AUTOMATED REDUCTION OF THE SPECTRA OF SOME METALLIC-LINE STARS

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CONTENTS

ABSTRACT

A computer technique for measuring equivalent widths of absorption lines in stellar spectra, previously tested for main sequence stars, was adapted for the purpose of analysing spectra of metallic-line (Am) stars. The technique requires that a line list of expected lines is known and that the line profile is the same for all lines (excluding Balmer lines). The profile is, in fact, assumed to be Gaussian. A major advantage is that the continuum term in the basic equation is filtered out reducing the errors considerably.

Medium dispersion (17Å/mm) spectra of eight Am stars, obtained in the blue region of the spectrum with the 36" telescope at the Royal Observatory, Edinburgh, are analysed and the equivalent widths presented. The line list was constructed from previous research on Am stars. However, three of the stars, 88 Tau, ω Tau and σ^1Tau, were found to have broader lines than the others resulting in a blend problem when this list was used. Because of this a separate line list was constructed for these stars, badly blended lines being grouped together.

A curve of growth program was developed to perform a coarse analysis for the five narrow-lined stars. The results of this showed iron to be overabundant by a factor of up to four relative to the sun. The abundances of the other elements with respect to iron relative to a standard were similar to those found by other authors. The theory whereby a diffusion process operates in the layer below the outer convective zone could explain the abundance anomalies found.

Different velocity parameters, and hence microturbulent velocities, were found for the three ions FeI, FeII and TiII. It is suggested that this is due to microturbulence increasing outwards towards the star's surface particularly in the region above the outer convective zone. Microturbulence is found to be correlated with line width and hence, probably with macroturbulence.

Four stars, 68 Tau, 15 UMa, τ UMa and 60 Leo, are confirmed as Am. 28 And is found to be border-line.

INTRODUCTION

In the past, most attempts at spectrum reduction have been done by hand which necessarily involves a certain amount of subjectivity on the part of the researcher. This is true not only of measuring equivalent widths but also, and perhaps especially, of the subsequent curve of growth analysis.

Recently, Dr. Thompson, working at the Royal Observatory, Edinburgh, developed a computer program which, to a large extent, removes this subjectivity. Medium dispersion spectra of main sequence stars are measured digitally on a Joyce-Loebl microdensitometer and the program measures the equivalent widths of the absorption lines present. This program has been tested for F and G stars in the red region of the spectrum (THOMPSON (1966), (1971)). The advantage of such a program lies not only in the removal of subjectivity but also in allowing a large batch of spectra to be analysed at once.

The object of this particular piece of research was to apply this program to the problem of metallic-line stars.

Since these stars are near the main sequence and for other reasons described later in this introduction, it was felt that the program would require a minimum of adaptation while still providing interesting problems. To some extent metallic-line stars show enough variation within their group to render their treatment as a batch as problematic but not enough, it was thought, to make it impossible.

The region of the spectrum chosen for analysis was the blue ($\lambda\lambda$ 4150 - 5050 Å) since the frequency of absorption lines in this region approaches the optimum for the program. Attempts to extend the project to the red and infra-red regions were abandoned due to a scarcity of lines there.

In order to analyse the results from this program, a further program was written which performed a curve of growth analysis. This was applied fairly successfully to some previous results on metallic-line stars, mainly those of Praderie's work on η Lyr A (PRADERIE (1967)). Since then it has been refined somewhat and extended to produce a differential curve of growth analysis as well as a direct one.

To consider first the automatic reduction of the data and production of equivalent widths, it is best to look at the assumptions made and the mathematical techniques involved.

We can represent the relative intensity s(x) at any point in the spectrum by

$$s(x) = c(x) - \sum_j A_j f(x-a_j) + n(x) \quad \text{— Equ}^{\text{n}}\ 1.$$

where c(x) is the continuum intensity at x;

A_j is the equivalent width of the j^{th} line centred at a_j;

f(x) is the normalised line profile i.e.

$\int_{-\infty}^{\infty} f(x)dx = 1$;

and n(x) is the noise at x.

The basic assumptions made are twofold.

(a) It is required that a line list of expected lines and their positions be specified beforehand. The construction of such a list will be discussed in Chapter 2.

(b) The instrumental profile is the dominant factor in the observed profile so that all absorption lines in any spectrum, with the exception of the Balmer lines, can be considered to have the same profile f(x). This also ensures that blending is linear, since no attempt is made to resolve very close intrinsically blended lines.

Since we are using a dispersion of 17Å/mm the second assumption is valid.

The difficulties of continuum fitting have been discussed in THOMPSON (1971). There it was shown that any fitted continuum is a function of the initial line list, as well as being distorted by faint absorption bands, clustering of lines etc.

For this reason, it is useful to define the following step function, which will henceforth be referred to as the D-function.

$D(n) = -\frac{1}{2}$; $n = -2q-1,\ 2q + 1$;

$D(n) = -1$; $n = -2q,\ -2q + 1, ..-q-1,\ q + 1,\ q + 2, ..2q$;

$D(n) = 1$; $n = -q,\ -q + 1, ..q$;

$D(n) = 0$; elsewhere.

where q is predetermined - see Fig. I.1.

If we know roughly the expected half-half width in microdensitometer step lengths of the absorption lines in any spectrum,

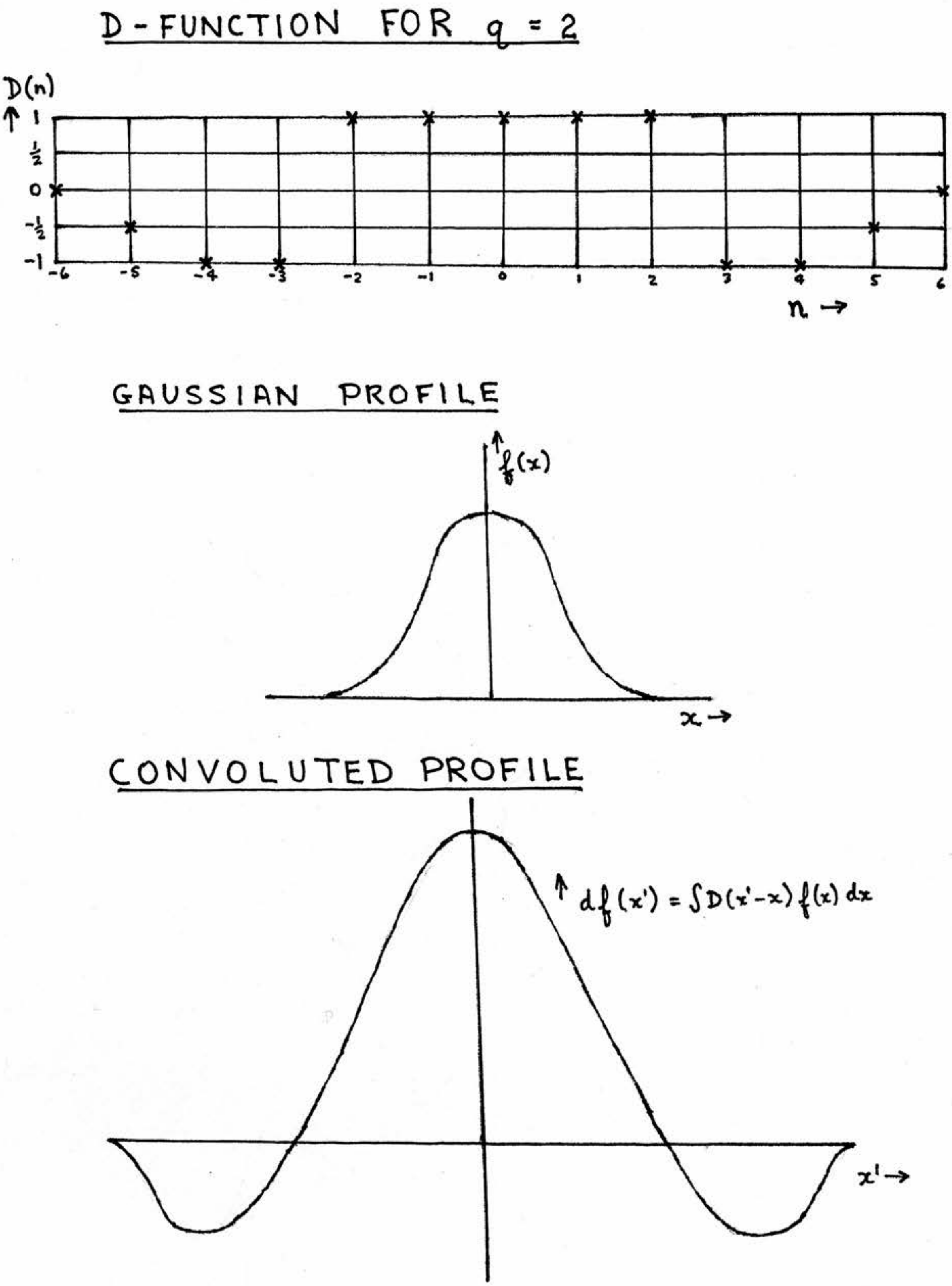

D-FUNCTION FOR q = 2
D(n)
1
½
0
-½
-1
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
n →
GAUSSIAN PROFILE
f(x)
x →
CONVOLUTED PROFILE
df(x') = ∫D(x'-x) f(x) dx
x' →

FIG. I.1

we set q equal to the integer nearest to this. Normally q is set to 2 or 3.

With this value of q we convolve the entire digitised spectrum with the D-function and from equation 1 with $x = x_i$ for the i^{th} step we obtain

$$\sum_1 D(x_1 - x_i)\, s(x_1) = -\sum_j A_j \sum_1 D(x_1 - x_i)\, f(x_1 - a_j) + \sum_1 D(x_1 - x_i)\, n(x_1) \qquad \text{Equ}^n\ 2.$$

Note that the continuum term is now absent.

The assumption that $\sum_1 D(x_1 - x_i)\, c\,(x_1) = 0$ is equivalent to regarding the continuum as linear over the full width of a line (since $\sum x\, D(x) = \sum D(x) = 0$). The absence of a continuum term enables us to treat each blend separately so that long distance coupling of lines is removed. The removal of a continuum is satisfying also since the effect of the exclusion of weak lines on the estimation of the continuum is not negligible. However, a price paid is that the convolution with the D-function increases the extent of the influence of a line and so somewhat increases the blending.

The effect of convolving the line profile with the D-function is shown in Fig.I.1. The D-function filters out low spatial frequencies such as continuum, absorption bands and wide wings from the high spatial frequencies i.e. lines. It should also be noted that the resultant spectrum is in fact a smeared version of the second derivative of the original spectrum (since, as a continuously defined D-function becomes increasingly smaller, the convoluted spectrum tends to $d^2 s(x)/dx^2$.

If we refer to equation 2 we see that there is one such equation for each step and since the number of unknowns, i.e. the number of equivalent widths to be measured, is very much less than this, we can solve by least squares for them.

To cut down the interaction of lines the equations used for estimating A_j are not strictly the normal equations but the following related set.

$$\sum_i f(x_i - a_j) \sum_1 D(x_1 - x_i)\, s(x_1) = -\sum_k A_k \sum_i f(x_i - a_k) \sum_1 D(x_1 - x_i)\, f(x_1 - a_j)$$

If we let $df(x_i - a_j) = \sum_1 D(x_1 - x_i)\, f(x_1 - a_j)$,

$$Z_j = -\sum_i f(x_i - a_j) \sum_l D(x_l - x_i)\, s(x_l) = \sum_l s(x_l)\, df(x_l - a_j),$$
$$\lambda_{jk} = \sum_i f(x_i - a_k) \sum_l D(x_l - x_i)\, f(x_l - a_j) = \sum_i f(x_i - a_k)\, df(x_i - a_j)$$

we get $Z_j = \sum_k A_k \lambda_{jk}$.

If λ_{jk} is thought of as a matrix element and λ^{jk} is the corresponding element in the inverse matrix, we have the solution

$$A_k = \sum_j \lambda^{jk} Z_j$$

The coefficient λ_{jk} is a measure of the interaction of the lines j and k and is a function of their separation $a_j - a_k$.

The expected variance and covariance of the equivalent widths is, as shown in THOMPSON (1971)

$$\mathrm{COV}(A_j, A_k) = \sigma^2 \sum_i \sum_l \lambda^{ji} \Lambda_{il} \lambda^{kl}$$
$$\text{where } \Lambda_{il} = \sum df(x - a_i)\, df(x - a_l)$$

and σ^2 is the expected variance of a single measured datum.

For a single isolated line

$$\mathrm{VAR}(A_j) = \sigma^2 \Lambda_{jj} / \lambda_{jj}$$

Thus we are able to compute the equivalent width and standard deviation of each line.

In equation 1 we have not specified f(x), the observed line profile. In order to do this we assume a Gaussian profile

$$\frac{1}{\beta\sqrt{\pi}}\, e^{-(x/\beta)^2}$$

One advantage of this function is that we need find only one parameter i.e. β. It was thought that there was insufficient information in the spectrum to obtain more than one parameter with any degree of accuracy. A dispersion formula was considered but the Gaussian function was chosen instead because the D-function in removing low spatial frequencies may trim the wings, so that we are left with the central core of the line. This central core which is due to random motion of the atoms, can be best represented by a Gaussian function.

It is possible by using a wider D-function to measure something closer to the original definition of equivalent width, (see WOOLLEY and STIBBS (1953) p.150), but by doing so one loses accuracy since the errors now become larger. In fact, according to Griffin (GRIFFIN (1969)) the wings of the instrumental profile of a spectrograph are in general so extensive that the original definition of equivalent width becomes, from a practical viewpoint, meaningless. It was therefore felt that the present treatment was justified, provided one was aware of these limitations. Thompson submits that the conventional method of measuring equivalent widths by hand is less amenable to such treatment since the procedure will have the same effect on all lines, whereas chopping off wings by hand may not be done uniformly.

Assuming a Gaussian profile for the line profile it remains to determine the value of β which best fits the data.

We can write equation 2 in the form

$$ds(x_i) = -\sum_j A_j df(x_i - a_j) + dn(x_i) \quad \text{— Equ}^{\text{n}}\ 3.$$

where $ds(x_i) = \sum_1 D(x_1 - x_i)\, s(x_1)$; $dn(x_i) = \sum_1 D(x_1 - x_i)\, n(x_1)$;
and $df(x_i - a_j) = \sum_1 D(x_1 - x_i)\, f(x_1 - a_j)$ as before.

If we take the auto-convariance function of equation 3,

$$\begin{aligned} A_{ds}(\tau) &= \sum_i ds(x_i - \tau)\, ds(x_i) \\ &= \sum_i \Big(\sum_j A_j df(x_i - \tau - a_j) \sum_k A_k df(x_i - a_k)\Big) \\ &\qquad + \sum_i dn(x_i - \tau)\, dn(x_i) \end{aligned}$$

where terms linear in n are ignored. So that

$$\begin{aligned} A_{ds}(\tau) &= \sum_i \Big(\sum_j \sum_k A_j A_k\, df(x_i - \tau - a_j)\, df(x_i - a_k)\Big) \\ &\qquad + \sum_i dn(x_i - \tau)\, dn(x_i) \\ &= \sum_j \sum_k A_j A_k \Big(\sum_i df(x_i - \tau - a_j)\, df(x_i - a_k)\Big) + \sum_i dn(x_i - \tau) dn(x_i) \\ &= \sum_j \sum_k A_j A_k\, \Delta(\tau + a_j - a_k) + \sum_i dn(x_i - \tau)\, dn(x_i) \quad \text{— Equ}^{\text{n}}\ 4. \end{aligned}$$

where $\Delta(\tau + a_j - a_k) = \sum_i df(x_i - \tau - a_j)\, df(x_i - a_k)$; $\left[= \Delta(\tau) \sum_j A_j^2 + \text{noise term for unblended lines}\right]$

It should be pointed out here that the noise term can be measured from the calibration spectra on the plate and subtracted out at this stage. A description of its calculation will be given in Chapter 2.

If a_{jk} is the separation between blended lines of equivalent widths A_j and A_k we have

$$A_{ds}(\tau) = \Delta(\tau) \sum_j A_j^2 + \sum A_j A_k\, \Delta(\tau + a_{jk}) + \sum A_j A_k\, \Delta(\tau - a_{jk}) + \sum \ldots + \sum \ldots \text{ etc.}$$

where the summations are over all pairs of blended lines with the same separation (e.g. in the first case over all pairs of lines with separation a_{jk}).

Taking the Fourier Transform of both sides we get

$$
\begin{aligned}
F(\zeta) = \mathcal{F}\{A\,ds(\tau), \zeta\} &= \sum_j A_j^2 \mathcal{F}\{\Lambda(\tau), \zeta\} \\
&+ \sum A_j A_k \mathcal{F}\{\Lambda(\tau + a_{jk}), \zeta\} + \sum A_j A_k \mathcal{F}\{\Lambda(\tau - a_{jk}), \zeta\} + \ldots \\
&= \mathcal{F}\{\Lambda(\tau), \zeta\} \sum_j A_j^2 + \sum A_j A_k e^{-i\zeta a_{jk}} \mathcal{F}\{\Lambda(\tau), \zeta\} \\
&+ \sum A_j A_k e^{i\zeta a_{jk}} \mathcal{F}\{\Lambda(\tau), \zeta\} + \ldots \\
&= \mathcal{F}\{\Lambda(\tau), \zeta\} \left\{ \sum_j A_j^2 + 2\sum A_j A_k \cos(\zeta a_{jk}) + \ldots \right\}
\end{aligned}
$$

— Equ$^{\underline{n}}$ 5.

The term on the left-hand side can be derived from the observed spectrum and $\mathcal{F}\{\Lambda(\tau), \zeta\}$ can be found for various values of β.

The term in brackets can be calculated knowing the equivalent widths and their separations. Therefore, before we can calculate β, we need to know the equivalent widths. This suggests an iterative process.

First of all a value of β is assumed and the initial equivalent widths calculated. We are then able to recalculate β i.e. that value which best fits equation 4. If necessary further approximations can be made until convergence is obtained.

Knowing the final value of β and hence the line profile we can recalculate the equivalent widths.

We have already mentioned why metallic-line stars are suitable for the application of the program. Having discussed the program other reasons now emerge. Metallic-line stars do not have large magnetic fields, otherwise the absorption line profile could not be considered fixed for all lines in a spectrum. Also, Am stars offer a region of the spectrum which gives the optimum line density (see Section 2.5). Furthermore, unlike Ap stars, a list of expected lines and line positions can be drawn up for Am stars, which requires little or no modification from star to star.

The problem of the metallic-line phenomenon has been one of considerable interest in recent years and a review of this topic appears in Chapter 1.

Having obtained theequivalent widths of these stars a curve of growth analysis was performed.

The theory of a curve of growth analysis is given in several standard text-books on stellar atmospheres e.g. WOOLLEY and STIBBS (1953), AMBARTSUMIAN (1958). There are two types of curve of growth analyses, namely the direct and differential methods. Let us consider, first of all, the direct curve of growth.

Looking at the problem from a purely mathematical viewpoint we have a set of lines, corresponding to a given ion, whose equivalent widths have been measured and whose gf values are known. If for the i^{th} line

$$y_i = \log_{10} W/\lambda$$
$$x_i = \log_{10} gf\lambda$$

where W is the equivalent width, λ the wavelength, g the statistical weight and f the oscillator strength, we have the mathematical relationship

$$y_i - b = f_t\left(x_i + a - c\chi_i\right) \quad \text{— Equ}^{n}\ 6.$$

where χ_i is the excitation potential, f_t is one of a family of theoretical curves which are generated by varying the damping constant, and a, b and c are unknowns. Physically they are

$$a = \log_{10} N_r k_o$$
$$b = \log_{10} V/c'$$
$$c = \theta_{exc.}$$

where V is the velocity parameter, c' the velocity of light, $\theta_{exc} = 5040/T_{exc}$, where T_{exc} is the excitation temperature, N_r is the total number of atoms in the r^{th} ionisation state per cm^3 and k_o is a factor which depends on the curve of growth method chosen (see 4.1).

For the moment we will assume that the damping constant is known and hence $f_t(x)$ defined. We are only interested in ions where the number of lines measured exceeds 3, so that we have more than 3 equations in 3 unknowns.

If a_o, b_o and c_o are the first approximations for these three parameters, we have by a Taylor expansion of equation 6

$$y_i - b_0 - \delta b = f_t(x_i + a_0 - c_0\chi_i) + \delta a\, f_t'(x_i + a_0 - c_0\chi_i) - \delta c\, f_t'(x_i + a_0 - c_0\chi_i)\chi_i .$$

Solving by least squares for δa, δb and δc we obtain

$$a = a_0 + \delta a ; \quad b = b_0 + \delta b ; \quad c = c_0 + \delta c .$$

Since the above Taylor expansion ignores powers of δa and δc higher than the first, we accept the new values of the parameters only as a second approximation. Providing the first approximations are reasonable, we would expect an iterative procedure based on the above method to converge to the required parameters.

Hence we can obtain the relative abundances of the ions and hence of the elements, the velocity parameter, and the excitation temperature of the atmosphere. It is possible to deduce other parameters from these. This will be described in later chapters (Chapters 4 & 5) along with the method for determining the damping constant.

This formed the basis of the original curve of growth program. It was found convenient to include weighting of individual lines and a mechanism for fixing any desired parameter or parameters while solving for the others.

In the case of the differential curve of growth program two stars are compared and the parameter differentials, Δa, Δb, Δc, are measured. Again it is possible to fix one or more of these and solve for the others. This program is best used in conjunction with the direct one and a fuller description will be given in Chapter 4.

Finally, the results obtained from such an analysis are compared with previous results and the various theories examined in the light of the new information in Chapter 6.

CHAPTER 1

REVIEW ON METALLIC-LINE STARS

In recent years metallic-line stars have been the subject of several reviews; SARGENT (1964), HACK (1966), ABT (1966) and CONTI (1970). Here an attempt is made to pick out the important features of each review under certain headings and compare them. Am stars are compared both with normal A stars and with Ap stars. The views of Sargent, Hack, Abt and Conti on the interpretation of the anomalies are incorporated. Also a discussion is given of more recent research not included in the above reviews.

1.1 Spectral features

The spectra of Am stars have:-

(a) weakened Ca and Sc lines, particularly the K line of CaII which is weak, sharp and deep.

(b) metallic lines which are strong; particularly ionised metals and rare earths. This effect runs through the usual spectral region for photography. SrII is noticeably strong.

Three different spectral types may be defined using the K line of CaII, or the Balmer lines of H, or the metallic lines. The K line gives the earliest estimate and the metallic lines the latest.

Conti defines the Am phenomenon as follows:

"The Am phenomenon is present in stars that have an apparent surface underabundance of Ca (and/or Sc) and/or an apparent over-abundance of the Fe group and heavier elements."

He suggests three groups:-

(a) stars with strong metallic lines and weak Ca(Sc).

(b) stars with weak Ca(Sc) only.

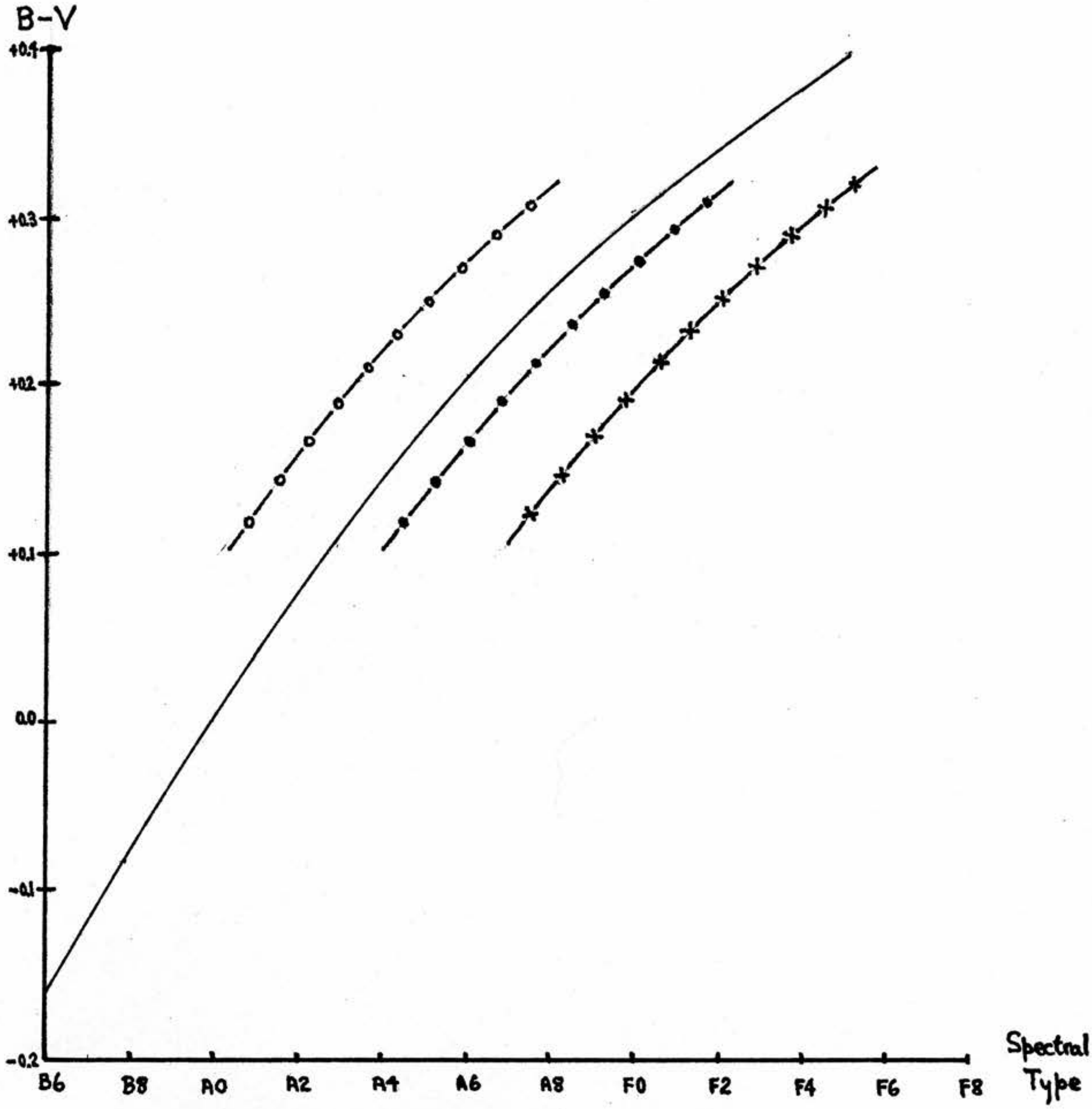
(c) stars with strong metallic lines only.

All other properties he considers secondary.

Other than this, no families seem to exist. Furthermore there seems to be no sharp division into A and Am.

1.2 Photometric features

Hack plots B-V against spectral type derived from the Balmer lines, the K line of calcium, and the metallic lines (Fig.1.1), and shows that while the Balmer spectral type is too late for the colour, it is closer to normal than the spectral types derived from the other means. This is due to the continuum being determined mainly by



Averaged curves with spectral type derived from
(a) K line —o—o
(b) hydrogen lines —•—•
(c) metallic lines —x—x
Main sequence is shown by continuous line.

FIG. 1.1

the continuous absorption coefficient of hydrogen.

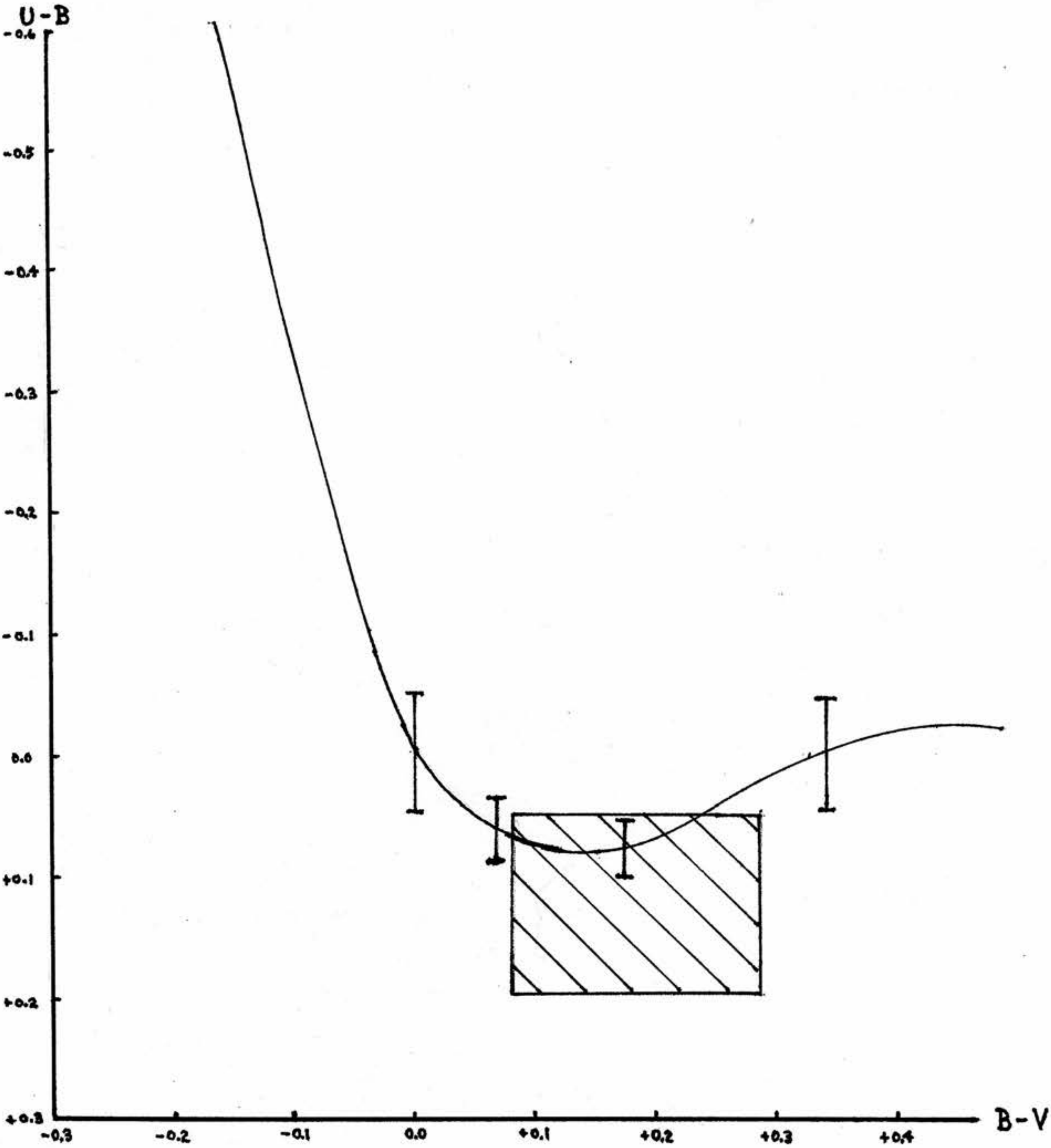
Hack also shows the position of Am stars on the two colour (U-B v. B-V) diagram (Fig. 1.2). BASCHEK and OKE (1965) showed that the U-B deficiency in Am stars shown by this diagram could be due to excessive line-blanketing. Stars with high "metallicity" deviate more than those with low, metallicity being defined as the difference between hydrogen and metallic spectral types.

Abt suggests that Strömgren's 4-colour system is well suited to Am stars since it has two filters (blue and yellow) unaffected by differential blanketing and one filter (violet) centred on the region (λ 4100Å) of heavy line absorption and can be used as a measure of the average strength of the metallic lines. Strömgren's metallic index m_1 can be determined and Abt plots m_1 against (i) Sp(M) - Sp(H); and (ii) Sp(H) - Sp(Ca). (Figs. 1.3 and 1.4). Fig. 1.3 shows that (a) 4-colour measures can be used to detect abnormal line absorption, and (b) not all Am stars have strong metallic lines. Fig. 1.4 shows (a) Am stars with strong metallic lines (right of diagram) with nearly normal or weak Ca lines, and (b) Am stars with nearly normal Ca lines (bottom of diagram) but with normal or strong metallic lines.

No obvious correlation exists between Ca and metallic line anomalies and Abt suggests that there exist continua in both characteristics from normal to abnormal. It is difficult from this diagram to distinguish the boundaries of the three subclasses defined by Conti (see 1.1).

