

ZONE ANALYSIS OF DRYING
AND OTHER PROCESSES
IN
PACKED BEDS OF SOLIDS

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CHAPTER INON-CATALYTIC GAS-SOLID REACTIONS IN CHEMICAL ENGINEERING1.0 Introduction

Contact between substances to allow exchange processes is a fundamental requirement in many chemical engineering operations. The contact of a gas with a solid in a bed of particles is one such important operation involving two phases. A bed is a solid matrix of particles containing voids through which a fluid may flow. The solid matrix may be fixed or moving or may be expanded (fluidised) owing to the drag of the fluid passing through the voids becoming equal to gravitational forces acting on the bed in its settled position. The types of bed are commonly referred to as fixed, moving or fluidised beds.

Gas-solid operations hold a dominant position in the field of reactor design owing especially to the number of processes using packed beds of catalyst. Non-catalytic gas-solid reactions are important for example in the metallurgical and cement industries whose problems are becoming regarded more and more as chemical engineering problems. However, the large number of catalytic processes has perhaps focussed attention upon this type of process to the detriment of the non-catalytic processes. This distribution of effort has been promoted also by the greater complexity of non-catalytic

reactions. In some way to remedy this situation this project is restricted to the analysis of non-catalytic gas-solid reactions. The essential feature of a non-catalytic gas-solid reaction is the creation of a reaction zone or zones which move into the unused material since at any one location the reacting solid becomes exhausted. This complicates the temperature and concentration profiles along the reactor and correspondingly increases the difficulty of the analysis. The moving bed equivalent to this moving zone system has the solid phase moving into the stationary reaction zone. In many industrial gas-solid reactions a number of reactions may occur either simultaneously or consecutively so that further complications arise. Any analysis therefore which can simplify such systems is of interest even if the assumptions made in carrying it out are valid in some systems only.

While it is of interest and important to understand the basic mechanisms of physical phenomena, an engineering analysis should aim at the prediction of essential design parameters rather than fundamental parameters. To build a working reactor for example, the essential design parameters are the cross sectional area and length of the reactor and the mean residence time the reactants must remain in the reactor to accomplish the desired exchange process.

TABLE I

Some Commercial Packed Bed Processes *

The directions of the bulk flows commonly used are shown with reference to an external observer G_s for the solid G for the gas - a stationary phase is indicated by the symbol (O)

	G	G_s
Heat transfer - pebble bed heaters and coolers	↑	↓
Limestone calcination in vertical kilns	↑	↓
Low } High) temperature coal carbonisation	↑	O
Zinc sulphide oxidation D-L sinter strand	↓	→
ZnO reduction in vertical retorts	↑	O
Calcium cyanamide from CaC_2 with N_2	↑	O
Lead } Copper) sulphides reduced to oxides	↑	↓
Copper sulphides to sulphates	↑	↓
Chain grate stokers	↑	→
Blast furnaces	↓	↓
Grain driers	→	↓
Ion exchange, adsorption, gas chromatography	↓	O

* see Kirk and Othmer - Encyclopedia of Chemical Technology

These design parameters are obtained from a mathematical simulation of the process. A complete simulation for non-catalytic processes is so complicated that it is unlikely that the equations can be solved and so simplifying assumptions must be made. For large diameter beds with approximately uniform flow over the cross sectional area it is frequently assumed that operation is adiabatic. The small diameters necessarily used in laboratory apparatus, further complicated by the very sudden temperature changes in non-catalytic processes, make the design of a suitable small scale apparatus very difficult.

1.1 Commercial Use of Packed Beds

Many industrial gas-solid reactions in which the solid reactants become exhausted take place in packed beds; for instance the oxidation of zinc sulphide, limestone calcination, and the reduction of copper ores. Table 1 lists some present industrial uses of packed bed systems and Venner (1955) gives a list of processes which might be carried out in moving packed beds in the future. A number of different flow arrangements for the phases are encountered which are also shown according to the notation given at the head of the table.

The details of the beds used vary but within such systems exothermic and endothermic zones may be identified

together with accompanying heat transfer regions. In vertical kilns for limestone calcination, for instance, the zones are well defined; the position of the burners, the gas recycle offtake and the exhaust gas ports marking respectively the end and beginning of the calcination zones and the solids preheating zone. In multihearth furnaces the zones are physically separate, the solid moving radially in each zone before dropping to the succeeding zone. Each zone is separately fixed for precise control of conditions. The design of such packed bed systems is however very much empirical still being largely based upon previous experience.

Preparation of the burden for a number of vertical shaft furnaces is carried out on Dwight-Lloyd sinter strands. In this system the top of the bed is ignited and then air is sucked down through the bed. An exothermic reaction supplies heat to the gas which transfers it deeper into the bed heating the solid and hence sustaining the reaction. The reaction zone then propagates towards the bottom of the bed. During this process the solid is carried along on the moving grate horizontally so that the reaction zone inclines at a slight angle to the horizontal when viewed externally. Fresh solid is introduced at one side of the grate and the exhausted solid removed at the other. The exothermic process may be inherent in the sintering operation, as in zinc sulphide oxidation, or the reaction heat may be supplied by mixing coke or coal with the

*
I In any gas- solid reaction system: regions may be distinguished in which separate processes take place; each region in which one process occurs is defined as a zone. The process may be a combination of phenomena or a single phenomenon. For instance, drying and heat transfer, the former process is a combination of heat and mass transfer phenomena, the latter process is a single phenomenon. The zone boundaries are indicated by characteristic flow parameters the boundary values of which are representative of the start and finish of the zone process.

raw mix. As the beds are generally wide (6ft.) compared to the depth of solid (8") used, the system is approximately adiabatic in a horizontal plane. Furthermore if the grate speed is slow compared with the velocity of propagation of the reaction zone parallel to gas flow, we may neglect the solid cross feed velocity perpendicular to the gas flow and consider the system as seen by an observer travelling on the grate; which is an unsteady state one dimensional system aligned parallel to the direction of gas flow. Systems of chain grate stokers are closely allied to the Dwight-Lloyd strand and similar considerations apply to them. Carman (1957) gives a survey of the different arrangements used in these operations. Similarly, the vertical shaft kilns described briefly above may be considered as one dimensional steady state systems with a counterflow of solid and gas.

Common to each of these systems however is that the temperature and reactant concentrations may be expressed as functions of distance through the bed and time, or distance alone. The shape of these profiles, of temperature and concentration v. distance, can provide the basis for an arbitrary division of the system into zones.* In some systems the zones so defined will be the same as those defined by the structure of the reaction vessel. Even when this does not occur the division into zones enables the

* see opposite

reaction and exchange processes to be considered individually rather than simultaneously.

In fig. (1) is given an estimated typical temperature profile through a bed of wet zinc sulphide undergoing oxidation at some time after the reaction is well established. The notation used in this figure will be retained throughout the manuscript for reference to profiles, that is, the plane of cross-section at the gas inlet to the bed is numbered zero and arbitrary zones are defined between successive numbered cross sectional planes.

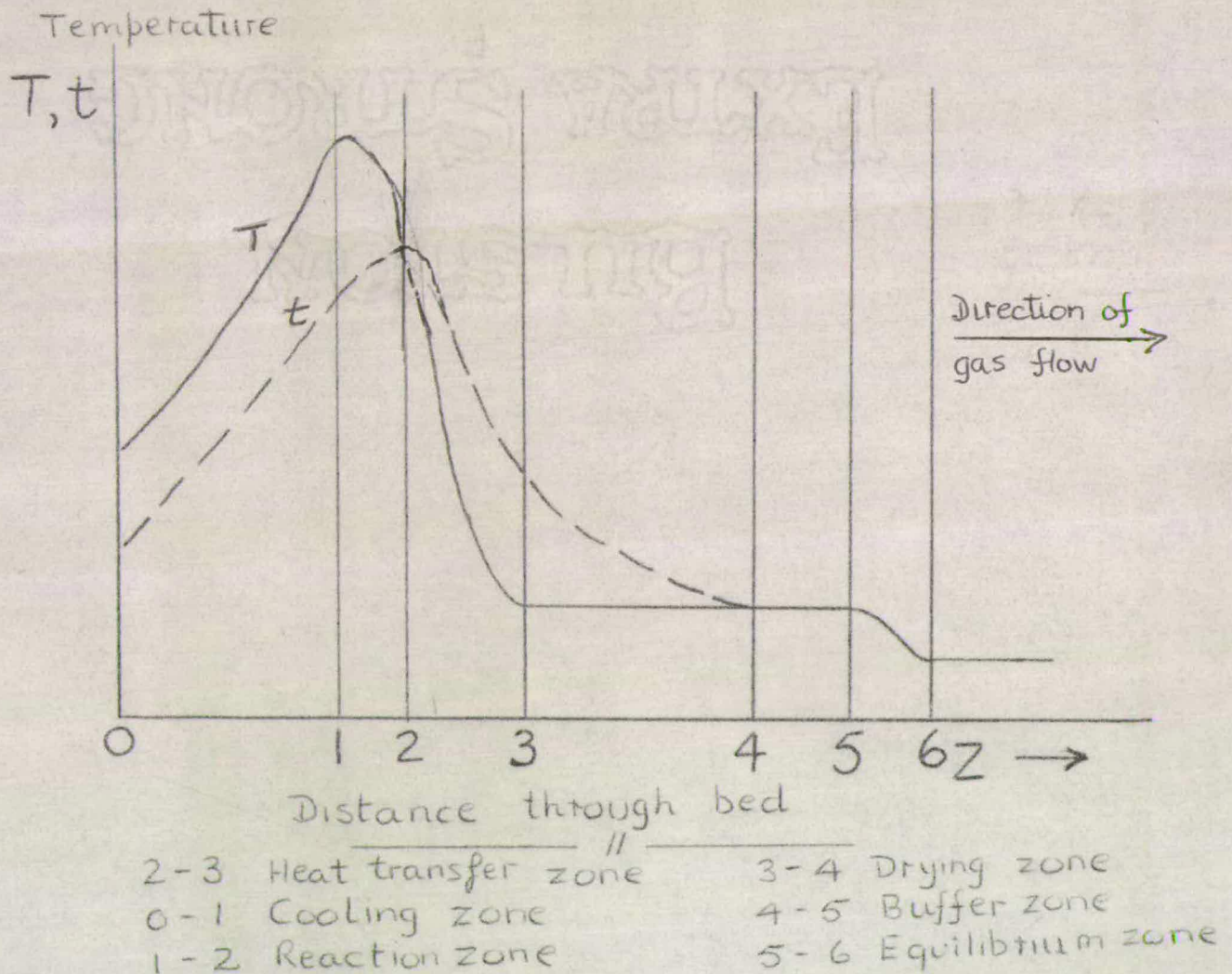


FIG 1

In a system containing both exothermic and endothermic zones the latter may delay the establishment of the former by keeping the temperature below that necessary for ignition. For instance in zinc sulphide oxidation the drying of the mix must be complete before oxidation can start (see fig. 1). The velocity of the drying zone may therefore be rate controlling. Many similar endothermic zones appear in industrial processes and the existence of a drying zone is common. This project has started therefore with the drying of water from porous particles by a high temperature air stream. These particles have the advantages of easy handling, controllable water content and well known drying kinetics.

1.2 Objects of this Investigation

Non-catalytic gas-solid reactions have been shown to be of considerable interest because of their commercial importance and difficulty of analysis. This project aims at their investigation for which a physical simulation is necessary. The processes which it is intended should be investigated are those occurring in moving zones in fixed beds or in the equivalent moving bed system. The beds which will be modelled are those in which one dimensional heat and mass balance equations may be written parallel to the direction of gas flow.

A complete study of a system also needs theoretical simulation of its processes so that the behaviour of the system

may be determined without reference to experiment. Such a full study would include prediction of the temperature and concentration profiles throughout the system. While in this project various theoretical models, for use with a zonal system, are proposed no attempt has been made to compare experimental and theoretical profiles. The theoretical simulation aims at predicting zone velocities which will be compared to experimentally measured velocities. The physical simulation will be an experimental apparatus designed to reflect as nearly as possible the conditions encountered in industrial processes so that any conclusions may apply to full scale plants.

1.3 Physical Simulation of Processes

For this analysis an apparatus which will suit a number of different systems has been designed and built. The aim is to reproduce in a small sample under adiabatic conditions the processes taking place in industrial reactions. The design enables the variables under study to be altered at will. The size of the samples is important in most non-catalytic processes as each sample can be used only once. Large samples are undesirable as they require a much larger parent batch of material: this batch may then be of heterogeneous composition or a number of different parent batches may be needed to complete the experiments with

consequent variation in composition of successive deliveries. In addition large quantities of some materials may not be available and the cost of the material and of the experiments increases. If small samples are used, say two pounds of solid, a number of different systems may be examined quickly at low cost once the apparatus is commissioned.

When a hot gas enters a region where an endothermic process is taking place a low temperature zone is created and as the endothermic reactants are exhausted the temperature rises again. This temperature rise moves as a wave through the bed and marks the end of the endothermic zone. In this situation the velocity of the temperature wave is a characteristic feature and in this project it will be measured and used as the endothermic zone velocity.

A heat transfer zone expands at constant velocity behind the endothermic zone, increasing in length as the latter zone moves through the bed.

1.4 Theoretical Simulation of Processes

An approximate analysis published by Beveridge (1963) assumed constant zone velocities, radial adiabaticity, axial symmetry and plug flow of gas. The zone velocities were then predicted from overall heat and mass balances on each

9a.

zone under pseudo-steady state conditions as viewed from a co-ordinate origin moving with the zone. The apparatus built will be used to test the validity of this approximate analysis by comparing predicted zone velocities to those measured experimentally. Making the same assumptions as Beveridge zone internal analyses are possible from which mass concentration and temperature profiles may be calculated. These analyses will be used to illustrate the method of using the approximate analysis in a computer design of a reactor.

An internal analysis will also be made of the heat transfer zone expanding at constant velocity and the result of this unsteady state analysis used to predict the boundary conditions at the inlet to the endothermic zone. A comparison of the experimental and predicted solid temperatures at this boundary will be made.

Analysis by individual zones has been found in the literature but the reported results are limited to the blast furnace. The formulation of balances was in numerical terms suitable for particular plants and no articles were found which set out a general treatment.

Various system parameters appear in the theoretical simulation, such as specific heats, and may have to be measured.

The mathematical models used in the internal analyses are simplified differential diffusion models derived from the equations set up for example by Singer and Wilhelm (1950).

1.5 Summary of Objectives

- (i) To build an experimental apparatus as an accurate physical model of industrial gas-solid non-catalytic processes and to measure the velocity of individual endothermic zones such as are found in industrial processes.
- (ii) To test the validity of a previously published analysis of such processes by comparison of predicted results with experimental ones.
- (iii) To measure any system parameter necessary for the above comparison.

CHAPTER 2

MATHEMATICAL MODELS OF NON-CATALYTIC FLUID-SOLID PROCESSES

To investigate a chemical process it is necessary to simulate it by a mathematical model which then may be used to predict the behaviour of the actual process. In any model certain system parameters, such as densities, specific heats and conductivities must be found from either theoretical or empirical correlations. Systems incorporating beds of particles may be examined to determine system parameters or to improve

upon the models available. Work in the former sphere complements that in the latter in the final analysis of a process. No attempt will be made to evaluate the literature on system parameters.

A number of mathematical models of beds of particles have been proposed: Lamb and Wilhelm (1963) visualised a hierarchy of models for various situations which can be grouped as :-

1. Deterministic

A. Continuum (diffusion model) formulated in terms of ordinary or partial differential equations.

B. Discrete (mixing cell models) finite stage models formulated in terms of finite difference equations.

2. Stochastic

A. Random perturbation of deterministic model.

B. Purely stochastic model.

The diffusion model is essentially for use when the flow properties, velocity, temperature and concentration, vary over distances of the magnitude of the bed diameter so that local variations may be neglected. This model is found in varying degrees of complexity, for instance, variation of properties in one direction only and in more than one; adiabatic operation and operation with a heat flux through the wall; and with and without plug flow of the phases.

Most success however has come from the one dimensional adiabatic version using plug flow of the phases and simple, first or zero order, reactions.

When void spaces and particles become large local variations in the flow properties begin to effect the temperature and concentration profiles. Systems exhibiting such behaviour, for instance very short beds, are usually analysed using model 1B. This assumes the bed is a matrix of void spaces each of which may be regarded as a mixing cell of characteristic length one particle diameter. This model has advantages when axial and radial mixing effects must be considered as the diffusion model may account for these only by "mixing" coefficients. To get an analytical solution to the diffusion model these must usually be assumed constant, whereas in short beds they are in fact a function of bed length. The stepwise finite difference model however can include the variation with length.

Both the deterministic models suffer from the disadvantage that they must assume a uniform solid matrix even if the packed bed is a random array of particles. Stochastic models attempt to predict the effect of the statistical distribution of void spaces on the flow property profiles. If sufficient statistical information about packed bed structure were available a purely stochastic model could be set up such as has been attempted by Haughey

and Beveridge (1966-1967). Lamb and Wilhelm (1963) considered a random distribution of void spaces as perturbation on a diffusion model to predict the effect of a non uniform particle matrix on the temperature and concentration patterns.

The diffusion model 1A is the most popular model because of its wide application to commercial systems where large tube to particle diameter ratios and approximately adiabatic conditions are found. This model will be used in the present project.

It was formulated, for example, by Singer and Wilhelm (1958) and when written in three dimensional co-ordinates their equations for the unsteady state heat balance can be given in the general form :

Fluid Phase

$$\text{div} (K_f \text{ grad } t) - \text{div } G'ct + h_v(T-t) = \delta \rho \frac{c \partial t}{\partial \theta} \quad (\text{A})$$

Solid Phase (assuming negligible internal temperature variation)

$$\text{div}(K_s \text{ grad } T) + \text{div } G_s c_s T - h_v(T-t) + R(-\Delta H) = (1-\delta) \rho_s c_s \frac{\partial T}{\partial \theta} \quad (\text{B})$$

where T, t are solid and gas temperatures respectively.

Singer and Wilhelm discuss fully the significance of the various terms in these equations. Owing to their complexity a solution to these equations is nigh on impossible, even numerically, except by simplification, and a large literature

has grown up around the different simplifications proposed to obtain solutions. Beveridge (1962) has surveyed the literature available until June of that year.

In a non-catalytic packed bed process a number of models may be needed as frequently reaction or exchange processes allowing different simplifications to the general model occur simultaneously. For instance in zinc sulphide oxidation, the oxidation reaction is accompanied by drying and pure heat transfer in different zones. Hence we are interested in particular in solutions in which there is a finite reaction rate at the surface, and heat and mass interphase transfer resistances taken separately and together, and combinations of these.

2.1 Systems without Reaction or Exchange Control at the Surface

The general equations for systems in which only heat transfer is occurring are equations A and B above with $R = 0$. The most general solutions available are those of Amundson which assume that axial and radial solid conduction is negligible. If axial conduction can be neglected we may say there is an almost complete general solution. The general solution equations however are very complex with resultant computational difficulties. The system equations with pure heat transfer are also equivalent to mass transfer under linear equilibrium at the interface and an external

boundary layer resistance.

More simplified situations which are special cases of the more general, such as no radial fluid temperature gradients, or plug flow of fluid are also available (Beveridge 1962). The axial boundary conditions for this problem at steady state were formulated by Wehner and Wilhelm (1956) and apply to both 1-D and 3-D systems. Bischoff (1961) generalised these conditions for any order of reaction and recently van Cauwenberghe (1966) has extended the analysis to unsteady state systems.

Of the other simplifications which can be made to the general equations the set which finds the most application is that which reduces the equations to their one dimensional form, either steady or unsteady state, with the dependent variable varying along the axis of the reactor. This is used for ion exchange, adsorption and desorption under linear isotherms besides pure heat transfer.

2.2 Systems including Rate Control at the Surface by Reaction or Exchange Processes

As their operation is often isothermal the non-catalytic fluid-solid ion exchange or adsorption processes may be regarded as special cases with only the mass balance of the general equations, together with the rate of sorption, applying. Some adsorption processes are not isothermal and here the heat

and mass balances must be coupled. The isothermal equations are of interest because of the wide application of such processes but in addition they exhibit some of the features of the more complicated reactions encountered in metallurgical operations and suggest methods of attack on their models.

The single phase mass transfer situation with linear equilibrium and external diffusion resistance has been considered above. Bischoff and Levenspiel (1962) contrast and compare diffusion models of different complexity for linear rate processes using the second moments, or variances, of the solution concentration distributions along the axis. They developed criteria for the selection of an adequate model for a system based upon its ratio of characteristic dimensions parallel and perpendicular to the flow of fluid. In general there is a complete body of theory for isothermal linear processes and recent work has concentrated upon making good minor deficiencies.

Sherman (1964) resolved the one dimensional unsteady state equations for first order reaction when calculating diffusion coefficients. In order to make the model simulate experimental conditions more accurately he used a quartic equation instead of a step function at start up. The quartic equation was a best fit approximation to the

observed input function. Chao and Hoelscher (1966) considered the same equations but included a term for active surface interchange i.e. an accumulation term for the solid concentration (viz.)

$$\frac{1}{Pe} \frac{\partial^2 m}{\partial z^2} - St' \frac{\partial m}{\partial z} = \delta \frac{\partial m}{\partial \theta} + (1-\delta) \frac{\partial M}{\partial \theta}$$

This corresponds to the isothermal gas-solid reaction with a zero order reaction.

Very little work is available for adiabatic or quasiadiabatic sorption processes. Amundson et al (1965) however have considered the transient one dimensional equations for adiabatic sorption with Langmuir kinetics. They show that for some isotherms the coupled heat and mass balances are reducible to a quasilinear form to which simple wave theory may be applied. Numerical integration of the reduced equations illustrates the saturation of a "clean" bed and the desorption of a saturated bed. The theory is for single solutes.

Getty and Armstrong (1964) examined the adiabatic adsorption of water on activated alumina but only fitted a multiple regression equation to their results and did not advance any theoretical model. This is useful for design but the equation will only be valid within the design space covered by the experiments.

The most general work on coupled heat and mass transfer with chemical reaction is that of Crider and Foss (1966) and Lee (1966). These papers gave the results of numerical solutions of various models. Crider and Foss investigated the effect of various parameters on packed bed operation. They solved the two dimensional finite stage model, the one dimensional finite stage model and the one dimensional plug flow differential unsteady state equations to evaluate respectively axial and radial mixing, axial mixing alone and plug flow operation. They used a second order reaction homogeneous in the fluid phase, varying the reaction rate and the thermal capacity of the packing.

Lee (1966) developed a technique based on Newton Raphson iteration to solve the one dimensional transient equations including axial mixing in the fluid for a first order reaction. The convergence of the iterative method is very rapid, three or four steps, and the results are compared with Liu and Amundson's (1963) for the cases of isothermal and adiabatic operation.

A number of workers, Liu (1963) Warden (1963) and Beyer (1964) also have analysed the stability of packed bed reactions using first order exothermic rate processes and have shown the existence of non-unique solutions. However although the above solutions are non linear and therefore of interest for adsorption processes, no work has been found which

attempts to analyse the situations which occur in non-catalytic gas-solid reactions. The only approach has been by Schulman (1963) but this too is a much simplified case.

Schulman (1963) formulated the one dimensional, plug flow, unsteady state equations for the combustion of carbon during catalyst regeneration. A term including heat loss at the wall is included and balances written for conservation of heat, carbon and oxygen. These partial differential equations were integrated numerically using a new technique of partial integration. The assumption was also made that the gas and solid temperatures were equal at any point in the bed. This is reasonable for catalyst regeneration but not for many non-catalytic reactions encountered, for instance, limestone calcination and zinc sulphide oxidation.

The complexity of the equations for this type of system has focussed considerable attention upon approximate solutions. A similar situation prevails for many non-linear adsorption equilibrium isotherms. An equilibrium isotherm being the distribution of solute between solid and fluid under steady state conditions.

2.3 Approximate Solutions

One of the most useful approximate solutions in adsorption is obtained if the bed is sufficiently long for the concentration profile to approach asymptotically a constant shape. This is only true for uniform flow

conditions, constant system parameters and "favourable" equilibrium. That is an equilibrium relation such that the adsorption front moves faster at higher concentrations of the solute so that the profile becomes self sharpening. This constant shape wave then moves through the bed at constant velocity. The concentration profile may be readily calculated under these conditions and the effluent concentrations determined. This approach has great utility as practical considerations usually demand "favourable" equilibrium.

Recently Cooney and Lightfoot (1965) have extended the proof of the existence of asymptotic solutions to the differential equations which describe single solute fixed bed exchange processes. The existence of these profiles was proved by Rosen (1952) for conditions of zero axial dispersion and finite boundary layer mass transfer resistance: the solutions are now extended to include axial dispersion by Cooney and Lightfoot (1965). Later they extended their results for single solutes to multicomponent systems of mutually interfering solutes (1966). For Rosen's case they consider different types of isotherms resulting from non-linear equilibria and extend the applicability of his work.

In another approximate method Houghton (1962) arranged the one dimensional, steady state, mass transfer equation

including axial diffusion and a non-linear kinetic expression into an equivalent integral equation. This was solved using an iterative method which converged in two or three terms. The original differential equation is :

$$\beta \frac{d^2 m}{dz^2} - 2 \frac{dm}{dz} - \lambda m^n = 0$$

which when rearranged becomes :

$$m(z) = 1 - \frac{\lambda}{2} \int_0^1 k(z, z') m^n(z') dz'$$

where z' is the reactor boundary.

The iterative method gives solutions for small values of λ . A solution is given for the case when the product, $\beta\lambda$, the group characterising the interaction between axial dispersion and the reaction, is small. The dispersion is considered as a perturbation on the first order differential equation which represents the reaction kinetics alone. A comparison with previous more exact solutions is given. This approximate method is useful to evaluate the interaction effect between dispersion and reaction. A poor result would suggest the use of a more sophisticated model.

These approximate solutions however apply to isothermal operation when only mass transfer need be considered. An approach which has found much application in blast furnace processes, where the severe temperature changes and

complicated simultaneous reactions preclude analyses such as above, is to divide the reactor into zones by planes of cross section perpendicular to the bed axis.

2.4 Zone Approach

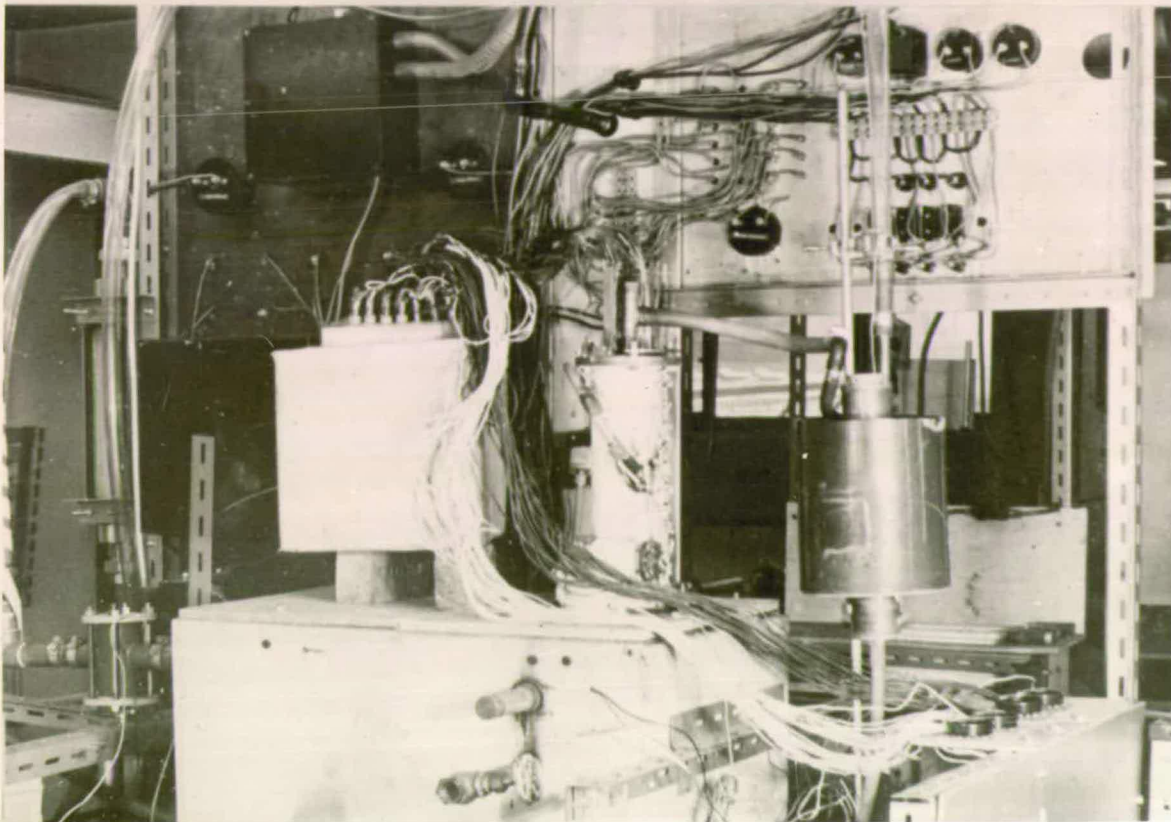
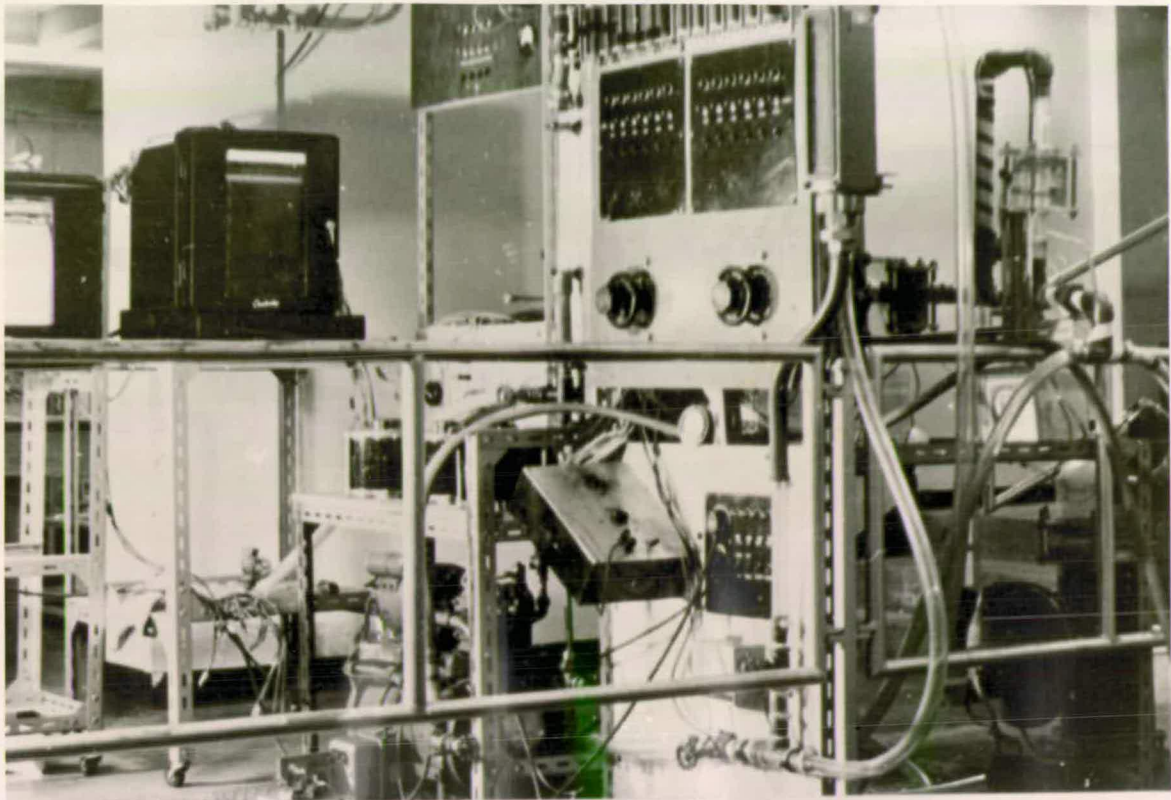
The zone approach builds a model of the process by combining separate simple models of parts of the process whereas other approaches try and simplify a model of the complete process. One is approximation by reduction of process variables, the other by separation of these variables.

A number of authors have divided the iron ore reduction process in a blast furnace into zones and used heat and mass balances carried out under steady state conditions sequentially upon each zone to determine the operating conditions. Ridgion (1961) reviews the various models proposed and advances a similar model of his own (1962). Dancoisne and Michaud also have used this approach (1959, 1962).

Schluter and Bitsianes (1961) analysed the mechanism of iron ore sintering by creating a combustion zone and then quenching the reaction in order to examine the sinter. One of the first uses of the zone approach was that of Voice and Wild (1956) and later Wild and Dixon (1961) who investigated the temperature profiles and zone speeds of endothermic zones produced when wet iron ore mixes and calcium carbonate were

sintered with coke. However none of these workers have formulated the general equations for this type of analysis or tried to apply it outside the limited operation of iron ore reduction.

Beveridge (1963) formulated the general balances for any zone under pseudo-steady state conditions and showed its applicability to reaction and exchange processes. The advantage of this work is its generality and that it throws emphasis on to the velocity of the zone as an important parameter in design. In addition the general theory has applications to the sorption processes described above and also to zone melting. In the former operation the asymptotic concentration wave may be considered to define an exchange zone and in the latter the zone is defined by the liquid solid interfaces. The general equations for zone propagation are simple to use and assume only constant zone velocity in addition to the assumptions made above, namely, one dimensional representation is possible, uniform flow and constant system parameters. In contrast to the other approximate methods these equations can handle solids flow cocurrent or countercurrent and may be extended to include axial diffusion and conduction. It has the disadvantage that constant velocity of the zone must be a realistic assumption and also the zone must be of constant width so that pseudo-steady state equations may be applied.



General Views of Apparatus.

Top - Front view of control panel.

Bottom - Back view of control panel and bed,
furnace box and thermocouples.

CHAPTER 3EXPERIMENTAL APPARATUS3.0 Introduction

The apparatus which is to be a model of the industrial systems outlined previously must use small samples of solid in its bed. This bed must remain radially adiabatic in spite of sudden temperature changes within it and in spite of its small diameter. In addition the axial temperature must be measured as a function of axial distance and time so that the movement of zones may be followed. For this it is only necessary to place thermocouples in known positions and record their output as a function of time.

The bed is cylindrical and adiabatic conditions are achieved by surrounding it with an annular bed in which the same processes occur at the same time and level. Because it is the test bed the centre bed is operated under fixed conditions throughout each run but the only restriction on the guard bed operation is that the temperatures along any radius be the same at the same time in both beds; how this is achieved does not matter.

A test bed within a guard bed was chosen when it was found impractical to use wall wound heaters alone to achieve adiabaticity in a single bed. The wall temperatures, controlled by such heaters, could not follow the sudden

temperature rises within the bed when the endothermic process was complete at any level. The guard bed method also has the advantage that once the technique of matching the temperature is mastered, it may be applied easily to different systems irrespective of the shape of temperature profile produced by a system as this is compensated for by the similar processes in both beds.

The requirements of adiabaticity considered above are independent of the system being modelled.

For each gas-solid system examined the apparatus must supply the gas and the heat for the endothermic process. Also the temperature, flow rate and composition of the inlet gas must be measured and any factor which may affect the propagation velocity must be controlled. The solid active components must similarly be measured and controlled.

The heat for the endothermic process is supplied by the gas stream heated outside the bed. This corresponds to heating the gas by passage through an exothermic zone before the endothermic one. The choice of variables to be controlled depends on the system being examined: the system chosen for this project was air/wet catalyst carrier and for this system the variables are gas inlet temperature, gas inlet moisture content, gas flow rate and solid moisture content.

This system was chosen because owing to the large heat

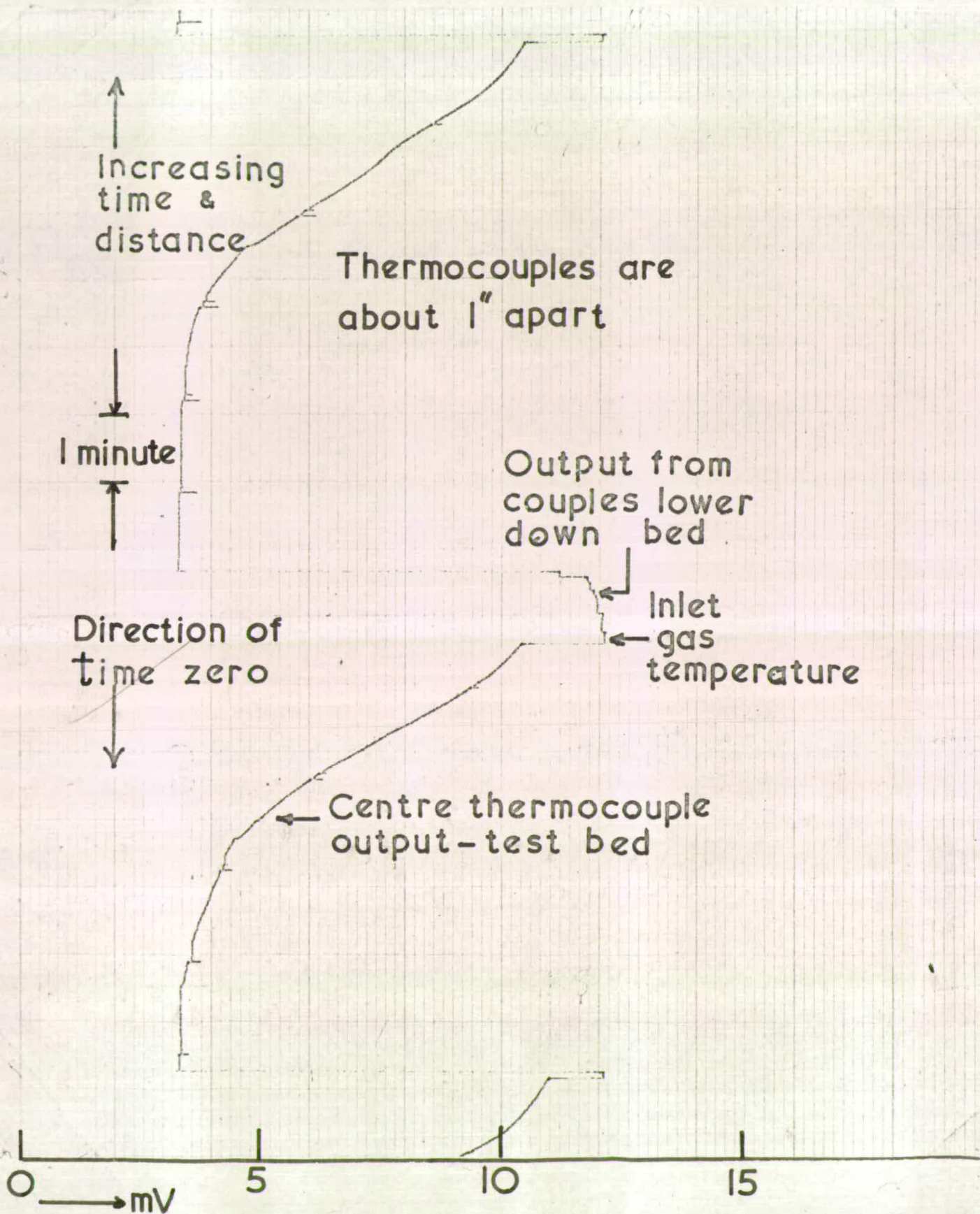


FIG 2 SINGLE POINT RECORDER TRACE

absorption on evaporation of the water the zone is well defined by the temperature profile (see fig. 2), the process takes place at temperatures easily attained, the particles may be used repeatedly and the water content of them can be varied. Water is often present in particles used for metallurgical operations as froth flotation is a common preliminary treatment and so the system has a practical industrial counterpart. Fig. 3 shows a block diagram of the apparatus.

3.1 Notes on Apparatus Design

The apparatus is designed to produce a controlled condition gas to through-dry a wet fixed bed of porous particles and to measure the velocity and form of the temperature wave caused by the drying. The condition of the gas entering the bed is determined by three variables, its moisture content on a dry basis (m_2) its dry bulb temperature (t_2) and its flow rate on an inert basis (G).

The inlet moisture content is controlled by a saturator and bypass unit after the air compressor. The temperature t_2 is controlled by the flow rate through a pipe heated by an electric element supplied from a Variac. The flow rate is varied by adjusting a bleed valve.

The bed is mounted on top of this furnace on a wide flange. The furnace tube diverges into a well, which is

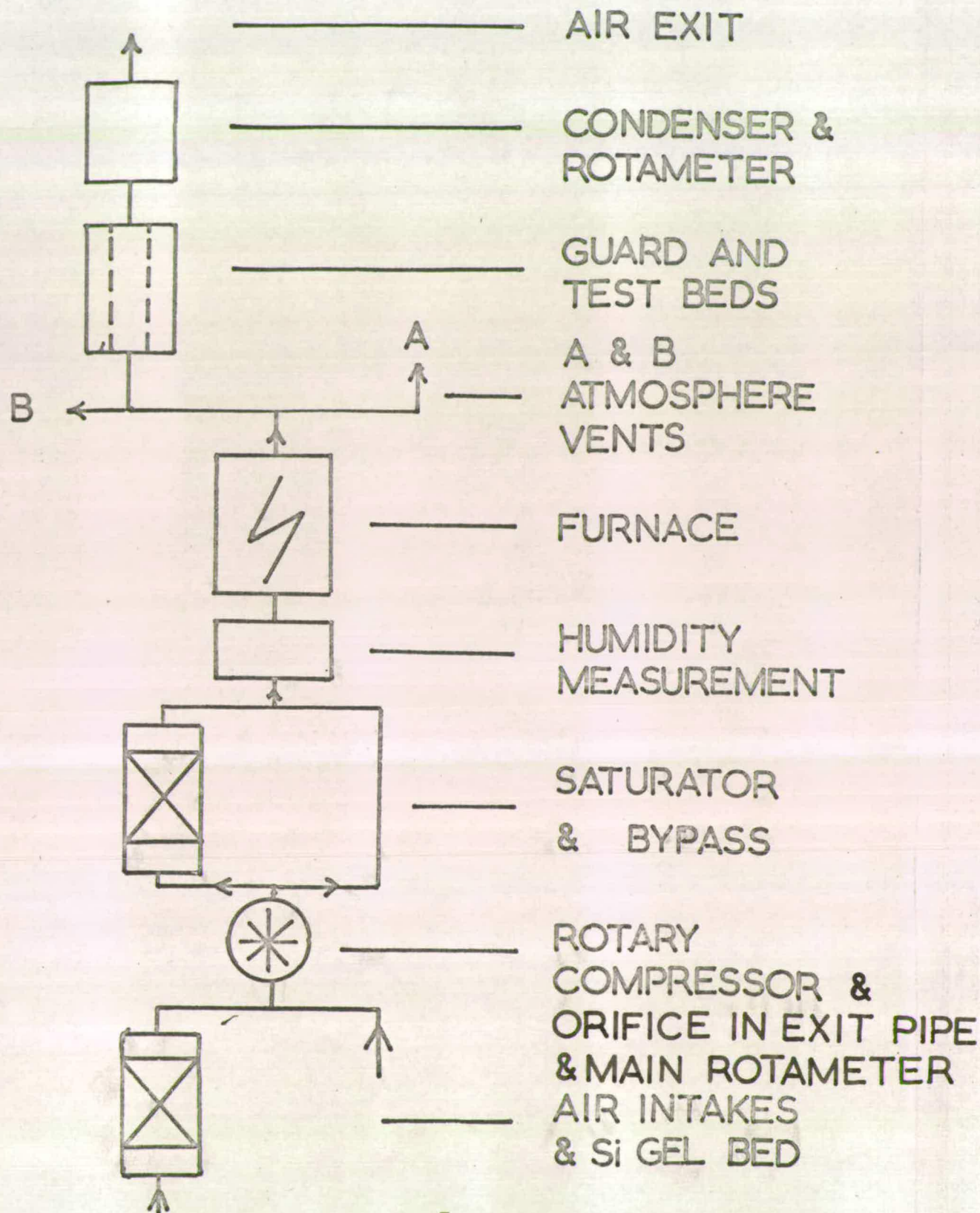


FIG 3
BLOCK DIAGRAM OF APPARATUS

set into the flange and filled with packing, the same diameter as the guard bed. The measuring part of the apparatus is considered separately from the gas conditioning apparatus which is taken in the order the gas flows through.

