EXPERIMENTAL AND THEORETICAL STUDIES OF THE STRUCTURAL BEHAVIOUR OF BRICKWORK CROSS-WALL SYSTEMS

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PRINCIPAL NOTATION

The following symbols have been used in this thesis:

A ..... Area of triangle; area of web or flange.

a ..... Torsional constant for a channel section.

a_j, a_k ..... X- coordinates of j and k points of a triangle.

$[B]$ ..... Matrix consisting of the coordinates of a triangle.

b ..... Width of flange of a channel; (length of return wall).

b_j, b_k ..... y - coordinates of j and k points of a triangle.

C ..... Torsional rigidity

c_1 ..... Integration constant.

D ..... Flexural rigidity of the flange in its plane.

$[D]$ ..... Matrix consisting of elastic constants of the material.

E ..... Modulus of elasticity.

e ..... Eccentricity

e_A ..... Shear Centre

$[F]^{(e)}_{\delta}$ ..... Matrix of corner forces of an element 'e' produci deflection 'δ'

G, G_1, G_2, G_3, G_4, G_5 ... Shear modulus

H ..... Total height of structure

h ..... Height of one storey.

I_1, I_2 ..... Moment of inertia of cross-walls.

I_z ..... Moment of inertia of the cross-section of the channel about z axis.

I_z^1 ..... Moment of inertia of the cross-section of web about z-axis.

i,j,k ..... Refer to three corner points of a triangle

K_1 ..... Torsional constant for an open section.

$[k]$ ..... Stiffness matrix for an element.

$[K]$ ..... Stiffness matrix for the complete structure.

L ..... Bay width.

ℓ ..... Length of cross-wall

M_t ..... Twisting moment

N ..... Number of samples

n ..... Number of storeys

P ..... Total lateral load

$P, P_1, P_2, P_3, P_4, P_5$ ..... Lateral load at storey level.

Q ..... Static moment of area of the cross-section of web about z axis.

Q_1 ..... -do- ; referred to flange.

$[R]$ ..... Column vector representing external forces.

R_x ..... Rigidity in x-direction

R_{ax} ..... Rigidity in x-direction; referred to wall A.

R_z ..... Rigidity in z direction.

R_{az} ..... Rigidity in z-direction; referred to Wall A.

s ..... Compressive force in the flange.

S_{xt} ..... Shear in x-direction due to torque.

S_{axt} ..... Shear in x-direction due to torque; referred to Wall A.

S_{zt} ..... Shear in z-direction due to torque.

S_{azt} ..... shear in z-direction due to torque; referred to Wall A.

t_w ..... Thickness of cross-wall.

t_f ..... Thickness of return wall.

u, v ..... Functions of x and y.

x, y, z ..... Cartesian coordinate system.

x ..... Values of strength.

$\bar{x}$ ..... Mean value of strength

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ ..... Constants of polynomial.

γ_{xy} ..... Shear strain in cross-wall.

γ_{zy} ..... Shear strain in return wall.

$\Delta, \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5$ ..... Horizontal deflections at slab levels.

$\delta, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5$ ..... Total deflections at storey levels of a multi-storey cross-wall Structure

$\varepsilon_x, \varepsilon_y$ ..... Strain in x and y.directions.

$[\varepsilon]$ ..... Matrix of strains

θ ..... Angle of twist per unit height; rotation at base of structure.

ν ..... Poisson's ratio

σ_D ..... Standard derivation.

$[\sigma]$ ..... Matrix of stresses

σ_1, σ_2 ..... Major and minor principal stresses.

σ_x, σ_y ..... stresses in x and y direction.

τ_{xy} ..... shear stress in web due to torsion.

τ_{zy} ..... shear stress in flange due to torsion.

ϕ ..... Angle of twist.

ABSTRACT

The purpose of the work reported in this thesis was to investigate whether existing shear wall theories could be used to analyse multi-storey brickwork buildings. A simple multi-storey cross-wall structure of relatively low rigidity was constructed in one-sixth-scale brickwork and tested under lateral loading. Analyses were carried out using the finite element method and a simplified theory was formulated for comparison with the experimental results. Good agreement between measured and calculated deflections was found, provided that appropriate values of elastic moduli for brickwork were employed in the calculations. The finite element method gave reasonably good estimates of the stresses in the cross-walls.

The effective width of return walls acting in conjunction with a pair of simple cross-walls was examined experimentally and by the finite element method. In the case considered, the limiting effective 'flange' width was found to be 0.35 of the storey height. Similar calculations indicate that the maximum effective floor slab width would be 0.5 of the bay width.

The work was further extended to investigate the effect of torsion on a cross-wall structure. A four-storey one-sixth-scale structure in brickwork was tested under lateral loading applied eccentrically. Analyses were carried out using the finite element method, theories of torsion for thin walled sections and a simplified

method of calculation. Reasonable agreement was found between the measured and calculated values of stresses and deflections.

CHAPTER 1

INTRODUCTION

1.1 GENERAL

The basic advantage of loadbearing brickwork construction is that brickwork elements can serve several functions simultaneously[24]. They provide structure, subdivision of space, thermal and acoustic insulation, fire and weather protection. In other types of construction, for example in a steel frame building all the above mentioned functions have to be provided separately with consequent complication in planning, design and construction. The material is comparatively cheap but durable and can be constructed without much capital investment by the builder in specialised equipment.

In using loadbearing brickwork as a structural medium, the general requirement is that the floor plan should repeat at each storey throughout the entire height of the building and that adequate stiffness be provided in all directions. The floor area should be divided into a relatively large number of rooms of small to medium size. As a result, this type of construction is suitable for types of buildings such as flats, maisonettes, hostels and other residential buildings where large clear areas are not required. Although the basic requirement in planning a building of this type is the repetition of floor plans and subdivision of floors into smaller to medium size rooms, the first one or two storeys may be of reinforced concrete table form of construction to provide for entrances and other facilities at foyer levels[11].

In a loadbearing brickwork structure the walls together with the floor slabs act as thin vertical plates, which, in addition to carrying vertical loads, also resist lateral loads due to wind and

other effects. The stability of walls in the weaker direction in a building of this type is often achieved by careful planning of the layout and utilization can be made of the lift shafts and staircases for restraining lateral forces.

From the experience of tall buildings in loadbearing brickwork constructed in Great Britain no insuperable foundation problems have been found[24]. In most cases a reinforced concrete raft 2 feet thick has been used. If the soil condition is very good spread footings for walls may be satisfactory. Whereas in the case of very poor soil condition piling with ground beams may be necessary.

Although brickwork is one of the oldest materials in use in the building industry, the design of structural brickwork, throughout the years, has developed on empirical basis. People used to design buildings on "rule-of-thumb" which resulted very thick walls. One of the finest examples of the earlier high-rise buildings constructed with loadbearing masonry walls is the sixteen storey Monadnock building in Chicago built in 1891. The 200 feet high walls were designed to carry both vertical and lateral loads and as a result of the approximate methods of analysis then available the walls are 6 feet thick at the base. However, this building seemed to be the ultimate in frameless building and a commemoratory plaque declares it to be: "The final triumph of traditional masonry construction".

With the advent of steel and reinforced concrete most multi-storey buildings were designed for framed construction. Brickwork ceased to be a structural medium and used for cladding and panel infilling. With the publication of a Standard Code of Practice in

1948, people came to know some research results on the structural behaviour of brickwork. The code was very conservative and as a result most architects did not prefer brickwork even by that time.

1.2 CROSS-WALL STRUCTURE

In planning multi-storey flats in brickwork shear walls are provided for the structural function of carrying lateral and vertical loads. In addition they do the non-structural function of dividing and enclosing space. This idea applied in planning leads to the system of cross-wall construction (Fig.1.1)[43]. All ideal cross-walls are shear walls, but not all shear-walls are cross-walls. A multi-storey brickwork building is thus normally composed of a number of wall units, each of which has two or more walls bonded together at right angles. These walls form equivalent webs and flanges in the unit. However, the behaviour of a wall acting as an equivalent web or flange depends entirely on the direction of application of the lateral load.

Wind load becomes a major criterion in designing a tall building. It has been recognized in modern times that masonry can participate in resisting the lateral load due to wind or other effects. In high-rise loadbearing brickwork building the lateral stiffness of the building is achieved by providing cross-walls. The rigidity of walls in a cross-wall structure towards lateral loading is much greater than the rigidity of a frame structure. In fact in a composite construction such as walls and frames one usually assumes that the full lateral load is carried to the ground by the walls alone.

Very unsymmetrical wall arrangements are not advisable in a

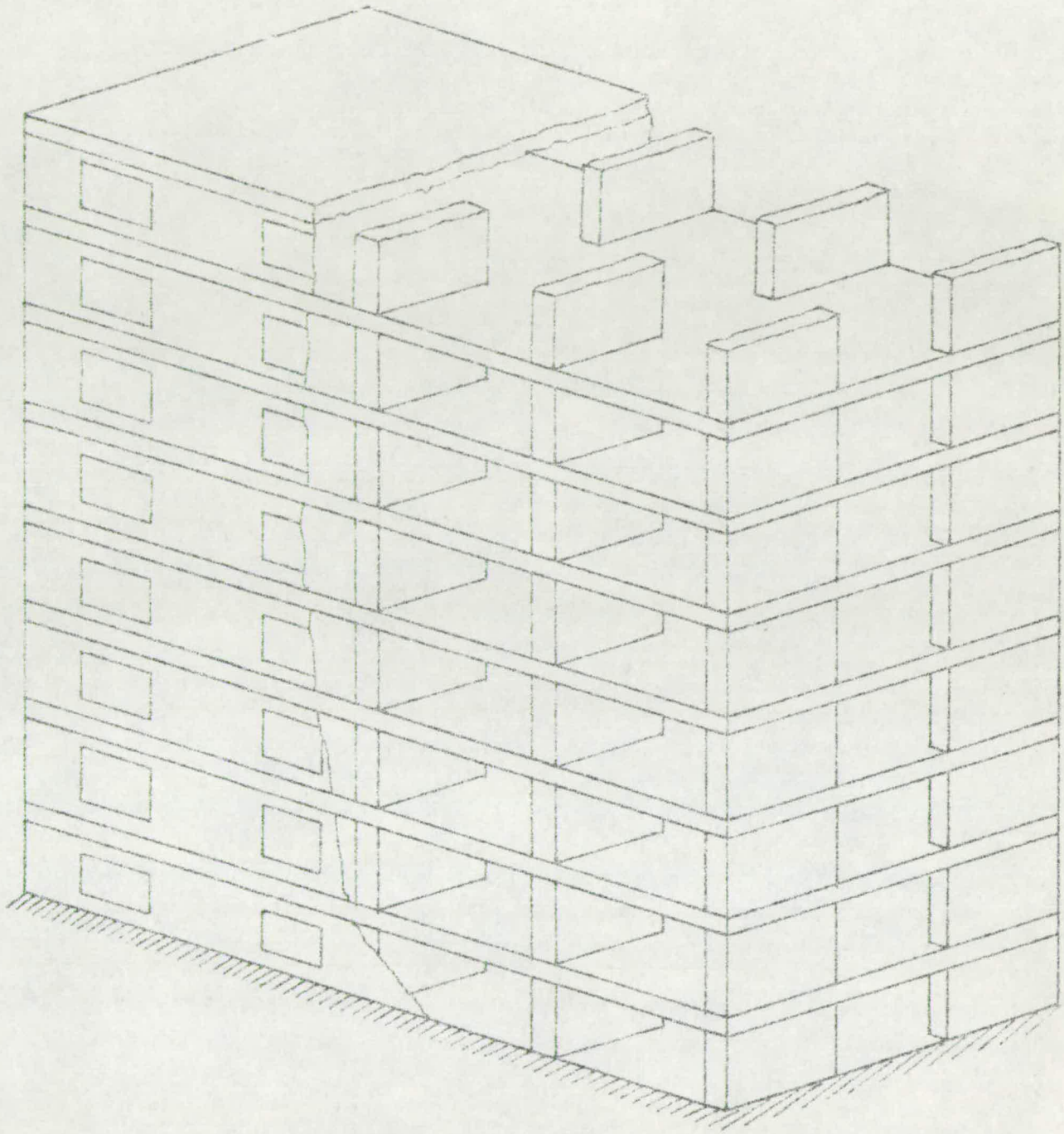


Fig. 1.1 IDEALISED BRICKWORK CROSS-WALL BUILDING.

brickwork building. However, even in a symmetrical building torsion may be developed accidentally from mere lack of symmetry of the wind action or earthquake disturbances and no design calculation involving strength and rigidity can be so perfect as to negate the possibility of torsion[46].

1.3 MODERN DEVELOPMENTS IN HIGH-RISE LOADBEARING BRICKWORK

From 1951 onwards frameless blocks in loadbearing brickwork have been erected in several European countries, Switzerland being the pioneer. Perhaps less exciting work has been done in Germany, France and Denmark[21].

The first scientific investigation on the strength of brickwork was conducted at EMPA (Swiss Federal Materials Testing and Research Institution)[48], which revolutionised the use of brickwork and laid the foundation for the design and construction of thirteen-storey apartment buildings built in Basle from 1951-1953. Other exciting examples[21] are eighteen-storey flats at Schwamendingen (Plate 1.1), built in 1957, ten-storey slab type block of flats in loadbearing brickwork at zug built in 1958, multi-storey flats at Luzern built during 1959-1961. The structure consists of fourteen storeys of loadbearing brickwork on a two-storey basement and heating room of reinforced concrete.

There has not been much exploitation of calculated brickwork in Great Britain as compared to Switzerland. The main reason is a basic lack of knowledge which lead to the conservative recommendations in the Code of Practice (C.P.111).

In Britain research in brickwork was first carried out at the Building Research Station which formed the basis of 1948 Code.

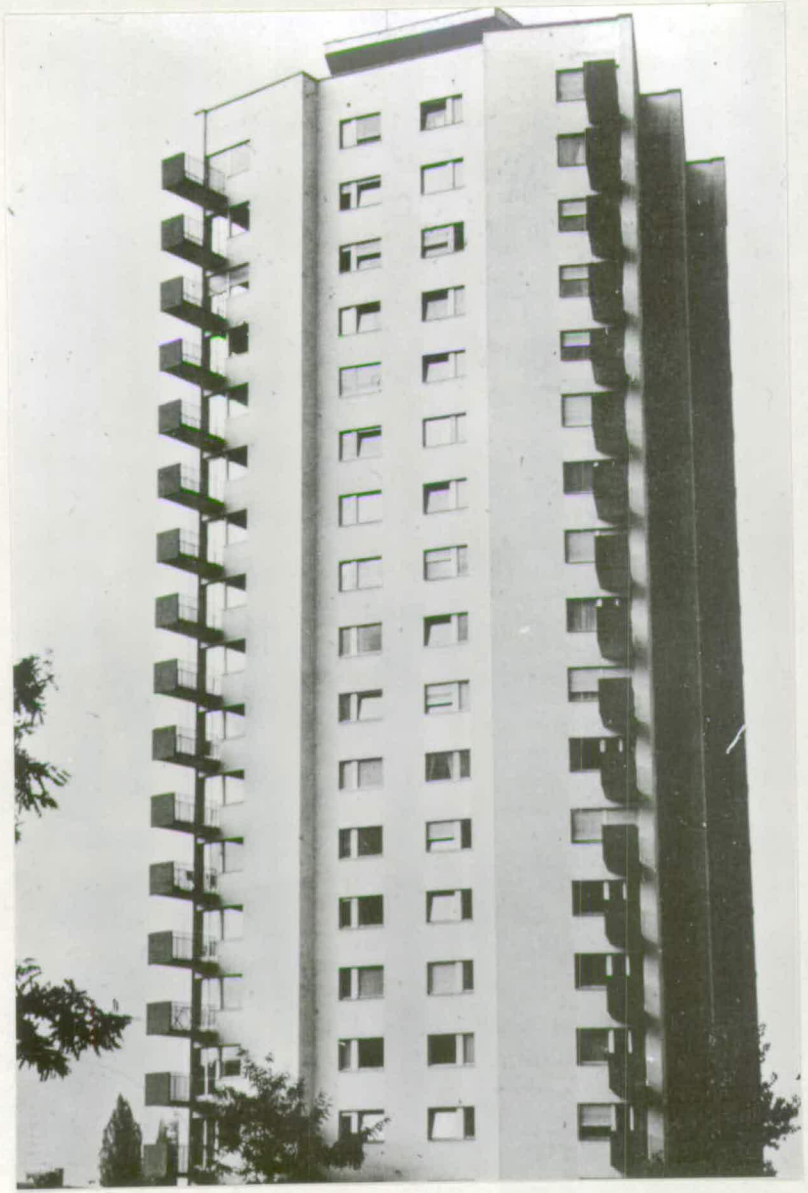


PLATE 1.1 - 18-STOREY BLOCK OF FLATS AT SCHWAMENDINGEN, SWITZERLAND

PLATE 1.2 - 14-STOREY RESIDENTIAL TOWER BLOCK FOR UNIVERSITY OF ESSEX

PLATE 1.3 - 16-STOREY TOWER BLOCK OF STUDY/ BED SITTING ROOMS FOR HAMILTON COLLEGE, SCOTLAND

PLATE 1.4 - FLATS FOR GREATER LONDON COUNCIL, BAYLIS ROAD, LAMBETH

In recent years extensive research into the properties of brickwork has been carried out[18,55] and much information has been obtained on fundamental and applied problems. By 1953 fair amount of data were available. However, these were not harnessed to any appreciable extent till 1960. The construction of a twelve storey flat at Birmingham and the Swiss experience had great impact. As a result the Code was revised in 1964. Gradually engineers and architects are selecting loadbearing brickwork for an increasing number of projects.

Presently there are several examples of tall buildings in loadbearing brickwork all over Great Britain[54]. The most interesting ones are fourteen-storey residential tower blocks at Essex University, near Colchester (Plate 1.2), Sixteen-storey tower block of study/bed sitting rooms for Hamilton College, Scotland (Plate 1.3), Lambeth flats for Greater London Council (Plate 1.4), Sixteen storey residential tower at the Beaconsfield Army School of Education, Hamilton House, in Kensington, London etc.

CHAPTER 2

A REVIEW OF PAST RESEARCH ON CROSS-WALL STRUCTURES AND THE OBJECT OF THE PRESENT INVESTIGATION

2.1 SHEAR TESTS ON SINGLE-STOREY STRUCTURES

The first tests on storey-height brickwork and blockwork walls in Great Britain were carried out by Simms[52] in the Building Research Station. Simms carried out tests applying racking load or a combination of racking and uniform vertical loads. Investigation was also made using various types of clay bricks and blocks and mortar joints.

In the U.S.S.R., Polyakov[40] in 1948 carried out major investigation of infilled frames subjected to racking loads at the Central Institute for Industrial Structures in Moscow. He found that the usual mode of failure of complete infills was by cracking of the mortar joint along the compressive diagonal. Polyakov proposed an analysis of infilled frames by assuming the infill to behave as diagonal bracing structs.

Benjamin and Willams[4] performed tests on a number of large scale models of single storey brick and reinforced concrete shear walls, with or without openings and on reinforced concrete shear wall. assemblies consisting of one or two storey shear walls connected by diaphragms. The object of the experimental investigation was to determine the influence on the strength and stiffness of various types of reinforcement round the openings. Based on their work, an analysis of walls containing openings has been suggested.[3] Similar work was reported elsewhere[53].

2.2 RESEARCH ON MULTI-STOREY BUILDINGS

In recent years, with increasing cost of land, the engineering and architectural trend has changed towards taller buildings, with more open spaces and gardens at ground level. Wind has a significant effect on such tall buildings. To resist lateral load due to wind or other effects cross-wall typesof construction, which carry both shear and vertical loads are very common. The theoretical research on shear wall structures has now become an important field in many universities.

In order to analyse multi-storey shear wall structures Chitty[15] considered a number of interconnected cantilever beams. The method assumes that the structure is composed of line elements so that the interconnecting beams bend about the centroid of the column. No restraint is put on the relative stiffness of column and beams. By assuming that the cross-beams have a point of contraflexure at mid-span, and do not deflect axially, the behaviour of the system can be defined by a single second order differential equation. In the simplest case of two-column frame axial deformation was considered but in general it was neglected. The equations relate forces in terms of continuously differentiable functions of height and their solution follows classical lines. A general solution form is stated and end conditions are applied to evaluate constants of integration. She found that the results obtained by the above method were in promising agreement with those obtained by slope-deflection and other theoretical methods and experimental measurements on models.

Green[22] treated the multi-storey coupled shear wall as an equivalent frame having relatively deep members. He applied the "portal method" of analysis assuming (a) lengths of piers and spandrels do not change (b) the inflection points of piers and spandrels must be at centres of their lengths. Green used a modified element stiffness to take account of shear as well as bending deformations. The floor slabs was assumed to be rigid in their own plane and a method was developed to give the redistribution of forces which must occur if all walls are to have the same deflection throughout the building irrespective of stiffness variations.

Frischmann, Prabhu and Toppler[20] suggested two different approaches based on methods of analysis for rigidly jointed frames, to deal with coupled shear walls. They analysed the structure by the equivalent column method where a multi-bay structure can be reduced to a single column by lumping together of all column stiffnesses, with a single beam restraint at every storey level, produced by the addition of the beam stiffnesses. Using a continuous distribution of restraint and loads, the problem was reduced to a single second order differential equation. From this equation column bending moments in the continuous system could be evaluated and then redistributed to the real discrete system.

The second method is the conventional frame analysis using the flexibility approach. The advantage of this method is that it allows use of different values of elastic constants assuming concrete of different strength or other material is used along the height of the building. The method produces a set of simultaneous equations and needs computer for solution. Since deformations due to shear and axial forces have been neglected, both the approaches seem to

underestimate deflections[39,48].

MacLeod[39] treated coupled shear walls as a wide column frame and used stiffness method[33] to obtain a solution. He also considered shear, axial and bending deformations. One of the techniques he used was the finite element method. The results obtained by the finite element method were closer to the experimental values compared to those obtained by the wide column frame. MacLeod found that the effect of axial deformation of beams in the horizontal direction is negligible but the corresponding effect of the walls is significant.

Amaratunga[2] also used "equivalent frame analysis" to investigate the behaviour of multi-storey shear walls containing a single large opening in each storey. He compared his results with those obtained from a photoelastic[28] investigation and found reasonable agreement between stresses for sections not too near the corners of the openings.

Beck[5,6], Rosman[44,45] and Eriksson[19] adopted the idea of continuous connection technique in an approximate analysis of multi-storey coupled shear wall problems, taking into account the finite depth of the shear walls and consequent rotation of the cross section.

The basic principle of this method is that the connecting beams will be replaced by a continuum of same stiffness from storey to storey as the connecting beams. The accuracy of this approximate method increases with the increase of floor numbers.

In Beck's paper shear force per unit length of the continuum is used as the statically redundant function instead of the integral shear force. This seems to lead to more complicated expressions

for integration constants and the numerical procedure becomes more complicated than that of Rosman's paper. However, assumptions made in both papers are same and hence both methods lead to the same results.

Beck considered two uniform coupled shear walls on a rigid foundation, subjected to uniformly distributed lateral load. He presented design graphs to enable bending moments, shear forces and deflections of the walls to be determined rapidly. Rosman used direct mathematical solution for a wall with one band or two symmetrical bands of openings with different load conditions. He also considered the cases where on the ground floor the walls are often resolved into two or three columns for easier traffic circulation.

Eriksson considered the general case of walls pierced by any number of regular set of openings. The general procedure was used to solve the particular case of three non-symmetrical coupled walls, subjected to any kind of distributed wind loading.

Soane[51] following Eriksson's work, derived a set of general shear condition equations and solved by the analogue computer.

Coull and Choudhury[13] following mainly Rosman's work, have presented design curves for the rapid evaluation of the stresses and maximum deflection for a system of symmetrical coupled shear walls with one band of openings and subjected to a uniformly distributed load.

Kazimi[32] used Grinter's grid analogy technique to predict the stress distribution in shear walls, with rectangular openings,

subjected to lateral loading. The results obtained were compared with those obtained from photoelastic tests on model walls, with or without opening, supported on either a grid of foundation or on elastic beams.

Michael[37] considered the coupling effect of floor beams on the separate walls and gave a general solution to the resulting differential equation. He gave expressions for wall bending moments, the beam shear forces and the wall displacements. He compared his theoretical results with published test values of Jenkins et.al.[29].

Jenkins and Harrison[29] gave an analysis of shear walls subjected to bending and torsion. They presented a stiffness matrix method[33] for bending analysis and an energy method[1] for torsion. The theoretical results were compared with experimental results from perspex models.

Khan and Sbarounis[31] presented a method of successive approximations for the analysis of shear walls combined with frames. They also investigated the influence of slab width on the rigidity of a cross-wall structure.

More recently Qadeer and Stafford Smith[43] have suggested a method to calculate the bending stiffness of slabs connecting shear walls. The slab was analysed as an elastic plate by using the finite difference method. The results are presented graphically and give the non-dimensional stiffness of the slab for varying slab proportions, wall openings and slab-to-wall lengths. The value for the slab stiffness in terms of bending stiffness or an "effective width"

can be derived from the curves. Having obtained the effective stiffness, the slabs may be assumed to act in an analogous fashion to the connecting beams in coupled shear walls.

2.3 MODEL WORK ON LOADBEARING BRICKWORK STRUCTURES

The first tests on model bricks and brickwork were carried out by Hendry and Murthy[25,35]. The object of their work was to develop a model method for the investigation of the strength of brick structures. Tests on full-size brick piers that had been carried out at the Building Research Station were repeated at 1/3 and 1/6 scales. They came to the conclusion that, for given strengths of brick and mortar, the strength of full-size brickwork can be reproduced with reasonable accuracy by means of model tests if the thickness of mortar joints is scaled down and the strength of 1-in cubes of mortar is considered in place of 2.78-in. cubes used in full-size tests.

Murthy[36] carried out a series of tests on storey-height structures and on a 3-bay, 3-storey high model constructed of one-sixth scale loadbearing brickwork to investigate the rigidity of brickwork structure to lateral load. Analysis was carried out using the method suggested by Benjamin[4].

Sinha[48] following Murthy's work carried out tests on single-storey shear panels with openings and on a five-storey one-sixth scale model structure subjected to lateral load. The ultimate load behaviour was consistent with the previous work done on shear panels. To compare the values obtained experimentally, he calculated deflections using existing shear wall theories and found that these could not be used with reasonable accuracy to calculate the rigidity

of brickwork structures. However, deflections calculated by a simple method based on measurements of shear deflections in single-storey structures were promising.

SCOPE OF THE PRESENT WORK

Previous work on loadbearing brickwork cross-wall structures[47] indicates that the existing shear-wall theories cannot be used to calculate the rigidity and strength of a brickwork building at least as they stand. However, deflections obtained by a simple method were promising if the appropriate values of the elastic moduli of brickwork were employed in the calculation. There are also doubts about the applicability of shear-wall theory because of the relatively large shear deflections expected, about the contribution of return walls and floor slabs and about the degree of fixity of wall/slab connections.

By introducing the effect of interaction between the walls and floor slabs, or connecting beams the structure may be shown to be stiffer and stronger against horizontal loads than if calculated by the traditional 'separate cantilever' method. However, the behaviour of a brickwork building towards lateral load is likely to be in between the above two extremes. The present work[30] is intended to clarify these problems and to establish whether available methods of analysis can give reasonable estimates of stresses and deflections in brickwork structures of simple form.

CHAPTER 3

THE FINITE ELEMENT METHOD FOR PLANE STRESS AND PLANE STRAIN PROBLEMS

3.1 INTRODUCTION

The finite element method is essentially a generalization of standard structural analysis procedures. The method appears to be conceived by Turner et al[59] in the year 1956 in the United States. It was originally developed in the aircraft industry in response to the need for a procedure which could provide a refined solution for extremely complex airframe analysis. Since its beginning in the aircraft industry, the method has becomes very popular in many other fields[38,61]. Principal researchers into the development and the application of the finite element method have been presently Zienkiewicz et al[62] in the United Kingdom and Clough[16,17] in the United States.

Within the past fifteen years the development of the digital computer has motivated renewed and more extensive development of numerical methods. The finite element idealization of a problem produces a highly indeterminate structural system. Because of the tremendous number of numerical operations which are inherent in such an analysis, the use of modern digital computer is essential.

3.2 THE PHILOSOPHY OF THE METHOD

In the finite element method the actual structure is represented by an approximate one. However, it depends on the geometry of the actual structure. In some cases it may not be necessary to make any approximation. The structure is divided into a finite number

of small elements which are assumed to be connected only at the nodal points. Application of the equations of equilibrium to the forces acting at these nodal points will lead to a number of simultaneous equations which can be solved by the computer. No approximation is required in the mathematical procedures. This is in fact the basic difference between the finite element and the finite difference methods. The finite difference method gives an approximate mathematical solution to the exact continuum whereas the finite element method gives an exact mathematical solution to an approximate continuum.

One of the major advantages of the finite element method is that it can cope with arbitrary material properties. By dividing the structure into small elements all material properties of the original system can be retained within the individual element. Of equal importance, is the facility to deal with structures of irregular shapes.

3.3 BASIC ASSUMPTIONS

The strains $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ in an element are usually assumed to be constant. This is required in order to maintain continuity between elements of the system. Since strains are constant, stresses $\sigma_x, \sigma_y, \tau_{xy}$, which act on the edges are also constant. These stresses are replaced by stress resultants which act at the corners of the element. Based on these assumptions, it is possible to determine the stiffness of a typical element, which is an expression for the corner forces resulting from unit corner displacements. After this relationship is developed standard methods of structural analysis[1,33] are employed to solve the complete structure.

3.4 THEORETICAL DEVELOPMENTS

3.4.1 GENERAL

The accuracy of the finite element method depends on how close the behaviour of the actual structure is represented by the approximate structure. Sound judgement is required in sub-dividing the structure. If boundary stresses are required finer subdivision should be used along such boundaries.

The number of different shaped elements should, however, be kept to a minimum. This will reduce the amount of initial computation of element stiffness characteristics.

3.4.2 DISPLACEMENT FUNCTION

Once the shape of the element has been selected different assumptions can be made concerning the behaviour of the element. There can be assumptions concerning the deflections, strains or stresses and are usually expressed by means of a displacement function. The selection of the displacement function is obviously of some importance in the final accuracy of results. For triangular elements the assumed displacement function is of the simplest type. For rectangular elements it is necessary to use a more complex function and accuracy in final results obtained is also more. The displacement function should be selected so that,

(a) it represents the displacement as closely as possible,

(b) no straining of the element occurs for rigid body displacements,

(c) if the nodal displacements are compatible with a constant strain condition, such a constant strain will in fact be obtained throughout the element.

(d) displacements should be continuous across the interface between elements.

Assuming

(1) Triangular elements

(2) No distributed loads

(3) No initial strains

(4) Displacements linear (Fig.3.1) and strains constant (Fig.3.2) within the element,

the displacement of any point within the element (Fig. 3.3) is defined in terms of $u(x,y)$ and $v(x,y)$ by equations of the following polynomials.

$$\left.\begin{aligned} u(x,y) &= u_i + \alpha_1 x + \alpha_2 y \\ v(x,y) &= v_i + \alpha_3 x + \alpha_4 y \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (3.1)$$

The displacement function satisfies the condition that strains are constant within an element.

The constants α_1, α_2, α_3 and α_4 can be determined by applying Eq (3.1) in the points $j(a_j,b_j)$ and $k(a_k,b_k)$

$$\left.\begin{aligned} u_j &= u_i + \alpha_1 a_j + \alpha_2 b_j \\ v_j &= v_i + \alpha_3 a_j + \alpha_4 b_j \\ u_k &= u_i + \alpha_1 a_k + \alpha_2 b_k \\ v_k &= u_i + \alpha_3 a_k + \alpha_4 b_k \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (3.2)$$

Solving for α_1, α_2, α_3 and α_4

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \frac{1}{a_j b_k - a_k b_j} \begin{bmatrix} b_j - b_k & 0 & b_k & 0 & -b_j & 0 \\ a_k - a_j & 0 & -a_k & 0 & a_j & 0 \\ 0 & b_j - b_k & 0 & b_k & 0 & -b_j \\ 0 & a_k - a_j & 0 & -a_k & 0 & a_j \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{bmatrix} \quad \ldots\ldots(3.3)$$

The strains within the element can be obtained from the assumed displacement function (Fig. 3.4).

From equation 3.1.

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= \varepsilon_x = \alpha_1 \\ \frac{\partial v}{\partial y} &= \varepsilon_y = \alpha_4 \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} &= \gamma_{xy} = \alpha_2 + \alpha_3 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.4)$$

Equation 3.4 shows that strains are constant within an element.

Combining equation 3.3 and equation 3.4 the strains can be expressed as a function of corner displacements,

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \frac{1}{a_j b_k - a_k b_j} \begin{bmatrix} b_j - b_k & 0 & b_k & 0 & -b_j & 0 \\ 0 & a_k - a_j & 0 & -a_k & 0 & a_j \\ a_k - a_j & b_j - b_k & -a_k & b_k & a_j & -b_j \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{bmatrix} \quad \ldots(3.5)$$

or in symbolic form

$$\left[\varepsilon\right] = \left[\,B\,\right]\left[\sigma\right]^{(e)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

Matrix $\left[B\right]$ consists of only the coordinates of the three corners of the triangle.

3.4.3 STRESS-STRAIN RELATIONSHIP

One of the advantages of the finite element method in two dimensional elasticity is that it can deal with structures of anisotropric material properties. The general relationship of stress-strain is as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \qquad \ldots\ldots\ldots\ldots (3.7)$$

The stress-strain relationship for an isotropic material in the state of plane strain for example is of the form

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \qquad \ldots(3.8)$$

or $$\left[\sigma\right] = \left[\,D\,\right]\left[\varepsilon\right] \qquad \ldots\ldots\ldots\ldots (3.9)$$

Matrix $\left[D\right]$ consists of only the elastic constants of the material in the element.

3.4.4 STRESS RESULTANTS

Since strains within the element are constant, it follows that stresses are also constant and in particular the stresses along the boundaries of the element are uniform. Stresses acting along the boundaries can be replaced by stress-resultants acting at corners (Fig. 3.5). The corner forces $\{F\}^{(e)}_{\delta}$ can thus be expressed in the following matrix form.

$$\begin{bmatrix} F_x^i \\ F_y^i \\ F_x^j \\ F_y^j \\ F_x^k \\ F_y^k \end{bmatrix} = \frac{1}{2} \begin{bmatrix} b_j - b_k & 0 & a_k - a_j \\ 0 & a_k - a_j & b_j - b_k \\ b_k & 0 & -a_k \\ 0 & -a_k & b_k \\ -b_j & 0 & a_j \\ 0 & a_j & -b_j \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad \ldots\ldots (3.10)$$

Comparing this equation with equation 3.5 it can be seen that the transpose of the matrix $[B]$ appears in equation 3.10.

Therefore

$$\{F\}^{(e)}_{\delta} = \frac{(a_j b_k - a_k b_j)}{2} [B]^T [\sigma] \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.11)$$

$$= (\text{area of the element}) [B]^T [\sigma]$$

3.4.5 ELEMENT STIFFNESS

Putting the values of $[\varepsilon]$ and $[\sigma]$ in equation (3.11)

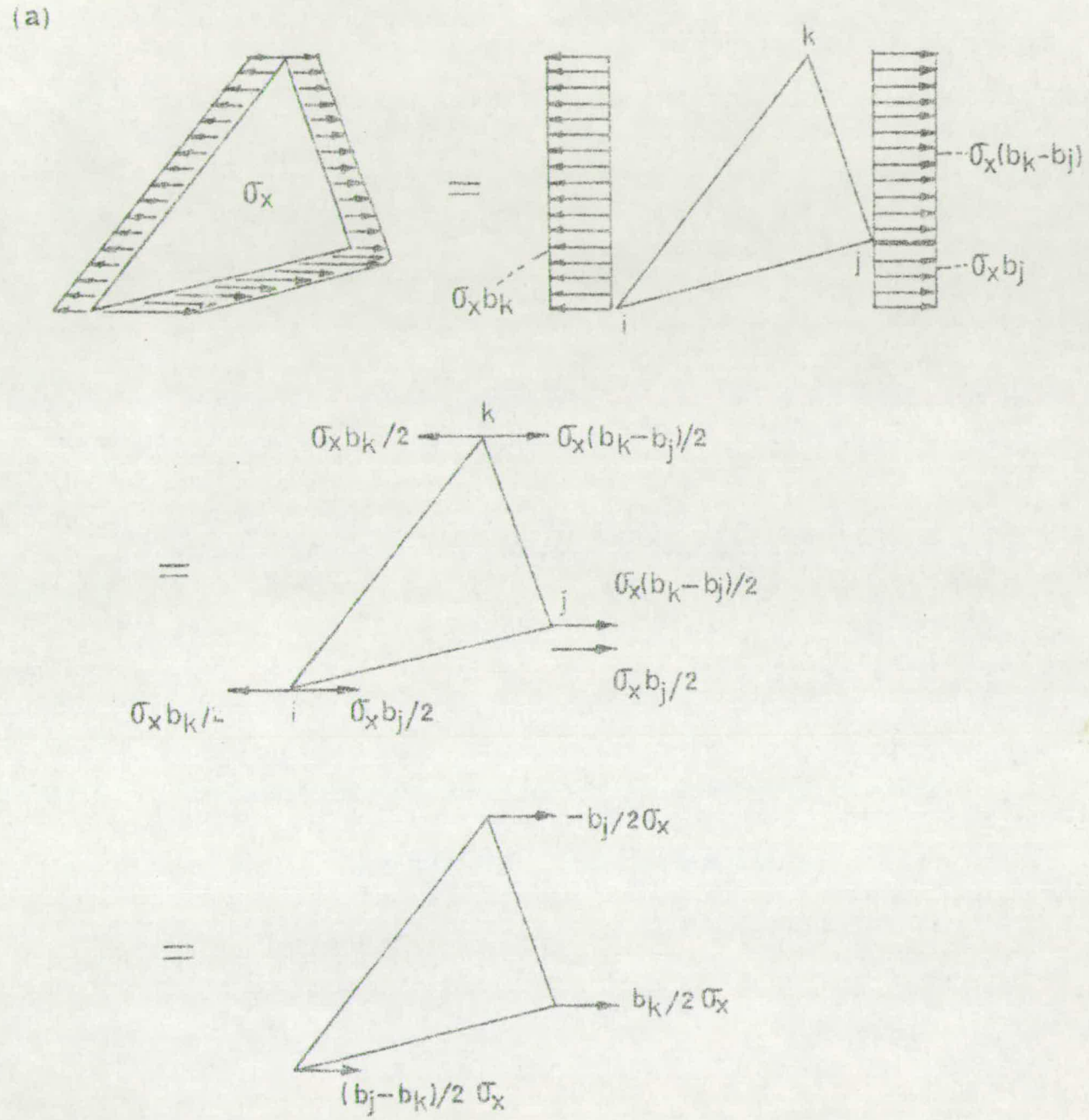
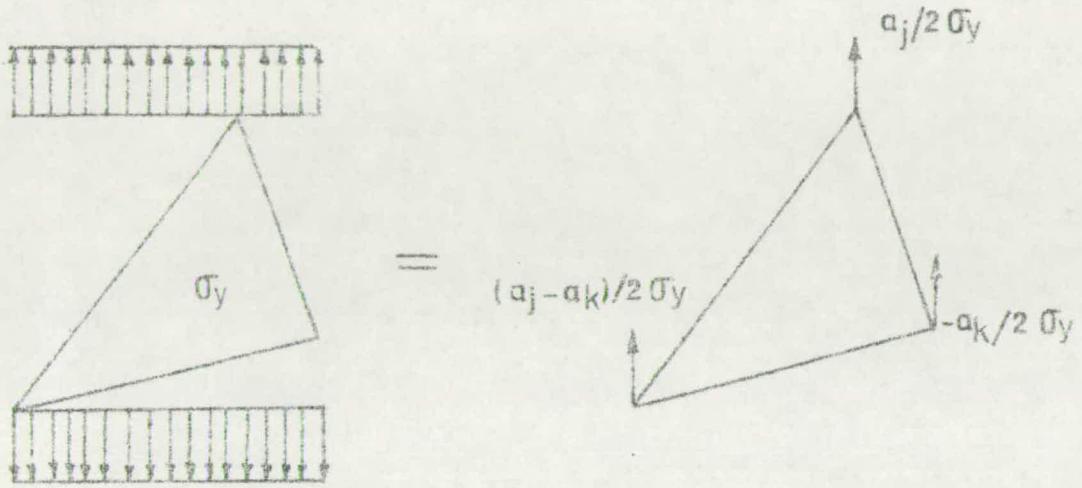


Fig. 3.5 STRESS RESULTANTS.

(b)



(c)

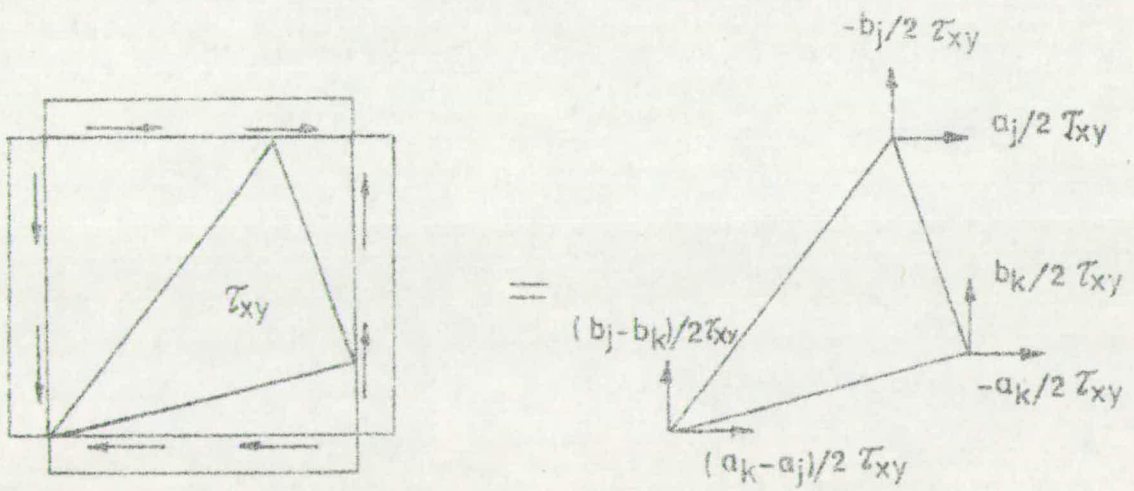


Fig. 3.5 STRESS RESULTANTS.

$$
\begin{aligned}
\underset{\delta}{[F]^{(e)}} &= A[B]^T[\sigma] \\
&= A[B]^T[D][\varepsilon] \\
&= A\underset{6\times3}{[B]^T}\underset{3\times3}{[D]}\underset{3\times6}{[B]}\underset{6\times1}{[\delta]^{(e)}} \\
&= \underset{6\times6}{[k]^{(e)}}[\delta]^{(e)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.12)
\end{aligned}
$$

where A = Area of the triangle,

and $[k]^{(e)} = A[B]^T[D][B] = A$ 6 x 6 stiffness matrix for the element.

Equation 3.12 represents a relationship between the nodal forces and the nodal displacements for one element. The elements of the matrix $[k]^{(e)}$ are calculated by the computer from the following data,

(1) Material constants

(2) Coordinates of nodal points

Using more sophisticated methods such as virtual work or minimum potential energy[16,17] a more general expression can be derived for the stiffness matrix.

$$[k]^{(e)} = \int [B]^T[D][B] \; d(\text{Volume}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3.13)$$

3.4.6 STIFFNESS OF COMPLETE STRUCTURE

The stiffness of the complete structure can be found by a systematic addition of stiffnesses of all elements in the system.

Let us consider equation 3.12 written in full.

$$\begin{bmatrix} F_i^{(e)} \\ F_j^{(e)} \\ F_k^{(e)} \end{bmatrix} = \begin{bmatrix} k_{ii}^{(e)} & k_{ij}^{(e)} & k_{ik}^{(e)} \\ k_{ji}^{(e)} & k_{jj}^{(e)} & k_{jk}^{(e)} \\ k_{ki}^{(e)} & k_{kj}^{(e)} & k_{kk}^{(e)} \end{bmatrix} \begin{bmatrix} \delta_i^{(e)} \\ \delta_j^{(e)} \\ \delta_k^{(e)} \end{bmatrix} \quad \ldots (3.14)$$

which really means

$$\begin{bmatrix} F_{ix}^{(e)} \\ F_{iy}^{(e)} \\ F_{jx}^{(e)} \\ F_{jy}^{(e)} \\ F_{kx}^{(e)} \\ F_{ky}^{(e)} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}_{ii}^{(e)} & \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}_{ij}^{(e)} & \begin{bmatrix} \\ \\ \end{bmatrix}_{ik}^{(e)} \\ \begin{bmatrix} \\ \\ \end{bmatrix}_{ji}^{(e)} & \begin{bmatrix} \\ \\ \end{bmatrix}_{jj}^{(e)} & \begin{bmatrix} \\ \\ \end{bmatrix}_{jk}^{(e)} \\ \begin{bmatrix} \\ \\ \end{bmatrix}_{ki}^{(e)} & \begin{bmatrix} \\ \\ \end{bmatrix}_{kj}^{(e)} & \begin{bmatrix} \\ \\ \end{bmatrix}_{kk}^{(e)} \end{bmatrix} \begin{bmatrix} \delta_{ix}^{(e)} \\ \delta_{iy}^{(e)} \\ \delta_{jx}^{(e)} \\ \delta_{jy}^{(e)} \\ \delta_{kx}^{(e)} \\ \delta_{ky}^{(e)} \end{bmatrix}$$

.....(3.15)

To illustrate the meaning of the elements in the above stiffness matrix, let

$$k_{ij}^{(e)} = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}_{ij}^{(e)} \quad \ldots\ldots\ldots\

Therefore, the general stiffness term 'K_{ij}' for the complete structure, which is the sum of the forces acting on all elements at nodal point 'i' due to unit displacements at nodal point 'j' is given by

$$K_{ij} = \sum_{(e)} k_{ij}^{(e)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.17)$$

K_{ij} exists only if 'i' equals to 'j', or if 'i' and 'j' are adjacent nodal points in the physical system.

