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THE SCATTERING AND FILTERING
of
HETEROGENEOUS X-RAYS
by
MATTER OF SMALL ATOMIC WEIGHT.

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PART I. THEORY OF EXPERIMENTS.

HISTORICAL INTRODUCTION.

After the discovery of X-rays by Rontgen[1] in 1895, the main properties of the rays were at once established, namely their power (a) to cause fluorescence, (b) to affect a photographic plate, (c) to ionize gases, (d) to penetrate, in varying degrees, matter placed in their path and (e) to cause such matter to become a source of secondary X-radiation. The problem of establishing the nature of the rays was not, however, an easy one. In the light of the physical knowledge of the period, and the earliest known properties of X-rays, these rays could conceivably have had either a corpuscular or a wave nature. That they would at a later period be explained partly in terms of corpuscular concepts and partly in terms of wave concepts could not at that time have been anticipated.

In 1897 G. Stokes[2] advanced the view that X-rays were thin pulses of electromagnetic radiation and were thus akin to light. J.J. Thomson, whose important researches on ions threw much light on the subject, laid the foundation for future work by giving mathematical form to the electromagnetic pulse theory[3], showing that the abrupt stoppage of a rapidly moving charged/

charged particle, (such as the cathode rays were known to be), would give rise to an electromagnetic pulse of definite energy. A much fuller treatment, given by him in 1903,[4] extended the theory to include the scattering of X-rays and the absorption of energy from X-rays by charged particles. The rate of energy flow per unit area for rays scattered from such a pulse by an ion was given as

$$\frac{1}{4\pi} \cdot \frac{e^2 f^2 \sin^2\theta}{V.\ OP^2} \quad - - - - - - - \quad (1)$$

where V is the velocity of light, OP the distance from the scattering ion, f the acceleration of the ion, e its charge in $\varepsilon.m.u.$ and θ the angle between the directions of the acceleration and the scattered ray. The total energy radiated per unit volume of scatterer was, for an assumed form of pulse, shown to be

$$\frac{4\pi}{3} \cdot \frac{n e^4 E}{m^2} \quad - - - - - - \quad (2)$$

where n is the number of ions in unit volume of scatterer, each of charge e and mass m, and where E is the energy in the primary pulse per unit area.

An equation for the loss of energy from the pulse on account of absorption of energy by charged particles was developed for an assumed form of pulse.

$$\frac{dE}{dx} = -hE \quad - - - - - - - \quad (3)$$

where x is thickness of absorbing material and h depends/

depends on the properties of this material and the character of the pulse.

Expression (2) shows the scattered energy to be independent of the pulse thickness (or quality) of the primary radiation, a condition frequently referred to subsequently as "perfect" scattering. In (3) h is shown to increase as the pulse thickness increases. Different degrees of penetrating power or hardness of X-rays are thus accounted for.

The wave properties of X-rays were first convincingly shown by C.G. Barkla in the polarization experiments of 1906[5]. The quality of the secondary rays when scattered by light elements had already been found by him to be closely similar to that of the primary rays.[6] The angular distribution of scattered rays from soft primary beams was shown by him to follow over a wide range of angle a ($1 + \cos^2\theta$) law obtained by a development of expression (1)[7]. In general, so long as the scattering material was of small atomic weight and the penetrating power of the radiation not too great, very satisfactory agreement was obtained by Barkla with the Stokes-Thomson theory of X-rays, and this theory, in consequence, gathered prestige. Barkla's early and fundamental experiments on total energy of scattered radiation indicated that the scattering ions were in all probability particles having/

having the charge to mass ratio of the cathode ray particles, later called electrons.

Other experiments provided evidence of a different kind of secondary radiation. It was observed that when the scattering material was of medium or ~~light~~ atomic high weight, the secondary radiations were usually and sometimes very markedly less penetrating than the primary beams by which the secondaries were excited[9]. The effect was described as a transformation of the primary radiant energy and was successfully interpreted as the X-ray counterpart of optical fluorescence. The work of Barkla (1906) and of Barkla and Sadler (1907 and 1908) showed that the transformation was due to the excitation of homogeneous radiations characteristic of the scattering material, and that the complete secondary radiations under these conditions consisted partly of the characteristic radiations and partly of scattered radiation conforming closely to the predictions of Thomson's theory. The two types of secondary radiation have sharply contrasted properties. The chief of these are given here in a comparative table.

SECONDARY RADIATION FROM ELEMENTS OF LIGHT ATOMIC WEIGHT.	SECONDARY RADIATION FROM ELEMENTS OF MEDIUM OR HEAVY ATOMIC WEIGHT.
(1) Penetrating power exactly or nearly the same as that of the exciting primary.	(1) Penetrating power different and usually much less penetrating than that of the exciting primary beam.
(2) Quality that of the primary, i.e. heterogeneous.	(2) Quality remarkably homogeneous.
(3) Total energy proportional to energy of primary and independent of penetrating power of primary, within wide limits. Total energy given by expression (2)	(3) Total intensity dependent on the penetrating power of the primary. Emission conditional on higher penetrating power of primary, following Stokes' Law.
(4) Intensity varying with azimuth. For soft radiations agreeing well with expression (1)	(4) Uniform intensity at all angles.
(5) Varying in intensity round a polarized beam.	(5) No variation in intensity round a polarized beam.

Distinction between the two types of secondary radiation was therefore in theory at least an easy matter.

The characteristic radiations varied in a serial manner with the atomic weight of the scattering substance, becoming more penetrating as the atomic weight of the scatterer increased. They were shown by Barkla to fall into two series, finally named the K and L series. For any element, the former was more penetrating than the latter. The K , L nomenclature was/

was advocated by Barkla as preferable to the original symbols, "as it is highly probable that series of radiations both more absorbable and more penetrating exist".[10] Strong indirect evidence of the existence of an M series less penetrating than the L series had already been observed. The position with regard to a J series more penetrating than the K series was somewhat different.

The hypothesis that a J series of radiations existed was of particular interest, for such a hypothesis would explain one persistent feature of the observations which had been noted even in the earliest papers. The Thomson theory predicted 'perfect' scattering of the primary beam, and without question, the secondary radiation from light elements was of a quality so closely dependent on the quality of the primary as to be regarded as 'perfect' to a first approximation. Nevertheless, careful comparison of the penetrating powers of primary beams and of secondary beams derived from these primaries seemed to indicate that the secondary was slightly the less penetrating of the two. Such results might be consistent with the Thomson theory, it was thought, if the presence in the secondary beam of a small proportion of characteristic radiation belonging to a J series could be established. The fact that the discrepancy in the

penetrating/

powers of primary beams and of secondary beams derived from these primaries seemed to indicate that the secondary was slightly the less penetrating of the two. Such results might be consistent with the Thomson theory, it was thought, if the presence in the secondary beam of a small proportion of characteristic radiation belonging to a J series ~~could be established. The fact that the discrepancy in the~~ penetrating power increased perceptibly, and the detectable primary polarization diminished, when the primary radiation was made more penetrating was also consistent with this hypothesis. There were, however, alternative hypotheses, namely, (i) that the softening was a spurious effect, introduced in some way by the conditions of the experiments, e.g. by tertiary rays, or (ii) that the scattering was actually less 'perfect' for harder rays. The discussion of this point begins in Barkla and Sadler's paper of 1908.

The suggestion that some transf~~erence~~ormation of radiant energy took place in scattering from ~~light elements~~ ~~air~~ was first made by Sagnac[11] in 1897, and by Townshend[12] in 1899. References to the question occur in papers by Barkla from 1904 onwards, but a thorough experimental investigation was not embarked upon by him until after the work on characteristic and corpuscular radiations had been brought to fruit. Certain results published by Barkla in/

in the Bakerian Lecture[13] (1917) focussed attention once more on the problem of the existence of J characteristic radiations. Observations of the ratio of the energy of K corpuscular radiation to the energy of K primary absorption showed first a rapid increase, and then a sudden diminution, as the wave length of the primary was decreased. The dependence on wave length was in itself strongly suggestive of the influence of a characteristic radiation, and the appearance of the graphs was equally so. Confirmatory observations[14] were obtained in experiments on the relative ionization of ethyl bromide and air, and in a number of other similar experiments. All indicated that the effect occurred in air. Professor Barkla was naturally persuaded that a new more penetrating series of radiations had been discovered, and expressed himself fairly definitely on the point in the Bakerian Lecture.

Meanwhile research in collateral fields of investigation began to throw light on the problem. The α - particle investigations of Lord Rutherford[15], and the attempt made by Bohr[16] to interpret atomic line spectra, led to the formulation of the very successful Bohr-Rutherford theory of the atom. Upon this theory, characteristic radiations arise from the emission of energy by electrons 'falling' within the atom from a higher to a lower energy level. The frequency of the emission/

emission was related by Bohr to the amount of energy released, thus: $h\nu = E_n - E_m$ in which **h** is Plank's constant, ν the emitted frequency, E_n the energy of the electron before its 'fall', and E_m the energy of the electron after its 'fall'. To account for the line spectra, Bohr introduced a quantization of electron orbits by equating the angular momentum of the electron about the nucleus to an integral multiple of $h/2\pi$, and thus derived the equations:-

$$r_n = \frac{n^2 h^2}{4\pi^2 Z e^2 m}$$

$$E_n = \frac{2\pi^2 e^4 Z^2 m}{n^2 h^2}$$

where e and m are the charge and mass of the electron, Z the nuclear charge in electron units, n the integral multiplier, r_n the radius of the electron orbit (assumed to be circular), and E_n the energy of the electron in this orbit. From these relations it follows that the maximum frequency is emitted when the energy release, $E_n - E_m$ is a maximum and that this condition is realised only if an electron 'fall' to the lowest energy level, that is to the orbit of $n = 1$ and of smallest radius. From this theory a value can be deduced for the Rydberg constant. The value so obtained agrees remarkably well with the value given by the optical line spectra of hydrogen and helium. By applying the same theory to the heavier atoms, excellent agreement is obtained with the/

the refined X-ray spectroscopic observations of Moseley[17] for the K series of characteristic radiations, provided that E_m be taken as the electron energy for the innermost orbit. If the implications of this theory are true and the K series of characteristic radiations are emitted by the 'fall' of 'planetary' electrons to the innermost orbit, no characteristic radiations of higher frequency can be accounted for by this theory. ~~and the possibility of~~ ~~series.~~ ~~a J series is thus ruled out~~. The Bohr-Rutherford theory of the atom has proved to be an entirely sound step in the development of atomic theory, and the passage of time has only confirmed more definitely the predictions based upon it regarding J radiations.

Nevertheless, the anomalies observed by Professor Barkla continued to be seen,[18] although it soon became evident that "with all the similarities to the production of X-ray fluorescence, striking differences have appeared". In particular, ~~The chief difference was that~~, under conditions which proved difficult to define, a sudden apparent increase in absorbability was produced in heterogeneous beams by filtering to a sufficient extent. The effect therefore came to be known as the J phenomenon, or as the J absorption discontinuity.

Of all the various experiments used by Professor Barkla to display and study the phenomenon, two ~~in particular~~ have been continuously employed over a long period. These are known as the scattering experiment

and/

and the filtering experiment. Both of these have given results different from what might have been expected. Since most of the present writer's work has been concerned with these two experiments, an account of our knowledge of scattering and filtering must now be given.

The theory of scattered X-rays.

J.J. Thomson's theory~~8~~, ~~7~~ as developed by Barkla treat~~ed~~s of the scattering of X-rays by free and independent electrons, and gives the intensity of scattering of an unpolarized incident beam. If the unpolarized incident beam has intensity I, then the intensity S due to scattering by a single electron, at distance r from the electron, and in a direction making angle ϕ with the direction of the primary beam, is given by

$$S = I\frac{e^4}{2r^2m^2c^4}(1+\cos^2\phi),$$

where e is the electronic charge in e.s.u., m is the mass of the electron, and c the velocity of light. When n independent electrons per cubic centimetre are scattering, the intensity becomes $S = I\frac{ne^4(1+\cos^2\phi)}{2r^2m^2c^4}$ per cc. of scatterer ~~per [illegible] scatterer~~

By integrating S over a closed surface surrounding the n electrons, the power P_S in the scattered radiation per cubic cm. of scatterer is evaluated as $P_s = I\frac{8\pi}{3}\frac{ne^4}{m^2c^4}$

Hence the scattering coefficient $\sigma = \frac{P_s}{I} = \frac{8\pi ne^4}{3m^2c^4}$

per ~~cubic~~ cm. of scattering material, a formula independent of wave length.

Now/

Now it appears that while the condition of freedom can be said to hold approximately for the orbital electrons of the atom*, the condition of independence is not so readily satisfied. For longer X-ray wave lengths the intra-atomic distances are comparable with the wave lengths and interference will take place between the scattered waves from neighbouring electrons. Indeed, should the wavelength be long by comparison with the diameter of the atom, the electrons will combine to scatter like a single charge of Ze, Z being the number of orbital electrons and e the electron charge. In this extreme case the intensity of scattering per atom will be Z times the intensity given by Z independent electrons. The well known fact that σ , the scattering coefficient, rises slightly with increasing wave length and more rapidly as the atomic number[19] of the scatterer increases has always been explained as a break down of the independence condition.

Just as the nature of an optical interference pattern can give information about the disposition of the sources of interfering light, so the distribution of scattered X-rays can give information about the disposition of scattering material,[20] i.e. electric charge, within the atom. This information appears in the form of a time average of charge density for a given distance from the atomic centre. By using refined observations of the scattering from different

materials/

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* I.e. in the early stages of the experimental work no discrepancy between theory and experiment called for a consideration of the influence of binding forces. ~~This point is not of much significance in the light of later theoretical development, since the quantum theory of X-ray scattering taking electron binding energy into consideration differs, on this account, from the classical theory by a factor which for long waves~~ is ~~practically unity.~~

materials, amongst which crystals and the inert gases are specially important, a great deal of information has been obtained about the extra-nuclear structure of atoms. In this way the 'three body problem', not to mention the many body problem, which twenty years ago seemed to block the study of atomic structure, has been short circuited.

But before this work was started, a new type of X-ray scattering known as incoherent scattering was discovered by A.H. Compton[21] in 1922. He observed that under spectroscopic examination the rays scattered from a monochromatic primary of wave length λ consist, in general, of two wave lengths, one, λ being the same as the primary wave length, and the other, λ' slightly longer. A theory of this scattering in terms of particle impacts and de Broglie wave mechanics was given by Compton in 1923. The change of wave length predicted was

$$\delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) = 0{\cdot}024(1 - \cos\phi)$$

where ϕ is the angle between the primary and the scattered rays, and the other symbols have their usual connotation. $\delta\lambda$ varies only with ϕ and is independent of the scattering material and of the primary wave length. The agreement between theory and experiment was excellent. (~~It may be noted that the slight departures from 'perfect' scattering may observed in early experiments may have been entirely due to the Compton effect~~.)

Under the stimulus of this discovery and in the light/

light of the new quantum mechanical and relativity conceptions, attempts were made to work out a more adequate theory of scattering. It had already been shown by Debye[22] (1916) and Thomson (1917) that the intensity of scattering from a group of electrons could be expressed as a function of $\frac{1}{\lambda}\sin\frac{\phi}{2}$. In 1927 Schrödinger[23] published a wave mechanical theory, based on de Broglie's[24] wave mechanics, for the scattering of X-rays from free electrons. This theory accounted for the production of incoherent scattering, the intensity of the incoherent scattered wave being $S = S_o(1+\alpha \,\mathrm{vers}\,\phi)^{-3} = S_o\left(\frac{\nu'}{\nu}\right)^3$ where S_o is the intensity of Thomson scattering, ϕ the angle between scattered and incident rays, and $\alpha = \frac{h\nu}{mc^2}$.

The incoherent scattered wave was shown to be plane polarised in the same plane as classically scattered waves. In 1929, using Dirac[25] wave mechanics, invariant with respect to Lorentz transformations, Klein and Nishina[26] arrived at the formula for the intensity of incoherent scattering:

$$S = S_o(1+\alpha\,\mathrm{vers}\,\phi)^{-3}\left[1 + \frac{\alpha^2\,\mathrm{vers}^2\phi}{(1+\cos^2\phi)(1+\alpha\,\mathrm{vers}\,\phi)}\right],$$

the additional term in this expression giving the intensity of an unpolarised ray.

Wentzel[27] and Sommerfeld[28] have carried the development of scattering theory to a more advanced stage by working out the case of scattering by electrons subject to binding forces. Their conclusions are most simply stated/

stated in terms of the electronic structure factor

$$f_{kk} = \iiint (\psi_k \psi_k^*) \cos (k_1 a \cos\alpha)\, d\tau ,$$

ψ_K being the wave function of order k, $\psi_K \psi_K^*$ the electric charge density distribution of order k, and $k_1 a \cos\alpha$ the phase corresponding to the volume element $d\tau$. Two summations of f_{KK} ~~for~~ over the Z electrons of the atom are required to express the scattering by atoms. The first of these is the atomic scattering factor F = $\left[\sum_Z f_{kk}\right]$, the second the incoherent scattering function $\sum_Z \left(1 - f_{KK}^2\right)$. Values for F have been computed by James and Brindlay[29] for most of the light atoms and ions. Values of $\sum_Z \left(f_{KK}^2\right)$ for the incoherent scattering function are given in Compton and Allison[30], page 782. The theory of Wenzel and Sommerfeld has been supplemented by Waller,[31] who develops an electron spin correction term, $-\sum\sum_{K \neq \ell} f_{K\ell}^2$, reducing slightly the modified intensity. The complete theory then gives the unmodified intensity S_U , and the modified intensity S_M as follows.

$$S_U = S_o F^2,$$

$$S_M = S_o (1 + \alpha \operatorname{vers} \phi)^{-3} \left\{ Z - \sum_Z f_{KK}^2 - \sum_{k \neq \ell}\sum f_{Kl}^2 + Z \frac{\alpha^2 \operatorname{vers}^2 \phi}{(1+\cos^2\phi)(1+\alpha \operatorname{vers} \phi)} \right\},$$

S_o being the Thomson scattering per electron.

These formulae agree with experiment as far as they have been tested. Observations of the scattering by the monatomic gases[32] agree with the predictions of total scattering (coherent and incoherent rays) given by the wave mechanics theory, assuming Hartree's model[33] of/

of the atoms. The coherent scattering alone, observed in crystal structure intensity measurements, also gives satisfactory agreement on the basis of Hartree's theory; and the diffuse scattering observed in crystal structure photographs has been shown by Wollan to agree with the predicted intensity of the incoherent rays. Polarization tests are satisfactory throughout the range of testing, but the 'hard' end polarization has not been tested so far. As to the intensity of the 'hard' end scattering, the results of Chao[34] and of Read and Lauritsen[35] are apparently in good agreement with the Klein Nishina formula.

Unfortunately, the data on F and on the incoherent scattering function are available only up to values of $\frac{1}{\lambda}\sin\frac{\phi}{2} = 1.1$. This gives information for scattering at 90° for wave lengths greater than 0.64 ÅU, and none for the rather important and difficult region $0.11 < \lambda < 0.64$ ÅU.

Calculated intensities of scattering per electron have been evaluated from the formula of Wentzel and Sommerfeld for the light scattering elements and compounds used in the present research, see Table VIII and Figure 30a. For reasons to be explained later (pages 18, 19) the recoil factor is treated as unity. Waller's term is not computed since its value is not known, but from data given by Waller in his paper, the term is not significant for wave lengths shorter than 1.2 Å.U. and may amount at most to a 10% reduction of the modified scattering of long waves. Here a 5% reduction at 1.6 Å.U. is taken as an ample allowance.

The predictions of theory as given by Table VIII for variation of intensities of 90° scattering with wave length are:

(1) for hydrogen, constant scattering ratio,

(2) for paraffin, carbon and filter paper, very slow increase of scattering as λ increases; all three giving the same scattering up to 1.2 Å.U. but diverging somewhat for longer wave lengths,

(3) for aluminium a more rapid increase of scattering as λ increases.

For heterogeneous beams the principle of superposition is applicable. This implies that when heterogeneous beams are scattered, the scattering of the whole beam should be the sum of the scattering of its homogeneous constituents. As penetrating constituents are removed/

removed from the beam the scattering should increase; as soft constituents are removed from it the scattering should diminish. The only factor not taken into account so far is the differential absorption of modified scattering (see p. 18a.), but this for thin scatterers should not be important.

The scattering experiment as performed by Barkla's research workers has been found to give under certain circumstances and especially but not exclusively with thin scatterers, what appears to be a constant scattering ratio over a wide range of penetrating powers and with very different beam constitutions. It has also been found to give what appears to be a decrease in scattering ratio as the primary beam is softened.

A discrepancy clearly exists between these interpretations of experimental facts and the predictions of theory. Other difficulties (to be described later) have arisen in absorption experiments. This situation has called forth the suggestion from Professor Barkla that the principle of superposition is not applicable to heterogeneous beams. The need for further investigation is clearly indicated. A study of the scattering experiment has therefore been undertaken by the writer, and forms the main part of the work reported in this thesis.

THEORY/

Theory of X-ray absorption.

Absorption by homogeneous beams.

The fundamental equation of X-ray absorption ~~is~~ ~~as originally derived by J.J. Thomson and experimentally substantiated,~~ ~~empirically derived and~~ gives the fractional loss of power when a monochromatic beam of wave length λ and power P_o passes normally through a sheet of matter of uniform thickness dx,

$$-\frac{dP}{P} = \mu dx,$$

where μ is a constant for a given absorbing material and/

and for the given wave length. Integrating this equation we obtain

$$P = P_o e^{-\mu x},$$

where P is the power after traversing thickness x of the absorber. This equation is usually applied in one of the two forms

$$P = P_o e^{-\frac{\mu}{\rho} \cdot \rho x}$$

$$\text{or} \quad \frac{\mu}{\rho} = \frac{1}{\rho x} \log_e \frac{P_o}{P},$$

for, the density ρ or the surface density ρx is easily determined, and μ/ρ the mass absorption coefficient is a constant for the chemical element used to absorb, and depends only on λ . This law was tested on homogeneous characteristic radiation by Barkla[36] and his collaborators in the early years of the century, and since then many further tests have been made on homogeneous beams given by single and double X-ray spectrometers.[37]

It is well known that the experimental conditions must be strictly controlled[38] if a unique value of μ/ρ is to be obtained for a homogeneous beam of definite wave length.

(1) The aperture system must be relatively small, for if the angle subtended at the ionisation chamber by the irradiated absorber is large, the measured intensities are increased by the scattered radiation received through this angle. The radiation will then appear 'harder' than if a narrower aperture system were employed.

(2)/

(2) The beam must be approximately parallel, otherwise the thickness of absorber traversed by the different rays is not uniform.

(3) The chemical purity of the absorber is important. This is specially true where aluminium (with a common impurity of iron) is used as the standard absorbing material.

The theory of the mass absorption coefficient, μ/ρ, is as follows ~~also empirical~~. Energy absorbed from a ~~the~~ primary monochromatic beam becomes (1) the energy absorbed by the photoelectric effect and used in ~~required for~~ the production of ions and of characteristic X-rays, called true absorption, and (2) the energy of the scattered X-rays. If μ is the fractional change of power per cm. of absorber traversed, σ the fraction of power scattered per unit thickness and τ the fraction of power used in ionising per unit thickness, then

$$\mu = \sigma + \tau$$

$$\text{or} \quad \mu/\rho = \frac{\sigma}{\rho} + \frac{\tau}{\rho}$$

M X

According to classical theory $\frac{\sigma}{\rho} = \frac{4}{3}\pi \frac{N}{m} \frac{e^4}{c^4}$ (e.s.u.) where N is Avogadro's number. Inserting known values of the constants, ~~Thomson's calculation~~ $\frac{\sigma}{\rho} = 0.2$. The experimental value varies, being less than 0.2 at the 'hard end' and increasing much above 0.2 for long waves and heavy atoms, where, in fact, the independence condition of classical ~~Thomson~~ scattering breaks down. If μ/ρ be plotted against λ, the form of the graph is approximately that of $\mu/\rho \propto \lambda^3$ [39]. The exact power can be obtained from a double logarithmic plot. At the K critical absorption wave length the value of μ/ρ drops suddenly on the long wave length side to a fraction of its value on/

still classical

on the short wave length side. The absorption jump ratio r is the ratio of $\frac{\mu}{\rho}$ on the short wave length side to $\frac{\mu}{\rho}$ on the long wave length side of the critical wave length. It gives a measure of the increase of true absorption on account of the K absorption. There are excellent tables of both $\frac{\mu}{\rho}$ and r in Compton and Allison pp. 799-802; 521-525; 528-529.

The approximately cubic relationship between $\frac{\mu}{\rho}$ and λ is of special interest in connection with intensity measurements of incoherent scattering. Let us consider a primary of intensity I and wave length λ, producing either a classical ~~standard wave~~ scattered ray of intensity S wave length λ or a modified ray of intensity $\left(\frac{\nu'}{\nu}\right)^3 . S$ wave length λ'. If the absorption of these two rays between scatterer and ionization chamber is negligible ~~T~~the intensity absorbed in an ionization chamber from ~~those scattered waves is~~ them is respectively $S(1 - e^{-\mu l})$ and $\left(\frac{\nu'}{\nu}\right)^3 . S . (1 - e^{-\mu' l})$*, μ and μ' being the linear absorption coefficients for the wave lengths λ and λ' in the ionized gas, l being the length of the ionization chamber. For wave lengths giving an appreciable intensity of modified scattering μ and μ' are small for the light gases. If moreover l is small (as it is in the chamber used here) the ratio of the absorptions of modified to classical scattered waves is

$$\left(\frac{\nu'}{\nu}\right)^3 \cdot \frac{(1 - e^{-\mu' l})}{(1 - e^{-\mu l})} \doteqdot \left(\frac{\nu'}{\nu}\right)^3 \frac{\mu'}{\mu} \doteqdot \left(\frac{\nu'}{\nu}\right)^3 \cdot \left(\frac{\lambda'}{\lambda}\right)^n$$

where n has a value of approximately 3. Thus the absorption ratio has a value approximately unity, and to/

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* Neglecting ionization chamber losses, which will be appreciably the same ~~for~~ fraction of both beams.

to a first approximation ~~ionization chamber measurements~~ classical and modified scattered radiations produce equal ionizations.* The scattered wave may then be treated as a ~~do not reveal the difference between modified and classical scattering.~~ pure classical scattered wave except when the absorption between scatterer and ionization chamber is sufficient to introduce measurably different reductions in the intensities of modified and unmodified rays. The latter condition will be referred to as differential absorption of modified rays.

Absorption of heterogeneous beams.

In the present work all the X-ray beams are heterogeneous. If in such a beam P_{o_λ} is the power per unit range of wave length of the radiation of wave length λ, the power per unit range of wave length P_λ of this constituent ~~then~~ after normal transmission through thickness x of absorbing matter is $P_{o_\lambda} e^{-\mu_\lambda x}$, where μ_λ is the linear absorption coefficient of the absorbing matter for wave length λ. The principle of superposition is now applied and the equation giving the power of the beam after transmission is

$$\int_\lambda P_\lambda \, d\lambda = \int_\lambda P_{o_\lambda} e^{-\mu_\lambda x} \, d\lambda .$$

The customary method of measuring the quality of heterogeneous beams is followed here. The beam is first measured in its ionization chamber against another beam as standard. Then the beam is intercepted by a thickness x of pure aluminium sufficient to halve the ionization. If i_λ is the ionization per unit power at wave length λ, the equation connecting intercepted and unintercepted ionizations is

$$\int_\lambda i_\lambda P_\lambda \, d\lambda = \int_\lambda i_\lambda P_{o_\lambda} e^{-\mu_\lambda x} \, d\lambda .$$

~~If~~ We write this

$$= e^{-\bar{\mu} x} \int_\lambda i_\lambda P_{o_\lambda} \, d\lambda ,$$

from which

$$\frac{\bar{\mu}}{\rho} = \frac{1}{\rho x} \log_e \frac{\int_\lambda i_\lambda P_{o_\lambda} \, d\lambda}{\int_\lambda i_\lambda P_\lambda \, d\lambda} ,$$

where/

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*Calculated deviations of the ratio from unity for SO_2 in the particular ionization chamber used here are given in Table IX b.

where ρ is the density of aluminium, we thus define a quantity $\bar{\mu}$, which depends, through P_{o_λ}, on the beam constitution, through i_λ on the measuring system, and on x. Hence we obtain an average mass absorption coefficient for the whole beam. Now i_λ is kept constant for the measuring system, and x is specified as that thickness which makes the ratio of integrated ionizations exactly 2. Then $\frac{\bar{\mu}}{\rho}$ has a unique value for any given beam constitution (although different beam constitutions may give the same value of $\frac{\bar{\mu}}{\rho}$). $\frac{\bar{\mu}}{\rho}$ is not however a fundamental quantity because it depends on the wave length sensitivity of the ionization chamber (which will be discussed later). In a practical determination of $\frac{\bar{\mu}}{\rho}$ the ratio of the ionizations may vary from 2, by 10 or 12 per cent. The error introduced on this account is always less than ~~1 per cent except for the 'softest' beams~~ the error of observation.

According to the above theory of absorption of X-ray beams by matter, the intensity of a homogeneous beam falls exponentially with increasing thickness of absorbing matter, and the intensity of a heterogeneous beam falls according to the sum of the exponentially decreasing intensities of the homogeneous constituents into which, in theory, it can be resolved. If ~~the~~ two beams have the same spectrum then the ratio of their intensities after passage through equal absorbing layers is constant. The filtering experiment used by Barkla examines the relative ionization, S/P produced by scattered and primary beams after passage through equal absorbing layers. The ratio S/P is plotted against thickness of absorbing layers.

A discussion of the predictions of theory for this experiment will be given at a later stage. It is sufficient for the present to say that the resulting graphs assume various forms, and that one, in particular, is obtainable which shows a sharp discontinuity of approximately 7% in S/P . This has been interpreted as a sudden increase in absorbability. Such a phenomenon is certainly incompatible with accepted theory. To account for it Professor Barkla has put forward the hypothesis already mentioned in relation to the scattering experiment that the principle of superposition breaks down for heterogeneous beams. The most careful study of this experiment is therefore necessary. Observations made by the writer will be reported.

POLARIZATION OF THE PRIMARY BEAM.

The heterogeneous primary radiation from an X-ray tube is polarized to a variable extent, the plane of polarization being perpendicular to the plane containing the primary ray and the cathode stream. This polarization arises because the decelerating forces acting on the cathode ray particles at the anticathode must act preponderatingly in a direction opposite to that of the cathode stream. Since 1905, when Barkla made the first observations of primary polarization, many experimenters have made similar measurements.[40] Latterly, the experiments have been conducted chiefly with a view to determining the distribution of the polarization in the tube spectrum, and the influence of target thickness. While the results agree on the whole, there are nevertheless some surprisingly conflicting ones: notably the differential filter experiment of P.A. Ross, and the thick filter experiment of P. Kirkpatrick. Fairly general agreement exists, however, on three points: (a) that under certain circumstances polarization is increased by filtering the primary beam, (b) with lower voltages on the tube more polarization is observed, and (c) with thin targets the total polarization is greater.

In the scattering experiments carried out by the writer, it was found necessary to apply corrections for polarization/

polarization. The theory of this correction will therefore now be given.

The quantum theory of X-ray scattering gives two scattered rays, the modified ray and the unmodified ray. Since for 90° scattering these two rays have the same distribution about the primary beam, and since in polarization measurements the two necessary observations involve exactly the same amount of absorbing matter between scatterer and ionization chamber, it is not necessary to treat the problem otherwise than as for purely classical scattering.

Let us consider first a monochromatic constituent, of constant intensity and of wave length λ , in the primary beam. Since the intensity is constant, the amplitude of the electric vector, E_λ , must be constant. To allow for the partial polarization of this constituent, the direction of E_λ must be supposed variable with time. If this direction be indicated by angle α , where α is the angle of inclination not greater than 90° between the electric vector and the plane containing X-ray beam and cathode stream, then all possible values of α lie between + 90° and -90°. The probability of this angle having a particular value α is a function of α , the form of which is not important except for the fact that, from the nature of the processes taking place at the anti-cathode, it must be symmetrical about $\alpha = 0$.

If/

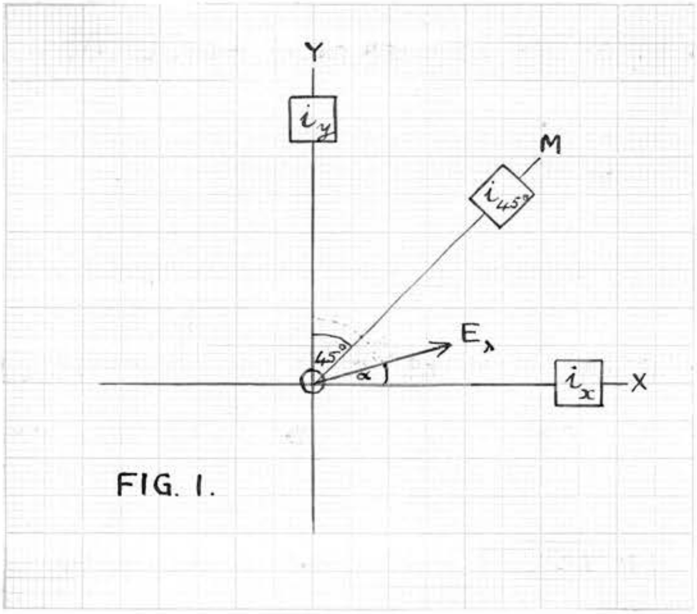


FIG. 1.

If this monochromatic wave, of electric vector E_λ and intensity $\frac{cE_\lambda^2}{4\pi}$, proceeds perpendicular to the paper (figure 1.), and scatters in the plane of the paper from an electron at O, then we have at distance r from O :

Intensity of scattered wave along $OX = \frac{cE_\lambda^2}{4\pi}\left(\frac{e^4 \sin^2\alpha}{2r^2m^2c^4}\right)$ C

" " " " $OY = \frac{cE_\lambda^2}{4\pi}\left(\frac{e^4 \cos^2\alpha}{2r^2m^2c^4}\right)$ C

Let there be two identical ionization chambers at equal distance from O along OX and OY, of equal sensitivity such that

K (ionization current) = Intensity of incident scattered wave, and if i_x and i_y be the respective ionization currents, then

Time average of $\frac{cE_\lambda^2}{4\pi}\left(\frac{e^4 \sin^2\alpha}{2r^2m^2c^4}\right) = K i_x$ C

Time average of $\frac{cE_\lambda^2}{4\pi}\left(\frac{e^4 \cos^2\alpha}{2r^2m^2c^4}\right) = K i_y$ C

For an unpolarized primary beam of the same intensity $\frac{cE_\lambda^2}{4\pi}$, producing an ionization current i in the Y ionization chamber, the corresponding equation would be

Time average of $\frac{cE_\lambda^2}{4\pi}\left(\frac{e^4 \cos^2\theta}{2r^2m^2c^4}\right) = Ki$ where θ has an equal probability for all values in the range Assuming etc

$+90^\circ \geqslant \theta > -90^\circ$

so that $Ki = \frac{E_\lambda^2}{16\pi}\left(\frac{e^4}{r^2m^2c^3}\right)$ C

Defining the fractional polarization p as the excess Why?

of Ki_y over Ki expressed as a fraction of Ki,

$$p = \frac{i_y - i}{i} \quad - - - - - - - - \quad (A)$$

Now $Ki_x + Ki_y$ = Time average of $\frac{E_\lambda^2 e^4}{8\pi r^2 m^2 c^3}\left\{\sin^2\alpha + \cos^2\alpha\right\}$

$$= \frac{E_\lambda^2 e^4}{8\pi r^2 m^2 c^3}$$

$$\therefore Ki_x + Ki_y = 2Ki$$

$$\therefore \quad i = \tfrac{1}{2}(i_x + i_y)$$

Substituting this value in equation (A) above

$$p = \frac{i_y - i_x}{i_y + i_x}$$

$$\text{also} \quad i_y = i(1+p)$$

$$i_x = i(1-p)$$

Suppose, now, that a third ionization chamber having exactly the same sensitivity as the other chambers is placed equally far from O along OM at 45° to the direction OY, and there receives an ionization current i_{45°.

Intensity scattered by polariz~~ation~~ed wave along OM is

$$\frac{c E_\lambda^2}{4\pi}\left(\frac{e^4}{2r^2 m^2 c^4}\sin^2(45^\circ - \alpha)\right)$$

$$= \frac{E_\lambda^2}{16\pi r^2 m^2 c^3}\left(1 - 2\sin\alpha.\cos\alpha\right)$$

And Ki_{45° = Time average of $\frac{E_\lambda^2 e^4}{16\pi r^2 m^2 c^3}\left(1 - 2\sin\alpha.\cos\alpha\right)$

But the probability of $+\alpha$ = the probability of $-\alpha$

Therefore the time average of $\sin\alpha\cos\alpha$ is zero, and

$$Ki_{45^\circ} = \frac{E_\lambda^2 e^4}{16\pi r^2 m^2 c^3} = Ki.$$

The/

The ionization in the chamber on OM is the same as if the primary beam were unpolarized.

