny by Han *et al.* (2016) concluded a

new multiphysics model is required to accurately relate the electrical and elastic properties of a porous rock using a single set of model parameters. In fact, Werthmüller *et al.* (2013) posited there is no known direct link between a rock’s velocity and resistivity.

In this chapter, we reformulate the electrical differential effective medium (DEM) model of Mendelson and Cohen (1982) to estimate the overall electrical conductivity of a rock comprised of ellipsoidal pores embedded in a solid matrix background. We use this model reformulation, combined with the elastic DEM model of Berryman (1992), to produce DEM expressions which relate the overall electrical and elastic properties of an isotropic, porous rock. The formulated electrical-elastic expressions are free from porosity terms and can be expressed in terms of the functions $Q^{(*2)}$, $P^{(*2)}$, and $R^{(*2)}$, proposed by Berryman (1980b, 1995). The single model parameter, effective pore aspect ratio, seems weakly sensitive to clay volume fraction when modelling the mixed sandstone data set of Han *et al.* (2011b). The uncertainty in shear modulus modelling also seems weakly sensitive to clay volume fraction in this data example. Predicting synthetic velocity well logs using both the proposed electrical-elastic DEM expressions, and the traditional DEM modelling method which estimates porosity as an intermediate step, shows the proposed method reduces velocity modelling error by up to 23%.

To begin, we reformulate and analyse the electrical differential effective medium (DEM) model of Mendelson and Cohen (1982) in terms of pores embedded in a matrix background, before proposing new electrical-elastic DEM expressions. We then test this cross-property DEM model’s performance on the laboratory electrical-elastic measurements of Han *et al.* (2011b), which are made on brine-flooded sandstone cores from multiple localities, having a range of clay and pore volume fractions. We estimate the optimal parameters which model the rock’s bulk and shear modulus from resistivity measurements, and investigate the effect of clay content on the model’s parametrisation and uncertainty. Following this,

we predict synthetic velocity logs and statistically compare the proposed and traditional cross-property DEM modelling methods.

4.3 Model Derivation

4.3.1 Electrical modelling

Maxwell Garnett (1904) popularised electrical inclusion modelling by proposing an effective medium approximation for the electrical properties of a material containing spherical inclusions. Bruggeman (1935) proposed an electrical DEM model for the same material, while self-consistent electrical modelling for spherical inclusions has been addressed by Bruggeman (1935), Landauer (1952), and Berryman (1995). Sen *et al.* (1981) proposed a DEM model which embedded fractal-layered, insulating, spherical grains into a background initially made of water and, in doing so, derived Archie's (Archie, 1942) first law with cementation exponent $m = 1.5$ and tortuosity $a = 1$.

The assumption of spherical grains existing in a rock is far from realistic in almost all cases, however analytical modelling while considering a rock's true grain shapes is largely untenable. The general ellipsoid, therefore, is a popular middle ground to approximate a rock's grain shape as it has enough form diversity to approximate spherical, platy, and needle-like grains, while still permitting first principles modelling of a rock's physical properties. Polder and Van Santeen (1946) estimated the average electrical properties of an isotropic medium containing randomly oriented ellipsoidal inclusions with the assumption of non-interacting inclusions. Frank (1963) calculated the effective electric field in a medium containing a single homogeneous ellipsoid with principal axes arbitrarily

aligned with respect to the incident electric field. Stroud (1975) treated the case of electrically anisotropic effective media with inclusions of arbitrary shape, while Mendelson and Cohen (1982) modelled the effective electrical properties of a medium containing many arbitrarily oriented ellipsoidal inclusions using a DEM method.

Arguably the most renowned aspect of the DEM model of Mendelson and Cohen (1982), abbreviated to “M&C”, is a subsidiary result: By setting the grain conductivity to zero, fluid conductivity to that of water, and letting grains be randomly oriented, M&C’s electrical DEM model takes the form of Archie’s (Archie, 1942) first law:

$$\sigma^* = \sigma_w \phi^m. \quad (4.1)$$

Parameter σ^* is the rock’s overall conductivity, σ_w is the saturating water’s conductivity, and m is the cementation factor. We note tortuosity factor a , a common coefficient of Archie’s first law (Glover, 2016), is unity in equation 4.1, as is also derived in the model of Sen *et al.* (1981) for spherical grains. Importantly, M&C showed m is a function of grain aspect ratio α , agreeing with observations that m depends on grain shape (e.g., Salem and Chilingarian (1999)).

M&C consider a rock with background material (i.e., pore fluid) volume fraction, or porosity, ϕ , background material conductivity σ_2 , and inclusion (i.e., grain) conductivity σ_1 . M&C’s DEM model in its most general form for inclusions of a single aspect ratio is then:

$$d\sigma^* = -\frac{d\phi}{\phi} \left\langle (\sigma_1 - \sigma^*) \left[1 + (\sigma^*)^{-\frac{1}{2}} \Theta_1 (\sigma^*)^{-\frac{1}{2}} (\sigma_1 - \sigma^*) \right]^{-1} \right\rangle; \quad (4.2)$$

where angled brackets denote an average over all inclusion orientations and Θ_1 is a 3×3 matrix, such that:

$$\Theta_1 = \mathbf{R}\mathbf{L}\mathbf{R}^{-1}. \quad (4.3)$$

Matrix Θ_1 , introduced by Frank (1963), contains the effective depolarisation factors of an ellipsoidal inclusion of phase i which is arbitrarily rotated with respect to an incident electric field. Matrix $\mathbf{L}$ is the diagonal matrix with the ellipsoid's three depolarisation factors L_p , $p \in \{1, 2, 3\}$, along its diagonal. Depolarisation factors L_p describe how a uniform, incident electrical field is perturbed by the presence of an ellipsoidal inclusion which has its p^{th} semi-major axis aligned with the field direction. Depolarisation factors depend on the ellipsoidal inclusion's aspect ratio and are reviewed and evaluated by Osborn (1945). Matrix $\mathbf{R}$ is an orthogonal matrix. In the case of randomly oriented ellipsoidal grains, $\mathbf{R} = \mathbf{I}$ and the rock's effective conductivity is isotropic, presuming an isotropic background and grain conductivity. In this chapter, we only consider the isotropic case and so set $\Theta_1 = \mathbf{L}$ henceforth. Importantly, we adopt the convention that the trace of $\mathbf{L}$ is unity (Mendelson and Cohen, 1982).

To derive equation 4.1, M&C make the approximation that grains are perfect resistors by setting $\sigma_1 = 0$ in equation 4.2 and obtain the differential equation:

$$\frac{d\sigma^*}{\sigma^*} = m \frac{d\phi}{\phi}; \quad (4.4)$$

with cementation factor m :

$$m = \frac{1}{3} \sum_{p=1}^3 \left\langle [1 - L_p]^{-1} \right\rangle. \quad (4.5)$$

Taking the average of equation 4.5 over all grain orientations and integrating with the boundary condition $\sigma^*(\phi = 1) = \sigma_2$ yields equation 4.1 with $\sigma_2 = \sigma_w$.

Our current aim is to reformulate this DEM model with reversed phases, embedding fluid-filled pores into a background of matrix material. This is demonstrated in Figure 4.1, which shows the M&C model and the model's reformulation we aim to derive here, highlighting the key difference that one formulation is grain supported while the other is matrix supported.

To reformulate the model of M&C as desired, we change the variable ϕ to $(1 - \phi)$ and interchange subscripts 1 and 2 in equation 4.2, obtaining:

$$d\sigma^* = -\frac{d(1 - \phi)}{(1 - \phi)} \left\langle (\sigma_2 - \sigma^*) \left[1 + (\sigma^*)^{-\frac{1}{2}} \Theta_2 (\sigma^*)^{-\frac{1}{2}} (\sigma_2 - \sigma^*) \right]^{-1} \right\rangle; \quad (4.6)$$

which evaluates to:

$$\frac{d\sigma^*}{\sigma^* - \sigma_2} = \bar{m} \frac{d(1 - \phi)}{(1 - \phi)}; \quad (4.7)$$

where:

$$\bar{m} = \frac{1}{3} \sum_{p=1}^3 \left\langle \left[1 + \left(\frac{\sigma_2}{\sigma^*} - 1 \right) L_p \right]^{-1} \right\rangle. \quad (4.8)$$

Equations 4.7 and 4.8 are a reformulation of equations 4.4 and 4.5 respectively, here with pores embedded into a background of matrix material.

To evaluate $d\sigma^*/d\phi$ from equation 4.7, we perform the change in variable:

$$\frac{d\sigma^*}{d\phi} = -\frac{d\sigma^*}{d(1-\phi)} . \quad (4.9)$$

Then, by equations 4.7 and 4.9, the reformulation of M&C's electrical DEM model with ellipsoidal pores embedded into a background of matrix material is:

$$\frac{d\sigma^*}{d\phi} = \frac{(\sigma_2 - \sigma^*) \bar{m}}{(1 - \phi)} . \quad (4.10)$$

By considering spheroidal inclusions, we drop the subscript p in depolarisation factors L_p , calling the principal depolarisation factor L , and the two degenerate factors $(1 - L)/2$, given the trace of $\mathbf{L}$ is unity.

Parameter $\bar{m}$ from equation 4.8 for randomly oriented spheroidal inclusions is thus:

$$\bar{m} = \frac{1}{3}\sigma^* \left[\frac{4}{\sigma^* + \sigma_2 + L(\sigma^* - \sigma_2)} + \frac{1}{\sigma^* - L(\sigma^* - \sigma_2)} \right] ; \quad (4.11)$$

The function $R^{(*2)}$ proposed by Berryman (1995) for the electrical Clausius-Mossotti and self-consistent approximation models with a single phase of randomly oriented spheroidal inclusions is defined as:

$$R^{(*2)} = \frac{1}{9} \left[\frac{4}{\sigma^* + \sigma_2 + L(\sigma^* - \sigma_2)} + \frac{1}{\sigma^* - L(\sigma^* - \sigma_2)} \right]; \quad (4.12)$$

$$= \frac{\bar{m}}{3\sigma^*}. \quad (4.13)$$

Thus, by equations 4.10 and 4.13, the DEM model of M&C can now be expressed in terms of the popular geometric function $R^{(*2)}$:

$$\frac{d\sigma^*}{d\phi} = 3\sigma^* \frac{(\sigma_2 - \sigma^*) R^{(*2)}}{(1 - \phi)}. \quad (4.14)$$

As an aside, we note the two-phase self-consistent electrical model for spheroidal pores of Berryman (1995) can be formulated in terms of $\bar{m}$:

$$\frac{\phi(\sigma_2 - \sigma^*) \bar{m}}{3\sigma^*} = 0. \quad (4.15)$$

The electrical DEM model for spherical inclusions derived by Bruggeman (1935) is (Berryman, 1995):

$$\frac{d\sigma^*}{d\phi} = \frac{(\sigma_2 - \sigma^*)}{(1 - \phi)} \frac{3\sigma^*}{(\sigma_2 + 2\sigma^*)}. \quad (4.16)$$

In the case of spherical inclusions, $L = 1/3$ and so, by equation 4.12:

$$R^{(*2)} [L = 1/3] = \frac{3\sigma^*}{\sigma_2 + 2\sigma^*}. \quad (4.17)$$

Combining equations 4.13, 4.16, and 4.17, we see the Bruggeman (1935) DEM model is:

$$\frac{d\sigma^*}{d\phi} = \frac{(\sigma_2 - \sigma^*)}{(1 - \phi)} \bar{m} [L = 1/3] . \quad (4.18)$$

which is the special case of equation 4.10 when $L = 1/3$. Thus, we have shown this reformulation of the M&C model is consistent with that of Bruggeman (1935) in the special case of spherical inclusions. This result is expected as the model of Mendelson and Cohen (1982) is consistent with that of Sen *et al.* (1981) in the case of spherical inclusions, which in turn is consistent with that of Bruggeman (1935).

We require equation 4.10 to obtain our proposed, joint electrical-elastic DEM expressions. For completeness, however, we integrate equation 4.10 here to obtain an expression analogous to Archie's first law as derived by M&C. For simplicity, we make the approximation that $\bar{m}$ in equation 4.11 is constant in σ^* by assuming $\sigma^* \ll \sigma_2$, as is often the case in rocks with fluid-filled pores. However, equation 4.7 can be integrated analytically with $\bar{m}$ as defined in equation 4.11.

Integrating both sides of equation 4.7 with boundary condition $\sigma^*(\phi = 0) = \sigma_1$, we find the analogous equation to M&C's expression of Archie's first law:

$$\frac{\sigma_2 - \sigma^*}{\sigma_2 - \sigma_1} = (1 - \phi)^{\bar{m}} . \quad (4.19)$$

To show equation 4.19 is consistent with M&C's expression of Archie's first law, we let σ_b and ϕ_b represent a rock's background material conductivity and volume fraction respectively. By invoking M&C's assumption that inclusion conductivity is zero and employing this new notation, equation 4.19 becomes:

$$\sigma^* = \sigma_b \phi_b^{\bar{m}_{MC}}; \quad (4.20)$$

where $\bar{m}$ has simplified to:

$$\bar{m}_{MC} = \frac{5 - 3L}{3(1 - L^2)}. \quad (4.21)$$

Equation 4.20 is identical to equation 4.1 as derived by M&C, where σ_b and ϕ_b are the conductivity and volume fraction of water. Parameter $\bar{m}_{MC}$ in equation 4.21 is exactly the expression for m proposed by M&C in equation 4.5 under the assumption of spheroidal inclusions. In contrast to Archie's first law as derived by M&C, it may be that equation 4.19 must be solved numerically as $\bar{m} = \bar{m}(\sigma_2, \sigma^*, \alpha)$.

4.3.2 Elastic modelling

Berryman (1992) presented a DEM model which estimates the effective elastic moduli of a dilute dispersion of randomly oriented ellipsoidal inclusions in a background material by the coupled equations:

$$\frac{dK^*}{d\phi} = \frac{(K_2 - K^*) P^{(*2)}}{(1 - \phi)}; \quad (4.22)$$

$$\frac{d\mu^*}{d\phi} = \frac{(\mu_2 - \mu^*) Q^{(*2)}}{(1 - \phi)}; \quad (4.23)$$

where K and μ denote bulk and shear moduli respectively. Functions $P^{(*2)}$ and

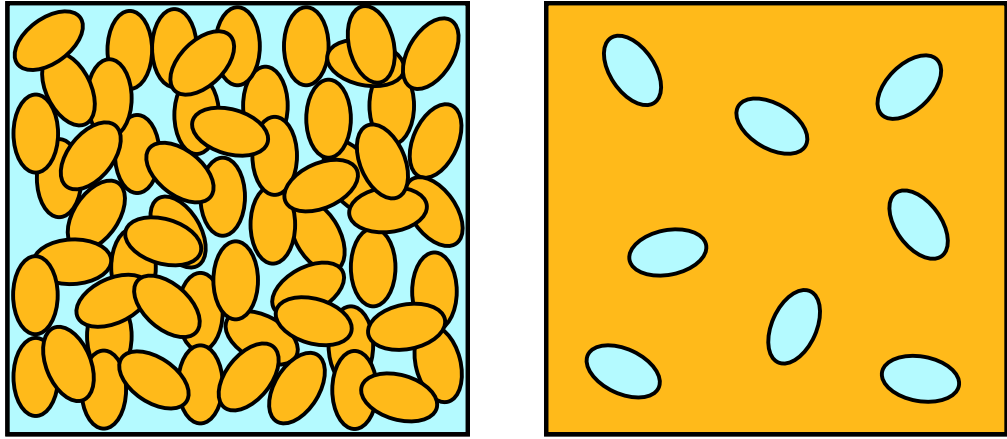


Figure 4.1: A comparison of the Archie's law model originally proposed by Mendelson and Cohen (1982) (left), and this chapter's rederived model (right), which embeds pores into matrix material. Both models have only a fluid and matrix phase, and do not account for the double layer or surface effects. Archie's law requires connected porosity (in 3D) and insulating grains. The model formulation proposed in this chapter, rather, requires slightly conductive matrix material but does not require pores to be connected.

$Q^{(*2)}$ are defined explicitly by Berryman (1980b) for the arbitrary spheroidal inclusion. Functions $P^{(*2)}$ and $Q^{(*2)}$ are similar to functions $\bar{m}$ and $R^{(*2)}$ in that they depend on the inclusion aspect ratio α , and the inclusion and the effective properties of the composite. Analogous to the boundary conditions of the reformulated electrical DEM model, equations 4.22 and 4.23 are solved with boundary conditions $K^*(\phi = 0) = K_1$ and $\mu^*(\phi = 0) = \mu_1$.

4.3.3 Electrical-elastic modelling

Dividing equations 4.22 and 4.23 by equation 4.10, (i.e., employing the chain rule), we obtain a set of coupled differential equations which relate the electrical and elastic properties of a two-phase, isotropic, linearly elastic, electrically conductive composite:

$$\frac{dK^*}{d\sigma^*} = \left(\frac{K_2 - K^*}{\sigma_2 - \sigma^*} \right) \frac{P^{(*2)}}{\bar{m}}; \quad (4.24)$$

$$\frac{d\mu^*}{d\sigma^*} = \left(\frac{\mu_2 - \mu^*}{\sigma_2 - \sigma^*} \right) \frac{Q^{(*2)}}{\bar{m}}. \quad (4.25)$$

Porosity is rendered a dummy variable when the chain rule is applied, meaning the resultant expressions contain no porosity terms.

By considering equation 4.13, equations 4.24 and 4.25 can be expressed in terms of $P^{(*2)}$, $Q^{(*2)}$, and $R^{(*2)}$ (Berryman, 1980b, 1992, 1995):

$$\frac{dK^*}{d\sigma^*} = \frac{1}{3\sigma^*} \left(\frac{K_2 - K^*}{\sigma_2 - \sigma^*} \right) \frac{P^{(*2)}}{R^{(*2)}}; \quad (4.26)$$

$$\frac{d\mu^*}{d\sigma^*} = \frac{1}{3\sigma^*} \left(\frac{\mu_2 - \mu^*}{\sigma_2 - \sigma^*} \right) \frac{Q^{(*2)}}{R^{(*2)}}. \quad (4.27)$$

The boundary conditions of equations 4.24 and 4.25, or 4.26 and 4.27, are:

$$K^*(\sigma^* = \sigma_1) = K_1; \quad (4.28)$$

$$\mu^*(\sigma^* = \sigma_1) = \mu_1. \quad (4.29)$$

The physical intuition behind the proposed electrical-elastic DEM expressions is shown in Figure 4.2. The key conceptual difference between these multiphysics DEM expressions and a typical electrical or elastic DEM model is that in this electrical-elastic method, the porosity added in each rock-building iteration is not $d\phi$, but an unknown volume fraction which increments the rock's effective conductivity by $d\sigma^*$. These cross-property DEM expressions are correct in both the high and low porosity limit even though porosity acts as a dummy variable.

4.4 Laboratory Data Modelling

To test the proposed electrical-elastic DEM modelling method, we use the electrical-elastic laboratory data set of Han *et al.* (2011b). This data set is comprised of simultaneous electrical and elastic laboratory measurements made on

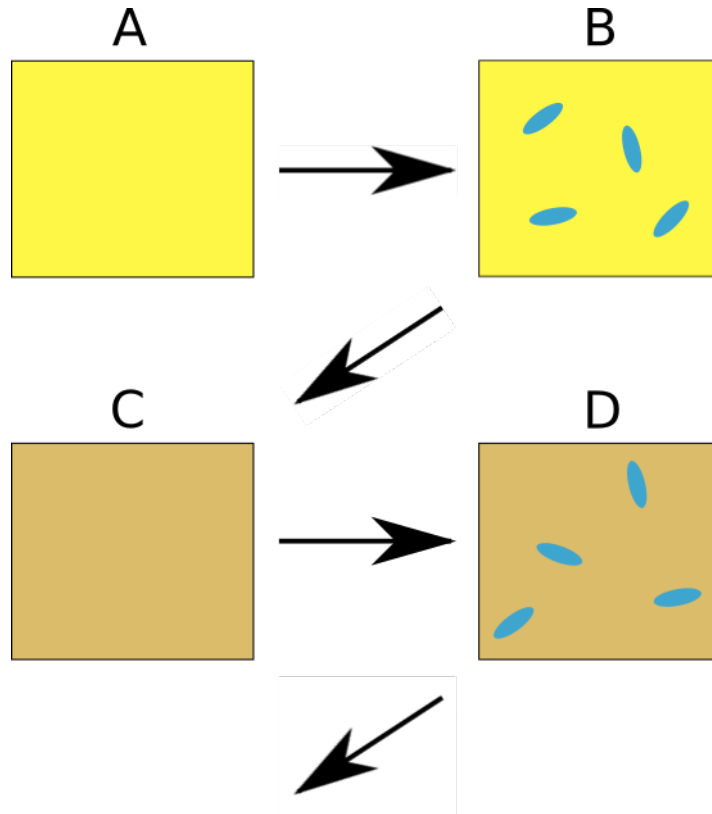


Figure 4.2: The physical interpretation of our proposed electrical-elastic DEM modelling method (Figure modified from Norris (1985)). State A) A homogeneous rock with a given conductivity, bulk modulus, and shear modulus; B) Ellipsoidal pores are added with such a volume that the rock's effective conductivity increments by $d\sigma^*$; C) A homogeneous rock is created with new effective elastic properties given its updated effective conductivity; D) More ellipsoidal pores are added to this new homogeneous rock, and the process continues until the desired effective conductivity is reached.

63 sandstone cores collected from the UK and China. At least eight minerals are present in the cores, including quartz, four clay minerals, two feldspar minerals, and carbonates. The experiments were conducted by Han *et al.* (2011b) with a saturating brine concentration of 35 g/L and a pore fluid pressure of 5 MPa. The electrical resistivity measurements had a low-frequency (2 Hz) source, as is of interest in CSEM studies. Core porosity was measured by helium porosimetry, and mineralogy was determined by whole rock X-ray diffraction. The samples were maintained at 19 ± 1 degrees Celcius throughout experiments. Brine-saturated compressional and shear wave velocities were measured at 1.0 MHz and 0.7 MHz respectively.

Velocities were measured at various pressures, ranging from 8-60 MPa (Han *et al.*, 2011b). We study the elastic measurements made at 20 MPa in this chapter. This is a relatively low pressure, and microcracks are likely to be present in these samples, compared to, say, experiments conducted at 60 MPa. The presence of these microcracks would increase the rocks effective ellipsoidal inclusion eccentricity (i.e. the distance between EPAR and unity would increase) when compared to a rock at higher pressure, which has only stiff pores. This may also lead to more prominent squirt flow effects being observed over a certain frequency band, as squirt flow is often attributed to fluid moving between microcracks and pores (Chapman, 2003). However, given the constant frequency of all experimental sources, we did not consider these factors as concerning.

We study only measurements from the subset of 30 cores which have quartz content greater than 60% volume fraction. We refer to this subset of measurements as the “Han” data. By considering only these measurements, we ensure mineralogy is quartz-dominated. Accordingly, we model all data assuming samples have a pure quartz matrix. The clay volume fraction is non-negligible in many samples, as is shown in Figure 4.3. By modelling the properties of clay-bearing rocks with the assumption of pure quartz matrix, we test the impact of clay content on the

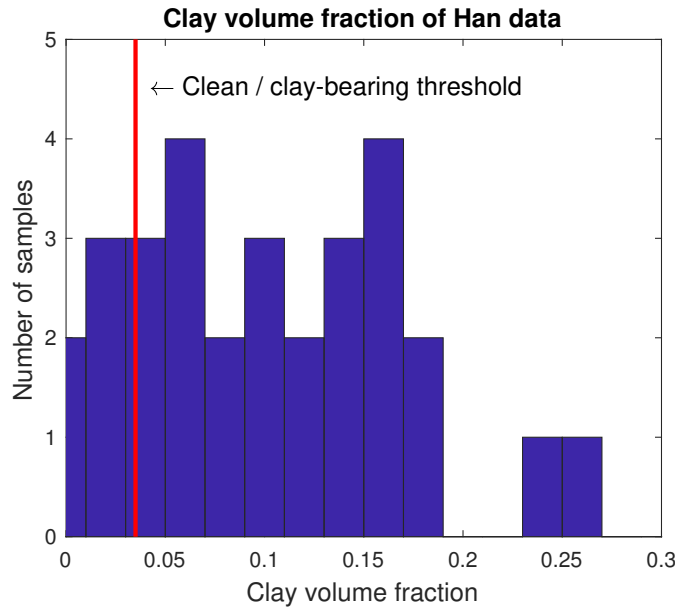


Figure 4.3: Histogram of the clay content for all samples in the Han data set. The red line at 3.5% clay volume fraction marks the arbitrary cut-off between clean and clay-bearing sandstone samples.

model's parametrisation. All electrical and elastic parameters used in modelling are shown in Table 4.1.

To investigate the effects of clay on the electrical-elastic model, we partitioned the Han data into clean and clay-bearing data by arbitrarily choosing a threshold of 3.5% clay volume fraction, as shown in Figure 4.3. This is of interest as the electrical properties of a rock are affected by the presence of clay through the double layer effect (Waxman and Smits, 1968), which the proposed electrical-elastic model does not account for. Also, the elastic moduli of a sandstone can also be significantly affected by only a small amount of present clay (Han *et al.*, 1986).

We refer to the model's parameter α as the effective pore aspect ratio (EPAR) when applied to real data, which is synonymous with the equivalent pore

Table 4.1: Physical parameters used in electrical-elastic modelling.

Constituent	K (GPa)	μ (GPa)	σ ($1/(\Omega\text{m})$)	Moduli source	Conductivity source
Quartz	36.6	45.5	10^{-5}	Mavko <i>et al.</i> (2009)	Han <i>et al.</i> (2011a)
Brine	2.29	0	1/0.213	Han <i>et al.</i> (2011a)	Han <i>et al.</i> (2011a)

Table 4.2: EPAR inversion results for the Han data set and its clean and clay-bearing subsets.

Data set	# Samples	α_{K_0}	α_{μ_0}	$s.d.(\alpha_{K_0})$	$s.d.(\alpha_{\mu_0})$	α_K^*	α_μ^*	α_K^*/α_μ^*
All samples	30	17.0	13.0	3.65	1.26	16.4	12.8	1.28
Clean (clay < 3.5%)	6	16.6	12.3	0.78	0.73	16.6	12.3	1.35
Clay-bearing (clay > 3.5%)	24	17.1	13.1	4.07	1.32	16.3	12.9	1.26

aspect ratio proposed by Fournier *et al.* (2011, 2014, 2018). We differentiate between *EPAR* in applied inclusion modelling and *inclusion aspect ratio* in theoretical inclusion modelling because real pores are not ideal spheroids, while the theoretical model’s inclusions are.

To test the electrical-elastic model’s performance on the Han data, we first invert equations 4.24, 4.25, 4.28, and 4.29 for the EPAR associated with each core sample, using the parameters in Table 4.1 and the measured elastic and electrical data. Rather than solving for a single α which satisfies both bulk and shear modulus measurements in some way, we solve for both the “bulk modulus EPAR”, α_K , and “shear modulus EPAR”, α_μ , for each sample. Fournier *et al.* (2011, 2014, 2018) describe α_K and α_μ as the EPARs which minimise the misfit between the modelled and measured bulk modulus and shear modulus data respectively.

In an ideal composite material with a dilute dispersion of ellipsoidal inclusions, α_K and α_μ of the medium are expected to be equal (Eshelby, 1957). In reality, deviations from the ideal ellipsoid (or “pore asperities”) cause α_K and α_μ to be generally unequal, as shown by Fournier *et al.* (2011, 2014, 2018) and Cilli and Chapman (in prep.; Chapter 5). Figure 4.4 demonstrates the effects of pore asperities on a rock’s effective bulk and shear moduli, and hence EPARs.

By inverting for both α_K and α_μ separately, we obtain end-member parameters which bound the solution space for any multi-objective optimisation which considers linear combinations of bulk and shear modulus modelling misfits. We use the root mean square error (RMSE) as the misfit metric in all inversions.

We display the inverted EPARs, α_K and α_μ , with their means, α_{K_0} and α_{μ_0} , and 95% confidence intervals in Figures 4.5 and 4.6. We also show inversion statistics in Table 4.2, including the number of samples in each data subset and the standard deviation of the inverted α_K and α_μ for each data subset. We allow the inversion

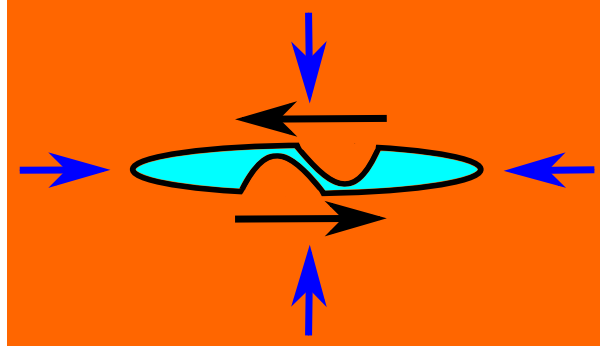


Figure 4.4: A quasi-ellipsoidal pore containing asperities. The interaction of asperities with one another under compressional (blue arrows) and shear (black arrows) forces leads to differences in effective bulk and shear moduli, and hence EPARs.

algorithm to search over the full model space $\alpha \in (0, \infty)$ and find it consistently favours modelling with prolate spheroidal pores, where $\alpha > 1$. The relationship between ellipsoids, prolate and oblate spheroids, and spheres is shown in Figure 4.7.

To calculate the optimal model parameters, α_K^* and α_μ^* , we used the approximate solutions, α_{K_0} and α_{μ_0} , as starting points in non-linear global optimisations which minimised the cumulative bulk or shear modulus misfit over all samples. These optimal EPARs are also shown in Table 4.2. There is a 1.8% difference in α_K^* and a 4.7% difference in α_μ^* between clean and clay-bearing model parametrisations. Thus, we conclude α_K^* and α_μ^* are weakly influenced by clay volume fraction in this data set. The ratio of α_K^* and α_μ^* is also shown in Table 4.2, and interestingly is not unity, as has been observed previously by Fournier *et al.* (2011, 2014, 2018) and Cilli and Chapman (in prep.; Chapter 5) in the case of elastic modelling.

We forward modelled resistivity-bulk modulus and resistivity-shear modulus trends using α_K^* and α_μ^* respectively, as shown in Figure 4.8. The propagated 95% confidence intervals from the sample-by-sample EPAR inversions of Figures

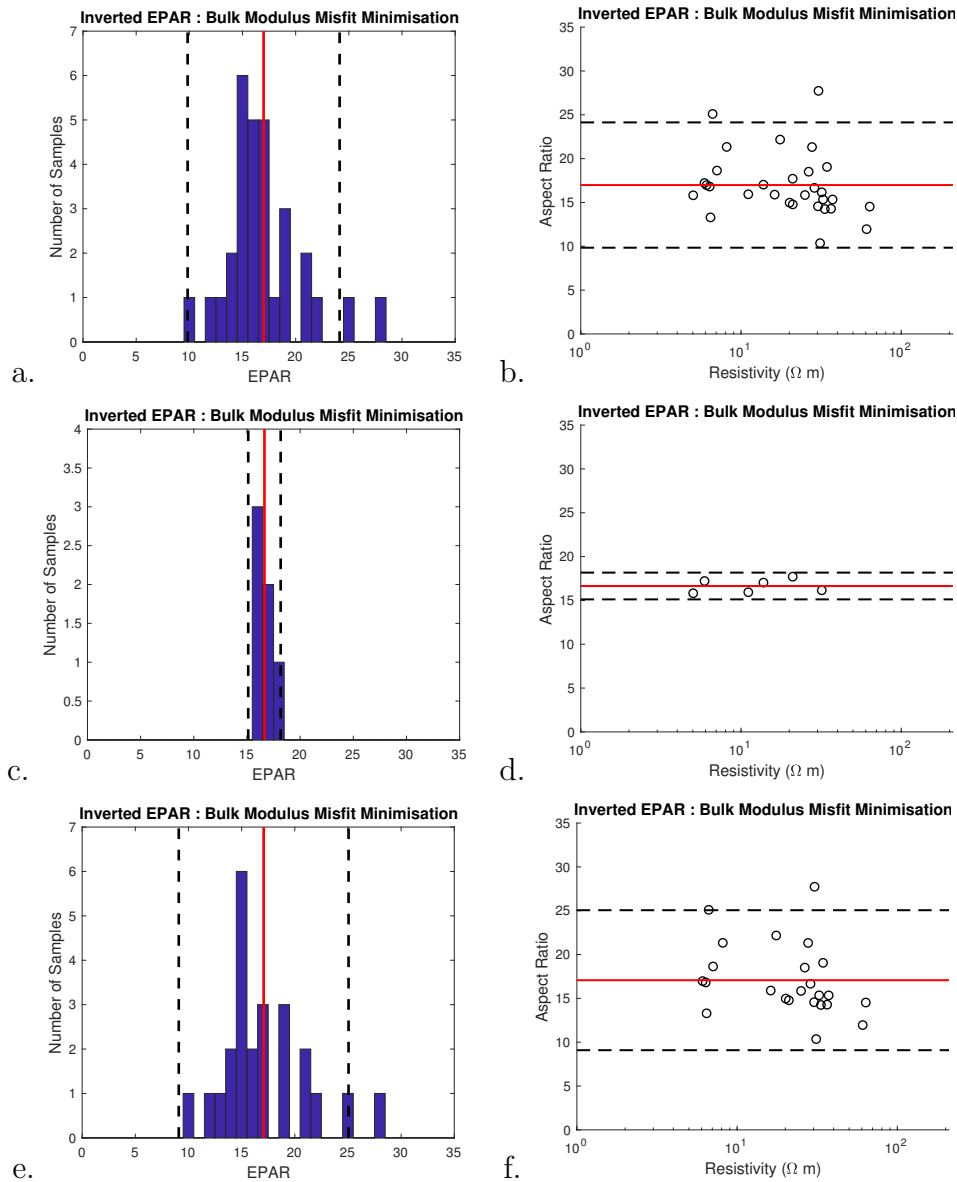


Figure 4.5: Inverted α_K for each core sample by minimising misfit between measured and modelled bulk modulus. Subfigures a and b show all data; c and d show clean data; and e and f show clay-bearing data. Subfigures a, c, and e show histograms of inversion results, while subfigures b, d, and f shows the inverted results against measured sample resistivity. The mean EPAR for each data set is shown (red line), as well as the solution's 95% confidence intervals (dashed black).

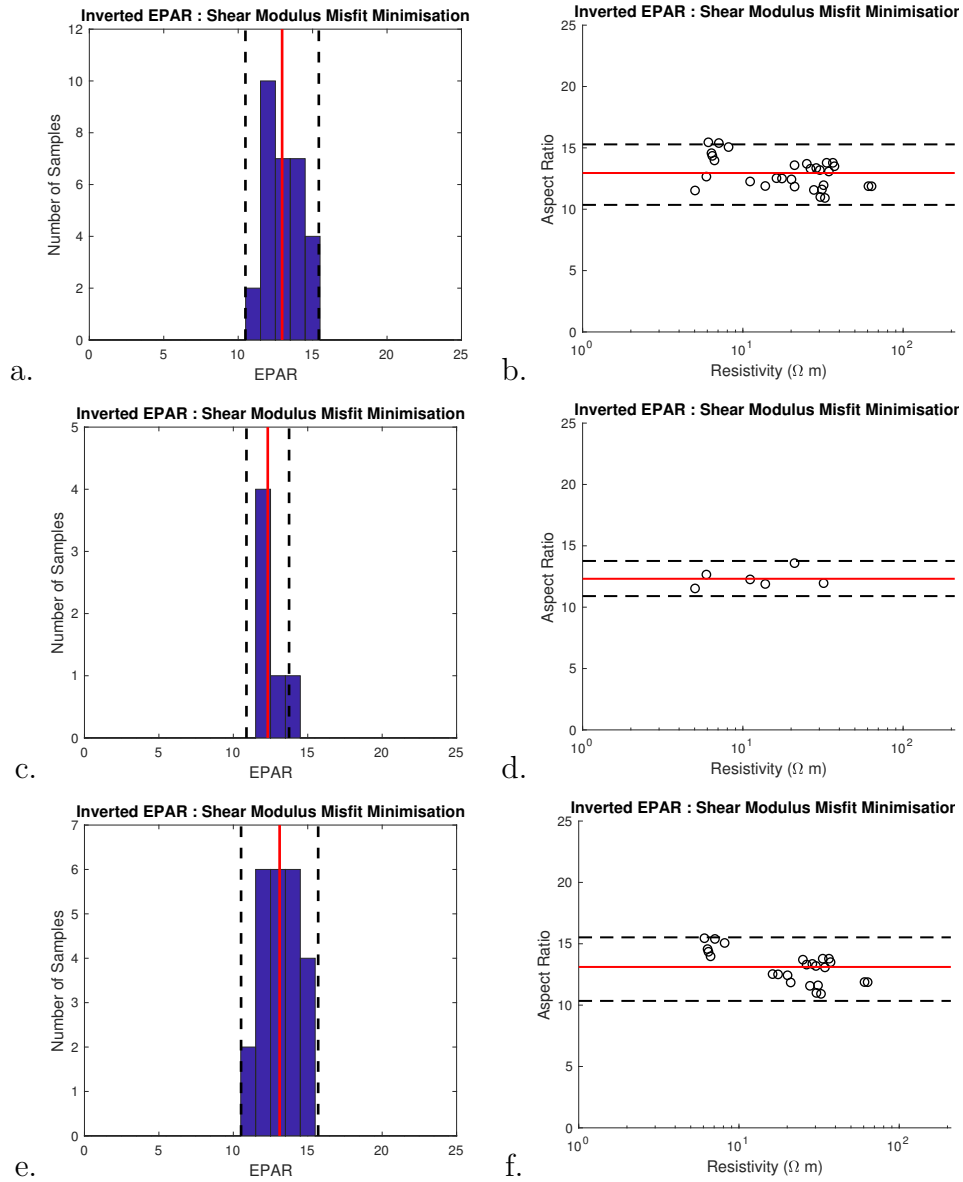


Figure 4.6: Inverted α_μ for each core sample by minimising misfit between measured and modelled bulk modulus. Subfigures a and b show all data; c and d show clean data; and e and f show clay-bearing data. Subfigures a, c, and e show histograms of inversion results, while Subfigures b, d, and f show the inverted results against measured sample resistivity. The mean EPAR for each data set is shown (red line), as well as the solution's 95% confidence intervals (dashed black).

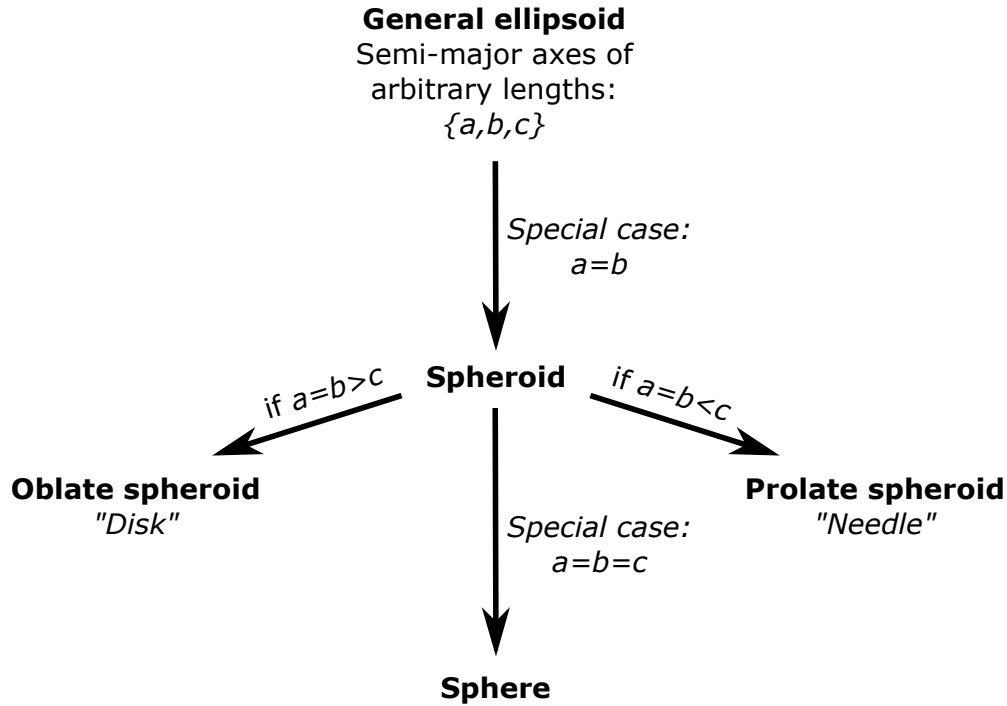


Figure 4.7: Nomenclature for the general ellipsoid and its special cases.

4.5 and 4.6 are also shown. The joint electrical-elastic Hashin-Shtrikman bounds (Carcione *et al.*, 2007) are displayed, with the proposed electrical-elastic modelling method obeying these bounds in all examples. Being asymptotically correct, the electrical-elastic DEM model converges to the water moduli at 100% porosity and the resistivity of water, and converges to the matrix moduli at zero porosity and the resistivity of matrix material. We note the matrix resistivity is never shown on Figure 4.8 and so the low-porosity, asymptotic modulus values (red diamonds) do not coincide with the forward modelled trend line in the Figure.

4.5 Synthetic Well Log Modelling

Estimating a rock's elastic properties from measured electrical properties typically requires the estimation of porosity as an intermediate step. Hence, modelling

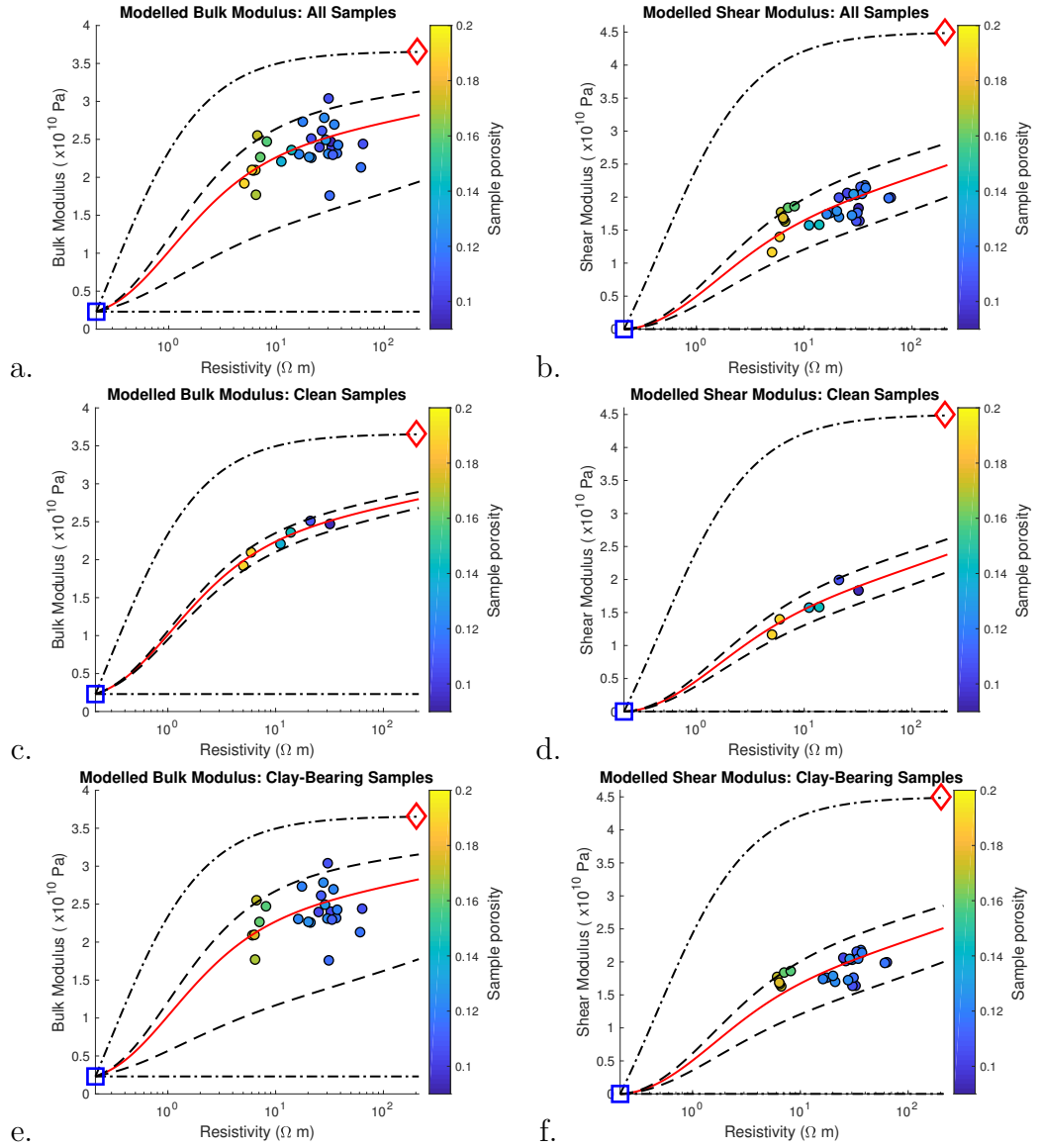


Figure 4.8: The electrical-elastic measurements (circles) of the Han data set and its clean and clay-bearing sample subsets are modelled by the proposed joint electrical-elastic DEM expressions (red trends) using optimal aspect ratios shown in Table 4.2. Bulk modulus (left column) and shear modulus (right column) are modelled using parameters α_K^* and α_μ^* respectively. The 95% confidence intervals on initial EPAR estimates, shown in Figures 4.5 and 4.6, are forward modelled (dashed curves). The electrical-elastic upper and lower Hashin-Shtrikman bounds are displayed (dot-dashed curves). The electrical-elastic model converges exactly to the properties of water (blue square) at 100% porosity and $\sigma^* = \sigma_2$. It also converges to the properties of matrix material when $\sigma^* = \sigma_1$ and porosity is zero (asymptotic modulus value shown by red diamond). Measured sample porosity (marker colour) is also shown for context.

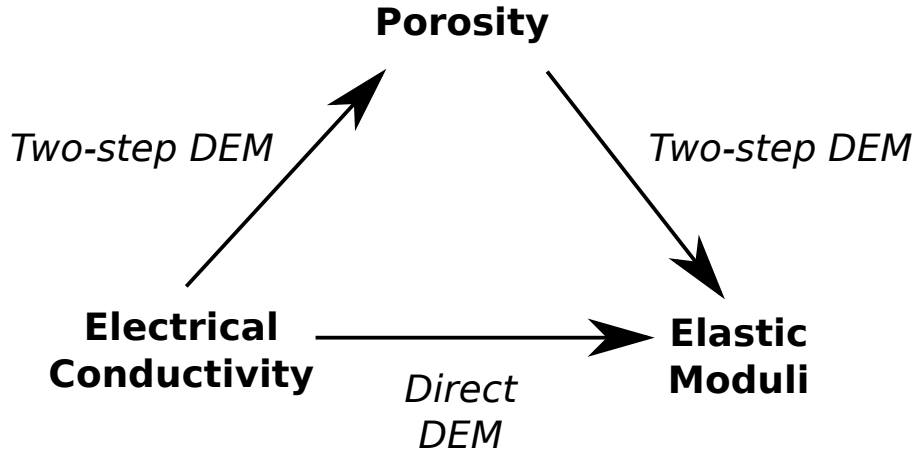


Figure 4.9: Schematic of two methods to estimate elastic moduli from conductivity measurements using DEM models. The traditional, two-step DEM method requires inverting for porosity from resistivity measurements using an electrical DEM model, before estimating elastic moduli given porosity using an elastic DEM model. The direct DEM method, proposed in this chapter, estimates elastic moduli directly from resistivity measurements with no need to estimate porosity.

error can be introduced into this process in two steps: in estimating porosity from electrical properties, and in estimating elastic moduli from porosity. It therefore stands to reason that performing a direct estimation of elastic moduli from resistivity, as proposed in this chapter, may reduce modelling error compared to the traditional, two-step method.

To test whether modelling error is reduced using the direct electrical-elastic DEM method over the two-step DEM method, we first generate synthetic well logs then use both methods to predict velocity logs from resistivity logs, as shown schematically in Figure 4.9. We perform this modelling with parameters pre-calibrated on the Han laboratory data, testing all model performances when calibrated on the full Han data set as well as the clean sandstone subset.

4.5.1 Well log construction

For consistency with the laboratory data of Han, we constructed synthetic well logs (Figure 4.10) using classic rock physics relations applicable to wet sandstones. We created a clean porosity log of 225 m measured depth (1500 samples at 0.15 m spacing), stepping from 5% to 35% porosity at evenly spaced depths. A synthetic measured porosity log was then generated from this clean log by adding Gaussian white noise with a standard deviation (s.d.) of 5% porosity, then assigning one in ten randomly selected samples to a uniform random value between 0% and 40% porosity.

We generated a measured formation factor log from this measured porosity log using Archie's (Archie, 1942) first law with cementation exponent $m = 2$ and a percolating porosity $\phi_0 = 0.02$, adding Gaussian white noise (s.d. = 1) to its natural logarithm.

The V_p log was created from the measured porosity log using the Raymer-Gardner-Hunt (Raymer *et al.*, 1980) relation plus Gaussian white noise (s.d. 100 m/s). From this V_p log, V_s logs were created using the Castagna (Castagna *et al.*, 1993) equation with added Gaussian white noise (s.d. 50 m/s), and density well logs were created by the Gardner (Gardner *et al.*, 1974) equation plus Gaussian white noise (s.d. 2 g/cc).

4.5.2 Well log prediction

Figure 4.9 shows a schematic of the two DEM methods compared in velocity log prediction. In the two-part DEM method, porosity was inverted for using the reformulated, pores-in-matrix electrical DEM model (equation 4.10). Given this estimated porosity log, the elastic DEM model (equations 4.22 and 4.23) was then

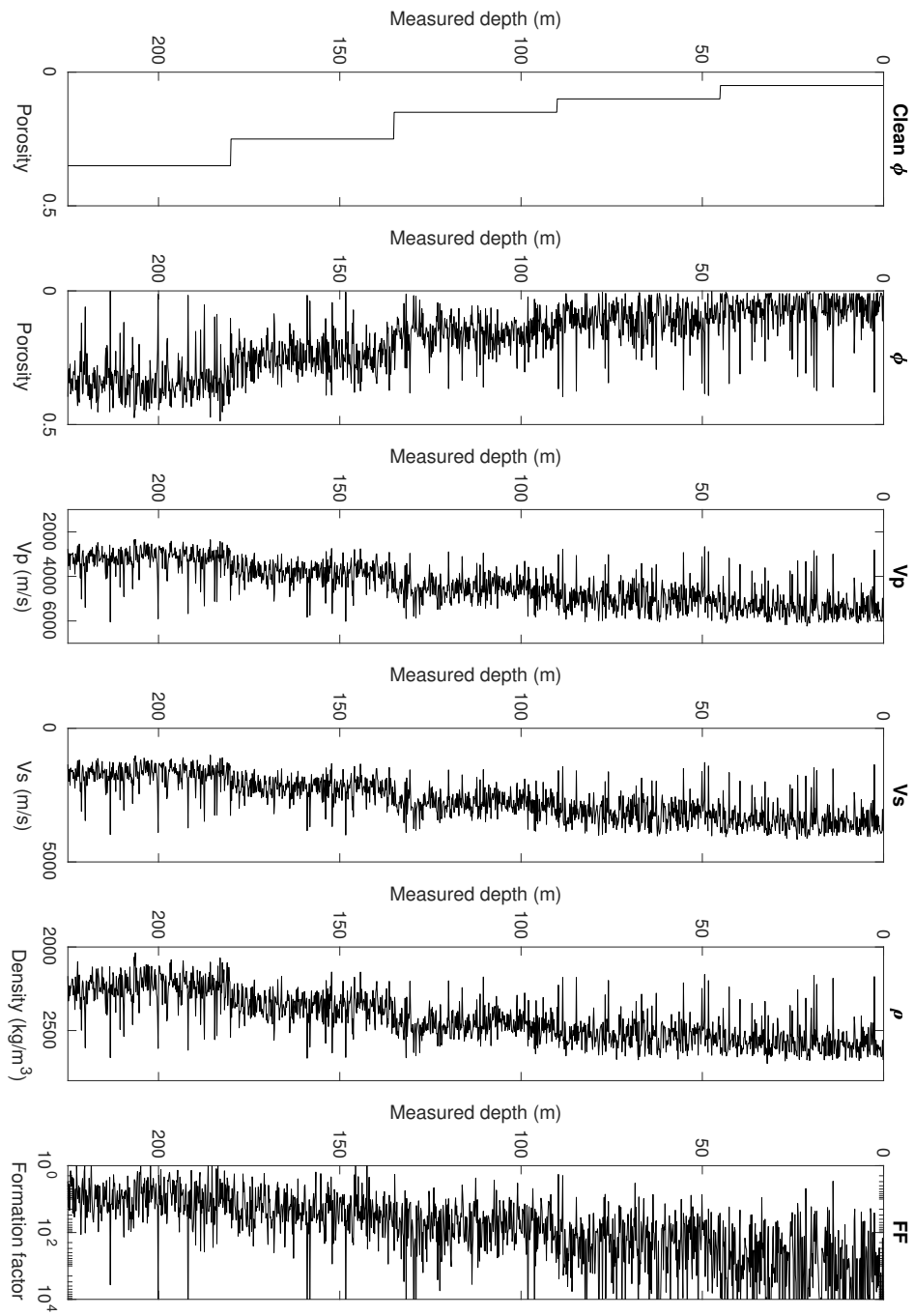


Figure 4.10: Synthetic well logs used in velocity prediction testing. From left to right: Track 1) Clean synthetic porosity used to generate all other logs; 2) Synthetic measured porosity log; 3-6) Synthetic measured V_p , V_s , density, and formation factor logs.

Table 4.3: EPARs calibrated on the full Han data set and the clean sample subset for well log modelling.

Data set	EPAR $(\phi \rightarrow K)$	EPAR $(\phi \rightarrow \mu)$	EPAR $(\phi \rightarrow \sigma)$	EPAR $(\sigma \rightarrow K)$	EPAR $(\sigma \rightarrow \mu)$
All samples	0.128	0.061	0.012	16.4	12.8
Clean samples	0.127	0.055	0.010	16.6	12.3

used to estimate the elastic moduli logs, before calculating velocity logs using the measured density log. Alternatively, the direct DEM method used equations 4.24 and 4.25 to estimate elastic moduli logs directly from resistivity measurements before converting to velocity logs. Using measured densities, we perform these tests twice, using EPARs pre-calibrated on both the full Han data set and the clean sample subset, as shown in Table 4.3.

Figure 4.11 shows the measured and predicted velocity logs when parameterised on clean sandstone measurements, while Figure 4.12 shows velocity log predictions when parameterised on measurements from all samples. Figures 4.13 and 4.14 are simply enhanced views of Figures 4.11 and 4.12 respectively. Interestingly, the direct DEM method generally predicts lower velocities than the two-step DEM method. There is a reduction in velocity modelling error when using the direct DEM method over the two-step DEM method, as shown in Table 4.4.

Hurvich and Tsai (1989) proposed a relative likelihood analysis method for model comparison, the Corrected Akaike Information Criterion, AIC_C , which is an extension of the Akaike Information Criterion (Akaike, 1973). In comparing models, the AIC_C favours models which minimise residuals, and those which have fewer parameters, as elaborated on in Appendix A. The calculated AIC_C

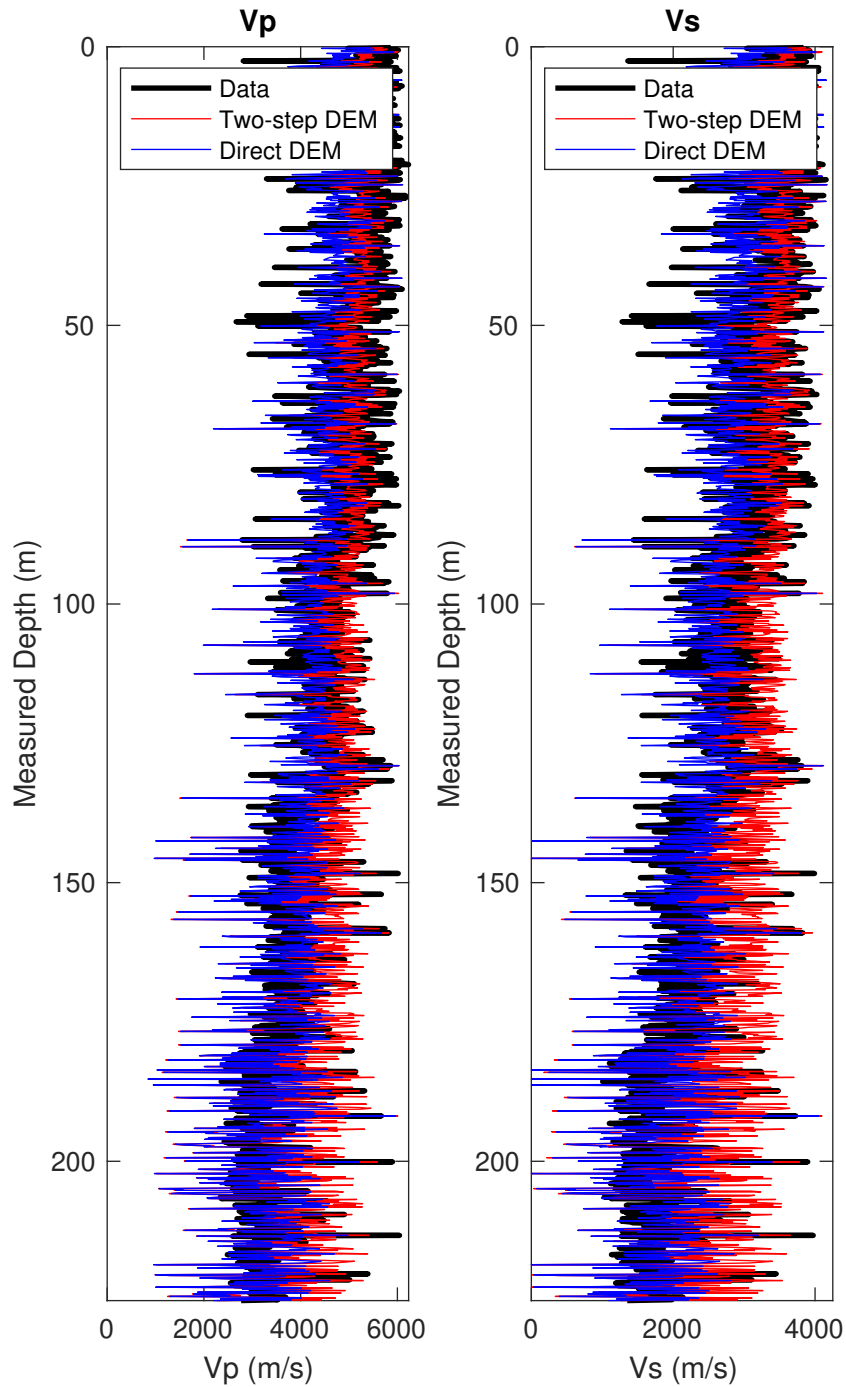


Figure 4.11: Synthetic velocity logs (black) and predicted well logs using the traditional, two-step (red) and direct (blue) DEM methods with parameters calibrated on the clean sandstone subset of the Han data set.

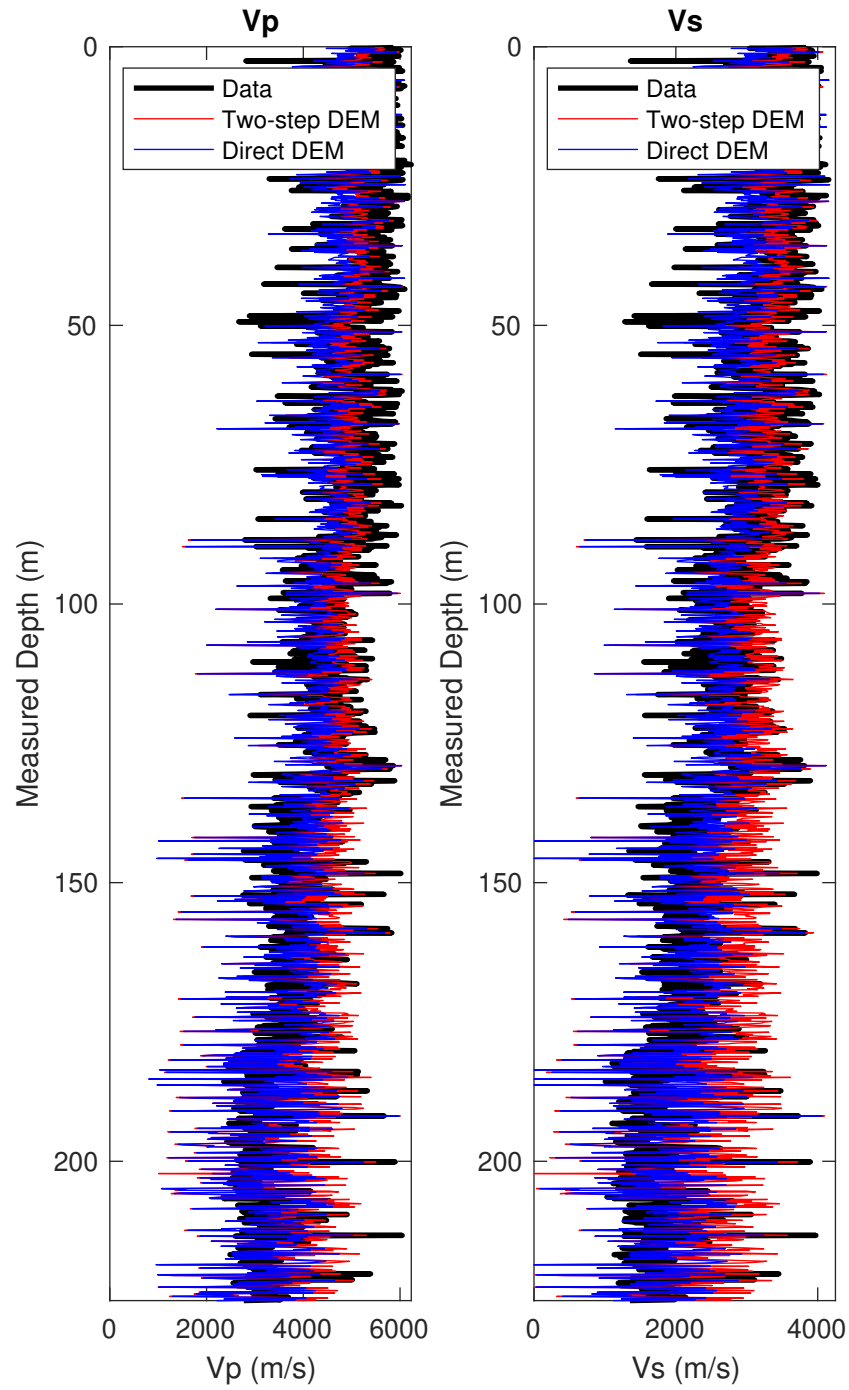


Figure 4.12: Synthetic velocity logs (black) and predicted well logs using the traditional, two-step (red) and direct (blue) DEM methods with parameters calibrated on all measurements in Han data set.

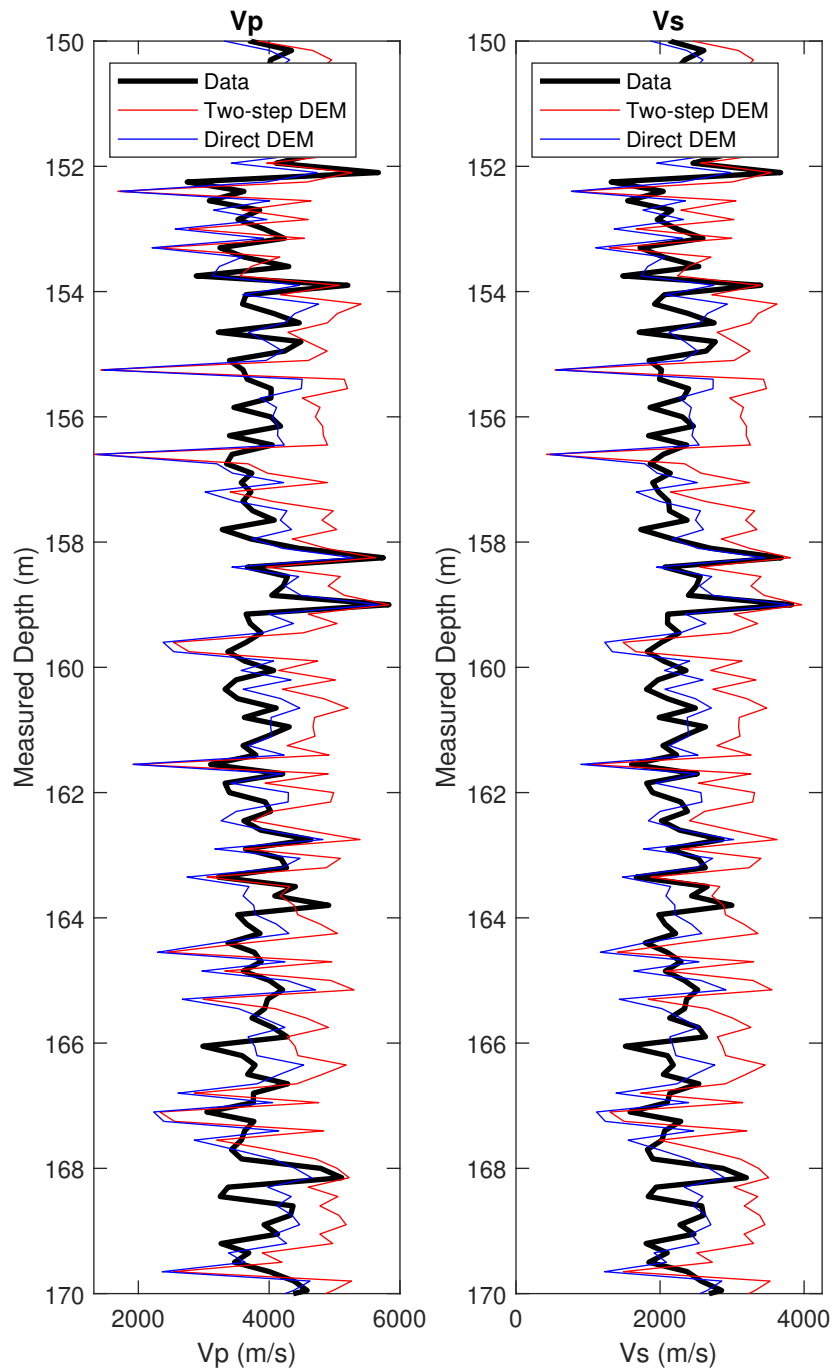


Figure 4.13: An enlarged view of Figure 4.11, showing synthetic velocity logs (black) and predicted well logs using the traditional, two-step (red) and direct (blue) DEM methods with parameters calibrated on the clean sandstone subset of the Han data set.

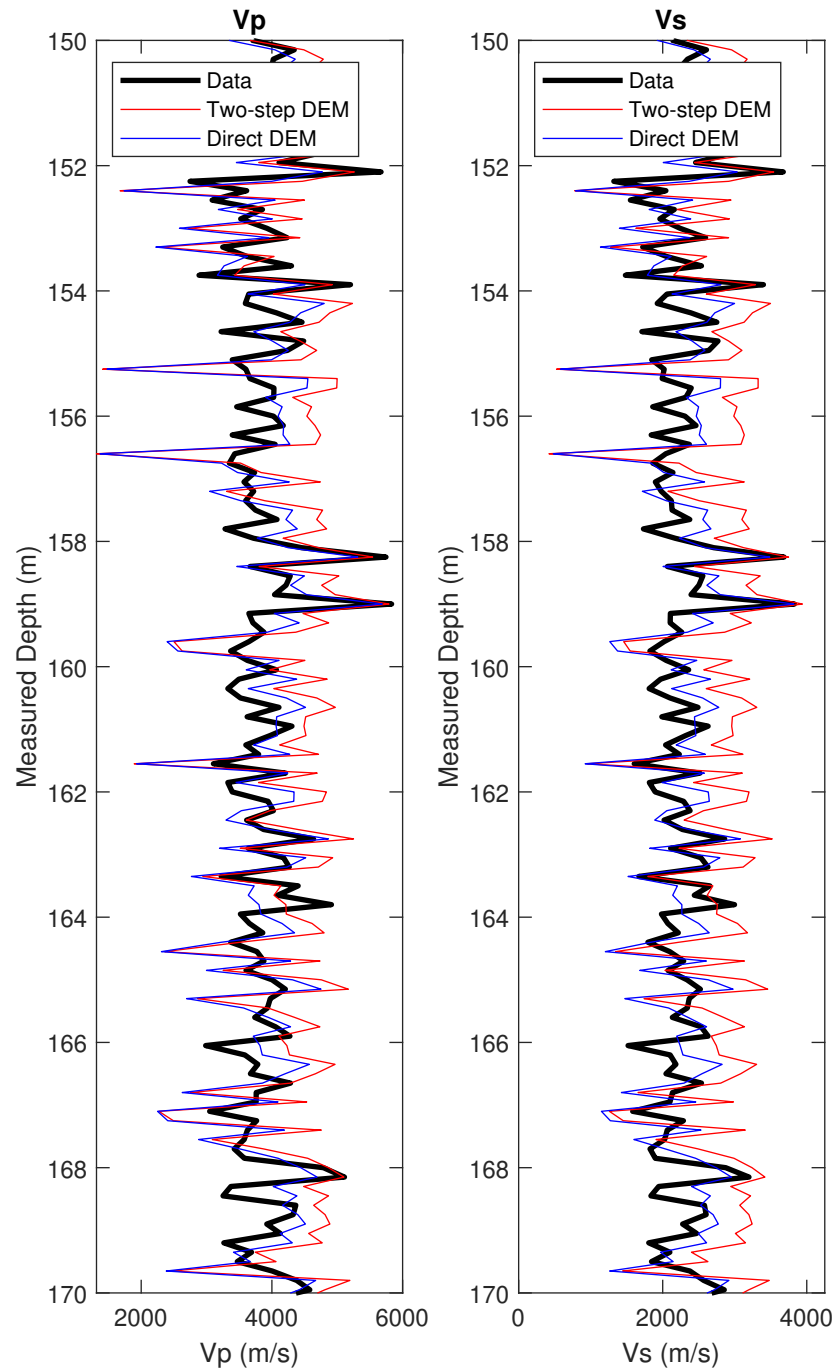


Figure 4.14: An enlarged view of Figure 4.12, showing synthetic velocity logs (black) and predicted well logs using the traditional, two-step (red) and direct (blue) DEM methods with parameters calibrated on all measurements in Han data set.

Table 4.4: Velocity log modelling comparison between direct and two-step DEM methods.

Data modelled	% RMSE decrease on two-step DEM (Full data params)	% RMSE decrease on two-step DEM (Clean params)	ΔAIC_C (Full data params)	ΔAIC_C (Clean params)
V_p	4.2	9.3	129	294
V_s	17.6	23.3	582	795

score for a given model is not in itself indicative of a model's performance. Rather, the difference in AIC_C scores between a candidate and reference model, ΔAIC_C , is the logarithm of the relative likelihood that the candidate model is better than the reference model in describing the data (Burnham and Anderson, 2002).