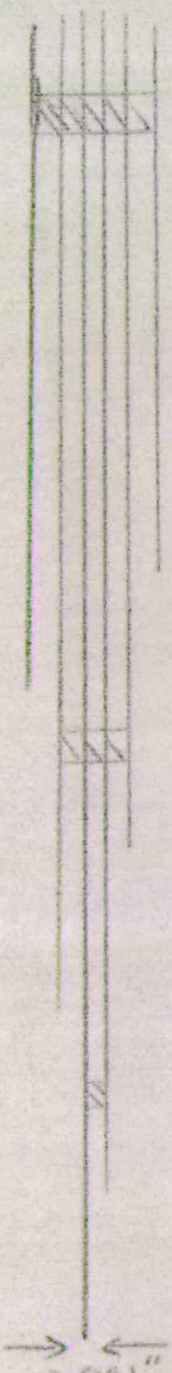
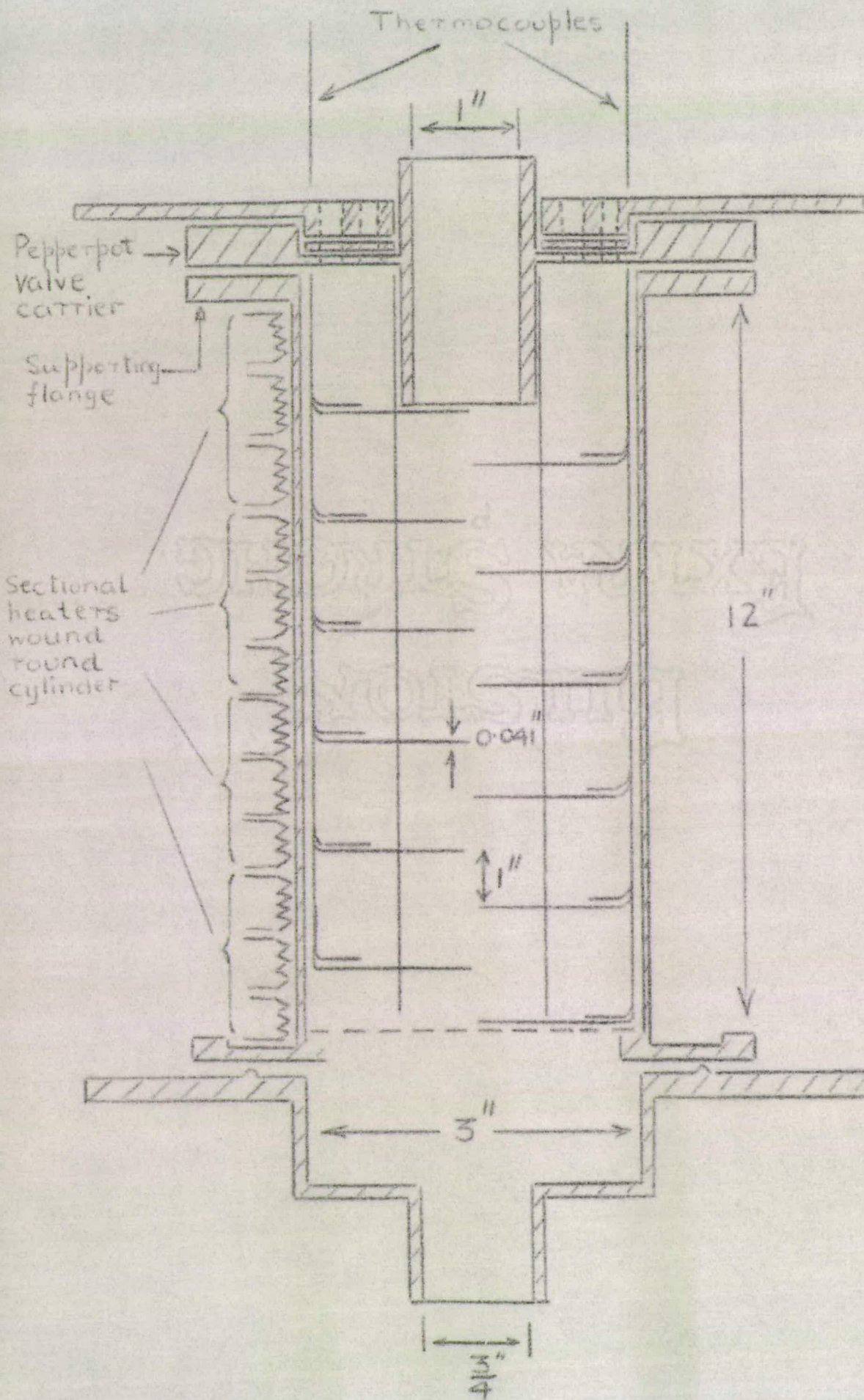
3.1.0 Gas Supply

The compressor draws air either directly from the surrounding atmosphere or through a silica gel bed. A pressure of between 4 and 7 psig is maintained in the line as far as an orifice place above the main flow rotameter. Interchangeable orifices allow the pressure to be maintained irrespective of the gas flow rate used in any particular run. This pressure is much greater than the back pressure variations caused by mounting the bed and stabilises the furnace exit temperature down stream of the orifice. Oil is filtered and flow fluctuations damped by passing the air through a glass wool filter and ballotini bed in series.

After the orifice the gas is divided between a wet packed column and a bypass. The amount of gas bypassed determines the moisture content of gas when the two streams join together again. A humidity sensor monitors the gas entering the furnace. For very high humidities this is replaced by a wet and dry junction thermocouple hygrometer.

The gas passes through the packed furnace pipe with a wire resistance element wound on a former on the outside.

Cross section through concentric beds



Arrangement of the thermocouple sheaths against side of bed

It can follow then alternative paths either to a vent to the atmosphere or to the mounting flange. The direction of flow is chosen by the position of a double cone valve which blocks either one of the pipes. The cone is mounted on a rod so that a simple push pull action switches the gas from the atmosphere vent to the test position. The pressure drop through the vent is adjusted to be the same as across the bed to minimise back pressure caused by operating the cone valve.

The pipe leading to the mounting flange, the flange itself and the well are heated to the furnace exit gas temperature with externally wound heaters. This makes an adiabatic gas path from the furnace to the base of the bed and so a step input in gas temperature is possible at the start of a run.

3.1.1 The Bed

The bed itself consists of a three inch inside diameter flanged brass tube supporting an inner one inch inside diameter "Fiberfrax" paper tube. Externally wound heater sections supply the heat capacity requirements of the brass bed and any external losses. Four Variacs each control the voltage applied to a group of three sections together with individual rheostats for fine adjustments. Trial runs to set the heaters to maintain adiabatic steady state conditions

are carried out before a statistical design experiment. The "Fiberfrax" tube has a heat capacity negligible compared to that of the solid bed and also it is an insulator which reduces longitudinal conduction. The bed is sealed to the flange with a damp asbestos gasket which is fresh each time the bed is mounted. A small ring is raised on the flange surface biting into the gasket to confirm the seal. The top of the brass tube carries the "Fiberfrax" test bed's exit pipe which passes the gas to atmosphere via a condenser and rotameter. The ice cooled condenser prevents flooding of the rotameter; the test bed exit gas is saturated; and as the gas leaves the condenser at $0-3^{\circ}\text{C}$ to a near approximation the rotameter is measuring the inert gas flow.

3.1.2 Velocity Measurement

The velocity of the temperature wave is measured by placing a number of thermocouples in the interstices of the bed at inch intervals along the axis and recording the time taken from the start of the run until the temperature rises, as each level dries out. The thermocouples are mineral insulated nickel-chromium/nickel-aluminium couples in a stainless steel sheath, by Pyrotenax. They have the hot junction bonded to the sheath and this forbids the use of a common return. The thermocouple outputs are recorded via a rotary switch on a single point recorder and directly

on a multipoint recorder.

At each level there are two thermocouples, one on the test bed axis and one in the centre of the guard bed and these are linked to adjacent points on the multipoint recorder. The rotary switch allows any one thermocouple in the beds or any of the four furnace thermocouples to supply the input signal to the single point recorder. The four furnace thermocouples measure the temperature of the furnace exit gas, the wall of the pipe leading to the flange, the junction of the well wall and the flange and the gas at the well centre just below the gauze at the bottom of the beds.

The temperature is also measured before each rotameter and at the humidity sensor for calibration purposes. All thermocouples have brazed junctions, protected by glass tubes, immersed in melting ice to provide the cold junction reference temperature. The protecting tubes are well separated to minimise variations in this temperature.

The apparatus is suitable for some systems as it stands but for others minor modifications would be necessary. For instance, if a carbonate decomposition were being studied the partial pressure of the carbon dioxide in the air stream would be an important variable, hence provision must be made for carbon dioxide injection and measurement. For this say,

the saturator and bypass could be replaced by a mixing valve system attached to a carbon dioxide cylinder. The bed and furnace are independent of the system being examined but the design of the other parts was dictated by the nature of the air-wet catalyst pellet system.

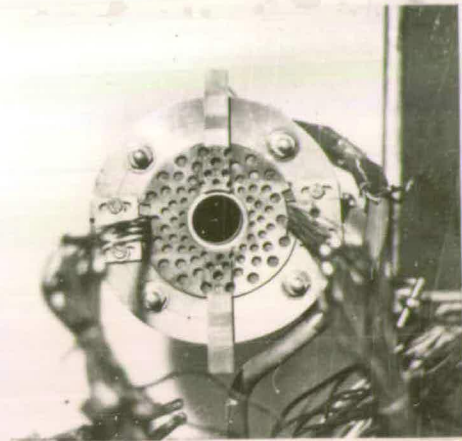
3.2 Technique of Operation

Before each run the furnace heater currents and total mass flow of gas are set and the apparatus warms up with the gas vented to atmosphere through A. The starting of the compressor and switching on of the heaters is accomplished with a time delay relay in the early hours of the morning. The values of the variables set are checked once steady state has been reached with the gas vented through A and no bed in place.

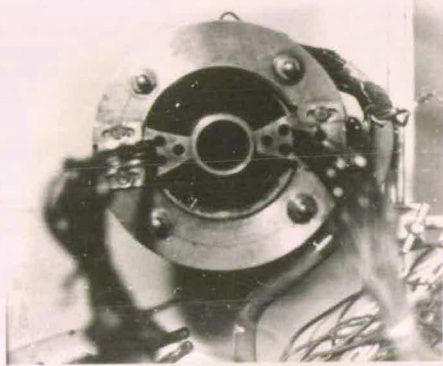
To equalise the temperature throughout the well packing, which distributes the gas flow over the bed cross-sectional area, the gas is then vented through a dummy bed until steady conditions are reached again. Pressure drop through the dummy bed is adjusted to be the same as through the actual bed so that temperature fluctuations due to flow fluctuations before and after the change over from the dummy to the actual bed are minimised. At the start of the run the dummy bed is replaced by the actual bed as quickly as possible with the gas vented to atmosphere through A during



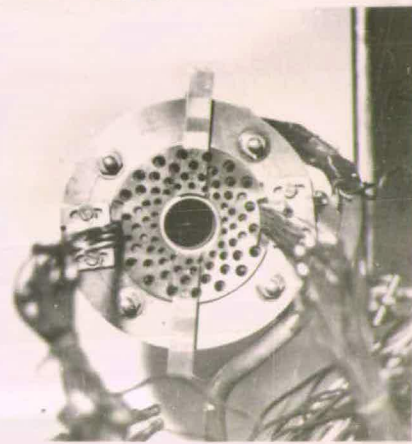
INTERNAL VIEW



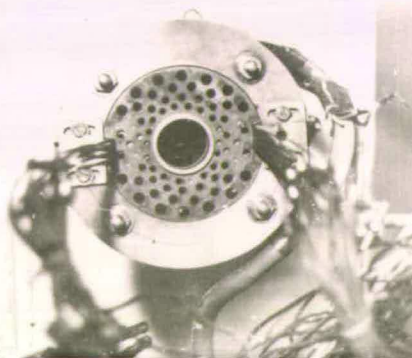
ROTATING PEPPERPOT SHUT



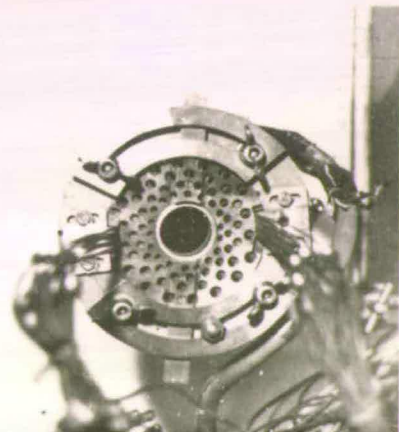
TOP FLANGE ALONE



ROTATING HALF OPEN



STATIONARY PEPPERPOT COMPLETE ASSEMBLY



VIEWS OF TEST AND GUARD BEDS

TOP ASSEMBLY AND THERMOCOUPLES

Views of pepperpot valve as mounted on top
flange of bed together with an internal
view of bed interior and thermocouples.

the seating of the bed.

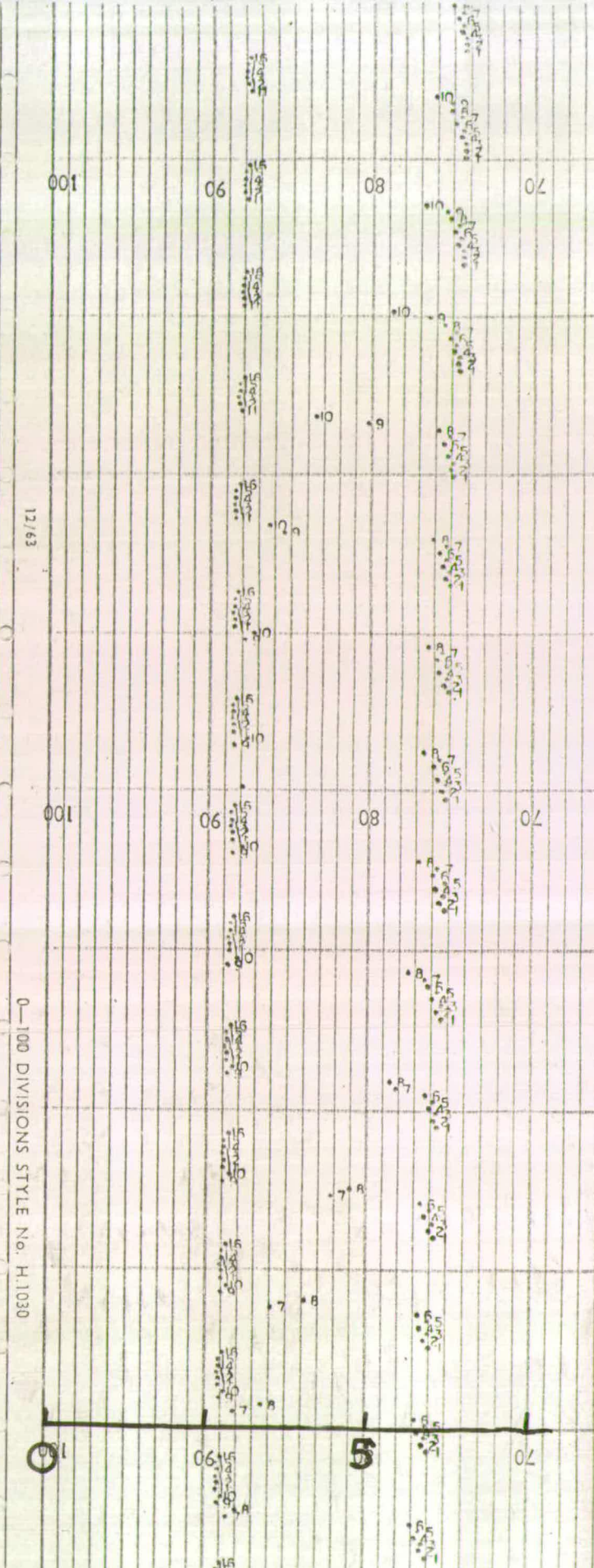
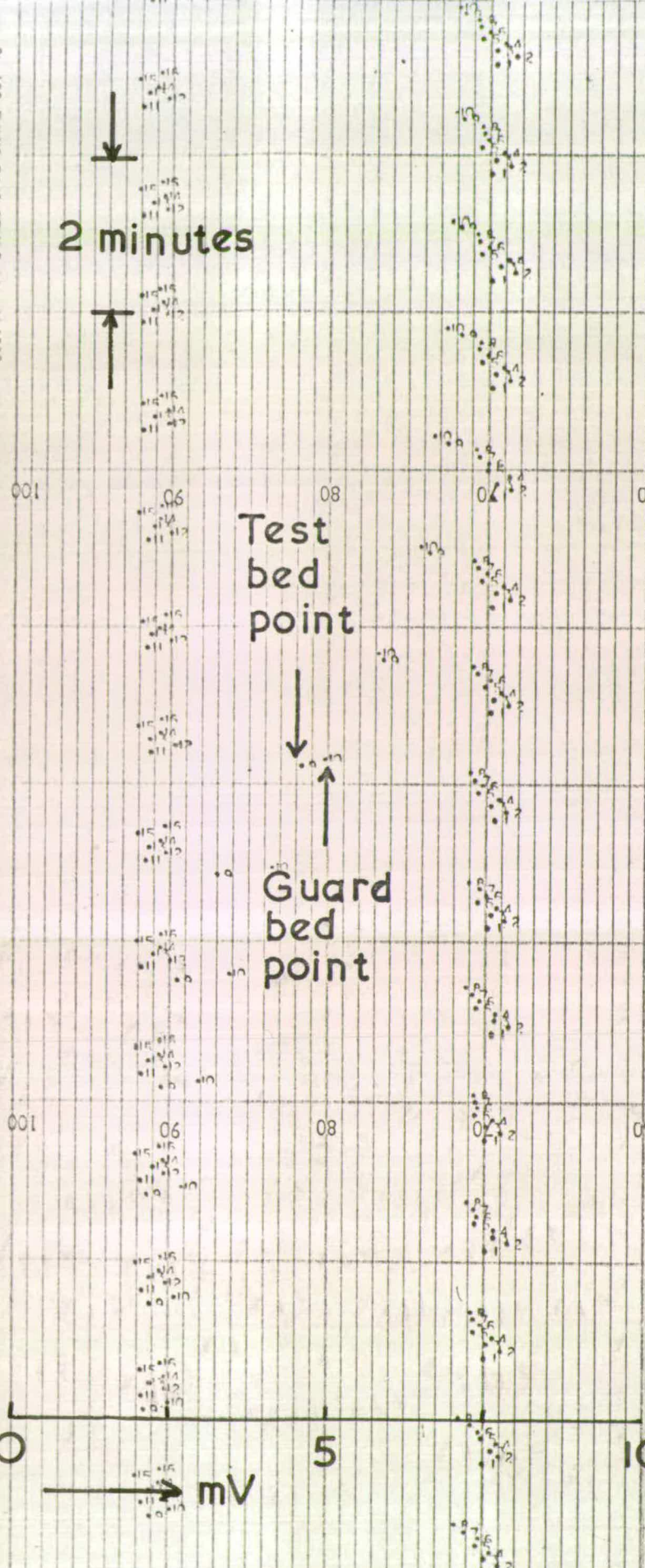
After the start of the run the profile and the velocity of the temperature wave must be maintained the same in the guard bed as in the test bed. If this aim is realised there will be no temperature difference across the test bed wall and adiabatic conditions are obtained.

The flow through the test bed must be maintained at a constant value, but that through the guard bed may be altered at will. A pepperpot valve at the exit to the guard bed can adjust the flow through it.* Also part of the total gas flow may be vented to atmosphere through a valve B between the exit from the gas heater and the entrance to the bed. Heaters are wound in axial sections round the outside of the guard bed to coincide roughly with the spaces between the thermocouples. These are set for a given gas inlet temperature to give no heat loss under final steady state conditions.

The temperature wave may be retarded in the guard bed by venting through B some of the total gas input and restricting the pepperpot to maintain the test bed flow; and accelerated vice versa. The limit of acceleration is fixed by the total flow available. The time of switching the wall heaters in affects the profiles in the guard bed and is a matter of experience.

Before a run is started the total gas input flow rate

* see photographs opposite



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FIG. 4-2 TYPICAL MULTIPOINT CHARTS

is set to supply gas slightly in excess of the requirement for equal gas flow per unit area of cross-section in the guard and test beds; the excess going to vent through B during the run. During the run the total gas flow and gas heater controls should not be touched as this affects the gas inlet temperature (t_2). In practice however slight adjustments can be made provided these compensate so that t_2 is unaltered.

The decision to change the velocity of the guard bed wave is based upon the difference between the thermocouple readings at the same level in the guard and test beds. As soon as this is greater than zero the guard bed velocity must be altered. The amount of gas vented through B depends upon the operator's assessment of the size of the difference between the velocities. As the temperatures at any level are monitored only every eighty seconds this assessment must be subjective and considerable experience is necessary to maintain adiabatic conditions throughout a run. A typical multipoint recorder charts showing the adjacent temperatures at successive levels is shown in fig. 4.

CHAPTER 4AN ANALYSIS BY A SEPARABLE ZONE TECHNIQUE

The complete continuum model for heat and mass transfer in a packed bed with chemical reaction was formulated by Singer and Wilhelm (1950) but as we have seen solutions are only possible for simplified versions. The simplifications used in this work are chosen from consideration of conditions in industrial packed beds. The main assumptions, of radial adiabaticity and angular symmetry perpendicular to gas flow, are acceptable in view of the relatively large cross-sections often employed commercially. Under these assumptions the model reduces to being one dimensional with respect to axial length; but still is solvable only for the simplest chemical rate expressions.

As has been shown it is possible to separate the complex processes occurring in many non-catalytic gas-solid reactions by postulating the existence of zones. The processes occurring within these zones, although individually complex, may be capable of solution as the number of processes considered together is reduced. The usual approach defines one zone as equivalent to the bed and so all processes must be included in one analysis.

The one dimensional model can be applied to individual

zones defined by the hypothesis of the separable zone approach and hence an internal analysis made of them. The definition of the zone limits fixes the zone boundary conditions, some or all of which may be known from the processes for which the boundary conditions define initiation or completion.

Analytical solutions are presented for zones defining the simpler processes and numerical solutions for those which would require unrealistic simplification to permit an analytical solution. Various zones associated with through circulation drying in fixed and moving beds are considered.

4.0 Moving Zone and Moving Bed Models

A zone of constant length moving through a packed bed at constant velocity is equivalent to a stationary bed of the same length with countercurrent flow of the gas and solid through it. This is seen intuitively if one imagines the origin moving with the zone (App. 2). The one dimensional heat and mass balances for each can then be represented by one model in which the unsteady state process of a moving zone may be represented by steady state equations. This may be called the pseudo-steady state viz,

$$K_f \frac{d^2 t}{dz^2} - (G - V\delta\rho) c \frac{dt}{dz} + h_v(T - t) = 0 \quad (1)$$

$$E_f \frac{d^2 m}{dz^2} - (G - V\delta\rho) \frac{dm}{dz} + k_v(M - m) = 0 \quad (2)$$

$$K_s \frac{d^2 T}{dz^2} + V(1 - \delta) \rho_s c_s \frac{dT}{dz} - h_v(T - t) + R(-\Delta H) = 0 \quad (3)$$

$$E_s \frac{d^2 M}{dz^2} + V(1 - \delta) \rho_s \frac{dM}{dz} - k_v(M - m) + R = 0 \quad (4)$$

Solutions to this general model will be presented for certain cases (§ 4.2). First let us consider the overall balance equations (see Chapter 2, Beveridge (1963)).

4.1 Overall Balances

One consequence of representing moving zones by countercurrent pseudo steady state equations is that the zone velocities may be predicted from overall balances on each zone. The derivation of these balances has been published previously but as they are the basis of this project it seems suitable that their final form be set out here.

In many systems zones are found which may be classified by the zone processes. For instance, heat transfer alone, heat transfer + drying and heat transfer + a chemical reaction are all common in industrial systems (see fig. 1). Let us look at the published balances as applied to heat transfer and heat transfer + drying as this system has been chosen for the model apparatus.

The overall balances will be used to predict zone velocities for comparison with experimentally measured velocities. The predicted velocities will depend on the accuracy with which the control parameters (m_2 t_2 M_4 G) values are known.

4.1.1 Constant Rate Drying Zone + Heat Transfer

Mass Balance

$$\frac{G - V\delta\rho}{V(1-\delta)\rho_s} = \frac{M_4 - M_2}{m_4 - m_2}$$

Heat Balance

$$\frac{G - V\delta\rho}{V(1-\delta)\rho_s} = \frac{C_s(T_2 - t_4) + M_4 L_{T4}}{(c + m_2 c_w)(t_2 - t_4)}$$

For the ease of general application the balances have been rearranged as functions of characteristic groups.

Drying Zone

$$V_R = \frac{(G - V\delta\rho)(c + c_w m_2)}{V(1-\delta)\rho_s C_s} = \frac{M_4}{C_s} \frac{(c + m_2 c_w)}{(m_4 - m_2)} \quad (5)$$

where $\frac{M_4}{C_s}$ is called group CSR - capacity solids ratio

and $\frac{c + m_2 c_w}{m_4 - m_2}$ " " CGR - " gas "

$$t_3 = \frac{(c + m_2 c_w) L_{T4} + T_4}{m_4 - m_2} \quad (6)$$

Heat Transfer Zone Balance

$$V_R = \frac{(T_2 - T_3)}{t_2 - t_3} \quad (7)$$

Tables have been produced on a digital computer for the dependent variables as functions of the characteristic groups (see Appendices) For this purpose the drying zone heat balance has been separated from that for the heat transfer zone as in equation (6) above. And M_2 is taken as zero to define the solid as being bone dry at the exit to the drying zone. The tables also include values of m_2 corresponding to the OGR values as sometimes m_2 is the more convenient parameter. An illustration of the tables' use is given in Appendix 5.

The simple overall balances allow the zone velocity to be related to the concentrations and temperatures at the zone boundaries. The velocity of the zone may then be determined without knowing the internal analysis of the zone processes. If a zone is not affected by other zones it will attain its maximum velocity but in a sequence of zones the later must travel at the speed of the earlier. However if the early zones in the sequence move faster than the later

ones an expanding zone will be created between the two: if vice versa is the case the temperature and concentration profiles through the zones will become self sharpening as the faster zones cannot move out of sequence to overtake the slower.

While these simple overall balances can predict the velocity of a sequence of zones they can supply no estimate of the zone length which is an essential parameter if the separable zone technique is to be used for design purposes neither can they link zones separated by an expanding zone. This information can only come from the internal analyses, which follow.

4.2 Internal Analyses

4.2.1 Zone with Pure Heat Transfer Process

The equations for the internal analysis of a zone without reaction become equations (1) and (3) given in the previous section with $R = 0$. Such zones appear in between zones in which two different processes are occurring when one is completed and the other not yet initiated. Regions of pure heat transfer are needed to balance any zone system such as is proposed here.

The solution to the equations depends upon whether the complete equations are solved or further simplifications made.

The complete differential equations may be combined to give a quartic equation which is integrable once at sight. The resulting cubic equation has an analytical solution and so the general solution becomes :

$$t = c_1 e^{\alpha z} + c_2 e^{\beta z} + c_3 e^{\gamma z} + c_4$$

similarly

$$T = c_1' e^{\alpha' z} + c_2' e^{\beta' z} + c_3' e^{\gamma' z} + c_4'$$

where α, β, γ and α', β', γ' are the roots of the cubic auxiliary equation which may have three real and separate, three real equal or one real and two imaginary roots. As the algebraic expression governing the type of roots is complicated it must be evaluated numerically for particular values of the coefficients in the original cubic. In a system in which axial gaseous diffusion and solids conduction are important particular numerical values must be substituted in the analytical solution before the profile may be specified, even in general shape.

If however we may neglect the axial diffusion and conduction terms in the equations we are left with two first order ordinary differential equations. This is the diffusion model which must be used in conjunction with the approximate zone theory as this assumes plug flow of gas and solid.

$$-\frac{G'c}{h_v D_p} \frac{dt}{dz} + (T - t) = 0 \quad (8)$$

$$\frac{V(1 - \delta) \rho_s c_s}{h_v D_p} \frac{dT}{dz} - (T - t) = 0 \quad (9)$$

These when combined into a second order equation have an immediate solution which characterises the temperature profile by only two dimensionless groups.

$$t = c_1 + c_2 e^{-St'(1 - V_R) z}$$

$$T = c_1 + c_2 V_R e^{-St'(1 - V_R) z}$$

where St' is a modified Stanton number $h_v D_p / G'c$

V_R is the ratio $G'c / V(1 - \delta) \rho_s c_s$

Once these two parameters are known the calculation of the zone length follows and hence the mean residence time necessary to establish given conditions in the zones before and after the heat transfer zone.

The modified Stanton number may be calculated from correlations of the j_H factor of Chilton and Colburn (1934) with modification for the volumetric heat transfer coefficient used in the definition of St' . A typical

representative of such correlations, by De Acetis and Thodos (1960) was used here as it was derived from experiments on a similar system. V_R is known from the effective throughputs of solid and gas. The profile shape is easily seen to be an exponential curve displaced from the origin by a constant amount.

Previous work by Singer and Wilhelm (1950) suggests that the axial terms may be neglected for high tube to particle diameter ratios and solids of low conductivity and relatively low temperature gradients through the bed. In some metallurgical processes the temperatures reached are high enough to warrant the inclusion of the axial terms in the internal analysis even though the metal ores may have low conductivities. For instance it is thought that axial conduction and radiation, carrying heat forward from the reaction zone, are responsible for the solid temperature in the pure heat transfer zone which precedes the reaction zone. Haughey (1966) has studied radiation heat transfer in this type of system.

4.2.2 Endothermic Zones

For reasons given previously by Beveridge (1963) the endothermic zones in any sequence are likely to move the slowest, and being possibly rate controlling their internal analysis is of some interest. Although the velocities of

individual zones may be predicted by the approximate approach using overall balances internal analysis of the zones is required to fix their lengths and temperature and mass concentration profiles.

Constant Rate Drying Zone

The important parameters in this zone are the temperature of the inlet gas and the approach to saturation at the exit. To achieve saturation at the exit to the zone, i.e. $t = T$, an infinite zone length must be provided; however should the gas and solid have some fractional approach to each other (say 0.01°C for instance) the length is practicable. If the moisture content of the solid phase is low so that the change in gas humid heat through the zone can be assumed negligible the first order differential equations are integrable at sight as the solid temperature remains constant at the wet-bulb temperature.

$$-\frac{Gc'}{h_v D_p} \frac{dt}{dz} + (T - t) = 0 \quad (10)$$

This has the general solution :

$$t - T = Ce^{-St'z}$$

and if the boundary conditions on the zone at planes (3) and (4) are included :

$$z = 0, \quad t = t_3; \quad z \rightarrow \infty, \quad T \rightarrow T_4$$

$$\frac{t - T_4}{t_3 - T_4} = e^{-St'z} \quad (11)$$

The similar solution for mass transfer is :

$$\frac{m - m_s}{m_3 - m_s} = De^{\frac{-kvDp}{G} \cdot z} \quad (12)$$

where m_s is the saturation moisture content of the gas at the wet-bulb temperature.

Above, the gas temperature profiles for the much simplified model used to predict zone velocities were considered.

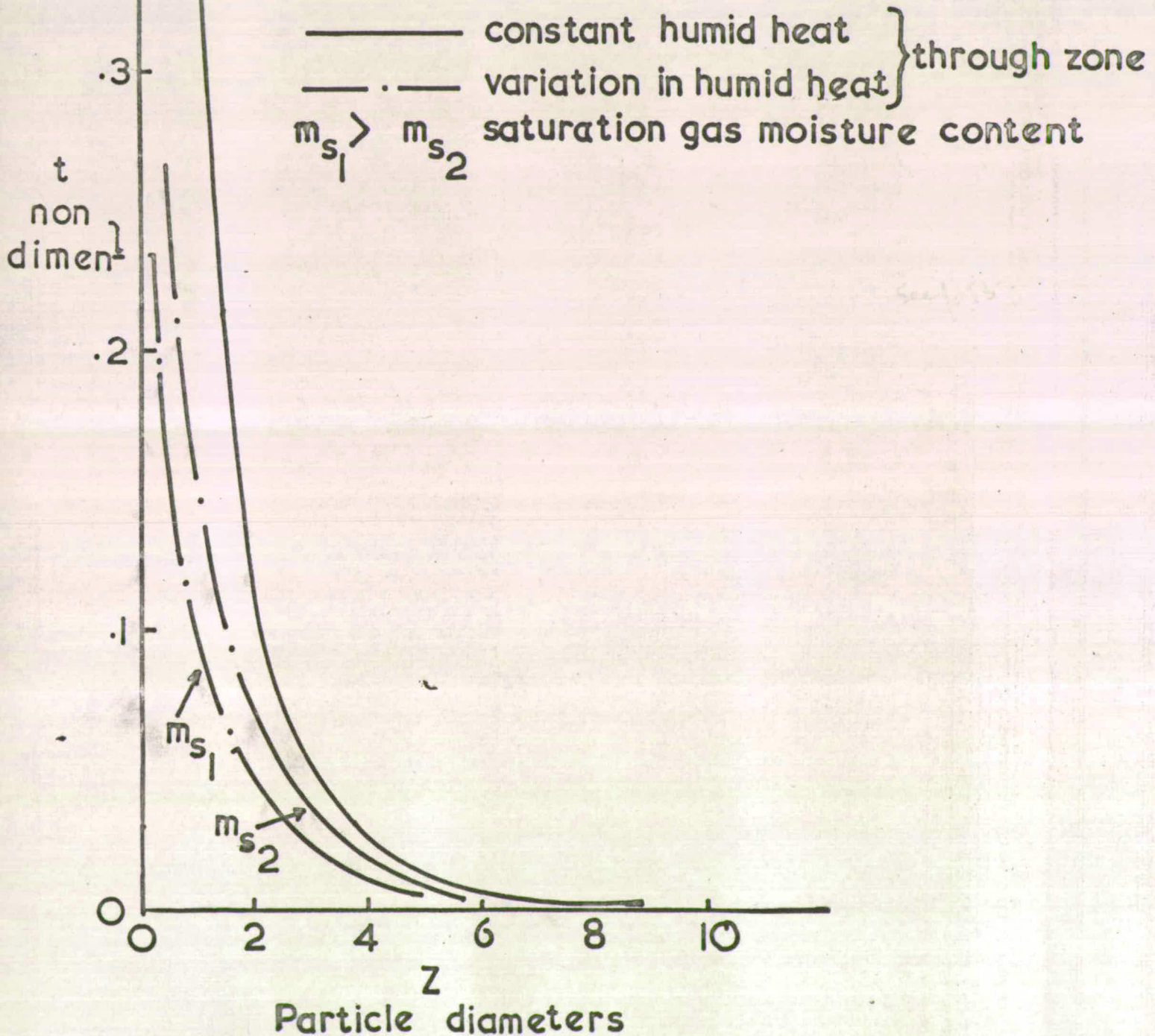
However when large amounts of water are to be evaporated the humid heat of the gas should not be assumed constant and an allowance made for the change in gas enthalpy due to the increase in the moisture content of the gas on passage through the zone. The differential equations have been modified to include this effect giving for the fluid phase

Heat Balance

$$- \left[\frac{Gc}{h_v Dp} + m \frac{Gc_w}{h_v Dp} \right] \frac{dt}{dz} + (T - t) = 0 \quad (13)$$

FIG 5
PLOT OF

GAS TEMPERATURE v DISTANCE THROUGH
DRYING ZONE



Mass Balance

$$-\frac{G'}{k_v D_p} \frac{dm}{dz} = (m - m_s)$$

When integrated the general solution for heat transfer may be compared with the solution to the more simplified equation presented earlier.

$$\frac{t - T}{t_3 - T} = \frac{(A_1 + B_1) e^{-z/A_1}}{A_1 + B_1 e^{-\Omega z}} \quad (14)$$

Qualitative comparison profiles, fluid temperature v distance along the bed are shown in fig. 5 indicating the effect of including the humid heat variation. However the calculated results upon which the qualitative profiles were based show that the humid heat variation may be ignored for practical purposes (see Appendix 3).

Equilibrium Zone

In this zone a gas almost saturated is being cooled and the solid previously at ambient temperature is heating up to the equilibrium temperature, the wet bulb temperature. As a saturated gas is being cooled mass transfer takes place from the gas to the solid. The variation in the gas humid heat and solid specific heat must be included in the differential heat and mass balances. These then become

complex and numerical solution is necessary which was considered outside the scope of the project. The overall balances on this zone are covered fully by Beveridge (1963).

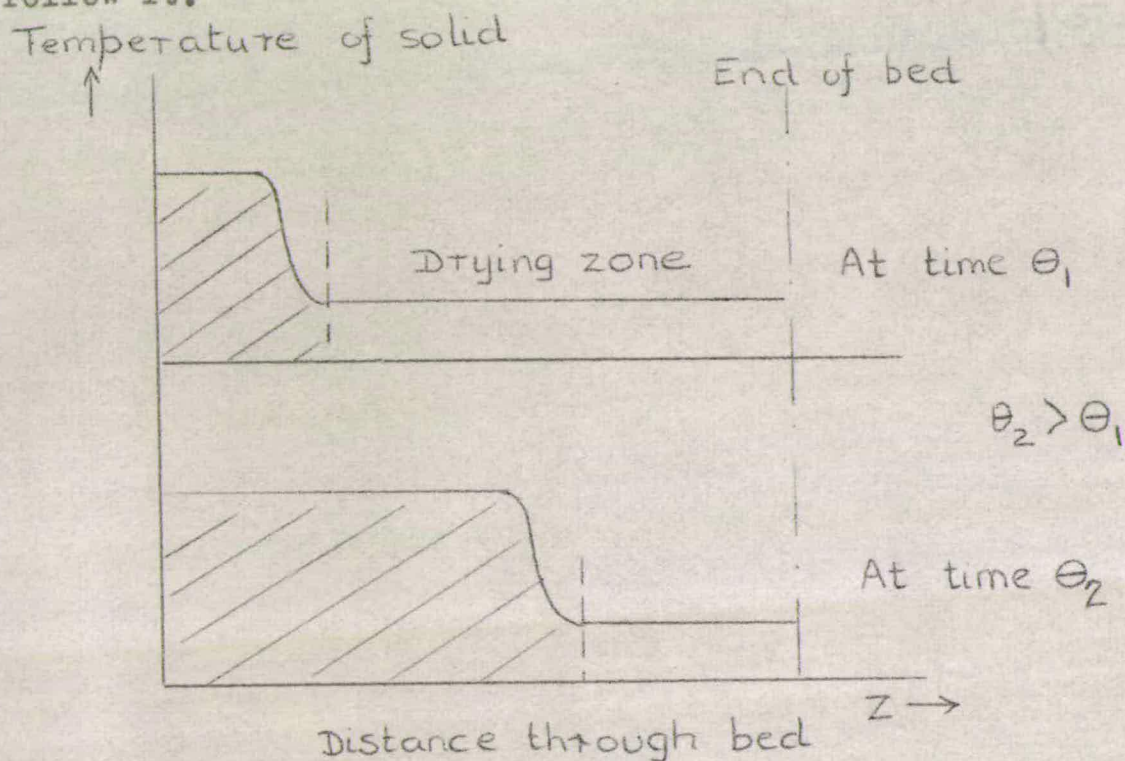
Buffer Zone

Beveridge (1963) showed that the potential velocity of the equilibrium zone was greater than that of the drying zone. As this zone precedes the drying zone it is not constrained by other zones and in a stationary bed will approach its potential maximum velocity. The drying zone therefore expands with plane (4) moving at the velocity of the equilibrium zone. However as the gas and solid temperatures in the drying zone very rapidly approach each other, for most of this expanding zone the gas and solid temperatures are very nearly equal. Beveridge therefore postulated the constant width drying zone and a buffer zone in which the gas and solid temperatures are everywhere equal. In strict reality the buffer zone is that part of the drying zone where as the length tends to infinity the gas temperature tends to the solid temperature and the temperature gradients are infinitesimally small. However the results of this project bear out the buffer zone postulate (see Chapter 5).

4.2.3 Model for Present Experimental System

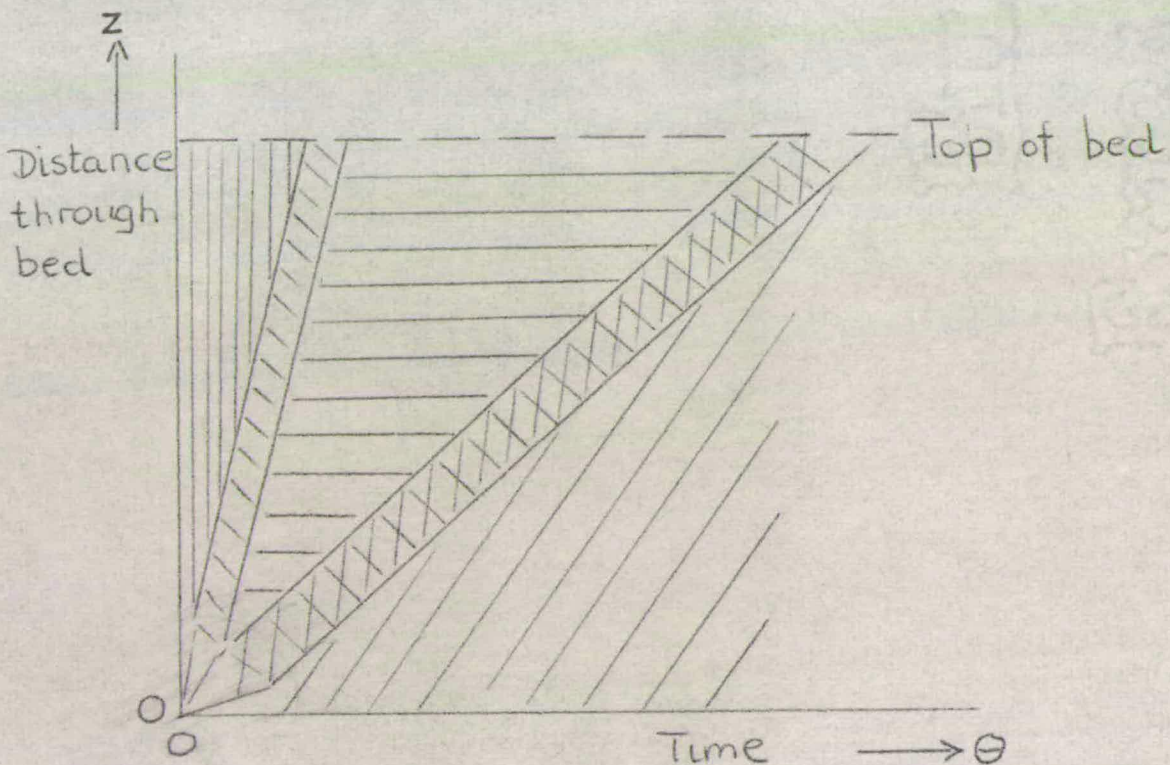
In any moving zone system which caters for zones

moving at different speeds, heat transfer regions must be specified which expand or contract. For instance in the stationary bed used in this project a drying zone moves through the bed causing an expanding heat transfer zone to follow it.



Temperature profiles through experimental bed
at successive times

Showing this behaviour on a distance v time graph we plot the time for the temperature wave front to reach successively higher thermocouple levels.



The cross hatched zone represents the endothermic drying zone moving at constant velocity and with constant width. The vertical shading represents the fresh wet solid and the horizontal shading the buffer zone expanding between the drying and equilibrium zone. The left to right shaded portion represents the constant velocity, constant width equilibrium zone and the right to left shading represents the heat transfer zone expanding behind the drying zone.

Expanding Zone

This expanding zone can be represented by the one

Expanding Zone Boundary Conditions

At expanding boundary $z = V\theta$ $Y' = 0$ $\theta = \theta_0$ $T = T_3$

At fixed boundary $z = 0$ $Z' = 0$ $\theta = \theta_0$ $t(0) = t_2$

As $z \rightarrow \infty$ $Y' \rightarrow \infty$ T is finite

N.B. t at $\theta = 0$ & $z = 0$ unknown as discontinuity

dimensional unsteady state heat transfer equations. If back diffusion of the gas and conduction through the solid can be ignored and the rearmost zone expands at constant velocity, these unsteady state equations may be solved using Laplace Transforms. As this technique treats initial value problems the independent variable z is transformed using the substitution $Y = z - V\theta$ so that the known boundary conditions at the expanding boundary become initial conditions at $Y = 0$.^{*} Graphically the effect of the transformation is to expand the right to left shaded portion of the z/θ plot above to occupy the whole quadrant.

Fluid Heat Balance

$$- \frac{Gc}{h_v D_p} \frac{\partial t}{\partial z} + (T - t) = \frac{\delta \rho c}{h_v} \frac{\partial t}{\partial \theta} \quad (15)$$

Solid

$$- (T - t) = \frac{(1 - \delta) \rho_s C_s}{h_v} \frac{\partial T}{\partial \theta} \quad (16)$$

These become on transformation

$$\frac{\partial t}{\partial z'} + (T - t) = -\phi \frac{\partial t}{\partial Y'} \quad (17)$$

$$- (T - t) = \frac{\partial T}{\partial Y'} \quad (18)$$

where $\phi = \frac{(G - V\delta\rho)c}{V(1 - \delta)\rho_s C_s}$

* see opposite

The analytical solution obtained is not immediately usable as it involves integrals derived from the convolution theorem. Three solutions of interest arise from these heat balances on the two phases; solid and fluid temperatures as functions of distance, and hence the fluid condition at the expanding boundary. Also arising from the general solutions are the trivial solutions at the inlet to the expanding zone. The derivation of these solutions is given in the appendix.(3)

General Solutions

Fluid Phase

$$\begin{aligned}
 t e^{Y'} = \frac{(T_3 - t_2)}{Y'} \beta \left\{ \int_0^{z'} e^{-\alpha \tilde{\tau}} q \cdot \beta (\tilde{\tau}^2 + 2\beta \tilde{\tau}) \frac{1}{2} \left[q (\tilde{\tau}^2 + 2\beta \tilde{\tau})^{1/2} \right] \right. \\
 \int_0^{z' - \tilde{\tau}} e^{-\alpha \tilde{\tau}} q (\tilde{\tau}^{-1}) \frac{1}{2} I_1(q \tilde{\tau}) d\tilde{\tau} d\tilde{\tau} \\
 \left. + \int_0^{z'} e^{-\alpha \tilde{\tau}} \cdot q \cdot \tilde{\tau}^{-1} I_1(q \tilde{\tau}) d\tilde{\tau} \right\} + t_2 e^{Y'} \quad (19)
 \end{aligned}$$

Solid Phase

$$Te^{Y'} = (T_3 - t_2)\beta q \int_0^{z'} e^{-\alpha\tilde{\tau}}(\tilde{\tau}^2 + 2\beta\tilde{\tau})^{-1/2} I_1(q(\tilde{\tau}^2 + 2\beta\tilde{\tau})^{1/2}) d\tilde{\tau} + T_3 + t_2(e^{Y'} - 1) \quad (20)$$

Fluid Conditions at Expanding Boundary $Y' = 0$

$$t_3 = \frac{T_3 - t_2}{2} \int_0^{z'} q \cdot e^{-\alpha\tilde{\tau}} \cdot \tilde{\tau}^{-1} I_1(q\tilde{\tau}) d\tilde{\tau} + t_2 \quad (21)$$

The last of these three equations is integrable analytically to give an infinite series. The first two may be integrated numerically. As the series solution is itself complicated the summation must be carried out numerically but this is an easier task than the numerical integration would be.

The general one dimensional unsteady state equations although stated here in terms of temperature are valid for any general concentration parameter and as the form of the solutions remains the same the integrals have been calculated and graphed as functions of their generalised parameters for future use.