There are thus two methods of correcting for polarization, that is, of obtaining the ionization current i. These are given by the equations

$$i = i_{45^\circ}$$
$$i = \frac{1}{2}(i_x + i_y)$$

The first and best method is to observe the 90° scattering in a direction at 45° to the plane of polarization. The second is to observe the 90° scattering in the two directions OX and OY, the ratio of the ionization currents being absolutely determined, and to calculate the mean of the observations.

If the principle of superposition holds, both methods are applicable to heterogeneous beams in which polarization is distributed in any manner through the spectrum of the beam. The fractional polarization, p is, in theory, not constant for different wave lengths in a heterogeneous beam, but increases for the higher frequencies. The polarized energy in the primary is therefore harder than the primary beam as a whole.

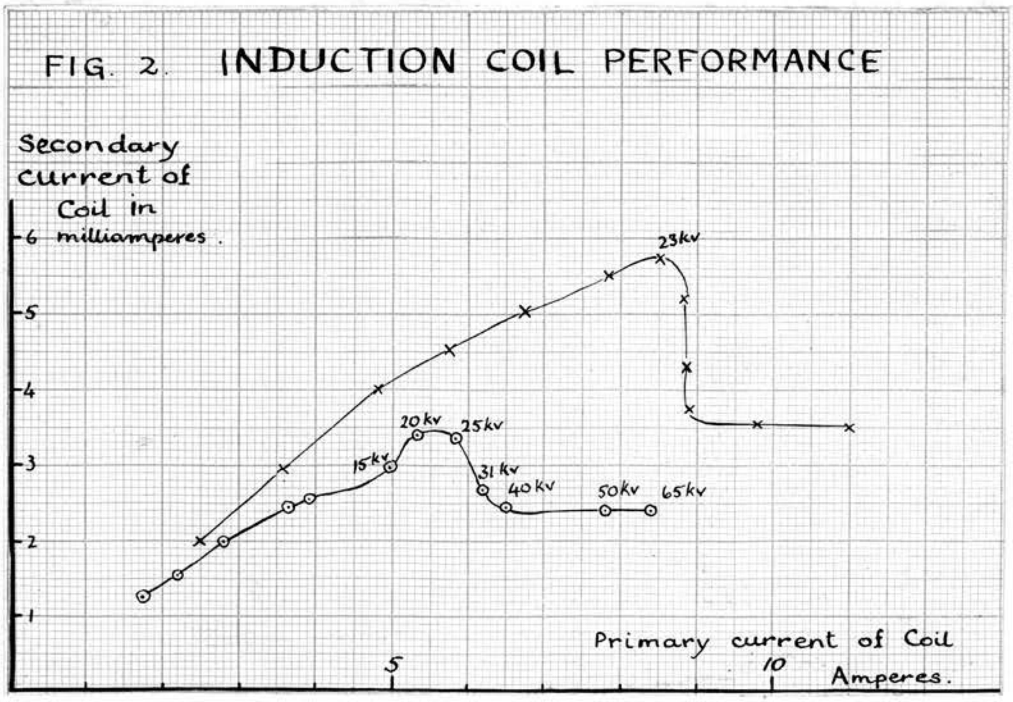


FIG. 2. INDUCTION COIL PERFORMANCE

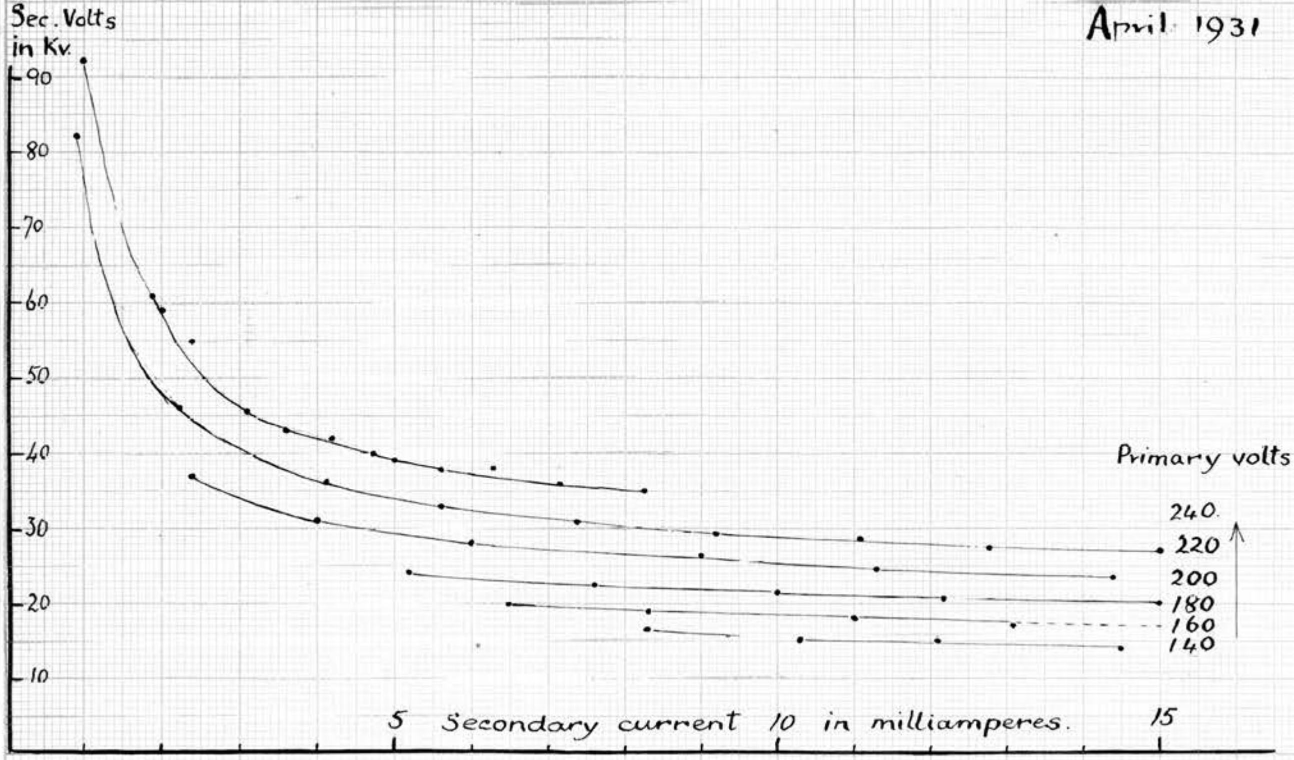
PART II. ACCOUNT OF APPARATUS.

Sources of High Tension.

Induction Coils. Three induction coils were used. Two were eight inch spark gap coils and one a six inch spark gap coil. The coils were excited by voltages of 20, 30, or 40 volts D.C. obtained from 100 ampere-hour accumulators. In series with the primary winding were a rheostat and a mains driven mercury gas make and break, the latter being controlled in frequency by a lamp resistance and rheostat. On rare occasions the mechanical make and break on the coil was substituted for the mercury gas break. The only available method of measuring the potential produced by the coil was to use the point to point coil spark gap, but this was found most unreliable when calibrated against a reliable ball spark gap. No measurements of potential were made and reliance was placed on measuring the penetrating power of the radiation.

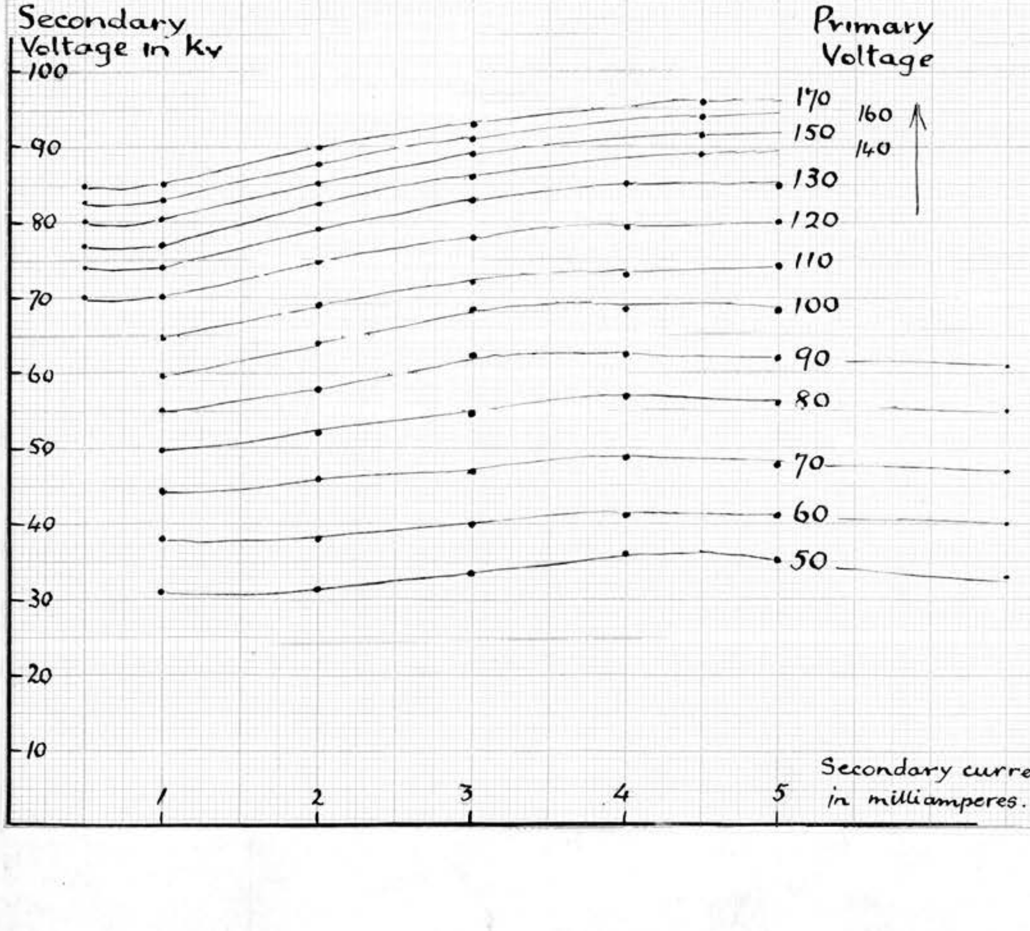
When the Muller tube (1) was used with an 8 inch induction coil, a calibration of the secondary circuit was possible. This showed that the range of secondary voltages was from 20 kv to 65 kv, and that the secondary currents varied from one to 6 milliamperes, but as will be seen on the graph (fig. 2) there was a part of the range/

FIG. 3 CALIBRATION CURVES OF TRANSFORMER (1)



April 1931

FIG. 4 CALIBRATION CURVES OF TRANSFORMER (2)



February 1932

range showing rapid variations. This was thought unsuitable for further work and transformer (1) was obtained to replace the induction coil.

Transformer (1), fed from 230 volt A.C. mains through an autotransformer, produced low voltages, i.e. nothing above 40 kv for all tube currents from 4 milliamperes upwards. Over this range the load characteristic was fairly good. For smaller currents the voltage varied rapidly with secondary current, ~~and this was determined~~ which was in turn controlled by the temperature of the hot cathode in the X-ray tube. ~~The latter depended on~~ This temperature was maintained by the current from a twelve volt battery. The unavoidable small fluctuation of ~~which~~ this battery made it difficult to work reliably at high voltages. The calibration of th~~is~~e transformer is shown in fig. 3.

Transformer (2) had a 5 KW output, was centre earthed and had an excellent load characteristic, tested from 0.5 milliamperes to 5.0 milliamperes for high voltages and to 8 milliamperes for secondary voltages up to 60 kv. It was driven off 230 volt A.C. mains through a rheostat with coarse and fine adjustment. This reliable source of potential was used from February 1932 till the end of the experimental work.

X-ray Tubes.

Low pressure gas filled tubes. Eight different tubes of this type were used during the first eighteen months of research. Most of these were 7" diameter bulbs with relatively/

relatively thin glass walls. One was 5" diameter and one an old and still smaller tube. All had tungsten anticathodes. The "softer" tubes inclined to be unreliable and sometimes hardened considerably during the course of a few observations. Under these circumstances the observations had to be discarded. In using these tubes the usual difficulty was found, that under the most favourable conditions, only a small range of $\overline{\mu}/\rho$ could be experimented upon with accuracy.

Muller Tube (1) This was a useful vacuum tube with a tungsten anticathode rated at 3 Kilowatts and normally operated much below its rating. It was designed to have a spot focus of considerable power. This was obtained by using a filament wound in a cylinder of small radius ~~line cathode~~ from which a well focussed sheet of electrons was directed towards the face of the anticathode. The plane of the latter was perpendicular to the plane of the cathode stream and inclined at nearly 90° to that stream. Viewed in a direction perpendicular to the cathode stream and in its plane, the focal line appeared as a circular spot. The anticathode was cooled by water circulating ~~water~~ from a small tank built into the tube. Unlike the gas tube, which was run intermittently, the Muller tube was continuously run. Since the readings were inclined to be irregular when the water was boiling hard, this condition was avoided as far as possible. The Muller tube had comparatively thick glass walls and could not be worked below/

FIG. 5(a) APERTURE SYSTEMS

FIG 5(a) USED FROM 22·2·29
TO 15·7·30

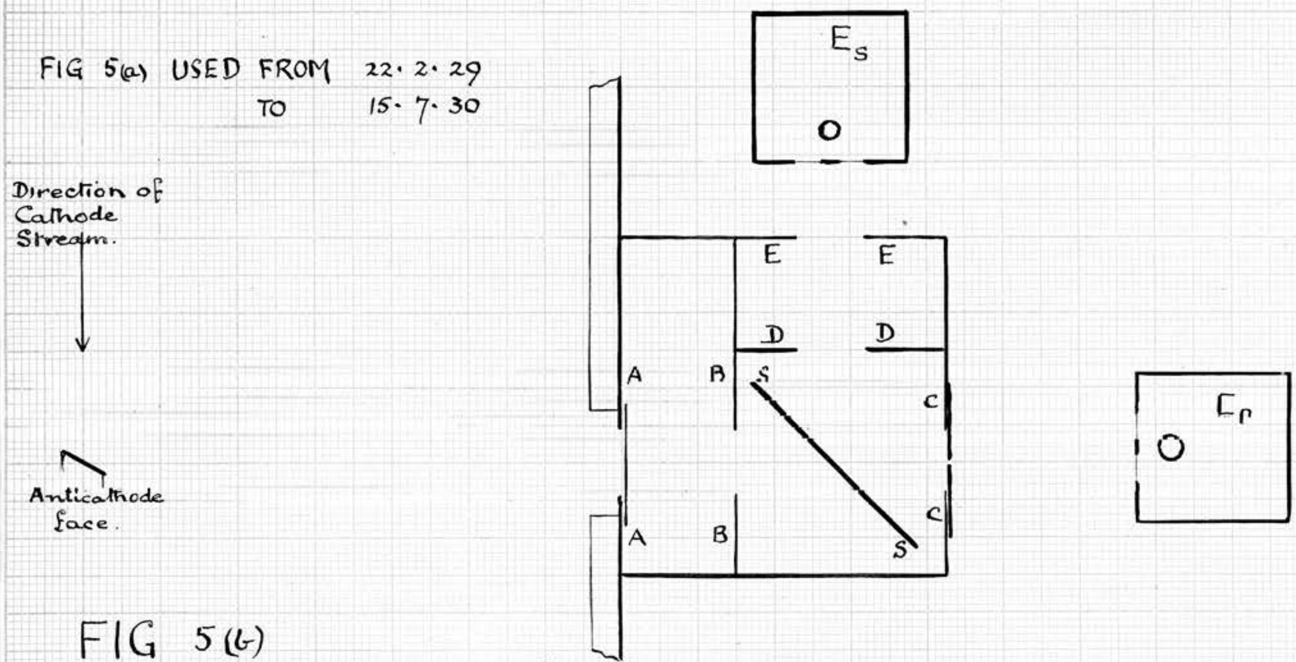
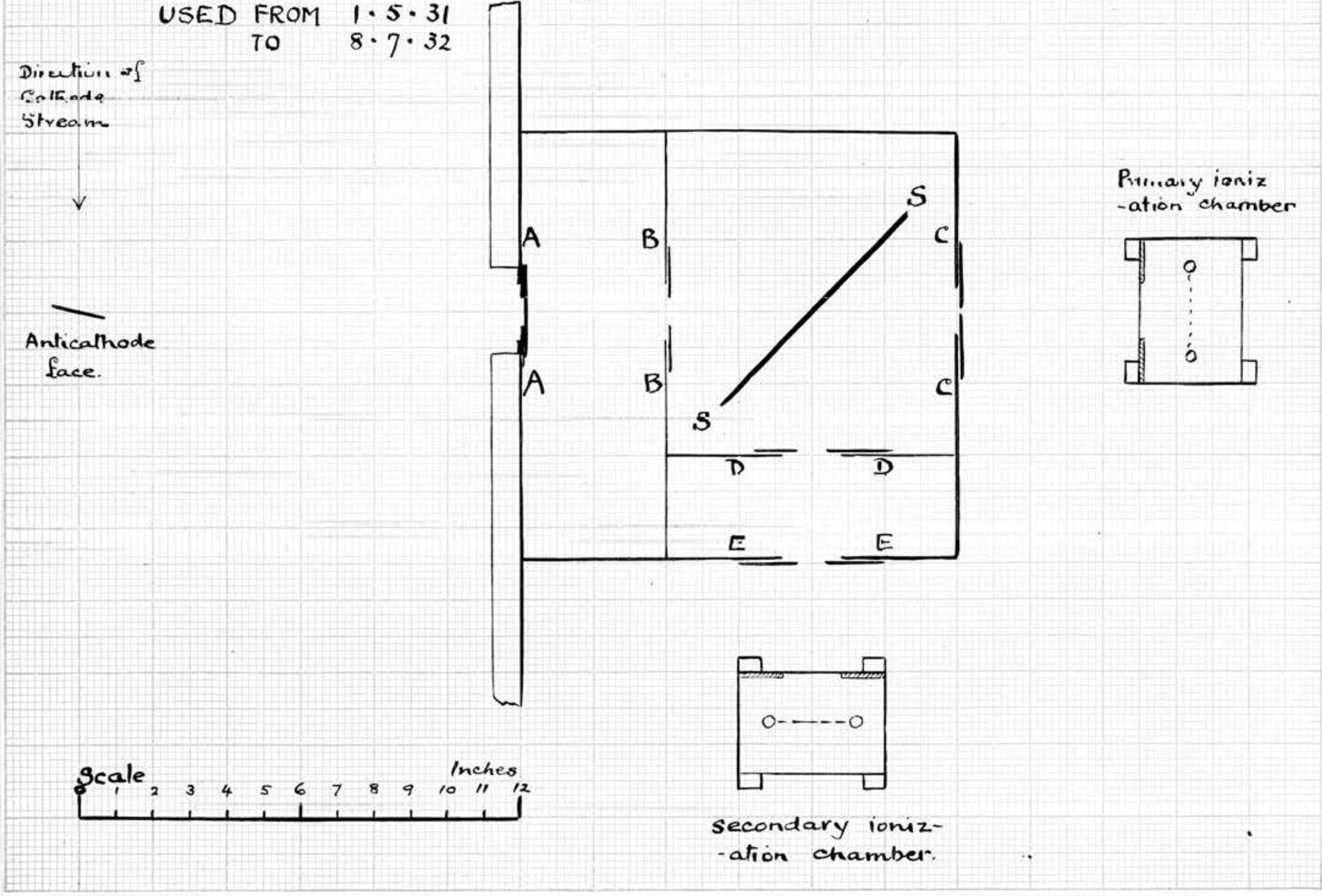


FIG 5(b)
USED FROM 1·5·31
TO 8·7·32



below 30 kv. ~~The radiation produced by it was on the whole, 'harder' than that produced by other vacuum bulbs~~.

The Coolidge Tube was a hot cathode vacuum tube with a molybdenum anticathode. The anticathode ran red hot and was cooled by its heat radiation. The glass walls of this tube were somewhat thinner than those of the Muller (1) or Muller (2), but the main difference in output was caused by the employment of an anticathode of lower atomic number.

The Andrews Tube (1) was a hot cathode vacuum tube with a window of specially thin glass. The tube was designed to give a good output of 'soft' radiation ~~under high potentials~~ and had a tungsten anticathode.

The Andrews Tube (2) was an improved form of the previous tube. It gave a still 'softer' radiation ~~under high potentials~~ and like Andrews Tube (1) had a tungsten anticathode.(See Fig. 14)

The Muller Tube (2) was a 3 KW tube of the same type exactly as the Muller tube (1). It had been somewhat longer in use, and on that account, the glass walls had a coloured deposit. In operation the difference between the two Muller bulbs was inappreciable.

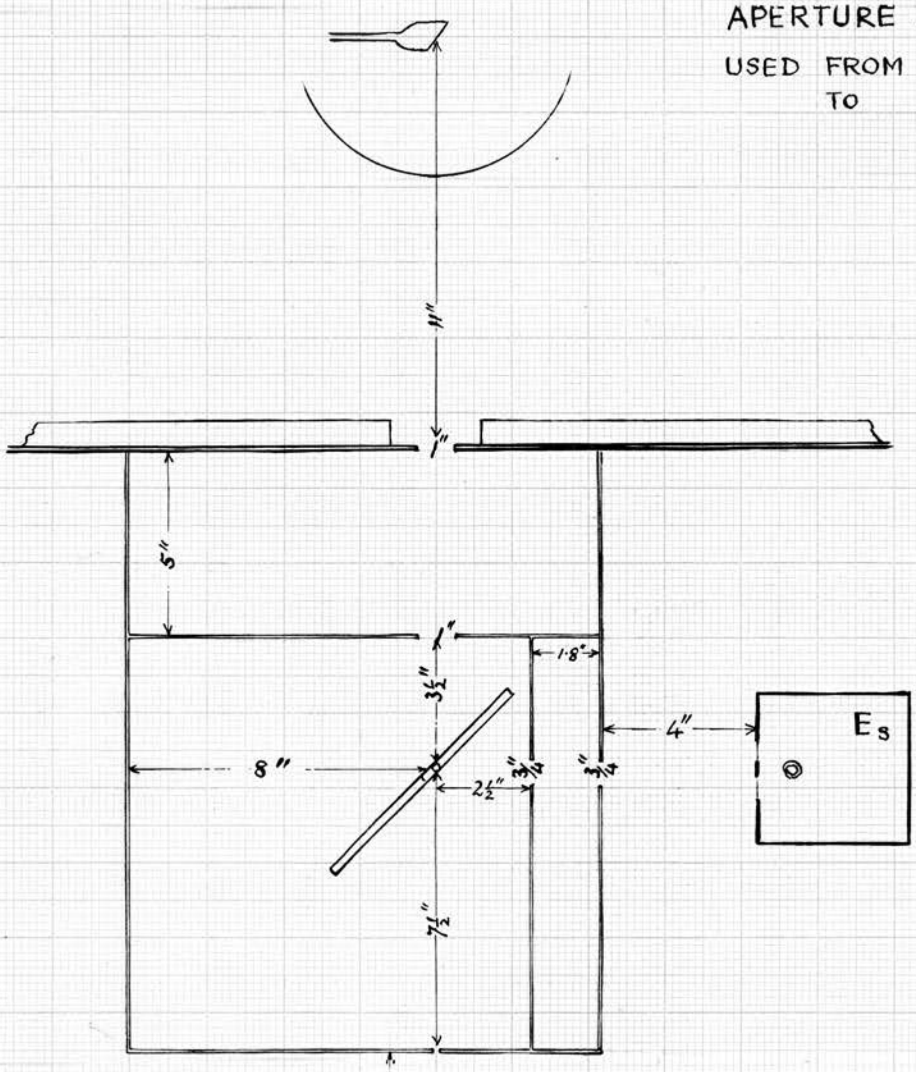
Aperture Systems.

The X-ray tube was mounted on an insulating stand inside a wooden box whose entire outer surface was covered with lead of surface density 7 lbs per square foot. This provided ample protection for the measuring systems/

FIG 6

APERTURE SYSTEM

USED FROM 14·11·32
TO 10·3·33



Note: the single figure box should extend to include Ep and scale.

FIG 7.

APERTURE SYSTEM
USED FROM 28·4·33
TO 30·7·34
ANGLE OF SCATTERING 90° ± 9°

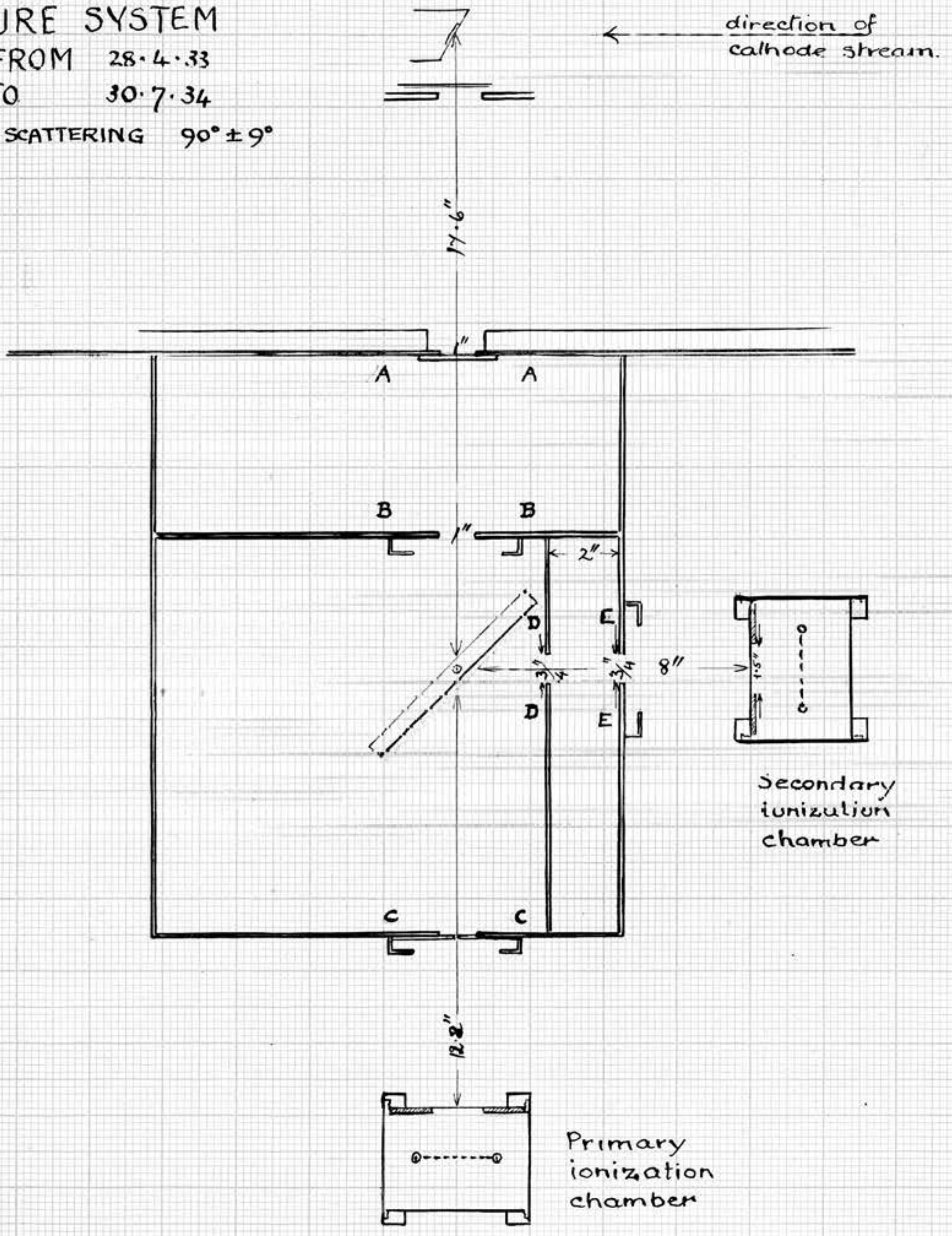
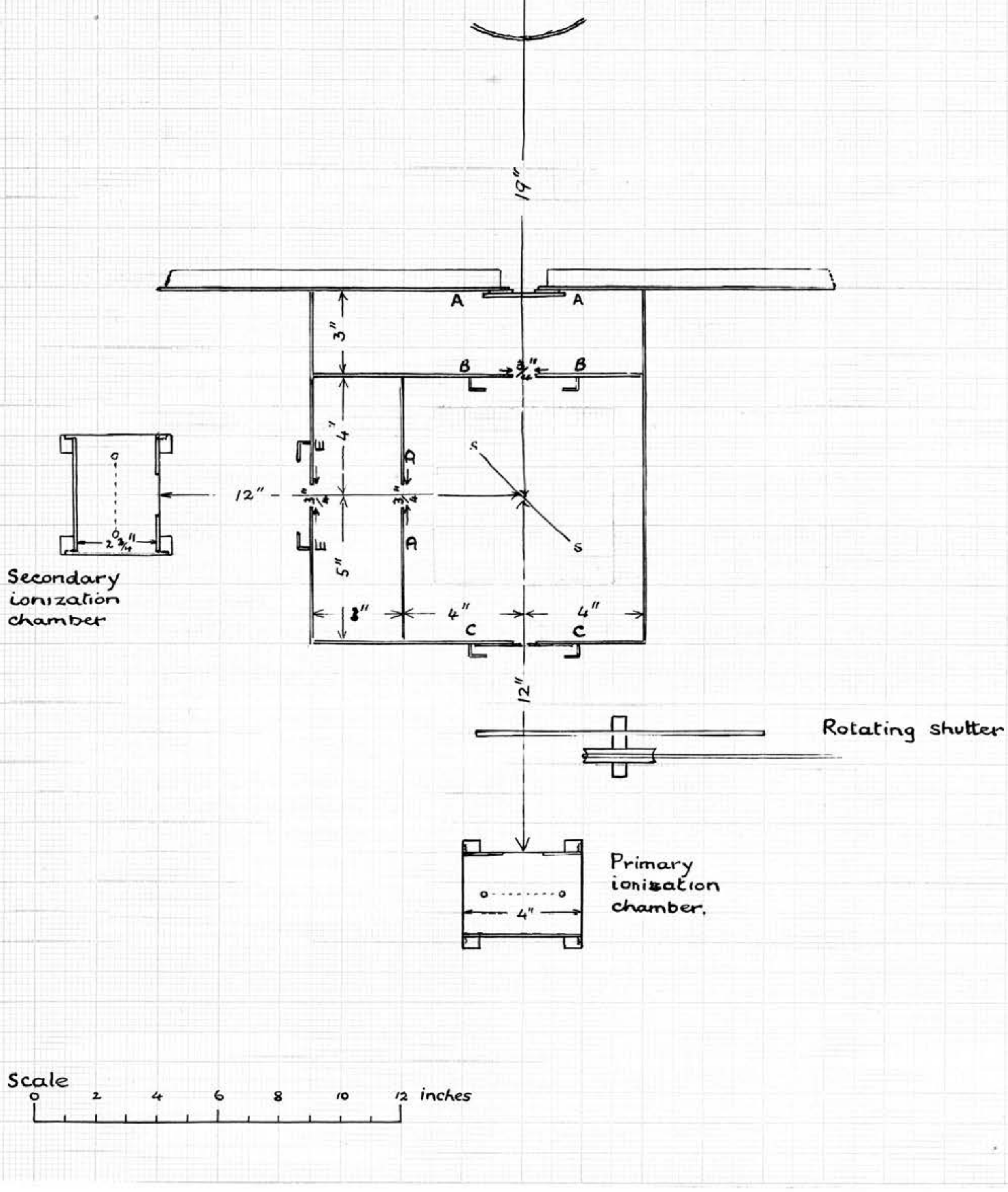


FIG. 8 APERTURE SYSTEM

USED FROM 11·12·34
TO 31·3·36
ANGLE OF SCATTERING 90° ± 5·6°



systems. A 2" hole AA in the box and lead covering admitted an X-ray beam through an aperture system to the scatterer SS and thence to the primary or secondary measuring instruments. A lead shutter was used to shut off the X-rays and all the apertures could be reduced in size by inserting lead screens with smaller holes. The electrodes in the ionization chambers were screened by lead from the direct X-rays. Now the primary beam intensity was of the order of 1000 times the secondary beam intensity, while the sensitivity of the measuring instruments was approximately the same for primary and secondary when cubical box electroscopes were used as ionization chambers, and approximately in the ratio of 1 to 10 when separate ionization chambers were used. It was therefore necessary to use a very small primary aperture after the scatterer to equalise the power entering the primary and secondary measuring instruments. This aperture was of the order of 1 mm in diameter. With so small an aperture it was necessary to ensure by the use of fluorescent screens or X-ray film exposures that the alignment of X-ray focal spot, apertures and Primary measuring instrument was satisfactory. Such a test was always made. Filter holders were mounted at the aperture before the scatterer and at all the subsequent apertures.

Sections of all aperture systems (made by cutting with a horizontal plane through the anticathode focal spot) are given in figs. 5 to 8, which are dated.

Measuring Instruments and the Problems of Measurement.

While there is no other instrument so sensitive, quick and convenient as the ionization chamber for the comparison of X-ray beam intensities, it is nevertheless not easy to interpret ~~ensure that~~ the observations ~~made by them properly record the facts~~. The difficulties met with in the present work arise from a number of causes and principally from the following:-.

~~(1) The disparity in intensity of the primary and secondary beams which are to be compared. As already mentioned the primary intensity is of the order of 1000 times the secondary intensity. The density of ions in the primary chamber is therefore correspondingly greater than the ion density in the secondary chamber. Care must therefore be taken that saturation potential conditions are maintained in the primary chamber for the greatest primary ionization currents.~~

(~~2~~1) The extreme range of power which it is desired to measure with a single ionization chamber in the course of a single experiment) of the order of 100 to 1. The extreme practicable range of power measurement in the experiments here reported was 15 to 1.*

(~~3~~2) The greatest difficulty met with in interpreting the observations arises from the wide range of wave lengths existing in some of the heterogeneous beams. It would be ideal to obtain a true measurement of power. This is not practicable in beams which include all wave-lengths from, say, 2.5 Å U to 0.11 Å U.

Cubical/

---

* The ratio of duration of longest allowable to shortest allowable readings.

Cubical box gold leaf electroscopes.

The instruments employed were made of brass protected on the outside with lead and lined on the inside with aluminium and three thicknesses of filter paper. Their over all dimensions were four inches each way. The insulation between case and electrode was of sulphur. The brass electrode and its attached gold leaf were shielded by lead from the direct radiation. The window of the electroscope was of aluminium 0.01 cm. thick, backed by 3 sheets of filter paper. The ionized gas was, of course, air.

In the older type of apparatus where these electroscopes were used to measure the relatively feeble output from gas filled X-ray tubes driven by induction coil, the conditions for satisfactory measurement were well known and easily attained. A potential sufficiently high to operate the gold leaf system efficiently (about 200 volts) was also sufficient to give saturation potential. Then, provided that precautions were taken to eliminate corpuscular radiation from heavy atoms in electrode or chamber walls ~~effects~~, the observations were reliable. But with the powerful output of transformer driven, hot cathode, water-cooled X-ray tubes, the difficulty is no longer to obtain sufficient power but to avoid overloading the primary ionization chamber. A test under working conditions is required to ensure that, under maximum input, saturation potential conditions are maintained. To carry out this test, the beam intensities are gradually increased, other conditions being/

being kept constant, and observations are made of the ratio of secondary ionization S to primary ionization P. This ratio remains constant until saturation potential conditions fail to be maintained in the primary chamber. Further increase in intensity causes the ratio S/P to rise. If while carrying out this test the time taken to register a standard deflection of 10 divisions in the primary is recorded, a useful guide is obtained as to the minimum time allowable for the standard deflection.

Ionization chambers.

The ionization chambers used were cylindrical, 4" in diameter, and 2.6" deep. They were made of brass, lined with aluminium foil 0.01 cm. thick, and within that with three sheets of filter paper. The window was of aluminium 0.01 cm. thick, backed by a lead aperture 1.25" diameter, and this in turn backed with three sheets of filter paper. The electrode was a circular ring 3" in diameter of brass wire with silk net made conducting ~~net~~ stretched over it, the whole mounted parallel to the chamber ends. The electrode lead was insulated with Ebonite ~~sulphur~~, and leaks from it were avoided by using a guard ring. The chambers were gas tight and fitted with gas tight taps. Sulphur dioxide at atmospheric pressure was used as the ionizing gas. It was sufficiently free from admixture with air ~~and its purity was sufficient~~ to give 10.8 to 11.0 times the ionization of air. The voltages acquired were measured by a cubical box electroscope for/

FIG 9(a) WORKING VOLTAGE TEST FOR IONIZATION CHAMBER.