Cilli and Chapman (in prep; Chapter 5) adapted the AIC_C model comparison rule of thumb of Burnham and Anderson (2004) to DEM modelling, proposing that there is compelling evidence that the candidate model is the better model for describing the measured data when $\Delta AIC_C > 10$. The ΔAIC_C scores for all modelling is shown in Table 4.4. As $\Delta AIC_C \gg 10$ for all tests, we conclude there is compelling evidence by log-relative likelihood analysis that the direct estimation of velocity from resistivity logs is the statistically preferable method.

4.6 Discussion

The proposed electrical-elastic DEM modelling method's preference to approximate pores using prolate spheroids contrasts with typical elastic modelling flows,

where oblate spheroidal pores are commonly used. Modelling elastic properties with disk-shaped, oblate spheroidal inclusions in a DEM framework coincides with the lower Hashin-Shtrikman bounds (Norris, 1985), while modelling with prolate spheroids does not. This may suggest that prolate spheroids may only be preferable for electrical-elastic modelling so long as the rock's elastic moduli are sufficiently high.

Figures 4.5 and 4.6 show a large scatter in α_K for the clay-bearing data sets. The EPAR parameter in a DEM model acts as a fitting parameter, absorbing inaccuracies in other model parameters and the independent variable. The presence of scatter in EPAR when inverted from clay-bearing samples but not clean samples indicates variability in other model parameters due to the presence of clay is being accounted for by the EPAR parameter.

The scatter in inverted EPARs (Figures 4.5 and 4.6) for clay bearing samples is significantly lower in the inversion for α_μ than for α_K . This suggests there is higher modelling uncertainty due to clay content in estimating the effective bulk modulus than shear modulus. Table 4.2 shows the standard deviation in inverted α_{μ_0} for clean samples and the full data set are very similar. It therefore seems the uncertainty in modelled shear modulus is weakly sensitive to clay content in the Han data.

We can expect open microcracks in the samples at 20 MPa effective pressure. As different samples will have different volumes and arrangements of microcracks, the inverted EPARs of each sample should be affected differently by their presence. However, the clean samples all have a very similar EPARs, even at this low effective pressure. This may suggest that the presence of microcracks has only a small effect on a rock's EPARs. It thus may also follow that the deviation in EPARs we have claimed is due to clay content is, in fact, due to clay content as suspected, and not due to variations in microcracking across samples.

The presented model accounts for an isotropic stiffness and conductivity, where the electrical and elastic properties are fully described by scalar moduli, rather than the full electrical resistivity and stiffness tensors. There seems to be no obvious reason why this method couldn't be extended to the anisotropic case, which would be of great interest, as it was shown by North and Best (2014) that even visually isotropic sandstones can have up to 25% anisotropy due to syn-depositional and post-depositional compaction processes.

I have presented an electrical-elastic DEM model which is tested by predicting a rock's elastic properties from electrical measurements. However, I do not predict electrical properties from elastic measurements. This is because the model comprises of coupled differential equations which are solved numerically; one equation relates bulk modulus, while the other relates shear modulus, to electrical measurements. Inverting these coupled differential equations to solve for electrical conductivity from both measured bulk and shear moduli is significantly more difficult, if at all possible, compared to the modelling of elastic moduli presented here. I have not investigated the possibility of inverting these equations to solve for electrical properties from elastic data in this thesis.

Predicting velocities in synthetic well logs using the direct electrical-elastic DEM expressions is shown to result in lower error than the two-step DEM approach. However, all synthetic well logs in this chapter were created with mutually uncorrelated noise. We have not tested how the model will perform in the case of noise which affects both the rock's electrical and elastic properties in a correlated manner. To test the performance of this model in the presence of physically realistic correlated noise, velocity prediction should be tested as is done here but on real well log data.

Model comparison using the AIC_C metric is typically applied when two competing models have a different number of parameters. We have applied it in comparing

the two-step and direct DEM methods assuming both methods have one parameter, EPAR. If the two-step method's AIC_C score were calculated assuming two parameters (one EPAR for each DEM step), the model comparison tests would be even more supportive of the direct DEM method as the most likely best model to use.

Thermal conductivity of a porous rock can also be modelled by DEM theory (Gao and Zhou, 2006). The conceptual idea of “chain rule DEM” presented in this chapter can be extended to modelling thermal-related cross-property relationships of porous rocks (or more generally, a range of composite materials). Appendix B shows theory for the estimation of a porous rock's thermal-electrical, thermal-elastic, and elastic-electrical properties using the chain rule DEM concepts presented in this chapter.

4.7 Conclusion

We argue that reformulating a pre-existing electrical effective medium model allows us to derive new electrical-elastic DEM expressions which effectively model public domain, clay-bearing sandstone measurements. Many electrical-elastic modelling workflows require an estimation of porosity as an intermediate step, however the proposed multiphysics DEM expressions require no porosity information for evaluation and, in turn, allow for reduced error in predicting synthetic velocity well logs from resistivity measurements when compared to a more typical approach which estimates porosity as an intermediate step. Being a DEM model, these proposed expressions are correct in the high and low porosity limit, unlike many empirical electrical-elastic models. Modelling V_p/V_s from resistivity measurements has been challenging to date due to the difficulty of shear modulus modelling from electrical measurements, however the

proposed electrical-elastic DEM modelling method models both V_p and V_s data for laboratory measurements and synthetic well logs, showing potential for improved V_p/V_s modelling from resistivity data. Although a rock's elastic moduli and resistivity are known to be affected by clay, it seems the multiphysics DEM model's single parameter is only weakly affected by clay volume fraction in the measured laboratory data, showing promise for robust multiphysics modelling in lithologies with unknown or variable clay content.

Chapter 5

Modelling the Elastic and Electrical Properties of Rocks with Complex Pore Geometries

5.1 Abstract

Geophysicists depend on rock physics relationships to interpret seismic velocity and resistivity in terms of rock porosity, but it has proven difficult to capture the effect of pore geometry on such relations through simple and easy to apply formulae. Inclusion modelling relates moduli to porosity through an effective pore aspect ratio but often fails to account for observed trends, whereas empirical relations can be hard to extrapolate beyond their range of validity, often giving incorrect results in the low and high porosity limits. We show that

introducing a power law relationship between porosity and effective pore aspect ratio allows inclusion models to reproduce 7 published velocity-porosity and resistivity-porosity relationships. The method produces resistivity-porosity and velocity-porosity relationships which approximate pre-existing empirical relations, providing a first principles basis for extrapolation to other cases of interest. We obtain an analogue for the Castagna and Pickett relationships for wet, calcitic rocks, which is valid in the low and high porosity limits, giving rise to a new, physically derived V_p/V_s versus porosity model. Deviation of resistivity from Archie's law is related to systematic change of pore shape with porosity, and we derive a new relation which fits carbonate data as well as the Humble equation while being physically consistent at high porosity.

5.2 Introduction

An ongoing challenge in rock physics modelling is understanding how elastic and electric properties vary with porosity for various rock types. For electrical resistivity, Archie's (Archie, 1942) law is widely believed to produce acceptable results in clean sandstones (Glover *et al.*, 2000). Significant progress has been made in modelling velocity-porosity trends in the siliciclastic environment (e.g., Raymer *et al.* (1980); Han *et al.* (1986); Vernik and Nur (1992); Dvorkin and Nur (1996)), while carbonates can be more complex (Anselmetti and Eberli, 1993, 1999).

A range of tools, including empirical, bounding, and inclusions methods, has been developed to produce modulus-porosity trends. Empirical methods (E.g., Pickett (1963); Han *et al.* (1986); Castagna *et al.* (1993)) are useful but challenging to extrapolate beyond their pre-calibrated rock types. Bounding average (E.g., Hill (1952)) and modified bound (E.g., Nur *et al.* (1995, 1998)) methods

can yield comparable accuracy to more sophisticated models (Zimmerman, 1991a; Man and Huang, 2011), but can suppress important dependencies on microstructure. Inclusion-based models (E.g., Eshelby (1957); Kuster and Toksöz (1974); Berryman (1980b); Norris (1985)) are often not preferred since the advantages of having a physics-based approach can be outweighed by the unrealistic assumptions made about the pore geometry. Practical application of the above techniques has gained in popularity through Rock Physics Template analysis (Ødegaard and Avseth, 2003; Avseth and Ødegaard, 2004; Avseth *et al.*, 2005), which has become routine in reservoir geosciences (Avseth *et al.*, 2006, 2010; Gupta *et al.*, 2012; Carcione and Avseth, 2015).

The rock physics trends of interest are controlled by pore geometry (Saleh and Castagna, 2004; Xu and Payne, 2009). Hence, to obtain realistic trends for real rocks we have to characterise pore geometry, which is a prevailing challenge in carbonate rock physics (Anselmetti and Eberli, 1993, 1999; Eberli *et al.*, 2003; Fournier *et al.*, 2018).

Several works consider models in which pore or grain aspect ratio varies systematically with porosity (Kazatchenko *et al.*, 2004; Markov *et al.*, 2004; Kazatchenko *et al.*, 2006). A piecewise-constant aspect ratio in porosity was proposed by Kazatchenko *et al.* (2004). Quadratic trends were considered by Aquino-López *et al.* (2011, 2015), while Ellis and Kirstetter (2018) proposed a logarithmic trend.

This chapter argues for the adoption of a power law relationship between pore or grain aspect ratio and porosity. We show the power law relationship, combined with a differential effective medium (DEM) model, fits 7 elastic and electrical data sets with lower misfit than the single aspect ratio DEM model. Additionally, this power law model provides a replacement for the empirical Pickett (Pickett, 1963) and Castagna (Castagna *et al.*, 1993) $V_p - V_s$ relations for wet calcite, which is derived from first principles and correct in both the high and low porosity limit.

A new, first-principles $V_p/V_s - \phi$ model for porous rocks also follows from this power law model. Furthermore, the presented power law model approximates the empirical resistivity-porosity model of Focke and Munn (1987) for carbonates, and has comparable accuracy to the Humble (Winsauer *et al.*, 1952) equation in the range of measured data while being correct at high porosities like Archie's (Archie, 1942) first law. Through this power law model, we also infer the observed, non-monotonic formation factor-porosity trends in carbonate rocks are the result of an interplay between changing pore shape and proportion of resistive material with porosity.

We begin by overviewing the rock physics models considered in this chapter, before inverting three elastic data sets (Verwer *et al.*, 2008; Bakhori, 2010; Fournier *et al.*, 2011) for their porosity-dependent effective pore aspect ratios. Parameterising a power law relation for each data set, we forward model bulk and shear modulus trends for each data set, as well as $V_p - V_s$ and $V_p - V_s - \phi$ trends, which we compare with the Pickett (Pickett, 1963) and Castagna (Castagna *et al.*, 1993) empirical relations. We then invert four electrical (Focke and Munn, 1987) data sets for effective grain aspect ratio and parameterise the corresponding power law relation with porosity. We forward model cementation factor and formation factor trends and compare results with Archie's (Archie, 1942) first law, the Humble (Winsauer *et al.*, 1952) equation, and the empirical trends of Focke and Munn (1987). Throughout this chapter, we compare our model's efficiency with the typical, single aspect ratio model using the Corrected Akaike Information Criterion (Hurvich and Tsai, 1989).

5.3 Modelling Approaches

Rock physics trends are generally studied using collections of samples. In this chapter, we model the relationships between elastic moduli or resistivity and porosity using a number of these collections, each containing laboratory measurements made on many carbonate core samples. We model the data's effective electrical and elastic properties using the differential effective medium (DEM) theory (Mendelson and Cohen, 1982; Berryman, 1992). DEM models are constructed by iteratively adding a small volume of ellipsoidal inclusions into a background material, homogenising this composite's physical properties, and setting this new homogenised material as the background material for the subsequent iteration until the desired inclusion volume fraction is attained. As inclusions are added, the assumption of dilute inclusions becomes less realistic, meaning DEM modelling uncertainty increases with increasing inclusion concentration.

5.3.1 Elastic modelling

The elastic DEM model is described by the coupled differential equations (Berryman, 1992):

$$(1 - \phi) \frac{d}{d\phi} [K^*(\phi)] = (K_2 - K^*(\phi)) P^{(*2)}; \quad (5.1)$$

$$(1 - \phi) \frac{d}{d\phi} [\mu^*(\phi)] = (\mu_2 - \mu^*(\phi)) Q^{(*2)}; \quad (5.2)$$

with the initial conditions $K^*(0) = K_1$ and $\mu^*(0) = \mu_1$. Subscript 1 refers to background properties, while subscript 2 refers to inclusion properties. Thus, in

the case of ellipsoidal pores embedded into a mineral background, K_1 and μ_1 are the mineral bulk and shear moduli; K_2 and μ_2 are the pore fluid bulk and shear moduli; K^* and μ^* are the porous rock's effective bulk and shear moduli; ϕ is the porosity; and P and Q are geometric functions dependent on inclusion aspect ratio, α (Berryman, 1980b). The superscript (*2) in equations 5.1 and 5.2 indicates their base is to be calculated assuming inclusions are embedded in a background material made of the effective medium.

In practice, we call the inclusion aspect ratio, α , the effective pore aspect ratio, which is synonymous with the “equivalent pore aspect ratio” or EPAR, as proposed by Fournier *et al.* (2011, 2014, 2018). EPAR is the model parameter which quantifies the contribution of a rock's pore geometry to its effective elastic moduli.

5.3.2 Electrical modelling

Mendelson and Cohen (1982) used a DEM framework to derive expressions for the overall resistivity of a material consisting of arbitrarily oriented ellipsoidal inclusions in a background of conductive material. By making further assumptions that the inclusions are perfectly resistive and the background is initially water, they derived Archie's (Archie, 1942) first law:

$$F = \phi^{-m}; \quad (5.3)$$

where $F = \sigma_w/\sigma$, with water conductivity σ_w , effective conductivity σ , and cementation factor m . We note equation 5.3 is missing the coefficient a presented

in the more general Humble (Winsauer *et al.*, 1952) equation (e.g., see Glover (2016)):

$$F = a\phi^{-m}, \quad (5.4)$$

suggesting $a = 1$ in the model of Mendelson and Cohen (1982).

Salem and Chilingarian (1999) showed by analysis of well log data that m is strongly dependent on the shape of rock grains and pores. Further, Focke and Munn (1987) showed cementation factor can be non-constant across a range of porosities in carbonates. This dependence of m on pore geometry has been investigated throughout the literature (Mendelson and Cohen, 1982; Nigmatullin *et al.*, 1992; Glover *et al.*, 2000; Glover, 2010). The derivation of Archie's first law by Mendelson and Cohen (1982) showed cementation factor m is a function of grain aspect ratio through depolarisation factors L_p , for $p \in 1, 2, 3$ corresponding to each of the ellipsoidal grain's semi-major axes. Depolarisation factors are scaling parameters which relate the background electrical potential field to the perturbation potential field caused by the presence of the uncharged, conducting ellipsoid.

The potential of a time-independent, uniform electric field $\mathcal{E}$ along the x axis, without loss of generality, is:

$$\varphi_0 = -\mathcal{E}x. \quad (5.5)$$

The presence of an ellipsoid with its p^{th} semi-major axis aligned with the electrical field's x axis causes a perturbation potential field (Landau *et al.*, 2013):

$$\varphi' = \frac{\mathcal{E}xV}{4\pi L_p r^3}; \quad (5.6)$$

in the overall electrical potential field $\varphi = \varphi_0 + \varphi'$, where V is the ellipsoid's volume and r is the distance from the ellipsoid's centroid. If the field and ellipsoid's coordinate systems don't coincide, equation 5.6 is written in tensor form:

$$\varphi' = \frac{\mathcal{E}_i k V}{4\pi L_{ik} r^3}; \quad (5.7)$$

where $\mathcal{E}_i$ is the electrical field aligned with its coordinate system's i^{th} axis, k represents position on the k^{th} axis in the ellipsoid's coordinate system, and L_{ik} is a tensor such that L_p are its three eigenvalues. It can be shown mathematically that $\sum L_p = 1$ (Landau *et al.*, 2013).

The expression for cementation factor m derived by Mendelson and Cohen (1982) is:

$$m = \frac{1}{3} \sum_{p=1}^3 \left\langle (1 - L_p)^{-1} \right\rangle; \quad (5.8)$$

where angled brackets $\langle \cdot \rangle$ denote the average over the distribution of grain aspect ratios present. By assuming spheroidal inclusions, Mendelson and Cohen (1982) made the simplification $L_1 = L$ and $L_2 = L_3 = (1 - L)/2$ in equation 5.8 and averaged over all inclusion orientations for a single grain aspect ratio to produce:

$$m = \frac{5 - 3L}{3(1 - L^2)}; \quad (5.9)$$

as was also found by Gelius and Wang (2008) and Han *et al.* (2015). Extending the elastic model naming convention of Fournier *et al.* (2011, 2014, 2018), we abbreviate effective grain aspect ratio to “EGAR” throughout this chapter.

5.4 Description of Data

We investigate seven public domain laboratory data sets which come from carbonate outcrop, surface borehole, and well cores in various global localities. The data have varied porosity ranges, diverse pore network architectures, and are approximately monomineralic. Three of these laboratory data sets have elastic measurements and four have electrical measurements.

We model three of the four public domain elastic data sets investigated by Kittridge (2014). These carbonate laboratory data sets are from Verwer *et al.* (2008), Bakhorji (2010), and Fournier *et al.* (2011), which we will refer to as the “Verwer”, “Bakhorji”, and “Fournier” data sets respectively for brevity. Appendix C and Kittridge (2014) present further details on these data sets. For elastic modelling, we use only dry measurements made on the subset of cores comprised of approximately 100% calcite in the Bakhorji and Fournier data sets, and 100% dolomite in the Verwer data set. This experimental design allows us to perform all elastic modelling assuming a two-phase rock, composed of a monomineralic matrix and dry pore space. In doing this, we minimise modelling uncertainties due to errors in matrix and fluid compositions.

We use the carbonate data of Focke and Munn (1987) for our electrical modelling tests, as described in Appendix C. We refer to these data as the “FM” data for brevity. These measurements are made on limestones with intergranular porosity; dolostones with intergranular porosity; sucrosic dolomites with intercrystalline

porosity; and moldic oolitic limestones and dolostones. Following Focke and Munn (1987), we model the first three rock types as one data set due to their petrophysical similarities, and model the moldic carbonates as three separate data sets, partitioned into permeability classes: $0 \leq k < 0.1$ mD; $0.1 \leq k < 1$ mD; and $1 \leq k < 100$ mD.

5.5 Elastic Modelling

5.5.1 Bulk modulus modelling

To investigate the relationship between EPAR and porosity in elastic DEM modelling, we first calculated a measured effective bulk and shear modulus for each core using the laboratory-measured P and S wave velocities, and bulk density. With known mineralogy and porosity from experimental data, and mineral moduli shown in Table 5.1, we inverted for the EPAR of each core sample individually by minimising the l_2 -distance between the measured and modelled bulk modulus using equations 5.1 and 5.2.

We represented all pores as oblate spheroidal inclusions, which can approximate cracks when α is small (Walsh, 1965), and coincide exactly with the Reuss (1929) and Hashin-Shtrikman (Hashin and Shtrikman, 1963) lower bounds when $\alpha = 0$ (Norris, 1985).

We displayed the inverted EPARs against measured sample porosity ϕ on a log-log scale, as shown in Figure 5.1. The central observation underpinning this study is the observed linear trend. We placed a line of best fit through each data set's inverted EPARs, having the form:

Table 5.1: Elastic parameters used in modelling.

Constituent	Bulk modulus (GPa)	Shear modulus (GPa)	Density ($\times 10^3 \text{ kg/m}^3$)	References
Calcite	76.8	32.0	2.71	Simmons (1965)
Dolomite	94.9	45.0	2.87	Humbert and Plicque (1972)
Water	2.3	0	-	Han <i>et al.</i> (2011a)
Air	$1.01 \times 10^5 \text{ Pa}$	0	1.29 kg/m^3	Mavko <i>et al.</i> (2009)

$$\log_{10} \alpha = C + \xi \log_{10} \phi. \quad (5.10)$$

We also display each linear fit's 95% confidence interval on C and ξ for each data set, calculated from the linear regressions' covariance matrices.

It follows from equation 5.10 that a best-fitting constant C and gradient ξ become parameters $\{\Gamma, \xi\}$ in the power law:

$$\alpha = \Gamma \phi^\xi; \quad (5.11)$$

where $\Gamma = 10^C$.

As the inverted EPARs in Figure 5.1 are not independent of ϕ , we used the solution parameters $\{\Gamma_0, \xi_0\}$ taken from the line of best fit for each data set as the starting point in a non-linear inversion to find the true solution, $\{\Gamma^*, \xi^*\}$. Solutions $\{\Gamma_0, \xi_0\}$ and $\{\Gamma^*, \xi^*\}$ are shown in Table 5.2 for all data sets, where we see only small updates in solution parameters.

We forward modelled best-fitting $\phi - K$ trends with equations 5.1, 5.2, and 5.11 using the optimal parameters $\{\Gamma^*, \xi^*\}$. We also calculated the best-fitting Single- α , α_{DEM}^* , for each data set and forward modelled the corresponding $\phi - K$ trends for comparison (Figure 5.2). We see in Figures 5.2a and 5.2b that our new model, or “power law” model, is more accurate than Single- α DEM, particularly at low porosities. In fact, the percentage decrease in elastic modelling error by using the power law DEM model over Single- α DEM model is calculated in terms of the residual sum of squares (RSS) statistic, and is seen in Table 5.2 to be over 60% in the Bakhorji data. Figure 5.2c is an example of the power law model collapsing to a Single- α DEM model, with $\xi^* \approx 0$, and $\Gamma^* \approx \alpha_{DEM}^*$ (Table 5.2).

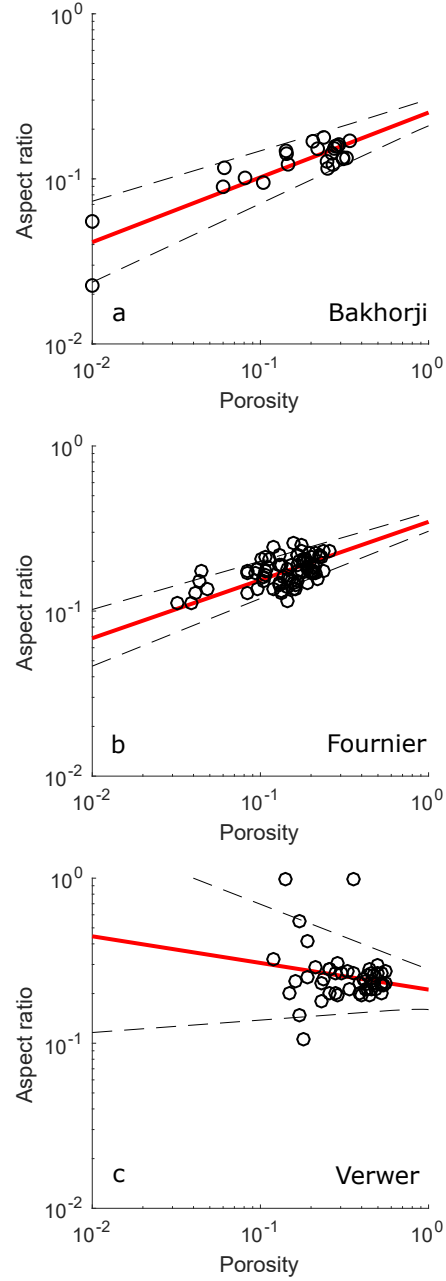


Figure 5.1: Inverted EPARs (circles) from bulk modulus data. Lines of best fit (solid red) and their 95% confidence intervals (dashed black) are shown. Subfigures show a) the Bakhorji dataset; b) the Fournier dataset; and c) the Verwer dataset.

Table 5.2: Elastic modelling results.

Elastic inversion	n	$\{\Gamma_0, \xi_0\}$	$\{\Gamma^*, \xi^*\}$	α_{DEM}^*	Valid ϕ	% SSR decrease on Single- α DEM	Log-relative likelihood ΔAIC_c
Bakhorji (2010) K -inversion	24	$\{0.252, 0.393\}$	$\{0.257, 0.387\}$	0.13	All ϕ	60.5	19.7
Bakhorji (2010) μ -inversion	24	$\{0.273, 0.561\}$	$\{0.257, 0.521\}$	0.12	All ϕ	48.8	13.6
Fournier <i>et al.</i> (2011) K -inversion	80	$\{0.347, 0.352\}$	$\{0.277, 0.238\}$	0.17	All ϕ	22.1	17.9
Fournier <i>et al.</i> (2011) μ -inversion	80	$\{0.379, 0.562\}$	$\{0.296, 0.439\}$	0.13	All ϕ	41.6	40.8
Verwer <i>et al.</i> (2008) K -inversion	51	$\{0.211, -0.116\}$	$\{0.231, -0.046\}$	0.25	$\phi > 10^{-13}$	0.7	-1.9
Verwer <i>et al.</i> (2008) μ -inversion	51	$\{0.185, 0.024\}$	$\{0.204, 0.140\}$	0.17	All ϕ	4.3	0.0

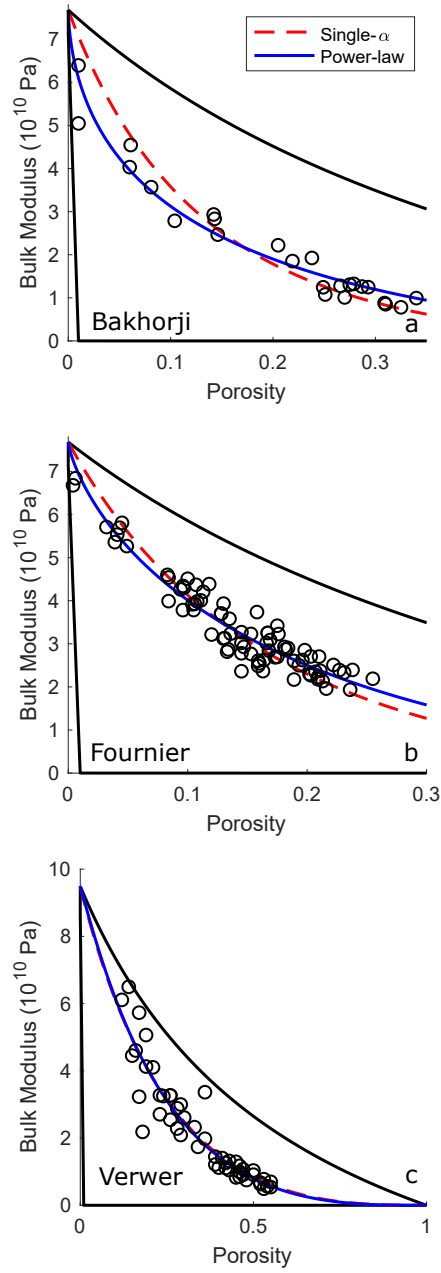


Figure 5.2: Forward modelled bulk modulus from power law DEM model (solid blue), and optimal Single- α DEM (dashed red), as well as measured data (circles), and the Hashin-Shtrikman bounds (solid black bounding curves). Subfigures show a) the Bakhorji dataset; b) the Fournier dataset; and c) the Verwer dataset. Notice the power law and Single- α trends are almost identical in the Verwer data.

To quantitatively establish the preferred model for each data set, we use the Corrected Akaike Information Criterion (Hurvich and Tsai, 1989), as reviewed in Appendix A. Table 5.2 shows the log-relative likelihood (ΔAIC_C) for all bulk modulus elastic modelling comparisons, which is greater than ten for the Bakhorji and Fournier data sets. Following the model selection convention discussed in Appendix A, we conclude there is compelling evidence for the use of the power law model in these cases. In modelling the Verwer data, when the power law model approximates the special case of a Single- α DEM model, both models generate a similar $\phi - K$ trend (Figure 5.2c) but the Single- α model has fewer parameters. The corresponding ΔAIC_C is -1.9, which supports the use of the Single- α model with high uncertainty of which model is better.

There is a theoretical possibility, given certain model parameters, that modelled EPAR can be greater than unity, which would be inconsistent with our modelling assumption of oblate spheroids. As such, we show the range of porosities where the power law model is valid for each elastic data set in Table 5.2.

5.5.2 Shear modulus and V_p - V_s modelling

A shortcoming of elastic inclusion modelling is the practical inability to model both a rock's bulk and shear modulus using the same EPAR. This is observed by Fournier *et al.* (2011, 2014, 2018), and is usually attributed to the presence of asperities in pores. In fact, Fournier *et al.* (2014, 2018) investigate the relationship between bulk and shear modulus EPARs using inclusion models and exploit this relationship to effectively characterise different lithologies. In this section, we first mathematically relate the bulk and shear modulus EPARs of a rock before deriving V_p - V_s and V_p/V_s - ϕ models based on elastic DEM theory and the proposed power law relationship.

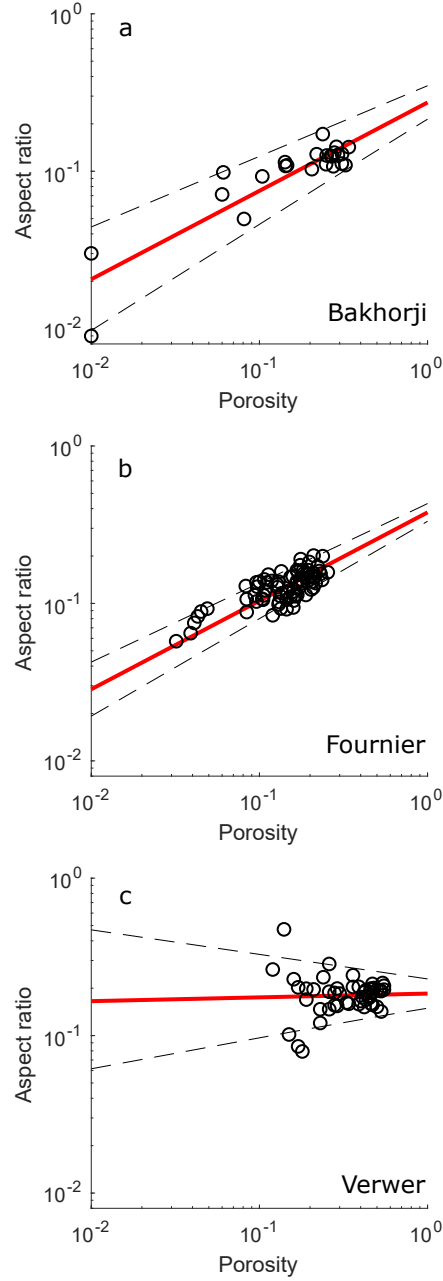


Figure 5.3: Inverted EPARs (circles) from shear modulus data. Lines of best fit (solid red) and their 95% confidence intervals (dashed black) are shown. Subfigures show a) the Bakhorji dataset; b) the Fournier dataset; and c) the Verwer dataset.

We inverted for the shear modulus model parameters $\{\Gamma_\mu, \xi_\mu\}$ of the three elastic data sets by the same method as bulk modulus inversion but minimising shear modulus misfit. Initial and final shear modulus inversion solution parameters are displayed in Table 5.2, denoted by subscript “0” and superscript “*” respectively. Figure 5.3 shows the parameterised linear $\phi - \alpha$ trends on a log-log plot after shear modulus inversion. Forward modelling $\phi - \mu$ trends generates more accurate fits over standard, Single- α DEM methods, in the Bakhorji and Fournier data sets, as seen in Figure 5.4. Comparing the proposed power law model and the best-fitting, Single- α DEM in terms of log-relative likelihoods, there is compelling evidence that the power law model is the best model for use on the Bakhorji and Fournier shear modulus data, with $\Delta AIC_C > 10$ (Table 5.2). It is approximately equally likely the power law and Single- α DEM models are the best model by the AIC_C metric for the Verwer data set, with $\Delta AIC_C = 0.0$.

We denote the rock’s porosity-dependent bulk and shear modulus EPARs by $\alpha_K(\phi)$ and $\alpha_\mu(\phi)$ respectively. Similarly, $\{\Gamma_K, \xi_K\}$ and $\{\Gamma_\mu, \xi_\mu\}$ are their respective model parameters. From equation 5.11, the ratio of $\alpha_K(\phi)$ and $\alpha_\mu(\phi)$ is:

$$\alpha_\mu(\phi) = \frac{\Gamma_\mu}{\Gamma_K} \phi^{\bar{\xi}} \alpha_K(\phi) ; \quad (5.12)$$

where $\bar{\xi} = \xi_\mu - \xi_K$.

In Table 5.2, we observe Γ_μ^* and Γ_K^* are similar for the Bakhorji and Fournier data sets, implying α_K and α_μ are similar in the high porosity limit in these datasets. Given this similarity, we modelled V_p - V_s - ϕ relationships for the Fournier and Bakhorji data using the approximation:

$$\alpha_\mu(\phi) \approx \phi^{\bar{\xi}} \alpha_K(\phi) . \quad (5.13)$$

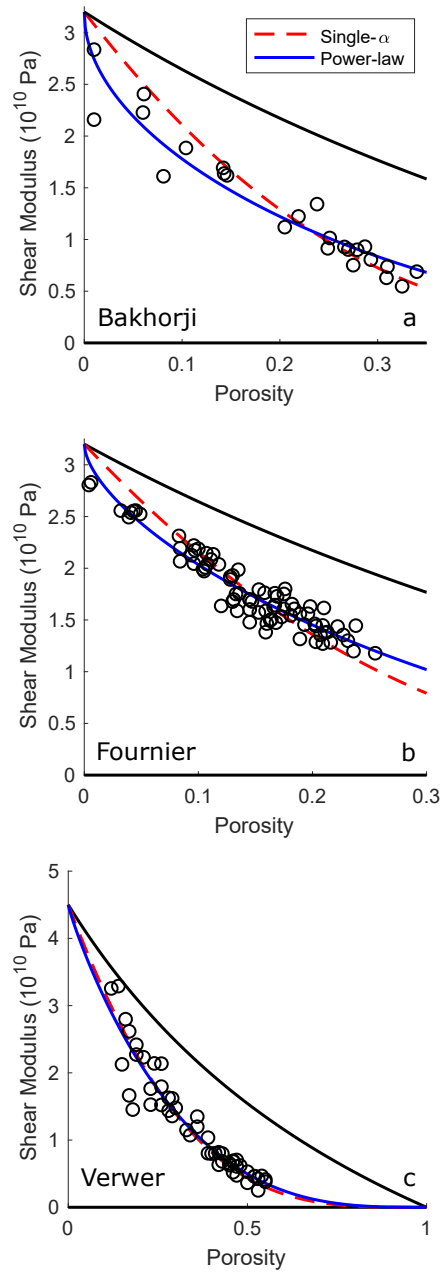


Figure 5.4: Forward modelled shear modulus from power law DEM model (solid blue), and optimal Single- α DEM (dashed red), as well as measured data (circles), and the Hashin-Shtrikman bounds (solid black bounding curves). Subfigures show a) the Bakhorji dataset; b) the Fournier dataset; and c) the Verwer dataset. Notice the power law and Single- α trends are almost identical in the Verwer data.

Figure 5.5 shows the inverted bulk and shear modulus EPARs for each core sample and the forward modelled α_μ - α_K trend (equation 5.13) for the Bakhorji (left) and Fournier (right) data sets. We forward modelled effective bulk and shear modulus trends using the elastic DEM model, followed by dry V_p - V_s trends for each data set using densities from Table 5.1. Water-saturating the dry V_p - V_s trends using Gassmann (1951) fluid substitution, we compare the model's behaviour with the empirical relations of Pickett (1963) and Castagna *et al.* (1993) in Figure 5.5. The proposed model evidently approximates the empirical models in the range of the data, while having the added benefits of being asymptotically correct in the high and low porosity limits, and based on first principles.

5.6 Electrical Modelling

In this section, we examine if an electrical power law relation can produce a more appropriate electrical model for rocks with complex pore geometries than a Single- α electrical DEM model. Substituting equation 5.11 into equations 5.9, and 5.3 assuming rock grains are oblate spheroids, we obtain a new cementation factor and formation factor which depends on both porosity and effective grain aspect ratio, EGAR. Namely:

$$F = \phi \left[\frac{3\phi^{-2\xi} \left(1 - \frac{\arcsin \left[\Gamma \phi^\xi \sqrt{\frac{\phi^{-2\xi}}{\Gamma^2} - 1} \right]}{\sqrt{\frac{\phi^{-2\xi}}{\Gamma^2} - 1}} \right)}{\Gamma^2 \left(\frac{\phi^{-2\xi}}{\Gamma^2} - 1 \right)} - \frac{\Gamma^2 \phi^{2\xi} \left(\sqrt{\frac{\phi^{-2\xi}}{\Gamma^2} - 1} - \arcsin \left[\Gamma \phi^\xi \sqrt{\frac{\phi^{-2\xi}}{\Gamma^2} - 1} \right] \right)^2}{\left(\Gamma^2 \phi^{2\xi} - 1 \right)^3} + 1 \right]^{-1}. \quad (5.14)$$

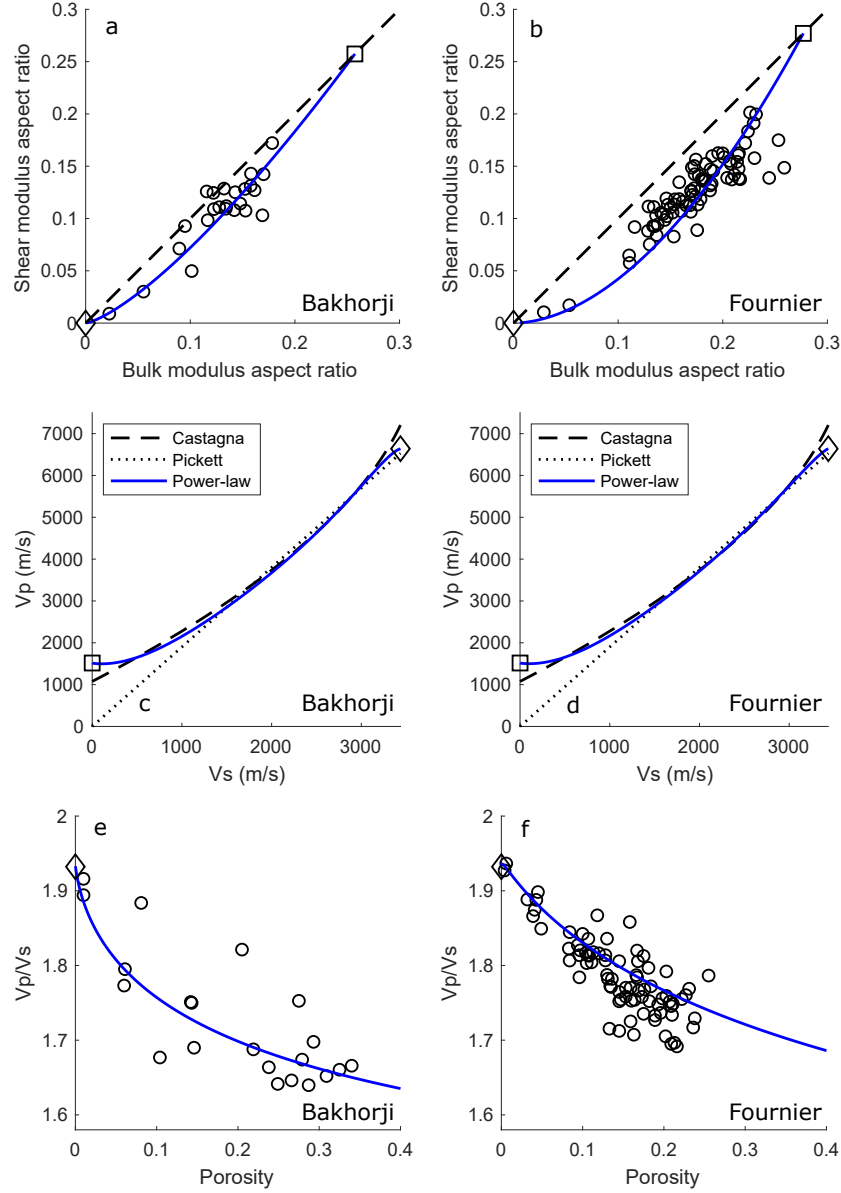


Figure 5.5: Top row) α_μ - α_K trends (solid) and inverted EPARs from laboratory measurements (circles) are shown with a 1:1 line (dashed) for reference. Middle row) Power law DEM V_p - V_s trends (solid) are shown with the Castagna *et al.* (1993) (dashed) and Pickett (1963) (dotted) empirical relations. Bottom row) Power law DEM V_p/V_s - ϕ trends (solid) are shown with laboratory measurements (circles). Diamond markers represent 100% mineral material, while square markers denote 100% fluid. Subfigures a, c, and e show the Bakhorji dataset, while Subfigures b, d, and f show the Fournier dataset.

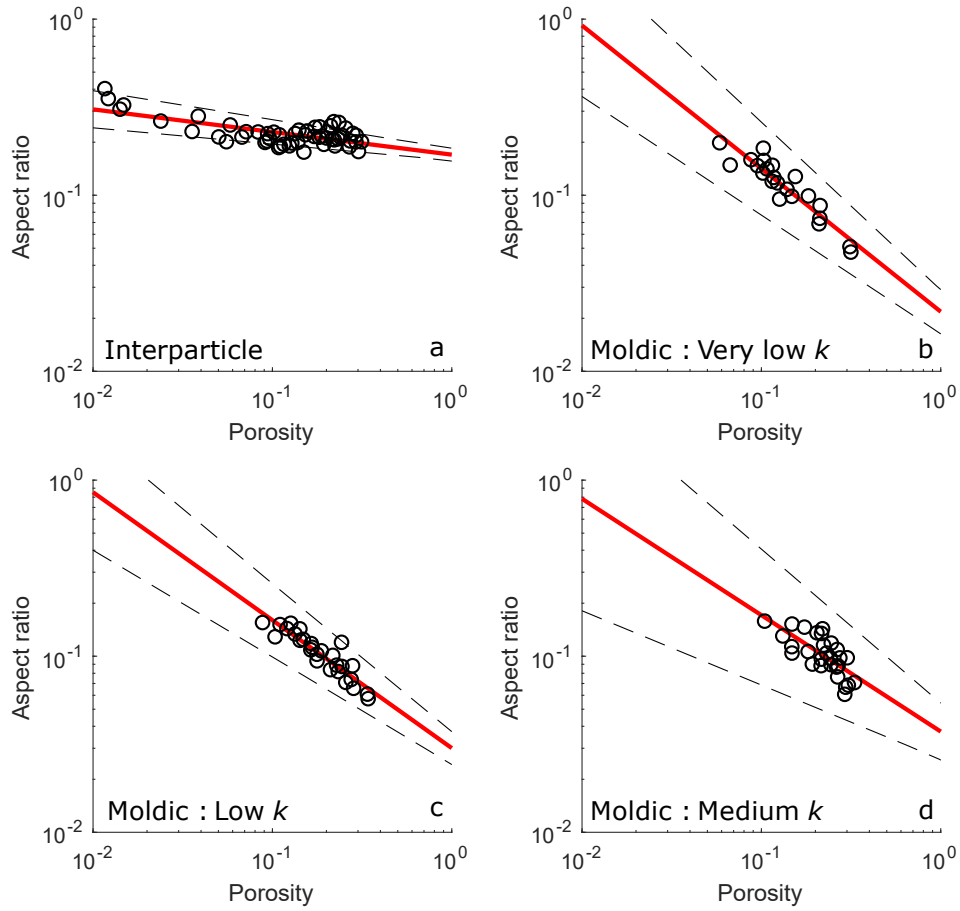


Figure 5.6: Inverted EGARs from the FM data using the electrical DEM model of Mendelson and Cohen (1982). Lines of best fit and their 95% confidence intervals are shown. Subfigures show a) Interparticle porosity; b) Moldic porosity with $0 \leq k < 0.1$ mD; c) Moldic porosity with $0.1 \leq k < 1$ mD; and d) Moldic porosity with $1 \leq k < 100$ mD.

Our EGAR inversion workflow was similar to that of elastic inversion, inverting equation 5.14 for the approximate then true model parameters, $\{\Gamma_0, \xi_0\}$ and $\{\Gamma^*, \xi^*\}$, respectively. We minimised the l_1 -norm of the logarithm of formation factor misfits to reduce preferential fitting at low porosities.

Figure 5.6 shows the calculated EGAR for all datasets and the best fitting line on a log-log scale. Parameterising the proposed power law using these lines of best fit, we forward model formation factor and cementation factor trends for each electrical dataset, as seen in Figures 5.7 and 5.8. The power law model is seen to fit both the formation and cementation factor data more accurately than the best-fitting Archie's law trends while approximating the empirical models of Focke and Munn (1987). We notice the scatter in medium permeability formation factor measurements (Figure 5.7d) leads to all models providing a qualitatively comparable fit. The Humble equation may provide a better fit to the data in Figure 5.8, however it is incorrect in the high porosity limit. As $\xi < 0$ for all four modelled data sets, and as grains are assumed to be oblate spheroids, the power law model is only valid on porosities above that where $\alpha = 1$. When $\alpha = 1$, $m = 3/2$ and the power law model reduces to that of Sen *et al.* (1981) for spherical grains, indicated by an empty black square in Figure 5.7.

Table 5.3 summarises the electrical inversion results, with a 50% to 85% improvement in SSR error on Archie's law across all FM data sets. All modelling log-relative likelihoods were much greater than 10, meaning there is compelling evidence supporting the use of the power law model on all electrical data sets.

The points of inflexion and turning points in the power law forward modelled formation factor trends are not present in Archie's law but are key features in the empirical models of Focke and Munn (1987). We infer these special points are due to the competing effects of inclusion geometry and pore volume fraction on a porous rock's overall resistivity. A porous rock's resistivity decreases with

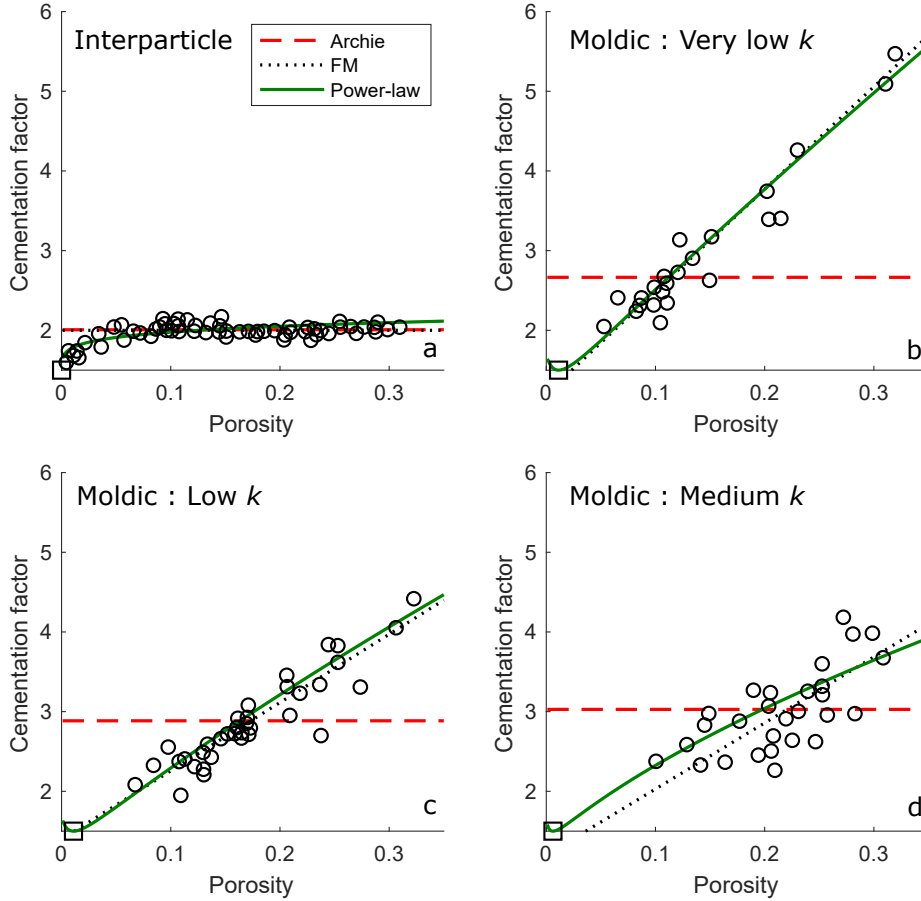


Figure 5.7: Forward modelled cementation factor for the FM data using the power law DEM model (solid), Archie's first law (dashed), and Focke and Munn's empirical relationships (dotted). The power law model's valid porosity lower bound is also shown (square). Subfigures show a) Interparticle porosity; b) Moldic porosity with $0 \leq k < 0.1$ mD; c) Moldic porosity with $0.1 \leq k < 1$ mD; and d) Moldic porosity with $1 \leq k < 100$ mD.

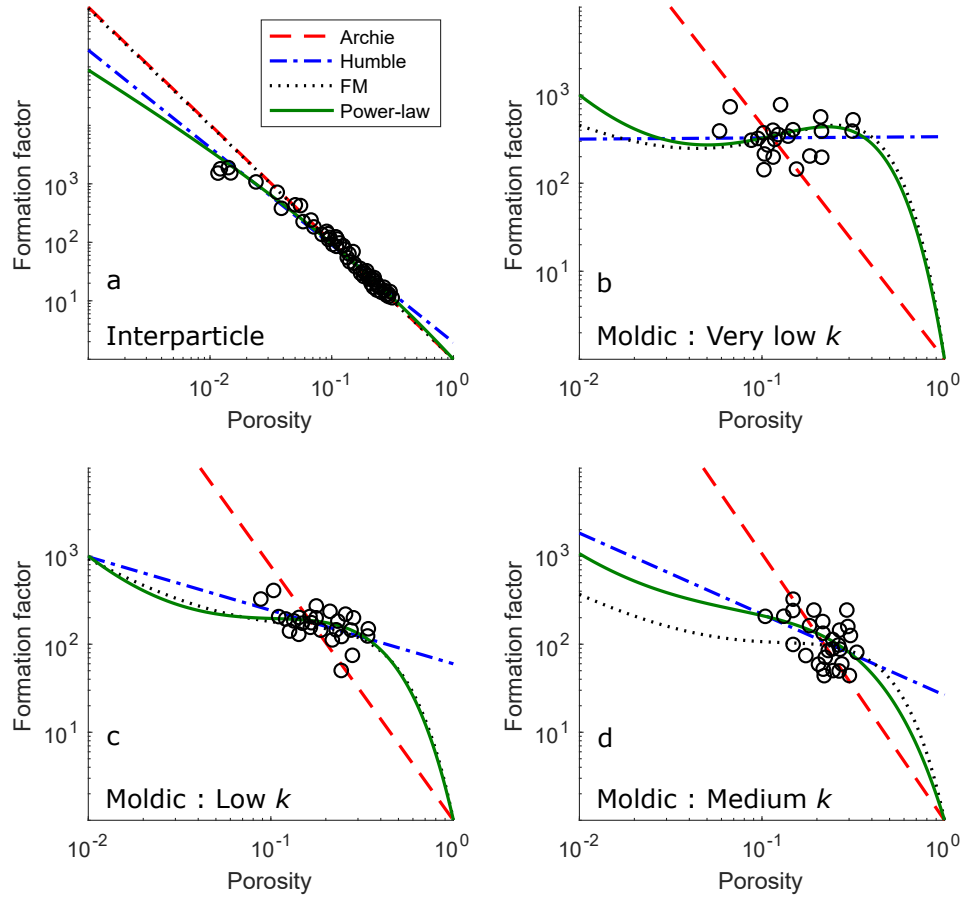


Figure 5.8: Forward modelled formation factor for the FM data using the power law DEM model (solid), Archie's first law (dashed), the Humble equation (dot-dashed), and Focke and Munn's empirical relationships (dotted). Subfigures show a) Interparticle porosity; b) Moldic porosity with $0 \leq k < 0.1$ mD; c) Moldic porosity with $0.1 \leq k < 1$ mD; and d) Moldic porosity with $1 \leq k < 100$ mD.

Table 5.3: Electrical modelling results.

Electrical inversion	n	$\{\Gamma_0, \xi_0\}$	$\{\Gamma^*, \xi^*\}$	m_{Archie}^*	d_{Humb}^*	m_{Humb}^*	Valid ϕ	% SSR decrease on Archie's Law	Log-relative likelihood ΔAIC_C
FM data sets 1-3	55	$\{0.170, -0.129\}$	$\{0.173, -0.122\}$	2.01	1.97	1.67	$\phi > 10^{-6}$	68.8	61.8
FM data set 4a	22	$\{0.022, -0.813\}$	$\{0.018, -0.895\}$	2.67	338	-0.01	$\phi > 0.011$	85.6	40.0
FM data set 4b	27	$\{0.030, -0.727\}$	$\{0.026, -0.808\}$	2.89	59.8	0.61	$\phi > 0.011$	83.5	46.1
FM data set 4c	28	$\{0.037, -0.661\}$	$\{0.036, -0.650\}$	3.03	26.3	0.92	$\phi > 0.006$	52.5	18.3

increasing porosity due to a reduction in the amount of insulating material. The resistivity of a rock comprised of ellipsoidal grains, however, increases with grain eccentricity, as shown by Mendelson and Cohen (1982). These two effects compete in the FM data, where inclusions become more eccentric with increasing porosity, leading to the non-monotonic formation factor trends observed in Figure 5.8. These special points can be solved for analytically through the differential equations derived in Appendix D. The location of all special points for the four FM data sets are also solved for in Appendix D.

5.7 Discussion

We have presented a modified DEM model which fits 7 public domain elastic and electrical data sets more accurately than the typical DEM modelling approach. This improved fitting, however, is at the expense of an extra model parameter, which we have justified using log-relative likelihood analysis. Model parameters ξ and Γ both have physical interpretation. Parameter ξ signifies the rate at which EPAR or EGAR changes with porosity. It follows that ξ may be an indicator of how a rock is affected by the physical processes which alter pore geometry such as diagenesis; a quality which has not been quantified by other inclusion models. Parameter Γ indicates the limiting EPAR or EGAR when $\phi \rightarrow 1$, and therefore indicates the expected stiffness of a rock at high porosities.

We have selected data with highly constrained mineralogy and fluid content to minimise errors in EPAR or EGAR inversion. The extension of this work to multimineralic and multifluid rocks may have larger modelling errors as additional, mixing models must also be used. The proposed power law electrical model is not designed to account for the double-layer effect (Waxman and Smits, 1968), as all solid phases are assumed to be perfect insulators.

The presented power law model has the same number of model parameters as the critical porosity model of Mukerji *et al.* (1995). This power law model can act as an approximate critical porosity model when $\xi < 0$, as well as the Single- α DEM model when $\xi = 0$. The power law model's form when $\xi > 0$ cannot be approximated by the typical critical porosity model, which may make it the preferable model choice in the case of unknown critical porosity.

The sign of parameter ξ^* is positive in the elastic case, and negative in the electrical case. This is due to the elastic model being constructed with inclusions of fluid being embedded into a background of matrix, while the electrical model is constructed with inclusions of grain material being embedded into a background of fluid. Further, all examples here have $\xi = 0$.

A major drawback of modelling with the Humble equation is that it is non-physical when $\phi \rightarrow 1$. The proposed electrical power law DEM model addresses this issue. It models carbonate data with comparable accuracy to the Humble equation (Figure 5.8) and uses the same number of model parameters, but is correct when $\phi \rightarrow 1$, like Archie's first law.

Our claim of a non-constant relationship between porosity and EPAR contrasts with that of Fournier *et al.* (2018), who conclude EPAR is constant in mineralogy and porosity for carbonates with a given dominant pore type. Fournier *et al.* (2018), however, do observe a change in EPAR with porosity for a given carbonate facies (e.g., in Offshore Brazilian Spherulites). Further, Fournier *et al.* (2018) show diagenetic alteration in carbonates, such as vug-forming dissolution, leads to altered EPARs. Our findings may therefore be consistent with the foundational works of Fournier *et al.* (2011, 2014, 2018) if the samples we have investigated are diagenetically altered or differ in dominant pore type across different porosities.

An alternative rock physics model popular for modelling carbonate rocks is the

Extended Biot Theory (EBT) of Sun (1994, 2000, 2004). EBT has separate model parameters which describe the bulk and shear modulus frame flexibility, analogous to inclusion modelling. Under certain assumptions, we can analytically relate the model parameters of EBT to Γ and ξ , as shown in Appendix E. In doing this, we create a new, theoretical link between inclusion modelling and EBT.

Although a power law between inclusion aspect ratio and porosity is based on observation throughout this study, it can be derived from first principles under certain assumptions, as shown in Appendix F. Namely, this power law can arise theoretically when a rock possesses a fractal pore geometry, which is often the case in sandstones and carbonates (Krohn, 1988), is modelled using DEM theory.

5.8 Conclusion

We argue that introducing a power law relationship between porosity and aspect ratio improves the efficiency of modelling the variation of elastic and electrical properties with porosity. Much interpretation of velocity or resistivity in terms of porosity depends on a small number of empirical relationships, which are known to break down in many important cases. Our power law leads to alternative relationships which are derived from first principles, reproduce the empirical relations over much of the porosity range, and are exactly correct in the asymptotic limits. This provides a basis for extrapolating the empirical relationship to different geological conditions, as well as an alternative in situations where the empirical models are known to fail, as is the case with Archie's first law in many carbonates. Use of the power law model to link elastic and electrical properties would require a data set with both measurements, but we hope the proposed models are a step towards multiphysics modelling from first principles.

5.9 Acknowledgements

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Chapter 6

Brazilian Pre-Salt Carbonate Rock Physics Modelling Using Facies- and Porosity-Dependent Effective Pore Aspect Ratios

6.1 Summary

The physical characteristics of Brazilian Pre-salt hydrocarbon carbonate reservoirs pertinent to production can be significantly dependent on the carbonate facies present. Deep water, sub-salt coring for facies interpretation is technically challenging and expensive, however, so facies categorisation using well log

data is desirable. The often similar mineralogies and fluid contents across carbonate facies within a reservoir can make their categorisation from geophysical data challenging. I hypothesise the dependence of a Brazilian Pre-salt carbonate rock's effective elastic moduli on its pore space architecture as a function of porosity may be different for each facies. I test this hypothesis by modelling the facies-dependent bulk and shear modulus of two Brazilian Pre-salt well logs using an elastic differential effective medium (DEM) model and an empirical power law relationship between effective pore aspect ratio (EPAR) and porosity. Using this facies-dependent power law DEM model significantly reduces modelling error for all facies compared to modelling with a single EPAR for each facies, as well as modelling with a single EPAR fitted to the full data set. Log-relative likelihood analysis using the Corrected Akaike Information Criterion provides a parsimony-promoting quantification of the evidence that a candidate model is most likely the best descriptor of a data set, compared to some other reference model, and shows that there is compelling evidence that the power law DEM model is the best model for 7 of 10 carbonate facies when modelling bulk moduli, while at least 4 of 10 facies have compelling evidence that the power law model is the best model for estimating shear moduli. Inaccuracies in the assumed mineral and fluid moduli, however, introduce significant uncertainty into these shear modulus modelling results. Forward modelling velocity logs, I find V_p and V_s logs are both modelled with lowest average misfit when using the facies-dependent power law DEM model. I propose a simple combination of the two power law DEM model parameters which I find may be useful in facies categorisation from well log data in the case of the bulk modulus modelling parameters. I do not, however, reach a conclusion on the usefulness of this new combination of power law parameters in facies categorisation when calculated from shear modulus modelling parameters as the experimental errors were too large to be meaningful due to the inaccurate estimation of mineral and fluid shear moduli.

6.2 Introduction

Brazilian Pre-salt carbonate reservoirs can have multiple carbonate facies present within the reservoir interval (Al-Maghlouth *et al.*, 2017). Given the different depositional and diagenetic history across carbonate facies, a rock's pore space architecture, and hence reservoir properties such as permeability and pore connectivity, can be facies-dependent. It is thus in the interest of exploration and production to understand which carbonate facies are present at depth in a Brazilian Pre-salt reservoir. Brazilian Pre-salt reservoirs, however, are typically situated in deep water beneath complex overburden, and so are technically challenging and expensive to core. It is therefore desirable to be able to estimate the probable facies present in a reservoir using well log measurements.

Although a reservoir's facies may have different depositional and diagenetic histories, which can lead to different pore space architectures and characteristic arrangements of carbonate mud, matrix, and grains, they are often made of similar mineralogies. Effective medium models could be useful in estimating carbonate facies type from well log data, however typical model input variables include the fluid and matrix materials' elastic properties, which can be similar between facies.

Effective medium models often have parameters which capture the effect of pore space architecture on the medium's gross elastic moduli, for example, the frame modulus in the Gassmann model, or the effective pore aspect ratio (EPAR) in inclusion models. This parameter thus seems to be the most promising model feature for discrimination between Brazilian Pre-salt carbonate facies from elastic measurements.

In an inclusion model, the mapping from pore space architecture to a rock's EPAR is many-to-one. For example, a massive rock with a few large, randomly oriented fractures and a rock with the same matrix and pore fluid elastic moduli,

but containing a random distribution of many small cracks rather than large fractures, can possess the same EPAR in a low-frequency inclusion model. The way a facies' pore space architecture changes with a change in pore volume fraction, however, is something which depends on how the rock's mud, matrix, and grains are affected by diagenetic forces. These diagenetic effects could include dissolution, precipitation, dolomitisation, compaction, and may vary from facies to facies. Thus, it seems it may be plausible to categorise a carbonate facies by a characteristic change in its relative frame stiffness as a function of porosity, as opposed to its pore space architecture at any given porosity. In the case of inclusion modelling, this effect is modelled using a porosity-dependent EPAR.

In this chapter, I show that modelling Brazilian Pre-salt elastic properties and velocities from well log data using a power law relation between porosity and EPAR in a DEM model, parameterised differently for each carbonate facies, has a lower modelling error than modelling using a facies-dependent constant aspect ratio and a constant aspect ratio calculated on the full data set, regardless of facies. Further, I investigate whether the model parameters which quantify the facies-dependent porosity-EPAR power law relation for each facies can be used to classify carbonate facies from well log measurements and a pre-calibrated classification scheme.

Given well log data from two Brazilian Pre-salt carbonate reservoir intervals, I show both bulk and shear modulus modelling errors are reduced by using a facies-dependent power law DEM model. However, significant uncertainty in the modelled mineral moduli lowers confidence in these results, with a number of measured samples lying outwith the shear modulus Hashin-Shtrikman bounds calculated using the modelling mineral and fluid moduli. I use the corrected Akaike Information Criterion as a means of model comparison to determine whether the power law or Single-EPAR DEM relations are the best models to describe each facies, finding there is compelling evidence that 7 of 10 facies

are best modelled by the power law relation in bulk modulus modelling, while there is compelling evidence that at least 4 of 10 facies are best modelled by the power law relation in shear modulus modelling, with the incorrect rock constituent compositions leading to increased modelling error.

Forward modelling velocity well logs, I show the facies-dependent power law DEM model predicts V_p and V_s logs with lower average error than the facies dependent and full data Single-EPAR models. I propose a new parameter Ξ which is the ratio of constant and gradient terms of the line which is fitted through EPAR and porosity on a log-log scale to generate each facies' power law model parameters. I test parameter Ξ as a candidate combination of porosity-dependent EPAR model parameters to classify carbonate facies from well log data. I find there seems to be potential to classify facies from Ξ when calculated with model parameters used to model bulk modulus data, however the large uncertainty in shear modulus parameters due to inaccurately estimated mineral shear moduli in modelling leads to very large errors in Ξ calculated from shear modulus derived parameters. Interestingly, I find a qualitative trend between a rock's mud content and its bulk modulus modelled Ξ value. I do not investigate the effects of micrite specifically on Ξ as I have no information of micrite content for the samples. Rather, I consider only the relationship between mud content and Ξ as mud content can be estimated using the interpreted facies at each sample point and the Dunham carbonate classification scheme.

I begin this chapter by briefly reviewing the data processing workflow preceding the rock physics inversion of well log data, followed by facies-dependent elastic inversion of Brazilian Pre-salt carbonate well log data. Using the inverted model parameters, I forward model porosity-elastic modulus trends and analyse elastic modelling statistics. I then forward model velocity logs using the calibrated rock physics models, before investigating the feasibility of Brazilian Pre-salt

facies classification using the porosity-EPAR power law relationship's two model parameters.

6.3 Elastic Inversion

I modelled the elastic properties of Brazilian Pre-salt Wells A and B, with V_p and V_s well logs shown in Figures 6.1 and 6.2. Geological core interpretations were provided by project sponsors for both wells. The 10 most populous interpreted (carbonate) facies are shown in Figures 6.1 and 6.2, and listed in Tables 6.1 and 6.2. Well B was more densely sampled with core interpretations. In fact, an interval of over 20 meters is without geological interpretation in Well A.

To perform facies-dependent elastic modelling using the well logs and geological interpretations of Wells A and B, I interpolated the interpreted facies across both wells onto the well logs' regular 15 cm sampling grid using a nearest neighbour algorithm. I did not know how the locations for core interpretation were chosen; for example, whether interpreted depths marked the tops of new geological units, the middles of units, or points of interest. As such, I introduced an arbitrary threshold of 0.5 m such that any point on the well logs' sampling grids further than this distance from a core interpretation was assigned no facies and discarded from my elastic modelling experiment. The final number of interpolated well log samples corresponding to each facies are shown in Tables 6.1 and 6.2.

With a carbonate facies assigned to each well log sample, I inverted for each sample's effective pore aspect ratio (EPAR) α using the differential effective medium (DEM) model (Berryman, 1992). The input for each sample's inversion was its bulk density, V_p , V_s , and neutron porosity, from well logs. I estimated the fluid in place was inviscid oil, with a bulk modulus of 2.183 GPa (Batzle and Wang,

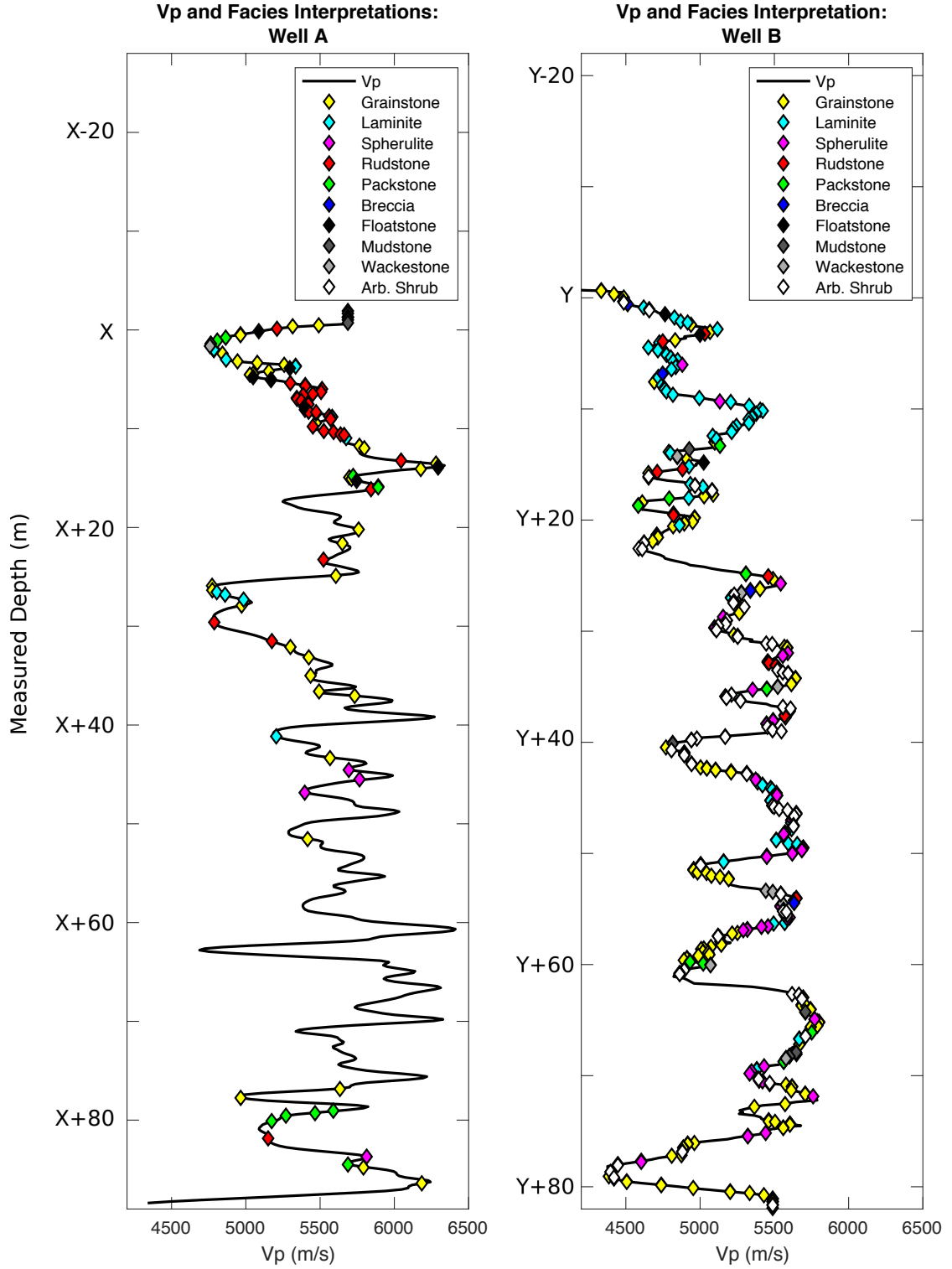


Figure 6.1: V_p logs for Wells A and B. Markers indicate core interpretations of the 10 most populous facies, as used in this study.

