SEISMIC REFRACTION NETWORKS
IN THE STUDY OF THE EARTH'S CRUST

by

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VOLUME 1

Thesis submitted for the degree of
Doctor of Philosophy
in the University of Edinburgh

April 1967

SYNOPSIS

This dissertation deals with the interpretation of the crust as a series of layers, each of uniform velocity but not necessarily plane.

The ray paths within the structure are defined by Snell's law from the velocity ratios. By observing travel-times between several explosions and several seismometers, and relating these to the ray paths for critically-refracted waves, the structure can be interpreted as a characteristic "time-term" to each interface for each site and a propagation velocity for each layer.

Where the number of observations exceeds the number of unknowns, a "best-fitting" solution is obtainable by least-squares techniques, and further refinements permit correction for the effect of dipping structure and curved ray paths.

The network configuration is not restricted by the analysis to formal patterns (e.g. straight lines), and figures of merit are available for evaluation of the quality of fit, the contribution of each observation, and the effectiveness of the areal spread. Comparisons of the suitability of various possible configurations are made, and statistical expressions for the uncertainties of the solution are presented.

Techniques are developed for the preliminary assessment of data and the refinement of solutions by a progression of computer-aided judgments, and illustrated by examples. A computer programme has been developed in Atlas Autocode.

Practical aspects of field equipment are discussed, with various considerations of signal quality in relation to background noise.

The method is applied to data from four separate projects in the British Isles and Scandinavia. The results indicate that the upper portion of the crust in the British Isles departs considerably from uniform layering, but an intermediate refractor (of velocity 7.3 km/sec.) is clearly observed in the south of the region; a lower refractor is observed, although the velocity is not well determined. In Scandinavia a 3-layer model gives a satisfactory approximation, with a well-defined basin structure.

Keywords:

Seismic

Refraction

Networks

Crust

CONTENTS

VOLUME 1

VOLUME 2 : APPENDICES

FIGURES

1. INTRODUCTION

The objective in seismic studies of the Earth's crust is to obtain a picture of the distribution of materials so that their evolution may be understood, or so that exploitable natural resources such as gas or oil may be located.

Basically the parameters which can be measured are the propagation velocities for various types of wave-motion; gradients and discontinuities in the velocity distribution may be recognised by certain characteristic features.

In refraction work, the directly-observed quantities are not the velocities and depth of layers, but simply the shot-to-station distances and the times of arrival of various waves which an experienced seismologist identifies on the seismogram as being associated with certain ray paths.

An essential concept in such work is the "marker horizon", which will be referred to here as simply "refractor". Typically this is a boundary (more or less horizontal), where the velocity changes discontinuously from a lower value in the over-lying medium to a higher value in the under-lying medium.

The wave system associated with such a refractor is a lateral wave in the lower medium and a head wave in the upper; the ray path appropriate to the onset of each wave-packet is that defined by critical refraction into the lower medium, i.e. propagation along the boundary at the velocity of the lower medium.

Where other discontinuities or gradients of velocity occur above the refractor under consideration, the ray path through these is defined by refraction in accordance with Snell's Law. (see, for example, Nettleton, 1940).

If only a single well-defined plane refractor existed, the interpretation would be a relatively simple matter, but usually in practice a number of non-plane refractors are involved, some distinct and some much less so (corresponding to discontinuities with only slight velocity contrast or to gradual transition zones). The tasks are then to recognise the main features of the structure, to identify the wave arrivals associated with each refractor, and from these to map the surface of each refractor.

The methods most commonly used for analysis of such refraction data rely on fitting a straight line to the graph of travel-time against distance, corresponding to the assumption of plane layering (not necessarily restricted to horizontal).

For a profile shot in only one direction (i. e. "unreversed") the slope of the line gives the reciprocal of apparent velocity.

For a profile shot in both directions (i. e. "reversed"), two separate apparent velocities are derived, and these can be combined to give the true velocity and the dip of the refractor.

If horizontal layering is assumed, the intercept on the time axis corresponds to twice the time taken for the wave to propagate between the refractor and the surface, and gives a measure of the depth to the refractor, provided that the velocity distribution in the material above the refractor can be estimated.

Similarly for a dipping layer, the time intercept relates to the structure under the reference (zero-distance) point.

The following problems arise in interpreting refraction data, especially when attempting to combine the results of several surveys over an extended area:

The assumption of plane-layering implicit in most methods of interpretation is clearly not valid for extended areas. Even if the deeper refractors can be reasonably approximated by plane-layering, the travel-times for such refractors will vary irregularly if the shallower refractors depart from plane layering, so that fitting of a straight line to the data is bound to be unsatisfactory.

While individual projects may be based on formal patterns (such as a "reversed profile" along a straight line), in general the geographical distribution of survey points will be a somewhat random two-dimensional spread. The method of analysis should be capable of making controlled use of data from such a distribution.

A reliable indication is needed for the uncertainties, which are influenced both by the quality of observations and by the suitability of the pattern of shots and stations.

The refractor velocity may vary to some degree across the region.

The "time-term" method is a generalised form of solution which provides a very satisfactory way of dealing with departure from plane horizontal layering, and also with two-dimensional spreads.

The statistical treatment of the uncertainties in refraction profiles has been thoroughly discussed by Steinhart and Meyer (1961), but regrettably these techniques are still widely neglected. For the time-term method similar statistical techniques are available, and in addition figures of merit may be derived for the effectiveness of the spread of survey points and the contribution of individual observations.

Regional variations in refractor velocity remain an embarrassment, but no more so with the time-term method than with straight-line methods. In general for deeper structures the variations are small enough to be manageable, whereas for shallower structure (which is of interest mainly in seismic prospecting) the velocity variation may well be the most important feature of the structure. Such conditions (provided that the survey points lie along a profile) may be effectively dealt with by the "Plus-Minus Method" (Hagedoorn, 1959).

A similar approach may be used in the time-term method, for each case where suitable alignments of survey points are available, to give local estimates of velocity after an average velocity for the entire region has been derived in a first approximate solution, as discussed in Section (2. 6) (Willmore, Herrin, and Meyer, 1963).

The key feature in the mapping of non-plane refractors in the "time-term", being the time taken for the critically-refracted wave to propagate between the refractor and a survey point on the surface. A more rigorous definition is given in Section (2. 2. 1).

The total travel-time from a shot to a station can then be partitioned into three sections: the shot time-term, the time in the lower medium, and the station time-term. In this form the observational data are used to build up a set of simultaneous equations, whose solution is then a matter of conventional mathematics.

By suitably specifying the time-term in relation to the geometry of the ray path, its value is made relatively insensitive to dip of the refractor, and an iterative procedure may be applied to allow for the effect of dip.

For each refractor, at each survey point, the time-term has a unique value which enters into the travel-time of all connections to that survey point via that refractor.

The task of interpretation then takes the following form: Several refractors have been recognised, and for each refractor a set of observations has been selected (each observation consisting of a pair of values: the distance from shot to station, and the corresponding travel-time). A model is to be calculated to account as closely as possible for the observed travel-times.

At this point it should be emphasized that the information directly involved in the observation from such a refraction survey is not the depth but rather the time-term for each survey point (a pair of time-terms, plus a lower-medium travel-time, in each observation). Fitting a model defined as time-terms is quite a direct process, whereas fitting a model in term of depths is artificially complicated. After the model has been fitted as a set of time-terms and refractor velocities, the subsequent stage of converting it into a set of depths is completely independent of the fitting process.

Each time-term can be translated as a figure for depth only if the velocity distribution above the refractor at that point can be estimated. However a shot point or station which yields only observations of a deeper refractor still makes effective contribution to the time-term network even in the absence of information regarding the shallower layers; the time-terms and velocity are determined from the analysis without directly involving the thickness or velocity distribution of the shallower layers.

The practical objection may well be raised that "time-terms" do not constitute the positive answers which are sought, and that the proper end-point is the model in terms of depths and velocities.

On the other hand, the merit of the time-term concept lies in the fact that the greatest sources of uncertainty in the chain of interpretation are those which come after the fitting of the time-terms; there may well be pecularities in the velocity distribution of the material above the refractor, but these do not enter into the fitting of the time-terms.

If more information subsequently becomes available on such a velocity distribution, it modifies only the translation of the time-terms into depths and need not involve a re-working of the time-term solution.

An "observational equation" can be written for each connection between a shot and a station, in the form:

$$t_{ij} = a_i + b_j + \Delta_{ij}/v \qquad (1)$$

where t_{ij} is the observed travel time and Δ_{ij} the distance, between the i^{th} shot and the j^{th} station; a_i and b_j are the time-terms of the shot and station respectively; v is the refractor velocity.

In this way a set of simultaneous equations can be built up, and a solution is required to give the unknown time-terms a_i and b_j and the velocity v.

Due to the inherent indeterminacy of the system (discussed in more detail in Section (2.7)), it is necessary to assign a value arbitrarily to one of the time-terms.

If there are m shots and n stations, with all shots observed at all stations, the total number of equations would be m x n, while the number of unknowns is m + n (a time-term for each survey point except one, and a refractor velocity). In practical situations it is usually not possible to observe this maximum number of connections, but in general there will be more equations than unknowns. Consequently it becomes possible to calculate a "best-fitting" solution by least-squares procedure.

2. BASIC RELATIONSHIPS

2.1 Assumptions

In order to interpret a set of refraction data, a number of basic assumptions must be made concerning the structure:

One or more distinct refractors extend throughout the area of the survey.

The wave system due to the refractor consists of a head wave in the medium above the refractor and a lateral wave in the medium below it. Continuously-refracted waves that penetrate ever more deeply as the shot-station separation increases can be handled by an extended version of the analysis.

Slope and curvature of the refracting surface must be small. Corrections may be applied to allow for the influence of slope and curvature if necessary after a preliminary solution has indicated the trend of the structure.

Over the entire area of the survey, the material immediately underlying each refractor must have no major horizontal variations of velocity. After a first approximation on this basis, the solution may be refined to reveal regional trends of velocity.

Within the critically-refracted ray cone under each survey point, velocity varies only with depth (perpendicular-to-refractor).

2.2 Definition of Time-term

2.2.1 Single-Refractor Case

The travel-time, t, for the head wave from a shot to a station at a distance Δ is to be expressed in the form

$$t = a_i + b_j + \Delta/v_2 \qquad (2)$$

where a_i and b_j are time-terms characteristic of the shot i and the station j respectively, and v_2 in the velocity in the lower medium.

It is important to understand that this way of partitioning the travel-time does not correspond to treating the path simply in the three separate sections delineated by the "offset points" (the points at which the critically refracted ray passes through the refractor).

The distance Δ is not the distance between offset points but the full distance between survey points.

The time-term represents the difference in travel-times between the following two paths:

(i) the slant path through the upper medium for the critically-refracted wave between survey point and refractor (segment AB in Figure (1))

(ii) a path lying entirely in the lower medium, defined by the projection of this slant path on to the refractor (segment AC in Figure (1)).

It has been shown (Willmore and Bancroft, 1961) that the case of non-plane layering is satisfactorily handled by referring the measurements to perpendiculars from each survey point to the refractor.

Let Δ = distance between the feet of the perpendiculars dropped from the shot and the station to the refractor

h = depth to the refractor (measured along the perpendicular)

v_1 = velocity in upper medium

v_2 = velocity in lower medium

Θ = angle between the perpendicular and the critically refracted ray path, i. e. $\Theta = \sin^{-1}\left(v_1/v_2\right)$

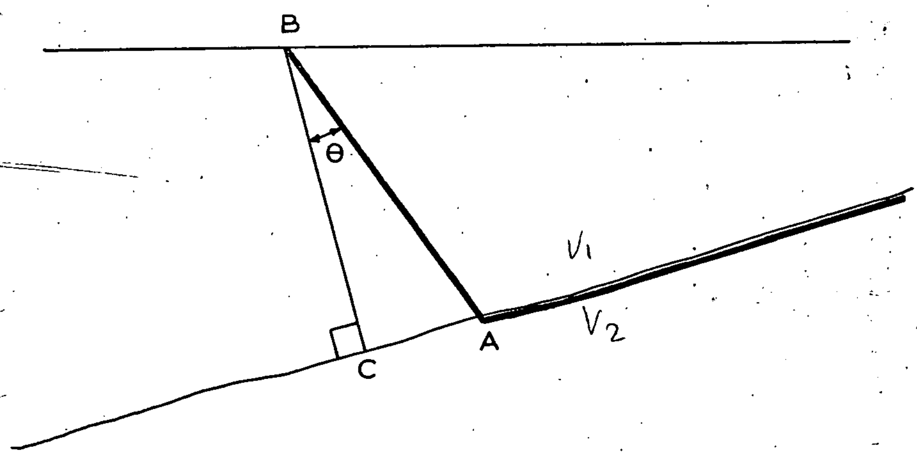


Figure (1). Ray path for definition of time-term.

The time-term is given by:

$$T = \frac{AB}{v_1} - \frac{AC}{v_2}$$

which reduces to the forms: $T = \frac{h}{v_1}\cos\Theta$ (3)

$$\text{or} \quad T = \frac{h}{v_1}\sqrt{1 - ({}^{v_1}/{}_{v_2})^2} \qquad (4)$$

$$\text{or} \quad T = \frac{h}{v_1 v_2}\sqrt{(v_2)^2 - (v_1)^2} \qquad (5)$$

$$\text{or} \quad T = h\sqrt{(\frac{1}{v_1})^2 - (\frac{1}{v_2})^2} \qquad (6)$$

Of these four forms, the first one is strongly preferred because of the ease with which a physical significance may be attached, and this is of particular value in extension to multi-layered cases.

The time which would be required if the wave travelled directly along the perpendicular BC between survey point and refractor is simply given by $(\frac{h}{v_1})$. It is then convenient to regard the factor (cos Θ) in the expression for the time-term as a correction for the slant path.

For waves which are not critically refracted at the first refractor but pass through to a deeper path, Θ is defined from the geometry of the path, and the expression $(\frac{h}{v_1} \cos \Theta)$ then gives the time lost in the upper layer.

2.2.2 Multiple-refractor case

By the use of a suitable notation, the inherent simplicity of the expression for the time-term can be retained.

Let Z_r = thickness of r^{th} layer

v_r = velocity of r^{th} layer

Θ_{rn} = angle of ray passing through r^{th} layer to be critically refracted into n^{th} layer.

$$\Theta_{rn} = \sin^{-1}(v_r/v_n)$$

$$\cos\Theta_{rn} = \sqrt{1 - (v_r/v_n)^2}$$

Then the time-term to the n^{th} layer (i.e. to the $(n-1)^{th}$ refractor) is given by:

$$T_{n-1} = \frac{Z_1}{v_1}\cos\Theta_{1n} + \frac{Z_2}{v_2}\cos\Theta_{2n} + \frac{Z_3}{v_3}\cos\Theta_{3n} \\ \ldots\ldots\ldots + \frac{Z_{n-1}}{v_{n-1}}\cos\Theta_{(n-1)n}$$

or simply:

$$T_{n-1} = \sum_{r=1}^{r=n-1} \frac{Z_r}{v_r}\cos\Theta_{rn} \qquad (7)$$

For the ray path through any given arrangement of layers the angles Θ_{rn} are defined from the velocity ratios by Snell's Law, and each term in the above summation then represents the time lost in the corresponding layer, i.e. the difference between the time along the slant path and the time along the path projected on to the deepest refractor.

2. 2. 3 General case of a known velocity distribution

In the general case where the velocity distribution with depth in the material overlying the refractor is known, the time-term is calculated in a similar manner.

Let v_h = velocity at depth h.

v_n = velocity of base refractor, at a depth H.

Θ_h = angle of ray at depth h for critical refraction into base refractor, i. e. $\Theta_h = \sin^{-1}(v_h/v_n)$.

Then time-term, T, to the base refractor is given by

$$T = \int_0^H \frac{\cos \Theta_h}{v_h} dh \qquad (8)$$

or alternatively

$$T = \int_0^H \left(\frac{1}{v_h}\right) \sqrt{1 - (v_h/v_n)^2}\ dh \qquad (9)$$

These general expressions would also be applicable to structures involving low-velocity material underlying material of higher velocity. However,in the present work the available data were insufficient to warrant a deeper investigation of this aspect.

2.3 Interpretation as Depth

Section (2.2) has dealt with the expressions defining the time-terms when the layer thicknesses (together with the velocities) are known.

The practical requirement in interpretation is the reverse of this: given a model in the form of time-terms and refractor velocities, to derive the structure in terms of layer thicknesses.

2.3.1 Single-refractor case

For a single refractor the conversion is perfectly straightforward.

From Equation (3):-

$$h_1 = T_1 v_1 \frac{1}{\cos \Theta_{12}}$$

2.3.2 Two-refractor case

From Equation (7) the time-term to the deeper refractor is:

$$T_2 = \frac{Z_1}{v_1} \cos \Theta_{13} + \frac{Z_2}{v_2} \cos \Theta_{23}$$

= (time lost in first layer)

+ (time lost in second layer)

from which the thickness Z_2 can be calculated once the value of Z_1 is known.

2.3.3 Multiple-refractor case

The least confusing approach is to work from Equation (7), treating the time-term to each refractor as the sum of the times lost in each of the layers above it.

The interpretation must start from the uppermost layer and work downwards. The time-terms to the first refractor yield values for the thickness of the first layer directly.

The thickness values in turn lead to values for the time lost in the first layer by waves critically refracted at the second refractor. The difference between (time-term to the second refractor) and the corresponding (time lost in the first layer) must be (time lost in the second layer) for each survey point, giving the thickness of the second layer by use of a conversion factor $\left(\frac{v_2}{\cos \Theta_{23}}\right)$.

Proceeding to critical refractions at the third refractor, the time lost in each of the first two layers is calculated for the new path angles from the thicknesses already derived, and so on.

By using this approach, the quantities involved at each stage have a simple physical significance which can be kept in mind throughout. In contrast, the expression for the depth is quite unwieldy if written out in full as a function of the time-terms and velocities. For example, the depth to the second refractor involves:

$$h_2 = \frac{V_2}{\sqrt{1 - V_2^2/V_3^2}}\, T_2 - \left[V_2 \frac{\sqrt{1 - V_1^2/V_3^2}}{\sqrt{1 - V_2^2/V_3^2}} - V_1 \right] \frac{T_1}{\sqrt{1 - V_1^2/V_2^2}}$$

2.4 Available Information on Velocity

In calculating the thickness of layers from the values of the time-terms, it is necessary to have an estimate of the mean velocity for waves passing through each layer in a vertical direction.

The velocity which emerges from the least-squares procedure relates primarily to the head waves which propagate along the refractor in a near-horizontal direction, i.e. the velocity of the material immediately below the discontinuity.

If the structure involves either anisotropy or gradient of velocity, then the least-squares estimate may not be sufficiently close to the velocity for the near vertical path, and may require to be judiciously increased in the light of the evidence.

There is evidence that in some crustal materials anisotropy may cause the vertical velocity to be as much as 10 % greater than the horizontal velocity (Richards, Utrecht 1966). If reflected arrivals can be observed, they provide an estimate of the vertical velocity.

For a layer in which the velocity increases with depth, the refraction observation at longer ranges involve waves following a slightly curved path and penetrating to some depth in the layer. Such waves show an apparent velocity slightly higher than those waves which remain in the uppermost part of the layer, but in general not so high as those near-vertical waves which pass completely through to the lower layer.

2.5 Dipping Structures and Curved Ray Paths (Figure (2))

In applying the time-term solution, it is not known in advance whether dip of the refractor will make a significant contribution. Taking the distances between shots and stations, and the observed travel times, a first approximation yields a set of time-terms, a refractor velocity, and a set of residuals. The structure may then be plotted to provide estimates of the dip under each survey point.

For the second approximation close estimates of the relevant distances are given by the distances between the feet of perpendiculars from each survey point to the estimated refractor; to allow for the effect of the difference in angle of emergence when substantial dip occurs, the observed travel-times are each modified by an amount calculated from the first estimates of time-terms and dip.

The correction to the distance is simply

$$\delta\Delta = H \cos \Theta_2$$

where H and Θ_2 are the first estimates of depth and angle of incidence in the lower medium, respectively.

The correction to the observed travel-times is obtained by multiplying the first estimate of time-term by a factor of the form

$$\left(\frac{\cos \Theta_1}{\cos \Theta}\right) - 1$$

$$\text{or} \quad \sqrt{\frac{(1 - r_2 \sin^2 \Theta_2)}{1 - r^2}} - 1$$

where r = velocity ratio, v_1/v_2

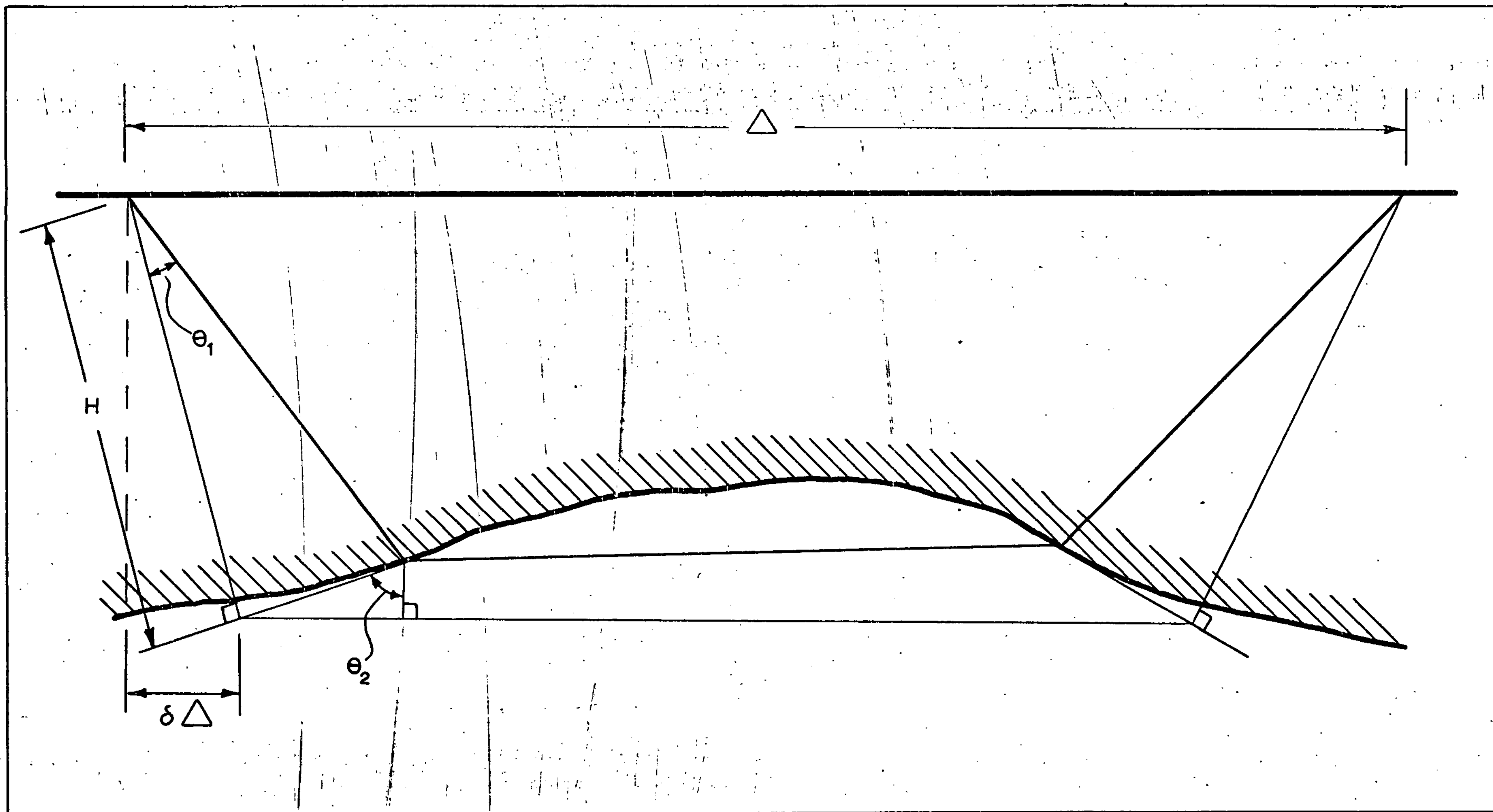


Figure (2)   Ray path for dipping structures

The second approximation using these modified distances and travel-times yields a set of residuals by which one may judge whether it is significantly better than the first.

A similar treatment may be applied in the case where a substantial gradient of velocity in the lower medium causes curvature of the ray paths, giving a steeper angle of emergence than for the critical-refraction path discussed above.

Such a gradient of velocity would also lead to a variation of mean velocity with distance, and this may be taken into account in the course of the least-squares fitting, as mentioned in Section (3. 3).

For the anticlinal structures shown in Figure (2), the ray path can be readily defined by Snell's Law, and should correspond to a significant transmission of energy by the head wave, whereas for a synclinal structure one might well expect the energy in the head wave to be negligible. However, in view of the fact that satisfactory observations have been obtained over synclinal structures, it must be argued that sufficient curvature or diffraction of the rays occurs in the lower medium.

An interesting example arises in the Jutland-Skagerrak Project. It had been noted that for shots at the southern end of the profile(in the Little Belt), Pn was not observed at distances beyond 200 km in Jutland whereas recordings were obtained at stations in

Norway at distances of 325 - 360 km.

This anomalous behaviour has been attributed by Hirschleber et al (1966) to attenuation of energy in the thick sediment cover of Jutland. It may, however, be due to curvature of the refractor. Figure (3) shows the general trend of the structure derived from the time-term solution. Shots fired at A were observed at C but not at B, and it is suggested that the structure beneath B is sufficiently synclinal to produce a slight shadow zone, while beneath C the trend is anticlinal, giving a slight enhancement of the head wave energy.

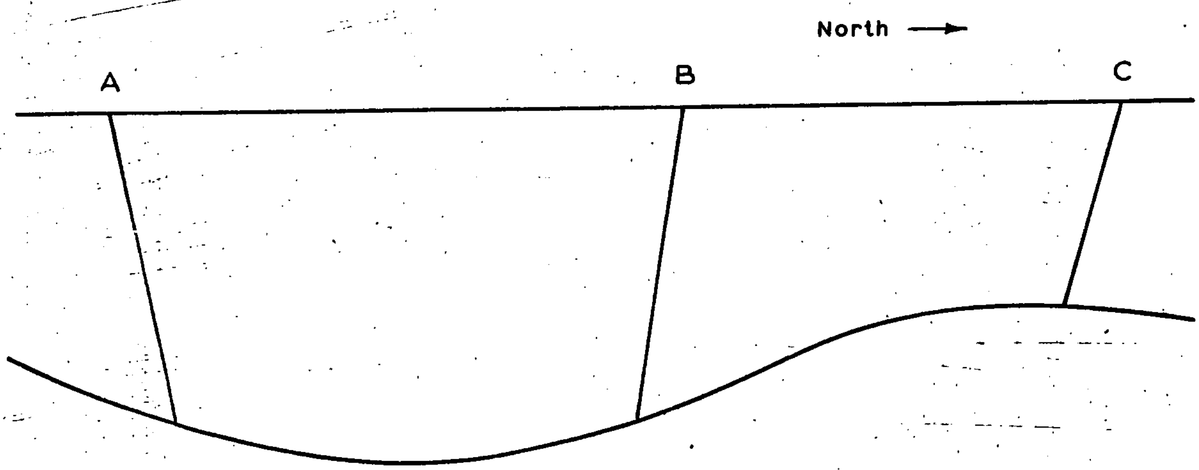


Figure (3) Structural trend of the Jutland-Skagerrak profile.

2.6 Local Velocity Estimation

Given a suitable pattern of survey points, it becomes possible to derive estimates of the local refractor velocity.

Willmore, Herrin and Meyer (1963) have discussed the case of a pair of stations approximately in line with a given shot (or similarly a pair of shots in line with a given station). It is assumed that the data form part of a sufficiently large set to give good statistical control over the time-terms.

To be more specific, this means that the effect of random observational errors should be small; it is also implied that a preliminary determination using a constant refractor velocity over the entire area will yield sufficiently reliable estimates of the time-terms without being unduly disturbed by systematic regional variations in velocity.

For such a pair of stations the calculated time-terms b_j are combined with the observed travel-times t_{ij} and known distances Δ_{ij} to give an estimate of the local velocity v^1 from the relation:

$$v' = \frac{(t_{11} - b_1) - (t_{12} - b_2)}{\Delta_{11} - \Delta_{12}}$$

Since this approach does involve the possibility that regional velocity variations might be partially masked by the errors which they would introduce into the time-terms, it is worth noting that a purer estimate of velocity is given from the configuration discussed by Jeffreys (1935), using four survey points: a pair of stations bracketed by a pair of shots as shown in Figure (4) (or similarly for a pair of shots bracketed by a pair of stations).

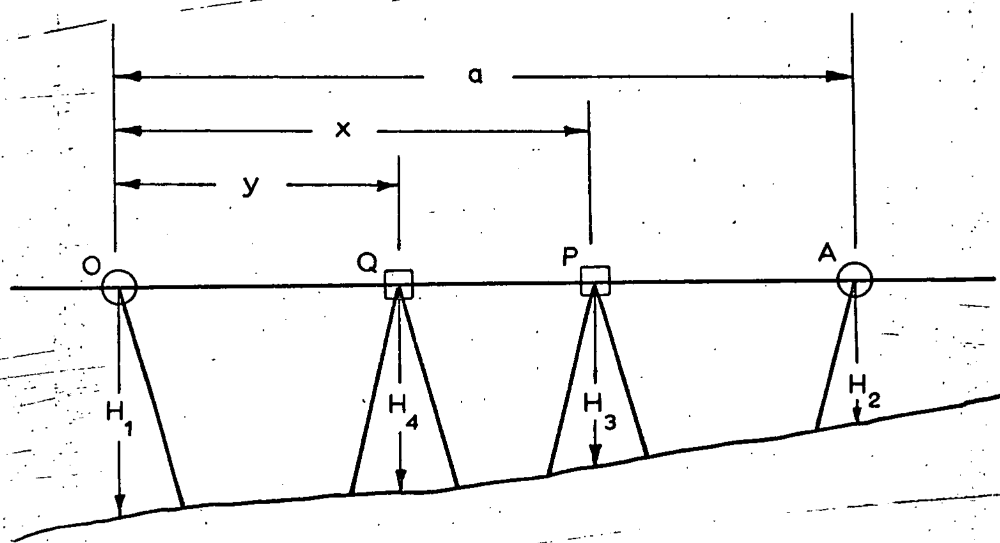


Figure (4) Ray path for local velocity determination.

Let η = conversion factor from depth to time-term

(e.g. time-term at site O = $\eta\ H_1$)

v = velocity of lower medium

Then travel-time T_{OP} from O to P is given by:

$$T_{OP} = \frac{x}{v} + \eta\ (H_1 + H_3)$$

and so: $$T_{OP} - T_{OQ} = \frac{x - y}{v} + \eta\ (H_3 - H_4)$$

and $$T_{AQ} - T_{AP} = \frac{x - y}{v} - \eta\ (H_3 - H_4)$$

by addition: $$T_{OP} - T_{OQ} + T_{AQ} - T_{AP} = \frac{2(x - y)}{v}$$

from which v is determined directly without the need for evaluating an estimate of the time-terms.

