

DYNAMIC ALONG-WIND RESPONSE OF TALL BLUFF
STRUCTURES IN STRONG WIND

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PREFACE

This thesis is the result of a three-year research work for the degree of doctor of philosophy in the Department of Civil Engineering and Building Science, University of Edinburgh since April 1976.

Papers by the author published in that period, which are included in an appendix of the thesis, are as follows :

1. "Height dependence of root-coherence in the natural wind", Co-author R. Royles, *Proc. Int. Seminar Safety of Structures under Dynamic Loading*, June 1977, Tapir Publishers, Trondheim, Norway, 1978;
2. "Further consideration of the height dependence of root-coherence in the natural wind", Co-author R. Royles, *Building and Environment*, vol. 13, no. 3, 1978;
3. "A transducer for determining dynamic drag and lift coefficient in wind tunnels", Co-author R. Royles, *Strain*, vol. 14, no. 4, Oct. 1978.

It is declared that the thesis has been composed by the author himself, and all works and results in the thesis have been carried out and achieved solely by him under the supervision of Dr. Rodney Royles, unless otherwise stated.

Edinburgh, January 1979

ABSTRACT

Since a statistical approach to the wind loading problem was proposed in the early 1960's, there have been a number of gust response approaches developed. However, some simplification or approximation employed in those approaches seems to lead to inaccurate predicted results in some circumstances.

An improvement of the gust response approach has been attempted in this work. Firstly a flexible mathematical model of natural turbulence characteristics has been suggested as a result of reviewing recent works. Formulae suggested include various parameters which allow the height dependence of power spectral density and co-coherence of the longitudinal turbulence component to be taken into account. For mathematical convenience the height dependence of wind characteristics has been expressed in terms of a power law profile. These height dependent expressions with appropriate parameters have been incorporated with a method for gust response prediction.

Secondly the dynamic force coefficient concept has been employed to improve the conventional stochastic prediction theory for gust response. The coefficient has been evaluated experimentally by using a two-dimensional single degree of freedom system model in a partial boundary layer wind tunnel. Experimental results for the static drag coefficient showed some relevance to previous works. Experimental results for the dynamic along-wind force coefficient have been reduced into

an empirical form, with the section aspect ratio and the reduced wind speed as variables, in terms of its ratio to the static drag coefficient.

A computer program for the gust response prediction has been developed and convenient chart diagrams have been presented for practical applications. Effects of the variation of parameters used in the wind characteristics' model have been examined numerically and the significant role of the dynamic along-wind force coefficient in the gust response prediction discussed.

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NOTATION

Principal notations used in the thesis are listed as follows,

A	= dimension of model in y-direction
$A_1, A_2, A_3, \dots$	= longitudinal measurement positions in wind tunnel
B	= width of a structure or model
B	= background excitation factor
C_D	= drag coefficient
C_D	= static drag coefficient
$\tilde{C}_D$	= dynamic along-wind force coefficient
C_L	= lift coefficient
C_M	= mass coefficient
$C_{u_1, u_2}(f)$	= co-coherence function of u_1, u_2
D	= depth of a structure or model (in x-direction)
$E []$	= average operation
E	= gust power factor
$F(t)$	= instantaneous force
$F_n(t)$	= generalised force of the n-th mode
f	= frequency
$\tilde{f}$	= normalised frequency $(= \frac{f \cdot L_1(z)}{\bar{U}(10)})$
f^*	= modified frequency $(= \sqrt{f^2 + \{ \frac{\bar{U}(10)}{k_2 \cdot L_1(z_m)} \}^2})$
f_o	= natural frequency of single degree of freedom (S.D.O.F.) system
f_n	= natural frequency of the n-th mode

G	= gust factor
g	= gravity acceleration ($\approx 9.8 \text{ m/sec}^2$)
H	= height of a structure
i	= indicator of x, y, z or $1, 2, 3, \dots$, or $\sqrt{-1}$
J	= constant defined by D/B for $\tilde{C}_D$
j	= indicator of $1, 2, 3, \dots$
$K_{\frac{1}{6}}(\), K_{\frac{5}{6}}(\)$	= modified Bessel functions of the second kind of order denoted by the subscript
k	= spring constant of S.D.O.F. system
k'_n	= effective spring constant of the n -th mode
k_1, k_2	= constants
$k_y(z_m), k_z(z_m)$	= horizontal and vertical decay constants based on $\bar{U}(z)$
$k_{ly}(z_m), k_{lz}(z_m)$	= horizontal and vertical decay constants based on $\bar{U}(10)$
$k_{hy}(z_m), k_{hz}(z_m)$	= horizontal and vertical decay constants based on $\bar{U}(H)$
L_i	= length scale of turbulence in i -direction where $i = x, y, z$
$L(z)$	= length constant based on $\bar{U}(z)$
$L_1(z)$	= length constant based on $\bar{U}(10)$
$L_h(z)$	= length constant based on $\bar{U}(H)$
$L_0, L_1, L_2, \dots$	= lateral measurement positions in wind tunnel
M	= mass of S.D.O.F. system
M_{T_n}	= total mass of the n -th mode
$m(z)$	= mass of a structure per unit area
N	= number of degree of freedom
$P(t)$	= net wind pressure or local along-wind force per area
$\bar{P}(z)$	= mean of $P(t)$

$p(t)$	= fluctuating component of $P(t)$
p	= peak factor
$q_n(t)$	= generalised co-ordinate of the n -th mode
R_e	= Reynolds number
$R_{u_1 u_2}(f)$	= root-coherence function of u_1, u_2
R	= resonance amplification factor
$R_{F_n}(\tau)$	= auto-correlation coefficient of F_n
$R_u(\tau)$	= auto-correlation coefficient of $u(t)$
$R_{u_1 u_2}(\tau)$	= cross-correlation coefficient of u_1, u_2
r_i	= separation in i -direction where $i = x, y, z$
r	= roughness factor
$S_{F_n}(f)$	= power spectral density of F_n
$S_u(f)$	= power spectral density of $u(t)$
$S_{u_1 u_2}(f)$	= cross spectral density of u_1, u_2
$S_\delta(f)$	= power spectral density of $\delta(t)$
S	= size reduction factor
T	= duration of process or averaging time
t	= time
$U(t)$	= instantaneous velocity or wind speed
$U'(t)$	= relative wind speed ($= U(t) - \dot{\delta}(t)$)
$\bar{U}(z)$	= mean wind speed
$\tilde{U}$	= reduced velocity or wind speed ($= \frac{\bar{U}}{f_o \cdot D}$)
$\tilde{U}_B$	= reduced velocity or wind speed based on B ($= \frac{\bar{U}}{f_o \cdot B}$)
$u(t)$	= fluctuating component of $U(t)$
u_1, u_2	= $u(t)$ at (y_1, z_1) and (y_2, z_2) respectively

$\dot{u}(t)$	= longitudinal wind acceleration
x	= longitudinal co-ordinate (along-wind direction)
y, y_1, y_2	= lateral co-ordinate
z_1, z_2, z_3	= vertical measurement positions in wind tunnel
z, z_1, z_2	= vertical co-ordinate
z_o	= roughness length
z_G	= gradient height
z_m	= geometric mean of z_1, z_2 ($= \sqrt{z_1 z_2}$)
z_r	= reference height (≈ 10 m)
α	= power law exponent of mean wind speed profile
α_D	= power law exponent of k_{ly}, k_{lz} profile
α_L	= power law exponent of L_1 profile
α_μ	= power law exponent of modal shape
β	= power index in $S_u(f)$
$\Gamma()$	= gamma function
γ	= average structural mass density
$\Delta(t)$	= longitudinal instantaneous displacement
$\bar{\Delta}(z)$	= mean displacement
$\delta(t)$	= fluctuating component of $\Delta(t)$
$\dot{\delta}(t)$	= longitudinal velocity of response
$\ddot{\delta}(t)$	= longitudinal acceleration of response
ζ_S	= structural damping ratio (fraction of critical)
ζ_n	= structural damping ratio of the n-th mode
ζ_A	= aerodynamic damping ratio
ζ_T	= total damping ratio ($= \zeta_S + \zeta_A$)
θ	= angle of attack
λ	= geometrical scale factor

$\mu_n(z)$	=	modal shape of the n-th mode
ν	=	kinematic viscosity, effective frequency
ξ	=	reduced frequency ($= \frac{1}{\tilde{U}_B}$)
ρ	=	air mass density ($\approx 1.2 \text{ kg/m}^3$)
σ_u	=	r.m.s. value of $u(t)$
σ_δ	=	r.m.s. value of $\delta(t)$
$\sigma_{\ddot{\delta}}$	=	r.m.s. value of $\ddot{\delta}(t)$
τ	=	lag time
$ \chi(f) ^2$	=	mechanical admittance function

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CHAPTER 1 INTRODUCTORY REVIEW OF GUST RESPONSE APPROACHES

1.1 INTRODUCTION

In the past ten years there has been a growing interest in the dynamic response of structures due to turbulent wind forces throughout the world including Britain and Japan. The 'background' — approaches in the two countries are rather different.

Since Japanese islands are situated in a strong earthquake region, the seismic engineering has been one of the most important aspects as far as the structural design of tall buildings is concerned. All highrise buildings have been designed against strong earthquake motions. The wind effects have not been considered to be the most significant factor in determining the main structure of buildings. The design forces of the earthquake have always been considered to be much greater than the extreme wind forces until the late 1960's when the super-highrise type of building appeared.

The earthquake motion has its strong power in a high frequency range, in most cases higher than 1 Hz, while the natural wind turbulence has its significant power in a very low frequency range compared with that of the earthquake. The higher the building becomes, the lower is the natural frequency or the fundamental frequency. Moreover recent industrial developments help buildings to be lighter by using light gross density materials. These tendencies are advantageous for aseismic design but not for wind resistance. When a building was designed against severe earthquake, it had been considered to be

strong enough against the extreme wind forces. Even a very conservative way of estimation of wind forces had been quite sufficient in Japan until the late 1960's. However, nowadays for buildings that have the height of 200m or more, the wind loads appear to be greater than seismic load even in the strong earthquake region. The critical wind speed for the galloping type oscillation could be near the same order of the design wind speed for a super-highrise building. The oscillation due to Karman vortex shedding which is commonly taken into account for tall chimney design must be investigated also for certain low natural frequency buildings.

At the same time when demands for more precise wind response analysis of tall buildings grew in Japan, a prediction method for the wind response based on a stochastic treatment was proposed and developed by Davenport in 1967. Since then there have been several modifications and improvements suggested both theoretically and empirically. Quite a few environmental or atmospheric measurements have been achieved and also many actual measurements of building response in strong winds have been reported. Most of these reports have been collected and discussed in the past four international symposia on wind effects on buildings and structures and other symposia. And yet for practical purposes an enormous number of natural wind measurements with a standard equipment system are still required to establish a general expression for the natural wind characteristics. Also numerous actual measurements of building response are needed to confirm the empirical or theoretical prediction methods because of the

large number of parameters involved and the random nature of the wind itself.

The dynamic response of tall building in strong wind cannot be explained sufficiently, although many trials and modifications of gust response factors have been carried out by pioneers referred to in this chapter. In the following sections the development of the gust response approach in the last fifteen years is reviewed and some areas of difficulties are highlighted. The aims of this thesis are also set out.

1.2 STOCHASTIC APPROACH FOR DYNAMIC RESPONSE OF TALL BUILDINGS TO WIND EXCITATION

1.2.1 General Review

Wind loading has been considered in the design procedure of buildings and structures since the end of last century. One of the most common references is probably due to Sherlock⁽¹⁾, who introduced the concept of 'gust factor' in 1947. Since then a reasonable instantaneous velocity and therefore a reasonable maximum pressure or force as well as the mean velocity and pressure gradually have been considered. However, it took another few decades for wind dynamic effects on structures to be taken into account.

Traditional design procedures for wind forces appear to cause an overestimation particularly for large-scale wind exposed structures such as skyscrapers, long-span bridges, tall masts

and chimneys and so on because of the assumption of a constant gust factor irrespective of geometrical and structural properties. For example there are still no specified design criteria in the British Code of Practice⁽²⁾ for wind loads relating to dynamic response caused by turbulent wind or Karman vortex shedding.

In 1961, despite the paucity of basic information about natural wind characteristics, a situation which has been improved recently, Davenport^(3,4) proposed a statistical approach to predict the wind dynamic response of structures and developed a general prediction procedure in 1967⁽⁵⁾ based on his generalised expressions for wind characteristics⁽⁶⁾. This was one of the greatest achievements in the wind engineering field at that stage and enabled structural engineers to develop wind-resistant design in a more rational and economical way. The method in a simplified form is still current, although there have been some inaccurate simplifications which required slight modifications at later stages.

Quite a few stochastic concepts were employed to deal with the dynamic response of structures by means of spectral density distributions. The wind turbulence was treated not only in terms of the turbulence intensity but also the power spectral density, namely the frequency component of power of longitudinal turbulence, and the root-coherence or the space correlation factor in the frequency domain, which permit the dynamic properties of structures such as the natural frequency and the damping ratio, and the physical size of structures to be taken into account.

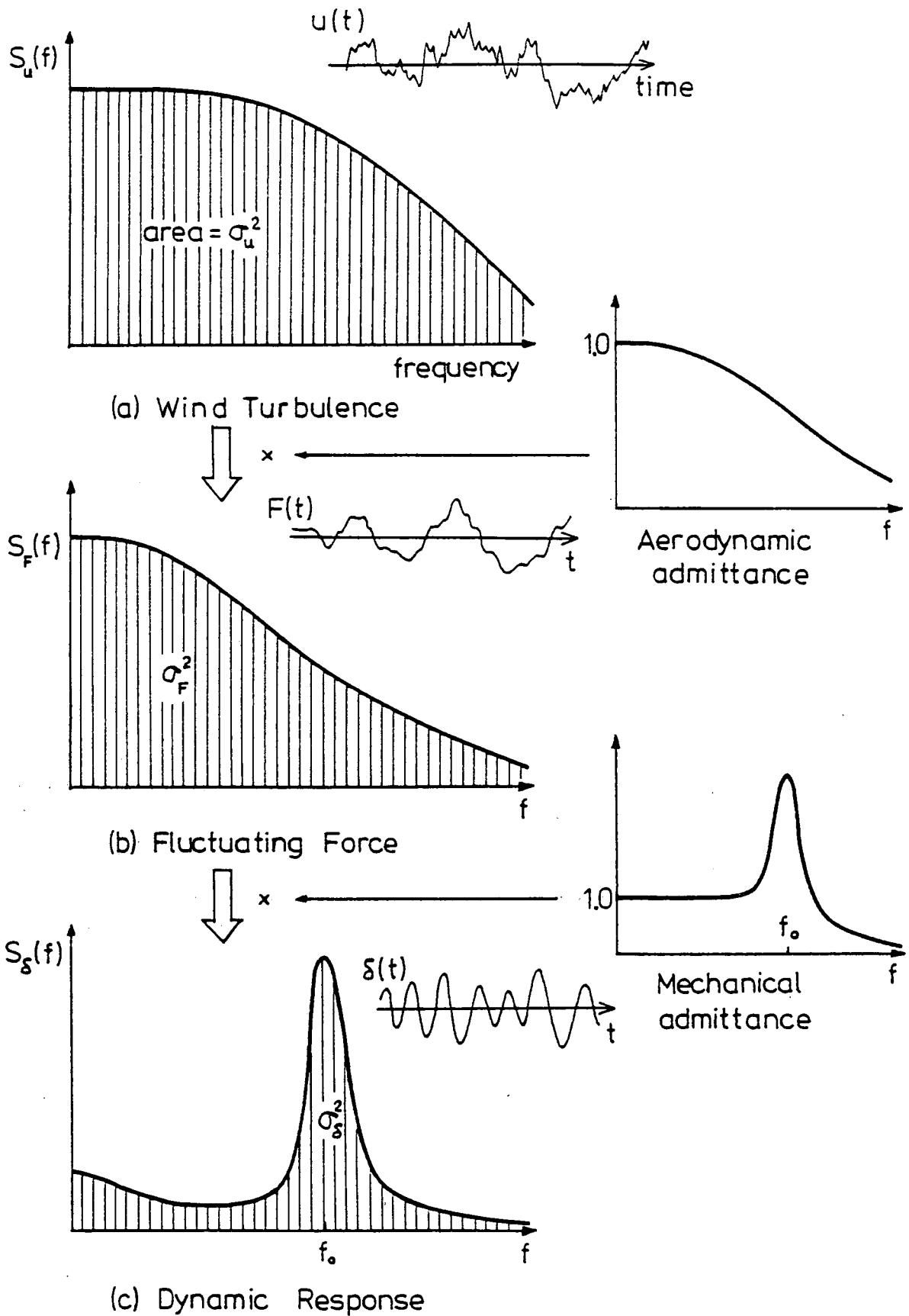


Figure 1-1 Schematic concept of dynamic wind response

The schematic concept is shown in Figure 1-1 in terms of spectral expressions as a stochastic treatment in the frequency domain. $S_u(f)$, $S_F(f)$ and $S_\delta(f)$ are the power spectral density of wind turbulence, dynamic force and response of a structure respectively. The force spectrum can be expressed as the product of wind spectrum and aerodynamic admittance. This process is conceptually analogous to the dynamic response process of random vibration⁽⁷⁾, ie, the spectral density of response can be obtained as the product of force spectrum and mechanical admittance.

In most cases structures were considered to be single degree of freedom (S.D.O.F.) systems as shown in Figure 1-1. This assumption though not totally correct is adequate to some extent for tall buildings as confirmed by recent measurements^(8,9,10).

Davenport also suggested⁽³⁾ that the value of drag coefficient could vary with the reduced frequency or the reduced wind speed. The drag coefficient contributing to the fluctuating force will be called the dynamic drag coefficient in this work to distinguish it from the ordinary one which will be called the static or mean drag coefficient for convenience. A general expression for the time variant resistance force, $F(t)$, acting on an object immersed in a fluctuating flow can be expressed as^(11,12),

$$F(t) = \frac{1}{2} C_D \cdot \rho \cdot B \cdot U(t) |U(t)| + C_M \cdot \rho \cdot A_0 \frac{dU(t)}{dt} \quad (1-1)$$

where $F(t)$ is the force per unit length,

$U(t)$ is the reference fluid velocity,

ρ is the fluid density,

B is the reference dimension of the object,

A_0 is the reference area of the object ($= \frac{\pi B^2}{4}$),

C_D is the drag coefficient, and

C_M is the mass coefficient.

Generally it can be expected for a type of flow with periodic fluctuation that both the drag and mass coefficients can be expressed as a function of reduced velocity, $\tilde{U} = U_{\text{ref}}/fB$, where f is the frequency of the fluid fluctuation, and U_{ref} is the reference velocity.

Keulegan and Carpenter⁽¹²⁾ presented some results of those coefficients for flat plates and circular cylinders and the former are shown in Figure 1-2, and Davenport's results⁽¹³⁾ in Figure 1-3. Keulegan and Carpenter used a standing wave to produce sinusoidal currents with a velocity amplitude, U_m , which was used as U_{ref} , while Davenport used a flow consisting of a mean velocity, $\bar{U}$, which was U_{ref} , and superimposed on it a sinusoidal fluctuation.

Since the basic condition of flow was entirely different, a direct comparison between the two sets of data does not seem to be possible. However, from Figures 1-2 and 1-3, one could expect that the drag and the mass coefficients for a fluctuating flow would not be constant but depend on the reduced velocity.

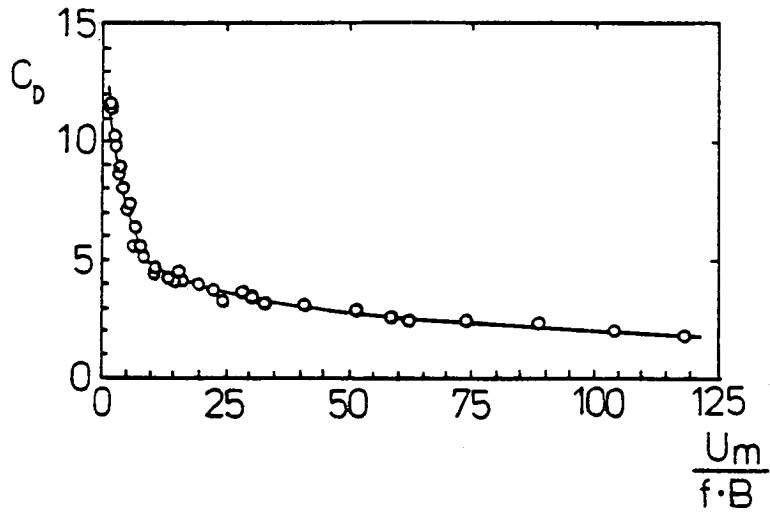


Figure 1-2(a) Variation of drag coefficient of plates after Keulegan and Carpenter⁽¹²⁾

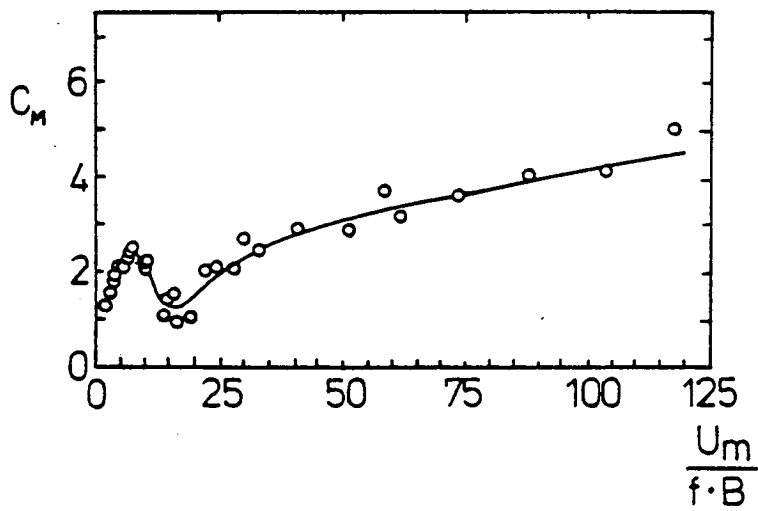


Figure 1-2(b) Variation of mass coefficient of plates after Keulegan and Carpenter⁽¹²⁾

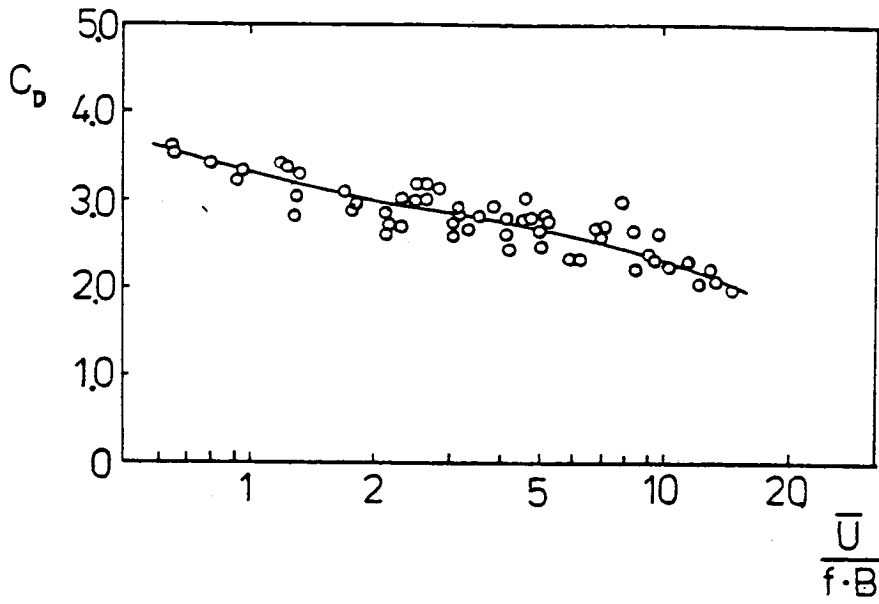


Figure 1-3(a) Variation of drag coefficient of plates after Davenport⁽¹³⁾

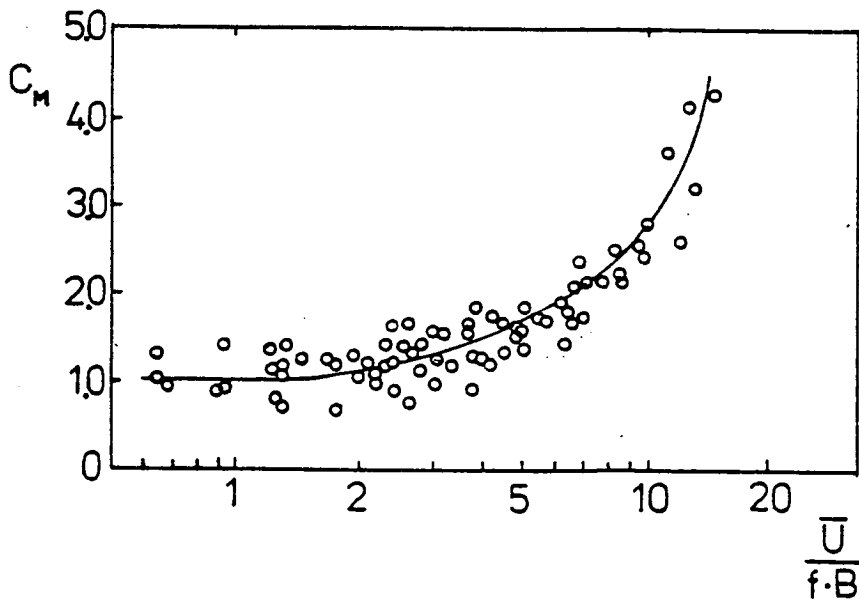


Figure 1-3(b) Variation of mass coefficient of plates after Davenport⁽¹³⁾

This evaluation of the dynamic drag and mass coefficients, of course, may not be directly applicable in the stochastic approach, since the natural wind turbulence is not a sinusoidal wave but a random fluctuation consisting of widely spread frequency components. Davenport, therefore, simply used the static value of the drag coefficient in his approach and neglected the mass coefficient effect assuming its role to be less significant.

Vellozi and Cohen⁽¹⁴⁾ also proposed a similar method to predict the dynamic response of structures in 1968. Their procedure is an interesting one drawing attention to the along-wind correlation which was not taken into account in Davenport's approach. However, the estimation of the aerodynamic admittance or the joint acceptance was inadequately made as pointed out by Vickery⁽¹⁵⁾ and Simiu⁽¹⁶⁾. In other words, the correlation between the windward and the leeward pressure was applied to have an effect on the correlation of the windward face pressure itself, and this led to a significant underestimation of the gust response factor.

Both Davenport, and Vellozi and Cohen used a simplification of the integral of root-coherence function, in order to evaluate the space correlation effect in the frequency domain according to Diedrich⁽¹⁷⁾, who applied the simplified integral in estimating the dynamic force on aeroplanes. This simplification would be quite reasonable for a line-like structure and even beneficial from the view point of applicability, since the integration is performed analytically without the aid of numerical computation,

but if this simplification is applied to structures with large surfaces there would be a significant error up to almost 20% according to Vickery⁽¹⁵⁾, which can be simply avoided by a numerical integral.

In 1970 Vickery⁽¹⁸⁾ himself proposed a gust factor approach following Davenport's one and made numerical comparisons with previous approaches. He made some modification to the root-coherence expression considering the height dependence with slightly different decay constants for the root-coherence function from Davenport's values.

The foregoing gust response approaches have been criticised by Simiu^(19,20) who has suggested the use of the logarithmic law for the mean wind speed profile rather than the power law profile which has been more commonly used and also to allow the power spectral expression to vary with height⁽²¹⁾. Also he gave some consideration to the decay constants for the root-coherence referring to data by Newberry et al⁽²²⁾.

The most interesting and important point that he made was that the along-wind pressure correlation should be taken into account and its value was surprisingly low, eg, between 0.2 and zero. In other words it could be stated that the dynamic drag coefficient should be significantly smaller than the static or mean drag coefficient because the drag coefficient is the integral of the pressure coefficient in the along-wind direction with respect to the surface of an object and the simple summation of the mean windward

and leeward pressure determines the mean drag force whereas the square root of the sum of the mean squares of windward and leeward pressure is the significant factor in establishing the dynamic drag force in cases of very low correlation between the pressures on these faces. Therefore if the same drag coefficient was used for the static and dynamic response as in the previous approaches, a significant overestimation would be inevitable. Some examples given in Simiu's study^(16,20) illustrate this point clearly.

Considerable progress has been achieved since Davenport's first proposal of the application of a stochastic approach, particularly as far as the interpretation of the natural wind characteristics is concerned simply because more information becomes available as time progresses. During the same period theoretical studies of wind response predictions in addition to the ones mentioned above have been made by several authors such as Harris (1963)⁽²³⁾, Etkin (1966)⁽²⁴⁾, Wyatt (1970)⁽²⁵⁾, MacDonald and Morgan (1971)⁽²⁶⁾, and Solnes and Sigbjornsson (1973)⁽²⁷⁾, all of whom used stochastic principles in one form or another.

However, the analysis of turbulent flow around a bluff body such as a cylindrical body with a rectilinear section is still a difficult problem especially in practical design situations, although some numerical simulations^(28,29) have been achieved within certain limits. Another purely theoretical approach has

been developed by Hunt^(30,31) to determine the turbulence distortion mechanism around a bluff body, and this, supported by experimental investigation by Bearman⁽³²⁾, could aid response prediction.

1.2.2 Categorisation of Existing and Proposed Gust Response Approaches

Since wind turbulence is a random phenomenon, the stochastic approach could be a most useful method for predicting the wind response of a bluff structure like a tall building whose most significant vibration appears to be the buffeting type rather than the Aeolian or galloping vibration. Therefore the approach in this thesis follows basically those gust response methods reviewed in the previous section, although a more precise interpretation of wind characteristics is intended.

The dynamic wind response of a structure can be expressed as a function of mainly four factors as follows,

$$\frac{\sigma_{\delta}}{\bar{\Delta}} = f(\mathcal{M}, \mathcal{R}, \mathcal{S}, \mathcal{Z}) \quad (1-2)$$

where $\bar{\Delta}$ is the mean response of structure,

σ_{δ} is the standard deviation of dynamic response,

$\mathcal{M}$ represents the structural dynamic characteristics,

$\mathcal{R}$ represents the normalised cross-spectral density of turbulence or root-coherence,

$\mathcal{S}$ represents the power spectral density of turbulence and turbulence intensity, and

$\mathcal{Z}$ represents the drag coefficient or pressure coefficients.

These four main factors have been taken into account by most stochastic approaches, although there might be some slight discrepancies among them. On the other hand numerous fully simulated model experiments have been carried out^(33,34,35) to predict the wind response of individual tall buildings following a theoretical approach. It is interesting to compare the wind response prediction approaches with respect to the treatment of these four factors. This comparison is shown in Table 1-1, indicating the relative position of the approach in this thesis to previous works.

Approach I is the gust response factor approach reviewed in the previous section. The most important problem at the moment is how to evaluate the correlation between windward and leeward face pressure on a body; an aspect which had been neglected until Simiu's suggestion. However, even if the along-wind correlation is evaluated, there remains a question about the leeward face pressure itself. Would it be the same value for the static and dynamic response? Would it be established for the various reduced wind speeds or the turbulence characteristics? The mass coefficient, C_M , is also neglected in this approach. Although the mass coefficient effects might be less significant in some circumstances, its significance should be examined quantitatively.

The entirely opposite method of attack is by means of a fully simulated model, approach IV. If perfect simulation is achieved, this could produce the best solution for the wind response

Table 1-1 Comparison of Approaches for Wind Response Prediction

APPROACH		I	II	III	IV
THEORETICAL OR EXPERIMENTAL		THEORETICAL	SEMI- THEORETICAL	SEMI- EXPERIMENTAL	EXPERIMENTAL
BASIC CONDITION		Same drag or pressure coefficient as the static value	Dynamic drag coefficient to be evaluated	Geometrically simulated model	Fully simulated model
FACTORS	M	T	T	T	M
	R	T	T	M	M
	S	T	T	M	M
	T	T	M	M	M
REFERENCES		Davenport ⁽⁵⁾ Vellozi and Cohen (14) Vickery ⁽¹⁸⁾ Simiu ⁽²⁰⁾	Solnes and Sigbjornsson (27) Cooper and Surry (41) <u>This thesis</u>	Vickery ⁽³⁸⁾ Ellis ⁽³⁶⁾ Saunders and Melbourne (37)	Davenport and Isyumov(33) Kato and Kanda (34) Fujimoto et al (35)

note : T shows that the factor is obtained basically from an established theory with some simplification.

M shows that the factor is to be obtained basically from a wind tunnel experiment.

prediction. In the case of a tall building or a structure of peculiar shape this approach could be essential prior to a stochastic analysis. However, the disadvantage of this method is that the result is rather restricted to the situation of the model and wind characteristics which are reproduced in the wind tunnel. Also it could be expensive and even difficult to reproduce an exactly simulated turbulence and dynamic model, although the fundamental rectilinear mode might be sufficient to represent the dynamic response mode for most highrise buildings.

In order to overcome this inefficiency a geometrically simulated model technique (approach III) has been proposed to predict the wind response by Ellis⁽³⁶⁾ and Saunders and Melbourne⁽³⁷⁾ in 1975; the work of the latter concentrated on the cross-wind oscillation. A similar approach was suggested by Vickery⁽³⁸⁾ in 1968. If the natural wind characteristics are reproduced in a wind tunnel, the normalised fluctuating force can be obtained by a specially designed transducer which has a geometrically simulated shape. Since the structural dynamics are well established to predict the response^(7,39), the actual response can be computed from the normalised fluctuating force. This concept was recommended also by Holmes⁽⁴⁰⁾. The difficulty of exact representation of the natural turbulence as in approach IV still remains here. Although the result is meant to be applicable to the different dynamic characteristics of structures, it is still restricted to the same geometrical shape of the model which usually tends to give a similar dynamic characteristic. There might not be as much difference between approaches III and IV in

terms of efficiency or applicability as would be initially expected, although approach III concentrated more on the generalised or normalised force rather than the response itself.

A semi-theoretical treatment, approach II, is put forward in this work bearing similarity with that of Solnes and Sigbjornsson⁽²⁷⁾ in 1973 and another one by Cooper and Surry⁽⁴¹⁾ in 1975; the latter is for the circular cylinder and only the possibility of evaluation of the dynamic drag coefficient was suggested in both cases.

The pressure coefficients or static drag coefficients used in approach I were empirically established values. However, since it has been revealed by McLaren, Sherratt and Morton⁽⁴²⁾ in 1969 and Lee^(43,44) in 1975 that both the turbulence intensity and scale have significant effects on the static drag coefficients, those effects on the dynamic drag coefficient also should be investigated. It would be more useful and convenient to evaluate the dynamic drag coefficient rather than to evaluate only the along-wind pressure correlation because the dynamic drag coefficient can be established as a function of reduced wind speed empirically so that it would permit the effect of Karman vortex shedding in turbulent flow to be taken into account and even the mass coefficient effect as well. Obviously the vortex shedding effect is more significant on the fluctuating lift forces, nevertheless, those effects on the along-wind response should not be ignored even in a highly turbulent flow. The mass coefficient is an independent factor from the dynamic drag coefficient but if

both coefficients are expressed as a function of the reduced frequency, ξ , (inverse of the reduced wind speed $\tilde{U}$), an equivalent combined dynamic drag coefficient, $\tilde{C}_D(\xi)$, which will be called the dynamic along-wind force coefficient in this work, can be defined to represent the two coefficients, for example as suggested by Davenport⁽³⁾,

$$\tilde{C}_D(\xi) = \sqrt{C_D^2(\xi) + \frac{\pi \xi^2}{4} C_M^2(\xi)} \quad (1-3)$$

As stated by Vickery⁽¹⁵⁾, "the most difficult problem encountered in the prediction of the response of a large bluff structure to atmospheric turbulence has been that of determining the aerodynamic admittance function" which is sometimes called 'correlation function'⁽¹⁴⁾ or 'drag (lift) describing function'⁽⁴¹⁾. When this 'aerodynamic admittance' was measured experimentally it usually included both major contributing factors, namely, (i) the space correlation of on-coming fluctuating components of turbulence and (ii) the reduced frequency dependence of dynamic force coefficients. However, the former is a part of the wind characteristics and the latter is a general problem of evaluation of force coefficients and so it can be suggested that the two contributing factors should be treated and investigated separately. Works concentrated on the latter contributor, ie, combined effects of the mass coefficient and along-wind pressure correlation, have never been carried out separately from the space correlation effect of on-coming wind, so far as the author knows. Those effects could

be investigated and evaluated by using a relatively simple two-dimensional model.

Approach II is necessitated from the view point mentioned above, the importance of which has been either neglected or underestimated in approach I. Furthermore experiments in approach II can be made in a more efficient way than those of approaches III and IV since the variation of the dynamic along-wind force coefficient with reduced wind speed can be established for various possible parameters whereas the generalised force spectra in approach III are only applicable to individual cases and seem difficult to be evaluated in a standard form.

1.3 OBJECTIVES

As noted in the previous sections, in order to improve the prediction of wind induced response of structures it would be most desirable at the present moment to investigate the relation between a turbulent wind and a fluctuating wind force experimentally in terms of the dynamic along-wind force coefficient by choosing a sufficient number of parameters involved in that interaction, then to compute the stochastic response by applying the most recent information regarding the turbulent wind characteristics and the dynamic properties of individual structures.

The basic concepts and objectives of this thesis are outlined simply as follows,

- (i) a theoretical analysis for the prediction of dynamic response of tall bluff structures in strong winds is developed with an emphasis on the dynamic along-wind force coefficient;
- (ii) the turbulence characteristics of the natural wind are considered so as to establish more general and adequate expressions as a mathematical model for further applications;
- (iii) the dynamic along-wind force coefficient for bluff bodies is evaluated from a wind tunnel experiment by using a two-dimensional S.D.O.F. system model;
- (iv) a computer program to predict the along-wind response of tall bluff structures is developed in order to examine the effects of several parameters on the response and also to provide some charts of those parameters for practical applications.

CHAPTER 2 WIND RESPONSE OF BLUFF STRUCTURES

2.1 INTRODUCTION

The dynamic response of a slender structure exposed to an atmospheric turbulent wind has been treated as a statistical structural response problem by a number of investigators as reviewed in the previous chapter.

The theoretical analysis developed here basically follows an approach described by Etkin⁽²⁴⁾, which deals with the along-wind response of a vertical 'line-like' structure. In this work the approach is applied to the three-dimensional bluff structure and the reduced frequency dependence of the along-wind force coefficient is emphasised. A simplified result is compared with an energy method proposed by MacDonald and Morgan⁽²⁶⁾ and developed by Royles and Das (1974)⁽⁴⁶⁾.

The cross-wind motion involves more complex problems since it generally consists of three different causes; (i) fluctuating force associated with the lateral components of turbulence, (ii) vortex shedding which will be dominant in a resonance condition, (iii) aerodynamic negative damping which leads to galloping or self-exciting oscillations when the mean velocity exceeds the critical velocity.

A treatment similar to the along-wind response is investigated to describe the cross-wind response taking all three factors into account for cases in which the mean velocity is less than the critical one. In other words, the galloping oscillation itself is

excluded, although the linear approximation for the negative damping is considered, since it seems still unlikely that a design wind speed would exceed the critical wind speed of galloping oscillation for tall buildings^(37,47,48).

The approach can be applied also to the case of a general angle of attack yielding a combination of along-wind and cross-wind response.

2.2 DYNAMIC ALONG-WIND RESPONSE

2.2.1 Preliminaries

Firstly the relationship between the fluctuating drag force and the longitudinal turbulent component of the natural wind is discussed. As a basic assumption the stochastic process of this problem is postulated to be stationary.

The state of wind-structure interaction and a three-dimensional body (vertical cantilever-like structure) is shown in Figure 2-1. $\bar{U}(z)$ is the mean wind speed profile and $u(y, z, t)$ is the longitudinal fluctuating component. The structure has a multi-degree-of-freedom system of flexure or sway in the x-direction but the torsional motion is assumed to be small. $\bar{\Delta}(z)$ is the mean deflection of the structure associated with $\bar{U}(z)$ and $\delta(y, z, t)$ is the fluctuating motion associated with $u(y, z, t)$. If the torsional motion is neglected, $\delta(y, z, t) = \delta(z, t)$.

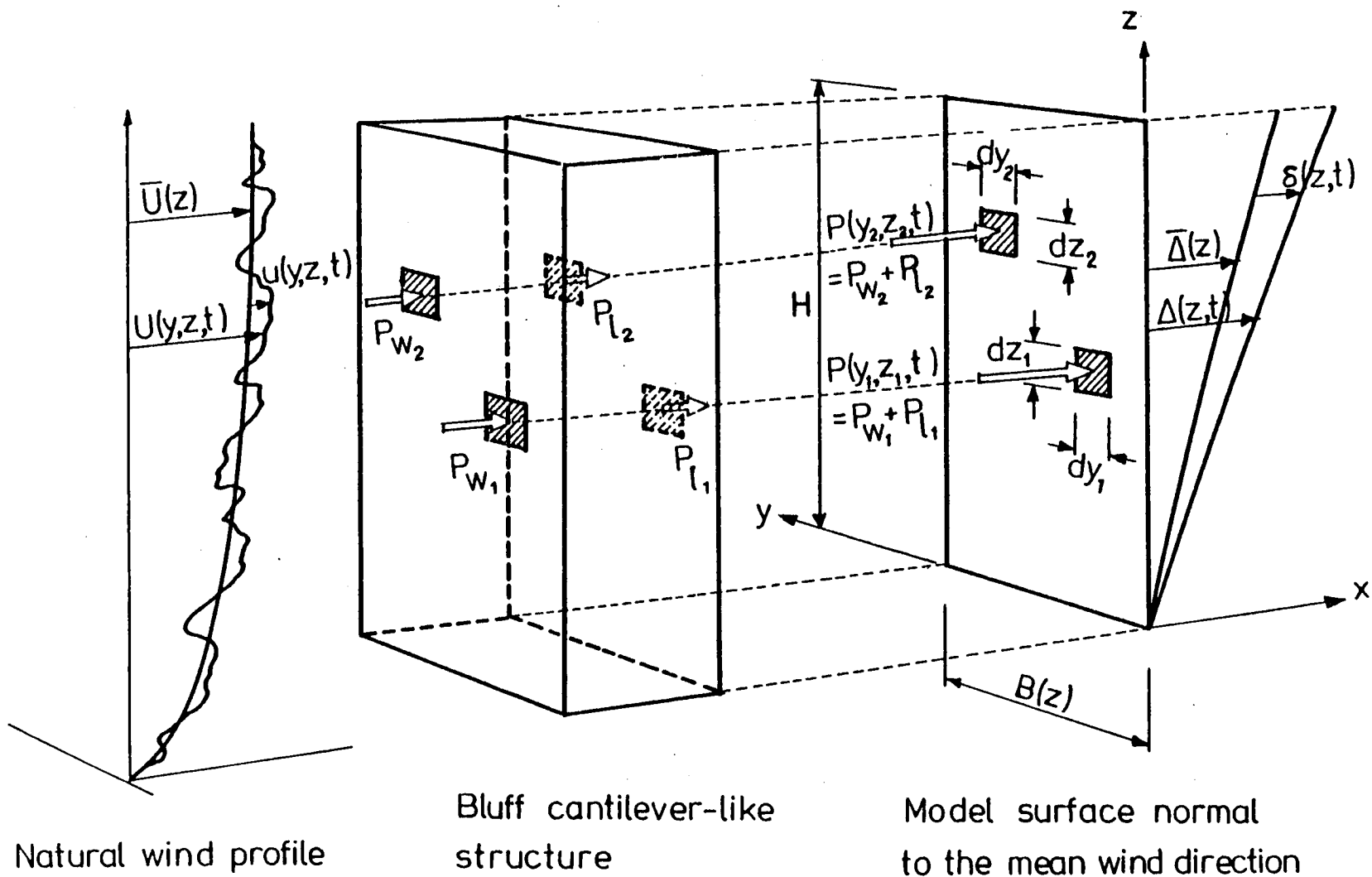


Figure 2-1 Schematic of wind-structure interaction

/ due to

The local instantaneous force on a structure is the net pressure which is the difference between pressures on the windward and leeward surface of the structure, $P(y, z, t)$, of which $\bar{P}(z)$ is the mean and $p(y, z, t)$ is the fluctuating part. Namely,

$$U(y, z, t) = \bar{U}(z) + u(y, z, t) \quad (2-1(a))$$

$$P(y, z, t) = \bar{P}(y, z) + p(y, z, t) \quad (2-1(b))$$

$$\Delta(z, t) = \bar{\Delta}(z) + \delta(z, t) \quad (2-1(c))$$

and the 'relative wind speed' is

$$U'(y, z, t) = U(y, z, t) - \dot{\delta}(y, z, t) \quad (2-2)$$

The assumption of a 'strip theory' relationship between the local drag and the local relative velocity for a two-dimensional body (see equation (1-1)) can be applied also to a three-dimensional body, by replacing the local drag with the net pressure, which can be considered to act through the structure on the idealised surface normal to the mean wind direction as shown in Figure 2-1, ie,

$$P(y, z, t) = \frac{1}{2} \rho C_D(y, z, \xi) U'^2(y, z, t) + \rho B(z) C_M(y, z, \xi) \dot{U}'(y, z, t)$$

$$(2-3)$$

where C_D is the drag coefficient, C_M is the mass coefficient;

$$\xi = \frac{f \cdot B(z)}{\bar{U}(z)}, \text{ the reduced frequency;}$$

$B(z)$ is the width of a structure; and

ρ is the air mass density.

Both C_D and C_M are considered to vary with position (y, z) and reduced frequency ξ . In the two-dimensional strip theory $B(z)$ is usually taken as $\frac{\pi}{4} B$ where B is the reference dimension of the object⁽³⁾ to give a reasonable order of the value C_M . But such a consideration may not be necessary in this work and so $B(z)$ is taken as the dimension of the body normal to the wind direction.

2.2.2 Fluctuating Load

It follows from equations (2-2) and (2-3) that

$$\begin{aligned} P(y, z, t) = & \frac{1}{2} \rho C_D(y, z, \xi) [U^2 - 2U \dot{\delta} + \dot{\delta}^2] \\ & + \rho B(z) C_M(y, z, \xi) (\dot{U} - \ddot{\delta}) \end{aligned} \quad (2-4)$$

Taking the time average of equation (2-4),

$$\begin{aligned} \bar{P}(y, z) = & \frac{1}{2} \rho C_{D_0}(y, z) (\overline{U^2} - 2\overline{U\dot{\delta}} + \overline{\dot{\delta}^2}) \\ & + \rho B(z) C_M(y, z) (\bar{\dot{U}} - \bar{\ddot{\delta}}) \end{aligned} \quad (2-5)$$

where C_{D_0} is the static or mean drag coefficient.

Since $\overline{\dot{U}}$ and $\overline{\ddot{\delta}}$ are both zero and

$$\overline{U\dot{\delta}} = \overline{(\overline{U} + u)\dot{\delta}} = \overline{\overline{U}\dot{\delta}} + \overline{u\dot{\delta}} = \overline{u\dot{\delta}}$$

with $\overline{U^2} = \overline{(\overline{U} + u)^2} = \overline{U^2} + 2\overline{u\overline{U}} + \overline{u^2} = \overline{U^2} + \overline{u^2}$

then $\overline{P}(y, z) = \frac{1}{2}\rho C_{D_0}(y, z)(\overline{U^2} - 2\overline{u\dot{\delta}} + \overline{u^2} + \overline{\dot{\delta}^2})$ (2-6)

Subtracting equation (2-6) from (2-4), the fluctuating part of the load can be obtained as,

$$\begin{aligned} p(y, z, t) = & \rho C_D(y, z, \xi) \overline{U} u + \rho B(z) C_M(y, z, \xi) \dot{u} \\ & - \rho C_D(y, z, \xi) \overline{U} \dot{\delta} - \rho B(z) C_M(y, z, \xi) \ddot{\delta} \\ & + \frac{1}{2}\rho C_D(y, z, \xi)(\dot{\delta}^2 - 2u\dot{\delta} + u^2) \\ & - \frac{1}{2}\rho C_{D_0}(y, z)(\overline{\dot{\delta}^2} - 2\overline{u\dot{\delta}} + \overline{u^2}) \end{aligned} \quad (2-7)$$

If the turbulence and the fluctuating motion of the structure are both small compared with $\overline{U}$, the second order terms in u and $\dot{\delta}$ can be neglected. Then equations (2-6) and (2-7) become as follows respectively,

$$\overline{P}(y, z) = \frac{1}{2}\rho C_{D_0}(y, z) \overline{U}^2 \quad (2-8)$$

$$\begin{aligned} p(y, z, t) = & \rho C_D(y, z, \xi) \overline{U} \cdot u + \rho B(z) C_M(y, z, \xi) \dot{u} \\ & - \rho C_D(y, z, \xi) \overline{U} \cdot \dot{\delta} - \rho B(z) C_M(y, z, \xi) \ddot{\delta} \end{aligned}$$

2.2.3 Static Deflection

The mean deflection of a structure of N degrees of freedom can be expressed by a matrix form as,

$$\bar{\Delta}(z_i) = \sum_{j=1}^N \alpha_{ij} \bar{\Phi}_j \quad (2-10)$$

where α_{ij} is an ith row element of the inverse of the stiffness matrix K,

$\bar{\Phi}_j$ is the total static force applied on the j-th position (or floor for a building) and

$$\bar{\Phi}_j = \int_{\frac{z_{j-1}+z_j}{2}}^{\frac{z_j+z_{j+1}}{2}} \int_0^{B(z)} \bar{P}(z, y) dy dz \quad (2-11)$$

where z_j indicates the reference height at the j-th position of structure and at the extremities, $z_0 = 0$, $z_N = z_{N+1} = H$.

Substituting equation (2-8) into (2-11),

$$\bar{\Phi}_j = \int_{\frac{z_{j-1}+z_j}{2}}^{\frac{z_j+z_{j+1}}{2}} \int_0^{B(z)} \frac{1}{2} \rho C_{D_0}(y, z) \bar{U}^2 dy dz \quad (2-12)$$

Assuming ρ , $B(z)$ and C_{D_o} are constant with y and z ,

$$\bar{\Phi}_j = \frac{1}{2} \rho C_{D_o} B \int_{\frac{z_{j-1}+z_j}{2}}^{\frac{z_j+z_{j+1}}{2}} \bar{U}^2(z) dz \quad (2-13)$$

It may not be quite correct to assume that C_{D_o} is constant with position. However it is possible to determine appropriate values of C_{D_o} in equation (2-13) which represent the value of $C_{D_o}(y, z)$ in equation (2-12) as an averaged value from empirical measurements.

2.2.4 Dynamic Deflection

The last two terms of equation (2-9) are not dependent on the turbulence but on the structural response. Consequently they are considered as the additional damping and mass in the equation of motion of a structure. Then the dynamic part of the along-wind net pressure $p'(y, z, t)$ for the force contribution can be written as,

$$p'(y, z, t) = \rho C_{D_o}(y, z, \xi) u \cdot \bar{U} + \rho B(z) C_M(y, z, \xi) \dot{u} \quad (2-14)$$

The dynamic displacement response $\delta(z, t)$ can be treated as a random function made up of components from the various independent modes of vibration⁽³⁹⁾. Then the equation of motion for the n -th mode, taking the aerodynamic damping and

additional mass terms mentioned above into account, can be written as follows :

$$\ddot{q}_n(t) + 2\zeta_{T_n}(2\pi f_n)\dot{q}_n(t) + (2\pi f_n)^2 q_n(t) = \frac{F_n(t)}{M_{T_n}} \quad (2-15)$$

where

q_n is the generalised displacement of the n-th mode;

$$\delta(z,t) = \sum_{n=1}^N \mu_n(z) \cdot q_n(t) ;$$

$\mu_n(z)$ is the n-th mode shape;

f_n is the n-th natural frequency;

$$M_{T_n} = \int_0^H \int_0^B \{m(z) + \rho B(z) C_M(y,z,\xi)\} \mu_n^2(z) dy dz ; \quad (2-16)$$

$m(z)$ is the mass of structure per unit surface area;

$\zeta_{T_n} = \zeta_n + \zeta_{A_n}$, total damping ratio;

ζ_n is the damping ratio of the structure of the n-th mode;

$$\zeta_{A_n} = \int_0^H \int_0^B \frac{\rho C_D(y,z,\xi) \bar{U}(z) \mu_n^2(z)}{4\pi f_n M_{T_n}} dy dz ; \quad \text{and} \quad (2-17)$$

F_n is the generalised force associated with the turbulence,

$$F_n(t) = \int_0^H \int_0^B p'(y,z,t) \mu_n(z) dy dz \quad (2-18)$$

For the most lightly damped structures the cross-coupling between modes is unlikely. Therefore the power spectral density of the response, $S_\delta(f)$, can be written as follows, as a solution of equation (2-15),

$$S_\delta(f) = \sum_{n=1}^N \mu_n^2(z) |\chi_n(f)|^2 S_{F_n}(f) \quad (2-19)$$

where

$$|\chi_n(f)|^2 = \frac{1}{(4\pi^2 M_{T_n})^2 \{ f^4 + f_n^4 + (4\zeta_{T_n}^2 - 2)f^2 f_n^2 \}}$$

Now the generalised force of the n -th mode $F_n(t)$ can be computed by substituting equation (2-14) into (2-18).

$$F_n(t) = \int_0^H \int_0^B \{ \rho C_D(y, z, \xi) \bar{U} \cdot u + \rho C_M(y, z, \xi) B(z) \dot{u} \} \mu_n(z) dz dy \quad (2-20)$$

In equation (2-20), $F_n(t)$ is expressed as a function of t and ξ , but since for lightly damped structures only the components of response in the narrow band of frequency around resonance in a particular mode will be of significance, therefore only the corresponding values of C_D and C_M need be taken into account. This will be adequate for small ζ_T (eg, $\zeta_T < 2\%$) where if the variation of $C_D(\xi)$ and $C_M(\xi)$ with ξ is not large (eg, less than 10%). If the variation of

$C_D(\xi)$ and $C_M(\xi)$ is large against $C_D(\xi_n)$ and $C_M(\xi_n)$ it can be recommended that $C_{D_{qs}}$ and $C_{M_{qs}}$, which are effective values at a low frequency range, for the quasi-static part should be employed. Then equation (2-20) can be rewritten accordingly.

$$F_n(t) = \int_0^H \int_0^{B(z)} \{C_{D_n}^*(y,z) u + C_{M_n}^*(y,z) \dot{u}\} dz dy \quad (2-21)$$

where

$$C_{D_n}^*(y,z) = \rho C_D(y,z,\xi_n) \bar{U}(z) \mu_n(z); \quad (2-21(a))$$

$$C_{M_n}^*(y,z) = \rho C_M(y,z,\xi_n) B(z) \mu_n(z); \quad \text{and} \quad (2-21(b))$$

$$\xi_n = \frac{f_n \cdot B(z)}{\bar{U}(z)}$$

Now $S_{F_n}(f)$ can be defined as a Fourier transform of the auto-correlation function of $F_n(t)$ as follows,

$$S_{F_n}(f) = 2 \int_{-\infty}^{\infty} R_{F_n}(\tau) e^{-i2\pi f\tau} d\tau \quad (2-22)$$

where

$$R_{F_n}(\tau) = E[F_n(t) \cdot F_n(t + \tau)] \quad (2-23)$$

In the following development $B(z)$ is treated as a constant with height. Substituting equation (2-21) into (2-23),

$$\begin{aligned}
 R_{F_n}(\tau) &= E \left[\int_0^H \int_0^B \{ C_{D_n}^* u(t) + C_{M_n}^* \dot{u}(t) \} dy dz \right. \\
 &\quad \left. \times \int_0^H \int_0^B \{ C_{D_n}^* u(t+\tau) + C_{M_n}^* \dot{u}(t+\tau) \} dy dz \right] \\
 &= E \left[\int_0^H \int_0^B C_{D_n}^* u(t) dy dz \cdot \int_0^H \int_0^B C_{D_n}^* u(t+\tau) dy dz \right. \\
 &\quad + E \left[\int_0^H \int_0^B C_{D_n}^* u(t) dy dz \cdot \int_0^H \int_0^B C_{M_n}^* \dot{u}(t+\tau) dy dz \right] \quad \times \\
 &\quad + E \left[\int_0^H \int_0^B C_{M_n}^* \dot{u}(t) dy dz \cdot \int_0^H \int_0^B C_{D_n}^* u(t+\tau) dy dz \right] \\
 &\quad + E \left[\int_0^H \int_0^B C_{M_n}^* \dot{u}(t) dy dz \cdot \int_0^H \int_0^B C_{M_n}^* \dot{u}(t+\tau) dy dz \right] \cdot
 \end{aligned}
 \tag{2-24}$$

In order to clarify the cross coupling effects between turbulent wind components at different positions in equation (2-24), the variables y, z are replaced by similar variables, y_1, z_1 and y_2, z_2 corresponding to integral expressions involving t and $t + \tau$. Then equation (2-24) can be rearranged by removing terms independent of time out of the time averaging operation $E[\]$.

$$\begin{aligned}
R_{F_n}(\tau) = & \int_0^H \int_0^H \int_0^B \int_0^B \{ C_{D_n}^*(y_1, z_1) C_{D_n}^*(y_2, z_2) E[u_1(t) \cdot u_2(t+\tau)] \\
& + C_{D_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2) E[u_1(t) \cdot \dot{u}_2(t+\tau)] \\
& + C_{M_n}^*(y_1, z_1) C_{D_n}^*(y_2, z_2) E[\dot{u}_1(t) \cdot u_2(t+\tau)] \\
& + C_{M_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2) E[\dot{u}_1(t) \cdot \dot{u}_2(t+\tau)] \} \\
& \times dy_1 dy_2 dz_1 dz_2
\end{aligned} \tag{2-25}$$

where $u_1(t)$, $u_2(t)$ are turbulent velocity components, u , at different positions (y_1, z_1) and (y_2, z_2) respectively.

Since $\frac{d}{dt} u(t+\tau) = \frac{d}{d\tau} u(t+\tau)$, differentials of the cross-correlation function of u_1 and u_2 can be written as follows,

$$R_{u_1 u_2}(\tau) = E[u_1(t) \cdot u_2(t+\tau)] = E[u_1(t-\tau) \cdot u_2(t)] \tag{2-26}$$

$$\begin{aligned}
\frac{d}{d\tau} R_{u_1 u_2}(\tau) &= \frac{d}{d\tau} \{E[u_1(t) \cdot u_2(t+\tau)]\} = E[u_1(t) \cdot \dot{u}_2(t+\tau)] \\
&= \frac{d}{d\tau} \{E[u_1(t-\tau) \cdot u_2(t)]\} = -E[\dot{u}_1(t-\tau) \cdot u_2(t)] \\
&= -E[\dot{u}_1(t) \cdot u_2(t+\tau)]
\end{aligned} \tag{2-27}$$

$$\begin{aligned}
 \frac{d^2}{d\tau^2} R_{u_1, u_2}(\tau) &= \frac{d}{d\tau} \{-E[\dot{u}_1(t) \cdot u_2(t+\tau)]\} \\
 &= -E[\ddot{u}_1(t) \cdot \dot{u}_2(t+\tau)]
 \end{aligned} \tag{2-28}$$

Since the cross-spectrum of turbulent velocities, $S_{u_1, u_2}(f)$, is a Fourier transform of the cross-correlation function, $R_{u_1, u_2}(\tau)$, similar to equation (2-22),

$$S_{u_1, u_2}(f) = 2 \int_{-\infty}^{\infty} R_{u_1, u_2}(\tau) e^{-i2\pi f\tau} d\tau \tag{2-29}$$

where

$$R_{u_1, u_2}(\tau) = \int_0^{\infty} S_{u_1, u_2}(f) e^{i2\pi f\tau} df \tag{2-30}$$

Hence

$$\begin{aligned}
 \frac{d}{d\tau} R_{u_1, u_2}(\tau) &= i2\pi f \int_0^{\infty} S_{u_1, u_2}(f) e^{i2\pi f\tau} df = i2\pi f R_{u_1, u_2}(\tau) \\
 &\tag{2-31}
 \end{aligned}$$

$$\begin{aligned}
 \frac{d^2}{d\tau^2} R_{u_1, u_2}(\tau) &= -4\pi^2 f^2 \int_0^{\infty} S_{u_1, u_2}(f) e^{i2\pi f\tau} df = -4\pi^2 f^2 R_{u_1, u_2}(\tau) \\
 &\tag{2-32}
 \end{aligned}$$

Equating equations (2-27) and (2-28) to (2-31) and (2-32) respectively,

$$\begin{aligned}
 E[u_1(t) \cdot \dot{u}_2(t+\tau)] &= -E[\dot{u}_1(t) \cdot u_2(t+\tau)] = i2\pi f R_{u_1, u_2}(\tau) \\
 &\tag{2-33}
 \end{aligned}$$

$$E[\dot{u}_1(t) \cdot \dot{u}_2(t+\tau)] = 4\pi^2 f^2 R_{u_1 u_2}(\tau) \quad (2-34)$$

Substituting equations (2-26), (2-33) and (2-34) into the integral expression (2-25),

$$\begin{aligned} R_{F_n}(\tau) = & \int_0^H \int_0^H \int_0^B \int_0^B \{C_{D_n}^*(y_1, z_1) C_{D_n}^*(y_2, z_2) \\ & + i2\pi f C_{D_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2) - i2\pi f C_{M_n}^*(y_1, z_1) C_{D_n}^*(y_2, z_2) \\ & + 4\pi^2 f^2 C_{M_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2)\} R_{u_1 u_2}(\tau) \cdot dy_1 dy_2 dz_1 dz_2 \end{aligned} \quad (2-35)$$

Then the power spectral density function of generalised force for the n-th mode can be computed by equation (2-22),

$$\begin{aligned} S_{F_n}(f) = & 2 \int_{-\infty}^{\infty} \int_0^H \int_0^H \int_0^B \int_0^B R_{u_1 u_2}(\tau) \{C_{D_n}^*(y_1, z_1) C_{D_n}^*(y_2, z_2) \\ & + i2\pi f [C_{D_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2) - C_{M_n}^*(y_1, z_1) C_{D_n}^*(y_2, z_2)] \\ & + 4\pi^2 f^2 C_{M_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2)\} dy_1 dy_2 dz_1 dz_2 \\ & \times \int_{-\infty}^{\infty} e^{-i2\pi f \tau} d\tau \end{aligned} \quad (2-36)$$

Since the terms within brackets { } are independent of the integral variable τ , the Fourier integral can be performed for the cross-correlation function as given in equation (2-29), then equation (2-36) can be rearranged as,

$$\begin{aligned}
 S_{F_n}(t) = & \int_0^H \int_0^H \int_0^B \int_0^B S_{u_1, u_2}(f) \{ C_{D_n}^*(y_1, z_1) C_{D_n}^*(y_2, z_2) \\
 & + i2\pi f [C_{D_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2) - C_{M_n}^*(y_1, z_1) C_{D_n}^*(y_2, z_2)] \\
 & + 4\pi^2 f^2 C_{M_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2) \} dy_1 dy_2 dz_1 dz_2
 \end{aligned}
 \tag{2-37}$$

If C_D and C_M are assumed to be constant with y or can be replaced by a representative constant value (a similar assumption was applied to the static C_D giving C_{D_o}) and can have the same profile with z ,

$$C_{D_n}^*(z_1) C_{M_n}^*(z_2) - C_{M_n}^*(z_1) C_{D_n}^*(z_2) = 0 \tag{2-38}$$

Consequently equation (2-37) can be simplified as,

$$\begin{aligned}
 S_{F_n}(f) = & \int_0^H \int_0^H \int_0^B \int_0^B S_{u_1, u_2}(f) \{ C_{D_n}^*(z_1) C_{D_n}^*(z_2) + 4\pi^2 f^2 C_{M_n}^*(z_1) C_{M_n}^*(z_2) \} \\
 & \times dy_1 dy_2 dz_1 dz_2
 \end{aligned}
 \tag{2-39}$$

2.2.4.1 Dynamic Along-Wind Force Coefficient Instead of using two coefficients $C_D(\xi)$ and $C_M(\xi)$ a dynamic along-wind force coefficient $\tilde{C}_D(\xi)$ can be defined as,

$$\tilde{C}_D(\xi) = C_D(\xi) \sqrt{1 + (2\pi f)^2 \frac{B^2 \cdot C_M^2(\xi)}{\bar{U}^2(z) \cdot C_D^2(\xi)}} \quad (2-40)$$

The different constant factor between equations (1-3) and (2-40) is due to the choice of reference dimensions A_0 and B in equation (1-3) and so the value of $C_M(\xi)$ will be different in the two definitions.

Then the definition equations (2-21(a) and (b)) can be rewritten accordingly,

$$\begin{aligned} \tilde{C}_{D_n}^*(z) &= \rho \tilde{C}_D(\xi_n) \bar{U}(z) \mu_n(z) \\ &= \sqrt{\{C_{D_n}^*(z)\}^2 + \{2\pi f C_{M_n}^*(z)\}^2} \end{aligned} \quad (2-41)$$

Note that if the variation of $\tilde{C}_D(\xi)$ is large, a quasi-static value $\tilde{C}_{D_{qs}}^*(z) = \rho \tilde{C}_{D_{qs}} \bar{U}(z) \mu_n(z)$ should be used for the quasi-static part of response, where $\tilde{C}_{D_{qs}}$ can be defined from equation (2-40) in which $C_D(\xi)$ and $C_M(\xi)$ are replaced by quasi-static values $C_{D_{qs}}$ and $C_{M_{qs}}$ respectively.

From the assumption given by equation (2-38),

$$\begin{aligned}
& \{C_{D_n}^*(z_1) \cdot C_{M_n}^*(z_2)\}^2 + \{C_{M_n}^*(z_1) \cdot C_{D_n}^*(z_2)\}^2 \\
& = 2 \cdot \{C_{D_n}^*(z_1) \cdot C_{D_n}^*(z_2) \times C_{M_n}^*(z_1) \cdot C_{M_n}^*(z_2)\} \quad (2-42)
\end{aligned}$$

Hence,

$$\begin{aligned}
& \tilde{C}_{D_n}^*(z_1) \cdot \tilde{C}_{D_n}^*(z_2) \\
& = \sqrt{\{C_{D_n}^*(z_1) C_{D_n}^*(z_2)\}^2 + \{4\pi^2 f^2 C_{M_n}^*(z_1) C_{M_n}^*(z_2)\}^2} \\
& \quad + 4\pi^2 f^2 [\{C_{D_n}^*(z_1) C_{M_n}^*(z_2)\}^2 + \{C_{M_n}^*(z_1) C_{D_n}^*(z_2)\}^2] \\
& = \sqrt{\{C_{D_n}^*(z_1) C_{D_n}^*(z_2)\}^2 + \{4\pi^2 f^2 C_{M_n}^*(z_1) C_{M_n}^*(z_2)\}^2} \\
& \quad + 2 \times 4\pi^2 f^2 C_{D_n}^*(z_1) C_{D_n}^*(z_2) C_{M_n}^*(z_1) C_{M_n}^*(z_2) \\
& = C_{D_n}^*(z_1) C_{D_n}^*(z_2) + 4\pi^2 f^2 C_{M_n}^*(z_1) C_{M_n}^*(z_2) \quad (2-43)
\end{aligned}$$

Then equation (2-39) becomes

$$S_{F_n}(f) = \int_0^H \int_0^H \int_0^B \int_0^B S_{u_1 u_2}(f) \tilde{C}_{D_n}^*(z_1) \tilde{C}_{D_n}^*(z_2) dy_1 dy_2 dz_1 dz_2 \quad (2-44)$$

Generally the natural wind is not a homogeneous turbulent flow and so the cross-spectral density function consists of real and imaginary parts. However, since the power spectrum of the

generalised force is a real function, the real part of the cross-spectrum which for convenience can be called the 'co-coherence' according to Harris⁽⁴⁹⁾, should be taken into account and the imaginary part $Q_{u_1 u_2}$ can be ignored if the assumption (2-38) is adopted and employed in (2-37).

Namely,

$$S_{F_n}(f) = \int_0^H \int_0^H \int_0^B \int_0^B C_{u_1 u_2}(f) \sqrt{S_{u_1}(f) S_{u_2}(f)} \tilde{C}_{D_n}^*(z_1) \tilde{C}_{D_n}^*(z_2) \times dy_1 dy_2 dz_1 dz_2 \quad (2-45)$$

where

$$S_{u_1 u_2}(f) = (C_{u_1 u_2}(f) - i Q_{u_1 u_2}(f)) \sqrt{S_{u_1}(f) S_{u_2}(f)}$$

Then the variance of the dynamic response can be written as follows,

$$\begin{aligned} \overline{\delta^2}(z) &= \int_0^\infty S_\delta(f) df \\ &= \sum_{n=1}^N \mu_n^2(z) \int_0^\infty |\chi_n(f)|^2 S_{F_n}(f) df \end{aligned} \quad (2-46)$$

For a tall building which has its fundamental natural frequency much greater than the peak frequency of the power spectral density of wind turbulence, it would be sufficient to consider only the first one or two modes of vibration, ie, $N \leq 2$. Van Koten⁽⁹⁾ has reported no higher mode components

other than the fundamental having any significance in his extensive dynamic measurements of wind response so far as tall buildings are concerned.

2.2.5 Simplification for Computation

Although the variance of the dynamic response can be computed directly from equations (2-45) and (2-46), the mechanical admittance function, $|\chi_n(f)|^2$, has a dominant peak at $f = f_n$ when the damping is low and so it would be more efficient to divide the integral with respect to the frequency into two parts for the computation of equation (2-46), ie,

$$\overline{\delta^2}(z) = \sum_{n=1}^N \mu_n^2(z) \left\{ \int_0^{f_{nE}} \frac{1}{k_n'^2} S_{F_{qs}}(f) df + \int_0^{f_{nU}} [|\chi_n(f)|^2 - \frac{1}{k_n'^2}] S_{F_n}(f) df \right\} \quad (2-47)$$

where f_{nE} , f_{nU} can be defined as ;

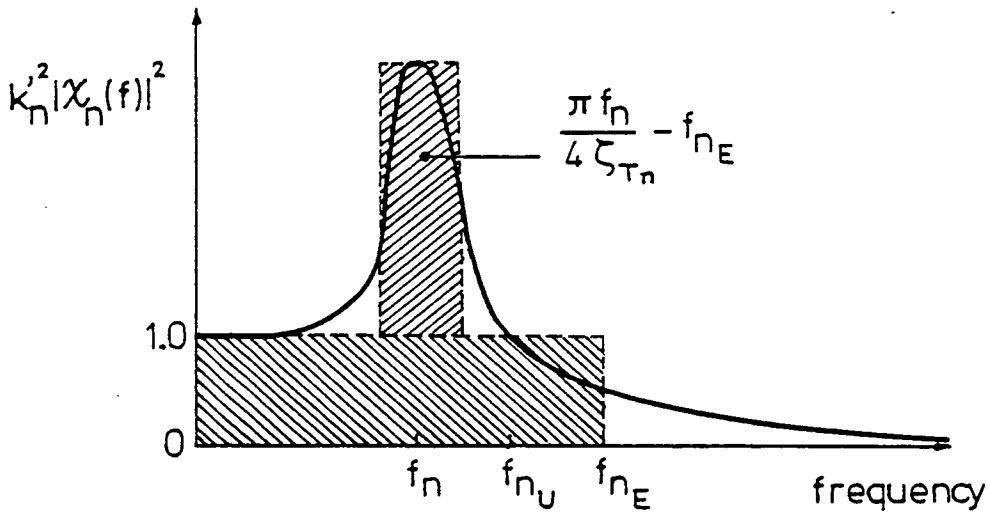
$$f_{nE} = f_{nU} + \int_{f_{nU}}^{\infty} k_n'^2 |\chi_n(f)|^2 df$$

$$k_n'^2 |\chi_n(f_{nU})|^2 = 1, \quad f_{nU} \neq 0 \text{ or } f_{nU} = \sqrt{2 - 4\zeta_n^2} \cdot f_n$$

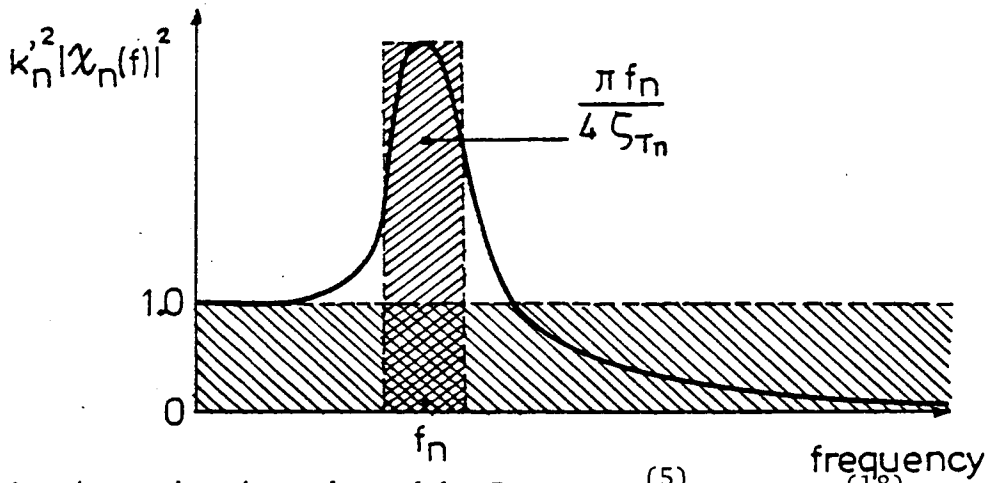
as shown in Figure 2-2(a),

$$k_n' = (2\pi f_n)^2 M_{T_n} \quad (\text{effective spring constant of the } n\text{-th mode})$$

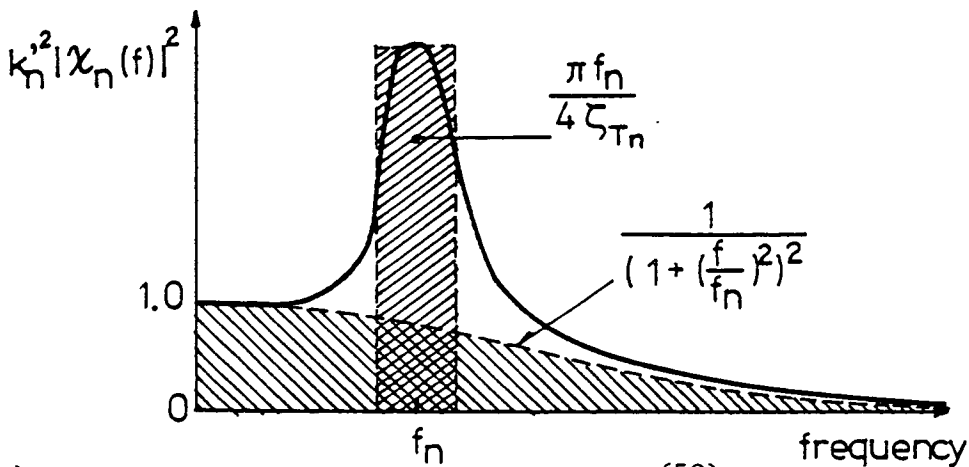
and $S_{F_{qs}}(f)$ is the quasi-static part of $S_{F_n}(f)$.



