

ABSTRACT OF THESIS

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Title of Thesis Polarisation Phenomena in High Energy Weak and Electromagnetic Interactions.

High energy lepton-hadron scattering experiments have served as a source of rich data in the investigation of elementary particle physics. Whether dealing with electromagnetic processes, such as electron-proton scattering, or in the realm of weak interactions via neutrino reactions, they have served to determine the form factors for the respective processes which contain the details of the strong interaction dynamics and knowledge of which is, naturally, of paramount importance. As yet, all experiments have been performed with unpolarised targets or beams; however the rapid strides forward in experimental techniques have now brought into reality the feasibility of polarisation measurements in such reactions and so in this thesis we examine in detail the information extractable with polarisation techniques from these processes.

We commence in Chapter 1 with a brief survey of high energy electromagnetic scattering and the motivation for studying polarisation phenomena. The detailed analysis is then first applied in Chapter 2 to elastic lepton-nucleon e.m. scattering and reveals the utility of certain configurations for determining the electric form factor G_E . Tests of electron-muon universality are also discussed.

Chapter 3 contains an analysis of polarisation effects in elastic colliding beam reactions and in nucleon-antinucleon annihilation into lepton pairs. It is seen these permit evaluation of the \bar{G}_M , \bar{G}_E form factors, which are an analytic continuation of /

of the form factors in lepton-nucleon scattering and concerning which little is known at present, experimentally or theoretically. Tests for electron-muon universality are again discussed here.

The restriction to elastic reactions is lifted in Chapter 4, where polarisation effects in inelastic scattering and colliding beam reactions are analysed. Besides permitting extraction of the W_1 form factor at small angles, not possible in unpolarised experiments, a sum rule due to Bjorken may be tested here as may be time-reversal invariance, both of great interest in themselves.

Finally we conclude in Chapter 5 with an investigation of neutrino reactions, employing a polarised nucleon target. A general discussion is given for inelastic reactions and then applied to the elastic case; with suitable configurations, four of the six form factors in the latter reaction may be determined. In particular it is possible to readily test for the presence of second class currents and also the validity of time-reversal invariance in weak interactions on which much attention is presently focused. The richness of polarisation studies is here quite clearly revealed.

POLARISATION PHENOMENA IN HIGH ENERGY

WEAK AND ELECTROMAGNETIC INTERACTIONS

Thesis

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CHAPTER I

INTRODUCTION

After the successful breakthrough in quantum electrodynamics at the end of the 1940's when the introduction of renormalisation theory enabled the problem of higher-order divergences to be overcome, it was only natural that attention should be directed towards elucidating the electromagnetic structure of the nucleon. A direct method of obtaining such information is to scatter high energy leptons* off a nucleon target and observe the energy and angular distributions of the scattered lepton. We are thus able in particular to determine the instantaneous charge distribution within the nucleon and hence "observe" its structure. Since the nucleon is a strongly interacting particle one might guess that its basic structure consists of a core of size $(\text{Nucleon Mass})^{-1} \sim 2 \times 10^{-14}$ cms. surrounded by a pion cloud extending over a distance $(\text{Pion Mass})^{-1} \sim 10^{-13}$ cms. Now a lepton of 1 GeV energy has a de Broglie wavelength of $\sim 10^{-14}$ cms. and so it may be used as a sensitive probe of the nucleon, since it possesses no strong interactions while its weak interactions are of negligible strength.

The production of high energy electron beams, commencing with the pioneering experiments of Hofstadter⁽¹⁾ at Stanford, initiated a vast experimental effort continuing up to the present time and has provided a wealth of details on the subject.

* By lepton we mean electron or muon.

To appreciate the interpretation of these experiments, we review briefly the theoretical basis of lepton-nucleon scattering.

Rosenbluth Formula

The basic process of lepton-nucleon scattering is believed to proceed as shown in the Feynman diagram of Fig. 1.

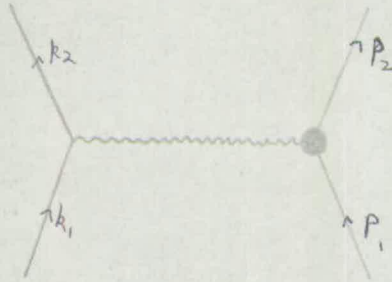


Fig. 1.

- k_1 = four momentum of initial lepton
- k_2 = " " " scattered lepton
- p_1 = " " " target nucleon
- p_2 = " " " recoil nucleon.

It is assumed only one-photon exchange term is important. Within such an assumption, the differential cross-section for lepton-nucleon scattering is given by the Rosenbluth formula⁽²⁾ :

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left\{ \frac{G_E^2(q^2) - \tau G_M^2(q^2)}{1 - \tau} - 2\tau G_M^2(q^2) \tan^2 \frac{\theta}{2} \right\} \quad (1.1)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2 \cos^2 \frac{\theta}{2} (4E_l^2 \sin^4 \frac{\theta}{2})^{-1}}{(1 + 2 \frac{E_l}{M} \sin^2 \frac{\theta}{2})} \frac{\alpha^2}{(-q^2)} \left(\frac{E_l'}{E_l}\right)^2 \cot^2 \frac{\theta}{2} ,$$

$$\tau = q^2/4M^2, \quad q^2 = \frac{-2E_l^2(1 - \cos\theta)}{1 + \frac{E_l}{M}(1 - \cos\theta)}$$

α is the fine-structure constant.

E_ℓ = initial lepton energy in the laboratory frame. E'_ℓ = scattered lepton energy.

θ = lepton-scattering angle in the laboratory frame.

q = Four-momentum transfer of lepton = $k_1 - k_2 = p_2 - p_1$.

The lepton mass has been put equal to zero which is adequate when in the range $E_\ell \geq 1$ GeV. Note that in our metric (see Appendix 1) q^2 is negative.

$G_E(q^2)$ and $G_M(q^2)$ are so-called form factors, scalar functions of q^2 only, which measure the longitudinal (coulomb) and transverse (magnetic) couplings respectively of the virtual photon to the nucleon. In some respect they are (in configuration space) a measure of the fourier transform of the electric and magnetic charge distributions in the nucleon (although this interpretation is not unambiguous⁽³⁾), and now generally bear the appellation the electric and magnetic form factor. These form factors contain entirely the effects of the strong interaction of the nucleon and their determination is hence of prime importance as any successful theory of strong interactions must satisfactorily explain their behaviour.

Experimental Status.

A detailed review of the experimental situation up to September 1969 has been given by Rutherglen at the Daresbury Conference⁽⁴⁾. The behaviour of the form factors is closely approximated by the "scaling law" and dipole fit:

$$G_{EP}(q^2) = G_{MP}(q^2)/\mu_P = G_{MN}(q^2)\mu_N = G_D G_{EN} = 0 \quad (1.2)$$

where $G_D = \frac{1}{(1 - q^2/0.71)^2}$ (Dipole formula), q^2 here being measured in units of $(\frac{\text{GeV}}{c})^2$. The P and N indices on the G's refer to proton and neutron, and $\mu_P(\mu_N)$ denotes the total magnetic moment of the proton(neutron).

Definite deviations from the dipole fit are well known, of up to +5% below $q^2 = -5(\frac{\text{GeV}}{c})^2$ and up to -15% at $-20(\frac{\text{GeV}}{c})^2$. The latest experimental results on electron-proton scattering⁽⁵⁾ at values of q^2 up to $-3.75(\frac{\text{GeV}}{c})^2$ are compatible with scaling, $G_E(q^2) = G_M(q^2)/\mu_P$ within the experimental errors.

Clearly an accurate knowledge of G_E and G_M , to as large values of q^2 as possible, is highly desirable. A number of phenomenological models and higher symmetry schemes differ considerably at high momentum transfers and more extensive and detailed data are necessary to establish if any one is capable of a good fit to all values of q^2 . In particular the status of the scaling law (which is predicted by SU(6)⁽⁶⁾) demands closer scrutiny.

However an examination of the Rosenbluth formula eq. (1.1), reveals the problems attendant in attempting to extract such data. The magnetic form factor G_M is multiplied by a factor τ which increases in proportion to q^2 and thus at high momentum transfers becomes very much the dominant term. In illustration, assuming the scaling relation eq. (2) to be true, the maximum percentage contribution of $G_E^2(q^2)$ to the $(G_E^2 - \tau G_M^2)$ term is 31% at $q^2 = -1(\text{GeV}/c)^2$, 15% at $q^2 = -2.5(\text{GeV}/c)^2$ and 10% at $q^2 = -4(\text{GeV}/c)^2$. At $q^2 = -20(\text{GeV}/c)^2$ it is less than 2%. The

The extraction of the G_E form factor thus becomes increasingly difficult as the momentum transfer increases and is inherent in the form of the Rosenbluth formula.

An alternative (and possibly superior) plot of the data has been recently employed. Using new form factors defined as $g_E = G_E (1 - q^2/0.71)^2$, $g_M = \frac{G_M}{u} (1 - q^2/0.71)^2$ and the kinematic factor $A = \epsilon/u^2$ where $\epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2} \right]^{-1}$ the Rosenbluth formula now assumes the form:

$$R_2 = (1 + A) \frac{d6}{d\Omega} / \left(\frac{d6}{d\Omega} \right)_{\text{dipole}} = g_M^2 + A g_E^2$$

where $\left(\frac{d6}{d\Omega} \right)_{\text{Dipole}}$ is obtained by substituting the dipole form factors Eq. (1.2) into Eq. (1.1).

As ϵ is limited to values between 0 and 1, A varies from 0 to $1/u^2\tau$.

Plotting R_2 against A therefore gives a straight line of slope g_E^2 and extrapolated intercept g_M^2 . If scaling is true, the slope and intercept will be equal; if the dipole relation also holds, both will equal unity. In this representation the statistical correlation between G_E and G_M is more easily seen. However at high q^2 , the range of values of A drops sharply, i.e. for $\tau = -1$, $0 < A < 0.13$; $\tau = -5$, $0 < A < 0.025$; and therefore measurement of the slope is very sensitive to small errors in $(d6/d\Omega)$.

The problem in obtaining accurate values of $G_E(q^2)$ at high q^2 is due basically to the existence of only G_E^2 and G_M^2 terms appearing separately in the Rosenbluth formula Eq. (1.1). The appearance of a cross term in $(G_E G_M)$ would resolve this difficulty

as measurement of it would yield directly both the magnitude and sign (relative to G_M) of G_E , it being presumed that G_M is already known from the slope of the Rosenbluth plot. Such a cross-term cannot arise in unpolarised lepton-nucleon scattering, where there is no interference between electric and magnetic scattering, but it can arise in polarisation experiments.

Up to now all scattering experiments have been performed with unpolarised beams and targets. However rapid developments in experimental techniques have made possible the realisation of good polarised targets, suitable for use even with electron beams. Thus polarisation measurements are now definitely feasible and in the next few years one may expect much effort to be concentrated in this direction with, one hopes, rewarding results.

A further stimulus arises from the anticipated expansion in proton accelerators with increasing yields of pion beams. As the pion decays dominantly into a muon plus neutrino, this in turn implies large flux of muon beams and enhances their suitability for scattering experiments which have been rather limited as yet. The muon appears to be purely a heavy lepton, and so plays an analogous role in extracting the nucleon form factors. Alternatively, such data may be used to establish the muon has no anomalous (i.e. non-electromagnetic) interactions other than that of a point lepton.

The muon possesses two distinct advantages over the electron:-
1) Because of its heavier mass, the magnitude of radiative corrections is considerably lowered and although computation of these corrections is a well understood (if tiresome) process⁽⁷⁾, it is naturally advantageous for them to be reduced to a minimum.

2) As a consequence of its birth from a pion by weak decay, the muon is polarised and so presents itself as a suitable candidate for polarisation experiments.

As a consequence of the reality of polarisation experiments, one is led to examine their theoretical foundations and see what information may be gleaned from them. Even a cursory glance indicates that they are a rich source of data which will add very considerably to our present state of knowledge on hadrons. Although motivation for such studies arose originally from the problem of determining the electric form factor G_E in elastic lepton-nucleon scattering, their influence ranges over a far wider field. Besides elastic lepton-nucleon scattering, they may be extended to inelastic lepton-nucleon processes where there is presently an intense experimental and theoretical program under way which has already produced some novel, exciting results. In particular, as will be shown later, use of a polarised target in inelastic scattering permits time-reversal invariance in electromagnetic interactions to be tested, which is certainly of prime importance and not feasible in elastic scattering experiments.

The development of intersecting storage rings has initiated wide studies, hitherto not feasible, such as determination of the pion form factor in the time-like region⁽⁸⁾. Theoretically such processes are closely correlated to lepton-hadron scattering events so it is natural to expect polarisation techniques will play an equally distinguished role here, as indeed they do.

Clearly the initiation of polarisation measurements in high-energy electromagnetic interactions is highly desirable from the

theoretical point of view, Experimentally, notwithstanding the development of suitable polarised targets, these will be difficult experiments to perform, fully taxing the skill and ingenuity of all concerned. However they will be done and the fact that they are in principle possible is, we believe, the most encouraging aspect and justifies the detailed theoretical analysis to be given in the following pages.

Before commencing the analysis, we wish to make two points. Firstly in this thesis we are concerned with extraction of form factors and not their theoretical interpretation. The latter is a subject in itself and no attempt will be made here to delve into speculative explanations. Discussion on such matters exists elsewhere in the literature.

Secondly, by the very tentative nature of the experimental situation on polarisation phenomena which is just being initiated, we shall on occasions be concerned with a circumstance which is presently purely theoretical. For example, there arises the case in which a recoil nucleon possesses longitudinal polarisation, i.e. along its direction of recoil. Present methods of detecting recoil polarisation using a second scattering are sensitive only to the transverse polarisation, and so such a measurement is not yet feasible. However this is not to say it will never be so. The ever-advancing scene in experimental techniques makes it possible in time detection of longitudinal recoil polarisation will be feasible and the appropriate measurements performed. Therefore, with this in mind, and for the sake of completeness, we give an extensive discussion of polarisation phenomena, although the experimental realisation is not yet in sight.

CHAPTER 2

POLARISATION IN ELASTIC LEPTON-NUCLEON SCATTERING

We commence with an investigation involving polarised nucleons, either as a target or in recoil. As first pointed out by Akhiezer et al,⁽⁹⁾ in the Born approximation (i.e. one photon exchange) the final lepton or nucleon is not polarised if the initial particles are unpolarised, and there is no asymmetry in the angular distribution when polarised leptons are scattered off an unpolarised target or in the scattering of unpolarised leptons from a polarised target. This is a direct consequence of parity conservation in electromagnetic interactions. What is required is a double-spin correlation, i.e. use of polarised lepton beam (and/or polarised target), and measurement of recoil lepton or proton polarisation. The respective cases most relevant to determining the electric and magnetic form factors will now be discussed.

1. Polarised Nucleon Target; Recoil Nucleon Polarisation
Determined⁽¹⁰⁾

The notation here is as for Fig. 1. In momentum space, the matrix element for scattering, m_2 , is proportional to the term:

$$m_2 \propto \langle P_2 | J_\mu | P_1 \rangle \frac{1}{q^2} \langle k_2 | j_\mu | k_1 \rangle .$$

The term $\langle k_2 | j_\mu | k_1 \rangle$ represents the matrix element (up to kinematic terms) for a lepton with four-momentum k_1 interacting with a photon and being scattered with momentum k_2 . Similarly for the nucleon term $\langle P_2 | J_\mu | P_1 \rangle$. J_μ and j_μ are operators

describing the nucleon and lepton currents respectively and $\frac{1}{2} \frac{1}{q}$ denotes the photon propagator where q is, as before, the four-momentum transfer $= k_1 - k_2 = P_2 - P_1$.

The lepton current having no structure

$$\langle k_2 | j_\mu | k_1 \rangle = e \bar{u}(k_2) \gamma_\mu u(k_1) \quad (2.1)$$

as for a point Dirac particle (e is electron charge).

The nucleon, electromagnetic vertex in its most general form is taken as

$$\begin{aligned} \langle P_2 | J_\mu | P_1 \rangle = & \bar{u}(P_2) \frac{1}{1-\tau} \left\{ (G_E - \tau G_M) \gamma_\mu \right. \\ & \left. + (G_M - G_E) \frac{i6uvq_\nu}{2M} \right\} u(P_1) \quad (2.2) \end{aligned}$$

where $\tau = q^2/4M^2$ and M is the nucleon mass. (For notation of the Dirac matrices, and Eq. (2.2), see Appendix 2). For ease of computation Eq. (2.2) may be re-written with the aid of the Dirac equation in the following form:

$$\langle P_2 | J_\mu | P_1 \rangle = \bar{u}(P_2) \left\{ A \gamma_\mu - B P_\mu \right\} u(P_1) \quad (2.3)$$

where $A = G_M$ $B = (G_M - G_E)/2M(1-\tau)$ $P_\mu = (P_{1\mu} + P_{2\mu})$.

Summing over the lepton spins, (see Appendix 2)

$$\begin{aligned} L_{u\lambda} &= \sum_{\text{spins}} \langle k_2 | j_\mu | k_1 \rangle^2 = \text{Tr} (K_2 + m_\ell) \gamma_\mu (K_1 + m_\ell) \gamma_\lambda \\ &= 4 \left[k_{2\mu} k_{1\lambda} + k_{2\lambda} k_{1\mu} - (k_1 k_2 - m_\ell^2) \delta_{\lambda\mu} \right] \quad (2.4) \end{aligned}$$

m_ℓ = lepton mass ; $A = \gamma_\mu A_\mu$ $\text{Tr} = \text{trace}$.

For the hadron term in the matrix element, we introduce projection operators to project out the required nucleon polarizations, and give

$$H_{u\lambda} = \langle P_2 | J_u | P_1 \rangle^2 = \text{Tr} (\not{P}_2 + M) \frac{(1 - \not{S}_B \not{y}_5)}{2} (\not{y}_u A - B \not{P}_u) (\not{P}_1 + M) \frac{(1 - \not{S}_A \not{y}_5)}{2} (\not{y}_\lambda A^* - B^* \not{P}_\lambda) \quad (2.5)$$

where * denotes complex conjugate, and S_A and S_B denote the operators for target and recoil nucleon respectively. A discussion of the properties of S_A and S_B is contained in Appendix 3.

Hermiticity (and/or time reversal invariance also) demands A and B to be purely real, i.e. $A = A^*$, $B = B^*$ (12).

Straightforward evaluation of $H_{u\lambda}$ using trace theorems of Appendix 1 gives the following result.

$$\begin{aligned} H_{u\lambda} = R_{u\lambda} - A^2 & \left\{ (S_B S_A) \left\{ P_{2u} P_{1\lambda} + P_{2\lambda} P_{1u} - (P_1 P_2 - M^2) \delta_{\lambda u} \right\} + \right. \\ & + (P_1 P_2 - M^2) \left\{ S_{Bu} S_{A\lambda} + S_{B\lambda} S_{Au} \right\} - S_B \cdot P_1 \left\{ S_{Au} P_{2\lambda} + S_{A\lambda} P_{2u} \right\} \\ & - S_A \cdot P_2 \left\{ S_{Au} P_{1\lambda} + S_{B\lambda} P_{1u} \right\} + S_B P_1 (S_A \cdot P_2) \delta_{\lambda u} \left. \right\} \\ & + B^2 P_u P_\lambda \left[- S_B \cdot S_A (P_1 P_2 + M^2) + S_B \cdot P_1 (S_A \cdot P_2) \right] \\ & + MAB \left[S_B \cdot S_A \left\{ P_u P_{1\lambda} + P_\lambda P_{1u} + P_u P_{2\lambda} + P_\lambda P_{2u} \right\} - \right. \\ & \left. - S_B \cdot P_1 \left\{ P_u S_{A\lambda} + S_{Au} P_\lambda \right\} - S_A \cdot P_2 \left\{ P_u S_{B\lambda} + P_\lambda S_{Bu} \right\} \right] \quad (2.6) \end{aligned}$$

$R_{u\lambda}$ represents non-spin part of $4H_{u\lambda}$, as in the Rosenbluth formula, and equals

$$\begin{aligned} R_{u\lambda} = \delta_{\lambda u} \left[\frac{q^2}{2} A^2 \right] + 2P_{1\lambda} P_{1u} & \left[(A - 2MB)^2 - B^2 q^2 \right] = \delta_{\lambda u} G_M^2 \frac{q^2}{2} \\ & + P_{1\lambda} P_{1u} \frac{2(G_E^2 - \tau G_M^2)}{1 - \tau} \quad (2.6a) \end{aligned}$$

Contracting (2.6) with eq. (2.4) yields the required result for

m^2 . The actual form of this may be deduced a priori from symmetry arguments and is perhaps instructive.

m^2 must depend on the variables S_A, S_B, P_1, P_2, k_1 and k_2 . By definition $S_B \cdot P_2 = 0$ and $S_A \cdot P_1 = 0$, while from four-momentum conservation $P_1 + k_1 = P_2 + k_2$ yields:- $S_B \cdot P_1 + S_B \cdot k_1 = S_B \cdot k_2$ and $S_A \cdot k_1 = S_A \cdot k_2 + S_A \cdot P_2$. Hence for $S_A(S_B)$ contracted with the four-momentum terms, there are only two independent terms: it turns out convenient to make the following choice. For $S_A, (S_A \cdot k_1)$ and $(S_A \cdot k_2)$; for $S_B, (S_B \cdot P_1)$ and $(S_B \cdot k_1)$. There exists 1 further independent term namely $(S_A \cdot S_B)$. Since S_A and S_B appear linearly in Eq. (2.5) they must do so in m^2 and parity conservation demands they appear as a product, i.e. $(S_B \cdot P_1)(S_A \cdot k_1)$ etc. Therefore the most general form for m^2 is as follows:

$$m^2 = a + b(S_B \cdot S_A) + c(S_B \cdot P_1)(S_A \cdot k_1) + d(S_B \cdot P_1)(S_A \cdot k_2) + e(S_B \cdot k_1)(S_A \cdot k_1) + f(S_B \cdot k_1)(S_A \cdot k_2) \quad (2.7)$$

where $a, b, c \dots f$ are scalar functions of $P_1 P_2, k_1$ and k_2 but independent of S_A and S_B .

Direct evaluation of m^2 yields such a form with the following values for the coefficients $a, b, c, \dots f$.

$$\begin{aligned} \omega \equiv a/4 &= \text{as for Rosenbluth} = \\ R_{\mu\lambda} \text{ (cf. eq. (2.6a)). } L_{\mu\lambda} &= -q^2 M^2 \left[\frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2 \right] \\ &= \left(\frac{d\sigma}{d\Omega} \right) \frac{1}{2} q^4 M^2 \left(\frac{E_e}{E_e - \tau} \right) \text{ from eq. (1.1)} \end{aligned}$$

$$b/4 = -2 \left[2(P_1 k_1)(P_1 k_2) - M^2(k_1 k_2) \right] \left[A^2 + 2B^2(P_1 P_2 + M^2) - 4MAB \right]$$

$$\begin{aligned} c/4 &= 2 \left\{ -A^2 [P_1 P_2 - M^2 - P_1 k_1 - P_1 k_2] + 2B^2 [2P_1 k_1(P_1 k_2) - M^2 k_1 k_2] \right. \\ &\quad \left. - 2ABM [P_1 k_1 + P_1 k_2 - k_1 k_2] \right\} \end{aligned}$$

$$d/4 = -4B \left\{ B \left[2P_1 k_1 (P_1 k_2) - M^2 (k_1 k_2) \right] + MA (k_1 k_2) \right\}$$

$$e/4 = -2A \left\{ A \left[P_1 P_2 - M^2 - P_1 k_1 - P_1 k_2 \right] + 2MB \left[P_1 k_1 + P_1 k_2 \right] \right\}$$

$$f/4 = -2A \left\{ A \left[P_1 P_2 - M^2 + P_1 k_1 + P_1 k_2 \right] - 2MB \left[P_1 k_1 + P_1 k_2 \right] \right\}$$

with A and B as given in eq. (2.3).

Collecting up terms (2.7) may be simplified to give⁽¹³⁾

$$\begin{aligned} \frac{m^2}{4} &= \omega + G_M^2 q^2 \left[S_B P_1 (S_A \cdot k_1) + (S_B \cdot k_1) + S_B \cdot k_1 (S_A \cdot k_2) \right] \\ &+ \left[G_M (G_M - G_E) q^2 / (1 - \tau) \right] \left[-S_B \cdot P_1 (S_A \cdot k_1) + S_B \cdot P_1 (S_A \cdot k_2) \right] \\ &+ \left[2G_M (G_E - \tau G_M) / (1 - \tau) \right] (P_1 k_1 + P_1 k_2) \left[S_B \cdot k_1 (S_A \cdot k_1) + S_B \cdot P_1 (S_A \cdot k_1) \right. \\ &\quad \left. - S_B \cdot k_1 (S_A \cdot k_2) \right] \\ &+ 2 \left[2P_1 k_1 (P_1 k_2) - M^2 k_1 k_2 \right] \left[-S_B S_A \left(\frac{G_E^2 - \tau G_M^2}{1 - \tau} \right) + 2S_B \cdot P_1 (S_A \cdot k_1) B^2 \right. \\ &\quad \left. - 2B^2 (S_B \cdot P_1) (S_A \cdot k_2) \right] \\ &\equiv \omega + M_S \end{aligned} \tag{2.8}$$

In both (2.7) and (2.8) terms involving the lepton mass have been ignored, which is an adequate approximation in the high energy region, i.e. lepton beam energies ≥ 1 GeV.

The recoil nucleon polarisation is given by

$$P = \frac{m^2 - m_{(S_B \rightarrow -S_B)}^2}{m^2 + m_{(S_B \rightarrow -S_B)}^2} = \frac{\text{Spin-part of eq. (2.8)} (\equiv M_S)}{\text{Non-spin part} (= \omega)} \tag{2.9}$$

We now apply eq. (2.9) to specific configurations

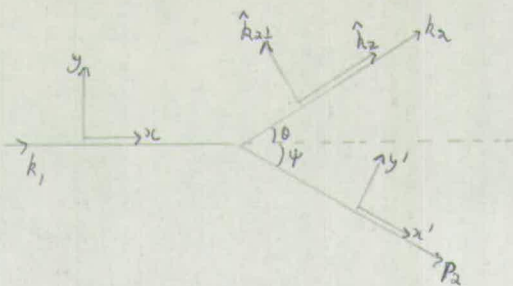


Fig. 2. Lepton-nucleon scattering in the laboratory frame. 4-momentum notation as for Fig. 1.

We work in the laboratory frame as depicted in Fig. 2; the kinematics and notation in this frame are discussed in Appendix 4.

i) Target nucleon polarised along \bar{k}_1 , direction of incoming lepton, recoil nucleon transversely polarised, i.e. perpendicular to \bar{P}_2 . In terms of right-handed axes of Fig. 2, this means \bar{S}_A along the x-axis and \bar{S}_B along y' axis. (Both the z and z' axes are coming out of the paper).

Since the target nucleon is at rest, $\bar{P}_1 = 0$, whence it immediately follows from $S_A \cdot P_1 = 0$ that $S_{AO} P_{10} = 0$, i.e. $S_{AO} = 0$, as $P_{10} = M$. Thus as $S^2 = S_{AO}^2 - \bar{S}_A^2 = -1$, $\bar{S}_A = 1$ and in component form $S_A = (0, \hat{x})$ where for any 4-vector $A = (A_0, \bar{A})$. $A^2 = A_0^2 - \bar{A}^2$; \hat{x} denotes a unit vector along the x-axis. (Similarly \hat{y} denotes a unit vector along the y axis, and in general \hat{A} denotes a unit vector along \bar{A}).

As S_B is transversely polarised, i.e. $\bar{S}_B \cdot P_2 = 0$ it likewise follows $S_{BO} = 0$ and $S_B = (0, \hat{y}')$.

Writing out the 4-momenta in component form

$$k_1 = (E_l, E_l \hat{x}), \quad k_2 = (E'_l, E'_l \hat{k}_l), \quad P_1 = (M, 0)$$

$$P_2 = (E_B, |\bar{P}_B| \hat{x}') \quad (2.10)$$

Direct contraction then yields the following results:

$$S_B \cdot P_1 = 0 \quad S_B \cdot k_1 = -E_\ell \sin \psi \quad S_A k_1 = -E_\ell$$

$$S_A k_2 = -E'_\ell \cos \theta \quad S_A \cdot S_B = -\sin \psi$$

where we have used $\hat{x} \cdot \hat{y}' = \sin \psi$ and $\hat{x} \cdot \hat{k}_2 = \cos \theta$.

Substituting these values into (2.8) and (2.9) then determines the polarisation P. Since the actual evaluation of (2.8) requires some involved juggling of kinematics, we present details of the algebra as an illustration of the techniques employed.

Direct substitution in (2.8) gives:

$$\begin{aligned} M_S (\equiv [M^2/4] - a) &= G_M^2 q^2 \left[-E_\ell \sin \psi (-E_\ell - E'_\ell \cos \theta) \right] \\ &+ \frac{2G_M(G_E - \tau G_M)}{1 - \tau} (2ME_\ell + q^2/a) (-E_\ell \sin \psi) (-E_\ell + E'_\ell \cos \theta) \\ &+ 2 \left[M^2 \frac{q^2}{2} + 2ME_\ell (ME_\ell + \frac{q^2}{2}) \right] \sin \psi \frac{(G_E^2 - \tau G_M^2)}{1 - \tau} \end{aligned}$$

where $p_1 k_1$, $p_1 k_2$ and $k_1 k_2$ are expressed in terms of E_ℓ and four-momentum transfer q as given in Appendix 4.

$$\begin{aligned} \therefore \frac{M_S}{-\sin \psi / (1 - \tau)} &= (1 - \tau) G_M^2 q^2 \left[E_\ell \left\{ -2E_\ell - \frac{q^2}{2E_\ell} \left(1 + \frac{E_\ell}{M} \right) \right\} \right] \\ &+ G_M (G_E - \tau G_M) \left(2ME_\ell + \frac{q^2}{2} \right) \frac{q^2}{2} \left(1 + \frac{E_\ell}{M} \right) \\ &- \left[M^2 q^2 + 2ME_\ell q^2 + 4M^2 E_\ell^2 \right] (G_E^2 - \tau G_M^2) \end{aligned} \quad (2.11)$$

where we have written $-E_\ell - E'_\ell \cos \theta = 2E_\ell + E_\ell - E'_\ell \cos \theta$ and used $E_\ell = E'_\ell \cos \theta = -q^2 (1 + \frac{E_\ell}{M}) / 2E_\ell$. (2.11) is clearly a quadratic expression in E_ℓ and simplifies to

$$\frac{M_S}{\sin\psi/(1-\tau)} = q^2 M^2 [G_E^2 + \tau G_M (G_M - 2G_E)] + 2q^2 M E_\ell [G_E^2 - G_E G_M + \tau G_M (G_M - G_E)] + 4M^2 E_\ell^2 [G_E^2 + \tau G_M (G_M - 2G_E)]$$

$$\therefore P = \frac{M_S}{\omega} = \frac{\sin\psi \left\{ \tau [G_E^2 + \tau G_M (G_M - 2G_E)] + \lambda \tau \frac{E_\ell}{M} [G_E^2 - G_E G_M + \tau G_M (G_M - G_E)] + \left(\frac{E_\ell}{M}\right)^2 [G_E^2 + \tau G_M (G_M - 2G_E)] \right\}}{-\tau(1-\tau) \left\{ \frac{G_E^2 - \tau G_M^2 \cot^2 \frac{\theta}{2} - 2\tau G_M^2}{1-\tau} \right\}} \quad (2.12)$$

Alternatively noting from eq. (1.1) that the curly bracket in the denominator in (2.12) is equal to $\left(\frac{d6}{d\Omega}\right) \left(-\frac{q^2}{a}\right) \left(\frac{E_\ell}{E_1}\right)^2$ where $d6/d\Omega$ is the differential cross-section summed over spins, we may rewrite (2.12) as

$$P \cdot \frac{d6}{d\Omega} \frac{1}{\sin\psi} = a^2 \left(\frac{E_\ell}{E_1}\right)^2 \left\{ \right\} / q^2 (\tau)(1-\tau) \quad (2.13)$$

where $\left\{ \right\}$ denotes the expression in such brackets in the numerator of eq. (2.12).

For fixed q^2 i.e. fixed τ , the expression in curly brackets in eqs. (2.12) and (2.13) is a polynomial of second order in the energy, E_ℓ . Therefore measurement of the transverse nucleon polarisation for three different values of the initial energy (E_ℓ) corresponding to a fixed value of q^2 is necessary and permits one to extract the following combination of form factors:

$$G_1 = G_E^2 + \tau G_M (G_M - 2G_E) \quad G_2 = [G_E^2 - G_E G_M + \tau G_M (G_M - G_E)] \quad (2.13a)$$

From these, we note $G_1 - G_2 = [-\tau + 1] G_M G_E$ and so the value of the cross-term is determined, whence G_E may be found.

ii) Target nucleon polarised perpendicular to \bar{k}_1 (i.e. along \hat{y})
 recoil nucleon longitudinally polarised, i.e. along \hat{x}' .

Here we have $S_A = (0, \hat{y})$, $S_B = (\frac{\bar{P}_B}{M}, \frac{E_B}{M} \hat{x}')$, giving
 the following terms $S_A \cdot k_1 = 0$, $S_A \cdot k_2 = -E'_\ell \sin \theta$, $S_B \cdot P_1 = \bar{P}_B$,
 $S_B \cdot k_1 = E'_\ell (\bar{P}_B - E_B \cos \psi)/M$, $S_A \cdot S_B = \frac{E_B \sin \psi}{M}$. Substituting
 in eq. (2.8), multiplying throughout by \bar{P}_B , using $\bar{P}_B \sin \psi = E'_\ell \sin \theta$
 and extracting the latter as a common factor throughout gives:

$$M_S \bar{P}_B = -G\tau^2 q^2 \frac{E'_\ell}{M} \left\{ q^2(\tau-1) + M(1-2\tau) \frac{q^2}{2E'_\ell} (1 + \frac{E'_\ell}{M}) \right\} + G\tau(G\tau - G\tau) q^4 + 2 \frac{E'_\ell}{M} G\tau(G\tau - \tau G\tau) (2M E'_\ell + q^2/2) \left\{ q^2(\tau-1) + M(1-2\tau) \frac{q^2}{2E'_\ell} (1 + \frac{E'_\ell}{M}) \right\}$$

$$+ \left[M^2 q^2 + 2ME'_\ell q^2 + 4M^2 E'^2_\ell q^2 \right] \left\{ \frac{-(1-2\tau)(G\tau^2 - \tau G\tau^2)}{1-\tau} - \frac{(G\tau^2 + G\tau^2 - 2G\tau G\tau) q^2}{2M^2} \right\}$$

(2.14)

where we have used $\bar{P}_B^2 = q^2(\tau - 1)$, $E_B = M(1-2\tau)$ and
 $\bar{P}_B \cos \psi = E'_\ell - E'_\ell \cos \theta = -q^2(1 + \frac{E'_\ell}{M})/2E'_\ell$. (See Appendix 4).

This simplifies to:

$$\frac{M_S \bar{P}_B}{E'_\ell \sin \theta} (1-\tau) = \left\{ M^2 q^2 \left[-G^2_E + 2\tau G_M G_E + \tau G^2_M - 2\tau^2 G^2_M \right] \right.$$

$$+ 2q^2 M E'_\ell \left[-G^2_E + G_M G_E + \tau G_M G_E - \tau G^2_M \right]$$

$$+ 4M^2 E'^2_\ell \left[-G^2_E + 2\tau G_M G_E - \tau G^2_M \right] \left. \right\}$$

(2.15)

$$P = \frac{E'_\ell \sin \theta}{\bar{P}_B} \frac{1}{-\tau(1-\tau)} \left\{ \frac{\tau \left[-G^2_E + 2\tau G_M G_E + \tau G^2_M - 2\tau^2 G^2_M \right] + 2\tau (E'_\ell/M) \left[-G^2_E + G_M G_E + \tau G_M G_E - \tau G^2_M \right] + (E'_\ell/M)^2 \left[-G^2_E + 2\tau G_M G_E - \tau G^2_M \right]}{\left[\frac{(G^2_E - \tau G^2_M) \cot^2 \theta}{1-\tau} - 2\tau G\tau^2 \right]} \right\}$$

$$\therefore P = \frac{q^2 \cot \frac{\theta}{2}}{2E_l \bar{P}_B} \frac{1}{(1-\tau)} \frac{\{ \}}{[]} \text{ or } P \cdot \frac{d\Omega}{d\Omega} = -a^2 \left(\frac{E_l'}{E_l} \right)^2 \frac{\cot \theta}{2} \times \left\{ \right\} \cdot \frac{1}{2E_l \bar{P}_B \tau (1-\tau)} .$$

Similarly to eq. (2.12) this is a quadratic expression in E_l , so again three measurements of P for different values of E_l but fixed q^2 are necessary to extract the following combination of form factors (14):

$$\begin{aligned} G_1 &= -G_E^2 + 2\tau G_M G_E + \tau G_M^2 - 2\tau^2 G_M^2 ; \\ G_2 &= -G_E^2 + G_M G_E + \tau G_M G_E - \tau G_M^2 ; \\ G_3 &= -G_E^2 + 2\tau G_M G_E - \tau G_M^2 \dots \end{aligned} \quad (2.16a)$$

Note G_2 and G_3 are the same (apart from a sign change) as G_1 and G_2 in eq. (2.13a) and therefore the cross-term is again determined by $G_2 - G_3 = (1-\tau)G_M G_E$.

(iii) Target nucleon polarised along y , recoil nucleon transversely polarised, i.e. along y' .

$$S_A = (0, Y), \quad S_B = (0, y') \quad \text{giving:}$$

$$\begin{aligned} S_A \cdot k_1 &= 0, \quad S_A \cdot k_2 = -E_l' \sin \theta, \quad S_B \cdot P_1 = 0, \quad S_B \cdot k_1 = -E_l \sin \psi, \\ S_A \cdot S_B &= -\cos \psi. \end{aligned}$$

Substituting in (2.8) and multiplying through by \bar{P}_B yields:

$$\begin{aligned} M_S \cdot \bar{P}_B &= \left\{ -\frac{M^2 q^2 \cdot q^2}{2E_l} \times \left(1 + \frac{E_l}{M} \right) \frac{(G_E^2 - \tau G_M^2)}{1-\tau} \right\} + \frac{E_l E_l'}{1-\tau} \left\{ G_M^2 q^2 (1-\tau) E_l' \sin^2 \theta \right. \\ &\quad \left. - G_M (G_E - \tau G_M) (q^2 + 4ME_l) E_l' \sin^2 \theta \right. \\ &\quad \left. - 4M^2 \frac{q^2}{2E_l} \left(1 + \frac{E_l}{M} \right) \frac{G_E^2 - \tau G_M^2}{1-\tau} \right\} \end{aligned}$$

$$\frac{M_S \bar{P}_B}{-q^2 \cdot q^2 / 2E_\ell (1 - \tau)}$$

$$= M^2 \left(1 + \frac{E_\ell}{M}\right) (G_E^2 - \tau G_M^2) + \frac{1}{2(1 - \cos\theta)} \left\{ G_M G_E (q^2 + 4ME_\ell) (1 + \cos\theta) - \left(1 + \frac{E_\ell}{M}\right) 4M^2 (G_E^2 + \tau G_M^2 \cos\theta) \right\}$$

$$= \left(1 + \frac{E_\ell}{M}\right) \frac{(1 + \cos\theta)}{1 - \cos\theta} \left\{ -M^2 [G_E^2 + \tau G_M^2] + \frac{(2ME_\ell + \frac{q^2}{2})}{1 + \frac{E_\ell}{M}} G_M G_E \right\}$$

$$\text{and } P = \left(1 + \frac{E_\ell}{M}\right) \cot^2 \frac{\theta}{2} \frac{2\tau}{\bar{P}_B E_\ell (1 - \tau)} \left\{ \right\} / \frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2 \quad (2.17)$$

$$\text{or } P \cdot \frac{d\sigma}{d\Omega} = \frac{-q^2 \left(\frac{E_\ell}{E}\right)^2 \left(1 + \frac{E_\ell}{M}\right) \frac{2\tau \cot^2 \frac{\theta}{2}}{\bar{P}_B E_\ell (1 - \tau)} \left\{ \right\}}{q^2}$$

From (2.17) we see now only two measurements of P for different values of E_ℓ but fixed q^2 are necessary to extract the following form factors:

$$G_1 = G_E^2 + \tau G_M^2, \quad G_2 = G_M G_E.$$

Hence determination of G_2 immediately gives the required cross-term.