The uncertainty of such local velocity estimates will be greater than that of the overall velocity estimate because of the much shorter spread of distance and the smaller number of observation involved, but where a number of independent estimates can be compiled for adjacent links of the network there is the possibility of recognizing broad regional trends.

If there is strong evidence of regional velocity variation, the time-term solution may be refined to take account of it, on similar lines to the method employed by Herrin and Taggart (1962) in epi-centre location. The region may be subdivided on a grid system, and an appropriate velocity correction factor assigned to each element of the grid. In drawing up the observational equations, for each connection the velocity correction factor will be given by the mean of all the values along that path. In this way the least-squares process may still be used to good effect.

2.7 The Inherent Indeterminacy

It should be noted that since each equation includes a pair of time-terms, one for a shot and one for a station, the relation between shot time-terms and station time-terms is not determined uniquely from these equations. An arbitrary constant could be added to all the shot time-terms and subtracted from all the station time-terms without altering the validity of the solution. It is necessary to assign one time-term arbitrarily in the first instance, and when the equations have been solved the relation between shot and station time-terms can be readjusted to minimise the apparent discontinuities of the structure or to concentrate them in the vicinity of known geological faults or anomalies.

In the initial assignment of the time-term, any convenient criterion may be applied. In the literature a possible source of confusion occurs, in that slightly different approaches have been employed in the two basic papers which set out the time-term method.

Willmore & Bancroft (1961) deal with the special case of a filled array (i. e. all shots observed at all stations), which can be handled by desk calculator. Some reduction in the amount of calculation is possible by specifying that the mean of all shot time-term should be zero.

Scheidegger & Willmore (1957) deal with the more general case of an unfilled array (i. e. some connections missing), for which it is virtually essential to use a large computer. It is then convenient to set one time-term to zero, which can permit a reduction of one in the dimensions of the matrix to be handled. However this apparent economy does involve a slight sacrifice of statistical information relating to the uncertainties of the solution, and it is preferable to retain the full dimensions of the matrices for this purpose.

The correct procedure for avoiding the indeterminacy is to arrange that at least one survey point contributes observations both as a shot point and as a station. This will mean that the solution yields values for the unadjusted time-terms, a_i as a shot and b_j as a station, giving directly the value of adjustment required to make them identical. Ideally a number of survey points should contribute in this way, so that a more accurate mean is available, together with an indication of the uncertainty.

The requirement as stated above may appear straightforward, but in practice with a limited number of shots and stations it is not always easy to arrange that the necessary interchanges are covered by reliably identified observations.

In this context, "reliable identification" implies restriction to first arrivals, backed by a sufficient number of well recorded later arrivals to permit the observations to be assigned to the correct refractor. (Observations near the crossover distance should not be included before a preliminary solution has provided additional evidence for identification.)

There is obviously a great deal to be gained by carrying out surveys in two stages, rather along the lines suggested by Gardner (1939). The first stage should be chiefly a reconnaissance to determine:

(i) the existence of well-defined refractors at the depth of interest

(ii) the refractor velocities

(iii) the crossover distances

The second stage can then be planned to provide connections at the most favourable distances, and by re-occupying some of the sites from the first stage, both sets of data may be incorporated in the final solution.

The minimum conditions for interchange on any one refractor may be re-stated as follows:

If site 1 is to provide interchange, then when a shot is fired at 1, a station 2 should observe it at a suitable distance; when a station is operated at 1, a shot at 3 should be observed by both stations 1 and 2 at suitable distances.

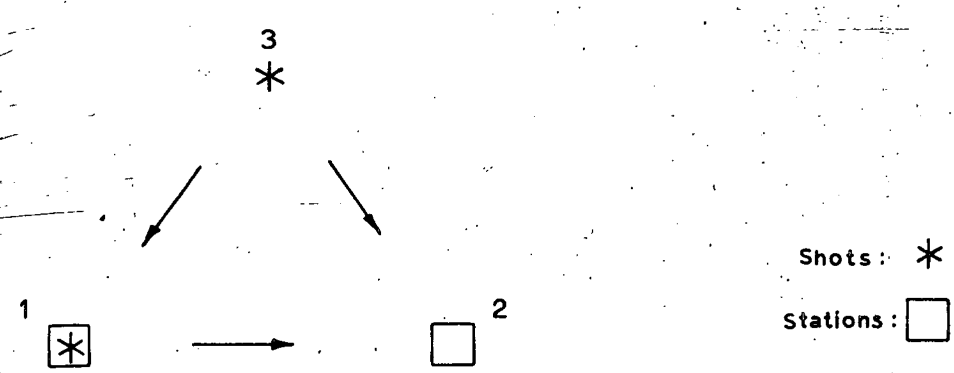


Figure (5). Arrangement of shots and stations for interchange.

This yields three observational equations:

$$t_{12} = a_1 + b_2 + \Delta_{12}/v$$

$$t_{31} = b_1 + a_3 + \Delta_{31}/v$$

$$t_{32} = b_2 + a_3 + \Delta_{32}/v$$

and since $a_1 = b_1$, the values of the time-terms are uniquely determined, provided that the value of v is already defined by other observations.

At this stage a further consideration arises: in order to reduce the possibility of error due to inclusion of an observation from a different refractor, or to non-linearity of the time-distance relationship, it is preferable that in the vital interchange observations the distance between shot and station should be roughly the same for each connection, leading to a pattern of equilateral triangles.

It might be argued that such a configuration does not give adequate control over the velocity determination (see Section (4. 4)), but there is no need to rely on the same basic groups of survey points to provide control of both interchange and velocity determination.

In fact the recommendation towards roughly equivalent distances for each connection is quite the reverse of the condition for optimum velocity determination, and a set of observations so spaced would be quite useless without the aid of other data to define the velocity. An ideal network layout would include both the "reversed-profile" configuration to determine velocity and the triangular configuration to determine interchange, and this fits in with the suggestion made earlier (page 27) that surveys should be carried out in two stages.

2.8 Graphical Representation

If a set of refraction data is plotted in the form of a graph of travel-time vs. distance, the distribution of points will be something similar to Figure (6). The general trend will be along a sloping line AB, but some scatter will be evident.

The assumption of a single plane layer corresponds to the expectation that the points should lie exactly on a straight line and that the scatter is due only to random errors. Accordingly a straight line AB is fitted by least squares, and the slope of the line gives an estimate of $1/v_2$, where v_2 is the velocity below the refractor.

For the further assumption of horizontal layering, the intercept time, OA is regarded as characteristic of all the survey points, and is converted into a figure for depth, h, by the relation:

$$t = 2\,{}^{h}/_{v_1} \sqrt{1 - \left({}^{v_1}/_{v_2}\right)^2}$$

which may be recognised as a statement that the intercept time is twice the time-term.

The possibility of dipping plane layering brings a demand for more thorough coverage; in conventional terms the profile must be "reversed" by shooting in both directions. The procedure is then to fit a separate least-squares line for each direction, giving an independent value of time intercept for each end of the profile, corresponding to twice the time-term in each case.

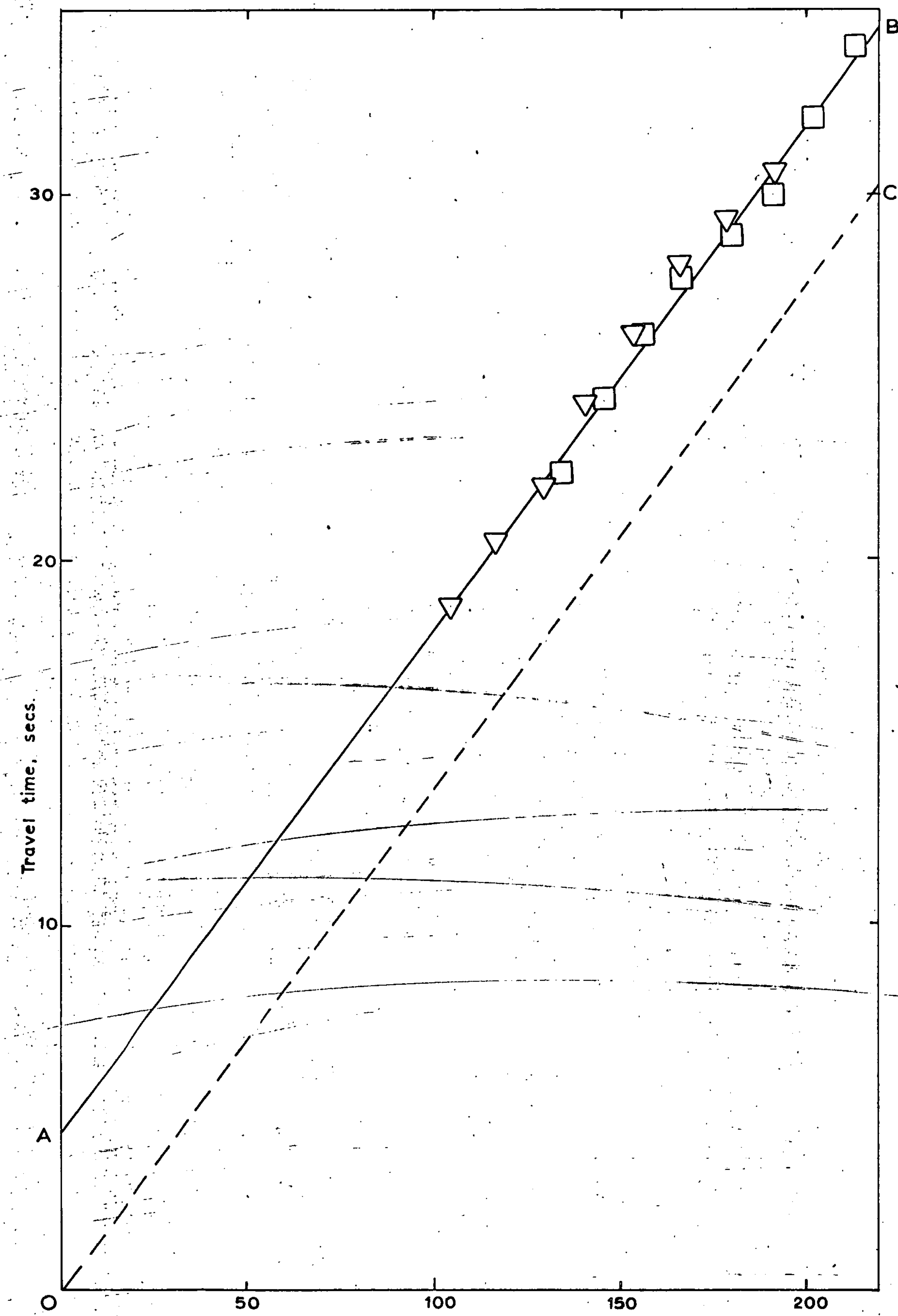


Figure (6) Specimen plot of travel time vs. distance

A useful insight into time-term interpretation is obtained by taking as the principal straight line not a line AB through the points but a line OC through the origin with a slope of $^1/_{v_2}$. Returning to the observational equations in the form

$$t_{ij} = a_i + b_j + \Delta_{ij}/_v$$

it is seen that for each observation the length of ordinate from the distance axis to the line OC corresponds to the term ($\Delta_{ij}/_v$), and the remainder of the ordinate between the plotted point and the line OC corresponds to the sum of the two time-terms ($a_i + b_j$).

If instead of plotting travel-time directly, the "reduced" travel-time ($t - \Delta/_v$) is used, the display will be as shown in Figure (7). Since the range of values of the reduced travel-times is quite small, it becomes possible to use a greatly enlarged vertical scale so that the scatter of data can be more clearly examined, and this is usually the motive for introducing such a form.

However the most useful aspect of the reduced travel-time display is that, since the contributions of the distances Δ_{ij} to the travel-times have been largely removed, the differences which remain between the reduced travel-times can be attributed mainly to differences of time-terms (plus observational errors).

If the exact value of the velocity were known, the reduced travel-times would then faithfully represent observations of pairs of time-terms, but it is not imperative that the exact value be used, since the effect of an error of velocity will be simply a tilting of

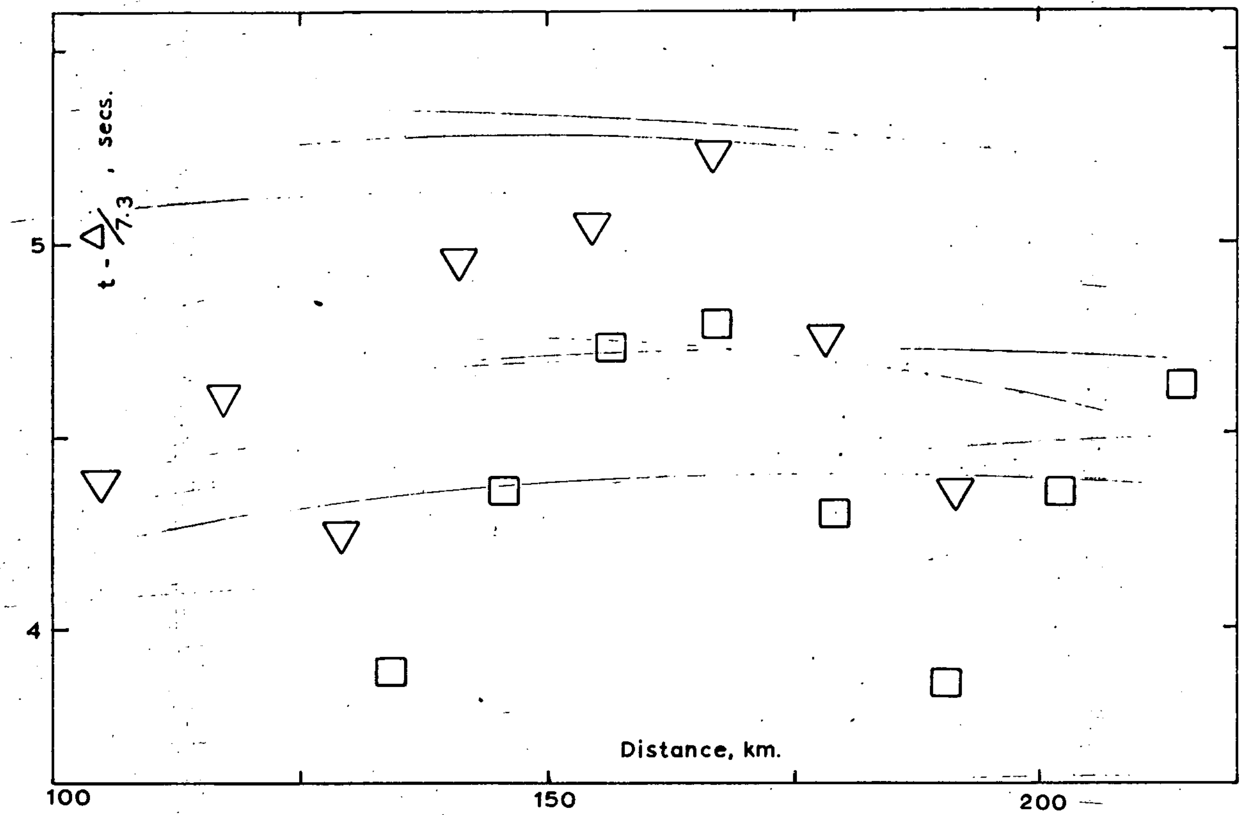


Figure (7) Specimen plot of reduced travel time vs. distance using exact velocity

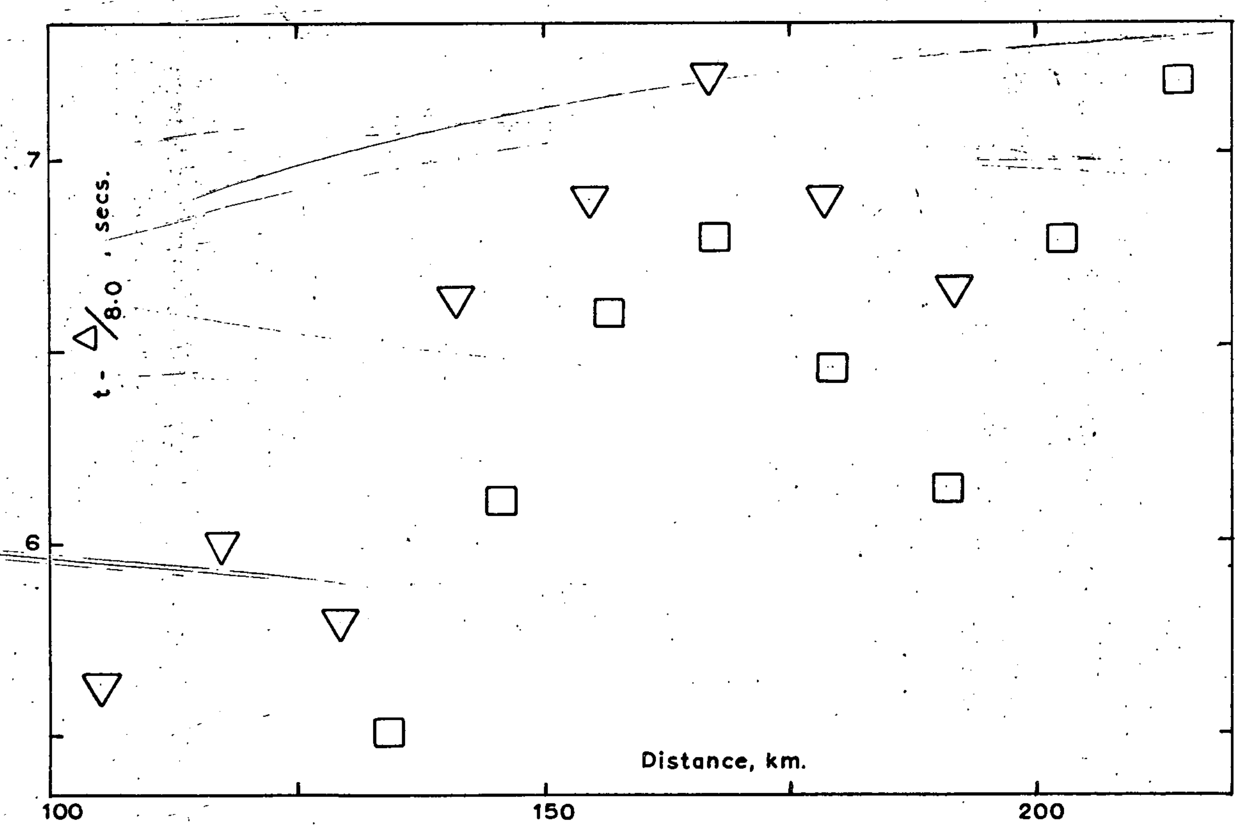


Figure (8) Specimen plot of reduced travel time vs. distance using velocity 10% high the picture, as shown in Figure (8). (Drawn for a 10 % error in velocity as compared with figure (7)). An approximate velocity chosen on a basis of keeping the reduced travel-time within a reasonable range will serve quite well to display the data for closer examination.

Bearing in mind that each reduced travel-time is an approximation to the sum of the time-terms of the shot and station involved, then for any one station the reduced travel-times from a number of shots will show differences which represent principally the differences in shot time-terms, since the same station time-term enters into each (and correspondingly for any shot observed at a number of stations).

At this stage there is an indication only of the sum of a shot time-term and a station time-term, with no way of telling how the time should be apportioned between them. This is the inherent indeterminacy which is discussed in detail in Section (2. 7).

Figure (9) shows a specimen set of data illustrating the way in which a subjective assessment of consistency may be obtained from the reduced travel-time display. The observations were obtained as first-arrivals from line 2 of the Irish Sea project, recorded at station BD and PC, forming approximately a reversed profile (Figure (10)).

The data have been reduced to an arbitrary velocity of 8.0 km/sec., and the general trend of the reduced points is a line

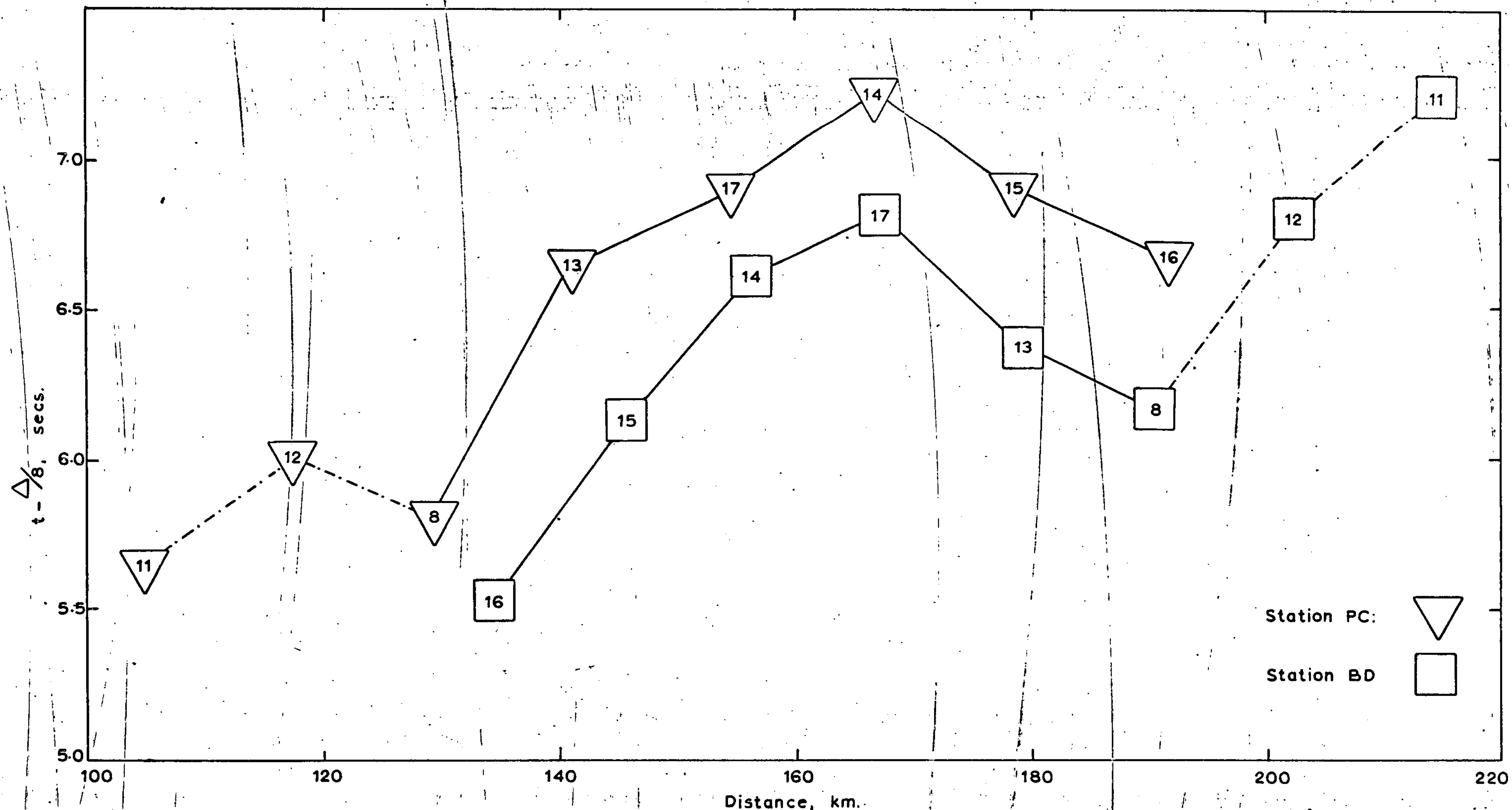


Figure (9) Specimen data for consistency testing of positive slope (approximately along the SW-NE diagonal of the diagram). This would suggest that the true refractor velocity is less than 8.0, and subsequent analysis of fuller data sets does in fact point to a velocity of 7.3 km/sec.

Allowing for this sloping trend, it is seen that the travel-times to station PC are consistently about $\frac{1}{2}$ sec. less than the travel-times to station BD, and this is borne out by a corresponding difference in the station time-term in the fuller solution.

If the shape of the profile through shots 8, 13, ~~17, 14~~, 15, and 16 is examined more closely (still keeping the sloping trend in mind), a distinctly curved feature can be recognised in the data from both stations. The pictures from the two stations do not match exactly, and this is the discrepancy which is represented by the residuals of the time-term solution, but for the six shots mentioned the agreement is quite close.

On turning attention to shot 11, however, a much clearer discrepancy emerges. If a line through points 8 and 16 is taken as reference for each station it is found that for station PC the observation of shot 11 is reasonably close to the line, whereas for station BD it is substantially later. Shot 12 also shows a similar tendency, but much less distinctly.

This would imply that for shot 11 (and possibly for shot 12), the two stations do not observe waves from the same refractor as first arrivals. It is quite likely (in the light of all the other available data) that the observations at distances less than 125 km represent first arrivals from a shallower refractor.

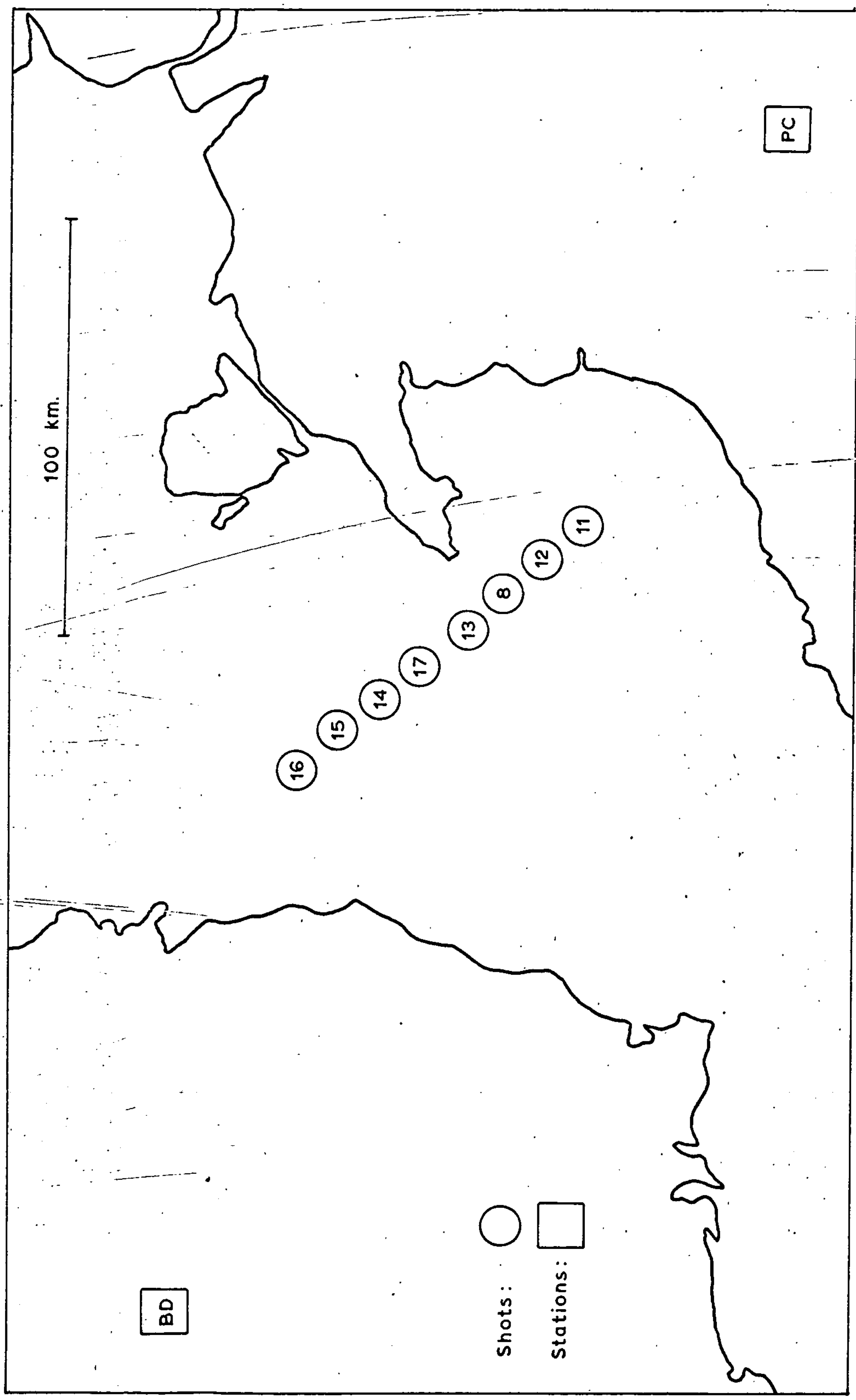


Figure (10) Arrangement of shots and stations for data of Figure (9)

2.9 Inspection of Data

In the case of least-squares fitting of a straight line, the residuals give a direct indication of the discrepancy for each observation, and an error in one observation would show up fairly clearly. However, in the case of the time-term solution, fitting a larger number of unknowns, an error in one observation tends to reflect in the residuals at other connected sites in the network, and the residuals do not give such a clear indication as for the straight-line fit.

It is therefore not ideal to commence the time-term solution by including all possible observations and to attempt subsequently to weed out the anomalous observations in the light of the residuals. This is especially true with weakly-connected networks, where erroneous observations could distort the pattern of residuals severely.

It is more satisfactory to commence with a smaller central set of fairly consistent observations, and to add the remaining data in small batches(checking for any sudden deterioration in the quality of the solution at each stage). In selecting such a central set, the aim should be to concentrate on those survey points which are most strongly interconnected. Before performing the time-term solution, it is desirable to have some form of preliminary inspection to permit the recognition of possibly anomalous observations, and especially to check against the possibility that some observations may have been assigned to the wrong refractor.

In the ideal case of a structure with plane horizontal layering such an inspection could readily be performed by plotting the travel-times or reduced travel-times against distance, when the points should lie on a straight line for each refractor (or a curved line if a velocity gradient were involved).

In the practical case of non-plane structures, the picture is by no means so simple, as the spread in the travel-times will often be influenced as much by the differences in time-terms as by the refractor involved when near a crossover point.

Willmore (1963) introduced the concept of "marker wave lead" as an aid in identification near the crossover point, but in the form given it would only be reliable for large strongly-connected data sets, since it involved using the suspect data in a first solution to provide the figures on which they were to be judged. Translated into physical terms, the use of the "marker wave lead" criterion suggests that the wave following the deeper path is not recognised as a first arrival unless its predicted arrival time is earlier than that for the shallower path by a finite "marker-wave lead". (The value may depend on various factors, e.g. the dominant frequencies of the two arrivals, the frequency response of the recording equipment, and the signal-to-noise ratio; in the example under discussion the value inferred was 0.13 seconds.)

From an examination of the plot of reduced travel-times vs. distance, it should be possible to recognise the lines corresponding to the important refractors, and to define the crossover distances.

For the preliminary interpretation the only observation used should be those first-arrivals which are well away from the vicinity of a crossover point. This yields estimates of time-terms, from which the expected arrival time can be calculated for each refractor at each of the suspect connections. A comparison of the expected arrival times, with the aid of the "marker-wave lead" criterion, should then clear up the doubtful identifications.

