Some Numerical Results for the Flat and Tall Floats

David Pizer

Wave Power Project University of Edinburgh

26 August 1994

1 Hydrodynamic Coefficients

Graphs are given for the added mass matrix, damping matrix and excitation force vector for the two floats discussed at the Hanstholm meeting.

The flat float has a diameter of $6m$, a height of $1.5m$ and is half submerged, giving a volume displacement of $21.2m^2$

The tall float has a diameter of $3.\dot{3}m$, a height of $5m$ and is also half submerged giving a volume displacement of $21.4m^2$.

Hydrodynamic coefficients are evaluated using the 3-D source distribution method. Discretisations for the two floats are shown in figures 1 and 2. I would expect the results to be accurate to within a few percent.

The reference point C^m (see figure 13) is located at the centre of the base of the float.

The complex excitation force, f, is defined by:

$$f(t) = Re[fe^{-i\omega t}] \tag{1}$$

and is with respect to a wave elevation ζ:

$$\zeta(t) = sin[kx - \omega t] \tag{2}$$

Hence, for heave in the long wave limit $f = -i/4\rho g\pi D^2$

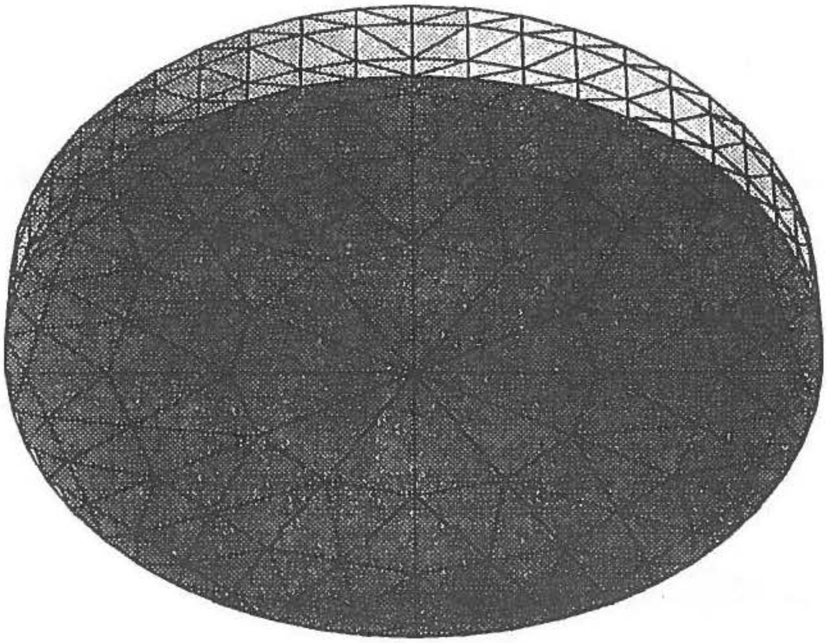

Figure 1: Discretisation for flat float

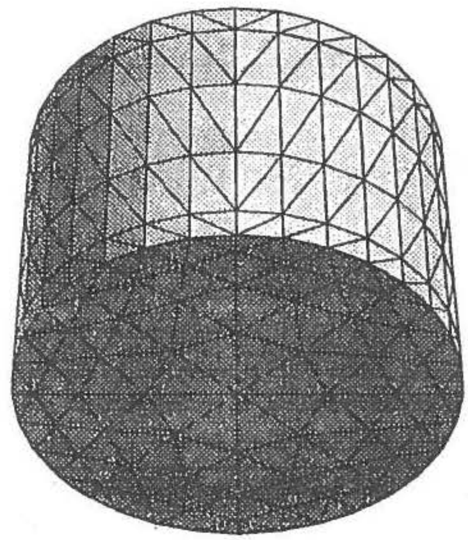

Figure 2: Discretisation for tall float

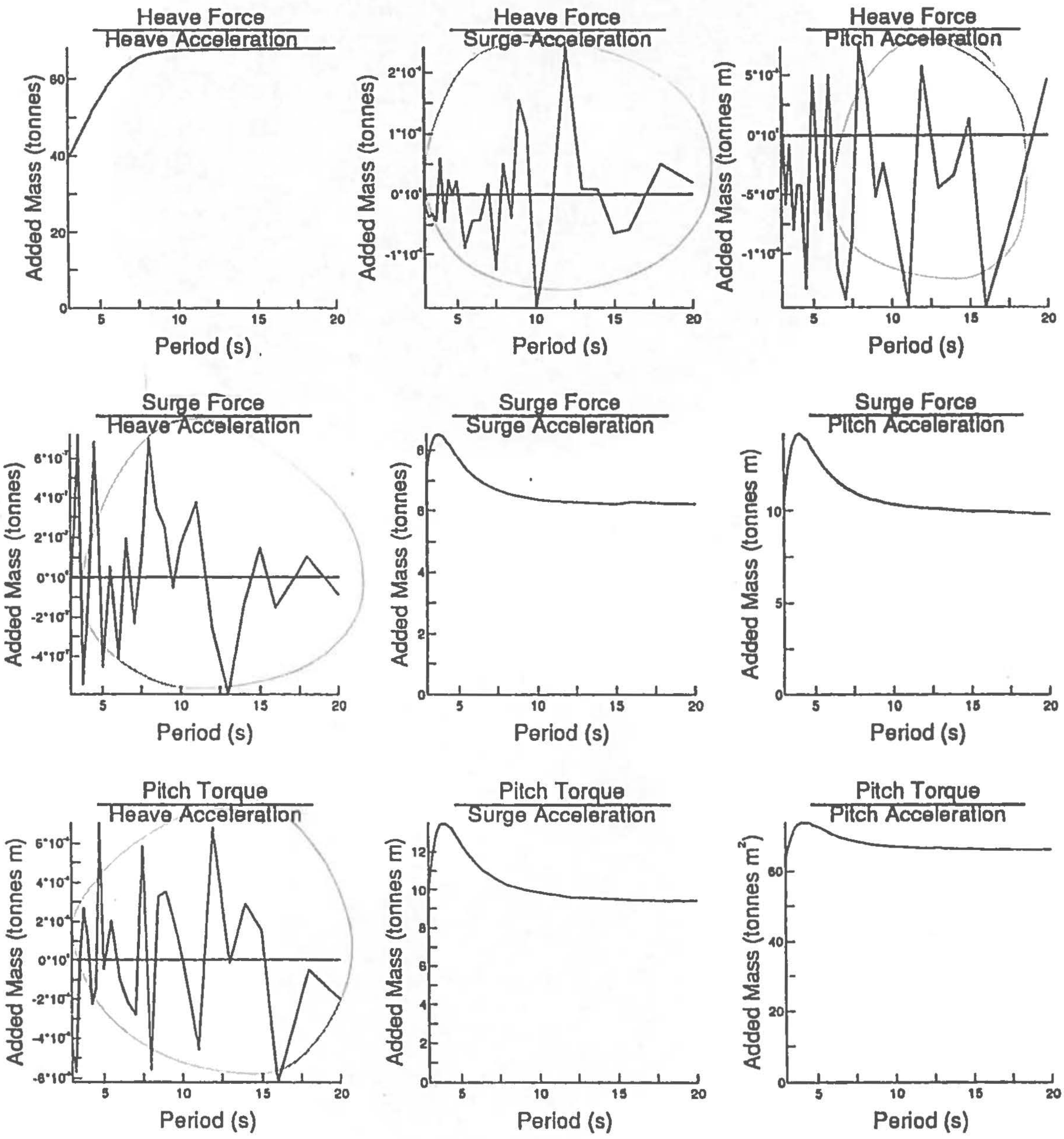

Figure 3: Added Mass matrix for flat float

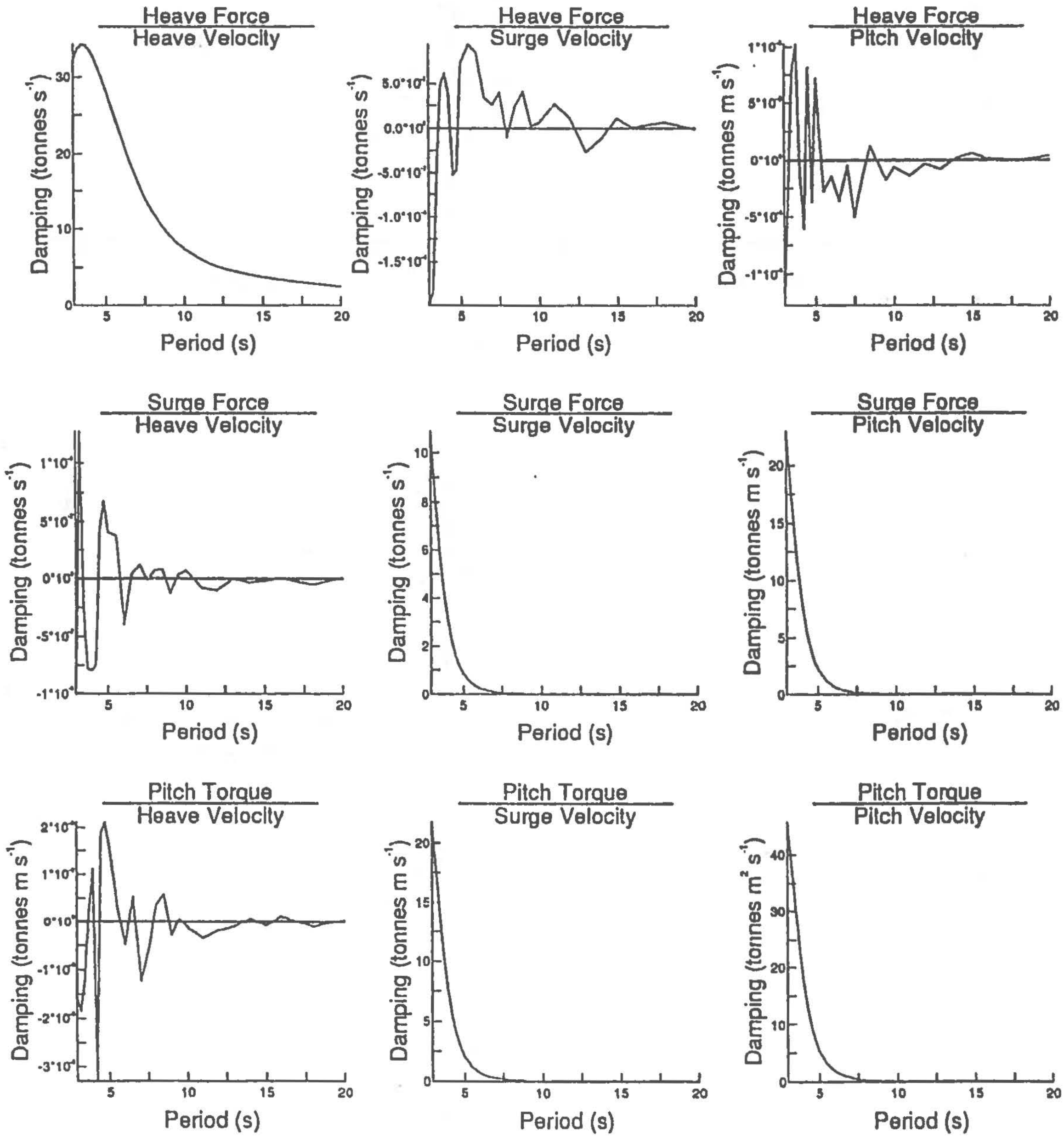

Figure 4: Damping matrix for flat float

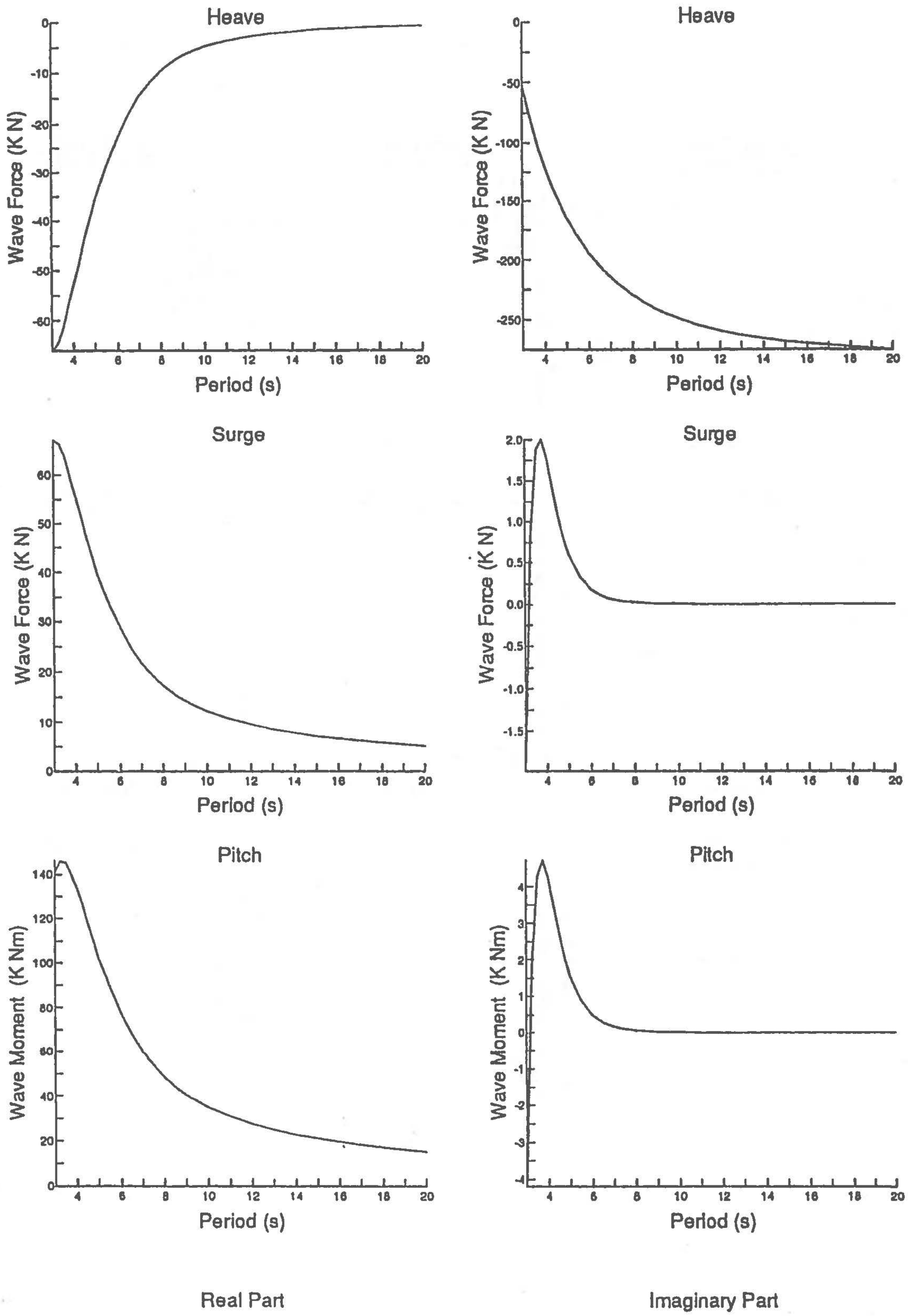

Figure 5: Wave excitation for flat float

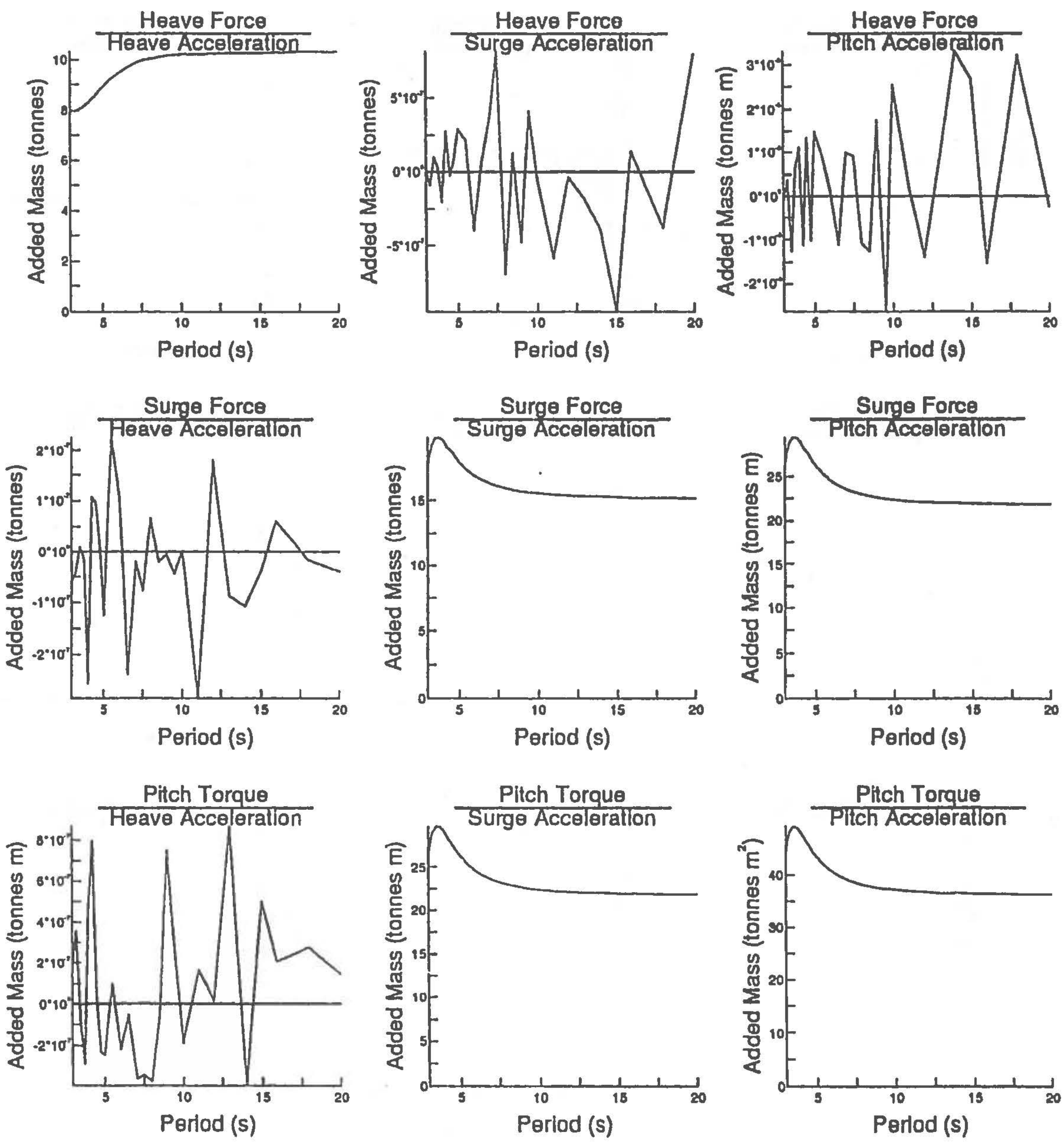

Figure 6: Added Mass matrix for tall float

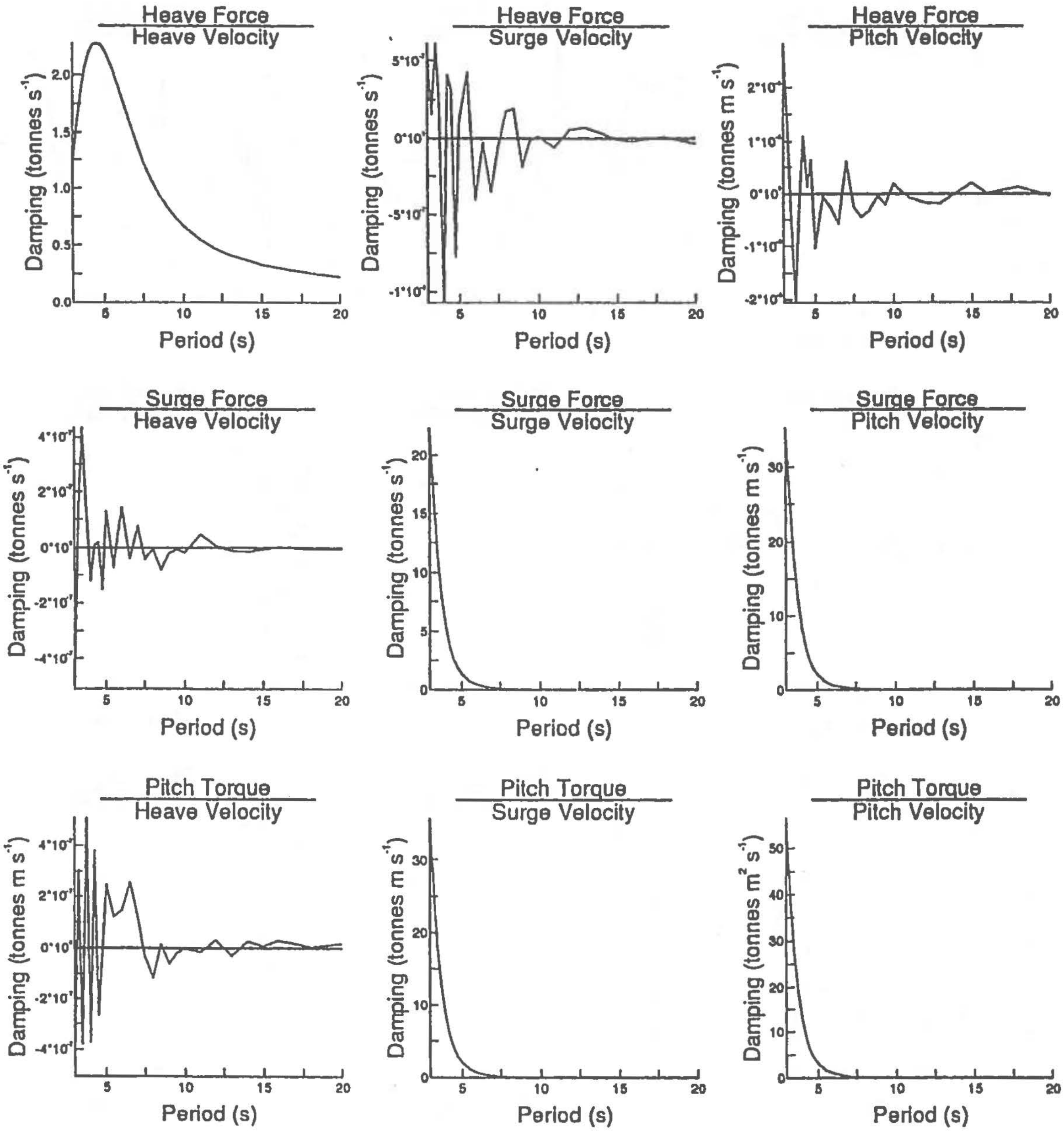

Figure 7: Damping matrix for tall float

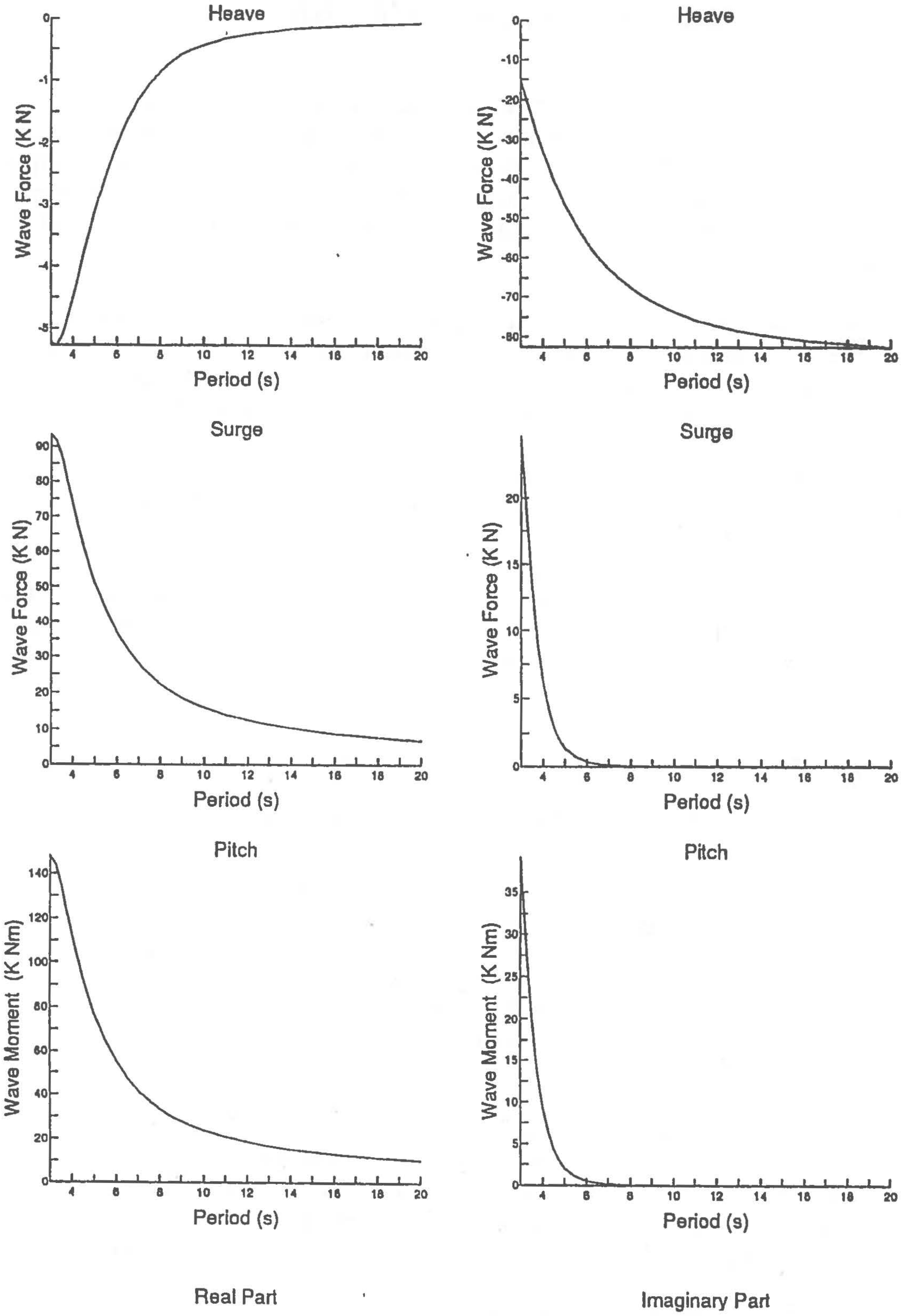

Figure 8: Wave excitation for tall float

