

STUDIES WITH A MAGNETIC β -PARTICLE
SPECTROMETER OF NOVEL DESIGN.

Thesis

for the degree of

DOCTOR OF PHILOSOPHY

submitted

by

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University of Edinburgh,
June, 1951.



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PREFACE

This work has been principally concerned with the construction and testing of a new type of magnetic β -spectrometer. The design was based upon an analogy between the electron optical properties of a prolate spheroidal magnetic field and image formation by a concave mirror surface.

In Section A an alternative model is discussed in relation to conventional magnetic spectrometers. This model depends on the suitable positioning of two refracting surfaces so as to correct the spherical aberration of each in a special sense. Energy discrimination is obtained by the selection of one, or two, meridional focal lines, resulting in a relationship between resolution and solid angle which involves powers of the latter. The focusing principle is not confined to image formation by lenses.

In Section B the spheroidal field spectrometer is described. It has been found that it is possible to obtain a high solid angle, of the order of 5% of 4π without undue sacrifice of resolution and that with care resolutions of $1/200$ may be used. A disadvantage is the small source diameter. The performance of the instrument is in agreement with the general predictions of Section A.

Section C concerns a measurement of the

continuous β -spectrum of Tl^{204} . A value of 764 ± 10 kev has been found and a spectrum shape in agreement over the range measured, with that predicted by the Fermi theory for a first forbidden transition with spin change of two units. This is not wholly in agreement with a previous measurement.

The initiation of this research was due to Dr. H.O.W. Richardson, who made the original design of the spectrometer. The work of Section B was done in collaboration with him, and I wish to thank him for the introduction to β -ray spectroscopy technique, for the design of much of the mechanical construction of the spectrometer and for his interest, advice and assistance generally during the taking of the measurements. Some of the work of Section B is to be described in a paper in the course of publication, together with a fuller discussion of the magnetic mirror analogy than is given here.

I also wish to thank the members of the workshop and technical staff for their help in setting up the apparatus.

SECTION A.

INTRODUCTION.

The spectroscopy of the radiations from β - active bodies has required the application of several distinct methods of investigation, partly because of technical difficulties and partly because of the different objects with which experiments have been conducted. Each of these methods is, generally speaking, of a limited range of usefulness and reliability and so the method chosen must depend on the substance to be investigated and the information it is desired to obtain. Nevertheless the analysis of the particle momentum by deflection in a suitable magnetic field is in most respects so far superior to other possible methods that it is employed almost exclusively in the general field of β -ray work, and most of the effort recently expended in technical advancement has been applied to the improvement of magnetic spectrometers.

The main interest, experimentally, lies in such questions as the elucidation of γ -ray level schemes by detection of either the natural γ -ray lines or the excited photo-electrons or Compton recoil electrons, and in the comparison with theoretical predictions of the shapes of continuous spectra, particularly in the case of forbidden transitions. Of these,

the first requires a fairly high, and in some cases very high, degree of energy discrimination, and both require a high efficiency of detection of the emitted particles, this question being particularly acute in the low energy region where very thin sources must be used to avoid errors due to self-absorption. The absorption method requires only weak sources but it is unable to give reliable spectrum shapes, or any detailed and precise information, and the cloud chamber, though capable of more precision, is very slow and laborious. The recent developments of the proportional counter and scintillation counter spectrometers as well as β -sensitive photographic emulsions have produced powerful weapons for weak activities, though the first and last are limited to fairly low energies (as hitherto applied) and all have resolving powers of the order of five per cent. or more.

The principal advantages of magnetic spectrometers are their greater resolving power and greater energy range, making it desirable to apply the method as widely as is permitted by other considerations, such as the solid angle of collection, which is, in general, no more than a very few per cent. The central problem of spectrometer design is to maintain or improve the resolving power while improving the solid angle, or at least retaining a value of the latter.

sufficiently high to make feasible measurements with low activities. The two aims are to a great extent conflicting; a gain in one factor, in a given instrument, can always be obtained by some sacrifice of the other, but many types of instrument have been designed to improve the ratio, generally by improving greatly the latter at some sacrifice of the former. Such an improvement is of great importance for a great many problems where highest resolution is not needed, and in particular in cases where events are investigated in coincidence schemes. The work to be described here concerns the development and investigation of a new type of spectrometer primarily intended to increase the solid angle considerably, while retaining moderately high resolution, and which has been discovered also to have some possibility of adjustment for high resolution.

The electron optical properties are unusual and will be discussed in Section A in relation to more conventional focusing systems and the experimental tests will be described in Section B.

I. CONVENTIONAL β -SPECTROMETERS.

The earliest β -spectrometer, due to Danysz (1911), was that known as the semi-circular type, in which for the first time magnetic deflection of the rays was accompanied by a focusing action. This has been more often used than any other system and still for many purposes retains a pre-eminent place.

Essentially, the particles move in one plane and are deflected by a transverse uniform field so that they take up circular orbits; rays of slightly different inclinations to a central ray (Fig.A3(a)), whose initial direction is at right angles to the line which passes through source and counter slit, are brought to a focus after the central ray has transversed 180° of arc. Rays of slight inclination to this median plane perform flat helical orbits and so the image from a point source becomes a short line. The width of this line is

$$\Delta \rho = 2\rho (1 - \cos \alpha)$$

where the source is assumed to have no width, so the resolving power may be expressed as

$$\frac{\Delta H \rho}{H \rho} = 2 (1 - \cos \alpha)$$

This means that for small α there is a high concentration of rays on a narrow counter slit and the resolving power is high, but $\Delta \rho$ increases very rapidly with increasing α and therefore any considerable increase of the aperture of the instrument

results in rapid deterioration of the resolving power. This is illustrated by the asymmetrical line shape (Fig. A3(b)) with its distinct tail to the lower energy side. The solid angle is further limited by the fact that rays of only small inclinations (5° in general) to the median plane can be accepted. This means that in practice the semi-circular focusing spectrometer is used where high resolution is required and solid angle is a minor consideration, or where absolute measurements are to be made, when the simplicity of the field is an advantage for calibration. Resolving powers of one in several hundreds are easily obtained, with the solid angle less than .1%.

The width of the focused image can be made less, and some focusing of the rays in a direction at right angles to the mid-plane can be obtained by shaping the pole pieces suitably. For instance, Richardson has shown (1947) that by inclining the pole-pieces to each other so that the rays move through a region where the field varies by a factor of 10, the width of the image is decreased and the component of the field parallel to the central plane results in some focusing effect on the spiralling rays, thus shortening the length of the image. This makes possible an improvement of the solid angle by a factor of 16 for the same resolving power when compared with a semi-circular instrument of similar dimensions.

Langer and Cook have shown (1948) that a similar effect on the normal 180° image can be obtained by using pole-pieces tapered radially. A considerable extension of this idea will be discussed later.

Focusing in 3 dimensions is most easily obtained by using an axially symmetric field as a lens system. There are two classes of operation which have been used by different workers, illustrated in Fig. A7(b) for the ordinary optical lens. If a sheaf of rays falls on the lens at a large aperture the spherical aberration results in the production of two focal lines, the "meridional" line M, normal to the meridional plane which results in a circle or "ring focus" surrounding the axis and is due to the intersection of adjacent rays on a caustic, and an axial line AB. Only in the paraxial zone of the lens do these coincide to give a point image I of O; in general the best image is the circle of least confusion Pq. The spherical aberration, in this case, (where rays of greatest initial inclination return first to the axis) is by convention positive. Since for a given sheaf selected by the ring baffle $B_1 B_2$ the position of M and Pq depends on the energy of the focused rays, either may be made the basis of a spectrometer; the former will be referred to as meridional focusing and the latter will be called axial focusing.

The earliest lens spectrometer, due to Tricker

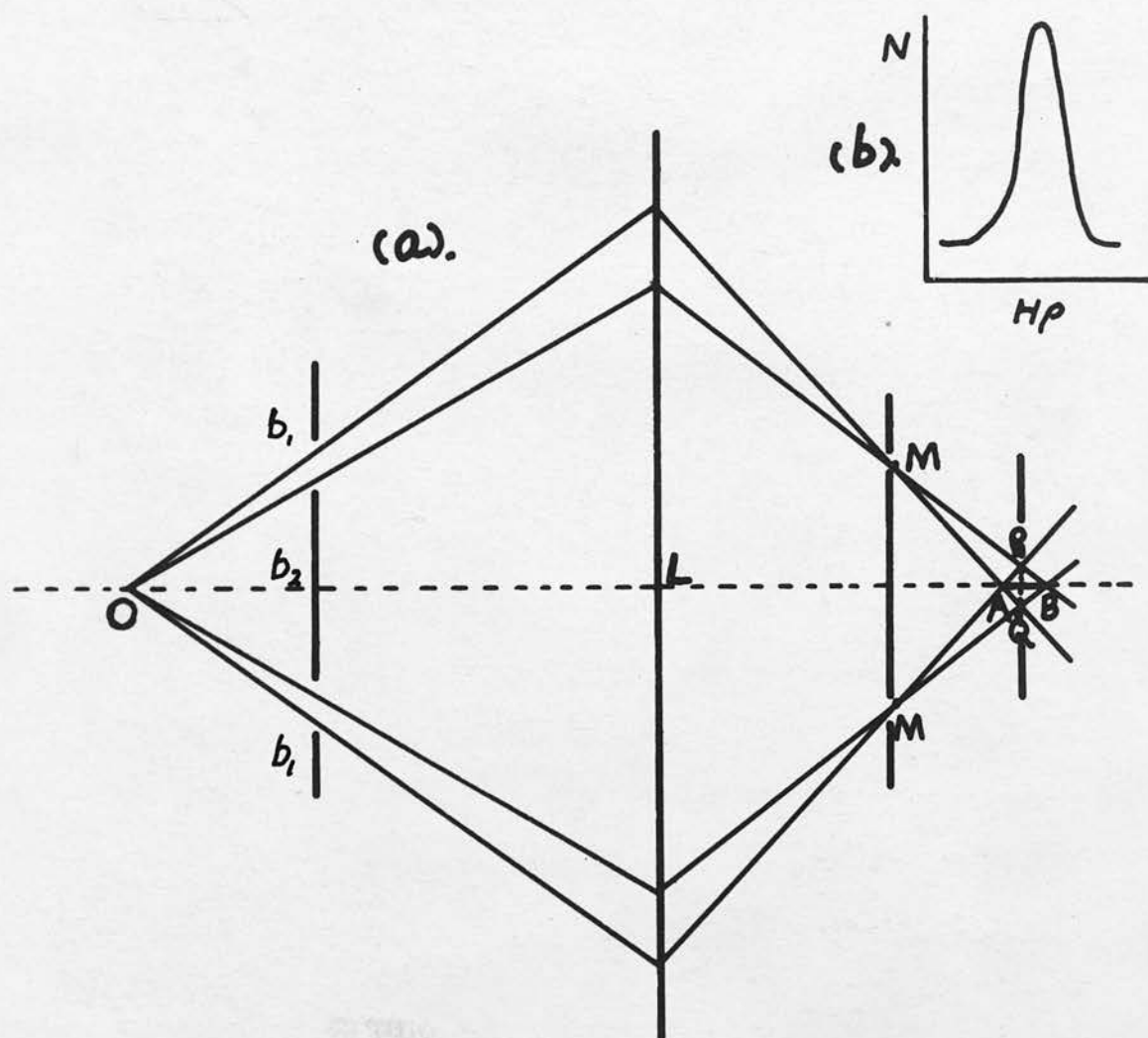


Fig.7 (a) Axial and meridional focusing in a thin lens
 (b) Typical line shape from axial focusing in a lens spectrometer

(1924) made use of the helical trajectories of particles in a uniform field produced by a long solenoid. There is no genuine image in this case, but rays return to the axis after a distance

$$Z = v \cos \theta \frac{2\pi m v}{He}$$

*v, e, m, = electron velocity, charge, mass
θ = emission angle
H = magnetic field*

so that for small θ there is a focusing effect. This type is now disused.

The first applications to β -spectroscopy of the short magnetic lens (Busch 1926) were made by Klemperer (1935), Cosslett (1940), and Siegbahn (1940) and many such spectrometers have been constructed since, almost invariably using axial focusing. The general result has been that the solid angle has been increased to a value of several per cent at a resolving power usually of 2-3% (or more) and it is not possible, as in the 180° case to obtain very high resolving power without very drastically reducing the solid angle, though the solid angle is readily increased at expense of resolution. The reason for this lies in the very high value of the spherical aberration in magnetic lenses, which have only a very small paraxial region. This cannot be used in spectroscopy (as is done in microscopy by stopping down the lens) since the low angle rays must be stopped off for energy discrimination, and so the normal procedure is to select the circle of least confusion by having a counter window equally large. The lack of high resolution then appears as being

due to the non-availability of the paraxial low aberration rays; once again this is illustrated by the line shape (Fig. A7(b)) which is usually roughly triangular.

It is of great importance to reduce the spherical aberration as much as possible. Since all magnetic lenses are positive in power it is not possible to correct the aberration by combining lenses as in the ordinary optical case, and Scherzer has said that the aberration can never be zero in a system of magnetic lenses, but of course it is always possible to reduce it by judicious lens design, i.e., as Siegbahn (1944) has pointed out, by reducing the gradient of the radial component of the field in the centre of the lens.

Glaser (1940) has shown that a field having infinite values at axial extremes with a concavity in the centre of the lens can actually give zero aberration, and that a considerable reduction is still obtained though the field remain finite at the extremes. Siegbahn (1946) has constructed a lens with such a characteristic shape of field by closing the ends of a long coil with plane pole pieces. He found this to result in an improvement by a factor of two in solid angle over a previous lens constructed by him, though the performance (2-3% solid angle, 3-5% resolving power) is still not remarkable. Several other spectrometers have been designed to have a

minimum in the centre of the lens by using two separated focusing coils (e.g. Waade-Halliday (1948)).

More recently Geneviève, Paquien & Grivet (1950) have treated this type of spectrometer as a case of two bell-shaped lens fields at a distance from each other with the source at the centre of one and the counter at the centre of the other. Glaser has already shown that for the bell-shaped field

$$H = \frac{H_0}{1 + \frac{z^2}{a^2}}$$

the aberration is a minimum when the source is at the centre of the lens (though the image is virtual) so that calculation of the trajectories (using Glaser's paraxial equations) in the double lens case leads to values for baffle radii, optimum separation etc. Agnew & Anderson (1949) have constructed such a spectrometer, using two focusing coils, one metre apart, and have obtained a very considerably improved performance - resolving power 1%, solid angle 1.4%. Genuine progress is therefore being made in this direction.

Meridional focusing has been much neglected until fairly recently, and only one lens spectrometer has actually been operated on this principle. This was constructed by Witcher (1941) who used the focal ring formed by a solenoid one metre long, with rays inclined at 18 degrees to the axis. The performance obtained was very moderate (resolving power 5%, solid angle 1%) and he did not give a detailed

theory of the instrument. Frankel has pointed out, however (1948), that such an instrument should, if operated with rays inclined at much larger angles to the axis ($30-60^\circ$), have a higher ratio of solid angle to resolving power than any other spectrometer, and in general, that the resolving power for given trajectories (at the same solid angle therefore) is increased as against the axial focusing case, simply since better definition is obtained with a narrow ring baffle at the intersection with the caustic than with a broad counter window at the disc of least confusion.

Du Mond (1949) and Persico (1949) have both produced theoretical treatments of this case, and have shown that for 45° it is possible to obtain, at a solid angle of 1% , resolving power as high as 0.3% . Very recently Keller, Koenigsberg & Paschin (1950) have produced a computation of the large angle trajectories in a thin magnetic lens and conclude that for this particular lens the use of the meridional focal ring gives an improvement of the solid angle by a factor of two for the same resolving power. It seems likely that meridional focusing will be used more often in future developments.

Thus it is seen that in general semi-circular focusing has been used for high resolving power, and the various types of lens for high collection power. The reason for this is that the lens, although possessing the advantage of space focusing, suffers

from the disadvantage that the image is formed by rays for which the spherical aberration is large since the "central" rays must be blocked off.

There is however one further type of focusing field which possesses some of the properties of both the fundamental types which have been discussed. This is the so-called double focusing spectrometer, due originally to Siegbahn & Svertholm (1946). The space focusing property of the radially inhomogeneous field has already been mentioned with respect to the 180° focus; if the field varies as $1/\sqrt{r}$ then one has the "betatron focusing" action, and the axial and radial focusing are of equal strength in that they produce a non astigmatic image after a rotation of angle $\pi\sqrt{2}$. This may be regarded as a lens with a curvilinear axis, since the image formation has all the properties of a Gaussian optical system. This means that the central rays are now used, giving high resolving power, along with the high solid angle provided by the space focusing; the line shape corresponds with the "semi-circular type" shape, as is to be expected. This instrument gives a resolving power of 1% or better with a solid angle of 1%, and the writer understands that Siegbahn has found it possible to obtain $R = 0.1\%$ though these figures have not been published.

The spectrometer whose investigation is to be discussed uses an axially symmetric lens field and

central rays, though of a rather special type, and achieves correction of the aberration, and even negative aberration, in a manner quite different from any discussed in this section.

III. SECOND IMAGE SPECTROMETERS.

The purpose of this section is two-fold, firstly to give an account of the original paper on the focusing properties of the prolate spheroidal field (Richardson, 1949), on which is based the experimental work of Section B, and secondly to discuss in more general terms the class of focusing action to which it belongs, to be referred to as second image focusing. Since two published papers on this subject have been written from quite different viewpoints they will be discussed in their original terms, first, to serve for reference to Section B.

(1) Prolate Spheroidal Field (Richardson 1949).

The starting point of the original paper was an attempt to improve the performance of the lens spectrometer by using larger angles of emission in a type of field which has some analogy with a cylindrical mirror.

In an axially symmetric magnetic field the trajectories of rays radiating from a point on the axis may be fully described in terms of their motion in a meridional (r, z) plane passing through the axis, and the rotation ϕ of this plane about the axis. The motion in this meridional plane is then governed by a (scalar) potential energy $V(r, z)$ in this plane. In this special case of trajectories originating on the axis, V is given by the azimuthal component of

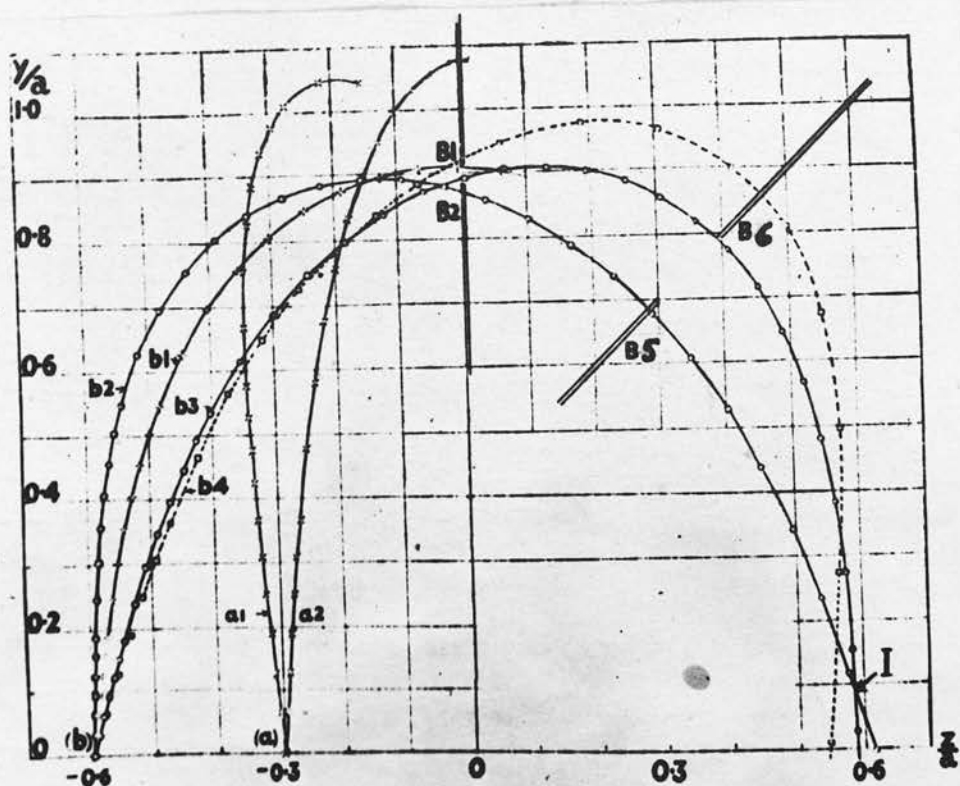
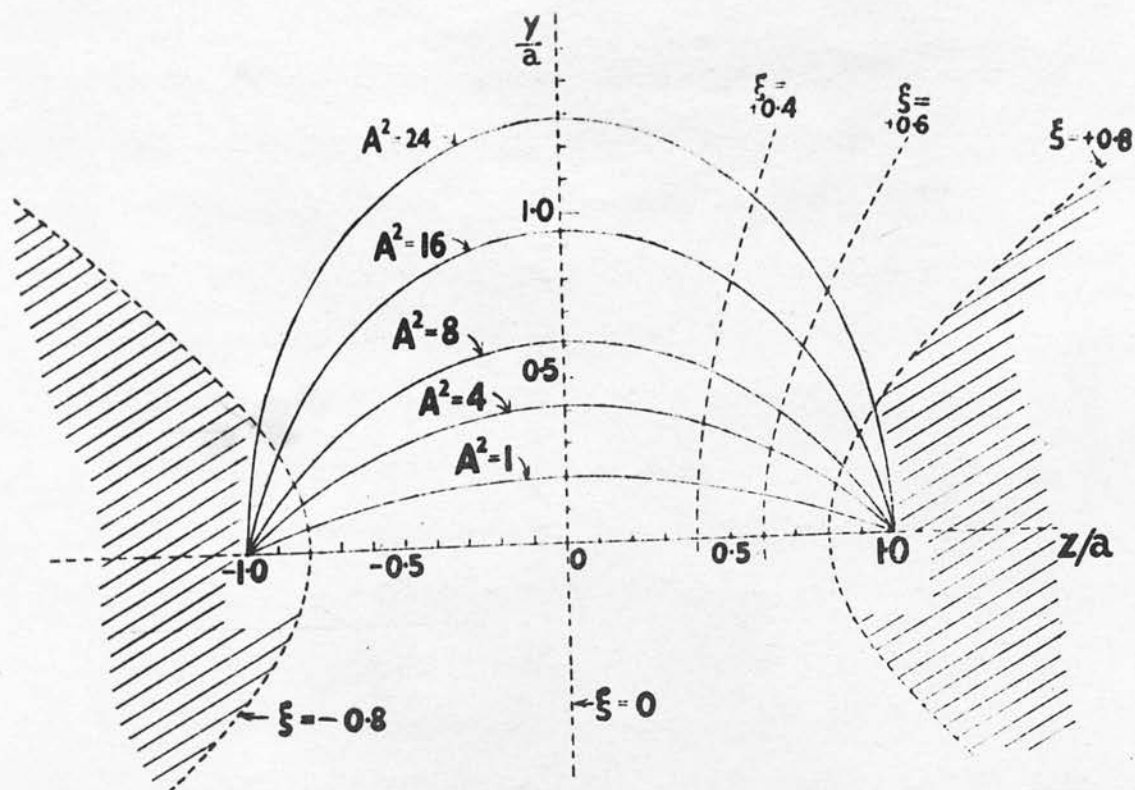


Fig. 15 Spheroidal magnetic field spectrometer
 (a) Countours of vector potential
 (b) Trajectories in the field as calculated by ray tracing
 In the model constructed "a" was chosen equal to 10 cms.

the vector potential $A(r,z)$ and is in fact proportional to A^2 . The contours of A for a lens field then form a potential barrier and the refraction of a family of rays of a particular momentum in the field may be pictured to some extent as a repulsion from one of the contours. In particular, if these contours are bowl shaped this repulsion may, for a range of angles of incidence, possess analogies with the reflection of a beam from a concave mirror, including some focusing action.

The field form chosen by Richardson was that in which the equipotentials are hyperboloids of revolution and the field lines lie on the orthogonal family of confocal ellipsoids - a convenient form for computations. The form of the contours of equal vector potential is shown in Fig. A15, showing, particularly for larger radii, a distinct concavity from which reflection properties might be expected. He shows that the contour for which both centres of curvature lie on the symmetry axis is that which crosses the mid-plane at a radius r equal to $.87a$, $2a$ being the focal distance of the conicoids, so that this particular potential barrier should have a true mirror analogy and form an axial image of an axial object, the object and image being symmetrically placed about the mid-plane. To obtain more precise information he computed (by the method of Goddard & Klemperer, 1944) trajectories for a family of rays crossing the mid-plane at near

this value of the radius. The result of this computation is shown in Fig. A15, where a family originating at $z = 0.6a$, and one inclined at $\theta = 74^\circ$ to 84° to this axis, is shown to give two meridional focal rings, one near the mid-plane as expected from the reflection hypothesis and one near the axis, suitable for detection by a small counter, at which the aberrations are much less, all three computed rays appearing to pass quite closely through the same point. Thus two defining slits $B_1 B_2$ and $B_3 B_4$ are needed, the first to provide the main momentum selection, passing no rays of lesser momentum than that of the central ray which just reaches the slit (a "central" ray being one which falls normally on the mid-plane), and the second to take advantage of any chromatic aberration in the formation of the second focal line. Other defining baffles $B_5 B_6$ are needed to stop rays of higher momentum such as b_4 which also manage to pass through $B_1 B_2$. (This ray shows chromatic aberration at the second crossover).

There are thus two contributions to the resolving power, but that to be obtained from the second slit depends on the minimisation of non-chromatic aberrations, including that due to the size of the source. The solid angle should be large since Fig. A15 shows a concentration on $B_1 B_2$ of width $.02a$ of 8.5% of the total emission. However, the actual solid

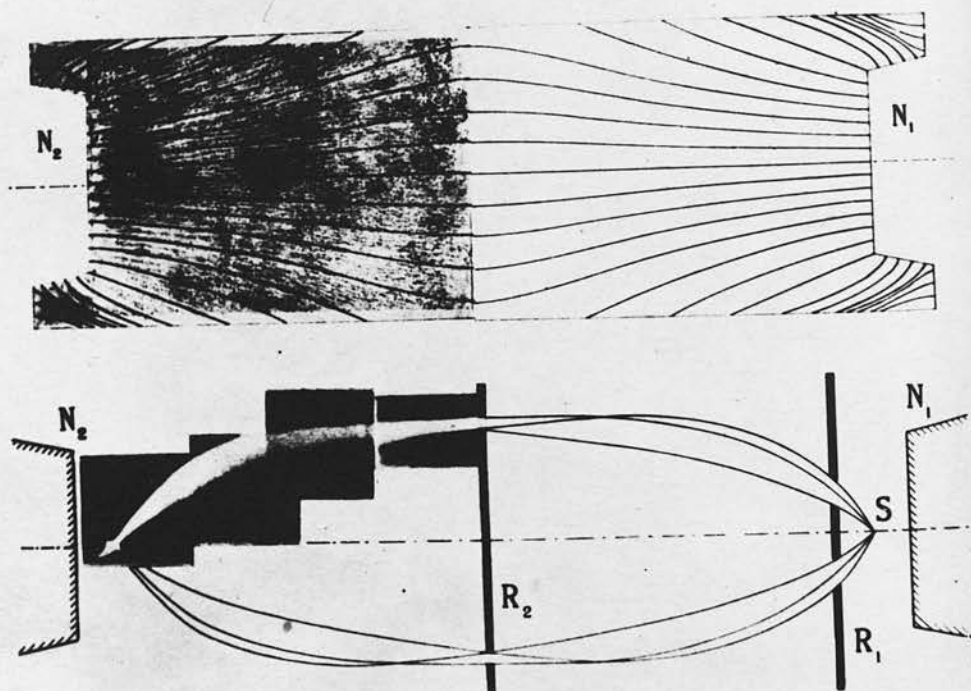
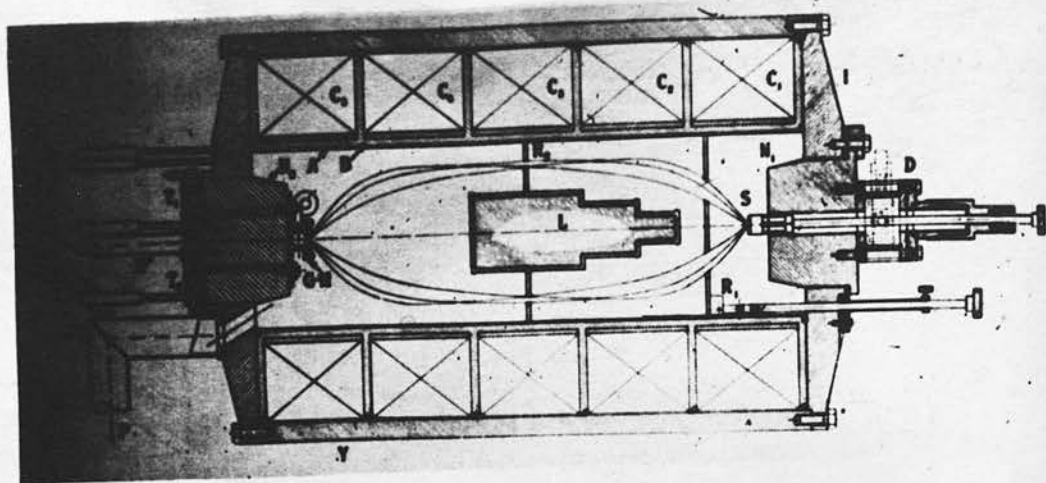


Fig. AI7 "Intermediate image" spectrometer
 (a) Construction
 (b) Field lines, and trajectories obtained by
 a photographic method

angle and resolving power likely to be obtained can only be determined experimentally (or else by a considerably laborious computation of a large number of trajectories which Richardson did not undertake).