(iv) Both target and recoil nucleon polarised perpendicular to the scattering planes, i.e. along \hat{z} and \hat{z}' respectively. Then $S_A = (0, \hat{z})$, $S_B = (0, \hat{z}')$ and all vector products vanish except $S_A \cdot S_B = -1$.

From (2.8) it immediately follows

$$M_S = [M^2 q^2 + 2ME_\ell q^2 + 4M^2 E_\ell^2] (G_E^2 - \tau G_M^2) / (1 - \tau)$$

$$\text{and } P = \frac{-1}{\tau} \left[\tau + 2\tau \left(\frac{E_\ell}{M}\right) + \left(\frac{E_\ell}{M}\right)^2 \right] \left\{ \frac{G_E^2 - \tau G_M^2}{1 - \tau} \right\} / \frac{(G_E^2 - \tau G_M^2)}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2 \quad (2.18)$$

$$\text{or } P \frac{d6}{d\Omega} = \frac{q^2}{q} \left(\frac{E_\ell'}{E_\ell} \right)^2 \frac{1}{\tau} \left[\tau + 2\tau \frac{E_\ell}{M} + \left(\frac{E_\ell}{M} \right)^2 \right] \left\{ \frac{G_E^2 - \tau G_M^2}{1 - \tau} \right\} .$$

Although only one measurement of P is here necessary to determine the form factor $G_1 = G_E^2 - \tau G_M^2$, the latter is dominated by G_M^2 for $\tau \gg 1$ and so is unsuitable for determining G_E (G_1 in fact occurs directly in the Rosenbluth formula, eq. (1.1)).

(v) Target nucleon polarised along direction of recoil nucleon, recoil nucleon transversely polarised, i.e. $S_A = (0, \hat{x}')$, $S_B = (0, \hat{y}')$.

We now have $S_A \cdot k_1 = -E_\ell \cos \psi$, $S_A \cdot k_2 = -E_\ell' \cos(\theta + \psi) = \bar{P}_B - E_\ell \cos \psi$, $S_B \cdot P_1 = 0$, $S_B \cdot k_1 = -E_\ell \sin \psi$, $S_A \cdot S_B = 0$. Substituting in (2.8) gives:

$$\begin{aligned} \frac{M_s \bar{P}_B}{-E_\ell \sin \psi} &= G_M^2 q^2 \left[q^2(\tau - 1) + q^2(1 + E_\ell/M) \right] + 2G_M(G_E - \tau G_M) [2ME_\ell + q^2/q] q^2 \\ &= q^2(4ME_\ell + q^2) G_M G_E . \end{aligned}$$

$$\therefore M_s = q^2(4ME_\ell + q^2)(-E_\ell E_\ell') \sin \theta G_M G_E / q^2(\tau - 1)$$

$$= \left\{ (4ME_\ell + q^2) q^2 (\cot \frac{\theta}{2}) / 2(\tau - 1) \right\} G_M G_E$$

$$\text{and } P = \frac{2 \cot \frac{\theta}{2} [\tau + E_\ell/M] G_M G_E}{(1 - \tau) \left\{ (G_E^2 - \tau G_M^2) \cot^2 \frac{\theta}{2} / (1 - \tau) \right\} - 2\tau G_M^2} \quad (15) \quad (2.19)$$

$$\text{or } P \frac{d6}{d\Omega} = \frac{-q^2}{q} \left(\frac{E_\ell'}{E_\ell} \right)^2 2 \cot \frac{\theta}{2} \frac{[\tau + E_\ell/M]}{1 - \tau} G_M G_E .$$

Clearly from eq. (2.19) a single measurement of the recoil polarisation is sufficient to determine the cross-term, and thus G_E . A disadvantage of this configuration is that the direction

of the target nucleon polarisation has to be changed for changes in the recoil nucleon scattering angle ψ . This may be overcome by fixing the angle ψ and then varying q^2 by altering the energy of the initial lepton beam. Such a technique is applicable whenever the target polarisation is correlated to the recoil direction and a similar analysis when correlated to the scattered lepton direction.

(vi) Target nucleon polarised perpendicular to the direction of the recoil nucleon, recoil nucleon longitudinally polarised, i.e. $S_A = (0, \hat{y}')$, $S_B = (\bar{P}_B \downarrow, \frac{E_B}{M} \hat{x}')$, giving

$$S_A \cdot k_1 = -E_\ell \sin \psi, \quad S_A \cdot k_2 = -E_\ell' \sin(\theta + \psi) = -E_\ell \sin \psi,$$

$$S_N \cdot P_1 = \bar{P}_B, \quad S_B \cdot k_1 = \frac{E_\ell}{M} (P_B - E_B \cos \psi), \quad S_A \cdot S_B = 0.$$

Substituting in eq. (2.8) yields:

$$\frac{M_S \times \bar{P}_B}{-E_\ell \sin \psi} = G_M^2 q^2 \left[q^2(\tau - 1) + \frac{2E_\ell}{M} \left\{ q^2(\tau - 1) + M(1 - 2\tau) \frac{q^2}{2E_\ell} \left(1 + \frac{E_\ell}{M} \right) \right\} \right]$$

$$- 2G_M(G_E - \tau G_M) [2ME_\ell + q^2/a] q^2$$

$$= -q^2(4ME_\ell + q^2) G_M G_E \quad (10)$$

This is exactly the same as for the previous case, except for the change in sign, and so the polarisation = $-P_y$ where P is as given in eq. (2.19). Although this configuration also directly measures $G_E G_M$, it has the defect of having to measure the longitudinal polarisation of the recoil nucleon.

(vii) Target nucleon polarised perpendicular to recoil nucleon direction, recoil nucleon transversely polarised,

i.e. $S_A = (0, \hat{y}')$, $S_B = (0, \hat{y}')$

Hence $S_A \cdot k_1 = S_A \cdot k_2 = -E_\ell \sin \psi$, $S_B \cdot P_1 = 0$, $S_B \cdot k_1 = -E_\ell \sin \psi$,
 $S_A \cdot S_B = -1$.

$\therefore M_S \times \bar{P}_B^2 = G_M^2 q^2 2E_\ell^2 E_\ell'^2 \sin^2 \theta - [M^2 q^2 + 2ME_\ell q^2 + 4M^2 E_\ell^2] q^2 (G_E^2 - \tau G_M^2)$
 $= G_M^2 \frac{q^2}{2} q^4 \cot^2 \frac{\theta}{2} - [-M^2 q^2 \cot^2 \frac{\theta}{2}] q^2 (G_E^2 - \tau G_M^2)$

$= q^4 M^2 \cot^2 \frac{\theta}{2} (G_E^2 + \tau G_M^2)$
 and $P = \cot^2 \frac{\theta}{2} \frac{(G_E^2 + \tau G_M^2)}{1 - \tau} / \frac{(G_E^2 - \tau G_M^2)}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2$ (Kabir ref. 10) (2.20)

One measurement of P at fixed q^2 determines $(G_E^2 + \tau G_M^2)$;
 however as this is dominated by the G_M term for $\tau \gg 1$, this configuration is unsuitable.

(viii) Target nucleon polarised perpendicular to direction of scattered lepton, recoil nucleon longitudinally polarised,
 i.e. $S_A = (0, \hat{k}_{2\perp})$, $S_B = (|\bar{P}_B|, \frac{E_B}{M} \hat{x}')$

Then $S_A \cdot k_1 = E_\ell \sin \theta$, $S_A \cdot k_2 = 0$, $S_B \cdot P_1 = \bar{P}_B$, $S_B \cdot k_1 = \frac{E_\ell}{M} (\bar{P}_B - E_B \cos \psi)$
 and substituting in (2.8)

$$\frac{M_S \times \bar{P}_B}{E_\ell \sin \theta} = G_M^2 q^2 \left[q^2 (\tau - 1) + \frac{E_\ell}{M} \left\{ q^2 (\tau - 1) + M(1 - 2\tau) \frac{q^2}{2E_\ell} \left(1 + \frac{E_\ell}{M} \right) \right\} \right]$$

$$+ G_M (G_M - G_E) q^2 \cdot q^2 = G_M^2 q^2 \left[2G_M \frac{(G_E - \tau G_M)(2ME_\ell + q^2/2)}{1 - \tau} + [M^2 q^2 + 2ME_\ell q^2 + 4M^2 E_\ell^2] \frac{[-E_\ell(G_E^2 - \tau G_M^2)]}{M(1 - \tau)} \right]$$

$= M^2 q^2 [-G_E^2 + \tau G_M^2 - 2\tau G_M G_E + 4\tau^2 G_M G_E - 2\tau^2 G_M^2] + 2ME_\ell q^2 [-G_E^2 - G_M G_E - \tau G_M^2 + 3\tau G_M G_E] + 4M^2 E_\ell^2 [-G_E^2 - \tau G_M^2 + 2\tau G_M G_E]$
 and $P = \frac{E_\ell \sin \theta}{\bar{P}_B (-\tau)} \left\{ \tau \frac{[-G_E^2 + \tau G_M^2 - 2\tau G_M G_E + 4\tau^2 G_M G_E - 2\tau^2 G_M^2]}{(G_E^2 - \tau G_M^2) \cot^2 \frac{\theta}{2}} + \frac{E_\ell}{M} \frac{[-G_E^2 - G_M G_E - \tau G_M^2 + 3\tau G_M G_E]}{1 - \tau} + \frac{E_\ell^2}{M^2} \frac{[-G_E^2 - \tau G_M^2 + 2\tau G_M G_E]}{1 - \tau} \right\}$

$$\text{or } P \frac{d\delta}{d\Omega} = \frac{-a^2}{q^2} \left(\frac{E'_\ell}{E_\ell} \right)^2 \frac{E_\ell \sin \theta}{P_B (-\tau)} \left\{ \right\}. \quad (2.21)$$

Eq. (2.21) being quadratic in E_p , again three measurements of P for different E_ℓ but fixed q^2 are necessary to extract the following form factors:

$$G_1 = -G_E^2 + \tau G_M^2 - 2\tau G_M G_E + 4\tau^2 G_M G_E - 2\tau^2 G_M^2;$$

$$G_2 = -G_E^2 - G_M G_E - \tau G_M^2 + 3\tau G_M G_E; \quad G_3 = -G_E^2 - \tau G_M^2 + 2\tau G_M G_E.$$

We note $G_3 - G_2 = (1 - \tau) G_M G_E$, whence the cross-term is determined.

It may be remarked that all correlations involving target and nucleon polarisations in orthogonal planes vanish, i.e. for $S_A = (0, \hat{z})$ and S_B in the scattering plane, or $S_B = (0, \hat{z}')$ and S_A in the scattering plane. As pointed out by Hey and Kabir⁽¹⁰⁾ this is a consequence of parity invariance. Moreover for the case when both polarisations are normal to the plane of scattering, the recoil nucleon polarisation does not involve $G_E G_M$ (see e.g. (2.18), and therefore in practice one will only be concerned with polarisation lying in the scattering plane.

Up to now it has been assumed that the target polarisation is fixed in direction. If, however, the latter is reversed and the difference in the recoil polarisation determined, i.e.

$M_S - (M_S \text{ with } S_A \rightarrow -S_A)$ with M_S given by eq. (2.8), then the polarisation effect is doubled, a considerable bonus to experimentalists.

v.B. In this chapter, there is considerable overlap with other published literature. Where a result has been previously obtained, the appropriate reference is given. Otherwise, and throughout the rest of the thesis, absence of an explicit reference indicates this represents the author's original work.

2. Polarised Nucleon Target, Polarised Lepton Beam ⁽¹⁶⁾

Summing over final lepton spin now we have:

$$L_{u\lambda} = \sum \langle k_2 | j_u | k_1 \rangle^2 = \text{Tr} (k_2 + m_\ell) \gamma_u (k_1 + m_\ell) \frac{1 - \beta_\ell \gamma_5}{2} \gamma_\lambda$$

$$= 2 \left[k_{2u} k_{1\lambda} + k_{2\lambda} k_{1u} - (k_1 k_2 - m_\ell^2) \delta_{u\lambda} + i m_\ell \epsilon_{u\lambda\alpha\beta} q_\alpha S_{\ell\beta} \right] \quad (2.22)$$

The notation is as for Fig. 1; m_ℓ denotes the lepton mass and S_ℓ the spin projection operator for the initial lepton. This operator is analogous to that for the nucleon, and determined by

$$S_\ell^2 = 1 \quad \text{and} \quad S_\ell \cdot k_1 = 0.$$

Summing over the final nucleon spin gives:

$$H_{u\lambda} = \sum \langle P_2 | J_u | P_1 \rangle^2 = \text{Tr} (\not{P}_2 + M) \gamma_u A - B \not{P}_1 (\not{P}_1 + M) \frac{1 - \beta_A \gamma_5}{2} \cdot$$

$$\cdot \left[\gamma_\lambda A^* - B^* P_\lambda \right]$$

$$\therefore \frac{H_{u\lambda}}{2} = R_{u\lambda} + i \epsilon_{u\lambda st} M A^2 (-q_s) S_{At} - i \left[\epsilon_{ustw} P_\lambda - \epsilon_{\lambda stw} P_u \right] \cdot$$

$$\cdot P_{2s} P_{1t} S_{Aw} AB \quad (2.23)$$

with A and B as defined in eq. (2.3), $R_{u\lambda}$ as given in eq. (2.6a), $P = P_1 + P_2$ and S_A is the spin-projection operator for the nucleon. M denotes the nucleon mass and $\epsilon_{\alpha\beta\gamma\delta}$ is the fourth-order antisymmetric tensor, defined in Appendix 1. Contracting (2.22) with (2.23) yields:

$$\frac{M^2}{4} = \omega + 2m_\ell (S_A^+ \cdot S_L) G_M (-M q^2 G_E) + 2m_\ell (S_A \cdot q) (S_L \cdot k_2) (-M G_M) \frac{[G_E - \tau G_M]}{1 - \tau}$$

$$- 4m_\ell M \tau (S_A \cdot q) (S_L \cdot P_1) \frac{G_M (G_M - G_E)}{1 - \tau} \quad (2.24)$$

where ω is as given in eq. (2.7).

We now apply (2.24) to specific configurations.

- (1) Target nucleon polarised along direction of lepton beam, initial lepton longitudinally polarised, i.e. $S_A = (0, \hat{x})$,
 $S_L = \left(\frac{E_\ell}{m_\ell}, \frac{E_\ell \hat{x}}{m_\ell} \right)$.

Therefore, $S_A \cdot k_1 = S_A \cdot k_2 = -E_\ell + E'_\ell \cos \theta$, $S_L \cdot P_1 = M \frac{E_L}{m_\ell}$,
 $S_L \cdot k_2 = E'_\ell \frac{E_\ell}{m_\ell} (1 - \cos \theta)$, $S_A \cdot S_L = \frac{E_\ell}{m_\ell}$.

Substituting in (2.24) gives

$$M_S(\text{spin part of 2.24}) = q^2 \frac{2M^3}{E_\ell(1-\tau)} \left[\tau G_M(G_E - \tau G_M) + \frac{E_\ell}{M} G_M(2G_E - G_M - \tau G_M) + \left(\frac{E_\ell}{M} \right)^2 G_M(G_E - \tau G_M) \right]$$

and $P = M_S/\omega$ (2.25)

This configuration is not useful however, as each of the three combinations of form factors in (2.25) is dominated by the G_M^2 term for $\tau \gg 1$.

- (ii) Target nucleon polarised along \hat{k} , as before, lepton beam transversely polarised, i.e. $S_A = (0, \hat{x})$, $S_L = (0, \hat{y})$

This gives $S_L \cdot P_1 = 0$, $S_L \cdot k_2 = -E'_\ell \sin \theta$, $S_A \cdot S_L = 0$ and from (2.24) we have

$$M_S = q^4 \cot \frac{\theta}{2} \frac{M m_\ell}{2E_\ell^2} \left(1 + \frac{E_\ell}{M} \right) \frac{(G_E - \tau G_M)}{1 - \tau} G_M \text{ and polarisation}$$

$$P = M_S/\omega.$$

Again this is unsuitable as the G_M term dominates for $\tau \gg 1$.

P is defined by
$$P = \frac{M^2 - M_{(S_L \rightarrow -S_L)}^2}{M^2 + M_{(S_L \rightarrow -S_L)}^2}$$

(iii) Target nucleon polarised perpendicular to \hat{k}_1 in the scattering plane, lepton beam longitudinally polarised, i.e.

$$S_A = (0, \hat{y}), \quad S_L = \left(\frac{E_\ell}{m_\ell}, \frac{E_\ell}{m_\ell} \hat{x}' \right)$$

$$\therefore S_A \cdot q = S_A \cdot k_1 - S_A \cdot k_2 = +E'_\ell \sin \theta, \quad S_L \cdot P_1 = E_\ell \frac{M}{m_\ell},$$

$$S_A \cdot k_2 = E'_\ell \frac{E_\ell}{m_\ell} (1 - \cos \theta), \quad S_A \cdot S_L = 0.$$

$$(2.24) \text{ then gives } M_S = E'_\ell \sin \theta M q^2 \left\{ G_E - \tau G_M - \frac{E_\ell}{M} (G_M - G_E) \right\} / (1 - \tau)$$

and the polarisation

$$P = \frac{q^2 \cot \frac{\theta}{2}}{2ME_\ell} \frac{1}{(1-\tau)} \left\{ \frac{[G_E - \tau G_M - \frac{E_\ell}{M} (G_M - G_E)] G_M}{(G_E^2 - G_M^2) \cot^2 \frac{\theta}{2} / (1-\tau) - 2\tau G_M^2} \right\}$$

$$\text{or } \frac{P_{d6}}{d\Omega} = \frac{-a^2}{2ME_\ell} \left(\frac{E'_\ell}{E_\ell} \right)^2 \cot \frac{\theta}{2} \left\{ \right\} / (1 - \tau) \quad (2.26)$$

Measurement of P for two values of initial energy E_ℓ , corresponding to fixed q^2 , then determines the form factors;

$G_1 = (G_E - \tau G_M) G_M$ and $G_2 = (G_M - G_E) G_M$ whence from G_2 , G_E may be found.

(iv) Target nucleon polarised as in (iii), initial lepton transversely polarised in the plane, i.e. $S_A = (0, \hat{y})$, $S_L = (0, \hat{y})$ which leads to $S_L \cdot P_1 = 0$, $S_L \cdot k_2 = -E'_\ell \sin \theta$, $S_A \cdot S_L = -1$.

From equation (2.24) we have thus

$$M_S = 2m_\ell M q^2 G_M \left\{ (1-\tau) G_E + \tau \frac{M^2}{E_\ell^2} \cot^2 \frac{\theta}{2} (G_E - \tau G_M) \right\} / (1 - \tau) \text{ and polariza-}$$

$$\text{tion } P = -2(m_\ell/M) \frac{1}{1-\tau} \left\{ G_M \frac{[(1-\tau) G_E + \tau (M^2/E_\ell^2) \cot^2 \frac{\theta}{2} (G_E - \tau G_M)]}{(G_E^2 - \tau G_M^2) \cot^2 \frac{\theta}{2} / (1-\tau) - 2\tau G_M^2} \right\} \quad \text{or}$$

$$\frac{P_{d6}}{d\Omega} = \frac{+a^2}{q^2} 2 \left(\frac{m_\ell}{M} \right) \left(\frac{E'_\ell}{E_\ell} \right)^2 \left\{ \right\} / (1 - \tau) \quad (2.27)$$

Therefore determination of P for two values of $\cot^2 \frac{\theta}{2} / E_L$ corresponding to fixed q^2 yields the two form factors

$G_1 = (1-\tau)G_E G_M$ and $G_2 = (G_E G_M - \tau G_M^2)$, G_1 being the required term.

(v) Both target nucleon and lepton beam polarised perpendicular to the scattering planes so $S_A = (0, \hat{z})$, $S_L = (0, \hat{z})$ and hence $S_A \cdot A_L = -1$ with all other terms vanishing. This gives

$$M_S = 2m_\ell q^2 G_M G_E \text{ and}$$

$$P = -2(m_\ell/M)G_M G_E \left[(G_E^2 - \tau G_M^2) \cot^2 \frac{\theta}{2} / (1-\tau) - 2\tau G_M^2 \right] \text{ or}$$

$$P \frac{d\sigma}{d\Omega} = \frac{q^2}{q^2} \left(\frac{E'_\ell}{E_\ell} \right)^2 \frac{2m_\ell}{M} G_M G_E \quad (17) \quad (2.28)$$

Thus one measurement of the polarisation suffices to determine $G_M G_E$. However this configuration has the disadvantage that the value of P is reduced by a factor (m_ℓ/M) compared to the case (iii) for longitudinally polarised leptons.

(vi) Target nucleon polarised along the direction of recoil nucleon, lepton beam longitudinally polarised, i.e. $S_A = (0, \hat{x}')$, $S_L = \left(\frac{E_\ell}{m_\ell}, \frac{E_\ell}{m_\ell} \hat{x} \right)$, whence $S_A \cdot q = + S_A \cdot P_2 = -\bar{P}_B$, $S_L \cdot P_1 = M \frac{E_\ell}{m_\ell}$, $S_L \cdot k_2 = E'_\ell \frac{E_\ell}{M_\ell} (1 - \cos \theta)$ and $S_A \cdot S_L = -\frac{E_\ell}{m_\ell} \cos \psi$. Therefore (2.24) gives $M_S \cdot \bar{P}_B = -Mq^4 G_M^2 \left(\tau + \frac{E_\ell}{M} \right)$ and $P = M_S/\omega$ with ω given by eq. (2.7). One measurement of P here directly determines G_M .

(vii) Target nucleon polarised as in (vi), lepton beam transversely polarised, i.e. $S_A = (0, \hat{x}')$, $S_L = (0, \hat{y})$ giving $S_L \cdot P_1 = 0$, $S_L \cdot k_2 = -E'_\ell \sin \theta$ and $S_A \cdot S_L = \sin \psi$. Substituting in (2.24) yields

$M_S \bar{P}_B = q^4 \frac{Mm_\ell}{E_\ell} \cot \frac{\Theta}{2} \tau G_M^2$ and $P = M_S/\omega$. Again this directly measures G_M .

(viii) Target nucleon polarised perpendicular to recoil nucleon direction, longitudinally polarised lepton. Now we have

$S_A = (0, \hat{y}')$, $S_L = (\frac{E_L}{m_\ell}, \frac{E_\ell}{m_\ell} \hat{x})$ whence it follows, $S_A \cdot q = S_A \cdot P_2 = 0$ and $S_A S_L = \frac{-E_L}{m_\ell} \sin \psi$ giving $M_S = 2E_\ell \sin \psi M q^2 G_M G_E$

$$\equiv \left[-M q^4 \cot \frac{\Theta}{2} / \bar{P}_B \right] G_M G_E$$

$$\text{and } P = -\frac{2E_\ell}{M} \sin \psi G_M G_E \left/ \left[\frac{(G_E^2 - \tau G_M^2)}{1 - \tau} \cot^2 \frac{\Theta}{2} - 2\tau G_M^2 \right] \right.$$

$$\text{or } P \frac{d6}{d\Omega} = \frac{q^2}{2} \frac{2E_\ell}{M} \sin \psi \left(\frac{E_\ell'}{E_\ell} \right)^2 G_M G_E \quad (2.29)$$

Thus one measurement of P here immediately yields the cross-term $G_E G_M$.

(ix) Target nucleon polarised as in (viii) but lepton beam transversely polarised. Therefore $S_A = (0, \hat{y}')$, $S_L = (0, \hat{y})$ and we have $S_A \cdot S_L = -\cos \psi$, whence (2.24) gives

$$M_S = 2M m_\ell q^2 \cos \psi G_M G_E = \left[-m q^4 \left(1 + \frac{E_\ell}{M} \right) m_\ell / E_\ell \bar{P}_B \right] G_M G_E$$

$$\text{and } P = -\frac{2m_\ell}{M} \cos \psi G_M G_E \left/ \left[\frac{(G_E^2 - G_M^2)}{1 - \tau} \cot^2 \frac{\Theta}{2} - 2\tau G_M^2 \right] \right.$$

$$\text{or } P \frac{d6}{d\Omega} = \frac{q^2}{2} 2 \left(\frac{m_\ell}{M} \right) \cos \psi \left(\frac{E_\ell'}{E_\ell} \right)^2 G_M G_E \quad (2.30)$$

Again only one measurement of P suffices to determine $G_M G_E$ but the magnitude of P is reduced by a factor (m_ℓ/E_ℓ) compared with case (viii).

(x) Target nucleon polarised along direction of scattered lepton, transversely polarised lepton, i.e. $S_A = (0, \hat{k}_2)$, $S_L = (0, \hat{y})$ and hence $S_A \cdot q = \cancel{S_A \cdot k_1} - S_A \cdot k_2 = -E_\ell \cos \theta + E'_\ell$, $S_L \cdot P_1 = 0$, $S_L \cdot k_2 = -E'_\ell \sin \theta$, $S_A \cdot S_L = -\sin \theta$. Eq. (2.24) thus gives $M_S = 2m_\ell M^2 \sin \theta \left[G_E + \tau \frac{M^2}{E_\ell^2} \frac{(1 - (E_\ell/M) \cos \theta)}{1 - \cos \theta} \frac{(G_E - \tau G_M)}{1 - \tau} \right] G_M$

and

$$P = -2(m_\ell/M) \sin \theta \left\{ G_E G_M + \tau \frac{M^2}{E_\ell^2} \frac{(1 - \frac{E_\ell}{M} \cos \theta)}{(1 - \cos \theta)(1 - \tau)} G_M (G_E - \tau G_M) \right\} / \left[\frac{(G_E^2 - \tau G_M^2)}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2 \right] \quad (2.31)$$

$$\text{or } P \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{q^2} 2 \left(\frac{m_\ell}{M} \right) \left(\frac{E'_\ell}{E_\ell} \right)^2 \sin \theta \left\{ \quad \right\} .$$

Two measurements of P for different E_ℓ but fixed q^2 permit one to extract the form factors $G_1 = G_E G_M$ and $G_2 = (G_M G_E - \tau G_M^2)$.

(xi) Target nucleon polarised perpendicular to direction of scattered lepton, longitudinally polarised lepton beam,

i.e. $S_A = (0, \hat{k}_2)$, $S_L = \left(\frac{E_\ell}{m_\ell}, \frac{E'_\ell}{m_\ell} \hat{x} \right)$, and so $S_A \cdot q = S_A \cdot k_1 = -E_\ell \sin \theta$, $S_A \cdot S_L = \frac{E_\ell}{m_\ell} \sin \theta$.

$$(2.24) \text{ gives } M_S = q^2 M E_\ell \sin \theta \left[-2G_E G_M + \tau G_M (G_E - G_M) + \frac{E_\ell}{M} (G_M - G_E) G_M \right] / 1 - \tau$$

$$\text{and } P = -(E_\ell/M) \sin \theta \left[\quad \right] / (1 - \tau) \left[\frac{(G_E^2 - \tau G_M^2)}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2 \right] \quad (2.32)$$

$$\text{or } P \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{q^2} \left(\frac{E_\ell}{M} \right) \left(\frac{E'_\ell}{E_\ell} \right)^2 \sin \theta \left[\quad \right] .$$

Two measurements of P as in the previous case determine the form factors $G_1 = \tau G_M (G_E - G_M) - 2G_E G_M$, $G_2 = G_M (G_M - G_E)$, whence G_E may be found.

(xii) Target polarised as in (xi), transversely polarised lepton beam. Here $S_A = (0, \hat{k}_{2\perp})$, $S_L = (0, \hat{y})$ whence $S_L \cdot P_1 = 0$, $S_L \cdot k_2 = -E_\ell \sin \theta$, $S_A \cdot S_L = -\cos \theta$, and (2.24) yields

$$M_S = 2Mm_\ell q^2 \cos \theta \left[G_E - \frac{E_\ell^2}{q^2} \frac{\sin^2 \theta}{\cos \theta} \frac{(G_E - \tau G_M)}{1 - \tau} \right] G_M$$

and

$$P = -2(m_\ell/M) \cos \theta \left\{ G_E G_M - \frac{E_\ell^2}{q^2} \frac{\sin^2 \theta}{\cos \theta} \frac{(G_E - \tau G_M) G_M}{1 - \tau} \right\} / \left[\frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2 \right] \quad (2.33)$$

Again two measurements of P for different E_ℓ and θ but fixed q^2 determine the form factors $G_1 = G_E G_M$, $G_2 = G_M (G_E - \tau G_M)$.

As with both target and nucleon polarisations measured, so here also all correlations involving S_A and S_L lying in transverse planes, i.e. over in the scattering plane and one perpendicular to it, vanish because of parity invariance. Note all effects due to transverse polarisation of the lepton beam are a factor (m_ℓ/M) or (m_ℓ/E_ℓ) smaller than those with longitudinal polarisation⁽¹⁹⁾. This is a general result for polarised leptons and implies in practice only ν beams will contribute significantly to the transverse polarisation. Again by reversing the target polarisation and measuring the change in polarisation P (i.e. $M_S - M_S$ (with $S_A \rightarrow -S_A$)) the total effect is doubled.

3. Polarised Nucleon Target, Recoil Lepton Polarised

Summing over the initial lepton spin,

$$L_{\mu\lambda} = \sum \langle k_2 | j_u | k_1 \rangle^2 = \text{Tr} (K_{2+m_\ell}) \frac{1 - \gamma_5 \gamma_L}{2} \gamma_u (K_{1+m_\ell}) \gamma_\lambda \dots (2.22)$$

where S_L now refers to the final lepton. Summing over the final nucleon spin gives again eq. (2.23). Therefore on contracting we find

$$\begin{aligned} \frac{m^2}{4} = & \omega + 2m_\ell (S_A \cdot S_L) G_M [-Mq^2 G_E] + 2m_\ell (S_A \cdot q) (S_L \cdot k_1) M G_M \frac{(G_E - \tau G_M)}{1 - \tau} \\ & - 4m_\ell M \tau (S_A \cdot q) (S_L \cdot P_1) \frac{G_M (G_M - G_E)}{1 - \tau} \quad (2.34) \end{aligned}$$

This is precisely the same as eq. (2.24) except for $(S_L \cdot k_2) \rightarrow -(S_L \cdot k_1)$

The following configurations are of interest:

- (1) Target nucleon polarised along direction of initial lepton beam, scattered lepton transversely polarised, i.e. $S_A = (0, \hat{x})$, $S_L = (0, \hat{k}_{2\perp})$ leading to $S_A \cdot q = S_A \cdot k_1 - S_A \cdot k_2 = -E_\ell + E'_\ell \cos \theta$, $S_L \cdot P_1 = 0$, $S_L \cdot k_1 = E_\ell \sin \theta$, $S_A S_L = \sin \theta$.

Eq. (2.34) then gives

$$M_S = M m_\ell q^2 \sin \theta \left[-G_E + \tau (2G_E - G_M) + \frac{E_\ell}{M} (G_E - \tau G_M) \right] G_M / (1 - \tau)$$

and

$$\therefore P^* = \frac{-(m_\ell/M) \sin \theta \left\{ -G_E G_M + \tau G_M (2G_E - G_M) + \frac{E_\ell}{M} \cos \theta (G_E - \tau G_M) \right\}}{(1 - \tau) \left[(G_E^2 - \tau G_M^2) \cot^2 \frac{\theta}{2} / (1 - \tau) - 2\tau G_M^2 \right]}$$

$$\text{or } P \frac{d\sigma}{d\Omega} = \frac{q^2}{2} \left(\frac{E'_\ell}{E_\ell} \right)^2 \frac{m_\ell}{M} \frac{\sin \theta}{(1 - \tau)} \{ \quad \} \quad (2.35)$$

Two measurements of P at fixed q^2 but different E_ℓ suffice to extract the form factors: $G_1 = -G_E G_M + \tau (2G_E G_M - G_M^2)$,

$$G_2 = G_E G_M - \tau G_M^2.$$

Note $G_2 - G_1 = 2(1 - \tau) G_M G_E$ which determines the cross-term.

* P defined here analogously as in (2.25)

(ii) Target nucleon polarised transverse to \hat{k}_1 , longitudinally polarised recoil lepton. Hence $S_A = (0, \hat{k}_{1\perp}) = (0, \hat{y})$,
 $S_L = \left(\frac{E'_l}{m_l}, \frac{E'_l}{m_l} \hat{k}_2 \right)$ which gives $S_A \cdot q = S_A \cdot k_1 - S_A \cdot k_2 = E'_l \sin \theta$,
 $S_L \cdot P_1 = ME'_l/m_l$, $S_L \cdot k_1 = E_l E'_l (1 - \cos \theta)/m_l$, $S_A \cdot S_L = -E'_l \sin \theta/m_l$,
 and (2.34) yields

$$M_S = Mq^2 E'_l \sin \theta \left[G_E + \tau(G_M - 2G_E) + \tau \frac{M}{E_l} \operatorname{cosec}^2 \frac{\theta}{2} (G_M - G_E) \right] G_M / 1 - \tau$$

and

$$P = \frac{2 \frac{M}{E_l} \cot \frac{\theta}{2} \left\{ G_E + \tau(G_M - 2G_E) + \tau \frac{M}{E_l} \operatorname{cosec}^2 \frac{\theta}{2} (G_M - G_E) \right\} G_M}{(1 - \tau) \left[(G_E^2 - \tau G_M^2) \cot^2 \frac{\theta}{2} / (1 - \tau) - 2\tau G_M^2 \right]}$$

$$\text{or } P \frac{d\sigma}{d\Omega} = - \frac{a^2}{2ME_l} \left(\frac{E'_l}{E_l} \right)^2 \frac{\cot \frac{\theta}{2}}{1 - \tau} \left\{ \right\} G_M \quad (2.36)$$

Again two measurements of P are necessary to yield the form factors $G_1 = G_M G_E + \tau(G_M - 2G_E)G_M$, $G_2 = G_M(G_M - G_E)$ whence G_E may be found.

(iii) Target nucleon polarised as for (ii), transversely polarised recoil lepton. i.e. $S_L = (0, \hat{k}_{2\perp})$ and hence $S_L \cdot P_1 = 0$, $S_L \cdot k_1 = E_l \sin \theta$, $S_A \cdot S_L = -\cos \theta$. This gives

$$M_S = Mm_l q^2 \left\{ -G_E + \tau G_M + \cos \theta \left[G_E + \tau(G_M - 2G_E) \right] \right\} G_M / 1 - \tau$$

and polarisation

$$P = \frac{-(m_l/M) \left\{ -G_E + \tau G_M + \cos \theta \left[G_E + \tau(G_M - 2G_E) \right] \right\} G_M}{(1 - \tau) \left[(G_E^2 - \tau G_M^2) \cot^2 \frac{\theta}{2} / 1 - \tau - 2\tau G_M^2 \right]} \quad (2.37)$$

Measuring P for two values of θ corresponding to fixed q^2 determines the form factors $G_1 = (-G_E + \tau G_M)G_M$,
 $G_2 = G_E G_M + \tau(G_M - 2G_E)G_M$. Note these also arise in case (i).

(iv) Target nucleon and recoil lepton both polarised transverse to the scattering plane: $S_A = S_L = (0, \hat{z})$ and $S_A S_L = -1$ with all other terms zero.

$$\therefore M_S = 2Mm_\ell q^2 G_M G_E \text{ and}$$

$$P = -2(m_\ell/M)G_M G_E \left/ \left[(G_E^2 - \tau G_M^2) \cot^2 \frac{\theta}{2} / 1 - \tau - 2\tau G_M^2 \right] \right. \quad (2.38)$$

A measurement of P thus directly determines $G_E G_M$ but is reduced in magnitude by (m_ℓ/M) factor compared to case (ii) with longitudinally polarised leptons.

(v) Target nucleon polarised along recoil nucleon direction,

longitudinally polarised recoil lepton, i.e. $S_A = (0, \hat{x}')$,

$S_L = \left(\frac{E'_\ell}{m_\ell}, \frac{E'_\ell}{m_\ell} \hat{k}_2 \right)$ and so $S_A \cdot q = S_A \cdot P_2 - S_A \cdot P_1 = -\bar{P}_B$,

$S_A S_L = -E'_\ell \cos(\theta + \psi) / m_\ell$.

Thus we have

$$M_S = \frac{q^4 \bar{P}_B \operatorname{cosec} \frac{2\theta}{2}}{4E_\ell (1-\tau)} \left\{ \tau G_M^2 \left[\frac{M}{E_\ell} + 1 - \cos\theta \right] - G_M^2 \right\} \text{ and}$$

$P = M_S / \omega$. Hence this directly measures G_M .

(vi) Target nucleon polarised as in (v), scattered lepton transversely polarised.

$S_A = (0, \hat{x}')$, $S_L = (0, \hat{k}_{2\perp})$ giving $S_A \cdot S_L = \sin(\theta + \psi)$

whence (2.34) yields

$$M_S = 2Mm_\ell q^2 \frac{E}{\bar{P}_B} \sin \theta (-\tau G_M^2) \text{ and } P = M_S / \omega. \text{ Again this}$$

directly measures G_M .

(vii) Target nucleon polarised transverse to recoil nucleon momentum, longitudinally polarised recoil lepton.

Here $S_A S_L = -\cos(\theta + \psi)$ and $M_S = \left[M m_\ell q^4 \left(1 - \frac{E_\ell}{M} \cos\theta\right) / E_\ell \bar{P}_B \right] G_M G_E$

$$\therefore P = -(m_\ell/M) \frac{q^2}{E_\ell \bar{P}_B} \left(1 - \frac{E_\ell}{M} \cos\theta\right) \frac{G_M G_E}{\frac{(G_E^2 - \tau G_M^2)}{1-\tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2}$$

$$\text{or } P \frac{d\delta}{d\Omega} = \frac{q^2}{E_\ell \bar{P}_B} \left(\frac{E'_\ell}{E_\ell}\right)^2 \frac{m_\ell}{M} \left(1 - \frac{E_\ell}{M} \cos\theta\right) G_M G_E \quad (2.40)$$

Again one measurement suffices to yield $G_M G_E$ but the value of P is smaller by a factor (m_ℓ/M) compared with case (vii).

(ix) Target nucleon polarised transverse to direction of recoil lepton, latter longitudinally polarised, i.e. $S_A = (0, \hat{k}_2)$, $S_L = \left(\frac{E'_\ell}{m_\ell}, \frac{E'_\ell}{m_\ell} \hat{k}_2\right)$. Therefore, $S_A q = S_A \cdot k_1 - S_A \cdot k_2 = E_\ell \sin\theta$ and $S_A S_L = 0$. (2.34) now gives

$$M_S = -M E_\ell q^2 \sin\theta \left\{ G_E - \tau G_M + \left[E_\ell/M + E_\ell(1-\cos\theta) \right] (G_M - G_E) \right\} / (1-\tau)$$

and

$$P = \frac{-E_\ell}{M} \sin\theta \frac{\left\{ G_E - \tau G_M + \left[E_\ell/M + E_\ell(1-\cos\theta) \right] (G_M - G_E) \right\} G_M}{(1-\tau) \left[\frac{G_E^2 - \tau G_M^2}{1-\tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2 \right]}$$

or

$$P \frac{d\delta}{d\Omega} = \frac{q^2}{2} \left(\frac{E'_\ell}{E_\ell}\right)^2 \frac{E_\ell}{M} \frac{\sin\theta}{1-\tau} \left\{ \right\} G_M \quad (2.41)$$

Two measurements of P are required here to extract the form factors:

$G_1 = (G_E - \tau G_M) G_M$, $G_2 = (G_M - G_E) G_M$, G being the one of interest.