The fluid condition at the expanding boundary has been

graphed as :

$$\frac{t_2 - t_3}{t_2 - T_3} = F_1 = \frac{k(\phi, z')}{\phi^{1/2}} \quad (22)$$



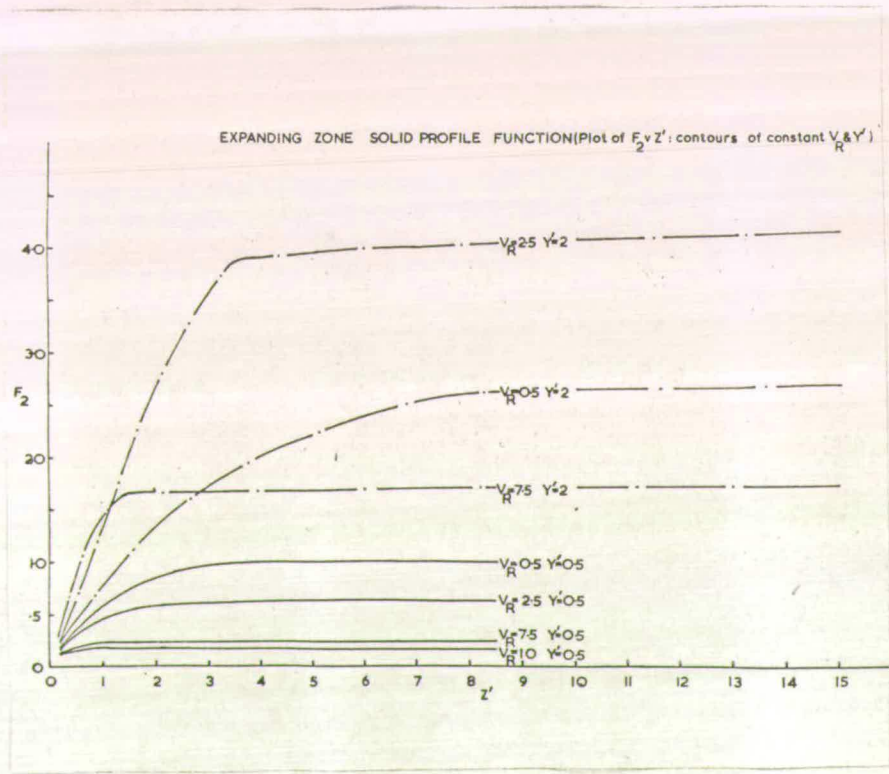
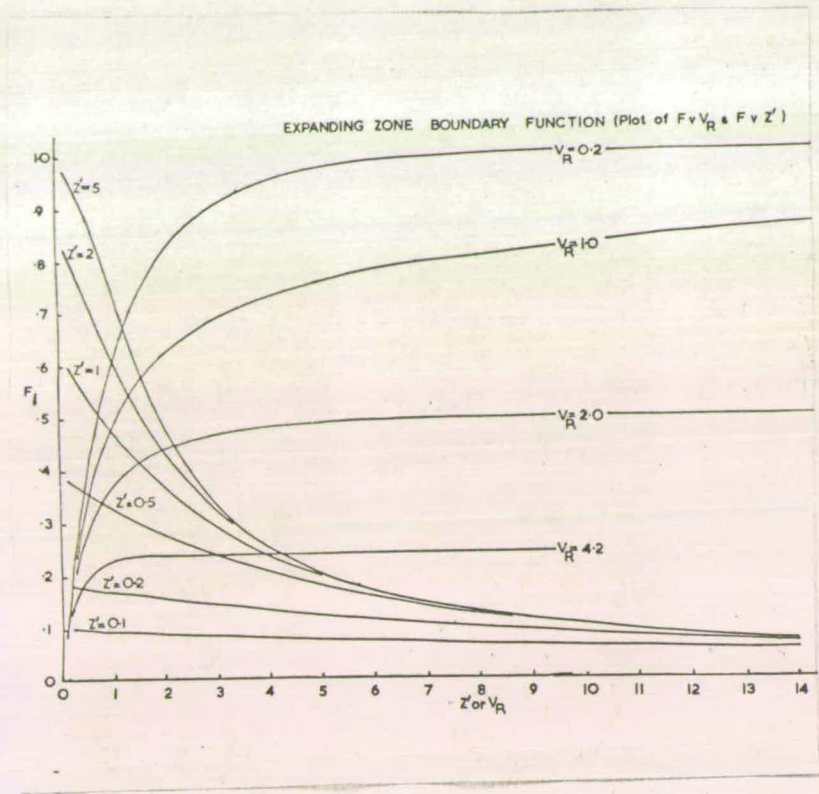


FIG 6

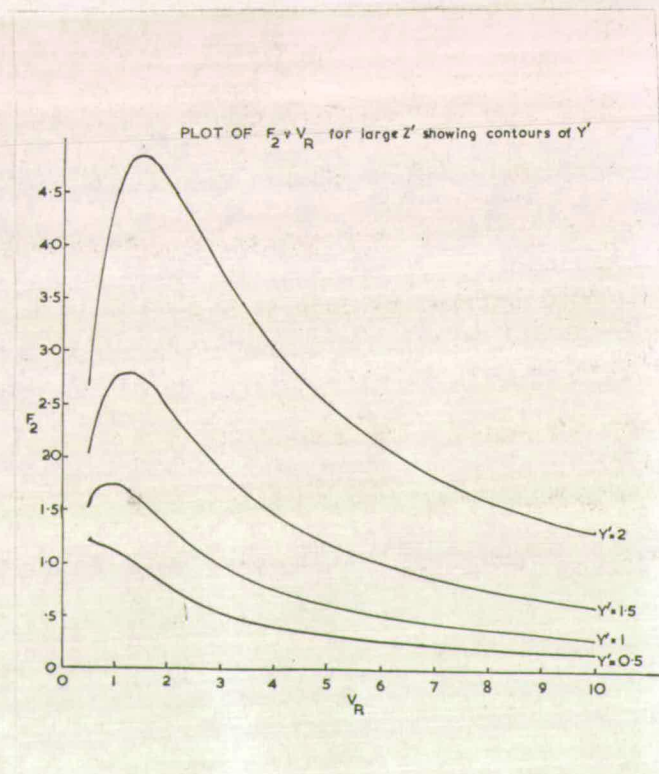
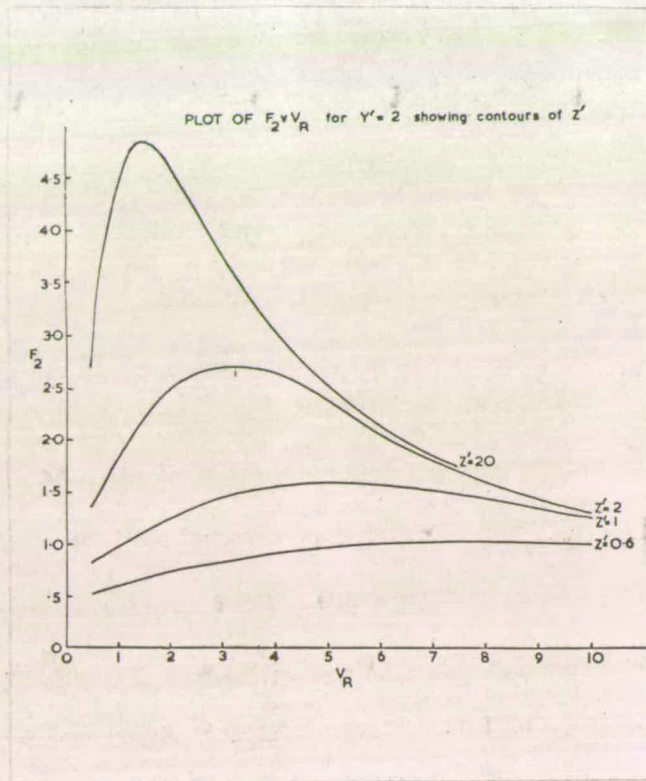


FIG 6 (cont.)

where $k(\phi, z')$ denotes the integral expression. Similarly the solid temperature function, as it is usually the solid temperature which is measured, has been calculated and graphed as:

$$\frac{e^{Y'}}{Y'} \left(\frac{t_2 - T}{t_2 - T_3} \right) \frac{1}{Y'} = F_2 \equiv \frac{k(\phi, z', Y')}{\phi^{1/2}} \quad (23)$$

The form of the functions is shown in fig. 6 from which it can be seen that as time and hence length of the zone increase the fluid exit conditions approach a constant value. By a suitable iterative procedure this function may be used to determine the inlet boundary conditions for a zone ahead of the expanding one. In this way, using graphical iteration, the inlet conditions to the drying zone were found and hence theoretical velocities for the zone calculated for comparison with those measured.

4.3 Overall Balances - A Modification

To include axial diffusion and conduction in the overall balances used in the approximate approach extra terms may be included. The extra terms are the temperature or concentration gradients across the zone boundaries. This is not difficult in itself; it is in evaluating the gradients without completing the whole zone internal analysis which presents the difficulty. Wehner and Wilhelm and their

successors showed how to attack this problem when they considered boundary conditions in flow reactors (1956, 1961, 1966).

If the zone system is divided into three zones

$$(i) \quad -\infty < z < a$$

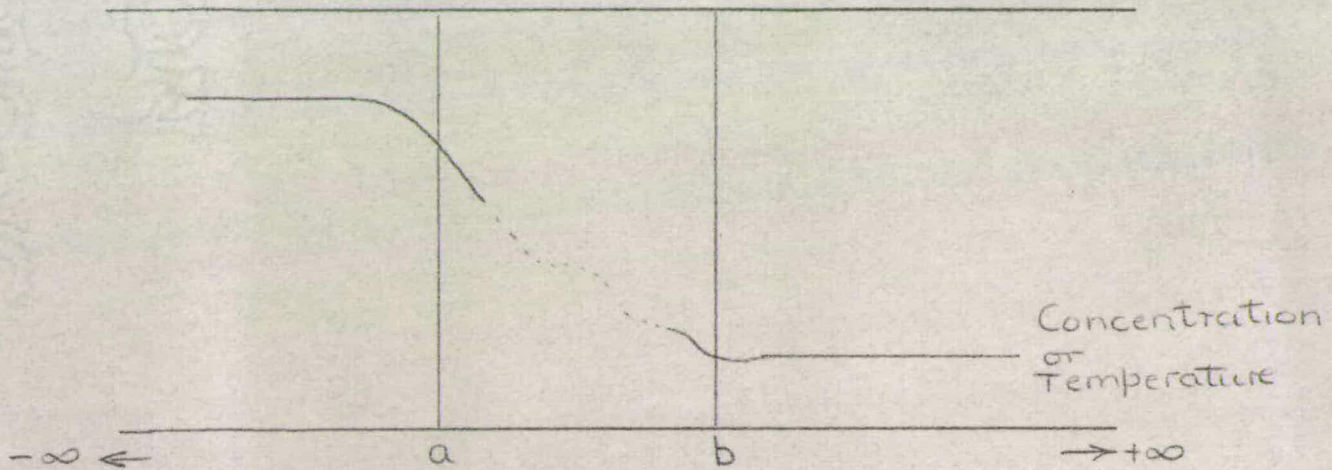
$$(ii) \quad a < z < b$$

$$(iii) \quad b < z < +\infty$$

and all reaction or exchange processes are confined to the zone $a \rightarrow b$, the differential mass and heat balances may be formulated for the semi-infinite end zones. These will be simpler than the balances for the central zone and their integration allows the temperature and concentration gradients to be determined at the zone boundaries.

As we have confined all exchange processes to the central zone there will be no interphase mass transfer in the terminal zones. Hence for a moving zone in a fixed bed there will be constant concentration of reactant in the solid in the end zones. It is possible that solids backmixing occurs in moving beds but very little is known about this. However eddy diffusion effects will be present in the fluid phase and hence the uniform bulk flow concentration profile has this effect superimposed on it which results in a concentration gradient in the fluid

across the zone boundary.



For heat transfer however there is the possibility of solid conduction together with eddy diffusion heat transfer and hence a gradient will exist in both the fluid and solid temperature profiles across the zone boundary. The overall heat and mass balances on the central zone $a \rightarrow b$ including axial effects are set out below.

Heat Balance

$$t_a - \frac{1}{Pe_f} \frac{dt_a}{dz} - \frac{1}{V_R} T_a - \frac{1}{Pe_f} \frac{dT_a}{dz} - t_b + \frac{1}{Pe_f} \frac{dt_b}{dz} + \frac{1}{V_R} T_b - \frac{1}{Pe_f} \frac{dT_b}{dz} - \frac{1}{V_R} \frac{(-\Delta H)}{C_s} (Mi_a - Mi_b) = 0 \quad (24)$$

where $Pe_f = \frac{G'cDp}{k_f}$ $Pe_s = \frac{G'cDp}{k_s}$ and $c = \sum m_i H_i$, the

total enthalpy of i species.

Mass Balance

$$m_{i_a} - \frac{1}{Pe_f} \frac{dm_{i_a}}{dz} - \frac{1}{V_R} \frac{C_s}{c} M_{i_a} - m_{i_b} + \frac{1}{Pe_f} + \frac{1}{V_R} \frac{C_s}{c} M_{i_b} - \frac{1}{V_R} \frac{C_s}{c} (M_{i_a} - M_{i_b}) = 0 \quad (25)$$

where Pe_f and Pe_s are the corresponding groups for mass transfer to those above, $\frac{G'Dp}{E_f}$ and $\frac{G'Dp}{E_s}$

The differential heat balances, and similarly the mass balance, take the simplified form below for the end zones.

Fluid Balance

$$\frac{1}{Pe_f} \frac{d^2 t}{dz^2} - \frac{dt}{dz} + St'(T - t) = 0 \quad (26)$$

Solid Balance

$$\frac{1}{Pe_f} \frac{d^2 T}{dz^2} + \frac{1}{V_R} \frac{dT}{dz} - St'(T - t) = 0 \quad (27)$$

The differential equations may be integrated and combined to give :

$$\frac{1}{Pe_f} \frac{dt}{dz} - t + \frac{1}{Pe_s} \frac{dT}{dz} + \frac{T}{V_R} = k \quad (28)$$

As $z \rightarrow \pm\infty$, $\frac{dt}{dz}$ and $\frac{dT}{dz} \rightarrow 0$; t and $T \rightarrow t(\pm\infty)$

and $T(\pm\infty)$

Hence

$$\text{For zone } -\infty < z < a \quad k = t(-\infty)(V_R^{-1} - 1) = T_A(V_R^{-1} - 1) \quad (29)$$

$$\text{" " } b < z < +\infty \quad k = T_c(V_R^{-1} - 1) \quad (30)$$

Substitution of (28) (29) and (30) in the overall heat balances (24) gives it the same algebraic form as the plug flow balance with the zone boundary conditions at finite values of z replaced by those at $\pm\infty$. Writing this in full with gas and condensible enthalpies written separately :

$$V_R \frac{C_s}{c} = \frac{\sum C_{i_b} m_{i_b} (T_A - t') - \sum C_{i_a} m_{i_a} (T_A - t') + (T_c - T_A) C_s + \Delta H (m_{i_a} - m_{i_b})}{\sum m_{i_a} H_{i_a} - \sum m_{i_b} H_{i_b} + C(T_A - T_c)} \quad (31)$$

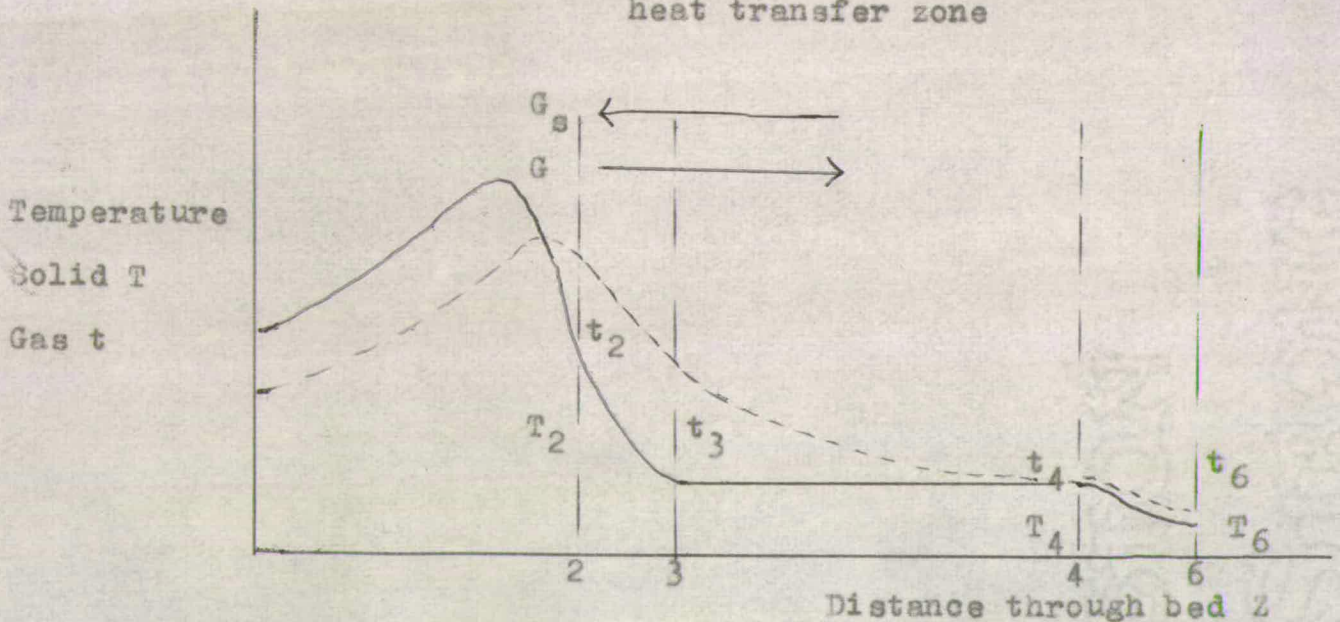
Similarly for the drying zone we may show that :

$$m_A = m(-\infty) = -m_{i_a} + \frac{E_f}{GD\rho} \frac{dm_i}{dz}; \quad m(+\infty) = m_b = m_c$$

TABLE 2

System Parameters Used in Computer Design

$C_s = 0.25$ Btu/lb $^{\circ}F$	specific heat of solid
$C_w = 1$ " " "	" " " water
$c_w = 0.45$ " " "	" " " vapour
$c = 0.25$ " " "	" " " air
$T_6 = 70^{\circ}F$	initial temp. of solid
$M_6 = 0.05$ lb H_2O /lb dry solid	" moisture content of solid
$M_2 = 0$ " " " " "	solid bone dry at drying zone exit
$m_2 = 0.01$ " " " " gas	inlet gas moisture content
$app = 0.0001^{\circ}F$	limit of approach of gas to solid temp. in drying zone
limit $t_2 = 3000^{\circ}F$	max upper limit of gas temp. inlet heat transfer zone
limit $T_2 = 1000^{\circ}F$	lower limit of solid temp. exit heat transfer zone



Estimated typical temperature profile for computer exercise steady state operation

and the conditions at $\pm \infty$ replace the conditions at the finite zone boundaries when axial effects are present.

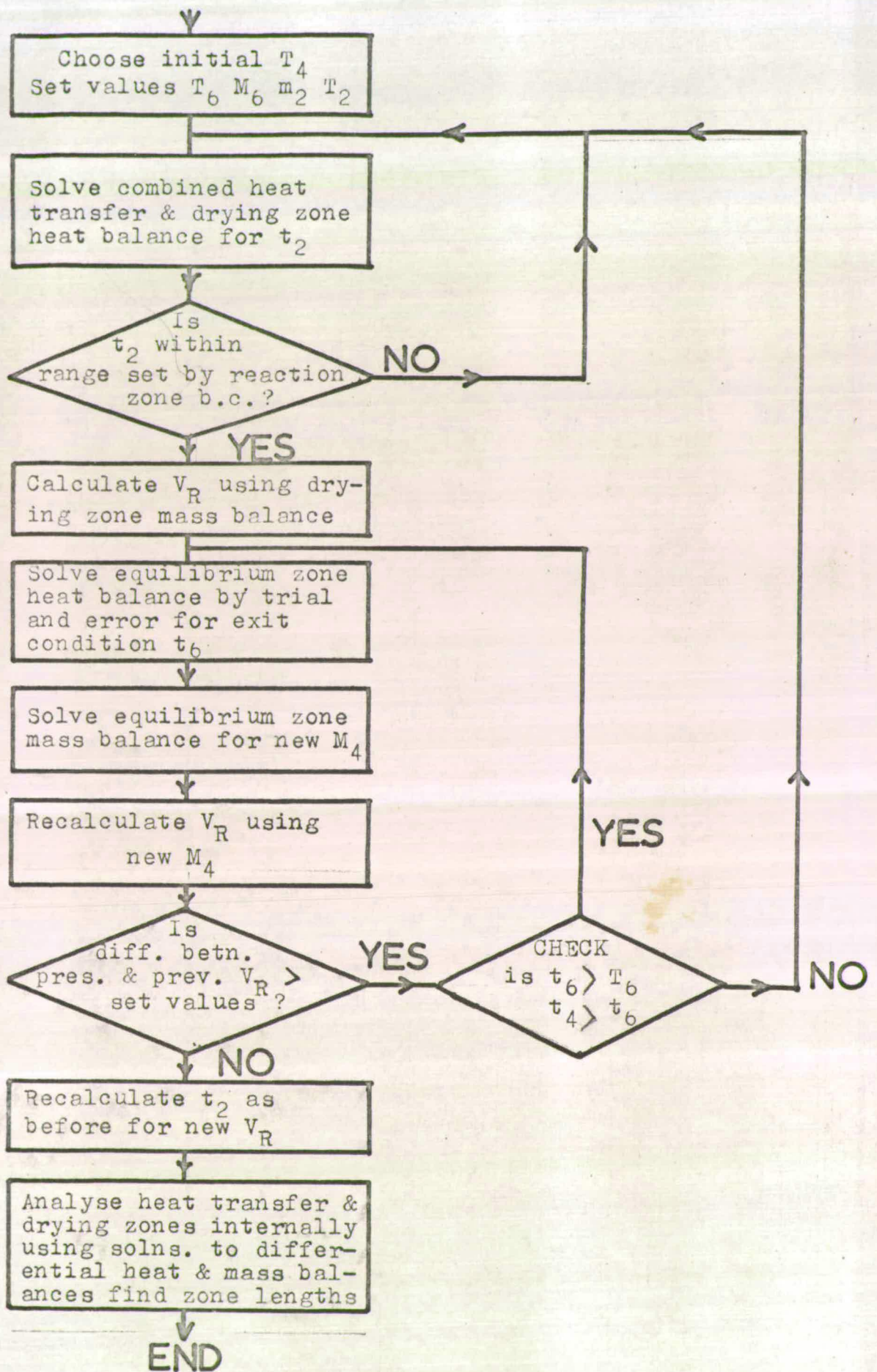
The overall mass balance then becomes :

$$\frac{V_R C_s}{c} = \frac{Mi_A - Mi_B}{mi_A - mi_b}$$

These zones may be analysed internally when the heat transfer analysis is the same as given in appendix (3) and the mass transfer analysis is as published by Wehner and Wilhelm (1956).

4.4 Application of the Separable Zone Technique

To illustrate the application of this technique, let us calculate the lengths of the heat transfer and drying zones which would be found in a pseudo steady state operation of a fixed bed, using the conditions which are typical of the boundaries of these zones when they occur on a Dwight Lloyd sinter strand during zinc sulphide oxidation. To illustrate also how the technique is suited to stepwise computer calculation the equilibrium zone has been constrained to remain in the bed as it would do in a moving bed operation. The cooling of the gas in this zone will deposit moisture whether the zone is constrained or not and so the constraint will have little effect on the lengths of the other two zones for the purposes of comparison with industrial values.



COMPUTER FLOW DIAGRAM OF
APPLICATION OF SEPARABLE ZONE TECHNIQUE

A typical value of the moisture content of the wet sulphide mix was chosen together with common values of other system parameters (see table 2). Also chosen were a typical value of the ignition temperature of the solid (T_2) and the gas temperature at the same point (t_2). These are the temperatures at the exit to the heat transfer zone and the inlet to the reaction zone in the typical temperature profile for this type of process (see fig. 1). The lengths of the heat transfer and drying zones were then calculated and compared to the depth of bed usually used on sinter strands.

As the zones are linked by the conditions at their common boundaries, each cannot be considered in isolation. The three zones were therefore linked by a trial and error method using the overall balances and once all the overall balances were satisfied, the internal analyses were used to calculate the lengths of the heat transfer and drying zones. The equations given in Chapter 4 were used. A block flow diagram of the calculation method is shown opposite and the results are given below, for three representative cases.

	t_{2F}°	T_{2F}°	t_{3F}°	T_{4F}°	t_{6F}°	V_R	l	l_D	$l + l_D$ (ins)
i	1029	1000	308	110	106	1.214	27.3	45.7	9"
ii	1390	1000	405	120	114	0.877	8.3	41.0	6"
iii	1857	1000	533	130	122	0.646	<u>5.4</u>	<u>36.9</u>	5"

particle
diameters

In result (i) the gas and solid temperatures are very nearly equal and either (ii) or (iii) is probably nearer the true values found in practice. This is supported by the fact that Dwight-Lloyd sinter strands usually operate with a bed depth of about 8 inches. Therefore the second and third results would seem to compare favourably with values found in commercial practice.

CHAPTER 5Results and Discussion5.0 Results5.0.1. Apparatus

An apparatus has been built for the study of unsteady state gas-solid systems such as are found in noncatalytic fixed bed processes. The apparatus was used to measure the velocities of moving zones in such systems while maintaining adiabatic conditions as found in industrial beds of large cross section. The apparatus was designed so that, with small changes in auxiliary equipment, it was suited to examining a number of different systems.

The problems involved in operating noncatalytic gas solid systems under adiabatic conditions when small samples are used were successfully overcome by surrounding the test bed by a guard bed in which the same processes occurred. (See Chapter 3.) The feasibility of this technique was proved in operation and velocity measurements on the gas solid system air-wet porous catalyst pellets.

5.0.2. Experimental and Theoretical Data

Zone velocities, which are a characteristic feature of moving zones in these systems, were measured for a drying zone moving through a deep bed of partially water saturated solid. The heat for drying was supplied solely by a hot air stream flowing through the bed in an axial direction.

The effect of four variables on the drying zone velocity (V)

Run No.	Time for temperature wave to reach each thermocouple level from time zero (mins)										
	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
W53	14.6	28.9	36.8	54.7	65.1	80.9	91.2	108.3	118.7	135.1	146.9
W56	16.9	31.4	46.5	61.1	74.0	90.7	107.1	126.7	143.1	161.9	176.1
W64	12.6	27.8	41.3	55.6	69.8	86.7	98.7	117.4	130.9	145.2	157.7
W65	19.8	39.5	58.8	80.0	101.0	120.2	142.5	175.2	199.0	223.5	241.2
W63	6.6	13.4	19.8	27.3	35.4	41.8	47.9	55.2	61.6	70.3	75.4
W61	5.8	13.6	19.8	27.5	34.0	41.5	47.7	55.5	62.0	70.0	78.2
W57	6.2	16.8	24.6	32.4	41.6	50.6	57.2	68.4	77.6	86.0	93.0
W67	9.0	17.4	26.2	33.5	42.4	51.1	58.7	68.7	76.7	85.1	94.9
W52	6.4	15.1	23.6	30.2	37.8	46.1	52.2	61.4	67.8	74.8	81.2
W66	10.2	20.4	29.9	42.3	51.7	61.9	70.0	-	93.3	105.9	118.3
W59	9.2	19.8	27.0	39.4	49.4	58.6	67.0	79.4	90.0	97.4	108.2
W58	10.6	25.6	35.6	47.8	59.0	68.6	77.0	92.4	106.0	120.6	134.6
W69	2.7	7.1	11.0	15.3	19.6	23.6	27.4	31.4	35.6	40.0	43.6
W55	3.6	7.6	12.4	16.6	20.8	24.8	29.4	34.8	39.7	45.0	49.3
W62	3.6	9.8	14.2	18.2	23.2	27.8	31.9	37.9	41.8	47.0	51.0
W60	4.2	11.0	15.6	20.8	26.1	31.8	-	43.4	47.6	53.8	58.8
W94	5.5	12.2	19.7	27.5	36.9	45.0	51.8	64.2	71.7	80.4	89.0
W95	3.8	10.2	15.6	21.1	26.5	33.2	37.7	45.3	51.3	58.7	64.3
W96	4.3	10.9	15.1	23.2	29.1	35.0	41.3	49.4	55.8	63.6	70.2
W97	5.5	15.8	23.8	33.4	43.4	52.2	63.6	76.1	85.6	98.6	109.0
W86	10.5	24.2	37.0	54.0	69.0	83.5	94.5	112.8	128.5	143.5	156.5
W91	15.5	36.5	51.8	72.2	86.5	110.5	126.8	157.8	177.2	203.2	220.0
W88	5.0	12.5	17.3	25.4	30.4	40.2	47.2	56.4	62.2	71.4	79.8
W90	6.1	14.6	21.6	28.5	35.7	45.3	51.1	60.8	68.1	76.4	82.9
W93	8.2	20.6	27.7	37.0	48.2	58.0	66.1	77.5	86.8	95.2	105.5
W92	7.4	18.4	26.2	37.4	47.2	56.7	66.4	74.8	84.4	92.3	101.4
W89	2.9	7.2	11.2	16.3	20.8	25.5	28.4	34.0	38.5	41.2	46.4
W87	3.6	8.8	12.8	17.6	22.2	26.4	31.2	36.8	42.1	46.8	52.8

C_{1,2,3}----- indicate thermocouple levels

C₁ is always time zero and
is at the bottom of the bed.

TABLE 3
EXPERIMENTAL DATA

Run No.	Levels	Scaled Values of Ind. Var.				Expt. V/G	Theor. V/G
		m	M	t	G		
W53	l	-1	-0.8	-1	-1	30.6	33.2
W56	m	1	-1.2686	-1	-1	25.2	29.0
W64	M	-1	0.6286	-1	-1	27.8	28.5
W65	mM	1.3	1	-1	-1	19.0	20.0
W63	t	-1	-0.8	1	-1	58.6	62.4
W61	mt	0.4	-0.7714	1	-1	57.2	62.6
W57	Mt	-1	1.5143	1	-1	46.8	52.3
W67	mMt	1.3	1.1429	1	-1	47.8	52.1
W52	G	-1	-0.8857	-1	1	33.8	33.9
W66	mG	1.75	-0.3571	-1	1	24.7	24.7
W59	MG	-1	1.4571	-1	1	25.5	26.4
W58	mMG	0.4	1.1714	-1	1	21.0	23.7
W69	tG	-1	-0.7314	1	1	61.7	63.3
W55	mtG	2.2	-1.3429	1	1	55.0	64.8
W62	MtG	-1	1.2514	1	1	53.8	52.8
W60	mMtG	0.9	1.4	1	1	46.7	51.8
W94		2	0.0857	1	-1	47.9	57.0
W95		1.1	-1.8120	1	-1	67.6	70.8
W96		1.1	0.1543	2	-1	61.6	71.1
W97		1.2	-0.9257	1	-2	56.6	63.7

Runs W52 - 69 are the original factorial design

Runs W94 - 97 are extra points to make up composite design

Variables are scaled to zero at the centre of the expt. range

	m	M	t	G
Base	0.01 <u>lb.water</u> lb.drygas	15.25 <u>lb.water</u> lb.drysolid	230 °F	320 <u>lb</u> hr.ft ²
Unit				
Range	0.01	1.75	65	75
Scaled	<u>m-0.01</u>	<u>M-15.25</u>	<u>t-230</u>	<u>G-320</u>
Var.	0.01	1.75	65	75
Units	V/G = <u>ins.hr.ft²</u> min.lb			

Run No.	Levels	Scaled Values of Ind. Var.				Expt. V/G	Theor. V/G
		m	M	t	G		
W86(3)	l	-1	-0.0114	-1	-1	27.0	30.8
W91	mM	0.92	1.3429	-1	-1	19.6	22.7
W88	mt	0.975	-0.2	1	-1	54.3	60.0
W90	Mt	-1	1.0114	1	-1	52.7	54.6
W93	mG	0.9	-0.7143	-1	1	26.2	30.7
W92	MG	-1	1.1429	-1	1	26.9	27.2
W89	tG	-1	0.0171	1	1	58.1	58.2
W87	mMtG	0.94	1.0857	1	1	52.1	52.1

W86(3) - 93 Runs for 1/2 replicate check experiment

W72		-1	-1.8114	0	-1	50.4	54.9
W73		-1	-2.0000	-1	0	35.4	38.6
W75		-1	-2.5771	0	0	53.4	57.5
W74		-1	-2.1943	1	0	73.5	76.6
W71		-1	-1.2400	0	1	50.4	51.8
W77		-1	-3.6686	-1	-1	46.6	49.6
W79		-1	-4.8349	1	-1	103.8	106.4
W80		-1	-4.9289	-1	0	63.1	62.9
W82		-1	-2.6857	1	0	75.5	75.1
W84		-1	-5.0000	0	1	86.5	88.4

W71 - 84 Extra runs performed as test runs during development of apparatus.

Height of thermocouples from bottom of bed (inches)

C1 = 0	C7 = 5.875
C2 = 0.8125	C8 = 6.75
C3 = 1.875	C9 = 7.9375
C4 = 2.8125	C10 = 8.8125
C5 = 3.8125	C11 = 9.875
C6 = 4.8125	C12 = 10.75

Solid used in the bed was catalyst support pellets supplied by Norton Abrasives Ltd., Welwyn Garden City. Their reference number SA 5103/6 - pellets 1/8" x 1/8" cylinders.

was examined, using experiments arranged according to a statistical design which is discussed in Appendix 4. Inlet gas temperature (t_2), inlet gas moisture content (m_2), gas flow rate (G) and solids' moisture content (m_4) were the variables altered.

For each set of experimental conditions, a theoretical "velocity" was also calculated. This response, chosen from a theoretical consideration of the zone overall balance was calculated as the quotient zone velocity/gas mass flow rate (V/G) (See Appendix 4). The theoretical and experimental responses were both correlated as polynomial functions of the four variables, separate polynomials being fitted to the experimental and theoretical responses. The polynomials are each duplicated for statistical reasons given in Appendix 4, but equations (33) and (35) correspond to the same set of experimental conditions as do (34) and (36).

Experimental Response

$$\begin{aligned} \frac{V}{G} = & 43.31 - 2.6m - 4.21M + 14.63t_2 + 0.92G - 0.66m^2 + 0.51m^2 \\ & - 1.81t^2 - 0.22G^2 + 0.23mM + 0.5mt - 0.32mG - 0.94Mt \\ & + 0.067MG - 0.034tG \end{aligned} \quad (33)$$

$$\begin{aligned} \frac{V}{G} = & 39.44 - 2.31m - 3.96M + 14.23t + 0.9G - 0.2m^2 + 0.94M^2 \\ & - 0.33t^2 + 0.53G^2 - 0.072mM + 0.087mt - 0.48mG - 1.50Mt \\ & - 0.068MG + 0.03tG \end{aligned} \quad (34)$$

Theoretical Responses

$$\begin{aligned} \frac{V}{G} = & 45.07 - 1.24m - 4.46M + 15.3t + 0.17G - 0.66m^2 + 0.46M^2 \\ & - 1.14t^2 + 0.08G^2 - 0.25mM + 1.25mt + 0.04mG - 0.98Mt \\ & - 0.36MG - 0.25tG \end{aligned} \quad (35)$$

$$\begin{aligned} \frac{V}{G} = & 42.27 - 1.56m - 4.16M + 15.47t + 0.23G - 0.11m^2 + 0.86M^2 \\ & - 0.518t^2 + 0.79G^2 - 0.007mM + 1.06mt + 0.12mG - 1.42Mt \\ & - 0.13MG - 0.17tG \end{aligned} \quad (36)$$

5.1 Discussion5.1.1 Apparatus

The main advantage of the bed design in the apparatus is that it can duplicate for small samples, conditions normally found in commercial reactors of large adiabatic cross section. The design of a bed within a guard bed gains over the other designs, such as larger cross section beds, lagged small beds, or beds compensated for heat loss by external heaters by enabling very small samples to be used and allowing adiabatic conditions to be maintained while reproducing processes whose temperature profiles change abruptly. In small lagged beds the heat capacity of the lagging approaches that of the particles; in the concentric beds system the wall may be very thin, as it is supported on both sides by the particles,

and of negligible heat capacity compared to the particles. And externally wound heaters cannot follow the abrupt temperature changes occurring in non-catalytic gas-solid processes.

Considerable dexterity and judgement are required in the operation of the apparatus described in Chapter 3. However, this skill can soon be acquired with practice. For very high temperature applications or processes producing very fast moving zones the apparatus would have to be modified and automated. For instance, it is not practicable to mount the bed by hand at gas inlet temperatures much above 500°C and if the zones move fast, some form of automatic control would be necessary for the guard bed and high speed recorders and so on. This would add to the cost of the apparatus but the bed design would still be attractive especially when the solid is expensive or in short supply.

Of the four independent variables the measurement of the inlet gas moisture was the most inexact. The measurement of gas moisture contents is very difficult in either near dry or near saturation conditions. Lorenzen (1941) shows how the accuracy of a wet and dry bulb thermocouple hygrometer falls off at low moisture contents. The humidity sensor was offscale at the low moisture contents and required frequent checking due to drifting at the high ones. The errors in the other variables were small by comparison. The error in

Table 4

Coeff.	Statistical Design + 10 pts		Statistical Design + 8 pts		
	99% Expt. Conf. Range	Theoretical Value	99% Expt. Conf. Range	Theoretical Value	
b_o	45.87 to 33.00	42.27	51.57 to 35.05	45.07	
b_m	0.6	-4.57	-4.08	-0.24	-1.24
b_M	2.32	-5.61	-5.99	-2.43	-4.46
b_t	16.17	12.302	16.31	12.95	15.30
b_G	2.90	-1.10	2.62	-0.78	0.17
b_m^2	2.42	-2.82	1.96	-3.29	-0.66
b_M^2	1.41	0.471	2.95	-1.94	+0.46
b_t^2	2.71	-3.37	1.66	-5.28	-1.14
b_G^2	3.59	-2.53	3.22	-3.66	0.08
b_{mM}	1.39	-1.54	1.94	-1.9	0.25
b_{mt}	1.79	-1.62	2.09	-1.08	1.25
b_{mG}	1.2	-2.15	1.17	-1.81	0.04
b_{Mt}	0.46	-2.55	0.53	-2.60	-0.98
b_{MG}	1.08	-1.09	1.99	-1.86	0.36
b_{tG}	2.04	-1.98	1.62	-1.69	-0.25

the temperature measurement is about $\pm 2\%$ at worst. The rotameter can be read to a similar accuracy. The solid sampling method was examined statistically to check that the sample moisture contents were representative of the bulk load into the bed. The error in the rotameter is about $\pm 5\%$ inherent in the instruments design.

5.1.2 Data

As the coefficients of the second order terms in the approximate polynomials are generally much smaller than those of the first order terms, the velocity of the temperature wave (defined as V/G) may be expressed approximately as a linear function of inlet gas moisture content (m), solid moisture content (M) and inlet gas temperature (t).

The accuracy of the predictions was assessed by comparing corresponding coefficients in the two pairs of polynomials (equations 33 to 36). From earlier experiments, it was concluded that the errors in the responses were normally distributed and so confidence limits were calculated for the coefficients. Table 4 shows both sets of coefficients together with the confidence range for the experimental ones. As the range includes the coefficients from the theoretical responses, we conclude that within experimental error the approximate zone theory predicts the experimentally found responses.

The size of the coefficient for G terms and for the

TABLE 5 Comparison of Experimental and Theoretical Results(*)

Comparison of Responses (V/G)

<u>Run</u>	<u>V/G (Exptl.) 10⁶</u> <i>ms. hr. ft²</i> <i>min. lb.</i>	<u>V/G (Theoret.) 10⁶</u> <i>ms. hr. ft²</i> <i>min. lb.</i>	<u>Diff %</u>
W52	338	339	0.3
W53	306	332	8.5
W54	617	633	2.6
W55	550	648	17.7
W56	252	287	13.9
W57	468	523	11.8
W58	210	237	12.9
W59	255	264	3.5
W60	467	518	10.9
W61	572	626	9.4
W62	538	528	1.9
W63	586	624	6.5
W64	278	285	2.5
W65	190	200	5.3
W66	247	247	0
W67	478	521	9.0

Factorial Mean Difference 7.3%
 Extra pts. " " 6.9%
 Overall " " 7.1%

TABLE 5a Comparison of Wet Bulb Temperatures

<u>Run</u>	<u>T_{WB} (Exptl.)</u> °F	<u>T_{WB} (theoretical)</u> °F
W 86 (3)	75	75
W 87	106	98.4
W 88	106	108.2
W 89	96	95.1
W 90	97	95.5
W 91	94	94.1
W 92	75	75.3
W 93	95	111.0
W 94	113	114.0
W 95	110	107.9
W 96	115	112.9
W 97	112	109.6

and quadratic terms relative to the range is small but the size of some of these coefficients indicates that the effect may become more pronounced if the range of the variables is widened. This does not however invalidate the theory as under much wider ranges of the variables, the assumptions of constant system parameters will not apply and the theory is intended as an essentially simple and approximate analysis not a fundamental one.

Tables 5 and 5a show both experimental and theoretical true velocities and the difference between them and a comparison of some measured and calculated wet bulb temperatures. The closeness of most of the theoretical velocities to those measured shows the validity of the separable zone approximate analysis technique. The mean difference between the velocities is shown at the foot of the Table.

The close agreement in most cases between the wet bulb temperatures indicates the soundness of the expanding zone analysis. It was found that three or four trials of the iterative numerical calculating technique gave the wet bulb temperature to the same accuracy as it could be measured; a very fast convergence which emphasises the utility of the expanding zone analysis. An example is given in Appendix 5 which illustrates this calculating technique.

The most likely source of the difference between the

theoretical and the experimental velocities is, as discussed in 5.1.1., the error in the measurement of the inlet gas moisture content. The conclusions from the statistical analysis of the data from the check experiment on sample solid moisture contents, were that none of the possible sources of variation considered did influence the moisture content of the samples as measured by the significance test at the 5% level. The samples were considered to be representative of the bulk load to the bed within random error. The effect of random error was reduced by taking five samples of each bed load and using the mean of their moisture contents as the moisture content of the bulk load.

5.1.3. Relation between Statistical Equations and Theoretical Model

The analysis of variance technique which was applied first to the results was a preliminary investigation to determine the relative magnitudes of the effects present. It indicated a more suitable dependent variable than the velocity itself (i.e. the quotient V/G) but was only used to examine the factorial design for snags, such as interaction effects.

Three mathematical models of the system (air/wet catalyst pellets) which was used in the experiments are presented. These are (i) a statistical polynomial model of the experimental results, (ii) a similar polynomial model of the theoretically predicted results for the same conditions as used

in the experiments and (iii) a theoretical algebraic mechanistic model which was used to predict the results for comparison with experiment.

The statistical polynomial model (ii) is essentially a version of the theoretical mechanistic model (iii) applying in the limited design space covered by the experiments. Similarly the polynomial model (i) is a model of the experimental results in the design space of the experiments. Therefore, within the experimental design space, comparison of models (i) and (ii) is equivalent to comparison of models (i) and (iii) i.e. it is a comparison of the experimental results with the theoretical model's results.

Outside the experimental design space the model (iii) is the only one which, strictly, is valid as it is not restricted in derivation to one region of variable space. Hence if it is desired to predict zone velocities in another region of the variable space model (iii) should be used. Some confidence could be placed in this model as it has been tested over part of the variable space but complete confidence could not be placed in it unless it had been tested over the whole space.

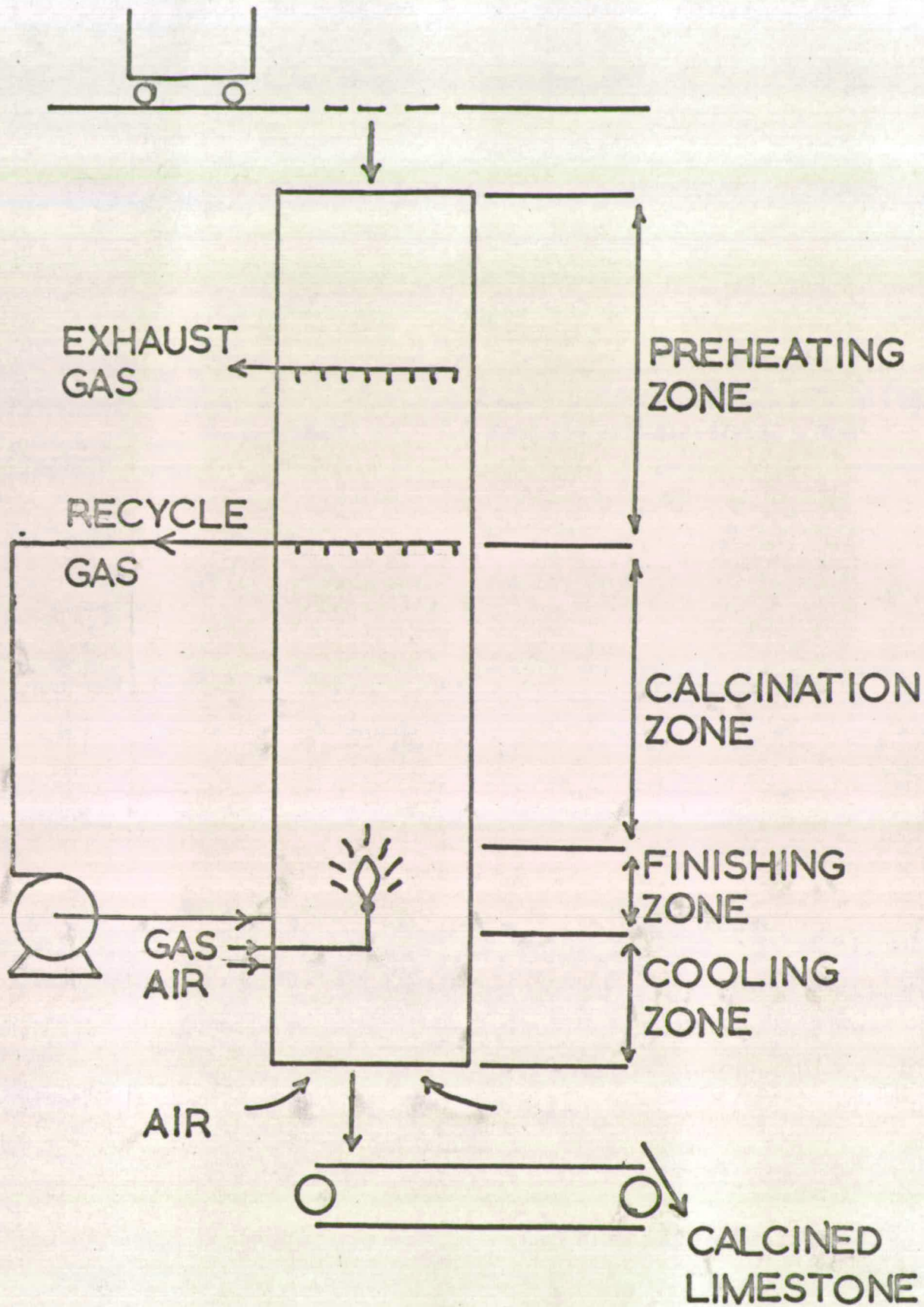
Furthermore the polynomial is only correct at the individual points in the variable space which were used in its derivation. Therefore once again the model (iii) should be used except at one of these points, although the

polynomial would be expected to be a very near approximation to model (iii) at points within the design space.

In general the polynomial models are useful for comparing experimental results with those predicted by the theoretical model over a region of variable space, but at individual points in the space the theoretical model should be used.