As has been more specifically pointed out recently (Braid and Richardson, 1951), the aberration of the second image is actually negative (and the choice of a smaller value of r could lead to a corrected point image on the axis). It is possible to regard the correction of the aberration as a result of a combination of lens and mirror action since electron mirrors, as opposed to electron lenses, have negative spherical aberration (Cosslett, 1940). However, the mirror analogy, though highly convenient, is obviously not fundamental and it will not be pursued further here (it is discussed more fully in the paper mentioned) and instead the correction for aberration will be discussed later in terms of a pure refraction model.

(2) The "Intermediate Image" Spectrometer:

This name has been applied by Slatis and Siegbahn (1949, 1950) to a lens field which, planned originally as a further step towards the reduction of the aberration by the method already mentioned in A II, (that of using a lens with a minimum in the field strength between object and image) has been found to

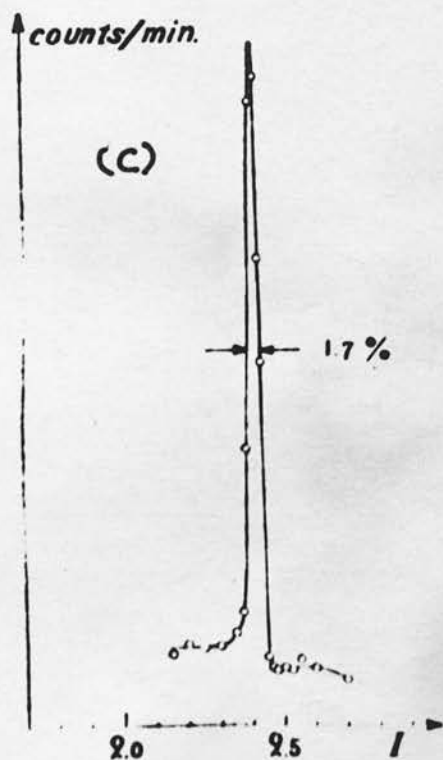
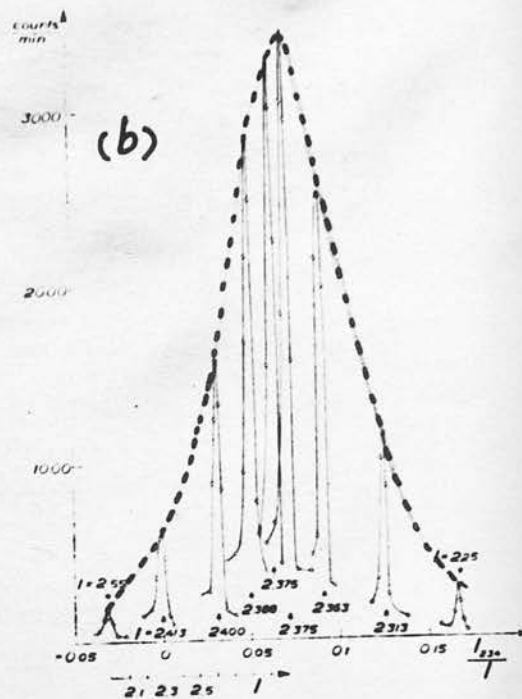
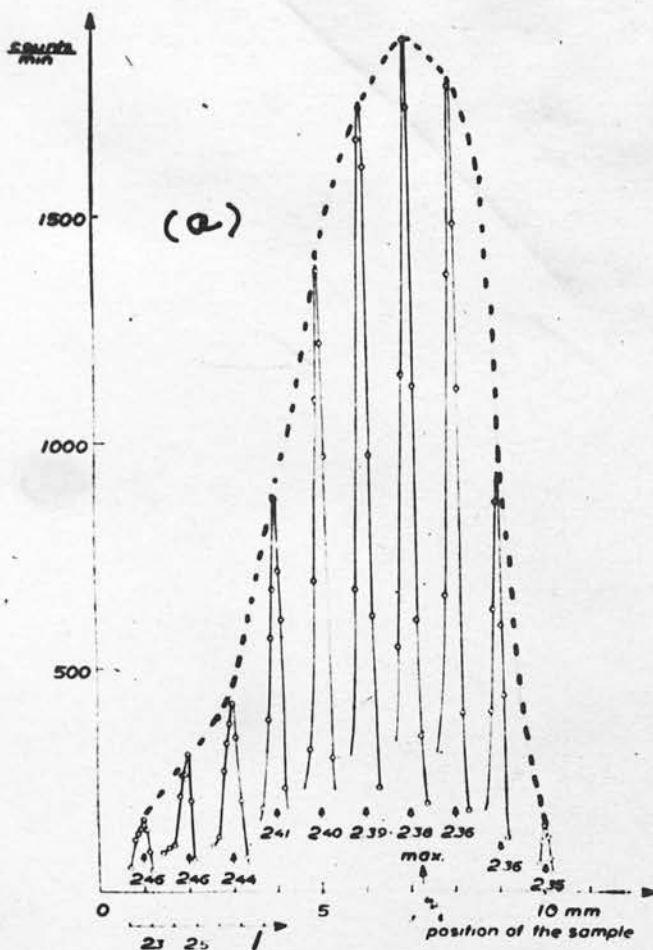


Fig. A18 "Intermediate image" spectrometer
 (a) Effect of motion of source along axis
 (b) Effect of varying ratio of coil currents
 (i.e. field shape). Peak of curve corresponds
 to a distinct minimum in the axial value of the
 field.
 (c) Typical line shape.

give trajectories showing the same general characteristics as those discussed in (1), i.e., a first ring focus in the mid-plane with a second image on or near the axis.

The field was formed by a long coil system in an iron-cased lens (Fig. A17), the ratio of currents in the different coils being adjusted to obtain the desired field gradient. For the case when the field had a very distinct minimum between the object and image, Fig. A 17 shows the field lines and the trajectories as obtained by a photographic method. The only essential difference between them and the trajectories in the spheroidal field is that in this case the regions of greatest deviation are considerably separated from the cross-over, near which, in fact, the paths are almost straight. This means that the reflection analogy cannot be applied.

with a 2 mm. diameter source and a width of 2mm. for the main selecting slit a resolving power of 1.7% was obtained, while if the mid-plane slit was widened to 8 mm., the resolving power was 4% and the solid angle 8% of 4π .

Figures A18(a) and (b) show two sets of curves from Slätis and Siegbahn's paper, showing the effect on the transmission (i.e. the maximum counting rate at the peak of the F-line of Thorium B) of variation of the field gradient for a given radius of central baffle, obtained by varying the ratio of the currents

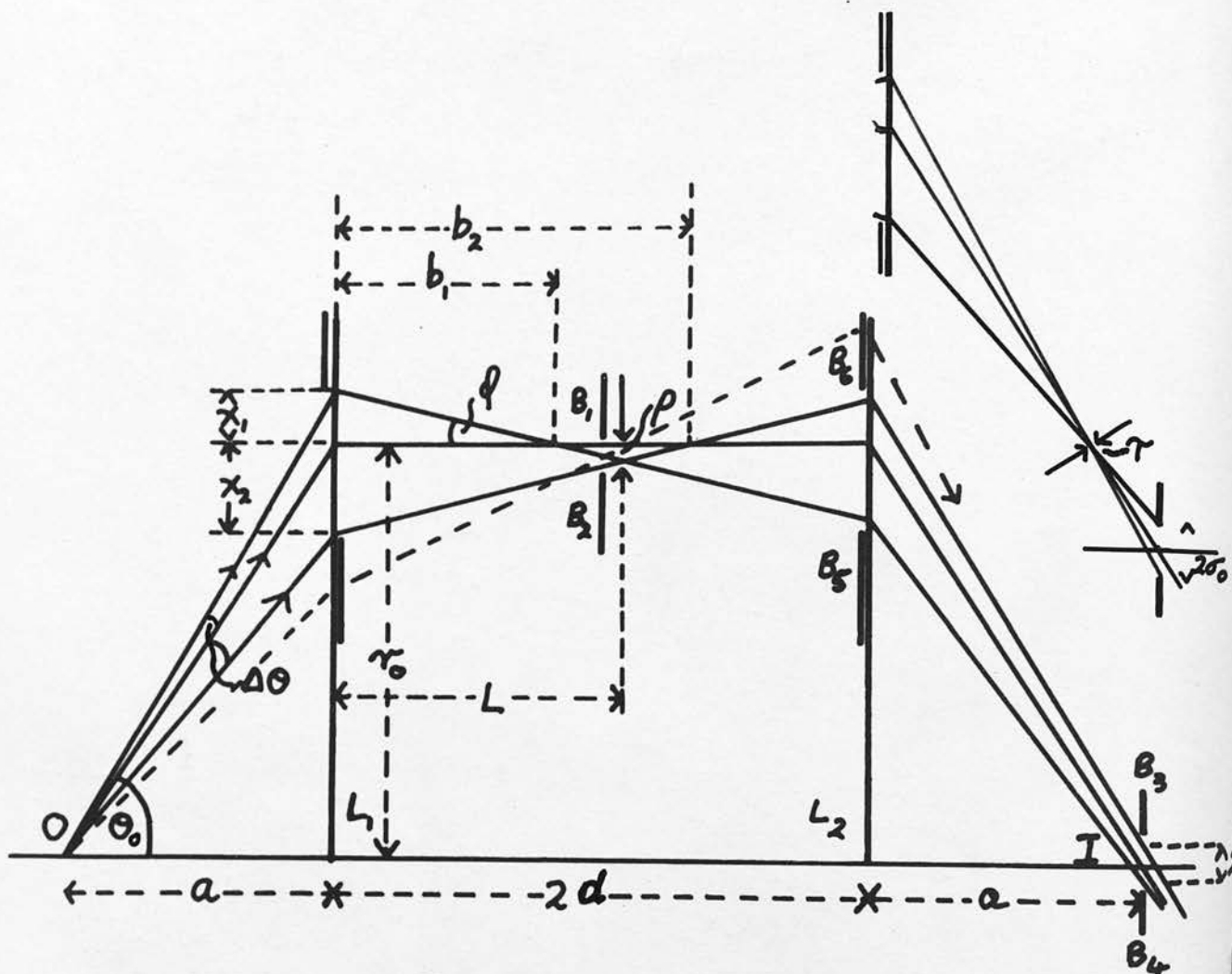


Fig.A19 "Second Image" focusing in the case of two thin lenses

in the magnetizing coils and also the source position along the axis, from which it is obvious that a motion of a very few millimetres reduces the transmission to a very small fraction of the maximum value. The former is simply the complementary effect to choosing the correct baffle radius for a given field¹; the latter is more interesting and important and will reappear later.

(3) Double Lens Model:

The general similarity of the focused trajectories in these two cases leads the writer to suggest that they were both forms of a more general form of focusing system of which the extreme example was the two refracting surfaces (Braid & Richardson, 1951) of a thick lens, or, which is the same thing, two ideal thin lenses. If these two lenses are both assumed to have positive aberration then and we consider a narrow pencil of large initial inclination which is deviated by the ^{first} lens so that it becomes parallel to the axis, then this sheaf will have somewhere a meridional focal line of the peculiar type that it is divorced from a genuine axial line (since the latter is partly virtual). If the distance between the lenses is now chosen to be twice the distance from the first to the focal line, then a true axial point image will be produced by the second lens, i.e., the aberration has been corrected for a

¹ As in (1).

wide angle sheaf. It is easy to see that if the distance between the two lenses is increased the aberration of the first will be over-compensated by that of the second and so the final image will be split again into two focal lines, the aberration being now negative. The former case corresponds in principle to the adjustment used in the Slätis and Siegbahn spectrometer and the latter to that proposed by Richardson for the spheroidal field instrument.

Such a model for a β -spectrometer had previously been proposed, unknown to the writer, in a letter by Bothe (1950) and a further paper is to be published by him on the subject. (Grivet (1950) has also described the trajectories in the Slätis and Siegbahn spectrometer in terms of a double lens). The priority of Bothe's work is acknowledged and in the following simple discussion his approximate expressions for resolution and size of the final image are reproduced. He expresses the deviation experienced by each ray in terms of simple forms of dependence on a co-ordinate x , which is the distance from the axis relative to that of the "central" ray (which is rendered parallel to the axis) at which it meets the first lens, confining his treatment to rays of small initial inclination.

Here it is the angle of inclination, α , of each ray to the axis between the lenses which is expressed in terms of x . This is done to indicate

that the properties of the system are due to its symmetry rather than to the characteristics of lenses. No attempt is made to deduce detailed or precise results since very simple and idealized assumptions regarding the deviation of the rays are sufficient to demonstrate the general form of behaviour (in the meridional plane).

(1) Image formation:

In Fig. A19 rays are described by x , the radial co-ordinate, relative to the central ray, of the point at which they meet the first lens. The subscripts 1 and 2 denote respectively rays of positive and negative x which have equal and opposite values of Q ; such rays will be referred to as forming a pair. The sheaf of rays is made up of such pairs plus the central ray for which Q is zero. The rays of a pair intersect the central ray on the axis after passing through the second lens only if they intersect each other exactly mid-way between the lenses.

The various symbols to be used are shown in Fig. A19. Q and $\Delta\theta$ are assumed small throughout. $\Delta\theta$ is given in terms of x by

$$\Delta\theta = \frac{x \cos^2\theta}{a}$$

The inclination Q may be expressed in terms of x ,

$Q = f(x)$. Consider now some simple forms of $f(x)$ in the case of a point source at 0.

Note: Throughout, the deviation of a ray at a lens will be expressed in terms of position of incidence only and effects due to varying angle of contact neglected.

(a) Suppose $Q = \alpha x$

(i.e. the tangent to $f(x)$ at $x = 0$).

Then $b_1 = b_2 = \frac{x}{Q} = \frac{1}{\alpha}$ (independent of x)

Therefore, $L = \frac{b_1 + b_2}{2} = \frac{1}{\alpha}$

Thus if $2d = 2L$, $\rho = 0$, and $\sigma = 0$

This is the "paraxial" case for very small $\Delta\theta$.

(b) Suppose $Q = \alpha_1 x$, x positive)
 $= \alpha_2 x$, x negative)

Therefore, $b_1 = \frac{1}{\alpha_1}$: $b_2 = \frac{1}{\alpha_2}$

Therefore, $L = \frac{\alpha_1 + \alpha_2}{2\alpha_1\alpha_2}$

Since $x_2 = x_1 + 2\rho$

$$\alpha_1 x_1 - \alpha_2 x_2 = 0$$

$$\rho = \frac{\alpha_1 - \alpha_2}{\alpha_2} x$$

Again if $2d = 2L$, $\sigma = 0$

This means that a corrected point image may be obtained, no matter what the form of $f(x)$ for any given pair of rays by choosing L properly. σ will then pass twice through zero, over the range $\Delta\theta$.

(c) Suppose $Q = \alpha x + \beta x^2$

Therefore, $x_2 = x_1 + 2\rho$

$$\left. \begin{aligned} \alpha x_1 + \beta x_1^2 - \alpha x_2 + \beta x_2^2 &= 0 \end{aligned} \right\}$$

Neglecting ρ^2 ,

$$\rho \approx \frac{\beta}{\alpha} x^2 \left(1 + \frac{\beta}{\alpha} x - \frac{\beta^2}{\alpha^2} x^2 + \dots \right)$$

$$\approx \frac{\beta}{\alpha} x^2 \text{ (c.f. Bothe)}$$

Therefore, $\rho = \frac{\beta}{\alpha} (\Delta\theta)^2 \frac{a^2}{\cos^4\theta} \sim (\Delta\theta)^2$

Now $b = \frac{x}{\alpha} = \frac{1}{\alpha} (1 - \frac{\beta}{\alpha} x + \frac{\beta^2}{\alpha^2} x^2 - \frac{\beta^3}{\alpha^3} x^3 + \dots)$

$$L = \frac{1}{\alpha} \left\{ 1 + \frac{1}{2} \frac{\beta}{\alpha} (x_2 - x_1) + \frac{1}{2} \frac{\beta^2}{\alpha^2} (x_1^2 - x_2^2) - \dots \right\}$$

$$= \frac{1}{\alpha} \left\{ 1 + \frac{\beta}{\alpha} \rho + \frac{\beta^2}{\alpha^2} (x^2 + \rho x + 2\rho^2) - \dots \right\}$$

$$\approx \frac{1}{\alpha} (1 + 2 \frac{\beta^2}{\alpha^2} x^2)$$

$$\therefore L \approx \frac{1}{\alpha} \left\{ 1 + 2 \frac{\beta^2}{\alpha^2} \frac{a^2 (\Delta\theta)^2}{\cos^4 \theta} \right\}$$

If $2d = 2/\alpha$, then $(d - L) = -2 \frac{\beta^2}{\alpha^3} x^2$

At the second lens this results in an error

$$\Delta x = 2(d - L)\alpha$$

$$= -4 \frac{\beta^2}{\alpha^3} x^2 (\alpha x + \beta x^2)$$

(writing explicitly the equations for a ray of negative x at the first lens).

The deviation introduced at the second lens now corresponds to that of a ray incident on the lens at $(x - \Delta x)$ in the "paraxial" case. Since it has a different inclination φ , the relative change in inclination θ is

$$d\theta = (\varphi_x - \varphi_{x-\Delta x})$$

$$= (\alpha + 2\beta x) \Delta x$$

$$= -(\alpha + 2\beta x) \frac{4\beta^2}{\alpha^3} x^2 (\alpha x + \beta x^2)$$

Therefore, $\sigma \approx \frac{a}{\cos^2 \theta} d\theta$

$$\approx \frac{4\beta^2}{\alpha} a \frac{x^3}{\cos^2 \theta}$$

Therefore, $\sigma \approx \frac{4\beta^2}{\alpha} a^4 \frac{(\Delta\theta)^3}{\cos^8 \theta} \sim (\Delta\theta)^3$

$$\rightarrow 4 \frac{\beta^2}{\alpha} a x^3 \text{ for small } \theta \text{ (cf. Bothe).}$$

Thus the size of the final image depends on the cubic term in $\Delta\theta$ i.e., the aberration is zero in the second order of approximation.

In practice, of course, $f(x)$ would contain important contributions from higher powers of x ; the effect of this would be in general to cause a rapid increase in the aberration as $\Delta\theta$ increases. It is sufficient for the present purpose to consider the quadratic case.

(ii) Chromatic aberration:

Suppose that a counter window placed at the image position has a diameter 2σ . The chromatic effects may be estimated by considering the change $\Delta\sigma$ in σ consequent on a change Δp of the momentum p .

Let the change in deviation D of a ray be given by

$$\frac{\Delta D}{D} = -F \left(\frac{\Delta p}{p} \right)$$

Therefore, $\Delta D \approx -F\theta_0$.

This gives a change of the position of incidence on the second lens $\Delta x \approx 2\alpha \Delta D \approx \frac{2}{\alpha} F\theta_0$,

(where Δx is positive if Δp is positive and we are dealing with a ray for which x was negative at the first lens).

The total change in inclination of the ray after passing through the second lens (as compared with that of a ray of momentum p and co-ordinate $(x+\Delta x)$ for which σ has already been derived) is

$$d\theta \approx -(\alpha + 2\beta x) \Delta x + 2\Delta D$$

where the first factor is due to the geometry of the rays between the lenses as distinct from the intrinsic change in deviation.

This becomes $|d\theta| = |F\theta_0 \frac{4\beta x}{\alpha}|$ where the
 sign is positive) for Δp positive) for rays
) negative)
 negative)

having x positive at the first lens, and vice versa for rays having x negative at the first lens. This means that for increased momenta the image disc at I becomes smaller while for decreased momenta it becomes larger. This is to be expected since in the one case the geometrical arrangement of the rays approaches more closely to the ideal focusing condition and in the other departs further from it. The largest possible ΔD which is permitted by the central baffles B, B_2 is roughly

$$|\Delta D| = \rho \alpha = \beta x^2$$

$$\therefore |F| = \frac{\beta x^2}{\theta_0}$$

$$\therefore |\Delta \sigma| = \left| \frac{4\beta^2}{\alpha} x^3 \frac{a}{\cos^2 \theta} \right| = |\sigma| \quad \text{(as compared with the ray incident at } x + \Delta x \text{)}$$

Thus the size of the image formed by the range of momentum allowed to pass the baffles B, B_2 (with varying degrees of efficiency) varies from a point to a disc of radius $2\sigma_0$ where σ_0 is the image radius for the momentum most efficiently focused.

The expression shows that $\Delta \sigma$ is zero in the case where β is zero, i.e., it is only in virtue of the other aberration that the chromatic effect exists. To some extent O and I approximate to aplanatic points.

(iii) Resolution and solid angle:

It is readily seen that the contribution to the

resolution from the counter aperture at I is nil for increasing momenta but in the case of decreasing momenta those rays will fail to pass through, for which Δp is such that they strike the second lens at a greater value of x than those rays of momentum p for which $\sigma = \frac{1}{2} \sigma_0$ where σ_0 is the radius of the aperture. Since σ is a function of x^3 this effect is not large. All central rays which pass $B_1 B_2$ pass also $B_3 B_4$.

The resolving power of the spectrometer, then, must be mainly due to the ring slit $B_1 B_2$. All momenta below that for which the central ray just passes B_2 are completely excluded; all momenta above that for which the fringing pair cross close to the edge of B_1 are excluded (provided that baffles of the type $B_5 B_6$ are placed so as to stop higher momentum rays such as that shown dotted). Since the concentration of rays on this slit depends on x^2 the line shape will correspond to what has loosely been called the "semi-circular type", i.e., the high energy side of the line will be much the sharper and the low energy tail will increase as the slit $B_1 B_2$ is widened.

The changes ΔD in the deviation, necessary for complete cut-off are

$$\Delta D = \frac{p}{\alpha} \approx p\alpha = \beta x^2 \quad (\text{Increasing } p)$$

$$\text{and } \Delta D = \frac{p \cos^2 \theta}{\alpha} + \alpha p - (\beta p^2) \quad (\text{Decreasing } p)$$

where the extra terms in the latter are due to the motion of the central ray to a smaller radius.

Thus the total width of the line is

$$\Delta D = \rho \frac{\cos^2 \theta}{a} + 2\alpha \rho$$

If the fractional momentum width of the base of the line is taken as a measure of the resolving power R , then in the paraxial approximation for an electron lens in which the deviation experienced by a particle is proportional to the reciprocal of the square of the momentum

$$R = \frac{\Delta p}{p} = -\frac{1}{2} \frac{\Delta D}{D}$$

This gives an expression for R in the limit $\theta \rightarrow 0$.

$$\frac{\Delta p}{p} \approx \frac{1}{2} \frac{\rho}{\theta_0} \left\{ \frac{1}{a} + 2\alpha \right\}$$

$$\text{Therefore, } R \approx \frac{1}{2} \frac{\beta \alpha^2 (\Delta \theta)^2}{\alpha \cos^4 \theta_0 \cdot \theta_0} \left\{ \frac{1}{a} + 2\alpha \right\} \sim (\Delta \theta)^2$$

If $1/a$ can be neglected this becomes essentially Bothe's expression. It does not strictly apply to the wide-angle case, but it is sufficient for the present purpose which is to show the dependence of R on a power of $\Delta \theta$ at least as high as the square.

The dependence of the solid angle on θ is given by

$$\Omega \sim \frac{1}{2} \sin \theta \cdot \Delta \theta$$

This shows that the use of a wide angle sheaf gives the possibility of considerable gain in solid angle provided that the increase in aberrations due to the increase in angle does not more than offset the

$\sin \theta$ term. The resolution and solid angle are therefore linked by a quadratic relationship; this again means that the resolution is high for very narrow slits but deteriorates rapidly as these are widened. The quadratic relationship is simply that which Frankel (1948) pointed out specifically for meridional focusing in the solenoid spectrometer, as opposed to the roughly linear relationship obtained with axial focusing in a thin lens.

By setting the inter-lens distance so that a second meridional ring is formed near I a further contribution (of the same nature) is given to the resolution including the possibility of cutting out many of the unwanted central rays passing through $B_1 B_2$.

(iv) Use of negative aberration:

The width of the ring so formed is much smaller than the diameter of the disc "image"; the following calculation gives its dependence on $\Delta\theta$, though not accurately on the other parameters since it forms the end of a long series of approximations.

Suppose the inter-lens distance is set at

$$2d = \frac{2}{\alpha} + 2 \frac{\beta^2 x_1^2}{\alpha^3}$$

i.e., greater than the paraxial value by as much as it has been less in calculations to date. The effect of this is to cause the rays to cross near I (which is still situated at the same distance as formerly from the second lens) and give an "inverted" disc of radius σ_0 (c.f. (i)) to a first approximation.

$\sim \frac{2}{\alpha} \quad 2d?$

The cross-over of the standard rays is in the form of a small triangle whose position is $\approx \frac{\sigma}{x}$ distant from I along the central ray, i.e., the radius of the focal ring is $\frac{\sigma}{x} \sin \theta_0$.

The vertices of the triangle are $\frac{\sigma}{x}$ and $\frac{\sigma}{x + 2\rho}$ distant from I so that the altitude and therefore the width of the focal line (perpendicular to the central rays) is, crudely,

$$\begin{aligned} \tau &\approx \sigma \left(\frac{1}{x} - \frac{1}{x + 2\rho} \right) \cdot \Delta\theta \\ &\approx \frac{\sigma}{x} \left(\frac{2\rho}{x + 2\rho} \right) \cdot \Delta\theta \end{aligned}$$

therefore $\tau \approx (\Delta\theta)^4$ c.f. $\rho \sim (\Delta\theta)^2$

so that the selecting slit here can be very narrow indeed. The resolution is correspondingly increased, in particular many central rays being cut out.

Estimating the resolution as before, in terms of the total angular range of deviation of the family of momentum $p \pm \Delta p$ completely filling the slit we see that

$$\Delta D \sim \frac{\tau}{\ell}$$

where ℓ is a composite length since now both lenses contribute to the resolution, though the main contribution still comes from the first lens if $\frac{2}{\alpha} > \alpha$

Therefore, $R \sim (\Delta\theta)^4$

Since the solid angle is unchanged the introduction of the second ring slit results in a considerable improvement in performance.

(v) Contracurrence and source size:

It is easy to show that the system is contra-current. If the source O is moved distance Δa_1 , along the axis, then the point of intersection of a ray with the axis in the image space moves a contra-current distance Δa_2 given by

$$\Delta a_2 \approx \frac{a}{\cos \theta \cdot \sin \theta} \left\{ -(\alpha + 2\beta x) \Delta x + \Delta Q \right\}$$

where $\Delta x = \frac{2}{\alpha} \Delta Q$ and $\Delta Q = \frac{\Delta a \cdot \cos \theta \cdot \sin \theta}{a}$

Therefore $\Delta a_2 \approx \Delta a_1 \left(1 + \frac{4\beta x}{\alpha} \right)$

i.e., for the central ray the motion of O and I are equal and opposite; for other rays the term in x (which refers to the second lens) causes the image size to alter as it moves. The effect of this is that the instrument rapidly becomes defocused. This defocusing takes place for all momenta since all central rays move equally. For a given energy an outside limit to the amount of motion is set by the central ring baffle. This is roughly $\Delta a_1 = \frac{a\alpha\rho}{\cos \theta \cdot \sin \theta}$ which is small because of the product of α and ρ . If α and $\frac{1}{\alpha}$ are equal then an outside limit to the source size which can contribute to the counting rate is $\rho / \cos^2 \theta$, and of course no good focusing is obtained unless the source is extremely small. The maximum source diameter is even less if σ_0 is small, or if a second focal ring is used at I .

