COMPONENTS OF RISK FOR INVESTMENT TRUSTS

BY

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ABSTRACT

Risk assessment is a topical subject in the investment trust sector. Several fund management groups have started issuing risk gradings for their investment trusts. Money Management and Micropal now publish volatility figures and in March 1995, the Association of Investment Trust Companies issued a factsheet on risk for private investors which proposed the use of volatility in the risk assessment of investment trusts.

This paper develops a model which splits the variance of total returns to shareholders into three components. The relative importance of each of these components is then estimated for different time intervals using historical data. The results emphasise the importance of the investor's time horizon. This suggests that there should be separate management group risk gradings for private investors with different time horizons. For most investors, standard deviation of net asset value total return is a more appropriate measure of risk than volatility.

1. INTRODUCTION

An investment trust is a UK company whose assets consist of a portfolio of shares or other securities. Some are designed primarily for investors seeking the benefits of a well-diversified portfolio of equities for a relatively modest outlay. Others provide a vehicle for investment in some specialist area such as a particular geographical region or a specific sector of industry.

Ultimate responsibility for running the affairs of an investment trust lies with the board of directors, but day-to-day management is normally delegated to professional investment managers. These investment managers will usually be members of a management group rather than directly employed by the investment trust. The group may be involved in the management of other investment trusts together with other types of funds such as pension funds or unit trusts.

In common with any other company, an investment trust has a fixed capital structure which must contain share capital but which may also include loan capital. Some investment trusts, known as "splits", have innovative capital structures which attempt to match the risk, income or other preferences of different types of potential investor. There are over 250 conventional investment trusts and over 60 splits. This paper is concerned purely with conventional trusts.

As the ordinary shares of an investment trust must be listed on the Stock Exchange, the procedure for dealing in the shares is the same as for any other listed shares. So investors who wish to purchase or sell investment trust shares do so at prices which reflect the supply and demand for the shares rather than the underlying net assets of the company.

Nevertheless, investors generally regard investment trust shares as essentially claims on assets, and investment trust analysts watch the relationship between the value of the investment trust shares and the value of the underlying net assets very carefully.

The net asset value (NAV) of an investment trust is obtained by deducting prior capital from the value of underlying assets, and is normally expressed on a per share basis. NAVs are published monthly with some investment trusts even publishing the figures daily.

Published NAVs are generally considered to be reasonably accurate but if there is a significant proportion of unquoted investments (including property) held, there is bound to be some uncertainty as to the true value of underlying assets. There has also been some debate as to what value should be deducted for prior capital. Trust fund managers have argued in the past that investment trusts should be treated as continuing businesses so that prior capital should be deducted at current market value. But pressure from institutional investors, who often regard investment trusts as potential take-over candidates, has led to NAVs invariably being calculated nowadays with prior capital deducted at nominal value. If there are convertibles or warrants outstanding, adjustments can be made on a per share basis to give a "fully diluted" figure if required i.e. convertibles are assumed to be converted and warrants are treated as exercised if dilution of NAV would occur.

Investment trust analysts pay particular attention to the discount (or premium) to NAV. Discount to NAV may be defined as NAV less share price, expressed as a percentage of NAV. If the investment trust shares and their underlying NAVs are priced efficiently, a discount (or premium) to NAV implies that assets held indirectly through the investment trust are less (or more) valuable than if they were held directly.

Discounts vary widely from one trust to another and also vary over time, but attempts to explain these variations have met with no more than limited success (see, for example, Draper and Paudyal, 1992). It seems that there are many factors which influence investment trust discounts and the importance of these factors vary over time. There is also evidence that discounts partly reflect inefficient market pricing of the investment trust shares (Cheng, Copeland and O'Hanlan, 1994; Levis and Thomas, 1995).

As discounts vary over time, part of the return from an investment trust share is due to changes in the discount. Thus, discount variation over time contributes to the risk of investment trust shares. But how important is this effect? And are discount changes over a period related to returns from the underlying net assets of the trust over the same period? If there is a positive (or negative) correlation between discounts and returns then this will reduce (or increase) risk. This paper seeks to answer these important questions.