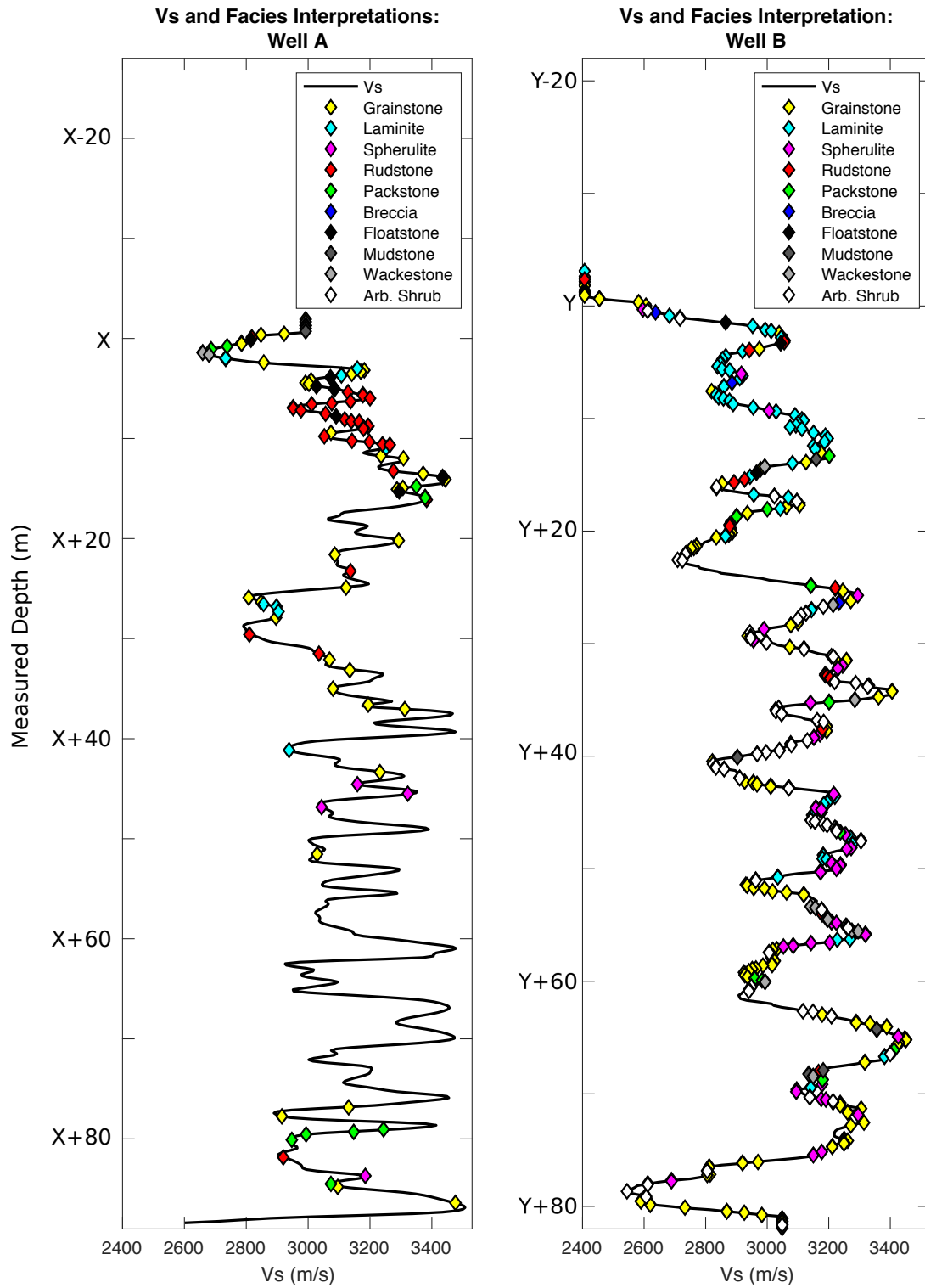


Figure 6.2: V_s logs for Wells A and B. Markers indicate core interpretations of the 10 most populous facies, as used in this study.

1992; Boyd *et al.*, 2015). With no mineralogy logs, I assumed all samples had a mineralogy of 95% calcite and 5% dolomite; an estimate provided by a Brazilian Pre-salt reservoir petrologist who had analysed thin sections from many of the core samples shown in Figures 6.1 and 6.2. I modelled the bulk modulus of calcite and dolomite as 76.8 GPa and 94.9 GPa respectively, and their shear moduli as 32.0 GPa and 45.0 GPa respectively (Mavko *et al.*, 2009). I calculated the effective matrix moduli by the Voigt-Reuss-Hill average of constituent minerals, as was done by Xu and Payne (2009) in their popular carbonate effective medium modelling workflow.

I specifically inverted for each sample's bulk modulus EPAR, α_K , and shear modulus EPAR, α_μ . Parameter α_K is the EPAR which minimises the misfit between a sample's measured and modelled bulk modulus. Similarly, α_μ is the EPAR which minimises the misfit between a sample's measured and modelled shear modulus. Figures 6.3 and 6.4 show the inverted bulk and shear modulus EPARs respectively for each modelled Brazilian Pre-salt facies. The inverted bulk modulus EPARs seem to follow a linear trend in Figure 6.3 on a log-log scale, suggesting a power law relationship may be appropriate to model the relationship between porosity ϕ and bulk modulus EPAR in Pre-salt carbonates, which can be denoted by:

$$\alpha = \Gamma \phi^\xi. \quad (6.1)$$

I used a line of best fit through the inverted bulk modulus EPARs on a log-log scale to calculate bulk modulus EPAR parameters Γ_K and ξ_K , which are shown in Table 6.1 for each facies. The 95% confidence intervals shown in Figure 6.3 do not contain within them the horizontal line for any facies, suggesting qualitatively the power law model may be more appropriate than the more typical Single- α

method, where a constant EPAR is prescribed for all porosities, either for all samples in a well, or using a different, Single-EPAR for every facies.

All facies except Breccia, Floatstone, and Mudstone demonstrate a deviation from the linear trend below $\phi \approx 0.01$ in Figure 6.3. The solution space's lower bound for the inversion of EPAR was 10^{-3} . These samples are evidently sitting on the inversion's solution space lower bound and correspond to rocks with low effective bulk modulus given their composition. In practice, these samples may lie beneath the lower Hashin-Shtrikman bound, where $\alpha = 0$, which would suggest the modelled fluid and matrix moduli are stiffer than those present in the reservoir rock. A small amount of soft material such as clay in the rock's matrix could lower the rock's effective bulk modulus and result in the lowering of α_K if it is not accounted for in the modelled matrix modulus. Diagenetic effects such as differential cementation and dissolution alter particular minerals preferentially, depending on the system's thermodynamic and geochemical state (Henrich and Wefer, 1986; Kenter *et al.*, 1997; Wen *et al.*, 2016). It may be that soft minerals such as clay are present when the rock's porosity is very low, but is removed from the rock as porosity increases. If this effect is not due to the presence of softer materials at lower porosities, it seems the first order power law approximation of the porosity-EPAR relationship is inappropriate at very low porosities, and a higher order relation should be considered.

Figure 6.4 shows the analogous results for the inverted α_μ of each sample. There are many samples which have an inverted shear modulus EPAR of ~ 1 . This suggests a number of samples have too great a shear stiffness to be modelled using the modelled matrix and fluid shear moduli, and thus lie above the Hashin-Shtrikman upper bound. This could be caused by, for example, a higher proportion of dolomite or quartz in the matrix than was modelled, both of which have a higher shear modulus than calcite (Mavko *et al.*, 2009). However, having no extra information available on the reservoir's mineralogy, I fit a linear trend

through all samples as shown in Figure 6.4 to calculate shear modulus EPAR parameters Γ_μ and ξ_μ , which are shown in Table 6.2.

The DEM model is only valid where $0 \leq \alpha \leq 1$. I therefore calculate the valid domain for the bulk modulus and shear modulus power law model given each facies' parameterisation, as shown in Tables 6.1 and 6.2.

6.4 Forward Modelling Elastic Moduli

I calculated the best constant bulk and shear modulus EPARs for each facies, the “facies-dependent Single- α_K /Single- α_μ ”, or α_{Facies}^* , as those which minimised the forward modelled misfit in bulk or shear modulus respectively over all samples for a given facies. Similarly, I calculated the best constant bulk and shear modulus EPARs for all samples, the “full data Single- α_K /Single- α_μ ”, or α_{Full}^* , as those which minimised the forward modelled misfit in bulk or shear modulus respectively over all well log samples, regardless of interpreted facies. I then compared the modelling performance of the facies-dependent power law DEM model with the Single- α DEM models.

For every facies, I used parameters Γ_K and ξ_K in Table 6.1 to calculate the porosity-dependent bulk modulus EPAR α_K by equation 6.1. Likewise, I calculated the porosity-dependent α_μ for all facies using Γ_μ and ξ_μ from Table 6.2. I then used these porosity-dependent EPAR trends to forward model porosity-bulk modulus trends (Figure 6.5) and porosity-shear modulus trends (Figure 6.6) for every facies. I also forward modelled the optimal facies-dependent and full data Single- α trends for comparison, and displayed the Hashin-Shtrikman bounds for reference, as seen in Figures 6.5 and 6.6.

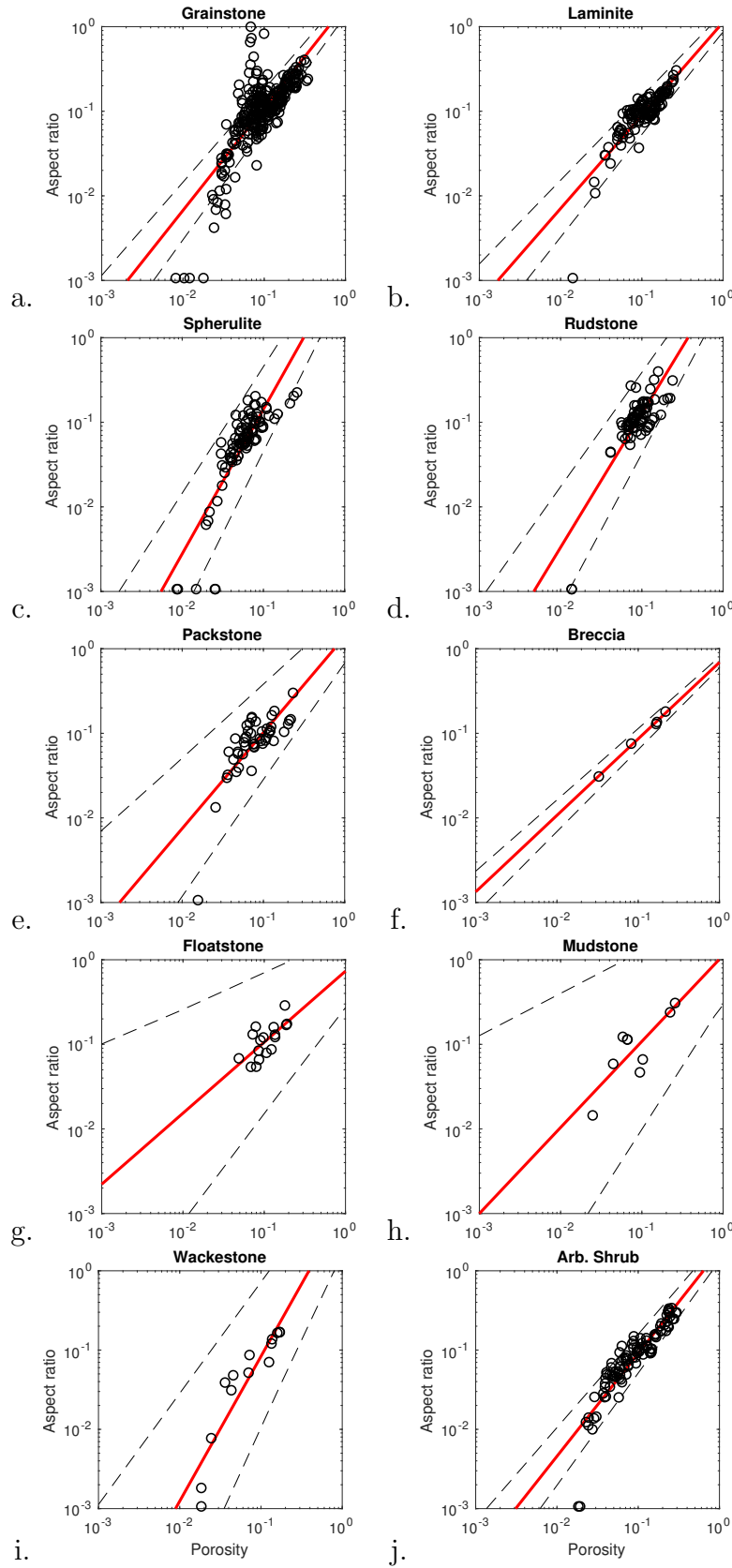


Figure 6.3: Inverted bulk modulus EPARs (circles) from minimising misfit in modelled and measured bulk modulus for the 10 most populous interpreted Pre-salt facies over Wells A and B. Lines of best fit (red) are used to calculate Γ_K and ξ_K . 95% confidence intervals on the linear fit are shown (dashed black).

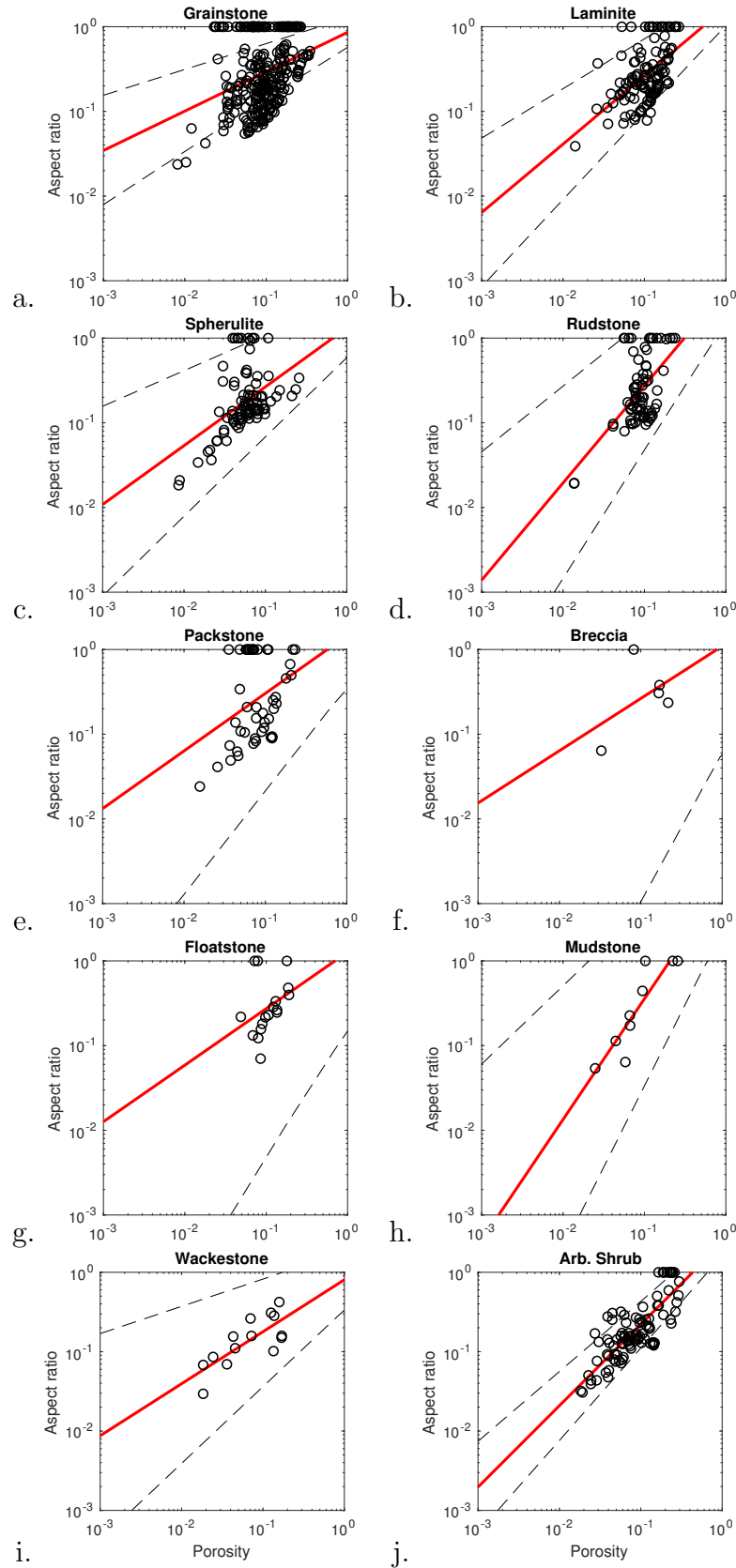


Figure 6.4: Inverted shear modulus EPARs (circles) from minimising misfit in modelled and measured shear modulus for the 10 most populous interpreted Pre-salt facies over Wells A and B. Lines of best fit (red) are used to calculate Γ_μ and ξ_μ . 95% confidence intervals on the linear fit are shown (dashed black).

Figure 6.5 shows samples at low porosities falling below the lower Hashin-Shtrikman bounds in most facies, as was speculated above upon analysis of inverted bulk modulus EPARs. The forward modelled power law model has more versatility in its form when compared to the Single- α methods, leading to a reduction in the sum of the squared residuals (SSR) error of between 28.5% and 99.3% in bulk modulus modelling when compared to facies-dependent Single- α modelling for each facies, and 31.5% and 99.4% when compared to full data Single- α modelling, as shown in Table 6.1. Interestingly, the reduction in SSR error between full-data and facies-dependent Single- α modelling is notably less, ranging between 0.1% and 25.14% depending on facies.

The power law DEM model fits data with smaller misfit in bulk modulus modelling, however, has an extra model parameter. I use the Corrected Akaike Information Criterion (AIC_C) (Akaike, 1973; Hurvich and Tsai, 1989) for model comparison. AIC_C analysis, reviewed in Appendix A, is used to calculate the value ΔAIC_C , which is the logarithm of the relative likelihood that one candidate model is the best descriptor of a data set compared to a reference model. As described in Appendix A, I follow the convention that if the difference in AIC_C values, ΔAIC_C , between the power law DEM model and another candidate model is $\Delta AIC_C > 10$, there is compelling evidence that the power law DEM model is the best model to use to describe the observed data. Table 6.1 shows there is compelling evidence that the power law model is the best model to use in modelling porosity-bulk modulus data for all facies except the Breccia, Floatstone, and Mudstone facies, when compared to both the facies-dependent and full data Single- α models. The power law model is still likely the best model for the Breccia, Floatstone, and Mudstone facies, however there is substantial evidence the Single- α models may be the better models as $\Delta AIC_C < 10$.

Figure 6.6 shows samples above the upper Hashin-Shtrikman bounds in most facies, as suspected from inspection of inverted shear modulus EPARs. As with

the bulk modulus power law model, the forward modelled shear modulus power law model has more versatility in its form when compared to the Single- α methods. This leads to a reduction in SSR error of between 1.5% and 60.9% in shear modulus modelling when compared to facies-dependent Single- α modelling for each facies, and 2.0% and 70.6% when compared to full data Single- α modelling, as shown in Table 6.2. The reduction in SSR error between full-data and facies-dependent Single- α modelling ranged between 0.46% and 26.35%, depending on the facies.

Table 6.2 shows there is compelling evidence that the power law model is the best model to use in modelling porosity-shear modulus data for the Grainstone, Laminite, Rudstone, and Arb. Shrubs facies when compared with the facies dependent Single- α model. There is also compelling evidence the power law model is the best model to use in modelling the Spherulite facies when compared with the full data Single- α model. In the case that $\Delta AIC_C < 0$, such as in the Breccia and Packstone shear modulus modelling, the Single- α model is more likely the best model to use. This is true even though there is a reduction in modelling error by using the power law model, as the power law model is penalised in log-relative likelihood analysis for having an extra parameter.

6.5 Forward Modelling Velocities

With parameters Γ , ξ , α_{Facies}^* , and α_{Full}^* calculated for both bulk and shear modulus modelling over all facies, I used the porosity log to forward model the bulk and shear modulus of each measured well log sample given its interpreted facies. With these modelled bulk and shear modulus logs and the measured bulk density logs, I forward modelled synthetic, facies-dependent V_p and V_s logs using the power law and Single- α models, as shown in Figures 6.8 and 6.7.

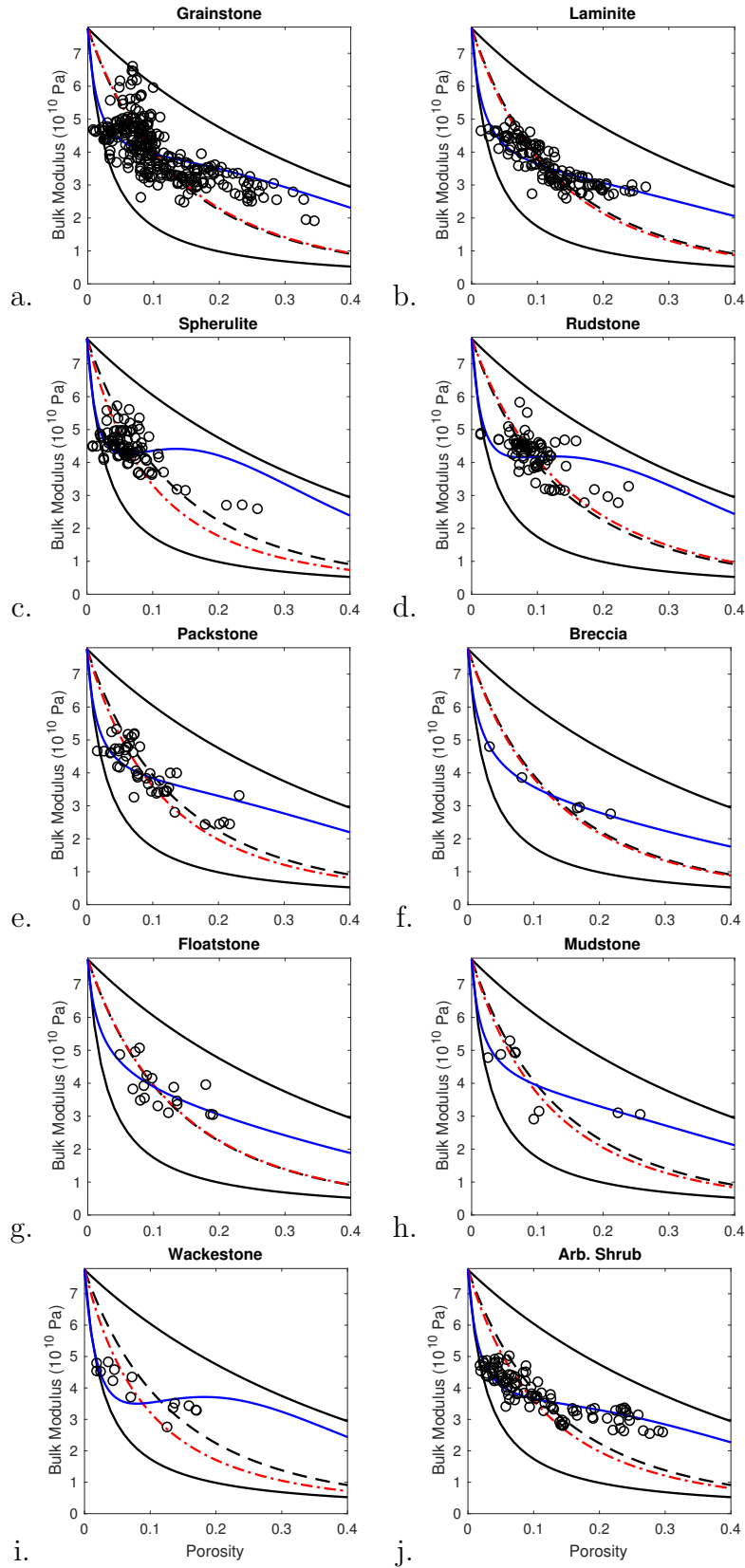


Figure 6.5: Measured bulk modulus data from Wells A and B (circles) with best fitting power law DEM model (solid blue), facies-dependent Single- α DEM (dot-dashed red), and full data set Single- α DEM (dashed black). Hashin-Shtrikman bounds are shown (solid black).

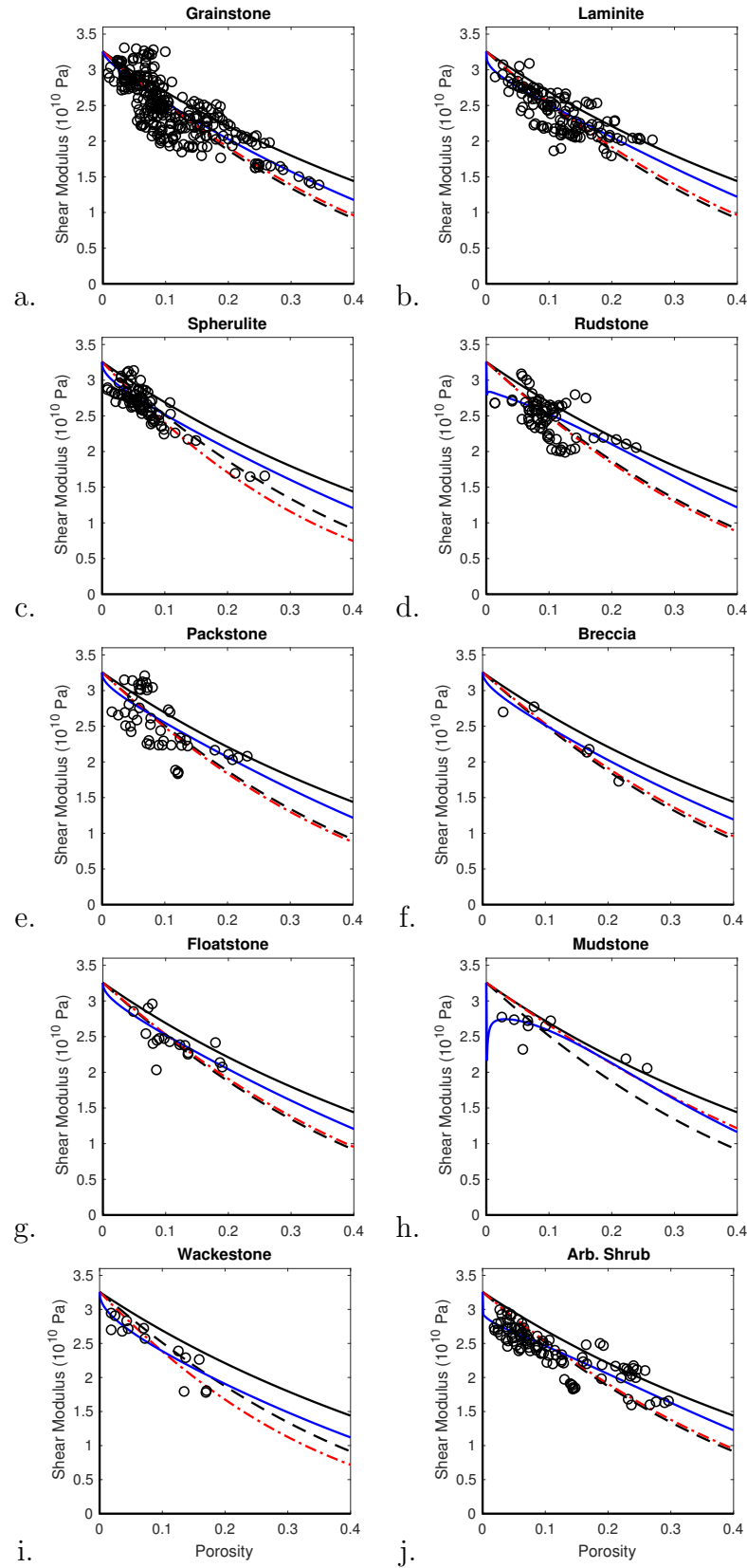


Figure 6.6: Measured shear modulus data from Wells A and B (circles) with best fitting power law DEM model (solid blue), facies-dependent Single- α DEM (dot-dashed red), and full data set Single- α DEM (dashed black). Hashin-Shtrikman bounds are shown (solid black).

Facies	n	$\{\Gamma, \xi\}$	α_{Facies}^*	α_{Full}^*	Valid domain	Facies-dep. Single- α ΔAIC_C	Full data Single- α ΔAIC_C	Δ SSR on facies-dep. Single- α (%)	Δ SSR on full data Single- α (%)	Δ SSR between facies-dep. & full data Single- α (%)	Ξ
Grainstone	281	{1.79, 1.21}	0.11	0.11	$\phi < 0.62$	164.5	165.5	44.7	44.9	0.32	0.21
Laminite	123	{1.13, 1.10}	0.10	0.11	$\phi < 0.90$	137.7	140.8	67.9	68.7	2.50	0.05
Spherulite	90	{7.30, 1.71}	0.07	0.11	$\phi < 0.31$	61.3	87.4	50.6	63.0	25.14	0.51
Rudstone	76	{4.82, 1.58}	0.12	0.11	$\phi < 0.37$	23.4	26.6	28.5	31.5	4.18	0.43
Packstone	48	{1.39, 1.13}	0.09	0.11	$\phi < 0.75$	21.1	28.5	38.5	47.4	14.35	0.13
Breccia	5	{0.69, 0.90}	0.10	0.11	All ϕ	5.1	5.2	99.3	99.4	0.93	-0.18
Floatstone	17	{0.73, 0.84}	0.11	0.11	All ϕ	8.3	8.3	48.4	48.4	0.10	-0.16
Mudstone	9	{1.13, 1.02}	0.09	0.11	$\phi < 0.89$	2.3	2.6	54.4	56.2	3.80	0.05
Wackestone	14	{5.59, 1.83}	0.07	0.11	$\phi < 0.39$	17.2	19.9	76.9	81.0	17.61	0.41
Arb. Shrub	101	{1.80, 1.30}	0.09	0.11	$\phi < 0.63$	179.3	185.8	83.4	84.4	6.31	0.05

Table 6.1: Bulk modulus modelling results.

Facies	n	$\{\Gamma, \xi\}$	α_{Facies}^*	α_{Full}^*	Valid domain	Facies-dep. Single- α ΔAIC_C	Full data Single- α ΔAIC_C	Δ SSR on facies-dep. Single- α (%)	Δ SSR on full data Single- α (%)	Δ SSR between facies-dep. & full data Single- α (%)
Grainstone	281	{0.86, 0.47}	0.28	0.26	All ϕ	19.5	22.1	7.4	8.2	0.90
Laminite	123	{1.69, 0.81}	0.29	0.26	$\phi < 0.52$	29.1	31.5	22.4	23.9	1.92
Spherulite	90	{1.31, 0.69}	0.19	0.26	$\phi < 0.67$	4.5	24.2	7.1	25.4	19.67
Rudstone	76	{3.84, 1.15}	0.24	0.26	$\phi < 0.31$	15.5	16.1	20.8	21.3	0.72
Packstone	48	{1.45, 0.68}	0.24	0.26	$\phi < 0.58$	-1.5	-1.3	1.5	2.0	0.46
Breccia	5	{1.09, 0.62}	0.29	0.26	$\phi < 0.87$	-19.7	-19.5	6.1	9.6	3.74
Floatstone	17	{1.27, 0.67}	0.28	0.26	$\phi < 0.70$	0.3	0.5	17.4	18.3	1.15
Mudstone	9	{9.52, 1.43}	0.76	0.26	$\phi < 0.21$	3.7	6.2	60.9	70.6	24.86
Wackestone	14	{0.81, 0.66}	0.18	0.26	All ϕ	-0.3	4.0	19.2	40.5	26.35
Arb. Shrub	101	{2.34, 1.03}	0.28	0.26	$\phi < 0.44$	74.5	75.1	53.2	53.5	0.66

Table 6.2: Shear modulus modelling results.

Modelling error statistic	Full data Single- α	Facies dep. Single- α	Power law
Mean V_p error (%)	5.95	6.11	4.01
Median V_p error (%)	4.91	4.94	3.22
Mean V_s error (%)	4.39	4.40	3.74
Median V_s error (%)	3.68	3.53	2.88

Table 6.3: Average V_p and V_s modelling errors calculated from forward modelled and measured velocities at each well log sample in Wells A and B.

The percentage in V_p and V_s modelling errors for the three models are also shown in Figures 6.8 and 6.7. The power law model seems to match the measured well log data more closely and have lower modelling error than the Single- α models in Well B, however it is not clear by inspection whether there is a V_p and V_s modelling improvement in Well A by using the power law model. The discontinuity of the forward modelled well logs, particularly noticeable in Well A, is due to the estimation of facies-dependent model parameters and hence forward modelled well logs only within 0.5 m from all core interpretation locations.

The average V_p and V_s modelling errors over both wells for the three models are shown in Table 6.3. Both the mean and median statistic for V_p and V_s data sets show the power law model has the smallest modelling error on average. Interestingly, the full data and facies-dependent Single- α models have similar errors by all measures, which may support using the full data Single- α over the facies-dependent Single- α for its convenience.

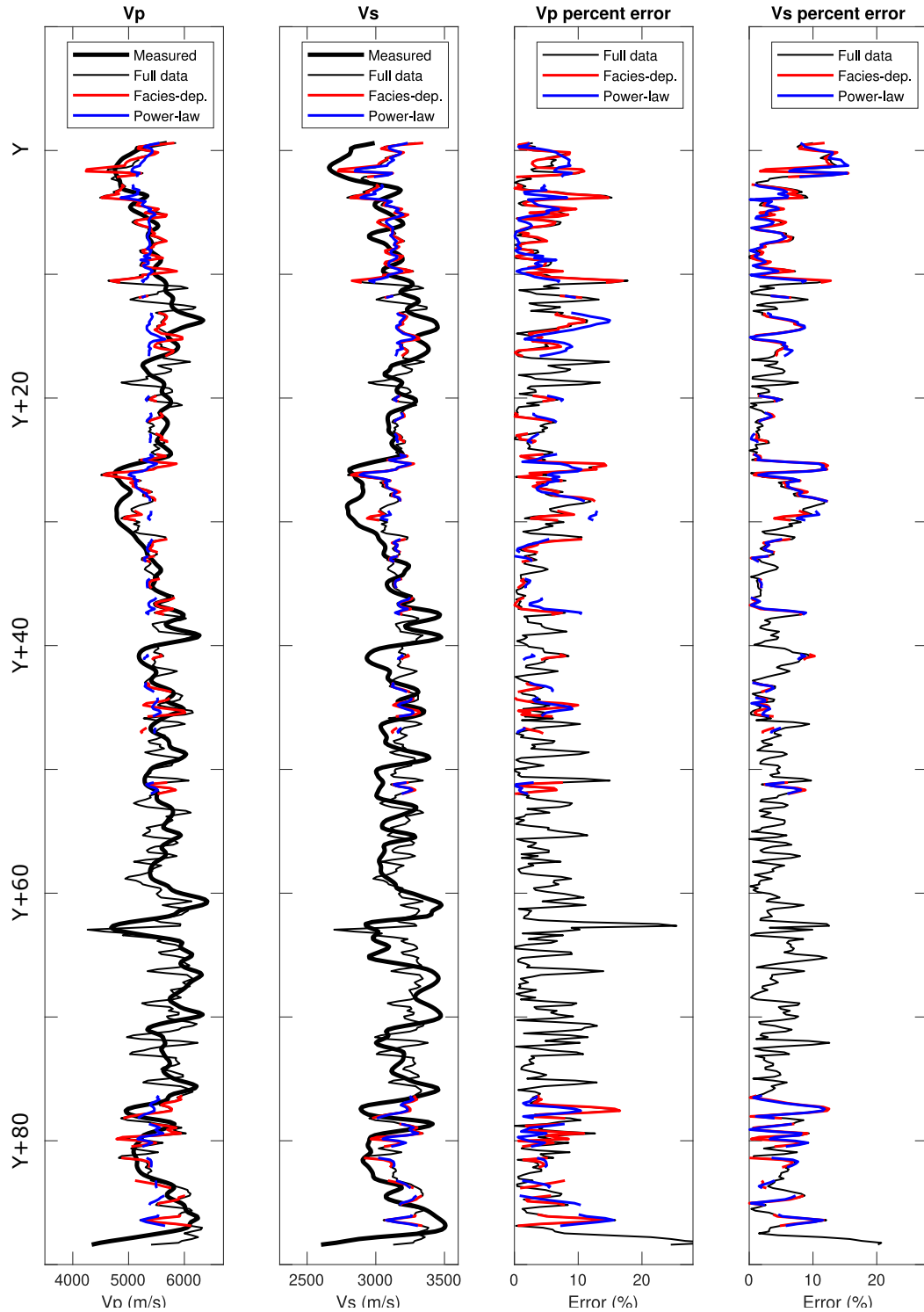


Figure 6.7: Measured and forward modelled V_p and V_s well logs from Well A, and associated modelling errors. Tracks from left to right are: V_p ; V_s ; Error in modelled V_p ; Error in modelled V_s . The measured log (thick black) is compared with forward modelled velocities using the facies-dependent power law DEM (blue), facies-dependent single- α DEM (red), and full data set single- α DEM (thin black) models.

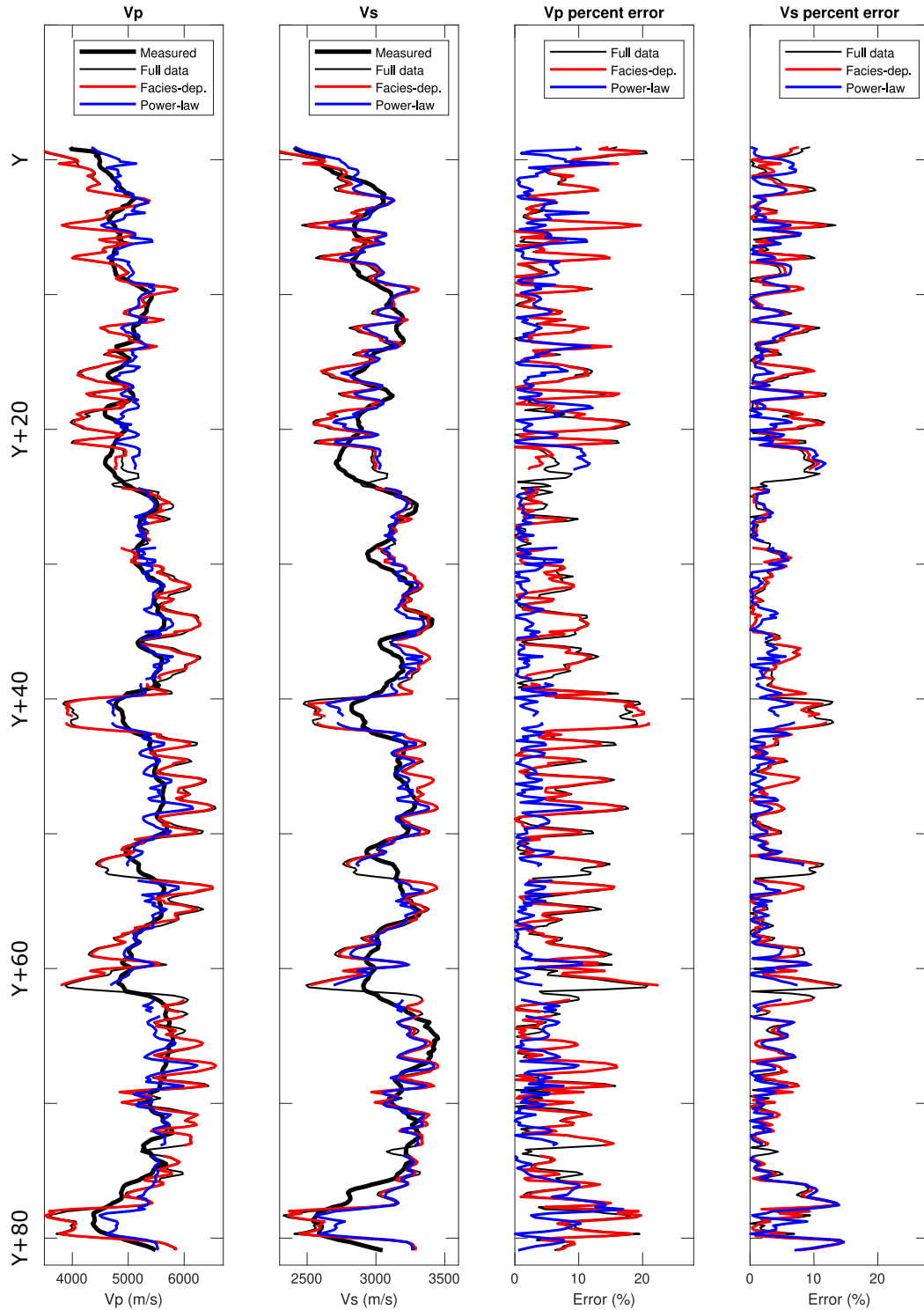


Figure 6.8: Measured and forward modelled V_p and V_s well logs from Well B, and associated modelling errors. Tracks from left to right are: V_p ; V_s ; Error in modelled V_p ; Error in modelled V_s . The measured log (thick black) is compared with forward modelled velocities using the facies-dependent power law DEM (blue), facies-dependent single- α DEM (red), and full data set single- α DEM (thin black) models.

6.6 Facies Classification by Γ and ξ

Model parameters Γ and ξ have physical interpretation in describing how the contribution of a rock's pore space architecture to its effective bulk or shear modulus changes with porosity. I hypothesise that the change in pore space architecture with porosity and hence its contribution to a rock's overall bulk or shear modulus may differ between carbonate facies. Given this hypothesis, I examined whether the power law model's parameters Γ and ξ could be used to differentiate between carbonate facies from only well log data and a pre-calibrated facies classification model.

I tested this hypothesis using bulk modulus modelling parameters Γ_K and ξ_K , as well as shear modulus modelling parameters Γ_μ and ξ_μ . Model parameters Γ and ξ were not effective individually in differentiating between facies. Thus, I considered simple combinations of Γ and ξ , including additive and multiplicative operations. I found the ratio of $\log_{10} \Gamma$ and ξ calculated from both bulk and shear modulus modelling to provide distinct values for each facies within $[-1, 1]$, although not limited to this interval theoretically. I considered a base-10 logarithm of Γ , rather than Γ itself, in this calculation as $\log_{10} \Gamma$ and ξ are the constant and gradient terms in the best fitting line in the porosity-EPAR relationships, as seen in Figures 6.3 and 6.4. I thus propose the parameter Ξ :

$$\Xi = \frac{\log_{10} \Gamma}{\xi}; \quad (6.2)$$

as a candidate parameter to distinguish between the 10 Pre-salt facies analysed in this study from well log data. The calculated Ξ for all facies are shown in Tables 6.1 and 6.2. I also calculated 95% confidence intervals on Ξ by propagating errors

obtained from the covariance matrix of each facies' linear fit through its porosity-EPAR plot, as shown in Figures 6.3 and 6.4.

Figure 6.9 and Table 6.1 shows parameter Ξ differs with facies when calculated with bulk modulus power law parameters. The small errors associated with certain facies are encouraging for the prospect of facies classification using well log data and a pre-calibrated dictionary of Ξ values. For example, given the data in this analysis, it seems a Wackestone, Rudstone, or Spherulite could be distinguished with confidence from a Breccia or Laminite. It seems possible the errors on Ξ could be reduced further by training the facies classification model on more Brazilian Pre-salt data, especially from the same reservoir.

Figure 6.10 and Table 6.2 show parameter Ξ when calculated with shear modulus power law model parameters. While the estimated value of Ξ is seen to change between facies in this modelling, a larger error is generally associated with this Ξ than in bulk modulus modelling, which is the propagation of error primarily associated with estimating shear modulus EPARs given samples which lie outside the Hashin-Shtrikman bounds. Thus, we can't infer from this experiment whether parameter Ξ may be useful in Brazilian Pre-salt carbonate facies categorisation when calculated using a porosity-dependent shear modulus EPAR parameterisation.

The Dunham (1962) classification of carbonate rocks specifies floatstones are matrix supported with large clasts, mudstones are mud supported with less than 10% grains, wackestones are mud supported with more than 10% grains, packstones are grain supported with mud, grainstones are grain supported with no mud, and rudstones are grain supported with large clasts. In Figure 6.9, we see a qualitative trend that bulk modulus parameterised Ξ increases with mud content for the rocks defined by the Dunham classification, observing the matrix supported floatstone has the lowest Ξ value, with mud supported rocks having a higher Ξ ,

and grain supported rocks having the highest Ξ . The exception to this trend is the wackestone, which has a relatively high Ξ even though it is mud supported. A possible qualitative trend between grain size and Γ may be observable, with the large grained floatstone and rudstone having the most negative and positive Ξ values respectively of all rocks in the Dunham classification. As Γ is an indicator of a rock's stiffness relative to the Hashin-Shtrikman bounds at high porosity, and ξ is an indicator of how its stiffness changes due to changes in pore space architecture with porosity, I speculate the possible mud-content and grain-size related trends in Ξ are linked to how mud, matrix, and grain supported rocks change pore space structure differently with changing porosity; for example, how muddy material may be preferentially removed by physical processes over grains or matrix.

6.7 Discussion

The presence of shear modulus samples above the upper Hashin-Shtrikman bound may be explained by the underestimation of the rock's constituent elastic properties in modelling. If the inaccuracy is in the modelled matrix material's shear modulus, the benefits of the proposed power law DEM model may not be undermined. By increasing the modelled matrix shear modulus, the gradient ξ_μ of the line of best fit through shear modulus EPARs as a function of porosity on a log-log scale may increase, thus moving further from zero and hence justifying the use of power law model further. If the EPARs corresponding to samples above the Upper Hashin-Shtrikman bound are estimated with the correct matrix moduli in such a way that the line of best fit has a less positive gradient, there would be more support for a Single- α DEM model over the proposed power law model.

By forward modelling velocity logs, the strength of the facies-dependent power law

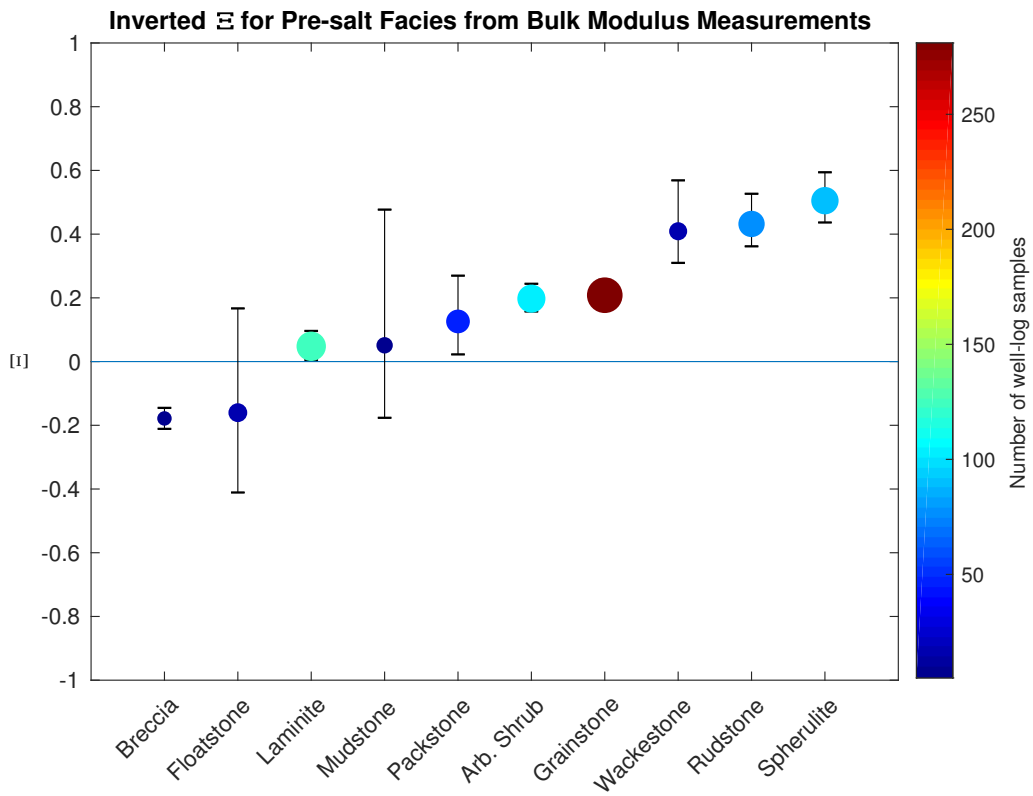


Figure 6.9: Inverted Ξ for each Pre-salt facies using bulk modulus model parameters Γ_K and ξ_K . Facies are ordered from left to right by increasing Ξ . Both warmer colours and larger markers indicate a larger sample size for each facies. 95% confidence intervals in fitting Γ_K and ξ_K for each facies are propagated as error bars.

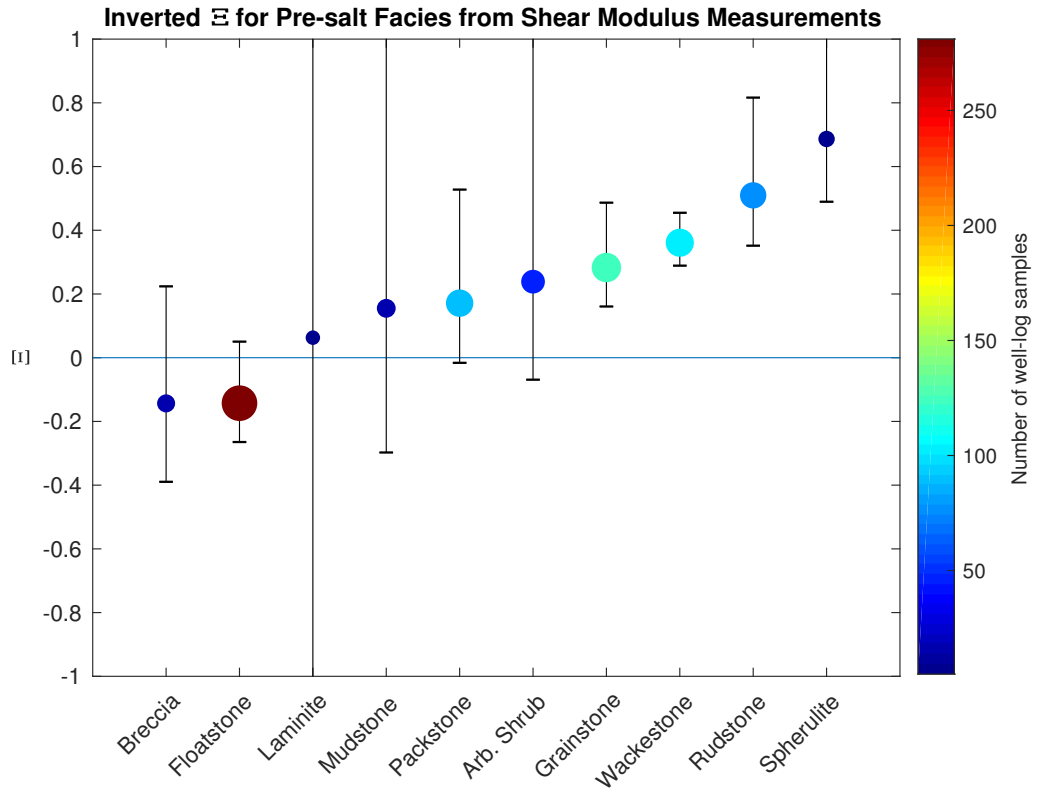


Figure 6.10: Inverted Ξ for each Pre-salt facies using shear modulus model parameters Γ_μ and ξ_μ . Facies are ordered from left to right by increasing Ξ . Both warmer colours and larger markers indicate a larger sample size for each facies. 95% confidence intervals in fitting Γ_μ and ξ_μ for each facies are propagated as error bars.

model is obviated, particularly the estimation of V_p for Well B. Estimating V_s logs from the power law DEM model shows a smaller improvement than estimating V_p logs. This may be because the forward modelled V_p logs are calculated using both the estimated bulk modulus and shear modulus and thus benefit from the significant reduction in modelling error from the power law model in bulk modulus modelling. The forward modelled V_s log, however, does not depend on the estimated bulk modulus and hence is more affected by the significant modelling errors present in all shear modulus models. The improvement on modelling velocities in Well B seems to be more significant than that of Well A. This may be due to the fact that the majority of samples used to estimate Γ and ξ are from Well B.

Equation 6.2 shows parameter Ξ only exists when $\xi \neq 0$. Single- α DEM models necessarily have $\xi = 0$. Thus, the proposed parameter Ξ can only be used for facies characterisation with the proposed power law DEM model, and not Single- α DEM models.

6.8 Conclusion

Predicting probable carbonate facies from well log data using inclusion-based elastic effective medium theories can be challenging as there can be a many-to-one mapping from a rock's facies type to its effective pore aspect ratio (EPAR) at any given porosity. To circumvent this difficulty and attempt to classify carbonate facies from well log data, I hypothesise that different facies have a different porosity-EPAR relationship and it may be possible to use these relationships for facies classification and facies-dependent elastic modelling. I find an empirical power law relation is suitable to model the bulk and shear modulus porosity-EPAR relationship for two Brazilian Pre-salt wells. This power law relationship decreases

modelling misfit in all facies compared to single EPAR modelling, however the power law model is only more efficient in 7 of the 10 facies. Furthermore, the shear modulus power law model is only the most efficient in 4 of the ten facies, however there is low confidence in these results as a number of samples lie above the upper Hashin-Shtrikman bound for the chosen model parameters.

I ask the question whether the power law model's parameters can be used to discriminate between carbonate facies using well log data, and propose a well log-derived parameter as a potential candidate to do so. Calibrating this parameter for each facies on a larger data set would be required to thoroughly test its ability to predict facies.

Chapter 7

Brazilian Pre-Salt Fracture Characterisation from Walkaround VSPs

7.1 Summary

Reservoir fracture characterisation from VSP data can assist in fluid flow modelling, however assumptions underpinning typical methods can be inappropriate for deviated wells beneath complex overburden. Fracture characterisation by shear wave splitting analysis requires fast and slow shear wave velocity analysis, which can be challenging beneath complex overburden, while azimuthal attenuation analysis by the spectral ratio method commonly assumes a simple overburden or a simple well log trajectory. To characterise fractures in a Brazilian Pre-salt

reservoir interval, I derive an interval relative azimuthal attenuation model based on the method of Maultzsch (2005) which estimates the difference in attenuation as a function of azimuth between two contiguous VSP receivers in an arbitrarily deviated well. I use a stochastic optimisation regime at each receiver in the reservoir interval to match the calculated interval relative azimuthal attenuation from VSP data to the azimuthal attenuation predicted by the fractured, multi-fluid, viscoelastic rock physics model of Jin *et al.* (2018), estimating the fracture strike, dip, and density throughout the reservoir interval. I then use a Gaussian mixture model to estimate the fracture dominance of the two apparent fracture sets as a function of depth. Testing the sensitivity of the model of Jin *et al.* (2018) to key parameters, it seems the model is highly sensitive to water saturation and fracture density, however, is less sensitive to the relative permeability model used and patchiness of the saturating fluid when water saturation is held at the approximated 0.15.

7.2 Introduction

Understanding fracture characteristics in reservoirs is desirable as subsurface fluid flow can be affected by the presence of conducting or non-conducting fractures (e.g., Rogers (2003); Barton (2007); Clifford *et al.* (2005)). Fractures in a reservoir can have a large influence on permeability anisotropy (Snow, 1969), and can also affect a reservoir's anisotropic seismic response (Leary *et al.*, 1990; Liu and Martinez, 2013) which, in turn, affects seismic quantitative interpretation and imaging. Fracture characterisation by Vertical Seismic Profile (VSP) data analysis allows geophysicists to tighten constraints on the orientation and density of fracturing in a reservoir, leading to more informed reservoir and fluid flow modelling.

The inversion of observed seismic anisotropy using fractured rock physics models has become a popular means of fracture characterisation as the presence of aligned fractures in a rock can lead to seismic anisotropy. Typical anisotropy inversion methods for fracture characterisation include shear wave splitting (Crampin, 1985; Winterstein and De, 2001; MacBeth, 2002; Mueller, 1991), S-wave azimuthal amplitude analysis (Mueller, 1991), S-wave azimuthal travel time and velocity analysis (Lewallen *et al.*, 2008), P-wave azimuthal amplitude analysis (Corrigan, 1990; Rüger, 1997, 1998; Maultzsch, 2005; Maultzsch *et al.*, 2007), and P-wave azimuthal travel time and velocity analysis (Grechka and Tsvankin, 1998; Li, 1999; Wang and Li, 2005).

Maultzsch (2005) and Maultzsch *et al.* (2005) proposed a method of fracture characterisation using multicomponent VSP which considers changes in P-wave attenuation with azimuth. This method, however, is developed with the assumption that the VSP receiver's overburden is simple, which is not the case in many reservoirs, such as the Brazilian Pre-salt reservoirs. Maultzsch (2005) and Maultzsch *et al.* (2005) use the fractured squirt flow model of Chapman (2003) to model azimuthal attenuation and estimate fracture orientation, assuming the presence of a single fracture set and a single phase fluid, which may be inaccurate in a real reservoir setting.

In this section I extend the relative attenuation method of Maultzsch (2005) and Maultzsch *et al.* (2005) to be suitable for fracture characterisation using an array of VSP receivers in deviated wells beneath complex overburden. In the application of this model on real Pre-salt VSP data, I use a two-fluid, patchy saturation squirt flow model to model fracture orientation and density, and present a statistical modelling method which allows the approximate characterisation of multiple fracture sets' physical properties. I also show that, at low water saturations, this fracture characterisation workflow has low sensitivity to the parameterised relative permeability model and fluid patchiness, but is sensitive to fracture density.

To begin, I overview my walkaround VSP pre-processing workflow which preceded inversion for fracture characteristics. I then extend the work of Maultzsch (2005) and Maultzsch *et al.* (2005) on azimuthal relative attenuation modelling and develop a technique to calculate the relative attenuation as a function of azimuth over an interval which is suitable for an array of VSP receivers beneath complex geological overburden in an arbitrarily deviated well. After this, I perform the first fracture characterisation by VSP inversion using the fractured squirt flow model of Jin *et al.* (2018) for two viscous, immiscible fluids. I perform this fracture characterisation as a function of depth and display the new fracture characterisation parameters as pseudologs. Finally, I perform the first sensitivity tests for the model of Jin *et al.* (2018) in a walkaround VSP inversion setting and analyse the results.

7.3 Modelling Relative Attenuation

7.3.1 Estimating seismic attenuation from multicomponent VSPs

The fracture characterisation method of Maultzsch (2005) is based on the spectral ratio method (Johnston, 1981; Hauge, 1981; Pujol and Smithson, 1991; Liu *et al.*, 1993; Dasgupta and Clark, 1998; Hackert *et al.*, 2001), which considers how the amplitude of a wave recorded over an interval changes with frequency to estimate attenuation. A measure of a material's seismic attenuation, $1/Q(\omega)$, at angular frequency ω can be defined as the energy lost by a propagating wave per angular cycle, ΔE , normalised by its peak strain energy E . That is:

$$\frac{1}{Q(\omega)} = -\frac{\Delta E}{2\pi E} . \quad (7.1)$$

As wave amplitude $|A|$ is proportional to $\sqrt{E}$, it follows that attenuation can be defined in terms of the wave's change in amplitude and its peak amplitude over a cycle as:

$$\frac{1}{Q(\omega)} = -\frac{\Delta|A|}{\pi|A|} . \quad (7.2)$$

If $|A| = |A_0|$ at time $t = 0$, and dissipates by a fraction of $\pi/Q(\omega)$ every wave cycle, the wave amplitude at time t is:

$$|A(t)| = |A_0| \left(1 - \frac{\pi}{Q(\omega)}\right)^n \quad \text{for } t = 2n\pi/\omega . \quad (7.3)$$

Then by the definition:

$$\exp(a) = \lim_{n \rightarrow \infty} (1 + a/n)^n ; \quad (7.4)$$

we see:

$$|A(t)| = |A_0| \exp \left[-\frac{\omega t}{2Q(\omega)} \right] \quad \text{for large } t . \quad (7.5)$$

Equation 7.5 is derived assuming a high-frequency wave, or ray, travelling through a homogeneous medium with constant velocity and attenuation. We can relax these assumptions slightly by considering a high frequency wave travelling through

a medium with spatially-variant velocity and attenuation. In this case, we can define the causal attenuation operator (Kurita, 1968; Båth, 1974; Matheney and Nowack, 1995) as:

$$t^*(\omega) = \int \frac{ds}{Q(\omega, s)v(\omega, s)}; \quad (7.6)$$

along the ray with arc-length parameterised by s . Using this operator, we can extend equation 7.5 to account for path-dependent velocity and attenuation:

$$|A(\omega)| = |A_0| \exp \left[-\frac{\omega t^*(\omega)}{2} \right]. \quad (7.7)$$

Following Båth (1974), Matheney and Nowack (1995) suggest that, along with medium-dependent attenuation effects, the amplitude $A(\omega, x)$ of a seismic wave recorded at position x is also a function of the wave's source signature $S(\omega)$, its source radiation pattern $B(\theta_1)$, its local source and receiver inhomogeneity effects, respectively $C_1(\omega, \theta_1)$ and $C_2(\omega, \theta_2, x)$, where θ_1 and θ_2 are the source and receiver ray angles, the receiver's instrument response $I(\omega)$, and the path geometric spreading $G(\omega, x)$. The amplitude term $|A_0|$ is then modelled by Båth (1974) as the product of these factors in the frequency domain:

$$|A_0| = |S(\omega)B(\theta_1)C_1(\omega, \theta_1)C_2(\omega, \theta_2, x)I(\omega)G(\omega, x)|. \quad (7.8)$$

Thus, equation 7.7 becomes:

$$|A(\omega, x)| = |S(\omega)B(\theta_1)C_1(\omega, \theta_1)C_2(\omega, \theta_2, x)I(\omega)G(\omega, x)| \exp \left[-\frac{\omega t^*(\omega)}{2} \right]. \quad (7.9)$$

We now consider the seismic signal from a given source recorded on two receivers with instrument responses $I_0(\omega)$ and $I_1(\omega)$, located at positions x_0 and x_1 such that the ray travels through receiver position x_0 where it is recorded, before being recorded at x_1 . The ratio of the recorded amplitude spectra in this case is:

$$\frac{|A(\omega, x_1)|}{|A(\omega, x_0)|} = \frac{|S(\omega)B(\theta_1)C_1(\omega, \theta_1)C_2(\omega, \theta_2, x_1)I_1(\omega)G(\omega, x_1)| \exp \left[-\frac{\omega t_1^*(\omega)}{2} \right]}{|S(\omega)B(\theta_1)C_1(\omega, \theta_1)C_2(\omega, \theta_2, x_0)I_0(\omega)G(\omega, x_0)| \exp \left[-\frac{\omega t_0^*(\omega)}{2} \right]}. \quad (7.10)$$

where the subscript on function t^* denotes the integral in equation 7.6 is to be calculated over the ray between the source and the receiver of the subscribed index.

As the source is shared for the recorded signals and the ray path is identical for both signals until passing the location of the first receiver, functions $S(\omega)$, $B(\theta_1)$, and $C_1(\omega, \theta_1)$ cancel out in equation 7.10. Assuming both receivers have identical instrument responses, the $I(\omega)$ terms also cancel out in equation 7.10. Finally, we make a further assumption that the local receiver inhomogeneity effects $C_2(\omega, \theta_2, x_i)$ are equal at the receivers and thus cancel out. For this assumption to be true, we must assume either the material is homogeneous, the rays are straight, or both, depending on the how C_2 varies with ω and θ_2 .

With these assumptions in place, equation 7.10 simplifies to:

$$\frac{|A(\omega, x_1)|}{|A(\omega, x_0)|} = \frac{G(\omega, x_1) \exp \left[-\frac{\omega t_1^*(\omega)}{2} \right]}{G(\omega, x_0) \exp \left[-\frac{\omega t_0^*(\omega)}{2} \right]}. \quad (7.11)$$

We now make a further assumption that geometrical spreading $G(\omega, x)$ is frequency-independent and take the logarithms of both sides of equation 7.11 to obtain:

$$\ln \frac{|A(\omega, x_1)|}{|A(\omega, x_0)|} = \ln \frac{G(x_1)}{G(x_0)} - \left(\frac{t_1^*(\omega)}{2} - \frac{t_0^*(\omega)}{2} \right) \omega. \quad (7.12)$$

Further simplifying equation 7.12, we assume the rock has frequency-independent attenuation and velocity, which upscale to some constant, frequency-independent average values $1/\bar{Q}_i$ and $\bar{v}_i$ respectively over the i^{th} ray path. Parameter t^* therefore also becomes frequency independent by equation 7.6. We must note the significance of using a frequency-independent, average velocity and attenuation in this calculation. In an attenuating medium, velocity is always frequency dependent. However, I will obtain velocities from analysing seismic travel times on VSP data, which means only a single, frequency-independent value for velocity (say, an “average” value) can ever easily be inferred. I will then solve for attenuation $1/\bar{Q}_i$ given this known $\bar{v}_i$, as will be shown. Thus, the assumed frequency independence of velocity necessarily leads to the frequency independence of attenuation in this derivation of the spectral ratio method.

Evaluating the integral in equation 7.6 and changing the dependent variable from angular frequency to temporal frequency f , we find:

$$\ln \frac{|A(f, x_1)|}{|A(f, x_0)|} = \ln \frac{G(x_1)}{G(x_0)} - (a_1 x_1 - a_0 x_0) f \quad \text{with} \quad a_i = \frac{\pi}{\bar{Q}_i \bar{v}_i} \text{ for } i = 0, 1. \quad (7.13)$$

We see equation 7.13 is a linear equation in f with gradient:

$$b_{ij} = a_i x_i - a_j x_j; \quad (7.14)$$

for ray paths i and j .

By approximating the high-frequency travel time t_i from source to receiver i as $t = x_i/\bar{v}_i$, we see:

$$b_{ij} = -\pi \left(\frac{t_j}{\bar{Q}_j} - \frac{t_i}{\bar{Q}_i} \right). \quad (7.15)$$

Thus, by measuring travel times at both receivers, one can calculate the gradient of the linear relationship between frequency and the logarithm of the ratio of the amplitude spectra then use equation 7.15 to calculate $\bar{Q}_j$ with a known $\bar{Q}_i$.

7.3.2 Azimuthal attenuation estimation by the spectral ratio method

Much of the derivation of the spectral ratio method depends on both receivers' recorded signals coming from the same source and travelling along the same ray path until transecting the first receiver's position. Rather than analysing a signal from a single source received at two distinct receivers to estimate attenuation between the receivers, however, I wish to estimate the change in attenuation with incidence azimuth about a single receiver by comparing the signals incident on the receiver from multiple sources located at different azimuths.

To make the spectral ratio method valid for azimuthal attenuation estimation, we assume all shots have identical source signatures. We also assume the source radiation pattern is constant over all emanating ray angles. Further, we assume all sources' local effects are identical and constant over all emanating ray angles. We impose the assumption that the receiver's local attenuation effects are constant over all incident ray angles and that the receiver's instrument response is constant with time, so all shots recorded over a given time period are recorded with the same instrument response. Other prior assumptions, such as frequency-independent geometrical path spreading, remain in place. Finally, the rock's upscaled velocity is assumed to be isotropic, which is a strong and admittedly contradictory assumption in the case of fractured rocks, where analysing azimuth-dependent travel-times and hence seismic anisotropy is a popular method for fracture characterisation.

With these assumptions in place, the spectral ratio method presented in equations 7.13 to 7.15 can also be applied to azimuthal attenuation estimation, where t_i corresponds to the travel time for the i^{th} shot incident on a single receiver, while $\bar{Q}_i$ is the average attenuation along the ray path from the i^{th} shot to this receiver. Figure 7.1 shows a schematic of rays incident on a single receiver arriving from different azimuths.

We must note these assumptions are highly restrictive and are almost impossible to realise in a marine geophysical field experiment. The seismic source apparatus used throughout the acquisition of the walkaround seismic data studied in this thesis was unchanged throughout the survey. I did not have information on the source's radiation pattern or the consistency of the source depth while shooting the walkaround survey. Also, I had no information on the sea state during acquisition, which can alter the source ghost and thus the early-arriving seismic energy. These possible temporal (and hence azimuthal) variations in the seismic source are a

major contributor to uncertainty in the estimation of azimuthal relative Q for real data using the spectral ratio method.

7.3.3 Interval relative azimuthal attenuation estimation

In practice, estimating the relative azimuthal attenuation $\Delta 1/Q = 1/\bar{Q}_1 - 1/\bar{Q}_0$ may be less error prone than estimating the absolute azimuth-dependent attenuation $1/\bar{Q}_1$ due to uncertainty in the reference attenuation $1/\bar{Q}_0$. Maultzsch (2005) and Maultzsch *et al.* (2005) showed analysing relative attenuation about a VSP receiver can be used to estimate azimuthal anisotropy and thus can be used for fracture characterisation. This method, however, assumes the recorded incident rays from different shots have similar incidence angles, and estimates attenuation over the entire ray path between source and receiver, rather than the geological interval of interest, such as the reservoir interval.

As such, the method of Maultzsch (2005) and Maultzsch *et al.* (2005) does not extend readily to the case of complex overburdens. Asymmetric overburden, for example in the case of salt structures above VSP receivers, can significantly alter first arrival incidence angle as a function of azimuth, regardless of the simplicity of the receivers' local geologies. Extending the azimuthal attenuation spectral ratio method of Maultzsch (2005) to account for each ray's incident angle in this calculation, I make the method more applicable to attenuation estimation beneath complex overburdens. Furthermore, I calculate an interval relative azimuthal attenuation, $1/\Delta Q^I$, which is the relative azimuthal attenuation occurring over an interval between two VSP receivers, focusing analysis to the properties of the reservoir interval and removing the effects of the overburden.

Interval relative azimuthal attenuation in a vertical VSP well

To develop this interval relative azimuthal attenuation method, we first consider the simplest case of a vertical VSP well, where the interval of interest is bounded by two VSP receivers, one exactly above the other. Consider a body of rock with a seismic receiver placed distance x vertically beneath the top of the interval to be analysed, as shown in Figure 7.1. Assuming the rock has isotropic velocity v , the vertical travel time t through the interval to the receiver is $t = x/v$.

Now consider a downgoing incident wave, denoted by subscript i , which travels through the interval and is recorded at the receiver with polar angle of incidence θ_i , as shown in Figure 7.1. Assuming straight rays in this homogeneous interval, this wave has travelled distance $x_i = x \sec(\theta_i)$ and takes time $t_i = t \sec(\theta_i)$ to pass through the interval. Similarly, some other arbitrary reference wave, denoted by subscript r and incident on the receiver at polar angle θ_r , would travel distance $x_r = x \sec(\theta_r)$ and take time $t_r = t \sec(\theta_r)$ to pass through the interval, as shown in Figure 7.1. The travel times of these two rays are evidently linked by the relation:

$$t = t_i \cos(\theta_i) = t_r \cos(\theta_r) . \quad (7.16)$$

Thus:

$$t_i = t_r \frac{\cos(\theta_r)}{\cos(\theta_i)} . \quad (7.17)$$

Letting Q_i^I and Q_r^I be the average (interval) attenuation over the i^{th} and r^{th} ray paths through interval I respectively, equation 7.15 becomes:

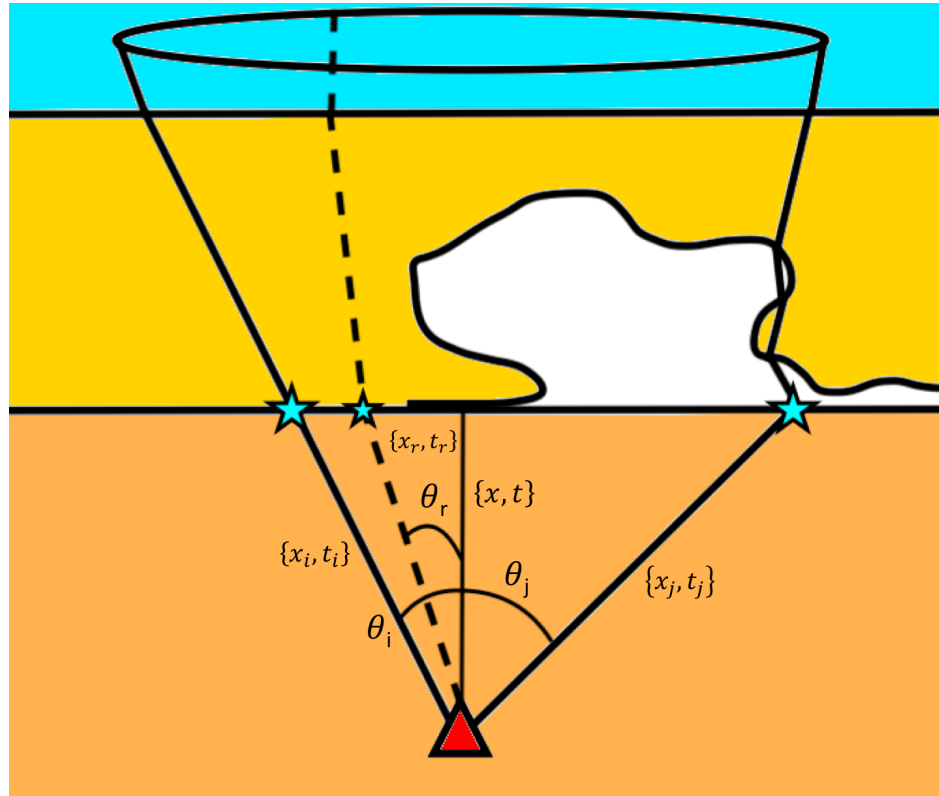


Figure 7.1: Schematic ray diagram for the interval relative azimuthal attenuation model for complex overburden. Given a walkaround seismic survey acquisition (top circle), incidence angles θ from seismic waves i, j, r (black rays) on a VSP receiver (red triangle) can be significantly different, even in locally homogeneous rock (orange section), due to the presence of an asymmetric, complex overburden (yellow section containing white salt body). The proposed models strip off effects of overburden (everything above blue stars) and account for incidence angle variation with azimuth when estimating the interval relative azimuthal attenuation.

$$b_{ri} = -\pi \left(\frac{t_i}{Q_i^I} - \frac{t_r}{Q_r^I} \right); \quad (7.18)$$

$$= -\pi \left(\frac{t_r \frac{\cos(\theta_r)}{\cos(\theta_i)}}{Q_i^I} - \frac{t_r}{Q_r^I} \right); \quad (7.19)$$

$$= -\pi t_r \left(\frac{\cos(\theta_r)}{\cos(\theta_i) Q_i^I} - \frac{1}{Q_r^I} \right). \quad (7.20)$$

By equation 7.20, we express attenuation for the i^{th} azimuth with respect to that of a reference azimuth over interval I as:

$$\frac{1}{Q_i^I} = \frac{\cos(\theta_i)}{\cos(\theta_r)} \left[\frac{1}{Q_r^I} - \frac{b_{ri}}{\pi t_r} \right]. \quad (7.21)$$

We can now relate two arbitrary ray paths i and j incident on receiver x at polar incidence angles θ_i and θ_j respectively, through the third, reference ray path r , as shown in Figure 7.1. The difference in attenuation for ray paths i and j over I takes the form:

$$\frac{1}{\Delta Q_{ij}^I} = \frac{1}{Q_j^I} - \frac{1}{Q_i^I} = \sec(\theta_r) \left[\frac{\cos(\theta_j) - \cos(\theta_i)}{Q_r^I} + \frac{\cos(\theta_i) b_{ri} - \cos(\theta_j) b_{rj}}{\pi t_r} \right]. \quad (7.22)$$

It also follows from equation 7.22 that for three arbitrary traces indexed i , j , and k ;

$$\frac{1}{\Delta Q_{ik}^I} = \frac{1}{\Delta Q_{ij}^I} + \frac{1}{\Delta Q_{jk}^I}; \quad (7.23)$$

allowing us to transform between interval relative azimuthal attenuation with respect to any reference trace without expensive calculation. Finally, we see from equation 7.22 that $1/\Delta Q_{ii}^I = 0$ for arbitrary trace i , meaning that, for every receiver, at least one shot (the reference shot) should have $1/\Delta Q^I = 0$ exactly.