2 Some Power Absorption Calculations

Two control strategies are considered for power absorption in heave: i) the optimal complex control, ii) the optimal real control. These are described in Joule 1, Annex report B1, Device Fundamentals Hydrodynamics, by Evans and Linton. In both cases an amplitude constraint is imposed on the heave excursion. The floats are free to move in sway and pitch, however these motions are uncoupled with heave and therefore do not effect the power calculations.

These results assume that the float reacts against an infinite mass (ie is attached to the sea bed) and are therefore not valid for the two slack moored devices, except *perhaps* for very short wave periods. The results should be valid for the two tight moored devices at wave amplitudes for which linearity and engineering limitations are not exceeded.

Graphs are given for: the amplitude of the heave control force; the excursion amplitudes for heave, surge and pitch; the power absorbed; the imaginary power; the relative capture width and the hypotenuse power ratio.

The heave control force and the excursion amplitudes for surge and pitch are dependent on the mass distribution of the floats. The weight of the floats are one fifth of the buoyancy force, with the centre of mass at the centre of the floats and pitch moment of inertia of $2886 Kgm^2$

The amplitude constraint is chosen to be half the height of the floats.

Results are presented for five incident wave amplitudes ranging from 0.35 m to 1.41 m.

2.1 Optimal Complex Control

Here the constraint is imposed in terms of an absolute amplitude, rather than in terms of a constraint on the ratio of the heave amplitude with the wave amplitude as presented in Evans and Linton.