(a) Approximation proposed for integral of $|\chi_n(f)|^2$



(b) Approximation adopted by Davenport⁽⁵⁾, Vickery⁽¹⁸⁾



(c) Approximation adopted by van Koten⁽⁵⁰⁾

Figure 2-2 Various approximations for integral of $|\chi_n(f)|^2$

Furthermore the integral of the resonance part (shown by  in Figure 2-2) can be approximated by a function of ζ_n for a lightly damped structure, since it is known that

$$\int_0^{\infty} \frac{dr}{(1 - r^2)^2 + 4\zeta^2 r^2} = \frac{\pi}{4\zeta}$$

where r is the frequency ratio, $r = f/f_n$.

When the area AR_n is defined as,

$$AR_n = \int_0^{f_{nU}} \left[|\chi_n(f)|^2 - \frac{1}{k_n'^2} \right] df \quad (2-48)$$

it can be approximated by using a numerically obtained value of $f_{nE} \simeq 1.75 f_n$.

$$\begin{aligned} AR_n &= \int_0^{\infty} |\chi_n(f)|^2 df - \int_0^{f_{nU}} \frac{df}{k_n'^2} - \int_{f_{nU}}^{\infty} |\chi_n(f)|^2 df \\ &= \frac{1}{k_n'^2} \left(\frac{\pi f_n}{4\zeta_n} - f_{nE} \right) \\ &\simeq \frac{1}{k_n'^2} \left(\frac{\pi}{4\zeta_n} - 1.75 \right) f_n \end{aligned} \quad (2-49)$$

where $f_{nU} \simeq \sqrt{2} f_n$.

Since the resonance part has a very narrow band of frequency, $S_F(f)$ in the integral of resonance part can be replaced with a constant $S_F(f_n)$ without losing accuracy.

Hence equation (2-47) becomes,

$$\overline{\delta^2}(z) \simeq \sum_{n=1}^N \mu_n^2(z) \frac{1}{k_n^{1/2}} \left\{ \int_0^{1.75f_n} S_{F_{qs}}(f) df + \left(\frac{\pi}{4\zeta_{T_n}} - 1.75 \right) f_n S_{F_n}(f_n) \right\} \quad (2-50)$$

If only the fundamental modal component is significant, the response at the top, $\overline{\delta^2}(H)$, can be written as,

$$\overline{\delta^2}(H) \simeq \frac{1}{k_n^{1/2}} \left\{ \int_0^{1.75f_1} S_{F_{qs}}(f) df + \left(\frac{\pi}{4\zeta_{T_1}} - 1.75 \right) f_1 S_{F_1}(f_1) \right\} \quad (2-51)$$

where $S_{F_{qs}}(f)$ and $S_{F_1}(f)$ can be obtained from equation (2-45) with $\tilde{C}_{D_{qs}}^*$ and $\tilde{C}_{D_1}^*$ respectively defined by equation (2-41).

2.2.6 Comparison with Energy Method

An alternative way of simplifying the resonance part of the response is described in order to compare the final form with that deducible from an energy method proposed by MacDonald and Morgan (1971)⁽²⁶⁾.

In the energy method the wind turbulence component was treated as a simple harmonic fluctuation at the resonance frequency, f_1 , with a narrow band Δf_1 which is defined from the

mechanical admittance function $|X_1(f)|^2$ as follows,

$$\Delta f_1 \cdot |X_1(f_1)|^2 = \int_0^{f_{1U}} \left[|X_1(f)|^2 - \frac{1}{k_n^2} \right] df \quad (2-52)$$

Within the narrow band of frequency the power spectral density of turbulence can be treated as constant, ie,

$$S_u(f) = \frac{A_w^2}{\Delta f_1} \quad (2-53)$$

The standard deviation of the response at the top of a structure at resonance part, A_s , ($=\sqrt{\delta^2(H)}$ at resonance) can be written as follows from equation (2-47),

$$A_s = \left\{ \int_0^{f_{1U}} \left[|X_1(f)|^2 - \frac{1}{k_n^2} \right] S_{F_1}(f) df \right\}^{\frac{1}{2}} \quad (2-54)$$

Substituting equations (2-52) and (2-45) into (2-54),

$$A_s = \left\{ \Delta f_1 |X_1(f_1)|^2 \int_0^H \int_0^H \int_0^B \int_0^B C_{u_1 u_2}(f) \sqrt{S_{u_1}(f) S_{u_2}(f)} \right. \\ \left. \times \tilde{C}_{D_1}^*(z_1) \tilde{C}_{D_1}^*(z_2) dy_1 dy_2 dz_1 dz_2 \right\}^{\frac{1}{2}} \quad (2-55)$$

Assuming that the horizontal correlation is unity (line-like structure) and the power spectral density is constant with height;

$$C_{u_1 u_2}(f) = C_{u_1 u_2}(z_1, z_2, f) \quad (2-56)$$

$$S_{u_1}(f) = S_{u_2}(f) = S_u(f) \quad (2-57)$$

and that the mean wind speed profile can be expressed by a power law;

$$\bar{U}(z) = \bar{U}(H) \left(\frac{z}{H} \right)^\alpha \quad (2-58)$$

and that the additional mass term is neglected and constant C_D is used for $\tilde{C}_D(\xi_1)$.

Then equation (2-16) becomes,

$$M_{T_1} = M_1 = B \int_0^H m \mu^2(z) dz, \quad (2-59)$$

equation (2-17) becomes

$$\zeta_{T_1} = \zeta_1 + \int_0^H \frac{\rho C_D B \bar{U}(z) \mu_1^2(z)}{4\pi f_1 M_1} dz, \quad (2-60)$$

and equation (2-41) becomes

$$\tilde{C}_{D_1}^*(z) = \rho C_D \bar{U}(z) \mu_1(z) \quad (2-61)$$

Substituting equations (2-53) and (2-56) to (2-61) into equation (2-55),

$$\begin{aligned}
A_s &= \left\{ \Delta f_1 \frac{1}{[2\zeta_{T_1} M_1 (2\pi f_1)^2]^2} \right. \\
&\quad \times \left. \int_0^H \int_0^H C_{u_1 u_2}(z_1, z_2, f_1) B^2 \frac{A_w^2}{\Delta f_1} \rho^2 C_D^2 \bar{U}(z_1) \bar{U}(z_2) \mu(z_1) \mu(z_2) dz_1 dz_2 \right\}^{\frac{1}{2}} \\
&= \frac{\bar{U}(H) B A_w \rho C_D \left\{ \int_0^H \int_0^H C_{u_1 u_2}(z_1, z_2, f_1) \mu_1(z_1) \mu_1(z_2) \left(\frac{z_1}{H}\right)^\alpha \left(\frac{z_2}{H}\right)^\alpha dz_1 dz_2 \right\}^{\frac{1}{2}}}{2M_1 (2\pi f_1)^2 \left\{ \zeta_1 + \frac{\rho C_D B \bar{U}(H) \int_0^H \mu_1^2(z) \left(\frac{z}{H}\right)^\alpha dz}{2M_1 (2\pi f_1)} \right\}} \\
&= \frac{A_w}{2\pi f_1} \cdot \frac{\left\{ \int_0^H \int_0^H C_{u_1 u_2}(z_1, z_2, f_1) \mu_1(z_1) \mu_1(z_2) \left(\frac{z_1}{H}\right)^\alpha \left(\frac{z_2}{H}\right)^\alpha dz_1 dz_2 \right\}^{\frac{1}{2}}}{\frac{2\zeta_1 (2\pi f_1)}{\rho C_D \bar{U}(H)} \int_0^H \mu_1^2(z) dz + \int_0^H \mu_1^2(z) \left(\frac{z}{H}\right)^\alpha dz}
\end{aligned}$$

(2-62)

This final form is exactly the same with the result developed by means of the energy method. A summary of the development and modification of the energy method is shown in appendix 1. Since the same result can be derived, there is no evidence of the superiority or advantage of the energy method as it was claimed previously⁽²⁶⁾.

2.3 DYNAMIC CROSS-WIND RESPONSE

For some slender structures it is also possible to apply a similar approach to that described in the previous section for the estimation of cross-wind response due to turbulent wind.

2.3.1 Buffeting Load Due to v Component

The relationship between the fluctuating lateral or cross-wind force and the lateral turbulent component of the natural wind is shown in Figure 2-3.

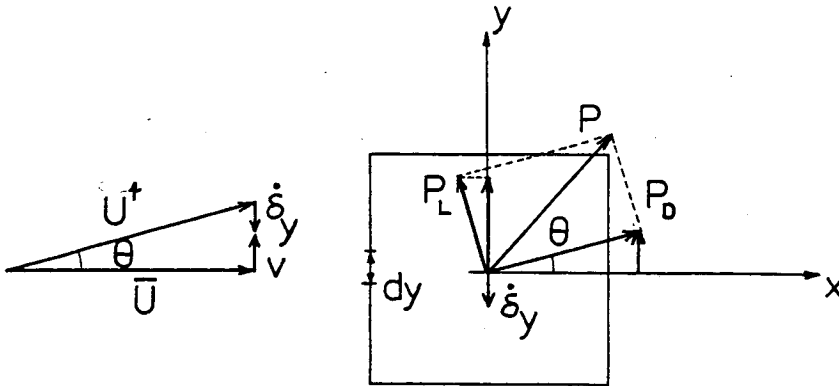


Figure 2-3 Cross-section of laterally vibrating structure in a floor

P_L and P_D are the lift and drag forces (acting on $dy \, dz$) due to $\vec{U}^+$, $\vec{U}^+$ is an instantaneous relative velocity, and $\dot{\delta}_y$ is the cross-wind response velocity.

When the mean wind direction is normal to the surface of a structure, the mean lateral deflection will be zero.

Similar to equation (2-1),

$$P(y, z, t) = \bar{P}(z) + P(y, z, t), \quad \bar{P}(z) = 0$$

$$\Delta_y(z, t) = \delta_y(z, t) \quad (2-62)$$

The relative wind speed has an instantaneous angle of attack $\theta(y, z, t)$,

$$\theta(y, z, t) = \frac{v(y, z, t) - \dot{\delta}_y(z, t)}{\bar{U}(z)} \quad (2-63)$$

The y-component of the fluctuating net pressure, P_y , can be written as,

$$\begin{aligned} P_y &= P_L \cos \theta + P_D \sin \theta \\ &= \frac{1}{2} \rho C_L \bar{U}^2 \cos \theta + \frac{1}{2} \rho C_D \bar{U}^2 \sin \theta \\ &= \frac{1}{2} \rho C_{F_y}(\theta) \bar{U}^2 \end{aligned} \quad (2-64)$$

where $C_{F_y}(\theta) = C_L \cos \theta + C_D \sin \theta$.

C_{F_y} can be assumed to be a polynomial function of θ around $\theta = 0$ and may be approximated using the first derivative when θ is small.

$$C_{F_y}(\theta) \approx \frac{\partial}{\partial \theta} (C_{F_y})_{\theta=0} \cdot \theta + C_{F_y}(0) \quad (2-65)$$

where

$$\begin{aligned} \frac{\partial}{\partial \theta} (C_{F_y})_{\theta=0} &= \left[\frac{\partial}{\partial \theta} C_L \cdot \cos \theta + \sin \theta \cdot C_L + \frac{\partial}{\partial \theta} C_D \cdot \sin \theta + C_D \cos \theta \right]_{\theta=0} \\ &= \frac{\partial}{\partial \theta} (C_L)_{\theta=0} + C_D \end{aligned} \quad (2-66)$$

$$\text{and } C_{F_y}(0) = 0$$

Substituting equations (2-65), (2-66) into (2-64).

$$\begin{aligned} P_y &= \frac{1}{2} \rho \left(\frac{\partial}{\partial \theta} C_L + C_D \right)_{\theta=0} \cdot \theta U^3 \\ &\approx \rho {}_1C_y(\xi) \bar{U} \{v(y, z, t) - \dot{\delta}_y(z, t)\} \end{aligned} \quad (2-67)$$

$$\text{where } {}_1C_y(\xi) = \frac{1}{2} \left(\frac{\partial}{\partial \theta} C_L + C_D \right)_{\theta=0}$$

Equation (2-67) corresponds to equation (2-9) of the along-wind response. The second term in the bracket of equation (2-67) is the aerodynamic damping term.

The power spectral density of the generalised fluctuating lift force, ${}_1S_{F_y}(f)$ associated with the lateral turbulent component, v , can be obtained similar to equation (2-45) as follows,

$$\begin{aligned}
 {}_1S_{F_{y_n}}(f) &= \int_0^H \int_0^H \int_0^B \int_0^B C_{v_1 v_2}(f) \sqrt{S_{v_1}(f) S_{v_2}(f)} {}_1C_{y_n}^*(z_1) {}_1C_{y_n}^*(z_2) \\
 &\quad \times dy_1 dy_2 dz_1 dz_2
 \end{aligned}
 \tag{2-68}$$

where $C_{v_1 v_2}(f)$ is the co-coherence of v_1 and v_2 ,

$S_{v_1}(f)$ $S_{v_2}(f)$ are the power spectral densities of v_1 and v_2 respectively, and

$${}_1C_{y_n}^*(z) = \rho_1 C_y(\xi_n) \bar{U}(z) \mu_n(z) \tag{2-69}$$

2.3.2 Fluctuating Load Due to Vortex Shedding

When the cross-wind response is discussed, the vortex shedding effect must be taken into account. If the turbulence intensity is high (eg, ~20% or more), its effect can be observed in a fairly wide range of the reduced velocity around the critical velocity⁽³⁷⁾. Although there is no theoretical approach established for this type of oscillation, it may be possible to express the generalised force spectrum, $S_{F_{y_n}}(f)$, analogously to equation (2-68) taking the longitudinal turbulent characteristics into account since the main contributor of vortex shedding is the longitudinal velocity, not the lateral component.

$$2S_{F_{y_n}}(f) = \int_0^H \int_0^H \int_0^B \int_0^B C_{u_1 u_2}(f) \sqrt{S_{u_1}(f) S_{u_2}(f)} 2C_{y_n}^*(z_1) C_{y_n}^*(z_2) \times dy_1 dy_2 dz_1 dz_2 \quad (2-70)$$

where

$$2C_{y_n}^*(z) = \rho 2C_{y_n}(\xi_n) \bar{U}(z) \mu_n(z)$$

$2C_{y_n}$ is the equivalent dynamic lift coefficient for vortex shedding.

2.3.3 Dynamic Cross-Wind Deflection

If the relationship between along-wind and cross-wind turbulent characteristics is known as ,

$$C_{u_1 u_2}(f) \sqrt{S_{u_1}(f) S_{u_2}(f)} = \psi(f) C_{v_1 v_2}(f) \sqrt{S_{v_1}(f) S_{v_2}(f)} \quad (2-71)$$

where $\psi(f)$ is a relating factor between cross-spectral densities of along-wind and cross-wind component,

both the cross-wind buffeting and vortex shedding effect can be expressed in one equation.

Since the power spectral density of generalised lift force is the summation of those associated with cross-wind buffeting and vortex shedding, ie,

$$S_{F_{y_n}}(f) = {}_1S_{F_{y_n}}(f) + {}_2S_{F_{y_n}}(f)$$

analogous to equation (2-41) $\tilde{C}_{y_n}^*(z)$ can be defined and then,

$$S_{F_{y_n}}(f) = \int_0^H \int_0^H \int_0^B \int_0^B C_{v_1 v_2}(f) \sqrt{S_{v_1} S_{v_2}}(f) \tilde{C}_{y_n}^*(z_1) \tilde{C}_{y_n}^*(z_2) \\ \times dy_1 dy_2 dz_1 dz_2 \quad (2-72)$$

where

$$\tilde{C}_{y_n}^*(z) = \sqrt{\{ {}_1C_{y_n}^*(z) \}^2 + \psi(f) \{ {}_2C_{y_n}^*(z) \}^2}$$

If the value of $\tilde{C}_{y_n}^*(z)$ is established empirically, the cross-wind response can be computed in a similar manner to that indicated in equation (2-50) or (2-51). If the fundamental modal component is predominant, the variance of the cross-wind response displacement may be written as,

$$\overline{\delta_y^2(H)} \simeq \frac{1}{k_1^{1/2}} \left\{ \int_0^{1.75f_1} {}_1S_{F_{y_1}}(f) df + \left(\frac{\pi}{4\zeta_{T_1}} - 1.75 \right) f_1 S_{F_{y_1}}(f_1) \right\} \quad (2-73)$$

where $\zeta_{T_1} = \zeta_1 + \zeta_{A_{L_1}}$,

$$\zeta_{A_{L_1}} = \int_0^H \int_0^B \frac{\rho_1 C_y(\xi_1) \bar{U}(z) \mu_n^2(z)}{2(2\pi f_1) M_1} dy dz \quad (2-74)$$

Equation (2-74) is the cross-wind aerodynamic damping which tends to be a negative value depending on the cross-sectional shape of the structure.

The reason why ${}_1S_{F_{y_1}}$ is used instead of $S_{F_{y_1}}$ for the quasi-static part of equation (2-73) is that the vortex shedding effect is only significant if the design velocity is close to the critical velocity and the contribution to the background excitation (quasi-static part) is less significant because of its limited band width in the frequency range.

2.4 INFLUENCE OF DIFFERENT ANGLES OF ATTACK

When the wind has a certain angle of attack, θ , to the surface of a structure as shown in Figure 2-4, the response either in the x-or y-direction of the model is the combination of results described in preceeding sections.

Although this is a simple extension of along-wind and cross-wind response prediction, it is important especially for the comparison with the actual measurements of wind response of a structure.

One additional matter to this case is that a steady lift force and its fluctuating component due to the longitudinal turbulence must be taken into account. And the vortex shedding effect is reduced in most cases.

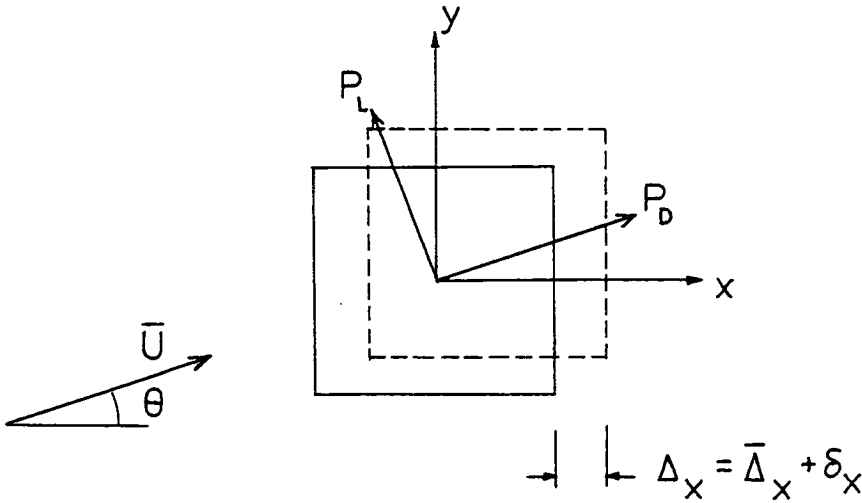


Figure 2-4 Cross-section in a flow with an angle of attack of mean velocity

If the coupling effect between δ_x and δ_y is neglected, the variance of displacement response, $\delta_x^2(z)$, can be obtained from,

$$\overline{\delta_x^2(z)} = \sum_{n=1}^N \mu_n^2(z) \int_0^\infty |\chi_n(f)|^2 \{S_{F_{D_n}}(f) \cos \theta + S_{F_{L_n}}(f) \sin \theta\} df \quad (2-75)$$

where

$$|\chi_n(f)|^2 = \frac{1}{(4\pi^2 M_{T_n})^2 \{f^4 + f_n^4 + (4\zeta_{T_n}^2 - 2)f^2 f_n^2\}} ;$$

$$M_{T_n} = \int_0^H \int_0^B \{m(z) + \rho B(z) C_M(y, z, \xi) \cdot \cos \theta\} \mu_n^2(z) dy dz ; \quad (2-76)$$

$$\zeta_{T_n} = \zeta_n + \int_0^H \int_0^B \frac{\rho \bar{U}(z) \mu_n^2(z) \{C_D(y, z, \xi_n) \cos \theta + C_Y(y, z, \xi_n) \sin \theta\}}{4\pi f_n M_{T_n}} dy dz ; \quad (2-77)$$

$$S_{F_{D_n}}(f) = \int_0^H \int_0^H \int_0^B \int_0^B C_{u_1 u_2}(f) \sqrt{S_{u_1}(f) S_{u_2}(f)} \tilde{C}_{D_n}^*(z_1) \tilde{C}_{D_n}^*(z_2) \\ \times dy_1 dy_2 dz_1 dz_2 \quad (2-78)$$

(power spectral density of generalised drag force of the n-th mode) ;

$\tilde{C}_{D_n}^*(z) = \rho \tilde{C}_D(\xi_n) \bar{U}(z) \mu_n(z)$, $\tilde{C}_D(\xi_n)$ is a dynamic along-wind force coefficient at θ ;

$$S_{F_{L_n}}(f) = \int_0^H \int_0^H \int_0^B \int_0^B C_{v_1 v_2}(f) \sqrt{S_{v_1}(f) S_{v_2}(f)} \tilde{C}_{L_n}^*(z_1) \tilde{C}_{L_n}^*(z_2) \\ \times dy_1 dy_2 dz_1 dz_2 ; \quad (2-79)$$

(power spectral density of generalised lift force of the n-th mode) ;

$$\tilde{C}_{L_n}^*(z) = \sqrt{\{ {}_1\tilde{C}_{L_n}^*(z) \}^2 + \psi(f) \{ {}_2\tilde{C}_{L_n}^*(z) \}^2 + \psi(f) \{ {}_3\tilde{C}_{L_n}^*(z) \}^2} ;$$

$${}_1\tilde{C}_{L_n}^*(z) = \rho {}_1C_L(\xi_n) \bar{U}(z) \mu_n(z), \quad {}_1C_L(\xi_n) = \frac{1}{2} \left(\frac{\partial}{\partial \theta} C_L + C_D \right)_{\theta} ;$$

${}_2\tilde{C}_{L_n}^*(z) = \rho {}_2C_L(\xi_n) \bar{U}(z) \mu_n(z)$, this term should be zero for quasi-static part when the integral in equation (2-75) is performed.

${}_3C_n^*(z) = \rho {}_3C_L(\xi_n) \bar{U}(z) u_n(z)$, ${}_3C_L(\xi_n)$ is a dynamic cross-wind force coefficient at θ .

Because a number of parameters are involved (eg, ξ , θ , turbulence intensity, turbulence scale and geometrical shape of structure, etc), the evaluation of the dynamic along-wind and cross-wind force coefficients might not be an easy task. However, once these coefficients were evaluated the prediction of wind response would be positively improved.

CHAPTER 3 FURTHER CONSIDERATION OF TURBULENCE IN
RELATION TO STRUCTURAL RESPONSE

3.1 INTRODUCTION

In order to estimate the total fluctuating load acting on a structure due to the turbulent wind it is essential to formulate expressions for the dynamic wind characteristics such as the turbulence intensity, the power spectral density, the cross-spectral density and so on. In most previous works, however, the turbulence in the natural wind was assumed to be homogeneous, in other words, the characteristics of turbulence were assumed to be independent of position or height⁽⁶⁾. Although it is widely admitted that the wind characteristics are nearly uniform horizontally, there is no reason for them to be uniform vertically.

When the profile of mean wind speed and the root mean square (r.m.s.) value of the fluctuating component are established, the most important problem for the wind-loading estimation is how to express the power spectral density and the root-coherence (or the normalised cross-spectral density) of the fluctuating component. As this work concentrates on the along-wind response, only the longitudinal component of turbulence is discussed in this chapter.

Some expressions for the power spectral density as a function of height have been suggested^(21,51). Despite the derivation from similar data sources forms of those empirical expressions vary considerably from case to case. Therefore it is logical to seek a general expression with parameters for its form and height dependence.

The height dependence of root-coherence, however, has hardly been taken into account for reasons of simplicity and convenience in practical applications. One exception to this pattern is the use of a wind speed averaged between two points having some vertical separation suggested by Vickery⁽¹⁸⁾. A more flexible form has been suggested by Sfintesco and Wyatt⁽⁵²⁾, but both expressions were given implicitly and so need to be examined with recent measurement data available. An alternative approach suggested by Engineering Sciences Data Unit (ESDU)⁽⁵³⁾ assumes homogeneous isotropic turbulence to exist in the first instance and develops an expression for root-coherence which incorporates some allowance for the variation of length scale of turbulence with height and yields a complicated function based on Harris' modified Bessel function expression⁽⁴⁹⁾.

In following sections 3.2, 3.3 and 3.4, the mean wind speed and turbulence intensity profile, the power spectral density distribution and coherence functions in natural winds are discussed respectively and more flexible expressions are suggested for practical applications. Some consideration of the height dependence of the coherence function have been published as papers by Royles and the author (June 1978 and November 1978); see appendix 4 paper 1 and 2.

In the final section, 3.5, the turbulence characteristics of a wind tunnel used in this work are presented and the applicability of expressions suggested in the foregoing sections is examined.

3.2 MEAN WIND SPEED AND TURBULENCE INTENSITY PROFILE

There have been quite a few works conducted to determine an hourly mean wind speed profile. It was believed that the logarithmic law⁽⁵⁴⁾ could be theoretically derived but only appropriate to the profile for mean wind speed near the ground, ie, only for the lower part of boundary layer. On the contrary the power law profile can provide a better fit to measured data over an extensive range of height. Although a recent work by Deaves and Harris⁽⁵⁵⁾ suggests that a modified logarithmic law (logarithmic-polynomial law or logarithmic-linear law as a simplification) could be the best answer to the mean wind speed profile expression, the power law was chosen in this work for mathematical convenience at the later stage of analysis, and is written as,

$$\bar{U}(z) = \bar{U}(z_r) \left(\frac{z}{z_r} \right)^\alpha \quad (3-1)$$

where $\bar{U}(z)$ is the hourly mean wind speed at height z ;

z_r is the reference height and $z_r = 10\text{m}$ is taken conventionally;

α is the empirical power exponent.

The profile could be improved if instead of the height z from the ground surface the effective height measured from the 'displacement plane' which is somewhat higher than the actual surface as recommended by ESDU⁽⁵⁶⁾ is used, however, also from mathematical convenience the displacement plane was assumed to be

the actual surface in this work. The values of α are to be chosen to give a consistent fit to measured data in a wide range of different terrains⁽⁵⁷⁾. For example, $\alpha = 0.15, 0.22$ and 0.33 can be used in representing conditions over a typical open flat field, a suburban area and a heavily built-up city respectively.

The upper limit of height within which the profile equation (3-1) can hold is defined as the gradient height, z_G . The mean wind speed can be represented by a constant value $\bar{U}(z_G)$ at higher level than z_G . The gradient height, z_G , is also dependent on the roughness of terrain, which can be defined by a parameter called the roughness length, z_0 .

Relationships between α and z_0 , and z_G and z_0 can be established by mathematical expressions. The former relationship may be written by matching the modified logarithmic law and equation (3-1) as⁽⁵⁴⁾,

$$\frac{1.16 + \frac{7.0}{\bar{U}(z_G)}}{\alpha} = \ln \left(\frac{150}{z_0} \right)$$

where z_0 is in m and $\bar{U}(z_G)$ is in m/s.

Alternatively taking $\bar{U}(z_G) = 30$ m/s as a typical case since the departure from the value of $\bar{U}(z_G)$ will not have a significant effect on the relation between α and z_0 ,

$$z_0 = 150 \exp \left(- \frac{1.39}{\alpha} \right) \quad (3-2)$$

where z_0 is in m.

The gradient height varies slightly with the gradient wind speed and also the lack of information on the profile at very high levels makes it difficult to produce a standard relationship between z_G and z_0 . One simple formula recommended by ESDU⁽⁵⁶⁾ is as follows,

$$z_G = 1000 z_0^{0.18}$$

However, this equation gives considerably higher values of z_G compared with those summarised by Davenport⁽⁵⁷⁾, and therefore a modified form may be suggested within the range of scattered data examined by ESDU as,

$$z_G = 900 z_0^{0.18} \quad (3-3)$$

where z_G and z_0 are in m.

In most previous works relating to the wind response of structures the r.m.s. value of longitudinal fluctuating component, $\sigma_u(z)$, has been a constant value with height. Sfintesco and Wyatt⁽⁵²⁾ suggested a simple power law for the variation of $\sigma_u(z)$ with height taking the advantage of the power law profile of mean wind speed. In various ranges of α (or z_0) an expression conforming well with the ESDU recommendations can

be written as follows,

$$\sigma_u(z) = \sigma_u(z_G) \left(\frac{z}{z_G} \right)^{-\alpha_T} \quad (3-4)$$

where $\alpha_T = 0.08$, $\sigma_u(z_G) \simeq 8\%$.

Since numerous amounts of data were examined in ESDU's study, it seems unnecessary to make comparisons with individual measurement data. However, usually more information is needed especially for rough terrain and so it may be worth quoting a record obtained at the Tokyo Tower, Tokyo, Japan by Soma⁽⁵⁸⁾. The quality of the data may not be high enough to discuss the validity of equation (3-4) but yet it seems a good example to demonstrate the height variation of $\sigma_u(z)$ as shown in Figure 3-1. Plots were made by the author from Soma's data. The power law profile index α was reported to be between 0.30 and 0.36 mainly depending on the wind direction in that area, ie, in a built-up area of a large city.

3.3 POWER SPECTRAL DENSITY DISTRIBUTION

There have been many measurements of spectra of wind speed reported in recent years. For higher frequency regions most measurements appear to confirm the Kolmogorov hypothesis⁽⁵⁹⁾, however, for lower frequency regions there are still considerable variations between established formulae for the power spectral density distribution of longitudinal turbulent component.

legend for Figs3-1& 2

Symbol	Data No.	U(253)
○	15	32.5 m/s
△	18	33.2
□	24	33.3

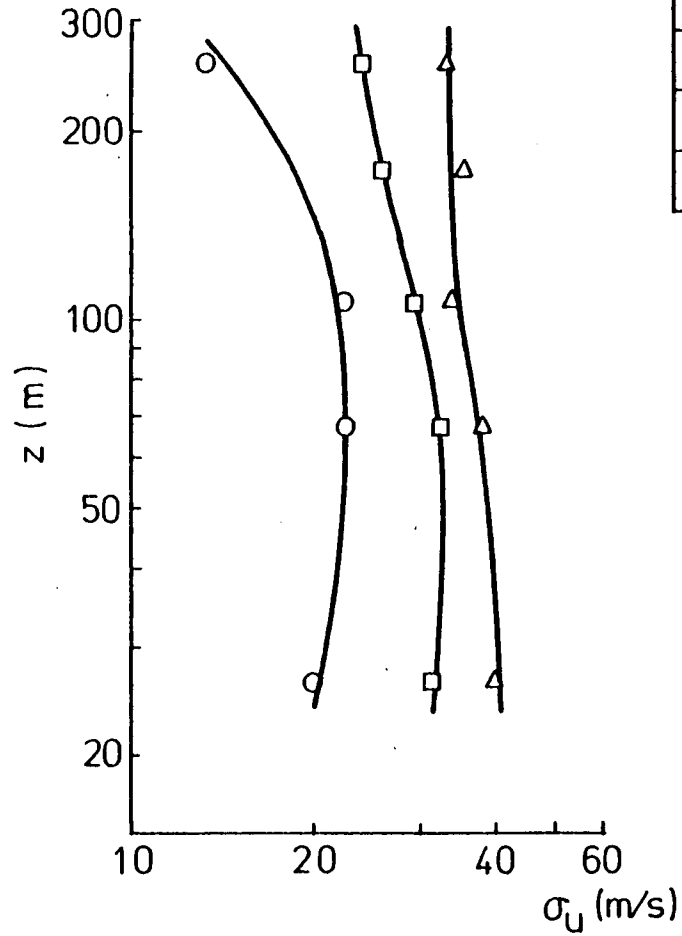


Figure 3-1 Height variation of $\sigma_u(z)$ at the Tokyo Tower⁽⁵⁸⁾

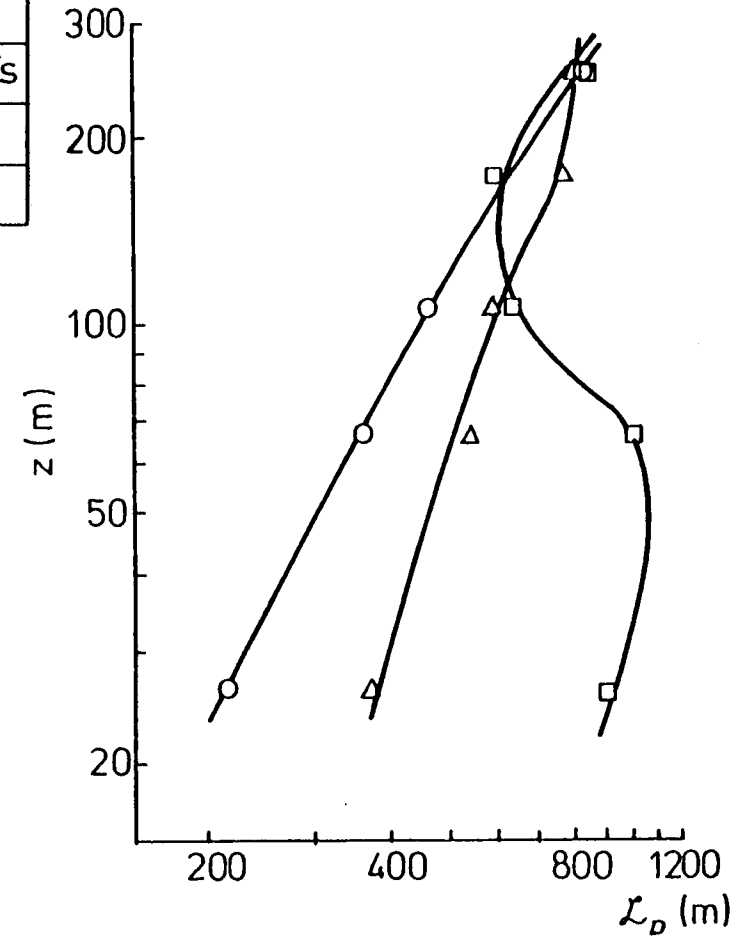


Figure 3-2 Height variation of length constant, L_D , at the Tokyo Tower⁽⁵⁸⁾

One of the well-known expressions for the power spectral density has a following height independent form proposed by Davenport⁽⁶⁾ as the basis of his approach to the prediction of wind response. The form is expressed as a reduced power spectral density, since the power of frequency component of turbulence on the logarithmic scale can be well demonstrated by this form.

$$\frac{f \cdot S_u(f)}{\sigma_u^2} = \frac{2}{3} \cdot \frac{\tilde{f}^2}{(1 + \tilde{f}^2)^{4/3}} \quad (3-5)$$

where $\tilde{f} = \frac{f \cdot L_D}{\bar{U}(10)}$, L_D = length constant = 1200 m.

This formula has been used quite often but further investigations suggest that it may not be appropriate in certain circumstances. The first point is that $S_u(f)$ in equation (3-5) has a zero value at $f = 0$ Hz in spite of the fact that the one-dimensional spectral density approaches a finite value when the frequency goes to zero⁽⁵⁹⁾. The second point is that equation (3-5) does not allow for height dependence. The length constant L_D appears to vary from site to site and to increase with height. One example is quoted here also from the Tokyo Tower data by Soma⁽⁵⁸⁾. From the best fit curves of equation (3-5) to three sets of data L_D was plotted against height in Figure 3-2 where some conversion was required to make L_D relative to a reference height ($z_r = 10$ m) wind speed, ie, estimated $\bar{U}(10)$ based on $\alpha = 0.35$. This kind of height dependence of L_D

seems to be commonly observed, although it is difficult to find a suitable height variation form only from this figure.

Hino⁽⁶⁰⁾ proposed an expression known as a von Karman spectrum based on the power law profile of mean wind speed and confirmed its consistency with the height dependence observed in measurements at Sale, Victoria, Australia and Brookhaven, Long Island, USA⁽⁶⁾. It may be written as⁽⁵¹⁾,

$$\frac{f \cdot S_u(f, z)}{\sigma_u^2(z)} = 0.475 \frac{\tilde{f}}{(1 + \tilde{f}^2)^{5/6}} \quad (3-6)$$

$$\text{where } \tilde{f} = \frac{f \cdot L_H(z)}{\bar{U}(10)} \quad \text{and} \quad L_H(z) = L_H(10) \left(\frac{z}{10} \right)^{1-4\alpha}.$$

Equation (3-6) has a similar form to that suggested by Harris⁽⁴⁹⁾ who used a length constant $L_H = \frac{1}{\sqrt{2}} \cdot 1800 \text{ m}$ to give a better fit to data obtained at a high level ($z = 166 \text{ m}$) in Rugby, UK. The relation between length constants in equations(3-5) and (3-6) can be expressed as $L_D = \sqrt{2} L_H$ so that both equations have a peak value at the same frequency.

Also a related expression according to Simiu⁽²¹⁾ can be written in a similar way to equation (3-6) as,

$$\frac{f \cdot S_u(f, z)}{\sigma_u^2(z)} = \frac{2}{3} \cdot \frac{\tilde{f}}{(1 + \tilde{f}^2)^{5/3}} \quad (3-7)$$

$$\text{where } \tilde{f} = \frac{f \cdot L_S(z)}{\bar{U}(10)} \quad \text{and} \quad L_S(z) = L_S(10) \left(\frac{z}{10} \right)^{1-\alpha}.$$

Equation (3-7) was derived from the balance of the energy dissipation based on the logarithmic law for the mean wind speed profile and the power exponent, α , is, therefore, an equivalent value for the logarithmic profile used by Simiu. The length constant value is also slightly different from that in equation (3-6), ie, $L_S = \sqrt{\frac{3}{2}} L_H$ is obtained for both equations (3-6) and (3-7) to have a peak value at the same frequency.

There may not be much difference between equations (3-6) and (3-7) at lower heights if appropriate length constant values are chosen according to Hino and Simiu respectively, but the variation of length constant is rather different in the two cases. Consequently a significant difference arises at higher levels. It is interesting to note that all three equations (3-5) to (3-7) have been compared with the same data sources (eg, Sale and Brookhaven) for their justification when they were proposed and yet rather different conclusions were made. According to Davenport⁽⁶¹⁾, the length constant was confirmed to be more or less invariant with height as supported by Bearman's study⁽⁶²⁾ on the height variation of the frequency giving the peak value of $f \cdot S_u(f)$.

What these different expressions mean is that from limited amounts of full scale data it is difficult to determine a definite expression in an empirical way. A more flexible expression, therefore, seems necessary and can be written as follows⁽⁶³⁾,

$$\frac{f \cdot S_u(f, z)}{\sigma_u^2(z)} = k_1 \frac{\tilde{f}}{(1 + \tilde{f}^\beta)^{5/3\beta}} \quad (3-8)$$

$$\text{where } \tilde{f} = \frac{f \cdot L_1(z)}{\bar{U}(10)}, \quad L_1(z) = L(10) \left(\frac{z}{10} \right)^{\alpha_L},$$

$$k_1 = \frac{\beta \Gamma(\frac{5}{3\beta})}{\Gamma(\frac{1}{\beta}) \Gamma(\frac{2}{3\beta})} \quad (\text{a constant for normalising purposes})$$

and α_L is the power law index of $L_1(z)$.

According to Fichtl et al⁽⁶³⁾, $\beta = 0.845$ in equation (3-8) was suggested from several sets of data by means of the least square method. $\beta = 2$ corresponds to equation (3-6), $\beta = 1$ corresponds to equation (3-7) and $\beta = 5/3$ corresponds to Panofsky-Lumley's⁽⁶⁴⁾ expression which is another well-known formula similar to that proposed by Kaimal et al⁽⁶⁵⁾. The Davenport formula (3-5) has some similarity to equation (3-8) with $\beta \approx 3.0$ as far as the peak value of the reduced power spectral density is concerned. The higher the value of β , the greater the peak value of $f \cdot S_u(f)$ becomes, ie, a lower value of β gives a rather flat shape for $f \cdot S_u(f)$. The peak value of $f \cdot S_u(f)$ occurs at $\tilde{f} = (1.5)^{1/\beta}$ in equation (3-8), whereas the peak value of equation (3-5) occurs at $\tilde{f} = \sqrt{3}$.

As noted in equation (3-8) $S_u(f)$ in that form has a constant value when the frequency approaches zero and decreases with

frequency linearly to the $5/3$ power of the frequency at a high frequency region (Kolmogorov spectrum). The value of β determines the form of the transient region of frequency in a way such that the higher value of β makes $S_u(f)$ close to two asymptotic straight lines on log-log co-ordinates, in other words within the small region of middle frequency $S_u(f)$ changes from a constant value to the Kolmogorov spectrum when β is large. Therefore β can be a convenient parameter which defines the form of reduced power spectral density instead of using several parameters in a polynomial form as employed by Simiu⁽²¹⁾. The role of β is well illustrated in Figure 3-3. Variations due to β are not so significant within the range between $\beta = 1.5$ to 3.0 .

Although the value of β is to be determined empirically, according to Harris⁽⁴⁹⁾ and ESDU⁽⁵⁶⁾, $\beta = 2.0$ may be most recommendable since otherwise the power spectral density does not become an even function of frequency which is a requirement for the mathematical consistency of the power spectral density.

For practical applications, it is also important to establish the values of length constant, $L_1(z)$, and its profile index, α_L .

It is sometimes convenient to rewrite the reduced spectral density equation (3-8) against $\bar{U}(z)$ instead of $\bar{U}(10)$ as,

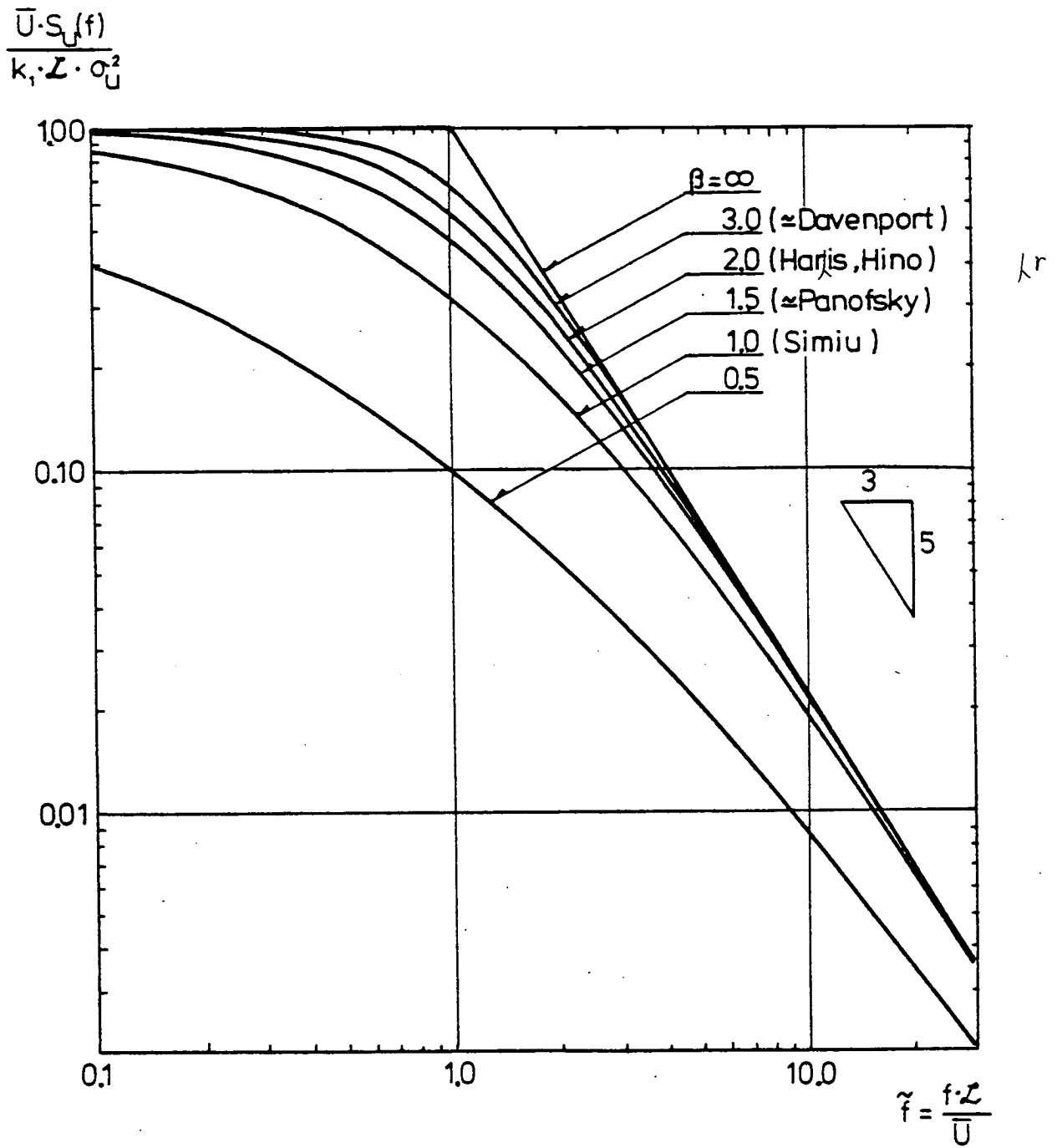


Figure 3-3 Variation of power spectral density with β

$$\frac{f \cdot S_u(f, z)}{\sigma_n^2(z)} = k_1 \frac{\tilde{f}}{(1 + \tilde{f}^\beta)^{5/3\beta}} \quad (3-9)$$

$$\text{where } \tilde{f} = \frac{f L(z)}{\bar{U}(z)} \quad \text{and} \quad L(z) = L(10) \left(\frac{z}{10} \right)^{\alpha_L + \alpha}$$

In the case of $\beta = 2.0$, if the homogeneous isotropic condition is assumed, the Fourier inverse transform of the normalised power spectral density (ie, the auto-correlation coefficient) can be obtained analytically⁽⁶⁶⁾ and from the integral of auto-correlation coefficient with respect to lag time (ie, the time scale) and Taylor's hypothesis the relation between the length scale, L_x , and the length constant, $L(z)$, can be established as⁽⁴⁹⁾,

$$L_x = 0.118 L(z) \quad (3-10)$$

It is suggested by ESDU⁽⁵⁶⁾ that the length scale, L_x , can be expressed as a function of z and z_G as,

$$L_x = 280(z / z_G)^{0.35} \quad (3-11)$$

Namely equation (3-11) means $\alpha_L + \alpha = 0.35$ but this form is rather different from both Hino's and Simiu's suggestions for the height dependence of $L(z)$ in equations (3-6) and (3-7) respectively.

Equation (3-6) represents the data obtained from Sale reasonably well, but according to Hino for $\alpha > 0.25$ then $\alpha_L (= 1 - 4\alpha)$ becomes negative, ie, L_1 decreases with height and it seems rather peculiar in comparison with most full scale data. In fact Duchene-Marullaz⁽⁶⁷⁾ presented the trend of increasing $L_1(z)$ for $\alpha = 0.33$ ($\alpha_L \approx 0.2$ can be estimated from his results) and also from the Tokyo Tower data⁽⁵⁸⁾ increasing $L_1(z)$ with height can be recognised in Figure 3-2.

On the contrary Simiu's expression (3-7) provides a rather high value of α_L throughout the range of terrain roughness. However, its consistency was only examined up to 150 m of height (most data quoted in Simiu's discussion⁽²¹⁾ are below 100 m). The extrapolation of the profile to higher levels with a high α_L value could mislead the general tendency rather seriously.

Another recent measurement in Hong Kong by Mackey and Ko⁽⁶⁸⁾ suggested a relatively higher power law exponent such as $\alpha_L + \alpha = 0.55$ and greater values of turbulence scale (eg, $L_x = 210$ m at $z = 10$ m); the power law index for the mean wind speed profile, α , was approximately 0.19.

Consequently, the following form for α_L can be suggested in this work as,

$$\alpha_L = 0.50 - \alpha \quad (3-12)$$

Equation (3-12) gives a value of α_L less than Simiu's value and greater than ESDU's recommendation in order to obtain a better agreement with recent measurements. Various proposed relations between α_L and α are shown in Figure 3-4 for comparison.

The values of $L(z)$ or $L_1(z)$ may be obtained from equations (3-10) and (3-11) but those for the rough terrain based on the equations above become significantly smaller than those for the smooth terrain. This is because the same turbulence scale is given at the gradient height irrespective of the terrain roughness. On the contrary, according to Deaves and Harris⁽⁵⁵⁾, the turbulence scale at the gradient height appears to be almost linear with the height. As a higher value of α_L is given by equation (3-12) in comparison with equation (3-11) a larger length constant than that obtained from equations (3-10) and (3-11) would be appropriate at the gradient height so as to have good agreement with ESDU recommendations at the height $z \approx 100$ m for which most data were obtained. The following form for $L(z)$ is suggested as,

$$L(z_G) = 3.3 z_G + 1500 \quad (3-13)$$

Then

$$L(10) = (3.3 z_G + 1500) \left(\frac{10}{z_G} \right)^{0.5} \quad (3-14)$$

where L , z_G are in m.

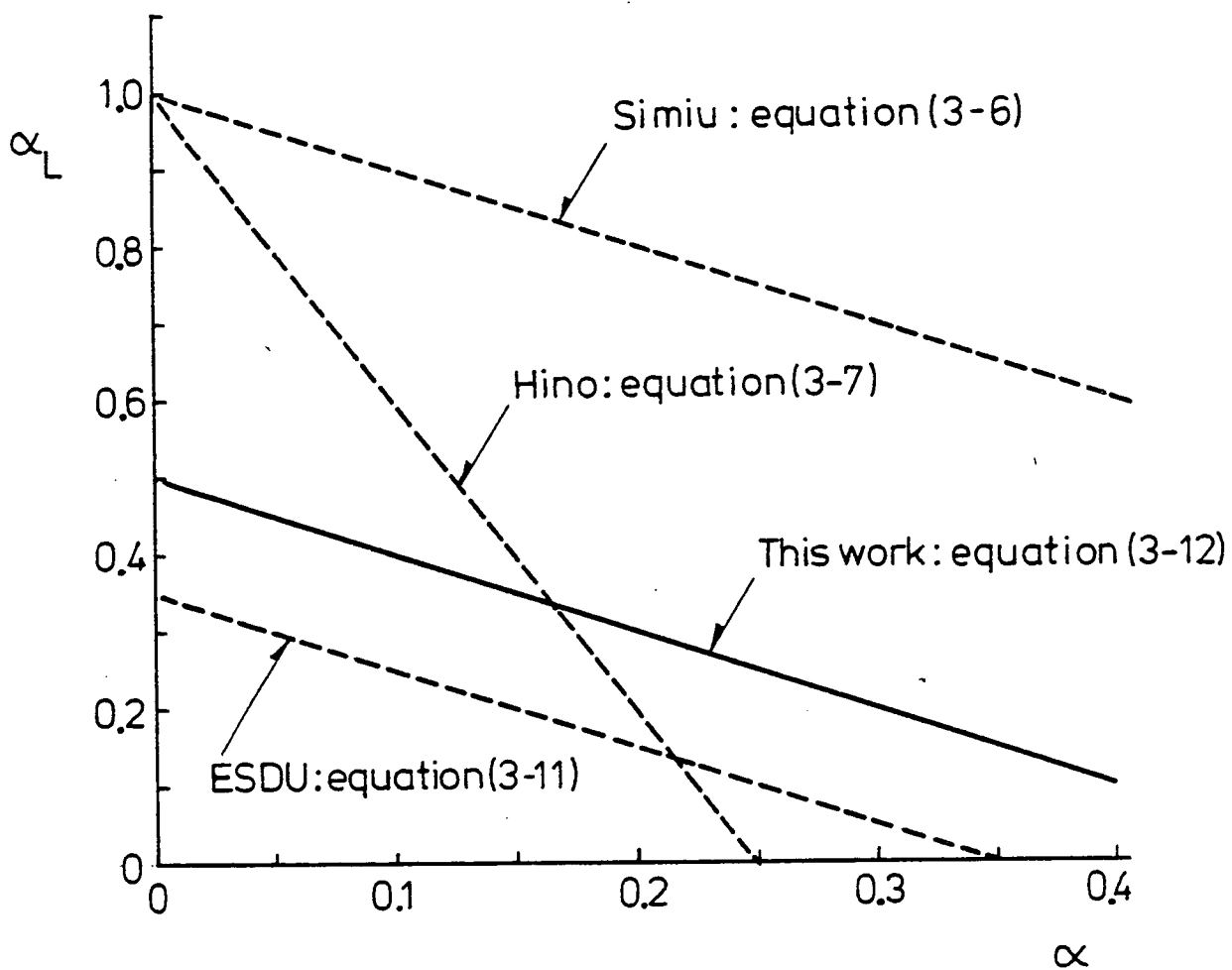


Figure 3-4 Proposals for $\alpha_L - \alpha$ relation

As discussed by ESDU⁽⁵⁶⁾, the accuracy was rather poor at higher levels (say more than 200 m) especially for the rough terrain and so the length constant value at gradient height should be considered as a hypothetical one and not well confirmed yet by measured data.

The reduced power spectral expression equation (3-9) together with (3-12) and (3-14) have good agreement with ESDU recommendations especially for smooth terrain. In order to confirm the adequacy of the proposed expressions for the reduced power spectral density comparisons with several available data are made and presented in Figure 3-5(a), (b) and (c). The height dependence of the power spectral density is well demonstrated by the proposed form with reasonable accuracy for both smooth (data from Sale⁽⁶⁾ and Rugby⁽⁴⁹⁾) and rough terrain (data from Nantes⁽⁶⁷⁾).

3.4 ROOT AND CO-COHERENCE FUNCTIONS

The real part of the cross-spectral density has a significant role in the wind response problem of a structure as shown in the previous chapter, but the quadrature component of the normalised cross-spectral density was assumed to be negligible compared with the co-component (ie, co-coherence) under normal circumstances⁽⁶⁾, and often the root-coherence has been used for the response prediction instead of the

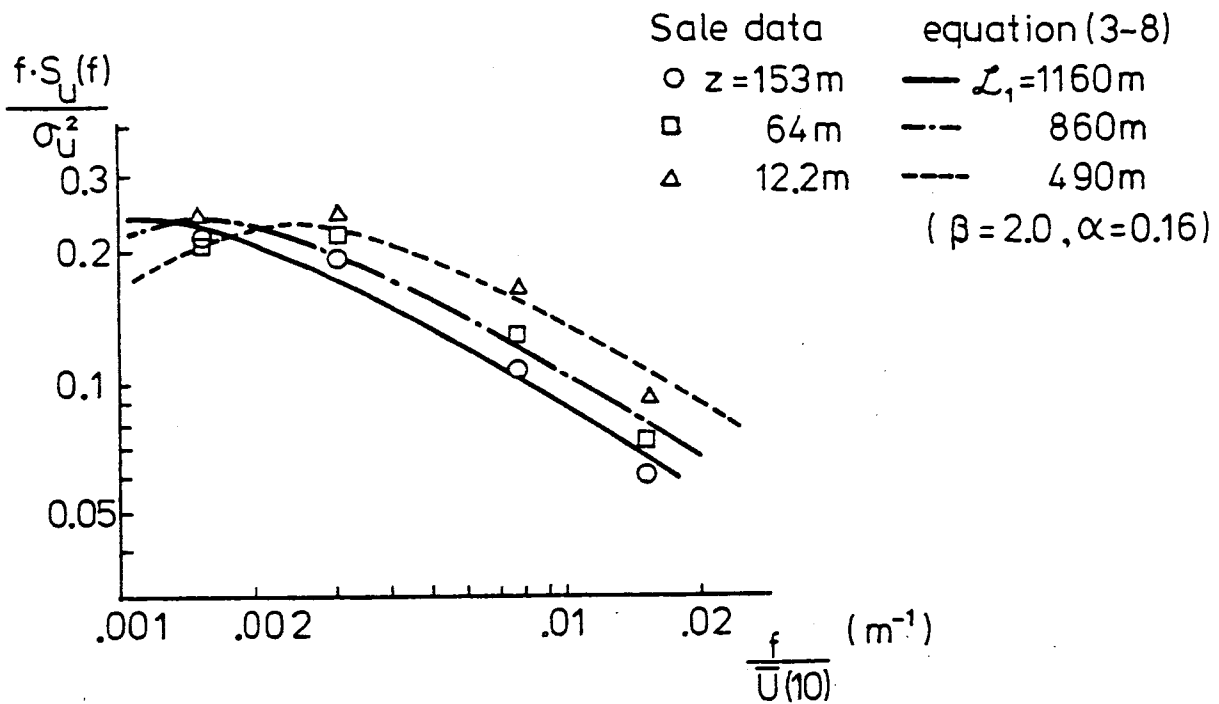


Figure 3-5(a) Height variation of reduced power spectrum after Davenport⁽⁶⁾

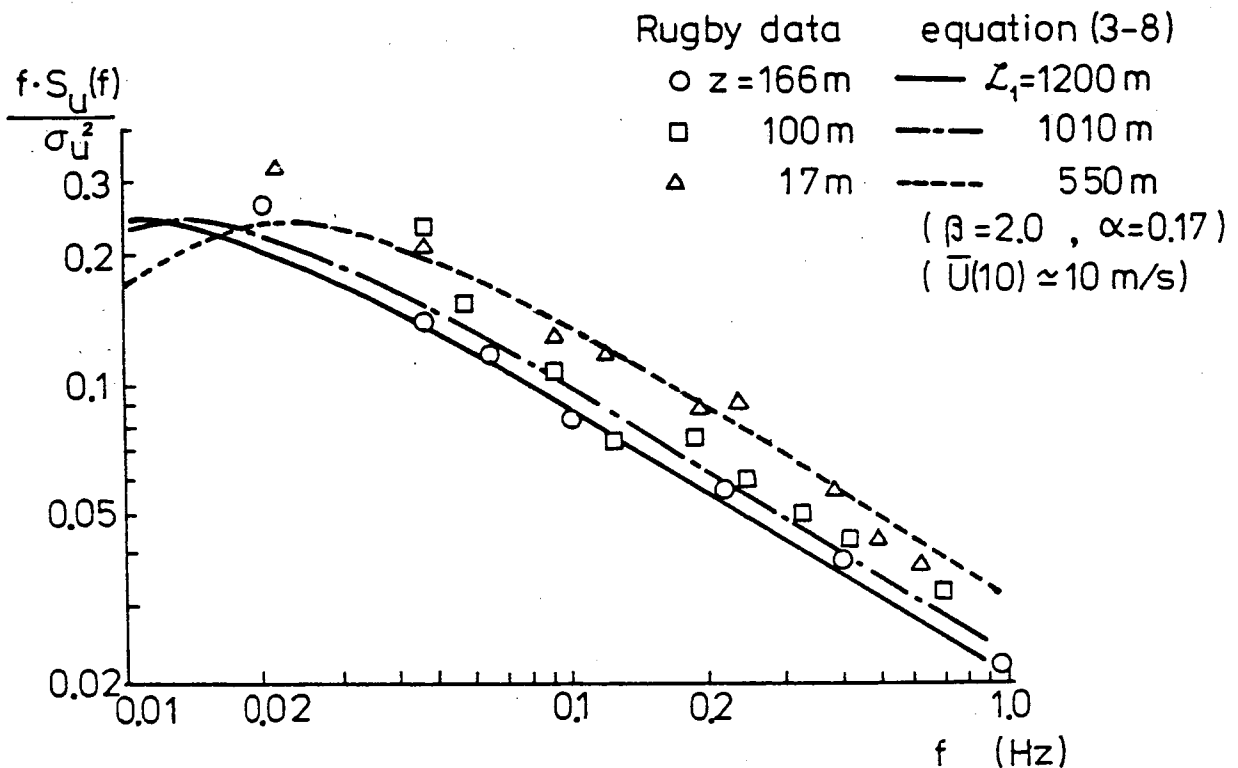


Figure 3-5(b) Height variation of reduced power spectrum after Harris⁽⁴⁹⁾

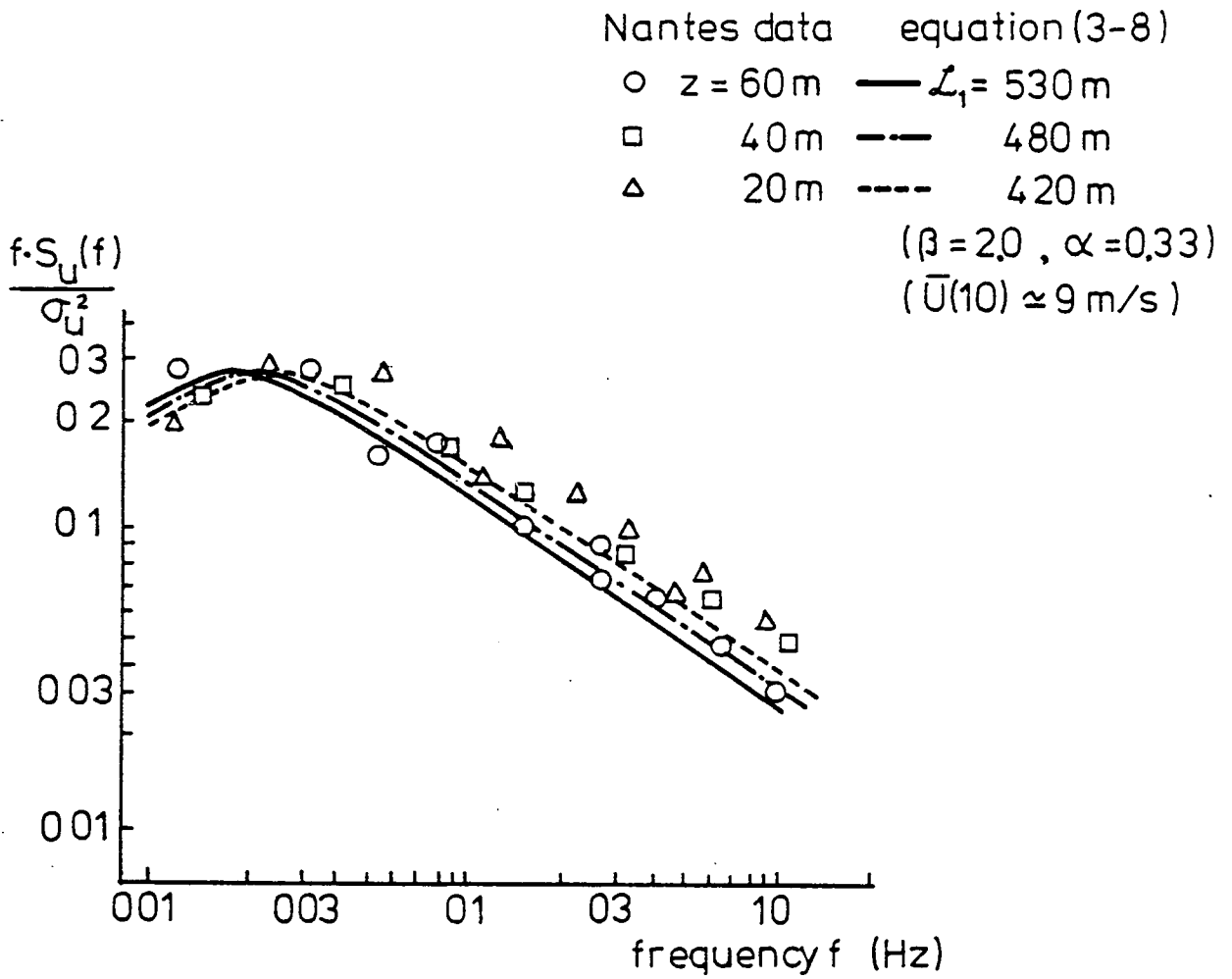


Figure 3-5(c) Height variation of reduced power spectrum
after Duchene-Marullaz⁽⁶⁷⁾

co-coherence⁽⁵⁾. In fact in the homogeneous isotropic flow both the root and co-coherence become identical but in full scale measurements the existence of the quadrature component was confirmed^(40,69) and so the co-coherence should be used but not the root-coherence.

However, the form of co-coherence probably can be the same as the root-coherence with slightly different parameter values. A commonly used formula for the root- or co-coherence can be written according to Vickery⁽¹⁸⁾, as follows,

$$C_{u_1 u_2}(y_1, y_2, z_1, z_2, f) = \exp\left(-\frac{\sqrt{k_y^2(y_2 - y_1)^2 + k_z^2(z_2 - z_1)^2}}{\frac{1}{2}(\bar{U}(z_2) + \bar{U}(z_1))} f\right) \quad (3-15)$$

where k_y , k_z are the horizontal and vertical decay constants respectively.

This form is a modified expression of a simple exponential decay formula suggested by Davenport⁽⁶⁾. However, equation (3-15) is considered to be still rather conservative for two reasons. Firstly the decay constants are given implicitly as invariant with height relative to the mean velocity, $\frac{1}{2}(\bar{U}(z_1) + \bar{U}(z_2))$. In Davenport's form they are constant relative to the reference wind speed $\bar{U}(10)$. Sfintesco and Wyatt⁽⁵²⁾ suggested that decay constants have a slightly different power law profile from the mean wind speed profile. Consequently a more flexible form for the decay constants seems to be necessary in order to allow for their height variation.

The second point is that the co-coherence value in equation (3-15) becomes unity at $f = 0$ irrespective of the separation distance. This tendency is rather inconsistent with empirical data, ie, the co-coherence appears to approach a value significantly less than unity when the frequency approaches zero, and its value at $f = 0$ decreases with the separation distance $|y_2 - y_1|$ or $|z_2 - z_1|$.

This latter point can be clearly indicated by Harris' theoretical form for the root-coherence function obtained from the power spectral expression equation (3-8) with $\beta = 2$ and constant L_1 assuming the homogeneous isotropic condition, as follows⁽⁵⁵⁾,

$$C_{u_1 u_2}(y_1, y_2, z_1, z_2, f) = \frac{2}{\Gamma(\frac{5}{6})} \left\{ \left(\frac{\eta}{2}\right)^{5/6} K_{5/6}(\eta) - \left(\frac{\eta}{2}\right)^{11/6} K_{1/6}(\eta) \right\} \quad (3-16)$$

$$\text{where } \eta = \frac{2\pi\sqrt{\{(y_2 - y_1)^2 + (z_2 - z_1)^2\}} (1 + \tilde{f}^2)}{L_1 \sqrt{\bar{U}(z_1) \bar{U}(z_2)}} \cdot \bar{U}(10), \quad \tilde{f} = \frac{f \cdot L_1}{\bar{U}(10)}$$

and $K_{5/6}$, $K_{1/6}$ are the modified Bessel functions of the second kind of the order 5/6, 1/6 respectively.

Equation (3-16) could be modified to take the height dependence into account, as suggested by ESDU⁽⁵³⁾, and also the different decay tendency in the y- and z-directions

(anisotropic condition) could be introduced by applying different ratios of L_y/L_x and L_z/L_x to the modification of L_1 . The modified Bessel function form itself is complicated and not as convenient as the exponential decay form for practical applications.

Considering these points a flexible decaying expression for the root-or co-coherence is suggested as,

$$C_{u_1 u_2}(y_1, y_2, z_1, z_2, f) = \exp \left(- \frac{\sqrt{k_{1y}^2(z_m) (y_2 - y_1)^2 + k_{1z}^2(z_m) (z_2 - z_1)^2}}{\bar{U}(10)} \cdot f^* \right) \quad (3-17)$$

$$\text{where } k_{1y}(z_m) = k_y(10) \left(\frac{z_m}{10} \right)^{-\alpha_{Dy}}, \quad k_{1z}(z_m) = k_z(10) \left(\frac{z_m}{10} \right)^{-\alpha_{Dz}};$$

$$z_m = \sqrt{z_1 \cdot z_2}, \quad \text{a geometric mean of } z_1 \text{ and } z_2;$$

$$f^* = \sqrt{\left(\frac{\bar{U}(10)}{k_2 \cdot L_1(z_m)} \right)^2 + f^2}, \text{ the modified frequency;}$$

and k_2 is a constant.

The consistency of equation (3-17) is discussed both theoretically and empirically in papers published previously (appendix 4 paper 1 and 2), for the case of $k_2 = 1.0$. It seems a popular choice to take k_2 as unity from the analogous expression of η in equation (3-16). However, the decay

tendency of equation (3-16) is slightly different from the exponential curve employed in equation (3-17) and so it may be reasonable to determine the value of k_2 from the comparison of both equations at $f = 0$.

The comparison was made in Table 3-1 for the case of $z_1 = z_2 = 10$ m and $k_y(10) = 9.0$. This decay constant value can be obtained theoretically for the first postulation of $k_2 = 1.0$ as shown in appendix 4 paper 2 based on the same assumption as that employed for the derivation of equation (3-16).

When the value of k_2 is determined by equating equations (3-17) to (3-16), it seems to vary depending on the separation $(y_2 - y_1)/L_1$ although the variation at smaller separations is not significant as shown in Table 3-1.

The first postulation value $k_2 = 1.0$ in equation (3-17) gives a reasonable representation of equation (3-16) at $f = 0$ but its slight underestimation can be avoided by using a conservative value greater than 2.0. When the separation is large the coherence becomes insignificant and so the range of $(y_2 - y_1)/L_1$ or $(z_2 - z_1)/L_1$ of interest lies below 0.05. Therefore $k_2 = 2.0$ to $\sqrt{10}$ can be suggested in this work for practical applications. Although it is slightly conservative at larger separations the co-coherence value at $f = 0$ by equation (3-17) is considered to be much improved by comparison with earlier exponential expressions such as equation (3-15).