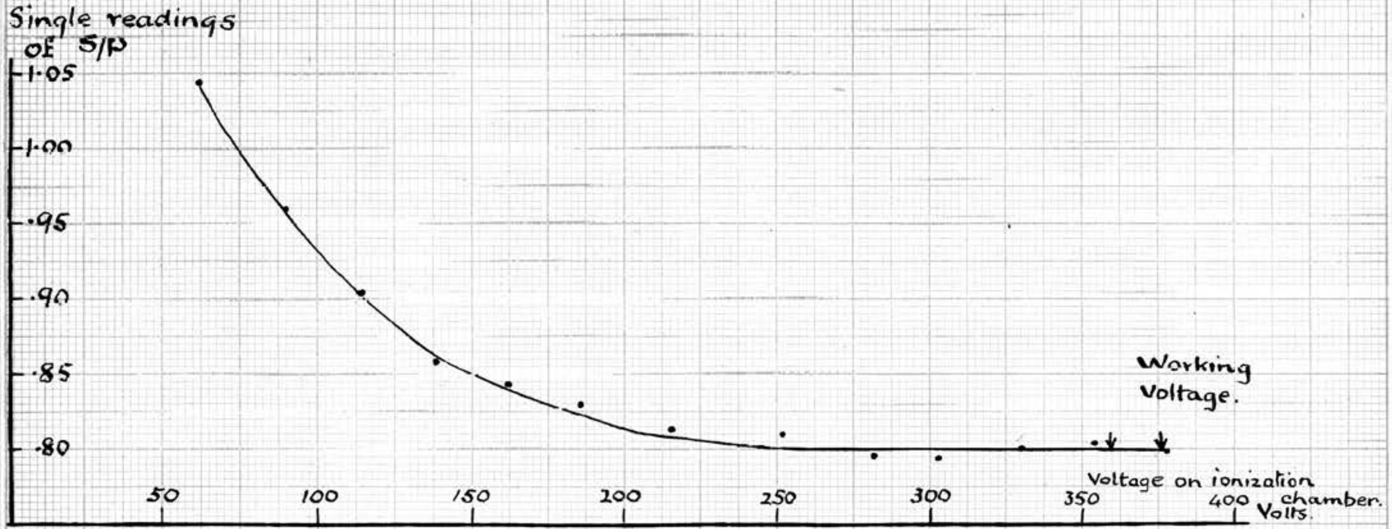
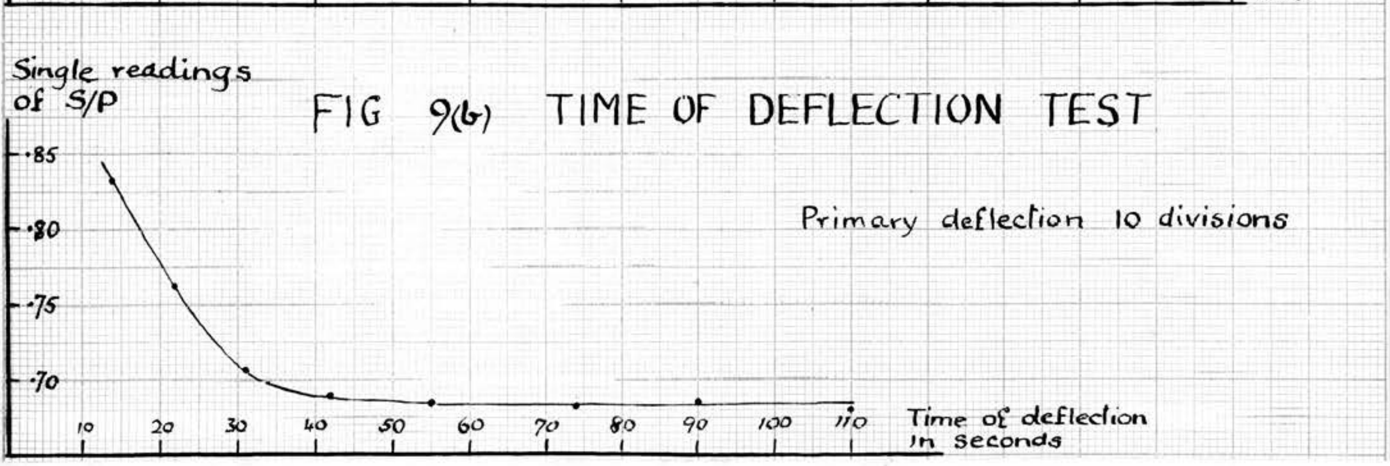


FIG 9(b) TIME OF DEFLECTION TEST



Primary deflection 10 divisions

for the primary and by a Wilson tilted electroscope for the secondary.

A preliminary test was made to determine the best working voltages for these chambers. The X-ray tube was operated at 95 kv. and 5 ma. with a scatterer, which was about to be experimented with at the time, of 1.1 cm. paraffin wax and 0.008 cm. Ag. The secondary ionization chamber was working normally and the potential V_p on the primary ionization chamber was varied. For each value of V_p the ratio S/P was determined, and the observed values of S/P graphed against V_p. The ~~A typical~~ graph is shown in fig. 9a. From this graph it can be seen that in the range $378 > V_p > 360$ saturation potential conditions are fully satisfied. V_p was kept within this range throughout the experiments. The working conditions varied so greatly, however, that a subsidiary check was necessary; one that could be made concurrently with each observation. A time of standard deflection test was therefore carried out, as for the electroscopes, and the results are shown in fig. 9b. From this graph saturation conditions are evidently assured for deflection times of 8 or more seconds per division. In practice no readings taking less than 12 seconds per division were used unless the beams were strongly filtered, and even then never less than 8 seconds per division.

|| diffusion?

Wave/

Wave length response of the ionization chambers.

When ionization chambers are employed for the comparison of powers in heterogeneous beams, consideration must be given to the wave length response in the particular ionization chamber employed. If the power of the radiation is totally absorbed, then, since the energy required to form an ion pair is a constant for a given gas, the power will be proportional to the ionization current. But if the power is not totally absorbed, then the fraction of energy absorbed is different for different wave lengths. A true interpretation of the observations is then possible only if due allowance can be made for the wave length response. In the present paper the correction for cubical electroscopes will not be discussed, since these were not used for the important experiments.

If a parallel beam of wave length λ and of linear absorption coefficient μ_λ in SO_2 penetrates the front window and lining of the ionization chamber and reaches the SO_2 with power P, then after passing through thickness l of SO_2 its power will be $P.e^{-\mu_\lambda l}$. The power removed by the gas from the beam is then $P(1 - e^{-\mu_\lambda l})$ But ~~not all~~ this power is not all used in forming ions. ~~absorbed by the SO_2 on account of ion formation~~. Some may leave the chamber as ~~escape in the form of~~ characteristic radiation ~~from~~ of the SO_2, some ~~in the form of~~ as classically scattered radiation, some in modified scattered radiation, and there may be a net balance/

balance, positive or negative, of corpuscular radiation passing into the walls. A discussion of this question is given in Compton and Allison pp. 492-500, where the loss of energy due to incoherent scattering is computed along with that due to classical scattering as if no modification took place, and where corpuscular "end effects" are assumed to be negligible. The formula derived is that the absorbed power is

$P\,(1 - e^{-\mu_\lambda l})\,(1 - K$ radiation loss - scattering loss).

The K radiation loss is $\frac{\omega_K}{\nu}\left(\frac{r-1}{r}\right)\sum_f' \nu_f z_f e^{-\tau_f d}$,

Where ω_K is the fluorescence yield,

ν is the frequency of wave length λ,

r is the absorption jump ratio,

z_f being the fraction of the K characteristic quanta of frequency ν_f,

d a kind of average path length of the K radiation in the chambers and

τ_f the true absorption of the line ν_f in the gas.

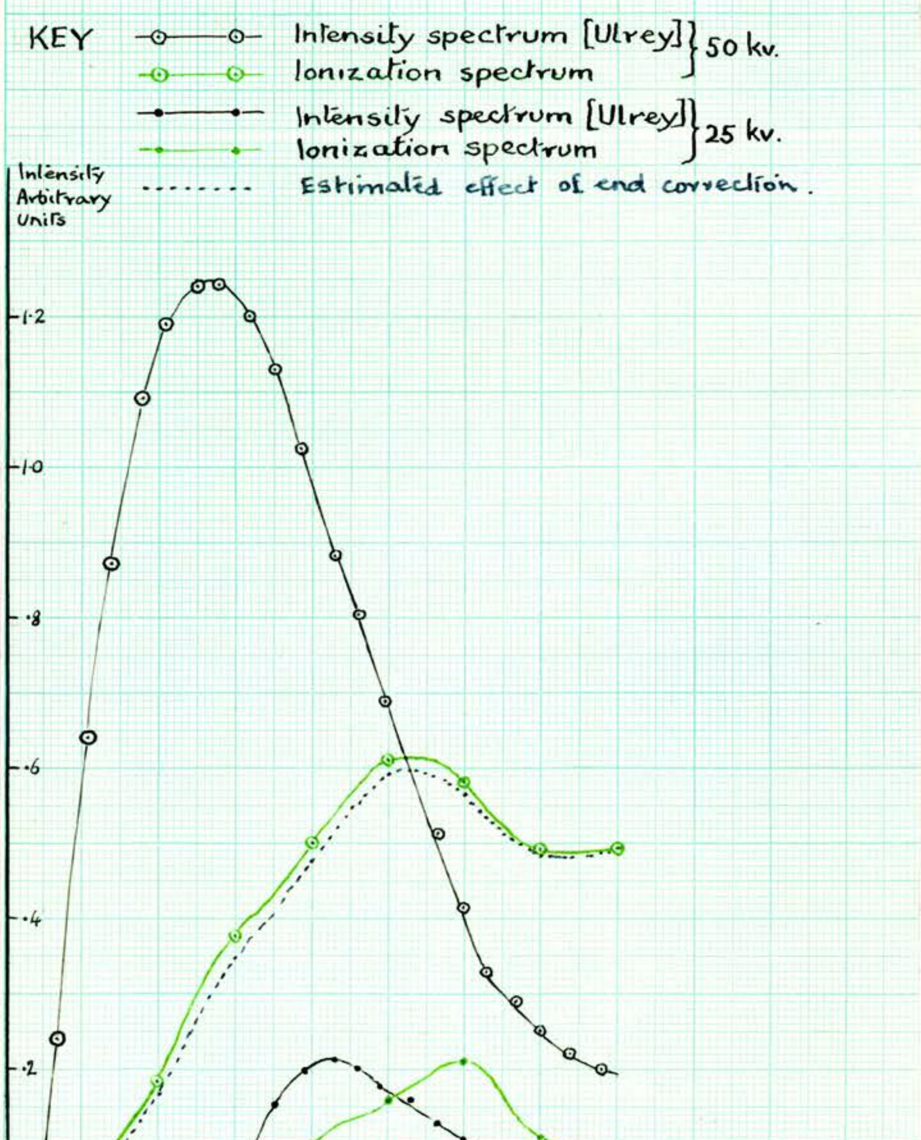
The scattering loss is given as $\frac{\sigma_\lambda}{\mu_\lambda} \cdot e^{-\tau_\lambda d}$.

Excellent agreement with experiment has been obtained, using this correction, by Allison and Andrew[41] who employed an ionization chamber 28 cm. long and 7 cm. in diameter for wavelengths between .6 and 1.5 ÅU ionizing a variety of gases.

In the present case ~~experiments~~ the chamber is short, and ~~but~~
the end effect is then appreciable at the hard end. An estimated end
~~as the difference in atomic number between the material~~
correction factor is shown on the lowest line of Table $\overline{X}$. The rest
~~of the surface and the ionized gas is not great, corpuscular~~/

FIG. 10 WAVE LENGTH SENSITIVITY OF THE IONIZATION CHAMBER

WITHOUT END CORRECTION



~~corpuscular effects, although they will involve a small loss, are considered to be negligible~~. ~~The results of~~ of this table shows the computation of the formula ~~are~~ given ~~in Table X~~ above, , the data being obtained from the useful Tables given in Compton and Allison. The loss due to K radiation is everywhere negligible compared with that due to scattering. The fraction of power absorbed by the ionization chamber is given (a) excluding and (b) including the estimated end correction factor.

The effect of the power loss is worked out graphically in fig. 10 for two beams. The spectral intensities are those given by Ulrey[42] for radiation from a tungsten target under 50 and 25 **kv** respectively. The intensity which would be absorbed from these two beams in the ionization chambers under discussion is shown in green ink. The fact that the ionization chamber is more sensitive at the soft end is clearly shown. The peak of the absorption curve occurs at a longer wave length than the peak of the spectral intensity curve, and this shift is greater for the curve of greater X-ray tube tension. Table X shows that the chamber is 300 to 400 times as sensitive to radiation of λ = 1.6 ÅU as it is to radiation of λ = 0.2 ÅU.

The 'soft' end of the spectrum is not drawn in fig. 10. The reason for this is that in the course of the experiments to be described, different thicknesses of scatterer are used, varying from 10 sheets of paper of surface density 0.064 gm. per sq. cm. to 1.9 cm. of paraffin wax of surface density 1.77 gm. per sq. cm. Using/

Using as thick a scatterer as 1.9 cm of paraffin wax the spectrum at the 'soft' end is virtually reduced to zero intensity about 1.2 ÅU (without considering the absorption of the X-ray tube window, or of other matter in the path of the rays). Using a thinner scatterer, say of 0.3 cm. paraffin wax, the spectrum extends to 2.5 or 3 ÅU (again excluding the other absorbers in the X-ray paths). The extent of the spectrum at the 'soft' end must therefore be regarded as most variable.

Aperture effects and ionization.

The great disparity in cross section of the beams entering primary and secondary ionization chambers was thought to require consideration. The primary aperture was therefore altered and made to consist of many (about 30), very small, well spaced pinholes. This caused the scattering ratio to diminish rapidly as the primary beam was hardened, suggesting that the greatly multiplied aperture edges were filtering a considerable proportion of the primary beam. ~~This affected the observations so much that~~ No primary apertures less than 1·2 mm. diameter were subsequently used. In the later stages of the work a ¼" primary aperture was used, the primary beam intensity was reduced and at the same time interrupted by a mechanically driven rotating shutter. Precautions with this device were required to avoid inaccurate readings due to effects of equal or approximate synchronism between the frequency of the mains (50 cycles / sec,) and that of the interrupter.

Gold leaf electroscopes used for measuring change of potential.

Gold leaf electroscopes were used to measure the potentials acquired by the electrodes of the ionization chambers. Observations of gold leaf deflections were made/

made by long focus microscopes in terms of eyepiece scale divisions. Readings were estimated to tenths of a division on a tenth millimetre scale.

A cubical box electroscope was used with the primary. Since it is convenient to keep the case of the electroscope at earth potential, this case and the walls of the primary ionization chamber were kept at earth potential and the full potential of 360 volts required to maintain saturation potential in the ionization chamber was therefore used on the electrode. A gold leaf was mounted which gave with this potential a sensitivity of about 1 division per volt. For the secondary ionization chamber, a Wilson Tilted electroscope was used. In this instrument it is convenient to keep ~~the gold leaf and~~ the case at and operate the gold leaf from earth potential and to maintain a constant potential difference of 200 volts between ~~these~~ case and ~~the~~ adjustable plate. The electrode of the secondary ionization chamber was therefore operated from ~~at~~ earth potential, and the walls insulated and kept at 360 volts. The potential of the electroscope plate was kept at - 200 volts. The sensitivity of the Wilson Tilted electroscope can be raised or lowered by adjusting the distance between plate and leaf, and it also depends upon the plate voltage and tilt. As the sensitivity is increased however it tends to become more variable on account of slight changes in one or other of the important factors. For this reason precautions must/

must be taken to keep battery voltages as far as possible free from temperature variations throughout the day. In practice it is found better in the long run to work at a low sensitivity which will keep relatively constant than to sacrifice constancy to sensitivity. During the course of the experiments the sensitivity of this electroscope was therefore kept between 10 and 12 divisions per volt. The sensitivity was tested at intervals against a standard Weston cell.

The Gold leaf problems.

Beaten gold leaf is partially but not uniformly crystalline, and from this cause irregularities may arise in its bending under electrostatic stress. The narrower and more sensitive leaves show this very readily. The leaf motion must therefore be tested by a calibration carried out with both increasing and decreasing potentials. The calibration curve of an unsatisfactory leaf is shown in fig. 11 and of a satisfactory one in fig.12. The same calibration provides a test for the linearity of the scale of the instrument. The scale of the cubical box electroscope was always linear over more than the working range, but the test in the tilted electroscope was found to be necessary. If the scale was not sufficiently linear it could be improved by slightly lowering the sensitivity.

Correction for leak in electroscopes.

Using the relatively high potentials on the ionization chambers that were found to be necessary, electroscope/

electroscope leaks took place sufficient to require correction. The leaks were not large but were sufficient to introduce an error of 1 to 3 per cent on the longest readings. It was considered preferable to observe the leaks and apply the corrections to readings of long duration (15 to 25 minutes) rather than to alter the conditions by excessive raising of the X-ray tube current. The rate of leak was usually observed in the morning, at mid-day and in the late afternoon.

PART III. EXPERIMENTAL EVIDENCE.

Before entering upon a description of the main series of experiments, one preliminary experiment will now be reported, because it gave the first evidence of certain unexpected facts which it is the object of this thesis to establish.

Section I

A.

COMPARISON OF $\bar{\mu}/\rho$ OF THE PRIMARY BEAM WITH $\bar{\mu}/\rho$ OF THE SECONDARY BEAM SCATTERED AT 90° FROM LIGHT SCATTERERS.

This experiment was originally carried out using an induction coil and Muller X-ray tube (1) with a tungsten anticathode. The tube was mounted with its axis horizontal. The primary of the induction coil was energised by a 30 volt battery through a rheostat. The voltage applied to the tube was varied by changing the rheostat adjustment and thereby primary X-ray beams of different penetrating power were produced. These passed in succession through the system of apertures, (fig. 5(a)) containing the scatterers at SS, the aperture~~s~~ centres being in a horizontal plane. The primary radiation passed to E_P and radiation scattered by SS passed to electroscope E_S, the angle of scattering being $90° \pm 12°$. With short exposures measurements were made of $\alpha = S/P$, $\beta = S/P'$, and $\gamma = S'/P$

S being the deflection of E_S due to the X-ray exposure

P " " " " E_P in the same " "

S'/

S' being the deflection of E_S when a 50% absorption filter of aluminium thickness Y cm. is placed at EE.

P' " " " " E_P when a 50% absorption filter of aluminium thickness x is placed at CC.

Then $\left(\frac{\mu}{\rho}\right)_P = \frac{1}{\rho x} \log_e \left(\frac{P}{P'}\right) = \frac{1}{\rho x} \log_e \left(\frac{\beta}{\alpha}\right)$

And $\left(\frac{\mu}{\rho}\right)_S = \frac{1}{\rho x} \log_e \left(\frac{S}{S'}\right) = \frac{1}{\rho x} \log_e \left(\frac{\alpha}{\gamma}\right)$

In experiments by earlier workers $\left(\frac{\bar{\mu}}{\rho}\right)_S$ was found to be almost the same as $\left(\frac{\bar{\mu}}{\rho}\right)_P$ and practically indistinguishable from it towards the low frequency end of the range of investigation. This was interpreted as showing good agreement with the Thomson theory of scattering. On the Compton theory of scattering however, identity of penetrating power in primary and secondary beams is not expected, for within the range of wave lengths of these experiments an appreciable intensity of incoherent scattering is predicted* from light elements, and the relative intensity of incoherent radiation increases for the shorter wave lengths. For comparison with experiment the values of $\left(\frac{\mu}{\rho}\right)_S$ corresponding to integral values of $\left(\frac{\mu}{\rho}\right)_P$ (both measured in aluminium) have been worked out for totally modified homogeneous beams. They are as follows:

*See values of $Z - \Sigma f^2_{KK}$ - KN. in Table VIII

$\left(\frac{\mu}{\rho}\right)_P$	$\left(\frac{\mu}{\rho}\right)_S$	$\frac{\left(\frac{\mu}{\rho}\right)_S - \left(\frac{\mu}{\rho}\right)_P}{\left(\frac{\mu}{\rho}\right)_P}$
1	1.2	20.0%
2	2~~1~~.25	12.5
3	3.35	11.7
4	4.4	10.0
5	5.45	9.0
6	6.5	8.3
7	7.55	7.9
8	8.6	7.5
9	9.65	7.2

This experiment cannot be regarded as a very accurate one. The quantities $\left(\frac{\bar{\mu}}{\rho}\right)_S$ and $\left(\frac{\bar{\mu}}{\rho}\right)_P$ are indirectly obtained, and an error in the observation of S/P will cause errors in opposite directions in $\left(\frac{\bar{\mu}}{\rho}\right)_S$ and $\left(\frac{\bar{\mu}}{\rho}\right)_P$. The observations are, moreover, very sensitive to unsteadiness either in the high tension supply or in the filament current of the X-ray tube. The results, however, although unexpected, show a fair consistency. Early results are given in Table I(a), (b), (c) and (d). In these tables the calculated values of $\frac{\left(\frac{\bar{\mu}}{\rho}\right)_S - \left(\frac{\bar{\mu}}{\rho}\right)_P}{\left(\frac{\bar{\mu}}{\rho}\right)_P}$ for 100% incoherent scattering at 90° for a homogeneous primary beam of $\frac{\mu}{\rho}$ equal to that observed are also given.

In these experiments it appeared that $\left(\frac{\bar{\mu}}{\rho}\right)_S$ was considerably greater than $\left(\frac{\bar{\mu}}{\rho}\right)_P$; the observed difference being greater even than the maximum possible for equivalent homogeneous beams ~~on the Compton theory of~~ with totally modified scattering. In particular the 'softening' showed to a surprising extent at the 'soft' end. Since it is possible that apparent softening of the secondary beam might be due either to scattering at angles differing/

differing appreciably from 90° or to some L characteristic radiations entering the secondary electroscope from lead screening or from filters, the following changes in the apparatus were made.

(1) The aperture AA was narrowed down from 2" diameter to 7/8" diameter, and at the same time the aluminium filters for the secondary beam were placed at DD instead of at EE. From the upper part of Table I(e) it will be seen that the results were not much affected by this change.

(2) All lead screening which could be suspected of affecting the secondary electroscope was covered with thick pads of filter paper. The readings were again not appreciably affected by this alteration (see the lower part of Table I(e).)

(3) A still smaller primary aperture 1/2" diameter was used, and the lead ~~opposite the secondary electroscope~~ previously covered with filter paper was removed. The observations were unchanged.

All attempts to explain the softening as a spurious effect having failed, the remaining hypotheses were either that some unsuspected source of error existed or that the results were genuine. In the latter case there were two alternative explanations: either the softening of the secondary was greater than any theory predicted, or the effect was due to a disproportionate hardness in the primary, showing to a certain extent in/

in all the observations, and most markedly at the 'soft' end of the range. If penetrating constituents of the primary beam were for any reason not proportionally represented in the secondary beam, the primary would be correspondingly 'harder' than the secondary derived from it. Now such disproportionate scattering is to be expected if the 'hardest' constituents of the primary beam are partially polarised in a vertical plane; a condition which is known to exist. The influence of polarisation was not immediately tested. A year later the same experiment was repeated after the high tension apparatus had been changed from induction coil to transformer (1), and the electroscopes replaced by ionization chambers. The Muller X-ray tube used was the same as in the earlier experiments. The results (given in Table I(f)) were not materially different from those obtained with the less reliable apparatus.

In passing, attention is drawn to three observations out of the total of 66 recorded in Table I, namely two in Table I(e) and one in Table I(f) which give $\left(\frac{\bar{\mu}}{\rho}\right)_p = \left(\frac{\bar{\mu}}{\rho}\right)_s$. These observations will be referred to subsequently in the thesis.

The above results were not in agreement with observations made by Barkla at a much earlier period, nor with those (unpublished) made ~~at~~ more recently by Mr. Sale. They were for this reason discredited at the time they were made. It should be noted however that the tube used in previous experiments was a gas tube.

meaning?

SECTION II. SCATTERING EXPERIMENTS.

Turning now to what have usually been referred to in the laboratory as the "horizontal line" experiments, a general survey of the observations made by the writer will be given.

In this experiment the ratio S/P of the ionizations produced by the scattered and primary beams is measured for a series of beams of different average penetrating power. The scattered beam was always observed at 90° to the primary. Beams of different penetrating power were obtained by varying tube voltage. The value of S/P as ordinate is plotted against observed values of μ/ρ as abscissa. The descriptive name "horizontal line" experiment comes from the fact that the experiment, as ordinarily carried out in the laboratory, gives a graph having a horizontal portion. Before the present work was begun this experiment had been performed by many research workers and reports of some of their observations had been made. As already explained this experiment gave results apparently inconsistent with accepted theory. Much obscurity remained however, the position being as follows. Results under given conditions (that is, on one particular apparatus used with every controllable condition the same) were constant. Different experimental conditions led to a variety of results. Experiments in which some/

X [illegible]

some controllable condition was varied showed on one apparatus that the particular condition was significant and on another that it was not so.* Such circumstances suggest that the real controlling condition is eluding the observer because it is not one of the directly controllable conditions. Under such circumstances the only profitable course is to conduct the experiment over as wide a range of conditions as possible in the hope of finding a clue to the elusive variable.

This experiment was performed many times, over a long period, during which many variations were made in the conditions. The main conditions which can be varied or controlled are:-

(1) Type of high tension supply.
(2) Type of X-ray tube.
(3) Filtering of primary beam.
(4) Tube current.
(5) Atomic number of tube target.
(6) Scatterer material.
(7) Scatterer thickness.

In the earlier stages of the work it was impossible to say which of the conditions were important. But as the body of experimental information grew, certain generalisations became possible and it appeared probable that the experimental results were determined, not by any/

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* Evidence of this has been given as recently as 1939, the suggestion being put forward that the experimental results may depend on the room in which the experiment is performed.[43]

any one or two factors, but by a number of factors, all of which were operative together, sometimes with one factor predominant, sometimes with others.

Altogether 157 experiments of this type were carried out. Of these, 109 were hot cathode tube experiments, 89 of which are admitted for reporting here. The 20 discarded experiments include:-

(a) the experiments carried out with transformer (1) (on account of the transformer's bad low current performance) - 2 in number;

(b) experiments carried out with pinhole primary apertures or otherwise special conditions - 10 in number;

(c) experiments in which the ionization chambers were not in good working order - 8 in number.

The gas tube experiments totalled 48, all carried out during the first fifteen months of research. Of these, 13 are used for comparison with the later more accurate work, and further useful information is derived from many of the others. All the admitted experiments carried out with paper or paraffin wax scatterers are included in Tables II and III pp. excepting a few with 16 sheets of paper, and certain experiments on filtered primary beams which will be reported differently.

The/

The observations were made by admitting the radiation through the aperture system* till a standard deflection was obtained on one of the electroscopes and the deflection of the other electroscope then taken. The time required for a single reading was anything from 2 (or under special circumstances a little less) to 25 minutes. Repetition of the quickly taken readings was an easy matter, but the time available in a working day set limitations on the repetitions possible for the lengthy readings. No reading of S/P was treated as fully established unless there were at least two observations and for the faster readings three was the usual number, four or five being occasionally taken. Values of $\overline{\mu/\rho}$ were usually taken once only. The maximum reading error in S/P can be taken as 2 per cent, and the reading error of $\overline{\mu/\rho}$ about 4 per cent. If the observations were pushed to the limit at the soft end, (25 minutes duration), the reading error for the last point includes the leak correction reading and amounts to 3 per cent in S/P and up to 5 per cent in $\overline{\mu/\rho}$. Repeated observations were usually but not invariably within these limits. The plotted points are considered to be accurate to 1% for all points except possibly the softest radiation point, and for this a 2% allowance is ample.

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* Cf. date of experiment with date of aperture systems in figures 5 to 8.

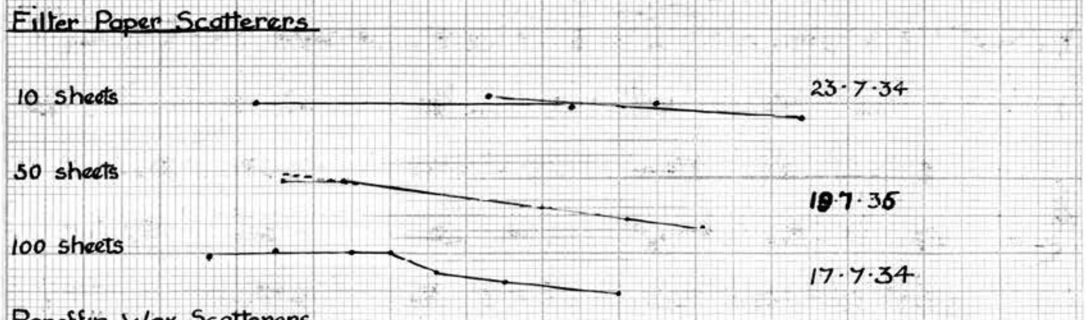

FIG 13. SCATTERING EXPERIMENT.

Results of the experiments.

In the tables of scattering experiments, Table II pp. 113-116 gives an analysis of all the experiments made with wax scatterers. Six different thicknesses were used and the experiments are grouped together first according to scatterer thickness and then according to the X-ray tube. Within each group the experiments are arranged in the order of performance. Table III gives a similar analysis for scatterers of filter paper, grouped first according to scattering thickness and then according to X-ray tube. Within each group the order is that of date of performance except in IIIb and the lower group of IIIc where the experiments are arranged according to the size of X-ray tube currents, with the smallest tube current at the top of the group.

One graph for each scatterer is given in fig.13 by way of illustration and to explain how the analysis given in the tables records the essential facts of the observations. All these graphs are taken from Muller tube experiments using similar currents, so as to be as far as possible comparable with one another. The obvious features of the graph are a horizontal portion for lower values of $\overline{\mu/\rho}$, and at greater values of $\overline{\mu/\rho}$ a portion where S/P decreases for increasing $\overline{\mu/\rho}$. In certain of the graphs the horizontal portion, if it exists at all, does so for so short a range of $\overline{\mu/\rho}$ or for such low values of $\overline{\mu/\rho}$ that it is not observed.

In/

In other graphs not shown in the figure the horizontal portion continues throughout the entire range of observation.

These results are not new, for the scattering experiment has been carried out by many workers over a long period. In their reports, however, emphasis has till recently been placed on the fact of the horizontal portion.[44] In the present view the general shape of the graph is the essential thing. The analysis has therefore been carried out and tabled to show the range of the horizontal portion (column 4), the value of μ/ρ at the change from horizontal line to slope (column 5), the range over which S/P decreases (column 6) and an estimate of the rate of decrease (column 7). In general the slopes observed are small.

In all the experiments of Tables II and III the average penetrating power was changed by varying the primary voltage. Apart from the unavoidable filtering caused by (1) the tube walls, (2) the scatterer, and (3) the relatively small air path (70 cm), the primary beam was unfiltered. The current through the X-ray tube could not be kept at one constant value throughout the range of the experiment because the intensity of the X-ray beam varies enormously when the voltage on the tube is varied, the ionizations for 100 kv. and 30 kv. being approximately in the ratio of 100 to 1. For reasons/

reasons of accuracy, readings taking less than 2 minutes* or more than 25 minutes were not admissable. It was therefore necessary to raise the current when using low voltages, the high currents being usually 3 to 6 times the lower current. So long as the measuring system was operating properly these current changes did not affect the value of S/P on the horizontal portion of the graph, and the change therefore seemed quite permissible, but it appeared that large current variations had a definite influence on the sloping part of the graph.

It was also found necessary, at times, to lower the current at the extreme high voltage point, in order to keep within the saturation potential conditions for the primary ionization chamber. This change of current was slight and had no undesirable influence on the value of S/P.

Before proceeding to the analysis of the observations given in Tables II and III it is felt to be necessary to emphasise that when this work was begun all attempts at controlled experiments had led to inconclusive results. The main aim in the present series of experiments which was carried out under Professor Barkla's supervision was to accumulate a large/

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* or with well filtered beams 80 seconds.

large body of experimental evidence covering as wide a range of conditions as possible. This was done in the hope of discovering significant influences. The straight forward scattering experiment, however, although it was carried out with many variations in the conditions, did not indicate clearly any important influence, and it was from a modified form of the scattering experiment, performed only a few times that a line of development arose. The somewhat inconclusive evidence of the straightforward scattering experiment will be discussed before the modified experiment is reported.

I. THE RESULTS OF THE EXPERIMENT DEPEND ON THE TUBE CURRENT.

When the results of the scattering experiment were analysed and grouped according to (1) scatterer material (2) scatterer thickness and (3) tube, it was evident that within certain of the large groups there was considerable variation. Especially in the largest group of all - in Table IIIb - Scatterer, Filter paper, thickness, 50 sheets, Tube, Muller (2) the graphs varied from entirely horizontal to entirely sloping. When this group was arranged roughly in order to tube currents it was seen that all the entirely sloping graphs were/

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were obtained with low currents. With medium and high currents all graphs showed a horizontal portion, only four out of the ten graphs showing a sloping portion. Other groups were therefore examined for signs of a similar tendency.

The groups containing suitable material for this examination were:

	TUBE	SCATTERER
(1)	Muller (2)	Filter Paper 10 sheets
(2)	Muller (2)	Filter Paper 50 sheets
(3)	Andrews (2)	Filter Paper 50 sheets
(4)	Muller (2)	Filter Paper 100 sheets.

In group (1) the scatterer was so thin that low currents were not practicable. Moreover the interpretation was difficult in this group because almost invariably there was a high point in the graph, as noted in Table IIIa. and illustrated in the first graph of figure 13. Such high points were not observed in any other group and it is thought that they may be connected with the thinness of the scatterer. Group (1) experiments could therefore not be used. In groups 2. 3. and 4. it is seen that with the lower currents the sloping portion of the graph develops at a lower value of $\overline{\mu/\rho}$.

2./

2. Muller tube (2) and scatterer of 50 sheets of filter paper.

With tube currents ranging from about 0.2 to 1.5 ma, the three experiments show no horizontal portion.

With medium currents of about 0.4 to 6.0 ma, a definite extended horizontal portion is seen in all six experiments up to μ/ρ = 3.25 at least, in one case the limit is at 4.4, in two cases there is no sign of the break as far as μ/ρ = 5.0, and in one, none as far as μ/ρ = 5.75.

With high currents reaching 8 or 9 ma, breaks are seen at 4.0(?) in one experiment, at 5.0 in another, and, in the other two, no bend is seen although the graphs extend to μ/ρ = 4.9 and μ/ρ = 6.25.

3. Andrews tube (2) and scatterer of fifty sheets of filter paper.

The 2 experiments with lower currents show a break at μ/ρ = 3.5. In 3 experiments with medium currents no break is seen up to μ/ρ = 5.1, 5.4 and 7.0 respectively.

4. Muller tube (2) and scatterer of 100 sheets of filter paper.

In 3 experiments with small currents and 2 experiments with medium currents the break occurs at 2.5. One experiment was horizontal up to 2.5 and not carried further. Two experiments with large tube currents showed/

showed the break at $\overline{\mu/\rho}$ = 3.25 and 3.55 respectively.

These comparisons suggest that there is a tendency for the curve to break into a slope at lower values of $\overline{\mu/\rho}$ as the tube currents are reduced.

II. THE RESULTS OF THE EXPERIMENTS DEPEND IN PART ON THE X-RAY TUBE.

To show that this is the case, experiments must be carried out with different tubes, other conditions being maintained as far as possible the same. This at once raises difficult questions, for while it is a straightforward matter to conduct the comparison with identical scatterers and with tubes having anticathodes of the same atomic number, it is not possible to say what current for the one tube gives comparable conditions with a definite current for the second tube. The only possible procedure is to compare different tubes working under their own average operating conditions. Comparison of the results obtained with the Andrews tube (2) and the Muller tubes (1) or (2) is obtained from Tables IIa, IIIb and IIIc.

From/

From Table IIa. Scatterer - Paraffin Wax,
Thickness 0.3 cm.

2 Experiments with Andrews tube.(2)	S/P constant throughout the observed range $2.40 < \overline{\mu/\rho} <$ 6.25.
4 Experiments with Muller tube (2).	S/P decreasing throughout the observed range $2.25 < \overline{\mu/\rho} <$ 5.1.

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From Table IIIb. Scatterer - Filter Paper,
50 sheets. Only the low current values
lead to distinct differences.

2 Experiments with Andrews tube (2)	S/P constant to $\overline{\mu/\rho}$ = 3.5.
3 Experiments with Muller tube (2)	S/P not constant above $\overline{\mu/\rho}$ = 2.2. and possibly not above 1.8

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From Table IIIc. Scatterer - Filter Paper,
100 sheets.