(x) Target nucleon polarised as in (ix), but recoil lepton transversely polarised, i.e. $S = (0, \hat{k}_{2\perp})$. Thus $S_A \cdot S_L = -1$

and we have

$$M_S = 2m_\ell M q^2 \left\{ G_E - \cos^2 \frac{\Theta}{2} \left[1 + \frac{E_\ell}{M} (1 - \cos \Theta) \right] (G_E - \tau G_M) / (1 - \tau) \right\} G_M$$

$$\text{and } P = 2(m_\ell/M) \frac{\left\{ G_E G_M - \left[\cos^2 \frac{\Theta}{2} \left(1 + \frac{E_\ell}{M} (1 - \cos \Theta) \right) (G_E - \tau G_M) G_M / (1 - \tau) \right] \right\}}{\left[(G_E^2 - \tau G_M^2) \cot^2 \frac{\Theta}{2} / (1 - \tau) - 2\tau G_M^2 \right]}$$

$$\text{or } P \frac{d\sigma}{d\Omega} = \frac{q^2}{q^2} \left(\frac{E_\ell}{E_\ell} \right)^2 2 \left(\frac{m_\ell}{M} \right) \{ \quad \} \quad (2.42)$$

This enables one to determine the form factors, $G_1 = G_M G_E$, $G_2 = G_M (G_E - \tau G_M)$ but P is reduced by (m_ℓ/E_ℓ) in magnitude compared with that for longitudinally polarised leptons, case (ix).

Again all correlations involving one particle polarised in the scattering plane and one polarised transverse to it vanish due to parity invariance.

4. Polarised Lepton Beam, Recoil Nucleon Polarised ⁽²⁰⁾

Summing over the lepton spins gives $L_{u\lambda} \equiv \text{eq. (2.24)}$.

Summing over the initial nucleon spin,

$$H_{u\lambda} = \sum \langle P_2 | J_u | P_1 \rangle^2 = \text{Tr}(\not{P}_2 + M) \frac{1 - \not{P}_B \gamma_5}{2} (\not{y}_u A - B \not{P}_u) (\not{P}_1 + M) (\gamma_\lambda \not{A}^* - B \not{P}_\lambda^*)$$

$$H_{\mu\lambda}/2 = \mathcal{R}_{\mu\lambda} - iMA^2 \epsilon_{\mu\lambda st} q_s S_{Bt} + ABi \left[P_\lambda \epsilon_{ustw} - P_u \epsilon_{\lambda stw} \right] P_{2s} S_{Bt} P_{1w} \quad (2.43)$$

A and B as in (2.3), $\mathcal{R}_{\mu\lambda}$ as in (2.6a), $P = P_1 + P_2$ and S_B now refers to the recoil nucleon. Contracting yields $\frac{m^2}{4} =$

$$= \omega + 2m_\ell (S_B \cdot S_L) q^2 G_M (-MG_E) - 2m_\ell (S_B \cdot q) (S_L \cdot k_2) MG_M \frac{[G_E + \tau(G_M - 2G_E)]}{1 - \tau} + 4m_\ell M \tau (S_B \cdot q) (\not{L} \cdot P_1) \frac{G_M (G_M - G_E)}{1 - \tau} = \omega + \eta_s \quad (2.44)$$

with ω defined in (2.7). Polarisation $P = \left[\frac{m^2 - m_{(S_S \rightarrow -S_B)}^2}{m^2 + m_{S_B \rightarrow -S_B}^2} \right] = \eta_S / \omega$

(i) Recoil nucleon longitudinally polarised, as is lepton beam.

i.e. $S_B = \left(\frac{|\bar{P}_B|}{M}, \frac{E_\ell}{M} \hat{x}' \right)$, $S_L = \left(\frac{E_\ell}{m_\ell}, \frac{E_\ell}{m_\ell} \hat{x} \right)$ giving

$$S_B \cdot q = -S_B \cdot P_1 = -\bar{P}_B \quad \text{and} \quad S_L \cdot S_B = \frac{E_\ell}{M m_\ell} (\bar{P}_B - E_B \cos \psi)$$

$$\therefore M_S = M q^4 \left[\tau + \frac{E_\ell}{M} \right] G_M^2 / \bar{P}_B \quad \text{and} \quad P = -q^2 \left[\tau + \frac{E_\ell}{M} \right] G_M^2 / M \bar{P}_B \left\{ \frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2 \tau G_M^2 \right\} \quad (2.45)$$

This directly measures G_M .

(ii) Recoil nucleon polarised in (i), transverse polarised lepton beam.

i.e. $S_L = (0, \hat{y})$, giving $S_L \cdot P_1 = 0$, $S_L \cdot k_2 = -E'_\ell \sin \theta$ and $S_L \cdot S_B = E_B \sin \psi / M$. (2.44) then yields

$$M_S = -q^4 \cot \frac{\theta}{2} M m_\ell \tau G_M^2 / E_\ell \bar{P}_B \quad \text{and}$$

$$P = -q^2 (m_\ell / M) \cot \frac{\theta}{2} \tau G_M^2 / E_\ell \bar{P}_B \left\{ \frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2 \tau (G_M)^2 \right\} \\ = -2 (m_\ell / M) \sin \psi \tau G_M^2 / \{ \}$$

Again G_M is directly determined.

(iii) Recoil nucleon transversely polarised, longitudinal polarised lepton beam.

$S_B = (0, \hat{y}')$, $S_L = (E_\ell / m_\ell, E_\ell / m_\ell \hat{x})$. Hence $S_B \cdot q = -S_B \cdot P_1 = 0$ and $S_L \cdot S_B = -E_\ell \sin \psi / m_\ell$.

$$\therefore M_S = -q^4 M \cot \frac{\theta}{2} G_M G_E / \bar{P}_B \quad \text{and} \quad P = \frac{q^2 \cot \frac{\theta}{2}}{M \bar{P}_B} \frac{G_M G_E}{\frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2 \tau G_M^2}$$

$$\text{or } P \frac{d6}{d\Omega} = \frac{-q^2 (E'_\ell / E_\ell)^2}{M \bar{P}_B} \cot \frac{\theta}{2} G_M G_E \quad (2.46)$$

Thus one measurement of P here immediately gives $G_M G_E$.

(iv) Recoil nucleon polarised transversely, transverse polarised lepton beam, i.e. $S_L = (0, \hat{y})$.

Hence $S_L \cdot S_B = -\cos \psi$ and $M_S = -q^4 m_\ell (1 + \frac{M}{E_\ell}) G_M G_E / \bar{P}_B$

$$\therefore P = q^2 \frac{(m_\ell/M)}{M \bar{P}_B} (1 + \frac{M}{E_\ell}) \frac{G_M G_E}{\frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2}$$

$$\text{or } P \frac{d\sigma}{d\Omega} = \frac{-q^2}{M \bar{P}_B} \left(\frac{E_\ell'}{E_\ell}\right) \frac{m_\ell}{M} (1 + \frac{M}{E_\ell}) G_M G_E \quad (2.47)$$

While this also determines $G_M G_E$, the magnitude of P is lower by factor (m_ℓ/M) compared to case (iii).

(v) Both lepton and nucleon polarised perpendicular to the scattering plane. $S_L = (0, \hat{z})$, $S_B = (0, \hat{z}')$ and $S_L \cdot S_B = -1$ with all other terms zero.

$$\therefore M_S = 2m_\ell M q^2 G_M G_E \quad \text{and} \quad P = -2(m_\ell/M) \frac{G_M G_E}{\frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2}$$

$$\text{or } P \frac{d\sigma}{d\Omega} = \frac{q^2}{2} \left(\frac{E_\ell'}{E_\ell}\right)^2 2 \frac{m_\ell}{M} G_M G_E \quad (2.48)$$

Again the value of P is of order (m_ℓ/M) less compared with case (iii).

5. Both Final Particles Polarised

Summing over lepton spins gives $L_{\mu\lambda} = \text{eq. (2.22)}$.

Summing over the initial nucleon spin gives $H_{\mu\lambda} \equiv \text{eq. (2.43)}$.

\therefore Contracting gives

$$\frac{m^2}{4} = \omega + 2m_\ell (S_B \cdot S_L) q^2 G_M (-MG_E) + 2m_\ell M (S_B \cdot q) (S_L \cdot k_1) G_M \frac{(G_E + (G_M - 2G_E)\tau)}{1 - \tau} + 4m_\ell M \tau (S_B \cdot q) (S_L \cdot P_1) G_M (G_M - G_E) / (1 - \tau) \quad (2.49)$$

(i) Recoil nucleon longitudinally polarised, likewise for scattered lepton. Here $S_B \cdot q = -S_B \cdot P_1 = -\bar{P}_B$

$$S_B S_L = \frac{E'_\ell}{m_\ell} \frac{1}{M} (\bar{P}_B - E_B \cos(\theta + \psi)) \quad \text{and (2.49) gives}$$

$$M_S = q^6 \operatorname{cosec}^2 \frac{\theta}{2} G_M^2 \left[1 - \tau + \tau \cos \theta - \frac{M}{E_\ell} \tau \right] / 4E_\ell \bar{P}_B$$

$$\therefore P = \tau \frac{q^2}{E_\ell \bar{P}_B} G_M^2 \frac{\left\{ 1 - \tau + \tau \cos \theta - \frac{M}{E_\ell} \tau \right\}}{\left(\frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2 \right)}$$

$$\text{or } P \frac{d\delta}{d\Omega} = \frac{-q^2}{E_\ell \bar{P}_B} \left(\frac{E'_\ell}{E_\ell} \right)^2 G_M^2 \{ \} \quad (2.50)$$

which measures G_M .

(ii) Recoil nucleon as in (i), recoil lepton transversely polarised.

$$S_B \cdot q = -S_B \cdot P_1, \quad S_B \cdot S_L = \frac{E_B}{M} \sin(\theta + \psi), \quad \text{and}$$

$$M_S = 2Mq^2 E_\ell \sin \theta m_\ell (\tau G_M^2) / P_B \quad \text{and} \quad P = M_S / \omega.$$

Again this directly measures G_M .

(iii) Recoil nucleon transversely polarised, longitudinal polarised recoil lepton,

$$\text{i.e. } S_B = (0, \hat{y}'), \quad S_L = \left(\frac{E'_\ell}{m_\ell}, \frac{E'_\ell}{m_\ell} \hat{k}_2 \right) \quad \text{giving } S_B \cdot q = -S_B \cdot P_1 = 0,$$

$$\text{and } S_B \cdot S_L = -E'_\ell \sin(\theta + \psi) / m_\ell.$$

∴ (2.49) yields $M_S = 2Mq^2 E_\ell \sin \psi G_M G_E \equiv -Mq^4 \cot \frac{\theta}{2} / \bar{P}_B$

and polarisation $P = \frac{-2(E_\ell/M) \sin \psi \left[\equiv q^2 \cot \frac{\theta}{2} / \bar{M} \bar{P}_B \right] G_M G_E}{(G_E^2 - \tau G_M^2) \cot^2 \frac{\theta}{2} (1 - \tau) - 2\tau G_M^2}$

or $P \frac{d6}{d\Omega} = \alpha^2 \left(\frac{E'_\ell}{E_\ell} \right)^2 \frac{2}{M} \frac{\sin \psi}{q^2} \left[\equiv \frac{\cot^2 \left(\frac{\theta}{2} \right)}{\bar{M} \bar{P}_B} \right] G_M G_E \quad (2.51)$

Hence the cross-term is immediately extractable.

(iv) Recoil nucleon as in (iii) but recoil lepton transversely polarised. Now $S_B \cdot q = 0$ and $S_L \cdot S_B = -\cos(\theta + \psi)$,

∴ $M_S = \left[q^4 m_\ell (M/E_\ell - \cos \theta) / \bar{P}_B \right] G_M G_E$ and

$$P = \frac{4\tau m_\ell}{\bar{P}_B} \frac{G_M G_E (M/E_\ell - \cos \theta)}{\frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2}$$

or $P \frac{d6}{d\Omega} = -\alpha^2 \left(\frac{m_\ell}{M} \right) \frac{[M/E_\ell - \cos \theta]}{M \bar{P}_B} \left(\frac{E'_\ell}{E_\ell} \right)^2 G_M G_E \quad (2.52)$

The magnitude of P is here a factor (m_ℓ/M) lower compared with case (iii).

(v) Both final particles polarised perpendicular to the scattering plane. i.e. $S_B \cdot S_L = -1$ and $M_S = 2Mm_\ell q^2 G_M G_E$ or

$$P = -2(m_\ell/M) \frac{G_M G_E}{\frac{G_E^2 - \tau G_M^2}{1 - \tau} \cot^2 \frac{\theta}{2} - 2\tau G_M^2} \quad (2.53)$$

Similar comment here applies as for (iv).

Fig. 3.

Plot of eq. (219) assuming the 'scaling' fit

$G_E = G_M/\mu$ here done for the proton ($\mu = 2.79$)

$$P = 2 \cot \frac{\theta}{2} \left(\tau + \frac{E_L}{M} \right) G_M G_E / \left(G_E^2 - \tau G_M^2 \right) \cot^2 \frac{\theta}{2} - 2\tau(\tau-1) G_M^2$$

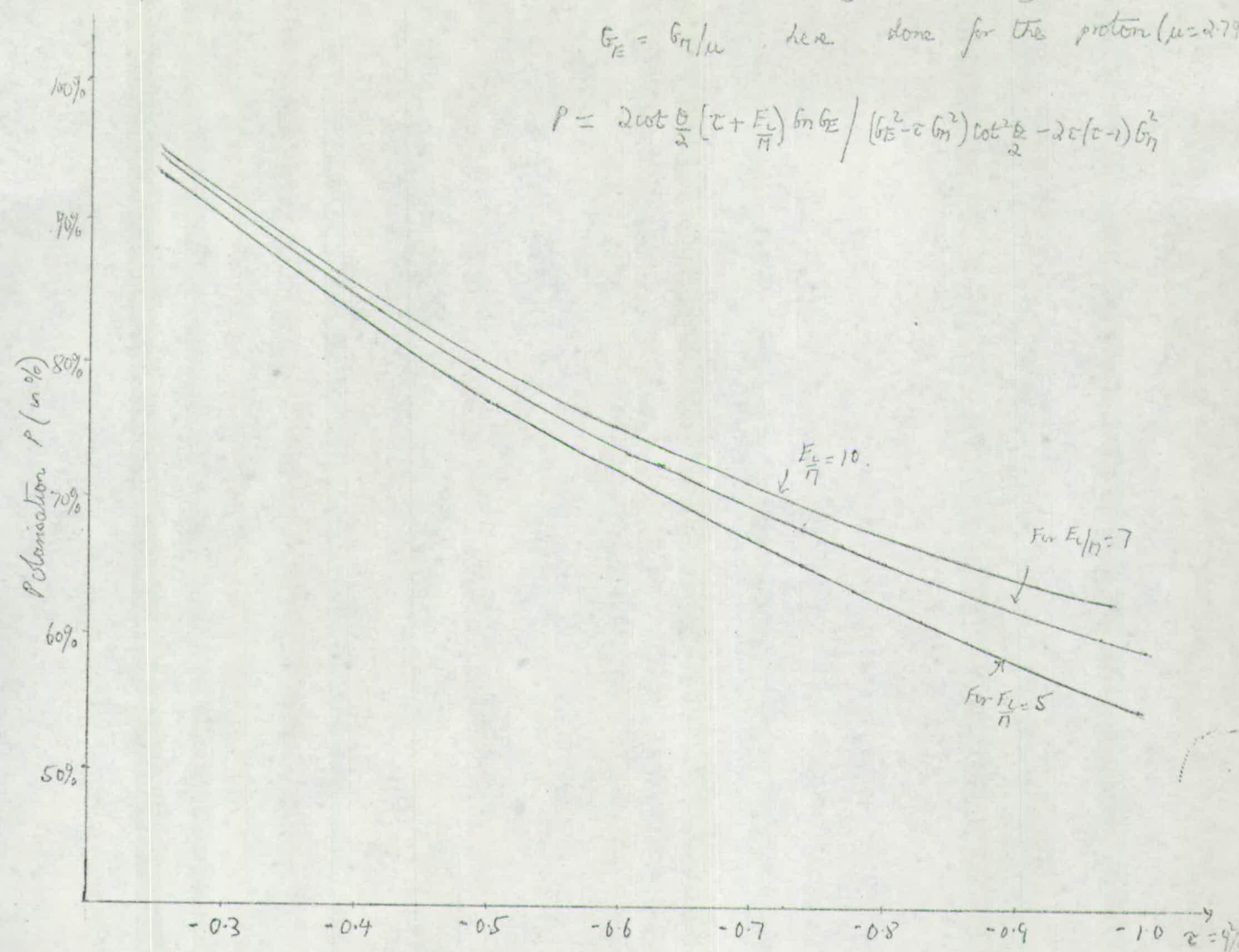
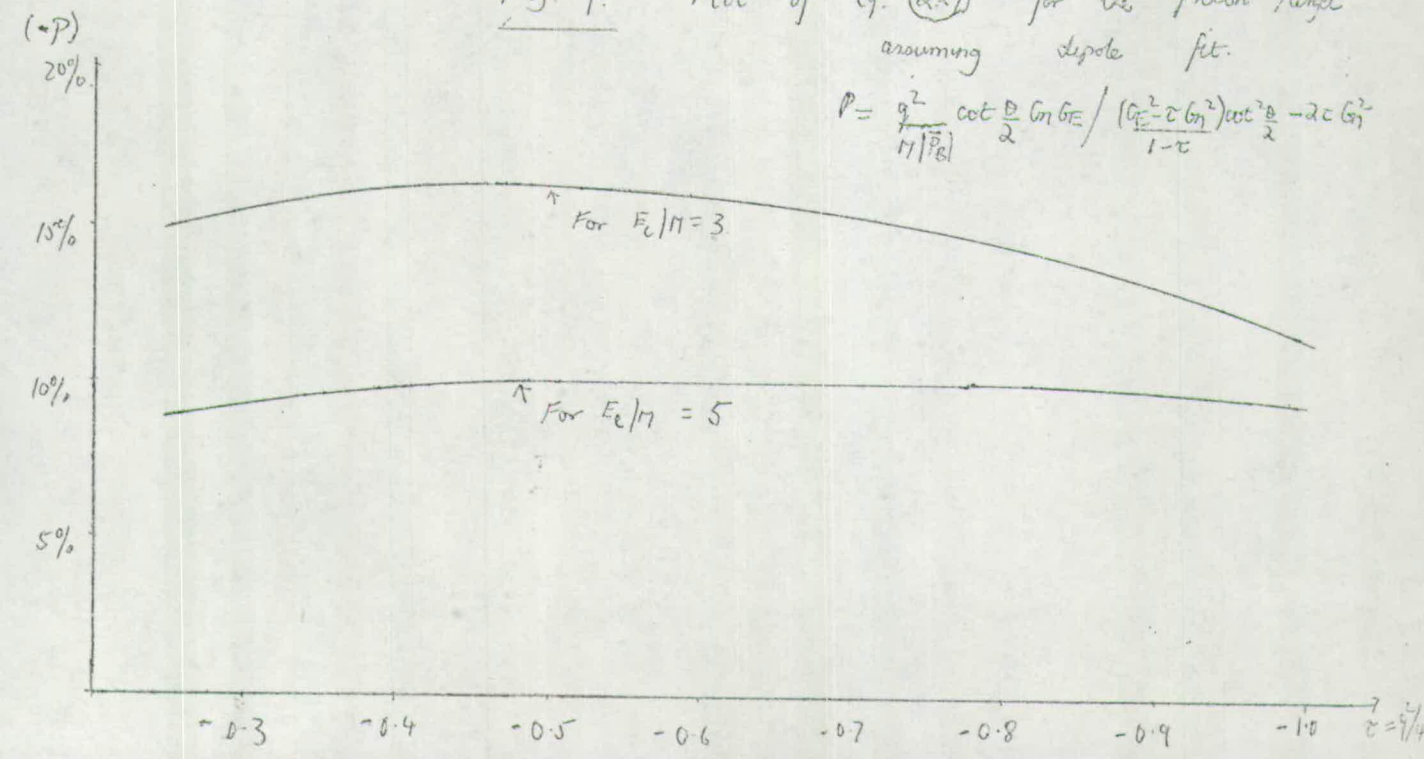


Fig. 4. Plot of eq. (229) for the proton target assuming dipole fit.

$$P = \frac{g^2}{M^2 |F_E|^2} \cot \frac{\theta}{2} G_M G_E / \left(\frac{G_E^2 - \tau G_M^2}{1-\tau} \right) \cot^2 \frac{\theta}{2} - 2\tau G_M^2$$



There is one more possible configuration, namely that when both initial and final leptons are polarised. Since both nucleon spins are summed over, the factor $H_{\mu\lambda} = \sum \langle P_2 | J_\mu | P_1 \rangle^2$ is exactly the same as for unpolarised nucleons, i.e. (2.6a) and involves only the form factors G_M^2 and $(G_E^2 - \tau G_M^2)$, but no cross term. Therefore such measurements add nothing new to information obtainable from unpolarised scattering and are not considered any further here⁽²¹⁾.

As an illustration of the utility of polarisation measurements, Figs. 3 and 4 represent plots of some of the most favourable cases, namely eq. (2.19) and eqs. (2.29), (2.39), (2.46) respectively. (The last three are given by the same expression). It is seen that appreciable polarisation can arise, particularly for (2.19), and thus such measurements are definitely feasible, even when for (2.19) the target polarisation is quite small.

Test of Electron-Muon Universality

All the formulae derived in the previous sections apply to point leptons satisfying the Dirac equation. While the latter is known to be true for electrons to a very high approximation, it has not been extensively checked for muons. The mysterious large mass difference between muons and electrons, which has so far defied any theoretical explanation, prompts one to examine if the muon possesses any difference in electromagnetic structure.

From general Lorentz invariance considerations and gauge invariance, the most general form for the muon-photon vertex

$i_s(22)$

$$\langle k_2 | j_u | k_1 \rangle = e \bar{u}(k_2) \left[H_1(q^2) \gamma_u + H_2(q^2) \frac{16_{uv} q_v}{2m_\ell} \right] u(k_1) \quad (2.54)$$

where $\bar{u}(k_1)$ is the positive energy free field spinor for a particle of momentum k_1 etc. H_1 and H_2 are form factors, functions of q^2 only, which describe any possible lepton structure. The static value of H_2 (i.e. at $q^2 = 0$) is equal to the anomalous magnetic moment of the muon, that is the part of it not attributable to a point lepton. Since the latest experiments confirm excellent agreement of the theoretical value for a point lepton⁽²³⁾, we may take $H_2 = 0$ to a very high approximation. Thus the vertex for the muon-photon differs only from that for the electron-photon by a possible multiplication factor $H_1(q^2)$.

There arises one further difference with muons, namely their mass. For electrons their mass has been neglected in deriving all formulae so far, which is quite satisfactory since $m_\ell/M < 10^{-3}$. For muons however $m_\ell/M \simeq 0.1$ and so mass effects cannot be neglected a priori. It turns out that the inclusion of the particle's mass gives rise to correction terms of the form $(m_\ell/M)^2$, $(m_\ell/E_\ell)^2$ or (m_ℓ^2/ME_ℓ) . For the high energy region $E_\ell > 1$ GeV, such terms amount to a correction of less than 1% and thus at the present level of experimental accuracy, may be safely neglected.

Hence the only possible effect arising from muons beams is a multiplicative factor $H_1(q^2)$ and since the muon-photon vertex $= \langle k_2 | j_u | k_1 \rangle$ is squared to yield the total matrix element, all formulae previously derived will have a factor

$[H_1(q^2)]^2$ in front. Thus the test for muon-structure will be quite clear. From the electron-scattering experiments the nucleon form factors will be determined; they will then be used as input in the muon experiments and a q^2 -dependent structure sought in $P_{\text{muon}}/P_{\text{electron}}$ ⁽²⁴⁾. Note that since all spins are not summed over, polarisation measurements are more sensitive to details of lepton structure than unpolarised experiments ⁽²⁵⁾, which have already put quite stringent limits on muon structure.

CHAPTER 3

POLARISATION IN COLLIDING BEAMS AND NUCLEON-ANTINUCLEON ANNIHILATION REACTIONS

The rapid development of colliding-beam experiments naturally leads one to investigate the effect of polarisation phenomena. We now discuss in detail the analysis of such reactions.

$l + \bar{l} \rightarrow N + \bar{N}$

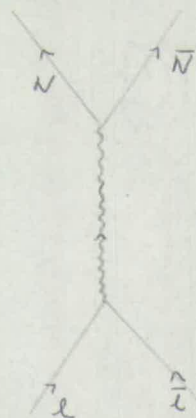


Fig. 5 depicts the Feynman diagram for lepton-antilepton annihilation into a nucleon, antinucleon pair. This process is related to the elastic scattering case $l + N \rightarrow l + N$ by crossing symmetry, which states the matrix element for the former process is related to that for the latter

by the following substitutions:

$l + N \rightarrow l + N$

$l + \bar{l} \rightarrow N + \bar{N}$

k_1	=	initial lepton 4-momentum	\rightarrow	l_1	where	l_1	=	initial 4-momentum of the lepton
k_2	=	final	"	"	\rightarrow	$-l_2$	"	l_2 = initial 4-momentum of the antilepton
P_1	=	initial nucleon	"	"	\rightarrow	$-t_2$	"	t_2 = final 4-momentum of the antinucleon
P_2	=	final	"	"	\rightarrow	t_1	"	t_1 = final 4-momentum of the nucleon.

∴ Summing over all lepton spins

$\sum \langle 0 | j_\mu | l_1 l_2 \rangle^2 = -4 [l_{2\mu} l_{1\lambda} + l_{2\lambda} l_{1\mu} - (l_1 l_2) \delta_{\mu\lambda}] \quad (3.1)$

on substitution from eq. (2.4).

Summing over all hadron spins,

$$\sum \langle t_1 t_2 | J_\mu | 0 \rangle^2 = \delta_{\mu\lambda} \bar{G}_M^2 q^2 / 2 + t_{1\mu} t_{1\lambda} \frac{2[\bar{G}_E^2 - \tau \bar{G}_M^2]}{1 - \tau} \quad (3.2)$$

on substitution from (2.6a), where form factors $\bar{G}_E(q^2)$ are defined by

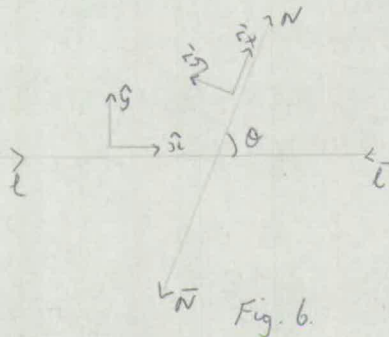
$$\langle t_1 t_2 | J_\mu | 0 \rangle = \frac{\bar{u}(t_1)}{1 - \tau} \left[(\bar{G}_E - \tau \bar{G}_M) \gamma_\mu + (\bar{G}_M - \bar{G}_E) i \frac{6 u_\nu v_\nu q_\nu}{2M} \right] u(t_2) \quad (3.3)$$

q = four-momentum transfer, here = $l_1 + l_2 = t_1 + t_2$.

The connection between the form factors for elastic scattering, defined in eq. (2.2) and those in (3.3) will be discussed below.

Contracting (3.2) with (3.1) gives the matrix-element squared for the reaction and in the centre of mass (cm) frame, i.e. colliding beams, see Fig. 6, leads to a differential cross-section

$$\frac{d\sigma}{d\Omega} = \frac{q^2}{16E^2} \beta \left[\bar{G}_M^2 (1 + \cos^2 \theta) + \frac{\bar{G}_E^2}{\tau} \sin^2 \theta \right] \quad (26) \quad (3.4)$$



where E is the incoming energy of either lepton in the c.m. frame;

$\beta = \bar{P}_f / E$ when \bar{P}_f is nucleon momentum = $(E^2 - M^2)^{1/2}$; and θ is the angle

between the lepton and nucleon directions as shown in Fig. 6.

$$t = q^2 / 4M^2 \quad \text{and} \quad q^2 = (l_1 + l_2)^2 = 4E^2 \quad (3.4a)$$

Since q^2 is independent of θ , (3.4) may be immediately integrated to give a total cross-section

$$\sigma = 2\pi\alpha^2 \beta \left[\bar{G}_M^2 + \frac{\bar{G}_E^2 - \tau \bar{G}_M^2}{3\tau} \right] / q^2 \quad (3.5)$$

The expression in brackets [] in (3.4) may be rewritten

as $\left[2\bar{G}_M^2 + \frac{\sin^2\theta(\bar{G}_E^2 - \tau\bar{G}_M^2)}{\tau} \right]$. Thus from measurement of the angular distribution, one may determine the form factors \bar{G}_M^2 and $(\bar{G}_E^2 - \tau\bar{G}_M^2)$ analogously to the Rosenbluth plot (1.1). We see again however for high q^2 i.e. $\tau \gg 1$, the extraction of the \bar{G}_E form factor will not be easy as the \bar{G}_M terms dominate, and resort to polarisation measurements must be made to determine the cross-term $\bar{G}_M\bar{G}_E$.

Before analysing the details of polarisation effects, the connection between the form factors in (3.3) and those in elastic scattering (eq. (2.2)) may be noted. In elastic scattering, the form factors are functions of q^2 which is there negative and lies in the range $-\infty < q^2 < 0$. In colliding beams, on the other hand, q^2 is positive and lies in the range $q^2 \geq 4M^2$. It turns out that $G(q^2)$ and $\bar{G}(q^2)$ are essentially the same complex function $G(z)$ (z is a complex variable) with the property $G(z) = G(q^2)$ for $q^2 < 0$ and $G(z) = \bar{G}(q^2)$ for $q^2 > 0$. In other words, the $\bar{G}_E(q^2)$, $\bar{G}_M(q^2)$ form factors are the analytic continuation of the $G_E(q^2)$, $G_M(q^2)$ form factors into the region $q^2 \geq 4M^2$ (27).

There is one notable difference between the two sets of form-factors, however. The hermiticity condition which demanded that G_E and G_M be real, fails for the colliding beam case since it merely gives $\langle t_1 t_2 | J_\mu | 0 \rangle = \langle 0 | J_\mu | t_1 t_2 \rangle^*$ and the process $\langle t_1 t_2 | J_\mu | 0 \rangle$ (creation of $N\bar{N}$) is different from $\langle 0 | J_\mu | t_1 t_2 \rangle$ (annihilation of $N\bar{N}$ pair). Time-reversal arguments also fail here (28). Thus \bar{G}_M and \bar{G}_E are in general complex, which leads to a distinct polarisation effect, as will be shown below.

1) Polarised Antinucleon Produced, Polarised Lepton Beam

Making the appropriate substitutions, given above (3.1), in eqs. (2.22) and in (2.23), gives

$$L_{\mu\lambda} = \sum \langle 0 | j_{\mu} | l_1 l_2 \rangle^2 = 2 \left[-l_{2\mu} l_{1\lambda} - l_{2\lambda} l_{1\mu} + (l_1 l_2) \delta_{\mu\lambda} + i m_L \epsilon_{\mu\lambda\alpha\beta} q_{\alpha} S_{L\beta} \right] \quad (3.6)$$

where $q = l_1 + l_2 = t_1 + t_2$ and S_L the spin projection operator for the initial lepton and

$$\begin{aligned} \frac{H_{\mu\lambda}}{2} &= \frac{1}{2} \sum \langle t_1 t_2 | J_{\mu} | 0 \rangle^2 = R_{\mu\lambda} + i \epsilon_{\mu\lambda st} M A^{-2} (-q_s S_t^{\bar{N}}) - i \left[\epsilon_{\mu s v \omega} t_{\lambda} - \epsilon_{\lambda s v \omega} t_u \right] t_{2s} t_{1v} S_{\omega}^{\bar{N}} \text{Re}(\overline{AB}^*) \\ &+ \left[\epsilon_{u s v \omega} t_{\lambda} + \epsilon_{\lambda s v \omega} t_u \right] t_{2s} t_{1v} S_{\omega}^{\bar{N}} \text{Im}(\overline{AB}^*) \end{aligned} \quad (3.7)$$

$t_1 = t_1 + t_2$, $S^{\bar{N}}$ denotes the spin-projection operator for the antinucleon, and Re and Im refer to the real and imaginary parts of (\overline{AB}) . The last term in (3.7) vanishes in the elastic scattering case but for this process is a priori, unknown. On contraction with (3.6) it contributes to the matrix element squared $|Im|^2$ an amount $4I_s$, where I_s

$$\begin{aligned} I_s &= -4 \text{Im}(\overline{AB}^*) \left[(l_1 t_1) \epsilon_{u s v \omega} l_{2u} + (t_1 l_2) \epsilon_{u s v \omega} \right] t_{2s} t_{1v} S_{\omega}^{\bar{N}} \\ &= 4 \text{Im}(\overline{AB}^*) \left\{ l_1 t_1 \left[-E(\bar{l}_2 \cdot \bar{t}_2 \times \bar{S}^{\bar{N}}) + E(\bar{l}_2 \cdot \bar{t}_1 \times \bar{S}^{\bar{N}}) \right] \right. \\ &\quad \left. + l_2 t_1 \left[-E(\bar{l}_1 \cdot \bar{t}_2 \times \bar{S}^{\bar{N}}) + E(\bar{l}_1 \cdot \bar{t}_1 \times \bar{S}^{\bar{N}}) \right] \right\} \end{aligned} \quad (3.8)$$

For a polarisation lying in the scattering plane eq. (3.8) vanishes; however for polarisation normal to the scattering plane

$$I_S = 4\text{Im} \left[\frac{(\bar{G}_M^* - \bar{G}_E^*) \bar{G}_E}{1 - \tau} \right] \beta^2 E^5 \sin 2\theta / M \quad (3.9)$$

Noting that $\omega = R_{\mu\lambda} L_{\mu\lambda} / 2 = 4E^4 \left[\bar{G}_M^2 (1 + \cos^2 \theta) + \bar{G}_E^2 \sin^2 \theta / \tau \right]$
 $= \left(\frac{d6}{d\Omega} \right) \frac{64E^6}{\alpha^2 \beta^2}$ from (3.4)

we see then that the antinucleon has a polarisation normal to the scattering plane (unit normal in the direction $\bar{t}_1 \times \bar{t}_1$) of amount $P = \left[m^2 - m_{(S\bar{N} \rightarrow S\bar{N})}^2 \right] / m^2 + m_{S\bar{N} \rightarrow S\bar{N}}^2$, here $= I_S / \omega$

$$\text{i.e. } \frac{d6}{d\Omega} = \frac{-1}{16} \frac{\text{Im}(\bar{G}_M \bar{G}_E^*)}{1 - \tau} \frac{\alpha^2 \beta^3}{E^2} \left(\frac{E}{M} \right) \sin 2\theta \quad (29) \quad (3.10)$$

This is independent of the initial lepton being polarised.

Measurement of P thus immediately determines $\text{Im}(\bar{G}_M \bar{G}_E^*)$.

Contracting the remainder of (3.7) with (3.6) yields

$$\begin{aligned} \frac{M^2}{4} &= \omega + 2m_\ell (S^{\bar{N}} \cdot S_L) (-Mq^2) \text{Re}(\bar{G}_M \bar{G}_E^*) + 2m_\ell (S^{\bar{N}} \cdot q) (S_L \cdot t_2) \frac{M}{\tau - 1} \\ &\cdot \left[-\tau \bar{G}_M^2 + (2\tau - 1) \text{Re}(\bar{G}_M \bar{G}_E^*) \right] + 4m_\ell (S^{\bar{N}} \cdot q) (S_L \cdot t_1) \frac{M}{\tau - 1} \\ &\cdot \left[\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] \\ &= \omega + M_S \end{aligned} \quad (3.11)$$

We now apply (3.11) to specific configurations.

(1) Longitudinally polarised lepton, antinucleon likewise

polarised. i.e. $S_\ell = \left(\frac{E}{m_\ell}, \frac{E}{m_\ell} \hat{x} \right)$, $S^{\bar{N}} = \frac{1}{M} \left[|P_F|, E(-\hat{x}') \right]$

$$S^{\bar{N}} \cdot q = \bar{P}_F \frac{2E}{M}, \quad S_L \cdot t_2 = \frac{2E^2}{m_\ell}, \quad S_L \cdot t_1 = \frac{E(E - P_F \cos \theta)}{m_\ell},$$

$$S^{\bar{N}} \cdot S_L = \frac{E(P_F + E \cos \theta)}{M m_\ell}.$$

$$\therefore M_S = -8E^4 \cos \theta \bar{G}_M^2, \quad \text{and the polarisation } P = M_S / \omega =$$

$$= \frac{-2 \cos \theta \bar{G}_M^2}{\bar{G}_M^2(1+\cos^2 \theta) + \bar{G}_E^2 \sin^2 \theta / \tau} \quad \text{or} \quad P \frac{d\delta}{d\Omega} = \frac{-a^2 \beta \cdot \cos \theta (\bar{G}_M^2)}{8E^2} \quad (3.12)$$

(ii) Longitudinally polarised lepton, antinucleon transversely polarised in the plane. i.e. $S_L = (\frac{E}{m}, \frac{E}{m} \hat{x})$, $S^{\bar{N}} = (0, -\hat{y}')$. Thus $S^{\bar{N}} \cdot q = 0$ and $S^{\bar{N}} \cdot S_L = -(E/m) \sin \theta$ and (3.11) gives $M_S = 2EMq^2 \sin \theta \operatorname{Re}(\bar{G}_M \bar{G}_E^*)$.

Hence polarisation

$$P = 2\left(\frac{M}{E}\right) \sin \theta \frac{\operatorname{Re}(\bar{G}_M \bar{G}_E^*)}{\bar{G}_M^2(1+\cos^2 \theta) + \bar{G}_E^2 \sin^2 \theta / \tau} \quad \text{or}$$

$$P \frac{d\delta}{d\Omega} = \frac{a^2}{8} \left(\frac{M}{E}\right) \sin \theta \frac{1}{E^2} \beta \operatorname{Re}(\bar{G}_M \bar{G}_E^*) \quad (3.13)$$

Hence one measurement of P in cases (i) and (ii) suffices to yield the value of \bar{G}_M^2 and $\operatorname{Re}(\bar{G}_M \bar{G}_E^*)$ respectively.

(iii) Transversely polarised lepton, longitudinally polarised antinucleon. i.e. $S_L = (0, \hat{y})$, $S^{\bar{N}} = \frac{1}{M} (|\bar{P}_f|, -E \hat{x}')$ and $\therefore S_L \cdot t_2 = 0$, $S_L \cdot t_1 = -\bar{P}_f \sin \theta$, $S^{\bar{N}} \cdot S_L = (E/M) \sin \theta$, giving

$$M_S = -2m_l E q^2 \sin \theta \bar{G}_M^2 \quad \text{and} \quad P = -2(m_l/E) \sin \theta \bar{G}_M^2 / \left[\bar{G}_M^2(1+\cos^2 \theta) + \bar{G}_E^2 \sin^2 \theta / \tau \right] \quad (3.14)$$

Again this measured \bar{G}_M^2 .

(iv) Transverse polarised lepton, transverse polarised \bar{N} .

i.e. $S^{\bar{N}} \cdot S_L = \cos \theta \quad \therefore M_S = -2Mm_l q^2 \cos \theta \operatorname{Re}(\bar{G}_M \bar{G}_E^*)$ and

$$P = -2\left(\frac{M}{E}\right) \left(\frac{m_l}{E}\right) \cos \theta \operatorname{Re}(\bar{G}_M \bar{G}_E^*) / \left[\bar{G}_M^2(1+\cos^2 \theta) + \frac{\bar{G}_E^2}{\tau} \sin^2 \theta \right]$$

$$\text{or} \quad P \frac{d\delta}{d\Omega} = \frac{-a^2 \beta}{8E^2} \left(\frac{M}{E}\right) \left(\frac{m_l}{E}\right) \cos \theta \operatorname{Re}(\bar{G}_M \bar{G}_E^*) \quad (3.15)$$

Note that whilst this also measures $\operatorname{Re}(\bar{G}_M \bar{G}_E^*)$ it is smaller by

a factor (m_ℓ/E) compared with (3.13).