Table 3-1 Comparison between Root-coherence Expressions
 (3-16) and (3-17) at Zero Frequency and
 $z_2 = z_1 = 10 \text{ m}$

$\frac{y_2 - y_1}{L_1}$		0.001	0.002	0.01	0.02	0.05	0.1	0.2
η		0.0063	0.0126	0.063	0.126	0.314	0.63	1.26
equation (3-16)		0.9995	0.998	0.976	0.942	0.793	0.551	0.225
equation (3-17) with $k_2 =$	1.0	0.991	0.982	0.913	0.835	0.638	0.406	0.165
	2.0	0.995	0.991	0.956	0.913	0.800	0.638	0.406
	$\sqrt{10}$	0.997	0.994	0.972	0.944	0.867	0.752	0.565
k_2 obtained by equating (3-17) to (3-16)		18.0	9.0	3.7	3.0	1.9	1.5	1.2

There is a certain mathematical relationship between the root-coherence and the turbulence scale in either the y- or z-direction. The turbulence scale, L_i , can be defined as,

$$L_i = \int_0^{\infty} R_u(r_i) dr_i \quad (3-18)$$

where $i = x, y, z$, $r_i = i_2 - i_1$ and

$R_u(r_i)$ is the cross-correlation coefficient in the i-direction with no lag time.

In a homogeneous isotropic flow

$$L_x = 2L_y = 2L_z \quad (3-19)$$

Since $R_u(r_i)$ is the Fourier inverse transform of the cross-spectral density with zero lag time, the turbulence scale can be obtained as (see appendix 4 paper 2),

$$L_i(z_m) = \frac{k_1 \cdot L_1(z_m)}{k_{1i}(z_m)} \left(\frac{z_m}{10} \right)^{-\alpha} D_i^{+\alpha} \int_0^{\infty} \frac{d\tilde{f}}{(1+\tilde{f}^\beta)^{5/3\beta} \left(\frac{1}{k_2^2} + \tilde{f}^2 \right)^{\frac{1}{2}}} \quad (3-20)$$

where $i = y$ or z .

The integral term in equation (3-20) has a numerical value. From equations (3-10), (3-19) and (3-20) $k_i(10)$ or $k_{1i}(10)$ can be obtained. For example,

$$\beta = 2.0 \quad k_2 = 1.0 \quad \text{yields} \quad k_i(10) = 9.0$$

$$\beta = 2.0 \quad k_2 = \sqrt{10} \quad \text{yields} \quad k_i(10) = 16.5$$

The latter value will be slightly overestimated due to the conservative estimate of k_2 .

In the natural turbulent flow the turbulence scale in the y-or z-direction is less than a half of the longitudinal one, L_x , as exemplified by full scale data^(49,67,68), and values of $k_i(10)$ obtained from the homogeneous isotropic assumption of equation (3-19) would be rather underestimated especially at the lower levels of the boundary layer such as $z = 10$ m.

The decay constants have been taken as invariable with height in Davenport's approach^(4,5) with $k_{1y} = 20$, $k_{1z} = 7$. A number of full scale measurements showed reasonable agreement with the vertical decay constant value according to his later confirmation⁽⁶¹⁾. However, more recent data quoted below suggest firstly that there is less difference between values of k_{1y} and k_{1z} and secondly that the significant height dependence of k_{1y} and k_{1z} exists.

Examples of decay constants from full scale measurements by Shiotani and Iwatani⁽⁶⁹⁾, Chuen⁽⁷⁰⁾ and Duchene-Marullaz⁽⁶⁷⁾ are plotted against a geometric mean height, z_m , in Figure 3-6; Chuen's results requiring some conversion to make them relative to a reference height wind speed $\bar{U}(10)$. Similar decay constants have been deduced from Harris' data obtained at Rugby⁽⁴⁹⁾ and are plotted in the same figure.

Suitable power law exponents α_{Dy} and α_{Dz} for each set of data were estimated by means of the least square method and are listed in Figure 3-6 together with the power law exponent, α , for the mean wind speed profile. Decay constant values plotted in the figure are based on the root-coherence except Harris' data and so they are slightly underestimated for the co-coherence function.

It appears rather difficult to establish a general formula for the decay constants only from these data. However it may be summarised that the decay constant at $z_r = 10$ m is smaller the rougher the terrain and the power law exponent, α_D , increases with terrain roughness. The difference between the horizontal decay constant and the vertical one is very small as far as examples in Figure 3-6 are concerned and a suggested expression for the decay constants may be written as follows,

$$\left. \begin{aligned} k_y(z) &= 7.0 \left(\frac{z}{z_G} \right)^{-\alpha_{Dy} + \alpha} \\ k_z(z) &= 6.0 \left(\frac{z}{z_G} \right)^{-\alpha_{Dz} + \alpha} \end{aligned} \right\} \quad (3-21)$$

and so

$$\left. \begin{aligned} k_{1y}(z) &= k_y(10) \left(\frac{z}{10} \right)^{-\alpha_{Dy}} = 7.0 \cdot \left(\frac{10}{z_G} \right)^{-\alpha_{Dy} + \alpha} \left(\frac{z}{10} \right)^{-\alpha_{Dy}} \\ k_{1z}(z) &= k_z(10) \left(\frac{z}{10} \right)^{-\alpha_{Dz}} = 6.0 \left(\frac{10}{z_G} \right)^{-\alpha_{Dz} + \alpha} \left(\frac{z}{10} \right)^{-\alpha_{Dz}} \end{aligned} \right\} (3-22)$$

$$\alpha_{Dy} = \alpha_{Dz} = 0.3 + \frac{\alpha}{2} \quad (3-23)$$

Typical examples of equation (3-22) and (3-23) for cases $\alpha = 0.15$ and 0.33 are plotted in Figure 3-6 in comparison with Vickery's suggestion⁽¹⁸⁾. Since the simple average mean wind speed was employed in Vickery's expression equation (3-15) the value for vertical decay constant will be comparable with one defined in equation (3-17) only for a case with relatively small separation. Although individual full scale plots in Figure 3-6 may not be well represented by equations (3-22) and (3-23) tendencies found from those sets of plots can be satisfactorily explained by the suggested equations.

Values obtained from the Nantes area seem to be rather smaller than those deduced from the suggested expressions, but some previous findings in similar rough terrains indicate greater values than those from Nantes; eg, $k_{1y} \approx 7.7$ at $z \approx 125$ m (α assumed to be 0.35) was reported in the windward surface pressure measurements of a tall building by Dalglish et al⁽⁷¹⁾ and $k_{1z} \approx 5.0$ at $z_m \approx 100$ m from the Tokyo Tower

Empirical data

vertical	α_{Dz}	horizontal	α_{Dy}	α	ref.
Δ	0.47	—	—	0.17	(49)
$\circ$	0.45	$\oplus$	—	0.14	(69)
$\square$	0.40	$\boxplus$	0.68	0.19	(70)
$\diamond$	0.62	$\diamond$	0.69	0.33	(67)

Suggested formula equation(3-22) and (3-23)

vertical	horizontal	α	$\alpha_{Dz} = \alpha_{Dy}$
1 ———	2 ———	0.33	0.47
3 ———	4 ———	0.15	0.38
1 - - - -	2 - - - -	0.33	} Vickery(18)
3 - - - -	4 - - - -	0.15	

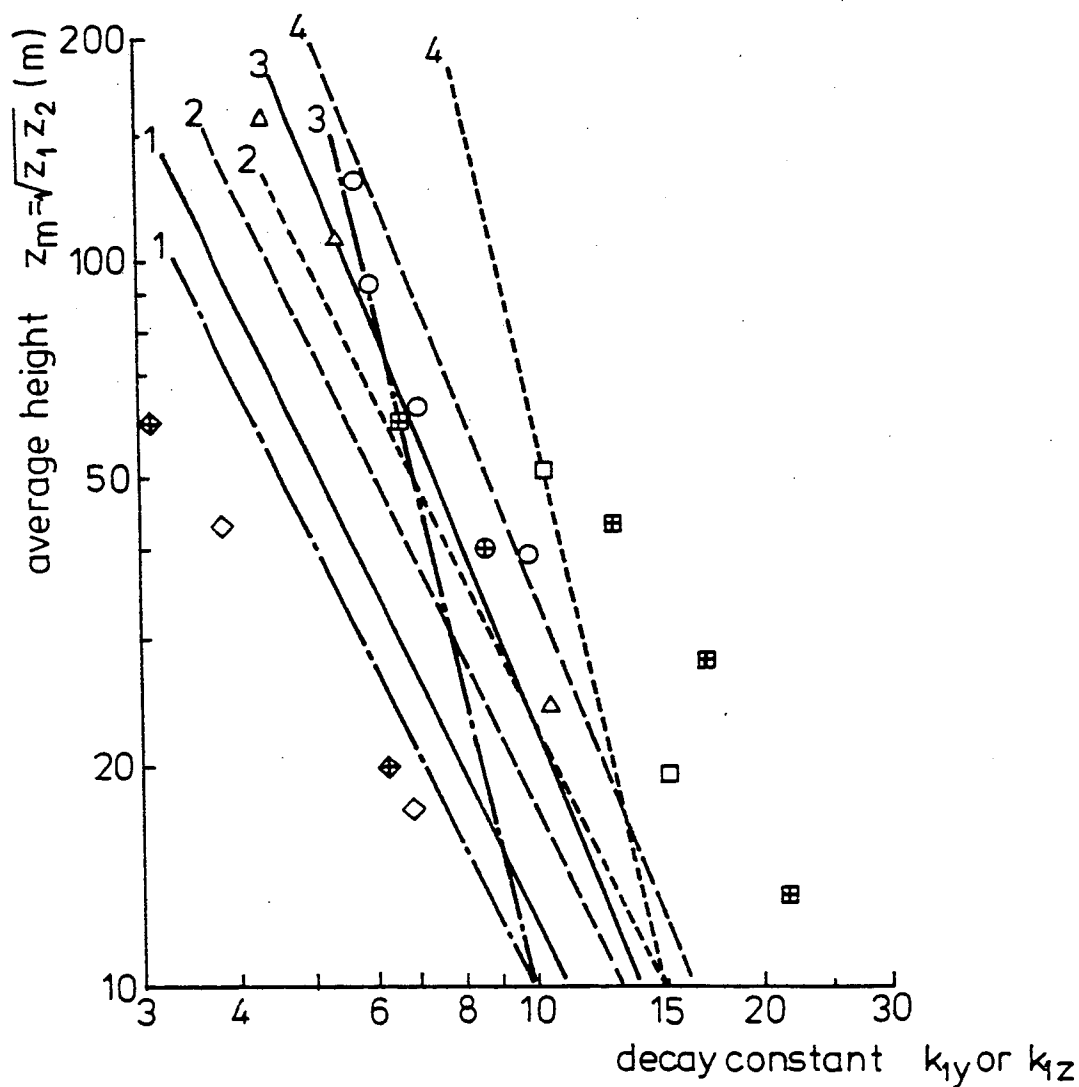


Figure 3-6 Variation of decay constants with height

measurements by Soma⁽⁵⁸⁾. The latter value fits fairly well to the suggested equations.

The decay constant values at gradient height (7.0 and 6.0 for k_y and k_z respectively in this case) are also rather hypothetical ones due to the lack of information at a very high level in the boundary layer.

It is interesting to note the closeness of vertical and horizontal decay constant values from recent empirical data in Figure 3-6, contrasting them with those in previous works such as,

$$k_{1z} = 7.0, \quad k_{1y} = 20.0 \text{ due to Davenport}^{(5)} \text{ and}$$

$$k_z = 10.0, \quad k_y = 16.0 \text{ due to Vickery}^{(18)},$$

the latter also as shown in Figure 3-6.

3.5 TURBULENT WIND IN A PARTIAL BOUNDARY LAYER WIND TUNNEL

Prior to the wind tunnel experiments, wind speed measurements were carried out to establish the wind characteristics in the tunnel to be used. Although the foregoing mathematical model was deduced basically from full scale measurements, it is a tool to discuss the wind characteristics in the tunnel relative to those in nature. The results are presented in terms of the formulae suggested in the previous sections.

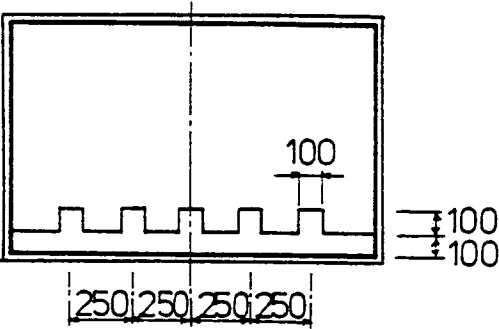
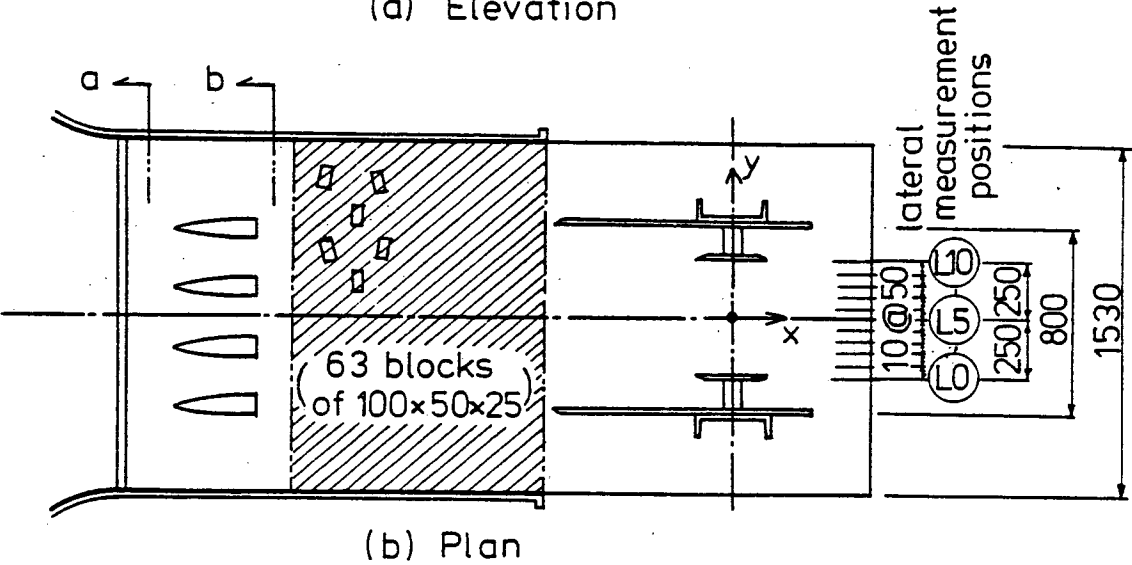
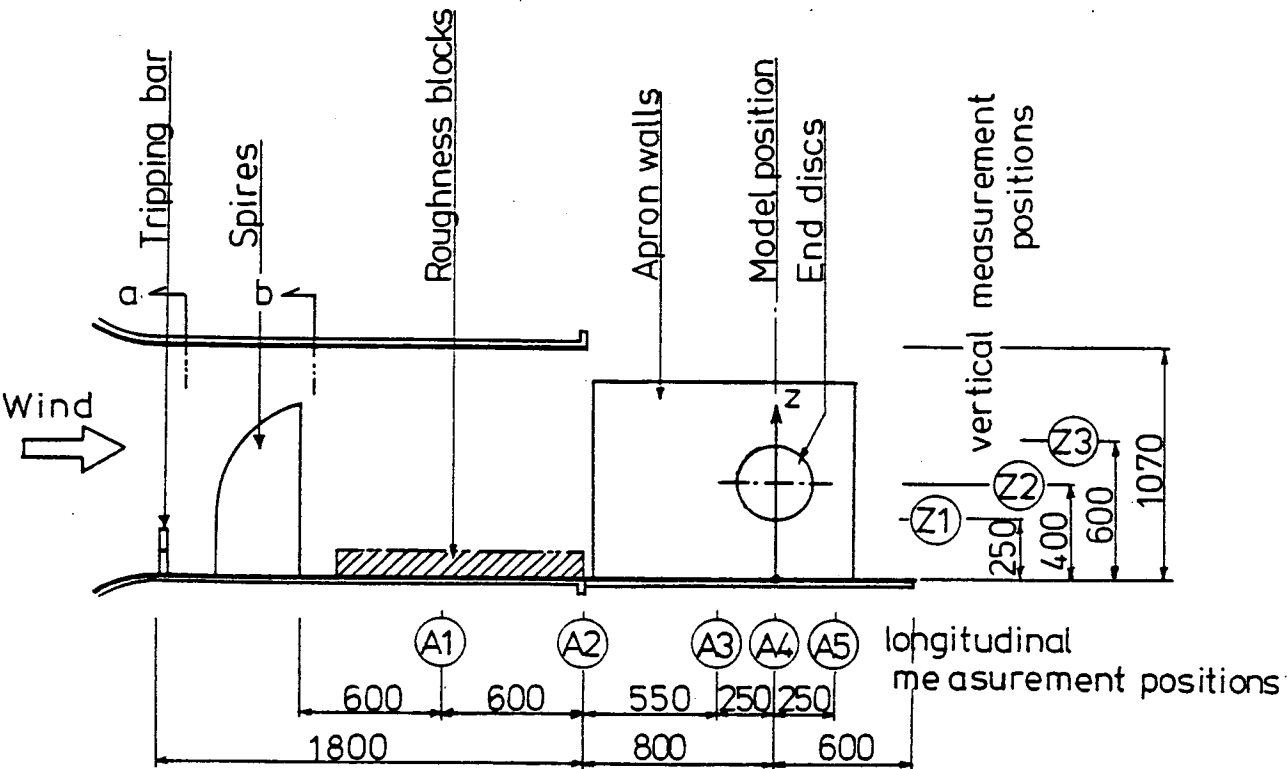
3.5.1 Wind Tunnel Configuration and Instrumentation

The wind tunnel in the Department of Civil Engineering and Building Science, University of Edinburgh, is an open jet type with an apron attached to the outlet. An overall sketch is shown in appendix 2. The total length is 8.9 m and the dimensions of outlet are: width = 1.53 m and height = 1.07 m. The maximum velocity of the airflow was continuously adjustable up to 9.0 m/s.

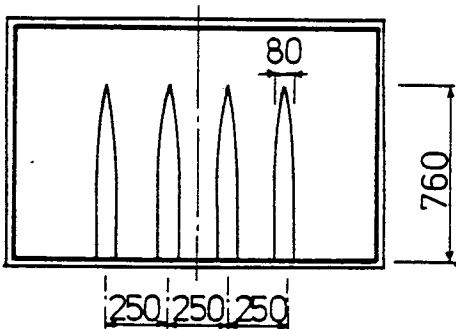
For this type of relatively short fetch tunnel it is recommended that a spire-roughness method^(72,73) should be employed in order to create a shear velocity profile with a reasonable exponent and simulated turbulence with its scale as large as possible.

There is an alternative method of generating large scale turbulence by means of coarse grids, especially for a two-dimensional model study, as demonstrated by Bearman⁽³²⁾ and Lee^(43,44). However the former method was preferred since the process of generating turbulence in a boundary layer is similar to the actual atmosphere and so it was hoped to provide a better representation of the natural turbulence in this way.

The arrangement of a trip bar, spires and roughness blocks in the tunnel section are illustrated in Figure 3-7 together



(c) Section a



(d) Section b

unit: mm

Figure 3-7 Wind tunnel arrangement

with the measurement position arrangement. Five longitudinal positions were chosen in order to examine the development of the wind profile; A1 at 600 mm downstream from the spires, A2 at 1200 mm downstream from the spires, A3 at 250 mm upstream of the model centre, A4 at model centre (2000 mm from the spires) and A5 at 250 mm downstream of the model centre.

Turbulence produced by a similar roughness arrangement has been applied to the investigation of dynamic response of cantilever structures⁽⁷⁴⁾.

The measurements were carried out at the start of the two-dimensional model experiment (chapter 4) with the model parts removed, ie, with the apron walls, frames to support the model and end discs which are shown in Figure 3-7.

The longitudinal mean wind speed was measured by means of a pitot-static tube (2.5 mm diameter) with a micromanometer (Furness Controls Ltd) at five longitudinal (A1 to A5), eleven lateral (L0 to L10) and three vertical positions (Z1, Z2, Z3) to find out the overall state of the flow (see Figure 3-7).

The vertical profile for the wind speed was measured by a miniature hot wire probe (DISA type 55 P11) with an associated anemometer (DISA type 55 D05) and lineariser (DISA type 55D15) at five longitudinal positions on the tunnel centre line. The calibration was repeated with the pitot-static tube system before

and after each hot wire probe measurement. The output from the lineariser was split into two parts - one passing to a damping circuit to yield the mean signal and the other to an analogue correlator (DISA type 55 D70) to provide the r.m.s. signal and both were recorded on a pen chart recorder (Watanabe six-channel type).

Measurements for correlation and spectral analysis were made around the model centre position. The space correlation coefficients in both vertical and lateral directions were obtained from the correlator mentioned above using two anemometer probes and circuitry. The auto-correlations were obtained similarly from the correlator with a time delay unit (DISA type 55 D75) and a sweep drive unit (DISA type 52 B01).

Signals from the two probe circuits with various separation distances at two steps of wind speed were recorded simultaneously by a data recorder (RACAL STORE 14). Recorded data were digitised in two different ways, one at sampling rates of 200 and 100 Hz for each set of data and the other using simultaneous digitisation of two channels at a sampling rate of 312.5 Hz. Both digitisations were made by a PDP 11 system. The former gave 2048 digitised data for each channel from within a sampling period of approximately 10 and 20 sec respectively and the latter gave 10240 digitised data for each channel from within a sampling period of approximately 33 sec. A computer program for obtaining the mean and r.m.s. values, power spectral density,

and real and imaginary parts of normalised cross-spectral density was developed by employing an established Fast Fourier Transform (FFT) subprogram⁽⁷⁵⁾. The program and its flow chart are shown in appendix 3. Normalised standard errors estimated⁽⁷⁶⁾ were 25% and 16% for 2048 data input and 10240 data input respectively. Output covered frequency components from approximately 0.3 Hz to 50 Hz for 20 sec of 2048 data, from 0.6 Hz to 100 Hz for 10 sec of 2048 data and from 0.5 Hz to 160 Hz for 33 sec of 10240 data.

3.5.2 Mean Wind Speed and Turbulence Profile

Overall profiles of mean wind speed in the tunnel section are presented in Figure 3-8. Decelerated portions can be clearly observed at positions A1 and A2, but around the model position the profile is satisfactorily uniform except very near the end discs (lateral positions L0, 1 and 9, 10 in Figure 3-8).

The vertical profile is a typical partial boundary layer profile as shown in Figure 3-9. The profile around the model position appears to be made up of two layers; the upper layer is reasonably stable with a power law index $\alpha \approx 0.33$ up to $z = 600$ mm but the lower layer is still in the developing state with a lower value of $\alpha \approx 0.18$.

The roughness length, z_0 , and shear velocity, u^* , were obtained by fitting $\bar{U}(z) = 2.5 u^* \ln \left(\frac{z}{z_0} \right)$ to the data and

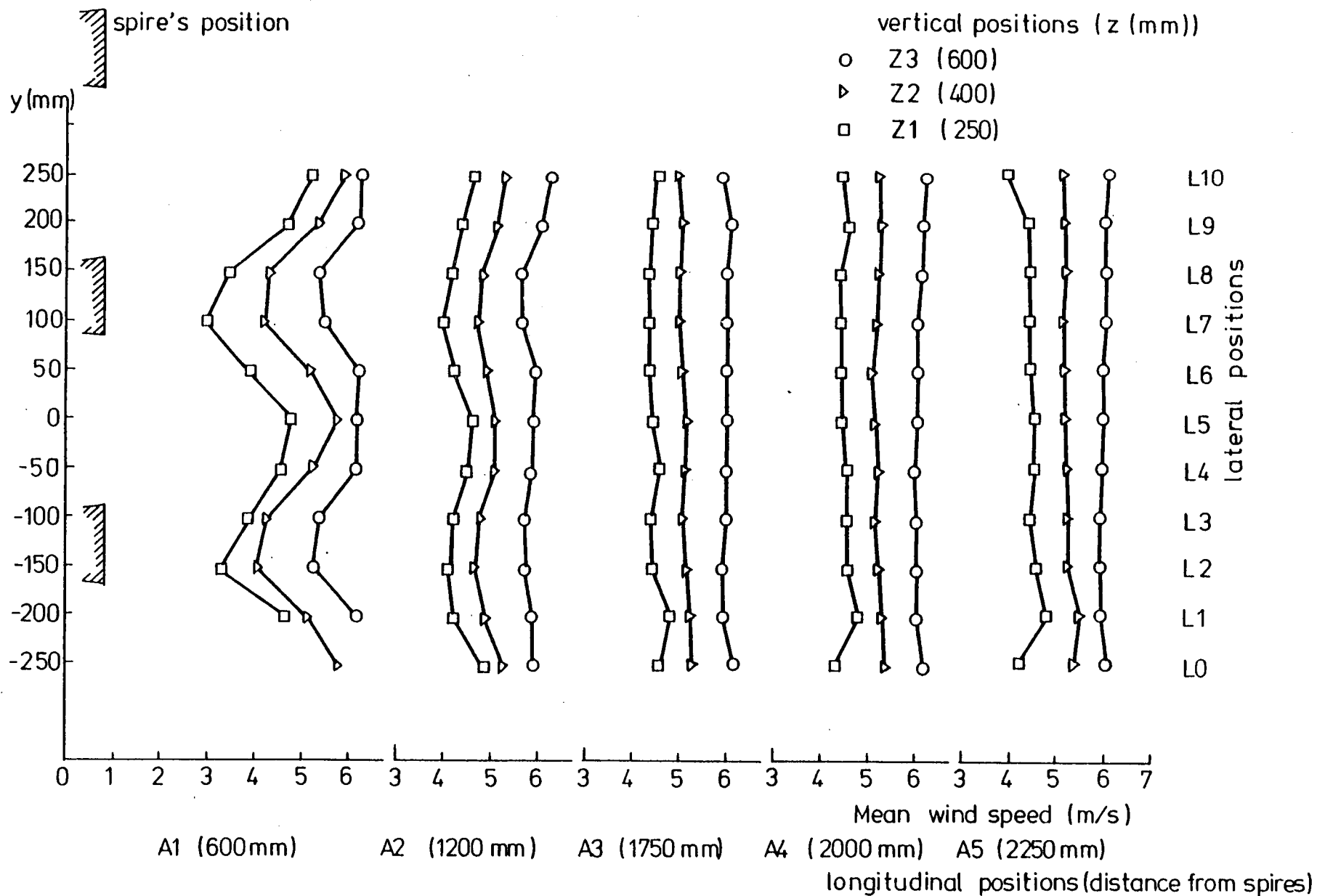


Figure 3-8 Lateral profiles of mean velocity in wind tunnel

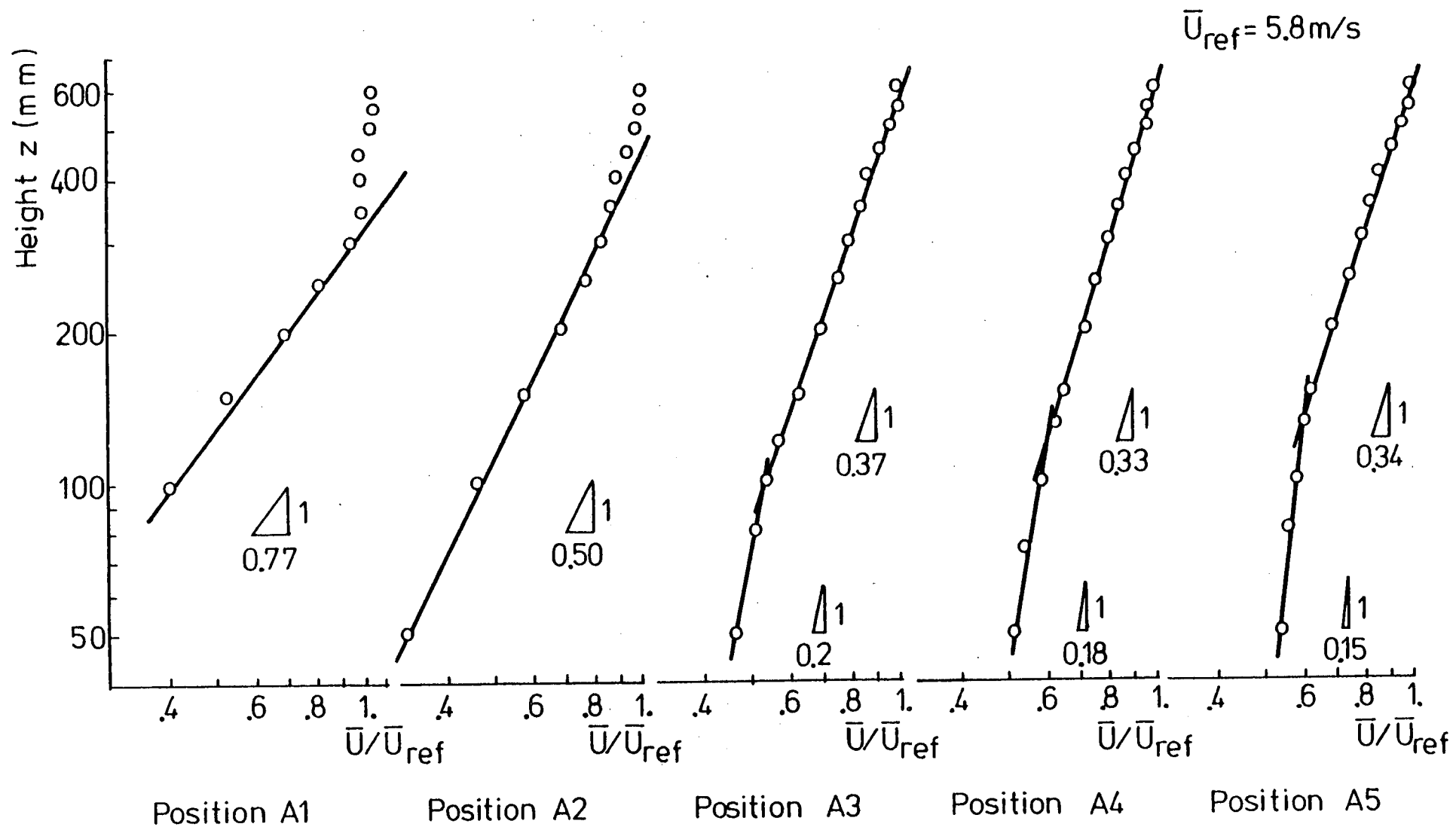


Figure 3-9 Vertical profiles of mean velocity in wind tunnel

found to be,

$$z_o = 6.0 \text{ to } 9.0 \text{ mm at } u^* = 0.5 \text{ to } 0.9 \text{ m/s}$$

for the upper layer profile at the model centre position.

By comparison with the full scale value $z_o = 2.0 \text{ m}$ for $\alpha = 0.33$ (see equation (3-2)) the scale reduction ratio $1/\lambda \approx 1/300$ can be estimated for this upper portion of mean wind speed profile, while from the corresponding value of $z_G = 1000 \text{ m}$ (in full scale) to the boundary layer thickness (600 mm) $1/\lambda \approx 1/1500$ may be deduced. However as noted in section 3.2, suggested values of z_G differed from case to case^(56,61) and the boundary layer thickness in this tunnel may not correspond directly to the gradient height in full scale since the shear velocity profile was created not only by the surface roughness as in nature but also by spires. Therefore consistent scale reduction ratio values may not be strictly required for these parameters z_o and z_G .

The turbulence profiles are presented in terms of the local turbulence intensity in Figure 3-10. A peculiar profile at position A1 generated by spires becomes reasonably smoothed at positions A3 to A5. The order of turbulence intensity is slightly smaller than that occurring naturally but similar to that at higher levels (say $z \geq 100 \text{ m}$) over a rough terrain⁽⁵⁶⁾.

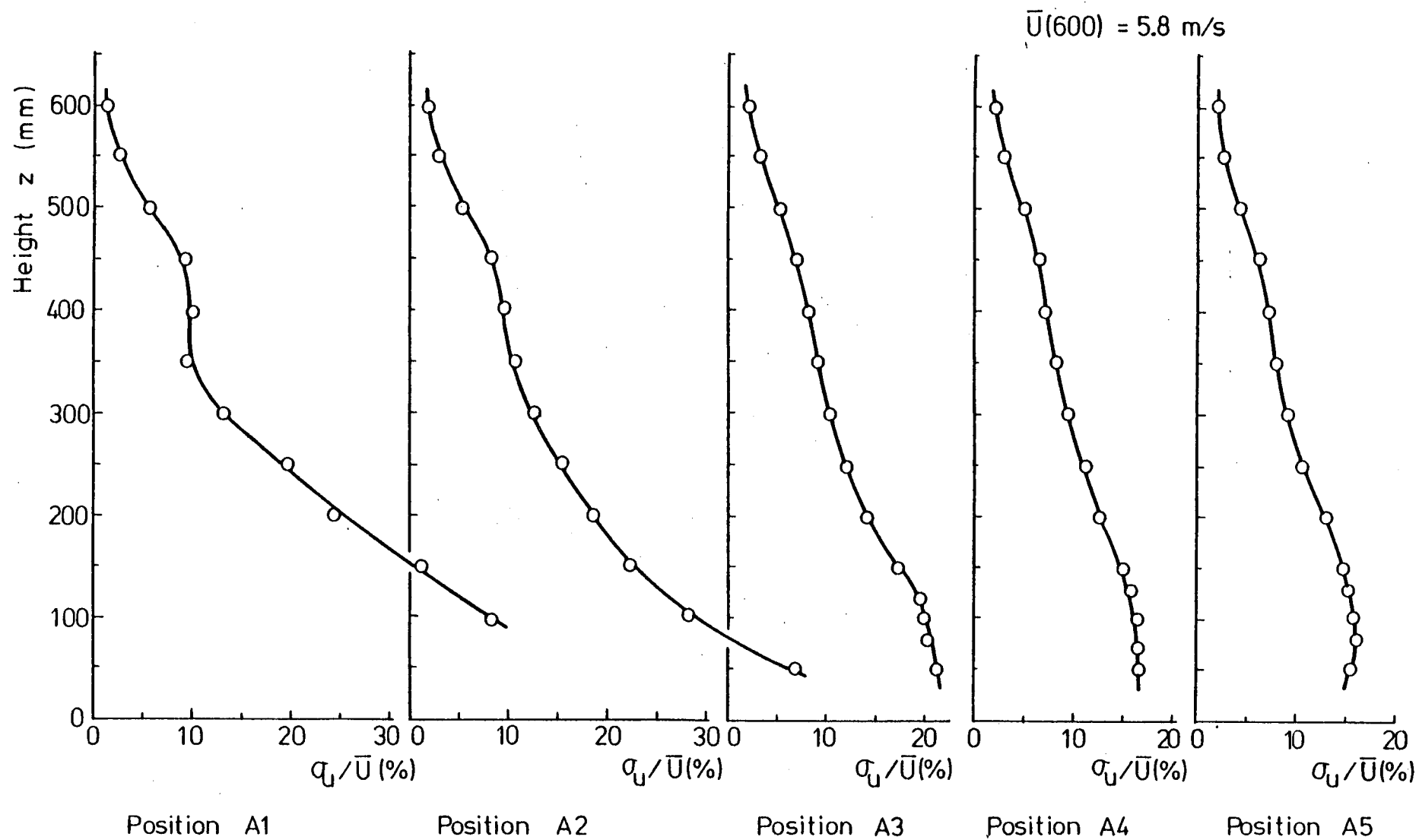


Figure 3-10 Local turbulence intensity profiles in wind tunnel

It is evident from the results that the turbulence intensity rapidly decreases with height. It appears to decrease more rapidly than that of the natural wind; eg, $\alpha_T \approx 0.25$ between $z = 150$ to 400 mm whereas $\alpha_T = 0.08$ in equation (3-4).

In the along-wind direction the turbulence intensity also decreases. However the decreasing rate around the model position at vertical positions Z1 ($z = 250$ mm) and Z2 ($z = 400$ mm) are approximately 0.3% per 10 cm and 0.15% per 10 cm respectively.

The values of turbulence intensity in Figure 3-10 were underestimated slightly since the analogue correlator used in these measurements employed a high pass filter with a nominal threshold of 0.5 Hz (eg, approximately 70% of the full scale signal at 1.0 Hz was passed - see calibration curve in appendix 2 Figure A2-2). Therefore the digital spectral analysis was carried out using the direct tape recordings of the wind speed, ie, the average value of $\sigma_u(z)/\bar{U}(z)$ was 14.0% at Z1 and 9.9% at Z2 of the model centre position A4. Differences due to mean wind speed levels (approximately 3 m/s and 6 m/s at $z = 600$ mm) were found to be approximately 0.4% for the former and 0.6% for the latter and in both cases the lower wind speed made the turbulence intensity slightly larger.

3.5.3 Power Spectral Density Distribution

Power spectral density distributions of the longitudinal turbulence component were computed for various heights and lateral positions around the model centre. Typical examples are presented in Figure 3-11 (a) position 1 ($z = 250$ mm), and (b) position 2 ($z = 400$ mm). Results were determined from 10240 data obtained from each of two probes at any one level.

Best fit curves expressed by equation (3-9) with $\beta = 2$ are shown in the same figure. Within a satisfactory deviation all results are well represented by equation (3-9) with suitable length constant $L(z)$. Differences due to wind speed levels were insignificant and differences between power spectral density plots at different lateral positions were also insignificant within the examined range (± 100 mm from the model centre position L5). Computational results from the two different digitisation systems were also very alike, although the larger the number of sectors within a record data, the less scattered were the results.

The length constant $L(z)$ obtained from the best fit power spectral form is plotted in Figure 3-12. It is clearly seen that the length constant decreased with height between 150 and 500 mm in this experiment, whereas the full scale measurement results indicated an opposite trend as noted in the length constant profile given by equation (3-9). This peculiarity was presumably caused by the turbulence generating system.

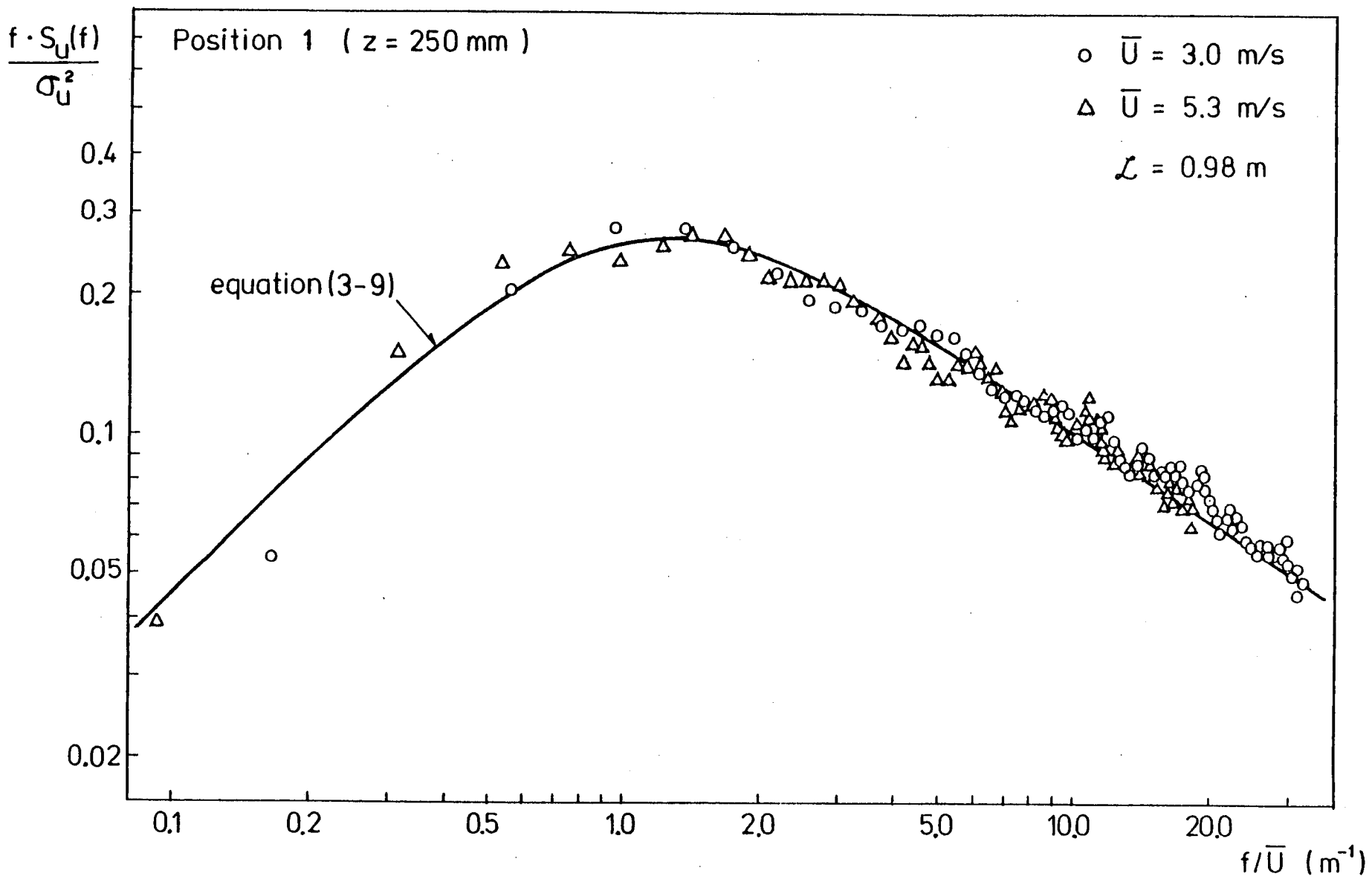


Figure 3-11(a) Height variation of reduced power spectra in wind tunnel around position Z1

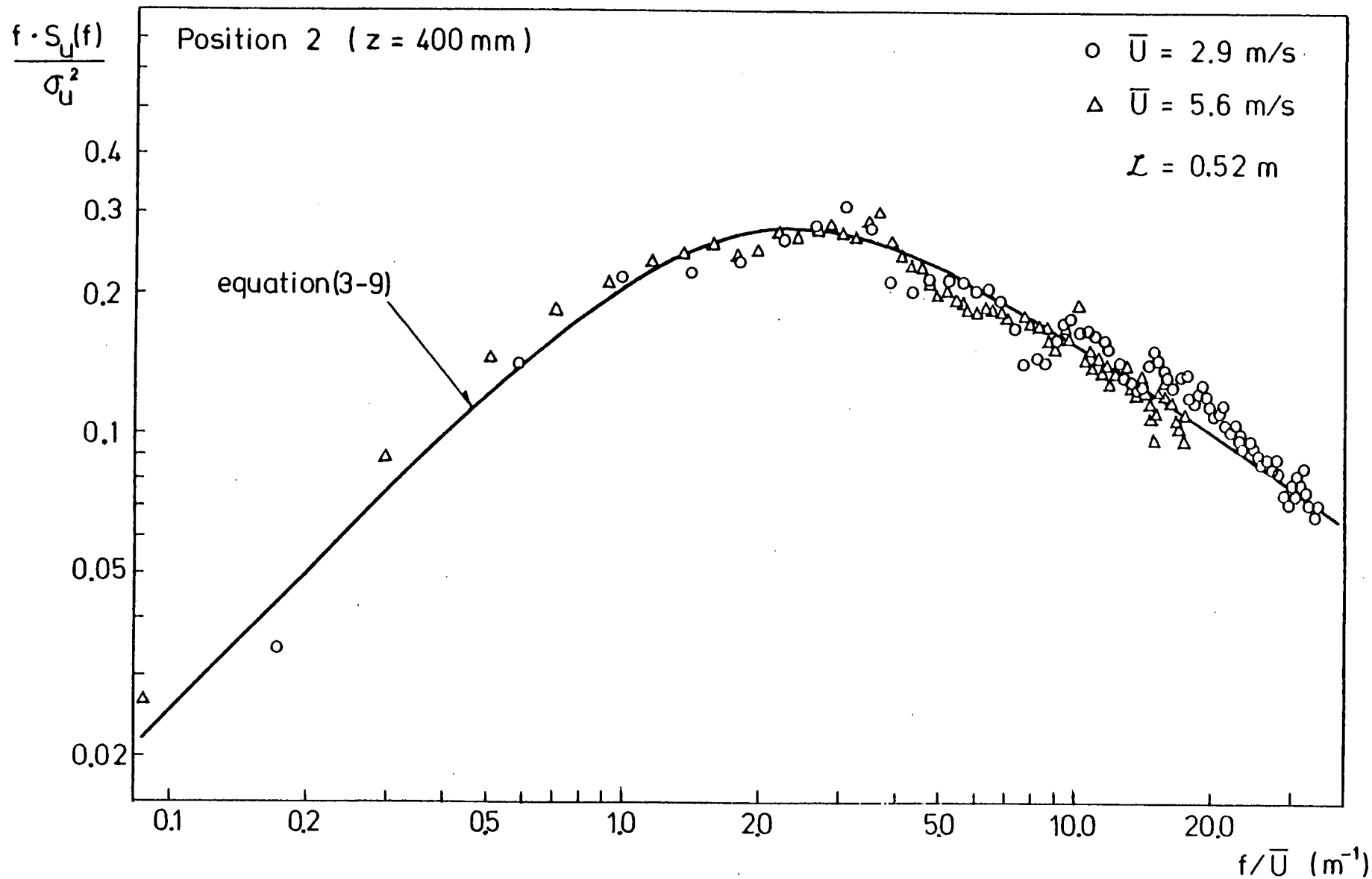


Figure 3-11(b) Height variation of reduced power spectra in wind tunnel around position Z2

The upper portion of a spire ($z \geq 400$ mm) had a width of the order of 50 mm while the width of the lower portion approached 80 mm and blocks on a trip bar near to the tunnel floor had a dimension of 100 mm. These values are similar to the turbulence scales at corresponding levels obtained from $L(z)$ and equation (3-10), ie, assuming a homogeneous isotropic condition $L_x = 115$ mm at Z1 and 61 mm at Z2 with $L(Z1) = 980$ mm and $L(Z2) = 520$ mm respectively.

From a comparison with corresponding length constant values in a full scale mathematical model given by equations (3-9) and (3-13), ie, $L(z) = 4300$ m at $z = 0.8 z_G$, $L(z) = 2400$ m at $z = 0.25 z_G$ for $\alpha = 0.33$, estimated scale reduction ratio $1/\lambda$ is approximately,

$$1/8000 \quad \text{at } z \approx 500 \text{ mm} \quad \text{and} \quad 1/1800 \quad \text{at } z \approx 150 \text{ mm}$$

in the wind tunnel at model centre position A4.

3.5.4 Correlation and Coherence Functions

The turbulence scale can be obtained from the integral of the space correlation coefficient. The longitudinal turbulence scale, L_x , was obtained from the auto-correlation coefficient based on Taylor's hypothesis, ie, as the product of mean velocity and time scale defined as the integral of the auto-correlation coefficient with respect to the lag time from

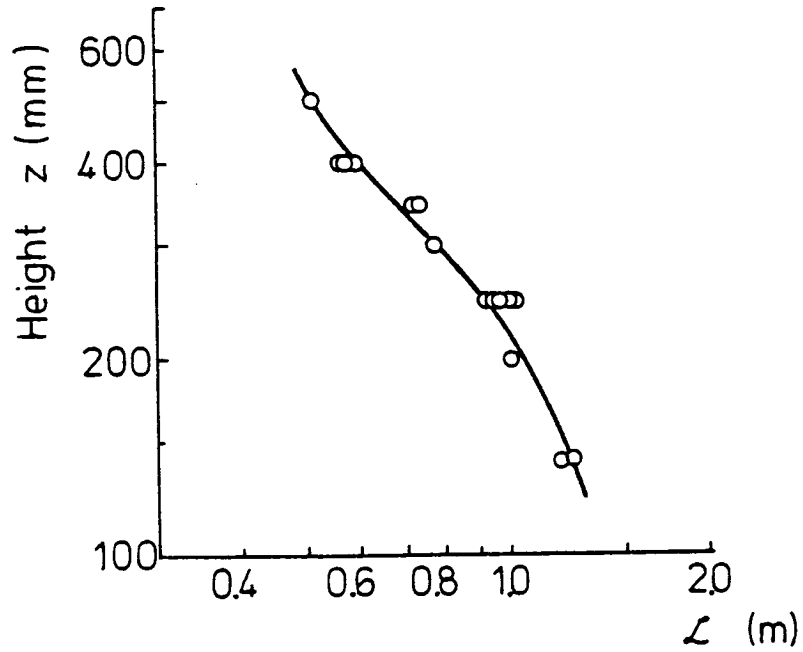


Figure 3-12 Height variation of length constant $L(z)$ in wind tunnel

Table 3-2 Turbulence Scales in Wind Tunnel (mm)

Position	L_x^*	L_x	$L_{z\uparrow}$	$L_{z\downarrow}$	L_y
Z2 (z = 400 mm)	61	65	35	39	28
Z1 (z = 250 mm)	115	105	48	49	38

note : L_x^* was obtained by fitting the power spectral density form.
 $L_{z\uparrow}$ indicates upward scale and $L_{z\downarrow}$ indicates downward scale.

zero to the first zero crossing point. The integration was achieved by means of a planimeter.

The longitudinal turbulence scale values found in this way were $L_x = 105$ mm and 65 mm at $z = 250$ and 400 mm respectively. These values are in satisfactory agreement with those deduced from the power spectral densities.

Lateral and vertical turbulence scales, L_y and L_z respectively, are obtained directly from the integral of space correlation (no lag time). Results are listed in Table 3-2. The ratio L_x/L_z is of the order of 2.0 and L_x/L_y is nominally 2.0 at the higher position Z2 and 3.0 at the lower position Z1.

In homogeneous isotropic flow those ratios become 2.0 and so it may be stated that the flow condition at the higher position Z2 was nearly homogeneous isotropic but at the lower level flow was anisotropic as might be expected in a natural turbulent flow near the ground.

Some typical examples of auto-correlation coefficients and space correlation coefficients are presented in appendix 2.

Real and imaginary parts of normalised cross-spectral densities, ie, co- and quad-coherence, were computed from wind speed measurement records around two positions; 1 (A4, L5, Z1) and 2 (A4, L5, Z2) of model centre A4 at two levels of wind speed. Imaginary components were small especially at the lower frequency range and since only real components have a dominant significance

on wind-structure interaction typical co-coherence results are presented.

Figure 3-13 shows computed co-coherence plots against frequency with (a) vertical separations in position 1, (b) horizontal separations in position 1, (c) vertical separations in position 2 and (d) horizontal separations in position 2. Results in Figure 3-13(a) are from the higher wind speed records and the others are from the lower ones.

Equation (3-17) may be rewritten for these co-coherence measurements as follows,

$$C_{u_1 u_2}(r_i, f) = \exp\left(-\frac{k_i(z_m)r_i}{\bar{U}(z_m)} \cdot \sqrt{\left(\frac{\bar{U}(z_m)}{k_2 L(z_m)}\right)^2 + f^2}\right) \quad (3-24)$$

where r_i is either vertical ($i = z$) or horizontal ($i = y$) separation distance and $z_m = \sqrt{z_2 \cdot z_1}$.

Firstly a co-coherence curve by equation (3-24) with a best fit decay constant $k_i(z_m)$ and a theoretical k_2 which is shown in Table 3-1 was obtained as shown in Figure 3-13. All values of $k_i(z_m)$ found are listed in appendix 2 Table A2-1. Then average values of $k_i(z_m)$ are calculated for the two positions 1 and 2, ie, $k_z = 10.8$ and $k_y = 10.8$ around position 1 and $k_z = 9.2$ and $k_y = 9.4$ around position 2.

Taking $k_2 = 2.0$ as a standard value, curves with average decay constants mentioned above are then superimposed on each

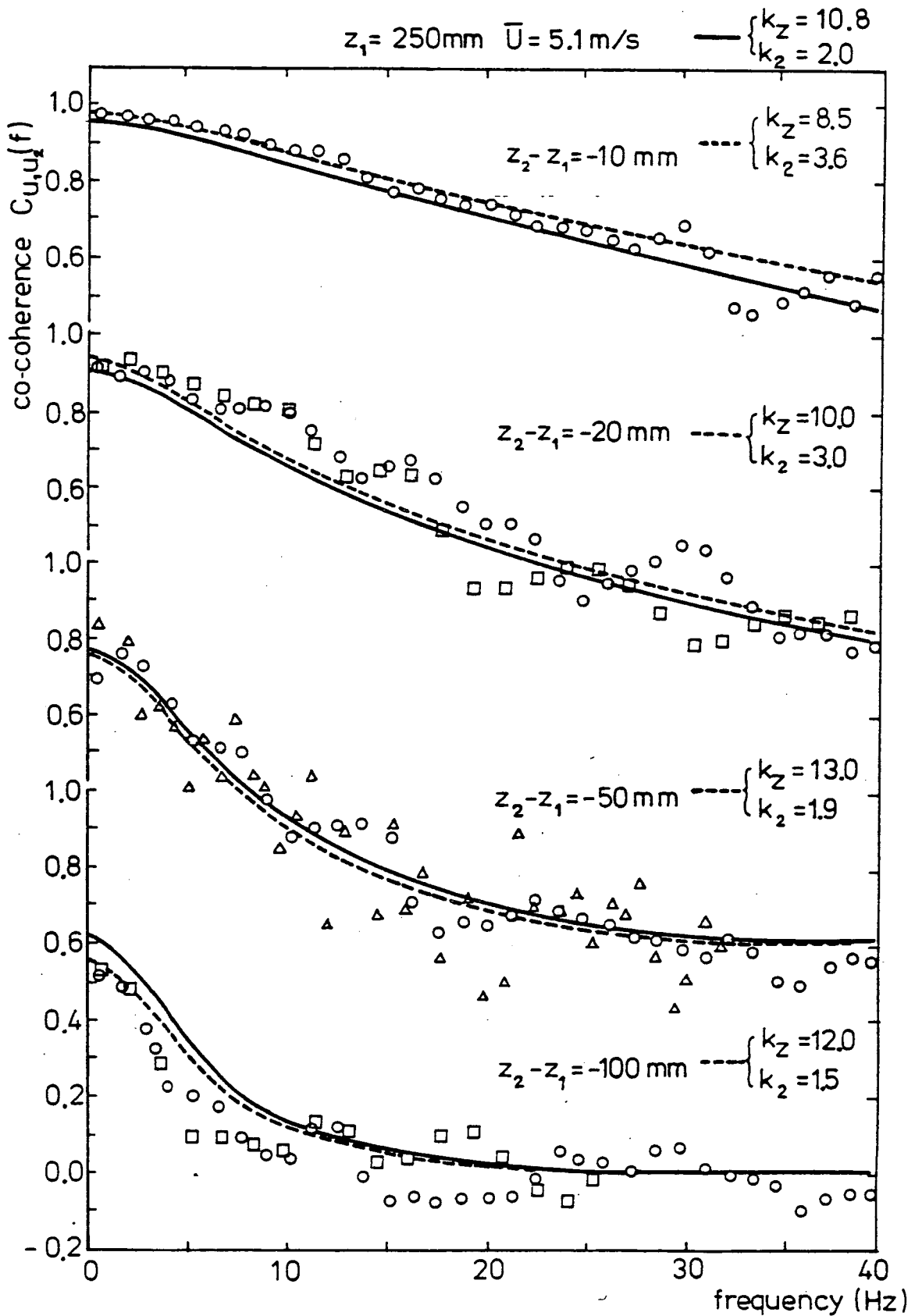


Figure 3-13(a) Co-coherence results in wind tunnel with vertical separations at position 1

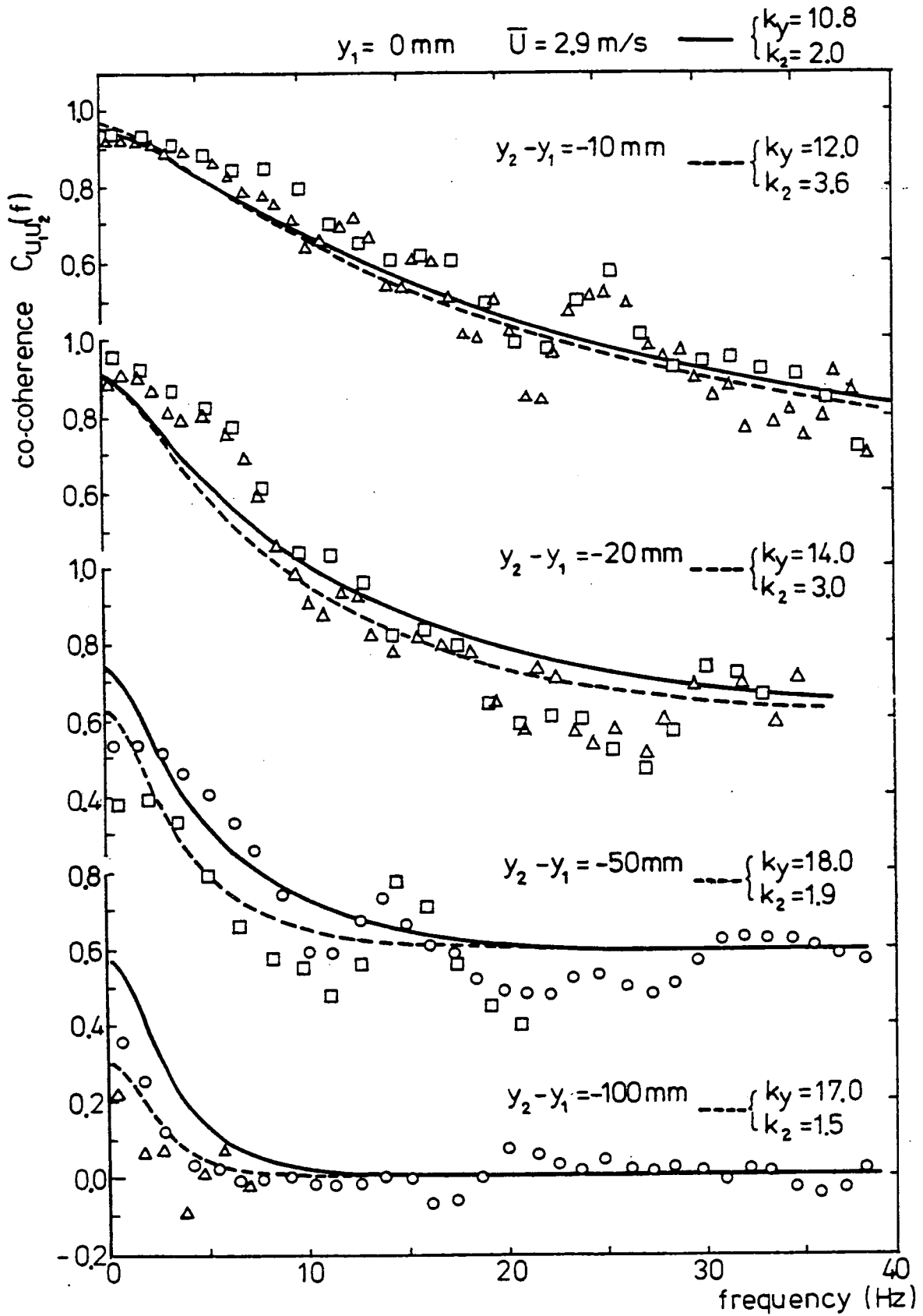


Figure 3-13(b) Co-coherence results in wind tunnel with horizontal separations at position 1

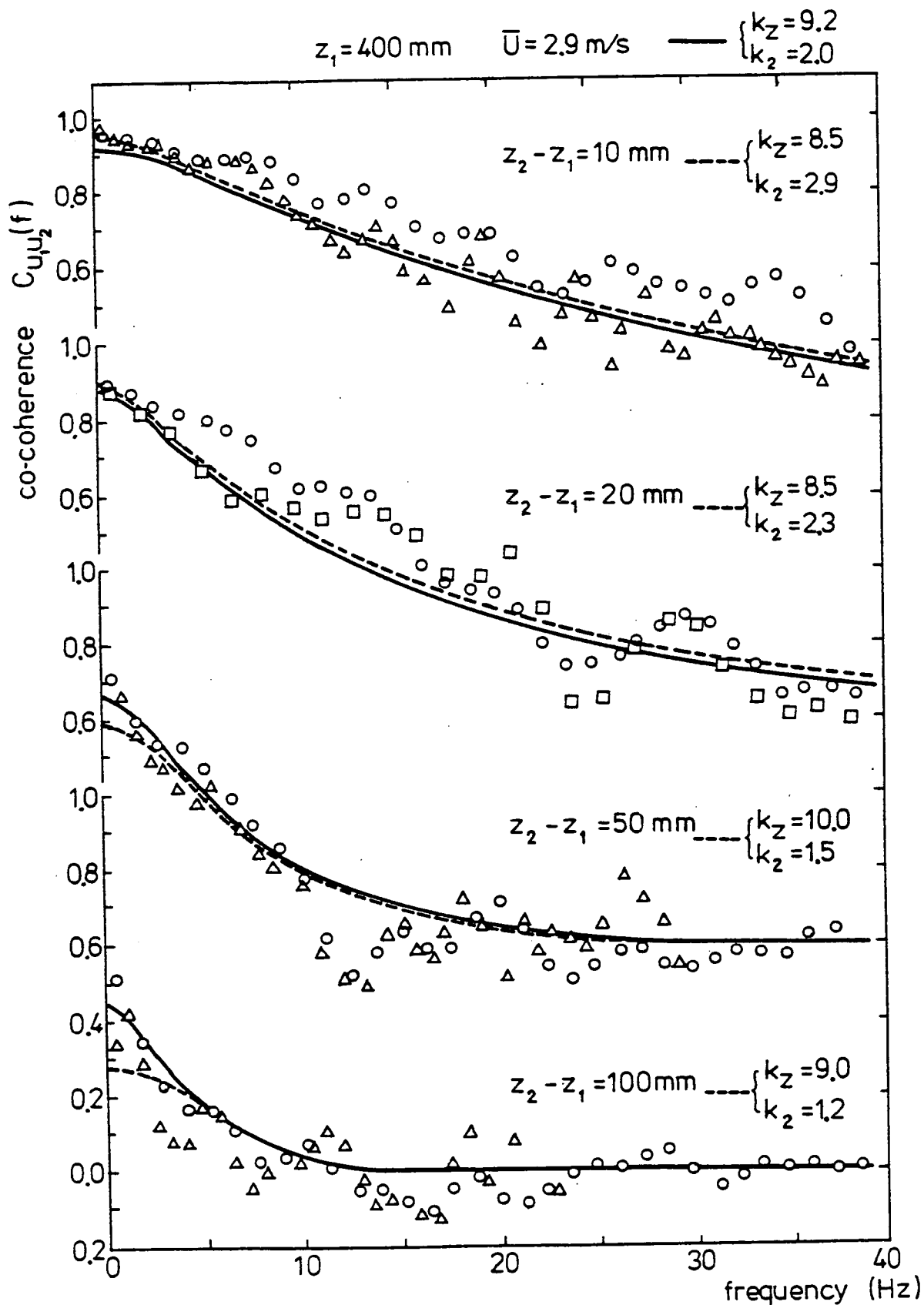


Figure 3-13(c) Co-coherence results in wind tunnel with vertical separations at position 2

Experimental results

- from 33sec 2×10240 data
- △ from 20sec 2×2048 data
- from 10sec 2×2048 data

Empirical curves equation (3-24)

- average k_z or k_y and $k_2=20$
- - - individual best fit

$y_1=0\text{ mm}$ $\bar{U}=2.9\text{ m/s}$ — $\begin{cases} k_y=9.4 \\ k_2=2.0 \end{cases}$

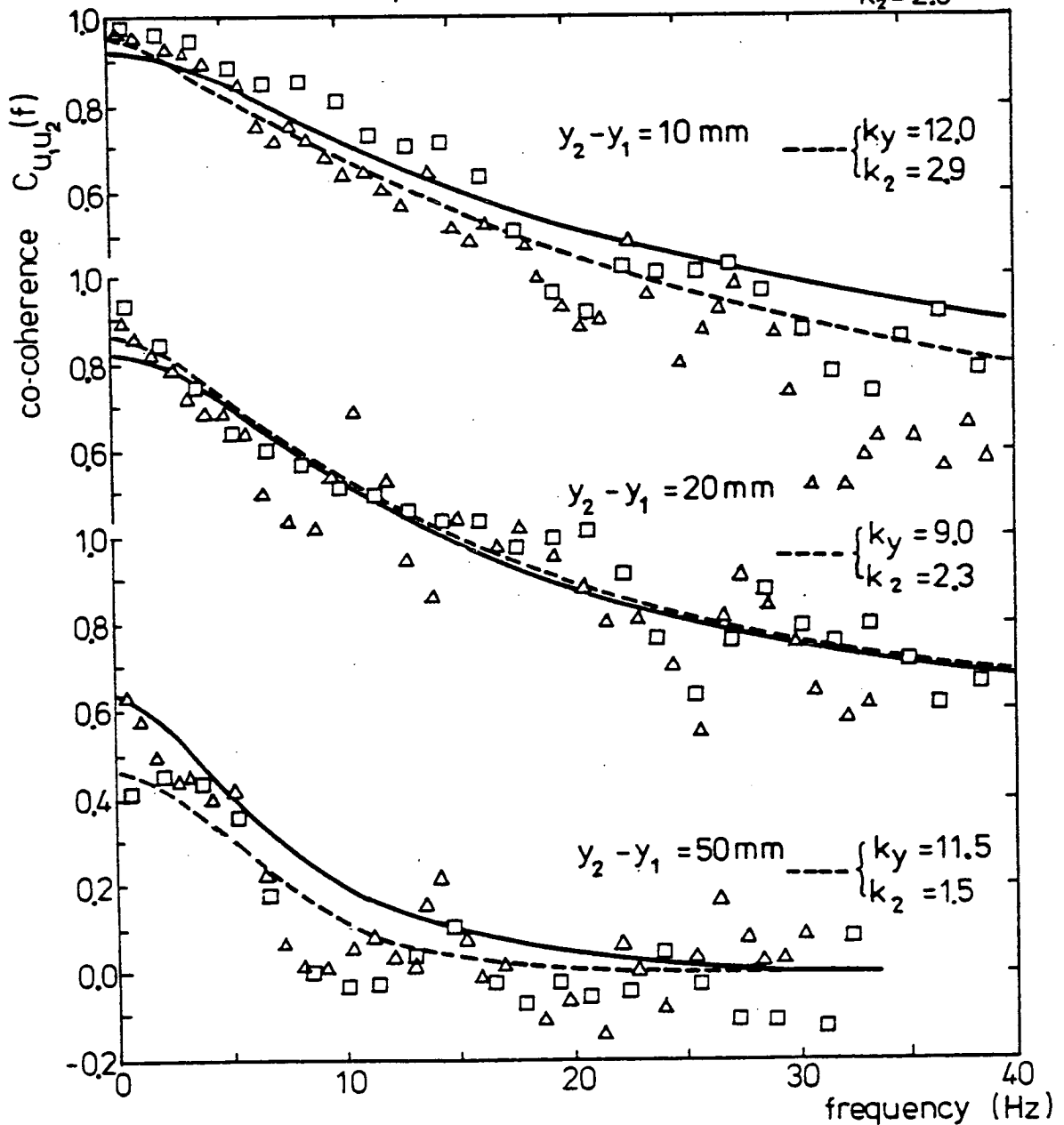


Figure 3-13(d) Co-coherence results in wind tunnel with horizontal separations at position 2

figure (Figure 3-13(a) to (d)). It can be seen that not only are the best fit curves in reasonably good agreement with experimental results but also the standard curves and so the latter can be considered to be applicable in the following experiment (chapter 4).

The zero frequency values of co-coherence are very well represented by equation (3-24) with a theoretical k_2 . Another commonly observed feature is that a smaller decay constant tends to give a better fit in the case of shorter separations at low frequencies. A similar tendency was pointed out by Newberry et al⁽²²⁾ in some full scale measurements. These results seem to confirm that Harris' theoretical expression equation (3-16) has some relevance in this partial boundary layer wind tunnel.

The results of large separation are considerably scattered and might be stated to have some peculiar form particularly in Figures 3-13(b) and (d). One of the main causes is due to the normalised standard error of the computation process which could be reduced by increasing the number of data or taking an average of more samples. However, results giving a low co-coherence value (eg, of the order of ± 0.2) tend to be much more scattered than those giving higher co-coherence values. This is observed commonly in full-scale measurements^(49,58,67). Some results⁽⁷⁷⁾ from grid-generated low turbulence indicate that experimental coherence plots are very well represented by equation (3-16) without such a large scatter.

3.5.5 Discussion

Based on the experimental evidence presented in the previous sections, it seems clear that the turbulence characteristics of natural wind are difficult to simulate fully in a partial boundary layer wind tunnel.

For example the mean wind speed profile seems to be made up of two layers as observed in other spire-roughness generated boundary layers⁽⁷³⁾. The lower portion of the layer ($z \leq 150$ mm) has a profile similar to that of smooth terrain ($\alpha \approx 0.18$). Although its effect on the response of tall structures may not be significant in comparison with that of the higher portion of the layer, this profile is not ideal for a three-dimensional cantilever type model experiment.

The higher portion of the layer has considerably smaller turbulence scale. For example the turbulence scale at $z \approx 500$ mm is less than a quarter of the size required by a similarity condition estimated from the lower portion of layer ($z \approx 150$ mm). This is because the decreasing trend of $L(z)$ or the turbulence scale, L_x , which is entirely opposite to the atmospheric turbulence characteristics (see Figure 3-5). Similar trends can be seen in the middle half portion of the profile for L_x obtained in the boundary layer in a larger fetch wind tunnel^(41,73), and also even in some full scale measurements, eg, in 0.05 to 0.12 times gradient height portion of length scale profile in Rugby measurement⁽⁵⁵⁾.

However as far as the middle half part of the layer (between $z \approx 150$ and 400 mm) is concerned the turbulence characteristics in this partial boundary layer can be representative of that of the natural wind over rough terrain ($\alpha = 0.33$) with a scale reduction ratio $1/\lambda \approx 1500$ to 2000 except for the decreasing trend of $L(z)$ with height mentioned above.

The experimental results for power spectral densities around $z_1 = 250$ mm (position 1) and 400 mm (position 2) fit well to a von Karman type spectral form given by equation (3-9). Also from the fact that the ratio between turbulence scales L_x/L_z and L_x/L_y are of the order of 2.0 , the homogeneous isotropic assumption seems to have some relevance. This suggests that the modified exponential expression equation (3-24), in which the zero frequency value is based on Harris' form of equation (3-16), may well be representative of the experimental results. Large amounts of experimental data on the co-coherence function have been examined and found to fit to equation (3-24) with appropriate k_2 and k_y or k_z values and most of them can be represented by a standard form with $k_2 = 2.0$ and an average value of k_z or k_y for positions 1 and 2.

Scatter in the results may be explained by the fact that the turbulence was not developed fully but still developing at the model centre position and that a direct influence of wakes behind vertical spires persisted. This could be the reason why

the peculiar wave form in the co-coherence is more significant in the case of horizontal separation. However, considering that the root- or co-coherence in full scale measurements is usually more highly scattered than that in grid generated turbulence, such a peculiarity or scattering of results at low coherence values may be a common feature of nonhomogeneous anisotropic turbulence with high intensity ($\sigma_u/\bar{U} \geq 10\%$) in a boundary layer.

The inconsistency of the simulated turbulence characteristics in the turbulent flow described in this chapter may not be recommendable for three-dimensional or fully simulated model studies such as required by approaches III and IV in Table 1-1. However since the power spectral density and co-coherence function of the longitudinal fluctuating component are stable and do not vary so rapidly in any direction around the model centre position, they can be represented by average formulae such as equations (3-9) and (3-24) with appropriate parameters within a satisfactory limit. Therefore the turbulence produced in this partial boundary layer is considered to be applicable to certain types of two-dimensional model study.

More research has to be performed in order to produce better simulated turbulence presumably by solving the problems of insufficient roughness fetch and turbulence scale, which is needed for more general applications.

CHAPTER 4 EXPERIMENTAL EVALUATION OF TWO-DIMENSIONAL
DYNAMIC ALONG-WIND FORCE COEFFICIENT FOR
BLUFF BODIES

4.1 INTRODUCTION

The drag coefficient is one of the most important factors in an investigation of the interaction mechanism between wind speed and along-wind forces. As the results of the empirical achievements of previous investigators some factors have been taken into account to determine the drag coefficient. For example the values of drag coefficient are specified in terms of the aspect ratio H/B (height by width) and the section aspect ratio D/B (depth by width) in the British Code of Practice⁽²⁾ as well as in most other countries' standard codes. However, those values have been established basically from smooth flow wind tunnel experiments and the turbulence effects have seldom been taken into account in the most current design procedures as far as the drag coefficient is concerned.