The procedure outlined optimistically in the preceding paragraph takes for granted an abundance of data. In particular, it implies that each survey point involved in the doubtful ranges of observation (i. e. near crossover distances) is also covered by at least two other connections for each refractor, giving unambiguous first arrivals. These conditions are difficult to achieve in practice with a limited number of stations, especially if the structure turns out to be more complex than first indicated (as in the case of the Irish Sea Project).

In the discussion of graphical representation, Section (2. 8), a method has been suggested for making a somewhat subjective assessment of the trend of time-terms from those observations at reliable ranges and using this to check the more doubtful observations.

If sufficiently large and well-connected sets of data are available to warrant it, the method can be applied in a more formal way by the computer, or at least it can form the basis for a series of computer-aided judgements.

Such an approach might proceed along the following lines: Starting from a reliable distance range, successive time-term solutions are performed, gradually embracing a wider spread of distance. At each step the standard deviation of one time observation is compared with that from the previous step. A sudden worsening of this figure would signify the inclusion of unsatisfactory observations, and a sudden change in velocity should also be regarded with suspicion.

The newly-added observations may reveal their inconsistency by having larger residuals, but this is not necessarily so; if they have upset the velocity estimate, the largest residuals may then instead attach to some of the original satisfactory observations.

As an illustration, the specimen data which have been examined by the graphical technique in Section (2. 8) have also been computer-processed as short sets selected in various ways (data sets 2066 to 2072 inclusive), and the solutions are listed in Appendix B.

Data set 2069 starts with 8 observations, from which sets 2070, 2071, and 2072 are built up to 12, 14, and 16 observations respectively. The calculated velocity decreases slightly at each stage, but without an abrupt step. However, the standard deviation of one time observation increases by 50 % in set 2072 with the inclusion of shot 11, which would appear to be unsatisfactory.

This is the effect one would expect to see if the shortest-range observation of shot 11 (1048) km) does not relate to the same refractor as the remaining observations. Note that the doubtful pair of observations do not produce the largest residuals, and also that they do make a large contribution to the velocity determination (the column headed 'DIJX2', see Section (4.4) for discussion).

Since it is rather difficult to compare the statistical features of solutions involving different small numbers of observations, it is instructive to take three sets each using the same number of observations (12) from 6 adjacent shots (data sets 2066, 2067, and 2068).

The velocity for set 2066 is lower than for the other two, but this can hardly be regarded as significant in the light of the standard deviation of velocity. However, a comparison of the three values for standard deviation of one time observation again points to set 2066 as less satisfactory than the others.

3. SOLUTION OF THE OBSERVATIONAL EQUATIONS

3.1 Introduction

The foregoing sections have shown how the relationship between observed travel-times t_{ij} and distances Δ_{ij} can be written as a system of simultaneous equations of the form:

$$t_{ij} = a_i + b_j + \Delta_{ij}/v$$

(where a_i and b_j represent time-terms for shots and stations respectively, and v represents the refractor velocity).

The next step is a purely arithmetical process: to assign values to the individual time-terms and to the velocity, in such a way as to give the "best possible fit".

The criterion normally adopted is that the sum of the squares of the residuals should be minimised. The residual δ_{ij} is defined as the difference between the observed and calculated travel-times:

$$\delta_{ij} = t_{ij} - \Delta_{ij}/v - a_i - b_j$$

Matrix algebra provides a convenient notation for defining the necessary manipulations of arrays of numbers in performing the solution.

Scheidegger and Willmore (1957) have suggested a system for the solution of the observational equation, based on the "Cracovian" method of Banachiewicz. This may be advantageous when the calculations are to be carried out on a hand machine.

However, with the availability of digital computers it is now perhaps more important to adhere to the widely-accepted notation of orthodox matrix algebra, for which most computer installations have libraries of sub-routines.

The observational equations:

$$t_{ij} = a_i + b_j + \Delta_{ij}/_v$$

can be represented in matrix notation as:

$$[t] = [p].[x] + [\Delta].{}^1/v \qquad (11)$$

$[t]$ and $[\Delta]$ are column matrices of dimensions 1 x k, containing the observational data, where k is the number of observations.

$[x]$ is a column matrix of dimensions 1 x (m + n), containing the time-terms, one for each survey point, where m and n are the numbers of shots and stations respectively.

$[p]$ is a rectangular matrix of dimensions (m + n) x k, containing the coefficients of the time-terms in the observational equations. Each row of the matrix corresponds to one observational equation, with the coefficient unity for the particular shot and station involved and zero for the remaining shots and stations.

3.2 Inclusive Treatment for Velocity

The most direct approach is to treat $^1/_v$ as an ordinary unknown like each of the (m + n) time-terms.

In this case the column matrix $[\triangle]$ is made the $(m + n + 1)^{th}$ column of the rectangular matrix $[p]$.

$^1/_v$ is made the $(m + n + 1)^{th}$ entry in the column matrix $[x]$, which takes dimensions 1 x (m + n + 1).

There are then a total of k equations to be solved for a total of (m + n + 1) unknowns; the equations are represented as:

$$[p].[x] = [t] \qquad (12)$$

The operation of assigning the arbitrary value of zero to the last time-term (as discussed in Section (2.7) on the inherent indeterminacy) may be performed by striking out the appropriate column of the matrix $[p]$, thereby reducing the dimension of the matrix to be handled. However, to do so would involve the sacrifice of some information which later is relevant to the uncertainties of the solution, and a more tidy approach is to add one more "observational equation" stating simply:

$$b_n = o$$

The condition of minimising the sum of squares of residuals is defined by a set of (m + n + 1) normal equations, obtained by multiplying across by the transpose of the matrix $[p]$, which will be denoted by $[p^t]$:

$$[p^t].[p].[x] = [p^t].[t] \qquad (13)$$

The solution to this set of equations is given by multiplying across by the inverse of $[p^t].[p]$.

or, writing $[p^t].[p] = [q]$ (14)

and defining the inverse matrix $[q^{-1}]$ such that:

$$[q^{-1}].[q] = I \quad \text{(unit matrix)} \tag{15}$$

then:

$$[x] = [q^{-1}].[p^t].[t] \tag{16}$$

This represents a listing of the solutions for the m + n time-terms and the velocity, as direct numerical values:

$a_1 =$

$a_2 =$

⋮

$a_m =$

$b_1 =$

$b_2 =$

⋮

$b_m =$

$\frac{1}{v} =$

3.3 Separate Treatment for Velocity

There are, however, various advantages to be gained from a somewhat different approach to the solution, reserving the velocity for separate treatment. In particular, it gives an insight into the way in which the velocity determination is influenced by the distribution of shots and stations.

The observational equations are rearranged as:

$$a_i + b_j = t_{ij} - \Delta_{ij}/v \qquad (17)$$

or in matrix notation

$$[p].[x] = [t] - [\Delta].\frac{1}{v} \qquad (18)$$

where, as at the start of the preceding section:

$[p]$ is a rectangular matrix of dimensions $(m + n) \times k$

$[x]$ is a column matrix of dimension $1 \times (m + n)$

$[t]$ and $[\Delta]$ are column matrices of dimension $1 \times k$

In the same way as before, the least-squares solution is given by:

$$[x] = [q^{-1}].[p^t].[t] - [q^{-1}].[p^t].[\Delta].\frac{1}{v} \qquad (19)$$

This represents a listing of the solutions for the m + n time-terms, and in each case the right-hand side contains two terms, one having the coefficient unity and one having the coefficient $\frac{1}{v}$.

When the time-terms in this form are substituted in the observational equations, residuals are obtained:

$$\delta_{ij} = t_{ij} - \Delta_{ij}/_{v} - a_i - b_j \qquad (20)$$

These may each be regrouped as two terms, having coefficients unity and $\frac{1}{v}$:

$$\delta_{ij} = c_{ij} - d_{ij}\left(\frac{1}{v}\right) \qquad (21)$$

The condition of minimising the sum of squares of these residuals is given by:

$$\frac{1}{v} = \frac{\sum c_{ij} d_{ij}}{\sum d_{ij}^{2}} \qquad (22)$$

The value of $\frac{1}{v}$ so obtained may now be substituted in the expressions derived above for the time-terms, to give numerical values, and likewise for the residuals.

If there is some doubt as to whether or not the value of velocity derived from the least-squares procedure is the most suitable on other grounds, alternative values may be substituted in order to observe the effect on the values of time-terms and residuals.

In some data where the overall spread of the survey network is unsuitable for determination of velocity, it may be more satisfactory to use a value of velocity obtained from an independent source.

A further advantage of the separate treatment for velocity is that the solution may readily be extended to cover cases where the velocity is not constant but is a function of distance.

The case of a linear increase in velocity with distance has been discussed in detail by Smith et al (1966), in an analysis of the data from the Lake Superior project. However, the results were inconclusive, in that the computed value for the rate of increase of velocity was of the same order as the uncertainty.

3.4 Weighting of Data

The technique which has been outlined so far has assumed that all data are to be given equal weight. In practice it may be useful to weight the data according to some criterion of reliability (e. g. timing accuracy or legibility of onset). A similar requirement arises when certain connections have been observed more than once.

The procedure is quite straightforward: each observational equation is multiplied across by its assigned weight, or in the case of multiple observations the appropriate observational equations are added together.

The effect is that the relevant elements of the $[p]$ matrix are changed from unity to the assigned weight or the number of multiple observations, with corresponding changes in the $[t]$ and $[\Delta]$ matrices.

3.5 Singly-connected survey points

Any survey point which is tied into the main network by only a single connection (whether repeated or not) makes no contribution to the least-squares fitting, since the residual for that connection can always be made zero by assigning a suitable value to the time-term.

When a set of data has been selected for analysis, the procedure is to set aside any singly-connected observations until the least-squares fitting has defined the velocity and the time-terms for the main set. The singly-connected observations are then used to calculate the time-terms for the corresponding survey points.

4. UNCERTAINTIES AND FIGURES OF MERIT

4.1 Introduction

Conventional refraction methods have developed around various basic configurations of shot and station layout which have inherently strong control over the velocity determination.

One of the great advantages of time-term analysis is that full use can be made of a distributed network of survey points, without insistence on formal patterns such as lines or arcs.

This is not to say that formal patterns should be abandoned; in fact the results may be more conveniently presented if the survey points are laid out as a series of lines rather than distributed randomly. Furthermore it becomes possible to build up a series of local velocity estimates from such lines. (See Section (2.6)).

It becomes of great importance when using data from a distributed network to have a means of assessing the contribution of each connection and the effectiveness of the overall spread.

Established techniques of statistical analysis enable estimates to be made of the uncertainties in the values of parameters which have been fitted by least-squares.

It cannot be too strongly emphasized that such estimates, based on the spread of the residuals, represent only the minimum uncertainties, on the assumption that the errors are genuinely random. The presence of systematic errors may lead to erroneous parameter values without producing an appropriate indication in the uncertainty estimates.

An example of this is discussed in Section (6.4.2), where a preliminary analysis neglecting the effects of dip yields an apparently satisfactory velocity determination. However, the structure involves a basin-shaped feature, which tends to make the calculated value of velocity rather low; a second approximation with corrections for dip produces a new value of velocity differing from the first by an amount much greater than the estimated uncertainty.

The standard error of v is obtained from the standard error of $\frac{1}{v}$ by the relation:

$$\sigma(v) = (v^2).\ \sigma\left(\frac{1}{v}\right) \qquad (23)$$

4.2 Confidence Limits

For the simple case of a random variable normally distributed with variance σ^2, it can be shown that there is a 95 % probability that any observation taken at random from such a population will not differ from the mean by more than $\pm 1.96\sigma$. In other words, 95 times out of 100 the difference between the mean and such a randomly chosen observation will be no more than approximately twice the standard error of the mean.

In this way it is possible to arrive at estimates of "Confidence Limits" for the value of each of the parameters fitted by least-squares technique. If the true value of the variance σ^2 were known in each case, then "95 % confidence limits" could be stated as $\pm 1.96\sigma$.

In practice the variance of the errors (the differences between the true and observed values of the observed quantities) is not known directly, but a reasonable estimate is obtained on the basis of the variance of the residuals (the differences between the calculated and observed values of the observed quantities).

Defining the "number of degrees of freedom" as the number of observational equations minus the number of unknowns, then when the number of degrees of freedom is very large the variance of errors of the data population as a whole can be reliably estimated on the basis of the residuals of the data sample under investigation.

When the number of degrees of freedom is small, however, the limited data gives a less reliable estimate of the variance of the errors. Allowance is made for this in deriving the confidence limits by replacing the factor 1.96 by a factor 't' (known as "Student's t") which depends on the number of degrees of freedom and is greater than 1.96.

Tables of Student's t for various degrees of freedom and levels of probability are to be found in most textbooks on statistics. As an indication of the trend, some representative values are listed below for the 95 % confidence level:

deg. of f.	40	20	10	6	4	3	2	1
t	1.98	2.09	2.23	2.45	2.78	3.18	4.30	12.70

In general where the number of degrees of freedom exceeds 10, there is little benefit in referring to a table of Student's t, and a round figure of 2 is sufficiently accurate. Most of the data sets in the present investigation fall in this category when the variance of the data is assumed to be uniform at all survey points, but small numbers of degrees of freedom do arise in the course of treating the data from each survey point as having come from a different statistical population.

4.3 Statistical expressions for straight-line solution

Suitable forms of the expressions for the fitting by least-squares of a conventional straight-line solution to a set of travel-time data have been detailed by Steinhart and Meyer (1961).

For a set of N observations (t_i, x_i) to be approximated by the line:

$$T = t_o + \lambda . X$$

the least-squares estimates of the parameters t_o and λ are given by

$$t_o = \frac{\Sigma x_i \; \Sigma x_i t_i - \Sigma t_i \; \Sigma x_i^2}{(\Sigma x_i)^2 - N \Sigma x_i^2} \qquad (24)$$

$$\lambda = \frac{\Sigma x_i t_i - \frac{\Sigma x_i t_i}{N}}{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}} \qquad (25)$$

Estimates of the standard error of t_o and λ are obtained from the variances $(S_t)^2$ and $(S_\lambda)^2$.

$$(S_t)^2 = \frac{\Sigma (T - t_i)^2}{N-2} \qquad (26)$$

$$(S_\lambda)^2 = \frac{(S_t)^2}{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}} \qquad (27)$$

The numerator in $(S_t)^2$ is the sum of squares of residuals, and the denominator is the number of degrees of freedom, i.e. (number of observations) minus (number of unknowns).

The denominator in $(S_\lambda)^2$ is a quantity related to the effectiveness of the shot/station pattern in controlling the determination of velocity. This can be more readily appreciated by re-arranging it as follows:

$$\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N} = \Sigma (x_i - \bar{x})^2$$

where $\bar{x}$ = mean x_i = $\frac{\Sigma x_i}{N}$

Note that in these expressions for the straight-line solution, S_λ relates to the standard error of the apparent velocity, and account is not taken of the basic indeterminacy of an unreversed profile. An extension of the analysis to the case of dipping layers observed by reversed profiles is given by Steinhart and Meyer (1961).

4.4 Statistical expressions for time-term solution

It is instructive to compare the expressions quoted above for the straight-line solution with the corresponding ones for the time-term method (Scheidegger and Willmore, 1957; Willmore and Bancroft, 1961).

$$\sigma^2(t) = \frac{\Sigma \, \delta_{ij}^{\,2}}{mn - m - n} \qquad (28)$$

$$\sigma^2\left(\frac{1}{v}\right) = \frac{\sigma^2(t)}{\Sigma \, d_{ij}^{\,2}} \qquad (29)$$

The numerator of $\sigma^2(t)$ is again the sum of squares of residuals, and the denominator has the general form of (number of degrees of freedom). The first term, mn, is appropriate only in the case when all stations observe all shots; in practice there are usually some connections missing, and the number of observations is denoted by k (less than mn). The expression for σ^2 (t) then becomes:

$$\sigma^2(t) = \frac{\Sigma \, \delta_{ij}^{\,2}}{k - m - n} \qquad (30)$$

4.5 Uncertainty of velocity

In Equation (29), the denominator of the right-hand side has the same significance as the corresponding term in the straight-line solution (Equation (27)), namely, the effectiveness of the survey network in controlling the determination of velocity.

For the case when all m shots are observed at all n stations, each term d_{ij} in the summation has the form:

$$d_{ij} = \Delta_{ij} - \frac{1}{m}\sum_j \Delta_{ij} - \frac{1}{n}\sum_i \Delta_{ij} + \frac{1}{mn}\sum_i \sum_j \Delta_{ij} \qquad (31)$$

or d_{ij} = (distance from shot i to station j)

- (mean distance for all stations which observed shot i)

- (mean distance for all shots observed by station j)

+ (mean distance for all observations)

For the case when some connections are missing the expression for d_{ij} is less simple and involves the inverse matrix, but it retains the same basic structure: a combination of the shot-station distance Δ_{ij} with <u>weighted</u> means for the three remaining terms.

If the stations are all close together, the first two terms will almost cancel each other, and so will the last two. Similarly if the shots are all close together, the first and third terms will almost cancel and so will the second and fourth. This shows that for proper control of velocity it is essential to have an adequate spread of both shots and stations.

In the extreme case of a single shot observed at a number of stations (or a single station observing a number of shots) it becomes impossible to distinguish between the effects of velocity and regional trend of time-terms (in other words, dip of the refractor). This indeterminacy is duly recognized by the fact that $\sigma^2 (\frac{1}{v})$ becomes infinite as the "spread effectiveness "term Σd_{ij}^2 becomes zero.

The velocity derived from such data is the apparent velocity, and differs from the true velocity in the refractor if the magnitude of the time-terms is correlated with distance from the source. Even when the stations are well distributed in azimuth and distance with respect to the source, there is the possibility that a dome or basin in the marker layer could give rise to a correlation between the time-terms and distance. In such cases the trend of time-terms is absorbed into the apparent velocity, and the residuals show only the experimental errors and the departures of individual time-terms from the mean trend. This mean trend corresponds to approximating the general form of the refractor by a cone rather than a plane.

This extreme case of a single shot observed at a number of stations does arise from time to time (e. g. in the disposal of a dump of surplus explosive). It is important that the inherent weakness of such a system be recognised, together with a means of alleviating the weakness: several of the stations should observe an additional shot at some distance from the first. This treatment can be applied even to cases where the main shot has already been fired and the recording stations closed down; provided several of the sites can be re-occupied at a later date for one or more additional shots, the effectiveness of the spread can be built up. For similar reasons it is important that when surveys are being extended to neighbouring areas, some previously-used sites should be re-occupied.

4.6 Comparison of various network configurations

If the number of observations of given quality is very large, the variance $\sigma^2(t)$ tends to a constant value. Then from the expression for the variance of $\frac{1}{v}$:

$$\sigma^2\left(\frac{1}{v}\right) = \frac{\sigma^2(t)}{\Sigma\, d_{ij}^2} \qquad (32)$$

it is seen that the effectiveness of each observation in reducing the uncertainty in $\frac{1}{v}$ can be assessed by its contribution to $\Sigma\, d_{ij}^2$. This provides the type of figure of merit which is needed for measuring the effectiveness of the spatial distribution of the survey network.

The technique is quite rigorous, and shows clearly the weakness of various unsuitable configurations.

The quantity Σd_{ij}^2 may be used for comparison of the effectiveness of different data sets, or the quantity d_{ij}^2 may be used for comparison of the contributions of individual observations within a data set. If one observation is discarded from a data set, the contributions of the remaining observations are liable to be considerably re-arranged.

Some rather surprising results emerge from the application of the criterion, which is best illustrated by a few examples. In each case the same comments would apply equally to a transposition of the network, with each station replaced by a shot, and each shot by a station.

Figure (11a)
Example 1

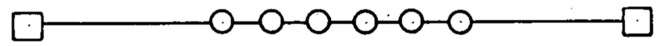


Figure (11b)
Example 2

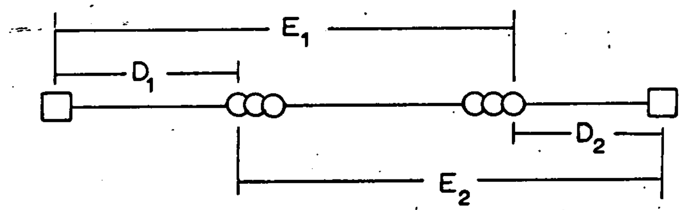


Figure (11c)
Example 3

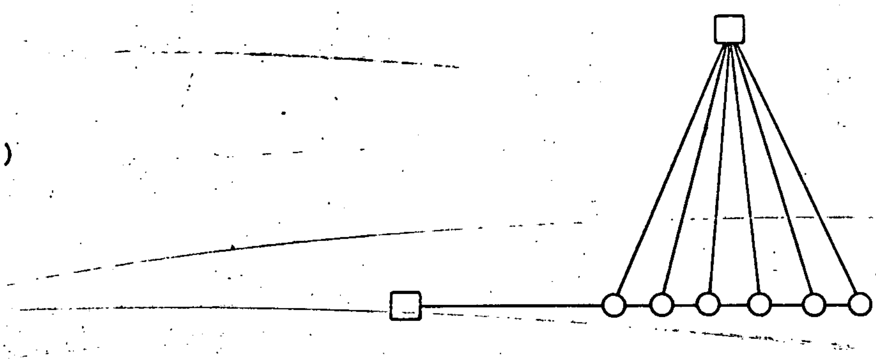


Figure (11d)
Example 4

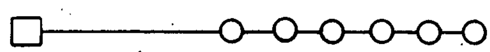


Figure (11e)
Example 5

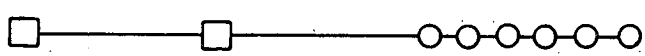


Figure (11f)
Example 6

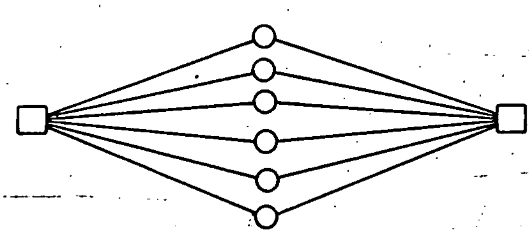


Figure (11g)
Example 7

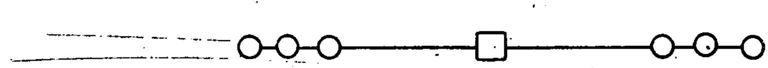


Figure (11) Specimen network configurations

It should be noted that even if the standard deviation of one time observation, $\sigma(t)$, were satisfactorily small, the uncertainty of each time-term is a function also of the standard error of velocity, $\sigma(\frac{1}{v})$, and in consequence the time-terms are not adequately determined if $\sigma(\frac{1}{v})$ is large.

Example 1: Figure (11a). A line of shots, with a station at each end on the extension of the line. This is the traditional "reversed-profile" configuration, and gives ideal control of velocity. The d_{ij} terms have substantial values, leading to a low value for $\sigma(\frac{1}{v})$. Connections to the shots nearest the centre of the line make the smallest contribution to Σd_{ij}^2 and consequently to the velocity determination, while those nearest the ends make the greatest contribution.

Example 2: Figure (11b). Because of the smallness of the contribution from shots near the centre in the previous example, it is tempting to suggest that the most effective way to deploy a given number of shots is to concentrate them at the ends of the shot line. Dimensions D_1 and D_2 would in this case be the shortest distance which will give first arrivals from the refractor of interest, and dimensions E_1 and E_2 the longest distance (limited by either signal strength or the appearance of an earlier arrival from another refractor). However, the distributed line, Figure (11a), represents a much more sound arrangement, since in general the maximum and minimum distances are not reliably known in advance; furthermore the distributed line provides a more satisfactory check for the existence of suitable refractors.

Example 3: Figure (11c). One station broadside to a line of shots, and one station at the end of the line. The d_{ij} terms, while smaller than for the previous two examples, are acceptably large, and adequate control of velocity determination is obtained. The effectiveness of the pattern, as indicated by the value of Σd_{ij}^2, is approximately one-quarter of that for example 1.

Example 4: Figure (11d). A single station at one end of a line of shots. Each d_{ij} term becomes zero and $\sigma(\frac{1}{v})$ becomes infinite. No control over velocity. An interpretation is possible only if the layering is assumed to be both plane and horizontal.

Example 5: Figure (11e). Two stations at the same end of a line of shots (with all sites lying along a straight line.) Each d_{ij} term becomes zero. (The four terms in the d_{ij} expression no longer cancel in pairs as they would for a single-station arrangement like the previous example, but the sum of the terms is zero). Again $\sigma(\frac{1}{v})$ becomes infinite.

It is interesting to observe that this configuration could be utilised only if the layering could be assumed plane (though not necessarily horizontal); otherwise it is no more effective than Example 4, regardless of the station separation.

Example 6: Figure (11f). A pair of stations broadside to a line of shots. Once more each d_{ij} term becomes zero, and $\sigma(\frac{1}{v})$ infinite. No control over velocity.

Example 7: Figure (11g). A single station in the centre of a line of shots. No control over velocity, but the data could be salvaged by an assumption of plane-layering as with Example 5.

It is difficult to define in one brief phrase the characteristic which should be emphasized for best control over velocity. It can be seen that the minimum network involves two shots and two stations. What is needed is that the range interval between the two shots as observed from the first station should be as different as possible from the range interval as observed from the second station.

For example, if shot A is further than shot B from station C, then station D should be situated so that shot B is further from it than shot A. Alternatively in terms of azimuths: the azimuth of the first station with respect to the line AB should be nearly 0° and that of the second station nearly 180°.

For a pair of shots and a pair of stations it is easy to visualize this approach leading to the reversed profile configuration, but for an extended network it is more difficult, and then the Σd_{ij}^2 criterion comes into its own.

4.7 Uncertainties of time-terms

4.7.1 Uniform data quality

On the assumption that the distances are accurately known and that the errors are therefore concentrated in the time observations, the expression for $\sigma^2(t)$, Equation (30), leads to an estimate of "standard deviation of one time observation".

The derivation of estimates of standard error for each time-term involves the application of appropriate weights for each survey point. This weighting function should take account not only of the number of direct connections for each survey point but also of the number of indirect connections between that point and the remainder of the network.

A convenient feature of matrix algebra is that the required weighting function becomes available in the course of the calculation, as the elements of the leading diagonal of the inverse matrix $[q^{-1}]$, assuming that the data from all survey points are of uniform quality (i. e. uniform variance).

Estimates of standard error for the individual time-terms are obtained by multiplying the general standard deviation $\sigma(t)$ by the square-root of the appropriate term $[q_{ii}^{-1}]$.

The relative values of standard errors derived in this way depend only on the corresponding $[q_{ii}^{-1}]$, reflecting the amount of data contributed by that survey point and the manner in which it is linked with the remainder of the data. The <u>overall</u> fit of the solution is reflected by $\sigma(t)$, which enters into each standard error, and there is no direct indication of the relative quality of data for different survey points.

4.7.2 Non-uniform data quality

Since in practice the data from some survey points will be of much higher quality than those from others, there may be some justification for an alternative statistical approach which treats the data from each survey point as coming from a different statistical population (Berry and West, 1966).

This takes no account of the manner in which the data are interconnected, but it offers advantages in dealing with survey points of widely differing quality (e. g. stations with high local noise level or poor timing accuracy). At the very least it provides a basis for comparison of data quality between different survey points.

The standard deviation of data, σ_t, for a survey point t is given by

$$(\sigma_t)^2 = \frac{\sum_s R_{st}^2 \gamma_{st}}{\left(\sum_s \gamma_{st}\right) - 1} \qquad (33)$$

using the notation:

$\sum_s$ = summation over all survey points

R_{st} = residual for connection between survey points s and t

γ_{st} = 1 when connection between s and t is observed

= 0 " " " " " " " not observed

or simply: $(\bar{\sigma}_t)^2$ = (sum of squared residuals for all connections to survey point t) ÷ ((number of connections to survey point t) - 1).

For a survey point t connected by N observations of variance $(\bar{\sigma}_t)^2$, the standard error of time-term, σ_t, will be given by:

$$\sigma_t = \frac{\bar{\sigma}_t}{\sqrt{N}}$$

$$= \frac{\bar{\sigma}_t}{\sqrt{\sum_s \gamma_{st}}} \qquad (34)$$

As a means of comparing the quality of fit for various solutions, Berry and West used an expression corresponding to Equation (30):

$$\sigma^2 = \frac{\text{sum of squared residuals for all observations}}{\text{number of degrees of freedom}}$$

4.7.3 Influence of velocity uncertainty on time-terms

The normal procedure for least-squares fitting of a line of the form:

$$T = t_o + \lambda X$$

to a set of N observations, (t_i, x_i), leads to estimates of the standard error of slope, S_λ, and the standard error of intercept, S_t.

The time-term method, as a generalisation of the above, leads to corresponding estimates of the standard error of slope, $\sigma(\frac{1}{v})$, and the standard error of time-term, σ(t.t.).

In attempting to derive from this information an estimate of the uncertainty of each time-term, it is important to bear in mind the influence of the velocity uncertainty. If the velocity were known, the uncertainty of the time-term would be derived directly from the standard error of time-term, but on the other hand when the velocity is poorly determined the uncertainty of time-terms is dominated by the effect of the velocity uncertainty.

For the straight-line solution, the contribution of the velocity uncertainty to the uncertainty of intercept will depend on $\bar{x}$, (the mean distance) and S_λ, (the standard error of slope).

$$S'_t = \bar{x}.S_\lambda \qquad (35)$$

The combined uncertainty is given by:

$$(S_{to})^2 = (S_t)^2 + (S'_t)^2 \qquad (36)$$

For the time-term analysis, the relevant mean distance is the mean for all connections involving the survey point under consideration, i.e.:

$$\bar{\Delta} = \frac{\sum_i \Delta_{ij} \gamma_{ij}}{\sum_i \gamma_{ij}} \quad \text{for a station} \qquad (37)$$

or

$$\bar{\Delta} = \frac{\sum_j \Delta_{ij} \gamma_{ij}}{\sum_j \gamma_{ij}} \quad \text{for a shot} \qquad (38)$$

where γ_{ij} = 1 when connection between i and j is observed.

= 0 " " " " is not observed

The contribution of the velocity uncertainty to the time-term uncertainty is given by:

$$\sigma'(t) = \bar{\Delta}\cdot\sigma\left(\frac{1}{v}\right) \qquad (39)$$

and the combined uncertainty by:

$$(\sigma_o(t.t.))^2 = (\sigma(t.t.))^2 + (\sigma'(t))^2 \qquad (40)$$

4.7.4 Uncertainty in interchange

On account of the inherent indeterminacy, discussed in Section (2.7), it is necessary initially to assign an arbitrary value to one time-term and subsequently to adjust the calculated time-terms by adding a constant term to each of the shot time-terms and subtracting it from each of the station time-terms, choosing the value which gives the most satisfactory continuity between shot and station time-terms.

Ideally the network should contain a number of survey points which serve as shot points for some connections and as stations for others. Each such point provides an estimate of the adjustment term required, and by combining a number of these separate estimates a more accurate overall figure is obtained, together with an indication of the uncertainty of the adjustment term.