5.2. Some General Considerations.

The simplicity of the separable zone technique imposes limitations on its applicability. Any system to which the technique is to be applied must be represented at least approximately by a 1-D set of equations for the differential heat and mass balances. The division of the system into zones is another approximation and this demands well established zones such as are shown in the estimated typical profile fig. 1 (See Chapters 1 and 2). The internal analyses of the zones are restricted further to systems in which the temperature and concentration gradients inside the particles may be neglected.



LINE DIAGRAM OF VERTICAL LIMEKILN
(Centre burner, gas fired)

In a general non-catalytic gas-solid process there may be both exothermic and endothermic zones; the separable zone technique will apply to both if the assumptions of constant velocity and width are satisfied. This system is most likely when a slow endothermic zone precedes a faster exothermic one, so that a self sharpening profile is produced. However, a countercurrent gas-solid system constrains the zones to be of constant width and the separating zone analysis is particularly applicable.

For instance calcium carbonate decomposition is often carried out in vertical kilns in which the zones are of constant width. In these kilns the solid flows vertically downward, settling under gravity, and it passes through zones as shown opposite. The zones are well defined by the structure of the kiln. The calcining zone, for instance, is limited to the length of the kiln in which the hot flue gas is in contact with the limestone, that is between the entry points and the recycle offtake. The other zones are similarly defined.

The separation of the zones for internal analysis, while imposing these restrictions has advantages in that each internal analysis can be examined separately and the overall balances can be used to link a number of such analyses. This should be useful with computer techniques which are especially suited to stepwise calculations. A very simple example of

this method of approach is given in Chapter 4.4. This technique also lends itself to steady state optimisation as the effects of the various component processes on the end products can be easily followed. The separation of the processes occurring, into individual zones, does not remove the necessity for internal analysis of these zones and here the various simplified approaches discussed in Chapter 2 will be useful.

5.3 Recommendations for Future Work

The separable zone technique should be applied to more complicated zone systems to determine its range and limits of useful application. The range covered by the present statistical designs could be extended, still using the drying process, and the effect of this on the interactions possible between the various factors examined. A natural development of this endothermic work would be to investigate a true endothermic reaction, for instance, a carbonate decomposition, and after this to experiment with systems of exothermic and endothermic, or a sequence of endothermic, zones. Experiments on some of the newer applications of moving beds suggested by Venner (1955) could then be tried.

For this the apparatus should be fabricated in materials which can withstand much higher temperatures than those used here, so that the operating technique for examining other

complicated systems reacting at high temperatures (above 1000°C say) such as some metallurgical processes. can be developed. For instance, limestone calcination and zinc sulphide oxidation show complicated sequential zone systems.

Adiabatic sorption and desorption problems are another field, hardly touched as yet, which are suitable for the separable zone technique and the concentric bed apparatus. This work would be at low temperatures with a consequent simplification of construction expense and control difficulties.

The simple example, of a computer approach to a design problem, which was given in Chapter 4.4 could be extended and compared with some industrial situations. If this proved to be a useful and feasible design method then academic and industrial research on single reactions would have more immediate application as work by different authors could be combined for industrial use using the separable zone technique.

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APPENDIX 1Nomenclature

A	constant coefficient in differential equation; cross sectional area
A_1	constant group $G'(c_p + m_4 c_w)/h_v D_p$
B	constant coefficient in differential equation
B_1	" group $(m_3 - m_4) G' c_w / h_v D_p$
b	estimate of true coefficient β in statistical polynomial
C	specific heat of solid or liquid with subscripts which indicate application (Btu/lb °F)
c	ditto for gas or vapour (Btu/lb °F)
c_k	constants of integration ($k = 1, 2, 3 \dots$)
D	differential operator; constant coefficient
d.f	statistical degrees of freedom
E, F	constant coefficients
F	variance ratio
$f()$	function of ()
G	gas mass flow rate (lb/hr. ft ²); $G' = G - V\delta\rho$
	modified gas mass flow rate; subscript s solid mass flow rate
$g()$	function of ()
h_v	volumetric heat transfer coefficient (Btu/hr. ft ² °F)
H	enthalpy
i, j, k	integers
I_1	modified Bessel Function of the first kind
j_H, j_D	heat and mass transfer correlation expressions

- k_v volumetric mass transfer coefficient (lb/hr. ft² Δ concⁿ)
 k_s, k_f axial solid and gas thermal conductivities
 $k(\phi, z')$, $k(\phi, z', Y')$ integrals used in representing the results of internal analysis of an expanding heat transfer zone
 l length of a zone with subscripts indicating which zone, no subscript for heat transfer zone
 \mathcal{L} Laplacian Operator
 m, M concentration on an inert basis of active component in gas and solid respectively (lb/lb inerts)
 m_s gas moisture content at saturation
 n integer
 P, P_0 constant coefficients
 p variable in Laplace Transform equations
 q constant " " " "
 Q, R " coefficients
 S, s variables in Laplace Transform equations
 t, T gas and solid temperatures
 \bar{t}, \bar{T} " " " " transformed by Laplace transformation
 V zone velocity or in a countercurrent gas-solid system mean solids velocity (ft/hr)
 $W(i, j)$ experimental run numbers (i, j integers)
 Y, Y' independent variables in expanding zone equations
 $Y = \alpha_1(z - V \theta)$; $Y' = h_v D_p Y / V(1 - \delta) / \rho_s c_s \alpha_1$

$$Z, Z' \quad \text{ditto} \quad Z = \alpha_2(z); \quad Z' = Zh_{vD_p}/G'c\alpha_2$$

$$z = \bar{y}/D_p$$

Statistical Matrices (See Appendix 4)

X matrix of independent variables

X' transpose of X

B vector of coefficients

Y " " responses

D design matrix

$T = X(X.X')^{-1}$ transforming matrix

$C = (X.X')$ precision matrix

Dimensionless Groups

$Re_p = \frac{D_p G}{\mu}$ particulate Reynolds Number

$Pe_s, Pe_f = \frac{k_s}{G'cD_p}, \frac{k_f}{G'cD_p}$ axial Peclet Numbers for heat transfer solid and fluid respectively

$Pe'_s, Pe'_f = \frac{E_s}{G'D_p}, \frac{E_f}{G'D_p}$ ditto for mass transfer

$St' = \frac{h_v D_p}{G'c}$ modified Stanton Number for heat transfer

$V_R = \frac{G - V\delta\rho}{V(1-\delta)\rho_s C_s}$ velocity ratio

Greek Symbols

α root of equation

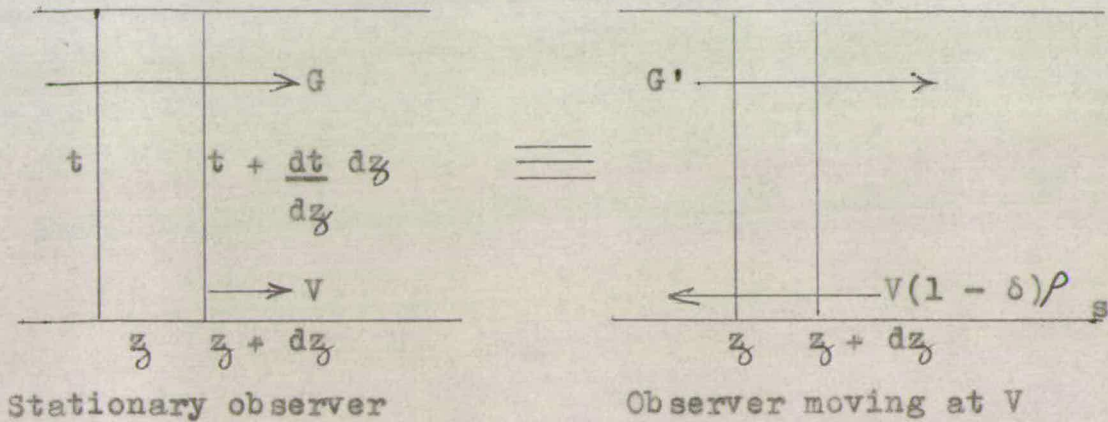
β " " " ; variable in Laplace Transform (see text)

β_i coefficients in statistical polynomials ($i = 1, 2, 3 \dots$)

δ, δ'	root of equation
$\delta_1 \delta_2$	statistical coefficients of skewness and excess
δ	fractional voids
ρ	gas density
ρ_s	solid bulk density
Θ	time
ϕ	V_R
Φ, γ	independent variables used in expanding zone analysis
Ω	group $k_v D_p / G'$

Subscripts

i, j, k	integers
i, o	inlet, outlet
p	constant pressure
w	water
WB	wet bulb
s	solid

APPENDIX 2Transformation to Moving Origin

Consider gas mass flows to be constant. The fluid heat balance over an element of bed length dz is an unsteady state one dimensional partial differential equation with respect to a stationary origin.

i.e. Fluid Heat Balance

$$- G_c \frac{\partial t}{\partial z_j} + h_v (T - t) = \delta \rho c \frac{\partial t}{\partial \theta} \quad (1)$$

Solid Phase Balance

$$- h_v (T - t) = (1 - \delta) \rho_s c_s \frac{\partial T}{\partial \theta} \quad (2)$$

Now define $z' = z_j - V\theta$ $\theta' = \theta$ i.e. origin moving at velocity V

$$\text{Then } \frac{\partial t}{\partial z_j} = \frac{\partial t}{\partial z'} \cdot \frac{\partial z'}{\partial z_j} + \frac{\partial t}{\partial \theta'} \cdot \frac{\partial \theta'}{\partial z_j} = \frac{\partial t}{\partial z'}$$

$$\frac{\partial t}{\partial \theta} = \frac{\partial t}{\partial Z'} \frac{\partial Z'}{\partial \theta} + \frac{\partial t}{\partial \theta'} \frac{\partial \theta'}{\partial \theta} = \frac{\partial t}{\partial \theta'} - V \frac{\partial t}{\partial Z'}$$

and if we write $G = G' + V\delta\rho$ and substitute the above equations in (1) and (2) above

$$- (G' + V\delta\rho) c \frac{\partial t}{\partial Z'} + h_V(T - t) = \delta\rho c \frac{\partial t}{\partial \theta'} - V \frac{\partial t}{\partial Z'}$$

$$- h_V(T - t) = (1 - \delta) \rho_s c_s \frac{\partial T}{\partial \theta'} - V \frac{\partial T}{\partial Z'}$$

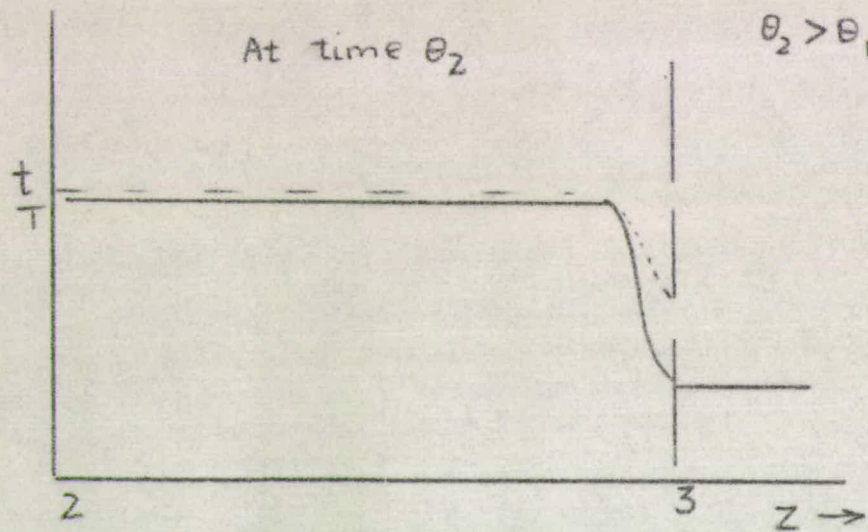
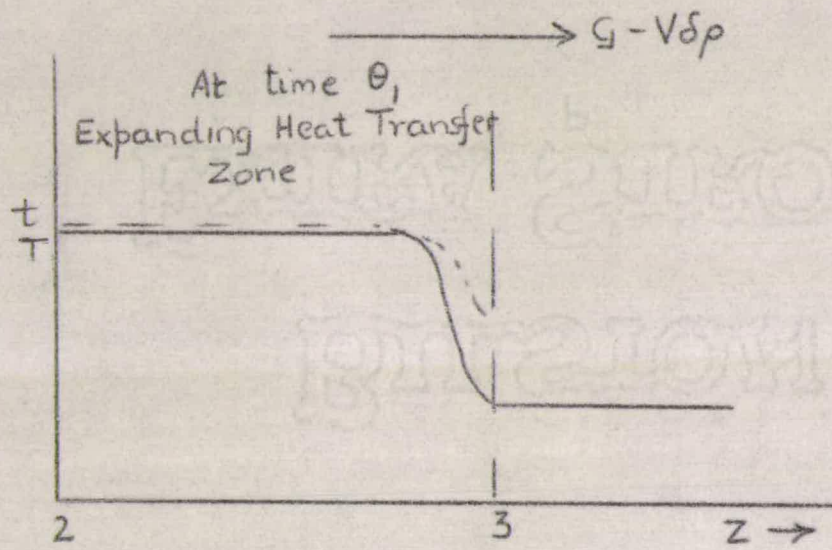
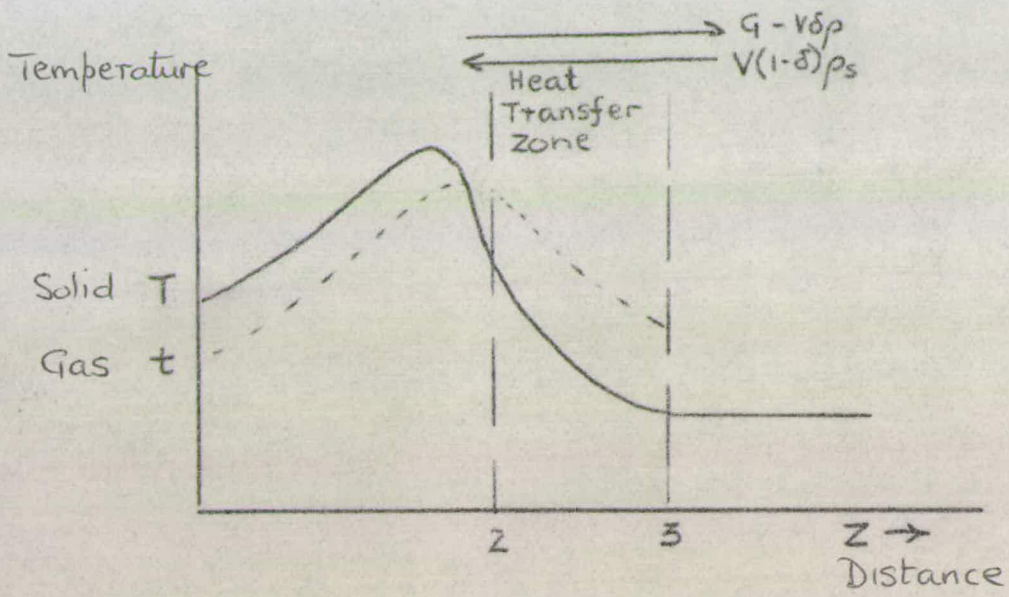
Rearranging :

$$- (G - V\delta\rho) \frac{dt}{dZ'} + h_V(T - t) = 0$$

$$V(1 - \delta) \rho_s c_s \frac{dT}{dZ'} - h_V(T - t) = 0$$

since $\frac{\partial t}{\partial \theta'}$ and $\frac{\partial T}{\partial \theta'}$ are zero by definition in new coordinate

system (quasi stationary state).



APPENDIX 3Internal Analyses

The zones for which detailed internal analyses are given are defined in terms of the estimated typical profile (see Chapter 2). Analyses are given for a pure heat transfer zone and for a constant drying rate zone. The former is also applicable to the equilibrium zone under certain assumptions.

The estimated typical profile is that suggested for steady state conditions. It can also be applied to a fixed bed system when the heat and mass transfer equations may be arranged in pseudo steady state form. An internal analysis is also derived for a heat transfer zone expanding at constant velocity from zero width as this applies to the present experimental system. This zone is also defined in terms of an estimated typical profile similar to that above, but under unsteady state conditions. See estimated profiles opposite.

3.1 Internal Analysis of a Zone with only Heat Transfer taking place under Pseudo Steady State Conditions

The heat transfer zone is defined in terms of the estimated typical profile as that zone between planes (2) and (3). In this zone heat transfer takes place by bulk convection from the gas to the solid. For the pseudo steady state analysis, the zone is regarded as stationary with countercurrent flow of gas and solid through it. The

equations of Singer and Wilhelm (1950) which apply are obtained by setting $R = 0$ in equations (1) and (2) in Chapter 4. The equations however, are still second order and as they are simultaneous equations they may be combined to give the quartic differential equation A3.3.

$$K_f \frac{d^2 t}{dz^2} - G'_c \frac{dt}{dz} + h_v(T - t) = 0 \quad A3.1$$

$$K_s \frac{d^2 T}{dz^2} + V(1 - \delta) \rho_s C_s \frac{dT}{dz} - h_v(T - t) = 0 \quad A3.2$$

Combining the above in dimensionless form :

$$\frac{d^4 t}{dz^4} + P \frac{d^3 t}{dz^3} - Q \frac{d^2 t}{dz^2} - R \frac{dt}{dz} = 0 \quad A3.3$$

which is immediately integrable to give :

$$\frac{d^3 t}{dz^3} + P \frac{d^2 t}{dz^2} - Q \frac{dt}{dz} - Rt = c_4 \quad A3.4$$

$$\text{where } P = \frac{Pe_s}{V_R} - Pe_f ; \quad Q = St' \left[Pe_s + \frac{Pe_s Pe_f}{St' V_R} - Pe_f \right]$$

$$R = Pe_s \left[Pe_f St' + \frac{1}{V_R} \right]$$

The general solution to the quartic differential equation depends on the roots of the cubic auxiliary equation from A3.4.

$$D^3 + PD^2 - QD - R = 0$$

which may have real or complex roots. To decide which roots apply for any given system, the dimensionless groups which make up the constant coefficients must be evaluated numerically because the roots are stated in terms of P, Q and R. Perry (4th Edition).

The general solution has the following form

$$t = c_1 e^{\lambda_1 z} + c_2 e^{\lambda_2 z} + c_3 e^{\lambda_3 z} + c_4 \quad A3.5$$

where λ_1 λ_2 λ_3 are the roots of the auxiliary equation. These roots may be all three real and equal, one real and two complex or all three real and distinct. The choice depends on the magnitudes of P, Q and R.

These equations are useful in the approximate zone theory when axial effects, eddy diffusion and conduction, must be included in the internal analysis of the zone (see Chapter 4). However, as the keynote of this separable zone technique is simplicity, the plug flow differential balances are more useful. The plug flow balances are obtained by deleting the second order terms in A3.1 and A3.2 to give first order equations. Combination of these two balances results in a second order equation whose solution depends on only two dimensionless groups.

Adiabatic Heat Transfer without Axial EffectsHeat Balance - Both Phases

$$- G'c \frac{dt}{dz} + V(1 - \delta) \rho_s C_s \frac{dT}{dz} = 0 \quad A3.6$$

Fluid Phase Alone

$$- G'c \frac{dt}{dz} + h_v(T - t) = 0 \quad A3.7$$

Differentiating A3.7 and eliminating $\frac{dT}{dz}$ by substitution from A3.6

$$\frac{G'c}{h_v D_p} \cdot \frac{d^2 t}{dz^2} + \left(1 - \frac{G'c}{V(1 - \delta) \rho_s C_s} \right) \frac{dt}{dz} = 0$$

from which integration gives

$$t = c_1 + c_2 e^{-P_0 z} \quad A3.8$$

where $P_0 = St'(1 - V_R)$

and

$$T = c_1 + c_2 V_R e^{-P_0 z} \quad A3.9$$

Boundary Conditions

$$\begin{array}{lll} z = 0 & t = t_2 & T = T_2 \\ z = 1 & t = t_3 & T = T_3 \end{array}$$

Hence we may write four equations for the conditions of each phase at the boundaries and discuss the conditions they impose upon the general solutions and their use.

General Conditions

$$z = 0 \quad t_2 = c_1 + c_2 e^{-P_0 l} \quad \text{A3.10}$$

$$z = 1 \quad t_3 = c_1 + c_2 e^{-P_0 l} \quad \text{A3.11}$$

$$z = 1 \quad T_3 = c_1 + c_2 V_R e^{-P_0 l} \quad \text{A3.12}$$

$$z = 0 \quad T_2 = c_1 + c_2 V_R \quad \text{A3.13}$$

Subtracting A3.13 from A3.12 and A3.11 from A3.10

$$T_2 - T_3 = c_2 V_R (1 - e^{-P_0 l}) ; \quad t_2 - t_3 = c_2 (1 - e^{-P_0 l})$$

$$\frac{t_2 - t_3}{T_2 - T_3} = \frac{1}{V_R} \quad \text{A3.14}$$

Now we can say $t_2 \geq t_3$ $T_2 \geq T_3$

$t_2 \geq T_2$ $t_3 \geq T_3$

If $t_2 > T_2$ and $t_3 > T_3$ then $V_R \leq 1$

Let us put $t_2 = T_2 + \alpha$ and examine conditions as $\alpha \rightarrow 0$

From A3.12
$$e^{-P_0 l} = \frac{T_3 - c_1}{c_2 V_R}$$

and substituting in A3.11

$$c_1 = \frac{V_R t_3 - T_3}{V_R - 1}$$

and so from A3.10

$$c_2 = t_2 - \frac{V_R t_3 - T_3}{V_R - 1}$$

If we back substitute for c_1 c_2 in A3.12 we get l as a function of V_R and known temperatures

$$e^{-P_0 l} = \frac{t_2(V_R - 1) - V_R t_3 + T_3}{T_2 - t_3}$$

Rearranging and substituting for V_R from A3.14

$$e^{-P_0 l} = \frac{\alpha}{t_3 - T_3} \quad \text{A3.15}$$

where $P_0 = St'(1 - V_R)$

If $\alpha = 0$ and $t_3 > T_3$ then from A3.15

$$e^{St'(1 - V_R)l} = 0$$

$$(1 - V_R)l = -\infty$$

and since if $\alpha = 0$ $V_R > 1$ then $P_0 < 0 \therefore l = \infty$

If $\alpha > 0$ and $t_3 = T_3$ then from A3.15

$$e^{St'(1 - V_R)l} = \infty$$

$$(1 - V_R)l = \infty$$

and since $\alpha > 0$ $V_R < 1$ then $P_0 > 0 \therefore l = \infty$

and if $\alpha = 0$ and $t_3 = T_3$ $e^{-P_0 l} = \frac{0}{0} \therefore l$ indeterminate

Therefore, even for approximation purposes, no matter how closely the gas and solid temperatures at the zone

boundaries approach each other they must never be set equal.

$$\text{In general} \quad St'(1 - V_R)l = \ln \frac{\alpha}{t_3 - T_3}$$

$$l = \frac{l}{St'(1 - V_R)} \ln \frac{\alpha}{t_3 - T_3}$$

and the conditions for sensible solutions are

$$\text{for } V_R \geq 1 \quad \alpha > 0 \quad \text{and } t_3 > T_3$$

$$\text{if } V_R < 1 \quad \text{then } l > 0 \quad \text{if } \alpha > t_3 - T_3$$

$$\text{and } V_R > 1 \quad " \quad l > 0 \quad \text{if } \alpha > t_3 - T_3$$

A corollary is as $\alpha \rightarrow 0$; $c_1 \rightarrow t_2$ and $c_2 \rightarrow 0$

Limiting Case

As the general solution to the heat transfer equations includes $(1 - V_R)$ in the exponential exponent the limiting case of $V_R \rightarrow 1$ should be examined as this could arise in practice. From the four equations resulting from the application of the boundary conditions A3.10 A3.13 the length of the zone may be written as

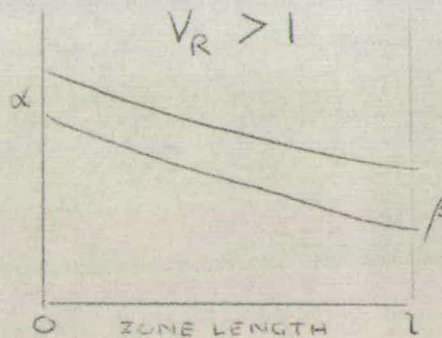
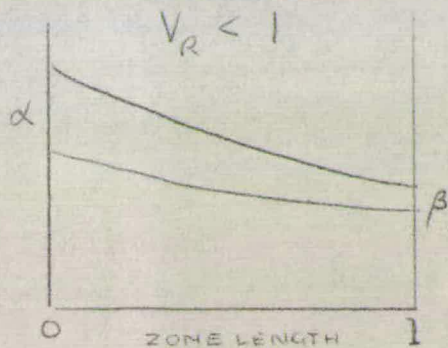
$$l = \frac{l}{P_0} \ln \frac{t_2 - T_2}{t_3 - T_3} = \frac{l}{P_0} \ln \frac{\alpha}{\beta} \quad \text{A3.16}$$

$$\text{As } l > 0 \quad \text{we may say } V_R < 1 \quad \alpha > \beta$$

$$V_R > 1 \quad \alpha < \beta$$

i.e. the temperature profiles must have the form in the

sketches below



As $V_R \rightarrow 1$, $\alpha \rightarrow \beta$ and l tends $\rightarrow 0/0$ indeterminate. This case may be examined using L'Hopital's rule. Using the overall balance on the heat transfer zone

$$V_R = \frac{T_2 - T_3}{t_2 - t_3}$$

to substitute in A3.16 we may write this as

$$l = \frac{t_2 - t_3}{St'} \ln \frac{\alpha}{\beta}$$

Now we may regard $t_2 - t_3$ as a fixed permissible temperature change in a given situation and so

$$l = \frac{t_2 - t_3}{St'} \frac{f_1(\alpha)}{f_2(\alpha)}$$

$$\text{and } f_1'(\alpha) = \frac{1}{\alpha} \quad f_2'(\beta) = 1$$

by L'Hopital's Rule

$$\lim_{\alpha \rightarrow \beta} l = \frac{t_2 - t_3}{St'} \lim_{\alpha \rightarrow \beta} \frac{f_1'(\alpha)}{f_2'(\alpha)} = \frac{t_2 - t_3}{St'} \cdot \frac{1}{t_2 - T_2}$$

3.2 Internal Analysis of Constant Drying Rate Zone

The following differential heat and mass balances are formulated from those of Singer and Wilhelm using the assumptions previously discussed.

In addition, it is assumed that in a zone in which constant rate drying is taking place

1. Solid temperature is constant.
2. The specific heat of the gas is constant

throughout the zone.

3. The rate of evaporation from the solid surface is given by $R_{\text{evap}} = k_v(m_s - m)$

4. No temperature or concentration gradients exist inside the particles.

5. Isobars are superimposed on enthalpy/temp. plot and $T' = T$ reference (temp.).

Fluid Phase - Heat Balance

$$- Gc \frac{dt}{dz} + h_v(T - t) = 0 \quad \text{where } c = c_p + c_{wm_3} \quad \text{A3.17}$$

Solid Phase

$$- (T - t) = -R_{\text{ev}} L_{\text{T}} \quad \text{A3.18}$$

Fluid Phase - Mass Balance

$$- G' \frac{dm}{dz} + k_v(m_4 - m) = 0 \quad \text{A3.19}$$

Solid Phase

$$-k_v(m_4 - m) = -R_{ev} \quad A3.20$$

where $m_4 = m_s$ as at the drying zone boundary plane (4) the gas is very nearly 100% saturated.

As T is constant the integrated equations are in dimensionless form $z = z/Dp$

$$t - T = Ce^{-\frac{h_v Dp}{G' c} z}$$

$$m_4 - m = De^{-\frac{k_v Dp}{G'} z}$$

Whence inserting the boundary condition $Z = 0, m = m_3, t = t_3$

$$\frac{t - T}{t_3 - T} = e^{-St'z}$$

$$\frac{m_4 - m}{m_4 - m_3} = e^{-\frac{k_v Dp}{G'} z} \quad A 3.21$$

If it is assumed that the specific heat of the gas is not constant and allowance is made for the change in enthalpy due to the change in gas moisture concentration along the length of the zone, the balances A4.1 and 2 are modified.

Fluid Phase - Heat Balance

$$-G \left[C_p + c_w m(z) \right] \frac{dt}{dz} + h_v (T - t) = 0 \quad A3.22$$

Solid Phase

$$RL_T - h_v(T - t) = 0 \quad A3.23$$

Fluid Phase - Mass Balance

$$- G' \frac{dm}{dz} + k_v(m_4 - m) = 0 \quad A3.24$$

Substituting for m from the integrated form of the mass balance A3.24 into the fluid heat balance A3.22 we get in dimensionless form

$$-\left\{ \frac{G'c_p}{h_v D_p} + \left[m_4 + (m_3 - m_4) e^{-\frac{k_v D_p}{G'} z} \right] \frac{G'c_w}{h_v D_p} \right\} \frac{dt}{dz} + (T - t) = 0$$

Rearranging

$$-\left[\frac{G'(c_p + m_4 c_w)}{h_v D_p} + (m_3 - m_4) e^{-\frac{k_v D_p}{G'} z} \frac{G'c_w}{h_v D_p} \right] \frac{dt}{dz} + T - t = 0$$

$$\text{i.e. } (A_1 + B_1 e^{-\Omega z}) \frac{dt}{dz} + T - t = 0 \quad A3.25$$

Separating variables in A3.25 and integrating :

$$\frac{dt}{t - T} = - \frac{dz}{A_1 + B_1 e^{-\Omega z}}$$

$$\ln(t - T) = - \int \frac{dz}{A_1 + B_1 e^{-\Omega z}} + C$$

Using the substitution $u = e^{-\Omega z}$ and separating partial fractions, the right hand side is integrated

$$\ln(t - T) = -\frac{1}{A_1} \left[z + \frac{1}{\beta} \ln(A_1 + B_1 e^{-\Omega z}) \right] + C$$

$$t - T = \frac{c' e^{-z/A_1}}{(A_1 + B_1 e^{-\Omega z})^{\Omega A_1}} \quad A3.26$$

At $z = 0$ $t = t_3$

$$t_3 - T = \frac{c'}{(A_1 + B_1)^{\Omega A_1}}$$

$$\text{and } \Omega A_1 = \frac{k_v D_p}{G'} \cdot \frac{G'(c_1 + m_4 c_w)}{h_v D_p} = \frac{k_v (c_p + m_4 c_w)}{h v} \approx 1$$

for the system air-water (Lewis Relation)

$$\text{i.e. } \underline{t} = \frac{t - T}{t_3 - T} = \frac{(A + B) e^{-z/A_1}}{A_1 + B_1 e^{-\Omega z}} \quad A3.27$$

Check $z = 0$ $\underline{t} = 1$

$z \rightarrow \infty$ $\underline{t} \rightarrow 0$

Comparison of Profiles

If we assume some typical values of system parameters for the constant rate drying of ceramic catalyst pellets we may compare the profiles from the above solutions for various values of m_s the saturated moisture content of the gas corresponding to different wet bulb temperatures in the drying zone.

$$\text{Assume } c_p = 0.25 \text{ Btu/lb dry gas } ^\circ\text{F}$$

$$c_w = 0.5 \text{ Btu/lb vap. } ^\circ\text{F}$$

$$m_3 = m_2 = 0 \text{ lb H}_2\text{O/lb dry gas}$$

$$G' = 400 \text{ lb dry gas/ft}^2 \text{ hr}$$

$$\delta = 0.6 \quad D_p = 0.0066 \text{ ft}$$

$$\mu = 0.055 \text{ lb/ft hr}$$

Hence using the correlations of De Acetis and Thodos (1960)

$$j_H = \frac{1.1}{\text{Re}_p^{0.41} - 0.15} \quad j_D = \frac{0.725}{\text{Re}_p^{0.41} - 0.15}$$

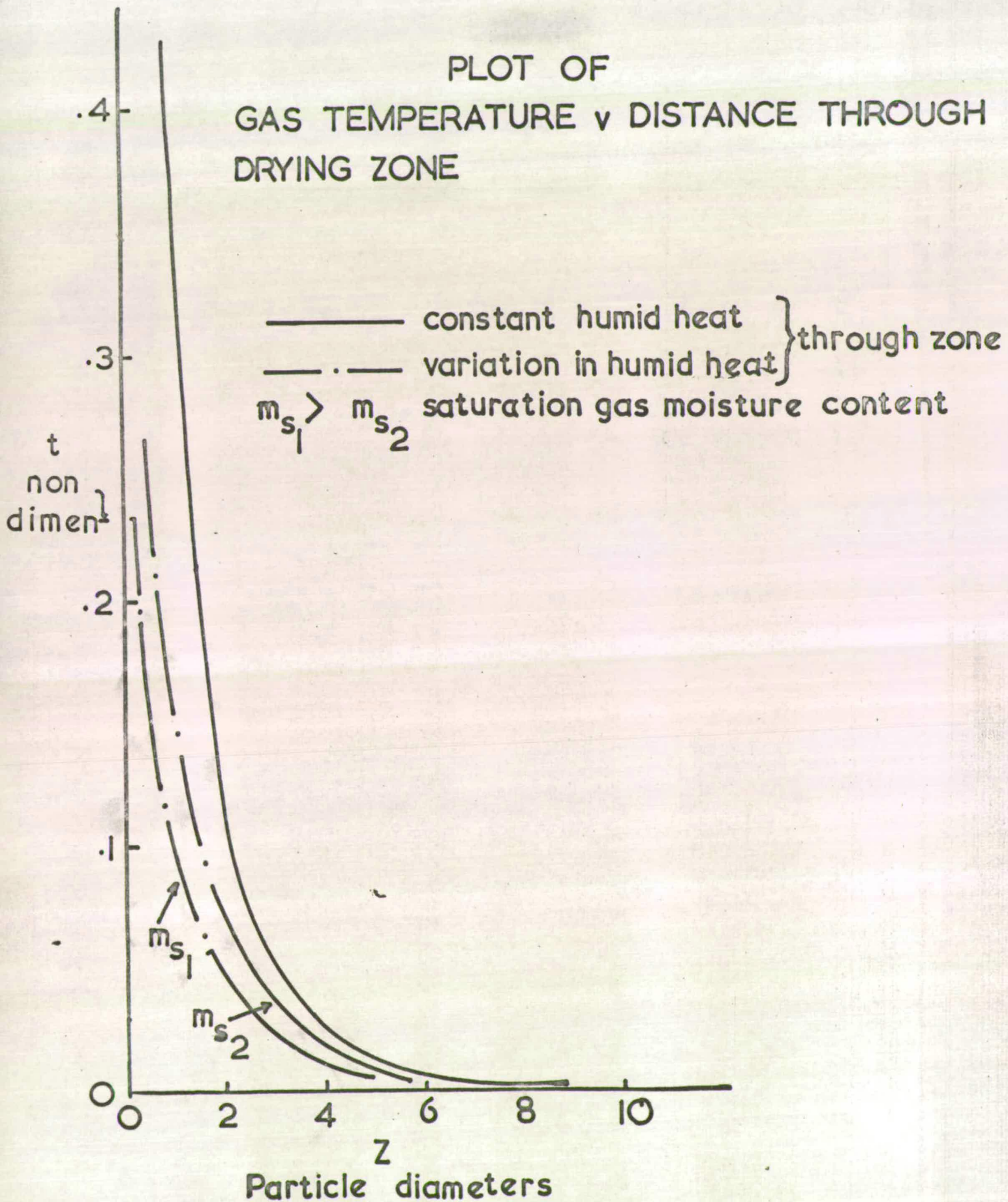
$$\text{St}' = \frac{h_v D_p}{G'(c_p + m_2 c_w)} = 1.06$$

$$G'(c_p + m_2 c_w)$$

$$\frac{k_v D_p}{G'} = 0.76$$

$$\text{and let us assume } \text{St}' = \frac{h_v D_p}{G'(c_p + m_2 c_w)} = \frac{h_v D_p}{G(c_p + m_4 c_w)} = \text{St}''$$

PLOT OF
GAS TEMPERATURE v DISTANCE THROUGH
DRYING ZONE



z	$\underline{t} = e^{-St'z}$	$\underline{t} = \frac{(c_p + m_2 c_w) e^{-St'z}}{(c_p + m_4 c_w) + (m_2 - m_4) c_w e^{-\frac{k_v D_p}{G} z}}$		
		$m_4 = m_3 = 0.01$	0.05	0.1
0	1	1	1	1
1	0.34650	0.3425	0.3295	0.3118
2	0.12000	0.1182	0.1114	0.1038
4	0.01440	0.01412	0.01367	0.0121
8	0.00020	0.000196	0.0001818	0.000167
10	0.00003	0.0000275	0.0000254	0.0000234

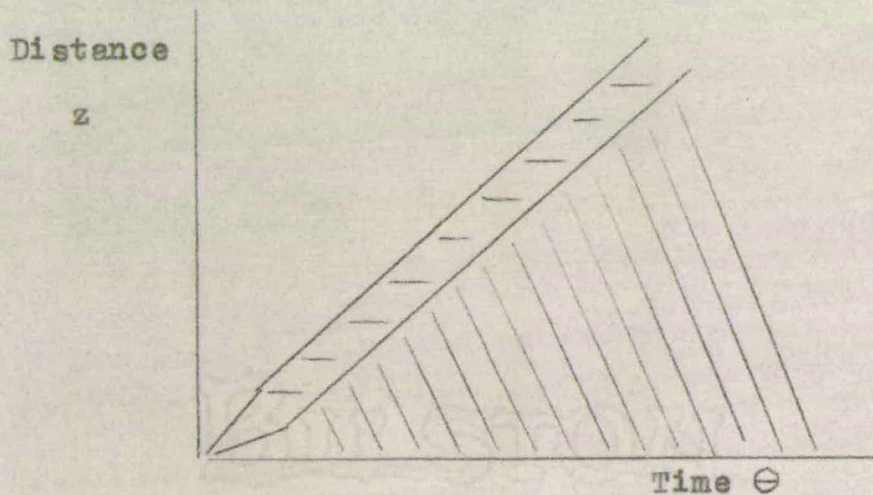
An expanded plot showing the effect of the different solutions is given in the figure opposite although the difference between them can be seen from the table to be negligible for practical purposes.

3.3 Heat Transfer in an Expanding Zone

Introduction

In any moving zone system which caters for zones moving at different speeds heat transfer regions must be specified which either expand or contract. For instance, a hot gas passing through a packed bed accompanied by an endothermic reaction causes an expanding heat transfer zone to follow the reaction zone.

It is possible to show this behaviour on a distance v time graph, plotting the time to reach various points in the bed for the zone.



The shaded portion represents the heat transfer zone expanding with increasing time. The shaded band represents the reaction zone moving at constant velocity.

The analysis of an expanding heat transfer zone presented below will apply only to a zone expanding at constant velocity. Back diffusion in the fluid and back conduction in the solid are neglected. The heat transfer processes can then be represented by the unsteady state equations in one dimension.

Method of Solution

The 1 - D U.S.S. equations for heat transfer in a packed bed :

Fluid Phase

$$- Gc \frac{\partial t}{\partial z} + h_v(T - t) = \delta \rho c \frac{\partial t}{\partial \theta} \quad A3.28$$

Solid Phase

$$- h_v(T - t) = (1 - \delta) \rho_s c_s \frac{\partial T}{\partial \theta} \quad A3.29$$

Putting the coordinate of distance in dimensionless form these equations become

$$- \frac{Gc}{h_v D_p} \frac{\partial t}{\partial z} + (T - t) = \frac{\delta \rho c}{h_v} \frac{\partial t}{\partial \theta} \quad A3.30$$

$$- T + t = \frac{(1 - \delta) \rho_s c_s}{h_v} \frac{\partial T}{\partial \theta} \quad A3.31$$

where $Z = \frac{z}{D_p}$

Boundary Conditions

$t = t_2$	$z = 0$	$\theta = 0$	initial value
$T = T_3$	$z = V\theta$	$\theta = 0$	end value

NB $t @ \theta = 0$ unknown as a discontinuity
 $z = 0$

However, by changing the independent variable we can convert t_2 and T_3 into initial values and as an initial value problem the equations can be solved using Laplace Transforms.

Change of Variable

$$t = f(z, \theta)$$

$$dt = \left. \frac{\partial t}{\partial z} \right|_{\theta} dz + \left. \frac{\partial t}{\partial \theta} \right|_z d\theta \quad \text{A3.31}$$

New independent variables Y, Z are chosen where

$$Y = \alpha_1 \left[\frac{V\theta}{D_p} - z \right] \quad \text{A3.32}$$

$$Z = \alpha_2 z \quad \text{A3.33}$$

which are dimensionless and let $\alpha_1 = \alpha_2 = 1$

Then

$$z = \bar{\phi}(Z, Y)$$

$$\theta = \bar{\gamma}(Z, Y)$$

$$dz = \left. \frac{\partial \bar{\phi}}{\partial Z} \right|_Y dZ + \left. \frac{\partial \bar{\phi}}{\partial Y} \right|_Z dY \quad \text{A3.34}$$

$$d\theta = \left. \frac{\partial \bar{\gamma}}{\partial Z} \right|_Y dZ + \left. \frac{\partial \bar{\gamma}}{\partial Y} \right|_Z dY \quad \text{A3.35}$$

Substituting in A3.31 from A3.34 and A3.35

$$dt = \left. \frac{\partial t}{\partial z} \right|_{\theta} \left[\left. \frac{\partial \bar{\phi}}{\partial Z} \right|_Y dZ + \left. \frac{\partial \bar{\phi}}{\partial Y} \right|_Z dY \right] + \left. \frac{\partial t}{\partial \theta} \right|_z \left[\left. \frac{\partial \bar{\gamma}}{\partial Z} \right|_Y dZ + \left. \frac{\partial \bar{\gamma}}{\partial Y} \right|_Z dY \right]$$

Hence

$$\left. \frac{\partial t}{\partial Z} \right|_Y = \frac{\partial t}{\partial z} \cdot \frac{\partial \bar{\phi}}{\partial Z} + \frac{\partial t}{\partial \theta} \cdot \frac{\partial \bar{\gamma}}{\partial Z} \quad \text{A3.36}$$

$$\text{and } \left. \frac{\partial t}{\partial Y} \right|_Z = \frac{\partial t}{\partial z} \cdot \frac{\partial \bar{\phi}}{\partial Y} + \frac{\partial t}{\partial \theta} \cdot \frac{\partial \gamma}{\partial Y} \quad \text{A3.37}$$

$$\text{Now } \bar{\phi} = \frac{z}{\alpha_2} \quad \left| \quad \gamma = \frac{Y D_p}{\alpha_1 V} + \frac{D_p z}{V} = \frac{Y D_p}{\alpha_1 V} + \frac{z}{\alpha_2 V} \right.$$

$$\left. \frac{\partial \bar{\phi}}{\partial z} \right|_Y = \frac{1}{\alpha_2} ; \quad \left. \frac{\partial \bar{\phi}}{\partial Y} \right|_z = 0 \quad \left| \quad \frac{\partial \gamma}{\partial Y} \right|_z = \frac{D_p}{\alpha_1 V} ; \quad \frac{\partial \gamma}{\partial z} \Big|_Y = \frac{D_p}{\alpha_2 V} \right.$$

Hence A3.36 and A3.37 above become

$$\left. \frac{\partial t}{\partial z} \right|_Y = \frac{\partial t}{\partial z} \cdot \frac{1}{\alpha_2} + \frac{\partial t}{\partial \theta} \cdot \frac{D_p}{\alpha_2 V} \quad \text{A3.38}$$

$$\left. \frac{\partial t}{\partial Y} \right|_z = \frac{\partial t}{\partial z} \Big|_0 = 0 + \frac{\partial t}{\partial \theta} \cdot \frac{D_p}{\alpha_1 V} \quad \text{A3.39}$$

Substituting from A3.38 and A3.39 into original equations A3.34 and 35 :

$$- \frac{G_c}{h_v D_p} \alpha_2 \frac{\partial t}{\partial z} - \alpha_1 \frac{\partial t}{\partial Y} + (T - t) = \frac{\delta \rho c}{h_v} \frac{\alpha_1 V}{D_p} \frac{\partial t}{\partial Y}$$

$$- (T - t) = \frac{(1 - \delta) \rho_s c_s}{h_v} \frac{\alpha_1 V}{D_p} \frac{\partial T}{\partial Y}$$

Collecting terms we get

$$- \frac{G_c}{h_v D_p} \cdot \alpha_2 \cdot \frac{\partial t}{\partial z} + (T - t) = - \left[\frac{G_c}{h_v D_p} \cdot \alpha_1 - \frac{\delta \rho c}{h_v D_p} \alpha_1 V \right] \frac{\partial t}{\partial Y}$$

A3.40

$$-(T - t) = \frac{V(1 - \delta)\rho_s c_s}{h_v D_p} \alpha_1 \frac{\partial T}{\partial Y} \quad \text{A3.41}$$

Now let us redefine independent variables again as :

$$Z' = \frac{Z}{\frac{Gc}{h_v D_p} \alpha_2} \quad Y' = \frac{Y}{\frac{V(1 - \delta)\rho_s c_s}{h_v D_p} \alpha_1}$$

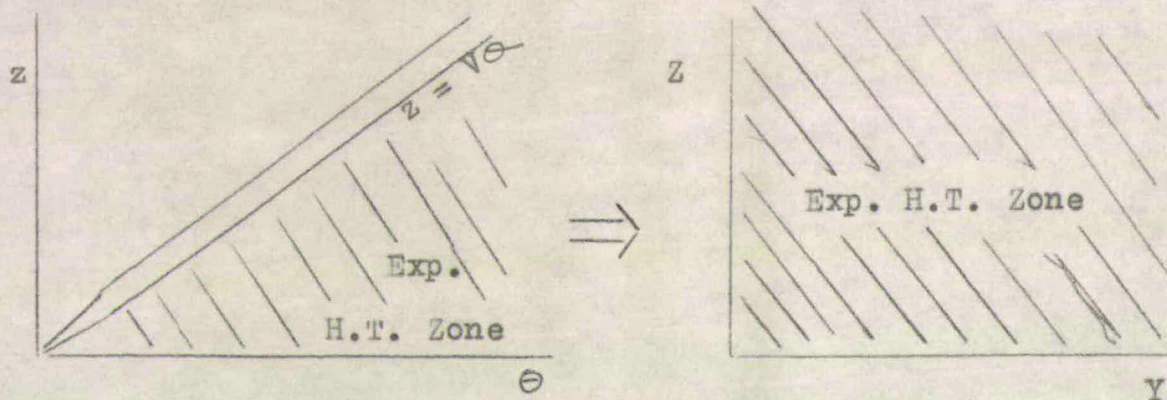
equations A3.40 and A3.41 become :

$$-\frac{\partial t}{\partial Z'} + (T - t) = -\phi \frac{\partial T}{\partial Y'} \quad \text{A3.42}$$

$$-(T - t) = \frac{\partial T}{\partial Y'} \quad \text{A3.43}$$

where $\phi = \frac{(G - V\delta\rho)c}{V(1 - \delta)\rho_s c_s}$ which is always > 0

The change of independent variable affects the z/θ plot so that the expanding zone is now represented by the full quadrant.



$$\begin{aligned} \text{since } z = 0 & \quad Z = 0 & \quad Y = \alpha_1 V \theta \\ z = V \theta & \quad Y = 0 \end{aligned}$$

We now solve equations A3.42 and A3.43 using a Laplace Transform defined w.r.t. Z'

$$\text{i.e. } \mathcal{L} [f(Z')] = \int_0^{\infty} e^{-sZ'} f(Z') dz' = \bar{F}(s)$$

now transforming A3.42 and A3.43 we get :

$$- [s\bar{t}(s) - t(0)] + \bar{T}(s) - \bar{t}(s) = -\phi \frac{d}{dY'} \bar{t}(s) \quad (\text{L1})$$

$$- [\bar{T}(s) - \bar{t}(s)] = \frac{d}{dY'} \bar{T}(s) \quad (\text{L2})$$

Differentiate (L2) w.r.t. Y'

$$\frac{d^2 \bar{T}}{dY'^2} + \frac{d\bar{T}}{dY'} = \frac{d\bar{t}}{dY'} \quad (\text{L3})$$

Substituting for $\frac{d\bar{t}}{dY'}$ in (L3) from (L1)

$$\frac{d^2 \bar{T}}{dY'^2} + \frac{d\bar{T}}{dY'} = -\frac{1}{\phi} \left[t(0) - (s+1)\bar{t} + \bar{T} \right]$$

Substituting for \bar{t} from (L2)

$$\frac{d^2 \bar{T}}{dY'^2} + \frac{d\bar{T}}{dY'} = -\frac{1}{\phi} \left[t(0) - (s+1) \left(\frac{d\bar{T}}{dY'} + \bar{T} \right) + \bar{T} \right]$$

$$\frac{d^2 \bar{T}}{dY'^2} + \left(1 + \frac{s+1}{\phi} \right) \frac{d\bar{T}}{dY'} - \frac{s}{\phi} \bar{T} = -\frac{t(0)}{\phi} \quad (\text{L4})$$

$$\bar{T} = \frac{T_3 - t_2}{s} \exp \left\{ \frac{Y'}{2\phi} \left[s + 1 - \phi - \sqrt{s + (\phi + 1)^2 - 4\phi} \right] \right\} + \frac{t_2}{s}$$

Let us change variable operator S to p where $p = S + (\phi + 1)$

\therefore (L5) becomes :

$$\bar{T} = \frac{T_3 - t_2}{s} \exp \left\{ \frac{Y'}{2\phi} \left[p - \sqrt{p^2 - 4\phi} - 2\phi \right] \right\} + \frac{t_2}{s}$$

$$\text{i.e. } \bar{T}e^{Y'} = \frac{T_3 - t_2}{s} \exp \left\{ \frac{Y'}{2\phi} \left[p - \sqrt{p^2 - 4\phi} \right] \right\} + \frac{t_2}{s} e^{Y'} \quad (\text{L6})$$

Now we know

$$\mathcal{L}^{-1} \left\{ \exp \left[\frac{Y'}{2\phi} \left(p - \sqrt{p^2 - 4\phi} \right) \right] - 1 \right\}$$

\therefore rearrange (L6) in this form :

$$\bar{T}e^{Y'} = \frac{T_3 - t_2}{s} \left[\exp \left\{ p - \sqrt{p^2 - 4\phi} \right\} \frac{Y' - 1}{2\phi} \right] + \frac{T_3 - t_2}{s} + \frac{t_2}{s} e^{Y'}$$

then let $\beta = \frac{Y'}{2\phi}$ $k^2 = p^2 - q^2$ where $q^2 = 4\phi$

$$\text{i.e. } \bar{T}e^{Y'} = \frac{T_3 - t_2}{s} \exp \left[\beta(p - k) \right] - 1 + \frac{T_3 + t_2(e^{Y'} - 1)}{s} \quad (\text{L7})$$

The 1st and 3rd terms may be inverted directly and the 2nd term using the convolution theorem.