For a small enough wave amplitude, optimal power absorption is obtained without motions reaching the constraint. In such cases the control coefficient is the complex conjugate of the impedance, see Evans and Linton, equation (79). As the wave amplitude is increased the constraint is eventually reached. For larger wave amplitudes the optimal control is obtained by adding extra damping in order to limit the motions to the imposed constraint.

A large hypotenuse power ratio implies that, over a cycle, large amounts of power is being put into and taken out of the motion of the float, compared with the relatively small time-averaged power absorbed. Losses in the power exchange mechanisms will be amplified and the predicted power absorption will be unattainable. It is thought that hypotenuse power ratios greater than about three are impractical for efficient power absorption, although this figure will greatly depend on the efficiency of the power exchange mechanisms.

Results for the flat and tall buoy in heave are shown in figures 9 and 11.

The tall float has a surge/pitch resonance at about 4.5 seconds resulting in large predicted excursions. In practice these would be reduced due to non-linear effects and viscous damping.

For both floats the constraint is reached at most frequencies. Both have the same power absorptions, although the tall float undergoes larger displacements and has much larger imaginary powers (except at the heave resonance where it is unity).

2.2 Optimal Real Control

A real control coefficient corresponds to applying damping only. For a small enough wave amplitude the optimal control coefficient is the modulus of the impedance, see Evans and Linton, equation (82). The imaginary powers are therefore zero and the hypotenuse power ratios are one.

Results for the flat and tall buoy in heave are shown in figures 10 and 12.

As expected, at the heave natural periods the power absorbed with real control is the same as with complex control. Away from resonance the power absorbed using real control is greatly reduced from that of complex control. For the flat float case, at a wave amplitude of 0.5 m real control produces less than half the power from complex control at a hypotenuse power ratio of 2. The same is true for the tall float at 1.41 m wave height.