Therefore for the complete structure we can write

$$\left[F \right]_{\delta} = \left[K \right] \left[\delta \right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.18)$$

where

$\left[\delta \right]$ = A column vector of all nodal displacements

$\left[K \right]$ = Complete stiffness matrix for the structure

$\left[F \right]_{\delta}$ = Forces at nodes compatible with $\left[\delta \right]$

3.4.7 EXTERNAL FORCES

If the external forces at the nodes are represented by the column vector $\left[R \right]$ then by equilibrium

$$\left[R \right] = \left[F \right]_{\delta}$$

$$\text{i.e.} \left[R \right] = \left[K \right] \left[\delta \right]$$

$$\text{or} \quad \left[\delta \right] = \left[K \right]^{-1} \left[R \right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.19)$$

3.4.8 COMPUTER PROGRAM

The procedure for the preparation of input data for the program has appeared in Appendix I. Using these data the computer will calculate

$[B]$ and $[B]^T$ from coordinates

$[D]$ from material constants

$[k]^{(e)}$ for each element

$[K]$ for complete structure

$[\delta]$ for each node using external forces and equation (3.19).

From these values of $[\delta]$ the computer will calculate the stresses at the centre of gravity of each element.

CHAPTER 4

EXPERIMENTAL AND THEORETICAL INVESTIGATIONS OF THE BEHAVIOUR OF A FIVE-STOREY MODEL CROSS-WALL STRUCTURE IN BRICKWORK SUBJECTED TO LATERAL LOADING

4.1 INTRODUCTION

The work described in this chapter[30] represents an extension of an earlier study of the strength and rigidity of a multi-storey cross-wall structure[47] carried out at model scale. This previous work indicated that shear wall theory based on the continuum method of analysis[5,13,14,19,41,44] would not give an accurate estimate of the deflections of the structure but a simple method of calculation would do so provided that appropriate values of the elastic moduli of brickwork were employed in the calculation. These values were found to vary with precompression, and thus from storey to storey of the structure. A simple multi-storey cross-wall structure of relatively low rigidity was constructed in one-sixth-scale brickwork and tested under lateral loading. Analyses were carried out using the finite element method and a simple method for comparison with the experimental results formulated.

4.2 MATERIALS

4.2.1 BRICKS

One-sixth scale model bricks were used for the construction of the walls of the structure. The bricks were from two batches, the details of strength of which are given in the following table,

TABLE 4.1

Crushing Strength (lbf/in^2) of 1/6th Scale Model Bricks

	Batch 1	Batch 2	Batch 1 & 2 Combined
Sample Size	54	54	108
Mean crushing Strength ($\bar{x}$)	3442	3744	3593
Standard Deviation $\sigma_D = \sqrt{\left[\frac{\Sigma (x - \bar{x})^2}{N - 1}\right]}$	372	408	417
Coefficient of Variation $\frac{\sigma_D}{\bar{x}} \times 100$	10.8%	10.9%	11.6%

In table 4.1,

σ_D standard deviation, expressed in the same unit as x.

x the values of strength

$\bar{x}$ Mean value of strength

N Number of samples

4.2.2 SAND

Fine sand such as Leighton Buzzard No.19, the grading curve of which is shown in Fig. 4.1 was used for mortar.

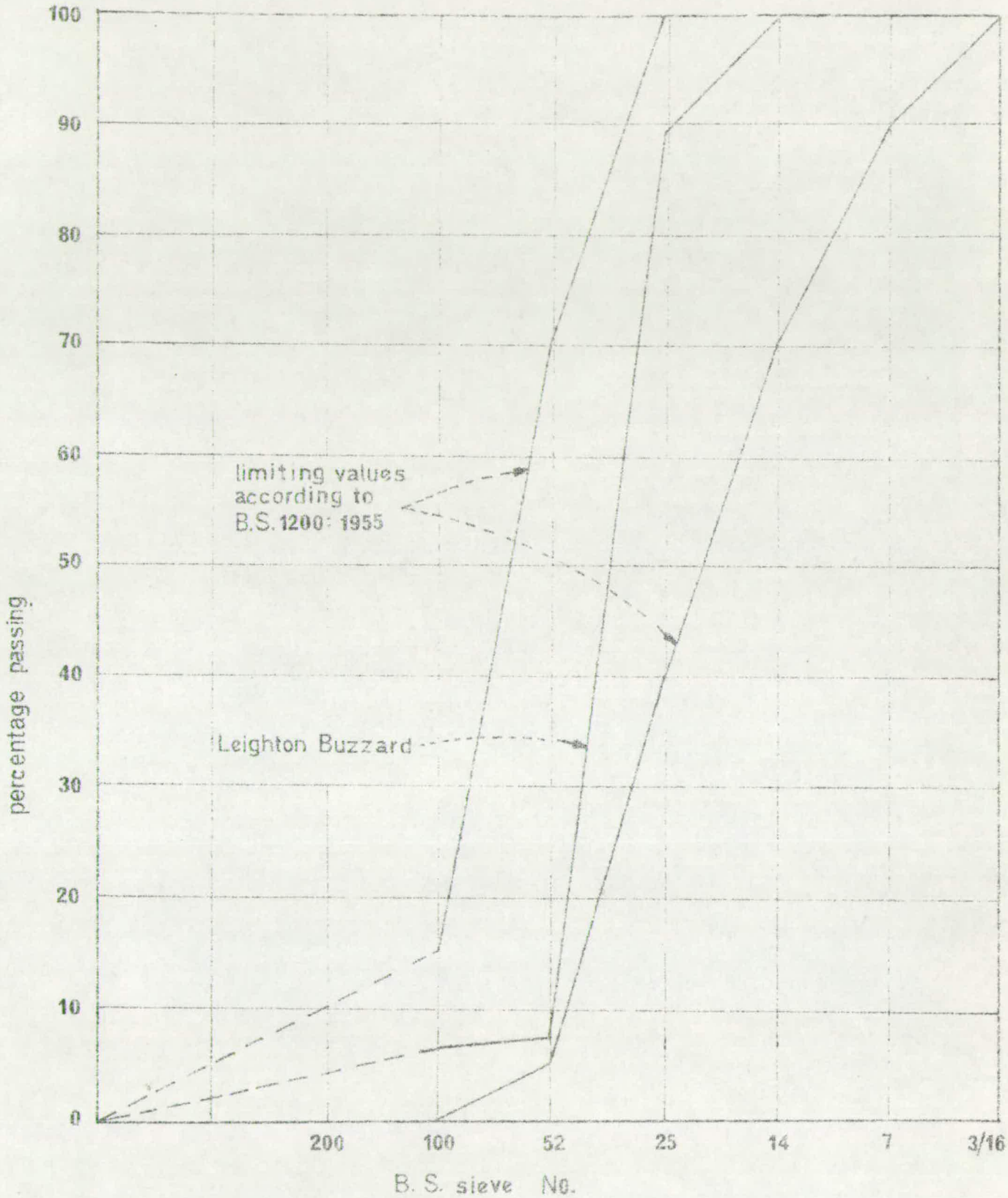


Fig. 4.1 GRADING CURVE OF LEIGHTON BUZZARD SAND (SAMPLE No. 19).

4.2.3 CEMENT

Rapid hardening cement "Ferrocrete" was used for all tests. The average compressive strength of 2.78 in. cement mortar cubes was found to be 4,780 lbf/in^2 at the age of 7 days against the minimum of 4,000 lbf/in^2 recommended by the British Standard Institution[8].

4.2.4 MORTAR

The mortar used was 1:4 cement/sand by weight (approx. 1:3 by volume). The crushing strength of 1-in. mortar cubes were as follows:

TABLE 4.2

Crushing strength (lbf/in^2) of 1-in Mortar Cubes

Range	Mean	Standard Deviation	Coefficient of variation	Nos.
1344-3718	2154	388	18%	60

4.3 CONSTRUCTIONAL DETAILS

4.3.1 LAYOUT

Figures 4.2(a) and 4.2 (b) are respectively the plan and elevation of the model structure. Figure 4.3 is the typical plan of a high-rise loadbearing brickwork building of simple cross-wall type[24]. It may be assumed that the model structure represents a pair of cross-walls shown within the broken lines in the prototype building. Each storey in the model consists of two cross-walls, two sections of external wall and the floor slab.

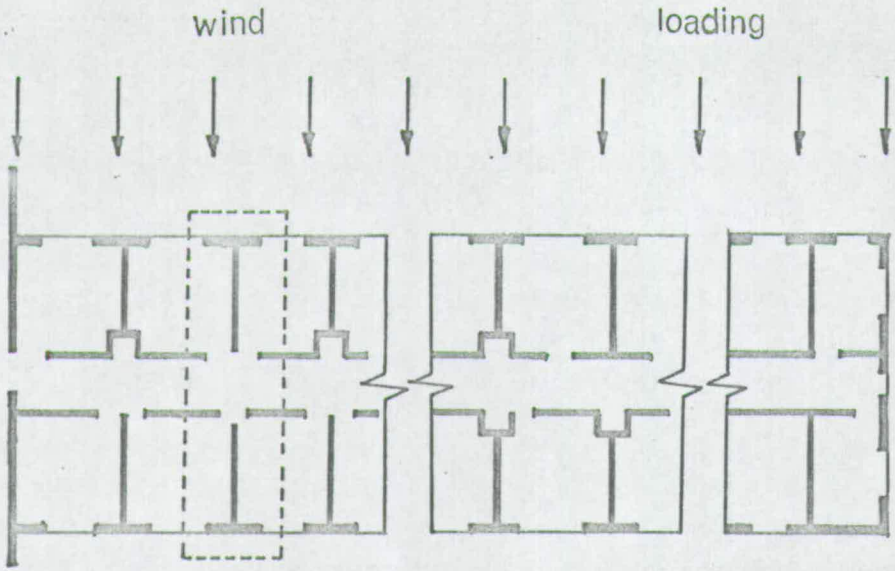
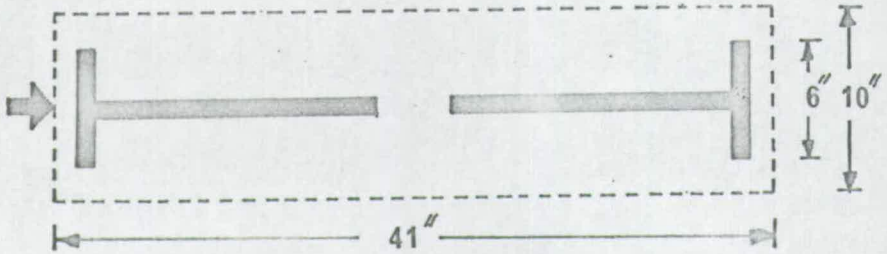
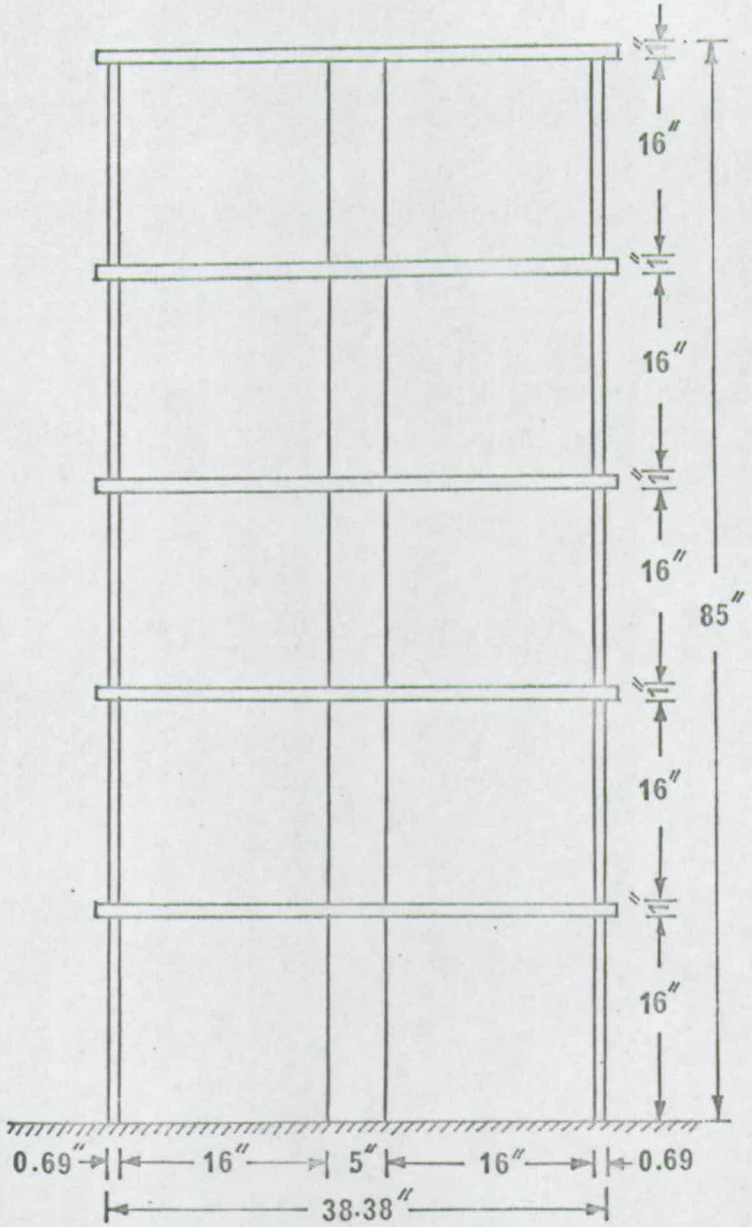


Fig.4.3 BANGOR UNIVERSITY COLLEGE.



(a) PLAN



(b) FRONT ELEVATION

Fig.4.2 (a) PLAN AND (b) ELEVATION OF MODEL STRUCTURE.

4.3.2 METHOD OF CONSTRUCTION

The first storey was constructed in-situ but additional storeys were constructed by first building the walls in jigs. The cross-walls were constructed first, keeping brick keys at every third or fourth course to strengthen the joint between the cross-wall, and the short lengths of external wall built at right angles to the cross-walls. Similar mortar was used for jointing walls and slabs. Figure 4.4 shows an isometric view of the model structure with the roof slab omitted.

4.3.3 R.C. SLAB

Precast reinforced concrete slabs were used for floors. The concrete was 1:1:2 by weight having maximum size of aggregate 3/16in. One inch square mesh of $\frac{1}{8}$ in. diameter bars was used for top and bottom reinforcement. The mean average crushing strength of 6 in. dia. 12 in. height cylinders was 6734 lbf/in^2. The modulus of elasticity of concrete[10] was found to be 4.4×10^6 lbf/in.^{2} (Fig. 4.5). Assuming Poisson's ratio for concrete as 0.15, the calculated value of the shear modulus was 1.9×10^6 lbf/in.^{2} and this value was used in the analysis.

4.4 TESTING EQUIPMENT

4.4.1 LOADING FRAME

A specially designed loading frame[36,48] was used for applying lateral load. (Plate 4.3).

The base of the structure was cemented in $1\frac{1}{4}$ in. steel channels which are welded to a base frame of 4 x 2 in. channels and 4 x $1\frac{1}{2}$ in. I-sections. These were cemented to the concrete floor to avoid

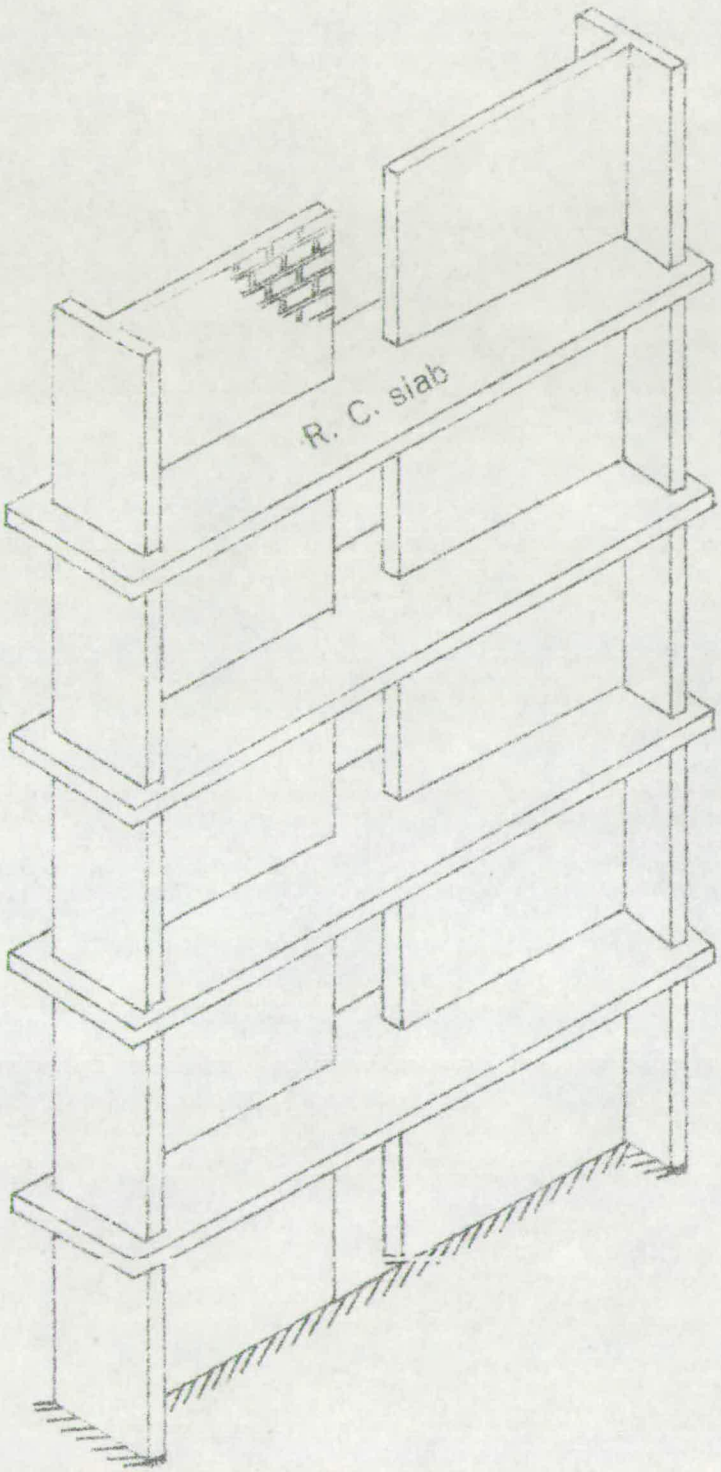


Fig.4.4 TYPICAL CONSTRUCTION OF BRICKWORK STRUCTURE.

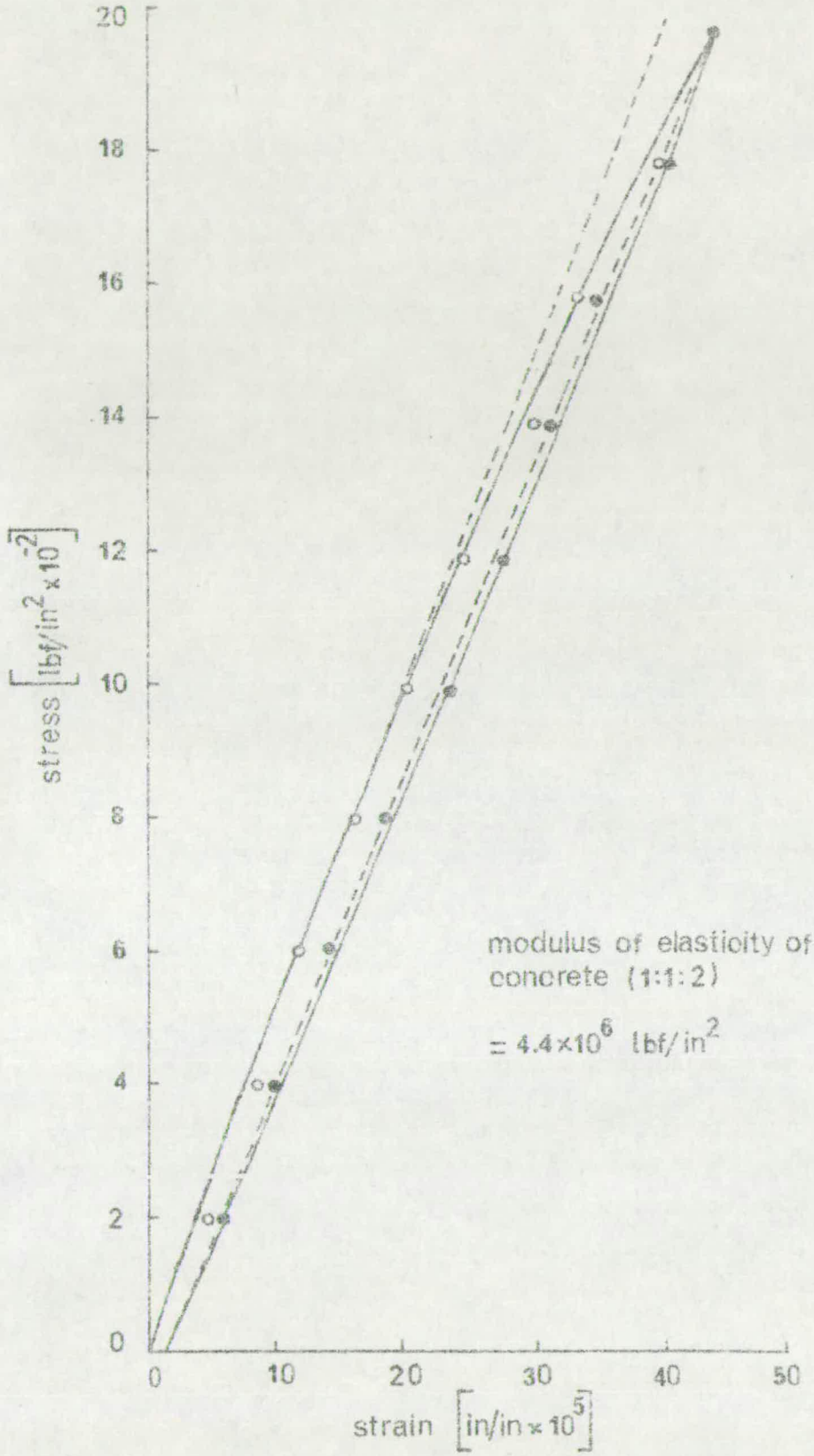


Fig.4.5 STRESS/STRAIN CURVE FOR CYLINDER, 6"ø x 12".

rotation. However, in the first attempt it was not successful to check rotation completely. Three vertical members of 8 x 4 in. I-section interconnected by 4 x 2 in. channels at each floor level were connected to the base structure and supported by 4 x 2 in. channel section inclined stays. The frame has a total capacity of 8 tons horizontal load. This was specially designed to test one-sixth scale structures as high as up to five storeys by applying horizontal load simultaneously to all storeys.

4.4.2 LOADING BEAM AND LOAD-MEASURING APPARATUS

The racking load was applied by a 6 ton hydraulic jack on a semi-circular hinge at the centre of the loading beam. The loading beam was $\frac{3}{4}$ x $\frac{3}{8}$ x 7 in. long of high tensile steel and was supported on $\frac{1}{2}$ in. dia. rollers spaced 6 in. apart (Plate 4.1). The load was transmitted to the slab through the rollers seating on a thin piece of plywood at the end of the slab.

For measuring the racking load a reference beam of $\frac{3}{4}$ x $\frac{1}{4}$ x 7 in. long was connected to the loading beam by two pins 6 in. apart. A dial gauge was mounted on the reference beam to measure the central deflection of the loading beam due to the applied load (Plate 4.1). The apparatus was calibrated in an Instron testing machine and calibration curves are shown in Fig. 4.6.

4.5 SIMULATION OF DEAD LOAD STRESSES

Being a model structure of 1/6th scale it was necessary to increase the dead weight stresses in the material to six times in order to simulate the effect of gravity load in the prototype structure[34]. This was achieved by hanging lead billets from the

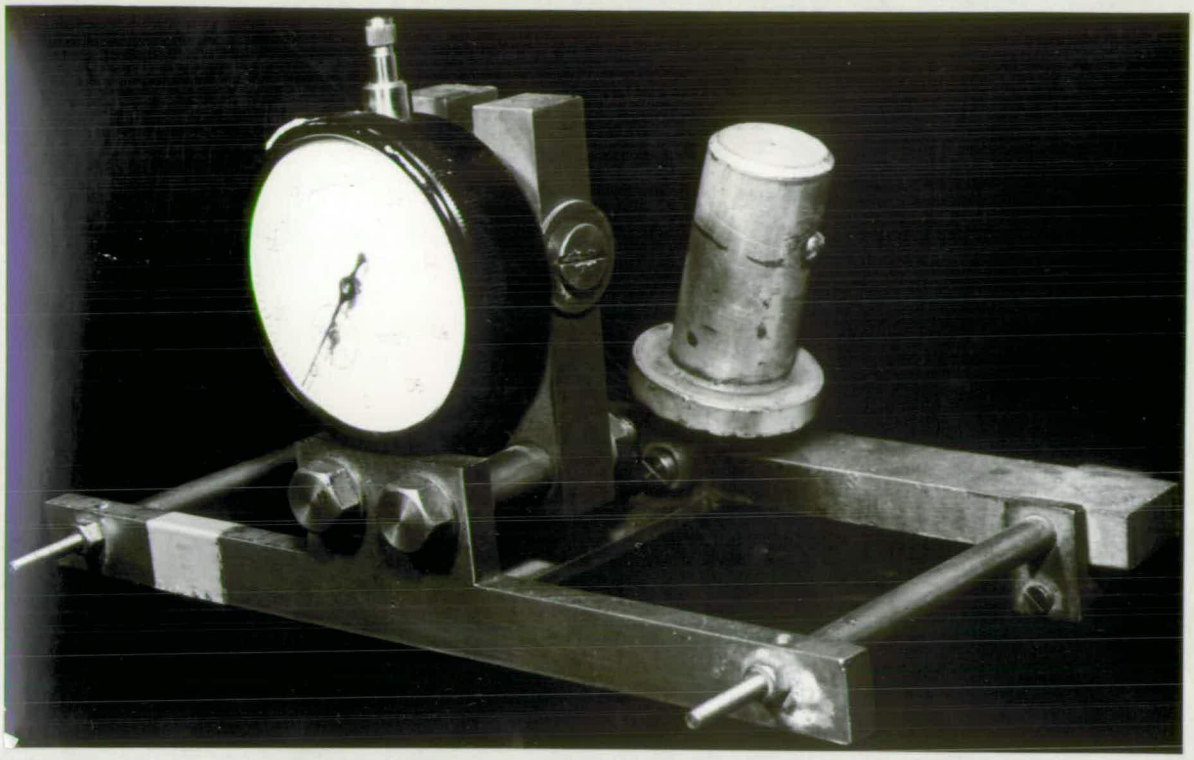


PLATE 4.1 - LOAD MEASURING APPARATUS

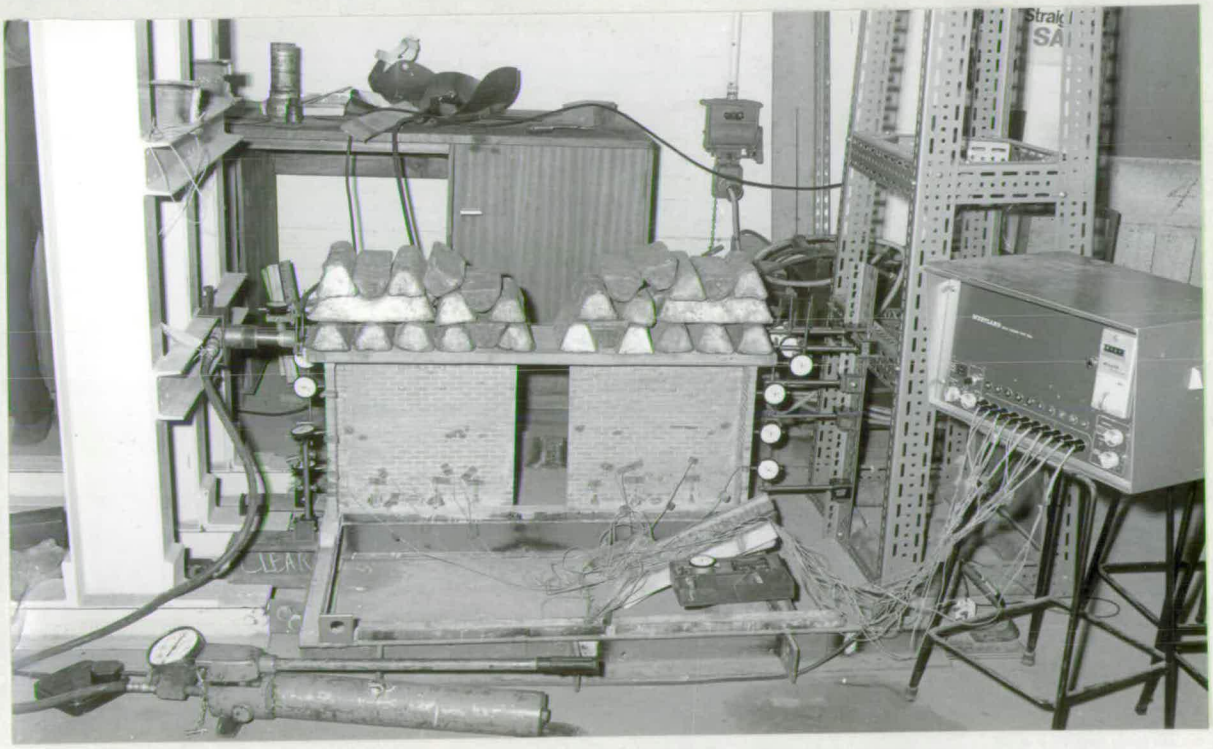


PLATE 4.2 - TEST ON SINGLE-STOREY STRUCTURE

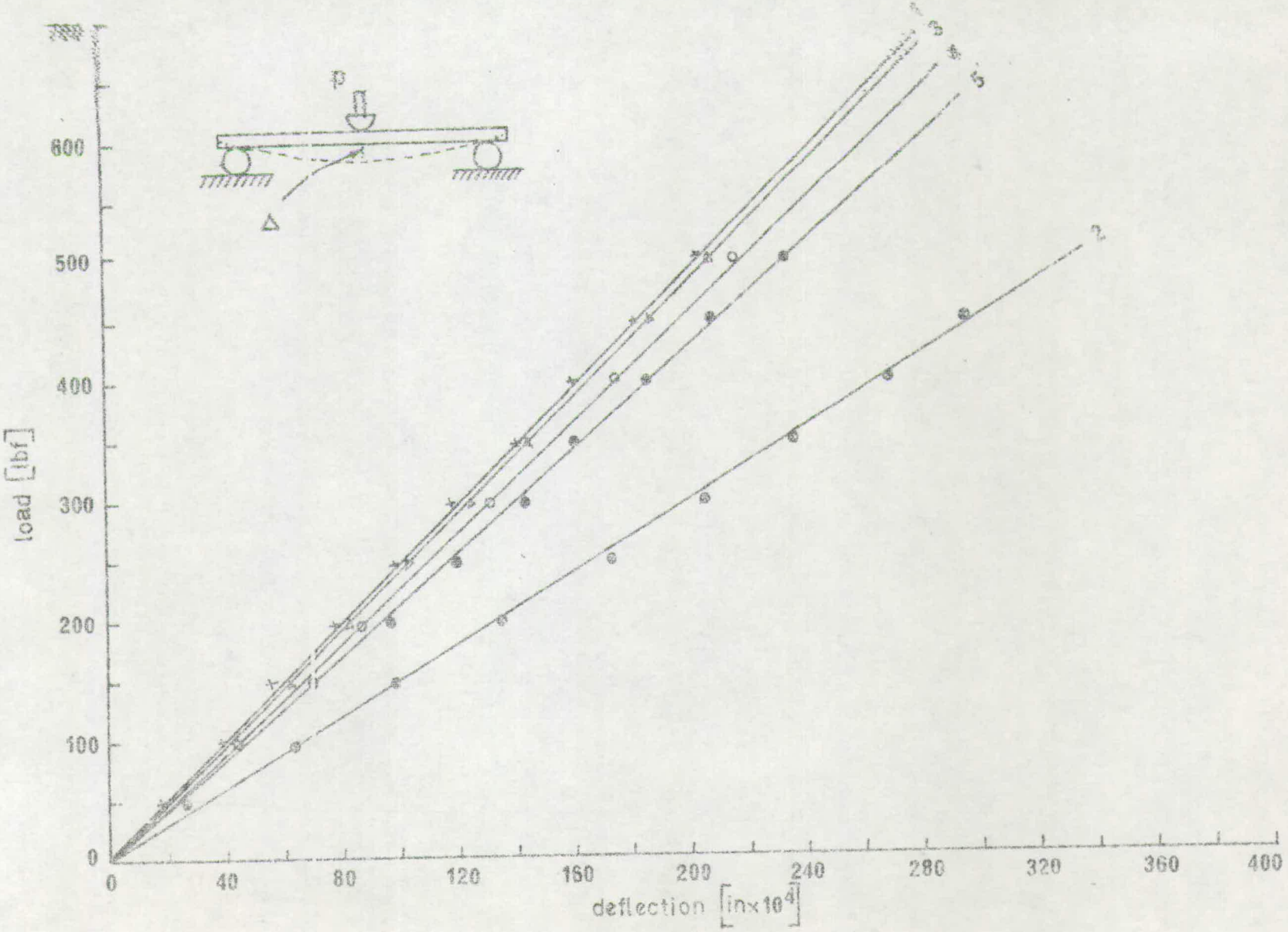


Fig.4.6 CALIBRATION CURVES FOR LOAD MEASURING BEAMS.

walls and by putting lead billets on the floor slabs (Plates 4.3 and 4.4). In this way the estimated precompressions were 50, 40, 30, 20 and 10 lbf/in^2. respectively at 1st, 2nd, 3rd, 4th and 5th storey.

4.6 EXPERIMENTAL INVESTIGATIONS

When the first storey was completed preliminary tests were carried out on it [30]. These tests were done after 28 days of curing time even though Ferrocrete is a rapid hardening cement. Racking load was applied at the floor level by a hydraulic jack (Plate 4.2) through the loading device described earlier. For preliminary tests loads were kept below one-fifth to one-quarter of the expected ultimate value. Deflections were measured at floor level and at quarter, half and three-quarter height of the storey. Figure 4.7 shows the load deflection curves for different precompressions. These curves were used later on to calculate the values of effective shear modulus of brickwork subjected to lateral load and precompression.

In testing the five storey structure loads were applied by jacks one at each floor level. All loads from 1st to 4th storey were equal; the load on the roof slab was equal to half of the load applied at other slab levels. These point loads may be considered to be equivalent to the wind load acting uniformly over the whole height of the building. Deflections were measured at each floor level. Figure 4.8 shows the load/deflection curves at various floor levels. Keeping the load at about quarter of the expected ultimate value the experiment was repeated three times. No significant differences were observed. Finally, the structure was loaded to destruction (Plates 4.3 and 4.4). The ultimate strength was found to be about 50 lbf/in.^{2}. As may be seen from Fig. 4.9 this is consistent with

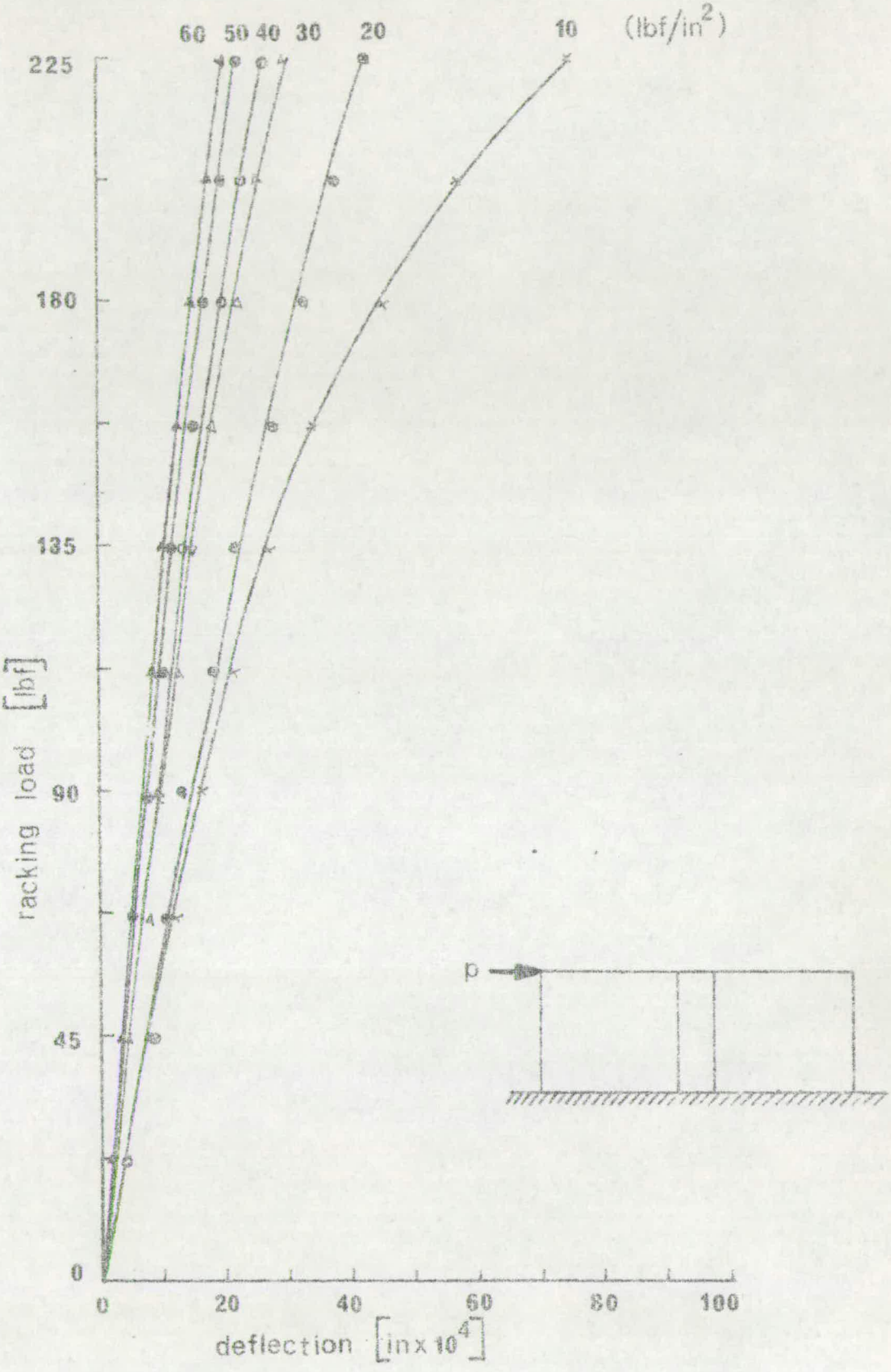
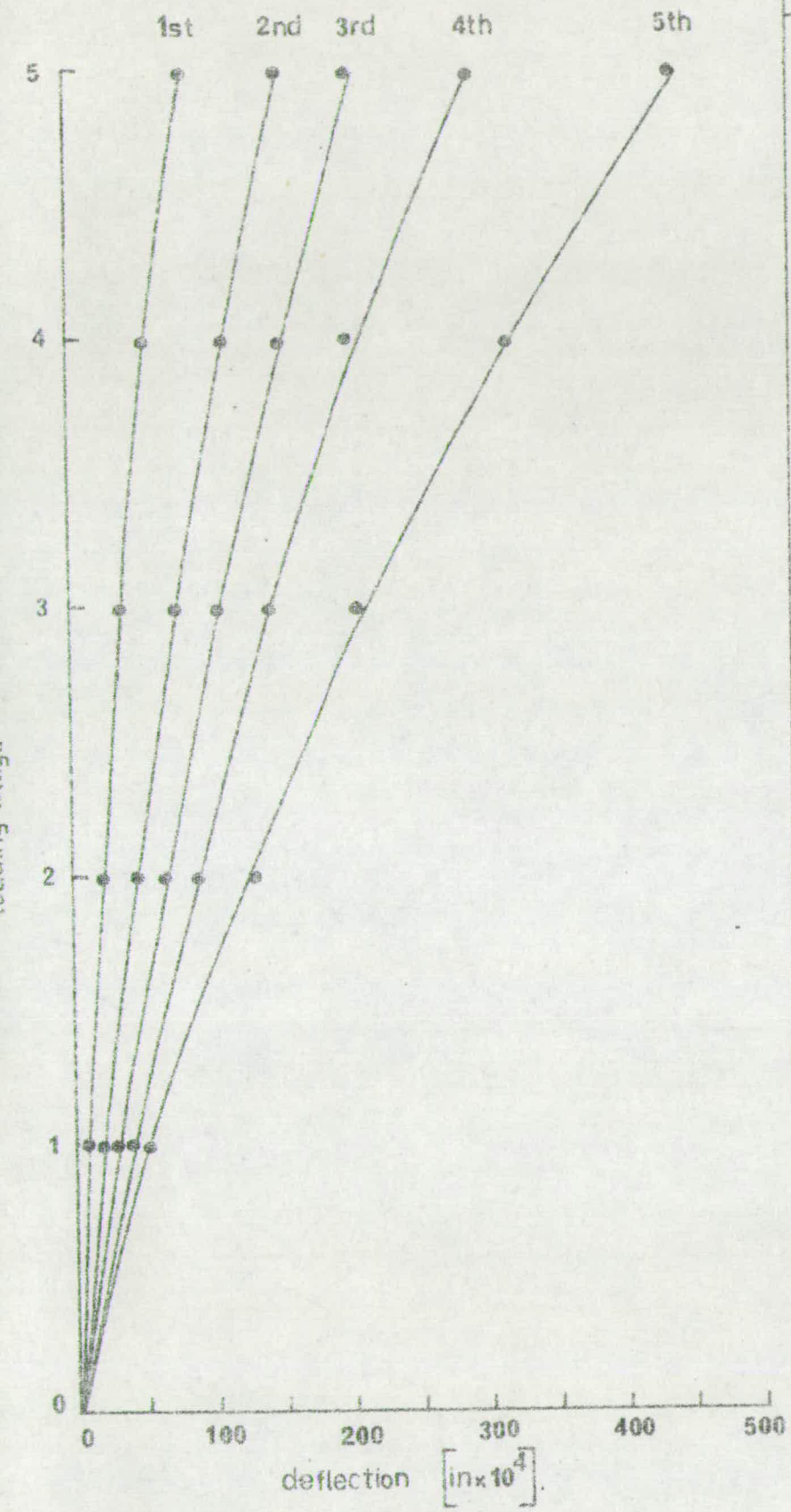


Fig.4.7 LOAD/DEFLECTION CURVES AT DIFFERENT PRECOMPRESSION.



1	2	3	4	5	stage
10	20	30	40	50	5h
20	40	60	80	100	4h
20	40	60	80	100	3h
20	40	60	80	100	2h
20	40	60	80	100	h
90	180	270	360	450	total (lbf)

Fig.4.8 LOAD/DEFLECTION CURVES AT VARIOUS FLOOR LEVELS.