(4) Discussion.

The contracurrence of object and image is a property which makes it possible in the case of the spheroidal field to use the analogy of reflection from a mirror. Though the model discussed has more obvious affinities with the Slätis and Siegbahn instrument the same general considerations hold for the other, which must be considered as two contiguous immersion lenses.

It is obvious that such lens systems have no value in the field of electron optics since no image can be formed of an extended object. Fundamentally this is due to the failure to satisfy the sine condition.

Poor image formation is not necessarily a disadvantage in β -spectroscopy, and here the possibility of obtaining a high solid angle at good resolution is attractive. It arises from the possible use of a much wider angle for the sheaf of rays than has been customary, and from the quadratic (or quartic) relationship between resolution and solid angle which comes from the use of meridional focusing. This is clearly seen in the present case to arise from the presence in the sheaf of central rays, i.e., rays for which the aberration is zero. Such rays are not present in axial focusing in a lens spectrometer, and their presence here is simply due to the selection of the meridional focal ring, the formation of which is

free of aberration to a higher order of approximation in $\Delta\theta$. The use of successive meridional images raises the order.

The treatment has been very simple and has not depended in any fundamental fashion on the properties of lenses, and therefore the same results apply to one-dimensional systems where the bending of the rays occurs into two places or nearly symmetrical regions. The exact nature of the formulae, of course, depends on the geometry and $f(x)$. A simple and perfect one-dimensional case is the uniform field where the "ring focus" appears at 180° rotation and the image coincides with the source. If the field strength were to fall off in some fashion with distance from the source position as measured in a direction parallel to the line joining the source and 180° image, the source and image would move apart. The introduction then of a slight asymmetry would produce the equivalent of the second meridional line, i.e., an asymmetrically sited cross-over.

The possible applications of this "second image" focusing seem to lie in the field where high resolution at good solid angle is required, and where it is possible to tolerate a small source. If an extended source is used then, of course, in addition to the deterioration of meridional focusing already mentioned there will be large comatic aberrations due to the skew rays.

SECTION B.

I. INTRODUCTION.

The general form of spectrometer required to take advantage of the focusing properties of the prolate spheroidal field is clear; the necessary field shape must be produced between suitably shaped pole-pieces in a magnet which should preferably have a high degree of axial symmetry, the baffle system must consist of two annular selecting slits, one near the mid-plane of radius approximately $9a$ ¹, one encircling the axis at a radius of $0.10a$ and two ring baffles sited where the sheaf is wide, so as best to delimit the fringes of the sheaf and help to cut out the rays of different momentum which nevertheless pass through the first slit, and the counter window must also be arranged to encircle the axis so that the rays entering almost normally may most easily pass through.

But the actual dimensions and siting of these components cannot immediately be taken to be precisely those resulting from the theoretical survey since the second ring focus must be sensitive to small errors in computation, and to any departure of the field shape, as realized in practice, from the truly spheroidal. Also it is not to be assumed without further question that the particular family

¹ In the construction a was chosen to be 1 cm.

of trajectories chosen by Richardson for computation corresponds to the optimum conditions of operation, though it is necessary to base the construction upon them.

For these reasons (and others to be mentioned later) it was decided to proceed first with a preliminary experiment in which some variation of operating conditions was possible, the variable factor being principally the radius of the baffle B_1, B_2 as it is obviously here that there is most scope for adjustment of the position of the second cross-over to coincide with a previously fixed counter slit. The discussion of this procedure, however, will be deferred until a description of the construction of the main body of the spectrometer has been given in B II.

This construction was based on the theoretical discussion (Richardson 1949) which has been described in AIII, the scale chosen being that which makes $a = 10$ cms.

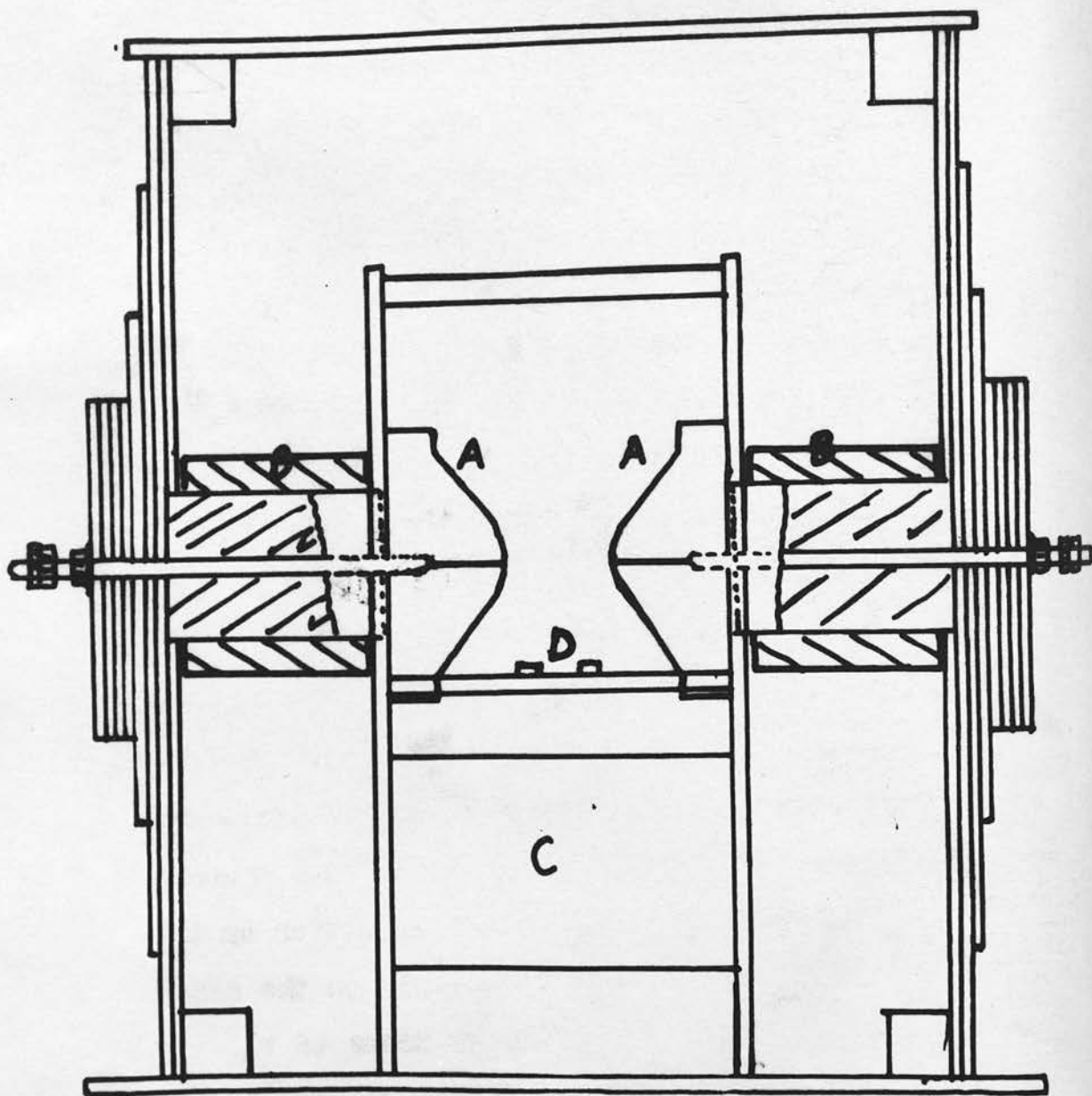


Fig.B3 Spectrometer magnet showing, A pole pieces, B magnetising coils, C supporting cradle, D tank rails.

II. CONSTRUCTION OF SPECTROMETER AND ACCESSORIES.

The magnet, spectrometer tank and supporting frame had been designed by H.O.W. Richardson and manufactured, the first by Colvilles Ltd., and the others by H.M. Torpedo Establishment, Greenock, before the writer joined the project; their construction and modification took place during the course of the work being described.

(1) The Magnet:

This is as shown in Fig. B3. The end-plates are dodecagons of 4 ft. diameter, situated 3 ft. apart, into which are set the 7" diameter axial cores carrying the magnetising coils. A 1" rod passes through the end-plate and the core and screws into the pole-piece so that the pole, core and end-plate may be drawn into good magnetic contact, and this provides the main support for the pole-piece. Additional support is provided by a brass cradle which also serves as a bearing surface for a number of bolts to impose a small amount of motion on the poles to adjust them to be accurately on the axis, testing of this symmetry being by means of two axial $\frac{1}{8}$ " holes bored through the poles, (normally plugged with short soft iron rods). The magnetic return circuit is completed by twelve side-plates (two of which are hinged to give access to the interior) giving the whole a barrel-shaped appearance, and the

flux density round this circuit is roughly equalized by the use of the end-discs shown. The magnetising coils were taken from a disused dynamo and contain about 3,000 turns of d.c.c. copper wire. They were impregnated with glyptal resin and a copper sheet was bolted tightly round the circumference bearing the water-cooling pipes, one coil of which also passes between two of the end discs. It is important to note that the cross-sectional area of the poles is greater than that of the cores ~~the circuit~~ so that magnetic saturation effects from this source must occur long before they do in the poles themselves and so even at the highest fields able to be used the shape of the field must remain unaffected.

(2) Shaping of Pole-Pieces:

Ideally the prolate spheroidal field should be produced by pole-pieces of hyperboloidal shape, but since the maximum diameter of the poles is 30 cms. (their separation being 16 cms.) there must be a considerable leakage of flux round the edges, accentuated by the proximity of the end and side-plates. This meant that the pole-shape had to be specially chosen so as to maintain a fairly close fit to the desired field over the volume through which the rays move. This was done by studying the field distributions given by different pole shapes in an electrolytic tank, where, as is

well known, the flow of current through the electrolyte is governed by Laplace's equation so that if a model of a magnetic system is made, replacing iron (idealized to have infinite permeability) by conducting metal at an electric potential corresponding to its magnetic potential then the lines of current flow correspond to lines of magnetic force and the equipotential surfaces are the same in the two cases. One has then the possibility of determining the equipotentials simply by dipping into the electrolyte a current probe connected in a potential measuring circuit, or the potential gradient (i.e., the magnetic field strength) by using two electrodes close to each other and registering the potential between them. More refined methods are possible (and necessary for high precision), but these two were sufficient in this investigation.

Simplifications are possible in a problem possessing planes of symmetry for the lines of force either meet these normally or lie in them. In the former case one half of the system may be represented by a plane collecting electrode at the symmetry plane, and in the latter by an insulating plane, e.g., in the case of a system with axial symmetry it is possible to represent the three dimensional system by a tank tilted so that the electrolyte surface intersects the symmetry axis, the floor of

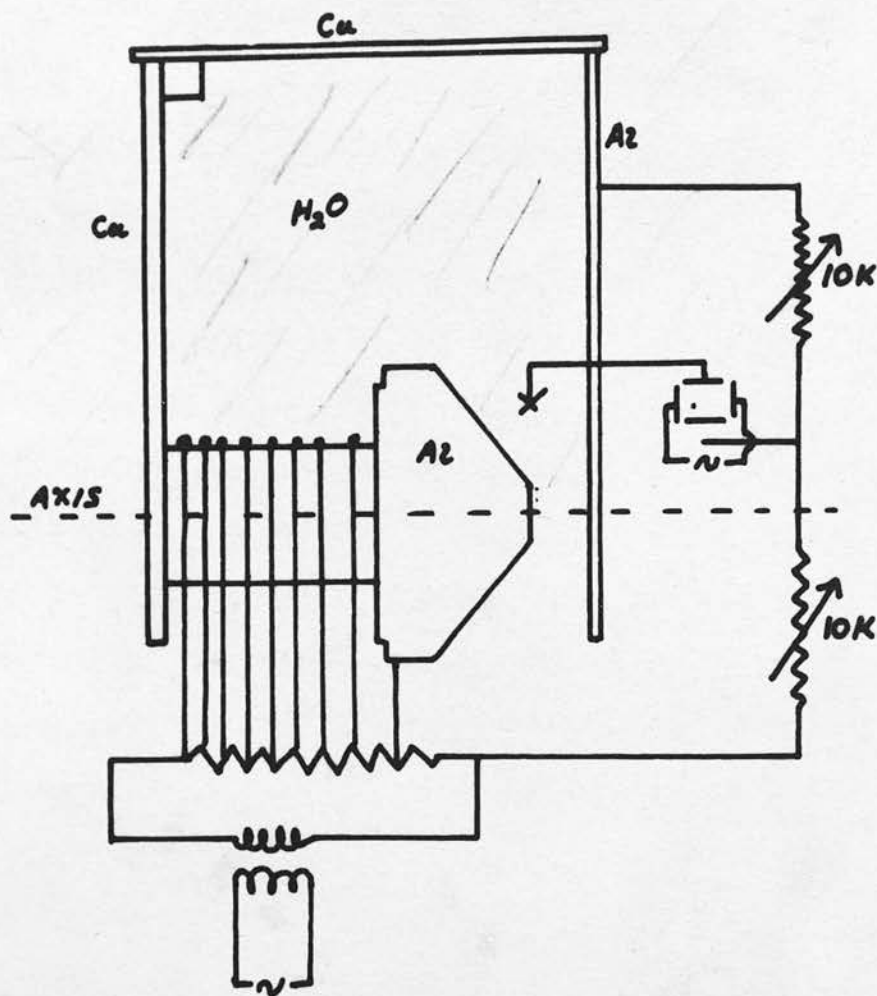


Fig.B6 Diagram of electrolytic tank

the tank being non-conducting. Advantage was taken of both these simplifications, a plane electrode being placed in the mid-plane and the tank being tilted to an angle of 30 degrees, so that the model consisted of a full size representation of one pole piece and core and that part of the end and side-plates intersected between planes through the axis and mid-points of adjacent sides of the dodecagon.

The tank was a large wooden box lined with tin-plate and supported on an axle so that it could be tilted readily to the correct angle. The floor was of glass, and on it rested the model consisting of full size half section of one of the poles and cores, made from teak, the end and side plates made of wood covered with copper foil and the central plane of aluminium sheet. The pole-piece was covered with thin aluminium foil which was smoothed on and stuck by wax. The potential gradient along the coil was made linear by supplying appropriate voltages to ten equally spaced electrodes along it, a preliminary comparison of equipotentials so obtained with the field as obtained by compass plotting in the magnet itself showing that this assumption was correct. The current supply was from the 50 cycle mains, stepped down through a variac to 10 volts and the different potentials were tapped off a 70Ω resistance, full voltage being applied between the pole and the mid-plane electrode (which was electrically connected to the end and side plates).

The equipotentials were detected by means of a single probe connected in a Wheatstone bridge circuit, using two 10,000 Ω P.O. boxes (non inductive to 50 c/s) and a cathode ray tube as detector. The probe was carried by a good pantograph of unit magnification so that its position could be transferred to a sheet of paper for record. The probe itself was a piece of fine wire fixed centrally in a glass tube gripped in the pantograph by a collet so that it could be finely adjusted only just to touch the surface of the electrolyte, which was ordinary tap water. Possible sources of error lay in surface tension effects and in phase angle effects which are, according to some writers, more pronounced when low frequencies are used and which might be expected to be more serious with aluminium which can have rectifying properties in a surface layer in an electrolyte.

Surface tension is only serious near the axis where the water layer is very shallow. This was minimised by the large scale of the model, but no attempt was made to take accurate readings very close to the axis. By careful wetting, the water could be made to intersect the true axis accurately.

A certain small phase angle effect was found, meaning that a perfect zero could not be obtained on the C.R.T. screen even in the absence of pick-up. A frequency of 1,000 c/s was tried, slightly decreasing

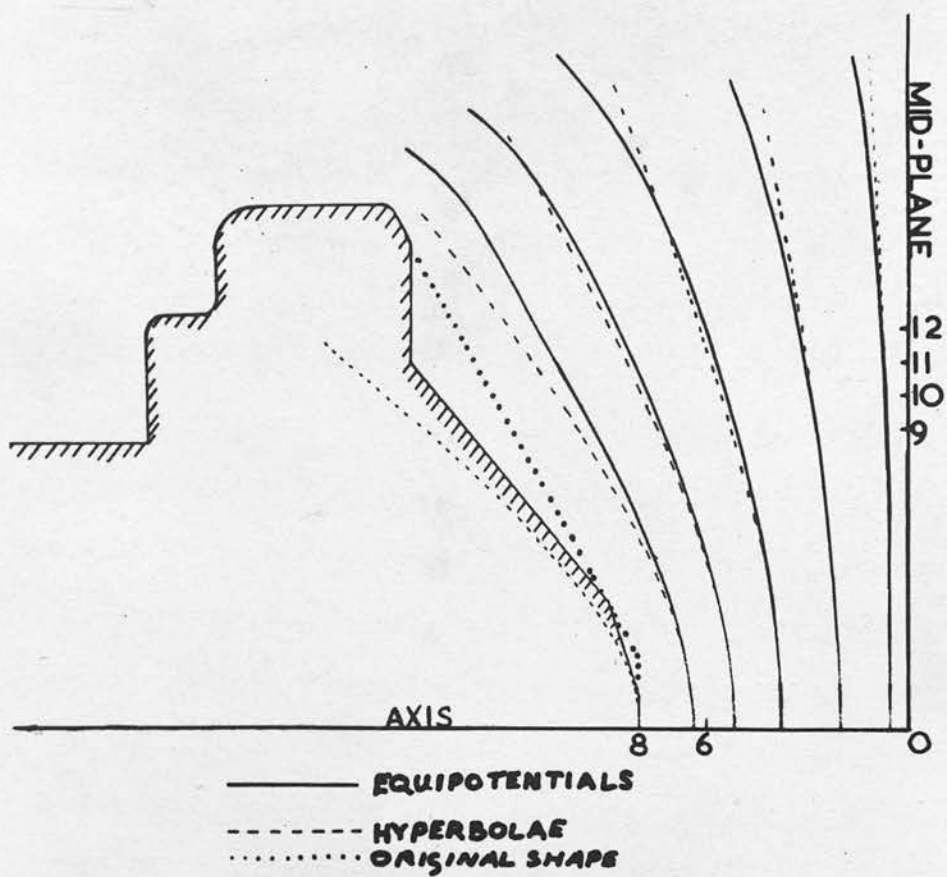


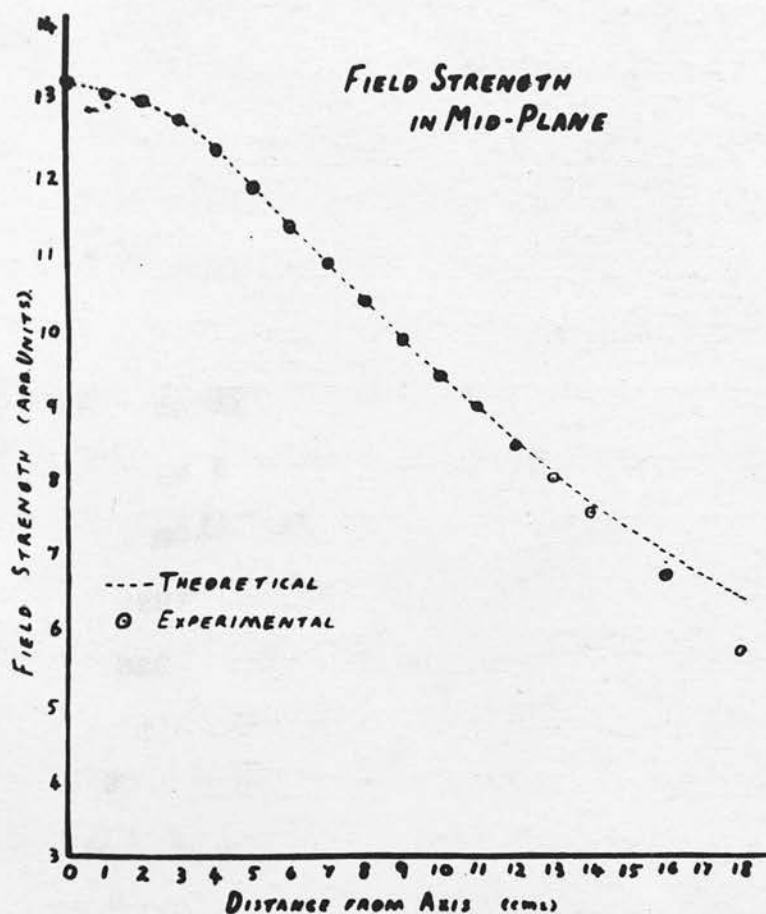
Fig.B8 Comparison of measured equipotentials with the hyperbolae of the ideal field

the phase angle without changing the position of a standard equipotential. Similarly, coating the aluminium surfaces with graphite made no change (the current density was in any case kept very low, no more than 10 ma passing through the whole tank, so that electro-chemical reactions were unlikely).

It thus seemed that there was no error in the position of the equipotentials due to the use, in particular, of aluminium or low frequency, and since the phase angle seemed fairly constant throughout the tank it could be taken that it contributed no error. (It may have occurred uniformly at a surface, or in the circuit).

The method, then, appeared to be sufficiently reliable to within the precision of measurement; this was quite high, as an equipotential could be located to within a small fraction of a millimetre (the limit of mechanical uncertainty).

The original pole shape was that shown on Fig. B8, and the procedure for arriving at the correct shape was to plot a large number of the equipotentials, recording them simultaneously on drawing paper for record and tracing paper for ready comparison with a family of standard hyperbolae and previous equipotentials obtained with other pole shapes. The wooden pole was then altered by carving as seemed best till a good agreement with the desired field was obtained, the final shape being built up of conical sections for easy machining. Fig. B8 shows



r (cm)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H ₀	13.2	13.1	12.9	12.7	12.3	11.8	11.3	10.8	10.3	9.8	9.3	8.9	8.4	7.9	7.4	7.3
H ₁		13.2	13	12.7	12.2	11.7	11.2	10.8	10.2	9.9	9.4	8.9	8.3	7.8	7.5	7.2
H ₂	13.2	13.0	12.9	12.7	12.3	11.8	11.3	10.8	10.3	9.8	9.3	8.9	8.4	7.9	7.5	7.2

Fig.B9 Ideal and measured field strengths in the mid-plane

the final shape and the degree of agreement between five typical equipotentials and the corresponding hyperbolas. The general form of the pole is of a rough hyperbola near the axis with a guard ring at large radii. The agreement with the spheroidal field is quite good over the region in which the electrons move, and it was decided that the fit was sufficiently good in view of the range of adjustment to be allowed in the spectrometer design.

The field strength in the mid-plane was measured by measuring the potential between the mid-plane and the probe, moved radially at a fixed distance from it (0.8 cms). The result of this measurement is shown in Fig. B9 where the variation of H with radial distance from the axis as measured by this method is compared with the calculated curve for the perfect spheroidal field. Once again the agreement is good out to a value of r beyond that reached by the electrons. A further check was made by measuring with a coil and fluxmeter the field in this plane obtained in the magnet when the poles had been given this shape. The table (Fig. B9) shows the agreement between the values of H_0 (the calculated field), H_1 (that obtained in the tank), and H_2 (that obtained by the fluxmeter in the magnet), (the accuracy in this case certainly is not better than 1%). The three are normalised on $r=3$ cms, as in the case of H_1 , readings nearer the axis are not reliable.

When mounted the poles were lined up accurately using the adjusting bolts already mentioned with $\frac{1}{2}$ " silver steel rods in the axial holes as guides. At one stage a search coil mounted on an axle running in these holes was swung round to test the axial symmetry of the field itself. This was found to be correct to within one part in a thousand, the estimated accuracy of the method, though this did require careful lining up.

(3) Current Supply:

Current is supplied to the magnet from a 100 volt bank of accumulators connected in a floating battery circuit with the D.C. mains, a copper oxide rectifier being connected "in reverse" across the coils as a safety device against surge voltages. The voltage developed by the current across a standard resistance is measured by a Tinsley vernier potentiometer. A linear field versus current relationship is maintained up to 5 amps and 1300 oersted (on the axis mid-way between the poles) and then slight saturation in the core begins to show, so that the departure from linearity at 7 amps is 12%. Since the field shape may safely be taken as unaltered the magnet can be used to focus β -rays up to 1.5 mev.

Fine control of the magnet current is by a 500-2,500 Ω shunt in parallel with the magnet coils (whose resistance is 7 Ω when fed in parallel). No

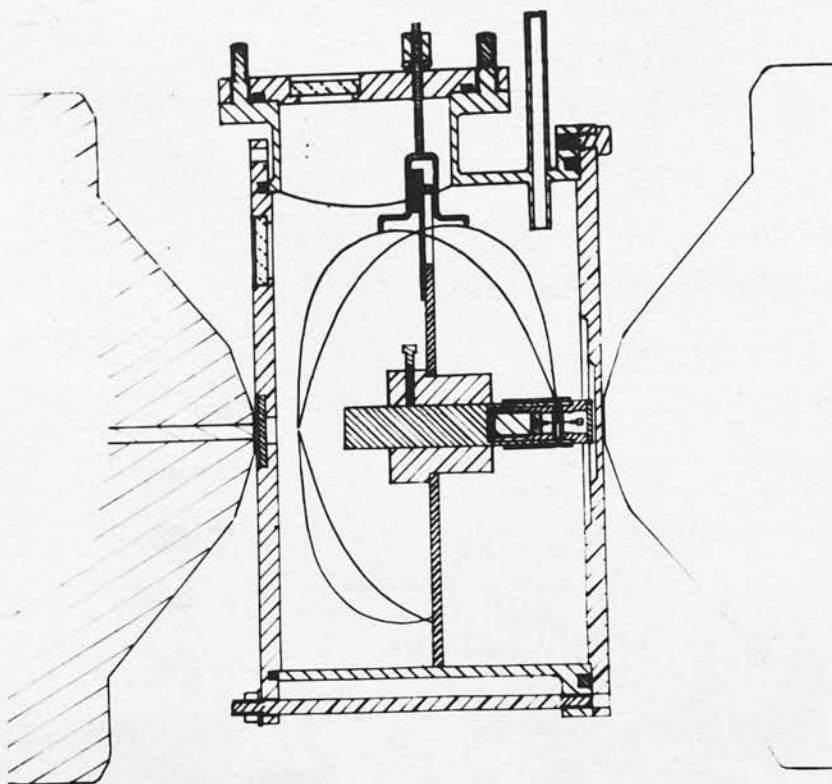


Fig. BII Spectrometer tank showing preliminary short slit with radial motion and wing baffles

automatic stabilization of the current nor direct measurement of the magnetic field has as yet been employed. It has been found that with the floating battery circuit current stability is good, and a reproducible field v. current relationship can be maintained by keeping on a standard hysteresis cycle.

(4) The Vacuum System:

The spectrometer tank (shown in Fig. B11) is cylindrical with two end-doors drawn against rubber gaskets. A source door is at one side and the whole stands on four adjusting screws on two rails on the supporting cradle. The diffusion pump (Metrovac O3B) is screwed on directly underneath the main chamber so as to obtain high pumping speeds. Other smaller pipes through the main wall provide connections for gauges, counter connections, H.T. leads, etc. The backing pump (Metrovac Rotary D2) may be connected via a 3-way stop-cock to the main chamber, counter filling line or source gate valve, and a second stop-cock connects the counter to the main vacuum or filling line. The system is almost entirely of metal, use being made of O rings and converted petrol pipe unions where possible.