MacLaren et al⁽⁴²⁾ and Lee^(43,44) have presented their experimental results regarding the turbulence effects on the drag coefficients which revealed not only that the turbulence intensity whose effects are compared with the information of the Engineering Sciences Data Unit (1971)⁽⁷⁸⁾ but also the turbulence scale has very significant effects on the determination of drag coefficient values.

When the dynamic response of a structure is discussed there are more factors involved in the evaluation of dynamic along-wind forces. They are, for example, the mass coefficient, the along-wind correlation of fluctuating pressures and the vortex shedding effects, and furthermore the variation of drag coefficient itself with reduced frequency could be of significance. In order to take all

of those factors into account the dynamic along-wind force coefficient concept was employed in this research as described in chapter 2, and evaluated experimentally by using a two-dimensional S.D.O.F. system model as a transducer.

In the following sections the theoretical development defines the dynamic along-wind force coefficient for this experiment (section 4.2) and the outline of experiment shows the range of various parameters to be taken into account (section 4.3). Some parts of the basic description of the transducer for determination of the coefficient have already been published as a paper by Royles and the author (October 1978); see appendix 4 paper 3.

Experimental data were reduced by computer into the static drag and the dynamic along-wind force coefficients and compared with some existing results available (section 4.4). The validity of the results is also discussed from the aspect of possible errors caused during testing and finally results are reduced to a simple general expression of the dynamic along-wind force coefficient (section 4.5).

4.2 DYNAMIC DRAG RESPONSE OF S.D.O.F. MODEL

This section describes a basic theory for the wind tunnel experiment to determine the static drag and the dynamic along-wind force coefficients by means of a two-dimensional S.D.O.F. model as a transducer.

The model has a single degree of freedom in the along-wind direction with a natural frequency f_0 and a damping ratio ζ_S . The equation of motion under a turbulent wind force $F(t)$ can be written according to the theory described in section 2.2. From equation (2-15),

$$\ddot{\Delta}(t) + 2\zeta_T(2\pi f_0) \dot{\Delta}(t) + (2\pi f_0)^2 \Delta(t) = \frac{F(t)}{M} \quad (4-1)$$

where $\Delta(t)$ is the displacement of the model in the along-wind direction;

M is the mass of the model;

$$\zeta_T = \zeta_S + \zeta_A;$$

ζ_A is the aerodynamic damping and

$$\zeta_A = \frac{\rho C_D \bar{U} A \cdot B}{4\pi f_0 M} = \frac{\rho C_D \tilde{U}}{4\pi \gamma}, \quad \tilde{U} = \frac{\bar{U}}{Df_0} \quad (4-2)$$

A , B and D are the lateral, vertical and along-wind dimensions of the model as shown in Figure 4-1; namely, the part of two-dimensional cylinder (length = A) is to be the dynamic model to respond against the wind. The remainder of it is fixed and causes the two-dimensional flow around the model.

In the general theory the additional mass term is taken into account but it was neglected in this experiment since the air mass density, ρ , was much smaller than the average model mass density, γ ,

($\approx 300 \times \rho$). The drag coefficient used in equation (4-2) should be the dynamic one but it is not known beforehand and so the static drag coefficient was used to obtain the theoretical aerodynamic and total damping ratio of equation (4-1) conventionally.

The characteristics of the turbulent wind were assumed to satisfy a condition that the changing of the mean wind speed, $\bar{U}$, turbulence intensity, $\frac{\sigma_u}{\bar{U}}$, turbulence scale, L_x , horizontal and vertical decay constants k_y , k_z and the form of power spectral density $S_u(f)$ are small within the dimensions of model so that the effects of variation of those factors are negligible.

Following the same procedure developed from equation (2-15) to (2-45), the power spectral density of the fluctuating force on the model can be expressed as follows,

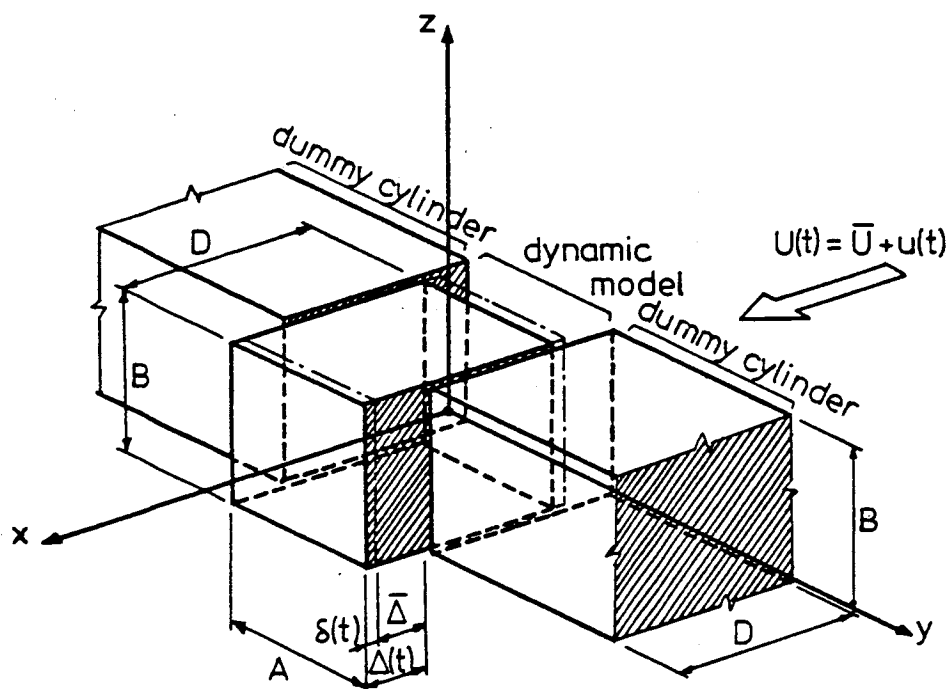


Figure 4-1 Co-ordinates of model

$$S_F(f) = \int_0^B \int_0^B \int_0^A \int_0^A C_{u_1 u_2}(f) S_u(f) (\rho \tilde{C}_D \bar{U})^2 dy_1 dy_2 dz_1 dz_2 \quad (4-3)$$

Then the variance of the dynamic response can be written as,

$$\begin{aligned} \sigma_\delta^2 &= \int_0^\infty S_\delta(f) df \\ &= \int_0^\infty |X(f)|^2 S_F(f) df \end{aligned} \quad (4-4)$$

where

$$|X(f)|^2 = \frac{1}{k^2 \left\{ \left(1 - \left(\frac{f}{f_0} \right)^2 \right)^2 + 4\zeta_T^2 \left(\frac{f}{f_0} \right)^2 \right\}} ;$$

k is the spring constant, ie, $2\pi f_0 = \sqrt{\frac{k}{M}}$

The integral in equation (4-4) can be divided into two parts, ie, quasi-static and resonance part as it was described in section 2.2.5. For a lightly damped system the resonance part is predominant (ie, $\frac{\pi}{4\zeta_T} \gg 1.75$), and the effect of frequency dependence of $\tilde{C}_D$ in the quasi-static part becomes much less than that of the resonance part, and so $\tilde{C}_D$ in the quasi-static part can be approximated conventionally by a static value C_{D_0} which can be obtained from the mean response force $\bar{F}$ as,

$$C_{D_0} = \frac{\bar{F}}{\frac{1}{2}\rho\bar{U}^2 A \cdot B} \quad (4-5)$$

where $\bar{F} = k \cdot \bar{\Delta}$ and $\bar{\Delta}$ is the mean response displacement.

Then substituting equation (4-3) into (4-4) and applying the simplification of the integral equation (2-50),

$$\begin{aligned} \sigma_\delta^2 = & \frac{1}{k^2} \left\{ \int_0^{1.75f_0} \int_0^B \int_0^B \int_0^A \int_0^A C_{u_1 u_2}(f) S_u(f) (\rho C_{D_0} \bar{U})^2 dy_1 dy_2 dz_1 dz_2 df \right. \\ & + \left(\frac{\pi}{4\zeta_T} - 1.75 \right) f_0 S_u(f_0) \int_0^B \int_0^B \int_0^A \int_0^A C_{u_1 u_2}(f_0) \\ & \left. \times (\rho \tilde{C}_D \bar{U})^2 dy_1 dy_2 dz_1 dz_2 \right\} \end{aligned}$$

and so,

$$\tilde{C}_D^2 = \frac{k^2 \sigma_\delta^2 - (\rho C_{D_0} \bar{U})^2 \int_0^{1.75f_0} \int_0^B \int_0^B \int_0^A \int_0^A C_{u_1 u_2}(f) S_u(f) dy_1 dy_2 dz_1 dz_2 df}{\left(\frac{\pi}{4\zeta_T} - 1.75 \right) f_0 S_u(f_0) \rho^2 \bar{U}^2 \int_0^B \int_0^B \int_0^A \int_0^A C_{u_1 u_2}(f_0) dy_1 dy_2 dz_1 dz_2}$$

or

$$\left(\frac{\tilde{C}_D}{C_{D_0}} \right)^2 = \frac{k^2 \sigma_\delta^2 - 4 \cdot \bar{F}^2 \frac{\sigma_u^2}{U^2} \int_0^{1.75 f_0} \int_0^1 \int_0^1 \int_0^1 C_{u_1 u_2}(f) \frac{S_u(f)}{\sigma_u^2} dy'_1 dy'_2 dz'_1 dz'_2 df}{\left(\frac{\pi}{4 \zeta_T} - 1.75 \right) f_0 \frac{S_u(f_0)}{\sigma_u^2} \cdot 4 \bar{F}^2 \cdot \frac{\sigma_u^2}{U^2} \int_0^1 \int_0^1 \int_0^1 \int_0^1 C_{u_1 u_2}(f_0) dy'_1 dy'_2 dz'_1 dz'_2}$$

(4-6)

where $y' = \frac{y}{A}$ and $z' = \frac{z}{B}$ are normalised co-ordinates.

Thus the dynamic along-wind force coefficient $\tilde{C}_D$ or $\tilde{C}_D/C_{D_0}$ can be computed from $k \sigma_\delta$, $\bar{F}$, $\frac{\sigma_u}{U}$, $C_{u_1 u_2}$, $S_u(f)$, ζ_T and f_0 all of which are measurable. The computer program for performing the integration in equation (4-6) was developed and is shown in appendix 3.

4.3 WIND TUNNEL EXPERIMENT

4.3.1 Model Design Concept

A two-dimensional S.D.O.F. system model with light damping was designed and employed as a transducer in order to evaluate dynamic and static along-wind forces on a bluff body.

A diagrammatic view of the model positioned centrally in its shrouds and separated by a narrow air gap from dummy shrouds of the same cross-section is shown in Figure 4-2 with the model mounted horizontally and transversely across a wind tunnel. It was decided that the model should be laid horizontally rather than vertically for convenience of immersion in an air flow with different turbulence characteristics, ie by changing the vertical position of the model in the wind tunnel.

The characteristics of turbulent wind in the wind tunnel to be used were described in a previous section, 3-5.

The model was to consist of a mass supported by two pairs of plate springs to allow non-rotational deflection in the x-direction. Also some facility for varying the angle of attack of the flow on the model surface was desired for the convenience of further application such as the evaluation of dynamic lift coefficient and the investigation of the effects of different angles of attack.

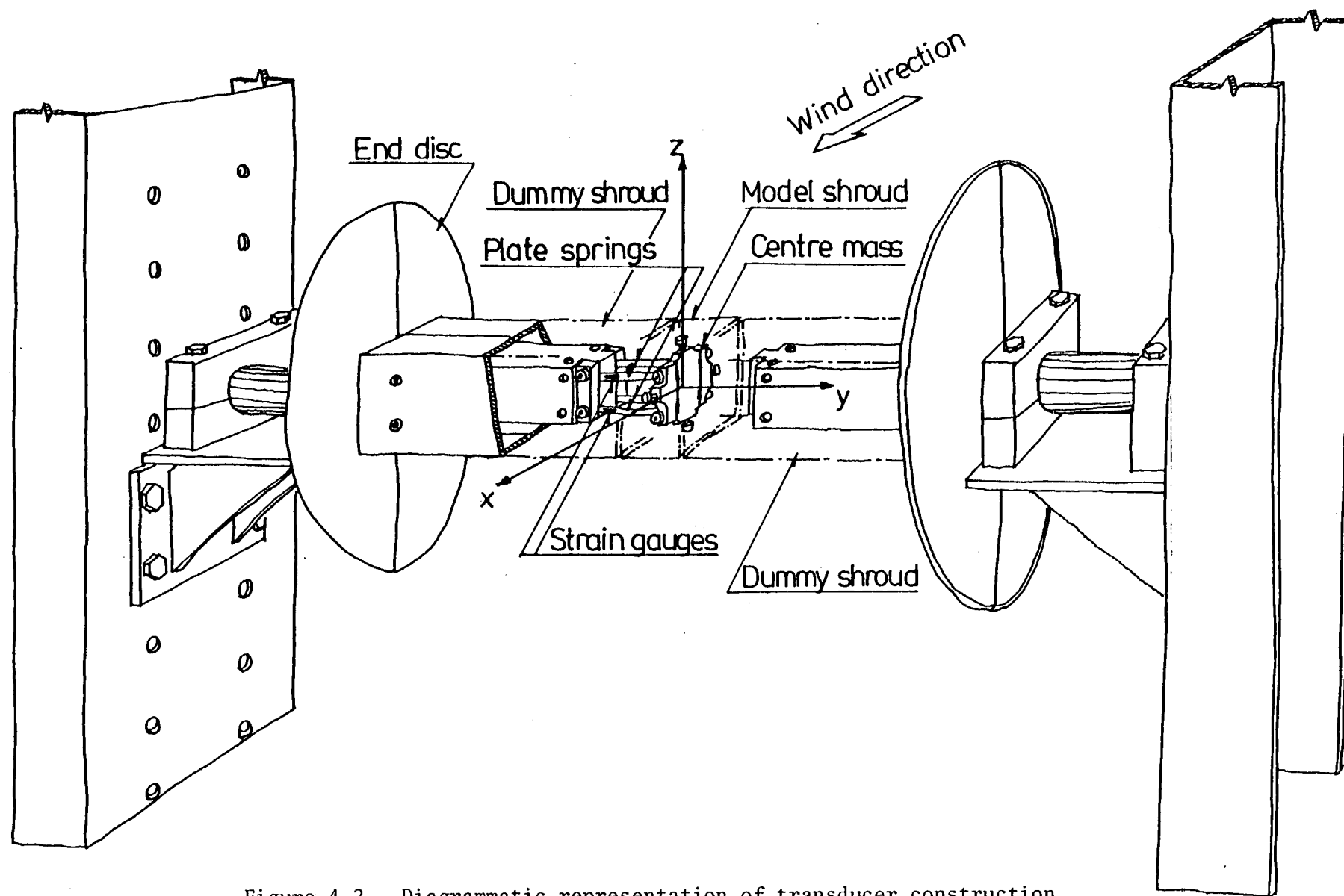


Figure 4-2 Diagrammatic representation of transducer construction

Measurement of the response of a model could be achieved in at least three different ways. They are : (i) electric wire or foil resistance strain gauges placed at the base of the double curvature columnar spring plates; (ii) a miniature light-weight accelerometer located on the vibrating mass; (iii) a small detached type transducer monitoring translation of the mass relative to its base. Lack of availability of accurate and stable instrumentation for (ii) and (iii) led to the adoption of alternative (i).

Generally there are severe design restrictions for this type of model due to limitations in wind tunnel facilities. For example, it is difficult to produce large scale turbulence in wind tunnels and models must be appropriate to the turbulence scale in accordance with the similarity law, in other words the smaller the turbulence scale in the wind tunnel the smaller the models should be. By contrast from the aspect of instrumentation and accurate construction it is better for a model to be of large dimensions.

It is one of the aims of this experiment to represent various parameters over a wide range covering actual situations as extensively as possible. A number of dynamic along-wind force and static drag coefficient values are intended to be obtained using parameters such as seven different shapes, five different spring systems, four wind speed levels and two positions of height.

4.3.2 Variable Parameters and Design Requirements

For the wind tunnel simulation of a full scale structure several non-dimensional parameters must be satisfied⁽⁷⁹⁾. These can be summarised as follows for the two-dimensional interaction between wind speed and forces.

1. $\tilde{U} = \frac{\bar{U}}{f_o \cdot D}$: reduced wind speed,
2. $\frac{\gamma}{\rho}$: mass density ratio between structure and air,
3. $\frac{\sigma_u}{\bar{U}}$: local turbulence intensity,
4. $\frac{L_x}{D}$: ratio of turbulence scale to the reference dimension of a structure,
5. $\frac{D}{B}$: section aspect ratio,
6. $R_e = \frac{\bar{U} \cdot D}{\nu}$: Reynolds number.

The range of each parameter depends on the type of structure under consideration. If a modern tall building of height greater than 100 m is taken as an example, from the possible range of values of the physical quantities involved the range of values of the parameters can be estimated as follows,

$$\tilde{U} \approx 2.0 \text{ to } 20.0 ; \frac{\gamma}{\rho} \approx 150 \text{ to } 500 ;$$

$$\frac{\sigma_u}{\bar{U}} \approx 8\% \text{ to } 30\% ; \frac{L_x}{D} \approx 0.5 \text{ to } 20 ; \dots$$

$$\frac{D}{B} \approx 0.2 \text{ to } 5.0 ; R_e \approx 10^7$$

If a lower building is of interest, a smaller reduced velocity and a wider range of section aspect ratio should be used. When a small dimension structure such as a chimney or tower is in question a greater reduced velocity and a larger turbulence scale ratio should be employed.

The average mass density of the model was fixed at approximately 400 Kg/m^3 . Although some modern buildings have a lower mass density value according to Jeary and Spark's report⁽⁸⁰⁾, its effect on aerodynamic forces may not be as significant as the effects of other parameters. The use of a higher value of the average mass density, γ , eliminates the need to account for the additional mass term effects in the equation of motion (4-1). Seldom have such effects been considered in aerodynamic studies of civil engineering structures. In fact the contribution of air mass to the total mass involved in the inertia of a structure is usually small and can be neglected except in the case of very light structures.

Two positions were chosen for the models (position 1 : 250 mm above the apron floor and position 2 : 400 mm) to give an opportunity of varying the local turbulence intensity between 10% and 14% and the longitudinal turbulence scale, L_x , between 61 mm and 115 mm. These values were obtained at positions 2 and 1 respectively, see Table 3-2.

The turbulence scale ratio, L_x/D , was considered to be varied at least around unity, since a case of $L_x/D \sim 1.0$ could be expected to produce a different air flow condition around a body from other cases such as $L_x/D \ll 1.0$ and $L_x/D \gg 1.0$. This can be explained as the oncoming turbulence is distorted in a different manner when the dimension of the body is similar to the scale of turbulence⁽³²⁾, and this effect can be clearly seen also in the variation of static drag coefficient values^(42,43,44). The range of L_x/D was chosen accordingly to be between 0.5 and 2.0 in this experiment, although it is desirable to extend this parameter up to 10.0 or more to confirm a genuine trend at $L_x/D \gg 1.0$ probably by using a larger wind tunnel.

Seven shapes of the model were selected to provide different B/D and L_x/D values. The lower limit of D was determined by the physical size of components available for model construction and its upper limit was constrained by the requirement that the wind characteristics should not change significantly over the along-wind dimension of model.

Since the air flow around a bluff body is not influenced much by the Reynolds number its effects can be neglected. The range of R_e value in this experiment was between 1.2×10^4 and 8.0×10^4 .

Considering the desirable range of reduced wind speed and the maximum available velocity in the wind tunnel the range of natural frequency for the model was determined. Different spring systems were designed to provide those natural frequencies for the known dimensions and masses of models taking into account such conditions as:

- (a) maximum deflection should be well within the order of the model dimension D ;
- (b) spring axial displacement (y -direction in Figure 4-2) should be less than the gap of separation between the dynamic model and the dummy shroud;
- (c) maximum strain should be below the proportional limit of the spring material; and
- (d) minimum force should produce a significant signal relative to background noise.

4.3.3 Design Feature of Model

As noted previously five spring systems were selected. Each system was designed to meet the conditions mentioned in the previous section. Both ends of a spring plate were clamped firmly so that it would deform in double curvature. A closer view of the dynamic model is shown in Figure 4-3. Some typical calculations and drawing details for the model are shown in appendix 2.

Circular discs were attached at each end of the model assembly, see Figures 4-2 and 4-3, to channel the flow past the model, the discs being sufficiently remote from the model for the flow in its vicinity to be undisturbed by them. The necessity for this type of disc might not be as essential for bluff bodies as it is for a circular cylinder, according to Lee⁽⁸¹⁾.

The whole system was firmly fixed to a rigid frame of steel channels (305 mm x 102 mm [$\approx$ 46.8 Kg/m) and steel equal angles (76 mm x 76 mm EA @ 10.57 Kg/m). The frame was placed on four rubber vibration insulators independent of the wind tunnel body. A pair of apron walls were attached to the inner faces of the frame posts so that the flow approaching the model assembly was affected only by the end disc plates and not by the frame members.

The frame permitted the vertical position of the model assembly to be varied relative to the floor of the wind tunnel. The whole frame and assembly could be moved into and out of position at the mouth of the tunnel on a hydraulic trolley.

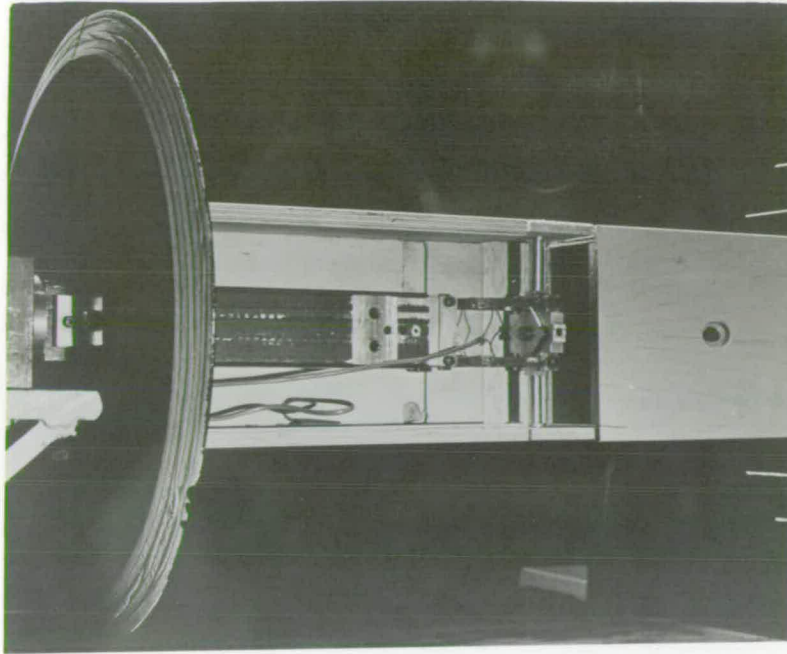


Figure 4-3 View of transducer with shrouds on one-side removed

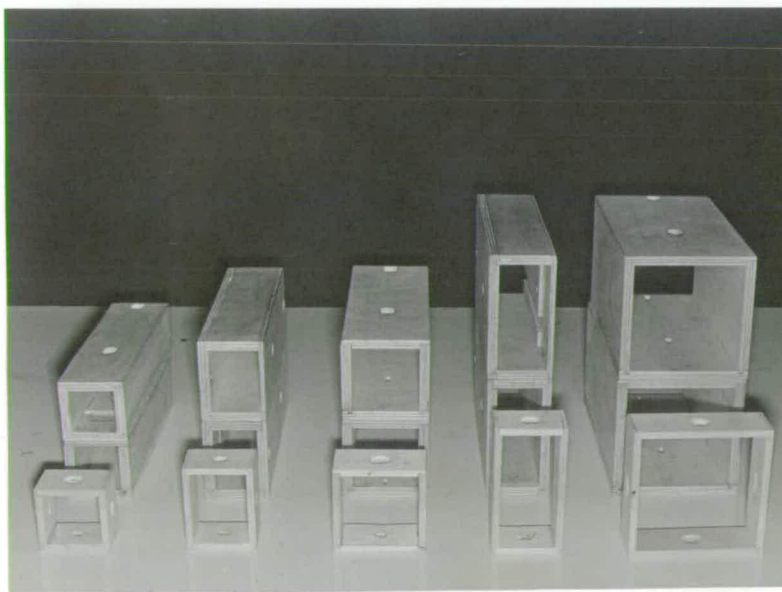


Figure 4-4 Array of shrouds (from left to right shape I, II and III, IV, V and VI, and VII)

The core part of model was made of aluminium and the model shroud was made of balsa wood to minimize weight which could be raised by fixing some additional known steel plate weights on to the centre mass enabling a fairly constant average model structural density to be maintained.

The dummy shrouds were made of plywood having exactly the same sectional profile as that of the corresponding model shroud. Figure 4-4 (reference Figure 5 in paper 3 appendix 4) shows the array of model and dummy shrouds employed with the transducer, the former being in the foreground.

For rectangular sections the long side of shrouds and the short side of those are interchangeable so that two sets of rectangular section shrouds can provide four different D/B settings, ie shape II and III and shape V and VI as shown in Figure 4-4.

Shape I, IV and VII have square cross sections with different dimensions which vary the turbulence scale ratio L_x/D from approximately 0.5 to 2.0.

The total range of masses and densities achieved are given in Table 4-1 together with actual dimensions for the cross sections of models. The weight includes that of the centre core mass, model shrouds, spacers, bolts, washers and a half of spring plates.

Table 4-1 Dimensions of Model

Shape	Length	Width	Depth	D/B	Mass	Average Density
	A (mm)	B (mm)	D (mm)			
I	40	60	60	1.0	54.6	0.380
II	40	80	60	0.75	80.4	0.418
III	40	60	80	1.33	80.4	0.418
IV	40	80	80	1.0	100.5	0.410
V	40	120	60	0.5	116.1	0.403
VI	40	60	120	2.0	116.1	0.403
VII	40	120	120	1.0	229.1	0.398

note : Values of mass and average density are those for spring system A. Differences due to other types of spring system are approximately up to +1.2g or -1.6g.

4.3.4 Measurement System

As mentioned in section 4.3.1 electric resistance strain gauges were used to measure the dynamic response of the model. As a most suitable size, a gauge length of 3 mm was chosen for the plate springs used (minimum width 5 mm). Foil gauges were preferred to wire gauges for this type of dynamic transducer because of their longer fatigue life. The gauges employed were T.M.L. type FLA-3-11 with resistance of 120 ± 0.3 ohms and gauge factor of 2.08. The position of each gauge on a spring plate was 5 mm from its centre to the clamp at the fixed end of the shaft. The gauges were bonded according to the manufacturer's recommendations using a cyano-acrylate (C.N.) adhesive. After bonding all gauges were coated with a P.V.C. coating (VYCOAT ACA 60) for environmental protection.

Each gauge was wired to a terminal strip bonded on to the corresponding plate spring and two gauges were used in the active arm of a half bridge circuit with two similar dummy gauges in the temperature compensating arm. Bridge completion was internal to a strain gauge transducer meter (SANGAMO type 56-NT9). The dummy gauges were fixed in the same manner as the active ones to a similar piece of spring plate located inside the dummy shrouds for temperature compensation during test measurements. The bridge excitation was via an oscillator supply at 5 V r.m.s. with carrier frequency of 5 kHz.

The output from the transducer was amplified to a level suitable for recording. The zero shift during a test was checked by a digital indicator visually and by an oscillograph recording before and after each test run and was found to be negligible.

All data were recorded by an ultra violet recorder (Honeywell 6-channel Visicorder type 1706).

During each test a reference wind speed was measured at a specific point upstream by means of a pitot-tube and a micro-manometer as described in section 3.5.1. From that reference wind speed reading a wind speed at the model centre when the model was hypothetically removed can be estimated according to the previously established mean wind speed profile.

4.3.5 Calibration

The dynamic characteristics for an actual test were established by the calibration of a model in three independent ways.

(1) Displacement - Strain Relationship:

Load was applied manually in the same direction as the air flow (x-direction in Figure 4-2). Horizontal displacements up to 4 mm were measured by dial gauge. This maximum displacement was well in excess of the estimated maximum deflection of a model

under test conditions. The loading was repeated four times to obtain a linear relationship between displacement and strain from the four sets of readings. Strain was indicated by the deflection of the light beam on the oscillograph record.

(2) Load - Strain Relationship:

After rotating the model 90° about its axis load was applied vertically by means of known dead weights. The maximum load was 0.9 N comprising 0.5 N self weight (without shrouds and additional steel plate weights) and 0.4 N of additional weights. This load was in excess of the estimated maximum wind force which was approximately 0.8 N for the largest model in the highest wind speed. The calibration was repeated four times and a mean of the four sets of results determined. Strain was represented again by the deflection of the light beam on the oscillograph record.

The two types of calibration, (1) and (2), enable a force-displacement curve to be deduced for a particular spring system and model shape yielding the spring constant of a system.

(3) Free Vibration Test:

Free vibration tests were performed on a mass-spring system before and after each set of response measurements. The natural frequency and damping ratio of a mass-spring system were computed from a 100 wave decaying curve. The natural frequency was compared with the corresponding value calculated using the measured mass and spring constant obtained from calibrations (1) and (2). Agreement between these two methods of determining natural frequency for a system was generally good. The dynamic characteristics

of the model are summarised in Table 4-2. The discrepancies between those values of positions 1 and 2 were due to slightly different positions of clamp setting.

Test arrangement for calibrations and some typical calibration results are shown in appendix 4 paper 3.

All calibration curves for both displacement-strain and load-strain showed a good linearity over the full range of calibration.

For one spring system in combination with the range of seven model shapes the calibrations and actual test runs lasted almost two days and the values of spring constant, k , in Table 4-2 demonstrate the degree of repeatability and stability achieved except the case of position 1 spring system A in which a resetting the spring system took place between Shape IV and V of the model. Most of the differences between the spring constants deduced from static calibrations and those from free vibration tests were within 5%; the difference in terms of the natural frequency being 2.5%.

The free vibration records confirmed that the natural frequency did not vary with amplitude of deflection or over the period of a test run.

Damping ratio, ζ , is another contributor to the linearity of the dynamic system and was obtained from the free vibration records. Mean values of ζ were calculated from every 20-wave decay out of 100-wave records, namely from five values of each before

Table 4-2 Dynamic Properties of Model

Spring System	Shape	Position 1			Position 2		
		Natural Frequency f_o (Hz)	Spring Constant k (N/m)	Damping Ratio ζ (%)	Natural Frequency f_o (Hz)	Spring Constant k (N/m)	Damping Ratio ζ (%)
A	I	18.8	762. (770.)	0.30 to 0.35	17.6	668. (705.)	0.25 to 0.30
	II	15.3	776. (771.)	0.20 to 0.24	14.7	685. (708.)	0.27 to 0.35
	III	15.3	776. (771.)	0.22 to 0.33*	14.7	685. (708.)	0.19 to 0.26
	IV	13.65	736. (735.)	0.45 to 0.47	13.1	681. (705.)	0.32
	V	12.0	659. (670.)	0.25 to 0.29	12.0	660. (704.)	0.20 to 0.28
	VI	12.1	670. (670.)	0.70 to 0.85	12.2	682. (704.)	0.40 to 0.70*
	VII	8.3	623. (660.)	0.35 to 0.39	8.5	653. (698.)	0.20 to 0.26
B	I	13.0	360. (374.)	0.40 to 0.45	13.1	366. (368.)	0.50
	II	10.6	355. (365.)	0.30	10.9	374. (375.)	0.18
	III	10.6	355. (369.)	0.25	11.0	380. (375.)	0.60 to 0.65
	IV	9.2	334. (360.)	0.15 to 0.34*	9.7	368. (365.)	0.46 to 0.53
	V	8.7	345. (366.)	0.22 to 0.39*	9.0	369. (370.)	0.40 to 0.44
	VI	8.6	337. (363.)	0.23 to 0.24	9.0	369. (370.)	0.42
	VII	6.15	340. (368.)	0.45	6.35	363. (365.)	0.39 to 0.41
C	I	8.55	155. (155.)	0.45 to 0.55	8.6	157. (170.)	0.46 to 0.50
	II	7.0	154. (169.)	0.40 to 0.50	7.1	158. (168.)	0.43 to 0.45
	III	7.0	154. (169.)	0.40 to 0.50	7.1	158. (168.)	0.34 to 0.38
	IV	6.25	153. (150.)	0.55 to 0.56	6.3	156. (160.)	0.40 to 0.50
	V	5.75	151. (158.)	0.45 to 0.60	5.8	156. (160.)	0.33 to 0.39
	VI	5.75	151. (158.)	0.65 to 0.69	5.85	160. (160.)	0.65 to 0.73
	VII	4.00	146. (155.)	0.53 to 0.60	4.13	154. (165.)	0.44 to 0.48
D	I	6.65	95. (102.)	1.10 to 1.50	6.6	94. (101.)	1.75 to 2.0
	II	5.53	96. (101.)	1.00 to 1.30	5.3	92. (99.)	0.70 to 0.85
	III	5.51	95. (100.)	0.95 to 1.10	5.3	92. (99.)	1.25 to 1.35
	IV	4.92	95. (97.)	1.21	4.70	88. (95.)	0.68 to 0.75
	V	4.48	91. (98.)	1.03 to 1.06	4.32	87. (96.)	0.66 to 0.72
	VI	4.50	92. (98.)	0.92 to 0.93	4.41	88. (96.)	1.10 to 1.20
	VII	3.10	88. (96.)	0.91 to 0.94	2.94	86. (93.)	0.65 to 0.80
E	I	22.3	1099. (1027.)	0.58 to 0.65	21.4	1012. (1020.)	0.14 to 0.17
	II	17.2	981. (1070.)	0.30 to 0.50*	17.8	1040. (1050.)	0.16 to 0.40*
	III	17.2	981. (1070.)	0.52 to 0.65	17.8	1040. (1045.)	0.16 to 0.29*
	IV	16.3	1070. (1040.)	0.21	15.9	1018. (1020.)	0.14 to 0.15
	V	15.1	1053. (1040.)	0.18 to 0.19	14.65	992. (1010.)	0.15 to 0.18
	VI	15.2	1067. (1040.)	0.28	14.65	992. (1010.)	0.16 to 0.18
	VII	10.85	1073. (1035.)	0.18	10.4	986. (1010.)	0.14 to 0.18

Note : (1) * indicates considerable variation of the structural damping ζ with the amplitude level of response;

(2) values are obtained from dynamic calibration (3);

(3) k in () is obtained from static calibrations (1) and (2).

and after test run and ranged from 0.15 to 2.0% of critical.

In general ζ remained reasonably constant with amplitude, for example in most cases the standard deviation of the variation of ζ falls within 10%. However some cases were found in which ζ varied considerably with amplitude of response. Under such circumstances the r.m.s. level of the wind excited response could be multiplied by a peak factor $\sqrt{2}$ and compared with the corresponding amplitude level of the free vibration record and the value of ζ appropriate to that level could be established from the decay in the same region. This could be considered as a reasonable estimate of the mean value of ζ for the particular response and was used in the determination of dynamic along-wind force coefficient from that response record.

4.3.6 Test Procedure

The method of test was simple and straightforward. After the flow had settled down and the velocity had been measured, a recording was taken on the oscillograph trace. Each test run lasted only a few minutes but before a test series an initial warm-up period of an hour was allowed.

In order to change parameters efficiently and obtain necessary and sufficient calibrations for each combination of different parameters, tests were carried out according to a flow chart shown in Figure 4-5. The general view of experiment is shown in Figure 4-6 (reference Figure 1 in paper 3, appendix 4).

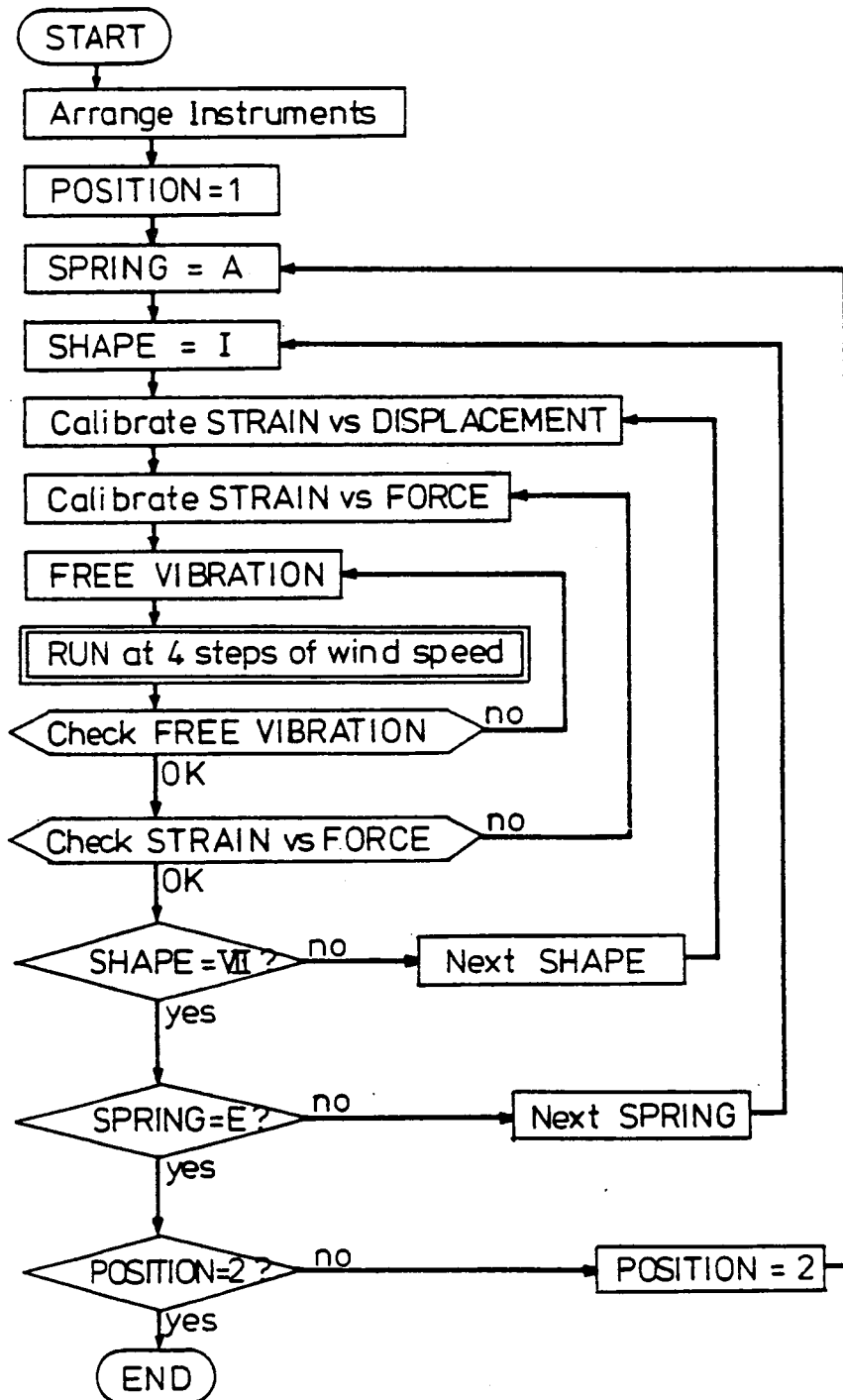


Figure 4-5 Wind tunnel test flow chart

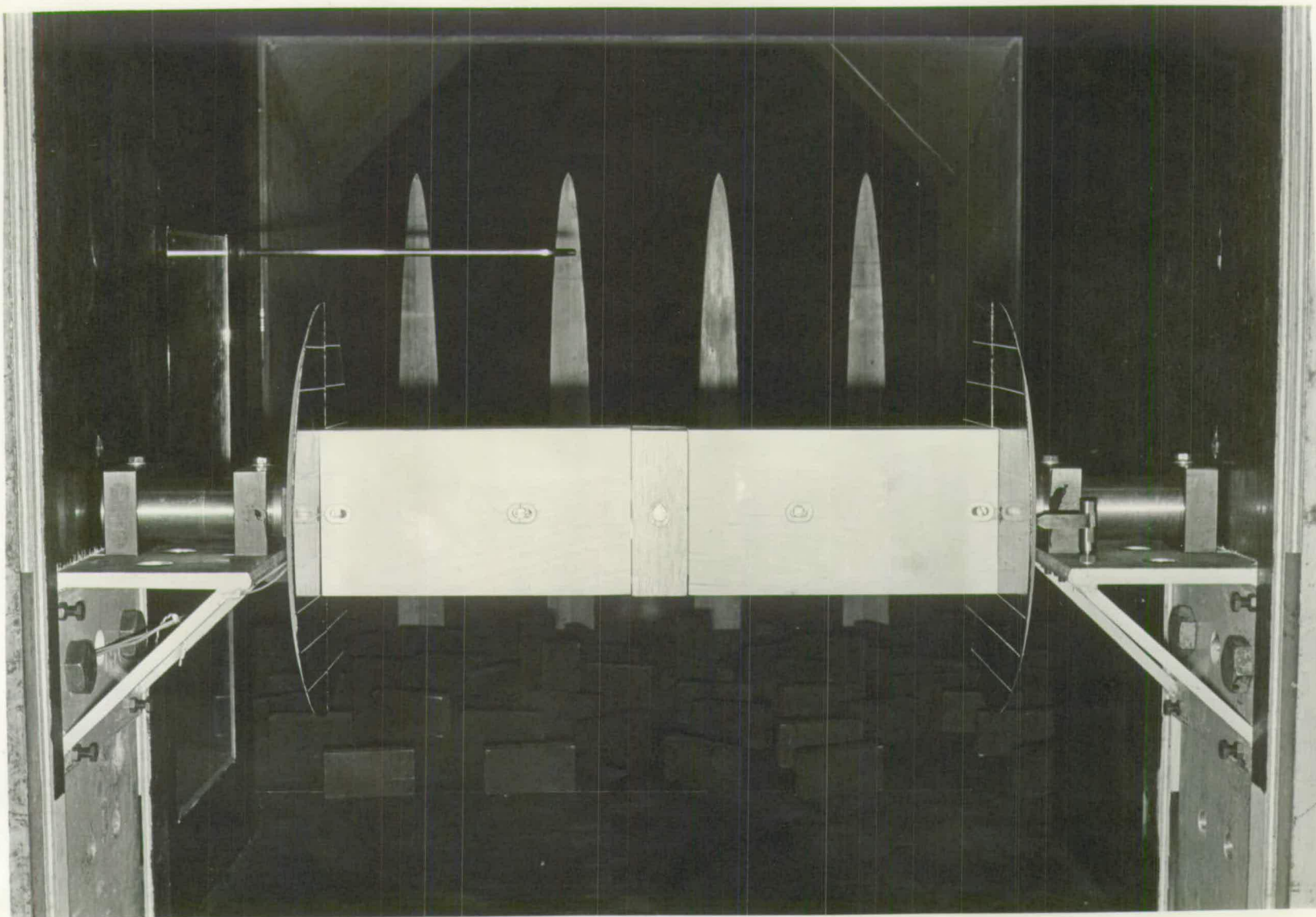


Figure 4-6 General view of experiment

4.3.7 Data Gathering and Data Reduction

Since the dynamic model was a S.D.O.F. system, the output signal from the oscillograph could be represented by a sinusoidal wave whose amplitude varies randomly. The response of the dynamic model to wind excitation can be defined by values of mean and root mean square (r.m.s.) response if the power spectral density and the frequency distribution are known. The power spectral density of the response has a typical narrow band distribution form around a centre frequency being the natural frequency of the model. The power spectral density in the lower frequency range and the probability distribution are mainly dependent on the excitation, namely the wind characteristics, if the dynamic system holds its linearity.

The record period to be analysed was chosen to be 100 times the natural period of the dynamic model* so that the peak factor, p , could be expected to be fairly uniform where

$$p = \frac{\Delta_{\max} - \bar{\Delta}}{\sigma_{\delta}} \quad (4-7)$$

and $\Delta_{\max}$ is the maximum response displacement in that recorded period.

From the oscillograph records a set of 100 waves, which are considered to be stationary, is taken for data gathering. Each peak value Δ_{pi} and bottom value Δ_{Bi} out of 100 waves were read manually to be reduced into a file of digital data from which the

* Such a record length should be more than sufficient to establish the stationary process for the tunnel data.

mean and r.m.s. response were calculated. The mean value $\bar{\Delta}$ can be written as a mean of i-th centre values $\bar{\Delta}_i$ as

$$\bar{\Delta} = \frac{1}{100} \sum_{i=1}^{100} \bar{\Delta}_i = \frac{1}{100} \sum_{i=1}^{100} \frac{\Delta_{Pi} + \Delta_{Bi}}{2} \quad (4-8)$$

Assuming each wave has a sinusoidal form, ie, the standard deviation of i-th wave, σ_{δ_i} , is,

$$\sigma_{\delta_i} = \frac{1}{\sqrt{2}} \cdot \frac{\Delta_{Pi} - \Delta_{Bi}}{2}$$

Then the r.m.s. value σ_{δ} for a 100 wave record can be obtained as follows,

$$\sigma_{\delta} = \left[\frac{1}{200} \sum_{i=1}^{100} \left\{ \frac{(\Delta_{Pi} - \bar{\Delta})^2}{2} + \frac{(\Delta_{Bi} - \bar{\Delta})^2}{2} \right\} \right]^{\frac{1}{2}} \quad (4-9(a))$$

$$\approx \left[\frac{1}{100} \sum_{i=1}^{100} \frac{(\Delta_{Pi} - \Delta_{Bi})^2}{8} \right]^{\frac{1}{2}} \quad (4-9(b))$$

when $\bar{\Delta} \approx \frac{\Delta_{Pi} + \Delta_{Bi}}{2}$

If either the response is nonstationary or the lower frequency component of the power spectral density is significant compared with the resonance part, equation (4-9(b)) could cause some underestimation. However as the dynamic system has a very low damping and so the resonance part is always predominant, therefore the error caused by the simplification of equation (4-9(b)) can be expected to be very small. In fact some typical numerical

examinations confirmed that the differences between the r.m.s. values obtained from equations (4-9(a)) and (4-9(b)) were within 1%.

From mean and r.m.s. response values the static drag coefficient and the dynamic along-wind force coefficient values were computed from equations (4-5) and (4-6) respectively with known wind characteristics and the dynamic characteristics of the model. In order to obtain the aerodynamic damping ratio the static drag coefficient value was taken for C_D in equation (4-2) in the first instance.

However it seems desirable to avoid the uncertainty of theoretical estimation of the aerodynamic damping ratio using equation (4-2). Direct estimation of the total damping ratio was attempted, ie the total damping ratio was computed according to the auto-correlation method⁽⁸²⁾ from response records. Then the aerodynamic damping was obtained from the total damping by subtracting the known structural damping. This value of aerodynamic damping was compared with the conventional theoretical value based on equation (4-2).

In order to see the random nature of response from a different aspect, the probability distribution was also examined for each response record. A computer program for the probability distribution and the total damping ratio of response was developed and is shown in appendix 3.

4.4 EXPERIMENTAL RESULTS

4.4.1 Random Nature of Response

A typical example from the oscillograph recordings is given in Figure 4-7. Typical probability distributions of records are shown in Figure 4-8. The normal distribution fits reasonably well to these results.

Response records were obviously random vibrations as shown in Figures 4-7 and 4-8 and the random nature appeared clearly in the variation of aerodynamic damping. Measured aerodynamic damping values ζ_A' obtained from the auto-correlation method are plotted against the reduced velocity $\tilde{U}$ in terms of the ratio to the theoretical value ζ_A in Figure 4-9; (a) for position 1 and (b) for position 2.

Although values are scattered over a fairly wide range, a general decreasing trend with $\tilde{U}$ is quite evident irrespective either of shape or position. This seems to suggest some variation of the dynamic drag coefficient with $\tilde{U}$, in other words the static value C_{D_o} used in equation (4-2) to obtain the theoretical ζ_A should be replaced by the dynamic drag coefficient which is expected to decrease with increasing reduced velocity as shown in both Keulegan and Carpenter's ⁽¹²⁾ and Davenport's ⁽¹³⁾ results.

A steep decreasing trend of ζ_A'/ζ_A with $\tilde{U}$ is significant between $\tilde{U} = 1$ and 10 and then relatively uniform values mostly less than unity were obtained. More than 10 negative values out of

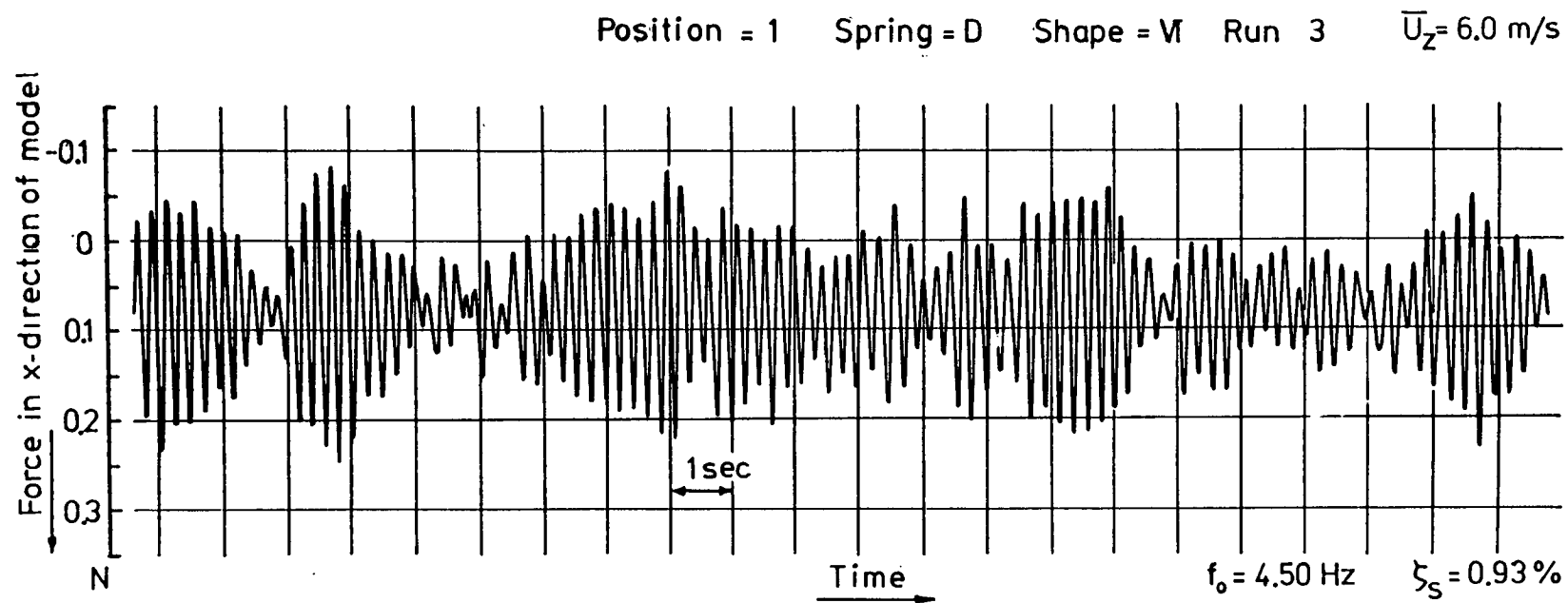


Figure 4-7 Typical example of wind induced response record

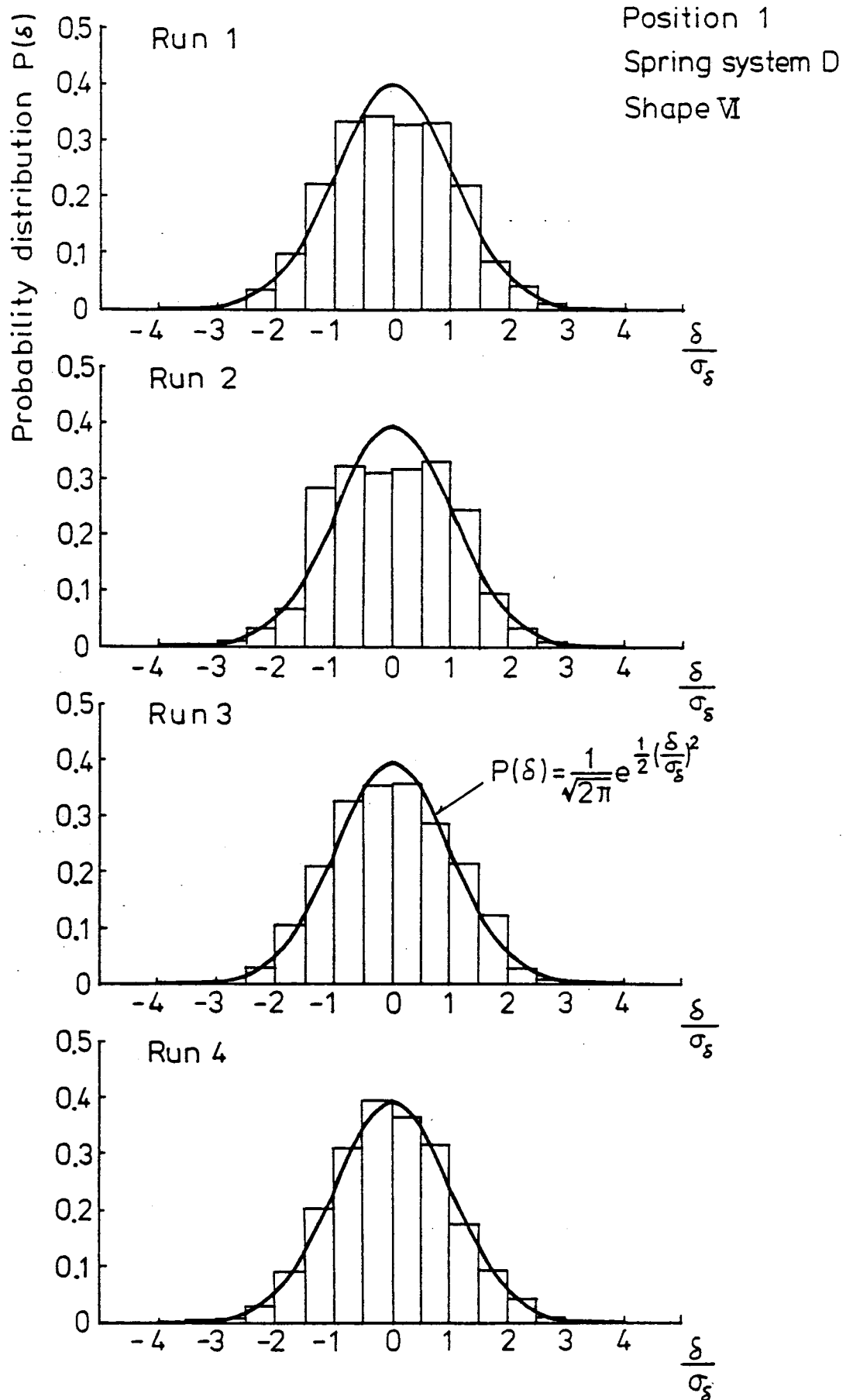


Figure 4-8 Typical probability distributions of response

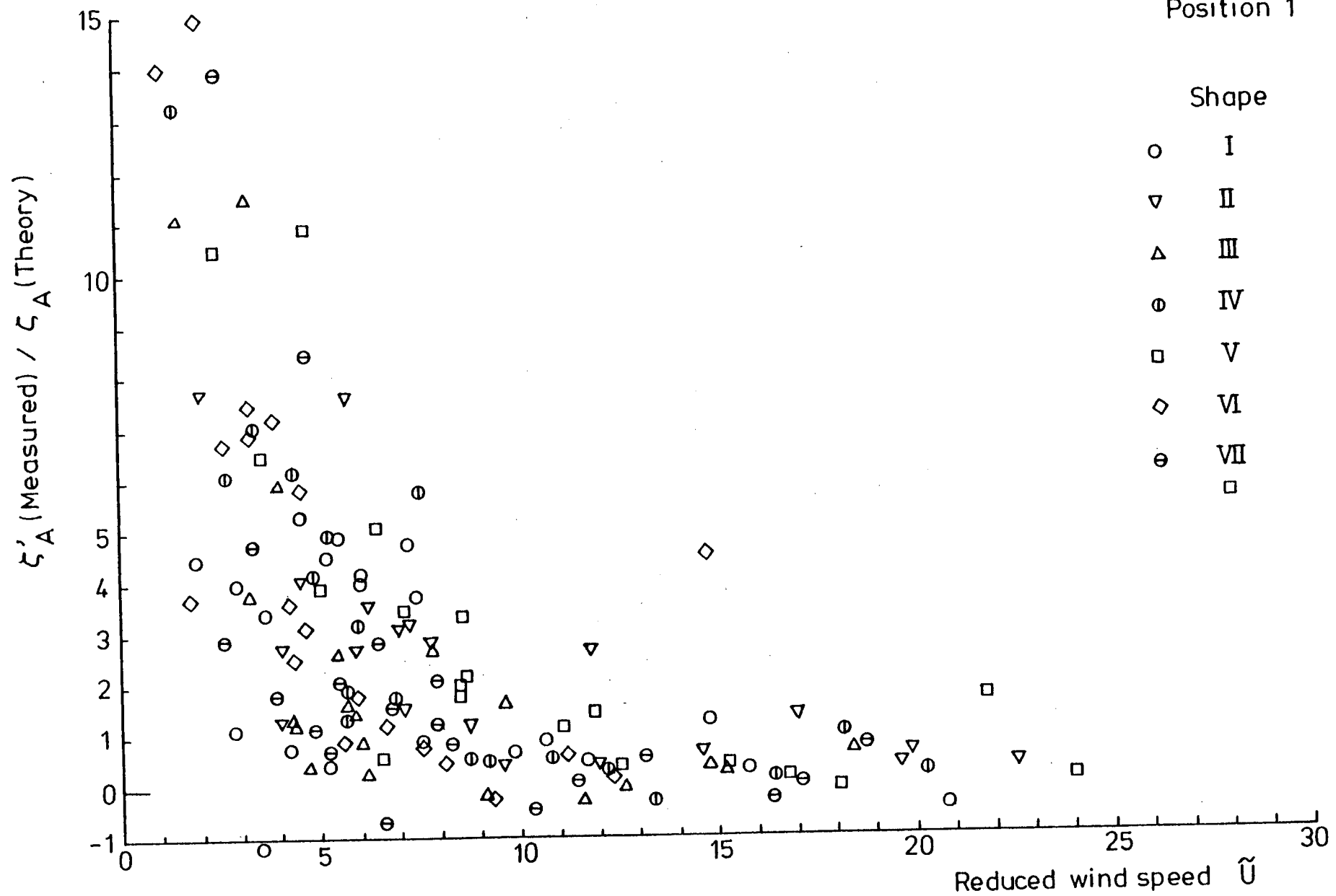


Figure 4-9(a) Variation of measured ζ'_A at position 1

Position 2

Shape

- I
- ▽ II
- △ III
- ⊙ IV
- V
- ◇ VI
- ⊖ VII

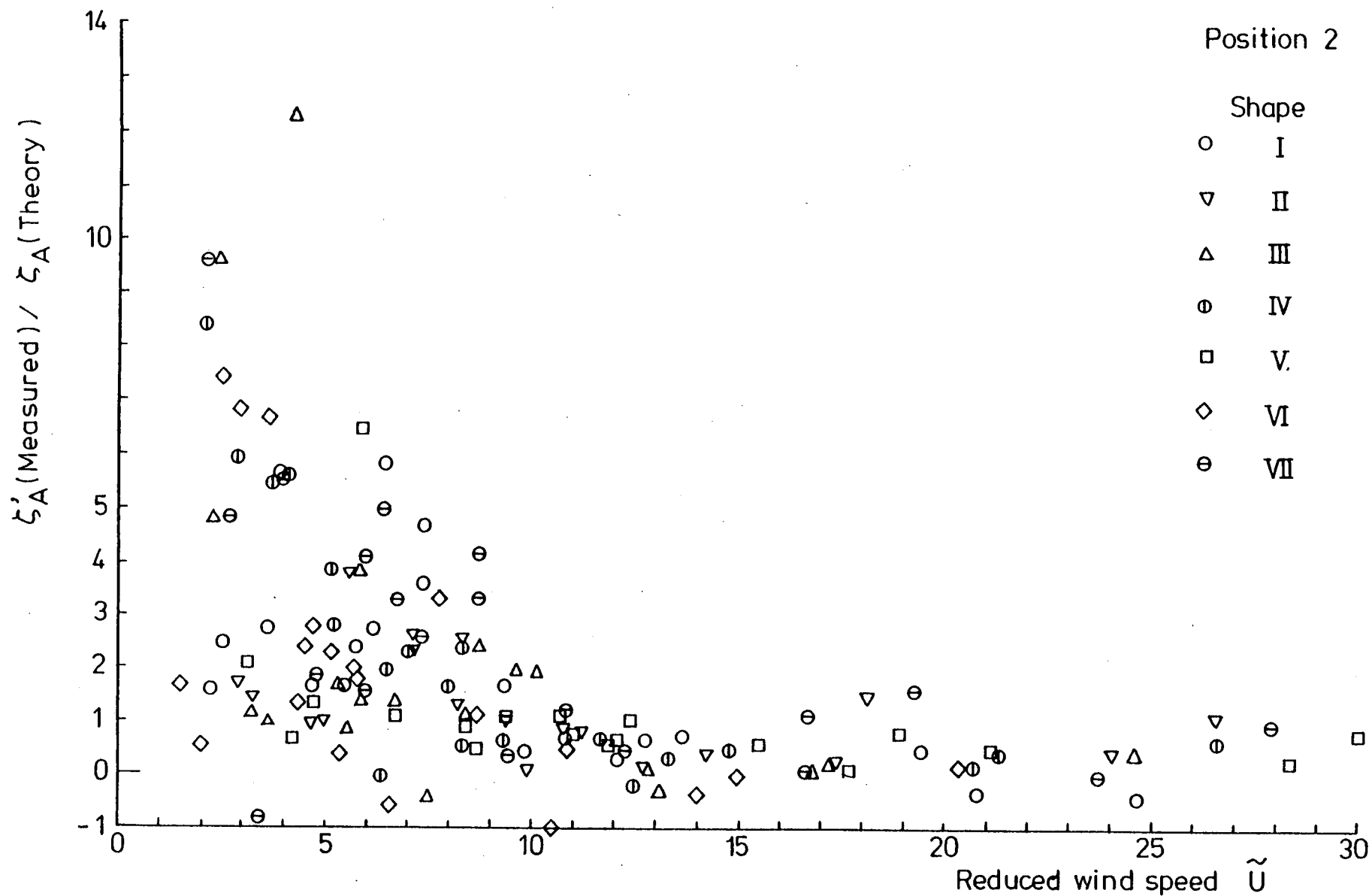


Figure 4-9(b) Variation of measured ζ'_A at position 2

each 140 results were obtained. Although there seems to be evidence of the occurrence of negative damping in practical cases of along-wind response observed by Davenport⁽¹³⁾, negative values in this experiment would be mainly due to the random nature of response.

The peak factor, p , is also another indicator of the random vibration and fairly uniform values around 3.0 were obtained, whereas the theoretical value can be calculated based on the normal probability distribution as follows⁽⁸³⁾,

$$p = \sqrt{2 \log_e v_o T} + \frac{0.577}{\sqrt{2 \log_e v_o T}} \quad (4-10)$$

where T is the duration of process, and

$$v_o^2 = \frac{\int_0^\infty f^2 s_\delta(f) df}{\int_0^\infty s_\delta(f) df}$$

Assuming the resonance part is predominant, ie $v_o = f_o$
 $p = 3.2$ is obtained for $T = 100 \times \frac{1}{f_o}$.

Individual experimental results are listed in appendix 2
 Table A2-2.

4.4.2 Static Drag Coefficient

Since the static drag coefficient, C_{D_o} , is considered to be constant with the reduced velocity, values are listed after taking an average of 20 data (4 wind speeds and 5 spring systems) in Table 4-3. Those values can be compared with existing results in two different ways; one with respect to the section aspect ratio D/B and the other with respect to the turbulence scale ratio L_x/D for a square cylinder.

The former comparison is made among results of Nakaguchi et al (1968)⁽⁸⁴⁾, Bearman and Trueman (1972)⁽⁸⁵⁾ and Vickery (1968)⁽³⁸⁾ in Figure 4-10. Nakaguchi et al and Bearman and Trueman presented very similar results by using two dimensional models in a smooth flow. Although the results of present test show lower values than those because of the existence of high turbulence, the tendency such that C_{D_o} decreases with D/B between 0.75 and 2.0 is in a very good agreement with that of the smooth flow results.

Vickery's results were obtained from a similar turbulent flow to this test except that he used a water tunnel and three-dimensional models. In order to make a fair comparison results of $H/B = 10$ are extracted from his results and plotted in Figure 4-10. Generally the agreement is good except at the lowest D/B .

Table 4-3 Static Drag Coefficients

Shape	Position	$\overline{C_{D_0}}$	$\pm$ Standard Deviation	(%)
I	1	1.84	± 0.08	(4.6)
	2	1.88	± 0.10	(5.2)
II	1	1.92	± 0.09	(4.5)
	2	2.01	± 0.11	(5.5)
III	1	1.69	± 0.11	(6.8)
	2	1.64	± 0.11	(6.9)
IV	1	1.76	± 0.12	(6.6)
	2	1.76	± 0.08	(4.8)
V	1	1.70	± 0.12	(7.0)
	2	1.56	± 0.10	(6.5)
VI	1	1.54	± 0.09	(5.8)
	2	1.51	± 0.13	(8.3)
VII	1	1.68	± 0.11	(6.8)
	2	1.49	± 0.10	(6.9)

note : (1) Basic information on turbulence is as follows :

<u>position</u>	$\sigma_u/\bar{U}$	L_x
1	14%	61. mm
2	10%	115. mm

(2) Sample averaging number = 20

(3) Individual values of C_{D_0} are shown in appendix 2

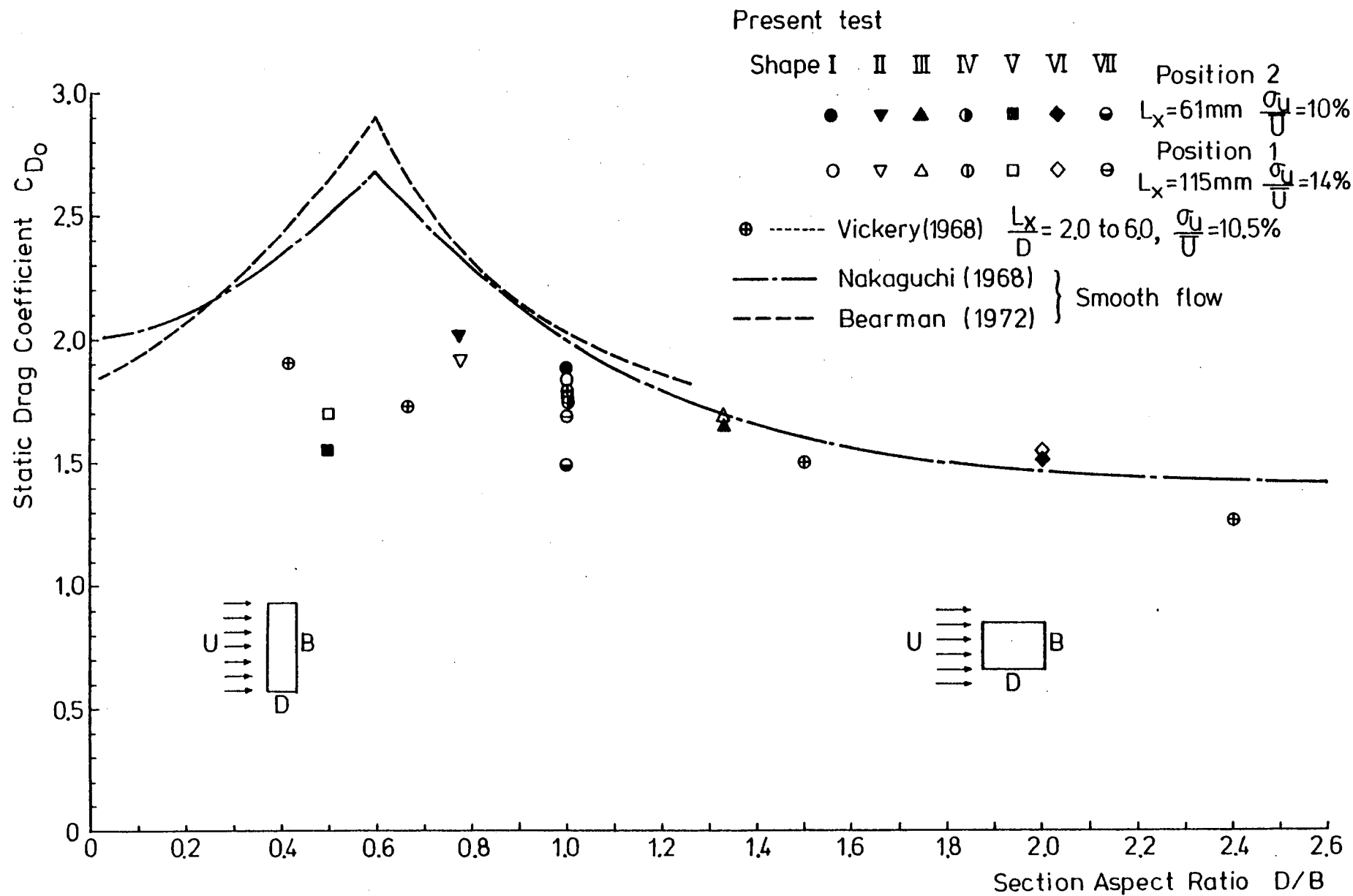


Figure 4-10 Variation of static drag with section aspect ratio

In this experiment shape II ($D/B = 0.75$) produced the largest C_{D_o} for both positions 1 and 2. The difference due to positions (ie stemming from different turbulence intensities and scale) is small for rectangular cylinders but a significant variation can be pointed out for a square cylinder and plotted in Figure 4-11 in comparison with McLaren et al's⁽⁴²⁾ and Lee's results⁽⁴⁴⁾.

Both positions 1 and 2 showed an increasing tendency with L_x/D . Results at position 2 are in reasonable agreement with both McLaren et al's and Lee's values but C_{D_o} values of the higher turbulence intensity ($\frac{\sigma_u}{\bar{U}} = 14\%$) appear to be considerably higher than those of Lee.