6 Experiments with Andrews tube (2)	S/P constant up to $\overline{\mu/\rho}$ = 4.0 or greater. In 3 out of 4 experiments, S/P decreasing after $\overline{\mu/\rho}$ = 4.5.
8 Experiments with Muller tube (2)	S/P not constant beyond $\overline{\mu/\rho}$ = 2.5 unless very large currents are used and even then not constant above 3.5.

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Unfortunately full details about the material and construction of the tubes are not available. The best guide to the difference in performance between the Andrews tube and the Muller tube is to plot $\overline{\mu/\rho}$ against tube kilovolts for each in turn. Two such graphs are given/

FIG 14 (a)

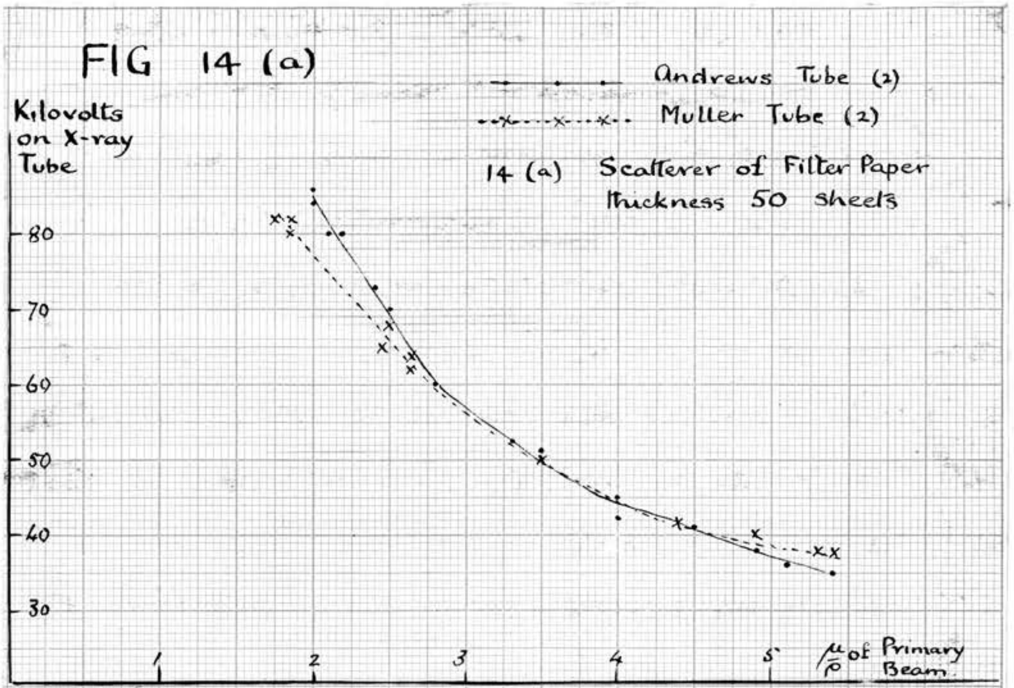
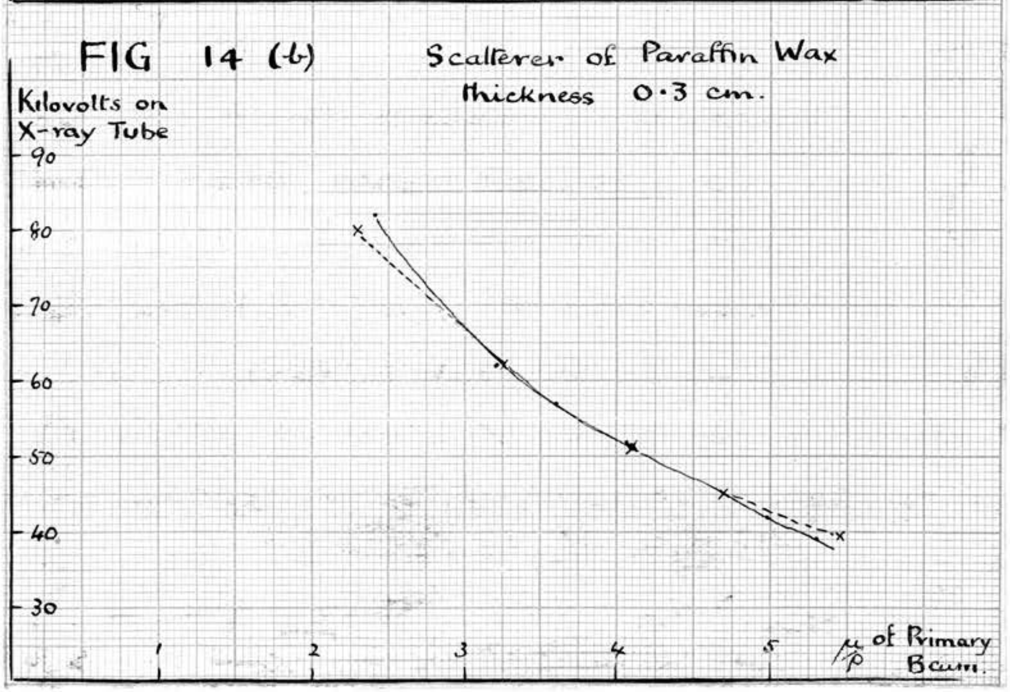


FIG 14 (b)



given in figures 14(a) and 14(b). They show that the Andrews tube is comparatively 'softer' than the Muller at the 'hard' end and 'harder' than the Muller at the 'soft' end. The difference between the tubes is, however, slight, and apparently negligible in the middle range of kilovoltage. The shape of the scattering graph is therefore not thought to be determined by differences in the penetrating power of radiation from the two tubes when working at the same kilovoltage.

The origin of these differences was not further investigated.

> The second conclusion is that the limit of constant S/P depends in part on the X-ray tube employed. It is always at a lower value of $\overline{\mu/\rho}$ for the Muller tubes than for the Andrews tube.

III. THE RESULTS OF THE EXPERIMENT DEPEND ON THE THICKNESS OF THE SCATTERER.

The influence of scatterer thickness is seen by comparing the results obtained using a particular bulb and a particular scattering material, and a series of different thicknesses. The following tables are extracted from Tables II and III pp. 113-119

Taking the results of filter paper scatterers first, the grouping according to the size of tube currents, we have the following comparisons.

SCATTERERS OF FILTER PAPER, MULLER TUBE (2), HIGH CURRENTS.

50 sheets 0.32 gm/sq.cm.	4 experiments	S/P constant to μ/ρ = 4 or higher.
100 sheets 0.64 gm/sq.cm.	2 experiments	S/P constant to μ/ρ = 3.25 or 3.55.

SCATTERERS OF FILTER PAPER, MULLER TUBE (2), MEDIUM TUBE CURRENTS.

50 sheets 0.32 gm/sq.cm.	6 experiments.	S/P constant to μ/ρ = 3.25 or higher.
100 sheets 0.64 gm/sq.cm.	2 experiments	S/P constant to μ/ρ = 2.5.

SCATTERERS OF FILTER PAPER, MULLER TUBE (2), LOW CURRENTS.

50 sheets 0.32 gm/sq.cm.	3 experiments	S/P not constant above μ/ρ = 1.8 in one 2.2 (?)
100 sheets 0.64 gm/sq.cm.	4 experiments	S/P constant to μ/ρ = 2.5

SCATTERERS OF FILTER PAPER, ANDREWS TUBE (2), MEDIUM CURRENTS.

50 sheets 0.32 gm/sq.cm.	3 experiments	S/P constant over the observed range to μ/ρ = 5.1, 5.4 and 7.0 respectively.
100 sheets 0.64 gm/sq.cm.	4 experiments	S/P constant to μ/ρ = 4, and in one case to 5.4.

As has already been shown, the limit of constant S/P comes at a higher value of $\overline{\mu/\rho}$ when the current is increased. The above table shows that for a given current this limit tends to occur at a higher value of $\overline{\mu/\rho}$ for the thinner scatterers. Using the Andrews tube (2), the limit of constant S/P is reached for the thicker scatterer only. The exception seen in the low current observations with the Muller tube will be discussed later. Similar results are seen when using paraffin wax scatterers:-

SCATTERER OF PARAFFIN WAX. MULLER TUBE (2)
PRIMARY APERTURE 6.55 mm. ROTATING SHUTTER IN USE.

1.1 cm. 1.01 gm/sq.cm.	3 experiments Low and medium currents.	S/P constant to $\overline{\mu/\rho}$ = 3.0.	
1.9 cm. 1.77 gm/sq.cm.	3 experiments Low and medium currents.	S/P decreasing from the first point at $\overline{\mu/\rho}$ = 1.8.	Average slope - 3.4.

SCATTERER OF PARAFFIN WAX, ANDREWS TUBE (2)
MEDIUM CURRENTS.

0.3 cm. 0.27 gm/sq .cm.	2 experiments	S/P constant to last observed point at $\overline{\mu/\rho}$ = 5.0 to 6.0.	
1.25 cm. 1.15 gm/sq.cm.	2 experiments	S/P decreasing from the first observed point at $\overline{\mu/\rho}$ = 1.8.	Average slope - 2.8.

SCATTERER OF PARAFFIN WAX. COOLIDGE TUBE.
MEDIUM CURRENTS.

0.3 cm. 0.27 gm/sq.cm.	2 experiments	S/P constant to $\overline{\mu/\rho}$ = 3.5 to 5.0
0.85 cm. 0.79 gm/sq.cm.	1 experiment	S/P decreasing from the first observed point at $\overline{\mu/\rho}$ = 2.9.
1.1 cm. 1.01 gm/sq.cm.	1 experiment	S/P constant to $\overline{\mu/\rho}$ = 2.3

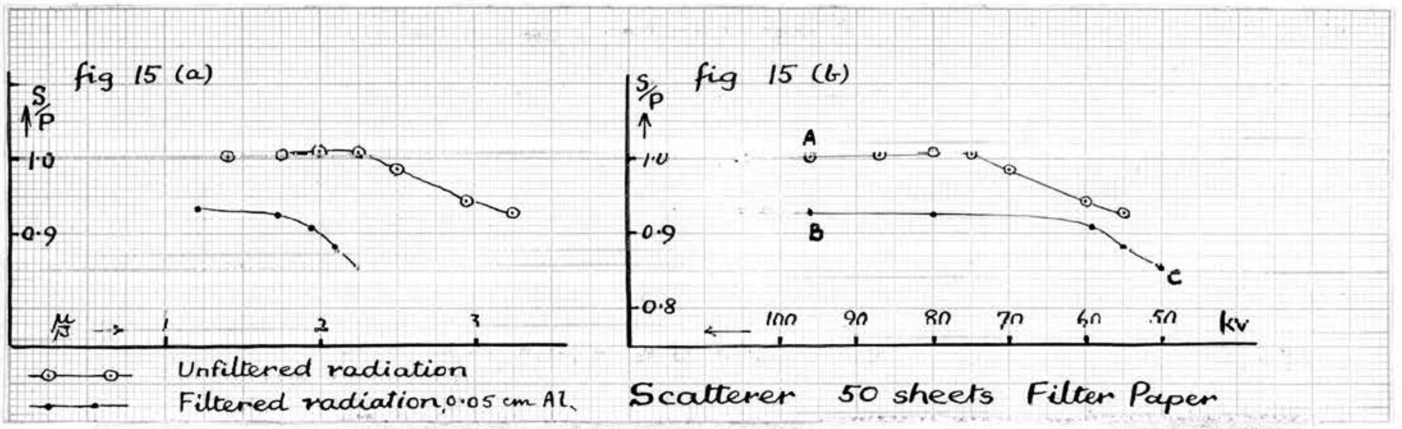
Working with a paraffin wax scatterer the experiments carried out with the Andrews tube and the Coolidge tube agree in showing more extended horizontal portions with the thinner scatterers. In the Muller tube experiment the graphs are all sloping, and no comparison is possible about the particular values of where the slope begins.

<u>The findings in this section are as follows:</u> Six out of seven comparisons consistently show the limit of constant S/P at a higher value of $\overline{\mu/\rho}$ for thinner scatterers than for thicker scatterers. In the seventh comparison, that in which filter paper, Muller tube and low currents were used, (i) the observations are/

are apparently inconsistent with the other six sets of results, (ii) the observations are nevertheless well substantiated since there are several experiments with each of the two thicknesses, (iii) a possible explanation of the apparent inconsistency is that where there are so many influences at work, it is still possible that some variable other than the scatterer thickness is here the more influential one.

In addition to the agreement shown in the comparisons, there is a general tendency for the graph to slope when thick scatterers are used. This bears out very well the view that increasing the thickness of the scatterer makes the limit of constant S/P change to a lower value of $\overline{\mu/\rho}$. A wide range of surface densities has been used, from 0.27 to 1.77 gm/sq. cm. and with the highest surface densities no observations of a region of constant S/P could be obtained.

This completes the account of the observations on the simple scattering experiment. We pass now to consider a modified form of this experiment in which the primary beam was filtered before reaching the scatterer.



KV.	$\overline{\mu}/\rho$	FILTERED S/P.	FILTERED AND INTERCEPTED S/P.
96	1.22	0.933	0.883
80	1.73	0.926	0.882
59	1.95	0.909	0.876
55	2.10	0.882	0.847
50	2.25	0.855	0.826

KV.	$\overline{\mu}/\rho$	UNFILTERED PRIMARY S/P.
96	1.4	1.002
87	1.75	1.004
80	2.0	1.010
75	2.25	1.007
70	2.5	0.984
60	2.95	0.943
55	3.25	0.928

IV. THE EFFECT OF FILTERING THE X-RAY BEAM BEFORE IT REACHES THE SCATTERER.

Although only a few experiments with filtered rays were made, they have that greater reliability which has often been remarked upon by experimenters using filtered beams. The separate readings seem to be more consistent than do those made with unfiltered radiation. The importance of these experiments lies in the fact that by their somewhat striking results the main clue to the problem under investigation was suggested.

1. The first experiment was carried out on 7.7.33 with 50 sheets of paper and with Coolidge tube operated at 1 ma; ionization chambers were in use. The beam was filtered by 0.05 cm. aluminium placed at aperture BB (fig. 7). On the two immediately preceding days the experiment using unfiltered beams had been performed with the same current, 1 ma, the results on these two days being consistent. No change of any kind had been made in the tube or scatterer.

The graphs are compared in fig. 15(a) where μ/ρ is abscissa and in fig. 15(b) where kilovoltage on the X-ray tube is abscissa. The contrast between the filtered and unfiltered beams is clear and the graphs show that S/P is constant to about μ/ρ = 2.25 for unfiltered and to about 1.8 for filtered radiation. When the results are/

KV.	$\overline{\mu}/\rho$	UNFILTERED PRIMARY S/P.
80	1.97	1.000
72	2.27	0.967
64	2.75	0.973
52	3.55	0.942
46	4.15	0.946
40	4.7	0.933
34	5.4	0.907

KV.	$\overline{\mu}/\rho$	FILTERED S/P.
90	1.4	0.93
68	2.05	0.90
57	2.5	0.88
41	3.55	0.823

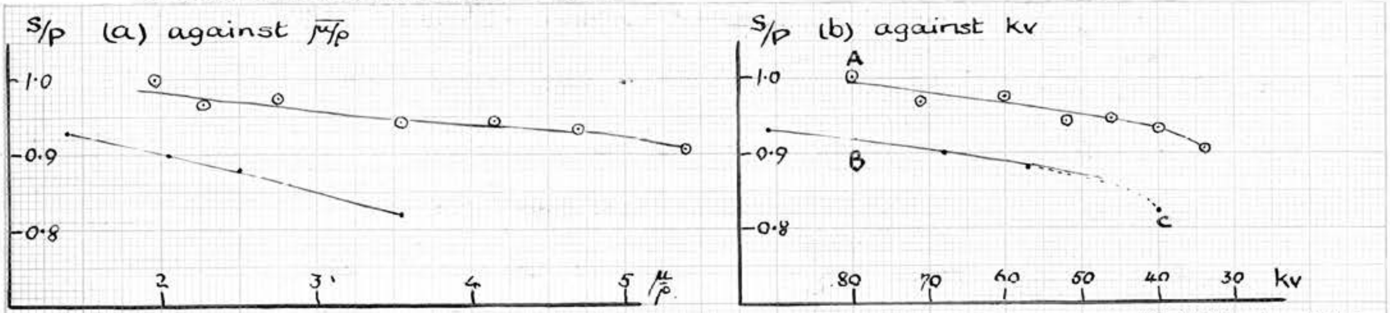


FIG 16 Scatterer Paraffin Wax, thickness 1·25 cm.

-o—o- Unfiltered radiation
—•—•— Filtered radiation 0·059 cm Al.

FIG 17(a)

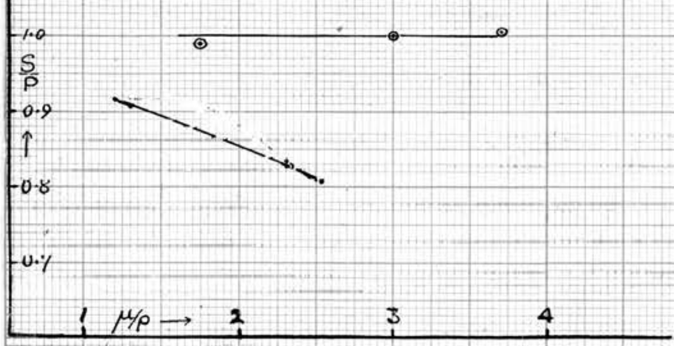
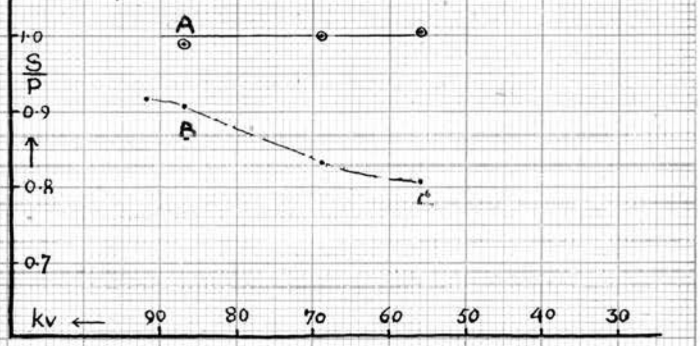


FIG 17(b)

3:6:35



-o—o- UNFILTERED RADIATION
—•—•— RADIATION FILTERED BY 0·125 CM. Al.

SCATTERER FILTER PAPER 10 SHEETS
MULLER TUBE (2) TUBE CURRENT 1-4 milliamperes.

DATA

UNFILTERED RADIATION

KV ON TUBE	$\overline{\mu}/\rho$	S/P
87	1·75	0·990
69	3·00	1·000
56	3·70	1·005

FILTERED RADIATION

KV ON TUBE	$\overline{\mu}/\rho$	S/P
92	1·2	0·916
87	1·3	0·906
69	2·3	0·832
56	2·55	0·807

are plotted against kilovoltage on the tube, S/P is constant to about 75 kilovolts for unfiltered beams, and for the filtered beams a rapid decrease in S/P sets in about 60 kv.

2. On 7.5.34 a filtered radiation experiment was performed using a paraffin wax scatterer 1.25 cm. thick (1.15 gm/sq.cm) and the Andrews tube (2). On the two immediately preceding experimenting days the experiments had been conducted with unfiltered radiation under exactly similar conditions and with consistent results. No alteration in tube or scatterer was made, but for the experiment with filtered radiation a thickness of 0.059 cm. aluminium was inserted at BB (fig. 7). With the unfiltered radiation the limit of constant S/P was already at too low a value of $\overline{\mu/\rho}$ to be observed. The influence of filtering on the sloping portion of the graph is seen in fig. 16. When plotted against $\overline{\mu/\rho}$, S/P for the filtered beams showed an increased slope throughout the observed range, but plotted against kilovoltage the slope is somewhat lessened above 56 kv and increased below this limit.

3. Two further experiments of this kind were made. On 3.6.35 in the afternoon, an experiment with filtered primary beams was conducted using a filter paper scatterer, 10 sheets thick, the Muller tube (2) and a filter of 0.125 cm. aluminium. On the morning of that day, the unfiltered radiation had been used under identical conditions. The two experiments of 3.6.35 are shown in

fig./

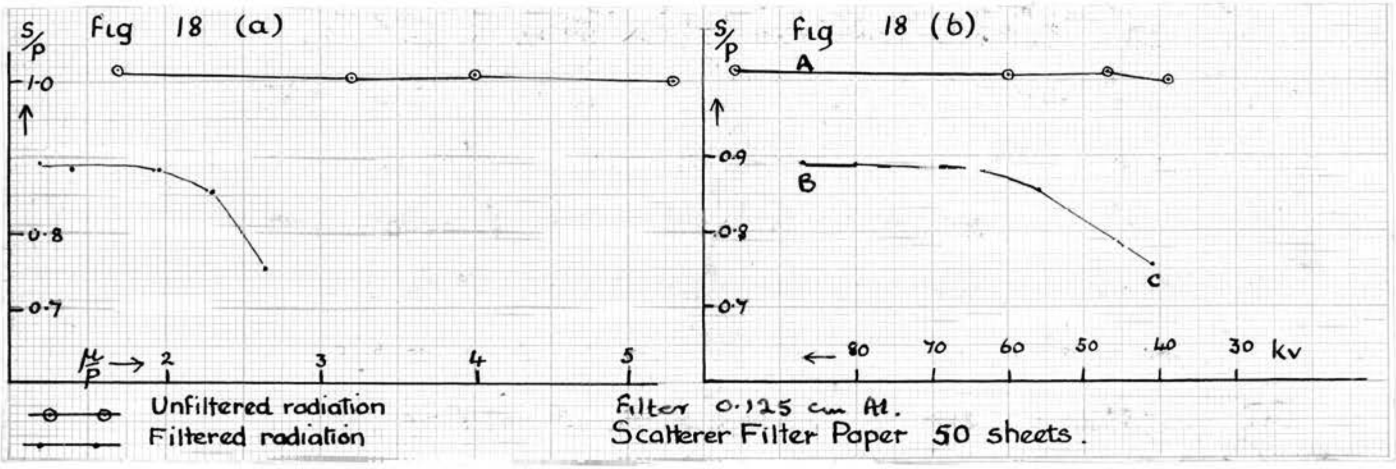


Fig 18 (a) Fig 18 (b)

⊙—⊙ Unfiltered radiation
•—• Filtered radiation

Filter 0·125 cm Al.
Scatterer Filter Paper 50 sheets.

Kv	$\overline{\mu/\rho}$	Unfiltered Primary S/P.	Kv.	$\overline{\mu/\rho}$	Filtered S/P.
96	1.7	1.017	87	1.2	0.906 0.897
60	3.2	1.005	80	1.4	0.89
53	4.0	1.009	69	1.96	0.888
39	5.3	1.00	56	2.3	0.858
			41	2.65	0.755

fig. 17(a) plotted against $\overline{\mu/\rho}$ and in fig. 17(b) plotted against kilovoltage. In this experiment the slope does not appear on the graph for unfiltered radiation while it does so on the graph for filtered radiation.

4. On the following day, 4.6.35 the experiment was carried out with 50 sheets of filter paper as scatterer, with the Muller tube (2) unaltered in any way, and with a filter 0.125 cm. of aluminium. This experiment shows a drop of fully 16½ per cent in the value of S/P for 41 kv as compared with that for 68.5 kv; the corresponding change in $\overline{\mu/\rho}$ from 2.65 to 1.96 being small and equivalent to monochromatic beams of $\overline{\mu/\rho}$ = 0.56 and $\overline{\mu/\rho}$ = 0.5 AU.

Discussion of foregoing results.

These observations on filtered primary rays throw light on the problem of the simple scattering experiment. This experiment, especially in the range of observation which yields constant values of S/P has been taken by all workers in this research school to give a true law of scattering for variation of penetrating power of heterogeneous beams. Assuming that the experiment does so, an interpretation has to be obtained for the experiments with filtered radiation which are now under discussion.

evidence? X

The observations show that if the radiation produced with highest voltage is filtered before the scatterer/

scatterer, the scattering ratio falls (from point A to point B in the figures). The filtering removes a large part of the softest rays. A fall of scattering ratio resulting from this hardening process is in accordance with accepted theory, for, as Table VIII shows, (a) total 90° scattering (modified and unmodified) per unit primary intensity decreases slightly as wave length decreases and (b) the proportion of modified scattering increases as wave length decreases. (b) implies a reduction of S/P because in practice the secondary beam is subject to filtering subsequent to scattering and before reaching the gas in the ionization chamber.

If we pass down the filtered radiation graph from B to C in the figures, that is, from the highest to the lowest voltage point, the primary beam is deprived of its hardest constituents and the remaining softer radiations must have on the whole higher 90° scattering per unit primary intensity and a smaller proportion of modified scattering. The scattering ratio ought to rise on account of both changes. But it does not, and in fact falls by a considerable amount; the least observed fall was 8% and the greatest 15%.

If on the other hand we discard accepted theory and look for some other explanation, there is only the view put forward by Barkla that heterogeneous beams do/

do not obey the same laws of scattering and absorption as homogeneous beams. Applying this to the observations it appears that the most homogeneous beam has the smallest scattering ratio. But the laws of scattering which are known to be obeyed by homogeneous beams were first established by Barkla using heterogeneous beams: discrepancies of 8 to 15% in the observations would not have been passed by him.

It is therefore urgently necessary to enquire whether there is not a disturbing influence at work in the scattering experiment which is accentuated when the rays are filtered. The essential question is what influence besides (a) and (b) above can cause the ratio of S/P to fall with filtering before the scatterer. We look for an influence that becomes more potent when the voltage on the tube is lowered. Polarization in the primary beam was thought to be a possible influence. It should be noted that all experimental work was done with the apparatus so oriented that the scattered beam was observed in a direction perpendicular to the plane of primary polarization.

Further, if it can be established that polarization is present to such an extent that it influences the value of S/P, then since observable polarization is known to vary with voltage on the tube, the scattering experiment as performed can no longer be taken to give the fundamental law of scattering.

V. THE TEST FOR THE INFLUENCE OF PRIMARY POLARIZATION.

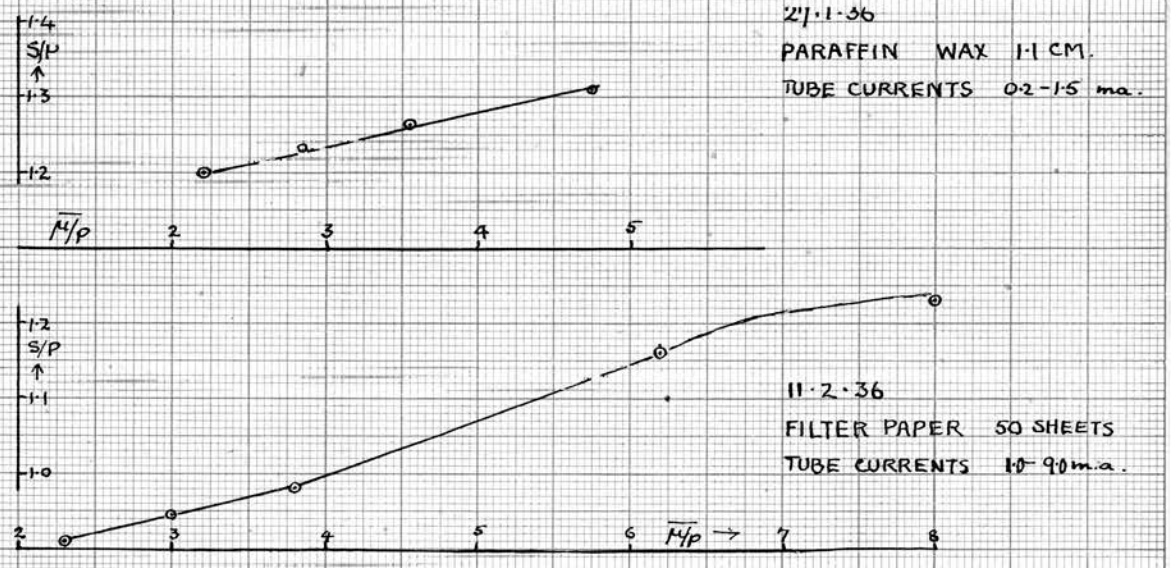
No further progress could be made in the investigation until a test had been carried out for the hypothesis that the observed ratio S/P was influenced by polarization present in the primary beam. There are two ways of conducting such a test. (i) to keep the source of X-rays fixed and rotate the measuring system and (ii) to keep the measuring system fixed and rotate the source. The first method is greatly to be preferred if the whole measuring apparatus can be designed from the start to be rotated. If this is not practicable, the next best degree of precision is obtained by rotating the X-ray tube. Since the measuring system was in good order, but fixed, the easiest way to adapt the apparatus for the test was to alter the mounting of the X-ray tube so that its axis could be rotated about the primary X-ray beam and firmly pinned in position at 0°, 45° and 90° to the horizontal. Since the constitution of the primary beam varies with direction from the focal spot, the axis of rotation must coincide with the ray passing through the small primary aperture CC (fig. 8). To ensure that the primary ray remained as far as possible unaltered, a ray direction had to be fixed by marking the outside of the X-ray tube. Various methods were tried, the most satisfactory being to mark a 'point' with either a fine cotton thread cross or a fine ink circle. The alignment could then be tested optically by switching on the tube filament to act as a source of light. The slightly scarred surface of the focal spot on the anticathode scatters the light and looks bright. The X-ray tube in its rotating holder was placed so that, seen through the primary aperture, the thread cross appeared central on the focal spot as a bright background. The whole was viewed by a telescope (with cross wires in the eyepiece) focussed upon the surface mark. The orientation of the axis of rotation was deemed satisfactory if the line defined by the two crosses (in the telescope eyepiece and on the tube surface) appeared to pass through the centre of the bright spot, both when the tube axis was horizontal and when (after rotation through 90°) it was in the vertical position. Using this method of alignment and a primary aperture CC fig. 8, of 1.2 mm diameter certain variations in the values of S/P, attributable to imperfect alignment and amounting to at most ± 3 per cent, were found to occur. These were relatively unimportant, because they did not affect the general shape of the graph. Larger errors from this source had indeed been anticipated.

In January 1936 an attempt was made to reduce residual errors caused by imperfect alignment of the axis of rotation. The primary aperture was increased in/

in diameter from 1.2 mm. to 6.55 mm. To ensure saturation potential conditions being maintained in the primary ionization chamber, the tube currents were kept to the lower ranges and the former ratio of ionization currents in primary and secondary chambers was restored by using an interrupter in the primary beam, mounted between CC and the chamber. This took the form of a rotating shutter which permitted only four or five per cent of the radiation to pass through to the primary chamber. The shutter introduced a new kind of error, a variability in the individual observations due to effects of synchronism between the speed of rotation of the shutter and the mains frequency (the high tension in the X-ray tube being unrectified). This proved to be a most annoying trouble, the only cure being to keep varying the speed of rotation of the shutter. Since the shutter was driven by a hot air engine, a gradually changing frequency was possible. The compensating advantage was that a fairly good estimate of total polarization could be made. After this change the alignment errors appeared to be about ±1 per cent of S/P.

The simplest test for the presence of polarization was to carry out the scattering experiment for unfiltered radiation with the X-ray tube axis in a vertical position, i.e. the cathode stream vertical (the plane of/

FIG 19 SCATTERING EXPERIMENT WITH THE TUBE VERTICAL



27·1·36

PARAFFIN WAX 1·1 CM.

TUBE CURRENTS 0·2-1·5 ma.

11·2·36

FILTER PAPER 50 SHEETS

TUBE CURRENTS 1·0-9·0 m.a.

of observation being a fixed horizontal one). If the graph of S/P against $\overline{\mu/\rho}$ was then different from that when the tube was in the normal (i.e. horizontal) position, polarization was present. The first set of observations showed markedly the effect of tube orientation. Altogether 18 experiments were performed with the tube vertical. In no case was S/P found to be constant over a range of $\overline{\mu/\rho}$ but invariably increased as $\overline{\mu/\rho}$ increased.

Sometimes the graph was a straight line and sometimes it bent upwards as $\overline{\mu/\rho}$ increased (fig. 19). The latter was thought to be a possible counterpart to the downward bend observed with the tube horizontal. The eighteen experiments were therefore analysed to show the ranges of $\overline{\mu/\rho}$ observed and the corresponding rates of increase of S/P (see Table IV pp. 120-1)

These experiments were conducted exactly as were the original scattering experiments except for the difference in tube orientation. No change at all was made in the measuring system. Careful attention was given to alignment. Under these conditions any differences introduced into the observations by the tube rotation must be attributed to polarization in the primary beam. Here we are not concerned with a difference on a small scale. The over all increase in S/P in an experiment with the tube vertical varied from 5% to/

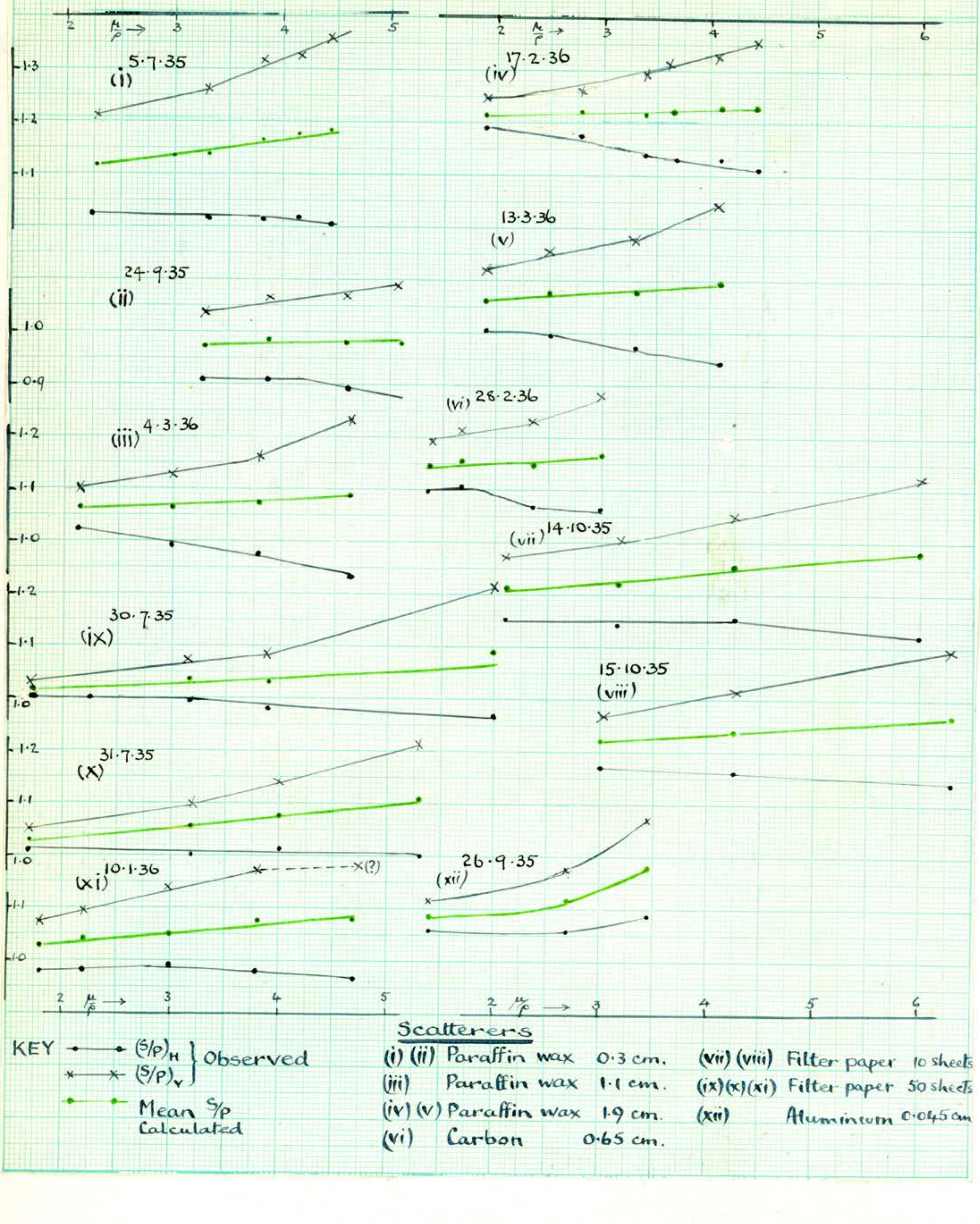
to 20%, values in every case greater than could be accounted for in terms of experimental error.

VI. COMPARISON OF SCATTERING PARALLEL AND PERPENDICULAR TO THE CATHODE STREAM.

The question of the observability of polarization effects in the scattering experiment was not now doubted. It was however desirable to compare the scattering curves with the tube first in one position and then in the other. Such an experiment was not easy to conduct because of the limited time available in a working day. The alignment of the tube had to be examined before each part of the experiment. Although great care was taken with alignment, trouble sometimes developed on account of the tube slipping slightly after it was set in the vertical position, apparently when the high tension was first put on. This shift was never observed in the horizontal position. A check on the alignment was always made after a run of observations as well as before the run, but it was found that if observable slip had taken place, it showed itself in the readings. An example of this will be shown in the next section. Any trouble of this kind prevented the successful completion of the day's work.