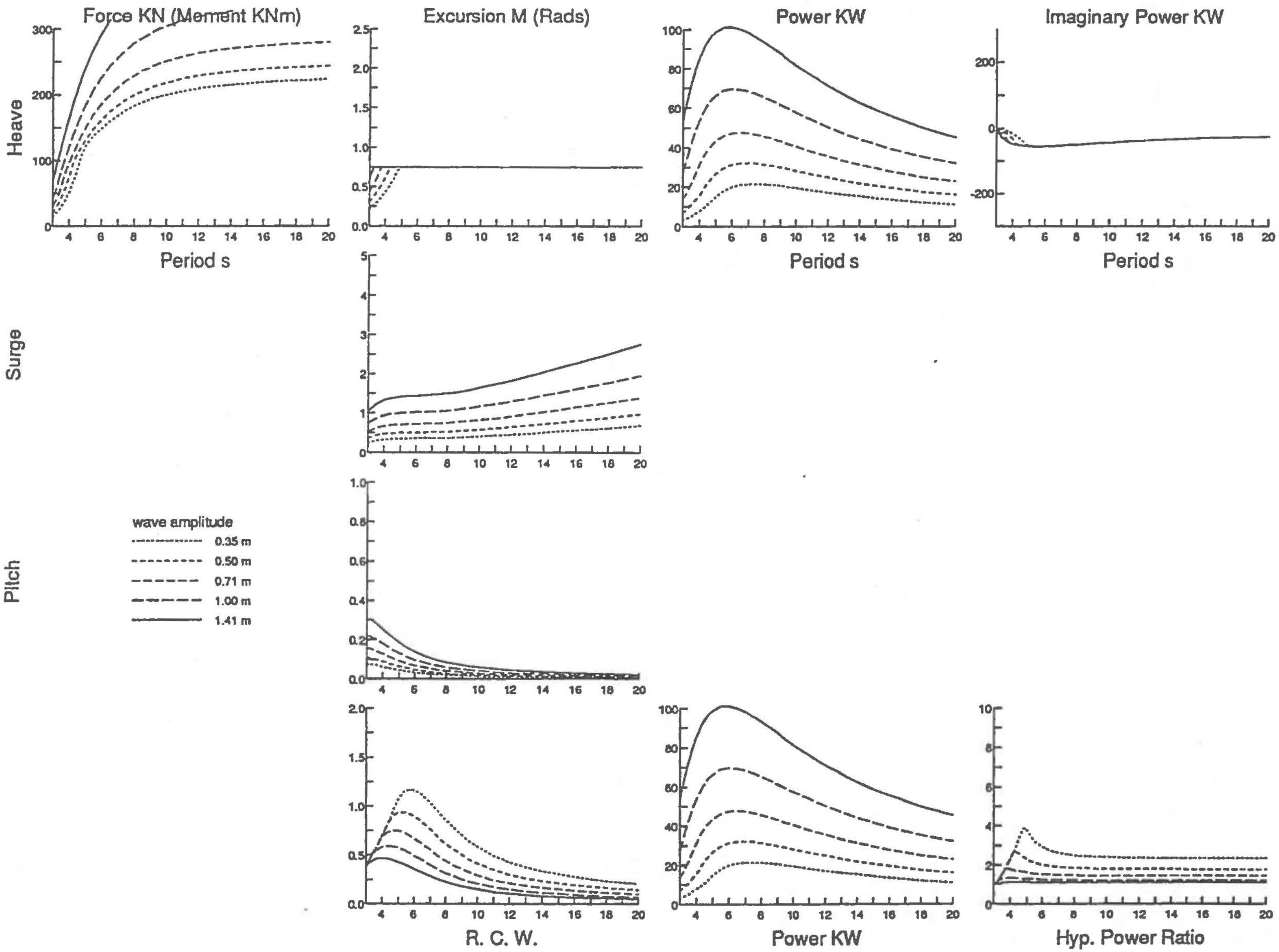

Figure 9: Power absorption for flat float with optimal complex control in heave

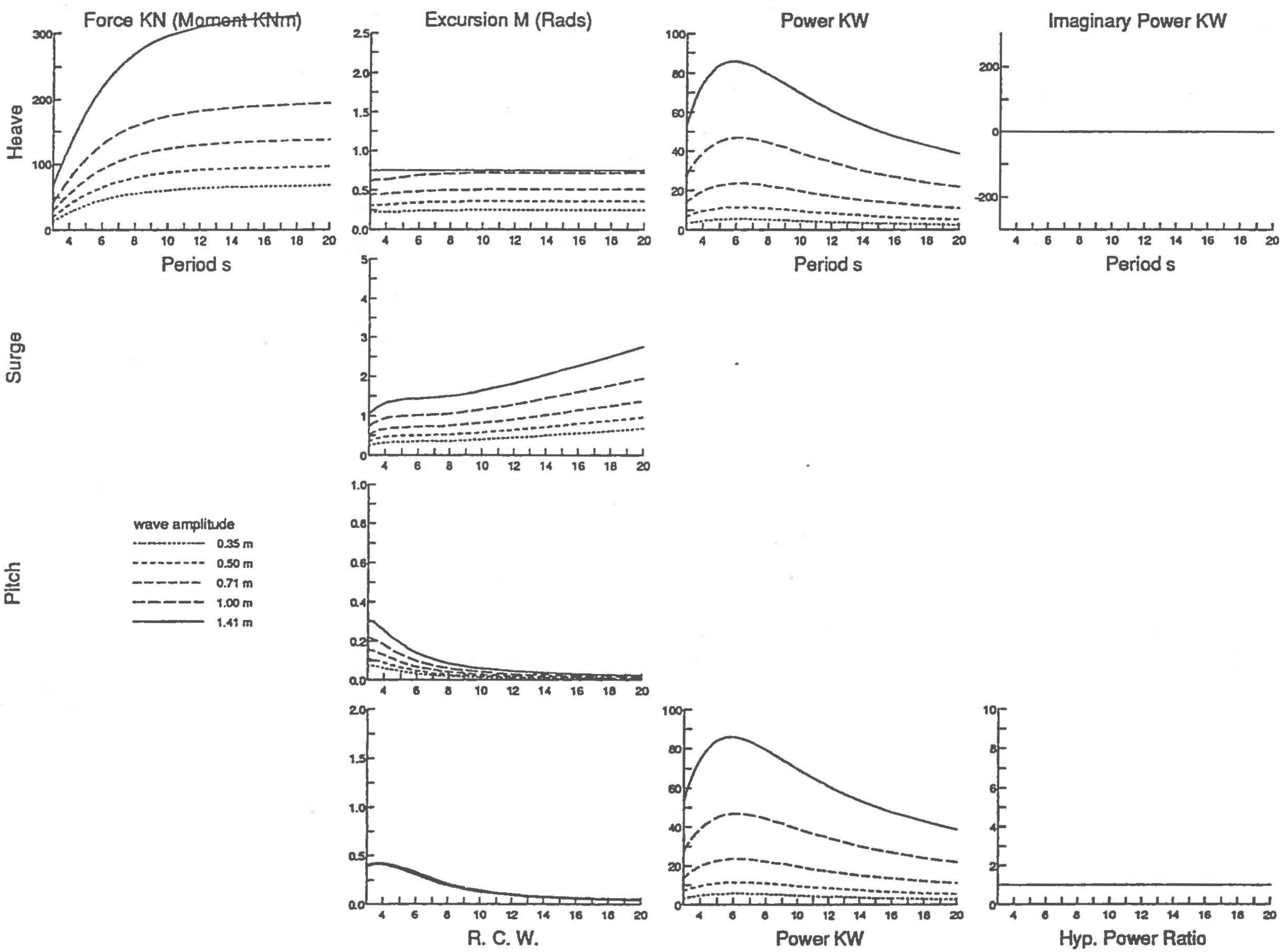

Figure 10: Power absorption for flat float with optimal real control in heave

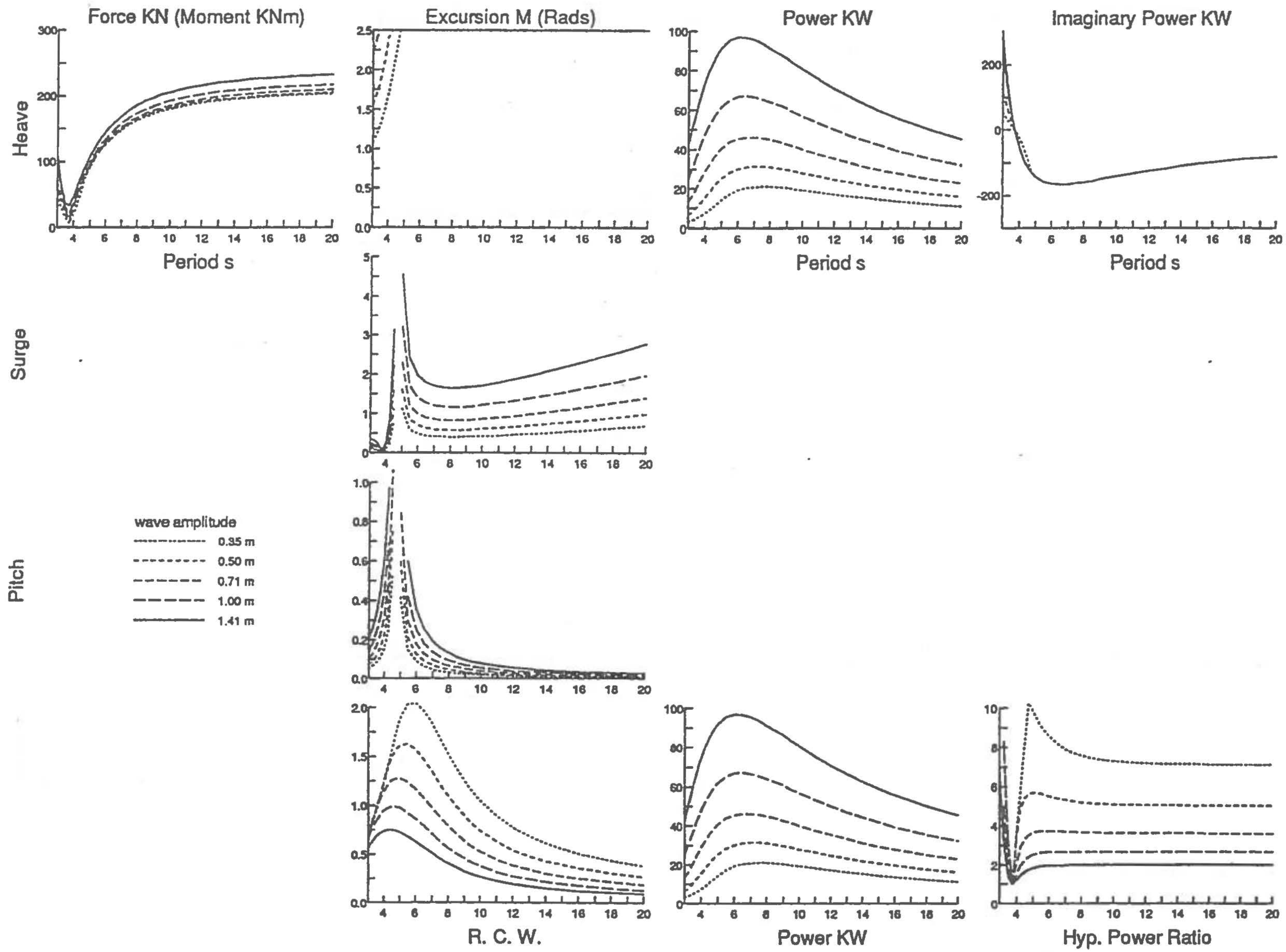

Figure 11: Power absorption for tall float with optimal complex control in heave

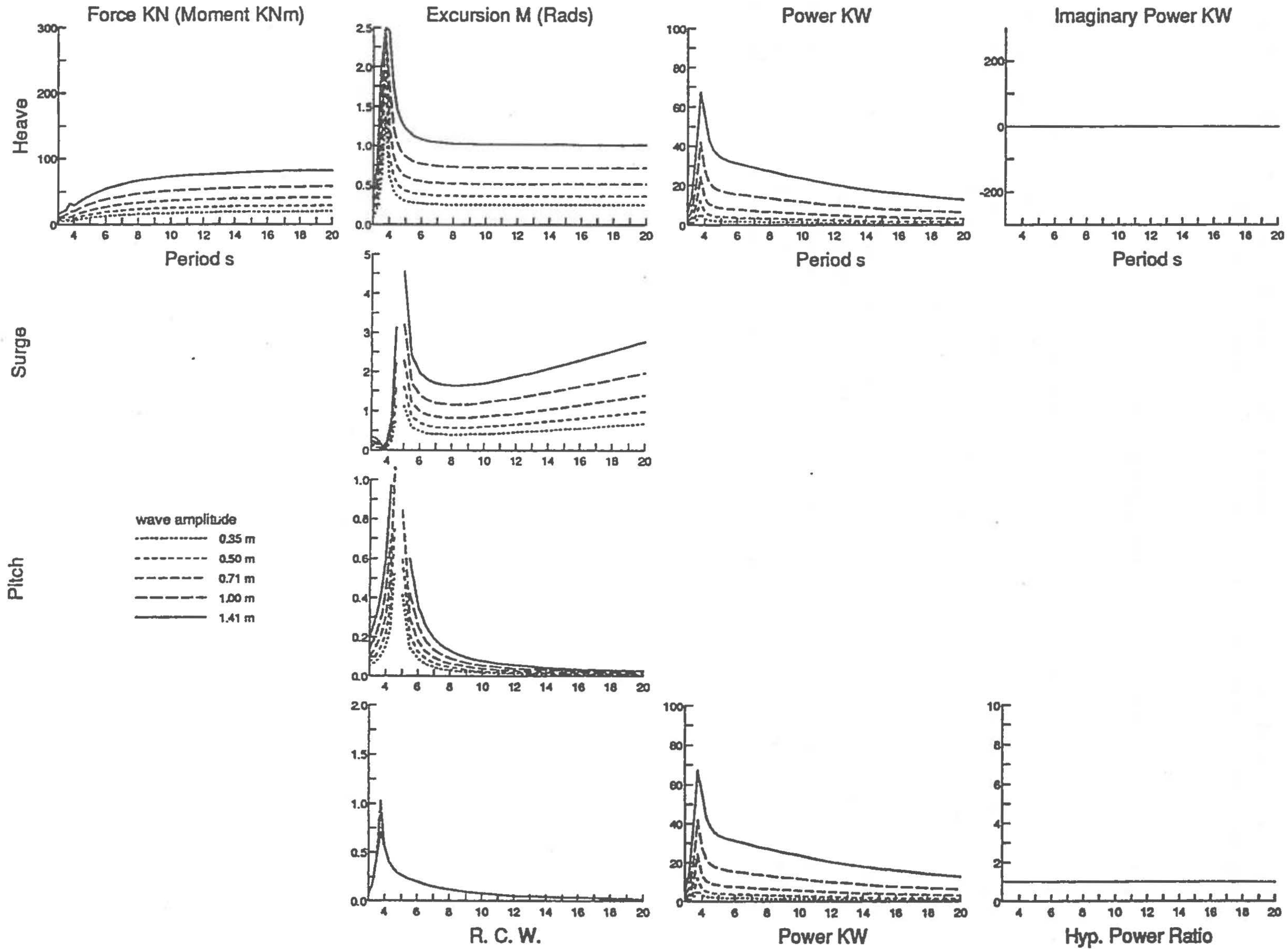

Figure 12: Power absorption for tall float with optimal real control in heave

A Formulation of Optimal Power Absorption

The equations of motion for a floating body are presented in the following four subsections. Details of the formulation may be found in my PhD thesis.

A.1 Description of the Motions

The formulation is generalised in that displacements are defined with respect to an arbitrarily specified reference point C, rather than the centre of gravity or a point in the equilibrium of the free-surface. Moments are evaluated with respect to the instantaneous position of the reference point — rather than its equilibrium position.

The six degrees of freedom of the rigid body motion are defined by ξ_i, $i = 1 \ldots 6$, where $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3) = \underline{C^m C}$ represents the translational displacements of surge, sway and heave, and $\boldsymbol{\alpha} = (\xi_4, \xi_5, \xi_6)$ represents the rotational displacements of roll, pitch and