The low and high vacua are measured continuously by Pirani gauges and for leak hunting and rarer use there is an ionization gauge whose current is amplified by a 1 valve D.C. current amplifier. The

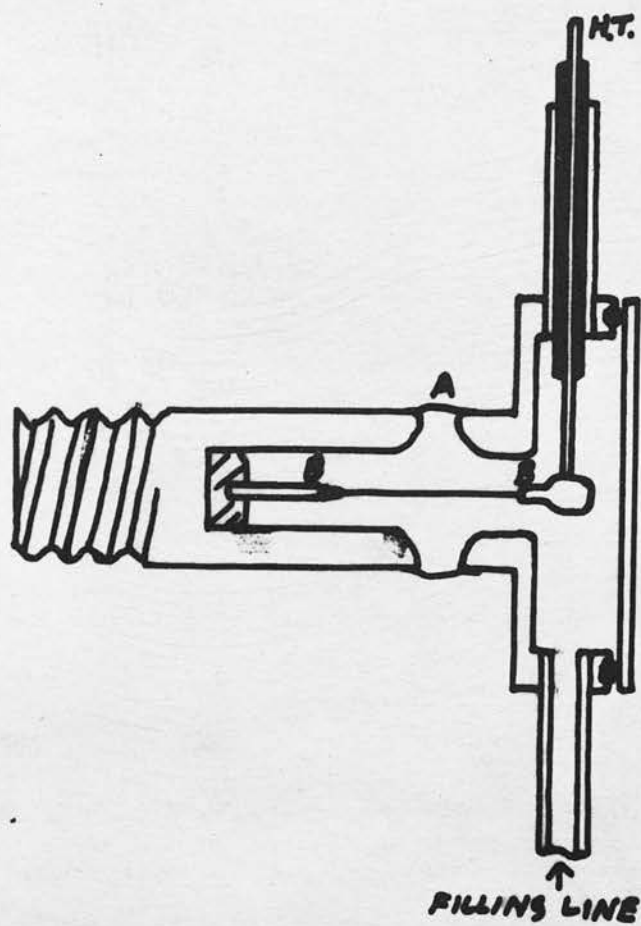


Fig.BI2 Geiger counter showing, A thin window site, B wire supports

best vacuum obtained has been 2.10^{-5} cms Hg (ionization gauge) but the normal operating point is 10^{-4} cms Hg, or rather better.

(5) Geiger Counter:

The geiger counter (Fig. B12) underwent many modifications in the course of time but its general design remained unaltered, and so the final model will be described first, reserving till later the significant differences of an earlier model. The counter body is a hollow brass cylinder of $\frac{3}{8}$ " internal diameter and $\frac{1}{8}$ " wall thickness; at one end this becomes solid screwed rod for locating purposes and at the other it widens out to a chamber 1" in diameter and $\frac{1}{4}$ " in depth to provide access to the interior for assembly and through whose walls pass copper tubes for the filling gas connection and the H.T. voltage lead. The end of this chamber is sealed by a disc drawn by screws against a rubber gasket. The window slit with its plane normal to the counter axis was milled and bevelled at an angle of 30° so as to leave three supporting bridges, one main one $\frac{1}{4}$ " across and two subsidiaries, obstructing a total of 70° of arc, the full width of the slit being 2.2 mm.

The counter wire lies along the axis, being supported at one end by a polystyrene plug and at the other by a clamp formed by the H.T. lead which is a thin brass rod waxed into a quartz insulator through the wall of the end chamber. The effective

counting length of the wire is limited to 0.5 cms. opposite the window by threading it through fine "uno" tubing which elsewhere reduces the field below the critical value. The advantage of this is to limit the counting region to that through which the focused β -rays will pass and so reduce the γ -ray background from the source which is only 12 cms. distant.

The counter is connected via flexible tubing and a gasket joint to a large reservoir of some 100 cc. capacity in the filling line; this is necessary to obtain stable characteristics since the counter volume is only 2-3 cc. This connects via the filling stop-cock either to the main vacuum for evacuation by the diffusion pump or to the filling line, the normal filling being 2 cms. alcohol and 8-10 cms. argon.

The principal difficulty lay in providing a suitable window. This had to be wrapped round the cylinder so that the two ends met on the main bridge, and then clamped and sealed. The substance eventually adopted after tests with mica and polystyrene film was nylon film, nominally .5 mgms/cm² thick. This was much stronger and more easily handled than mica of similar thickness though it had the disadvantage of stretching considerably under pressure. It was found that such nylon windows would stand up to 20 cms pressure with perfect reliability and some windows survived accidental imposition of 60 cms.

pressure, though individual samples varied greatly.

The nylon was first coated with a film of formvar made by evaporating on a glass quarter-plate a solution of 1% formvar in ethylene dichloride and floating off on water. Such films as made by standard technique were found to be 30-50 $\mu\text{gms}/\text{cm}^2$ thick, and their purpose was to stop up small pin-holes in the nylon. The counter body was then covered with a very thin film of silicone vacuum grease and the nylon carefully smoothed on, the edges and the join being painted over with a solution of "Durofix" in amyl acetate which gave a strong mechanical and vacuum tight seal and pulled out remaining wrinkles in the nylon through surface tension. The whole was now given mechanical strength by a system of clamping rings whose pressure was transmitted to the nylon through a gasket of .013" thick balloon rubber which also provided an additional vacuum seal. The effective window width left by this arrangement was 2 millimetres. Each window was tested under water at 20 cms. pressure. In the first model of the counter, used in the work described in BIII the window was of 5 mgms/cm^2 aluminium foil which was sealed with black wax. This was never a satisfactory arrangement.

With a filling of 2 cms. alcohol and 10 cms. argon the threshold for counting was about 1,000 volts. A counter with such a peculiar cathode shape and low length to diameter ratio would not normally be expected to give a good plateau, due to large

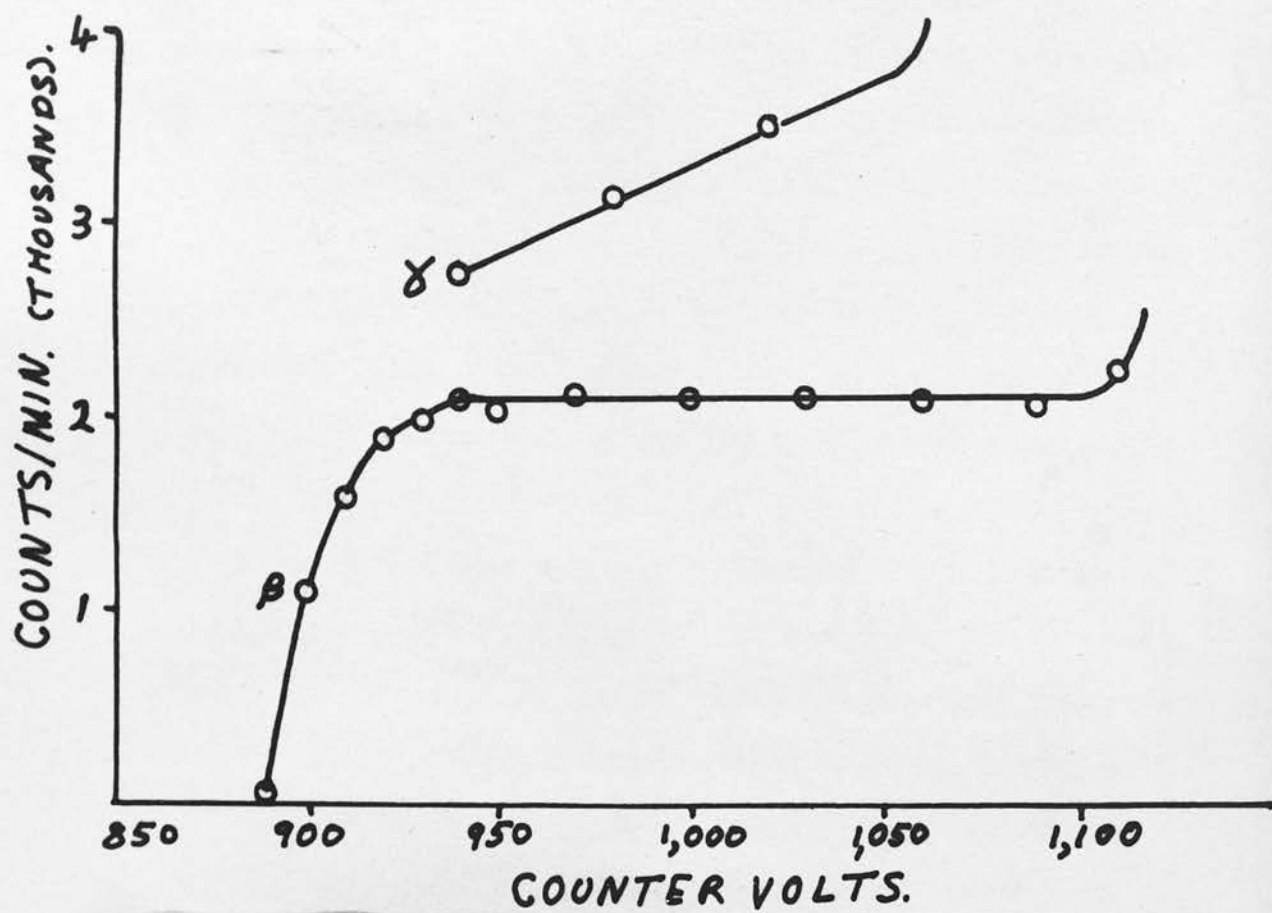


Fig.BI5 Geiger counter plateau

increases in the effective counting volume with increasing voltage, and such appears the case when a plateau is plotted with a γ -ray source which produces ion pairs throughout the counter volume, a slope being obtained of 0.25% per volt (Fig. B15). But for β -rays as focused in the spectrometer which initiate counts in a sharply defined region away from irregular changing fields the plateau is extremely flat - less than .01%/volt over a range of 160 volts. This shows that it is possible to use very unconventionally shaped counters with excellent results if the operating conditions are carefully defined. It also shows that the common practice of standardising a β -counter with a γ -ray source is not necessarily reliable.

The background was found to be 6 per min. which is rather high for such a small counter; it was however a genuine background from the counter itself. It is possible that some slight rough point contributed some of this as it was found that any roughnesses, particularly on the ends of the "unc" tubing, lead to a high background and all parts were in consequence very highly polished.

In general the shape of the plateau deteriorated with time, an effect which was due partly no doubt to slow leakage, but which was mainly attributed to slow absorption of the alcohol filling in the polystyrene and the greased rubber gaskets. This could to a large

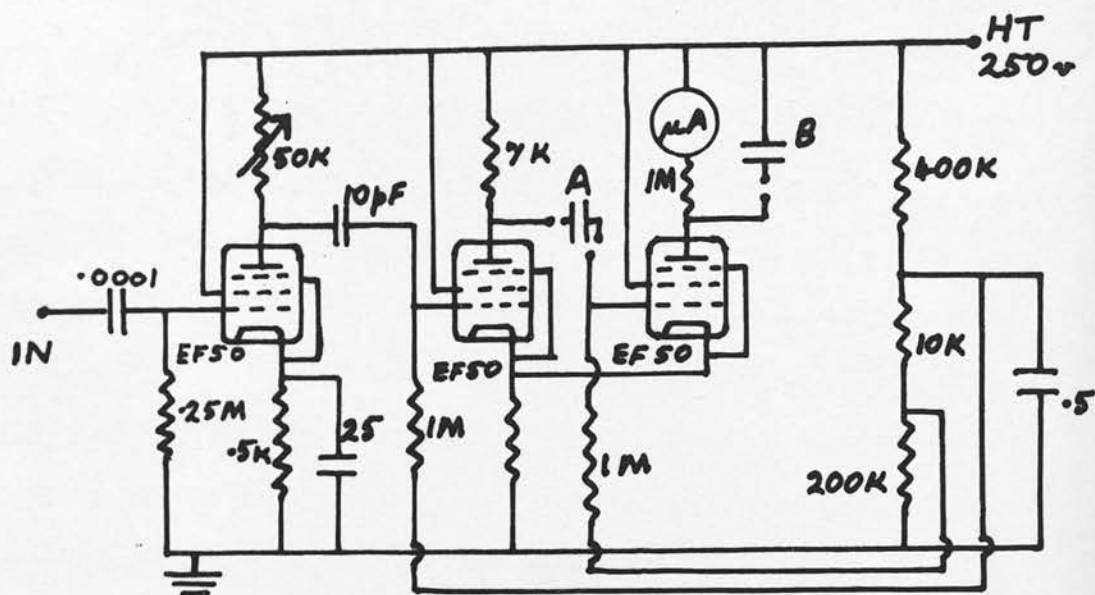


Fig. BI6 Counting rate meter circuit. Counting range is set by A, values of 10, 100, 1000 pf. giving full scale readings 1,200, 12,000 120,000 c.p.m. Statistical accuracy is set by B which has values 1 to 8mf.

extent be controlled by refraining from evacuating rigorously the counter between fillings.

(6) Counting Equipment:

The main counting equipment was manufactured by Dynatron Radio Co. and supplied from A.E.R.E. Harwell. The H.T. voltage is supplied from a type 1007 power unit through a type 1007 potentiometer unit. The counter pulses are fed through a type 200 probe to a simple amplifier and then to a type 1009A scaler and a counting rate meter.

This latter was of great value in the adjustment of the spectrometer, and in all cases where speed and ease were more important considerations than the highest accuracy. It is based on a simple "flip-flop" trigger circuit, the R.C. tank circuit being in the anode circuit of the second (normally non-conducting) valve. Three ranges were provided by varying the breadth of the pulse passing in this valve by changing the input time constant. This extremely simple circuit gave a reliable, linear response. As only relative readings were ever required of the meter long term calibration stability was not needed.

A three-way switch enabled the counting unit to be switched to three different sets of counters - the

β -counter already described, a γ -counter fixed in a standard position on the spectrometer tank near the source, and thin window α - and β -counters mounted in a paxolin box assembly with a shelf system and a

number of $\frac{1}{8}$ " aluminium sheets with small apertures so that the activity of a source could be measured under standard conditions using a narrow beam of rays.

A 2 mc. radium source fixed in a brass cylinder could be located in a standard position to standardise the counters.

The description given has been that of the final arrangement. Any significant deviation during the course of early experiments will be mentioned as appropriate in the rest of the text.

III. PRELIMINARY MEASUREMENTS WITH A SHORT SLIT.

The first investigations of the focusing properties were made with only a very short segment of the annular slit $B_1 B_2$, whose effective radius could easily be varied. Such a procedure was necessary because as has already been pointed out Richardson's calculations do not necessarily describe the best focusing conditions and are also subject to small errors whose effect on the position of the second focal ring may be quite serious. Variation of the radius permits most readily the adjustment of this ring to be coincident with the counter slit. A further advantage of a short slit is that any disturbing effects of axial-symmetry of field or baffles are removed, and therefore the optimum performance of the instrument may be obtained more reliably than with a full annular slit. In practice it was found that the question of such symmetry was of very great importance. The measurements with the short slit, then may be resolved into two parts, a pilot measurement to determine the optimum radius of $B_1 B_2$ for a final model, and a study of the upper limit of performance of the focusing.

(1) The Baffle System:

The arrangement of the baffles and slits is shown in Fig. B11. The short slit assembly was of $\frac{1}{16}$ " brass carrying a slit initially 1" in length and later blocked off to $\frac{1}{2}$ ", machined to a radius of 9 cms.

so that it could be moved over a range of several millimetres while still remaining a good approximation to a circular segment. This assembly moved in guides in the $\frac{1}{8}$ " brass centre-plate over a portion cut away to permit the passage of the electrons, and its position was set by means of a brass rod passing through a Wilson seal in the source door, whose protruding length gave a measure of the radius. To this centre-plate, which was itself adjusted to be in the correct plane and position by screwed fixing rods to one end-plate of the tank, was fixed an axial brass rod which carried both the counter and second ring slit which was a brass tube 2 cms. in diameter with a slit milled to a width of 0.5 mm., only three very narrow bridge pieces being left for supports. This tube slid freely over the supporting rod so that it could be set accurately over the window, and it was fixed at a distance of 12 cms. from the source. The counter was the early model with the 5 mg/cm² thick (aluminium) window. It was desirable that the baffles B_5 and B_6 should have their radii altered to correspond with that of the central slit. This was arranged by fixing them to the slit unit. They are constructed as the "wing" baffles shown in Fig. B11, adjusted so that their edges just touched each side of the computed beam of Fig. A15, their shape being such as to limit any glancing scattering to the region at the very edge. Since their axial distance

from B_1, B_2 was only 1 and 1.5 cms. in the two cases, they continued to cut the edges of the beam efficiently for all radii to which the short slit was moved.

The source holder was a $\frac{1}{8}$ " thick aluminium plate which slipped into guides in a large $\frac{1}{8}$ " aluminium plate fixed $\frac{1}{8}$ " from the left hand end-plate of the tank. The sources were fixed to a piece of mica clipped over a 1" diameter hole in the holder over which stretched fine cross-wires for centering purposes.

The axis of this system was made coincident with that of the field by means of the source cross wires, centre marks on the axial brass bar and the counter end-disc, (one of the tank end-plates being removed) and sights or pointed silver steel rods pushed through the $\frac{1}{4}$ " axial holes in the poles. The adjustment was made by means of levelling screws and set screws on the supporting cradle rails. A further screw, fitted with a micrometer head and scale enabled the tank to be moved along the axis over a range of about 2 cms. The correct position of the tank could therefore be reproduced with precision.

The sources used to study the focusing were of thorium active deposit made by recoil collection through a defining mask of mica on graphited mica or platinum from an open source of radiothorium. For the smallest sources the deposit was on the ground and highly polished end of a piece of

$\frac{1}{3}$ mm. platinum wire, shielded when exposed so as to reduce the deposit on the sides of the wire. The latter type of source was located in a $\frac{1}{3}$ mm. hole in a piece of $\frac{1}{16}$ inch aluminium stuck with vacuum plasticine to the mica backing of the source holder, while larger sources, deposited on discs of mica or platinum, were fixed directly to this mica, being centred on the cross-wires with the aid of a transparent scintillation screen of barium platino-cyanide on glass (which detected the α -activity).

(2) Adjustment Transmission Curves:

It is apparent that for the second ring image to fall on the counter slit the source and counter must occupy their correct positions relative to the field, and this adjustment was made using the axial motion previously mentioned. The current was set to focus a momentum range within which there was a flat region of the continuous spectrum free of conversion lines, the region $H\epsilon = 2,200$ of the thorium active deposit being suitable, and the spectrometer was moved along the axis till a maximum counting rate was obtained. This procedure is simply that of selecting for a given radius r of the slit B, B_2 , the particular momentum and position of the source and slit which gives the best concentration of rays on the second slit. If the spectrometer is moved sufficiently far in either direction from the maximum counting rate position the counting rate falls to that of the γ -ray

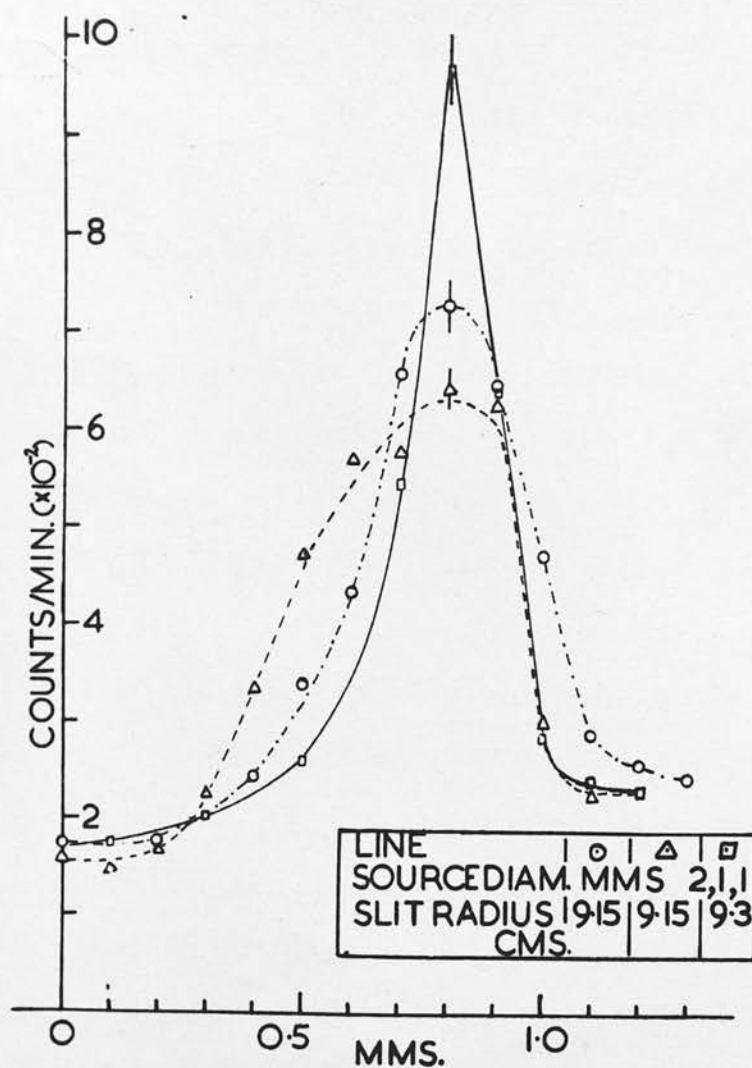


Fig.B22 Transmission curves showing effect of source size and central baffle radius

background since no β -rays at all can reach the slit. The range over which they can do so is small, little more than .5 mm, the width of the counter slit, showing that the process is that of moving across the second slit a narrow beam containing the momentum range allowed to pass by the first slit.

A curve of counting rate versus position of the tank, and obtained in these circumstances, will be referred to as a "transmission curve" as it has been found that the maximum counting rate of these curves is a measure of the transmission or solid angle of the spectrometer for a given state of adjustment. Also, since the spectrometer will, for given slit widths, be adjusted both for best resolution and highest solid angle when the concentration of the rays on the slits is highest, the best resolving power may also be expected to be obtained by setting the spectrometer for maximum transmission in this sense (i.e. for given slit widths $B_1 B_2$ and $B_3 B_4$ and in the absence of interfering effects from the other baffles). The transmission curves were therefore of great value throughout the work that follows in addition to their use for initial adjustment.

Three of them are shown in Fig. B22 where it is seen that the maximum transmission T and also the breadth of the curve Δ , depend on the radius of the main slit and of the source. This is to be expected,

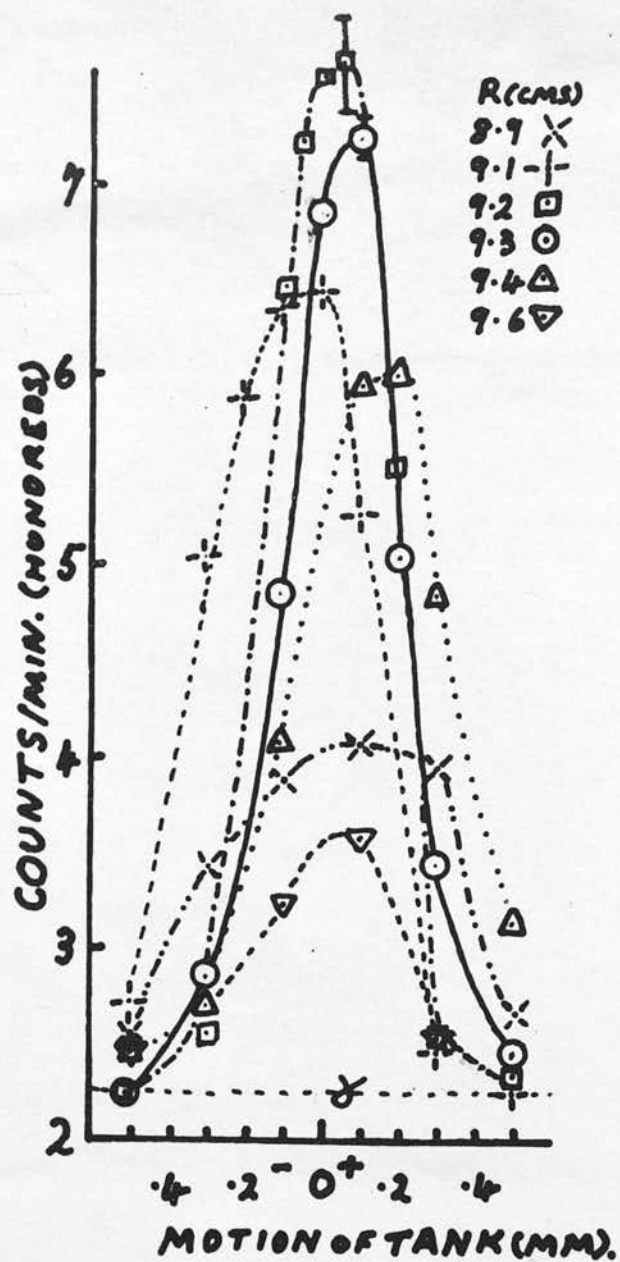


Fig.B23 Transmission curves for different radii of central baffle

since the aberrations at the second ring slit must depend on these factors.

(3) Choice of Optimum Radius:

The effect on the transmission curves of changing the radius of the main slit may be seen in Fig. B23 which was obtained, using a 1 mm. diameter source deposited in platinum in the course of the search for the optimum radius. The width of $B_1 B_2$ was .95 mm. The peak counting rate T and the reciprocal half-width of the line $\frac{1}{\Delta}$ increase till they pass through a maximum for radius r near 9.3 cms. The following table, referring to these curves shows clearly the variation with radius of transmission and resolving power.

r	Δ mm.	T	R
8.9	.4	2	$\frac{1}{56}$
9.1	.26	4.1	$\frac{1}{77}$
9.2	.19	5.1	$\frac{1}{85}$
9.3	.16	4.7	$\frac{1}{70}$
9.4	.23	3.1	
9.6	.26	1.6	

where T is the genuine β -ray count in arbitrary units and R is the fractional half-width of the F-line of ThB ($H_p=1385$) (actually taken with a different source with the same spectrometer adjustment).

These figures clearly show that the best radius lies close to 9.3. The fact that the transmission curve for $r = 9.2$ has a slightly higher peak value than that of $r = 9.3$, by 8%, is probably due to some secondary effect of baffles $B_5 B_6$ as $r = 9.3$ gives the sharper curve and the best resolving power. The broadening out of the curves as the radius is increased or decreased from this value is very marked, the height decreasing at the same time. This shows clearly that the beam itself at the counter slit is broadened considerably, and the second ring focus moves to a different radius. The eventual large decrease in total area under the curves must be due in the main to poorer concentration of the first ring focus on $B_1 B_2$, and to a smaller extent, on the effect of the baffles $B_5 B_6$ which will cut out more of the beam as it becomes more diffuse. The evidence that optimum radius lies between 9.2 and 9.3 was not all quite so clear as the above, for with other sources slightly varying results were obtained, due presumably to small differences in the centering and uniformity of distribution of the sources themselves. Thus it was sometimes found (as will be seen later) that $r = 9.4$ gave a much better relative performance than is shown above, approaching that of $r = 9.3$.

The conclusion was therefore that the best

value of the optimum radius was close to 9.3 cms. but that there was a short range of r about this value (of less than 2 mm.) over which there was some uncertainty, most probably due to a slow change of the figure of merit in the neighbourhood of the correct value, and also due to possible lack of precision in source setting, as will appear later.