It was also necessary to work as quickly as possible over both parts of the curve in the hope of finding/

FIG. 20. POLARISATION CORRECTION EXPERIMENTS.



KEY
•——• $(S/P)_H$ } Observed
×——× $(S/P)_v$ }
•——• Mean S/P Calculated

Scatterers

(i) (ii)	Paraffin wax	0·3 cm.
(iii)	Paraffin wax	1·1 cm.
(iv) (v)	Paraffin wax	1·9 cm.
(vi)	Carbon	0·65 cm.
(vii) (viii)	Filter paper	10 sheets
(ix)(x)(xi)	Filter paper	50 sheets
(xii)	Aluminium	0·045 cm

finding time at the end of the day to check up on the first point of the first graph. This necessitated a third setting of the tube and was possible on only a few occasions. The range of variation of $\overline{\mu/\rho}$ was naturally restricted, readings taking more than 15 minutes being prohibitively long.

The observations in all twelve experiments of this group are given in Table V. The ratios $(S/P)_H$ and $(S/P)_V$ are the values of S/P when the tube axis is horizontal and vertical respectively, and the scattering parallel and perpendicular, respectively, to the cathode stream. The twelve graphs are shown in fig. 20.

For reasons already explained, the accuracy of the points is good in graphs i, ii, vii, viii, ix, x and xii, the accuracy of separation between the $(S/P)_V$ and $(S/P)_H$ graphs is good in graphs iii, iv, v, vi and xi. The points in graph xi are particularly badly affected by stroboscopic error.

In the experiments with thick scatterers, 1.1 cm. and 1.9 cm. Paraffin Wax, $(S/P)_H$ is sloping steeply and shows no sign of constancy except possibly at the very beginning of graph iv. In these graphs $(S/P)_V$ slopes steeply upward. A bend in $(S/P)_V$ (Graph iii) appears. Graph viii for 10 sheets of paper also shows no horizontal part of $(S/P)_H$ but here $\overline{\mu/\rho}$ is not taken to low values. All other graphs show $(S/P)_H$ constant over a certain/

certain range. The accuracy of the experiments is not sufficient to determine whether or not the bend occurs at the same values of $\overline{\mu/\rho}$ in $(S/P)_H$ and $(S/P)_V$. To determine this it is desirable to use a more accurate method. Suggestions regarding this will be made at a later stage.

It follows from the theory given on pp.26-30, that the fraction of polarized energy and the value of S/P corrected for polarization can both be obtained from these observations, provided that the beams measured in the two positions of the tube are properly comparable, i.e. if the axis of rotation of the tube truly coincides with the ray through the centres of the primary apertures. The polarized energy is $\frac{(S/P)_V - (S/P)_H}{(S/P)_V + (S/P)_H}$, and the corrected value of S/P is $\frac{1}{2}\{(S/P)_V + (S/P)_H\}$.

If however the alignment is not quite perfect, the secondary beams in the two cases are standardised against slightly different primaries, the secondaries themselves being inappreciably affected since the secondary aperture is large. An error of a few per cent will therefore enter the measurement of $(S/P)_V$ relative to $(S/P)_H$. Since $(S/P)_V$ and $(S/P)_H$ are nearly the same, the polarized energy, depending on their differences will be rather inaccurate, but the corrected value of S/P will only carry half the error of $(S/P)_V$. The mean value of S/P is calculated in the tables and plotted/

plotted on the graph. These mean values suggest that the true S/P value increases very slowly as $\overline{\mu/\rho}$ increases, at a rate whic
is within experimental error independent of the slope of $(S/P)_H$.

The experiments of the present section provide a better test of the observability of primary polarization than do those of the preceding section. Besides showing the different shapes of the two graphs they show $(S/P)_V > (S/P)_H$. They also give an (S/P) graph corrected for polarization.

Regarding the accuracy of the observations the following points are of importance. In the early experiments with small aperture a repetition of the process of tube alignment and reading of S/P gave values varying from the mean S/P by ± 3%. This error affects only the absolute magnitude of S/P and not appreciably the shape of the graph. But it also affects the separation between the graphs of $(S/P)_V$ and $(S/P)_H$. This shows in measurements of fractional polarization, which vary from 1.3% to 8% at the hard end for small aperture observations. This corresponds to observed separations of 2.6% and 16.5% or a reading variability of ± 3.5%. This is in agreement with the resetting variability.

In January 1936 the large aperture was introduced to reduce this source of error. Repetition of alignment then gave S/P constant to ± 1%, i.e. of the order of accuracy of the observational error with the tube fixed.

A/

A table of the polarization observations made with this aperture is given here abstracted from Table V.

Scatterer	Paraffin Wax.			Carbon.	Filter Paper.
Thickness	1.1 cm.	1.9 cm.	1.9 cm.	0.65 cm.	50 sheets
Experiment No.	III	IV	V	VI	XI
Tube Kilovolts					
100	-	-	-	4	5
80-90	4	3	5	5	5
59-64	6	5	8	7	6
50-52	9	9	10	9	9
45	-	10	-	-	-
43	-	-	-	-	10
40	14	11	15	-	-
33	-	13	-	-	-

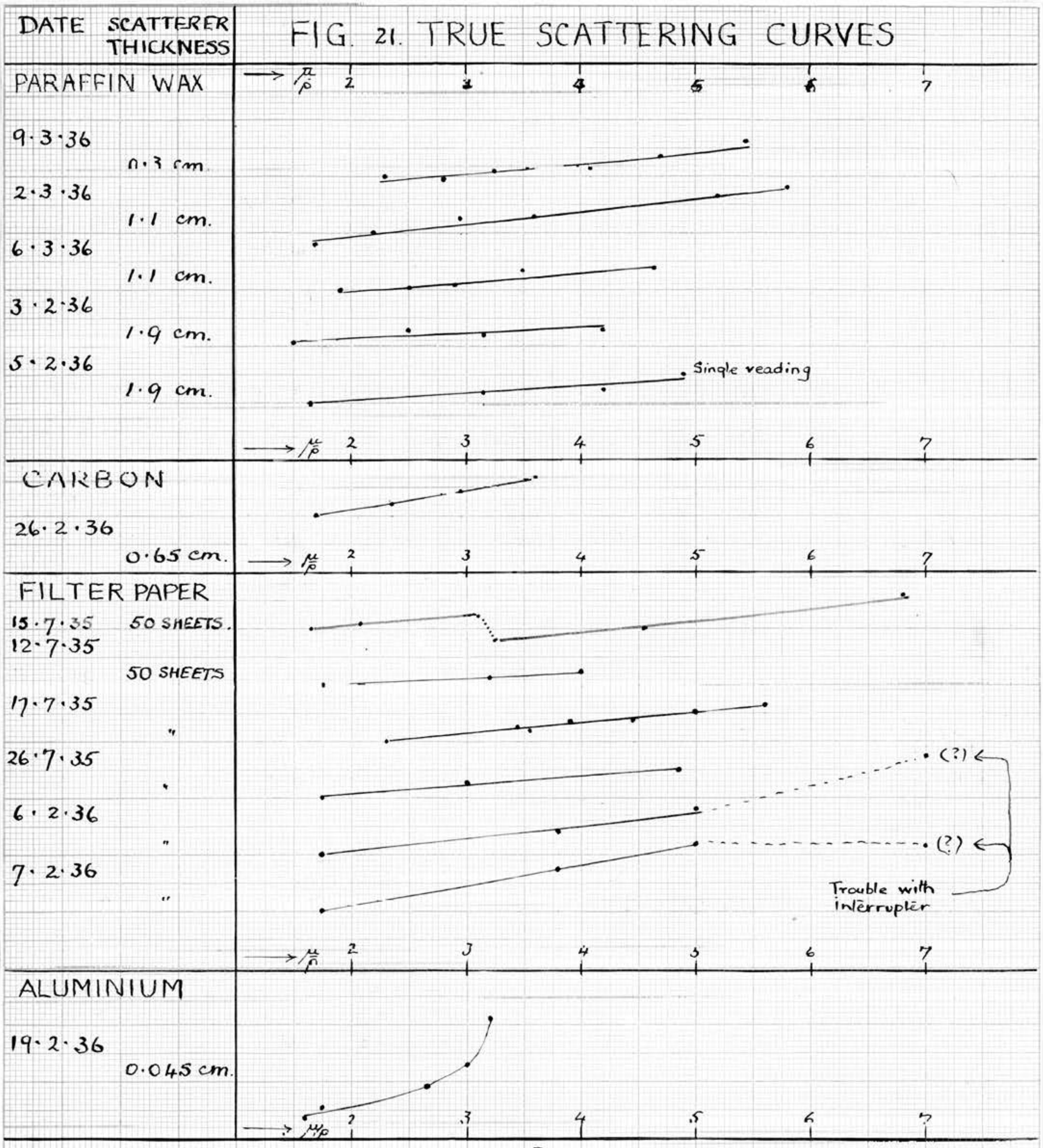
Polarization values for 80-90 tube kilovolts give separations between 6% and 10% corresponding to reading variations of ± 1% agreeing with the previous estimate.

The large aperture polarization values are therefore considered to be accurate to ± 1% at the hard end, and not quite so accurately at the soft end.

It is clear from figure 20 and Table V. that variation of scatterer thickness does not result in variation of corrected S/P within the experimental limits of accuracy. One tube only has been used. Significant variations in tube currents have not been made.

VII./

FIG. 21. TRUE SCATTERING CURVES



The vertical scale for S/p is in each case 1 cm = 0·1 units

The observed ratio S/p is reduced to the same scale in all cases by making S/p at the low absorption coefficient end equal to unity

VII. THE TRUE SCATTERING OBSERVED AT 45° TO THE CATHODE STREAM.

By far the most accurate method of observing the true scattering curve is to make the observations in such a plane that the polarization does not cause errors in the scattering curve, as explained in the theoretical section of this report, p.30 . To do so the axis of the X-ray tube must be set in a plane perpendicular to the primary beam, at an angle of 45° to the plane of observation and with the focal spot of the tube centred in that plane. 13 experiments of this kind were made. The results are tabulated on p.126 Table VI and graphed in fig. 21. Four of the experiments were conducted with 1.2 mm. primary aperture and 9 with 6.5 mm. aperture and rotating shutter. These will now be separately discussed for accuracy.

Small aperture experiments.

One of these (15.7.35) shows a break at the change of current. After completing the run on this day the alignment was found to be faulty, and this was considered to be a likely reason for the break. If the alignment was imperfect for the primary beam, a small increase in the size of the focal spot accompanying the increase of current would account for the drop in S/P. This type of error is associated with the use of a small aperture.

Large/

Large aperture and rotating shutter experiments.

These were done on days in 1936 when time was too short for an experiment with tube both vertical and horizontal. Stroboscopic error was involved and shows itself markedly in 6.2.36 and 7.2.36. Attention was given to keep this source of error in check. The rotating shutter was driven by a small hot air engine heated by bunsen. The speed of shutter rotation was constantly varied by varying the heat. This kept the stroboscopic error relatively small. The test for this is the standard deviation of observations for single points. For example, the standard deviations for the points of the first graph of figure 21 (9.3.36) are given in the accompanying table.

$\overline{\mu/\rho}$ value of point.	No. of Readings.	Percentage Standard Deviation of observations.
2.3	3	2.3
2.8	2	0.9
3.25	4	0.2
4.1	4	1.2
4.7	4	0.6
5.45	2	0.4

In practice readings at a single point had a standard deviation not greater than 2.5% and usually not greater than 1.5%. Under these conditions the stroboscopic error can be regarded as small and the observations/

observations reliable to about 2% over the main part of the graphs with rather higher errors for very soft radiation (i.e. $\overline{\mu/\rho} > 6.$) The accuracy is thought to be somewhat better than this in most of the graphs, and the slope errors are evaluated allowing 1% at hard end and 2% at soft end in S/P with errors of 0.05 and 0.10 in $\overline{\mu/\rho}$ at hard and soft ends respectively.

Two of the curves with 50 sheets of paper (6.2.36 7.2.36.) were done with considerably higher currents than the others. Significant variations were not thereby introduced. The graphs for paraffin wax show no variation with variation of scatterer thickness.

This set of observations is considered to be the best evidence obtained in the course of the research carried out by the author as to the intensity of scattered X-rays produced by heterogeneous primary beams from scatterers of small atomic number. The graphs show that as the voltage on the X-ray tube is decreased, and the penetrating power of the rays thus diminished, the intensity of the scattered rays increases slowly, and to the accuracy of observation steadily. This is precisely what the accepted theory of X-ray scattering would lead us to expect if the heterogeneous beam is scattered and absorbed according to the sum of its homogeneous constituents; for then, the scattering of the 'softer' heterogeneous beams should approximate to the scattering of homogeneous beams of longer wave length. The results given here are not in agreement with/

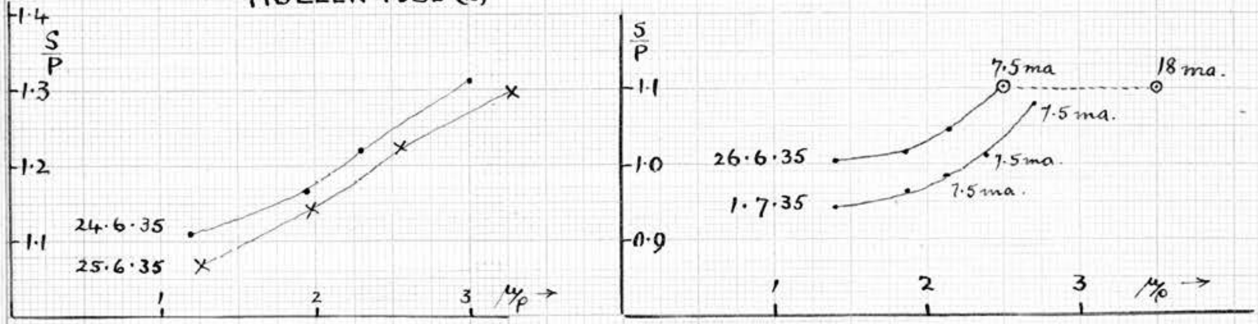
with those obtained by James Reekie, given in the Ph.D. Thesis (Edinburgh 1937) "The Scattering and Absorption of Heterogeneous X-Radiation", where on pages 68 *et seq* the author concludes that, after applying a correction for polarization, the scattering ratio S/P is constant.

VIII. CONFIRMATORY EXPERIMENTS.

1. Experimentally it was possible to check the idea that the 45° observations do in fact give results in agreement with the other main accepted law of scattering: namely, that the scattering increases more rapidly with increasing wave length as the atomic number of the scatterer is increased. Experiments were therefore carried out with scattering sheets of carbon (atomic number 6) 0.65 cm. thick. These gave corrected scattering curves and 45° scattering curves which agreed well with each other and which were very similar to the results for paraffin wax ($C_n H_{2n+2}$). Comparison experiments were carried out with aluminium (atomic number 13). These gave corrected scattering curves and 45 scattering curves which were both concave upwards. This shows that the scattering from aluminium increases more rapidly with increase of μ/ρ than the scattering from the element carbon, or for that matter from the oxygen (atomic number 8) in the filter paper scatterer/

FIG 22 (a) TUBE VERTICAL : FILTER BEFORE SCATTERER OF 0·125 CM ALUMINIUM

MULLER TUBE (2)



SCATTERER 50 SHEETS OF FILTER PAPER

TUBE CURRENTS 16 ma.

SCATTERER 0·3 CM PARAFFIN WAX

TUBE CURRENTS 7·5 ma.

DATA

24·6·35		25·6·35	
S/ρ	μ/ρ.	S/ρ	μ/ρ
1·21	1·2	1·17	1·25
1·265	1·95	1·245	1·95
1·32	2·3	1·325	2·55
1·41	3·0	1·4	3·3

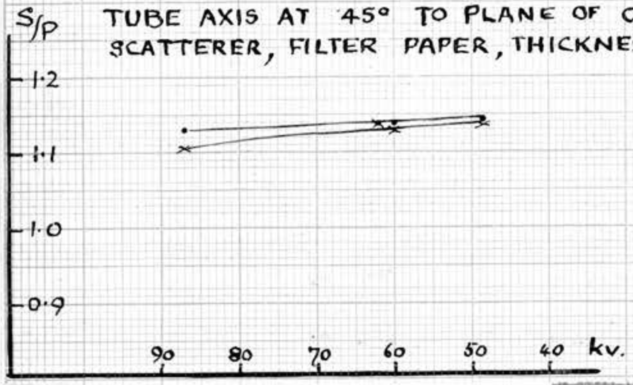
DATA

	26·6·35			1·7·35	
	S/ρ	μ/ρ		S/ρ	μ/ρ.
7·5 ma	1·005	1·4	7·5 ma	0·945	1·4
"	1·02	1·87	"	0·965	1·87
"	1·045	2·15	"	0·985	2·12
(7·5 ma.)	1·10	2·5	"	1·01	2·4
(18 ma)	1·10	3·5	"	1·08	2·7.

FIG 22(b) FILTERED RADIATION MULLER TUBE (2)

TUBE AXIS AT 45° TO PLANE OF OBSERVATION

SCATTERER, FILTER PAPER, THICKNESS 50 SHEETS.



OBSERVATIONS

kilo-volts	S/P Unfilt-ered	S/P Filt-ered	% decrease in S/P
87	1·13	1·105	2%
62	1·136	1·136	0%
60	1·137	1·128	¾%
48·5	1·144	1·137	¾%

. UNFILTERED

x FILTERED BY 0·125 CM. AL.

FURTHER OBSERVATIONS

Scatterer Paraffin Wax thickness 0·3 cm

			KV.	S/P	S'/P'	DECREASE IN S/P
Filtered	by	0·16 cm	80	1·028	0·998	3%
"	"	0·32 cm	80	1·028	0·977	5%

scatterer. See figs. 20(VI) and (XII) and 21.

2. Since experiments on filtered radiation gave rise to the suggestion that polarization errors were occurring, similar experiments performed with the X-ray tube vertical or at an angle of 45° to the horizontal plane of observation ought to provide another useful confirmation. With the tube vertical the following experiments were made.

2 experiments, Muller tube (2) Filter 0.125 cm aluminium.	Scatterer of filter paper, 50 sheets thick.
2 experiments, Muller tube (2) Filter 0.125 cm. aluminium.	Scatterer of paraffin wax, 0.3 cm. thick.

The results of these experiments were qualitatively satisfactory, but quantitatively they varied considerably with the current through the tube. This is in agreement with the observations already noted pp. 61-64 There it is found that at the 'soft' end, the shape of the uncorrected scattering graph was sensitive to tube current, tending to rise with rising tube current.

In the present experiment the graph tends to fall with rising tube currents. The fact that this change occurs in opposite directions when the tube is in horizontal and vertical position indicates that the effect is indeed due to polarization and that with larger tube currents there is less polarization in the primary beam.* This is not unlikely. It is known that the focussing in the tube is influenced to some extent by/

---

* This precludes the explanation of the rise of S/P in the horizontal case as a lack of saturation in the primary ionization chamber.

by a space charge effect in the filament emission and that a shield is mounted behind the tube filament as a partial control on space charge. With large currents some modification is therefore possible in the focussing of the cathode stream. The magnitude of any effect of this kind cannot of course be estimated and the point could not be tested with the type of tube in use.

3. It is therefore thought that a direct test on filtered radiation with the tube at 45° would be the best. A single experiment was conducted both with and without a filter of aluminium, 0.125 cm thick, the scatterer being 50 sheets of filter paper. The results, plotted against kilovoltage, are shown in fig. 22(b). The graph shows that S/P is slightly reduced by filtering the rays, and rather more reduced at the 'hard' end, than elsewhere. Here, with no polarization present, we see the true reduction of S/P caused by factors (a) and (b) discussed on p. 74 . Using a scatterer of paraffin wax containing a high proportion of hydrogen the reduction due to factor (b) should be greater. The observations shown for paraffin wax seem to agree.

4. Attempts were also made to find out whether, if the radiation at the 'hard' end was filtered, the resulting changes in S/P depended appreciably on polarization. Using 100 kv. on the tube, 50 sheets of filter paper as the scatterer, and 0.125 cm. aluminium as the filter/

filter, it was found that the orientation of the tube made no difference to the observed decrease, 2½ per cent, in the value of S/P. Whatever the polarization of the 'hard' radiation, it was appreciably constant throughout a considerable amount of filtering.

IX. DISCUSSION.

It is now possible to discuss the whole work carried out on the scattering experiment. It has been shown that polarization is present in the primary beam to such an extent and with such a distribution in the spectrum that it causes the 90° scattering graph taken in the plane of polarization to deviate from that taken in the perpendicular plane. The corrected 90° scattering ratio deduced from these observations shows a slowly increasing scattering ratio as $\overline{\mu/\rho}$ increases. This is qualitatively in agreement with accepted theory, as evaluated for the different scattering materials in Tables VIII and IX, Figures 30A and 30B, provided heterogeneous beams scatter according to the sum of their homogeneous components. If the scattering ratio is observed in a plane at 45° to the plane of polarization the results are again in agreement with accepted theory. In Table VI a comparison is obtained in the fifth and sixth columns between variation of corrected S/P by the two methods. The errors are evaluated on the/

the basis of 1% at hard end and 2% at soft end as error in S/P and 0.05 at hard end and 0.1 at soft end in $\overline{\mu/\rho}$. Comparison shows good consistency. The observations indicate that the true variation of S/P with $\overline{\mu/\rho}$ is a small increase as $\overline{\mu/\rho}$ increases. No change of slope is evident such as is seen in $(S/P)_H$. ~~(using light scatterers and beams of medium penetrating power~~).

Reverting to sections I to III pp. 61-70 it is now possible to say that the form of graph there obtained (horizontal line and slope) is dependent on tube orientation and that its main features disappear when polarization is corrected for. It would therefore seem that the variations of graph form studied in these sections depend on variations in primary polarization. The observations indicate that observable primary polarization depends to some extent on (i) tube currents (ii) tube construction and (iii) thickness of scatterer used. These points could all be directly tested. The connection between observable polarization and the results reported on pp. 48-53 could also be tested. For this purpose certain refinements in experimental method are desirable.

The chief experimental limitations and inaccuracies are as follows.

(1) limited range of penetrating powers observed.

(2) limited accuracy of alignment when small primary aperture was used.

(3) Stroboscopic error when large primary aperture was used.

These/

These restrictions of range and accuracy were imposed by the conditions of the experiment. The substitution of constant high tension for unrectified high tension would immediately eliminate error (3) and, by reducing the minimum time of readings, greatly extend the range of observation. Limitation (2) could be partly removed by using a rotating measuring system, but if constant high tension were available the large aperture arrangement used here would be equally good.

SECTION II. FILTERING EXPERIMENTS.

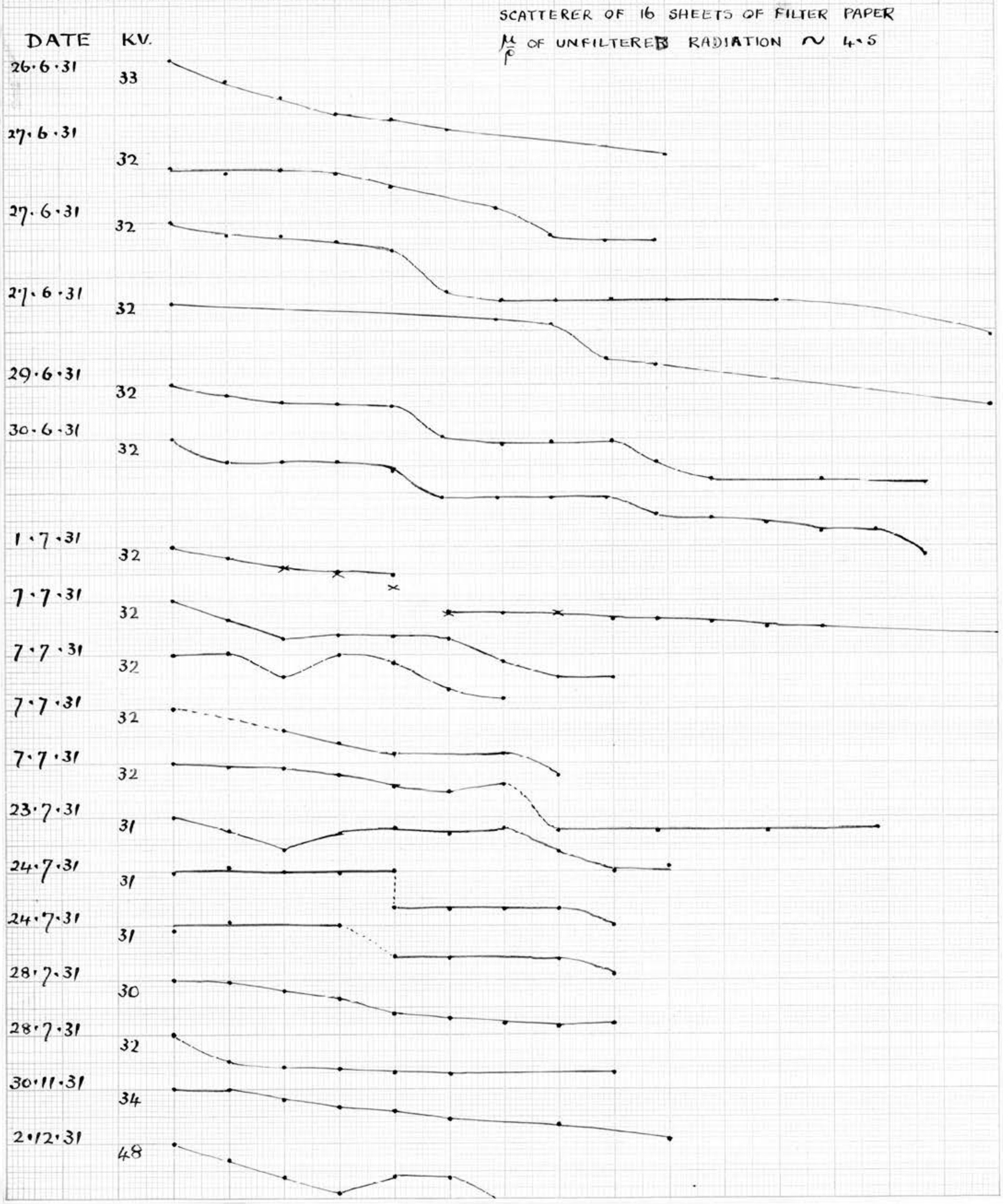
I. FILTERING OF SCATTERED RADIATION.

Much of the work done by Professor Barkla and his research students during the period since 1924 consisted in studying the effects of filtering the primary and secondary beams simultaneously. The experiment which was used by these workers, known as the "filtering experiment", gave under certain circumstances most unexpected results. The experiment compared the secondary beam with the primary beam after each had been transmitted through equivalent filters. If in such an experiment the two beams had equal penetrating power, the scattering ratio should remain constant; if the secondary was somewhat 'softer' than the primary, the scattering ratio should fall. The latter result was frequently obtained, but under certain conditions, the nature of which remained obscure, the ratio S/P was constant throughout a range of increasing thickness of filter, and then, on still further increasing the thickness of the filter, S/P assumed a new and lower value. This, in turn, remained relatively constant for a certain further range of filtering. The observer plotted S/P against filtering thickness; the graph in the first case being smooth, and in the second showing a discontinuity where S/P changed abruptly(18).

Some of the experiments carried out by the present writer/

FIG 23.

FILTERING EXPERIMENTS MULLER TUBE (1) AND TRANSFORMER (1)



The vertical scale is the same for all the graphs, namely, 1 cm = 0.1 units of S/P. Each graph has the values of S/P reduced to the same scale, with S/P unfiltered = 1.0.

The horizontal scale is also the same for all graphs, namely, 1 cm = 0.01 cm Aluminium.

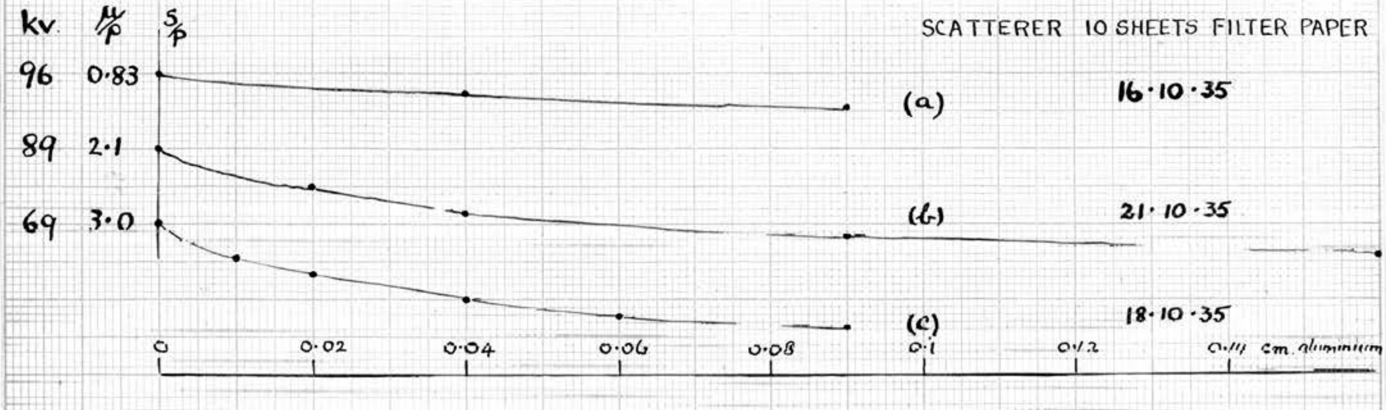
writer were filtering experiments of this type. Since, as stated above, the conditions for producing the discontinuity were not known, the procedure adopted in the earlier stages of the research was to try the experiment, and if the discontinuity did not appear, to make variations on the conditions in the hope of its doing so. Proceeding in this manner, one perfect and many nearly perfect examples of the discontinuity were obtained, fig. 23.

All the filtering experiments performed by the writer, with the exception of three, were undertaken before the polarization correction had been studied. All were conducted with the cathode stream of the X-ray tube parallel to the direction of the observed scattered beam. All are therefore reported here as a minor contribution to the thesis, and the purpose of reporting them is to bring them into relation with the results of the scattering experiment. For this purpose, the only filtering experiments which have much value are those which belong to definite series.

In all, about 86 filtering experiments were carried out, 25 with scatterers of paraffin wax and 61 with scatterers of filter paper. Of the 25 experiments with wax scatterers, 8 carried out with a scatterer of 0.3 cm, and 10 with a scatterer of 1.1 cm will be reported. The other seven were done with a pinhole primary aperture. Of the 61 experiments with scatterers of filter paper/

FIG 24 FILTERING EXPERIMENTS MULLER TUBE (2)

WHERE FILTERING DOES NOT INCREASE OBSERVED POLARIZATION (a)
THE DROP IN S/p CAUSED BY FILTERING IS SMALL



SCATTERER 10 SHEETS FILTER PAPER

THE VERTICAL SCALE IS 1 cm = 0·1 units of S/p
THE UNFILTERED POINT IS TAKEN WITH S/p = 1·00.

OBSERVATIONS

kilovolts	0	·01	·02	·04	·06	·09	·16	·32	cm aluminium
(a) 96	1·0	–	–	0·975	–	0·953	–	–	
(b) 89	1·0	–	0·949	0·915	–	0·882	0·858	0·823	
(c) 69	1·0	0·953	0·933	0·900	0·877	0·863	–	–	

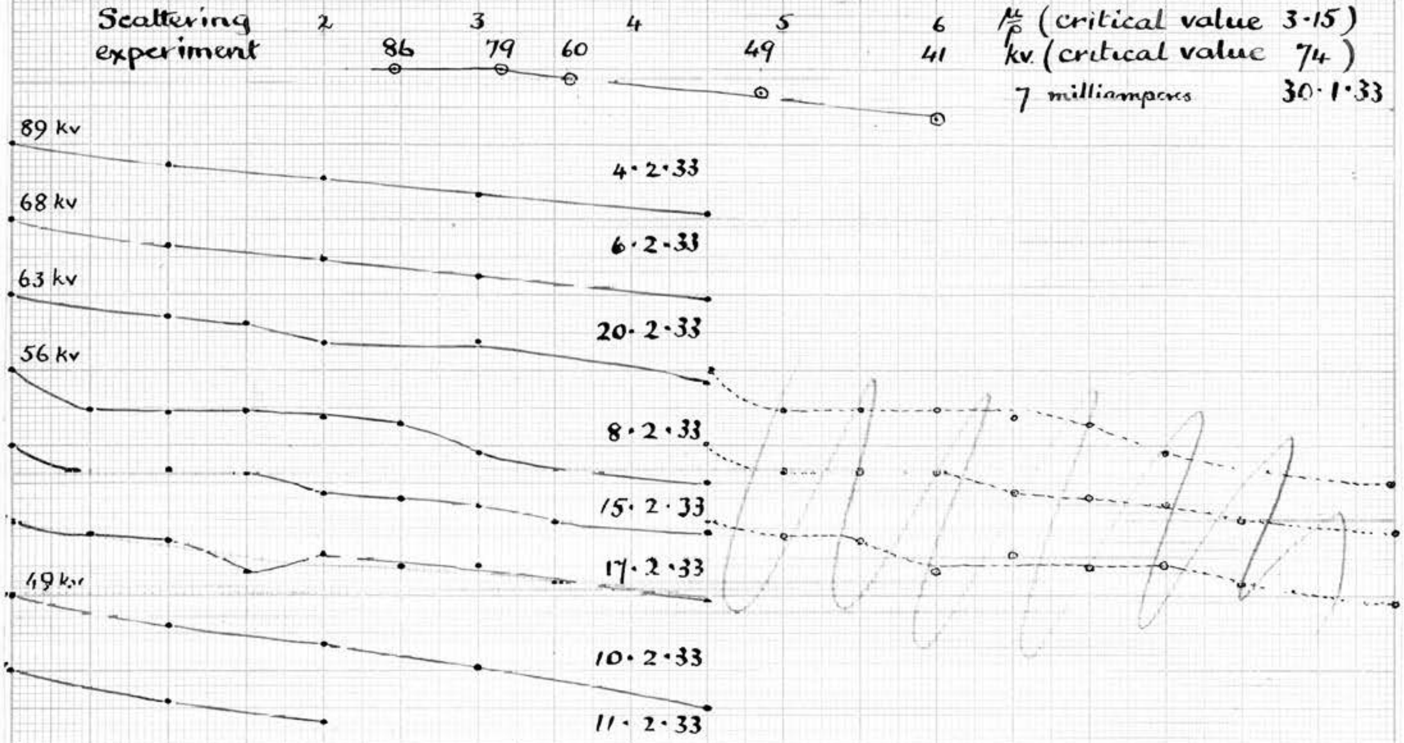
paper, only a few, showing typical or specially interesting graphs are reported, namely, those already seen in fig. 23 together with a few high voltage experiments which were amongst the latest and most accurate observations made, see fig. 24. The filter paper experiments include 22 done with transformer (1) which worked reliably over only a short range of voltage, and 17 done with gas tube and induction coil, where the voltage was not measured. The remaining observations show no long series. This experiment, it may be stated, can be performed with high accuracy, the ~~reading~~ error in S/P for any point being about 1%. If the source of high potential and the X-ray tube are both steady, the filtering experiment is more accurate than any other reported here.

For the two paraffin series, the observations, all reduced to the same standard, are given in Table VII and Fig. 25. For each series of filtering experiments scattering experiments ~~In the figure a scattering graph is drawn~~ were performed under closely similar conditions. The tube was not altered in any ~~above the filtering curves in each group. This scatter-~~ way. The scatterer was untouched and similar tube currents were used. One of ~~ing graph is chosen to correspond as closely as pos-~~ these scattering experiments is shown with each filtering series. All the ~~sible to the conditions of the filtering experiments~~, experiments are dated. (~~i.e. the same tube~~, ~~the same scatterer and similar currents~~).