Section 2 of the paper looks at the fundamentalist approach to comparing the risk of different conventional investment trusts. Section 3 considers statistical measures of risk based on historic return data. Section 4 reviews relevant UK academic research. Section 5 carries out empirical tests of a model which splits the variance of total returns to shareholders into three components; in particular, the relative importance of the three components are estimated for different time horizons using historic data. Section 6 is the conclusion.

2. RISK ASSESSMENT BASED ON THE FUNDAMENTALS

Investment analysts normally use a "fundamental" approach to risk assessment of conventional investment trusts. This involves consideration of the investment objectives of the trust and a number of financial ratios.

2.1 Investment objectives

Whether a trust is a "generalist" or a "specialist" is important. Specialisation can take many forms but is principally by geographical region (e.g. Japan, Europe), by sector (e.g. technology, financials) or by style (e.g. income growth, smaller companies). Specialists are normally considered to be higher risk than the more broadly-based general trusts as the latter are cushioned against unexpected events affecting a particular industry or geographical region. Some underlying markets are inherently more volatile (e.g. emerging markets or smaller companies) and, for the geographical specialists, currency movements add to the risk. Furthermore, specialist trusts are subject to fashion so that discounts are more volatile and it is a generally held view among practitioners that as an area swings in and out of fashion, changes in the discount tend to exaggerate underlying NAV movements.

2.2 Financial ratios

The more important financial ratios used by analysts in comparing the risk of different investment trusts include gearing, relative discount range, portfolio yield, percentage of assets in different geographical regions, and percentage of total assets which are unquoted.

Investment trusts may issue fixed-income capital and thereby acquire the benefits and risks of gearing. As holders of fixed-income capital are normally entitled to repayment of a fixed amount of capital in a liquidation, any increase or decrease in the value of underlying assets is wholly attributable to the ordinary shareholders. Thus, there is increased volatility of NAV with higher levels of gearing. The term gearing is normally taken to mean the ratio of total assets to shareholders funds; sometimes fixed-interest and cash investments are deducted from total assets as this effectively reduces the level of gearing.

The magnitude of the discount range over a given period (e.g. the last year) compared with that which is typical for the sector gives an indication of risk associated with changes in discount. Analysts may also judge that a discount which is high (or low) relative to its recent range may be more likely to fall (or rise) with obvious implications for risk in the short term.

A high (or low) portfolio yield is generally associated with low (or high) risk, where portfolio yield may be defined as gross revenue expressed as a percentage of gross underlying assets. Examination of a simple dividend discount model (see the Appendix) confirms this generally held view, as does the empirical evidence (Lyle, 1983). Portfolio yield will, of course, be highly correlated with the investment trust shareholders' dividend yield.

For general trusts, the degree of international diversification can be assessed by observing the percentage of assets in the different geographical regions. Correlations between the returns from shares held in the "world" market should be less than those between the returns from shares confined to a particular domestic equity market such as that of the UK, so international diversification should reduce risk. International diversification involves foreign currency exposure but changes in exchange rates can offer protection against (unanticipated) higher relative UK inflation as in the long run exchange rates tend to mirror relative inflation in their economies. Foreign currencies may even be regarded as assets in their own right which can be used to reduce the overall portfolio risk. However, currency exposure can be managed independently of the underlying portfolio and this may be carried out with the aim of boosting returns rather than reducing risk.

Uncertainty as to true net asset value is greatly influenced by the percentage of total assets which are unquoted. Directors' valuations of unquoted investments may be unreliable and historic to some extent, only changing when "something happens", such as a share stake changing hands.

2.3 Risk gradings within management groups

Several fund management groups have started issuing risk gradings for their investment trusts. They involve a broad grading of trusts into categories on consideration of their investment objectives and a number of financial ratios such as those described in section 2.2. These categories might be: well above average risk, above average risk, average risk etc.