4.4.3 Dynamic Along-wind Force Coefficient

The dynamic along-wind force coefficient was computed from equation (4-6) and values were plotted against the reduced wind speed, $\tilde{U}$, in terms of its ratio to the static drag coefficient ($\tilde{C}_D/C_{D_o}$) in Figure 4-12 (a) to (g) for shapes I to VII respectively. Results are rather scattered especially in the lower reduced wind speed range. Very high values of $\tilde{C}_D/C_{D_o}$ ($>> 1.0$) appeared consistently in the range of $\tilde{U}$ (< 5.0) and converged to around unity at higher $\tilde{U}$ values. This trend is common to all the shapes of cross-section examined and confirms the tendency noted in the measured aerodynamic damping ratio plots in Figure 4-9. The difference between positions 1 and 2 is not significant even for

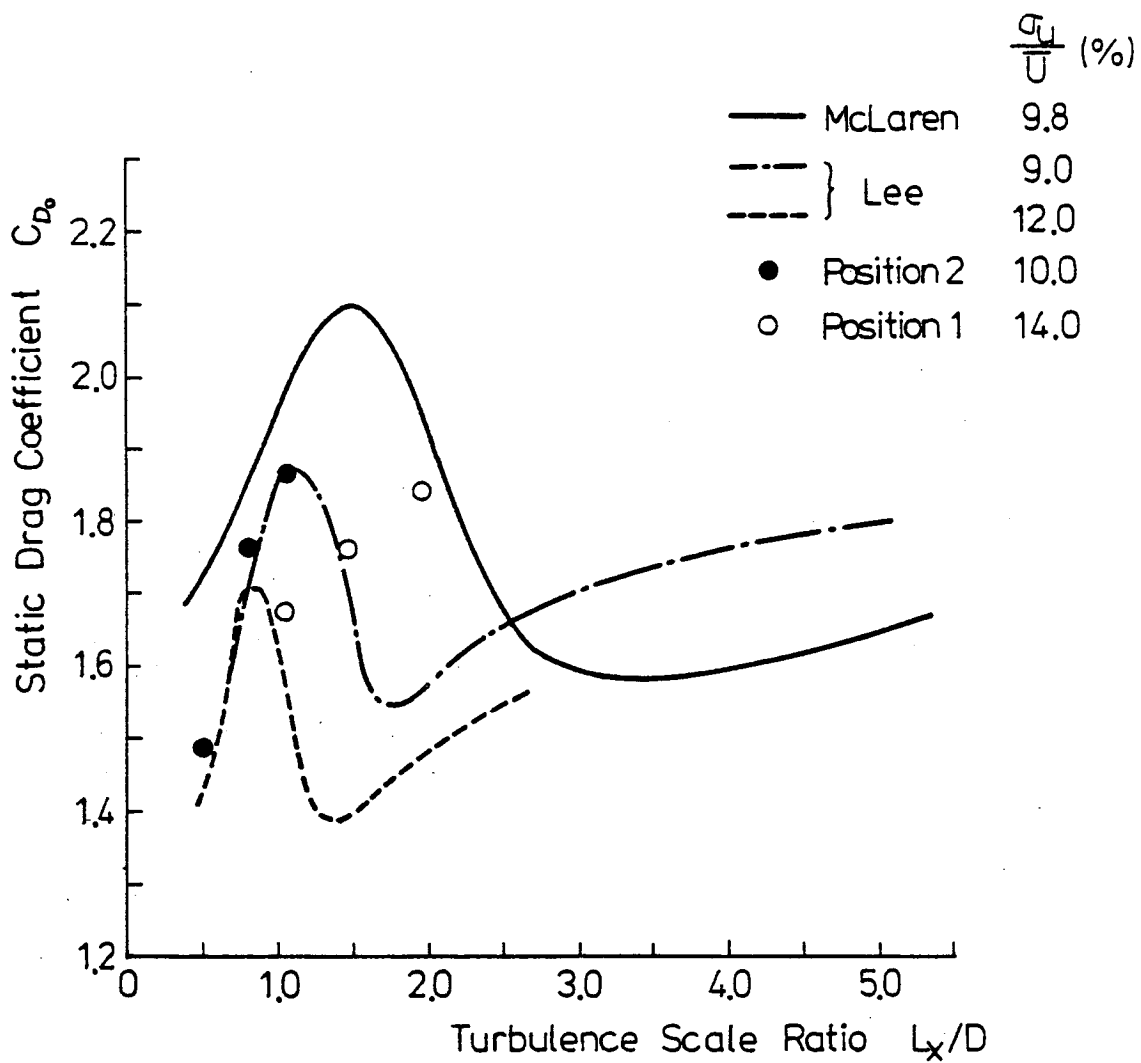


Figure 4-11 Variation of static drag with longitudinal turbulence scale

shape VII, whose static coefficients are rather different in the two positions.

Also the inverse of Strouhal number, $\tilde{U}_s = \frac{\bar{U}}{2f_s \cdot D}$ where f_s is the vortex shedding frequency, according to Nakaguchi et al⁽⁸⁴⁾ is shown in Figure 4-12 in order to indicate the vortex shedding effect on the along-wind response, but it is difficult to point out its significance in these results.

One of the causes of scattered results is probably due to variations in aerodynamic damping and the slightly inaccurate theoretical representation of it by equation (4-2). This can be improved by using a measured total damping ratio from the auto-correlation method rather than a theoretical postulation. Corrected dynamic along-wind force coefficient $\tilde{C}_D'$ values were computed accordingly.

Results after the aerodynamic damping ratio, ζ_A , correction are plotted against the same reduced wind speed $\tilde{U}$ in three group figures, Figure 4-13 (a), (b) and (c) for square cylinders, short rectangular ($D/B < 1.0$) and long rectangular ($D/B > 1.0$) cylinders respectively. Results are much less scattered than those of Figure 4-12 and make it possible to represent them by a single curve for each shape. Differences between positions 1 and 2 are again negligible. It is interesting to note that even an exceptionally low $\tilde{C}_D/C_{D_o}$ (≈ 0.3) obtained at $\tilde{U} = 28$ as shown in Figure 4-12 (e) falls within a reasonable range of variation as $\tilde{C}_D'/C_{D_o} = 0.75$ after ζ_A correction.

Shape I

Position 1, 2

$$\frac{L_x}{D} = 1.92, 1.01$$

Spring system

A ○ ●

B ◇ ◆

C △ ▲

D ▽ ▼

E □ ■

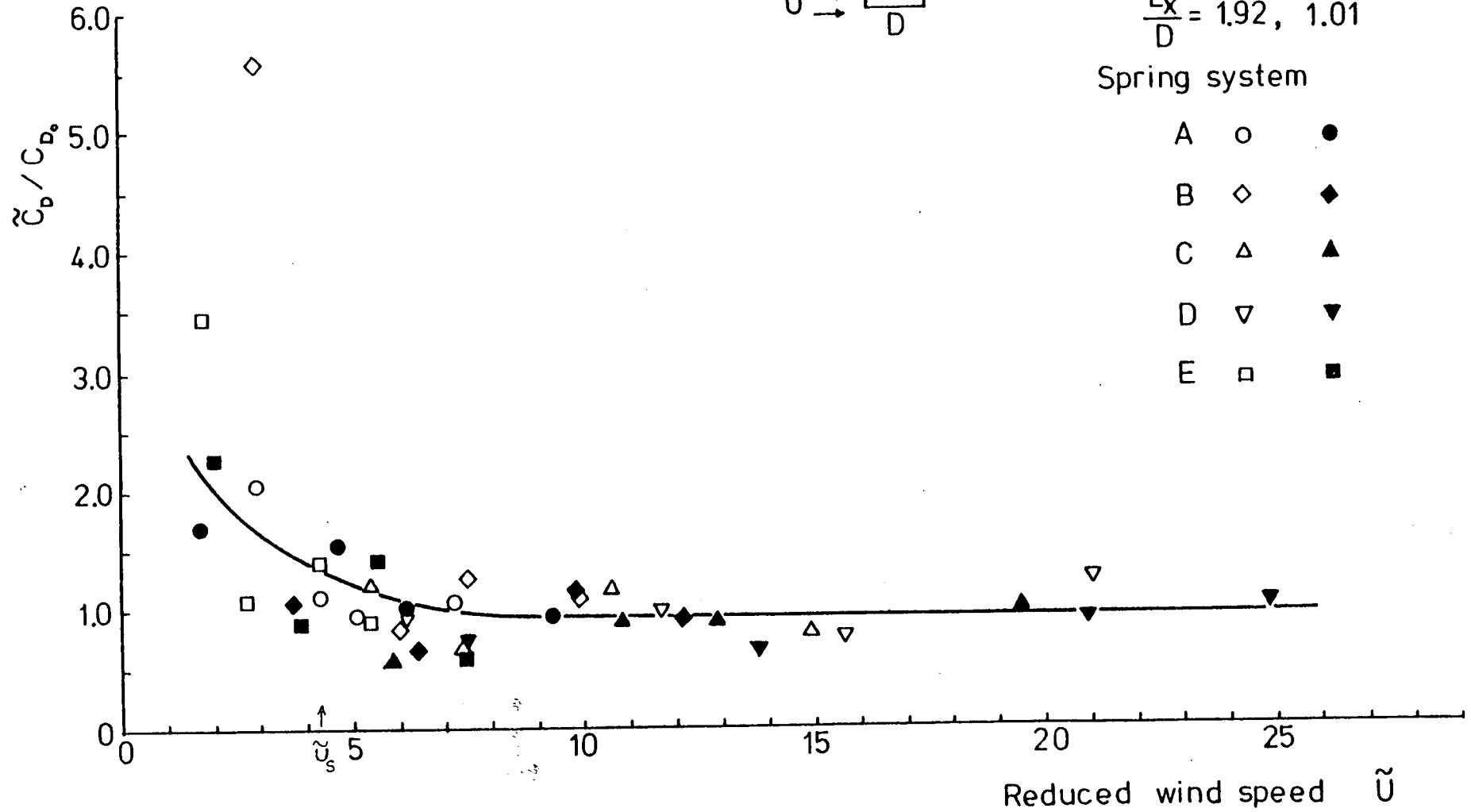


Figure 4-12(a) Variation of $\tilde{C}_D$

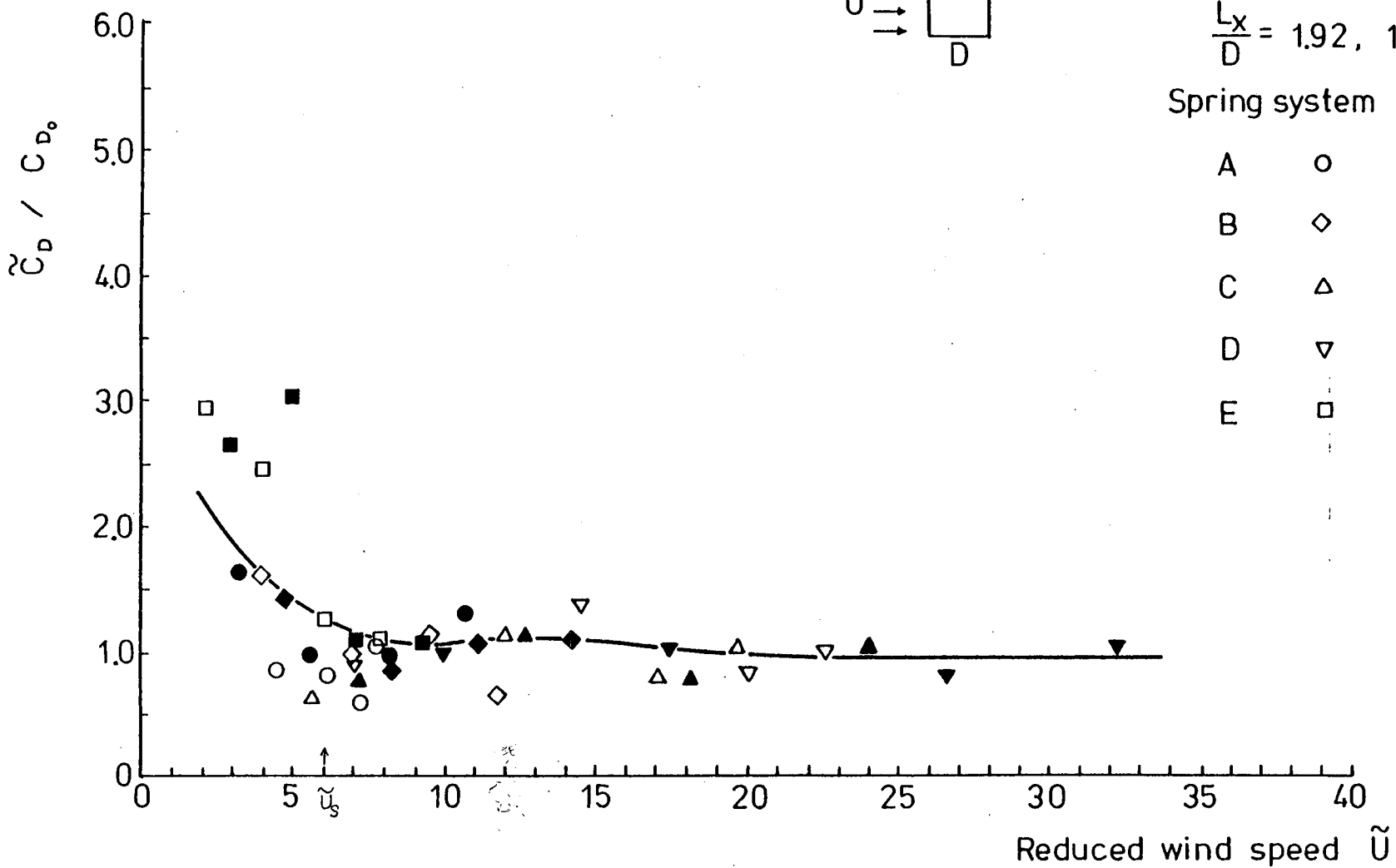


Figure 4-12(b) Variation of $\tilde{C}_D$

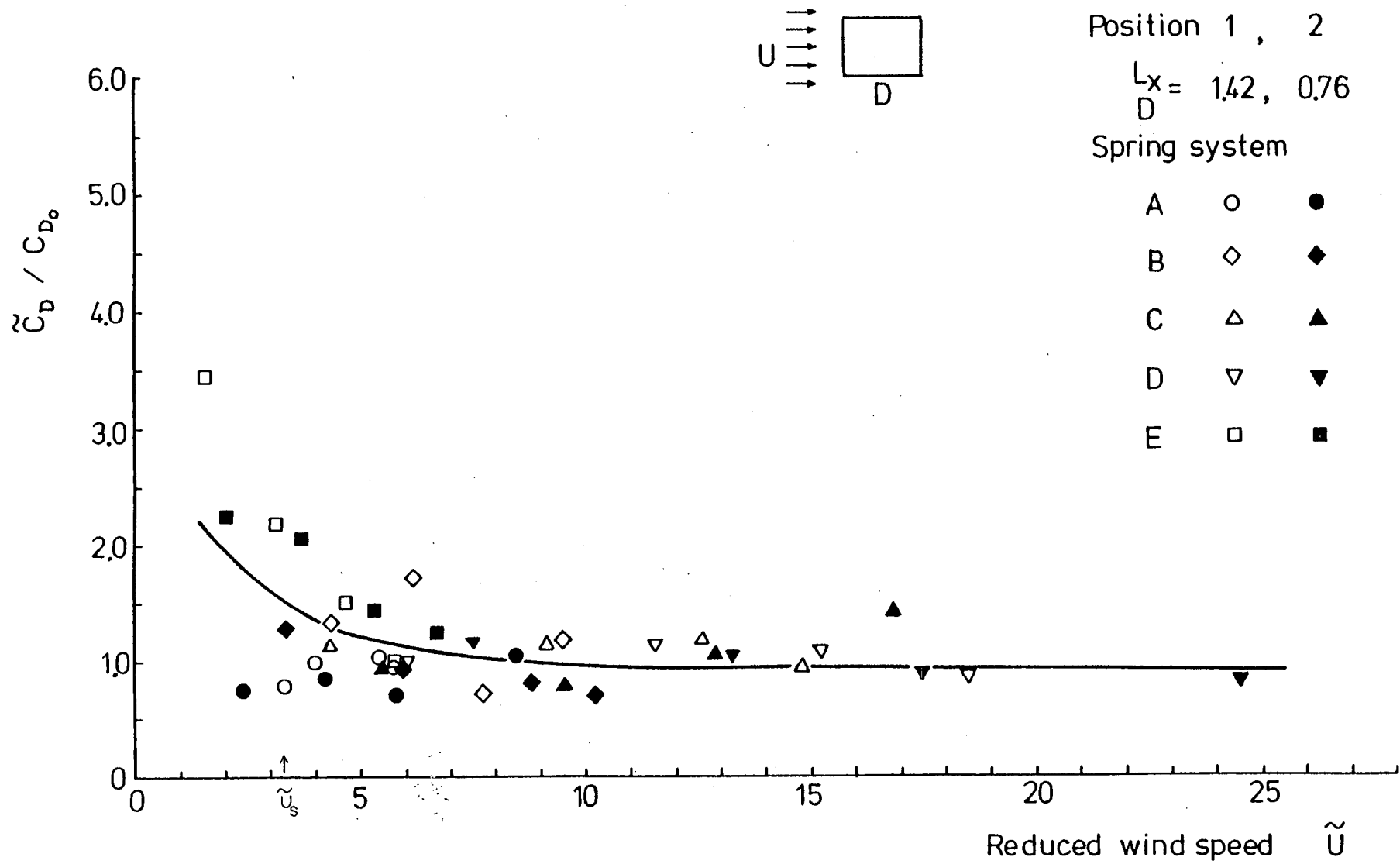


Figure 4-12(c) Variation of $\tilde{C}_D$

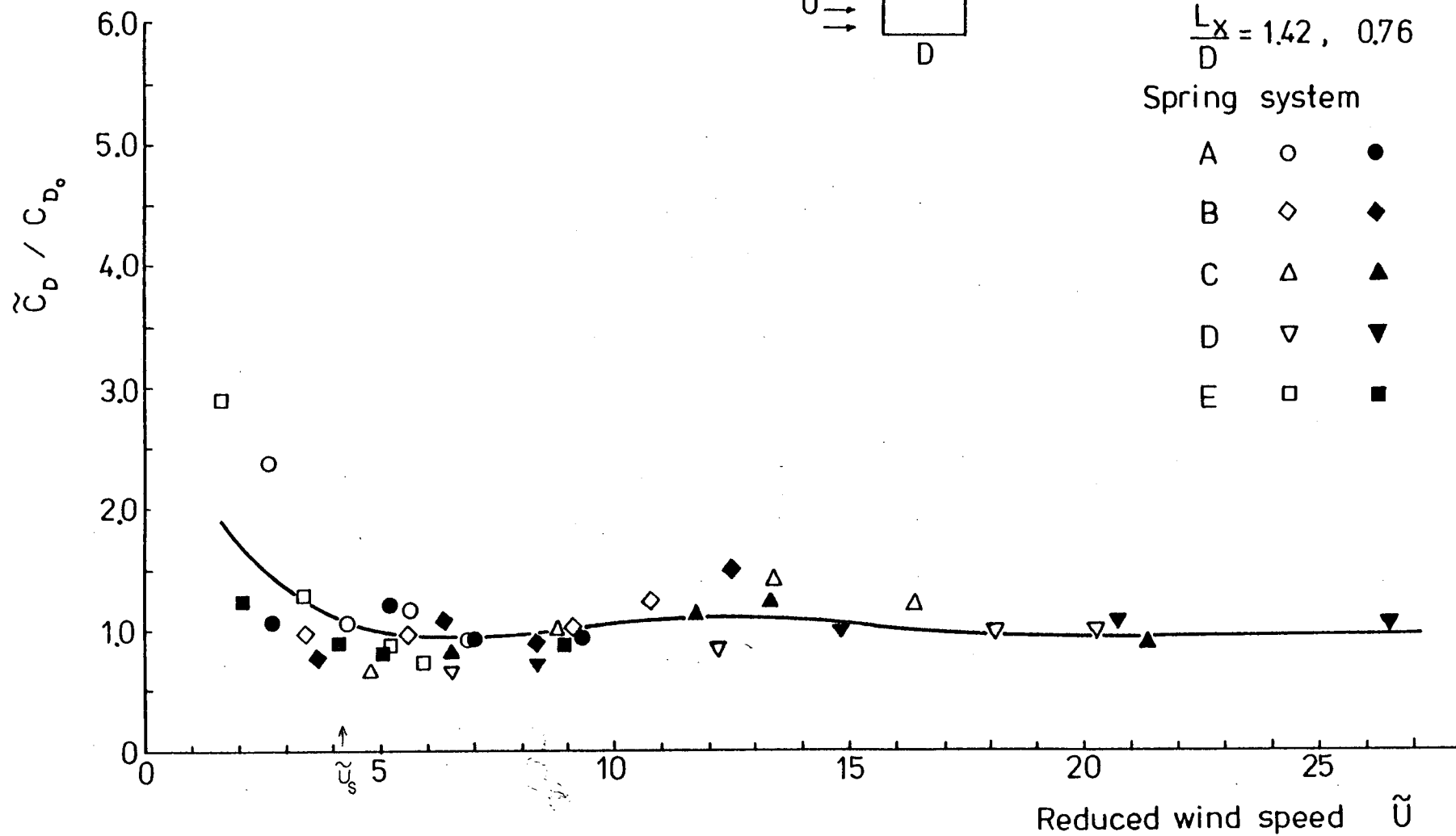
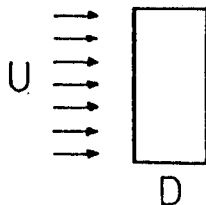


Figure 4-12(d) Variation of $\tilde{C}_D$



Shape V
 Position 1, 2
 $\frac{L_x}{D} = 1.92, 1.01$

Spring system

A	○	●
B	◇	◆
C	△	▲
D	▽	▼
E	□	■

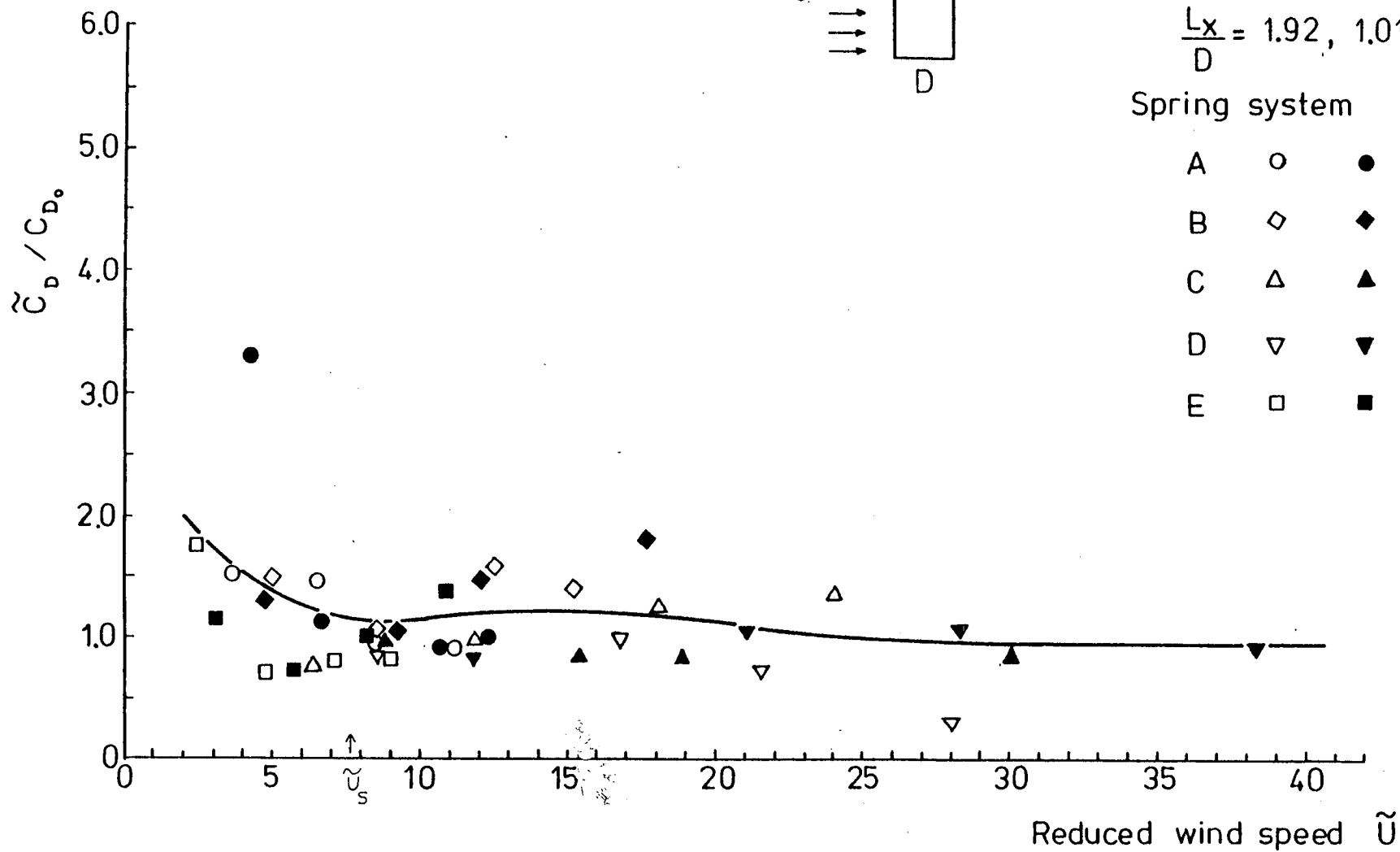


Figure 4-12(e) Variation of $\tilde{C}_D$

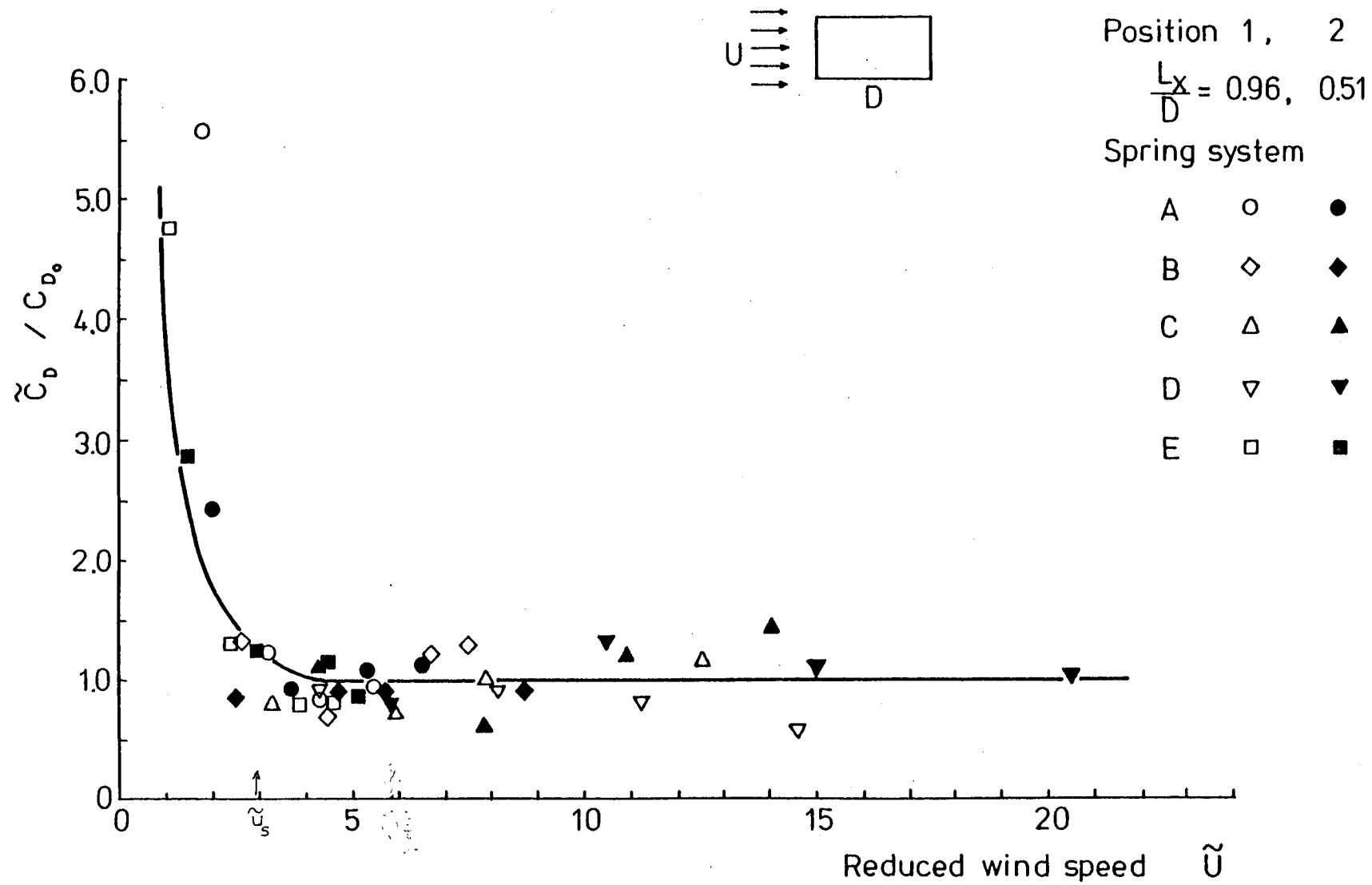


Figure 4-12(f) Variation of $\tilde{C}_D$

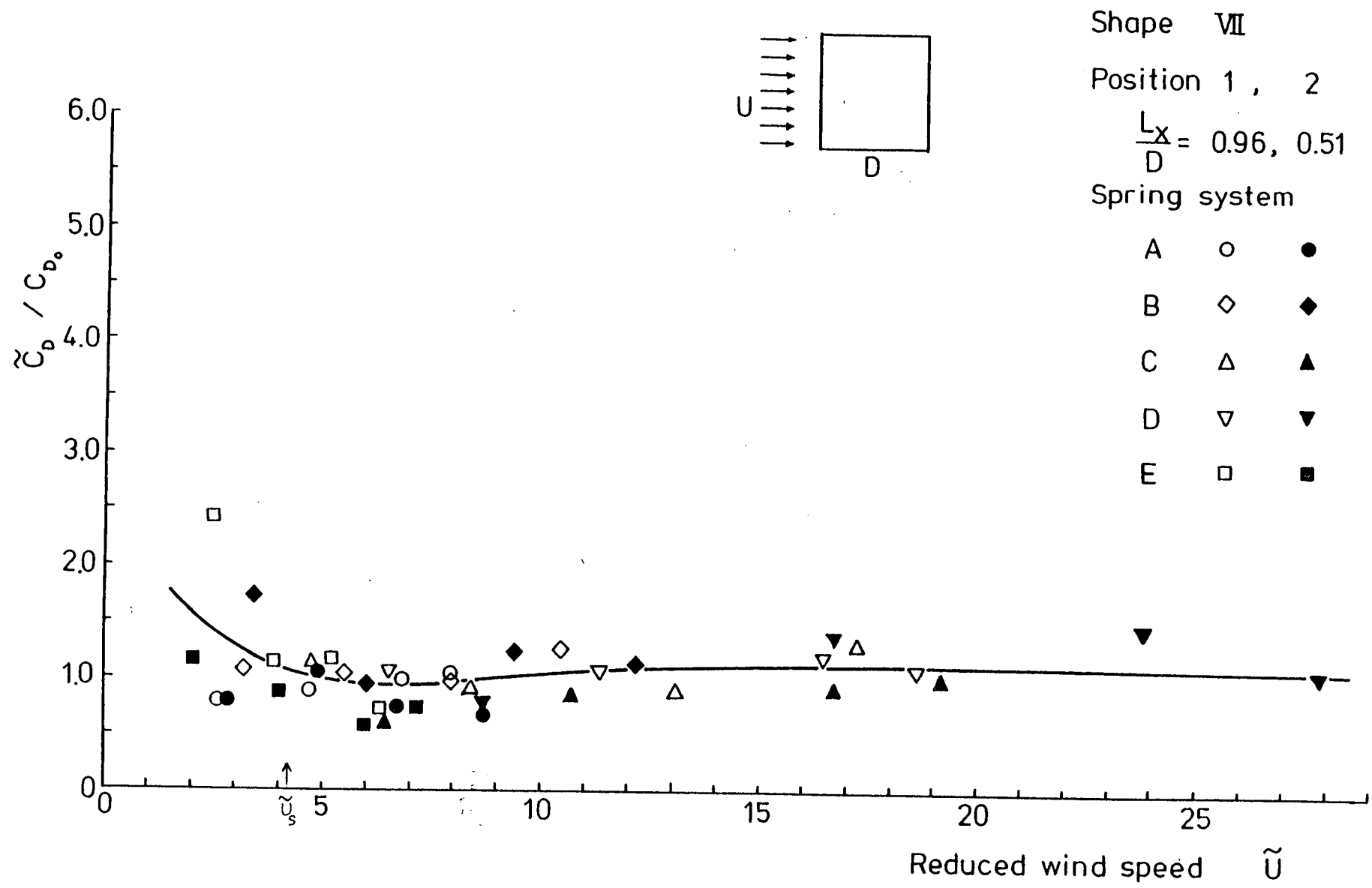


Figure 4-12(g) Variation of $\tilde{C}_D$

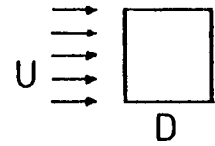
The maximum value of $\tilde{C}_D'/C_{D_o}$ is found to be 7.0 at the lowest, reduced velocity, $\tilde{U} = 1.1$, for $D/B = 2.0$, but this is not an exceptional value. $\tilde{C}_D'/C_{D_o}$ values are as large as 3 or more around $\tilde{U} = 2.0$ for any shape and position. At the lower range of $\tilde{U}$ examined in this experiment the smaller D/B becomes, the higher $\tilde{C}_D'/C_{D_o}$ values tend to be. By contrast at the higher range of $\tilde{U}$ ($10 < \tilde{U} < 25$) $\tilde{C}_D'/C_{D_o}$ values become almost constant (≈ 0.8) irrespective of D/B . This seems to indicate that the dynamic along-wind force coefficient, $\tilde{C}_D$, has a very similar variation pattern to that of the static drag coefficient, C_{D_o} , against the turbulence intensity, scale and the section aspect ratio at higher $\tilde{U}$ ranges.

The tendency of $\tilde{C}_D'/C_{D_o}$ v.s. $\tilde{U}$ is discussed in detail in the following section. Its overall validity is also examined.

4.5 DISCUSSION

4.5.1 Sources of Error

Possible sources of error are discussed to examine the validity of results obtained in the previous section. The accuracy of static drag coefficient values can be estimated from the standard deviation of 20 samples as shown in Table 4-3 and is of the order of 7.0% for most cases. This can be expected to be caused by some manual reading error of reference wind



Position 1, 2

Shape I ○

IV ○ ●

VII ○ ●

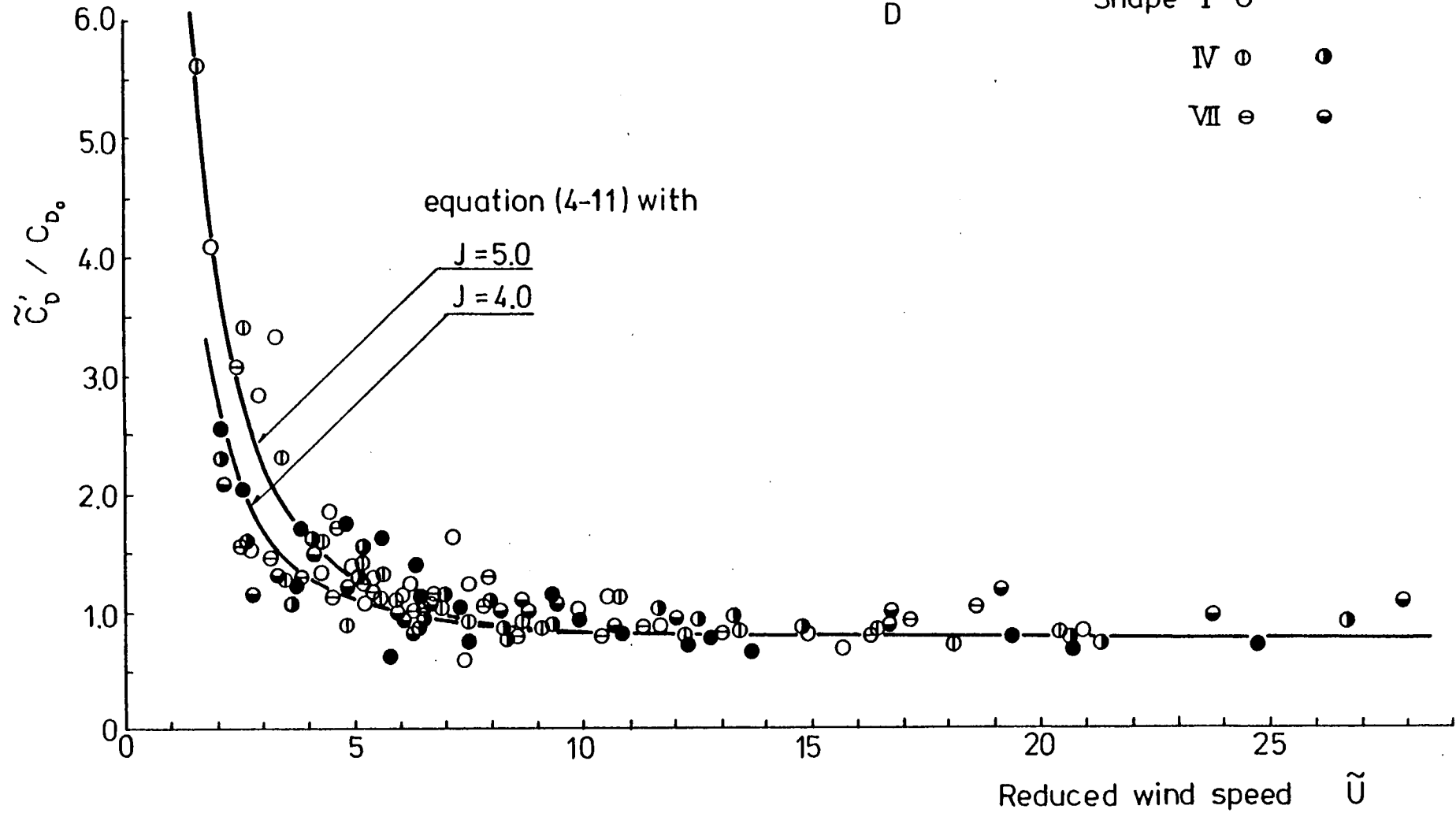


Figure 4-13(a) Variation of $\tilde{C}_p'$ for square cylinder
(after ζ_A correction)

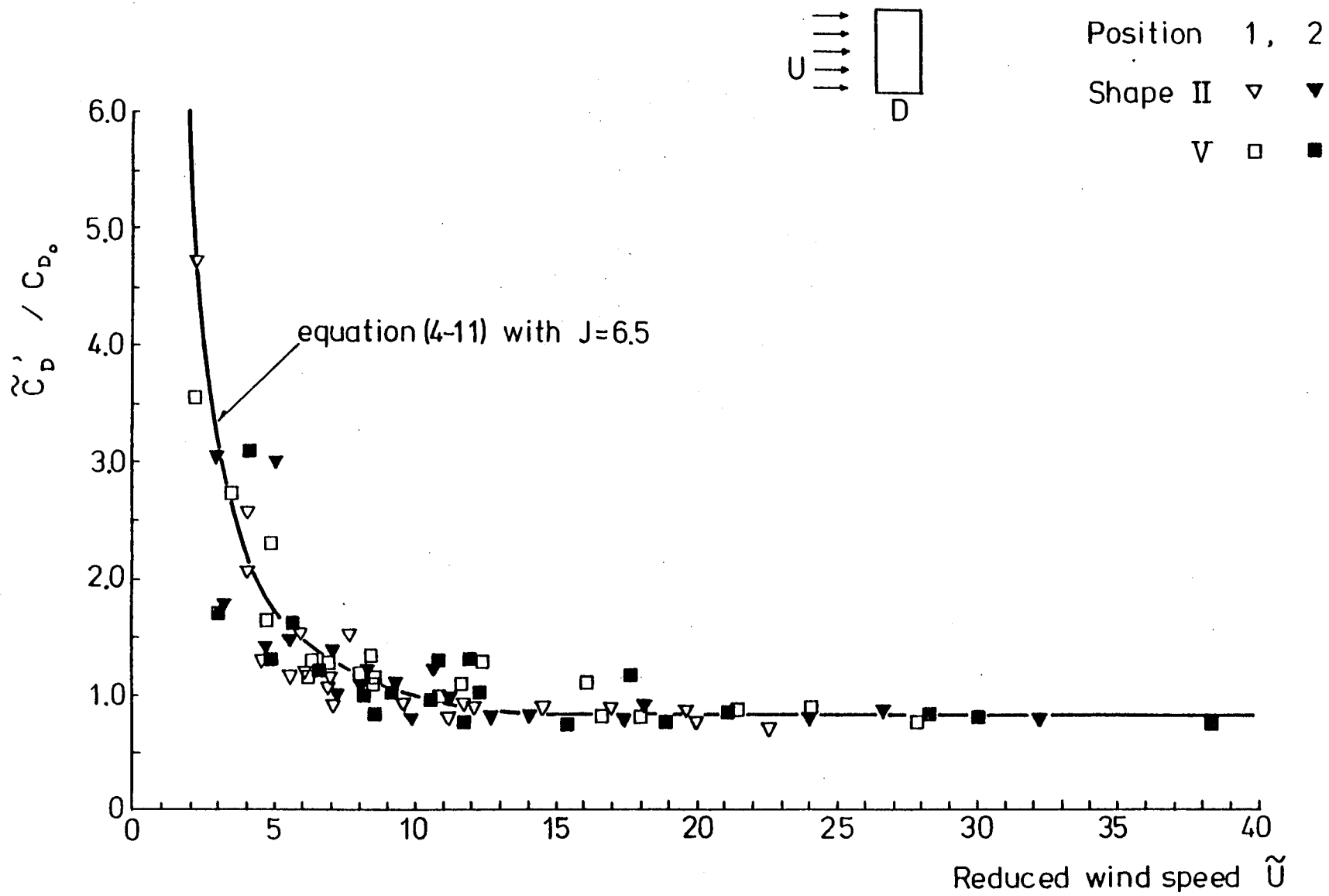


Figure 4-13(b) Variation of $\tilde{C}_D'$ for short rectangular cylinder
(after ζ_A correction)

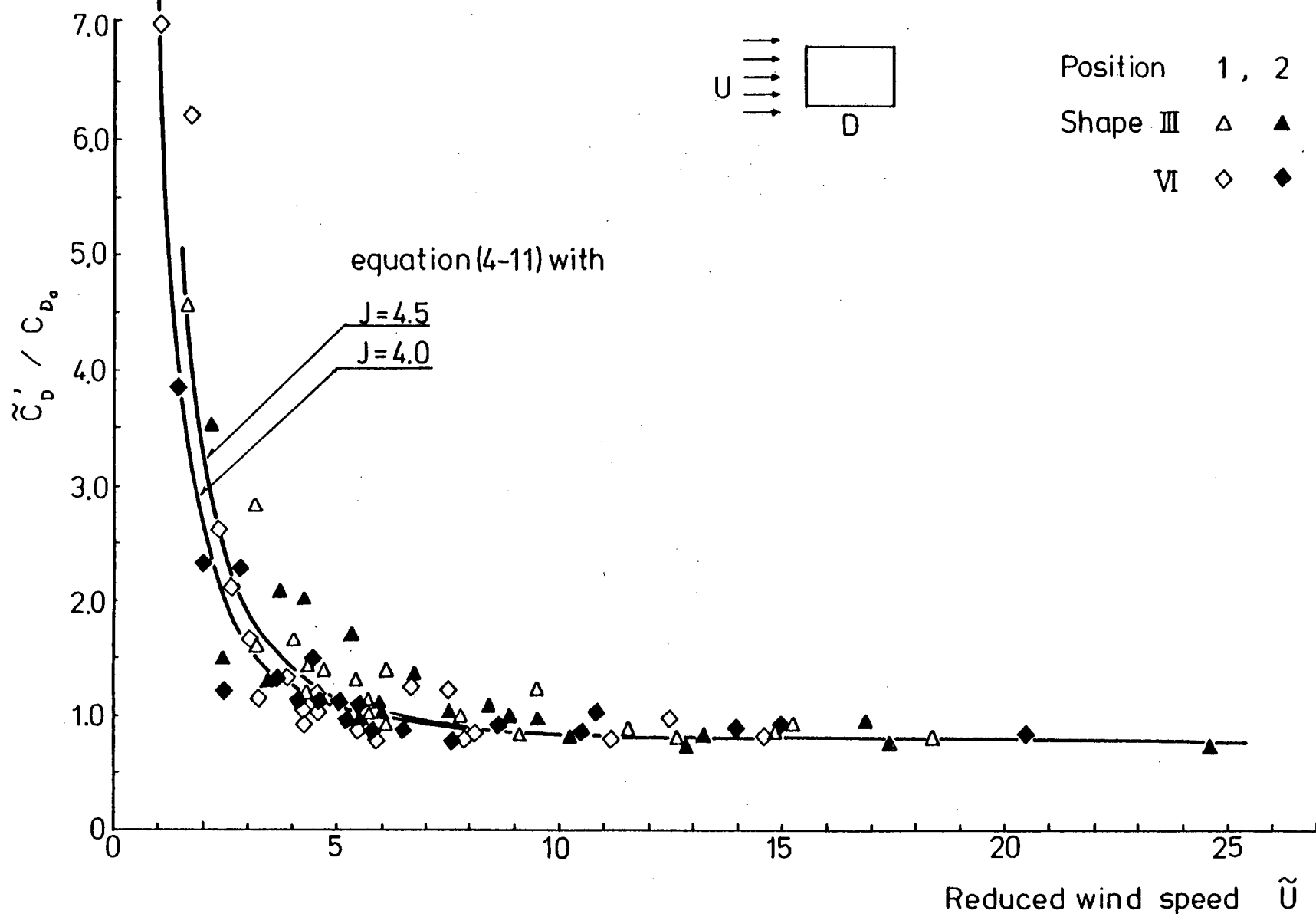


Figure 4-13(c) Variation of $\tilde{C}_D'$ for long rectangular cylinder
(after ζ_A correction)

pressure measurements and output records. For instance, a slight variation of the mean wind speed profile or the value of power law index, α , could arise since a 3% variation of mean wind speed could promote a 10% variation of α .

Allowing 7.0% variation for the static drag coefficient value there are still considerable discrepancies between the present results and Lee's as shown in Figure 4-11. One can expect that the different turbulence characteristics might cause these discrepancies since Lee's results were obtained in grid-generated turbulence, whereas the present tests used a partial boundary layer turbulence. However considering the fact that a drastic change of C_{D_o} around $L_x/D = 1.0$ was commonly observed, it would be difficult to draw a general conclusion for the evaluation of the static drag coefficient in a turbulent flow at the moment. It is evident that the C_{D_o} value is sensitive to both the turbulence intensity and scale around $L_x/D = 1.0$, but more supporting data would be necessary with experiments in a more extensive range of $\frac{\sigma_u}{U}$ and L_x/D in order to discuss their effects quantitatively.

It was expected in previous sections that there would be more parameters contributing to the evaluation of the dynamic along-wind force coefficient rather than to that of the static drag coefficient. In fact $\tilde{C}_D/C_{D_o}$ plots in Figure 4-12 have widely spread variations despite the fact that the error caused by reference wind speed calibration was eliminated by taking the ratio

against C_{D_0} . However $\tilde{C}_D/C_{D_0}$ does not appear to be sensitive to $\sigma_u/\bar{U}$ and L_X/D as the static drag coefficient is and also the scattered variation of $\tilde{C}_D/C_{D_0}$ seems due mainly to the variation of aerodynamic damping. This can be seen in Figure 4-13 where reasonably consistent variations of $\tilde{C}_D'/C_{D_0}$ were deduced from two groups of scattered data in Figure 4-9 for ζ_A'/ζ_A and Figure 4-12 for $\tilde{C}_D/C_{D_0}$.

The range of accuracy of $\tilde{C}_D/C_{D_0}$ and $\tilde{C}_D'/C_{D_0}$ may be estimated as follows,

- (i) the order of reading error from the oscillograph output records will be approximately 2% since record amplitudes lay between ± 5 and ± 50 mm of the chart with a trace line width ≈ 0.2 mm;
- (ii) the order of error in turbulence intensity will be approximately 5% which is the order of difference due to corrected analogue estimation and digital computation (see section 3.5.2);
- (iii) the variation of decay factors k_z and k_h for $C_{u_1 u_2}(f)$, the length constant L and power spectral form index β will be of the order of 15% but the contribution of those parameters to the evaluation of $\tilde{C}_D$ is relatively low, eg a typical variation of $\tilde{C}_D$ due to a 15% deviation of k_z , k_y and β is approximately 3% and the variation caused by a 15% deviation of L is approximately 5%;

- (iv) the order of error in f_o and ζ_s will be approximately 3% and 15% respectively as noted in section 4.3.5 and the expected error in $\tilde{C}_D$ due to those will be approximately 1% and 8% respectively;
- (v) it seems difficult to estimate the order of error in ζ_T' computed by the autocorrelation method, but it will be similar to that of ζ_s , ie 15%, which causes 8% error $\times$ of $\tilde{C}_D'$. The error in ζ_T based on ζ_A obtained from equation (4-2) could be as high as 30% (causing a 15% error in $\tilde{C}_D$) for the higher $\tilde{U}$ range because of uncertainty about the dynamic drag coefficient in the equation, and even higher for the lower $\tilde{U}$ range.

Then the overall error due to possible causes, predominantly from the error source (v), can be expected to be approximately 10% as a square root of the summation of individual errors for $\tilde{C}_D'/C_{D_o}$ results and 20% for $\tilde{C}_D/C_{D_o}$ results at $\tilde{U} \approx 5.0$ to 25.0. This order of error seems reasonable in comparison with the variation of actual plots in Figures 4-13 and 4-12.

It is interesting to note that random variation appears more significantly in the aerodynamic damping values of Figure 4-9 rather than in the dynamic along-wind force coefficient values of Figure 4-13.

4.5.2 Further Application

From final results plotted in Figure 4-13 an empirical expression for $\tilde{C}_D/C_{D_o}$ between $\tilde{U} = 2.0$ and 25.0 can be proposed as follows,

$$\frac{\tilde{C}_D}{C_{D_o}} = \left\{ 0.6 + \left(\frac{J}{\tilde{U}} \right)^3 \right\}^{\frac{1}{2}} \quad (4-11)$$

where J is a constant defined empirically by D/B.

Best fit J values are tabulated with the section aspect ratio D/B in Table 4-4.

Table 4-4 Variation of J with D/B

D/B	0.5	0.75	1.0			1.33	2.0
Shape	II	V	I	IV	VII	III	VI
J	6.5	6.5	5.0	5.0	4.0	4.5	4.0

Equation (4-11) with corresponding J values in Table 4-4 are shown in Figure 4-13 so as to represent the experimental results. The greater the section ratio, the smaller the J values as clearly seen in Figure 4-13 and Table 4-4.

The constant 0.6 in the square root of equation (4-11) was determined from a consistent value of $\tilde{C}_D'/C_D \approx 0.8$ at the higher reduced wind speed range ($\tilde{U} > 15$) irrespective of D/B. The power index 3 inside the square root of equation (4-11) was taken to

provide best fit curves at the lower reduced wind speed range. Since the mass coefficient effect on $\tilde{C}_D$ will be represented by a power index 2 if $C_M \approx \text{constant}$ (see equation (2-40)), the power index 3 means that the variation of the dynamic drag coefficient is very significant. This trend can be indicated by Keulegan and Carpenter's results more clearly than Davenport's ones as shown Figures 1-2(a) and 1-3(a). However it is not possible to state which effect, C_M or variation of C_D itself, is predominant on $\tilde{C}_D$ only from the present results, even though the aerodynamic damping ratio plots in Figure 4-9 suggest the significance of the variation of C_D .

The vortex shedding effect was neglected in this form since its significance was not apparently confirmed by this two-dimensional bluff body experiment and hence it can be expected to be less significant for three-dimensional blocks⁽³⁸⁾.

The proposed form is expressed as a function of $\tilde{U}$ based on the along-wind dimension D , but it can be modified easily to an expression with $\tilde{U}_B$ based on a cross-wind dimension, B , as $\tilde{U}_B = \bar{U}/(f_o \cdot B)$ as sometimes conventionally used. Then equation (4-11) becomes

$$\frac{\tilde{C}_D}{C_{D_o}} = \left\{ 0.6 + \left(\frac{J_B}{\tilde{U}_B} \right)^3 \right\}^{\frac{1}{2}} \quad (4-12)$$

where $J_B = J \times \frac{D}{B}$.

For example $J_B = 3.25, 5.0$ and 8.0 when $D/B = 0.5, 1.0$ and 2.0 respectively. Equation (4-12) demonstrates D/B effect on the evaluation of $\tilde{C}_D$ clearly and can be compared with flat plate results ($D/B \approx 0$) by Davenport (1961)⁽³⁾ in Figure 4-14 as an extrapolation of the present results.

As shown in Figure 1-3, C_D and C_M were evaluated by Davenport for a flat plate by using a sinusoidally fluctuating velocity. If the along-wind pressure correlation was assumed to be unity, $\tilde{C}_D$ could be expressed by equation (1-3). However, Simiu pointed out as reviewed in section 1.2.1, the along-wind pressure correlation was considered to play an important role in the evaluation of $\tilde{C}_D$. Simiu recommended the along-wind pressure correlation to be 0.0 or 0.2⁽¹⁶⁾ which leads to $\tilde{C}_D \approx 0.71 C_{D_0}$ and $0.78 C_{D_0}$ respectively. He modified his proposals for this correlation to a more conservative value of 0.4⁽²⁰⁾ which leads to $\tilde{C}_D \approx 0.84 C_{D_0}$. The former values seem to be more consistent with his quotations^(45,86). However, even in the high reduced wind speed range, the mass coefficient effects should not be neglected, for example, $\tilde{C}_D \approx 1.1 C_{D_0}$ at $\tilde{U} \geq 15$ can be estimated from Davenport's results. Therefore taking both the mass coefficient effect and the variation of dynamic drag coefficient from Davenport's results and assuming zero along-wind pressure correlation for both C_D and C_M , another empirical form for $\tilde{C}_D$ may be written as,

$$\frac{\tilde{C}_D}{C_{D_0}} = 0.71 \frac{C_D}{C_{D_0}} \sqrt{1 + \frac{\pi^4}{4\tilde{U}^2} \cdot \frac{C_M^2}{C_D^2}} \quad (4-13)$$

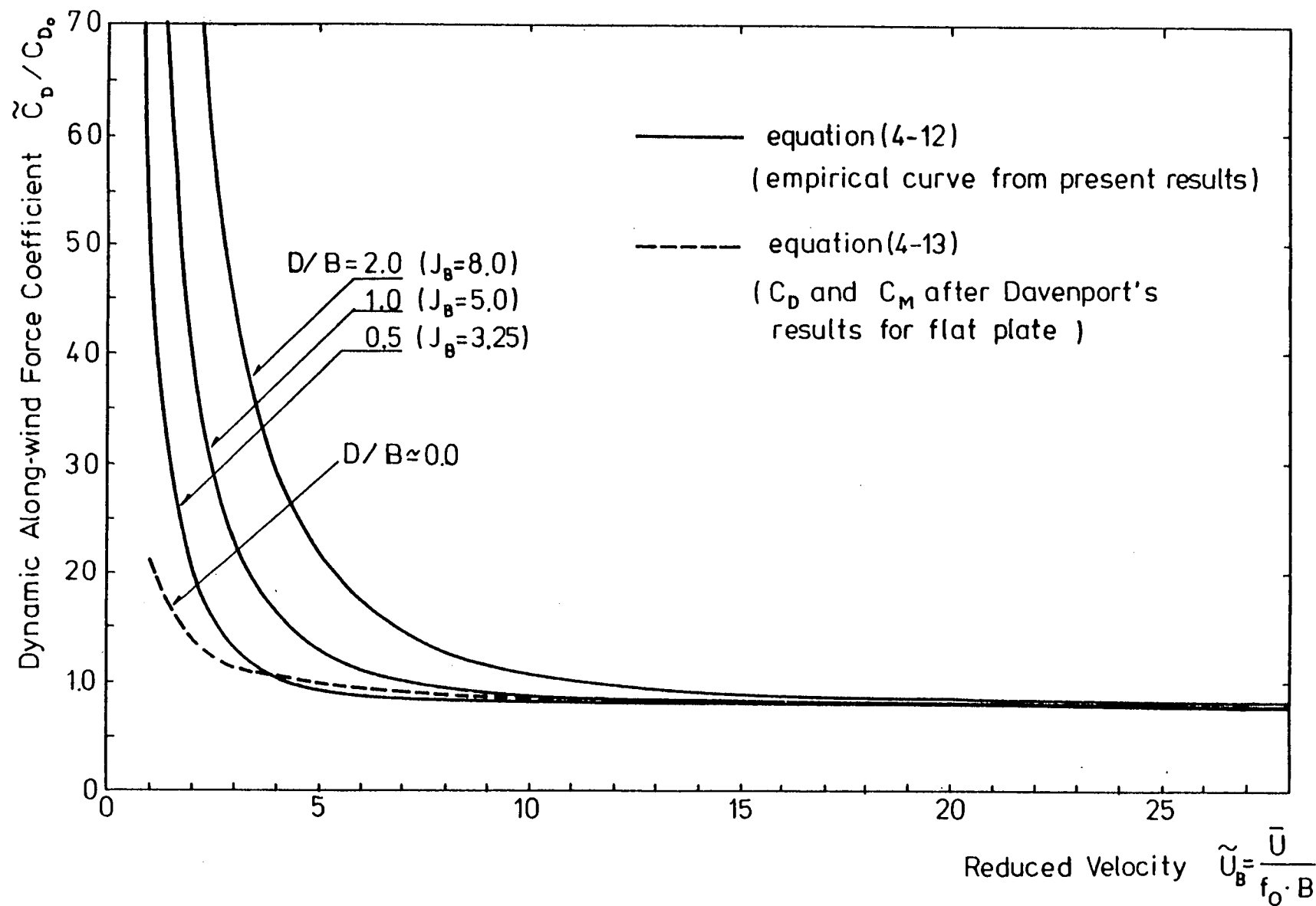


Figure 4-14 Proposed expression for $\tilde{C}_D$ against $\tilde{U}_B$ and $\frac{D}{B}$ presented in normalised form

This yields $\tilde{C}_D \approx 0.78 C_{D_o}$ at $\tilde{U} > 15$ giving a reasonable consistency with equation (4-12). At the lower reduced wind speed range, the variation of the proposed form of equation (4-12) due to different D/B (between 0.5 and 2.0) is in a very good agreement with the extreme case of D/B = 0.0 represented by equation (4-13).

Although parameters employed in this experiment are limited especially for L_x/D in comparison with actual situations, the dynamic along-wind force coefficient evaluated appears to be consistent in an extensive range of the reduced velocity with five different D/B values. In other words equation (4-12) seems reasonably consistent with suitable J values except shape VII. The value of J found for shape VII appear to be comparatively smaller than those of shape I or IV (D/B = 1.0) and that of III with a larger D/B. Also experimental results of $\tilde{C}_D/C_{D_o}$ at higher $\tilde{U}$ values for shape VII are less agreeable with equation (4-11) than those for other shapes. This might be caused by a smaller turbulence scale ratio L_x/D .

Since the experimental results of the dynamic along-wind force coefficient were well represented by equation (4-12) for the range of L_x/D around unity with two different turbulence intensities despite a significant variation of C_{D_o} revealed in the same range of turbulence characteristics, equation (4-12) could be applicable to cases in a more extensive range of L_x/D and turbulence intensity. The latter could be another factor in determining $\tilde{C}_D$,

but the difference due to that was not significant in the range of $\sigma_u/\bar{U} = 10\%$ and 14% . This may well be expected from Davenport's investigation⁽¹³⁾, ie the dynamic drag and mass coefficient was found to be fairly constant with different levels of sinusoidal amplitude of velocity.

The sharp increase of the dynamic along-wind force coefficient at low reduced velocities may be explained by the nature of power spectral density of leeward face pressure. It has been reported in quite a few pressure measurements^(71,22,86,87) of a full scale structure that the normalised power spectral density of pressure at a leeward surface has considerably greater values in the higher frequency range than that of pressure at a windward surface; the latter being rather similar to that of the natural undisturbed wind. This indicates that at the relatively low frequency range the dynamic drag coefficient has a value similar to or less than the static one if the along-wind correlation is poor and that, at the higher frequency range the dynamic drag force coefficient could be considerably higher than the static drag coefficient, because of the existence of a high wake component in that range (namely the lower reduced velocity range).

Distinctive features of drag or along-wind force between a two-dimensional cylinder and a three-dimensional body are mostly due to the difference in wake pressure or leeward face pressure. It is recognised that three-dimensional effects such as the lower aspect ratio (H/B) and the shear velocity profile tends to decrease the significance of wake pressure contribution to the total force⁽³⁸⁾.

This can be explained because the regular vortex shedding in the wake of a two-dimensional body is unlikely to occur near the top of a three-dimensional body and also the velocity profile prevents regular vortices from being as significant near the bottom as in cases of uniform flow. When there is a likelihood of regular vortex shedding being maintained the static drag coefficient could increase as noted by Bearman and Trueman⁽⁸⁵⁾. (N.B. the peak value of C_{D_0} at $D/B \approx 0.6$ in Figure 4-10).

In turbulent flow regular vortex shedding is rather unlikely. Nevertheless it seems reasonable to state that the leeward pressure will be more significant in the case of two-dimensional flow than three-dimensional flow. Therefore if the sharp increase of the dynamic along-wind force coefficient is mostly due to the higher frequency component of leeward face pressure, this effect will be reduced when applied to the three-dimensional problem similar to the vortex shedding effect. However it is difficult to discuss the relationship between the dynamic along-wind force coefficient and the leeward face pressure in more detail at the present moment. The applicability of the present findings to full scale wind-structure interaction has to be investigated further in future work.

CHAPTER 5 APPLICATION TO DESIGN OF TALL BUILDINGS
AGAINST STRONG WIND

5.1 INTRODUCTION

Gust response approaches reviewed in section 1.2 still seem to be insufficient to provide a most general applicability. This is partly because those approaches adopted more or less fixed formulae for wind characteristics with limited variation of parameters and also because the variation of the dynamic drag and mass coefficients and the along-wind pressure correlation were not fully taken into account.

Since more flexible forms for wind characteristics suggested in chapter 3 and the experimental evaluation of the dynamic along-wind force coefficient achieved in chapter 4 improved the situation mentioned above, it is necessary and useful to develop a gust response prediction procedure incorporating all possible parameters for practical applications.

Although it is generally important to discover some unknown physical phenomena, it is sometimes more important for engineering researchers to make their results readily applicable by practising engineers (in this case civil and structural designers) to their works. A basic design procedure emanating from this research is described in section 5.2 as a guide to subsequent sections.

A prediction method is developed in section 5.3 as a simple extension of the basic theory of along-wind response in chapter 2. The computation process of gust response can be divided into

several parts each of which can be represented by a specific factor. These factors are presented in section 5.4 in chart form for practical convenience using various parameters similar to Vickery's presentation⁽¹⁸⁾.

Vickery's work has been appreciated because of its treatment of the turbulence scale, the decay constants in the coherence function and the shape of the vibrational mode as variables, all of which were assigned constant values in previous works^(5,14). However, further availability of meteorological information requires a more flexible form for the computation of a gust factor. Therefore in this work new height dependent parameters of turbulence scale, turbulence intensity and decay constants and also a parameter determining the form of the power spectral expression are introduced.

In sections 5.5 and 5.6 the effects on dynamic along-wind response of the height dependence of the wind characteristics and the variation of dynamic along-wind force coefficient are discussed. Numerical comparison with previous approaches proposed by Davenport⁽⁵⁾, Vickery⁽¹⁸⁾ and Simiu⁽¹⁶⁾ is also made to clarify those effects mentioned above.

5.2 DESIGN PROCEDURE

A rational sequence from wind data to structural aspects can be summarised as follows⁽⁵²⁾,

1. definition of windiness of region from macro-meteorological data;
2. determination of micro-meteorological features (wind structure) of a site within that region, concerning factors : (a) terrain roughness, (b) topography and (c) orientation of structure;
3. response analysis based on aerodynamic coefficients and statistical parameters;
4. probabilistic interpretation of prediction to meet design code requirements.

Each procedure should be treated in a stochastic manner bearing the probability of occurrence in mind. In procedure (1) it is nowadays quite common to use a design wind speed from a statistical extrapolation of meteorological data, eg, a maximum expected wind speed in 50 years⁽⁸⁸⁾. The return period for the design speed may well depend on the social or economical situation. It is preferable to use at least two periods for design practice; one is longer such as 50 or 100 years mainly depending on the expected life of a structure and the other is

shorter such as 10 or 20 years depending on the life of a secondary member and fatigue effects. A third period may need to be considered in respect of the susceptibility of building users to structural response and this could be of the order of a 1- to 2-year return period.

For example it could happen in a highrise residential building that the maximum wind speed in a year causes an unbearable discomfort to residents although the maximum response predicted from the maximum expected wind speed in 50 years is well within the limits of structural safety. The return period for the shorter one is also a problem of social or psychological factors not purely an engineering one. The determination of the return period is out of the scope of this thesis and so it will be sufficient to point out the existence of this problem.

Currently the design wind speed considered is a 3-sec gust speed in the British Code of Practice⁽²⁾. The risk of exceedance in the 50-year period is 0.63 and annually 0.02. However the period of 3 sec is only determined conventionally by the response ability of anemometers. As shown in Van der Hoven's spectrum⁽⁸⁹⁾ over an extensive range of frequency (10^{-3} to 10^3 cycles/hour), the spectral gap exists between 1 to 6 cycles/hour, namely inverse of 1 hour to 10 minutes of period. This order of period is quite long enough to be far from the interesting range of frequency where the wind-structure interaction is in question. On the contrary the period 3 sec which was used to calculate design wind speed is well within the range of frequency of interest

and furthermore the measured value could have been influenced by the type of anemometer used. From the points mentioned above the one-hour (or at the shortest ten-minute) mean wind speed should be considered for determination of the design wind speed in order to continue following procedures in which the stationary process is postulated.

Although a large amount of information is readily available, factors like (b) or (c) in procedure (2) are seldom taken into account for determining wind structures for design. Fully probabilistic treatment for those factors and also for pressure or force coefficients or even for aerodynamic damping in procedure (3) is most desirable and yet highly complicated. A more general probabilistic assessment than the common procedure has been proposed by Mayne and Cook⁽⁹⁰⁾, applying the probability distribution of extreme hourly mean wind speed and pressure coefficient together with extreme value analysis to some particular cases.

In most gust response approaches the probabilistic treatment is only concentrated on deducing the maximum response from the mean and standard deviation (r.m.s.) value, eg, by adopting equation (4-10). Further establishment of information regarding the probability distribution of various parameters will be required for a fully probabilistic treatment in a whole design procedure.

In the following sections the prediction of maximum response of typical tall buildings is discussed based on the mathematical model of turbulence characteristics in a natural wind suggested in chapter 3.

5.3 PREDICTION OF ALONG-WIND RESPONSE

In order to reduce the expression for the power spectral density of generalised force described in chapter 2 to a form suitable for computation with various parameters, numerous simplifications and approximations are necessary.

The meaning of simplifications employed to represent the natural wind characteristics was discussed in chapter 3, and those simplified formulae are summarised by using as a reference height the structure height, H , as follows:

- (i) The mean wind speed profile can be expressed in a power law form as,

$$\bar{U}(z) = \left(\frac{z}{H} \right)^\alpha \bar{U}(H) \quad (5-1)$$

The mean velocity value here is an hourly mean, so that the whole process can be assumed to be reasonably stationary.

- (ii) The r.m.s. value of the longitudinal turbulent component can be expressed also in a similar form,

$$\sigma_u(z) = \left(\frac{z}{H}\right)^{-\alpha_T} \sigma_u(H) \quad (5-2)$$

- (iii) The reduced power spectral density of the longitudinal turbulent component can be expressed in a general form using a reference height H as,

$$\frac{f \cdot S_u(f, z)}{\sigma_u^2(z)} = k_1 \frac{\tilde{f}(z)}{(1 + \tilde{f}(z)^\beta)^{5/3\beta}} \quad (5-3)$$

where

$$\tilde{f}(z) = \frac{f \cdot L_h(z)}{\bar{U}(H)}, \quad L_h(z) = L(H) \left(\frac{z}{H}\right)^{\alpha_L} = L_1(z) \left(\frac{H}{10}\right)^\alpha,$$

$$L_1(z) = L(10) \left(\frac{z}{10}\right)^{\alpha_L} \quad \text{and} \quad k_1 = \frac{\beta \Gamma\left(\frac{5}{3\beta}\right)}{\Gamma\left(\frac{1}{\beta}\right) \Gamma\left(\frac{2}{3\beta}\right)}$$

- (iv) The co-coherence function of the longitudinal turbulent component can be expressed in a modified exponential form, ie,

$$C_{u_1 u_2}(y_1, y_2, z_1, z_2, f) = \exp\left(-\frac{f \sqrt{k_{hy}^2(z_m)(y_1 - y_2)^2 + k_{hz}^2(z_m)(z_1 - z_2)^2}}{\bar{U}(H)}\right)$$

(5-4)

$$\text{where } f^* = \sqrt{f^2 + \left(\frac{\bar{U}(10)}{k_2 L_h(z)} \right)^2}, \quad z_m = \sqrt{z_1 z_2},$$

$$k_2 = \sqrt{10} \quad (\text{see section 3.4}),$$

$$k_{hy}(z_m) = k_y(H) \left(\frac{z_m}{H} \right)^{-\alpha_D} \text{ and } k_{hz}(z_m) = k_z(H) \left(\frac{z_m}{H} \right)^{-\alpha_D}$$

Further simplifications to be made for a structure are as follows :

- (v) The response of the structure in the fundamental mode is dominant for both mean and fluctuating deflections and the structure is free-standing with a fundamental mode shape, $\mu(z)$, which can be approximated by a power law form with z (the torsional vibration is ignored), ie,

$$\mu(z) = \left(\frac{z}{H} \right)^{\alpha_\mu} \quad (5-5)$$

- (vi) The projected area of the structure is rectangular in shape with a constant width B (height H).

Substituting expressions (5-3) and (5-4) into (2-45) the power spectral density of generalised force $S_{F_n}(f)$ can be expressed as follows,

$$S_{F_n}(f) = \int_0^H \int_0^H \int_0^B \int_0^B \exp \left\{ \frac{-f^* \sqrt{k_{hy}^2(z_m)(y_1 - y_2)^2 + k_{hz}^2(z_m)(z_1 - z_2)^2}}{\bar{U}(H)} \right\} \cdot \sigma_u(z_1) \sigma_u(z_2)$$

$$\times k_1 \sqrt{\frac{\frac{L_h(z_1)}{\bar{U}(H)}}{\left[1 + \left\{\frac{f \cdot L_h(z_1)}{\bar{U}(H)}\right\}^{\beta \cdot 5/3\beta}\right]} \cdot \frac{\frac{L_h(z_2)}{\bar{U}(H)}}{\left[1 + \left\{\frac{f \cdot L_h(z_2)}{\bar{U}(H)}\right\}^{\beta \cdot 5/3\beta}\right]} \cdot \rho^2 \cdot \tilde{C}_{Dn}^2}$$

$$\times \bar{U}(z_1) \bar{U}(z_2) \mu(z_1) \mu(z_2) dy_1 dy_2 dz_1 dz_2$$

(5-6)

where $n = 1$ for resonance part and q.s. for quasi-static part.

The dynamic along-wind force coefficient value can be determined from an established $\tilde{C}_D - \tilde{U}$ relation. For example from Figure 4-13 or 14 if the reduced wind speed, $\tilde{U}$, is known a $\tilde{C}_D$ value can be estimated. For the resonance part the fundamental frequency f_1 will be used and for the quasi-static part a representative value f q.s., which gives the maximum $f \cdot S_u(f, \frac{2}{3}H)$ value in equation (5-3), ie,

$$\frac{f \text{ q.s.} \cdot L_h(\frac{2}{3}H)}{\bar{U}(H)} = \left(\frac{3}{2}\right)^{\frac{1}{\beta}}$$

may be used in order to calculate the corresponding $\tilde{U}$ value.

A value of mean wind speed of $\bar{U}(\frac{2}{3}H)$ could be considered

representative of the dominant mean wind speed over the height of a structure, ie, $\frac{2}{3} H$ is a location removed from influences at the top or bottom of the structure.