(v) Both lepton and antinucleon polarised transverse to the scattering plane, i.e. $\bar{S}^{\bar{N}} \cdot S_L = -1$ and $M_S = 2Mm_\ell q^2 \text{Re}(\bar{G}_M \bar{G}_E^*)$, $P = M_S/\omega$. We recall there is also the contribution from eq. (3.10), but as this is independent of S_L , it may be separately determined. Note all contributions with \bar{S}_L transverse to the scattering plane and $\bar{S}^{\bar{N}}$ in the plane vanish due to parity invariance.

2. Polarised Antinucleon Produced, Polarised Antilepton Beam

$H_{\mu\lambda}$ is again given by eq. (3.7) and $L_{\mu\lambda}$ is given also by eq. (3.6). On contraction they yield

$$\begin{aligned} m^2/4 &= \omega + 2m_\ell (S^{\bar{N}} \cdot S_L) (-Mq^2) \text{Re}(\bar{G}_M \bar{G}_E^*) \\ &+ 2m_\ell (S^{\bar{N}} \cdot q) (S_L \cdot t_1) \frac{M}{\tau-1} \left[-\tau \bar{G}_M^2 + (2\tau-1) \text{Re}(\bar{G}_M \bar{G}_E^*) \right] \\ &+ 4m_\ell (S^{\bar{N}} \cdot q) (S_L \cdot t_1) \frac{M\tau}{\tau-1} \left[\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] \end{aligned} \quad (3.16)$$

where S_L now refers to the antilepton.

(i) Antilepton longitudinally polarised, antinucleon likewise polarised, i.e. $S_L = (\frac{E}{m_\ell}, -\frac{E}{m_\ell} \hat{x})$.

$$\therefore S_L \cdot t_1 = 2E^2/m_\ell, \quad S_L \cdot t_1 = \frac{E}{m_\ell} (E + P_f \cos \theta),$$

$$S_L \cdot S^{\bar{N}} = \frac{E}{m_\ell} \frac{1}{M} (\bar{P}_f - E \cos \theta).$$

(3.16) then gives $M_S = m^2/4 - \omega = 8E^4 \cos \theta \bar{G}_M^2$ and

Polarisation, $P = -$ eq. (3.12). (ρ defined as above (3.10) = τ/ω)

(ii) Antilepton longitudinally polarised, antinucleon transversely polarised in the plane, i.e. $S_L = (0, -\hat{y})$, $\bar{S}^N \cdot q = 0$, $S_L \cdot S^N = (E/m_\ell) \sin \theta$ and (3.16) $M_S = -8ME^3 \sin \theta \operatorname{Re}(\bar{G}_M \bar{G}_E^*)$.
 ∴ Polarisation $P = -$ (eq. 3.13)).

Similarly for the other cases, i.e.

For transverse polarised antilepton, antinucleon longitudinally polarised $P = -$ (eq. (3.14))

For transverse polarised antilepton, antinucleon transversely polarised $P = -$ (eq. (3.15))

but for both \bar{l} and \bar{N} transverse to the plane

$$P = + 2Mm_\ell q^2 \operatorname{Re}(\bar{G}_M \bar{G}_E^*) / \omega.$$

The difference in sign arises from the fact that the lepton and antilepton travelling in opposite directions, have their polarisation vectors anti-parallel to one another, except when both are transverse to the scattering plane.

3. Polarised Nucleon Produced, Polarised Lepton Beam

Making the appropriate substitutions in eqs. (2.43) and (2.23) gives $L_{\mu\lambda} =$ eq. (3.6) and $H_{\mu\lambda} =$ eq. (3.7) where in the latter $S^{\bar{N}} \rightarrow S^N$, the spin projection operator for the nucleon. On contraction we have

$$\begin{aligned} \mathcal{M}^2/4 = & \omega + 2m_\ell (S^N \cdot S_L) (-Mq^2) \operatorname{Re}(\bar{G}_M \bar{G}_E^*) + 2m_\ell (S^N \cdot q) (S_L \cdot \ell_2) \left[\tau \bar{G}_7^2 \right. \\ & \left. - \operatorname{Re}(\bar{G}_M \bar{G}_E^*) \right] \frac{M}{\tau-1} - 4m_\ell (S^N \cdot t_1) \frac{M}{\tau-1} \left[\bar{G}_M^2 - \operatorname{Re}(\bar{G}_M \bar{G}_E^*) \right] \\ & - I_S \text{ (defined in eq. (3.8))} \end{aligned} \quad (3.17)$$

On evaluating (3.17) for the various configurations, we find the polarisation $P' = -P$ for polarised leptons and antinucleon

set-up eqs. (3.12) - (3.14) $\left\{ \rho' = \left(m^2 - \frac{m^2}{s^N \rightarrow -s^N} \right) / m^2 + m^2_{s^N \rightarrow -s^N} \right\}$

= +P for polarised antilepton and antinucleon set-up eq. (3.16)

The equality for P for the cases of polarised lepton and nucleon, and polarised antilepton and antinucleon is a consequence of charge conjugation invariance. Note only when both the lepton and nucleon are polarised normal to the scattering plane (in the sense $\vec{k}_1 \times \vec{P}_1$ is normal direction) is $P = P$ for eq. (3.15). Again due to the form factors being complex, there is a polarisation of the nucleon normal to the scattering plane = - polarisation for antinucleon case eq. (3.10). This relation is a consequence of C.P.T. invariance⁽²⁹⁾.

4. Polarised Nucleon, Polarised antilepton Beam

The transition from lepton to antilepton beam introduces a minus sign in the value of P (compare cases (1) and (2)), except of course for polarisations normal to the scattering plane. Thus this case is equivalent to that for polarised antinucleons and leptons case (i), as follows, also from charge conjugation invariance⁽³⁰⁾.

Test for electron-muon universality

The most direct test as such is obviously to employ colliding muon beams and seek a q^2 -dependent structure in $P_{\text{muon}}/P_{\text{electron}}$, analogously to the elastic scattering case. Proposals for such beams are indeed being considered and may be realised in the near future⁽³¹⁾. However there is also another means of testing universality, and that is in the *reaction*



$e + \bar{e} \rightarrow \mu$ (muon) + $\bar{\mu}$ (antimuon). The vertex $\langle \mu\bar{\mu} | J_\mu | 0 \rangle$ is related to that for $N\bar{N} | J_\mu | 0$ by putting $\bar{G}_M(q^2) = \bar{G}_E(q^2) = \bar{H}_1(q^2)$ i.e. $\langle \mu\bar{\mu} | J_\mu | 0 \rangle = \bar{u}(t_1) \gamma_\mu U(t_2) \bar{H}_1(q^2)$. (See eq. (3.31) and (2.54).

Thus all the formulae derived previously for $e + \bar{e} \rightarrow N + \bar{N}$ will apply for $e + \bar{e} \rightarrow \mu + \bar{\mu}$, with the substitution $\bar{G}_M(q^2) = \bar{G}_E(q^2) = \bar{H}_1(q^2)$ and $M \rightarrow m_\mu$, muon mass. This gives for the ratio P (for $e + \bar{e} \rightarrow \mu + \bar{\mu}$) / P (for $e + \bar{e} \rightarrow e$ (electron) + \bar{e} (positron)) = $[(\bar{H}_1(q^2))]^2$, and deviations of H from unity for $q^2 \geq 4m_\mu^2$ will be an indication of muon structure. Again the muon mass has been neglected here, which is justified for $E \geq 1$ GeV since it introduces an error of less than 1%.

Determination of the form factors \bar{G}_M and \bar{G}_E is of great interest since at present their behaviour is terra incognita. Theoretical models relying on extrapolation of the G_M, G_E form factors, are very speculative and no reliable results have been established⁽³²⁾. One property that is believed to be true concerns the threshold value of the form factors and states $\bar{G}_M(+4M^2) = \bar{G}_E(4M^2)$ ⁽³³⁾; the 'dipole' fit for the elastic scattering form factors violates this condition when extrapolated into the $q^2 > 0$ region (it predicts $\bar{G}_M(4M^2) = (\text{Total magnetic moment of particle}) \times \bar{G}_E(4M^2)$) and it will be very interesting to see if in fact the dipole behaviour is not obeyed and the threshold condition confirmed⁽³⁴⁾.

Experimental measurements of polarisation phenomena are again favoured by the very appreciable values of P possible. This is illustrated in Figs. 7 and 8 depicting eqs. (3.12) and (3.13) respectively for reasonable values of the form factors, Only if

Fig 7

Plot of eq. (3.12); the — curves are plotted for the case $\bar{G}_E = \bar{G}_H / \mu$ (for the proton). The — curves correspond to $\bar{G}_E = \bar{G}_H$.

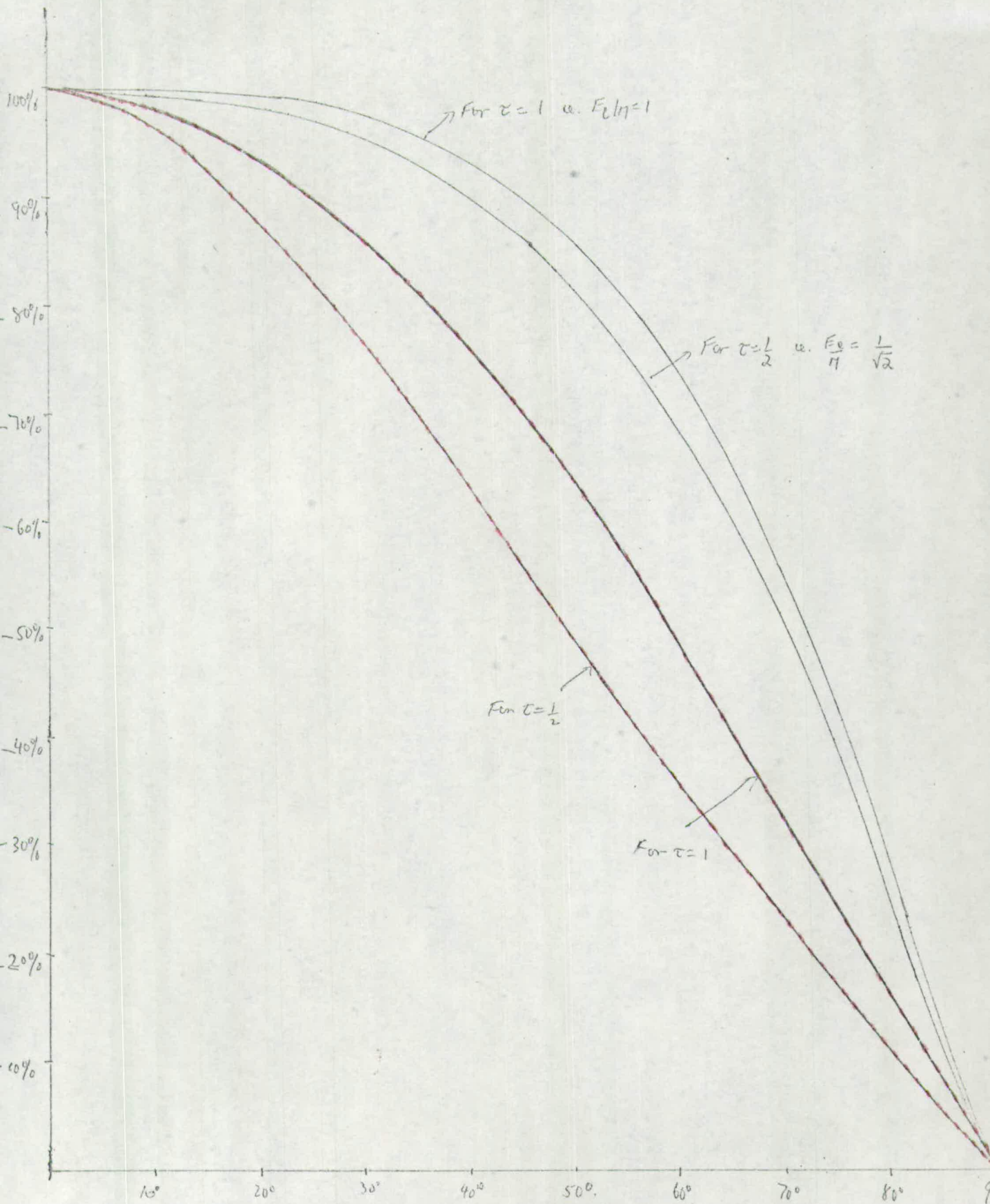
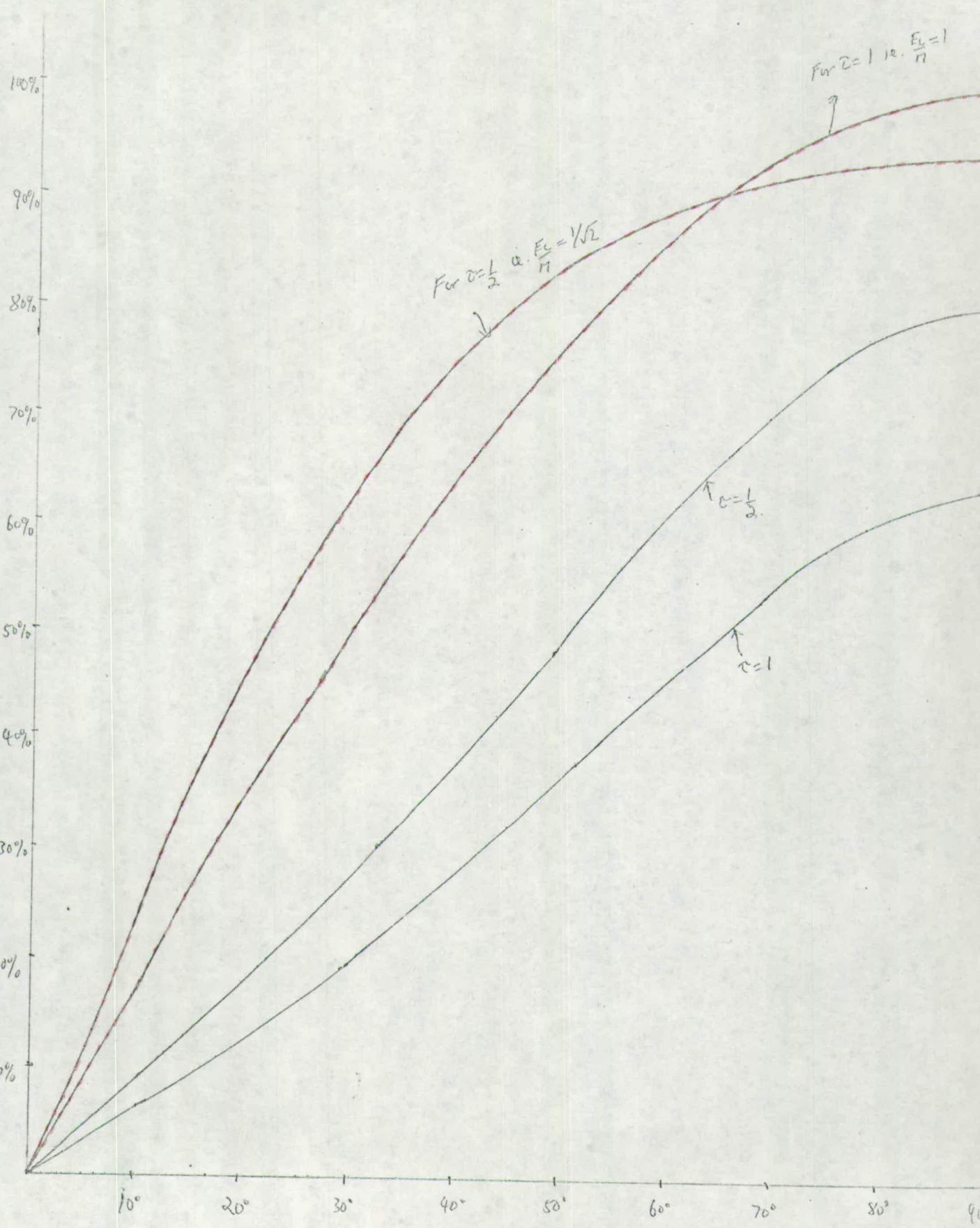


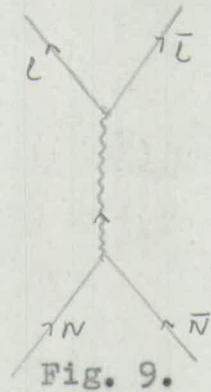
Fig. 8

Plot of eq. (3.13) — curves are for $\bar{G}_E = \bar{G}_M/\omega$ (for proton)
 — " " " $\bar{G}_E = \bar{G}_M$



\bar{G}_M decreases much more rapidly than \bar{G}_E will the situation be experimentally unfavourable, though not impossible.

(B) Nucleon-antinucleon annihilation, $N + \bar{N} \rightarrow \ell + \bar{\ell}$



This is the reverse of lepton-antilepton annihilation and proceeds via the Feynman diagram of Fig. 9. From crossing it is connected to elastic scattering by the following substitutions

$$\ell + N \rightarrow \ell + N$$

$$N + \bar{N} \rightarrow \ell + \bar{\ell}$$

k_1	\rightarrow	$-\ell_2$	where	ℓ_2	=	final antilepton 4-momentum
k_2	\rightarrow	ℓ_1	"	ℓ_1	=	" Lepton "
P_1	\rightarrow	t_1	"	t_1	=	initial nucleon "
P_2	\rightarrow	$-t_2$	"	t_2	=	" antinucleon "

leading to a differential cross-section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E_c^2} \frac{1}{\beta} \left[\bar{G}_M^2 (1 + \cos^2\theta) + \frac{\bar{G}_E^2}{c} \sin^2\theta \right] \quad (35) \quad (3.18)$$

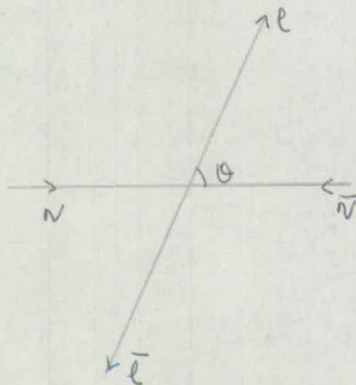


Fig. 10

where E_c is the incoming energy of either nucleon in the (c.m.) centre of mass frame; $\beta = \bar{P}_f/E$, where \bar{P}_f is the nucleon momentum $= (E_c^2 - M^2)^{1/2}$ and θ is the angle between the nucleon and lepton in the c.m. frame (see Fig. 10)

In the laboratory frame, with the nucleon at rest and an incoming anti-nucleon beam of energy E_B ,

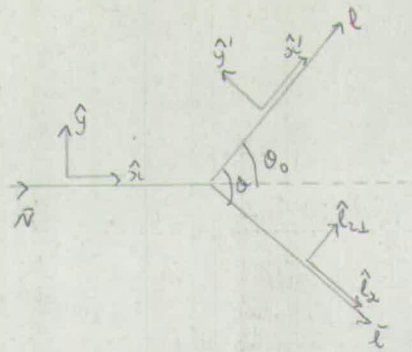


Fig. 11

the differential cross-section becomes

$$\frac{d\sigma}{d\Omega_0} = \frac{a^2 E_e^2}{q^2 M \bar{P}} \left\{ \bar{G}_M^2 + \cot^2 \frac{\theta}{2} \frac{\bar{G}_E^2 - \tau \bar{G}_M^2}{2t(t-1)} \right\} \quad (3.19)$$

where $d\Omega_0 = 2\pi d(\cos \theta_0)$; E_e is the final lepton energy $= q^2/2(M+E_B - |\bar{P}| \cos \theta_0)$; \bar{P} = antinucleon momentum $= (E_B^2 - M^2)^{1/2}$;

$q = 4$ momentum transfer $= t_1 + t_2 = \ell_1 + \ell_2$; and θ and θ_0 are as depicted in Fig. 11. ($\tau = q^2/4M^2$).

$\bar{G}_M(q^2)$ and $\bar{G}_E(q^2)$ are the form factors defined by

$$\langle 0 | J_\mu | t_1 t_2 \rangle = \frac{\bar{M}(t_2)}{1-\tau} \left[(\bar{G}_E - \tau \bar{G}_M) \gamma_\mu + (\bar{G}_M - \bar{G}_E) \frac{i6 \bar{M} v^a v^b}{2M} \right] u(t_1) \quad (3.20)$$

and are equivalent to those for colliding beams, c.f. eq. (3.3).

(3.18) is readily integrated to give the total cross-section

$$\sigma = \frac{a^2 \pi}{M(E^2 - M^2)^{1/2}} \left\{ \bar{G}_M^2 + \frac{\bar{G}_E^2 - \tau \bar{G}_M^2}{3t} \right\}. \quad \text{Here } E \text{ is the antinucleon}$$

energy in the laboratory frame.

From (3.18) or (3.19) it is evident that the \bar{G}_M and \bar{G}_E form factors may be determined but again at high q^2 the \bar{G}_M term dominates and only polarisation measurements will determine the cross-term $\bar{G}_M \bar{G}_E$. We now discuss specific configurations, working in the laboratory frame (nucleon at rest). For kinematic details see Appendix 5.

(1) Polarised nucleon target, polarised lepton produced

Effecting the substitutions given above (3.18), in eqs.

(2.22) and (2.23) gives

$$L_{\mu\lambda} = \sum \langle \ell_1 \ell_2 | \bar{J}_\mu | 0 \rangle^2 = 2 \left[-\ell_{2\mu} \ell_{1\lambda} - \ell_{2\lambda} \ell_{1\mu} + (\ell_1 \ell_2) \delta_{\mu\lambda} - i m_\ell \epsilon_{\mu\lambda\alpha\beta} q_\alpha S_{L\beta} \right] \quad (3.21)$$

$$H_{\mu\lambda}/2 = \frac{1}{2} \sum \langle 0 | \bar{J}_\mu | t_1 t_2 \rangle^2 = R_{\mu\lambda} + i \epsilon_{\mu\lambda st} - M \bar{A}^2 (q_s S_t^N) + i \left[\epsilon_{usv\omega} t_\lambda - \epsilon_{\lambda sv\omega} t_\mu \right] t_{2s} t_{1v} S_\omega^N \text{Re}(\bar{A} \bar{B}^*) - \left[\epsilon_{usv\omega} t_\lambda + \epsilon_{\lambda sv\omega} t_\mu \right] t_{2s} t_{1v} \cdot S_\omega^N \text{Im}(\bar{A} \bar{B}^*) \quad (3.22)$$

S^N now refers to the spin projection operator for the nucleon.

The last term in (3.22) contributes to the matrix element squared, an amount $4I_s$ where

$$I_s = 4 \text{Im}(\bar{A} \bar{B}^*) \left\{ (t_1 \ell_1) \epsilon_{usv\omega} \ell_{2\mu} + (t_1 \ell_2) \epsilon_{usv\omega} \ell_{1\mu} \right\} t_{2s} t_{1v} S_\omega^N = -4M \text{Im}(\bar{A} \bar{B}^*) \left(2ME_\ell - \frac{q^2}{2} \right) \left[\bar{S}^N \bar{t}_1 \times \bar{t}_2 \right] \quad (3.23)$$

For \bar{S}_N lying in the scattering plane (3.23) vanishes; however if it is normal to the scattering plane (3.23) is a maximum.

Noting $\omega = R_{\mu\lambda} L_{\mu\lambda}/2$

$$= \left(\frac{q^4}{2} \right) \left[\bar{G}_M^2 + \cot^2 \frac{\theta}{2} \frac{(\bar{G}_E^2 - \tau \bar{G}_M^2)}{2\tau(\tau-1)} \right]$$

we see then that a nucleon polarisation normal to the plane (in the same sense as $(\bar{t}_2 \times \ell_1)$ leads to an azimuthal distribution

$$\frac{d\delta}{d\Omega_0} = \frac{a^2}{2q^2} \frac{E_\ell^2}{M_P} \left[\bar{G}_M^2 + \cot^2 \frac{\theta}{2} \frac{(\bar{G}_E^2 - \tau \bar{G}_M^2)}{2\tau(\tau-1)} + \frac{2 \text{Im}(\bar{G}_M \bar{G}_E^*)}{q^2 t} \cdot \left(-\tau + \frac{E_\ell}{M} \right) \left(\bar{S}^N \cdot \bar{t}_2 \times \bar{t}_1 \right) \right] \quad (3.24)$$

The notation is as for (3.19). Measurement of the spin-term thus determines $\text{Im}(\bar{G}_M \bar{G}_E^*)$. Contracting the remainder of (3.22) with (3.21) yields

$$\begin{aligned} \frac{m^2}{4} &= \omega + 2m_\ell (S^N, S_L) \text{Re}(\bar{G}_M \bar{G}_E^*) (-Mq^2) + 2m_\ell (S^N \cdot q)(S_L \cdot t_2) \cdot \\ &M \left[\tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] / (\tau - 1) - 4m_\ell (S^N \cdot q)(S_L \cdot t_1) \cdot \\ &M \tau \left[\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] / (\tau - 1) \equiv \omega + M_S \end{aligned} \quad (3.25)$$

This is exactly the same as for the colliding beams case, eq. (3.17) and is a consequence of time-reversal invariance.

(ii) Target nucleon polarised along direction of incoming anti-nucleon, ($\bar{t}_2 = \bar{P}$), longitudinally polarised lepton produced,

i.e. $S^N = (0, \hat{x})$ (since it is at rest), $S_L = \left(\frac{E_\ell}{m_\ell}, \frac{E}{m_\ell} \hat{x}' \right)$.

Hence $S^N \cdot q = S^N t_2 = -\bar{P} \cdot S_L \cdot t_1 = M \frac{E_\ell}{m_\ell}$, $S_L \cdot t_2 = E'_\ell E_\ell (1 - \cos \theta) / m_\ell$

$$\text{Im} \bar{G}_M, S_L \cdot S^N = -\frac{E_\ell}{m_\ell} \cos \theta$$

and (3.25) gives $M_S = q^2 E_\ell \left[-|\bar{P}| + 2M \cos \theta_0 \right] \bar{G}_M^2$. Polarisation

$P = M_S / \omega$. This directly measures \bar{G}_M . Here $P = \frac{(m^2 - m_{(S_L \cdot S^N)}^2)}{(m^2 + m_{(S_L \cdot S^N)}^2)}$

for cases (i) to (x)

(ii) Target polarised as in (i) but lepton transversely polarised,

i.e. $S_L = (0, \hat{y}')$. Then $S_L t_1 = 0$, $S_L t_2 = E'_\ell \sin \theta$, $S^N S_L = \sin \theta$

where E'_ℓ is the antilepton laboratory energy yielding

$M_S = -(2m_\ell M \sin \theta_0) q^2 \tau \bar{G}_M^2$ and $P = M_S / \omega$. Again \bar{G}_M is deter-

mined here.

(iii) Target nucleon polarised transversely to \bar{P} , longitudinally

polarised lepton, i.e. $S^N = (0, \hat{y})$

$$\therefore S^N \cdot q = S^N \cdot t_2 = 0 \quad \text{and} \quad S^N \cdot S_L = -(E_\ell/m_\ell) \sin \theta_0,$$

whence (3.25) yields

$$M_S = 2m_\ell q^2 \sin \theta_0 \operatorname{Re}(\bar{G}_M \bar{G}_E^*) \quad \text{and} \quad P = \left(\tau - \frac{\bar{P}}{2M} \cos \theta_0\right)^{-1} \frac{\operatorname{Re}(\bar{G}_M \bar{G}_E^*) \sin \theta_0}{\left[\bar{G}_M^2 + \cot^2 \frac{\theta_0}{2} \frac{(\bar{G}_E^2 - \tau \bar{G}_M^2)}{2\tau(\tau-1)}\right]} \quad (3.26)$$

$$\text{or } P \frac{d6}{d\Omega} = a^2 (8M \bar{P})^{-1} \tau \sin \theta_0 \operatorname{Re}(\bar{G}_M \bar{G}_E^*) / \left(\tau - \frac{\bar{P}}{2M} \cos \theta_0\right)^3$$

One measurement of P thus immediately yields $\operatorname{Re}(\bar{G}_M \bar{G}_E^*)$.

(iv) Target nucleon polarised as in (iii), transversely polarised lepton. Now $S^N \cdot S_L = -\cos \theta_0$, $M_S = 2m_\ell M q^2 \cos \theta_0 \operatorname{Re}(\bar{G}_M \bar{G}_E^*)$ and

$$P = (m_\ell/M) \cos \theta_0 \operatorname{Re}(\bar{G}_M \bar{G}_E^*) / \tau \left[\bar{G}_M^2 + \cot^2 \frac{\theta_0}{2} \left(\frac{\bar{G}_E^2 - \tau \bar{G}_M^2}{2\tau(\tau-1)}\right)\right] \quad (3.27)$$

This also measures $\operatorname{Re}(\bar{G}_M \bar{G}_E^*)$ but is reduced by (m_ℓ/M) factor compared with (3.26).

(v) Both target nucleon and lepton polarised transversely to the scattering plane, i.e. $S^N \cdot S_L = -1$ and $P = \text{eq. (3.27)}/\cos \theta_0$. Similar comment applies here as there.

(vi) Target nucleon polarised perpendicular to direction of produced lepton, the latter being longitudinally polarised, i.e. $S^N = (0, \hat{y}')$.

So $S^N \cdot q = |\bar{P}| \sin \theta_0$, $S^N \cdot S_L = 0$ and (3.25) gives

$$M_S = M q^4 \left\{ \tau \bar{G}_M^2 - \operatorname{Re}(\bar{G}_M \bar{G}_E^*) - \frac{E_\ell}{M} \left[\bar{G}_M^2 - \operatorname{Re}(\bar{G}_M \bar{G}_E^*) \right] \right\} / |\bar{P}|$$

$$\therefore \text{Polarisation } P = \left[\frac{2M}{|\bar{P}|} \right] \left\{ \tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) - \frac{\tau}{(\tau - \frac{|\bar{P}|}{2M} \cos \theta_0)} [\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)] \right\} / \left[\bar{G}_M^2 + \cot^2 \frac{\theta}{2} \frac{(\bar{G}_E^2 - \tau \bar{G}_M^2)}{2\tau(\tau - 1)} \right]$$

$$\text{or } P \frac{d\theta}{d\Omega} = \lambda \alpha^2 [16M^2(\tau - 1)]^{-1} (\tau - \frac{|\bar{P}|}{2M} \cos \theta_0)^{-2} \{ \dots \} \quad (3.28)$$

Two measurements of P are now required for fixed q^2 (or τ) but varying θ_0 to extract the form factors $G_1 = \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)$, $G_2 = \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)$ whence from G_2 the cross-term may be found. There is the disadvantage here that the target polarisation direction has to be altered for changes in θ_0 ; one cannot fix θ_0 and vary \bar{P} since the latter is fixed by fixed q^2 , i.e. $\bar{P}^2 = q^2(\tau - 1)$.

(vii) Nucleon polarised as in (vi), transversely polarised lepton, i.e. $S^N \cdot S_L = -1$.

$$\therefore M_S = 2M_\ell q^2 \left\{ \text{Re}(\bar{G}_M \bar{G}_E^*) + \sin^2 \theta_0 [\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)] \right\} \text{ and } P = M_S / \omega.$$

This is of order (m_ℓ/M) smaller than (3.28).

(viii) Target nucleon polarised along direction of produced anti-lepton, the lepton being transversely polarised, i.e.

$$S^N = (0, \hat{e}_2) \text{ (see Fig. 11).}$$

Then $S^N \cdot q = -|\bar{P}| \cos(\theta - \theta_0)$ and $S^N \cdot S_L = \sin \theta$ giving

$$M_S = 2Mm_\ell \left[q^2 \text{Re}(\bar{G}_M \bar{G}_E^*) - E'_\ell |\bar{P}| (\theta - \theta_0) \frac{(\tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*))}{\tau - 1} \right]$$

\therefore Polarisation $P =$

$$\frac{1}{\tau} \left(\frac{m_\ell}{M} \right) \frac{\left\{ \text{Re}(\bar{G}_M \bar{G}_E^*) - |\bar{P}| \left[4M(\tau - \frac{|\bar{P}|}{2M} \cos(\theta - \theta_0)) \right]^{-1} [\tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)] \right\}}{\bar{G}_M^2 + \cot^2 \frac{\theta}{2} \frac{(\bar{G}_E^2 - \tau \bar{G}_M^2)}{2\tau(\tau - 1)}} \quad (3.29)$$

$$\text{or } P \frac{d6}{d\Omega} = \frac{\alpha^2}{4} \left(\frac{m}{M} \right) (M/|\bar{P}|)^{-1} \left(\tau - \frac{|\bar{P}|}{2M} \cos \theta_0 \right)^{-2} \{ \quad \} .$$

Two measurements of P for varying $(\theta - \theta_0)$ but fixed q^2 determines the form factors $G_1 = \text{Re}(\bar{G}_M \bar{G}_E^*)$, $G_2 = \tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)$ but once again the nucleon polarisation direction has to be changed as $(\theta - \theta_0)$ varies.

(ix) Target nucleon polarised transverse to antilepton momentum, longitudinally polarised lepton, i.e. $S^N = (0, \hat{e}_{2L})$. Thus $S^N \cdot q = -|\bar{P}| \sin(\theta - \theta_0) = -E_\ell \sin \theta$, $S^N \cdot S_L = -E_L \sin \theta / m_\ell$ giving

$$M_S = q^2 M E_\ell \sin \theta \left\{ \tau \left[2\text{Re}(\bar{G}_M \bar{G}_E^*) - \bar{G}_M^2 \right] - \text{Re}(G_M G_E^*) \right. \\ \left. + (E_\ell / M) \left[\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] \right\} / \tau - 1$$

$$\therefore P = \left(2 \left[\tau - \frac{|\bar{P}|}{2M} \cos \theta_0 \right] \right)^{-1} \times$$

$$\left\{ \frac{\tau \left[2\text{Re}(\bar{G}_M \bar{G}_E^*) - \bar{G}_M^2 \right] - \text{Re}(\bar{G}_M \bar{G}_E^*) + \tau \left[\tau - \frac{|\bar{P}|}{2M} \cos \theta_0 \right] \left[\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right]}{\left[\bar{G}_M^2 (\tau - 1) + \cot^2 \frac{\theta}{2} (\bar{G}_E^2 - \tau \bar{G}_M^2) / 2\tau \right]} \right\}$$

$$\text{or } P \frac{d6}{d\Omega} = \alpha^2 (8M/|\bar{P}|)^{-1} \tau \sin \theta \{ \quad \} / \left(\tau - \frac{|\bar{P}|}{2M} \cos \theta_0 \right)^3 \quad (3.30)$$

From (3.30) one extracts the form factors

$$G_1 = \tau \left[2\text{Re}(\bar{G}_M \bar{G}_E^*) - \bar{G}_M^2 \right] - \text{Re}(\bar{G}_M \bar{G}_E^*), \quad G_2 = \tau \left[\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right]$$

and noting $G_1 + G_2 = (\tau - 1)\text{Re}(\bar{G}_M \bar{G}_E^*)$ fixes the value of the cross-term.

(x) Nucleon polarised as in (ix), lepton transversely polarised. Here $S^N S_L = -\cos \theta$,

$$\therefore M_S = -2m_e q^2 \left\{ \tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) - \cos \theta \left[\tau (2\text{Re}(\bar{G}_M \bar{G}_E^*)) - \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] \right\} / 2(\tau - 1)$$

and polarisation P

$$= \frac{-(m_e/M) \left\{ \tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) - \cos \theta \left[\tau (2\text{Re}(\bar{G}_M \bar{G}_E^*)) - \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] \right\}}{2\tau \left[\bar{G}_M^2 (\tau - 1) + \cot^2 \frac{\theta}{2} (\bar{G}_E^2 - \tau \bar{G}_M^2) / 2\tau \right]} \quad (3.31)$$

$$\text{or } P \frac{d\sigma}{d\Omega} = -\frac{a^2}{8} (m_e/M) (M | \bar{P})^{-1} \left(\tau - \frac{|P|}{2M} \cos \theta_0 \right)^{-2} \{ . \}$$

Two measurements of P at different θ but fixed q^2 determines the form factors $G_1 = \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)$,
 $G_2 = \tau [2\text{Re}(\bar{G}_M \bar{G}_E^*) - \bar{G}_M^2] - \text{Re}(\bar{G}_M \bar{G}_E^*)$, $\therefore G_1 + G_2 = 2(\tau - 1)\text{Re}(\bar{G}_M \bar{G}_E^*)$.

The nucleon polarisation direction has again to be varied as θ changes. Note both (3.31) and (3.29) are suppressed because of the (m_e/M) factor.

All correlations involving the nucleon polarised in the scattering plane and the lepton polarised normal to it, vanish because of parity invariance. Reversing the target polarisation and measuring the change in the lepton polarisation, i.e. $M_S - M_S$ (with $S^N \rightarrow -S^N$) doubles the polarisation effect.

2. Polarised nucleon target, polarised antilepton produced.

Analogously to the colliding beams case, eq. (3.16), the matrix element is given by substituting $(S_L \cdot \ell_2) \rightarrow (S_L \cdot \ell_1)$ in eq. (3.25), where S_L now refers to the antilepton.

$$\begin{aligned} \therefore M_S &= 2m_\ell (S^N \cdot S_L) \text{Re}(\bar{G}_M \bar{G}_E^*) (Mq^2) + 2m_\ell (S^N \cdot q)(S_L \cdot \hat{e}_1) M \left[\tau \bar{G}_M^2 \right. \\ &\quad \left. - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] / (\tau - 1) - 4m_\ell (S^N \cdot q)(S_L \cdot \hat{t}_1) M \tau \left[\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] / \tau - 1 \end{aligned} \quad (3.32)$$

(i) Nucleon polarised along direction of \bar{P} , longitudinally polarised antilepton, i.e. $S_L = \left(\frac{E'_\ell}{m_\ell}, \frac{E'_\ell}{m_\ell}, \hat{e}_2 \right)$,

$$\therefore S_L \cdot \hat{t}_1 = \frac{ME'_\ell}{m_\ell}, \quad S_L \cdot \hat{e}_1 = \frac{E'_\ell}{m_L} (1 - \cos \theta), \quad S^N \cdot S_L = \frac{-E'_\ell}{m_\ell} \cos(\theta - \theta_0)$$

and gives $M_S = q^4 E'_\ell \left[(\tau - 1) \bar{G}_M^2 - \frac{|\bar{P}|}{2M} \cos \theta_0 \bar{G}_M^{-2} \right] / |\bar{P}|$,

$P = M/\omega = -P$ for lepton case (1)(i). For this section, polarisation $P = \frac{[m^2 - m^2_{(S \rightarrow -S)}]}{m^2 + m^2_{S \rightarrow -S}}$.

(ii) Nucleon polarised as for (i), transversely polarised antilepton, i.e. $S_L = (0, \hat{e}_{21})$, $S_L \cdot \hat{t}_1 = 0$, $S_L \cdot \hat{e}_1 = -E'_\ell \sin \theta$, $S^N \cdot S_L = -\sin(\theta - \theta_0)$ and $M_S = 2Mm_\ell q^2 \sin(\theta - \theta_0) \tau \bar{G}_M^2$.

Measurement of $P = M_S/\omega$ then determines \bar{G}_M .