Merril Lynch do risk gradings for Flemings and Gartmore but these rely quite heavily on quantitative inputs such as historic volatility. NatWest Securities do risk gradings for Kleinwort Benson, Ivory & Sime and Foreign & Colonial; their approach is based more on subjective judgement. The risk gradings are generally for the benefit of private investors.

3. STATISTICAL MEASURES OF RISK

Statistical measures of risk based on historic returns are increasingly being used in evaluating investment trust risk, the most commonly used measure being standard deviation (or variance) of monthly share returns (often known as "volatility"). Money Management and Micropal now publish volatility figures for investment trusts on a regular basis.

In March 1995, the Association of Investment Trust Companies issued an investor's guide to understanding risk entitled "Making risk work for you". This was written mainly for private investors. The guide said that volatility gives a very useful, but only partial, assessment of risk. It recommended that volatility be used in conjunction with other aspects of an investment trust which indicate the level of risk involved. Nevertheless, the guide said that volatility tends to remain roughly constant for a long time and, being a routine formula, can easily be defended against the criticism of being biased. It should also be noted that many of the other criteria for assessing risk, such as investment objectives, specialisation and gearing will be reflected in historic volatility, so there is some degree of double counting if they are used in addition to volatility.

Most institutional investors hold multi-asset portfolio to meet long-term liabilities. For such investors, it is the interaction between assets and actuarial liabilities which in the long run determine the level of risk. But while the allocation of funds between different asset categories may take account of the nature of liabilities, equity portfolio managers are often more concerned with short-term asset performance or fear of underperforming the "herd", with little regard for the institution's liabilities. This implies that statistical measures of risk based on historic returns may be relevant to institutions as well as private investors.

Standard deviation (or variance) of returns, known as "total risk", can be calculated using time intervals other than one month. After all, the investor's time horizon will typically be much greater than one month.

Sometimes, total risk is separated into two components, "market risk" and "specific risk", as described in Section 3.1. An alternative model, which splits total risk into three components, is proposed in Section 3.2. The two models are then compared in Section 3.3.

3.1 The Market Model

It is well known that when a stock market goes up, most shares within that market tend to increase in price, and when a stock market goes down, most shares tend to decrease in price. Sharpe (1963) suggested that this common response to market changes could be written mathematically as:

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it} \qquad (1)$$

where

R_{it} is the return on the ith share in period *t*.

R_m is the return on the market index in period *t*.

α_i is the constant return unique to share *i*.

β_i is a measure of the sensitivity of the return on share i to the return on the market index.

e_{it} is the random residual error in period *t*, assumed to be independently and normally distributed with zero mean and constant variance.

Equation (1) describes what is known as the Market Model. It requires that the only common factor affecting all securities is the return on the market index. All shares, to a greater or lesser extent, tend to move with the market.

Although β_i is generally defined in terms of monthly returns, it really reflects relationships among expectations about the values of fundamental economic variables over the long term.

It follows from equation (1) that:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma^2(e_i) \qquad (2)$$

where

σ_i^2 is the variance of return on share *i*.

σ_m^2 is the variance of return on the market index.

$\sigma^2(e_i)$ is the variance of the error term.

The parameters α_i, β_i and $\sigma^2(e_i)$ for share *i* may be estimated by studying the historical relationship between the returns on share *i* and the returns on the market index. R_i is plotted against R_m for a number of periods (say every month for 5 years) and a "best fit" line is drawn through the points using regression analysis. The gradient of the line is an estimate of β_i and the intercept with the y-axis is an estimate of α_i. The scatter of points about the regression line represents the residual variation in returns after removing the market effect.

The first term on the right hand side of equation (2), $\beta_i^2 \alpha_m^2$, known as *systematic* or *market* risk, is related to fluctuations of the market as a whole and cannot be eliminated by diversification.

The second term on the right hand side of equation (2), $\sigma^2(e_i)$, known as *non-systematic* or *specific* risk, can be eliminated by diversification. This type of risk is unique to the company or its industry, or is related to other factors such as company size or dividend yield.

3.2 Proposed model for components of risk.

We now develop an alternative model which splits the variance of total returns to shareholders into three components. It is similar to a model derived by Sharpe and Sosin (1974) for US closed-end investment companies.