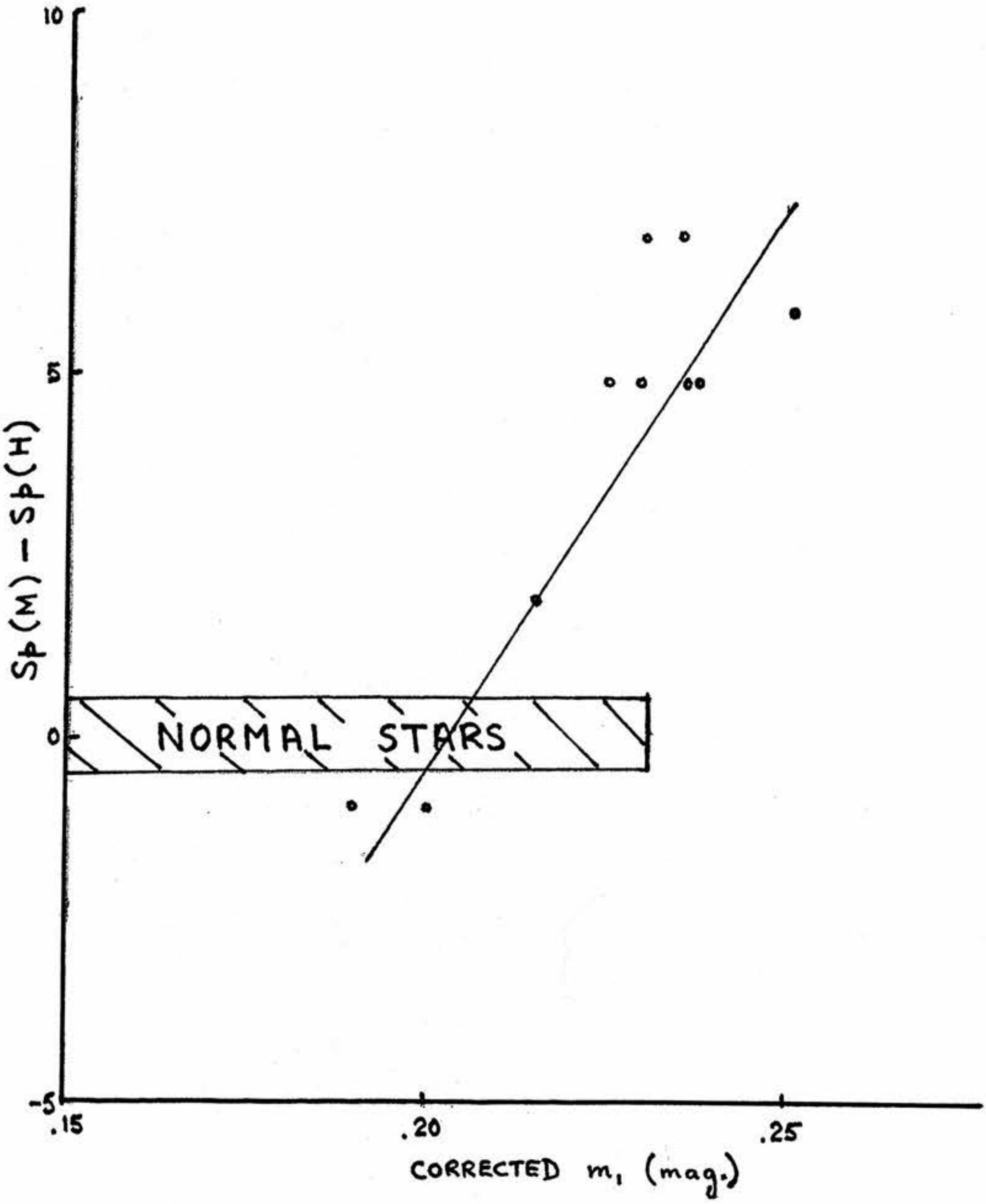
Strömgren's index $[m_1]$ (STRÖMGREN (1966)) is related to m_1 but is reddening and gravity independent. HENRY (1969) has used this in conjunction with an index, k, which measures the strength of the K line. He plots Δk against $\Delta[m_1]$ (Fig. 1.5) where each is the deviation from normal. Conti uses this as evidence for the existence of his three subclasses although again it is difficult to see any discontinuities which would define these. Nevertheless, his suggestion that two separate processes are involved, one accounting for the deficiency of Ca (Sc) and the other for the overabundance of metals, seems reasonable as the anomalies would appear to be independent.

Henry claims that his diagram is the only known method to show a clear distinction between Am and A stars. While this can be regarded



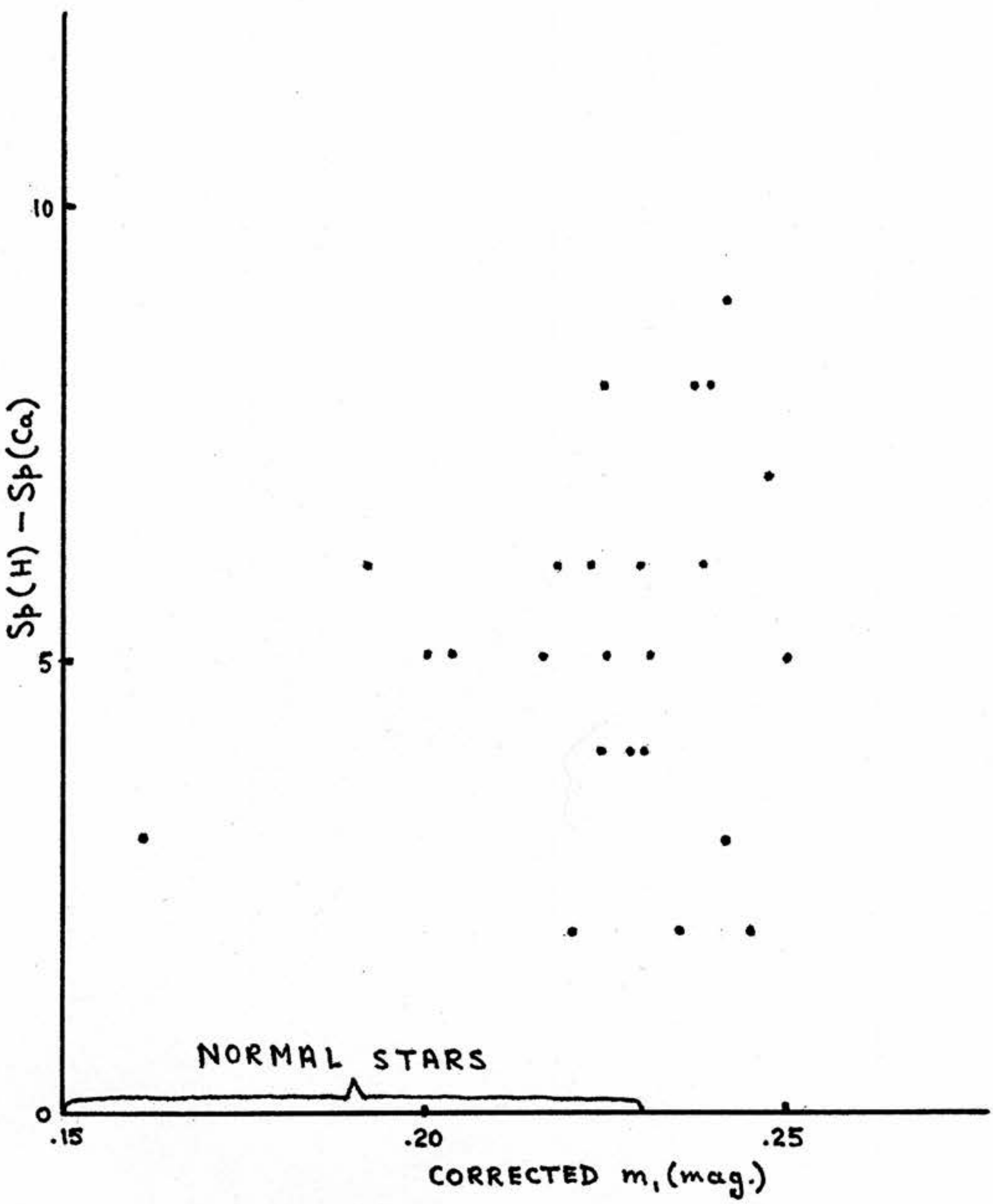
The shaded area represents the area occupied by Am stars. The error bars represent the scatter of main sequence stars included by Hack.

FIG. 1.2



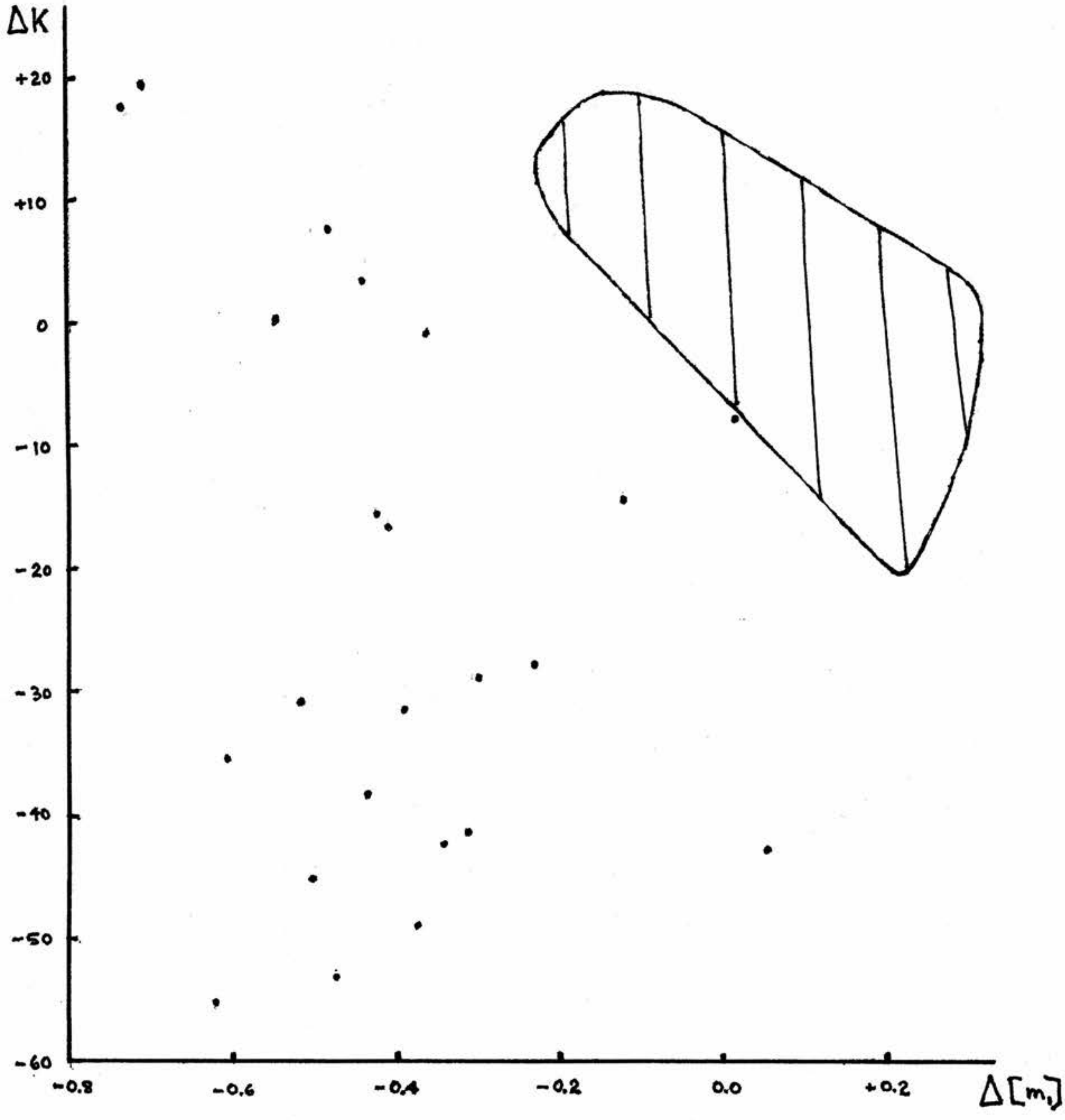
The difference in spectral types in tenths of a class is plotted against Strömgren and Perry's measures of the metallic index corrected for differences in the strength of Hδ which lies within the filter transmission band. The scatter about the line is of the order expected for visual estimates of spectral type.

FIG. 1.3



The difference between spectral types determined visually from hydrogen and calcium lines is plotted against Strömgren's metallic index corrected for differences in the strength of Hδ.

FIG. 1.4



The dots represent Am stars listed by COWLEY et al. (1969). The shaded area represents the area occupied by normal A stars according to Henry.

FIG. 1.5

as a very useful test for Am stars, it should not be assumed that such a sharp division exists since the classification for some stars seems to be somewhat doubtful. Most of the doubtful cases, however, occur close to the boundary line.

1.3 Binaries

ABT (1961) investigated 25 Am stars and found that 22 of them were spectroscopic binaries. From this he concluded that all Am stars are members of spectroscopic binaries by suggesting that the remaining three could have been pole-on binary systems.

He plotted orbital period against frequency per factor of 4 in period for 20 Am stars and suggested that there was a dip near 100 days. Apart from the somewhat contrived nature of this graph, the statistical significance of such a conclusion, where just one more star in a certain range would remove the dip, is nil. It is however significant that 14 stars have periods of less than 100 days.

From 55 A4-F2 stars studied, Abt found only 17 spectroscopic binaries. None of these had periods of less than 100 days.

ABT and BIDELMAN (1969), in a study of late A stars known to be short-period binaries, showed that all binaries with periods greater than $2\frac{1}{2}$ days were Am, while several binaries with periods less than $2\frac{1}{2}$ days were not Am. Abt suggests that short-period Am binaries are slowed by tidal interaction and become Am as a result. Some binaries with very short periods may rotate too rapidly to be Am.

CONTI (1968) has noted that the percentage of short period binaries along the main sequence from B to G is approximately constant if Am stars are included. If they are not, there is a conspicuous gap. The conclusion he draws is that Am stars may be unevolved and, if this is the case, the Am phenomenon cannot be the result of an evolutionary stage.

1.4 Rotational velocities

Am stars have relatively sharp lines and the measured rotational velocity is low for that spectral type or colour ($v \sin i < 100$ kms/sec). Metallicism could be masked by broadening of the spectral lines but would still be noticeable at 150 kms/sec. Also the difference in spectral type indicated by the Balmer and K lines would still be detected in shallow-line spectra. This suggests that Am stars are slow rotators. Furthermore the mass functions of the 25 stars in Abt's sample suggest random orientation of orbital axes. Since we would expect orbital and rotational axes to be aligned, these stars would not appear to be viewed pole-on.

Abt plots equatorial rotational velocity against frequency/ 50 kms/sec (Fig.1.6). He notes that there are no Am stars with rotational velocity $>$ 100 kms/sec. and no A stars with rotational velocity $<$ 50 kms/sec.

DEUTSCH (1967) has suggested that $V < 100$ kms/sec is a necessary and sufficient condition that an A type star be Am. Conti adds that while this is true for late A stars, there are some slow rotators among early A stars which are not Am.

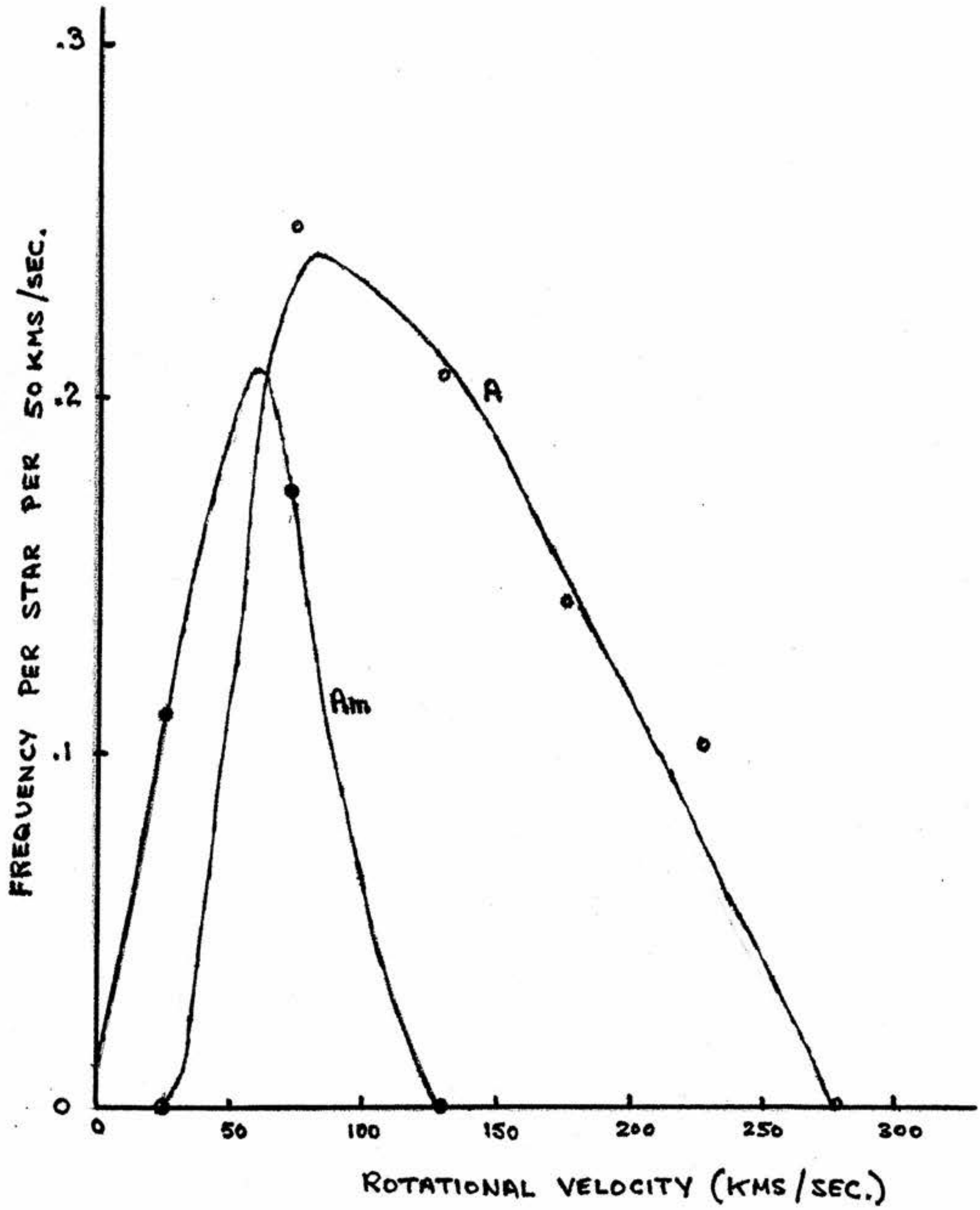
Hack plots v sin i against spectral type given by the metallic lines (Fig. 1.7). She notes that there are no stars with zero projected velocity and suggests that the lines are broadened mainly by macroturbulence. She adds that quantitative analyses of atmospheres suggest higher microturbulence than Ap stars and considers it reasonable to suggest that stars of higher microturbulence should have higher macroturbulence.

1.5 Magnetic fields.

BABCOCK (1958) carried out a survey of the magnetic fields of Am stars from which JASCHEK and JASCHEK (1962) and SARGENT (1964) conclude that all Am stars have small magnetic fields of the order of at most a few hundred gauss.

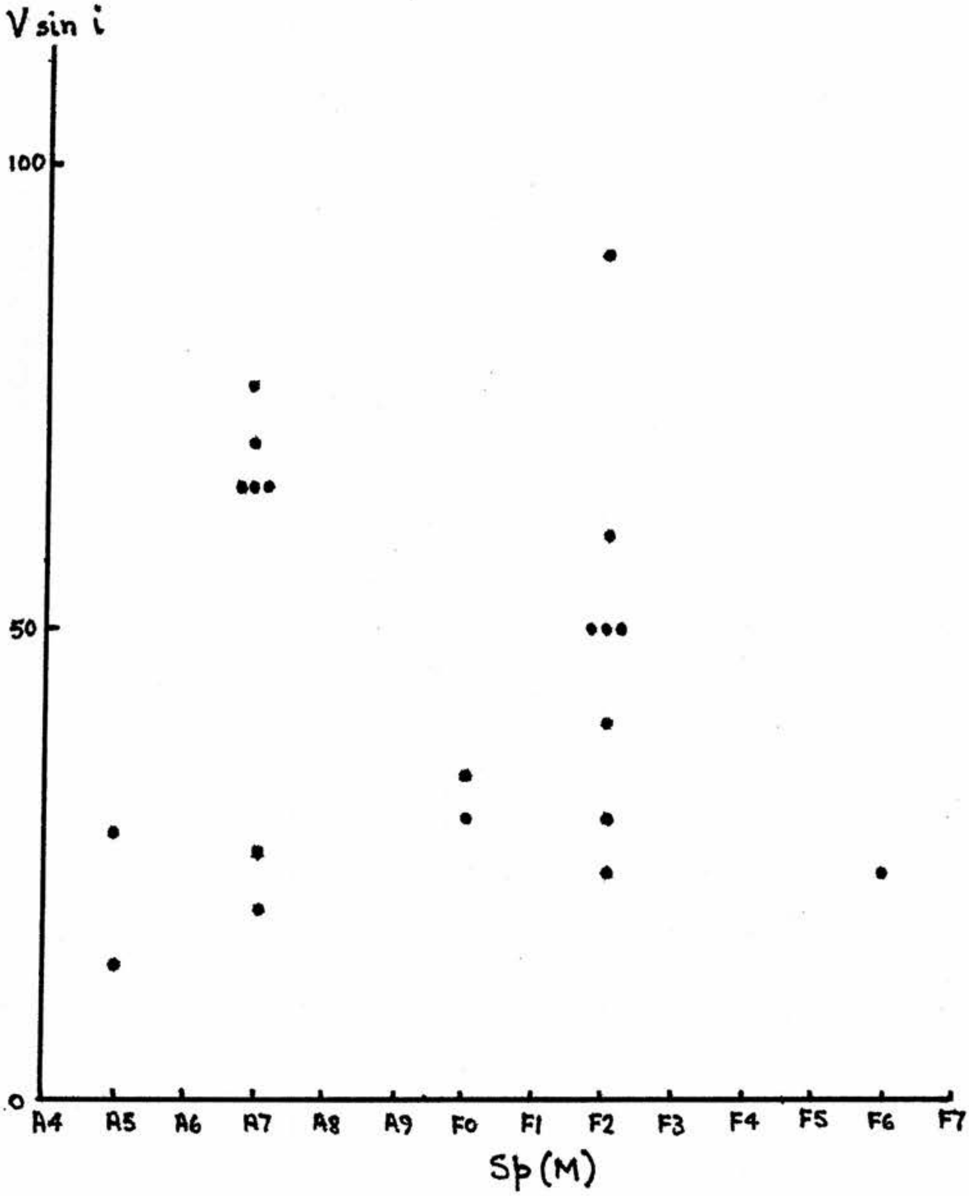
1.6 Evolutionary properties

Sargent points out that when Am stars occur in wide binaries with normal companions they may be secondary or primary, earlier or later than their companion. Companion stars include A and F types and even a K2 giant, but no B type stars. This, he concludes, is evidence that Am stars are not extremely young objects. The most recent survey of binary systems containing B stars by MURPHY (1969) shows no Am companions but only seven late A type companions. As only 10% of A stars are Am, Conti concludes that the statistics for the absence of B type companions among Am stars are not significant.



The equatorial rotational velocity distributions for 34 Am and 55 A stars with known V sin i. The ordinate assumes a relative frequency of 28 Am to 72 A stars. This quoted frequency of Am stars is now known to be somewhat high but the graph is left unaltered for the purpose of clarity.

FIG. 1.6



Projected rotational velocity is plotted against the spectral type derived from the metallic lines.

FIG. 1.7

Hack shows the positions on the colour-magnitude diagram of Am stars where the absolute magnitude is derived from tabulated trigonometric parallaxes. She also plots Am stars in the clusters Praesepe, Coma Berenicis and Hyades (Fig. 1.8). Her conclusion is that the field and Praesepe Am stars are ½ mag. above the main sequence while the Hyades and Coma B Am stars are on the main sequence. This, she adds, is difficult to explain in evolutionary terms since Praesepe and Hyades have ages 8.7×10^8 years while the age of Coma B is 6×10^8 years.

In this survey it appears that she decided to ignore the effect of line blanketing although she corrected in the case of two Ap stars in the same article. Abt points out that this effect may increase B-V making the stars seem older. Also the sample chosen, being taken in the main from two clusters of the same age, means that the age of maximum frequency for Am stars, quoted by Hack as 8.7×10^8 years, must be viewed with reservations.

Using the Strömgren 4-colour system, nearly independent of line blanketing, Abt concludes that, while zero-age Am stars do exist, in general Am stars are considerably older than zero-age.

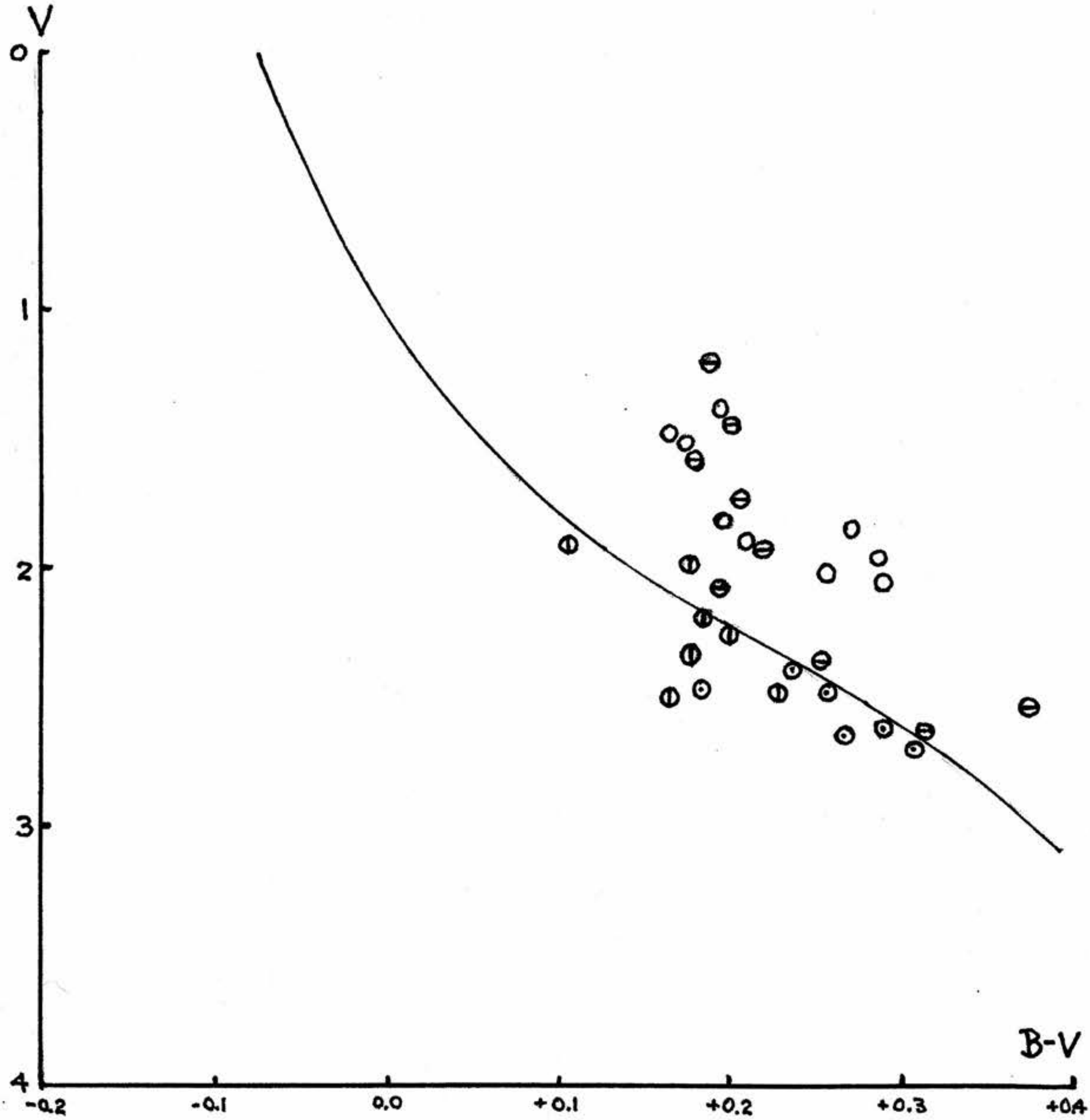
CONTI and VAN DEN HEUVEL (1970) have concluded that the statistics for the search for Am stars in young clusters are insufficient. They have found two Am stars in M7, which has several B type stars while PESCH (1967) has discovered two Am stars in M67, an old cluster.

JASCHEK and JASCHEK (1967) have used proper motion studies to determine the age of Am stars and Conti concludes that Am stars are neither very old nor very young although individual Am stars may be either.

1.7 Analysis and Interpretation

The question arises if the anomalies in Am stars are due to abnormal chemical compositions of the atmospheres or if they are due to some other mechanism.

One such mechanism is suggested by BÖHM-VITENSE (1960). She suggests that Am stars have atmospheres distended by turbulence probably supported by magnetic fields and that $T(\tau)$ is abnormal. There is some evidence for this as Am stars have been found to have weak magnetic fields and the atmospheric pressure is low.



H-R diagram for Am stars. The symbols mean

o Am stars ; field.
⊖ Am stars in Praesepe.
⦶ Am stars in Coma Berenicis.
⊙ Am stars in Hyades.

FIG. 1.8

She argues that the magnetic pressure could conceivably be comparable to the gas pressure. It has not been shown, however, that this explanation can fully account for the anomalies in Am stars.

CONTI (1965) finds that (i) models with effective gravity decreasing outwards through the atmosphere. (ii) models with a higher boundary temperature than a scaled solar $T(\tau)$, and (iii) models with a high helium content (80%) in the atmosphere, are unable to explain the abundance anomalies, particularly Ca.

Pecker-Wimel's suggestion (PECKER-WIMEL (1953)) that Am stars have helium-poor atmospheres in radiative equilibrium fails to explain the weakness of the K line and the strength of the metallic lines, although it gives the Balmer discontinuity as observed.

VAN DEN HEUVEL (1968) has suggested that Am stars are the result of evolution in close binaries. In his theory the primary evolved first and deposited its material on the surface of the secondary which became Am. The original primary would now be a white dwarf. Conti raises several objections to this theory, e.g. the probable presence of Am stars in young clusters.

Various curve of growth and model atmosphere analyses have been carried out on Am stars. Later authors agree with GREENSTEIN (1948) that Am stars have abnormally large values of microturbulent velocity for dwarfs and a low spectroscopic surface gravity, and with VAN T'VEER-MENNERET (1963) who, using a model atmosphere analysis of 63 Tau, found that the abundance anomalies are real and that the only physical atmospheric peculiarity is the high turbulent velocity. Thus it now appears that all attempts to explain the abundance anomalies by abnormal atmospheres have only historical interest.

Sargent chooses four stars: 8 Com, 15 Vul, τ UMa and 63 Tau; and finds (a) normal abundances of Mg, Al, Ti, V, Mn, Fe and Co; (b) deficiencies of Ca and Sc by factors of about four; (c) overabundances of Ni and Zn by about the same factor; and (d) differences from star to star in the behaviour of Sr, Y, Zr, Ba and the rare earths. He concludes that although the abundance anomalies are real, they are small, or when large are limited to elements (e.g. the rare earths) which usually have either weak or few lines, so that the metallicity seems to be caused more by a general increase in line strength due to large turbulent velocities than by abundance anomalies.

Sargent suggeststhat the abnormal compositions are the result of surface, non-thermal nuclear reactions, a hypothesis which was put forward by FOWLER, BURBIDGE and BURBIDGE (1955) to explain the anomalies of Ap stars. SMITH (1971), however, in an analysis of several Am stars, has found abundance correlations among the enhanced elements which do not follow from known nucleosynthetic processes.

JENSEN (1962) and BABCOCK (1963) have suggested that local abundance peculiarities arise due to migration of certain ions in the magnetic gradients on the star's surface. The observed abundance anomalies, however, would appear to be too large to be explained in this way.

PRADERIE (1968) has put forward diffusion and gravitational separation as the reason for the abnormal surface composition. The process by which heavier elements would sink from the visible atmosphere, according to her theory, does not operate in Am stars, so that Am stars are, in fact, the normal ones. SCHATZMAN (1969) has added that even a little turbulence mixing would impede gravitational separation.

MICHAUD (1970) has suggested diffusion as an explanation for Ap stars. Radiation pressure, in the case of certain elements, will more than compensate for the effects of gravity and push these elements outwards resulting in an overabundance. Other elements will not be able to absorb sufficient radiation to counter gravity and gravitation settling will lead to a deficiency. This is dependent on the strong magnetic fields of Ap stars stabilizing the photospheres. The smaller fields in Am stars would mean that the outer zone would be convective as in normal A stars.

WATSON (1970) and SMITH (1971) have independently suggested that the diffusion process suggested by Michaud takes place in the radiative zone below the outer convective zone in Am stars. Unlike normal A stars, the magnetic fields in Am stars will inhibit circulation which otherwise would dominate diffusion. A problem is that the radiative zone may not be large enough for the effects of convection to be sufficiently reduced.

1.8 Relationships with other star classes - Ap and δ Sct.

To consider, first of all, the relationship between Ap and Am stars, it is best to look at the differences and similarities.

(a) Ap and Am stars are found in adjacent parts of the HR diagram.

(b) Abundance anomalies, although probably real in both, are less pronounced in Am stars.

(c) Deficiency of Ca is not so apparent in Ap stars.

(d) Redder Ap stars have oxygen deficiency. This is not true for Am stars. Further, the overabundance of Ni in Am is not found in Ap stars.

(e) Ages of both range from $2 \times 10^7 - 10^9$ years. In general, Am stars appear to be older, although this may be a line blanketing effect.

(f) Percentage of spectroscopic binaries is 3 times smaller than normal for Ap stars; 2-3 times larger for Am stars.

(g) Percentage of visual binaries for both is the same as for normal stars.

(h) Small projected rotational velocities are found for both. Am stars are slowly rotating objects. Ap stars <u>may</u> be seen pole-on. See (f) and (g).

(i) Ap stars include a large number of spectrum variables. Spectrum variables among Am stars are not common.

(j) Both have magnetic fields, the fields in the Ap stars being much the stronger.

(k) Am stars have large microturbulence and probably large macroturbulence. Ap stars have low microturbulence.

(l) Balmer contours are roughly similar to those of main sequence stars of about the same colour. However, sharp cores are present in some Ap stars. This effect is not noticed in Am stars. In Am stars the intensity of H δ is slightly low for colour. Intensity of H δ in Ap stars is similar to normal main sequence stars of the same colour.

Although there are basic similarities between the classes, there are sufficient differences to suggest that different processes are responsible, or, if the same process is involved, that it takes place in different physical conditions e.g. the outer convection zone may be absent in Ap stars.

EGGEN (1963) and others have drawn attention to a possible relationship between Am stars and δ Sct variable stars. Both groups occur in the same region of the HR diagram and have strong metallic lines. One δ Sct star, δ Delphini, has been analysed, using model atmospheres, by REIMERS (1969) who showed fairly normal abundances of Fe, Ca and Sc. BESSELL(1969), using a curve of growth analysis, found abundances which were not too dissimilar from these but concluded that the star was Am. This anomaly has not yet been explained.

BREGER (1969), in detailed photometric study of variability in A and F stars, found many δ Sct stars, but only one out of eight Am stars studied was variable. This star, HR114, was later shown at high dispersion to be a normal F2 V star (MILTON and CONTI (unpublished)). Breger has quoted the instability gap for δ Sct stars as cutting the main sequence between F2 and A4. The lack of variables among Am stars has therefore prompted Conti to suggest that the phenomena may be mutually exclusive.

1.9 Discussion

The main differences between Am and normal A stars are listed below:-

(i) The spectra of Am stars are abnormal.

(ii) Am stars are much slower rotators than normal A stars.