Employing normalised co-ordinates: $y = By'$ and $z = Hz'$, and substituting equations (5-1) (5-2) and (5-5) into (5-6), it may be rewritten as,

$$S_{F_n}(f) = \left\{ \frac{\tilde{C}_{D_n} \cdot \rho \cdot \bar{U}^2(H) \cdot B \cdot H}{(1 + \alpha - \alpha_T + \alpha_\mu)} \right\}^2 \cdot \frac{\sigma_u^2(H)}{\bar{U}^2(H)} \cdot \frac{S_u(f \cdot H)}{\sigma_n^2(H)}$$

$$\times \Psi^2(\alpha, \alpha_T, \alpha_\mu, \alpha_D, \alpha_L, \tilde{f}(H), f_B, f_H)$$

(5-7)

where

$$\Psi^2(\alpha, \alpha_T, \alpha_\mu, \alpha_D, \alpha_L, \tilde{f}(H), f_B, f_H) =$$

$$(1 + \alpha - \alpha_T + \alpha_\mu)^2 \int_0^1 \int_0^1 \int_0^1 \int_0^1 z_1'^{\alpha - \alpha_T + \alpha_\mu} z_2'^{\alpha - \alpha_T + \alpha_\mu}$$

$$\times \exp \left[-\sqrt{\left\{ 1 + \frac{z_1'^{-\alpha_L} \cdot z_2'^{-\alpha_L}}{10 \tilde{f}^2(H)} \right\} \left\{ f_B^2 (y_1' - y_2')^2 + f_H^2 (z_1' - z_2')^2 \right\} z_1'^{-\alpha_D} z_2'^{-\alpha_D}} \right]$$

$$\times \left\{ 1 + \tilde{f}^\beta(H) \right\}^{5/3\beta} \cdot \sqrt{\frac{z_1'^{\alpha_L}}{\left[1 + \left\{ \tilde{f}(H) z_1'^{\alpha_L} \right\}^\beta \right]^{5/3\beta}} \cdot \frac{z_2'^{\alpha_L}}{\left[1 + \left\{ \tilde{f}(H) z_2'^{\alpha_L} \right\}^\beta \right]^{5/3\beta}}}$$

$$\times dy_1' dy_2' dz_1' dz_2'$$

(5-8)

$$\text{and } f_B = \frac{f \cdot B \cdot k_y(H)}{\bar{U}(H)}, \quad f_H = \frac{f \cdot H \cdot k_z(H)}{\bar{U}(H)}$$

$$\text{If } \beta \approx 2 \text{ and } \frac{z_1'^{2\alpha_L} + z_2'^{2\alpha_L}}{2} \approx \sqrt{z_1'^{2\alpha_L} \cdot z_2'^{2\alpha_L}}, \text{ then}$$

$$\begin{aligned} & \left[1 + \left\{ \tilde{f}(H) z_1'^{\alpha_L} \right\}^\beta \right]^{5/3\beta} \left[1 + \left\{ \tilde{f}(H) z_2'^{\alpha_L} \right\}^\beta \right]^{5/3\beta} \\ & \approx \left[(1 + \left\{ \tilde{f}(H) z_1'^{\alpha_L} \right\}^2) (1 + \left\{ \tilde{f}(H) z_2'^{\alpha_L} \right\}^2) \right]^{5/6} \\ & = \left[1 + \tilde{f}^2(H) \{ z_1'^{2\alpha_L} + z_2'^{2\alpha_L} \} + \tilde{f}^4(H) (z_1' z_2')^{2\alpha_L} \right]^{5/6} \\ & \approx \left[1 + \tilde{f}^2(H) \cdot 2\sqrt{z_1'^{2\alpha_L} \cdot z_2'^{2\alpha_L}} + \tilde{f}^4(H) (z_1' z_2')^{2\alpha_L} \right]^{5/6} \\ & = \left[1 + \tilde{f}^2(H) (z_1' z_2')^{\alpha_L} \right]^{5/3} \approx \left[1 + \tilde{f}^\beta(H) (z_1' z_2')^{\beta\alpha_L/2} \right]^{10/3\beta} \end{aligned}$$

Then the term of height dependence of power spectral density in equation (5-8) may be approximated as,

$$\begin{aligned} & \left[1 + \tilde{f}^\beta(H) \right]^{5/3\beta} \sqrt{\frac{z_1'^{\alpha_L}}{\left[1 + \left\{ \tilde{f}(H) z_1'^{\alpha_L} \right\}^\beta \right]^{5/3\beta}} \cdot \frac{z_2'^{\alpha_L}}{\left[1 + \left\{ \tilde{f}(H) z_2'^{\alpha_L} \right\}^\beta \right]^{5/3\beta}}} \\ & \approx z_1'^{\alpha_L/2} z_2'^{\alpha_L/2} \left[\frac{1 + \tilde{f}^\beta(H)}{1 + \left\{ \tilde{f}(H) z_1'^{\alpha_L/2} z_2'^{\alpha_L/2} \right\}^\beta} \right]^{5/3\beta} \end{aligned} \quad (5-9)$$

$$\simeq \begin{cases} (z_1' z_2')^{\alpha_L/2} & \text{when } \tilde{f}(H) \ll 1 \\ (z_1' z_2')^{-\alpha_L/3} & \text{when } \tilde{f}(H) \gg 1 \end{cases} \quad (5-10)$$

The approximation $\frac{z_1'^{2\alpha_L} + z_2'^{2\alpha_L}}{2} \simeq \sqrt{z_1'^{2\alpha_L} z_2'^{2\alpha_L}}$ may not hold

when α_L is of the order of unity and either z_1 or $z_2 \ll H$.

However in such cases $|z_1 - z_2|$ becomes of the order of H and so the co-coherence value becomes small (eg, if $|z_1 - z_2| = 100$ m and $f = 0.1$ Hz, $C_{u_1 u_2}(y_1, y_2, z_1, z_2, f) \leq 0.05$), therefore an error due to the approximation used above would have a very small effect on the result of the integration for $S_{F_n}(f)$. Moreover the simplified form given by equation (5-9) converges to the same form as the original one at both extreme cases of $\tilde{f}(H) \ll 1$ and $\tilde{f}(H) \gg 1$ as shown by equation (5-10) irrespective of α_L and β .

Substituting equation (5-9) into (5-8),

$$\begin{aligned} & \psi^2(\alpha, \alpha_T, \alpha_\mu, \alpha_D, \alpha_L, \tilde{f}(H), f_B, f_H) \\ &= (1 + \alpha - \alpha_T + \alpha_\mu)^2 \int_0^1 \int_0^1 \int_0^1 \int_0^1 (z_1' \cdot z_2')^{\alpha - \alpha_T + \alpha_\mu + \frac{\alpha_L}{2}} \\ & \times \exp \left[-\sqrt{\left\{ 1 + \frac{(z_1' \cdot z_2')^{-\alpha_L}}{10\tilde{f}^2(H)} \right\} \left\{ f_B^2 (y_1' - y_2')^2 + f_H^2 (z_1' - z_2')^2 \right\} (z_1' z_2')^{-\alpha_D}} \right] \\ & \times \left[\frac{1 + \tilde{f}^{\beta(H)}}{1 + \left\{ \tilde{f}(H) (z_1' z_2')^{\alpha_L/2} \right\}^{\beta}} \right]^{5/3 \beta} dy_1' dy_2' dz_1' dz_2' \end{aligned} \quad (5-11)$$

Although function $\Psi^2(\)$ given by equation (5-11) has eight variables, f_B and f_H are significant variables and others have only minor effects. The variation of α , α_T , α_μ , α_D , α_L and $\tilde{f}(H)$ was neglected in Vickery's approach by taking an average value of $(1 + \alpha + \alpha_\mu)$ and assuming $\alpha_T = \alpha_L = 0$, $\alpha_D = \alpha$ and eliminating $\tilde{f}(H)$. In the following, the function is conventionally abbreviated as $\Psi^2(f_B, f_H)$.

The mean deflection of the structure $\bar{\Delta}(H)$ and the r.m.s. deflection $\sigma_\delta(H)$ at the top of structure can be obtained assuming the same deflection mode, $\mu(z)$, for each as follows,

$$\bar{\Delta}(H) = \frac{1}{1 + 2\alpha + \alpha_\mu} \left\{ C_{D_0} \cdot \rho \frac{\bar{U}^2(H)}{2} B \cdot H \right\} \cdot \frac{1}{k}, \quad (5-12)$$

$$\sigma_\delta(H) = \frac{1}{1 + \alpha - \alpha_T + \alpha_\mu} \left\{ C_{D_0} \cdot \rho \frac{\bar{U}^2(H)}{2} B \cdot H \right\} \cdot \frac{2\sigma_u(H)}{\bar{U}(H)}$$

$$\times \left\{ \int_0^\infty \left(\frac{\tilde{C}_{Dn}}{C_{D_0}} \right) \frac{S_u(f, H)}{\sigma_u^2(H)} \Psi^2(f_B, f_H) |\chi(f)|^2 df \right\}^{\frac{1}{2}} \quad (5-13)$$

where C_{D_0} is the static drag coefficient,

$$k' = (2\pi f_1)^2 M_1 \text{ (effective spring constant)}$$

$$M_1 = \frac{1}{1 + 2\alpha_\mu} H \cdot B \cdot D \cdot \gamma \text{ (generalised mass for the fundamental mode)}$$

γ = average density of structure,

$$|\chi(f)|^2 = \frac{1}{k'^2 \left\{ \left(1 - \left(\frac{f}{f_1}\right)^2\right)^2 + 4\zeta_T^2 \left(\frac{f}{f_1}\right)^2 \right\}},$$

$\zeta_T = \zeta_S + \zeta_A$ (total damping ratio) ,

ζ_S = structural damping ratio for the fundamental mode ,

$$\zeta_A = \frac{1 + 2\alpha_\mu}{1 + \alpha + 2\alpha_\mu} \frac{\tilde{C}_{D1} \bar{U}(H)\rho}{4\pi f_1 D \gamma} \quad (\text{aerodynamic damping})$$

(5-14)

(see equation (2-17)).

Although the density of a structure may not be uniform but generally vary with height, the value of γ can be determined to give the appropriate generalised mass M_1 , as,

$$\gamma = \frac{1 + 2\alpha_\mu}{H \cdot B \cdot D} \int_0^H \int_0^B m(z) \mu^2(z) dy dz$$

where $m(z)$ = mass per unit area on the projected surface.

The ratio of r.m.s. to mean deflection is deduced from equations (5-12) and (5-13), ie,

$$\frac{\sigma_{\delta}(H)}{\bar{\Delta}(H)} = \frac{1 + 2\alpha + \alpha_{\mu}}{1 + \alpha - \alpha_T + \alpha_{\mu}} \cdot \frac{2\sigma_u(H)}{\bar{U}(H)}$$

$$\times \left\{ \int_0^{\infty} \left(\frac{\tilde{C}_{D_n}}{C_{D_0}} \right)^2 \frac{S_u(f, H)}{\sigma_u^2(H)} \psi^2(f_B, f_H) |\chi(f)|^2 df \right\}^{\frac{1}{2}}$$

(5-15)

The integral in equation (5-15) with respect to frequency can be approximated as described in section 2.2.5.

$$\begin{aligned} & \int_0^{\infty} \left(\frac{\tilde{C}_{D_n}}{C_{D_0}} \right)^2 \frac{S_u(f, H)}{\sigma_u^2(H)} \psi^2(f_B, f_H) |\chi(f)|^2 df \\ &= \int_0^{1.75f_1} \left(\frac{\tilde{C}_{D_{qs}}}{C_{D_0}} \right)^2 \frac{S_u(f, H)}{\sigma_u^2(H)} \psi^2(f_B, f_H) df \\ &+ \left(\frac{\tilde{C}_{D_1}}{C_{D_0}} \right)^2 \frac{f_1 \cdot S_u(f_1 \cdot H)}{\sigma_u^2(H)} \left(\frac{\pi}{4\zeta_T} - 1.75 \right) \psi^2(F_B, F_H) \end{aligned}$$

(5-16)

where

$$F_B = \frac{f_1 \cdot B \cdot k_y(H)}{\bar{U}(H)}, \quad F_H = \frac{f_1 \cdot H \cdot k_z(H)}{\bar{U}(H)}$$

Then equation (5-15) can be rewritten as follows,

$$\frac{\sigma_{\delta}(H)}{\bar{\Delta}(H)} = r \left\{ \left(\frac{\tilde{C}_{D_{qs}}}{C_{D_o}} \right)^2 \cdot B + \left(\frac{\tilde{C}_{D_1}}{C_{D_o}} \right)^2 R \cdot E \cdot S \right\}^{\frac{1}{2}} \quad (5-17)$$

$$\text{where } r = \frac{1 + 2\alpha + \alpha_{\mu}}{1 + \alpha - \alpha_T + \alpha_{\mu}} \cdot \frac{2\sigma_u(H)}{\bar{U}(H)} \quad (\text{roughness factor}) \quad (5-17(a))$$

$$B = \int_0^{1.75f_1} \frac{S_u(f, H)}{\sigma_u^2(H)} \psi^2(f_B, f_H) df \quad (\text{background excitation factor}) \quad (5-17(b))$$

$$= \text{function } (\alpha, \alpha_T, \alpha_{\mu}, \alpha_D, \alpha_L, \tilde{f}_1(H), L_B, L_H)$$

$$\tilde{f}_1(H) = \frac{f_1 \cdot L(H)}{\bar{U}(H)}, \quad L_B = \frac{B \cdot k_y(H)}{L(H)}, \quad L_H = \frac{H \cdot k_z(H)}{L(H)}$$

$$R = \frac{\pi}{4\zeta_T} - 1.75 \quad (\text{resonance amplification factor}) \quad (5-17(c))$$

$$E = \frac{f_1 \cdot S_u(f_1, H)}{\sigma_u^2(H)} = k_1 \frac{\frac{f_1 \cdot L(H)}{\bar{U}(H)}}{\left[1 + \left\{ \frac{f_1 \cdot L(H)}{\bar{U}(H)} \right\}^{\beta 5/3\beta} \right]} \quad (\text{gust power factor}) \quad (5-17(d))$$

$$S = \psi^2(\alpha, \alpha_T, \alpha_{\mu}, \alpha_D, \alpha_L, \tilde{f}_1(H), F_B, F_H) \quad (\text{size reduction factor}) \quad (5-17(e))$$

The expected value of maximum response $\Delta_{\max}(H)$ may be obtained from the r.m.s. value multiplied by the peak factor, p , defined in equation (4-10), namely,

$$\Delta_{\max}(H) = \bar{\Delta}(H) + p \cdot \sigma_{\delta}(H) \quad (5-18)$$

where

$$p = \sqrt{2 \ln v_0 T} + \frac{0.577}{\sqrt{2 \ln v_0 T}}, \quad v_0 = \left\{ \frac{\int_0^{\infty} f^2 S_{\delta}(f) df}{\int_0^{\infty} S_{\delta}(f) df} \right\}^{\frac{1}{2}}$$

$S_{\delta}(f)$ is the power spectral density of response displacement, ie,

$$S_{\delta}(f) = \begin{cases} S_{F_{qs}}(f) \times \frac{1}{k^{1/2}} & \text{at } f \ll f_1 \text{ or } f_1 \ll f \\ S_{F_1}(f) \times |\chi(f)|^2 & \text{at } f \approx f_1 \end{cases}$$

$T = 3600 \text{ sec (1 hour)}$, and

v_0 is of the order of f_1 if the response spectrum has a sharp resonance peak.

The r.m.s. acceleration, $\sigma_{\delta}''(H)$, can be obtained from the power spectrum of r.m.s. displacement response as follows,

$$\sigma_{\delta}''(H) = \left\{ \int_0^{\infty} (2\pi f)^4 S_{\delta}(f) df \right\}^{\frac{1}{2}} \quad (5-19)$$

The maximum acceleration can be estimated in a similar manner to equation (5-18) but v_o can be replaced by f_1 without losing accuracy since the lower frequency components of the power spectral density of acceleration response has much less significance than that of displacement response. Namely,

$$\ddot{\delta}_{\max}(H) = p_a \ddot{\sigma}_{\delta}(H) \quad (5-19)$$

where

$$p_a = \sqrt{2 \ln f_1 T} + \frac{0.577}{\sqrt{2 \ln f_1 T}}$$

A computer program to predict the wind response of a tall structure in the along-wind direction was developed. Necessary input data (various parameters employed in equation (5-17)) and available output information are summarised as follows,

Input data :

H, B, D, f_1, γ and α_u for a structure;

$\bar{U}(z_r), \alpha, \sigma_u(z_r)/\bar{U}(z_r), \alpha_T, L(z_r), \alpha_L, \beta, k_y(z_r),$

$k_z(z_r), \alpha_D, z_r$ for wind characteristics; and

$C_{D_o}, \tilde{C}_{D_{qs}}/C_{D_o}, \tilde{C}_{D_1}/C_{D_o},$

Output available :

$\bar{U}(H)$, $\sigma_u(H)/\bar{U}(H)$, $L(H)$, $k_y(H)$, $k_z(H)$ for confirmation of input data;

$\bar{\Delta}(H)$, $\sigma_\delta(H)$, $\sigma_{\ddot{\delta}}(H)$, $\Delta_{\max}(H)$, $\ddot{\delta}_{\max}(H)$ as a computation result; and

r , B , E , S , G as intermediate factors,

where

$$G = \frac{\bar{\Delta}_{\max}(H)}{\bar{\Delta}(H)} = 1 + p \frac{\sigma_\delta(H)}{\bar{\Delta}(H)} \quad (\text{gust factor}).$$

The reference height, z_r , can be chosen as an arbitrary height either conventional 10 m or height of a structure H or gradient z_G depending on the situation of available information about wind characteristics.

Results are presented in a tabular form with different structural damping ratio values. Since it is difficult to predict damping ratio of structures⁽⁹⁾, standard values such as $\zeta_s = 0.01$ and 0.02 are chosen as a default.

The computer program is shown in appendix 3 together with a typical example of input and output. The time consuming part of the program is the numerical integral of equation (5-11), which was developed as a subroutine by applying Simpson's rule⁽⁹¹⁾ to perform the numerical integration efficiently.

5.4 SIMPLIFIED APPROACH

The computer program mentioned in the previous section enables users to obtain an answer to the prediction of along-wind response of a tall structure in a strong wind. However information obtained from such an answer is rather limited to an individual case and may not be sufficient in practical circumstances, eg, generally a design procedure needs many feed backs and may require several iterations of a computation. It would be more convenient, therefore, for practical applications if intermediate factors such as r , B , E , and S are obtainable from chart diagrams. Furthermore such diagrams would help to interpret the contribution of parameters employed in this work firstly to the intermediate factors and secondly to the response prediction.

Roughness factor, r , can be obtained if profile indices α , α_T , α_μ and turbulence intensity factor, $2\sigma_u(H)/\bar{U}(H)$ are known. The latter is illustrated in Figure 5-1, which has a modified form from that presented by Vickery⁽¹⁸⁾ and the turbulence profile index $\alpha_T = 0.08$ was taken to provide a better fit to recent available measurements. Three typical cases of terrain, ie, URBAN, SUBURBAN and OPEN COUNTRY, are shown and the agreement with turbulence intensity values recommended by ESDU⁽⁵³⁾ is fairly good.

Background excitation factor, B , is plotted against parameters L_H and L_B with four combinations of different values

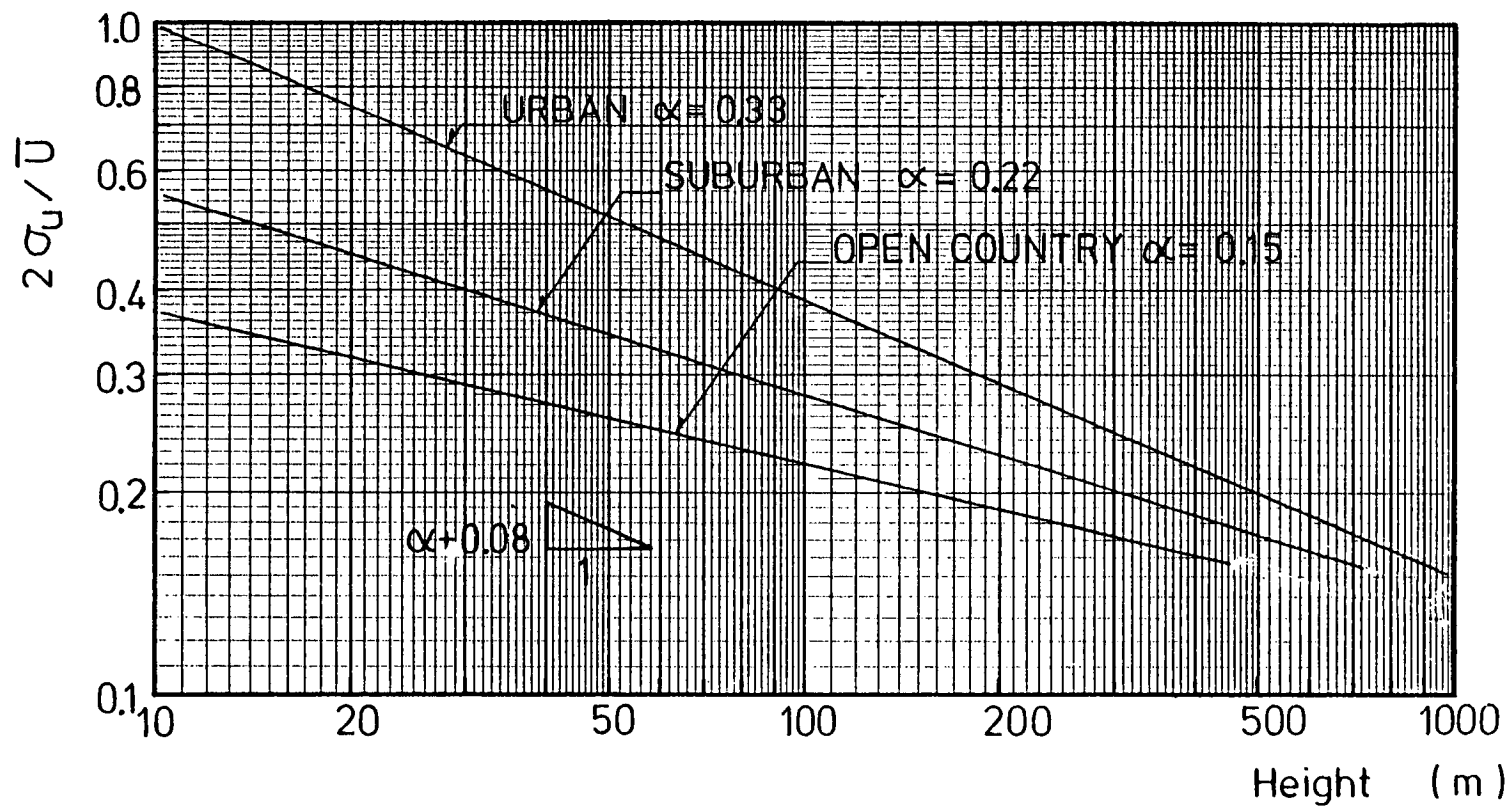


Figure 5-1 Turbulence intensity factor $\frac{2\sigma_u(H)}{\bar{u}(H)}$

of the length constant profile index α_L and the decay constant profile index α_D in Figure 5-2(a) to (d). The effect of the variation of $\alpha_\mu + \alpha - \alpha_T$ upon B is tolerably small as described by Vickery⁽¹⁸⁾ and so an average value 1.2 was taken. Further simplifications employed here were $\tilde{f}_1 = 10.0$ and $\beta = 2.0$, whose effects on B are also considered to be small. Possible error caused by these simplifications will be discussed later. As typical values, $\alpha_L = 0$ and 0.5 , and $\alpha_D = 0$ and 0.4 were taken. The variation of B with different α_L and α_D values is approximately 5% to 20% and so the B value for any case of α_L between 0 and 1.0 and α_D between 0 to 0.8 can be obtained by a linear interpolation or extrapolation with a reasonably accuracy.

In previous approaches^(5,14,18), instead of taking L_B and L_H as parameters to define B , L_H and $\frac{B}{H}$ or $\frac{Bk_y}{Hk_z}$ were used. However, L_B and L_H are preferred in this work for two reasons. Firstly $\frac{B}{H}$ is not an independent factor of L_H , in other words, for a very high value of L_H a higher $\frac{B}{H}$ is unlikely in a practical situation and so is a lower $\frac{B}{H}$ for a very low value of L_H . Therefore in order to cover the same range with $L_B = 0.01$ to 10.0 and $L_H = 0.01$ to 10.0, the range of $\frac{B}{H} \approx 0.0001$ to 100 must be provided. Secondly for a fixed value of $\frac{B}{H}$ the slope of B against L_H becomes as much as twice of that of B for a fixed value of L_B . This means that a reading error for the former could be as high as twice that for the latter. Therefore a chart diagram of B with L_B and L_H seems easier to be read in comparison with that with L_H and $\frac{B}{H}$ or $\frac{Bk_y}{Hk_z}$.

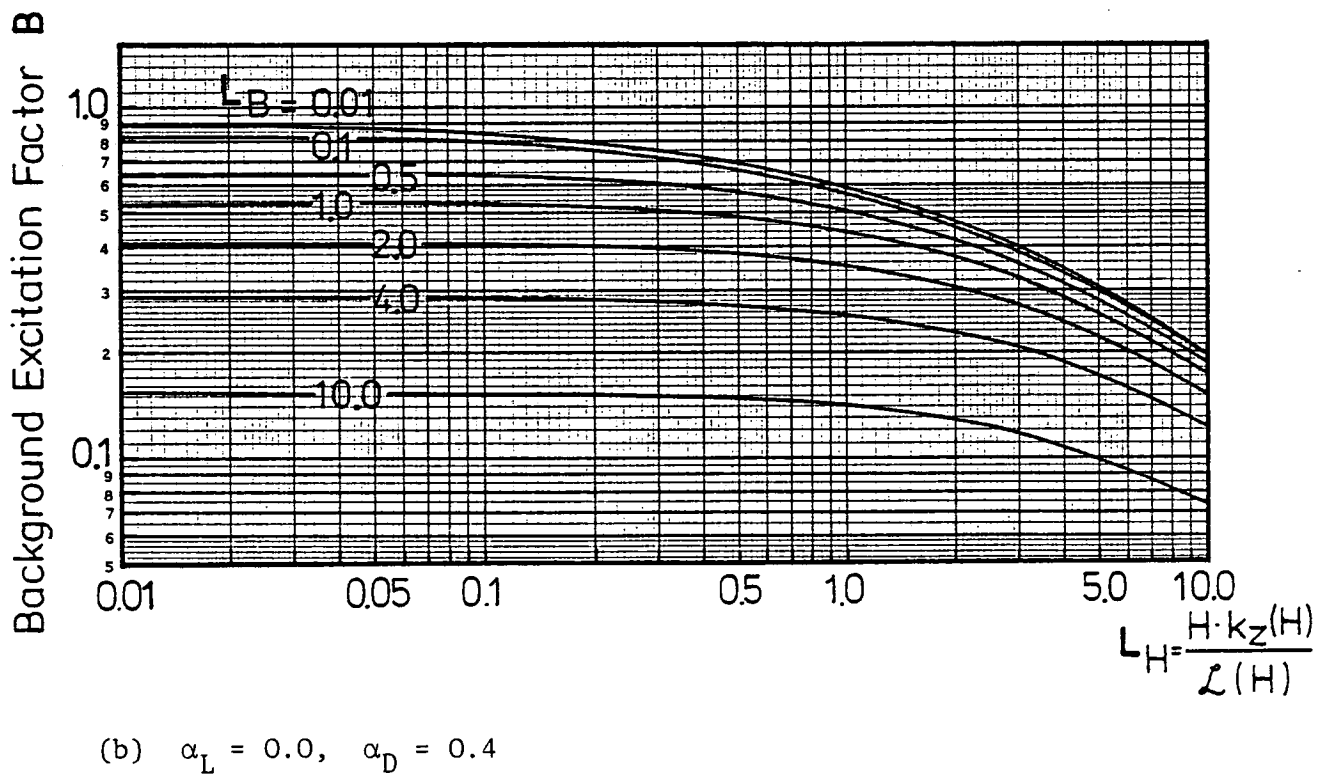
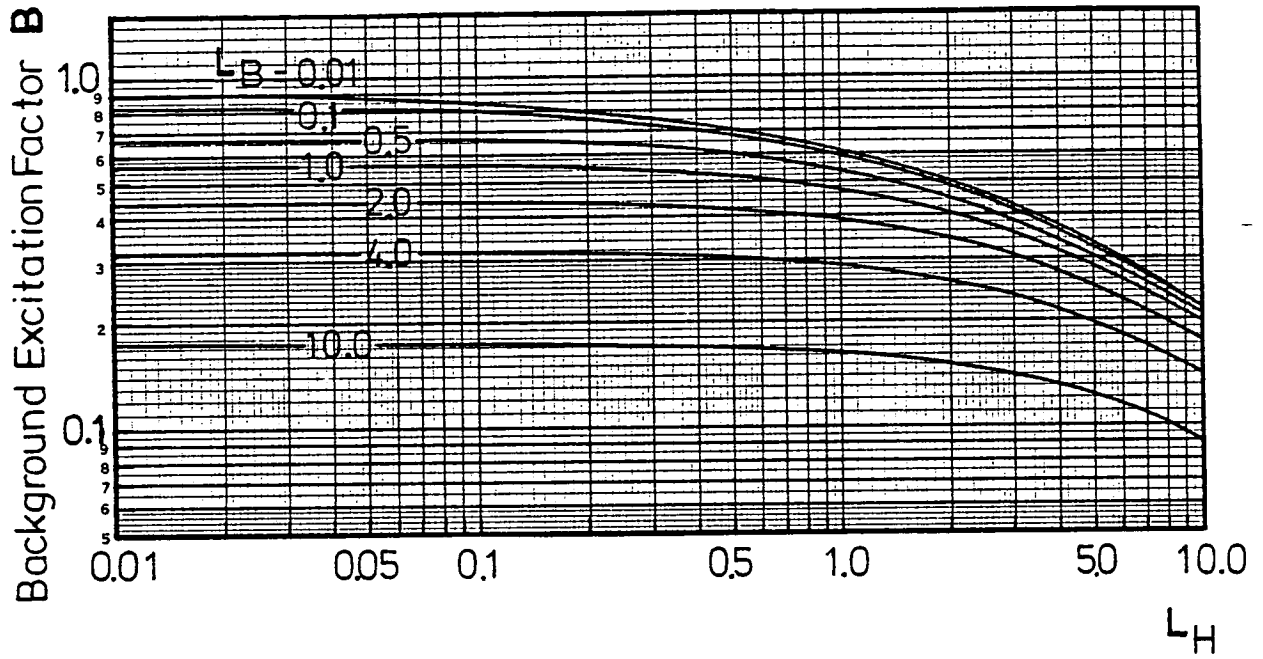
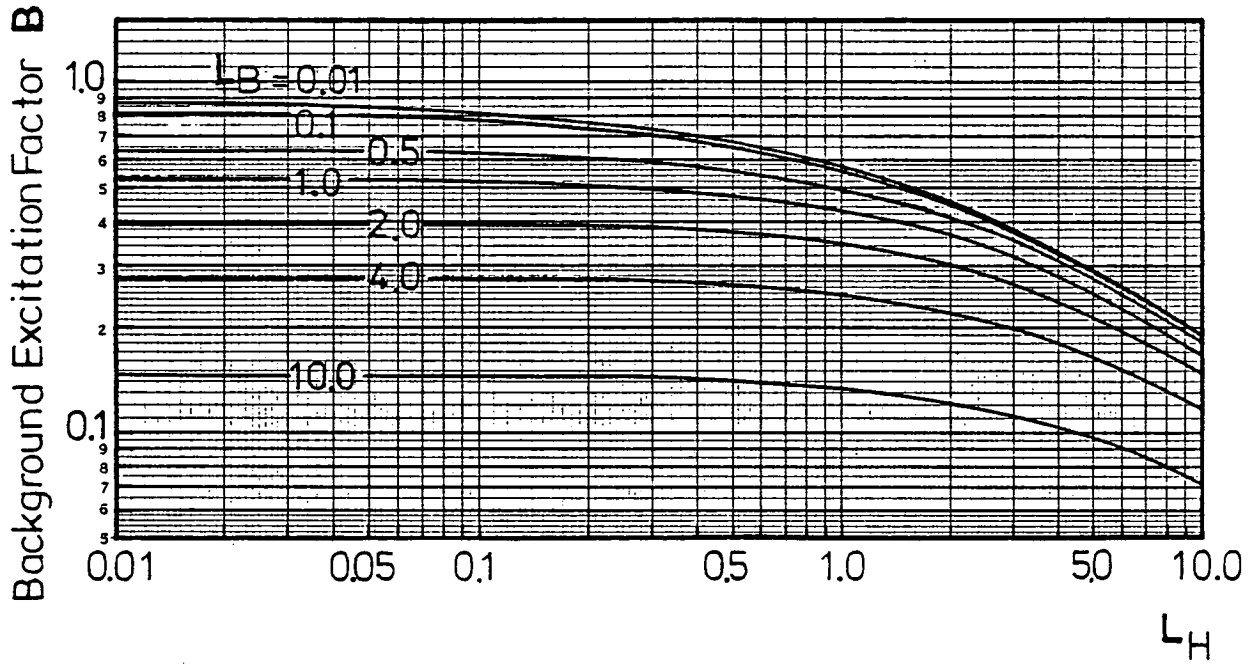
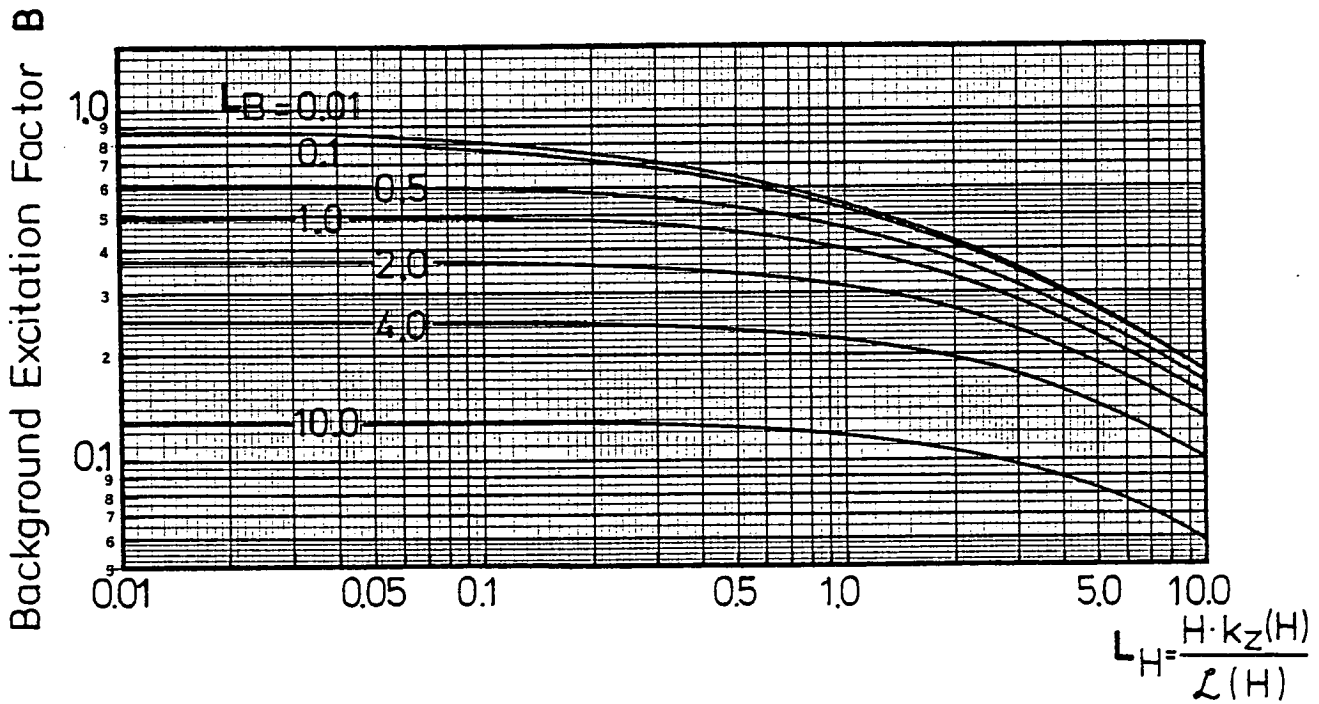


Figure 5-2 Background excitation factor B v.s. L_B and L_H



(c) $\alpha_L = 0.5, \alpha_D = 0.0$



(d) $\alpha_L = 0.5, \alpha_D = 0.4$

Figure 5-2 (continued)

Gust power factor, E , which is equivalent to a reduced power spectral density $S(f_1, H)$, is plotted against $\tilde{f}_1(H)$ with various values of spectral form index β in Figure 5-3.

Size reduction factor, S , is presented against parameters F_B and F_H in Figure 5-4(a) to (d) similar to the chart for B in Figure 5-2.

Wind characteristics parameters are summarised in Table 5-1 for three typical terrains. Suggested values are those extracted from the development in chapter 3.

A numerical example of prediction of gust factor, G , is demonstrated for a typical tall building 'O' which has dimensions of :

$$H = 250\text{m}, \quad B = 40\text{m}, \quad D = 30\text{m}, \quad f_1 = 0.15, \quad \gamma = 200 \text{ kg/m}^3,$$

$$\zeta_s = 1.0\%, \quad \alpha_\mu = 1.0.$$

In this demonstration the terrain SUBURBAN is taken for an example, ie, $\alpha = 0.22$. The basic design hourly mean wind speed is 50 m/s at the gradient height ($z_G = 700 \text{ m}$). The wind parameters at the height of the building are as follows :

$$\bar{U}(H) = 50 \times \left(\frac{250}{700} \right)^{0.22} = 40 \text{ m/s}$$

$$L(H) = 3800 \times \left(\frac{250}{700} \right)^{0.5} = 2270 \text{ m (see equation (3-9))}$$

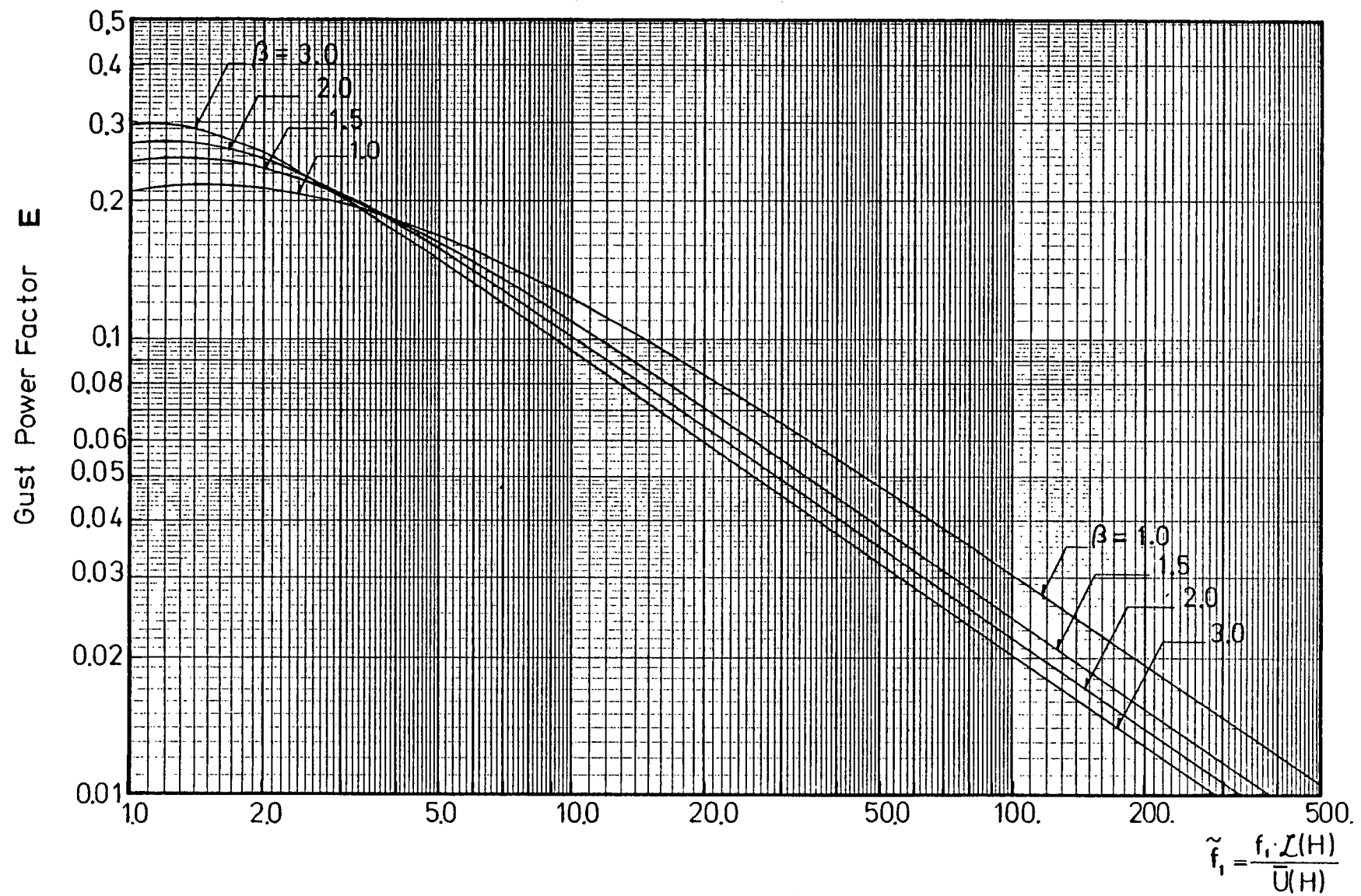


Figure 5-3 Gust power factor E

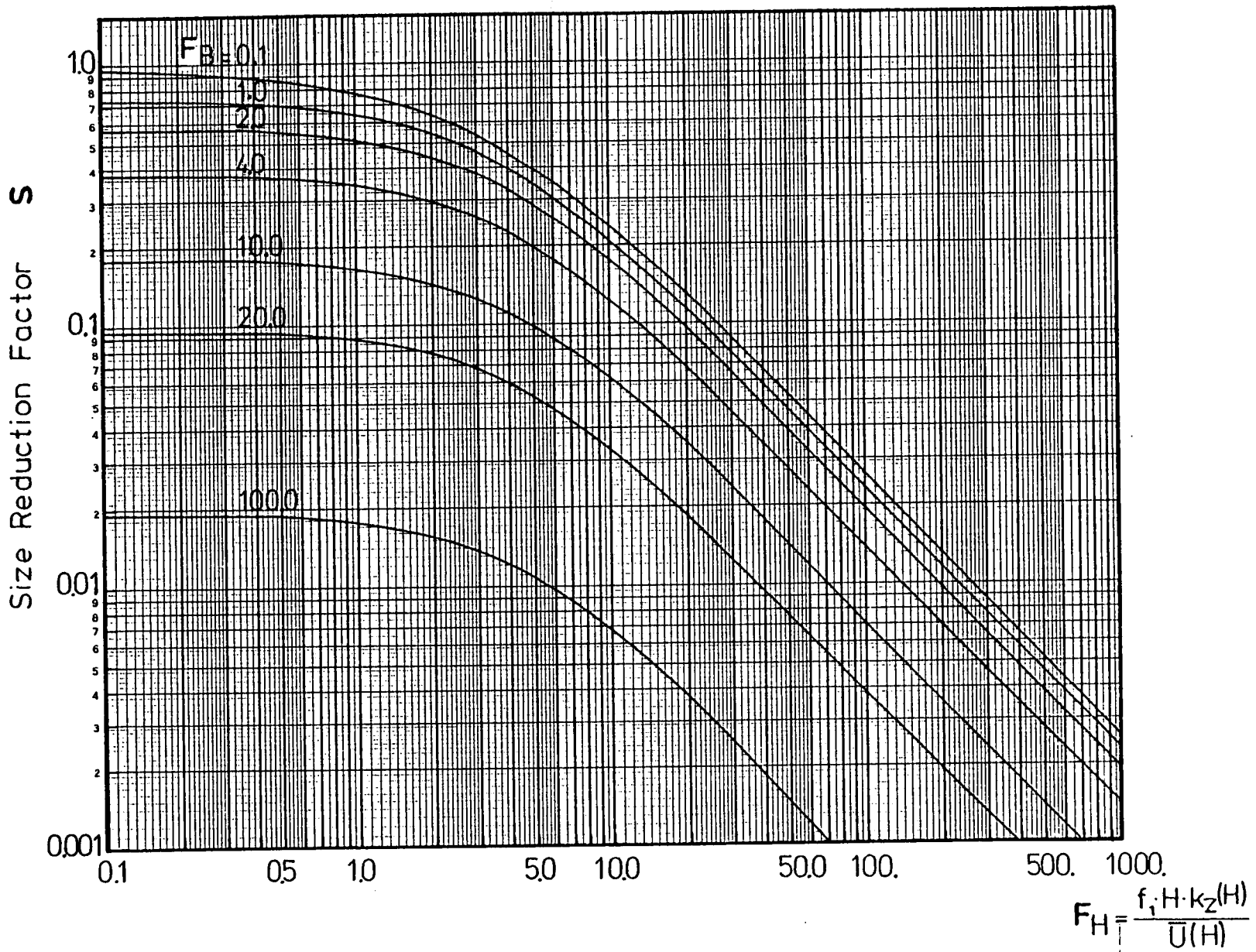


Figure 5-4 Size reduction factor S v.s. F_B and F_H (a) $\alpha_L = 0.0$, $\alpha_D = 0.0$

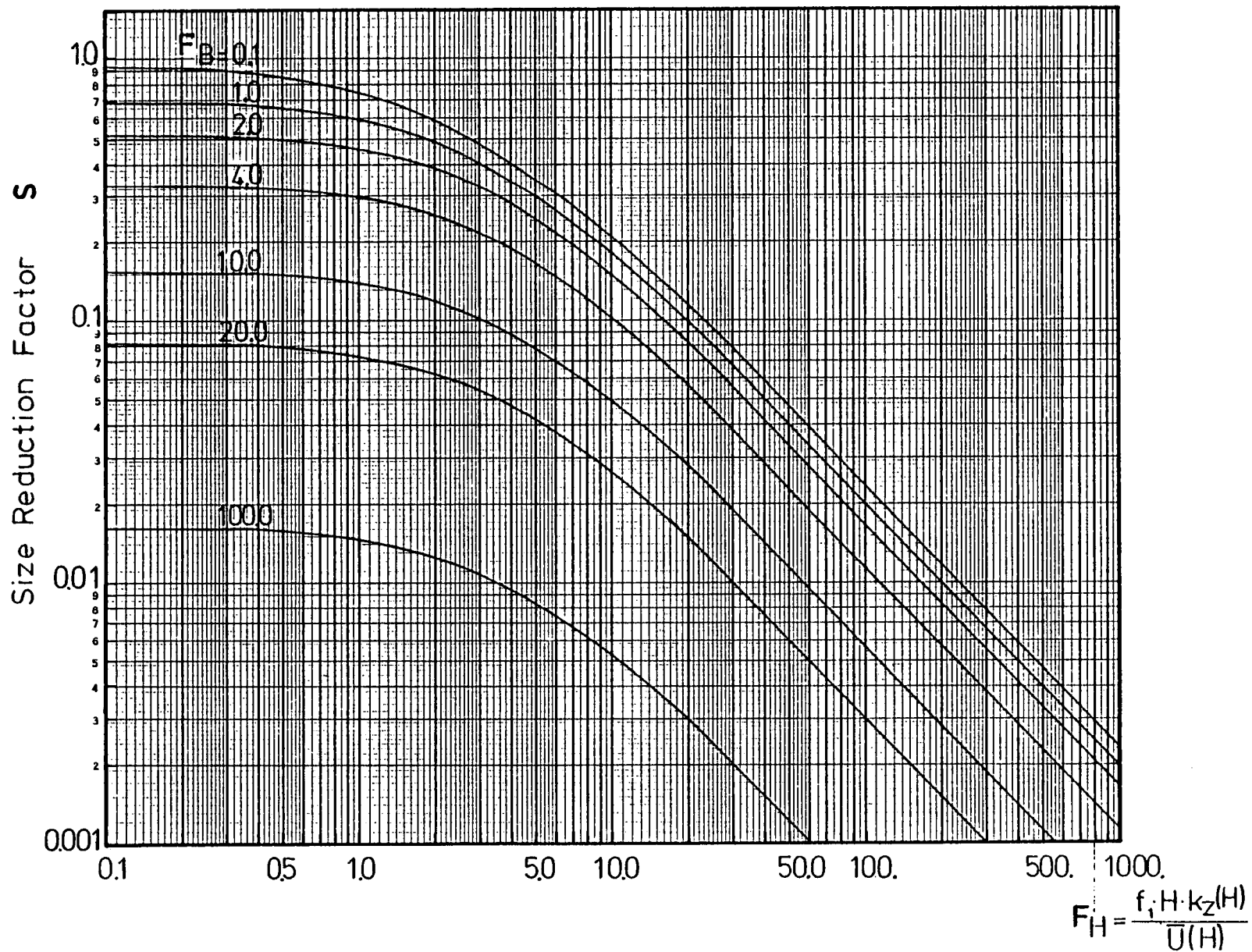


Figure 5-4

(continued)

(b) $\alpha_L = 0.0, \alpha_D = 0.4$

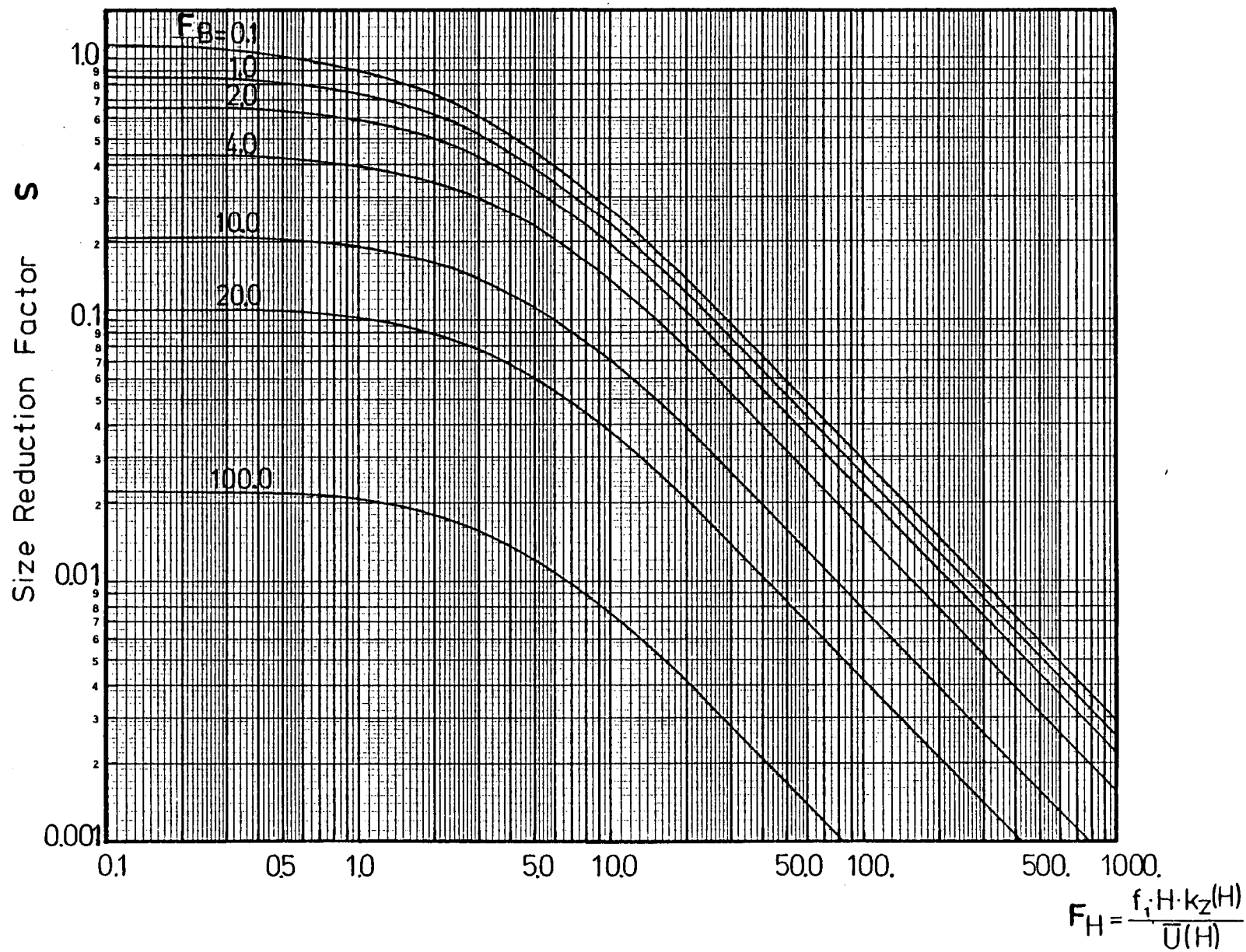


Figure 5-4

(continued)

(c) $\alpha_L = 0.5$, $\alpha_D = 0.0$

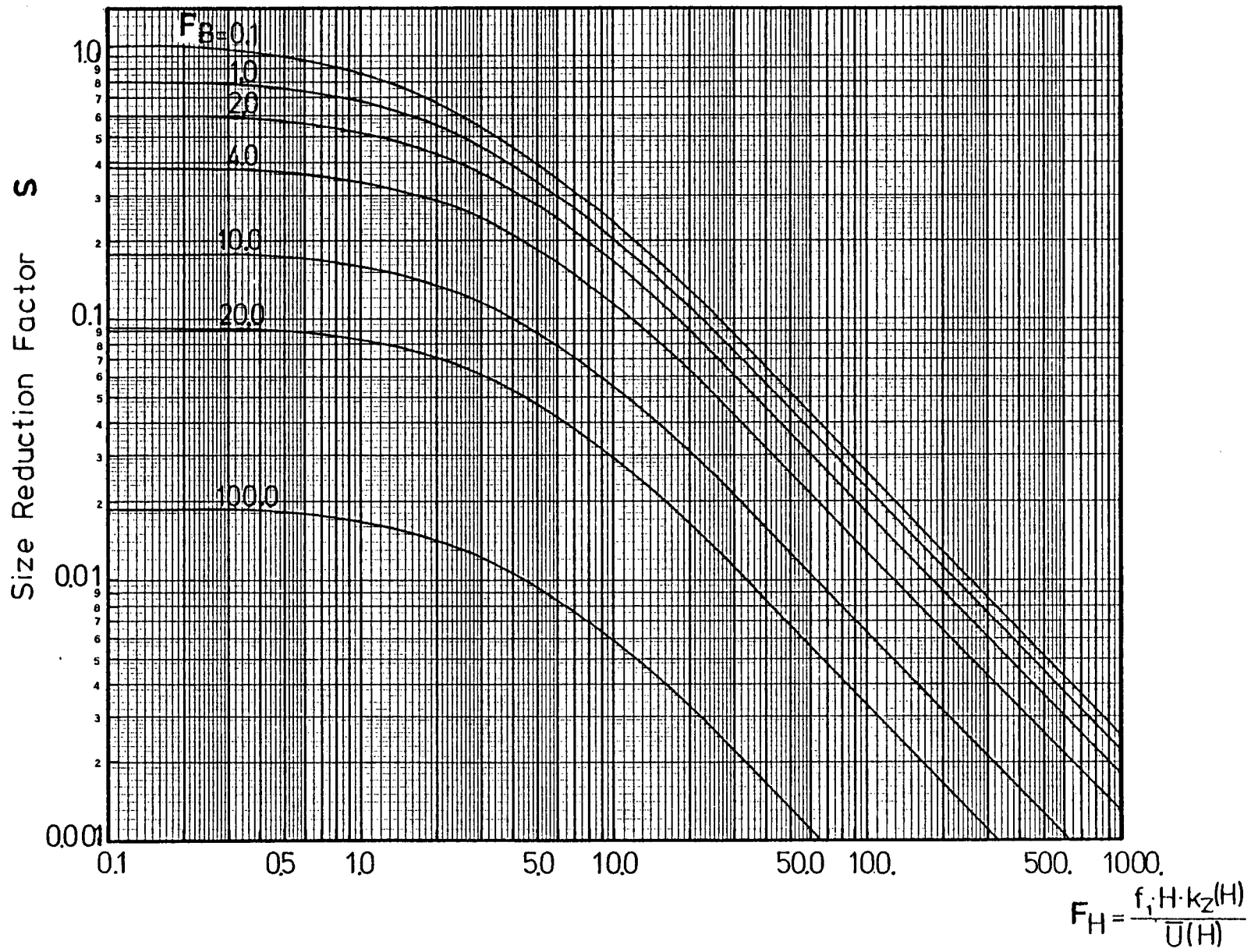


Figure 5-4 (continued) (d) $\alpha_L = 0.5$, $\alpha_D = 0.4$

Table 5-1 Suggested Values of Wind Characteristics Parameters

Terrain	OPEN	SUBURBAN	URBAN	Related equation
α	0.15	0.22	0.33	(3-1)
z_o (m)	0.015	0.3	2.0	(3-2)
z_G (m)	400	700	1000	(3-3)
α_T	0.08	0.08	0.08	(3-4)
L (z_G) (m)	2800	3800	4800	(3-13)
L (10) (m)	440	460	480	(3-14)
α_L	0.35	0.28	0.17	(3-8) & (3-12)
$\alpha_L + \alpha$	0.50	0.50	0.50	(3-9)
k_y (z_G)	7.0	7.0	7.0	} (3-21)
k_y (10)	16.3	15.7	13.0	
k_z (z_G)	6.0	6.0	6.0	
k_z (10)	14.8	13.4	11.2	
$-\alpha_D$	-0.38	-0.41	-0.47	(3-17) & (3-23)
$-\alpha_D + \alpha$	-0.23	-0.19	-0.14	(3-21)

$$\left. \begin{aligned} k_z(H) &= 6.0 \times \left(\frac{250}{700} \right)^{-0.19} = 7.3 \\ k_y(H) &= 7.0 \times \left(\frac{250}{700} \right)^{-0.19} = 8.5 \end{aligned} \right\} \text{(see equation (3-21))}$$

Parameters for determining factors r , B , E , S , R , are :

$$\frac{2\sigma_u(H)}{\bar{U}(H)} = 0.215 \quad \text{from Figure 5-1}$$

$$L_H = \frac{H \cdot k_z(H)}{L(H)} = \frac{250 \times 7.3}{2270} = 0.80,$$

$$L_B = \frac{B \cdot k_y(H)}{L(H)} = \frac{40 \times 8.5}{2270} = 0.150$$

$$\tilde{f}_1 = \frac{f_1 \cdot L(H)}{\bar{U}(H)} = \frac{0.15 \times 2270}{40} = 8.5$$

$$F_H = L_H \cdot \tilde{f}_1 = 6.84, \quad F_B = L_B \cdot \tilde{f}_1 = 1.27$$

Then

$$r = \frac{1 + 2\alpha + \alpha_\mu}{1 + \alpha + \alpha_\mu - \alpha_T} \cdot \frac{2\sigma_u(H)}{\bar{U}(H)} = 1.14 \times 0.215 = 0.245$$

$$B_{(\alpha_L = 0, \alpha_D = 0.4)} = 0.58 \quad \text{from Figure 5-2(b)}$$

$$B_{(\alpha_L = 0.5, \alpha_D = 0.4)} = 0.54 \quad \text{from Figure 5-2(d)}$$

By means of linear interpolation

$$B = 0.58 - \frac{0.58 - 0.54}{0.5} \times 0.28 = 0.56$$

(see Table 5-1 suburban terrain for α_L).

Similarly from Figure 5-4(b) and (d)

$$S = 0.23 + \frac{0.26 - 0.23}{0.5} \times 0.28 = 0.247$$

From Figure 5-3,

$$E = 0.111 \quad \text{at} \quad \tilde{f}_1 = 8.5 \quad \text{and} \quad \beta = 2.0.$$

As an effective reduced wind speed for determining the dynamic along-wind force coefficient the mean wind speed at two thirds height of building is taken, ie, $\tilde{U} = \frac{\bar{U}(2/3H)}{f_o D}$. Then coefficients may be estimated as,

$$C_{D_o} = 1.4^{(78)}, \quad \tilde{C}_{D_{qs}} = 0.8 C_{D_o} \quad \text{and} \quad \tilde{C}_{D_1} = 1.05 C_{D_o}$$

from equation (4-11) in which $\tilde{U}(2/3H) = 8.13$, $J = 6.5$.

From equation (5-14) the aerodynamic damping ratio, ζ_A , can be obtained assuming an air mass density of 1.2 kg/m^3 as,

$$\zeta_A = \frac{1 + 2 \times 1.0}{1 + 0.22 + 2 \times 1.0} \cdot \frac{1.05 \times 1.4 \times 40 \times 1.2}{4 \times \pi \times 0.15 \times 30 \times 200} = 0.0058$$

Then the resonance amplification factor, R , is

$$R = \frac{\pi}{4 \times (0.01 + 0.0058)} - 1.75 = 48.0$$

From equation (5-17),

$$\begin{aligned} \frac{\sigma_{\delta}(H)}{\bar{\Delta}(H)} &= 0.245 \left\{ 0.8^2 \times 0.56 + 1.05^2 \times 48.0 \times 0.247 \times 0.111 \right\}^{\frac{1}{2}} \\ &= 0.328 \end{aligned}$$

When the peak factor $p = 3.7$ is used for $v_0 T \simeq 500$ (see equation (5-18)), the gust factor, G , becomes

$$G = 1 + p \frac{\sigma_{\delta}(H)}{\bar{\Delta}(H)} = 2.21$$

In the following section numerical comparisons with previous approaches are presented and effects of newly employed parameters on the gust response prediction are discussed.

5.5 NUMERICAL COMPARISON WITH PREVIOUS WORKS

Firstly a comparison is made with Vickery's work⁽¹⁸⁾. Dimensions of buildings to be taken for this case study are listed in Table 5-2.

Table 5-2 Description of Buildings for Case Studies

Building	$H_{(m)}$	$B_{(m)}$	$D_{(m)}$	f_1 (Hz)	ζ (%)	γ (kg/m ³)
A	46	46	30	1.0	1.0	150
B	152	61	45	0.2	1.0	150
C	366	61	45	0.1	1.0	150

Gust factors are calculated in a similar manner to the previous numerical example for building 'O' and the results are listed in Table 5-3. For two typical terrains a large city ($\alpha = 0.33$; 0.35 in Vickery's case) and an open field ($\alpha = 0.15$; 0.16 in Vickery's case) were chosen. The design wind speed values at building height were taken as the same as Vickery's case so as to make an easy comparison. These values are not far from those obtained from the gradient wind speed $\bar{U}(z_G) = 50$ m/s, as seen in Table 5-3. For the same purpose the peak factor $p = 3.50$ ⁽¹⁸⁾ was taken, although it is a slightly underestimated value for a one-hour mean wind speed.

Values of the dynamic along-wind force coefficient were chosen based on the experimental results in the previous chapter but reasonably conservative values were taken in this case study, since the applicability of two-dimensional model results to the three-dimensional actual situation is not established.

Table 5-3 Calculation of Gust Factors for Three Typical Buildings

BUILDING	A		B		C	
H (m)	46		152		365	
B (m)	46		61		61	
f_1 (Hz)	1.0		0.2		0.1	
terrain	URBAN	OPEN	URBAN	OPEN	URBAN	OPEN
α	0.33	0.15	0.33	0.15	0.33	0.15
$\bar{U}(H)$ (m/s)	20 (18)	35 (34)	30 (27)	40 (43)	40 (34)	45 (49)
$L(H)$ (m)	1030	950	1870	1730	2900	2695
α_L	0.17	0.35	0.17	0.35	0.17	0.35
$k_z(H)$	9.2	9.9	7.8	7.5	6.9	6.1
$k_y(H)$	10.8	11.5	9.1	8.7	8.1	7.2
α_D	0.47	0.38	0.47	0.38	0.47	0.38
$\tilde{f}_1 = \frac{f_1 L(H)}{\bar{U}(H)}$	51.5	27.1	12.5	8.65	7.25	5.94
$L_H = \frac{H \cdot k_z}{L(H)}$	0.41	0.48	0.63	0.66	0.87	0.83
$L_B = \frac{B \cdot k_y}{L(H)}$	0.48	0.57	0.30	0.31	0.17	0.16
$F_H = \frac{H \cdot k_z \cdot f_1}{\bar{U}(H)}$	21.1	13.0	7.9	5.7	6.3	5.0
$F_B = \frac{B \cdot k_y \cdot f_1}{\bar{U}(H)}$	24.7	15.4	3.75	2.65	1.23	0.98
$\frac{1+2\alpha+\alpha_\mu}{1+\alpha+\alpha_\mu-\alpha}$	1.18	1.11	1.18	1.11	1.18	1.11
$2 \frac{\sigma_u(H)}{\bar{U}(H)}$	0.53	0.265	0.33	0.20	0.23	0.165
r	0.625	0.294	0.390	0.222	0.271	0.183
B	0.59	0.53	0.57	0.53	0.55	0.53
E	0.035	0.053	0.088	0.111	0.126	0.142
S	0.013	0.031	0.140	0.220	0.255	0.335
$\tilde{C}_{D_{qs}}/C_{D_0}$	0.8	0.8	0.8	0.8	0.8	0.8
$\tilde{C}_D/C_{D_0}$	3.0	2.0	1.5	1.3	1.00	0.95
ζ_A	0.0015	0.002	0.0035	0.0048	0.0074	0.0084
R	66.5	63.7	56.4	51.3	43.4	40.9
$RES_x(\tilde{C}_D/C_{D_0})^2$	0.27	0.42	1.56	2.12	1.39	1.76
G	2.76	2.00	2.90	2.22	2.25	1.93
Vickery (18)						
B	0.71	0.67	0.68	0.60	0.64	0.51
S	0.010	0.025	0.065*	0.101	0.140	0.190*
RES	0.022	0.081	0.316*	0.79	1.00	2.20*
G	2.67	1.83	2.30*	1.97	2.26	2.15*

note : (1) $\bar{U}(H)$ values in () are those estimated from $\bar{U}(z_G) = 50$ m and equation (3-1).
(2) * values are corrected by the author according to Vickery's figures⁽¹⁸⁾.

Corresponding with Vickery's study values for factors B, S and $R \cdot E \cdot S$ were also tabulated together with gust factor, G, values.

Background excitation factor, B, is smaller in this study except for the case of building 'C' in an open terrain. This is mainly due to the smaller length constant values for the lower level of the boundary layer. The integral limit employed in equation (5-17(b)) also reduces the value of B as compared with Vickery's case where the limit was infinity. However this effect on the computation of B is approximately less than 2% in most cases.

Size reduction factor, S, on the contrary, is generally greater in this study especially for taller buildings. The main reason for this is due to the adoption of smaller values for decay constants. Particularly for building 'C', S values in this work are approximately 1.8 times those in Vickery's study.

The effect of the dynamic along-wind force coefficient is clearly demonstrated by this comparison. In particular G values for building 'B' are considerably greater than those in Vickery's work because of the greater $\tilde{C}_D/C_{D_0}$. However for building 'A' in spite of the greater $\tilde{C}_D/C_{D_0}$ such as 3.0 and 2.0 for an urban and open terrain respectively the gust factor does not increase so much as in the case of building 'B' since the resonance component of response has less significance compared

with that of the quasi-static. Gust factor values for building 'C' were reduced by the effect of the dynamic along-wind force coefficient for the quasi-static response.

Although the difference between G values in this study and in Vickery's for building 'C' is not particularly evident, if similar values for the decay constants are used the difference would be more emphatic.

A further comparison is made with gust factor approaches by Davenport⁽⁵⁾, Vickery⁽¹⁸⁾ and Simiu⁽¹⁶⁾. Two building examples 'B' and 'C' and terrain conditions are the same but two cases of structural damping, ie, 1% and 2% and also cases when the dynamic along-wind force coefficient is taken as constant (equal to the static drag coefficient) are added to the previous comparison. The results are listed in Table 5-4.

Generally Vickery's values appear to be greater than those of Davenport's. This is mainly due to Vickery's modification of the co-coherence expression with smaller horizontal decay constants. Simiu's values are small compared with the others. This is simply because of his very low along-wind pressure correlation coefficients. His later suggestion for the height dependence of power spectral density was not taken into account in the computation of these values. If this effect is considered, the gust factors become smaller since the peak frequency of the power spectrum tends to be significantly lower

Table 5-4 Numerical Comparison of Computed Gust Factors

Building	Terrain	Structural Damping	Davenport ⁽⁵⁾	Vickery ⁽¹⁸⁾	Simiu ⁽¹⁶⁾		Section 5.4	
					N(f) = 0	N(f) = 0.2	$\tilde{C}_D = C_{D_o}$	$\tilde{C}_D$ as Table 5-3
'B' H = 152 m B = 61 m f ₁ = 0.2 Hz	URBAN	0.01	2.28	2.33 [*]	1.95	2.04	2.58	2.90
		0.02	2.10	2.20 [*]	1.86	1.94	2.35	2.53
	OPEN	0.01	1.93	1.97	1.69	1.76	2.07	2.22
		0.02	1.78	1.88	1.63	1.69	1.89	1.98
'C' H = 365 m B = 61 m f ₁ = 0.1 Hz	URBAN	0.01	2.48	2.26	1.90	1.98	2.32	2.25
		0.02	2.08	2.09	1.78	1.85	2.13	2.04
	OPEN	0.01	2.03	2.15 [*]	1.82	1.90	2.01	1.93
		0.02	1.95	1.96 [*]	1.68	1.75	1.85	1.77

note: * values were corrected by Simiu⁽¹⁶⁾

N(f) is the along-wind pressure correlation

at heights greater than 100 m, than that of Davenport's spectrum according to Simiu's presentation⁽²¹⁾.

Gust factor values in the column for $\tilde{C}_D = C_{D_0}$ in Table 5-4 demonstrate the effects of a difference in wind characteristics parameters, whereas the difference between G values in the last two columns of that table demonstrates the effect of the dynamic along-wind force coefficient.

Obviously differences between gust factors in this study and those in previous approaches are due to the combined effects of various wind characteristics parameters and the dynamic along-wind force coefficient. Nevertheless the latter's effect seems significant as noted in Table 5-4 as well as 5-3. If $\tilde{C}_D/C_{D_0} = 1.5$ is taken for building 'B' in an urban terrain for example, although this is a conservative value as compared with experimental results in the previous chapter, gust factor values increase approximately 15% for the case of $\zeta_S = 1.0\%$ and 8% for that of $\zeta_S = 2.0\%$ compared with cases of $\tilde{C}_D = C_{D_0} = \text{const.}$

Effects of individual parameters are discussed in the following section in more detail.

5.6 DISCUSSION

A parametric study is developed by making full use of the chart diagrams and some computational examples.

5.6.1 Influence of Structural Characteristics Parameters

Mode shape parameter, α_μ , was taken as 1.0 in this study. The variation of this parameter causes slight differences in roughness factor, r . From equation (5-17(a)) the variation of r can be exemplified with various α_μ and α in Table 5-5. Values are normalised by the standard case of $\alpha_\mu = 1.0$.

Table 5-5 Variation of r with α_μ

$\alpha_\mu \backslash \alpha$	0.5	1.0	1.5	2.0
0.15	1.02	1.0	0.98	0.97
0.33	1.04	1.0	0.98	0.96

Since factors B and S are almost independent of α_μ , the variation of r with α_μ can represent the variation of $\sigma_\delta(H)/\bar{\Delta}(H)$ with α_μ estimated from equation (5-17). For instance S decreases only slightly with α_μ , ie, about 3% for 50% variation of α_μ at $F_H = 5$, $F_B = 5$ and less than 1% at $F_H < 1$, $F_B < 1$. As far as a suitable power law representation of the fundamental mode shape of a structure is possible, the deviations from it seem to have an insignificant effect on the gust response prediction. In most cases of tall buildings $\alpha_\mu = 1.0$ will provide a reasonable representation of the

fundamental mode shape of a structure for $0.5 < \alpha_\mu < 2.0$.

The variation of α_μ also causes slight differences in the aerodynamic damping ratio value. The effect can be estimated from equation (5-14). Its order is slightly greater than those shown in Table 5-5, eg, the aerodynamic damping ratios calculated assuming $\alpha_\mu = 0.5$ and 1.5 differ approximately 5% from that calculated assuming $\alpha_\mu = 1.0$. However the effect of aerodynamic damping on the dynamic response is less significant than that of r . As a typical case 5% departure from an aerodynamic damping ratio value causes less than 0.5% of variation of $\sigma_\delta(H)/\bar{\Delta}(H)$ in the numerical example of building 'O' in section 5.4.

The average mass density of a structure, γ , is another factor in determining the aerodynamic damping ratio, ζ_A , but since, as pointed out above, ζ_A has little influence on the response ratio $\sigma_\delta(H)/\bar{\Delta}(H)$ the effect of γ on this ratio will likewise be small. By contrast the equivalent spring constant, k_1' , is calculated from the natural frequency, f_1 , and the total mass based on γ , and therefore the value of γ should be determined reasonably accurately in order to compute the actual response, $\sigma_\delta(H)$ or $\bar{\Delta}(H)$.