(iii) Nucleon polarised transverse to \bar{P} , longitudinally polarised antilepton. $S^N \cdot q = 0$, $S^N \cdot S_L = E'_\ell \sin(\theta - \theta_0)/m_\ell = E'_\ell \sin \theta_0 / m$

$$\begin{aligned} \therefore M_S &= -2ME'_\ell q^2 \sin \theta_0 \text{Re}(\bar{G}_M \bar{G}_E^*) \text{ and Polarisation } P \\ &= -(\tau - |\bar{P}| \cos \theta_0)^{-1} \frac{\sin \theta_0 \text{Re}(\bar{G}_M \bar{G}_E^*)}{\left[\bar{G}_M^2 + \cot^2 \frac{\theta}{2} \left(\frac{G^2_{E-\tau G^2_M}}{2(\tau-1)} \right) \right]} = - (3.26) \quad (3.33) \end{aligned}$$

(iv) Nucleon polarised as in (iii), transversely polarised antilepton $S^N \cdot S_L = -\cos(\theta - \theta_0)$

$$\therefore M_S = 2Mm q^2 \cos(\theta - \theta_0) \text{Re}(\bar{G}_M \bar{G}_E^*) \text{ and}$$

$$P = \frac{1}{\tau} \left(\frac{m_\ell}{M} \right) \cos(\theta - \theta_0) \operatorname{Re}(\bar{G}_M \bar{G}_E^*) / \left[\bar{G}_M^2 + \cot^2 \frac{\theta}{2} \left(\frac{\bar{G}_E^2 - \tau \bar{G}_M^2}{2\tau(\tau-1)} \right) \right] \quad (3.34)$$

Although (3.34) directly measures the cross-term, it is reduced by the factor (m_ℓ/M) compared with (3.33).

(v) Both nucleon and antilepton polarised transverse to the scattering plane, i.e. $S^N \cdot S_L = -1$

$$\therefore M_S = 2m_\ell M q^2 \operatorname{Re}(\bar{G}_M \bar{G}_E^*) \text{ and } P = (3.34) / \cos(\theta - \theta_0).$$

(vi) Nucleon polarised along momentum direction of produced lepton, antilepton transversely polarised, i.e. $S^N = (0, \hat{x}')$, $S_L = (0, \hat{z}_{2\perp})$ giving $S^N \cdot S_L = -\sin \theta$, $S^N \cdot q = P \sin \theta_0$.

\therefore (3.32) yields

$$M_S = 2Mm_\ell \sin \theta \left[q^2 \operatorname{Re}(\bar{G}_M \bar{G}_E^*) + |\bar{P}| E_\ell \cos \theta_0 (\tau \bar{G}_M^2 - \operatorname{Re}(\bar{G}_M \bar{G}_E^*)) / (\tau - 1) \right]$$

and

$$P = (m_\ell/M) \frac{(\sin \theta)}{\tau} \frac{\operatorname{Re}(\bar{G}_M \bar{G}_E^*) + |\bar{P}| \cos \theta_0 \left[4M(\tau - \frac{|\bar{P}|}{2M} \cos \theta_0)^{-1} (\tau \bar{G}_M^2 - \operatorname{Re}(\bar{G}_M \bar{G}_E^*)) / (\tau - 1) \right]}{\bar{G}_M^2 + \cot^2 \frac{\theta}{2} (\bar{G}_E^2 - \tau \bar{G}_M^2) / 2\tau(\tau - 1)}$$

(3.35)

The measurement of P at different θ_0 but fixed q^2 enables one to extract the form factors $G_1 = \operatorname{Re}(\bar{G}_M \bar{G}_E^*)$, $G_2 = \tau \bar{G}_M^2 - \operatorname{Re}(\bar{G}_M \bar{G}_E^*)$. Similar comment applies here as for (iv).

(vii) Nucleon polarised transverse to lepton direction, antilepton longitudinally polarised, i.e. $S^N = (0, \hat{y}')$ and $S^N \cdot q = S^N \cdot t_2 = \bar{P} \sin \theta_0$, $S^N \cdot S_L = E_\ell' \sin \theta / m_\ell$.

(3.32) gives

$$M_S = M|\bar{P}| q^2 \sin \theta_0 \left[\tau(\bar{G}_M^2 - 2\text{Re}(\bar{G}_M \bar{G}_E^*)) + \text{Re}(\bar{G}_M \bar{G}_E^*) \right. \\ \left. - (E'_\ell/M)(\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)) \right] / \tau - 1$$

$$\text{and } P = 2 \left(\frac{\tau-1}{\tau} \right)^{1/2} \sin \theta_0 \frac{ \left\{ \tau [\bar{G}_M^2 - 2\text{Re}(\bar{G}_M \bar{G}_E^*)] + \text{Re}(\bar{G}_M \bar{G}_E^*) - \tau \left[\tau - \frac{|P|}{2M} \cos(\theta - \theta_0) \right]^{-1} [\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)] \right\} }{ [\bar{G}_M^2 (\tau-1) + \cot^2 \frac{\theta}{2} (\bar{G}_E^2 - \tau \bar{G}_M^2) / 2\tau] } \quad (3.36)$$

$$\text{or } P \frac{d\theta}{d\Omega} = \alpha^2 (8M^2)^{-1} \sin \theta \left\{ \quad \right\} / \left[\tau - \frac{|P|}{2M} \cos \theta_0 \right]^2$$

From (3.36) one determines the form factors

$$G_1 = \tau \left[\bar{G}_M^2 - 2\text{Re}(\bar{G}_M \bar{G}_E^*) \right] + \text{Re}(\bar{G}_M \bar{G}_E^*) , \quad G_2 = \tau \left[\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right]$$

whence we have $G_2 - G_1 = (\tau-1)\text{Re}(\bar{G}_M \bar{G}_E^*)$. However as $(\theta - \theta_0)$ varies so the nucleon polarisation direction must be changed,

(viii) Nucleon polarised as in (vii), antilepton transversely polarised. Now $S^N \cdot S_L = -\cos \theta$ and

$$M_S = -2Mm_\ell q^2 \left\{ \tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) - \cos \theta \left[\tau (2\text{Re}(\bar{G}_M \bar{G}_E^*) - \bar{G}_M^2) - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] \right\}$$

∴ polarisation $P \equiv$ eq. (3.31) and comments thereafter apply here too.

(ix) Nucleon polarised transverse to antilepton momentum direction, antilepton longitudinally polarised, i.e. $S^N = (0, \hat{e}_{2\perp})$

$$S^N q = -|\bar{P}| \sin(\theta - \theta_0), \quad S^N \cdot S_L = 0.$$

$$\therefore M_S = -ME_\ell \sin \theta q^2 \left[\tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) - (E'_\ell/M)(\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)) \right] / \tau - 1$$

and polarisation P

$$= - \left[2 \left(\tau - \frac{|P| \cos \theta_0}{2M} \right) \right]^{-1} \frac{ \left\{ \tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) - \tau \left[\tau - \frac{|P|}{2M} \cos(\theta - \theta_0) \right]^{-1} [\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)] \right\} }{ [\bar{G}_M^2 (\tau-1) + \cot^2 \frac{\theta}{2} (\bar{G}_E^2 - \tau \bar{G}_M^2) / 2\tau] }$$

$$\text{or } P \frac{d\theta}{d\Omega} = -\alpha^2 (8M|\bar{P}|)^{-1} \tau \sin \theta \left\{ \quad \right\} / \left[\tau - \frac{|P|}{2M} \cos \theta_0 \right]^3 \quad (3.37)$$

From (3.37) one determines the form factors $G_1 = \tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)$
 $G_2 = \tau [\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)]$, $\therefore G_1 - G_2 = (\tau - 1) \text{Re}(\bar{G}_M \bar{G}_E^*)$ fixes
 the value of the cross-term.

(x) Nucleon polarised as in (ix), antilepton transversely
 polarised. $S^N \cdot S_L = -1$.

$$\therefore M_S = 2m_e M \left[q^2 \text{Re}(\bar{G}_M \bar{G}_E^*) + E_e |\bar{P}| \sin(\theta - \theta_0) \left\{ \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right\} / \tau - 1 \right]$$

and

$$P = \left(\frac{m_e}{M} \right) \frac{\left\{ \text{Re}(\bar{G}_M \bar{G}_E^*) + |\bar{P}| \sin(\theta - \theta_0) \left[4M \left(\tau - \frac{E_e}{2M} \cos \theta_0 \right) \right]^{-1} \left[\tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right] / \tau - 1 \right\}}{\tau \left[\bar{G}_M^2 + \cot^2 \frac{\theta}{2} (\bar{G}_E^2 - \tau \bar{G}_M^2) / 2\tau(\tau - 1) \right]} \quad (3.38)$$

From here may be extracted the form factors $G_1 = \text{Re}(\bar{G}_M \bar{G}_E^*)$ and
 $G_2 = \tau \bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*)$ but note (3.38) is reduced compared to
 (3.37) because of the (m_e/M) factor. For both cases (ix) and
 (x), the target polarisation has to be changed in direction as
 θ_0 and $(\theta - \theta_0)$ varies.

For all correlations considered here, reversing the target
 polarisation and measuring the change in antilepton polarisation
 again doubles the effect.

3. Polarised Antinucleon Beam, Polarised Lepton Produced.

This may be evaluated by effecting the appropriate substitutions
 in (2.22) and (2.43); however it is quicker to note time-reversal
 invariance immediately equates the matrix element squared for this
 case to the colliding beams case, eq. (3.11).

$$\begin{aligned} \therefore \frac{M^2}{4} &= \omega + 2m_\ell (S^{\bar{N}} \cdot S_L) (-Mq^2) \text{Re}(\bar{G}_M \bar{G}_E^\lambda) + 2m_\ell (S^{\bar{N}} \cdot q)(S_L \cdot t_2) \frac{M}{\tau-1} \\ &\cdot \left[-\tau \bar{G}_M^2 + (2\tau-1) \text{Re}(\bar{G}_M \bar{G}_E^\lambda) \right] - I_S + 4m_\ell (S^{\bar{N}} \cdot q)(S_L \cdot t_1) \\ &\cdot M(\tau/(\tau-1)) \left[\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^\lambda) \right] = \omega + M_S - I_S \quad (3.40) \end{aligned}$$

where $S^{\bar{N}}$ denotes the spin-projection operator for the antinucleon ($S^{\bar{N}} \cdot p_2 = 0$) and ω and I_S as defined below (3.23).

(i) Antinucleon longitudinally polarised as is the lepton, i.e. $S^{\bar{N}} = (|\bar{P}| \cdot E_B \hat{x})/M$ where E_B is the antinucleon incoming laboratory energy, i.e. $E_B^2 = \bar{P}^2 + M^2$.

Thus we have $S^{\bar{N}} \cdot q = S^{\bar{N}} \cdot p_1 = \bar{P}$, $S^{\bar{N}} \cdot S_L = E_\ell \left[|\bar{P}| - E_B \cos \theta_0 \right] / Mm_\ell$

and (3.40) yields $M_S = q^2 E_\ell \left[-|\bar{P}| \bar{G}_M^2 + 2M\tau \cos \theta_0 \bar{G}_M^2 \right]$ and

$P' = P$ for case (1)(i). Polarisation $P' = \left[m^2 - m_\ell^2 \right] / \left[m^2 + m_\ell^2 \right]$

with m^2 given by (3.40).

(ii) Antinucleon polarised as in (i) but lepton transversely polarised, i.e. $S^{\bar{N}} \cdot S_L = E_B \sin \theta_0 / M$

$\therefore M_S = -2Mm_\ell q^2 \sin \theta_0 \bar{G}_M^2$ and $P' = P$ for case (1)(ii).

When the antinucleon is polarised transverse to its momentum \bar{P} , $S^{\bar{N}} = (0, 1) = S^N$ for case (i), thus P' for such configurations is equal to that in case (1), i.e.

Antinucleon transversely polarised, lepton longitudinally polarised

$$P' = \text{eq. (3.26)}$$

" " " " " transversely polarised

$$P' = \text{eq. (3.27)}$$

Antinucleon and lepton both polarised normal to the scattering

plane, $P' = \text{eq. (3.27)} / \cos \theta_0$

4. Polarised Antinucleon, Polarised Antilepton Produced.

\mathcal{M}^2 for this case is given by substituting $(S_L \cdot \ell_2) \rightarrow (S_L \cdot \ell_1)$ in (3.40) (see similarly (3.22)). Again on evaluating we find the polarisation P is equal to that for the corresponding configuration in case (2), i.e. for antinucleon transversely polarised, antilepton longitudinally polarised $P = \text{eq. (3.33)}$ etc.

5. Polarised Nucleon Target and Polarised Antinucleon Beam

Making the appropriate substitutions in (2.4) and (2.6) and contracting together yields

$$\begin{aligned} \mathcal{M}^2 = & a' + b'(S^{\bar{N}} \cdot S^N) + c'(S^{\bar{N}} \cdot t_1)(S^N \cdot \ell_1) + d'(S^{\bar{N}} \cdot t_1)(S^N \cdot \ell_2) \\ & + e'(S^{\bar{N}} \cdot \ell_1)(S^N \cdot \ell_1) + f'(S^{\bar{N}} \cdot \ell_1)(S^N \cdot \ell_2) \end{aligned}$$

analogously to (2.7) where the coefficients $a', b', \dots f'$ are given by

$$a'/4 = (q^4/2) \left[\bar{G}_M^2 + \cot^2 \frac{\theta}{2} (\bar{G}_E^2 - \tau \bar{G}_M^2) / 2\tau(\tau - 1) \right]$$

$$b'/4 = 2 \left[-2(t_1 \ell_1)(t_1 \cdot \ell_2) + M^2(\ell_1 \ell_2) \right] \left\{ -A^{-2} + 2\bar{B}^2(t_1 t_2 - M^2) + 4M \text{Re}(\overline{AB}^*) \right\}$$

$$\begin{aligned} c'/4 = & 2 \left[-\bar{A}^2(t_1 t_2 + M^2 + t_1 \ell_2 - t_1 \ell_1) - 2\bar{B}^2(-2t_1 \ell_1(t_1 \ell_2) + (\ell_1 \ell_2)M^2) \right. \\ & \left. + 2M \text{Re}(\overline{AB}^*)(-t_1 \ell_1 + t_1 \ell_2 + \ell_1 \ell_2) \right] \end{aligned}$$

$$d'/4 = 4 \left[-\bar{B}^2(-2t_1 \ell_1(t_1 \ell_2) + M^2 \ell_1 \ell_2) + M \text{Re}(\overline{AB}^*)(\ell_1 \ell_2) \right]$$

$$e'/4 = 2 \left[-\bar{A}^2(-t_1 \ell_2 + t_1 \ell_1 - t_1 t_2 - M^2) + 2M \text{Re}(\overline{AB}^*)(t_1 \ell_1 - t_1 \ell_2) \right]$$

$$f'/4 = 2 \left[-\bar{A}^2(-t_1 \ell_2 + t_1 \ell_1 + t_1 t_2 + M^2) + 2M \text{Re}(\overline{AB}^*)(t_1 \ell_1 - t_1 \ell_2) \right]$$

Notation is as above (3.18), $\bar{A} = \bar{G}_M$ and $\bar{B} = (\bar{G}_M - \bar{G}_E) / 2M(1 - \tau)$.

The above expression for \mathcal{M}^2 simplifies to

$$\begin{aligned}
 M^4/4 = & \omega + \bar{G}_M^2 q^2 \left[(-s^{\bar{N}} \cdot t_1)(s^N \cdot \ell_1) + s^{\bar{N}} \cdot \ell_1 (s^N \cdot \ell_1) - s^{\bar{N}} \cdot \ell_1 (s^N \cdot \ell_2) \right] \\
 & + \left(\bar{G}_M^2 - \text{Re}(\bar{G}_M \bar{G}_E^*) \right) (q^2/1-\tau) \left\{ s^{\bar{N}} \cdot t_1 (s^N \cdot \ell_1) + s^{\bar{N}} \cdot t_1 (s^N \cdot \ell_2) \right\} \\
 & + 2 \left[\text{Re}(\bar{G}_M \bar{G}_E^*) - \tau \bar{G}_M^2 \right] (t_1 \ell_1 - t_1 \ell_2) / (1-\tau) \left\{ s^{\bar{N}} \cdot t_1 (s^N \cdot \ell_1) \right. \\
 & \left. - s^{\bar{N}} \cdot \ell_1 (s^N \cdot \ell_1) - s^N \cdot \ell_1 (s^N \cdot \ell_2) \right\} \\
 & + 2 \left[-2(t_1 \ell_1)(t_1 \ell_2) + M^2(\ell_1 \ell_2) \right] \left\{ -s^{\bar{N}} \cdot s^N (\bar{G}_E^2 - \tau \bar{G}_M^2) / (1-\tau) \right. \\
 & \left. - 2\bar{B}^2 (s^{\bar{N}} \cdot t_1) s^N \cdot \ell_1 - 2\bar{B}^2 (s^{\bar{N}} \cdot t_1) (s^N \cdot \ell_2) \right\} \equiv \omega + M_S
 \end{aligned} \tag{3.41}$$

We now apply (3.41) to some specific configurations.

(i) Target nucleon polarised along direction of incoming anti-nucleon, the latter being transversely polarised,

i.e. $s^N = (0, \hat{x})$. $s^{\bar{N}} = (0, \hat{y})$

$\therefore s^N \cdot \ell_1 = -E_\ell \cos \theta_0$, $s^N \cdot \ell_2 = -E'_\ell \cos(\theta - \theta_0) = -|\bar{P}| + E_\ell \cos \theta$,
 $s^{\bar{N}} \cdot t_1 = s^{\bar{N}} \cdot s^N = 0$, $s^{\bar{N}} \cdot \ell_1 = -E_\ell \sin \theta_0$ and (3.41) gives

$$M_S = (s^{\bar{N}} \cdot \ell_1) 4ME_\ell \left[|\bar{P}| - 2M\tau \cos \theta_0 \right] \text{Re}(\bar{G}_M \bar{G}_E^*)$$

and polarisation $P = \left[m^2 - m^2_{s^{\bar{N}} \rightarrow -s^{\bar{N}}} \right] / \left[m^2 + m^2_{s^{\bar{N}} \rightarrow -s^{\bar{N}}} \right] = M_S/\omega =$

$$-\sin \theta_0 \left[\tau - \frac{|\bar{P}|}{2M} \cos \theta_0 \right]^{-2} \left\{ \left(\frac{|\bar{P}|}{2M} - \tau \cos \theta_0 \right) \text{Re}(\bar{G}_M \bar{G}_E^*) \right\} /$$

$$\left[\bar{G}_M^2 + \cot^2 \frac{\theta}{2} (\bar{G}_E^2 - \tau \bar{G}_M^2) / 2\tau(\tau - 1) \right]$$

$$\text{or } P \frac{d\sigma}{d\Omega} = \frac{-\alpha^2 \sin \theta_0 \tau (4M |\bar{P}|)^{-1}}{\left[\tau - \frac{|\bar{P}|}{2M} \cos \theta_0 \right]^4} \tag{3.42}$$

One measurement of P therefore immediately determines the term

$\text{Re}(\bar{G}_M \bar{G}_E^*)$.

(ii) Target nucleon polarised normal to antinucleon momentum,

the latter being longitudinally polarised, i.e. $S^N = (0, \hat{y})$,

$S^{\bar{N}} = (\frac{\bar{P}}{M}, \frac{E_B}{M} \hat{x})$. Thus $S^{\bar{N}} \cdot t_1 = \bar{P}$, $S^{\bar{N}} \cdot \ell_1 = E_\ell (\bar{P} - E_B \cos \theta_0)$

$S^N \cdot \ell_1 = -S^N \cdot \ell_2 = -E_\ell \sin \theta_0$ and $S^{\bar{N}} \cdot S^N = 0$, whence (3.41)

yields

$M_S = (S^N \cdot \ell_1) 4ME_\ell \left[|\bar{P}| - 2M \tau \cos \theta_0 \right] \text{Re}(\bar{G}_M \bar{G}_E^*)$ and $P = \text{eq. (3.42)}$.

Again this directly measures the cross-term.

(iii) Target nucleon polarised as in (ii), antinucleon transversely

polarised, i.e. $S^{\bar{N}} \cdot S^N = -1$

$\therefore M_S = M^2 q^2 \cot^2 \frac{\theta}{2} \left[\tau \bar{G}_M + G_E^2 \right] / \tau - 1$ and $P = M_S / \omega$.

(iv) Both nucleon and antinucleon polarised transverse to scatter-

ing plane. Then $M_S = +M^2 q^2 \cot^2 \frac{\theta}{2} (\bar{G}_E^2 - \tau \bar{G}_M^2) / (\tau - 1)$ and $P = M_S / \omega$.

For both (iii) and (iv) the \bar{G}_M term dominates at high q^2 .

(v) Nucleon polarised along lepton momentum direction, antinucleon

transversely polarised, i.e. $S^N = (0, \hat{x}')$, $S^{\bar{N}} = (0, \hat{y})$

$\therefore S^N \cdot \ell_1 = -E_\ell$, $S^N \cdot \ell_2 = -\bar{P} \cos \theta_0 + 2E_\ell$, $S^N \cdot S^{\bar{N}} = -\sin \theta_0$

and (3.41) gives

$M_S = 4M^2 E_\ell (-E_\ell \sin \theta_0) \left\{ \tau^2 \bar{G}_M^2 - \tau \bar{G}_M^2 + \tau \bar{G}_E^2 - \bar{G}_E^2 + \frac{|\bar{P}|}{M} (1-\tau) \cos \theta_0 \text{Re}(\bar{G}_M \bar{G}_E^*) \right.$
 $\left. - (\tau - 1) \cos^2 \theta_0 \left[\tau \bar{G}_M^2 - \tau 2 \text{Re}(\bar{G}_M \bar{G}_E^*) + \bar{G}_E^2 \right] \right\}$

\therefore

$P = \frac{-1}{2} \left[\tau - \frac{E}{2\gamma} \cos \theta_0 \right]^{-2} \sin^2 \theta_0 \left\{ \tau^2 \bar{G}_M^2 - \tau \bar{G}_M^2 + \tau \bar{G}_E^2 - \bar{G}_E^2 + \frac{|\bar{P}|}{M} (1-\tau) \cos \theta_0 \text{Re}(\bar{G}_M \bar{G}_E^*) - (\tau - 1) \cos^2 \theta_0 \left[\tau \bar{G}_M^2 - 2\tau \text{Re}(\bar{G}_M \bar{G}_E^*) + \bar{G}_E^2 \right] \right.$
 $\left. \bar{G}_M^2 + \cot^2 \frac{\theta}{2} \left(\frac{\bar{G}_E^2 - \tau \bar{G}_M^2}{2\tau(\tau - 1)} \right) \right\}$

$$\text{or } P \frac{d6}{d\Omega} = -a^2 \sin \theta_0 \tau (8M |\bar{P}|)^{-1} \left[\tau - \frac{|\bar{P}|}{2M} \cos \theta_0 \right]^{-4} \{ \quad \} \quad (3.43)$$

Three measurements of P at varying θ_0 but fixed q^2 is here required to extract the form factors: $G_1 = t^2 \bar{G}_M^2 - (\bar{G}_M^2 - \bar{G}_E^2) \tau - \bar{G}_E^2$, $G_2 = \text{Re}(\bar{G}_M \bar{G}_E^*)$, $G_3 = \tau [\bar{G}_M^2 - 2\text{Re}(\bar{G}_M \bar{G}_E^*)] + \bar{G}_E^2$. There is the disadvantage however that the nucleon polarisation direction has to be changed as θ_0 varies.

(vi) Nucleon polarised transverse to lepton momentum, antinucleon transversely polarised, i.e. $S^N = (0, \hat{y}')$ and $S^N \cdot \hat{e}_1 = 0$,

$S^N \cdot \hat{e}_2 = S^N \cdot \hat{e}_3 = |\bar{P}| \sin \theta_0$, $S^N \cdot S^{\bar{N}} = -\cos \theta_0$. Then we have

$$M_S = M^2 q^2 \cot^2 \frac{\theta}{2} \left[-\frac{|\bar{P}|}{M} \text{Re}(\bar{G}_M \bar{G}_E^*) + \cos \theta_0 \left\{ \tau (-\bar{G}_M^2 + 2\text{Re}(\bar{G}_M \bar{G}_E^*)) - \bar{G}_E^2 \right\} \right]$$

$$\therefore P = \cot^2 \frac{\theta}{2} \left\{ \frac{-\frac{|\bar{P}|}{M} \text{Re}(\bar{G}_M \bar{G}_E^*) + \cos \theta_0 \left[\tau (-\bar{G}_M^2 + 2\text{Re}(\bar{G}_M \bar{G}_E^*)) - \bar{G}_E^2 \right]}{2\tau \bar{G}_M^2 + \cot^2 \frac{\theta}{2} (\bar{G}_E^2 - \tau \bar{G}_M^2) / \tau - 1} \right\}$$

$$\text{or } P \frac{d6}{d\Omega} = \frac{a^2 (8M |\bar{P}|)^{-1} \cot^2 \frac{\theta}{2} \{ \quad \}}{\left[\tau - \frac{|\bar{P}|}{2M} \cos \theta_0 \right]^2} \quad (3.47)$$


Two measurements of P at different θ_0 but fixed q^2 determines the form factors: $G_1 = \text{Re}(\bar{G}_M \bar{G}_E^*)$ and $G_2 = \tau [-\bar{G}_M^2 + 2\text{Re}(\bar{G}_M \bar{G}_E^*)] - \bar{G}_E^2$.

(vii) Nucleon polarised along antilepton momentum direction, antinucleon transversely polarised, i.e. $S^N = (0, \hat{e}_2)$, thus

$$S^N \cdot \hat{e}_1 = -E_\ell \cos \theta = -|\bar{P}| \cos(\theta - \theta_0) + E_\ell', \quad S^N \cdot \hat{e}_2 = -E_\ell',$$

$S^N \cdot S^{\bar{N}} = \sin(\theta - \theta_0)$ when (3.41) yields

Fig 12.

Plot of eq. (3.2b) The plain pencilled curve corresponds to the fit with $\bar{G}_E = \bar{G}_M$; the  curve is the result for a scaling fit $\bar{G}_E = \bar{G}_M/\mu$ (for proton)

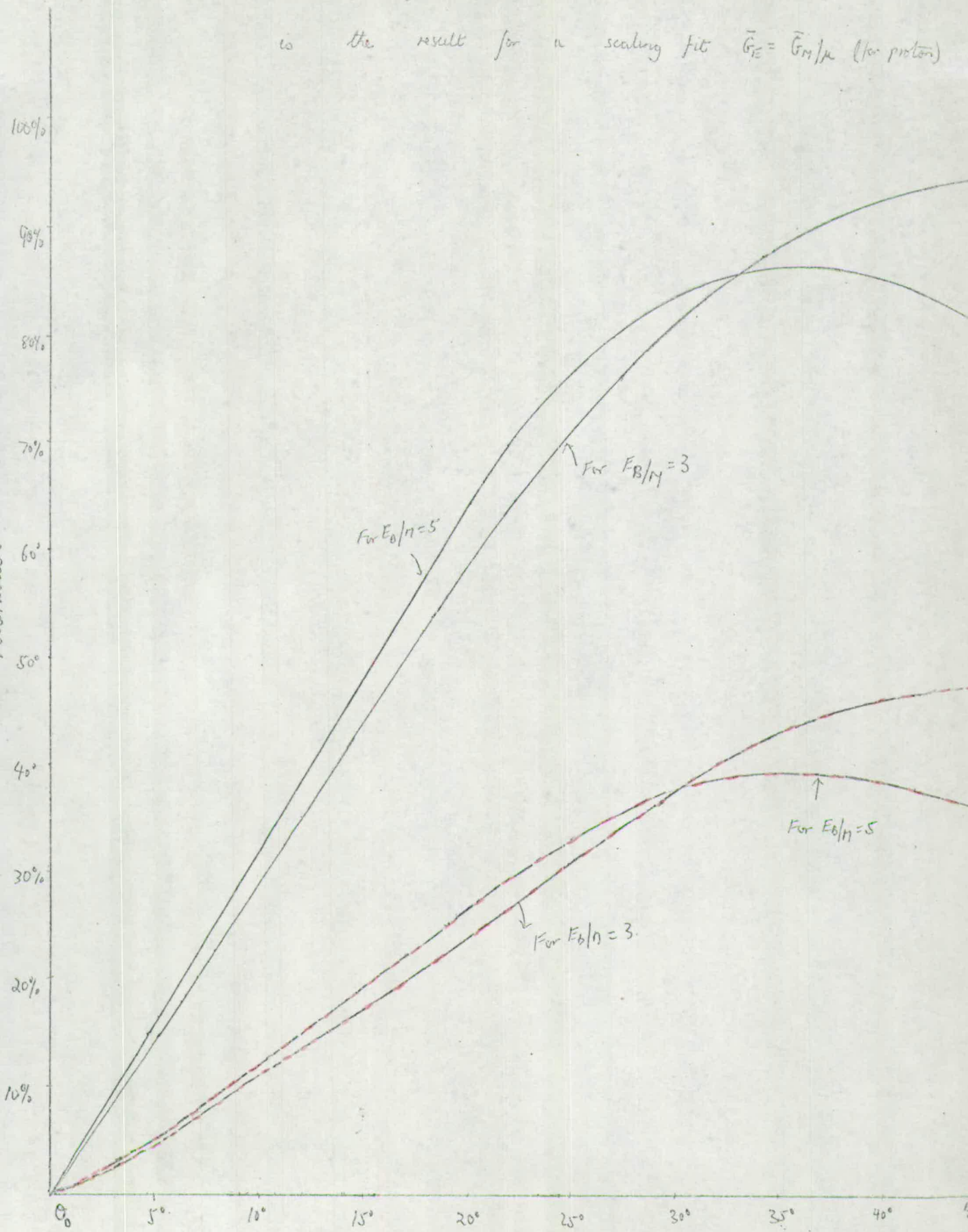
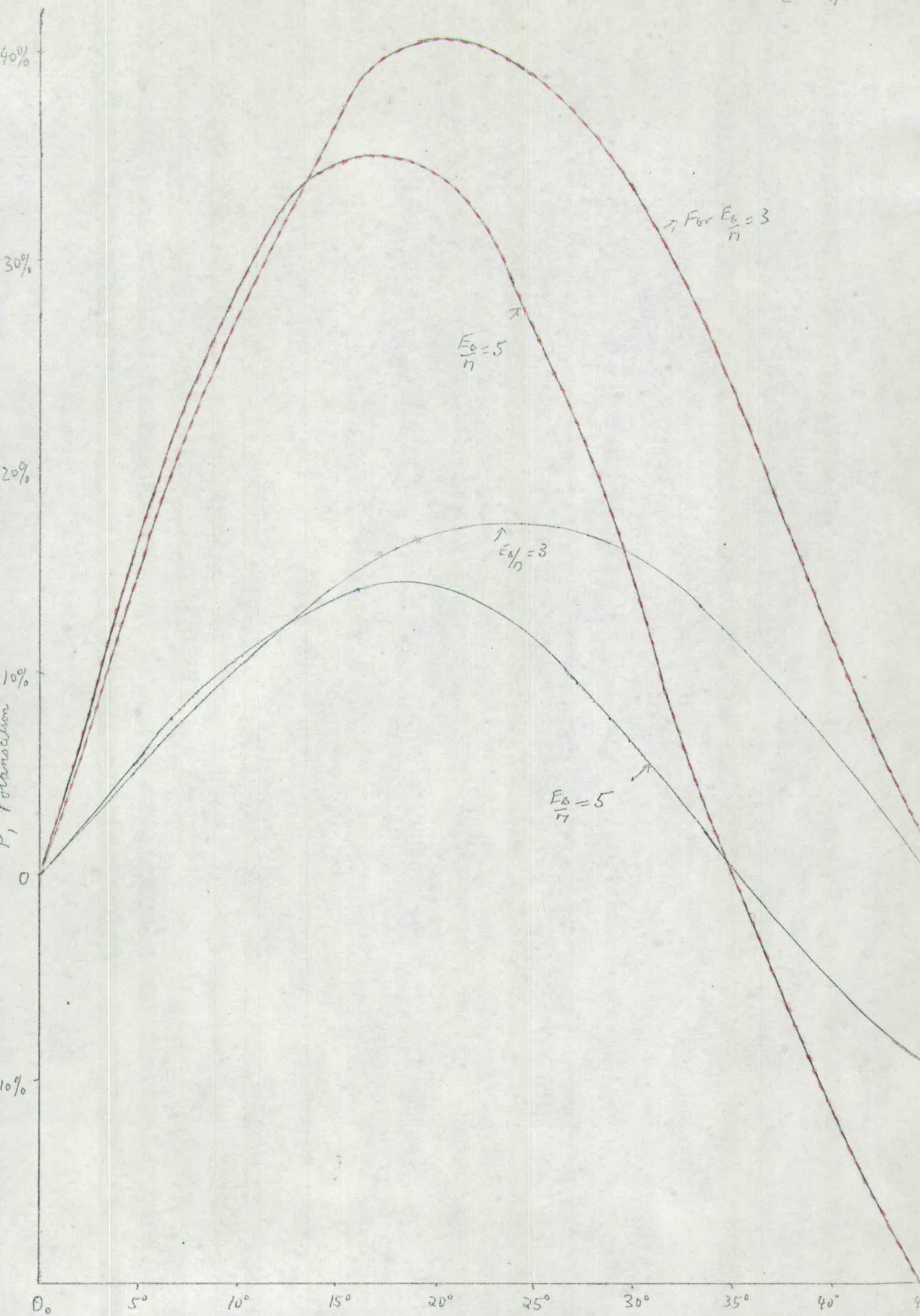


Fig 13. Plot of (3.43) — corresponds to $\bar{v}_E = \bar{v}_T / \mu$
 — " " " $\bar{v}_E = \bar{v}_T$



$$\text{or } P \frac{d6}{d\Omega} = a^2 (8M |\bar{P}|)^{-1} \tau \left[2\tau - 1 - \frac{|\bar{P}|}{M} \cos \theta_0 \right] \sin \theta_0 \left\{ \dots \right\} / \left[\tau - \frac{|\bar{P}|}{2M} \cos \theta_0 \right]^4 \quad (3.45)$$

As in case (v) three determinations of P at fixed q^2 are necessary here to extract the form factors;

$$G_1 = \tau^2 \bar{G}_M^2 + \tau (\bar{G}_E^2 - \bar{G}_M^2) - \bar{G}_H^2, \quad G_2 = \text{Re}(\bar{G}_M \bar{G}_E^*),$$

$$G_3 = \text{Re}(\bar{G}_M \bar{G}_E^*) - \bar{G}_M^2.$$

Note in configuration (5) with both nucleon and antinucleon polarised, the possibility of the form factors being complex does not introduce any additional azimuthal distribution to (3.41), in contrast to when one of the nucleons is polarised (see (3.24) and (3.40)).

Electron-muon universality

This may be directly tested by comparing the reactions $N + \bar{N} \rightarrow \mu(\text{muon}) + \bar{\mu}$ (antimuon) and $N + \bar{N} \rightarrow e$ (electron) + \bar{e} (positron). Analogously to the discussion at the end of the section on colliding beams, muon structure introduces an additional multiplicative factor $[\bar{H}_1(q^2)]^2$ in all the above formulae, i.e. (3.18) to (3.45), and therefore the ratio P (for $\bar{N}\bar{N} \rightarrow \mu\bar{\mu}$) / P (for $\bar{N}\bar{N} \rightarrow e\bar{e}$) = $[\bar{H}_1(q^2)]^2$ measures any deviation from universality. (As before, the muon mass is here neglected.)

Figs. 12 and 13 depict eqs. (3.26) and (3.42) for plausible values of the form-factors and indicate the appreciable values of polarisation possible.

With the advent of high energy colliding beams, such reactions appear certainly more favourable experimentally than nucleon-antinucleon annihilation. The latter has an enormous

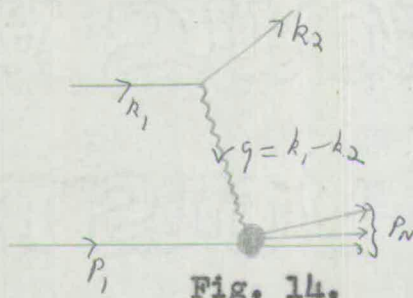
background of strong interaction particles which makes detection of the electromagnetic process $N + \bar{N} \rightarrow l + \bar{l}$ a formidable achievement. Nevertheless by extensive counter controls, it has proved possible to distinguish the $N + \bar{N} \rightarrow l + \bar{l}$ reaction amidst the background effects and, though no such event has yet been detected, the feasibility of measurement with this process is certainly a possibility (36).

CHAPTER 4

POLARISATION EFFECTS IN INELASTIC SCATTERING AND
COLLIDING BEAM REACTIONS

A. Inelastic Lepton-Nucleon Scattering

In Chapter 2 we discussed polarisation phenomena in elastic lepton-nucleon scattering. We now extend the analysis to inelastic scattering, i.e. the reaction $l + N \rightarrow l + \text{"Anything"}$, where "anything" denotes all possible hadronic states. Experimentally only the final lepton is detected, all final hadronic states being summed over.



The reaction is assumed to proceed through one photon exchange, as shown in Fig. 14. k_1 and k_2 denote the lepton's initial and final 4-momentum, p_1 the target nucleon 4-momentum and p_N the sum of final hadron 4-momenta.

As for elastic scattering, the lepton-photon vertex is described by $\langle k_2 | \bar{J}_\mu | k_1 \rangle = \bar{u}(k_2) \gamma_\mu u(k_1)$, therefore summing over both spins,

$$L_{\mu\lambda} = \sum \langle k_2 | \bar{J}_\mu | k_1 \rangle^2 = 2 \left[k_{2\mu} k_{1\lambda} + k_{2\lambda} k_{1\mu} - (k_1 k_2) \delta_{\lambda\mu} + m_l^2 \delta_{\mu\lambda} \right] \quad (4.1)$$

m_l denotes the lepton mass.

The hadron vertex $\sum \langle P_N | J_\mu | P_1 \rangle^2$ is a priori unknown; however it can only depend on the variables P_1, P_N and q (four-momentum transfer) as a second rank tensor, and Lorentz and gauge invariance restrict its form to be ⁽³⁷⁾

$$\begin{aligned}
 H_{\mu\lambda} &= \sum_{\text{init. \& final states}} \sum (2\pi)^4 \delta^4(P_1 + q - P_N) \langle P_N | J_\mu | P_1 \rangle^2 \\
 &= -W_1(q^2, \nu) \left[\delta_{\mu\lambda} - \frac{q_\mu q_\lambda}{q^2} \right] + \frac{W_2(q^2, \nu)}{M^2} \left[P_{1\mu} - \frac{(P_1 \cdot q) q_\mu}{q^2} \right] \left[P_{1\lambda} - \frac{(P_1 \cdot q) q_\lambda}{q^2} \right]
 \end{aligned}
 \tag{4.2}$$

Contracting gives differential cross-section:

$$\frac{d^2\sigma}{d\Omega' dE'_l} = \frac{4\alpha^2 (E'_l)^2}{q^4} \cos^2 \frac{\theta}{2} \left[W_2 + 2W_1 \tan^2 \frac{\theta}{2} \right]
 \tag{4.3}$$

where E_l and E'_l denote the lepton's initial and final energy, $d\Omega' = 2\pi d(\cos \theta)$, $q^2 = 4E_l E'_l \sin^2 \frac{\theta}{2}$, θ is the angle through which the lepton is scattered (all these referring to the lab. frame, i.e. nucleon at rest) and $\nu = p_1 \cdot q / M = q_0 = E_l - E'_l$ (M is nucleon mass). $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ are the unknown form-factors, and are functions of q^2 and ν .

There is presently a vast experimental and theoretical effort under way to measure and interpret the behaviour of these form-factors ⁽³⁸⁾. From the form of (4.3) it is clear in principle that one measures the angular distribution of $d^2\sigma/d\Omega' dE'_l$ at fixed q^2 and ν , whence W_1 and W_2 are determined analogously as in the Rosenbluth plot (eq. (1.1)). In practice, as yet the data has been very largely confined to small θ which implies that one is measuring W_2 . Even this depends on the value of a certain ratio $R = 6_S/6_T$, where

$\sigma_S(q^2, \nu)$ and $\sigma_T(q^2, \nu)$ are the total cross-sections for the absorption of longitudinal and transverse virtual photons on nucleons⁽³⁸⁾, and to determine R requires large angle data. Unquestionably such data will be forthcoming in the near future, thus permitting complete separation of the W_1 and W_2 form factors. However polarisation measurements permit such a separation even at small angles as we now discuss.