Where only one interchange point is available, the uncertainty of the adjustment term is not known. In practice an even less satisfactory situation often arises where true interchange does not exist, and the adjustment term has to be decided by subjective assessment of time-terms for several adjacent (rather than coincident) survey points.

The uncertainty of the interchange process is usually dominated by the uncertainties of the time-terms for the interchange stations. An assessment of the build-up of uncertainties may often prove to be a very sobering exercise.

In general the time-terms after adjustment should not be significantly negative, but if this appears to be a limiting condition the reliability of the interchange observations requires careful scrutiny. A negative time-term could be the result of an error in shot time (or station time), or in the case of a survey point for which the connections are poorly distributed in azimuth it could be the result of a positional error or a local velocity anomaly.

4.8 Uncertainty of depth

In so far as the principal end-product of a crustal investigation is a model in terms of depths, this is the aspect of the solution which stands in greatest need of realistic estimates of uncertainty.

Although suitable analytical techniques have been available in the literature for a number of years (e.g. Steinhart and Meyer, 1961), the practice of publishing results with no indication of the uncertainties is still all too common. In some cases the reason for the oversight is simple enough: even a conservative estimate of uncertainties may be sufficient to show that the "results" are not statistically significant.

The depth calculation involves a combination of derived quantities (each with its own uncertainty) so that in the final answer the build-up of uncertainties may be intolerable, even when the separate steps of the analysis yielded acceptably small uncertainties.

In general the uncertainties in refractor velocities are relatively small (less than 5%), and the principal sourcesof uncertainty in depth are the uncertainties in time-terms. In the estimation of depth uncertainties it is usually therefore permissible to neglect the velocity uncertainty in comparison with the time-term uncertainty.

As a basis for discussion, the expressions relating depth to time-term and velocities are summarized below with a suitable notation.

Consider a model with three refractors, i. e. four layers having velocities v_1, v_2, v_3, and v_4. Let the layer thicknesses be Z_1, Z_2, and Z_3 respectively, and the time-terms be T_1, T_2, and T_3 respectively.

Using a double-suffix notation:

t_{rs} = time lost in the r^{th} layer by the wave critically refracted in the s^{th} layer.

Θ_{rs} = angle of ray in the r^{th} layer for the wave critically refracted in the s^{th} layer.

The time-depth conversion involves a number of terms of the form $\frac{v}{\cos\Theta}$, which can be regarded as "slanted velocities". For convenience these are denoted as follows:-

$$\frac{v_1}{\cos\Theta_{12}} = A$$

$$\frac{v_1}{\cos\Theta_{13}} = B \qquad \frac{v_2}{\cos\Theta_{23}} = C$$

$$\frac{v_1}{\cos\Theta_{14}} = D \qquad \frac{v_2}{\cos\Theta_{24}} = E \qquad \frac{v_3}{\cos\Theta_{34}} = F$$

then:

$$t_{12} = T_1$$

$$Z_1 = A.T_1 \qquad (41)$$

$$t_{13} = \frac{A}{B} T_1$$

$$t_{23} = T_2 - t_{13} = T_2 - \frac{A}{B} T_1$$

$$Z_2 = C.t_{23} = C.T_2 - \frac{A.C}{B} T_1 \qquad (42)$$

$$t_{14} = \frac{A}{D} T_1$$

$$t_{24} = \frac{C}{E} t_{23} = \frac{C}{E} T_2 - \frac{AC}{BE} T_1$$

$$t_{34} = T_3 - t_{24} - t_{14}$$

$$= T_3 - \frac{C}{E} T_2 - \left(\frac{A}{D} - \frac{AC}{BE}\right) T_1$$

$$Z_3 = F.t_{34}$$

$$= F.T_3 - \frac{CF}{E} T_2 - \left(\frac{AF}{D} - \frac{ACF}{BE}\right) T_1 \qquad (43)$$

$$Z_1 + Z_2 = \left(A - \frac{AC}{B}\right) T_1 + C.T_2 \qquad (44)$$

$$Z_1 + Z_2 + Z_3 = \left(A - \frac{AC}{B} - \frac{AF}{D} + \frac{ACF}{BE}\right) T_1$$

$$+ \left(C - \frac{CF}{E}\right) T_2 + F.T_3 \qquad (45)$$

Now let the uncertainties of the time-terms be δ_1, δ_2, and δ_3 respectively. Note that it is important to distinguish the sign of each term in the uncertainty expressions, since the uncertainties are not always cumulative. For example, an error which tends to increase the estimate of the thickness of the first layer will tend to decrease the estimate of the thickness of the second layer (the proportion depending on the velocity ratios).

The uncertainties in the thicknesses of the three layers will then be:

$$\delta(z_1) = \pm(A.\delta_1) \tag{46}$$

$$\delta(z_2) = \pm\left(-\frac{AC}{B}\,\delta_1\right) \quad \pm(C.\delta_2) \tag{47}$$

$$\delta(z_3) = \pm\left(\left(-\frac{AF}{D} + \frac{ACF}{BE}\right)\delta_1\right) \quad \pm\left(-\frac{CF}{E}\,\delta_2\right) \quad \pm(F.\delta_3) \tag{48}$$

and the corresponding uncertainties in the depths to the three refractors will be:

$$\delta(z_1) = \pm(A.\delta_1) \tag{49}$$

$$\delta(z_1 + z_2) = \pm\left(\left(A - \frac{AC}{B}\right)\delta_1\right) \quad \pm(C.\delta_2) \tag{50}$$

$$\delta(z_1 + z_2 + z_3) = \pm\left(\left(A - \frac{AC}{B} - \frac{AF}{D} + \frac{ACF}{BE}\right)\delta_1\right) \quad \pm\left(\left(C - \frac{CF}{E}\right)\delta_2\right) \quad \pm(F.\delta_3) \tag{51}$$

5. PROGESSIVE REFINEMENTS OF SOLUTIONS

5.1 Introduction

The selection and rejection of observations is a matter which must be approached with great care, since there is a temptation to concentrate on those observations which support one's prejudices and to suppress those which offer contradictions.

There is no simple set of formal rules which can be guaranteed to produce ideal solutions, especially for small data sets. Instead the approach is of necessity highly subjective, involving a progression of computer-aided judgements.

To illustrate the subtle differences upon which judgements are based, Appendix B contains detailed listings of not only the various final solutions obtained but also a selection of the earlier solutions which led to them.

5.2 Structural Model and Phase Identification

The first question to be asked is whether or not sufficient evidence is available to define the structural model and to permit reliable identification of phases. If some ambiguity exists, then it may be advisable to try alternative identifications and to compare the quality of fit for the different solutions.

If first-arrivals from two different refractors have been wrongly grouped into a single set, then it might be expected that this should show up as a trend of residuals with distance (with the observed trav el-times being later than predicted in the middle of the range and earlier at the extremes of the range). However, if the spread of survey points is unfavourable, it may well happen that such trends are absorbed as a bias in the values of time-terms for a few stations which have contributed the observations at the extremes of the distance range.

A similar trend would arise if the structure involves a velocity gradient, so that for distant observations the waves follow a deeper path at a greater mean velocity than for the short-range observations.

When there is a suspicion of such a trend, it may be possible to subdivide the data into two sets, for longer and shorter distances, to test whether significantly different estimates of velocity are produced.

A possible source of systematic error is that the distant observations are liable to be read late, because of the progressive diminution of the energy of the refracted wave with distance. The trend of residuals with distance due to this effect will be the opposite of that due to velocity gradient or mixed refractors.

As an example, it is instructive to compare the solutions for data sets 2039 and 2050. The former represents all the available observations attributable to Pg for the Irish Sea project, and it is far from satisfactory. The typical uncertainty of a time-term ($\pm$ 1.0 sec.) is slightly greater than the typical value of a time-term (say 0.8 - 0.9 sec. after adjustment to match shots 91 and 92 with station 3).

An examination of the distribution of residuals by distance and size reveals that the three largest values occur at the longer distances, and the application of an arbitrary distance limit of 200 km would eliminate them.

The remaining observations having been re-processed as data set 2050, a more satisfactory solution is obtained. The residuals and uncertainties are much reduced, and the distribution of residuals with distance is rather more uniform.

The control of the velocity distribution has unfortunately been weakened (since the long-range connections are those which would make the greatest contribution), and yet in spite of this the uncertainty of velocity is reduced (because of the greater reduction of the residuals).

A check on the quality of the original seismograms for the connections over 200 km confirms that the signal-to-noise ratio is indeed poor, and it is therefore quite possible that the arrivals have been read late. If this were the case, then the truncated data set would be expected to yield a higher estimate of velocity, but since instead it yields a lower estimate there are some grounds for postulating a velocity gradient.

This idea is supported by the fact that even the truncated data set 2050 still shows some signs of a similar trend of residuals with distance: the observations beyond 168 km give predominantly negative residuals.

Unfortunately these observations are so scanty and of such poor quality that an attempt to perform a more sophisticated solution would hardly be justified.

As an example of an unsatisfactory distribution attributable to the combination of data from two different refractors, data set 2075 is composed of first-arrival observations of the Irish Sea and Seagull shots at distances beyond 125 to 130 km. The lower limit of distance had been chosen, after careful examination of all first-arrival data, as representing the cross-over distance between arrivals from an upper refractor (with a velocity around 6.1 km/sec.) and those from some deeper refractor (for which the velocity should be determined by the large data set).

The velocity calculated, 7.73 km/sec., is suspiciously low to be Pn, but the velocity uncertainty is reassuring at first sight.

However, the distribution of residuals shows undeniable trends with distance, the larger negative residuals being concentrated at the extremities of the distance range while the positive residuals occur mainly at intermediate distances.

In the light of this, it is clearly unrealistic to attribute all the observations to a single refractor of reasonably uniform velocity extending over the entire region. Once such a fundamental assumption of the time-term approach has been violated, the solution cannot be trusted in any respect: the uncertainty derived for the velocity is quite spurious, and the incorrect estimate of velocity leads to a distorted set of values for time-terms.

This trend of residuals could be due to a velocity gradient, but a more likely explanation is that the observations relate to two different refractors. Data sets 2040 and 2048 comprise basically the same observations, subdivided geographically by region into southern and northern groups respectively, and these give two quite different velocities, 7.27 and 8.09 km/sec., with much smaller residuals and a more uniform distribution with distance.

The distribution of residuals with distance for data set 2048 is not, however, a very reliable basis on which to judge the suitability of the crustal model, since only two stations are involved and the areal spread of survey points is far from ideal; a trend with distance could easily be absorbed as a bias in the time-terms.

5.3 Velocity determination

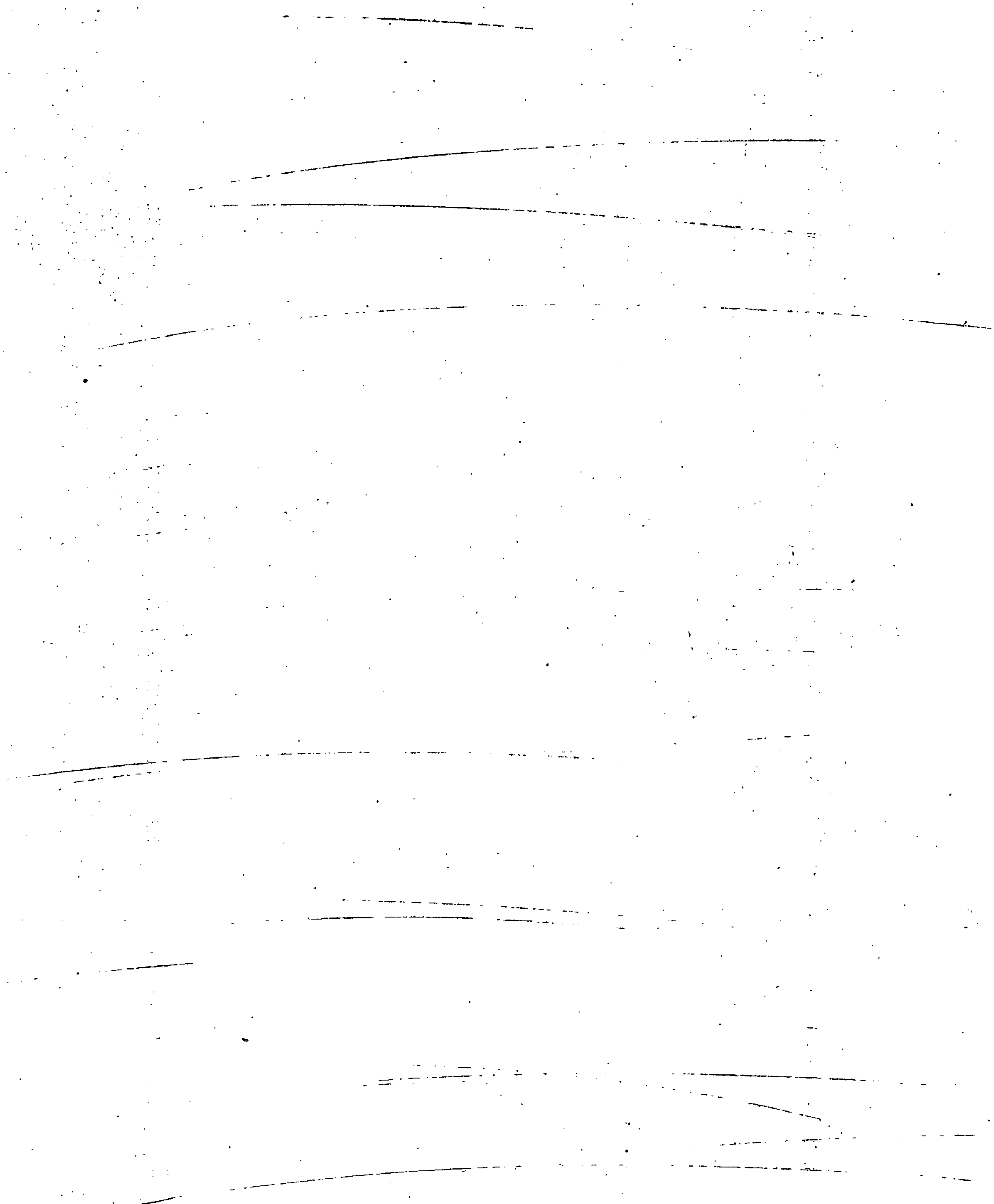
Having made reasonable attempts to confirm that the data belong to a single refractor of more or less constant velocity, it is still not possible to proceed to a detailed examination of the residuals until the velocity determination has been scrutinised and approved; any factor which produces an erroneous estimate of velocity will severely distort the pattern of residuals.

The nature of the least-squares fitting process for velocity places the greatest weight on observations with the widest spread of distances, and unfortunately this criterion leads one to gather up a motley assortment of second arrivals and weak onsets.

It is necessary to check whether each of these heavily weighted connections is of a quality to justify the reliance placed on it, and the tabulation of d^2_{ij} (DIJX2) in the solution provides a convenient guide to the relative weighting.

It might even be desirable in some cases to introduce an additional weighting factor to take account of the reliability of each observation, but in general it is sufficient to exclude any dubious observation from the initial solutions and include them subsequently on the condition that they do not disturb the character of the solution.

There is one category of dubious observation even more unwelcome than second-arrivals and weak onsets: those in the vicinity of the cross-over distance. When two broadly similar phases arrive almost simultaneously they can easily interfere in such a way that both of the true onsets are lost and the first readable arrival is much too late for either.



5.4 Residuals

When the most reasonable structural model has been chosen, and a fairly reliable estimate of velocity obtained, attention can be turned to the individual residuals.

Apart from some influence of velocity error (which is expected to be small), the residuals for the connections to any given survey point reflect the extent of the disagreement between the separate estimates of the time-term for that point, derived from the corresponding observational equations. The sum of all such residuals for any given point is zero.

Because of the way each observational equation contains two time-terms, an error in any one observation spreads into the two time-terms concerned and then (to a smaller degree) into the time-terms of each point connected with the first two, and so on. Consequently in the process of progressive refinement it is advisable to move cautiously and to make only a few changes in the data set between successive runs of the solution.

At this stage it may be appropriate to reconsider one of the major assumptions underlying the use of least-squares methods, namely that the distribution of errors is approximately normal.

One can easily envisage ways in which systematic trends could disturb the distribution; in addition to the deterioration in signal-to-noise ratio with increasing distance already mentioned, there is generally a tendency to read late rather than early, leading to a slightly skew distribution. If this tendency applied equally to all observations it would result in a uniform error in all the time-terms and would not be reflected in the residuals, but it applies mainly to the weaker onsets and will vary from one observer to another.

A rough indication of the error distribution may be obtained from the distribution of the residuals. (One would like to know the discrepancies between the observed travel-times and the true travel-times, whereas the residuals represent the discrepancies between the observed travel-times and the best available estimates of the true travel times.)

The distribution of residuals offers a guide to the uniformity of the data quality. An approximately normal distribution suggests that there is little prospect of dramatic improvement by the rejection of a few observations, while a very irregular distribution suggests the presence of gross errors which one may be able to recognize and eliminate.

If the residuals indicate an approximately normal distribution, this provides a quantitative basis for accepting or rejecting observations. A residual twice as great as the standard deviation of one observation would be expected to occur once in 20 observations, whereas a residual three times the standard deviation would be expected to occur only once in 1000 observations and one would hope to obtain a better solution by rejecting it.

The tabulation of "standard deviation of data" for each survey point is a convenient pointer to the observations requiring closer examination. Starting with the survey point whose data shows the largest standard deviation, the distribution of residuals is weighed up for each point in turn.

When a point is connected by as few as three or four observations it is hardly meaningful to speak of a "normal" distribution of residuals, but even then it is possible to distinguish characteristics of an unsatisfactory distribution: if one residual has a rather large value, and the remainder are of the opposite sign and smaller value (and similar to each other in value) then the first one should be regarded with some suspicion.

The fact that a particular observation does not fit well into the solution is hardly sufficient grounds for suppressing it. In some cases it may represent the last voice raised in protest against an erroneous but rather plausible solution.

When the original seismograms are available, it is advantageous to check them again for signal quality at this stage. It may turn out that from a group of contradictory observations, some of the onsets are so poorly defined that they cannot stand up to a challenge. On the other hand it may turn out that the one observation which had been earmarked for rejection is in fact the only one clear enough to be trusted.

The observations which should be retained in the solution at all costs are strong arrivals away from the cross-over points, while no such scruples need apply to emergent onsets, confused second-arrivals, and weak long-range observations.

The objective should be not so much to minimise the residuals by the suppression of all inconvenient observations, but rather to include the maximum possible number of observations without unduly increasing the residuals and the standard deviation of one observation.

The ultimate test of the consistency of any one observation with the main body of data rests on the comparison of the solutions with and without the observation in question. If the addition of new observations makes little change in the standard deviations it can be accepted that they are consistent with the main data.

6. PRACTICAL CONSIDERATIONS IN REFRACTION SURVEYS

6.1 General Planning

The usual preference is to have shots fired in water rather than on land, because of the more efficient coupling of explosive energy, with the observing stations on land for operating convenience and because of the more favourable background noise levels.

For interchange purposes, to enable the relation between shot time-terms and station time-terms to be unambiguously determined, it is required that at least one site should serve as a shot point for some observations and as a station for others.

The most satisfactory way of achieving this (provided that the signal-to-noise ratio is acceptable) is to operate a hydrophone station on a site which has been used as a shot point (or to fire a shot on a site previously occupied by a hydrophone station).

If hydrophone stations cannot be used, the best compromise is to arrange a station on land and a shot in water as close together as possible. Such a compromise is by no means ideal, since the boundaries between areas of land and water are liable to coincide with structural irregularities.

In choosing sites for field stations, a careful balance must be struck between many factors. A very thorough discussion of the problems has been given by Carder (1963), although perhaps more relevant to permanent teleseismic observations than to temporary field stations.

By far the most important consideration is the availability of bed-rock at a convenient depth; the sites should be an adequate distance from local sources of noise in the seismic frequency-band, e.g. trees, streams, roads, railways; access by vehicle may be necessary for unloading equipment, testing, or changing batteries; communication by telephone may be of value if there is a genuine need to relay information about the shot programme.

It is common practice to carry out refraction work with short-run recording equipment, relying on direct communication between shot-firing and recording teams or on a pre-arranged firing schedule. This imposes severe limitations on the programme, and it is far more satisfactory to have continuously-recording stations with independent timing facilities, operating throughout the days when shots are being fired.

Since ship time is more expensive than explosive, and also since a spread of both shots and stations is essential to velocity determination, the most efficient method of utilisation is for the ship to proceed along a line firing shots at intervals. The location of the shots need not be specified precisely in advance, and the time of firing is completely at the discretion of the shot-firing team. The only communication necessary to the recording team is notification of the completion (or major postponement) of the shot programme.

6.2 Field equipment

The system design requirements for field equipment have been discussed in detail in an earlier publication (Parks, 1966). The principal features relevant to the use of such equipment for refraction surveys are outlined below.

Recording on magnetic tape allows considerable flexibility in choosing suitable pass-bands and sensitivity on playback, and high timing accuracy is possible by using a fast paper speed on the output pen-recorder.

Each recording unit incorporates a crystal-controlled time service, producing time marks at 1-second intervals with coding to identify the hour, the minute, and the day. Broadcast time signals are also recorded on the tape to provide calibration of the internal timing.

The equipment is weather-proofed so that it may be installed in the open without additional protection. The main recording unit for each station is intended to be sited where it can be reached by a vehicle of the Land-Rover type for servicing.

The seismometers and associated electronics are not dependent on vehicle access, and may be sited at a distance from the recording unit. For distances up to a few hundred metres, cable links may be employed, or for longer distances (up to 100 km) radio links are available, although limited to line-of-sight propagation.

Power is supplied by ordinary lead-acid accumulators which are readily obtainable throughout the world. Mains power may be utilised, where available, to save labour in re-charging batteries, but normally a refraction survey involves only a few day's operation and there is no justification for limiting the choice of site for the sake of obtaining mains power.

The equipment is designed for continuous operation, requiring attention only at intervals of two days for changing tapes and batteries.

6.3 Background Noise

Seismic background noise may be divided into two main classes of origin: natural (mainly microseismic) and man-made (mainly vehicular traffic).

6.3.1 Spectral content

Seismologists accustomed to teleseismic observation tend to be unduly pessimistic about the level of natural background noise at coastal stations (or, worse still, island stations), and it is important to recognize the difference in requirements for local explosion studies.

Arrivals from events at teleseismic distances are mainly at frequencies of 1 Hz. or lower, and in this range the noise is mainly due to microseisms which one would expect to be more serious near exposed coasts.

At distances of the order of several hundred kilometres, on the other hand, the frequencies involved are mainly above 4 Hz., a band where microseismic effects are scarcely significant. Consequently, sites which are too noisy for teleseismic observation may well be perfectly satisfactory for local explosion studies. This was found to be the case when recordings were made of nuclear explosions in the Pacific (D. S. Carder, personal communication).

Man-made background noise unfortunately occurs mainly in the frequency band which is of greatest interest in explosion studies, approximately 2 to 10 Hz.

Comparative spectral analyses of background noise for four temporary stations are shown in Figure (12), for the morning of 22nd September 1965. Details of the world weather situation for that time have not been investigated, but the background noise on that day was quite typical of the preceding month, and showed no significant short-term changes (i.e. over an interval of 4 hours).

The two features which stand out are (a) that the noise level in the band 2 to 8 Hz. is considerably less than at lower frequencies, and (b) that the differences in noise (of presumably microseismic origin) at individual stations show some correlation with the proximity to bodies of water which could contribute microseism activity.

There are grounds for believing that the dominant frequency of microseisms is related to the extent of the body of water in which they are developed.

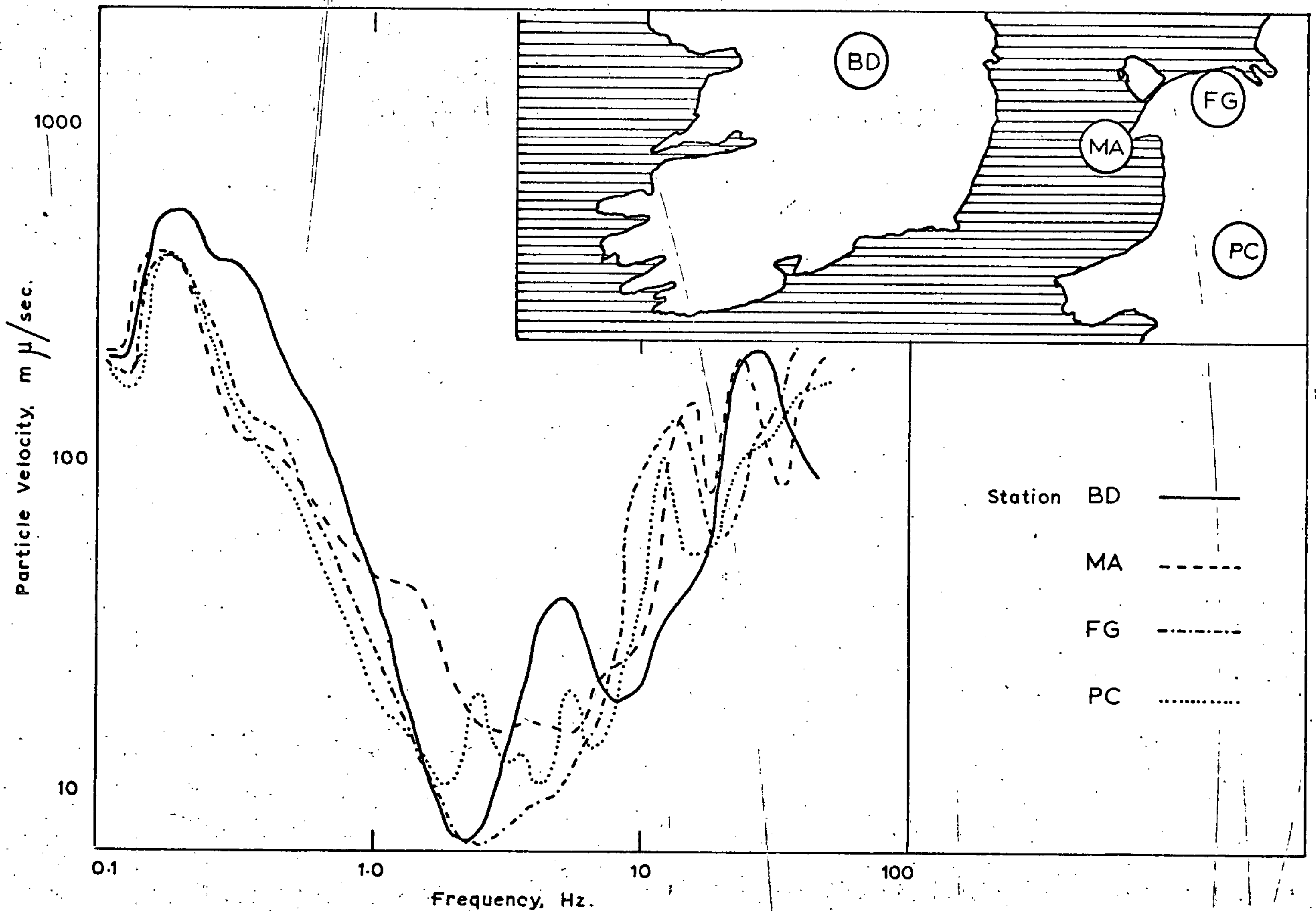


Figure (12) Spectral Analyses of Background Noise

Station BD, which is nearest to the Atlantic Ocean, shows most activity in the band 0.2 to 1.0 Hz., which is typical of oceanic microseisms.

Station MA, on a headland in the Irish Sea, shows a greater activity around 2 Hz., which is more typical of a small shallow sea.

Stations FG and PC, more remote from both the Atlantic Ocean and the Irish Sea, are both quieter in the microseismic range.

To keep the matter in proper perspective, it should be noted that the range in level between the quietest and noisiest of these four sites was only a factor of 2 at most.

No explanation is available for the broad peak around 5 Hz. in the spectrum for station BD, but the sharp increase above 9 - 10 Hz. for all stations may be partly attributable to instrument noise, especially to cross-talk.

6.3.2 Variation with time of day

A further consideration on background noise level is the possibility of variation with time of day if a significant proportion of the noise is man-made.

At Eskdalemuir and the temporary stations used in 1965, there was no significant difference between day-time and night-time levels. However at the 1966 temporary station NM in East Anglia, the noise level was found to rise to an intolerable level during the day, but to remain quite acceptable during the night, as shown in Figure (13). For comparison, the typical level at one of the 1965 stations is indicated.

This possibility of a "night window" has considerable bearing on the question of the most useful charge size, since the extra trouble involved in preparation of a large shot usually restricts such operations to day-time.

For example, at a station such as NM a signal-to-noise improvement of the order of 10 times is obtainable by firing at night. If reliance were placed instead on increasing the charge size by a factor of 10, the signal-to-noise improvement would be only of the order of 3 times.

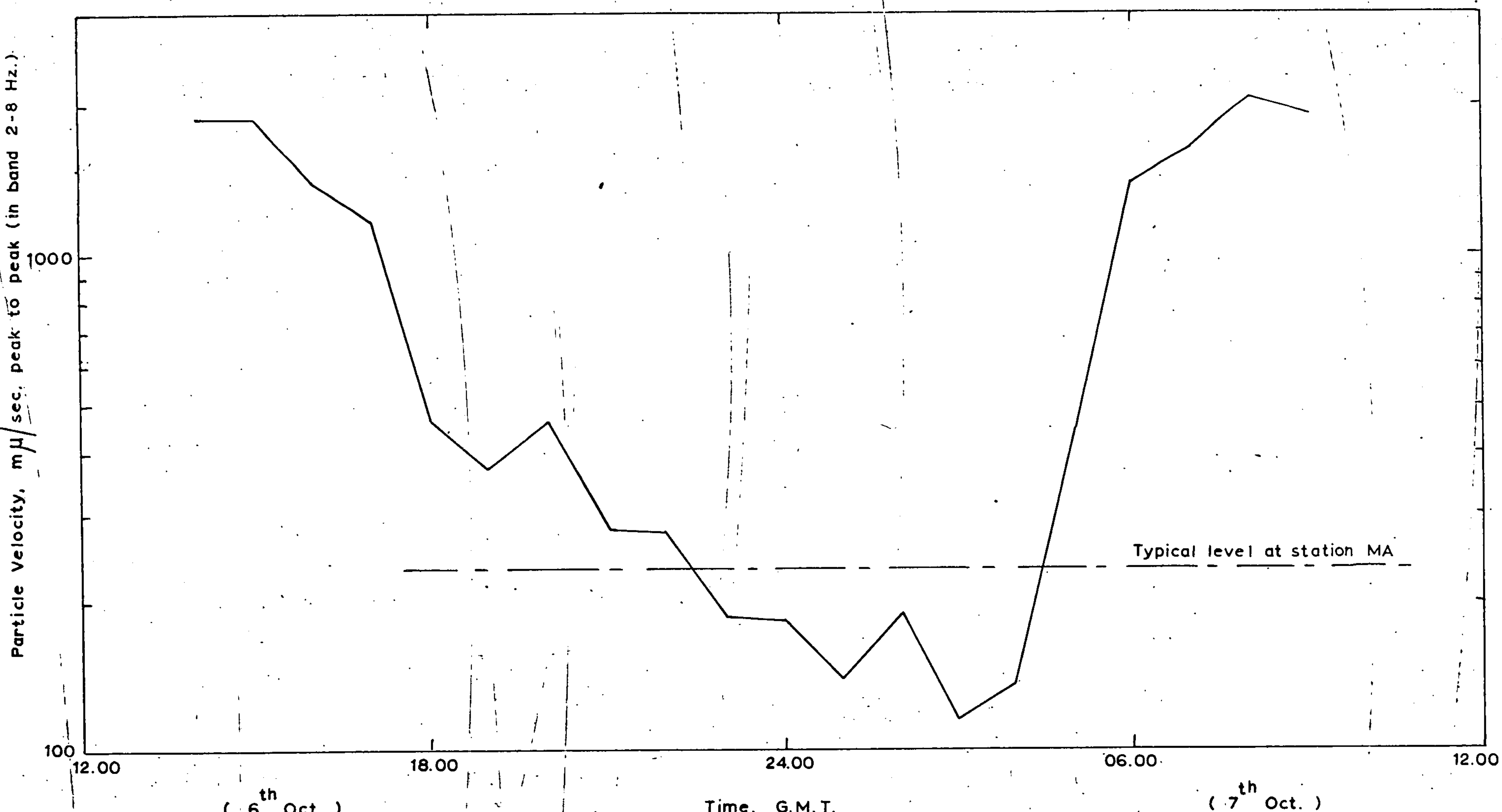


Figure (13) Variation of Background Noise with Time of Day

6.3.3 Threshold of Detection

If an assessment of background noise is to be of value, some estimate of expected signal strength is required.