Let $\frac{1}{s}$ be denoted by $\bar{F}(s)$

$e^{\beta(p - k)} - 1$ " " $\bar{g}(p)$

The inversion operation is :

$$\mathcal{L}^{-1} \left[\bar{F}(s) \cdot \bar{g}(p) \right]$$

and separately these invert to :

$$\mathcal{L}^{-1} \bar{f}(s) = f(z') = 1$$

$$\mathcal{L}^{-1} \bar{g}(p) = e^{-\alpha z'} g(z') = e^{-\alpha z'} \beta q (z'^2 + 2\beta z')^{-1/2} I_1 \left[q (z'^2 + 2\beta z')^{+1/2} \right]$$

The convolution expression is thus :

$$\int_0^{z'} f(\tau) e^{-\alpha(z' - \tau)} g(z' - \tau) d\tau = \int_0^{z'} f(z' - \tau) e^{-\alpha\tau} g(\tau) d\tau$$

which becomes

$$\beta q \int_0^{z'} e^{-\alpha\tau} (\tau^2 + 2\beta\tau)^{-1/2} I_1 \left[q(\tau^2 + 2\beta\tau)^{1/2} \right] d\tau$$

and when the whole equation (L7) is inverted we get :

$$T e^{Y'} = (T_3 - t_2) \beta q \int_0^{z'} e^{-\alpha\tau} (\tau^2 + 2\beta\tau)^{-1/2} I_1 \left[q(\tau^2 + 2\beta\tau)^{1/2} \right] d\tau$$

$$+ T_3 + t_2 (e^{Y'} - 1) \quad A3.44$$

This equation (A3.44) is the equation of the solid temperature (T) profile with distance and time, for a zone expanding at constant velocity.

Similarly, we can find a profile equation for t , the gas

temperature :

Substituting in (L2) for $\bar{T}(s)$

$$\begin{aligned} \bar{t}(s) &= \bar{T}(s) + \frac{d\bar{T}}{dY'} \\ &= \frac{T_3 - t_2}{s} \exp \left\{ \frac{Y'}{2\phi} \left[s + 1 - \phi + \sqrt{[s + (\phi + 1)]^2 - 4\phi} \right] \right\} + \frac{t_2}{s} \\ &\quad + \frac{T_3 - t_2}{s} \frac{1}{2\phi} \left[s + 1 - \phi - \sqrt{[s + (\phi + 1)]^2 - 4\phi} \right] \\ &\quad \exp \left\{ \frac{Y'}{2\phi} \left[s - \phi + 1 - \sqrt{[s + (\phi + 1)]^2 - 4\phi} \right] \right\} \end{aligned}$$

$$\begin{aligned} \text{i.e. } \bar{t}(s) &= \left\{ \frac{T_3 - t_2}{s} \exp \frac{Y'}{2\phi} \left[s + 1 - \phi - \sqrt{[s + 1 + \phi]^2 - 4\phi} \right] \right\} \\ &\quad \left\{ 1 + \frac{1}{2\phi} \left[s + 1 - \phi - \sqrt{(s + 1 + \phi)^2 - 4\phi} \right] \right\} + \frac{t_2}{s} \end{aligned}$$

Put $p = s + 1 + \phi$

$$\begin{aligned} e^{Y'\bar{t}} &= \left\{ \frac{T_3 - t_2}{s} \exp \frac{Y'}{2\phi} \left[p - \sqrt{p^2 - 4\phi} \right] \right\} \left\{ 1 + \frac{1}{2\phi} \left[p - \sqrt{p^2 - 4\phi} - 2\phi \right] \right\} \\ &\quad + \frac{t_2 e^{Y'}}{s} \end{aligned}$$

$$\text{Put } \beta = \frac{Y'}{2\phi} \quad k^2 = p^2 - q^2 \quad q^2 = 4\phi$$

$$e^{Y'\bar{t}} = \frac{T_3 - t_2}{s} \frac{\beta}{Y'} (p - k) e^{\beta(p - k)} + \frac{t_2 e^{Y'}}{s} \quad (\text{L8})$$

$$= \frac{(t_3 - t_2)\beta}{Y'} \left[\frac{p - k}{s} (e^{\beta(p - k)} - 1) + \frac{p - k}{s} \right] + \frac{t_2 e^{Y'}}{s} \quad (L9)$$

we know inverse of exponential \therefore require

$$\mathcal{L}^{-1} \frac{p - k}{s} = \int_0^{Z'} e^{-\alpha \tilde{\tau}} \mathcal{L}^{-1}(p - k) d\tilde{\tau}$$

Require $\mathcal{L}^{-1}(p - k)$

$$\text{Now } p - k = p - k \cdot \frac{p + k}{p + k} = \frac{p^2 - k^2}{p + k}$$

$$\text{but } k^2 = p^2 - q^2 \therefore p - k = q^2(p + k)^{-1} = q^2 s^{-1} \quad (\text{Bateman})$$

$$\mathcal{L}^{-1}(p - k) = q^2 \cdot q^{-1} \cdot I_1(qZ')$$

$$\therefore \mathcal{L}^{-1} \left(\frac{p - k}{s} \right) = \int_0^{Z'} e^{-\alpha \tilde{\tau}} q \tilde{\tau}^{-1} I_1(q \tilde{\tau}) d\tilde{\tau}$$

We can now apply the convolution theorem again to invert product which includes exponential. The inverted solution becomes :

$$te^{Y'} = \frac{(t_3 - t_2)\beta}{Y'} \left\{ \int_0^{Z'} e^{-\alpha \tilde{\tau}} q \beta (\tilde{\tau}^2 + 2\beta \tilde{\tau})^{-1/2} I_1 \left[q(\tilde{\tau}^2 + 2\beta \tilde{\tau})^{1/2} \right] \right. \\ \left. \int_0^{Z' - \tilde{\tau}} e^{-\alpha \tilde{\tau}} q \tilde{\tau}^{-1} I_1(q \tilde{\tau}) d\tilde{\tau} d\tilde{\tau} + \int_0^{Z'} e^{-\alpha \tilde{\tau}} q \tilde{\tau}^{-1} I_1(q \tilde{\tau}) d\tilde{\tau} \right\} \\ + t_2 e^{Y'}$$

This equation (A3.45) is the equation for the gas temperature profile in an expanding heat transfer zone expanding with constant velocity. These expressions are too complicated for analytical integration and will require numerical solution for the complete profile. However, it is of more interest to examine the behaviour at the boundaries to the zone as this will tell us how the heat transfer zone affects preceding and succeeding zones.

Boundary Behaviour

$Y' = 0$ - moving boundary

$$t_3 = \frac{T_3 - t_2}{2\phi} \int_0^{Z'} e^{-\alpha\tau} \tau^{-1} I_1(q\tau) d\tau + t_2 \quad \text{A3.46}$$

$$T_3 = T_3$$

At $Z' = 0$ - fixed boundary

$$t_2 = t_2$$

$$T_2 = \frac{T_3 + t_2(e^{Y'} - 1)}{e^{Y'}} \quad \text{A3.47}$$

And we can rearrange these equations as :

$$\frac{t_2 - t_3}{t_2 - T_3} = \frac{K(\phi, Z')}{\phi^{1/2}} = F_1$$

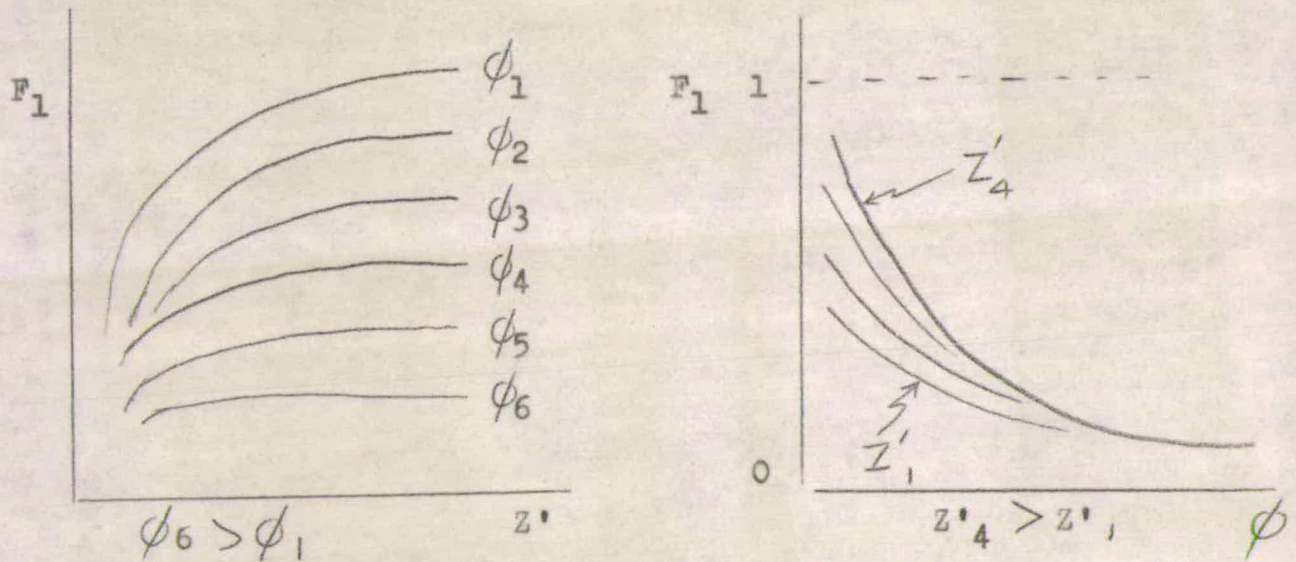
$$\frac{t_2 - T_3}{t_2 - T_2} = e^{Y'} \quad \text{or} \quad T_2 = \frac{T_3}{e^{Y'}} + t_2 \left(\frac{e^{Y'} - 1}{e^{Y'}} \right)$$

i.e. $t_3 \rightarrow T_3$ as $\phi, Z' \rightarrow \infty$ (large ϕ, Z')

$T_2 \rightarrow t_2$ " $Y \rightarrow \infty$ (large Y')

$$\text{where } K(\phi, Z') = \int_0^{Z'} e^{-\alpha \tilde{\tau}} \tilde{\tau}^{-1} I_1(q \tilde{\tau}) d\tilde{\tau}$$

For utility the information on gas temperatures is produced as a plot of F_1 v ϕ for different Z' and F_1 v Z' for different ϕ



Integration of Convolution Expression

We require to integrate :

$$\int_0^{Z'} e^{-\alpha \tilde{\tau}} \tilde{\tau}^{-1} I_1(q \tilde{\tau}) d\tilde{\tau}$$

$$\text{where } \alpha = \phi + 1 \quad q = 2\phi^{1/2}$$

$$\phi = V_R = \frac{G'c}{V(1-\delta)\rho_s c_s}$$

Let us change variable by putting $\tau = z \cdot \tilde{\tau}'$ and dropping primes

$$\int_0^1 e^{-(\phi + 1)z \cdot \tilde{\tau}} \tilde{\tau}^{-1} I_1(2\phi^{1/2} z \cdot \tilde{\tau}) d\tilde{\tau}$$

The general term for the modified Bessel function is :

$$\sum_{k=0}^{\infty} \frac{(qz \cdot \tilde{\tau}/2)^{2k+1}}{k! k+1!}$$

Hence the general term of the integral becomes :

$$\sum_{k=0}^{\infty} \left(\frac{qz}{2}\right)^{2k+1} \frac{\tilde{\tau}^{2k} e^{-(\phi+1)z \cdot \tilde{\tau}}}{k! k+1!}$$

Expanding this and integrating term by term :

$$\begin{aligned} & \frac{qz}{2} \cdot \frac{e^{-(\phi+1)z \cdot \tilde{\tau}}}{-(\phi+1)z \cdot \tilde{\tau}} \Bigg|_0^1 \\ & + \left(\frac{qz}{2}\right)^3 \frac{e^{-(\phi+1)z \cdot \tilde{\tau}}}{2! 1!} \left\{ \frac{\tilde{\tau}^2}{-(\phi+1)z \cdot \tilde{\tau}} - \frac{2\tilde{\tau}}{[-(\phi+1)z \cdot \tilde{\tau}]^2} + \frac{2 \cdot 1}{[-(\phi+1)z \cdot \tilde{\tau}]^3} \right\} \Bigg|_0^1 \\ & \text{-----} \\ & + \left(\frac{qz}{2}\right)^{2k+1} \frac{e^{-(\phi+1)z \cdot \tilde{\tau}}}{k! k+1!} \left\{ \frac{\tilde{\tau}^{2k}}{-(\phi+1)z \cdot \tilde{\tau}} - \frac{2k \tilde{\tau}^{2k-1}}{[-(\phi+1)z \cdot \tilde{\tau}]^2} \right. \\ & \left. + \frac{2k(2k-1)\tilde{\tau}^{2k-2}}{[-(\phi+1)z \cdot \tilde{\tau}]^3} + \text{-----} - \frac{2k(2k-1) \text{-----} 2k-(2k-1)}{[-(\phi+1)z \cdot \tilde{\tau}]^{2k+1}} \right\} \Bigg|_0^1 \end{aligned}$$

This expansion in series can be written as :

$$- \sum_{k=0}^{\infty} \left(\frac{qZ}{2} \right)^{2k+1} \frac{e^{-(\phi+1)Z} \tilde{\gamma}}{k! (k+1)!} \left[\sum_{n=0}^{2k} \frac{D^n(\tilde{\gamma})}{(\phi+1)^{n+1} Z^{n+1}} \right] \Big/ \Big|_0^1$$

where D^n is the differential operator of order n .

Putting in limits this becomes :

At $\tilde{\gamma} = 1$

$$- \sum_{k=0}^{\infty} \left(\frac{qZ}{2} \right)^{2k+1} \frac{e^{-(\phi+1)Z}}{k! (k+1)!} \left[\sum_{n=0}^{2k} \frac{2k!}{(\phi+1)^{n+1} Z^{n+1} 2k-n!} \right]$$

At $\tilde{\gamma} = 0$

$$- \sum_{k=0}^{\infty} \left(\frac{qZ}{2} \right)^{2k+1} \frac{2k!}{(\phi+1)^{2k+1} Z^{2k+1}} \frac{1}{k! (k+1)!}$$

\therefore integrated expression is :

$$\sum_{k=0}^{\infty} \left(\frac{qZ}{2} \right)^{2k+1} \frac{1}{k! (k+1)!} \left[\frac{2k!}{(\phi+1)^{2k+1} Z^{2k+1}} - e^{-(\phi+1)Z} \sum_{n=0}^{2k} \frac{2k!}{(\phi+1)^{n+1} Z^{n+1} 2k-n!} \right]$$

Since for computation this involves $0! = 1$ write as :

$$\sum_{k=1}^{\infty} \left(\frac{qZ'}{2}\right)^{2k+1} \frac{1}{k! k+1!} \left[\frac{2k!}{(\phi+1)^{2k+1} Z'^{2k+1}} - \frac{e^{-(\phi+1)Z'}}{(\phi+1)Z'} \right]$$

$$\left[\sum_{n=0}^{2k} \frac{\Gamma(2k-n)}{(\phi+1)^{n+1} Z'^{n+1}} + 1 \right] + \frac{qZ'}{2} \left[\frac{1}{(\phi+1)Z'} - \frac{e^{-(\phi+1)Z'}}{(\phi+1)Z'} \right]$$

i.e. compute 1st term $k = 0$ separately.

A similar approach is not practicable for the integral in the expression for the solid temperature profile A3.44 and so this was integrated numerically using a repeated Simpson's Rule method. A standard library subroutine was used for the integration. This was supplied by the Edinburgh University Computer Unit.

The results of this numerical integration were displayed graphically as a plot of $F_2 \propto \frac{k(\phi, Z', Y')}{\phi^{1/2}}$ similar to the plot

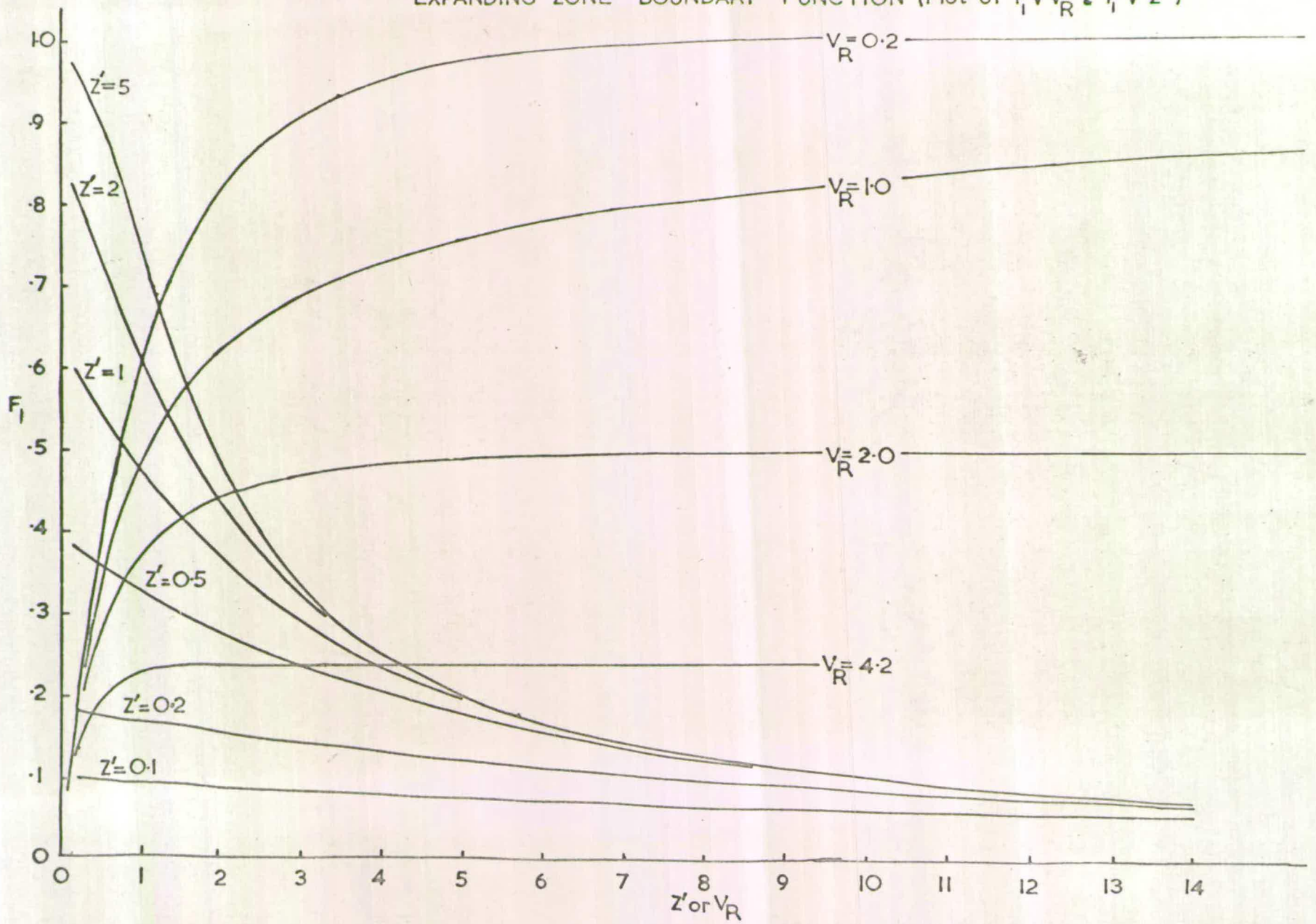
used for the integrals resulting from the boundary condition at $Y' = 0$

where

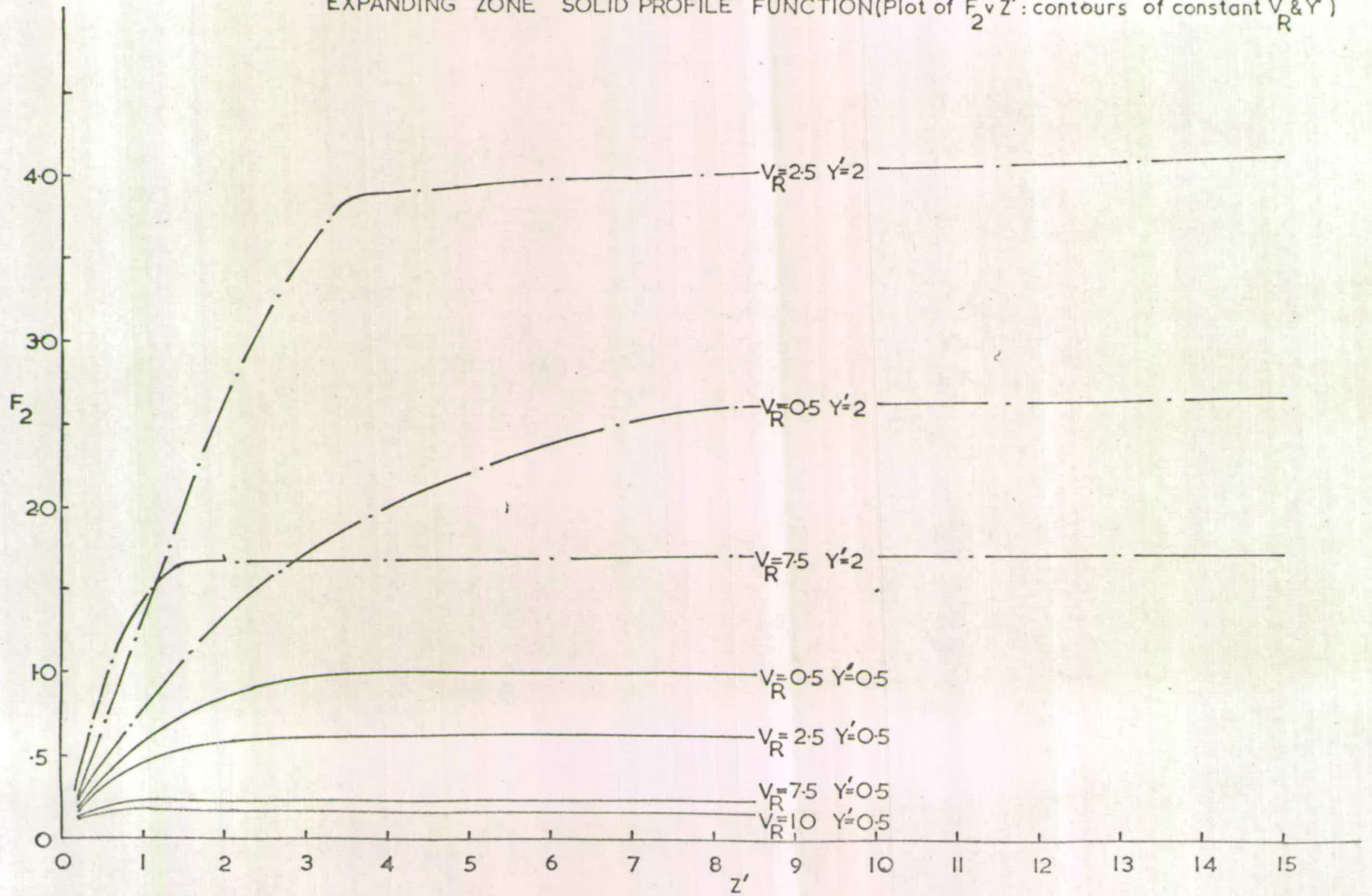
$$\frac{e^{Y'}}{Y'} \left(\frac{t_2 - T}{t_2 - t_3} \right) - \frac{1}{Y'} \equiv F_2 = \frac{k(\phi, Z', Y')}{\phi^{1/2}}$$

see overleaf for plots of F_1 & F_2

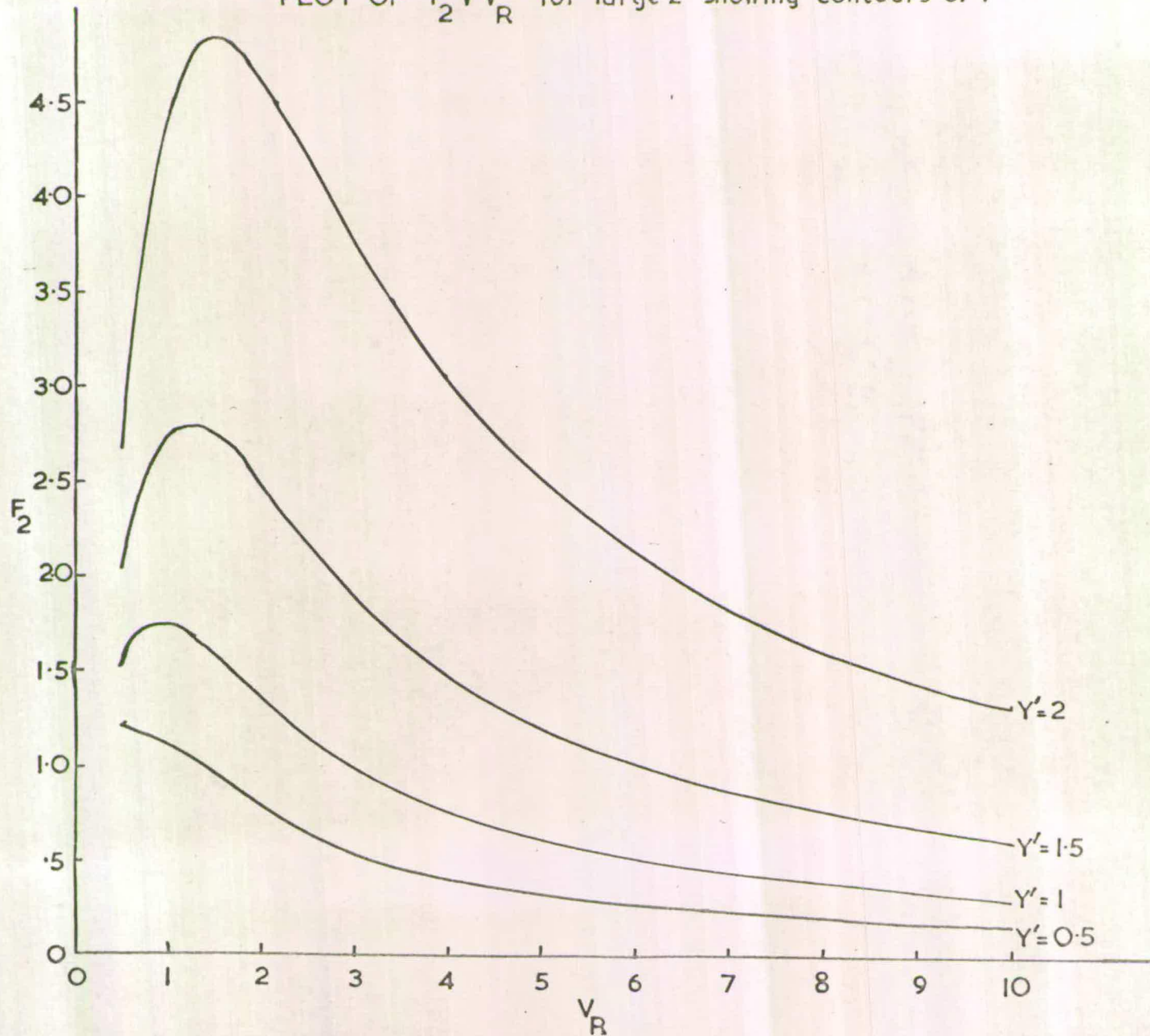
EXPANDING ZONE BOUNDARY FUNCTION (Plot of F_I vs V_R & F_I vs Z')



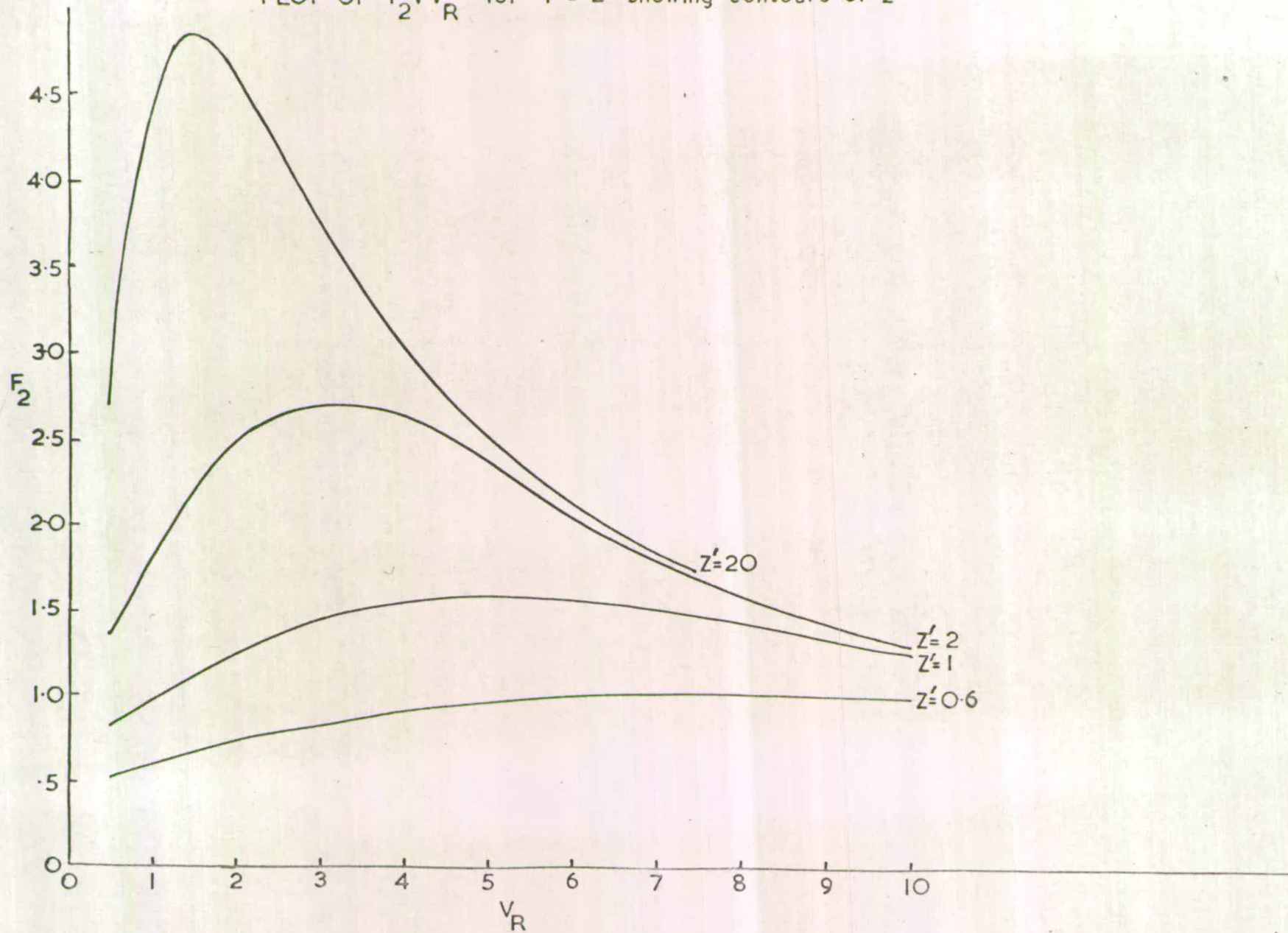
EXPANDING ZONE SOLID PROFILE FUNCTION (Plot of $F_2 v Z'$: contours of constant V_R & Y')



PLOT OF $F_2 \text{ v } V_R$ for large Z' showing contours of Y'



PLOT OF $F_2 v V_R$ for $Y' = 2$ showing contours of Z'



APPENDIX 4Analysis of Results4.0 Introduction

As the approximate analysis which this project is examining assumes constant system parameters in any zone, four variables control the velocity of the drying zone. A factorial design was chosen as the most efficient method of assessing the effect of these variables because fewer experiments are needed than if one variable is altered at a time and interactions between variables, if any, are revealed by this design. A two level design was chosen as a three level design requires eighty-one experiments which was too many for the time available, taking into consideration the preparatory runs associated with each experimental run (see Chapter 3). The resultant matrix of velocities or other characteristic responses must be analysed to show the effect of each of the four variables.

The responses may be analysed using the analysis of variance technique to show the importance of the various factors and the presence of any interactions between them. Alternatively, a polynomial may be fitted to the response surface represented by the experimental response matrix and compared with the polynomial fitted to the matrix of theoretical responses which have been calculated for the same

experimental conditions.

Inherent in the analysis of variance technique is an estimate of the experimental error due to random sources and also the revelation of any unexpected interaction between supposedly independent variables. For instance, a first analysis of variance showed a large interaction between gas inlet temperature and gas flow rate; this led to simplification of the work as it suggested that V/G was the true response, not velocity alone. Polynomial representation gains in the handling of the experimental data as matrix operations are easy using a digital computer. Once the operations have been programmed, the inclusion of additional observations and the fitting of different response surfaces involves no extra labour.

4.0.1 Two Level Factorial Design

A four factor two level design requires sixteen experiments for each full replicate and eight for each half replicate (Davies 1963, Moroney 1962). The design points for the full replicate are represented in the design space by the vertices of a hypercube. The variables, known as the independent variables, affecting the drying zone velocity are gas inlet moisture content (m), solids inlet moisture content (M), gas inlet temperature (t) and gas mass flow rate (G).

In calculating the analysis of variance results the method of Yates (1937) was used for ease of computation. Writing the combinations of the variables in their standard order they are

1, m, M, mM, t, mt, Mt, mMt, G, mG, MG, mMG, tG, mtG, MtG, mMtG

where the presence of a letter in a combination indicates that that factor is present at the higher of the two levels.

The polynomial representation of the response surface was initially of second degree, ignoring quadratic effects, but examination of the coefficients of the interaction terms suggested that a better fit might be obtained by including these.

Polynomial without Quadratic Terms

$$V/G = \beta_0 + \beta_1 m + \beta_2 M + \beta_3 t + \beta_4 G + \beta_{12} mM + \beta_{13} mt + \beta_{14} mG \\ + \beta_{23} Mt + \beta_{24} MG + \beta_{34} tG \quad A4.1$$

Polynomial including Quadratic Terms

Equation A4.1 with four extra terms

$$- - - + \beta_{11} m^2 + \beta_{22} M^2 + \beta_{33} t^2 + \beta_{44} G^2 \quad A4.2$$

Hence eleven coefficients must be estimated to fit equation A4.1 and fifteen to fit equation A4.2.

In general if D is the matrix showing the combinations of

individual values of the independent variables in each separate run, called the design matrix, X is the matrix of independent variables required for fitting an equation of given degree, where X is formed from the cross product terms in A4.1 and A4.2 above.

The estimates of the true coefficients β are the elements of a column vector B formed as

$$B = T.Y$$

where Y is the column vector of responses and $T = (X^T X)^{-1} X^T$ and is called the transforming matrix.

The basic full replicate factorial design using two levels does not provide sufficient observations to include the quadratic terms in the polynomial. The design is therefore augmented with extra points to form a composite design (Davies 1963). In this case, a non central composite design was compiled with four extra points extending from the vertex $(1, -1, 1, -1)$ of the hypercube. It is usual to scale the independent variables in the design to a zero value at half their range i.e. the upper level becomes $+1$; the lower -1 . Scaling the variables in this way, produces an orthogonal design so that the main effect's coefficients are uncorrelated with each other and the correlation between the interaction terms' coefficients is a minimum. Similar factorial designs were used to determine the effect of qualitative variables on some of the measured system parameters. For instance, the

effect of vibration on the packed density of the solid was examined and also possible sources of variation in the sampling scheme for solid moisture content.

4.0.2 Choice of Range of Independent Variables

The choice and zero level of the variables were chosen for practical reasons. For instance, it was found that overnight soaking in cold water gave a reproducible moisture content of the solid particles for the lower level, and boiling and dumping into cold water, a similarly reproducible higher value.

Levels and Ranges Used

	m	M	t	G
Zero	0.01 lb water /lb dry gas	15.25 $\frac{\text{lb water}}{\text{lb dry solid}}$	230°F	320 lb /hr ft ²
Range	0.01	1.75	65	75

The higher level of m corresponds to saturation at 25°C and the levels of (t) and (G) were chosen to give runs of about 1 to 2 hours duration.

4.1 Analysis of Data

Once the experimental design has been completed, each run executed represents a single response point in the design space. This response is obtained from a trace produced by the single pen recorder. The complete chart trace consists of

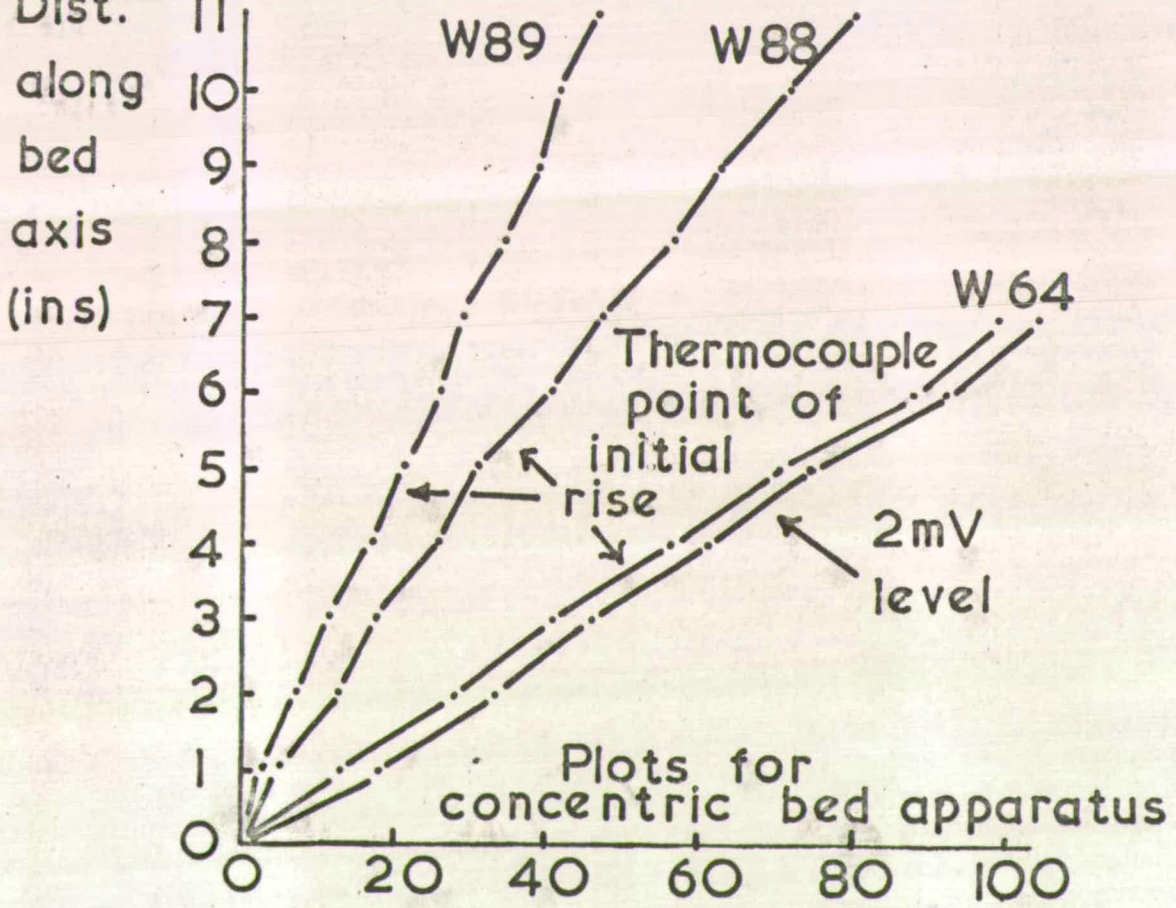
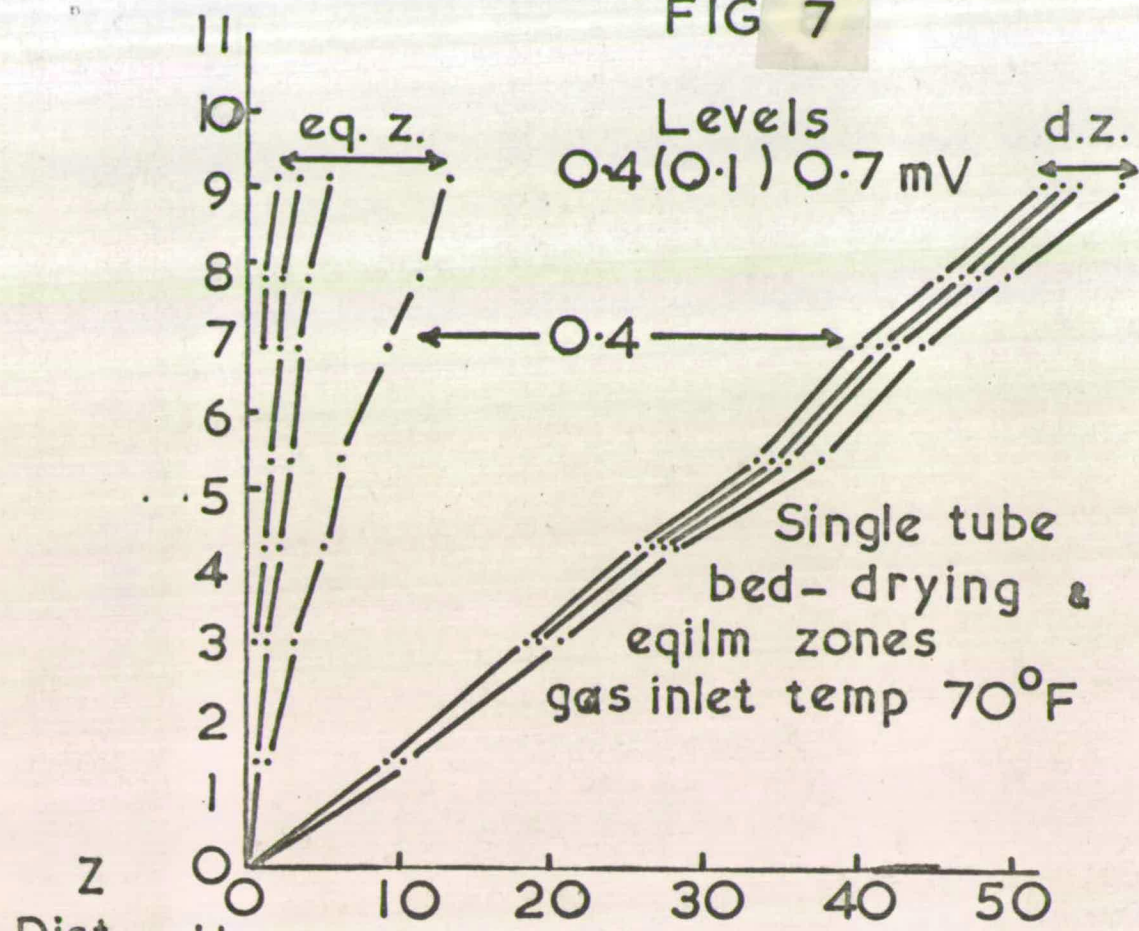
successive records from individual thermocouples starting with the thermocouple at the base of the bed. The temperature measured by each thermocouple is recorded while that section of the bed is at the wet bulb temperature and continuously until the temperature has risen almost to the gas inlet temperature, when the recorder is switched to monitor the next thermocouple.