1. Both leptons polarised

The hadron vertex is again given by eq. (4.2). For the lepton vertex we have

$$L_{\mu\lambda} = \text{Tr} (K_2 + m_\ell) \frac{(1 - \not{S}_0 \gamma_5)}{2} \gamma_\mu (K_1 + m_\ell) \frac{(1 - \not{S}_L \gamma_5)}{2} \gamma_\lambda,$$

where S_L and S_0 are the spin projection operators for the initial and final leptons respectively ($S_L \cdot k_1 = S_0 \cdot k_2 = 0$).

Evaluating gives $L_{\mu\lambda} =$

$$\begin{aligned} & (S_L \cdot S_0) \left[\delta_{\mu\lambda} (k_1 k_2 - m_\ell^2) - k_{2\mu} k_{1\lambda} - k_{2\lambda} k_{1\mu} \right] + \left[S_{0\lambda} S_{L\mu} + S_{0\mu} S_{L\lambda} \right] (m_c^2 - k_{1\lambda} k_{2\mu}) + (S_0 \cdot k_1) \\ & \cdot \left[S_{L\mu} k_{2\lambda} + S_{L\lambda} k_{2\mu} \right] + S_L \cdot k_2 \left[S_{0\lambda} k_{1\mu} + S_{0\mu} k_{1\lambda} \right] - (S_0 \cdot k_1) (S_L \cdot k_2) \delta_{\mu\lambda} \\ & + k_{2\mu} k_{1\lambda} + k_{2\lambda} k_{1\mu} - (k_1 k_2) \delta_{\mu\lambda} + m_\ell^2 \delta_{\mu\lambda} \end{aligned}$$

and contracting yields

$$\begin{aligned} m^2 = & W_1 (2k_1 k_2 - 4m_\ell^2) + (W_2/M^2) \left[2(P_1 k_1) P_1 k_2 - (k_1 k_2 - m_\ell^2) M^2 \right] \\ & + W_1 2m_\ell^2 (S_L \cdot S_0) + (W_2/M^2) \left[S_L \cdot S_0 \left\{ -2(P_1 k_1) (P_1 k_2) - M^2 \frac{q^2}{2} \right\} \right. \\ & + \frac{q^2}{2} 2S_0 \cdot P_1 (S_L \cdot P_1) (S_L \cdot P_1) + 2(S_0 \cdot k_1) (S_L \cdot P_1) P_1 k_2 + 2(S_0 \cdot P_1) \cdot \\ & \left. (S_L \cdot k_2) P_1 k_1 - (S_0 \cdot k_1) (S_L \cdot k_2) M^2 \right] = R + M_S \text{ (spin-dependent term)} \end{aligned} \tag{4.4}$$

(i) Both leptons longitudinally polarised, i.e.

$$S_L = \left(\frac{|\bar{P}_L|}{m_l}, \frac{E_l}{m_l} \hat{k}_1 \right), \quad S_O = \left(\frac{|\bar{P}'_L|}{m_l}, \frac{E'_l}{m_l} \hat{k}_2 \right) \text{ where } \bar{P}_L \text{ and } \bar{P}'_L$$

denote the lepton's initial and final 3-momentum. Then

$$S_L \cdot S_O = \left[\frac{|\bar{P}_L| |\bar{P}'_L| - E_l E'_l \cos \theta}{m_l^2} \right], \quad S_L \cdot P_1 = M |\bar{P}_L| / m_l, \quad S_O \cdot P_1 = M |\bar{P}'_L| / m_l,$$

$$S_O \cdot k_1 = \left[\frac{E_l |\bar{P}'_L| - |\bar{P}_L| E'_l \cos \theta}{m_l} \right] \text{ and}$$

$$S_L \cdot k_2 = \left[\frac{E'_l |\bar{P}_L| - |\bar{P}'_L| E_l \cos \theta}{m_l} \right].$$

$$\therefore (4.4) \text{ yields } M_S = 2W_1 \left[\frac{|\bar{P}_L| |\bar{P}'_L| - E_l E'_l \cos \theta}{m_l^2} \right] + W_2 \left[\frac{|\bar{P}_L| |\bar{P}'_L|}{m_l^2} + \frac{E_l E'_l \cos \theta}{m_l^2} + \frac{m_l^2 \cos \theta}{m_l^2} \right].$$

Ignoring lepton mass terms, this gives polarisation $P = \left[\frac{m^2 - m'^2}{(S_0 \rightarrow S_0)} \right] / \frac{m^2 + m'^2}{(S_0)}$

i.e. $P = M_S / R = 1!$ (Consequence of helicity conservation)

(ii) Initial lepton longitudinally polarised, recoil lepton transversely polarised, i.e. $S_O \cdot P_1 = 0$, $S_O \cdot k_1 = |\bar{P}'_L| \sin \theta$,

$$S_O \cdot S_L = E_l \sin \theta / m_l.$$

$$(4.4) \text{ gives } M_S = m_l E_l \sin \theta \left[2W_1 - \frac{W_2}{2} \left(\frac{E'_l}{E_l} \right) \left\{ \left(1 + \frac{E_l^2}{E'_l{}^2} \right) \cos \theta + 2 \right\} \right]$$

where we have used the approximation $|\bar{P}_L| = E_l - (m_l^2 / 2E_l)$ and similarly for \bar{P}'_L

$$\therefore P = \frac{m_l \sin \theta \left[2W_1 - \frac{W_2}{2} \left(\frac{E'_l}{E_l} \right) \left[\left(1 + \frac{E_l^2}{E'_l{}^2} \right) \cos \theta + 2 \right] \right]}{2E_l \left[W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2} \right]}$$

$$P = \frac{m_l \sin \theta \left[2W_1 - \frac{W_2}{2} \left(\frac{E'_l}{E_l} \right) \left[\left(1 + \frac{E_l^2}{E'_l{}^2} \right) \cos \theta + 2 \right] \right]}{2E_l \left[W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2} \right]}$$

P =

$$\text{or } P \left(\frac{d^2 6}{d\Omega' dE'_l} \right)_{\text{unpol.}} = - \alpha^2 (q^2)^{-1} (m_l / E_l) \cot \frac{\theta}{2} \left\{ \quad \right\} \quad (4.5)$$

Thus two measurements of P at different values of $(E'_l/E_l) \cdot \left\{ \left[1 + (E_l^2/E_l'^2) \right] \cos \theta + 2 \right\}$ but fixed q^2 and v determines the W_1 and W_2 form factors. Note $d^2 6/d\Omega' dE'_l$ in (4.5) refers to the unpolarised differential cross-section, eq. (4.3).

(iii) Initial lepton transversely polarised, recoil lepton longitudinally polarised. Now $S_L \cdot P_1 = 0$, $S_L \cdot k_2 = - \bar{P}' \sin \theta$ and $S_L \cdot S_0 = - E'_l \sin \theta / m_l$.

$$\therefore M_S = m_l E'_l \sin \theta \left[-2W_1 + \frac{W_2}{2} \left(\frac{E_l}{E'_l} \right) \left\{ 2 + \left(1 + \frac{E_l'^2}{E_l^2} \right) \cos \theta \right\} \right]$$

using the same approximation as in (ii), so

$$P = \frac{m \sin \theta \left\{ -2W_1 + \frac{W_2}{2} (E_l/E'_l) \left[2 + \left(1 + \frac{E_l'^2}{E_l^2} \right) \cos \theta \right] \right\}}{2 E_l \left[W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2} \right]}$$

$$\text{or } P \left(\frac{d^2 6}{d\Omega' dE'_l} \right) = \alpha^2 \left(\frac{m_l}{E_l} \right) \cot^2 \frac{\theta}{2} (4E_l^2 \sin^2 \frac{\theta}{2})^{-1} \left\{ \quad \right\} \quad (4.6)$$

Analogously as in (ii), two measurements of P at different values of $(E_l/E'_l) \left[2 + \left(1 + \frac{E_l'^2}{E_l^2} \right) \cos \theta \right]$ but fixed q^2 and v permits extraction of W_1 and W_2 . Because of the m_l factor in (4.5) and (4.6), in practice these will only apply to muons. However this factor is partly counterbalanced by the $\cot \frac{\theta}{2}$ term for small θ . Thus for $\theta = 6^\circ$ and 10° (as appropriate to experiments⁽³⁸⁾) $\cot \frac{\theta}{2} = 19$, and 11.4 respectively; for $\theta = 5^\circ$ $\cot \frac{\theta}{2} = 23$, and so for $E_l = 10$ (GeV/c), $(m_l/E_l) \cot \frac{\theta}{2} \approx \frac{1}{5} \left(\frac{1}{9} \right)$ for $\theta = 6^\circ$ (10°). Hence (4.5) and (4.6) are appropriate to small angles only. Because of the extra E_l^{-2} factor in (4.6),

which increases rapidly with E_l , (4.5) is the more favourable configuration.

(iv) Both leptons transversely polarised, i.e.

$$S_L \cdot P_1 = S_O \cdot P_1 = 0, \quad S_L \cdot k_2 = -|\bar{P}'_L| \sin \theta, \quad S_O \cdot k_1 = |\bar{P}_L| \sin \theta \text{ and} \\ S_L \cdot S_O = -\cos \theta.$$

(4.4) yields

$$M_S = W_1(-2m_l^2 \cos \theta) + W_2 [E_l E'_l \cos \theta + |\bar{P}_L| |\bar{P}'_L|] \\ \approx \quad \quad \quad + W_2 \frac{E_l E'_l}{q^2} (1 + \cos \theta) = W_1(-2m_l^2 \cos \theta) \\ - W_2 \frac{q^2}{2} \cot^2 \frac{\theta}{2}$$

The W_2 term completely dominates the first term especially for small θ (true for muons as well as electrons), so to an error of less than 1%,

$$\text{polarisation } P = \frac{\cos^2 \frac{\theta}{2} W_2}{W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2}}$$

$$\text{or } P \left(\frac{d^2 \sigma}{d\Omega' dE'_l} \right) = \frac{4a^2 (E'_l)^2}{q^4} \frac{1}{2} = \frac{a^2}{E_l^2 \sin^4 \frac{\theta}{2}} W_2 \quad (4.7)$$

So one measurement of P immediately gives W_2 , and again small angles are favoured.

(v) Both leptons polarised transversely to the plane formed by the vectors $|\bar{P}_L| |\bar{P}'_L|$, i.e. $S_L \cdot S_O = -1$ all other terms = 0

so we have $M_S = -2m_l^2 W_1 - W_2 [E_l E'_l + |\bar{P}_L| |\bar{P}'_L| \cos \theta]$. As in (iv) m_l^2 terms are negligible, thus $M_S \approx -E_l E'_l (1 + \cos \theta) \frac{1}{2}$ and polarisation $P = -P$ for (4.7). Similar comments apply here as there.

2. Target nucleon polarised, initial lepton polarised

The target nucleon being polarised introduces another variable S_A , its spin projection operator. As usual $S_A \cdot P_1 = 0$

and $S_A^2 = -1$. This means $H_{\mu\lambda}$ as defined in (4.2) but not summed over the nucleon spin now depends on the variables q , P , and S_A ; Lorentz invariance, parity conservation and gauge invariance dictate general form to be

$$\begin{aligned}
 H_{\mu\lambda} = & W_1(q^2, \nu) \left[\delta_{\mu\lambda} - (q_\mu q_\lambda)/q^2 \right] + \frac{W_2(q^2, \nu)}{M} P'_{1\mu} P'_{1\lambda} + \\
 & + \frac{W_3(q^2, \nu)}{M} i \epsilon_{\mu\lambda st} S_{As} q_t \\
 & + \frac{W_4(q^2, \nu)}{M^3} i \left[P'_{1\lambda} \epsilon_{\mu st\omega} - P'_{1\mu} \epsilon_{\lambda st\omega} \right] P_{1s} S_{At} q_\omega \\
 & + \frac{W_5(q^2, \nu)}{M^3} \left[P'_{1\lambda} \epsilon_{ust\omega} + P'_{1\mu} \epsilon_{\lambda st\omega} \right] P_{1s} S_{At} q_\omega \quad (4.8)
 \end{aligned}$$

where $P'_{1\mu} = P_{1\mu} - \left(\frac{P_1 \cdot q}{q^2} \right) q_\mu$.

The form factors W_3 , W_4 and W_5 are each functions of q^2 and $\nu = (P_1 \cdot q)/M$ only, W_1 and W_2 are as defined in (4.2).

Summing over the final lepton spin,

$L_{\mu\lambda} = \sum \langle k_2 | J_\mu | k_1 \rangle^2 \equiv$ eq. (2.22), and contracting with (4.8) we find

$$\begin{aligned}
 \frac{m^2}{2} = \frac{H_{\mu\lambda} L_{\mu\lambda}}{2} = & R + 2(m_\ell/M) S_A \cdot S_L \left[-q^2 (W_3 + W_4) + W_4 \nu^2 \right] \\
 & - 2(m_\ell/M^2) S_A \cdot q (S_L \cdot P_1) \nu W_4 + 2(m_\ell/M) S_A \cdot q (S_L \cdot q) [W_3 + W_4] \\
 & - 2W_5 \left[(E_\ell + E'_\ell)/M \right] (\bar{S}_A \cdot \bar{k}_1 \times \bar{k}_2) \equiv R + M \quad (4.9)
 \end{aligned}$$

where \bar{k} denotes the three-momentum vector of k etc. and R as in (4.4).

We see that the W_5 form factor introduces an azimuthal distribution solely due to the nucleon polarisation, and which violates time-reversal invariance. Thus detection of a $(\vec{S}_A \cdot \vec{k}_1 \times \vec{k}_2)$ correlation (maximum when the target is polarised normal to the $\vec{k}_1 - \vec{k}_2$ plane) is a direct test of time-reversal invariance in electromagnetic interactions, a test not possible in elastic scattering⁽³⁹⁾.

We now apply (4.4) to specific configurations.

(i) Target nucleon polarised along direction of lepton beam, initial lepton longitudinally polarised.

$$\text{i.e. } S_A \cdot q = S_A \cdot k_1 - S_A \cdot k_2 = -E_\ell + E'_\ell \cos \theta; \quad S_L \cdot P_1 = (ME_\ell)/m_\ell;$$

$$q = -S_L \cdot k_2 = -E_\ell E'_\ell (1 - \cos \theta)/m_\ell = q^2/2m_\ell, \quad S_A \cdot S_L = -E_\ell/m_\ell$$

whence (4.9) gives

$$M_S = (q^2/M) \left[-vW_4 + (E_\ell + E'_\ell \cos \theta)(W_3 + W_4) \right], \text{ therefore the}$$

$$\text{polarisation } P = \frac{[m^2 - m_{(S_L \rightarrow -S_L)}^2]}{m^2 + m_{(S_L \rightarrow -S_L)}^2} = M_S/R =$$

$$\frac{-2 \sin^2 \frac{\theta}{2} \left\{ -vW_4 + (E_\ell + E'_\ell \cos \theta) [W_3 + W_4] \right\}}{M \left[W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2} \right]} \quad (4.0)$$

$$\text{or } P \left(\frac{d^2\sigma}{d\Omega' dE'_\ell} \right) = \frac{-a^2 \{ \dots \}}{2ME_\ell^2 \sin^2 \frac{\theta}{2}} \quad (4.10)$$

Thus two measurements of P at different values of $(E_\ell + E'_\ell \cos \theta)$ but fixed q^2 and v permits one to extract the form factors

$$G_1 = W_4 \quad \text{and} \quad G_2 = W_3 + W_4.$$

(ii) Nucleon polarised as in (i), lepton transversely polarised,

$$\text{i.e. } S_L \cdot P_1 = 0 = S_A \cdot S_L, \quad S_L \cdot k_2 = -E'_\ell \sin \theta$$

$$\therefore M_S = 2(m_\ell/M) E'_\ell \sin \theta (-E_\ell + E'_\ell \cos \theta)(W_3 + W_4) \quad \text{and}$$

$$P = \frac{(m_\ell/M) \sin \theta \{(-E_\ell + E'_\ell \cos \theta)(W_3 + W_4)\}}{E_\ell [W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2}]} \quad \text{or}$$

$$P \left(\frac{d^2\sigma}{d\Omega' dE'_\ell} \right) = \frac{a^2}{2E_\ell^2} \cot \frac{\theta}{2} (1 + \cot^2 \frac{\theta}{2}) \frac{m_\ell}{M} (1 + \cos \theta \frac{E'_\ell}{E_\ell})(W_3 + W_4) \quad (4.11)$$

This immediately determines $(W_3 + W_4)$; the factor (m_ℓ/M) is offset by the $\cot \frac{\theta}{2}$ terms for small θ .

(iii) Target nucleon polarised transversely to incoming lepton momentum, lepton longitudinally polarised. Here $S_L \cdot S_A = 0$ and $S_A \cdot q = -S_A \cdot k_2 = E'_\ell \sin \theta$, giving

$$M_S = E'_\ell \sin \theta [-2E_\ell \nu W_4 + q^2(W_3 + W_4)]/M$$

$$\therefore P = \frac{\sin \theta \{q^2(W_3 + W_4) - E_\ell \nu W_4\}}{2ME_\ell [W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2}]} \quad \text{or}$$

$$P \frac{d^2\sigma}{d\Omega' dE'_\ell} = \frac{a^2}{4E_\ell^2} \cot \frac{\theta}{2} (1 + \cot^2 \frac{\theta}{2}) \frac{\{ \quad \}}{ME_\ell} \quad (4.12)$$

Two measurements of P at different E_ℓ suffice to give the form factors $G_1 = W_3 + W_4$ and $G_2 = W_4$.

(iv) Both target nucleon and lepton transversely polarised, i.e. $S_L \cdot P_1 = 0$, $S_L \cdot k_2 = -E_\ell \sin \theta$, $S_A \cdot S_L = -1$.

$$\text{Hence } M_S = -2(m_\ell/M) [-q^2(W_3 + W_4) + \nu^2 W_4 + (E'_\ell \sin \theta)^2 (W_3 + W_4)]$$

$$\therefore P = \frac{-(m_\ell/M)}{E_\ell E'_\ell} \frac{\{-q^2(W_3 + W_4) + \nu^2 W_4 + (E'_\ell \sin \theta)^2 (W_3 + W_4)\}}{W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2}}$$

$$P\left(\frac{d^2\sigma}{d\Omega'dE'_l}\right) = \alpha^2\left(\frac{m_l}{M}\right) \frac{1}{E_l^2 \sin^2 \frac{\theta}{2}} \left\{ \frac{\dots}{q^2} \right\} \quad (4.13)$$

Determination of P at two values of $E'_l \sin \theta$ for fixed q^2 and ν enables one to extract the form factors

$$G = -q^2(W_3 + W_4) + \nu^2 W_4, \quad G_2 = W_3 + W_4. \quad \text{Similarly to (ii), the } (m_l/M) \text{ factor is offset by } P \sin^2 \frac{\theta}{2}^{-1} \text{ term for small } \theta.$$

(v) Both nucleon and lepton polarised normal to the $\bar{k}_1 - \bar{k}_2$ plane. i.e. $S_A \cdot S_L = 1$, all else zero. Therefore

$$M_S = -2(m_l/M) \left[-q^2(W_3 + W_4) + W_4 \nu^2 \right] \quad \text{and}$$

$$P = \text{eq. (4.13) without the } (E'_l \sin \theta)^2.$$

(vi) Nucleon polarised along direction of recoil lepton momentum, initial lepton longitudinally polarised, i.e.

$$S_A \cdot q = S_A \cdot k_1 - S_A \cdot k_2 = -E_l \cos \theta + E'_l, \quad S_A \cdot S_L = -E_l \cos \theta / m_l$$

$$\text{giving } M_S = \frac{q^2}{M} \left[\nu W_4 + (W_3 + W_4)(E_l \cos \theta + E'_l) \right]$$

$$\therefore \text{Polarisation } P = \frac{-2 \sin^2 \frac{\theta}{2}}{M} \left\{ \frac{+\nu W_4 + (E_l \cos \theta + E'_l)(W_3 + W_4)}{W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2}} \right\}$$

$$\text{of } P(d^2\sigma/d\Omega'dE'_l) = -\alpha^2 \left\{ \dots \right\} / 2ME_l^2 \sin^2 \frac{\theta}{2} \quad (4.14)$$

Two measurements of P for varying $(E_l \cos \theta + E'_l)$ determines the form factors W_4 and $(W_3 + W_4)$.

(vii) Nucleon polarised as in (vi) but initial lepton transversely polarised. Here $S_L \cdot P_1 = 0$, $S_L \cdot q = -S_L \cdot k_2 = E'_l \sin \theta$, $S_A \cdot S_L = -\sin \theta$ whence (4.9) yields

$$M_S = 2(m_l/M) \sin \theta \left[q^2(W_3 + W_4) - \nu^2 W_4 + E'_l(E'_l - E_l \cos \theta)(W_3 + W_4) \right]$$

$$\therefore P = \frac{(m_\ell/M)}{E_\ell E'_\ell} \sin \theta \frac{\{ +q^2(W_3+W_4) - \gamma^2 W_4 + E'_\ell(E'_\ell - E_\ell \cos \theta)(W_3+W_4) \}}{(W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2})}$$

$$\text{or } P \frac{d^2 G}{d\Omega' dE'_\ell} = -2a^2 \left(\frac{m_\ell}{M} \right) \frac{\cot \frac{\theta}{2}}{E_\ell^2} \frac{\{ \}}{q^2} \quad (4.15)$$

This permits the extraction of the form factors

$G_1 = q^2(W_3+W_4) - \gamma^2 W_4$; $G_2 = W_3 + W_4$. For small θ , the $\cot \frac{\theta}{2}$ term compensates for the (m_ℓ/M) factor.

(viii) Nucleon polarised transverse to recoil lepton momentum, initial lepton longitudinally polarised, i.e. $S_A \cdot q = S_A \cdot k_1 = E_\ell \sin \theta$, $S_A \cdot S_L = E_\ell \sin \theta / m_\ell$ so that $M_S = -(E_\ell/M) \sin \theta [q^2(W_3+W_4) + 2E'_\ell \gamma W_4]$

$$\text{Hence } P = \frac{-\sin \theta}{2ME'_\ell} \frac{\{ q^2(W_3 + W_4) + 2E'_\ell \gamma W_4 \}}{(W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2})}$$

$$\text{or } P \frac{d^2 G}{d\Omega' dE'_\ell} = \frac{-a^2}{ME_\ell} \cdot \cot \frac{\theta}{2} \frac{\{ \}}{q^2} \quad (4.16)$$

which measures the same form factors as (vi).

(ix) Nucleon polarised as in (viii), lepton transversely polarised.

Now $S_A \cdot S_L = -\cos \theta$

so $M_S = (m_\ell/M) [-q^2(W_3+W_4) + \cos \theta \{ q^2(W_3+W_4) - 2W_4 \gamma^2 \}]$

$$\therefore P = \frac{(m_\ell/M)}{2E_\ell E'_\ell} \frac{\{ -q^2(W_3+W_4) + \cos \theta [q^2(W_3+W_4) - 2W_4 \gamma^2] \}}{W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2}} \quad (4.17)$$

3. Target nucleon polarised, final lepton polarised

m^2 is now given also by (4.9) where S_L refers to the final lepton

(i) Target nucleon polarised along lepton beam momentum, final lepton longitudinally polarised, i.e. $S_L \cdot P_1 = ME'_l/m_l$, $S_L \cdot q = S_L \cdot k_1$

$= E_l E'_l (1 - \cos \theta)/m_l = -q^2/2m_l$, $S_A \cdot S_L = -E'_l \cos \theta/m_l$. Then (4.9) gives

$$M_S = (q^2/M) [-vW_4 + (E_l + E'_l \cos \theta)(W_3 + W_4)] \therefore P = eq. (4.10).$$

(ii) Target polarised as in (i), lepton transversely polarised, i.e.

$$S_L \cdot P_1 = 0, \quad S_L \cdot k_1 = E_l \sin \theta, \quad S_A \cdot S_L = \sin \theta.$$

$$\therefore M_S = 2(m_l/M) \sin \theta [-q^2(W_3 + W_4) + v^2 W_4 + E_l (-E_l + E'_l \cos \theta)(W_3 + W_4)]$$

$$\text{and } P = (m_l/M) \frac{\sin \theta}{E_l E'_l} \frac{\{-q^2(W_3 + W_4) + v^2 W_4 + E_l (-E_l + E'_l \cos \theta)(W_3 + W_4)\}}{[W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2}]}$$

$$\text{or } \frac{Pd^2 6}{d\Omega dE'_l} = -2\alpha^2 \left(\frac{m_l}{M}\right) \frac{\cot^2 \frac{\theta}{2}}{E_l^2} \frac{\{ \}}{q^2} \quad (4.18)$$

From here one obtains the form factors $G_1 = -q^2(W_3 + W_4) + v^2 W_4$, $G_2 = W_3 + W_4$.

(iii) Target nucleon polarised transversely to incoming lepton

momentum, final lepton longitudinally polarised, i.e. $S_A \cdot q = -S_A \cdot k_2$

$$= E'_l \sin \theta, \quad S_A \cdot S_L = -E'_l \sin \theta/m_l,$$

$$\text{so } M_S = -E'_l \sin \theta [-q^2(W_3 + W_4) + 2E_l v W_4]/M, \quad \therefore P = eq. (4.12).$$

(iv) Nucleon and final lepton transversely polarised, i.e.

$$S_A \cdot S_L = -1, \quad S_L \cdot k_1 = E_l \sin \theta.$$

$$\text{Here } M_S = -2(m_l/M) [-q^2(W_3 + W_4) + v^2 W_4 + E_l E'_l \sin^2 \theta (W_3 + W_4)]$$

$$\therefore P = \frac{-(m_l/M)}{E_l E'_l} \frac{\{-q^2(W_3 + W_4) + v^2 W_4 - q^2 \cos^2 \frac{\theta}{2} (W_3 + W_4)\}}{[W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2}]}$$

$$\text{or } P \frac{d^2 6}{d\Omega dE'_l} = \alpha^2 \left(\frac{m_l}{M}\right) \frac{1}{E_l^2 \sin^2 \frac{\theta}{2}} \frac{\{ \}}{q^2} \quad (4.19)$$

This determines the same form factors as in (ii).

(v) Both nucleon and lepton polarised normal to the $\bar{k}_1 - \bar{k}_2$ plane. Exactly as (v) with initial lepton polarised.

(vi) Target nucleon polarised along direction of final lepton, latter being longitudinally polarised, i.e. $S_A \cdot S_L = -E'_l/m_l$ so (4.9) yields $M_S = q^2 \left[\nu W_4 + (W_3 + W_4)(E_l \cos \theta + E'_l) \right] / M$.

∴ Polarisation $P = \text{eq. (4.14)}$.

(vii) Nucleon polarised as in (vi), final lepton transversely polarised. Here $S_A \cdot S_L = 0$, $S_L \cdot P_1 = 0$.

$$\therefore M_S = 2(m_l/M)(E'_l - E_l \cos \theta)E_l \sin \theta (W_3 + W_4)$$

$$\text{so } P = \frac{(m_l/M)}{E'_l} \sin \theta \frac{\{(E'_l - E_l \cos \theta)(W_3 + W_4)\}}{(W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2})}$$

$$\text{or } P \frac{d^2 \sigma}{d\Omega' dE'_l} = \frac{-2a^2}{q^2} \left(\frac{m_l}{M}\right) \cot \frac{\theta}{2} \frac{\{ \}}{E_l} \quad (4.20)$$

One measurement of P fixes the value of $(W_3 + W_4)$,

(viii) Nucleon polarised transverse to final lepton momentum, lepton longitudinally polarised, i.e. $S_A \cdot S_L = 0$, $S_A \cdot q = S_A \cdot k_1 = E_l \sin \theta$ yielding $M_S = -(E_l/M) \sin \theta \left[q^2(W_3 + W_4) + 2E'_l \nu W_4 \right]$.

∴ $P = \text{eq. (4.16)}$.

(ix) Nucleon polarised as in (viii), final lepton transversely polarised. Now $S_A \cdot S_L = 1$. Therefore $M_S = 2(m_l/M) \left[q^2(W_3 + W_4) - W_4 \nu^2 + E_l^2 \sin^2 \theta (W_3 + W_4) \right]$

$$\text{and so } P = \frac{(m_l/M)}{2E_l E'_l} \frac{\{ q^2(W_3 + W_4) - W_4 \nu^2 + E_l^2 \sin^2 \theta (W_3 + W_4) \}}{W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \theta}$$

$$\text{or } P \frac{d^2\sigma}{d\Omega' dE'_\ell} = a^2 \left(\frac{m_\ell}{M} \right) \frac{1}{E_\ell^2 \sin^2 \frac{\theta}{2}} \left\{ \frac{1}{q^2} \right\} \quad (4.21)$$

This determines the same form factors as (4.15).

Bjorken⁽⁴¹⁾ has given a sum rule involving the initial lepton and nucleon spins parallel (and antiparallel) and along the direction of the incoming lepton, i.e. case (i). Even when polarisation measurements are done, this may be difficult to measure; however we see that once the functions W_3 and W_4 are known, (4.10) describes this configuration, and thus polarisation measurements with any of the other configurations treated enables one to determine Bjorken's case. So far no measurements with a polarised target have been made but the availability and use of such a target very recently to test time-reversal invariance⁽³⁹⁾ clearly indicates data will not be long in forthcoming.

B. Inelastic Colliding Beams

We now examine the process $\ell + \bar{\ell} \rightarrow N + \text{"Anything"}$, which is an extension of the elastic case treated in Chapter 3. This reaction proceeds as shown in Fig. 15.

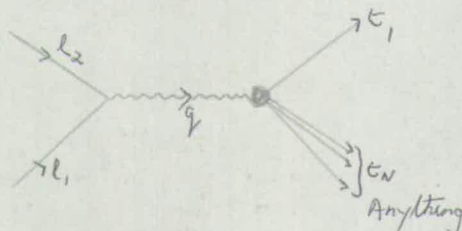


Fig. 15

$\ell_1(\ell_2)$ is the lepton (antilepton) four momentum.

t is the emerging nucleon four-momentum.

$q = \text{four-momentum transfer} = \ell_1 + \ell_2.$

Summing over lepton spins, $L_{\mu\lambda} = \langle 0 | j_\mu | \ell_1 \ell_2 \rangle^2 = e^2 (2.4)$

Analogously to (4.2) for the hadron vertex we have

$$H_{\mu\lambda} = \sum \sum (2\pi)^4 \delta^4(q-t_1-t_2) \langle t_1 t_N | J_\mu | 0 \rangle^2 = \bar{W}_1(q^2, \nu) \left[\delta_{\mu\lambda} - \frac{q_\mu q_\lambda}{q^2} \right] + \frac{\bar{W}_2}{M^2}(q^2, \nu) \left[t_{1\mu} - \frac{(P_1 q) q_\mu}{q^2} \right] \left[t_{1\lambda} - \frac{(P_1 q) q_\lambda}{q^2} \right] \quad (4.22)$$

giving in the colliding beam frame a cross-section,

$$\frac{d^2\sigma}{dE_1 d(\cos\theta)} = \frac{4\pi\alpha^2}{q^4} \frac{M^2}{\sqrt{q^2}} \left(1 - \frac{q^2}{2}\right)^{\frac{1}{2}} \left[2\bar{W}_1(q^2, \nu) + \frac{2M}{q^2} \left(1 - \frac{q^2}{2}\right) \frac{\bar{W}_2(q^2, \nu) \sin^2 \theta}{2M} \right] \quad (4.23)$$

The form factors \bar{W}_1 and \bar{W}_2 are functions of q^2 and $\nu = t_1 \cdot q/M$, and are related to the inelastic scattering ones by crossing which gives $H_{\mu\lambda}(q, t_1) = -H_{\mu\lambda}(q, -P_1)$ i.e. $\bar{W}_1(q^2, \nu) = -\bar{W}_1(q^2, -\nu)$; $\nu \bar{W}_2(q^2, \nu) = -\nu \bar{W}_2(q^2, -\nu)$. E_1 is the energy of the detected nucleon and θ is the angle which the proton momentum \vec{P} makes with the axis defined by colliding lepton beams (see Fig. 16).

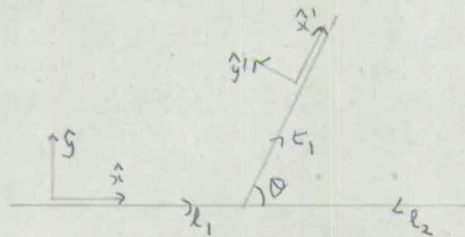


Fig. 16.

1. Both leptons polarised

Eq. (4.17) refers to unpolarised particles, when both leptons are polarised, the appropriate substitutions $P_1 \rightarrow -t_1$, $k_1 \rightarrow \ell_1$, $k_2 \rightarrow -\ell_2$ in (4.4) yields

$$m^2 = -\bar{W}_1(q^2, \nu) [-2\ell_1 \ell_2 - 4m_\ell^2] + \frac{\bar{W}_2}{M^2}(q^2, \nu) \left[-2(t_1 \ell_1)(t_1, \ell_2) + (t_1 \ell_2 + m_\ell^2) \eta^2 \right] + \bar{W}_1(q^2, \nu) 2m_\ell^2 (s_L \cdot s_0) + [(s_0 \cdot \ell_1)(s_L \cdot \ell_2) M^2 + q^2 (s_0 \cdot t_1)(s_L \cdot t_1) - 2(s_0 \cdot t_1)(s_L \cdot \ell_2) t_1 \ell_1 - 2(s_0 \cdot \ell_1)(s_L \cdot t_1) t_1 \ell_2 + s_L s_0 \{ 2t_1 \ell_1 (t_1 \ell_2) - \eta^2 q^2 \}] \frac{\sqrt{2}}{M} \sqrt{q^2 \nu}$$

$$\equiv Q + M_S \quad (\text{spin-dependent term}) \quad (4.24)$$

$S_L(S_O)$ refers to the lepton (antilepton).

(i) Both leptons longitudinally polarised, i.e.

$$S_L = \left(\frac{\bar{k}}{m_\ell}, \frac{E}{m_\ell} \hat{x} \right), \quad S_O = \left(\frac{\bar{k}}{m_\ell}, \frac{-E}{m_\ell} \hat{x} \right),$$

where E is the incoming energy of either lepton and \bar{k} the corresponding 3-momentum (thus $E^2 = (\bar{k})^2 + m_\ell^2$). This gives

$$S_L \cdot t_1 = (E_1 |\bar{k}| - |\bar{P}| E \cos \theta) / m_\ell, \quad S_L \cdot t_2 = 2E |\bar{k}| / m_\ell,$$

$$S_O \cdot t_1 = (E_1 |\bar{k}| + |\bar{P}| E \cos \theta) / m_\ell, \quad S_O \cdot t_1 = 2E |\bar{k}| / m_\ell$$

$$S_L \cdot S_O = (\bar{k}^2 + E^2) / m_\ell^2, \quad \text{whence (4.18) yields}$$

$$M_S = \bar{W}_1 (4E^2 - 2m_\ell^2) + \frac{\bar{W}_2}{M^2} \left\{ 2E^2 (M^2 + E_1^2) - 2|\bar{P}|^2 \cos^2 \theta (E^2 + m_\ell^2) \right\}$$

\therefore Polarisation $P = M_S / Q$

$$= \frac{\left\{ \left(1 + \frac{v^2}{q^2}\right) \bar{W}_2 + 2\bar{W}_1 - \left(\frac{v^2}{q^2} - 1\right) \cos^2 \theta \bar{W}_2 \right\}}{2\bar{W}_1 + \frac{2Mv}{q^2} \left(1 - \frac{q^2}{v^2}\right) \sin^2 \theta \frac{v\hbar\lambda}{2\gamma}}$$

$$\text{or } P \frac{d^2\sigma}{dE_1 d(\cos\theta)} = \alpha^2 \pi \frac{4M^2 v^2}{q^4} \frac{1}{(q^2)^{1/2}} \left(1 - \frac{q^2}{v^2}\right)^{1/2} \left\{ \right\} \quad (4.25)$$

where we have employed the identities $q^2 = 4E^2$, $E_1 = Mv/\sqrt{q^2}$, $\bar{P}^2 = M^2 \left(\frac{v^2}{q^2} - 1\right)$ and neglected lepton mass terms in M_S . Thus two measurements of P at different θ but fixed q^2 and v determine the form factors $G_1 = \left(1 + \frac{v^2}{q^2}\right) \bar{W}_2 + 2\bar{W}_1$; $G_2 = \bar{W}_2$

(ii) Lepton longitudinally polarised, antilepton transversely polarised, i.e. $S_O = (0, -\hat{y})$. Hence $S_O \cdot t_1 = \bar{P} \sin \theta$,

$S_0 \cdot l_1 = 0, S_L \cdot S_0 = 0$, whence (4.18) yields

$$M_S = -4m_\ell E |\bar{P}|^2 \sin \theta \cos \theta \bar{W}_2 / M^2$$

$$\therefore P = \frac{-2m_\ell \sin 2\theta \left(\frac{v^2}{q^2} - 1\right) \bar{W}_2}{\sqrt{q^2} \left[2\bar{W}_1 + \frac{2Mv}{q^2} \left(1 - \frac{q^2}{v^2} - \frac{v\bar{W}_2}{2M} \sin^2 \theta \right) \right]}$$

$$\text{or } P \frac{d^2 \delta}{dE_1 d(\cos \theta)} = \frac{-8\pi \alpha^2 m_\ell M^2 \sin 2\theta}{\sqrt{q^2} q^4} \left(\frac{v^2}{q^2} - 1\right)^{3/2} \bar{W}_2 \quad (4.26)$$

which directly determines \bar{W}_2 . (4.26) is reduced by a factor of order $m_\ell (v^2/q^2 - 1)^{3/2}/v$ compared with (4.25) but for large v the $()^{3/2}$ term offsets the (m_ℓ/v) term.

(iii) Lepton transversely polarised, antilepton longitudinally polarised, i.e. $S_L = (0, \hat{y})$. Hence $S_L \cdot l_2 = 0 = S_L \cdot S_0$, $S_L \cdot t_1 = -|\bar{P}| \sin \theta$, giving

$$M_S = -4m_\ell E |\bar{P}|^2 \sin \theta \cos \theta \bar{W}_2 / M^2$$

$$\therefore P = \text{eq. (4.20).}$$

(iv) Both leptons transversely polarised, i.e. $S_0 \cdot S_L = 1$,

$S_0 \cdot l_1 = S_L \cdot l_2 = 0$. (4.18) then gives

$$M_S = 2m_\ell^2 \bar{W}_1 + \left\{ 2|\bar{P}|^2 \bar{W}_2 \left[-E^2(1 - \cos^2 \theta) + m_\ell^2 \cos^2 \theta \right] \right\} / M^2.$$

The \bar{W}_2 term completely dominates the \bar{W}_1 term so neglecting m_ℓ^2 gives

$$P = \frac{-2 \sin^2 \frac{\theta}{2} \left(\frac{v^2}{q^2} - 1\right) \bar{W}_2}{\sqrt{q^2} \left[2\bar{W}_1 + \frac{2Mv}{q^2} \left(1 - \frac{q^2}{v^2}\right) - \frac{v\bar{W}_2}{2M} \sin^2 \theta \right]} \quad \text{or}$$

$$P \frac{d^2 \delta}{dE_1 d(\cos \theta)} = -8\pi \alpha^2 \frac{M^2}{q^4} \sin^2 \frac{\theta}{2} \left(\frac{v^2}{q^2} - 1\right)^{3/2} \bar{W}_2 \quad (4.27)$$

(v) Both leptons polarised transverse to the plane formed by $\bar{\mathbf{k}}$ and $\bar{\mathbf{P}}$. Now $S_L \cdot S_O = -1$, all else zero.

$$\therefore M_S = 2m_\ell^2 \bar{W}_1 - \left\{ 2 |\bar{\mathbf{P}}|^2 \bar{W}_2 \left[E^2 (1 - \cos^2 \Theta) + m_\ell^2 \cos^2 \Theta \right] \right\} / M^2$$

and neglecting m_ℓ^2 terms gives $P = \text{eq. (4.27)}$.