Some of the early published work on amplitudes has been concerned with the maximum amplitude or the total energy of a phase, but in the present application the significant quantity is the strength of the onset, since it is this earliest discernable arrival which yields the travel-time relevant to refraction measurements.

The advent of underground nuclear test explosions has within the last few years given a fresh impetus to the study of the scaling laws for amplitude, and at the same time has provided material for study, in the form of accurately observed events yielding a high signal-to-noise ratio so that reliable measurements could be made for the first one or two half-cycles of the onset.

The calculation of amplitude is a complex process involving a great many factors, and has been dealt with at length by Werth, Herbst, and Springer (1962), Springer (1966A), and Springer (1966B). The problem may be considered in three parts: the source, the propagation, and the receiver.

At the source there is an extremely wide variation in the degree of coupling for underground shots, depending on the material in which the shots are fired and the arrangement of the charge (e.g. in a cavity or in a drill-hole, tamped by dry earth or by mud, etc.).

Underwater shots have a much higher coupling efficiency and could be expected to give more repeatable figures for amplitude.

Simple energy considerations would lead one to expect a relationship of the form:

$$a = KW^{0.5}$$

where a = particle velocity

W = charge weight

K = a constant

but in the case of underground shots this relationship is not found to hold, due to the complex mechanisms of deformation and of energy radiation.

Evidence presented by Gaskell (1956) and O'Brien (1957B) suggests that if the relationship is written in the form

$$a = KW^{n}$$

the value of n should be around 1.0 (for underground shots) rather than 0.5.

Data from a substantial number of underground nuclear tests investigated by Springer (1966B) strongly support a value $n = 0.8$. He was particularly interested in the coupling efficiency, and showed that this could vary by as much as a factor of 30 between granite on the one hand and low-porosity alluvium on the other. Certain materials chosen for especially low coupling (e.g. pumice, diatomite, and ashfall) could even be a factor of 5 lower than alluvium.

The studies mentioned so far have all dealt with underground shots, and although a considerable quantity of data must by now have been accumulated from underwater shots, comparatively little has been published on the matter.

Båth and Tryggvason (1962) suggested values:

$n = 0.75$ for P_1, and $n = 1.0$ for P_2, from underwater shots.

A more detailed study by Burkhardt (1963) led to values:

$n = 0.9$ for underground shots

$n = 0.7$ for underwater shots less than 10 g.

$n = 0.5$ for underwater shots greater than 10 g.

The propagation is dominated by the effect of geometrical spreading of the energy in three-dimensional space, making the amplitude inversely proportional to the square of the distance (O'Brien, 1957A). There will also be an exponental term due to attenuation, but for the propagation paths relevant to crustal studies the effect of attenuation is small compared to that of spreading.

At the receiver end there are uncertainties of coupling efficiency similar to those at the source. Some materials such as alluvium may actually increase the amplitude, due to resonance effects (Carder, 1963) but in general the seismometer will have been sited on hard bed-rock and the coupling efficiency will be high and predictable.

From the number of explosions already recorded around the British Isles, it is possible to make an approximate check on the

the scaling relationships between amplitude, charge size, and distance.

Most of the recordings were not of sufficiently high signal-to-noise ratio to permit critical measurement of the first half-cycle of the onset, and a more empirical approach was adopted.

It was considered that the maximum lateness which could be tolerated (if the observations were to remain acceptable for time-term analysis) was of the order of 0.25 sec. Records were taken from the most quiet station available (EKA), so that the true onset time could be reliably recognized. An estimate was made of the maximum signal amplitude during the first 0.25 sec. after the onset. It was then assumed that the maximum permissible noise level for detection of the onset was half of this signal level.

An advantage of using EKA records was that the signals from a number of seismometers could be compared and averaged, so that a meaningful estimate was possible even when the signal-to-noise ratio on each seismometer was around unity.

Attention was concentrated on Pn arrivals at distances beyond 160 km, since the signal-to-noise ratio at shorter distances is usually adequate. The line of Noordzee shots conveniently covers distances between 160 and 350 km from EKA.

The observed signal strength depends on source conditions which varied from shot to shot, and if this variation were systematic it could seriously affect the validity of the interpretation. Consequently it comes as a pleasant surprise to find reasonable agreement between the simple theory and the observations.

Figure (14) shows the variation of threshold level with distance, before normalizing for charge weight. Comparison of the large and small shots around 260 km supports proportionality between amplitude and square-root of charge weight, and comparison of the small shots around 170 and 260 km supports proportionality between amplitude and inverse square of distance.

Figure (15) shows the variation of threshold level with distance, after normalizing for charge weight on this basis, and Figure (16) the variation with charge weight, after normalizing for distance. In each case the straight line represents the relation

$$a = (1.74 \times 10^5).(W)^{0.5}.(d)^{-2}$$

where a = threshold level, millimicrons/sec.

W = charge size, kg

d = distance, km.

The agreement is as close as one could hope to achieve with these data.

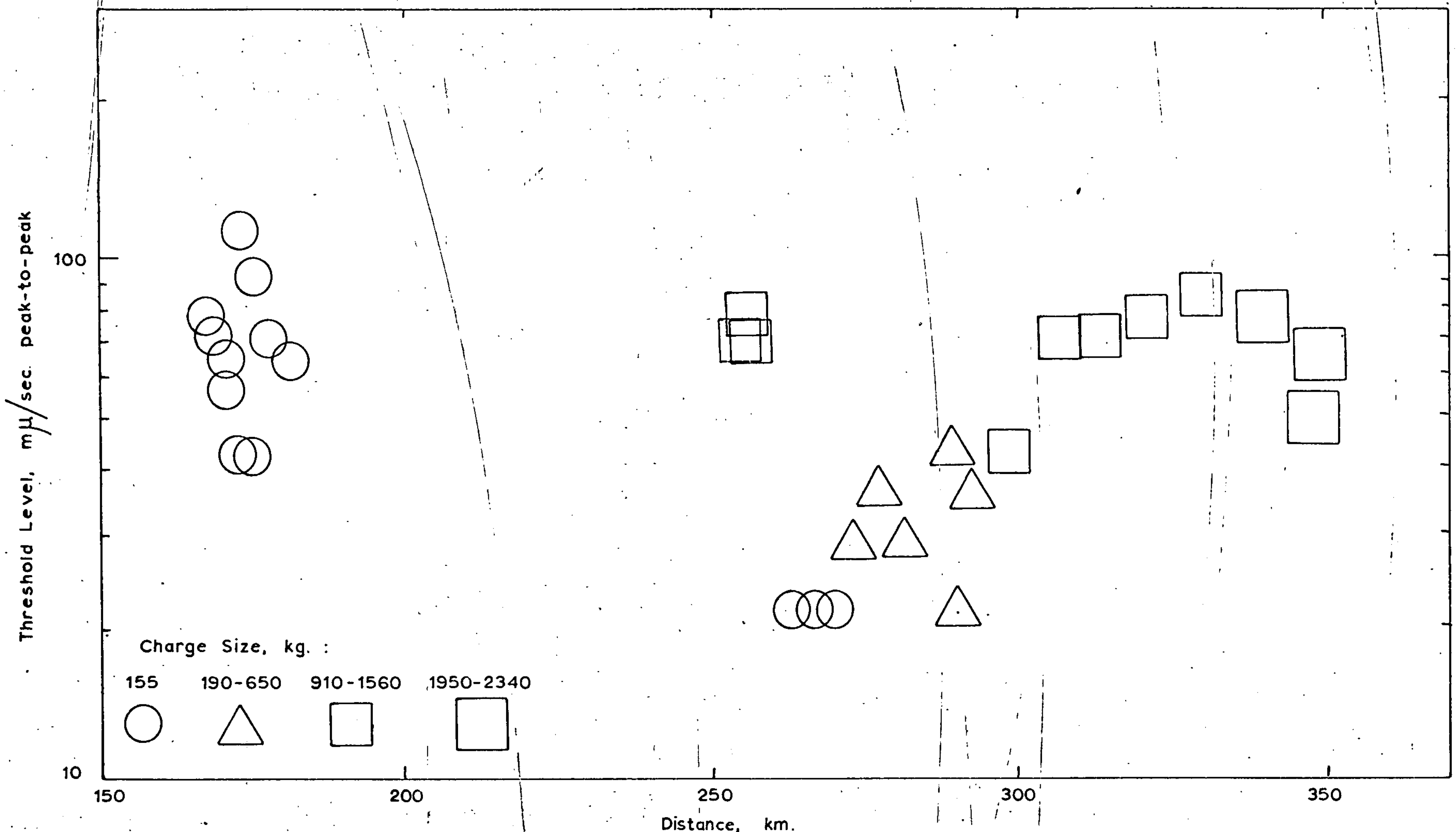
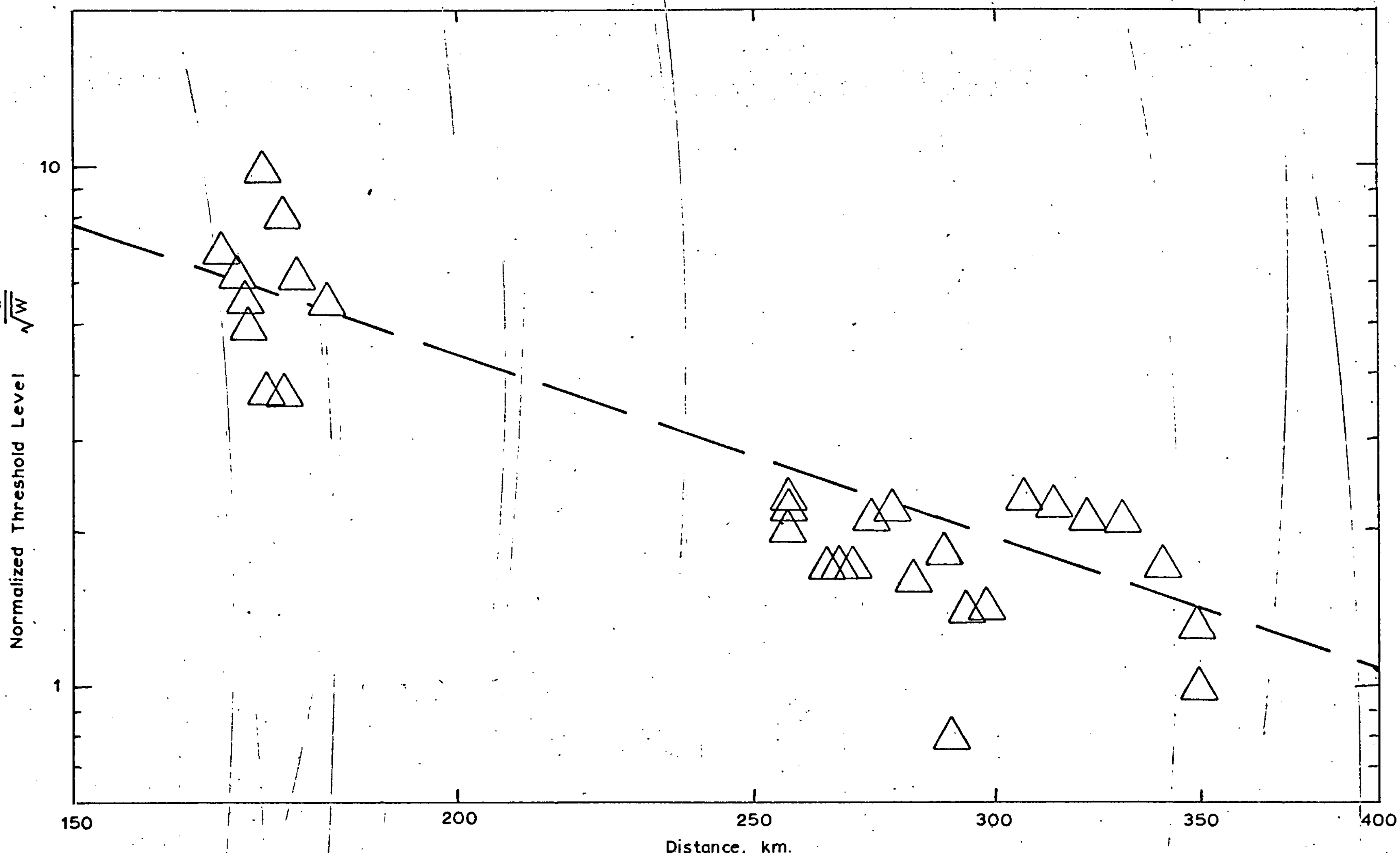


Figure (14) Variation of Threshold Level with Distance



Figure(15) Variation of Threshold Level with Distance (normalized for Charge Weight)

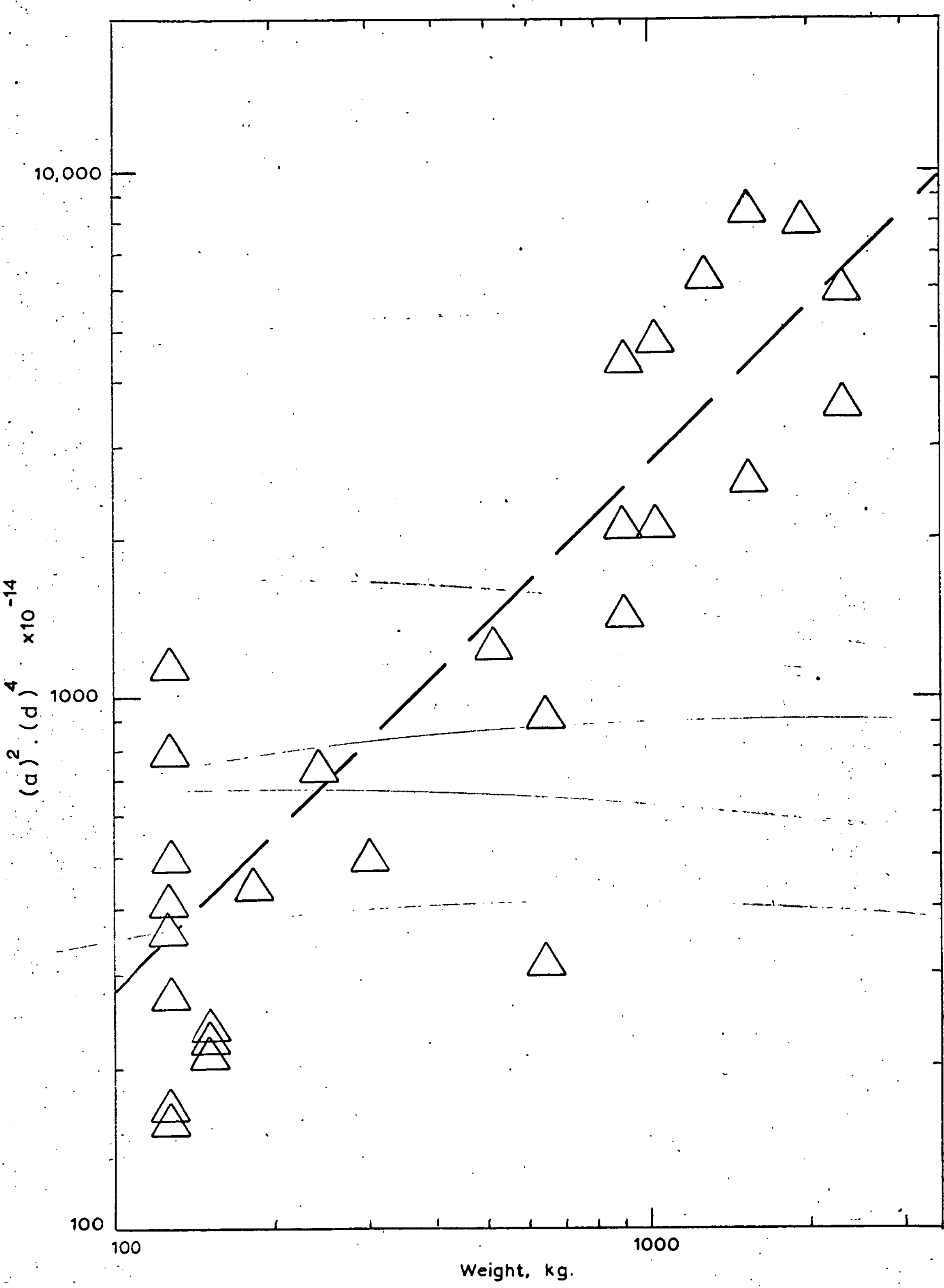


Figure (16)

Variation of (Threshold Level)^2 with Charge Weight
(normalized for Distance)

If some more reliable data were available, a less crude method of fitting would obviously be desirable, and least-squares technique could be applied to amplitude data as readily as to travel-time data (Willmore, Herrin, and Meyer 1963). This approach has recently been used in the estimation of magnitude of underground explosions (Douglas, 1966), with a mathematical treatment closely similar to that discussed in Section (3).

6.3.4 Second and later arrivals

It is customary in refraction surveys to make use of not only the first arrivals but also any later arrivals which give reasonably clear onsets, provided that the travel-times are in accord with the postulated crustal model.

The practice is not to be recommended, especially for stations using only a single instrument, because of the extreme complexity and irregularity of the signal following the first arrival. The excuse usually put forward for using later arrivals is that this makes a greater amount of data available, but in many cases the additional data do more harm than good, as a realistic estimate of the uncertainties may reveal!

There is also the danger of choosing only those later arrivals which lend support to some pre-conceived theory, while ignoring others which might be unfavourable to it. Some form of significance test is called for; in a few cases one may be satisfied that practically all the available observations have been utilised without ambiguity, but if a significant number of observations have been set aside, or if alternative identifications are equally plausible, then a more sophisticated test may be necessary.

Willmore (1949) has suggested a statistical method for testing the significance of a line drawn through such a selected group of points, by comparing the number of points lying within a defined distance of the line with the number which might be expected if the points were randomly distributed.

6.3.5 Signal-generated noise

As research continues into the nature of the seismic signal following the first arrival, it becomes increasingly clear that more elaborate arrangements of seismometers are desirable for recognition of the later arrivals.

Much of the important development in this field is due to the U.K.A.E.A. group at Blacknest. Even without advanced processing techniques, a comparison of records from a number of adjacent seismometers of a spaced array (typically 1 - 2 km apart) readily reveals the inadequacy of a single seismometer for later arrivals. Often a single record shows an arrival with all the characteristics of a genuine phase, when from the records of the neighbouring seismometers it is clearly not coherent.

The large spaced arrays were developed specifically for processing by "velocity-filtering", which enables a fuller investigation to be made of the azimuth and apparent velocity of the energy arriving at the array at any given instant.

Unpublished work by the Blacknest group (H.I.S. Thirlaway, personal communication) has shown that much of the complexity in the "coda" following the first P onset is due to mode conversion of the P-wave energy at structural irregularities in the vicinity of the station. The term "signal-generated noise" is used to describe the effect.

Another investigation by the same group (F.A. Key, personal communication) has dealt with a comparison of records obtained at a temporary station (NM) in East Anglia and at the permanent array station (EKA) in southern Scotland. During field operations in connection with the Dutch Noordzee project (see map, Figure (26)), in October 1966, station NM recorded (on a 3-component short-period set) an event in Novaya Zemlaya, which was also recorded at EKA.

The records from the two stations show striking differences which are attributed to the geological features of the sites - EKA on hard bed-rock, NM on low-velocity sedimentary layers. The EKA record is simple in character, with a short distinct initial P pulse followed by a low amplitude coda, whereas at NM, due to the multiple layering and subsequent reverberations and mode conversions, the energy in the initial pulse is spread into an extended coda, with attenuation of the peak amplitude. The PcP phase, however, arriving almost vertically, is much less affected by the local structure, and the character of the record is quite similar at both stations.

Correlation of the 3-component records for NM indicates that the particle motion corresponds to true P motion for only 0.4 seconds after the first onset, before becoming confused.

6.3.6 Optimum depth of firing

The material in this section is due entirely to A.M. Bancroft (personal communication). It is included here an account of its relevance to the planning of refraction surveys, since it deserves to be more widely studied.

Bancroft investigated the relation between depth of firing and "detec tion probability" for a total of 77 shots in the Lake Superior project recorded at 11 U.S.A. stations. Dividing the shots into groups by depth intervals of 15 fathoms, he defined the "detection probability" as the ratio of the number of successful observations to the number of possible connections in each group.

The results, as shown in Figure (17), indicate a surprisingly pronounced peak in the vicinity of 80 fathoms. A change of 30 fathoms (either increase or decrease) in depth is apparently sufficient to halve the detection probability.

Before these deductions could be confidently applied to surveys in other areas, it would be necessary to look into the conditions of the experiment in more detail. Two questions which immediately spring to mind are:

(i) What was the total water depth in relation to depth of firing?

(ii) What were the factors influencing the choice of of depth for firing? Perhaps the shot-firing party used a formula based on charge size and weather conditions!

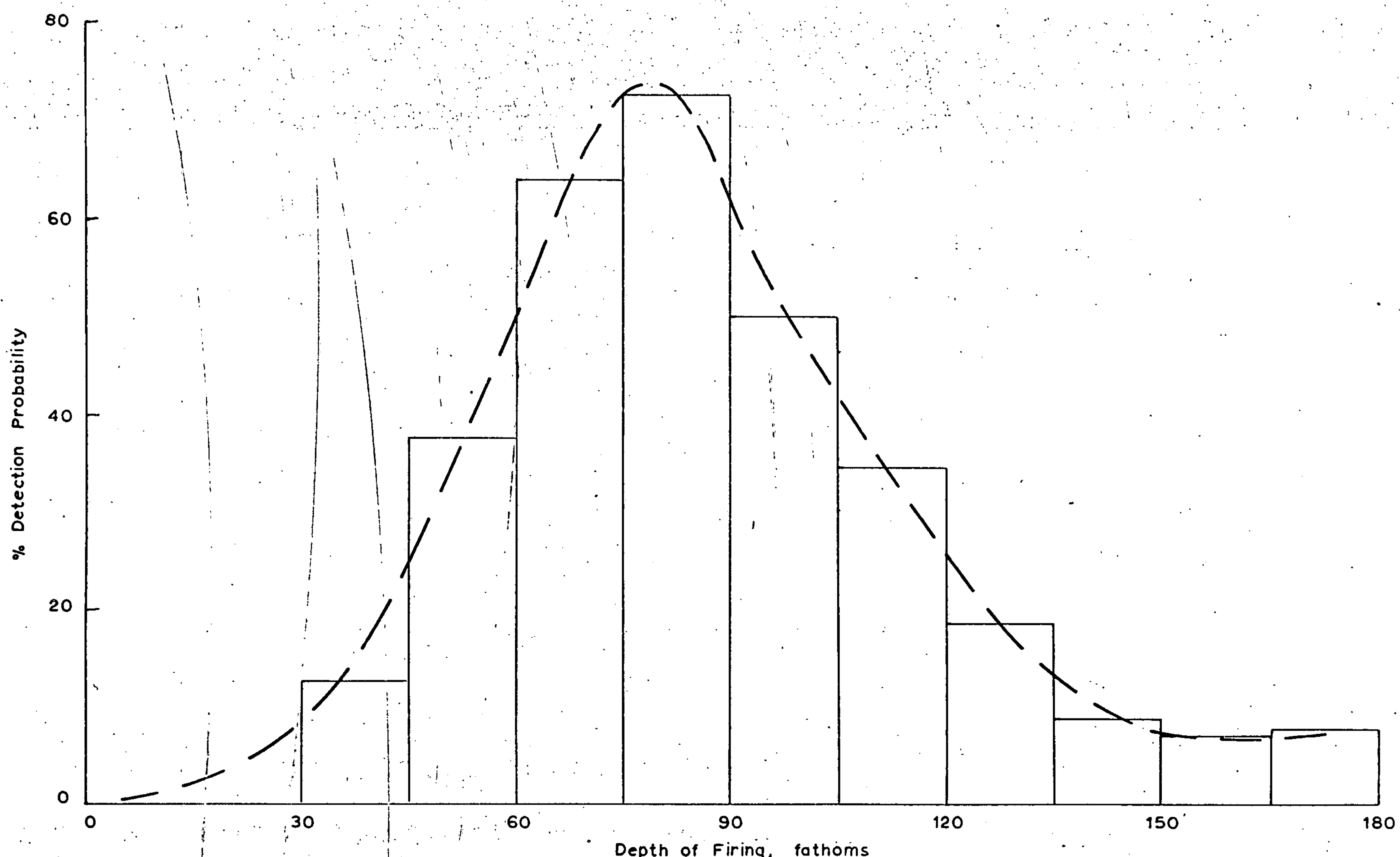


Figure (17) Relation between Depth of Firing and Detection Probability

7. APPLICATION OF THE TIME-TERM METHOD

7.1 U.K.A.E.A. Experiments

Several programmes of test explosions have been carried out by UKAEA around the British Isles, and all the relevant data have been generously made available to Edinburgh for further study.

The material falls broadly into two categories: before and after the commissioning of the permanent array station at Eskdalemuir. The early material (mainly direct recordings on paper chart) is generally characterized by the low timing accuracy, and is not of great value for estimation of crustal thickness. The later material, known as the Seagull II project, using magnetic tape recording, is of much higher quality as regards both timing and signal-to-noise ratio, and is very suitable for the present type of study.

At this stage it is appropriate to point out a major difference in emphasis between the UKAEA work and the present investigations. The purpose of the UKAEA programme was to obtain information on amplitudes and signal-to-noise ratios for events of known energy at various distances, and on recording conditions at various station sites; the laterseries was planned to provide calibration material for the Eskdalemuir array, in the form of known events at selected distances and azimuths.

For investigation of the crust as a layered structure where velocities and thicknesses are to be estimated, there are two specific requirements additional to those for amplitude studies: timing accuracy and a suitable distribution of shots and stations.

The later UKAEA material has been used as the basis of a paper on crustal structure (Agger & Carpenter, 1964) which extracts the maximum amount of information from the travel-time data but is severely limited by the shortcomings of the distribution of shots and stations. However, apart from these inherent limitations, the data and the deductions drawn from them are quite compatible with the subsequent work in the Irish Sea area.

Some of the shots were recorded at a second station, Rookhope, with a borehole seismometer, but unfortunately the line joining the two stations is approximately broadside to the majority of the shots, and therefore extremely poor control is obtained over the velocity. The arrangement of survey points is shown in Figure (18). If it had been possible to include a third station in Wales or Ireland to observe the shots in the Irish Sea from the reverse direction, much more reliable determinations of velocity would have been possible.

Because of the scarcity of observations at shorter distances, little could be deduced about the shallow layers, and a P-wave velocity of 4.7 km/sec. was assigned "somewhat subjectively on available geological and geophysical evidence".

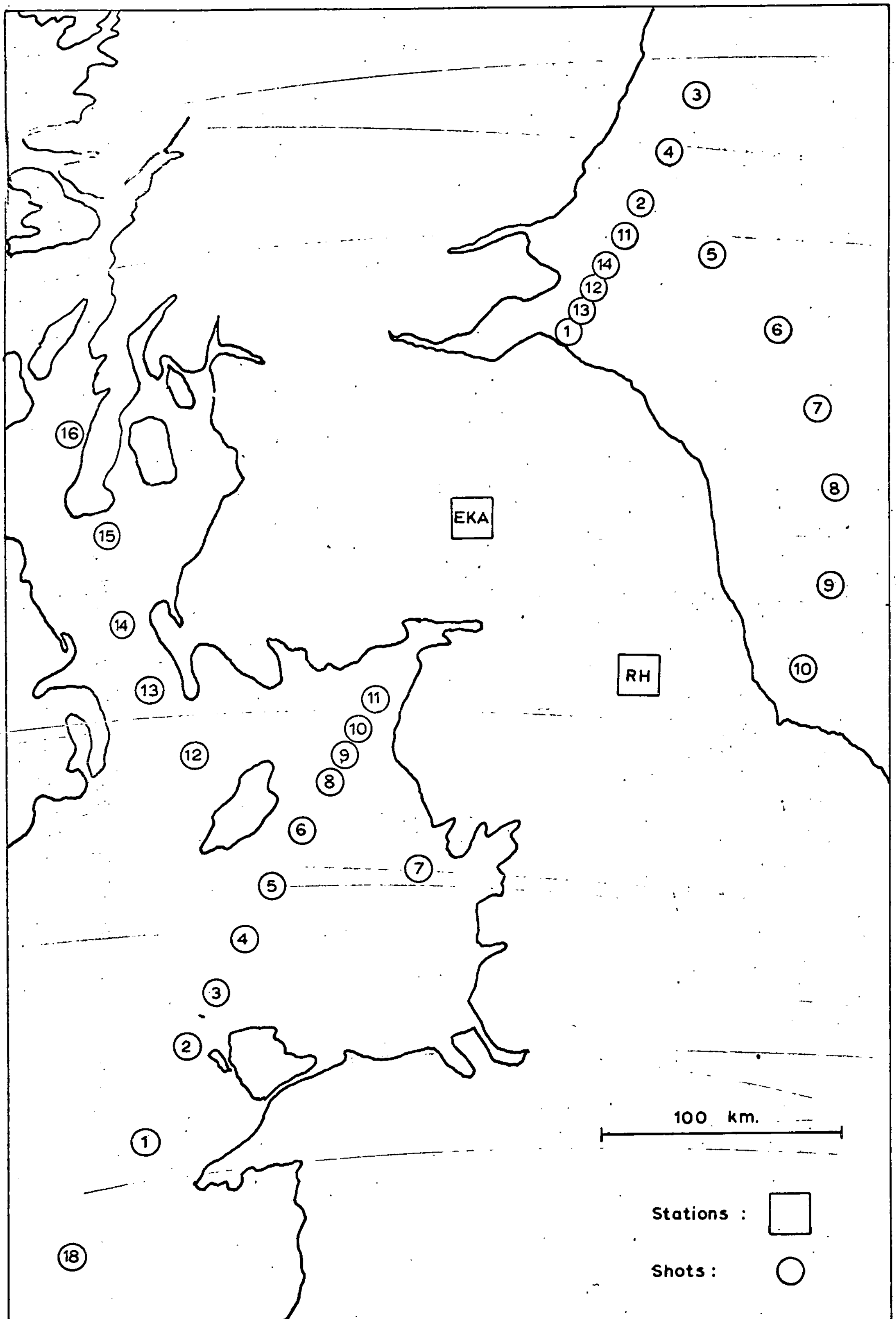


Figure (18) Arrangement of shots and stations for SEAGULL II Project

In view of the co-operation received from the UKAEA group, particularly in making their data and calculations available, it may seem rather churlish to criticise the validity of their published results.