yaw which transform the $\mathbf{i}, \mathbf{j}, \mathbf{k}$, vectors onto the $\mathbf{i}', \mathbf{j}', \mathbf{k}'$, vectors. For a given motion of the body $\boldsymbol{\xi}$ will depend on the choice of C, whereas $\boldsymbol{\alpha}$ is independent of C. See figure 13.

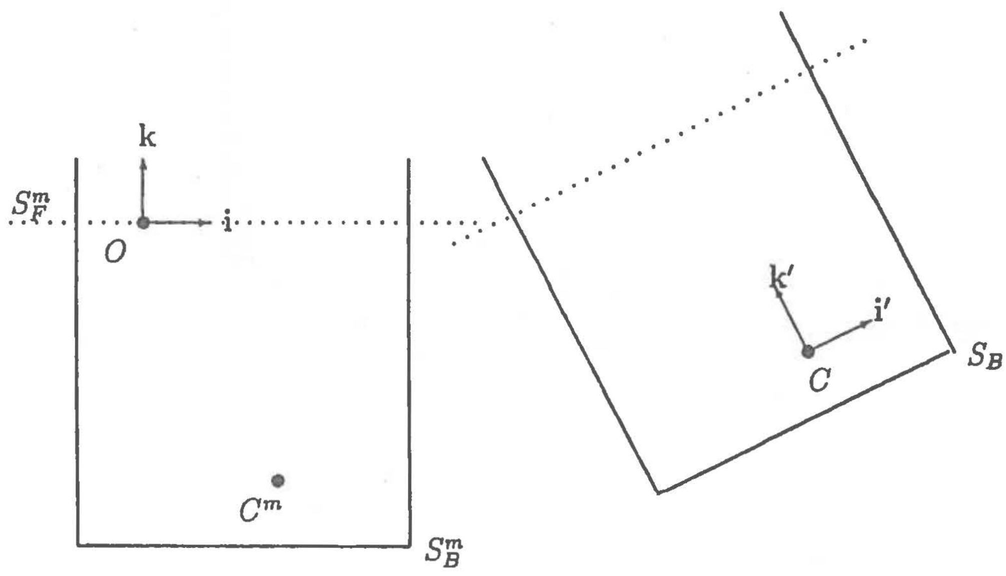

Figure 13: Definition of $\boldsymbol{\xi}$ and $\boldsymbol{\alpha}$

A.2 Equations of Motion

The equations of motion are derived from Newton's laws of motion and the properties of rigid body motion. Neglecting non-linear terms gives:

$$\mathbf{F}^{(0)} + \mathbf{F}_G^{(0)} + \mathbf{F}_C^{(0)} = 0 \quad (3)$$
$$\mathbf{F}^{(1)} + \mathbf{F}_G^{(1)} + \mathbf{F}_C^{(1)} = m\ddot{\xi}^{(1)} + m\ddot{\alpha}^{(1)} \times \mathbf{x}'_G \quad (4)$$
$$\mathbf{M}^{(0)} + \mathbf{M}_G^{(0)} + \mathbf{M}_C^{(0)} = 0 \quad (5)$$
$$\mathbf{M}^{(1)} + \mathbf{M}_G^{(1)} + \mathbf{M}_C^{(1)} = m\mathbf{x}'_G \times \ddot{\xi}^{(1)} + I_c\dot{\omega}^{(1)} \quad (6)$$

Where $\mathbf{F}$ and $\mathbf{M}$ are the force and moment due to the water, $\mathbf{F}_G$ and $\mathbf{M}_G$ are due to gravity, and $\mathbf{F}_C$ and $\mathbf{M}_C$ are the control force and moment. m is the mass of the device, I_c is its inertia tensor with respect to the reference point, C, the superfix $^{(0)}$ denotes zeroth order (equilibrium) quantities, and $^{(1)}$ denotes linear dynamic quantities.