2. Polarised lepton beam, polarisation of emitted nucleon measured.

For the hadron-photon vertex, proceeding exactly as in the elastic scattering case (compare (4.8)) we have

$$\begin{aligned} H_{\mu\lambda} = & -\bar{W}_1(q^2, \nu) \left[\delta_{\mu\lambda} - (q_\mu q_\lambda / q^2) \right] + \frac{\bar{W}_2(q^2, \nu)}{M^2} t_{i\mu}^i t_{i\lambda}^i \\ & + \frac{\bar{W}_3(q^2, \nu)}{M} i \epsilon_{\mu\lambda st} S_{As} q_t + \frac{\bar{W}_4(q^2, \nu)}{M^3} \left[t_{i\lambda}^i \epsilon_{\mu st \omega} - t_{i\mu}^i \epsilon_{\lambda st \omega} \right] \cdot \\ & \cdot t_{ls} S_{At} q_\omega + \frac{\bar{W}_5(q^2, \nu)}{M^3} \left[t_{i\lambda}^i \epsilon_{\mu st \omega} + t_{i\mu}^i \epsilon_{\lambda st \omega} \right] t_{ls} S_{At} q_\omega \end{aligned} \quad (4.28)$$

(S_A now refers to emitted nucleon)

$$\begin{aligned} \therefore \frac{m^2}{2} = \frac{H_{\mu\lambda} L_{\mu\lambda}}{2} = & Q + 2(m_\ell/M) S_A \cdot S_L \left[-q^2 (\bar{W}_3 + \bar{W}_4) + \bar{W}_4 \nu^2 \right] \\ & - 2(m_\ell/M^2) S_A \cdot q (S_L \cdot t_1) \nu \bar{W}_4 + 2(m_\ell/M) S_A \cdot q (S_L \cdot q) \left[\bar{W}_3 + \bar{W}_4 \right] \\ & + 2\bar{W}_5 (\bar{S}_A \cdot \bar{\mathbf{k}} \times \bar{\mathbf{P}}) (P_1 \ell_1 - P_1 \ell_2) \left[(q^2)^{1/2} / M^3 \right] \end{aligned} \quad (4.29)$$

(Compare with (4.9)). The last term above

= $-2\bar{W}_5 (q^2/M^2) (\nu^2/q^2 - 1)^{1/2} \cos \Theta \left[\bar{S}_A \cdot \bar{\mathbf{k}} \times \bar{\mathbf{P}} \right]$ violates time-reversal invariance so detection of a $\left[\bar{S}_A \cdot \bar{\mathbf{k}} \times \bar{\mathbf{P}} \right]$ correlation tests this symmetry in electromagnetic interaction.

(i) Emitted nucleon longitudinally polarised as is the incident lepton, i.e. $S_L = (|\vec{k}|, E\hat{x})/m_\ell$, $S_A = (|\vec{P}|, E, \hat{x}')/M$ and hence, $S_L \cdot q = 2E|\vec{k}|/m_\ell$, $S_L \cdot t_1 = (|\vec{k}|E_1 - E|\vec{P}| \cos \theta)$, $S_A \cdot q = 2E|\vec{P}|/M$, $S_A \cdot S_L = (|\vec{k}|/|\vec{P}| - E E_1 \cos \theta)/Mm_\ell$. (4.22) then gives M_S (spin-dependent part) = $(q^2/M^2) M_Y \cos \theta \bar{W}_3$. Therefore

$$\text{Polarisation } P = \frac{2(v/M)\cos \theta \bar{W}_3}{2\bar{W}_1 + \frac{2M_Y}{q} \left(1 - \frac{q^2}{v^2}\right) \frac{v\bar{W}_2}{2M} \sin^2 \theta} \quad \left[\equiv (m^2 - m_{(S_A \rightarrow S_L)}^2) / m^2 + m_{(S_A \rightarrow S_L)}^2 \right]$$

$$P \frac{d^2\sigma}{dE_1 d(\cos \theta)} = \frac{4\pi\alpha^2}{q^2} \left(\frac{2M_Y}{q}\right)^{1/2} \cos \theta \bar{W}_3 \quad (4.30)$$

where extra lepton mass terms have been neglected (as in the following sections too). This then directly measures \bar{W}_3 .

(ii) Nucleon polarised as in (i), lepton transversely polarised.

i.e. $S_L \cdot q = S_L \cdot t_2 = 0$, $S_L \cdot t_1 = -|\vec{P}| \sin \theta$, $S_A \cdot S_L = -(E_1/M) \sin \theta$ giving $M_S = 2(m_\ell/M) \sin \theta \sqrt{q^2} \bar{W}_3$. Therefore

$$P = \frac{4(m_\ell/M) v \sin \theta \bar{W}_3}{\sqrt{q^2} \left[2\bar{W}_1 + \frac{2M_Y}{q} \left(1 - \frac{q^2}{v^2}\right) \frac{v\bar{W}_2}{2M} \sin^2 \theta \right]} \quad \text{or}$$

$$P \frac{d^2\sigma}{dE_1 d(\cos \theta)} = 4 \left(\frac{m_\ell}{M}\right) \frac{\pi\alpha^2}{q^2} \left(\frac{2M_Y}{q}\right)^2 \left(1 - \frac{q^2}{v^2}\right)^{1/2} \sin \theta \bar{W}_3 \quad (4.31)$$

This is a factor (m_ℓ/M) smaller than (4.30).

(iii) Emitted nucleon transversely polarised, incident lepton longitudinally polarised. i.e. $S_A = (0, \hat{y})$ $\therefore S_A \cdot q = 0$,

$S_A \cdot S_L = (E/m_\ell) \sin \theta$ giving $M_S = \left\{ \sqrt{q^2} \left[-q^2(\bar{W}_3 + \bar{W}_4) + v^2 \bar{W}_4 \right] / M \right\} \sin \theta$

$$\therefore P = \frac{2 \left[\sqrt{q^2}/M \right] \sin \theta \left\{ -\bar{W}_3 + \bar{W}_4 \left(\frac{v^2}{q^2} - 1 \right) \right\}}{2\bar{W}_1 + \frac{2M_Y}{q} \left(1 - \frac{q^2}{v^2}\right) \frac{v\bar{W}_2}{2M} \sin^2 \theta} \quad \text{or}$$

$$P \frac{d^2\sigma}{dE_1(\cos \theta)} = \frac{4\pi\alpha^2}{q^2} \left(\frac{2M\nu}{q}\right) \left(1 - \frac{q^2}{\nu^2}\right)^{1/2} \sin \theta \{ \} \quad (4.32)$$

One measurement of P here directly determines $-\bar{W}_3 + \bar{W}_4 \left(\frac{\nu^2}{q^2} - 1\right)$.

(iv) Both nucleon and lepton transversely polarised. Now

$$S_A \cdot S_L = -\cos \theta, \text{ so}$$

$$M_S = -2(m_\ell/M)q^2 \cos \theta \left[-\bar{W}_3 + \bar{W}_4 \left(\frac{\nu^2}{q^2} - 1\right)\right]. \text{ Hence,}$$

$$P = -4\left(\frac{m_\ell}{M}\right) \frac{\cos \theta \left\{-\bar{W}_3 + \bar{W}_4 \left(\frac{\nu^2}{q^2} - 1\right)\right\}}{2\bar{W}_1 + \frac{2M\nu}{q^2} \left(1 - \frac{q^2}{\nu^2}\right) \frac{\nu M^2}{2M} \sin^2 \theta} \quad \text{or}$$

$$P \frac{d^2\sigma}{dE_1 d(\cos \theta)} = \frac{-4\pi\alpha^2}{q^2} 4\left(\frac{m_\ell}{M}\right) \frac{M^2}{q^2} \left(\frac{\nu^2}{q^2} - 1\right)^{1/2} \cos \theta \{ \} \quad (4.33)$$

This measures the same form factor as (4.32) but is reduced by the (m_ℓ/M) factor.

(v) Both nucleon and lepton polarised normal to the $\bar{k} - \bar{P}$ plane. i.e. $S_A \cdot S_L = -1$. $\therefore P = \text{eq. (4.33)}/\cos \theta$.

3. For the antilepton beam polarised and the emitted nucleon polarisation measured, the above formula (4.30) - (4.33) are multiplied by a minus sign.

All the formulae in this chapter apply equally well to electrons or muons. Hence a comparison of the two respective cases serves to test for the presence of any muon structure.

CHAPTER 5

WEAK INTERACTIONS

Polarisation Effects in High Energy Neutrino Scattering

So far in this thesis, we have been concerned with electromagnetic interactions and polarisation phenomena therein. In this last chapter we turn to the field of weak interactions, where there is perhaps even a richer structure to explore, and examine the role played by polarisation effects here.

The generation of high energy neutrino beams in the last few years has made possible a study of high energy weak scattering, analogous to that for electromagnetic lepton-nucleon scattering⁽⁴³⁾. Such measurements permit the determination of certain form factors which describe the transition between the initial nucleon target and the final hadron target. A number of these form factors also arise in the semi-leptonic decays of certain baryons, and in principle, with sufficiently accurate data could be extracted from an analysis of these decays. However in such decays one is naturally constrained to a small region of momentum transfer determined by the mass difference between the initial and final particle states. It is the unique virtue of neutrino reactions that the range of momentum transfer may be extended indefinitely, constrained only by the limiting energy of the incoming beam, and thus one may study weak interactions at very high energies.

Presently there exists a theory of weak interactions which

describes purely leptonic and semi-leptonic processes very well at low energies^{(44)*}. (The situation in non-leptonic decays is still rather unclear). However it is expected on general grounds, from unitarity bounds, that this theory cannot hold at sufficiently high energies so there is great interest as to the actual behaviour of weak effects in this region. In particular one may explore for the existence of the so-called intermediate vector boson which has been postulated to mediate weak interactions⁽⁴⁵⁾ analogously to the photon being the mediator of electromagnetic interactions.

The enormous difficulties in handling neutrino beams, plus the relatively low rate for such reactions with present accelerators, has resulted in rather meagre experimental data so far⁽⁴³⁾. However the operation of the new 300 GeV/c accelerators, with a greatly increased neutrino flux, will be expected to radically improve the situation and provide sufficient data as to enable a detailed theoretical analysis to be made. This naturally stimulates interest in polarisation effects, which should also be experimentally feasible, and greatly enrich our knowledge of weak interactions.

Polarisation phenomena in two body neutrino reactions of the form $\nu(\bar{\nu}) + N \rightarrow e(\bar{e}) + Y$ have been widely discussed in the literature⁽⁴⁶⁾. Here $\nu(\bar{\nu})$ denotes the neutrino (antineutrino), N the nucleon target, $e(\bar{e})$ the emerging lepton (antilepton), and Y the final baryon state, i.e. N, Σ, Λ . Not so is the situation for inelastic neutrino reactions of the form $\nu(\bar{\nu}) + N \rightarrow e(\bar{e}) +$

"Anything", where "Anything" denotes a sum over all possible final

* This of course, refers only to the first order in perturbation theory. The calculation of higher orders raises profound problems and is an unsolved question.

hadronic states. These reactions are the analogue of the inelastic lepton-nucleon scattering process (see Chapter 4 A.) and certain theoretical models predict a close affinity in the behaviour of the two processes⁽⁴⁷⁾. Study of the inelastic reaction is thus of considerable interest in itself and in this chapter we investigate the influence of polarisation effects here. In particular the final lepton polarisation and the employment of a polarised nucleon target is discussed. In the latter case we simplify to the two body reaction, where interest has so far been largely concentrated on the final particle polarisations. The feasibility of employing polarised targets clearly indicates such a study is now opportune.

Inelastic Neutrino Reactions

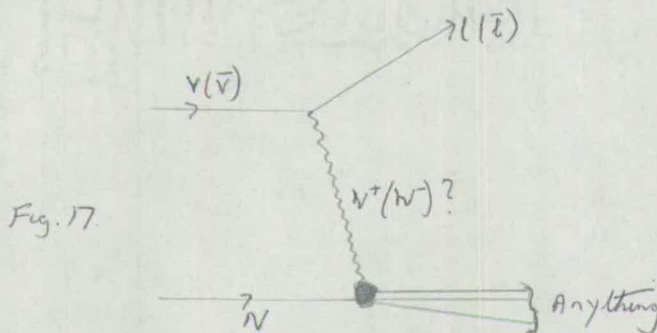


Fig. 17 depicts the process $\nu(\bar{\nu}) + N \longrightarrow e(\bar{e}) + \text{"Anything"}$. The analogy to inelastic lepton scattering (see Fig. 14) is clearest if the weak interaction is mediated by an intermediate vector boson $W^+(W^-)$ as shown. However this boson may well not exist, in which case the weak interaction may be a point interaction corresponding to an infinite mass for the W boson.

We use the following conventions:-

k_1 (k_2) denotes the neutrino (final lepton) 4-momentum.

P_1 " " nucleon 4-momentum,

P_N " total 4-momentum of final hadronic state.

Thus $P_1 + k_1 = P_2 + k_2$. The four-momentum transfer

$$q = k_1 - k_2 = P_N - P_1.$$

For an unpolarised lepton, the lepton-neutrino weak vertex is given by

$$\begin{aligned} L_{\mu\lambda} &= \sum \langle k_2 | j_\mu | k_1 \rangle^2 = \text{Tr}(\not{k}_2 + m_e) \gamma_\mu (1 + \gamma_5) (\not{k}_1) \gamma_\lambda (1 + \gamma_5) \\ &= 8 \left[k_{2\mu} k_{1\lambda} + k_{2\lambda} k_{1\mu} - (k_1 k_2) \delta_{\mu\lambda} + i \epsilon_{\mu\lambda\alpha\beta} k_{2\alpha} k_{1\beta} \right] \end{aligned} \quad (51)$$

where m_e is the final lepton mass and $+(-)$ refers to neutrino (antineutrino) beam respectively. j_μ is the weak leptonic current operator. The hadron vertex $H_{\mu\lambda} = \sum_N \langle P_N | J_\mu | P_1 \rangle^2$, where J_μ is the weak hadronic current operator, is a second-rank tensor depending on P , q and P_N only and Lorentz invariance dictates its form to be:

$$\begin{aligned} H_{\mu\lambda} &= -W_1^v(q^2, \nu) \delta_{\mu\lambda} + \frac{W_2^v(q^2, \nu)}{M^2} P_{1\mu} P_{1\lambda} + \frac{iW_3^v(q^2, \nu)}{M^2} \epsilon_{\mu\lambda\gamma\delta} P_{1\gamma} q_\delta \\ &\quad + \frac{W_4^v(q^2, \nu)}{M^2} q_\mu q_\lambda \\ &\quad + \frac{W_5^v(q^2, \nu)}{M^2} [P_{1\mu} q_\lambda + P_{1\lambda} q_\mu] + \frac{iW_6^v(q^2, \nu)}{M^2} [P_{1\mu} q_\lambda - P_{1\lambda} q_\mu] \\ &= R_{\mu\lambda} \end{aligned} \quad (52)$$

Here $\nu = (P_1 \cdot q)/M$ and M is the target nucleon mass. The six form factors $W_1^v \dots W_6^v$ are functions of q^2 and ν only, and are the counterpart of those in inelastic lepton-nucleon scattering

(see eq. (4.2)). Only two form factors arise there as conservation of the lepton current demands $W_4^V = W_5^V = W_6^V = 0$, while parity invariance implies $W_3^V = 0$.) Contracting (5.1) and (5.2) gives the differential cross-section

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{G^2}{4\pi E_\ell^2} \left[a_1(P_1 k_1)^2 + a_2(P_1 k_1) + a_3 \right] \text{ with } a_1, a_2, a_3 \text{ given}$$

as follows:

$$a_1 = 2W_2^V/M^2, \quad a_2 = \left[-2W_2^V M\nu + 2W_3^V q^2 - 2W_5^V m_\ell^2 \right] / M^2,$$

$$a_3 = (k_1 k_2) \left[2W_1^V - W_2^V + (2W_3^V)/M + (W_4^V m_\ell^2)/M^2 \right].$$

Neglecting the lepton mass the above simplifies to

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{G^2}{2\pi} \left[\frac{E'_\ell}{E_\ell} W_2^V \cos^2 \frac{\Theta}{2} + 2W_1^V \sin^2 \frac{\Theta}{2} + \frac{2W_3^V}{M} (E_\ell + E'_\ell) \sin^2 \frac{\Theta}{2} \right] \quad (48)$$

(5.3)

E_ℓ denotes the incident neutrino energy in the laboratory frame, E'_ℓ the final lepton energy and Θ the lepton-neutrino scattering angle. G is the weak interaction Fermi constant measured in β -decay $\simeq 10^{-5}/M^2$ (49). From (5.3) it is clear at small Θ that one measures W_2^V , at large angles, W_1^V and W_3^V .

In the above analysis, we have summed over the target nucleon and final lepton spins. We now consider polarisation effects arising from these respective cases.

(1) Final lepton polarisation determined

As the nucleon spin is summed over, $H_{\mu\lambda}$ is again given by eq. (5.2). For the lepton-neutrino vertex we now have

$$\frac{L_{\mu\lambda}}{8} = \frac{1}{8} \text{Tr} (X_{2+m_\ell})(1-\gamma_L \gamma_5) \gamma_\mu (1+\gamma_5) X_{1\lambda} \gamma_\lambda (1+\gamma_5) = R_{\mu\lambda} \mp m_\ell [S_{4\mu} k_{1\lambda} + S_{L\lambda} k_{1\mu} - (S_L \cdot k_1) \delta_{\mu\lambda}] - i m_\ell \epsilon_{\mu\lambda\alpha\beta} S_{L\alpha} k_{1\beta} \quad (5.4)$$

with $R_{\mu\lambda} = \text{eq. (5.1)}/8$ Contracting together we find,

$$\frac{m^2}{8} = R (\equiv R_{\mu\lambda} H_{\mu\lambda}) + b_1 (S_L \cdot P_1) + b_2 (S_L \cdot k_1) + b_3 \epsilon_{\mu\lambda\alpha\beta} P_{1\mu} q_\lambda S_{L\alpha} k_{1\beta} \quad (5.5)$$

(Pas, ref. 46. gives this in a slightly different form)

where

$$b_1/m_\ell = \mp \left[\frac{W_5^V}{M^2} (q^2 - m_\ell^2) + \frac{2W_2^V}{M^2} (P_1 \cdot k_1) \right] - \frac{W_3^V}{M^2} (q^2 - m_\ell^2)$$

$$b_2/m_\ell = \mp \left[-2W_1^V + W_2^V + W_4^V (m_\ell^2/M^2) + 2M \frac{W_5^V}{M^2} - 2(P_1 \cdot k_1) \frac{W_3^V}{M^2} \right] + 2(P_1 \cdot k_1) \frac{W_3^V}{M^2}$$

$$b_3/m_\ell = 2W_3^V/M^2$$

The upper (lower) sign here refers to neutrino (antineutrino) reaction.

(i) Lepton longitudinally polarised, i.e. $S_L \cdot P_1 = M E_\ell^*/m_\ell$,

$S_L \cdot k_1 = E_\ell (E_\ell^* - |\vec{P}_\ell^*| \cos \theta)/m_\ell$, \vec{P}_ℓ^* being the lepton 3-momentum

i.e. $(E_\ell^*)^2 = (\vec{P}_\ell^*)^2 + m_\ell^2$. Then (5.5) gives

$$M_S (\text{spin part of (5.5)}) = 2E_\ell E_\ell^* \left[\pm W_1^V 2\sin^2 \frac{\theta}{2} \pm W_2^V \cos^2 \frac{\theta}{2} - \frac{2W_3^V}{M} (E_\ell^* E_\ell^*) \sin^2 \frac{\theta}{2} \right] \text{ ignoring extra}$$

lepton mass term.

∴ Polarisation = ± 1 . Here $P = [m^2 - m_{\ell \rightarrow S}^2] / [m^2 + m_{\ell \rightarrow S}^2] = \gamma_S/R$.

This expresses the fact that the lepton itself has the two-component coupling properties of the neutrons in the limit $m_\ell = 0$.

(ii) Lepton transversely polarised in the scattering plane.

Hence

$S_L \cdot P_1 = 0$, $S_L \cdot k_1 = |\bar{P}'_\ell| \sin \theta$ giving

$$M_S = m_\ell |\bar{P}'_\ell| \sin \theta \left[+(-2W_1^V + W_2^V + W_4^V \frac{m_\ell^2}{M^2} + \frac{2\nu}{M} W_5^V) + \frac{2E_\ell}{M} (W_3^V + W_5^V) \right]$$

$$\therefore P = \frac{m_\ell \sin \theta \left\{ +(-2W_1^V + W_2^V + W_4^V \frac{m_\ell^2}{M^2} + \frac{2\nu}{M} W_5^V) + \frac{2E_\ell}{M} (W_3^V + W_5^V) \right\}}{2E'_\ell \left[W_2^V \cos^2 \frac{\theta}{2} + 2W_1^V \sin^2 \frac{\theta}{2} + \frac{2W_3}{M} (E_\ell + E'_\ell) \sin^2 \frac{\theta}{2} \right]}$$

$$\text{or } \frac{Pd^2_6}{dq^2 d\nu} = \frac{G^2 m_\ell}{4\pi E_L} \sin \theta \{ \cdot \}, \text{ using } |\bar{P}'_\ell| = E'_\ell \quad (5.6)$$

Two measurements of P at differing values of E_ℓ , but fixed q^2 and ν determines the form factors $G_1 = -2W_1^V + W_2^V + W_4^V (m_\ell/M)^2 + \frac{2\nu}{M} W_5^V$, $G_2 = W_3^V + W_5^V$. Note however the rapid decrease in (5.6) due to the m_ℓ/E_ℓ factor; ($\frac{d^2_6}{dq^2 d\nu}$ therein refers to eq. (5.3)).

(iii) Lepton polarised normal to the lepton-neutrino scattering plane.

Here $(\bar{S}_L \cdot \bar{k}_1 \times \bar{k}_2) = E_\ell |\bar{P}'_\ell| \sin \theta \simeq E_\ell E'_\ell \sin \theta$ and

$$M_S = -2W_6 (m_\ell/M) (\bar{S}_L \cdot \bar{k}_1 \times \bar{k}_2)$$

$$\therefore P = -W_6^V \left(\frac{m_\ell}{M} \right) \frac{(\bar{S}_L \cdot \bar{k}_1 \times \bar{k}_2) \cdot (E_\ell E'_\ell)^{-1}}{\left[W_2^V \cos^2 \frac{\theta}{2} + 2W_1^V \sin^2 \frac{\theta}{2} + \frac{-2W^V}{M} (E_\ell + E'_\ell) \sin^2 \frac{\theta}{2} \right]}$$

$$\text{or } \frac{Pd^2_6}{dq^2 d\nu} = \frac{-G^2}{2\pi} \left(\frac{m_\ell}{M} \right) \frac{(\bar{S}_L \cdot \bar{k}_1 \times \bar{k}_2)}{E_\ell^2} W_6^V \quad (5.7)$$

Note that the correlation term $(\bar{S}_L \cdot \bar{k}_1 \times \bar{k}_2)$ violates time-reversal invariance, and hence detection of such a correlation acts as a test for this symmetry in weak interaction processes⁽⁵¹⁾.

(2) Target Nucleon polarised

As lepton spins are summed over, $L_{\mu\lambda}$ is given by eq. (5.1). For the hadron vertex, there is now an independent extra vector S_A , the nucleon spin projection operator, and so Lorentz invariance yields

$$\begin{aligned}
 H_{\mu\lambda} = \sum_{\text{final spins}} \langle P_1^V | J_\mu | P_1 \rangle^2 &= K_{\mu\lambda} + \frac{W_7^V}{M} (S_A \cdot q) \delta_{\mu\lambda} + \frac{W_8^V}{M^3} (S_A \cdot q) P_{1\mu} P_{1\lambda} \\
 &+ \frac{W_9^V}{M^3} (S_A \cdot q) q_\mu q_\lambda + \frac{W_{10}^V}{M^3} (S_A \cdot q) [P_{1\mu} q_\lambda + P_{1\lambda} q_\mu] \\
 &+ \frac{W_{11}^V}{M^3} (S_A \cdot q) i [P_{1\mu} q_\lambda - P_{1\lambda} q_\mu] + \frac{W_{12}^V}{M} [P_{1\mu} S_{A\lambda} + P_{1\lambda} S_{A\mu}] + \frac{W_{13}^V}{M} i [P_{1\mu} S_{A\lambda} - P_{1\lambda} S_{A\mu}] \\
 &+ \frac{W_{14}^V}{M} [q_\mu S_{A\lambda} + q_\lambda S_{A\mu}] + \frac{W_{15}^V}{M} i [q_\mu S_{A\lambda} - q_\lambda S_{A\mu}] \\
 &+ \frac{W_{16}^V}{M} i \epsilon_{\mu\lambda st} q_s S_{A_t} + \frac{W_{17}^V}{M} i \epsilon_{\mu\lambda st} P_{1s} S_{A_t} + \frac{W_{18}^V}{M^3} i [\epsilon_{\mu stw} P_{1\lambda} - \epsilon_{\lambda stw} P_{1\mu}] \\
 &+ \frac{W_{19}^V}{M^3} [\epsilon_{ustw} P_{1\lambda} + \epsilon_{\lambda stw} P_{1\mu}] q_s P_{1t} S_{A_\omega} \\
 &+ \frac{W_{20}^V}{M^3} i [\epsilon_{ustw} q_\lambda - \epsilon_{\lambda stw} q_\mu] q_s P_{1t} S_{A_\omega} \\
 &+ \frac{W_{21}^V}{M^3} [\epsilon_{ustw} q_\lambda + \epsilon_{\lambda stw} q_\mu] q_s P_{1t} S_{A_\omega}
 \end{aligned} \tag{5.8}$$

$K_{\mu\lambda}$ is as defined in (5.2).

Note the slight complication compared with eq. (4.8)! In the electromagnetic process $W_7^V, W_8^V, \dots, W_{15}^V$, vanish because they give

rise to parity-violating terms, while gauge invariance demands W_{17}^V , W_{20}^V and W_{21}^V are also zero. Neither parity nor gauge invariance holds true for weak processes and therefore there is no reason a priori why such terms should vanish. We mention once more all the form factors in (5.8) are functions of q^2 and ν only.

Contracting together we find

$$\frac{m^2}{8} = R + \frac{A}{M} (S_A \cdot k_1) + \frac{B}{M} (S_A \cdot k_2) + \frac{C}{M} \epsilon_{ustw} P_{1\mu} S_{As} k_{2t} k_{1\omega} \equiv R + M_S \quad (5.9)$$

where $A = a_1 + a_2 E_\ell + a_3 E_\ell^2$, with

$$a_1 = (q^2 - m_\ell^2) \left[W_7^V + W_{8/2}^V + W_9^V (m_\ell^2 / 2M^2) \right] \mp (q^2 + m_\ell^2) \left[W_{16}^V + W_{18}^V \right] - 2W_{14} m_\ell^2 - 2M\nu \left[W_{12}^V \mp W_{17}^V \right] \mp 2W_{18}^V \nu^2 ;$$

$$a_2 = 2 \left[\nu (-W_8^V \mp W_{18}^V) + M(W_{12}^V - W_{10}^V (m_\ell^2 / M^2) \mp W_7^V) \right] ; \quad a_3 = 2W_8^V .$$

$B = b_1 + b_2 E_\ell + b_3 E_\ell^2$ with

$$b_1 = - (q^2 - m_\ell^2) \left[W_7^V + W_{8/2}^V - W_9^V (m_\ell^2 / 2M^2) \right] \mp (q^2 - m_\ell^2) \left[W_{16}^V - W_{18}^V \right] ;$$

$$b_2 = 2 \left[\nu (W_8^V \mp W_{18}^V) + M(W_{12}^V \mp W_{17}^V) + W_{10}^V (m_\ell^2 / M) \right] ; \quad b_3 = -2W_8^V ;$$

$$\text{and } C = 2 \left[\mp W_{13}^V + W_{19}^V (2E_\ell - \nu) / M - (m_\ell^2 / M^2) W_{15}^V \right] .$$

The upper and lower signs refer to an incident neutrino or anti-neutrino respectively; k is as defined in (5.5).

(i) Target nucleon polarised normal to the lepton-neutrino scattering plane. This is given by the last term in M_S , and we have

their extraction will be an extremely difficult task anyway.

(ii) Target nucleon polarised along direction of incoming

neutrino beam. Here $S_A \cdot k_1 = -E_\ell$, $S_A \cdot k_2 = -E'_\ell \cos \theta$.

Hence $M_S = -A \left[\frac{E_\ell}{M} + B \frac{E'_\ell}{M} \cos \theta \right]$.

Using the kinematic relations

$$E_\ell - E'_\ell \cos \theta = \nu + E'_\ell (1 - \cos \theta) = \nu - \frac{q^2}{2E_\ell}, \text{ yields}$$

$$M_S = -\frac{q^2}{ME_\ell} \left[a + bE_\ell + cE_\ell^2 + dE_\ell^3 \right], \text{ where} \quad (5.12)$$

$$a = (-q^2/2) \left[W_7^V + W_8^V/2 + W_{13}^V + W_{16}^V \right];$$

$$b = \nu(W_7^V + \frac{3}{2}W_8^V + W_{16}^V + 2W_{18}^V) + M(W_{12}^V + W_{17}^V);$$

$$c = 2 \left[-W_8^V/2 + W_{16}^V - \frac{2M\nu}{q^2} (W_{12}^V + W_{17}^V) - \frac{\nu^2}{q^2} W_8^V \right];$$

$$d = \frac{2}{q^2} \left[2M(W_{12}^V + W_{17}^V) + \nu W_8^V \right].$$

$$\therefore \text{Polarisation } P = \frac{-2\sin^2 \frac{\theta}{2}}{ME_\ell} \frac{\left\{ a + bE_\ell + cE_\ell^2 + dE_\ell^3 \right\}}{\left[W_2^V \cos^2 \frac{2\theta}{2} + 2W_1^V \sin^2 \frac{2\theta}{2} + 2W_3^V \frac{(E_\ell + E'_\ell)}{M} \sin^2 \frac{2\theta}{2} \right]}$$

$$\text{or } \frac{Pd^2\sigma}{dq^2 d\nu} = \frac{-G^2}{4\pi} \frac{q^2}{ME_\ell} \frac{\left\{ \right\}}{E_\ell^2} \quad (5.13)$$

Thus, in the notation of (5.10) we have

$$\frac{1}{2} \left[P_\nu \frac{d^2\sigma}{dq^2 d\nu} + P_{\bar{\nu}} \frac{d^2\sigma}{dq^2 d\nu} \right] = -\frac{G^2}{4\pi} \frac{q^2}{ME_\ell^3} \left\{ -\frac{q^2}{2} [W_7^V + W_8^V] + E_\ell \left[(W_7^V + \frac{3}{2}W_8^V)\nu + MW_{12}^V \right] + E_\ell^2 \left[-W_8^V - \frac{4M\nu}{q^2} W_{12}^V - \frac{2\nu^2}{q^2} W_8^V \right] + E_\ell^3 \left[\frac{4M}{q^2} W_{12}^V + \frac{2\nu}{q^2} W_8^V \right] \right\} \quad (5.14)$$

$$\frac{1}{2} \left[P_V \frac{d^2 \bar{6}}{dq^2 dv} - P_{\bar{V}} \frac{d^2 \bar{6}}{dq^2 dv} \right] = -\frac{G^2 v^2}{4\pi M E_\ell^3} \left\{ q^2 \frac{v}{2} (W_{18}^V - W_{16}^V) + E_\ell [(W_{16}^V - 2W_{18}^V)v + M W_{17}^V] + E_\ell^2 [-2W_{16}^V - \frac{4Mv}{q^2} W_{17}^V] + E_\ell^3 [\frac{4M}{q^2} W_{17}^V] \right\} \quad (5.15)$$

We see now that 4 measurements of P at fixed values of q^2 and v , but varying E_ℓ , are necessary to extract the requisite form factors in (5.14) and (5.15). This is the price one has to pay for the rapid increase in complexity compared to the analogous electromagnetic process, i.e. eq. (4.8). From (5.14) one determines the form factors $G_1 = W_7^V + W_8^V/2$; $G_2 = (W_7^V + \frac{3}{2}W_8^V)v + MW_{12}^V$, $G_3 = -W_8^V(1 + 2v^2/q^2) - \frac{4Mv}{q^2} W_{12}^V$, $G_4 = \frac{2}{q^2} [2MW_{12}^V + vW_8^V]$ which over determines W_7^V , W_8^V and W_{12}^V . From (5.15) one determines the form factors $G_1 = W_{18}^V - W_{16}^V$; $G_2 = (W_{16}^V - 2W_{18}^V)v + MW_{17}^V$; $G_3 = -2W_{16}^V + \frac{2Mv}{q^2} W_{17}^V$; $G_4 = (4M/q^2)W_{17}^V$, which overdetermines W_{16}^V , W_{17}^V and W_{18}^V . Note in the high energy limit $E_\ell \rightarrow \infty$, (5.14) and (5.15) reduce to

$$\frac{1}{2} \left[P_V \frac{d^2 \bar{6}}{dq^2 dv} + P_{\bar{V}} \frac{d^2 \bar{6}}{dq^2 dv} \right]_{E_\ell \rightarrow \infty} = \frac{-6^2}{2\pi} \left[2W_{12}^V + \frac{v}{M} W_8^V \right];$$

$$\frac{1}{2} \left[P_V \frac{d^2 \bar{6}}{dq^2 dv} - P_{\bar{V}} \frac{d^2 \bar{6}}{dq^2 dv} \right]_{E_\ell \rightarrow \infty} = \frac{-G^2}{\pi} W_{17}^V.$$

(iii) Target nucleon polarised perpendicular to neutrino beam momentum. i.e. $S_A \cdot k_1 = 0$ and $S_A \cdot k_2 = -E_\ell' \sin \theta$, whence (5.9) gives $M_S = -(E_\ell'/M) \sin \theta B$.

$$\therefore \text{Polarisation } P = \frac{-(2ME_\ell)^{-1} \{ b_1 + b_2 E_\ell + b_3 E_\ell^2 \sin \theta \}}{\left[W_2^V \cos^2 \frac{2\theta}{2} + 2W_1^V \sin^2 \frac{2\theta}{2} + 2W_3^V \frac{(E_\ell + E_\ell')}{M} \sin^2 \frac{2\theta}{2} \right]}$$

or

$$\frac{Pd^2_6}{dq^2 d\nu} = \frac{G^2}{2\pi} \left(\frac{q^2}{4ME_\ell} \right) \cot \frac{\theta}{2} \frac{\{ \}}{E_\ell^2} \quad (5.16)$$

with b_1, b_2, b_3 as defined in (5.9). Hence we have

$$\frac{1}{2} \left[P_V \frac{d^2_6}{dq^2 d\nu} + P_{\bar{V}} \frac{d^2_7}{dq^2 d\nu} \right] = \frac{G^2}{2\pi} \left(\frac{q^2}{4ME_\ell} \right) \frac{\cot \frac{\theta}{2}}{E_\ell^2} \left\{ - (W_7^V + W_{8/2}^V) q^2 + 2E_\ell (\sqrt{W_8^V} + MW_{12}^V) - 2W_8^V E_\ell^2 \right\} \quad (5.17)$$

$$\frac{1}{2} \left[P_V \frac{d^2_6}{dq^2 d\nu} - P_{\bar{V}} \frac{d^2_7}{dq^2 d\nu} \right] = \frac{G^2}{2\pi} \left(\frac{q^2}{4ME_\ell} \right) \frac{\cot \frac{\theta}{2}}{E_\ell^2} \left\{ - (W_{16}^V - W_{18}^V) q^2 + 2E_\ell [-\sqrt{W_{18}^V} + MW_{17}^V] \right\} \quad (5.18)$$

This configuration is clearly more favourable experimentally than (ii) as only three measurements of P for (5.17) and two for (5.18) are necessary. From (5.17) one may extract the form factors $G_1 = W_7^V + W_{8/2}^V$; $G_2 = \sqrt{W_8^V} + MW_{12}^V$; $G_3 = W_8^V$, whence W_7^V , W_8^V and W_{12}^V are separately fixed. (5.18) yields form factors $G_1 = W_{16}^V - W_{18}^V$; $G_2 = -\sqrt{W_{18}^V} + MW_{17}^V$; however these are not sufficient to separately determine W_{16}^V , W_{17}^V and W_{18}^V , which is the drawback in this configuration.

(iv) Target nucleon polarised along the direction of the final lepton momentum. Now $S_A \cdot k_1 = -E_\ell \cos \theta$, $S_A \cdot k_2 = -E'_\ell$ and hence $M_S = -[AE_\ell \cos \theta + BE'_\ell]/M$,

$$\text{or } M = \frac{-q^2}{nE'_\ell} [a' + b'E'_\ell + c'E_\ell^2 + d'E_\ell^3] \quad \text{where}$$

$$a' = (q^2/2) \left[W_7^V + \frac{W_8^V}{2} + W_{16}^V + W_{18}^V \right];$$

$$b' = \nu (W_7^V + \frac{3}{2}W_8^V + W_{16}^V) + M(W_{12}^V + W_{17}^V) ;$$

$$c' = (W_8^V + 2W_{16}^V) + \frac{4M\nu}{q} (W_{12}^V + W_{17}^V) + \frac{2\nu^2}{q} W_8^V ;$$

$$d' = \frac{2}{q} [2M(W_{12}^V + W_{17}^V) + \nu W_8^V]$$

$$\therefore \text{Polarisation } P = \frac{-2\sin^2 \frac{\theta}{2}}{ME_\ell} \frac{\{ a' + b'E_\ell' + c'E_\ell'^2 + d'E_\ell'^3 \}}{[W_2^V \cos \frac{2\theta}{2} + 2W_1^V \sin \frac{2\theta}{2} + 2W_3^V \frac{(E_\ell + E_\ell')}{M} \sin \frac{2\theta}{2}]}$$

$$\text{or } P \frac{d^2 6}{dq^2 d} = -\frac{G^2}{\pi} \sin^2 \frac{\theta}{2} \frac{\{ \}}{ME_\ell} \quad (5.20)$$

This gives

$$\begin{aligned} \frac{1}{2} \left[P_V \frac{d^2 6}{dq^2 d\nu} + P_{\bar{V}} \frac{d^2 \bar{6}}{dq^2 d\nu} \right] &= -\frac{G^2}{\pi} \frac{\sin^2 \frac{\theta}{2}}{ME_\ell} \left\{ q^2/2(W_7^V + W_8^V/2) \right. \\ &+ E_\ell' [\nu(W_7^V + \frac{3}{2}W_8^V) + MW_{12}^V] + E_\ell'^2 [W_8^V + \frac{4M\nu}{q} W_{12}^V + \frac{2\nu^2}{q} W_8^V] \\ &\left. + E_\ell'^3 [\frac{4M}{q} W_{12}^V + \frac{2\nu}{q} W_8^V] \right\} \quad (5.21) \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \left[P_V \frac{d^2 6}{dq^2 d\nu} - P_{\bar{V}} \frac{d^2 \bar{6}}{dq^2 d\nu} \right] &= -\frac{G^2}{\pi} \frac{\sin^2 \frac{\theta}{2}}{ME_\ell} \left\{ -q^2/2(W_{16}^V + W_{18}^V) \right. \\ &+ E_\ell' [-\nu W_{16}^V + MW_{17}^V] + E_\ell'^2 [-2W_{16}^V + \frac{4M\nu}{q} W_{17}^V] \\ &\left. + E_\ell'^3 (\frac{4M}{q} W_{17}^V) \right\} \quad (5.22) \end{aligned}$$

Analogously to case (ii), four measurements of the polarisation are here required. From (5.21) one determines the form factors

$$G_1 = W_7^V + (W_8^V/2); \quad G_2 = \nu(W_7^V + \frac{3}{2}W_8^V) + MW_{12}^V ;$$

$$G_3 = W_8^V (1 + \frac{2\nu^2}{q^2}) + \frac{4M\nu}{q^2} W_{12}^V \quad \text{and} \quad G_4 = 2MW_{12}^V + \nu W_8^V \quad \text{which}$$

again overdetermines W_7^V , W_8^V and W_{12}^V ; while (5.22) yields
 $G_1 = W_{16}^V + W_{18}^V$; $G_2 = -\nu W_{16}^V + MW_{17}^V$; $G_3 = -2W_{16}^V + \frac{4M\nu}{2} W_{17}^V$
 and $G_4 = W_{17}^V$ which also overdetermines W_{16}^V , W_{17}^V and W_{18}^V .