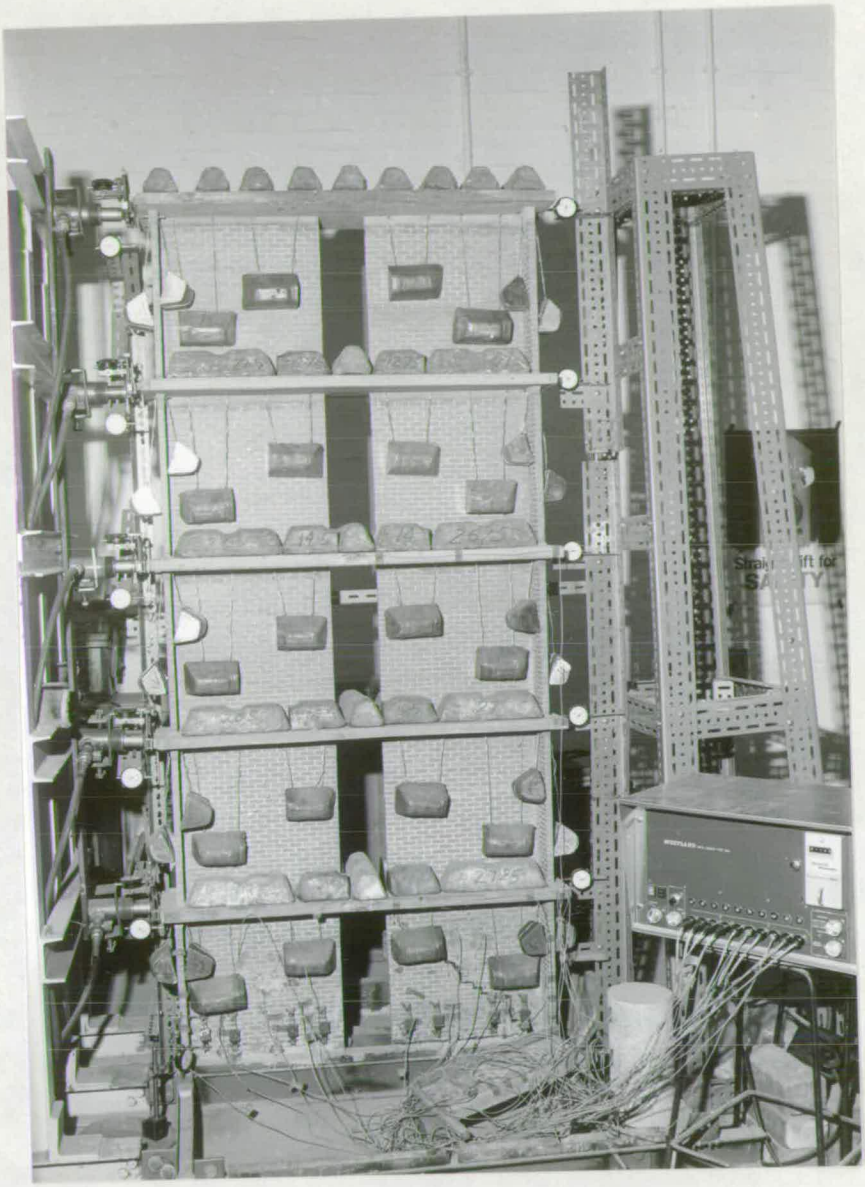


PLATE 4.3 - DETAILS OF TEST ON FIVE STOREY MODEL SHOWING FAILURE CRACK IN LOWERMOST STOREY OF CROSS-WALL

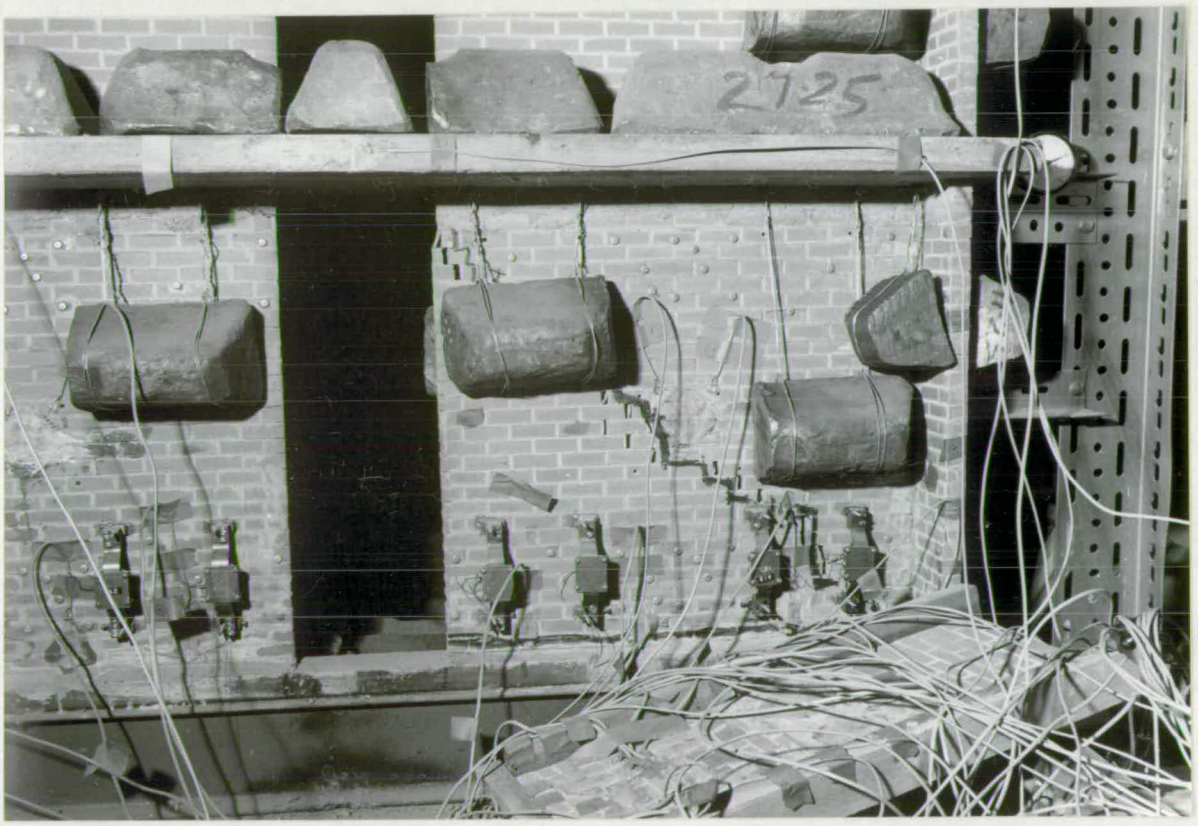


PLATE 4.4 - DETAILS OF ACTIVE AND DUMMY GAUGES

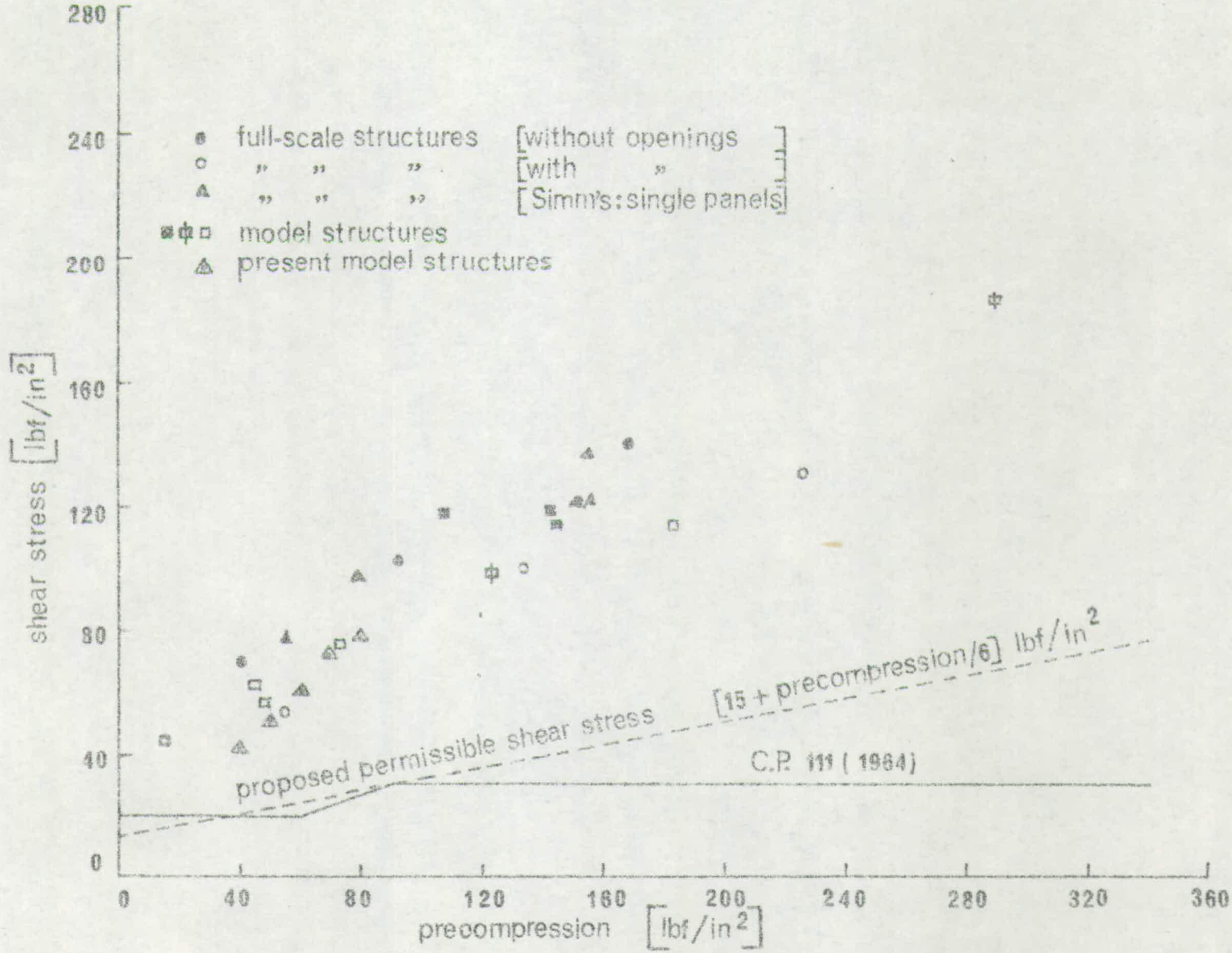


Fig.4.9 VARIATION OF SHEAR STRENGTH OF BRICKWORK WITH PRECOMPRESSION.

previous test results in model and full scale structures[26].

A dial gauge was placed near the base in order to check whether there was any rotation of the base frame. The rotation measurements are embodied in Fig. 4.10, from which it will be seen that at higher loads rotation is very significant. The experimental results when compared with the analytical ones (Fig. 4.11) were corrected for appropriate amount of rotation. Fig. 4.12 shows typical deflection pattern for shear wall and frame.

4.7 MEASUREMENT OF STRAINS

Electric resistance strain gauges[27] were used to measure strains (Plate 4.4). A description of the type of strain gauges and the strain recording equipment has appeared in Appendix II.

4.8 DESIGN LOADING AND FACTOR OF SAFETY

According to C.P.3: Chapter V (1952)[9] the basic wind load for an equivalent prototype building of 42'-6" height is 17 lbf/ft.^{2} (exposure D). In addition, the shear walls are supposed to resist a lateral load equal to $2\frac{1}{2}\%$ of the total load carried by the return walls, which is equivalent to 1.6 lbf/ft.^{2}. Therefore the structure must be designed to resist a lateral load of 18.6 lbf/ft.^{2} on the exposed area. The total load of 180 lbf. applied to the model structure is equivalent to the above pressure in a prototype building assuming bay width equal to 8 ft. The ultimate failure load was about 1000 lbf. giving a factor of safety of 5.5. According to C.P.111 (1964)[7], the factor of safety comes to be 3 and it reduces to about 2.3 when the proposed revised permissible shear stress is used (Fig. 4.9).

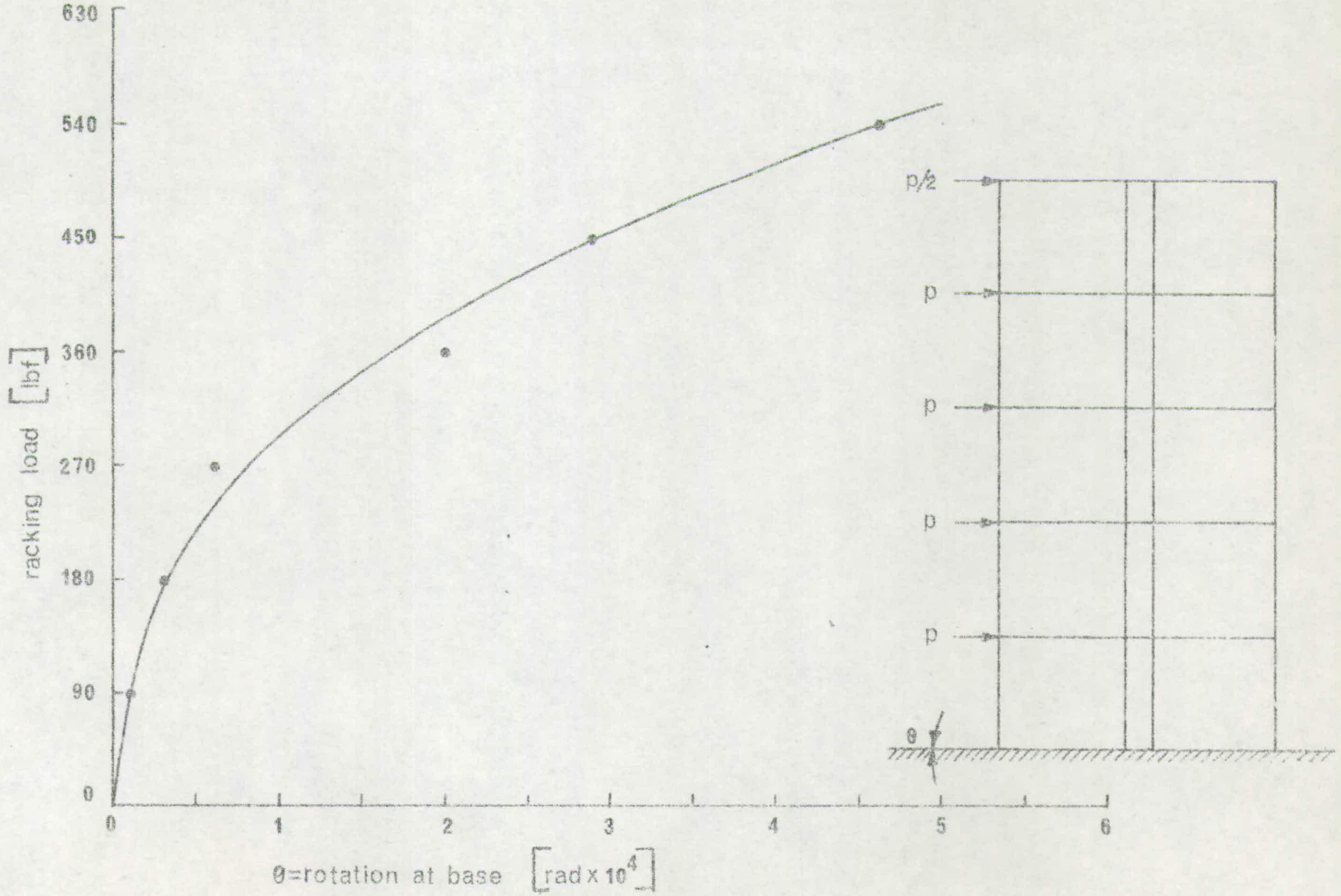


Fig.4.10 ROTATION AT BASE OF STRUCTURE .

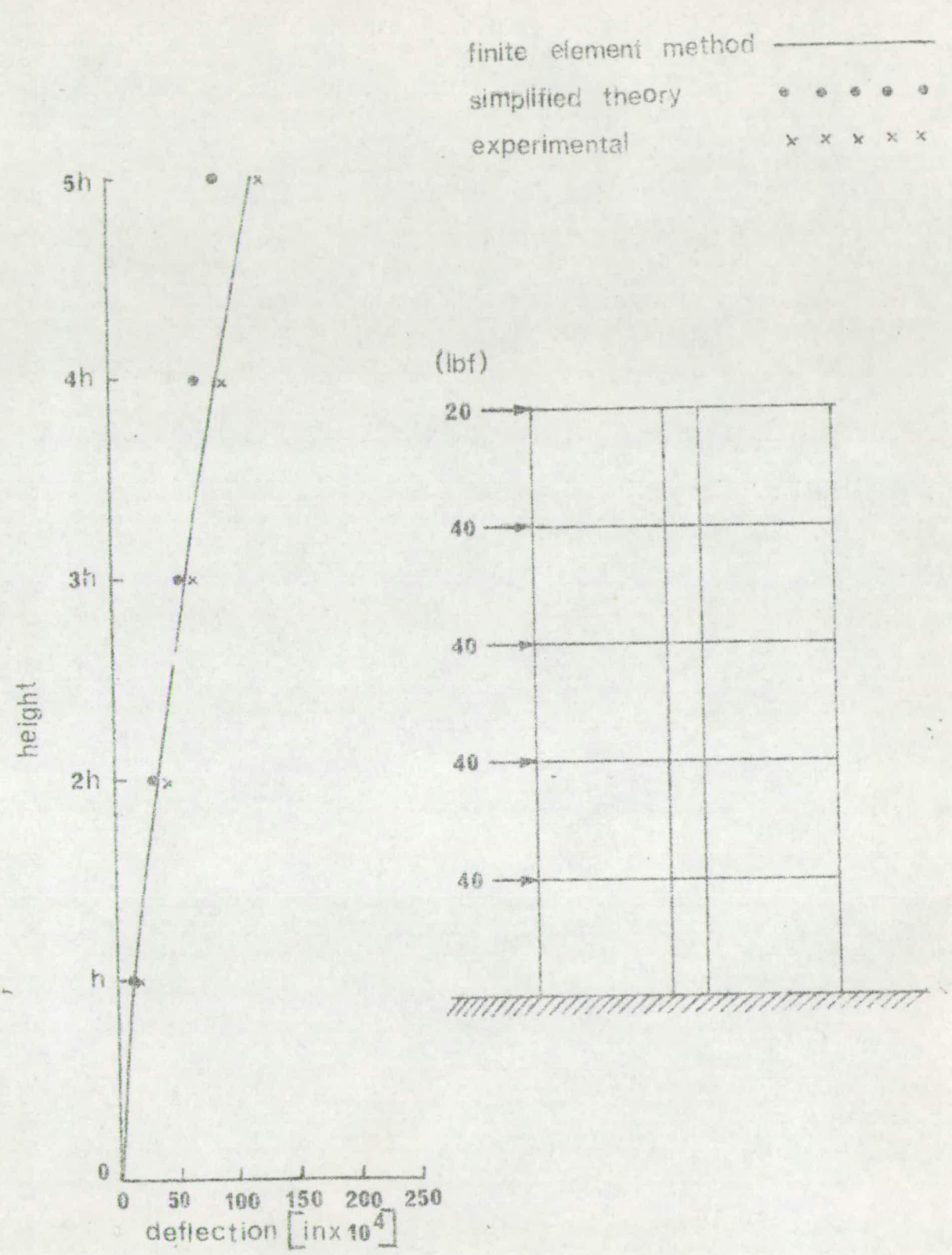
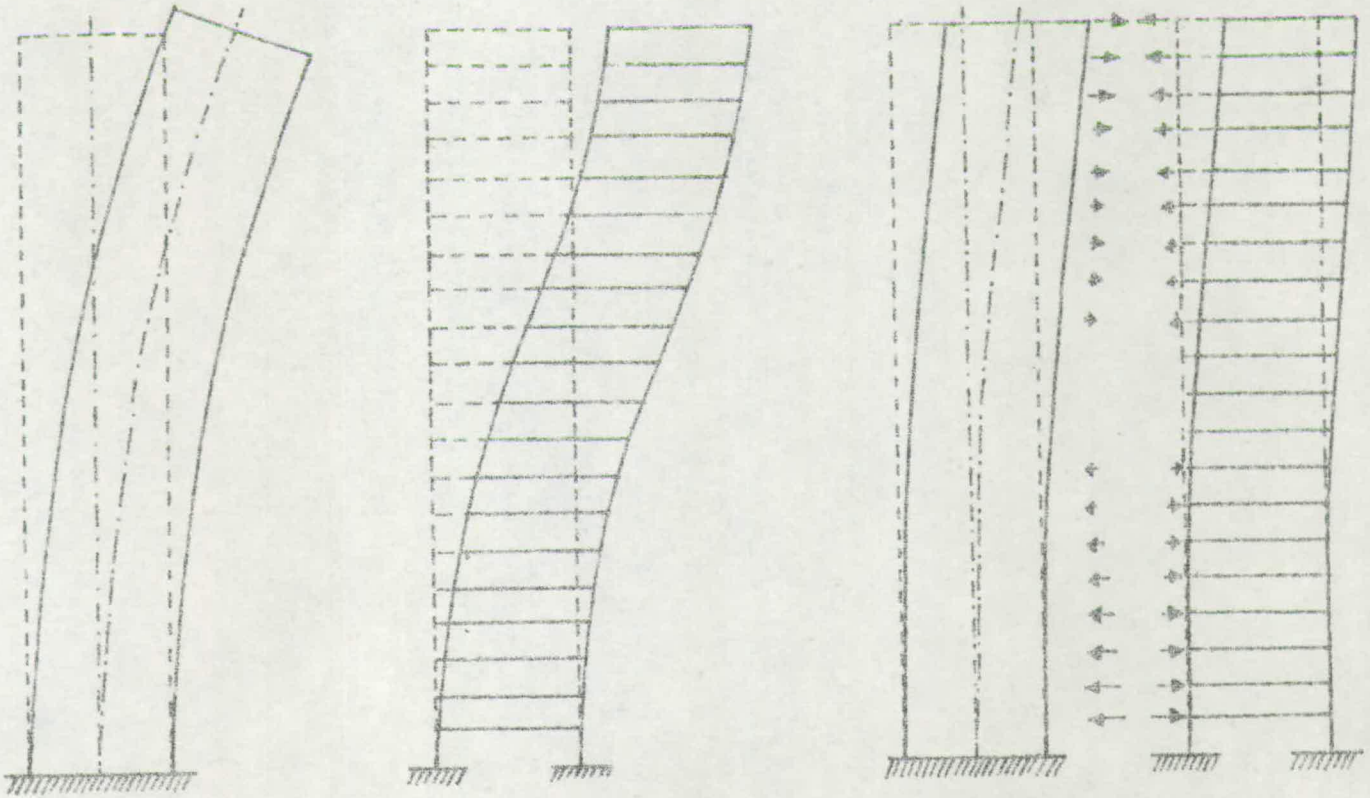


Fig.4.11 COMPARISON OF DEFLECTION OBTAINED FROM EXPERIMENTAL AND ANALYTICAL METHODS.



(a) shear wall

(b) frame

(c) combination of shear wall and frame

Fig.4.12 DEFLECTION PATTERN DUE TO LATERAL LOADING.

In the light of Ronan Point Tribunal report, the British Standard Institution issued a draft for the revision of the wind loading clauses in C.P.3: Ch.V: 1952.

According to this draft, the basic wind speed for Edinburgh area is about 48 metre/sec. i.e. 108 m.p.h. In order to find the design wind speed this basic wind speed must be multiplied by three other factors. They are as follows:

1. Topography factor which can be taken as 0.9, assuming the building to be constructed in a built-up-area.
2. Ground roughness, building size and height factor. This may be taken as 0.95 for 2 sec. gust and surface category 3.
3. Building life factor. This is taken as 1.1 assuming probability level 0.63 and the life of the building about 100 years.

So the final design speed comes out to be about 100 m.p.h. which is equivalent to a pressure of 25.6 lbf/ft.^{2}, instead of 17 lbf/ft.^{2} calculated previously. Therefore the factor of safety given by the previous C.P.3: Ch.V: 1952 will be reduced to about 3.5.

It must, however, be emphasised that the model tested represents a building in construction, in other words before occupation. No live load was considered in the analysis. In fact, lateral resistance will increase when live load is present as it will increase the precompression.

4.9. THEORETICAL ANALYSIS

4.9.1 DETERMINATION OF THE VALUES OF SHEAR MODULUS

As mentioned earlier, preliminary tests were carried out on the storey height structure. Following Benjamin[3] the values of G were calculated from the following equation using the curves of Fig. 4.7.

$$\Delta = \frac{ph^3}{12\,EI} + \frac{1.2\,ph}{AG} \qquad \text{.....(4.1)}$$

where

Δ measured deflections from the linear portions of the curves (Fig. 4.7).

p racking load

h storey height

$E = 2G(1 + \nu)$, $\nu = 0.1$ (assumed)

ν Poisson's ratio

$I = I_1 + I_2$ moment of inertia of the shear walls

$A = A_1 + A_2$ area of shear walls, flange neglected

G shear modulus

The variation of shear modulus with precompression is illustrated in figure 4.13. Similar variation in the case of concrete observed by other researcher[60] is shown in Fig. 4.14.

4.9.2 SIMPLIFIED THEORY

The theory suggested earlier[47] was used to calculate deflections. Deflection at each storey was calculated separately from the following equation.

$$\Delta = \frac{(P_1 + P_2 + P_3 + P_4 + P_5)h^3}{12\,EI} + 1.2\,\frac{(P_1 + P_2 + P_3 + P_4 + P_5)h}{AG}$$

.....(4.2)

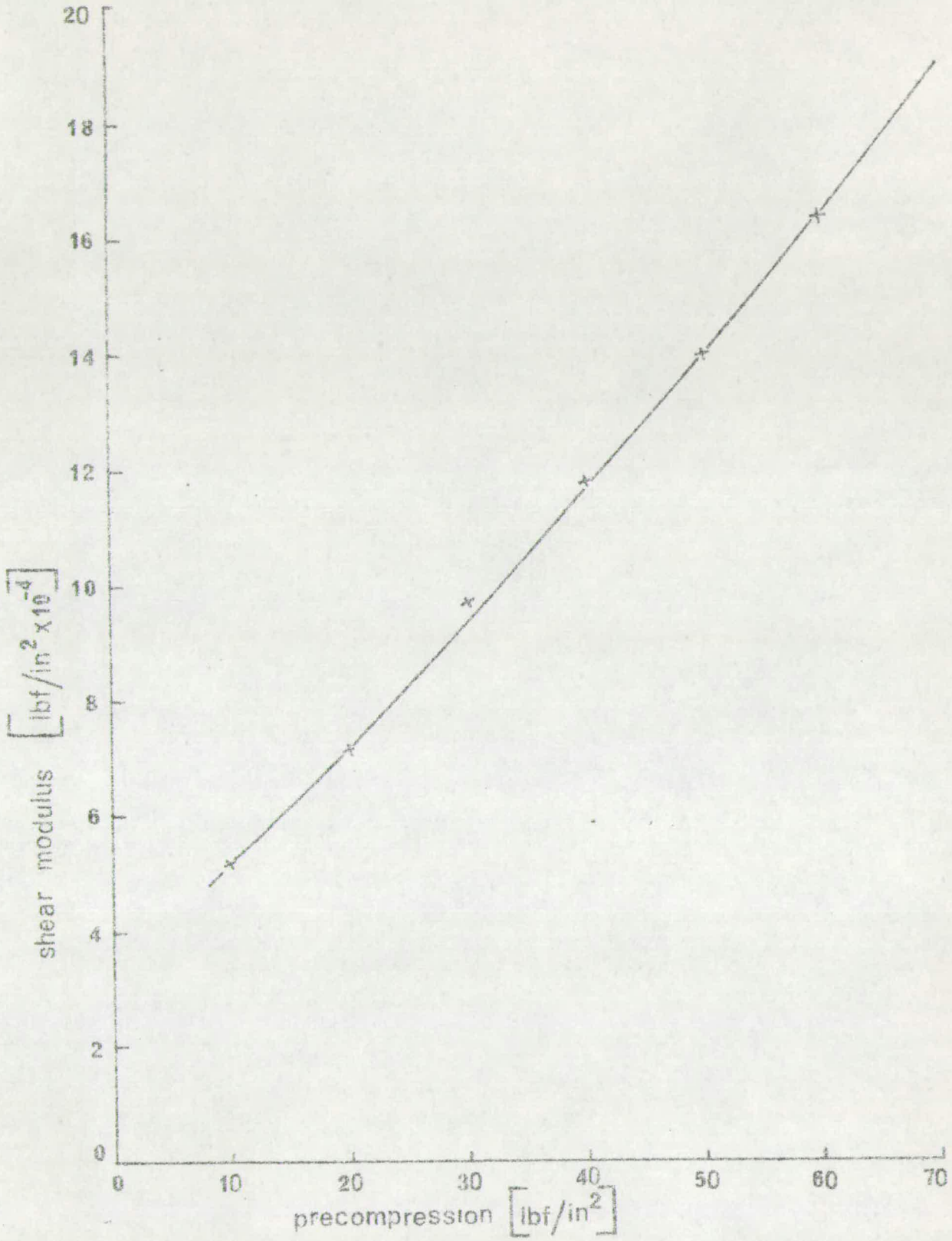


Fig.4.13 VARIATION OF SHEAR MODULUS WITH PRECOMPRESSION.

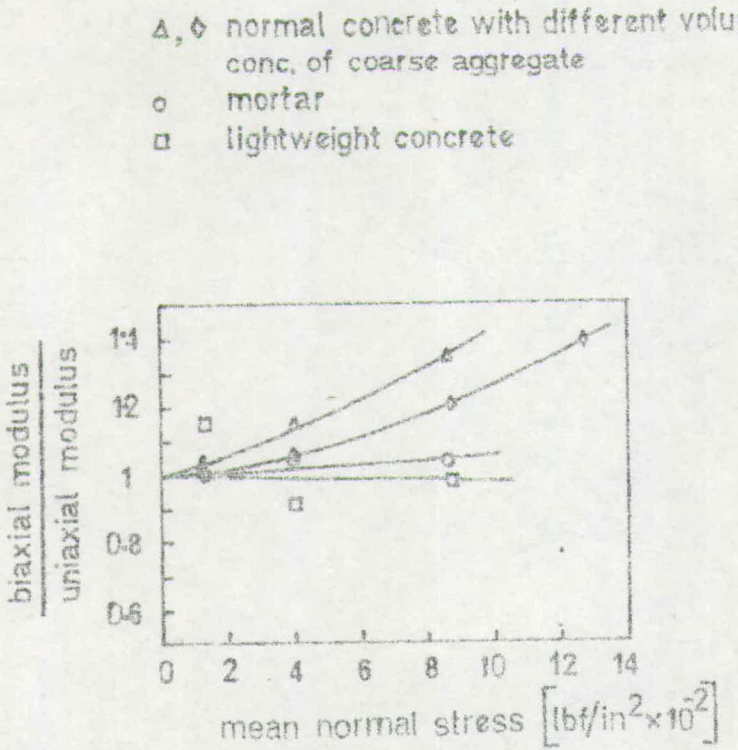


Fig. 4.14 VARIATION OF ELASTIC MODULUS WITH BIAXIAL STRESS.

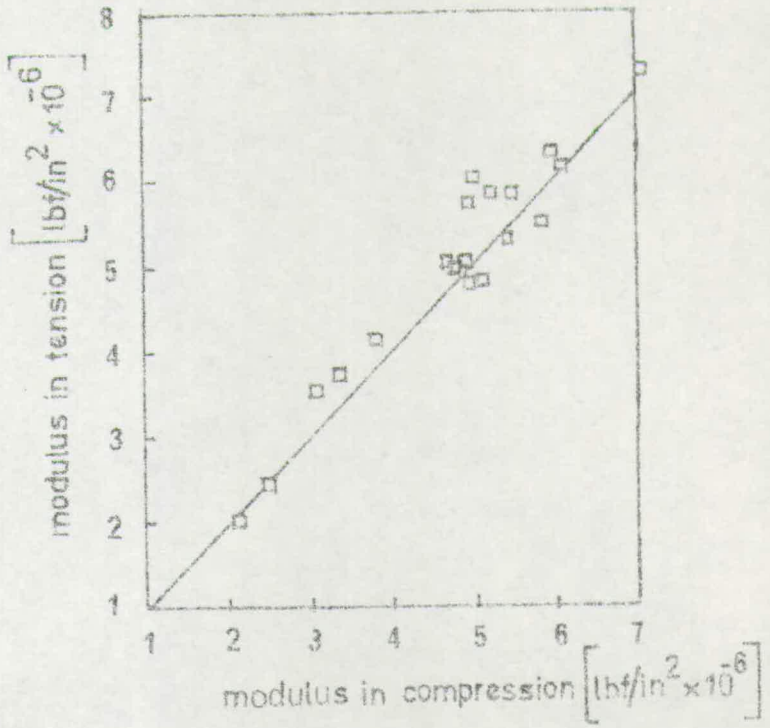


Fig. 4.15 STATIC ELASTIC MODULUS UNDER UNIAXIAL STRESS.

On the right hand side of equation 4.2 the only variables are E and G (known for each storey). Assuming values of Δ calculated from equation 4.2 as

$$\Delta_1, \Delta_2, \Delta_3, \Delta_4 \text{ and } \Delta_5$$

the deflections of the structure will be as follows:-

Deflection at first storey level, $\delta_1 = \Delta_1$

" " 2nd " " $\delta_2 = \Delta_1 + \Delta_2$

" " 3rd " " $\delta_3 = \Delta_1 + \Delta_2 + \Delta_3$

" " 4th " " $\delta_4 = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4$

" " 5th " " $\delta_5 = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5$

The results are compared with the experimental ones in figure 4.11.

4.9.3 THE FINITE ELEMENT METHOD

Although the above theory is convenient and simple for calculating the rigidity of a multi-storey brickwork structure, the need for a more sophisticated theory cannot be overlooked, especially to find the distribution of stresses. Earlier work[47] suggested that the existing shear wall theories, cannot be used, at least as they stand, for calculation of the rigidity of brickwork structures (Fig. 4.16).

The author has therefore analysed the model structure by the finite element method[61,62], the principles and theoretical developments of which were discussed in Chapter 3.

The "Stress" program for the Univac 1108 computer at the National Engineering Laboratory, East Kilbridge, Scotland was used.

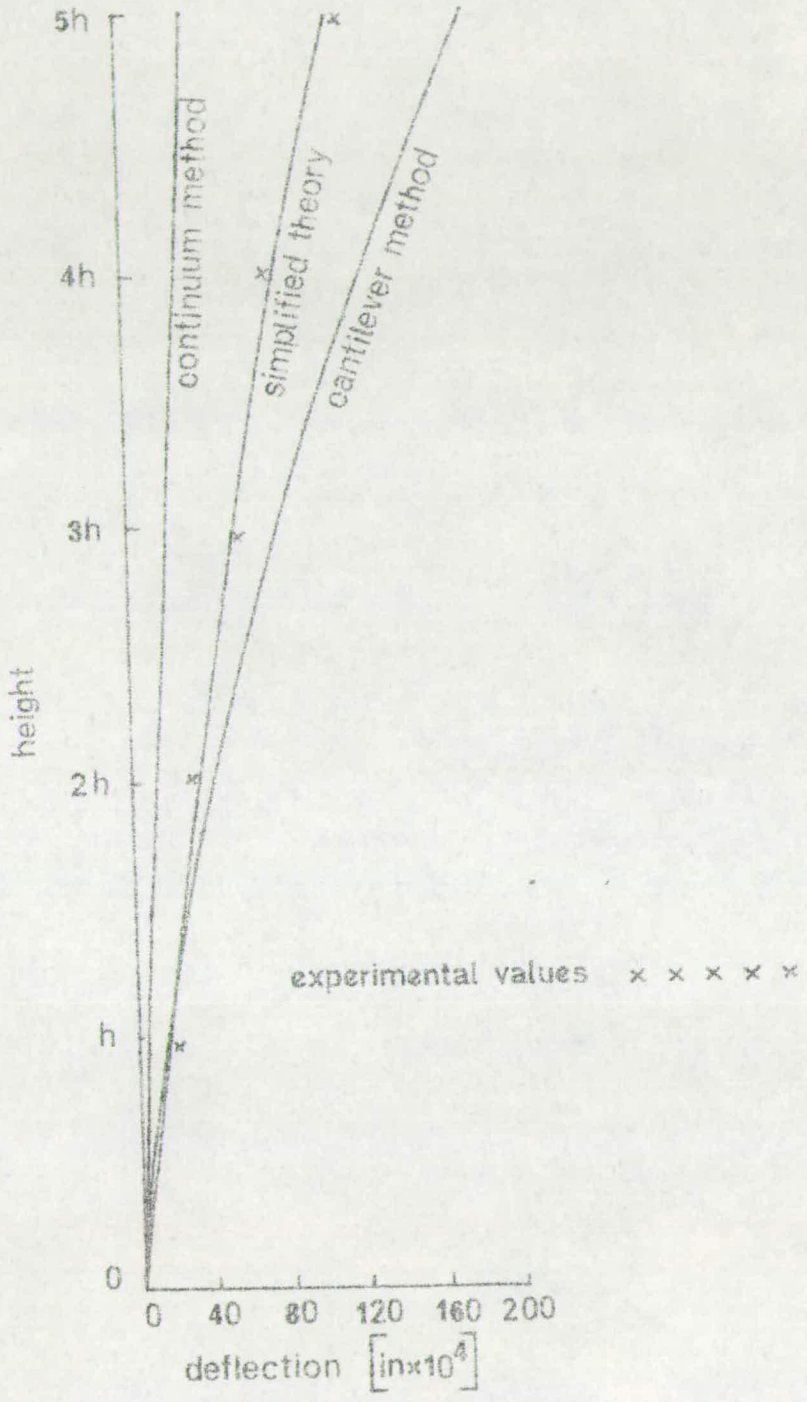


Fig.4.16 DEFLECTIONS OBTAINED BY EXPERIMENTAL AND ANALYTICAL METHODS.

A description of the"Stress"program and the procedure for the preparation of input data has appeared in Appendix I. Since the model structure was not of uniform thickness throughout, it was necessary to reduce the width of the flanges (i.e. the lengths of external walls) and the slabs to that of the cross-walls. To do this the E and G values of the corresponding elements were increased by appropriate factors. In doing so, the final values of deflections are not affected but stresses are to be reduced by the same factors to get the correct ones. The values of E and G for elements of each storey are those calculated from equation 4.1 and Fig. 4.7.

Figure 4.17 shows how the structure has been divided into triangular finite elements.

Deflections obtained by the finite element method are plotted in figure 4.11 for comparison with the experimental results. Figures 4.18 and 4.19 show the distribution of vertical stresses in sections $x_1 - x_1$ and $x_3 - x_3$ respectively. The distribution of shear at different sections has been shown in Fig. 4.20. The principal stresses were plotted using "Calcomp" plotter. Figure 4.21 shows the automatic plotting of principal stresses.

4.9.4 AN ANALYSIS OF A MULTI-STOREY FULL-SCALE CROSS-WALL STRUCTURE IN BRICKWORK USING THE FINITE ELEMENT METHOD

After obtaining reasonable results in the analysis of the model structure, the finite element method was used to analyse a prototype building built at Torphin quarry, Edinburgh (Plate 4.5). This is a five-storey brickwork building with R.C. slabs for floors and roof. Tests on this building were carried out by Maurenbrecher

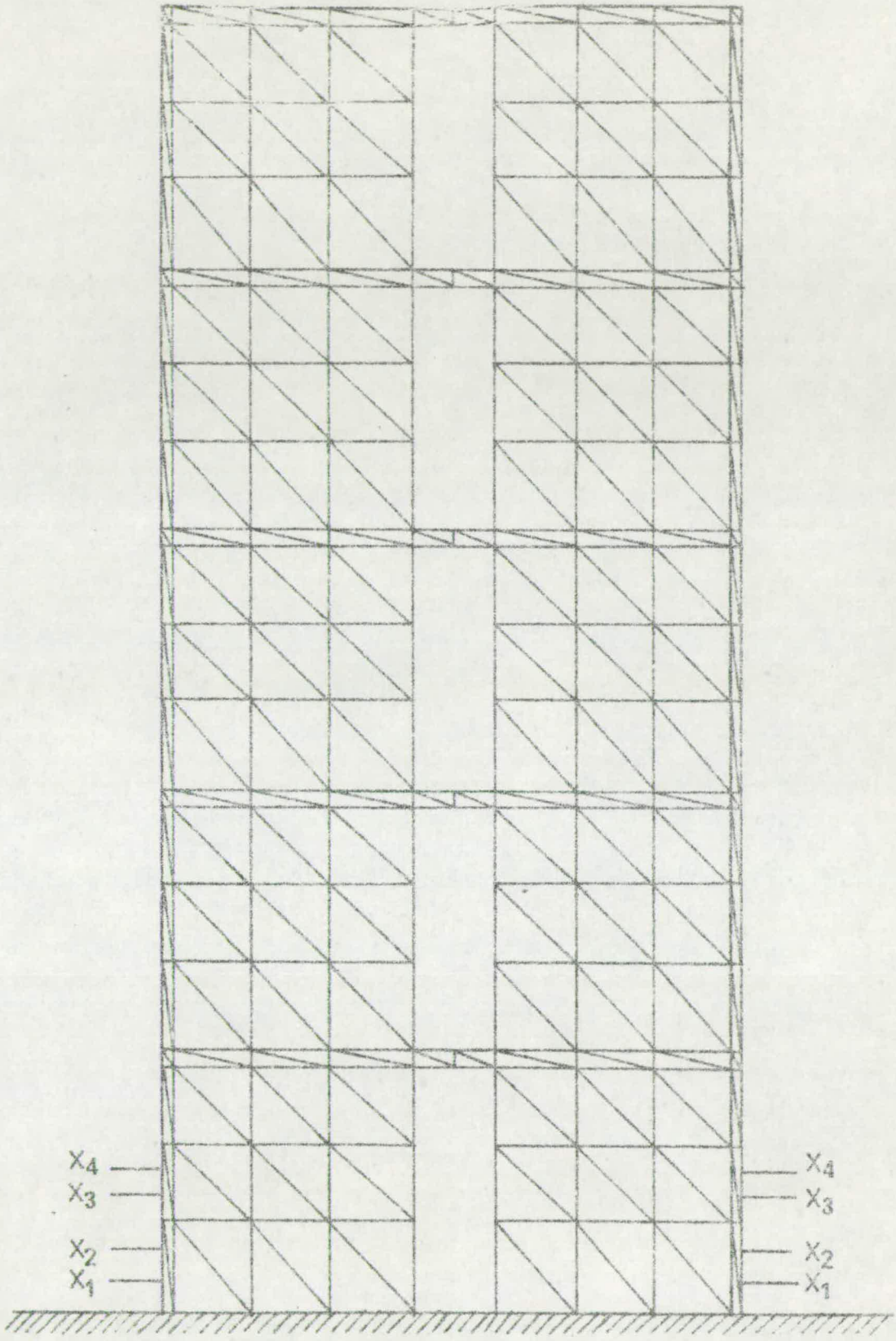


Fig.4.17 APPROXIMATE STRUCTURE DIVIDED INTO FINITE ELEMENTS.

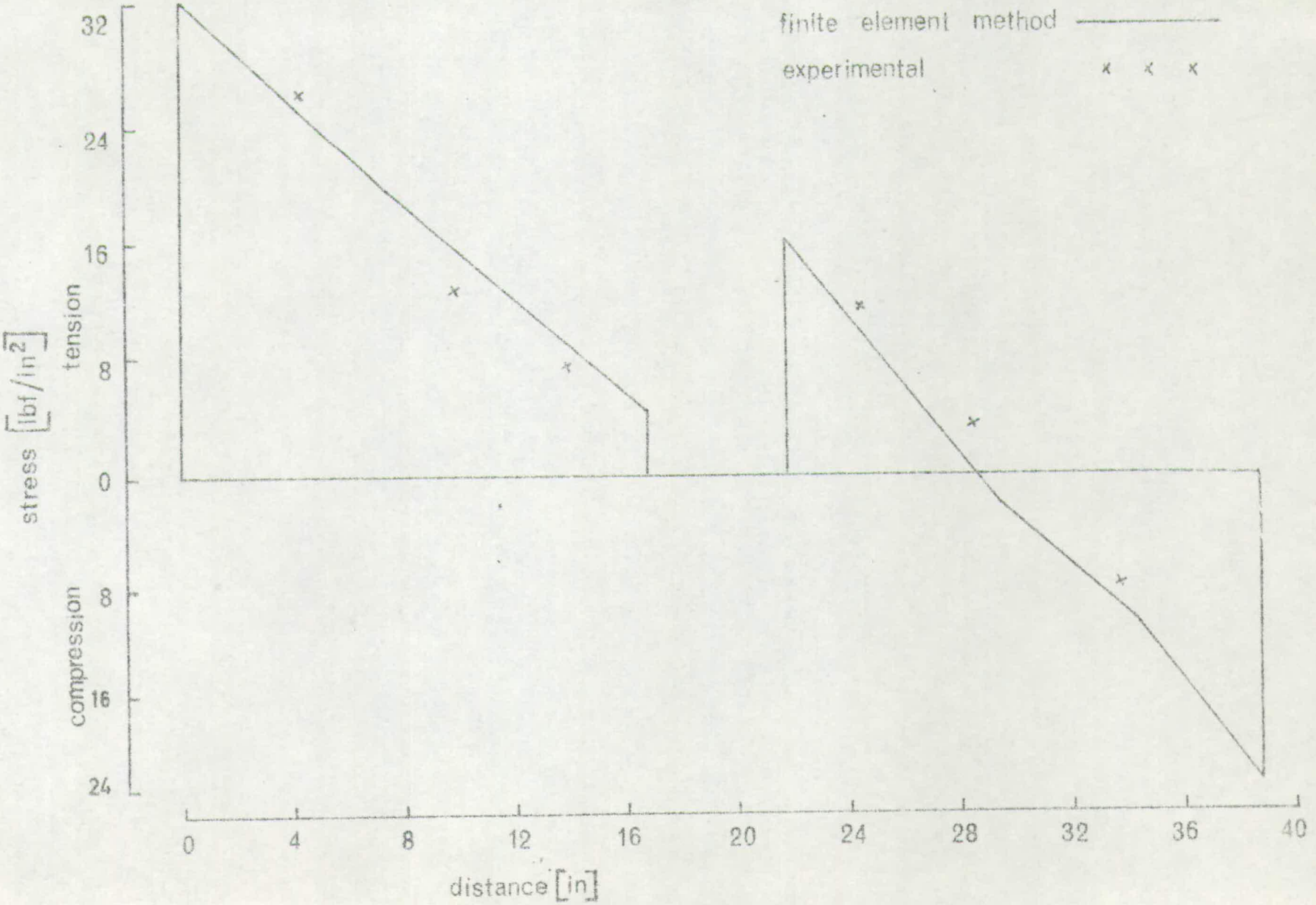


Fig.4.18 VERTICAL STRESS AT SECTION $X_1 - X_1$.

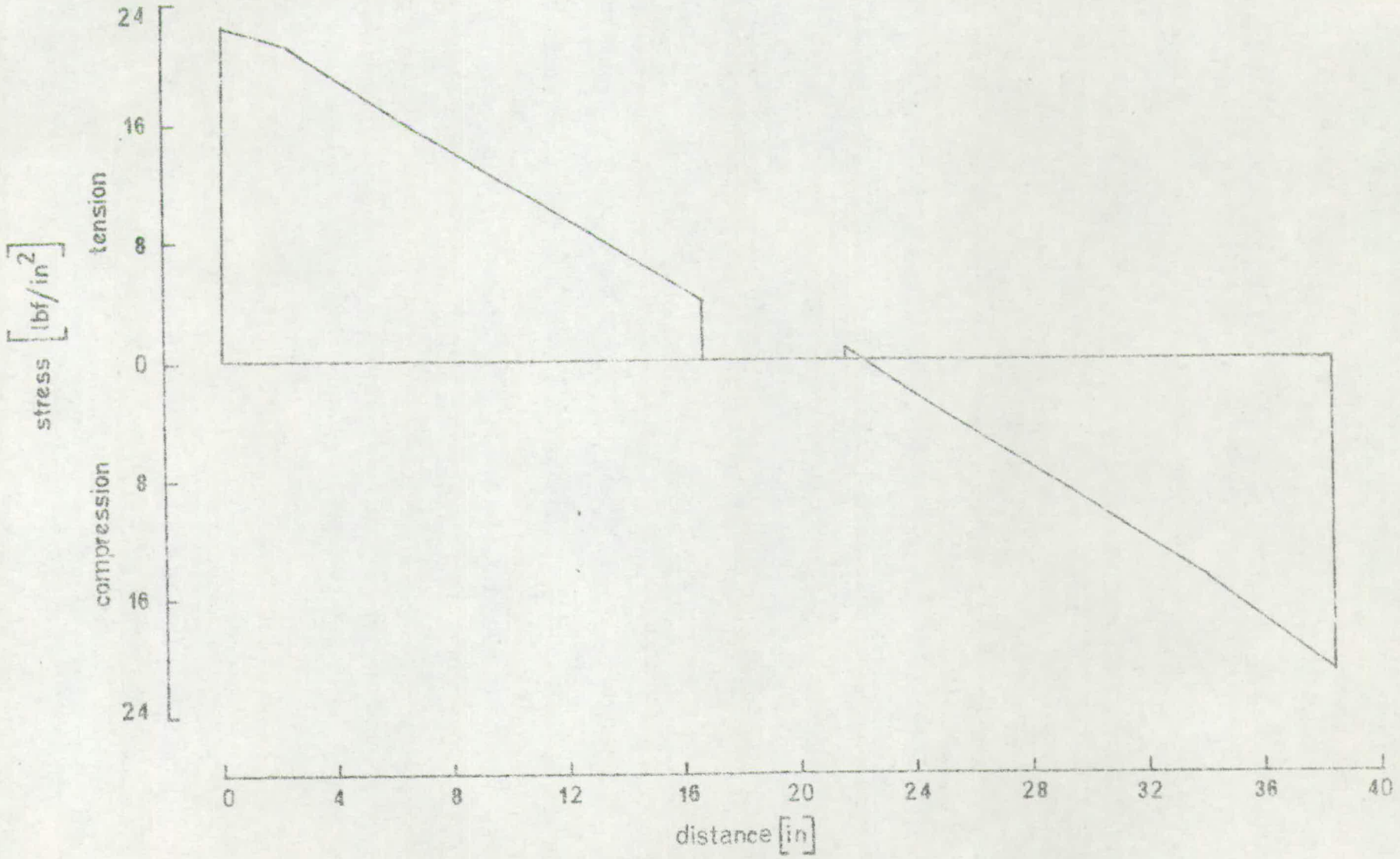


Fig.4.19 VERTICAL STRESS AT SECTION $X_3 - X_3$.