Let P_t = share price of investment trust at time t

A_t = net asset value per share at time t

$r_t = \frac{P_t}{A_t}$ (= 1-discount) which we call "the ratio"

R_t^p = share price total return in period t

R_t^A = net asset value total return in period t

By definition of r_t,

$$P_t = A_t . r_t$$

and

$$P_{t-1} = A_{t-1} . r_{t-1}$$

Hence
$$\frac{P_t}{P_{t-1}} = \frac{A_t}{A_{t-1}} . \frac{r_t}{r_{t-1}}$$

Assuming that dividends are added to P_t and to A_t in the appropriate months to avoid discontinuities,

$$1+R_t^p = \left(1+R_t^A\right)\frac{r_t}{r_{t-1}}$$

Taking natural logarithms of both sides,

$$\log_e\left(1+R_t^p\right)=\log_e\left\{\left(1+R_t^A\right)\frac{r_t}{r_{t-1}}\right\}$$

It is helpful to take logarithms as the returns become additive over time and their distributions are more symmetric.

Hence

$$\log_e\left(1+R_t^p\right)=\log_e\left(1+R_t^A\right)+\left(\log_e r_t-\log_e r_{t-1}\right)$$

So the share price total return is equal to the net asset value total return plus the return on "the ratio".

The standard statistical formula for the variance of the sum of two random variables gives:

$$\begin{aligned}Var\left\{\log_e\left(1+R_t^p\right)\right\} &= Var\left\{\log_e\left(1+R_t^A\right)\right\}+Var\left\{\log_e r_t-\log_e r_{t-1}\right\}\\ &\quad +2Cov\left\{\log_e\left(1+R_t^A\right),\left(\log_e r_t-\log_e r_{t-1}\right)\right\}\end{aligned} \qquad (3)$$

Thus, the variance of share price total return has been split into the following three components.

a) Variance of net asset value total return.

b) Variance of the return on "the ratio".

c) Covariance between net asset value total returns and returns on the "the ratio".

3.3 Comparison of the two models for risk assessment

The appropriate model for the risk assessment of different investment trusts depends on the particular investor in question.

For a small UK private investor holding only one investment trust, it is total risk which is important. Market risk, however defined, is not a relevant measure of risk to such an investor and thus the Market Model is inappropriate. The alternative model proposed in section 3.2 is more relevant as the importance of discount variation can be considered in the light of the investor's time horizon. Short term investors will clearly be more concerned about discount variation than long term investors, because discounts are likely to be "mean reverting". Consideration of the covariance between net asset value total returns and returns on "the ratio" is also relevant to all such investors, whether short-term or long-term.

For those small UK investors able to contemplate modest diversification within the investment trust sector, the appropriate measure of risk will lie somewhere between total risk and market risk, where market risk is in relation to a general UK equity market index. The exact positioning between the two will depend on the risk unique to the investment trust industry. Thus, market risk is relevant for such an investor, being a lower bound for the appropriate measure of risk. But total risk is also relevant and therefore so is the alternative model for its components.

For institutions holding diversified equity portfolios, it may seem natural to use the Market Model for risk assessment. But what should be used for the market index? The FT All Share Index or the FT Actuaries World Index would be candidates for UK institutions. However, the risk associated with investments held in an equity portfolio should not be considered in isolation. Other asset categories such as bonds and property, together with all other potential asset categories, should be taken into account. There is normally no simple solution to the question of which index to use and this is a major problem for the Market Model approach.

4. RELEVANT UK ACADEMIC RESEARCH

A number of UK research papers have touched on the question of risk for investment trusts, but the main emphasis of each of these papers has generally concerned other matters.

Corner and Matatko (1982) considered the monthly returns on 92 investment trusts over the period 1974 to 1979 and obtained the following results.

	Average $\hat{\beta}$	Average total risk (S.D., % per month)
Net asset value total returns	0.66	5.8
Share price total returns	0.94	8.6

So for *both* risk measures, underlying portfolios were much less risky than the corresponding investment trust shares.