Note in the high energy limit, $E_\ell \rightarrow \infty$, (5.21) and (5.22) reduce to

$$\frac{1}{2} \left[P_V \frac{d^2 \bar{6}}{dq^2 d\nu} + P_{\bar{V}} \frac{d^2 \bar{6}}{dq^2 d\nu} \right]_{E_\ell \rightarrow \infty} = -\frac{G^2}{\pi} 2 \sin^2 \frac{\theta}{2} \frac{E_\ell^2}{q} (2W_{12}^V + \nu W_8^V);$$

$$\frac{1}{2} \left[P_V \frac{d^2 \bar{6}}{dq^2 d\nu} - P_{\bar{V}} \frac{d^2 \bar{6}}{dq^2 d\nu} \right]_{E_\ell \rightarrow \infty} = -\frac{G^2}{\pi} 4 \sin^2 \frac{\theta}{2} \frac{E_\ell^2}{q} W_{17}^V.$$

(v) Target nucleon polarised perpendicular to final lepton momentum.
 i.e. $S_A \cdot k_1 = E_\ell \sin \theta$, $S_A \cdot k_2 = 0$ $\therefore M_S = \frac{E_\ell}{M} \sin \theta$ (A)
 from (5.9)

$$\text{so } P = (2ME_\ell^2)^{-1} \frac{\sin \theta \left\{ a_1 + a_2 E_\ell + a_3 E_\ell^2 \right\}}{\left[W_2^V \cos^2 \frac{\theta}{2} + 2W_1^V \sin^2 \frac{\theta}{2} + 2W_3^V \left(\frac{E_\ell + E_\ell'}{M} \right) \sin^2 \frac{\theta}{2} \right]}$$

$$\text{or } P \frac{d^2 \bar{6}}{dq^2 d\nu} = \frac{G^2}{4\pi} \sin \theta \frac{\{ \}}{ME_\ell} \quad (5.23)$$

This gives

$$\frac{1}{2} \left[P_V \frac{d^2 \bar{6}}{dq^2 d\nu} + P_{\bar{V}} \frac{d^2 \bar{6}}{dq^2 d\nu} \right] = \frac{G^2}{4\pi} \frac{\sin \theta}{ME_\ell} \left\{ q^2 (W_7^V + (W_8^V/2)) - 2M\nu W_{12}^V + 2E_\ell (-\nu W_8^V + MW_{12}^V) + 2E_\ell^2 W_8^V \right\} \quad (5.24)$$

$$\frac{1}{2} \left[P_V \frac{d^2 \bar{6}}{dq^2 d\nu} - P_{\bar{V}} \frac{d^2 \bar{6}}{dq^2 d\nu} \right] = \frac{G^2}{4\pi} \frac{\sin \theta}{ME_\ell} \left\{ -(W_{16}^V + W_{18}^V) q^2 - 2M\nu W_M^V - 2W_{18}^V \nu^2 + 2E_\ell [\nu W_{18}^V + MW_{17}^V] \right\} \quad (5.25)$$

As for case (iii), only three measurements of P for (5.24) and two for (5.25) are here required. The former yield the form factors $G_1 = q^2(W_7^V + (W_8^V/2)) - 2M W_{12}^V$, $G_2 = -\nu W_8^V + MW_{12}^V$; $G_3 = W_8^V$ whence W_7^V , W_8^V and W_{12}^V are separately fixed. (5.25) however yields only $G_1 = -(W_{16}^V + W_{18}^V)q^2 - 2\nu(MW_{17}^V + \nu W_{18}^V)$ and $G_2 = \nu W_{18}^V + MW_{17}^V$, which does not individually determine W_{16}^V , W_{17}^V and W_{18}^V . Both cases (iv) and (v) possess the disadvantage that the orientation of the nucleon polarisation has to be changed as the lepton scattering angle θ varies.

The general discussion above applies to inelastic neutrino reactions. Although such processes are undoubtedly of great interest, there is at present little theoretical knowledge of the form factors defined in (5.8). On the other hand the above discussion is immediately applicable to elastic reactions also, where the theoretical groundwork is much firmer. As the latter processes will unquestionably be thoroughly explored once polarisation techniques are feasible experimentally, we conclude with a discussion on the information extractable from them with a polarised target.

Elastic neutrino reactions

By this we mean reactions of the type $\nu(\nu) + N \rightarrow \ell(\ell) + Y$, where Y is a baryon state, i.e. N , Λ or Σ . Summing over the lepton spin, the lepton-neutrino vertex is again given by eq. (5.1).

For the hadron vertex we now have, as a general consequence of Lorentz invariance⁽⁵²⁾:

$$\langle P_2 | J_\mu | P_1 \rangle = \frac{G}{\sqrt{2}} \bar{u}(P_2) \left\{ f_1 \gamma_\mu + i \frac{f_2}{M} \sigma_{\mu\nu} q_\nu + \frac{f_3}{M} q_\mu + g_1 \gamma_\mu \gamma_5 + i \frac{g_2}{M} \sigma_{\mu\nu} q_\nu \gamma_5 + \frac{g_3}{M} q_\mu \gamma_5 \right\} u(P_1) \quad (5.26)$$

where $u(P_1)$ is the free-field Dirac spinor for a particle of momentum P_1 , etc. and P_2 is the momentum of the Υ baryon (spin $\frac{1}{2}$). (5.26) is the general extension of eq. (2.2) for parity-violating reactions, and the form factors $f_1, f_2, \dots, g_1, \dots, g_3$ are each functions of q^2 only, where q = four-momentum transfer = $k_1 - k_2 = P_2 - P_1$. (M is the nucleon mass). G is as defined in (5.3).

Using the Dirac equation (5.26) may be rewritten as

$$\langle P_2 | J_\mu | P_1 \rangle = \frac{G}{\sqrt{2}} \bar{u}(P_2) \left\{ g_V \gamma_\mu - 2P_{1\mu} \frac{f_V}{M} + q_\mu \frac{(h_V - f_V)}{M} + g_A \gamma_\mu \gamma_5 - 2P_{1\mu} \frac{f_A \gamma_5}{M} + q_\mu \gamma_5 \frac{(h_A - f_A)}{M} \right\} \quad (5.27)$$

where we have

$$g_V = f_1 + \left(1 + \frac{M_Y}{M}\right) f_2; \quad f_V = f_2, \quad h_V = f_3; \quad g_A = g_1 - \left(1 - \frac{M_Y}{M}\right) g_2; \quad f_A = g_2, \quad h_A = g_3$$

This form is simpler for computational work. M_Y here denotes the baryon Υ mass.

From (5.27) summing over the final baryon spin yields

$$H_{\mu\lambda} = \sum \langle P_2 | J_\mu | P_1 \rangle^2 \text{ of the form in (5.8) with the form factors } W_1^V \dots W_{21}^V \text{ as given in Appendix 6.}$$

We see immediately the configuration (5.9) with the target polarised normal to the scattering plane determines the form factors

$$W_{13}^V = M \left\{ 2M_Y \text{Im}(g_V g_A^*) + (M_Y^2 + M^2 - q^2)/M \text{Im}(g_A f_V^* - g_V f_A^*) \right\}$$

$$W_{15}^V = (M_Y^2 + M^2 - q^2)/2 \left\{ \text{Im}(g_V h_A^* - g_A h_V^* - g_V f_A^* + g_A f_V^*) \right\}$$

$$W_{19}^V = 2M^2 \left\{ \text{Im}(g_V f_V^* - g_A f_A^*) \right\} \quad \text{where Im denotes the}$$

imaginary part of,

Time reversal invariance demands that each of the form factors $g_V, g_A, \dots, h_V, h_A$ be real⁽⁵³⁾, and hence measurement of the normal polarisation is of prime interest. Further if the conserved vector current hypothesis (CVC)⁽⁵⁴⁾ holds good, g_V and f_V are both real so in the high energy limit $E_e \rightarrow \infty$ one determines $W_{19}^V \propto \text{Im}(g_A f_A^*)$.

For polarisations lying in the scattering plane, cases (ii) to (v), enables one to extract the form factors

$$W_7^V = -2M^2 \text{Re}(g_V g_A^*) : \quad W_8^V = 4M^2 \left[2\text{Re}(f_V f_A^*) - \text{Re}(g_V f_A^*) + \text{Re}(g_A f_V^*) \right]$$

$$W_{12}^V = M^2 \left[2\text{Re}(g_V g_A^*) + \frac{M^2 + M_Y^2 - q^2}{2M^2} \cdot \text{Re}(g_V f_A^* - g_A f_V^*) \right]$$

$$W_{16}^V = -M^2 \left[g_V^2 + g_A^2 \right]$$

$$W_{17}^V = M \left[(M_Y - M)g_V^2 - (M_Y + M)g_A^2 \right] \quad W_{18}^V = 2M^2 \left[\text{Re}(-g_V f_V^* + g_A f_A^*) \right]$$

where Re denotes the real part of.

W_{16}^V and W_{17}^V plus W_7^V determine the g_V and g_A form factors. The former can then be compared with the predictions of CVC when the final baryon state is a nucleon too, i.e. reactions

$v + n \rightarrow e^- + p, \quad \bar{v} + p \rightarrow e^+ + n$ where $n(p)$ denotes a neutron

(proton). CVC implies that

$$h_V = 0 : g_V = \left[F_1^V + (\mu_P - \mu_N) F_2^V \right] \left(1 + q^2/M_W^2 \right)^{-1} \cos \theta_c :$$

$$\frac{f_V}{M} = (\mu_P - \mu_N) \frac{F_2^V}{2M} \left(1 + q^2/M_W^2 \right) \cos \theta_c$$

(5.28)

where μ_P and μ_N denotes the anomalous magnetic moment of the proton and neutron respectively. θ_c is the Cabbibo angle, and $F_2^V(q^2)$ are the isovector electric and magnetic form factors normalised to $F_1^V(0) = F_2^V(0) = 1$ (55). The factor $(1 + q^2/M_W^2)^{-1}$ takes account of the possible existence of an intermediate vector boson of mass M_W .

Determination of the g_A form-factor will serve to test the theoretical estimates based on dispersion relation methods (56). In particular it will be interesting to see if g_A has a similar q^{-4} fall-off for large q as the dipole fit for the electromagnetic form factors

In the limit $q^2 \rightarrow 0$, SU(3) symmetry predicts the g_V and g_A form factors for the reactions $v(\bar{v}) + N \rightarrow \ell(\bar{\ell}) + Y(\bar{Y})$ given their values for the non-strangeness changing $\Delta S = 0$ reactions $v(\bar{v}) + n(p) \rightarrow \ell(\bar{\ell}) + p(n)$ (57). This is in fact the inverse of the decay processes $Y(\bar{Y}) \rightarrow N + \ell(\bar{\ell}) + \bar{v}(v)$ where such an analysis has been made (58), and will serve as a separate test of the Cabbibo theory and determination of the angle θ_c . It must be pointed out however that the strangeness changing $\Delta S = 1$ processes are suppressed by a factor $\sin^2 \theta_c \approx (20 - 30)^{-1}$ and will therefore require much more data for analysis than the $\Delta S = 0$ reactions.

The form factors W_8^V , W_{12}^V and W_{18}^V fix the values of f_V and f_A

its being presumed g_A and g_V are known from W_7^V , W_{16}^V and W_{17}^V . Besides testing CVC for f_V in $S = 0$ reactions, the determination of the f_A form factor will be of great interest. The $\Delta I = 1$ rule, that the weak current J_μ and its conjugate are isovector components belonging to the same isotriplet, implies (up to common phase) that g_V , g_A , f_V , h_A are real and f_A , h_V are imaginary⁽⁵⁹⁾. Thus in conjunction with time-reversal invariance, which implies all form factors are real, leads to $f_A = h_V = 0$. This has been elegantly expressed by Weinberg⁽⁶⁰⁾ in terms of the G-parity transformation properties of the currents; g_V , g_A , f_V , h_A are termed first class form factors, while f_A and h_V are the second class form factors. Thus polarisation measurements in the scattering plane are especially well suited to establish whether there exists such a real second class axial form factor f_A , a feat otherwise not at all easy to establish⁽⁶¹⁾. The existence of h_V serves as a test for CVC and second class currents, while knowledge of h_A would indicate if a generalised Goldberger-Treiman type relation held true⁽⁶²⁾.

Finally there arises the question of the form factors h_V and h_A . These form factors appear in W_9^V , W_{10}^V , W_{14}^V and W_{15}^V ; however in each case they are multiplied by a factor m_c^2 and therefore their determination will be correspondingly difficult. This is a problem inherent in all attempts to determine these form factors⁽⁶³⁾.

With this final discussion we conclude our investigation of polarisation phenomena in high energy weak and electromagnetic interactions. It is clear that this is a rich field of study,

whose dividends should certainly repay the effort required to initiate these difficult and taxing experiments. With the production of a polarised target and polarised beams now very much a reality, such data should be forthcoming in the very near future.

Appendix 1

Throughout this thesis, we adhere to the metric and gamma matrix conventions used by J. Bjorken and S. Drell in their book, "Relativistic Quantum Mechanics" (McGraw-Hill, New York, 1964 - especially Appendix A, p. 281.)

Metric

Space-time coordinates (t, x, y, z) are denoted by the contravariant four vector

$$x^\mu \equiv (x^0, x^1, x^2, x^3) \equiv (t, x, y, z) = (t, \vec{x})^*$$

N.B. Natural units ($\hbar = c = 1$) are used, where $\hbar = \text{Planck's constant}/2\pi$ and $c = \text{velocity of light}$. The covariant four-vector $x_\mu \equiv (x_0, x_1, x_2, x_3) \equiv (t, -x, -y, -z) = g_{\mu\nu} x^\nu$ in vacuo where metric tensor $g_{\mu\nu} = g^{\mu\nu}$ is given by

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The four-momentum of a particle of mass m is defined by

$$P^\mu = (E, P_x, P_y, P_z) = [(\vec{P}^2 + m^2)^{\frac{1}{2}}, \vec{P}] \quad \text{so that } P^2 = P^\mu P_\mu = E^2 - \vec{P}^2 = m^2.$$

Scalar product of two four-momenta is

$$P_1 \cdot P_2 = P_1^\mu P_{2\mu} = P_{1\mu} P_2^\mu = E_1 E_2 - \vec{P}_1 \cdot \vec{P}_2 \quad \text{and similarly for any two four-vectors.}$$

Dirac Matrices and Spinors

The γ matrices in the Dirac equation satisfy the anti-commutation relations $\gamma^\mu \cdot \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$ and are related to the Dirac matrices α^γ and β by

$$\gamma^r = \beta \alpha^r \quad (r = 1, 2, 3) \quad \gamma^0 = \beta.$$

A standard representation is as follows: let 1 and σ^r ($r = 1, 2, 3$) denote the 2×2 unit and Pauli spin matrices

$$1 = \begin{pmatrix} +1 & 0 \\ 0 & +1 \end{pmatrix} \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Then

$$\gamma^r = \begin{pmatrix} 0 & \sigma^r \\ -\sigma^r & 0 \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Frequently used combinations are $\sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$ and $\gamma^5 = i\gamma^0 \gamma^1 \gamma_2 \gamma_3 = \gamma_5$. In this representation

$$\sigma^{ij} = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} \quad \text{with } i, j, k = 1, 2, 3 \text{ in cyclic order;}$$

$$\sigma^{0i} = i \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \quad \gamma^5 = \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Also common is the product of a γ matrix with an ordinary four-vector A .

$$\gamma_\mu A^\mu = \not{A} = \gamma^0 A - \vec{\gamma} \cdot \vec{A}.$$

The positive energy spinor for a particle of physical four momentum P and spin variable s is denoted by $\mu(P, s)$ and satisfies the free field Dirac equation.

* A bar over a letter always denotes the three vector, except when denoting the adjoint spinor $\bar{\mu}(p, s)$.

$$(\not{P} - m)\mu(P, s) = 0 .$$

The adjoint spinor $\bar{\mu}(P, s) = \mu^\dagger(P, s)\gamma^0$ (where \dagger denotes hermitean conjugate) satisfies

$$\bar{\mu}(P, s)(\not{P} - m) = 0 .$$

The positive energy spinor is normalised according to Feynman's convention (R.P. Feynman, "Quantum Electrodynamics", New York, W.A. Benjamin, Inc., 1962)

$$\bar{\mu}(P, s)u(P, s) = 2m$$

and the positive energy projection operator is given by

$$\Lambda_+(P) = \sum_s \mu(P, s)\bar{\mu}(P, s) = \not{P} + m .$$

Similar corresponding formulas for negative-energy spinors are given in the Appendix of Bjorken and Drell's book.

Note the following properties of the γ matrices:

$$\begin{aligned} (\gamma^0)^\dagger &= \gamma^0, & (\gamma^r)^\dagger &= -\gamma^r, & (\gamma^0)^2 &= 1, & (\gamma^r)^2 &= -1, \\ (\gamma^5)^\dagger &= \gamma^5, & (\gamma^5)^2 &= -1, & \gamma_\mu\gamma_5 + \gamma_5\gamma_\mu &= 0. \end{aligned}$$

Trace theorems

The trace of an odd number of γ matrices vanishes.

$$\text{Tr } \gamma_5 \neq 0, \quad \text{Tr } 1 = 4, \quad \text{Tr } \not{a}\not{b} = 4(a \cdot b)$$

$$\begin{aligned} \text{Tr } \not{a}_1\not{a}_2 \dots \not{a}_n &= (a_1 \cdot a_2) \text{Tr } \not{a}_3 \dots \not{a}_n - (a_1 \cdot a_3) \text{Tr } \not{a}_2\not{a}_4 \dots \not{a}_n + \dots \\ &+ (a_1 \cdot a_n) \text{Tr } \not{a}_2 \dots \not{a}_{n-1}. \end{aligned}$$

$$\begin{aligned} \text{In particular } \text{Tr } \not{a}_1\not{a}_2\not{a}_3\not{a}_4 &= 4[(a_1 \cdot a_2)(a_3 \cdot a_4) - (a_1 \cdot a_3)(a_2 \cdot a_4) \\ &+ (a_1 \cdot a_4)(a_2 \cdot a_3)] \end{aligned}$$

$$\text{Tr } \gamma_5 \not{a}\not{b} = 0 \quad \text{Tr } \gamma_5 \not{a}\not{b}\not{c}\not{d} = 4i \epsilon_{\alpha\beta\gamma\delta} a^\alpha b^\beta c^\gamma d^\delta$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is fourth-order antisymmetric tensor, equal to +1 for $(\alpha, \beta, \gamma, \delta)$ and an even permutation of $(0, 1, 2, 3)$; is -1 for an odd permutation, and is 0 if two indices are the same.

Although not trace theorems the following relations are often required:

$$g_{\mu\nu} g_{\mu\nu} = 4, \quad g_{\mu\alpha} g_{\alpha\nu} = g_{\mu\nu}$$

$$\epsilon_{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} = - \begin{vmatrix} g_{\beta p} & g_{\beta q} & g_{\beta r} \\ g_{\gamma p} & g_{\gamma q} & g_{\gamma r} \\ g_{\delta p} & g_{\delta q} & g_{\delta r} \end{vmatrix} = -g_{\beta p}(g_{\gamma q}g_{\delta r} - g_{\gamma r}g_{\delta q}) \\ + g_{\beta q}(g_{\gamma p}g_{\delta r} - g_{\gamma r}g_{\delta p}) \\ - g_{\beta r}(g_{\gamma p}g_{\delta q} - g_{\gamma q}g_{\delta p})$$

whence it follows

$$\epsilon_{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} = -2 \begin{vmatrix} g_{\gamma q} & g_{\gamma r} \\ g_{\delta q} & g_{\delta r} \end{vmatrix} = -2(g_{\gamma q}g_{\delta r} - g_{\gamma r}g_{\delta q}) .$$

Appendix 2

Nucleon and Lepton Electromagnetic Vertices

From Dirac's equation it follows that the interaction of the lepton with the electromagnetic field gives a lepton current j_μ , whose matrix element between initial and final states is given by the 4-vector,

$$\langle k_2 | j_\mu | k_1 \rangle = e \bar{\mu}(k_2) \gamma_\mu \mu(k_1) \quad (\text{A.1})$$

where $\mu(k)$ is the positive energy, free field spinor for a particle of four-momentum k . This coupling is purely electrodynamic and takes no account of strong interaction. Hence for the nucleon-photon vertex it has to be modified and a general analysis based on Lorentz invariance, parity and gauge invariance shows the most general form for such a vertex assumes the form:

$$\langle P_2 | J_\mu | P_1 \rangle = e \bar{\mu}(P_2) \left\{ F_1(q^2) \gamma_\mu + F_2(q^2) \frac{16 \mu_\nu q_\nu}{2M} \right\} \mu(P_1) \quad (\text{A.2})$$

where P_2 and P_1 represent the final and initial four-momentum, and F_1 and F_2 are form factors which are functions of q^2 only. In the static limit $q^2 = 0$, $F_1(0) = 1$, and $F_2(0) =$ anomalous magnetic moment of the nucleon (in units of $e/2M$).

Although F_1 and F_2 appear directly in (A.2) it has been generally accepted that they are not of fundamental significance. In particular the linear combination of form factors given by

$$G_M = F_1 + F_2 \quad G_E = F_1 + \tau F_2 \quad \text{where } \tau = q^2/4M^2 \quad (\text{A.3})$$

appears to possess more physical significance and all data is now discussed in terms of them. It is with these form factors that we are concerned.

Inverting (A.3) $\Rightarrow F_2 = (G_M - G_E)/(1-\tau)$ and $F_1 = (G_E - \tau G_M)/(1-\tau)$ and substituting these values of F_1 and F_2 in (A.2) gives eq. (22).

Use is now made of the following identity

$$\bar{u}(P_2) i \gamma_{\mu\nu} q_{\nu} u(P_1) = 2M \bar{u}(P_2) \gamma_{\mu} u(P_1) - \bar{u}(P_2) [P_{1\mu} + P_{2\mu}] u(P_1),$$

which yields eq. (23).

Evaluating m^2

In computing the value of the square of the matrix element for the scattering process, we have to evaluate

$$L_{\mu\lambda} = \sum_{\text{lepton spins}} \langle k_2 | j_{\mu} | k_1 \rangle^2 \quad \text{and similarly for the nucleon vertex.}$$

$L_{\lambda\mu}$ is determined by inserting a positive energy projection operator $\Lambda_{+}(k) = \sum_s u(k,s) \bar{u}(k,s) = (\not{k} + m)$, and then we

have (see Feynman, "Quantum Electrodynamics" p. 112)

$$L_{\mu\lambda} = \text{Tr} [\not{k}_2 + m] \gamma_{\mu} [\not{k}_1 + m] \gamma_{\lambda}$$

which when evaluated with the use of trace theorems in Appendix 1 gives eq. (2.4).

This is a special case of the general form:

$$\begin{aligned} \sum_{\text{spins}} |\bar{u}(f) T u(i)|^2 &= \sum (\bar{u}(f) T u(i)) (\bar{u}(i) \tilde{T} u(f)) \\ &= \text{Tr} (f + m) T (\not{i} + m) \tilde{T} \end{aligned}$$

+ as referred to on p. 113 here.

where $\tilde{T} = \gamma_0 T^\dagger \gamma_0$, \dagger denotes hermitean conjugate.

In particular $\tilde{\gamma}_\mu = \gamma_\mu$, $\tilde{g}_{\mu\nu} = g_{\mu\nu}$, $\tilde{\gamma}_5 = -\gamma_5$, $\tilde{\gamma}_\mu \gamma_5 = \gamma_\mu \gamma_5$.

For the hadron vertex eq. (2.3) , when summing over nucleon spins this gives

$$\sum_{\text{spins}} H_{\mu\lambda} = \text{Tr} (\not{P}_2 + M)(A\gamma_\mu - B\gamma_\mu \gamma_5)(\not{P}_1 + M)(\gamma_\lambda \tilde{A} - \tilde{B} \gamma_\lambda \gamma_5) .$$

Evaluation of this gives $R_{\mu\lambda}$ the term relevant to the Rosenbluth formula, whose value is given in eq. (2.6a).

When polarisation of the nucleons is involved, obviously one must not sum over all spin states. To effect this we introduce spin projection operators (analogously to positive energy projection operators) which are discussed in Appendix 3.

Appendix 3

Spin Projection Operators for Spinors

In non-relativistic quantum mechanics, the spin projection operator is given by $P_{\pm} = (1 \pm 6)/2$ where $6 = \begin{pmatrix} 6_3 & 0 \\ 0 & 6_3 \end{pmatrix}$ and 6_3 is the Pauli matrix defined in Appendix 1. This projects out of an arbitrary state the spin-up (+) or spin-down (-) amplitude. For the Dirac equation we require the relativistic analogue to this, and this turns out to be $\Sigma(S) = (1 - \not{s}\gamma_5)/2$ where s is general spin vector normalised to $s^2 = -1$.

The following property of S immediately ensues. Since $\Sigma(S)$ and $\Lambda_+(p)$, the spin and energy projection operators must commute, being independent generators, we have

$$-\not{s}\gamma_5 = -\not{s}\gamma_5 = +\not{s}\gamma_5 \quad \text{using the anti-commutative property of } \gamma_5.$$

Thus $\not{s}\not{s} + \not{s}\not{s} = 0$ or, since in general $\not{a}\not{b} + \not{b}\not{a} = 2(a \cdot b)$ from the basic anti-commutation relation of the γ matrices, it follows $(S \cdot P) = 0$.

We can now immediately verify in the rest frame of the particle, the covariant operator $\Sigma(S)$ reduces to the familiar non-relativistic one $P_{\pm} = (1 \pm 6)/2$. In the rest frame $\vec{p} = 0$ so it follows from $S \cdot P = 0$ that $S_0 = 0$ and $s^2 = s_0^2 - \vec{s}^2 = -\vec{s}^2 = -1$. Therefore we choose \vec{s} a unit vector along the z direction, i.e. $\vec{s} = (0, 0, \pm 1)$, $\not{s} = -\vec{\gamma} \cdot \vec{s} = \mp \gamma_3$ and $(1 - \not{s}\gamma_5)/2 = (1 \pm \gamma_3\gamma_5)/2 = (1 \pm 6)/2$. Clearly for the general case, $\Sigma(S)$ is the covariant generalisation of the non-relativistic case. It possesses the following properties:

$$\Sigma(S) + \Sigma(-S) = 1, \quad \Sigma(S) \cdot \Sigma(-S) = 0$$

$$|\Sigma(S)|^2 = \Sigma(S)$$

Using the spin projection operator, we may proceed similarly to Appendix 2 to evaluate $|\bar{\mu}(f)T\mu(i)|^2$ when spins are not summed over. This introduces $\Sigma(S)$ for initial and final particles and leads to the general expression:

$$|\mu(f)T\mu(i)|^2 = \text{Tr}(\not{f} + m) \frac{1 - \not{\beta}_f \gamma_5}{2} T(\not{\chi} + m) \frac{1 - \not{\beta}_i \gamma_5}{2} \tilde{T}$$

Applying this to the hadron vertex of eq. (2.3) gives immediately (2.5). When only one of the spin correlations is involved we sum over the unobserved spin; thus i.e. for polarised initial particle, final polarisation summed over,

$$\begin{aligned} \sum_{S_f} |\mu(f)T\mu(i)|^2 &= \sum |\mu(f)T\mu(i)|^2 + \sum |\mu(f)T\mu(i)|^2_{(S_f \rightarrow -S_f)} \\ &= \text{Tr}(\not{f} + m) T(\not{\chi} + m) \frac{1 - \not{\beta}_i \gamma_5}{2} \tilde{T} \end{aligned}$$

The two properties $S^2 = -1$, $S \cdot P = 0$ determine the general value of S for arbitrary configurations of S . Written out in terms of components,

$$S^2 = S_0^2 - \bar{S}^2 = -1 \quad \text{and} \quad S \cdot P = S_0 E - \bar{S} \cdot \bar{P} = 0$$

The second property may be rewritten $S_0 = \bar{S} \cdot \bar{\beta}$ where $\bar{\beta} = \bar{P}/E$. Combining this with $S^2 = -1$ gives $\bar{S} = [1 - (\hat{S} \cdot \bar{\beta})^2]^{-1/2}$.

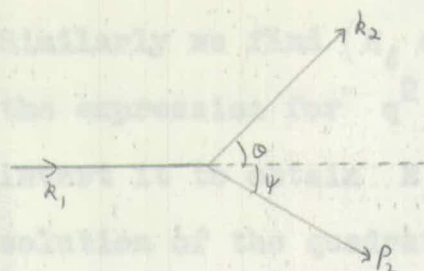
Consider two specific cases now:

a) $\bar{S} \cdot \bar{P} = 0$ i.e. transverse polarisation of the particle. This immediately gives $S_0 = 0$ since $E \neq 0$ and $|\bar{S}| = 1$.

∴ $S = (0, 1)$ in component form.

b) $\vec{S} \cdot \vec{\beta} = \beta$ i.e. longitudinal particle polarisation, spin vector parallel to the momentum (right handed particle). Then $|\vec{S}| = [1 - (\beta)^2]^{-1/2} = E/m$ and $S_0 = |\vec{P}|/m$. Similarly for the spin polarisation antiparallel to the momentum (left-handed particle), we have $\vec{S} \cdot \vec{\beta} = -\beta$ giving $|\vec{S}| = E/m$ and $S_0 = -|\vec{P}|/m$.

Appendix 4



We consider the kinematics of lepton-nucleon elastic scattering as shown in the figure.

$k_1 \equiv (E_l, E_l, \hat{k}_1)$ = four-momentum of incoming lepton (We take lepton mass = 0)

$P_1 \equiv (M, 0)$ = " " " target nucleon

$k_2 \equiv (E'_l, E'_l, \hat{k}_2)$ = " " " scattered lepton

$P_2 \equiv (E_B, \vec{P}_B, P_2)$ = " " " recoil nucleon.

Energy-momentum conservation gives $P_1 + k_1 = P_2 + k_2$; the 4-momentum transfer $q = k_1 - k_2 = P_2 - P_1$. These yield the following terms: $P_1 k_1 = ME_l$, $P_1 k_2 = ME'_l = P_1 \cdot (k_1 - q) = ME_l + q^2/2$.
 $q^2 = -2k_1 k_2 = -2E_l E'_l (1 - \cos \theta)$. Also $q^2 = 2M^2 - 2P_1 P_2 = 2M(M - E_B)$
 i.e. $E_B = -q^2/2M + M = M(1 - 2\tau)$ where $\tau = q^2/4M^2$. The last term leads to $\vec{P}_B^2 = E_B^2 - M^2 = q^2(\tau - 1)$.

Explicitly writing out the energy-momentum eqs. $E_l + M = E'_l + E_B$

$$E_l = E'_l \cos \theta + \vec{P}_B \cos \psi; \quad E'_l \sin \theta = \vec{P}_B \sin \psi.$$

Eliminating \vec{P}_B and ψ these equations may be solved to give

$$E'_l = E_l / \left[1 + \frac{E_l}{M} (1 - \cos \theta) \right].$$

$$\begin{aligned} \text{Hence } E_l - E'_l \cos \theta &= E_l \left[1 - \frac{\cos \theta}{1 + \frac{E_l}{M} (1 - \cos \theta)} \right] \\ &= \frac{E_l (1 - \cos \theta) (1 + (E_l/M))}{\left[1 + \frac{E_l}{M} (1 - \cos \theta) \right]} \end{aligned}$$

$$\text{and } q^2 = -2E_l E'_l (1 - \cos \theta) = -2E_l^2 (1 - \cos \theta) / \left[1 + \frac{E_l}{M} (1 - \cos \theta) \right]$$

whence we note we can express $(E_l - E'_l \cos \theta) = -q^2(1 + \frac{E_l}{M})/2E_l$.
 Similarly we find $(E_l \cos \theta - E'_l) = q^2(1 - \frac{E_l}{M} \cos \theta)/2E_l$. From
 the expression for q^2 in terms of E_l and $\cos \theta$, we may
 invert it to obtain E_l in terms of q^2 and $\cos \theta$, as the
 solution of the quadratic equation

$$4E_l^2 \sin^2 \frac{\theta}{2} + \frac{2E_l}{M} \sin^2 \frac{\theta}{2} q^2 + q^2 = 0, \quad \text{with solution}$$

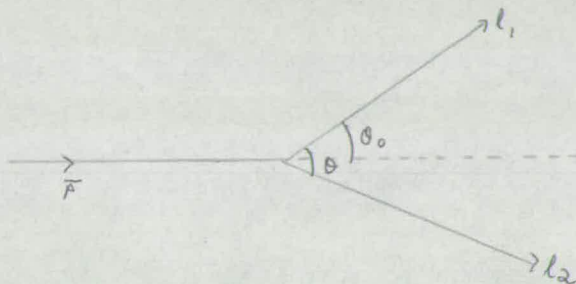
$$E_l = -M\tau + \frac{1}{2}\sqrt{q^2} (\tau - \operatorname{cosec}^2 \frac{\theta}{2})^{1/2} \therefore \tau + \frac{E_l}{M} = \left[\tau (\tau - \operatorname{cosec}^2 \frac{\theta}{2}) \right]^{1/2}$$

where the positive value in front of the square root has been
 taken as $(\tau + \frac{E_l}{M}) > 0$.

From the relations $q^2 = -2E_l E'_l (1 - \cos \theta)$,
 $E'_l \sin \theta = \bar{P}_B \sin \psi$, eliminating E'_l yields the connection
 between θ and ψ , namely $\bar{P}_B \sin \psi = -q^2 \cot \frac{\theta}{2} / 2E_l$.

APPENDIX 5

Kinematics of Nucleon-antinucleon annihilation into leptons.



ℓ_1 and ℓ_2 denote the four-momentum of the lepton and antilepton respectively. E_B is the incoming laboratory energy of the antinucleon and \bar{p} its 3-momentum, i.e. $E_B^2 = \bar{p}^2 + M^2$. The four-momentum transfer $q = \ell_1 + \ell_2 = t_1 + t_2$, where $t_1(t_2)$ is the nucleon (antinucleon) four-momentum. (Lepton mass terms are here neglected). It follows immediately that $q^2 = 2\ell_1 \cdot \ell_2 = 2E_\ell E'_\ell (1 - \cos \theta) = 2M(M + E_B)$. $E_\ell(E'_\ell)$ represents the lepton (antilepton) laboratory energy, and θ is depicted above.

Energy conservation gives $M + E_B = E_\ell + E'_\ell$ (i)

Momentum " " " $\bar{p} = E_\ell \cos \theta_0 + E'_\ell \cos(\theta - \theta_0)$ (ii)

$E_\ell \sin \theta_0 = E'_\ell \sin(\theta - \theta_0)$ (iii)

Solving for E_ℓ , we find

$$E_\ell = \frac{q^2}{2[M + E_B - |\bar{p}| \cos \theta_0]} \quad \text{and} \quad E'_\ell = \frac{q^2}{2M} \frac{(E_B - |\bar{p}| \cos \theta_0)}{(M + E_B - |\bar{p}| \cos \theta_0)}$$

Alternatively eliminating θ_0 ,

$$E'_\ell = \frac{q^2}{2[M + E_B - |\bar{p}| \cos(\theta - \theta_0)]} ; E_\ell = \frac{q^2 [E_B - |\bar{p}| \cos(\theta - \theta_0)]}{2M [M + E_B - |\bar{p}| \cos(\theta - \theta_0)]}$$

Note also the relations

$$|\bar{p}| \cos \theta_0 = E_\ell + E'_\ell \cos \theta . \quad |\bar{p}| \sin \theta_0 = E'_\ell \sin \theta$$

$$|\bar{p}| \cos(\theta - \theta_0) = E_\ell \cos \theta + E'_\ell . \quad |\bar{p}| \sin(\theta - \theta_0) = E_\ell \sin \theta$$

whence we have $\tan \theta = |\bar{p}| \sin \theta_0 / [|\bar{p}| \cos \theta_0 - E_\ell]$

and from $q^2 = 2M(M + E_B)$ it follows $E_B = M(2t-1)$ and

$$\bar{p}^2 = q^2(t-1).$$

Inverting the expression for E_ℓ gives $(E_\ell)^{-1} q^2 = 4M \left[t - \frac{|\bar{p}|}{2M} \cos \theta \right]$
and similarly for E'_ℓ .

APPENDIX 6

The expression for the W_1^V (eq. (5.8)) are given here for the elastic case in terms of the g_V, g_A etc. form factors defined in (5.27).

$$\delta \left(\nu - \left[\frac{M^2 - \eta^2 - q^2}{2} \right] \right) W_1^V = (\Delta^2 - q^2) g_V^2 / 2 + (\Sigma^2 - q^2) g_A^2 / 2.$$

$$W_2^V = 2M^2 \left\{ (g_V - f_V \frac{\Sigma}{M})^2 + (g_A - f_A \frac{\Delta}{M})^2 - \frac{q^2}{M^2} (f_V^2 + f_A^2) \right\}$$

$$W_3^V = 2M^2 \operatorname{Re}(g_V g_A^*)$$

$$W_4^V = M^2 \left\{ -W_2^V / 2M^2 + 2W_5^V / M^2 - \left| g_V + \frac{\Delta h_V}{M} \right|^2 - \left| g_A + \frac{\Sigma h_A}{M} \right|^2 + (\Sigma^2 + \Delta^2 - q^2)(h_V^2 + h_A^2) / M^2 \right\}$$

$$W_5^V = M^2 \left\{ W_2^V / 2M^2 + \operatorname{Re} \left[(g_V \frac{\Sigma}{M} - f_V \frac{(\Sigma^2 - q^2)}{M^2}) h_V^* - (g_A^* \frac{\Sigma}{M} + \frac{(\Delta^2 - q^2)}{M^2} h_A^*) f_A + \frac{\Delta}{M} (g_V^* f_V - g_A^* h_A^*) \right] \right\}$$

$$W_6^V = M^2 \operatorname{Im} \left\{ (g_V \frac{\Sigma}{M} - f_V \frac{(\Sigma^2 - q^2)}{M^2}) h_V^* - (g_A^* \frac{\Sigma}{M} + \frac{(\Delta^2 - q^2)}{M^2} h_A^*) f_A + \frac{\Delta}{M} (g_V^* f_V - g_A^* h_A^*) \right\}$$

$$W_7^V = -2M^2 \operatorname{Re}(g_V g_A^*) = -W_3^V : \delta(\nu) W_8^V = 4M^2 \operatorname{Re} [2f_V f_A^* - g_V f_A^* + g_A f_V^*]$$

$$W_9^V = 2M^2 \operatorname{Re} [h_V h_A^* + f_V f_A^* - f_V h_A^* - h_V f_A^*]$$

$$W_{10}^V = M^2 \operatorname{Re} [4f_V f_A^* - 2(f_V h_A^* + f_A h_V^*) + g_V (h_A^* - f_A^*) - g_A (h_V^* - f_V^*)]$$

$$W_{11}^V = M^2 \operatorname{Im} [-2(f_V h_A^* + f_A h_V^*) + g_V (h_A^* - f_A^*) - g_A (h_V^* - f_V^*)]$$