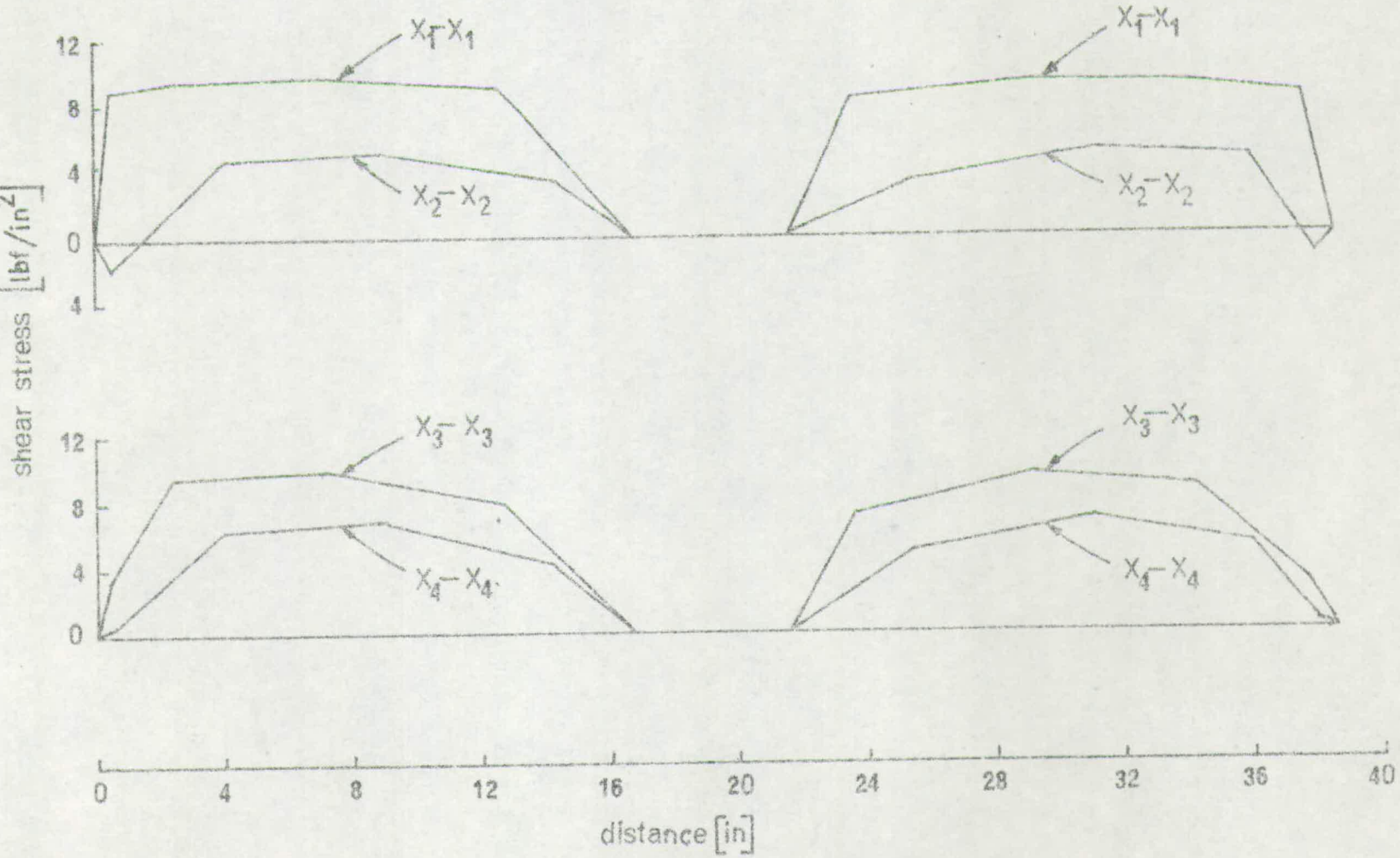


Fig.4.20 DISTRIBUTION OF SHEAR AT DIFFERENT SECTIONS.

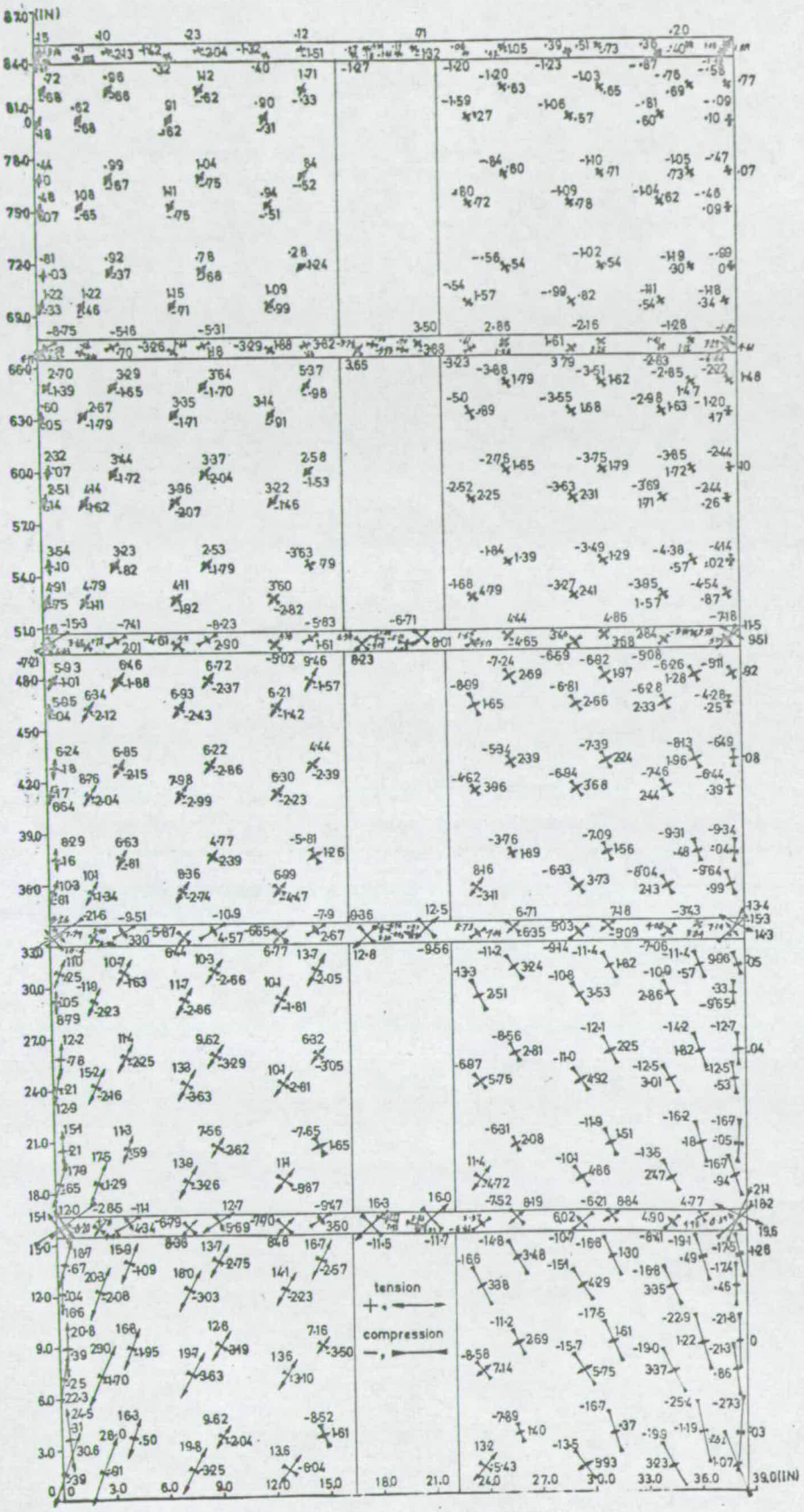


Fig. 4.21 AUTOMATIC PLOTTING OF PRINCIPAL STRESSES.

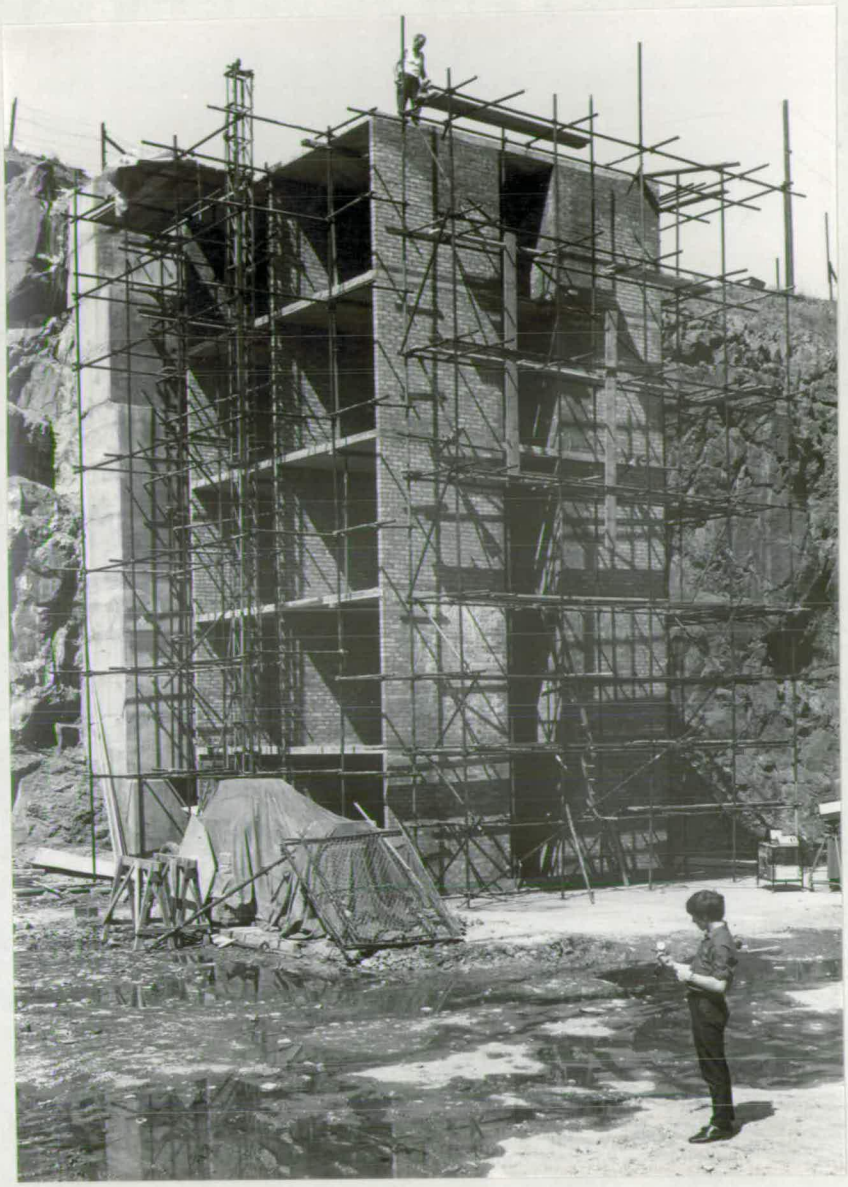


PLATE 4.5 - FULL-SCALE FIVE-STOREY CROSS-WALL STRUCTURE AT TORPHIN QUARRY, EDINBURGH

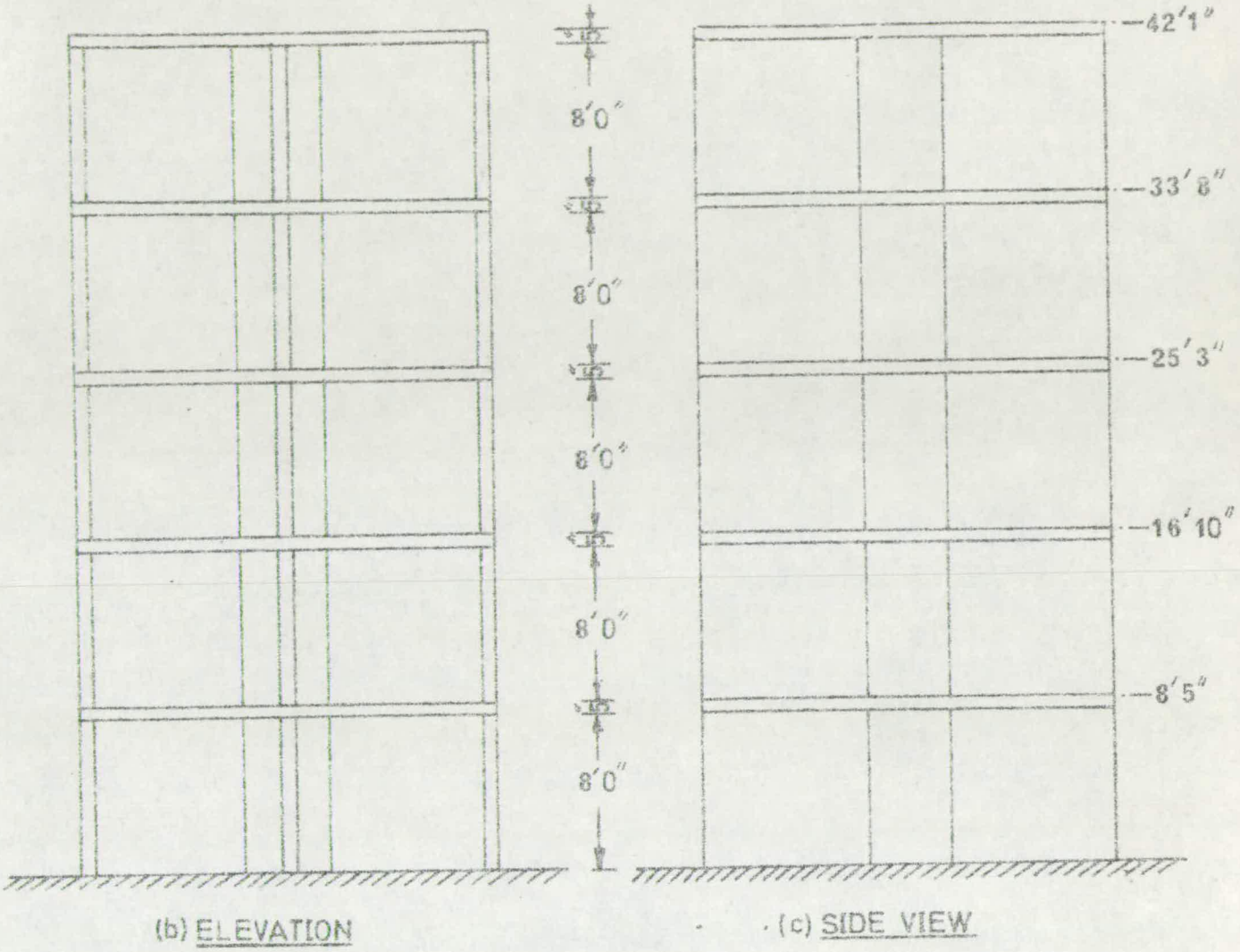
and Sinha under the direction of Professor A.W. Hendry to investigate the strength and rigidity of brickwork cross-wall structures subjected to lateral loading and to verify the model work done in this line in the Department of Civil Engineering and Building Science. The details of construction and testing of this building have been reported elsewhere[49].

Figure 4.22 shows the plan and elevation of the full-scale building. Since the building is symmetrical and load was applied symmetrically, half of the building was included in the analysis assuming half of the load would act on it. The appropriate width of slab and return walls (shown in broken lines on the plan) were included in the analysis. The building was analysed at an applied shear load of 10.2 lbf per sq. in. The estimated precompressions in the full-scale building were 67, 53, 39, 26 and 13 lbf/in^2 at 1st, 2nd, 3rd, 4th and 5th storey respectively. The values of G used were from Fig. 4.13.

Figure 4.23 shows the finite element subdivision of the structure. Deflections calculated are plotted in Fig. 4.24 which also shows the measured values. Vertical stresses at section $x_1 - x_1$ and distribution of shear at different sections are shown in Figs. 4.25 and 4.26 respectively.

4.10 DISCUSSION OF RESULTS

From figure 4.11 it is found that the deflection curve obtained from the finite element solution is reasonably close to the experimental results. The discrepancy of about 10% can be ascribed to the fact that the deformation characteristics of brickwork depend on many factors including brick and mortar properties, workmanship etc.



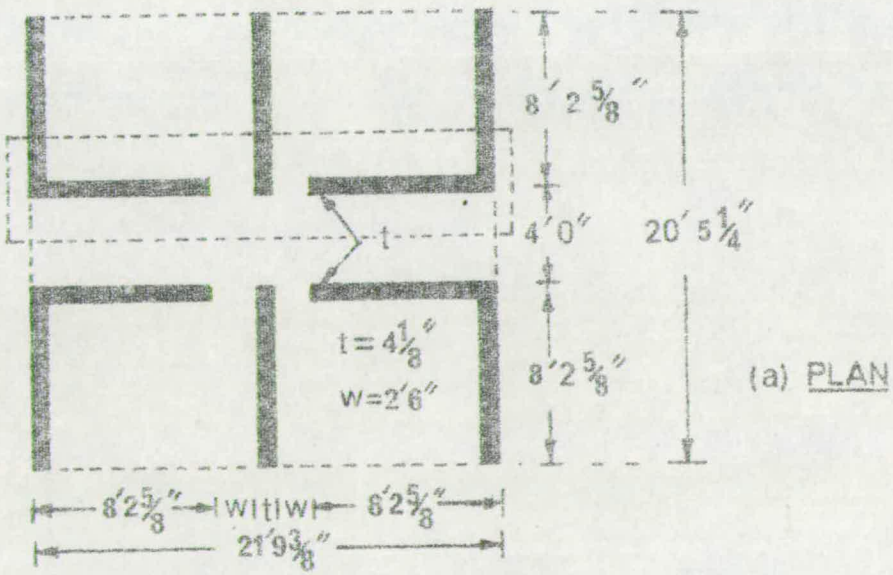


Fig.4.22 (a) PLAN (b) ELEVATION AND (c) SIDE VIEW OF A FULL-SCALE BUILDING.

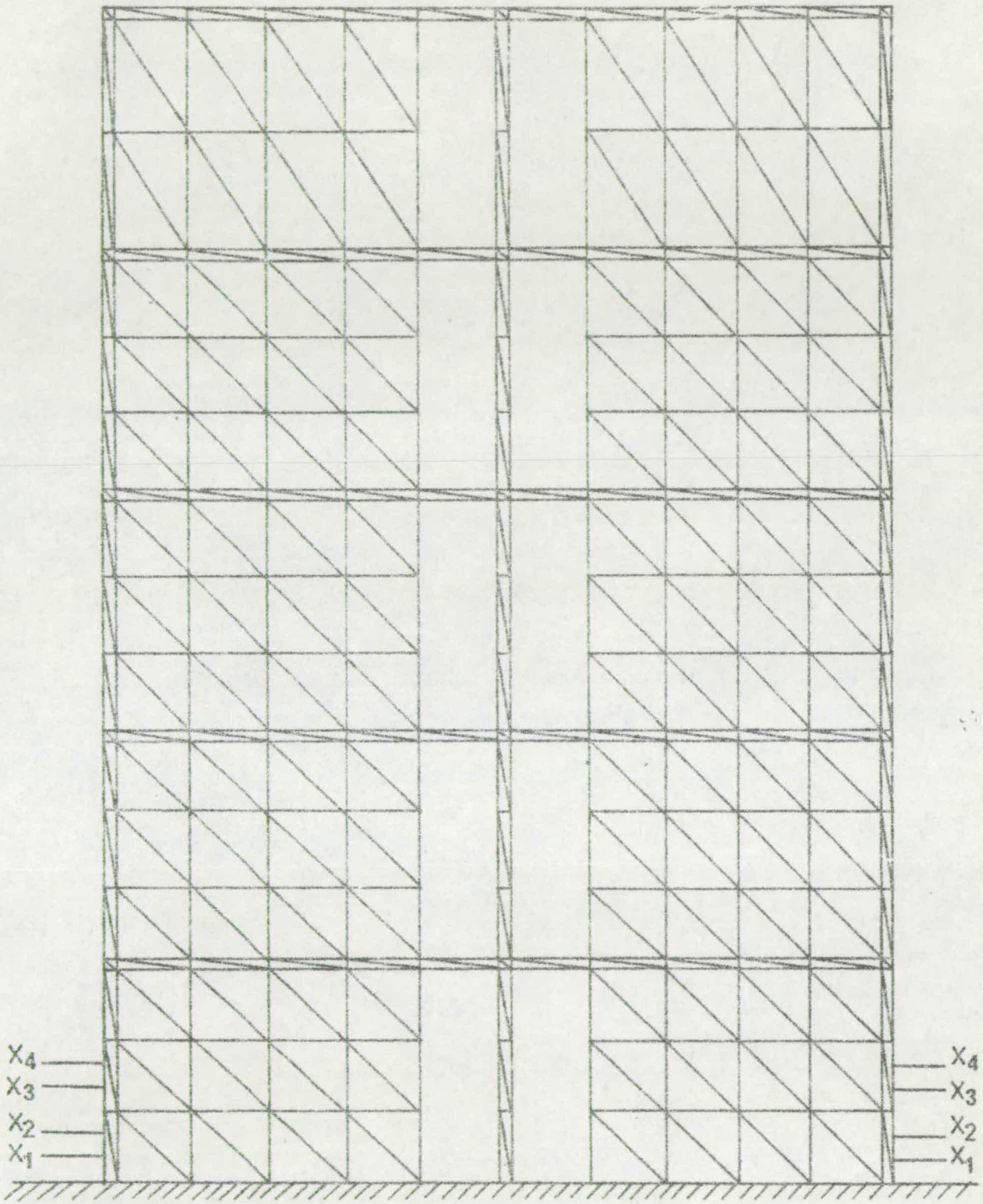


Fig.4-23 FULL-SCALE STRUCTURE DIVIDED INTO FINITE ELEMENTS.

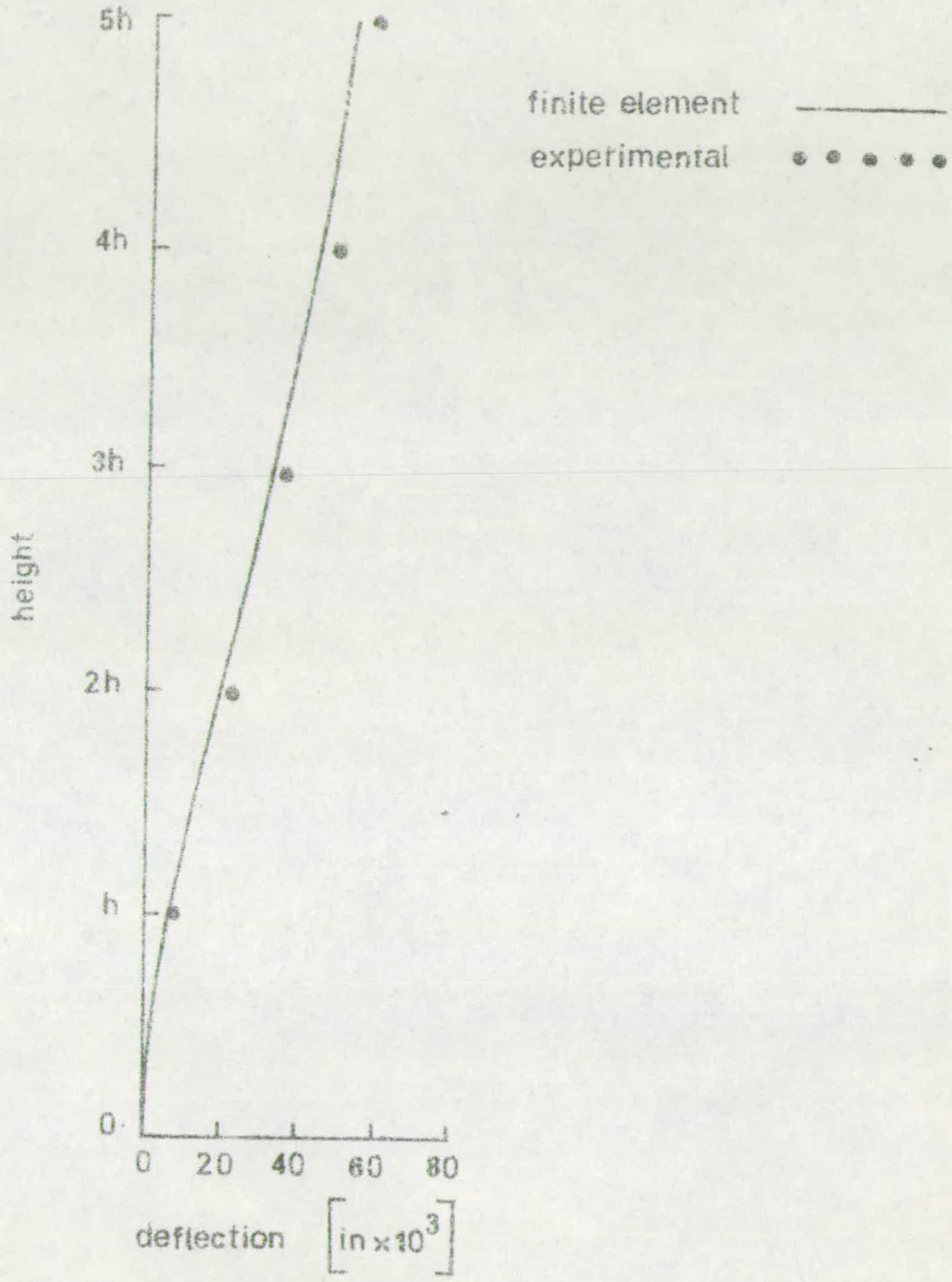


Fig.4.24 DEFLECTIONS FOR A FULL-SCALE BUILDING

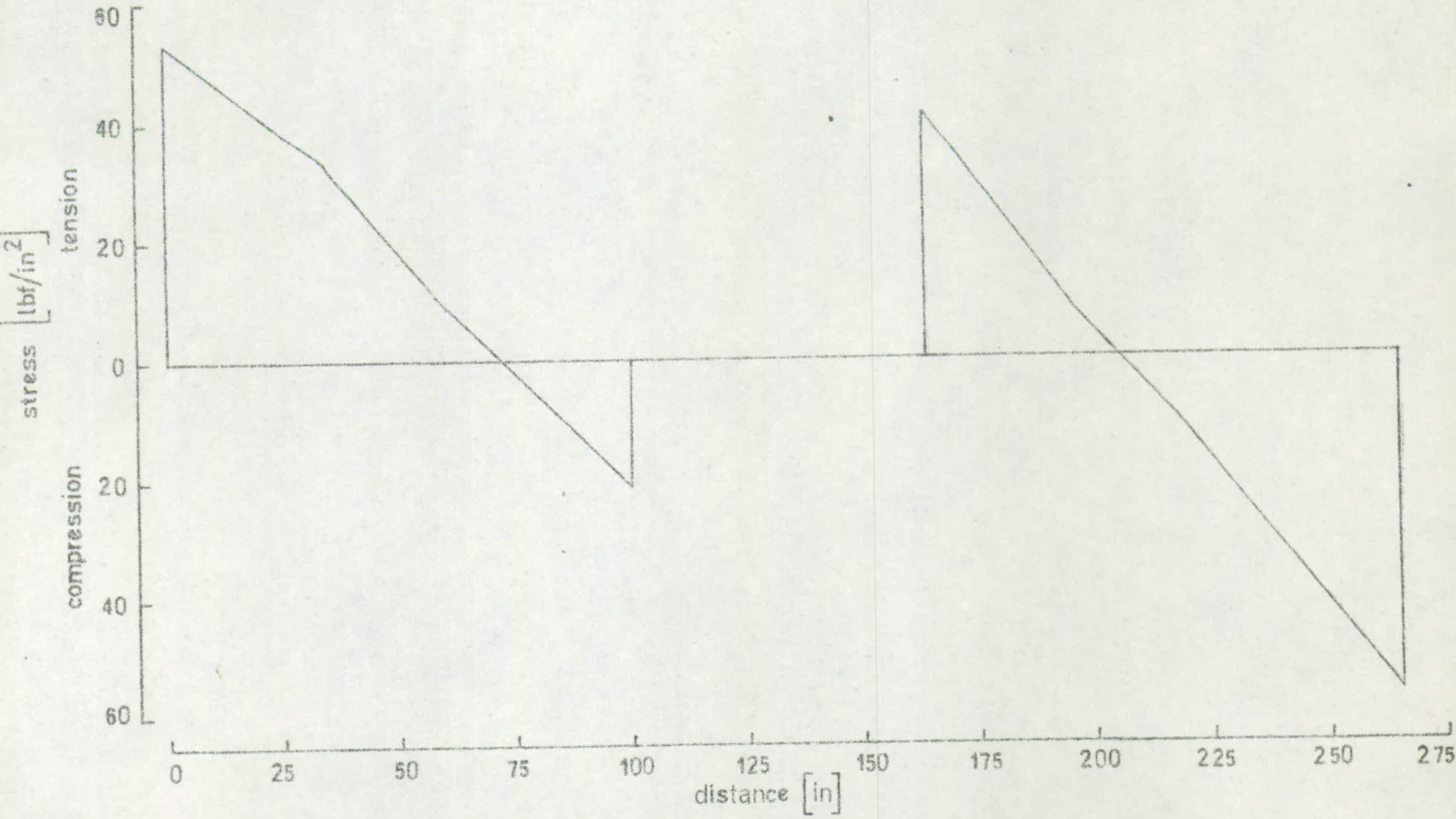


Fig.4.25 VERTICAL STRESS AT SECTION X_1-X_1.

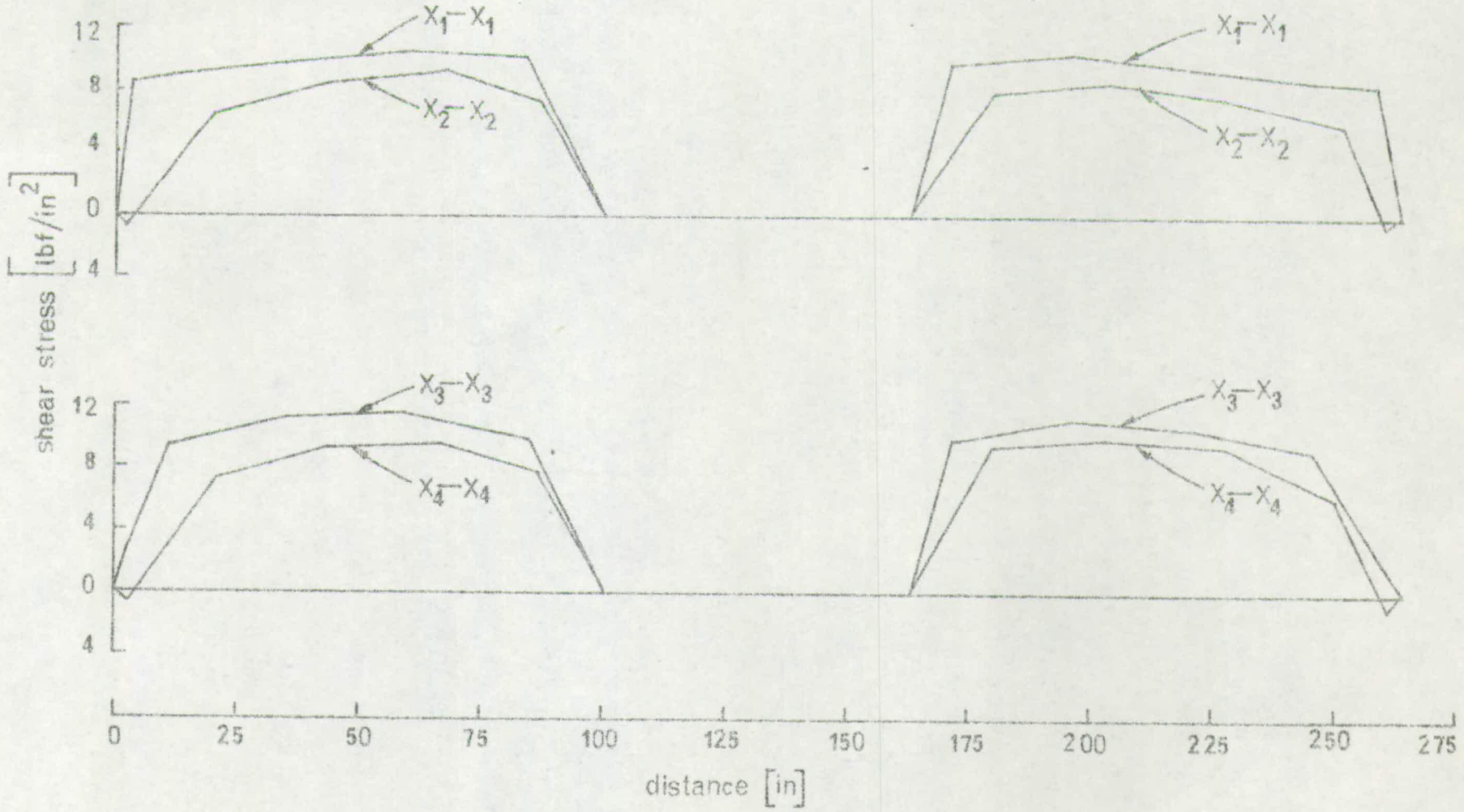


Fig.4.26 DISTRIBUTION OF SHEAR AT DIFFERENT SECTIONS.

The calculated deflections (Fig. 4.24) for the full-scale building differ from measured ones by about 13%, but the nature of both curves for model and full-scale are similar. They deviate slightly from the pattern of the idealised deflection curve for shear walls (Fig. 4.12), probably because of the presence of the openings.

From Figure 4.18 it is found that the calculated stresses are also close to the experimental values. However, there is a kink in the theoretical stress line in the compression zone; this may be because the section is taken very near to the fixed end where the assumptions of the stress distribution are satisfied only at nodal points.

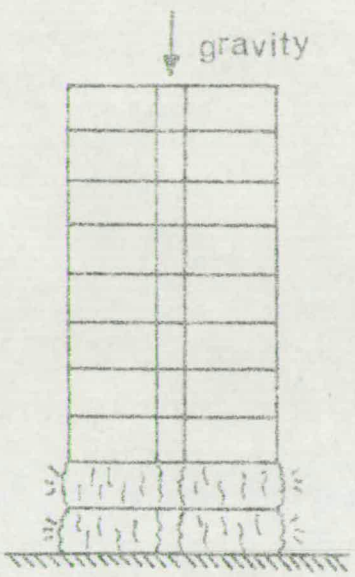
All the curves of the distribution of shear or vertical stresses for the model and the full-scale structure were drawn by joining the stress values at certain points, viz. the centres of gravity of the elements. Therefore making finer mesh more points for stresses could be obtained and hence a better prediction of stress distribution.

Recent research[23] on shear walls with openings shows that the distribution of stresses depends mainly on (1) type of loading (2) stiffness of connecting beams (3) width of opening. It has been found that as the width of opening increases coupled shear walls will tend to behave as independent cantilevers. Whereas they will tend to behave as a composite one as the width of opening decreases. These facts were observed in the present investigation. For the model structure the vertical stresses (Figs. 4,18, 4.19) showed the tendency of the structure to behave as a composite cantilever; because the width of opening was comparatively narrow.

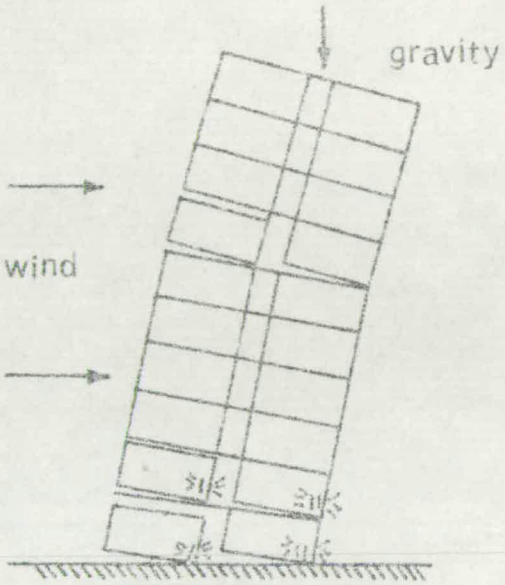
Whereas in the case of the full-scale structure (Fig. 4.25) the coupled walls showed the tendency of behaving independently.

The shear strength of brickwork constitutes bond shear and frictional shear, the latter being a function of precompression. As a result the shear strength of brickwork increases with precompression (Fig. 4.9). The shear modulus also varies with precompression. This was noticed in concrete also[60]. However, the variation was not as much as found in brickwork. For concrete with normal aggregate, the elastic constants were found to be a function of the mean normal stress under biaxial compression (Fig. 4.14)[60]. The increase appears to be dependent upon the volume concentration of coarse aggregate and contrasts with the equality of elastic modulus values obtained under uniaxial compression and tensile states of stress (Fig. 4.15). The variation of elastic modulus with mean normal stress confirmed that a particle interaction effect occurred. In brickwork also this interaction effect may occur between brick and mortar which increases with precompression.

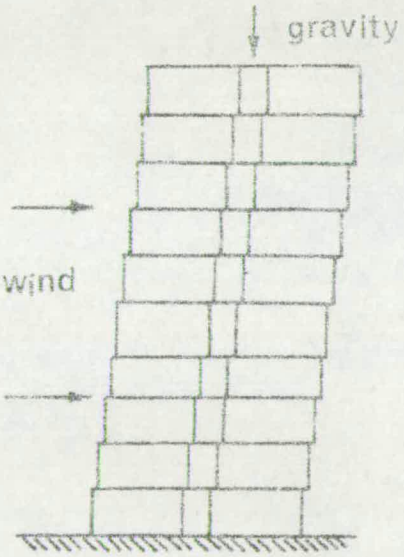
Failure of the multi-storey model structure was found to be in the first storey (Plates 4.3 and 4.4) due to the breakdown of bond and frictional resistance at the brick/mortar interface, leading to diagonal cracks stepping through the vertical and horizontal mortar joints. In one panel the diagonal crack was not very prominent. This may be because of the failure of the joint between 1st floor slab and wall. In any case the failure pattern is more or less similar to the case (d) of Fig. 4.27[11].



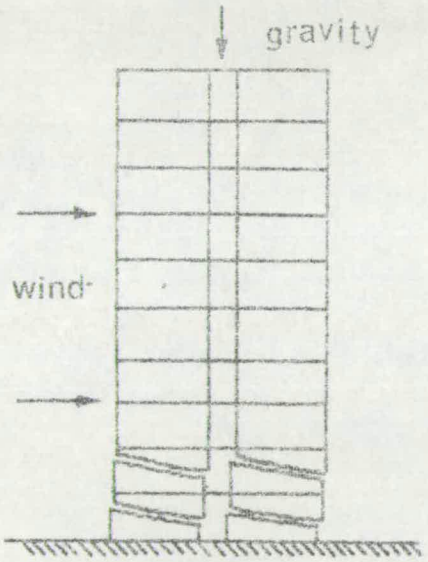
(a) compression in lower storeys



(b) tension or compression (at ends of walls)



(c) sliding of walls at floor levels



(d) shear failure in lower storeys

Fig.4.27 POSSIBLE MODES OF FAILURE IN MULTI-STOREY LOADBEARING BRICKWORK CONSTRUCTION.

4.11 CONCLUSIONS

It is possible to study the behaviour of a full-scale multi-storey brickwork structure subjected to lateral loading by carrying out tests on model structures under laboratory conditions. Assuming uniform thickness of wall and bricks of similar strength throughout the height, the failure in a multi-storey brickwork cross-wall structure occurs in the lowermost storey. Failure is due to the breakdown of bond and frictional resistance at the brick/mortar interface, leading to diagonal cracks stepping through the vertical and horizontal mortar joints. In some cases, however, the joint between slab and wall may fail.

The finite element method can be used to calculate the rigidity and stresses in a brickwork structure with a reasonable accuracy, provided that appropriate values of shear modulus are employed in the calculation. Shear modulus of brickwork is assumed to be a function of precompression and hence it will vary along the height of the building.

CHAPTER 5

INVESTIGATIONS INTO THE INFLUENCE OF THE WIDTH OF RETURN WALL AND SLAB-WIDTH ON THE RIGIDITY OF A CROSS-WALL STRUCTURE

5.1 INTRODUCTION

In planning a typical loadbearing brickwork building, the walls are frequently combined to form wall units, each unit consisting of two or more walls oriented at right angles. A wall may act either as a web or a flange depending upon the direction of lateral loading. The flange, referred as the return wall is equally effective in carrying vertical loads. However, in resisting lateral load only an effective width rather than the whole acts with the web. This effective width of return wall should be included in calculations of the area and moment of inertia in the analysis of a cross-wall structure.

Using stress functions Timoshenko[56,57] has shown for a wide flange T-beam on continuous simple supports how bending stresses decline along the flanges and has derived a formula for the effective flange width. The British Code of Practice on reinforced concrete[10] has suggested slightly different formulae. From the experience of tests on a perspex model of one of the University of Essex tower blocks 1 Soane[50] has suggested slightly higher values.

Table 5.1

Effective flange width

Source	T - beam	L - beam
1. Timoshenko	$0.363h + t_w$	
2. C.P.114	$0.33h + t_w$	$0.17h + t_w$
3. C.P.114	$12t_f + t_w$	$4t_f + t_w$
4. S.C.P.I.	$12t_f + 0.16\,nh$	$6t_f + 0.062nh$
5. Soane	$14t_f + t_w$	$7t_f + t_w$
6. Value suggested	$0.35h$	

In table 5.1,

h	storey height
t_w	thickness of web
t_f	thickness of and flange
n	No. of storeys above the section in question

Brickwork construction produces "shear wall-slab structures", consisting of continuous reinforced concrete slabs spanning between loadbearing walls which resist lateral load as well as gravity loading[12]. In analysing a building of this type due to lateral forces it is often difficult to decide how much width of the slab will act as a beam element with the load bearing walls which act as columns.

Khan and Sbarounis[31] considered the problem in the analysis of column slab structures. They divided an idealized slab-column element into a grid of intersecting beams, with assumed boundary conditions.

Investigation was carried out for an applied moment at the centre. Simultaneous equations for deflection and slope at all points of intersection were developed which took into account the flexural as well as torsional stiffness. Theoretical results were compared with those obtained experimentally on an isolated plate. However, boundaries were discontinuous and hence more severe than would occur in an actual building.

In order to find the effective slab-width Soane[50] carried out a model test on a column and slab structure model. First he analysed the structure with a general program using various slab-widths acting as beams. Theoretical column moments and deflections were compared with measured values. Optimum fit was obtained using effective width as half of the bay width. He also used the strain energy method to calculate the moment carried in the slabs by performing a graphical integration of the strain profile on a full bay width.

Table 5.2

Effective slab widths

Method	L_e/L
1. Khan and Sbarounis	0.395
2. Equivalent frame (Soane)	0.5
3. Strain energy (Soane)	0.622
4. Value suggested	0.5

In table 5.2,

L bay width

L_e effective slab width

5.2 PLANS OF SIMPLE STRUCTURES INVESTIGATED

A simple single-storey brickwork structure of one-sixth scale model was selected[30]. The structure consisted of a couple of shear walls with two external return walls connected by a precast R.C. slab. For investigation of the influence of the length of return wall the slab-width was kept constant varying the length of external wall and in the other case the length of the return wall was kept constant, whereas slab-width was varied. Figure 5.1 shows details of the structures investigated.

5.3 ANALYSIS

As mentioned earlier in Chapter 4, the "Stress" program was used to analyse the structures. Figure 5.2 is the finite element subdivision of the structure showing numbering of nodes, elements and partitions for the preparation of the input data. Programs using different flange width (length of external wall) and slab-width were run. Figure 5.3 shows the deflection curves,

(a) when length of return wall varies,

(b) when width of slab varies.

From these results Fig. 5.4 and Fig. 5.5 were drawn. These figures show the variation of deflection/rigidity with flange-width and slab-width respectively. Figure 5.6 is the automatic plotting of the principal stresses, for a single-storey structure subjected to a lateral load at the slab level.