Lyle (1983) made the point that higher betas for investment trust shares compared with their underlying portfolios, means that discounts tend to move against the market. That is, when the market falls (or rises), discounts widen (or narrow). He talked of the "implied discount beta" $\hat{\beta}_D$ given by:

$$\hat{\beta}_D = \hat{\beta}_E - \hat{\beta}_p$$

where $\hat{\beta}_E$ is the beta for the shares and $\hat{\beta}_p$ is the beta for the net asset value.

Lyle also looked at the relationship between underlying asset portfolio yields and risk. Annual average gross yields on portfolio assets were grouped into high, medium and low categories for 97 investment trusts over the period 1971 to 1980. Corresponding figures for average beta and average total risk were calculated. A statistically significant but rather weak relationship between portfolio yield and risk was revealed for both measures of risk. The higher the yield the lower the risk.

Capstaff (1989) carried out a survey of the risk perceptions of investment analysts in respect of equities generally. The standard deviation measure of total risk was shown to be highly significant. Analysts appeared to consider all the risk which surrounds equities, not just a systematic element of it. Overall, the "fundamentalist" approach to risk analysis was emphasised.

Armitage and Whittaker (1990) looked at monthly data over the period January 1988 to July 1989 for the following investment trust subsectors: UK non-specialists; North America; Europe; Japan; and Far East. They discovered a strong inverse relationship between the size of the sub-sector average discount and the relevant stock market index. In other words, as the relevant stock market fell (or rose), the average discount widened (or narrowed). So the "double whammy" effect which had been observed by other researchers for the investment trust sector as a whole, also applied to specialist sub-sectors and the relevant national stock market taken on their own.

Draper and Paudyal (1991) regressed monthly changes in the average discount (DD_t) on the 41 largest trusts against the monthly total return on the FT-Actuaries All Share Index (RM_t) over the period 1975 to 1986. They came up with the following relationship:

$$DD_t = 0.897 - 0.781RM_t - 0.198DD_{t-1}$$

(1.69) (-10.53)* (-3.26)*

$R^2 = 0.41$

An asterisk indicates significance at the 5% level, with t-statistics shown in brackets. Once again there is a clear negative relationship between changes in the average discount and returns on the market. In addition, a change in the average discount in one month tends to be followed by a change in the average discount in the opposite direction in the following month.

Chang, Copeland and O'Hanlon (1994). This paper was mainly concerned with showing that excess returns can be earned by selling low discount trusts and buying high discount trusts. As part of the study, they considered monthly discount changes for the 63 investment trusts in the FT-Actuaries All Share Index over the period 1985 to 1989. They observed high negative autocorrelation in discount changes of -0.234 on average. All but 5 of the 63 trusts had negative autocorrelation coefficients, and almost half were significantly different from zero at the 5% level.

5. EMPIRICAL TESTING OF THE PROPOSED MODEL

This section describes an empirical investigation to determine the importance of the three components of risk in the model proposed in Section 3.2.

5.1 Data

The data was kindly provided by NatWest Securities.

The sample consisted of the 40 largest trusts as at 31 December 1982 (i.e. those with market capitalisation greater than £25m at that time) which survived until the end of 1993. Monthly net asset value total returns and discount to net asset values for the month-end, were obtained for the period January 1983 to December 1993.

5.2 Results

Table 1 shows the results for monthly returns and monthly discounts. Columns (1), (2) and (3) give the figures for the three terms on the right hand side of equation (3). Columns (4), (5) and (6) then give the percentage of total risk due to each of the three terms. Summary statistics are then calculated by taking an average of the figures in columns (4), (5) and (6).

Tables 2 and 3 show the results for the same analysis, but with the data at three-monthly intervals and six-monthly intervals respectively.

Columns (4) and (5) of Table 1 show that, with monthly data, variation in the discount contributes much more to variance of share price total return than does variation in the returns

of the underlying portfolio. With three-monthly data (Table 2), variation in the discount and variation in the net asset value total returns seem to be roughly of equal importance. But with six-monthly data (Table 3), variation in the discount is much less important than variation in the net asset value total returns.