However, since the southern part of their network overlaps the area of the Irish Sea project, a comparison of results can hardly be avoided, and immediately the question of confidence limits arises.

Although Agger and Carpenter (1964) paid due respect to the use of 95 % confidence limits in relation to the straight-line interpretation, they were less than thorough in dealing with the time-term interpretation, and the basic data sets which they used have therefore been re-processed as sets 2056 and 2063 for Pn and Pg respectively, and the solutions are listed in Appendix B.

The figures which they quoted for confidence limits on the velocities would appear to be too low, by a factor of 3.

The time-terms and uncertainties are summarized in Section (B4), and the weakness of the solution is painfully evident.

Even the operation of assigning the arbitrary constant (Section (2.7)) lapses into indeterminacy. It may appear to be satisfactory to invoke the condition that the time-terms are everywhere positive, and to use this to define the arbitrary constant β from the Pg time-terms of station EKA and shot 13 West by writing:

$$0.72 \geq \beta \geq 0.62$$

but when the uncertainties are included, the condition can be seen to be of scant value in determining the value of β :

$$(0.72 \pm 1.02) \geq \beta \geq (0.62 \pm 1.21)$$

The uncertainties quoted are those which take account of the velocity uncertainty. As an indication of the contribution of the velocity uncertainty, the summary of Pg time-terms also lists the values which would be obtained by increasing the velocity by only $1\frac{1}{2}$ % (half of its confidence limit) and readjusting to give EKA the small positive value favoured by Agger and Carpenter. This interpretation could hardly be said to be less consistent with the data, yet it represents quite a change in the thickness of the upper layer.

At this stage one might well consider using other geophysical and geological evidence to estimate the thickness of the upper layer, and indeed such a procedure is more likely to produce a reliable answer!

The uncertainties in the Pn time-terms are so large that it is unrealistic to draw a detailed profile of depths. The apparently interesting feature of "a large Pn time-term for shot 1 West" can hardly be regarded as significant; shot 1 differs from the mean of the two neighbouring points (shots 2 and 18) by only 0.77 sec., while the uncertainty of each time-term is ± 0.52 sec. (even without considering the velocity uncertainty, which increases the uncertainty for shot 1 to ± 1.64 sec.).

7.2 The Irish Sea Project

7.2.1 General Plan

In 1965 an explosion project for investigation of crustal structure was undertaken jointly by the Universities of Edinburgh and Birmingham.

The Birmingham group had previously conducted short-range seismic surveys in the area, and they proposed to extend the scale of operations to Pg and Pn ranges. The original intention was that the Birmingham sonobuoy equipment would be used for two stations in the middle of the shot network to observe at short range, while the Edinburgh field equipment would be used for land-based stations at longer range, one in Ireland and three in Wales.

For a limited operation, the configuration of shot and station networks must be carefully chosen in the light of any available information on the structure of the crust (the number of major layers, and the approximate crossover distances). In this case the only relevant previous work was that of the UKAEA group (Agger and Carpenter 1964): a three-layer model was suggested, with velocities of approximately 4.7 km/sex., 6.1 km/sec. (identified as Pg) and 8.0 km/sec. (identified as Pn). Their upper layer velocity was not observed directly, the figure of 4.7 km/sec. being only a subjective estimate. However, the other two layers were covered by a satisfactory number of observations, the 6.1 largely as second arrivals and the 8.0 as first arrivals, with a cross-over distance around 120 km.

On the basis of the Esdalemuir records it was expected that observations of Pn could be obtained up to 250 km from a standard depth-charge (300 lbs).

To meet the requirements of interchange, shots were to be fired on the sites to be occupied by the sonobuoy stations. As there was a possibility that the signal-to-noise ratio might not be sufficient to permit Pn to be observed at first-arrival distances on these stations, one of the land-based stations was sited on a headland and two shots were fired in the sea as close as practicable to the site. To complete the interchange there were also to be several shots at sufficient distance to give Pn as a first arrival.

Unfortunately, owing to force of circumstance the plans had to be curtailed at rather short notice, with the result that the data are much less complete than one would wish. The shots planned for the three most northerly locations had to be omitted in response to an official objection on the possibility of interference with fishing grounds. Consequently there were no shots at distances beyond the cross-over distance from the interchange station.

Due to rearrangement of the shooting dates, it was not possible for the sonobuoy stations to be used, so that first-arrival Pg observations were obtained at only one station.

The arrangement of shots and stations is shown in Figure (19). It is now clear that operation of an additional station on the outskirts of Dublin would have been of exceptional value in completing the short-range coverage, but at the time this was not possible due to lack of equipment.

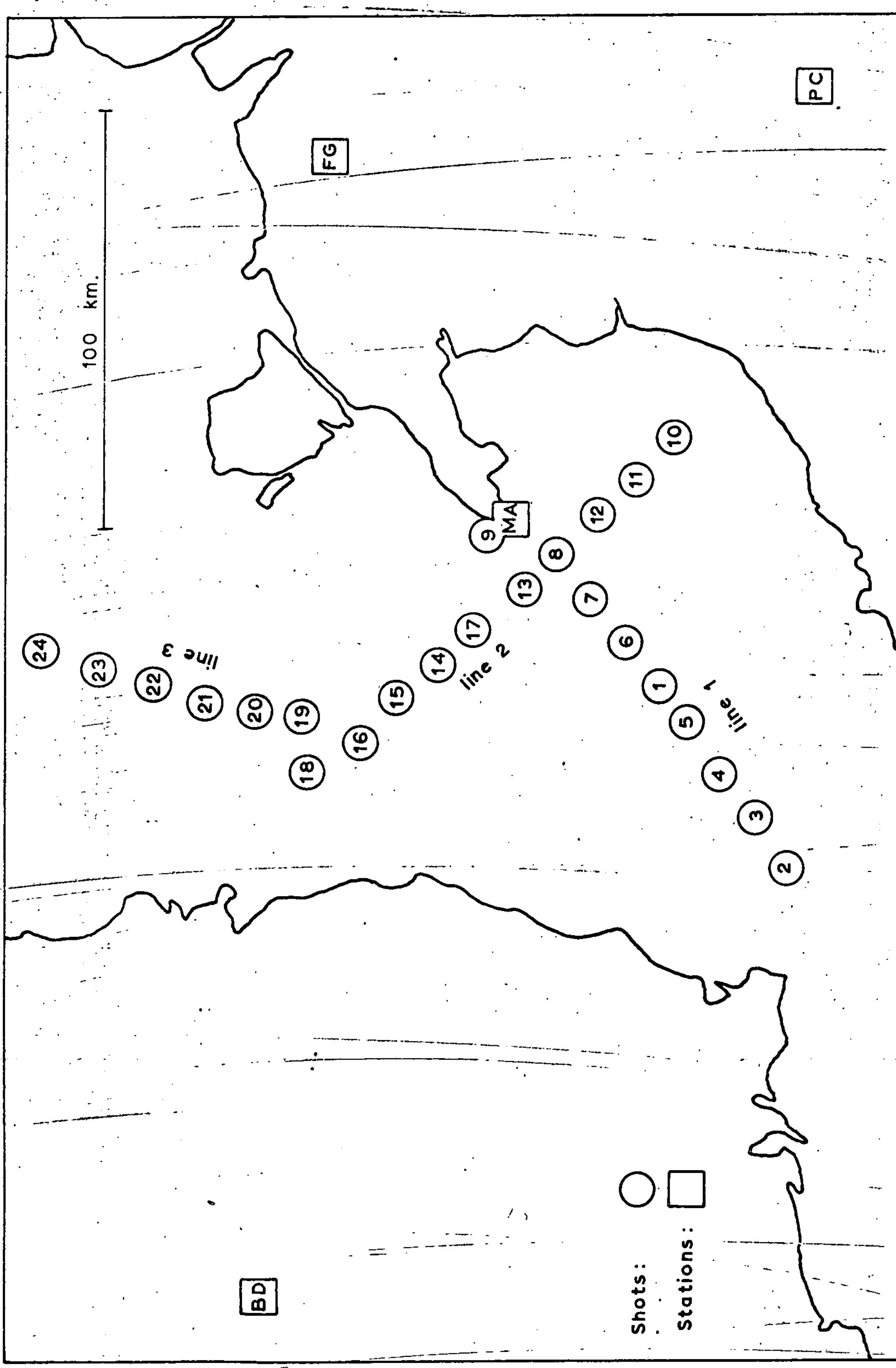


Figure (19) Arrangement of shots and stations for IRISH SEA project

7.2.2 Field Arrangements

Two independent systems were employed for timing the shot instants, to guard against the possibility of break-down.

A high-speed photographic chart recorder on the ship recorded the output from a seismometer placed on deck, together with time-marks from a crystal-controlled time encoder which was checked against BBC time-signals.

The second system used the ship's radio transmitter to relay the output from the Precision Depth Recorder. This radio signal was received by the nearest land-based station (MA), and recorded on tape with independent time-marks from a second time encoder.

Although the recording stations were operated continuously and did not need to rely on communication with the shot-firing party, the ship's transmitter was used to make advance announcements of the time of each shot. In fact the only essential message was the confirmation of completion of the shot-firing programme.

All charges were fired on the sea bottom, since the available water depths were considerably less than the value of 80 fathoms suggested by A. M. Bancroft (personal communication) as most favourable for detection at longer ranges. (see Section (6.3.6)).

The depth-charge firing mechanism was set to give its maximum time delay (approximately 90 sec.) to permit the ship to reach a safe distance before detonation.

Both timing systems observed the instant when the water wave reached the ship; a correction for the travel-time from charge to ship was applied by measuring the time interval between dropping the charge and detecting the water-wave, and combining this with estimates of the ship's speed and the water-wave velocity.

7.2.3 Correlation of 3-component records

Each of the temporary stations was equipped with a 3-component set, and originally it was planned that correlation techniques would be used for studying the particle motion as an aid to the identification of later arrivals (and especially in distinguishing between P and S waves).

Unfortunately the special unit developed for the purpose was not completed in time to be used extensively. However, with a prototype version it was possible to process the records from station MA for the shots in line 1, as these were all at approximately the same azimuth, 225°.

Figure (20) shows travel-times vs. distance for all first arrivals at distances up to 130 km, with later arrivals for line 1 picked by 3-component correlation. The significance of the various arrivals showing P-wave characteristics would require further study, but the correlation technique proves to be of great value in distinguishing the first S-wave onset from amongst late P-wave motion.

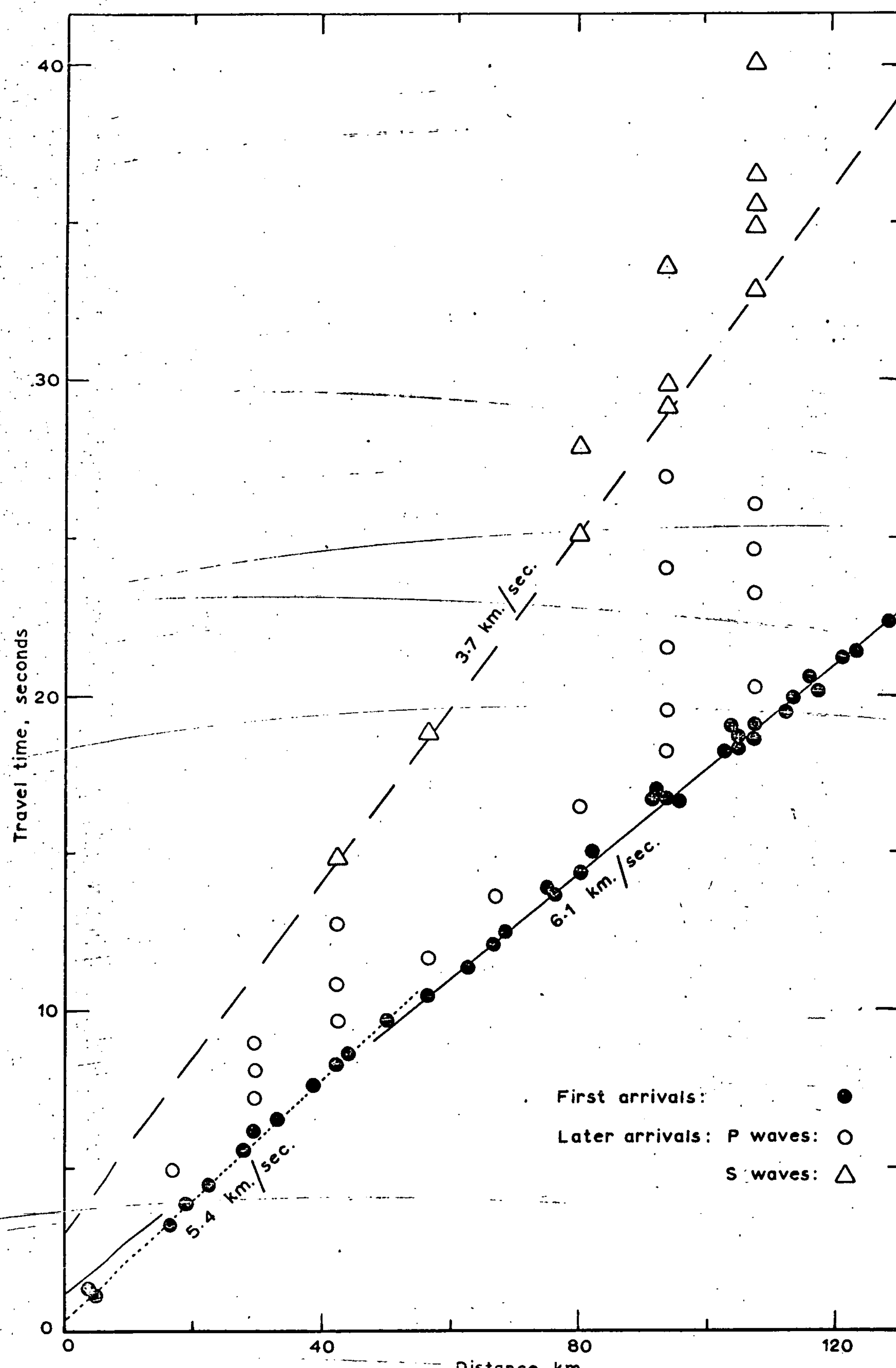


Figure (20) Travel times for distances < 130 km. for IRISH SEA project

7.2.4 Discussion of results

A preliminary inspection of the first-arrival data by the graphical technique discussed in Section (2.8) indicated a cross-over point in the vicinity of 125 km, and it was expected that the two refractors involved would correspond to those of velocities 6.1 and 8.0 km/sec. found by Agger and Carpenter (1964).

The surprising feature is that for the four southern stations, BD, FG, MA, and PC, the first arrivals beyond this cross-over point (and second arrivals at shorter distances) do not show the expected Pn velocity around 8 km/sec. The travel-times have considerable scatter if treated by a straight-line solution, but a time-term treatment gives a much more satisfactory fit; the velocity indicated is 7.3, and the uncertainties are so small that there can be little doubt that a well-defined refractor of this velocity occurs throughout the area.

For the two northern stations, EKA and RH, on the other hand, the Irish Sea shots give reasonable agreement with a velocity around 8 km/sec. as do the long-range observations from the Seagull project.

Unfortunately, the Irish Sea shots are all practically broadside to the line joining EKA and RH, giving extremely poor control over velocity, and consequently these shots add very little information on Pn velocity to that already derived from the Seagull data. Data set 2048 combines all the available observations which appear to relate to Pn from both EKA and RH.

The observations from the four southern stations relating to the 7.3 velocity are covered in data sets 2040, 2043, and 2044. Shots 10, 11 and 12 gave weak arrivals at all stations (attributed to an unfavourable combination of shallow water and thick sediments), so these were omitted from the first run, data set 2040. Observations of shot 12 were then added to make data set 2043, giving a solution consistent with 2040 and therefore accepted as satisfactory. The inclusion of shot 11 to make data set 2044 showed a distinct deterioration and was rejected, while the observations of shot 10 were so inconsistent that they could be rejected immediately.

It is considered that these results represent an intermediate refractor of velocity 7.28 km/sec., extending under the southern part of the region (i.e. under the Irish Sea shots and stations BD, FG, MA, and PC) but not continuing as far north as station EKA and RH. Beneath this a refractor of velocity 8.09 km/sec. is apparently continuous throughout the entire region. No first - arrival observations at the four southern stations have been identified as relating to this refractor, but if the time-terms of these stations are similar to those of the southern shots (typically 3.4 - 3.5 sec.) then this deepest refractor could be expected to show only as second arrivals at the ranges of observation (up to230 km).

If the intermediate refractor were continuous also under the northern stations, one would expect it to show as a second arrival at the longer range at EKA. A number of the records do show a second arrival which could be so identified, but in view of the general unreliability of later arrivals it is felt that no great weight could be attached to this evidence. Velocity-filtering would not provide a great improvement in this particular case, as the array dimensions were originally chosen to suit longer ranges of observation.

The first arrivals between 50 and 125 km, and second arrivals beyond 125 km, have been processed as data set 2039, giving a velocity of 6.30 km/sec. but rather poor quality of fit. The exclusion of observations beyond an arbitrary limit of 200 km gives a velocity of 6.14 km/sec. with some improvement in the fit, but it is still far from satisfactory.

This poor quality of fit is in part due to the inadequate signal-to-noise ratio of the records obtained. However, it is interesting to note that the scatter of the time-terms about a local mean is substantially less than the estimated uncertainty. It may be that much of the difficulty arises from the use of a relatively simple model in an area where the structure is known to be complicated (W. Bullerwell, personal communication). It is probable that the uppermost 10 km of the crust in the British Isles involves a high degree of velocity variation both horizontally and vertically, and that severe faulting and folding violate the basic assumption of a more or less uniform refractor, continuous under the area.

Distances less than 50 km were covered by observations from only a single station, MA, so that a full time-term analysis was not possible for them. However, a velocity of 5.4 km/sec. fits reasonably well, and station MA was known to be sited directly on rock of this velocity. This would seem to imply the presence of a refractor of 5.4 km/sec., overlain by sediments of lower velocity giving time-terms corresponding to the reduced travel-times shown in Figure (21); station MA is taken to have zero time-term, and apparently shot 8 also has zero time-term, i.e. the 5.4 km/sec. material is exposed.

The time-terms along the three lines of shots are shown in Figures (22), (23), and (24). Of the three refractors, only the intermediate one is reasonably well determined, and although a basin-shaped trend can be recognized in the middle of line 2 it is not possible to say with confidence whether a basin structure exists in the upper and lower refractors. The geographical distribution of time-terms for the intermediate refractor is shown in Figure (25).

In view of the uncertainties it is not considered that detailed profiles of depth along each line are worth presenting. By taking regional averages, the following values are estimated as typical:

	Depth	Velocity
Upper refractor:	5.3 ± 0.9 km	6.14 ± 0.11 km/sec.
Intermediate refractor:	25.0 ± 0.9 km	7.28 ± 0.05 km/sec.
Lower refractor:	30.7 ± 2.4 km	8.09 ± 0.35 km/sec.

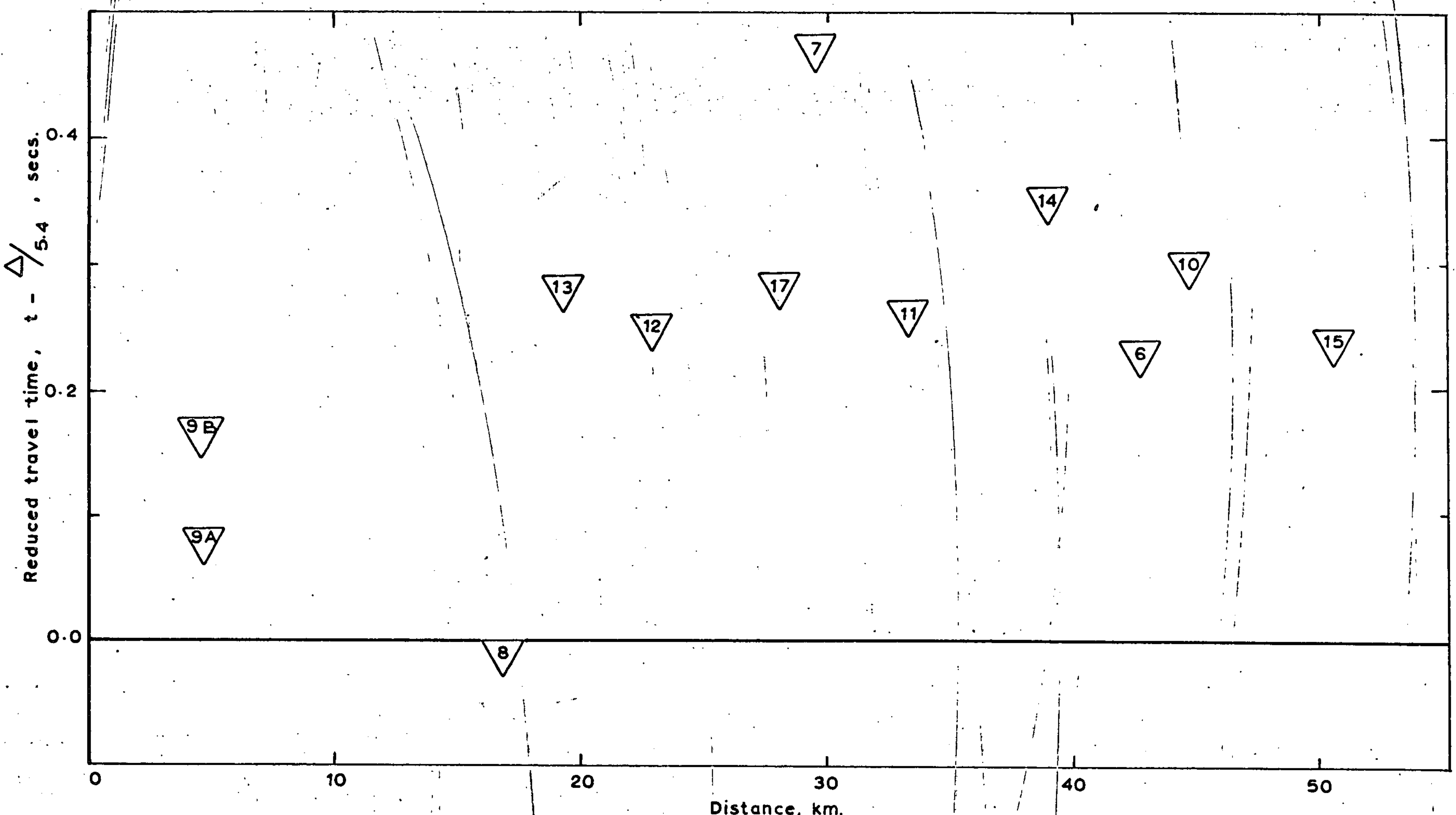


Figure (21) Reduced travel times for distances < 50 km.