5.4 EXPERIMENTAL VERIFICATION

In order to verify the analytical results tests on single-storey structures with different width of flange (Plates 5.1, 5.2 and 5.3)

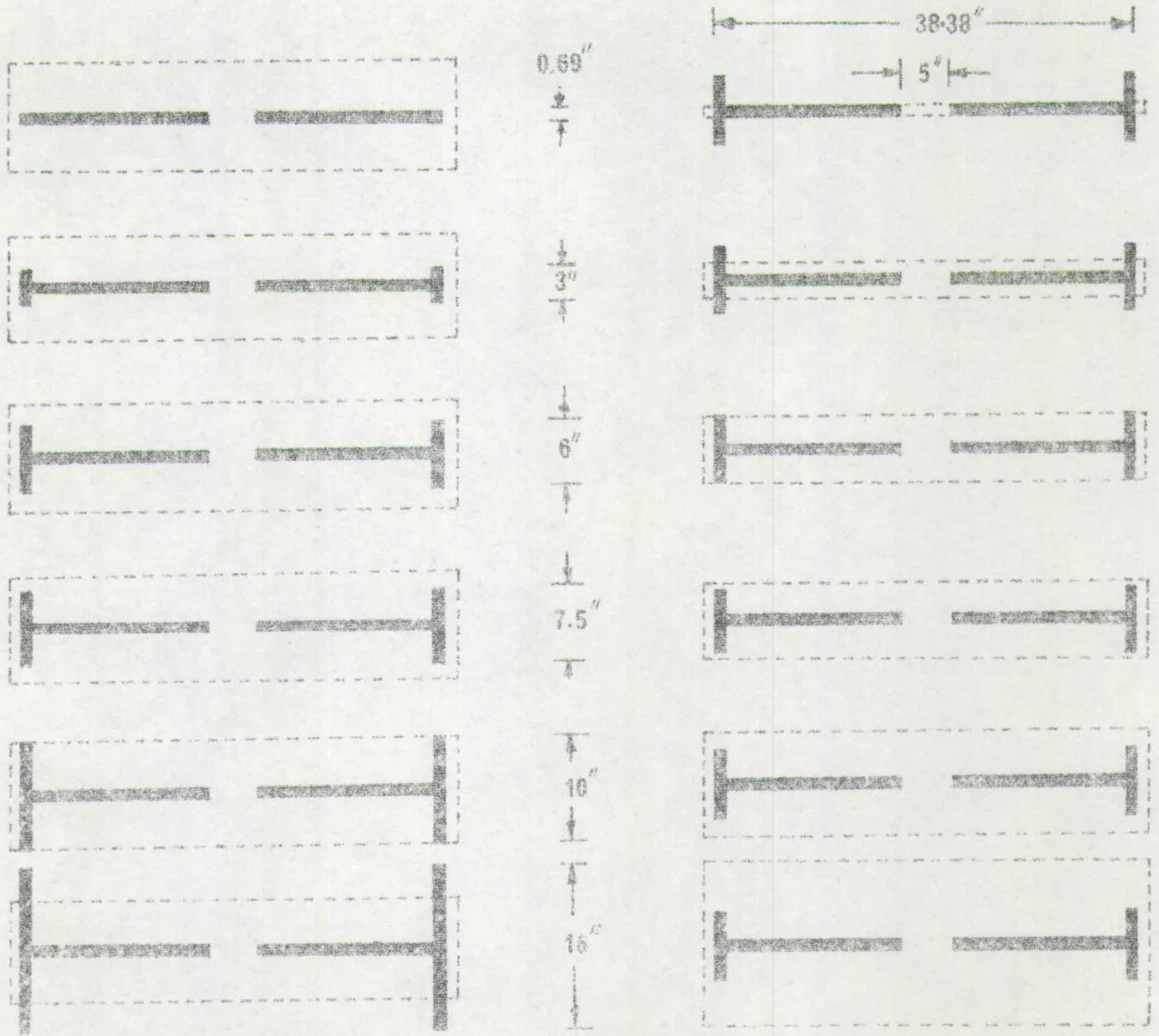


Fig.5.1 STRUCTURES ANALYSED TO INVESTIGATE INFLUENCE OF FLANGE WIDTH AND SLAB WIDTH ON RIGIDITY.

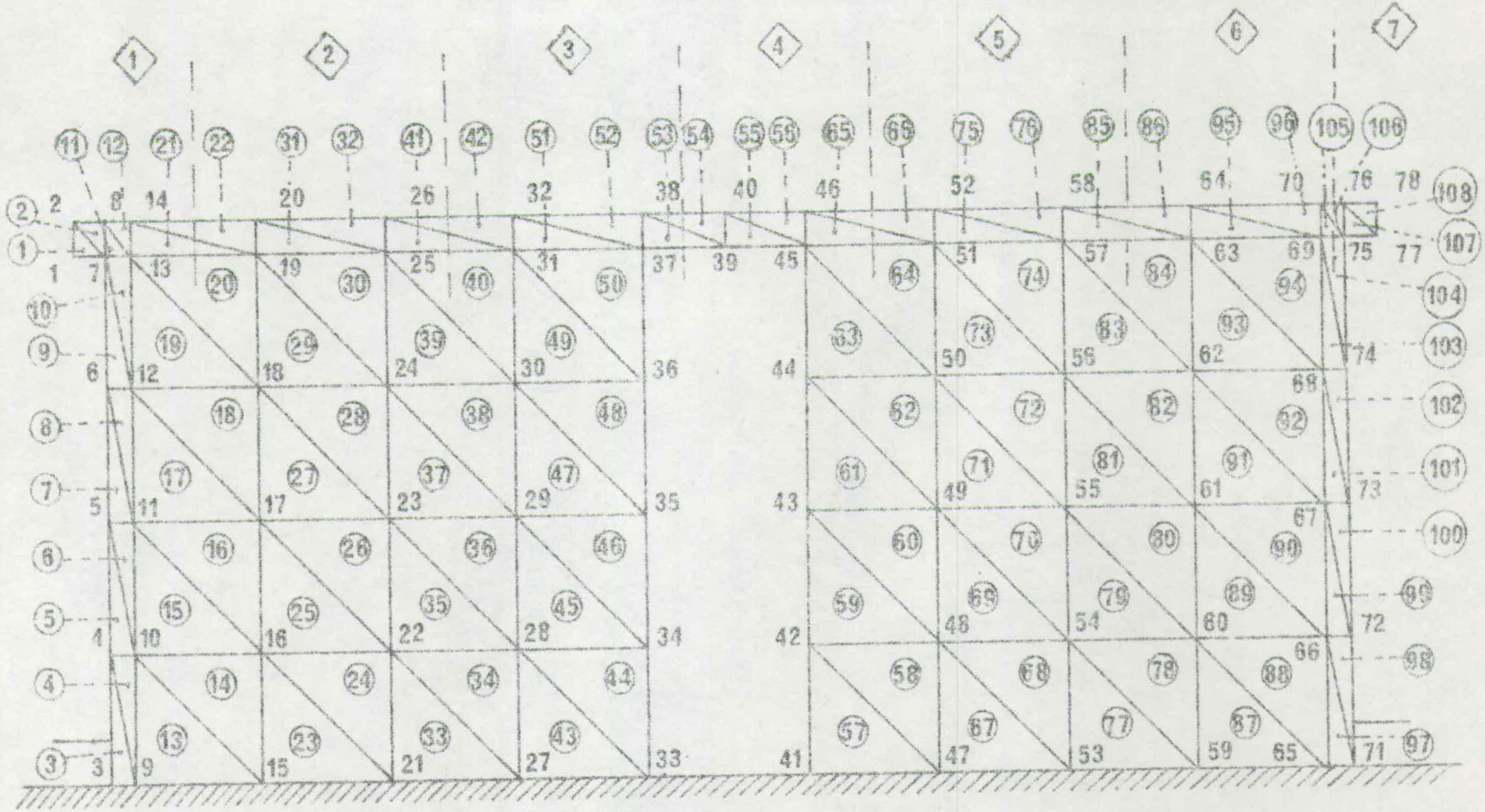


Fig.5.2 STRUCTURE DIVIDED INTO FINITE ELEMENTS SHOWING NUMBERING OF NODES, ELEMENTS AND PARTITIONS.

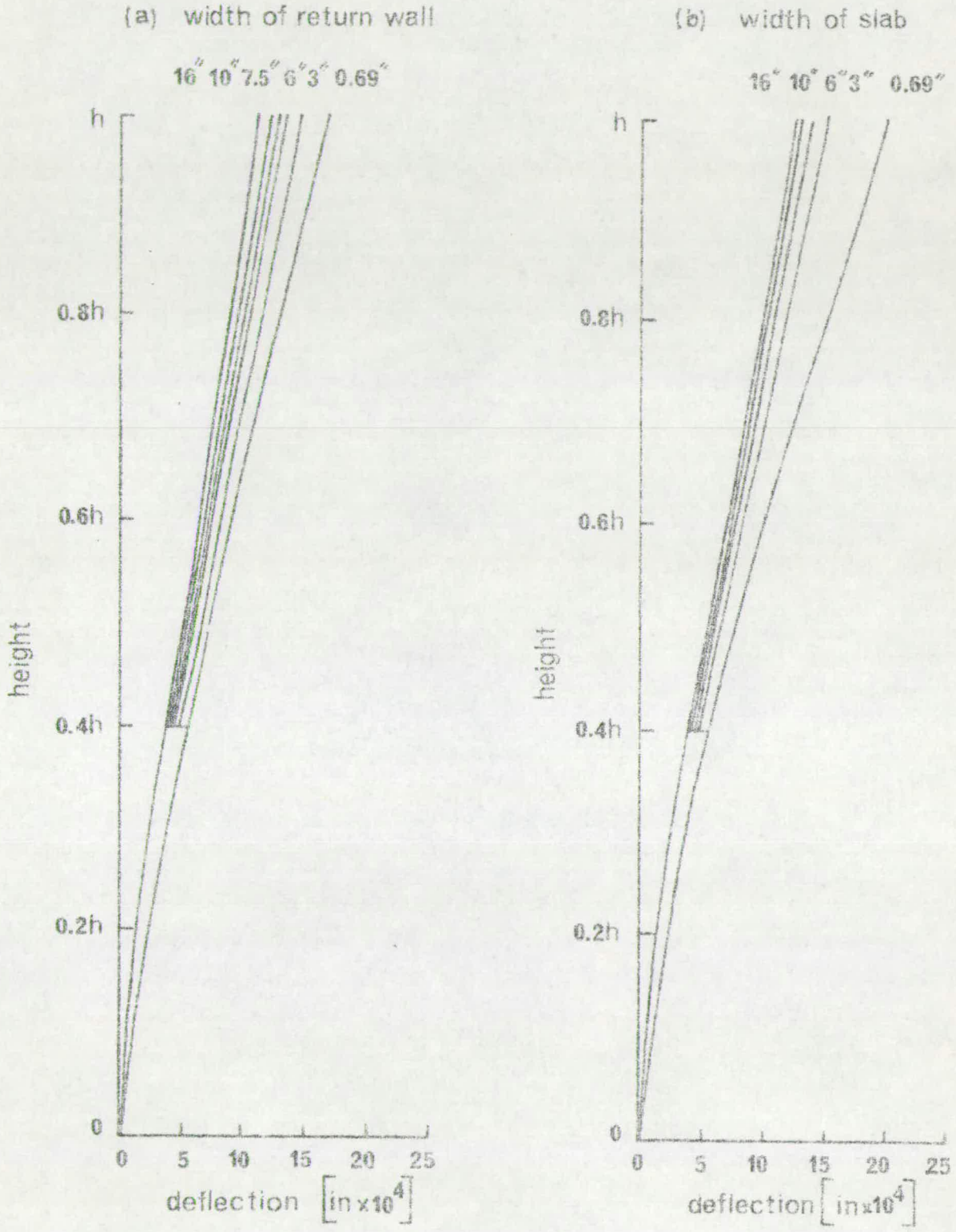


Fig.5.3 DEFLECTIONS OF STRUCTURES HAVING DIFFERENT (a) FLANGE WIDTH (b) SLAB WIDTH.

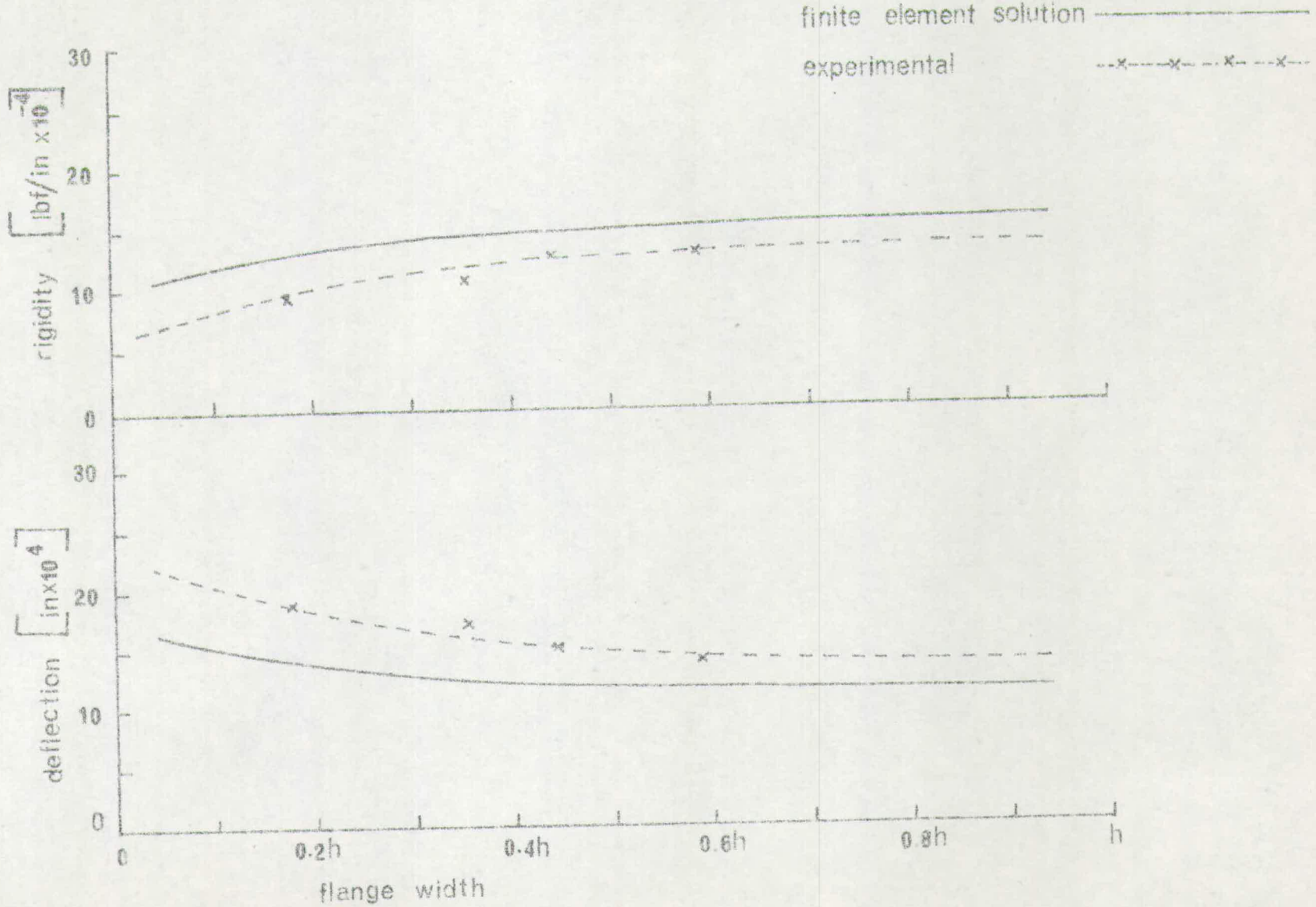


Fig.5.4 VARIATION OF DEFLECTION/RIGIDITY WITH FLANGE WIDTH.

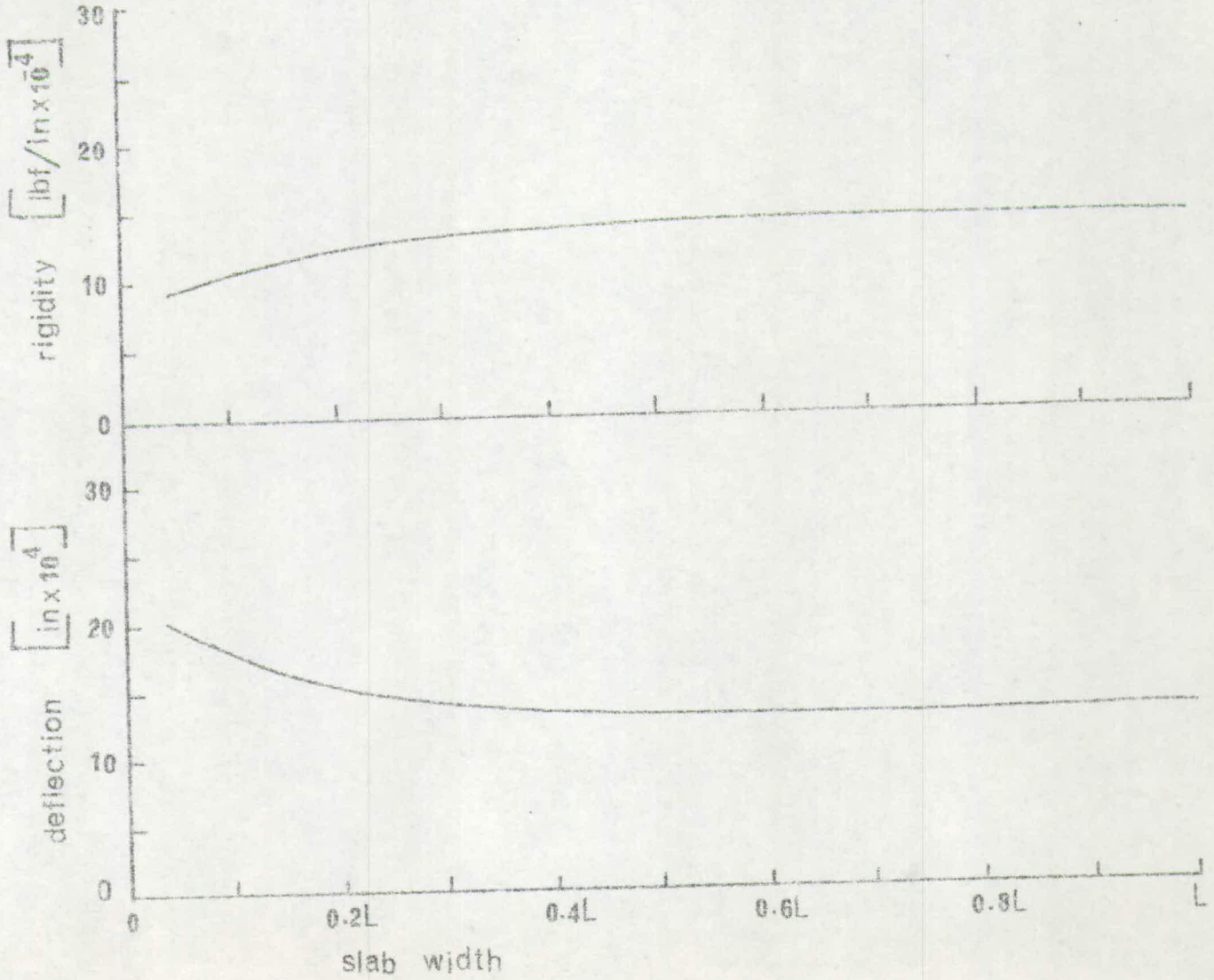


Fig.5.5 VARIATION OF DEFLECTION/RIGIDITY WITH SLAB WIDTH.

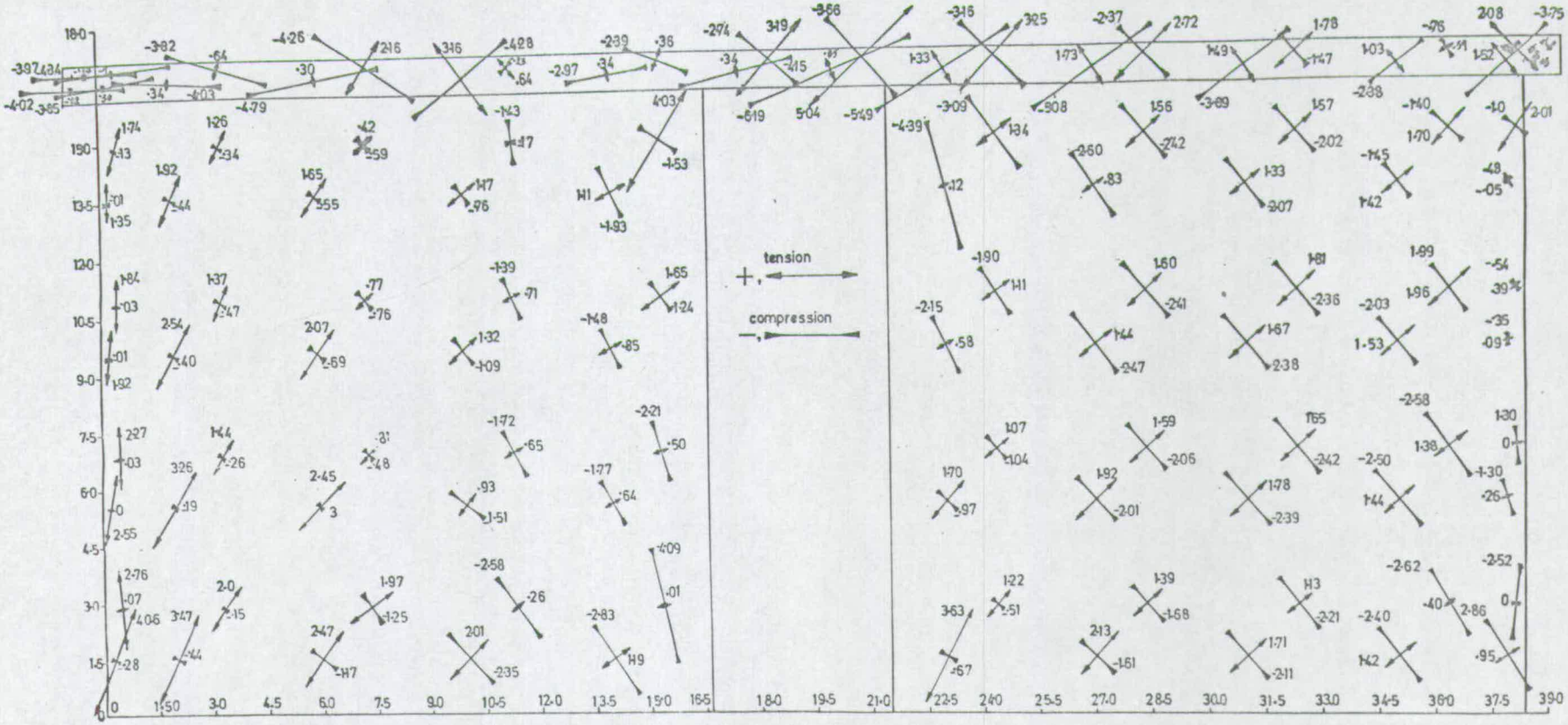


Fig.5.6 AUTOMATIC PLOTTING OF PRINCIPAL STRESSES.

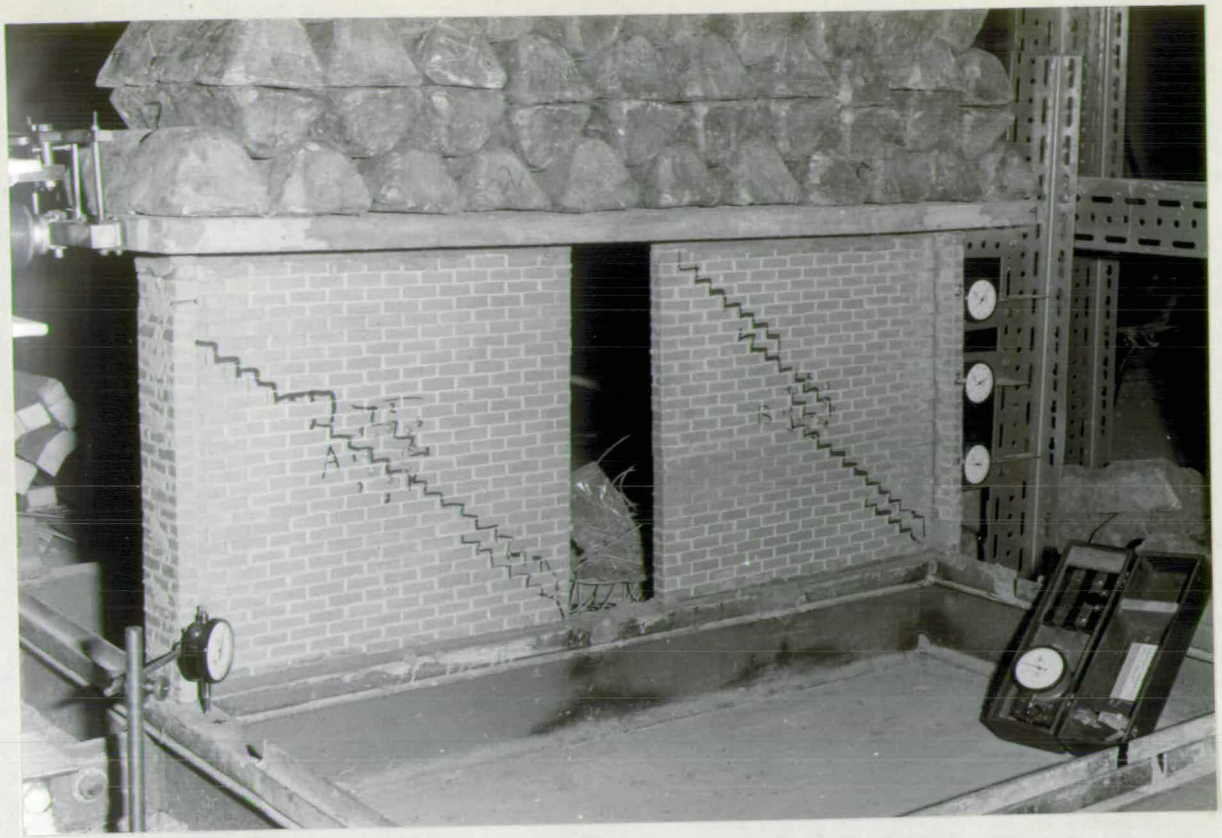


PLATE 5.1 - FAILURE OF A SINGLE-STOREY STRUCTURE, FLANGE 3 IN.

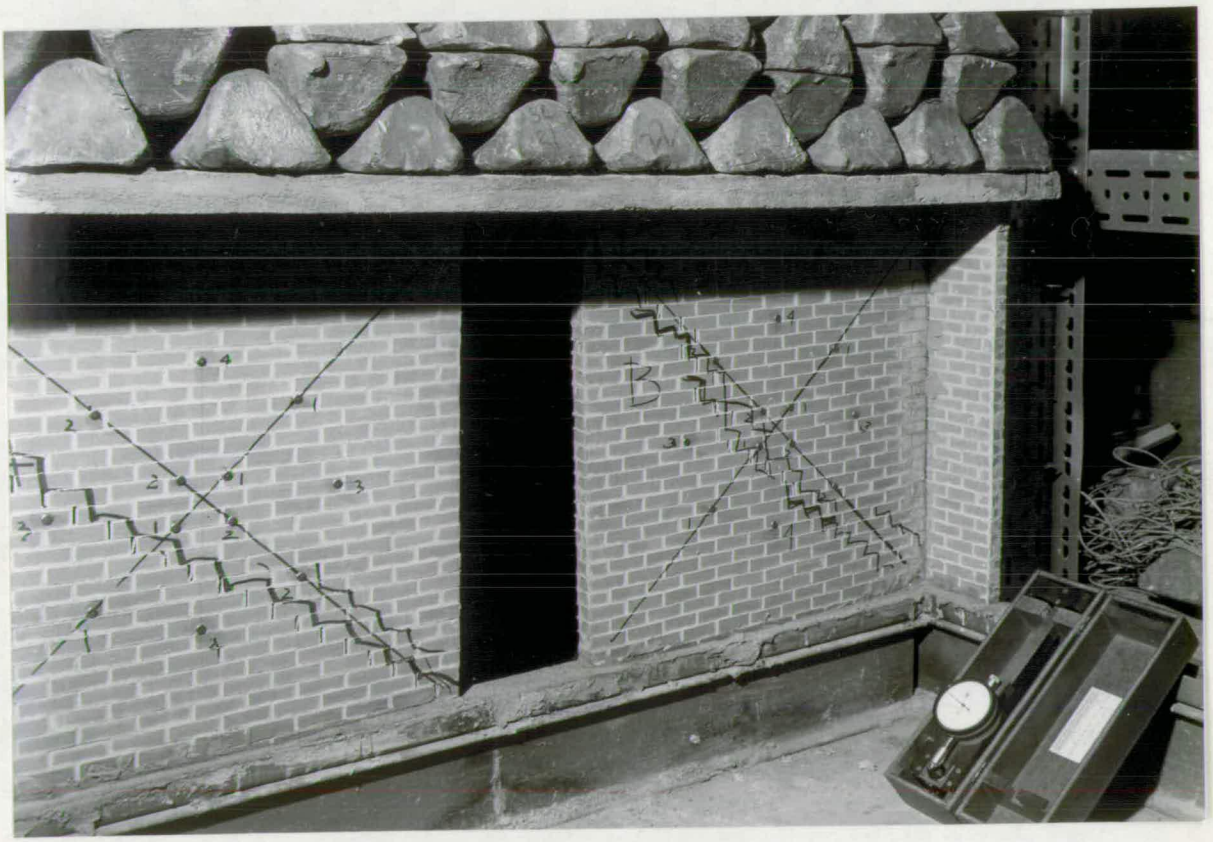


PLATE 5.2 - FAILURE OF SINGLE-STOREY STRUCTURE, FLANGE 7.5 IN.

PLATE 5.3 - FAILURE OF SINGLE-STOREY STRUCTURE, FLANGE 10 IN.

were carried out. Model structures with flange width 3", 6", 7.5" and 10" were tested. Precompressive stress was kept constant in each case, thereby keeping the shear modulus constant.

Figures 5.7 and 5.8 show respectively the calibration curve for the load measuring beam used in the test and load/deflection curves for the structures. The experimental results are compared in Fig. 5.4. The variation of the diagonal strains with applied shear for the structure with 10" flange is shown in figure 5.9.

5.5 DISCUSSION OF RESULTS

From figure 5.4 it can be reasonably assumed that the effective width of flange in the case of a Tee-section is about 0.35h, where h is the storey height. Similarly from figure 5.5 the effective width of slab is about 0.5L , where L is the bay width. These results are similar to those obtained elsewhere (Tables 5.1 and 5.2). Although the theoretical and the experimental results in Fig. 5.4 and 5.5 differ by about 20%, they do not effect the above assumptions.

The results shown in Table 5.1 and 5.2 are approximate ones. They should be quite reasonable for design purposes. However, in a structure having complex units of walls and presence of holes in the slabs may have significant influence on the effective bay width. The effective width of slab will also depend to some extent on the relative thickness of walls and slabs.

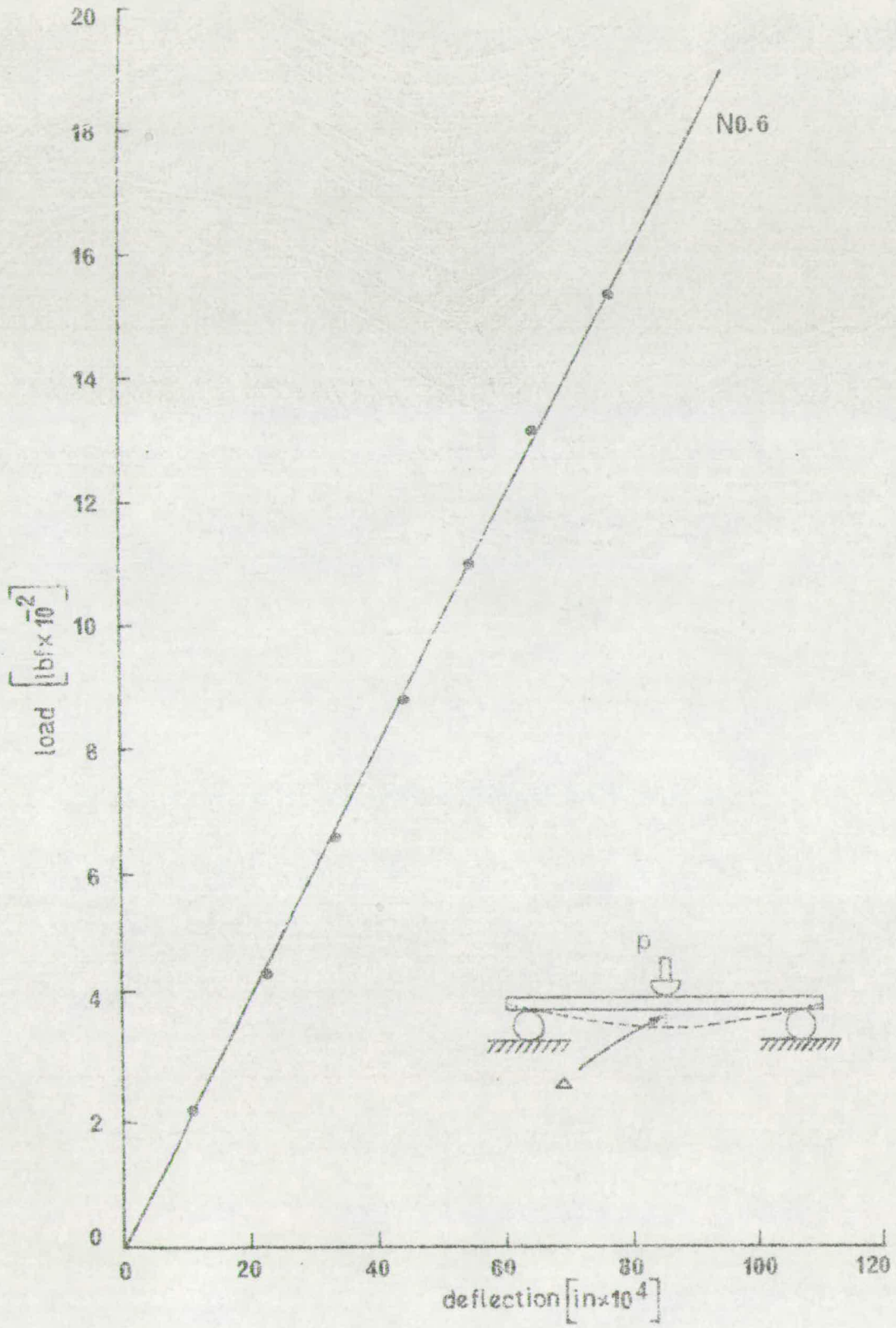


Fig.5.7 CALIBRATION OF LOAD MEASURING BEAM No.6.

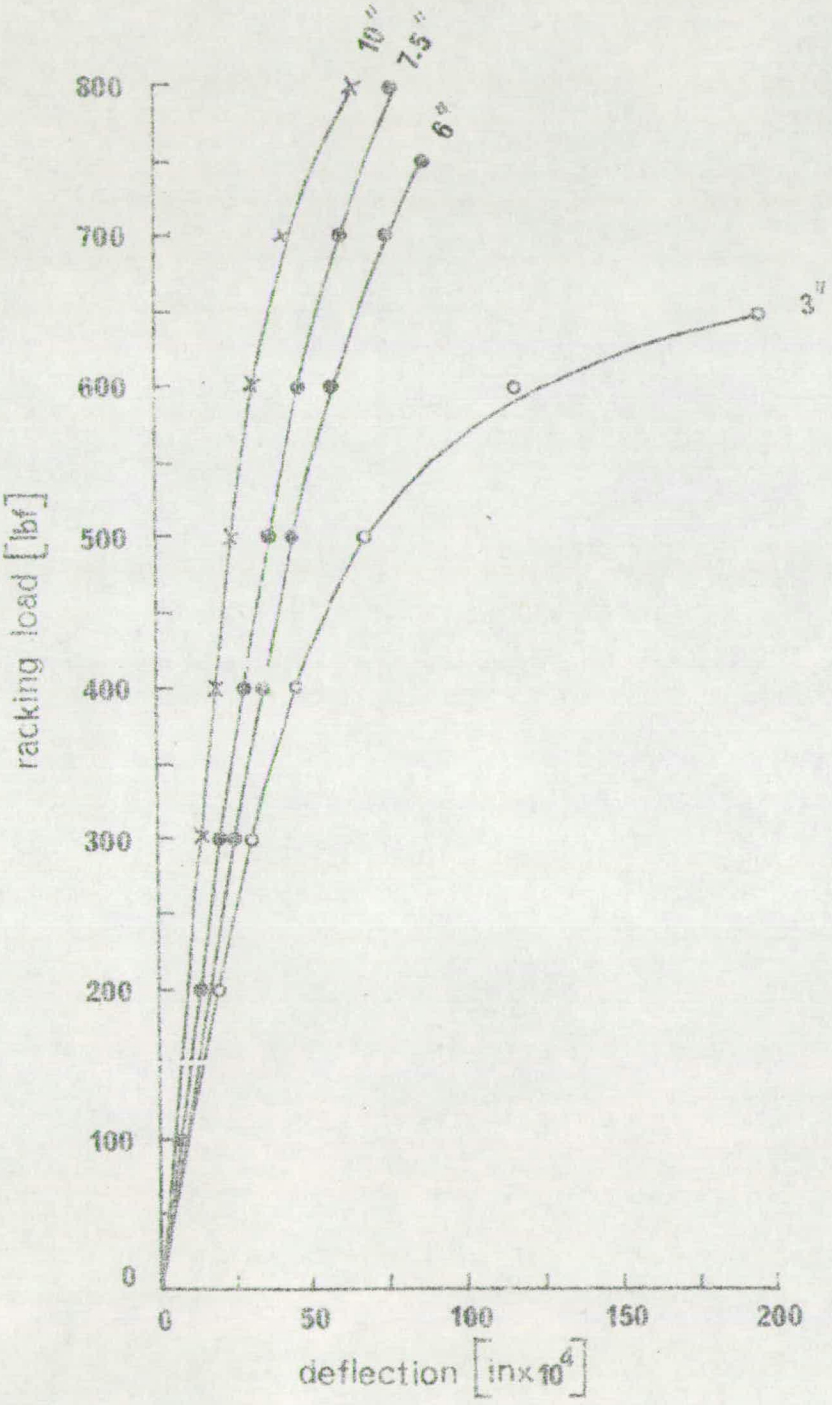


Fig.5.8 LOAD/DEFLECTION CURVES FOR STRUCTURES HAVING DIFFERENT FLANGE WIDTH.

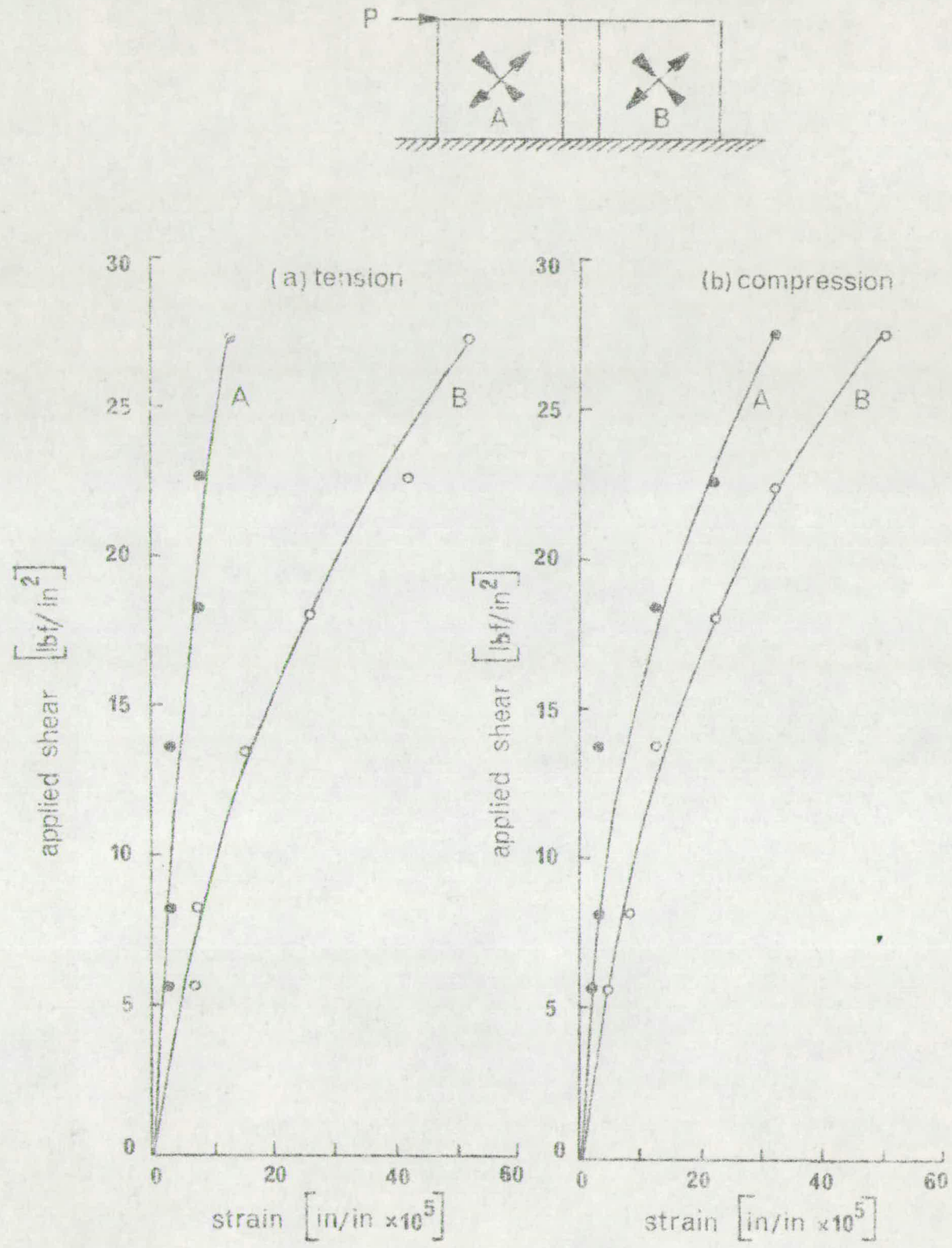


Fig.5.9 RELATIONSHIP BETWEEN APPLIED SHEAR AND DIAGONAL STRAIN.

5.6 CONCLUSIONS

The influence of the width of external return wall and the slab width on the rigidity of a brickwork cross-wall structure can be studied by carrying out shear tests on single-storey model structures. The finite element method can also be used to investigate the effective width of external return walls and slabs. For the simple structure investigated the effective "flange" width was 0.35 of the storey height and the effective slab width was 0.5 of the bay width.

When precompression is high the failure of shear walls would occur in diagonal cracks passing through both bricks and mortar, in which case the ultimate strength of shear walls depends on the diagonal tensile strength of brickwork. However, if the pre-compression is not fairly high, diagonal cracks may not develop.

CHAPTER 6

AN EXPERIMENTAL AND THEORETICAL INVESTIGATION OF THE BEHAVIOUR OF A FOUR-STOREY MODEL STRUCTURE IN BRICKWORK UNDER LATERAL LOADING APPLIED ECCENTRICALLY

6.1 INTRODUCTION

In tall buildings, in order to economise means of providing the required lateral stability concrete shafts are used. In high-rise loadbearing construction also brickwork shafts can be used in the similar way as in a concrete building. They are generally in the form of lift or staircase shafts and perforated by vertical bands of openings for doors and windows. When the principal axis of the building coincides with that of the shaft, usually there will be no torsion developed due to lateral load on the building except in the case of lack of symmetry of the wind action or earthquake disturbances. However, torsion occurs when the building axis fails to coincide with that of the shaft. The distance between the two axes multiplied by the amount of lateral load is the torsional load. The building must be designed to resist this load in addition to and simultaneously with the lateral load.

Based on the theories of thin walled bars and perforated shear walls Rosman[46] has developed an analytical method for perforated concrete shafts of tall buildings subjected to torsion.

Jenkins and Harrison[29] gave an analysis of shear walls subjected to bending and torsion. For analysis of torsion they used the principles of theorem of minimum potential energy. The method is simple

and requires summation of a number of series which can be produced for any number of storeys. They neglected the bending of flanges. The results obtained, were, however, not very promising.

Michael[37] based on the theory of warping displacements in thin section members gave a method for analysis of torsion walls in a tall building. His analytical result gave good agreement when compared with published experimental results of Jenkins etal[29].

Taranath and Morice[58] conducted tests on simple perspex and mortar models of "flag structures" and "open-box structures", subjected to transverse loading. For analysis they used the method of virtual work. A good agreement between theory and experiment was obtained.

Murthy and Hendry[34] carried out tests on an unsymmetrical model cross-wall structure in brickwork subjected to lateral loading. The torsional rigidities at the second floor and roof level of the three-sty. structure calculated on the basis of single-storey theory[3] were found to be higher compared with actual rigidities. The error involved in single-storey theory seemed to increase with the increase of lateral load.

In order to investigate the torsional rigidity of a brickwork cross-wall structure the author carried out tests on a four storey model structure under lateral load applied eccentrically. Analyses were carried out using,

(1) the finite element method for direct lateral load,

(2) theories of torsion for thin walled sections,

(3) a simplified method based on shear distribution on single-storey structure.

6.2 MATERIALS

One-sixth scale model bricks were used for the construction of the torsion walls. The bricks were from a batch, the details of strength of which are given in the following table.

Table 6.1

Crushing Strength (lbf/in^2) of 1/6 scale Model Bricks

Number in Sample	Mean Crushing Strength	Standard Deviation	Co-efficient of variation
46	3865	311	8.0%

The mortar used was 1:4 cement/sand by weight (1:3 approx. by volume). Fine sand such as Leighton Buzzard No.19 (Fig.4.1) and rapid hardening cement (Ferrocrete) were used. The mean crushing strength of 1-in. mortar cubes was found to be 2095 lbf/in^2.

Precast reinforced concrete slabs were used for floors. The concrete was 1:1:2 by weight having the maximum size of aggregate 3/16 in. One-inch-square mesh of 1/8 in. dia. bars were used for top and bottom reinforcement. The mean average crushing strength of cylinders of 6-in. dia. 12 in. height, was 6900 $lbf/in.^2$.