(iii) Many of the Am stars are found in binary systems.

For B types, single stars or members of wide doubles have large rotational velocities; close doubles have small rotational velocities. Abt suggests that perhaps A and Am stars have this property. As a possible reason he suggests equipartition of angular momentum between rotational and orbital motion. In A stars, however, orbital angular momentum is so much greater than rotational that it renders this suggestion improbable. Because of this Abt proposes tidal interaction as the reason. This would have the greatest effect during the evolutionary contraction phase when protostar surfaces are close together. This, of course, will only account for Am stars with binary orbital period less than about 100 days. For the rest Abt suggests that perhaps there is a closer, undetected companion or else some other mechanism is responsible.

Sargent concludes that both Am and Ap stars have abnormal chemical compositions due to non-thermal, surface nuclear reactions. Abt suggests that rapid rotation destroys this mechanism or dilutes abnormal material by convective mixing.

The trend of recent thought, however, seems to be towards diffusion and separation processes. Conti concludes that these might have an important bearing on the understanding of the Am phenomenon and, if anything, their importance has increased since his review was published.

CHAPTER 2

OBSERVATIONS AND METHOD OF REDUCTION

2.1 Choice of Stars

Eight metallic-line stars were chosen for detailed analysis. The choice was based on the following factors:-

(i) availability in the sky

(ii) magnitude

(iii) suitability for telescope e.g. no visual binaries sufficiently close to upset the autoguider

(iv) range of metallic-line stars from 'very peculiar' to 'almost normal' should be covered.

The suitability of the reduction program to deal with these stars was not taken directly into account although it was noted that ABT (1961) lists 88 Tau as a double-line binary, which would be extremely difficult to deal with, if this were the case. However, no corroboration was found in the literature and an examination of the spectrum proved negative. Also 68 Tau is listed in several catalogues as not being metallic-line. For example BERTAUD (1959) lists it as A2 V with some metallic-line properties. Hence this could be taken as a case of an 'almost normal' star.

The stars chosen appear below:-

Star	R.A.(1970)	Decl.(+)	M_v	B-V	U-B
28 And	$0^h28^m33^s$	29°36'	5.20	0.26	0.08
68 Tau	$4^h23^m45^s$	17°52'	4.30	0.05	0.07
88 Tau	$4^h34^m01^s$	10°05'	4.25	0.18	0.11
ω Tau	$4^h15^m30^s$	20°31'	4.96	0.22	0.10
σ^1 Tau	$4^h37^m27^s$	15°44'	5.07	0.14	0.16
15 UMa	$9^h06^m45^s$	51°43'	4.47	0.27	0.11
τ UMa	$9^h08^m27^s$	68°38'	4.67	0.36	0.15
60 Leo	$11^h00^m44^s$	20°21'	4.41	0.05	0.05

R.A. and Decl. are calculated from the Yale Bright Star Catalogue (1962) positions for 1900. M_v, B-V, U-B are calculated by averaging the values given in Pub. U.S.Naval Obs. Second Series Vol. XXI (BLANCO et al. (1968)).

2.2 Observations

The spectra were taken with the 36" telescope at the Royal Observatory, Edinburgh at a dispersion of 17Å/mm. Observations were made over three distinct periods, namely 7th December, 1967-19th December, 1967, 27th February, 1968-8th March, 1968 and also on 25th March, 1969. The spectra were taken on Eastman-Kodak IIaO emulsion which gave a measureable wavelength range of roughly 4050 Å - 5050 Å. The spectra were uniformly widened ½ mm and the projected spectrograph slit width was 27 μ. At least 3 spectra were taken of each star. The plates were calibrated in a separate step-slit spectrograph a few hours after exposure and brush developed in Kodak D19b developer for 4 minutes at about 20°C. A list of plates and spectra obtained appears below:

Date	Plate No.	Spectra	Exposure Times
7.12.67	1	28 And, ω Tau, σ' Tau, 15 UMa	$89^m 78^m 79^m 41^m$
8.12.67	2	28 And, 88 Tau, τ UMa	$129^m, 83^m, 113^m$
9.12.67	3	15 UMa, τ UMa, 60 Leo	$45^m, 50^m, 43^m$
11.12.67	4	28 And	63^m
13.12.67	5	68 Tau	38^m
15.12.67	6	τ UMa	93^m
17.12.67	7	28 And, 88 Tau	$146^m, 79^m$
19.12.67	8	ω Tau, 68 Tau, σ' Tau	$92^m, 56^m, 139^m$
27.2.68	9	88 Tau	61^m
27.2.68	10	τ UMa, 15 UMa, 60 Leo	$85^m, 69^m, 67^m$
28.2.68	11	σ' Tau, 60 Leo	120^m, 58^m
2.3.68	12	68 Tau, 88 Tau	$32^m, 48^m$
6.3.68	13	ω Tau	112^m
8.3.68	14	ω Tau, τ UMa	$98^m, 42^m$
25.3.69	15	60 Leo	120^m

* was not used due to plate defects.

2.3 Spectra Read to Magnetic Tape

The reduction of the data was made on the Observatory's Elliot 4130 computer.

The information from the spectra was recorded on paper tape using a digitised Joyce-Loebl microdensitometer with a projected slit width of 12½ μ and a step interval of 12½ μ.

Preliminary tracings were made of each spectrum and the corresponding calibration spectra and it was decided which slits on the calibration spectra should be used in order to form a grid of intensity as a function of density and wavelength in order to contain the spectrum for the wavelength range required. A strong line near the centre of the stellar spectrum was chosen to serve as a position indicator. Similarly, the calibration spectra were superposed with lines from an Hg lamp, and the strongest Hg line was used as position indicator. Using a travelling microscope on the Hg lines, the linearity of the calibration dispersion was tested and it was found to be linear to within the error of the experiment. The dispersion of the calibration spectra was found to be 0.68 Å/step. The slit and step length for the calibration spectra were the same as for the stellar spectra.

The wavelength range to be examined on the stellar spectrum was decided upon and knowing the approximate dispersion (17Å/mm), the number of steps to be measured on either side of the position indicator was calculated. Similarly, the number of steps to be measured on either side of the Hg line on the calibration spectra in order to cover the above wavelength range was calculated.

The relevant calibration slits were then recorded by setting up on the strongest line, stepping back manually to the starting position and recording the required number of steps. The corresponding stellar spectrum or spectra was then recorded in the same way using the chosen line.

As well as recording the stellar spectrum and the corresponding calibrations, fog measures were obtained by taking a number of readings along a part of the plate close to the stellar spectrum or between calibration spectra. To test if the machine was drifting, several readings were taken from a fixed point on the plate (e.g. near a mark or blemish). In both these cases the slit and step length were the same as before. Also in order to convert to density it is necessary, first of all, to convert to centimetres on the wedge scale. To do this the wedge was placed several times at appropriate positions at either end of the scale and a manual reading taken.

It should also be noted that since calibration and stellar spectra are run at different periods of the day (e.g. morning and afternoon) or even on different days, it is necessary to adjust the level of the calibration to allow for drifting, possible changes in scale, etc. To do this several slit steps were re-measured, at a prescribed number of steps from the strongest Hg line, on the flat part of each calibration spectrum before the stellar spectrum was read and the calibration scaled accordingly.

After each run the spectrum was scanned using the manual control to find blemishes. The readings on the screw were noted on either side of each blemish. Since the start position and the position on the screw of the indicator line had also been noted, these could be fed into the computer as sections of the scan to be ignored.

The Joyce-Loebl tapes were then transferred to a magnetic tape file.

2.4 Standard Data

Before the equivalent width program can be run, certain information about the spectra must be written to magnetic file so that it can be called down easily by the program when required. This information, which will be referred to as the standard data, includes a list of star and plate names, an approximate dispersion curve, the wedge constants, the intensity ratio of the calibration slits, etc. The calculation of some of these parameters involves a certain amount of experimentation.

For example, in determining the approximate dispersion curve, the 'findlines' procedure (see 2.7) was used to find the positions of possible lines on an uncalibrated spectrum. An attempt was made to identify these and a polynomial fit of wavelength against step number was carried out using an orthogonal polynomial technique based on a paper by FORSYTHE (1957). As well as calculating the polynomial coefficients the program chooses the optimal degree using Student's t-test.

In the case of the wedge constants, which are the factors used to convert distance on the Joyce-Loebl wedge scale to density, it was found that the wedge was linear to within 2% which can be ignored.

The intensity ratios of the calibration slits were known from previous experiments.

2.5 Line List

In addition to the standard data, the program requires that a list of lines, which would be expected to appear in the spectra to be investigated, be stored on magnetic tape. In order to construct such a list, previous work done on these and other similar stars was studied and from this a list of elements and multiplets which had at least one member with an equivalent width greater than some nominal value was drawn up. The next step was to study these multiplets in Miss Moore's Multiplet Tables (MOORE (1945)) and to decide, on the basis of the quoted intensity and the equivalent widths of any members of the multiplet given in previous lists, which members of the multiplet to include in the list. A line was included in this way if its expected equivalent width was greater than some fixed value.

It should be noted at this point that the construction of the line list is one of the major parts of the project. The reason for this is that digitised Joyce-Loebl output contains a finite amount of information about the spectrum and hence including too many lines would be trying to extract information which, in fact, is not obtainable. It is best to vary as little as possible from the optimum number of lines (about one every ten steps). The number of steps per line is roughly the range over which a line affects the convoluted spectrum. To exceed this would create a serious blending problem. If fewer lines are chosen we are throwing out useful information. Thus the choice of lowest acceptable expected equivalent width is very important and all lines must be vetted individually to see if they meet the requirements. Lines fainter than the lowest acceptable equivalent width are ignored.

In the analysis of metallic-line stars, four lists were chosen as references:

(i) Praderie's analysis of η LyrA (PRADERIE (1967)).

(ii) Dominion Observatory List for 15 Vul, 68 Tau (WRIGHT et al. (1964)).

(iii) Greenstein's analysis of τ UMa (GREENSTEIN (1948)).

(iv) Swensson's analysis of Procyon (SWENSSON (1946)).

The wavelength range chosen was 4050 Å - 5050 Å.

(i) is not very comprehensive e.g. misses out blends, but η LyrA is metallic-line and the whole spectral range is covered.

(ii) is comprehensive and both stars are metallic-line but the list only covers the range 4050 Å - 4520 Å.

(iii) is not very comprehensive although the star is metallic-line and the wavelength range is covered.

(iv) is comprehensive and the wavelength range is covered but the star is actually F0 and not metallic-line. However, since the metallic-lines are in general fairly typical of F-type stars, this list can be put to good use.

(i), (ii) and (iii) were used for finding likely multiplets and line strengths in the wavelength range 4050 Å - 4520 Å and comparing these results with (iv), we were able to calibrate (iv) for various ions and use (i), (ii) and (iv) for the remainder of the spectrum. From this it was expected that the list for the low wavelength end should be the better. This was justified when other lines were found using the 'findlines' program (see 2.7) and when these were added a more comprehensive list was obtained.

The criterion for the inclusion of a multiplet was that it should have a line with an expected equivalent width of at least 20mÅ. The critical level for inclusion of a line in such a multiplet was also taken as 20mÅ.

After a preliminary run it was found that too many blends occurred and in the second case the level was raised to 30mÅ.

2.6 Calculation of Noise

Before the commencement of the main program, the expected noise at each density must be determined. To do this all the calibration spectra in a particular batch of plates are used and a polynomial of r.m.s. noise against Baker density is fitted. In this process the program scans the spectrum and throws out blemishes, defined to be where the deviation from a running mean exceeds the expected noise by a given factor(taken as 3.2).

From equation 4 (see introduction), we have for a spectrum without lines

$$Ads(\tau) = \sum_i dn(x_i - \tau)\, dn(x_i)$$
$$= \sigma^2 \sum_i D(x_i - \tau)\, D(x_i) \quad \text{for white noise}$$

so that σ^2, the expected variance of a single datum can be defined by $Ads\,(0)/\sum_i D^2(x_i)$ which can be computed in short sections for which the average density is also found and converted to Baker density as in 2.7.

If the noise is known as a function of density, this can be fed in as predata when the equivalent width program is being run, thus enabling the standard deviation for each equivalent width to be calculated.

This also enables us to calculate the function $\sum_i dn(x_i - \tau).dn(x_i)$ used in determining the line profile (see introduction). The process described is dependent on the slit and step length being the same for the calibration and stellar spectra.

2.7 Calibration and Line Finding

When the various preliminaries have been completed, the next stage is to run the main program as far as what is called the 'findlines' stage. This part consists of four main sections,
(i) converting the Joyce-Loebl readings to Baker density
(ii) using the calibration spectra to form an intensity grid
(iii) calibrating the corresponding stellar spectrum (or spectra)
(iv) finding the position of the main absorption lines in each spectrum.

Taking these in turn:
(i) Using the wedge constants, the readings are first converted into relative densities. Each set of fog readings is then averaged and subtracted from the appropriate calibration and stellar spectra. This is permissible since, for the plates used, the fog was uniform across the plate and any slight deviations from this occurred at the extremities of the plate where no readings of any kind were taken.

This gives the photographic density D, which is related to the transparency T, i.e. the ratio of the light transmitted through the spectrum to that transmitted through the clear plate, by

$$D = \log_{10} 1/T$$

The Baker density, Δ, (BAKER (1949)) is related to T by

$$\Delta = \log_{10} \frac{1-T}{T}$$

Hence the photographic densities can be converted to Baker densities for all spectra. Δ was preferred to T since it is an approximately linear function of $\log_{10} I$ over the density range used, I being the light intensity to which the plate was exposed.

(ii) The calibration spectra corresponding to the various slits are convolved with the D-function (see introduction) with q taken as the expected $\frac{1}{2}$-$\frac{1}{2}$ width in steps of the Hg lines.

The program finds the strongest Hg line for each of the slits taken to be the highest point of the convoluted spectrum, and then fixes the wavelength scale based on the dispersion given in the standard data and the position of the line. It then omits the Hg lines and any blemishes found in the same way as in the calculation of noise (see 2.6).

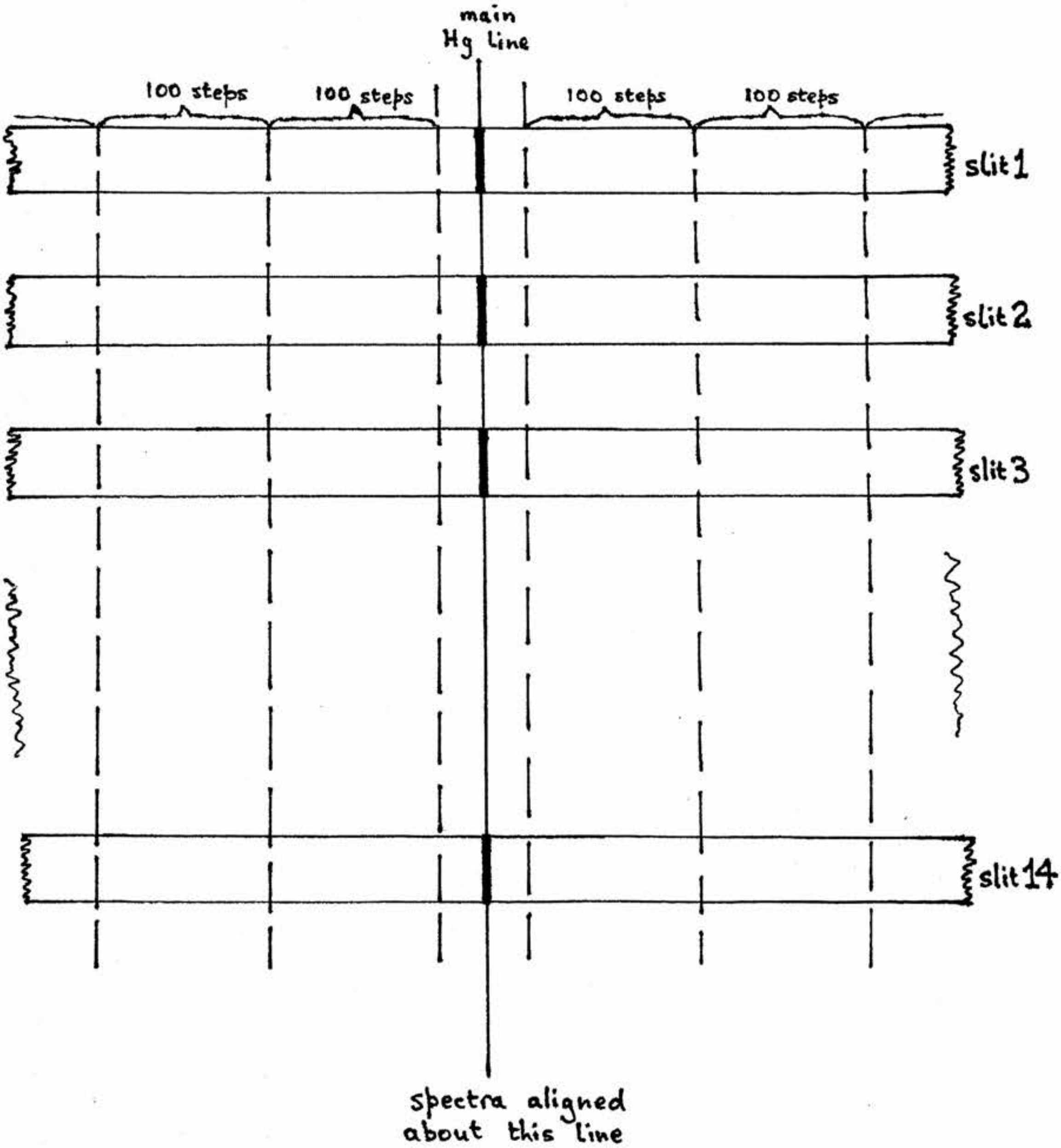
The spectrum is divided into blocks of a set number of steps (in most cases 100). Fig. 2.1 shows how the spectra appear. For each block (corresponding to a range of wavelengths), the Baker densities are averaged and a polynomial of second degree fitted to a plot of mean density against logarithmic slit intensity ratio. Therefore, given any density within the range of the slits used, the relative intensity can be determined.

(iii) Thus we have formed a grid of intensity as a function of density and wavelength which can be used to calibrate the stellar spectrum.

(iv) Once this has been completed, the calibrated spectrum is convolved with the D-function with q taken as the expected $\frac{1}{2}$-$\frac{1}{2}$ width in steps of a spectral line (i.e. 2 or 3). A level is then set so that any point in the convoluted spectrum below this is considered to be in a line. The spectrum is then scanned and a list of positions of these points together with the approximate wavelength derived from the predata is output. If two points appear together their mean position is output; if more than two appear together a best-fit parabola is drawn through them and the position of the minimum is output. A measure of the strength and width of the line is also output and these help to determine whether or not the line is 'real'. Thus we have a list of the positions and approximate wavelengths of possible lines.

The initial critical level chosen was 3.5 times the expected noise and this was then varied from spectrum to spectrum so that a sufficient number of lines was found in each. In no case was this parameter taken as less than 2.0.

From time to time distortions occur due to, for example, the



All 14 slits on the spectrograph are represented in this diagram. In practice only those necessary to cover the density range of the spectrum are used, which usually is about eight or nine.

FIG. 2.1

upside down fitting of a parabola. These are infrequent and are either marked, as in the example chosen, or are easy to detect.

The procedure is fairly insensitive to its parameters i.e. the value of q and the critical level.

2.8 Dispersion Curve

The next stage is to identify these lines using the prepared line list and to select lines which could be used in setting up an accurate dispersion curve for each spectrum.

For this purpose a fiduciary list of lines was drawn up. To qualify for this list a line had to be unblended and positively identified for at least four spectra. The lines found by the 'findlines' procedure were split into three groups:

(i) fiduciary lines : lines which appeared on the fiduciary list.

(ii) possible and blended lines : lines which did not appear in (i) but were either blended or identified for less than four spectra.

(iii) unidentified lines.

The next stage of the program uses lines in (i) to set up a polynomial dispersion curve for each spectrum. The output includes the r.m.s. deviation of these points from the curve and also the individual residuals. For group (ii) lines we get the deviation of the position of the found line from the nearest line or lines in the line list, and for group (iii) lines we get the wavelength of each line.

At this point unidentified lines which gave rise to the same (or very similar) wavelengths in more than one spectrum were studied in some detail. In some cases it was found that a certain line in the multiplet tables or the Solar Spectrum (MOORE et al. (1966)) corresponded to the unidentified line and if, after some consideration, it could be justified, this line was included in the line list. The multiplet, of which this new line was a member, was also vetted to see if any other lines should be included. It was noted particularly in the wavelength range 4550 Å - 5050 Å, where the majority of unidentified lines were found, that sometimes two or more lines of the same multiplet appeared as new lines. This alone justified their inclusion. The new line list was read onto magnetic tape to replace the old one.

A new fiduciary list was chosen on the following basis:-

If a line occurred within one step of its position on the line list significantly more frequently ($\sim 2\frac{1}{2}$ times) than it occurred outside this region, and the line was not within 0.5 Å of another line, it was included. The reason for this was that although a line may not be an obvious blend from the line list, one or more adjacent weak lines, not quite strong enough to be included in the line list, could significantly alter its position.

The new fiduciary list was used to set up a new polynomial dispersion curve for each spectrum. The polynomial coefficients were output along with the r.m.s. deviation and the individual residuals. If these were acceptable we could proceed.

In a few cases it was necessary to look further at the lines forming the dispersion curve of individual spectra to see if perhaps one misidentification might be significantly increasing the r.m.s. deviation.

2.9 Line Profile

The next stage in the program is to find the line profile assumed to be the same for all except the Balmer lines. A description of the method used appears in the introduction. There it was seen that a profile was first of all assumed and the equivalent widths calculated.

Before calculating these, the regions to be omitted (i.e. Balmer lines and blemishes) are put to scrap and, if necessary, the wavelength range may be altered. Herein lies the main difference between the initial calculation of equivalent widths in order to determine the profile and the final equivalent widths determination, as described in 2.10.

In the initial calculation one can omit portions of the spectrum likely to give trouble e.g. parts where blending is particularly predominant. These omitted regions need not be the same as in the final analysis where, for example, we would wish to include all blends.

When the equivalent widths have been calculated, the line profile (i.e. the best value of β) is found. It happened that three stars 88 Tau, σ' Tau and ω Tau had values of β larger than had been anticipated (see Chapter 3). Because of the blending problem, the program would not run to completion when these values were input. In these three cases it was found necessary to choose 50 mÅ as the lower expected limit for the inclusion of a line and to combine blended lines in some places by averaging their wavelengths and inputting them as one line. This meant having a different line list for these three stars, which, in turn, meant that the program lost some of its beauty. For, instead of treating all eight metallic-line stars as one batch, we were now dealing with two separate groups.

2.10 Final Equivalent Widths

In any case, using the value of β thus found, we can recalculate theequivalent widths for each spectrum.

As before, omitted regions are put to scrap, and since it was found that the first 100 Å yielded little about the spectrum due to overcrowding of lines and low density, the wavelength range was reduced to 4150 Å - 5050 Å. The program divides steps into substeps enabling the positioning of line centres to be done more accurately. One step was divided into 15 such substeps.

Blends are dealt with in the following way. If the lines interact sufficiently, the sum and difference rather than the separate equivalent widths are calculated. We thus get at least one parameter (the sum) which has some meaning. The degree of interaction necessary for this to happen is fed in as predata. If the lines are too close to be separated at all (i.e. within one substep of each other) the sum of the equivalent widths alone is output.

These equivalent widths are found for a spectrum which has not been rectified (i.e. continuum intensity put to 100%). In order to correct for this we divide the spectrum into regions and fill in the lines using the known equivalent widths and assumed profile. A best-fit polynomial continuum is now fitted to each section and the factor, by which each equivalent width should be multiplied in order to rectify, is calculated. The error involved in introducing a continuum at this stage is much less than that when using the concept from the start. We can now find the rectified equivalent widths.

The 'filled-in' spectrum is then scanned by the line-finding process and any residual lines are output. The critical level for finding these lines was taken as 5.0 times the expected noise.

Another program is now used to compare the spectra of each star and average the equivalent widths. From this we obtain parameters which test if two spectra of the same star are significantly different.

This program can be used to test the values of β found for each spectrum. By changing β we can change the values of the equivalent widths on average by a constant factor (see THOMPSON (1971)). Hence if the weighted average of the equivalent widths is significantly different in one spectrum of a star from the other spectra of that star, we can alter β slightly in that spectrum so as to achieve consistency for that star.

In forming the averages of the equivalent widths there is a process which will reject equivalent widths which vary by a significant factor from the mean. This factor was taken as 3.7. In the case of blends where the sum and difference of equivalent widths are found, only the sum is averaged. This includes the case where the individual equivalent widths are found on one spectrum while the sum and the difference are found on another. There is also a facility in this program for comparing stars.

The final output gives wavelengths, equivalent widths and standard deviations for each star.

CHAPTER 3

EQUIVALENT WIDTHS

3.1 Dispersion Curve

The number of lines used in fitting the polynomial dispersion curve varied from star to star. Due to large residuals and apparent blending in the three stars 88 Tau, ω Tau and σ' Tau, the number used had to be cut considerably. The reason for the high residuals became apparent when the 'find β ' procedure was run and showed that these stars have relatively broad lines (see 3.2). Between 27 and 42 lines were used for these stars and between 42 and 88 for the others. In all cases a third degree polynomial was chosen by the computer.

The r.m.s. deviations, averaged over each star, predict the variation in line width found in the next section. In step units these are 60 Leo : 0.44; 68 Tau : 0.45; 15 UMa: 0.52; τ UMa: 0.52; 28 And: 0.56; 88 Tau: 0.68; σ' Tau: 0.77; ω Tau: 0.79, (c/f. Table 3.1) which roughly corresponds to 0.10 Å for the first five and 0.15 Å for the remainder.

3.2 Find β

The assumed line profile was defined in the introduction as

$f(x) = \frac{1}{\beta\sqrt{\pi}} e^{-x^2/\beta^2}$ which is a Gaussian function with standard deviation $\beta/\sqrt{2}$. When the 'find β ' procedure (see 2.9) was run with an initial value of $\beta = 2.4$, definite results were obtained only for the stars 68 Tau, 28 And, 60 Leo, τ UMa and 15 UMa. It was at once apparent that this value was too small for the others and various higher values were then attempted for these until values of β were obtained for all eight stars.

After running the equivalent width program and comparing different spectra of the same star it was apparent that a better agreement could sometimes be obtained by altering the value of β in one spectrum. An estimate of the degree of alteration required could be deduced from the disparity in equivalent widths and on average was less than 0.2 steps. The figures quoted in Table 3.1 are the final values of β used.

These results confirmed the suspicions which arose in the dispersion curve fitting and forced the construction of two separate line lists for the two groups of stars (see 2.9).

Results of 'Find β' Procedure - Table 3.1

Star	$\bar{\beta}$	Plate No.	β
68 Tau	2.1	5	2.1
		8	2.1
		12	2.1
28 And	2.3	1	2.5
		4	2.1
		7	2.2
60 Leo	2.4	3	2.2
		10	2.5
		11	2.5
		15	2.5
τ UMa	2.6	3	2.4
		10	2.6
		14	2.7
15 UMa	2.9	1	2.8
		10	3.0
88 Tau	3.9	7	3.7
		9	4.3
		12	3.7
ω Tau	4.5	1	4.5
		8	4.8
		13	4.3
σ^1 Tau	5.4	1	5.3
		8	5.4
		11	5.4

β and $\bar{\beta}$ are quoted in step units.
(1 step $\sim$ 0.2 Å)

3.3. Equivalent Widths

Table 3.2 contains the equivalent widths of the eight Am stars. The table has been divided into three sections for convenience. An explanation of the column headings follows:

For each star we quote

E computed equivalent width in milliangstroms (mÅ).

ΔE standard error of the equivalent width in mÅ.

N number of spectra used in its determination.

In the first section we quote the line list wavelength, identification and the multiplet number as given in Miss Moore's Multiplet Tables (MOORE (1945)). N1 is the number of the line in the first line list and N2 is the number of the line in the shortened second line list. Where two or more lines in N1 are combined to form a single line in N2, the number of this line appears against each line in the original list. A line in N1 omitted in N2 is shown by a dash.

In sections 2 and 3 a line is identified merely by its number under N1 or N2.

For closely blended lines the sum of the equivalent widths appears under the first line of the blend. The other lines in the blend are marked with an asterisk. If no equivalent width has been found, a dash appears.

LINE LIST | 15 UMa | τ UMa

	Identi-fication	Mult. No.	N1	N2	15 UMa E	15 UMa ΔE	15 UMa N	τ UMa E	τ UMa ΔE	τ UMa N
4150.258	FeI	695	1	1	-			-		
4150.970	ZrII	42	2	1	-			-		
4151.000	CrII	163	3	1	-			-		
4151.970	CeII	2	4	-	-			-		
4151.980	LaII	40	5	2	-			-		
4152.172	FeI	18	6	-	-			-		
4153.906	FeI	695	7	3	179	162	1	-		
4154.502	FeI	355	8	3	335	80	1	-		
4154.812	FeI	694	9	3	*			-		
4156.083	NdII	10	10	-	290	76	1	-		
4156.240	ZrII	29	11	4	*			-		
4156.803	FeI	354	12	4	-39	109	1	-		
4157.788	FeI	695	13	5	97	59	1	-		
4158.798	FeI	695	14	6	-36	78	1	-		
4159.180	-	-	15	6	233	87	1	269	28	2
4161.200	ZrII	42	16	7	256	55	1	339	25	3
4161.796	SrII	3	17	7	229	56	1	90	27	3
4163.644	TiII	105	18	8	321	52	1	361	31	3
4164.800	FeI	418	19	-	-63	42	1	21	29	3
4165.606	CeII	10	20	9	269	51	1	217	33	3
4167.266	MgI	15	21	10	281	53	1	360	33	3
4167.862	FeI	599	22	-	-98	48	1	-45	30	3
4170.860	CrII	181	23	11	187	41	2	247	26	3
4170.906	FeII	482	24	11	*			*		
4171.897	TiII	105	25	12	291	102	1	-		
4171.904	FeI	650	26	12	*			-		
4171.920	CrII	18	27	-	*			-		
4172.126	FeI	649	28	12	*			-360	329	2
4172.749	FeI	19	29	12	85	93	2	328	73	2
4173.450	FeII	27	30	13	294	66	2	254	23	3
4174.088	TiII	105	31	-	117	49	2	159	41	2

Table 3.2

	Identi-fication	Mult. No.	N1	N2	15 UMa E	15 UMa ΔE	15 UMa N	τ UMa E	τ UMa ΔE	τ UMa N
4174.917	FeI	19	32	14	68	42	2	-		
4.75.640	FeI	354	33	14	245	78	1	169	36	2
4176.571	FeI	695	34	15	-35	105	1	152	50	2
4177.321	NdII	10	35	-	546	123	1	14	389	2
4177.540	YII	14	36	16	*			470	343	2
4177.597	FeI	18	37	16	*			*		
4178.855	FeII	28	38	17	345	34	2	482	38	2
4179.430	CrII	26	39	17	247	36	2	100	41	2
4181.500	CrII	181	40	18	365	36	2	224	96	2
4181.758	FeI	354	41	18	*			184	108	2
4182.384	FeI	476a	42	18	234	49	2	192	53	2
4183.435	VII	37	43	19	46	220	1	-59	177	2
4183.998	FeI	-	44	19	339	332	1	2371	1116	2
4184.252	GdII	15	45	19	*			-1471	854	2
4184.329	TiII	21	46	19	*			*		
4184.895	FeI	355	47	19	124	238	1	-429	228	2
4186.599	CeII	1	48	20	-168	141	2	57	82	3
4186.700	ZrII	97	49	20	*			*		
4187.044	FeI	152	50	20	607	181	2	302	110	3
4187.802	FeI	152	51	21	180	46	2	221	28	3
4188.737	FeI	-	52	21	209	36	2	65	26	3
4191.436	FeI	152	53	22	336	34	2	482	27	3
4192.070	NiII	10	54	22	-9	34	2	-52	27	3
4195.337	FeI	693	55	23	226	37	2	380	26	3
4196.218	FeI	693	56	24	208	38	2	232	38	3
4196.533	FeI	418	57	24	*			11	42	3
4198.310	FeI	152	58	25	422	88	2	563	54	3
4198.645	FeI	693	59	25	83	113	2	-96	65	3
4199.098	FeI	552	60	25	162	70	2	217	45	3
4200.930	FeI	689	61	26	58	76	1	237	32	3
4202.031	FeI	42	62	27	309	109	1	401	40	2
4202.350	VII	25	63	-	142	112	1	121	40	3
4203.300	FeI	418	64	28	118	71	1	8	26	3
4203.987	FeI	355	65	28	118	70	1	286	26	3

	Identi-fication	Mult. No.	N1	N2	15 UMa E	ΔE	N	τ UMa E	ΔE	N
4205.050	EuII	1	66	29	301	122	1	511	39	3
4205.080	VII	37	67	29	*			*		
4205.546	FeI	689	68	29	91	181	1	-83	60	3
4206.702	FeI	3	69	-	378	74	1	250	29	3
4207.130	FeI	352	70	30	-63	84	1	68	32	3
4208.610	FeI	689,696	71	31	237	80	1	178	31	3
4208.990	ZrII	41	72	31	104	35	2	73	29	3
4210.352	FeI	152	73	32	299	65	1	295	24	3
4211.880	ZrII	15	74	33	157	30	2	125	24	3
4213.650	FeI	355	75	34	94	33	2	182	22	3
4215.524	SrII	1	76	35	452	33	2	580	21	3
4216.186	FeI	3	77	35	131	29	2	92	21	3
4217.551	FeI	693	78	36	153	34	2	283	22	3
4219.364	FeI	800	79	37	223	31	2	299	20	3
4220.347	FeI	482	80	37	85	30	2	163	20	3
4222.219	FeI	152	81	38	194	30	2	315	21	3
4224.176	FeI	689	82	39	-81	170	2	175	62	3
4224.509	FeI	689	83	39	389	203	2	106	105	3
4224.850	CrII	162	84	-	-156	158	2	92	79	3
4225.460	FeI	693	85	40	276	63	2	306	32	3
4226.426	FeI	352	86	41	319	28	2	258	46	3
4226.728	CaI	2	87	41	*			147	46	3
4227.434	FeI	693	88	41	217	41	2	365	25	3
4229.760	FeI	41	89	42	36	28	2	139	20	3
4233.167	FeII	27	90	43	742	115	1	534	51	3
4233.250	CrII	31	91	43	*			*		
4233.608	FeI	152	92	43	-340	141	1	-173	71	3
4235.140	MnI	23	93	-	267	146	2	162	196	3
4235.290	MnI	23	94	44	*			-223	276	3
4235.730	YII	5	95	44	37	143	2	560	212	3
4235.942	FeI	152	96	44	*			-187	137	3
4238.027	FeI	689,696	97	45	13	78	2	-3	51	3
4238.380	LaII	41	98	-	243	114	2	275	68	3
4238.816	FeI	693	99	45	11	82	2	90	50	3