If we assume there is a VSP receiver at both the top and bottom of interval I throughout this derivation, the presented formulation of interval relative azimuthal attenuation in equation 7.22 assumes the VSP receivers are vertically aligned. When a VSP well is non-vertical, the interval attenuation between receivers differs as a function of incidence polar angle, incidence azimuth, well trajectory azimuth, and well deviation angle.

7.3.4 Interval relative azimuthal attenuation estimation in an arbitrarily deviated VSP

To derive an expression for interval relative azimuthal attenuation over the interval between two receivers in a deviated well, I first derive the difference in distance x_i an incident plane wave, denoted by index i , travels through the interval to arrive at the receivers. This difference in distance which a wave travels between receivers can then be converted to the difference in arrival time t_i across receivers, which is then substituted into equation 7.18, extending equation 7.22 for this more general case. I assume an isotropic velocity in these derivations, however this is not required.

I first consider the case of two VSP receivers situated in a deviated well having a given trajectory azimuth such that the incident plane wave's normal vector is coplanar with the plane formed by the two VSP receivers and a third point vertically above or below one of the receiver (see Figure 7.2). After this, I calculate

x_i for the general case where the incident wave has arbitrary incidence angle and azimuth with respect to the well trajectory's azimuth and deviation angle.

Deviated VSP well with “in-plane” arrivals

Consider the scenario demonstrated in Figure 7.2 where a VSP well is deviated from the vertical by angle ψ_D and an incident wave i is arriving at incidence angle θ_i with its propagation vector lying in the plane formed by the two receivers and any point lying vertically above or below either receiver. That is, the wave is arriving “in-plane” on the receivers. The direct distance between the two receivers is x_R .

As shown in Figure 7.2, rotating the entire experiment by ψ_D replicates the simpler, vertical well case solved previously. After this rotation, the wave's incidence angle is transformed to $\tilde{\theta} = \theta + \psi_D$. Thus, we see the deviated well problem with in-plane incident waves has the solution:

$$x_i = x_R \cos(\theta + \psi_D) . \quad (7.24)$$

To check this makes intuitive sense, physical reasoning would require the following to hold:

$$x_i = x_R, \quad \text{if } \tilde{\theta} = 0; \quad (7.25)$$

$$x_i = 0, \quad \text{if } \tilde{\theta} = \pm \frac{\pi}{2}; \quad (7.26)$$

$$x_i = -x_R, \quad \text{if } \tilde{\theta} = \pi . \quad (7.27)$$

These conditions are satisfied by equation 7.24, and we have thus solved the problem for in-plane incident waves on a deviated VSP well.

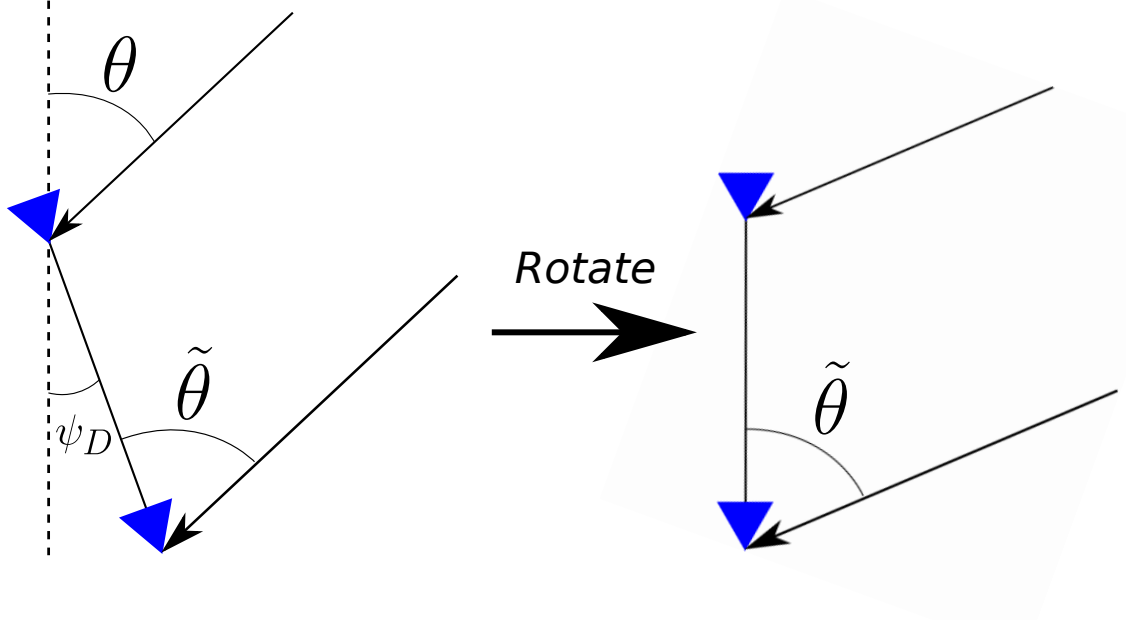


Figure 7.2: Two parallel, incident rays (arrows) from a plane wave on two receivers (triangles) at incidence angle θ from vertical in a well deviated by ψ_D (left). Rotating the entire system by ψ_D , we emulate the vertical well problem with a modified incidence angle $\tilde{\theta} = \theta + \psi_D$ (right).

Deviated VSP well with arrivals from arbitrary direction

Equation 7.24 showed the difference in distance travelled by an incident plane wave when its propagation vector is coplanar with the well's trajectory. We now consider the case when the azimuth of the deviated well's trajectory, Φ_D , is not co-planar to the azimuth of the incident plane wave's propagation vector, Φ_i .

Again, by physical reasoning, the following should hold true:

$$x_i = x_R \cos(\theta + \psi_D), \quad \text{if } \Phi_i - \Phi_D = 0; \quad (7.28)$$

$$x_i = x_R \cos(\theta), \quad \text{if } \Phi_i - \Phi_D = \pm \frac{\pi}{2}; \quad (7.29)$$

$$x_i = x_R \cos(\theta - \psi_D), \quad \text{if } \Phi_i - \Phi_D = \pi. \quad (7.30)$$

It therefore seems x_i can be modelled by:

$$x_i = x_R \cos(\theta + f(\psi)) ; \quad (7.31)$$

where $f(\psi_D)$ is the apparent well deviation given the incident plane wave's incident azimuth. Indeed, the apparent well deviation is the projection of the well's trajectory onto the plane containing the propagation vectors of the plane wave and the vector pointing vertically. Thus, $f(\psi_D)$ performs a rotation in azimuth to calculate this projection (Figure 7.3).

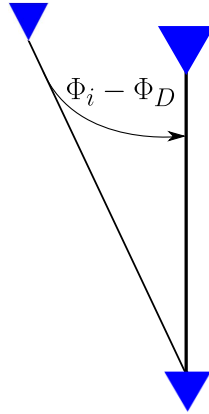


Figure 7.3: The apparent well deviation $f(\psi_D)$ is the deviation of the well from the perspective of the incidence plane wave. A rotation in azimuth is performed in equation 7.32 to calculate the apparent well deviation, as shown here.

Mathematically, we see:

$$f(\psi) = \psi_D \cos(\Phi_i - \Phi_D) ; \quad (7.32)$$

and the solution to the case of the arbitrary well deviation and trajectory, and arbitrary incident wave azimuth and polar angle is:

$$x_i = x_R \cos(\theta + \psi_D \cos(\Phi_i - \Phi_D)) . \quad (7.33)$$

We note this solution satisfies all expectations derived from physical reasoning presented here, as well those presented in the special cases of a vertical well and a deviated well with in-plane arrivals.

With a constant, isotropic interval velocity, it follows from equation 7.33 that:

$$t_i = t_R \cos[\theta_i + \psi_D \cos(\Phi_i - \Phi_D)] ; \quad (7.34)$$

where t_i is the difference in the wave's arrival times upon the receivers, and t_R is the direct travel time from one receiver to the other.

Equations 7.18 and 7.34 together show that the interval relative azimuthal attenuation between two VSP receivers for two plane waves i and j incident at polar angle θ_p and azimuth Φ_p for $p \in \{i, j\}$ in a well with arbitrary deviation angle ψ_D and trajectory azimuth Φ_D , given a reference wave r , travelling through a homogeneous, isotropic medium, is:

$$\frac{1}{\Delta Q_{ij}^I} = \frac{R}{Q_r^I} \left[\frac{1}{J} - \frac{1}{I} \right] + \frac{1}{t_R \pi} \left[\frac{b_{ri}}{I} - \frac{b_{rj}}{J} \right]; \quad (7.35)$$

where t_R is the direct travel time between the two receivers and R , I , and J are defined as:

$$R = \cos [\theta_r + \psi_D \cos (\Phi_r - \Phi_D)] ; \quad (7.36)$$

$$I = \cos [\theta_i + \psi_D \cos (\Phi_i - \Phi_D)] ; \quad (7.37)$$

$$J = \cos [\theta_j + \psi_D \cos (\Phi_j - \Phi_D)] . \quad (7.38)$$

7.4 VSP Data Review

7.4.1 VSP data overview

Data from two wells, Wells A and B, were available for this research. Both wells penetrated the same Pre-salt reservoir. Measurements from Well A consisted of VSP data, well log data, and core interpretation within the reservoir interval. Measurements from Well B consisted only of well log data and core interpretation within the reservoir interval. As such, VSP analysis was performed only in Well A.

The VSP data from Well A spanned the post-salt, salt, (Pre-salt) reservoir, and pre-reservoir intervals. Both the walkaway and walkaround VSP surveys were shot on the same receiver array, with 40 ungimballed 3-component VSP receivers

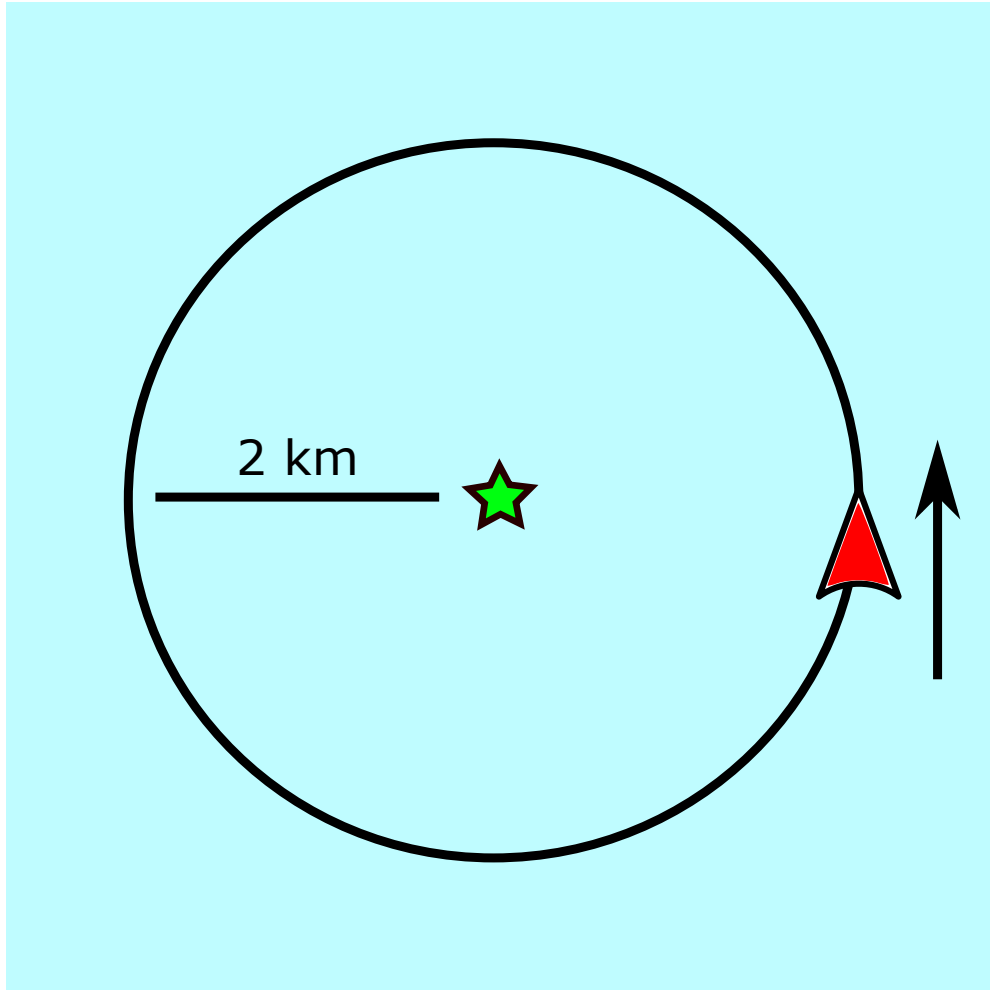


Figure 7.4: Plan view of the VSP walkaround acquisition geometry. The ship (red arrowhead) sailed in a circle of radius 2 km around the well head (green star), with 40 VSP receivers positioned in the well. Shots were spaced 40 m apart and the seismic data was recorded at 2 ms sample rate with a trace length of 8 s.

positioned at 15 m intervals in measured depth. The well was deviated at 18.5 degrees from vertical. The receivers were placed at depths Z to $Z+594$ m TVDSS. The walkaway line sailed at azimuth N115 (NW-SE). The walkaway seismic line was 12 km long and centred on Well A, with shots fired at approximately every 40 m. The walkaround line had 2 km radius, also with a 40 m shot spacing, as shown in Figure 7.4. All VSP data was recorded at a 2 ms sample rate with a trace length of 8 s. The receivers were located in the Salt interval and Pre-salt intervals, numbered 1 to 5 from shallow to deep. Interval “Pre-salt 2”, containing VSP receivers 23 to 28, was the reservoir interval and the focus area for fracture characterisation by VSP inversion.

7.5 Walkaround VSP Data Pre-Processing

Figure 7.5 shows the walkaround VSP processing flow implemented before inversion for azimuthal attenuation. After loading the walkaround VSP data into MATLAB, I rotated all data from the 40 ungimballed receivers to vertical. I calculated the rotation-to-vertical operators for each receiver using the walkaway VSP data, which was recorded on the same receiver array. These operators were found as the 3D rotation operators which maximised signal energy in a window containing the first-arriving, near-offset P-waves in the receiver’s vertical component. I also rotated the horizontal components to be oriented N-S and E-W for convenience throughout the processing flow. Figure 7.6 shows a common receiver gather after these rotations for receiver 25, which is situated in the reservoir interval. Figure 7.6 shows first arrival polarity flips and extinctions in the two horizontal components, and a converted S-wave arriving after the P-wave in all three components. The highly variable travel time with azimuth is due to a large

salt diapir being located on one side of the well, which is clearly visible in Figure 1.5.

With the walkaround VSP data rotated to vertical, I then manually reversed incorrect trace polarities. Using source and receiver locations from survey geometry, azimuth values were assigned to each trace to be used in azimuthal anisotropy analysis. Assigning azimuths in this way assumes that the seismic waves are incident on receivers at the same azimuth as that of the direct line between their source and receiver. In the case of VSP receivers in a Pre-salt reservoir, this assumption is unlikely to be accurate, however no velocity model was available for ray tracing in this project.

I flattened the downgoing P-wave first arrivals in time across the receiver array for each shot using a linear moveout (LMO) correction. On these LMO-corrected gathers, I used a 2D median filter to remove non-flattened events such as S-waves, upgoing P-waves, and some multiples, from the seismic records.

Following this, I estimated the polar incidence angle for each trace. I windowed the first arriving P-wave on each trace using a 100 ms Blackman window, before calculating the windowed signal's power in its vertical component and across all three components. With the total and vertical powers known, the incidence angle θ_k for the k^{th} shot upon any given receiver can be found by recognising that power P_k recorded on the vertical receiver changes with θ_k by $P_k(\theta_k) = P_{k_0} \cos^2(\theta_k)$, where P_{k_0} is the wave's three-component power. Hence, θ_k is:

$$\theta_k = \arccos \left[\sqrt{\frac{P_k(\theta_k)}{P_{k_0}}} \right]. \quad (7.39)$$

The geological markers in Figure 7.7 highlight the effect of an asymmetric salt overburden on the recorded seismic incidence angles in the Pre-salt reservoir.

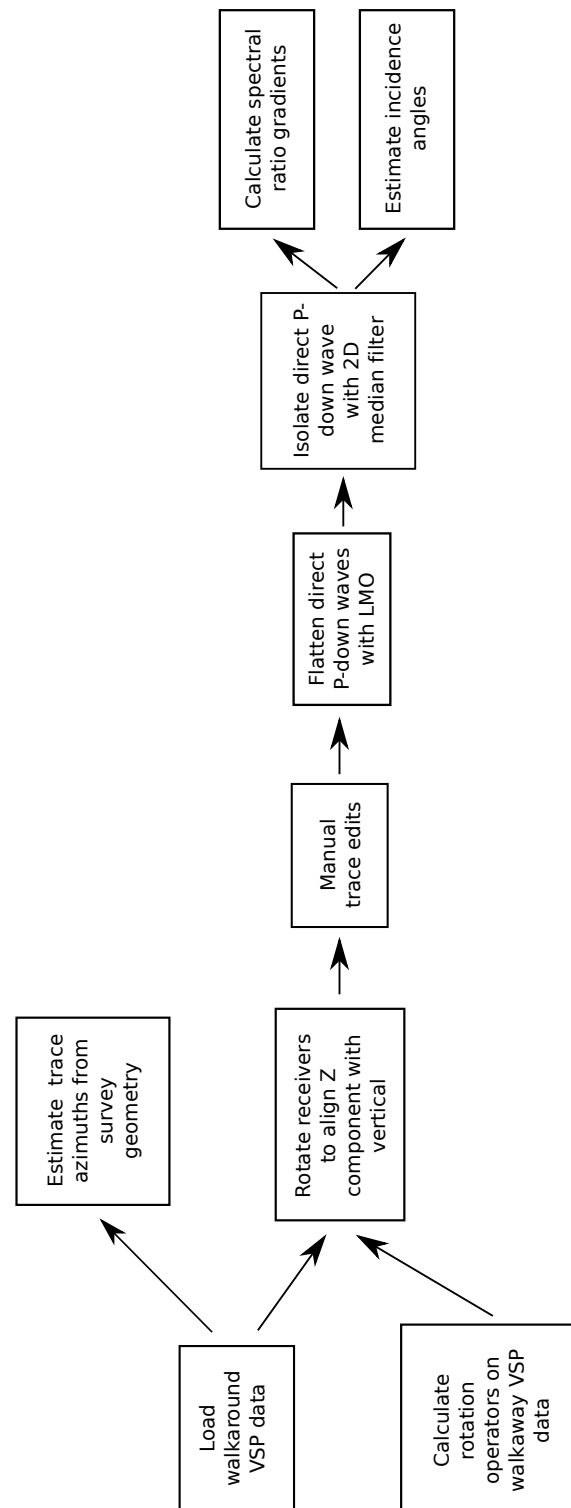


Figure 7.5: Walkaround VSP processing flow.

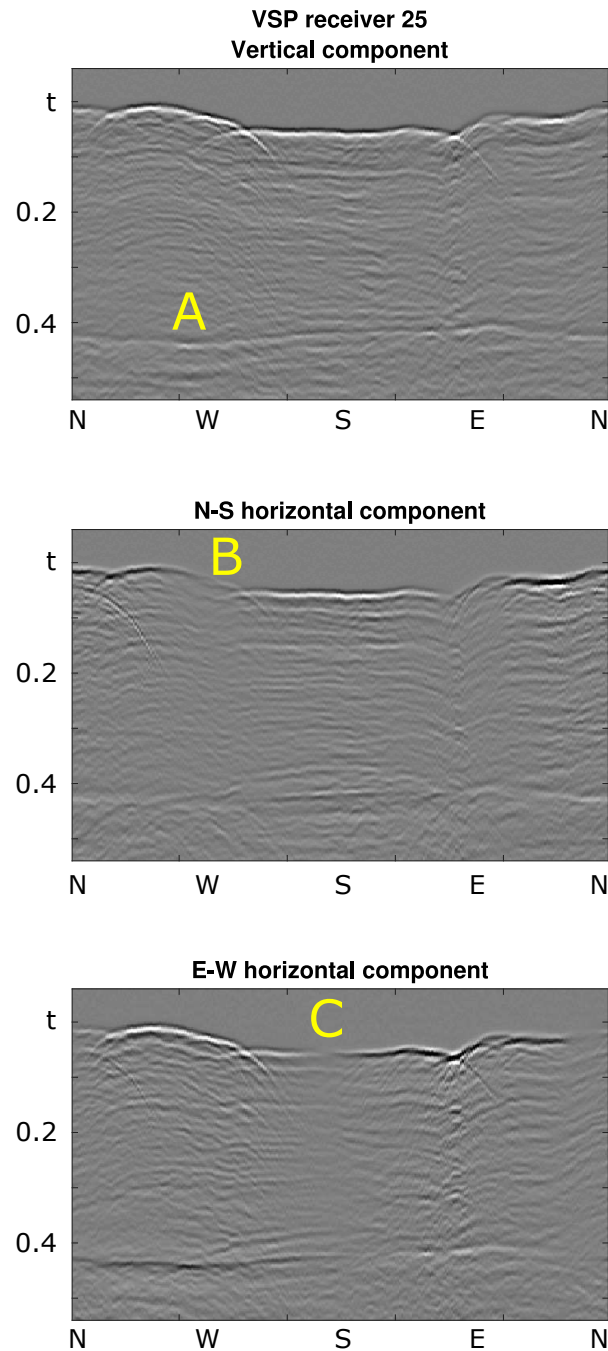


Figure 7.6: The three component walkaround VSP receiver gather for receiver 25, situated in the reservoir interval. The vertical component shows a converted shear wave arriving (A). The two horizontal components show extinctions every π radians or azimuth (B and C), with their extinctions $\pi/2$ radians out of phase from one another. These extinctions occur when the incidental P-wave is travelling perpendicular to a receiver component. The first arrival polarity shift which occurs across these extinctions is also as expected. The highly variable first arrival travel times with azimuth is due to the presence of a large salt diapir above one side of the well, as seen in Figure 1.5.

Estimating the interval attenuation between two receivers requires the direct travel time t_R between two receivers, by equation 7.35. Knowing *a priori* that the receiver spacing was 15 m measured depth and the well was linearly deviated, I used the upscaled compressional velocity log (see Figure 7.8 and Table 7.2) to obtain P-wave velocities for estimating t_R between every receiver.

Calculated azimuth-dependent incidence angle for all shots and receivers

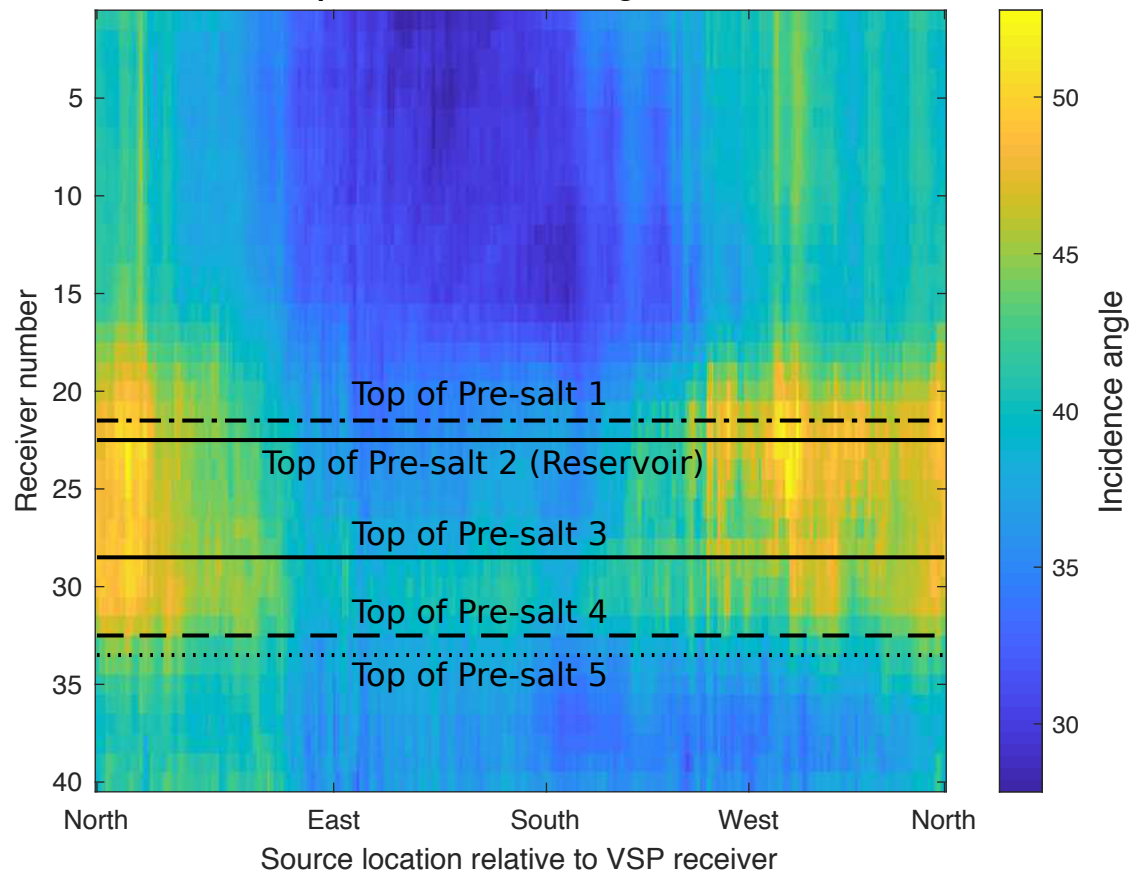


Figure 7.7: Incidence angle (degrees) calculated from VSP data for every source-receiver pair using equation 7.39 with approximate geological markers shown. VSP inversion is performed in the reservoir interval, bounded by solid lines.

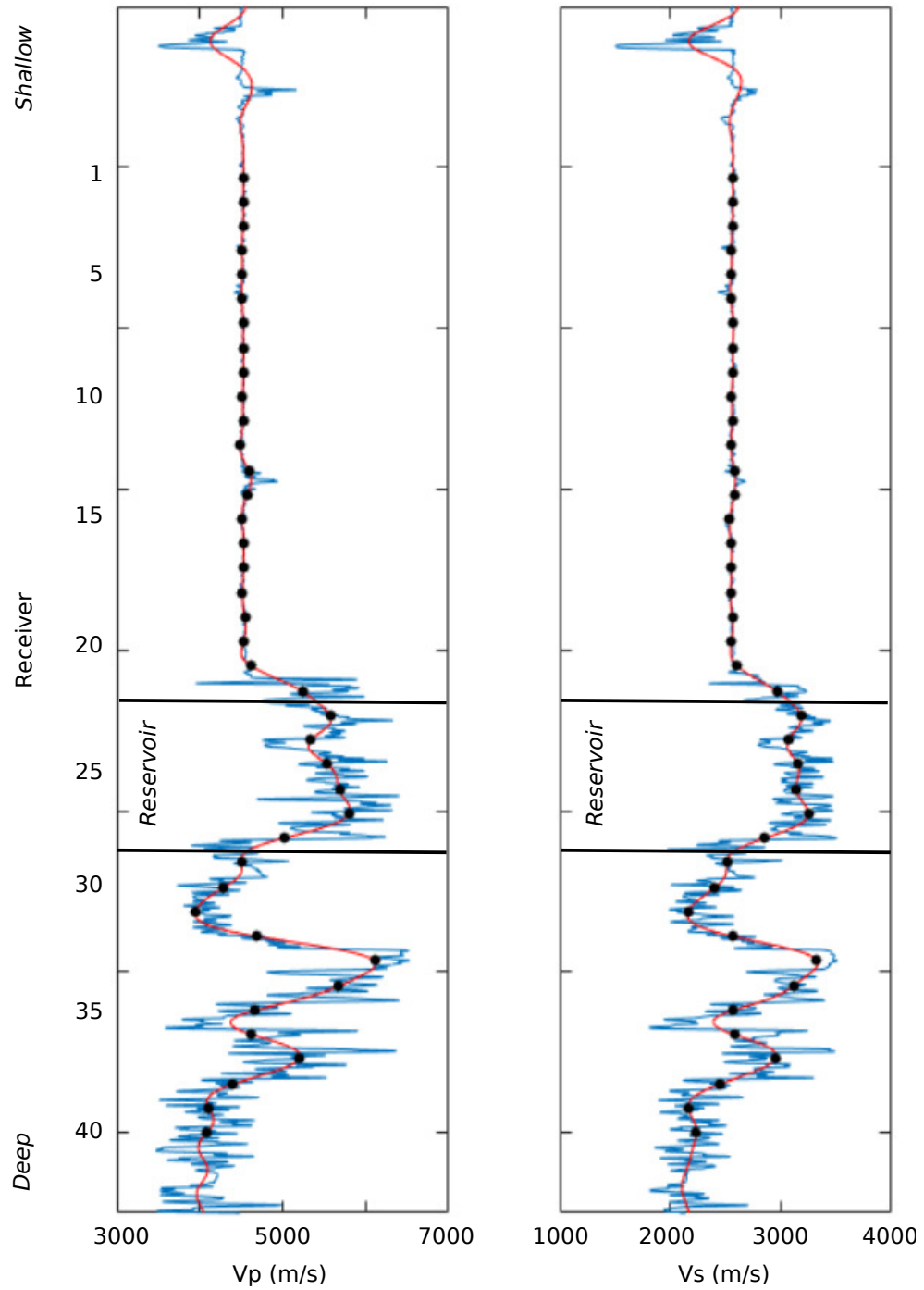


Figure 7.8: Raw (blue) and upscaled (red) V_p (left) and V_s (right) well logs. All receiver locations are shown with black points. The reservoir interval (receivers 23-28) is bounded by horizontal black lines. The upscaled velocities at each receiver location are used in equations 8.5 to 8.8 to calculate Thomsen's parameters from δ_{VSP} and η_{VSP} .

7.6 Calculating Pre-Salt Interval Relative Azimuthal Attenuation from VSPs

To estimate interval relative azimuthal attenuation using equations 7.35 to 7.38, I arbitrarily set $1/Q_r^I = 1/100$ and arbitrarily chose the reference trace r for each given VSP receiver.

After establishing a reference shot, and calculating t_R and Q_r^I between all pairs of contiguous receivers, I calculated b_{kl} for all shots k and l over all receivers using the spectral ratio method. Namely, by fitting a line through the logarithm of the ratio of the amplitude spectra for every combination of shots k and l , for every receiver. The reference gradient terms, b_{ri} and b_{rj} , were taken as those where shot k was the pre-selected reference shot and $l = i \vee j$. The well's trajectory was known to be deviated from the vertical by $\psi_D = 18.5$ degrees with the trajectory azimuth $\Phi_D = S26.8W$.

With known b_{ir} , b_{jr} , t_R , $1/Q_r^I$, ψ_D , Φ_D , θ_i , and Φ_i for all shots over all receivers, I calculated interval relative azimuthal attenuation between each pair of two contiguous receivers using equations 7.35 to 7.38. Using equation 7.23, I transformed the interval relative azimuthal attenuation for each receiver to be expressed in terms of the reference shot which set this relative attenuation to have a minimum value of 0.

Once interval relative azimuthal attenuation was calculated for every shot over all receivers, it was smoothed with a moving mean in azimuth using a 15 shot, or approximately 20-degree, window. This was followed by a moving mean filter across receivers, using a 3-receiver, or 45 metres MD, window.

A selection of the 40 receivers' interval relative azimuthal attenuation signals,

which span the Salt, Pre-Salt 1, Pre-Salt 2 (reservoir), Pre-Salt 3, Pre-Salt 4, and Pre-Salt 5 geological intervals, is shown in Figures 7.9 to 7.14. The gap in these signals at the North bearing is a result of smoothing.

To understand the effect of well deviation on calculated interval relative azimuthal attenuation, I also calculated this relative attenuation with $\psi_D = 0$, effectively employing the vertical well model of Subsection 7.3.3. Figure 7.15 shows the difference in interval relative attenuation when calculated assuming a vertical or deviated well on VSP receiver 25, holding all other model parameters constant. There is reduced asymmetry using the deviated well model, which may be desirable if no prior information about reservoir fracturing is available.

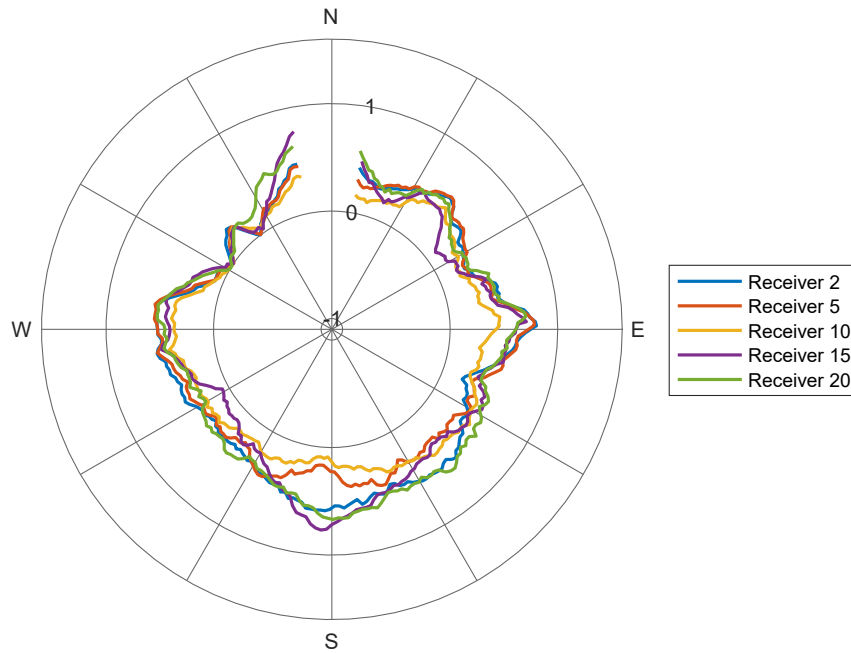


Figure 7.9: Interval relative attenuation calculated for selected receivers in the Salt interval.

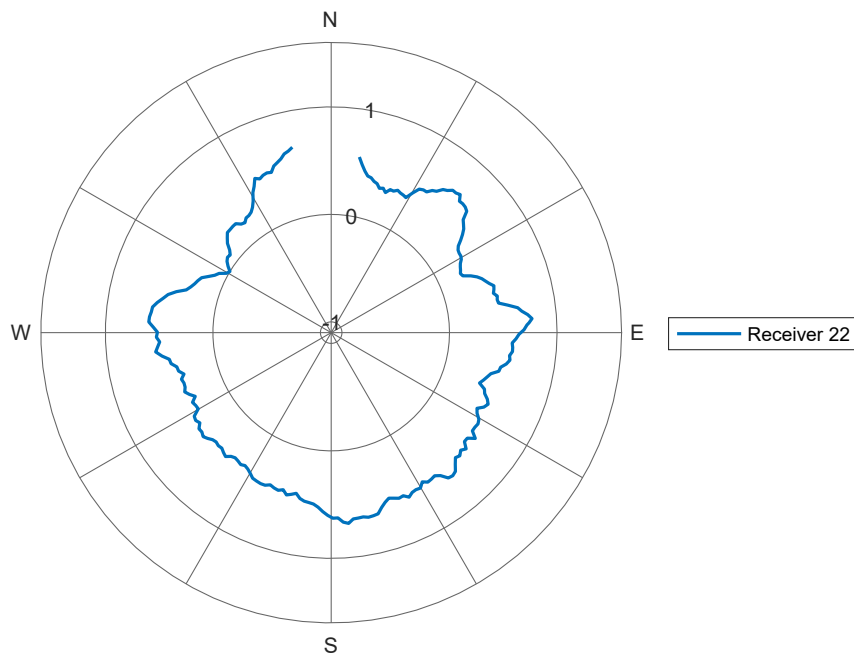


Figure 7.10: Interval relative attenuation calculated in interval “Pre-salt 1”.

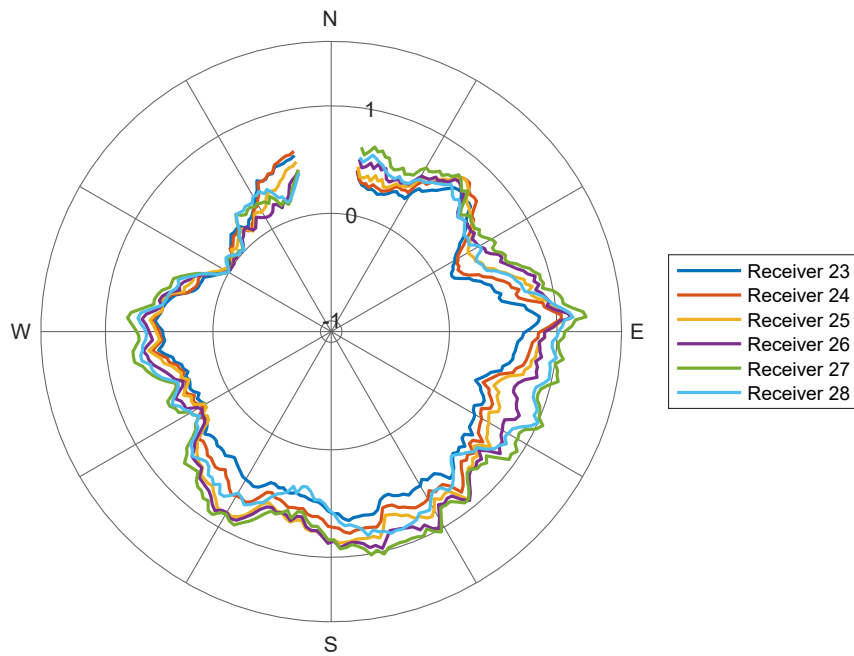


Figure 7.11: Interval relative attenuation calculated in the reservoir interval “Pre-salt 2”.

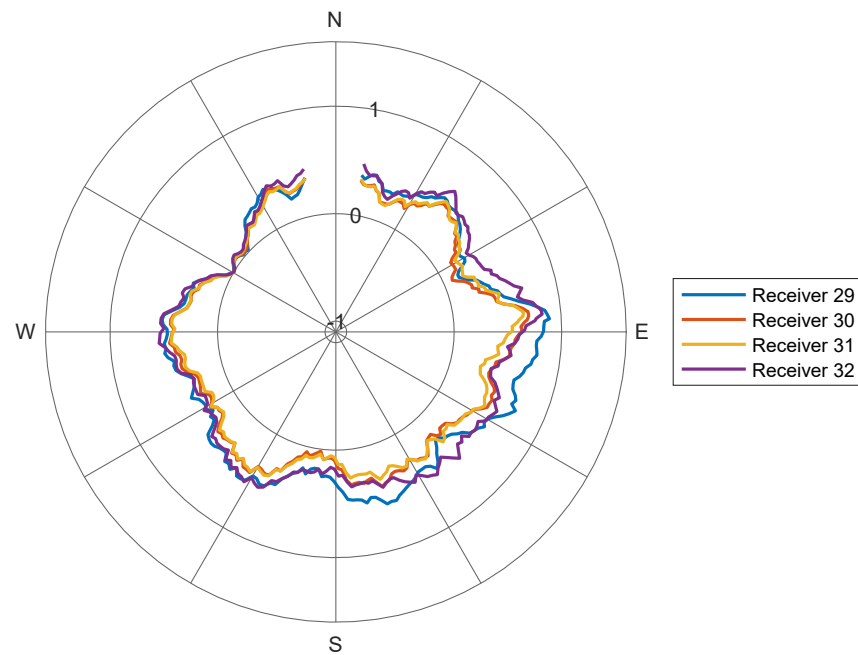


Figure 7.12: Interval relative attenuation calculated in interval "Pre-salt 3".

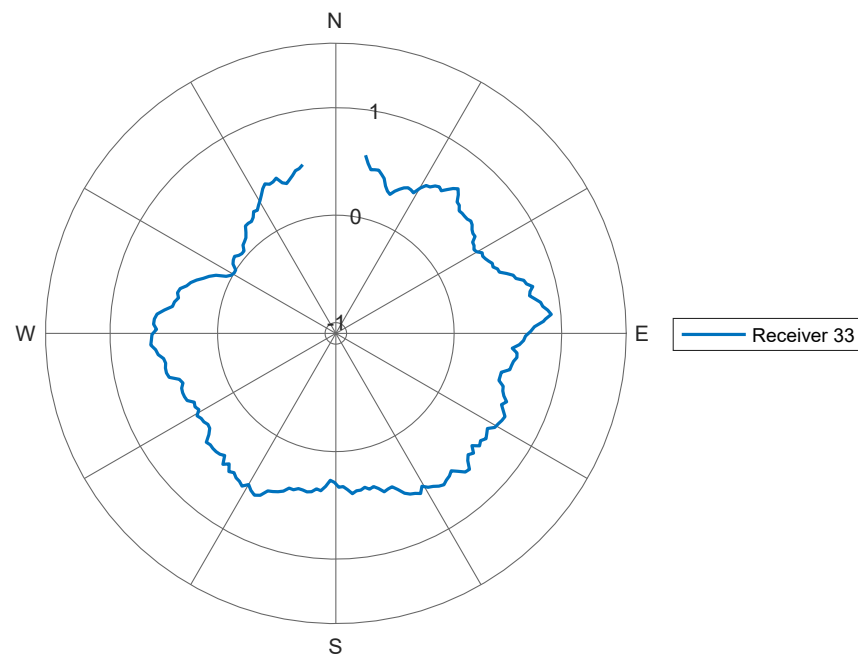


Figure 7.13: Interval relative attenuation calculated in interval "Pre-salt 4".

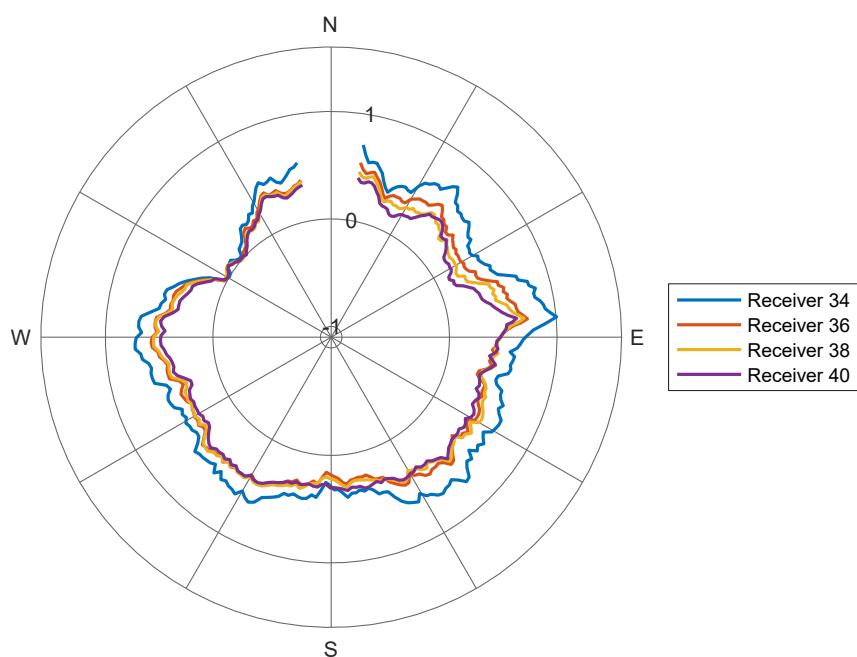


Figure 7.14: Interval relative attenuation calculated in interval “Pre-salt 5”.

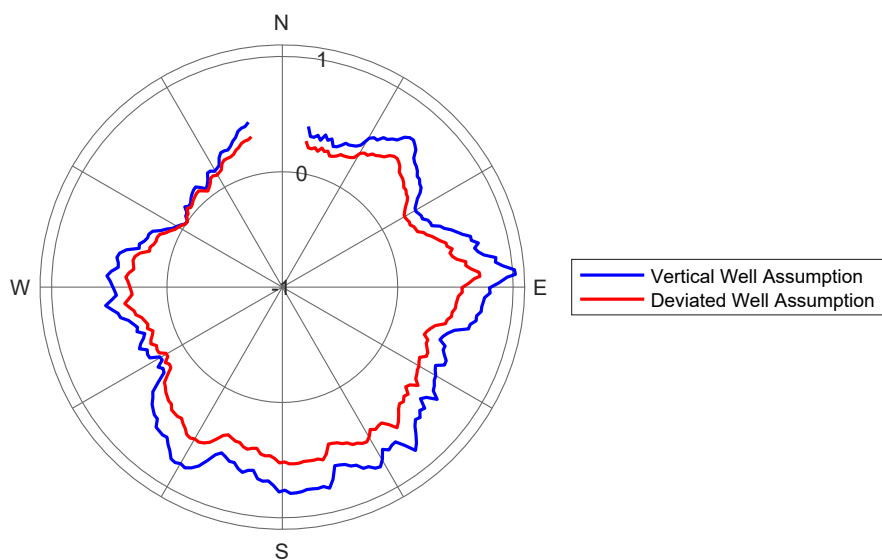


Figure 7.15: Interval relative attenuation calculated using data from VSP receiver 25 within the reservoir interval, assuming a vertical and deviated well trajectory.

7.7 Modelling 3D Anisotropic Attenuation Surfaces

Jin *et al.* (2018) proposed a squirt-flow model which calculated the viscoelastic stiffness tensor for fractured rocks containing two immiscible, patchy fluids, as reviewed in Chapter 3. I used this model to estimate the Pre-salt reservoir interval's seismic attenuation as Brazilian Pre-salt reservoirs can contain multiple fluids (Boyd *et al.*, 2015) and are possibly fractured. After discussions with Pre-salt petrological specialists, I assumed the matrix phase was a mixture of calcite and dolomite. I assumed the liquid phase was a mixture of viscous water and viscous oil (Boyd *et al.*, 2015). All model parameters are listed in Table 7.1.

Parameter	Value	Source
Calcite density	2710 kg/m ³	Mavko <i>et al.</i> (2009)
Dolomite density	2880 kg/m ³	Mavko <i>et al.</i> (2009)
Calcite-dolomite proportion	95%/5%	Estimated
Solid phase density	2719 kg/m ³	Inferred
Effective density	Variable	Upscaled <i>RHOB</i> log
V_p at reference frequency	Variable	Upscaled <i>dtc</i> log
V_s at reference frequency	Variable	Upscaled <i>dtc</i> log
Ref. rock porosity	Variable	Upscaled <i>NPHI</i> log
Crack aspect ratio	10^{-4}	Arbitrary
Fracture aspect ratio	10^{-4}	Arbitrary
Ref. rock crack density	0	Arbitrary
Ref. rock fracture density	0	Arbitrary
Ref. rock fracture length	0 m	Arbitrary
Ref. rock water saturation	0.15	Boyd <i>et al.</i> (2015)
Water viscosity	0.5 cP	Isdale and Morris (1972)

Water density	1 kg/m ³	Estimated
Water bulk modulus	2.3×10^9 Pa	Estimated
Oil viscosity	1 cP	Boyd <i>et al.</i> (2015)
Oil density	0.882 kg/m ³	Boyd <i>et al.</i> (2015)
Oil bulk modulus	2.2×10^9 Pa	Batzle and Wang (1992)
Fracture fluid water saturation	0.15	Boyd <i>et al.</i> (2015)
Crack density	0	Arbitrary
Fracture density	0.1	Arbitrary
Fracture aperture	10 cm	Arbitrary
Total porosity	Variable	Upscaled <i>NPHI</i> log
Relative permeability of water	$k_{Water} = S_w^3$	Jin <i>et al.</i> (2018)
Relative permeability of oil	$k_{Oil} = (1 - S_w)^2$	Jin <i>et al.</i> (2018)
Reference frequency	10 Hz	Arbitrary
Query frequency	30 Hz	Arbitrary
Patch parameter q	$q_0 = 0.957$	Arbitrary
Relaxation parameter τ	10^{-4} s	Arbitrary

Table 7.1: Parameters used in interval relative azimuthal attenuation modelling.

All depth-dependent parameters are shown in Table 7.2, estimated from well logs upscaled using a moving mean with a 3 m window. The maximum distance between an upscaled well log sample and a VSP receiver locations was less than 0.05 m for all receivers. With all model parameters defined in Tables 7.1 and 7.2, I calculated the 5 independent transverse isotropic viscoelastic stiffness tensor components, C_{11} , C_{33} , C_{12} , C_{13} , and C_{55} , using a MATLAB toolbox provided by Jin *et al.* (2018).

Once all stiffness tensor components were calculated, I regularly sampled 4000 points on the unit sphere using the algorithm of Deserno (2004), and solved

the Christoffel equation, as reviewed in Chapter 2, to find the slownesses in the direction of all vectors pointing from the origin to these sample points. Using the calculated slownesses, I calculated the P-wave quality factor Q for every slowness direction as the ratio of the real and imaginary parts of the squared P-wave velocities (Carcione and Cavallini, 1995). That is to say, I found Q for each slowness direction using the relationship (Carcione and Cavallini, 1995):

$$Q = \frac{\text{Re}[V^2]}{\text{Im}[V^2]}. \quad (7.40)$$

The P-wave attenuation for each slowness direction was then the reciprocal of these calculated quality factors.

The discretised P-wave attenuation surface for a viscoelastic stiffness tensor calculated using the model of Jin *et al.* (2018) and the model parameters of receiver 25 in the reservoir interval (See Tables 7.1 and 7.2) is shown in Figure 7.16. We note this attenuation surface is calculated for a rock with fractures in the horizontal plane, as is evaluated by the model of Jin *et al.* (2018). To model vertical fractures, for example, the evaluated stiffness tensor must be rotated by 90 degrees about an axis lying in the horizontal plane.

7.8 VSP Inversion for Fracture Characterisation

The fracture properties of interest in this reservoir fracture characterisation case study were fracture strike, dip, and fracture density. To find these at each receiver, I simultaneously scaled and rotated the modelled attenuation surface of Jin *et al.*

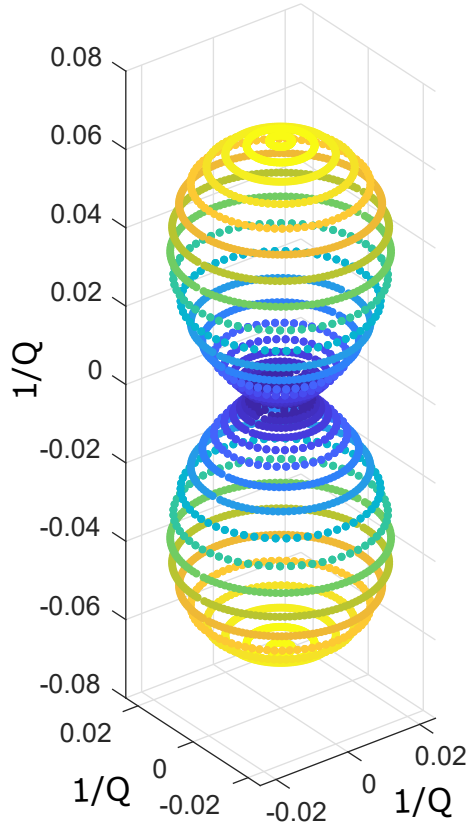
Discretised P-wave 1/Q surface from the model of Jin et al. (2018)

Figure 7.16: Discretised P-wave attenuation surface calculated by solving the Christoffel equation for all slowness directions pointing from the origin to points on the regularly spherically sampled unit sphere given a stiffness tensor from the viscoelastic model of Jin *et al.* (2018) and the model parameters for receiver 25. The radial distance of a point to the origin indicates the material's attenuation in that direction, as does each point's colour, with cool colours representing low attenuation (where P-waves propagate parallel to fracture planes) and warm colours representing high attenuation (where P-waves propagate perpendicular to fracture planes).

(2018) using 3D rotation matrices, then interpolated its intersection through the horizontal plane, making a closed, continuous curve. The fracture dip and strike were then the rotation angles which minimised the l_1 -misfit between the calculated interval relative azimuthal attenuation from VSP data, as seen in Figure 7.11, and the modelled interval relative azimuthal attenuation surface's intersection through the horizontal plane. I chose to use the l_1 -misfit as the inversion error metric its best-fitting solution is less influenced by any outlying samples when compared to, say, the l_2 -misfit. The fracture density indicator was the radial scaling applied to the attenuation surface which minimised this misfit. This is a fracture density indicator, as opposed to fracture density, as the modelled data is being matched to relative attenuation measurements. For the use of this fracture density indicator to be correct, there is an assumption that a change in fracture density results in a radial scaling of the attenuation surface which is constant in all directions. This may not be true, however by making this assumption, I removed the need to solve the Christoffel equation within every optimisation iteration for a new fracture density, speeding up the optimisation significantly. To test whether this assumption is appropriate, I performed modelling sensitivity analysis to fracture density in Section 7.10.

Being an inversion for interval properties, the solution parameters were descriptive of the 15 m of rock above the receiver at which they were calculated. I solved for the three model parameters using a bounded simulated annealing algorithm, running 100 independent simulations per receiver.

Receiver number	V_p (m/s)	V_s (m/s)	ϕ	ρ (kg/m ³)
23	5633	3194	0.09	2626
24	5004	2911	0.11	2602
25	5342	3032	0.08	2579
26	5633	3094	0.08	2625
27	5570	3132	0.07	2632
28	5940	3481	0.02	2721

Table 7.2: Receiver-dependent fracture characterisation input parameters in the reservoir interval. Parameters are from each receiver's nearest upscaled well log sample.

7.9 Results

7.9.1 Inversion results and analysis

Figure 7.17 shows a polar histogram of all simulated annealing solutions for fracture strike at each receiver in the reservoir interval. Figure 7.18 shows all strike and fracture density indicator solutions in the reservoir interval. Both Figure 7.17 and Figure 7.18 show depth-dependent solution strikes. Figure 7.18 show little variation in the fracture density indicator with depth, the median and mean of which are shown in Table 7.3 for each receiver.

The inverted fracture dip angle magnitude in the reservoir interval is shown in Figure 7.19 and the mean and median dip values are tabulated in Table 7.3. All fracture dips were found to have magnitude less than 1 degree from vertical at all depths. The searchable dip solution space in the simulated annealing inversion was constrained by ± 30 degrees and there were no known in-built algorithmic biases or penalties which encouraged the inversion to favour fractures closer to vertical. The seismic incident angles ranged from 33 to 53 degrees over the reservoir interval and seismic sources were shot over 360 degrees of azimuth. This acquisition geometry and inversion specifications should be sufficient to resolve at least mild fracture dips if they are present in the data. For this reason, any present fracture sets indeed seem to be approximately vertical.

7.9.2 Multiple fracture interpretation by Gaussian mixture modelling

The apparently bimodal distribution of fracture azimuths seen in Figures 7.17 and 7.18 suggest multiple fracture sets may be present in the reservoir interval. Assuming *a priori* that two fracture sets are indeed being resolved by the inversion, and that the errors in the stochastic optimisation solutions were normally distributed, I estimated the average strike of each fracture set for every receiver by Gaussian mixture modelling (Reynolds, 2009). A Gaussian mixture model is a weighted superposition of Gaussian probability density functions, defined as (Reynolds, 2009):

$$p(\mathbf{x}|\lambda) = \sum_{i=1}^M w_i g(\mathbf{x}|\mu_i, \Sigma_i) ; \quad (7.41)$$

where $\mathbf{x}$ is a D -dimensional data vector and w_i are mixture weights for the M component Gaussian densities $g(\mathbf{x}|\mu_i, \Sigma_i)$ with $i = 1, \dots, M$. Component densities g take the form:

$$g(\mathbf{x}|\mu_i, \Sigma_i) = \frac{1}{(2\pi)^{D/2} |\Sigma_i|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{x} - \mu_i)' \Sigma_i^{-1} (\mathbf{x} - \mu_i) \right\} ; \quad (7.42)$$

where μ_i and Σ_i are the mean and covariances of the i^{th} Gaussian component respectively, while:

$$\lambda = \{w_i, \mu_i, \Sigma_i\} \quad i = 1 \dots M . \quad (7.43)$$

Being a probability density function, mixing weights w_i are normalised such that $\sum_{i=1}^M w_i = 1$.

For each receiver, I added the entire same fracture azimuth set of solutions to both the highest and lowest azimuths of the existing solution set (0 and 180 degrees respectively) to make a new set of solutions which spanned $[-180, 360]$ degrees of azimuth and contained the same solutions repeated three times. By tiling the solution set in azimuth, like so, the Gaussian mixture model could be fitted to periodic trends in the data which spanned the upper and lower azimuth solution bounds of 0 and 180 degrees. As such, I then fitted a Gaussian mixture probability density function, assuming 6 constituent Gaussian densities existed in this tiled solution set; two Gaussian curves corresponded to two independent fracture sets in every 180 degrees of azimuth.

Disregarding all 4 solutions found outside of azimuth range $[0, 180]$ degrees, the remaining two weights w_i became a numerical indicator of the fracture dominance for their corresponding fracture sets. Figure 7.20 shows histograms of inversion strike azimuth solutions for all receivers, as well as their best-fitted Gaussian mixture density functions and the corresponding estimated mean fracture azimuths.

Table 7.3 shows the two fracture sets' mean azimuths for each receiver, corresponding to the black stars in Figure 7.20, while Figure 7.21 shows a collection of measured and calculated Pre-salt well log tracks for context. The 21 receivers in the Salt, above Pre-salt interval 1, are not displayed as they are not considered in this inversion, having insufficient well log data. The first track shows the geological markers throughout the well, while the second shows the location of all Pre-salt VSP receivers in black, as well as the mean fracture strike as a function

Receiver number	Mean strike (Az. 1)	Mean strike (Az. 2)	w_1 (Az. 1)	w_2 (Az. 2)	Median scaling factor	Mean scaling factor	Median dip (arcmin)	Mean dip (arcmin)
23	N78.2W	N38.7W	0.641	0.359	7.22	7.20	11.7	-3.7
24	N67.5W	N19.9W	0.779	0.221	7.11	7.18	-10.6	13.4
25	N67.6W	N41.4W	0.723	0.277	7.34	7.38	-12.0	-8.96
26	N62.4W	N13.2W	0.147	0.853	6.90	6.95	-16.1	-91.5
27	N17.0W	N3.2W	0.161	0.839	7.67	7.74	-12.9	-69.5
28	N1.3W	N1.0W	0.201	0.799	7.60	7.58	-7.5	-44.9

Table 7.3: Fracture characterisation solution statistics.

of depth for both fracture sets, differentiated by both marker and colour. Track three shows the fracture dominance of both fracture sets, calculated from their Gaussian mixture modelling weights. We observe a transition in fracture dominance from the more westerly-striking fracture set to the more northerly-striking fracture set, noting there seems to be only one north-striking fracture set at the bottom of the reservoir interval. Tracks four to seven show the raw well logs which were upscaled and used for the receiver-dependent model parameters, as shown in Figure 7.8 and Table 7.2. Density and porosity logs were only available in the reservoir interval, as seen in Figure 7.21. Fracture characterisation was not performed in other parts of the well for this reason.

7.10 Viscoelastic Modelling Sensitivity Analysis

The viscoelastic model of Jin *et al.* (2018) has a large number of input parameters compared to common effective medium models. In this section, I test the sensitivity of this model to changes in fracture density, effective permeability model, water saturation, and patch parameter q in a VSP setting.

To test the model sensitivity, I selected a single solution for receivers 23 and 28. With this selected fracture strike, dip, and density scaling indicator for both receivers, I modelled azimuthal attenuation as described in Section 7.7. I performed this modelling multiple times, each time varying a single model parameter while holding all other parameters constant as declared in Table 7.1. The parameter values tested are shown in Table 7.4.

Figure 7.22 shows the variation of modelled azimuthal attenuation as a function of fracture density. In the model of Jin *et al.* (2018), azimuthal attenuation is a result of the squirting of two immiscible, viscous fluids between aligned fractures

Parameter	Test values
q	$q_0 = 0.957, 1$
Fracture density	0.01, 0.05, 0.1, 0.15
Water saturation	0, 0.15, 0.25, 0.50, 0.75, 1
Relative permeability model	Jin <i>et al.</i> (2018), B.C.: $l = 1/3, 1, 2, 7.3, \infty$

Table 7.4: Model sensitivity testing parameters assuming fluid is an immiscible mixture of oil and water. Abbreviation B.C. denotes the Brooks-Corey (Brooks and Corey, 1964) relative permeability model, where different values of model parameter l are tested. The relative permeability model of Jin *et al.* (2018) is stated in Table 7.1

and surrounding, randomly oriented cracks and stiff pores. The low fracture density test, with value 0.01, shows an almost circular azimuthal attenuation pattern, with magnitude close to zero at all azimuths. This result is intuitive as no anisotropic squirt flow is expected when no fractures are present, and hence no azimuthal attenuation is expected. A very high fracture density of 0.15 is seen to increase the amplitude and alter the shape of the azimuthal attenuation pattern, as can be expected.

Figure 7.23 shows the variation of modelled azimuthal attenuation as a function of patch parameter q . Parameter q , proposed by Papageorgiou and Chapman (2017) and reviewed in Chapter 3, has a theoretical lower bound of $q = q_0 = K_i/K_j$ where K_i is the bulk modulus of the softer fluid, and K_j is that of the stiffer fluid, and an upper bound of 1. In the case of oil and water with similar bulk moduli (as opposed to, say, water and gas), q_0 is close to unity, as seen in Table 7.4. Because

of this, the variation in modelled azimuthal attenuation between the lower and upper bound of q is small, as seen in Figure 7.23. The introduction of gas into this composite fluid, for example, may significantly lower q_0 and potentially alter the modelled attenuation properties to a greater extent.

Figure 7.24 shows the variation of modelled azimuthal attenuation as a function of water saturation S_w . We see the azimuthal attenuation takes high amplitudes at both $S_w = 0$ and $S_w = 1$, with a modelled minimum occurring at $S_w = 0.5$ and maximum at $S_w = 0.25$. This result may seem counter-intuitive, but is a result of complex fluid mobility behaviour occurring across different water saturations due to the fluids' particular relative permeabilities, an effect which is observed and discussed by Papageorgiou *et al.* (2018). Given the importance of water saturation in a reservoir's economic viability, these results suggest water saturation sensitivity analysis should be heavily considered in reservoir fracture characterisation workflows, possibly justifying the use of the model of Jin *et al.* (2018).

Figure 7.25 shows how modelled azimuthal attenuation varies with the choice of implemented relative permeability model. The Brooks-Corey (Brooks and Corey, 1964) model is a commonly-used, empirical relative permeability model based on capillary effects. The Brooks-Corey parameter l is a measure of the diversity of pore sizes in the model, with higher l corresponding to the presence of fewer pore sizes. It seems the modelled attenuation has low sensitivity to the permeability model given our data and model parameterisation. All receivers in the reservoir interval have an estimated water saturation of 0.15 (see Table 7.1), so it may be that all tested relative permeability models behave similarly at this low water saturation, and the model of Jin *et al.* (2018) may be more sensitive to relative permeability model choice at water saturations closer to 0.5, say.

7.11 Discussion

The 3D attenuation surface derived from the Christoffel equation which corresponds to a rock with two sets of fractures may not necessarily be equal to the sum of the attenuation surfaces corresponding to two rocks, each with one fracture set embedded in it. Because of this, the method used herein to solve for the location and the fracture density indicator of two fracture sets in a rock is only an approximation to the more correct approach, which would solve for both fracture sets simultaneously by modelling a tilted monoclinic stiffness tensor, as opposed to the TTI stiffness tensor. Evaluating this tilted monoclinic inversion would have an expected increase computation cost over the current approach. Furthermore, the model of Jin *et al.* (2018) would first need to be extended theoretically to include multiple fracture sets.

The model of Jin *et al.* (2018) has many parameters, as seen in Table 7.1, most of which are chosen arbitrarily or estimated. I have tested the sensitivity of azimuthal attenuation for four of these model parameters, however many remain untested. There is a possibility of error in the use of upscaled well logs for receiver-dependent model parameters, as well as the unavoidable assumptions I made about rock and fluid composition. Most of these parameters, however, should primarily affect the radial amplitude of any modelled azimuthal attenuation curves, as opposed to the curves' geometrical features, for example the occurrence

of special points. It is these geometrical features which are predominantly used to solve for fracture strike and dip, and so my estimates of fracture azimuth and dip may still be useful in spite of the many error-prone modelling assumptions, estimations, and arbitrary but necessary decisions made.

I have assumed *a priori* when analysing inversion outputs by Gaussian mixture modelling that there are exactly two fracture sets present in the reservoir, which is a possible source of error. Indeed, there seems to be only one fracture set present at the bottom of the reservoir interval. One statistical approach to decide on the optimal number of Gaussian components for the mixture model is by use of the Akaike Information Criterion (Leroux, 1992), which is discussed in depth in Appendix A.

7.12 Conclusion

In this chapter, I have inverted for the fracture strike, dip, fracture density, and fracture dominance of two fracture sets in a Brazilian Pre-salt reservoir using multiple stochastic inversions given interval relative attenuation estimates from walkaround VSPs, and the fractured, viscoelastic model of Jin *et al.* (2018). To perform this inversion, I developed a new method of calculating interval relative azimuthal attenuation which is suitable for VSP data recorded in arbitrarily deviated wells beneath complex overburden such as salt structures. Testing the sensitivity of the fractured viscoelastic model of Jin *et al.* (2018) to four key parameters, it appears azimuthal attenuation is highly sensitive to water saturation and fracture density, and is less sensitive to patch parameter q and the relative permeability model employed.

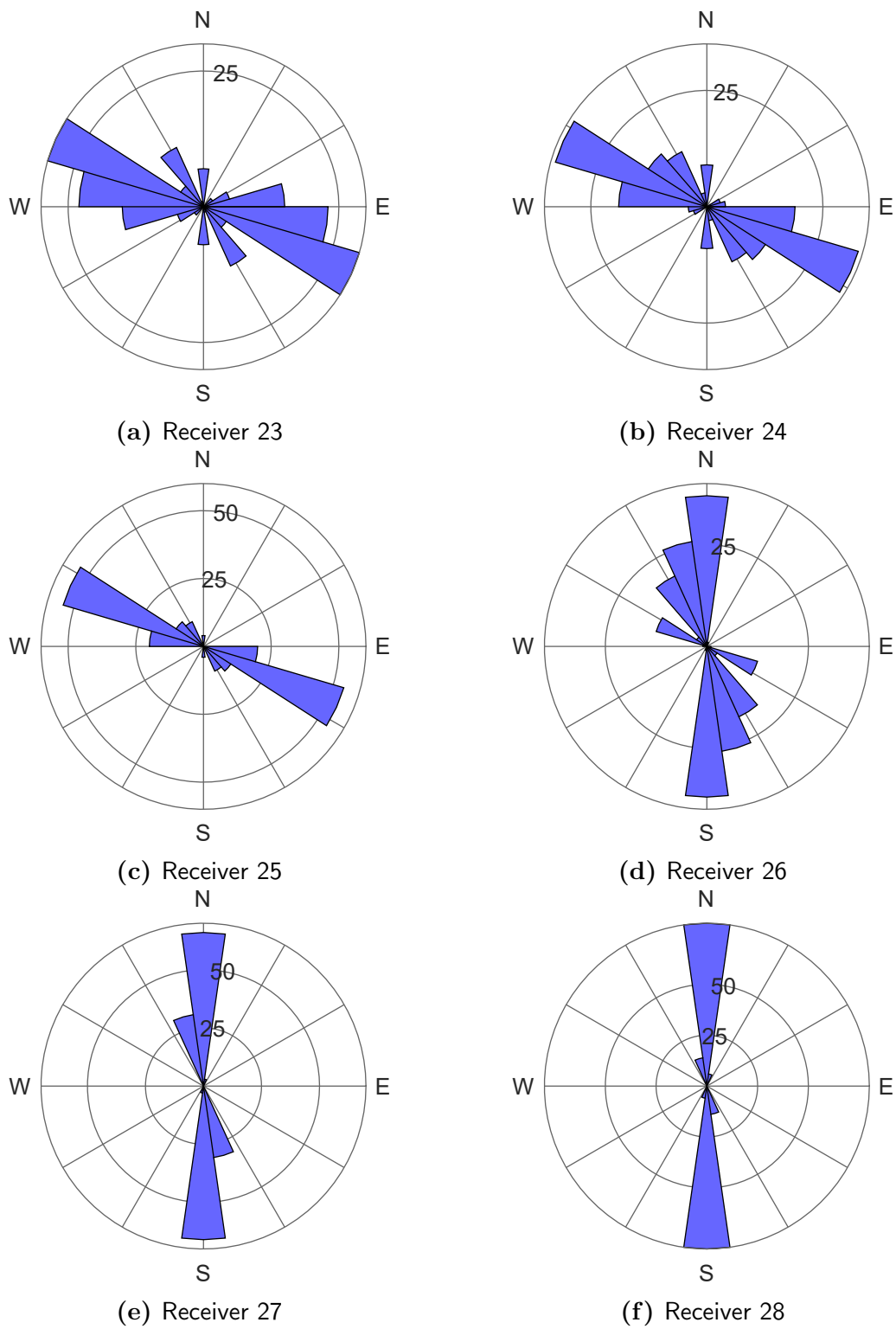


Figure 7.17: Polar histogram of inverted fracture strike azimuths. Note the varying radial axis scaling across subfigures.