The effect of variation of f_1 on the response prediction can be estimated from the chart of size reduction factor, S , Figure 5-4 and gust power factor, E , Figure 5-3. A 20% increment of f_1 causes up to 40% reduction of S at $\tilde{f}_1 \geq 20$ since both F_H and F_B increase by 20%, and up to 20% reduction of S at $\tilde{f}_1 \approx 5$. The same increment of f_1 causes approximately $\frac{2}{3} \times 20\%$ reduction of E at $\tilde{f}_1 \geq 5$. The resonance part of response is linear to the product of

E and S and so its variation due to 20% deviation in $\tilde{f}_1$ is approximately up to 60% and 30% at $\tilde{f}_1 \geq 20$ and $\tilde{f}_1 \approx 5$ respectively. If the quasi-static part and the resonance part of response are of the same order of magnitude, the variations of $\sigma_\delta(H)/\bar{\Delta}(H)$ due to a 20% deviation in $\tilde{f}_1$ will be reduced to about 15% and 8% at $\tilde{f}_1 \geq 20$ and $\tilde{f}_1 \approx 5$ respectively. However in most practical cases of design procedure the value of f_1 can be estimated within the accuracy of 10% or less.

A simplification was employed for the chart diagram of background excitation factor B, ie, $\tilde{f}_1$ was assumed to be 10.0 in order to determine the integral limit. Within the range of $\tilde{f}_1$ between 5.0 and 50.0 the variation of B is less than 2% against that obtained from $\tilde{f}_1 = 10.0$ for both L_B and L_H between 0.2 and 1.0. The order of variation is very small in comparison with the effect of f_1 on the S value.

The structural damping ratio, ζ_S , is usually difficult to predict at the structural design stage. A conventional value $\zeta_S = 1\%$, however, seems to be fairly reasonable for tall buildings from recent measurements of structural dynamic characteristics reported by Jeary and Sparks⁽⁸⁰⁾. Tall chimneys and masts may have smaller values such as $\zeta_S = 0.4\%$ and some R.C. buildings may be expected to have value as high as $\zeta_S = 2.0\%$. Generally for larger amplitudes of response a greater ζ_S may be expected from the non-linearity of the dynamic system of a structure. Effects of the variation of ζ_S on the gust response prediction will be significant as noted in equation (5-17) especially

when the resonance part is dominant, ie, for relatively low natural frequency structures. The structural damping ratio value should be chosen with a safe margin at the stage of practical design, ie, it is better to underestimate its value in the first instance.

The higher vibrational modes were neglected in this study. If the natural frequency for the second mode, f_2 , is assumed to be 2.5 times that of the fundamental one, f_1 , the contribution of the higher mode to the total response deflection may reach the order of 1% of the fundamental component according to Simiu⁽²⁰⁾. However for some types of structures which have f_2 very close to f_1 , the effect of the higher mode should be examined in detail.

5.6.2 Influence of Turbulence and Mean Wind Speed Profiles

Values suggested for $\sigma_u(H)/\bar{U}(H)$ in this work are similar to Vickery's values at around $H \approx 100$ m but larger for $H \ll 100$ m and smaller for $H \gg 100$ m than Vickery's since the profile index $\alpha_T = 0.08$ was employed compared with Vickery's $\alpha_T = 0$. On the other hand if the same value of $\sigma_u(H)/\bar{U}(H)$ was used, the roughness factor, r , becomes approximately 4% greater than Vickery's value as seen in equation (5-17(a)). Therefore the adoption of $\alpha_T = 0.08$ together with appropriate $\sigma_u(H)/\bar{U}(H)$ values makes the r value greater by approximately 8% for a building with $H \approx 50$ m, 4% for a building with

$H \approx 100$ m, similar for a building with $H \approx 200$ m and smaller by approximately 5% for a building with $H \approx 300$ m relative to the case with the assumption $\alpha_T = 0$.

The variation of the mean wind speed profile index, α , also changes the value of r , ie, for $\alpha_\mu = 1$ and $\alpha_T = 0.08$, the value of $(1 + \alpha_\mu + 2\alpha)/(1 + \alpha_\mu + \alpha - \alpha_T)$ varies from 1.11 to 1.18 with α from 0.15 to 0.33, and the local turbulence intensity $\sigma_u(H)/\bar{U}(H)$ varies from 0.13 to 0.26 at $H = 50$ m, 0.11 to 0.19 at $H = 100$ m, 0.095 to 0.145 at $H = 200$ m and 0.085 to 0.125 at $H = 300$ m. The effect of variation of α upon r is greater the lower the building height. When α varies from 0.15 to 0.33, r increases approximately 100% at $H \approx 50$ m and 50% at $H \approx 300$ m.

However, most of the turbulence characteristics parameters are dependent on the terrain roughness which was represented by α in this approach, and so the variation of r estimated from changing α values does not directly indicate the effect upon the structural response.

The design wind speed, $\bar{U}(H)$, value also has some effect on the gust response prediction. An increase in $\bar{U}(H)$ reduces the values of $\tilde{f}_1$ and F_H , F_B and so tends to increase the resonance part of response. This effect on factors E and S is the same as that caused by decreasing f_1 .

In the following the effects of turbulence characteristics parameters other than local turbulence intensity are discussed.

Those values can be estimated from α according to a mathematical model suggested in chapter 3 (or as extracted in Table 5-1). However if any local information about the strong wind at a structural site is available, values of those parameters may be substituted directly according to the particular information.

5.6.3 Influence of Power Spectral Density Parameters

The form of the power spectral density may be defined by an index β . The variation of form with $\beta = 0.5$ to 3.0 is illustrated in Figure 3-3 and in Figure 5-3; the latter in terms of the normalised reduced power spectral density. The resonance part of response is dependent on gust power factor, E , which decreases with increasing β for $\tilde{f}_1 > 4$. $\beta = 1.0$ and 3.0 give approximately 30% greater and 10% less values of E than that obtained assuming $\beta = 2.0$ at $\tilde{f}_1 > 10$. The variation of E with β over the same range at $\tilde{f}_1 \approx 3$ to 5 is less than $\pm 6\%$. The greater $\tilde{f}_1$ makes smaller E which reduces the contribution of the resonance excitation to the total. Therefore the effect of β on the gust response will be reduced considerably less than those figures mentioned above (eg, less than 10% in most cases).

On the contrary the background excitation factor, B , increases with β but the effect of the variation of β on B is negligible or as small as $\pm 2\%$ compared with that obtained using $\beta = 2.0$, within the range of β between 1.0 and 3.0 .

Therefore the difference of $\sigma_{\delta}(H)/\bar{\Delta}(H)$ due to von Karman spectral form ($\beta = 2.0$) and Simiu's form ($\beta = 1.0$) will be for example at most 10% for a building similar to 'B' and less by approximately 5% for a building similar to 'A' or 'C'.

Another important parameter for the power spectral density is the length constant $L_1(z)$ in equation (3-8) or $L_H(z)$ in the convenient form for gust response prediction equation (5-3). The value of $L_H(z)$ can be defined by a reference value $L(H)$ and the power law profile index α_L . Firstly the effect of the variation of $L(H)$ is examined. It has the same effect as f_1 on the gust power factor, E , namely $E \propto (L(H))^{-2/3}$ at the higher frequency range, eg, $\tilde{f}_1 \geq 5$. Increasing $L(H)$ also causes the reduction of background excitation factor, B . From the slope of curves in Figure 5-2 $B \propto (L(H))^{1/3}$ may be deduced for cases examined in the numerical case study for the buildings 'A, B, C'.

Consequently if the resonance part of response is assumed to be predominant, presumably for a tall building, a 20% increment of $L(H)$ decreases the response by as much as 6%, and if the quasi-static part is assumed to be predominant, presumably for a building with a relatively high natural frequency, a 20% increment of $L(H)$ increases the response by 3%.

A numerical example of the variation of the structural response due to different $L(H)$ values for building 'O' (see section 5.4) is given in Table 5-6. Values are normalised by a standard value (ie, $L(H) = 2270$ m as used in the previous example).

Table 5-6 Variation of Structural Response with $L(H)$ for Building 'O'

$L(H)$ (m) (ratio to 2270)		1135 (0.5)	1820 (0.8)	2270 (1.0)	2720 (1.2)	3170 (1.4)	3620 (1.6)
$\zeta_S = 1.0\%$	$\sigma_\delta(H)$	1.16	1.05	1.00	0.96	0.93	0.90
	$\sigma_{\ddot{\delta}}(H)$	1.22	1.07	1.00	0.94	0.89	0.86
	G	1.09	1.03	1.00	0.98	0.96	0.95
$\zeta_S = 2.0\%$	$\sigma_\delta(H)$	1.13	1.04	1.00	0.97	0.95	0.93
	$\sigma_{\ddot{\delta}}(H)$	1.23	1.06	1.00	0.94	0.89	0.85
	G	1.07	1.02	1.00	0.99	0.98	0.97

Then the effect of the variation of α_L is examined assuming a constant $L(H)$ value. As can be seen from Figure 5-2, differences of the background excitation factor, B , due to $\alpha_L = 0.0$ and 0.5 are approximately 10% (less when $\alpha_L = 0.5$) irrespective of α_D at $L_B = L_H = 1$ and less than that for $L_B < 1.0$ and $L_H < 1.0$.

From Figure 5-4 differences of the size reduction factor, S , due to $\alpha_L = 0.0$ and 0.5 are approximately 10 to 13% (greater when $\alpha_L = 0.5$) for $\alpha_D = 0.0$ at F_B and F_H values between 1.0 and 10.0. Differences of S for $\alpha_D = 0.4$ becomes slightly greater such as 11 to 15% in the same range of F_B and F_H .

Therefore by linear interpolation it can be estimated that around an average value of $\alpha_L = 0.3$, a 50% increment of α_L causes up to 2% reduction in $\sigma_\delta(H)/\bar{\Delta}(H)$ if the quasi-static part is assumed to be predominant and up to 3% increase if the resonance part is assumed to be predominant.

5.6.4 Influence of Co-coherence Function Parameters

Parameters to be considered in this section are decay constants, k_z , k_y ; their power law profile index, α_D , and a constant, k_2 , defined in equation (5-4).

Effects of the variation of $k_z(H)$ and $k_y(H)$ on the structural response $\sigma_\delta(H)/\bar{\Delta}(H)$ can be estimated in a similar manner to that discussed previously using the curves in Figures 5-2 and 5-4. It can be seen from Figure 5-2 that a 20% increment of $k_z(H)$ causes approximately a 5% reduction of B at $L_H \approx 1.0$ and a smaller variation of B for $L_H < 1.0$. Similarly from Figure 5-4 a 20% increment of $k_z(H)$ causes approximately a 3% reduction of S at $F_H \approx 1.0$ and a 15% reduction of S at $F_H \approx 10.0$. The variation of B and S due to different $k_y(H)$ is similar over ranges of L_B and F_B corresponding to those of L_H and F_H .

Consequently it can be expected that a 20% increment of $k_z(H)$ will reduce the structural response $\sigma_\delta(H)/\bar{\Delta}(H)$ by approximately 8% or less for a relatively low-rise building such as 'A' and 5% for a very tall building such as 'C'.

The effect of $k_y(H)$ is also very similar to that of $k_z(H)$ for a relatively low-rise building but as a building becomes taller both L_B and F_B tend to be much smaller than L_H and F_H respectively and so B and S become much less sensitive to $k_y(H)$ values as seen in Figures 5-2 and 5-4. For example a 20% increment of $k_y(H)$ seems only to reduce the value of $\sigma_\delta(H)/\bar{\Delta}(H)$ by about 2% in the case study of building 'O'.

Effects of the variation of α_D on $\sigma_\delta(H)/\bar{\Delta}(H)$ can be estimated in a similar manner to that of α_L . As can be seen from Figure 5-2, differences of B due to $\alpha_D = 0.0$ and 0.4 are approximately 8% (the value of B being lower for $\alpha_D = 0.4$) irrespective of α_L at $L_B \simeq L_H \simeq 1$ and less than that at $L_B < 1.0$ and $L_H < 1.0$.

From Figure 5-4, differences of S due to $\alpha_D = 0.0$ and 0.4 are approximately 8% (the lower values being given for $\alpha_D = 0.4$) at $F_B \simeq F_H \simeq 1.0$ and 20% at $F_B \simeq F_H \simeq 10.0$. Then around an average value of $\alpha_D = 0.4$ for example, a 50% increment of α_D seems to cause up to a 2% reduction of $\sigma_\delta(H)/\bar{\Delta}(H)$ if the quasi-static part is predominant and a 5% reduction if the resonance part is predominant.

A constant, k_2 , which was introduced to give a consistency for the zero frequency value of co-coherence was chosen to be $\sqrt{10}$ in this study. Its deviation from $\sqrt{10}$ was found to have an insignificant effect on computation of both B and S in most

practical cases. In a commonly used exponential function such as equation (3-15) k_2 is infinity. A reasonable lowest estimation of k_2 is unity as discussed in section 3.4. The difference of B due to those extreme cases of $k_2 = \infty$ and 1.0 was found to be less than 1% for three cases of typical building ('A', 'B' and 'C') by a numerical study. A significant difference of B due to applying the modified frequency to the co-coherence function could only occur in a situation where L_H and L_B are very large such as $L_H \approx L_B \gg 1.0$, ie, a structure having very large dimensions.

The effect of the adoption of modified frequency on S value is negligible if $\tilde{f}_1 \gg \frac{1}{k_2}$ as seen in equation (5-4).

5.6.5 Influence of Variation of Dynamic Along-Wind Force Coefficient

The dynamic along-wind force coefficient for the quasi-static response, $\tilde{C}_{D_{qs}}$, was taken as 0.8 times of the static drag coefficient, C_{D_o} . This value 0.8 seems to have some correspondence to the poor along-wind pressure correlation even at a low frequency range reported by Lam Put⁽⁴⁵⁾ and van Koten⁽⁸⁵⁾. Considering that $f \rightarrow 0$ could represent the static state, it may be expected $\tilde{C}_{D_{qs}}/C_{D_o}$ tends to unity when the frequency approaches zero. This could mean that the effective value of $\tilde{C}_{D_{qs}}/C_{D_o}$ lies between 0.8 and 1.0. However a very low frequency component was not investigated in the experimental study of chapter 4. The further specification of a $\tilde{C}_{D_{qs}}$ value should

be possible with the aid of more experimental works.

The most significant effect of the variation of dynamic along-wind force coefficient was demonstrated by gust factor results for building 'B' in Tables 5-3 and 5-4. Although $\tilde{C}_{D1}/C_{D0}$ values were taken conservatively compared with experimental findings in the previous chapter, considerably large gust factors were obtained; namely in the case of $\zeta_S = 1.0\%$ G values are approximately 20% and 10% greater for urban terrain and open terrain respectively than those obtained by Vickery and Davenport, and approximately 40% and 20% for urban terrain and open terrain respectively than those obtained by Simiu. Calculation results for building 'C' have a coincidental agreement with Simiu's results especially for open terrain.

5.6.6 Concluding Remarks

The summary of effects of individual parameters on the structural response $\sigma_\delta(H)/\bar{\Delta}(H)$ discussed in the foregoing sections is listed in Table 5-7. Effects are standardised by the 20% variation of individual parameters so that their significance of contribution to the response can be seen comparatively. All parameters are treated as independent of each other except α , since the value of α influences $L(H)$, α_L , $k_y(H)$, $k_z(H)$ and α_D according to the mathematical model suggested in chapter 3 (see also summarised Table 5-1).

Table 5-7 Effects of 20% Variation of Individual Parameters on
the Computation of $\sigma_{\delta}(H)/\bar{\Delta}(H)$

Building with $\zeta_s = 1\%$		B		C	
Terrain		URBAN	OPEN	URBAN	OPEN
Parameters	α_{μ}	- 1.5	- 1.0	- 1.5	- 1.0
	γ	- 2.5	- 3.0	- 4.0	- 4.5
	f_1	- 14	- 15	- 13	- 12
	ζ_s	- 6.0	- 5.5	- 4.5	- 4.5
	α	+ 18	+ 12	+ 16	+ 9
	α^*	+ 18	+10.5	+14.5	+ 8
	α_T	+ 0.5	+ 0.5	- 1.0	- 1.0
	β	- 3	- 2	- 2	- 1
	$L(H)$	- 4	- 5	- 4	- 5
	α_L	+ 0	+ 1	+ 0	+ 1
	$k_z(H)$	- 7	- 6	- 6	- 5
	$k_y(H)$	- 3	- 2	- 1.5	- 1
	α_D	- 2	- 2	- 3	- 3
	$\tilde{C}_D$	+ 5	+ 4	+ 5	+ 4
	$\tilde{C}_{D_{qs}}$	+ 16	+ 17	+ 15	+ 17
	$\tilde{C}_{D_1}$				

note: sign + indicate that increment of parameter increases $\sigma_{\delta}/\bar{\Delta}$

sign - indicate that increment of parameter decreases $\sigma_{\delta}/\bar{\Delta}$

unit is %

Therefore the combined effect of α itself and other parameters which change due to a 20% variation of α is shown as the effect of α^* to distinguish the effect of α on the roughness factor. The latter means that the value of other α -dependent parameters mentioned above are treated independent of α and are to be determined by additional local information. Since the secondary effects of those parameters due to 20% variation of α are small, there is not much difference between the effect of α and that of α^* .

Before closing this chapter the significance of the determination of design wind speed in gust response prediction requires examination. If the gradient wind speed is assumed to be the same over the various terrains, the ratio of the design wind speed at building height in open terrain to that in urban terrain becomes :

1.87 (1.75) at $H = 46$ m, 1.61 (1.33) at $H = 152$ m
and 1.38 (1.13) at $H = 365$ m, where values in () indicate those according to Vickery's study⁽¹⁸⁾.

Since the mean or static deflection is linear to the square of the design wind speed, the relative maximum deflection in open terrain to that in urban terrain becomes :

2.53 (2.08), 1.98 (1.52) and 1.63 (1.22) for buildings 'A', 'B' and 'C' respectively; values in () are due to Vickery's in Table 5-3.

It may be stated that the maximum deflection is not particularly sensitive to the terrain roughness for a very tall building; eg, 10% variation of α , which will be a reasonable estimate of error for any building site in usual circumstances, causes approximately 10% (4%) change of the maximum deflection for a very tall building with $H \approx 350$ m and 15% (9%) for a building with $H \approx 150$ m; values in () according to Vickery.

It seems evident that for a relatively low-rise building such as building 'A' with $H = 46$ m it is more important to estimate the appropriate design wind speed value rather than to increase the accuracy of gust response prediction. On the contrary for a tall building with $H > 150$ m, a precise prediction of the gust factor is as important as the design wind speed estimation.

In order to obtain a reasonably accurate gust factor prediction according to this approach, by using either the computer program or chart diagrams, it is essential to make a reliable estimation for f_1 , α , $\bar{U}(H)$ and $\tilde{C}_D/C_{D_0}$. Other wind characteristics parameters such as β , $L(H)$, $k_y(H)$ and $k_z(H)$ have less significant effects on the gust factor prediction than those mentioned above. Height dependence parameters such as α_μ , α_T , α_L and α_D have almost insignificant effects on it. However, this does not mean that effects of the height dependence of wind characteristics are negligible but does mean that average values employed in this study can be representative for a fairly wide variation of α_μ , α_T , α_L and α_D .

It is evident that the dynamic along-wind force coefficient increases significantly when the reduced wind speed approaches zero from the experimental results. However the direct applicability of those results to the full scale structure may not be straight forward as demonstrated in these case studies. The evaluation of $\tilde{C}_D$ for three-dimensional bodies should be the next step in the development of this semi-theoretical gust factor approach and has to be investigated by a future study. Nevertheless the author believes that the present two-dimensional experimental results would have some similarity in the full scale wind-structure interaction. If so the underestimation of gust factors caused by previous conventional methods could be serious.

CHAPTER 6 CONCLUSIONS

6.1 OVERALL DISCUSSION

There have been a number of gust response approaches available since Davenport's first attempt⁽⁴⁾ with a limited amount of information regarding natural wind characteristics. As time progresses more and more information about natural turbulence is becoming available and some mathematical models for strong natural wind have been suggested^(53,55,56) showing satisfactory consistency with extensive full scale measurements.

However, there still seems to be a gap between available information about natural turbulence and the gust response approaches. This is basically due to the lack of flexibility in both the mathematical model of strong wind and the gust response approach. In order to conquer this problem a more flexible mathematical model for strong natural wind has been proposed in chapter 3 with adjustable parameters bearing in mind the possible application to gust response prediction.

Recent full scale measurements indicate that the r.m.s. value of turbulence component, power spectral density and root-or co-coherence all have height dependence. They were expressed in a height invariant form in the first step of the gust response approach. Expressions suggested in this study for those quantities are well representative to their height dependence with a suitable power law form and have been examined to be consistent with recent theoretical and empirical works.

One of the important points in recent gust response approaches is the appreciation of the role played by the along-wind pressure correlation. Although its significance seems rather serious according to Simiu⁽¹⁶⁾, there has been some difficulty in establishing an appropriate value of the correlation, presumably because of lack of supporting evidence. Furthermore there has been an unsolved question regarding the values of dynamic drag and mass coefficients since Davenport's simplification, ie, the dynamic drag coefficient was taken as the same as the static value and the mass coefficient was neglected in spite of his experimental evidence⁽¹³⁾ which clearly indicated the variation of those dynamic coefficients with reduced frequency (the inverse of reduced velocity). Therefore in this work the dynamic along-wind force coefficient concept was introduced taking account of the effects mentioned above, namely the along-wind pressure correlation and the variation of dynamic drag and mass coefficients.

The dynamic along-wind force coefficient was successfully evaluated by using a two-dimensional S.D.O.F. model in a partial boundary layer wind tunnel. The turbulence characteristics in the wind tunnel were found not to simulate closely full scale turbulence. Nevertheless the power spectral densities and co-coherence functions obtained from the hot-wire measurements in the tunnel were fairly well represented by the same formulae as suggested in the full scale mathematical

model. Due to the limitation of wind tunnel facilities, the range of the turbulence scale examined in the experiment was small compared with actual situations of tall buildings. However cases examined for the turbulence scale greater than the model dimension seem to indicate some similarity with full scale situations where the turbulence scale can be as much as ten times the dimension of a building.

Within the range of the numerous parameters examined, values of static drag coefficients were in agreement with previous works^(42,43,44) and revealed the significance of the effect of turbulence intensity and scale.

Values of the dynamic along-wind force coefficient in terms of its ratio to the static drag coefficient, ie, $\tilde{C}_D/C_{D_0}$ obtained in the experiment fall in a general form expressed as a function of the section aspect ratio D/B and the reduced wind speed $\tilde{U} = \bar{U}/f_0 D$. At higher reduced wind speeds ($\tilde{U} > 10$) experimental results were in good agreement with those estimated from Simiu's recent proposals⁽²⁰⁾ for along-wind pressure correlation, in other words most values of $\tilde{C}_D/C_{D_0}$ converge around 0.8.

On the contrary at a lower reduced wind speed range ($\tilde{U} < 5.0$) an entirely opposite trend was revealed. Values of $\tilde{C}_D/C_{D_0}$ increased considerably when $\tilde{U}$ decreased. The maximum value of $\tilde{C}_D/C_{D_0}$ obtained in this experiment was nearly 7.0 at $\tilde{U} \approx 1.0$ for $D/B = 2.0$.

Rapid increases of $\tilde{C}_D/C_{D_0}$ may be explained by the nature of leeward pressure spectra which tend to have considerable power in the high frequency range (ie, corresponding to the lower reduced wind speed) than that of the wind spectra. The trend of $\tilde{C}_D/C_{D_0}$ to increase with frequency was indicated by the drag and mass coefficient measurements of Keulegan and Carpenter⁽¹²⁾ and Davenport⁽¹³⁾.

The effect of the variation of dynamic along-wind force coefficient on gust response prediction was investigated numerically by a computer program developed for practical applications. Effects of the variation of wind parameters also were examined. As noted in the numerical comparisons, values of the gust factor were found to be insensitive to most of the individual wind parameters. Although the height variations of r.m.s. longitudinal turbulence, power spectral density and co-coherence are supported substantially by recent full scale measurements, their effects on gust response predictions appear to be not particularly significant in most cases.

On the contrary, the variation of dynamic along-wind force coefficient or the along-wind pressure correlation in Simiu's study⁽¹⁶⁾ was found to have a distinct effect on the gust response prediction. Adopting the dynamic along-wind force coefficient based on the two-dimensional S.D.O.F. model

experiment, a case study of a building having slender dimension and a low fundamental frequency resulted in a similar gust factor value to that of Simiu's work. However, some other cases examined have indicated considerably larger gust factors due to a high dynamic along-wind force coefficient at the resonance part of response. It is interesting also that for a building with relatively high fundamental frequency the gust factor does not increase so much despite a very high dynamic along-wind force coefficient since the gust energy, ie, the reduced power spectral density, at the fundamental frequency drops considerably and so the resonance part becomes less significant than in the case of a building with a low natural frequency.

The author believes that the evaluated dynamic along-wind force coefficient has some relevance as their extrapolation coincides with Davenport's results. However its applicability to three-dimensional bodies is another problem and has to be investigated by a further extensive study.

Present experimental results indicate that gust factors suggested by Simiu could cause a serious underestimation of response for certain types of buildings (eg, height between 100 m and 200 m) and at the same time they confirm Simiu's gust factors for very tall slender buildings; in such cases gust factors by Davenport and Vickery seem to be rather over-estimated.

Although Simiu suggested that gust factors for most cases were overestimated by Davenport and Vickery due to the fully correlated along-wind pressure assumption and additionally for very tall buildings due to the height independent form of power spectral density, it seems difficult to accept this without more supporting empirical evidence. In fact some full scale measurements of building response⁽⁸⁾ and experimental results were represented within satisfactory limits by Davenport's and Vickery's approach, eg, within 20% of variation according to Vickery⁽¹⁸⁾. Also from van Koten's study⁽⁹⁾ based on a method of gust factor prediction similar to Davenport's one it is difficult to find a definite over-estimation trend of previous theoretical approaches. These facts may indicate that the ratio of dynamic along-wind force coefficients to the static one, $\tilde{C}_D/C_{D_0}$, will be considerably greater than unity in a lower reduced frequency range (say $\tilde{U} \leq 5.0$) where most full scale measurements have so far been carried out.

6.2 CONCLUSIONS

In line with the objectives of this work the following conclusions may be drawn :

- (i) The dynamic along-wind force coefficient was introduced to improve the conventional stochastic analysis for

the prediction of the dynamic response of tall bluff structures in strong winds.

- (ii) A simplification of the stochastic approach when applied to resonance response leads to the same result as can be derived from an energy method by MacDonald⁽²⁶⁾, the latter permitting the establishment of only the resonance component of the along-wind response.
- (iii) A similar development to (i) was applied to the cross-wind response of a bluff structure under both the case of a normal wind and one with a general angle of attack.
- (iv) A mathematical model for the longitudinal turbulent component of strong natural wind was proposed with flexible height dependent expressions.
- (v) Turbulence characteristics in a partial boundary layer wind tunnel were established and found to be stable though height dependent.
- (vi) Effects of turbulence scale and intensity on the static drag coefficient were found similar to those revealed by MacLaren⁽⁴²⁾ and Lee^(43,44).
- (vii) The dynamic along-wind force coefficients were evaluated in terms of their ratio to the static drag coefficient and expressed as a function of the section aspect ratio D/B and the reduced wind speed $\tilde{U}$.

- (viii) A gust response prediction program was developed incorporating the results achieved in (iv) and (vii) and presented in a chart diagram form for practical application.
- (ix) The variation of the dynamic along-wind force coefficient appears to have a significant effect on the prediction of dynamic wind response of tall buildings as was examined numerically by a computer method described in chapter 5.
- (x) By comparison with the numerical work in chapter 5 based on two-dimensional experimental results, gust factor procedures due to Davenport⁽⁵⁾ and Vickery⁽¹⁸⁾ could cause a considerable overestimation of response for a very tall building ($H > 200$ m). Similarly procedures due to these authors^(5,18) and Simiu⁽²⁰⁾ could lead to serious underestimation of the response of a medium tall building ($H \approx 100$ m).

Information obtained in this work for the dynamic along-wind force coefficient, $\tilde{C}_D$, is still limited from the aspect of both validity and applicability. Present experimental results may only be considered as a first step for the evaluation of $\tilde{C}_D$. However since the variation of $\tilde{C}_D$ with the reduced wind speed $\tilde{U}$ was found to be significant at least in the present experimental condition, there is a pressing need to incorporate this factor together with realistic wind characteristics parameters in the practical prediction procedure.

6.3 SUGGESTIONS FOR FUTURE RESEARCH

As noted repeatedly, in order to improve the gust response prediction it is important to establish a realistic mathematical model for strong natural wind. The model suggested in this work can be recommended until more reliable information is available. Values for the power spectral density form parameter, β , the length constant, $L_1(z)$, and the decay constants, $k_{1z}(z_m)$, $k_{1y}(z_m)$, in the co-coherence function need to be substantiated especially for the rougher terrain and at higher levels of boundary layer (say $z \geq 200$ m). As the height of a building becomes greater, the importance of wind characteristics at higher level increases. The mathematical model of strong wind is deduced by assuming the nature of wind at the gradient height and yet most available data concentrated on around the height between 20 and 200 m. Consequently the nature of wind at the gradient height is usually extrapolated from available data. Information about wind characteristics at higher levels will help also to make a more realistic mathematical model.

The applicability of the dynamic along-wind force coefficient evaluated in this work to the three-dimensional large body has to be investigated as an extension of this work. And also a further extension of parameters in two-dimensional model studies will provide more consistency when values of the dynamic along-wind force coefficient are applied to the practical

response prediction procedure. Particularly cases for the greater turbulence scale to the object dimension ($L_x/D \gg 1.0$) and the lower reduced velocity ($\tilde{U} \approx 1.0$) are of interest.

It is also very interesting to evaluate the along-wind force coefficient concentrating on the quasi-static value. It may be possible to obtain $\tilde{C}_D/C_{D_0}$ values at very high reduced velocity by using a very low natural frequency model, but an alternative way can be considered such that by using a rather highly damped ($\zeta \approx 50\%$ or more) dynamic model the average quasi-static $\tilde{C}_D/C_{D_0}$ will be evaluated from its response.

The relationship between leeward face pressure spectra and the dynamic along-wind force coefficient values will be another important key to solve the general problem of the wind-structure interaction.

Full scale measurements of structural response in strong winds are always very necessary to verify any kind of theoretical or empirical experimental prediction. The difficulty usually lies in measuring the mean or static component and the appropriate reference wind speed. Nevertheless the variation of dynamic along-wind force coefficient with the reduced wind speed should be investigated by extensive full scale measurements which alone can decide the degree of validity and applicability of the present results.

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APPENDIX 1 MODIFIED ENERGY METHOD

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A1.1 INTRODUCTION

A procedure for the prediction for gust response from the energy equilibrium was proposed by MacDonald and Morgan⁽²⁶⁾ in 1971. The advantage and significance of the energy method was studied extensively by MacDonald⁽⁹²⁾. Some practical applications based on the procedure have been attempted by Royles and Das^(46,74) since that time.

Generally in order to solve the dynamic problem it is necessary to find a solution from the direct equilibration of forces or energy, ie, either by solving the differential equation of motion or alternatively from the energy equilibrium (eg, Hamilton's principle)⁽⁹³⁾. A good illustration of the latter can be seen in a study of galloping vibration amplitude for which an energy method was developed effectively by Novak⁽⁹⁴⁾.

The original form of the energy method solution due to MacDonald was rather different from that of an ordinary spectral approach. However, in this appendix it is shown that the same expression for the dynamic resonance component of response can be deduced from the MacDonald method as is obtainable by the spectral analysis of chapter 2.

A1.2 BASIC ASSUMPTIONS

The basic assumptions employed for the energy method are, according to MacDonald⁽²⁶⁾,

- (i) the structure is a cantilever type and has a fundamental frequency f_0 and a modal shape $\mu(z)$,
- (ii) only the resonance component is transferred between wind and structure, and the quasi-static part is excluded,
- (iii) both wind turbulence and structure vibrational component at f_0 can be represented by sinusoidal fluctuations having effective amplitudes A_w and A_{st} at the top of structure ($z = H$) respectively,
- (iv) transfer energy is a maximum when turbulent and structural velocities are in phase,
- (v) A_w is constant with height and the profile of mean wind speed, $\bar{U}(z)$, is represented by a power law form with an index α ,
- (vi) the space correlation across the structural width B is unity (ie, an ideal line-like structure).

A1.3 ANALYTICAL PROCEDURE

From assumptions (i) and (iii), the dynamic deflection of a structure may be written as,

$$\delta = A_{st} \mu(z) \sin 2\pi f_o t \quad (A1-1)$$

Hence, the velocity of a structure is,

$$\dot{\delta} = 2\pi f_o A_{st} \mu(z) \cos 2\pi f_o t \quad (A1-2)$$

The dynamic component of the associated pressure on the structure at height z , $p(z, t)$, can be written (see section 2.2.2)

$$p(z, t) = \rho C_d \bar{U}(z) (u(z, t) - \dot{\delta}) \quad (A1-3)$$

where ρ is the air mass density, C_d is the drag coefficient (which is treated as a constant with frequency to avoid complexity), and $u(z, t)$ is the longitudinal turbulence component, which may be expressed as,

$$u(z, t) = A_w \cos 2\pi f_o t \quad (A1-4)$$

ie, in phase with $\dot{\delta}$.

Then from assumption (iv),

$$p(z, t) = \rho C_d \bar{U}(z) [A_w - 2\pi f_o A_{st} \mu(z)] \cos 2\pi f_o t \quad (A1-5)$$

The force on an element strip dz across width B of a structure at height z is,

$$dF = p(z, t) B dz \quad (A1-6)$$

and the work done on the element in moving a small distance $d\delta$ is,

$$\begin{aligned} dE_e &= dF \cdot d\delta = p(z, t) B \dot{\delta} dt dz \\ &= \rho B C_d \bar{U}(z) [A_w - 2\pi f_o A_{st} \mu(z)] 2\pi f_o A_{st} \mu(z) \\ &\quad \times \cos^2 2\pi f_o t dz dt \end{aligned} \quad (A1-7)$$

Introducing the power law profile for $\bar{U}(z)$,

$$\begin{aligned} dE_e &= \rho B C_d \bar{U}(H) \left(\frac{z}{H} \right)^\alpha 2\pi f_o A_w A_{st} \mu(z) \cos^2 2\pi f_o t dz dt \\ &\quad - \rho B C_d \bar{U}(H) \left(\frac{z}{H} \right)^\alpha (2\pi f_o)^2 A_{st}^2 \mu^2(z) \cos^2 2\pi f_o t dz dt \end{aligned} \quad (A1-8)$$

The first term in equation (A1-8) is related to both turbulence and structural response and the second term is only related to the structural response component.

When the input energy in one cycle is considered the space correlation effect must be taken into account. In the previous development^(26,46), equation (A1-8) was multiplied by the coherence function, $C_{u_1 u_2}$, or its equivalent and integrated with respect to f from 0 to H . However as noted from the nature of $C_{u_1 u_2}$, $C_{u_1 u_2}$ is not a function of one variable z but a function of the two positions z_1 and z_2 (in some cases $|z_1 - z_2|$), and since this input energy is defined at a frequency f_0 (ie, in the frequency domain), the space correlation effect should be taken into account by the square root of the double integral expression as developed in equation (2-55).

Then the input energy in one cycle is,

$$\begin{aligned}
 E_e = & \int_0^{1/f_0} \left\{ \int_0^H \int_0^H \rho B C_d \bar{U}(H) \cdot 2\pi f_0 \cdot A_w A_{st} \left(\frac{z_1}{H} \right)^\alpha \mu(z_1) \right. \\
 & \times \cos^2 2\pi f_0 t C_{u_1 u_2}(z_1, z_2, f_0) \cdot \rho B C_d \bar{U}(H) \cdot 2\pi f_0 \cdot A_w A_{st} \\
 & \times \left. \left(\frac{z_2}{H} \right)^\alpha \cdot \mu(z_2) \cos^2 2\pi f_0 t \, dz_1 \, dz_2 \right\}^{\frac{1}{2}} dt \\
 & - \int_0^{1/f_0} \int_0^H \rho B C_d \bar{U}(H) (2\pi f_0)^2 A_{st}^2 \left(\frac{z}{H} \right)^\alpha \mu^2(z) \cos^2 2\pi f_0 t \, dz \, dt
 \end{aligned}$$

If ρ , B and C_d are assumed to be constant with height, the integration for one cycle gives,

$$\begin{aligned}
 E_e = & \rho B C_d \bar{U}(H) (2\pi f_o)^2 A_{st} \left[\left\{ \int_0^H \int_0^H \left(\frac{A_w}{2\pi f_o} \right)^2 C_{u_1 u_2}(z_1, z_2, f_o) \right. \right. \\
 & \times \left. \left(\frac{z_1}{H} \right)^\alpha \mu(z_1) \cdot \left(\frac{z_2}{H} \right)^\alpha \mu(z_2) dz_1 dz_2 \right\}^{\frac{1}{2}} - A_{st} \int_0^H \mu^2(z) \left(\frac{z}{H} \right)^\alpha dz \right] \\
 & \times \frac{1}{2f_o}
 \end{aligned}$$

(A1-9)

The corresponding damping energy per cycle can be obtained by equation (A1-10).

$$E_d = \int_0^H \int_0^{1/f_o} c \dot{\delta} d\delta dz = \int_0^H \int_0^{1/f_o} c (\dot{\delta})^2 dt dz$$

(A1-10)

where c is the damping coefficient for the elemental height of the structure dz .

Substituting for $\dot{\delta}$ from equation (A1-2), gives,

$$\begin{aligned}
 E_d &= \int_0^H \int_0^{1/f_o} \left\{ c (2\pi f_o)^2 A_{st}^2 \mu^2(z) \cos^2 2\pi f_o t \right\} dt dz \\
 &= (2\pi f_o)^2 \cdot \frac{1}{2f_o} \cdot A_{st}^2 \int_0^H c \mu^2(z) dz
 \end{aligned}$$

$$E_d = (2\pi^2 f_o) A_{st}^2 \cdot c_g$$

where c_g is the generalised damping coefficient for the whole structure and can be represented as,

$$c_g = \int_0^H c \mu^2(z) dz = \zeta \cdot 4\pi f_o \cdot M$$

where

$$M = \int_0^H \int_0^B m \mu^2(z) dy dz \text{ (the generalised mass of the structure)}$$

Hence,

$$E_d = (2\pi^2 f_o) A_{st}^2 \cdot \zeta 4\pi f_o M = \zeta (2\pi f_o)^2 \cdot 2\pi A_{st}^2 M$$

(A1-11)

The steady state condition is given by equating (A1-9) to (A1-11). After a little rearrangement assuming that B is constant, the effective amplitude of response A_{st} can be expressed as follows,

$$A_{st} = \frac{A_w}{2\pi f_o} \cdot \frac{\left\{ \int_0^H \int_0^H C_{u_1 u_2}(z_1, z_2, f_o) \mu(z_1) \mu(z_2) \left(\frac{z_1}{H}\right)^\alpha \left(\frac{z_2}{H}\right)^\alpha dz_1 dz_2 \right\}^{\frac{1}{2}}}{\int_0^H \mu^2(z) \left(\frac{z}{H}\right)^\alpha dz + \frac{2\zeta(2\pi f_o)}{\rho \bar{U}(H) C_d} \int_0^H m \mu^2(z) dz}$$

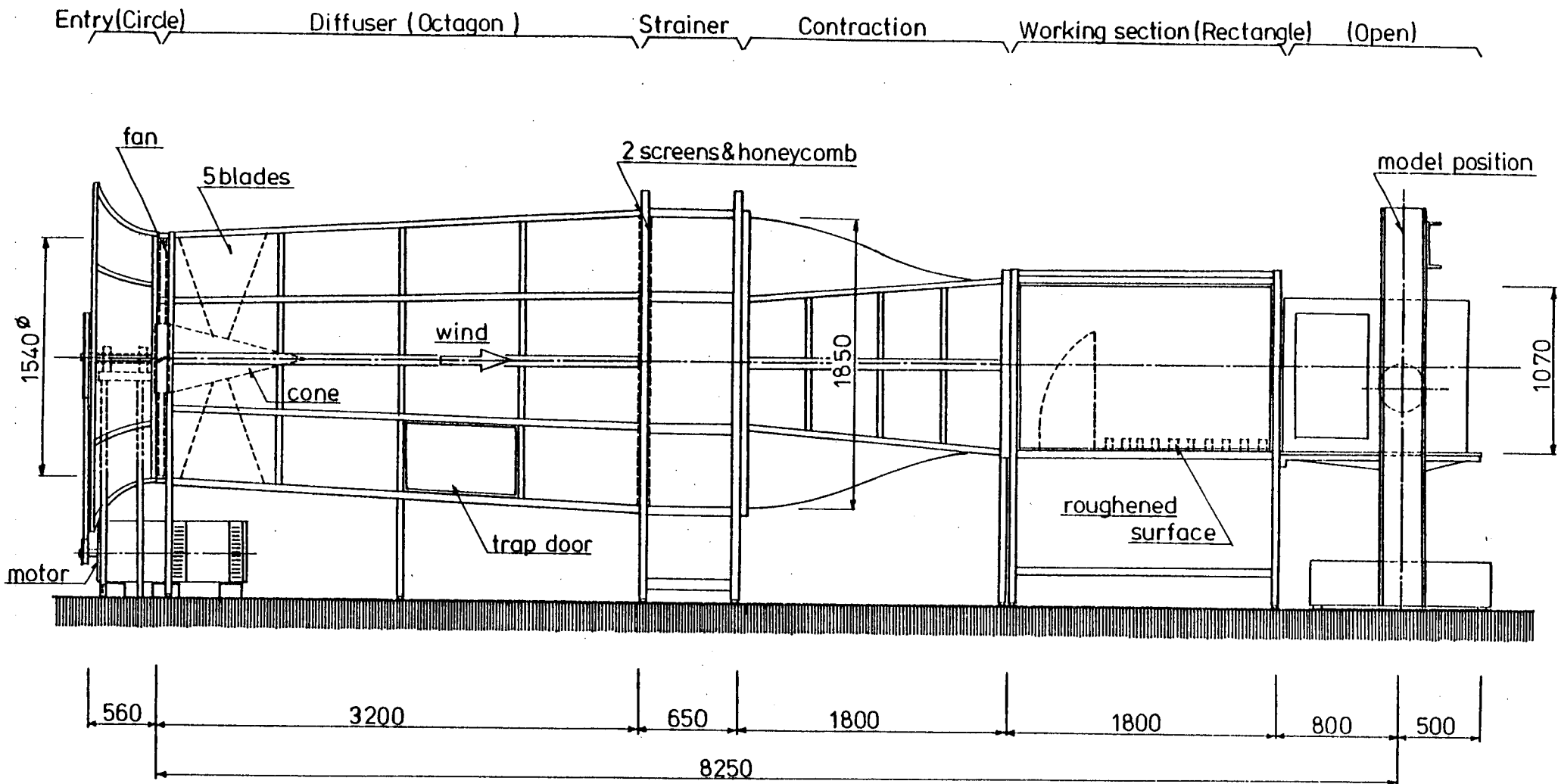
(A1-12)

which agrees with equation (2-62).

The square root of double integral in the numerator of equation (A1-12) has rather different value from the single integral result in the original method, and so this modification seems to be rather essential.

APPENDIX 2 EXPERIMENTAL DETAILS

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A2-2

unit:mm

Figure A2-1 Elevation of wind tunnel

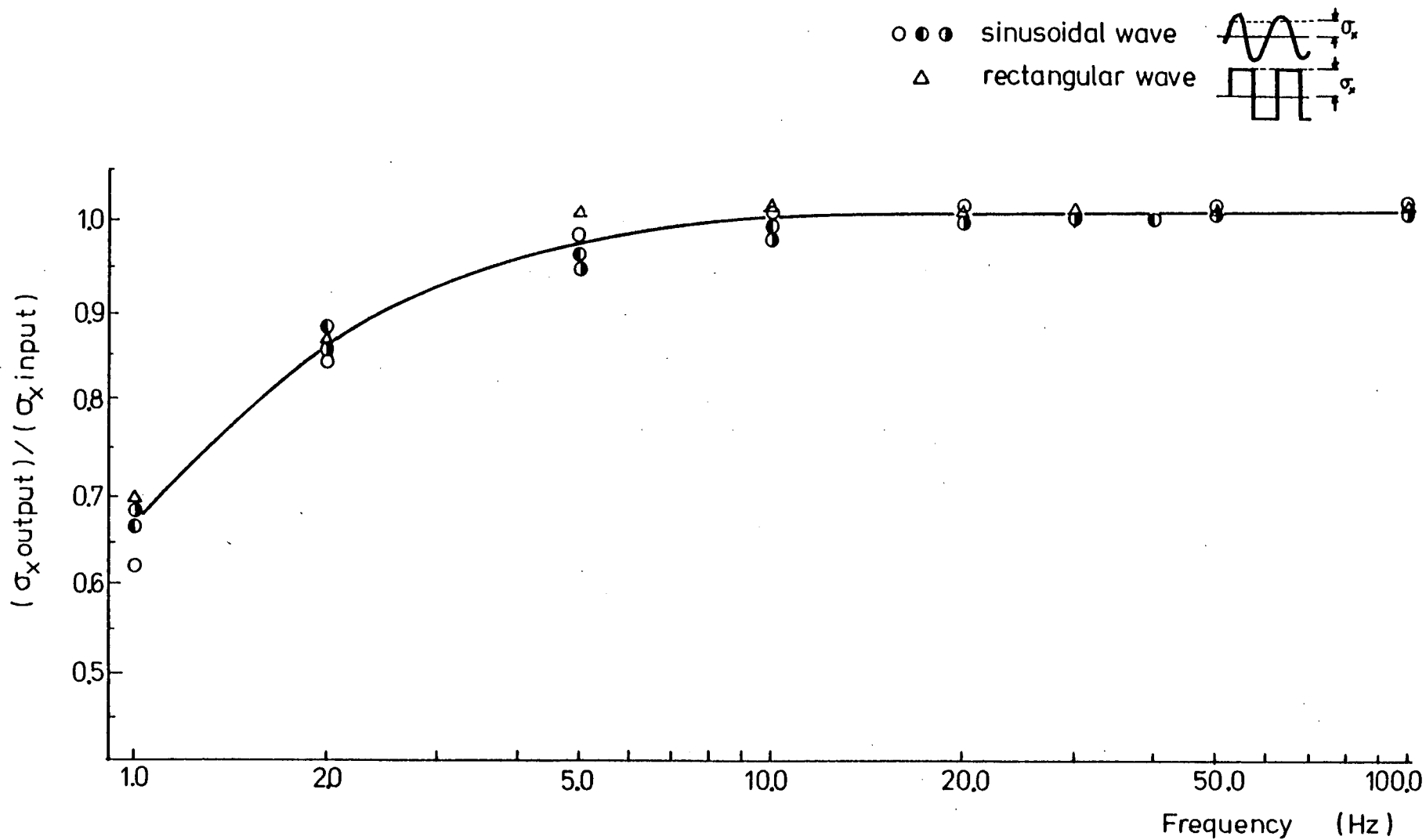


Figure A2-2 Frequency response calibration of analogue correlator

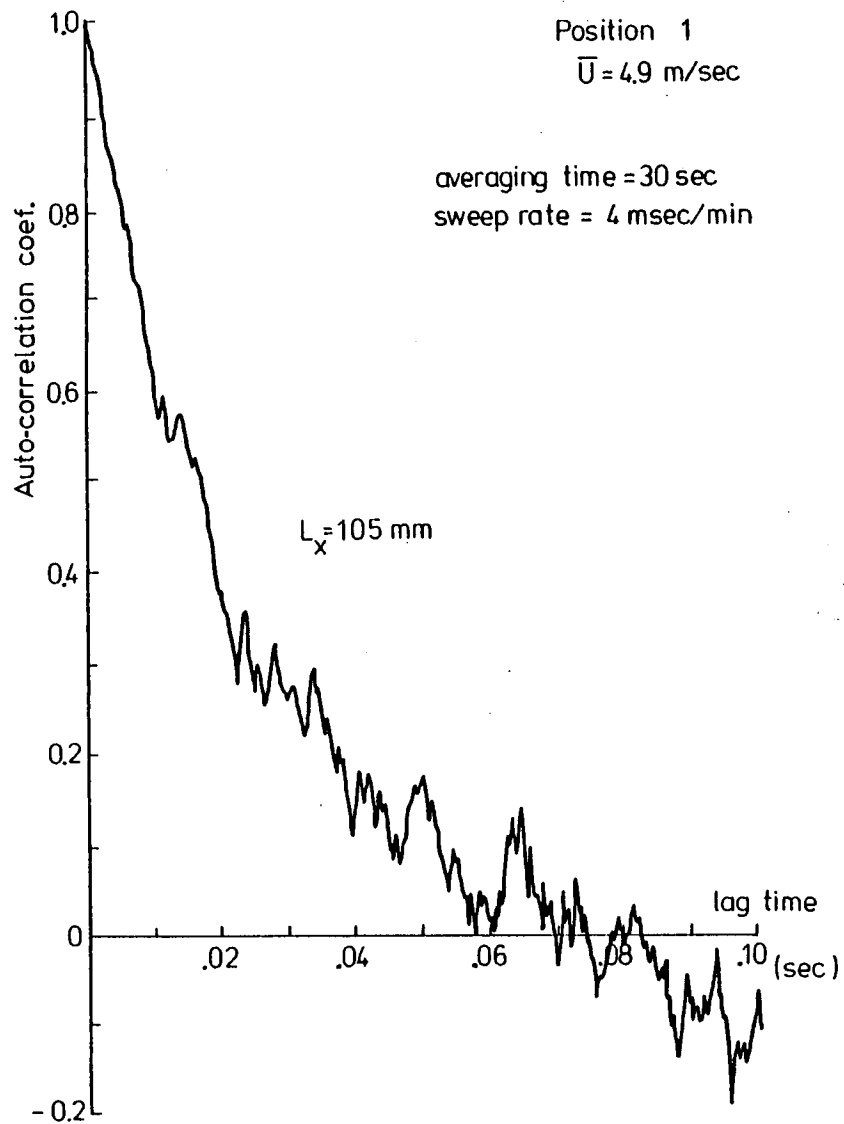


Figure A2-3(a) Measured auto-correlation coefficient at position 1

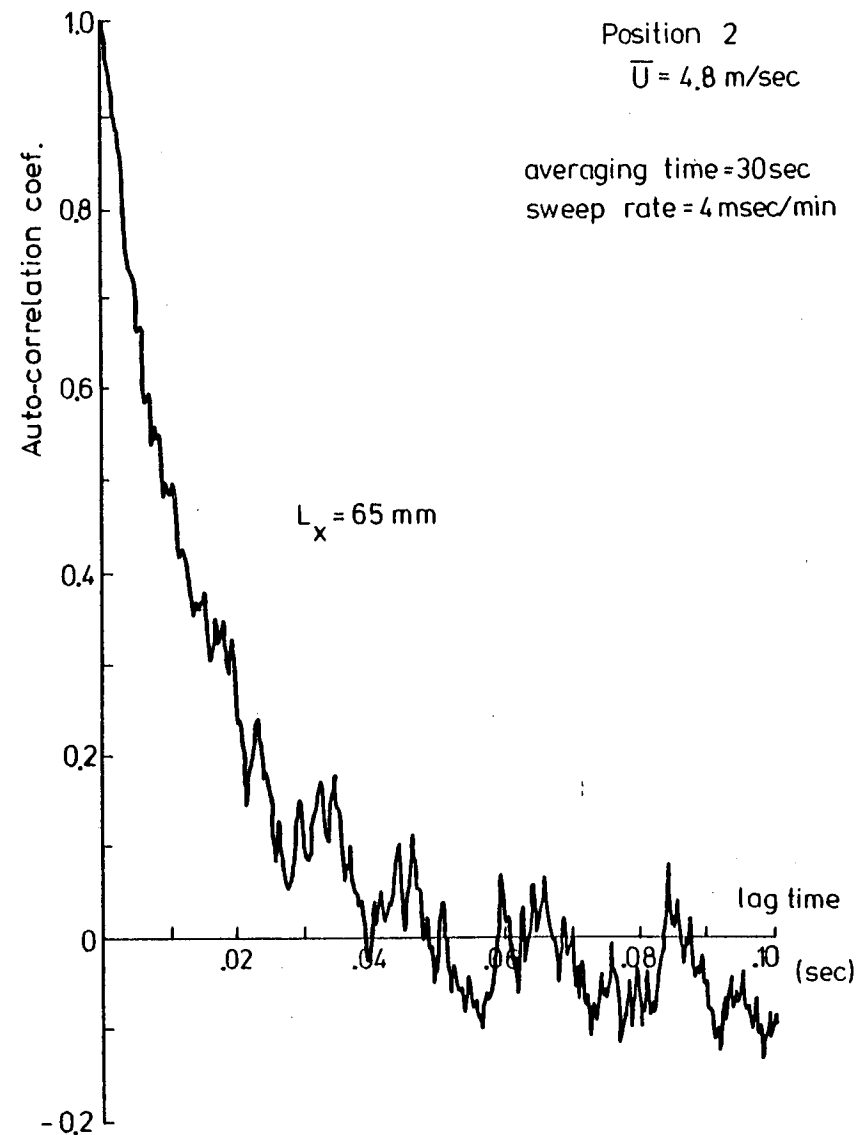


Figure A2-3(b) Measured auto-correlation coefficient at position 2

Position 1

 $y_1 = 0 \text{ mm}$, $z_1 = 250 \text{ mm}$

averaging time = 30 sec

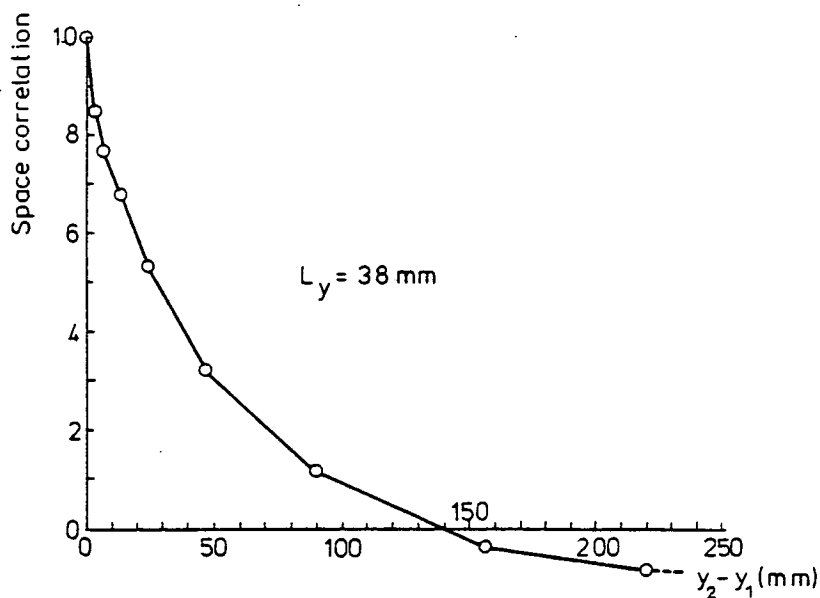
 $\bar{U} = 4.3 \text{ m/s}$ 

Figure A2-4(a) Measured space correlation coefficients with horizontal separations at position 1

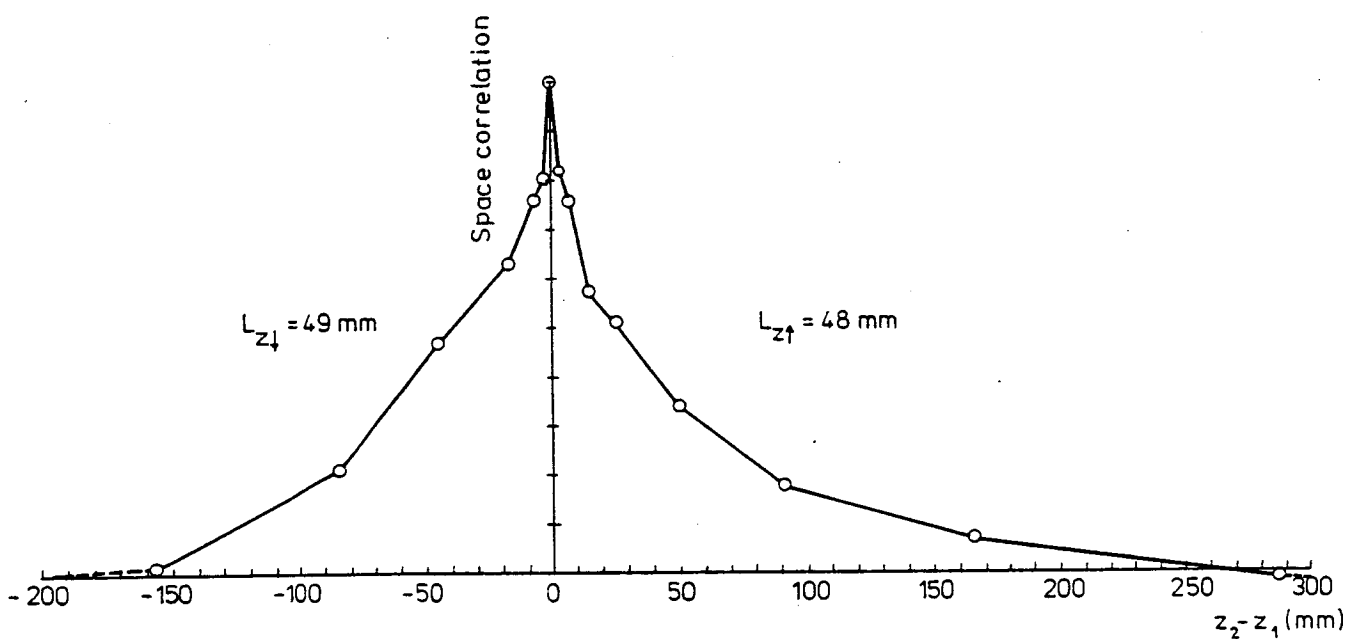


Figure A2-4(b) Measured space correlation coefficients with vertical separations at position 1

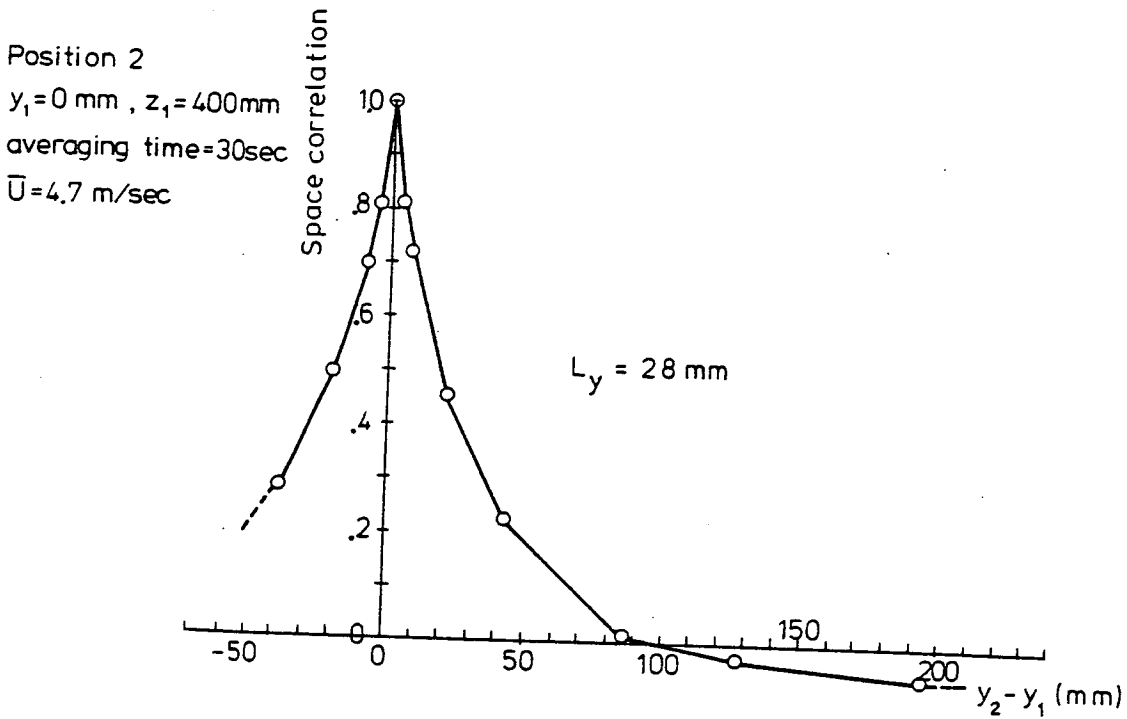


Figure A2-4(c) Measured space correlation coefficients with horizontal separations at position 2

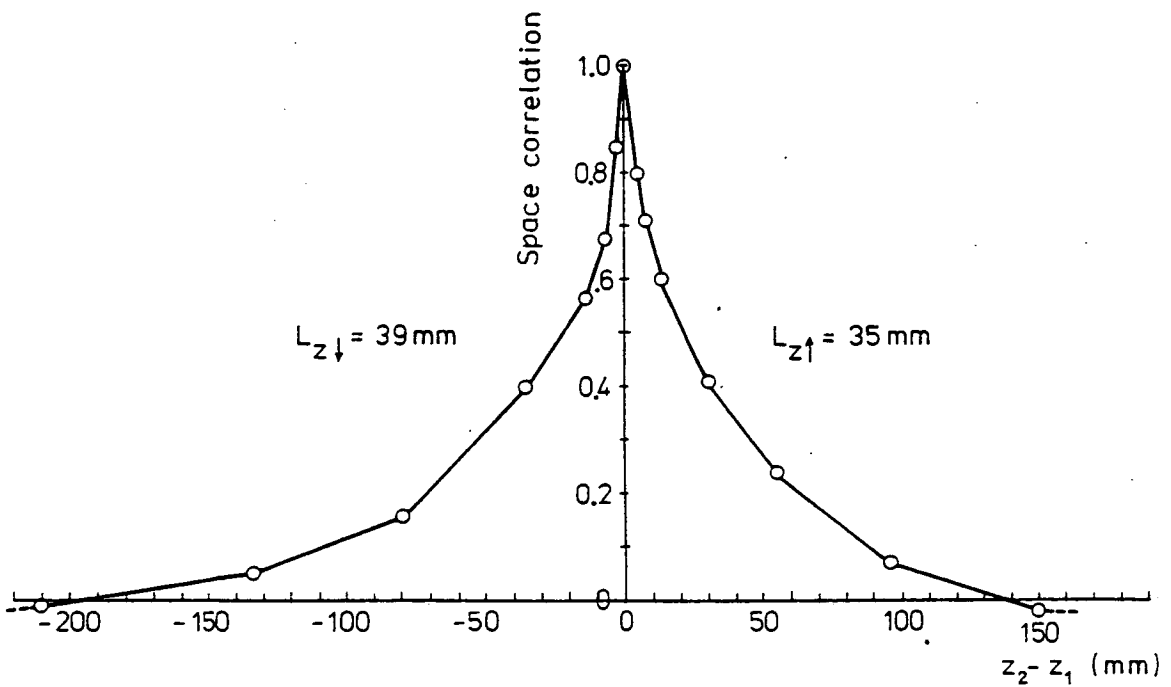


Figure A2-4(d) Measured space correlation coefficients with vertical separations at position 2

Table A2-1 Variation of Decay Constants of Co-coherence Function
in Wind Tunnel

Position	1 ($z_1 = 250$ mm)		2 ($z_1 = 400$ mm)	
Velocity	low (≈ 3.0 m/s)	high (≈ 6.0 m/s)	low (≈ 3.0 m/s)	high (≈ 6.0 m/s)
Vertical Separation *1				
100	11.0	10.8	9.0	—
($z_2 - z_1$) 50	10.0	12.0	10.0	9.0
(mm) 20	12.0	10.5	8.5	9.0
10	9.5	7.5	8.5	7.0
-10	11.5	8.5	11.5	6.5
-20	12.0	10.0	8.5	10.0
-50	12.0	13.0	13.5	7.5
-100	12.5	12.0	8.5	—
average of []	10.8		9.2	
Horizontal Separation *2				
($y_2 - y_1$) 50	12.0	12.0	11.5	12.5
(mm) 20	10.5	10.5	9.0	8.5
10	13.5	8.0	12.0	7.5
-10	12.0	8.0	11.5	7.0
-20	14.0	10.0	9.0	11.5
-50	18.0	10.0	11.5	11.0
-100	17.0	—	—	—
average of []	10.8		9.4	

note : *1 $y_2 = y_1 = 0$ mm

*2 $y_1 = 0$ mm, $z_2 = z_1$

(see Figure 3-7 for co-ordinates)

A2.3 DETAIL OF MODEL DESIGN

A2.3.1 Example of Design Calculation

Design of mass

Based on a desired overall structural mass density of 400 kg/m^3 .

Shape I (smallest size)

Required weight	$6.0 \times 6.0 \times 4.0 \times 0.4 = 57.6 \text{ g}$	
Aluminium core part	$4.2 \times 4.2 \times 0.8 \times 2.7 = 38.1 \text{ g}$	
Balsa shroud	$4.0 \times (5.4 + 5.4) \times 2 \times 0.2 = 17.3 \text{ g}$	
Bolts, washers etc.	2.0 g	
Total	57.4 g	
	(O.K.)	

Shape VII (largest size)

Required weight	$12.0 \times 12.0 \times 4.0 \times 0.4 = 230.4 \text{ g}$	
Aluminium core part	38.1 g	
Balsa shroud	$4.0 \times (11.4 + 11.4) \times 2 \times 0.2 = 36.5 \text{ g}$	
Spacers, bolts, washers etc.	8.0 g	
Sub-total	82.6 g	
Necessary additional weight	147.8 g	
Steel plate $2.5 \times 25 \times 25 \text{ mm}$	12.3 g	
Required number of plates	12	
Total	$82.6 + 12 \times 12.3 = 230.2 \text{ g}$	(O.K.)

Design of plate spring for shape II, spring system A .

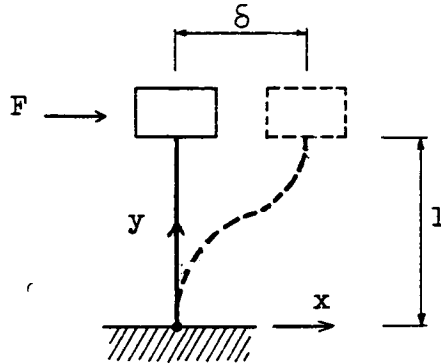


Figure A2-5 Plate spring deflection

Deflection $\delta = \frac{F l^3}{12 E I}$

where E = Young's modulus , I = Moment of inertia .

Spring constant $k = \frac{F}{\delta} = \frac{12 E I}{l^3}$

Four plate springs are used.

$$I = 4 \times \frac{b t^3}{12} , \text{ then } k = \frac{4 E b t^3}{l^3}$$

where b is the width of a spring plate,

t is the thickness of a spring plate.