	Identification	Mult. No.	N1	N2	15 UMa			τ UMa		
					E	ΔE	N	E	ΔE	N
4239.725	MnI	23	100	-	265	27	2	239	96	3
4239.735	FeI	416	101	46	*			*		
4239.912	CeII	2	102	-	*			39	96	3
4242.380	CrII	31	103	47	273	26	2	293	19	3
4243.368	FeI	906	104	-	49	27	2	96	21	3
4244.800	NiII	9	105	-	7	42	2	47	27	3
4245.258	FeI	352	106	48	143	40	2	238	23	3
4246.090	FeI	906	107	49	51	37	2	119	22	3
4246.829	ScII	7	108	49	173	39	2	337	22	3
4247.432	FeI	693	109	49	112	43	2	166	24	3
4248.228	FeI	482	110	50	86	43	2	167	24	3
4248.676	CeII	1	111	-	22	45	2	128	28	3
4250.125	FeI	152	112	51	269	33	2	340	22	3
4250.790	FeI	42	113	51	231	28	2	217	18	3
4251.733	GdII	15	114	52	74	26	2	71	18	3
4252.620	CrII	31	115	52	148	30	2	253	19	3
4254.346	CrI	1	116	53	297	31	2	303	20	3
4256.212	FeI	690	117	54	185	29	2	218	19	3
4257.659	MnI	23	118	-	284	83	2	30	53	3
4258.050	ZrII	15	119	55	52	58	2	342	36	3
4258.155	FeII	28	120	55	*			*		
4259.988	FeI	689	121	56	166	32	2	347	59	1
4260.479	FeI	152	122	56	280	30	2	328	21	3
4261.920	CrII	31	123	57	202	31	2	298	21	3
4263.590	LaII	84	124	58	87	40	2	98	25	3
4264.209	FeI	692	125	-	29	52	2	118	30	3
4264.743	FeI	993	126	-	-53	69	2	-70	37	3
4265.260	FeI	993,994	127	59	97	51	2	80	29	3
4265.924	MnI	23	128	-	-107	39	2	-48	24	3
4267.830	FeI	482	129	60	29	31	2	19	21	3
4269.280	CrII	31	130	61	154	31	2	247	21	3
4271.159	FeI	152	131	62	235	29	2	267	24	2
4271.764	FeI	42	132	62	182	28	2	348	22	3
4273.317	FeII	27	133	63	179	28	2	262	20	3

	Identi-fication	Mult. No.	N1	N2	15 UMa E	15 UMa ΔE	15 UMa N	τ UMa E	τ UMa ΔE	τ UMa N
4274.803	CrI	1	134	64	238	25	2	254	16	3
4275.570	CrII	31	135	64	160	23	2	91	15	2
4278.128	FeII	32	136	65	92	25	2	108	15	3
4279.480	FeI	993	137	66	82	26	2	119	15	3
4280.490	GdII	15	138	67	123	39	2	69	50	3
4280.789	SmII	46	139	67	*			193	100	3
4281.009	SmII	-	140	-	*			-39	69	3
4282.406	FeI	71	141	68	210	29	2	257	16	3
4283.010	CaI	5	142	68	29	28	2	51	16	3
4284.210	CrII	31	143	69	132	39	2	201	21	3
4284.683	NiI	86	144	-	65	34	2	66	18	3
4285.445	FeI	597	145	70	143	36	2	142	21	3
4286.976	FeI	976	146	71	61	29	2	58	19	3
4287.893	TiII	20	147	71	193	26	2	224	19	3
4289.364	CaI	5	148	72	-28	57	2	-106	101	1
4289.721	CrI	1	149	72	371	66	2	518	41	3
4290.222	TiII	41	150	72	246	41	2	201	29	3
4291.466	FeI	3,41	151	73	55	23	2	47	19	3
4294.101	TiII	20	152	74	263	25	2	390	21	3
4294.128	FeI	41	153	74	*			*		
4296.050	LaII	53	154	75	-11	56	2	198	44	3
4296.567	FeII	28	155	75	271	34	2	282	26	3
4296.680	CeII	2	156	-	*			*		
4296.040	FeI	520	157	76	65	26	2	99	23	3
4298.986	CaI	5	158	77	189	23	2	18	49	3
4299.242	FeI	152	159	77	*			318	48	3
4300.052	TiII	41	160	77	298	34	2	244	32	2
4301.928	TiII	41	161	78	308	118	2	-681	132	3
4302.191	FeI	520	162	78	*			1547	214	3
4302.527	CaI	5	163	78	-157	188	2	-938	151	3
4303.166	FeII	27	164	79	303	94	2	743	75	3
4303.573	NdII	10	165	79	-33	68	2	-292	59	3
4305.447	SrII	3	166	80	193	27	2	355	61	3
4305.455	FeI	476	167	80	*			*		

	Identification	Mult. No.	N1	N2	15 UMa E	15 UMa ΔE	15 UMa N	τ UMa E	τ UMa ΔE	τ UMa N
4305.715	ScII	15	168	-	*			-33	57	3
4306.724	CeII	1	169	-	29	29	2	-87	27	3
4307.741	CaI	5	170	81	329	22	2	-238	137	3
4307.900	TiII	41	171	81	*			506	136	3
4307.906	FeI	42	172	81	*			*		
4309.036	FeI	849	173	82	-56	89	2	-396	112	3
4309.382	FeI	414	174	82	356	67	2	957	204	3
4309.620	YII	5	175	82	*			-285	135	3
4312.861	TiII	41	176	83	167	30	1	306	26	1
4314.084	ScII	15	177	84	170	111	1	229	96	2
4314.289	FeII	32	178	-	*			579	121	2
4314.979	TiII	41	179	84	283	72	1	165	34	2
4315.087	FeI	71	180	84	*			*		
4316.052	GdII	43	181	-	-120	187	1	-35	25	2
4316.807	TiII	94	182	85	292	437	1	133	24	2
4317.320	ZrII	40	183	85	-360	821	1	-23	24	2
4318.652	CaI	5	184	86						
4318.936	SmII	27	185	86						
4320.745	ScII	15	186	87						
4320.965	TiII	41	187	-						
4325.010	ScII	15	188	88						
4325.607	NiI	86	189	-						
4325.765	FeI	42	190	88						
4327.125	GdII	15	191	89						
4327.920	FeI	597	192	-						
4330.264	TiII	94	193	90						
4330.708	TiII	41	194	90						
4333.760	LaII	24	195	91						
4334.153	SmII	27	196	_						
4337.049	FeI	41	197	92						
4337.330	TiII	94	198	-						
4337.916	TiII	20	199	92						
4340.468	H(γ)	1	200	93						
4342.179	GdII	15	201	94						

NO MEASURES DUE TO BROAD WINGS OF Hγ

	Identi-fication	Mult. No.	N1	N2	15 UMa E	15 UMa ΔE	15 UMa N	τ UMa E	τ UMa ΔE	τ UMa N
4344.291	TiII	20	202	95	NO MEASURES DUE TO BROAD WINGS OF Hγ					
4346.558	FeI	598	203	96						
4350.465	SmII	46	204	-						
4350.834	TiII	94	205	-						
4351.764	FeII	27	206	97						
4352.737	FeI	71	207	97						
4358.169	NdII	10	208	-						
4358.730	YII	5	209	98						
4359.585	NiI	86	210	-						
4359.740	ZrII	79	211	99						
4362.100	NiII	9	212	100						
4367.581	FeI	414	213	101	244	21	2	-		
4367.657	TiII	104	214	101	*			-		
4369.404	FeII	28	215	-	105	33	2	62	15	2
4369.774	FeI	518	216	102	153	31	2	130	16	2
4370.960	ZrII	79	217	103	50	31	2	44	17	2
4371.330	CI	14	218	103	23	31	2	58	18	2
4374.455	ScII	14	219	104	177	69	2	207	35	3
4374.825	TiII	93	220	104	200	52	2	128	26	3
4374.940	YII	13	221	104	*			*		
4375.932	FeI	2	222	105	107	22	2	110	13	3
4379.238	VI	22	223	106	-31	22	2	30	14	3
4379.780	ZrII	88	224	106	75	23	2	13	14	3
4382.167	CeII	2	225	107	40	26	2	30	15	3
4383.547	FeI	41	226	108	480	98	1	312	23	3
4384.330	FeII	32	227	109	-1185	1210	1	170	105	3
4384.643	MgII	10	228	109	949	559	1	135	80	3
4384.722	VI	5,22	229	109	*			*		
4385.381	FeII	27	230	109	658	1249	1	262	35	3
4386.858	TiII	104	231	110	-			178	18	3
4387.897	FeI	476	232	111	84	23	1	44	17	3
4388.412	FeI	830	233	111	153	30	1	221	19	3
4389.974	VI	22	234	112	-			1	25	3
4390.585	MgII	10	235	112	-			361	74	3

	Identification	Mult. No.	N1	N2	15 UMa E	ΔE	N	τ UMa E	ΔE	N
4390.954	FeI	414	236	112	-			28	57	3
4390.977	TiII	61	237	-	-			*		
4394.057	TiII	51	238	113	85	33	2	182	21	2
4395.031	TiII	19	239	113	264	68	1	-407	246	2
4395.228	VI	22	240	-	*			694	206	2
4395.288	FeI	828	241	-	*			*		
4395.848	TiII	61	242	114	36	76	1	179	50	2
4398.020	YII	5	243	115	141	20	2	204	18	3
4399.767	TiII	51	244	116	195	34	2	285	70	2
4400.355	ScII	14	245	116	4	81	1	-74	358	2
4400.575	VI	22	246	-	*			226	444	2
4401.293	FeI	828	247	117	300	130	1	-515	1392	2
4401.447	FeI	350	248	-	*			800	1231	2
4401.547	NiI	86	249	-	*			*		
4403.350	ZrII	79	250	118	125	22	2	164	36	1
4404.680	VII	30	251	-	288	23	2	353	41	1
4404.752	FeI	41	252	119	*			*		
4406.641	VI	22	253	-	52	31	2	24	29	3
4407.637	VI	22	254	-	729	1939	1	-52	101	3
4407.678	TiII	51	255	-	*			*		
4407.714	FeI	68	256	120	*			*		
4408.204	VI	22	257	-	348	1125	1	182	1107	2
4408.419	FeI	68	258	120	*			118	798	2
4408.511	VI	22	259	-	*			*		
4408.844	PrII	4	260	-	-1313	5019	1	112	513	2
4409.220	TiII	61	261	-	616	1979	1	1006	243	3
4409.519	TiII	61	262	-	*			-453	139	3
4410.516	NiI	88	263	-	43	36	2	130	22	3
4411.080	TiII	115	264	121	194	29	2	114	22	3
4411.936	TiII	61	265	-	82	27	2	84	17	3
4413.600	FeII	32	266	122	112	28	2	73	19	3
4414.540	ZrII	79	267	123	121	85	2	274	50	3
4414.879	MnI	22	268	123	282	48	2	-775	128	3
4415.125	FeI	41	269	123	*			1066	112	3

	Identi-fication	Mult. No.	N1	N2	15 UMa E	15 UMa ΔE	15 UMa N	τ UMa E	τ UMa ΔE	τ UMa N
4415.559	ScII	14	270	123	-78	59	2	-183	39	3
4416.817	FeII	27	271	124	166	18	2	190	13	3
4417.718	TiII	40	272	125	233	24	2	187	15	3
4418.340	TiII	51	273	125	10	32	2	80	19	3
4418.784	CeII	2	274	-	100	29	2	89	19	3
4421.949	TiII	93	275	126	41	30	2	38	21	2
4422.570	FeI	350	276	126	182	21	2	262	14	2
4422.590	YII	5	277	126	*			*		
4423.858	FeI	830	278	127	62	20	2	155	15	2
4425.441	CaI	4	279	128	32	21	2	56	15	2
4427.312	FeI	2	280	129	209	18	2	226	14	2
4430.197	FeI	472	281	-	126	23	2	189	17	2
4430.618	FeI	68	282	130	77	23	2	74	17	2
4433.223	FeI	830	283	131	52	31	2	55	20	3
4433.793	FeI	825	284	131	89	20	2	165	51	3
4433.991	MgII	9	285	-	*			-42	47	3
4434.960	CaI	4	286	132	110	22	2	99	14	3
4435.688	CaI	4	287	132	65	26	2	167	16	3
4436.352	MnI	22	288	-	54	25	2	-34	16	3
4436.931	FeI	516	289	-	24	27	2	66	17	3
4438.353	FeI	828	290	-	-7	22	2	22	16	3
4439.130	FeII	32	291	-	-56	21	2	-34	19	3
4440.450	ZrII	79	292	133	-29	23	2	39	23	2
4441.730	TiII	40	293	134	53	25	2	78	24	2
4442.343	FeI	68	294	134	96	39	2	121	33	2
4442.990	ZrII	88	295	135	169	23	2	339	101	2
4443.197	FeI	350	296	135	*			-157	102	2
4443.802	TiII	19	297	135	262	36	2	342	34	2
4444.559	TiII	31	298	136	91	22	2	179	20	2
4444.563	FeII	201	299	136	*			*		
4446.842	FeI	828	300	137	73	16	2	119	11	3
4447.722	FeI	68	301	137	134	16	2	148	11	3
4450.320	FeI	476	302	-	169	19	2	81	38	3
4450.487	TiII	19	303	138	*			183	39	3
4451.545	FeII	—	304	-	97	19	2	97	14	3

	Identification	Mult. No.	N1	N2	15 UMa			τ UMa		
					E	ΔE	N	E	ΔE	N
4451.566	NdII	50	305	-	*			*		
4451.586	MnI	22	306	139	*			*		
4453.005	MnI	22	307	-	-6	20	2	40	14	3
4454.383	FeI	350	308	140	-137	117	1	-29	48	2
4454.781	CaI	4	309	140	422	50	2	373	29	3
4454.800	ZrII	40	310	140	*			*		
4455.258	FeII	-	311	-	-32	115	1	-13	48	2
4455.887	CaI	4	312	141	60	27	2	114	18	3
4457.420	ZrII	79	313	142	10	25	2	12	16	3
4458.101	FeI	992	314	-	-16	22	2	118	14	3
4459.037	NiI	86	315	-	224	21	2	264	14	3
4459.121	FeI	68	316	143	*			*		
4460.213	CeII	2	317	144	-17	22	2	31	16	3
4461.654	FeI	2	318	145	276	46	2	298	30	3
4461.989	FeI	471, 825,902	319	145	-49	67	2	43	38	3
4462.460	NiI	86	320	-	94	51	2	131	29	3
4462.985	NdII	50	321	-	-9	29	2	35	19	3
4464.458	TiII	40	322	146	76	54	1	-229	95	3
4464.677	MnI	22	323	-	*			427	78	3
4464.773	FeI	472	324	-	*			*		
4466.554	FeI	350	325	147	135	20	2	216	14	3
4468.493	TiII	31	326	148	188	20	2	164	14	3
4469.381	FeI	830	327	148	254	46	1	323	54	1
4470.138	MnI	22	328	-	-136	167	1	-108	283	2
4470.390	VII	30	329	149	*			71	168	2
4470.483	NiI	86	330	-	*			*		
4470.864	TiII	40	331	149	260	167	1	622	203	1
4472.921	FeII	37	332	150	72	17	2	243	13	3
4476.021	FeI	350	333	151	173	16	2	217	12	3
4476.082	FeI	830	334	151	*			*		
4478.657	SmII	-	335	-	3	16	2	-37	12	3
4479.612	FeI	828,848	336	152	61	18	2	98	13	3
4481.228	MgII	4	337	153	392	22	2	364	14	3
4482.171	FeI	2	338	154	216	26	2	223	16	3
4482.257	FeI	68	339	154	*			*		

	Identification	Mult. No.	N1	N2	15 UMa			τ UMa		
					E	ΔE	N	E	ΔE	N
4482.750	FeI	828	340	154	-60	41	2	-51	26	3
4484.227	FeI	828	341	155	135	22	2	148	15	3
4485.679	FeI	830	342	156	4	21	2	74	15	3
4486.909	CeII	57	343	157	-2	20	2	-14	14	3
4488.319	TiII	115	344	158	62	18	2	33	13	3
4489.185	FeII	37	345	158	143	17	2	154	12	3
4491.401	FeII	37	346	159	142	17	2	152	12	3
4493.530	TiII	18	347	160	26	16	2	13	12	3
4494.568	FeI	68	348	161	175	17	2	205	12	3
4496.429	PrII	4,25	349	-	47	54	2	163	31	3
4496.862	CrI	10	350	-	129	33	2	10	45	2
4496.960	ZrII	40	351	162	*			*		
4496.989	MnII	17	352	-	*			*		
4498.897	MnI	22	353	163	43	17	2	80	12	3
4501.270	TiII	31	354	164	258	15	2	275	11	3
4502.220	MnI	22	355	-	27	14	2	93	11	3
4508.283	FeII	38	356	165	188	16	2	239	11	3
4510.210	MnII	17	357	166	5	16	2	14	11	3
4515.337	FeII	37	358	167	181	15	2	194	11	3
4517.530	FeI	472	359	-	63	15	2	22	11	3
4518.300	TiII	18	360	168	26	16	2	69	11	3
4520.225	FeII	37	361	169	187	16	2	256	11	3
4522.634	FeII	38	362	170	301	15	2	382	11	3
4525.142	FeI	826	363	171	170	16	2	224	11	3
4526.466	CrI	33	364	172	125	17	2	178	11	3
4528.472	CeII	1	365	-	232	30	2	259	24	2
4528.510	VII	56	366	173	*			*		
4528.619	FeI	68	367	173	*			*		
4529.562	FeI	987	368	-	72	24	2	138	14	3
4530.034	MnII	17	369	-	29	20	2	-2	13	3
4531.152	FeI	39	370	174	123	16	2	294	11	3
4533.966	TiII	50	371	175	299	17	2	142	25	3
4534.166	FeII	37	372	175	*			145	26	3
4535.721	CrI	33,276	373	176	65	16	2	171	11	3

	Identi-fication	Mult. No.	N1	N2	15 UMa			τUMa		
					E	ΔE	N	E	ΔE	N
4541.523	FeII	38	374	177	138	15	2	115	11	3
4544.009	TiII	60	375	-	39	20	2	27	13	3
4544.619	CrI	33	376	-	36	24	2	52	14	3
4545.144	TiII	30	377	178	62	22	2	96	14	3
4545.956	CrI	10	378	-	29	16	2	42	12	3
4547.234	NiI	146	379	179	106	17	2	143	12	3
4549.467	FeII	38	380	180	393	15	2	227	33	3
4549.622	TiII	82	381	180	*			247	33	3
4554.033	BaII	1	382	181	259	15	2	263	11	3
4555.020	CrII	44	383	182	115	17	2	149	12	3
4555.890	FeII	37	384	182	353	22	2	-114	80	3
4556.129	FeI	410, 820, 974	385	182	*			351	71	3
4556.169	CrI	173	386	-	*			*		
4558.659	CrII	44	387	183	197	15	2	249	11	3
4562.360	CeII	1	388	184	23	16	2	34	12	3
4563.761	TiII	50	389	185	178	16	2	245	12	3
4564.592	VII	56	390	-	28	14	2	53	10	3
4565.684	FeI	554	391	186	147	15	2	175	11	3
4568.789	FeI	554	392	187	1	14	2	45	10	3
4571.971	TiII	82	393	188	300	18	2	284	16	3
4572.277	CeII	1	394	188	*			99	16	3
4574.240	FeI	554	395	-	8	18	2	-24	12	3
4574.724	FeI	115	396	-	46	18	2	112	12	3
4576.331	FeII	38	397	189	155	15	2	178	11	3
4580.050	LaII	53	398	190	71	16	2	107	13	3
4580.055	FeII	26	399	-	*			*		
4580.056	CrI	10	400	-	*			*		
4580.619	NiI	146	401	-	-3	17	2	-13	17	3
4582.835	FeII	37	402	191	127	15	2	130	11	3
4583.829	FeII	38	403	191	228	16	2	257	11	3
4585.871	CaI	23	404	192	-12	15	2	1	11	3
4588.217	CrII	44	405	193	194	15	2	241	11	3
4589.890	CrII	44	406	194	117	16	2	131	11	3
4589.961	TiII	50	407	194	*			*		

	Identi-fication	Mult. No.	N1	N2	15 UMa			τ UMa		
					E	ΔE	N	E	ΔE	N
4592.090	CrII	44	408	195	156	38	2	197	24	3
4592.529	NiI	98	409	-	78	25	2	87	16	3
4592.655	FeI	39	410	195	*			*		
4595.363	FeI	594	411	-	50	15	2	103	10	3
4596.059	FeI	820	412	-	80	15	2	110	11	3
4598.122	FeI	554	413	196	97	16	2	139	11	3
4600.190	VII	56	414	-	130	15	2	-65	27	3
4600.372	NiI	98	415	197	*			300	26	3
4602.944	FeI	39	416	198	82	15	2	104	11	3
4604.994	NiI	98	417	199	75	15	2	153	11	3
4607.655	FeI	554, 969	418	200	73	14	2	72	10	3
4611.285	FeI	826	419	201	107	15	2	147	11	3
4613.210	FeI	554	420	202	103	15	2	44	28	3
4613.373	CrI	21	421	202	*			117	27	3
4616.640	CrII	44	422	203	176	14	2	175	10	3
4618.830	CrII	44	423	204	159	18	2	145	12	3
4619.294	FeI	821	424	204	126	17	2	139	12	3
4620.513	FeII	38	425	205	138	14	2	109	10	3
4625.052	FeI	554	426	206	85	14	2	110	10	3
4626.188	CrI	21	427	206	59	15	2	116	10	3
4628.160	CeII	1	428	207	49	16	2	87	11	3
4629.336	FeII	37	429	207	216	13	2	230	10	3
4630.125	FeI	115	430	-	32	14	2	69	10	3
4632.830	FeI	820	431	-	41	15	2	103	11	3
4632.915	FeI	39	432	208	*			*		
4634.110	CrII	44	433	208	168	14	2	180	10	3
4635.328	FeII	186	434	-	75	15	2	112	11	3
4637.512	FeI	554	435	209	69	17	2	71	11	3
4638.016	FeI	822	436	209	91	17	2	129	11	3
4643.468	FeI	820	437	210	45	14	2	67	10	3
4646.174	CrI	21	438	211	110	15	2	148	11	3
4647.437	FeI	409	439	212	122	14	2	183	11	3
4648.659	NiI	98	440	213	132	14	2	194	11	3

	Identi-fication	Mult. No.	N1	N2	15 UMa E	ΔE	N	τ UMa E	ΔE	N
4652. 158	CrI	21	441	214	48	14	2	98	10	3
4654.501	FeI	39	442	-	135	14	2	232	19	3
4654.628	FeI	554,821	443	215	*			*		
4656.974	FeII	43	444	216	125	14	2	174	10	3
4663.700	FeII	44	445	217	84	14	2	120	10	3
4666.750	FeII	37	446	218	93	40	2	326	45	3
4666.994	NiI	146	447	-	*			-156	64	3
4667.459	FeI	822	448	218	240	78	2	245	50	3
4668.070	FeI	826	449	218	49	32	2	107	18	3
4668.142	FeI	554	450	218	*			*		
4669.174	FeI	821	451	219	65	18	2	95	10	3
4670.170	FeII	25	452	-	-56	18	2	-104	68	3
4670.404	ScII	24	453	220	*			159	58	3
4670.483	VI	39	454	220	*			*		
4673.169	FeI	820	455	221	42	14	2	58	10	3
4678.852	FeI	821	456	222	120	14	2	193	10	3
4680.138	ZnI	2	457	223	44	15	2	148	11	3
4682.320	YII	12	458	224	77	14	2	102	10	3
4686.218	NiI	98	459	225	37	14	2	71	10	3
4690.146	FeI	820	460	226	-32	14	2	36	10	3
4691.414	FeI	409	461	227	35	20	2	114	10	3
4699.340	-	-	462	228	49	14	2	35	10	3
4702.983	MgI	11	463	229	148	14	2	136	10	3
4703.808	NiI	133	464	230	37	13	2	90	10	3
4704.958	FeI	821	465	231	2	15	2	33	11	3
4707.281	FeI	554	466	232	66	16	2	92	11	3
4709.092	FeI	821	467	-	79	18	2	72	12	3
4709.715	MnI	21	468	-	-22	18	2	26	11	3
4710.286	FeI	409	469	233	43	18	2	49	12	3
4714.421	NiI	98	470	234	169	15	2	204	11	3
4715.778	NiI	98	471	235	92	14	2	109	11	3
4722.159	ZnI	2	472	236	91	14	2	124	10	3
4727.405	FeI	821	473	237	181	43	2	143	25	3
4727.476	MnI	21	474	-	*			*		

	Identi-fication	Mult. No.	N1	N2	15 UMa E	15 UMa ΔE	15 UMa N	τ UMa E	τ UMa ΔE	τ UMa N
4727.851	NiI	146	475	-	-135	57	2	-16	33	3
4728.555	FeI	822	476	238	105	18	2	102	12	3
4731.439	FeII	43	477	239	152	14	2	175	10	3
4733.596	FeI	38	478	240	45	14	2	42	10	3
4735.846	FeI	1042	479	241	6	13	2	25	10	3
4736.780	FeI	554	480	242	186	13	2	189	10	3
4741.533	FeI	346	481	243	-5	14	2	28	10	3
4744.387	FeI	-	482	244	39	15	2	64	10	3
4745.129	FeI	67	483	-	34	14	2	25	12	2
4745.806	FeI	821, 1068	484	245	28	15	2	98	10	3
4748.120	ScII	48	485	246	87	14	2	106	10	3
4752.426	NiI	132	486	247	80	15	2	143	10	3
4754.042	MnI	16	487	248	160	21	1	143	11	3
4755.728	MnII	5	488	-	74	15	2	116	10	3
4756.519	NiI	98	489	249	110	14	2	141	9	3
4761.526	MnI	21	490	-	28	17	2	85	11	3
4762.376	MnI	21	491	250	83	13	2	164	9	3
4762.410	CI	6	492	-	*			*		
4763.790	FeII	50	493	-	60	41	1	50	19	2
4763.840	TiII	48	494	-	*			*		
4763.950	NiI	146	495	-	*			*		
4764.535	TiII	48	496	251	31	48	1	113	23	2
4765.859	MnI	21	497	-	66	16	2	80	11	3
4766.430	MnI	21	498	252	39	16	2	110	10	3
4768.397	FeI	384	499	253	17	14	2	124	10	3
4771.702	FeI	67	500	-	78	14	2	97	10	3
4771.720	CI	6	501	-	*			*		
4772.770	FeII	31	502	-	57	15	2	57	10	3
4772.817	FeI	38, 467	503	254	*			*		
4779.986	TiII	92	504	255	91	13	2	134	9	3
4783.420	MnI	16	505	256	128	13	2	192	9	3
4786.541	NiI	98	506	257	182	14	2	211	20	2
4786.810	FeI	467	507	257	*			29	21	2
4788.757	FeI	588	508	-	43	14	2	71	12	3

	Identi- fication	Mult. No.	N1	N2	15 UMa			τ UMa		
					E	ΔE	N	E	ΔE	N
4789.654	FeI	753	509	258	105	13	2	126	10	3
4792.390	TiII	48	510	259	36	13	2	62	10	3
4800.652	FeI	1042	511	260	62	14	2	65	10	3
4805.105	TiII	92	512	261	157	14	2	189	10	3
4806.996	NiI	163	513	262	53	16	2	95	11	3
4808.520	NiI	114	514	263	93	16	2	88	11	3
4810.534	ZnI	2	515	264	74	15	2	130	10	3
4811.999	NiI	130	516	-	21	21	2	22	14	3
4812.350	CrII	30	517	265	60	20	2	67	13	3
4817.773	FeI	67	518	266	17	14	2	48	12	2
4823.310	YII	22	519	-	149	14	2	18	27	3
4823.516	MnI	16	520	267	*			184	28	3
4824.130	CrII	30	521	267	112	20	2	140	13	3
4829.028	NiI	131	522	268	81	15	2	173	10	3
4831.183	NiI	111	523	269	77	16	2	126	11	3
4832.704	NiI	146	524	270	39	16	2	72	11	3
4835.862	FeI	1068	525	-	41	55	2	51	32	3
4836.220	CrII	30	526	271	50	42	2	131	23	3
4836.270	NiI	114	527	-	*			*		
4838.519	FeI	687	528	272	56	16	2	83	11	3
4839.549	FeI	588	529	272	41	14	2	67	10	3
4840.329	FeI	1068	530	272	44	16	2	114	11	3
4843.155	FeI	687	531	273	18	15	2	93	11	3
4848.240	CrII	30	532	274	161	16	2	194	11	3
4852.560	NiI	130	533	275						
4854.870	YII	22	534	276						
4855.414	NiI	130	535	276						
4855.540	FeII	25	536	-						
4855.683	FeI	687	537	-						
4856.190	CrII	30	538	277						
4857.382	NiI	111	539	278						
4859.120	FeI	1068	540	-						
4859.748	FeI	318	541	279						
4860.200	CrII	30	542	279						

NO MEASURES DUE TO BROAD WINGS OF Hβ.