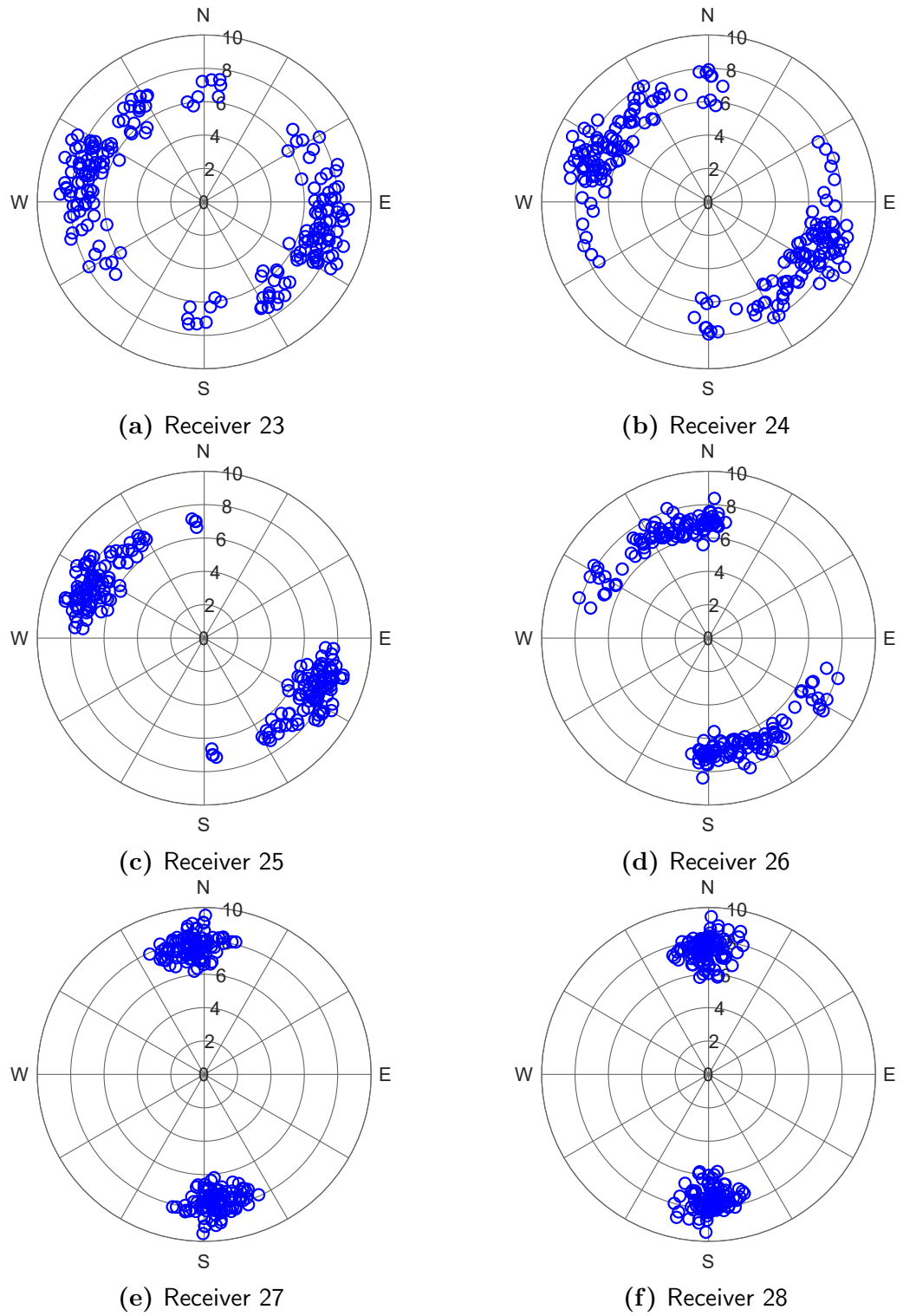


Figure 7.18: Inverted fracture strikes and their corresponding fracture density indicators, signified by each solution's radial magnitude.

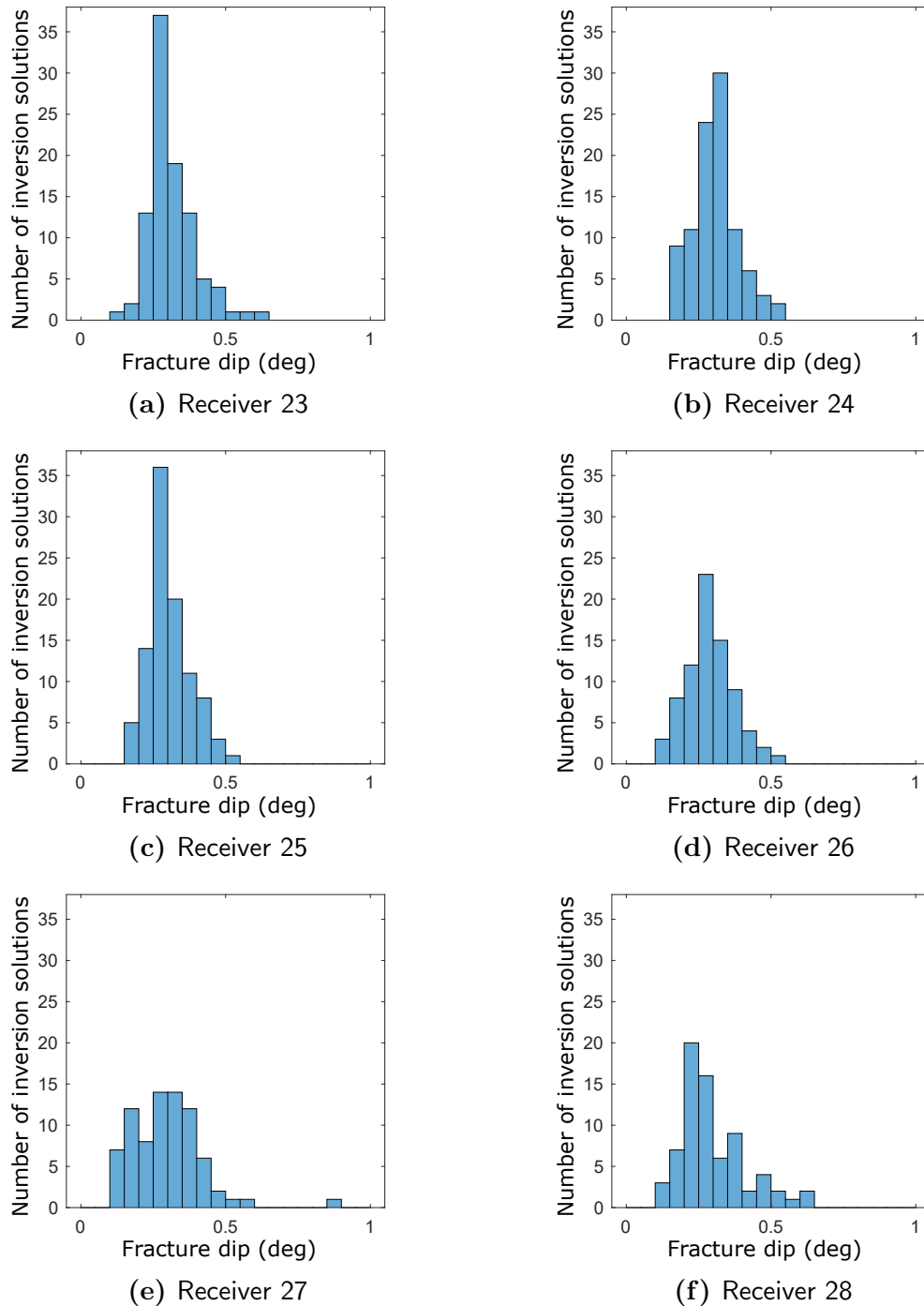


Figure 7.19: Histogram of inverted fracture dip magnitudes in degrees. We see all fractures at all depths appear approximately vertical.

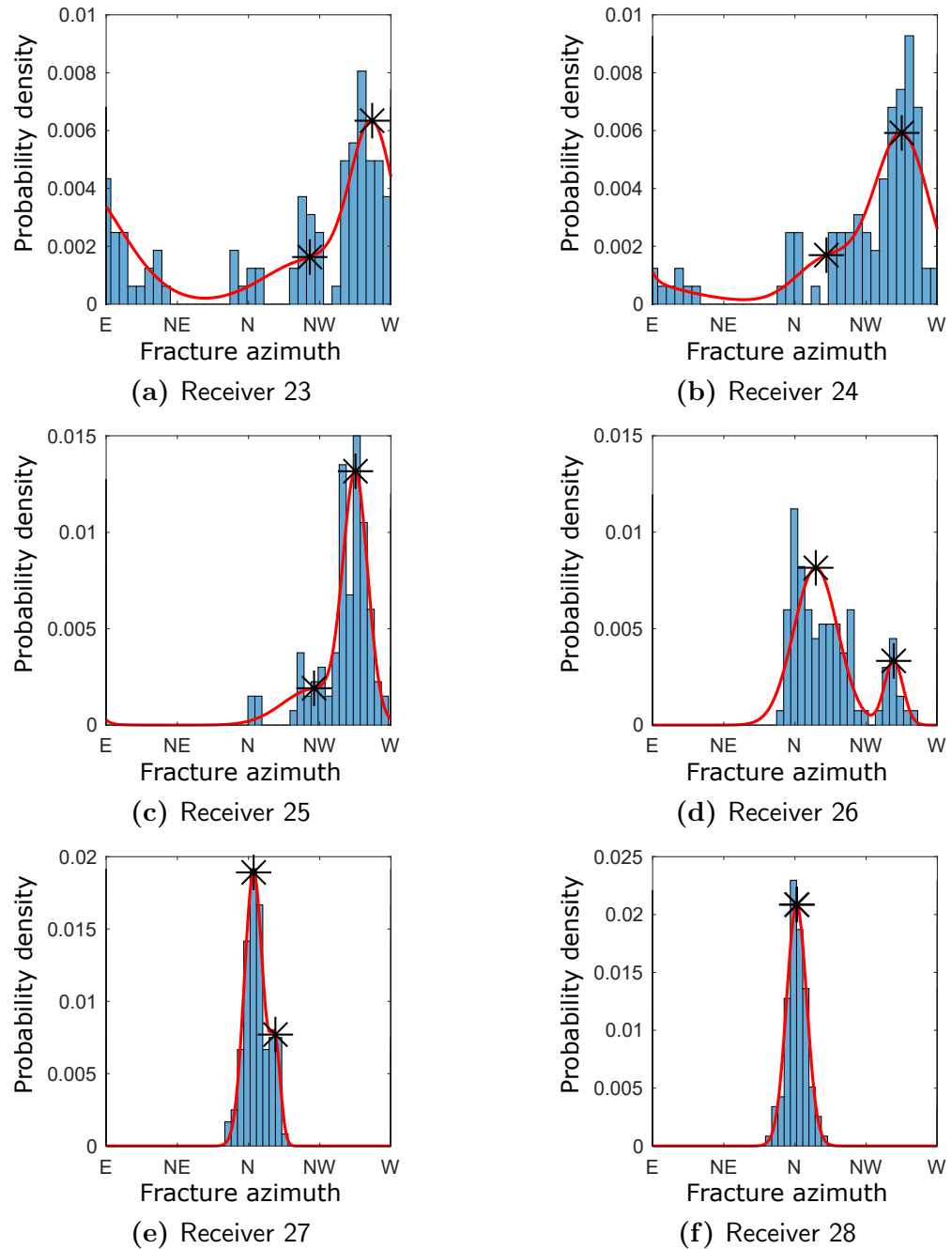


Figure 7.20: Histogram of inverted fracture strike azimuths and best-fitting 2-component Gaussian mixture probability density functions. The two Gaussian mixture modelled mean fracture azimuths for each receiver are shown with black stars.

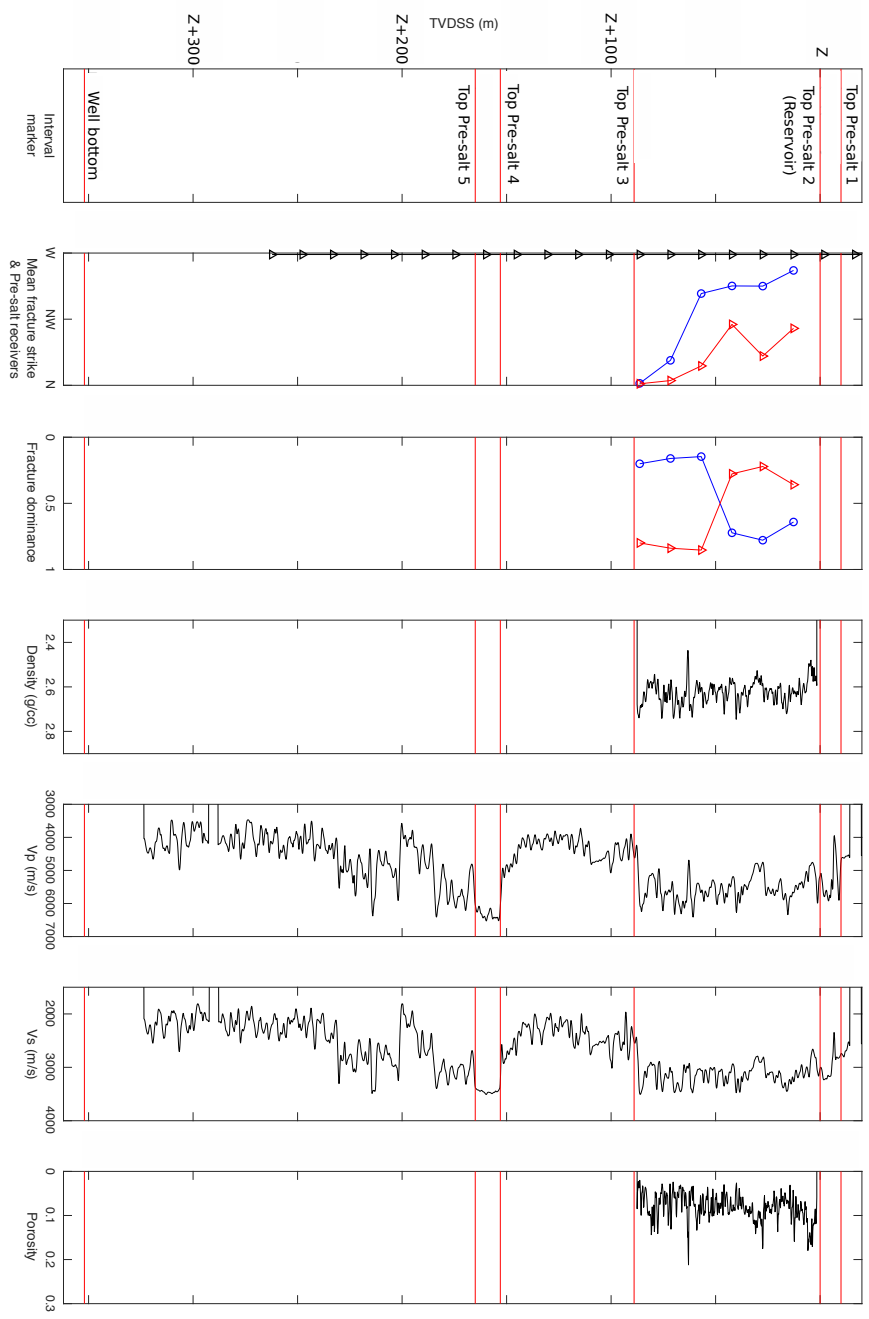


Figure 7.21: Well A's fracture inversion results and measured well logs. The trend with blue circles represents the inverted fracture set with a more Westerly strike, while the red triangle trend represents that with a more Northerly strike. Note VSP inversion only occurred where all well logs were available for model parameterisation. Also, the 21 VSP receivers positioned in the Salt interval are not displayed here. Black triangles show the VSP receiver locations.

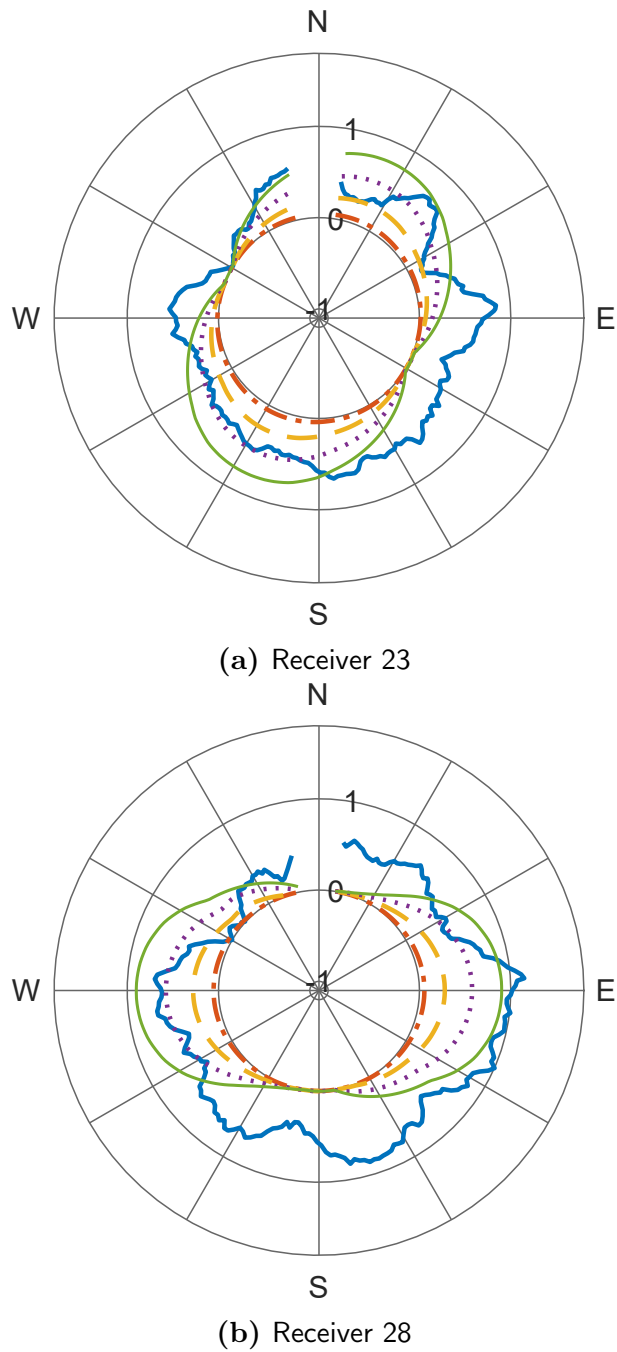
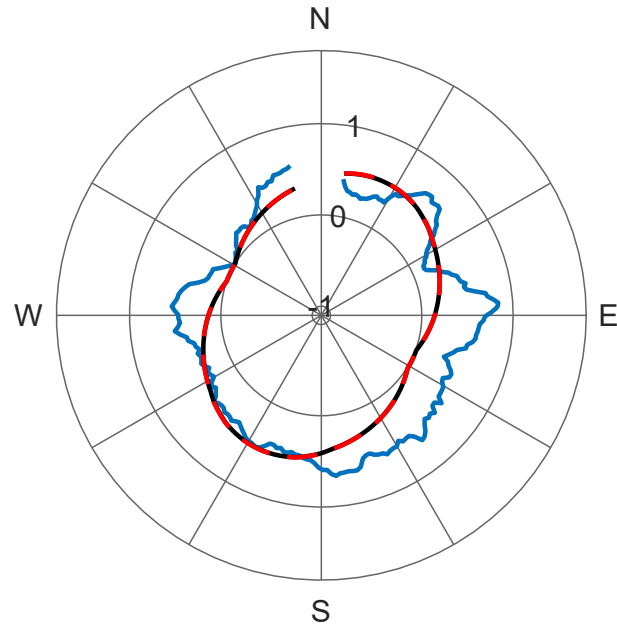
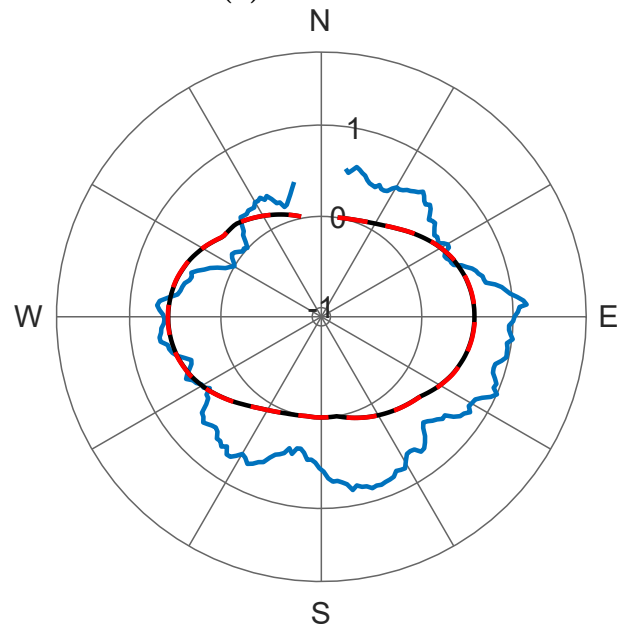


Figure 7.22: Azimuthal attenuation sensitivity testing for fracture density, holding all other parameters constant, for VSP receivers 23 and 28. Modelled attenuation with fracture density 0.01 (dot-dashed orange), 0.05 (dashed yellow), 0.10 (dotted purple), and 0.15 (solid green) are compared with the inverted attenuation from measured data (solid blue).

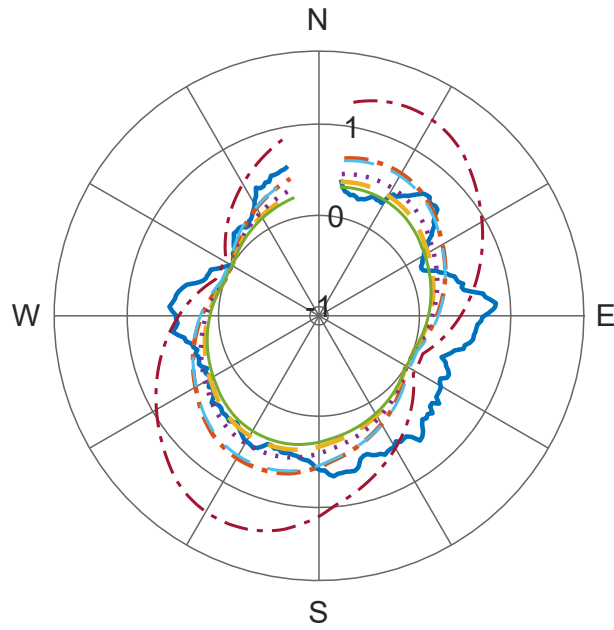


(a) Receiver 23

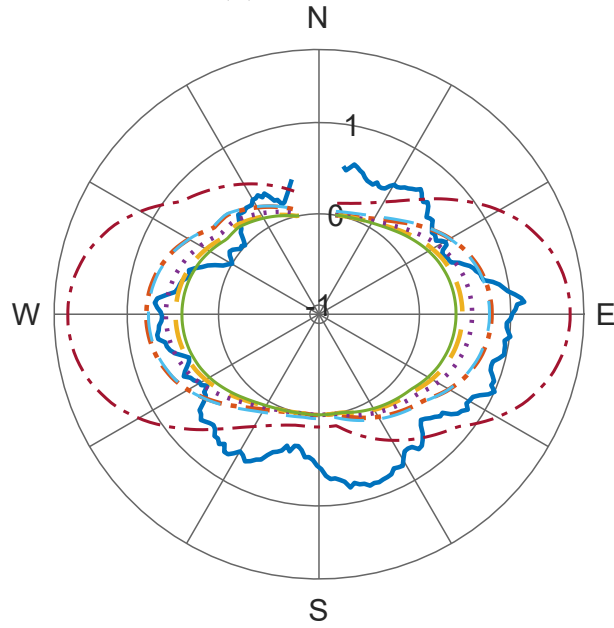


(b) Receiver 28

Figure 7.23: Azimuthal attenuation sensitivity testing for patch parameter q , holding all other parameters constant, for VSP receivers 23 and 28. Modelled attenuation with patch parameter $q = q_0$ (solid black), and $q = 1$ (dashed red) are compared with the inverted attenuation from measured data (solid blue).

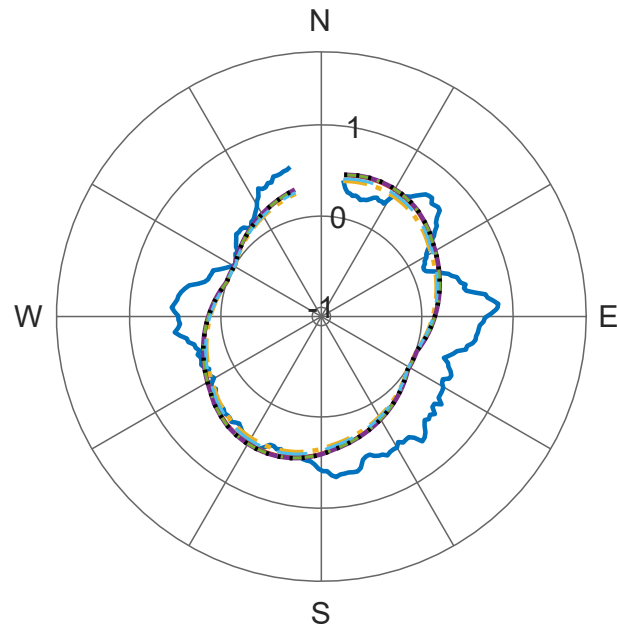


(a) Receiver 23

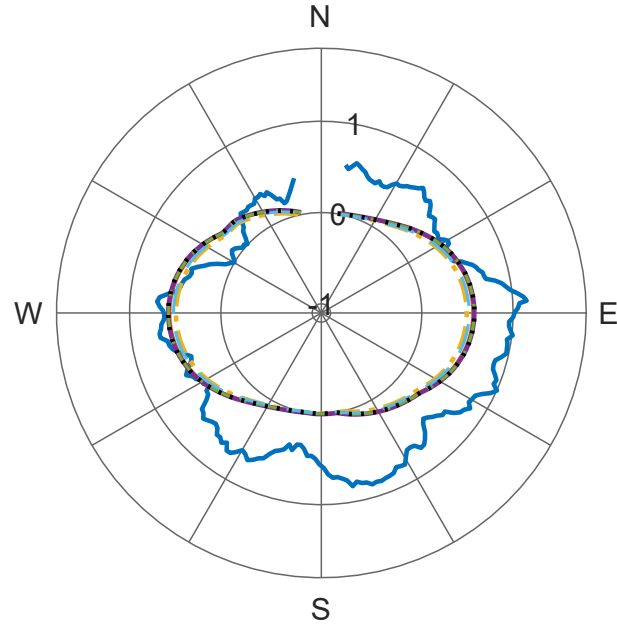


(b) Receiver 28

Figure 7.24: Azimuthal attenuation sensitivity testing for water saturation, holding all other parameters constant, for VSP receivers 23 and 28. Modelled Interval relative attenuation with water saturations $S_w = 0$ (dot-dashed thick orange), $S_w = 0.15$ (dashed thick yellow), $S_w = 0.25$ (dotted purple), $S_w = 0.5$ (solid green), $S_w = 0.75$ (dashed thin blue), and $S_w = 1$ (dot-dashed thin red), are compared with the inverted attenuation from measured data (solid blue).



(a) Receiver 23



(b) Receiver 28

Figure 7.25: Azimuthal attenuation sensitivity testing for relative permeability models, holding all other parameters constant, for VSP receivers 23 and 28. Modelled relative Interval relative attenuation given the relative permeability models of Jin *et al.* (2018) (solid orange), Brooks-Corey with $l = \infty$ (thick dot-dashed yellow), $l = 7.3$ (thick solid purple), $l = 2$ (thin dot-dashed green), $l = 1$ (thin dashed blue), and $l = 1/3$ (dotted black), are compared with the inverted attenuation from measured data (solid blue).

Chapter 8

Pre-Salt Tilted Transverse Isotropy Estimation from Walkaway VSPs

8.1 Summary

Understanding a reservoir's seismic tilted transverse isotropy (TTI) can lead to improved seismic imaging and permeability modelling. The TTI stiffness tensor components or corresponding anisotropic parameters can be estimated by inverting the Christoffel equation using the slowness-polarisation technique, given multicomponent VSP data which contains either P- and S-wave measurements, or only P-wave measurements as input. Testing both types of inversion, I find both solvers are effective in modelling public domain VSP measurements

from a shale reservoir. However, the P-wave only inversion method is more suitable for modelling the anisotropic properties of the case-study Brazilian Pre-salt reservoir, where identifying first-arriving S_V -waves is challenging, leading to high uncertainty in estimated S-wave slowness and polarisation measurements. Estimating anisotropic parameters in the Brazilian Pre-salt reservoir using the P-wave only algorithm, it seems the reservoir interval has a tilted TI symmetry axis with both a negative ϵ and δ . This result may be consistent with the findings of Chapter 7, which established the possible presence of vertical fractures in the reservoir interval, that could potentially lead to slower vertical than horizontal P-wave velocities, and hence a negative ϵ . The conversion from the P-wave only slowness-polarisation algorithm's native anisotropic parameters to the Thomsen parameters requires additional vertical V_s measurements, which I obtain from upscaled well logs. Testing sensitivity to vertical V_s , I find the estimated anisotropic parameters in the reservoir interval remain significantly different to the surrounding geological intervals, even in the case of $\pm 20\%$ uncertainty.

8.2 Introduction

By understanding and modelling a reservoir's elastic transverse isotropy (TI), improvements can be made in seismic imaging (Alkhalifah *et al.*, 1996; Fletcher *et al.*, 2009), and in understanding permeability anisotropy (Rasolofosaon and Zinszner, 2002). A typical way to estimate a reservoir's TI anisotropy is by inverting the Christoffel equation (Chapter 2, equation 2.38) for the reservoir's effective stiffness tensor using measured slowness and polarisation data (White *et al.*, 1983; De Parscau, 1991; Hsu *et al.*, 1991). This is typically achieved using measurements from multicomponent VSP receivers situated in the reservoir with active seismic sources on a walkaway VSP acquisition geometry (Gaiser,

1990; Horne and Leaney, 2000). The resultant estimation of the reservoir's TI anisotropy can then be interpreted for insights into formation strike and dip, layering, and even the presence of intrinsically anisotropic rocks, such as shales.

By inverting the Christoffel equation using slowness-polarisation methods, I find negative Thomsen's (Thomsen, 1986) parameters, δ and ϵ , in the reservoir interval. Even in the case of up to 20% uncertainty in shear wave vertical velocities, there seems to be a significant difference between the δ and ϵ parameters in the reservoir interval, and in the geological intervals above and below. Negative ϵ values in the reservoir interval may indicate the presence of vertical fracturing, which would seem consistent with findings from Chapter 7, which also predicted vertical fracturing using an independent set of measurements and different inversion methods.

In this chapter, I first pre-process seismic data from 40 3-component VSP receivers situated in Well A, penetrating the Brazilian Pre-salt reservoir, in preparation for TI inversion. I then implement the slowness-polarisation inversion algorithms of both Horne and Leaney (2000) and Grechka and Mateeva (2007) to solve for vertical and tilted TI (VTI and TTI, respectively) anisotropy, assessing their performance using the shale VSP measurements of Horne and Leaney (2000). Finding inversion using shear-wave measurements to be unstable in the Pre-salt VSP data, I choose to use the P-wave only slowness-polarisation algorithm of Grechka and Mateeva (2007) in estimating the TTI anisotropy at all receivers in the Pre-salt VSP well. After estimating the Thomsen parameters at every VSP receiver in Well A, I test the parameters' sensitivities to vertical shear wave velocity, which is estimated throughout this chapter using upscaled well logs.

8.3 Inversion Algorithms

The Christoffel equation (Chapter 2, equation 2.38) is the relationship between a plane wave's slowness and polarisation directions at a given point in an elastic medium possessing stiffness tensor C_{ijkl} and density ρ . Thus, if we measure a wave's slowness and polarisation information, we can invert for the rock's density-normalised stiffness tensor components, or other measures of anisotropy, such as the Thomsen parameters.

Hsu *et al.* (1991) and De Parscau (1991) both showed four density-normalised VTI parameters, c_{11} , c_{13} , c_{33} , and c_{55} , can be solved for uniquely given slowness and polarisation measurements from both P- and fast S-waves. In the case of weak anisotropy, one can equivalently solve for parameters V_{p0} , V_{s0} , ϵ , δ , where V_{p0} and V_{s0} are the vertical P- and S- wave velocities, and ϵ and δ are the Thomsen (1986) parameters (Grechka and Mateeva, 2007). Horne and Leaney (2000) presented an algorithm to invert the Christoffel equation for stiffness tensor components using P- and S-wave slowness and polarisation measurements extracted from multicomponent VSP data, solving for four independent model parameters in the special case of weak TI anisotropy.

8.3.1 Horne and Leaney's slowness-polarisation algorithm

The algorithm of Horne and Leaney (2000) solves for the triclinic stiffness tensor using seismic data acquired on an array of multicomponent receivers. To do this, the iterative algorithm finds the stiffness tensor components which generate a propagation direction which models a polarisation direction most accurately, given the measured slownesses and polarisations. At each receiver, the first-arriving S- and P-waves' polarisation directions are estimated by hodogram analysis, and

the wave's vertical slowness is estimated from the difference in travel time when arriving at receivers.

With slownesses and polarisation directions estimated at each receiver, the following algorithm is implemented:

1. Solve the Christoffel equation (Chapter 2, equation 2.38) for the P-wave polarisation direction p_i , using the initial propagation direction n_i which is initially assumed to be the measured polarisation direction p_0 .
2. Calculate the difference in measured and solved polarisation direction, $\Delta_i = p_i - p_0$.
3. Update and renormalise propagation direction such that $n_{i+1} = (n_i + \Delta_i) / |n_i + \Delta_i|$.
4. Solve the Christoffel equation for polarisation using this new propagation direction. Stop iterating if the polarisation direction is close enough to the measured polarisation. If not, iterate from step 2.

To quantify VTI symmetry under the assumption of weak anisotropy, Horne and Leaney invert for the four model parameters:

$$\mathbf{m} = \left(\epsilon_P, \epsilon_A, \frac{V_p}{V_{p0}}, \frac{V_s}{V_{s0}} \right)^T ; \quad (8.1)$$

Where ϵ_P and ϵ_A are the anisotropic parameters of Carrion *et al.* (1992):

$$\epsilon_P = \frac{c_{11} - c_{33}}{c_{11} + c_{33}}; \quad (8.2)$$

and

$$\epsilon_A = \frac{(c_{11} - c_{55})(c_{33} - c_{55}) - (c_{13} + c_{55})^2}{(c_{11} - c_{55})(c_{33} - c_{55})}. \quad (8.3)$$

Parameter ϵ_P (Carrion *et al.*, 1992) is similar to the parameter ϵ of Thomsen (1986) in that it considers a normalised difference in horizontal and vertical P-wave stiffnesses, and hence velocities, while ϵ_A is the anellipticity parameter, which is related to the anellipticity of the rock's slowness surface.

Parameters V_p/V_{p0} and V_s/V_{s0} are the ratio of the final and initial P and S wave velocities in the direction of the TI medium's symmetry axis (e.g., the vertical velocities in a VTI medium).

In my inversion for the TTI stiffness tensor, I also include a symmetry axis dip angle parameter θ , which is used to rotate the stiffness tensor every iteration of the inversion algorithm. In practice, I solved for the 5 model parameters using a simulated annealing algorithm.

8.3.2 Grechka and Mateeva's slowness-polarisation algorithm

This algorithm of Horne and Leaney (2000) is effective in estimating anisotropy where broad polar angular coverage and easily separable P- and S-wave measurements are available. In the case of complex overburdens, however, first arriving shear waves may be more challenging to isolate in seismic records, and shear wave slownesses may be difficult to estimate. These challenges led Williamson and Maocec (2001) then Zheng and Pšenčík (2002) to investigate estimating elastic

anisotropy using only P-wave measurements. Zheng and Pšenčík (2002) showed this is theoretically possible in the case of weak anisotropy; a result which was tested by Gomes *et al.* (2004) on VSP data. Zheng and Pšenčík (2002) were able to remove the need for S-wave measurements in the anisotropy inversion algorithm in the case of weak anisotropy by absorbing all mathematical S-wave dependencies into the model's fitting parameters, which were thus solved for using only P-wave measurements.

Grechka and Mateeva (2007) showed this combining of parameters to produce an approximate expression of the Christoffel equation with no explicit S-wave terms is also valid in the case of moderate to strong anisotropy. They concluded P-wave slowness-polarisation VSP data can be described by the equation:

$$q(\psi) \approx \frac{\cos \psi}{V_{p0}} (1 + \delta_{VSP} \sin^2 \psi + \eta_{VSP} \sin^4 \psi) ; \quad (8.4)$$

where q is the P-wave's vertical slowness and ψ is the P-wave polarization angle from vertical. The algorithm solves for anisotropy parameters η_{VSP} and δ_{VSP} which are defined as (Grechka and Mateeva, 2007):

$$\eta_{VSP} = (2f_0 - 1) \eta ; \quad (8.5)$$

$$\delta_{VSP} = (f_0 - 1) \delta ; \quad (8.6)$$

where η is the anellipticity parameter presented by Alkhalifah and Tsvankin (1995):

$$\eta = \frac{\epsilon - \delta}{1 + 2\delta}; \quad (8.7)$$

and f_0 is defined as:

$$f_0 = \frac{1}{1 - \frac{V_{s0}^2}{V_{p0}^2}}. \quad (8.8)$$

Equation 8.4 shows that only two parameters, δ_{VSP} and η_{VSP} , are required to estimate the anisotropic dependence of q , removing V_{s0} as an independent parameter.

In practice, I convert η_{VSP} and δ_{VSP} to η and δ , and hence ϵ , using equations 8.5 to 8.8. This conversion requires both vertical P- and S-wave slowness measurements, which I obtain from upscaled velocity well logs at the location of each receiver.

8.4 VSP Pre-Processing

Walkaway VSP measurements were acquired on 40 VSP receivers situated in Well A, which was deviated at 18.5 degrees from vertical. As described in Chapter 7, the 3-component, ungimballed receivers were spaced 15 m MD and spanned the Salt and Pre-salt intervals. The walkaway line sailed at azimuth N115 (NW-SE), was 12 km long, and centred on Well A, with 231 shots fired at approximately every 40 m. The source and receiver positions are shown in Figure 8.1, while each trace's offset, as calculated from SEG-Y headers, is shown in Figure 8.2. The data were recorded at a 2 ms sample rate with a trace length of 8 s. The reservoir interval, "Pre-salt 2", contained VSP receivers 23 to 28.

A full length gather from a receiver's Z-component is shown in Figure 8.3, while the same gather is shown with a zoom on the first arriving P-wave in Figure 8.4. The asymmetry in the first-arriving P-wave is due to the asymmetrical salt diapirism above the well.

After loading the 3-component SEG-Y data, I performed receiver rotation to orient the Z-component to vertical and a horizontal component to radial. I did this by finding the orientation which maximised the P-wave and fast S-wave energies in the vertical and radial components respectively within a small window surrounding the downgoing, P-wave first arrival at zero offset. After receiver rotation, I performed manual polarity corrections as recommended by Gaiser (2016) so all traces had consistent, conventional polarities in the radial-transverse coordinate system.

I then picked downgoing P-wave velocities on all shot gathers which were used to apply a linear moveout (LMO) correction to each gather, flattening the first-arriving, downgoing P-wave events. After LMO-correction, the downgoing P-wave energy was separated from all other energy using F-K filtering (Embree *et al.*, 1963; Yilmaz, 1987), as shown in Figure 8.5. Using the same method, I picked velocities for, then isolated the downgoing S-, upgoing S-, and upgoing P-waves over all shot gathers.

Using the radial and transverse components for each rotated gather, I generated upgoing and downgoing P-wave and S-wave hodograms from the wave energy within a 100 ms window centred on the first arrival, as shown in Figure 8.6. These hodograms were then used to calculate the polarisation direction for each recorded wavefield.

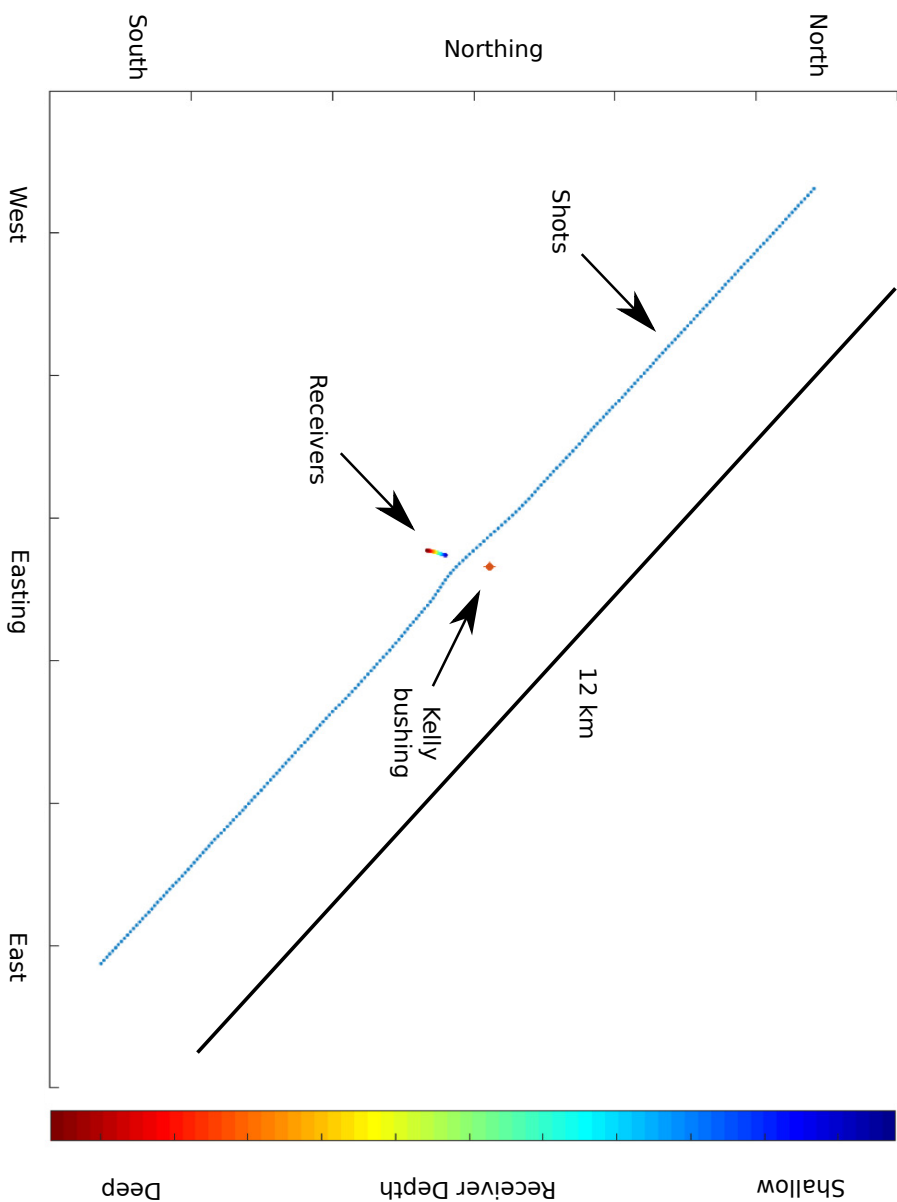


Figure 8.1: Map view of the walkaway sail line and receiver geometry. Blue points show locations of shots, spaced 40 m apart. The orange diamond shows the Kelly bushing location, while the rainbow coloured points show receiver locations down the well, with blue representing shallow and red representing deep. The well's deviated trajectory is visible from this perspective.

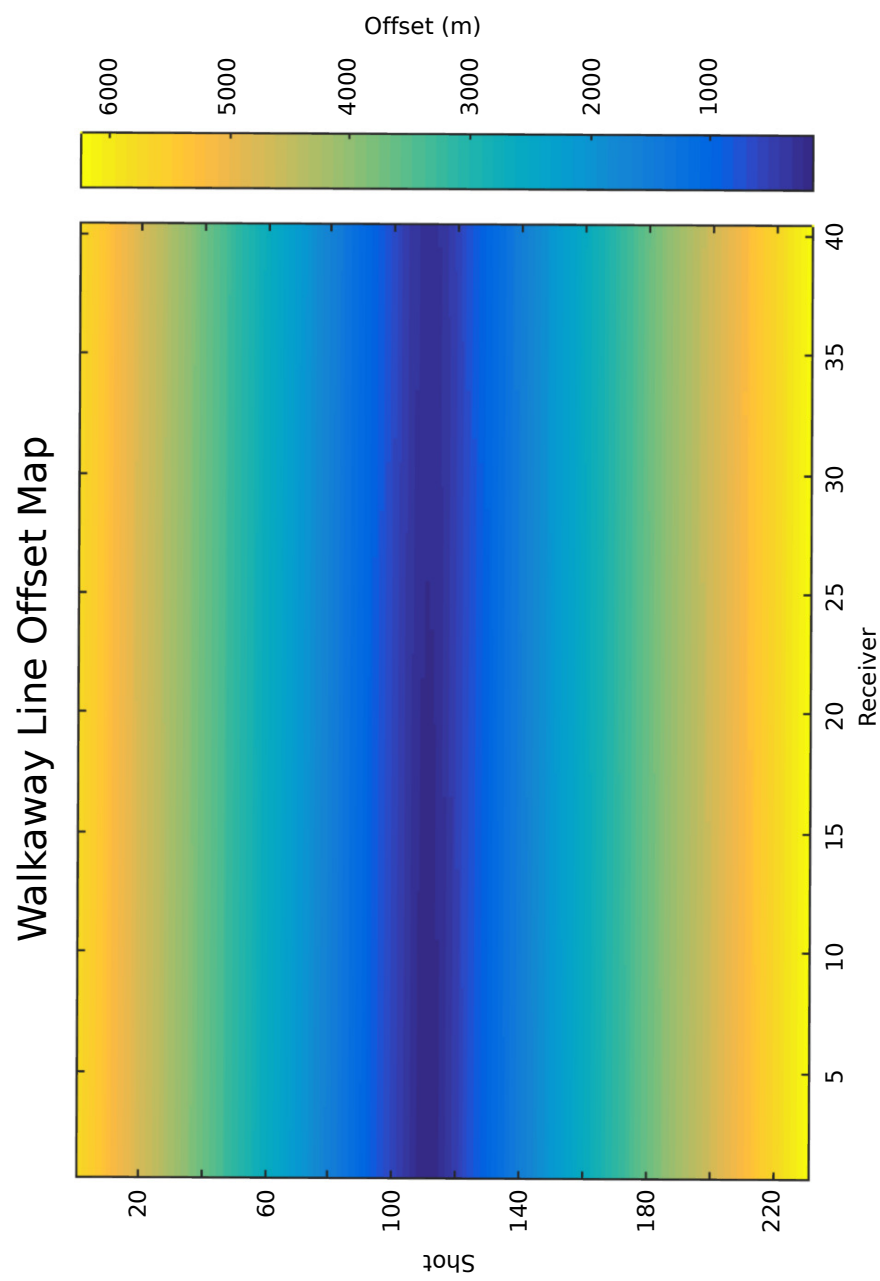


Figure 8.2: An offset map for all source-receiver pairs calculated from survey geometry information in the seismic SEG-Y headers. A maximum offset of over 6 km was achieved in the survey, allowing for waves incident with large polar angles to be recorded. Shooting with large offsets in subsalt seismic surveys can also allow more energy to penetrate beneath the salt, either by avoiding salt bodies altogether, or accounting for the potentially large changes in ray direction when crossing salt-sediment boundaries.

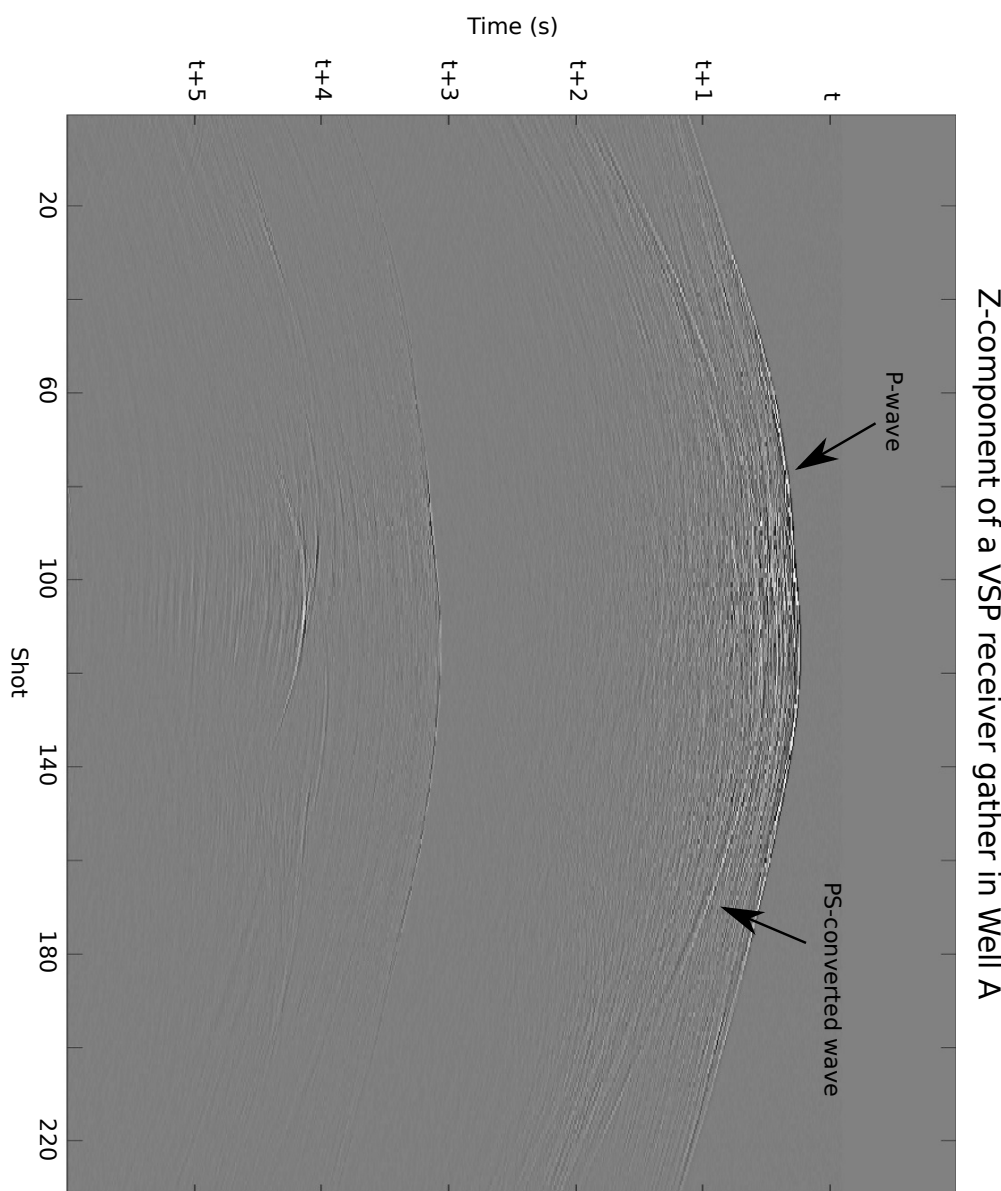


Figure 8.3: The Z-component of a raw receiver gather in Well A. The first arriving P-wave and a P-to-S converted wave (e.g., from the top or base of salt) are marked.

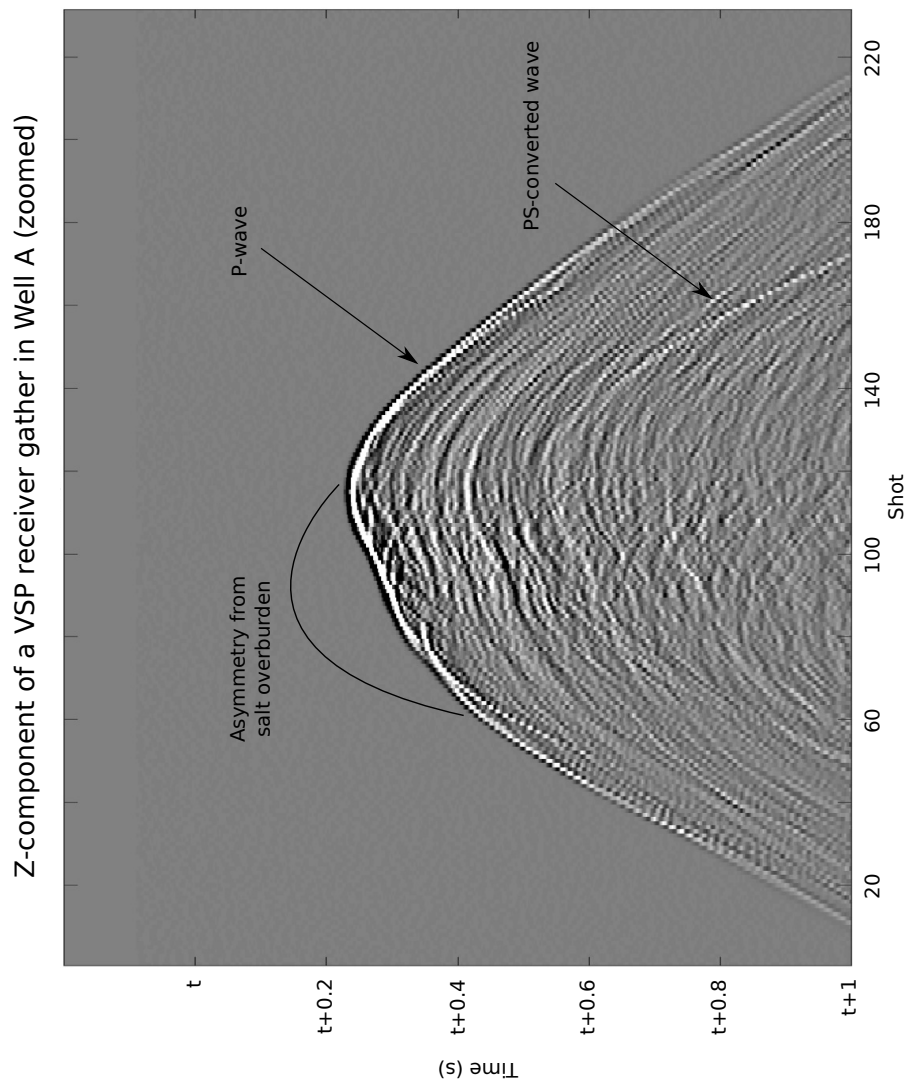


Figure 8.4: The Z-component of the raw receiver gather shown in Figure 8.3, zoomed in on the first arriving P-wave. The first arriving P-wave and a P-to-S converted wave (e.g., from the top or base of salt) are marked. The gather is asymmetrical about zero offset due to the presence of an asymmetrical salt overburden.

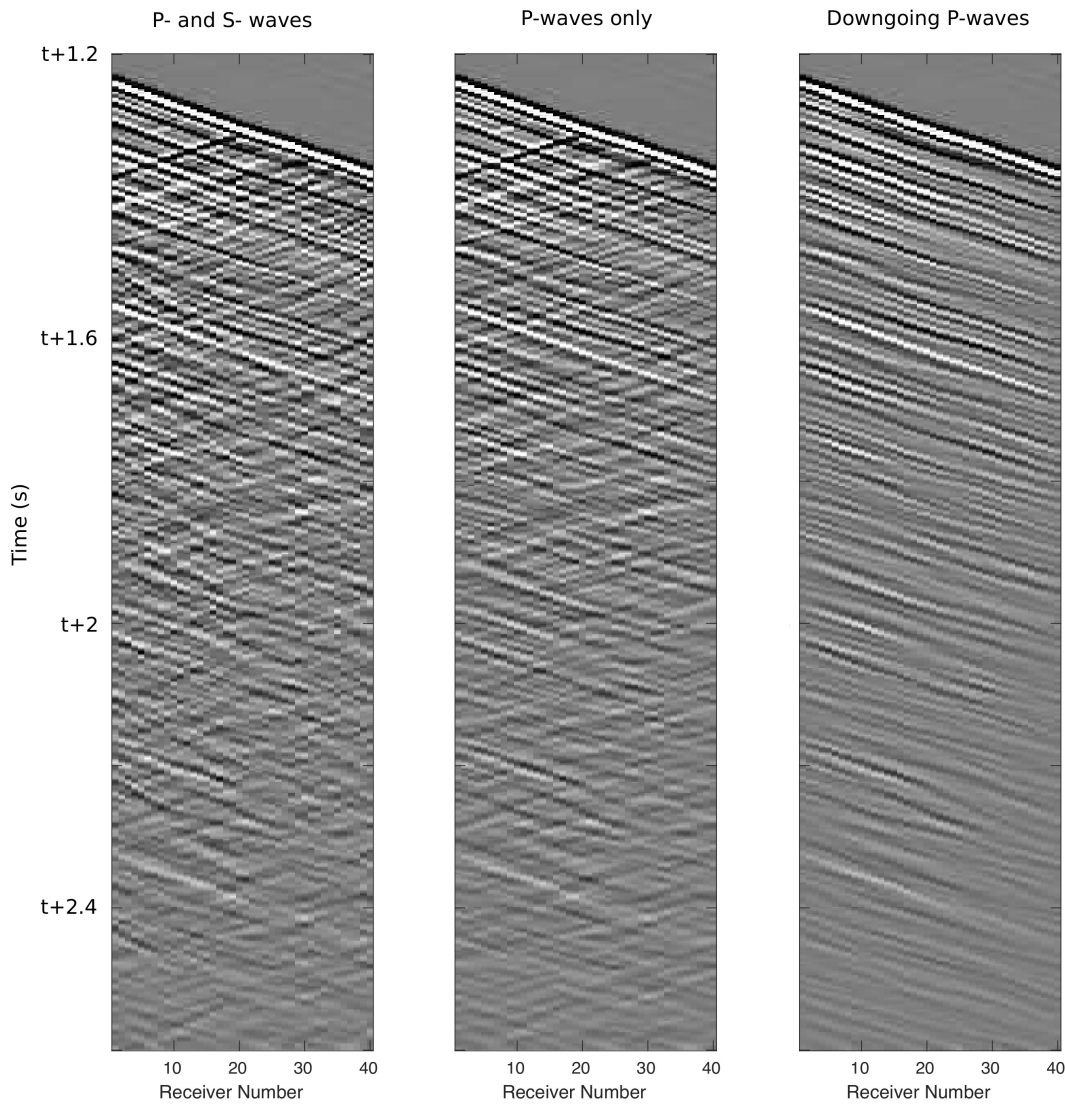


Figure 8.5: Example Pre-salt shot gathers showing stages of wavefield separation.

A shot gather (left) after receiver rotation and polarity correction contains upgoing and downgoing P- and S-waves. Separating P- and S-waves in the F-K domain produces a gather of upgoing and downgoing P-waves (centre). Removing upgoing energy in the F-K domain produces a downgoing P-wave gather (right). All other individual wavefields' gathers are created similarly. Minor artefacts are visible late in the downgoing P-wave gather, however the slowness-polarisation method only uses energy near the first arrival, making this tolerable.

I smoothed each of the four wavefields' picked interval velocities in the shot-receiver domain, then calculated interval slownesses as their reciprocals, as shown in Figure 8.6.

8.5 VSP Inversion for Tilted Transverse Isotropic Parameters

I implemented the algorithm of Horne and Leaney (2000) to invert for Thomsen's parameters ϵ and δ , given input slownesses and polarisations calculated from multicomponent VSP measurements. I also included a dip angle parameter in the inversion by incorporating of a cosine rotational transform which allowed the algorithm to solve for the formation dip by rotating the stiffness tensor within the plane of the walkaway sail line. I tested my inversion implementation in both VTI and TTI mode using the data of Horne and Leaney (2000), as shown in Figure 8.7a. The predicted anisotropic parameters are $\epsilon = 0.19$ and $\delta = -0.04$, which seems reasonable for shale (Sayers, 2004), and an estimated dip angle of 4 degrees.

I assessed the slowness-polarisation measurements extracted from Well A's walkaway VSP measurements, which are shown in Figure 8.8 for receiver 1 in the salt interval, and receiver 28 in the Pre-salt 2 (reservoir) interval.

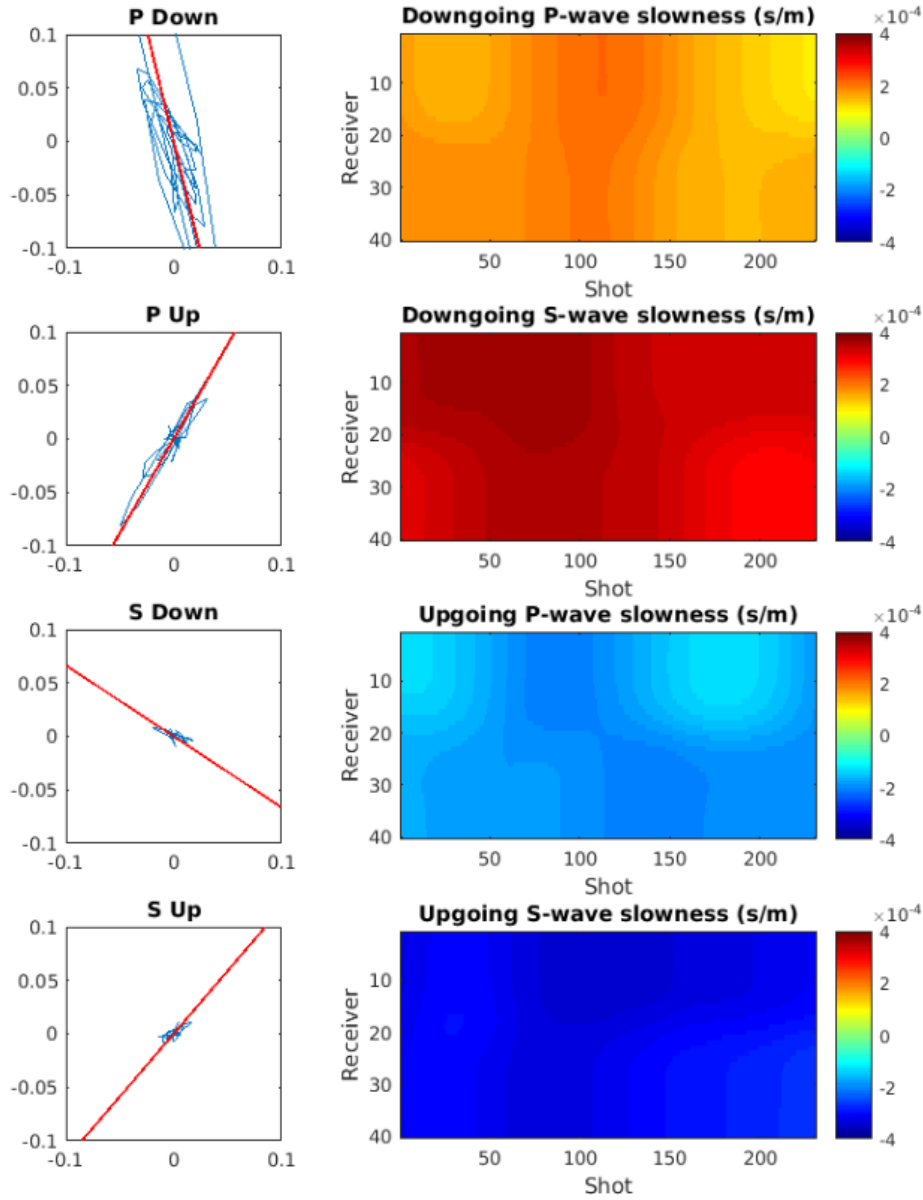


Figure 8.6: Typical hodograms and slowness fields for 4 separated wavefields. The gradient angle of the best fitting line (red) through recorded hodograms for each wavefield (left) provide the polarisation angles for the slowness-polarisation inversion input, while slownesses for each trace are from the slowness fields shown (right), calculated as the reciprocal of picked velocities after smoothing in the shot-receiver domain.

When I tried using the inversion algorithm of Horne and Leaney (2000) on these VSP measurements, results were unstable due to a large scattering of S-wave measurements, as seen in Figure 8.8. This observed scatter in the S-wave measurements can be expected in receivers beneath complex overburden, (for example, subsalt) (Grechka and Mateeva, 2007), where multiple S-waves can arrive coincidentally, making separation of the “correct” S_V -wave unachievable. Because of this, I chose to invert the Pre-salt VSP data for TTI parameters using the “P-wave only” slowness-polarisation algorithm of Grechka and Mateeva (2007).

I implemented the P-wave only slowness-polarisation algorithm, first modelling the data of Horne and Leaney (2000) to check the code’s performance, as seen in Figure 8.7b. I then performed 200 inversions given measurements made at each VSP receiver using a simulated annealing algorithm with a randomly varying starting point for each simulation. I chose the optimal solution for each receiver as that which minimised the l_1 -misfit between modelled and measured slownesses, and used upscaled measured velocity well logs to convert from the inversion output parameters to the conventional Thomsen parameters by equations 8.5 to 8.8, as shown in Figure 7.8.

Figure 8.9 shows the estimated δ , ϵ , dip angle, and inversion misfit for all receivers. Figure 8.10 shows all 200 simulation solutions for all receivers and their associate inversion misfit. A clear change in ϵ and δ is observed when moving from the salt (receiver 20) to the Pre-salt 1 interval (receiver 21) and the Pre-salt 2 (reservoir) interval (receiver 23) in Figure 8.9, where the values become negative. The TTI’s dip angle is also significant (mostly greater than 10 degrees from the vertical), arguably justifying the use of a TTI algorithm, rather than VTI inversion. The values of ϵ and δ in some cases have magnitude greater than 0.3, which could be considered too high for the assumption of weak to moderate anisotropy (Thomsen, 1986). These large magnitudes of values may be accurate; however, they may also

be due to uncertainty in measured velocities, either from VSP data or the upscaled well logs used in converting to Thomsen's parameters. It thus seems sensitivity analysis is an appropriate step for better understanding their accuracy.

Equation 8.8 shows the dependence of the Thomsen parameter solutions on vertical shear velocity. To test the sensitivity of the Thomsen parameter estimates to V_{s0} through equations 8.5 to 8.8, I calculated Thomsen parameters for all receivers multiple times using the V_{s0} calculated from upscaled well logs with an added uniform random error of $\pm 20\%$, showing results in Figure 8.11. Parameters ϵ and δ are both seen to be sensitive to V_{s0} . However, the reservoir interval is still visibly different in anisotropic properties to the formations above and below. Thus, it seems that even with significant errors in estimated shear wave velocity, the reservoir interval can still be recognised as having a qualitatively different anisotropic symmetry to its surrounding intervals.

8.6 Discussion

Performing fracture characterisation in Chapter 7 showed two non-orthogonal, vertical fracture sets may be present in the reservoir interval. In the presence of vertical fracturing, the non-vertical P-wave velocity can be slower than the vertical P-wave velocity, as waves must transect fractures when travelling non-vertically. This can result in negative ϵ values. The negative ϵ values predicted by slowness-polarisation inversion therefore seem consistent with the hypothesis that vertical fractures may be present in the reservoir interval.

The anisotropic parameters estimated in this section both have moderate to large magnitudes in the reservoir interval, being greater than 0.3 at some depths (Figure 8.9). The distribution of solutions when accounting for $\pm 20\%$ uniform random

error in V_{s0} , however, all have at least one quartile with magnitudes less than 0.3 (Figure 8.11). Equations 8.5 to 8.8 show that a systematic increase in V_{s0} leads to a systematic decrease in both ϵ and δ . Therefore, the systematic overprediction of V_{s0} through its acquisition, pre-processing, or application to the estimation of the Thomsen parameters, could lead to more negative values.

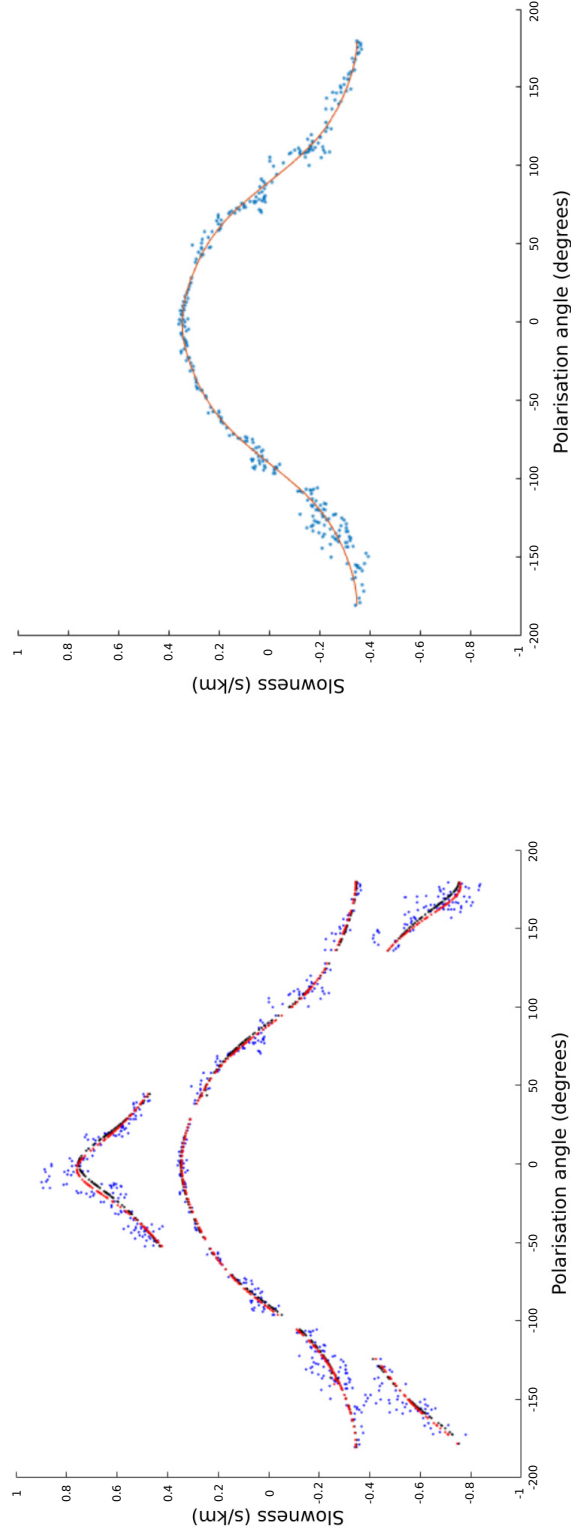
The TTI dip angles estimated in this chapter are not the rock's true dip angle, but the projection of the true dip angle onto the walkaway sail line. This means the dip angle may be larger in reality than what is calculated, and the rock's true Thomsen parameters may also be different to what is estimated here.