(4) Resolving Power:

The resolving power was investigated for different source sizes, and for two different values of the width of the slit B, B_2 . For most of these measurements and for all of those with the highest resolving power the short slit was stopped down to $\frac{1}{8}$ " in length. The performance was found to be very critical with respect to source size and proper centering and the best values, which will be quoted, were obtained after very careful adjustment using the optimum radius. An indication of the effect of source size has already been given in Fig. B22 where it is seen to broaden the transmission curve. The shape of the F-line of Thorium B for different adjustments is shown in Fig. B26, 27. With $B, B_2 = 2.2$ mm. values of the resolving power obtained were $\frac{1}{40}$ for a 2 mm. diameter source, and $\frac{1}{75}$ with a 1 mm. diameter source. When the width of B, B_2 was narrowed to 0.95 mm. there was a further improvement, to $\frac{1}{150}$ with a 1 mm. source (Fig. B27). The best resolving power was obtained with $B, B_2 = .95$ mm and with sources

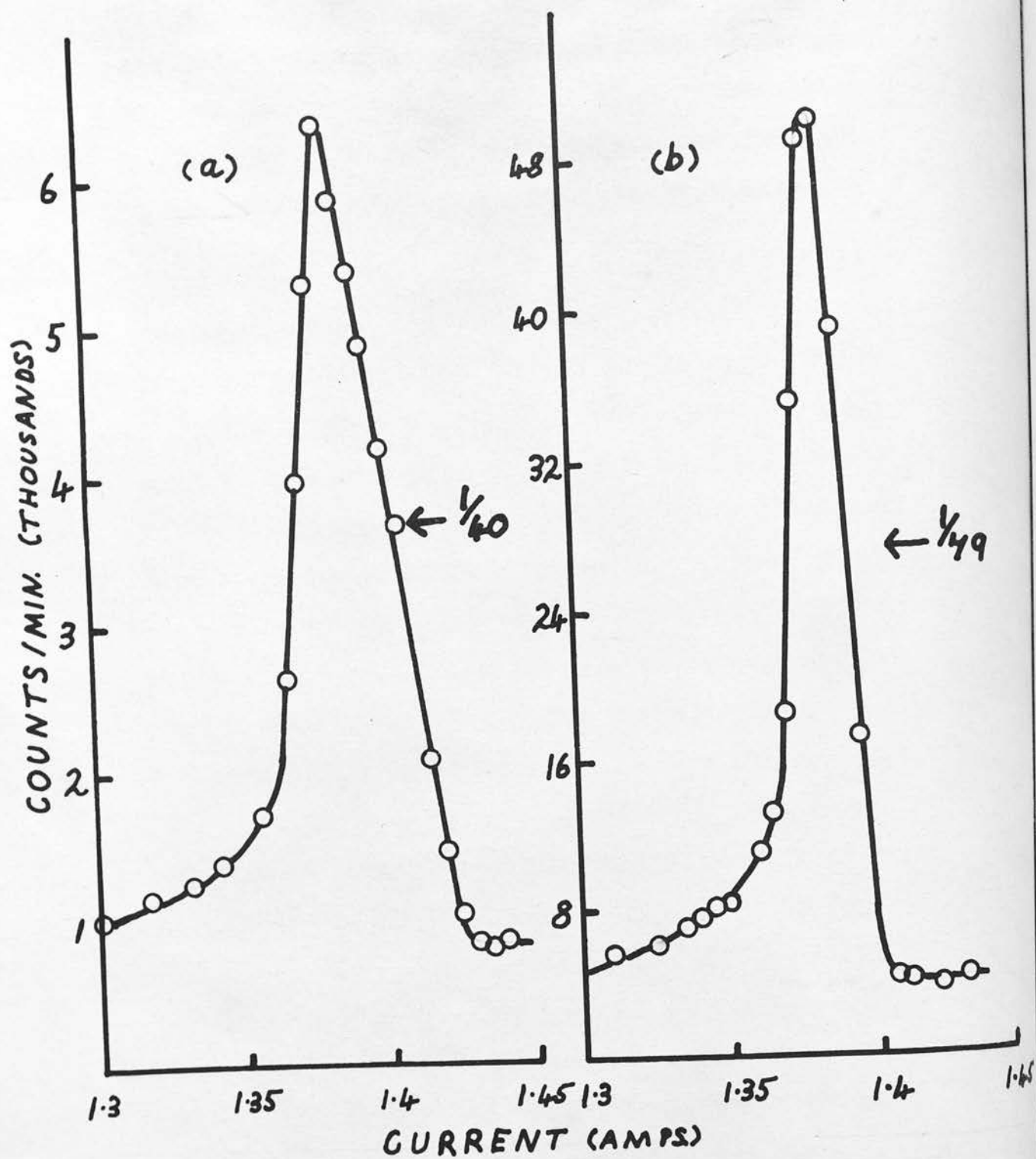


Fig.B26 F line of ThB with central baffle 2.2mm. wide (a) source diam. 2mm. (b) source diam. 1mm.

approximating closely to a point. These were obtained by depositing the activity as already mentioned, on the ground and polished end of a piece of platinum wire .33 mm. in diameter. The results thus obtained were rather erratic ^{due possibly to} non-uniformity of the source deposit, and varied from values of R of $\frac{1}{200}$ to almost $\frac{1}{300}$. Such lines are shown in Fig. B27. They show that the resolution limit of the spectrometer is high if the mid-plane slit is narrow and non chromatic aberration due to the source size can be kept low; in fact the size of the source appears as the most important factor in the matter of resolving power. The variation in results with the point sources is most probably due to differences in the precise location of the source in its holder, where any slight slackness in the locating hole could give a small error in position. A further possible source of error lay in the possibility of deposition of activity on the sides of the wire. This was avoided on the whole very successfully, and sources could be examined for this effect by using the α -particle "shadow" method with a scintillation screen; a further precaution was to allow the wire to protrude only very slightly from its locating hole, thereby cutting off any β -rays from the side. It has been found in general in all the investigations with the spectrometer that if consistent results are to be obtained great care must be taken that the activity is uniformly deposited on the source, and that the centering is exact.

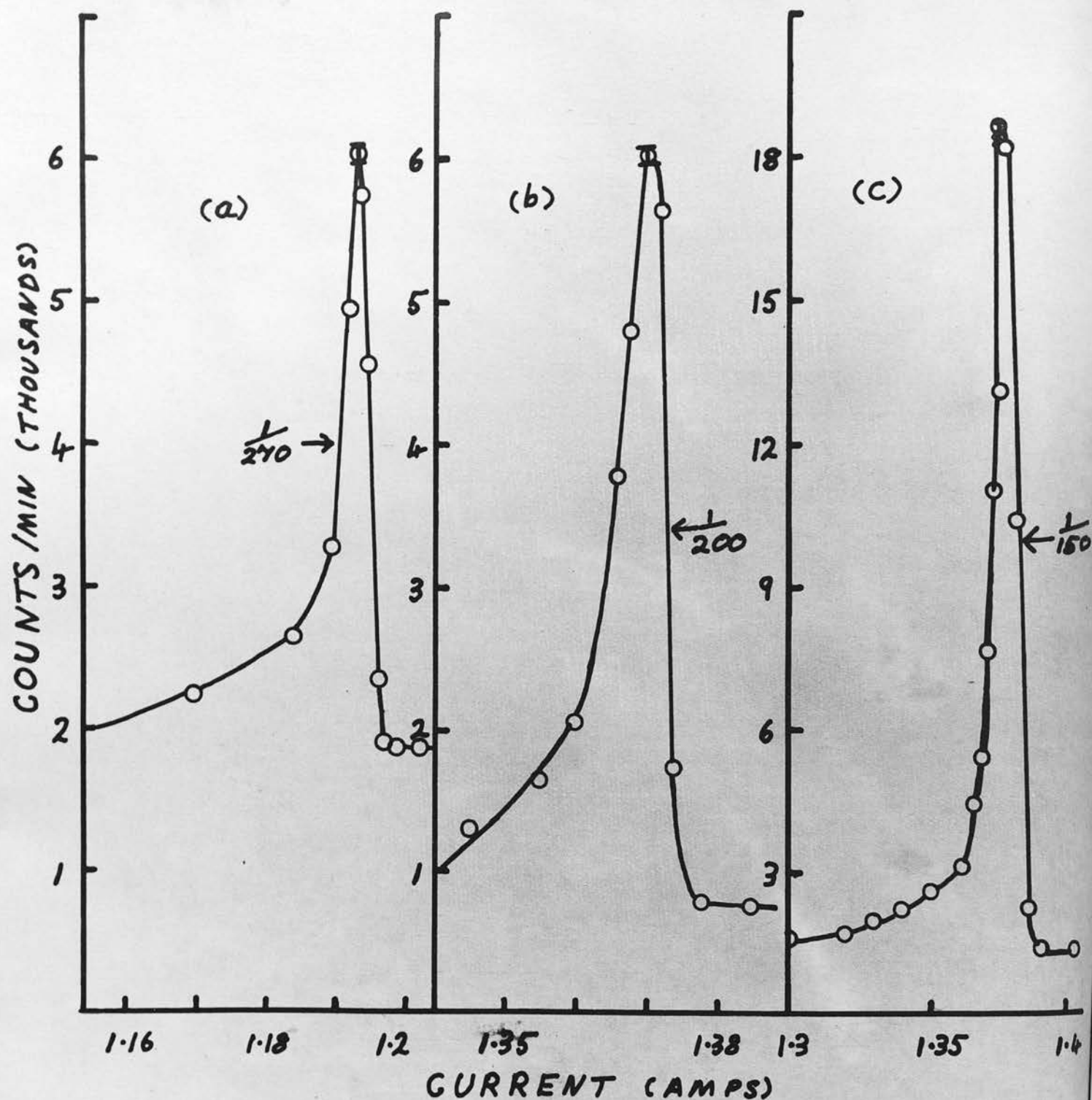


Fig.B27 F line of TnB with central baffle 1mm.
 wide (a) source diam. .33mms. baffle diam. 9.4cms
 (b) " " .33mms. " " 9.3cms
 (c) " " 1mm. " " 9.3cms

The best values of the resolving power obtained were:-

Source Diameter	•33	1	2
B_1, B_2	•95	•95, 2•2	2•2
R	$\frac{1}{270}$	$\frac{1}{150}, \frac{1}{79}$	$\frac{1}{40}$

B_3, B_4 being 0•5 millimetres.

It was also found that altering the source-counter distance over a range of two millimetres did not affect the resolving power, showing that as this distance varied, a slightly different momentum was selected in the transmission adjustment, but that the aberrations were not appreciably altered, for such a small change.

Solid angle estimates were made by comparing the counting rate at the peak of the F line where there are •3 conversion β -rays per disintegration of ThB, with the total source strength which was found by counting the 8•5 cm. α -rays of ThC-C" with a Rosenblum spark counter, kindly loaned by Dr. Connor of this department (Connor, 1949). This was at the time the only available α -ray counter; it was not particularly suitable for this measurement as the sources were always weak when measured, which meant, since the counter has a solid angle of approximately 10^{-4} of a sphere, counting for very long periods. The counter operates over only a very limited range about the peak of the Bragg curve and so over long counting periods errors due to changes in temperature and general atmospheric conditions are introduced.

For the line of Fig. B26 (obtained with a 1 mm. diameter source, $B_1 B_2$ being 2.2 mm.) a value for the solid angle of 3.1% was found, to which is to be added a correction of perhaps 10% for absorption in the aluminium counter window. (This figure is that to be obtained with a full 360 degrees azimuth). Unfortunately a variety of circumstances conspired to make the values obtained for the solid angle in other cases less reliable than that just quoted.

Several sources were compared in total activity with this source by means of the γ -ray background in the

β -counter and these were rendered valueless by a high spurious counting rate which developed undetected at the time in the counter and counting circuit, not at that stage in its final form. Others were rendered doubtful because of the possible inaccuracies in the source strength measurement of the very weak sources with the spark counter. However, an analysis of a number of these dubious results seemed to show that a value of the solid angle for the .33 mm. source with $B_1 B_2 = 0.95$ mm. lay between 1 and 2%.

Thus the preliminary measurements showed that the spectrometer was capable of high resolutions of $\frac{1}{200}$ or more provided that asymmetries of field and baffle system could be kept low, but that such high values could only be obtained by using extremely small sources. The solid angle in such circumstances could be more than 1%, and it would

be possible to obtain a solid angle of several per cent at a resolution of $\frac{1}{70}$, using a larger source.

It was also clear that the design of ^{the} final model should be based on a radius of 9.3 cms., but that, in view of the apparent critical dependence on axial symmetry and the possibility of small errors in this value, it would be best to allow still in the construction of a final model for some variation.

IV. COMPLETE SPECTROMETER, ADJUSTED FOR HIGHEST RESOLVING POWER.

The final model of the spectrometer was designed in the light of the conclusions of the last section. That is to say, it was decided that the radius of the main slit should be 9.3 cms., but that it would be desirable to allow for a slight range of variation so as to be able to check the previous result and set very accurately on the best value if it should be desired to produce very high resolution with a fine slit. It was also desirable to allow the possibility of adjusting the centering of the source and main slit during operation (that is without breaking the vacuum), and thus to observe immediately the effect of any adjustment and correct any errors directly.

(1) Construction of Final Model.

This problem was dealt with by Dr. Richardson, and the general mechanical design of the complete slit system was due in its main essentials to him. A schematic drawing of the assembly is shown in fig. B31. The basis is an expanding annular slit which is supported by a gun metal supporting "spider". This spider is fixed in place by rods screwed into the source-end end-plate of the spectrometer tank; it carries a massive lead γ -ray shield which is contoured to fit the beam of β -rays, and a screwed

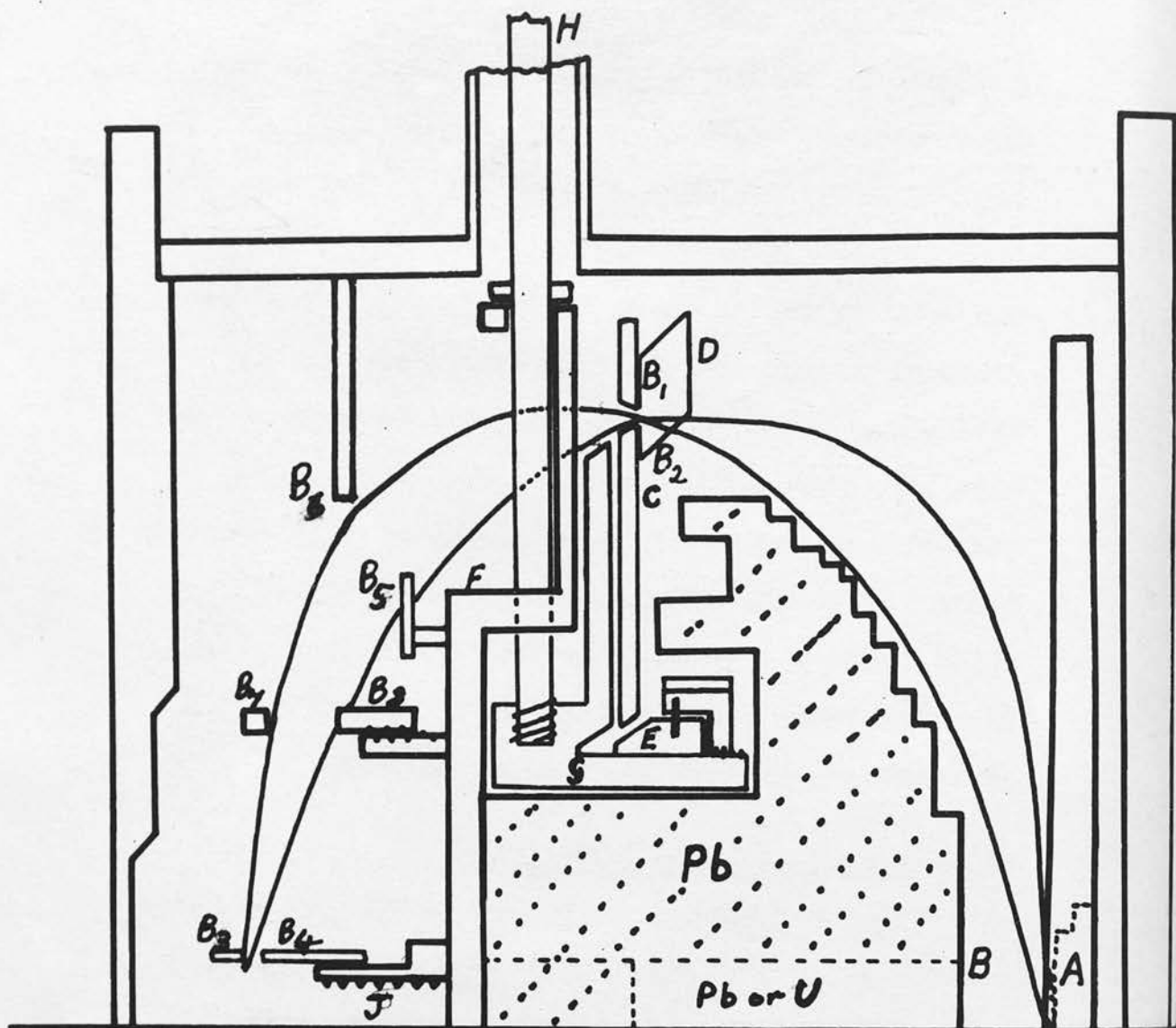


Fig.B3I Complete baffle system showing, A source slide and holder, B lead shielding, C sector of expanding annulus, D section of spiral baffle, E collett with driving wheel, F supporting spider, G annulus base plate, H supporting rod, J counter location.

flange to locate the counter and the counter slit. There is a 1 inch diameter hole along the axis of this block to accommodate a rod of uranium to improve the γ -ray shielding. This was made two inches long and there was one inch of lead between it and the counter to stop γ -rays from the uranium itself; these contributed only 3 per minute to the counter background so that there was a considerable gain from the use of the uranium with a strong γ -ray source. If desired it could easily be removed and replaced with a lead rod. The spider is cut away save for three supporting struts so as to allow the passage of the β -rays, and it bears the weight of the main slit $B_1 B_2$ which is in the form of an expanding annulus made up of 18 sectors. The slit unit is built up from a base-plate against which the sectors are held by springs; it also provides the seating for a collett, driven by a wheel, which advances on a screw. This drives the collett towards the base-plate so that it bears against the sectors of the annulus and pushes them out against a spring tension. Friction in the motion of the sectors is reduced to a minimum by having them slide, not actually on the base-plate, but on nickel silver rods retained in slots between the sectors and the base-plate. The collett driving wheel is turned by a rod which, passing through a vacuum gland in the source door, engages in castellations in the circumference. The

whole unit is supported from a single suspension on the spider and the supporting rod engages a driving rod passing to a wheel outside the tank in such a way that the annulus may be raised or lowered by turning this wheel. A phosphor-bronze spring holds the annulus base-plate against the spider so that it always remains in a vertical plane. A second screw, operated by a driving rod through a Wilson seal in the source door, provides horizontal motion of the annulus.

The "outside" and "inside" parts of each sector of the ring slit are held together by means of $\frac{1}{16}$ " aluminium paddle baffles which are turned to make an angle of 30 degrees with the plane of the annulus. Since the rays cross the mid-plane at this angle the baffles present to neutrons only their edge but for positrons, spiralling in the opposite direction, they completely block the sectors of the annulus. The figure of 30 degrees for the angle at which the rays cross the mid-plane was checked roughly by constructing the trajectory of a ray with a fine wire carrying a current. If the current is sufficiently strong to heat the wire and remove kinks, the wire takes up the path of an electron in a magnetic field. Such a test, using a length of 46 gauge copper wire, showed that the figure of 30 degrees is closely correct.

The source is mounted in a sliding bar made of brass, $\frac{1}{4}$ " thick, 2" high and 12" long. This passes

through a gate valve connected to the backing pump via the main stop-cock. The gate-valve is held against a rubber ring making a vacuum seal with the source door by three screws in such a way that the slide and the gate valve may be turned about two of them as a vertical axis. This means, in effect, that the end of the source slide (and therefore the source) may be moved horizontally towards or away from the mid-plane, and this is done by a screw passing through the end-plate of the tank and making contact with the slide. Vertical motion of the source is provided by raising, through a screw and cam, a bracket into which the end of the slide fits. These motions are allowed by the elasticity of the rubber gaskets sealing the slide and gate valve. The third co-ordinate in the source motion is ^{provided} by controlling the length to which the slide penetrates into the chamber. The source itself is located in some suitable fashion in a 1" hole in the slide. For .33 mm. diameter sources on the end of a fine wire this was done by having a fine central hole in an aluminium bridge piece.

The baffles B_5 and B_6 were of $\frac{1}{8}$ " aluminium, B_5 being attached to the spider and B_6 being in the form of a ring fitting closely inside the walls of the tank and fixed by long set screws relative to the spider.

The counter now installed was the final model described earlier.

The walls of the chamber were lined with aluminium

and the surface of the lead block was coated with graphite so as to reduce scattering from these surfaces. Scattering effects should be negligible, however, in an instrument using such a complex system of trajectories and so many baffles. (Owen (1949) has pointed out recently that scattering becomes an insignificant factor even with simple ray paths if the beam is defined by a large number of baffles).

The spectrometer was first adjusted with very narrow slits B_1B_2 and B_3B_4 which were chosen to be 0.6 mm. and .33 mm. respectively. This was done with a view to determining the optimum performance in respect of resolving power that could be expected with the full azimuth in operation. Such a procedure had the advantage, regarded as a stage in the setting up of a working spectrometer, that it would test severely the axial asymmetries which might arise in practice.

Sources used in this case were all .33 mm. "point" sources.

(2) Variation of Radius:

The first concern was to check the validity of the previous result concerning the optimum radius. This was done as before by examining the transmission curves and the shape of the F-line for different radii, and in the course of this investigation some

alterations were made in the positions of the other baffles. The situation, however, was not so simple as in the previous case since baffles B_5 and B_6 were now fixed and so could not be made to correspond to the radius of the central annulus if this was changed. A more important difficulty, however, was the fact that the expanding annulus when assembled proved incapable of compassing a radius of greater than 9.3 cms. and therefore the readings were perforce confined to values of the radius less than this.

Many of the observations were made by means of the counting rate meter which made it possible to make a quick survey of the transmission curves and the height of the F-line above background, which was found in general to be accurate measure of the resolution, the scaler being brought into operation only when precise information was required for record.

It was immediately found that the high resolution found with the short slit was not reproducible, a value of $\frac{1}{100}$ being normal for a .33 mm. source as opposed to over $\frac{1}{200}$ in the preliminary work. This will be discussed later; at first this was regretfully accepted after a careful check of the centering adjustments.

There were two possible sets of transmission curves to be obtained, that already mentioned, in

which the whole spectrometer is moved, and that where the source alone is moved. The two are shown compared in Fig. B37, where it is seen that the letter (A) is twice as broad as the former. (B). This is because in the former there is added to the contracurrent motion of the source and image an equal concurrent motion of the counter. The second type was in the end used solely for the work with the spectrometer because of its greater simplicity. The effect of activity on the sides of a wire source is shown by Fig. B37b. Source (a) was deposited on a platinum disc. This shows that the "tail" usually found with transmission curves is due simply to this cause. The curve (b) of this figure is as broad as that of Fig. B22, although it was taken with much narrower slits. This will be discussed later under aberrations due to asymmetry.

A family of type (b) curves for different values of the radius is shown in Fig. B38, where, as before, the curve becomes less tall and more broad with decreasing radius, the best values being at $r = 9.3$ cms., confirming that the optimum radius is not less than this value.

r	9.3	9.28	9.26	9.22	9.17	9.13
T	5.5	5.2	4.8	4.3	4.1	3.5
Δ (mm.)	0.28	0.3	0.38	0.45	0.52	0.6

The counting rate meter confirmed that the line

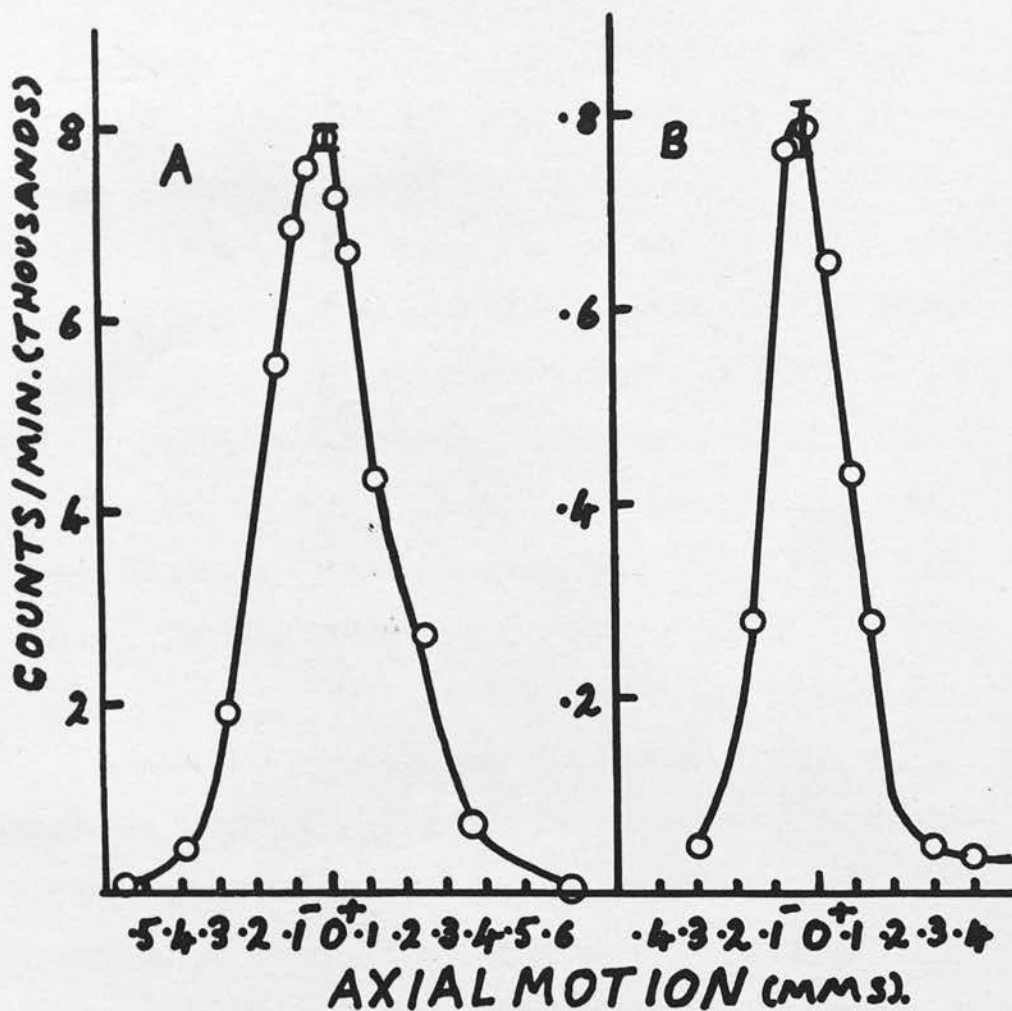


Fig.B37 Transmission curves (a) moving source alone
 (b) moving whole tank
 Positive motion is towards the counter, and stray activity on the side of the wire source in (b) gives a tail not found with a disc source (a)

height, and therefore the resolving power, was highest at $r = 9.3$ cms. These figures were obtained with B_5 $\frac{1}{4}$ " further to the left than it is shown in Fig. A15. It was moved $\frac{1}{4}$ " to the right so as to play its part in improving the resolving power.

With this arrangement decreasing the radius actually improved the resolving power. This was due to the action of B_5 in narrowing the width of the beam as the radius was decreased.

r	9.3	9.25	9.2	9.15
T	5.1	4.5	3.5	2.5
R	$\frac{1}{82}$	$\frac{1}{83}$	$\frac{1}{100}$	$\frac{1}{106}$

(T is in arbitrary units)

This result, of course, was to be expected as the action of B_5 is effectively that of reducing the width of the main slit $B_1 B_2$. It means also that any interpretation of results obtained with different radii is difficult, but it was quite apparent that $r = 9.3$ was better than any lesser radius. (In any case the improved resolution at the lower radii could only be obtained at the expense of reduced solid angle.)

It was possible to vary the source-counter distance over a small range (some 2 cms.) simply by moving the tank to different axial positions.