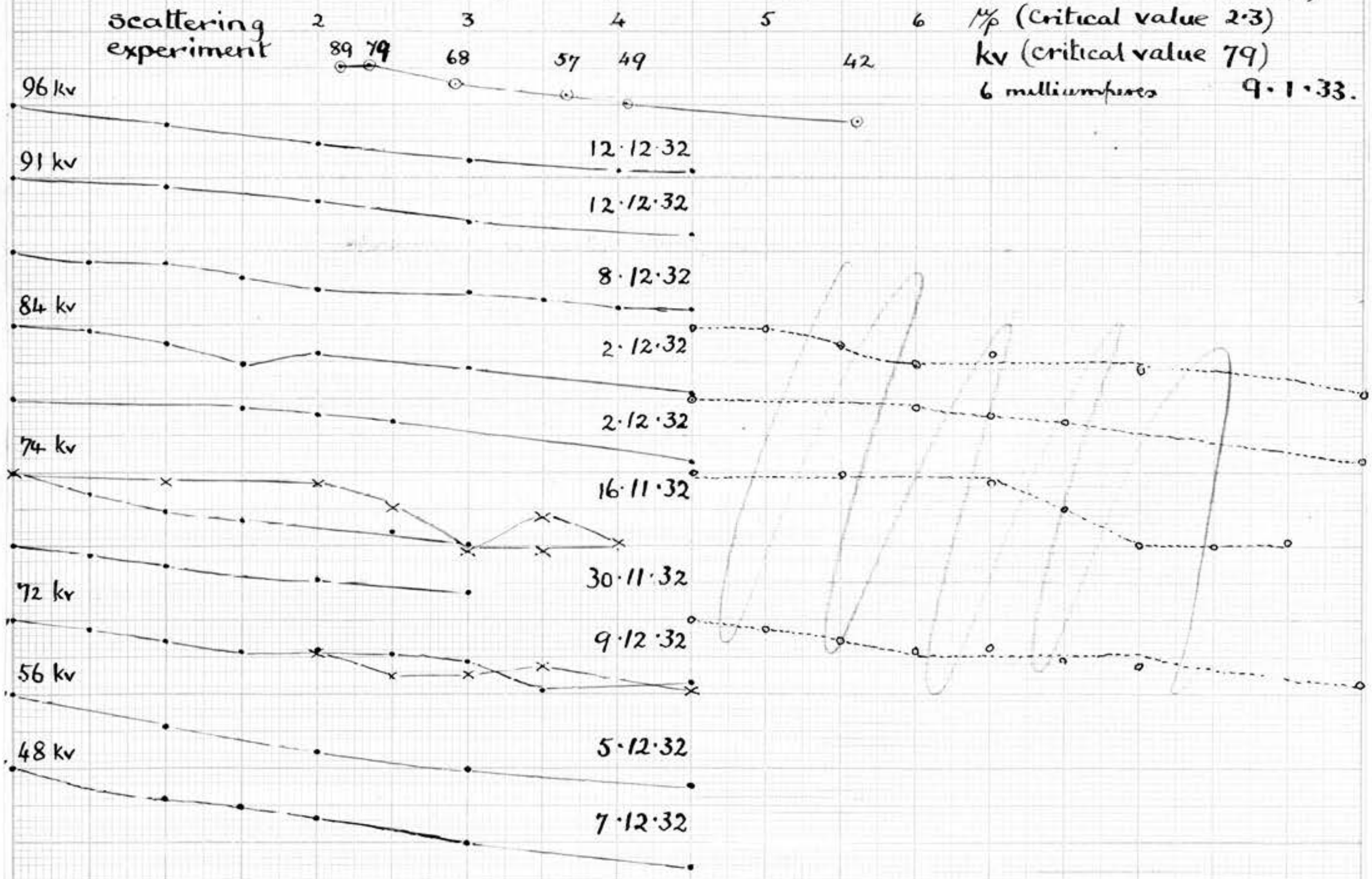
Let us now call the value of $\bar{\mu}/\rho$ at the limit of constant S/P in the scattering curve the critical value of $\bar{\mu}/\rho$, and the corresponding voltage on the X-ray tube the critical voltage. It would appear that when the filtering experiment is performed on radiation produced/

FIG. 25. FILTERING EXPERIMENTS

Scatterer of Paraffin Wax (thickness 0·3 cm) Coolidge Tube and Transformer (2)



Scatterer of Paraffin wax (1·1 cm) Coolidge tube and transformer (2)



WHERE POINTS MARKED WITH A DOT APPEAR ON THE SAME GRAPH AS POINTS MARKED WITH A CROSS THE CROSS POINTS WERE OBTAINED LATER IN THE DAY. SCALES AS FOR FIG 23.

produced by voltages considerably below the critical voltage, smooth and rapidly falling curves result. It also appears that when the radiation is produced by voltages much above the critical voltage, the filtering curves are again smooth but fall less rapidly. For voltages about the middle of the range, which are, moreover, in the neighbourhood of the critical voltage, some curious and variable curves are obtained.

All that can be said about these middle graphs at present is:

(1) In none of the series graphs has a perfect example of the discontinuity occurred. The nearest approach to it is in the experiment of 16.11.32, taking the afternoon points (marked with a cross), omitting one high value.

(2) Certain characteristics of the discontinuous graph do occur; Some graphs showing horizontal or almost horizontal portions ~~the attempt to impress a discontinuous form upon some of the graphs is fairly successful, (see the right hand graphs)~~.

(3) The results strongly suggest that there is a connection between the tube voltage required to produce a discontinuous filtering curve, and the critical voltage in the scattering experiment.

(4) Since in the scattering experiments already reported, a considerable amount of variability was found in the critical value of $\bar{\mu}/\rho$, then, <u>if the discontinuity is actually connected with the critical/</u>

critical point on the uncorrected scattering graph, variability is also to be anticipated where the discontinuity occurs.

II. THEORETICAL CONSIDERATION OF THE FILTERING EXPERIMENT.

It has already been pointed out that if in this experiment the primary and secondary beams are identical in spectrum then the observed ratio S/P must be constant for all thicknesses of filter. This statement Presupposes that the measuring system is working properly. The condition for identical spectra may be analytically given. If P_λ and S_λ be the intensity per unit range of wave length at wave length λ in the primary and secondary beams respectively, then the spectra are identical if the ratio $\frac{S_\lambda}{P_\lambda}$ is constant for all values of λ. If the experimental ratio S/P is not constant for all thicknesses of filter the beam constitutions are not identical. This is the observed condition.

According to the theory of scattering given on pp. 10-15 the two beams should not have identical spectra, for the following reasons.

(1) Total 90° scattering rises as wave length increases (see Table VIII).

(2) Some of the 90° scattered rays are modified in wave length (see Table VIII).

(3)/

(3) If the spectrum of polarized primary energy differs from the spectrum of total primary energy the spectrum of the 90° scattered beam differs from that of the total primary beam unless the scattered beam is observed at 45° to the plane of primary polarization.

The observation of inconstant S/P is therefore so far in agreement with theory.

Before attempting an exact comparison between theory and experiment other influences affecting the observed ratio of S/P must be considered.
The response of the ionization chamber varies rapidly with wave length and for beams of different spectra this response must be included in calculations of ionization current. It will be assumed that the conditions of observation are such that tertiary and obliquity scattering is negligible.

At the present stage it is not possible to develop the theory to the stage of comparison between theory and experiment for the following reasons.
Firstly , the primary beam spectrum is not known.
Secondly, the experiments here reported were made before the polarization error had been studied and the scattering was observed at 90° to the plane of primary polarization. It is now known that observable polarization is present in all the primary beams studied, and/

and also that filtering increases the observable polarization in all except the most penetrating beams. From observations reported in the previous section pp. 71-73 filtering may be expected to introduce variations in S/P from zero with hard beams up to perhaps 10% with soft beams for a filter thickness of 0.09 cm. Al. It is therefore possible that with soft beams the greater part of the fall in S/P is attributable to polarization.

The evidence reported pp. 79a-80a suggests that graphs departing from the smooth falling form are obtained from radiation near the bend on a "horizontal line" experiment. Making full allowance for experimental error, the existence of the bend cannot be doubted. When this experiment was corrected for polarization the graphs showed no bend, and the observed slope was within the limits of error the same whether the uncorrected graph was horizontal or sloping. The bend is thus observed by the writer to disappear when polarization is corrected for, and must therefore be considered to be produced by the polarization error. If then the filtering experiment has special features associated with the bend in the scattering experiment, it is necessary to enquire whether these features are not also produced by primary polarization.

Not tested in well marked case

No rigorous demonstration

It is therefore desirable to conduct experiments to test for a connection between primary polarization and/

and filtering discontinuities, and also to observe the true filtering curves from scattered beams at 45° to the plane of primary polarization. The experiments conforming most nearly to these conditions are the high voltage experiments especially the experiment with 96 KV. Fig. 24. At high voltages observable polarization does not increase with filtering up to .125 cm. Al. Now no high voltage experiment done by the writer ever gave other than a perfectly smooth curve, and moreover one with a relatively small fall in S/P. Other observers have given voltage discontinuity relations which are in agreement with the present observations, notably J.S. Kay, who observed the discontinuity most definitely formed at 60 KV. None of the available evidence is then in contradiction to the view that the anomalous filtering curves may be explained in terms of polarization.

III./

~~discontinuity does not imply an abrupt change in the behaviour of the rays.~~

~~(3) They locate the discontinuity near the critical value of $\bar{\mu}/\rho$ in the scattering experiment.~~

III. OBSERVATIONS OF "INTERCEPTED" RADIATION.

One other experiment will now be shortly reported. This experiment, designed by Professor Barkla to give more information about the scattered radiation, compares the observed ratio of S/P in the "scattering experiment" with the ratio S/P observed with equivalent filters of a particular thickness inserted at apertures CC and EE. The latter ratio is called the intercepted ratio and will be referred to as S'/P' . Both unintercepted and intercepted ratios are observed for a series of beams of different penetrating power and the ratios are plotted against $\bar{\mu}/\rho$ for the unintercepted beam. A series of intercepted curves for different thicknesses of interceptor gives exactly the same information as a series of filtering experiments taken for different voltages. The advantage of working both experiments is the gain in relative accuracy. We may also compare the intercepted curve with that obtained in the scattering experiment by using a single equivalent filter before the scatterer. The result of the two latter experiments should differ only in so far as the scattering differs from/

from pure coherent (or classical) scattering. Two sets of results will be reported, both of which come from the writer's early researches using gas tube and induction coil. The apparatus was not very reliable and the value of the observations was not clear at the time they were made, but in the light of subsequent work they clearly have value.

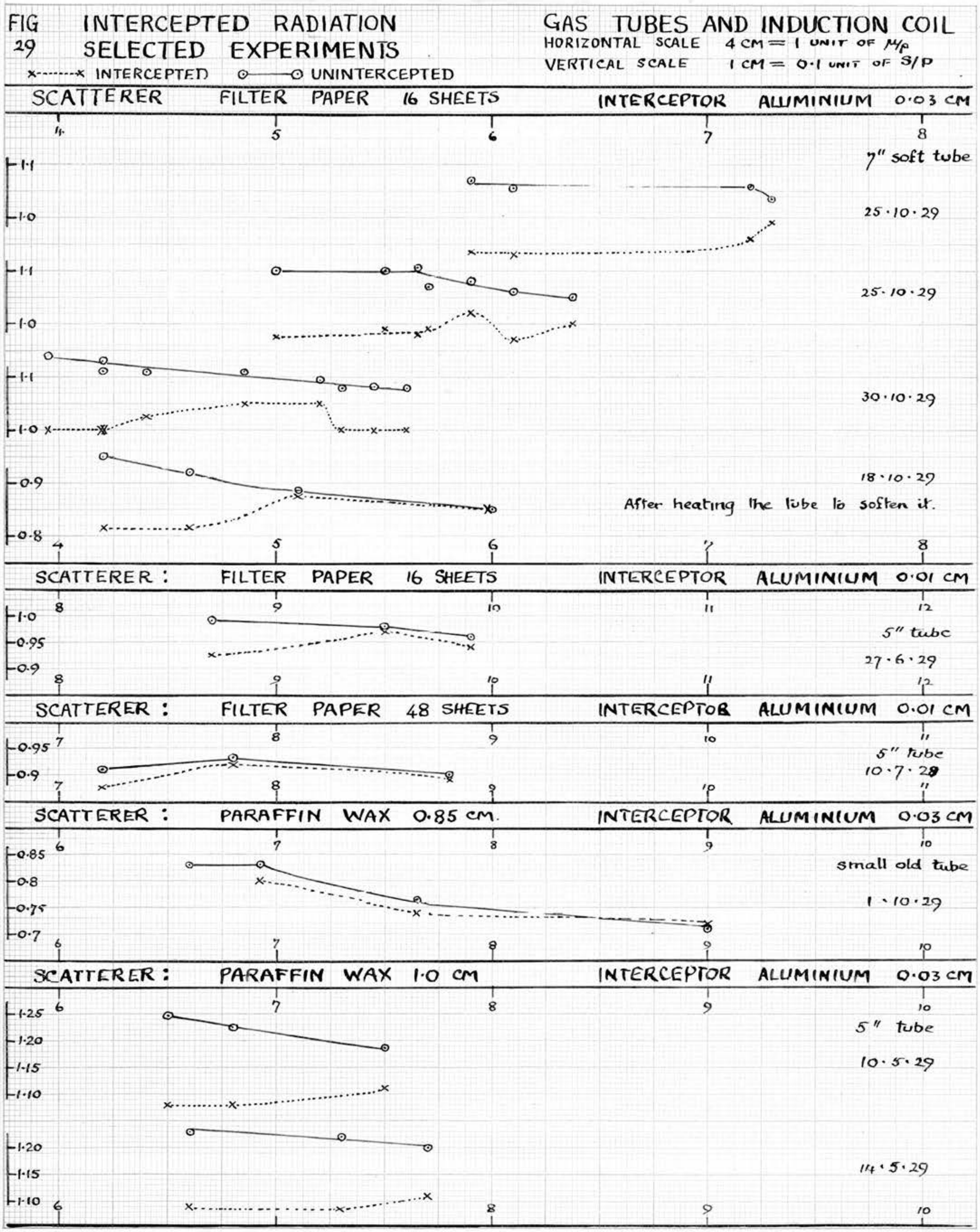
Gas tubes, although they have rather a short range and are somewhat erratic in use, nevertheless have one great advantage, namely, that it is easier with them than it is with the ordinary thick-walled hot cathode tubes to get the extremely 'soft' radiation of $\bar{\mu}/\rho$ = 9 to 12. The short range graphs from several experiments, covering different ranges of $\bar{\mu}/\rho$ can be put together. One would not have much confidence in such syntheses taken by themselves, but since they give results which are typically the same as those of the long range experiments, some reliance can be placed upon them.

Results with Paraffin Wax Scatterers.

Tube	Thickness of Scatterer	Range of $\bar{\mu}/\rho$ Where S/P Constant	Range of $\bar{\mu}/\rho$ Where S/P decreases	
'Hard' 7" Tube	1.5 cm.	1.5 to 2.8	2.8 to 4.5	
	1.0 cm.	----------	6.5 to 7.5	
'Softer' 7" Tube	0.85 cm.	4.6 to 5.3	5.3 to 5.5	
	0.85 cm.	4.0 to 5.0	5.0 to 6.0	
'Softest' thin glass Tube	0.85 cm.	----------	7.0 to 9.0	S/P = S'/ P' at $\bar{\mu}/\rho$ = 9.0

FIG 29 INTERCEPTED RADIATION SELECTED EXPERIMENTS

GAS TUBES AND INDUCTION COIL



Working with an exceedingly 'soft' tube, and with filter paper scatterers of 16 sheets thickness, observations were made of the percentage change in S/P caused by intercepting with 0.01 cm. Al. These observations are given in the following table, and in figure 28 where percentage fall of S/P is plotted against $\bar{\mu}/\rho$ of the primary beam. In making such observations on 'soft' radiation, the tube has to be controlled by the ~~sparkers. Considering the unsteadiness of this~~ method of control, and the impossibility of ensuring that the radiation giving the S/P value was effectively the same as that giving the $\bar{\mu}/\rho$ value, the results show a fair degree of regularity. The intercepted ratio S'/P' tends to the same value as the unintercepted ratio S/P as the radiation becomes 'softer'. The experiments show that when this tendency shows itself, S/P is decreasing, and S'/P' is simultaneously rising, (fig 29), or at least, not falling so rapidly as S/P. No great significance can be attached to any single experiment shown in Fig. 29. The radiation from these 'soft' tubes varied from day to day, as the density of the gas in the tube varied. Its polarized component is not likely to have been at all constant. Furthermore, the dependence of polarization on tube voltage is not likely likely to be the same as in the hot cathode tubes. The observations do, however, give a certain amount of confirmation to the view put forward on page 81 *et seq.*

GAS TUBE/

Two pages over.

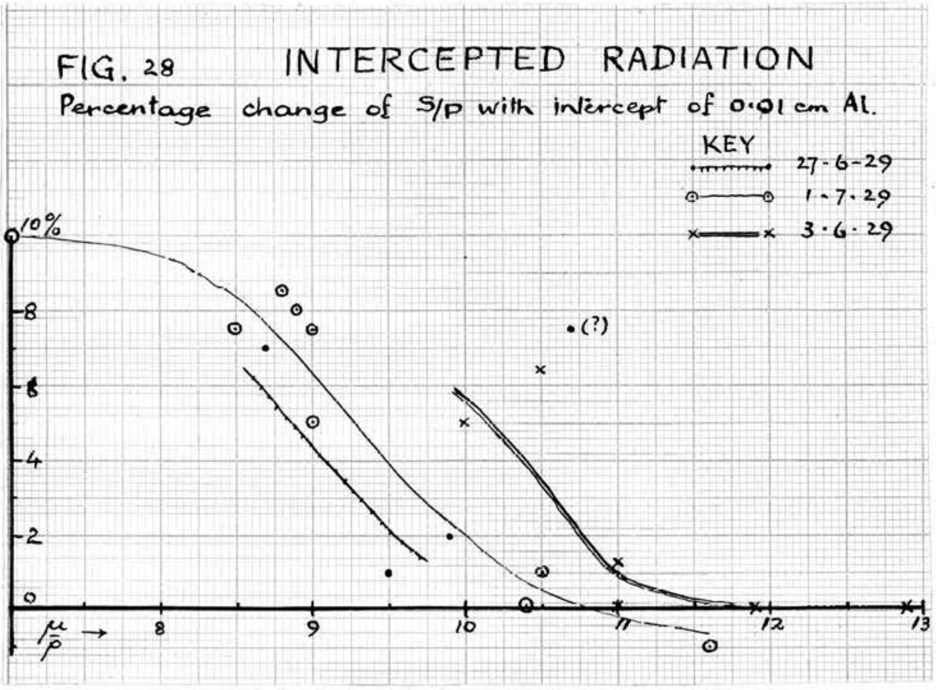


FIG. 28 INTERCEPTED RADIATION

Percentage change of S/p with intercept of 0·01 cm Al.

GAS TUBE AND INDUCTION COIL GIVING VERY 'SOFT' X-RAYS.	SCATTERER OF FILTER PAPER, 16 SHEETS.

DATE	Primary $\bar{\mu}/\rho_{Al}$	Percentage change in S/P with intercept of 0.01 cm. aluminium.
3.6.29.	10.0	5.0%
	10.5	6.4
	11.0	1.3
	11.0	0
	11.9	0
	12.9	0
1.7.29	7.0	10.0%
	8.5	7.5
	8.8	8.5
	8.9	8.0
	9.0	7.5
	9.0	5.0
	10.4	0
	10.5	1.0
	11.6	-1.0
27.6.29	8.7	7.0%
	9.5	1.0
	9.9	2.0
	10.7	7.5 (?)
18.10.29 Intercept of 0.03 cm.	4.2	14.0%
	4.6	12.0
	5.1	0
	6.0	0
10.7.29 48 sheets	7.2	3.0%
	7.8	1.0
	8.8	1.0

the sparkers. Considering the unsteadiness of this method of control, and the impossibility of ensuring that the radiation giving the S/P value was effectively the same as that giving the μ/ρ value, the results show a fair degree of regularity. The intercepted ratio S'/P' tends to the same value as the unintercepted ratio S/P as the radiation becomes 'softer'. The experiments ~~shows~~ show that when this tendency shows itself, S/P is decreasing, and S'/P' is simultaneously rising, (fig 29), or at least, not falling so rapidly as S/P. No great significance can be attached to any single experiment shown in Fig. 29. The radiation from these 'soft' tubes varied from day to day, as the density of the gas in the tube varied. Its polarized component is not likely to have been at all constant. Furthermore, the dependence of polarization on tube voltage is not likely to be the same as in the hot cathode tubes.

The observations are however consistent with others made in quite different experiments. It is possible now to compare the μ/ρ experiments reported on pp.41-45, those of the filtering experiments and those just reported, called interception experiments. In all these experiments filtering aluminium sheets were inserted after the scatterer in both primary and secondary beams. The results in each case can be summarised as follows.

The μ/ρ experiments.

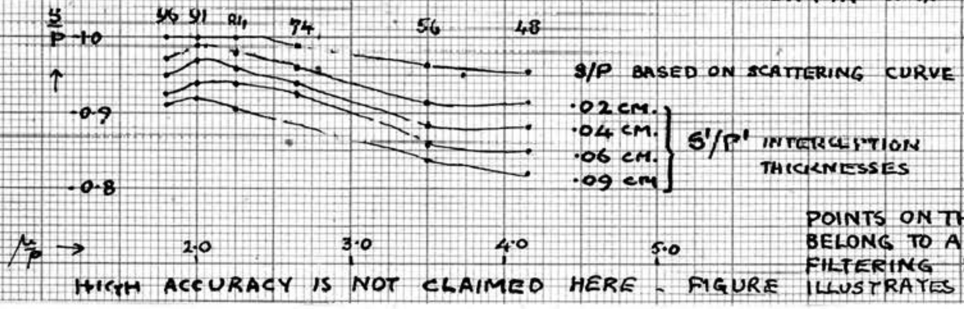
The scattered beam was found usually to be anomalously soft by comparison with the primary, that is, far softer than can be explained by total Compton modification of wave length for equivalent homogeneous beams. This can be translated into terms of the interception experiment by saying that, using a filter thickness sufficient to halve the primary ionization current, the ratio S/P suffers a decrease greater than the Compton modification seems able to account for.
At the time the observations were made it was suggested that primary polarization concentrated towards the high frequency end of the primary spectrum might be at work. But this was not tested.

In the same experiments however, an apparently opposite effect was observed in three out of sixty six observations. The fractional increase in μ/ρ (Table I.) was in these observations 0, 0 and -2%. In these observations the insertion of filtering aluminium sufficient to halve the primary ionization current halved the secondary current also, and in the third case did not reduce the secondary quite so much as the primary. Translating these results into the terms of the interception experiment S/P is equal, or even slightly less than S′/P′ .

The/

FIG 26. FROM TABLE VII (b) OR. FIG 25(b)
INTERCEPTION GRAPH RECONSTRUCTED FROM SELECTED FILTERING GRAPHS

PARAFFIN WAX 1·1 CM.



POINTS ON THE SAME ORDINATE BELONG TO A PARTICULAR FILTERING EXPERIMENT.

HIGH ACCURACY IS NOT CLAIMED HERE - FIGURE ILLUSTRATES DISCUSSION p107

The filtering experiment.

The results of this experiment obtained by the writer can be seen in figures 23-25 pp. 78a-80a. They can be translated at once into terms of the interception experiment by plotting the observations from any one experiment on a vertical line whose abscissa is the μ/ρ value of the unfiltered beam. If the relative values of S/P at unfiltered points for all experiments are known, correct interception graphs are then obtained by joining together points of equal intercepting thicknesses of aluminium. The filtering graph and the interception graphs are thus merely different ways of presenting the same observations. (see Fig. 26.)

If then the results of 27.6.31 and 24.7.31 (figure 23) were shown on intercepting graphs all intercepting curves up to 0.03 cm. Al would show S/P = S′/P′. Other graphs in figure 23, e.g. 28.7.31 (second curve) show an initial fall in S/P followed by constancy of S/P for several tenth millimetres of filtering aluminium. This implies concurrence of interception graphs. Attention is drawn to three curves in figure 23 and to at least one in figure 25 in which S/P rises with increasing filtering. This result has not yet been found by others and was at first thought to be due to errors of observation, but such/

such points were found to be repeatable. Observations of the same phenomenon were made in other experiments, e.g. Fraction increase of $\mu/\rho = -2\%$, (Table Ie), and the softest point of experiment 1.10.29 (figure 29) (These two examples are of course within the error of observation and are therefore not proved.) The observations of the third point of 23.7.31 and of the fourth point of 2.12.31 (fig. 23) were fully established.

Interception experiment.

The graphs on figure 29 are thus consistent with observations made in other experiments. The importance of the interception experiment is that it shows the condition $S/P = S'/P'$ to be approached gradually and not by discontinuity. This is demonstrated in both figures 28 and 29, the observations of figure 28 having been abstracted from the data of figure 29.

If, now, it is true, as experimental results earlier in this thesis show, that when S'/P' is much lower than S/P the lowering is largely due to primary polarization, then the first possible explanation of the condition $S'/P' = S/P$ is that the large polarization effect is for some reason reduced. It is certainly possible that the unpolarized characteristic radiations of anticathodes should give rise to anomalies in/

in the spectrum of polarization. It would hardly be possible to test this idea without the use of some method of beam analysis, either by spectroscope or by the less sensitive differential filter method. The latter method would have the advantage from the point of view of Professor Barkla's theory of retaining the use of heterogeneous beams.

PART IV. CONCLUSIONS.

At the time the writer's research began there were two major problems to be investigated. The first was a problem regarding the law of scattering of heterogeneous X-ray beams. The second was a problem regarding the law of absorption of the same beams. Conclusions had been reached by Professor Barkla with regard to both problems, and his conclusions are most simply stated in his own words: "A complex beam has "an activity which is not the sum of the activities of "its constituents. Such a beam has properties "depending on something analogous to temperature of "the radiation as a whole". Such a view is not in agreement with the current theories of X-ray scattering and absorption. In these theories no postulate of "frequency coherence" is made. It is therefore necessary to examine with the greatest care the experimental evidence which demands such a radical overhaul of accepted theory. The evidence upon which Professor Barkla relies most strongly is given in the papers by Barkla and White (1917), Barkla and Sale (1923), Barkla and Khastgir (1925 to 1927), Barkla and Watson (1926), and Barkla and Mackenzie (1925 to 1926), Barkla and Kay (1933) and recent Edinburgh Theses. In these papers a great variety of experiments is reported, the majority belonging either to the scattering type or the filtering type.[18, 43, 44.]

The/

The experiments performed by the writer have belonged mainly to the simplest forms of these two types, the scattering angle being always 90°. As the work continued the experimental conditions were progressively improved, errors due to obliquity of X-ray paths, tertiary scattering, lead L.radiation, faulty alignment and dissimilar apertures being gradually reduced. The arrangement of the apparatus was that usually adopted in the laboratory. The X-ray tube was mounted with both the cathode stream and the normal to the anticathode face in a horizontal plane, and the observations made on the radiations were taken in the same plane, the direction of the primary beam being perpendicular to the cathode stream.

SCATTERING EXPERIMENT.

With this arrangement of apparatus it was found that the results of the experiments showed large variations in the form of the graph depending on the factors given under I to IV below.

I. The current through the X-ray tube. At the 'soft' end, S/P is higher with higher currents.

II. The type of X-ray tube used. The feature in the tube construction which caused the variations was not determined.

III./

III. The thickness of the scatterer. The thicker the scatterer the shorter is the range of **mass absorption coefficient** over which S/P remains constant.

IV. The degree of filtering to which the primary beam is subject before it reaches the scatterer. With a thick filter the ratio of unfiltered S/P was reduced by as much as 25% for the lowest voltage on the tube. This remarkable result led to a test of the influence of the partial polarization of the primary beam, and this in turn determined a course of further investigation which produced the following results.

V. Test for polarization effects. The X-ray tube was rotated through 90° about the primary X-ray beam as axis, the cathode stream being then vertical. The result of the scattering experiment was quite different with the tube in this position, the value of $(S/P)_V$ rising as the X-ray beam was made less penetrating. ~~The results were shown to depend, as with the previous tube mounting, on~~

~~(i) current through the X-ray tube. (p. 76)~~

~~(ii) thickness of scatterer. (p. 70)~~

~~(iii) filtering before the scatterer. (p. 76)~~

These results led to the conclusion that correction for polarization was necessary.

VI. Polarization correction experiment. This was carried out by using the tube in both positions and observing $(S/P)_H$ and $(S/P)_V$ on the same day. From these/

these observations the corrected value, $\overline{S/P}$, was obtained as the mean of $(S/P)_V$ and $(S/P)_H$. The corrected ~~values~~ graphs of S/P were ~~almost~~ within the limits of error independent of (1) the particular form of the $(S/P)_H$ graph ~~current through the tube, except where alignment errors occurred~~, and (2) the thickness of the scatterer. ~~These results were therefore taken as true scattering curves for the heterogeneous beams, subject only to the minor errors introduced by the method of observation~~.

VII. Accurate observations of the ~~true~~ 90° scattering of heterogeneous beams uninfluenced by polarization. Since it has been established that partial polarization of the primary beam ~~was~~ is the main disturbing influence ~~upon~~ in the scattering experiments, it followes that the best determinations of the scattering ratio would be made with the axis of the tube orientated at an angle of 45° to the plane of observation. This method was accordingly employed. The results show satisfactory agreement with these made previously by the less accurate method. ~~They were~~ In particular, within the limits of experimental error they showed no ~~variation~~ ~~moreover, in qualitative agreement with the accepted~~ measurable variation with scatterer thickness. The observations also ~~theory for the intensity of scattering in the following~~ show that ~~respects~~.

~~(1)~~ the scattering ratio increases with wave length, and that ~~(2)~~ the increase of the scattering ratio with wave length itself increases as the atomic number of the scatterer increases.

Comparison of these results with those of other observers.

The results, uncorrected for polarization, are in good/

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~~good~~ general agreement with those of Mrs Sale, S.R. Khastgir[18] ~~and~~ J.S. Kay, J. Reekie and M.A.M. Wilson. The influence of Scatterer thickness on the critical point of the "horizontal line" graph confirms the work of Kay and Reekie. No polarization correction was applied by Mrs Sale, ~~or~~ S.R. Khastgir J.S. Kay or M.A.M. Wilson. A polarization correction was, however, applied by J. Reekie, from data supplied by H.K. Pal. Reekie carried out this correction on the understanding that the amount of polarization in the primary beam "varied little for different tubes and scatterers". Such a statement is, for primary beams characterized by mass absorption coefficient, not in agreement with the findings in the present work, where polarization was experimentally corrected. The validity of Reekie's correction is therefore open to question. The corrected results given by him do not agree with those obtained in the present investigation.

Quantitative comparison of theory with experiment.

The quantitative comparison of theory and experiment is not easily achieved, for the reasons that,

(1) Beam constitution is not known.

(2) Data are not available for the computation of the scattered intensities for the shorter wave lengths between 0.1 and 0.64 ÅU.

(3) The wave length sensitivity of the ionization chamber must be applied, and this is in the present case a purely theoretical correction.

In Table VIII and its accompanying graph, the final figures are given of a calculation which takes us/

us some way towards a comparison between theory and experiment. The calculation has been carried out to obtain for homogeneous beams of various wave lengths, an estimate of relative ionization produced by the secondary rays scattered at 90° , per unit intensity of primary, per scattering electron. Table VIII is computed from the tables given in Compton and Allison. ~~F gives the intensity of coherent scattering relative to Thomson scattering. Z - f KN gives all the terms in the expression for the intensity of incoherent scattering except the electron spin term due to Waller. This function is not multiplied by the recoil factor~~ , ~~since this is approximately annulled by the proportionally greater ionizing power of the incoherent ray, (see p.)~~ ~~The sum of these two rows~~ It gives ~~a total~~ estimates of total scattering per atom and per electron (relative to Thomson scattering) as observed by ionization methods. ~~The scattering per electron is thence derived~~. The calculation is made for hydrogen, carbon, oxygen and aluminium. Values for paraffin wax and cellulose are computed according to the formulae C_nH_{2n} (approx.) and $C_6H_{10}O_5$. Table IX (a) gives the ionization ~~spectrum~~ observed in the particular ionization chamber used, relative to the ~~per unit~~ intensity of primary beam, and has been obtained by applying Table X to the values of Table VIII.

To complete the comparison of observed and calculated values for a heterogeneous beam, we make the usual assumption that the homogeneous components into which the heterogeneous beam can be resolved, act independently. Integration over the intensity spectrum of the heterogeneous beam is then required. This integration has not been attempted because/

because the necessary infomation as to the intensity spectrum is not available. The effect of the fimal integration is bound to be a general lowering of the slopes of the graphs given in fig. 30, while the relative slopes as between one scatterer and another should not be greatly changed. So far as it goes, comparison is satisfactory. /

FILTERING EXPERIMENTS.

I. A study of the filtering experiment was made with a variety of apparatus, and over a considerable period of time. On 86 different occasions the experiment was performed. On one only of all these occasions was the result a perfect example of the discontinuity. On the majority of occasions, however, the graph of S/P against filtered thickness was not a smooth curve. Graphs with definite horizontal portions were common. According to the results obtained by the writer the discontinuity is not an effect which either "occurs or does not occur." Results intermediate between the smooth and the discontinuous are readily obtained. A series of filtering experiments shows smooth curves for tube voltages much above or much below the critical voltage for the scattering experiment. In the neighbourhood of the critical voltage the graphs ~~show~~ tend to depart from the smooth form. It has been shown that it is possible for S/P ~~a tendency to be discontinuous.~~ to rise with increase of filtering, an observation not hitherto made.

II. ~~THE SCATTERING EXPERIMENT WITH FILTERED RADIATION.~~
THEORETICAL CONSIDERATION OF THE FILTERING EXPERIMENT.

~~This experiment gives the scattering experiment~~ Theoretical consideration shows that the results of the ~~counterpart of the constant S/P condition in the~~ filtering experiment cannot be interbreted until they are conducted ~~filtering experiment.~~ ~~It shows (1) that the constancy~~ in such a manner that polarization effects are eliminated.
~~of~~/

~~constancy of S/P does not set in abruptly and (2) that constant S/P in the filtering curve is related to that range of the scattering experiment where S/P is not constant.~~

III. <u>INTERCEPTED BEAM EXPERIMENTS</u>. Observations with gas tubes on very 'soft' radiation show (1) that the condition ~~constancy of~~ S/P = S'/P' does not set in abruptly and (2) that it is related to a range of the scattering experiment in which S/P is falling and in which S'/P', the intercepted ratio is simultaneously rising.

~~An interesting agreement is shown with the observation of the voltage dependence of the discontinuity noted by J.S. Kay.[18]~~

The results of the μ/ρ experiment, the filtering experiment and the interception experiment are compared. The significance of the condition S/P = S'/P' is discussed and it is shown that no interpretation can be given to this observation until the primary polarization spectrum has been studied.

AGREEMENT WITH THE RESULTS OF OTHER WORKERS.

General agreement is obtained with the work of others. An interesting agreement is shown with the observation of the voltage dependence of the discontinuity noted by J. S. Kay. The observation S/P = S'/P' observed by many workers has been confirmed.

COMPARISON BETWEEN THEORY AND EXPERIMENT.

It has been pointed ~~but~~ that no useful ~~confirmation~~ comparison can be made between theory and experiment until observations free from polarization errors are available.

With regard to the major problems of the J phenomenon, the following are the final conclusions of the present researches. *No indication~~s~~ has been obtained of a fundamentally discontinuous process at work in either the scattering or the absorption of heterogeneous X-ray radiations*. All the evidence obtained, whether from the scattering experiments which formed the main line of investigation and which were exhaustively examined, or from other experiments, which were regarded as subsidiary at the time they were performed, consistently points to fundamentally continuous processes underlying all the changes in the observed quantities even in cases of apparent discontinuity.

The ~~true~~ scattering ratio freed from polarization errors is shown to rise slowly as the penetrating power of the heterogeneous primary beam decreases, and is not a constant over long ranges of penetrating power, as has been previously published.

Professor Barkla has under certain circumstances found the ratio of ionizations produced by primary and secondary beams to be unchanged by equivalent filtering. This fact has been repeatedly observed in the present work, but the circumstances bringing the condition about have been shown to do so by a gradual process.

Reasons are here given for regarding the discontinuity which can appear in the filtering experiment as not fundamentally discontinuous. These reasons are (a) the rarity of the phenomenon in its extreme form, (b)/

(b) the very frequent occurrence of intermediate forms, (c) strong indications that the discontinuous observations arise in close connection with the "critical point" in the scattering experiment (d) the smooth approach to the condition $S/P = S'/P'$, seen in the interception experiment which is physically the same as the filtering experiment. The experimental evidence supporting reason (c) is not strong enough to give proof beyond doubt, and further experimental work Eliminating polarization is certainly required. Nevertheless, such evidence as is available is entirely consistent with other experimental facts.

TABLE I. COMPARISON OF MASS ABSORPTION COEFFICIENTS OF PRIMARY AND SCATTERED BEAMS.

(a) Scatterer of 64 sheets of Filter Paper.

Muller X-Ray tube (1) and Induction Coil.