Contribution from the covariance term (Column 6) is generally positive reflecting the fact that discounts tend to widen (or narrow) when markets fall (or rise). This is true for one-monthly, three-monthly and six-monthly data. The reason for this effect is not clear but may be to do with liquidity or investor sentiment.

Figures 1,2 and 3 give the summary statistics (y-axis) against time intervals for the data running from one-monthly up to annually (x-axis). Note that the results for longer time intervals are less reliable than those for shorter time intervals as the former are based on less data points.

Overall, the results emphasise the importance of the investor's time horizon in the risk assessment of investment trusts.

6. CONCLUSION

In assessing total risk, short-term investors (with time horizon up to 3 months, say) should be mainly concerned with discount variation whereas longer-term investors (with time horizon over six months, say) should be mainly concerned with variation in net asset value total returns. This suggests that most private investors should be primarily concerned about variation in net asset value total return rather than volatility in assessing risk.

It is clear that discounts tend to widen (or narrow) when the underlying net asset value falls (or rises). This "double whammy" effect is relevant to both short-term and long-term investors. Investigation of the underlying reason for this effect is an interesting area for further research.

Another implication of the results is that investment trusts are more risky than unit trusts with the same underlying portfolio. This is due both to discount variation and to the "double whammy" effect. But this is not to say that unit trusts are better investments than corresponding investment trusts. Investment is about expected returns as well as risk. If the two additional components of risk for investment trusts that have been identified are reflected in the discount itself, investment trusts will offer higher expected returns than corresponding unit trusts. Indeed, if it is short-term risk which is reflected in market prices, very long-term investors are getting higher expected returns from investment trusts with relatively little extra risk

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APPENDIX. SENSITIVITY MEASURES FOR INVESTMENT TRUSTS

The concept of "sensitivity measures" for equities was developed by Adams and Booth (1995). In this appendix, we derive sensitivity measures for investment trusts in terms of the underlying portfolio yield.

Assume that dividends received by investment trust shareholders are paid annually, the next dividend is due in exactly one year from now, and dividends grow at a constant real rate g. Assume further that the market is efficient under rational expectations so that the present value of future dividends is equal to the market price. Taxation is ignored.

Then

$$P_0 = \frac{D_1}{1+j}\sum_{k=0}^{\infty}\left(\frac{1+g}{1+j}\right)^k$$

where

P_0 is the current share price

D_1 is the estimated dividend payable one year from now

j is the real discount rate.

Hence

$$P_0 = \frac{D_1}{j-g} \qquad (4)$$

Also

$$\frac{D_1}{P_0} = j - g$$

$$\text{i.e Dividend yield} = j - g \qquad (5)$$

Define *sensitivity to real discount rate* as

$$S_j = \frac{-\partial P_0}{\partial j} \cdot \frac{1}{P_0}$$
$$= \frac{1}{j-g} \text{ using equation (4)}$$

Then from equation (5),

$$S_j = \frac{1}{\text{dividend yield}} \qquad (6)$$

Now assume that costs (including management expenses and transaction costs) grow at a constant rate g, represent a proportion m of revenue received by the fund, and are a "dead weight" loss to the fund. Assume further that the investment trust has no prior capital and that all earnings are paid out as dividends.

Then if d_0 is the discount to NAV,

$$\frac{1-m}{1-d_0} \cdot \text{Portfolio yield} = \text{dividend yield}$$

Hence from equation (6),

$$S_j = \frac{1-d_0}{1-m} \cdot \frac{1}{\text{Portfolio yield}} \qquad (7)$$

Now define *sensitivity to real dividend growth* as

$$S_g = \frac{\partial P_0}{\partial g} \cdot \frac{1}{P_0}$$

Then using a similar approach to above, it can be shown that:

$$S_g = \frac{1-m}{1-d_0} \cdot \frac{1}{\text{Portfolio yield}} \qquad (8)$$

Note that both sensitivity to real discount rate and sensitivity to real dividend growth depend on portfolio yield. The greater the portfolio yield the lower the sensitivity.