An arbitrary zero is chosen as the instant when the bed is mounted on the flange, the same for all runs, and the distance is measured from this point to the point of initial rising of the temperature on successive thermocouple traces. These intervals can be converted to time intervals as the chart moves at constant speed. A graph is plotted of the height of the thermocouples above the bed base versus the time interval. The slope of this graph gives the velocity of the temperature wave and hence of the rear boundary of the endothermic drying zone.

The falling rate period is assumed to be insignificant in the drying of these particles and as a consequence of this the initial rising point on each thermocouple marks the point at which the solid moisture content reaches zero. This is substantiated by the sharp temperature rise away from the wet bulb temperature and by the agreement between measured and calculated theoretical velocities.

The plots of distance v. time (z/θ plot) are useful

FIG 7



⊖ Time to reach corr. dist.(mins)

to give an indication of any anomalies in a run (see fig. 7) but two separate responses can characterise a run. One is the reciprocal of the run's total time (until the last thermocouple trace starts to rise) and the other is the gradient of the best straight line on the z/θ plot, calculated using least squares method. Both these have advantages; the former takes into account the operator's skill in controlling unexpected changes in conditions which may cause velocity variations over part of the run even though, through compensating, the run finishes on time, the latter is the best estimate of the velocity including local variations and the least squares technique allows confidence limits to be calculated for the velocity. For showing up the effects present the analysis of variance was performed using the total time parameter; for fitting the polynomial, the response used was the least squares velocity divided by the gas mass flow rate (V/G). These least squares velocities were calculated using a SIRIUS digital computer and the program was arranged to include computation of confidence limits and the linear correlation coefficient of z with θ .

A large number of runs was completed, in addition to those for the experimental design, under varying conditions. Those runs for which full information was available were included in the polynomials using the methods of Plackett (1952) to modify the original matrices. The effect of

slightly extending the range of the variables outside the design space is shown in the resulting polynomials.

Measurement of Independent Variables

The gas flow rotameters were calibrated using charts supplied by the manufacturers. The thermocouple e.m.f.'s were converted to temperatures using manufacturers tables. These two measurements were the most accurate of those of the independent variables and their errors were estimated at ± 5 and $\pm 2\%$ respectively.

The moisture content of the solid in each run was determined by taking samples immediately before the run. To ensure no systematic error was introduced into the moisture content measurement by the sampling method a qualitative factorial experiment was set up to examine the effect of possible causes of variation. A similar approach was used when measuring the packed bulk density of the solids. Then the whole design was repeated in tubes of different diameters to test for any wall effect.

Discussion of Analysis

The analysis of variance was applied first to the experimental results from the full factorial design to determine which of the independent variables had significant effects on the velocity of the drying zone. A polynomial function of the four independent variables was fitted to the

Least Squares technique of Gauss. Here the matrix of velocity results appears as a vector of responses. The response may be any variable characteristic of the drying zone velocity.

At each point in the design space a theoretical response was calculated using the adiabatic expanding heat transfer zone equations to calculate the drying zone inlet temperature and the overall balance on this latter zone to determine its velocity. An example is given in Appendix 5. A polynomial, similar to the other, was then fitted to the theoretical response vector. The discussion of the theory and experiment are based upon comparison of these two polynomials.

Analyses of Variance

The mean squares resulting from the analyses are given in Table 5. The response chosen for Analysis 1 was the reciprocal of the total time taken for the drying zone to travel the length of the bed (time until 12th centre thermocouple started to rise $\Theta_{c 12}$). As coding the data does not affect the analysis, this is equivalent to using the velocity of the zone.

The residual in this single replication of the design is only based on one degree of freedom. Comparing mean squares also based upon one degree of freedom with this

TABLE 6

Results of Applying Bartlett's Test to Analysis of Variance

Interaction	Mean Sq.	d.f.	log M.S
	000.759	1	1.8802
	109.600	1	2.0399
	33.200	1	1.5211
	12.770	1	1.1062
	77.900	1	1.8915
	16.580	1	1.2196
	3.463	1	0.5395
	5.891	1	0.7702
	84.680	1	1.9279
	82.710	1	1.9176
	8.236		0.9158
	<hr/>		<hr/>
	435.848		13.795

$$= \sum (d.f._i) \log_{10}(M.S._i)$$

where i refers to each interaction effect

d.f. associated with sum of individual $(M.S)_i = 11$

$$\text{i.e. } (M.S)_T = 39.62$$

$$\log (MS)_T = 1.5979$$

$$(d.f) \log (MS)_T = 17.5769$$

Bartlett's test compute :

$$M = 2.303((d.f) \log_{10}(MS)_T - \sum (d.f._i) \log_{10}(MS_i))$$

$$= 8.86 \quad \text{N.S. at 1\% level (see Davies 1962)}$$

residual is likely to lead to erroneous conclusions as, using the variance ratio, F test the value the ratio must exceed under these conditions is excessively large.

It is usual therefore to pool the highest order interactions to get an estimate of the error variance based on more degrees of freedom. This procedure is only valid however if the highest order interactions really are non-significant and comparison of these with the residual encounters the difficulty mentioned above. Fortunately, Bartlett's Test is available which is designed to show up any heterogeneity of a set of mean squares assumed to be homogeneous. In testing the effects mean squares in these analyses whenever mean squares were pooled to obtain an improved estimate of the error variance Bartlett's Test has been carried out (Table 6).

In analysis 1, the three factor interactions were pooled to get an improved estimate of the error variance. Comparison of the two factor mean squares with this error shows that four of the six two factor interactions are significant, one at the 20% level, two at the 5% level and one at the 0.1% level of significance. While it is quite possible that the mean square significant at the 20% level arose by chance and even that the two significant at the 5% level did so, it is inconceivable that a mean square so large as to be highly significant at the 0.1% level arose by

chance. Our first conclusion is that there is a very strong interaction effect on the velocity from t and G and possibly much smaller interactions due to M_4m_2 with G . Testing the significance of the mean square due to the effect of the main variables, shows them all to be highly significant at the 0.1% level.

This interaction effect complicates the response but consideration of the theory and examination of the responses suggested that V/G might not exhibit this interaction. In analysis 2, the response variable is the reciprocal time divided by G , the mass flow rate and coded for ease of computation. Once again the three factor interactions were pooled and the two factor interactions compared with this estimate of the error variance using the F - test. In this case, only one interaction showed significance and only at the 20% level. As this mean square could possibly have arisen by chance, the two factor interactions were added to the pool and this set of mean squares subjected to Bartlett's Test, which showed no evidence of heterogeneity. The main effects were compared with this estimate based on eleven degrees of freedom, and it was found that now the variable G had no significant effect upon the response V/G .

Shortly after completing this factorial design, the pores of the catalyst carrier pellets began silting up and they would not absorb enough water to allow the design levels

to be maintained. At the same time a routine check on the Wayne Kerr humidity transducer disclosed that the element had disintegrated and the values of m_2 measured for the factorial design were suspect. New particles were bought and a new transducer element was supplied, and so to check that the results of the previous analyses were valid, a half replicate of the original design was performed. The results of this analysis 3 (see specimen analysis of variance Appendix 5) showed the main effects m , M , t to be highly significant, and the G effect to have a low significance. As the interactions are to some extent mixed with the main effects in a half replicate, this analysis confirmed the analyses 1 and 2. In analysis 3 the interactions were pooled with an estimate of the variance obtained from test runs on the new particles to give greater number of degrees of freedom to the estimate.

In conclusion, we may say that the analyses of variance showed there was no strong interaction between any of the independent variables affecting the response, and that the independent variable G has no significant effect upon the response V/G . We must also note however the possibility of a weak interaction between M and t and an effect from the variation of G influencing the response slightly.

4.2 Response Surfaces

It is possible to fit either an orthogonal or a non-orthogonal polynomial to any given response vector. This means that the levels of the independent variables are held or assumed to be held in the design at values which allow the estimates of the polynomial coefficients to be uncorrelated. In the designs used the levels varied about their assumed orthogonal levels of ± 1 but the true values were known and so either type of polynomial could have been fitted. In general, the use of the true non-orthogonal levels will result in the variance of the estimates of the coefficients being greater than for the orthogonal design and the estimates will be correlated to some degree. However the standard error will be less than for the orthogonal design.

For these experiments a second degree polynomial without quadratic interaction terms was fitted and then as the size of the linear interaction terms' coefficients were only one tenth of the main effects coefficients, it was decided to extend the original design to a non central composite design from the vertex (1, -1, 1, -1) by one unit and include quadratic interaction terms. The choice of the new levels as ± 2 meant that some degree of orthogonality was lost even if the original design was regarded as orthogonal.

As the fitting of the polynomial was carried out on a digital computer, polynomials were fitted using both the orthogonal design levels and the true non-orthogonal levels. As expected a slight reduction in the standard error was found for the non-orthogonal design and the estimates were correlated. However the difference in the variances of the estimates of the coefficients was very slight, and the correlation coefficient was always very small between the estimates (order 0.1) so as the non-orthogonal design was a truer representation of the experimental conditions, the experiment and theory were compared on the basis of the second degree polynomial fitted to the non-orthogonal design for four factors.

A semi orthogonal design with three factors was also fitted for use in representing the response surface on an isometric drawing. This assumes the coefficient of G is zero. Although G was found non significant the value of the coefficient found by fitting all four factors is the best estimate we have and should be retained for a true representation of the experimental results. However for an approximate graphical representation of the surface, the assumption of a zero coefficient will not be seriously in error.

Before we can put limits on the error in the coefficients an assumption must be made about the error distribution.

TABLE 7

Error Distribution

First Set of Test Runs

Coefficient of skewness $\gamma_1 = -0.738$

" " excess $\gamma_2 = 0.38$

Probability of "students" t being different from the normal distribution value is given in Davies (1963)

Normal distribution $t = 2.776$ 4 d.f. Probability of exceeding this value is 5% (double sides test)

For the values of γ_1 γ_2 given above this becomes :

Prob. $t > 2.776$ is 6.41% due to skew

" " $>$ " " 4.78% " " excess

Second Set of Test Runs

Coefficient of skewness $\gamma_1 = 0.5060$

" " excess $\gamma_2 = -1.2556$

Prob. $t > 2.776$ is 5.725 due to skew

" " $>$ " " 5.65 " " excess

4.2.1 Error Distribution

Two sets of five and six runs respectively were available which had been carried out at the same points in the design space. The coefficients of skewness and excess of these sets were calculated to determine the departure of the distributions about their means from normality. The coefficients are given in Table 7 and indicate that the assumption that error was normally distributed would not be seriously in error.

However, the non normality of the distribution has very different effects upon different statistical tests. For instance, a departure from normality has a far greater effect upon the comparison test for variances than that for the comparison of means (Davies 1963) and the effect upon the comparison of means decreases with an increase in the number of degrees of freedom. The data in Table 7 shows that the departure from normality evinced by these two sets of data would result in a true probability of reaching a given value of t , differing only about 1.5% from the normal probability. This is only true for the comparison of a mean with its standard value, for a comparison of variances the difference in probabilities is much more serious. Putting confidence limits on the estimates of the polynomial coefficients is equivalent to this former test but analysis of the variances of the coefficients, for

TABLE 8

Standard errors of orthogonal and non-orthogonal designs
and effect of extra points on main effects coefficients

Design	Orthogonal	Non-orthogonal	
	Std. error	Std. error	d.f.
2nd degree - linear	5.06	3.88	5 E
linear interaction only	4.17	0.29	T
2nd degree composite	5.06	3.36	5 E
as above + quadratic terms	4.17	0.96	T
+ 8 extra points	5.0	2.38	13 E
	3.06	0.902	T
+ 10 " "	6.64	2.60	15 E
	4.47	1.75	T

Non-orthogonal Coefficients

Composite Design

m_2	M_4	t_2	G	
-2.345	-4.469	14.241	0.875	E
-1.493	-4.427	15.457	0.207	T

Composite Design + 8 extra points

-2.162	-4.209	14.628	0.917	E
-1.243	-4.462	15.303	0.174	T

Composite Design + 10 extra points

-2.313	-3.963	14.234	0.900	E
-1.560	-4.156	15.473	0.232	T

E = Experimental Response

T = Theoretical "

instance to test if any were significantly different from zero, might be greatly in error. The coefficients' estimates were therefore compared using the t - test to evaluate confidence limits

$$\text{i.e. } \beta_1 \longrightarrow b_1 \pm \text{t.s.} \sqrt{c^{11}}$$

The level of confidence chosen to compare the coefficients was the 99.5% level i.e. the probability that the true coefficient lies outside the confidence interval is 0.5%.

4.2.2 Responses and Fitted Polynomials

Table 8 shows how the main coefficients in the fitted second degree polynomial altered as the number of points included in the fitted equation was increased. It also shows how the standard errors decrease as the number of degrees of freedom upon which the error estimate is based is increased on the second degree equation. The second set of coefficients have been corrected for the effect of including the responses from the half replicate check experiment. The third set includes the effect of ten runs done as test runs when checking the apparatus. The effect of the whole eighteen runs could not be included as this would raise the rank of the correction matrix above that of the original independent variables matrix.

The standard error of the data including the ten extra responses is greater than that when only eight are included even though the estimate is based on two more degrees of freedom. This is probably because the half replicate is within the original design space and close to the defined points whereas the ten extra responses are outside the design space; in particular, they are taken from a region of very low solids moisture content. The final choice of the fitted polynomials for comparison of theory and experiment is these last four.

Table 3 shows the experimental responses and the corresponding theoretical responses. The theoretical response is usually within about 10% of the theoretical. In Table 2 are given the estimates of the second degree polynomial coefficients and their corresponding effects. The confidence interval indicated by the limits following the experimental estimates embraces the theoretical coefficients and their confidence intervals.

The confidence intervals for the interaction effects are very large compared to the values of the coefficients themselves which places considerable doubt upon their values. However, as the preliminary analysis of variance showed that the interaction effects were insignificant or at best very small, this is understandable. The estimates are retained as they are the best we have.

The intervals for the coefficients from the original design and the half replicate are mutually inclusive of the coefficients when the design equation is applied slightly outside the design space.

We may conclude that the similarity of the results for the predicted V/G and the measured V/G as compared by their fitted second degree polynomials indicates that overall balances on the drying zone have predicted the velocity of the zone to within the experimental error.

4.3 Measurement of System Parameters

The packed density of the solid particles and their specific heat were required for the theoretical calculation of the drying zone velocities. The specific heat was measured using the method of mixtures repeated a number of times and using the mean value. The packed density was measured by filling a known volume with the solids and weighing. It was thought that a variation in the voidage of the bed and hence in the packed density might have been introduced by the vibration of the bed. The possibility of a wall effect in the bed was also investigated.

The effect of vibration was analysed qualitatively using a two level two factor factorial experiment; two levels of vibration frequency and of duration of vibration, replicated twice. The design was repeated in tubes of three different

TABLE 9

Measurement of System Parameters

Experimental Factorial Design Analysis

Bulk Density Measurement

	1" Dia Tube		1.5" Dia Tube		2" Dia Tube	
	Mean Sq.	Sig.	Mean Sq.	Sig.	Mean Sq.	Sig.
f	703	20%	5151	20%	15488	N.S.
θ	136	N.S.	78	N.S.	3698	N.S.
f θ	6	N.S.	36	N.S.	13284	N.S.
error	239		2700		12387	
Mean b.d.	80.45 lb/ft ³		81.57 lb/ft ³		80.45 lb/ft ³	

f = vibration frequency, θ = vibration time

Mean Bulk Density = 80.8 lb/ft³

Sample Scheme Test

Loading technique T, Time of taking sample θ , Size of sample S, Time interval before weighing I

Treatment	Mean Square	Significance
T	2.5998	N.S.
θ	12.9241	"
T θ	1.1148	"
S	3.1370	"
ST	0.6360	"
S θ	0.0315	"
I	1.5034	"
IT	0.4364	"
I θ	0.0944	"
IS	0.1679	"

Error variance = residual = 40.9 based on 32 d.f.

Conclusion - no significant variation in moisture content of sample introduced by sources considered

diameters. An analysis of variance of the results showed no significant effects and all the responses were pooled to give a mean estimate (see Table 9).

Another important parameter measured was the moisture content of the solids which was determined by sampling immediately prior to the run. It was thought that the technique of loading the samples into the bottles, the size of the sample taken, and the time of selection of the samples relative to the run and the time interval before weighing the wet samples might affect the values obtained for the moisture content. Accordingly, these factors were investigated qualitatively with a four factor, two level factorial design replicated three times. The levels chosen were the extremes encountered in practice and an analysis of variance of the results showed no significant effects (Table 9). The sample mean values for the runs were then assumed representative of the bulk populations used in the runs.

APPENDIX 5Specimen Calculations5.1 Specimen Heat Balances

Although precautions were taken during the design and the operation of the apparatus to maintain an adiabatic test bed to check the efficacy of these precautions a heat balance was carried out. A run was selected at random and the heat balance carried out over an arbitrary time interval.

Then

$$\left[\text{Net heat input in the gas} \right]_{\theta_1}^{\theta_2} = \left[\text{Increase in heat content of solid + bed walls} \right]_{\theta_1}^{\theta_2} + \text{heat losses}$$

which can be expressed as

$$G \int_{\theta_1}^{\theta_2} \left[c_p(t_i - t_o) + m_i H_i - m_o H_o \right] d\theta = \sum_j \rho_j c_{s,j} \int_{\theta_1}^{\theta_2} \int_0^1 T dz d\theta$$

$$\text{Now } H_i = c_w(t_{WB_i} - t') + L_{tWB_i} + c_w(t_i - t_{WB_i})$$

$$H_o = c_w(t_{WB_o} - t') + L_{tWB_o} + c_w(t_o - t_{WB_o})$$

If we balance these heat quantities once the run is established m_o has reached, and is constant, at the saturation value at the bed wet bulb temperature ($t_{WB_o} = t_o$).

Similarly t_i , m_i are kept constant and hence we may simplify the above equations.

$$G \left[c_p(t_i - t_o) + m_i \left[C_w(t_{WB_1} - t') + L_{tWB_1} + c_w(t_i - t_{WB_1}) \right]_1 \right. \\ \left. - m_o \left[C_w(t_{WB} - t') + L_{tWB} \right]_9 \right] \Delta\theta = \sum_j \rho_j C_{s_j} (\theta_2 - \theta_1) \int_0^1 T dz \\ \text{compts.}$$

Of these parameters we may obtain t_{WB_1} from m_i , t_i and t_o from the chart records of the run and also $(\theta_2 - \theta_1)$.

The system parameters $\rho_j C_{s_j}$ are found from experiment and

when j refers to the walls, the design of the bed allows us to neglect ρ and C_s . To find the integral we measure the temperatures at each thermocouple level along the length of the bed at θ_1 and θ_2 and integrate graphically the area between the T/Z profiles which are drawn.

Details of the calculations are given overleaf. The balances were carried out for a run with an inlet moisture content of zero (W89) and a run with an inlet moisture content at 0.0194 (W88) (± 1) levels in the statistical design.

	W88 ($m_i=+1$)	W89(-1)
Heat reqd. for drying	26.9 Btu	157.9 Btu
Net heat in the gas (including enthalpy change during drying)	4.70 Btu	1.52 Btu
Increase in bed heat content	0.52 Btu	1.87 Btu
Heat unaccounted for	4.18 Btu	0.35 Btu
% Heat required for drying	15.5%	0.22%

As the heat unaccounted for is small compared to that which is required to perform the drying, we conclude that the system described earlier may reasonably be represented by an adiabatic model.

Calculations

W88

System parameters $(1 - \delta)/\rho_s = 80 \text{ lb/ft}^3$ $A = 0.00769 \text{ ft}^2$
 $l = 10.75 \text{ ins}$ $C_s = 0.26 \text{ Btu/lb}$
 $c = 0.25 \text{ Btu/lb air } ^\circ\text{F}$ $\text{solid } ^\circ\text{F}$
 $c_w = 0.445 \text{ Btu/water vap. } ^\circ\text{F}$

Measured parameters $m_i = 0.0194$ $m_o = 0.0545$
 $t_i = 295^\circ\text{F}$ $t_o = 107^\circ\text{F}$
 $t_{WB_i} = 76^\circ\text{F}$ $t_{WB_o} = 107^\circ\text{F}$

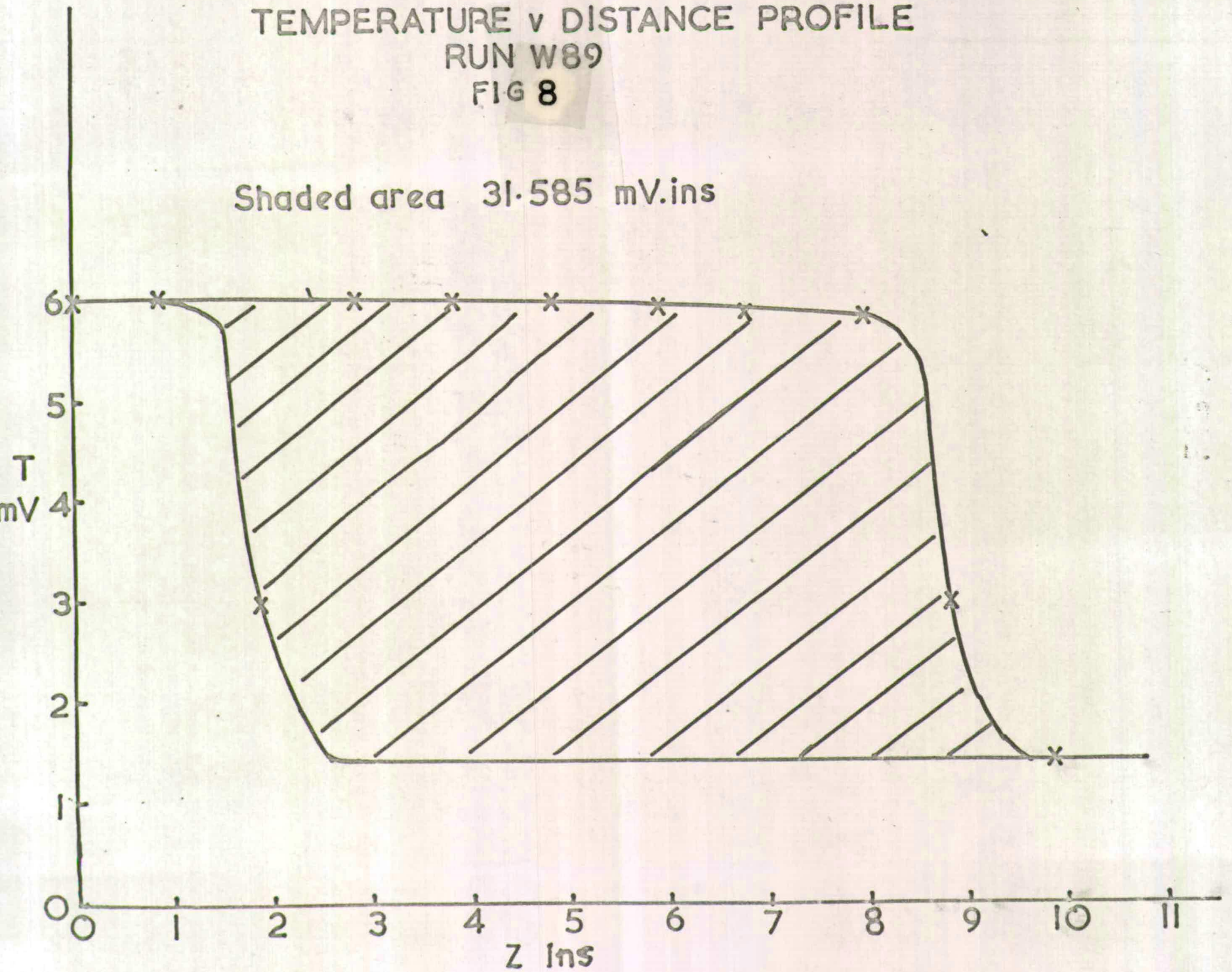
$$C_w(t_{WB_i} - t') = 44.59 \text{ Btu/lb wat. vap.} \quad Lt_{WB_i} = 1050.6 \text{ Btu/lb}$$

$$C_w(t_{WB_o} - t') = 74.95 \quad " \quad " \quad " \quad Lt_{WB_o} = 1033.3 \quad "$$

$$\Delta\theta = 23 \text{ mins.} \quad G = 245 \text{ lb/ft}^2\text{hr.}$$

TEMPERATURE v DISTANCE PROFILE
RUN W89
FIG 8

Shaded area 31.585 mV.ins



Net heat into system in gas Q_{ni}

$$Q_{ni} = G \left\{ c(t_1 - t_0) + m_i \left[C_w(t_{WB_1} - t') + L_{t_{WB_1}} + c_w(t_1 - t_{WB_1}) \right] - m_o \left[C_w(t_{WB_0} - t') + L_{t_{WB_0}} \right] \right\} \Delta\theta A$$

$$= \underline{4.70 \text{ Btu}}$$

Net heat required for exchange process = 26.9 Btu

See fig. 8 for T/z graphs at θ_1, θ_2

Area between curves $\int_0^1 Tdz = 0.728 \text{ mVft}$

Increased enthalpy of bed = $\rho_s C_s A (\theta_2 - \theta_1) \int_0^1 Tdz$

$$= 80 \cdot 0.26 \cdot 0.00769 \cdot \frac{23}{60} \cdot \frac{0.728}{0.0225}$$

$$= \underline{0.52 \text{ Btu}}$$

W 89

Measured parameters $t_1 = 295^\circ\text{F}$ $t_0 = 95^\circ\text{F}$
 $m_i = 0$ $m_o = 0.0365$
 $G = 395 \text{ lb/ft}^2\text{hr}$ $\Delta\theta = 30.9 \text{ mins}$

$$C_w(t_{WB_0} - t') = 62.98 \quad L_{t_{WB_0}} = 1040.1 \text{ Btu/lb}$$

$$Q_{ni} = G \left\{ c(t_1 - t_0) - m_o (C_w(t_{WB_0} - t') + L_{t_{WB_0}}) \right\}$$

$$= \underline{1.52 \text{ Btu}}$$

Net heat required for exchange process = 157.9 Btu

From fig. 3 of T/z graphs at θ_1, θ_2

Area between curves $\int Tdz = 2.63 \text{ mV.ft}$

Increased enthalpy of bed = $\rho_s c_s A (\theta_2 - \theta_1) \int Tdz$

$$= 80 \cdot 0.26 \cdot 0.00769 \cdot \frac{30.9}{60} \cdot \frac{2.63}{0.0225}$$

$$= \underline{1.87 \text{ Btu}}$$

5.2 Specimen Analysis of Variance - Analysis of Variance 3

(Yates Method) Repeat 1/2 replicate

Treatment	Response					Mean	Variance	Effect	Sig.
	V/G	(1)	(2)	(3)	(4)	Square	Ratio F		
$m_1M_1t_1G_1$	2.770	5.3441	9.5185	28.8074	58.6658	-	-	-	
$m_1M_2t_1G_1$	2.5671	4.1744	19.2889	29.8584	-5.6356	1.9850	50.126	M_4	****
$m_2M_1t_1G_1$	2.3191	9.7595	9.7865	-2.5918	-3.9160	0.9584	24.202	m_2	****
$m_2M_2t_1G_1$	1.8553	9.5294	20.0719	-3.0438	-0.1102	0.0759.10 ⁻²	-		
$m_1M_1t_2G_1$	5.3706	5.4696	-0.6737	-1.3998	20.0558	25.1397	634.84	t_2	****
$m_1M_2t_2G_1$	4.3889	4.3169	-1.9181	-2.5162	-1.3248	0.1096	2.962	M_4t_2	*
$m_2M_1t_2G_1$	5.2329	10.7177	-1.4817	-0.2086	0.7288	0.0332	-		
$m_2M_2t_2G_1$	4.2965	9.3542	-1.5621	0.0984	0.2354	0.0346.10 ⁻¹	Pooled		
$m_1M_1t_1G_2$	3.1255	-0.2099	-1.1697	9.7704	1.0510	0.0690	1.743	G	N.S.
$m_1M_2t_1G_2$	2.3441	-0.4638	-0.2301	10.2854	-0.4520	0.0128	-		
$m_2M_1t_1G_2$	2.5086	-0.9817	-1.1527	-1.2444	-1.1164	0.0779	2.105	m_2G	N.S.
$m_2M_2t_1G_2$	1.8083	-0.9364	-1.3635	-0.0804	0.3070	0.0589.10 ⁻¹	Pooled		
$m_1M_1t_2G_2$	5.7537	-0.7814	-0.2539	0.9396	0.5150	0.0166	-		
$m_1M_2t_2G_2$	4.9640	-0.7003	0.0453	-0.2108	1.1640	0.0847	Pooled		
$m_2M_1t_2G_2$	5.0633	-0.7897	0.0811	0.2992	-1.1504	0.0827	Pooled		
$m_2M_2t_2G_2$	4.2909	-0.7724	0.0173	-0.0638	-0.3630	0.0824.10 ⁻¹	Pooled		
	58.6658	-	-	-	-	28.5880			
W	15.6996	31.2909	17.1496	24.6072	75.5344				
X	13.1078	27.3749	35.8806	24.3968	-6.8352				
Y	16.4511	-2.7627	-2.4952	9.7648	-0.7008				
Z	13.4073	-2.8729	-1.5310	9.9304	0.6560				
X + W	28.8074	58.6658	53.0302	49.0040	68.6992				
Z + Y	29.8584	-5.6356	-4.0262	19.6952	-0.0448				
X - W	-2.5918	-3.9160	18.7310	-0.2104	-				**** F reaches 0.1% sig. level
Z - Y	-3.0438	-0.1102	0.9642	0.1656	-				*** " " 1.0% " "
X+W+Z+Y	58.6658	53.0302	49.0040	68.6992					** " " 5.0% " "
(X-W) + (Z - Y)	-5.6356	-4.0262	19.6952	-0.0448					* " " 20.0% " "
									N.S. F not significant
									- F less than 1

Check

3 factor interactions pooled with residual to give revised estimate of error variance of 0.0370 based on 5 d.f.

Plot (1) - t_3 & V_R v T_4 for
exptl. values of m_2 t_2 M_4
Plot (2) - boundary condn.
integral for moving bound'y
of expanding zone h.t. zone

Assume $t_3 = t_2$ & $z' = 0$

From
plot (2) find
 V_R corres.
to t_3

From
plot (2) find
 F_1 corres.
to z' and V_R

From F_1 and intersection
of linear t_2 v T_4 plot
with original t_3 v T_4 find
a corresponding V_R

Is
accuracy
sufficient

YES
END

Increase
 z' by 1

NO

FLOW DIAGRAM
THEORETICAL CALCULATION OF V/G

5.3 Specimen Calculation of Theoretical Response

Method

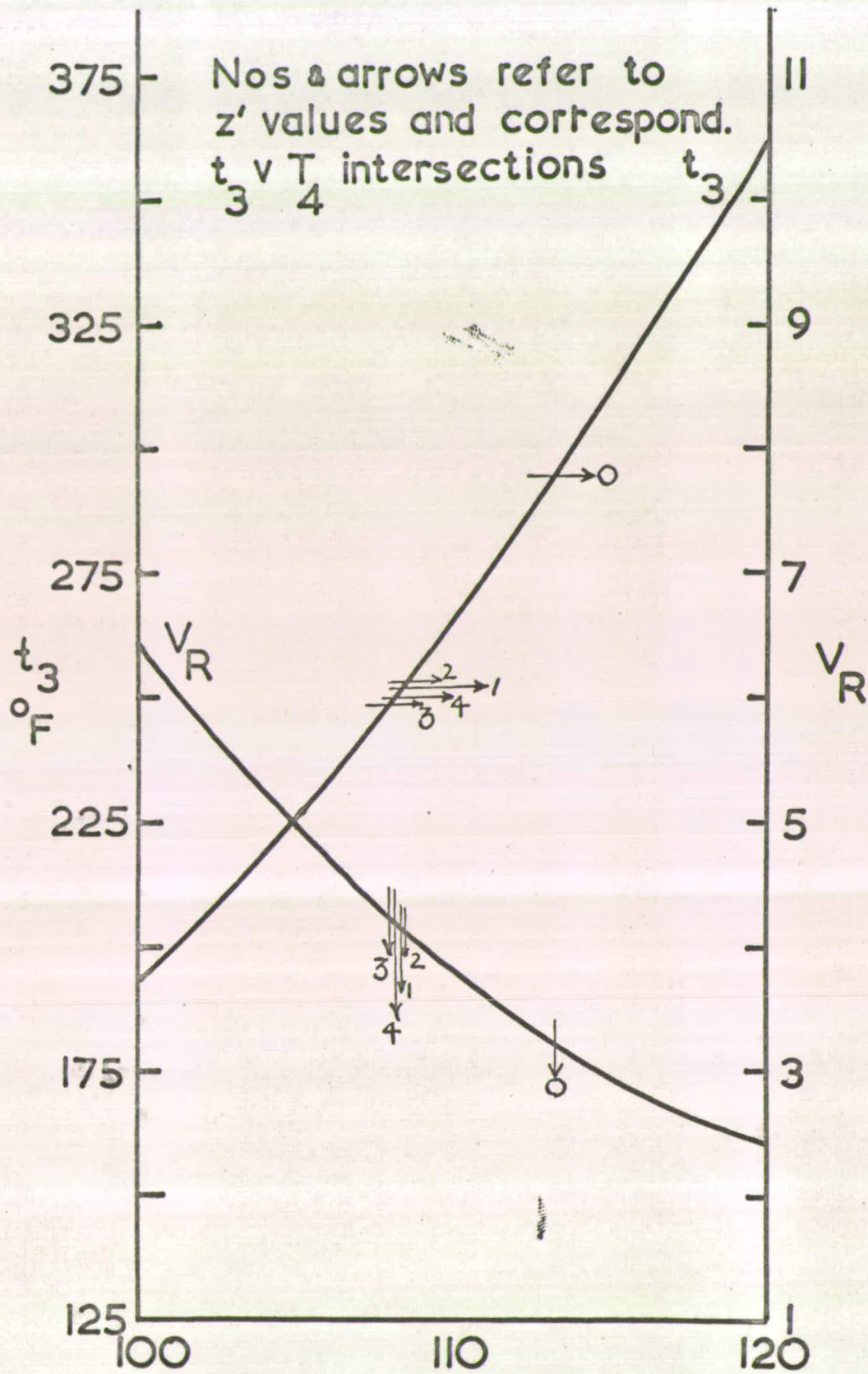
The technique is one of graphical iteration using the tables computed from the overall balances on the drying zone given in Chapter 4 together with the results from the internal analysis of the expanding zone applied at the expanding boundary ($Y' = 0$).

- (1) Construct from the computer tables plots of t_3 , the drying zone gas inlet temperature and V_R , the velocity ratio corresponding to this temperature versus T_4 , the wet bulb temperature of the wet solid in the drying zone. Do this for the values of the independent variables set for a given experimental run ($m M t$).
- (2) Assume, at $z' = 0$, a value of $t_3 = t_2$ and find the corresponding V_R . Use this value of V_R to find a value for F_1 at $z' = 1$ from a plot of the results of the expanding zone analysis. From the definition of F_1 , $t_3 = F_1 T_4 + (1 - F_1) t_2$ so plot t_3 v T_4 and from where this plot cuts original t_3 v T_4 plot find a new t_3 and hence V_R . Use this V_R to get a new F_1 and repeat until a constant value of V_R is reached. Hence from the definition of V_R calculate the theoretical response V/G .

Specimen Calculation for Run W88

Independent variable values :

m_2	t_2	M_4
0.020 lb moist/lb dry gas	295°F	0.149 lb moist/lb dry solid



T_4 °F

FIG 9

t_3 & V_R v T_4 for run W88

Hence $M_4/c_s = 0.573$

From computer tables draw fig. 9

Assume $z = 0$, $t_3 = t_2 = 295^\circ\text{F}$ and we know by definition of F_1 that $t_3 = F_1 T_4 + (1 - F_1)t_2$

Graphical Iteration

z	t_3	V_R	F_1	t_3	T_4	New V_R	T_4
0	295	3.2	0.226	250.9	100		
				253.2	110	4.1	
1	-	4.1	0.223	251.5	100		
				253.7	110	4.07	
2	-	4.07	0.244	247.4	100		
				249.8	110	4.17	
3	-	4.17	0.240	248.2	100		
				250.6	110	<u>4.15</u>	<u>108.2</u>

Hence, for W88 calculated $V_R = 4.15$

$$\text{i.e. } \frac{G(c + m_2 c_w)}{V(1 - \delta) \rho_s c_s} = 4.15 \quad (V\delta \rho \text{ negligible cf } G)$$

$$\frac{V}{G} = 0.0006 \quad \text{diff } 10\%$$

Experimentally determined $\frac{V}{G} = 0.000543$

Similar calculation for run W89 gave calc. $\frac{V}{G} = 0.000582$

diff 0.17%

exptl. $\frac{V}{G} = 0.000581$

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COMPUTER TABLES

(see chapter 4)

 T_4 = Wet bulb temperature $M_4 = m_4$ = saturation moisture content of air at T_4

APP FAHR is redundant

 $T_3 = t_3$ = gas temperature inlet drying zone V_R = velocity ratio group $(G - V\delta\rho)c / V(1 - \delta)\rho_s C_s$ CGR = capacity gas ratio $(c + m_2 c_w) / (m_4 - m_2)$ CSR = capacity solids ratio M_4 / C_s $m_2 = m_3$ = gas moisture content inlet drying zone

M4

T4

APP FAHR

0.0157

70.0000

I

T3	VR	CGR	M2	CSR
136.3022	1.5901	15.9014	0.0000	
131.9735	1.7012	17.0121	0.0010	
127.6603	1.8285	18.2847	0.0020	
123.3625	1.9757	19.7573	0.0030	
119.0801	2.1481	21.4812	0.0040	
114.8129	2.3527	23.5267	0.0050	
110.5610	2.5993	25.9930	0.0060	
106.3242	2.9025	29.0248	0.0070	
102.1024	3.2842	32.8418	0.0080	
97.8955	3.7795	37.7946	0.0090	
93.7036	4.4479	44.4785	0.0100	
72.9642	35.5678	355.6779	0.0150	0.10

136.3022	3.1803	15.9014	0.0000	
131.9735	3.4024	17.0121	0.0010	
127.6603	3.6569	18.2847	0.0020	
123.3625	3.9515	19.7573	0.0030	
119.0801	4.2962	21.4812	0.0040	
114.8129	4.7053	23.5267	0.0050	
110.5610	5.1986	25.9930	0.0060	
106.3242	5.8050	29.0248	0.0070	
102.1024	6.5684	32.8418	0.0080	
97.8955	7.5589	37.7946	0.0090	
93.7036	8.8957	44.4785	0.0100	
72.9642	71.1356	355.6779	0.0150	0.20

136.3022	4.7704	15.9014	0.0000	
131.9735	5.1036	17.0121	0.0010	
127.6603	5.4854	18.2847	0.0020	
123.3625	5.9272	19.7573	0.0030	
119.0801	6.4444	21.4812	0.0040	
114.8129	7.0580	23.5267	0.0050	
110.5610	7.7979	25.9930	0.0060	
106.3242	8.7074	29.0248	0.0070	
102.1024	9.8525	32.8418	0.0080	
97.8955	11.3384	37.7946	0.0090	
93.7036	13.3436	44.4785	0.0100	
72.9642	106.7034	355.6779	0.0150	0.30

136.3022	6.3606	15.9014	0.0000	
131.9735	6.8048	17.0121	0.0010	
127.6603	7.3139	18.2847	0.0020	

T3	VR	CGR	M2	CSR
123.3625	7.9029	19.7573	0.0030	
119.0801	8.5925	21.4812	0.0040	
114.8129	9.4107	23.5267	0.0050	
110.5610	10.3972	25.9930	0.0060	
106.3242	11.6099	29.0248	0.0070	
102.1024	13.1367	32.8418	0.0080	
97.8955	15.1178	37.7946	0.0090	
93.7036	17.7914	44.4785	0.0100	
72.9642	142.2712	355.6779	0.0150	0.40

136.3022	7.9507	15.9014	0.0000	
131.9735	8.5061	17.0121	0.0010	
127.6603	9.1423	18.2847	0.0020	
123.3625	9.8787	19.7573	0.0030	
119.0801	10.7406	21.4812	0.0040	
114.8129	11.7634	23.5267	0.0050	
110.5610	12.9965	25.9930	0.0060	
106.3242	14.5124	29.0248	0.0070	
102.1024	16.4209	32.8418	0.0080	
97.8955	18.8973	37.7946	0.0090	
93.7036	22.2393	44.4785	0.0100	
72.9642	177.8390	355.6779	0.0150	0.50

136.3022	9.5409	15.9014	0.0000	
131.9735	10.2073	17.0121	0.0010	
127.6603	10.9708	18.2847	0.0020	
123.3625	11.8544	19.7573	0.0030	
119.0801	12.8887	21.4812	0.0040	
114.8129	14.1160	23.5267	0.0050	
110.5610	15.5958	25.9930	0.0060	
106.3242	17.4149	29.0248	0.0070	
102.1024	19.7051	32.8418	0.0080	
97.8955	22.6768	37.7946	0.0090	
93.7036	26.6871	44.4785	0.0100	
72.9642	213.4067	355.6779	0.0150	0.60

136.3022	11.1310	15.9014	0.0000	
131.9735	11.9085	17.0121	0.0010	
127.6603	12.7993	18.2847	0.0020	
123.3625	13.8301	19.7573	0.0030	
119.0801	15.0369	21.4812	0.0040	
114.8129	16.4687	23.5267	0.0050	
110.5610	18.1951	25.9930	0.0060	
106.3242	20.3173	29.0248	0.0070	
102.1024	22.9893	32.8418	0.0080	
97.8955	26.4562	37.7946	0.0090	
93.7036	31.1350	44.4785	0.0100	
72.9642	248.9745	355.6779	0.0150	0.70

T3	VR	CGR	M2	CSR
I36.3022	I2.7211	I5.9014	0.0000	
I31.9735	I3.6097	I7.0121	0.0010	
I27.6603	I4.6278	I8.2847	0.0020	
I23.3625	I5.8059	I9.7573	0.0030	
I19.0801	I7.1850	21.4812	0.0040	
I14.8129	I8.8214	23.5267	0.0050	
I10.5610	20.7944	25.9930	0.0060	
I06.3242	23.2198	29.0248	0.0070	
I02.1024	26.2735	32.8418	0.0080	
97.8955	30.2357	37.7946	0.0090	
93.7036	35.5828	44.4785	0.0100	
72.9642	284.5423	355.6779	0.0150	0.80

I36.3022	I4.3113	I5.9014	0.0000	
I31.9735	I5.3109	I7.0121	0.0010	
I27.6603	I6.4562	I8.2847	0.0020	
I23.3625	I7.7816	I9.7573	0.0030	
I19.0801	I9.3331	21.4812	0.0040	
I14.8129	21.1740	23.5267	0.0050	
I10.5610	23.3937	25.9930	0.0060	
I06.3242	26.1223	29.0248	0.0070	
I02.1024	29.5576	32.8418	0.0080	
97.8955	34.0151	37.7946	0.0090	
93.7036	40.0307	44.4785	0.0100	
72.9642	320.1101	355.6779	0.0150	0.90

I36.3022	I5.9014	I5.9014	0.0000	
I31.9735	I7.0121	I7.0121	0.0010	
I27.6603	I8.2847	I8.2847	0.0020	
I23.3625	I9.7573	I9.7573	0.0030	
I19.0801	21.4812	21.4812	0.0040	
I14.8129	23.5267	23.5267	0.0050	
I10.5610	25.9930	25.9930	0.0060	
I06.3242	29.0248	29.0248	0.0070	
I02.1024	32.8418	32.8418	0.0080	
97.8955	37.7946	37.7946	0.0090	
93.7036	44.4785	44.4785	0.0100	
72.9642	355.6779	355.6779	0.0150	1.00

I36.3022	I7.4916	I5.9014	0.0000	
I31.9735	I8.7133	I7.0121	0.0010	
I27.6603	20.1132	I8.2847	0.0020	
I23.3625	21.7331	I9.7573	0.0030	
I19.0801	23.6294	21.4812	0.0040	
I14.8129	25.8794	23.5267	0.0050	
I10.5610	28.5923	25.9930	0.0060	
I06.3242	31.9272	29.0248	0.0070	
I02.1024	36.1260	32.8418	0.0080	

T3	VR	CGR	M2	CSR
97.8955	41.5741	37.7946	0.0090	
93.7036	48.9264	44.4785	0.0100	
72.9642	391.2457	355.6779	0.0150	1.10

136.3022	19.0817	15.9014	0.0000	
131.9735	20.4145	17.0121	0.0010	
127.6603	21.9416	18.2847	0.0020	
123.3625	23.7088	19.7573	0.0030	
119.0801	25.7775	21.4812	0.0040	
114.8129	28.2320	23.5267	0.0050	
110.5610	31.1916	25.9930	0.0060	
106.3242	34.8297	29.0248	0.0070	
102.1024	39.4102	32.8418	0.0080	
97.8955	45.3535	37.7946	0.0090	
93.7036	53.3742	44.4785	0.0100	
72.9642	426.8135	355.6779	0.0150	1.20

136.3022	20.6719	15.9014	0.0000	
131.9735	22.1158	17.0121	0.0010	
127.6603	23.7701	18.2847	0.0020	
123.3625	25.6845	19.7573	0.0030	
119.0801	27.9256	21.4812	0.0040	
114.8129	30.5847	23.5267	0.0050	
110.5610	33.7909	25.9930	0.0060	
106.3242	37.7322	29.0248	0.0070	
102.1024	42.6944	32.8418	0.0080	
97.8955	49.1330	37.7946	0.0090	
93.7036	57.8221	44.4785	0.0100	
72.9642	462.3813	355.6779	0.0150	1.30

136.3022	22.2620	15.9014	0.0000	
131.9735	23.8170	17.0121	0.0010	
127.6603	25.5986	18.2847	0.0020	
123.3625	27.6603	19.7573	0.0030	
119.0801	30.0737	21.4812	0.0040	
114.8129	32.9374	23.5267	0.0050	
110.5610	36.3902	25.9930	0.0060	
106.3242	40.6347	29.0248	0.0070	
102.1024	45.9786	32.8418	0.0080	
97.8955	52.9124	37.7946	0.0090	
93.7036	62.2700	44.4785	0.0100	
72.9642	497.9491	355.6779	0.0150	1.40