Observed $\frac{\mu}{\rho}$ in Aluminium of Primary Beam	Observed $\frac{\mu}{\rho}$ in Aluminium of Scattered Beam	Observed Fractional Increase of $\frac{\mu}{\rho}$ Al	Calculated Fractional Increase of $\frac{\mu}{\rho}$ Al for Modified Scattered Beams
2.3	2.8	22%	12%
2.5	3.1	24	12
2.5	3.1	24	12
2.7	3.2	19	12
3.1	3.8	23	11
4.1	4.3	5	10
4.5	5.4	20	9
5.3	5.9	11	9
5.7	6.9	21	9

(b) Scatterer of 32 sheets of Filter Paper.

Muller X-Ray Tube (1) and Induction Coil.

Observed $\frac{\mu}{\rho}$ in Aluminium of Primary Beam	Observed $\frac{\mu}{\rho}$ in Aluminium of Scattered Beam	Observed Fractional Increase of $\frac{\mu}{\rho}$ Al.	Calculated Fractional Increase of $\frac{\mu}{\rho}$ Al for Modified Scattered Beams
3.0	3.8	27%	12%
3.0	3.6	20	12
3.1	3.7	19	11
3.2	3.9	22	11
3.7	4.2	18	10
3.9	4.3	10	10
4.5	5.4	20	9
4.6	5.3	15	9
4.7	5.3	13	9
4.9	6.6	35	9
4.9	5.9	20	9
5.4	8.0	48	9

TABLE I. (contd.) COMPARISON OF MASS ABSORPTION COEFFICIENTS OF PRIMARY AND SCATTERED BEAMS.

(c) Scatterer of 16 sheets of Filter Paper.

Muller X-Ray Tube (1) and Induction Coil.

Observed μ/ρ in aluminium of Primary Beam	Observed μ/ρ in Aluminium of Scattered Beam	Observed Fractional Increase of μ/ρ Al.	Calculated Fractional Increase of μ/ρ Al. for Modified Scattered Beams.
3.8	4.9	29%	10%
4.1	4.9	20	10
4.2	5.2	24	10
4.4	5.4	23	10
4.9	6.6	35	9
5.0	6.0	20	9
5.2	6.6	27	9
6.1	7.9	29	8
6.3	7.6	21	8
6.6	7.8	18	8

(d) Scatterer of 0.045 cm. of Aluminium

2.3	2.9	26%	12%
2.7	3.3	22	12
2.7	3.4	26	12
3.1	4.0	29	11

(e) Scatterer of 16 sheets of Filter Paper.

Muller X-Ray Tube (1) and Induction Coil.

Observed μ/ρ in Aluminium of Primary Beam	Observed μ/ρ in Aluminium of Scattered Beam	Observed Fractional Increase of μ/ρ Al.	Calculated Fractional Increase of μ/ρ Al for Modified Scattered Beams
Aperture AA reduced to 7/8" diameter and Filtering sheets of aluminium moved further from E_s			
4.3	5.7	32%	10
4.5	5.6	24	9
4.5	5.5	22	9
4.6	5.2	13	9
Aperture AA reduced to ½" diameter and Possible sources of Lead L radiation removed. Low current of 3.0 milliamperes.			
2.6	3.5	35%	12%
2.7	3.5	30	12
2.9	4.0	38	12
3.0	4.3	43	12
3.2	3.8	19	11
3.8	4.8	26	10
4.1	4.1	0	10
4.9	4.8	-2	9

TABLE 1. (contd.) COMPARISON OF MASS ABSORPTION COEFFICIENTS OF PRIMARY AND SCATTERED BEAMS.

(f) Scatterer of 16 sheets of Filter Paper.

Muller X-Ray Tube (1) and Transformer.

Observed μ/ρ in Aluminium of Primary Beam.	Observed μ/ρ in Aluminium of Scattered Beam.	Observed Fractional Increase of μ/ρ al	Calculated Fractional Increase of μ/ρ Al for Modified Scattered Beams.
4.2	5.0	19%	10%
4.6	5.2	13	9
4.7	5.3	13	9
4.7	5.4	15	9
4.7	5.4	15	9
5.1	5.9	16	9
5.4	6.3	17	9
5.7	6.5	14	9
5.8	6.3	9	8
5.9	6.3	7	8
5.9	6.4	9	8
6.0	6.6	10	8
6.1	7.2	18	8
6.2	6.8	10	8
6.2	7.2	16	8
6.3	7.4	17	8
7.4	8.3	12	8
7.6	7.6	0	8
7.6	8.2	8	8

TABLE II. SCATTERING OF UNFILTERED PRIMARY BEAMS AT 90° TO THE PRIMARY AND PARALLEL TO THE CATHODE STREAM.

(a) SCATTERER OF PARAFFIN WAX OF THICKNESS 0.3 Cm. (0.27 gm/sq. cm.)

Date	Source of X-rays	Tube Current in Milli-amperes.	Range of μ/ρ Al for which S/P is observed constant.	Limit of Constant. S/P	Range of μ/ρ Al for which S/P is observed to decrease.	Fractional Increase of S/P per unit μ/ρ Al
30.1.33)	Coolidge Tube and Transformer(2)	(7.0	1.80 to 3.15	3.5	3.50 to 6.00	-2.5
1.2.33)		(7.0	2.25 to 4.50	4.5 to 5.0	5.00 to 6.00	-3.3
14.3.34)	Andrews Tube (2) and Transformer(2)	(1.5 to 4.0	2.40 to 6.25	——— not	observed	———
27.3.34)		(1.5 to 4.0	2.40 to 5.00 (possibly to 5.60)	——— not	observed	———
5.7.35)*	Muller (2) Tube and Transformer(2)	(0.4 to 1.2	——— ? ———		2.25 to 4.75	-0.5
8.7.35)		(0.4 to 1.2	—— not observed ——		2.50 to 4.90	-1.3
24.9.35)		(1.0 to 1.35	—— not observed ——		3.30 to 5.10	-1.9
25.9.35)		(0.3 to 1.3	—— not observed ——		2.10 to 4.70	-0.5

* Readings here not repeated, hence interpretation for such a small slope is uncertain.

SUMMARY. SCATTERER OF PARAFFIN WAX THICKNESS 0.3 c.m. (0.27 gm. ~~/~~sq. cm.)

2 experiments	Coolidge Tube and Transformer (2)	$\frac{S}{P}$ constant for range $3.5 < \mu/\rho < 5.0$ decreasing thereafter at about 3% per unit μ/ρ
2 experiments	Andrews Tube(2) and Transformer (2)	S/P constant throughout observed range $2.40 < \mu/\rho <$ 6.2
4 experiments	Muller Tube(2) and Transformer (2)	S/P decreasing throughout observed range $2.25 < \mu/\rho <$ 5.1 at about 1% per unit μ/ρ

TABLE II. SCATTERING OF UNFILTERED PRIMARY BEAM AT 90° TO PRIMARY AND PARALLEL TO THE CATHODE STREAM.

(b) SCATTERER OF PARAFFIN WAX OF THICKNESS 0.85 cm. (0.79 gm/sq.cm.)

Date	Source of X-rays	Tube Current in Milli-amperes	Range of μ/ρ_{Al} for which S/P is observed constant.	Limit of Constant S/P	Range of μ/ρ_{Al} for which S/P is observed to decrease	Fractional Increase of S/P per unit μ/ρ_{Al}	Remarks.
24.9.29	5" Gas Tube Rectifying Gas Valve, Induction Coil and Mains driven Gas Break	not	____ not	observed ____	5.40 to 6.65	-4.4	Tested for stray L radiation
25.9.29			4.7 to 5.1	5.1 ?	5.10 to 5.50	-2.2	
25.9.29			4.0 to 4.8	?	5.30 to 6.00	-10.0	
(30.9.29	same with softeners operating	meas-	5.3 to 6.5	____ not	observed	____	
(1.10.29	same with softer bulb	-ured	6.6 to 7.0	7.0 ?	7.00 to 9.00	-7.0	
23.6.33	Coolidge Tube and Transformer(2)	5.0	____ not	observed ____	2.90 to 5.60	-2.2	
18.7.34	Muller (2) Tube and Transformer(2)	0.4 to 0.7	____ not	observed ____	1.70 to 3.90	-2.2	
19.7.34		0.2 to 0.7	____ not	observed ____	1.25 to 3.90	-2.1	

Note: 4 experiments are omitted from Gas Tube section because conditions were unreliable.

SUMMARY. SCATTERER OF PARAFFIN WAX THICKNESS 0.85 cm. (0.79 gm/sq.cm.)

3 experiments	Gas tube and valve, Induction coil and mains break.	S/P constant for $\mu/\rho < 5.0$ and decreasing $\mu/\rho > 5.0$
2 experiments	Soft gas tube and valve Induction coil and mains break.	S/P constant for $\mu/\rho < 6.5$ and decreasing $\mu/\rho > 7.0$.
1 experiment	Coolidge tube and transformer(2).	S/P decreasing throughout observed range $2.90 < \mu/\rho < 5.60$ at 2% per unit μ/ρ
2 experiments	Muller tube and transformer(2)	S/P decreasing throughout observed range $1.25 < \mu/\rho < 3.90$ at 2% per unit μ/ρ

TABLE II. SCATTERING OF UNFILTERED PRIMARY BEAM AT 90° TO PRIMARY AND PARALLEL TO THE CATHODE STREAM.

(c) SCATTERER OF PARAFFIN WAX OF THICKNESS 1.1 cm. ($\overset{1.01}{\cancel{10.1}}$ gm/sq. cm.)

Date	Source of X-rays	Tube Current in Milli-amperes.	Range of μ/ρ_{Al} for which S/P is observed constant	Limit of Constant S/P	Range of μ/ρ_{Al} for which S/P is observed to decrease	Fractional Increase of S/P per unit μ/ρ_{Al}	Remarks.
12.5.32	Muller tube (1)	5.0	——	not observed —	1.95 to 7.30	-3.6	
16.5.32	and transformer	5.0	——	not observed —	2.80 to 7.70	-4.0	
2.6.32	(2)	5.0	——	not observed —	3.00 to 4.60	-5.0	
9.1.33	Coolidge Tube and Transformer(2)	6.0	1.6 to 2.3	2.3	2.30 to 5.60	-2.5	
17.1.36	Muller (2)	0.6 to 4.3	1.7 to 2.7	3.0	3.45 to 4.25	-4.0	Large primary aperture and interrupted beam.
21.1.36	Tube and	0.4 to 4.0	1.6 to 2.85	3.25	3.25 to 5.4	-3.0	
4.3.36	transformer(2)	0.2 to 2.0	2.15 to 2.8	2.9	2.9 to 4.65	-4.6	

SUMMARY. SCATTERER OF PARAFFIN WAX. THICKNESS 1.1 cm. ($\overset{1.01}{\cancel{10.1}}$ gm/sq.cm.)

3 experiments Muller tube and transformer(2) S/P decreasing throughout observed range $1.95 < \frac{\mu}{\rho} < 7.70$ at 4% per unit $\frac{\mu}{\rho}$.

1 experiment Coolidge tube and transformer(2) S/P constant $1.6 < \frac{\mu}{\rho} < 2.3$ decreasing $2.3 < \frac{\mu}{\rho} < 5.60$ at 2.5% per unit $\frac{\mu}{\rho}$

3 experiments Muller tube and transformer (2) S/P constant $1.6 < \frac{\mu}{\rho} < 2.9$ decreasing $3.25 < \frac{\mu}{\rho} < 4.4$ at 3.0% per unit $\frac{\mu}{\rho}$

TABLE II. SCATTERING OF UNFILTERED PRIMARY BEAM AT 90° TO PRIMARY AND PARALLEL TO THE CATHODE STREAM.

(d) SCATTERERS OF PARAFFIN WAX OF THICKNESSES 1.25 cm, 1.5 cm and 1.9 cm.

Date	Source of X-rays	Tube Current in Milli-amperes.	Range of $\frac{\mu}{\rho}_{Al}$ for which S/P is observed constant.	Limit of Constant S/P	Range of $\frac{\mu}{\rho}_{Al}$ for which S/P is observed to decrease.	Fractional Increase of S/P per unit $\frac{\mu}{\rho}$	Remarks.
2.5.34	Andrews (2) and	0.5 to 3.0	not observed		1.95 to 5.40	-2.8	Scatterer Thickness 1.25 cm.
4.5.34	transformer(2)	2.7 to 6.0	not observed		1.75 to 4.70	-2.7	
25.2.29	Gas tube rectifying)	not	1.45 to 2.60	not observed			Scatterer Thickness
27.2.29	valve with in-) duction coil)	measured	1.45 to 2.70	not observed			
13.3.39	and gas break.)		not observed		2.67 to 4.20	-1.2	1.5 cm.
29.1.36	Muller (2)	0.4 to 1.7	not observed		1.90 to 4.20	-3.1	Scatterer Thickness 1.9 cm.
17.2.36	tube and	1.0 to 3.0	not observed		1.90 to 4.40	-4.0	Large primary aperture and interrupted beam.
13.3.36	transformer(2)	0.7 to 3.0	not observed		1.90 to 4.10	-3.0	

SUMMARY SCATTERER OF PARAFFIN WAX THICKNESS 1.25 cm. (1.15 gm/sq.cm).

2 experiments Andrews tube(2) and transformer(2) S/P decreasing throughout observed range $1.75 < \frac{\mu}{\rho} < 5.4$ at 2.8% per unit $\frac{\mu}{\rho}$

SUMMARY SCATTERER OF PARAFFIN WAX THICKNESS 1.5 cm. (1.40 gm/sq.cm.)

3 experiments Gas tube, valve, induction coil and mains break. S/P constant $1.45 < \frac{\mu}{\rho} < 2.6$ decreasing $2.7 < \frac{\mu}{\rho} < 4.2$ at 1.2% per unit $\frac{\mu}{\rho}$

SUMMARY SCATTERER OF PARAFFIN WAX THICKNESS 1.9 cm. (1.77 gm/sq.cm.)

3 experiments Muller tube(2) and transformer(2) S/P decreasing throughout observed range $1.9 < \frac{\mu}{\rho} < 4.4$ at 3.4% per unit $\frac{\mu}{\rho}$

TABLE III. SCATTERING OF UNFILTERED PRIMARY BEAMS AT 90° TO PRIMARY AND PARELLEL TO THE CATHODE STREAM.

(a) SCATTERER OF FILTER PAPER OF THICKNESS 10 SHEETS. (0.064 gm/sq.cm.)

Date	Source of X-rays	Tube Current in Milli-amperes.	Range of μ/ρ_{Al} for which S/P is observed constant.	Limit of Constant S/P	Range of μ/ρ_{Al} for which S/P is observed to decrease.	Fractional Increase of S/P per unit μ/ρ	Remarks.
20.7.34	Muller(2)	0.4 to 4.0	1.6 to 3.15	3.25	3.25 to 5.2	-3.4	Initial S/P high at μ/ρ = 3.5.
23.7.34	Tube and Trans-former(2)	0.6 to 2.0	1.6 to 3.15	3.2	3.2 to 5.2	-1.7	S/P high at μ/ρ = 3.15 ~~high~~.
24.7.34		0.7 to 3.0	1.6 to 4.0	4	4.0 to 4.6	-2.0	? variations not elucidated.
25.7.34		0.9 to 3.7	1.7 to 5.1	-----	no decrease observed		
26.7.34		0.9 to 5.0	1.55to 3.25	3.25	3.25 to 5.75	-1.7	S/P high at μ/ρ = 3.3 and 3.7
27.7.34		1.0 to 5.0	1.55 to 5.5	-----	no decrease observed		S/P high at μ/ρ = 3.7
31.5.35		1.0 to 4.0	1.75 to 3.0	-----	no decrease observed		
36.6.35		1.0 to 7.5	1.75 to 3.7	-----	no decrease observed		S/P high at μ/ρ = 3.7
11.10.35		0.5 to 2.5	1.70 to 3.0	3.0	3.0 to 4.7	-5.0	S/P varying at μ/ρ = 3.
14.10.35		2.0 to 7.0	2.1 to ?	4.25?	4.25 to 6.0	-2.0	S/P high at μ/ρ = 4.25
15.10.35		4.5 to 6.5	3.0 to 4.25	4.25	4.25 to 6.25	-1.0	

SUMMARY. SCATTERER OF FILTER PAPER 10 SHEETS. (0.064 gm/sq. cm.)

11 experiments with Muller Tube (2) and Transformer (2) All graphs show S/P constant up to μ/ρ = 3 and all but four show S/P decreasing by μ/ρ = 4.25. In seven out of the eleven experiments S/P is higher by one or two per cent than the constant value of S/P, the high value occurring in the neighbourhood of the bend on the graph.

TABLE III. SCATTERING OF UNFILTERED PRIMARY BEAMS AT 90° TO THE PRIMARY AND PARALLEL TO THE CATHODE STREAM.

(b) SCATTERER OF FILTER PAPER OF THICKNESS 50 SHEETS. (0.32 gm/sq.cm.)

Date	Source of X-rays	Tube Current in Milli-amperes.	Range of $\overline{\mu/\rho}_{AL.}$ for which S/P is observed constant.	$\overline{\mu/\rho}_{AL}$ Limit of Constant. S/P	Range of $\overline{\mu/\rho}_{AL.}$ for which S/P is observed to decrease	Fractional Increase of S/P per unit $\overline{\mu/\rho}_{AL.}$	Remarks.
22.2.33	Coolidge	6.0 to 6.5	2.0 to 2.5	2.5	2.5 to 4.2	-0.9	Electroscope measurement.
24.2.33	Tube and	6.5	1.5 to 2.6	2.6	2.6 to 5.4	-2.9	
1.3.33	Transformer(2)	6.0	1.5 to 3.4	3.4	3.4 to 5.4	-2.0	
5.7.33		1.0	1.4 to 2.25	2.25	2.25 to 3.25	-8.0	Ionisation Chamber Measurement
6.7.33		1.0	1.4 to 2.2	2.2	2.2 to 2.9	-6.0	
17.7.33		1.0	1.9 to 2.7	2.75	2.75 to 5.75	-3.3	
21.7.33	Andrews (1) Transformer(2)	1.0	1.6 to 3.25	3.25	3.25 to 5.25	?	
4.10.33	Andrews	0.3 to 1.3	2.2 to 3.5	3.5	3.5 to 6.1	-1.3	low currents
11.10.33	tube(2) and	0.15 to 1.0	2.0 to 3.5	3.5	3.5 to 4.55	-5.0	
11.12.33	Transformer(2)	0.4 to 2.0	2.1 to 5.4	--------	not observed	----------	medium currents
8.1.34		4.0 to 4.5	4.0 to 7.0	--------	not observed	----------	
12.1.34		1.0 to 4.0	2.1 to 5.1	--------	not observed	----------	
23.7.35	Muller	0.2 to 1.0	--- not observed	---	1.85 to 4.8	-2.1	low currents
24.7.35	tube(2) and	0.2 to 1.0	--- not observed	---	1.85 to 4.8	-1.8	
19.7.35	Transformer(2)	0.16 to 1.18	-- not observed	with certainty.	1.8 to 4.6		see fig. 13.
27.2.35		0.4 to 2.0	1.0 to 5.0	-------	not observed	----------	medium currents.
13.3.35		0.4 to 4.0	1.5 to 3.9	-------	not observed	----------	
29.3.35		0.4 to 6.5	1.85 to 5.75	-------	not observed	----------	
10.1.36		0.6 to 3.6	1.7 to 3.0	3.25(?)	3.5 to 4.75	-1.0	
18.12.34		0.4 to 4.0	1.75 to 4.35	4.4	4.5 to 5.3	-3.3	
1.3.35		0.4 to 6.0	1.5 to 5.0	-------	not observed	----------	
13.1.36		0.6 to 9.2	1.8 to 4.0	4.0(?)	4.0 to 8.8	-2.8	high currents
27.3.35		1.0 to 8.0	1.5 to 5.0	5.0	5.0 to 5.7	-1.8	
19.12.34		0.3 to 9.0	1.6 to 6.25	-------	not observed	----------	
20.12.34		0.4 to 9.0	1.85 to 4.9	-------	not observed	----------	

TABLE III. SCATTERING OF UNFILTERED PRIMARY BEAMS AT 90° TO THE PRIMARY AND PARALLEL TO THE CATHODE STREAM.

(c) SCATTERERS OF FILTER PAPER OF THICKNESS 100 SHEETS. (0.64 gm/sq.cm.)

Date	Source of X-rays	Tube Current in Milli-amperes.	Range of μ/ρ_{Al} for which S/P is observed constant.	Limit of Constant S/P	Range of μ/ρ_{Al} for which S/P is observed to decrease	Fractional Increase of S/P per unit μ/ρ_{Al}	Remarks.
20.3.34	Andrews	1.0 to 2.5	1.7 to 4.0	4.0	4.0 to 5.4	-7.0	
22.3.34	Tube(2) and	2.5 to 5.0	1.5 to 5.4	-			
27.3.34	Transformer(2)	2.5 to 4.0	1.85 to 4.5	4.5?	4.5 to 5.4	-1.0	
27.4.34		1.3 to 6.5	1.5 to 4.0	4.0?	4.0 to 5.5	-2.0	
30.4.34		1.3 to 3.0	1.85 to 3.15	no further observations			
14.5.34		1.0 to 5.0	1.85 to 3.5	no further observations			
12.7.34	Muller (2)	0.6	1.3 to 2.5	2.5	2.5 to 3.95	-6.0	Small tube currents.
13.7.34	Tube and	0.6	1.3 to 2.5	no further observations			
16.7.34	Transformer(2)	0.4 to 0.7	1.3 to 2.5	2.5	2.5 to 3.25	-5.0	
17.7.34		0.7	1.3 to 2.5	2.5	2.5 to 4.0	-3.0	
6.6.34		0.6 to 8.0	1.75 to 2.5	2.5	2.5 to 4.0	-4.0	Medium tube currents
22.6.34		0.62 to 8.0	1.75 to 2.5	2.5	2.5 to 4.0	-4.0	
30.5.34		1.2 to 13.7	1.3 to 3.25	3.25	3.25 to 4.45	-7.0	Large tube currents.
1.6.34		1.8 to 13.5	1.35 to 3.55	3.55	3.55 to 4.2	-2.0	

SUMMARY. SCATTERERS OF FILTER PAPER 100 SHEETS (0.64 gm/sq.cm).

6 experiments with Andrews tube(2) and transformer(2) show S/P constant up to μ/ρ = 4 or greater
In three out of 4 experiments S/P is decreasing after μ/ρ = 4.5.

8 experiments with Muller tube(2) and transformer(2) show S/P not constant beyond μ/ρ = 2.5 unless very large currents are used and even then not constant above 3.5.

TABLE IV. SCATTERING OF UNFILTERED PRIMARY BEAMS AT 90° TO THE PRIMARY AND AT 90° TO THE CATHODE STREAM.

(a) SCATTERERS OF PARAFFIN WAX. MULLER TUBE (2) and TRANSFORMER (2)

Date	Tube Current in Milli-amperes.	Thickness of Scatterer cm.	Range(a) of μ/ρ_{Al} for which S/P increases slowly	Range(b) of μ/ρ_{Al} for which S/P increases more rapidly	Fractional increase in S/P per unit μ/ρ_{Al}		Average Fractional Change.	
					Range(a)	Range(b)	Range(a)	Range(b)
4.7.35	0.4 to 1.05	0.25	1.9 to 3.7	not observed	4.7 ±	-		
5.7.35	0.4 to 1.2	0.3	2.25 to 3.35	3.35 to 4.5	3.5 ±	7.2 ±		
23.9.35	0.3 to 1.0	0.3	2.0 to 4.75	not observed	2.4 ±	-	3.3%	7.2%
24.9.35	1.0 to 1.35	0.3	3.3 to 5.1	not observed	2.8 ±	-		
27.1.36	0.2 to 1.5	1.1	2.2 to ?	? to 4.75	3.7 ±		3.7	
4.3.36	0.2 to 2.0	1.1	2.15 to 2.9	2.9 to 4.65	2.1 ±	6.9 ±		
30.3.36	1.0 to 8.0	1.1	1.9 to 3.2	3.2 to 5.0	1.4 ±	5.4 ±	1.5	7.4
1.4.36	0.5 to 4.0	1.1	2.15 to 3.5	3.5 to 4.5	0.9 ±	9.8 ±		
17.2.36	1.0 to 3.0	1.9	not observed	1.9 to 4.4	-	5.2 ±	-	5.1
13.3.36	0.7 to 3.0	1.9	not observed	1.9 to 4.1	-	5.1 ±		

SUMMARY. SCATTERERS OF PARAFFIN WAX.

4 experiments using paraffin wax scatterers of 0.3 cm. thickness or less. No experiment shows S/P constant. Three experiments show S/P increasing at a constant rate throughout the observed range $2.0 < \mu/\rho < 5.0$. In one experiment S/P appears to increase a little more rapidly for $\mu/\rho > 3.4$.

4 experiments using paraffin wax scatters of 1.1 cm. thickness. In all experiments S/P increases with μ/ρ and in three experiments the increase is more rapid for $\mu/\rho > 3.5$.

2 experiments using paraffin wax scatters of 1.9 cm. thickness. In both experiments S/P increases rapidly with μ/ρ over the range $1.9 < \mu/\rho < 4.1$.

TABLE IV. SCATTERING OF UNFILTERED PRIMARY BEAMS AT 90° TO THE PRIMARY AND 90° TO THE CATHODE STREAM.

(b) SCATTERERS OF FILTER PAPER MULLER TUBE (2) AND TRANSFORMER (2)

Date	Tube Current in Milli-amperes	No. of scatter-:ing sheets	Range(a) of $\frac{\mu}{\rho_{Al}}$ for which S/P increases slowly.	Range(b) of $\frac{\mu}{\rho_{Al}}$ for which S/P increases more rapidly.	Fractional increase in S/P per unit $\frac{\mu}{\rho_{Al}}$		Average Fractional change	
					Range (a)	Range (b)	Range(a)	Range(b)
14.10.35	2.0 to 7.0	10	2.1 to ?	? to 6	3.7±			
15.10.35	4.5 to 6.5		3.0 to 4.25	4.25 to 6.25	3.6 ±	3.8 ±	3.6	3.8
21.6.35	0.5 to 4.0	50	1.9 to 3.0	not observed	4.0 ±	-	3.3	6.7
29.7.35	0.3 to 1.75		2.3 to 3.0	3.0 to 4.85	3.3 ±	4.1 ±		
30.7.35	0.2 to 1.5		3.0 to 4.0	4.0 to 5.0	0.9 ±	12.0 ±		
31.7.35	0.25 to 1.5		1.8 to 4.0	4.0 to 5.0	3.7 ±	5.6 ±		
10.1.36	0.6 to 3.6		1.8 to 3.0	3.0 to 4.7	3.0 ±	4.0 ±		
11.2.36	0.3 to 9.0		2.3 to 3.8	4.0 to 6.0	5.1 ±	8.0 ±		

SUMMARY. SCATTERERS OF FILTER PAPER.

2 experiments using 10 sheets of filter paper show no constancy of S/P but a definite increase as $\frac{\mu}{\rho}$ increases from $\frac{\mu}{\rho} = 2.1$ to $\frac{\mu}{\rho} = 6.25$.

6 experiments using 50 sheets of filter paper show no constancy of S/P but S/P increasing between $\frac{\mu}{\rho} = 1.8$ and $\frac{\mu}{\rho} = 6.0$, the increase being slow for range $1.8 < \frac{\mu}{\rho} < 3.0$ and more rapid for the range $4.0 < \frac{\mu}{\rho} < 6.0$.

TABLE V (a) POLARISATION CORRECTION EXPERIMENTS.

Primary Voltage in Volts	Tube Current in Milli-amperes	Estimated Voltage on tube in Kilovolts	Primary $\frac{\mu}{\rho}_{Al}$	$(S/P)_H$	$(S/P)_V$	Mean S/P	$\overline{G}$ Fractional Increase in mean S/P per $\frac{\mu}{\rho}_{Al}$ unit	Polarization per cent
(i) Scatterer of Paraffin Wax Thickness 0.3 cm. 5:7:35.								
170	0.4	85	2.28	1.027	1.211	1.119	%	8
100	0.8	59	3.34	1.022	1.259	1.140	2.5	10
85	1.2	53	3.83	1.019	1.316	1.167		13
80	1.2	50	4.15	1.018	1.326	1.172	±1.0	13
75	1.2	48	4.45	1.008	1.354	1.181		15
(ii) Scatterer of Paraffin Wax Thickness 0.3 cm. 24:9:35.								
100	1.0	59	3.3	0.909	1.035	0.972	%	6
85	1.0	52	3.9	0.909	1.065	0.987	-0.5	8
70	1.0	45	4.6	0.893	1.068	0.980	±1.3	9
60	1.35	39	5.1	0.874	1.086	0.980		11
(iii) Scatterer of Paraffin Wax Thickness 1.1 cm. 4:3:36.								
150	0.2	80	2.15	1.026	1.107	1.066	%	4
100	0.5	59	3.0	0.993	1.123	1.058	-0.8	6
80	0.8	50	3.8	0.978	1.160	1.069	±0.9	9
60	2.0	39	4.65	0.939	1.233	1.086		14

Much bigger pol. than others
[illegible] 6 8 29

TABLE V (b) POLARISATION CORRECTION EXPERIMENTS.

Primary Voltage in Volts	Tube Current in Milli-amperes	Estimated Voltage on tube in Kilovolts	Primary $\frac{\mu}{\rho}$ Al.	$(S/P)_H$	$(S/P)_V$	Mean S/P	$\overline{G}$ Fractional Increase in mean S/P per unit $\frac{\mu}{\rho}$	Polarization per cent.
(iv) Scatterer of Paraffin Wax Thickness 1.9 cm. 17:2:36.								
150	1.0	80	1.9	0.890	0.945	0.916	%	3
100	1.0	59	2.8	0.875	0.965	0.920		5
80	1.3	50	3.4	0.832	0.995	0.913	0.5	9
70	1.3	45	3.7	0.830	[1.01]	0.920		10
60	3.0	40	4.1	0.830	1.038	0.933	±1.0	11
50	3.0	33	4.45	0.810	1.047	0.928		13
(v) Scatterer of Paraffin Wax Thickness 1.9 cm. 13:3:36.								
150	0.7	80	1.9	1.005	1.115	1.060	%	5
100	2.0	64	2.5	0.992	1.160	1.076	1.5	8
80	2.0	52	3.3	0.970	1.181	1.076		10
60	3.0	40	4.1	0.945	1.243	1.094	±1.1	14
(vi) Scatterer of Carbon Thickness 0.65 cm. 28:2:36.								
180	0.25	100	1.42	1.095	1.189	1.142	%	4
150	0.3	80	1.7	1.105		1.157	1.7	5
140	~~0.3~~ 0.2	80	1.7		1.210			
100	~~0.2~~ 0.7	59	2.36	1.062	1.230	1.146	± 1.5	7
80	~~0.7~~ 1.1	50	3.0	1.065	1.282	1.173		9

TABLE V c POLARIZATION CORRECTION EXPERIMENTS. MULLER TUBE AND TRANSFORMER (2).

Primary Voltage in Volts	Tube Current in Milli-amperes	Estimated Voltage on tube in Kilovolts	Primary $\frac{\mu}{\rho}_{Al}$	$(S/P)_H$	$(S/P)_V$	Mean S/P	$\overline{G}$ Fractional Increase in mean S/P per unit $\frac{\mu}{\rho}$	Polarization per cent
(ix) Scatterer of Filter Paper Thickness 50 sheets 30:7:35.								
190	0.2	100	1.7	.998	1.024	1.011	%	1.3
145	0.3	80	2.25	.996			1.8	
100	0.3	59	3.2	.994	1.074	1.034	±0.6	4
80	1.1	50	3.9	.978	1.085	1.030		5
60	1.5	39	6.0	[.968]	1.210	1.089		11
(x) Scatterer of Filter Paper Thickness 50 sheets 31:7:35.								
190	0.25	100	1.70	1.016	1.053	1.034	%	1.8
100	1.0	59	3.2	1.004	1.110	1.057		5
75	1.3	48	4.0	1.010	1.141	1.075	2.1	6
60	1.5	38	5.3	[1.00]	1.218	1.109	±0.7	10
(xi) Scatterer of Filter Paper Thickness 50 sheets 10:1:36.								
180	0.6	100	1.8	0.980	1.075	1.027	%	5
140	0.6	90	2.2	0.982	1.096	1.039	1.7	5
100	2.0	64	3.0	0.990	1.114	1.052	±0.9	6
80	2.0	52	3.8	0.980	1.173	1.076		9
65	2.3	43	4.7	0.970	1.188	1.079		10

TABLE V d. POLARIZATION CORRECTION EXPERIMENTS. MULLER TUBE AND TRANSFORMER (2).

Primary Voltage in Volts	Tube Current in Milli-amperes	Estimated Voltage on tube in Kilovolts	Primary	$(S/P)_H$	$(S/P)_V$	Mean S/P	Fractional Increase in mean S/P per unit	Polarization per cent
(vii) Scatterer of Filter Paper Thickness 10 sheets 14:10:35.								
150	2.0	85	2.1	.950	1.070	1.010	%	6
100	2.0	64	3.2	.934	1.104	1.019		8
70	5.0	49	4.25	.956	1.152	1.054	-1.6	9
57	7.0	39	6.0	.925	1.220	1.072	±0.6	14
(viii) Scatterer of Filter Paper Thickness 10 sheets 15:10:35.								
100	4.5	68	3.00	.970	1.065	1.017	%	5
70	5.5	49	4.25	.960	1.116	1.038		8
55	6.5	38	6.25	.937	~~1.337~~ 1.192	1.064	-1.4	12
							±0.8	
(xii) Scatterer of Aluminium Thickness 0.045 cm. 26/9/35.								
180	1.5	100	1.4	1.159	1.216	1.188	%	2.5
100	3.0	67	2.7	1.157	1.275	1.216	3.7	5
70	4.0	49	3.45	1.185	1.372	1.278	±1.1	7

N.B. This graph is concave upwards.

TABLE VI. SCATTERING OF UNFILTERED PRIMARY BEAMS AT 90° TO THE PRIMARY AND AT 45° TO THE CATHODE STREAM.

Date	Muller Tube Current in Milli-amperes	Scattering Material its thickness and Density	Observed Range of μ/ρ_{Al}	Observed Fractional Increase of S/P per unit μ/ρ_{Al}	Corresponding values of $\overline{G}$ from Table V.	
9.3.36	1.0 to 2.3	0.3 cm.	2.30 to 5.45	2.0 ± 0.9%)	(2.5 ± 1.0%	(5.7.35).
					(0.5 ± 1.3%	(24.9.35)
2.3.36	0.2 to 1.2	Paraffin Wax	2.2 to 5.8	1.4 ± 1.2%)	0.8 ± 0.9%	(4.3.36)
6.3.36	0.4 to 2.8	1.1 cm.	1.9 to 4.65	1.5 ± 1.5)		
3.2.36	0.3 to 3.0	Paraffin Wax	1.5 to 4.2	0.8 ± 0.9)	(0.5 ± 1.0%	(17.2.36)
5.2.36	0.3 to 9.0	1.9 cm.	1.65 to 4.2	1.0 ± 1.3)	(1.5 ± 1.1%	(13.3.36)
12.7.35*	0.25 to 2.0	Filter Paper	1.8 to 4.0	0.7 ± 1.1)	(1.8 ± 0.6%	(30.7.35)
15.7.35*	0.2 to 2.0	50 sheets	1.65 to 6.8	1.1 ± 1.7)	(2.1 ± 0.7%	(31.7.35)
17.7.35*	0.45 to 1.21		2.3 to 5.6	1.9 ± 0.6)	(1.7 ± 0.9%	(10.1.36)
26.7.35*	0.2 to 1.0	0.32 gm/sq.cm.	1.74 to 4.85	1.6 ± 0.7)		
6.2.36	0.3 to 9.0		1.75 to 7.0?			
7.2.36	0.3 to 9.0		1.75 to 7.2?			
28.2.36	0.4 to 2.5	Carbon 0.65 cm 0.94 gm/sq.cm.	1.7 to 3.75	3.6 ± 1.7	1.7 ± 1.5%	(28.2.36)
19.2.36	1.0 to 3.0	Aluminium 0.045 cm. 0.12 gm/sq.cm.	1.5 to 3.4	5.0 ± 4.0) S/P graph) is concave) upwards)	(3.7 ± 1.1% (S/P graph is concave (upwards	(26.9.35)

(with Aluminium slopes are observed for internal part of the graphs)

* In these experiments the revolving primary shutter was not in use.

SUMMARY. Every experiment recorded in this table shows a small rate of increase of S/P with increase of $\overline{\mu/\rho}$. The error of observation in the slope is in three cases equal to or greater than the observed slope. 10 cases give certain slope. The values of G are consistent with corresponding values of .

TABLE VII DATA OF FILTERING EXPERIMENTS. COOLIDGE TUBE AND TRANSFORMER (2).

(a) Scatterer of Paraffin Wax, thickness 0.3 cm.