IRISH SEA project

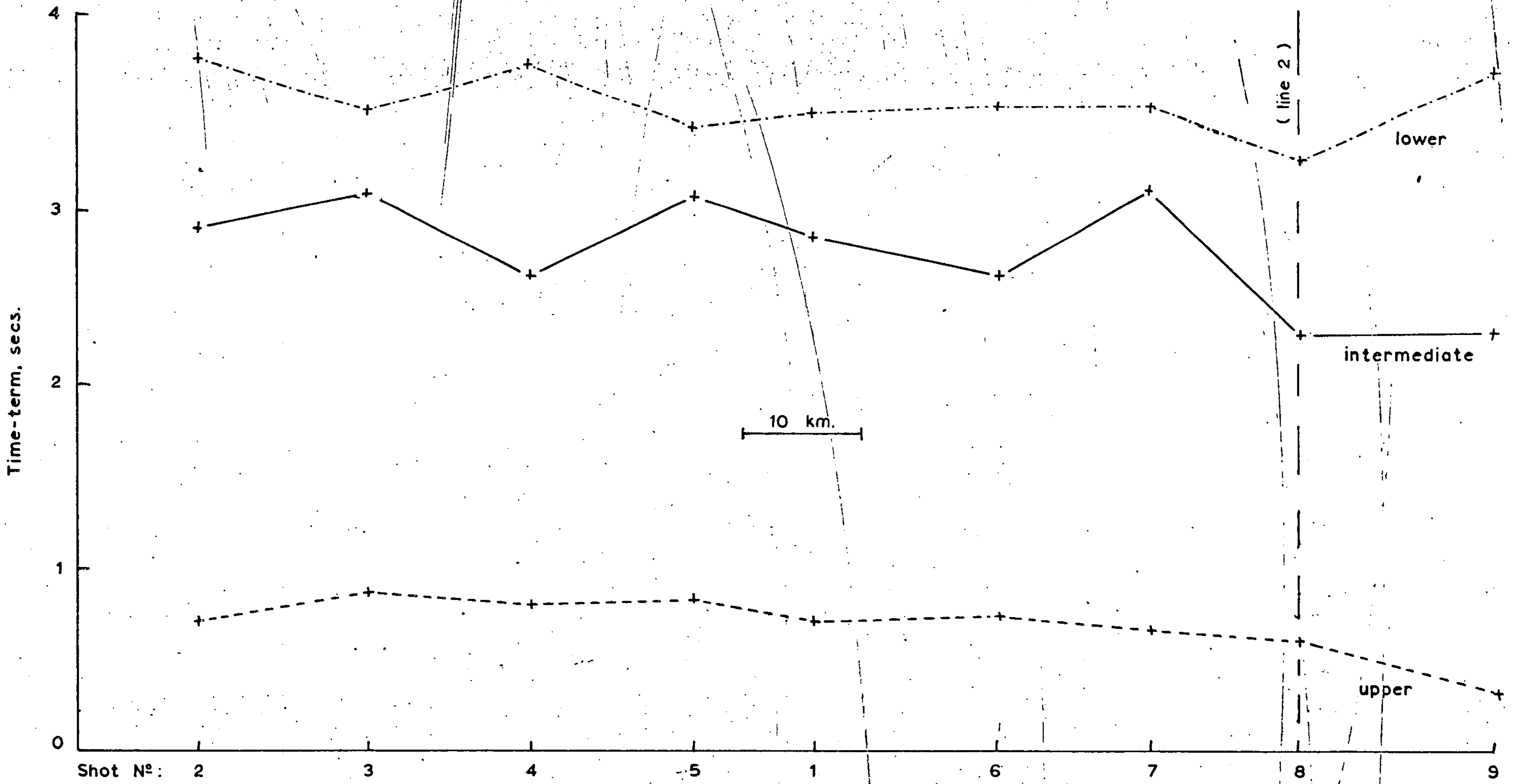


Figure (22) Time terms along Line 1, IRISH SEA

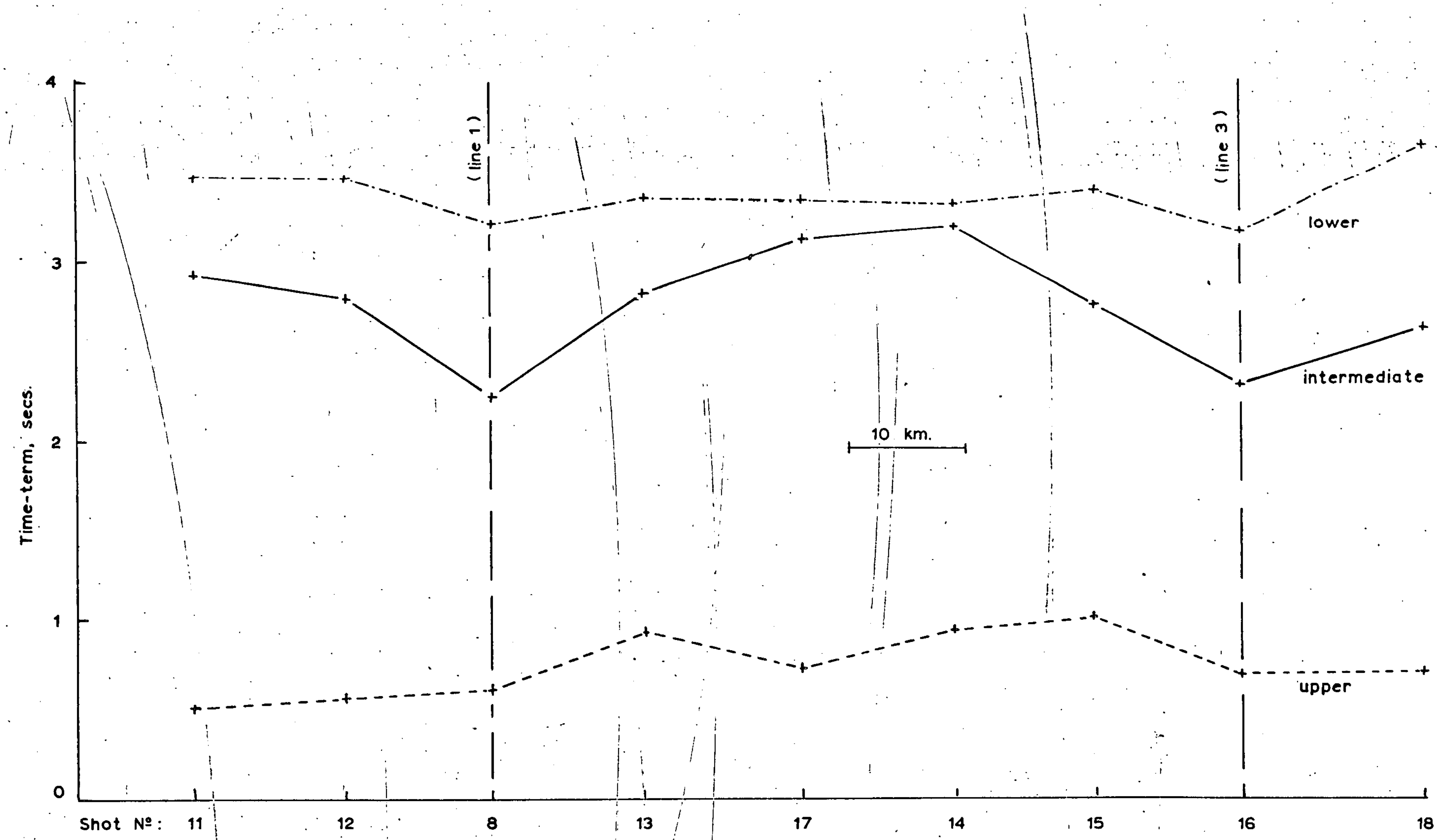


Figure (23) Time-terms along Line 2, IRISH SEA

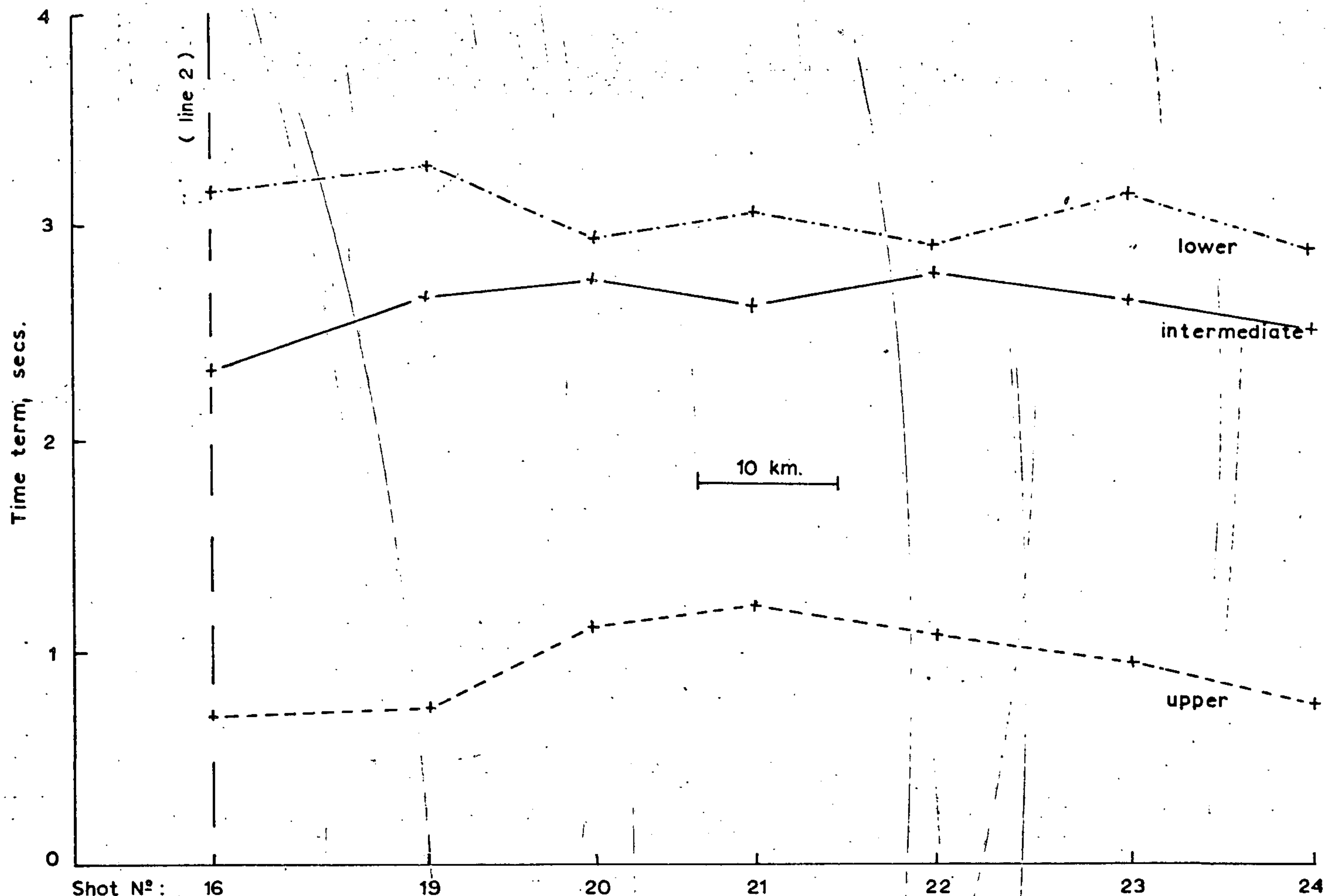


Figure (24) Time terms along Line 3, IRISH SEA

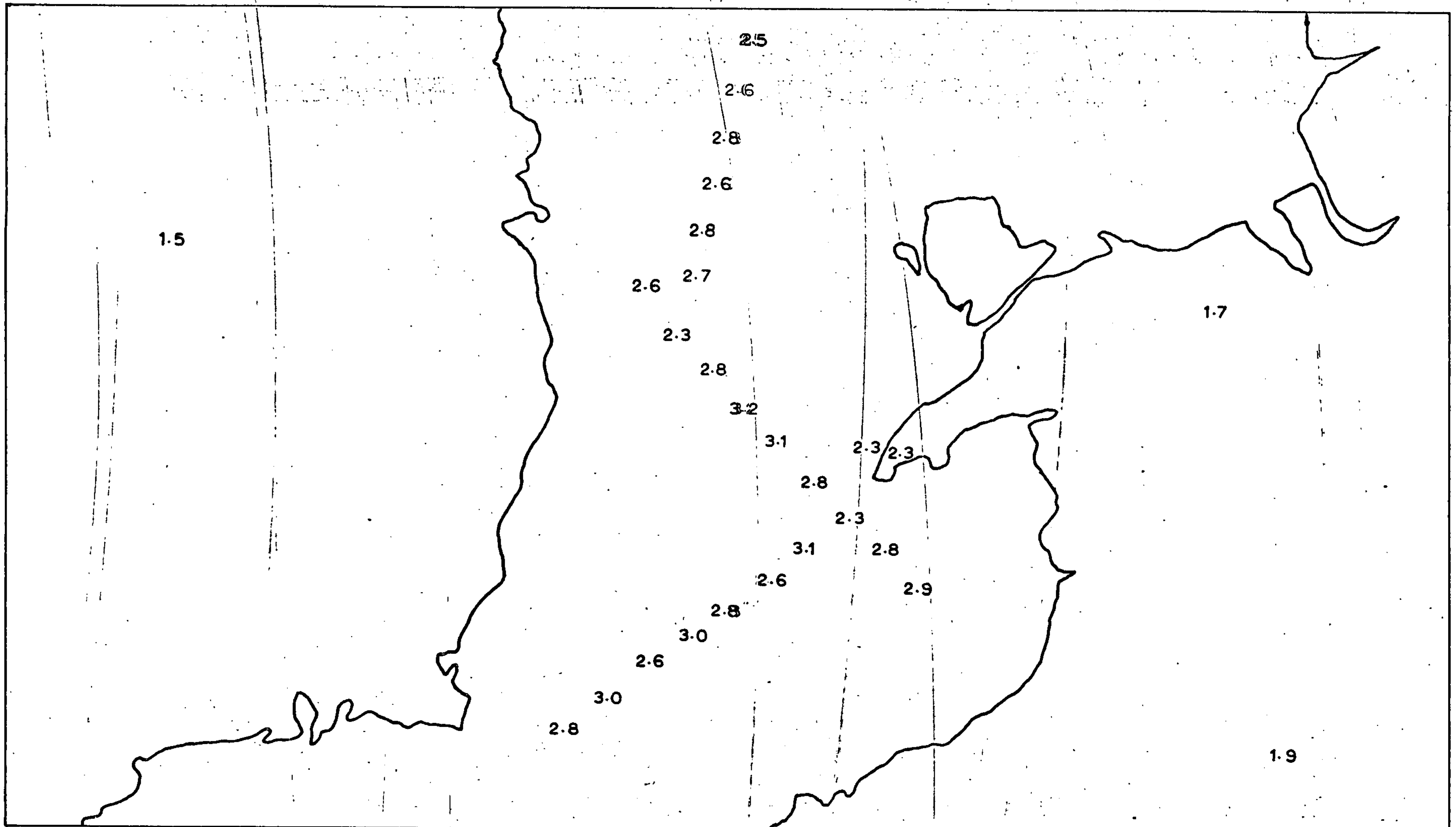


Figure (25) Distribution of time-terms for intermediate refractor, IRISH SEA project

7.3 The Nordzee project

During the summer of 1965, a refraction survey was conducted by the Vening Meinesz Institute of Utrecht, on a profile of approximately 180 km length off Flamborough Head (Figure (26)).

The "velocity-depth" method of shooting was employed, where the shooting and listening ships are progressively moved away from a common reference point. The advantage claimed is that it requires only half as many observations as a conventional reversed profile; at the price of obtaining no information on dip of the strata, it should provide a detailed picture of the distribution of velocity with depth, directly under the reference point. The great weakness is that it relies on plane-layering under the reference point, and gives no indication of the uncertainties (except by producing implausible velocities!)

If the assumption of plane-layering holds true, the interpretation should stand by itself without reliance on observations from other stations. For the upper layers the 1965 data yielded apparently reasonable velocity values, but for the M-discontinuity the data is less satisfactory; insistence on plane-layering would lead to a velocity in excess of 9 km/sec., and on the other hand if curvature is permitted the amount of curvature required to modify the velocity estimate to a more acceptable value is suspiciously high. In such cases additional observations from other stations become of great value. The Eskdalemuir array recorded all the shots of 300 lbs or larger, and although there is not an adequate data set to determine the Pn velocity reliably

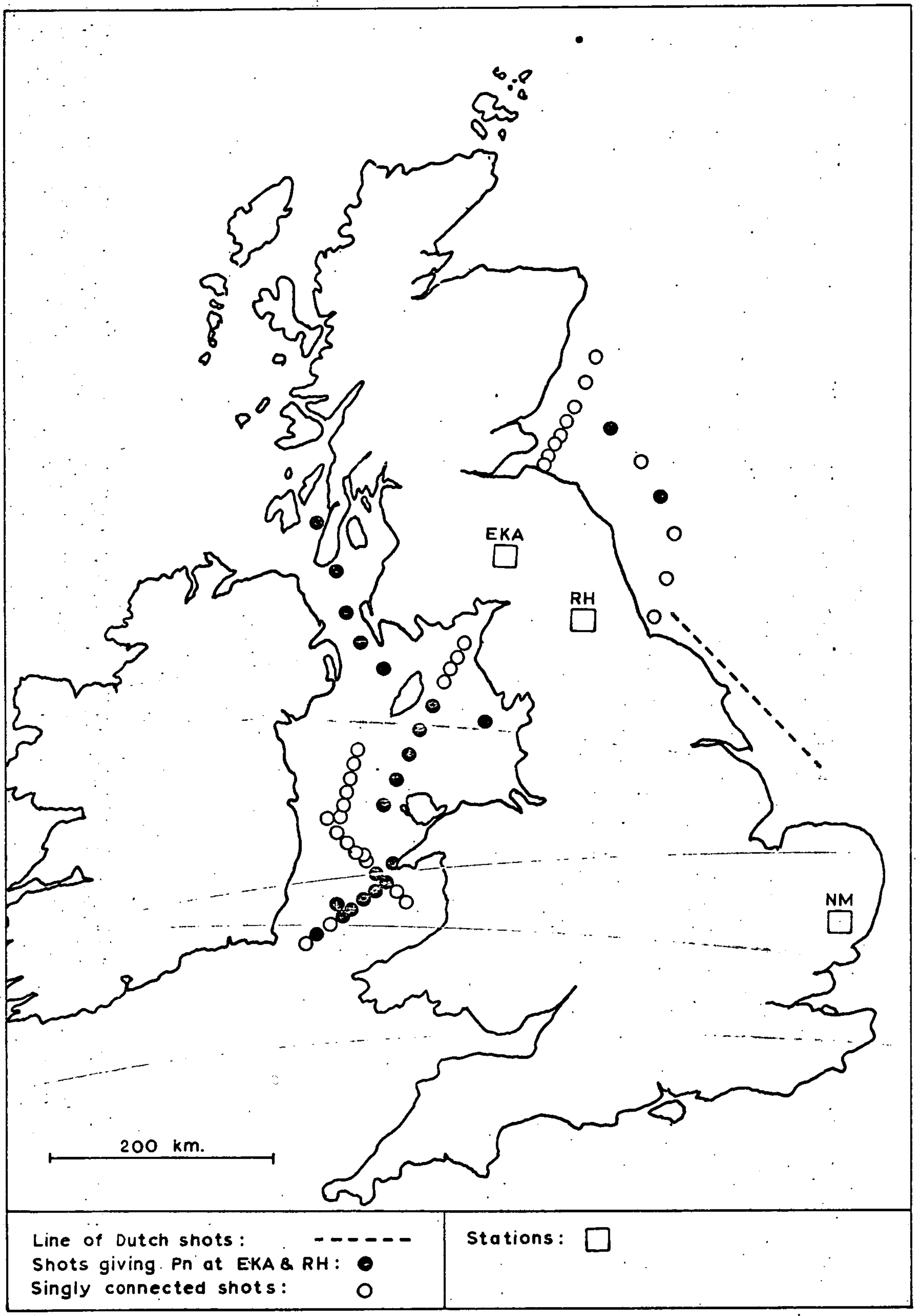


Figure (26) Arrangement of shots and stations relating to the NOORDZEE project it indicates that the depth to the M-discontinuity is reasonably uniform under the profile; to be more exact, it indicates that the Pn time-terms are either constant or changing smoothly with distance along the line.

Some of the larger shots were observed at two of the temporary stations in Wales, at Foel Gasyth and Painscastle, but the signal-to-noise ratio was rather poor and it is probable that the first readable motion on the records does not represent the true onset.

It may be that the breakdown of the velocity-depth method in the case of the M-discontinuity is simply due to scarcity of observations. As seen from Eskdalemuir, the cross-over distance for first arrivals of Pg and Pn is in the region of 170 km in the North Sea (in contrast to 120 km for the northern Irish Sea). The maximum range of observation in the velocity-depth data is only of this order, and in general the recognition of onsets near any cross-over point is somewhat unreliable as the two arrivals tend to interfere.

During October 1966 a further programme of explosions was carried out by the Utrecht group over part of the same line, with the objective of recording reflections from the M-discontinuity. As this offered a possibility of obtaining much-needed data to determine the Pn velocity, a temporary station was installed near Needham Market, in East Anglia, and it was hoped that the bore-hole seismometer at Rookhope would also be operated.

Since Eskdalemuir and Rookhope had both previously observed Pn for shots from the west, even a single shot from the east observed at both stations would have strengthened the velocity determination enormously. On the other hand for the Eskdalemuir/Needham Market pair, since no recordings had previously been made from the latter site it would have been necessary to observe at least two shots (some-distance apart) at both stations to give any velocity control.

In the end the exercise proved almost completely fruitless. The Rookhope station was unserviceable throughout the three weeks of the shots, due to a mechanical breakdown. The Needham Market site had an unacceptably high level of background noise during the daytime when the shots were being fired (as discussed in more detail in Section (6. 3. 2)). During the night-time the noise level dropped to a satisfactory value, but by the time sufficient data had been collected to prove this point it was too late to arrange permission from the Dutch authorities for shooting at night.

From the 1965 recordings at Eskdalemuir it was possible to estimate the maximum tolerable noise threshold for recognition of the onset, at various distances and charge sizes, and from the 1966 recordings at Needham Market it was possible to confirm the order of magnitude by comparing the peaks of those signals which were visible with similar ones from the Eskdalemuir recordings.

At present the most that can be done with the EKA travel-time observations is to apply the velocity of 8.09 km/sec. obtained from the

Seagull and Irish Sea shots (observed at EKA and RH), with a time-term of 3.04 sec. for EKA. This indicates a fairly uniform time-term value of 3.7 over the whole Noordzee line, compared with 3.3-3.4 sec. for the Irish Sea. In due course when estimates of the velocity distribution in the overlying material are available from the Dutch group, it may be possible to give a reasonable estimate of the crustal thickness along the profile.

7.4 The Jutland-Skagerrak project

7.4.1 General plan and published interpretation

During 1962, 1964, and 1965, a crustal refraction project has been carried out jointly by groups from Norway, Denmark and Germany, involving a long line of stations in Denmark with shots in the sea at each end and stations in Norway, as shown in Figure (27).

The principal source of difficulty in interpreting the refraction data is the presence of substantial thicknesses of low-velocity material under some of the survey points and not under others. This gives rise to considerable scatter if all the observations are combined to form a single plot of travel-time vs. distance for fitting by straight lines (corresponding to an assumption of plane layering across the entire area of the survey).

An interpretation in terms of straight-line fitting has been given by Hirschleber, Hjelme, and Sellevoll (1966). In it the above difficulty was recognized, and a method put forward to deal with it, taking the data shot by shot or station by station and assuming parallel travel-time curves.

This approach, if pursued further, would indeed lead to the technique already developed as the "Time-term Method", but in its elementary form it did not enable full use to be made of the data, and the resulting crustal model was unnecessarily sketchy.

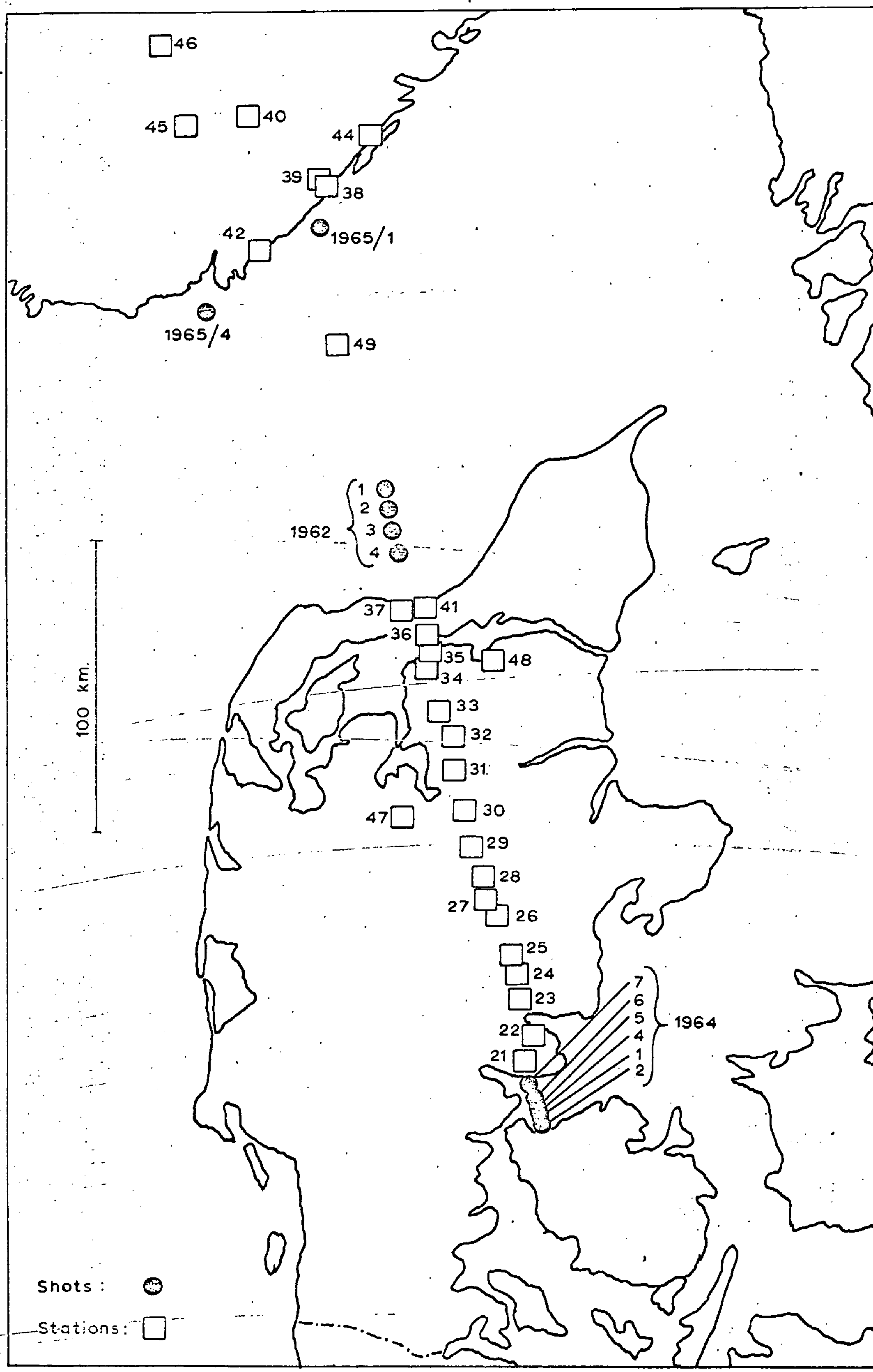


Figure (27) Arrangement of shots and stations for JUTLAND - SKAGERRAK project

The data turn out to be ideally suited to treatment by the time-term method, and the various observations which had been difficult to relate together do combine very satisfactorily, as indicated by low values for uncertainty of velocity and time-terms.

Hirschleber et al (1966) treated the observations relating to the shallower structure in two groups identified as "Pg" and "Pb". However, since the observations attributed to Pg comprise two short unreversed profiles (from which the true velocity cannot be determined), it was considered that there were not sufficient grounds for subdividing the data into two groups unless some other evidence of inconsistency emerged during processing.

7.4.2. Time-term interpretation

For the preliminary time-term analysis discussed here the structure was assumed to consist of an "upper refractor" (data sets 2053 and 2054), and a "lower refractor" (data sets 2034 and 2055) corresponding to the M-discontinuity.

The observations at station 49 for the lower refractor were at too short a distance to give critical refraction, and these were interpreted as wide-angle reflections to give an estimate of depth.

The striking feature which emerges is a major basin-shaped structure, as shown by the time-terms for the upper and lower refractors in Figure (28).

The first question to be considered is whether this basin exists only in the upper refractor with a flat lower refractor, or whether the latter has also some curvature. Even if the material overlying the upper refractor were assumed to consist entirely of very low-velocity material (say 1.8 km/sec.) this would not account for the variation in Pn time-terms sufficiently to make the lower refractor flat. It therefore seems more reasonable to apply an average velocity around 3.3 km/sec., and to calculate depths to a non-flat lower refractor; the results are shown in Figure (29).

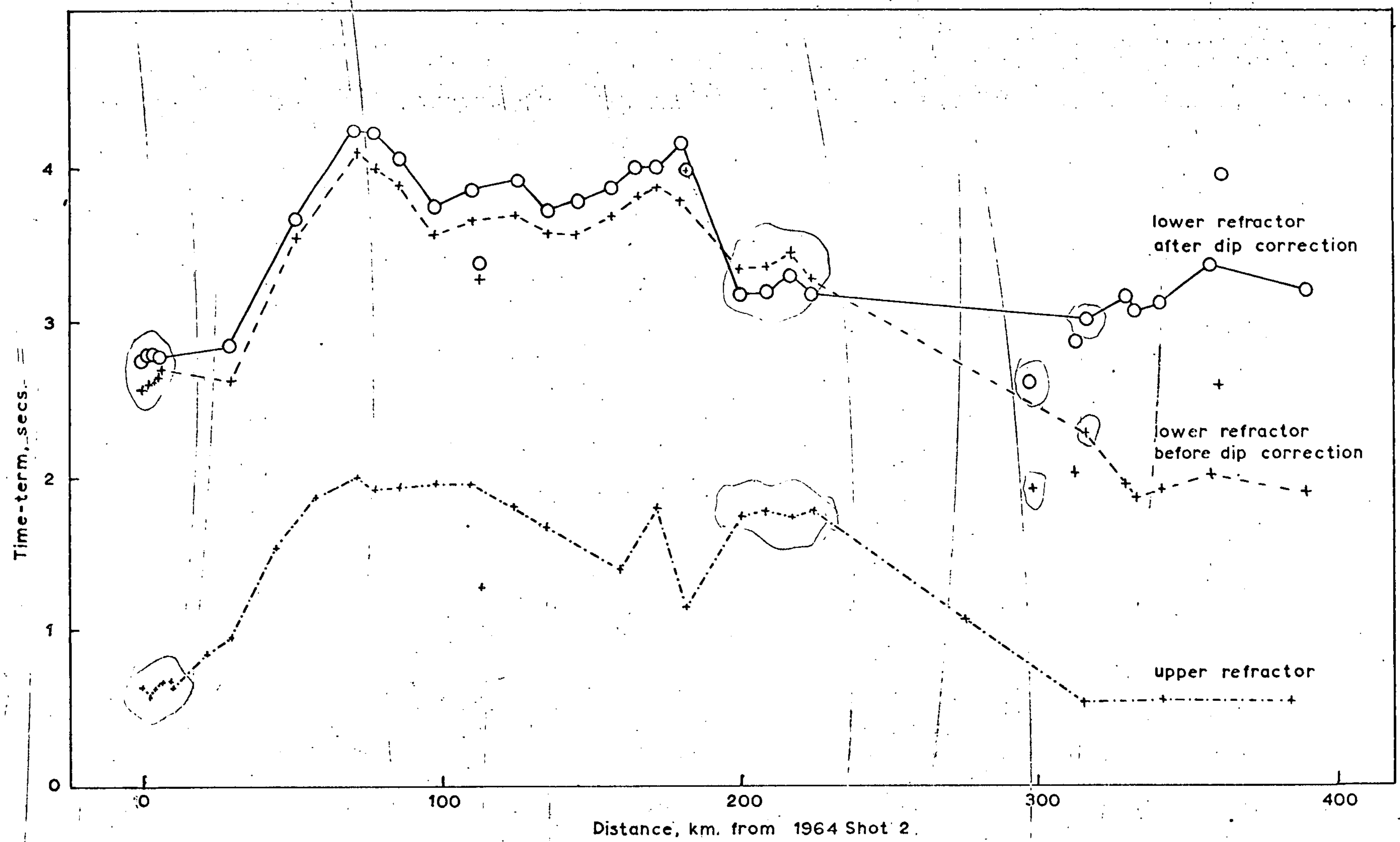


Figure (28) Time-terms along JUTLAND-SKAGERRAK profile

This first approximation presents two unexpected features: the Pn velocity of 7.8 km/sec. is much lower than that usually accepted for Northern Europe, and the crustal thickness at the Norwegian end is suspiciously small; other workers have suggested a crustal thickness of the order of 34-35 km as typical of Fennoscandia (Tryggvason, 1961; Sellevoll and Penttilä, 1964; Penttilä, 1965).

At this stage it must be recalled that the basic time-term method starts with the assumption that dip of the refractor is negligible. The presence of a major syniclinal structure would tend to produce an under-estimate of velocity, and this in turn would lead to low values of time-terms for those stations which are linked by only long-range connections.

The data present an interesting opportunity of applying the technique of correcting for dipping structure, as outlined in Section (2.5).

On the basis of a set of preliminary time-terms, depths to the lower refractor along the profile were calculated, and a smooth curve drawn to provide estimates of dip for each survey point, as shown in Figure (30).

Corrections to the travel-time and distance for each connection of data set 2034, were calculated, to make the revised data set 2055. (For the velocity ratios involved, the time corrections were in fact negligible compared to the distance corrections). This yields a more credible figure for Pn velocity, and the revised time-terms and depths are shown on Figures (28) and (29) respectively.

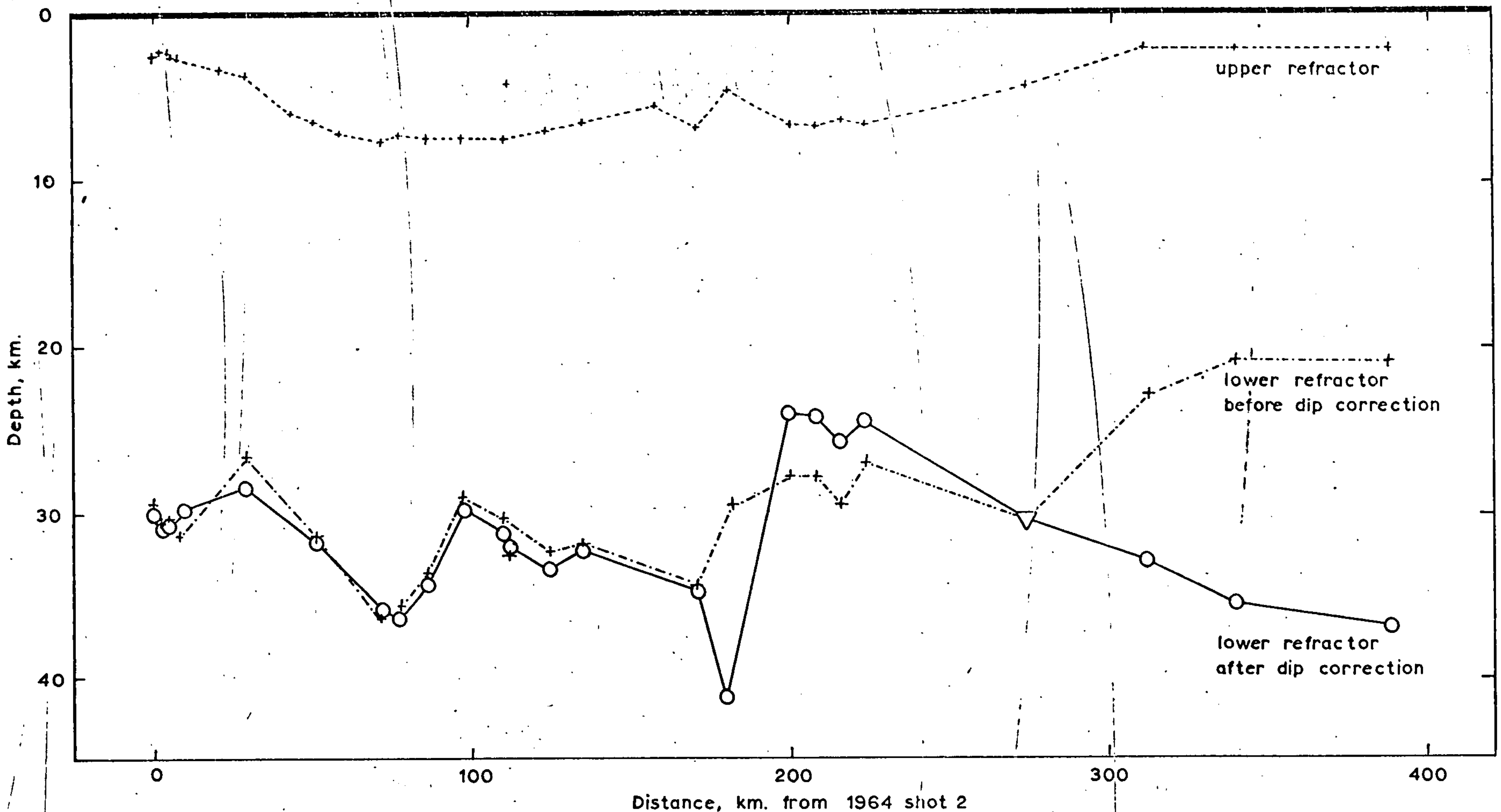


Figure (29) Depths along JUTLAND - SKAGERRAK profile

The structure for the southern part of the profile (the 1964 shots and Jutland stations with the exceptions of the most northerly) is not significantly altered, but the northern part now shows a fairly uniform dip of about 4^{o} northwards, with signs of some irregularity at the northern coast of Jutland.