The walkaway sail line runs perpendicular to the well's trajectory deviation, as shown in Figure 8.1. This results in the nearest offset shot recorded at the top receiver having a smaller offset than that recorded at the bottom receiver. Assuming a simple overburden, the shot with the minimum incidence angle recorded at the top receiver would therefore also have an incidence angle smaller than the shot with the minimum incidence angle recorded at the bottom receiver. This effect could lead to increased uncertainty with depth in estimating TTI parameters as the available near offset information lessens. However, I suspect this effect is less significant in this Brazilian Pre-salt case study than the drastic effect that the asymmetric salt overburden has on incidence angle, as seen in Chapter 7, Figure 7.7, for example.

8.7 Conclusion

Estimating the TTI parameters in a Pre-salt reservoir using both P- and S-wave measurements from multicomponent VSP data can be challenging due to uncertainty in multiple, coincident S-wave arrivals, and difficulty in distinguishing

between fast and slow shear waves, so I estimated a Brazilian Pre-salt reservoir's TTI parameters using a slowness-polarisation method that requires only P-wave measurements. The Pre-salt reservoir is shown to have a dip of 10 to 15 degrees and moderate to strong anisotropy, with Thomsen's parameters ϵ and δ both being less than -0.3 in some locations. Through sensitivity analysis, it is plausible that these large magnitudes could be due to inaccuracies in S-wave velocity measurements from upscaled well logs, or related to survey acquisition geometry. The negative ϵ values in the reservoir interval indicate the P-wave velocity perpendicular to the symmetry axis is slower than that in the parallel direction, which seems consistent with the results in Chapter 7 that suggest vertical fracturing may be present.



(a) Inversion algorithm of Horne and Leane (2000). (b) Inversion algorithm of Grechka and Mateeva (2007).

Figure 8.7: Modelled slowness-polarisation trends through the shale reservoir data of Horne and Leane (2000) using the VTI algorithm of Horne and Leane (2000) (Subfigure a, black), a modification of the algorithm of Horne and Leane (2000) for TTI inversion (Subfigure a, red), and the “P-wave only” TTI algorithm of Grechka and Mateeva (2007) (Subfigure b). All models predicted similar anisotropic parameters. Note shear polarisations are displayed $\pi/2$ radians out of phase for convenience.

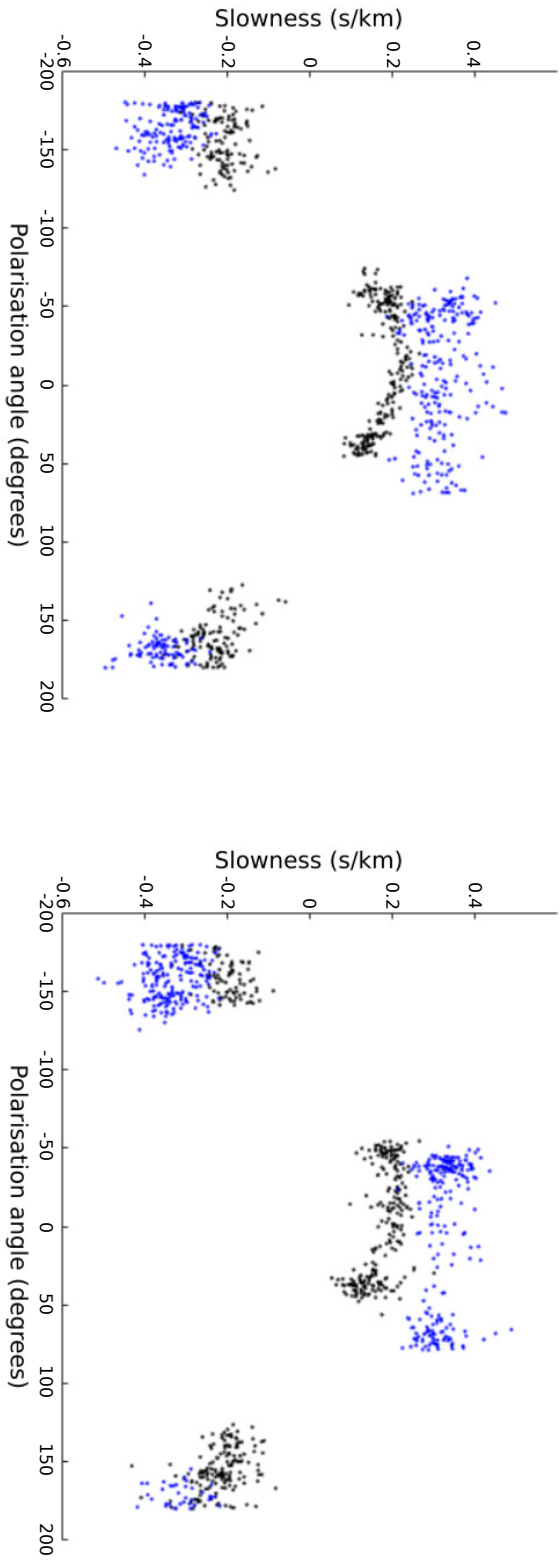


Figure 8.8: Calculated slownesses and polarisations for receivers 1 (salt) and 28 (reservoir) in Well A. P-wave measurements are in black while S-wave measurements are in blue. The measurements are more scattered than the data of Horne and Leaney (2000), shown in Figure 8.7. The scatter in S-wave measurements resulted in an unstable inversion using the method of Horne and Leaney (2000), leading to the use of the “P-wave only” inversion algorithm (Grechka and Mateeva, 2007) for estimating TTI parameters in the Pre-salt. Note shear polarisations are displayed $\pi/2$ radians out of phase for convenience.

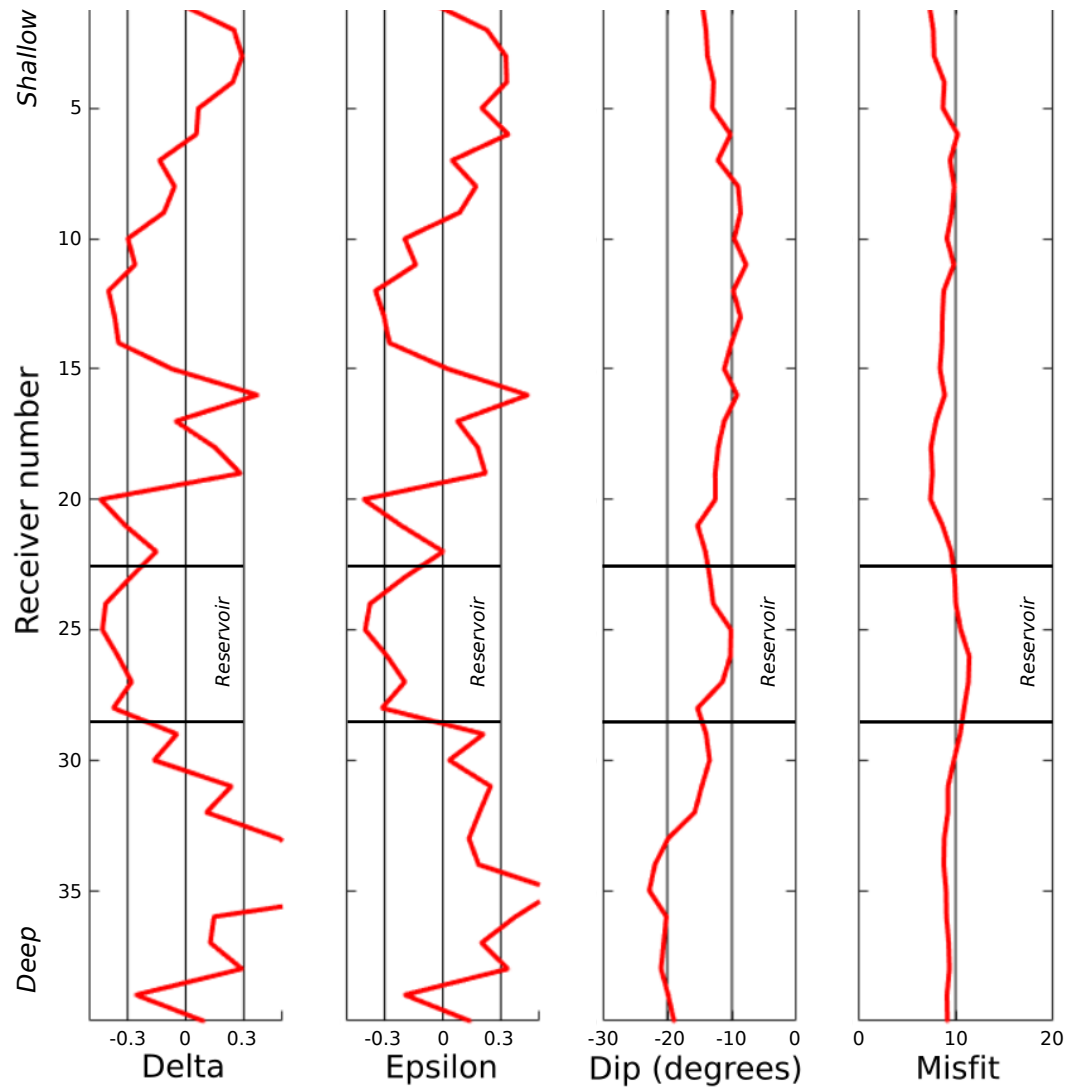


Figure 8.9: The single best solution parameters over all receivers. The negative epsilon values in the reservoir interval (receivers 23-28) suggest the horizontal P-wave velocity is slower than the vertical P-wave velocity, which may be attributed to (sub)vertical fracturing.

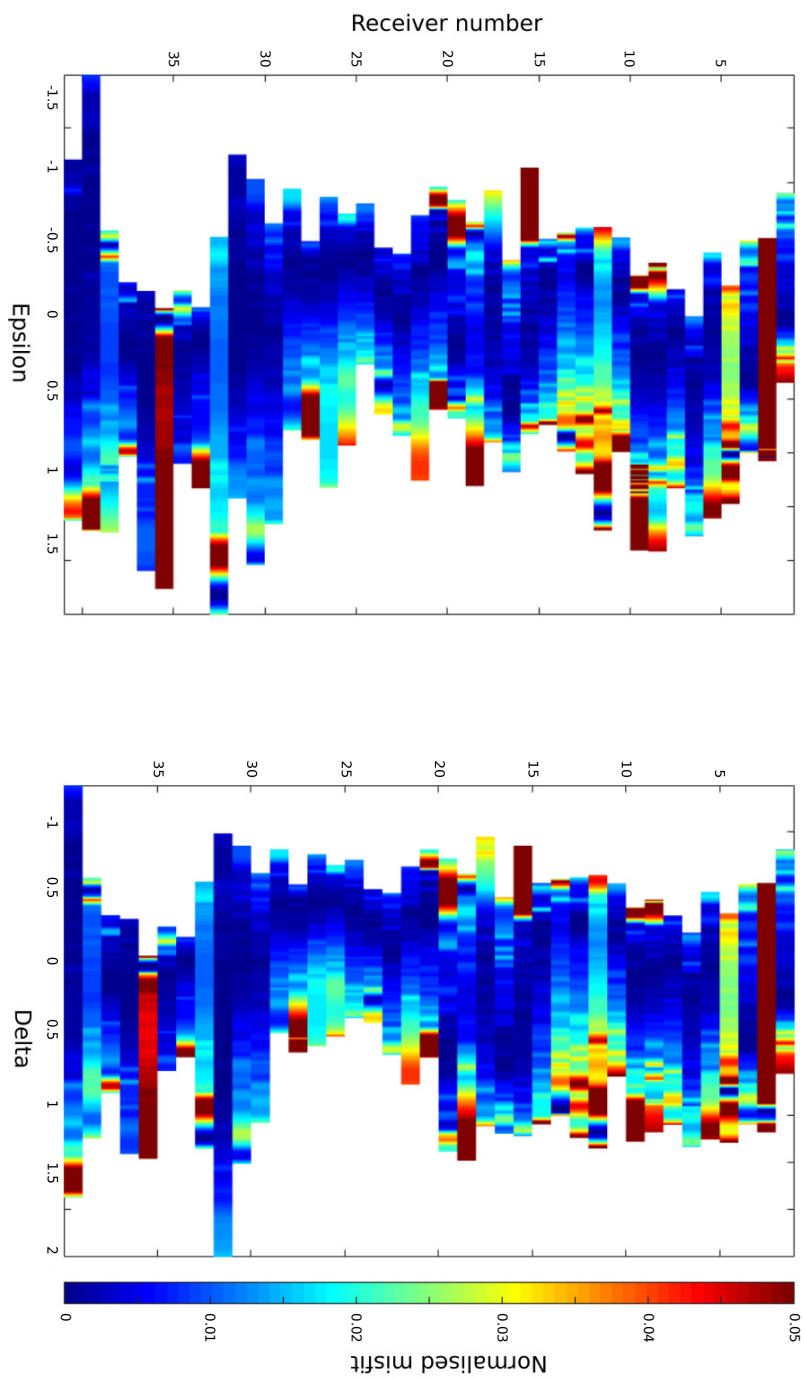


Figure 8.10: Solutions from all 200 simulations for each receiver's inversion coloured by normalised misfit.

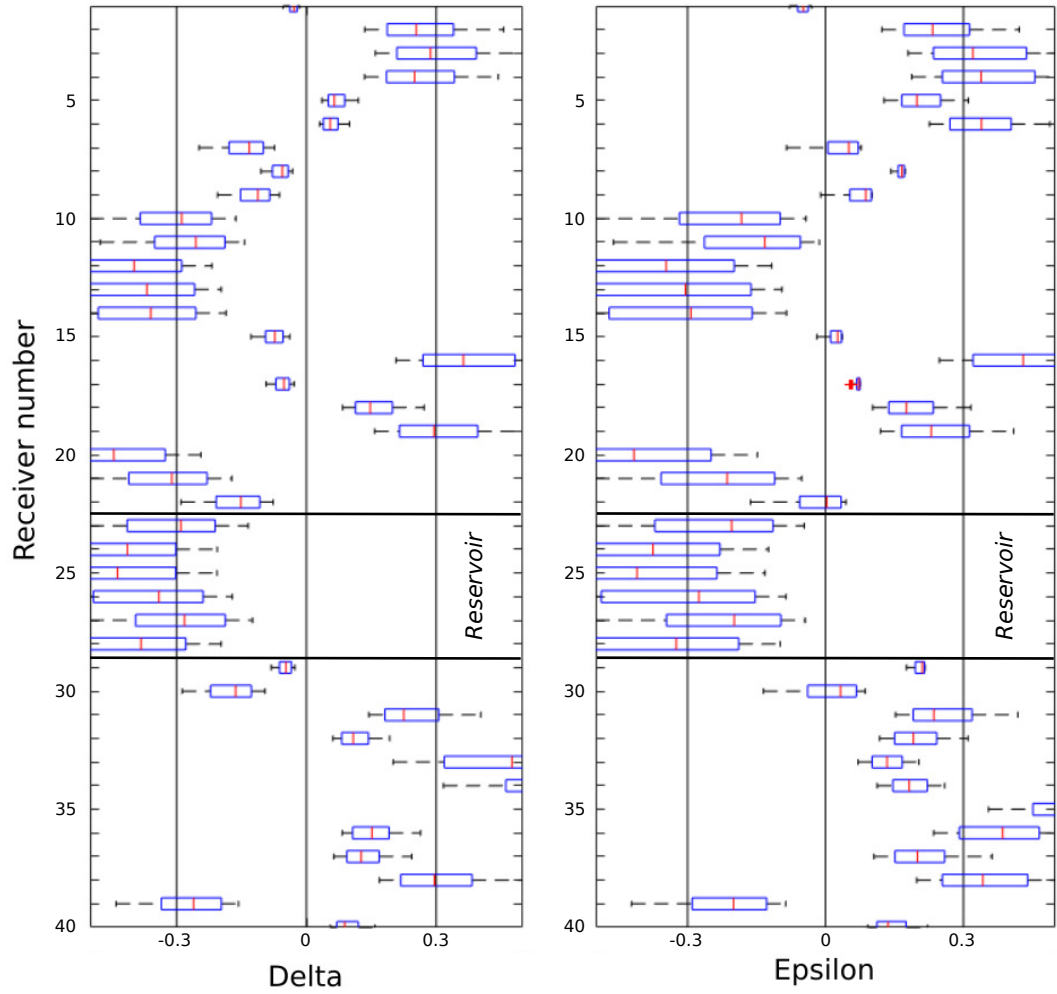


Figure 8.11: Sensitivity of final estimated anisotropic parameters to $\pm 20\%$ error in the upscaled V_{s0} well log. This V_{s0} term is used in equation 8.8 to convert from the inversion output parameters δ_{VSP} and η_{VSP} to the Thomsen parameters δ and ϵ .

Chapter 9

Model Integration

The slowness-polarisation inversion for the Brazilian Pre-salt reservoir's TI elastic symmetry using walkaway VSP data shows the presence of TTI symmetry with a symmetry axis dipping between 10 and 15 degrees from the vertical in the reservoir interval. This study also indicates Thomsen parameters ϵ and δ are negative in the reservoir interval; a characteristic commonly attributed to (sub)vertical fracturing, where the horizontal P-wave velocity is slower than the vertical.

The inversion of walkaround VSP data for fracture characterisation may indicate the presence of two, non-orthogonal, vertical fracture sets with depth-dependent orientation and fracture density in the reservoir interval. The presence of two non-orthogonal, vertical fracture sets in a background rock which has a TTI elastic symmetry could suggest the rock has an overall triclinic elastic symmetry, as shown schematically in Figure 9.1. However, I have not directly solved for the triclinic stiffness tensor parameters in this thesis, meaning we can only speculate as to whether this is the case.

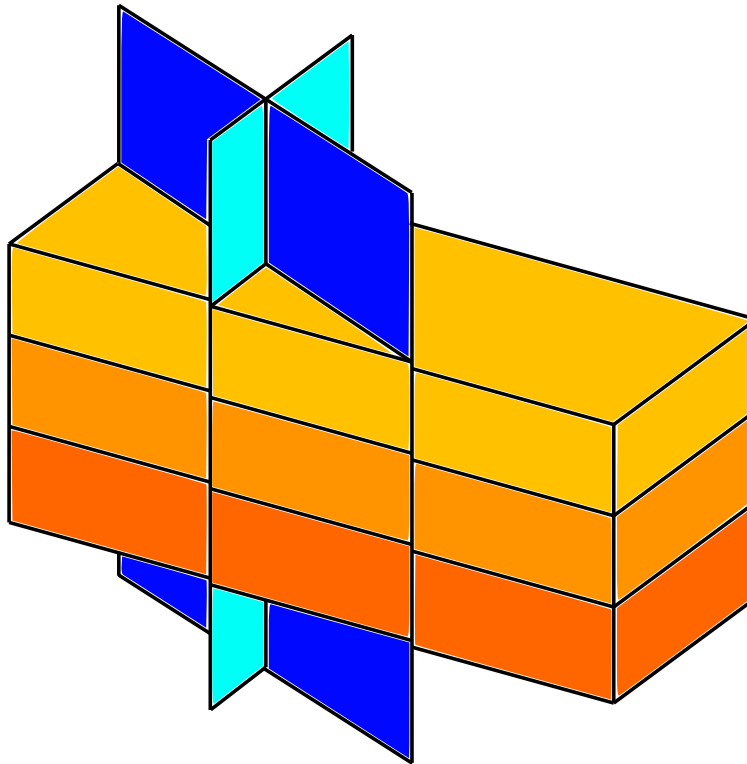


Figure 9.1: A possible interpretation of the seismic anisotropy observed in the reservoir interval. Two non-orthogonal, approximately vertical fracture sets are situated in a dipping reservoir rock.

A probable, single fracture set is resolved using walkaround VSP analysis at the bottom of the reservoir interval. The presence of a single, vertical fracture set overlain on a background rock with TTI symmetry could indicate the presence of an overall tilted orthorhombic or monoclinic elastic symmetry. Again, we can only speculate whether this is the case as I have not solved for the tilted orthorhombic or monoclinic stiffness tensor in this thesis.

The resolution of the anisotropic reservoir model obtained from VSP data is the receiver spacing of 15 m MD. Integrating well log rock physics modelling with the anisotropic reservoir model from VSP analysis can produce a reservoir model on the finer well log sampling scale which is informed on seismic anisotropy and

reservoir fracturing. I presented a new well log attribute Ξ in Chapter 6 which addressed the possibility of distinguishing between sets of carbonate facies using only well log data. By generating a Ξ pseudo-log using well log data, we may begin to investigate relationships between anisotropic parameters on the surface seismic scale and carbonate facies on the well log scale.

In Figure 9.2, I show pseudo-logs of the model parameters I have proposed throughout this thesis across the reservoir interval. The first track of Figure 9.2 shows the facies interpretation provided by this project’s sponsors. The locations of core interpretation are shown as black dots, while the distance-thresholded nearest-neighbour facies interpolation is shown in solid colour. Track 2 shows a synthetic pseudo-log of facies-dependent attribute Ξ , generated using the interpolated facies pseudo-log in track 1 and the parameterisation of Ξ from Chapter 6. The purpose of this track is to demonstrate what an ideal, predictive Ξ pseudo-log may look like in a blind test. Tracks 3 and 4 show the fracture dominance and orientation for two vertical fracture sets as a function of depth, while tracks 5, 6, and 7 show the calculated ϵ , δ , and formation dip. Tracks 8, 9, and 10 show the upscaled, measured well logs: density, V_p , and V_s . Together, these logs constitute my parameterisation of the reservoir interval’s fracture- and facies-dependent, anisotropic elastic properties.

A major limitation of the facies-dependent rock physics model presented in Chapter 6, and hence the integrated, anisotropic rock physics model presented here, is the location of geological core interpretations in the well. Core interpretation can be spatially infrequent, as seen in Figure 9.2. I have used a nearest neighbour interpolation to assign facies interpolations to all points on the well log sampling grid, then used a threshold of 0.5 m such that all points further than this distance from a core sample were not considered in the integrated rock physics modelling flow. Performing some form of interpolation and thresholding, as I have done, is necessary, given the lack of information on how core sampling locations were

chosen and how geological interpretation was performed. However, it is a clear source of error in this rock physics modelling workflow. This could be improved upon with knowledge of how core sample locations were selected and how the samples relate to the geology presented in the full well core.

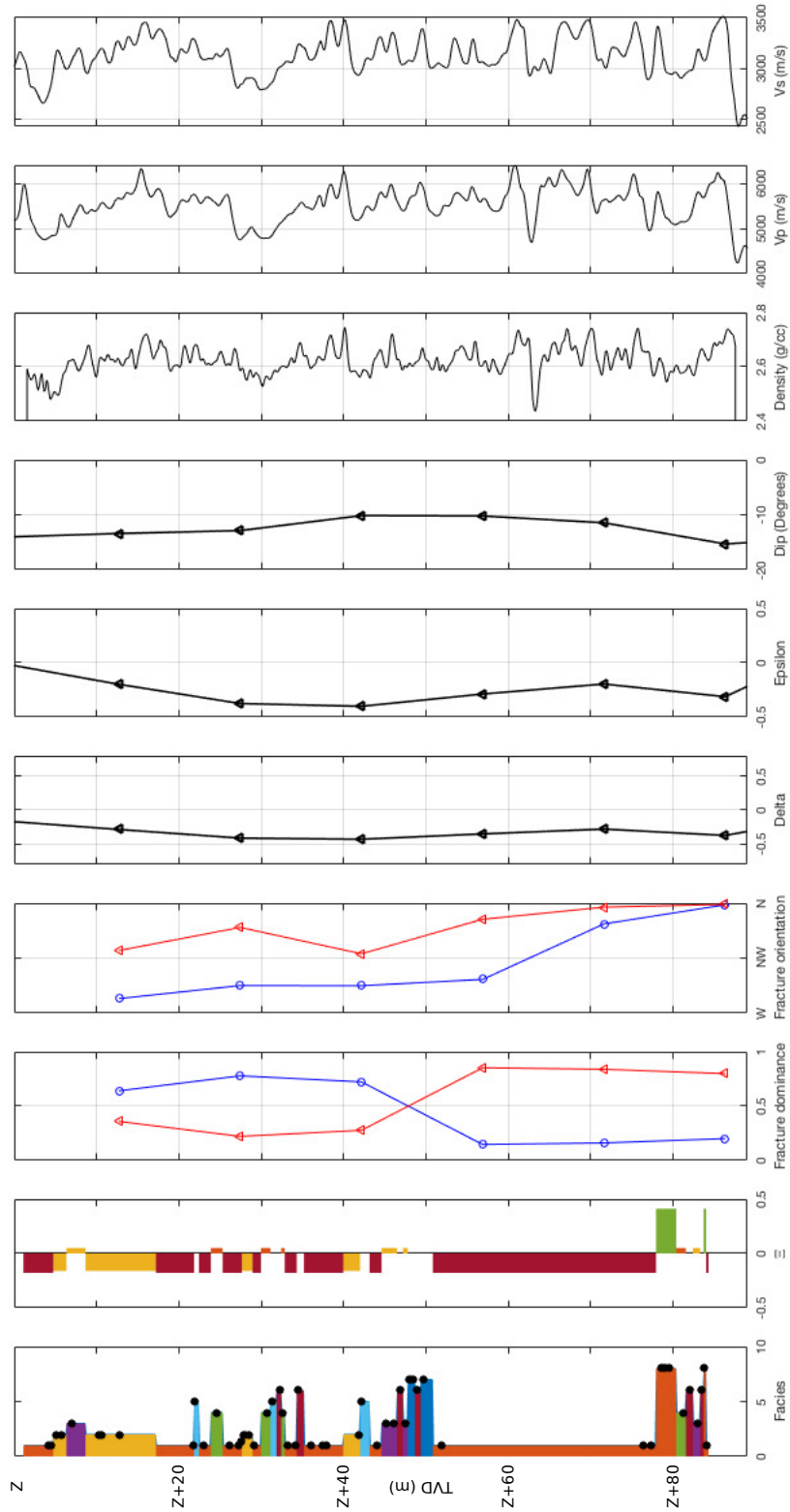


Figure 9.2: A collection of pseudo-logs summarising the proposed, integrated rock physics model over the Pre-salt 2 (reservoir) interval. From left to right: Track 1) Interpreted facies (black points) and nearest neighbour interpolation (filled colours). Colours arbitrarily denote different facies; 2) Synthetic Ξ pseudo-log made using facies interpretation in track 1 and parameters from Chapter 6. Colours arbitrarily denote different facies; 3) Fracture dominance; 4) Fracture orientation; 5-6) Thomsen's δ and ϵ ; 7) Formation dip; 8-10) Density, V_p , and V_s .

Chapter 10

Discussion

The reformulation of the electrical DEM model of Mendelson and Cohen (1982) presented in this thesis differs from the original formulation in that the reformulated model's mineral conductivity must be non-zero for evaluation. This allows the reformulated model to be correct in both the high and low porosity limit, unlike the formulation of Archie's first law by Mendelson and Cohen (1982), which is only correct in the high porosity limit as it assumed grain conductivity is zero.

The effective pore aspect ratio (EPAR) of the electrical-elastic DEM expressions presented seems constant in porosity when applied to wet, clay-bearing sandstone laboratory data. It is possible the observed power law relationship between EPAR or effective grain aspect ratio (EGAR) and porosity in electrical and elastic models is present only in carbonate data, where pore geometry may be more complex. It is also possible, however, that in rendering porosity a dummy variable through the derivation of the electrical-elastic DEM expressions, the EPAR's dependency on porosity in the electrical-elastic model is also removed. This could be investigated further by testing the electrical-elastic model on carbonates where the power

law relationship is known to exist when its electrical and elastic properties are modelled individually. Unfortunately, publicly available carbonate electrical-elastic laboratory data sets are scarce throughout the literature.

Mapping electrical resistivity to seismic velocities or elastic moduli on a reservoir scale typically requires the estimation of a porosity field away from well control, which can be error prone. The proposed electrical-elastic DEM expressions have the potential to avoid this issue as they have no porosity terms in their formulation. Rather than estimating a reservoir-scale porosity field, reservoir scale EPAR parameter fields would be produced when modelling with this DEM method. In Chapter 4, we have seen EPAR seems constant in porosity in clay-bearing wet sandstones from mixed localities, and Chapter 5 has shown EPAR varies by a power law with porosity in carbonates. Chapter 6 has shown us these parameters vary with facies. If we assume the relationship between porosity and EPAR is known for all facies, EPAR parameters could be solved for as a function of facies using well logs and core interpretation. This may allow the construction of EPAR parameter fields which could be guided by seismic interpretation and facilitate electrical-elastic modelling without requiring a reservoir-scale porosity field.

A major benefit of using the power law DEM model over empirical models is that it has the modelling accuracy of empirical relations, while being derived from first principles and correct in the high and low porosity limits. The power law DEM model, however, involves solving differential equations numerically and has multiple unknown parameters, and hence can be computationally expensive, or perhaps relatively tedious to implement. This contrasts with the simplicity of Archie's first law, the Humble equation, the Pickett relations and the Castagna relations. The use of empirical relations may therefore still be desirable over the power law model where speed and simplicity are preferable and the empirical relations are well-suited to the measured data.

I have shown parameter Ξ , a combination of ξ and Γ , can vary with carbonate facies, and have speculated this facies dependence arises from differences in how frame stiffness alters with porosity due to diagenetic effects, and how stiff different facies of carbonates are at high porosities. Parameter Ξ might be considered an attribute which indicates how a particular facies' frame stiffness changes with porosity. Fournier *et al.* (2014) found vug-forming dissolution correlated with an increasing EPAR with porosity, while Fournier *et al.* (2018) examined the correlation Brazilian Pre-salt carbonate petrographic features and EPAR. In a similar manner, recognising petrographical features which correlate with parameters Γ and ξ , or Ξ , could tie the change in EPAR with porosity to physical processes.

Facies-dependent inversion of well log data for power law parameters Γ and ξ was performed in this thesis assuming a constant mineralogy of 95% calcite and 5% dolomite throughout the reservoir. This assumption is unrealistic, however it was necessary, given the lack of available mineralogical information. This unrealistic assumption means the mineral modulus used in modelling is subject to error. The best mixing model to estimate the overall mineral modulus is also unknown and hence is another source of modelling error. Fluid modulus estimation was also subject to errors on its composition, however this effect is less significant in modelling the upscaled porous rock's upscaled moduli as both water and oil bulk moduli are typically an order of magnitude less than the mineral moduli. I have interpreted the facies-dependent variation in Γ and ξ , or alternatively Ξ , as indicators of how pore space architecture changes with porosity. However, these systematic differences between facies may be accounting for systematic differences in facies' mineralogies and fluid properties. This modelling ambiguity could be lowered by the use of mineralogy and water saturation logs.

The electrical and electrical-elastic effective medium models presented in Chapters 5 and 4 were not applied during the Brazilian Pre-salt reservoir characterisation

case study. However, one may be able to invert for facies-dependent effective pore or grain aspect ratio parameters using these electrical and cross-property models and resistivity logs. If this is possible, there may be scope to calculate parameter Ξ from electrical measurements. Thus, performing some kind of in situ facies characterisation using electrical or both electrical and elastic well logs seems plausible in principle by adapting the workflow of Chapter 6 to employ the electrical or electrical-elastic DEM models proposed in Chapters 4 and 5.

I have suggested the negative ϵ and azimuthal attenuation patterns observed in the reservoir interval may indicate fracturing. I have not addressed whether these fractures would be open or closed, although the squirt flow model of Jin *et al.* (2018) is derived assuming open fractures. There is also the possibility that the reservoir presents horizontal elastic symmetry axes without any fracturing, for example, caused by in situ stresses. Analysing image logs could provide further information on the state of reservoir fracturing.

The inversion for reservoir fracture orientation and density indicators using the spectral ratio method is founded on a number of assumptions which are not accurate in reality. I calculate the interval fracture characteristics of the reservoir between two contiguous receivers. Therefore, the effective seismic sources for each calculation are the seismic signals which would be measured along the incident ray path at the elevation of the receiver which the energy passes first. In the presented Pre-salt reservoir setting, where the overburden is severely asymmetric, the spectral ratio's assumption of identical sources is almost certainly violated. Regardless, the ability to window first-arriving P-waves in the Pre-salt VSP data made the spectral ratio method preferable over shear wave splitting analysis, where isolating fast and slow shear waves was prohibitively challenging.

I have shown the model of Jin *et al.* (2018) is sensitive to water saturation, which was unknown in my walkaround VSP analysis for fracture characterisation,

having no water saturation logs. I assumed water saturation in the entire reservoir interval was 15% (Boyd *et al.*, 2015). This constant water saturation throughout the reservoir is a significant source of error in this fracture characterisation study which could be improved upon with access to water saturation well logs. As a whole, the use of the model of Jin *et al.* (2018) in this thesis leads to a high uncertainty in fracture characterisation results. However, I suspect all fractured poroelastic models would have similarly high uncertainty in this setting, and the model of Jin *et al.* (2018) is probably more appropriate than other existing models as it accounts for multiple fluids with patchy saturation, which we know to exist in the reservoir.

The TTI elastic symmetry inversion of the Brazilian Pre-salt reservoir was performed using data from a single walkaway VSP line. Therefore, the symmetry axis' estimated dip angle is actually the true dip angle's projection onto the walkaway line's azimuth. It is thus possible the formation's true dip angle is larger than the estimated, apparent dip angle. Seismic images may be able to inform the true azimuth of the dipping symmetry axis if one makes the assumption that the geological strike direction seen on 3D seismic images is the same as the azimuth of the formation's TTI symmetry axis.

The walkaway and walkaround VSP data analyses presented in this thesis are based on inversion techniques underpinned by different, and sometimes contradictory assumptions. For example, the model of Jin *et al.* (2018) used in fracture characterisation is a viscoelastic model, while the slowness-polarisation inversion algorithm of Grechka and Mateeva (2007) used to characterise TTI symmetry is based on the (elastic) Christoffel equation. This difference in assumptions must be recognised when simultaneously interpreting these walkaway and walkaround VSP analyses to form a single, explanatory story.

The challenges of modelling the physical properties of Pre-salt carbonate reservoirs has led to the development of the power law DEM model and the interval relative azimuthal attenuation analysis methods presented in this thesis. Rather than being modelling techniques specifically designed for the Pre-salt, these new methods extend existing methods. Namely, the power law DEM model relaxes the typical assumption that EPAR remains constant in porosity. Similarly, the proposed interval relative azimuthal attenuation analysis method removes the assumption of constant seismic incident angle with azimuth and enables analysis given arbitrary well trajectories. As such, these methods may extend to geological settings other than the Brazilian Pre-salt.

Chapter 11

Conclusions

The aim of this thesis was to extend rock physics theory and methods for fractured carbonate reservoirs, with a particular focus on rocks with complex pore geometries, and those overlain by complex overburden.

I have presented a multiphysics model which estimates bulk and shear modulus from electrical resistivity, which I hope may lead to a new style of “porosity-free” reservoir modelling and joint electrical-elastic inversion. By estimating both bulk and shear moduli from resistivity, this multiphysics modelling method may provide a new way to estimate V_p/V_s -ratios from resistivity with no need for porosity information.

I have argued that electrical and elastic modelling efficiency improves when a power law relationship between porosity and inclusion aspect ratio is introduced. By accounting for this systematic change in aspect ratio in porosity, I have presented models which approximate electrical and elastic empirical models over much of the porosity range while being derived from first principles and exactly

correct in the asymptotic limits. These models provide alternatives for empirical models in situations where empirical models are known to fail, as is the case with Archie's first law in many carbonates. Furthermore, these power law models can, in principle, be used in different geological conditions, unlike many empirical relationships.

Fracture characterisation by VSP inversion is a challenge in Pre-salt reservoirs due to the azimuth-dependent incidence angle and attenuation caused by the asymmetric overburden, however these issues can be overcome to some extent by accounting for azimuth-dependent seismic incidence angle, well trajectory, and estimating an interval rather than full-ray attenuation. By presenting a stochastic inversion for a multiply-fractured Pre-salt reservoir using a fractured, multifluid model, I have demonstrated the possibility of two distinct fracture sets within the reservoir interval. This result, however, is based upon many assumptions in both the spectral gradient method and the model derivation and parameterisation.

Estimating the elastic polar anisotropic parameters of a Pre-salt reservoir can be error-prone due to a large scatter in shear wave velocity and polarisation measurements. However, using the P-wave only slowness-polarisation algorithm of Grechka and Mateeva (2007), I estimated Thomsen's parameters δ and ϵ to be negative in the reservoir interval. If these negative ϵ values are associated with the presence of fractures, there could be agreement between the walkaway and walkaround VSP analyses, indicating vertical fractures are present in the reservoir and are possibly superimposed on a tilted transverse isotropic background symmetry.

Appendix A

Corrected Akaike Information Criterion

The Akaike Information Criterion (AIC) (Akaike, 1973) is a model selection criterion based in Information theory which estimates the most likely amount of information lost when approximating measured data generated by a true, unknown model, with a candidate, fitted model. The AIC does this by estimating the fitted model's expected Kullback-Leibler divergence (Kullback and Leibler, 1951) from the true, unknown model which generates the measured data. Hurvich and Tsai (1989) formulate the AIC as:

$$AIC = n (\log \hat{s}^2 + 1) + 2 (m + 1) ; \quad (A.1)$$

where n is the number of samples, m is the number of model parameters, and $\hat{s}$ is the maximum likelihood estimate of the measured data's variance.

The AIC is biased in the case of small n , where it tends to favour models with larger m (Hurvich and Tsai, 1989). As our data sets are relatively small, we compare models using the Corrected Akaike Information Criterion (AIC_C), which is more accurate in small n . Hurvich and Tsai (1989) derive the AIC_C as:

$$AIC_C = AIC + \frac{2(m+1)(m+2)}{n-m-2}. \quad (\text{A.2})$$

We see the second, additive term on the right-hand side of equation A.2 goes to 0 when $n \gg m$, approximating the AIC , and is non-negligible when m and n are comparable. The difference, ΔAIC_C , in the AIC_C values of a reference and candidate model indicates the evidence for using one model over the other. It is the logarithm of the relative likelihood of the two models, conditional on the model parameters and residuals from the data (Burnham and Anderson, 2002). We thus refer to the ΔAIC_C as the *log-relative likelihood* throughout this thesis.

We compare the two-parameter, power law (superscript “PL”) model with the best single-parameter (superscript “DEM”) model using the ΔAIC_C , which we define as:

$$\Delta AIC_C = AIC_C^{\text{DEM}} - AIC_C^{\text{PL}}. \quad (\text{A.3})$$

The value of ΔAIC_C here indicates the evidence that the proposed power law model is more likely to be more efficient than the single-aspect ratio (“Single- α ”) DEM model. Burnham and Anderson (2002, 2004) provide useful rules of thumb for the interpreting the log-relative likelihood of competing models, analogous to the popular advice of Raftery (1996) or Jeffreys (1998) in the Bayesian model selection literature. Applied specifically to our formulation of ΔAIC_C , these guidelines suggest if:

1. $0 < \Delta AIC_C < 2$: Single- α DEM has substantial evidence as best model.
2. $4 < \Delta AIC_C < 7$: Single- α DEM has considerably less evidence.
3. $\Delta AIC_C > 10$: Single- α DEM has essentially no evidence.

We reframe these guidelines to focus on the power law model, proposing and discussing results in terms of the complimentary case:

4. $\Delta AIC_C > 10$: Power law model has compelling evidence as best model.

When $\Delta AIC_C < 0$, Single- α DEM is accepted as the best model and the magnitude of the log-relative likelihood is used to measure the evidence that the power law model is the best model under the guidelines of Burnham and Anderson (2002, 2004).

Appendix B

Extended Theoretical

Cross-Property Modelling by Chain Rule DEM

B.1 Single Property Modelling

The electrical DEM model of Mendelson and Cohen (1982) can be reformulated to estimate the overall electrical conductivity σ^* of rock with randomly oriented, electrically conductive spheroidal pores embedded in a homogeneous, electrically conductive matrix, as is presented in equation 4.14. Berryman (1992) presented a model which estimated this same rock's effective elastic moduli assuming linearly elastic phases (equations 4.22 and 4.23).

Gao and Zhou (2006) derived the overall thermal conductivity κ^* of a dilute dispersion of ellipsoidal inclusions in a background of matrix material:

$$\frac{d\kappa^*}{d\phi} = \frac{\kappa^*}{(1-\phi)} \left[\sum_{i=1}^3 \frac{\kappa^i - \kappa^*}{\kappa^* + L_i (\kappa^i - \kappa^*)} \right]. \quad (\text{B.1})$$

where κ^i is the ellipsoidal inclusion's thermal conductivity in its i^{th} axis, and L_i is the i^{th} depolarisation factor of the inclusion.

Considering a porous rock's thermal conductivity, we assume the thermal conductivity within pores is isotropic. We also assume inclusions are spheroids, leading to the variable substitution $L_1 = L$ and $L_2 = L_3 = (1-L)/2$. Denoting the inclusion phase with subscript 2 for consistency with electrical and elastic modelling notating, we see:

$$\frac{d\kappa^*}{d\phi} = 3\kappa^* \frac{(\kappa_2 - \kappa^*) \mathcal{J}^{(*2)}}{(1-\phi)}; \quad (\text{B.2})$$

where:

$$\mathcal{J}^{(*2)} = \frac{4}{\kappa^* + \kappa_2 + L(\kappa^* - \kappa_2)} + \frac{1}{\kappa^* - L(\kappa^* - \kappa_2)}. \quad (\text{B.3})$$

Notice the similarity of parameters $R^{(*2)}$ (equation 4.12) and $\mathcal{J}^{(*2)}$.

B.2 Cross-Property Modelling

B.2.1 Thermal-electrical modelling

To estimate the change in one modelled property with respect to another, we apply the chain rule to the differential equations describing various physical models'. The change in the electrical properties of a composite with respect to its thermal conductivity is thus:

$$\frac{d\sigma^*}{d\kappa^*} = \frac{\sigma^* (\sigma_2 - \sigma^*)}{\kappa^* (\kappa_2 - \kappa^*)} \frac{R^{(*2)}}{\mathcal{J}^{(*2)}}; \quad (\text{B.4})$$

with boundary condition $\sigma^* (\kappa^* = \kappa_1) = \sigma_1$. The model for thermal conductivity as a function of electrical conductivity is the reciprocal of equation B.4 using the boundary conditions $\kappa^* (\sigma^* = \sigma_1) = \kappa_1$.

B.2.2 Thermal-elastic modelling

Similarly, the change in the elastic moduli of a composite material with respect to its thermal conductivity is modelled by the coupled equations:

$$\frac{dK^*}{d\kappa^*} = \frac{1}{3\kappa^*} \left(\frac{K_2 - K^*}{\kappa_2 - \kappa^*} \right) \frac{P^{(*2)}}{\mathcal{J}^{(*2)}}; \quad (\text{B.5})$$

$$\frac{d\mu^*}{d\kappa^*} = \frac{1}{3\kappa^*} \left(\frac{\mu_2 - \mu^*}{\kappa_2 - \kappa^*} \right) \frac{Q^{(*2)}}{\mathcal{J}^{(*2)}}. \quad (\text{B.6})$$

with boundary conditions $K^*(\kappa^* = \kappa_1) = K_1$ and $\mu^*(\kappa^* = \kappa_1) = \mu_1$. The model for thermal conductivity as a function of bulk or shear modulus is less simple to obtain as equations B.5 and B.6 are coupled.

B.2.3 Electrical-elastic modelling

Repeated from Chapter 4 in this appendix for completion, the change in the elastic moduli of a composite material with respect to its thermal conductivity is modelled by the coupled equations:

$$\frac{dK^*}{d\sigma^*} = \frac{1}{3\sigma^*} \left(\frac{K_2 - K^*}{\sigma_2 - \sigma^*} \right) \frac{P^{(*2)}}{R^{(*2)}}; \quad (\text{B.7})$$

$$\frac{d\mu^*}{d\sigma^*} = \frac{1}{3\sigma^*} \left(\frac{\mu_2 - \mu^*}{\sigma_2 - \sigma^*} \right) \frac{Q^{(*2)}}{R^{(*2)}}. \quad (\text{B.8})$$

with boundary conditions $K^*(\sigma^* = \sigma_1) = K_1$ and $\mu^*(\sigma^* = \sigma_1) = \mu_1$. As in modelling thermal conductivity, the model for electrical conductivity as a function of bulk or shear modulus is less simple to obtain as equations B.7 and B.8 are coupled. Note the similarity between equations B.5 and B.6, and equations B.7 and B.8.

Appendix C

Power-Law DEM Data Review

We studied only subsets of the Verwer (Verwer *et al.*, 2008), Fournier (Fournier *et al.*, 2011), and Bakhorji (Bakhorji, 2010) elastic data sets to minimize the influence of confounding factors on our modelling results. We selected only dry measurements for elastic modelling made on approximately monomineralic samples. Table C.1, adapted from Kittridge (2014), shows data set details.

We studied the 51-sample subset from the Verwer data set which contained porosity, dry V_p , dry V_s , dry bulk density measurements, and had 100% dolomite composition to the nearest integer by XRD analysis. We modelled this data set assuming 100% dolomite mineralogy using the elastic parameters shown in Table 5.2. We used the dry core measurements of the Bakhorji data set at 20 MPa confining pressure from the loading stage of the loading-then-unloading experimental regime, as was done by Kittridge (2014). We studied the 24-sample subset from this Saudi-D reservoir data which contained at least 90% calcite by volume. The median composition of these samples was 99% calcite, so we modelled this data set using a 100% calcite mineralogy with the elastic parameters

Table C.1: Summary of elastic core data sets analysed (Adapted from Kittridge (2014)).

Source	Location	Type	#Spl	Lab velocity	Description	Additional descriptor(s)
Verver <i>et al.</i> (2008)	Cap Blanc of the Lluçmajor Platform, Mallorca	Outcrop and surface borehole (3)	250	Wet (deaired brine 3% NaCl) and dry; P _{eff} 10 MPa; P _{pore} 0.1 MPa; 1 MHz	Miocene; low- and high- Mg calcite, dolomite, aragonite. Mostly dolomitic	XRD, petrography, texture (granular, crystalline)
Bakhorji (2010)	Arab-D reservoir (Saudi Arabia) with seven wells	Wells	37	Wet and dry; P _{conf} 5-25 MPa (increasing, decreasing stress); ~1 MHz	Limestone or Dolomite (<1% noncarbonate)	Samples characterized as macro, micro, dual porosity. Petrography, SEM, mercury porosimetry
Fourrier <i>et al.</i> (2011)	Four outcrop locations South East France	Outcrop	80	Dry; P _{conf} 2.5, 5, 10, 20, and 40 MPa; P _{pore} 0.1 MPa; 1 MHz	Lower Cretaceous platform; microporous limestone	All grainstone texture, absence of intergranular, intercrystalline, or moldic porosity

shown in Table 5.2. We studied the dry, elastic measurements of the Fournier data set made at 20 MPa confining pressure on all 80 calcitic cores and modelled this data set with a 100% calcite mineralogy.

The resistivity data of Focke and Munn (1987) are laboratory measurements made on reservoir cores from offshore Qatar. The experimental pore fluid was simulated formation water and most plugs were made of clean carbonates. We digitized the data manually.

Appendix D

Analysis of Archie-Like Models

The points of inflexion and turning points in our forward modelled formation factor trends are not observed in typical Archie’s law, the Humble equation, or Single- α electrical DEM models but are key features in the empirical models of Focke and Munn (1987). We infer these special points arise due to the competing effects of inclusion geometry and pore volume fraction on a porous rock’s overall resistivity. Intuitively, we expect a porous rock’s resistivity to decrease with increased porosity due to a reduction in the volume fraction of insulating matrix material. The resistivity of a rock comprised of ellipsoidal grains, however, increases with increased grain eccentricity, as shown by Mendelson and Cohen (1982). These two effects compete in the FM data, where inclusions become more eccentric with increasing porosity, leading to the non-monotonic formation factor curves observed in Figure 5.8. By taking first and second derivatives of an Archie-like formation factor function with the form:

$$F(\phi) = \phi^{-m(\phi)}, \tag{D.1}$$

we derive the differential equation to solve for formation factor turning points:

$$m' + Am = 0; \quad (\text{D.2})$$

and inflexion points:

$$Bm'' - B(m')^2 \log \phi + Cm' - Cmm' \log \phi - Dm^2 - Dm = 0; \quad (\text{D.3})$$

with:

$$A = \frac{\phi^{-(m+1)}}{\log \phi}; \quad (\text{D.4})$$

$$B = \phi^{-m} \log \phi; \quad (\text{D.5})$$

$$C = 2\phi^{-(m+1)}; \quad (\text{D.6})$$

$$D = \phi^{-(m+2)}. \quad (\text{D.7})$$

The empirical model of Focke and Munn (1987) sees cementation factor m_{FM} take the form:

$$m_{FM}(\phi) = b\phi + c. \quad (\text{D.8})$$

Using equations D.2 to D.7, we solved for the existence and location of turning and inflexion points on all four FM data sets modelled in this thesis, using both the original parameterisations of equation D.8 published by Focke and Munn (1987),

and our power law DEM model's parameterisations as shown in Table 5.3. The solved special points are presented in Table D.1. Notably, this analysis correctly predicts the existence of special points which are non-existent on a log-log scale but indeed present on a linear scale.

Table D.1: Turning points (T. P.) and inflection points (I. P.) for Focke and Munn's (FM) empirical model (equation D.8) and our proposed power law (PL) model (equation 5.14), parameterized with $\{a, b\}$ or $\{\Gamma, \xi\}$ respectively on the four studied FM data sets.

Data set	Model	$\{b, c\} \vee \{\Gamma, \xi\}$	T. P. 1	T. P. 2	I. P. 1	I. P. 2
Grainstone	FM	$\{2, 0\}$	$\nexists$	$\nexists$	$\nexists$	$\nexists$
Grainstone	PL	$\{0.173, -0.122\}$	$\nexists$	$\nexists$	$\nexists$	$\nexists$
Moldic ($k < 0.1$ mD)	FM	$\{1.2, 12.86\}$	0.044	0.255	0.114	0.432
Moldic ($k < 0.1$ mD)	PL	$\{0.018, -0.895\}$	0.050	0.227	0.105	0.397
Moldic ($0.1 < k < 1$ mD)	FM	$\{1.4, 8.57\}$	$\nexists$	$\nexists$	0.173	0.353
Moldic ($0.1 < k < 1$ mD)	PL	$\{0.026, -0.808\}$	$\nexists$	$\nexists$	0.114	0.314
Moldic ($1 < k < 100$ mD)	FM	$\{1.2, 8.29\}$	$\nexists$	$\nexists$	0.146	0.396
Moldic ($1 < k < 100$ mD)	PL	$\{0.036, -0.650\}$	$\nexists$	$\nexists$	$\nexists$	$\nexists$

Appendix E

Extended Biot Theory and Power-Law DEM

In this appendix, we investigate the analytical relationship between the parameterisations of our power law DEM model and the Extended Biot Theory of Sun (1994, 2000, 2004). Huang and Sun (2017) summarise the Sun model, showing:

$$K_d = K_s (1 - \phi)^{\gamma_K} ; \tag{E.1}$$

$$\mu_d = \mu_s (1 - \phi)^{\gamma_\mu} ; \tag{E.2}$$

where subscript d denotes the rock's dry moduli and subscript s denotes the solid

rock moduli. Assuming the dry rock moduli can be modelled with an inclusion-based effective medium theory such as DEM, using the porosity-dependent EPAR expressed in equation 5.11, we find:

$$K_d(\alpha_K) = K_d(\phi; \Gamma_K, \xi_K) = K_s (1 - \phi)^{\gamma_K} ; \quad (\text{E.3})$$

$$\mu_d(\alpha_\mu) = \mu_d(\phi; \Gamma_\mu, \xi_\mu) = \mu_s (1 - \phi)^{\gamma_\mu} ; \quad (\text{E.4})$$

It follows that γ_K and γ_μ can indeed be expressed in terms of parameters $\{\Gamma_K, \xi_\mu\}$ and $\{\Gamma_\mu, \xi_\mu\}$ respectively by:

$$\gamma_K(\phi; \Gamma_K, \xi_K) = \frac{\log \left[\frac{K_d(\phi; \Gamma_K, \xi_K)}{K_s} \right]}{\log(1 - \phi)} ; \quad (\text{E.5})$$

$$\gamma_\mu(\phi; \Gamma_\mu, \xi_\mu) = \frac{\log \left[\frac{\mu_d(\phi; \Gamma_\mu, \xi_\mu)}{\mu_s} \right]}{\log(1 - \phi)} . \quad (\text{E.6})$$

To demonstrate, we calculate γ_K over $\phi \in (0, 1)$ using equation E.5 for all three data sets' bulk modulus parameterisations of the power law DEM model, as listed in Table 5.2. We display the results in Figure E.1a, marking the measured porosity of core samples for each data set along the modelled trend. Similarly, we calculate γ_μ over $\phi \in (0, 1)$ using equation E.6 for all three data sets' shear modulus parameterisations of the power law model, also listed in Table 5.2. These results are displayed in Figure E.1b.

Sun (2004) uses γ_K to discriminate and classify rocks into three possible pore structure types, depending on whether γ_K is “low”, “medium”, or “high”, where

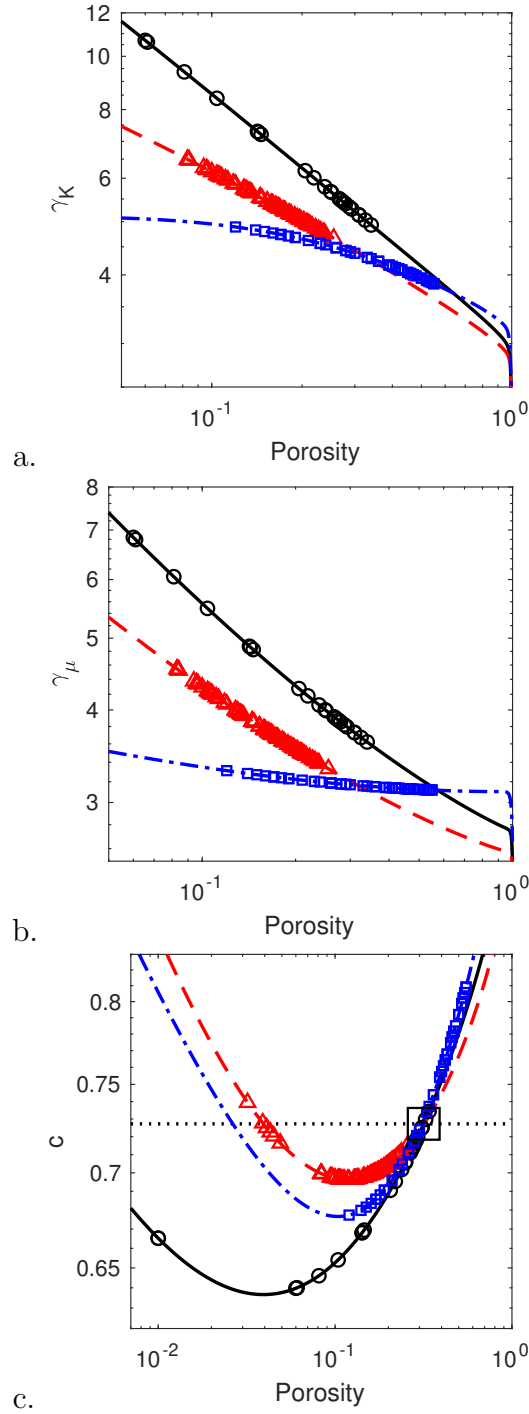


Figure E.1: Modelled γ_K , γ_μ , and c in Subfigures a, b, and c, respectively for the Bakhorji data (black solid curves and circular markers), Fournier data (blue dot-dashed curved and square markers), and Verwer data (red dashed curve and triangular markers). A dotted black line in c) shows the recommended value $c = 8/11$ for use in carbonates when c is otherwise unknown, as suggested by Sun (2004). Interestingly, all three modelled curves intersect this suggested value at $\phi \approx 0.32$ (large black square), which was not predictable by theory.

these three classes are calibrated project-specifically, given core interpretations. These three classes of pore structure types contain: 1) Moldic rocks with or without connected vugs and clean sandstones; 2) Mixtures of carbonate pore types, such as moldic, interparticle, and micropores, as well as shaly sandstones and sandy shales, and; 3) Most microporosity carbonates and siliciclastic rocks of a given porosity, including sandstones and shales. Because the cut-off γ_K values which delimit these three classes must be calibrated to geological interpretation, we do not classify our data into pore structure types here. Rather, we note our predicted γ_K and γ_μ trends are certainly in the expected range for porous carbonate rocks (E.g., see Karimpouli *et al.* (2013)) and so we gain confidence in the applicability of equations E.5 and E.6.

Sun (2004) also proposed the frame flexibility factor ratio c to be a physically meaningful parameter, which is defined as:

$$c = \frac{\gamma_\mu}{\gamma_K}. \quad (\text{E.7})$$

Parameter c quantifies the propensity of a rock's frame to undergo shear motion relative to its compressibility. This value is free to alter depending on pore space architecture, however it is suggested by Sun (2004) to be set to an average value of 8/11 if otherwise unknown. At this proposed value, the Poisson's ratio of the rock's skeletal frame under jacketed test is assumed to be equal or proportional to that of the rock's solid grain (Sun *et al.*, 2002). Figure E.1c shows c , the ratio of trends shown in Figures E.1a and E.1b. Interestingly, we see all three models converge at the suggested average $c = 8/11$, represented by a dotted, horizontal line, at $\phi \approx 0.32$, which was not predictable from our theory. This suggests the value $c = 8/11$ may indeed be an appropriate average over a number of carbonate rock types. Furthermore, the convergence of all three trends at $\phi \approx 0.32$ and $c = 8/11$ seems to imply the rocks from all three data sets have their skeletal

frame Poisson's ratio equal or proportional to their solid grain Poisson's ratio when $\phi \approx 0.32$. We also notice all c -trends intersect $c = 8/11$ at lower porosity values as well.

We can see from equations E.5 and E.6 the mapping from our proposed power law model parameters to the Sun model parameters exists and is real on the valid porosity domain of our power law model for all dry, porous rocks which are reasonably modelled by DEM and Extended Biot Theory. The mapping also seems bijective in the data examples shown, however we have not attempted to prove this. We note the mapping between parameterisations is porosity-dependent, even though each set of parameters are independent of porosity within their own modelling scheme. The inverse mappings of equations E.5 and E.6 are significantly more difficult to derive explicitly, if possible, due to the complexity of expressing EPAR in terms of effective elastic moduli.

Appendix F

ϕ - α Power-Law Emergence from Fractal Pore Geometry

To demonstrate how a power law relationship between porosity and EPAR can be derived theoretically in a DEM modelling framework, consider a range of length scales $l_1 \leq l \leq l_2$ over which a rock's pore geometry exhibits fractal behaviour, as has been observed in carbonates (e.g., Krohn (1988)). Furthermore, assume the smallest pore length scale in the rock is l_1 , often on the order of $20\text{\AA}$ in practice (Katz and Thompson, 1985; Krohn, 1988). We approximate a rock's porosity up to length scale l by a collection of ellipsoids with semimajor radius length $r(s)$ and number density $N(s)$ distributed continuously over length scales $s \in [l_1, l]$. Denoting EPAR with α and absorbing into $N(s)$ the normalizing factor required to convert from absolute pore volume to pore volume fraction, we express porosity as:

$$\phi(l) = \int_{l_1}^l \frac{4\pi}{3} N(s) r(s)^3 \alpha ds = \frac{4\pi}{3} \alpha G(l)^{-1}; \quad (\text{F.1})$$

where

$$G(l) = \left[\int_{l_1}^l N(s) r(s)^3 ds \right]^{-1}. \quad (\text{F.2})$$

From F.1, we see:

$$\alpha = \frac{3\phi(l)G(l)}{4\pi}. \quad (\text{F.3})$$

In the derivation of the elastic DEM theory's governing coupled differential equations (equations 5.1 and 5.2), a limit is taken where the volume of ellipsoids added to the effective background rock at every iteration of the rock-building process becomes infinitely small. Additionally, Norris (1985) suggests the size of the ellipsoids added every iteration should increase by at least an order of magnitude to ensure the correct proportion of matrix and pore space are replaced by any given ellipsoidal inclusion at every iteration. With this in mind, we consider an example scenario where $N(s) = N$ is a non-negative constant and $r(s)^3 = s^n$ for some sufficiently large n . Then:

$$G(l) = \frac{n+1}{N(l^{n+1} - l_1^{n+1})} \quad (\text{F.4})$$

$$\approx \frac{(n+1)l^{-(n+1)}}{N}; \quad (\text{F.5})$$

when $l^{n+1} \gg l_1^{n+1}$.

Katz and Thompson (1985) show a rock's porosity calculated over a range of pore scales upon which it possesses fractal geometry is expressed by:

$$\phi(l) = A \left(\frac{l_1}{l} \right)^{3-D}; \quad (\text{F.6})$$

where A is a constant, often set to unity, and D is the fractal dimension of the pore space such that $2 < D \leq 3$. Equation F.6 rearranges to:

$$l = l_1 \left(\frac{\phi}{A} \right)^{1/(D-3)}. \quad (\text{F.7})$$

Combining equations F.3, F.5, and F.7, we see a power law between α and ϕ emerges for a given parameter n and porosity length scale range $[l_1, l]$:

$$\alpha = J(n) \phi(l)^{\bar{D}(n)}; \quad (\text{F.8})$$

where:

$$J(n) = \frac{3(n+1)}{4\pi N l_1^{n+1} A^{(n+1)/(3-D)}}; \quad (\text{F.9})$$

and:

$$\bar{D}(n) = \frac{4+n-D}{3-D}. \quad (\text{F.10})$$

We note both $J(n)$ and $\bar{D}(n)$ are constant for a given n . Now suppose a rock possesses fractal pore geometry at small pore scales and Euclidean (non-fractal) geometry at large pore scales, as can occur in sandstone and carbonate reservoir rocks (Krohn, 1988). We partition its measured porosity ϕ into fractal porosity ϕ_F and Euclidean porosity ϕ_E such that $\phi = \phi_F + \phi_E$. Then we represent the fractal porosity's EPAR α_F over its associated porosity ϕ_F as is defined in equation F.8, and express the EPAR attributed solely to the porosity ϕ_E possessing Euclidean geometry as:

$$\alpha_E = F(\phi_E); \quad (\text{F.11})$$

for some unknown function $F(\phi_E)$.

Now we say this rock's measured EPAR, α , is some of its fractal and Euclidean EPARs, or:

$$\alpha = \langle \alpha_F, \alpha_E \rangle; \quad (\text{F.12})$$

where angled brackets here represent an unknown type of average. It follows from equations F.8 and F.11 that:

$$\alpha(\phi) = \langle J\phi_F^{\bar{D}}, F(\phi_E) \rangle; \quad (\text{F.13})$$

It is clear from equation F.13 that the observed, porosity-dependent behaviour of α depends on the proportion of pore space possessing fractal or Euclidean geometry; which kind of weighted average is used; all modelling parameters and assumptions; and function $F(\phi_E)$. Notwithstanding, it is plausible by equation

F.13 that $\alpha(\phi)$ could behave at least approximately like a power law if the small-scale porosity dominates the rock's measured porosity.

The analysis presented in this appendix shows how an approximate or exact $\phi - \alpha$ power law may arise from the DEM modelling of rocks with complex pore geometries. I do not claim that the existence of a fractal pore geometry, as presented in this appendix, is the sole cause of the observed power-law relationship. Rather, I've demonstrated that at least one seemingly plausible, physical explanation for the observed relationship exists.

Appendix G

Modelling the Electrical and Elastic Properties of Rocks with Complex Pore Geometries

Preface

This appendix contains an extended abstract for an oral presentation I delivered at the EAGE 2018 Annual Meeting in Copenhagen. This research and publication was performed by myself with contribution from Mark Chapman within the ordinary PhD supervisory capacity. The listed authors are Phil Cilli and Mark Chapman from the University of Edinburgh. All other required acknowledgements are addressed in the abstract.

Introduction

Perhaps the greatest challenge in rock physics modelling is to capture the complex effect of pore-geometry variations on the overall geophysical properties. Inclusion models should be a strong candidate for achieving this goal, as they explicitly model the effect of pore geometry. Unfortunately, this is achieved at the expense of making strong assumptions about the shapes of the pores, and the application of such models to the prediction of velocity-porosity and resistivity-porosity trends has been largely disappointing.

In this paper, we present the results of applying inclusion-based modelling to a range of published laboratory data of elastic and electrical properties. Our central claim is that the deficiencies of inclusion modelling arise not from the assumption of an ellipsoidal pore per se, but rather from the assumption that the aspect ratio does not vary with porosity. In fact, all the datasets which we analyse show evidence of an effective power law relationship between aspect ratio and porosity. This observation leads to a convenient and effective way to model the elastic properties, but also to a generalized version of Archie's Law, which captures the effect of pore geometry variations and successfully explains the published data. Applying our model to the dataset of Han et al. (2011), we demonstrate the effectiveness of our concepts by providing a consistent pore-space model which accounts for both elastic and electrical properties.

Elastic Modelling

The elastic properties of three public carbonate datasets (Verwer et al. 2008; Bakhorji 2010; Fournier et al. 2011) were investigated in this study. When the elastic moduli of a rock's components are known, the overall rock modulus in a DEM scheme depends on one free parameter, the pore aspect ratio. Therefore, we inverted for each sample's average pore aspect ratio by minimising bulk modulus misfit, given the measured mineralogy, pore content, porosity, P- and S-velocities, and density for each laboratory sample. Displaying the results against porosity (Figure 1a), we discovered a log-log-linear relation between inverted aspect ratio and porosity, inspiring the power-law model:

$$\alpha = \Gamma \phi^\xi \quad 1)$$

where parameter α is the pore aspect ratio, Γ indicates the relative stiffness of the rock in the limit of $\phi = 1$, and ξ is the rate at which aspect ratio, and the rock's relative stiffness at a given porosity, changes with porosity. We note a 1% change in porosity leads to a $\xi\%$ change in aspect ratio. Importantly, the proposed power law collapses to the standard, single aspect ratio model when $\xi = 0$.

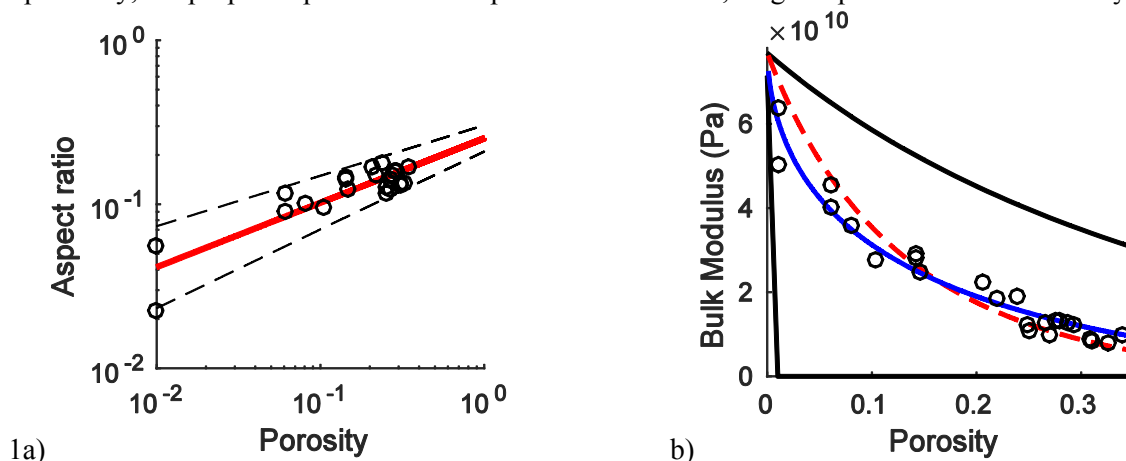


Figure 1. a) Inverted aspect ratio and porosity on log-log scale shows a linear trend, leading to the proposed power-law (Equation 1). Data are dry carbonate laboratory measurements (Bakhorji 2010) with calcite > 0.9 v/v at 20 MPa differential pressure. Dashed lines show the best fit 95% confidence interval; **b)** Forward modelled bulk modulus from the power-law model (blue) with the measured data, best single-aspect DEM trend (dashed red), and calcite-dry Hashin-Shtrikman bounds (black).

We estimated Γ and ξ by fitting a line of best fit through the inverted aspect ratio-porosity samples on a log-log scale (Figure 1a). The estimated Γ and ξ were then used as a starting point for an optimisation in Γ - ξ space by simulated annealing which minimises bulk modulus misfit. Figure 1b shows the forward model using the inversion's solution parameters, compared to the optimal DEM trend using a single aspect ratio. Our method decreases the l_2 -norm misfit by 37.1% on the optimal single aspect ratio DEM model. In addition to this, our model is shown to be more efficient on all tested data by Corrected Akaike Information Criterion analysis.

Inversion for aspect ratio by matching bulk or shear modulus measurements is known to yield different solutions (Fournier et al. 2014), with the relationship between them yet to be understood. Furthermore, we have found different solution couples $\{\Gamma, \xi\}$ for bulk or shear modulus inversions on the same samples. We have had promising success in tying our bulk and shear inversion solution parameters and hence modelled moduli by incorporating Pickett's (1963) empirical calcite and dolomite V_p - V_s relations into the proposed model and constitutive elasticity equations.

Electrical Modelling

The electrical response of a rock varies with pore space architecture (Chinh 2000). This is notable in carbonates, where Archie's (1942) cementation factor can range from 2 to above 5 (Focke and Munn 1987). Therefore, electrical modelling with a varying pore aspect ratio should improve resistivity modelling of rocks with complex porosities compared to single-aspect ratio models. We express our resistivity model in terms of Mendelson and Cohen's (1982) generalisation of Archie's law (Archie 1942) from DEM as a function of aspect ratio, in this case assuming oblate spheroidal inclusions:

$$F = \phi^{m(\phi; \Gamma, \xi)} ; \quad (2)$$

where F is formation factor. Exponent m is the porosity-dependent cementation factor:

$$m(\phi; \Gamma, \xi) = \frac{5-3 \left[\frac{\Lambda^2}{\Lambda^2-1} \left(1 - \frac{1}{(\Lambda^2-1)^{\frac{1}{2}}} \right) \arcsin \left(\frac{(\Lambda^2-1)^{\frac{1}{2}}}{\Lambda} \right) \right]}{3 \left(1 - \left[\frac{\Lambda^2}{\Lambda^2-1} \left(1 - \frac{1}{(\Lambda^2-1)^{\frac{1}{2}}} \right) \arcsin \left(\frac{(\Lambda^2-1)^{\frac{1}{2}}}{\Lambda} \right) \right]^2 \right)} ; \quad (3)$$

where Λ is the reciprocal of aspect ratio, which in our approach is $\Lambda = \Lambda(\phi; \Gamma, \xi) = \Gamma^{-1} \phi^{-\xi}$ by Equation 1. Using this Archie-like model formulation, we also obtain a new cementation factor model (Equation 3) which accounts for pore shape evolution with porosity. Analogous to the elastic model, the resistivity model collapses to Archie's law when $\xi = 0$, as cementation factor becomes constant.

To test our model, the carbonate resistivity data of Focke and Munn (1987) were accurately digitised. We inverted for aspect ratio by basis pursuit using Equations 1, 2 and 3 to find Γ and ξ . Cementation and formation factors were then forward modelled, and compared with Archie's law (Figure 2).