6.3 CONSTRUCTIONAL DETAILS

6.3.1 LAYOUT

Figures 6.1 (a), 6.1 (b) and 6.1 (c) are respectively the plan, front elevation and side elevation of the model structure. Figure 6.2 (a) shows typical torsion walls in a tall building[37]. Figure 6.2 (b) is an isometric view of the model structure with the roof slab omitted. Each storey in the model consists of two channel walls of brickwork and the R.C. floor slab.

6.3.2 METHOD OF CONSTRUCTION

All the walls of the model structure were constructed in situ. Two channel walls of brickwork for each storey were constructed one after another. Bricks were laid simultaneously for the web and the flanges for a channel. In other words the construction was monolithic. Plywood planks were used for webs to ensure plumbing. Plates 6.1 (a) and 6.1 (b) show the construction step by step one storey after another. Similar mortar was used for joining walls and slabs.

6.4 TESTING EQUIPMENT

A description of the loading frame and the load-measuring apparatus has already appeared in chapter 4, section 4.4 (Plate 4.1). Load-measuring beams Nos. 1,2,3 and 5 (Fig. 4.6) were used for applying lateral load on the four-storey model structure.

6.5 SIMULATION OF DEAD LOAD STRESSES

Since the structure was of one-sixth scale it was necessary to increase the dead weight stresses in the material to six times in order to simulate the effect of gravity load in the prototype

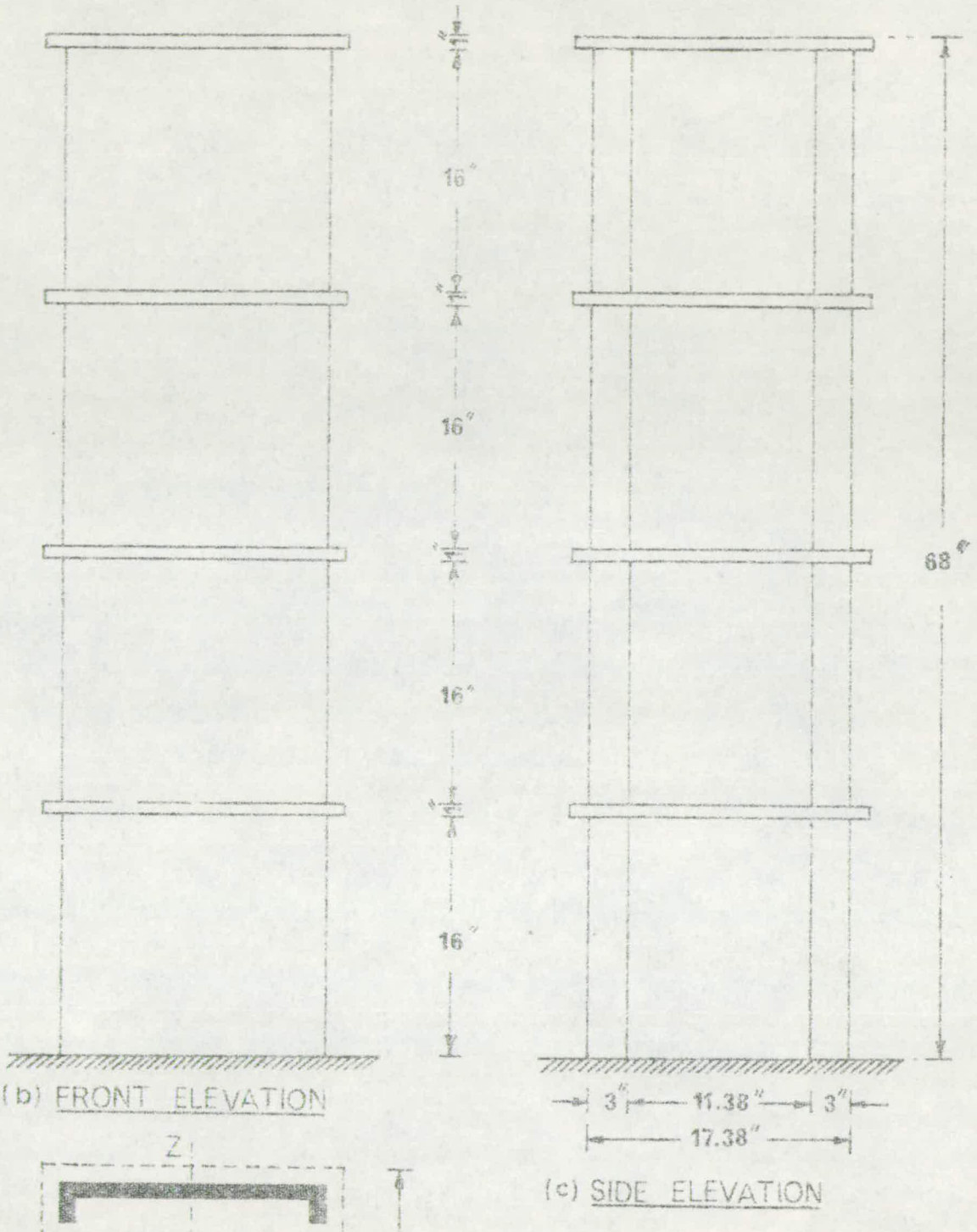
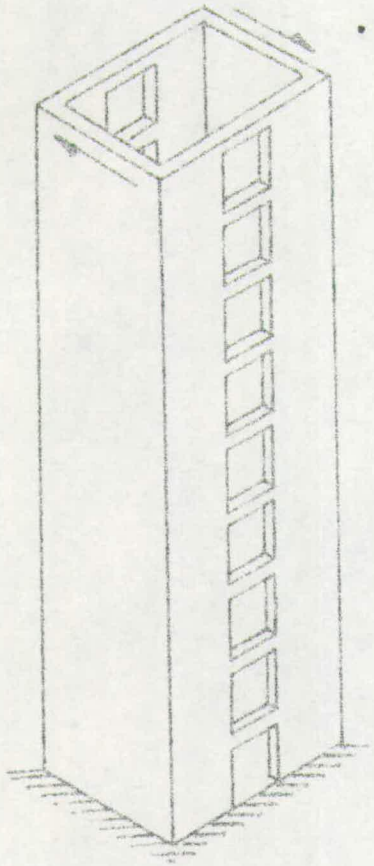
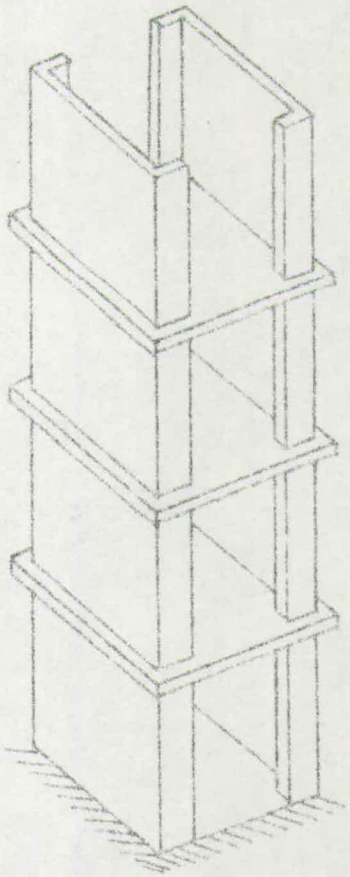


Fig.6.1 (a) PLAN, (b) FRONT ELEVATION AND (c) SIDE ELEVATION OF THE MODEL STRUCTURE.

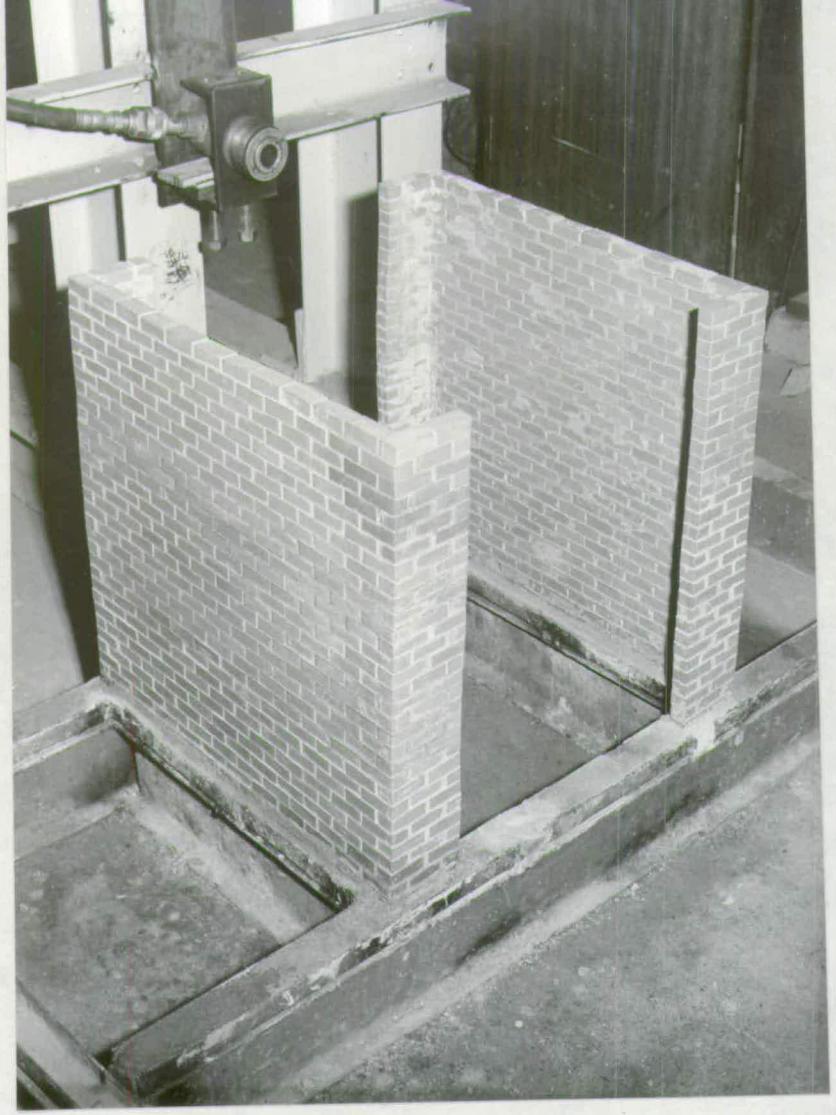


(a) torsion walls in a tall building

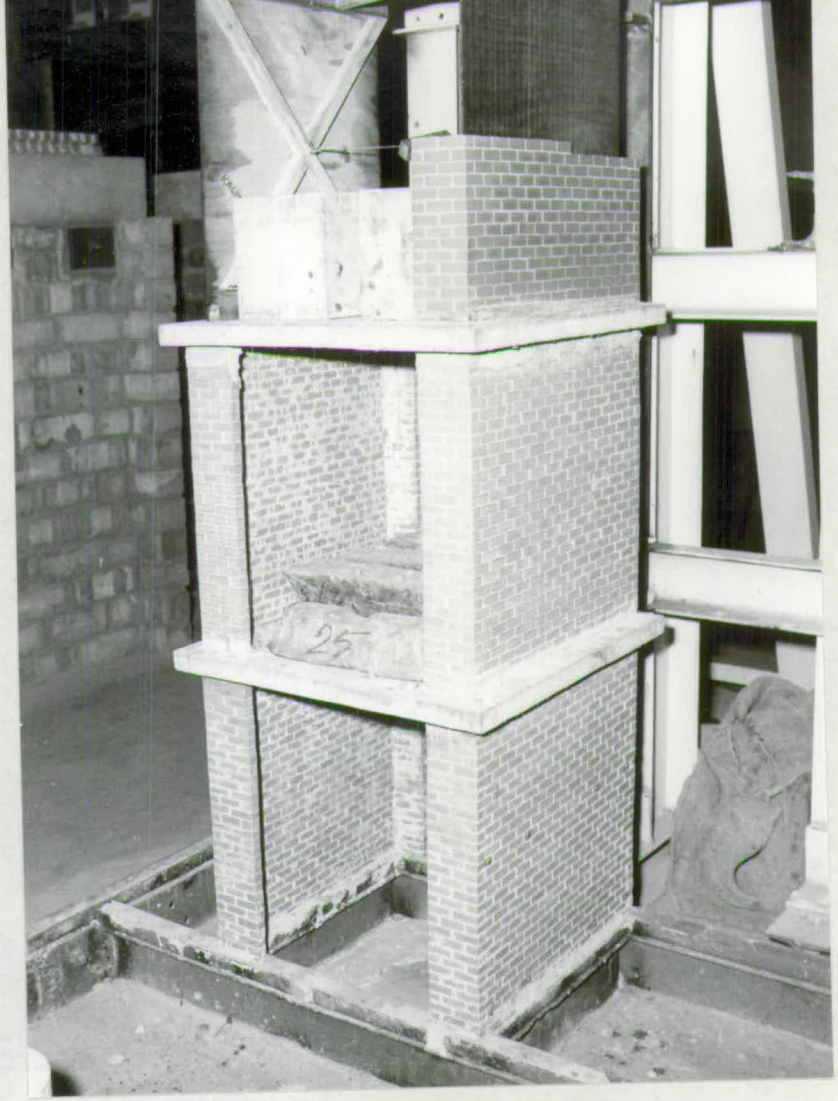


(b) model structure

Fig.6.2 (a) TYPICAL TORSION WALLS IN A TALL BUILDING (b) PRESENT MODEL STRUCTURE WITH THE ROOF SLAB OMITTED.



(a)



(b)

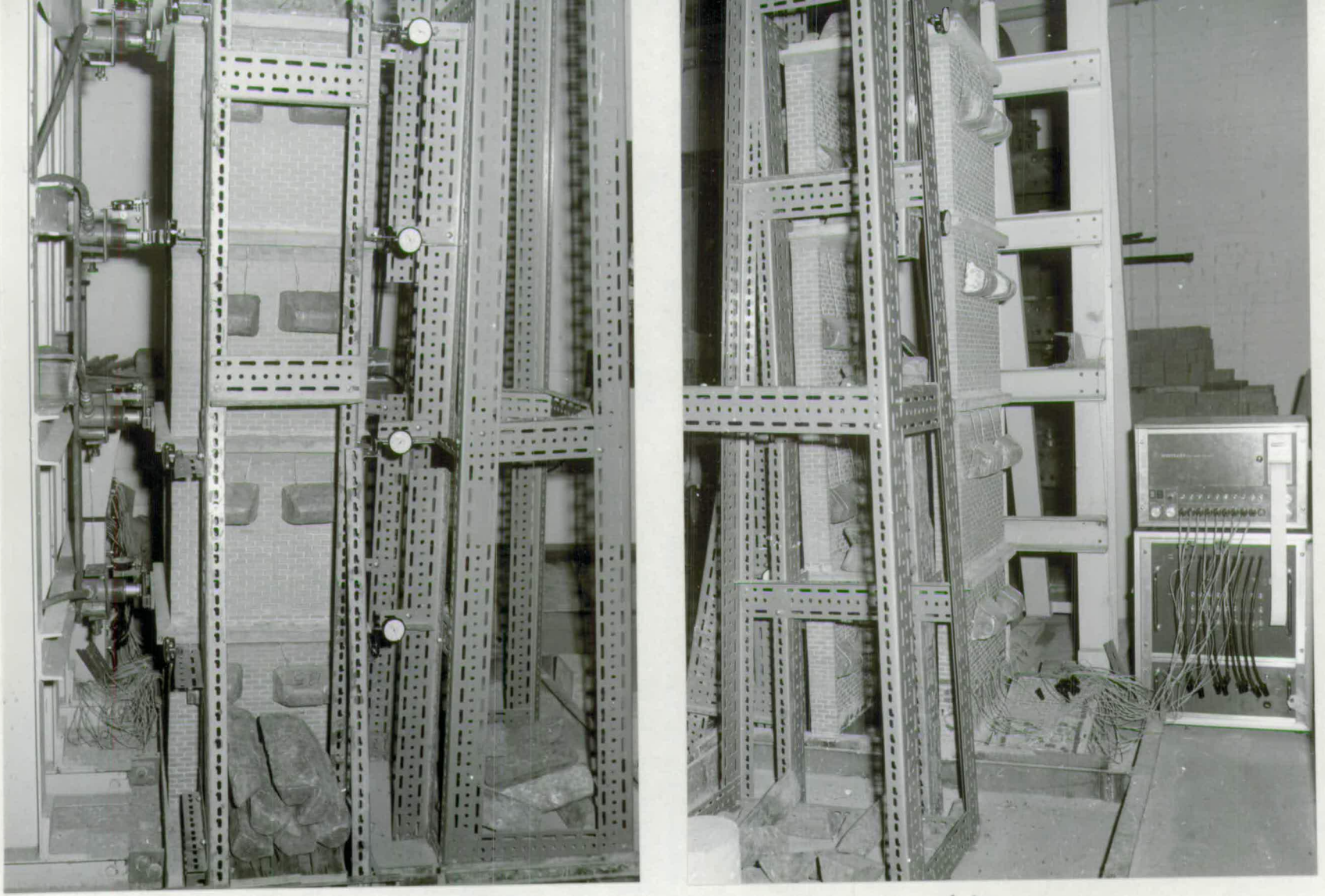
PLATE 6.1 - TORSION WALLS, SHOWING CONSTRUCTION IN SITU ONE STOREY AFTER ANOTHER

structure. This was done by hanging lead billets from the walls and by putting lead billets on the floor slabs (Plate 6.2). Thus the estimated precompressions were approximately 40, 30, 20 and 10 lbf/in^2 respectively at 1st, 2nd, 3rd and 4th storey.

6.6 EXPERIMENTAL INVESTIGATION

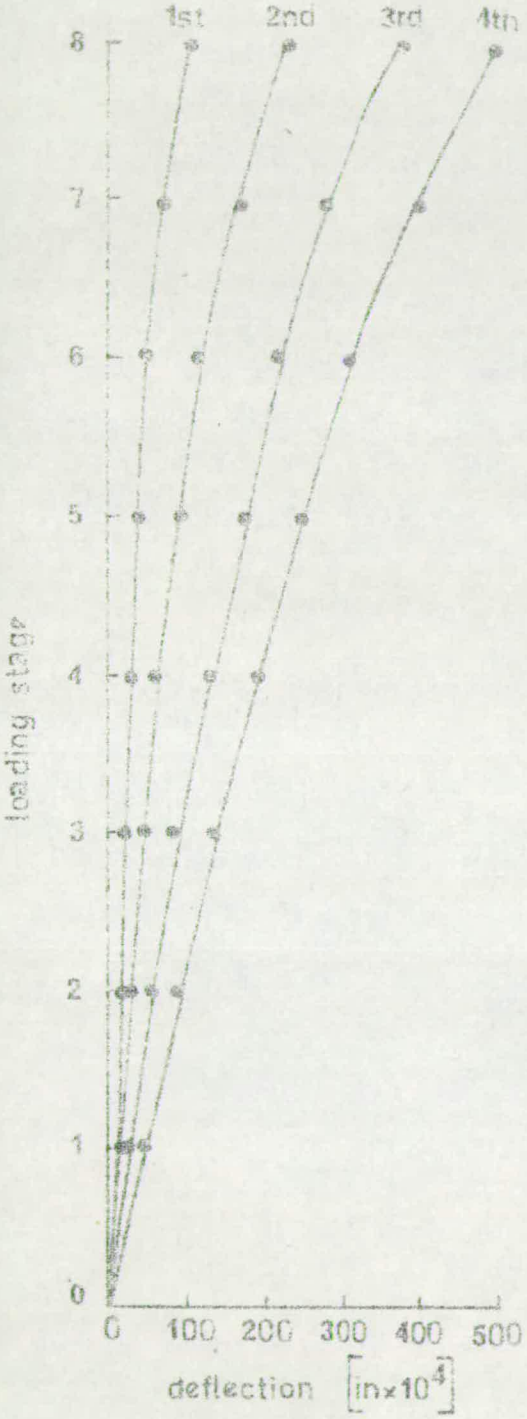
When the model was completed up to 4th storey, it was allowed to cure for 28 days before starting any test. In testing the four-storey structure loads were applied by hydraulic jacks one at each floor. The loads were applied at an eccentricity of 3 in. as illustrated in figure 6.1 (a).

All loads from 1st to 3rd storey were equal; the load on the roof slab was equal to half of the load applied at other slab levels. These point loads may be considered to be equivalent to the wind load acting uniformly over the whole height of the building, but unsymmetrically along the length of the building. Deflections were measured at each floor level, at four points, each $1\frac{1}{2}$ in. away from the corner of the slab, in x and z direction. Load/deflection curves at various floor-levels are shown in Figs. 6.3, 6.4 and 6.5. Keeping the load at about quarter of the expected ultimate value the experiment was repeated. No significant difference was observed. Finally, the structure was loaded to destruction (Plates 6.3 (a), 6.3 (b) and 6.3 (c)). The ultimate strength calculated from direct load and moment was found to be about 40 lbf/in^2 which is consistent with previous test results (Fig. 4.9).



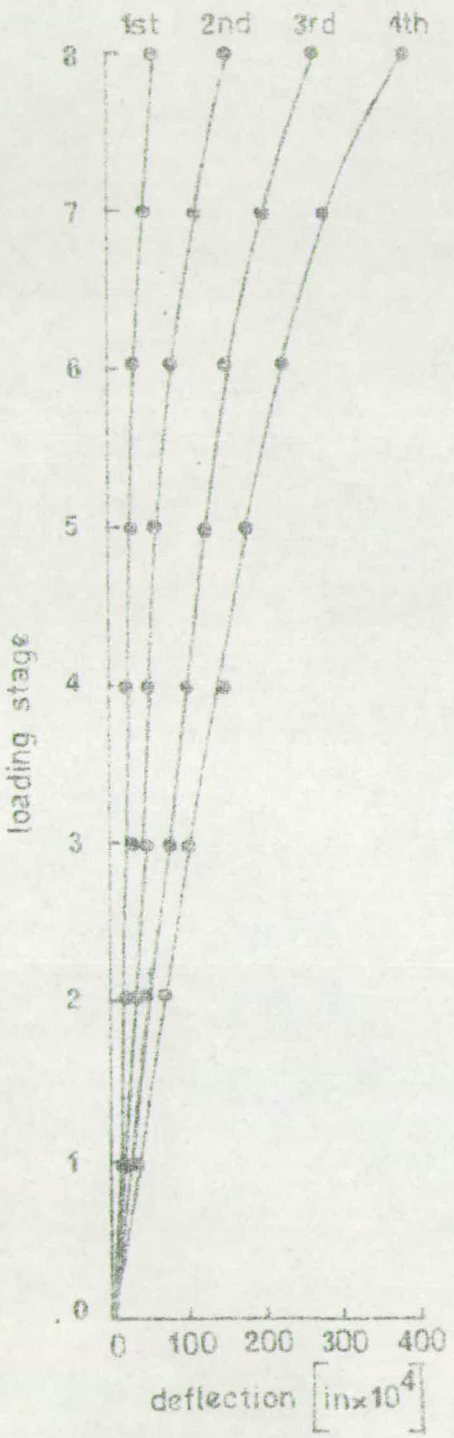
(a) (b)

PLATE 6.2 - FOUR-STOREY TORSION WALLS UNDER TEST



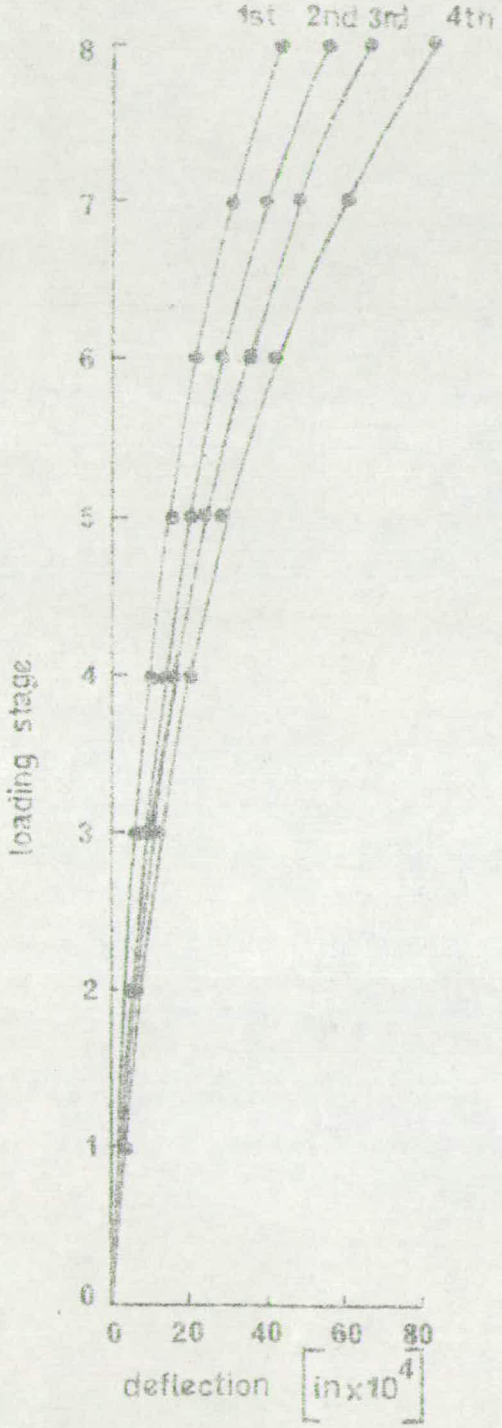
1	2	3	4	5	6	7	8	stage
5	10	15	20	25	30	35	40	4h
10	20	30	40	50	60	70	80	3h
10	20	30	40	50	60	70	80	2h
10	20	30	40	50	60	70	80	h
35	70	105	140	175	210	245	280	total (lbf)

Fig.6.3 LOAD/DEFLECTION CURVES FOR WALL 'A' AT VARIOUS FLOOR LEVELS (X-DIRECTION).



1	2	3	4	5	6	7	8	stage
5	10	15	20	25	30	35	40	4h
10	20	30	40	50	60	70	80	3h
10	20	30	40	50	60	70	80	2h
10	20	30	40	50	60	70	80	h
35	70	105	140	175	210	245	280	total (lbf)

Fig.6.4 LOAD/DEFLECTION CURVES FOR WALL 'B' AT VARIOUS FLOOR LEVELS (X-DIRECTION).



1	2	3	4	5	6	7	8	stage
5	10	15	20	25	30	35	40	4h
10	20	30	40	50	60	70	80	3h
10	20	30	40	50	60	70	80	2h
10	20	30	40	50	60	70	80	h
35	70	105	140	175	210	245	280	total (lbf)

Fig.6.5 LOAD/DEFLECTION CURVES AT VARIOUS FLOOR LEVELS (Z-DIRECTION).

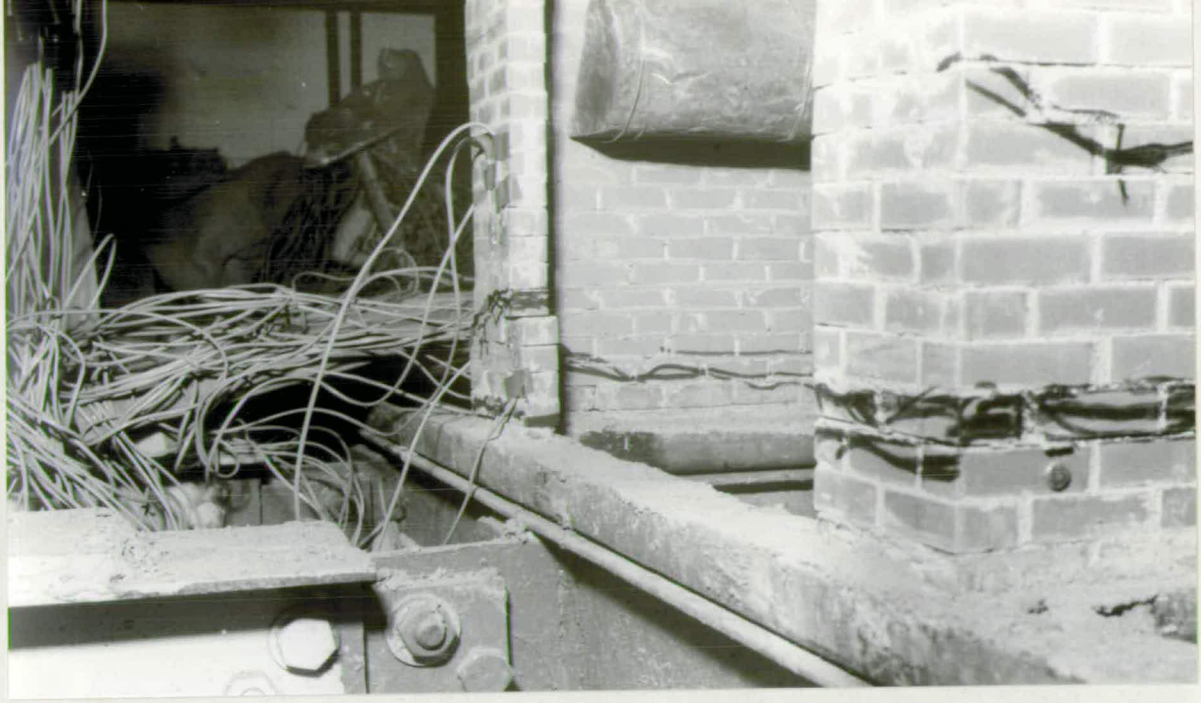


PLATE 6.3 (a) - FAILURE OF FOUR-STOREY TORSION WALLS, SHOWING FAILURE CRACKS IN CROSS-WALLS AND RETURN WALLS OF LOWERMOST STOREY, PHOTOGRAPHED FROM LOADED SIDE

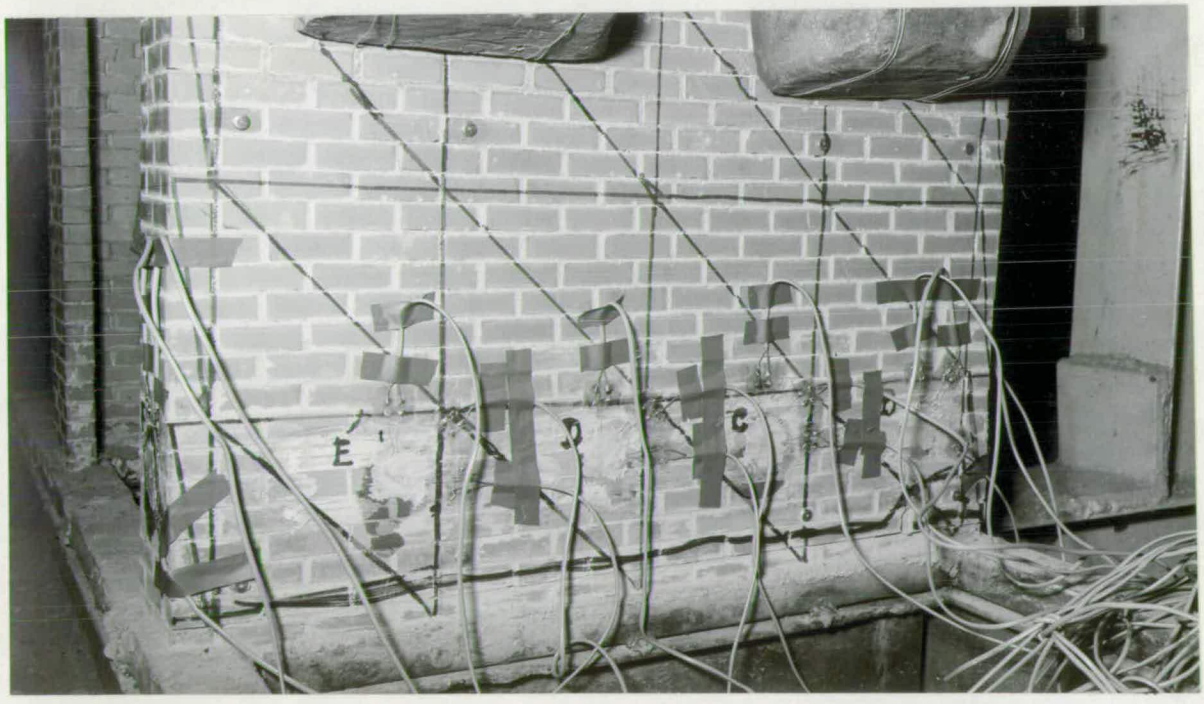


PLATE 6.3 (b) - FAILURE OF WALL A, ALSO SHOWING STRAIN GAUGES

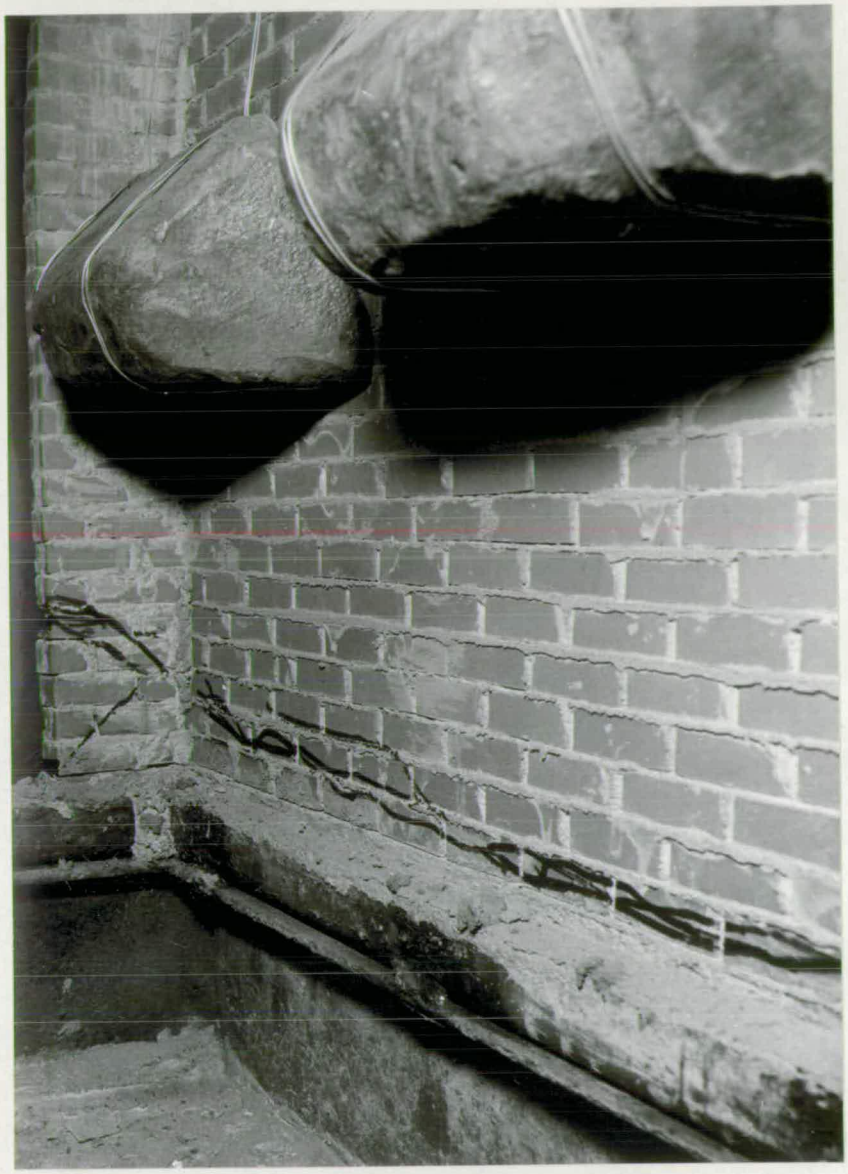


PLATE 6.3 (c) - WALL A AT FAILURE; PHOTOGRAPHED FROM INSIDE OF THE STRUCTURE

This time the base of the structure was very well cemented to the floor of the laboratory with strong cement mortar, and so it was possible to prevent rotation of the base. Never-the-less, a dial gauge was put near the base in order to see whether there was any rotation of the base frame. The dial gauge did not show any reading.

6.7 MEASUREMENT OF STRAINS

As before electric resistance gauges (Appendix II) were used to measure strains. Four gauges were fixed on the web of the highly stressed wall A at a horizontal section in the bottom storey. Two additional gauges, one on each of the flanges, were also fixed. Strains were recorded using the Data Logger Type 10SL (Appendix II).

6.8 ULTIMATE TESTS AT VARIOUS PRECOMPRESSIONS

After the failure of the model at first storey, it was supported on jacks placed under the first floor slab. The walls in the first floor were knocked down and the remaining portion of the structure (e.g. a three-storey model structure) was lowered on the base frame by means of a crane. Some mortar was put on the frame before lowering the slab on it. In addition to that the structure was held firmly by six bolts passing through the slab to the base frame. This structure was then tested in the similar way as the four-storey structure (Plates 6.4 (a), 6.4 (b), 6.4 (c)). The procedure was repeated until eventually the forth storey of the original model was tested as a single-storey structure (Plates 6.5 and 6.6). The maximum precompressive load used in the case of the three, two and single-structures were 60, 70 and 80 lbf/in.^{2} respectively. The

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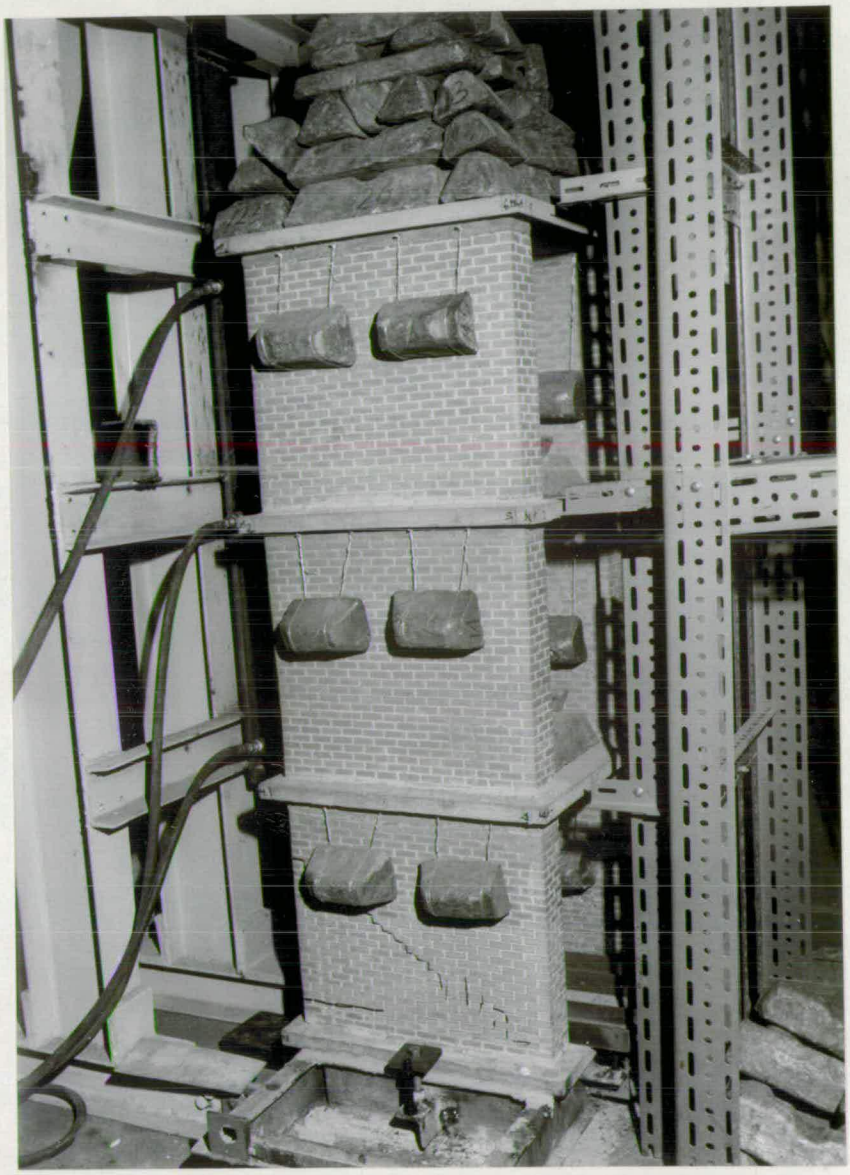
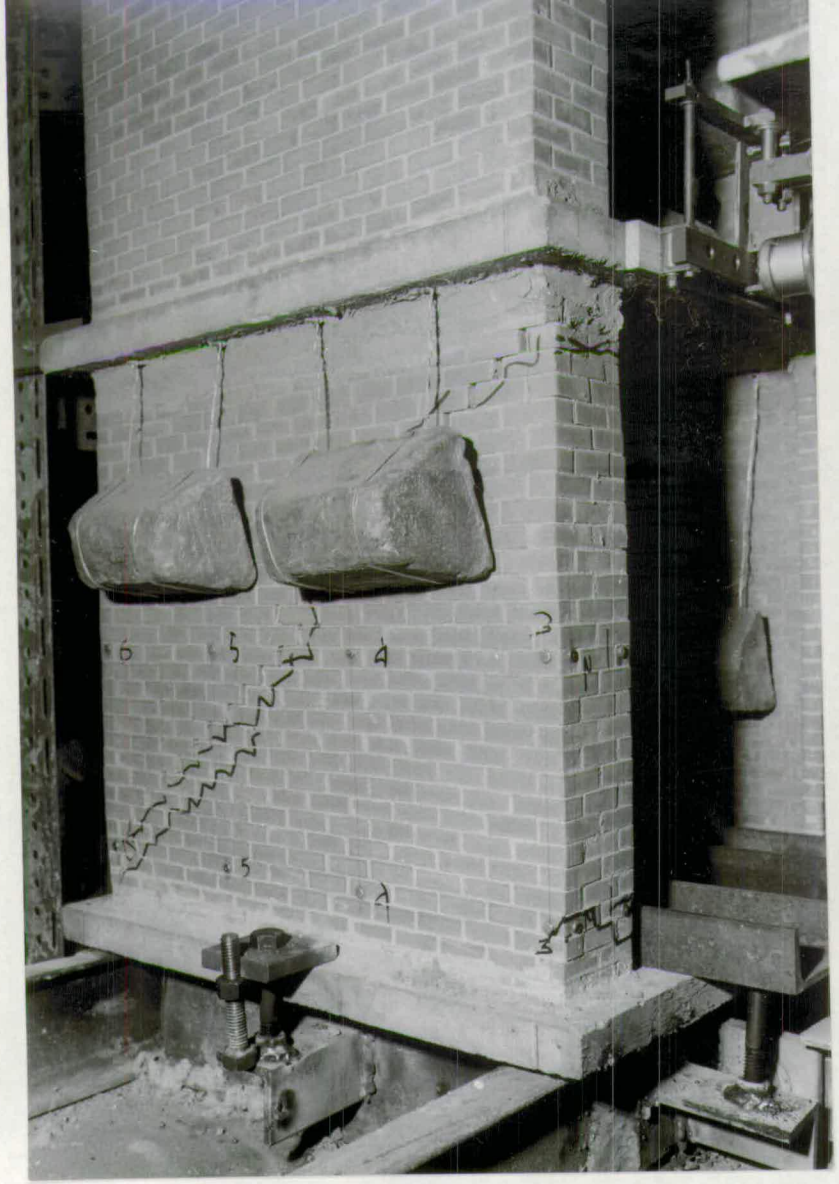
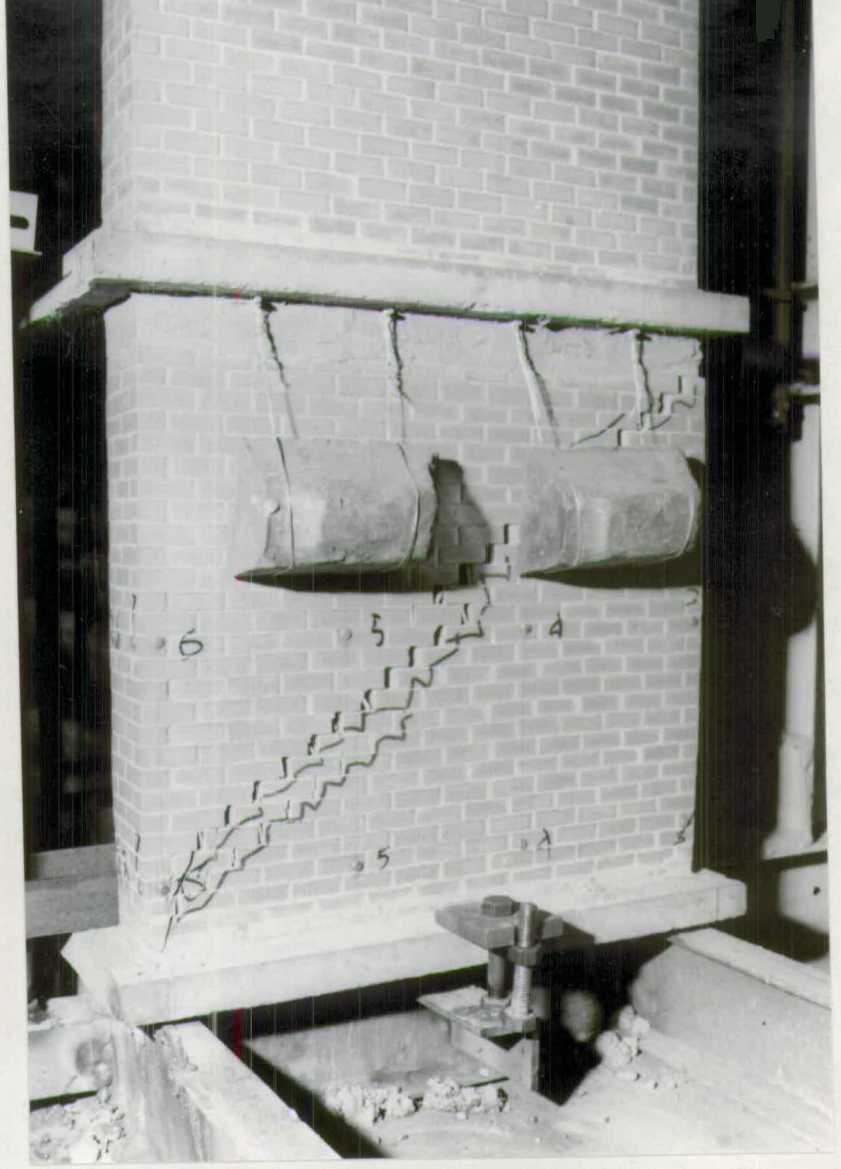