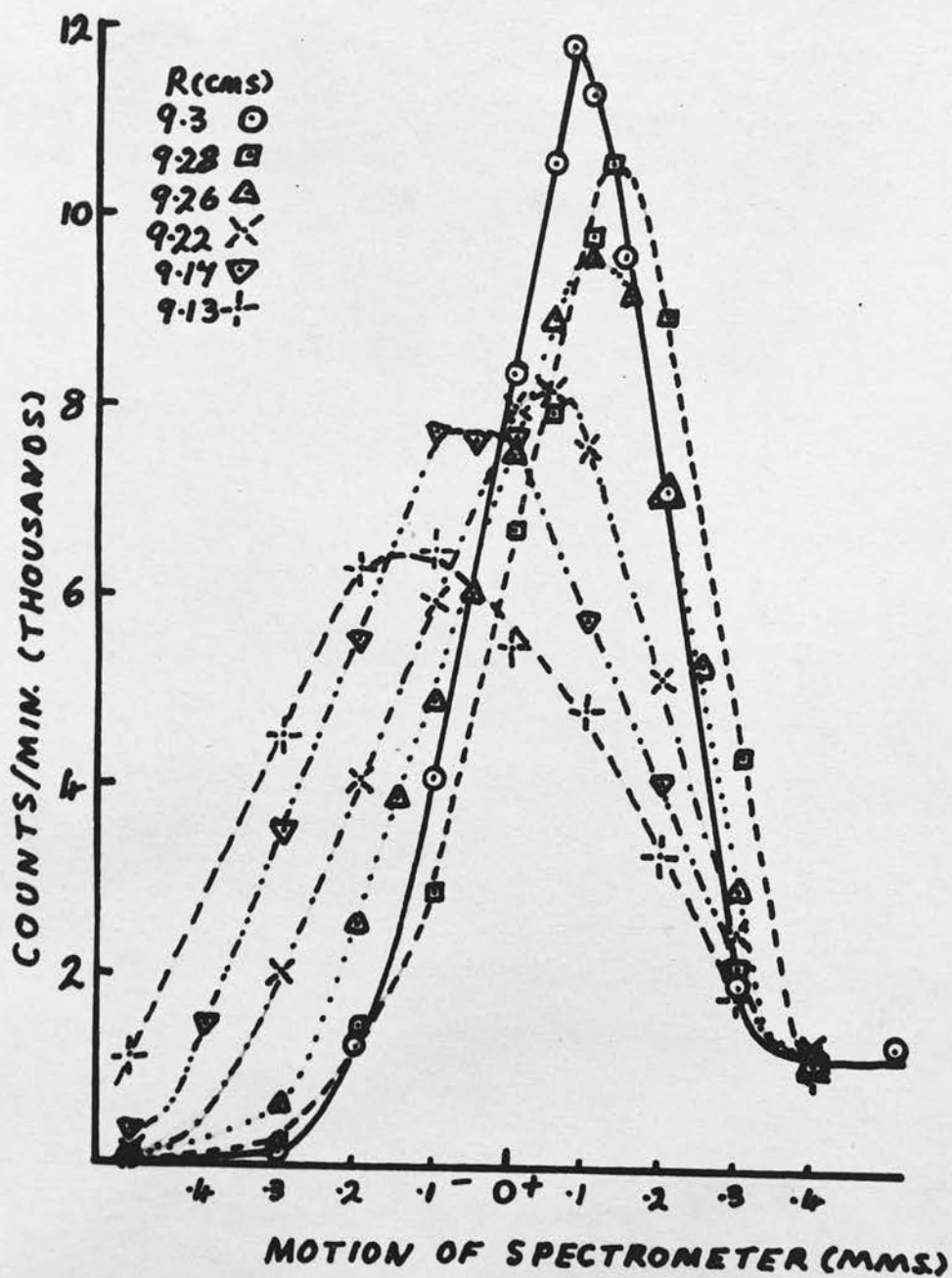


Fig.B38 Transmission curves with full azimuth showing effect of radius of central baffle

and then adjusting the source position to correspond with the (contracurrent) motion of object and image. Four different transmission curves obtained at different axial positions showed the same height and width (within 5%) so that this is as before a non-critical adjustment. The spectrometer tank was usually kept as far from the mid-plane as possible as a matter of convenience and to use the lowest angles of emission.

(3) Resolving Power and Solid Angle:

The source strengths were now measured using a thin-window E.H.M. 2 counter with the amplifier set below the threshold for β -counting. The window of the counter was 1.7 mg/cm^2 and the source was placed at a distance of 1 cm. from a hole 3 mm. in diameter to define the beam of α -particles passing into the counter. Under these conditions there was a very flat plateau 100 volts long, showing that α -particles were detected with 100% efficiency. This made solid angle measurements much more reliable than before; the remaining source^{of} inaccuracy was activity deposited on the sides of the wire; this was estimated to cause no more than 10% error as the sides of the wire were shielded on exposure to the radiothorium and were examined for activity with the scintillation screen and found in general to be almost free of any, when compared with the activity on the end.

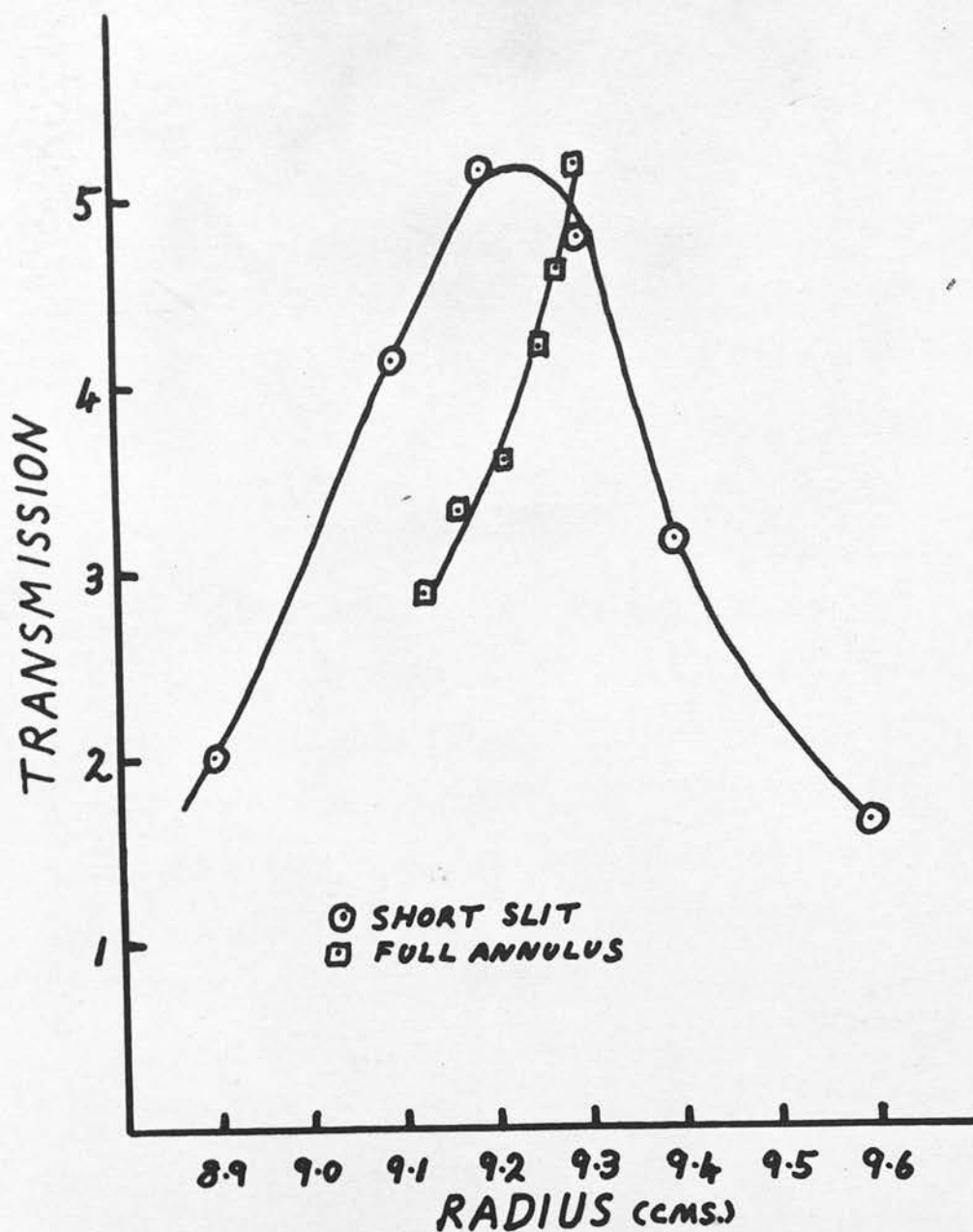


Fig.B39 Effect on transmission of radius of central baffle..Relative displacement is due to some difference in setting in this particular measurement with the short slit. A slight difference in slope can be attributed to the stationary baffles B₅B₆ in the case of the full annulus.

Fig. B40 shows a line obtained with the baffles in the sites just described. The half width is $\frac{1}{115}$ and the solid angle 0.8%; this was obtained only after considerable adjustment of the source centering to give best transmission as determined by the rate meter. (The value of the solid angle, of course, is reduced below that which would be obtained for a full 360° azimuth by the obstructions of the counter bridges (70°) and smaller amounts (a total of about 10% from the spider legs, edges of the "paddle" baffles etc.).

There was some interest in studying the performance with the baffle B_2 moved from the mid-plane towards the counter, because this should result in a reduction of the angle of emission from the source (a point of some importance), and so the annulus was moved 5 mm. to the right, this being the maximum distance set by the mechanical construction, and B_5 and B_6 were moved by the same amount. If the family of trajectories selected now remains the same we should expect that the solid angle should be slightly decreased because of the lesser concentration of rays on the slit and that the ^{effect on the} resolving power might be expected a reduction ~~in~~ because the central rays will be cut out, though the effects for such a small change should not be marked. The actual values obtained were $R = \frac{1}{106}$, $\Omega = 1.1\%$. This seems to indicate a slight change in the effect of $B_5 B_6$ but no more.

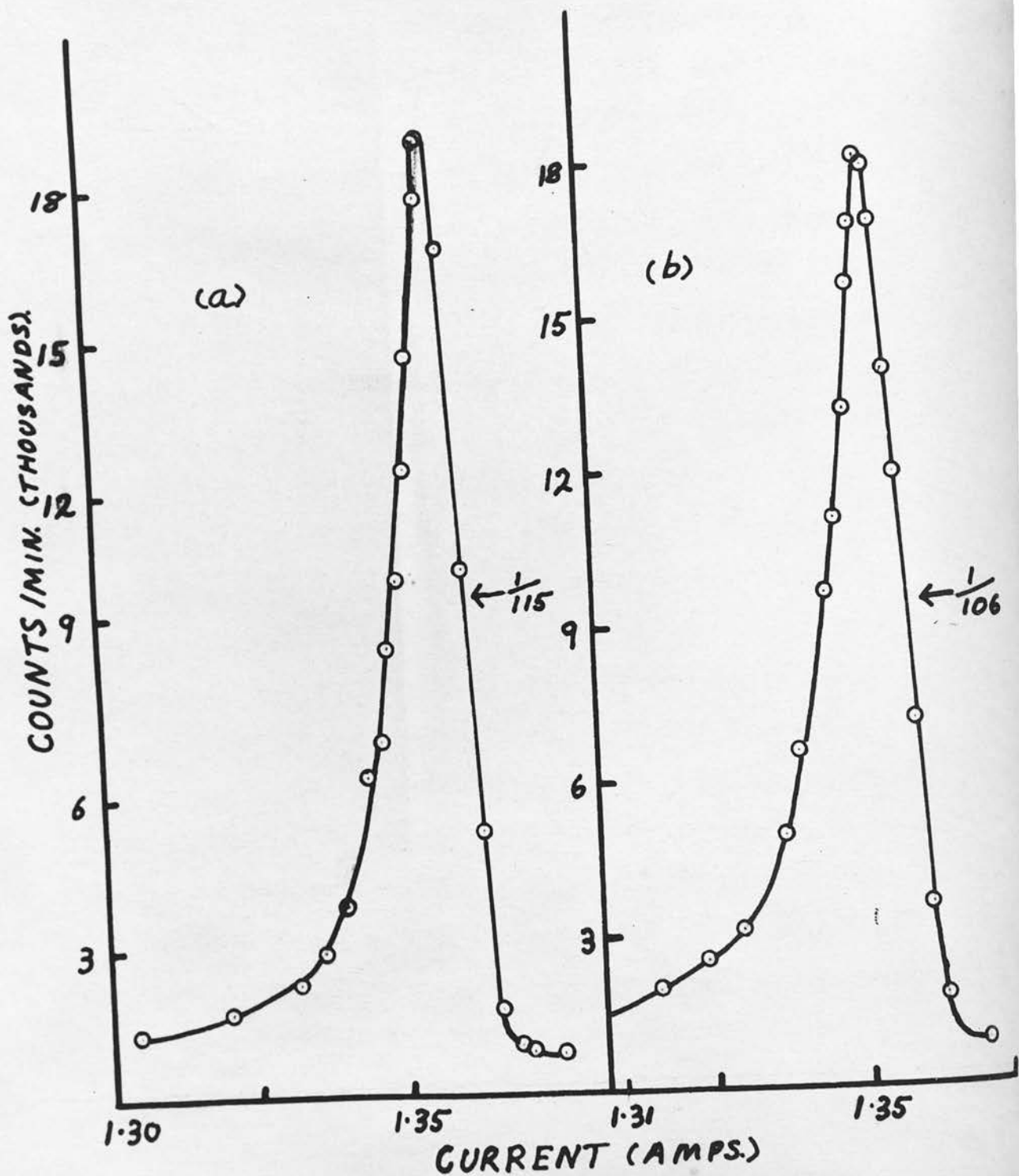


Fig.B40 F line of ThB taken with full annulus
In (b) the annulus was moved 5mm. towards the
counter

No sources of larger radius than .33 mm. were used, but Fig. B42 shows the effect of moving the source radially. It was obtained by moving the source slide out sideways in a radial motion of the source, when the longitudinal position was on the peak of a transmission curve. The rapid decrease in counting rate shows again that only small sources may be used.

Since this curve, like the transmission curves, was taken in a flat continuous spectrum, it is seen once again that rays, whatever their energy, are focused correctly only when the source is exactly at the correct position, meaning that scattered β -rays from material near the source will have only a small chance of reaching the counter.

(4) Effect of Asymmetry:

The resolving power and the width of the transmission curves differed so much from the preliminary results with the short slit that it was decided to investigate the matter by introducing a baffle into the spectrometer which could be rotated about the axis in such a way as to select different zones of the azimuth. This was made in the form of a slit encircling the axis at a radius of 5 cms. between $B_5 B_6$ and the counter, round which could be wrapped a strip

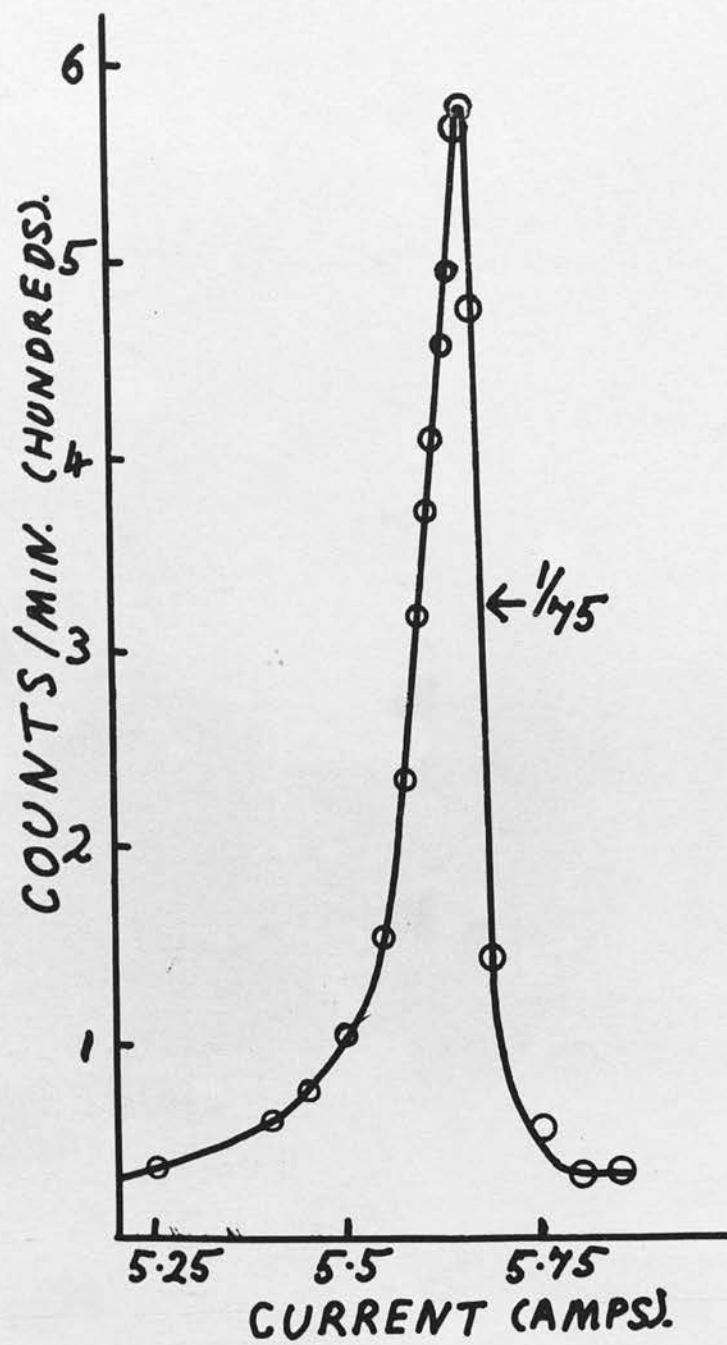


Fig.B41 A line of ThC, spectrometer as in Fig.40a

of aluminium to block off part of the azimuth as desired. The slit assembly screwed on to a fixing tube screwed to the spider and was rotated, changing both the azimuth and longitudinal position, by a rubber band passing round a pulley system operated from outside the vacuum. The slit itself was $\frac{1}{8}$ " in width, and a preliminary measurement made to see whether this by itself improved the resolving power, achieved only the rather disappointing result of $R = \frac{1}{120}$, and this was insensitive to the point of the beam selected by moving the annulus along the axis. This seemed to show that the bad performance was due to axial asymmetry rather than any inefficiency of the defining baffles. (It had been considered possible that the wing baffles on the short slits had been set badly. A slight error of the position of these baffles so near the first ring focus could be expected to have a considerable effect).

This conclusion was readily verified by blocking off all but one third of the rotating annulus. It was then found that different sectors gave different positions for the peak of the transmission curve, and focused on the F-line at slightly different currents. The rotating annulus was then arranged to have four short 1" slits, exposed one at a time in different quadrants, and it was found then that by setting the annulus off centre by only 1 mm. the transmission curves could be made to have quite distinctly different positions and the peak F-line was focused at

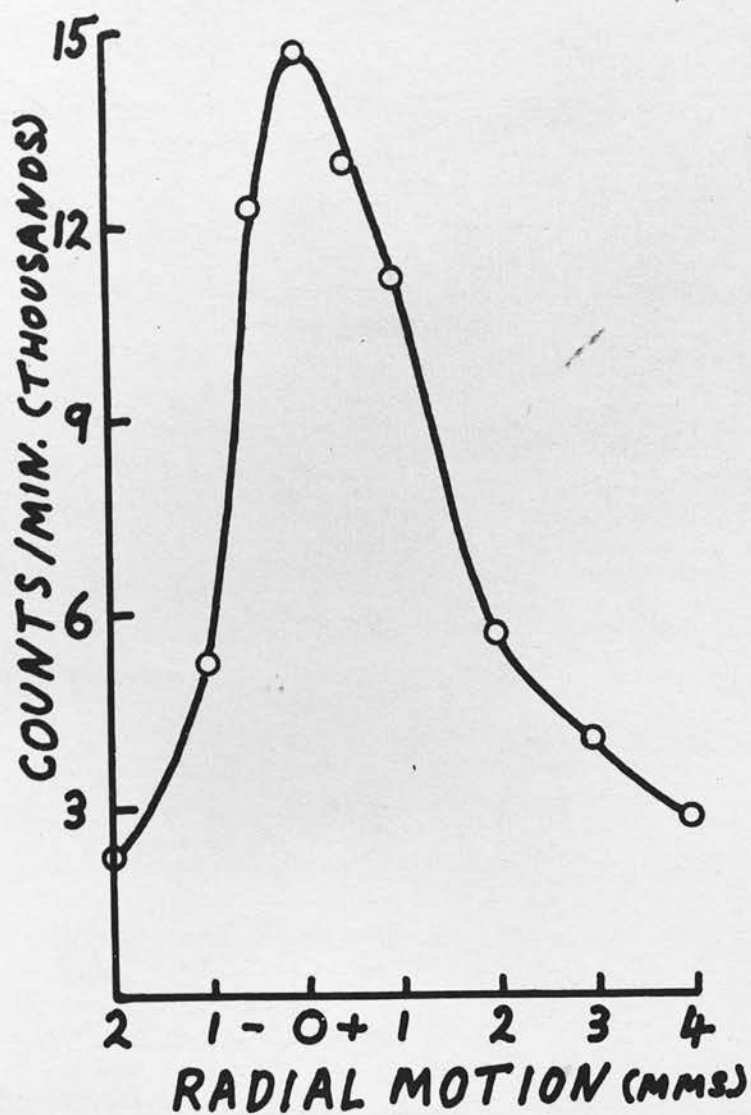


Fig.B42 Radial motion of source. The curve is asymmetrical because the source does not move exactly perpendicularly to the axis.

different currents.

Even careful centering of the annulus so as to reduce these effects did not produce higher resolution than had already been obtained.

It was at this stage that the axial symmetry of the field was checked by a rotating coil as described earlier, and the pole-pieces were very carefully lined up to remove any traces of asymmetry. It was also found that many of the brass components had a slightly ferromagnetic content (ordinary commercial brass contains up to 0.3% of iron). These were removed and replaced with copper. These precautions were found to produce no improvement.

When all other possibilities had been eliminated the residual factor was the centering of the source, although this could be done optically with fairly high accuracy. This had, of course, a considerable effect on the aberrations discussed and they could be reduced very considerably by using the source adjustments; nevertheless it did not seem to be possible (or at least it was not achieved) to obtain a resolving power better than $\frac{1}{115}$ with the full annulus. It is, of course, to be expected that the aberrations produced by a source off the axis would be large, but it was disquieting to find that the proper adjustment was so critical as to be impracticable. The effect of setting the annulus off the axis was considerable; one case, for instance, when the

annulus was set 2 mm. off the axis and the source was only a fraction of a millimetre off produced (perhaps with the aid of a little activity on the sides of the wire) a transmission curve which was more than twice as broad as normal, and showed a double-peaked top. It is interesting to note that in one case in the work with the short slit a defective mask caused a source to be produced with two concentrations of activity, and this produced a satellite F-line at a different value of the current.

The large values of the aberrations mean that the slits $B_1 B_2$ and $B_3 B_4$ are set to be less in width than is justified, so that they could be increased in width to some extent with no deterioration in resolution but with an increase in solid angle. Also the erratic nature of some of the results which has been commented upon is thus explained.

In Fig. B44 are shown lines obtained with the rotating annulus slit set at 100 degrees of azimuth and $\frac{1}{8}$ inch in width. That the adjustment is much better than in other cases quoted is shown by the increased resolution ($R = \frac{1}{145}$ for the I-Line), and the value of the solid angle ($\Omega = .27\%$ or $.8\%$ for 270° azimuth) which is greater than the value to be expected from a $\frac{1}{8}$ " slit selecting from a beam about twice as broad. (It must also be remembered of course that in this case the more central rays are being selected which by itself will lead to an improvement, and the effect is not purely one of reducing

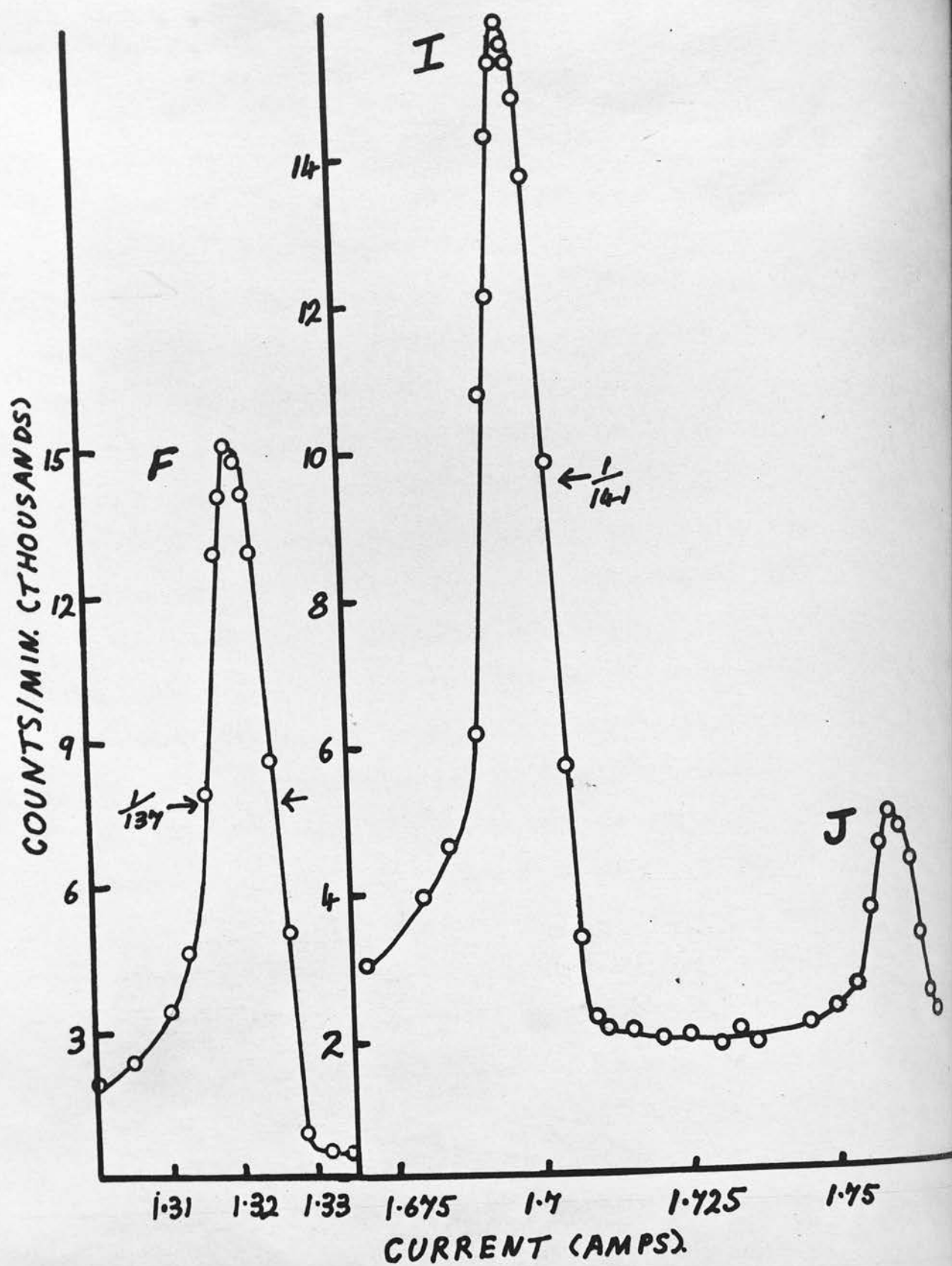


Fig.B44 F,I,J lines taken with 100 degrees azimuth
(relative current values are not significant)

azimuthal asymmetry; nevertheless, as has been already stated, a full azimuthal slit of this width did not give such an improvement).

(5) Axial Source:

The fact that the spectrometer focuses on the counter slit rays from only a short length of the axis means that if a source is used which is extended as a thin line along the axis then rays from only a short section will be focused - in fact the variation of solid angle efficiency along the source is then given by the transmission curve.

A fine radon tube kindly lent by Mr. D.A. Silverston was used to obtain Fig. B45, showing the shape of the .264 mev H-line of radium B, the half-width being $\frac{1}{40}$. The tube was 2 cms. long. The wall thickness was approximately 3.5 mgms/cm², which should account for about 10% of the line width. (White & Millington, 1928).