We find Archie's law somewhat adequate in modelling the carbonate grainstone data (Figure 2a,d), however when pore shape becomes highly heterogeneous and permeability is low (Figure 2b,c,e,f), cementation factor varies drastically from a constant value. Our method captures this cementation factor change with porosity by allowing pore shape to change as a power-law with porosity, unlike Archie's law and single-aspect ratio electrical DEM models.

Electrical-Elastic Modelling

The theoretical similarities of electrical and elastic DEM models led to the investigation of whether a single set of parameters $\{\Gamma, \xi\}$ could describe both the effective elastic and electrical properties of a rock. A subset of Han et al.'s (2011) electrical-elastic sandstone laboratory data which contained less

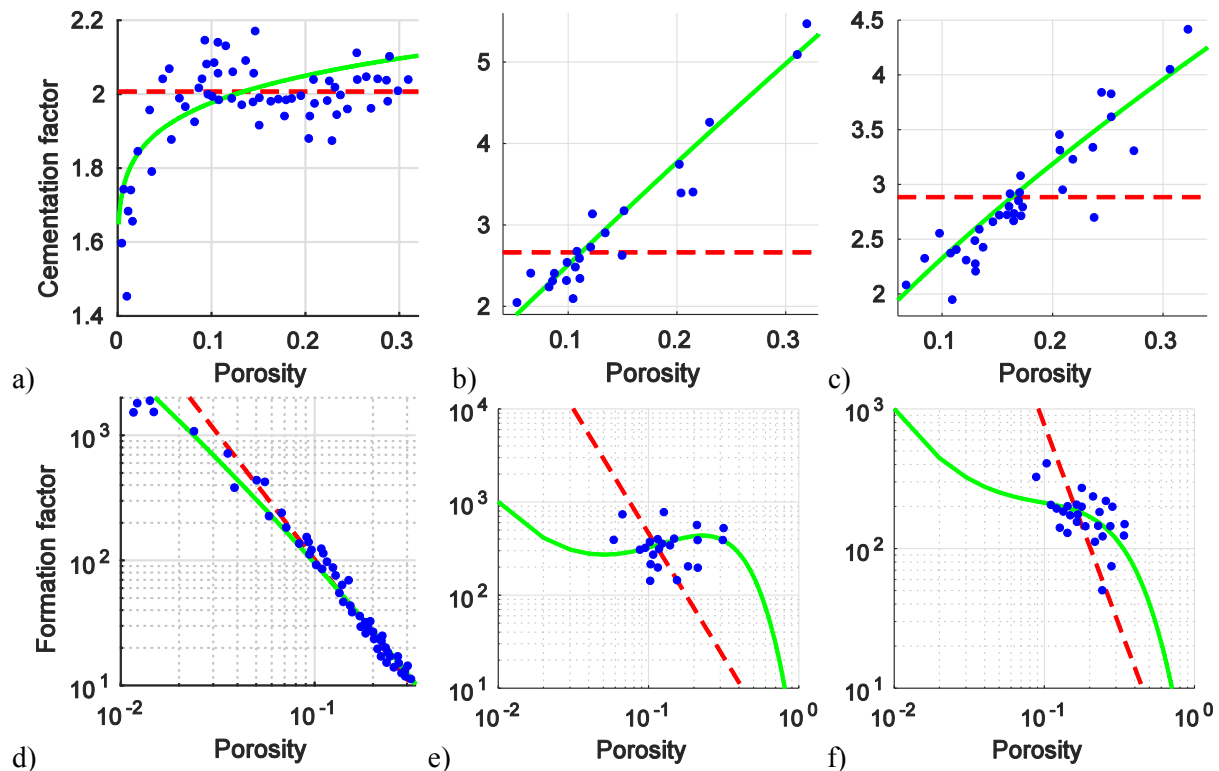


Figure 2 Focke and Munn (1987) carbonate cementation factor (2a-c) and formation factor (2d-f) measurements (blue) modelled by the presented model (green) and Archie's law (red dashed); **2a,d**) Lime and dolomite grainstones, and sucrosic dolomites; **2b,e**) Moldic carbonates ($k < 0.01$ mD); **2c,f**) Moldic carbonates ($0.01 < k < 1$ mD).

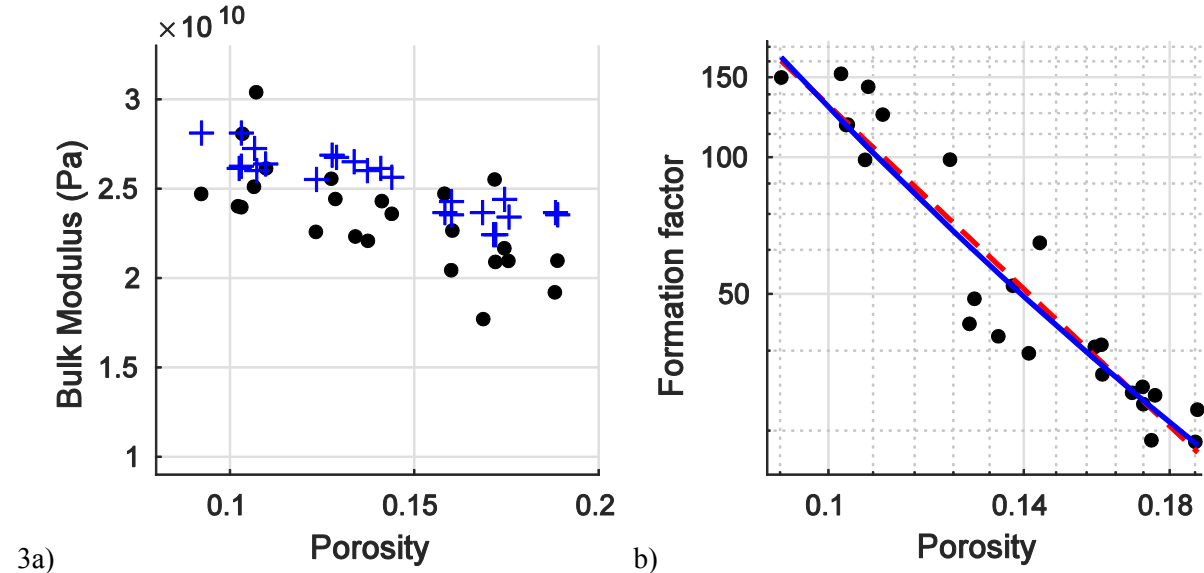


Figure 3 Electric-elastic sandstone measurements by Han et al. (2011). **a)** Elastic data (black points) compared with forward model from electrical inversion parameters (blue crosses); **b)** Electric data (black) compared with forward models by electrical inversion (blue) and Archie's law (red dashed).

than 10% clay and greater than 50% quartz by volume was used to model both effective electrical and elastic properties from both resistivity and bulk modulus measurements. We found the solution parameters by inversion of resistivity data were effective in modelling elastic data (Figure 3a) and also accurate in modelling resistivity data (Figure 3b).

Conclusions

The effective elastic and electrical properties of rocks with complex pore space architecture are challenging to model with standard effective medium theories which assume constant pore aspect ratio at all porosities. Observing that aspect ratio changes by power law with porosity on multiple data sets, we propose a generalisation to inclusion models which accounts for this changes. This simple model is physically intuitive, fits complex rocks with higher accuracy than existing methods in both electrical and elastic data, uncovers a new porosity-dependent cementation factor model, provides a new lead in unifying the electrical and elastic responses of a rock, and has potential to improve upon empirical links between effective bulk and shear modulus.

Acknowledgements

The authors would like to thank Andrew Curtis, Ian Main, and Rachel Wood for their helpful discussions, and Petrobras and Shell for sponsoring this work through the International Centre for Carbonate Reservoirs. We also thank Angus Best at the National Oceanography Centre, Southampton, and Tongcheng Han at the Petroleum University of China, Qingdao, and for providing their experimental data.

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Appendix H

AGU 2018 Annual Fall Meeting Presentation Abstracts

Preface

I presented three posters at the AGU 2018 Annual Fall Meeting in Washington D.C., USA. The published poster abstracts are as follows.

H.1 Modelling V_p/V_s Ratios in Rocks with Complex Pore Geometries

Authors: Phil Cilli, Mark Chapman

Modelling the elastic properties of rocks with complex pore geometries from first principles is challenging, yet it is an integral part of rock physics workflows in exploration and monitoring for stored carbon and hydrocarbon reservoirs. Simple and effective, empirical relations such as the Castagna or Pickett equations are therefore often used to relate P- and S-wave velocities of a given rock type, despite being inaccurate in the high and low porosity limits.

Cilli and Chapman (2018) showed differential effective medium (DEM) theory can accurately fit bulk modulus measurements from rocks with complex pore geometries when the effective pore aspect ratio (EPAR) parameter changes by a power law with porosity. In this work, we invert for EPAR using shear modulus data from the same, publicly available carbonate datasets, and find DEM's shear modulus EPAR also follows a power law with porosity. We observe bulk and shear modulus EPARs generally have different power law exponents for a given rock type but have equal EPAR in the high porosity limit.

With these results, we derive a simple, power law relationship between bulk and shear modulus EPARs as a function of porosity and forward model V_p and V_s trends using this relationship within a DEM scheme. After water-saturating our velocity trends with Gassmann fluid substitution, our model matches both the empirical Pickett and Castagna V_p - V_s relations very well for both wet limestones and dolomites.

Our inclusion-based V_p - V_s model is exactly correct when porosity goes to both zero and unity, unlike the aforementioned empirical relations. Additionally, we accurately model $V_p - V_s$ ratio as a function of porosity and examine its sensitivity to changes in EPAR. We suspect our model may be applicable over a wide range of porous rocks and may become a useful tool in understanding and modelling the elastic properties of rocks with complex pore geometries when empirical models are not suitable.

References:

Cilli, P., and Chapman, M. (2018, June). *Modelling the Elastic and Electrical Properties of Rocks with Complex Pore Geometries*. Paper presented at 80th EAGE Conference and Exhibition, Copenhagen, Denmark.

H.2 Digital Rock Physics in Four Dimensions: Simulating Geological Processes and Estimating the Response of Geophysical Properties

Authors: Jonathan Singh, Phil Cilli, Ian Main

Understanding the relationship between geophysical properties (e.g., seismic velocity and electrical resistivity) and porosity is fundamental to many rock physics models. However, the geological processes that dominate the formation of porosity, such as cementation and dissolution, will often occur over very long time-scales, making the experimental calibration of velocity-porosity trends challenging. Simulating such geological processes in 3D digital rocks and estimating elastic properties from the 3D volumes allows for velocity-porosity trends to be characterised without the long times required for laboratory experiments.

Here we simulate deposition of two carbonate clastic rocks, grainstone (near spherical grains) and coquina (shelly fragments), then simulate both cementation and dissolution. These simulations output a set of 3D volumes representing

rocks of varying porosity with known mineral and grain phases. Using the spatial phase information, combined with known velocity and densities of the relevant phase properties (we assume all mineral grains to be calcite, and porosity is fully saturated with fresh water) we create velocity and density models corresponding to each stage of cementation and dissolution. We then estimate seismic velocity from simulated wavefield propagation through each medium using the 3D staggered-grid finite difference method.

We use these estimated velocity-porosity trends to test the elastic model of Cilli and Chapman (2018), which extends differential effective medium theory with the claim that a rock's effective pore aspect ratio changes by power law with porosity. Inverting our digital elastic measurements for effective pore aspect ratio, we find our modelled rocks do follow this power law relationship. This validates the new rock physics model. Moreover, we see different effective pore aspect ratio-porosity trends for different rock types. This discovery paves the way to use the new rock physics model to link observed changes in effective pore aspect ratio to changes in porosity due to a wider range of geological processes, for example fracturing or compaction.

References:

Cilli, P., and Chapman, M. (2018, June). *Modelling the Elastic and Electrical Properties of Rocks with Complex Pore Geometries*. Paper presented at 80th EAGE Conference and Exhibition, Copenhagen, Denmark.

H.3 Location and Magnitude of Preferential Flow Path Formation in Carbonate Rocks, Forward Prediction from Pre-Experimental CT-Scan Using Machine Learning

Authors: Florent Brondolo, Phil Cilli, Ian Butler, Katriona Edlmann, Christopher McDermott

Understanding the mechanisms behind the development of preferential flow paths in carbonate rocks is important as they are related to several subsurface societal issues, e.g. carbon sequestration, groundwater resource competition, geo-energy including geothermal energy and energy storage applications. The dissolution processes during water/acid flooding cause channelisation which leads to the formation of flow paths called channels.

Over the past 50 years, several studies, performed in a variety of fluid-mineral systems, have managed to bind the effect of rock/water flooding by focusing on the general shape of the channel through an estimation of chemical and physical parameters, but few studies have been able to predict their location. Here we propose the approximation of the spatial distribution and magnitude of a channel by linking experimental and numerical work through image pre-processing and machine learning. The experimental work aims to investigate the mechanical and chemical controls on the optimum equilibrium for channel formation within carbonate travertine rocks using purified seawater as the injection fluid during core flood experiments under typical reservoir conditions. Each sample is CT-scanned before and after the experiment. The numerical simulation includes a pre-processing of the pre-experimental CT-scans followed by a training of the

neural network on the post-experimental channels data, formed as a result of the dissolution and erosion of carbonates during the experimental flooding. The main interest of this study relies on discarding chemical parameters during the forward prediction while demonstrating the strengths of machine learning applied within the sphere of Geosciences.

Early simulations show that, for heterogeneous rocks, we can accurately detect the location, the degree, and the spread of the dissolution pathway. Our findings confirm that the neural network has been able to highlight the key processes related to geometrical properties through image analysis. These processes are seconded by flow rate and effective stress. The neural network approach is enabling us to rank the most important features and facilitate the determination of the most likely location for the formation of a preferential flow path, and its magnitude.

Appendix I

Digital Rock Physics in Four Dimensions: Simulating Cementation and its Effect on Seismic Velocity

Preface

This appendix contains a manuscript which has been submitted to the Journal for Geophysical Investigation for publication. I am the second author on this work. This work was predominantly written and performed by first author, Jonathan

Singh. The listed authors are Jonathan Singh, Phil Cilli, Ian Main, Ola Hosa, who are all affiliated with the University of Edinburgh.

My contribution is the development of the power-law DEM model, which is presented in Chapters 5 and 6. I also wrote the rock physics inversion scripts and performed the inversion on the “Fournier” carbonate grainstone data set, which is shown in Chapter 5. I contributed to writing by reviewing the power-law DEM model, the inversion implementation and method, and the interpretation of the inverted EPARs for the digital coquina tests.

I.1 Abstract

Porosity exerts a strong control on the mechanical and hydraulic properties of rocks, but cannot be imaged directly from the surface. Therefore understanding the relationship between seismic velocity and porosity is a fundamental goal of many rock physics models. However, the geological processes that control porosity, such as cementation, will often occur over very long time scales, making the experimental calibration of velocity-porosity trends challenging. In contrast, simulating such geological processes numerically in 3D digital rocks and digitally estimating elastic properties from the 3D volumes allows for velocity-porosity trends to be characterised in a reasonable time frame.

Here we first simulate initial deposition of two carbonate sediments under gravity - grainstone (near spherical grains) and coquina (shell fragments), and then simulate their cementation. These simulations output a set of 3D volumes (or digital rocks) of varying controlled porosity with otherwise constant and known mineral and grain phases. Combining these models with the known velocity and densities of the relevant constitutive phases, we generate a set of velocity and

density models corresponding to each stage of cementation. These models are then used as input to a 3D acoustic staggered-grid finite difference simulation of wavefield propagation, from which we estimate bulk seismic velocity and calculate the estimated bulk modulus.

The resulting bulk modulus show realistic trends with porosity, all within the physical limits imposed by the Hashin-Shtrikman bounds. We observe anisotropy in the measured velocity (and bulk modulus) consistent with structural anisotropy due to the settling of elongate grains under gravity. We also observe a critical porosity effect in the coquina samples, where there is a sudden drop in bulk modulus at porosity $>30\%$, attributed to the complex ray paths through convex grain shapes. We use the resulting bulk velocity-porosity trends to test competing rock physics models, including one that accounts for varying effective pore aspect ratio with porosity. By inverting our digital elastic measurements for effective pore aspect ratio, we find our modelled rocks follow a power-law relationship between effective pore aspect ratio and porosity. The inversion results from digital rocks are also consistent with those obtained from a suite of laboratory carbonate grainstones. Finally, we investigate the effect of structural anisotropy from the resulting rock fabrics on the resulting elastic properties in three orthogonal directions using a sensitivity analysis. The results show the optimal model to be relatively insensitive to the degree of anisotropy in the fabric. Our approach paves the way to use the new rock physics model to link observed changes in effective pore aspect ratio to changes in porosity due to a wider range of geological processes, for example fracturing, dissolution and compaction.

I.2 Introduction

Geophysical properties such as seismic velocity and electrical resistivity, depend strongly on porosity, and the form of this relationship is fundamental to applications of imaging and modelling natural processes in the solid Earth. Specific examples include, the characterisation and monitoring of geomechanical processes such as compaction (Zimmer, 2004) or elucidating the process of rock fracturing (Pyrak-Nolte *et al.*, 1990). Seismic velocity is often used to monitor hydraulic processes where spatio-temporal changes in the seismic velocity may be caused by fluid flow and/or changes in effective stress during subsurface injection or production (Arts *et al.*, 2004; Brown, 2002; Guilbot and Smith, 2002; Stork *et al.*, 2018). Seismic velocity is also sensitive to diagenetic processes, such as cementation or dissolution (Dvorkin and Nur, 1996; Fabricius, 2003; Weil *et al.*, 2011).

Diagenesis is any physical, chemical, or biological alteration of sediments to form a sedimentary rock (Bathurst, 1972; Tucker and Wright, 1990). It strongly influences reservoir quality, with strong controls on porosity (Bjørlykke *et al.*, 1989; Moore, 1989), permeability (Bloch *et al.*, 2002; Nadeau, 1998), and wettability (Barclay and Worden, 2000). Understanding diagenetic processes, and how geophysical properties relate to them, is therefore important for hydrocarbon production, CO₂ injection, and groundwater resource management. However, measuring geophysical properties associated with diagenetic processes can be very challenging, even in a controlled laboratory environment. Diagenesis often occurs over very long time scales and in extreme conditions making it impractical to reproduce in the laboratory. In some experiments, microbial-induced calcite precipitation has been shown to accelerate cementation (Karol, 2003; Saneiyan *et al.*, 2018), but these methods require elaborate and expensive experimental apparatus. This practical problem is a prime motivation for using digital rocks, where geological or diagenetic processes such as cementation can be simulated in principle over

any time scale. A digital rock is a three-dimensional representation of a rock fabric, where individual phases within a rock (e.g., each mineral component and pore space/fluid) is known for each voxel in three dimensions (see Andr a *et al.* (2013a,b) for an extensive review of digital rock physics). Digital rock models are far more flexible and far less expensive to run than laboratory experiments, and can be used to test hypotheses and establish trends of evolving geophysical properties including specific diagenetic processes. Digital rock physics begins with a starting model from a three-dimensional image, most commonly obtained from high resolution x-ray micro-tomography, where data is segmented into individual phases (e.g., grain, pore, cement). Bulk rock properties can then be estimated, such as permeability (Martys *et al.*, 1999; Keehm, 2003), seismic velocity (Saenger *et al.*, 2000; Saenger, 2008; Arns *et al.*, 2002), thermal conductivity (Wiegmann and Zemitis, 2006) and electrical resistivity (Liu *et al.*, 2009; Zhan *et al.*, 2010). Rather than the x-ray imaging of real rocks, here we simulate rock deposition and cementation to form digital rocks.

There are several examples of process-based simulation of cementation in digital rocks, including isopachous and syntaxial cementation in both sandstones (Mousavi and Bryant, 2012; Latief *et al.*, 2010) and carbonates (Biswal *et al.*, 2007; Mousavi *et al.*, 2012). However, the physical properties investigated during cementation are most commonly transport properties such as permeability and electrical conductivity (e.g., Keehm *et al.* (2001)); to date there have been no applications of process-based cementation modelling for the estimation of elastic properties using digital rocks.

In order to interpret a change in the measured seismic velocity, a suitable rock physics model is required. There is an abundance of methods used in rock physics for the modelling of elastic moduli-porosity or velocity-porosity data. Empirical methods, such as Eberhart-Phillips *et al.* (1989); Gardner *et al.* (1974); Han *et al.* (1986), are entirely based on empirical matches to trends in observed

data. Thus, any inference of physical significance from using such models can be unreliable. Bounding models (Voigt, 1889; Reuss, 1929; Hashin and Shtrikman, 1963) and bounding average models (Hill, 1952) recognise the uncertainty of elastic moduli for a given porosity and therefore give a range of moduli, where the exact value depends on geometric factors of the medium. However, these bounds can be far too broad for many practical applications. Inclusion-based models such as Eshelby (1957); Mori and Tanaka (1973); Kuster and Toksöz (1974); Berryman (1992) assume an unrealistic, ellipsoidal inclusion shape embedded into a background material, but having this physics-based approach generally leads to more intuitive interpretation and model results that more closely match real-rock scenarios. Conventionally, inclusion models assume a constant inclusion shape to characterise an elastic modulus-porosity trend; recently, Cilli and Chapman (in prep.) explored the potential for a variable inclusion aspect ratio related to porosity by a power law, which proved a better fit to existing elastic moduli-porosity data than models which ignore the scaling of inclusion aspect ratios with porosity.

Here, we develop a process-based model for deposition and cementation in digital rocks, combined with the finite-difference simulation of wave propagation, and show that it is an effective method of developing understanding of changes in elastic properties. In addition we compare the Cilli-Chapman analytical rock physics model for a material with ellipsoidal inclusions with a commonly-used competing model. We compare these models using a statistical model selection method known as the corrected Akaike information criteria AIC_C (Akaike, 1973; Hurvich and Tsai, 1989), for their ability to capture the effects of varying cementation, cement type, and initial rock fabric without over-fitting. We also investigate the effect of underlying anisotropy on the relationship between porosity and elastic properties.

In this study, we describe the method used for generating digital rocks, including

the simulated deposition of the primary grains under gravity, followed by cementation. Following this we demonstrate the method of estimating the bulk seismic velocity and elastic modulus using a rotated-staggered-grid finite-difference numerical simulation (Saenger and Bohlen, 2004). We introduce the two existing inclusion models used for elastic moduli-porosity trends, and describe the method for selecting the most appropriate model (AIC_C), which is particularly important for comparing models with different numbers of model parameters. We compare results for grainstone (ellipsoidal grains) and coquina (shelly fragments) digital rocks, with two different types of cementation, and also investigate the effect of the initial structural anisotropy on the evolution of the elastic properties with cementation. Finally, we validate the model by comparing results from synthetic digital grainstone rocks with those of laboratory measured elastic data from Fournier *et al.* (2011).

I.3 Method

The methods used in this paper consist of a three part modelling approach. First is the process-based modelling of initial deposition and subsequent cementation to generate a suite of digital rocks. Anisotropy in the fabric is introduced due to the ellipticity of the clasts, and horizontal settling due to gravity. For a random horizontal orientation this results in bulk transverse isotropy. Secondly we model wavefield propagation through each digital rock to estimate bulk elastic properties. Finally we statistically compare the modelling performance of the Cilli-Chapman model (variable inclusion aspect ratio) and the commonly used differential effective medium theory model (constant inclusion aspect ratio) to the measured elastic data using the corrected Akaike information criterion.

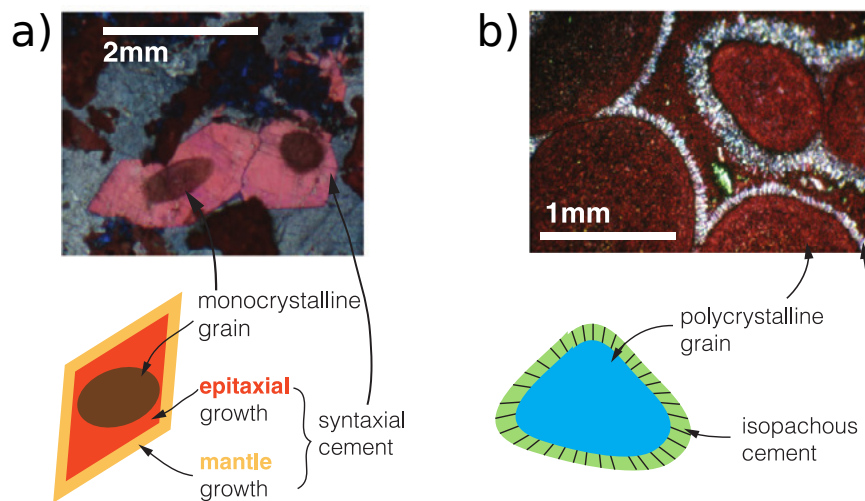


Figure I.1: a) Thin-section image and schematic illustrations of syntaxial cement growth, where cement grows in the rhombohedral crystal form of calcite, associated with monocrystalline (single crystal) grains b) Thin-section image and schematic illustrations of isopachous cement growth, where cement is precipitated evenly around polycrystalline grains. Figure adapted from Hosa and Wood (2017).

Table I.1: Range of Digital Rock Models

Initial Fabric	Cement	Number of Digital Rocks	Porosity Range	Degree of Anisotropy
Grainstone	Isopachous	6	6.6-38%	0.51
Grainstone	Syntaxial	20	18-38%	0.51
Shells	Isopachous	21	0.1-90%	0.55

I.3.1 Deposition and cementation of digital rocks

To generate digital rock models with different cement types, we follow the methodology described in detail by Hosa and Wood (2017). The process first involves the analysis of thin-section images, so that a realistic distribution of grain shapes and sizes are used for the initial deposition. The deposition and initial packing of grains is generated in a process-based simulation of grains falling and settling on one another in a 3D space under gravity. The model allows for heterogenous shapes and sizes in the grains, modelled with normal distributions. In this study we investigate two grain shapes: ellipsoidal grains and shells, representing those of end-member carbonate facies such as carbonate grainstones and coquinas respectively. The initial rock fabrics from the resulting deposition and settling due to gravity are shown as the high porosity slices (panels a and d) in Figures I.2, I.3 and I.4. The initial porosity for coquina ($\phi = 90\%$) is much higher than that of the grainstone ($\phi = 38\%$), which is a result of the difference in the geometries. Wadell (1932) defines the sphericity ψ as: $\psi = S_{sphere}/S_{grain}$, where S_{sphere} is the surface area of a sphere with the same volume of the grain, and S_{grain} is the actual surface area of the grain. Deviations from $\psi = 1$ (decreasing sphericity) results in higher porosity. The average sphericities for coquina and grainstone are $\psi_C = 0.04$ and $\psi_G = 0.14$, respectively.

After the simulation of the process of deposition, the resulting volume containing the grains is cropped and voxelised into a 300^3 voxel domain. The voxel sizes are $10 \mu m^3$ for the grainstones and $0.1 mm^3$ for the coquina, so the digital rock

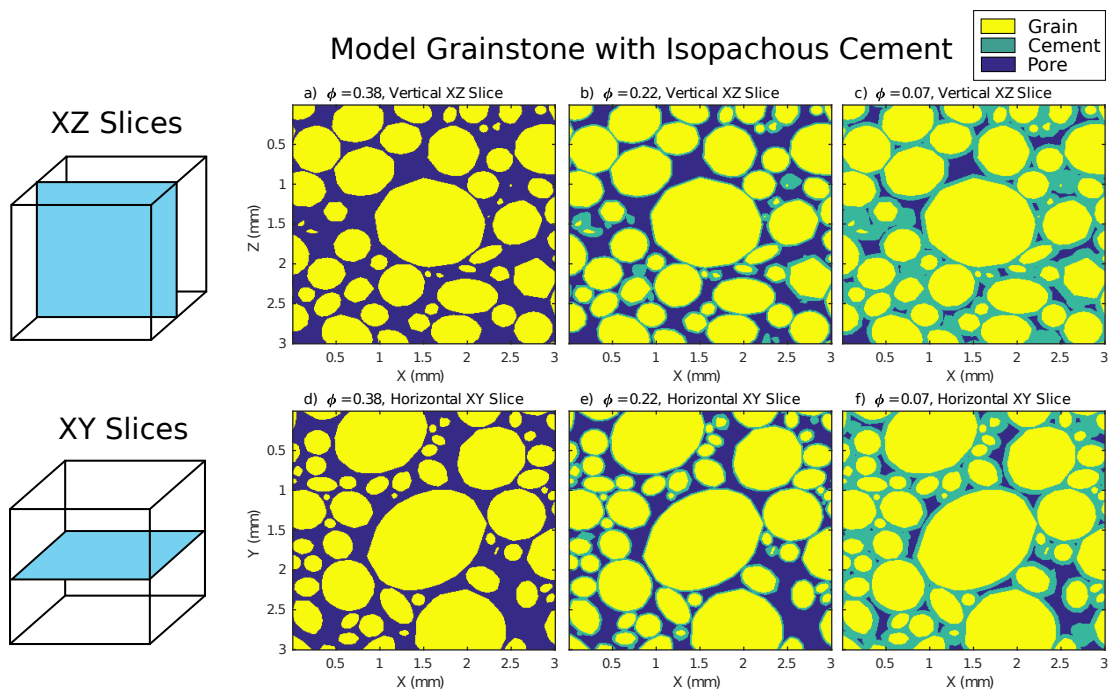


Figure I.2: Example vertical (a-c) and horizontal (d-f) slices through digital rock models at varying degrees of cementation. The model uses a grainstone morphology prior to cementation, which does not change with time (yellow clasts), and a syntaxial cement type that grows with time (green) into the pore space (blue).

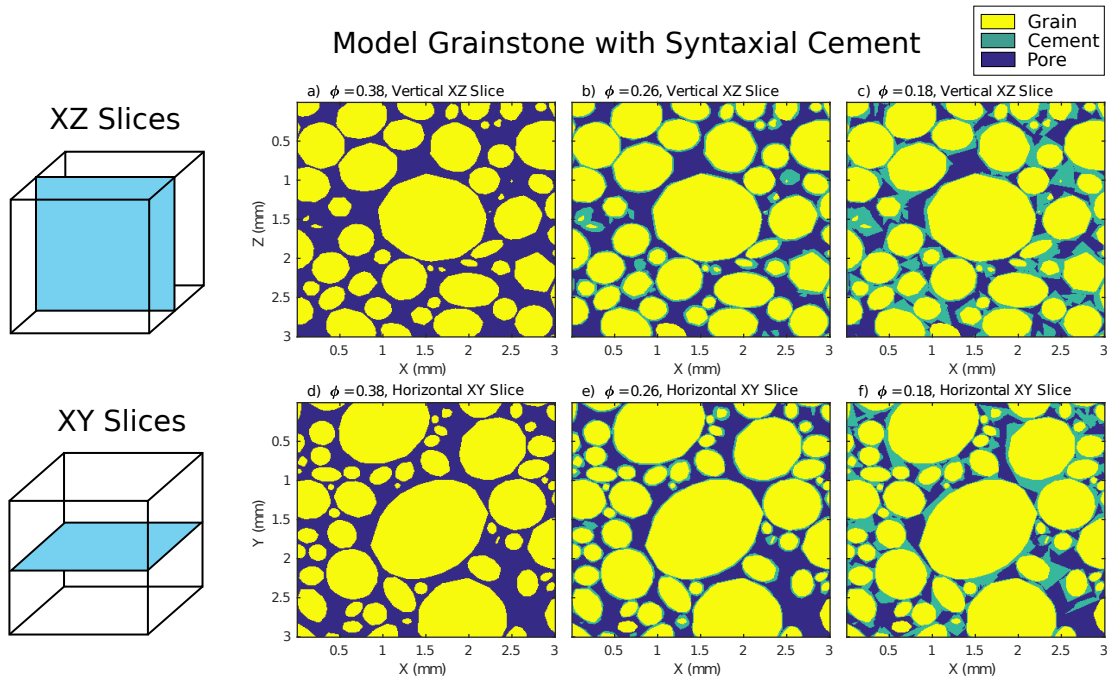


Figure I.3: Example vertical (a-c) and horizontal (d-f) slices through digital rock models at varying degrees of cementation, using a grainstone morphology prior to cementation and a isopachous cement type, which does not change with time (yellow clasts), and a syntaxial cement type that grows with time (green) into the pore space (blue).

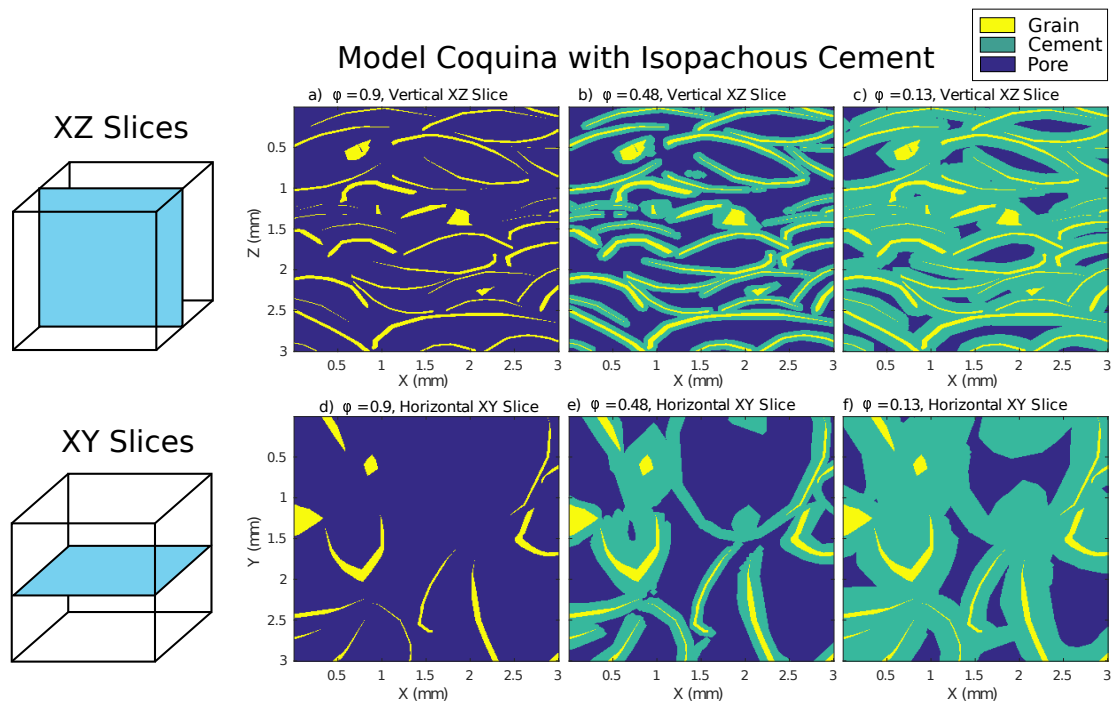


Figure I.4: Example vertical (a-c) and horizontal (d-f) slices through digital rock models at varying degrees of cementation, using a coquina morphology prior to cementation and a isopachous cement type, which does not change with time (yellow clasts), and a syntaxial cement type that grows with time (green) into the pore space (blue).

samples have the dimensions $3 \times 3 \times 3$ mm and $30 \times 30 \times 30$ mm, respectively. We then model calcite cement growth. The model considers two different grain types: polycrystalline and monocrystalline, which develop different types of early marine calcite cement - isopachous and syntaxial, respectively (Figure I.1). The syntaxial cement, which is associated with monocrystalline grains, is modelled in the shape of a parallelepiped (a prism whose all six faces are all parallelograms) and approximates the rhombohedral crystal form of calcite, which is a common calcite crystal form. In nature, syntaxial cement grows rapidly until euhedral crystal faces are reached (epitaxial growth) and approximate crystal form of calcite is achieved. The growth on the euhedral surfaces proceeds at a slower pace (mantle growth). Our method models the first, rapid stage of syntaxial growth. Isopachous cement, which develops on polycrystalline grains (consisting of many small crystals), is modelled as a layer of cement precipitated evenly around the grains. For both cement types, cementation progressed iteratively by adding a 1-voxel thick layer of cement in each iteration.

The modelled process of deposition and cementation for generation of digital rocks is greatly simplified compared to real-world scenarios, and has several underlying assumptions: several processes are ignored such as the agitation of deposited material and re-arrangement of deposited grains due to compaction, therefore initial porosities prior to cementation are unrealistically high. This is not considered to be a problem as the focus of this study is on the *change* in porosity and how seismic velocity responds. The cementation method used here assumes fully saturated media, that cementation is uniform throughout the rock, and cement grows on all grains. In reality, not all pore space is accessible by percolating fluids, and cementation is linked to the fluid dynamics within the pore space. The current implementation of our method includes the continuation of precipitation in pores even after they are cut off from the percolating pore

space. In reality, some sub-resolution porosity would likely allow the percolation of pore fluids but not to the extent we allow in our models.

To examine the effect of cement type, we generate a range of digital rock models using the same starting grainstone fabric assuming either polycrystalline or monocrystalline grains, therefore modelling both isopachous and syntaxial cementation. Representative slices through the grainstone models are shown in Figures I.2 and I.3. We also examine the effect of varying the initial rock deposition, where we use shell fragments representing a coquina, and model isopachous cementation. Two-dimensional slices through these models are shown in Figure I.4. The sections show horizontal alignment of elongate grains, and thus some emergent anisotropy. This may be due to finite size effects, for larger samples we might expect transverse isotropy in the fabric due to sedimentation under gravity. We quantify the degree of anisotropy DOA (Harrigan and Mann, 1984) of the initial fabrics prior to cementation as:

$$DOA = 1 - \frac{\min(\lambda)}{\max(\lambda)}, \quad (\text{I.1})$$

where $\min(\lambda)$ and $\max(\lambda)$ are the minimum and maximum eigenvalues of the data cloud obtained by mean intercept length analysis. Mean intercept length analysis determines the number of matrix voxels that intersect with a set of oriented rays sent through the volume at different angles. The mean number of intersecting voxels as a function of angle forms the data cloud then used in eigenvalue analysis. When $DOA = 0$, the fabric is perfectly isotropic, and the increases to $DOA = 1$, the fabric exhibits stronger anisotropy. Both coquina and grainstone initial fabrics exhibit some emergent structural anisotropy. The details of the ranges of models for each rock type are documented in Table I.1.

I.3.2 Estimating elastic properties

The digital rock models described in Table I.1, where every voxel has an assigned phase (either pore fluid, calcite grain or cement), are converted into corresponding 3D models of local velocity and density (assuming isotropic mineralogy for each phase). At this stage of the modelling we assume the properties of the precipitated cement to be the same as those of the calcite grains. The local velocity and density of calcite is taken from Mavko *et al.* (2009), to be $V_P = 6500$ m/s and $\rho = 2710$ kg/m³. We take the pore fluid velocity and density to be $V_P = 1500$ m/s and $\rho = 1000$ kg/m³. Each voxel is mapped to a regular grid of cells used as input to a three-dimensional finite-difference (FD) simulation of wavefield propagation (Moczo *et al.*, 2007). The FD method discretises the wave equation on a grid and replaces spatial derivatives by FD operators using neighbouring points. High-contrast discontinuities such as those between pores and mineral phases may cause instability problems on a staggered grid. We avoid these difficulties by implementing a rotated staggered grid technique (Saenger and Bohlen, 2004). We assume both point receivers and point sources as well as perfect transducer coupling, and use Ricker wavelets with central frequencies of 40 MHz and 4 MHz for the source time function in the grainstones and coquinas, respectively. The output is a set of synthetic seismograms from user-selected source and receiver locations. We treat these synthetic signals as if they were recorded in the laboratory, and estimate the bulk velocity from the origin time of the source signal, the arrival time of the first maximum (picked manually) and the known source-to-receiver distance. In order to measure any velocity anisotropy in the samples, we use three pairs of sources and receivers placed on opposite faces in orthogonal directions allowing for three measurements of velocity (V_P^X, V_P^Y, V_P^Z). We use the empirical relationship of Pickett (1963), which linearly relates P and S wave velocities in carbonates, to estimate S wave velocity from the P wave velocity (i.e., $V_S = f(V_P)$). The bulk and shear moduli are then calculated using

the following equations:

$$K = \rho \left(V_P^2 - \frac{4}{3} V_S^2 \right) \quad (\text{I.2})$$

$$\mu = \rho V_S^2 \quad (\text{I.3})$$

The density ρ is known for each model by taking the volumetric average of densities for all voxel phases in each model. The resulting estimates for P-wave velocities are shown in Figure I.5, where each subplot either represent a different initial rock fabric (coquina or grainstone), or a different type of cementation (isopachous or syntaxial). In each plot the bulk velocity lies between that of the pore fluid and the assumed local velocity of the grains and cement. For the coquina facies (Figure I.5a) we see a dramatic drop in bulk velocity V_P at a critical porosity of $\phi_C \approx 30\%$. The trend then flattens asymptotically to a lower bound, equivalent to the velocity of the pore fluid (1500 m/s). This most likely reflects a critical porosity effect; due to the thin and convex shape of the shells used coquina deposition (Fig. I.4a) there are very few grain-to-grain contacts at high porosity, thus the rock is effectively fluid-supported (i.e., a suspension). As cementation increases and porosity decreases below a critical porosity ($\phi < \phi_c$), the number of grain-to-grain contacts increases so that the rock becomes grain supported, and the velocity rapidly increases. The diagrams show bulk velocities measured in three directions. These synthetic data exhibit a drop in velocity at different porosities depending on orientation of the measurement. The initial fabric of grainstones use grain shapes which are much rounder than the shelly fragments of the coquina, and there are many grain-to-grain contacts. Consequently these digital rocks are always below ϕ_c and the curves in Figs. I.5b and c exhibit a much smoother response to increasing cementation. We see an exponential trend for isopachous cementation (Fig. I.5b) and a more linear one for syntaxial cementation, which is measured over a narrow porosity range (Fig. I.5c). Both grainstone samples exhibit anisotropy, where V_P^Z (measured in the direction of the vertical axes in the upper panels of Figs. I.2 and I.3) is approx. 500 m/s

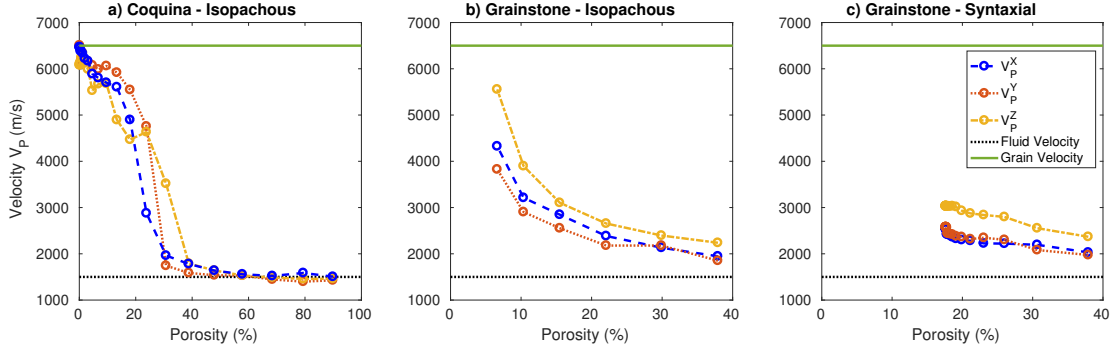


Figure I.5: Bulk velocity-porosity trends measured in three orthogonal components: V_P^X (blue), V_P^Y (red), V_P^Z (green), for a) Coquina (shelly fragments) with isopachous cementation, b) Grainstone (ellipsoids) with isopachous cementation, c) Grainstone with syntaxial cementation. The lower bound of fluid velocity (1500 m/s) and upper bound of calcite grain and cement velocity (6500 m/s) are denoted by horizontal lines.

faster than V_P^X and V_P^Y , as expected from the horizontal alignment in the fabric. The coquina facies also exhibits anisotropy, most notably in the critical porosity ϕ_C .

I.3.3 Rock physics modelling

Model elastic data estimated from synthetic digital rocks also provides an opportunity to test and validate existing analytical rock physics models. Here we test two existing analytical rock physics models: conventional differential effective medium theory (DEMT) as of Berryman (1992) and the Cilli-Chapman (CC) model (Cilli and Chapman; in prep.). In conventional DEMT, a small volume of spheroidal inclusions with aspect ratio α and known elastic properties, are iteratively added into an initially homogeneous background medium with known elastic properties. After each iteration of added inclusions, the average

properties of the average composite are calculated and used as background medium properties for the next iteration. When the the inclusion volume added per iteration becomes infinitesimally small, the elastic DEMENT model can be described by the following differential equations:

$$(1 - y) \frac{d}{dy} [K^*(y)] = (K_2 - K^*(y)) P^{(*2)} \quad (\text{I.4})$$

$$(1 - y) \frac{d}{dy} [\mu^*(y)] = (\mu_2 - \mu^*(y)) Q^{(*2)} \quad (\text{I.5})$$

with initial conditions $K^*(0) = K_1$ and $\mu^*(0) = \mu_1$, where K_1 and μ_1 are the inclusion bulk and shear moduli; K^* and μ^* are the rock's effective bulk and shear moduli; y is the inclusion volume fraction (i.e., porosity); P and Q are α -dependent geometric factors (Berryman, 1980b) and their superscript denotes they are to be calculated with ellipsoids of phase 2 embedded in the effective background material. Thus, the estimated effective moduli K_{DEMT}^* and μ_{DEMT}^* are functions of the bulk and shear moduli of the two phases, the porosity, and the inclusion aspect ratio. There are several assumptions and limitations to the application of the DEMENT method, for example that the bulk rock is assumed to be isotropic and inclusion shapes are idealised randomly-oriented ellipsoids. Also, the processes of incrementally adding inclusions to a background matrix is also entirely non-physical but rather a thought experiment, and does not represent the actual evolution of a rock's porosity.

In real rocks, pores are not ideal ellipsoids, and hence the inclusion aspect ratio is simply a parameter that quantifies the contribution of a rock's pore space architecture to the rock's overall elastic moduli. To make this distinction between the theoretical model inclusion aspect ratio and its applied physical interpretation, Fournier *et al.* (2011, 2014, 2018) refer to the applied DEMENT parameter α as 'equivalent' pore aspect ratio (EPAR). Similarly, Cilli and Chapman (in prep.)

synonymously label it the ‘effective’ pore aspect ratio. The EPAR for a given sample can be estimated by minimising the misfit between measured bulk modulus K_{meas} and the forward modelled moduli using equations I.4 and I.5:

$$\Psi(\alpha) = ||K_{meas} - K_{DEMT}^*(K_1, \mu_1, K_2, \mu_2, y, \alpha)||_2. \quad (I.6)$$

The EPAR is estimated by finding the aspect ratio α that minimises Ψ , where the bulk and shear moduli of the two phases (K_1, μ_1, K_2, μ_2) and the porosity (y) are known.

Cilli and Chapman (in prep.) investigated how a rock’s effective pore and grain aspect ratio (EPAR and EGAR) parameters changed with porosity for seven public domain datasets for elastic and electrical properties of carbonate rocks and concluded that these parameters vary with porosity as a power law, of the form:

$$\alpha = \Gamma \phi^\xi. \quad (I.7)$$

In the case of elastic modelling, where α represents EPAR, equation I.7 shows the model $\alpha = \Gamma = \text{constant}$ in the case of $\xi = 0$, consistent with the typical DEMT modelling approach.

By substituting equation I.7 into equations I.4 and I.5, the resultant bulk and shear moduli, K_{CC}^* and μ_{CC}^* , can be calculated as functions of the bulk and shear moduli of the two phases, the porosity, and two parameters Γ and ξ . The Cilli-Chapman model uses an extra model parameter than the standard, constant- α DEMT model, so its effectiveness cannot be compared to the standard method on the residuals alone. Cilli and Chapman (in prep.) use log-relative likelihood analysis that justifies the extra parameter used in fitting the data.

Given our digitally measured velocities and porosities, as well as the known fluid and matrix elastic moduli, we inverted equations I.4 and I.5 to estimate the EPAR of every rock sample. We inverted for the model parameters by minimising the misfit between each sample's measured (from Equation I.2) and modelled bulk modulus. We display the inverted EPARs for each data set as a function of porosity on a log-log scale in Figure I.6, and fit a line of best fit through each data set's inversion results. We calculated a close approximation $\{\Gamma_0, \xi_0\}$ of the true model parameters $\{\Gamma, \xi\}$ by the gradient and constant of these lines. As the inversion for each sample's EPAR was porosity-dependent, parameters $\{\Gamma_0, \xi_0\}$ are not necessarily equal to $\{\Gamma, \xi\}$. We thus performed a non-linear global optimisation to find the solution $\{\Gamma, \xi\}$ which minimise the l_2 -norm misfit of all samples by directly substituting equation I.7 into equations I.4 and I.5 with starting point $\{\Gamma_0, \xi_0\}$.

We use this inversion technique for four suite of bulk modulus-porosity data: 1) synthetically generated grainstones with syntaxial cementation, 2) synthetic grainstones with isopachous cementation, 3) synthetic coquinas with isopachous cementation, and 4) a subset of real-rock laboratory measured grainstone data from Fournier *et al.* (2011). This subset laboratory data consists of 80 outcrop samples, all exhibiting grainstone texture and with almost pure calcitic mineralogy. There is no presence of any intergranular, inter-crystalline, or moldic porosity in these samples, consistent with the definition of the grainstone end-member model examined here. In principle, these textures could be examined in future work.

I.3.4 Comparing competing models

Following from the inversions using both the constant- α differential effective medium theory model and the variable- α Cilli-Chapman model, it is then necessary to test these competing models. Here we use the corrected Akaike information criterion (AIC_C) (Akaike, 1973; Hurvich and Tsai, 1989), which is a statistical tool used for model selection based on information theory. The conventional AIC method penalises models with more modelling parameters (more complex models) and favours models with smaller misfits, thus dealing with the trade-off between data fit and model simplicity. This is necessary here because the conventional DGMT and the Cilli-Chapman models require different numbers of model parameters.

Let k be the number of parameters for a model, and $\hat{L}$ be the estimated likelihood function of the model (goodness of fit). The AIC value for a particular model is:

$$AIC = 2k - 2\ln(\hat{L}). \quad (I.8)$$

When the number of samples n is small, AIC becomes biased to models with more model parameters (McQuarrie and Tsai, 1998). As the datasets used here are relatively small, to therefore avoid this risk of overfitting, we use the corrected Akaike Information Criterion AIC_C (Hurvich and Tsai, 1989; Cavanaugh, 1997), defined as:

$$AIC_C = AIC + \frac{2k^2 + 2k}{n - k - 1}. \quad (I.9)$$

The absolute AIC_C value generally has little indication of the validity of a model (Burnham and Anderson, 2004); rather the difference between AIC_C values

(ΔAIC_C) for competing models is the significant measure, where $\Delta AIC_C = AIC_C^{DEMT} - AIC_C^{CC}$.

Burnham and Anderson (2002) provide a practical rule-of-thumb method for the interpretation of ΔAIC_C values, suggesting that: if $\Delta AIC_C < 2$, the variable- α CC model has “insufficient” evidence to accept as the best model; if $4 < \Delta AIC_C < 7$, the CC model has “good” evidence and constant- α DEMT has considerably less evidence as the best model; and if $\Delta AIC_C > 10$, the variable- α CC has “compelling” evidence to be the best model (constant- α DEMT has negligible evidence). Here we use an extra category for where $7 < \Delta AIC_C < 10$, the CC model has “substantial” evidence.

I.4 Results

We employ these guidelines for interpreting ΔAIC_C values and provide all inversion results, including the output parameters Γ and ξ from the Cilli-Chapman model and α_{DEMT} from conventional differential effective medium theory, summarised in Table I.2.

I.4.1 Grainstone

In order to test both the differential effective medium theory and Cilli-Chapman models described above, we first use elastic moduli estimated for grainstone samples using both syntaxial and isopachous cementation, as well as the laboratory measured grainstone data from Fournier *et al.* (2011). The results for both inversion techniques are shown as model fits to the data in Figure I.6. The grainstone samples with syntaxial cementation (Figures I.6a and b) show a strong power-law

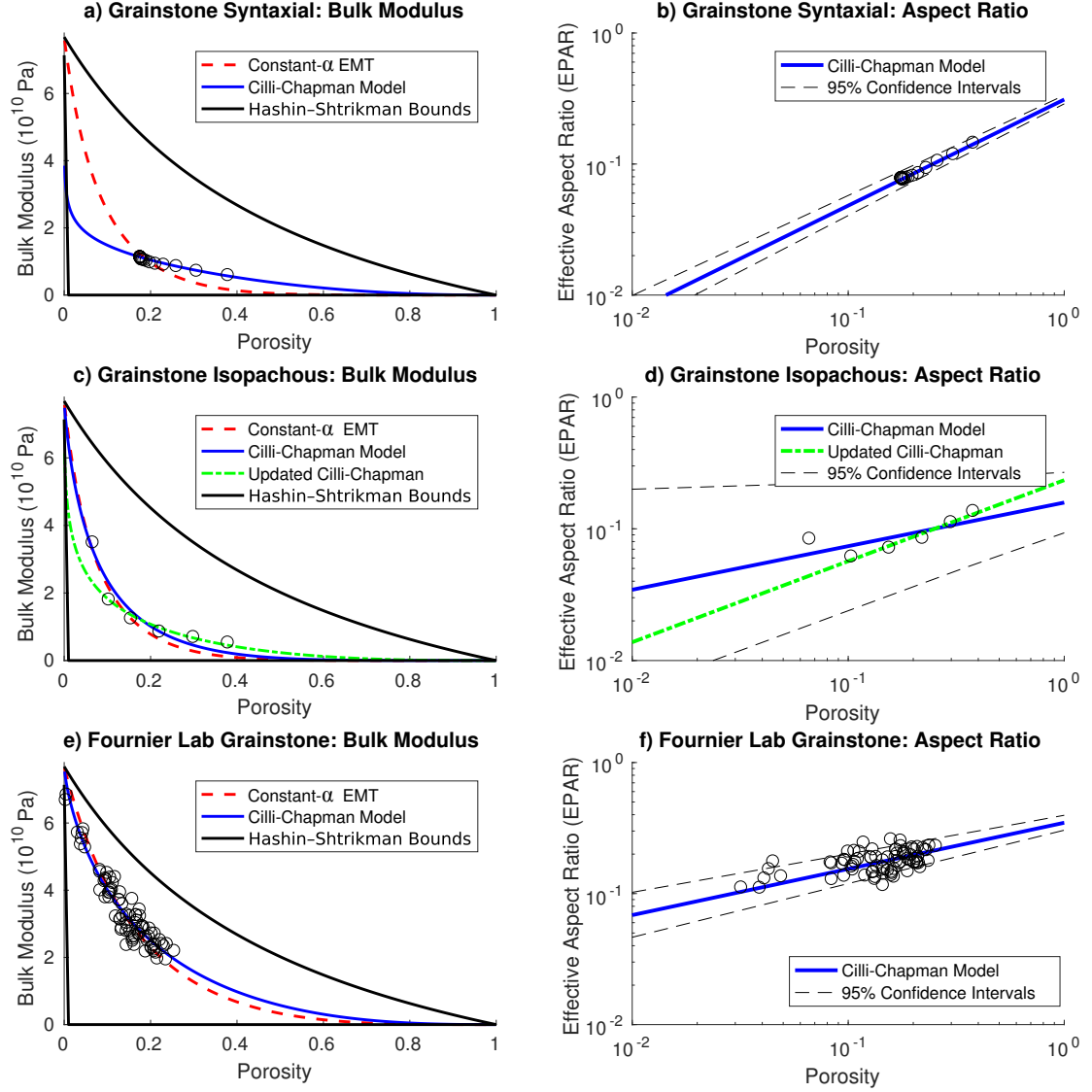


Figure I.6: Rock physics modelling results for grainstone data. The left panels are the modelled bulk modulus trends using a constant- α DEMT model (dashed red) and the variable- α Cilli-Chapman model (solid blue). The right panels show inverted EPAR as a function of porosity. The figure shows results for: a-b) syntaxial cementation, c-d) isopachous cementation, e-f) laboratory measured grainstone data from Fournier *et al.* (2011). For the isopachous cementation data (c-d), we include an updated model (green), where the low porosity is rejected in the regression as the data point has Cook's distance d_C equal to 2.650, well above the recommended threshold $d_C > 1$ (Cook and Weisberg, 1982).

Table I.2: Comparison of model inversion results

Figure	Data	ϕ Range	n	Γ	ξ	α_{DEMT}	ΔAIC_c	Preferred Model	Evidence
6(a-b)	Grainstone Syntaxial	18-37%	20	0.301	0.791	0.08	51.3	$\xi > 0$	Compelling
6(c-d)	Grainstone Isopachous	6.6-37%	6	0.014	0.139	0.07	-3.80	$\xi = 0$	Good
6(c-d)	Grainstone Isopachous (High ϕ range)	10.4-37%	5	0.213	0.562	0.07	9.13	$\xi > 0$	Substantial
6(e-f)	Fournier Lab Grainstones	0.4-26%	80	0.227	0.238	0.17	17.9	$\xi > 0$	Compelling
7	Coquina Isopachous	0.01-90%	21	0.359	0.147	0.23	-9.03	$\xi = 0$	Substantial
7	Coquina Isopachous (2x ϕ ranges)	0.01-21% 21-90%	13 8	0.727 0.844	0.279 1.757	0.23	53.9	$\xi > 0$	Compelling

relationship between EPAR and porosity (plotting linearly on a log-log scale). The data points for bulk modulus clearly cross-cut the forward modelled line (dashed red) using DEMT, which assumes a constant aspect ratio for all porosities. The Cilli-Chapman model (variable aspect ratio) provides a significant improvement in data fit in these cases. This improvement is expected to some degree, as the Cilli-Chapman model uses an additional modelling parameter, therefore an addition degree of freedom. As shown in Table I.2, despite the additional modelling parameter, the Cilli-Chapman model has compelling evidence as the best model ($\Delta AIC_C = 51.3$).

Grainstone samples using isopachous cementation shows a similar trend (Figures I.6c and d), where there is clear variation of EPAR with changing porosity. However the lowest porosity sample ($\phi = 6.6\%$) clearly does not follow the trend exhibited by the other samples. According to AIC_C , there is good evidence that DEMT is the best model to describe the data ($\Delta AIC_C = -3.8$). The low porosity sample can be identified as ‘highly influential’ to the model according to Cook’s distance, which is a statistical measure used to identify the influence of individual data points when performing least-squares regression (Cook, 1977). Cook’s distance of the $\phi = 6.6\%$ sample is $d_C = 2.65$, well above the recommended threshold for a sample to be considered highly influential, where $d_C > 1$ (Cook and Weisberg, 1982). This does not necessarily give grounds to reject the sample from the inversion, but indicates the data point should be examined more closely, and possibly indicates the limits of using digital rock physics at such low porosities (this is further discussed in Section I.5.2). For comparison, Figures I.6c and d include inversion results using Cilli-Chapman model for both the full porosity range (blue) and one excluding the low porosity sample (green). Where the low porosity value is excluded $\Delta AIC_C = 9.13$, which suggests stronger evidence for the Cilli-Chapman model as the preferred model.

Figures I.6e and f show inversion results for the laboratory measured ‘real-rock’

grainstone data. Compared with the synthetic digital rock results, the EPAR-porosity relationship exhibits a similar Γ value, but a significantly lower ξ value (gradient of best fit line through log-log EPAR-porosity plot). When ξ is small, the Cilli-Chapman model closer resembles the constant-aspect ratio DMT model. Despite the small variation in models, the improvement in data fit is regarded as statistically significant, where $\Delta AIC_C = 17.9$, due to the large number of data points. The difference between digital rock simulations and laboratory results is discussed in Section I.5.1.

I.4.2 Coquina

The estimated bulk modulus and the inverted Effective Pore Aspect Ratio (EPAR) values for the Coquina samples, using the same method as in Section I.4.1 are shown in Figure I.7. This data is using the velocity measurements taken only in the direction perpendicular to shell orientations (V_P^Z in Figure I.5a and the vertical direction in Figures I.4a-c). The coquina samples contain a very large porosity range ($\phi = 0.1 - 90\%$), and exhibit much more complex pore shapes in comparison with those in the grainstone samples. There appears to be two distinct EPAR-porosity trend-lines, separated into stages: I) $\phi < 0.21$ and II) $\phi > 0.21$ in Figure I.7b. This is interpreted as a critical porosity effect, where above a given porosity ϕ_C the rock can be treated as fluid supported, which also explains the abrupt drop in bulk modulus where $\phi > 0.21$ in Figure I.4a. Below this threshold the rock is grain or clast-supported. When the full range of porosity samples are included in the inversion the constant- α DMT model is statistically much stronger ($\Delta AIC_C = -9.03$) than a single power law relationship. However, when two separate inversion are performed for porosities both above and below $\phi_C = 0.21$, where two power law relationships doubles the number of model

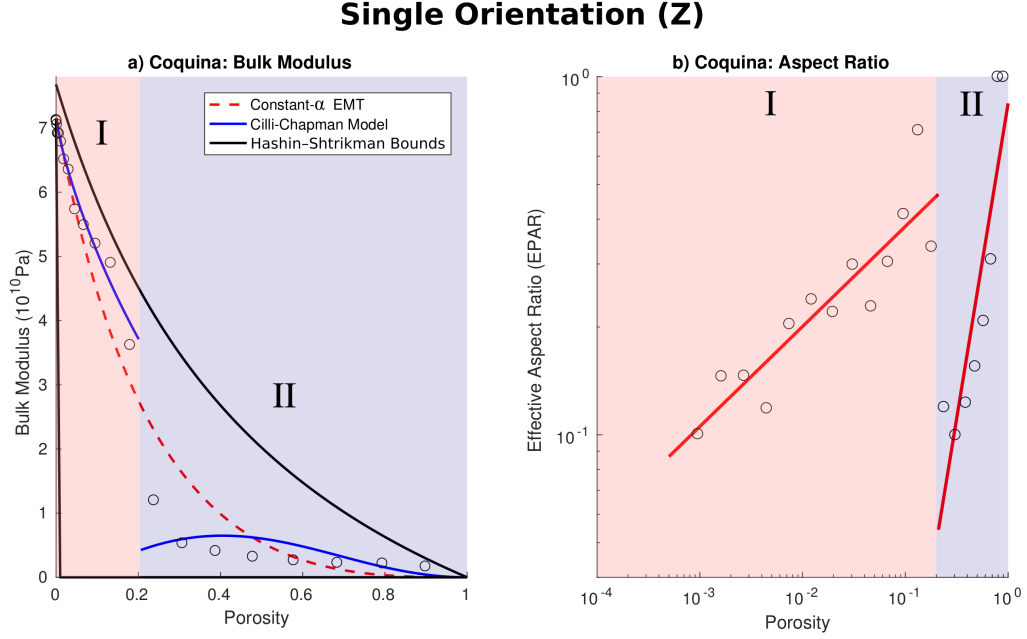


Figure I.7: Rock physics model comparison for Coquina samples, using bulk modulus inferred from velocities measured perpendicular to shell orientation (Z direction, vertical direction in Figures I.4a-c). a) Bulk modulus-porosity trend for a constant- α DEMT model (dashed red) and the variable- α Cilli-Chapman model (solid blue) separated into two separate porosity ranges (critical porosity $\phi_c = 0.21$) with two power-law relationships. b) Inverted aspect ratio α (circles) and two linear trend-lines: I) $\phi < 0.21$ and II) $\phi > 0.21$. Porosity range I is below critical porosity ($\phi < 0.21$) and exhibits the previously observed power-law (linear on log-log scale) relationship. Stage II is above critical porosity ($\phi > 0.21$), where inverted aspect ratio values drop initially, then increase to $\alpha = 1$.

parameters, $\Delta AIC_C = 53.9$, which suggests strong evidence for the two stage Cilli-Chapman model as the most appropriate.

The EPAR of a rock is related to its solid frame's stiffness. When $\alpha = 0$, the frame is as weak as possible and the rock's moduli coincide with the lower Hashin-Shtrikman (Hashin and Shtrikman, 1963) and Reuss (Reuss, 1929) bounds (Norris, 1985). A fluid-filled porous rock with a measured porosity above its critical porosity is, in effect, a suspension of solid material in a background of fluid and has elastic moduli coinciding with the lower Hashin-Shtrikman bound. In the case of the Coquina samples (Figure I.7), we see the rock's EPAR increasing with porosity in stage I, implying the matrix is becoming relatively more stiff as porosity increases. At the interface between stage I and stage II, however, the inverted EPARs plummet as the rock passes its critical porosity and becomes significantly softer. In stage II the rock is acting approximately as a suspension. We notice the Coquina's measured bulk modulus does not go exactly to the Hashin-Shtrikman lower bound, but rather seems to stay at an approximately constant value for all porosities above the critical porosity. The way in which the digital rocks have been generated, involves the deposition of falling grains and settling due to gravity, where grain-to-grain contacts must be present. Therefore the media cannot be true suspensions under gravity, though a critical porosity effect is still observed during dynamic wave propagation. Figure I.4 shows that even at 90% porosity, there are some load-bearing connected paths of matrix material from one side of the digital rock to the other under gravity. We speculate that if connected paths like this exist over all porosities, the bulk modulus will be higher than the Hashin-Shtrikman lower bound, as observed in Figure I.7. The bulk modulus remains approximately constant in the high porosity samples of Figure I.7 while the upper and lower Hashin-Shtrikman bounds converge with increasing porosity. The bulk modulus is hence becoming closer to the upper

Hashin-Shtrikman bound with increasing porosity in stage II, which is the cause of the observed increasing EPAR with porosity in this stage.

Both models are constructed assuming background medium, with spheroidal inclusions with a given aspect ratio α embedded within. Therefore using either DEMT or Cilli-Chapman models in suspensions, where the porosity is higher than the critical porosity ϕ_C , does not follow any physical intuition and does not allow for physical interpretation. It may therefore be more appropriate to consider a separate rock physics models for suspensions (e.g., Wood (1955)) in stage II.

I.4.3 Anisotropy

In the previous section measurements from a single orientation were used as input to the rock physics inversions for coquina samples. This was done to prevent the smoothing out of the critical porosity effect, as each orientation exhibits it's own distinct critical porosity. The difference in orientations is visible in Figure I.5a, where velocity abruptly drops at different porosities for each of the three orientations, to approximately the velocity of the pore fluid (1500 m/s). Using an average of all orientations for bulk modulus as input data to the rock physics inversions results in a different trend than that observed in the single-orientation inversion. Figure I.8 shows the inversion results using an average bulk modulus, where the inverted EPARs as a function of porosity appears to follow three separate power-law trends at different porosity stages: I) $< 11.5\%$; I+II) $11.4 - 41.75\%$; II) $> 41.75\%$. Stage I can be physically interpreted in the same way as before, where the medium is grain supported during wave propagation, and exhibits the usual power-law relationship between porosity and aspect ratio. Stage I+II is an intermediate phase, where two orientations exhibit a critical porosity effect. This intermediate stage is an artifact of the averaging

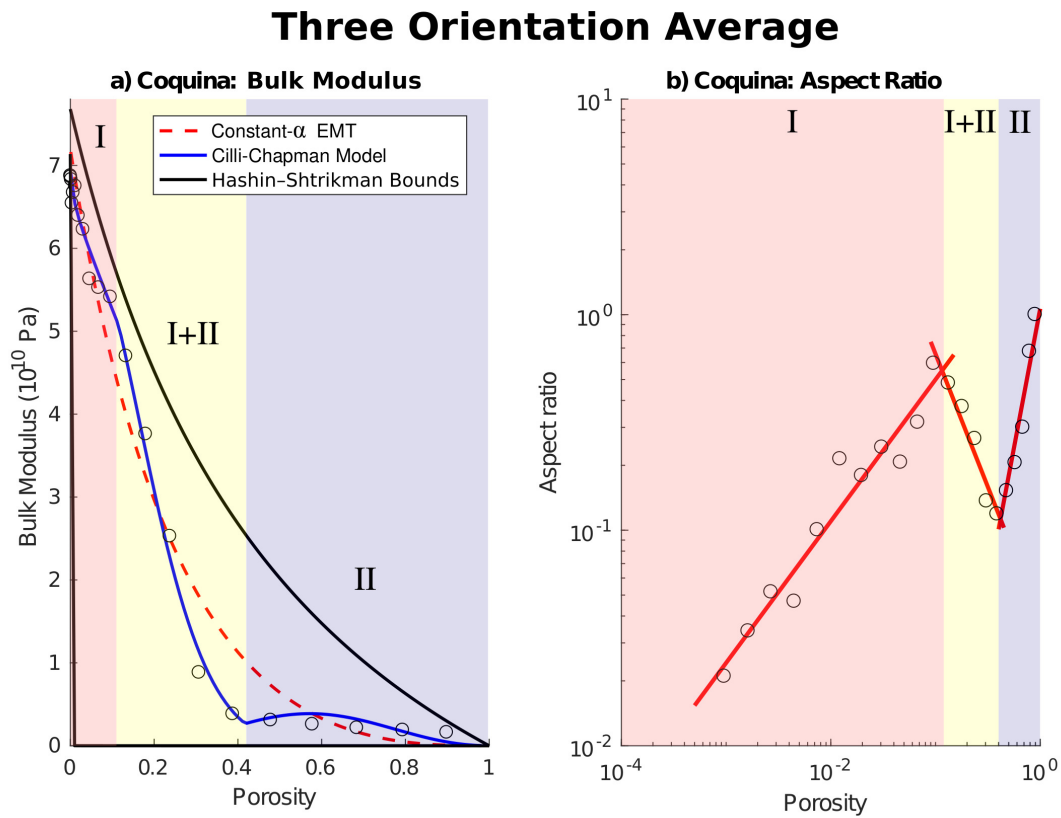


Figure I.8: Rock physics model comparison for Coquina samples, using data averaged across three orthogonal orientations. a) Bulk modulus-porosity trend for a constant- α DEMT model (dashed red) and the variable- α Cilli-Chapman model (solid blue) separated into stages at different porosity ranges: I) <11.5%; I+II) 11.4–41.75%; II) >41.75%, where the intermediate stage is an artefact of averaging across different critical porosities exhibited by each orientation (Fig. I.5a). b) Inverted aspect ratio α (circles) and three linear trend-lines.