DATE	TUBE VOLTAGE in kv.	S/P, observed after filtering through aluminium of thickness 0.0	0.01	0.62	0.03	0.04	0.05	0.06	0.07	0.08	0.09 cm.
4. 2.33	80	1.00		0.972		0.955		0.932			0.907
6. 2.33	68	1.00		0.965		0.948		0.925			0.893
20. 2.33	63	1.00		0.969	0.961	0.934		0.939			0.881
8. 2.33	56	1.00	0.947	0.942	0.944	0.937	0.928	0.890			0.850
15. 2.33	56	1.00	0.964	0.969	0.961	0.937	0.930	0.919	0.898		0.884
17. 2.33	56	1.00	0.982	0.974	0.931	0.953	0.937	0.939	0.915		0.892
10. 2.33	49	1.00		0.960		0.937		0.902			0.851
11. 2.33	49	1.00		0.953		0.931					

(b) Scatterer of Paraffin Wax, thickness 1.1 cm.

DATE	TUBE VOLTAGE	S/P, observed after filtering through aluminium of thickness 0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09 cm.
12.12.32	96	1.00		0.972		0.947		0.923		0.909	0.909
12.12.32	91	1.00		0.988		0.968		0.938			0.922
8.12.32	91	1.00	0.985	0.983	0.961	0.948		0.944	0.934	0.921	0.921
2.12.32	84	1.00	0.993	0.976	0.947	0.960		0.940			0.904
2.12.32	84	1.00	0.		0.986	0.978	0.967				0.913
16.12.32	74	1.00	0.971	0.947	0.934		0.919	0.901			
		1.00		0.987		0.981	0.951	0.891	0.940	0.892	0.904
30.11.32	74	1.00	0.986	0.971		0.952		0.936			
9.12.32	72	1.00	0.988	0.971	0.958	0.959	0.953	0.942	0.902		0.912
						0.957	0.923	0.927	0.937		0.903
5.12.32	56	1.00		0.952		0.921		0.898			0.877
7.12.32	48	1.00		0.959	0.949	0.932		0.898			0.866

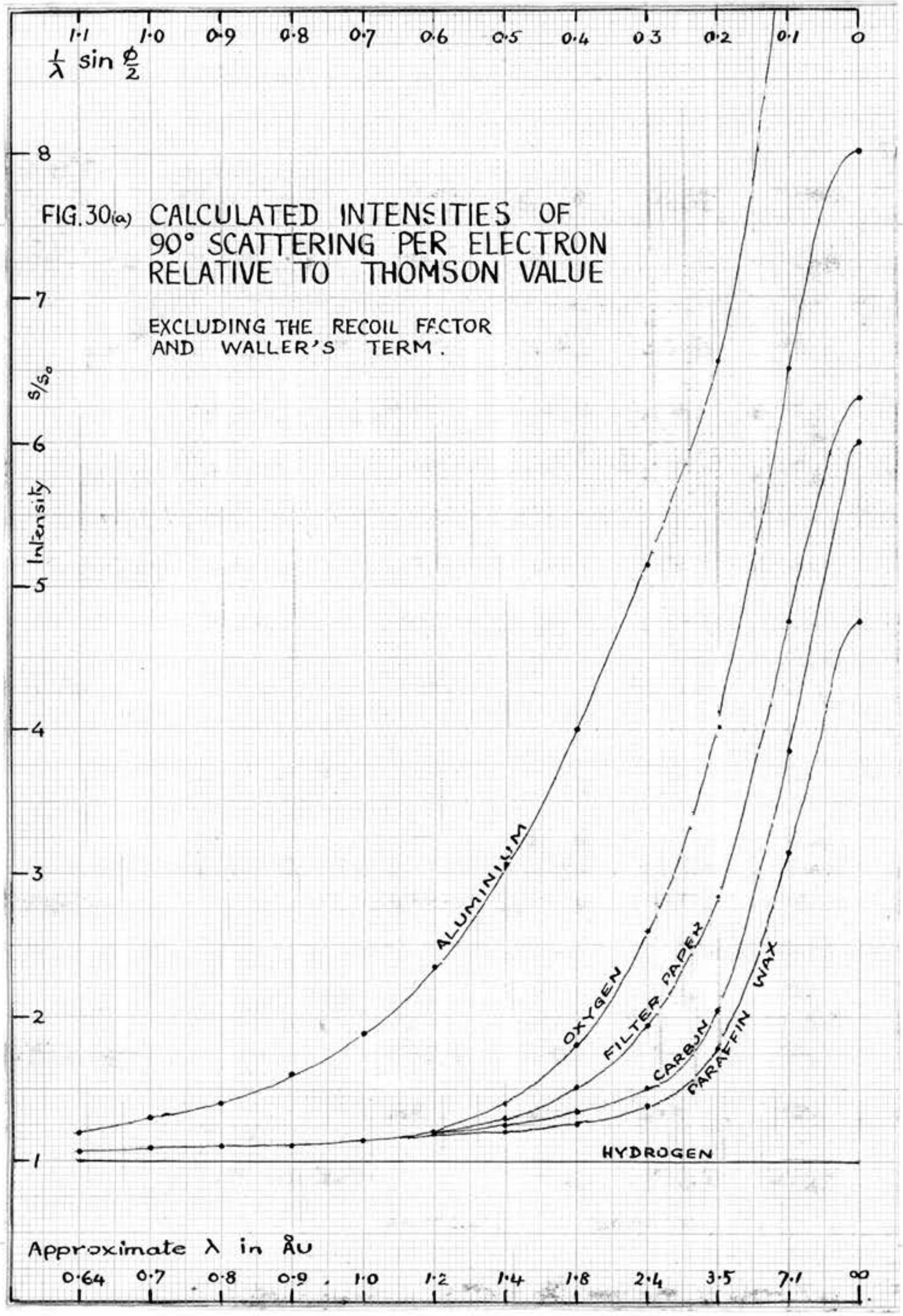


FIG. 30(a) CALCULATED INTENSITIES OF 90° SCATTERING PER ELECTRON RELATIVE TO THOMSON VALUE

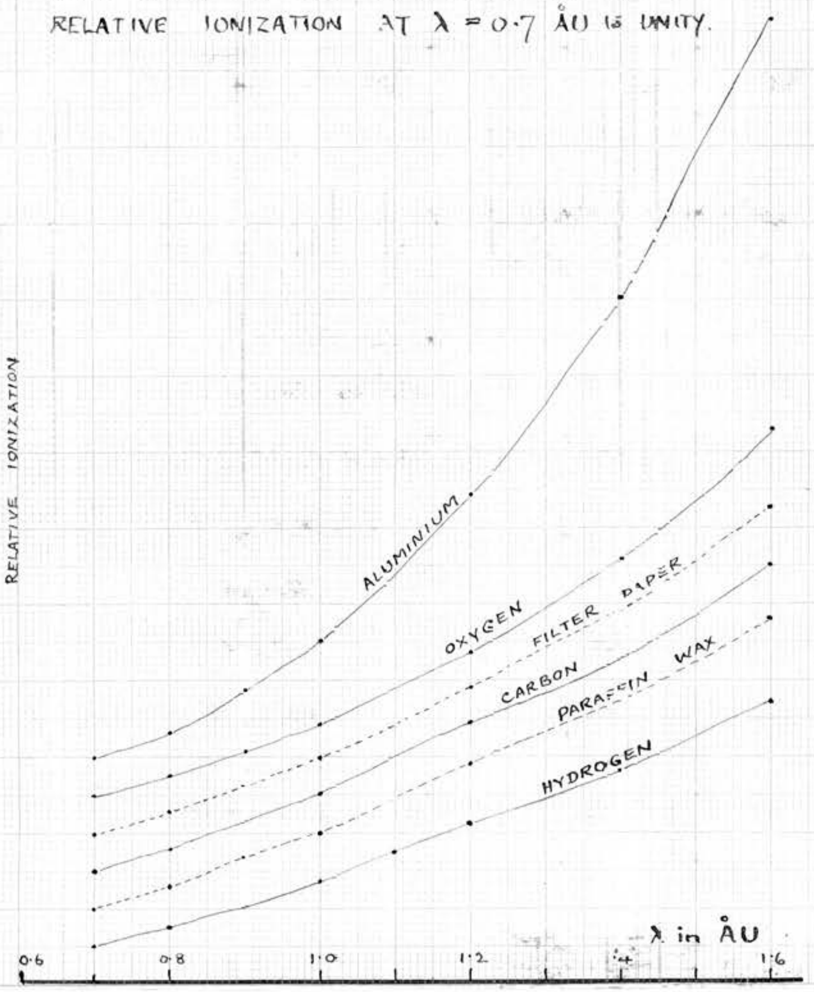
EXCLUDING THE RECOIL FACTOR AND WALLER'S TERM.

TABLE VIII. CALCULATED INTENSITIES OF SCATTERED X-RAYS AT VARIOUS WAVE LENGTHS.

	Wave Lengths in ÅU	∞	7.07	3.53	2.36	1.77	1.41	1.18	1.01	0.88	0.79	0.71	0.64
	$\{\sin \phi/2\}/\lambda$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
HYDROGEN	F_z^2	1.00	0.66	0.23	0.06	0.02	0	0	0	0	0	0	0
	$Z - \sum_1^Z f_{kk}^2$ + K.N.	0	0.34	0.77	0.94	0.98	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	Total / electron	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
CARBON	F_z^2	36.00	21.16	9.00	4.84	3.61	2.89	2.56	1.96	1.69	1.44	1.00	0.81
	$Z - \sum_1^Z f_{kk}^2$ + K.N.	0	2.00	3.30	4.20	4.50	4.70	5.00	5.20	5.30	5.41	5.51	5.61
	Total	36.00	23.16	12.30	9.04	8.11	7.59	7.56	7.16	6.99	6.85	6.51	6.42
	Total / electron	6.00	3.86	2.05	1.51	1.35	1.26	1.26	1.19	1.17	1.14	1.09	1.07
OXYGEN	F^2	64.00	50.41	28.09	15.21	8.41	4.84	3.24	2.56	2.25	1.96	1.82	1.56
	$Z - \sum f_{kk}^2$ + K.N.	0	1.60	4.10	5.60	6.00	6.40	6.50	6.70	6.81	6.91	7.11	7.21
	Total	64.00	52.01	32.19	20.81	14.41	11.24	9.74	9.26	9.06	8.87	8.93	8.77
	Total / electron	8.00	6.50	4.02	2.60	1.80	1.41	1.22	1.16	1.13	1.11	1.12	1.10
ALUMINIUM	F^2	169.00	121.00	80.10	60.06	43.56	30.25	20.25	13.69	9.61	7.02	5.29	4.00
	$Z - \sum f_{kk}^2$ + K.N.	0	2.80	5.20	7.00	8.50	9.60	10.41	10.81	11.11	11.31	11.51	11.62
	Total	169.00	123.80	85.30	67.06	52.06	39.85	30.66	24.50	20.72	18.33	16.80	15.62
	Total /electron	13.00	9.52	6.56	5.16	4.00	3.07	2.36	1.88	1.60	1.41	1.30	1.20
PARAFFIN WAX	Total per electron	4.75	3.15	1.79	1.38	1.26	1.20	1.19	1.15	1.12	1.11	1.06	1.05
FILTER PAPER	Total per electron	6.34	4.76	2.84	1.95	1.51	1.29	1.21	1.15	1.13	1.11	1.09	1.07

FIG 30(b) CALCULATED RELATIVE IONIZATIONS FOR X-RAYS SCATTERED AT 90° BY VARIOUS WAVE LENGTHS [IN THE PARTICULAR IONIZATION CHAMBER.]
VERTICAL SCALE 1 CM = 2 UNITS

TO MAKE THE GRADIENTS COMPARABLE ALL THE GRAPHS HAVE BEEN REDUCED TO A SCALE IN WHICH RELATIVE IONIZATION AT $\lambda = 0{\cdot}7$ ÅU IS UNITY.



These figures and the graph require the following additional correction:

(a) More accurate correction for the ionization due to the modified ray. This raises the ionization at $\lambda = 0.7$ ÅU by one per cent, and lowers it at $\lambda = 1.0$ ÅU by one per cent.

(b) Correction for electron spin term of Waller. This decreases the total intensities and ionizations by varying amounts, and at most by 5 per cent; the correction being appreciable on wave lengths greater than 1 ÅU.

After applying both corrections the maximum change on the aluminium graph is negligible at 0.7 ÅU and -6mm. at 1.6 ÅU. The maximum change on the hydrogen graph is negligible at 0.7 ÅU and -2 mm. at 1.6 ÅU.

TABLE IX (a). CALCULATED RELATIVE IONIZATIONS OF RADIATION SCATTERED AT 90° IN THE PARTICULAR IONIZATION CHAMBERS, AND SUBJECT TO CORRECTION FOR WALLER'S TERM AND TO FINE CORRECTION FOR THE MODIFIED RAY.
RELATIVE TO THE IONIZATION FOR WAVE LENGTH = 0·7 ÅU.

Wave length in AU	0.7	0.8	0.9	1.0	1.1	1.2	1.4	1.6
HYDROGEN	1.0	1.54	2.15	2.77	3.47	4.27	5.69	7.42
CARBON	1.0	1.62	2.32	3.03		4.95	6.57	8.99
OXYGEN	1.0	1.53	2.18	2.86		4.72	7.16	10.6
ALUMINIUM	1.0	1.69	2.76	3.98		7.88	13.1	20.3
PARAFFIN WAX	1.0	1.64	2.36	3.02		4.82	6.45	8.60
FILTER PAPER	1.0	1.59	2.25	2.93		4.78	6.77	9.54

TABLE IX (b). FINE CORRECTION FOR THE MODIFIED RAY. OBTAINED FROM THE DOUBLE LOGARITHMIC PLOT OF TABLE X. $C = (1 + \alpha \,\mathrm{vers})^{-3} \cdot \frac{\text{Fraction of power absorbed at } \lambda'}{\text{Fraction of power absorbed at } \lambda}$

Wave length in AU.	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.4	1.6
C	0.986	0.990	0.992	0.994	0.995	1.008	0.996	0.983	0.985	0.986	0.984	0.984	0.984

TABLE X WAVE LENGTH SENSITIVITY OF IONIZATION CHAMBER. (THEORETICAL)

(a) without, (b) with end correction factor (estimated).

λ ÅU		.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.4	1.6
$\frac{\sigma}{\rho}$ in SO_2		.16	.20	.24	.28	.32	.36	.41	.46	.50	.54	.60	.70	.80
$\frac{\mu}{\rho}$ in SO_2		.29	.56	1.10	1.95	3.10	5.35	8.5	12.0	15.8	20.5	26.5	39.	60.
$\frac{\sigma}{\mu}$		.55	.36	.22	.144	.103	.067	.049	.038	.032	.027	.023	.018	.013
e^{-4x}		.998	.996	.991	.981	.968	.944	.913	.877	.838	.793	.741	.647	.552
loss $\frac{\sigma}{\mu}e^{-4x}$		.55	.36	.22	.141	.100	.063	.044	.033	.027	.0214	.0170	.0116	.0072
K rad. loss		0	0	0	0	0	0	0	0	0	0	0	0	0
1 - total loss		.45	.64	.78	.86	.90	.937	.956	.967	.973	.979	.983	.988	.993
$1 - e^{-6.6\mu}$		.005	.010	.020	.036	.057	.097	.148	.203	.259	.323	.395	.524	.680
Fraction of Power Absorbed.	(a)	.0022	.0064	.0156	.031	.051	.091	.140	.196	.252	.316	.388	.518	.675
	(b)	.0016	.0054	.0143	.030	.048	.087	.136	.190	.247	.310	.380	.513	.670
End correction factor estimate		.71	.84	.91	.93	.95	.96	.97	.97	.98	.98	.99	.99	.993

TABLE XI UNPOLARIZED TERM IN THE INCOHERENT RADIATION. (90° SCATTERING.)

λ in ÅU	0.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.4	1.6
Klein N. Unpolarised	0.047	0.013	0.006	0.003	0.002	0.002	0.001	0.001	0	0	0	0	0	0

TABLE XII LOG OF SOURCES AND MEASURING SYSTEMS OF X-RAYS.

DATE	SOURCE OF HIGH TENSION	X-RAY TUBE	X-RAY POWER MEASUREMENT
10.11.28 to 12.3.30.	Induction coils (various) Series rheostat in primary circuit, Also mains driven mercury gas make and break.	Low pressure gas filled tubes (various) Where necessary used with single low pressure gas filled valve in series.	Cubical box air electroscopes with gold leaf.
7.5.30 to 15.12.30	As above.	Muller (1) 3KW. vacuum tube Hot cathode Water cooled anti-cathode with incorporated water tank.	As above
1.5.31 to 15.12.31.	Transformer (1) on A.C. mains through auto-transformer Output unrectified	As above	Ionisation chambers containing SO_2 Cubical electroscope for primary Wilson tilted for Secondary
26.2.32 to 8.7.32	Transformer (2) 5 Kw. on A.C. mains through series rheostat Output unrectified	As above	As above
14.11.32 to 10.3.33.	As above	Coolidge vacuum tube Hot cathode Hot anticathode	Cubical box air electroscopes with gold leaf.
28.4.33 to 18.7.33	As above	As above	Ionisation chambers containing SO_2 Cubical electroscope for primary Wilson Tilted for secondary.
21.7.33 to 24.7.33	As above	Andrews (1) vacuum tube Thin window	As above
25.9.33 to 14.5.34	As above	Andrews (2) vacuum tube Thin window	As above

TABLE XII (CONTD).

DATE	SOURCE OF HIGH TENSION	X-RAY TUBE	X-RAY POWER MEASUREMENT
30.5.34 to 25.10.35	As above	Muller (2) 3 Kw. vacuum tube Hot cathode Water cooled anti-cathode with incorporated water tank.	As above
8.1.36 to 31.3.36	As above	As above	As above with large primary aperture and rotating shutter.

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At Professor Barkla's request the accompanying reprint is submitted as evidence of further experience in applying the theory of scattered X-rays.

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The Crystal Structure of "Beta Alumina" $Na_2O \cdot 11 Al_2O_3$.

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of Edinburgh.

1. Introduction.

Alumina Al_2O_3 has been thought to possess a hexagonal form which has become known as "Beta Alumina". It has been shown recently, however, by chemical analysis[1] and from X-ray measurement of the unit cell and the density[2], that the true formula for "beta alumina" is $Na_2O \cdot 11\,Al_2O_3$.

Bragg, Gottfried and West[3] have attempted the X-ray analysis of this crystal on the assumption of a formula $\frac{1}{2} Na_2O \cdot 11\,\frac{1}{2} Al_2O_3$, agreeing with the best chemical analysis then available. They were not able to devise a structure which was completely satisfactory, but were led to suggest an ideal structure with a composition $Na_2O \cdot 11\,Al_2O_3$ to which "beta alumina" might tend. Since such a structure is now satisfactory from chemical analysis and from the density, a test can be made of its fit with X-ray intensities. This paper deals with the carrying out of such a test on $Na_2O \cdot 11\,Al_2O_3$ itself, and on the isomorphous $K_2O \cdot 11\,Al_2O_3$ which has been prepared by the Norton Company.

The unit cells of $Na_2O \cdot 11\,Al_2O_3$ and $K_2O \cdot 11\,Al_2O_3$ are hexagonal with $a_0 = 5.584$, $c_0 = 22.45$ Å, and $a_0 = 5.584$, $c_0 = 22.67$ Å, respectively. The space group is $C6/mmc$ (D_{6h}^4). The only point which is in doubt in the ideal structure suggested by Bragg, Gottfried and West is the position of the Na (or K) atoms in the mirror planes. If the origin is taken at one of the centres of symmetry the Na may be at either ($00\frac{1}{4}$) or ($\frac{2}{3}\frac{1}{3}\frac{1}{4}$). The present work suggests that the Na is actually upon the second of these two positions, so that the parameters of the structure become:

	x	y	z			x	y	z	
$2\,Na$ on	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	(d)	$12\,O_A$ on	$\frac{1}{6}$	$\frac{1}{3}$	.050	(k)
$2\,Al_A$ on	0	0	0	(a)	$4\,O_B$ on	$\frac{2}{3}$	$\frac{1}{3}$	.050	(f)
$4\,Al_B$ on	$\frac{1}{3}$	$\frac{2}{3}$	.022	(f)	$4\,O_C$ on	0	0	.144	(e)
$12\,Al_C$ on	$\frac{1}{3}$	$\frac{1}{6}$	.106	(k)	$12\,O_D$ on	$\frac{1}{2}$	$\frac{1}{2}$	.144	(k)
$4\,Al_D$ on	$\frac{1}{3}$	$\frac{2}{3}$	.178	(f)	$2\,O_E$ on	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	(c)

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1) Ridgway, Klein and O'Leary, Transactions of the Electrochemical Society, Vol. LXX, p. 71, 1936.

2) Beevers and Brohult, Z. Kristallogr. **95** (1936) 472.

3) Bragg, W. L., Gottfried, C. and West, J., Z. Kristallogr. **77** (1931) 255.

The last column in this table gives the designations of the sets of positions from Wyckoff, "Theory of Space Groups". The structure defined by these parameters is described in Section 4 of this paper.

2. Intensity Data.

Bragg, Gottfried and West give data which can be put into three sets, and are based on spectrometer observations, an extinction correction being applied. These sets are

1. the $(000l)$ intensities, and some planes like $(22\bar{4}l)$ with l even; which have similar intensities;
2. planes $(hki0)$ of the c-axis zone.
3. a few intensities $(h0\bar{h}l)$.

These intensities are from $Na_2O \cdot 11\,Al_2O_3$ and are probably quite as good as can be got from the thin cleavage plates which are the only experimental material available.

Further intensity results of a rather lower order of accuracy have been obtained by us from oscillation photographs. The main object of this work was to obtain a sufficiently complete set of intensities of the $(h0\bar{h}l)$ type to make a double Fourier synthesis on the $a-c$ plane practicable. The radiation used was $Cu\ K\alpha$ and the specimens were small chips of crystals of $Na_2O \cdot 11\,Al_2O_3$ and $K_2O \cdot 11\,Al_2O_3$, long in a direction parallel to one of the a-axes (the axis of rotation) and having cross-sections of 0.4×0.15 and 0.6×0.5 mm. respectively. Thirty-degree oscillations were employed and four photographs for each crystal were sufficient to give all the intensities with the exception of a few which could not be resolved. The equatorial layer lines were photometered after analysis and an attempt made to apply corrections for absorption in the specimens. The corrected intensities were divided by $\Theta = \frac{1 + \cos^2 2\theta}{\sin 2\theta}$ and the square roots found. These were multiplied by a suitable constant to make them comparable with the calculated F's. The Fourier synthesis performed with these values led to the structure given in Section 1.

3. Comparison of Observed and Calculated Intensities.

The intensities are calculated from the structure of Section 1, using the f-curves of James and Brindley. No heat motion correction is made, so that we may expect the observed F values to fall below the calculated ones, especially at the larger values of $\sin \theta/\lambda$. The general structure-factor for $C6/mmc$ is given in the "Internationale Tabellen" Vol. I, p. 303.

Since there are no atoms on the 24-fold set of equivalent positions (i. e. all the atoms are on the mirror planes parallel to the *c*-axis) this structure factor reduces to

$$4\cos(lz+\tfrac{1}{4}l)\{\cos[(h-k)x-\tfrac{1}{4}l]+\cos\,[(h+2k)x-\tfrac{1}{4}l] + \cos\,[(2h+k)x+\tfrac{1}{4}l]\}.$$

For the $(0k\bar{k}l)$ intensities this reduces still further to

$4\ \cos\ lz\ \{2\cos\ kx+\cos\ 2kx\}$ for l even, and

$4\ \sin\ lz\ \{2\sin\ kx+\sin\ 2kx\}$ for l odd.

The agreement for the intensities of Set 1 is shown in Table I. This agreement is improved if the *f*-curve for oxygen is made to approach zero more rapidly than the James and Brindley curve, in accordance with the curve suggested by Bragg and West[1]).

Table I. Planes of Set 1.

$(000l)$				$(22\bar{4}l)$			$(44\bar{8}l)$			$(60\bar{6}l)$		
l	$\sin\theta/\lambda$	F_{cal}	F_{obs}	$\sin\theta/\lambda$	F_{cal}	F_{obs}	$\sin\theta/\lambda$	F_{cal}	F_{obs}	$\sin\theta/\lambda$	F_{cal}	F_{obs}
2	.044	110	big	.362	54	54	.721	28	27	.625	32	31
4	.089	$\overline{106}$	118	.370	$\overline{53}$	58	.726	$\overline{28}$	30	.630	$\overline{32}$	36
6	.133	14	12	.385	1	16	.731	2	11	.638	1	10
8	.178	69	60	.401	51	31	.740	26	15	.650	31	16
10	.222	$\overline{60}$	82	.424	1	3.5	.752	$\overline{8}$	3.5	.662	$\overline{2}$	—
12	.266	$\overline{16}$	9	.448	$\overline{2}$	10	.769	$\overline{4}$	5.8	.680	$\overline{3}$	4.6
14	.311	$\overline{95}$	93	.476	$\overline{78}$	60	.786	$\overline{40}$	23	.700	$\overline{48}$	29
16	.355	$\overline{7}$	—	.507	$\overline{5}$	9	.802	$\overline{2}$	9	.718	$\overline{2}$	10
18	.399	51	48	.538	38	37	.824	22	17	.741	25	27
20	.448	112	62	.573	79	38	.848	56	26	.767	62	30
22	.488	3	8	.608	9	—	.870	10	—	.793	11	—
24	.532	$\overline{78}$	51	.644	$\overline{67}$	37	.900	$\overline{46}$	26	.823	$\overline{53}$	35
26	.576	$\overline{27}$	24									
28	.621	63	52									
30	.665	$\overline{31}$	15									
32	.710	$\overline{58}$	32									
34	.754	$\overline{14}$	—									

As shown by Bragg, Gottfried and West, the planes of the *c*-axis zone (Set 2) can be divided into four types, referred to by them as (a) (b), (c), and (d), the structure-factor being constant for any type, and the variation in F within a group being due to the variation in the *f*-values.

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1) Z. Kristallogr. **69** (1928) 139.

The agreement between calculated and observed values is best shown graphically by plotting F against sin θ/λ.[1])

Figure 1 shows that the agreement is very good, the observed F drops steadily below the calculated F owing to heat motion. The appearance of the graph is a definite improvement on a similar plot using the structure B' of Bragg, Gottfried and West.

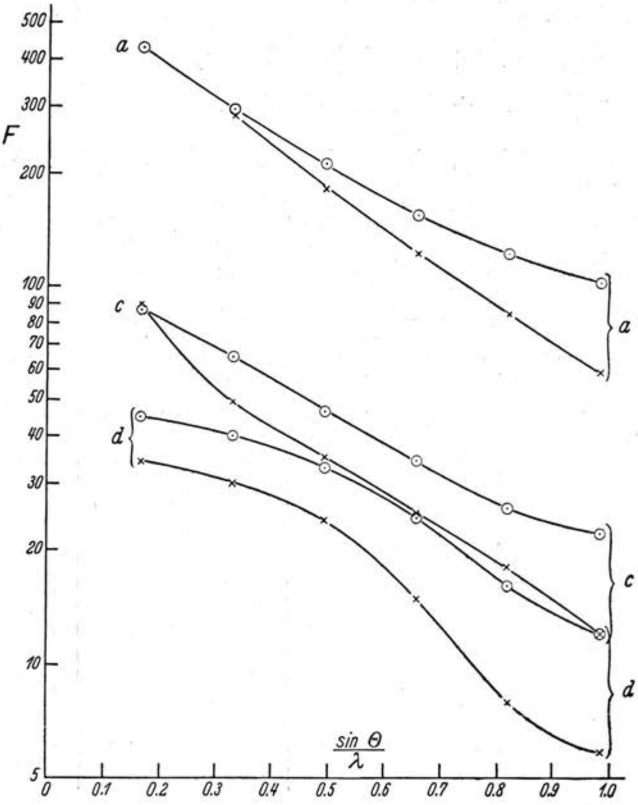

Fig. 1. The circles indicate calculated values, and the crosses observed values, of the F's of the planes of the c-axis zone (Set 2). The planes are divided into four types, each type giving a smooth curve. (Type (b) reflections are too small to be shown).

The intensities ($h0hl$)[2]) also agree very well indeed with the space group structure. The agreement is shown in Table II.

Finally the agreement of all the ($h0\bar{h}l$) intensities from the oscillation photographs is recorded in Table III, for $Na_2O \cdot 11\,Al_2O_3$ and $K_2O \cdot 11\,Al_2O_3$.

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1) Bragg, Gottfried and West, l. c., Table V, p. 272.

2) Bragg, Gottfried and West, l. c., Table VI, p. 272.

Table II. Planes of Set 3.

$h\,l$	1 1	1 2	1 3	2 2	2 4	2 6	3 6	4 4	4 8
$\sin\theta/\lambda$	.106	.113	.124	.212	.225	.248	.340	.424	.452
F_{cal}	11	35	21	40	41	95	60	15	29
F_{obs}	8	33	24	40	39	97	60	—	25

These intensities are definitely better with the Na and K atoms in the position $(\frac{2}{3}\frac{1}{3}\frac{1}{4})$ rather than in the other two-fold positions on the mirror-plane. The good general agreement of such a large number of planes as in Table III establishes the correctness of the structure. It was hoped from the observed F's contained in Table III to improve the parameters by the method of double Fourier synthesis parallel to an a-axis. Syntheses were carried out on both $Na_2O \cdot 11\,Al_2O_3$ and $K_2O \cdot 11\,Al_2O_3$ but no appreciable change of the parameters was suggested by them.

Table III. $(h0\bar{h}l)$ Intensities from Rotation Photographs.

Plane	$Na_2O \cdot 11\,Al_2O_3$		$K_2O \cdot 11\,Al_2O_3$		Plane	$Na_2O \cdot 11\,Al_2O_3$		$K_2O \cdot 11\,Al_2O_3$	
$h\,l$	Cal.	Obs.	Cal.	Obs.	$h\,l$	Cal.	Obs.	Cal.	Obs.
0 2	110	54	97	47	1 15	18	16	25	29
0 4	106	60	99	77	1 16	1	15	3	0
0 6	14	18	4	26	1 17	38	24	31	22
0 8	69	51	87	73	1 18	37	26	41	39
0 10	60	70	61	88	1 19	26	15	33	35
0 12	16	0	10	0	1 20	35	37	32	45
0 14	95	89	103	105	1 21	38	53	45	61
0 16	7	7	10	0	1 22	13	18	17	31
0 18	51	53	39	55	1 23	6	13	12	11
0 20	112	75	112	110	1 24	6	11		
0 22	3	12	3	0	2 0	42	48	47	55
0 24	78	55	68	50	2 1	61	53	52	65
1 0	17	8	23	12	2 2	40	38	36	47
1 1	11	9	0	20	2 3	38	18	46	40
1 2	35	24	42	41	2 4	41	37	46	45
1 3	21	24	10	0	2 5	87	62	95	68
1 4	26	14	32	0	2 6	95	73	100	72
1 5	2	7	8	0	2 7	79	63	71	57
1 6	10	22	4	0	2 8	51	49	47	42
1 7	94	60	104	67	2 9	35	17	42	26
1 8	22	29	17	22	2 10	47	45	43	34
1 9	15	28	6	19	2 11	104	78	111	64
1 10	38	32	43	33	2 12	18	29	14	0
1 11	1	0	9	0	2 13	127	100	110	89
1 12	6	10	10	23	2 14	135	90	139	100
1 13	27	35	20	36	2 15	11	0	18	0
1 14	30	26	33	35	2 16	5	0	10	0

Table III (Cont.)

Plane	$Na_2O\cdot 11Al_2O_3$		$K_2O\cdot 11Al_2O_3$		Plane	$Na_2O\cdot 11Al_2O_3$		$K_2O\cdot 11Al_2O_3$	
h l	Cal.	Obs.	Cal.	Obs.	*h l*	Cal.	Obs.	Cal.	Obs.
2 17	18	20	25	36	4 0	36	27	40	30
2 18	51	45	48	54	4 1	48	51	42	61
2 19	43	30	49	42	4 2	22	26	18	23
2 20	9	21	5	15	4 3	39	31	45	50
2 21	29	20	23	20	4 4	15	0	18	13
2 22	35	27	38	42	4 5	54	44	60	55
2 23	9	12	16	0	4 6	58	51	62	60
3 0	66	47	74	80	4 7	50	45	43	41
3 1	0	0	0	0	4 8	29	32	26	28
3 2	2	15	10	0	4 9	24	16	30	21
3 3	0	11	0	0	4 10	33	41	29	30
3 4	79	54	86	87	4 11	76	71	83	69
3 5	0	13	0	0	4 12	13	22	9	10
3 6	60	57	53	57	4 13	92	83	86	100
3 7	0	0	0	0	4 14	98	79	102	88
3 8	4	0	11	0	4 15	6	5	12	0
3 9	0	0	0	0	4 16	3	4	6	0
3 10	17	14	24	34	5 0	518	567	108	73
3 11	0	0	0	0	5 1				
3 12	47	37	54	38	5 2			19	9
3 13	0	7	0	0	5 3	10	9	4	0
3 14	12	5	19	9	5 4	11	12	15	15
3 15	0	10	0	0	5 5	0	8	6	0
3 16	32	17	39	36	5 6	7	12	3	0
3 17	0	7	0	8	5 7	51	50	58	49
3 18	29	20	36	30	5 8	15	29	11	21
3 19	0	0	0	0	5 9	8	22	1	13
3 20	31	41	24	28					

4. Description of the Structure.

The structure has been described briefly by Bragg, Gottfried and West[1]). It consists of blocks of cubic close-packed oxygen atoms of the thickness of four close-packed layers, adjacent blocks being held together by a layer of oxgen and alkali atoms which is not close-packed. The oxygens of the blocks are held together by aluminium atoms which have positions identical with the Al and Mg atoms in the structure of spinel, $MgAl_2O_4$. Figure 2 is a perspective drawing of the portion of the structure between the mirror planes (which are the top and bottom faces of the half of the unit cell shown).

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1) Z. Kristallogr. **77** (1931) 271.

The distance from the alkali atom in the position $(\frac{2}{3}\frac{1}{3}\frac{1}{4})$ to the (six) oxygens of the adjacent blocks is 2.89 Å, both for Na and K "beta". This distance is quite satisfactory for the K atom but is considerably too large for the Na atom. In consequence of this it was thought very probable that the Na atom might be at the other position available for it, viz. $(00\frac{1}{4})$ instead of $(\frac{2}{3}\frac{1}{3}\frac{1}{4})$, although it would there be in contact with only two oxygens. The position $(00\frac{1}{4})$ would also give a better distribution of atoms around the oxygens O_E (as has been pointed out by Westgren in a private communication), and a better "bond structure". However the intensity fit was distinctly better with Na on the $(\frac{2}{3}\frac{1}{3}\frac{1}{4})$ position, which was therefore taken to be the most probable. Fourier syntheses were carried out to see if the oxygens were closer round the alkali in the case of the Na "beta", but no appreciable parameter changes were indicated.

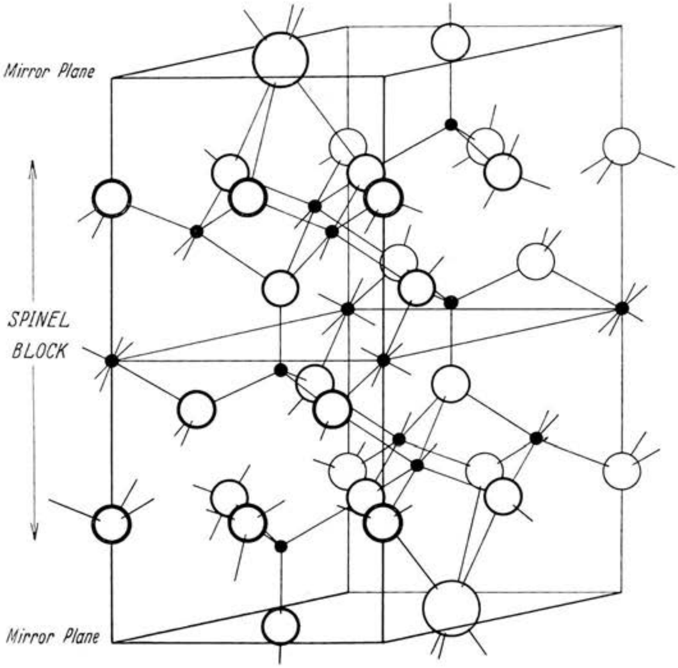

Fig. 2.

A perspective drawing of half of the unit cell. The large circles represent Na or K, the small ones oxygen, and the black dots aluminium atoms.

The distances from the *Al* atoms to oxygens in contact with them are 1.64; 1.64; 1.73; and 1.78 Å, for four co-ordinated aluminiums, and 1.84; 1.97; 2.05 Å for six co-ordinated aluminiums.

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