PLATE 6.4 (a) THREE-STOREY TORSION WALLS UNDER TEST



(b) Wall A



(c) Wall B

PLATE 6.4 - THREE-STOREY STRUCTURE, SHOWING FAILURE IN THE LOWERMOST STOREY

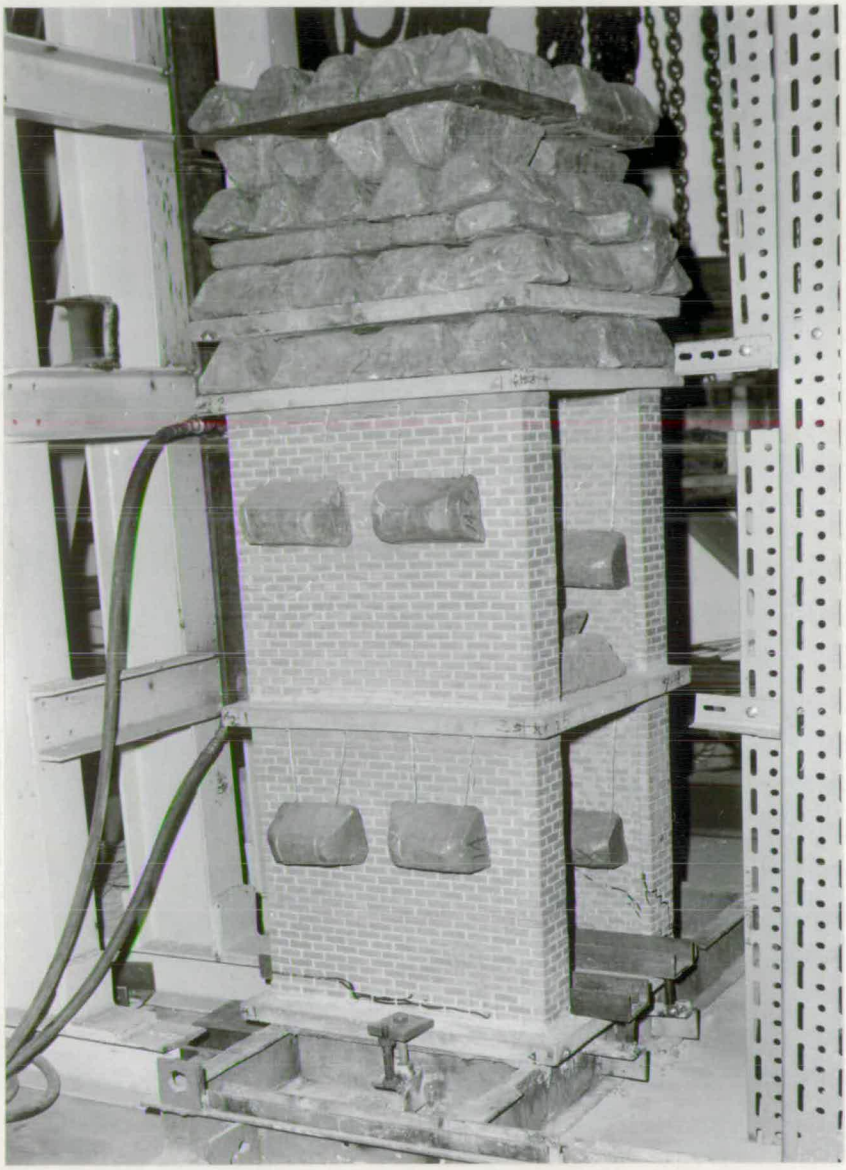
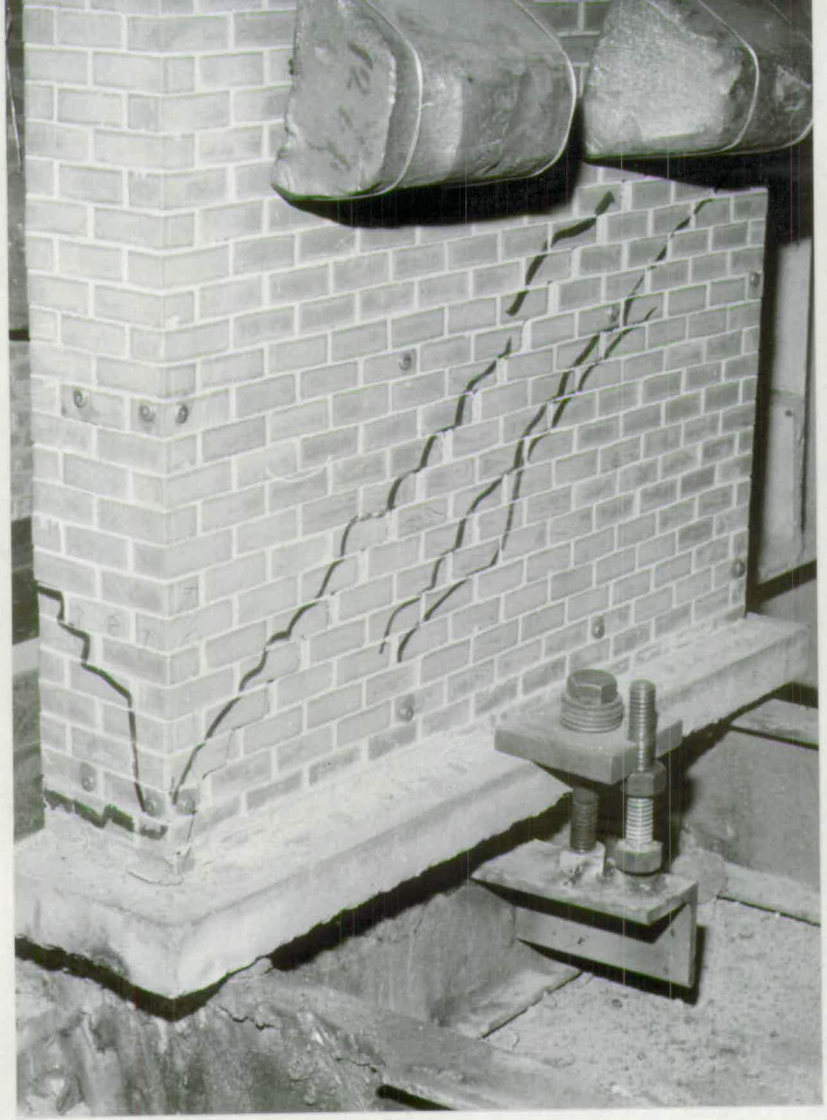
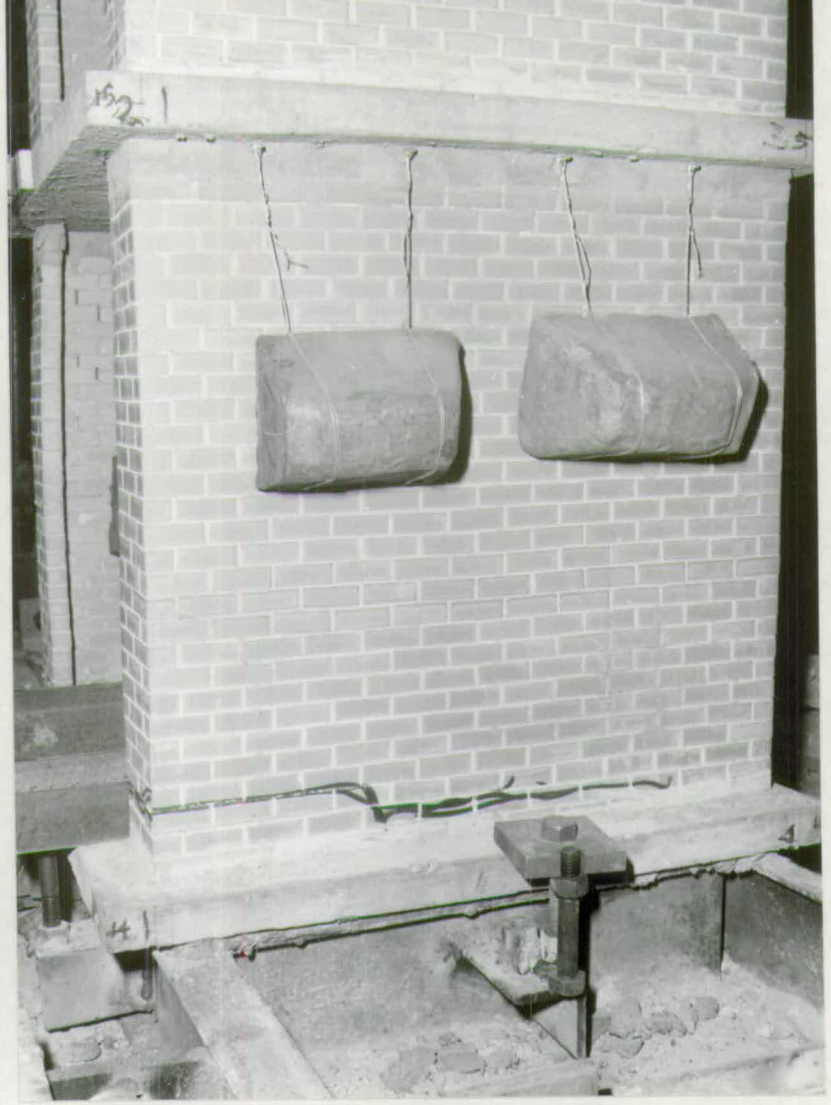


PLATE 6.5 (a) - TWO-STOREY TORSION WALLS UNDER TEST

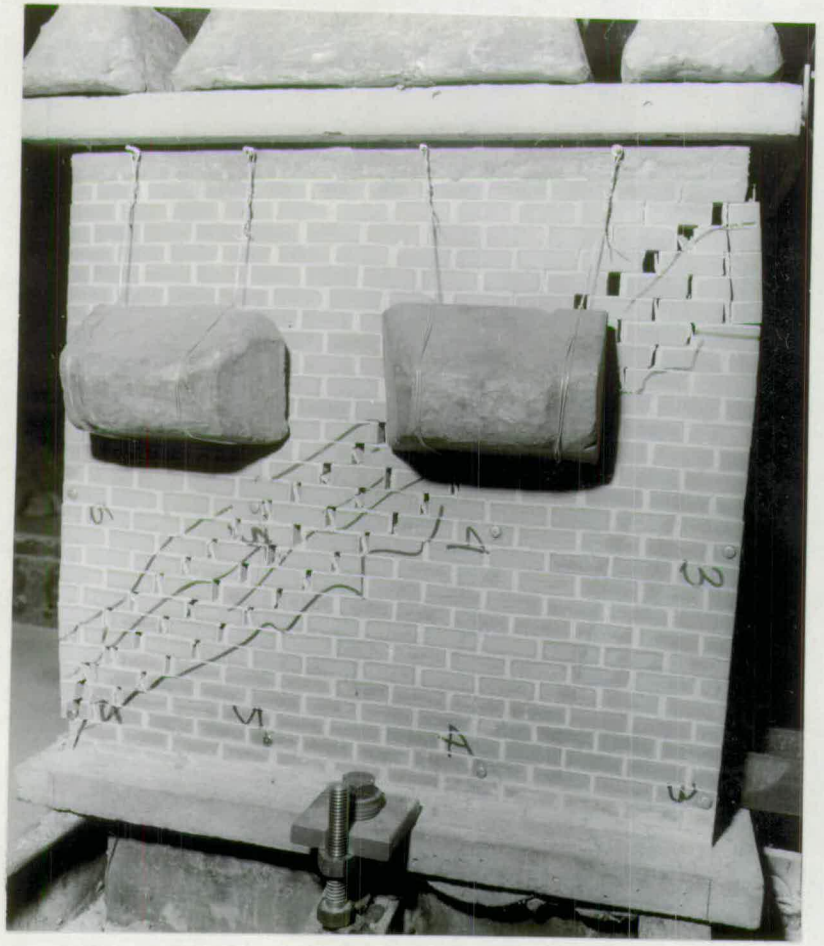


(b) WALL A



(c) WALL B

PLATE 6.5 - FAILURE IN LOWER STOREY OF A TWO-STOREY STRUCTURE



(a)

(b)

PLATE 6.6 - SINGLE-STOREY TORSION WALLS, SHOWING (a) FAILURE OF WALL A (b) FAILURE OF WALL B AND A FLANGE OF WALL A.

approximate ultimate shear at failure calculated from direct lateral load and twisting moment were 60, 72 and 77 lbf/in^2 for the three-storey, two-storey and single-storey structure respectively.

As may be seen from figure 4.9 of Chapter 4, these results are consistent with previous test results on model and full-scale structures[26].

6.9 THEORETICAL ANALYSIS

6.9.1 ASSUMPTIONS

From the experience of tests on the model structures and also as it is quite understandable, the deflection pattern of the structure will be as illustrated in figure 6.6. The lateral load has been split up into two direct and equal loads of half the original value acting on each of the channel, and a twisting moment 'pe' acting on the structure (Fig. 6.7). Since diaphragms (floor slabs) are reasonably rigid, it is further assumed that the moment 'pe' will be transmitted to the two channels carrying half by each channel. The effect of the twisting moment on Wall A will be additive to the direct shear whereas in the case of wall B it will be substractive.

6.9.2 SOLUTION FOR DIRECT LOADING

The finite element technique was used for analysis of wall A due to direct lateral loading. Solutions were obtained using the program mentioned earlier in Chapter 4. The E and G values were as those used in the analysis of the structures in Chapter 4, for respective precompression. The analysis was carried out for lateral loads 40 lbf. at each storey from 1st to 3rd and a load of 20 lbf. acting at the roof level of the model structure.

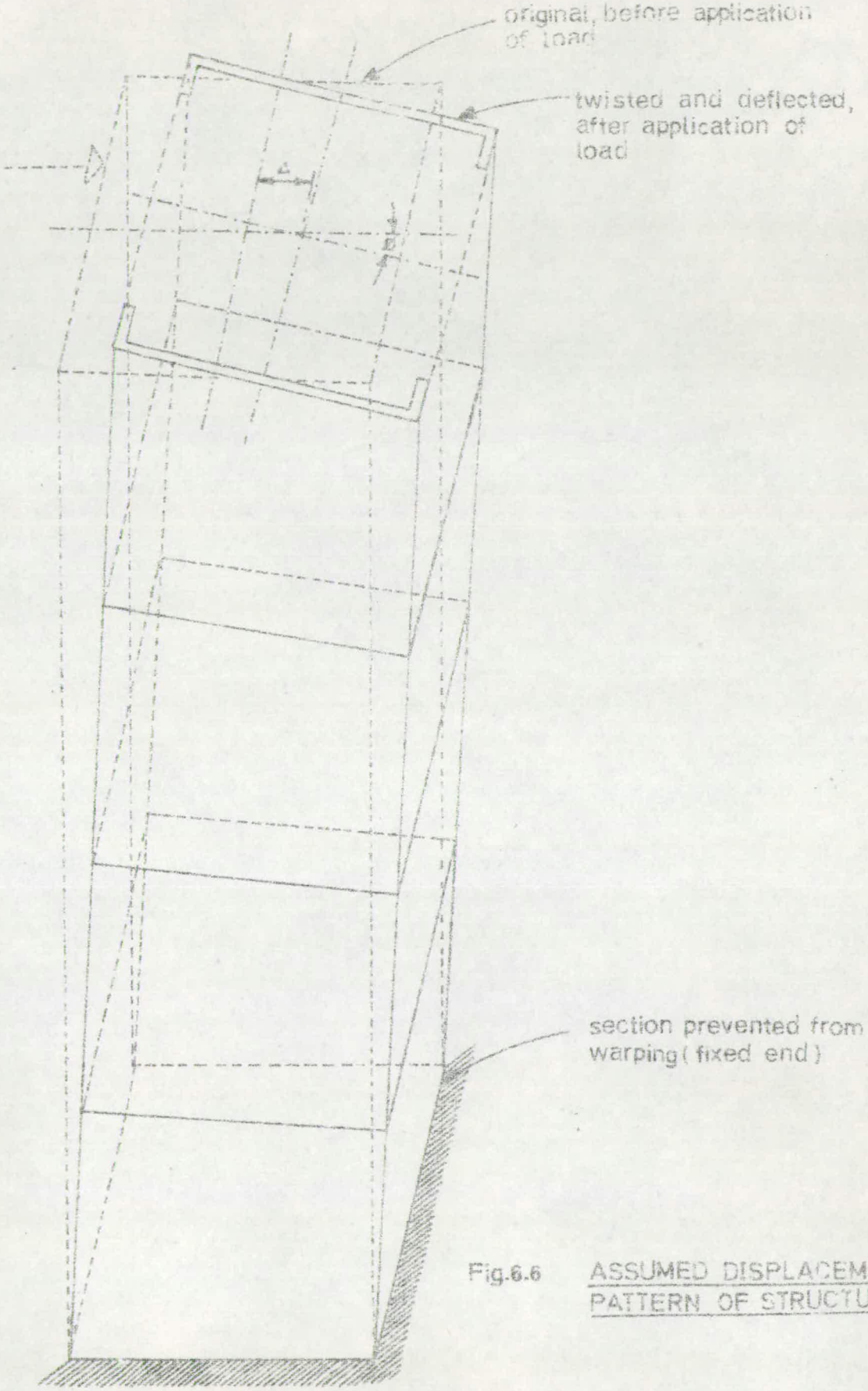


Fig.6.6 ASSUMED DISPLACEMENT PATTERN OF STRUCTURE.

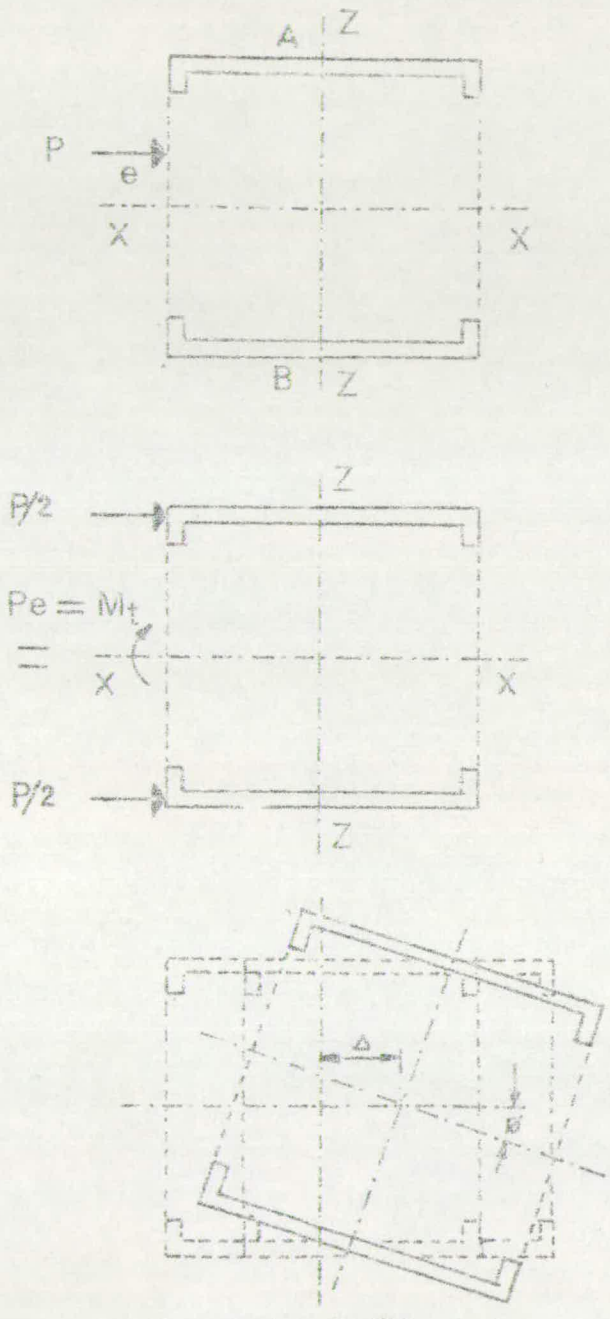


Fig.6.7 ECCENTRIC LOAD SPLIT UP INTO TWO DIRECT LOADS AND A MOMENT.

Figure 6.8 shows the finite element subdivision of wall A. Deflections obtained are shown in figure 6.9. Figure 6.10 shows vertical stress at section $X_1 - X_1$. Figure 6.11 is the automatic plotting of the principal stresses.

6.9.3 ANALYSIS FOR TWISTING MOMENT

6.9.3 (a) ANGLE OF TWIST

Referring to figure 6.7, the angle of twist per unit length for a channel section fixed at one end and subjected by a twisting moment at the other end is given by Timoshenko[57] as

$$\theta = \frac{M_t}{C}\left[1 - \frac{\text{Cosh}\left(\frac{H-Y}{a}\right)}{\text{Cosh}\left(\frac{H}{a}\right)}\right] \quad \ldots\ldots\ldots (6.1)$$

where,

$$\left.\begin{aligned} a^2 &= \frac{D\ell^2}{2C}\left(1 + \frac{t_w b^3}{4I_z}\right) \\ D &= \frac{Et_f b^3}{12} \\ C &= G \times K_1 \\ K_1 &= \frac{1}{3}\left(\ell t_w^3 + 2\, b t_f^3\right) \\ I_z &= \frac{t_w \ell^3}{12} + \frac{b t_f \ell^2}{12} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (6.2)$$

The angle of twist ϕ at any section of the channel can be found by integrating equation 6.1 with respect to the height of the structure.

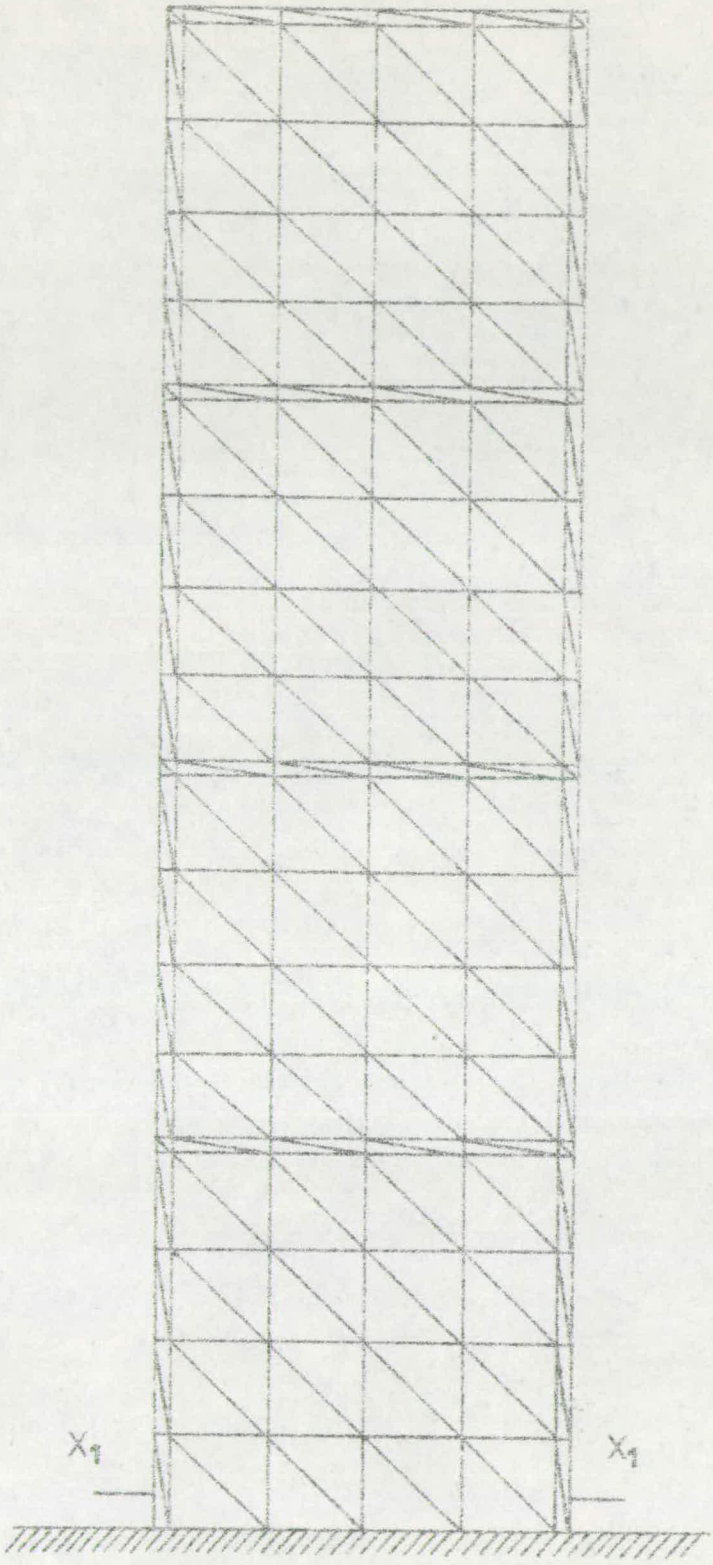


Fig.6.8 WALL A DIVIDED INTO FINITE ELEMENTS.

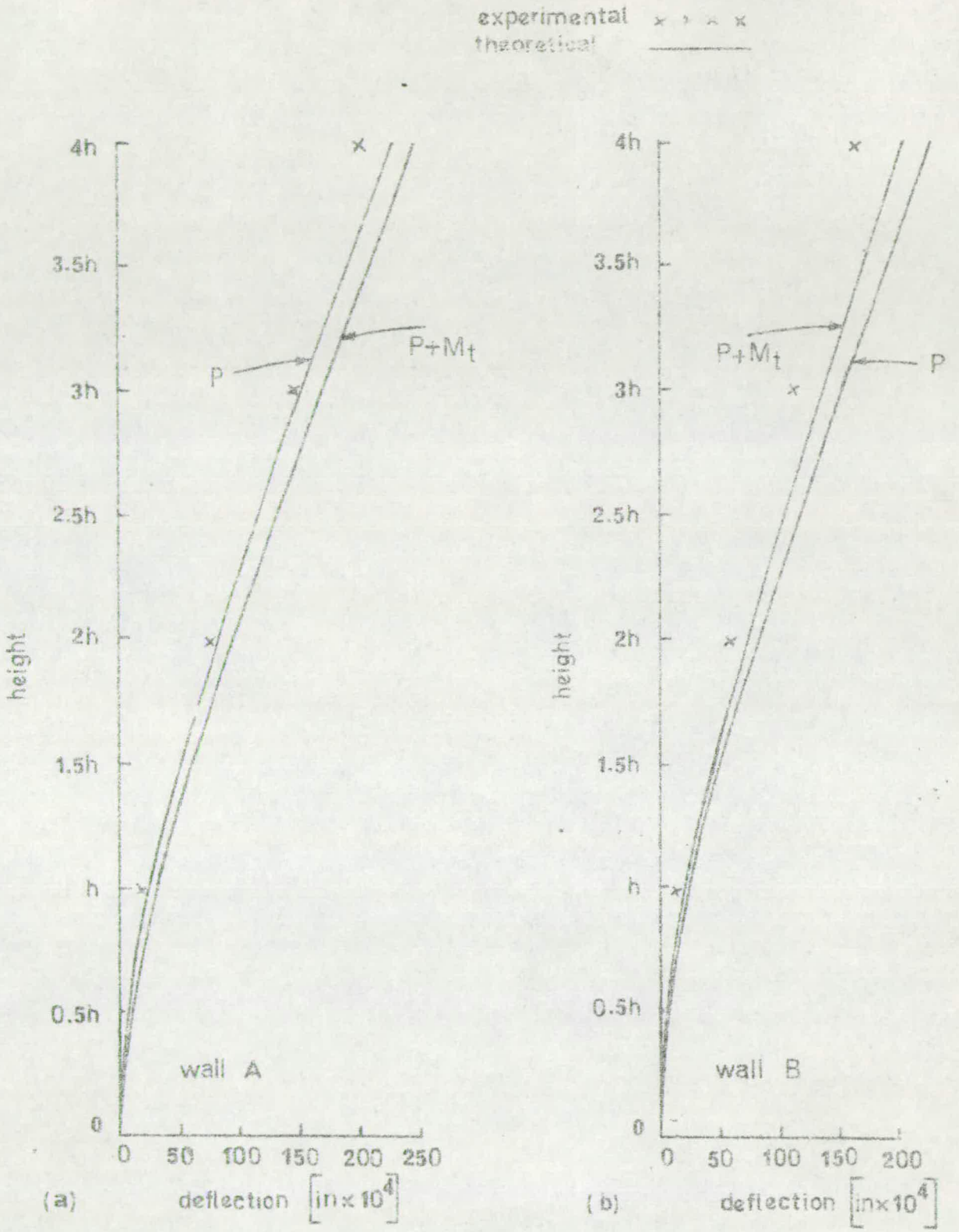


Fig.6.9 COMPARISON OF DEFLECTIONS OBTAINED FROM EXPERIMENTAL AND ANALYTICAL METHODS.

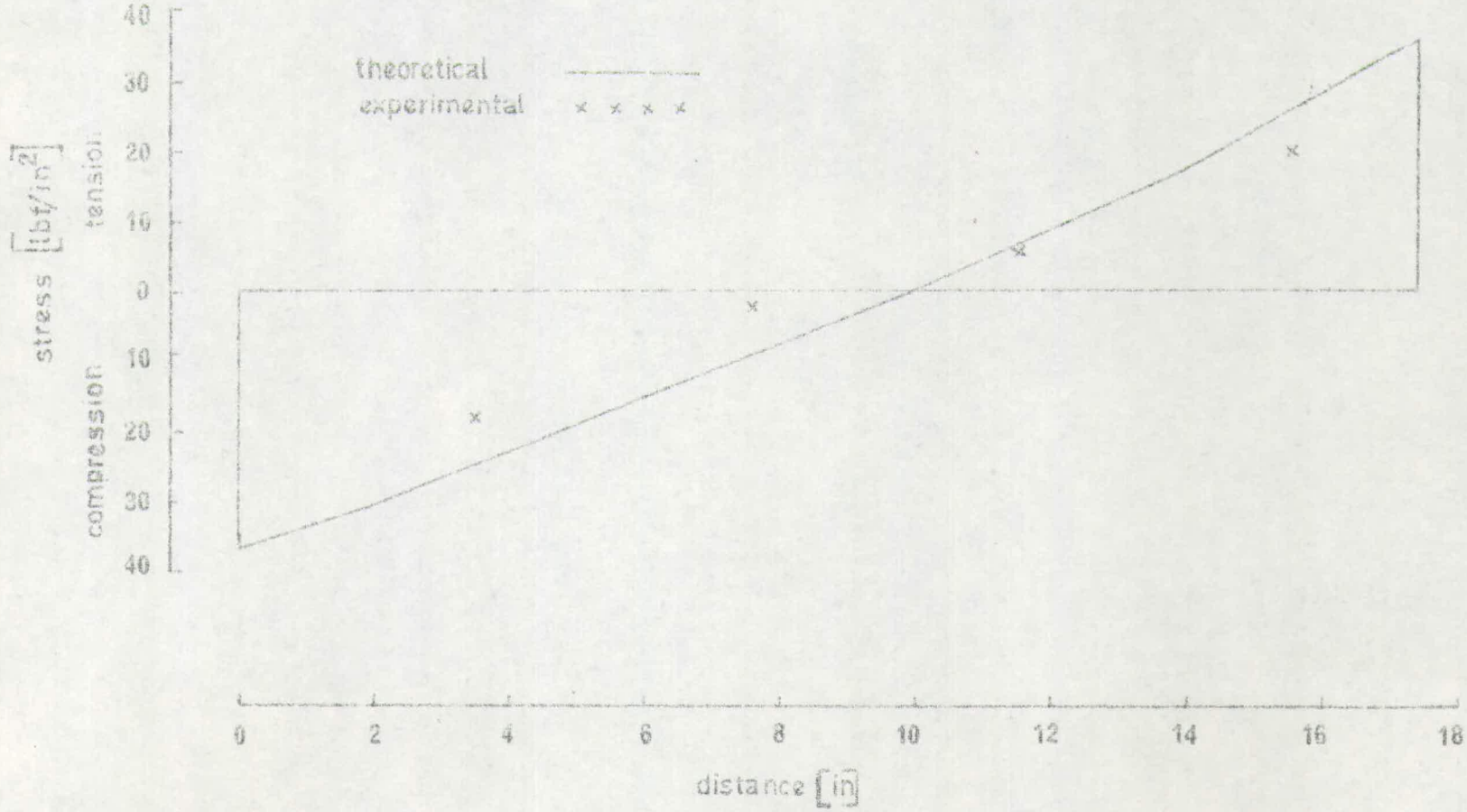


Fig.6.10 VERTICAL STRESS AT SECTION X_1-X_1.

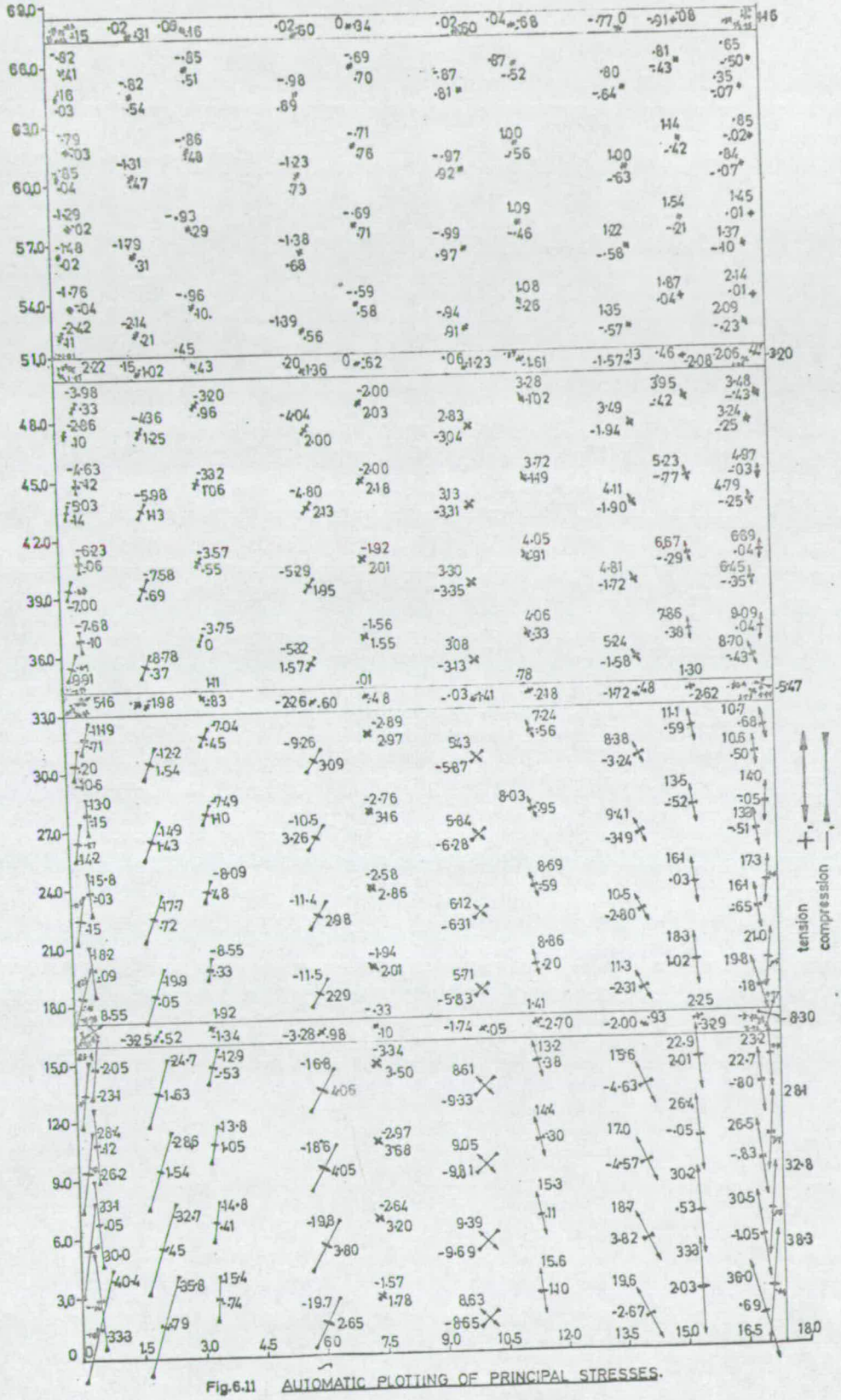


Fig.6.11 AUTOMATIC PLOTTING OF PRINCIPAL STRESSES.

Therefore,

$$\phi = \int \theta dy = \int \frac{M_t}{C} \left[1 - \frac{\cosh\left(\frac{H-Y}{a}\right)}{\cosh\left(\frac{H}{a}\right)} \right] dy \quad \ldots\ldots\ldots\ldots (6.3)$$

or

$$\phi = \frac{M_t}{c} \left[y + \frac{a \sinh\left(\frac{H-Y}{a}\right)}{\cosh\left(\frac{H}{a}\right)} \right] + c_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.4)$$

where c_1 is an integration constant,

Now $\phi = 0$ when $Y = 0$

Therefore,

$$c_1 = - \frac{M_t}{c} a \tanh\left(\frac{H}{a}\right)$$

Putting the value of c_1 in equation 6.4

$$\phi = \frac{M_t}{C} \left[y + \frac{a \sinh\left(\frac{H-Y}{a}\right)}{\cosh\left(\frac{H}{a}\right)} - a \tanh\left(\frac{H}{a}\right) \right] \ldots\ldots\ldots (6.5)$$

The values of ϕ due to four moments acting at each floor were calculated. The simple program for calculation has appeared in appendix I.

The values of angle of twist ϕ and deflection due to torque $\phi \times \frac{L}{2}$ are shown in figure 6.12.

6.9.3 (b) SHEAR STRESS IN WEB

The expression for the shear stress τ_{xy} as given by Timoshenko[57],

$$\tau_{xy} = \frac{ds}{t_w d_y} \left(1 - \frac{Q}{I'_z}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.6)$$

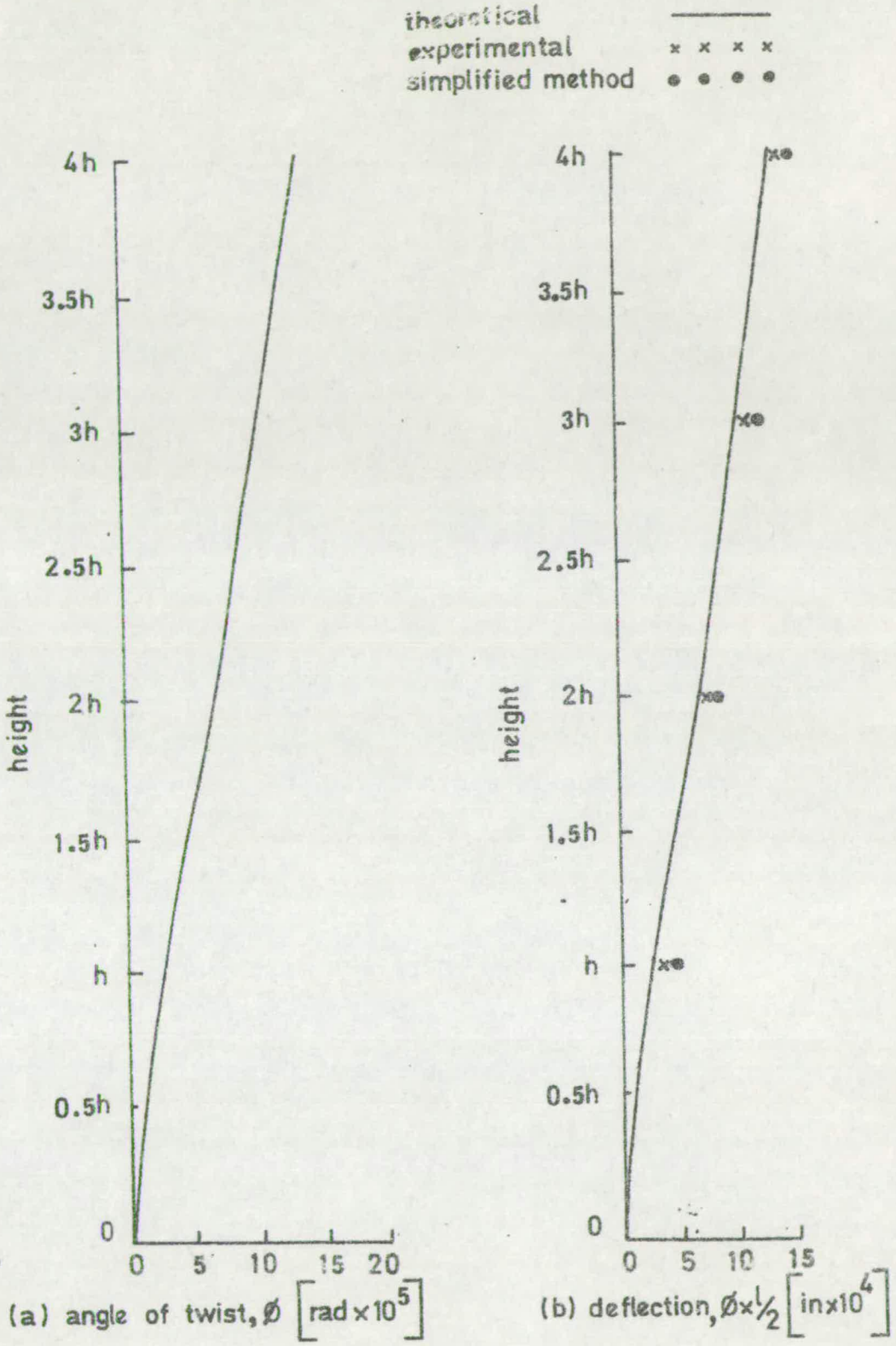


Fig.6.12 VARIATION OF (a) ANGLE OF TWIST AND (b) DEFLECTION WITH HEIGHT.

This expression can be represented in terms of the geometry of the structure and elastic constants of the material.

In equation 6.6

$$I_z' = \frac{t_w \ell^3}{12}$$

$$S = \frac{Eb^2 \ell^4 t_f t_w}{48 I_z} \cdot \frac{d^2\phi}{dy^2}$$

or $$\frac{ds}{dy} = \frac{Eb^2 \ell^4 t_f tw}{48 I_z} \cdot \frac{d^3\phi}{dy^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.7)$$

since $$\frac{d\phi}{dy} = \theta$$

Therefore, $$\frac{d^3\phi}{dy3} = \frac{d^2\theta}{dy2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.8)$$

Differentiating equation 6.1 with respect to y

$$\frac{d\theta}{dy} = \frac{M_t}{C}\left[0 - \frac{1}{\text{Cosh}\left(\frac{H}{a}\right)} \times \sinh\left(\frac{H-Y}{a}\right) \times \frac{1}{a}(0-1)\right]$$

or $$\frac{d^2\theta}{dy^2} = \frac{M_t}{C}\left[0 - \frac{1}{\text{Cosh}\left(\frac{H}{a}\right)} \times \text{Cosh}\left(\frac{H-Y}{a}\right) \times \frac{1}{a^2}(0-1)^2\right]$$

or $$\frac{d^2\theta}{dy2} = \frac{M_t}{C}\left[-\frac{1}{a^2} \times \frac{\cosh\left(\frac{H-Y}{a}\right)}{\cosh\left(\frac{H}{a}\right)}\right] \quad \ldots\ldots\ldots (6.9)$$

From equations 6.7, 6.8 and 6.9

$$\frac{ds}{dy} = \frac{Eb^2 \ell^4 t_f t_w}{48 I_z} \times \frac{M_t}{C}\left[- \frac{1}{a^2} \times \frac{\cosh\left(\frac{H-Y}{a}\right)}{\cosh\left(\frac{H}{a}\right)} \right] \ldots\ldots (6.10)$$

Substituting expression

for $\frac{ds}{dy}$ (Eq. 6.10) in equation 6.6

$$\tau_{xy} = \left(- \frac{M_t}{C t_w a^2} \times \frac{Eb^2 \ell^4 t_f t_w}{48 I_z} \right) \times$$

$$\left[\frac{\cosh\left(\frac{H-Y}{a}\right)}{\cosh\left(\frac{H}{a}\right)} \right] \times \left(1 - \frac{Q\ell}{I'_z} \right) \ldots\ldots\ldots\ldots\ldots\ldots (6.11)$$

where, $Q = t_w / 8 \times (\ell^2 - 4x^2)$

Equation 6.11 gives the distribution of shear stress in web due to twisting moment. Values of τ_{xy} were calculated using computer (appendix I). Figure 6.13 shows the distribution of shear stress in web at section $X_1 - x_1$.

6.9.3 (c) SHEAR STRESS IN FLANGE

The distribution of shear in flange[57] τ_{zy} is given by

$$\tau_{zy} = - \frac{1}{t_f} \frac{ds}{dy} \frac{b-z}{b} - \frac{E}{t_f} \cdot \frac{d^3\phi}{dy3} \cdot \frac{\ell Q1}{2} \ldots\ldots (6.12)$$

In the similar manner this expression can also be expressed in terms of dimensions of the structure and the elastic constants of the material.