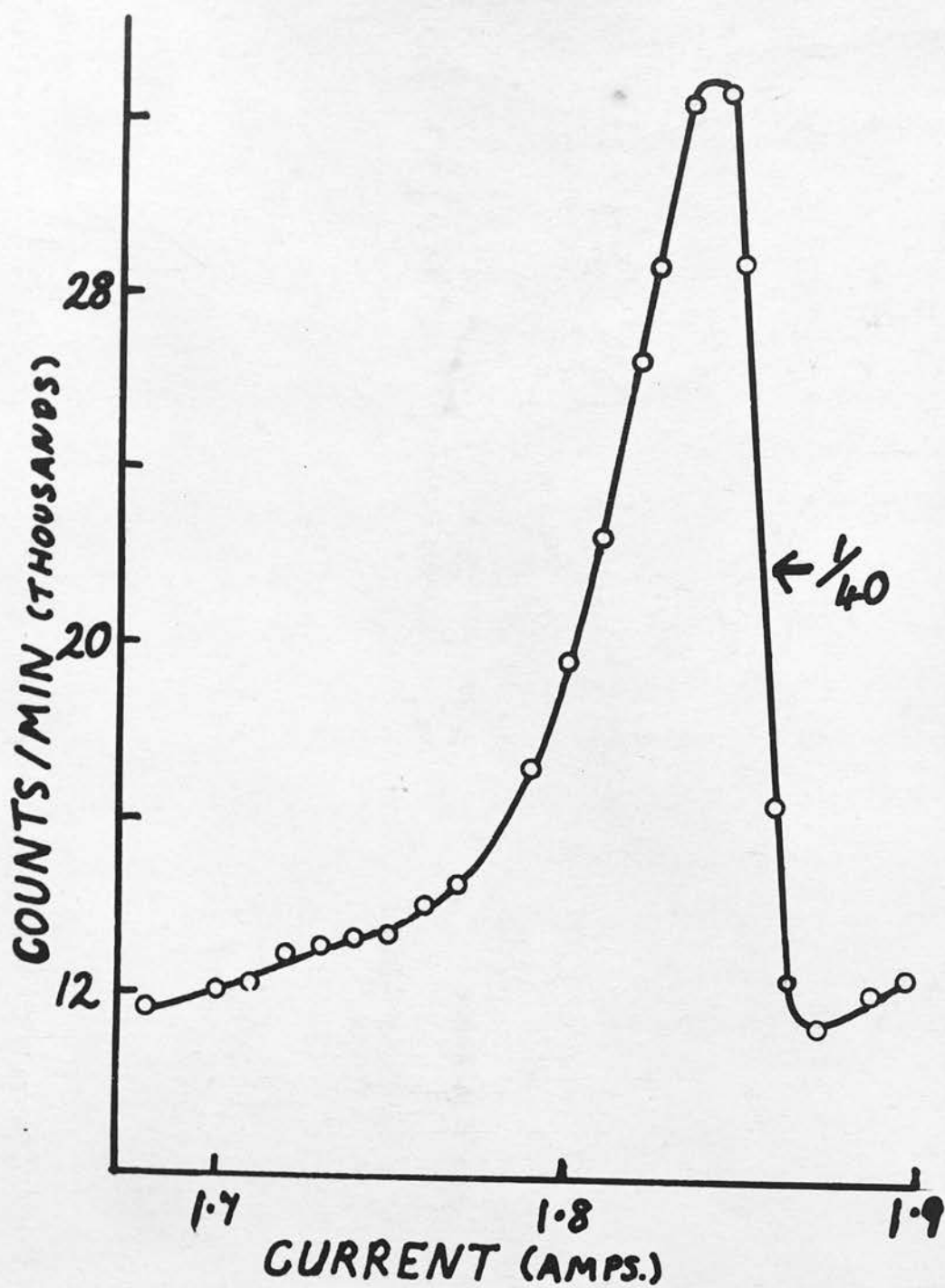


Fig.B45 H line of RaB.taken with a radon tube source

V. SPECTROMETER ADJUSTED FOR HIGH SOLID ANGLE.

In this case the spectrometer was adjusted with a view to making the measurements on the continuous spectrum of thallium to be described in Section C, and so the slit B_1, B_2 was moved 1 cm. from the mid-plane towards the counter in order to reduce the angle of emission. The other baffles B_5 and B_6 were removed by the same amount to the right. The centering adjustment of the annulus was removed and it was instead accurately centred on the spider and fixed with two dowel pins. The slit widths were set at $B_1, B_2 = 1.7$ mm., B_5, B_6 at 15 mm. and B_3, B_4 at 1.15 mm. ($\cdot 046$ "). The sources used were all deposited on a flat surface. In the case of those of $\cdot 33$ mm. and 1 mm. diameter they were deposited by recoil collection on platinum. Because of non-uniformity of deposition by this method, those of greater diameter were made by collecting the ThB on platinum, and dissolving it off in hydrochloric acid. A (diluted) drop was placed on a piece of mica mounted on a brass disc, and then the activity was precipitated as a sulphide by exposure to H_2S . The activity was confined to the desired region by means of silicone anti-wetting resin. The recoil activation method and the chemical method were both arranged to give sources accurately centred and the source holder was

basically an aluminium ring fitting into the hole in the source slide in which the sources were centred automatically, the holder itself being centred optically once for all. The various adjustments were not now so critical. (Also small asymmetries should have negligible effect with such slit widths). In particular the transmission curve for a small source had a flat top which was, of course, a great experimental convenience.

Transmission curves and R-lines obtained with different sources are shown in the figures. It is once again obvious that the best performance is obtained only for small sources, though the decrease (at first) of R is slow.

The values obtained for resolution and solid angle were:

Source diameter (mm)	*33	1	2	3
R	$\frac{1}{50}$	$\frac{1}{50}$	$\frac{1}{33}$	$\frac{1}{25}$
Ω	4.2%	4%	3.8%	3.2%

The main contribution to the width of the transmission curves is the width of the counter slit itself, and the aberrations of the image contribute to the slope of the sides of the curve. With the 3 mm diameter source the effect on the sides is pronounced.

In this condition, then, the instrument gave a very high collecting power (and it must be remembered that the figure given is the intrinsic solid angle reduced by obstructions; the solid angle for the full

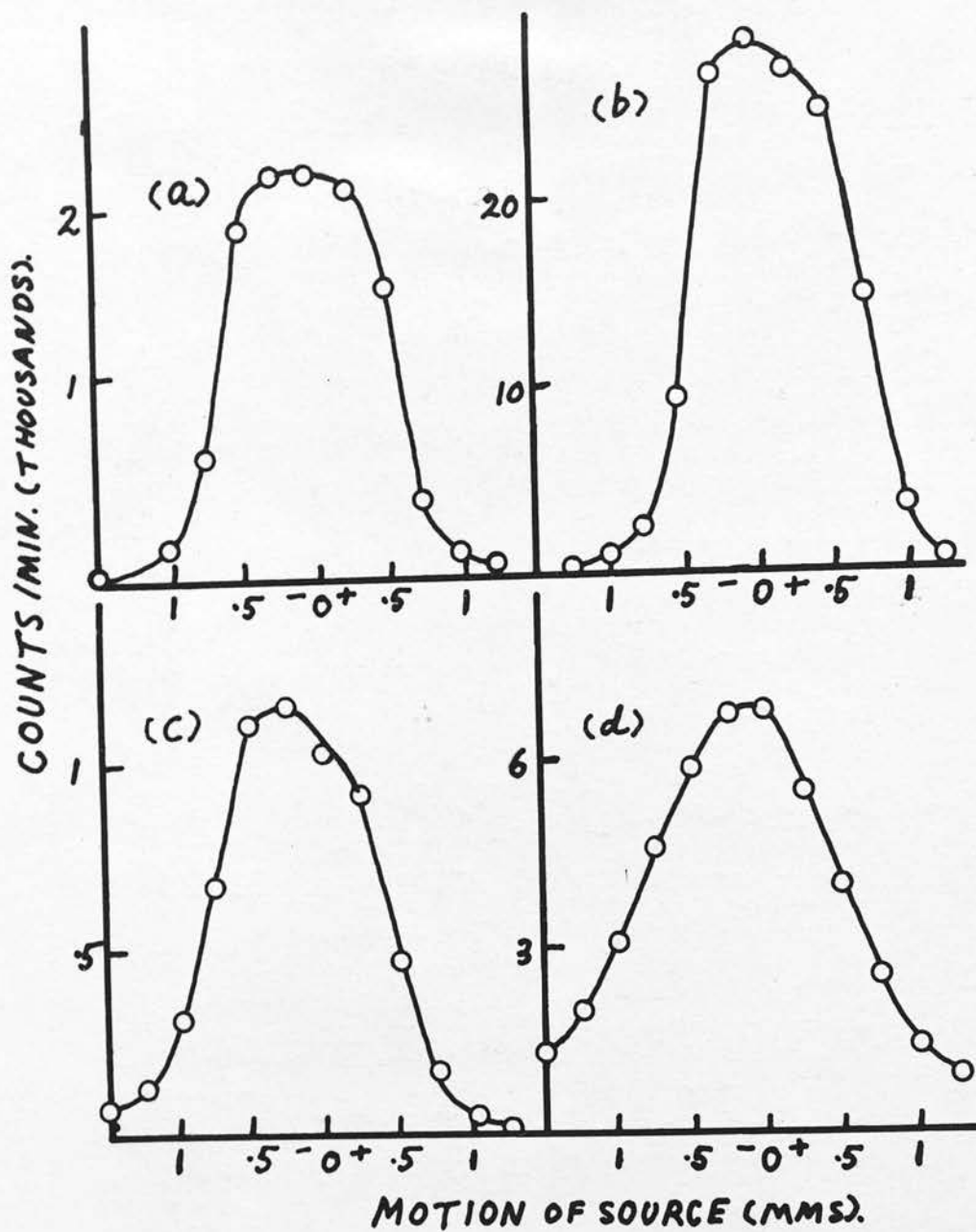


Fig.B47 Transmission curves with wide slits.
 Source diams. are (a) .33 (b) 1.0 (c) 2.0
 (d) 3.0 mms.

azimuth in the best case is 63), at quite good resolution. This should be of much value in cases where there is no objection to concentrating the activity on a small area.

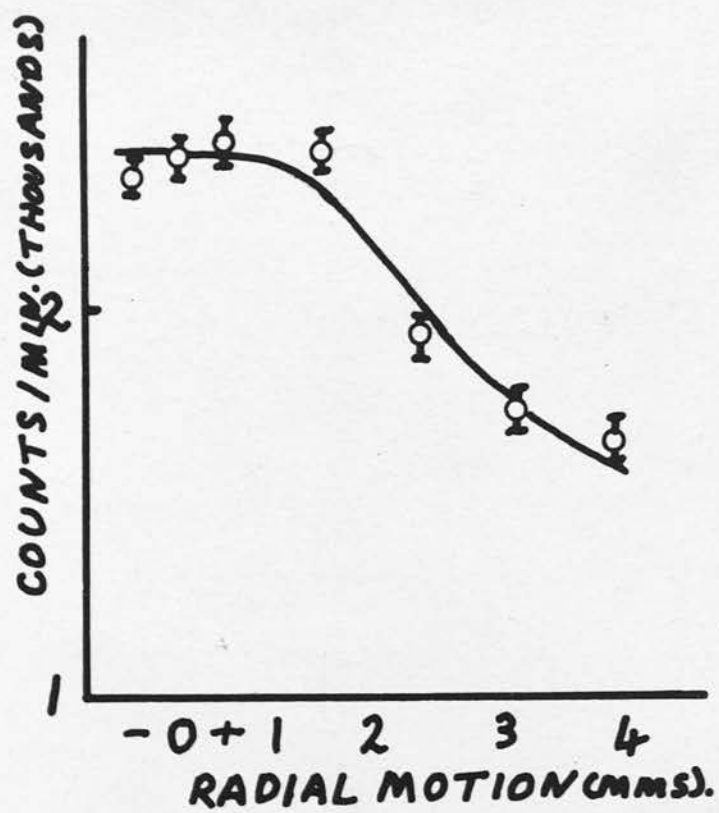


Fig.B48 Radial motion of source with wide slits

VI. DISCUSSION.

The results of IV and V show that the spheroidal field spectrometer possesses the general characteristics of the second image type of system discussed in Section A. Good performance is only to be obtained with very careful siting of the source and counter and only with small sources. It is thus confirmed that this type of lens field is of little value for the production of images in microscopy. It does not of course follow that such defects render it useless for β -spectroscopy where good image formation is not necessarily required.

The performance within these limitations is good, however, and resolutions of rather better than 1% may be obtained at a solid angle of more than 1%. At the more moderate resolution of $1/50$ a very high solid angle may be obtained, of about 6%, if the full 360° azimuth is used (as could be done with a scintillation counter, or any counter not requiring a thin window).

Nevertheless the limitation on source size is of great practical importance, and limits the range of usefulness. The study of continuous spectra, for instance, is rendered very difficult since it is necessary to use a thin and therefore weak source, a necessity intensified by the large angles of emission.

It seems to the writer that the principal application of this spectrometer may be to the study of Compton or photo-electrons ejected by γ -rays from a target. The disadvantage of the small size of the

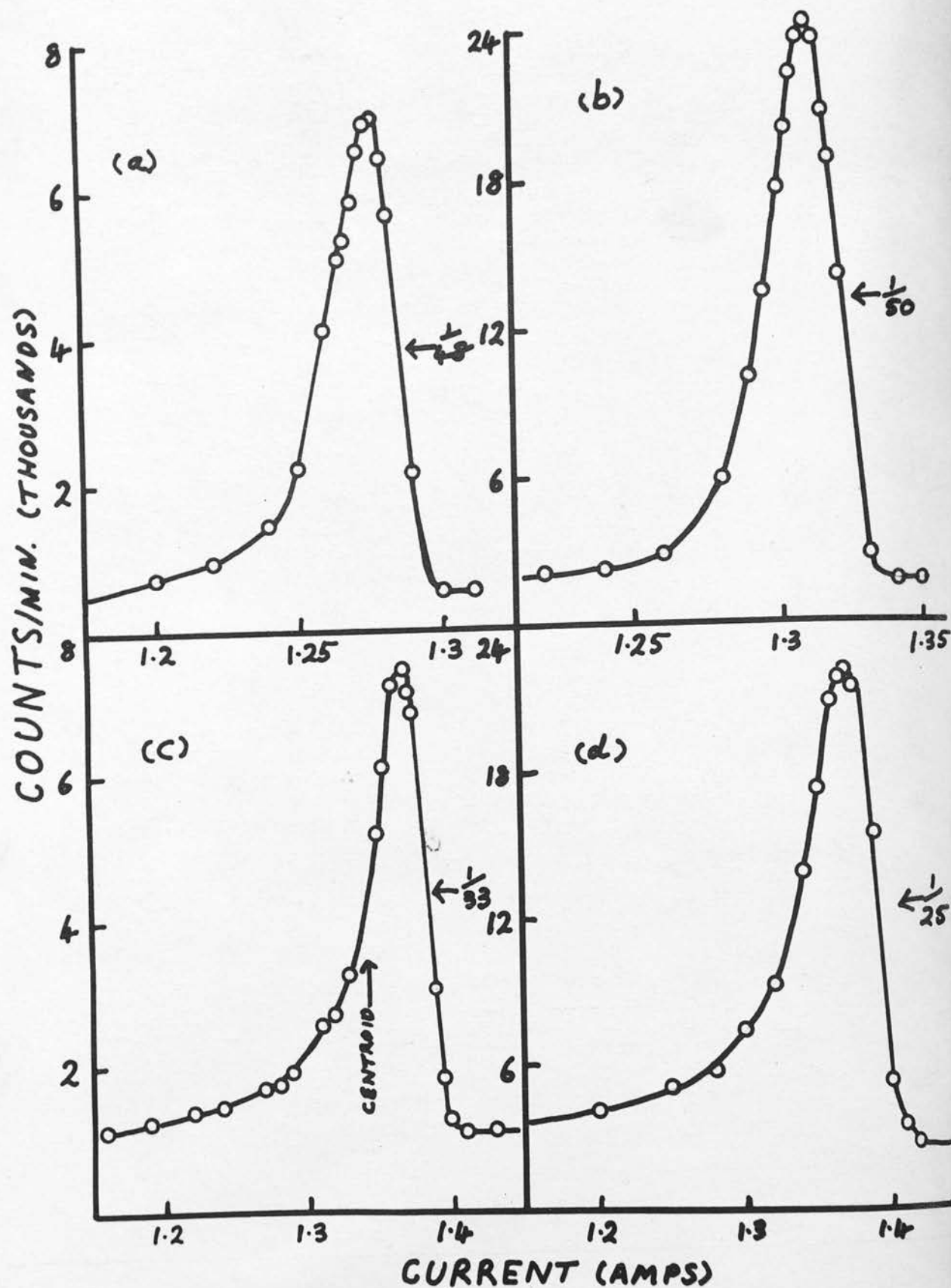


Fig.B49 F line of ThB with wide slits.
Source diams. are (a) .33 (b) 1.0 (c) 2.0
(d) 3.0 mm.

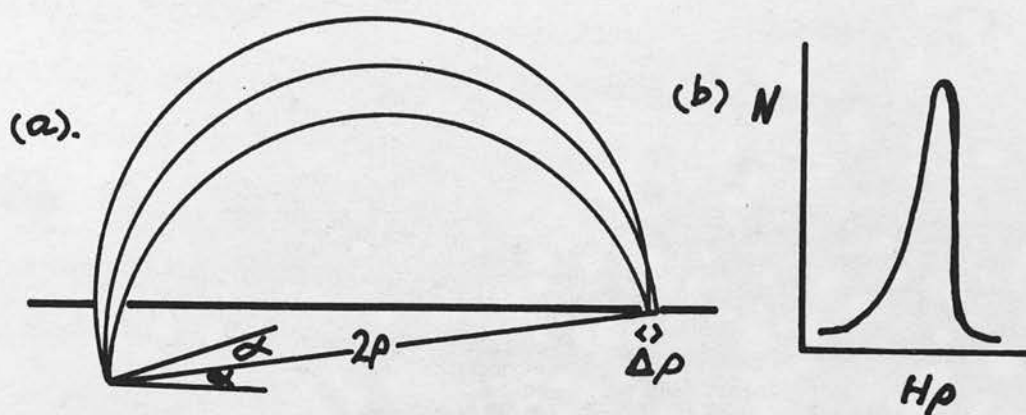


Fig. A3 (a) Semi-circular focusing; trajectories showing the order of crossing of the rays at the image
 (b) Typical line shape

target is offset to some extent by the possibility of placing the bombarding source (with its covering to stop the β -rays) at a small distance, say, 1 mm., from the target without having it contribute the usual background of Compton electrons from the β -ray absorber, save through deflection by the target. This would be a particular advantage in photo-electron measurements. A consequence of the large angle of emission is that if the geometrical conditions are carefully chosen it will be possible to reduce the upper limit of the Compton distribution from the normal value which corresponds to deflection of the quantum through zero angle or at most a very small angle to a much smaller value corresponding to deflection through a larger angle, thereby reducing considerably the background for medium energy photo-electron lines. This would only be of value of course if the target were sufficiently thin not to render the Compton distribution isotropic in direction through scattering.

Another interesting possibility opened by by the sharpness of the transmission curve is that of continuous activation of a source by α -recoil (e.g., Ellis, 1932) with this advantage over previous work in this direction that the parent source could be placed extremely close to the receiving target without contributing to the counting rate, thereby greatly improving the efficiency.

Since the area of the source depends on the square

of the linear dimensions of the spectrometer an increase in scale can provide a large improvement in respect of source strengths, and can enable full advantage to be taken of the large solid angle for all types of measurement. The existing magnet design allows the possibility of a scale increase of at least two, (subject to adjustment of the field shape over the larger volume).

SECTION C.

INTRODUCTION.

The spectrometer described in the last section has been used to study the continuous β -ray spectrum of the 2.7 year isotope of thallium, $_{81}\text{Tl}^{204}$. The spectrum of this substance is of more than average interest in the experimental verification of β -ray theory and it was considered worth while to attempt an investigation in view of this and the availability of a source of the material of fairly high specific activity and despite the disadvantages already mentioned of the spectrometer for this type of work.

This report describes the results obtained in the medium and high energy range of the spectrum, the stage to which external circumstances allowed the writer to carry the investigation. The results do not yet permit wholly conclusive deductions which must await some further measurements to be made after slight modifications to the spectrometer, but they do provide information of some interest.

Until recently almost all known continuous β -spectra yielded the shape predicted by the Fermi theory for allowed transitions,

$$N d\epsilon \sim F(z, \epsilon) \eta \epsilon (\epsilon_0 - \epsilon)^2 d\epsilon$$

even when it was known that the ft numbers showed that the transitions were forbidden in the empirical classification. No information about the fundamental interaction operator can be obtained from the allowed

distribution, but in the case of forbidden transitions some of the five possible interactions lead to distributions differing from the allowed (Konopinski, 1943). In some cases the exact shape cannot be predicted because of the presence in the formulae of more than one matrix element of unknown magnitude but others are uniquely expressed by a correction factor to the allowed shape which depends on the energy of the particle only (for a given z). The importance of Tl^{204} lies in that some previous measurements have shown it to have a spectrum differing distinctly from the allowed shape, though it has not been assigned to any one of the possible forbidden classes.

Tl^{204} decays with a 2.7 year half life (Seaborg & Perlman, 1948) to Pb^{204} . The transition is simple, as no γ -rays have been detected (Fajans & Voigt, 1941). Values of .80 mev (Hagemann, 1947) and .85 mev (Evans, 1950) using absorption methods, and more reliable values of 770 ± 10 kev (Pescocock, Jones & Overman, 1947), 783 ± 10 kev (Saxon & Richards, 1949) and 762 ± 7 kev (Wolfson, 1950 Unpublished) by spectrometer measurements, have been reported for the upper energy limit. The latest spectrometer measurements agree that the upper end of the distribution contains a surplus of particles as compared with the allowed distribution. Saxon & Richards found that down to an energy of 350 kev this surplus could be represented by the α -correction

factor which goes with first forbidden transitions for $\Delta J = \pm 2$ (yes) while at lower energies there was also a surplus which could not be accounted for by the α -factor. Wolfson found, using a much thicker source, that this factor represented the distribution above 550 kev. Peacock et al. apparently failed to observe this.

The ft value of Tl^{204} is $4.3 \cdot 10^9$ which would make it, on the customary classification, second forbidden, belonging to the 2C class (Konopinski).

II. ADJUSTMENT OF SPECTROMETER.

The adjustment of the spectrometer was as described in Section B, i.e., the width of the central annulus was 1.7 mm and that of the counter slit 1.15 mm. Sources of 2 mm. diameter were used as a compromise between resolution (3%) and workable counting rates. The counter window was 0.3 mgms/cm² nylon, formvar coated, and the counter plateau was 160 volts long with a slope of less than .01% per volt. The background counting rate was 6 per minute. This figure was determined as an average of more than ten thousand counts and checks were made throughout the measurements. Since measurements on a given source extended over a period of days account had to be taken of any slow change in counter characteristics so the vacuum source slide was put into operation to interchange the thallium sources with test ThB sources easily and the counter voltage was kept in the very flat region of the plateau. The counter was refilled when the characteristics had changed noticeably (in this connection see BI(5)).

Since no apparatus was available to measure the magnetic field accurately the magnet current was used as a measure of the momentum being focused, the F-line of ThB being used to calibrate the instrument. The maximum current required to focus the highest energy particles from Tl²⁰⁴ is 3.8 amps and the magnet field v. current relationship remains accurately linear up to values of the current of almost 5 amps.

Care was taken always to reproduce the same magnetic conditions by adopting a standard cycle of operations for changing the current, which was always increased from zero to a given value.

The current required to focus the 1385 H_e ThB F-line was 1.3675 amps for the peak of the line and 1.344 amps for the centroid. These figures apply for 2 mm ThB sources, and lead to the relations

$$H_e = 1013I$$

$$\text{and } H_e = 1030I \text{ respectively.}$$

There are reasons for preferring the centroid as a calibration point which will be discussed later, and also it is possible to locate it with less doubt than the peak of a broad line (see Fig. B49). Experimental broadening of the low energy line might of course lead to an error, but a check on the 1751 H_e I line of ThB gives a value of I consistent with the above relation to within 1 part in 500.

The current was measured by a Tinsley vernier potentiometer and kept constant to 1 part in 1,000 by manual adjustment. The momentum values are thus reliable within a fraction of a per cent and represent a negligible contribution to the experimental error.

III. PREPARATION OF SOURCES.

The source material was thallium oxide which had been activated in the high neutron flux of the Chalk River pile. Tl^{204} is formed by the capture of a neutron by the 30% abundant Tl^{203} and in the 2 years since activation short-lived activities from other isotopes had had time to die out. The specific activity was about 10 m.c. per gm.

Because of the dependence of the line shape on the shape, size and uniformity of sources, considerable care had to be taken in their manufacture and siting.

Since the source material was in the form of a fine powder it was decided to form the sources by deposition of fine particles from a suspension in a suitable liquid. Such a method would not be suitable in general for extremely thin sources but seemed reliable for the moderately thick sources used in this experiment. No simple chemical method seemed likely to produce the small, uniform, accurately shaped and placed sources required.

It was found that chloroform was, amongst the common liquids, most suitable. It possessed the double advantage that there seemed to be little or no tendency for the particles to aggregate and it evaporates rapidly so that a source could be built up of layers from different drops in a short time. Water was particularly bad because of aggregation, and it proved impossible to prepare a uniform source in the presence of the smallest trace of water over

the source area.

The oxide was ground in a mortar under chloroform and then the smaller particles were transferred by pipetting the upper layers of the liquid to a test tube where a further selection could be made after the heavier particles had settled to lower levels. In this way only the finest particles were used in the final source preparation.

The backing material was 0.3 mgm/cm^2 nylon, with a fine graphite conducting track to the edge of the source area to prevent source charging. This was stretched tightly across a brass ring which made a spring fit in a holder in the source slide so that its centre point was automatically centred in the spectrometer (given the initial adjustments).

This source ring also fitted into a dummy holder which indicated through the transparent nylon the correct site for the source disc. A piece of very thin mica had a 2 mm hole cleanly drilled in it and this was placed over the correct position, being fixed by surface tension between the slightly wetted mica and the nylon. (No water was allowed over the source area itself).

The thallium suspension was then placed in drops over the hole in the mica mask. Each drop spread over the mask and when it evaporated the encrustation round the edge which such a drop always leaves was left on the mask. The suspension was kept very dilute so that ten or more drops were needed to produce a

sufficiently strong source. The mica mask could then be lifted off, leaving a very well defined disc of active material, uniformly distributed over the area. This was covered with a film of formvar 30 to 50 micrograms/cm² thick. Not all sources were equally uniform but it was possible to judge this by eye using transmitted light and imperfect specimens were rejected.

The thickness of each source was estimated from its integrated counting rate found by using a thin-window counter.

IV. EFFECTS OF SOURCE THICKNESS.

In all β -ray work a source of error is the source thickness which results in the degradation of β -rays to lower energies. This leads to a spurious excess of particles in the low energy region and can only be minimised by using extremely thin sources - 50 micrograms/cm² or less, for best results. In this case sufficient intensity could only be obtained by using sources at least 150 μ gms/cm² thick. The difficulty is accentuated in this case by the large angle (74° mean) at which the rays are inclined to the axis as they leave the source. The effective thickness is therefore greater than the physical thickness by a factor of four.

With such source thicknesses it seemed unlikely that any information of value could be obtained in the low energy region unless special precautions were taken. The attempt to take such precautions was not successful to any marked degree due to practical difficulties, but there seems no reason why the method employed should not be successful with improved technique.

Owen & Primakoff (1950) have shown that it is possible to determine a correction factor for continuous spectra by observing the shape of monochromatic conversion lines of different energies as a function of source thickness. Then the observed counting rate M may be expressed in terms of the true counting

rate N by (neglecting a small term in $d^2N/d(H_e)^2$)

$$M = N + \{a(H_e, t) + b(H_e, t)\} dN/dH_e$$

where $a(H_e, t)$ represents the displacement of the peak of the line from the true position and $b(H_e, t)$ the position of the centroid relative to the peak. Even when the effect of thickness is negligible there is a correction due to the line shape of the spectrometer. If the calibration is made in terms of the centroid then it is sufficiently accurate to write

$$M = N + \Delta(H_e, t) \frac{dN}{dH_e}$$

where Δ represents the displacement of the centroid of the line with H_e , due to source thickness.