136.3022	23.8521	15.9014	0.0000	
131.9735	25.5182	17.0121	0.0010	
127.6603	27.4270	18.2847	0.0020	
123.3625	29.6360	19.7573	0.0030	
119.0801	32.2218	21.4812	0.0040	
114.8129	35.2900	23.5267	0.0050	

T3	VR	CGR	M2	CSR
110.5610	38.9894	25.9930	0.0060	
106.3242	43.5372	29.0248	0.0070	
102.1024	49.2627	32.8418	0.0080	
97.8955	56.6919	37.7946	0.0090	
93.7036	66.7178	44.4785	0.0100	
72.9642	533.5168	355.6779	0.0150	1.50

T₃/VR/T₄ TABLES ADIABATIC CONDITION^{NN}

M ₄		T ₄		APP	FAHR
0.0222		80.0000		I	
T ₃	VR	CGR	M ₂	CSR	
172.9927	1.1276	11.2761	0.0000		
168.6388	1.1830	11.8300	0.0010		
164.3005	1.2439	12.4388	0.0020		
159.9777	1.3111	13.1112	0.0030		
155.6703	1.3857	13.8575	0.0040		
151.3783	1.4691	14.6907	0.0050		
147.1016	1.5627	15.6270	0.0060		
142.8402	1.6687	16.6868	0.0070		
138.5938	1.7896	17.8961	0.0080		
134.3625	1.9289	19.2890	0.0090		
130.1461	2.0911	20.9109	0.0100		
109.2860	3.5805	35.8055	0.0150		
88.7884	11.9317	119.3169	0.0200	0.10	
172.9927	2.2552	11.2761	0.0000		
168.6388	2.3660	11.8300	0.0010		
164.3005	2.4878	12.4388	0.0020		
159.9777	2.6222	13.1112	0.0030		
155.6703	2.7715	13.8575	0.0040		
151.3783	2.9381	14.6907	0.0050		
147.1016	3.1254	15.6270	0.0060		
142.8402	3.3374	16.6868	0.0070		
138.5938	3.5792	17.8961	0.0080		
134.3625	3.8578	19.2890	0.0090		
130.1461	4.1822	20.9109	0.0100		
109.2860	7.1611	35.8055	0.0150		
88.7884	23.8634	119.3169	0.0200	0.20	
172.9927	3.3828	11.2761	0.0000		
168.6388	3.5490	11.8300	0.0010		
164.3005	3.7317	12.4388	0.0020		
159.9777	3.9333	13.1112	0.0030		
155.6703	4.1572	13.8575	0.0040		
151.3783	4.4072	14.6907	0.0050		
147.1016	4.6881	15.6270	0.0060		
142.8402	5.0060	16.6868	0.0070		
138.5938	5.3688	17.8961	0.0080		
134.3625	5.7867	19.2890	0.0090		
130.1461	6.2733	20.9109	0.0100		
109.2860	10.7416	35.8055	0.0150		
88.7884	35.7951	119.3169	0.0200	0.30	

T3	VR	CGR	M2	CSR
172.9927	4.5105	11.2761	0.0000	
168.6388	4.7320	11.8300	0.0010	
164.3005	4.9755	12.4388	0.0020	
159.9777	5.2445	13.1112	0.0030	
155.6703	5.5430	13.8575	0.0040	
151.3783	5.8763	14.6907	0.0050	
147.1016	6.2508	15.6270	0.0060	
142.8402	6.6747	16.6868	0.0070	
138.5938	7.1584	17.8961	0.0080	
134.3625	7.7156	19.2890	0.0090	
130.1461	8.3644	20.9109	0.0100	
109.2860	14.3222	35.8055	0.0150	
88.7884	47.7268	119.3169	0.0200	0.40

172.9927	5.6381	11.2761	0.0000	
168.6388	5.9150	11.8300	0.0010	
164.3005	6.2194	12.4388	0.0020	
159.9777	6.5556	13.1112	0.0030	
155.6703	6.9287	13.8575	0.0040	
151.3783	7.3454	14.6907	0.0050	
147.1016	7.8135	15.6270	0.0060	
142.8402	8.3434	16.6868	0.0070	
138.5938	8.9480	17.8961	0.0080	
134.3625	9.6445	19.2890	0.0090	
130.1461	10.4554	20.9109	0.0100	
109.2860	17.9027	35.8055	0.0150	
88.7884	59.6585	119.3169	0.0200	0.50

172.9927	6.7657	11.2761	0.0000	
168.6388	7.0980	11.8300	0.0010	
164.3005	7.4633	12.4388	0.0020	
159.9777	7.8667	13.1112	0.0030	
155.6703	8.3145	13.8575	0.0040	
151.3783	8.8144	14.6907	0.0050	
147.1016	9.3762	15.6270	0.0060	
142.8402	10.0121	16.6868	0.0070	
138.5938	10.7377	17.8961	0.0080	
134.3625	11.5734	19.2890	0.0090	
130.1461	12.5465	20.9109	0.0100	
109.2860	21.4833	35.8055	0.0150	
88.7884	71.5901	119.3169	0.0200	0.60

172.9927	7.8933	11.2761	0.0000	
168.6388	8.2810	11.8300	0.0010	
164.3005	8.7072	12.4388	0.0020	
159.9777	9.1778	13.1112	0.0030	
155.6703	9.7002	13.8575	0.0040	
151.3783	10.2835	14.6907	0.0050	

T3	VR	CGR	M2	CSR
147.1016	10.9389	15.6270	0.0060	
142.8402	11.6807	16.6868	0.0070	
138.5938	12.5273	17.8961	0.0080	
134.3625	13.5023	19.2890	0.0090	
130.1461	14.6376	20.9109	0.0100	
109.2860	25.0638	35.8055	0.0150	
88.7884	83.5218	119.3169	0.0200	0.70

172.9927	9.0209	11.2761	0.0000	
168.6388	9.4640	11.8300	0.0010	
164.3005	9.9511	12.4388	0.0020	
159.9777	10.4889	13.1112	0.0030	
155.6703	11.0860	13.8575	0.0040	
151.3783	11.7526	14.6907	0.0050	
147.1016	12.5016	15.6270	0.0060	
142.8402	13.3494	16.6868	0.0070	
138.5938	14.3169	17.8961	0.0080	
134.3625	15.4312	19.2890	0.0090	
130.1461	16.7287	20.9109	0.0100	
109.2860	28.6444	35.8055	0.0150	
88.7884	95.4535	119.3169	0.0200	0.80

172.9927	10.1485	11.2761	0.0000	
168.6388	10.6470	11.8300	0.0010	
164.3005	11.1950	12.4388	0.0020	
159.9777	11.8000	13.1112	0.0030	
155.6703	12.4717	13.8575	0.0040	
151.3783	13.2217	14.6907	0.0050	
147.1016	14.0643	15.6270	0.0060	
142.8402	15.0181	16.6868	0.0070	
138.5938	16.1065	17.8961	0.0080	
134.3625	17.3601	19.2890	0.0090	
130.1461	18.8198	20.9109	0.0100	
109.2860	32.2249	35.8055	0.0150	
88.7884	107.3852	119.3169	0.0200	0.90

172.9927	11.2761	11.2761	0.0000	
168.6388	11.8300	11.8300	0.0010	
164.3005	12.4388	12.4388	0.0020	
159.9777	13.1112	13.1112	0.0030	
155.6703	13.8575	13.8575	0.0040	
151.3783	14.6907	14.6907	0.0050	
147.1016	15.6270	15.6270	0.0060	
142.8402	16.6868	16.6868	0.0070	
138.5938	17.8961	17.8961	0.0080	
134.3625	19.2890	19.2890	0.0090	
130.1461	20.9109	20.9109	0.0100	
109.2860	35.8055	35.8055	0.0150	

T3	VR	CGR	M2	CSR
88.7884	119.3169	119.3169	0.0200	1.00
172.9927	12.4038	11.2761	0.0000	
168.6388	13.0130	11.8300	0.0010	
164.3005	13.6827	12.4388	0.0020	
159.9777	14.4223	13.1112	0.0030	
155.6703	15.2432	13.8575	0.0040	
151.3783	16.1598	14.6907	0.0050	
147.1016	17.1897	15.6270	0.0060	
142.8402	18.3555	16.6868	0.0070	
138.5938	19.6857	17.8961	0.0080	
134.3625	21.2179	19.2890	0.0090	
130.1461	23.0020	20.9109	0.0100	
109.2860	39.3860	35.8055	0.0150	
88.7884	131.2486	119.3169	0.0200	1.10

172.9927	13.5314	11.2761	0.0000	
168.6388	14.1960	11.8300	0.0010	
164.3005	14.9266	12.4388	0.0020	
159.9777	15.7334	13.1112	0.0030	
155.6703	16.6290	13.8575	0.0040	
151.3783	17.6289	14.6907	0.0050	
147.1016	18.7524	15.6270	0.0060	
142.8402	20.0241	16.6868	0.0070	
138.5938	21.4753	17.8961	0.0080	
134.3625	23.1469	19.2890	0.0090	
130.1461	25.0931	20.9109	0.0100	
109.2860	42.9666	35.8055	0.0150	
88.7884	143.1803	119.3169	0.0200	1.20

172.9927	14.6590	11.2761	0.0000	
168.6388	15.3790	11.8300	0.0010	
164.3005	16.1705	12.4388	0.0020	
159.9777	17.0445	13.1112	0.0030	
155.6703	18.0147	13.8575	0.0040	
151.3783	19.0980	14.6907	0.0050	
147.1016	20.3152	15.6270	0.0060	
142.8402	21.6928	16.6868	0.0070	
138.5938	23.2649	17.8961	0.0080	
134.3625	25.0758	19.2890	0.0090	
130.1461	27.1842	20.9109	0.0100	
109.2860	46.5471	35.8055	0.0150	
88.7884	155.1120	119.3169	0.0200	1.30

172.9927	15.7866	11.2761	0.0000	
168.6388	16.5620	11.8300	0.0010	
164.3005	17.4144	12.4388	0.0020	
159.9777	18.3556	13.1112	0.0030	
155.6703	19.4005	13.8575	0.0040	

T3	VR	CGR	M2	CSR
151.3783	20.5670	14.6907	0.0050	
147.1016	21.8779	15.6270	0.0060	
142.8402	23.3615	16.6868	0.0070	
138.5938	25.0545	17.8961	0.0080	
134.3625	27.0047	19.2890	0.0090	
130.1461	29.2753	20.9109	0.0100	
109.2860	50.1277	35.8055	0.0150	
88.7884	167.0437	119.3169	0.0200	1.40

172.9927	16.9142	11.2761	0.0000	
168.6388	17.7451	11.8300	0.0010	
164.3005	18.6583	12.4388	0.0020	
159.9777	19.6667	13.1112	0.0030	
155.6703	20.7862	13.8575	0.0040	
151.3783	22.0361	14.6907	0.0050	
147.1016	23.4406	15.6270	0.0060	
142.8402	25.0302	16.6868	0.0070	
138.5938	26.8441	17.8961	0.0080	
134.3625	28.9336	19.2890	0.0090	
130.1461	31.3663	20.9109	0.0100	
109.2860	53.7082	35.8055	0.0150	
88.7884	178.9754	119.3169	0.0200	1.50

M4

T4

APP FAHR

0.0310

90.0000

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T3

VR

CGR

M2

CSR

219.1322	0.8076	8.0762	0.0000	
214.7361	0.8361	8.3609	0.0010	
210.3557	0.8665	8.6651	0.0020	
205.9910	0.8991	8.9912	0.0030	
201.6420	0.9341	9.3415	0.0040	
197.3084	0.9719	9.7187	0.0050	
192.9903	1.0126	10.1262	0.0060	
188.6875	1.0568	10.5677	0.0070	
184.4000	1.1048	11.0477	0.0080	
180.1277	1.1571	11.5714	0.0090	
175.8705	1.2145	12.1450	0.0100	
154.8084	1.6092	16.0921	0.0150	
134.1122	2.3642	23.6420	0.0200	
113.7724	4.3870	43.8702	0.0250	
93.7801	27.5895	275.8951	0.0300	0.10

219.1322	1.6152	8.0762	0.0000	
214.7361	1.6722	8.3609	0.0010	
210.3557	1.7330	8.6651	0.0020	
205.9910	1.7982	8.9912	0.0030	
201.6420	1.8683	9.3415	0.0040	
197.3084	1.9437	9.7187	0.0050	
192.9903	2.0252	10.1262	0.0060	
188.6875	2.1135	10.5677	0.0070	
184.4000	2.2095	11.0477	0.0080	
180.1277	2.3143	11.5714	0.0090	
175.8705	2.4290	12.1450	0.0100	
154.8084	3.2184	16.0921	0.0150	
134.1122	4.7284	23.6420	0.0200	
113.7724	8.7740	43.8702	0.0250	
93.7801	55.1790	275.8951	0.0300	0.20

219.1322	2.4229	8.0762	0.0000	
214.7361	2.5083	8.3609	0.0010	
210.3557	2.5995	8.6651	0.0020	
205.9910	2.6974	8.9912	0.0030	
201.6420	2.8024	9.3415	0.0040	
197.3084	2.9156	9.7187	0.0050	
192.9903	3.0379	10.1262	0.0060	
188.6875	3.1703	10.5677	0.0070	
184.4000	3.3143	11.0477	0.0080	

T3	VR	CGR	M2	CSR
180.1277	3.4714	11.5714	0.0090	
175.8705	3.6435	12.1450	0.0100	
154.8084	4.8276	16.0921	0.0150	
134.1122	7.0926	23.6420	0.0200	
113.7724	13.1610	43.8702	0.0250	
93.7801	82.7685	275.8951	0.0300	0.30
219.1322	3.2305	8.0762	0.0000	
214.7361	3.3443	8.3609	0.0010	
210.3557	3.4661	8.6651	0.0020	
205.9910	3.5965	8.9912	0.0030	
201.6420	3.7366	9.3415	0.0040	
197.3084	3.8875	9.7187	0.0050	
192.9903	4.0505	10.1262	0.0060	
188.6875	4.2271	10.5677	0.0070	
184.4000	4.4191	11.0477	0.0080	
180.1277	4.6285	11.5714	0.0090	
175.8705	4.8580	12.1450	0.0100	
154.8084	6.4368	16.0921	0.0150	
134.1122	9.4568	23.6420	0.0200	
113.7724	17.5481	43.8702	0.0250	
93.7801	110.3581	275.8951	0.0300	0.40
219.1322	4.0381	8.0762	0.0000	
214.7361	4.1804	8.3609	0.0010	
210.3557	4.3326	8.6651	0.0020	
205.9910	4.4956	8.9912	0.0030	
201.6420	4.6707	9.3415	0.0040	
197.3084	4.8594	9.7187	0.0050	
192.9903	5.0631	10.1262	0.0060	
188.6875	5.2839	10.5677	0.0070	
184.4000	5.5238	11.0477	0.0080	
180.1277	5.7857	11.5714	0.0090	
175.8705	6.0725	12.1450	0.0100	
154.8084	8.0460	16.0921	0.0150	
134.1122	11.8210	23.6420	0.0200	
113.7724	21.9351	43.8702	0.0250	
93.7801	137.9476	275.8951	0.0300	0.50
219.1322	4.8457	8.0762	0.0000	
214.7361	5.0165	8.3609	0.0010	
210.3557	5.1991	8.6651	0.0020	
205.9910	5.3947	8.9912	0.0030	
201.6420	5.6049	9.3415	0.0040	
197.3084	5.8312	9.7187	0.0050	
192.9903	6.0757	10.1262	0.0060	
188.6875	6.3406	10.5677	0.0070	
184.4000	6.6286	11.0477	0.0080	

T3	VR	CGR	M2	CSR
180.1277	6.9428	11.5714	0.0090	
175.8705	7.2870	12.1450	0.0100	
154.8084	9.6552	16.0921	0.0150	
134.1122	14.1852	23.6420	0.0200	
113.7724	26.3221	43.8702	0.0250	
93.7801	165.5371	275.8951	0.0300	0.60

219.1322	5.6534	8.0762	0.0000	
214.7361	5.8526	8.3609	0.0010	
210.3557	6.0656	8.6651	0.0020	
205.9910	6.2938	8.9912	0.0030	
201.6420	6.5390	9.3415	0.0040	
197.3084	6.8031	9.7187	0.0050	
192.9903	7.0883	10.1262	0.0060	
188.6875	7.3974	10.5677	0.0070	
184.4000	7.7334	11.0477	0.0080	
180.1277	8.1000	11.5714	0.0090	
175.8705	8.5015	12.1450	0.0100	
154.8084	11.2644	16.0921	0.0150	
134.1122	16.5494	23.6420	0.0200	
113.7724	30.7091	43.8702	0.0250	
93.7801	193.1266	275.8951	0.0300	0.70

219.1322	6.4610	8.0762	0.0000	
214.7361	6.6887	8.3609	0.0010	
210.3557	6.9321	8.6651	0.0020	
205.9910	7.1930	8.9912	0.0030	
201.6420	7.4732	9.3415	0.0040	
197.3084	7.7750	9.7187	0.0050	
192.9903	8.1010	10.1262	0.0060	
188.6875	8.4542	10.5677	0.0070	
184.4000	8.8381	11.0477	0.0080	
180.1277	9.2571	11.5714	0.0090	
175.8705	9.7160	12.1450	0.0100	
154.8084	12.8736	16.0921	0.0150	
134.1122	18.9136	23.6420	0.0200	
113.7724	35.0961	43.8702	0.0250	
93.7801	220.7161	275.8951	0.0300	0.80

219.1322	7.2686	8.0762	0.0000	
214.7361	7.5248	8.3609	0.0010	
210.3557	7.7986	8.6651	0.0020	
205.9910	8.0921	8.9912	0.0030	
201.6420	8.4073	9.3415	0.0040	
197.3084	8.7468	9.7187	0.0050	
192.9903	9.1136	10.1262	0.0060	
188.6875	9.5109	10.5677	0.0070	
184.4000	9.9429	11.0477	0.0080	

T3	VR	CGR	M2	CSR
180.1277	10.4142	11.5714	0.0090	
175.8705	10.9305	12.1450	0.0100	
154.8084	14.4829	16.0921	0.0150	
134.1122	21.2778	23.6420	0.0200	
113.7724	39.4831	43.8702	0.0250	
93.7801	248.3056	275.8951	0.0300	0.90

219.1322	8.0762	8.0762	0.0000	
214.7361	8.3609	8.3609	0.0010	
210.3557	8.6651	8.6651	0.0020	
205.9910	8.9912	8.9912	0.0030	
201.6420	9.3415	9.3415	0.0040	
197.3084	9.7187	9.7187	0.0050	
192.9903	10.1262	10.1262	0.0060	
188.6875	10.5677	10.5677	0.0070	
184.4000	11.0477	11.0477	0.0080	
180.1277	11.5714	11.5714	0.0090	
175.8705	12.1450	12.1450	0.0100	
154.8084	16.0921	16.0921	0.0150	
134.1122	23.6420	23.6420	0.0200	
113.7724	43.8702	43.8702	0.0250	
93.7801	275.8951	275.8951	0.0300	1.00

219.1322	8.8838	8.0762	0.0000	
214.7361	9.1969	8.3609	0.0010	
210.3557	9.5317	8.6651	0.0020	
205.9910	9.8903	8.9912	0.0030	
201.6420	10.2756	9.3415	0.0040	
197.3084	10.6906	9.7187	0.0050	
192.9903	11.1388	10.1262	0.0060	
188.6875	11.6245	10.5677	0.0070	
184.4000	12.1524	11.0477	0.0080	
180.1277	12.7285	11.5714	0.0090	
175.8705	13.3595	12.1450	0.0100	
154.8084	17.7013	16.0921	0.0150	
134.1122	26.0062	23.6420	0.0200	
113.7724	48.2572	43.8702	0.0250	
93.7801	303.4847	275.8951	0.0300	1.10

219.1322	9.6915	8.0762	0.0000	
214.7361	10.0330	8.3609	0.0010	
210.3557	10.3982	8.6651	0.0020	
205.9910	10.7895	8.9912	0.0030	
201.6420	11.2098	9.3415	0.0040	
197.3084	11.6625	9.7187	0.0050	
192.9903	12.1514	10.1262	0.0060	
188.6875	12.6812	10.5677	0.0070	
184.4000	13.2572	11.0477	0.0080	

T3	VR	CGR	M2	CSR
180.1277	13.8856	11.5714	0.0090	
175.8705	14.5740	12.1450	0.0100	
154.8084	19.3105	16.0921	0.0150	
134.1122	28.3704	23.6420	0.0200	
113.7724	52.6442	43.8702	0.0250	
93.7801	331.0742	275.8951	0.0300	1.20

219.1322	10.4991	8.0762	0.0000	
214.7361	10.8691	8.3609	0.0010	
210.3557	11.2647	8.6651	0.0020	
205.9910	11.6886	8.9912	0.0030	
201.6420	12.1439	9.3415	0.0040	
197.3084	12.6343	9.7187	0.0050	
192.9903	13.1641	10.1262	0.0060	
188.6875	13.7380	10.5677	0.0070	
184.4000	14.3620	11.0477	0.0080	
180.1277	15.0428	11.5714	0.0090	
175.8705	15.7885	12.1450	0.0100	
154.8084	20.9197	16.0921	0.0150	
134.1122	30.7346	23.6420	0.0200	
113.7724	57.0312	43.8702	0.0250	
93.7801	358.6637	275.8951	0.0300	1.30

219.1322	11.3067	8.0762	0.0000	
214.7361	11.7052	8.3609	0.0010	
210.3557	12.1312	8.6651	0.0020	
205.9910	12.5877	8.9912	0.0030	
201.6420	13.0781	9.3415	0.0040	
197.3084	13.6062	9.7187	0.0050	
192.9903	14.1767	10.1262	0.0060	
188.6875	14.7948	10.5677	0.0070	
184.4000	15.4667	11.0477	0.0080	
180.1277	16.1999	11.5714	0.0090	
175.8705	17.0030	12.1450	0.0100	
154.8084	22.5289	16.0921	0.0150	
134.1122	33.0988	23.6420	0.0200	
113.7724	61.4182	43.8702	0.0250	
93.7801	386.2532	275.8951	0.0300	1.40

219.1322	12.1143	8.0762	0.0000	
214.7361	12.5413	8.3609	0.0010	
210.3557	12.9977	8.6651	0.0020	
205.9910	13.4868	8.9912	0.0030	
201.6420	14.0122	9.3415	0.0040	
197.3084	14.5781	9.7187	0.0050	
192.9903	15.1893	10.1262	0.0060	
188.6875	15.8515	10.5677	0.0070	
184.4000	16.5715	11.0477	0.0080	

T ₃	VR	CGR	M ₂	CSR
180.1277	17.3570	11.5714	0.0090	
175.8705	18.2175	12.1450	0.0100	
154.8084	24.1381	16.0921	0.0150	
134.1122	35.4630	23.6420	0.0200	
113.7724	65.8052	43.8702	0.0250	
93.7801	413.8427	275.8951	0.0300	1.50

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T3	VR	CGR	M2	CSR
277.7831	0.5834	5.8341	0.0000	
273.3223	0.5984	5.9842	0.0010	
268.8775	0.6142	6.1417	0.0020	
264.4486	0.6307	6.3071	0.0030	
260.0356	0.6481	6.4811	0.0040	
255.6383	0.6664	6.6642	0.0050	
251.2567	0.6857	6.8572	0.0060	
246.8906	0.7061	7.0610	0.0070	
242.5401	0.7277	7.2766	0.0080	
238.2049	0.7505	7.5048	0.0090	
233.8851	0.7747	7.7469	0.0100	
212.5132	0.9218	9.2185	0.0150	
191.5126	1.1334	11.3340	0.0200	
170.8737	1.4634	14.6345	0.0250	
150.5873	2.0503	20.5032	0.0300	
130.6445	3.3846	33.8463	0.0350	
111.0364	9.3980	93.9796	0.0400	0.10

277.7831	1.1668	5.8341	0.0000	
273.3223	1.1968	5.9842	0.0010	
268.8775	1.2283	6.1417	0.0020	
264.4486	1.2614	6.3071	0.0030	
260.0356	1.2962	6.4811	0.0040	
255.6383	1.3328	6.6642	0.0050	
251.2567	1.3714	6.8572	0.0060	
246.8906	1.4122	7.0610	0.0070	
242.5401	1.4553	7.2766	0.0080	
238.2049	1.5010	7.5048	0.0090	
233.8851	1.5494	7.7469	0.0100	
212.5132	1.8437	9.2185	0.0150	
191.5126	2.2668	11.3340	0.0200	
170.8737	2.9269	14.6345	0.0250	
150.5873	4.1006	20.5032	0.0300	
130.6445	6.7693	33.8463	0.0350	
111.0364	18.7959	93.9796	0.0400	0.20

277.7831	1.7502	5.8341	0.0000	
273.3223	1.7953	5.9842	0.0010	
268.8775	1.8425	6.1417	0.0020	
264.4486	1.8921	6.3071	0.0030	
260.0356	1.9443	6.4811	0.0040	

T3	VR	CGR	M2	CSR
255.6383	1.9993	6.6642	0.0050	
251.2567	2.0572	6.8572	0.0060	
246.8906	2.1183	7.0610	0.0070	
242.5401	2.1830	7.2766	0.0080	
238.2049	2.2514	7.5048	0.0090	
233.8851	2.3241	7.7469	0.0100	
212.5132	2.7655	9.2185	0.0150	
191.5126	3.4002	11.3340	0.0200	
170.8737	4.3903	14.6345	0.0250	
150.5873	6.1509	20.5032	0.0300	
130.6445	10.1539	33.8463	0.0350	
111.0364	28.1939	93.9796	0.0400	0.30

277.7831	2.3336	5.8341	0.0000	
273.3223	2.3937	5.9842	0.0010	
268.8775	2.4567	6.1417	0.0020	
264.4486	2.5229	6.3071	0.0030	
260.0356	2.5924	6.4811	0.0040	
255.6383	2.6657	6.6642	0.0050	
251.2567	2.7429	6.8572	0.0060	
246.8906	2.8244	7.0610	0.0070	
242.5401	2.9106	7.2766	0.0080	
238.2049	3.0019	7.5048	0.0090	
233.8851	3.0988	7.7469	0.0100	
212.5132	3.6874	9.2185	0.0150	
191.5126	4.5336	11.3340	0.0200	
170.8737	5.8538	14.6345	0.0250	
150.5873	8.2013	20.5032	0.0300	
130.6445	13.5385	33.8463	0.0350	
111.0364	37.5918	93.9796	0.0400	0.40

277.7831	2.9170	5.8341	0.0000	
273.3223	2.9921	5.9842	0.0010	
268.8775	3.0709	6.1417	0.0020	
264.4486	3.1536	6.3071	0.0030	
260.0356	3.2405	6.4811	0.0040	
255.6383	3.3321	6.6642	0.0050	
251.2567	3.4286	6.8572	0.0060	
246.8906	3.5305	7.0610	0.0070	
242.5401	3.6383	7.2766	0.0080	
238.2049	3.7524	7.5048	0.0090	
233.8851	3.8735	7.7469	0.0100	
212.5132	4.6092	9.2185	0.0150	
191.5126	5.6670	11.3340	0.0200	
170.8737	7.3172	14.6345	0.0250	
150.5873	10.2516	20.5032	0.0300	
130.6445	16.9231	33.8463	0.0350	

T3	VR	CGR	M2	CSR
III.0364	46.9898	93.9796	0.0400	0.50
277.7831	3.5004	5.8341	0.0000	
273.3223	3.5905	5.9842	0.0010	
268.8775	3.6850	6.1417	0.0020	
264.4486	3.7843	6.3071	0.0030	
260.0356	3.8886	6.4811	0.0040	
255.6383	3.9985	6.6642	0.0050	
251.2567	4.1143	6.8572	0.0060	
246.8906	4.2366	7.0610	0.0070	
242.5401	4.3659	7.2766	0.0080	
238.2049	4.5029	7.5048	0.0090	
233.8851	4.6482	7.7469	0.0100	
212.5132	5.5311	9.2185	0.0150	
191.5126	6.8004	11.3340	0.0200	
170.8737	8.7807	14.6345	0.0250	
150.5873	12.3019	20.5032	0.0300	
130.6445	20.3078	33.8463	0.0350	
III.0364	56.3878	93.9796	0.0400	0.60

277.7831	4.0839	5.8341	0.0000	
273.3223	4.1890	5.9842	0.0010	
268.8775	4.2992	6.1417	0.0020	
264.4486	4.4150	6.3071	0.0030	
260.0356	4.5367	6.4811	0.0040	
255.6383	4.6649	6.6642	0.0050	
251.2567	4.8001	6.8572	0.0060	
246.8906	4.9427	7.0610	0.0070	
242.5401	5.0936	7.2766	0.0080	
238.2049	5.2534	7.5048	0.0090	
233.8851	5.4229	7.7469	0.0100	
212.5132	6.4529	9.2185	0.0150	
191.5126	7.9338	11.3340	0.0200	
170.8737	10.2441	14.6345	0.0250	
150.5873	14.3522	20.5032	0.0300	
130.6445	23.6924	33.8463	0.0350	
III.0364	65.7857	93.9796	0.0400	0.70

277.7831	4.6673	5.8341	0.0000	
273.3223	4.7874	5.9842	0.0010	
268.8775	4.9134	6.1417	0.0020	
264.4486	5.0457	6.3071	0.0030	
260.0356	5.1848	6.4811	0.0040	
255.6383	5.3313	6.6642	0.0050	
251.2567	5.4858	6.8572	0.0060	
246.8906	5.6488	7.0610	0.0070	
242.5401	5.8212	7.2766	0.0080	
238.2049	6.0038	7.5048	0.0090	

T3	VR	CGR	M2	CSR
233.885I	6.1976	7.7469	0.0100	
212.5132	7.3748	9.2185	0.0150	
191.5126	9.0672	11.3340	0.0200	
170.8737	11.7076	14.6345	0.0250	
150.5873	16.4025	20.5032	0.0300	
130.6445	27.0770	33.8463	0.0350	
111.0364	75.1837	93.9796	0.0400	0.80

277.783I	5.2507	5.834I	0.0000	
273.3223	5.3858	5.9842	0.0010	
268.8775	5.5276	6.1417	0.0020	
264.4486	5.6764	6.307I	0.0030	
260.0356	5.8330	6.481I	0.0040	
255.6383	5.9978	6.6642	0.0050	
251.2567	6.1715	6.8572	0.0060	
246.8906	6.3549	7.0610	0.0070	
242.540I	6.5489	7.2766	0.0080	
238.2049	6.7543	7.5048	0.0090	
233.885I	6.9722	7.7469	0.0100	
212.5132	8.2966	9.2185	0.0150	
191.5126	10.2006	11.3340	0.0200	
170.8737	13.1710	14.6345	0.0250	
150.5873	18.4528	20.5032	0.0300	
130.6445	30.4616	33.8463	0.0350	
111.0364	84.5817	93.9796	0.0400	0.90

277.783I	5.834I	5.834I	0.0000	
273.3223	5.9842	5.9842	0.0010	
268.8775	6.1417	6.1417	0.0020	
264.4486	6.307I	6.307I	0.0030	
260.0356	6.481I	6.481I	0.0040	
255.6383	6.6642	6.6642	0.0050	
251.2567	6.8572	6.8572	0.0060	
246.8906	7.0610	7.0610	0.0070	
242.540I	7.2766	7.2766	0.0080	
238.2049	7.5048	7.5048	0.0090	
233.885I	7.7469	7.7469	0.0100	
212.5132	9.2185	9.2185	0.0150	
191.5126	11.3340	11.3340	0.0200	
170.8737	14.6345	14.6345	0.0250	
150.5873	20.5032	20.5032	0.0300	
130.6445	33.8463	33.8463	0.0350	
111.0364	93.9796	93.9796	0.0400	1.00

277.783I	6.4175	5.834I	0.0000	
273.3223	6.5827	5.9842	0.0010	
268.8775	6.7559	6.1417	0.0020	
264.4486	6.9379	6.307I	0.0030	

T3	VR	CGR	M2	CSR
260.0356	7.1292	6.4811	0.0040	
255.6383	7.3306	6.6642	0.0050	
251.2567	7.5429	6.8572	0.0060	
246.8906	7.7671	7.0610	0.0070	
242.5401	8.0042	7.2766	0.0080	
238.2049	8.2553	7.5048	0.0090	
233.8851	8.5216	7.7469	0.0100	
212.5132	10.1403	9.2185	0.0150	
191.5126	12.4674	11.3340	0.0200	
170.8737	16.0979	14.6345	0.0250	
150.5873	22.5535	20.5032	0.0300	
130.6445	37.2309	33.8463	0.0350	
111.0364	103.3776	93.9796	0.0400	I.10

277.7831	7.0009	5.8341	0.0000	
273.3223	7.1811	5.9842	0.0010	
268.8775	7.3701	6.1417	0.0020	
264.4486	7.5686	6.3071	0.0030	
260.0356	7.7773	6.4811	0.0040	
255.6383	7.9970	6.6642	0.0050	
251.2567	8.2287	6.8572	0.0060	
246.8906	8.4732	7.0610	0.0070	
242.5401	8.7319	7.2766	0.0080	
238.2049	9.0058	7.5048	0.0090	
233.8851	9.2963	7.7469	0.0100	
212.5132	11.0622	9.2185	0.0150	
191.5126	13.6008	11.3340	0.0200	
170.8737	17.5614	14.6345	0.0250	
150.5873	24.6038	20.5032	0.0300	
130.6445	40.6155	33.8463	0.0350	
111.0364	112.7755	93.9796	0.0400	I.20

277.7831	7.5843	5.8341	0.0000	
273.3223	7.7795	5.9842	0.0010	
268.8775	7.9842	6.1417	0.0020	
264.4486	8.1993	6.3071	0.0030	
260.0356	8.4254	6.4811	0.0040	
255.6383	8.6634	6.6642	0.0050	
251.2567	8.9144	6.8572	0.0060	
246.8906	9.1793	7.0610	0.0070	
242.5401	9.4595	7.2766	0.0080	
238.2049	9.7562	7.5048	0.0090	
233.8851	10.0710	7.7469	0.0100	
212.5132	11.9840	9.2185	0.0150	
191.5126	14.7341	11.3340	0.0200	
170.8737	19.0248	14.6345	0.0250	
150.5873	26.6541	20.5032	0.0300	

T3	VR	CGR	M2	CSR
130.6445	44.0001	33.8463	0.0350	
111.0364	122.1735	93.9796	0.0400	1.30

277.7831	8.1677	5.8341	0.0000	
273.3223	8.3779	5.9842	0.0010	
268.8775	8.5984	6.1417	0.0020	
264.4486	8.8300	6.3071	0.0030	
260.0356	9.0735	6.4811	0.0040	
255.6383	9.3298	6.6642	0.0050	
251.2567	9.6001	6.8572	0.0060	
246.8906	9.8855	7.0610	0.0070	
242.5401	10.1872	7.2766	0.0080	
238.2049	10.5067	7.5048	0.0090	
233.8851	10.8457	7.7469	0.0100	
212.5132	12.9059	9.2185	0.0150	
191.5126	15.8675	11.3340	0.0200	
170.8737	20.4883	14.6345	0.0250	
150.5873	28.7044	20.5032	0.0300	
130.6445	47.3848	33.8463	0.0350	
111.0364	131.5715	93.9796	0.0400	1.40

277.7831	8.7511	5.8341	0.0000	
273.3223	8.9763	5.9842	0.0010	
268.8775	9.2126	6.1417	0.0020	
264.4486	9.4607	6.3071	0.0030	
260.0356	9.7216	6.4811	0.0040	
255.6383	9.9963	6.6642	0.0050	
251.2567	10.2858	6.8572	0.0060	
246.8906	10.5916	7.0610	0.0070	
242.5401	10.9148	7.2766	0.0080	
238.2049	11.2572	7.5048	0.0090	
233.8851	11.6204	7.7469	0.0100	
212.5132	13.8277	9.2185	0.0150	
191.5126	17.0009	11.3340	0.0200	
170.8737	21.9517	14.6345	0.0250	
150.5873	30.7547	20.5032	0.0300	
130.6445	50.7694	33.8463	0.0350	
111.0364	140.9694	93.9796	0.0400	1.50

T₃/VR/T₄ TABLES ADIABATIC CONDITION

M ₄	T ₄	APP	FAHR	
0.0589	110.0000	I		
T ₃	VR	CGR	M ₂	CSR
353.2394	0.4241	4.2411	0.0000	
348.6833	0.4322	4.3220	0.0010	
344.1436	0.4406	4.4058	0.0020	
339.6202	0.4493	4.4926	0.0030	
335.1129	0.4583	4.5826	0.0040	
330.6218	0.4676	4.6759	0.0050	
326.1466	0.4773	4.7727	0.0060	
321.6873	0.4873	4.8732	0.0070	
317.2438	0.4978	4.9777	0.0080	
312.8162	0.5086	5.0864	0.0090	
308.4041	0.5199	5.1995	0.0100	
286.5758	0.5842	5.8423	0.0150	
265.1268	0.6650	6.6500	0.0200	
244.0472	0.7696	7.6958	0.0250	
223.3277	0.9103	9.1028	0.0300	
202.9589	1.1097	11.0974	0.0350	
182.9322	1.4145	14.1446	0.0400	
163.2390	1.9377	19.3768	0.0450	
143.8710	3.0457	30.4568	0.0500	
124.8202	6.9608	69.6079	0.0550	0.10
353.2394	0.8482	4.2411	0.0000	
348.6833	0.8644	4.3220	0.0010	
344.1436	0.8812	4.4058	0.0020	
339.6202	0.8985	4.4926	0.0030	
335.1129	0.9165	4.5826	0.0040	
330.6218	0.9352	4.6759	0.0050	
326.1466	0.9545	4.7727	0.0060	
321.6873	0.9746	4.8732	0.0070	
317.2438	0.9955	4.9777	0.0080	
312.8162	1.0173	5.0864	0.0090	
308.4041	1.0399	5.1995	0.0100	
286.5758	1.1685	5.8423	0.0150	
265.1268	1.3300	6.6500	0.0200	
244.0472	1.5392	7.6958	0.0250	
223.3277	1.8206	9.1028	0.0300	
202.9589	2.2195	11.0974	0.0350	
182.9322	2.8289	14.1446	0.0400	
163.2390	3.8754	19.3768	0.0450	
143.8710	6.0914	30.4568	0.0500	

T ₃	VR	CGR	M ₂	CSR
335.1129	2.2913	4.5826	0.0040	
330.6218	2.3379	4.6759	0.0050	
326.1466	2.3863	4.7727	0.0060	
321.6873	2.4366	4.8732	0.0070	
317.2438	2.4889	4.9777	0.0080	
312.8162	2.5432	5.0864	0.0090	
308.4041	2.5997	5.1995	0.0100	
286.5758	2.9211	5.8423	0.0150	
265.1268	3.3250	6.6500	0.0200	
244.0472	3.8479	7.6958	0.0250	
223.3277	4.5514	9.1028	0.0300	
202.9589	5.5487	11.0974	0.0350	
182.9322	7.0723	14.1446	0.0400	
163.2390	9.6884	19.3768	0.0450	
143.8710	15.2284	30.4568	0.0500	
124.8202	34.8039	69.6079	0.0550	0.50

353.2394	2.5447	4.2411	0.0000	
348.6833	2.5932	4.3220	0.0010	
344.1436	2.6435	4.4058	0.0020	
339.6202	2.6956	4.4926	0.0030	
335.1129	2.7496	4.5826	0.0040	
330.6218	2.8055	4.6759	0.0050	
326.1466	2.8636	4.7727	0.0060	
321.6873	2.9239	4.8732	0.0070	
317.2438	2.9866	4.9777	0.0080	
312.8162	3.0518	5.0864	0.0090	
308.4041	3.1197	5.1995	0.0100	
286.5758	3.5054	5.8423	0.0150	
265.1268	3.9900	6.6500	0.0200	
244.0472	4.6175	7.6958	0.0250	
223.3277	5.4617	9.1028	0.0300	
202.9589	6.6584	11.0974	0.0350	
182.9322	8.4868	14.1446	0.0400	
163.2390	11.6261	19.3768	0.0450	
143.8710	18.2741	30.4568	0.0500	
124.8202	41.7647	69.6079	0.0550	0.60

353.2394	2.9688	4.2411	0.0000	
348.6833	3.0254	4.3220	0.0010	
344.1436	3.0841	4.4058	0.0020	
339.6202	3.1448	4.4926	0.0030	
335.1129	3.2078	4.5826	0.0040	
330.6218	3.2731	4.6759	0.0050	
326.1466	3.3409	4.7727	0.0060	
321.6873	3.4113	4.8732	0.0070	
317.2438	3.4844	4.9777	0.0080	

T3	VR	CGR	M2	CSR
312.8162	3.5605	5.0864	0.0090	
308.4041	3.6396	5.1995	0.0100	
286.5758	4.0896	5.8423	0.0150	
265.1268	4.6550	6.6500	0.0200	
244.0472	5.3871	7.6958	0.0250	
223.3277	6.3720	9.1028	0.0300	
202.9589	7.7682	11.0974	0.0350	
182.9322	9.9012	14.1446	0.0400	
163.2390	13.5637	19.3768	0.0450	
143.8710	21.3197	30.4568	0.0500	
124.8202	48.7255	69.6079	0.0550	0.70

353.2394	3.3929	4.2411	0.0000	
348.6833	3.4576	4.3220	0.0010	
344.1436	3.5247	4.4058	0.0020	
339.6202	3.5941	4.4926	0.0030	
335.1129	3.6661	4.5826	0.0040	
330.6218	3.7407	4.6759	0.0050	
326.1466	3.8181	4.7727	0.0060	
321.6873	3.8986	4.8732	0.0070	
317.2438	3.9822	4.9777	0.0080	
312.8162	4.0691	5.0864	0.0090	
308.4041	4.1596	5.1995	0.0100	
286.5758	4.6738	5.8423	0.0150	
265.1268	5.3200	6.6500	0.0200	
244.0472	6.1566	7.6958	0.0250	
223.3277	7.2822	9.1028	0.0300	
202.9589	8.8779	11.0974	0.0350	
182.9322	11.3157	14.1446	0.0400	
163.2390	15.5014	19.3768	0.0450	
143.8710	24.3654	30.4568	0.0500	
124.8202	55.6863	69.6079	0.0550	0.80

353.2394	3.8170	4.2411	0.0000	
348.6833	3.8898	4.3220	0.0010	
344.1436	3.9653	4.4058	0.0020	
339.6202	4.0434	4.4926	0.0030	
335.1129	4.1243	4.5826	0.0040	
330.6218	4.2083	4.6759	0.0050	
326.1466	4.2954	4.7727	0.0060	
321.6873	4.3859	4.8732	0.0070	
317.2438	4.4799	4.9777	0.0080	
312.8162	4.5777	5.0864	0.0090	
308.4041	4.6795	5.1995	0.0100	
286.5758	5.2580	5.8423	0.0150	
265.1268	5.9850	6.6500	0.0200	
244.0472	6.9262	7.6958	0.0250	

T3	VR	CGR	M2	CSR
223.3277	8.1925	9.1028	0.0300	
202.9589	9.9876	11.0974	0.0350	
182.9322	12.7302	14.1446	0.0400	
163.2390	17.4391	19.3768	0.0450	
143.8710	27.4111	30.4568	0.0500	
124.8202	62.6471	69.6079	0.0550	0.90

353.2394	4.2411	4.2411	0.0000	
348.6833	4.3220	4.3220	0.0010	
344.1436	4.4058	4.4058	0.0020	
339.6202	4.4926	4.4926	0.0030	
335.1129	4.5826	4.5826	0.0040	
330.6218	4.6759	4.6759	0.0050	
326.1466	4.7727	4.7727	0.0060	
321.6873	4.8732	4.8732	0.0070	
317.2438	4.9777	4.9777	0.0080	
312.8162	5.0864	5.0864	0.0090	
308.4041	5.1995	5.1995	0.0100	
286.5758	5.8423	5.8423	0.0150	
265.1268	6.6500	6.6500	0.0200	
244.0472	7.6958	7.6958	0.0250	
223.3277	9.1028	9.1028	0.0300	
202.9589	11.0974	11.0974	0.0350	
182.9322	14.1446	14.1446	0.0400	
163.2390	19.3768	19.3768	0.0450	
143.8710	30.4568	30.4568	0.0500	
124.8202	69.6079	69.6079	0.0550	1.00

353.2394	4.6652	4.2411	0.0000	
348.6833	4.7542	4.3220	0.0010	
344.1436	4.8464	4.4058	0.0020	
339.6202	4.9419	4.4926	0.0030	
335.1129	5.0408	4.5826	0.0040	
330.6218	5.1435	4.6759	0.0050	
326.1466	5.2500	4.7727	0.0060	
321.6873	5.3605	4.8732	0.0070	
317.2438	5.4755	4.9777	0.0080	
312.8162	5.5950	5.0864	0.0090	
308.4041	5.7194	5.1995	0.0100	
286.5758	6.4265	5.8423	0.0150	
265.1268	7.3150	6.6500	0.0200	
244.0472	8.4654	7.6958	0.0250	
223.3277	10.0131	9.1028	0.0300	
202.9589	12.2071	11.0974	0.0350	
182.9322	15.5591	14.1446	0.0400	
163.2390	21.3145	19.3768	0.0450	
143.8710	33.5024	30.4568	0.0500	

T3	VR	CGR	M2	CSR
124.8202	76.5687	69.6079	0.0550	1.10
353.2394	5.0893	4.2411	0.0000	
348.6833	5.1865	4.3220	0.0010	
344.1436	5.2870	4.4058	0.0020	
339.6202	5.3912	4.4926	0.0030	
335.1129	5.4991	4.5826	0.0040	
330.6218	5.6111	4.6759	0.0050	
326.1466	5.7272	4.7727	0.0060	
321.6873	5.8479	4.8732	0.0070	
317.2438	5.9733	4.9777	0.0080	
312.8162	6.1037	5.0864	0.0090	
308.4041	6.2394	5.1995	0.0100	
286.5758	7.0107	5.8423	0.0150	
265.1268	7.9801	6.6500	0.0200	
244.0472	9.2350	7.6958	0.0250	
223.3277	10.9234	9.1028	0.0300	
202.9589	13.3168	11.0974	0.0350	
182.9322	16.9736	14.1446	0.0400	
163.2390	23.2521	19.3768	0.0450	
143.8710	36.5481	30.4568	0.0500	
124.8202	83.5295	69.6079	0.0550	1.20