	Identi-fication	Mult. No.	N1	N2	15 UMa			τ UMa		
					E	ΔE	N	E	ΔE	N
4861.332	H(β)	1	543	280	NO MEASURES DUE TO BROAD WINGS OF Hβ.					
4863.653	FeI	687	544	-						
4864.282	NiI	128	545	-						
4864.320	CrII	30	546	281						
4866.267	NiI	111	547	282						
4871.323	FeI	318	548	283						
4872.144	FeI	318	549	283						
4873.437	NiI	111	550	283						
4875.890	FeI	687	551	-	-28	19	2	9	12	3
4876.410	CrII	30	552	284	128	20	2	181	13	3
4878.132	CaI	35	553	285	98	18	2	193	12	3
4878.218	FeI	318	554	285	*			*		
4882.151	FeI	687	555	286	44	18	2	97	12	3
4883.690	YII	22	556	287	124	17	2	115	12	3
4884.570	CrII	30	557	-	2	17	2	58	12	3
4886.335	FeI	1066	558	-	-21	18	2	-2	12	3
4887.013	CrI	143	559	288	52	18	2	129	12	3
4888.651	FeI	1066	560	-	-76	57	2	-17	34	3
4889.009	FeI	67, 749	561	289	120	38	2	145	22	3
4889.113	FeI	985	562	289	*			*		
4890.762	FeI	318	563	290	164	17	2	202	12	3
4891.496	FeI	318	564	290	201	16	2	228	11	3
4893.780	FeII	36	565	291	21	16	2	75	11	3
4900.130	YII	22	566	292	229	16	2	225	11	3
4903.317	FeI	318	567	293	92	16	2	140	11	3
4904.413	NiI	129	568	294	73	16	2	156	11	3
4907.743	FeI	687	569	295	16	17	2	63	12	3
4910.027	FeI	687	570	-	166	61	2	234	35	3
4910.328	FeI	1068	571	-	43	41	2	-137	67	3
4910.570	FeI	1068	572	-	*			191	49	3
4911.205	TiII	114	573	296	94	25	2	76	15	3
4912.030	NiI	111	574	-	27	19	2	84	12	3
4913.970	NiI	132	575	297	71	18	2	96	12	3
4918.363	NiI	177	576	298	121	18	2	119	12	3

	Identi- fication	Mult. No.	N1	N2	15 UMa			τ UMa		
					E	ΔE	N	E	ΔE	N
4918.999	FeI	318	577	298	126	20	2	218	13	3
4920.509	FeI	318	578	299	178	19	2	269	14	3
4923.921	FeII	42	579	300	232	18	2	298	13	3
4924.776	FeI	114	580	301	56	17	2	81	13	3
4927.420	FeI	792	581	-	-24	22	2	45	14	3
4927.872	FeI	-	582	302	89	22	2	92	14	3
4930.331	FeI	985	583	303	54	20	2	113	13	3
4932.000	CI	13	584	304	29	22	2	18	15	3
4933.348	FeI	1065	585	-	107	29	2	115	18	3
4934.023	FeI	1068	586	-	263	20	2	305	13	3
4934.086	BaII	1	587	305	*			*		
4935.830	NiI	177	588	306	82	24	2	83	15	3
4937.337	NiI	114	589	307	11	23	2	75	15	3
4938.820	FeI	318	590	308	122	21	2	136	14	3
4941.920	NiI	114	591	309	60	19	2	-32	13	3
4945.458	NiI	145	592	310	76	18	2	86	13	3
4946.394	FeI	687	593	-	58	18	2	109	13	3
4950.112	FeI	687	594	311	-2	19	2	64	14	3
4952.646	FeI	1068, 1111	595	312	76	21	2	116	15	3
4953.204	NiI	111	596	-	7	21	2	23	15	3
4957.302	FeI	318	597	313	237	40	1	223	22	3
4957.603	FeI	318	598	313	149	40	1	312	30	2
4966.086	FeI	687	599	314	125	22	2	132	16	3
4967.899	FeI	1067	600	315	94	23	2	60	17	2
4969.927	FeI	1066	601	316	44	25	2	65	17	3
4971.354	NiI	274	602	317	71	26	2	52	18	3
4973.108	FeI	984	603	318	54	24	2	85	17	3
4978.606	FeI	966	604	319	25	26	2	55	18	3
4980.161	NiI	112	605	320	40	26	2	81	25	2
4982.507	FeI	1067	606	321	180	27	2	169	18	3
4983.258	FeI	1067	607	321	67	35	2	151	21	3
4983.855	FeI	1066	608	322	214	34	2	106	36	3
4984.126	NiI	143	609	322	*			95	36	3
4985.553	FeI	318	610	323	152	28	2	187	20	3

	Identi-fication	Mult. No.	N1	N2	15 UMa E	15 UMa ΔE	15 UMa N	τ UMa E	τ UMa ΔE	τ UMa N
4988.963	FeI	1066	611	324	105	26	2	131	25	2
4991.110	FeII	25	612	-	65	28	2	50	48	3
4991.277	FeI	1065	613	325	*			153	48	3
4993.355	FeII	36	614	326	-24	59	2	12	34	3
4993.687	FeI	1111	615	326	122	72	2	169	40	3
4994.133	FeI	16	616	326	18	48	2	54	28	3
4996.850	NiI	144	617	327	-49	32	2	66	21	3
4998.233	NiI	111	618	-	-35	35	2	75	24	3
4999.460	LaII	37	619	-	-27	31	2	88	22	3
5000.335	NiI	145	620	328	51	33	2	127	23	3
5001.871	FeI	965	621	329	109	32	2	167	22	3
5002.800	FeI	687	622	-	59	29	2	28	21	3
5005.720	FeI	984	623	330	50	39	2	104	26	3
5006.126	FeI	318	624	330	199	38	2	247	26	3
5007.289	FeI	966, 1065	625	331	156	31	2	166	23	3
5010.045	NiI	111	626	-	45	30	2	82	23	3
5010.961	NiI	144	627	332	56	30	2	22	24	2
5012.071	FeI	16	628	332	86	41	2	107	28	3
5012.464	NiI	111	629	-	124	43	2	127	29	3
5014.950	FeI	965	630	333	146	32	2	69	87	1
5017.591	NiI	111	631	334	175	37	1	66	24	3
5018.434	FeII	42	632	334	281	32	2	311	24	2
5022.244	FeI	965	633	335	40	34	2	139	25	3
5027.136	FeI	1065	634	336	57	45	2	83	30	3
5027.785	FeI	1110	635	-	59	66	2	118	39	3
5028.129	FeI	791	636	336	57	71	2	118	43	3
5029.623	FeI	718	637	-	9	43	2	-8	31	3
5030.784	FeI	585	638	337	49	41	2	111	47	3
5031.019	ScII	23	639	-	*			-94	47	3
5035.374	NiI	143	640	338	89	42	2	70	29	3
5035.961	NiI	145	641	338	158	38	2	249	27	3
5036.920	FeII	36	642	-	31	40	2	0	28	3
5039.266	FeI	687	643	339	23	41	2	84	29	3
5041.063	SiII	5	644	340	276	124	1	291	54	2

	Identification	Mult. No.	N1	N2	15 UMa			τ UMa		
					E	ΔE	N	E	ΔE	N
5041.074	FeI	16	645	340	*			*		
5041.620	CaI	34	646	340	81	82	1	63	180	1
5041.759	FeI	36	647	340	*			492	148	2
5049.825	FeI	114	648	341	-			-		

N1	28 AND E	ΔE	N	68 TAU E	ΔE	N	60 LEO E	ΔE	N
1	-			-			-		
2	-			-			-		
3	-			-			-		
4	-			-			-		
5	-			-			-		
6	764	91	1	-			337	120	1
7	558	57	3	-148	43	3	-1	48	2
8	-201	93	3	186	57	3	159	51	4
9	427	87	3	-188	60	3	-2	64	2
10	107	177	3	-441	113	3	-112	133	2
11	140	188	3	422	112	3	227	94	4
12	194	68	3	-100	43	3	-87	35	3
13	158	42	3	-44	25	3	-30	19	4
14	168	52	3	37	27	3	-3	24	4
15	-40	54	3	-73	28	3	32	25	4
16	163	40	3	92	21	3	94	18	4
17	93	41	3	30	21	3	80	18	4
18	247	41	3	152	21	3	122	18	4
19	-17	37	3	-15	19	3	-11	16	4
20	54	39	3	55	20	3	-11	18	4
21	159	40	3	116	20	3	39	18	4
22	43	37	3	-34	19	3	49	17	4
23	247	60	3	34	22	3	93	19	4
24	*			*			*		
25	291	184	3	-236	169	3	-4	275	3
26	*			*			*		
27	*			*			*		
28	19	162	3	465	187	3	187	308	3
29	107	101	3	-57	39	3	-4	60	3
30	240	60	3	223	22	3	215	26	4
31	-3	55	3	21	21	3	57	28	3
32	99	44	3	2	19	3	32	19	4
33	97	46	3	12	20	3	91	20	3
34	59	62	3	-34	29	3	24	29	3

N1	28 AND E	28 AND ΔE	N	68 TAU E	68 TAU ΔE	N	60 LEO E	60 LEO ΔE	N
35	542	491	3	80	266	3	24	231	3
36	-239	419	3	89	229	3	116	204	3
37	*			*			*		
38	152	38	3	139	20	3	125	18	4
39	147	41	3	74	21	3	146	21	3
40	37	88	3	49	43	3	-47	44	3
41	144	89	3	45	42	3	149	44	3
42	53	45	3	22	24	3	26	22	4
43	-8	112	2	43	38	3	147	176	2
44	353	553	2	37	188	3	-521	1047	2
45	61	445	2	37	147	3	464	801	2
46	*			*			*		
47	39	148	2	-13	52	3	126	248	2
48	-51	116	3	40	50	3	81	51	4
49	*			*			*		
50	332	169	3	73	76	3	-84	74	4
51	183	43	3	105	20	3	116	21	4
52	58	38	3	42	18	3	-18	18	3
53	218	33	3	114	17	3	106	15	4
54	-47	33	3	29	17	3	24	15	4
55	128	33	3	95	17	3	75	16	4
56	81	49	3	12	24	3	40	24	4
57	13	54	3	22	26	3	26	26	4
58	384	71	3	136	35	3	147	35	4
59	-141	81	3	34	40	3	-17	41	4
60	220	58	3	80	30	3	135	29	4
61	67	35	3	42	19	3	38	17	4
62	312	45	3	158	24	3	93	23	4
63	-19	46	3	-12	24	3	74	24	4
64	-14	36	3	-13	19	3	-25	14	4
65	147	31	3	58	17	3	48	16	4
66	52	46	3	93	25	3	139	24	4
67	*			*			*		
68	77	108	2	36	53	3	-137	28	4
69	89	37	3	-28	20	3	-31	18	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
70	85	36	3	66	20	3	38	19	4
71	29	39	3	5	21	3	-31	20	4
72	124	38	3	42	20	3	81	19	4
73	144	33	3	61	17	3	65	17	4
74	93	32	3	9	17	3	54	16	4
75	75	31	3	1	16	3	39	16	4
76	308	30	3	210	16	3	233	15	4
77	80	30	3	3	16	3	3	15	4
78	154	31	3	51	17	3	49	16	4
79	206	29	3	66	16	3	100	15	4
80	43	28	3	26	16	3	5	14	4
81	145	29	3	35	17	2	58	14	4
82	-34	79	3	52	44	3	21	40	4
83	246	132	3	18	71	3	38	68	4
84	-169	97	3	43	52	3	37	50	4
85	141	40	3	77	22	3	44	21	4
86	38	53	3	24	27	3	-9	29	4
87	240	52	3	141	27	3	114	29	4
88	136	32	3	104	17	3	119	17	4
89	29	26	3	9	15	3	18	13	4
90	260	67	3	229	41	3	302	34	4
91	*			*			*		
92	68	102	3	14	73	3	-83	48	4
93	23	295	3	-299	358	3	384	129	4
94	53	412	3	386	484	3	-585	183	4
95	-34	314	3	-134	353	3	513	141	4
96	193	196	3	134	210	3	-221	89	4
97	227	95	3	75	52	3	-40	33	4
98	-176	150	3	-43	65	3	28	44	4
99	254	130	3	15	50	3	56	32	4
100	-149	562	3	-251	210	3	9	73	3
101	*			*			*		
102	233	622	3	324	234	3	-2	73	3
103	137	25	3	145	15	3	93	13	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
104	21	27	3	5	16	3	10	14	4
105	30	33	3	53	18	3	2	18	4
106	84	29	3	11	16	3	34	16	4
107	57	28	3	33	15	3	44	15	4
108	263	27	3	54	15	3	-19	15	4
109	70	29	3	60	16	3	72	17	4
110	97	30	3	20	17	3	-5	17	4
111	-54	35	3	29	19	3	14	19	4
112	188	28	3	105	16	3	92	16	4
113	217	25	3	123	14	3	107	13	4
114	31	24	3	34	14	3	3	13	4
115	32	26	3	58	15	3	20	14	4
116	200	26	3	131	15	3	83	14	4
117	72	27	3	-22	16	3	17	14	4
118	212	71	3	-41	43	3	47	37	4
119	1	44	3	89	26	3	96	24	4
120	*			*			*		
121	137	27	3	20	16	3	41	15	4
122	186	26	3	119	15	3	148	15	4
123	152	26	3	101	15	3	114	14	4
124	30	29	3	32	17	3	51	16	4
125	17	33	3	-12	18	3	40	19	4
126	-11	40	3	21	21	3	-8	23	4
127	33	32	3	-1	18	3	-9	18	4
128	-49	27	3	22	16	3	19	15	4
129	30	25	3	21	15	3	-1	14	4
130	41	24	3	40	14	3	45	13	4
131	238	22	3	120	14	3	138	13	4
132	218	23	3	120	14	3	125	13	4
133	171	24	3	67	15	3	121	14	4
134	269	22	3	98	14	3	93	13	4
135	73	21	3	90	13	3	41	12	4
136	94	24	3	37	15	3	38	13	4
137	-14	25	3	7	15	3	30	14	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
138	151	85	3	18	53	3	-2	44	4
139	-271	161	3	29	97	3	31	84	4
140	206	114	3	-41	69	3	-41	59	4
141	201	27	3	78	16	3	77	14	4
142	108	25	3	3	15	3	-20	14	4
143	114	31	3	35	17	3	72	17	4
144	-18	26	3	27	14	3	-13	15	4
145	77	28	3	-7	16	3	5	16	4
146	29	26	3	-4	15	3	-20	14	4
147	160	26	3	87	15	3	24	14	4
148	286	46	3	40	25	3	-53	26	4
149	77	47	3	51	25	3	121	27	4
150	339	35	3	143	20	3	100	19	4
151	87	23	3	2	13	3	-13	13	4
152	213	22	3	174	13	3	155	13	4
153	*			*			*		
154	-48	44	3	40	30	3	-3	25	4
155	153	45	3	122	17	3	175	17	4
156	*			*			*		
157	-5	24	3	-10	18	3	18	13	4
158	86	42	3	54	30	3	54	25	4
159	134	40	3	68	23	3	24	24	4
160	160	26	3	173	21	3	183	18	3
161	47	126	3	417	120	3	197	68	4
162	276	201	3	-453	198	3	-120	112	4
163	-68	139	3	391	149	3	32	80	4
164	219	66	3	-29	70	3	149	39	4
165	-64	53	3	150	66	3	-46	30	4
166	112	43	3	-235	171	3	70	29	3
167	*			*			*		
168	50	41	3	408	232	3	-32	27	3
169	-14	20	3	43	23	3	-11	13	4
170	176	51	3	4	212	3	28	53	3
171	120	50	3	99	145	3	138	53	3

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
172	*			*			*		
173	118	52	3	132	69	3	-50	32	4
174	-125	95	3	-157	111	3	71	62	4
175	209	70	3	165	77	3	-5	45	4
176	214	48	3	108	22	3	175	34	2
177	277	538	3	24	89	3	-62	150	3
178	-33	656	3	37	103	3	-122	160	4
179	271	206	3	149	32	3	213	148	2
180	*			*			*		
181	11	350	3	2	23	3	37	103	3
182	96	588	3	33	23	3	-136	164	4
183	-155	1227	3	-505	2213	2	903	355	4
184									
185									
186									
187									
188									
189									
190									
191									
192									
193									
194									
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201									
202									
203									
204									
205									

NO MEASURES DUE TO BROAD WINGS OF Hγ

N1	28 AND E	ΔE	N	68 TAU E	ΔE	N	60 LEO E	ΔE	N
206	NO MEASURES DUE TO BROAD WINGS OF Hγ.								
207									
208									
209									
210									
211									
212									
213	114	20	2	87	12	3	61	11	4
214	*			*			*		
215	65	23	3	61	15	3	26	16	3
216	62	22	3	19	15	3	37	14	4
217	48	23	3	18	15	3	40	15	4
218	17	23	3	36	15	3	15	15	4
219	261	51	3	66	33	3	-41	31	4
220	48	45	3	23	47	3	138	27	4
221	*			*			*		
222	95	18	3	5	12	3	17	11	4
223	23	18	3	-37	12	3	22	11	4
224	23	18	3	66	12	3	36	11	4
225	12	22	3	-8	19	3	2	12	4
226	342	38	3	221	48	3	192	19	4
227	-56	173	3	-176	207	3	99	86	4
228	223	133	3	246	159	3	31	65	4
229	*			*			*		
230	229	57	3	184	76	3	95	29	4
231	124	25	3	105	25	3	39	13	4
232	95	20	3	3	15	3	31	11	4
233	34	21	3	34	14	3	21	12	4
234	50	17	3	-25	27	3	-17	19	3
235	161	113	2	190	95	3	176	56	3
236	-31	83	2	-40	66	3	-54	42	3
237	*			*			*		
238	123	26	3	68	26	3	19	17	2
239	305	398	3	-108	430	3	549	246	2

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
240	-89	336	3	233	359	3	-333	206	2
241	*			*			*		
242	151	72	3	88	84	3	-28	45	2
243	124	18	3	35	11	3	69	13	4
244	242	38	3	88	18	3	60	48	2
245	90	137	3	72	59	3	-182	218	2
246	12	168	3	-73	72	3	210	275	2
247	536	559	3	56	261	3	-563	890	2
248	-453	485	3	15	228	3	539	774	2
249	*			*			*		
250	55	24	3	28	13	3	-19	16	4
251	230	19	3	160	12	3	153	14	4
252	*			*			*		
253	3	29	3	-6	42	3	44	17	4
254	105	169	2	-80	184	3	71	96	3
255	*			*			*		
256	*			*			*		
257	-539	1125	2	595	959	3	-297	188	3
258	-126	243	3	-159	345	3	43	141	4
259	*			*			*		
260	101	329	3	-425	719	3	196	161	4
261	31	239	3	285	470	3	-97	143	4
262	50	131	3	-119	247	3	52	77	4
263	0	18	3	25	12	3	5	13	4
264	51	25	3	57	32	3	58	13	4
265	57	20	3	14	18	3	16	12	4
266	66	22	3	28	17	3	45	14	4
267	-80	60	3	23	48	3	-8	36	4
268	233	160	3	-68	134	3	11	92	4
269	42	142	3	194	118	3	156	80	4
270	139	54	3	-17	48	3	-56	29	4
271	172	18	3	125	16	3	138	10	4
272	208	19	3	106	14	3	105	12	4
273	90	19	3	26	13	3	14	13	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
274	4	21	3	13	15	3	16	14	4
275	-11	21	3	36	18	3	7	15	4
276	93	15	3	77	11	3	75	10	4
277	*			*			*		
278	46	16	3	15	12	3	8	11	4
279	99	16	3	24	11	3	-11	11	4
280	101	15	3	21	11	3	42	10	4
281	25	18	3	-8	13	3	23	12	4
282	72	18	3	34	13	3	34	12	4
283	54	23	3	40	15	3	15	14	4
284	5	51	3	-58	34	3	19	34	4
285	-2	48	3	100	33	3	28	31	4
286	134	16	3	-3	12	2	-8	10	4
287	54	14	3	8	11	3	34	11	4
288	2	17	3	-1	11	3	-4	11	4
289	-9	24	2	13	13	3	23	13	4
290	-32	22	2	5	12	3	34	13	3
291	-13	22	2	9	12	3	1	11	4
292	-7	23	2	12	14	3	28	12	4
293	44	24	2	45	14	3	21	12	4
294	103	27	2	31	16	3	49	15	4
295	65	75	2	110	46	3	14	44	4
296	57	74	2	-66	46	3	24	44	4
297	208	27	2	164	17	3	166	15	4
298	90	19	2	38	11	3	13	10	4
299	*			*			*		
300	59	16	2	8	10	3	-1	9	4
301	—			29	10	3	27	9	4
302	80	42	3	22	28	3	57	30	4
303	100	43	3	71	29	3	-14	32	4
304	58	17	3	45	13	3	37	14	4
305	*			*			*		
306	*			*			*		
307	19	20	3	25	17	3	-22	16	3

N1	28 AND E	ΔE	N	68 TAU E	ΔE	N	60 LEO E	ΔE	N
308	23	74	3	65	52	3	976	944	1
309	197	29	3	88	22	3	64	22	4
310	*			*			*		
311	53	63	3	-72	37	3	-762	763	1
312	54	31	3	7	19	3	-18	18	3
313	-28	19	3	4	14	3	-6	17	3
314	-12	16	3	-18	11	3	16	11	4
315	101	17	3	23	12	3	62	7	4
316	*			*			*		
317	-36	18	3	-12	12	3	-9	11	4
318	121	33	3	28	22	3	72	21	4
319	41	40	3	34	27	3	-20	26	4
320	-31	30	3	-6	21	3	56	20	4
321	-7	22	3	-16	16	3	-17	13	4
322	-114	121	3	-200	96	3	90	70	4
323	215	97	3	207	75	3	-15	57	4
324	*			*			*		
325	128	16	3	57	12	3	72	11	4
326	215	15	3	166	11	3	138	10	4
327	170	45	3	-35	41	3	130	31	3
328	-255	407	3	775	339	3	-278	262	4
329	161	235	3	-408	189	3	133	150	4
330	*			*			*		
331	217	179	3	-346	162	3	191	113	4
332	104	15	3	53	11	3	56	10	4
333	137	14	3	71	10	3	84	9	4
334	*			*			*		
335	-4	14	3	4	10	3	3	11	3
336	-33	15	3	-24	11	3	8	10	4
337	307	16	3	398	11	3	380	11	4
338	148	17	3	59	12	3	59	12	4
339	*			*			*		
340	-106	29	3	54	20	3	-24	19	4
341	8	17	3	34	12	3	45	11	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
342	8	16	3	-7	12	3	29	11	4
343	-46	16	3	-1	12	3	-10	11	4
344	91	15	3	41	11	3	34	10	4
345	93	14	3	100	10	3	84	9	4
346	82	14	3	99	10	3	110	9	4
347	14	14	3	12	11	3	26	9	4
348	116	14	3	81	12	3	75	9	4
349	46	40	2	-33	27	3	15	20	4
350	29	27	2	35	15	3	46	7	3
351	*			*			*		
352	*			*			*		
353	15	14	3	-19	12	3	-8	9	4
354	215	16	2	147	10	3	139	9	4
355	19	13	3	28	10	3	18	9	4
356	190	13	3	123	10	3	122	9	4
357	21	13	3	-10	10	3	-8	9	4
358	149	13	3	146	10	3	134	9	4
359	45	12	3	3	9	3	11	8	4
360	63	13	3	26	9	3	8	9	4
361	175	13	3	134	10	3	163	9	4
362	173	13	3	180	10	3	169	9	4
363	104	13	3	32	12	3	49	9	4
364	69	14	3	12	12	3	27	9	4
365	-			-157	119	3	-		
366	-			*			-		
367	-			*			-		
368	80	15	3	61	13	3	30	9	4
369	-18	15	3	3	15	3	7	10	3
370	78	13	3	45	13	3	19	9	4
371	230	28	3	153	20	3	64	19	4
372	14	28	3	55	20	3	128	20	4
373	60	13	3	-3	10	3	7	9	4
374	101	13	3	69	9	3	90	8	4
375	-14	14	3	11	10	3	30	10	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
376	51	15	3	16	10	3	-3	10	4
377	26	15	3	20	11	3	20	10	4
378	12	14	3	-1	10	3	6	9	4
379	20	14	3	44	10	3	-3	9	4
380	209	35	3	65	25	3	222	25	4
381	154	35	3	275	25	3	108	24	4
382	196	13	3	150	9	3	151	8	4
383	45	13	3	82	10	3	61	10	3
384	33	94	3	177	74	3	153	70	3
385	138	80	3	-35	62	3	-6	60	3
386	*			*			*		
387	149	12	3	172	9	3	165	8	4
388	54	14	3	6	10	3	-17	9	4
389	205	14	3	150	10	3	124	9	4
390	55	13	3	36	10	3	8	8	4
391	46	13	3	58	10	3	57	9	4
392	5	12	3	23	9	3	7	8	4
393	279	18	3	157	13	3	140	13	4
394	-17	18	3	-12	13	3	15	13	4
395	18	14	3	-9	10	3	14	9	4
396	16	14	3	-2	10	3	7	10	4
397	106	13	3	88	9	3	99	9	4
398	28	13	3	47	12	2	41	9	4
399	*			*			*		
400	*			*			*		
401	-7	13	3	-17	11	3	-3	9	4
402	45	12	3	84	9	3	61	9	3
403	213	13	3	166	9	3	199	8	4
404	76	12	3	-6	9	3	6	8	4
405	109	12	3	150	9	3	142	8	4
406	131	13	3	76	9	3	58	9	4
407	*			*			*		
408	134	25	3	43	19	3	93	18	4
409	-173	79	3	190	60	3	-11	57	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
410	195	70	3	-171	54	3	26	50	4
411	38	12	3	-11	9	3	13	8	4
412	60	12	3	43	9	3	30	8	4
413	65	12	3	27	9	3	18	8	4
414	-2	28	3	-3	20	3	-14	20	4
415	63	28	3	27	20	3	18	20	4
416	60	12	3	-8	9	3	12	9	3
417	57	12	3	40	9	3	22	8	4
418	19	12	3	3	9	3	11	8	4
419	66	12	3	30	9	3	43	8	4
420	-11	31	3	5	23	3	-0	22	4
421	76	31	3	-2	23	3	1	22	4
422	68	12	3	84	9	3	64	9	3
423	123	13	3	112	10	3	110	9	4
424	37	13	3	34	10	3	20	9	4
425	92	12	3	78	9	3	99	8	4
426	27	12	3	11	9	3	11	8	4
427	16	12	3	20	9	3	27	8	4
428	22	13	3	4	10	3	16	9	4
429	164	12	3	104	9	3	161	8	4
430	48	12	3	-57	10	2	1	8	4
431	32	12	3	-8	9	3	9	8	4
432	*			*			*		
433	100	12	3	110	9	3	108	8	4
434	43	12	3	64	9	3	69	9	4
435	17	13	3	11	9	3	3	9	4
436	49	13	3	30	9	3	36	11	3
437	52	12	3	13	9	3	12	9	3
438	59	12	3	18	9	3	2	8	4
439	48	13	3	21	10	3	23	9	4
440	37	12	3	50	9	3	49	8	4
441	40	11	3	11	9	3	10	8	4
442	11	38	3	-15	28	3	12	27	4
443	66	38	3	38	28	3	-5	27	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
444	83	11	3	60	10	2	61	8	4
445	3	11	3	47	9	3	25	8	4
446	217	63	3	-69	75	3	44	37	4
447	-118	85	3	164	97	3	11	50	4
448	115	66	3	-47	82	3	25	38	4
449	66	18	3	28	16	3	3	13	4
450	*			*			*		
451	37	12	3	16	13	2	-3	8	4
452	254	94	2	36	77	2	71	50	4
453	39	60	3	-13	58	2	-31	41	4
454	*			*			*		
455	6	11	3	20	9	3	15	8	4
456	89	12	3	21	9	3	46	8	4
457	-6	12	3	-2	9	3	19	8	4
458	8	11	3	18	9	3	13	8	4
459	14	11	3	3	9	3	4	8	4
460	18	12	3	1	9	3	-3	8	4
461	43	12	3	22	12	2	5	8	4
462	38	11	3	3	10	2	17	8	4
463	151	11	3	68	9	3	69	8	4
464	25	11	3	21	9	3	10	8	4
465	9	15	2	13	9	3	4	8	4
466	53	12	3	47	11	2	19	8	4
467	64	12	3	17	9	3	5	9	4
468	-36	12	3	15	9	3	14	8	4
469	44	12	3	3	9	3	-26	9	4
470	53	12	3	78	9	3	65	8	4
471	8	12	3	10	9	3	42	8	4
472	15	11	3	24	9	3	29	8	4
473	25	24	3	46	18	3	-43	18	4
474	*			*			*		
475	-2	35	3	-31	27	3	97	25	4
476	13	13	3	37	10	3	-24	9	4
477	65	11	3	71	8	3	93	8	4
478	44	11	3	-7	8	3	-22	8	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
479	8	11	3	-10	8	3	-13	8	4
480	74	11	3	26	9	3	10	9	3
481	-6	11	3	3	9	3	-3	8	4
482	27	11	3	18	9	3	9	8	4
483	13	11	3	15	9	3	4	7	4
484	32	11	3	5	9	3	-1	8	4
485	57	11	3	21	9	3	22	8	4
486	19	12	3	20	8	3	5	8	4
487	39	12	3	-3	10	2	9	9	4
488	36	11	3	24	8	3	27	9	3
489	-4	11	3	27	7	3	31	9	3
490	13	14	3	28	15	3	5	9	4
491	96	14	3	56	29	3	19	7	4
492	*			*			*		
493	-317	614	2	-265	1340	3	234	484	1
494	*			*			*		
495	419	646	2	346	1428	3	-220	512	1
496	16	42	2	27	72	3	28	33	1
497	27	15	3	1	19	3	21	8	4
498	42	12	3	-18	11	3	-4	8	4
499	46	11	3	10	8	3	9	8	4
500	55	12	3	14	8	3	6	8	4
501	*			*			*		
502	14	12	3	-8	8	3	-23	8	4
503	*			*			*		
504	89	11	3	52	9	3	48	9	3
505	84	11	3	33	8	3	-4	9	3
506	64	19	3	18	13	3	4	14	3
507	10	19	3	47	13	3	21	14	3
508	27	11	3	5	8	3	-3	9	3
509	32	11	3	8	8	3	6	9	3
510	33	11	3	-4	8	3	-16	8	4
511	5	11	3	-2	8	3	1	8	4
512	135	12	3	76	9	3	61	8	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
513	1	12	3	-8	9	3	20	8	4
514	-35	15	2	6	9	3	27	8	4
515	46	12	3	23	9	3	50	8	4
516	-9	16	3	-13	11	3	16	10	4
517	36	15	3	46	11	3	25	10	4
518	17	12	3	14	9	3	8	9	3
519	2	29	3	13	21	3	29	20	4
520	75	29	3	37	21	3	15	21	4
521	85	15	3	94	10	3	111	10	4
522	36	12	3	14	9	3	12	8	4
523	15	13	3	12	9	3	22	9	4
524	11	13	3	-1	9	3	1	9	4
525	55	38	3	-49	28	3	32	25	4
526	29	26	3	49	19	3	14	18	4
527	*			*			*		
528	39	13	3	-4	10	3	18	9	4
529	29	12	3	23	9	3	-11	8	4
530	28	13	3	15	10	3	-7	9	4
531	24	13	3	32	10	3	-6	9	4
532	101	14	3	65	11	3	84	11	3
533									
534									
535	NO MEASURES								
536									
537	DUE TO								
538									
539	BROAD WINGS								
540									
541	OF Hβ								
542									
543									
544									
545									
546									

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
547	NO MEASURES DUE TO								
548	BROAD WINGS OF Hβ								
549									
550				13	18	2			
551	-6	19	2	-19	11	3	52	22	1
552	79	20	2	88	11	3	64	23	1
553	136	15	3	34	11	3	8	7	4
554	*			*			*		
555	5	15	3	14	10	3	50	11	3
556	39	15	3	70	10	3	70	9	4
557	3	15	3	18	10	3	45	12	3
558	26	15	3	-3	12	2	9	9	4
559	40	15	3	18	12	2	-2	10	4
560	-41	41	3	-28	28	3	-24	26	4
561	106	32	3	144	92	3	33	20	4
562	*			-106	80	3	*		
563	106	15	3	73	10	3	71	10	4
564	146	14	3	71	9	3	98	9	4
565	-11	14	3	29	9	3	15	9	4
566	101	15	3	41	9	3	63	8	4
567	39	15	3	31	10	3	14	8	4
568	42	15	3	43	14	2	2	12	2
569	4	15	3	-7	9	3	-12	9	4
570	40	42	3	-60	27	3	-20	25	4
571	47	78	3	126	47	3	107	47	4
572	16	57	3	-77	34	3	-76	35	4
573	92	18	3	58	11	3	48	11	4
574	35	16	3	2	10	3	13	9	4
575	15	15	3	17	11	2	36	10	3
576	3	16	3	-1	10	3	14	9	4
577	108	16	3	72	10	3	74	9	4
578	150	17	3	94	10	3	84	9	4
579	234	17	3	165	10	3	165	9	4
580	16	16	3	25	11	2	24	9	4

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
581	1	19	3	9	11	3	11	10	4
582	37	19	3	11	11	3	29	13	3
583	65	18	3	-1	11	3	13	9	4
584	60	19	3	6	11	3	10	10	4
585	52	23	3	17	13	3	8	13	4
586	193	18	3	98	11	3	180	10	3
587	*			*			*		
588	15	19	3	-14	11	3	13	11	4
589	1	19	3	-19	11	3	18	11	4
590	94	19	3	20	11	3	12	10	4
591	22	18	3	2	11	3	1	10	4
592	24	18	3	9	11	3	10	10	4
593	45	18	3	13	11	3	-17	10	4
594	-7	19	3	-19	11	3	3	10	4
595	30	20	3	-16	11	3	12	10	4
596	-20	20	3	11	11	3	1	10	4
597	0	36	2	33	16	3	67	15	4
598	180	29	3	133	16	3	119	15	4
599	19	22	3	39	12	3	53	12	3
600	48	22	3	29	12	3	-12	11	4
601	13	24	3	10	13	3	4	12	4
602	17	24	3	16	13	3	-12	12	4
603	90	23	3	-9	13	3	37	13	3
604	2	25	3	28	13	3	16	12	4
605	-5	25	3	23	13	3	6	12	4
606	44	25	3	36	13	3	31	12	4
607	20	29	3	31	15	3	40	14	4
608	167	47	3	-22	22	3	-2	23	4
609	-66	49	3	52	24	3	76	24	4
610	52	27	3	10	14	3	56	13	4
611	76	27	3	-29	17	2	0	12	4
612	63	64	3	-24	32	3	46	31	4
613	24	64	3	60	32	3	-52	31	4
614	77	48	3	18	24	3	21	26	3

N1	28 AND			68 TAU			60 LEO		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
615	-21	53	3	2	26	3	18	30	3
616	65	39	3	-14	20	3	9	22	3
617	11	31	3	13	15	3	9	16	3
618	-59	34	3	-3	17	3	8	16	4
619	5	33	3	2	16	3	-16	15	4
620	-29	33	3	-12	16	3	15	15	4
621	41	33	3	26	16	3	110	15	4
622	-18	31	3	-40	15	3	19	14	4
623	-2	42	2	-23	17	3	-30	17	4
624	88	39	3	80	18	3	79	17	4
625	61	34	3	53	16	3	12	15	4
626	40	33	3	39	16	3	21	14	4
627	-2	34	3	-2	16	3	23	14	4
628	73	41	3	2	19	3	-12	18	4
629	-11	40	3	47	18	3	61	18	4
630	27	35	3	28	16	3	15	15	4
631	29	37	3	15	16	3	70	15	4
632	146	35	3	116	16	3	186	15	4
633	-33	37	3	39	17	3	38	16	4
634	46	45	3	61	18	3	16	18	4
635	15	58	3	-16	22	3	-23	24	4
636	30	64	3	107	25	2	84	26	4
637	67	44	3	40	19	3	31	19	4
638	34	76	3	59	29	3	89	30	4
639	126	76	3	-1	29	3	-50	30	4
640	-7	43	3	27	19	3	28	20	3
641	-62	42	3	74	18	3	67	17	4
642	-40	42	3	-14	20	3	39	18	4
643	44	50	2	-31	28	3	-5	19	4
644	-301	104	3	106	47	3	69	40	4
645	*			*			*		
646	1925	345	3	-182	127	3	-163	162	4
647	-1799	317	2	215	125	3	199	142	4
648	-			-			-		

N2	88 TAU E	ΔE	N	ω TAU E	ΔE	N	σ' TAU E	ΔE	N
1	-			-2190	1228	1	-		
2	-			2073	1006	1	-		
3	399	43	3	337	85	3	-		
4	314	34	3	181	66	2	-		
5	124	32	3	205	40	3	242	50	3
6	173	38	3	-102	58	3	-		
7	231	39	3	-32	52	3	-		
8	108	35	3	-94	46	3	-		
9	-27	31	3	-48	41	3	-		
10	62	32	3	203	39	3	-		
11	105	32	3	146	36	3	29	41	3
12	163	30	3	450	34	3	364	43	3
13	236	31	3	318	34	3	285	41	3
14	164	32	3	247	37	3	135	42	3
15	105	35	2	24	40	3	-12	55	3
16	263	33	3	409	41	3	352	49	3
17	313	35	3	459	36	3	325	38	3
18	208	35	3	505	36	3	458	50	2
19	252	33	3	439	35	3	364	74	1
20	200	29	3	317	32	3	438	50	2
21	174	28	3	344	30	3	220	83	1
22	162	27	3	270	30	3	-		
23	90	28	3	81	38	2	-		
24	99	29	3	265	34	3	-		
25	297	33	3	665	34	3	-		
26	43	34	3	29	36	3	-		
27	181	32	3	366	31	3	-		
28	47	36	3	78	35	3	-		
29	100	38	3	275	36	3	-		
30	-21	40	3	54	38	3	-		
31	44	37	3	-2	38	3	-		
32	6	35	3	135	36	3	-		
33	-52	37	3	-195	41	3	-		
34	-163	42	3	-108	48	3	-		

N2	88TAU			ω TAU			σ' TAU		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
35	161	39	3	261	47	3	-		
36	34	36	3	-104	47	3	-		
37	68	35	3	9	51	3	-		
38	101	31	3	12	44	3	-		
39	122	28	3	161	41	3	-		
40	63	24	3	162	29	3	-		
41	358	27	3	450	36	3	483	83	1
42	16	25	3	-145	42	1	-51	67	1
43	386	24	3	487	25	3	357	30	2
44	251	25	3	355	26	3	274	29	3
45	161	24	3	289	26	3	223	30	3
46	89	24	3	299	26	3	172	30	3
47	115	26	3	236	29	3	161	36	3
48	57	26	3	150	29	3	5	35	3
49	255	23	3	367	24	3	296	28	3
50	119	29	3	153	30	3	-1	34	3
51	295	30	3	368	31	3	268	37	2
52	83	32	3	158	33	3	46	32	3
53	161	34	3	255	35	3	134	39	2
54	76	31	3	64	33	3	-77	35	2
55	187	33	3	160	34	3	112	32	3
56	209	32	3	433	32	3	196	36	2
57	184	28	3	166	28	3	132	29	3
58	19	26	3	49	27	3	-36	28	3
59	19	29	3	62	32	3	-11	38	2
60	29	26	3	106	29	3	101	31	3
61	95	25	3	135	26	3	95	27	3
62	347	25	3	435	26	3	397	27	3
63	62	22	3	69	23	3	110	23	3
64	135	22	3	266	23	3	283	47	1
65	20	21	3	-109	26	3	-85	34	3
66	-27	20	3	42	24	3	-70	36	3
67	4	23	3	-56	27	3	-84	36	3
68	140	23	3	160	26	3	99	32	3
69	51	21	3	44	23	3	41	34	3

N2	88 TAU E	ΔE	N	ω TAU E	ΔE	N	σ' TAU E	ΔE	N
70	82	22	3	81	28	3	14	35	3
71	160	26	3	150	32	3	60	37	3
72	372	24	3	560	30	3	458	35	3
73	87	24	3	158	36	2	-24	38	3
74	205	25	3	169	32	3	132	39	3
75	157	23	3	201	27	3	166	33	3
76	-63	20	3	-27	22	3	-73	28	3
77	289	22	3	432	26	3	494	34	3
78	240	22	3	291	25	3	388	33	3
79	94	21	3	120	23	3	129	24	3
80	103	37	1	155	24	3	153	27	3
81	259	21	3	228	22	3	333	22	3
82	156	21	3	352	22	3	253	21	3
83	159	21	3	200	22	3	559	19	3
84	253	26	3	651	46	3	550	175	1
85	175	138	1						
86									
87									
88									
89									
90									
91									
92									
93									
94									
95									
96									
97									
98									
99									
100									
101	103	18	3	169	18	3	181	21	3
102	97	17	3	171	18	3	140	20	3
103	5	39	1	9	19	3	37	24	3

NO MEASURES DUE TO BROAD WINGS OF Hγ.