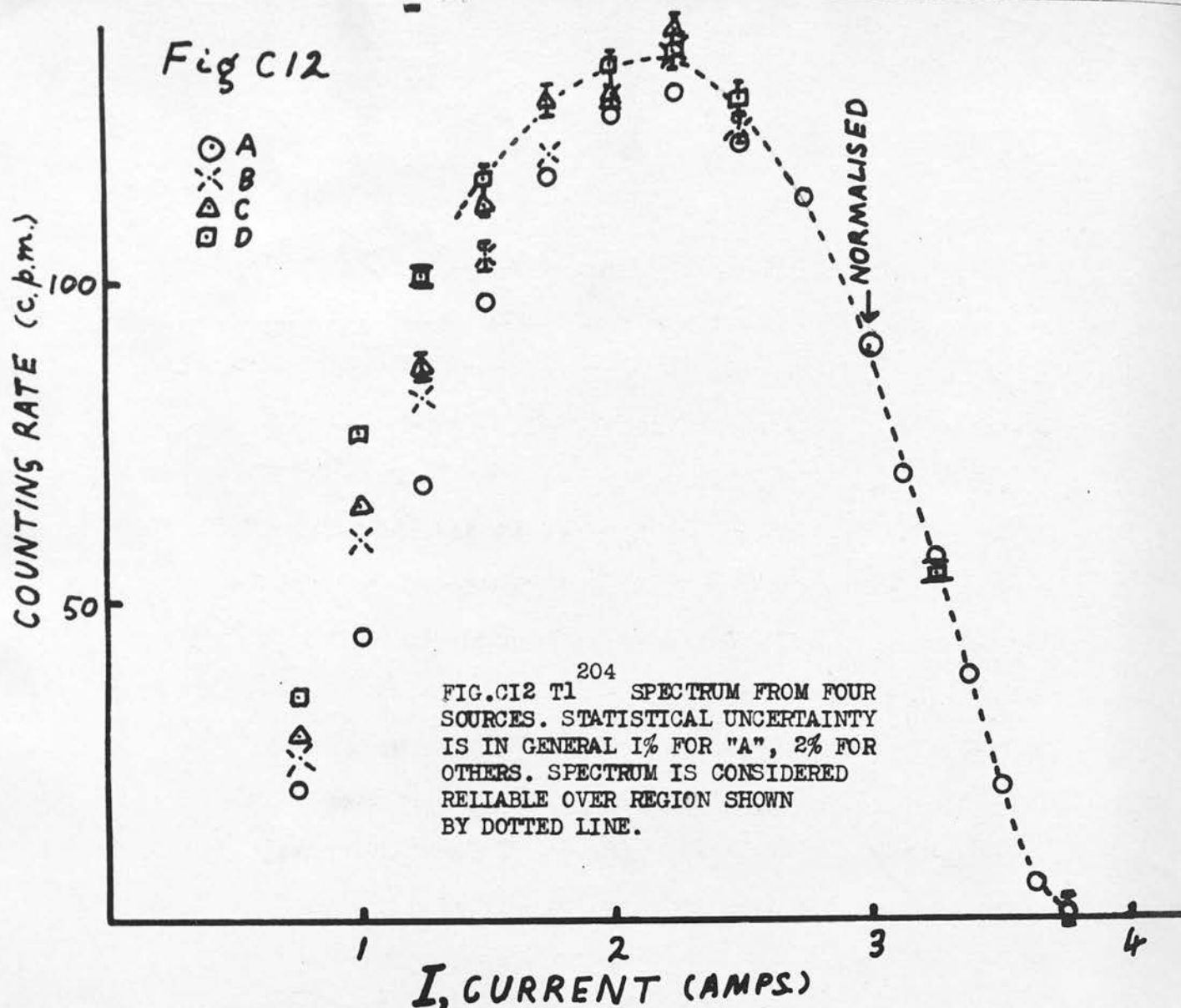
The initial attempt to apply this theory was to make sources of thallium oxide mixed with ThB activity, to measure the shape of the F and I conversion lines and thereby determine the value of Δ for that source, and then to measure the Tl^{204} spectrum after the decay of the ThB. This method was not successful because of the difficulty of obtaining sufficient ThB activity without using water at some stage in the chemical process for transferring the active deposit from the platinum foil on which it was collected to the thallium source material. The effect of water was, as already mentioned, to cause such tremendous non-uniformity in the sources as to render any correction factor determination quite unreliable, and the sources quite useless.

The difficulties encountered, however, should be surmountable, and then the method should provide a powerful means of dealing with thick sources of substances of medium and long lives.

It should also be possible to arrive at the value of the correction factor by measuring the displacement of the centroid as a result of laying different thicknesses of thallium oxide over a thin ThB source and then calculating from this the displacement for the case of uniform distribution throughout the heavy material. This was done, but the process of laying down the ThB source seemed to poison the surface so that the thallium oxide tended to aggregate, where it is of supreme importance in this method that it should not. The values of Δ obtained, therefore, were of little value, and were not used. It seems possible that here some of the trouble at least was due to the presence of the ionizing radiations; active thallium had shown some tendency to aggregate, more than inactive under the same conditions and if this were so the presence of a strong ThB source would very greatly increase the effect.

Direct activation of the thallium oxide powder by exposure in the open radiothorium source was rejected because of the contamination risk.

The possibility remains of taking a value for Δ from the curves given by Owen & Primakoff. Their source material was I¹³¹ and since they applied



I	TOTAL COUNTS				COUNTS/MIN - BG.				NORMALISED C/MIN			
	A	B	C	D	A	B	C	D	A	B	C	D
.75	2101	2061	2062	2087	21	18.6	13.6	6.9	21	26	28	35
1	4667	2250	2061	2266	45	31	27.3	15	45	60	65	77
1.25	6042	2089	2106	2977	69	43	42	19.6	69	82	87	101
1.5	5148	2008	2310	2252	97	54	54	22.6	97	104	112	116
1.75	5617	4321	2067	3210	116	62	62	25.5	116	119	128	128
2.0	5239	6215		2844	125	66		26.1	125	128		133
2.25	9536	4263	2032	3190	128	70	67	26.9	128	135	138	135
2.5	6807	4521		2672	120	64		25.6	120	122		
2.75	5402				111			17.4	111			
3.0	6469	4481	2087	2174	88	46	43		88	88	88	88
3.125	2240				69				69			
3.25	5306			1158	56			10.5	56			53
3.375	5306				37				37			
3.5	5318				20				20			
3.625	2048				6				6			
3.75	83				1				1			

their results to correct a spectrum of Cu^{64} taken with a 1 mgm source and found it to have an allowed shape as recently confirmed, it seems that they are reliable and may roughly be applied to the Tl^{204} case. From their curves **A** for a given thickness t is found as the difference in ordinates for two curves for thicknesses differing by t . In this way the effect of the source thickness is obtained apart from the geometrical properties of the spectrometer. (The line shape and width for the spectrometer is similar to that used by Owen and Primakoff).

Of course, the effective source thickness is four times the average thickness, only if there is rigorously no granular structure; with a wide angle beam such structure is not necessarily disadvantageous as in a conventional spectrometer.

In fig C16 are shown the results of such a correction in the case of a source of $.6 \text{ mgms/cm}^2$ effective thickness. In the column headed ΔH_p are the values of the displacement of the line centroid taken from Owen and Primakoff's paper, and in the next column **C** represents the calculated percentage correction to be applied to the $\frac{1}{\epsilon} \left(\frac{N}{I \alpha \theta} \right)^{\frac{1}{2}}$ plot of the spectrum

$$N d\epsilon = \alpha \theta(Z, \epsilon) \epsilon^2 (\epsilon - \epsilon_0)^2 d\epsilon$$

This should indicate the order of magnitude of the correction.

V. MEASUREMENTS.

Measurements were made on four sources which will be referred to as A, B, C and D. The approximate average effective thicknesses of these were 3, 1.5, 1.5 and .6 mgms/cm². A was extremely uniform as examined visually, D was almost as good but slight patchiness could be observed in B and C. In all four the general distribution of the source over the disc was good. Attention was mainly concentrated on A and D, A being counted over the whole range of the spectrum while most of the measurements with D were confined to the region below 350 kev. This was because of the low counting rates, the rates at the maximum of the spectrum being 128 c.p.m. with A and 26 c.p.m. with D. Check counts were made with all sources at 2.08 m.e.² energy for accurate normalizing purposes.

More than 5,000 counts per point were taken on most of the field values with source A and a minimum of 2,000 on the others and with sources B, C and D. (An exception is the last point shown at the upper end of the spectrum where only eleven counts over the background count were taken). This involved counting for periods of several hours per point so for each field value several counts of short duration were made and successive field values were chosen so as to minimize any effect of slow drift of counter characteristics even though these were tested

I	X _{PEAK} E	CONF. E	Θ		N/I		$\frac{1}{E(I\Theta)^{1/2}}$	
			Θ_D	Θ_A	A	D	$\frac{1}{E(I\Theta)^{1/2}}$	$\frac{1}{E(I\Theta)^{1/2}}$
.75	1.095	1.1	.946	.945	27.9	46.7	49	60
1	1.16	1.17	.925	.925	44.7	77	59	60
1.25	1.25	1.26	.895	.90	54.9	81	61	63
1.5	1.34	1.35	.87	.87	64.6	77.2	64	64
1.75	1.44	1.46	.84	.845	66.3	74.1	61	61
2.0	1.55	1.57	.815	.82	62.5	66.5	56	56
2.25	1.67	1.69	.78	.785	57.0	60	51	51
2.5	1.79	1.82	.76	.765	48.0		44	44
2.75	1.92	1.94	.735	.74	40.5		38	38.5
3.0	2.05	2.08	.715	.72	29.3	29.3	31	31
3.25	2.11	2.14	.705	.71	22.0		26	26.4
3.5	2.18	2.21	.70	.70	17.1	16.3	22.3	22.7
3.75	2.24	2.28	.69	.695	11.1		17.6	17.8
3.5	2.31	2.35	.68	.685	5.8		12.4	12.6
3.25	2.38	2.41	.67	.675	1.7		6.6	6.8
3.75	2.44	2.48	.665	.67	.25		1.5	1.5

Fig C14

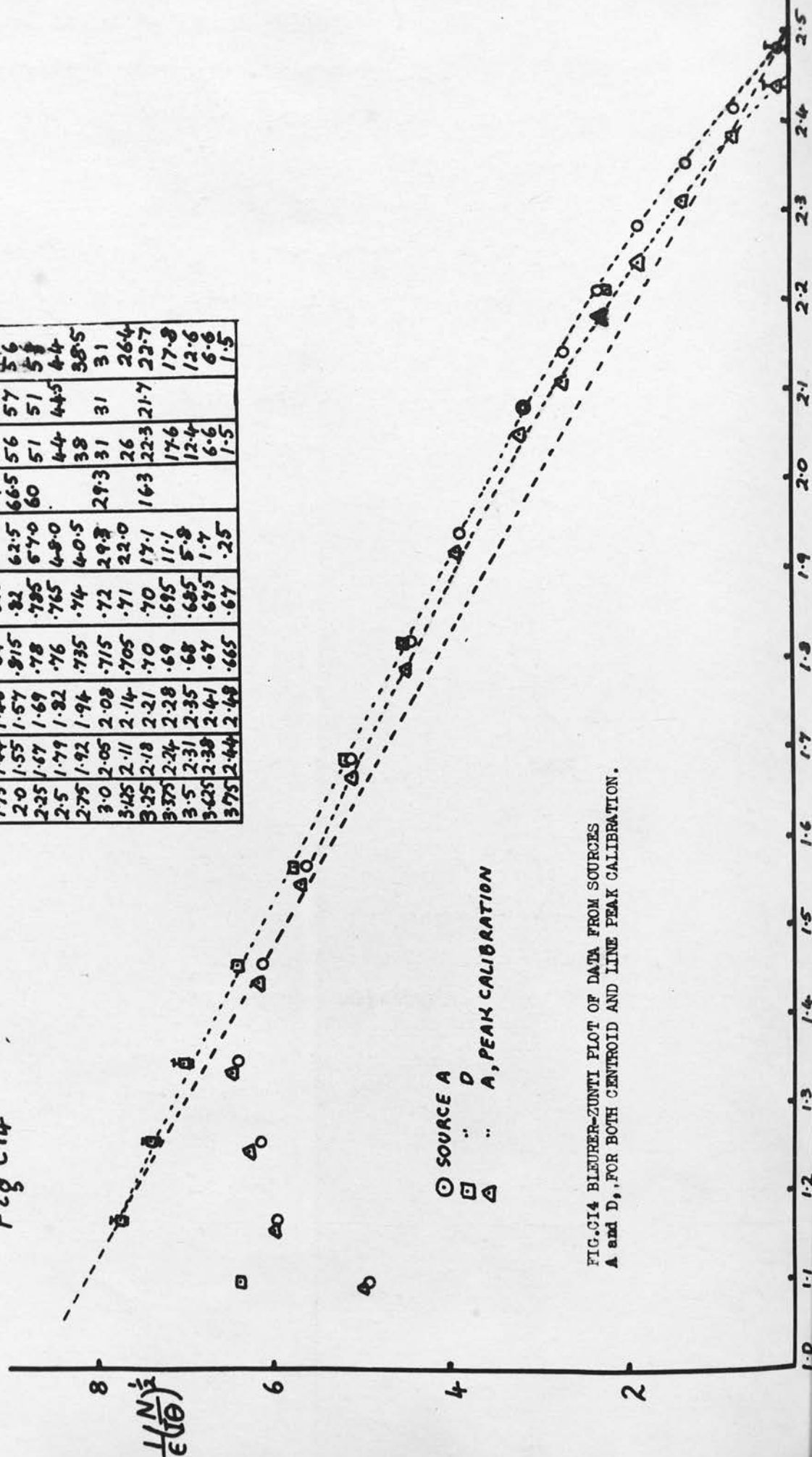


FIG.C14 BLEUER-ZUNTI PLOT OF DATA FROM SOURCES A and D, FOR BOTH CENTROID AND LINE PEAK CALIBRATION.

regularly.

The results are shown in Table C12 and fig. C12 where the ordinates are normalized at $I = 3.0$ amps.

The most obvious feature is that there is considerable disagreement between the different sources except over the upper-energy range. The disagreement is in the reverse direction from that to be expected from source thickness. It is certainly not to be explained as fluctuations in counter efficiency or errors in counting gear. The only possible explanation seems to be in terms of varying counter window thickness. This could be caused by an overdose of alcohol in the counter, causing it to condense on the window. Normally the window transmitted particles with energies of less than 30 kev (see A-line of fig B41), though on one occasion the A-line had not been observed, this being attributed at the time to a dirty source, falsely as it now appears.

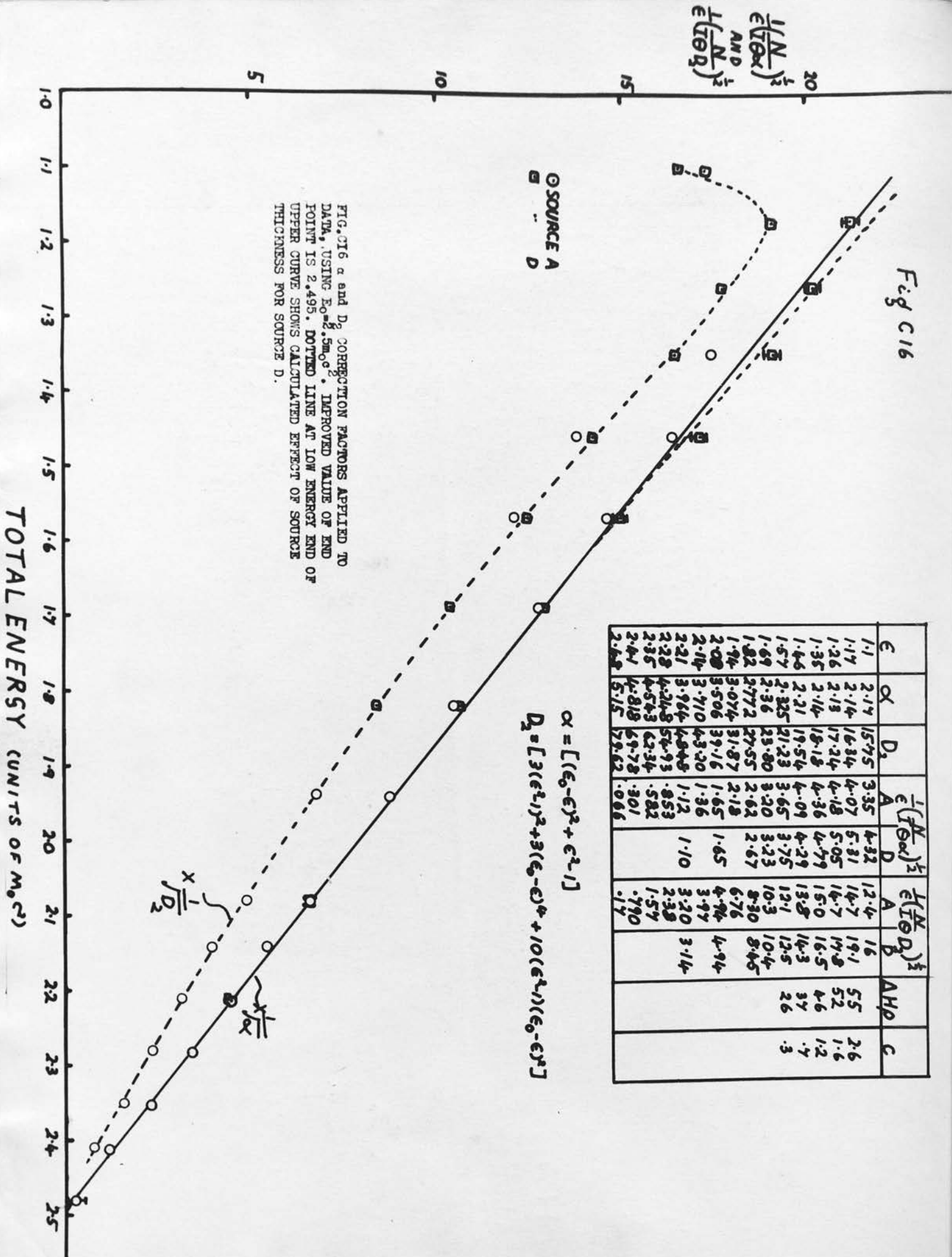
The sources A, B, C, D were measured in that order, the counter being filled several times. At the beginning of this period the counter had been filled by a rather different method in order to avoid a slow drift in characteristics which was thought to be due to absorption of the alcohol in the polystyrene. Instead of pumping out the counter for a long period to outgas the polystyrene, as had been normal, the normal filling mixture was put in almost immediately so as to keep the polystyrene saturated. This could supply an excess of alcohol

The procedure was returned to the previous one eventually, and D was counted with a normal filling procedure.

Unfortunately the effect was differently interpreted at the time and so no measurement was made which would say with certainty what the actual thickness was while D was being measured. It did not vary, however, during the one given measurement as the method of "scrambling" readings would have shown this up.

Although C and D gave different counting rates below $I = 1.5$ amps, they agree within statistical fluctuations at and above this value, (ignoring possible thickness effects). This would seem to indicate that at least down to this energy (175 kev) the figures from D are reliable. There would then be a reliable range from 175 kev up to the end point, using the values from source A at the upper end.

Fig C16



VI. INTERPRETATION OF MEASUREMENTS.

The results quoted have been compared with the predictions of the Fermi theory, using the version of the equation due to Bleuler and Zuntz (1946). Here the shape of the spectrum is given by

$$N d\epsilon = C(Z) \epsilon^2 (\epsilon_0 - \epsilon)^2 \theta(Z, \epsilon) d\epsilon$$

where $C(Z) \theta(Z, \epsilon)$ effectively replaces the Fermi function.

θ is given by them in graphical form in terms of ϵ for different values of Z so that values for any spectrum may be read off with a maximum error of two or three per cent. For an allowed spectrum a straight line should then be obtained by plotting ϵ against $\frac{1}{\epsilon} \cdot \left(\frac{N}{I \theta} \right)^{\frac{1}{2}}$ where N/I is obtained from the experimental counting rates by dividing each by the corresponding value of the magnetic field. ϵ is obtained from the values of magnetic current by means of the relations

$$\begin{aligned} H_c &= 1,013 I \text{ (peak of line calibration)} \\ \text{or } H_c &= 1,030 I \text{ (centroid of line calibration)} \\ \text{and } \epsilon &= \sqrt{1 + \eta^2}, \quad \eta = H_c / 1704. \end{aligned}$$

The results of this operation are shown in Fig. C14 and Table C14. The points fall on a curve with a distinct concavity to the energy axis at the upper end and tending towards the straight allowed line in the region of 200 kev. The spectrum thus differs significantly from the allowed shape.

The value of the extrapolated end-point is higher for the "centroid calibration" than for the

If the correction for a source thickness of 0.6 mgms/cm^2 is now applied to the distribution $\eta \propto N(\eta) d\eta$ one obtains the shape of curve which would have been observed for the first forbidden $\Delta J = \pm 2$ spectrum, and therefore a correction to be applied to the near straight line of Fig. C16. This gives the line shown dotted. There is almost agreement within the statistical error for all but the last two points which fall below because of window thickness (and probably the next point is affected by this also). It is not possible to say finally whether the slight rise above the dotted line is significant because of the uncertainties in the determination of the source thickness correction. In any case it is certainly considerably less than found by Sexon and Richards.

It therefore appears from these results that the measured spectrum shows agreement with the α -correction factor for $\Delta J = \pm 2$ and parity change down to an energy approaching $\frac{1}{2} \epsilon_0$ i.e., below 175 kev.

The energy of the end-point is measured to be $2.495 m_0 c^2$, i.e., 764 kev $\pm$ 10 kev.

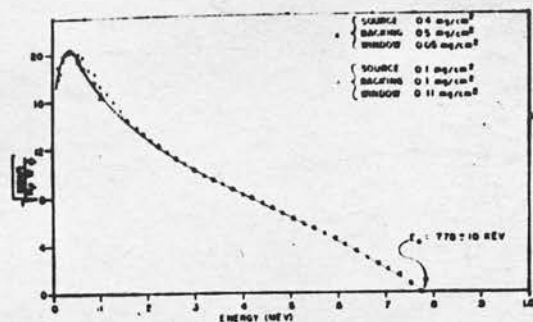


Fig. 1. Relativistic Fermi function Kurie plot for Tl^{204} (uncorrected).

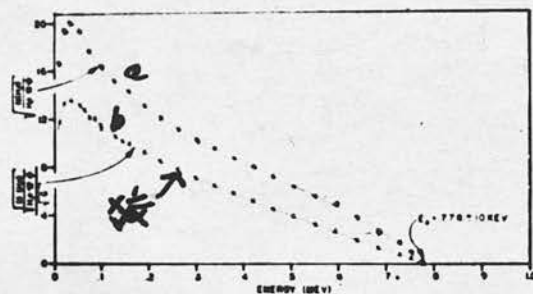


Fig. 2. Kurie plots for Tl^{204} . Upper curve, corrected for the 2 percent spectrometer resolution, lower curve, corrected for G .

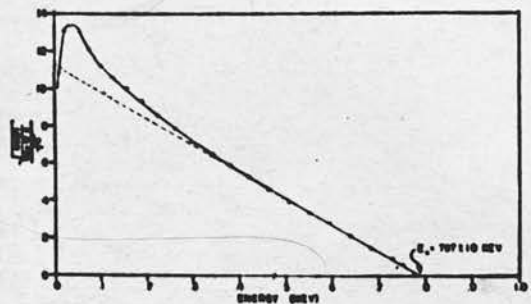


Fig. 3. Kurie plot for Tl^{204} . Corrections made both for resolution and G .

Fig.C2 Shape of Tl^{204} spectrum as reported by Saxon and Richards

- (i) Fermi plot
- (ii) (a) Corrected for spectrometer resolution
- (b) Corrected by α factor
- (iii) Both corrections applied

VII. DISCUSSION.

The value obtained for the end-point agrees well with the measurements of Wolfson (762 ± 7 kev) and Peacock, Jones & Overman (770 ± 10 kev), and rather less well with that of Saxon & Richards (783 ± 10 kev). The conclusions concerning the shape of the spectrum are in disagreement with those of the last-named authors. They found that the α -correction factor gave a linear slope down to 350 kev and that then there was a very considerable low energy excess (Fig. C2). It is not possible to explain this shape by a linear combination of invariants in the first forbidden approximation, since for such a large value of Z and small E_0 , all save the tensor and axial vector interactions with $\Delta J = \pm 2$ give essentially allowed shapes. It is in any case not possible to combine the α -factor with any other since for $\Delta J = \pm 2$ (yes) all other matrix elements vanish.

In the second forbidden approximation it has already been seen that no fit is obtained with the D_2 factor, i.e., tensor or axial vector with

$\Delta J = \pm 3$ (no). For $\Delta J = \pm 2$ (no) the scalar or axial vector interactions can have the general form required (Konopinski), or the combinations scalar and tensor, or vector and axial vector (Longmire, 1949), but only for values of Z not appreciably greater than the so-called critical value Z_c . For $Z \gg Z_c$

all the second forbidden correction factors decrease roughly as $(\epsilon_0 - \epsilon)^2$, so that it is not easy to understand the interpretation in terms of a second forbidden transition either.

There is other evidence, however, for a large number of nuclei have been found to which the α -factor applies. Shull & Feenberg (1949) have pointed out that for comparison the ft values should be multiplied by $(\epsilon_0^2 - 1)$ which is the average value of α . The table overleaf shows the ft and $(\epsilon_0^2 - 1)ft$ values for some of these nuclei.

The corresponding values for Tl^{204} , using $t=2.7y$, $\epsilon_0 = 2.5$ are $4.3.10^9$ and $2.3.10^{10}$. This is in close agreement with the other values of $(\epsilon_0^2 - 1)ft$, particularly in view of the fact that the nucleus has a much higher Z . This would make it seem highly probable that Tl^{204} belongs to the same class.

The evidence offered by the writer supports this view even when all due account is taken of the undoubted dubiety of the low energy points. In view of the internal consistency of the measurements with sources C and D down to 175 kev, it would seem justifiable to say that a good fit with the α -factor is obtained at least down to this value, even if the agreement below this is regarded as fortuitous. Saxon & Richards found that the surplus began at 350 kev, sources of .4 mgms and .1 mgm per sq. cm giving the same result. To explain this result in terms of source thickness, it would be

Nucleus : $^{38}_{17}\text{Cl}$

ft : $3.7.10^7$

$(\epsilon^2 - 1)$ ft : $0.45.10^{10}$

$^{42}_{19}\text{K}$

$1.1.10^8$

$0.7.10^{10}$

$^{86}_{37}\text{Rb}$

$3.8.10^8$

$0.8.10^{10}$

$^{89}_{38}\text{Sr}$

4.10^8

$0.6.10^{10}$

$^{90}_{38}\text{Sr}$

$1.4.10^9$

$0.4.10^{10}$

$^{91}_{38}\text{Sr}$

$1.2.10^8$

$0.6.10^{10}$

Nucleus : $^{90}_{39}\text{Y}$

ft : 1.10^8

$(\epsilon^2 - 1)$ ft : $0.3.10^{10}$

$^{91}_{39}\text{Y}$

$4.3.10^8$

$0.65.10^{10}$

$^{137}_{65}\text{Cs}$

$5.6.10^9$

$1.7.10^{10}$

necessary to postulate that both sources had an effective thickness roughly equal and very many times their average thickness.

It is hoped that workers on the Edinburgh spectrometer can eliminate the difficulty encountered in these measurements and confirm the results, and extend them, perhaps with the aid of the techniques outlined, with certainty into the region below $1.25 m_0 c^2$. The sources used in this measurement have been preserved for this purpose. In the meantime, there is evidence, statistical and experimental, that the decay of Tl^{204} is first forbidden, $\Delta J = \pm 2$ (yes). This is in line with recent β -ray work where it has been found that forbidden spectra of distinct shapes may be accounted for by the axial vector or tensor interactions where the spin change is one unit greater than the order of forbiddenness (e.g. Shull & Feenberg 1949, Wu, Feldman, 1950), or possibly by combinations of (2S, 2T) or (2V, 2A) in the case of Cl^{36} (Wu, Feldman, 1949), though it is possible that Cl^{36} really belongs to the aforementioned class (2A or 2T, $\Delta J = \pm 3$ (no)). (These combinations give agreement with the simple A or T interaction since, when ΔJ is one unit greater than the order of forbiddenness, all save that one matrix element vanishes).

It is a notable feature that the substances whose forbidden β -spectra have $\Delta J = \pm 2$ (yes) shape are very closely associated with the so-called magic

numbers in respect of their numbers of neutrons, the N numbers being 21(2), 23(2), 49(2), 51(2), 52(2), 53, 82 (6, Be¹⁰). Tl²⁰⁴, if its assignment to this class is confirmed, would be the first associated with the neutron magic number 126, (N=123). It is also only one proton removed from the proton number 82. What importance this has either in the theory of β -decay or shell structure remains to be seen.

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