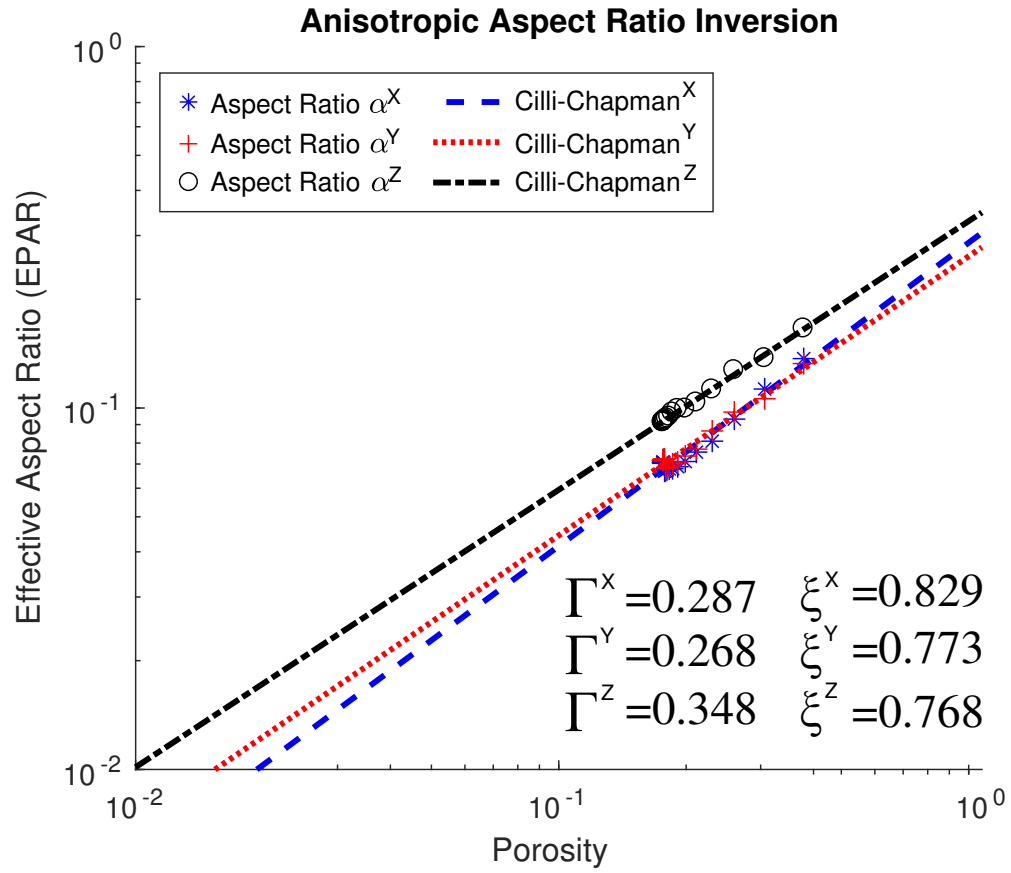


Figure I.9: Inverted aspect ratio α as a function of porosity, for grainstones with syntaxial cementation. Measurements are made using elastic data measured in three orthogonal directions (the input velocity measurements are shown in Figure I.5c). Γ (aspect ratio α at the porosity $\phi=1$ intersect) and ξ (gradient of the trend-line on log-log scale) for each orientation are labelled.

across the three orthogonal directions. In stage II, moduli from all orientations are close to the lower bound therefore all orientations can be treated as a suspension during dynamic wave propagation, thus stage II exhibits a similar aspect ratio-porosity trend to stage II for the single-orientation coquina inversion in Figure I.7b.

Figure I.5b and c highlight elastic anisotropy in the initial fabric of the grainstones, where velocity measured in the Z direction (vertical direction in Figures I.3 and I.2a-c) is approximately 500 m/s faster than V_P^X and V_P^Y . To further understand the effect of anisotropy on the rock physics model inversions, we invert for the independent EPAR parameters ($\alpha_X, \alpha_Y, \alpha_Z$) using bulk modulus calculated from the three separate orientations, using the grainstone samples with syntaxial cementation. The resulting inverted EPAR as a function of porosity are shown in Figure I.9. The results are consistent with a bulk seismic anisotropy that is transverse isotropic, where $V_P^Z > V_P^X \approx V_P^Y$ leading to $\alpha^Z > \alpha^X \approx \alpha^Y$ at all porosities in Figure I.9. This is consistent with the input model fabric due to sedimentation under gravity. Interestingly the effect of anisotropy is only on the Γ parameter in the Cilli-Chapman model (the theoretical inclusion aspect ratio α where porosity $\phi = 1$), whereas ξ (the gradient of the trend-line on a log-log scale) is approximately constant, independent of orientation and anisotropy. In this case, if bulk modulus is inferred from measured velocity along a single orientation (as is often the case in laboratory experiments), then the estimated ξ parameter can be assumed to be representative of the entire medium.

I.5 Discussion

I.5.1 Comparison of modelled and real data

When comparing the synthetic data from modelled digital rocks and those of the ‘real’ laboratory measured grainstones, a significant difference between the inverted power-law relationships is observed (Figure I.6), particularly the values of ξ (in Equation I.7). This value denotes the gradient of the aspect ratio-porosity trend-line on a log-log graph, and reflects the sensitivity of the EPAR to porosity changes. We interpret the high ξ values for digital rocks to be a product of considering cementation to be the only process occurring, essentially ignoring other physical effects. This results in much smoother trends with lower variance in the synthetic data compared to the data from real rocks. As porosity increases, the digital rocks show increasing EPAR, meaning the rock frame becomes relatively stiffer. However in reality, other physical effects also operate as porosity increase, such as the stiffness of grain contacts. Rocks with high porosity can be poorly consolidated, these samples would exhibit relatively low stiffness, therefore a low inverted EPAR. These different processes occurring and acting against each other essentially reduce the sensitivity of the effective pore EPAR to porosity (reducing ξ). A digital rock model incorporating changes in both cementation and grain contact stiffness could overcome this observed difference, but we leave this for future work.

I.5.2 Digital rocks at low porosity

In real rocks, pore structures are found to be fractals, i.e., the geometric features of the pore space are similar across all scales. At the smallest scales the estimation

of physical properties of porous rocks are hampered by the resolution of the 3D images, mainly due to the presence of unresolved microporosity. In digital rocks that attempt to emulate real ones, the smallest possible pore is equivalent to a single voxel with a cubic shape. Therefore digital rocks cannot replicate the fractal dimensions of a real rock, and particularly at low porosities, the pore shapes become unrealistic. This could be an explanation for the outlier result for the low porosity sample ($\phi = 6.6\%$) of the grainstones using isopachous cementation seen in Figures I.6e and d, which exhibits an EPAR higher than expected assuming a power-law relationship. As porosity decreases due to cementation, individual pore shapes become more cubic (closer to a sphere where $\alpha = 1$) and therefore relatively stiff. The inversion results from the laboratory measured grainstones (Figures I.6e and f) do not show this increase of EPAR at lower porosities indicating the phenomenon is an artifact of the finite bandwidth scaling of the digital rock. This can somewhat be overcome by increasing the resolution of the digital rocks, therefore increasing the porosity range where digital rock microstructure remains valid.

I.5.3 Critical porosity and beyond

Digital rock physics provides an insight into the elastic properties of media that either do not often occur in nature, or would be challenging to produce in laboratory experiments, such as the very high porosity of the initial coquina model prior to cementation. This has allowed us to observe critical porosity effects associated with a transition from grain-supported to fluid-supported textures during dynamic wave propagation. The critical porosity ϕ_C is generally defined as a threshold when porosity $\phi > \phi_C$, the medium can be treated as a fluid supported suspension, and when $\phi < \phi_C$ the medium is grain supported. However, we have previously stated that grain-to-grain contacts exist across all porosities

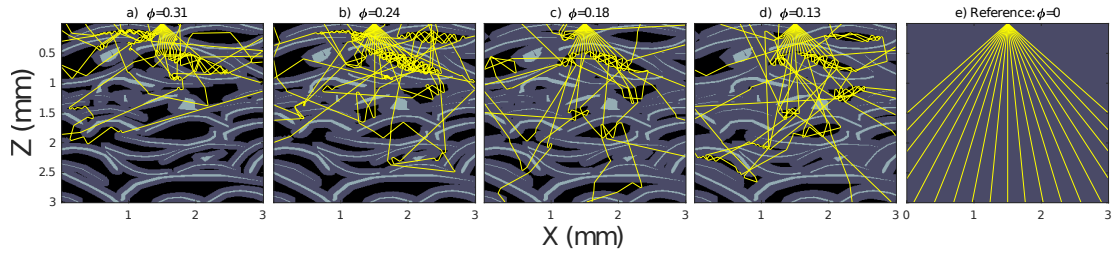


Figure I.10: 2D ray tracing (Margrave, 2000) results using the vertical slices through the center of coquina models with increasing cementation, over the porosity range that exhibits an abrupt increase in seismic velocity. Rays originate from the center-top of the models, and are shown for a 90° aperture with 5° intervals. Rays terminate when they leave the bounds of the velocity model.

under static load, though a critical porosity effect is still observed. To explore this further we take vertical slices through the center of four volumes, over the porosity range that exhibits the abrupt change in bulk modulus. We use the slices, converted into velocity models (following the same method described in Section I.3.2), as input into the ray-tracing software of Margrave (2000). This method takes a fan of rays from a user-selected source location, and calculates the ray path, which terminates when it leaves the bounds of the model. This process is to illustrate the difference in ray path complexity, rather than calculate absolute ray paths, as our finite-difference simulations are not limited to a 2D plane. The output shown in Figure I.10 gives an idea of how waves travel through each model. At high porosity (Figure I.10a) waves take very complex paths, with many internal reverberations. For a ray to traverse the full length of the medium (necessary for the measurement of seismic velocity with conventional experimental geometries) would require to take a complex path through grain and cement, or travel through a large amount of pore fluid. Contrast this with a lower porosity model (Figure I.10d), where rays are transmitted through the full length of the model, and paths are considerably simpler (fewer reflections). Therefore the critical porosity reflects

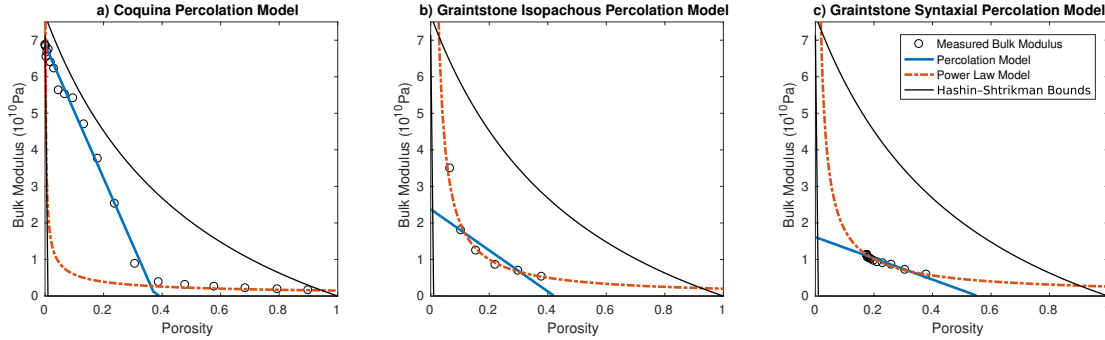


Figure I.11: Comparison of some alternative empirical models for fitting bulk modulus-porosity data: a percolation model using Equation I.10 (blue) and power law model using Equation I.11 (red) for the coquina (a), grainstones with isopachous (b) and syntaxial cement (c). The black lines represent the Hashin-Shtrikman bounds.

the transition between direct tortuous paths between the source and receiver on opposite sample boundaries. Independently, Walker *et al.* (2016) observe a step increase in the number of force chains at a critical stress when simulating simple shear in 3D media. In principle this could provide another possible explanation for critical porosity effect observed in the coquina, if it can be demonstrated to occur also under static gravitational load.

I.5.4 Alternative models

In this study we carry out an in-depth comparison of two physics-based rock physics inclusion models: conventional differential effective medium theory (Berryman, 1992) and the Cilli-Chapman model (Cilli and Chapman; in prep.). While it would not be feasible to statistically test all existing rock physics models, we believe it is worthwhile considering some empirical models, which give complementary insight into physical processes. Models arising from percolation theory

(Stauffer and Aharony, 1994), where bulk modulus K_{perc} can be modelled as:

$$K_{perc} = K_{max} \left(\frac{|\phi_c - \phi|}{\phi_c} \right)^m, \quad (\text{I.10})$$

Where K_{max} is the maximum measured bulk modulus, and m is a fitting parameter. Given the power-law nature of the Cilli-Chapman model, we also consider a simple empirical power law model for bulk modulus K_{pl} :

$$K_{pl} = K_0 \left(\frac{\phi}{\phi_0} \right)^n, \quad (\text{I.11})$$

where ϕ_0 is the initial (highest) porosity, K_0 is the initial (lowest) measured bulk modulus, and n is a fitting parameter. We invert for ϕ_c and m in Equation I.10 and n in Equation I.11 to find the best fitting models. We compare these models to the estimated bulk modulus data in Figure I.11. First we note that the percolation model performs relatively well for the coquina data below critical porosity (Fig. I.11a), exhibiting a near-linear trend. Percolation theory describes the connectivity of a network across a medium, where at a certain porosity threshold (or critical porosity), the connected network spans the entire medium. This physical process closely resembles the interpreted critical porosity effect observed in the coquina samples. The power law model performs well at porosities above ϕ_c though is clearly not appropriate at low porosities. The percolation model fails to fit the data for the grainstone with isopachous cement (Fig. I.11b), however the power law model fits the data relatively well. The grainstone with syntaxial cement data (Fig. I.11c) spans a narrow porosity range, thus both percolation and power law models fit the data well; there is not sufficient data to effectively assess the models. These empirical models can sometimes show impressive fits to the measured data, however offer limited ability to infer any physical significance. With the exception of one outlier noted above, the physics-based approach of the inclusion models considered here provides a more complete

description of the data, and does allow for a physical interpretation of the results observed at the bulk scale.

I.6 Conclusion

Measuring geophysical properties associated with geological processes, such as the measurement of seismic velocity during rock cementation, is very challenging in a laboratory environment. Such experiments require elaborate equipment and can require very long time scales making them expensive in terms of cost and time. By contrast, digital rock physics is an effective method for the estimation of seismic velocity as cementation is occurring, while remaining cheap, fast, and without the use of any intricate and complex laboratory equipment. The method could easily be extended to the study of other geological processes such as compaction, dissolution, or fracturing. The results presented here show realistic bulk modulus-porosity trends for a range of carbonate morphologies and cementation types. We observe anisotropy in the measured velocity (and elastic modulus) due to structural anisotropy caused by the settling of elongate grains under gravity. We also observe a critical porosity effect in the coquina samples, where bulk modulus above 30% porosity drops to the approximately the lower Reuss and Hashin-Shtrikman bounds. Digital rock physics also allows for the quantitative testing of competing rock physics models. For modelling bulk modulus-porosity data, a variable-aspect ratio model is statistically stronger than the conventionally used single-aspect ratio differential effective medium (DEMT) model for both grainstone and coquina digital rocks. The variable-aspect ratio model also remains robust to anisotropy where the conventional DEMT method fails. There are strong similarities between simulated data and real laboratory measured data for grainstones, validating both the digital rock physics approach, as well as the

variable aspect ratio rock physics model. Our results also highlight a significant critical porosity effect for the anisotropic ‘shelly’ fragments between phases that provide more direct or more tortuous path between the source and receiver at different stages of cementation.

I.7 Acknowledgements

The authors would like to thank Petrobras and Shell for their sponsorship of the International Centre for Carbonate Reservoirs (ICCR), and for permission to publish this work from the 4DRP, VSP, and RockType II projects. We also thank Rachel Wood, Mark Chapman and Satyan Singh for their helpful scientific input and support.

Appendix J

Predicting the Location and Magnitude of Wormhole Formation in Heterogeneous Carbonates by Machine Learning

Preface

This appendix is a preliminary version of work in preparation for submission to a high profile geoscience journal for publication. I am the second author on this work. This work was predominantly written and performed by first author, Florent Brondolo. The listed authors are Florent Brondolo, Phil Cilli, Ian B.

Butler, Andrew Fraser-Harris, Katriona Edlmann, and Chris McDermott, who are all affiliated with the University of Edinburgh.

My contribution to this paper is the development and implementation of the statistical neural network predictive model which are used to predict where carbonate wormholing occurs in laboratory samples. I also wrote the scripts and implemented the linear regression modelling for comparison with the neural network results. In the publication, I authored the literature review on neural networks, the neural networks implementation, and part of the interpretation of the modelling results.

J.1 Abstract

Understanding the mechanisms behind the development of preferential flow-paths in carbonate rocks is important as they are related to several subsurface societal issues, e.g. hydrocarbon production, carbon sequestration, groundwater resource competition, geo-energy including geothermal energy and energy storage applications. The dissolution processes during water/acid flooding cause channelisation which leads to the formation of flow-paths called channels. Over the past 50 years, several studies, performed in a variety of fluid-mineral systems, have managed to describe the effect of rock / water flooding by focusing on the general shape of the channel through an estimation of chemical and physical parameters, but few studies have been able to predict their location. Here we propose a new method to approximate the spatial distribution and magnitude of a channel by linking experimental and numerical work through image pre-processing and machine learning. The experimental work aims to investigate the mechanical and chemical controls on the optimum equilibrium for channel formation within carbonate travertine rocks using purified seawater as the injection fluid during core flood experiments

under typical reservoir conditions. Each sample is CT-scanned before and after the experiment. The numerical simulation includes a pre-processing of the pre-experimental CT-scans followed by a training of the neural network on channel data, formed as a result of the dissolution and erosion of carbonates during the experimental flooding. The main interest of this study relies on discarding chemical parameters during the forward prediction while demonstrating the strengths of machine learning applied within the sphere of Geosciences. Early simulations show that, for heterogeneous rocks, we can accurately detect the location, the degree, and the spread of the dissolution pathway. Our findings confirm that the neural network has been able to highlight the key variables related to geometrical properties through image analysis. These processes are seconded by flow rate and effective stress. The neural network approach is enabling us to rank the most important features and facilitate the determination of the most likely location for the formation of a preferential flow-path, and its magnitude.

J.2 Introduction

Artificial Neural Networks (ANNs) can be described as a brain-like processing tool which mimics 'qualitative reasoning' (Lapedes and Farber, 1988; Haykin, 1999). They are increasingly used in a variety of problem solving situations (Medicine, Bio-Informatics, Geosciences, etc.). The appealing characteristics of ANNs come from the non-linear aspect of the solving algorithms coupled with its learning capacities (Basheer and Hajmeer, 2000). Few studies have coupled X-ray Micro Tomography (XMT) and machine learning as a segmentation tool of 3D volumes (Cortina-Januchs *et al.*, 2011; Chauhan *et al.*, 2016) or for rock modulus estimations (Sonmez *et al.*, 2006). In this study, we investigate the likelihood

of channel propagation through carbonate samples of heterogeneous nature by comparing the ANN computed solutions to the experimental results.

Much work has been done on channel and karstification development and classification on variety of fluid-mineral systems, and acidity of percolating water is identified as a key control for carbonate dissolution resulting in the development of preferential flow-paths. Scientists try to recreate these effects by injecting fluids into core samples - often referred as core flooding. The dissolution processes during rock water/acid flooding lead to the formation of preferential flow-path(s) referred as channels or wormholes (Golfier *et al.*, 2002). Several authors have characterised wormhole propagation through carbonate samples as a function of the experimental injection rate and the fluid properties (Hoefner and Fogler, 1988; Steefel and Lasaga, 1990; Frick *et al.*, 1994a,b; Bazin *et al.*, 1995; Fredd and Fogler, 1996, 1998; Golfier *et al.*, 2002; McDuff *et al.*, 2010; Walle and Papamichos, 2015). For most experimental set-ups, the expected/generated channel(s) can be split into five main families: Compact dissolution, conical dissolution, dominant channel, ramified wormholes, and uniform dissolution. Numerical modelling studies have attempted to recreate these wormholes by including variables impacting the general shape and spread of the dissolution footprint, such as capillary pressure, permeability, velocity of the fluid, and kinetic transfer from the walls. Different numerical approaches have been assessed, from a very simplistic approach that consider a pre-existing cylindrical wormhole (Hung *et al.*, 1989; Wang *et al.*, 1993; Buijse, 1997; Huang *et al.*, 1997, 1999), to more complex models which focus on the grain scale (Hoefner and Fogler, 1988; Daccord *et al.*, 1989), on the fluid mechanics (Daccord *et al.*, 1993a,b; Frick *et al.*, 1994a), or on the continuum equations of mass and flow transfer (Liu *et al.*, 1997; Chen *et al.*, 1997). Most of these approaches display good qualitative results of channel geometry, and are partially backed by experimental output, but none of them works as a predictive tool of channel spatial distribution, spread and magnitude.

Our experimental work aims to investigate the mechanical and chemical controls on the optimum equilibrium for channel formation within carbonate travertine rocks using purified seawater as the injection fluid during core flood experiments under typical reservoir conditions. Each core sample is CT-scanned prior and after the experimental flooding. The numerical work includes a pre-processing of the pre-experimental CT-scans followed by a training of the ANN against the post-experimental channels data: we aim to understand and predict the occurrence, shape, and magnitude of wormhole formation in heterogeneous carbonate rocks using attributes extracted from CT scan prior to flooding.

We postulate that if a linear combination of these CT-extracted attributes can successfully predict a rock's spatial wormhole signature, the weights from the linear regression could be considered indicators of the influence of CT-extracted features on wormhole formation. Moreover, if a misfitted linear regression is unsuccessful in predicting wormhole signatures on a blind dataset, a more complex modelling tool, such as neural networks, is required to predict wormhole formation behaviour. The main interest of this study relies on voluntarily discounting chemical parameters while demonstrating the strengths of Artificial Intelligence applied within the sphere of Geosciences.

J.3 Materials and Methods

A set of four experiments have been conducted on highly heterogeneous travertine rocks using 3.8 cm diameter cores of variable lengths which have been flooded with a 30.5 g/L NaCl artificially made seawater. The dataset used in this section are the result sets of the four flooding experiment run on travertine core samples. Details of the experimental work is presented in a dedicated paper (Brondolo *et al.*, in prep.).

J.3.1 Rock sampling

In this study, we are focusing our work on the shrub layers, the definition of which was first introduced by Kitano (1963), and further developed in later studies that compared the structures as short and shrubby crystalline masses of CaCO_3 which expand upward by irregular branching (Chafetz and Folk, 1984; Pentecost, 1990; Guo and Riding, 1992, 1994, 1998; Chafetz and Guidry, 1999; Kele *et al.*, 2008; Jones and Renaut, 2010; Wright, 2012; Chafetz, 2013; Chitale *et al.*, 2015; Rezende and Pope, 2015; Wright and Barnett, 2015; Claes *et al.*, 2017; Erthal *et al.*, 2017). All samples have been sourced from a ~ 40 cm by ~ 40 cm rock bloc provided by PetroBras and cut from an quarry outcrop in Saturnia (Italy).

J.3.2 Rock preparation

The shrub layers within a 40 cm by 40 cm large block of travertine has been cored using a 38 mm diameter diamond impregnated cylindrical core drill. The plugs have then been edge trimmed using a circular saw. Both ends of each sample have been flattened and polished with an abrasive rotating disc. Fines have been removed by thoroughly sonicating each sample with distilled water. A fine layer of cement has been applied around the samples to prevent the confining elastomer sleeve of the Hassler cell extruding into the largest pores and rupturing.

J.3.3 Experimental procedure

The experimental rig used has been built and conceived for fracture flow study (Edlmann *et al.*, 2013; McCraw, 2016), and further modified for our needs. The

reduced framework of the pre- and post-experimental procedure is described in the steps below:

- Pre-experimental procedure
 1. Preparation and conditioning of the rock sample
 2. Pre-experimental XMT scan of the core sample
- Experimental procedure: the core is placed into a Hassler cell and connected into the flow rig, followed by a flooding until the permeability evolution reaches stability
- Post-experimental procedure
 1. Sonic bath of the sample using distilled water
 2. The core is dried at least 1 week in an oven at 65°C
 3. Post-experimental XMT scan of core sample

J.3.4 Experimental matrix and dissolution regimes

Injection of fluid into carbonate reservoir rocks creates changes in fluid dynamics and the reservoir stress state, leading to multi-physical changes where the pore network, chemistry, temperature, fluid and rock pressures impact the dissolution processes. In these alkaline environments, the fluid dynamic often lead to flow channelling due to the development of preferential flow-paths reinforced by the effect of local permeability and heterogeneities.

Our experimental plan consists of injecting an artificially made seawater of known chemistry and pH through travertine core samples of 3.8 cm diameter (D) and variable length (L). The four experiments have been carried under in-situ

geo-reservoir conditions of pressure, temperature and representative chemistry (temperature $T = 60$ °C). The effective stress used in this study refers to the work of Terzaghi (1951), while the injection rates used in this study are described by:

$$PV_{rate} = \frac{Q(t)}{V_p} \quad (J.1)$$

With $Q(t)$ the amount of fluid injected per minute logged (m^3) and V_p the volume of pore of the rock sample (m^3). The porosity is calculated prior to the experiments, using the triple weighting technique, and display an average value of ~ 7 %. The experimental matrix, displayed in table J.1, shows the four experimental scenarios. These four different experimental scenarios have been designed to pick the key processes of flow rate and effective stress that are involved during permeability recovery, and while the findings are not detailed in this chapter, we are using the results for our wormhole prediction tool.

		Pore volume rate		
		Low	Medium	High
Effective Stress	Low	0.2 PV/min 10 MPa		2.6 PV/min 10 MPa
	High		1 PV/min 40 MPa	2.5 PV/min 40 MPa

Table J.1: Pore volume rate (PV_{rate}) and effective stress (MPa) values for the four scenarios.

J.3.5 XMT acquisition

Equipment

A reconstructed volume provides us with a 3D map of absorbed light due to the difference in density of the matrix of a rock (Landis and Keane, 2010). For each core, pre- and post- experimental flooding tomographic data are acquired at 130 kV, and 25 W target power loading. Each scans consist of 2000 projections; each of 2 s duration, during a 360° revolution obtained on dry samples, prior and after the experimental flooding. The core samples have been cleaned for ~ 15 min in a sonic bath, after the coring/cutting phases, and after the flooding stage in order to drive the remaining seawater out the sample. These extra steps will allow us to obtain clearer outputs from the X-ray scanning. They are first recorded as 2D projected images, followed by a tomographic reconstruction into a 3D stacks.

Volume processing

A rotary stage is placed on top of an adjustable screw and facilitates the scanning of different parts of a sample by adjusting its elevation. The geometry of the cone-beam window of the electron tube source is approximately 3 cm wide. Due to the length of the cores being over 6 cm, each scan of a whole sample is split in three sections: top, middle and bottom acquisitions. They are first recorded as 2D projected images, followed by a tomographic reconstruction into a 3D stacks. Several studies provide further details about acquisition and reconstruction processes along with the underlying physics of CT-scanning (Landis and Keane, 2010; Salvo *et al.*, 2010). Reconstruction by filtered back projection uses Octopus v8.7 software (Dierick *et al.*, 2004), while post-processing data analysis are done through Fiji (Schindelin *et al.*, 2012) and Avizo® 9 functions.

Channel processing

The common effect of fluid injection into a carbonate rock sample is the generation of channels, which are preferential pathways which increase fluid conductivity and allow the injection fluid to flow more easily (Hoefner and Fogler, 1988; Fredd and Fogler, 1998; Golfier *et al.*, 2002). These damages can be detected through image processing by arithmetically comparing the pre- and post-experimental CT scans. The registration of the unaltered and altered dataset of each rock, followed by the processing of the differences between both stacks have been done using Avizo[®] 9. Two types of data have been gathered after each experiment:

- 3D volumes of pre-experimental scan - referred to as input data. This dataset is used for training the ANN and/or predicting the channel(s) location and magnitude.
- 3D volumes of dissolution channels - referred to as signature data. This dataset represents the true solution of channel(s) formation, and is used for training the ANN.

As a way to account for difference in samples sizes, we sub-sampled the 3D stacks into cuboids of variable side lengths (550 to 650 px large) and constant axial length of 710 slices. Both input and signature volumes of a same core sample are sub-sampled at the same location. In order to save further computational time, the 3D sub-sampled volumes are re-sliced a hundred times using the *Z re-slice* function in Fiji (Schindelin *et al.*, 2012), in two orthogonal directions with respect to the axial axis and the original orientation of the sample within the quarried bloc. Figure J.1 presents the workflow for XMT data acquisition.

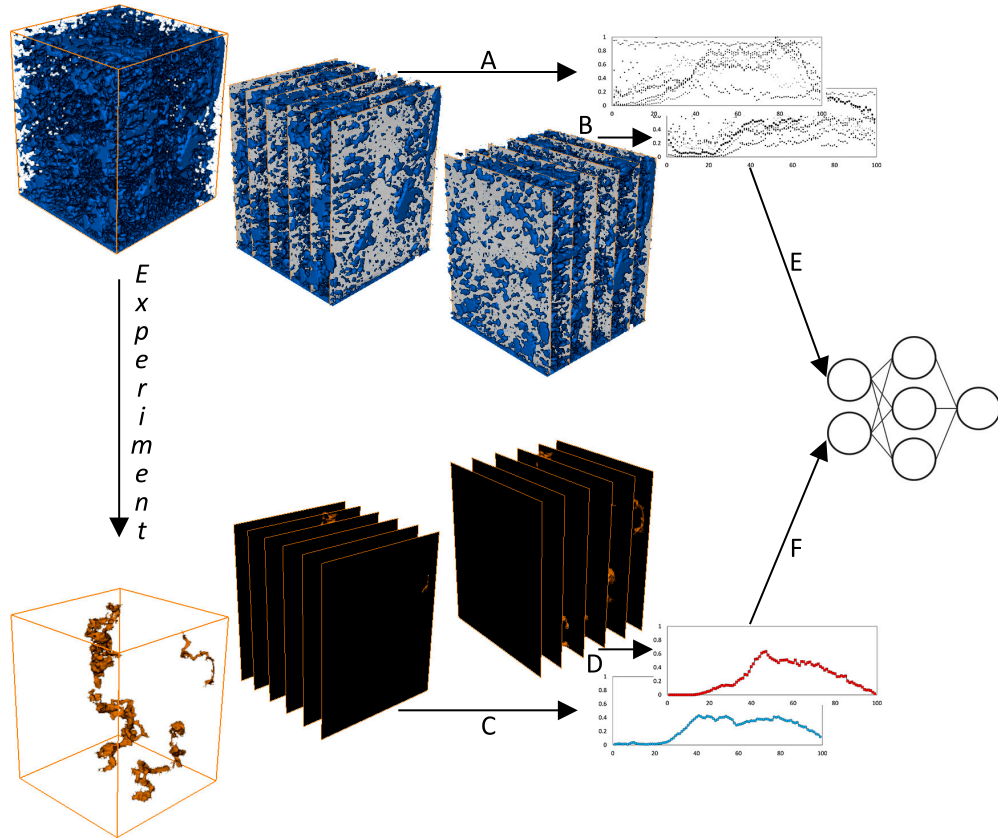


Figure J.1: Data acquisition workflow. The two sub-sampled cuboids are of the same size. Both stacks have been re-sliced a hundred times in two orthogonal directions. A & B: input data acquisition; C & D: signature data acquisition; E & F: input and signature data are attributes for the ANN.

J.4 Modelling

J.4.1 Input data pre-processing

The input data are geometric, physical, and simple statistical variables used as a structure for predicting material loss during experimental flooding. Extraction of the information involves a conversion from the 16 bit grey-scale sets 2D slices of pre-experimental scan to normalised 1D variables that can be evaluated by the ANN.

Formatting

A VBA batch coupled with Corel® X7 suite is used as a quick way to apply the same formatting to each 2D slice. The batch automatically and sequentially thresholds, smooths, vectorises, and resizes each image to its original size. Finally, it saves the new set of 2D binary slices under the SVG format.

Analysis

The analysis of the formatted 2D slices of input data is performed by a bespoke software written in C++ (ICCR-Macropore). For each formatted 2D slice, a set of nineteen relevant variables is collected through an in-depth image analysis:

1. Total pore area
2. Area of connected pores
3. Total Length connection
4. Median Length connection
5. Mean Length connection
6. Median pore area

7. Mean pore area	14. Number of pore larger than the mean size
8. Ratio of pore area	
9. Number of pores	15. Number of pore larger than the median size
10. Mean pore perimeter	16. Mean ellipse aspect ratio
11. Small ellipse perimeter	17. Mean distance between pores
12. Large ellipse perimeter	18. Pore Volume rate
13. Porosity	19. I/O connection

Most variables can be easily explained through image interpretation and basic mathematics (1, 6, 7, 9, 10, 13, 14, 15, 17), as well as variable 18 (PV_{rate}) which is calculated from equation J.1. The remaining variables (2, 3, 4, 5, 8, 11, 12, 16, 19) have been calculated through the use of functions developed for the ICCR-Macropore and using the ellipse shapes of each pore and the pore network (described below). The ICCR-Macropore has been developed as a 2D interpreter of pore network. This network, generated for each slice scanned, relies on the arrangement and overlap state of the 2D porosity: Each pore of a 2D slice is replaced by an ellipse of equivalent area, shape, and orientation. These ellipses are then enlarged by a factor controlled by the user, and set constant for the entire sample analysis. This factor, called the *area of influence*, is a computational way of representing the hydrogeological influence of a pore around its neighbourhood. Figure J.2 presents a simplified workflow for pore connection detection:

- The step (A) represents the area of influence applied around a pore through a multiplier of the original pore area.
- Case (B) shows two non-overlapping pores.

- Case (C) displays a case of overlapping ellipses.

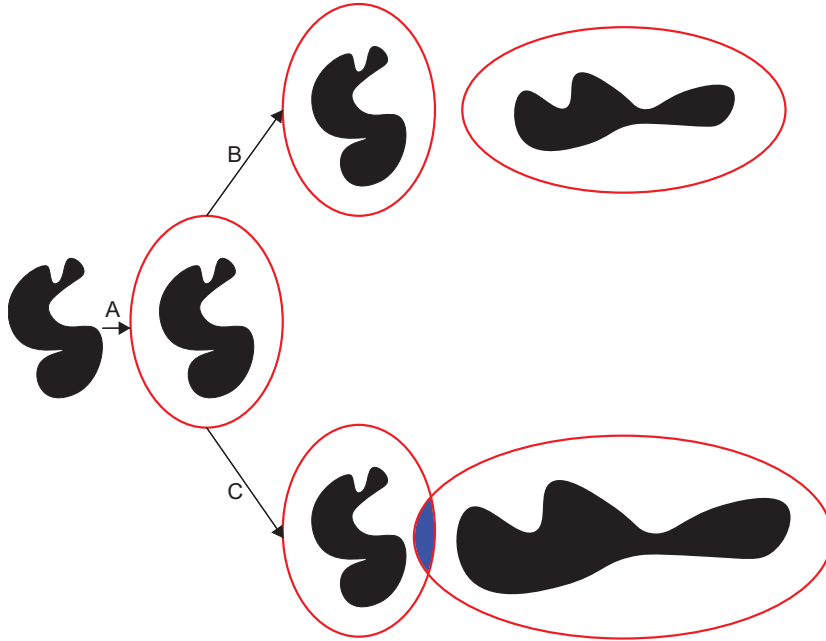


Figure J.2: Steps for the detection of a link between two pores (black shapes).

The overlapping function uses the geometrical attributes of an ellipse: Knowing that the sum of the distances between the foci $F_{1,2}$ and any point of the ellipse E_1 is a constant C_1 , here we assume that if the distance between foci of E_1 and any point of a second ellipse E_2 is less than C_1 , then E_2 overlaps E_1 , leading to the creation of a link between those two pores. A link is a 2D representation of a potential connection in 3D, while the 2D arrangement of a set of links symbolises a pore network. The analysis of this network allows the calculation of the variables 2, 3, 4, 5, and 19, while the analysis of the ellipses sets the variables 8, 11, 12, and 16.

Pore network analysis (variables 2, 3, 4, 5, 19)

Variables 2, 3, 4, 5, and 18 are determined as follow:

- Area of connected pores is the sum of the area of the pores which are part of a connected network.
- Total/ Median/ Mean length connection are basic mathematical calculations using the absolute length of every link from a 2D slice.
- I/O connection is a variable which is not directly used by the ANN, and further helps in determining the spatial location of a channel. A recursive function analyses the 2D network of links, and sets a Boolean to true if at least one path between bottom to top of the image is found; the Boolean is otherwise set to false.

Ellipse analysis (variables 8, 11, 12, 16)

The ellipse shape of a pore can resolve the following variables:

- Ratio of pore area represents the ratio between the area of the largest pore over the mean pore area of a 2D slice. This variable is used for excluding outliers.
- Small/ Large ellipse perimeter is a simple calculation of both the ellipse shape of a pore and its enlarged version.
- The mean aspect ratio is represented by the ratio of the minor axis b over the major axis a of an ellipse. All the ratios gathered per slice are averaged.

J.4.2 Signature data pre-processing

The signature data refers to the solution of wormhole magnitude and location. This signal is computed from the differential result between the pre- and post-experimental scans.

Formatting

The dissolution paths are isolated and sub-sampled using Avizo[®] 9. The cuboids are re-sliced and thresholded using Fiji (Schindelin *et al.*, 2012). The percentage of black and white area is calculated for each 2D slice using a batch based on the measure function under Fiji (Schindelin *et al.*, 2012), and is used as the true solution of the wormhole shape and size for a direction. The signature data is finally normalised so that the maximum percentage area of white equals 1 (presence of dissolution channel) while the minimum is equal or close to 0 (no dissolution detected).

J.4.3 Regression and neural network modelling

Neural networks are effective at fitting complex behaviours in data but their typical, inherent non-linearity makes the physical interpretation of their weights difficult or intractable in many cases. In this work we train both a linear regression and multiple hidden layer neural network on six datasets, corresponding to different experimental regimes as explained in section J.3.4. With this network trained, we then use it to predict the remaining two, blind datasets' spatial wormhole signatures, which correspond to the remaining experimental regime. For the four combinations of three training experimental regimes (6 datasets) and one blind test experimental regime (2 datasets), we performed a linear regression where eighteen normalised features were input and fitted to minimise the misfit when compared to the measured spatial wormhole signature extracted from before and after CT scans, as described in section J.4. We did not perform any regression or model training using the I/O feature, which was held aside for comparison, as seen in section J.4.1. The I/O features have not been used as an input data as these values have been pre-treated by a separate function and could have

biased the results. Instead, they are used as an alternative tool for wormhole detection and validation. After linear regression was parameterised, we performed modelling by artificial neural network. We designed a neural network using the MATLAB Deep Learning Toolbox, with three hidden layers, consisting of 11, 8, and 5 neurons in each layer between input and output (Hudson Beale *et al.*, 2018). All eighteen features were normalised, as discussed in section J.4.1, before input into the network. All transfer functions between the input and all hidden layers were hyperbolic tangent functions. The transfer function between the last hidden layer and the output layer was a linear function. Our experimental aim was to train our models on three experimental regimes (6 datasets), and predict wormhole formation on a fourth experimental regime (2 datasets). The blind test data we held aside in our neural network development was simply the two datasets out of 8 to be predicted. We used MATLAB's implementation of the Levenberg-Marquardt algorithm (Moré, 1978) to train our networks.

We randomly partitioned the data and features from the six training datasets into 74 % training data and 26 % validation data. We trained a neural network given this random partitioning of training and validation data, and forward-modelled the spatial wormhole signature on the two blind datasets, given this trained network. This workflow has been repeated 3000 times independently, each time training a new network given a different random partitioning of training and validation data from the same six datasets, and produced 3000 predictions of the two blind datasets' spatial wormhole signatures. These 3000 predictions have been made into a density plot which shows the most likely spatial wormhole signature, as well as the sensitivity of the network to different parameterisation conditions. We performed this workflow for all four combinations of three training experimental regimes (six datasets) and one prediction experimental regime (two datasets), allowing us to simulate four, independent experiments.

J.5 Results

J.5.1 Permeability and calcium evolution

This work focuses on the main controls behind each experimental result with regard to the evolution of the pore system's size, connectivity, and spread. A detailed temporal evolution of the rock matrix is assessed through calcium concentration analysis and permeability logging. Our ΔC_{Ca} displays positive values throughout each experiment ($\Delta C_{Ca} > 0$), implying that calcium removal occurred during flooding phases (Luquot *et al.*, 2016). This idea is reinforced by the pH measurements whose outlet values are consistently higher than the inlet ones.

The four core samples have been flooded at different pore volume rates and, therefore, at different injection rates (section J.3.4; table J.1). Figure J.3 shows the evolution of the permeability (J.3a) and calcium (J.3b) for the four experiments. The dimensionless number Ω in Figure J.3b represents the ratio of volume of calcium loss over the original volume of solid of the rock. We observe that both parameters are showing positive increase for all four core samples, which physically translates to the formation of preferential flow-paths.

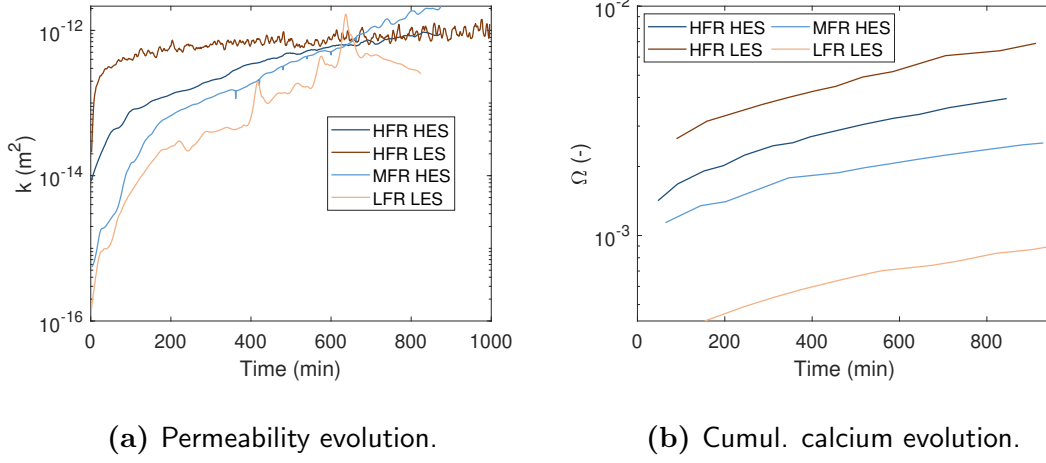


Figure J.3: Evolution of the permeability and calcium parameters over time. The alternation of curves correlates with the injection rates.

J.5.2 XMT outputs

Post-experimental results

Our experimental rig, as well as the experimental matrix do not enable us to acquire a temporal evolution of the core samples; instead we obtain the absolute difference in 3D volume of missing material by comparing the pre- and post-X-ray CT volumes. No porosity value has been calculated from CT-scan analysis as we focus this study on the detection and prediction of volume creation from flow through. The post-experimental networks, extracted through Avizo[®] 9 and using a label analysis parameter of 24 connections detection between voxels, was able to capture most of the connected voxels. We observe that as we increase the injection rate, we also increase the ramification of the system, while at lower injection rates we stay relatively far from a conical channel or face dissolution system, probably due to the weak acidity of our injection brine. Figures J.4, J.5, J.6, and J.7 present

the signal data plotted next to their respective 2D cross-section slice, from two orthogonal directions and for each of the four samples. The cross-plotting of the thresholded percentage area of black to white of a channel offers a good insight to the location, the spread, and the magnitude of the created pore space. The values of the signature data are cross-normalised between datasets.

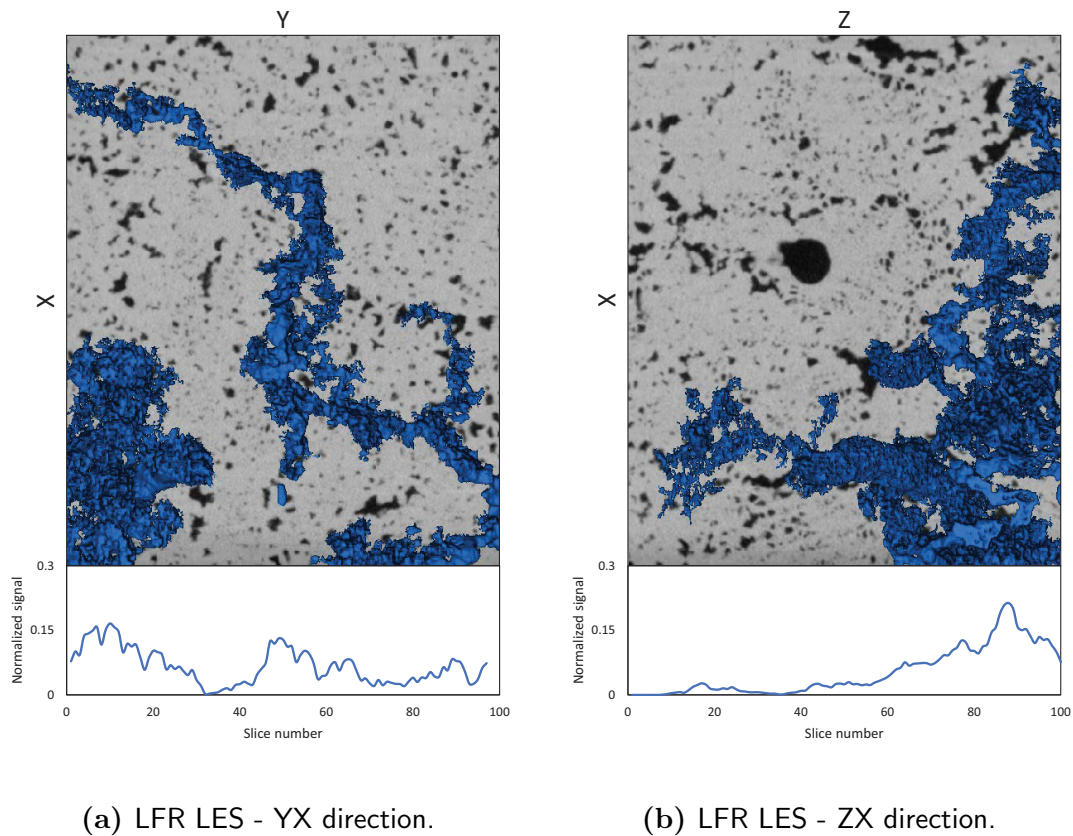


Figure J.4: Signal data plotted under their respective cross-sectional direction for the *LFR LES* experiment.

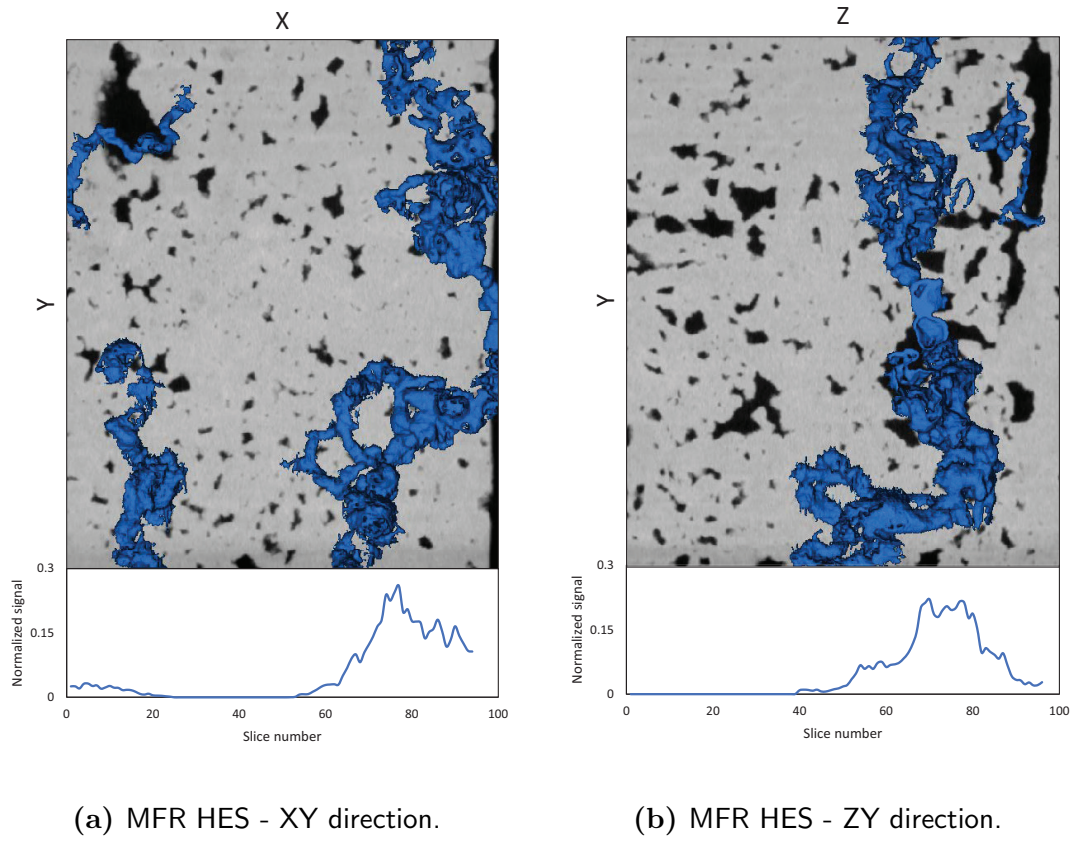
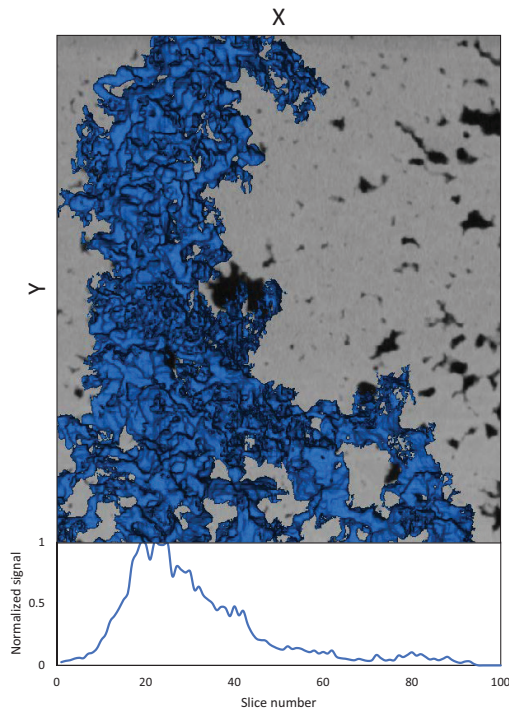
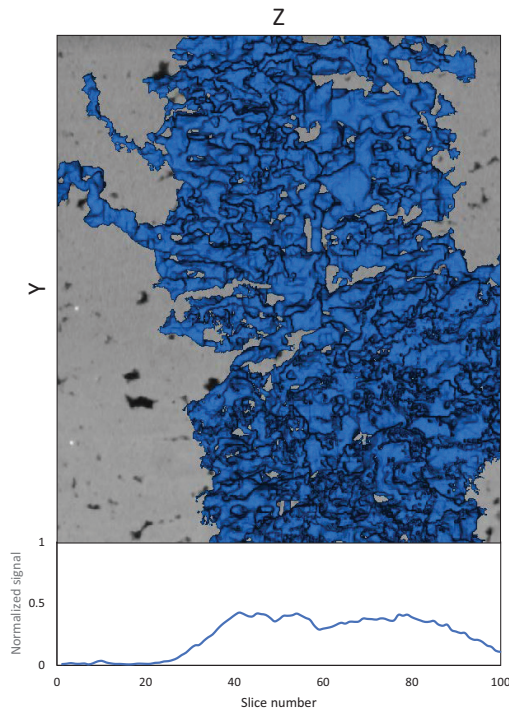


Figure J.5: Signal data plotted under their respective cross-sectional direction for the *MFR HES* experiment.

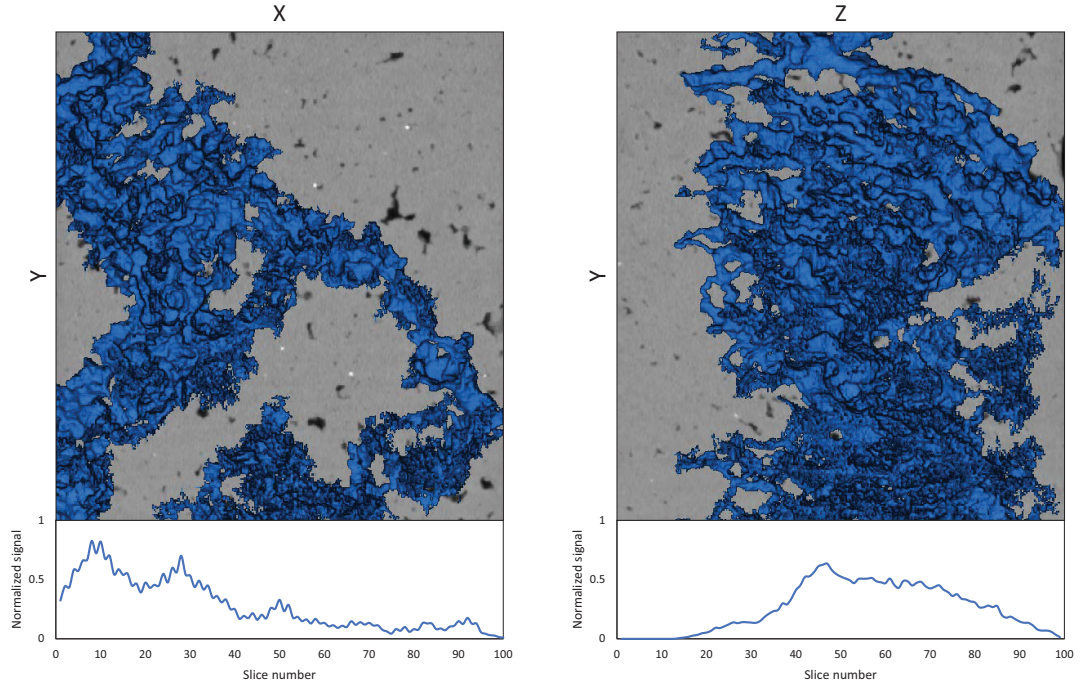


(a) HFR LES - XY direction.



(b) HFR LES - ZY direction.

Figure J.6: Signal data plotted under their respective cross-sectional direction for the *HFR LES* experiment.



(a) HFR HES - XY direction.

(b) HFR HES - ZY direction.

Figure J.7: Signal data plotted under their respective cross-sectional direction for the *HFR HES* experiment.

J.5.3 ANN outputs

The predicted spatial wormhole signatures from the fitted linear regression models, as seen in Figures J.9, J.10, J.11, and J.12, are displayed as white curves, with the signatures measured from CT scans displayed as red curves. It is clear that the linear regression model is generally ineffective at predicting spatial wormhole signatures. In Figure J.8, we show the linear regression weights for all features, as numbered in section J.4, over all four training scenarios. The last feature shown (feature 19), is the size of the constant term or bias in the linear regression. As all features are normalised before performing linear

regression, we propose that features with small magnitude weights are related to physical attributes which have little effect on rock's wormhole formation. By this reasoning, we interpret that features 2, 4, 6, 8, 16, and 18 all correspond to physical properties which have little influence on a rock's wormhole formation. These features are the area of connected pores, median length of connections, the median pore area, ratio of pore area, the mean ellipse aspect ratio, and the mean distance between pores repetitively. By the same reasoning as above, we interpret features with larger weights as proxies for the rocks' physical attributes which exert a stronger influence on wormhole formation, to first order. These features include numbers 9 to 12, which correspond to number of pores, mean pore perimeter, small ellipse perimeter, and large ellipse perimeter respectively. Of course, this reasoning of inferring feature influence on wormhole formation from linear regression weights is flawed as normalised features with large outliers may require large scalings to minimise their fitting residuals. Also, as we see in many cases, linear regression is in fact not a good predictor of wormhole formation on blind test data. Nevertheless, this analysis gives us a crude, qualitative first estimate of which features may or may not be important in wormhole formation prediction processes within heterogeneous rocks featuring macropores.

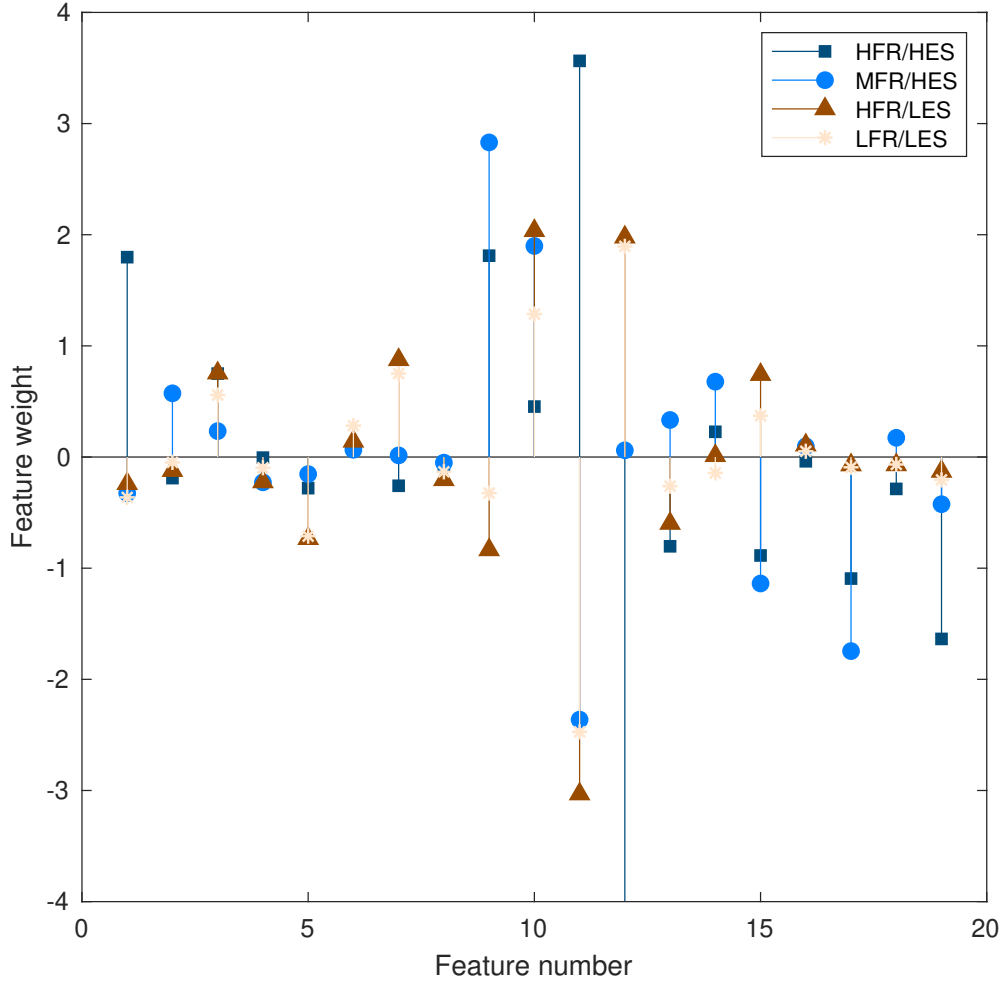


Figure J.8: Linear regression weights for all features, as numbered in Section J.4 and trained to predict four different blind test experimental regimes (2 datasets). The outlying weight on feature 12 is -6.22. The last weight is the linear regression's constant or bias.

The background density plots in Figures J.9, J.10, J.11, and J.12 show the distribution of blind predictions generated by the 3000 neural network simulations, given each combination of three training experimental regimes (six datasets) and one blind test experimental regime (two datasets). Intensity ranges from blue (very low confidence) to yellow (very high confidence), and the 'true' solutions

(signal data) have been plotted in red on top of each result. We see a generally effective prediction of wormhole location and magnitude by our neural network modelling approach for all four training and prediction experiments. There is a notable improvement on prediction accuracy over the linear regression modelling. The spread in the density plot at locations of high spatial wormhole signature indicates the sensitivity of neural network training to the particular, random segmentation of validation and training data. In fact, we designed our statistical approach of training and predicting networks multiple times to allow both the prediction of data and to obtain an understanding of modelling sensitivity in locations of interest. The red diamonds on Figures J.9, J.10, J.11, and J.12 indicate where the Boolean feature, I/O, is true, which indicates whether there is an existing pore space connection from sample input to output in the pre-flooded rock. This single feature is an effective predictor breakthrough identification. This implies wormholes are likely to occur where there is a pre-existing input-output connection in the rock before flooding. We note this feature only predicts the likelihood and location of breakthrough, rather than the magnitude of expected rock alteration. For this reason, the use of our neural network method is justified.

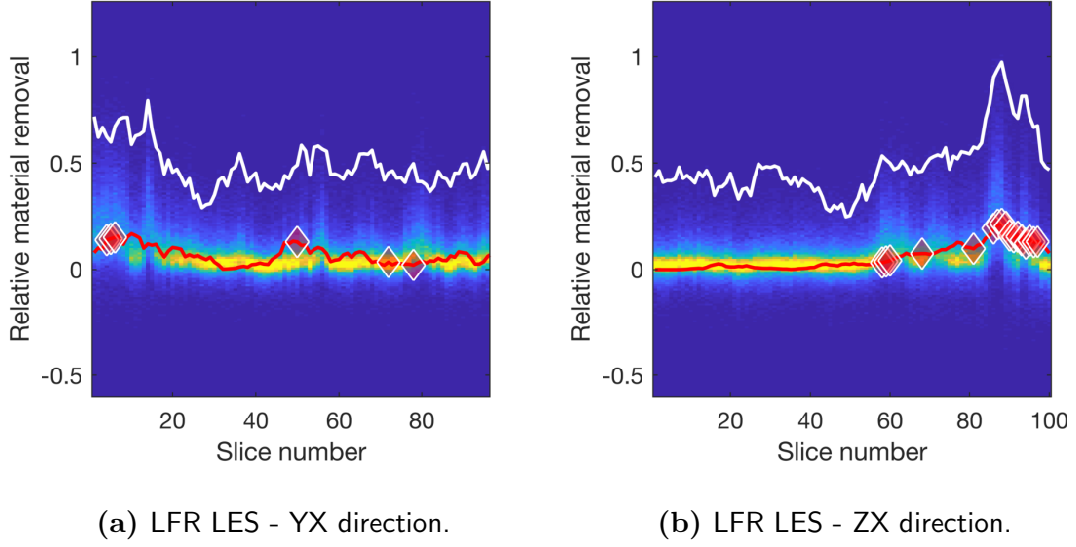


Figure J.9: Measured data (red) overlain on density plot of 3000 trained neural networks' blind test predictions for the *LFR LES* experiment.

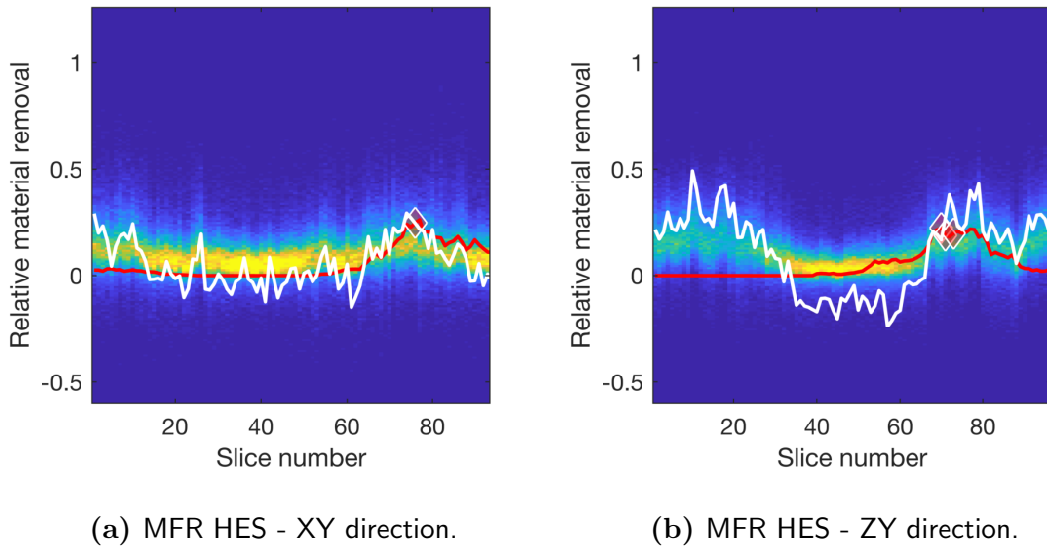


Figure J.10: Measured data (red) overlain on density plot of 3000 trained neural networks' blind test predictions for the *MFR HES* experiment.

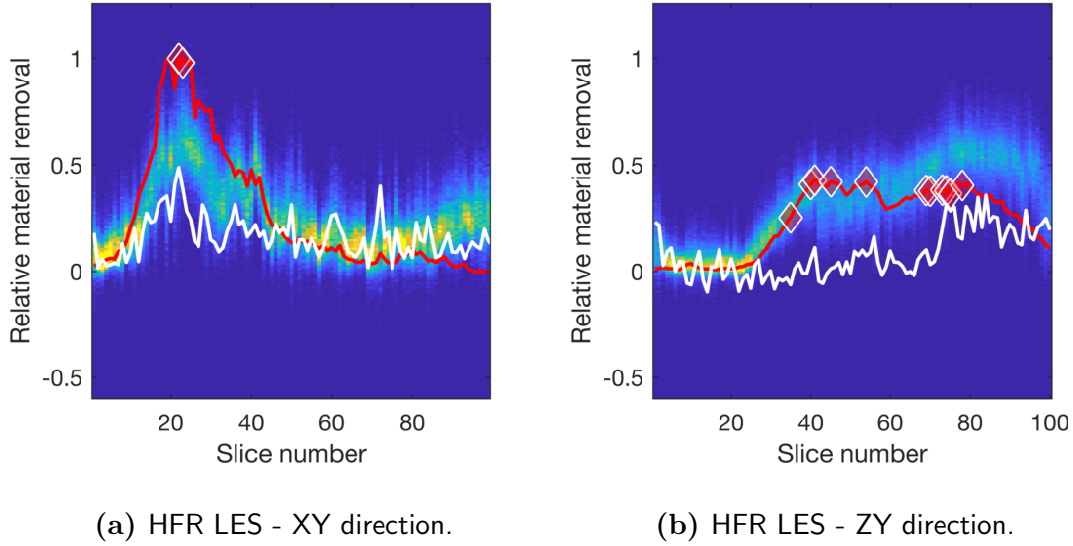


Figure J.11: Measured data (red) overlain on density plot of 3000 trained neural networks' blind test predictions for the *HFR LES* experiment.

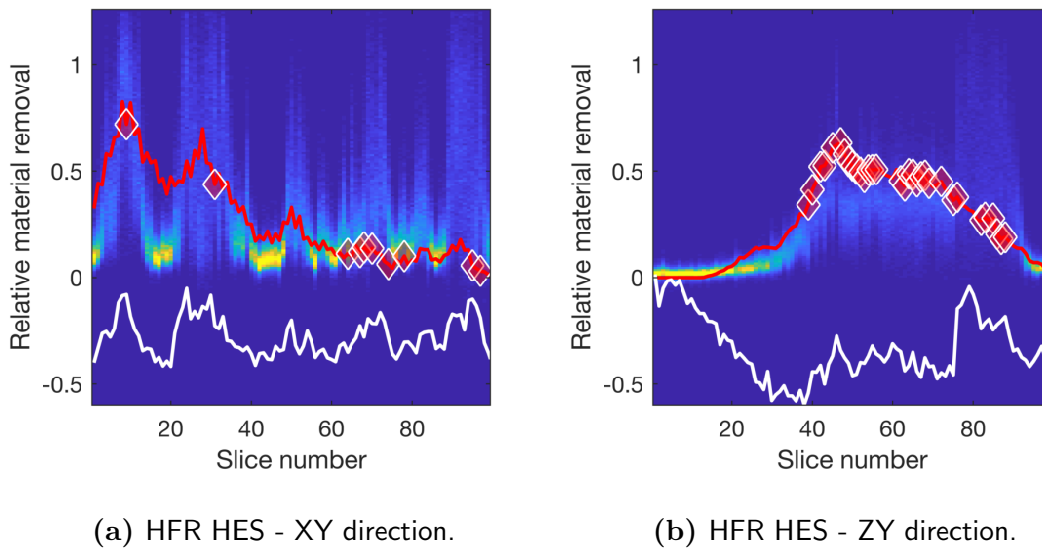


Figure J.12: Measured data (red) overlain on density plot of 3000 trained neural networks' blind test predictions for the *HFR HES* experiment.

Overall, the neural network method detects the location and extent of material removal under different flow rates, which we summarise as follows:

- *LFR LES*:
 - Figure J.9a: The main features are detected. The ANN has generated few peaks which are not featured by the solution, but it successfully displays them with a lower confidence.
 - Figure J.9b: The main area of interest (right) appears as one peak in this direction, and is clearly detected by the ANN. While the magnitude seems slightly uncertain, the location is precisely determined.
- *MFR HES*
 - Figure J.10a: The small amount of missing material shown by the red curve (left side, low magnitude) is resolved with a perceptible offset in the output magnitude. With high confidence, the main channel is detected on the right side. Its magnitude, even if close to the magnitude of the signal data, remains marginally lower.
 - Figure J.10b: The two main pore networks are easily identifiable (left and right sides). The dissolution channel (red curve, right side) is detected. The ANN shows quality in decision making, as it also offers the left side as a potential site for channel formation, but seems to display a lower - nonetheless still rather high - confidence.
- *HFR LES*
 - Figure J.11a: Overall, the main shape of the signal data is well resolved. The inaccuracy comes from the magnitude of the main channel which is not perfectly detected.
 - Figure J.11b: This cross sectional analysis displays the same issues as the cross-section J.12b: The spread and magnitude of the signal data

are accurately resolved, although the ANN does not properly determine its exact magnitude.

- *HFR HES*

- Figure J.12a: Input data for this cross-section are extremely scattered. Here the ANN remains doubtful across the sample. Nevertheless, and despite solving with highly distributed data, the ANN manages to find the two main peaks successfully (left side), as well as the general trend of the signal data, which presents a decrease from left to right.
- Figure J.12b: The spread of the main flow-path is clearly found (X axis) while the confidence of the ANN decreases as the intensity of missing rock increases. This is attributed to the fact that, even if the ANN catches the location and spread of the channel, the exact magnitude might be a challenge to resolve as the exact magnitude of a larger and more ramified channel might be linked to variables not yet accounted for by the ANN.

J.6 Conclusions

This study offers a new way of coupling Artificial Neural Network and CT images. A limited number of studies had already successfully linked these two tools as a segmentation method (Cortina-Januchs *et al.*, 2011; Chauhan *et al.*, 2016), and for rock modulus estimation (Sonmez *et al.*, 2006), but none used the non-linear solving aspect for dissolution prediction. Through this study, we have experimentally flooded four rock core samples and our results show that we have positively enhanced the pore connectivity, the porosity, and the permeability recovery through brine injection (Figure J.3). While it is commonly stated that the velocity of flow at the inlet of a core sample is the main factor for rock erosion

(Wang *et al.*, 1993; Frick *et al.*, 1994a; Bazin *et al.*, 1995), and our results do confirm this, this is only part of the story. We believe that the findings of our work show that spatial distribution of the permeability and porosity evolution can be predicted from the pore network information held by the rock sample; where the micro or macro heterogeneities of the porous medium drives the flow instabilities to direct flow and, as such, chemical removal, erosion towards zones of highest permeability (Hoefner and Fogler, 1988; Fredd and Fogler, 1998; Siddiqui *et al.*, 2006; Kalia and Balakotaiah, 2009; Izgec *et al.*, 2010) eventually leading to material loss. Our early results show that specific variables (Figure J.8) stand out of the ANN analysis, and validate that geometric factors linked to the porosity and pore shape of a rock contain, most of the time, the necessary data for predicting material loss during rock-water flooding (Figures J.9, J.10, J.11, and J.12 - red lines). If a linear combination of these CT-extracted attributes can successfully predict a rock's spatial wormhole signature, the weights from the linear regression could be considered indicators of the influence of CT-extracted feature in wormhole formation. Moreover, if a misfitted linear regression is unsuccessful in predicting wormhole signatures on a blind dataset, a more complex modelling tool, such as neural networks, is required to predict wormhole formation behaviour (Figures J.9, J.10, J.11, and J.12 - white lines). While this statement is valid in heterogeneous travertine rocks, we remain careful with other types of carbonate, or even other types of lithology which have not been tested in this study.

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