N2	88 TAU			ω TAU			σ' TAU		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
104	256	22	2	370	19	3	374	25	3
105	78	22	2	93	19	3	-83	25	3
106	13	18	3	5	23	2	-		
107	-70	19	3	-2	20	3	-3	43	1
108	168	17	3	257	18	3	255	43	1
109	160	19	3	341	19	3	328	43	1
110	4	18	3	122	18	3	-		
111	16	18	3	118	18	3	-		
112	46	18	3	183	22	2	-		
113	167	16	3	280	18	3	234	24	3
114	149	17	3	157	17	3	151	21	3
115	77	19	3	99	18	3	-25	25	2
116	176	18	3	250	18	3	211	21	3
117	100	18	3	214	18	3	76	22	3
118	3	17	3	-19	17	3	34	19	3
119	182	17	3	266	18	3	292	22	3
120	61	17	3	195	19	3	229	22	3
121	31	18	3	32	19	3	161	21	3
122	-34	17	3	-53	18	3	3	18	3
123	175	16	3	262	16	3	247	18	3
124	175	20	2	144	17	3	220	18	3
125	195	18	2	206	17	3	203	19	3
126	78	15	3	136	17	3	132	18	3
127	14	14	3	84	16	3	46	17	3
128	62	16	3	-35	16	3	-34	17	3
129	95	19	2	93	18	3	103	20	3
130	92	16	3	161	19	3	124	22	3
131	97	16	3	154	17	3	156	19	3
132	130	15	3	198	16	3	140	18	3
133	13	15	3	10	16	3	77	19	3
134	63	14	3	121	16	3	56	24	2
135	220	14	3	318	17	3	432	21	3
136	101	16	3	164	17	3	99	20	3
137	64	16	3	194	17	3	123	18	3

NZ	88 TAU			ω TAU			σ' TAU		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
138	101	15	3	211	17	3	242	23	3
139	17	16	3	84	17	3	13	25	3
140	123	16	3	317	18	3	216	29	2
141	88	15	3	111	16	3	73	24	2
142	48	16	3	89	17	3	23	26	2
143	109	16	3	276	18	3	214	28	2
144	10	16	3	16	17	3	19	27	2
145	147	18	3	343	21	3	288	30	2
146	75	18	3	175	22	3	92	26	3
147	61	21	2	180	22	3	73	20	3
148	136	17	3	298	20	3	266	19	3
149	38	17	3	162	18	3	68	18	3
150	53	17	3	113	18	3	82	21	3
151	92	15	3	203	17	3	145	20	3
152	3	15	3	119	17	3	1	19	3
153	445	15	3	482	16	3	665	19	3
154	29	16	3	119	17	3	19	20	3
155	37	16	3	45	17	3	39	19	3
156	-30	15	3	-12	16	3	57	19	3
157	-3	15	3	-45	18	3	-26	20	3
158	121	17	3	179	20	3	258	21	3
159	110	17	3	127	20	3	215	21	3
160	-6	15	3	-53	18	3	-17	19	3
161	90	16	3	186	18	3	194	20	3
162	55	16	3	56	18	3	117	18	3
163	19	16	3	40	16	3	26	17	3
164	182	15	3	259	16	3	276	17	3
165	162	14	3	210	14	3	207	15	3
166	34	14	3	-30	14	3	-11	15	3
167	143	14	3	200	16	3	214	18	3
168	33	15	3	69	16	3	58	18	3
169	128	17	3	167	16	3	225	17	3
170	189	20	2	301	18	3	327	19	3
171	67	16	3	188	17	3	207	18	3

N2	88 TAU E	ΔE	N	ω TAU E	ΔE	N	σ' TAU E	ΔE	N
172	6	15	3	65	16	3	37	17	3
173	159	16	3	227	17	3	320	18	3
174	52	16	3	63	17	3	122	19	3
175	224	14	3	339	15	3	410	17	3
176	-14	16	2	42	14	3	-30	15	3
177	91	14	3	146	14	3	161	16	3
178	-27	14	3	91	15	3	87	16	3
179	-14	14	3	82	14	3	19	14	3
180	357	14	3	442	15	3	512	15	3
181	146	13	3	281	14	3	246	15	3
182	233	14	3	335	14	3	343	16	3
183	174	14	3	185	15	3	284	17	3
184	-26	13	3	-13	14	3	-15	17	3
185	187	13	3	175	14	3	283	14	3
186	84	14	3	138	15	3	102	17	3
187	36	14	3	-18	15	3	5	18	3
188	208	13	3	291	14	3	302	16	3
189	117	13	3	124	14	3	193	16	3
190	72	13	3	72	15	3	141	17	3
191	164	18	2	230	17	3	334	18	3
192	-46	16	3	-70	17	3	-		
193	84	15	3	7	16	3	192	16	3
194	14	15	3	-14	16	3	21	16	3
195	81	14	3	146	16	3	131	17	3
196	5	14	3	-66	15	3	-36	16	3
197	-10	15	3	6	16	3	39	16	3
198	18	15	3	3	16	3	-22	16	3
199	18	14	3	49	16	3	44	20	3
200	-20	14	3	5	16	3	-42	24	2
201	13	13	3	82	14	3	57	16	3
202	11	13	3	52	14	3	45	19	2
203	54	15	3	146	15	3	132	17	3
204	138	13	3	227	13	3	204	15	3
205	66	13	3	80	13	3	64	15	3

N2	88 TAU			ω TAU			σ' TAU		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
206	2	16	2	111	14	3	89	15	3
207	109	13	3	149	14	3	238	16	3
208	77	12	3	117	13	3	113	15	3
209	25	12	3	33	13	3	1	15	3
210	11	13	3	36	14	3	46	16	3
211	37	14	3	102	15	3	73	18	3
212	4	12	3	109	14	3	67	17	3
213	20	13	3	110	15	3	114	19	3
214	-1	14	3	52	14	3	25	17	3
215	30	14	3	152	14	3	107	14	3
216	50	13	3	175	13	3	181	15	3
217	-5	12	3	42	13	3	28	16	3
218	94	13	3	267	13	3	172	16	3
219	23	12	3	19	13	3	-28	15	3
220	88	13	3	39	14	3	47	17	3
221	46	13	3	84	14	3	73	16	3
222	43	12	3	148	13	3	158	15	3
223	7	13	3	52	13	3	39	14	3
224	7	13	3	38	14	3	84	15	3
225	-16	12	3	2	13	3	59	15	3
226	-13	12	3	-11	13	3	18	15	3
227	8	12	3	41	13	3	14	15	3
228	23	12	3	73	13	3	66	15	3
229	122	15	3	86	21	3	177	25	3
230	12	14	3	83	20	3	79	30	2
231	48	14	3	-56	17	3	-16	20	3
232	39	14	3	42	15	3	39	16	3
233	1	13	3	3	14	3	0	16	3
234	73	12	3	144	13	3	139	15	3
235	32	12	3	34	13	3	18	14	3
236	24	12	3	80	13	3	29	14	3
237	-7	12	3	50	13	3	61	15	3
238	36	13	3	95	14	3	59	16	3
239	85	14	3	149	14	3	161	16	3

N2	88 TAU E	ΔE	N	ω TAU E	ΔE	N	σ' TAU E	ΔE	N
240	-28	14	3	34	13	3	25	14	3
241	-42	13	3	12	28	1	-3	17	3
242	47	13	3	146	14	3	127	17	3
243	-0	13	3	43	14	3	18	16	3
244	3	13	3	92	16	2	36	19	2
245	10	13	3	46	13	3	-10	14	3
246	37	13	3	90	14	3	86	15	3
247	-17	12	3	43	13	3	32	14	3
248	12	13	3	72	13	3	57	14	3
249	48	13	3	140	14	3	121	15	3
250	62	13	3	144	18	2	122	15	3
251	38	13	3	129	13	3	139	13	3
252	49	13	3	68	13	3	8	17	2
253	18	13	3	-4	14	3	10	14	3
254	25	12	3	29	13	3	36	14	3
255	69	12	3	65	13	3	57	15	3
256	67	12	3	109	13	3	71	16	3
257	87	12	3	168	14	3	123	16	3
258	25	13	3	140	14	3	76	16	3
259	21	12	3	74	13	3	49	15	3
260	25	12	3	32	12	3	15	14	3
261	55	13	3	117	13	3	109	14	3
262	-9	13	3	34	13	3	60	17	2
263	4	14	3	36	13	3	59	18	2
264	22	14	3	101	13	3	129	17	2
265	11	13	3	71	13	3	112	18	2
266	4	12	3	1	13	3	42	14	3
267	116	12	3	242	13	3	263	14	3
268	36	13	3	83	13	3	50	15	3
269	11	13	3	81	13	3	46	14	3
270	3	13	3	59	14	3	39	16	3
271	21	13	3	112	14	3	129	17	3
272	21	13	3	117	14	3	66	17	3
273	45	13	3	3	14	3	32	17	3

N2	88 TAU E	88 TAU ΔE	88 TAU N	ω TAU E	ω TAU ΔE	ω TAU N	σ' TAU E	σ' TAU ΔE	σ' TAU N
274	124	17	3	94	19	3	160	22	3
275									
276	NO MEASURES DUE								
277									
278	TO BROAD WINGS								
279									
280	OF H β.								
281									
282									
283									
284	99	14	3	156	14	3	197	17	3
285	88	14	3	98	14	3	87	17	3
286	-25	14	3	17	14	3	-7	17	3
287	123	14	3	57	15	3	79	18	3
288	-56	19	2	12	16	3	36	18	3
289	-12	15	3	2	15	3	7	16	3
290	149	15	3	231	16	3	204	18	3
291	40	14	3	-8	15	3	7	18	3
292	80	14	3	189	14	3	138	17	3
293	12	14	3	68	18	2	79	18	3
294	2	14	3	49	15	3	43	37	1
295	-29	14	3	-55	15	3	9	18	3
296	83	14	3	142	16	3	163	18	3
297	40	14	3	34	15	3	48	17	3
298	99	14	3	192	15	3	181	16	3
299	142	15	3	197	16	3	210	17	3
300	202	16	3	175	18	3	278	22	3
301	2	16	3	88	18	3	5	22	3
302	4	16	3	19	19	3	18	21	3
303	27	17	3	27	19	3	46	20	3
304	11	18	3	-37	20	3	38	19	3
305	180	19	3	238	20	3	256	20	3
306	-15	16	3	14	18	3	34	19	3
307	19	16	3	-11	20	2	78	20	2

N2	88 TAU			ω TAU			σ' TAU		
	E	ΔE	N	E	ΔE	N	E	ΔE	N
308	32	17	3	134	19	3	159	24	2
309	-24	16	3	-31	18	3	52	22	3
310	-17	16	3	22	17	3	82	19	3
311	35	17	3	37	18	3	43	19	3
312	10	17	3	6	18	3	72	20	3
313	205	16	3	331	17	3	324	19	3
314	63	18	3	110	19	3	-10	25	2
315	33	20	3	134	19	3	34	20	3
316	69	35	1	72	19	3	33	21	3
317	64	19	3	131	19	3	-5	20	3
318	67	19	3	75	20	3	-3	22	3
319	12	18	3	25	20	3	2	22	3
320	47	20	3	-9	25	2	27	24	3
321	91	21	3	214	23	3	125	29	3
322	117	19	3	241	24	2	89	29	2
323	115	20	3	154	22	3	81	27	3
324	37	21	3	68	23	3	24	28	3
325	20	23	3	110	24	3	59	28	3
326	64	22	3	90	25	3	108	29	3
327	24	22	3	-30	24	3	-2	29	3
328	73	21	3	-38	28	2	74	26	3
329	108	22	3	73	24	3	74	26	3
330	19	22	3	154	25	3	184	27	3
331	78	24	2	220	30	2	15	32	2
332	33	24	3	68	27	3	-24	30	3
333	64	25	3	19	28	3	-2	32	3
334	50	30	2	161	28	3	315	31	3
335	33	26	3	76	28	3	70	30	3
336	44	27	3	95	30	3	132	32	3
337	11	28	3	2	31	3	18	33	3
338	52	28	3	156	31	3	131	35	3
339	-29	31	3	38	33	3	63	36	3
340	79	32	3	158	33	3	218	35	3
341	-			-			-		

Notes on Table 3.2

(1) If a stellar spectrum was sufficiently different from the other spectra of that star to be of use, it was rejected. This happened when the value of β required for consistency among the spectra had to be altered by more than a fixed amount ($\sim$ 0.5 steps). The following three spectra were thus rejected.

ω TAU (Plate 14), 15 UMa (3) and τ UMa(6)

15 UMa (3) had easily the highest photographic density in the batch which would account for its rejection but the reasons for the other two were not obvious.

(2) In regions of high line density the information content of the results is not high. This applies to the range 4150 Å - 4250 Å and to various large blends at higher wavelengths. It also applies to these regions in the three broad-lined stars where, although the errors are fairly small, it must be remembered that one result may be the sum of the equivalent widths for several lines.

(3) A number of lines occur where the equivalent width, being negative, is a factor of two or three times greater in magnitude than the error. The significance of these will be discussed in 6.1. This section will also feature a comparison of these results with previous results.

(4) Some degree of inconsistency occurs between different spectra of the same star in the determination of equivalent widths of closely blended lines. The sums across the blend are consistent but not the individual equivalent widths, the disparity being greater than the predicted errors would suggest. The reason for this is that errors occur in the input wavelengths of the lines and a small positional differential arises when the dispersion curves are fitted. The exact effect of this cannot properly be estimated and it is not included in the standard error.

3.4 Residual Lines

The procedure for finding residual lines in the 'filled-in' spectrum (see 2.10.) was adapted to find emission as well as absorption lines. With the critical level fixed at 5.0 times the expected noise, a surprising number of lines were found. The reason for this, however, was obvious when the positions and strengths of these lines were studied.

Apart from the odd individual lines which occurred with a frequency of about one every three or four spectra, it was found that these lines occurred in groups in places where there were large blends. Very often they occurred in pairs or in threes where the algebraic sum of their line strengths was close to zero. This is explained when one considers the large errors involved in calculating the differences in equivalent widths in close blends. Often the difference found far exceeded the sum and yet was within the expected error. Thus one line was given a very large equivalent width while the other had a large negative value. Both of these would show in the running of this program. This effect disappeared to a large extent in the broad-lined stars where blends were eliminated by the grouping of lines.

Most of the individual lines found also originated in blends or in a region of the spectrum where the photographic density was weak.

CHAPTER 4

CURVE OF GROWTH ANALYSIS

4.1 Direct Curve of Growth

Various direct curve of growth methods are available, each making different assumptions. The adopted method, which has been used previously by MICZAIKA et al. (1956) for application to two Am stars, is based on a Milne-Eddington model and uses tables computed by WRUBEL (1949), from an exact solution for the residual intensity obtained by CHANDRASEKHAR (1947). The basic assumptions of such a model are that the ratio of continuous to total absorption is constant with respect to optical depth and that the Planck function $B_\nu(T)$ at frequency ν and optical depth τ_ν can be represented in the form $B_\nu(T) = B^{(0)} + B^{(1)}\tau_\nu$
where $B^{(0)}$ and $B^{(1)}$ are constants.

Wrubel computes $\log_{10} W/b'$ as a function of $\log_{10} \eta_o$ where b' is the Doppler width in frequency units and $\eta_o = \sigma_{\nu_o}/\kappa_{\nu_o}$, the ratio of the scattering coefficient to the continuous absorption coefficient, both obtained at the centre of the line. In fact

$$b' = V\lambda/c'$$

$$\text{and} \quad \eta_o = \{N_r \sqrt{\pi}\,\varepsilon^2/\rho\kappa_{\nu_o} m c' U(T) V\} gf\lambda e^{-\chi_i/kT_{exc.}}$$

where m and ε are the mass and charge of the electron, ρ is the mass density in the atmosphere, U(T) is the partition function of the element in question, and the other symbols are defined either in the introduction or in the above text.

Wrubel lists curves for various values of $B^{(0)}/B^{(1)}$ and for different values of the damping. Because of the spectral range involved it was decided to adopt the value 1/3 for $B^{(0)}/B^{(1)}$. The damping was determined from the running of the program (see 4.5).

To apply this method we require $\log_{10}W/\lambda$ and $\log_{10}gf\lambda$ for each line used. The first term was calculated from the computed equivalent width and the line list wavelength while for the second term tables of gf values were consulted. The tables used in the calculations were as follows:

CORLISS and TECH (1968) for FeI lines;
WARNER (1967) for FeII, TiII and CrII;
CORLISS and BOZMAN (1962) for all other ions.

If we now look at equation 6 of the introduction

$$y_i - b = f_t(x_i + a - c\chi_i)$$

we see that if the equation corresponds to a Wrubel curve and the points x_i and y_i to $\log_{10} gf\lambda$ and $\log_{10} W/\lambda$ respectively, we obtain a, b and c as stated i.e.

$$a = \log_{10} N_r k_o ; \quad b = \log_{10} V/c' ; \quad c = \theta_{exc} ;$$

where $k_o = \sqrt{\pi}\, \epsilon^2 / \rho K_{\nu_o} m c' U(T) V$.

It should be pointed out at this stage that since gf values are found from experiment, errors occur in the abscissa as well as in the ordinate. Because of this, methods other than an ordinary least squares fit were considered, e.g. one method minimised the sum of the squares of the perpendicular distances of the points from the curve. These were found to consume too much computer time and did not alter the results significantly. A least squares fit was therefore used.

If ε_i is the error in the i^{th} ordinate, then we are minimising $\sum_i w_i \varepsilon_i^2$ where w_i is the weight given to each point. w_i was chosen as W_i^2 / σ_i^2 where σ_i is the standard deviation of the equivalent width of the i^{th} line and W_i is, in this case, the equivalent width predicted by the gf value and the theoretical curve.

The lines to be used for any given ion are specified to the program by their numbers in the line list so that, in effect, we can choose which lines we wish. The program rejects any negative or zero equivalent widths so that all the required logarithmic values exist. Blended lines and others for which no equivalent widths were found are also ignored.

From the three constants a, b and c, found for each ion we can determine the following:-

(a) relative abundance of the ions, found directly from a;

(b) microturbulent velocity at the level of the stellar atmosphere at which the lines are formed, obtained directly from b;

(c) excitation temperature of the atmosphere, found directly from c;

(d) electron pressure in the atmosphere, found by applying Saha's equation to the relative abundance of the neutral and singly ionised states of an element.

A more detailed account of the determination of these factors together with the values obtained will be given in Chapter 5.

4.2 Differential Curve of Growth

The differential curve of growth is also based on equation 6 of the introduction. If we consider any line, we can write the equation as

$$\log W/\lambda = \log V/c' + f(\eta) \quad \text{— Equ}^{n}\ 1.$$

where f is the functional form of the curve of growth and

$$\eta = \text{constant} + a - \chi\theta$$

Differencing for the two stars gives

$$\delta \log W/\lambda = \log W_1/\lambda - \log W_2/\lambda = \delta \log V/c' + {}^{df}/_{d\eta}\,\delta\eta$$

$$\text{where } \delta\eta = \delta a - \chi\delta\theta,$$

so that the complete equation is

$$\delta \log W/\lambda = \delta \log V/c' + {}^{df}/_{d\eta}\,\delta a - \chi\, df/d\eta\,\delta\theta \quad \text{— Equ}^{n}\ 2.$$

In order to find the ${}^{df}/_{d\eta}$ factor we input a mean value for log V/c', find the mean log W/λ, and, using inverse interpolation in equation 1, obtain a value of η at which we evaluate the slope.

An initial value for a is also input but is used only for evaluating the weights.

Also an initial θ is input for each star which is used to evaluate the weights, and otherwise only to set $\delta\theta$ if this is not solved for.

Since equation 2 can be formed for each line, we can solve by least squares for any or all of the three parameter differentials.

$$\Delta a = \delta a; \quad \Delta b = \delta \log V/c'; \quad \Delta c = \delta\theta.$$

4.3 Statistical Limitations

Apart from the limitations of the curve of growth method due to the various inherent assumptions and approximations, there is also a statistical problem. From a distribution of points with possible errors in both co-ordinates, we are trying to obtain three parameters, which to a large extent are dependent on each other.

The error of the parameters and their correlations can be calculated from the output of both programs. It was found, using our data, that these errors and correlations were large when solving for three parameters. While this is mostly the fault of the errors, it is also due to a fundamental fault of the curve of growth in that the variables are inherently highly correlated. It would require data with extremely high signal to noise ratio to allow all three variables to be derived with good significance.

However, when just two parameters were solved for, the third being given, it was found that for FeI, FeII and TiII the errors and correlations were reasonable. The above applies to the direct curve of growth. For the differential curve it was found to be possible to solve, in the least square sense, for one parameter only.

The virtual breakdown of the least squares solution means that the parameters can vary over a wide range and still give an equally valid fit. They may not vary at random, of course, but must be related by the correlation coefficient. Consequently it is necessary to bring in other considerations to choose which solution one likes best. We have chosen to fix the temperature, since this can be deduced from colours.

As a further check on the independence of the two parameters solved for in the cases of FeI, FeII and TiII, we input values for V and solve for the abundance parameter. We thus can examine the effect a change in V will have both on the fit and on the abundances, although effectively we are still solving for two parameters.

4.4 Excitation Temperatures

It is possible to deduce the effective temperature of a main sequence star from its colour. From JOHNSTON (1966) we can obtain effective temperature against B-V and V-I for such stars. Also KOPYLOV (1963) has produced a table of excitation temperature against effective temperature for main sequence stars.

With these tables and the same curve-fitting procedure used in finding the dispersion curve, a polynomial fit was made of (i) θ_{eff} v. B-V ; (ii) θ_{eff} v. V-I ; (iii) θ_{exc} v. θ_{eff}. Using these in conjunction with one another it is possible to use the value of B-V or V-I to obtain θ_{exc} for any main sequence star, assuming that these colours are unaffected by reddening and line blanketing.

If the temperatures derived from this method were to be final, it would not be easy to justify the treatment of an Am star as main sequence in this context. We would expect deviations to occur but these we would hope to detect at some later stage.

The values of B-V used were those quoted in 2.1 while the V-I values were taken from JOHNSON et al. (1966).

Because of possible line blanketing effects it was decided to use V-I whenever possible. In the above reference V-I is measured for all but two of the stars. The values of V-I and B-V for each star along with the excitation temperatures derived are listed in Table 4.1.

4.5 Application of Programs

The lines included for each ion were chosen on the basis that the errors in the equivalent widths were not too high. If, for any star, the error in an equivalent width was greater than three times the lowest error for any line in that star, the line was rejected. This meant that all closely blended lines were excluded. Lines for which no gf value could be found were included but were used only in the differential curve of growth where the gf value is not required.

Shortage of lines in the various ion groups meant that only one parameter, the relative abundance, could be found for all but three elements. The initial runs were carried out using these groups, FeI, FeII and TiII, and all other ions were given the damping, temperature and velocity parameter of that one which they most closely resembled.

Two extra facilities of the program may be mentioned at this point. Firstly, we can fit curves to all five stars either fixing or varying the parameters and then for each line combine the residuals in the five stars to obtain a mean residual. Using a χ^2-test we also obtain the significance of the scatter of the five points about the mean residual. Secondly, an estimate of the error in the temperature of each star is output. This is obtained from a plot of the residual in the abscissa, R, against the excitation potential, χ, by measuring the gradient, α, of a fitted straight line, $R = \alpha\chi + \beta$.

The damping was found for FeI, FeII and TiII by fitting the various curves given by Wrubel to the points with only the temperatures fixed and comparing the residuals. Both the variances for the individual stars and a plot of mean residual against

DERIVED TEMPERATURES

STAR	TEMP (Θ exc)		B-V	V-I
28 And	0.83	*	0.26	-
68 Tau	0.66		0.05	0.10
88 Tau	0.78		0.18	0.29
ω Tau	0.82		0.22	0.37
σ^1 Tau	0.73	*	0.14	-
15 UMa	0.83		0.27	0.38
τ UMa	0.86		0.36	0.45
60 Leo	0.61		0.05	0.04

* Derived from B-V

Otherwise from V-I

As before $\Theta_{exc} = 5040/T_{exc}$ where T_{exc} is the excitation temperature.

Table 4.1

equivalent width were used in determining the best curve. If the variances were high, or if the graph was decidedly asymmetric about the equivalent width axis, the curve was rejected. The best curve in all three cases was the third one listed by Wrubel, which corresponds to a damping of log a' = -1.8, where a' is the ratio of the effective natural line width to the Doppler width. This value was adopted for all five stars.

The program was then run for each of these ions with fixed integral values, in kms/sec., for V, the velocity parameter, ranging from V = 1 to V = 8. The best value of V, to the nearest integer, was obtained by studying the variances.

A study of the individual lines was carried out for this value of V and lines with a consistently high deviation in one direction were thrown out. This would indicate that perhaps their gf values were wrong. Also lines which were very inconsistent from star to star were eliminated. This happened sometimes when a line was close to a cut-off point near a Balmer line.

The process was repeated with the new lists and with a smaller range of values for V, except that this time the best value was obtained to the nearest ½ km/sec. For the purposes of the direct curve of growth this value of the velocity parameter was accepted as final (see Table 4.2).

From the information we obtain about the temperature from these runs we ask if (a) the original temperatures are good enough; and (b) one temperature can be used for all ions in one star.

In order to answer these questions we look at the temperature corrections found by the program for each ion at the appropriate value of the velocity parameter. The error in this correction is extremely large for FeII. This is because the excitation potentials of all but one of the lines are very similar so that the straight-line fit involved depends too much on one point. As the velocity parameters found for FeII are intermediate between the other two, we can assume that if there is a difference in the depth at which the lines are formed giving rise to a significant difference in excitation temperature,

VELOCITY PARAMETER, V,
for FeI, FeII and TiII

STAR	V (in kms/sec)		
	FeI	FeII	TiII
28 And	2½	3½	5
68 Tau	2	3	6
15 UMa	3½	5½	6
τ UMa	3½	4½	6
60 Leo	2	3	6

Estimated statistical error, i.e. the error based purely on the statistics and ignoring the systematic errors due to the assumptions made, is of the order of between ½ and 1 km/sec. The largest errors occur in the cases of 68 Tau (FeI), 68 Tau (TiII) and 60 Leo (TiII) due to an absence of strong lines there.

Table 4.2

this will be apparent in a comparison of FeI and TiII. The corrections predicted for FeII were therefore ignored.

Table 4.3 shows the temperature correction found for each star from FeI and TiII, together with an estimate of its error. Only one of the ten differences quoted exceeds the error in magnitude and this by a factor of less than two. Statistically, therefore, the original estimates are very good and no good reason emerges for altering the temperature for any ion. In short, the method used is at least as good as the curve of growth method for estimating temperatures.

At this stage it was felt necessary to check the values found for the three parameters by running the differential curve of growth program. Since the techniques involved are somewhat different from the direct method (see 4.2), the two programs are sufficiently independent to indicate any discrepancies, should they occur.

The initial values for the program are those found by the direct method using the excitation temperatures derived in 4.4, i.e. the velocity parameters of Table 4.2 and the corresponding abundance parameters which can be found in Table 4.5 along with those of other ions. The lines used are the same as for the direct curve except that lines for which gf values could not be found can now be used. In every case, only one parameter was varied at a time. In Table 4.4 the values of Δa, Δb, and Δc obtained in each comparison are listed with the corresponding values found by the direct method. The differential curve of growth was not used for ions other than FeI, FeII and TiII.

To consider first the parameter Δb, it is apparent that the maximum disparity between the results of the two methods corresponds to a difference in V of less than 1 km/sec. When it is considered that, in the first place, the velocity parameters found were rounded off to the nearest $\frac{1}{2}$ km/sec. and that to obtain Δb_{Direct} we are differencing two of these, this disparity can easily be explained in terms of the errors involved and therefore ignored.

In the case of the temperature differential, Δc, the

	FeI		TiII	
	θ corr	error	θ corr	error
28 And	0.00	0.16	0.04	0.19
68 Tau	-0.06	0.27	-0.07	0.09
15 UMa	0.00	0.17	0.16	0.21
τ UMa	0.05	0.20	0.23	0.24
60 Leo	-0.09	0.20	-0.16	0.10

The temperature correction ($\theta_{corr} = \theta_{colour} - \theta_{C.O.G.}$) to be made to the temperature derived in 4.4, predicted for each star by the curves of growth for FeI and TiII, is listed together with its error. Although a hint of consistency is shown in that the corrections for any star have the same sign in both FeI and TiII, the errors are too large to make this very significant.

Table 4.3

RESULTS OF DIFFERENTIAL

CURVE OF GROWTH

FeI	Δa		Δb		Δc	
	Direct	Diff.	Direct	Diff.	Direct	Diff.
28And-68Tau	0.96	1.15	0.10	0.17	0.17	0.15
-15UMa	-0.08	-0.14	-0.15	-0.15	0.00	0.01
- τ UMa	-0.67	-0.78	-0.15	-0.17	-0.03	0.02
-60Leo	1.03	1.30	0.10	0.17	0.22	0.13
68Tau-15UMa	-1.04	-1.30	-0.25	-0.35	-0.17	-0.10
- τ UMa	-1.63	-1.70	-0.25	-0.28	-0.20	-0.18
-60Leo	0.07	0.14	0.00	0.02	0.05	0.01
15UMa- τ UMa	-0.59	-0.49	0.00	0.04	-0.03	-0.06
-60Leo	1.11	1.35	0.25	0.33	0.22	0.13
τ UMa-60Leo	1.70	1.87	0.25	0.30	0.25	0.18

FeII	Δa		Δb		Δc	
	Direct	Diff.	Direct	Diff.	Direct	Diff.
28And-68Tau	0.58	0.49	0.07	0.07	0.17	0.21
-15UMa	0.06	0.11	-0.18	-0.19	0.00	0.03
- τ UMa	-0.51	-0.47	-0.09	-0.07	-0.03	-0.03
-60Leo	0.57	0.51	0.07	0.05	0.22	0.25
68Tau-15UMa	-0.47	-0.40	-0.26	-0.23	-0.17	-0.19
- τ UMa	-1.09	-0.95	-0.18	-0.13	-0.20	-0.25
-60Leo	-0.01	0.01	0.00	0.00	0.05	0.03
15UMa- τ UMa	-0.62	-0.49	0.08	0.11	-0.03	-0.07
-60Leo	0.46	0.41	0.26	0.23	0.22	0.22
τ UMa-60Leo	1.08	1.06	0.18	0.15	0.25	0.25

Table 4.4

(Continued)

RESULTS OF DIFFERENTIAL

CURVE OF GROWTH

TiII	Δa		Δb		Δc	
	Direct	Diff.	Direct	Diff.	Direct	Diff.
28And-68Tau	0.88	0.81	-0.08	-0.10	0.17	0.18
-15UMa	0.13	0.08	-0.08	-0.10	0.00	0.03
- τUMa	-0.13	-0.12	-0.08	-0.10	-0.03	-0.04
-60Leo	1.04	1.05	-0.08	-0.07	0.22	0.17
68Tau-15UMa	-0.75	-0.75	0.00	-0.01	-0.17	-0.16
- τUMa	-1.00	-0.89	0.00	0.02	-0.20	-0.25
-60Leo	0.16	0.20	0.00	0.02	0.05	0.01
15UMa- τ UMa	-0.25	-0.16	0.00	0.00	-0.03	-0.12
-60Leo	0.91	1.00	0.00	0.00	0.22	0.21
τ UMa-60Leo	1.16	1.18	0.00	0.04	0.25	0.23

Table 4.4
(Continued)

values for FeII are included in the table since the inclusion of several lines with no gf value improved the excitation potential distribution. The results for all three ions are consistent within the expected errors.

So far we have not discussed the errors in the abundance parameter (for discussion see 4.6), so we cannot properly compare the Δ a values. For the moment we will regard them as separate sets of results.