In cases where the buoyancy force, $\mathbf{F}^{(0)}$, and moment, $\mathbf{M}^{(0)}$, are not balanced by the zeroth order gravity force, $\mathbf{F}_G^{(0)}$, and moment, $\mathbf{M}_G^{(0)}$, there must be other external forces, $\mathbf{F}_C^{(0)}$ and moments, $\mathbf{M}_C^{(0)}$ in order to satisfy the equilibrium condition.

A.3 Forces and Moments Due to Gravity

The gravity forces and moments are:

$$\mathbf{F}_G^{(0)} = -mg\mathbf{k} \quad (7)$$
$$\mathbf{F}_G^{(1)} = 0 \quad (8)$$
$$\mathbf{M}_G^{(0)} = -mg\mathbf{x}'_G \times \mathbf{k} \quad (9)$$
$$\mathbf{M}_G^{(1)} = -mg(\alpha^{(1)} \times \mathbf{x}'_G) \times \mathbf{k} \quad (10)$$
$$(11)$$

where $\mathbf{x}'_G$ is the centre of gravity.

A.4 Forces and Moments Due to the fluid

The hydrostatic ($\mathbf{F}^{(0)}$ and $\mathbf{M}^{(0)}$) and hydrodynamic ($\mathbf{F}^{(1)}$ and $\mathbf{M}^{(1)}$) forces and moments are given by:

$$\mathbf{F}^{(0)} = \rho g V\mathbf{k} \quad (12)$$
$$\mathbf{M}^{(0)} = \rho g V \begin{bmatrix} +y'_b \\ -x'_b \\ 0 \end{bmatrix} \quad (13)$$

$$\mathbf{F}^{(1)} = -\rho \int_{S_B^m} \mathbf{n}'\Phi_t^{(1)}\, dS - \rho g A[\xi_3^{(1)} + \xi_4^{(1)} y_f' - \xi_5^{(1)} x_f']\mathbf{k} \tag{14}$$

$$\begin{aligned} \mathbf{M}^{(1)} &= -\rho \int_{S_B^m} \mathbf{x}' \times \mathbf{n}'\Phi_t^{(1)}\, dS \\ &\quad -\rho g \begin{bmatrix} +Ay_f'\xi_3^{(1)} + (AL_{yy}' + Vz_B')\xi_4^{(1)} - AL_{xy}'\xi_5^{(1)} - Vx_B'\xi_6^{(1)} \\ -Ax_f'\xi_3^{(1)} - AL_{xy}'\xi_4^{(1)} + (AL_{xx}' + Vz_B')\xi_5^{(1)} - Vy_B'\xi_6^{(1)} \\ 0 \end{bmatrix} \end{aligned} \tag{15}$$

Where $\Phi^{(1)}$ is the linear velocity potential, A is the water-plane area, V is the volume displacement, x_B is the centre of buoyancy, L_{xy} ... are moments of the water-plane area and (x_f, y_f) is the centroid of the water-plane area. The surface integrals are performed over the equilibrium wetted surface of the body, S_B^m, and may be separated from the displacements to give added masses, dampings and wave excitation terms.

A.5 Power absorption

The power extracted from the body is the product of the control force with the device velocity:

$$P(t) = \mathbf{U}(t) \cdot \mathbf{F}_c(t) \tag{16}$$

where $\mathbf{F}_c$ is a six-vector representing the control forces and moments, and $\mathbf{U}$ is a six-vector for the translational and angular velocities.

It is assumed that all time-dependent quantities are sinusoidal, and may therefore be represented by complex variables. The above equations of motion may then be written as a single six-dimensional complex matrix equation:

$$\mathbf{F}_C = \mathbf{Z}\mathbf{U} - a\mathbf{X} \tag{17}$$

where $\mathbf{F}_C$ is a complex six-vector denoting the control forces and moments, $\mathbf{X}$ denotes the the excitation forces and moments for a wave of unit amplitude, a is the wave amplitude, $\mathbf{U}$ is the body velocity vector and $\mathbf{Z}$ is the impedance matrix which contains terms due to inertia, added mass, damping, hydrostatic restoring terms, and gravity restoring terms.

Under the assumption that $\mathbf{Z}$ is symmetric, the average power absorbed over a cycle, P, is then given by :

$$P = \frac{1}{4}[\mathbf{U}^*\mathbf{F}_E + \mathbf{F}_E^*\mathbf{U}] - \frac{1}{2}\mathbf{U}^*\mathbf{B}\mathbf{U} \tag{18}$$

A.6 Power absorption with uncontrolled degrees of freedom

We assume that the equation of motion (17) is re-written by removing the directions in which the device is fixed, to leave a system of n equations. Theses are ordered so that the first m equations correspond to controlled degrees of freedom, and the remaining $m-n$ degrees of freedom are un-controlled.

The equations of motion then become:

$$\begin{bmatrix} \mathbf{F}_c \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{cc} & \mathbf{Z}_{cf} \\ \mathbf{Z}_{fc} & \mathbf{Z}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{U}_c \\ \mathbf{U}_f \end{bmatrix} - a \begin{bmatrix} \mathbf{X}_c \\ \mathbf{X}_f \end{bmatrix} \quad . \tag{19}$$

Where c and f denote the controlled and uncontrolled partitions of vectors and matrices the obvious manner.

The uncontrolled rows of (19) gives:

$$\mathbf{U}_f = \mathbf{Z}_{ff}^{-1}(a\mathbf{X}_f - \mathbf{Z}_{fc}\mathbf{U}_c) \tag{20}$$

which is then substituted into the controlled rows of (19) to give:

$$\mathbf{F}_c = \mathbf{Z}_{cc}^{m}\dot{\mathbf{U}}_p - a\mathbf{X}_c^{m} \tag{21}$$

where the modified impedance is defined as $\mathbf{Z}_{cc}^{m} = \mathbf{Z}_{cc} - \mathbf{Z}_{cf}\mathbf{Z}_{ff}^{-1}\mathbf{Z}_{fc}$ and the modified excitation is defined as $\mathbf{X}_c^{m} = \mathbf{X}_c - \mathbf{Z}_{cf}\mathbf{Z}_{ff}^{-1}\mathbf{X}_f$

Equation (21) is now in the same form as equation (17). The optimal velocities, $\mathbf{U}_c$ may therefore be obtained using similar methods to the case of all degrees of freedom being controlled.