353.2394	5.5134	4.2411	0.0000	
348.6833	5.6187	4.3220	0.0010	
344.1436	5.7276	4.4058	0.0020	
339.6202	5.8404	4.4926	0.0030	
335.1129	5.9574	4.5826	0.0040	
330.6218	6.0786	4.6759	0.0050	
326.1466	6.2045	4.7727	0.0060	
321.6873	6.3352	4.8732	0.0070	
317.2438	6.4710	4.9777	0.0080	
312.8162	6.6123	5.0864	0.0090	
308.4041	6.7593	5.1995	0.0100	
286.5758	7.5949	5.8423	0.0150	
265.1268	8.6451	6.6500	0.0200	
244.0472	10.0045	7.6958	0.0250	
223.3277	11.8337	9.1028	0.0300	
202.9589	14.4266	11.0974	0.0350	
182.9322	18.3880	14.1446	0.0400	
163.2390	25.1898	19.3768	0.0450	
143.8710	39.5938	30.4568	0.0500	
124.8202	90.4903	69.6079	0.0550	1.30

353.2394	5.9375	4.2411	0.0000	
348.6833	6.0509	4.3220	0.0010	
344.1436	6.1682	4.4058	0.0020	
339.6202	6.2897	4.4926	0.0030	

T3	VR	CGR	M2	CSR
335.1129	6.4156	4.5826	0.0040	
330.6218	6.5462	4.6759	0.0050	
326.1466	6.6818	4.7727	0.0060	
321.6873	6.8225	4.8732	0.0070	
317.2438	6.9688	4.9777	0.0080	
312.8162	7.1209	5.0864	0.0090	
308.4041	7.2793	5.1995	0.0100	
286.5758	8.1792	5.8423	0.0150	
265.1268	9.3101	6.6500	0.0200	
244.0472	10.7741	7.6958	0.0250	
223.3277	12.7439	9.1028	0.0300	
202.9589	15.5363	11.0974	0.0350	
182.9322	19.8025	14.1446	0.0400	
163.2390	27.1275	19.3768	0.0450	
143.8710	42.6395	30.4568	0.0500	
124.8202	97.4511	69.6079	0.0550	1.40

353.2394	6.3616	4.2411	0.0000	
348.6833	6.4831	4.3220	0.0010	
344.1436	6.6088	4.4058	0.0020	
339.6202	6.7390	4.4926	0.0030	
335.1129	6.8739	4.5826	0.0040	
330.6218	7.0138	4.6759	0.0050	
326.1466	7.1590	4.7727	0.0060	
321.6873	7.3098	4.8732	0.0070	
317.2438	7.4666	4.9777	0.0080	
312.8162	7.6296	5.0864	0.0090	
308.4041	7.7992	5.1995	0.0100	
286.5758	8.7634	5.8423	0.0150	
265.1268	9.9751	6.6500	0.0200	
244.0472	11.5437	7.6958	0.0250	
223.3277	13.6542	9.1028	0.0300	
202.9589	16.6461	11.0974	0.0350	
182.9322	21.2170	14.1446	0.0400	
163.2390	29.0652	19.3768	0.0450	
143.8710	45.6851	30.4568	0.0500	
124.8202	104.4118	69.6079	0.0550	1.50

M4

T4

APP FAHR

0.0808

120.0000

I

T3

VR

CGR

M2

CSR

451.4163	0.3095	3.0952	0.0000	
446.7250	0.3140	3.1396	0.0010	
442.0505	0.3185	3.1852	0.0020	
437.3928	0.3232	3.2320	0.0030	
432.7517	0.3280	3.2799	0.0040	
428.1271	0.3329	3.3291	0.0050	
423.5191	0.3380	3.3797	0.0060	
418.9274	0.3432	3.4316	0.0070	
414.3520	0.3485	3.4849	0.0080	
409.7928	0.3540	3.5398	0.0090	
405.2498	0.3596	3.5961	0.0100	
382.7734	0.3904	3.9037	0.0150	
360.6875	0.4262	620	0.0200	
338.9821	0.4684	4.6844	0.0250	
317.6474	0.5190	5.1901	0.0300	
4 296.6738	0.5806	5.8062	0.0350	
276.0525	0.6573	6.5734	0.0400	
255.7745	0.7555	7.5552	0.0450	
235.8315	0.8856	8.8560	0.0500	
216.2150	1.0662	10.6615	0.0550	
196.9172	1.3336	13.3364	0.0600	
177.9304	1.7707	17.7074	0.0650	
159.2472	2.6137	26.1369	0.0700	
140.8602	4.9175	49.1751	0.0750	
122.7625	37.1332	371.3321	0.0800	0.10

451.4163	0.6190	3.0952	0.0000	
446.7250	0.6279	3.1396	0.0010	
442.0505	0.6370	3.1852	0.0020	
437.3928	0.6464	3.2320	0.0030	
432.7517	0.6560	3.2799	0.0040	
428.1271	0.6658	3.3291	0.0050	
423.5191	0.6759	3.3797	0.0060	
418.9274	0.6863	3.4316	0.0070	
414.3520	0.6970	3.4849	0.0080	
409.7928	0.7080	3.5398	0.0090	
405.2498	0.7192	3.5961	0.0100	
382.7734	0.7807	3.9037	0.0150	
360.6875	0.8524	4.2620	0.0200	
338.9821	0.9369	4.6844	0.0250	

T3	VR	CGR	M2	CSR
317.6474	1.0380	5.1901	0.0300	
296.6738	1.1612	5.8062	0.0350	
276.0525	1.3147	6.5734	0.0400	
255.7745	1.5110	7.5552	0.0450	
235.8315	1.7712	8.8560	0.0500	
216.2150	2.1323	10.6615	0.0550	
196.9172	2.6673	13.3364	0.0600	
177.9304	3.5415	17.7074	0.0650	
159.2472	5.2274	26.1369	0.0700	
140.8602	9.8350	49.1751	0.0750	
122.7625	74.2664	371.3321	0.0800	0.20

451.4163	0.9286	3.0952	0.0000	
446.7250	0.9419	3.1396	0.0010	
442.0505	0.9556	3.1852	0.0020	
437.3928	0.9696	3.2320	0.0030	
432.7517	0.9840	3.2799	0.0040	
428.1271	0.9987	3.3291	0.0050	
423.5191	1.0139	3.3797	0.0060	
418.9274	1.0295	3.4316	0.0070	
414.3520	1.0455	3.4849	0.0080	
409.7928	1.0619	3.5398	0.0090	
405.2498	1.0788	3.5961	0.0100	
382.7734	1.1711	3.9037	0.0150	
360.6875	1.2786	4.2620	0.0200	
338.9821	1.4053	4.6844	0.0250	
317.6474	1.5570	5.1901	0.0300	
296.6738	1.7419	5.8062	0.0350	
276.0525	1.9720	6.5734	0.0400	
255.7745	2.2666	7.5552	0.0450	
235.8315	2.6568	8.8560	0.0500	
216.2150	3.1985	10.6615	0.0550	
196.9172	4.0009	13.3364	0.0600	
177.9304	5.3122	17.7074	0.0650	
159.2472	7.8411	26.1369	0.0700	
140.8602	14.7525	49.1751	0.0750	
122.7625	111.3996	371.3321	0.0800	0.30

451.4163	1.2381	3.0952	0.0000	
446.7250	1.2559	3.1396	0.0010	
442.0505	1.2741	3.1852	0.0020	
437.3928	1.2928	3.2320	0.0030	
432.7517	1.3120	3.2799	0.0040	
428.1271	1.3317	3.3291	0.0050	
423.5191	1.3519	3.3797	0.0060	
418.9274	1.3726	3.4316	0.0070	
414.3520	1.3940	3.4849	0.0080	

T3	VR	CGR	M2	CSR
409.7928	1.4159	3.5398	0.0090	
405.2498	1.4385	3.5961	0.0100	
382.7734	1.5615	3.9037	0.0150	
360.6875	1.7048	4.2620	0.0200	
338.9821	1.8738	4.6844	0.0250	
317.6474	2.0760	5.1901	0.0300	
296.6738	2.3225	5.8062	0.0350	
276.0525	2.6294	6.5734	0.0400	
255.7745	3.0221	7.5552	0.0450	
235.8315	3.5424	8.8560	0.0500	
216.2150	4.2646	10.6615	0.0550	
196.9172	5.3346	13.3364	0.0600	
177.9304	7.0830	17.7074	0.0650	
159.2472	10.4548	26.1369	0.0700	
140.8602	19.6700	49.1751	0.0750	
122.7625	148.5329	371.3321	0.0800	0.40

451.4163	1.5476	3.0952	0.0000	
446.7250	1.5698	3.1396	0.0010	
442.0505	1.5926	3.1852	0.0020	
437.3928	1.6160	3.2320	0.0030	
432.7517	1.6400	3.2799	0.0040	
428.1271	1.6646	3.3291	0.0050	
423.5191	1.6898	3.3797	0.0060	
418.9274	1.7158	3.4316	0.0070	
414.3520	1.7425	3.4849	0.0080	
409.7928	1.7699	3.5398	0.0090	
405.2498	1.7981	3.5961	0.0100	
382.7734	1.9519	3.9037	0.0150	
360.6875	2.1310	4.2620	0.0200	
338.9821	2.3422	4.6844	0.0250	
317.6474	2.5950	5.1901	0.0300	
296.6738	2.9031	5.8062	0.0350	
276.0525	3.2867	6.5734	0.0400	
255.7745	3.7776	7.5552	0.0450	
235.8315	4.4280	8.8560	0.0500	
216.2150	5.3308	10.6615	0.0550	
196.9172	6.6682	13.3364	0.0600	
177.9304	8.8537	17.7074	0.0650	
159.2472	13.0685	26.1369	0.0700	
140.8602	24.5875	49.1751	0.0750	
122.7625	185.6661	371.3321	0.0800	0.50

451.4163	1.8571	3.0952	0.0000	
446.7250	1.8838	3.1396	0.0010	
442.0505	1.9111	3.1852	0.0020	
437.3928	1.9392	3.2320	0.0030	

T3	VR	CGR	M2	CSR
432.7517	1.9680	3.2799	0.0040	
428.1271	1.9975	3.3291	0.0050	
423.5191	2.0278	3.3797	0.0060	
418.9274	2.0590	3.4316	0.0070	
414.3520	2.0910	3.4849	0.0080	
409.7928	2.1239	3.5398	0.0090	
405.2498	2.1577	3.5961	0.0100	
382.7734	2.3422	3.9037	0.0150	
360.6875	2.5572	4.2620	0.0200	
338.9821	2.8106	4.6844	0.0250	
317.6474	3.1140	5.1901	0.0300	
296.6738	3.4837	5.8062	0.0350	
276.0525	3.9441	6.5734	0.0400	
255.7745	4.5331	7.5552	0.0450	
235.8315	5.3136	8.8560	0.0500	
216.2150	6.3969	10.6615	0.0550	
196.9172	8.0018	13.3364	0.0600	
177.9304	10.6245	17.7074	0.0650	
159.2472	15.6822	26.1369	0.0700	
140.8602	29.5050	49.1751	0.0750	
122.7625	222.7993	371.3321	0.0800	0.60

451.4163	2.1666	3.0952	0.0000	
446.7250	2.1978	3.1396	0.0010	
442.0505	2.2297	3.1852	0.0020	
437.3928	2.2624	3.2320	0.0030	
432.7517	2.2959	3.2799	0.0040	
428.1271	2.3304	3.3291	0.0050	
423.5191	2.3658	3.3797	0.0060	
418.9274	2.4021	3.4316	0.0070	
414.3520	2.4395	3.4849	0.0080	
409.7928	2.4778	3.5398	0.0090	
405.2498	2.5173	3.5961	0.0100	
382.7734	2.7326	3.9037	0.0150	
360.6875	2.9834	4.2620	0.0200	
338.9821	3.2791	4.6844	0.0250	
317.6474	3.6330	5.1901	0.0300	
296.6738	4.0643	5.8062	0.0350	
276.0525	4.6014	6.5734	0.0400	
255.7745	5.2886	7.5552	0.0450	
235.8315	6.1992	8.8560	0.0500	
216.2150	7.4631	10.6615	0.0550	
196.9172	9.3355	13.3364	0.0600	
177.9304	12.3952	17.7074	0.0650	
159.2472	18.2959	26.1369	0.0700	
140.8602	34.4226	49.1751	0.0750	

T3	VR	CGR	M2	CSR
122.7625	259.9325	371.3321	0.0800	0.70
451.4163	2.4762	3.0952	0.0000	
446.7250	2.5117	3.1396	0.0010	
442.0505	2.5482	3.1852	0.0020	
437.3928	2.5856	3.2320	0.0030	
432.7517	2.6239	3.2799	0.0040	
428.1271	2.6633	3.3291	0.0050	
423.5191	2.7038	3.3797	0.0060	
418.9274	2.7453	3.4316	0.0070	
414.3520	2.7880	3.4849	0.0080	
409.7928	2.8318	3.5398	0.0090	
405.2498	2.8769	3.5961	0.0100	
382.7734	3.1230	3.9037	0.0150	
360.6875	3.4096	4.2620	0.0200	
338.9821	3.7475	4.6844	0.0250	
317.6474	4.1520	5.1901	0.0300	
296.6738	4.6449	5.8062	0.0350	
276.0525	5.2587	6.5734	0.0400	
255.7745	6.0441	7.5552	0.0450	
235.8315	7.0848	8.8560	0.0500	
216.2150	8.5292	10.6615	0.0550	
196.9172	10.6691	13.3364	0.0600	
177.9304	14.1660	17.7074	0.0650	
159.2472	20.9095	26.1369	0.0700	
140.8602	39.3401	49.1751	0.0750	
122.7625	297.0657	371.3321	0.0800	0.80

451.4163	2.7857	3.0952	0.0000
446.7250	2.8257	3.1396	0.0010
442.0505	2.8667	3.1852	0.0020
437.3928	2.9088	3.2320	0.0030
432.7517	2.9519	3.2799	0.0040
428.1271	2.9962	3.3291	0.0050
423.5191	3.0417	3.3797	0.0060
418.9274	3.0884	3.4316	0.0070
414.3520	3.1364	3.4849	0.0080
409.7928	3.1858	3.5398	0.0090
405.2498	3.2365	3.5961	0.0100
382.7734	3.5134	3.9037	0.0150
360.6875	3.8358	4.2620	0.0200
338.9821	4.2160	4.6844	0.0250
317.6474	4.6710	5.1901	0.0300
296.6738	5.2256	5.8062	0.0350
276.0525	5.9161	6.5734	0.0400
255.7745	6.7997	7.5552	0.0450
235.8315	7.9704	8.8560	0.0500

T3	VR	CGR	M2	CSR
216.2150	9.5954	10.6615	0.0550	
196.9172	12.0028	13.3364	0.0600	
177.9304	15.9367	17.7074	0.0650	
159.2472	23.5232	26.1369	0.0700	
140.8602	44.2576	49.1751	0.0750	
122.7625	334.1989	371.3321	0.0800	0.90

451.4163	3.0952	3.0952	0.0000	
446.7250	3.1396	3.1396	0.0010	
442.0505	3.1852	3.1852	0.0020	
437.3928	3.2320	3.2320	0.0030	
432.7517	3.2799	3.2799	0.0040	
428.1271	3.3291	3.3291	0.0050	
423.5191	3.3797	3.3797	0.0060	
418.9274	3.4316	3.4316	0.0070	
414.3520	3.4849	3.4849	0.0080	
409.7928	3.5398	3.5398	0.0090	
405.2498	3.5961	3.5961	0.0100	
382.7734	3.9037	3.9037	0.0150	
360.6875	4.2620	4.2620	0.0200	
338.9821	4.6844	4.6844	0.0250	
317.6474	5.1901	5.1901	0.0300	
296.6738	5.8062	5.8062	0.0350	
276.0525	6.5734	6.5734	0.0400	
255.7745	7.5552	7.5552	0.0450	
235.8315	8.8560	8.8560	0.0500	
216.2150	10.6615	10.6615	0.0550	
196.9172	13.3364	13.3364	0.0600	
177.9304	17.7074	17.7074	0.0650	
159.2472	26.1369	26.1369	0.0700	
140.8602	49.1751	49.1751	0.0750	
122.7625	371.3321	371.3321	0.0800	1.00

451.4163	3.4047	3.0952	0.0000	
446.7250	3.4536	3.1396	0.0010	
442.0505	3.5037	3.1852	0.0020	
437.3928	3.5552	3.2320	0.0030	
432.7517	3.6079	3.2799	0.0040	
428.1271	3.6621	3.3291	0.0050	
423.5191	3.7177	3.3797	0.0060	
418.9274	3.7748	3.4316	0.0070	
414.3520	3.8334	3.4849	0.0080	
409.7928	3.8937	3.5398	0.0090	
405.2498	3.9558	3.5961	0.0100	
382.7734	4.2941	3.9037	0.0150	
360.6875	4.6882	4.2620	0.0200	
338.9821	5.1528	4.6844	0.0250	

T3	VR	CGR	M2	CSR
317.6474	5.7091	5.1901	0.0300	
296.6738	6.3868	5.8062	0.0350	
276.0525	7.2308	6.5734	0.0400	
255.7745	8.3107	7.5552	0.0450	
235.8315	9.7416	8.8560	0.0500	
216.2150	11.7277	10.6615	0.0550	
196.9172	14.6701	13.3364	0.0600	
177.9304	19.4782	17.7074	0.0650	
159.2472	28.7506	26.1369	0.0700	
140.8602	54.0926	49.1751	0.0750	
122.7625	408.4653	371.3321	0.0800	1.10

451.4163	3.7142	3.0952	0.0000	
446.7250	3.7676	3.1396	0.0010	
442.0505	3.8223	3.1852	0.0020	
437.3928	3.8783	3.2320	0.0030	
432.7517	3.9359	3.2799	0.0040	
428.1271	3.9950	3.3291	0.0050	
423.5191	4.0556	3.3797	0.0060	
418.9274	4.1179	3.4316	0.0070	
414.3520	4.1819	3.4849	0.0080	
409.7928	4.2477	3.5398	0.0090	
405.2498	4.3154	3.5961	0.0100	
382.7734	4.6845	3.9037	0.0150	
360.6875	5.1143	4.2620	0.0200	
338.9821	5.6213	4.6844	0.0250	
317.6474	6.2281	5.1901	0.0300	
296.6738	6.9674	5.8062	0.0350	
276.0525	7.8881	6.5734	0.0400	
255.7745	9.0662	7.5552	0.0450	
235.8315	10.6272	8.8560	0.0500	
216.2150	12.7938	10.6615	0.0550	
196.9172	16.0037	13.3364	0.0600	
177.9304	21.2489	17.7074	0.0650	
159.2472	31.3643	26.1369	0.0700	
140.8602	59.0101	49.1751	0.0750	
122.7625	445.5985	371.3321	0.0800	1.20

451.4163	4.0238	3.0952	0.0000	
446.7250	4.0815	3.1396	0.0010	
442.0505	4.1408	3.1852	0.0020	
437.3928	4.2015	3.2320	0.0030	
432.7517	4.2639	3.2799	0.0040	
428.1271	4.3279	3.3291	0.0050	
423.5191	4.3936	3.3797	0.0060	
418.9274	4.4611	3.4316	0.0070	
414.3520	4.5304	3.4849	0.0080	

T3	VR	CGR	M2	CSR
409.7928	4.6017	3.5398	0.0090	
405.2498	4.6750	3.5961	0.0100	
382.7734	5.0749	3.9037	0.0150	
360.6875	5.5405	4.2620	0.0200	
338.9821	6.0897	4.6844	0.0250	
317.6474	6.7471	5.1901	0.0300	
296.6738	7.5480	5.8062	0.0350	
276.0525	8.5455	6.5734	0.0400	
255.7745	9.8217	7.5552	0.0450	
235.8315	11.5128	8.8560	0.0500	
216.2150	13.8600	10.6615	0.0550	
196.9172	17.3373	13.3364	0.0600	
177.9304	23.0197	17.7074	0.0650	
159.2472	33.9780	26.1369	0.0700	
140.8602	63.9276	49.1751	0.0750	
122.7625	482.7318	371.3321	0.0800	1.30

451.4163	4.3333	3.0952	0.0000	
446.7250	4.3955	3.1396	0.0010	
442.0505	4.4593	3.1852	0.0020	
437.3928	4.5247	3.2320	0.0030	
432.7517	4.5919	3.2799	0.0040	
428.1271	4.6608	3.3291	0.0050	
423.5191	4.7316	3.3797	0.0060	
418.9274	4.8042	3.4316	0.0070	
414.3520	4.8789	3.4849	0.0080	
409.7928	4.9557	3.5398	0.0090	
405.2498	5.0346	3.5961	0.0100	
382.7734	5.4652	3.9037	0.0150	
360.6875	5.9667	4.2620	0.0200	
338.9821	6.5582	4.6844	0.0250	
317.6474	7.2661	5.1901	0.0300	
296.6738	8.1287	5.8062	0.0350	
276.0525	9.2028	6.5734	0.0400	
255.7745	10.5772	7.5552	0.0450	
235.8315	12.3984	8.8560	0.0500	
216.2150	14.9262	10.6615	0.0550	
196.9172	18.6710	13.3364	0.0600	
177.9304	24.7904	17.7074	0.0650	
159.2472	36.5917	26.1369	0.0700	
140.8602	68.8451	49.1751	0.0750	
122.7625	519.8650	371.3321	0.0800	1.40

451.4163	4.6428	3.0952	0.0000	
446.7250	4.7095	3.1396	0.0010	
442.0505	4.7778	3.1852	0.0020	
437.3928	4.8479	3.2320	0.0030	

T3	VR	CGR	M2	CSR
432.7517	4.9199	3.2799	0.0040	
428.1271	4.9937	3.3291	0.0050	
423.5191	5.0695	3.3797	0.0060	
418.9274	5.1474	3.4316	0.0070	
414.3520	5.2274	3.4849	0.0080	
409.7928	5.3097	3.5398	0.0090	
405.2498	5.3942	3.5961	0.0100	
382.7734	5.8556	3.9037	0.0150	
360.6875	6.3929	4.2620	0.0200	
338.9821	7.0266	4.6844	0.0250	
317.6474	7.7851	5.1901	0.0300	
296.6738	8.7093	5.8062	0.0350	
276.0525	9.8601	6.5734	0.0400	
255.7745	11.3328	7.5552	0.0450	
235.8315	13.2840	8.8560	0.0500	
216.2150	15.9923	10.6615	0.0550	
196.9172	20.0046	13.3364	0.0600	
177.9304	26.5612	17.7074	0.0650	
159.2472	39.2054	26.1369	0.0700	
140.8602	73.7626	49.1751	0.0750	
122.7625	556.9982	371.3321	0.0800	1.50

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T3	VR	CGR	M2	CSR
581.1546	0.2261	2.2609	0.0000	
576.2713	0.2286	2.2856	0.0010	
571.4055	0.2311	2.3108	0.0020	
566.5572	0.2336	2.3365	0.0030	
561.7261	0.2363	2.3626	0.0040	
556.9124	0.2389	2.3892	0.0050	
552.1157	0.2416	2.4164	0.0060	
547.3361	0.2444	2.4441	0.0070	
542.5735	0.2472	2.4723	0.0080	
537.8278	0.2501	2.5011	0.0090	
533.0988	0.2530	2.5304	0.0100	
509.7026	0.2686	2.6863	0.0150	
486.7129	0.2859	2.8594	0.0200	
464.1192	0.3053	3.0528	0.0250	
441.9114	0.3270	3.2702	0.0300	
420.0796	0.3516	3.5163	0.0350	
398.6143	0.3797	3.7973	0.0400	
377.5065	0.4121	4.1211	0.0450	
356.7473	0.4498	4.4984	0.0500	
336.3281	0.4944	4.9436	0.0550	
316.2406	0.5477	5.4768	0.0600	
296.4768	0.6127	6.1270	0.0650	
277.0289	0.6937	6.9374	0.0700	
257.8895	0.7976	7.9756	0.0750	
239.0512	0.9353	9.3534	0.0800	
220.5070	1.1270	11.2698	0.0850	
202.2501	1.4118	14.1176	0.0900	
184.2738	1.8794	18.7936	0.0950	
166.5717	2.7890	27.8904	0.1000	
149.1376	5.3298	53.2983	0.1050	
131.9654	51.8974	518.9742	0.1100	0.10

581.1546	0.4522	2.2609	0.0000	
576.2713	0.4571	2.2856	0.0010	
571.4055	0.4622	2.3108	0.0020	
566.5572	0.4673	2.3365	0.0030	
561.7261	0.4725	2.3626	0.0040	
556.9124	0.4778	2.3892	0.0050	
552.1157	0.4833	2.4164	0.0060	
547.3361	0.4888	2.4441	0.0070	

T3	VR	CGR	M2	CSR
542.5735	0.4945	2.4723	0.0080	
537.8278	0.5002	2.5011	0.0090	
533.0988	0.5061	2.5304	0.0100	
509.7026	0.5373	2.6863	0.0150	
486.7129	0.5719	2.8594	0.0200	
464.1192	0.6106	3.0528	0.0250	
441.9114	0.6540	3.2702	0.0300	
420.0796	0.7033	3.5163	0.0350	
398.6143	0.7595	3.7973	0.0400	
377.5065	0.8242	4.1211	0.0450	
356.7473	0.8997	4.4984	0.0500	
336.3281	0.9887	4.9436	0.0550	
316.2406	1.0954	5.4768	0.0600	
296.4768	1.2254	6.1270	0.0650	
277.0289	1.3875	6.9374	0.0700	
257.8895	1.5951	7.9756	0.0750	
239.0512	1.8707	9.3534	0.0800	
220.5070	2.2540	11.2698	0.0850	
202.2501	2.8235	14.1176	0.0900	
184.2738	3.7587	18.7936	0.0950	
166.5717	5.5781	27.8904	0.1000	
149.1376	10.6597	53.2983	0.1050	
131.9654	103.7948	518.9742	0.1100	0.20

581.1546	0.6783	2.2609	0.0000	
576.2713	0.6857	2.2856	0.0010	
571.4055	0.6932	2.3108	0.0020	
566.5572	0.7009	2.3365	0.0030	
561.7261	0.7088	2.3626	0.0040	
556.9124	0.7168	2.3892	0.0050	
552.1157	0.7249	2.4164	0.0060	
547.3361	0.7332	2.4441	0.0070	
542.5735	0.7417	2.4723	0.0080	
537.8278	0.7503	2.5011	0.0090	
533.0988	0.7591	2.5304	0.0100	
509.7026	0.8059	2.6863	0.0150	
486.7129	0.8578	2.8594	0.0200	
464.1192	0.9158	3.0528	0.0250	
441.9114	0.9810	3.2702	0.0300	
420.0796	1.0549	3.5163	0.0350	
398.6143	1.1392	3.7973	0.0400	
377.5065	1.2363	4.1211	0.0450	
356.7473	1.3495	4.4984	0.0500	
336.3281	1.4831	4.9436	0.0550	
316.2406	1.6430	5.4768	0.0600	
296.4768	1.8381	6.1270	0.0650	

T3	VR	CGR	M2	CSR
277.0289	2.0812	6.9374	0.0700	
257.8895	2.3927	7.9756	0.0750	
239.0512	2.8060	9.3534	0.0800	
220.5070	3.3810	11.2698	0.0850	
202.2501	4.2353	14.1176	0.0900	
184.2738	5.6381	18.7936	0.0950	
166.5717	8.3671	27.8904	0.1000	
149.1376	15.9895	53.2983	0.1050	
131.9654	155.6923	518.9742	0.1100	0.30

581.1546	0.9043	2.2609	0.0000	
576.2713	0.9142	2.2856	0.0010	
571.4055	0.9243	2.3108	0.0020	
566.5572	0.9346	2.3365	0.0030	
561.7261	0.9450	2.3626	0.0040	
556.9124	0.9557	2.3892	0.0050	
552.1157	0.9666	2.4164	0.0060	
547.3361	0.9776	2.4441	0.0070	
542.5735	0.9889	2.4723	0.0080	
537.8278	1.0004	2.5011	0.0090	
533.0988	1.0122	2.5304	0.0100	
509.7026	1.0745	2.6863	0.0150	
486.7129	1.1438	2.8594	0.0200	
464.1192	1.2211	3.0528	0.0250	
441.9114	1.3081	3.2702	0.0300	
420.0796	1.4065	3.5163	0.0350	
398.6143	1.5189	3.7973	0.0400	
377.5065	1.6484	4.1211	0.0450	
356.7473	1.7994	4.4984	0.0500	
336.3281	1.9774	4.9436	0.0550	
316.2406	2.1907	5.4768	0.0600	
296.4768	2.4508	6.1270	0.0650	
277.0289	2.7750	6.9374	0.0700	
257.8895	3.1903	7.9756	0.0750	
239.0512	3.7414	9.3534	0.0800	
220.5070	4.5079	11.2698	0.0850	
202.2501	5.6471	14.1176	0.0900	
184.2738	7.5174	18.7936	0.0950	
166.5717	11.1562	27.8904	0.1000	
149.1376	21.3193	53.2983	0.1050	
131.9654	207.5897	518.9742	0.1100	0.40

581.1546	1.1304	2.2609	0.0000	
576.2713	1.1428	2.2856	0.0010	
571.4055	1.1554	2.3108	0.0020	
566.5572	1.1682	2.3365	0.0030	
561.7261	1.1813	2.3626	0.0040	

T3	VR	CGR	M2	CSR
556.9124	1.1946	2.3892	0.0050	
552.1157	1.2082	2.4164	0.0060	
547.3361	1.2220	2.4441	0.0070	
542.5735	1.2361	2.4723	0.0080	
537.8278	1.2505	2.5011	0.0090	
533.0988	1.2652	2.5304	0.0100	
509.7026	1.3432	2.6863	0.0150	
486.7129	1.4297	2.8594	0.0200	
464.1192	1.5264	3.0528	0.0250	
441.9114	1.6351	3.2702	0.0300	
420.0796	1.7581	3.5163	0.0350	
398.6143	1.8986	3.7973	0.0400	
377.5065	2.0606	4.1211	0.0450	
356.7473	2.2492	4.4984	0.0500	
336.3281	2.4718	4.9436	0.0550	
316.2406	2.7384	5.4768	0.0600	
296.4768	3.0635	6.1270	0.0650	
277.0289	3.4687	6.9374	0.0700	
257.8895	3.9878	7.9756	0.0750	
239.0512	4.6767	9.3534	0.0800	
220.5070	5.6349	11.2698	0.0850	
202.2501	7.0588	14.1176	0.0900	
184.2738	9.3968	18.7936	0.0950	
166.5717	13.9452	27.8904	0.1000	
149.1376	26.6492	53.2983	0.1050	
131.9654	259.4871	518.9742	0.1100	0.50

581.1546	1.3565	2.2609	0.0000	
576.2713	1.3714	2.2856	0.0010	
571.4055	1.3865	2.3108	0.0020	
566.5572	1.4019	2.3365	0.0030	
561.7261	1.4176	2.3626	0.0040	
556.9124	1.4335	2.3892	0.0050	
552.1157	1.4498	2.4164	0.0060	
547.3361	1.4664	2.4441	0.0070	
542.5735	1.4834	2.4723	0.0080	
537.8278	1.5006	2.5011	0.0090	
533.0988	1.5182	2.5304	0.0100	
509.7026	1.6118	2.6863	0.0150	
486.7129	1.7157	2.8594	0.0200	
464.1192	1.8317	3.0528	0.0250	
441.9114	1.9621	3.2702	0.0300	
420.0796	2.1098	3.5163	0.0350	
398.6143	2.2784	3.7973	0.0400	
377.5065	2.4727	4.1211	0.0450	
356.7473	2.6990	4.4984	0.0500	

T3	VR	CGR	M2	CSR
336.3281	2.9661	4.9436	0.0550	
316.2406	3.2861	5.4768	0.0600	
296.4768	3.6762	6.1270	0.0650	
277.0289	4.1624	6.9374	0.0700	
257.8895	4.7854	7.9756	0.0750	
239.0512	5.6120	9.3534	0.0800	
220.5070	6.7619	11.2698	0.0850	
202.2501	8.4706	14.1176	0.0900	
184.2738	11.2762	18.7936	0.0950	
166.5717	16.7343	27.8904	0.1000	
149.1376	31.9790	53.2983	0.1050	
131.9654	311.3845	518.9742	0.1100	0.60

581.1546	1.5826	2.2609	0.0000	
576.2713	1.5999	2.2856	0.0010	
571.4055	1.6176	2.3108	0.0020	
566.5572	1.6355	2.3365	0.0030	
561.7261	1.6538	2.3626	0.0040	
556.9124	1.6725	2.3892	0.0050	
552.1157	1.6915	2.4164	0.0060	
547.3361	1.7109	2.4441	0.0070	
542.5735	1.7306	2.4723	0.0080	
537.8278	1.7507	2.5011	0.0090	
533.0988	1.7713	2.5304	0.0100	
509.7026	1.8804	2.6863	0.0150	
486.7129	2.0016	2.8594	0.0200	
464.1192	2.1370	3.0528	0.0250	
441.9114	2.2891	3.2702	0.0300	
420.0796	2.4614	3.5163	0.0350	
398.6143	2.6581	3.7973	0.0400	
377.5065	2.8848	4.1211	0.0450	
356.7473	3.1489	4.4984	0.0500	
336.3281	3.4605	4.9436	0.0550	
316.2406	3.8338	5.4768	0.0600	
296.4768	4.2889	6.1270	0.0650	
277.0289	4.8562	6.9374	0.0700	
257.8895	5.5829	7.9756	0.0750	
239.0512	6.5474	9.3534	0.0800	
220.5070	7.8889	11.2698	0.0850	
202.2501	9.8823	14.1176	0.0900	
184.2738	13.1555	18.7936	0.0950	
166.5717	19.5233	27.8904	0.1000	
149.1376	37.3088	53.2983	0.1050	
131.9654	363.2819	518.9742	0.1100	0.70

581.1546	1.8087	2.2609	0.0000	
576.2713	1.8285	2.2856	0.0010	

T3	VR	CGR	M2	CSR
571.4055	1.8486	2.3108	0.0020	
566.5572	1.8692	2.3365	0.0030	
561.7261	1.8901	2.3626	0.0040	
556.9124	1.9114	2.3892	0.0050	
552.1157	1.9331	2.4164	0.0060	
547.3361	1.9553	2.4441	0.0070	
542.5735	1.9778	2.4723	0.0080	
537.8278	2.0008	2.5011	0.0090	
533.0988	2.0243	2.5304	0.0100	
509.7026	2.1491	2.6863	0.0150	
486.7129	2.2876	2.8594	0.0200	
464.1192	2.4422	3.0528	0.0250	
441.9114	2.6161	3.2702	0.0300	
420.0796	2.8130	3.5163	0.0350	
398.6143	3.0378	3.7973	0.0400	
377.5065	3.2969	4.1211	0.0450	
356.7473	3.5987	4.4984	0.0500	
336.3281	3.9549	4.9436	0.0550	
316.2406	4.*14	5.4768	0.0600	
296.4768	4.9016	6.1270	0.0650	
277.0289	5.5499	6.9374	0.0700	
257.8895	6.3805	7.9756	0.0750	
239.0512	7.4827	9.3534	0.0800	
220.5070	9.0159	11.2698	0.0850	
202.2501	11.2941	14.1176	0.0900	
184.2738	15.0349	18.7936	0.0950	
166.5717	22.3124	27.8904	0.1000	
149.1376	42.6386	53.2983	0.1050	
131.9654	415.1794	518.9742	0.1100	0.80

581.1546	2.0348	2.2609	0.0000	
576.2713	2.0570	2.2856	0.0010	
571.4055	2.0797	2.3108	0.0020	
566.5572	2.1028	2.3365	0.0030	
561.7261	2.1263	2.3626	0.0040	
556.9124	2.1503	2.3892	0.0050	
552.1157	2.1748	2.4164	0.0060	
547.3361	2.1997	2.4441	0.0070	
542.5735	2.2251	2.4723	0.0080	
537.8278	2.2510	2.5011	0.0090	
533.0988	2.2774	2.5304	0.0100	
509.7026	2.4177	2.6863	0.0150	
486.7129	2.5735	2.8594	0.0200	
464.1192	2.7475	3.0528	0.0250	
441.9114	2.9431	3.2702	0.0300	
420.0796	3.1646	3.5163	0.0350	

T3	VR	CGR	M2	CSR
398.6143	3.4175	3.7973	0.0400	
377.5065	3.7090	4.1211	0.0450	
356.7473	4.0486	4.4984	0.0500	
336.3281	4.4492	4.9436	0.0550	
316.2406	4.9291	5.4768	0.0600	
296.4768	5.5143	6.1270	0.0650	
277.0289	6.2437	6.9374	0.0700	
257.8895	7.1781	7.9756	0.0750	
239.0512	8.4181	9.3534	0.0800	
220.5070	10.1429	11.2698	0.0850	
202.2501	12.7059	14.1176	0.0900	
184.2738	16.9143	18.7936	0.0950	
166.5717	25.1014	27.8904	0.1000	
149.1376	47.9685	53.2983	0.1050	
131.9654	467.0768	518.9742	0.1100	0.90

581.1546	2.2609	2.2609	0.0000	
576.2713	2.2856	2.2856	0.0010	
571.4055	2.3108	2.3108	0.0020	
566.5572	2.3365	2.3365	0.0030	
561.7261	2.3626	2.3626	0.0040	
556.9124	2.3892	2.3892	0.0050	
552.1157	2.4164	2.4164	0.0060	
547.3361	2.4441	2.4441	0.0070	
542.5735	2.4723	2.4723	0.0080	
537.8278	2.5011	2.5011	0.0090	
533.0988	2.5304	2.5304	0.0100	
509.7026	2.6863	2.6863	0.0150	
486.7129	2.8594	2.8594	0.0200	
464.1192	3.0528	3.0528	0.0250	
441.9114	3.2702	3.2702	0.0300	
420.0796	3.5163	3.5163	0.0350	
398.6143	3.7973	3.7973	0.0400	
377.5065	4.1211	4.1211	0.0450	
356.7473	4.4984	4.4984	0.0500	
336.3281	4.9436	4.9436	0.0550	
316.2406	5.4768	5.4768	0.0600	
296.4768	6.1270	6.1270	0.0650	
277.0289	6.9374	6.9374	0.0700	
257.8895	7.9756	7.9756	0.0750	
239.0512	9.3534	9.3534	0.0800	
220.5070	11.2698	11.2698	0.0850	
202.2501	14.1176	14.1176	0.0900	
184.2738	18.7936	18.7936	0.0950	
166.5717	27.8904	27.8904	0.1000	
149.1376	53.2983	53.2983	0.1050	

T3	VR	CGR	M2	CSR
131.9654	518.9742	518.9742	0.1100	1.00
581.1546	2.4870	2.2609	0.0000	
576.2713	2.5142	2.2856	0.0010	
571.4055	2.5419	2.3108	0.0020	
566.5572	2.5701	2.3365	0.0030	
561.7261	2.5989	2.3626	0.0040	
556.9124	2.6282	2.3892	0.0050	
552.1157	2.6580	2.4164	0.0060	
547.3361	2.6885	2.4441	0.0070	
542.5735	2.7195	2.4723	0.0080	
537.8278	2.7512	2.5011	0.0090	
533.0988	2.7834	2.5304	0.0100	
509.7026	2.9549	2.6863	0.0150	
486.7129	3.1454	2.8594	0.0200	
464.1192	3.3581	3.0528	0.0250	
441.9114	3.5972	3.2702	0.0300	
420.0796	3.8679	3.5163	0.0350	
398.6143	4.1770	3.7973	0.0400	
377.5065	4.5332	4.1211	0.0450	
356.7473	4.9482	4.4984	0.0500	
336.3281	5.4379	4.9436	0.0550	
316.2406	6.0245	5.4768	0.0600	
296.4768	6.7397	6.1270	0.0650	
277.0289	7.6312	6.9374	0.0700	
257.8895	8.7732	7.9756	0.0750	
239.0512	10.2887	9.3534	0.0800	
220.5070	12.3968	11.2698	0.0850	
202.2501	15.5294	14.1176	0.0900	
184.2738	20.6730	18.7936	0.0950	
166.5717	30.6795	27.8904	0.1000	
149.1376	58.6281	53.2983	0.1050	
131.9654	570.8716	518.9742	0.1100	1.10

581.1546	2.7130	2.2609	0.0000
576.2713	2.7427	2.2856	0.0010
571.4055	2.7730	2.3108	0.0020
566.5572	2.8038	2.3365	0.0030
561.7261	2.8351	2.3626	0.0040
556.9124	2.8671	2.3892	0.0050
552.1157	2.8997	2.4164	0.0060
547.3361	2.9329	2.4441	0.0070
542.5735	2.9667	2.4723	0.0080
537.8278	3.0013	2.5011	0.0090
533.0988	3.0365	2.5304	0.0100
509.7026	3.2236	2.6863	0.0150
486.7129	3.4313	2.8594	0.0200

T3	VR	CGR	M2	CSR
464.1192	3.6634	3.0528	0.0250	
441.9114	3.9242	3.2702	0.0300	
420.0796	4.2195	3.5163	0.0350	
398.6143	4.5567	3.7973	0.0400	
377.5065	4.9453	4.1211	0.0450	
356.7473	5.3981	4.4984	0.0500	
336.3281	5.9323	4.9436	0.0550	
316.2406	6.5721	5.4768	0.0600	
296.4768	7.3524	6.1270	0.0650	
277.0289	8.3249	6.9374	0.0700	
257.8895	9.5708	7.9756	0.0750	
239.0512	11.2241	9.3534	0.0800	
220.5070	13.5238	11.2698	0.0850	
202.2501	16.9412	14.1176	0.0900	
184.2738	22.5523	18.7936	0.0950	
166.5717	33.4685	27.8904	0.1000	
149.1376	63.9580	53.2983	0.1050	
131.9654	622.7690	518.9742	0.1100	1.20

581.1546	2.9391	2.2609	0.0000	
576.2713	2.9713	2.2856	0.0010	
571.4055	3.0040	2.3108	0.0020	
566.5572	3.0374	2.3365	0.0030	
561.7261	3.0714	2.3626	0.0040	
556.9124	3.1060	2.3892	0.0050	
552.1157	3.1413	2.4164	0.0060	
547.3361	3.1773	2.4441	0.0070	
542.5735	3.2140	2.4723	0.0080	
537.8278	3.2514	2.5011	0.0090	
533.0988	3.2895	2.5304	0.0100	
509.7026	3.4922	2.6863	0.0150	
486.7129	3.7173	2.8594	0.0200	
464.1192	3.9686	3.0528	0.0250	
441.9114	4.2512	3.2702	0.0300	
420.0796	4.5712	3.5163	0.0350	
398.6143	4.9364	3.7973	0.0400	
377.5065	5.3574	4.1211	0.0450	
356.7473	5.8479	4.4984	0.0500	
336.3281	6.4267	4.9436	0.0550	
316.2406	7.1198	5.4768	0.0600	
296.4768	7.9651	6.1270	0.0650	
277.0289	9.0186	6.9374	0.0700	
257.8895	10.3683	7.9756	0.0750	
239.0512	12.1594	9.3534	0.0800	
220.5070	14.6508	11.2698	0.0850	
202.2501	18.3529	14.1176	0.0900	

T3	VR	CGR	M2	CSR
184.2738	24.4317	18.7936	0.0950	
166.5717	36.2576	27.8904	0.1000	
149.1376	69.2878	53.2983	0.1050	
131.9654	674.6665	518.9742	0.1100	1.30

581.1546	3.1652	2.2609	0.0000	
576.2713	3.1998	2.2856	0.0010	
571.4055	3.2351	2.3108	0.0020	
566.5572	3.2710	2.3365	0.0030	
561.7261	3.3077	2.3626	0.0040	
556.9124	3.3449	2.3892	0.0050	
552.1157	3.3830	2.4164	0.0060	
547.3361	3.4217	2.4441	0.0070	
542.5735	3.4612	2.4723	0.0080	
537.8278	3.5015	2.5011	0.0090	
533.0988	3.5426	2.5304	0.0100	
509.7026	3.7608	2.6863	0.0150	
486.7129	4.0032	2.8594	0.0200	
464.1192	4.2739	3.0528	0.0250	
441.9114	4.5782	3.2702	0.0300	
420.0796	4.9228	3.5163	0.0350	
398.6143	5.3162	3.7973	0.0400	
377.5065	5.7695	4.1211	0.0450	
356.7473	6.2978	4.4984	0.0500	
336.3281	6.9210	4.9436	0.0550	
316.2406	7.6675	5.4768	0.0600	
296.4768	8.5778	6.1270	0.0650	
277.0289	9.7124	6.9374	0.0700	
257.8895	11.1659	7.9756	0.0750	
239.0512	13.0948	9.3534	0.0800	
220.5070	15.7778	11.2698	0.0850	
202.2501	19.7647	14.1176	0.0900	
184.2738	26.3111	18.7936	0.0950	
166.5717	39.0466	27.8904	0.1000	
149.1376	74.6176	53.2983	0.1050	
131.9654	726.5639	518.9742	0.1100	1.40

581.1546	3.3913	2.2609	0.0000	
576.2713	3.4284	2.2856	0.0010	
571.4055	3.4662	2.3108	0.0020	
566.5572	3.5047	2.3365	0.0030	
561.7261	3.5439	2.3626	0.0040	
556.9124	3.5839	2.3892	0.0050	
552.1157	3.6246	2.4164	0.0060	
547.3361	3.6661	2.4441	0.0070	
542.5735	3.7084	2.4723	0.0080	
537.8278	3.7516	2.5011	0.0090	

T3	VR	CGR	M2	CSR
533.0988	3.7956	2.5304	0.0100	
509.7026	4.0295	2.6863	0.0150	
486.7129	4.2892	2.8594	0.0200	
464.1192	4.5792	3.0528	0.0250	
441.9114	4.9052	3.2702	0.0300	
420.0796	5.2744	3.5163	0.0350	
398.6143	5.6959	3.7973	0.0400	
377.5065	6.1817	4.1211	0.0450	
356.7473	6.7476	4.4984	0.0500	
336.3281	7.4154	4.9436	0.0550	
316.2406	8.2152	5.4768	0.0600	
296.4768	9.1905	6.1270	0.0650	
277.0289	10.4061	6.9374	0.0700	
257.8895	11.9635	7.9756	0.0750	
239.0512	14.0301	9.3534	0.0800	
220.5070	16.9048	11.2698	0.0850	
202.2501	21.1765	14.1176	0.0900	
184.2738	28.1904	18.7936	0.0950	
166.5717	41.8357	27.8904	0.1000	
149.1376	79.9475	53.2983	0.1050	
131.9654	778.4613	518.9742	0.1100	1.50

M4

T4