In order to find the abundances of the remaining ions it was necessary to include them, according to the physical properties of their atoms, in one of the following three groups; (i) FeI group; (ii) FeII group; (iii) TiII group; and thus to run the program, fixing all but the abundance parameter accordingly. Because of the scarcity of lines available for these ions, no line was rejected purely on account of its residuals, but otherwise the conditions for the inclusion of a line were as for the other three ions. If an ion had less than four suitable lines, it was felt that no reasonable estimate of any parameter could be made. This reduced the list of ions which could be used to the following: CrI, NiI MnI, CaI, CrII, ZrII, YII and CeII.

The grouping was done with an open mind and not in accordance with any particular theory. Main considerations were ionisation potential, electron shell configuration and mean excitation potential of the appropriate absorption lines.

(i) All neutral elements were grouped with FeI. There was some doubt whether or not CaI should fall into this category, but it was decided to keep the neutral elements together while having reservations about CaI. This is shown in Table 4.5 by the use of brackets round the results for CaI.

(ii) Of the remaining four ions only CrII was grouped with FeII because of their close chemical resemblance.

(iii) The remaining three ZrII, YII and CeII were grouped with TiII.

Hence the complete groupings were as follows:

<u>FeI group</u>:

FeI, CrI, NiI, MnI, (CaI).

FeII group: FeII, CrII.

TiII group: TiII, ZrII, YII, CeII.

The results of the curve-fitting are given in Table 4.5 where the three constants a, b and c are listed for each ion together with the damping. The relative abundances and atmospheric parameters mentioned in 4.1, all of which may be derived from these constants, are calculated in the next chapter where sample curves are also displayed.

4.6 Errors Involved

A difficulty in curve of growth analysis is in estimating the error involved. In 4.3 the problems of curve of growth statistics were discussed and some estimate of these errors can now be made. It is much more difficult to estimate the systematic errors inherent in the actual method chosen, but a discussion of these will be made in 6.2.

We have already assigned an error of between ½ and 1 km/sec. to the values obtained for V and from a consideration of predicted temperature corrections we have seen that the curve of growth method for determining the temperature gives a fairly substantial error of between 0.10 and 0.25 in Θ_{exc}. Although it is expected that temperatures derived from colour will have smaller errors than this, the assumptions made for Am stars make an accurate estimate impossible. For this reason an upper limit of 0.10 will be quoted. It is apparent from the temperature errors quoted for the curve of growth method that the adoption of one excitation temperature for each star is the result of these large errors swamping any changes in excitation temperature from ion to ion.

In order to estimate the error in the abundance parameter, the program was re-run varying the other parameters by fixed amounts about the final values. It was found that an error of ½ km/sec. in V produced on average an error of 0.2 in the abundance parameter, while an error of 0.1 in Θ_{exc} resulted in an error of 0.3. Thus we can put forward 0.7 as the absolute outside limit on this parameter. If, however, we assume a standard error of 2/3 km/sec. in V and of 0.05 in Θ_{exc}, both of which seem reasonable, then the standard error of the abundance parameter

RESULTS OF DIRECT CURVE OF GROWTH (SUMMARY)

Damping in all cases: $\log a' = -1.8$

$c = \Theta_{exc}$

28 And:	0.83
68 Tau:	0.66
15 UMa:	0.83
τ UMa:	0.86
60 Leo:	0.61

$b = \log_{10} V/c'$

	'FeIgroup'	'FeII group'	'TiII group'
28 And:	-5.08	-4.93	-4.78
68 Tau:	-5.18	-5.00	-4.70
15 UMa:	-4.93	-4.74	-4.70
τ UMa:	-4.93	-4.82	-4.70
60 Leo:	-5.18	-5.00	-4.70

$a = \log_{10} N_r k_0$

ion	group	no. of lines	28And	68Tau	15UMa	60Leo	τUMa
FeI	FeI	113*	8.25	7.29	8.33	7.22	8.92
FeII	FeII	23*	9.89	9.31	9.78	9.32	10.40
TiII	TiII	18*	7.43	6.55	7.30	6.39	7.55
CrI	FeI	10	7.04	6.15	6.72	5.91	7.24
CrII	FeII	16	9.56	9.13	9.67	8.93	10.35
NiI	FeI	14	6.91	6.42	7.57	6.29	8.13
MnI	FeI	11	6.66	6.03	6.85	5.77	7.40
ZrII	TiII	4	6.10	5.54	6.32	5.55	6.36
YII	TiII	4	6.23	5.51	6.64	5.62	6.79
CeII	TiII	4	4.33	3.51	4.49	3.78	4.60
(CaI)	(FeI)	6	(7.49)	(5.73)	(6.69)	(6.29)	(7.20)

* number has been reduced by examining residuals

Table 4.5

becomes 0.3.

It is clear from a consideration of these error effects that, while the abundance parameter for any given ion correlates fairly well with the excitation temperature, these are in fact independent effects. This can also be demonstrated by giving all five stars the same temperature and velocity parameter and running the program for the abundances.

It is now possible to compare the results of the direct and differential curves of growth for the abundance differential, Δa. Again the agreement is within the expected error so we can conclude that we have support for the results obtained by the direct method.

CHAPTER 5

CURVE OF GROWTH RESULTS

5.1 Abundances

The abundance parameter, a, is given in 4.1 by $a = \log_{10} N_r k_o$ where $k_o = \sqrt{\pi}\,\varepsilon^2 / \rho K_{\nu_c}\, mc'\, U(T) V$ the symbols having been defined previously. Of these, only ρ, K_{ν_c}, U(T) and V are dependent on star and element, the remainder being known physical constants. We should note here that the dimensionless factor, η_o, is equal to $N_r k_o g f \lambda e^{-\chi_i/kT_{exc}}$. so that $N_r k_o$ has the dimension L^{-1} in the usual notation. (This can be checked using the above expression for k_o). Hence we must use the same units system as used for the gf λ values, namely c.g.s. Substituting the values for the physical constants we obtain $k_o = 0.014978 / \rho K_{\nu_o} U(T) V$

In order to obtain the relative abundances per hydrogen atom, we put $\rho = m_H N_H$ where m_H is the mass of the hydrogen atom and N_H the number of hydrogen atoms per cm^3. Substituting the value for m_H, we obtain

$$\log_{10} \frac{N_r}{N_H} = a - 21.95 + \log_{10} K_{\nu_o} + \log_{10} U(T) + \log_{10} V$$

It remains to evaluate the last three terms of this equation.

$\log_{10} V$ is found for each ion in each star by converting the value for the appropriate group in Table 4.2 to cms/sec. and taking the logarithm.

To estimate $\log_{10} K_{\nu_c}$ we assume that the continuous absorption coefficient is entirely due to the H^- ion and use the table in Astrophysical Quantities (ALLEN (1963)) of $\log_{10} a(H^-)$ as a function of temperature and wavelength. ν_c was defined as the frequency at the centre of the line in question but this we fix for all lines by evaluating $\log_{10} a(H^-)$ and hence $\log_{10} K_{\nu_c}$ at a wavelength of 4500 Å. The temperature of the star is taken as the effective temperature which is calculated from the colour (see 5.2). The interpolation used in the table in both directions was linear. Log K_{ν_c} is calculated from log $a(H^-)$ by correcting for the appropriate electron pressure which can be found as in 5.2 (e) since the K_{ν_c} terms cancel each other.

Another table in Astrophysical Quantities gives $\log_{10} U(T)$

for various ions for $\theta = 0.5$ and $\theta = 1.0$. The temperature in this case is the ionisation temperature also calculated in 5.2 from the colour. Again linear interpolation was used. In the cases of CeII and ZrII, not listed in this table, the values used are given in NBS Monograph 53 (CORLISS and BOZMAN (1962)) where a copper arc ($\theta \sim 1.0$) was used in the determination. In these cases the effect of increasing the temperature to that of the star in question cannot be estimated but since this was small for the other once ionised elements it is ignored.

Table 5.1 lists $\log N_r$ for the various ions ($\log N_H = 12.00$ scale).

5.2 Atmospheric Parameters

Several physical parameters of the atmosphere can be obtained from the constants a, b and c.

(a) Velocity Parameter, V

Since $b = \log_{10} V/c'$, we can obtain V in kms/sec. This in fact was done in 4.5 in obtaining three values of V for each star, V_{FeI}, V_{FeII} and V_{TiII}.

(b) Excitation Temperature, T_{exc}

Since $c = \theta_{exc} = 5040/T_{exc}$, we can calculate T_{exc}.

(c) Effective Temperature, T_{eff}

T_{eff} is not actually obtained from a, b or c, but is found directly from V-I (or B-V, where this is not available). Since the excitation temperatures found from colour were satisfactory and since this involved first calculating T_{eff}, we conclude that the errors in T_{eff} are, at most, of the same order as those in T_{exc}.

(d) Ionisation Temperature, T_{ion}

We calculate θ_{ion} directly from the colour using the empirical relationship $\theta_{ion} = (B-V) + 0.50$ of SARGENT and SEARLE (1964), previously used by HACK (1966) for application to Am stars.

(e) Electron Pressure, P_e

We can calculate P_e, the electron pressure, from the relative abundances of FeI and FeII using Saha's equation

$$\log_{10} \frac{N_{FeII}}{N_{FeI}} P_e = -\theta_{ion} I + 2.5 \log_{10} T_{ion} - 0.48 + \log_{10} \frac{2U_{FeII}(T)}{U_{FeI}(T)}$$

where I is the ionisation potential of FeI and the other symbols are as before.

ABUNDANCES

$Log_{10}N_r$ (scale of $logN_H = 12.00$)

	28 And	68 Tau	15 UMa	τ UMa	60 Leo
FeI	4.97	4.77	4.97	5.02	4.64
FeII	6.89	7.15	6.74	6.72	7.10
TiII	4.74	4.84	4.47	4.16	4.62
CrI	3.42	3.24	3.03	3.03	2.94
CrII	5.86	6.23	5.93	5.98	5.97
NiI	3.61	3.93	4.19	4.19	3.74
MnI	2.79	2.91	2.90	2.91	2.59
ZrII	3.25	3.73	3.32	2.77	3.68
YII	2.96	3.20	3.22	2.81	3.25
CeII	2.09	2.31	2.10	1.60	2.52
CaI	(2.95)	(1.88)	(2.08)	(2.08)	(2.38)

$Log_{10}N_r$ is listed for each ion in each star. N_r is the number of atoms of that ion/cm^3 on a scale of $logN_H = 12.00$. (N_H is the corresponding number density of hydrogen). The results for CaI are in brackets due to an uncertainty in the velocity parameter affecting the abundances. (see 4.5). For the remainder the estimated error is 0.45.

Table 5.1

We can also use CrI and CrII to determine P_e and since the error involved is of the same magnitude we take a straight mean.

(f) Microturbulent Velocity, ξ_t

The velocity parameter, V, is related to the microturbulent velocity, ξ_t, by $V^2 = \frac{2kT}{m} + \xi_t^2$
where in this case, m is the mass of the atom in question. ξ_t may be calculated from this equation but since the effect of the 2kT/m term is small compared with the errors involved, we will simply comment that for V = 2km/sec., ξ_t would be on average about ½km/sec. less than this. The effect of this term becomes reduced as V is increased and is negligible at V = 6km/sec.

The various atmospheric parameters mentioned in this section, apart from ξ_t, are listed for each star in Table 5.2.

5.3 Sample Curves

From the many curves of growth set up by the computer three have been chosen for display in Figs. 5.1, 5.2 and 5.3. One representative from each of the three ions, FeI, FeII and TiII, is shown, a different star being chosen each time. These are not necessarily the curves with the best fit, but are chosen because they show certain characteristics clearly.

For instance, the bottom left part of the graphs show, in general, more scatter about the chosen curve than is found elsewhere. This effect is expected since the errors are not proportional to the equivalent widths and we have taken logarithms. We have allowed for this in the weighting system (see 4.1). In fact where the equivalent width was less than the expected error the point was omitted on the displayed graph, partly because of its very low weight and partly because the inclusion of these points would require a scale change which would mask more important effects. In the graphs shown this applied only to a few points on the FeI curve and did not affect the others at all.

Fig. 5.3, the TiII curve for 68 Tau, is chosen because it exhibits the problems which arise when most of the points lie on the lower part of the curve. Uncertainties in velocity and abundance parameters are introduced although in this case there are just sufficient points round the corner to give some degree of confidence.

ATMOSPHERIC PARAMETERS

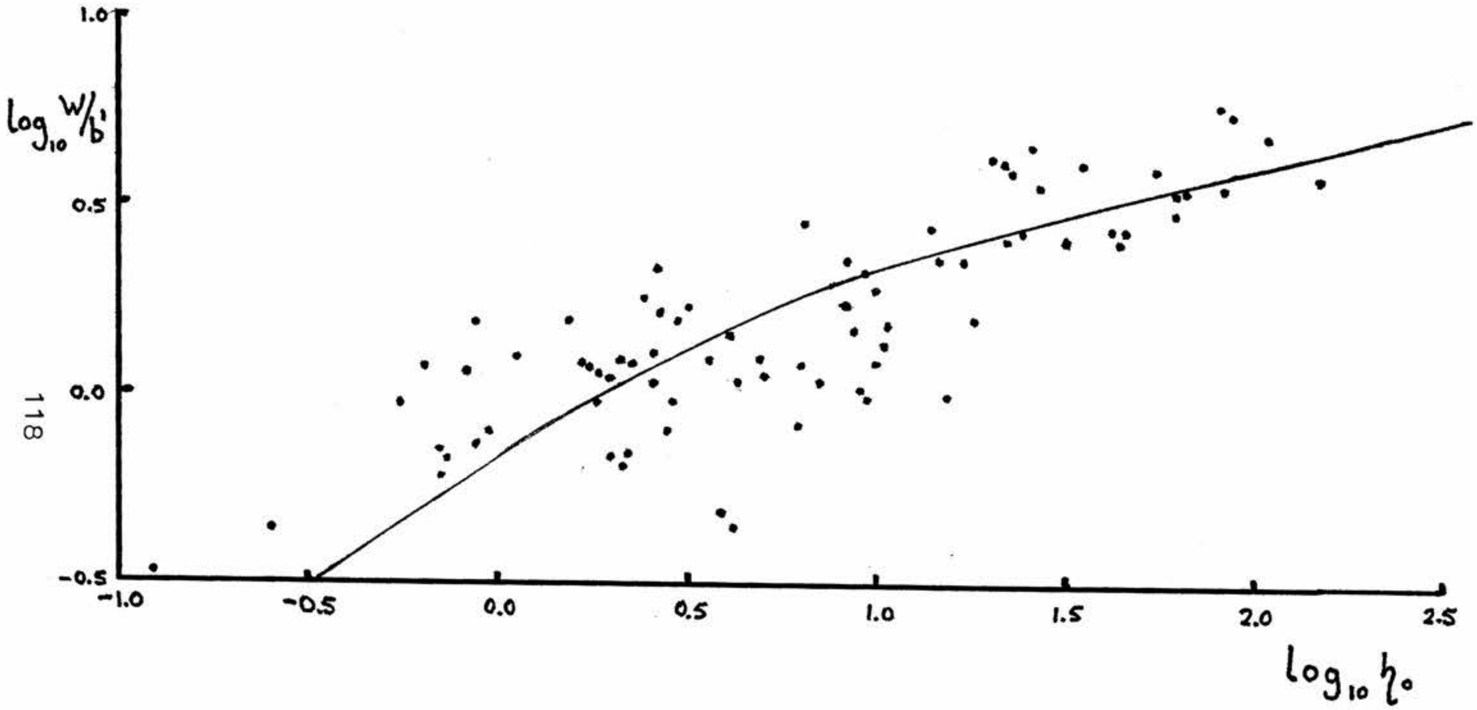
STAR	V_{FeI}	V_{FeII}	V_{TiII}	$T_{exc}(\theta_{exc})$	$T_{eff}(\theta_{eff})$	$T_{ion}(\theta_{ion})$	$\log_{10}P_e(Fe)$	$\log_{10}P_e(Cr)$	$\log_{10}P_e(mean)$
28 And	$2\frac{1}{2}$	$3\frac{1}{2}$	5	6100(0.83)	7400(0.68)	6600(0.76)	1.60	1.56	1.58
68 Tau	2	3	6	7600(0.66)	9000(0.56)	9200(0.55)	3.20	2.85	3.03
15 UMa	$3\frac{1}{2}$	$5\frac{1}{2}$	6	6100(0.83)	7400(0.68)	6500(0.77)	1.65	1.00	1.33
τ UMa	$3\frac{1}{2}$	$4\frac{1}{2}$	6	5900(0.86)	7100(0.71)	5900(0.86)	0.89	0.21	0.55
60 Leo	2	3	6	8300(0.61)	9500(0.53)	9200(0.55)	3.12	2.81	2.97
Average Error	$\frac{1}{2}$	$\frac{1}{2}$	1	500(0.05)	500(0.05)	500(0.05)	0.50	0.50	0.35

Values of the parameters are given for each star together with an estimate of the average error in each parameter. The velocity parameters are in units of kms/sec, the temperatures in degrees absolute and P_e is in dynes/cm^2. The values of $\log_{10}P_e$ derived from Fe and Cr agree within the expected errors.

Table 5.2

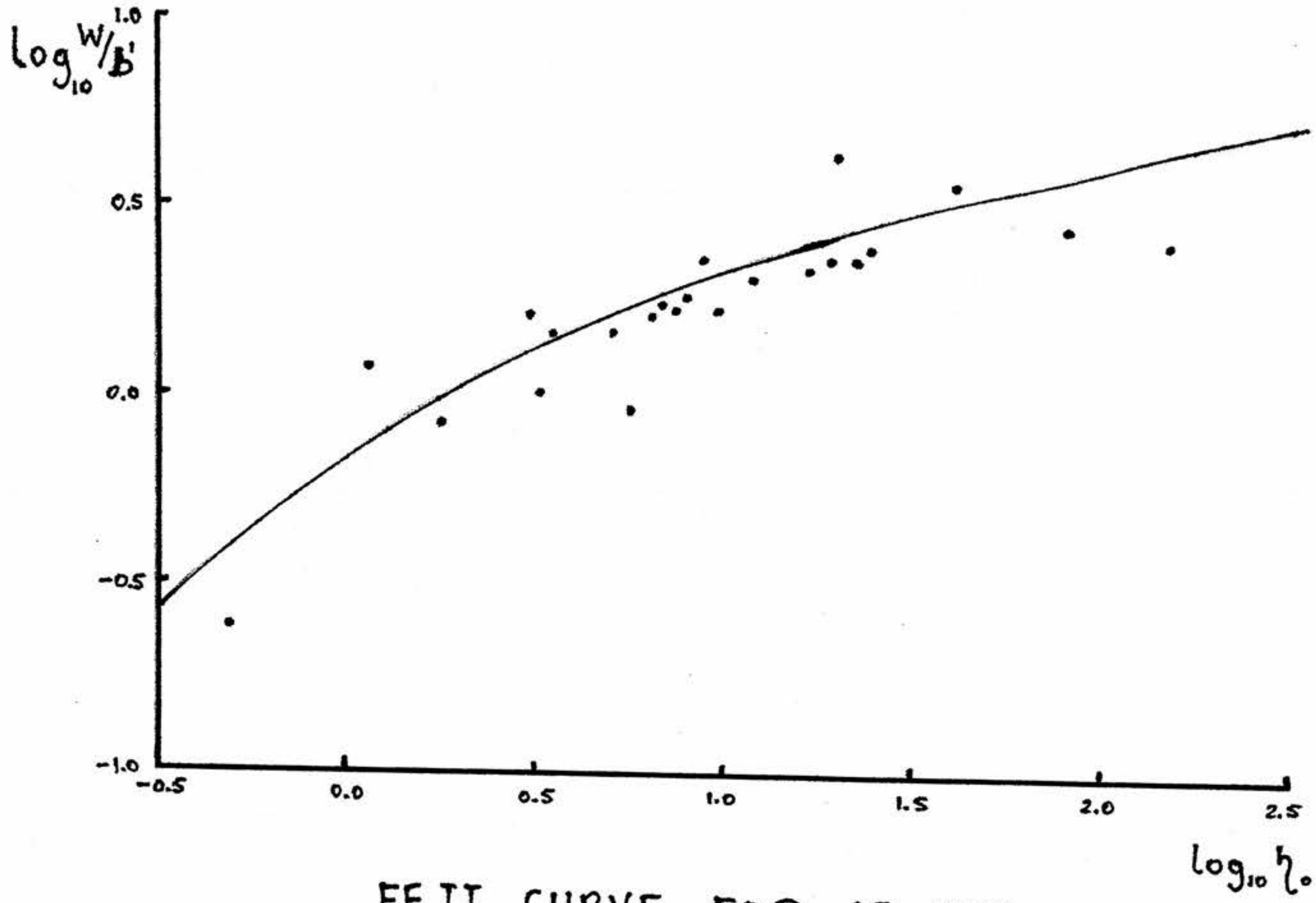
Fig. 5.2, the FeII curve for 15 UMa, demonstrates another difficulty when most of the points lie on the flat part of the curve of growth. Although three or four points lie on the straight line part these have larger errors than the rest which makes abundance determination difficult. Note that the velocity parameter will be largely unaffected but small deviations in this will give errors in the abundance parameter of a greater magnitude than previously predicted (see 4.6).

Fig. 5.1, the FeI curve for 28 And, shows a fair distribution of points between the two sections and fortunately this is the case for most curves. In fact, only 68 Tau (FeI) and 60 Leo (TiII) are similar to the first case and τ UMa (FeII) to the second.



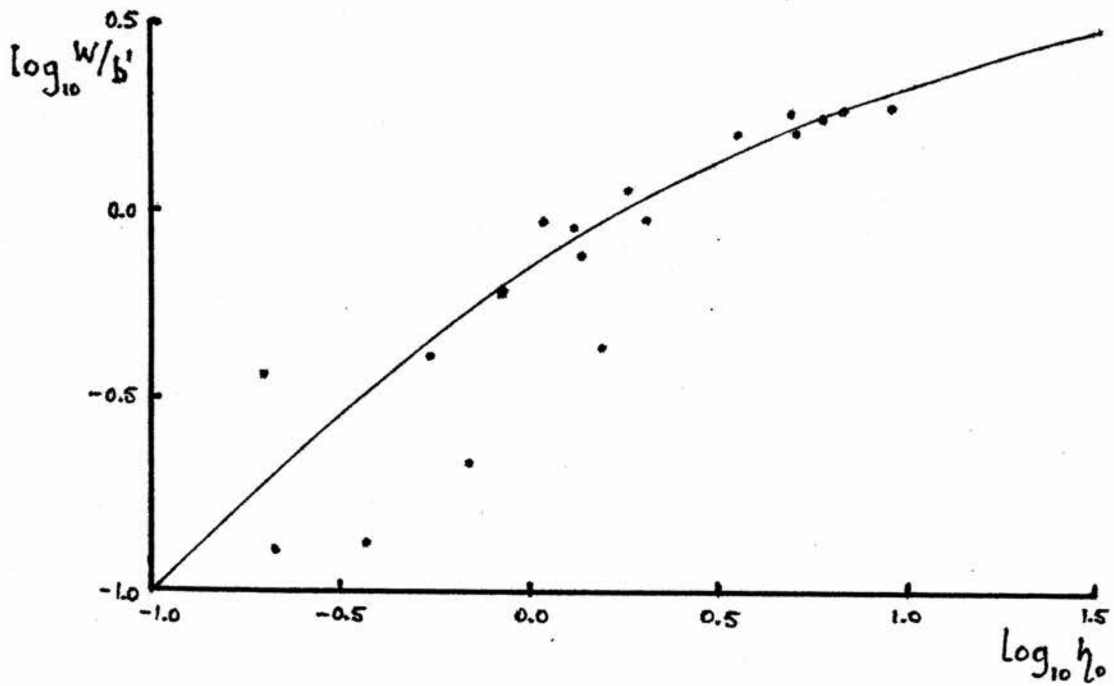
FE I CURVE FOR 28 AND

FIG. 5.1



FE II CURVE FOR 15 UMA

FIG. 5.2



TI II CURVE FOR 68 TAU

FIG. 5.3

CHAPTER 6

DISCUSSION

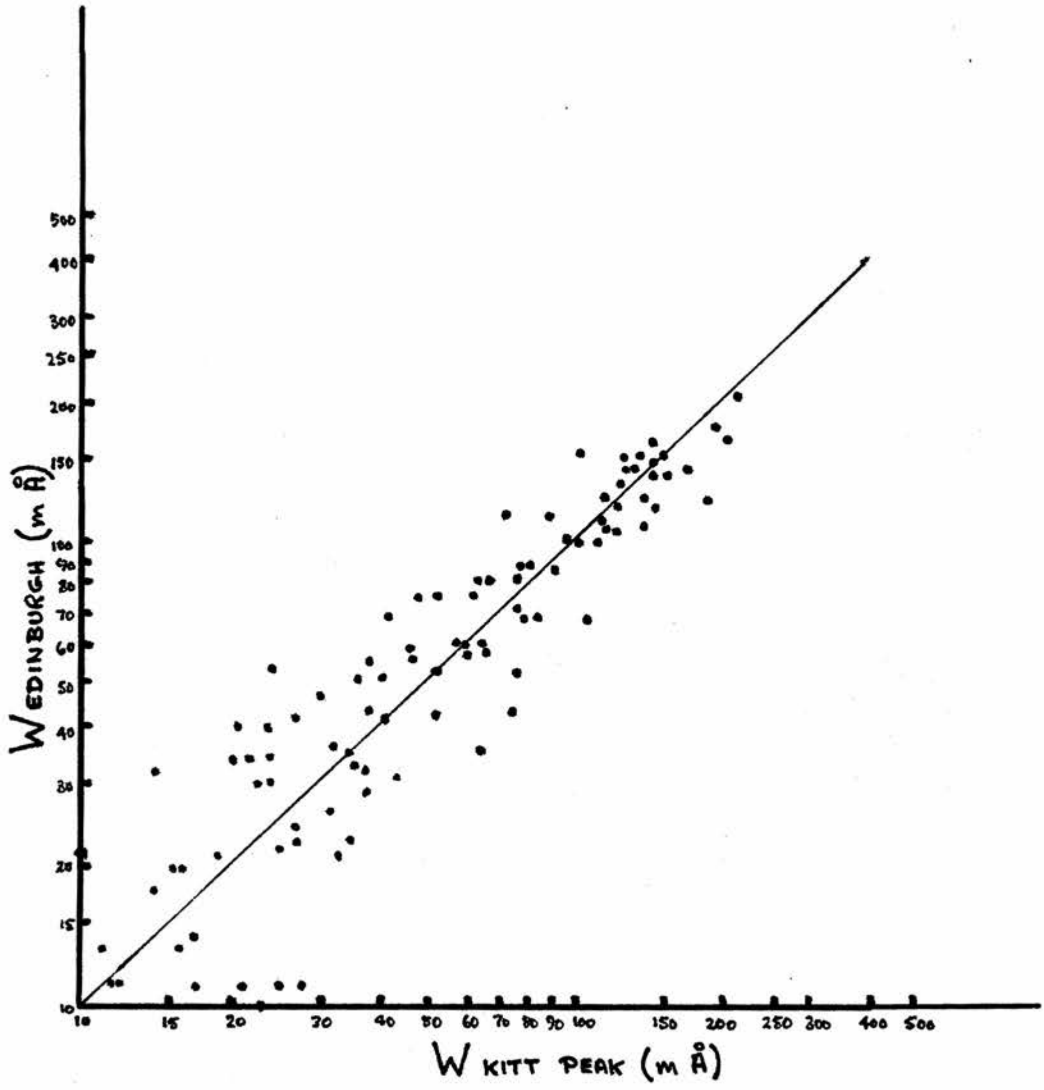
6.1 Evaluation of equivalent width results.

In Fig. 6.1 the equivalent width results for 68 Tau are compared with those of SMITH (1971 a) obtained at the Kitt Peak National Observatory. In the present analysis lines which may be non-existent in a particular stellar spectrum appear in the line list and often will give zero or negative equivalent widths. Smith, on the other hand, omits such lines. Hence, in Fig. 6.1, we ignore all lines of less than 10mÅ in either analysis. The graph is represented on a logarithmic scale in both axes mainly for display purposes as this has the effect of grouping the points. Also, in analyses by hand, one expects the large equivalent widths to have larger errors, which would cause the points to fan out at the top and clarity would be lost. Lines included in the graph have standard errors of 40mÅ or less in the present analysis.

In Fig. 6.2 our results for 68 Tau are further compared with those of WRIGHT et al. (1964) obtained at the Dominion Astrophysical Observatory, Victoria. For comparison purposes, only those lines appearing in the first graph are chosen, the number being reduced considerably since the Dominion wavelength range does not go higher than 4520 Å.

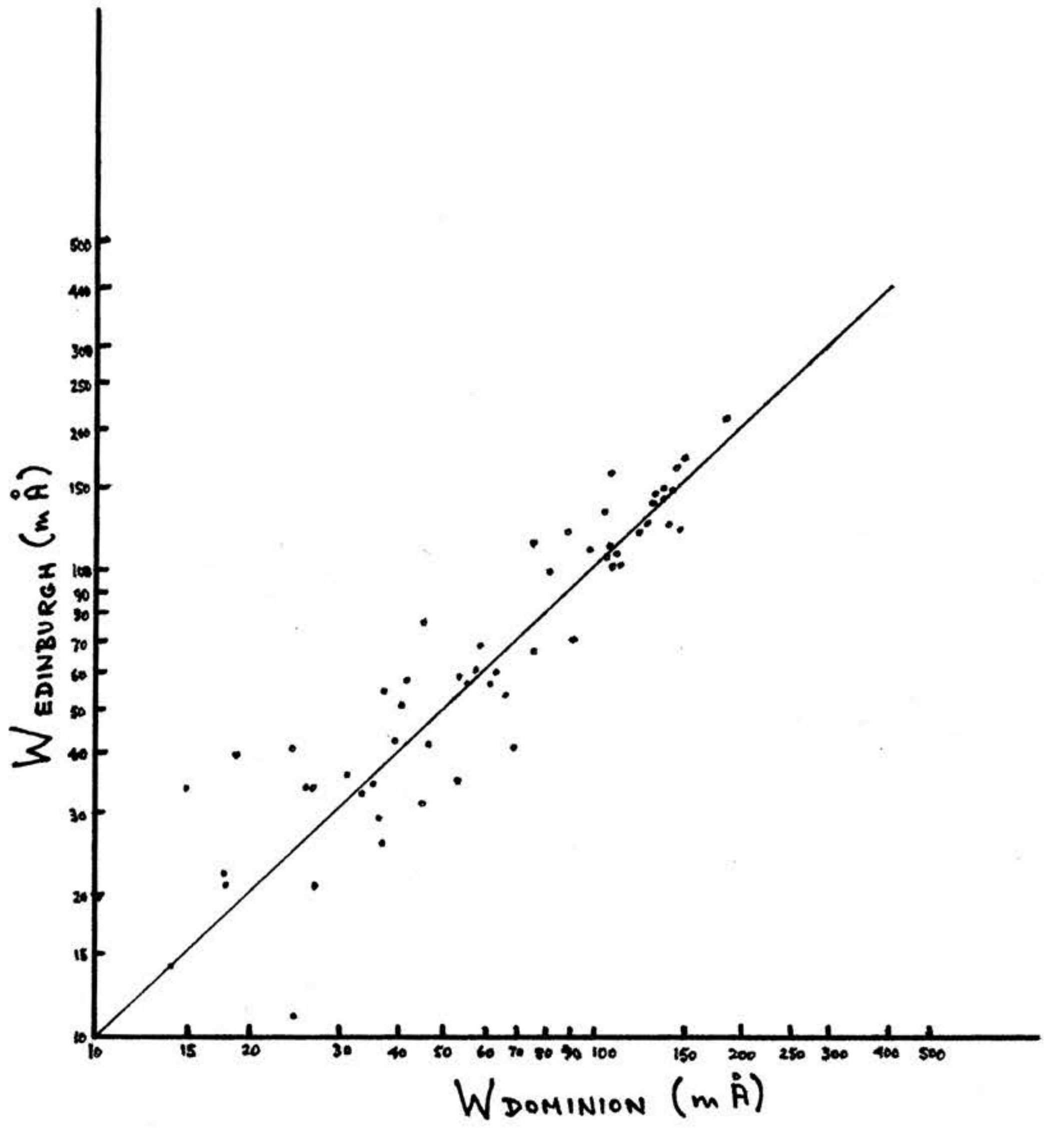
Using the same display and criteria as in Fig. 6.1, our results for 28 And are compared with those of Smith in Fig. 6.3.

The first two graphs show that the results for 68 Tau are reasonably consistent with previous work. The consistency between the Edinburgh results and those of either Kitt Peak or the Dominion Observatory is of the same order as the consistency between the latter two as shown by SMITH (1971 a). The Edinburgh result is intermediate between the other two for about one-third of the lines used as is to be expected. This is excellent agreement when one considers the Edinburgh dispersion of 17.0 Å/mm compared with 8.9 Å/mm for the Kitt Peak spectra and 3.4 Å/mm for the spectra obtained at the Dominion Observatory.