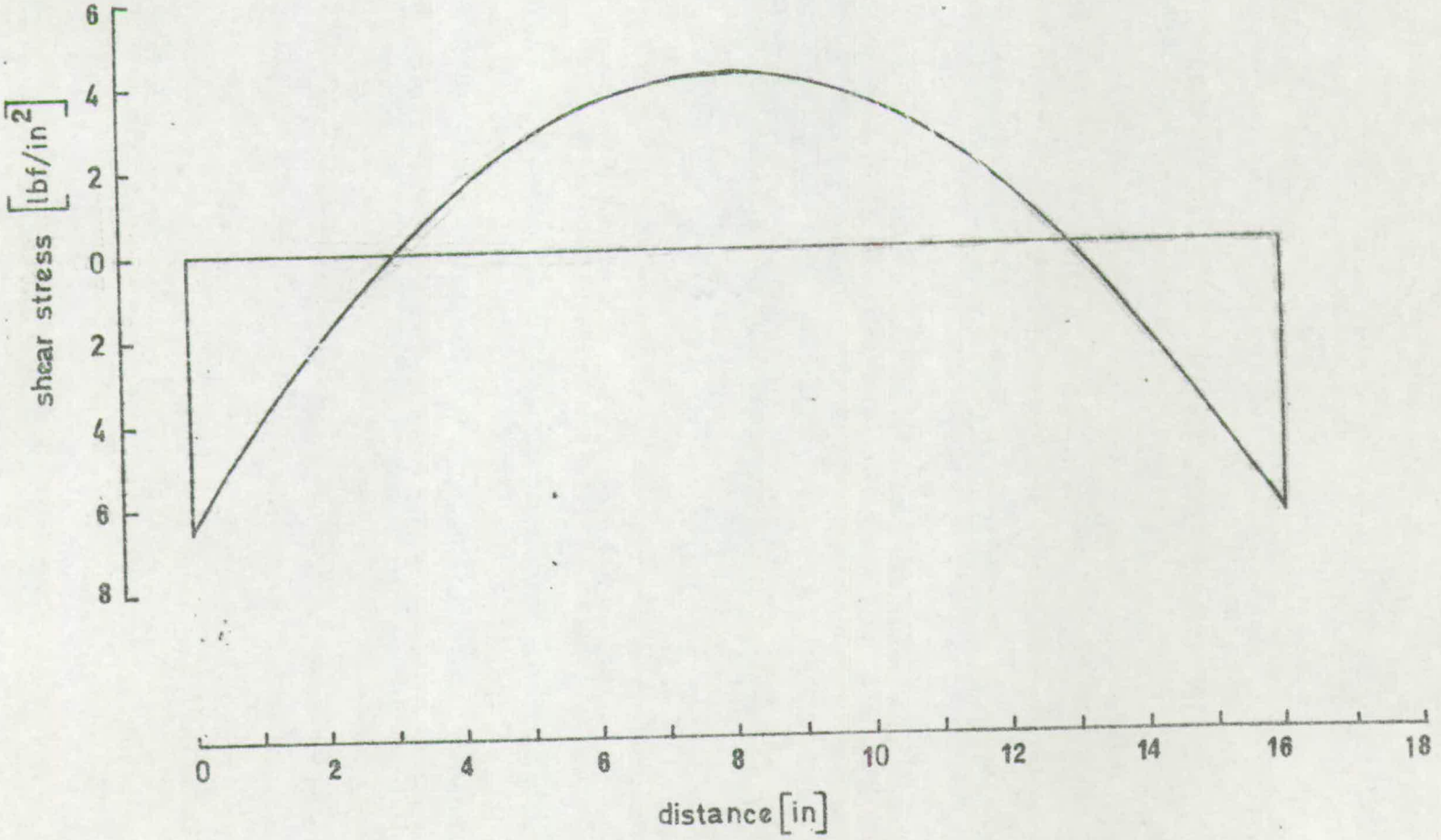


Fig.6.13 SHEAR STRESS IN WEB AT SECTION X1 – X1 DUE TO MOMENT.

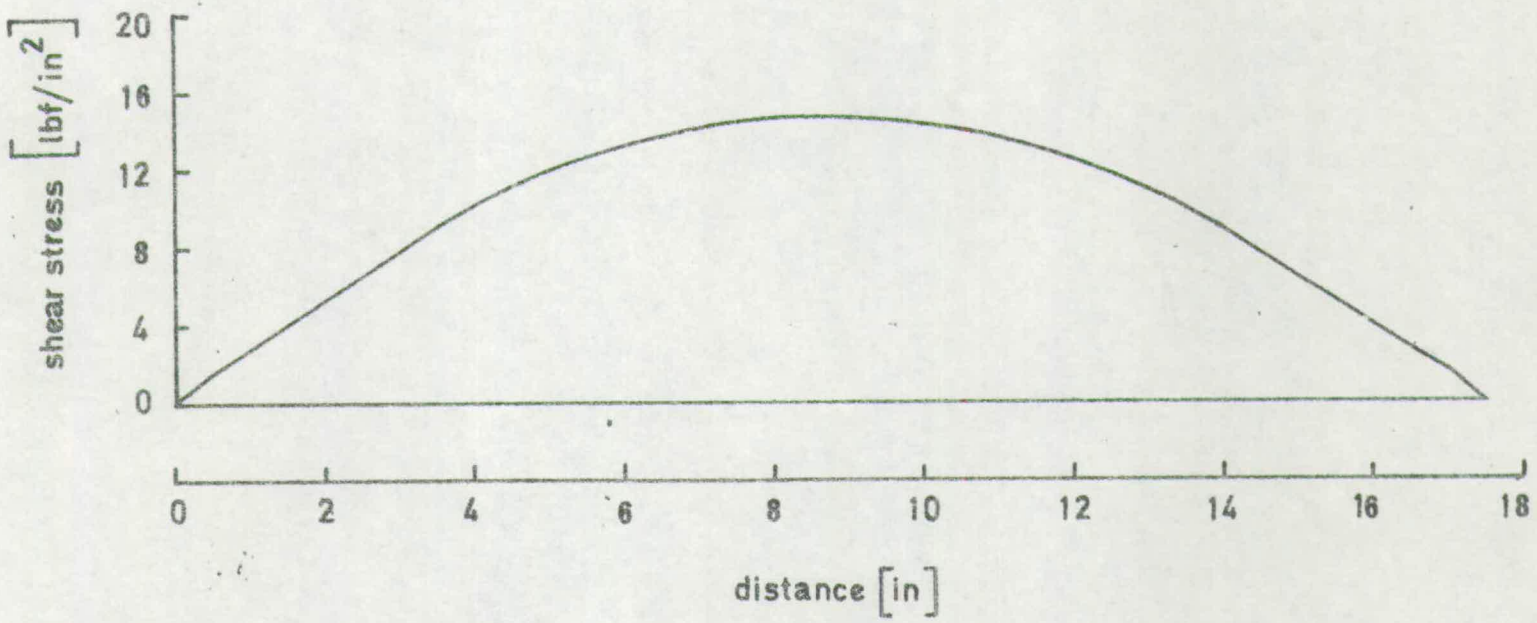


Fig.6.14 SHEAR STRESS AT SECTION X_1–X_1 DUE TO DIRECT LATERAL LOAD AND TORSION.

Using the expressions for $\frac{ds}{dy}$ and

$\frac{d^3\phi}{dy3}$ already derived

$$\tau_{zy} = - \frac{1}{t_f} \times \frac{Eb^2 \ell^4 t_f t_w}{48 I_z} \times \frac{M_t}{C} \left[- \frac{1}{a^2} \times \frac{\text{Cosh}\left(\frac{H-Y}{a}\right)}{\text{Cosh}\left(\frac{H}{a}\right)} \right]$$

$$\times \frac{b-z}{b} - \frac{E}{t_f} \times \frac{M_t}{C} \left[- \frac{1}{a^2} \times \frac{\cosh\left(\frac{H-Y}{a}\right)}{\cosh\left(\frac{H}{a}\right)} \right] \times \frac{\ell}{2} Q_1$$

$$\text{or} \quad \tau_{zy} = \left(\frac{M_t E}{c t_f a^2} \right) \times \left[\frac{\cosh\left(\frac{H-Y}{a}\right)}{\cosh\left(\frac{H}{a}\right)} \right] \times$$

$$\left(\frac{b^2 \ell^4 t_f t_w}{48 I_z} \times \frac{b-z}{b} + \frac{\ell Q_1}{2} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.13)$$

6.9.3 (d) A SIMPLIFIED METHOD FOR CALCULATION OF DEFLECTIONS DUE TO TWISTING MOMENT

The method of shear distribution[3,53] provides a simple means of analysing single-storey cross-wall structures. In this method the connecting diaphragm (floor slab) is assumed to be rigid in its plane and shear is distributed in the wall elements of the structural system according to their relative rigidities.

The expression for the relative rigidity[53] is

$$R = \frac{1}{\frac{h^3}{3I} + \frac{2.64h}{A}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.14)$$

For Wall A (Fig. 6.15)

$h = 17$ in.

$I_{Azz} = 523$ in.^{4}

$A = 11$ in.^{2}

Therefore the relative rigidity of Wall A in the direction x - x

$$R_{Ax} = 0.145$$

For flange of Wall A

$h = 17$ in.

$I_{Axx} = 7.5$ in.^{4}

$A = 3.2$ in.^{2}

Relative rigidity of flange of Wall A in direction z - z (Eq. 6.14)

$$R_{AZ} = 0.0043$$

Shear centre of Wall A (Fig. 6.15)

$$e_A = \frac{b}{2\left(\frac{a}{6b}\right) + 1}$$

where $a = 16.69$ in.

$b = 2.66$ in.

Therefore, $e_A = 0.65$ in.

Since the structure is symmetrical about both the principal axes, the shear centre (Fig. 6.15) of the assembly coincides with the point of intersection of axes x-x and z-z.

Adding product of rigidity and distance square from shear centre of the assembly for each wall of the structure,

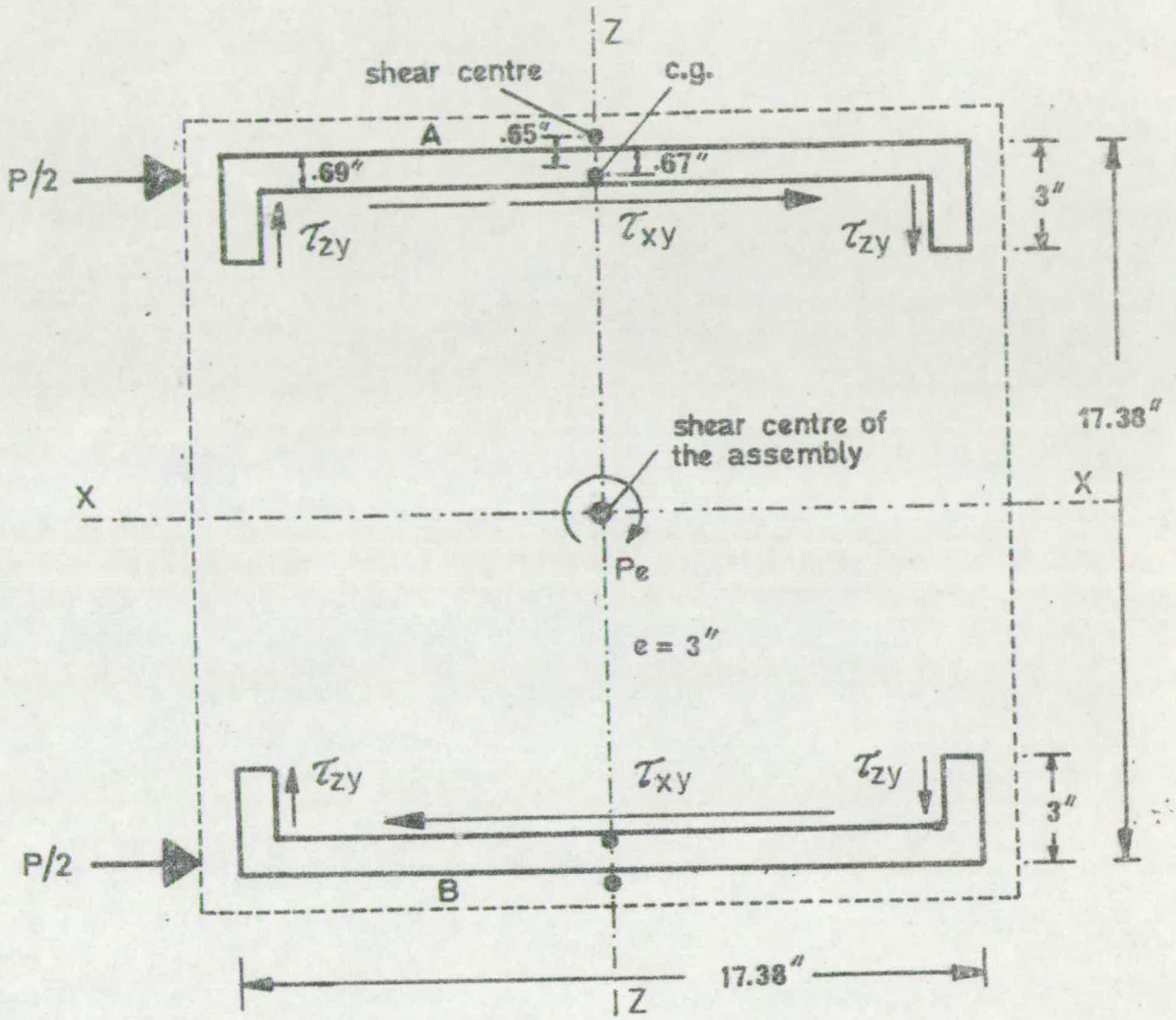


Fig.6.15 ARRANGEMENT OF TORSION WALLS.

$$\sum R_x Z^2 + \sum R_z x^2 = 24.7 \quad \text{.....(6.15)}$$

The shear developed on the cross-section of any shear-wall in directions x - x or z - z is assumed to be proportional to the distance from the shear centre of the assembly of shear-walls, when rigidity remains constant, and is also proportional to the rigidity of a given shear-wall at a given distance from the shear-centre.

The shear developed on the cross-section of the wall in the direction x - x due to torsional moment M_t is given by[53]

$$S_{xt} = \frac{M_t R_x Z}{\sum R_x z^2 + \sum R_z x^2} \quad \text{.....(6.16)}$$

For Wall A

$z = 9.0$ in.

$M_t = 420$ lb.in. (at 1st storey)

Therefore, shear on cross-section of Wall A at 1st storey in the direction of x - x

$$S_{Axt} = 22 \text{ lbf.}$$

Similarly shear developed at 2nd, 3rd, and 4th storey will be 16, 10 and 3 lbf. respectively.

Knowing the values of shear, the deflection at each storey was calculated separately by using the following formula,

$$\Delta = \frac{(P_1 + P_2 + P_3 + P_4)h^3}{12EI} + 1.2\,\frac{(P_1 + P_2 + P_3 + P_4)h}{AG} \quad \text{.... (6.17)}$$

Assuming values of Δ calculated from Eq. 6.17 be $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 the deflections of the structure will be as follows.

1st storey $\quad \delta_1 = \Delta_1$

2nd " $\quad \delta_2 = \Delta_1 + \Delta_2$

3rd " $\quad \delta_3 = \Delta_1 + \Delta_2 + \Delta_3$

4th " $\quad \delta_4 = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4$

These values are plotted in Fig. 6.12 for comparison with those obtained by other methods.

Shear on flange of Wall A due to M_t in z-z direction can also be calculated from the following formula.

$$S_{zt} = \frac{M_t R_z X}{\sum R_x Z^2 + \sum R_z X^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (6.18)$$

6.10 DISCUSSION OF RESULTS

From figure 6.12 it is seen that the calculated deflections due to twisting moment are reasonably close to the experimental values. Deflections calculated by the simplified method are also close. This is, however, an approximate method and quite satisfactory in the case of a single storey structure.

The method of analysis based on theory of warping of thin walled sections seems to be satisfactory for a simple brickwork cross-wall structure subjected to a twisting moment, provided the interaction of the floor slabs could be introduced in the calculations. In the

present case the floor slabs have been assumed to be of same thickness as that of the wall. To do this the elastic constants for slabs have been increased proportionately, and an average value of G for brickwork and concrete has been used in the calculation.

For analysis due to direct load the values of elastic constants are same as those used for analysis of structures in chapter 4 (Fig. 4.13).

Neglecting the effect of slabs, in other words assuming a channel of brickwork throughout the whole length without any cross diaphragm, the values of angle of twist are much higher than the experimental results. The reason may be that the floor slabs may have significant influence on the torsional rigidity of the structure.

In discussion of a paper (Taranath and Morice)[58], Harrison[29] has commented upon the important influence of cross diaphragms restraining structural members subjected to twisting. Figure 6.16 shows the experimental results presented by him. A channel, 0.25 in. perspex with 3.0 in. flanges and a 5.5 in. web, 89.0 in.long, fitted with cross diaphragms, 0.25 in. thick at 4.94 in centres was subjected to a pure torque. It was found that the effect of the diaphragms was to double the torsional rigidity of the channel. The influence of individual diaphragm can be included in the analysis of torsion using Vlasov's theory of thin walled beams.[58]

Figure 6.9 shows the deflection curves obtained by the finite element method. This figure also shows the deflections due to both direct load and twisting moment for Walls A and B. The calculated

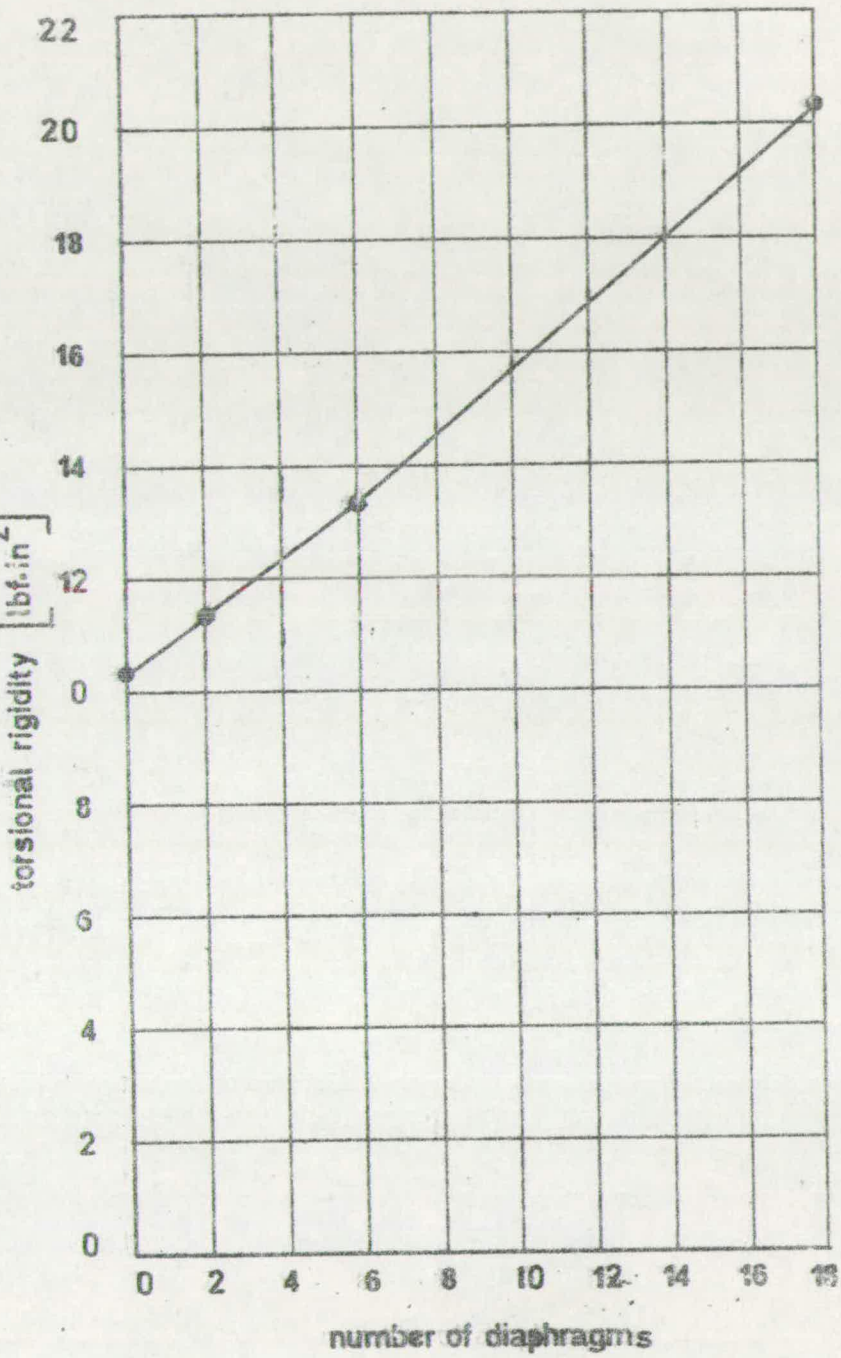


Fig.6.16 INFLUENCE OF CROSS DIAPHRAGMS ON TORSIONAL RIGIDITY.

deflections differed from the experimental ones by about 18%. However, in the case of symmetrical loading (chapter 4) a better agreement was obtained. The structure seemed to be stronger than expected analytically. This may be because of the interaction of floor slabs connecting the walls. The difference of 18% is not so bad in view of the fact that the rigidity of a brickwork structure is influenced by several variables such as brick strength, mortar strength, workmanship etc. The ultimate strength of the structure, calculated from direct load and twisting moment, was about 40 lbf/in.^{2}. This is consistent with previous results (Fig. 4.9, chapter 4).

The vertical stress distribution at section $x_1 - x_1$ of Wall A is shown in figure 6.10. Although theoretically the neutral axis should coincide with the axis of symmetry, it seems to be shifted by a little distance, probably because of the finite element discretization of the continuum. The measured values of stresses are also shown in figure 6.10.

The distribution of shear in web of Wall A at section $x_1 - x_1$ due to twisting moment is shown in figure 6.13. The approximate distribution of shear at section $x_1 - x_1$ due to twisting moment and direct load can be obtained from the principle of superposition (Fig. 6.14).

A large portion of shear due to twisting moment will be taken by the flanges of the channel. In the present case, however, the width of flange (length of external return wall) was very small compared to the main shear wall. The shear stress in flange,

due to twisting moment, can be calculated from equation 6.13 in the similar way as the calculation of shear in web from equation 6.11.

It may be anticipated that in designing a brickwork structure adequate stiffness has to be provided so that the cross-walls can withstand additional stresses due to torsion in addition to those produced by the lateral loads. Torsion may be developed, however, small in magnitude, by mere lack of symmetry of wind loading despite the fact that the building is a symmetrical one.

6.11 CONCLUSIONS

Under eccentric lateral loading which produces torsion in addition to direct lateral load, failure of a multi-storey brickwork cross-wall structure would take place in the lowermost storey due to resultant shear produced by the direct load and torsion. If the precompression is high cracks may go diagonally in the panel, due to breakdown of bond between bricks and mortar. Cracks also develop in the external return walls particularly towards the loaded side along with those in the main cross-walls.

The stresses and deflections of a simple brickwork cross-wall structure subjected to eccentric lateral loading can be calculated separately by splitting the problem into two parts e.g. analysis (a) due to direct lateral load and (b) due to

twisting moment. The finite element method can be used for solution due to direct lateral load and theories of torsion for thin walled section for analysis due to twisting moment. The final result can be obtained from the principle of superposition. The results obtained in the present case are reasonably accurate.

CHAPTER 7

GENERAL CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

7.1 CONCLUSIONS

The conclusions reached from the investigations presented in this thesis are summerised as follows:

(a) The rigidity and stresses in a brickwork structure can be calculated reasonably accurately by analytical solutions such as the finite element method provided that G is taken to be a function of precompression, i.e. G will vary along the height of the building.

(b) Under biaxial state of stress that is when precompression is applied normal to the shearing interface, the shear modulus and rigidity of a brickwork structure increase with increase of precompression.

(c) The shear modulus remains fairly constant at a lower stress value, after which it decreases with increase of shear stress.

(d) Failure in multi-storey brickwork cross-wall structures occurs in the first storey assuming uniform thickness of wall and the same brick throughout. Failure is due to the breakdown of bond and frictional resistance at the brick/mortar interface, leading to diagonal cracks stepping through the vertical and horizontal mortar joints. In some cases it may occur at the joint between slab and wall.

(e) The failure of a multi-storey brickwork structure under

eccentric lateral loading which produces torsion in addition to direct lateral load would take place due to resultant shear produced by direct load and torsion.

(f) By applying sufficient precompression, if the lateral load is applied, failure of shear walls would occur in diagonal cracks passing through both the bricks and mortar, in which case the ultimate strength of shear walls depends on the diagonal tensile strength of brickwork. However, if the precompression is not fairly high, diagonal cracks may not develop.

(g) In multi-storey loadbearing brickwork cross-wall structures failure occurs first in the main cross-walls of lowermost storey leaving the external return walls unaffected. This may be because of the fact that the shear due to direct lateral load resisted by these walls is negligible. However, when lateral load is applied at an eccentricity, the torsional shear produced causes the return walls to fail along with the main cross-walls.

(h) The stresses and deflections of a simple multi-storey brickwork structure subjected to lateral load and torsion can be calculated with reasonable accuracy by using the finite element method and theories of torsion for thin walled sections. However, further work is necessary to find a more refined solution which will take into account factors such as the interaction of cross diaphragms (floor slabs).

(i) In coupled shear walls the stresses are very much dependent on the width of the opening. As the width decreases, the coupled

walls tend to behave as a composite cantilever, whereas they will tend to behave as individual cantilever if the width of the opening is more. There are, however, other factors affecting the distribution of stress such as stiffness of connecting beam, type of loading etc.

(j) The ultimate strength of a brickwork structure subjected to lateral loading will not be very much dependent on the strength of individual brick unit. It may, however, play a significant role in resisting vertical compressive load, where failure is normally governed by the tensile strength of brickwork and where cracks pass through both mortar and bricks. Under lateral loading, in most cases, cracks develop through brick/mortar interface, except in the case, where the precompression is high, cracks will develop through brick/mortar joint and through bricks.

(k) The strength and rigidity of a full-scale brickwork structure subjected to lateral load can be reproduced with reasonable accuracy by means of model tests.

(l) The finite element method can be used to investigate the effective width of external return walls and slabs. For the simple structure investigated the effective "flange" width was 0.35 of the storey height and the effective slab width was 0.5 of the bay width.

(m) The effective "flange" width and slab width can also be studied experimentally by carrying out shear tests on model structures using different width of "flange" and slab.

7.2 FUTURE WORK

Although the modulus of rigidity of brickwork has been found to increase with increase of precompression, further work may be carried out on full-scale brickwork wall panel to investigate the magnitude of such variations. This could be done by carrying out lateral loading tests on a full-scale single panel cantilever wall with or without a slab on it, under laboratory conditions.

The stiffness of connecting beams(or floor slab) has significant influence on the behaviour of coupled shear-walls. Tests using slabs of different thicknesses may be carried out to investigate these effects. These effects can also be studied theoretically using the existing programme for the finite element method.

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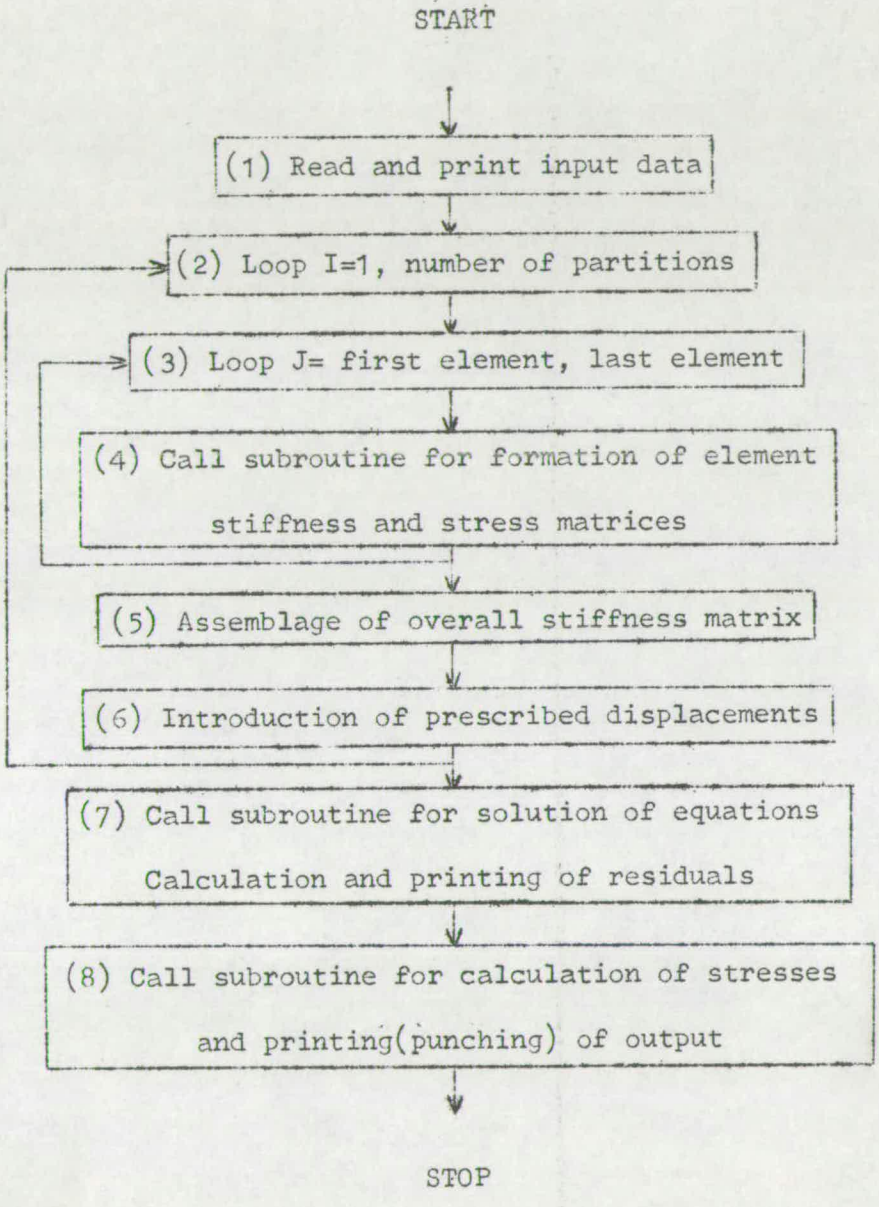
APPENDIX I

1. DESCRITPION OF "STRESS" PROGRAM

The "stress" program for the Univac 1108 computer, National Engineering Laboratory, East Kilbride, which was used for the analysis was written in Fortran V programming language. This program is for two dimensional plane stress or plain strain using triangular elements and was originally developed by Zienkiewicz and Cheung[62]. Detailed description, including notes on partitioning, preparation of data, etc., is given in a Computer Program Report ("Textbook" Program VARORT), University of Wales, Swansea, August 1967. This program with slight modifications, if necessary, can be used for any kind of two dimensional plane stress or plain strain problem.

The whole program constitutes eight subroutines excluding another subroutine for plotting the principal stresses. These subroutines are combined by a main program, the flow chart of which is given in the next page. The print out consists of deflections at each nodal point and stresses at the centre of gravity of each element. Punched machine cards can also be obtained for plotting of the principal stresses using "Calcomp" plotter.

2. FLOW CHART OF MAIN PROGRAM



3. PROCEDURE FOR PREPARATION OF INPUT DATA

	Description		Notation	Format
1.	1 Card			
	Columns			
	1 - 4	Number of problems to be run in one execution of program	NPROB	I4
2.	1 Card			
	Columns			
	1 - 4	Number of partitions (≤ 25)	NPART	I4
	5 - 8	Number of nodal points (≤ 350)	NPOIN	I4
	9 -12	Number of elements (≤ 450)	NELEM	I4
	13 - 16	Number of nodal points with prescribed displacements (boundary points) (≤ 120)	NBOUN	I4
	17 - 20	Number of load vectors (If points with concentrated loads are present, they together form one load vector (≤ 4)	NCOLN	I4
	21 - 24	Number of different elastic properties (≤ 20)	NYM	I4

25 - 28	0: plane strain 1: plane stress	NP	I4
29 - 32	Number of degree of freedom per node	NFREE	I4
33 - 48	Force per unit volume in y direction	DENSIT	F16.8
49 - 64	Force per unit volume in x-direction	EARTH	F16.8

3. 1 Card for each nodal point (in ascending order)

Columns			
1 - 16	x - coordinate of nodal point		F16.8
17 - 32	y - coordinate of nodal point		F16.8

4. 1 Card for each element (in ascending order)

Columns			
1 - 4	Element number		I4
5 - 8	Nodal points in	NOD	I4
9 - 12	Clockwise or anti-		I4
13 - 16	clockwise direction		I4
17 - 20	Elastic property number	NEP	I4

21 - 36	Angle which the x-axis of orthotropy makes with the global axis	AN	F16.8
37 - 52	Thickness of element	THICK	F16.8

5. 1 Card for each nodal point with prescribed displacement columns

1 - 4	Nodal point number	NF	I4
5 - 8	Displacement in x - direction 0: fixed 1: free	NB	I4
9 - 12	Displacement in y direction 0: fixed 1: free	NB	I4
13 - 28	Prescribed value of displacement in x - direction	BV	F16.8
29 - 44	Prescribed value of displacement in y - direction	BV	F16.8

6. 1 Card for each partition (in ascending order)

Columns			
1 - 4	First element in the partition	NSTART	I4
5 - 8	Last element in the partition	NEND	I4
9 - 12	First nodal point in the partition	NFIRST	I4
13 - 16	Last nodal point in the partition	NLAST	I4

7. 1 Card for each elastic property

Columns			
1 - 16	Young's modulus in x - direction	E1	F16.6
17 - 32	Young's modulus in y - direction	E2	F16.6
33 - 48	Poisson's ratio in x direction	P1	F16.6
48 - 64	Poisson's ratio in y - direction	P2	F16.6
65 - 80	shear modulus	GE	F16.6

8. 1 Card

Columns			
1 - 4	Number of nodal points with concentrated loads	NCONC	14

9. 1 Card for each nodal point with concentrated load

Columns

1 - 4	Nodal point number		I4
5 - 20	Load in x - direction	U	F16.8
21 - 36	Load in y - direction	U	F16.8

4. PROGRAM FOR CALCULATION OF ANGLE OF TWIST

E.R.C.C. IMP(AA) COMPILER/IBM 360/50 COMPUTER

```
%LIST
ICIE 025/009/KALITA/ANGLE OF TWIST
%BEGIN
%RELSLONG
.
.
.
%REAL MT,C,G,K1,D,E,T,B,K2,L,A
%REAL%ARRAY K4(1:4),M(1:4),K3(1:4,1:69),N(1:4,1:69),%C
AN(1:4,1:69)
%INTEGER I,J,P,S,H,Y
READ(MT);READ(G);READ(K1);READ(E);READ(T);%C
READ(B);READ(L)
C=G*K1
D=(E*T*(B**3))/12
K2=(T*(L**3))/12+(B*(T**2))/2
A=SQRT((D*(L**2))/(2*C)*(1+(T*(L**3))/(4*K2)))
%CYCLE I=17,17,68
H=I
P=H/17
K4(P)=H/A
M(P)=-A*(EXP(K4(P))-EXP(-K4(P)))/%C
(EXP(K4(P))+EXP(-K4(P)))
%CYCLE J=0,1,68
```

```
Y=J
S=Y+1
K3(P,S)=(H-Y)/A
N(P,S)=Y+(A*(EXP(K3(P,S))-EXP(-K3(P,S))))/%C
(ExP(K4(P))+EXP(-K4(P)))
AN(P,S)=MT/C*(M(P)+N(P,S))
%REPEAT
%REPEAT
NEWLINE
%CYCLE I=17,17,68
%CYCLE J = 0,1,68
PRINT FL(AN((I/17),(J+1)),6)
NEWLINE
%REPEAT
NEWLINES (4)
%REPEAT
%ENDOFPROGRAM
```

5. PROGRAM FOR CALCULATION OF SHEAR STRESSES DUE TO TWISTING MOMENTS

E.R.C.C. IMP(AA) COMPILER/IBM 360/50 COMPUTER

```
%LIST
ICIE 025/010/KALITA/SHEAR STRESS IN WEB
%BEGIN
%REALSLONG
.
.
.
%REAL C,G,K1,D,E,T,B,K2,L,A,M,MT,Y
%REAL%ARRAY K3(1:4),K4(1:4),N(1:4),Q(1:4,1:9),K5(1:4,1:9),%C
R(1:4,1:9),TXY(1:4,1:9),PTXY(1:1,1:9),K10(1:4),K11(1:4)
%INTEGER I,J,P,S,H,X
READ(G);READ(K1);READ(E);READ(T);READ(B);READ(L);%C
READ(MT);READ(Y)
C=G*K1
D=(E*T*(B**3))/12
K2=(T*(L**3))/12+(B*(T**2))/2
A-SQRT((D*(L**2))/(2*C)*(1+(T*(L**3))/(4*K2)))
M=-MT/(C*T*(A**2))*(E*(B**2)*(L**4)*(T**2))/(48*K2)
%CYCLE I=17,17,68
H=I
P=H/17
K3(P)=(H-Y)/A
K4(P)=H/A
K10(P)=(EXP/K3(P))+EXP(-K3(P)))/2
```

```
K11(P)=(EXP(K4(P))+EXP(-K4(P)))/2
N(P)=K110(P)/K11(P)
%CYCLE J=0,1,8
X=J
S=X+1
Q(P,S)=(T/8)*((L**2)-4*(X**2))
K5(P,S)=(T*(L**3))/12
R(P,S)=1-((Q(P,S))*L)/K5(P,S)
TXY(P,S)=M*N(P)*R(P,S)
%REPEAT
%REPEAT
NEWLINE
%CYCLE I=17,17,68
%CYCLE J=0,1,8
PRINT FL(TXY((I/17),(J+1)),6)
NEWLINES(2)
%REPEAT
NEWLINES(6)
%REPEAT
NEWLINES(15)
%CYCLE J=0,1,8
PTXY(1,J+1)=TXY(1,J+1)+TXY(2,J+1)+TXY(3,J+1)+TXY(4,J+1)
PRINT FL(PTXY(1,J+1),6)
NEWLINES(2)
%REPEAT
%ENDOFPROGRAM
```

APPENDIX II

1. TYPE OF STRAIN GAUGES

Polyester gauges manufactured by the Tokyo Sokki Kenkyujo Co. Ltd. were used for the measurement of strains. These are electrical resistance strain gauges. Types PL-10 and PR-10 were used.

2. METHOD OF FIXING

P-2 Adhesive supplied by the manufacturers was used for bonding the gauges. Before bonding the surface was cleaned with acetone, precoated with P-S adhesive and then made smooth and plane with sandpaper: P-S adhesive was also used for coating bonded gauges to make them waterproof.

3. RECORDING INSTRUMENT

A Data Logger type 10SL was used to record strains. The unit is capable of calibration to give readings directly in engineering units or in units of micro-strain. Plate II-1 shows a photograph of the Data Logger.

The unit embodies the following main features.

(1) A choice of three strain gauge energising supplies, selectable to values of 2.5V, 5.0V and 10.0V, with a maximum total load of 2A at 10.0V. The energising supplies are short circuit proof.

(2) Ten input channels, each with its own input socket and a ten turn potentiometer for balancing the external strain gauge bridge (Fig. II-1).

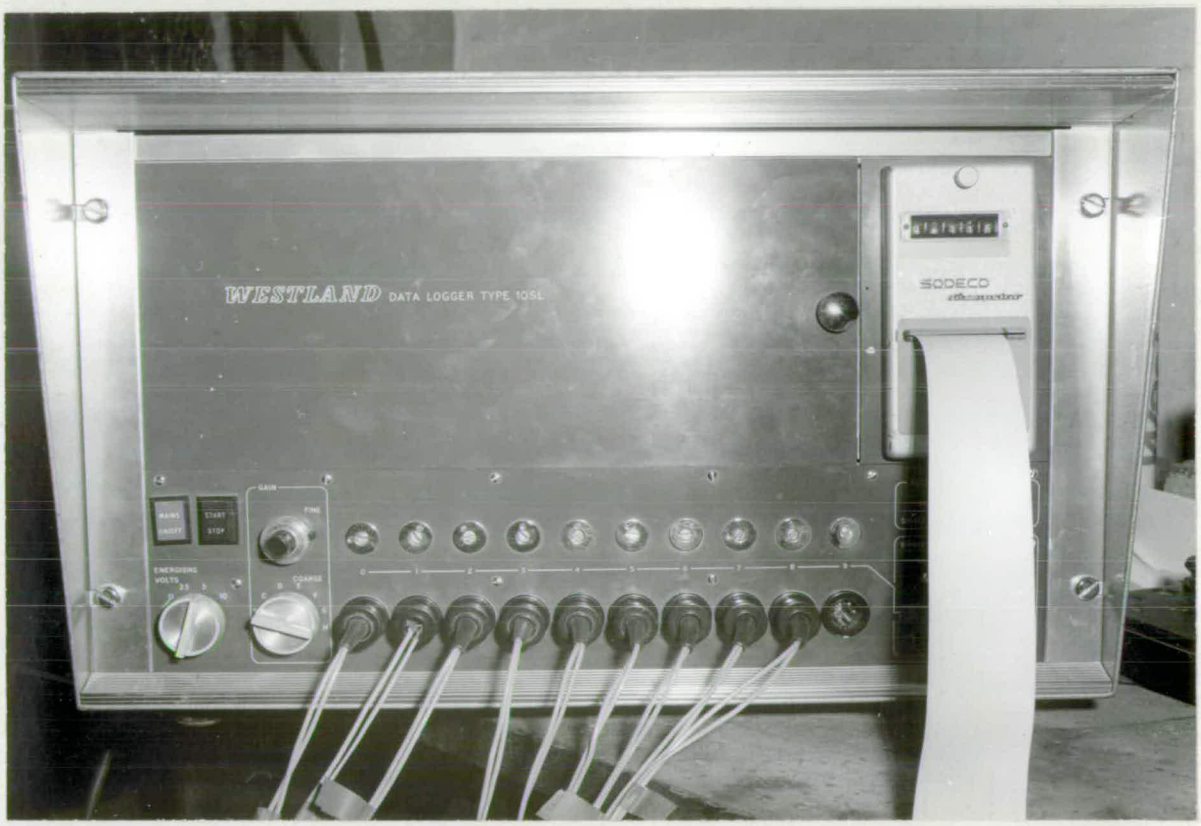


PLATE II-1 DATA LOGGER TYPE 10SL

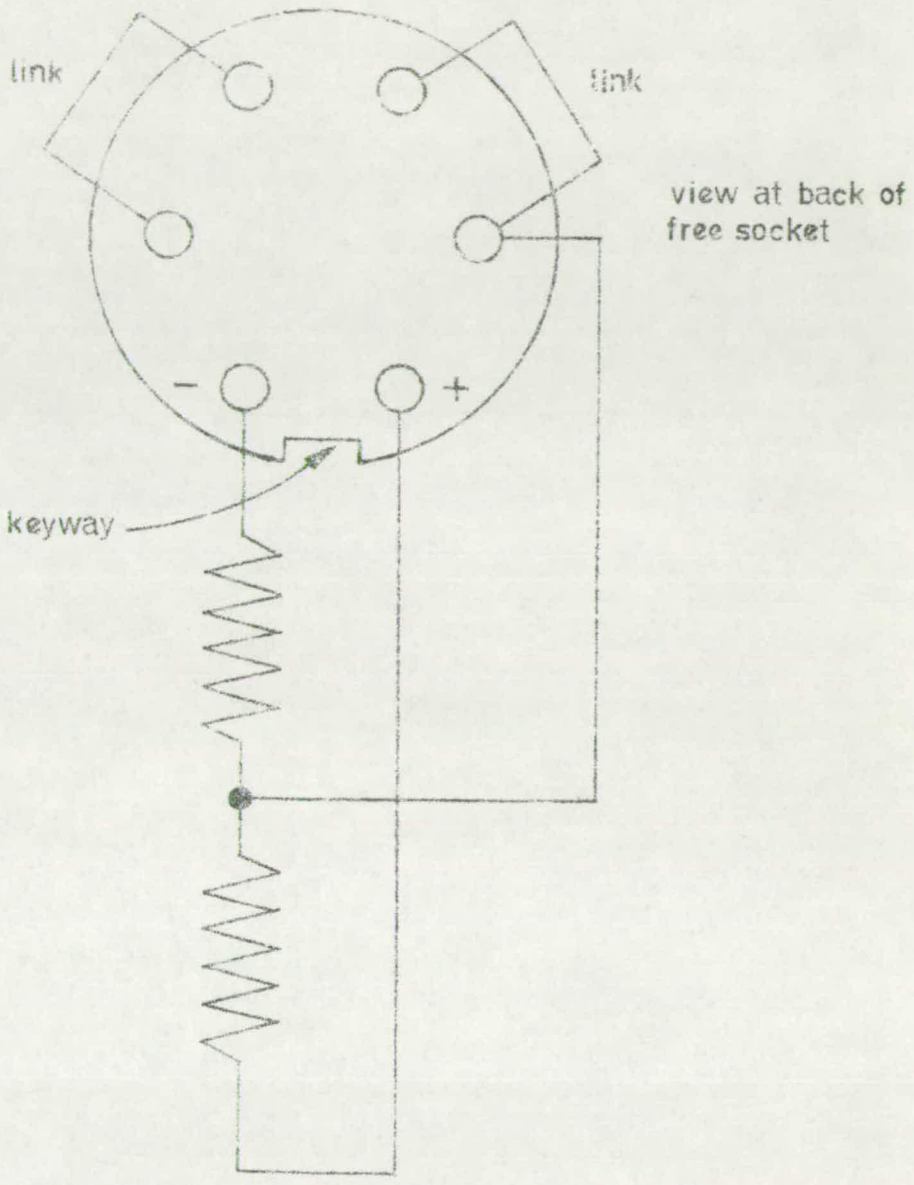


Fig.II-1 CONNECTIONS FOR HALF BRIDGE WORKING.

(3) An analogue to digital converter (A.D.C.) with a full scale range of $\pm$799 units. The A.D.C. is driven by a differential amplifier which is comprised of a number of balanced amplifier transistor pairs with two main feedback loops. The linearity and gain stability of the amplifier is defined by passive feedback resistors.

(4) A gain control comprised of a coarse scale switch and a ten turn potentiometer for fine control between the steps of gain provided by the coarse switch.

(5) An internal strain gauge bridge, one half of which is utilized as a dummy bridge for operation with external half bridge. The other half is unbalanced by the equivalent of 300 microstrain in two active arms.

(6) A pointer drive unit to drive a Sodeco printing giving a print out of channel number, polarity and digital output.

(7) A control system which provides the selection of channels and operating mode signals.