The revised picture should be regarded with caution at this stage. The structure indicated for the northern part is quite different from that which formed the basis of the dip corrections, implying that a further iteration is necessary.

The irregularity associated with the most northerly Jutland stations may be the result of errors in phase identification; this portion of the profile poses a number of difficulties in interpretation for both the original straight-line solution and the time-term solution.

Although the revised Pn velocity is more satisfactory than the earlier figure, the suggestion of a discontinuity in the lower refractor at the north coast of Jutland is rather implausible. One would expect a feature of this extent to be also evident from gravity data.

The whole question of phase identification would benefit from a fresh assessment now that some approximation to the structure is available. The preliminary time-term runs relied heavily on the identifications assigned by Hirschleber et al (1966) with some minor alterations (apart from the merging of their "Pg" and "Pb" groups). It should now be advantageous to review the identifications in the light of the cross-over distances suggested by the preliminary model.

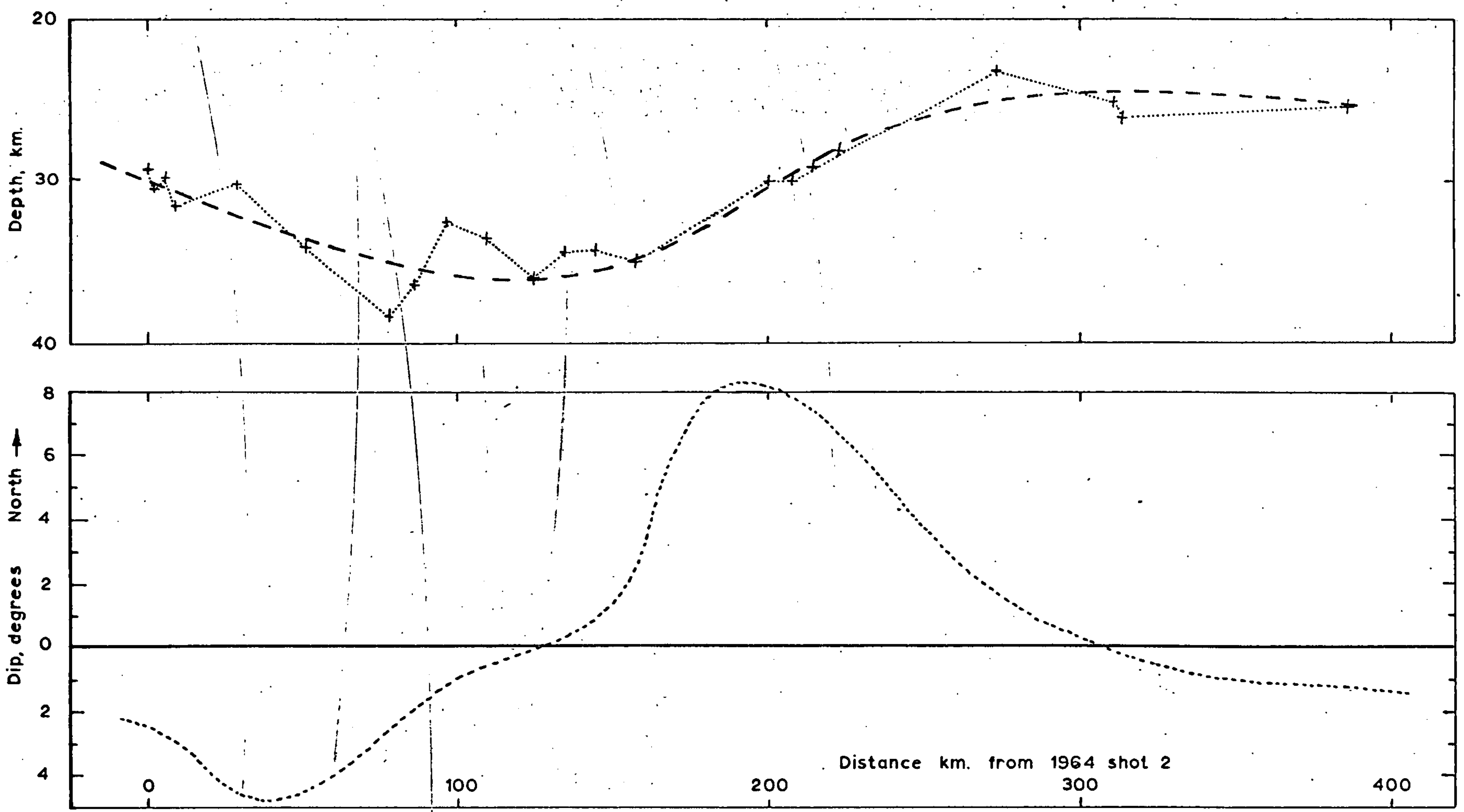


Figure (30) Preliminary estimate of dip of lower refractor, JUTLAND-SKAGERRAK profile

Additional data have recently been made available on the 1964 shots from observations with seismic reflection equipment on a site at a short distance to the south. These should provide a basis for reversed-profile investigation of the refractor identified as 6.1 km/sec. by the previous authors. It was also mentioned that observations were made by two groups from Hanover on a line from the 1964 shots toward the Danish-German border, and if the data from these is combined with the work already discussed the velocity distribution should be much better understood.

In the translation of time-terms into depth, a uniform velocity of 3.3 km/sec. has been used for the material overlying the upper refractor at all survey points. For the final interpretation, it should be possible to estimate the velocity distribution for each survey point separately in the light of local geological and geophysical evidence.

8. REVIEW OF CRUSTAL STRUCTURE

8.1 The Crust as a layered structure

The central assumption of the time-term method is the existence of continuous refracting horizons in the crust. Dipping structure and variations of velocity (either in horizontal or vertical directions) need present no serious difficulty, provided that the survey network is adequate both in number of sites and in distribution.

It is appropriate to reconsider the validity of this assumption in the light of the various refraction studies which have been discussed.

The Jutland-Skagerrak interpretation is on the whole very satisfactory, as indicated by the statistical estimates of uncertainty, and it would seem that the simplified 3-layer model which has been used is a reasonable approximation.

There are indications that a 4-layer model (or perhaps a 3-layer model with a vertical velocity gradient) would be an improvement, and the additional data now available should enable this solution to be further refined.

The British Isles interpretation, on the other hand, leaves a great deal to be desired. In particular the uncertainty in the time-terms to the upper refractor is of the same order as the average value of the time-terms, 0.6 sec.

It is, perhaps, significant that the scatter of the calculated values of time-term about a local mean is substantially less than the statistical estimate of uncertainty, and this could well be the result of horizontal variations of velocity. The analysis is rather sensitive to such variations on account of the relatively long range of observation, around 140 km; a change of only 3% in velocity is sufficient to change the travel time by over 0.6 sec.

On considering the possible composition of the "upper refractor", it is clear that there is a wide range of materials which may occur above and below it, and that the composition is liable to differ widely from one part of the region to another, if the surface geology is any guide.

If a discontinuity is definable at all (and in some cases there may not be an abrupt transition of velocity), the most reasonable division would seem to lie in sedimentary and low-grade metamorphic rocks for the overlying material, and high-grade metamorphic and igneous rocks for the underlying material.

Evidence both from refraction profiles and from direct velocity measurements suggests that the range of variation in velocity can be of the order of 15 - 20% (Day et al, 1956; Griffiths, King, and Wilson, 1961; Blundell, King, and Wilson, 1964).

Further evidence is provided in the recently published Tectonic Map of Great Britain and Northern Ireland (Institute of Geological Sciences (1966)), which shows sections of the upper 6 km of the crust. Even in areas where the structural interpretation is partly conjectural, such sections do provide a useful indication of the degree of complexity to be expected. (Figures (31) and (32)).

Having recognized the possibility of velocity variation in the upper refractor, one might question whether the "intermediate refractor" could be explained as regional variation of Pn velocity.

The intermediate velocity of 7.3 km/sec. is derived from observations at only the southern stations, and the velocity of 8.1 from observations at only the northern stations. If regional variation is to account for this it would need to be in excess of 5% per 100 km. Such variations have been observed in the western United States (Ryall, 1962; Herrin and Taggart, 1962), but it is considered unlikely that the deep crustal structure in the British Isles is equally complicated.

Some additional support for the existence of an intermediate refractor is given by Key, Marshall, and McDowall (1964), who studied the EKA array records of two local earthquakes (distances 190 km and 500 km). Their phase identifications include both P and S arrivals attributed to an intermediate refractor.

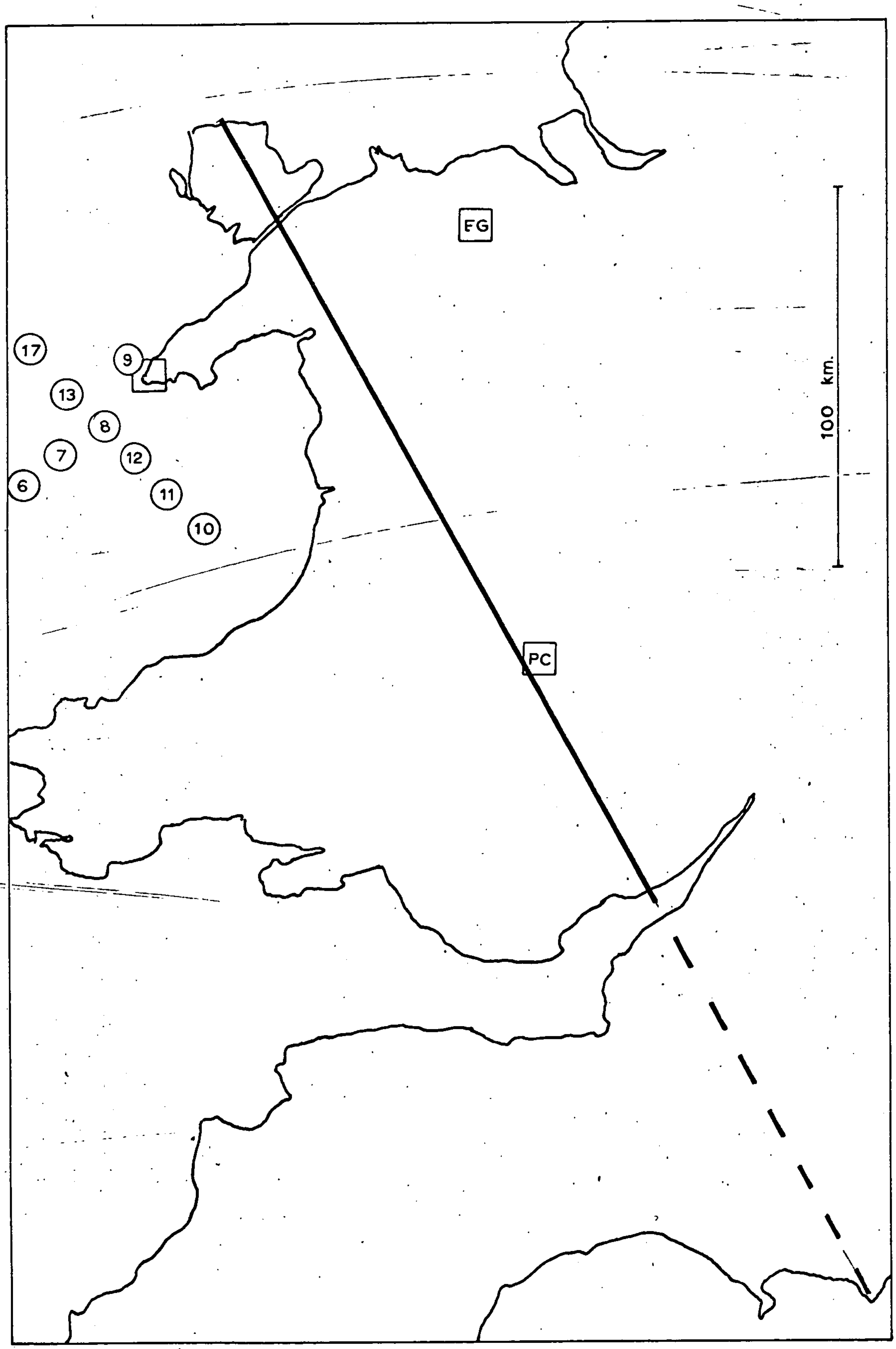


Figure (31) Map showing position of crustal section (Figure 32) and adjacent IRISH SEA sites

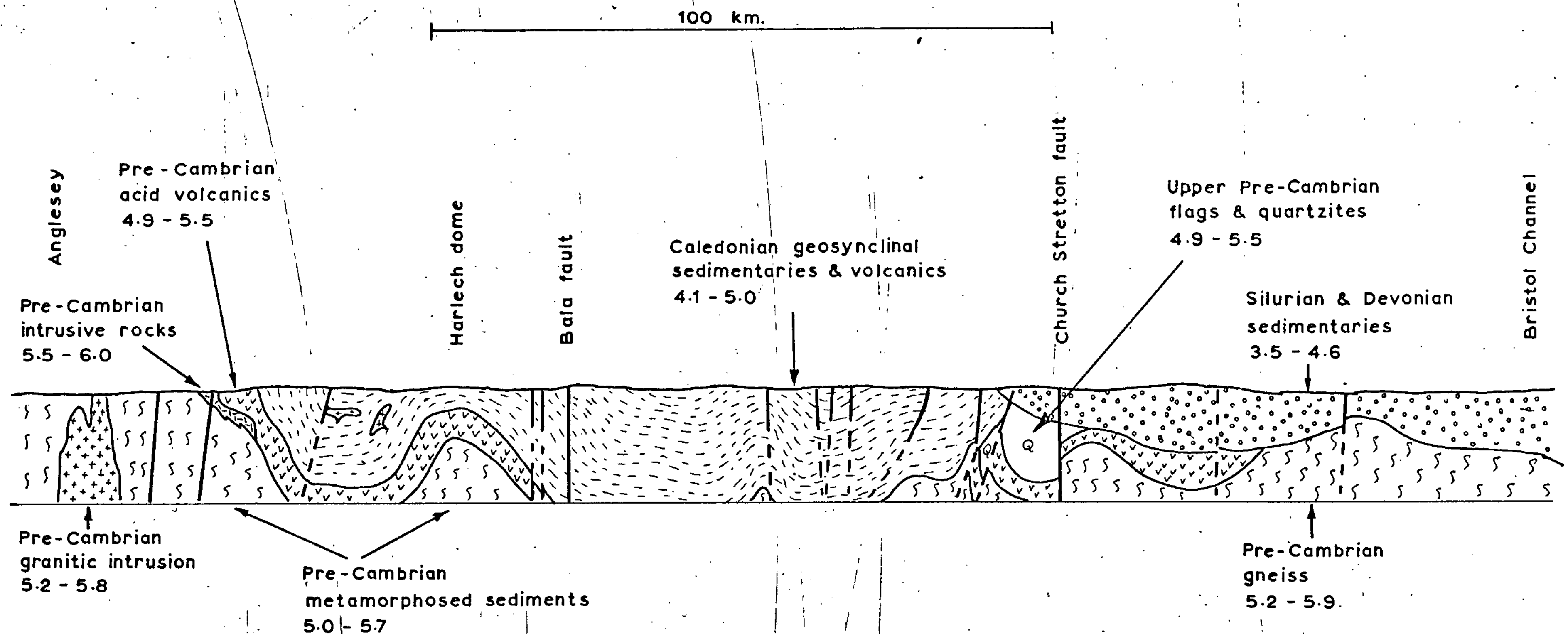


Figure (32) Crustal section across Wales to a depth of 6 km. (Based on I.G.S. Tectonic Map) indicating typical ranges of velocity (km. sec.)

8.2 The North Sea link

During the 1965 Norwegian shot programme, several of the large shots at Grimstad, off the south coast, were recorded not only at the Danish and Norwegian stations but also at the EKA array station, at distances of around 790 km.

Since the Pn time-terms for the shots and station are now known from the local time-term networks, it becomes possible to obtain a good estimate of the Pn velocity under the North Sea. This leads to a value of 8.23 $\pm$ 0.07 km/sec., which is in reasonable agreement with the values obtained for the British Isles and Scandinavia separately.

These shots formed part of a reversed profile of length 1400 km, from which another accurate determination of Pn velocity should be available in due course for comparison.

8.3 Future Activity

The various explosion projects conducted around the British Isles have so far failed to give a satisfactorily close estimate of Pn velocity, because of the poor distribution of survey points.

The most direct way to improve this situation would be to fire several shots on the eastern extension of a line from EKA to RH, at a suitable distance to give Pn as reliable first arrivals at both EKA and RH (which have already recorded Pn from a westerly direction).

By extending the scope to a profile in the North Sea, a station in East Anglia should provide observations to reverse the profile. This was the intention in establishing station NM for the 1966 Noordzee shots; on that occasion it was unsuccessful because of the high level of background noise during the day, but there is every reason to believe that a similar programme with shots fired at night would be successful.

A further possibility would be to install a temporary station near Waterford (a site which has already been used in refraction work and is to be used again in the future), and to fire a fresh series of shots along the Western Line of the Seagull project to form a reversed profile. This should provide additional insight into the nature of the intermediate refractor.

As regards the upper refractor in the British Isles, the chief problem appears to be scarcity of observations. If station MA were reoccupied and a fresh station established near Dublin while shots were repeated on the sites of the 1965 Irish Sea shots, this would be of great benefit. If it were also possible to use sonobuoy stations in the middle of each line as originally envisaged, a much stronger set of data would be obtained.

Since the upper part of the crust is admittedly very non-uniform, there would be considerable advantage in concentrating attention on a simple profile with closely-spaced survey points. There is also something to be said for taking the line of a profile which has already been published as of tectonic interest, namely the Welsh profile of the Tectonic Map (Institute of Geological Sciences (1966)). This passes close to station PC and to the Harlech dome area where considerable research has already been carried out by the University of Birmingham.

In due course it might prove possible to reverse the profile from shots in the Bristol Channel and English Channel. (Note that the time-term method accommodates survey points lying some distance off the profile without difficulty). Quarry blasts might also be utilised if of adequate size.

From the information gathered on threshold levels so far it appears that 300 lb depth charges are scarcely adequate for ranges greater than 200 km. If faced with a choice between a few large shots and a greater number of standard depth charges, then while the former might be of value in obtaining a better determination of

Pn velocity, one feels that a greater need at present is for the more comprehensive picture of the upper crust which could be obtained from the latter.

In the Jutland-Skagerrak area, the Skagerrak section is not so well covered as the Jutland section. It should be of great value to fire additional series of shots to fill in the gaps, re-occupying some of the stations previously used. This would also provide an opportunity of arranging suitable interchange connections, e.g. shots on the site of the hydrophone station 49 and close to station 44.

9. REVIEW OF LITERATURE

When one looks into the historical development of the "time-term" concept, the predominant impression is of continuous neglect - perhaps largely a consequence of inadequacy of information exchange.

On the one hand, as early as 1935 the basic features of the technique for dealing with departures from plane-layering over extended areas had already been outlined. On the other hand, as recently as 1964 leading experts on refraction surveying have gone so far as to state:"The time-term approach almost completely sacrifices the opportunity available in detailed reversed profilesto observe the propagation of waves and to relate these waves to crustal properties". (Pakiser and Steinhart, 1964).

However, even at the time when the above statement was published, there was beginning to take shape a striking demonstration of the power of the time-term technique, which was to persuade Steinhart to revise his attitude. A very extensive joint refraction project had been undertaken during 1963 in the Lake Superior region, involving a total of 78 shots and 55 stations (Steinhart, 1964). In due course the bitter fact emerged that "detailed reversed profiles" were not adequate to cope with the complexities of the structure; eventually there were published two papers which succeeded in interpreting the data by the aid of the time-term method, and Steinhart was one of the joint authors. (Smith, Steinhart, and Aldrich, 1966), (Berry and West, 1966).

Most of the development of the time-term idea has taken place in the field of seismic exploration of fairly shallow structures, while studies of earthquakes and deeper crustal structure have tended to accept the limitation of plane-layering, even though it is unnecessary (and often manifestly inappropriate).

The foundations of the time-term method were laid down in an early description of seismic refraction techniques by Edge & Laby (1931), as part of the official report of the Imperial Geophysical Experimental Survey. Considering the scarcity of relevant published material at that time, this work represented a very substantial advance.

The technique of "fan" shooting (nowadays referred to as "arc" shooting), was employed for recognition of departures from plane-layering in tracing the course of deep leads in the bed-rock surface.

To obtain unambiguous estimates of depth to a non-plane refractor, the "method of differences" was introduced, involving observations of the travel-times between three points in a straight line.

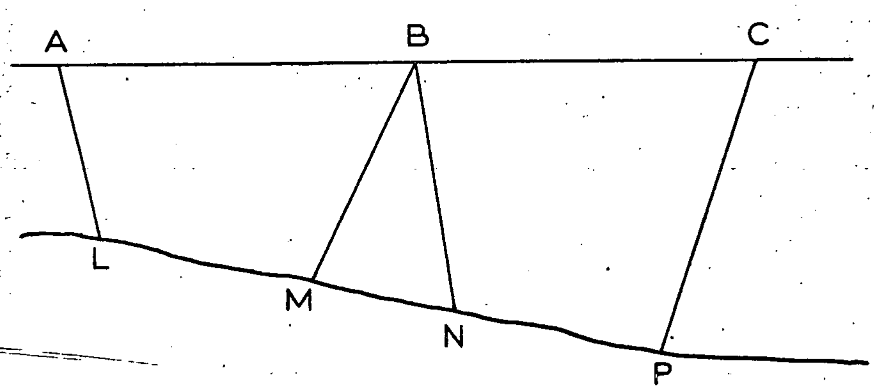


Figure (33). Ray paths considered in the "method of differences".

The three paths for which the travel time must be observed are ALMB, BNPC, ALPC. Then the time for the wave to travel between point B and the refractor is given by

$$T_{MB} + T_{BN} + T_{MN} = T_{ALMB} + T_{BNPC} - T_{ALPC} \qquad (52)$$

To simplify the calculation, it was first assumed that the ratio between the velocities above and below the refractor was so great that the path in the upper medium could be taken as approximately vertical, and therefore the term T_{MN} could be neglected; further T_{MB} would be taken equal to T_{BN}. Provided the velocity in the upper medium is known, the depth under point B is obtained directly from T_{MB} (or T_{BN}).

By continuing the process with a fourth point, the depth under point C may be determined, and similarly for further points.

An estimate of the velocity in the lower medium may be obtained from

$$V = \frac{\text{Distance NP}}{T_{BNPC} - T_{BN} - T_{PC}}$$

Once this is known, an estimate may be made for the term T_{MN} which had been neglected, and a correction applied.

There was, however, no real need for the assumption of vertical paths in the upper medium (which has been invoked by various other workers subsequently), as the arithmetic remains simple even with inclined ray paths. The quantity which has since been defined as the "time-term" is equivalent to

$$T_{MB} + \frac{1}{2}(T_{MN}) \quad \text{or} \quad T_{BN} + \frac{1}{2}(T_{MN})$$

i.e. half the left-hand side of equation (52).

By 1940 there were two papers which together contained in essence most of the features which make up the time-term method as now presented: (Bullard et al, 1940), (Gardner, 1939).

Bullard and his co-workers were concerned with relatively shallow structure (at depths less than 500 metres), and they found that in general the assumption of plane layering was satisfactory for the short spreads employed; in fact the strata were so nearly horizontal (the maximum dip being 3.9°), that unreversed profiles were adequate for the majority of the work.

As a consequence, there was not felt to be a great need for extension of the main analysis to non-plane structure. This was, to say the least, unfortunate, since they had a suitable technique available and did in fact apply it to a type of "sedimentary correction", as outlined below.

They recognised the value of partitioning the total travel time into a time-term at each end and a main-layer travel time in the middle. Since they were dealing in this case with a upper-layer velocity considerably less than that of the lower layer, they were able to assume that the path in the upper layer would be vertical, without introducing significant error.

If it had been necessary to allow for velocity ratios nearer unity, the authors had already derived an expression for the time spent in passing through any layer, in the form:

$$t = \frac{h}{v_1}\sqrt{1 - {V_1}^2/{V_2}^2}$$

The sedimentary corrections were determined by firing a small charge at each end site of a line of N geophones in turn and observing the travel-time to each of the other (N - 1) sites. This yielded 2(N - 1) observational equations of the form:

$$t_{ij} = a_i + b_j + \Delta_{ij}/v$$

(following the notation of section (2.2)).

It was stated that "if an approximate value of v is known from more distant shots these 2(N - 1) equations determine the 2N constants" (the a_i and b_j). It is surprising that they should have overlooked the fact that v would be very well determined from the same equations without the need for more distant shots.

The idea of surveying extended areas by a combination of arc and profile shooting, which had been suggested by Jones (1934), was more fully developed by Gardner (1939).

The latter recognised the usefulness of a triangular configuration of profiles in resolving the inherent indeterminacy between shot and station time-terms (see Section (2.7)) to obtain absolute values for the time-terms, while on the other hand an arc configuration yields more directly an indication of the relative values of the time-terms. By combining the two systems in an overlapping pattern, absolute values could be derived for the time-terms of all survey points.

Gardner also advocated that investigations should proceed in two distinct stages: preliminary shooting to determine whether or not suitable refractors are present and to obtain some necessary information on velocities and cross-over distances, followed by detailed mapping operations with a network of survey points chosen in the light of this information.

Gardner's proposals became severely complicated as a consequence of his inordinate preoccupation with "offset points" rather than survey points. (The"offset point" is the point where the critically-refracted ray path passes through the refractor).

He suggested that in planning a network of inter-connected profiles (or arcs), one should arrange for the common points of the net to be offset points on the refractor instead of shot points or stations on the surface (i.e. the survey points should be so positioned that the same offset points would apply for several observations from different azimuths). He regarded this as a worthwhile refinement, but on closer examination the weight of argument would seem to be against it.

The offset points could be held fixed only if the structure in the vicinity were already known.

In practical terms, for a substantial field operation it would be highly inconvenient to occupy all the sites involved for fixed offset points, or to move the survey points for each observation.

With small-scale seismic prospecting it is usually a relatively easy matter to adhere to a formal geometric pattern of survey points, but with large-scale crustal studies this may often prove to be impossible.

Both for fixed survey points and fixed offset points, there there is an implicit assumption of horizontal uniformity, i.e. velocity varying only with depth (perpendicular to the refractor) within the critically-refracted ray cone. In the case of a fixed survey point the apex of this cone is uppermost, at the survey point, whereas for a fixed offset point the apex is downwards, at the offset point. Since, in general, materials near the surface tend to be more varied than those at depth, the assumption of horizontal uniformity will be more valid for fixed survey points than for fixed offset points.

In the contributions of both Gardner (1939) and Bullard et al (1940), the refractor velocity was considered to be adequately defined by straight-line fitting of the travel-time data (i.e. an assumption of plane-layering), although Jeffreys (1935) had already shown how the velocity could be determined quite directly from the data while permitting the structure to depart from plane-layering.

The most significant advance in the time-term method came with the extension to a general network of survey points, yielding a number of observational equations greater than the number of unknowns and employing a least-squares technique to derive a "best-fitting" solution. (Scheidegger and Willmore, 1957; Willmore and Bancroft, 1961).

In this treatment the refractor velocity was assumed to be constant (i.e. uniform horizontally across the entire area; and unaffected by the range of observation).

For structures where the velocity increases uniformly with depth the ray paths are arcs of circles, and the travel-time vs. distance relation is a curve rather than a straight line (Willmore, Hales, and Gane, 1952).

The modification of the time-term method to fit such a curve has been dealt with by Smith, Steinhart, and Aldrich (1966), who applied it to the data of the Lake Superior project. However, their results indicated that the effect was not statistically significant for their data.

Whereas the least-squares solution for the time-terms involves a smaller number of unknowns than the total number of observations (giving improved accuracy, together with statistical information on the uncertainties), an alternative approach makes use of all the observations directly to give detailed measurements of the local velocity in the refractor between pairs of survey points, in addition to values for the individual time-terms (Hagedoorn, 1959; Hawkins, 1961).

The "Plus-Minus" method of Hagedoorn is essentially a refinement of the "method of differences" originally put forward by Edge and Laby (1931). The most important advance was the recognition of the inherent simplicity of derivation for both the time-terms and the local velocity in the special case of a reversed profile for which the end-to-end travel-time is observed.

By picturing the wave path in terms of wavefronts rather than rays, Hagedoorn's approach is of value in giving an appreciation of the propagation mechanism and the volume of material involved in transporting the energy which constitutes the onset.

In the application of time-term technique to practical field surveys, a leading part has been taken by the Dominion Observatory, Ottawa (Willmore and Scheidegger, 1956; Milne and White, 1960; Willmore, 1963; Willmore, 1965.). However, the value of the technique has not been widely recognized, and to date no complete project has been planned and executed to meet its specific requirements.

The 1965 Irish Sea project was planned in such a way, but it is at present incomplete because of the scarcity of observations at short ranges.

The most complete demonstrations of time-term analysis have come from the Lake Superior project (Berry and West, 1966; Smith, Steinhart and Aldrich, 1966). Although certain features such as distribution of survey points and provision for interchange observations fall short of the ideal, this is compensated to some extent by the unusually large amount of data.

In Scandinavia there is now the prospect that data from a number of separate experiments can be combined to give a comprehensive time-term survey of the area.

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ABSTRACT OF THESIS

Name of Candidate: RAYMOND PARKS

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Title of Thesis: Seismic Refraction Networks in the study of the Earth's Crust.

This dissertation deals with the interpretation of the crust as a series of layers, each of uniform velocity but not necessarily plane.

The ray paths within the structure are defined by Snell's Law from the velocity ratios. By observing travel-times between several explosions and several seismometers, and relating these to the ray paths for critically-refracted waves, the structure can be interpreted as a characteristic "time-term" to each interface for each site and a propagation velocity for each layer.

Where the number of observations exceeds the number of unknowns, a "best-fitting" solution is obtainable by least-squares techniques, and further refinements permit correction for the effect of dipping structure and curved ray paths.

The network configuration is not restricted by the analysis to formal patterns (e.g. straight lines), and figures of merit are available for evaluation of the quality of fit, the contribution of each observation, and the effectiveness of the areal spread. Comparisons of the suitability of various possible configurations are made, and statistical expression for the uncertainties of the solution are presented.

Techniques are developed for the preliminary assessment of data and the refinement of solutions by a progression of computer-aided judgements, and illustrated by examples. A computer programme has been developed in Atlas Autocode.

Practical aspects of field equipment are discussed, with various considerations of signal quality in relation to background noise.

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Use other side if necessary.

The method is applied to data from four separate projects in the British Isles and Scandinavia. The results indicate that the upper portion of the crust in the British Isles departs considerably from uniform layering, but an intermediate refractor (of velocity 7.3 km/sec) is clearly observed in the south of the region; a lower refractor is observed, although the velocity is not well determined. In Scandinavia a 3-layer model gives a satisfactory approximation, with a well-defined basin structure.

KEYWORDS: SEISMIC
REFRACTION
NETWORKS
CRUST