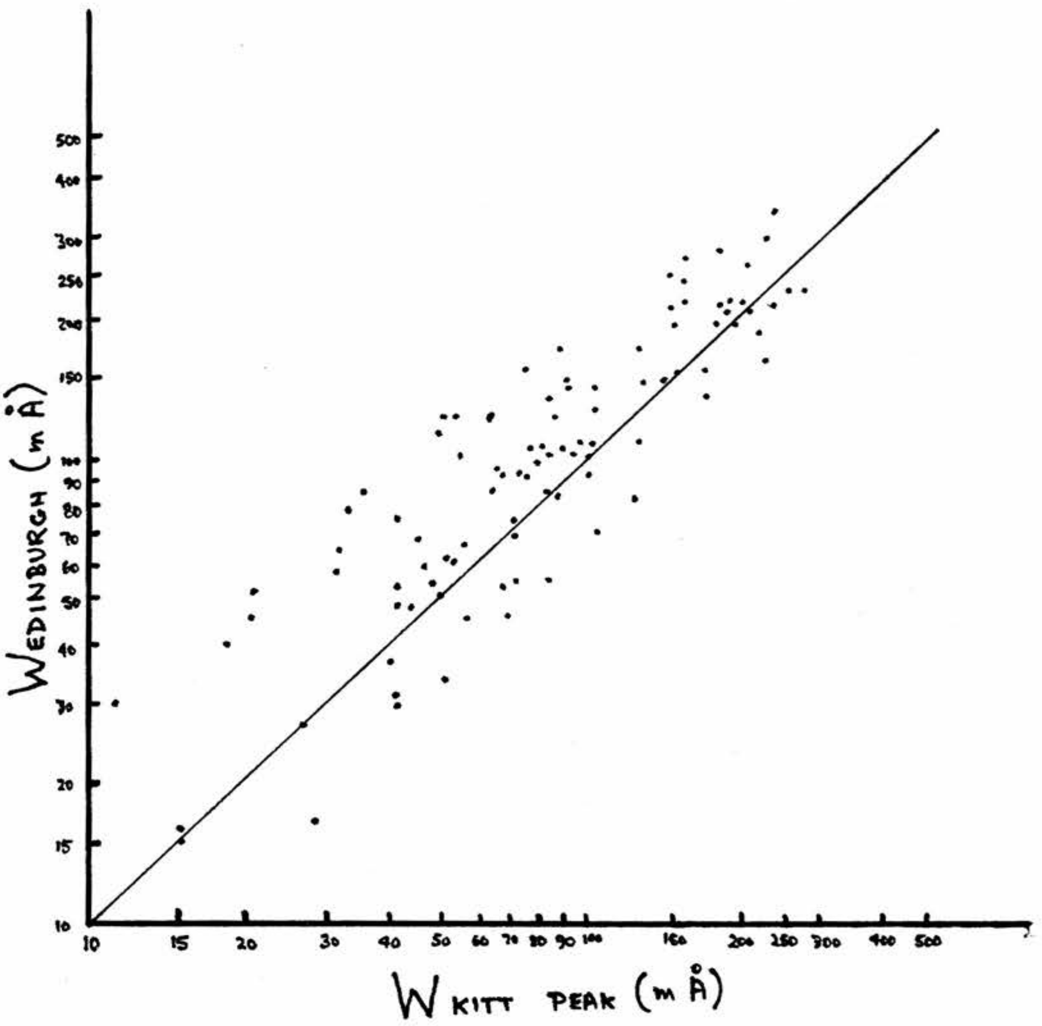
Comparison of results for 68 Tau.

FIG. 6.1



Comparison of results for 68 Tau.

FIG. 6.2



Comparison of results for 28 And.

FIG. 6.3

The consistency between the Edinburgh and Kitt Peak results for 28 And is not quite so good. Although the same general distribution as obtained in the other two graphs is found for most of the points, there are a number of points where the Edinburgh value is significantly larger. Whereas the mean plotted equivalent width for 68 Tau for both Edinburgh and Kitt Peak was identically 69.8mÅ, the 28 And figures show a mean of 118.1 m Å for Edinburgh and 99.9 m Å for Kitt Peak.

The difference in dispersion could possibly have some effect but in view of the results for 68 Tau the answer would appear to lie elsewhere. The lines of 28 And are slightly broader (see Table 3.1) than those of 68 Tau and have, on average, larger equivalent widths. The method used by Smith for both stars, i.e. approximating the lines by triangular profiles, would chop off the wings of such lines and while our program adopts the Doppler profile relevant more to the core of the absorption lines than the wings, there will at least be some contribution from the wings which would account for the larger equivalent widths.

For 16 Ori, an Am star with many strong lines and a high microturbulent velocity (ξ_t = 7.2 kms/sec, according to Smith) which could be indicative of broad lines (see 6.3), Smith obtained a similar effect when he compared his own results with those of CONTI (1965). The method of measurement used by Conti, based on a mean equivalent width-central intensity relationship calibrated by 20-30 lines and direct measurement of all very strong lines, takes more account of wings. On average, Conti's equivalent widths were found to be larger.

The equivalent widths found for the three broad-lined stars cannot properly be compared with past results. In previous analyses by hand of these stars use has been made of the triangular profile and other techniques which tend to ignore broad wings. It is the contention of the author that more sophisticated techniques are required. Whereas the results shown in Table 3.2 for these stars may be to some extent a bit crude and inconveniently presented due to combination of lines etc., it is at least an attempt to solve the large blend problem which exists.

One surprising and perhaps rather disappointing feature of the results was the large number of negative equivalent widths. Of the equivalent widths found 17.1 % were negative in stars for which the first line list was used while 11.7% were negative for the broad-lined stars requiring the second line list.

When these negative results were examined more closely it was found that three stars, 28 And, 68 Tau and 60 Leo, had very similar statistics. On average 13.0 % of the lines in these stars were negative but within one standard deviation. A further 5.8 % were between one and two standard deviations, 1.6 % between two and three and 0.6 % greater than three. This is nearly a Gaussian distribution. The same applies to 15 UMa where the figures were 14.3 %, 5.7 %, 0.9 % and 0.1 %. Hence the majority of negative equivalent widths can be explained by lines being absent or having very low equivalent widths.

This explanation is more feasible than it would at first appear. Each line was chosen on the basis that it would be present in at least one star. Since from previous results certain lines were found to be obviously present in some stars but apparently absent in others it would follow that in some stars there would be a fairly substantial distribution of equivalent widths around zero.

This is not the complete answer as the distribution was not Gaussian for τ UMa whose figures were 3.8 %, 3.8 %, 1.5 % and 1.7 %. Close examination showed that most of the high negatives occur in close blends where the sum of the equivalent widths in the blend is positive. Similar effects have previously been mentioned in note (4) after Table 3.2 and in 3.4. The reason is probably slight discrepancies in the position of the line as predicted by the dispersion curve. As no blended lines were used in the curve of growth analysis,the complementary large positive values do not affect the abundances etc.

Occasionally an undetected blemish on the spectrum, e.g. speck of dust, might occur and this might also produce a negative result.

Hence most, perhaps all, of the negative equivalent widths found for the narrow-lined stars can be explained.

In the case of the three broad-lined stars, 88 Tau, ω Tau and

σ' Tau, the distribution, taken on average, was 5.0 %, 3.5 %, 1.9 % and 1.3 %. As in the case of τ UMa, this cannot be explained entirely by absent lines. It was seen earlier, however, that the r.m.s. deviation for fiduciary lines in the polynomial dispersion curve was higher for these stars so that the position of weak, unblended lines might occasionally be in error. Blended lines would not present the same problem in this case since by treating blends as one line the resulting lines would be broad enough for positional errors to be swamped. A study of the results revealed that of 22 lines giving rise to negative equivalent widths of greater than two standard deviations in at least one star, only one was a combination of blended lines.

6.2 Evaluation of curve of growth results

One of the most interesting aspects of examining the results of the curve of growth program is to use them to determine the reliability of the curve of growth technique itself.

The statistical errors involved have already been discussed in 4.5. These errors of course are not entirely independent of the assumptions made as, for example, ignoring the depth dependence of η will lead to a certain amount of scatter as well as, in all likelihood, a systematic error. However, when errors in equivalent widths and gf values are taken into account along with the general assumptions made for all curves of growth, it becomes apparent that sophistication in curve of growth analysis will not improve results significantly. It is for this reason that a simple curve of growth method was chosen.

If anything, the large errors found using the automated curve of growth have cast even greater doubts on the usefulness of coarse analysis. It is the contention of the author that the general picture is all that one can obtain from a curve of growth analysis and that results obtained are just pointers in this direction.

With this in mind, we must now look at the various correlations and trends in the results to obtain this picture. We first look at the microturbulent velocities found. HACK (1955) has suggested that in Am stars different microturbulent velocities should be computed for neutral and once ionised elements. This view has been largely ignored in recent years. However the results of Table 5.2

indicate that different velocity parameters, and hence microturbulent velocities, should be computed for at least three different ion groups. Not only are the differences found large, but the fact that in all five stars $V_{FeI} < V_{FeII} < V_{TiII}$ indicates that this effect is significant.

HACK (1966) has listed abundances and atmospheric parameters for ten Am stars based on a re-analysis of previous work. We will compare our results with these and also those of SMITH (1971).

Below, microturbulent velocities found by these authors for the stars being investigated are compared with the mean microturbulent velocity found by giving each ion a weight according to the number of lines used.

STAR	$\overline{\xi}_t$	ξ_t(others)	Reference
28 And	2.4	5.0	SMITH
68 Tau	2.2	5.0	SMITH
15 UMa	3.8	-	
τ UMa	3.7	3.8	HACK
60 Leo	2.1	-	

The mean value for τ UMa is in excellent agreement with Hack's value which is based on GREENSTEIN (1948), while the mean values found for 28 And and 68 Tau are somewhat lower than those found by Smith. It is thought that this anomaly is mainly due to the method of determination used in each case as well as the statistical errors therein.

It is interesting to note that other values listed by these authors include

88 Tau, ξ_t = 7.0 (HACK);
ξ_t = 6.0 (SMITH).

ω Tau, ξ_t = 7.0 (HACK).

These are both broad-lined stars of the present investigation and the results support Hack's view that large microturbulence and large macroturbulence are correlated.

The mean microturbulent velocities found for the narrow-lined stars are in good agreement with those found by CHAFFEE (1970) for normal A stars of the same excitation temperature so that these stars at any rate would not appear to have microturbulent velocities significantly higher than normal. SMITH (1971) has also reached this conclusion and has suggested that in the past confusion over what is 'normal' led to microturbulent velocities for Am stars being considered high.

For the ten Am stars listed by Hack $\log_{10}P_e$ ranges from 0.12 to 2.4 with a mean of 1.29. For the five stars in Table 5.2 the range is from 0.55 to 3.03 with a mean of 1.89. Only one star, τ UMa, is common to both and in each case it has the lowest value. Although these results agree within the expected errors it would appear that our values for $\log_{10}P_e$ are consistently larger than those listed by Hack.

Hack has shown by computing two sets of results for ξ Cep that the method of deriving θ_{ion} adopted here would be expected to give larger values for $\log_{10}P_e$ than the method using $(\theta_* - \theta_\odot)_{exc} = (\theta_* - \theta_\odot)_{ion}$ which had previously been used in most cases. This could explain the discrepancies between the two sets of results. We have chosen the B-V method using hindsight since the results of the other method show characteristics of an extended atmosphere which, for reasons given in 1.7, is not in keeping with recent thinking.

The effective temperatures adopted by Smith for 28 And and 68 Tau agree well within the quoted errors with the temperatures adopted here. This is not altogether surprising as both methods were based on colour.

One of the project stars, 28 And (HR114), previously thought to be Am, has been classed as normal F2V by MILTON and CONTI (unpublished) (see 1.8). In fact SMITH (1971) used it as a standard. However, the iron abundance obtained here for 28 And is greater than twice the solar abundance of GOLDBERG et al. (1960) and is comparable with that of the other project stars.

In other respects, e.g. the abundances of the other elements with respect to iron, 28 And would appear to be fairly normal. We will therefore regard it as normal with some Am characteristics.

The iron abundances found are, on average, just over twice that of the sun and this factor ranges from about $1\frac{1}{2}$ to just less than four. This general conclusion is similar to that found by Smith but, compared with his values, iron is overabundant in 28 And by a factor of about four. This could be partly a result of the difference in mean microturbulent velocities but is in the main due to the actual differences in equivalent widths (see Fig. 6.3 and text). As we would expect if this were the case, in a similar comparison for 68 Tau we find an overabundance of iron but by a factor of less than two.

The effective error in the abundance parameter in these comparisons has been reduced due to the similar methods used in obtaining the temperature. In order to minimise this error in the general case it is necessary to calculate the abundances relative to iron and compare with a standard. Due to possible difficulties in the construction of a separate line list for a standard star we did not analyse one as such but, with some reservations, choose 28 And for the reasons given.

Table 6.1 shows the total abundances of the elements calculated using Saha's equation except for Fe and Cr where direct summation was possible. Table 6.2 lists the abundances with respect to iron relative to 28 And. The latter also appears in graph form in Fig. 6.4. As well as minimising the abundance parameter error for the other elements, this method gives some meaning to the results for calcium. Since the same assumptions were made for both Fe and Ca in both stars, their effects tend to cancel out.

The results support what others have shown in the past. Compared with Smith's results Mn and Ni are slightly overabundant, Ca and Ce slightly underabundant, while the other four elements are about the same. All the results for τ UMa are within 0.2 of those listed by Hack apart from Ca, Mn and Zr. Ca is twice as abundant in our results which can possibly be explained by the choice of standard. Mn and Zr are also two and eight times as abundant respectively. In the case of Zr our value is much closer to the values listed by Hack and Smith for other stars.

$Log_{10}N_{El}$ (scale of $log_{10}N_H$ = 12.00)

	28 And	68 Tau	15 UMa	τ UMa	60 Leo
Fe	6.90	7.15	6.75	6.73	7.10
Cr	5.86	6.23	5.93	5.98	5.97
Ni	5.22	5.99	5.96	5.98	5.86
Mn	4.94	5.58	5.21	5.20	5.32
Ti	4.74	4.84	4.47	4.16	4.62
Zr	3.25	3.73	3.32	2.77	3.68
Y	2.96	3.20	3.22	2.81	3.25
Ce	2.09	2.31	2.10	1.60	2.52
Ca	(6.26)	(5.47)	(5.55)	(5.63)	(6.03)

Total abundances for the elements are listed above.

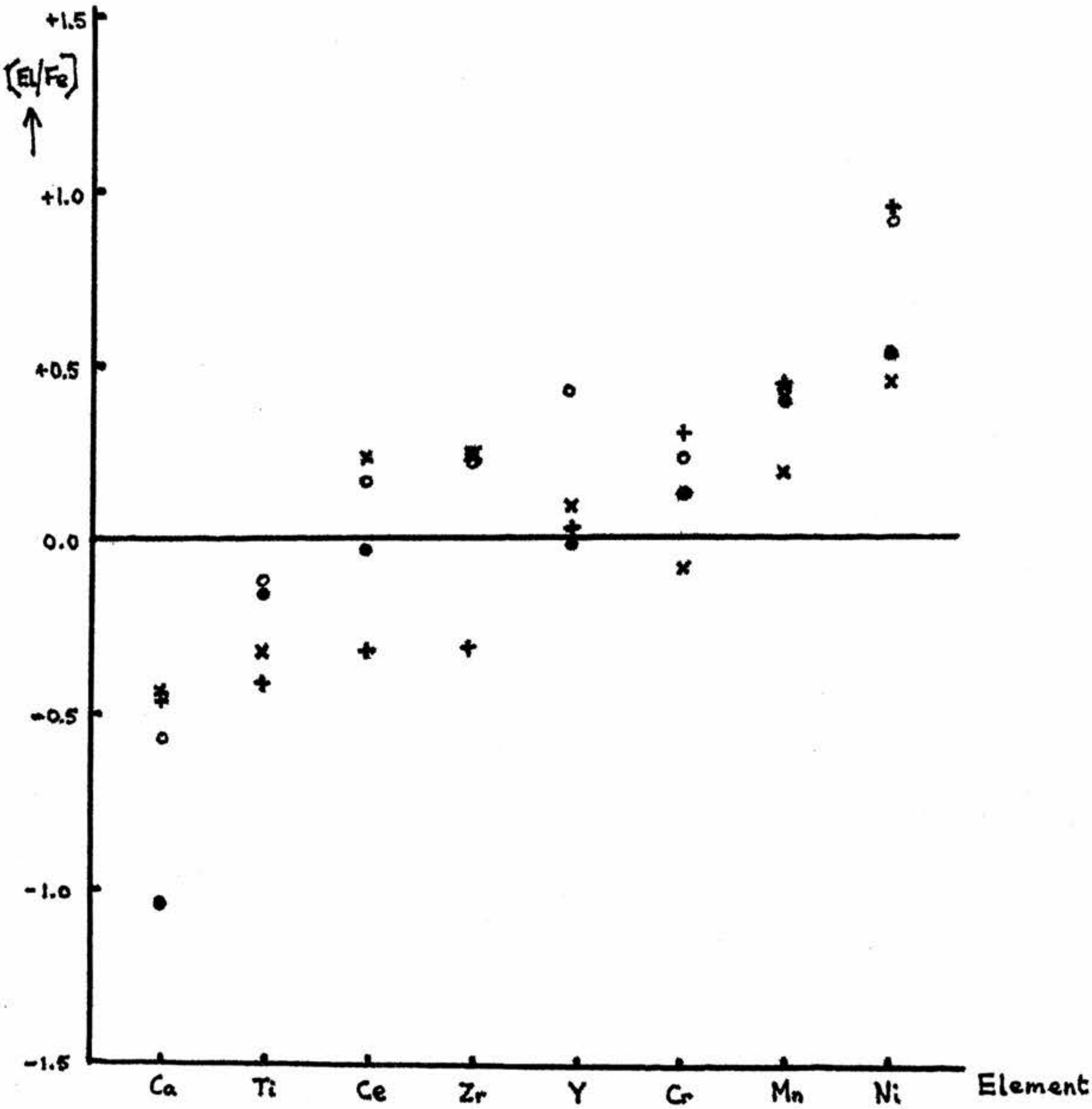
Table 6.1

[El/Fe]

	68 Tau	15 UMa	τ UMa	60 Leo
Cr	0.12	0.22	0.29	-0.09
Ni	0.52	0.89	0.93	0.44
Mn	0.39	0.42	0.43	0.18
Ti	-0.15	-0.12	-0.41	-0.32
Zr	0.23	0.22	-0.31	0.23
Y	-0.01	0.41	0.02	0.09
Ce	-0.03	0.16	-0.32	0.23
Ca	-1.04	-0.56	-0.46	-0.43

Abundances of the elements with respect to iron relative to 28 And are listed above.

Table 6.2



[El/Fe] is the logarithmic abundance ratio of the element with respect to iron relative to 28 And.
The symbols used are:

• 68 Tau   o 15 UMa   + τ UMa   × 60 Leo

FIG. 6.4

6.3 Astrophysical Significance

At the moment the most plausible theory for the origin of the Am phenomenon is that of diffusion in the layer below the outer convection zone. It has been shown by WATSON (1970) and SMITH (1971) how this theory can explain most of the abundance anomalies found in Am stars. STICKLAND and WHELAN (1972), using model stellar envelope calculations, have shown that the range of effective temperatures for which this phenomenon could take place agrees exactly with that range in which Am stars are found. For cooler stars the element separation zone vanishes while for hotter stars another radiative layer has developed in the outer convective zone preventing the metallicism reaching the photosphere.

The theory is therefore an attractive one and the abundances found here for the elements present no further problems. The diffusion velocities, calculated for representative conditions by Watson, for Fe(0.0 cm^{-4}/sec), Ni(2.1), Ti(-0.2) and Ca(-0.6) correlate well with the $\{El/Fe\}$ values displayed in Table 6.2.

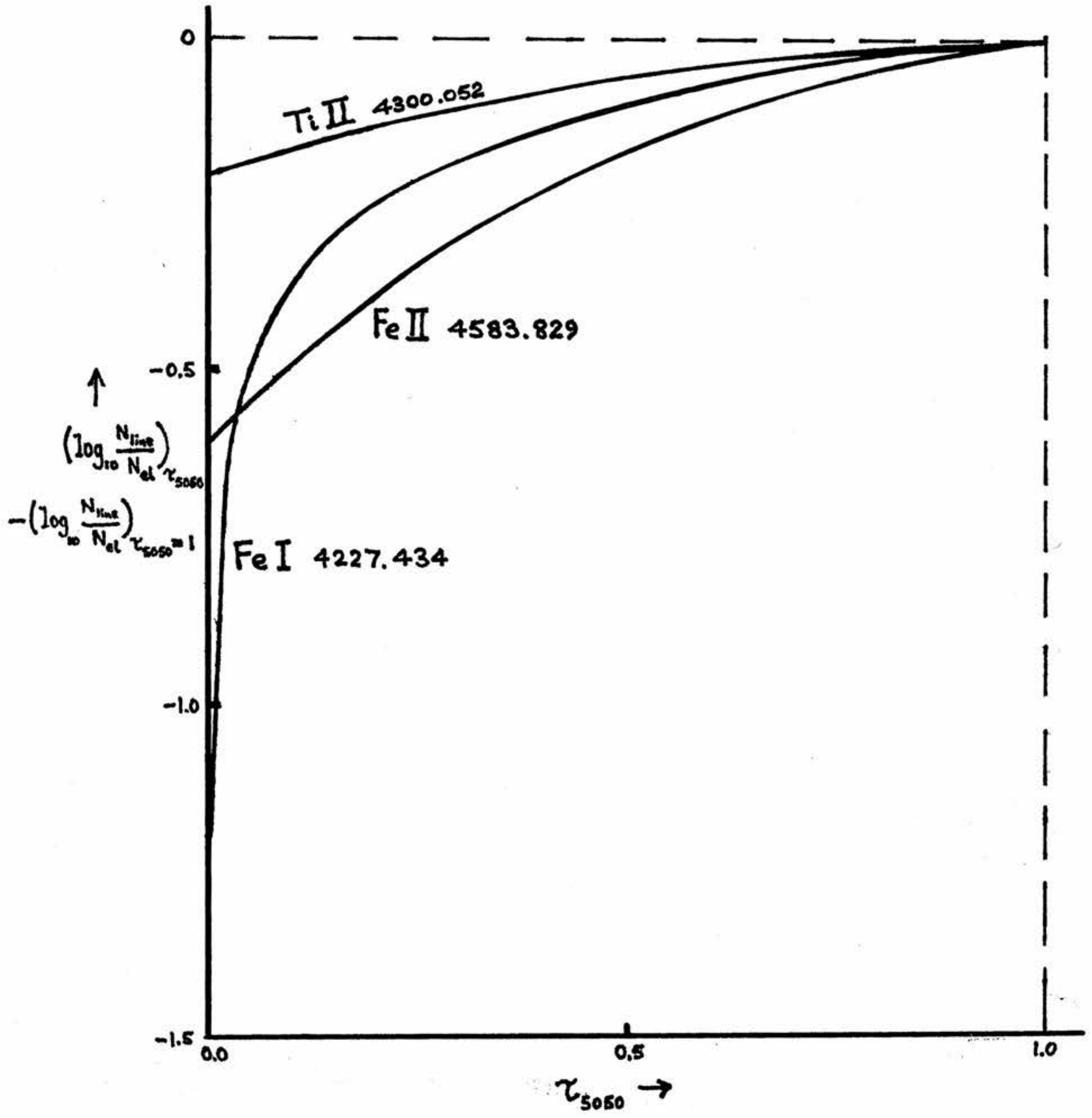
The diffusion process does not operate in normal A stars presumably because circulation velocities dominate diffusion in these stars. This may be due to the effect of the small magnetic fields found in Am stars or else to the small rotational velocities or to both.

In order to investigate if the different microturbulent velocities found for FeI, FeII and TiII is a real effect physically we refer to the model atmosphere analysis of 63 Tau by VAN T'VEERMENNERET (1963) who found different values of microturbulent velocity for neutral and singly ionised elements. Using his non-grey model for 63 Tau, $\log_{10} N_{Line}/N_{el}$, the logarithmic ratio of the number of atoms capable of absorbing the line to the total number of atoms of the element, was computed as a function of τ_{5050}, the monochromatic optical depth at λ 5050Å, for various lines of the three ions. (See e.g. ALLER (1963), p. 119).

In Fig. 6.5 we plot the results for three representative lines, one for each ion. The ordinate of the graph was chosen as

$$(\log_{10} N_{Line}/N_{el})_{\tau_{5050}} - (\log_{10} N_{Line}/N_{el})_{\tau_{5050}=1.0}$$

so that relative effects can be compared.



The logarithmic ratio of the number of atoms capable of absorbing the line indicated to the total number of atoms of the appropriate element relative to the ratio at $\tau_{5050} = 1.0$ is plotted against τ_{5050} for representative lines of Fe I, Fe II and Ti II.

FIG 6.5

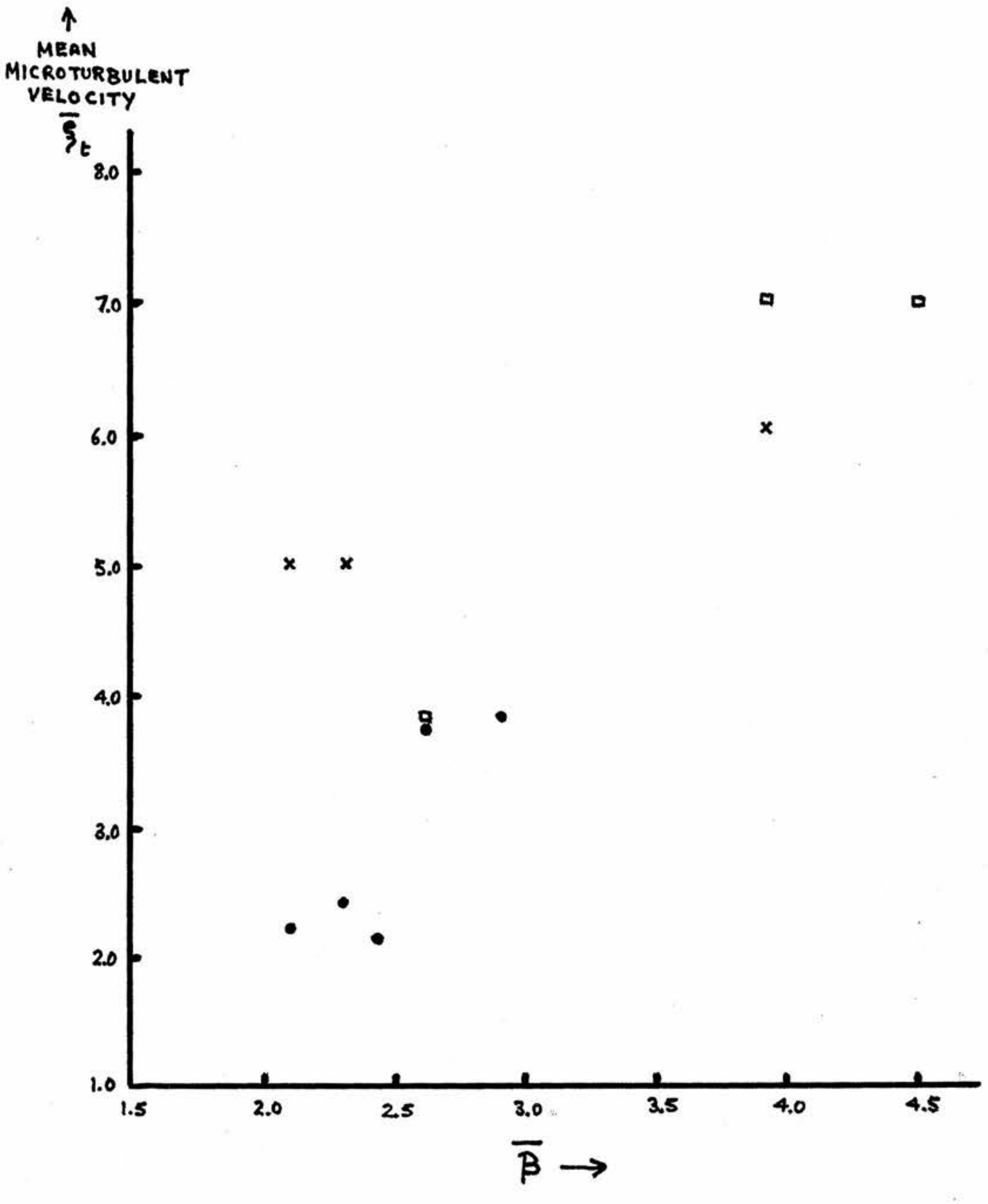
The lines chosen are all fairly strong lines from the flat part of the curve of growth. Since all FeII lines except for one fairly weak line have roughly the same excitation potential the choice of line is not critical. In the case of TiII the chosen line has a similar excitation potential to most strong lines, and indeed to most lines, used in the curve of growth. One or two other lines with different excitation potentials were examined and were found to exhibit an effect somewhere between that of the FeII and TiII lines chosen. Since such lines were rare it was not thought that they would have much effect on the microturbulent velocities obtained. Examination of other strong FeI lines with different excitation potentials gave fairly similar curves to the one displayed. An intermediate example was chosen.

The diagram shows that a possible explanation of the results is that microturbulent velocity increases outwards towards the star's surface particularly in the region above the outer convection zone (we would expect this to set in at about $\tau_{5050} = 0.25$). This would certainly explain the large disparity between the results for FeII and TiII and could possibly also explain the smaller disparity between FeI and FeII.

We note that for all three ions there are both strong and weak lines with similar excitation potentials and that there is no corrèlation between strength of line and wavelength. Hence the result does not appear to be due to a relative shift arising from errors in gf values measured under different conditions. This possibility was examined particularly in the case of TiII lines since this ion has fewest points on the flat part of the curve of growth.

An examination of the excitation potentials of the lines used in the abundance determination of the other ions shows that if a stratification effect is responsible then the allocation of these ions into groups (see 4.5) was done correctly. This includes the case of neutral calcium.

As mentioned in 1.4, Hack has suggested that the lines are broadened mainly by macroturbulence and that macroturbulence and microturbulence are correlated. In Fig. 6.6 $\bar{\xi}_t$, the mean



Mean microturbulent velocity is plotted against $\bar{\beta}$, the mean value of the line width parameter for the project stars. (no value of $\bar{\xi}_t$ was found for σ' Tau.) The symbols mean:

- • present analysis.
- × according to SMITH (1971).
- □ listed by HACK (1966).

FIG. 6.6

microturbulent velocity, is plotted for each star against $\bar{\beta}$, the mean value of the line width parameter listed in Table 3.1. Also included are values of ξ_t listed by HACK (1966) and SMITH (1971). (see also 6.2). From the graph we see that, in general, microturbulent velocity increases with line width. This trend is also noticeable for each set of results.

Of the five stars analysed we have confirmed that four of these τ UMa, 15 UMa, 68 Tau and 60 Leo are Am. Doubts had arisen about the last two due to the different classifications given to 68 Tau in the literature and the position of 60 Leo on the boundary of Henry's diagram. In many ways these two stars are alike. They have iron abundances slightly higher than the others while their [El/Fe] ratios are less extreme than those of τ UMa and 15 UMa. As they also have higher temperatures than the others we conclude that iron abundance and [El/Fe] ratio may be correlated with temperature. The fifth star, 28 And, would appear to be border-line. 28 And and 68 Tau were not included in Henry's diagram.

6.4 Suggestions for future research

For three stars the program failed to function in the normal way. These stars had broader lines than at first imagined and a major reorganisation of the project was necessary in order to obtain something of interest. Two questions arise here. Is it possible to develop the program to deal with such stars? If not, can the results obtained here be used to deduce astrophysical conclusions about these stars?

The former is a fairly deep question both logically and mathematically. Others have obtained results from these stars measuring equivalent widths by hand. However the significance of broad wings would tend to be ignored in hand reduction. Computer reduction on the other hand reveals the tremendous blend problem that occurs in the spectra of such stars. Because of the complexities involved it may be more profitable to look at the second question. Since the data at hand is in a somewhat jumbled form it seems that a model atmosphere approach, or at least one in which equivalent widths are constructed from initial parameters, is necessary.

One disadvantage of the program at the moment is that it can only work at moderate dispersions. This means that in some cases more accurate results may be obtained by others working by hand at higher dispersions. To overcome this it is desirable to develop the program for these dispersions. This would be a major undertaking as the basic assumption that all lines have the same profile could not be made. However it would be an extremely interesting and most worthwhile project.

As pointed out before, the curve of growth merely serves to indicate trends and anomalies in stellar atmospheres. Therefore it is necessary to perform model atmosphere analyses based on the conclusions of a curve of growth analysis. It is hoped that, in the future construction of such, attention will be paid to certain aspects emerging from the present analysis, particularly the various microturbulent velocities obtained from different ions. Also more thought is required as to why this should be the case.

Finally, more research is required on the effect of magnetic fields, circulation and other relevant phenomena before the adopted theory, or any other, can be fully accepted.

6.5 Conclusion

The computer method for determining equivalent widths of stellar absorption lines has been adapted to metallic-line stars with a fair degree of success. The results obtained agree fairly well with those of previous authors and where they differ it is arguable that the computer method is in fact the more accurate. The advantages of removal of subjectivity and reduction in batches are clear despite the problems that arise in line list construction and subsequent blending.

One of the main problems is including lines which are not present in all stars. Although these lines will be obvious from a statistical analysis of the resulting equivalent widths it may introduce a blending problem which does not exist. Also the restriction to medium dispersion spectra is not completely satisfactory. The problem of negative equivalent widths has largely been explained by absent lines and anomalies in close blends which are thought to be due to slight positional discrepancies although this has not been fully understood. The failure of the program to work for broad-lined stars using the general line list was disappointing but seems to be a real problem not just confined to computer reduction.

On the credit side the results in the main are satisfactory and show that the program is sufficiently flexible to be adapted to most types of star. Also, since the line lists have been tested thoroughly, future spectra of Am stars could be reduced fairly quickly.

The deficiencies of the curve of growth technique as a method for analysing equivalent widths were shown by a statistical analysis of the results from the curve of growth computer program. Because of this the temperature parameter was derived from colour and most conclusions were based on subsequent correlations and error statistics. The most important of these was the different velocity parameters, and hence microturbulent velocities, which were found for different ions in the same star. This effect had previously been suggested for different ionisation levels but has been ignored in other recent investigations.

The values found for the mean microturbulent velocity either agreed with or were lower than those of other authors and were of the same order as recent values for normal stars of the same excitation temperature. The values of $\log_{10}P_e$ found were, on average, slightly higher than those listed by Hack for similar stars. This was entirely due to the methods used, the adopted method being considered the more realistic.

Although iron was found to be overabundant in 28 And by a factor of more than two relative to the sun, the abundances of the other elements with respect to iron were normal. For this reason 28 And was chosen as a standard. In general the over-abundance of iron relative to the sun was by a factor of just over two on average, this factor ranging from $1\frac{1}{2}$ to just less than four. The abundances of the other elements with respect to iron relative to our standard gave similar results to those found in the past.

Diffusion and gravitational separation in the radiative layer at the base of the convective zone could account for the abundances found, particularly the $[El/Fe]$ ratios. This process would not operate in normal A stars as circulation velocities would dominate diffusion.

It is suggested that microturbulent velocity increases outwards towards the star's surface particularly in the region above the outer convection zone giving rise to different velocity parameters for FeI, FeII and TiII. Mean microturbulent velocity is correlated with line width suggesting correlation between microturbulence and macroturbulence.

Finally, of the five stars analysed, τ UMa, 15 UMa, 68 Tau and 60 Leo are confirmed as Am, 28 And being a borderline case.

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