The k required is obtained from the mass, m , and the required natural frequency, f_o ,

eg $f_o = 15 \text{ Hz}$

$$m = 6.0 \times 8.0 \times 4.0 \times 0.4 = 76.8 \text{ g} = 0.0768 \text{ kg}$$

$$\begin{aligned} k_{\text{req}} &= (2 \pi f_o)^2 \times m \\ &= (94.2)^2 \times 0.0768 = 682 \text{ N/m} \end{aligned}$$

Assuming $E = 21 \times 10^{10} \text{ N/m}^2$, $l = 0.06 \text{ m}$ and $b = 0.01 \text{ m}$,

$$t^3 = \frac{k l^3}{4 E b} = \frac{682 \times 0.06^3}{4 \times 21 \times 10^{10} \times 0.01} = \frac{0.147}{8.4 \times 10^9} = 17.5 \times 10^{-12}$$

$$t = 2.59 \times 10^{-4} \text{ m} \longrightarrow t = 0.254 \text{ mm}$$

Check the natural frequency

spring system A : $t = 0.254 \text{ mm}$, $b = 10.0 \text{ mm}$

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{4 E b t^3}{m l^3}}$$

$$= \frac{1}{\pi} \sqrt{\frac{21 \times 10^{10} \times 0.01 \times 0.000254^3}{0.0768 \times 0.06^3}} = 14.5 \text{ Hz}$$

spring system B : $t = 0.254 \text{ mm}$, $b = 5.0 \text{ mm}$

$$f_o = \frac{1}{\pi} \sqrt{\frac{21 \times 10^{10} \times 0.005 \times 0.000254^3}{0.0768 \times 0.06^3}} = 10.25 \text{ Hz}$$

Check the deflection of model (shape VII ... largest size, spring system A)

Assuming $U = 6.0 \text{ m/s}$

$$\bar{F} = C_{D_0} \cdot \frac{1}{2} \bar{U}^2 A = 2.0 \times \frac{1}{2} \times 1.2 \times 6.0^2 \times 0.12 \times 0.04$$

$$= 0.207 \text{ N}$$

$$\delta_{\text{static}} = \frac{\bar{F}}{k} = \frac{0.207}{680} = 3.0 \times 10^{-4} \dots\dots 0.3 \text{ mm}$$

Assuming $\delta_{\text{dynamic(rms)}} \simeq \frac{1}{3} \delta_{\text{static}}$,

then for an elastic system

$$\delta_{\text{max}} = \delta_{\text{static}} + p \times \delta_{\text{dynamic(rms)}}$$

Adopting a peak factor, p , in the range 3 to 4 ,

$$\delta_{\text{max}} \simeq 0.6 \text{ to } 1.0 \text{ mm}$$

Check the stress and strain

Base bending moment of the single spring A per leaf (4 leaves in all)

$$M = \frac{1}{4} \frac{F l}{2} = \frac{0.207 \times 0.06}{8} = 1.55 \times 10^{-3} \text{ N.m}$$

$$Z = \frac{0.01 \times (2.54 \times 10^{-4})^2}{6} = 1.07 \times 10^{-10} \text{ m}^3 \text{ (section modulus)}$$

Stress

$$\sigma_{\text{static}} = \frac{M}{Z} = \frac{1.55 \times 10^{-3}}{1.07 \times 10^{-10}} = 1.45 \times 10^7 \text{ N/m}^2 \dots 14.5 \text{ N/mm}^2$$

$$\sigma_{\text{max}} \approx 30 \text{ to } 50 \text{ N/mm}^2 \quad (\text{cf permissible stress in bending tension for tempered steel } 400 \text{ N/mm}^2)$$

Strain

$$\epsilon_{\text{static}} = \frac{\sigma_{\text{static}}}{E} = \frac{1.45 \times 10^7}{21 \times 10^{10}} = 1.19 \times 10^{-4} \dots 120 \mu\text{strain}$$

$$\epsilon_{\text{max}} \approx 150 \text{ to } 250 \mu\text{strain}$$

Check the deflection of support arm (square hollow section)

$$I = 10.4 \times 10^{-8} \text{ m}^4$$

Uniformly distributed load ω

$$\omega = 2.0 \times \frac{1}{2} \times 1.2 \times 6.0^2 \times 0.12 = 5.18 \text{ N/m}$$

$$\delta_{\text{max}} = \frac{\omega l^4}{8 E I} = \frac{5.18 \times 0.25^4}{8 \times 21 \times 10^{10} \times 10.4 \times 10^{-8}} = 1.2 \times 10^{-6} \text{ m}$$

negligible compared with

$$\delta_{\text{static}} = 3.0 \times 10^{-4} \text{ m of the model .}$$

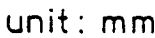


Figure A2-6 Dynamic model detail (shape II)



unit : mm

Figure A2-7 Shaft clamp detail

Table A2-2(a) Results of Wind Tunnel Experiment (Position 1)

SHAPE	SPRING	A				B			
I	$\bar{U}$	2.9	4.4	5.1	7.2	3.4	6.0	7.5	9.9
	C_{D_0}	1.78	1.80	1.83	1.79	1.78	1.75	1.75	1.75
	p	2.6	2.5	2.6	3.0	1.7	2.5	2.3	2.8
	ζ_A'/ζ_A	4.00	5.35	3.52	4.75	-1.33	3.04	0.91	0.77
	C_{D_0}'/C_{D_0}	2.06	1.13	0.99	1.06	5.58	0.83	1.28	1.08
	C_D'/C_{D_0}	2.85	1.86	1.40	1.69	3.36	1.11	1.26	1.02
II	$\bar{U}$	4.5	6.1	7.7	7.1	4.0	7.0	9.5	11.8
	C_{D_0}	1.94	1.80	1.85	1.90	1.79	1.81	1.86	1.85
	p	2.5	2.4	2.3	3.0	2.2	2.6	2.7	3.1
	ζ_A'/ζ_A	4.05	3.57	2.86	3.28	2.75	1.56	0.38	2.70
	C_{D_0}'/C_{D_0}	0.86	0.81	1.05	0.60	1.62	0.99	1.15	0.65
	C_D'/C_{D_0}	1.32	1.22	1.51	0.94	2.06	1.11	0.95	0.94
III	$\bar{U}$	3.2	4.0	5.4	5.7	4.3	6.1	7.7	9.5
	C_{D_0}	1.65	1.58	1.63	1.63	1.61	1.50	1.61	1.60
	p	2.7	2.6	2.9	3.0	2.6	2.2	2.7	2.9
	ζ_A'/ζ_A	11.5	5.90	2.62	1.45	1.30	.024	2.68	1.62
	C_{D_0}'/C_{D_0}	0.76	1.00	1.04	0.95	1.34	1.72	0.73	1.19
	C_D'/C_{D_0}	1.58	1.65	1.28	1.02	1.42	1.38	1.00	1.27
IV	$\bar{U}$	2.6	4.3	5.6	6.9	3.5	5.6	9.1	10.8
	C_{D_0}	1.51	1.73	1.72	1.76	1.73	1.75	1.92	1.94
	p	3.5	2.6	2.9	2.7	2.2	2.9	2.7	2.5
	ζ_A'/ζ_A	6.10	6.17	1.86	1.74	3.43	1.52	0.51	0.56
	C_{D_0}'/C_{D_0}	2.38	1.02	1.16	0.93	0.97	0.96	1.01	1.26
	C_D'/C_{D_0}	3.40	1.63	1.31	1.03	1.28	1.14	0.83	1.14
V	$\bar{U}$	3.6	6.5	8.5	11.1	5.0	8.5	12.5	15.2
	C_{D_0}	1.74	1.80	1.74	1.67	1.79	1.68	1.54	1.71
	p	2.6	2.3	2.6	2.9	3.5	2.4	2.4	2.4
	ζ_A'/ζ_A	6.50	0.57	1.86	1.16	3.95	2.00	0.42	0.36
	C_{D_0}'/C_{D_0}	1.56	1.47	0.96	0.93	1.48	1.05	1.57	1.40
	C_D'/C_{D_0}	2.73	1.30	1.16	0.97	2.32	1.31	1.27	1.10
VI	$\bar{U}$	1.8	3.2	4.3	5.5	2.6	4.5	6.7	7.5
	C_{D_0}	1.57	1.51	1.58	1.54	1.48	1.53	1.57	1.58
	p	2.3	3.1	3.4	2.5	2.4	2.5	2.3	2.3
	ζ_A'/ζ_A	3.72	6.90	2.50	0.85	6.70	5.80	1.20	0.90
	C_{D_0}'/C_{D_0}	5.58	1.23	0.84	0.95	1.32	0.68	1.22	1.28
	C_D'/C_{D_0}	6.21	1.61	0.94	0.94	2.13	1.17	1.28	1.24
VII	$\bar{U}$	2.6	4.7	6.8	7.9	3.2	5.4	7.9	10.4
	C_{D_0}	1.69	1.77	1.71	1.56	1.52	1.52	1.63	1.60
	p	2.6	2.6	2.5	2.7	3.0	2.9	2.6	2.4
	ζ_A'/ζ_A	14.0	8.45	1.59	2.14	4.75	2.10	1.29	-.44
	C_{D_0}'/C_{D_0}	0.81	0.89	1.01	1.08	1.09	1.03	1.05	1.34
	C_D'/C_{D_0}	1.59	1.70	1.13	1.32	1.46	1.19	1.11	0.78

Table A2-2(a) (continued)

C				D				E			
5.2	7.3	10.6	14.8	6.1	11.6	15.6	20.9	1.9	2.7	4.2	5.3
1.89	2.04	1.93	1.88	1.88	1.80	1.94	1.95	1.88	1.81	1.79	1.71
2.9	2.9	3.1	2.7	2.9	2.7	3.4	2.6	2.5	3.0	2.6	3.0
0.42	3.70	0.94	1.24	4.10	0.51	0.36	-0.45	4.55	1.14	0.78	4.91
1.19	0.63	1.15	0.81	0.95	0.97	0.75	1.30	3.45	1.05	1.36	0.92
1.08	0.63	1.14	0.88	1.24	0.89	0.66	0.85	4.10	1.60	1.32	1.32
5.6	12.0	17.0	19.6	7.0	14.5	19.9	22.5	2.1	4.0	5.9	7.8
1.89	1.98	1.93	1.93	1.93	1.89	1.95	1.92	1.89	2.11	2.02	2.10
3.3	2.8	3.0	2.3	3.3	3.1	3.5	3.3	2.9	2.4	2.5	2.7
7.65	0.39	1.32	0.40	3.20	0.65	0.58	0.34	7.70	1.26	2.70	1.17
0.63	1.14	0.82	1.04	0.89	1.38	0.82	1.03	2.97	2.46	1.28	1.08
1.18	0.93	0.89	0.84	1.09	0.89	0.75	0.68	4.72	2.58	1.53	1.12
4.3	9.1	12.6	14.8	6.0	11.5	15.2	18.4	1.6	3.1	4.7	5.7
1.65	1.73	1.74	1.73	1.70	1.69	1.83	1.71	1.57	1.90	1.92	1.87
2.9	3.2	2.8	2.8	2.6	2.8	2.6	2.7	3.9	2.4	2.9	2.6
1.25	-0.17	-0.06	0.33	0.91	-0.33	0.33	0.70	11.2	3.76	0.45	1.58
1.15	1.16	1.19	0.96	0.97	1.13	1.09	0.88	3.45	2.22	1.51	0.99
1.19	0.81	0.80	0.83	0.94	0.88	0.94	0.83	4.55	2.82	1.39	1.08
4.8	8.7	13.4	16.4	7.5	12.2	18.1	20.4	1.6	3.4	5.2	5.9
1.81	1.77	1.93	1.90	1.75	1.76	1.81	1.79	1.69	1.53	1.62	1.74
3.1	2.3	2.4	2.8	2.9	3.4	2.9	3.2	2.7	2.5	2.8	2.7
4.15	0.54	-0.30	0.13	5.80	0.35	1.12	0.29	13.3	7.10	4.95	3.20
0.66	1.00	1.42	1.20	0.65	0.86	0.97	0.98	2.91	1.29	0.84	0.74
0.89	0.91	0.82	0.85	0.91	0.78	0.70	0.82	5.61	2.32	1.42	1.10
6.4	11.9	18.1	24.0	8.6	16.8	21.6	28.0	2.4	4.8	7.1	8.6
1.81	1.86	1.86	1.80	1.60	1.70	1.76	1.80	1.43	1.48	1.65	1.66
2.5	2.7	2.7	2.6	3.4	3.1	2.8	4.3	3.0	2.7	2.6	2.8
5.10	1.44	-0.05	0.04	3.35	0.11	1.75	5.66	10.5	10.9	3.43	2.15
0.76	0.98	1.26	1.37	0.85	0.98	0.74	0.30	1.78	0.69	0.80	0.85
1.21	1.09	0.78	0.86	1.06	0.77	0.85	0.75	3.54	1.65	1.25	1.12
3.3	5.9	7.9	12.5	4.3	8.1	11.2	14.6	1.1	2.4	3.9	4.6
1.68	1.62	1.64	1.63	1.69	1.53	1.52	1.53	1.40	1.36	1.41	1.49
2.8	2.5	2.4	3.1	2.9	2.7	2.7	3.0	2.2	3.0	2.7	2.4
7.45	1.74	-0.27	0.21	3.54	0.41	0.69	4.56	14.0	15.0	7.10	3.12
0.79	0.73	1.00	1.15	0.89	0.90	0.83	0.55	4.75	1.30	0.78	0.80
1.15	0.80	0.78	0.99	1.05	0.83	0.79	0.83	7.00	2.62	1.34	1.07
4.8	8.4	13.1	17.2	6.5	11.3	16.4	18.6	2.5	3.8	5.2	6.3
1.65	1.71	1.77	1.71	1.78	1.84	1.90	1.85	1.57	1.54	1.61	1.62
2.3	2.7	3.2	2.6	3.0	2.9	2.3	2.9	2.0	2.4	2.2	2.9
1.11	0.88	0.62	0.07	-0.70	0.06	-0.31	0.78	2.90	1.79	0.70	2.85
1.14	0.91	0.91	1.33	1.07	1.07	1.23	1.10	2.42	1.14	1.19	0.70
1.15	0.89	0.82	0.94	0.98	0.88	0.80	1.04	3.10	1.32	1.25	1.00

Table A2-2(b) Results of Wind Tunnel Experiment (Position 2)

SHAPE	SPRING	A				B			
I	$\bar{U}$	2.6	4.7	6.2	9.3	3.7	6.4	9.8	12.1
	C_{D_0}	1.66	1.73	1.71	1.81	1.98	1.84	1.84	1.80
	p	2.3	2.3	2.4	2.9	2.7	3.0	2.7	3.3
	ζ_A'/ζ_A	2.27	1.65	2.73	1.64	2.72	5.85	0.45	0.30
	C_{D_0}'/C_{D_0}	1.67	1.53	1.03	0.97	1.04	0.65	1.14	0.90
	C_D'/C_D	1.97	1.73	1.40	1.14	1.26	1.08	0.98	0.75
II	$\bar{U}$	3.2	5.6	8.2	10.7	4.7	8.3	11.2	14.2
	C_{D_0}	2.09	2.19	1.98	1.94	2.00	1.89	1.98	2.01
	p	2.2	3.0	2.6	3.0	2.3	4.1	2.0	2.4
	ζ_A'/ζ_A	1.48	3.75	1.32	0.83	0.95	2.55	0.82	0.39
	C_{D_0}'/C_{D_0}	1.64	0.96	0.97	1.31	1.43	0.85	1.05	1.11
	C_D'/C_D	1.78	1.48	1.06	1.24	1.42	1.21	0.98	0.82
III	$\bar{U}$	2.4	4.2	5.8	8.4	3.4	5.9	8.8	10.2
	C_{D_0}	1.88	1.53	1.50	1.50	1.68	1.57	1.59	1.53
	p	2.7	2.6	3.2	3.2	2.5	3.2	2.7	2.8
	ζ_A'/ζ_A	9.6	12.3	3.80	1.04	1.16	1.38	2.42	1.97
	C_{D_0}'/C_{D_0}	0.74	0.84	0.72	1.06	1.30	0.93	0.82	0.68
	C_D'/C_D	1.47	2.01	1.09	1.07	1.31	0.97	0.99	0.79
IV	$\bar{U}$	2.7	5.2	7.0	9.3	3.7	6.3	8.3	12.5
	C_{D_0}	1.58	1.69	1.66	1.70	1.83	1.70	1.70	1.67
	p	2.6	3.1	2.4	3.2	2.7	2.6	2.6	2.3
	ζ_A'/ζ_A	5.30	2.80	2.40	0.69	5.45	-0.08	0.52	-0.21
	C_{D_0}'/C_{D_0}	1.06	1.20	0.93	0.95	0.76	1.08	0.86	1.52
	C_D'/C_D	1.60	1.56	1.15	0.88	1.08	0.86	0.78	0.94
V	$\bar{U}$	4.2	6.7	10.7	12.3	4.7	9.3	12.0	17.7
	C_{D_0}	1.42	1.51	1.44	1.46	1.44	1.50	1.52	1.52
	p	1.7	2.6	2.7	2.5	2.4	3.0	2.9	2.2
	ζ_A'/ζ_A	0.71	1.16	1.14	1.02	1.31	1.03	0.56	0.10
	C_{D_0}'/C_{D_0}	3.31	1.15	0.94	1.00	1.32	1.04	1.48	1.83
	C_D'/C_D	3.11	1.20	0.98	1.01	1.37	1.05	1.31	1.22
VI	$\bar{U}$	2.0	3.7	5.3	6.5	2.5	4.7	5.7	8.7
	C_{D_0}	1.27	1.32	1.35	1.37	1.56	1.56	1.52	1.52
	p	2.3	2.8	2.7	2.5	2.6	2.4	2.6	2.7
	ζ_A'/ζ_A	0.33	6.70	0.35	-0.62	7.45	2.76	1.90	1.12
	C_{D_0}'/C_{D_0}	2.43	0.92	1.07	1.12	0.83	0.88	0.87	0.90
	C_D'/C_D	2.32	1.31	1.00	0.89	1.21	1.08	0.98	0.93
VII	$\bar{U}$	2.8	4.9	6.7	8.7	3.4	6.0	9.4	12.2
	C_{D_0}	1.48	1.43	1.39	1.40	1.36	1.48	1.47	1.46
	p	2.8	2.6	3.2	2.9	2.0	3.0	2.2	2.8
	ζ_A'/ζ_A	4.80	1.76	3.32	4.16	-0.91	1.57	0.40	0.48
	C_{D_0}'/C_{D_0}	0.79	1.07	0.74	0.69	1.73	0.95	1.25	1.16
	C_D'/C_D	1.19	1.24	1.07	1.12	1.34	1.04	1.06	0.98

Table A2-2(b) (continued)

C				D				E			
5.8	10.8	12.8	19.4	7.4	13.7	20.7	24.6	2.2	3.9	5.5	7.4
1.96	1.91	1.93	1.86	1.96	1.98	1.99	1.95	1.93	1.90	1.97	1.89
2.5	3.5	3.1	2.8	3.8	3.1	3.2	2.9	2.1	3.0	2.1	2.9
2.37	0.70	0.64	0.49	3.60	0.73	-0.39	-0.46	1.60	5.65	1.61	4.65
0.53	0.89	0.89	1.00	0.64	0.67	0.90	1.08	2.30	0.88	1.40	0.57
0.67	0.81	0.79	0.80	0.77	0.65	0.63	0.73	2.56	1.68	1.65	1.06
7.1	12.7	18.1	24.0	9.9	17.4	26.6	32.2	2.9	5.0	7.1	9.3
1.82	1.92	1.86	1.87	2.16	2.12	2.15	2.09	2.25	2.03	1.98	1.94
2.7	2.6	3.2	3.0	3.0	3.3	2.8	3.0	2.2	2.1	2.5	2.7
2.36	0.12	1.47	0.38	0.08	0.21	1.16	0.31	1.71	0.96	2.65	1.03
0.77	1.14	0.79	1.05	0.99	1.06	0.79	1.05	2.65	3.05	1.09	1.11
0.96	0.81	0.94	0.79	0.79	0.81	0.85	0.79	3.05	3.00	1.39	1.11
5.5	9.5	12.8	16.8	7.5	13.2	17.4	24.5	2.2	3.7	5.3	6.7
1.67	1.57	1.56	1.55	1.61	1.70	1.67	1.69	1.87	1.80	1.70	1.68
2.4	2.9	2.7	2.7	2.6	2.9	2.8	3.9	2.3	2.3	2.6	2.9
0.81	1.97	0.02	0.08	-0.41	-0.35	0.21	0.37	4.78	1.00	1.70	1.36
0.99	0.80	1.04	1.43	1.19	1.04	0.88	0.82	2.28	2.10	1.45	1.25
0.95	0.97	0.70	0.94	1.02	0.82	0.75	0.71	3.52	2.10	1.69	1.35
6.5	11.7	13.3	21.3	8.3	14.8	20.7	26.6	2.1	4.1	5.1	8.0
1.77	1.79	1.84	1.77	1.77	1.90	1.88	1.88	1.69	1.78	1.76	1.86
2.6	2.7	3.1	2.9	3.4	2.9	2.8	3.3	2.6	2.6	2.6	2.8
1.92	0.68	0.28	0.40	2.42	0.49	0.19	0.60	8.36	5.60	3.85	1.68
0.79	1.13	1.23	0.90	0.69	0.99	1.06	1.05	1.22	0.88	0.81	0.91
0.93	1.03	0.96	0.71	0.84	0.86	0.79	0.91	2.33	1.63	1.31	1.10
8.6	15.4	18.9	30.1	11.8	21.1	28.3	38.3	3.1	5.8	8.3	10.9
1.48	1.50	1.52	1.49	1.54	1.66	1.67	1.70	1.68	1.67	1.65	1.74
2.3	3.2	2.9	3.3	2.7	2.8	3.5	2.9	2.7	2.5	3.0	2.9
0.45	0.59	0.80	0.88	0.58	0.47	0.28	0.62	2.08	6.45	0.97	0.77
0.93	0.83	0.83	0.83	0.81	1.04	1.10	0.92	1.17	0.73	1.00	1.39
0.80	0.72	0.78	0.80	0.74	0.88	0.82	0.74	1.74	1.54	0.99	1.28
4.3	7.8	10.9	14.0	5.8	10.5	15.0	20.5	1.5	2.9	4.5	5.1
1.37	1.51	1.46	1.43	1.53	1.63	1.56	1.65	1.65	1.68	1.62	1.66
2.4	2.8	3.3	2.3	2.7	2.8	2.7	2.9	2.1	2.5	2.2	2.8
1.28	3.36	0.37	-0.44	1.76	-1.0	-0.02	0.17	1.66	6.80	2.41	2.30
1.11	0.60	1.20	1.43	0.80	1.31	1.10	1.02	2.89	1.26	1.12	0.88
1.13	0.77	1.05	0.89	0.85	0.87	0.91	0.84	3.84	2.29	1.46	1.14
6.4	10.7	16.7	19.2	8.7	16.7	23.8	27.9	2.2	4.0	6.0	7.2
1.48	1.50	1.43	1.39	1.55	1.71	1.69	1.68	1.37	1.55	1.53	1.53
4.2	3.3	2.9	2.9	3.1	3.0	2.4	2.5	3.5	2.6	2.7	2.3
5.00	1.14	1.12	1.56	3.30	0.06	-0.01	0.90	9.60	5.56	4.13	2.57
0.58	0.85	0.92	1.01	0.79	1.44	1.51	1.10	1.16	0.88	0.60	0.75
0.89	0.90	0.91	1.22	1.05	1.05	0.99	1.06	2.13	1.51	1.02	1.06

APPENDIX 3 COMPUTER PROGRAMS

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A3.5 Prediction of Wind Response	A3-15

A3-1 GENERAL

Computer program listings are shown in this appendix. Programs for spectral analysis of two simultaneous data were developed and one typical version is shown as SAWS 06 together with its basic flow chart diagram in Figure A3-1. Program SAWS 06 requires two sets of 10240 data as the input and the power spectral density and the normalised cross-spectral density (real and imaginary components) are available as the output (see section 3.5).

Program CADRA 02 was developed to obtain the static drag and the dynamic along-wind force coefficients (see section 4.2). Input data consist of four sets of response results (mean and r.m.s. value in force) which were calculated from recordings at four steps of wind speed.

Program ACCAL 02 was developed to obtain the probability distribution, the autocorrelation coefficient and the damping ratio estimate from response data. Input data (2048) was created from 200 crest and trough values obtained from manual reading, by assuming a sinusoidal wave between peaks (see section 4.3.7).

Finally program WREAN 01 is shown together with a flow chart diagram for subroutine NIRCF and with an example of INPUT and OUTPUT. Subroutines NIRCF and NIPSD were similar to those employed in program CADRA 02 where the turbulence parameters

were assumed to be constant. In program WREAN 01 the height dependence of power spectral density and co-coherence function was taken into account (see section 5.3).

Approximate CPU times for these programs on Edinburgh computer (ICL 4-75 in the Edinburgh Regional Computer Centre) are as follows,

SAWS 06	122 sec
CADRA 02	23 sec
ACCAL.02	18 sec
WREAN 01	41 sec

A3.2 SPECTRAL ANALYSIS

FILE IDENTIFIER : SAWS06

```

C   SPECTRAL ANALYSIS OF WIND SPEED
C   SUBROUTINE WAS EMPLOYED FROM NAG LIBRARY FOR FFT OF COMPLEX VALUES
C           MARK 6           18TH MAY 1978
C           MARK 5           10TH APRIL 1978   JUN KANDA
C           MARK 1           DATE 1ST FEB. 1978   JUN KANDA
C           MARK 4           27TH FEB 1978   JUN KANDA
C   X: RAW DATA(10240)
C   Y: RAW DATA(10240)
C   R: SAMPLING RATE PER SECOND
C   X1,Y1: NORMALIZED DATA (1024)
C   A,B: DATA & RESULTS OF SUBROUTINE
C   C(I),D(I): REAL COMP. OF FFT OF X1,Y1
C   C(II),D(II): IMAG COMP. OF FFT OF X1,Y1
C   N=2**M: NUMBER OF DATA IN SUBROUTINE (M=10)
C   SX,SY: POWER SPECTRAL DENSITY OF X,Y
C   SXY: CROSS SPECTRAL DENSITY OF X,Y
C   CXY: CO-COMP. OF SXY
C   QXY: QUAD-COMP. OF SXY
C   RXY: ROOT COHERENCE FUNCTION OF X,Y
C   OEF: OVER-ESTIMATE FACTOR BY ALIASING EFFECT
C
C   SPECIFICATION STATEMENTS
C
C       INTEGER  I,II,M,M1,N,N1,N2,N3,N4,N5,J,J1,K/1/,IC,
1           I1,I2,I3,I4,LX(1040),LY(1040)
C       REAL*8   TITLE(7),X,Y,X1(1040),Y1(1040),A(1040),B(1040),C(1040),
1   D(1040),CW(12),
1           SX(140)/140*0.0/,SY(140)/140*0.0/,SXY(140)/140*0.0/,
1           SA(140),CXY(140)/140*0.0/,QXY(140)/140*0.0/,XI,YI,R,R1,
1           XMEAN/0.0/,XSD/0.0/,YMEAN/0.0/,YSD/0.0/,OEF
C       LOGICAL TRAN
C       M=10
C       M1=M+1
C       N=2**10
C       N1=1040
C       N2=2**(M-1)
C       N3=N+1
C       N4=N+2
C       N5=2**(M-3)
C       N6=N5-1
C
C   INPUT STATEMENTS
C
C       READ (5,999) TITLE
C       READ (5,992) R
10  IF (K.EQ.11) GO TO 80
C       WRITE (6,997) K
C       X1(N3)=0.0
C       Y1(N3)=0.0
C       X1(N4)=0.0
C       Y1(N4)=0.0
C       K=K+1
C       READ (5,996) (LX(I),LY(I),I=1,1024)

```

```

DO 20 I=1,N
  X1(I) = LX(I)
  Y1(I) = LY(I)
  X1(N3)=X1(I)+X1(N3)
  Y1(N3)=Y1(I)+Y1(N3)
20 CONTINUE

```

```

C
C  CALCUTATE MEAN & STANDARD DEVIATION
C

```

```

  X1(N3) = X1(N3) /1024.
  Y1(N3) = Y1(N3) /1024.
  DO 30 I=1,N
    X1(N4) = (X1(I)-X1(N3))*2 + X1(N4)
    Y1(N4) = (Y1(I) - Y1(N3))*2 + Y1(N4)
30 CONTINUE
  XMEAN=XMEAN+X1(N3)/10.
  X1(N4)=DSQRT(X1(N4)/1023.)
  XSD =XSD +X1(N4)/10.
  YMEAN=YMEAN+Y1(N3)/10.
  Y1(N4)=DSQRT(Y1(N4)/1023.)
  YSD =YSD +Y1(N4)/10.

```

```

C
C  COSINE TAPER FILTER & NORMALIZATION
C

```

```

      PAI/102.3 = 0.03071
DO 40 I=1,N
  IF (I.LT.104) GO TO 41
  IF (I.GT.1121) GO TO 42
  A(I)=(X1(I)-X1(N3))/X1(N4)
  B(I)=(Y1(I)-Y1(N3))/Y1(N4)
  GO TO 40
41  XI=I
  A(I)=(X1(I)-X1(N3))/X1(N4) * 0.5 *(1.-DCOS((XI-1.)*0.03071))
  B(I)=(Y1(I)-Y1(N3))/Y1(N4) * 0.5 *(1.-DCOS((XI-1.)*0.03071))
  GO TO 40
42  XI=I
  A(I)=(X1(I)-X1(N3))/X1(N4)*0.5*(1.-DCOS((512.-XI) *0.03071))
  B(I)=(Y1(I)-Y1(N3))/Y1(N4)*0.5*(1.-DCOS((512.-XI) *0.03071))
40 CONTINUE

```

```

C
C  FAST FOURIER TRANSFORM
C

```

```

  TRAN = .TRUE.
  CALL C06ABF(A,B,N1,TRAN,M1,CW)
  A(N3) = A(1)
  B(N3) = B(1)
  DO 50 I=1,N2
    J=N-I+2
    II=N2+I
    C(I) = (A(I) + A(J)) * 0.5
    D(II) = (A(J) - A(I)) * 0.5
    C(II)= (B(I) - B(J)) * 0.5
    D(I)= (B(I) + B(J)) * 0.5
50 CONTINUE

```

```

C
C  CALCULATION OF POWER & CROSS SPECTRA
C  ADJUSTMENT FACTOR DUE TO COSINE FILTER IS 1/0.875
C

```

```

  R1=1./0.875*2/R
DO 51 I=1,N2
  II= N2+I
  A(I) = ((C(I))**2 + (C(II))**2) * R1
  B(I) = ((D(I))**2 + (D(II))**2) * R1
  A(II)= (C(I)*D(I) + C(II)*D(II)) * R1

```

```

      B(II) = (C(I)*D(II) - C(II)*D(I)) * R1
51 CONTINUE

```

```

C
C   FREQUENCY SMOOTHING & SEGMENT AVERAGING
C

```

```

      DO 60 I=1,N5
        I1=4*I-3
        I2=4*I-2
        I3=4*I-1
        I4=4*I
        SX(I) = (A(I1)+A(I2)+A(I3)+A(I4))/40. +SX(I)
        SY(I) = (B(I1)+B(I2)+B(I3)+B(I4))/40. +SY(I)
        I1=I1+N2
        I2=I2+N2
        I3=I3+N2
        I4=I4+N2
        CXY(I) = (A(I1)+A(I2)+A(I3)+A(I4))/40. +CXY(I)
        QXY(I) = (B(I1)+B(I2)+B(I3)+B(I4))/40. +QXY(I)
60 CONTINUE
      GO TO 10

```

```

C
C   FINAL CURVE SMOOTHING
C

```

```

80 WRITE (6,998) (TITLE(I),I=1,6)
      DO 71 I=2,N6
        A(I) = 0.5*SX(I) + 0.25*( SX(I+1)+SX(I-1) )
        B(I) = 0.5*SY(I) + 0.25*( SY(I+1)+SY(I-1) )
        C(I) = 0.5*CXY(I) + 0.25*( CXY(I+1)+CXY(I-1) )
        D(I) = 0.5*QXY(I) + 0.25*( QXY(I+1)+QXY(I-1) )
71 CONTINUE
      DO 72 I=2,N6
        SX(I) = A(I)
        SY(I) = B(I)
        CXY(I) = C(I)
        QXY(I) = D(I)
72 CONTINUE
      DO 70 I=1,N5
        IF (I.EQ.1) WRITE (6,995) XMEAN ,YMEAN,XSD,YSD
        SA(I) = DSQRT(SX(I)*SY(I))
        CXY(I) = CXY(I)/SA(I)
        QXY(I) = QXY(I)/SA(I)
        SX(130) = SX(130) + SX(I)
        SY(130) = SY(130) + SY(I)
      IF (I.GE.83) GO TO 70

```

```

C
C   ALIASING EFFECT CORRECTION
C

```

```

      XI=I
      OEF = 0.0000384*(XI/2.)**2.54 + 1.00
      SX(I) = SX(I)/OEF
      SY(I) = SY(I)/OEF
      XI = (4.*XI-2.5)*R/1024.
      A(I) = SX(I) *XI
      B(I) = SY(I) *XI
      WRITE (6,994) XI,SX(I),SY(I),A(I),B(I),CXY(I),QXY(I)
70 CONTINUE
      SX(130) = SX(130)*R*4./1024.
      SY(130) = SY(130)*R*4./1024.
      WRITE (6,993) SX(130),SY(130)
      STOP

```

```

C
C   FORMAT STATEMENTS
C

```

```

999 FORMAT (6A4,1A3)

```



```

998 FORMAT (4(1X/ ),15X,37H***** SPECTRAL ANALYSIS RESULTS ***** /
1 40X,5A4,1A3)
997 FORMAT (1H , I3)
996 FORMAT ( 10I5)
995 FORMAT (40X,7HXMEAN = ,1F5.1,2X,7HYMEAN = ,1F5.1 /
1 40X,7HXS.D. = ,1F5.1,2X,7HYS.D. = ,1F5.1 //
1 12X,14HPOWER SPECTRUM,23X,14HCROSS SPECTRUM /1X,
1 5HFREQ.,3X,1HX,10X,1HY,
1 5X,8H FREQ* SX,4X,7HFREQ*SY,4X,5HCO- & ,1X,14HQUAD-COHERENCE /)
994 FORMAT (1H ,10(1X,F4.1,1X,6(F7.4,3X))/1X/)
993 FORMAT (1X/4X,3HSUM, F7.4,3X,F7.4 //1X,15HEND OF ANALYSIS )
992 FORMAT ( 1F7.1)
END

```

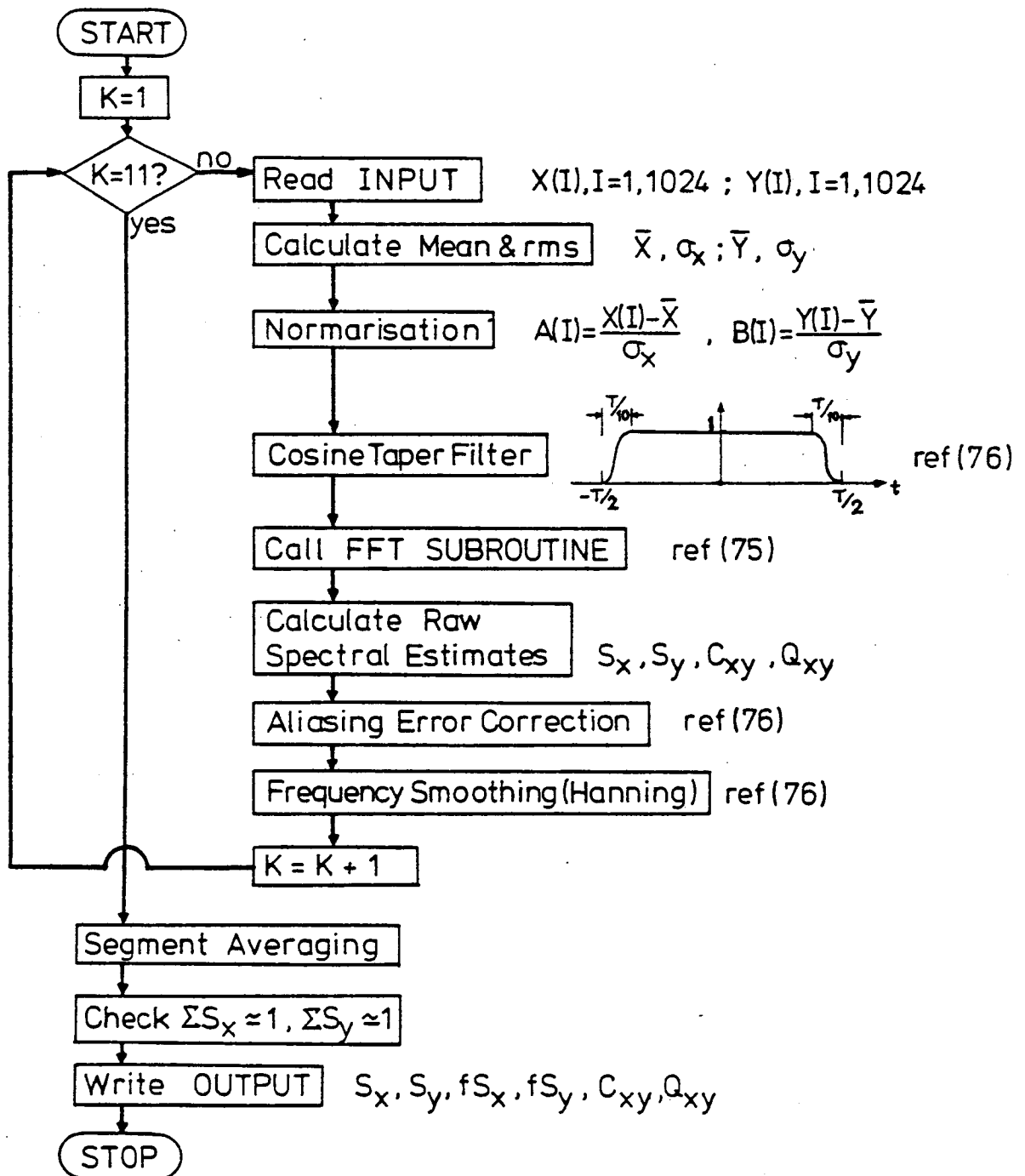


Figure A3-1 Basic flow chart for program SAWS 06

A3.3 DYNAMIC ALONG-WIND FORCE COEFFICIENT

FILE IDENTIFIER : CADRA02

```

C   CALCULATION OF DYNAMIC DRAG-COEFFICIENT
C       MK2                               8TH AUG 1978 JUN
C       MK1                               17TH FEB 1978   JUN KANDA
C   N   : DATA IDENTIFIER
C   PR   : REFERENCE WIND PRESSURE (N/M**2) AT Z=60CM
C   A   : LENGTH OF MODEL (=40MM)
C   B   : WIDTH OF MODEL  (=60,80,120MM)
C   D   : DEPTH OF MODEL  (=60,80,120MM)
C   PF   : PROFILE FACTOR  PF=(Z/60)**ALFA
C   TURB : TURBULENCE INTENSITY  TURB=DUZ/UZ (%)
C   UZ   : WIND SPEED AT Z (M/S)
C   DUZ  : STANDARD DEVIATION OF FLUCTUATING COMPONENT OF UZ
C   UZN  : NONDIMENSIONAL WIND SPEED (=UZ/(D*FO) )
C   SU(F): POWER SPECTRAL DENSITY
C           F*SU(F)/DUZ**2 = K1*X/(1+X**BETA)**5/(3*BETA)
C           WHERE X= F*SLE/UZ, K1=CONS.
C   RU(F): ROOT COHERENCE FUNCTION
C           RU(F)= EXP(-SQRT((DKH*Y)**2+(DKV*Z)**2) *FM )
C   TEMP : AIR TEMPERATURE IN WIND TUNNEL
C   PRES : ATMOSPHERIC PRESSURE IN WIND TUNNEL
C   FO   : NATURAL FREQUENCY OF MODEL (HZ)
C   F    : FREQUENCY (HZ)
C   FM   : MODIFIED FREQUENCY FOR RU(F)
C   ZETA : DAMPING RATIO (=SZETA+AZETA) (%)
C   DERA : DENSITY RATIO (STRD/AIRD)
C   STRD : DENSITY OF STRUCTURE (G/CM**3)
C   AIRD : DENSITY OF AIR (KG/M**3)
C
C   DF : MEAN DRAG FORCE (0.01N)
C   DDF : DYNAMIC DRAG FORCE (STANDARD DEVIATION) (0.01N)
C
C       INTEGER N,SHAP,IB,ID , I/O/,J,K,L,M
C       READ (5,998) N1,N2
998  FORMAT (2I5)
C       READ (5,999) PR,ZETA,DF,DDF,SHAP,PF,TURB,SLE,DKH,DKV,TEMP,PRES,
1    F0,STRD
999  FORMAT (F5.1,3F5.2,15,3F5.2,4F5.1,2F5.2)
C
C   PRELIMINARY CALCULATION
C
C       AIRD= 1.2059*(PRES/760.)*((TEMP+273.15)/293.15)
C       DERA= STRD/(AIRD*0.001)
100  I=I+1
C       TURB= TURB*0.01
C       PZ= PR*(PF)**2
C       A=0.04
C       B=0.06
C       D=0.06
C       IF (SHAP.EQ.2.OR.SHAP.EQ.4) B= 0.08
C       IF (SHAP.EQ.3.OR.SHAP.EQ.4) D= 0.08
C       IF (SHAP.EQ.5.OR.SHAP.EQ.7) B= 0.12
C       IF (SHAP.EQ.6.OR.SHAP.EQ.7) D= 0.12

```

```

DF= DF*0.0098
DDF= DDF*0.0098
DCF = DF/(PZ*A*B)
UZ= SQRT(2.*PZ/AIRD)
UZN= UZ/(D*FO)
12.5664= 4*PAI
AZETA= DCF*UZN/(12.5664*DERA)
ZETA= AZETA+ZETA*0.01
XMAX = 1.6*FO*SLE/UZ
FM = SQRT ((UZ/SLE)**2*0.1 + FO**2)
AF = A*FM*DKH/UZ
BF = B*FM*DKV/UZ
HDF= A*DKH
VDF= B*DKV

```

NUMERICAL INTEGRAL OF AERODYNAMIC ADMITTANCE AT FO

```

X1=FO*SLE/UZ
BETA=2.
CONS=0.47548
Q=-5./(3.*BETA)
FSU=CONS*X1*(1.+X1**BETA)**Q
CALL NIRCF (AF,BF,ADAD)
AEF= SQRT (ADAD*FSU*(0.78539-1.75*ZETA)/ZETA) * TURB

```

NUMERICAL INTEGRAL OF POWER SPECTRAL DENSITY TAKING AERODYNAMIC ADMITTANCE INTO ACCOUNT WITH RESPECT TO F

```

CALL NIPSD (XMAX,SLE,HDF,VDF,BGEF,BETA,CONS)
BEF= SQRT(BGEF) *TURB

```

CALCULATION OF DYNAMIC DRAG COEFFICIENT

```

DCF : DRAG COEFF.
DDCF: DYNAMIC DRAG COEFF.
RDD : DDCF/DCF
RDD = SQRT((DDF)**2. -(2.*DF*BEF)**2.) / (2.*DF*AEF)
DDCF= RDD*DCF

```

OUTPUT STATEMENTS

```

B=B*100.1
D=D*100.1
ZETA=ZETA*100.1
IB=B
ID=D
AZETA=AZETA*100.1
TURB=TURB*100.0
IF (I.NE.1) GO TO 101
WRITE(6,991) N1,N2
991 FORMAT (1H //12X,27H***** CADRA RESULTS FROM RD ,I3,3H = ,I3,
1 2X,5H***** )
101 WRITE(6,992) IB,ID,FO,ZETA
992 FORMAT (1H /2X,21HMODEL CHARACTERISTICS /2X,2HB= ,I2,6HCM: D=,
1 12,8HCM FO= ,F5.2,10HHZ ZETA=,F4.2,8HX(TOTAL) )
WRITE (6,993) UZ,UZN,AZETA,TURB
993 FORMAT(1H /2X,20HWIND CHARACTERISTICS /2X,3HUZ=,F4.1, 3HM/S,
1 7X,4HUZ*=,F4.1,4X,6HAZETA=,F4.2,1HX,8X.5HT.I.=,F5.1,1HX )
WRITE (6,994) DCF,AEF,DDCF,RDD,BEF
994 FORMAT (1H /2X,7HRESULTS /2X,5HDCF = ,F5.2,28X,10HRE.EX.FA = ,
1 F6.4/2X,5HDDCF= , F5.2,3H ( ,F4.2,7H * DCF) 14X,
1 10HBG.EX.FA.= , F6.4//15X,5H***** ,1X/ )

```

```

C   SEQUENTIAL ANALYSIS OR END OF ANALYSIS
C
C   IF (I.EQ.4) GO TO 200
C   READ (5,997) PR,ZETA,DF,DDF
997  FORMAT (F5.1,3F5.2 )
C   GO TO 100
200  WRITE (6,995)
995  FORMAT (1H ,1X, 15HEND OF RESULTS )
C   STOP
C   END

C   SUBROUTINE NIPSD (XMAX,SCLE,HDF,VDF,BGEF,BETA,CONS)
C   MODIFIED AS A SUBROUTINE OF CADRA 13TH FEB 1978
C   NUMERICAL INTEGRAL OF POWER SPECTRAL DENSITY FUNCTION
C   TO OBTAIN BACKGROUND EXCITATION FACTOR
C    $X = (\text{FREQUENCY}) * (\text{SCALE LENGTH}) / (\text{WIND SPEED})$ 
C    $S(X) = \text{POWER SPECTRAL DENSITY FUNCTION}$ 
C   BETA = POWER INDEX FOR  $S(X)$ 
C   K1 = CONSTANT FOR NORMALIZING PURPOSE (GAMMA FUNCTION OF BETA)
C
C   DIMENSION X(50),S(50),ADAD(50)
C
C   STEP = 2.0
C   XM = XMAX*0.4
C   XMIN = XMAX*0.01
C   SET VALUES OF X
C   X(1) = 0.000001
C   X(2) = XMIN*0.5
C   X(3) = XMIN
C   N = 3
110  M = N+2
C   X(M) = X(N)*STEP
C   N = M
C   IF (X(N).LT.XM) GO TO 110
C   MM = N+2
C   X(MM) = XMAX
C   MN = MM-1
C   DO 120 N = 4, MN, 2
C   NL = N-1
C   NR = N+1
120  X(N) = (X(NL)+X(NR))*0.5
C
C   SET VALUES OF POWER SPECTRAL DENSITY & AERODYNAMIC ADMITTANCE
C   P = BETA
C   Q = -5./(3.*BETA)
C   DO 130 N = 1, MM
C   AF = SQRT(0.25+ (X(N))**2 ) * HDF/SCLE
C   BF = SQRT(0.25+ (X(N))**2 ) * VDF/SCLE
C   CALL NIRCF(AF,BF,ADAD(N))
130  S(N) = (1.+X(N)**P)**Q*ADAD(N)
C
C   NUMERICAL INTEGRAL
C   AREA = 0.
C   DO 140 N = 2, MN, 2
C   NL = N-1
C   NR = N+1
140  AREA = (S(NL)+S(NR)+4.*S(N))*(X(NR)-X(NL))+AREA
C   BGEF = AREA*CONS/6.
C
C   RETURN
C   END

```

```

C      SUBROUTINE NIRCF(AF,BF,ADAD)
C      MODIFIED AS A SUBROUTINE OF CADRA      13TH FEB 1978
C      IMPROVED      19 OCT 77
C      NUMERICAL DOUBLE INTEGRAL OF ROOT-COHERENCE FUNCTION
C
C      A=(WIDTH)*(FREQUENCY)*(HORIZONTAL DECAY FACTOR)/(WIND SPEED)
C      B=(HEIGHT)*(FREQUENCY)*(VERTICAL DECAY FACTOR)/(WIND SPEED)
C      P=POWER INDEX OF PROFILE ( MODE & WIND SPEED )
C      P=0.0
C      NUMBER OF DIVISION ACCORDING TO AF OR BF
C      IF( AF.LE.0.1) GO TO 171
C      XKI=20.
C      GO TO 172
171  XKI=8.
172  A=(AF)**2
170  IF (BF.LE.0.1) GO TO 173
C      XKJ=20.
C      GO TO 174
173  XKJ=8.
174  B=(BF)**2
C      NQ=0
C      XJ=XKJ
C      H=(0.001/XKJ)**P
103  SUMB=0.
110  XJ=XKI
C      G=1.
C      SUMA=0.
C      SUMMATION WITH RESPECT TO I(ODD & EVEN) FOR ANY J (START WITH K)
C      BTERM=B*(XJ/XKJ)**2
111  SUMA=SUMA+4.*G*H*EXP(-SQRT(A*(XI/XKI)**2+BTERM))
C      XI=XI-1.
C      G=8.+8.*(XKI-XI-1.)
C      SUMA=SUMA+4.*G*H*EXP(-SQRT(A*(XI/XKI)**2+BTERM))
C      XI=XI-1.
C      IF(XI=0.5) 112,5,5
C      G=10.*(XKI-XI)
C      GO TO 111
112  G=10.*XKI-2.
C      XI=0.0105*XKI
C      IF J=0 GO TO 150 (FINAL ROUND)
C      IF (XJ=0.5) 150,6,6
C      6 SUMA=SUMA+2.*G*H*EXP(-SQRT(A*(XI/XKI)**2+BTERM))
C      IF (XJ=XKJ) 7,120,120
C      7 XJ=XJ-1.
C      IF J=EVEN GO TO 9, IF ODD GO TO 8, IF 0 GOTO 140
C      IF(NQ=0) 8,130,8
C      8 CALL SUBA(XJ,XKJ,P,SA)
C      H=4.*((0.001*XJ/(XKJ*XKJ))**P+((XKJ-XJ)/XKJ)**P)+SA
C      NQ=NQ+1
C      GO TO 113
120  XJ=XKJ-1.
C      H=4.*((0.001*XJ/(XKJ*XKJ))**P+(1./XKJ)**P)
C      GO TO 113
130  CALL SUBB(XJ,XKJ,P,SB)
C      IF(XJ=0.5) 140,9,9
C      9 H=2.*((0.001*XJ/(XKJ*XKJ))**P+((XKJ-XJ)/XKJ)**P)+SB
C      NQ=NQ+1
C      GO TO 113
140  H=(0.001/XKJ)**(2.*P)+1.+SB
C      XJ=0.0105*XKJ
C      SUMMATION WITH RESPECT TO J (AFTER ONE ROUND OF I)
113  SUMB=SUMB+SUMA/(9.*XKI*XKI)

```

```

C   GO BACK TO SUMMATION WITH RESPECT TO I FOR J=1
      GO TO 110
150  SUMA=SUMA/2.+G*H*EXP(-SQRT(A*(XI/XKI)**2+BTERM))

      SUMC=(SUMB+SUMA/(9.*XKI*XKI))*(1.+P)**2/(9.*XKJ*XKJ)
      ADAD=SUMC
198  CONTINUE
C   OUTPUT STATEMENTS
163  FORMAT(3X,4HB = ,3X,F7.3/1X,4HA = )
164  FORMAT(1X,F8.3,2X,F5.4)
199  CONTINUE
      RETURN
      END

C
C   SUBROUTINE STATEMENTS
C   PART OF 'H' FOR ODD J (NOT K=1)
      SUBROUTINE SUBA(XJ,XK,P,SA)
      XN=1.
      SA=0.
203  SA=SA+8.*((XJ+XN)*XN/(XK*XK))**P

      IF(XN=XK+XJ+1.5) 201,202,202
201  XN=XN+1.
      GO TO 203
202  RETURN
      END

C
C   PART OF 'H' FOR EVEN J (NOT K)
      SUBROUTINE SUBB(XJ,XK,P,SB)
      XN=1.
      SB=0.
      XXJ=(XK-XJ)/2.
213  IF(XN-XXJ+0.5) 211,212,212
211  SB=SB+16.*((XJ+2.*XN-1.)*(2.*XN-1.)/(XK*XK))**P
      +4.*((XJ+2.*XN)*(2.*XN)/(XK*XK))**P
      XN=XN+1.
      GO TO 213
212  SB=SB+16.*((XJ+2.*XN-1.)*(2.*XN-1.)/(XK*XK))**P
      RETURN
      END

```

A3.4 AUTO-CORRELATION AND FREQUENCY DISTRIBUTION

FILE IDENTIFIER : ACCAL02

```

C      FREQUENCY DISTRIBUTION AND
C      COMPUTATION OF AUTO-CORREATION VIA FFT
C      SUBROUTINE WAS EMPLOYED FROM NAG LIBRARY FOR CIRCULAR CONVOLUTION
C      MARK 2      15 JUNE 1978      JUN KANDA
C      X:  RAW DATA (2048)
C      A,B :  NORMALIZED DATA (2048 + 2048ZEROS )
C      N=2**M  NUMBER OF DATA IN SUBROUTINE (M=12)
C
C      SPECIFICATION STATEMENTS
C
C      INTEGER I,J(10)/10*0/,K(10)/10*0/,L,M,N,N1,N2,N3,N4,
1      LX(2048)
C      REAL*8 TITLE(7),X,X1(2100)/2100*0.0/,A(4100)/4100*0.0/,B(4100)
1      /4100*0.0/,C(4100),
1      D(2500),SCALE,YI,YJ,YK
C      LOGICAL TRAN
C      M=12
C      N=2**M
C      N1=4100
C      N2=2** (M-1)
C      N3=N2+1
C      N4=N2+2
C
C      INPUT STATEMENTS
C
C      READ(5,999) TITLE
C      WRITE(6,998) (TITLE(I),I=1,7)
C      READ(5,996) LX
C
C      MEAN & STANDARD DEVIATION
C
C      DO 10 I=1,N2
C          X1(I)=LX(I)
C          X1(N3)=X1(I)+X1(N3)
10  CONTINUE
C          X1(N3)=X1(N3)/2048.
C      DO 20 I=1,N2
C          X1(N4)=(X1(I)-X1(N3))*2 +X1(N4)
20  CONTINUE
C          X1(N4)=DSQRT(X1(N4)/2047.)
C
C      NORMALIZATION
C
C      DO 30 I=1,N2
C          A(I)=(X1(I)-X1(N3))/X1(N4)
C          B(I)=A(I)
30  CONTINUE
C
C      FREQUENCY DISTRIBUTION
CC
C      DO 100 I=1,N2
C          X=A(I)

```

```

      IF(X.LE.-4.5) J(1)=J(1)+1
      IF(X.LE.-4.0.AND.X.GT.-4.5) J(2)=J(2)+1
      IF(X.LE.-3.5.AND.X.GT.-4.0) J(3)=J(3)+1
      IF(X.LE.-3.0.AND.X.GT.-3.5) J(4)=J(4)+1
      IF(X.LE.-2.5.AND.X.GT.-3.0) J(5)=J(5)+1
      IF(X.LE.-2.0.AND.X.GT.-2.5) J(6)=J(6)+1
      IF(X.LE.-1.5.AND.X.GT.-2.0) J(7)=J(7)+1
      IF(X.LE.-1.0.AND.X.GT.-1.5) J(8)=J(8)+1
      IF(X.LE.-0.5.AND.X.GT.-1.0) J(9)=J(9)+1
      IF(X.LE.0.0.AND.X.GT.-0.5) J(10)=J(10)+1
      IF(X.LE.0.5.AND.X.GT.0.0) K(1)=K(1)+1
      IF(X.LE.1.0.AND.X.GT.0.5) K(2)=K(2)+1
      IF(X.LE.1.5.AND.X.GT.1.0) K(3)=K(3)+1
      IF(X.LE.2.0.AND.X.GT.1.5) K(4)=K(4)+1
      IF(X.LE.2.5.AND.X.GT.2.0) K(5)=K(5)+1
      IF(X.LE.3.0.AND.X.GT.2.5) K(6)=K(6)+1
      IF(X.LE.3.5.AND.X.GT.3.0) K(7)=K(7)+1
      IF(X.LE.4.0.AND.X.GT.3.5) K(8)=K(8)+1
      IF(X.LE.4.5.AND.X.GT.4.0) K(9)=K(9)+1
      IF(X.GT.4.5) K(10)=K(10)+1
100  CONTINUE
      WRITE(6,894) J,K
      WRITE(6,893)
894  FORMAT(4I3,12I4,4I3 /)
893  FORMAT (5X,3H-4S,3X,3H-3S,5X,3H-2S,5X,3H-1S,5X,4HMEAN,
1    5X,2H1S,6X,2H2S,6X,2H3S,4X,2H4S )

```

C
C
C

CIRCULAR CONVOLUTION VIA FFT

```

      TRAN=.TRUE.
      SCALE = 1.0
      CALL C06ACF(A,B,C,D,N1,M,N4,SCALE,TRAN)
      DO 40 I=1,100
        YI=I
        C(I) =C(I)/(2048.-YI)
40    CONTINUE
        WRITE(6,995) X1(N3),X1(N4)
        D(1)=0.0
        WRITE(6,994) (C(I),I=1,100,22)
        DO 50 I=2,5
          N=1+22*(I-1)
          PI=3.14159265358979
          YJ=I
          YJ=YJ-1.0
          D(I)=DLOG(C(1)/C(N))/(2.*PI*YJ)
50    CONTINUE
        WRITE(6,993) ( D(I),I=1,5)
        STOP
999  FORMAT (6A4,1A3)
998  FORMAT (1X/ ,10X,48H***** FREQUENCY DISTRIBUTION & AUTO-CORRELATIO.
1N ,5H***** / 35X,5HFROM , 6A4,1A3 //)
996  FORMAT (8I5)
994  FORMAT (5X,30HAUTO-CORRELATION PEAK VALUES /1H ,5F10.5)
993  FORMAT (5X,15HDAMPING RATIO /1H ,5F10.5)
995  FORMAT (1H ,5X,12HWHERE MEAN= ,10X,6HS.D.= /10X,2(F10.2,5X)/)
      END

```


A3.5 PREDICTION OF WIND RESPONSE

FILE IDENTIFIER : WREAN01

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C
C   PREDICTION OF DYNAMIC ALONG-WIND RESPONSE OF TALL BLUFF STRUCTURES
C
C           MARK 1           10 OCT  1978   JUN KANDA
C
C   ** NOTATION **
C   GENERAL
C       X   : ALONG-WIND CO-ORDINATE
C       Y   : LATERAL CO-ORDINATE
C       Z   : VERTICAL CO-ORDINATE
C       ZREF: REFERENCE HEIGHT( E.G. 10M OR GRADIENT HEIGHT )
C       F   : FREQUENCY (HZ)
C   FOR A STRUCTURE
C       H   : HEIGHT (M)
C       B   : BREADTH (M)
C       D   : DEPTH (M)
C       FO  : NATURAL FREQUENCY (HZ)
C       QM  : POWER EXPONENT OF MODAL SHAPE
C       ZETAS: DAMPING RATIO ( % OF CRITICAL )
C       DENS: AVERAGE DENSITY OF STRUCTURE (KG/M**3)
C
C   FOR WIND CHARACTERISTICS
C       UH  : T-MINUTE MEAN WIND SPEED AT Z=H (M/S)
C       T   : DURATION OF PROCESS ( AVERAGING PERIOD ) (MIN)
C       UR  : MEAN WIND SPEED AT Z=ZREF
C       QU  : POWER EXPONENT OF MEAN WIND PROFILE
C       TIH : TURBULENCE INTENSITY AT Z=H
C       TIR : TURBULENCE INTENSITY AT Z=ZREF
C       QT  : POWER EXPONENT OF TURBULENCE PROFILE
C       S(F): POWER SPECTRAL DENSITY OF TURBULENCE
C       CONS: CONSTANT FOR NORMALISING S(F)
C       BETA: POWER INDEX TO DEFINE THE FORM OF S(F)
C       X1  : F*SCALE/UH
C       SCALEH: LENGTH CONSTANT OF S(F) AT Z=H (M)
C       SCALER: LENGTH CONSTANT OF S(F) AT Z=ZREF (M)
C       QL  : POWER EXPONENT OF SCALE LENGTH
C       R(F): ROOT(OR CO)-COHERENCE FUNCTION OF TURBULENCE
C       DKHH: HORIZONTAL DECAY FACTOR OF R(F) AT Z=H
C       DKVH: VERTICAL DECAY FACTOR OF R(F) AT Z=H
C       DKHR: HORIZONTAL DECAY FACTOR AT Z=ZREF
C       DKVR: VERTICAL DECAY FACTOR AT Z=ZREF
C       QD  : POWER EXPONENT OF DECAY FACTOR PROFILE
C       FM  : SQRT(F**2+(UH/3.3SCALE)**2)
C
C   MISCELLANEOUS
C       UHN : UH/(FO*D)
C       XLB : B*DKHH/SCALEH
C       XLH : H*DKVH/SCALEH
C       XFB : B*FO*DKHH/UH
C       XFH : H*FO*DKVH/UH
C
C       SCD : STATIC OR MEAN DRAG COEFFICIENT
C       DCDQ: QUASI-STATIC DRAG COEFFICIENT ( RATIO TO SCD )
C       DCDR: DYNAMIC ALONG-WIND FORCE COEF. ( RATIO TO SCD )
C       ZETAA: AERODYNAMIC DAMPING RATIO
C       ZETAT: ZETAS+ZETAA
C
C   REAL*8   TITLE(8),C(4),QQ(3)

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      DIMENSION ZETAT(5),PF(5),PFA(5),XRMS(5),ARMS(5),XMAX(5),AMAX(5),
1      GF(5)
C
C  ** INPUT STATEMENT **
C
      READ(5,500) TITLE
      WRITE(6,600) (TITLE(I),I=1,8)
      READ(5,501) H,B,D,F0,DENS,QM
      READ(5,502) UR,QU,TIR,QT,SCLER,QL,BETA,DKHR,DKVR,QD,ZREF
      READ(5,503) SCD,DCDR,DCDR
500  FORMAT (8A4)
501  FORMAT (4F6.2,F6.0,F6.2)
502  FORMAT (F6.2,3F6.3,F6.0,6F6.2)
503  FORMAT (3F6.2)
C
C  ** PRELIMINARY CALCULATION **
C
      T=60.
      TIME=T*60.
      UH=UR*(H/ZREF)**QU
      TIH=TIR*(H/ZREF)**(-(QT+QU))
      TI =TIH*100.
      SCLEH=SCLER*(H/ZREF)**(QL+QU)
      IFAIL=0
      QQ1=5./(3.*BETA)
      QQ2=1./BETA
      QQ3=2./(3.*BETA)
      QQ(1)=QQ1
      QQ(2)=QQ2
      QQ(3)=QQ3
      C(1)=S14AAF(QQ(1),IFAIL)
      C(2)=S14AAF(QQ(2),IFAIL)
      C(3)=S14AAF(QQ(3),IFAIL)
C      WHERE S14AAF(X,IFAIL) IS A GAMMA FUNCTION ESTIMATE OF X
      C(4)=C(1)/(C(2)*C(3))
      CONS=C(4)
      CONS=CONS*BETA
      Q=-QQ1
      DKHH=DKHR*(H/ZREF)**(-QD+QU)
      DKVH=DKVR*(H/ZREF)**(-QD+QU)
      DERA=DENS/1.2059
C      WHERE 1.2059KG/M**3 IS AIR MASS DENSITY AT 20DEG.C & 760MMHG
      UHN=UH/(F0*D)
      PAI=3.14159265
      ZETAA=DCDR*SCD*UHN*(1.+2.*QM)/((1.+QU+2.*QM)*4.*PAI*DERA)
      AZETA=ZETAA*100.
      X1=F0*SCLEH/UH
      XFB=B*F0*DKHH/UH
      XFH=H*F0*DKVH/UH
      XLB=B*DKHH/SCLEH
      XLH=H*DKVH/SCLEH
C
C  ** RESPONSE CALCULATION **
C
      GUS=CONS*X1*(1.+X1**BETA)**Q
      ROF=2.*(1.+2.*QU+QM)*TIH/(1.+QU-QT+QM)
C
C      NUMERICAL INTEGRAL FOR SIZE REDUCTION FACTOR SRF AT F0
C
      QP=QU+QM-QT
      CALL NIRCF(XFB,XFH,QL,QD,QP,X1,BETA,ADAD)
      SRF=ADAD
      SRC=ADAD*DCDR**2
C
C      NUMERICAL INTEGRAL FOR BACKGROUND EXITATION FACTOR

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C      CALL NIPSD(XLB,XLH,QL,QD,QP,X1,BETA,BGEF,BEN2,BEN4)
      BGE=BGEF*CONS
      BEC=BGE*DCDQ**2
      BF2=BEN2*CONS*(DCDQ*UH/SCLEH)**2
      BF4=BEN4*CONS*(DCDQ*(2.*PAI*UH/SCLEH)**2)**2

C      MEAN DEFLECTION
C
C      EQM=H*B*D*DENS/(1.+2.*QM)
      EOK=(2.*PAI*F0)**2*EQM
      XMEAN=(SCD*1.2059*B*H*UH**2)/(2.*EOK*(1.+2.*QU+QM))

C      VARIATION OF DAMPING RATIO
C
      ZETAT(1)=0.01
      ZETAT(2)=0.01+ZETAA
      ZETAT(3)=0.02
      ZETAT(4)=0.02+ZETAA
      DO 10 I=1,4
        ZET=ZETAT(I)
        RES=SRC*GUS*(PAI/(4.*ZET)-1.75)
        RE2=RES*F0**2
        RE4=RES*(2.*PAI*F0)**4

C      PEAK FACTOR
C
      EQF=SQRT((BE2+RE2)/(BEC+RES))
      ENT=EQF*TIME
      PNT=SQRT(2.*ALOG(ENT))
      PEAKX=PNT+0.577/PNT
      PNTA=SQRT(2.*ALOG(F0*TIME))
      PEAKA=PNTA+0.577/PNTA
      PF(I)=PEAKX
      PFA(I)=PEAKA

C      RESULTS
C
      XSDX=ROF*SQRT(BEC+RES)
      XRMS(I)=XSDX*XMEAN
      ARMS(I)=ROF*XMEAN*SQRT(BE4+RE4)/9.8
      XMAX(I)=XMEAN+XRMS(I)*PEAKX
      AMAX(I)=ARMS(I)*PEAKA
      GF(I)=1.+PEAKX*XSDX
10  CONTINUE

C      ** OUTPUT STATEMENTS **
C
600  FORMAT(5(1X/ )6X,37H**** DYNAMIC WIND RESPONSE ANALYSIS ,
1    14HRESULTS **** ///1X,7HNAME : ,8A4 )
      WRITE(6,601) H,B,D,F0
601  FORMAT(1H0,19HSIZE OF STRUCTURE : ,2X,3HH = ,F5.0,1HM,3X,3HB = ,
1    F4.0,1HM,3X,3HD = ,F4.0,1HM,4X,3HF0= ,F4.2,2HHZ )
      WRITE(6,602) UH,TI,SCLEH,BETA,QU,QT,QL,DKHH,DKVH,QD
602  FORMAT(1H0,45HWIND CHARACTERISTICS AT Z=H ( ( ) SHOWS POWER ,
1    23H EXPONENT OF PROFILE ): //
1    6X,6HU(H) = ,F4.0,3HM/S,3X,6HT.I. = ,F4.0,1H%,4X,6HL(H) = ,
1    F6.0,1HM,2X,6HBETA = ,F4.1/11X,1H(,F4.2,1H),9X,2H(-,
1    F4.2,1H),11X,1H(,F4.2,1H) // 6X,17HDECAY FACTOR KY = ,
1    F5.1,4X,4HKZ = ,F5.1,4X,2H(-, F4.2,1H) )
      WRITE(6,612) SCD,DCDQ,DCDR
612  FORMAT(1H0,14HCOEFFICIENTS : ,7X,3(5HC =,F5.2,5X)/
1    22X,3HDST,12X,3HDQS,12X,3HDDR/ )
      WRITE(6,603) QM,XMEAN,AZETA
603  FORMAT(1H0,45HRESULTS: DEFLECTION X & ACCERALATION A AT Z=H

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1    / 30X,29H(MODAL SHAPE POWER EXPONENT = ,F4.1,2H ) //9X,
1    7HXMEAN = ,F7.4,1HM,5X,11HAERO.D.R. = , F5.3,1H% //
1    9X,13HDAMPING RATIO ,11X,4H1.0%,22X,4H2.0% //
WRITE(6,604) (XRMS(I),I=1,4)
604  FORMAT(1H /9X,11HAERODYNAMIC ,9X,1HN,9X,1HY,14X,1HN,9X,1HY/
1    11X,14HDAMPING EFFECT /
1    / 9X,9HX RMS (M) ,9X,2(F6.4,4X),2(4X,F6.4)//
WRITE(6,605) (XMAX(I),I=1,4)
605  FORMAT(9X,9HX MAX (M),9X2(F6.4,4X),2(4X,F6.4)//
WRITE(6,606) (ARMS(I),I=1,4)
606  FORMAT(9X,9HA RMS (G) ,9X,2(F6.4,4X),2(4X,F6.4)//
WRITE(6,607) (AMAX(I),I=1,4)
607  FORMAT(9X,9HA MAX (G),9X,2(F6.4,4X),2(4X,F6.4)//
WRITE(6,608) (GF(I),I=1,4)
608  FORMAT(9X,11HGUST FACTOR ,8X,2(F4.2,6X),5X,F4.2,6X,F4.2 /)
WRITE(6,609) (PF(I),I=1,4),(PFA(I),I=1,4)
609  FORMAT(9X,13HPEAK FACTOR X,6X,2(F4.2,6X),5X,F4.2,6X,F4.2/
1    /21X,1HA,6X,2(F4.2,6X),5X,F4.2,6X,F4.2 /)
WRITE(6,610) ROF,BGE,GUS,SRF
610  FORMAT(9X,16HROUGHNESS FACTOR,3X,F5.3,5X,16HBACKGROUND EX.F. ,
1    3X,F5.3//9X,15HGUST EX. FACTOR,4X,F5.3,5X,
1    16HSIZE RED. FACTOR ,3X,F5.3 ////)
WRITE (6,999)
999  FORMAT (1H0,20X,4(5H*****))
STOP
END
C  ** SUBROUTINE STATEMENTS **
C
C  SUBROUTINE NIPSD (A,B,SCPR,DEPR,TMPR,X1,BETA,BGEF,BEN2,BEN4)
C  MODIFIED AS A SUBROUTINE OF WREAN 6TH OCT 1978
C  NUMERICAL INTEGRAL OF POWER SPECTRAL DENSITY FUNCTION S(F)
C  FS(F)=X/(1+X**BETA)**(5/(3*BETA))
C  TO OBTAIN BACKGROUND EXCITATION FACTOR
C  X=(FREQUENCY)*(SCALE LENGTH)/(WIND SPEED)
C  BETA=POWER INDEX FOR S(F)
C  K1=CONST FOR NORMALIZING PURPOSE(GAMMA FUNCTION OF BETA)
C
C  DIMENSION X(50),S(50),S2(50),S4(50)
C
C  XMAX = 1.75*X1
C  STEP =2.0
C  XM =XMAX*0.4
C  XMIN=XMAX*0.01
C  SET VALUES OF X
C  X(1)=0.000001
C  X(2)=XMIN*0.5
C  X(3)=XMIN
C  N=3
110  M=N+2
C  X(M)=X(N)*STEP
C  N=M
C  IF(X(N).LT.XM) GO TO 110
C  MM=N+2
C  X(MM)=XMAX
C  MN=MM-1
C  DO 120 N=4,MN,2
C  NL=N-1
C  NR=N+1
120  X(N)=(X(NL)+X(NR))*0.5
C
C  SET VALUES OF POWER SPECTRAL DENSITY & AERODYNAMIC ADMITTANCE
C  P=BETA
C  Q=-5./(3.*BETA)
C  DO 130 N=1,MM

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      AF=A*X(N)
      BF=B*X(N)
      CALL NIRCF(AF,BF,SCPR,DEPR,TMPR,X(N),P,ADAD)
      S(N)=(1.+X(N)**P)**Q*ADAD
      S2(N)=S(N)*X(N)**2
130   S4(N)=S(N)*X(N)**4
C
C   NUMERICAL INTEGRAL
      AREA=0.
      AREA2=0.
      AREA4=0.
      DO 140 N=2,MN,2
      NL=N-1
      NR=N+1
      XSTEP=X(NR)-X(NL)
      AREA=(S(NL)+S(NR)+4.*S(N))*XSTEP+AREA
      AREA2=(S2(NL)+S2(NR)+4.*S2(N))*XSTEP+AREA2
140   AREA4=(S4(NL)+S4(NR)+4.*S4(N))*XSTEP+AREA4
      BGEF=AREA/6.
      BEN2=AREA2/6.
      BEN4=AREA4/6.
C
      RETURN
      END
C
C   SUBROUTINE NIRCF(AF,BF,QF,RF,P,X1,BETA,ADAD)
      MODIFIED AS A SUBROUTINE OF WREAN 6TH OCT 1978 FROM NIRCF
C
C   NUMERICAL FOUR TIMES INTEGRAL OF ROOT COHERENCE FUNCTION
C   R.C.F.=EXP(-SQRT((AF(Y2-Y1)**2+(BF(Z2-Z1)**2)
C   Y=Y2-Y1,   Z=Z2-Z1,   W=Z1
C   AF=(WIDTH)*FM*(HORIZONTAL DECAY FACTOR)/(WIND SPEED)
C   BF=(HEIGHT)*FM*(VERTICAL DECAY FACTOR)/(WIND SPEED)
C   FM=SQRT(0.1*(UR/SCLE)**2+F**2)
C   F=FREQUENCY(HZ)
C   P=POWER INDEX OF PROFILE (MODE SHAPE AND WIND SPEED)
C   R=POWER INDEX OF DECAY FACTOR PROFILE (NEGATIVE)
C   Q=POWER INDEX OF LENGTH SCALE OF PSD
C   HEIGHT DEPENDENCE OF PSD IS TAKEN INTO ACCOUNT
C   YK,ZK=NO. OF DIVISION FOR SUMMATION WITH RESPECT TO Y & Z
C   RESPECTIVELY
C   NQ,NQW=IDENTIFIER OF EITHER ODD OR EVEN NUMBER OF SUMMATION CYCLE
C
      ZFRO=0.00001
      R=RF
      Q=QF
C   DETERMINE K ACCORDING TO A AND B
      IF (AF.LE.1.0) GO TO 51
      YK=20.
      GO TO 53
51   YK=4.
53   ZEROY=0.21/YK
      A=(AF)**2
55   ZK=20.
58   ZERGO=0.21/ZK
      B=(BF)**2
      XH=X1**BETA
      X2=X1**2
      PP=BETA
      QQ=5./(3.*BETA)
C
C   SUMMATION STARTS FROM Z=Z2-Z1=1. TO ZEROZ
      HQ=0
      Z=1.
      W=ZERO
      H=(ZERGO/ZK)**(P+Q/2.)*(1.+XH)**QQ

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```

      SB=0.
110  Y=1.
      G=1.
      SA=0.
C    SUMMATION WITH RESPECT TO Y (ODD AND EVEN) FOR ANY Z
      ZW=(Z+W)*W
      BTERM=B*Z**2
      IF (Z.LE.ZEROZ) Z=ZERO
      WTERM=(ZW)**(-R)*(1+0.1/(X2*ZW**Q))
      IF (Z.LE.ZEROZ) Z=ZEROZ
111  SA=SA+4.*G*H*EXP(-SQRT((A*Y**2+BTERM)*WTERM))
      Y=Y-1./YK
      G=8.*(YK-Y*YK)
      SA=SA+4.*G*H*EXP(-SQRT((A*Y**2+BTERM)*WTERM))
      Y=Y-1./YK
      IF (Y.LE.ZEROY) GO TO 112
      G=10.*(YK-Y*YK)
      GO TO 111
112  Y=ZEROY
      G=10.*YK-2.
      SA=SA+2.*G*H*EXP(-SQRT((A*Y**2+BTERM)*WTERM))
C    CHANGE ONE STEP FOR W(=Z2)
      WLIM=1.-Z
      IF (W.GE.WLIM) GO TO 130
      W=W+1./ZK
      ZW=(Z+W)*W
      IF (Z.LE.ZEROZ) Z=ZERO
      IF (W.GE.WLIM) GO TO 123
      H=(ZW)**(P+Q/2.)*((1.+XH)/(1.+XH*ZW**(Q*PP/2.)))**QQ
      IF (NQ.EQ.0) GO TO 131
      H=H*8.
      GO TO 126
131  IF (NQW.EQ.0) GO TO 132
      NQW=NQW-1
      H=H*4.
      GO TO 126
132  NQW=NQW+1
      H=H*16.
      GO TO 126
130  IF (Z.LE.ZEROZ) GO TO 150
      NQW=0
      W=ZERO
120  IF (NQ.EQ.0) GO TO 121
      NQ=NQ-1
      GO TO 122
121  NQ=NQ+1
C    CHANGE ONE STEP FOR Z(=Z2-Z1)
122  Z=Z-1./ZK
C    FIRST OR LAST CYCLE FOR Z
123  ZW=(Z+W)*W
      H=(ZW)**(P+Q/2.)*((1.+XH)/(1.+XH*ZW**(Q*PP/2.)))**QQ
      IF (Z.LE.ZERO) GO TO 126
      IF (NQ.EQ.0) GO TO 125
      H=H*4.
      GO TO 126
125  H=H*2.
126  IF (Z.GT.ZEROZ) GO TO 113
      Z=ZEROZ
      H=H/2.
C    SUMMATION AFTER ONE CYCLE OF Y
113  SB=SB+SA/(9.*YK*YK)
C    GO BACK TO SUMMATION WITH RESPECT TO Y
      GO TO 110
150  SC=(SB+SA/(9.*YK*YK))*((1.+P)/(3.*ZK))**2
      ADAD=SC
      RETURN
      END

```

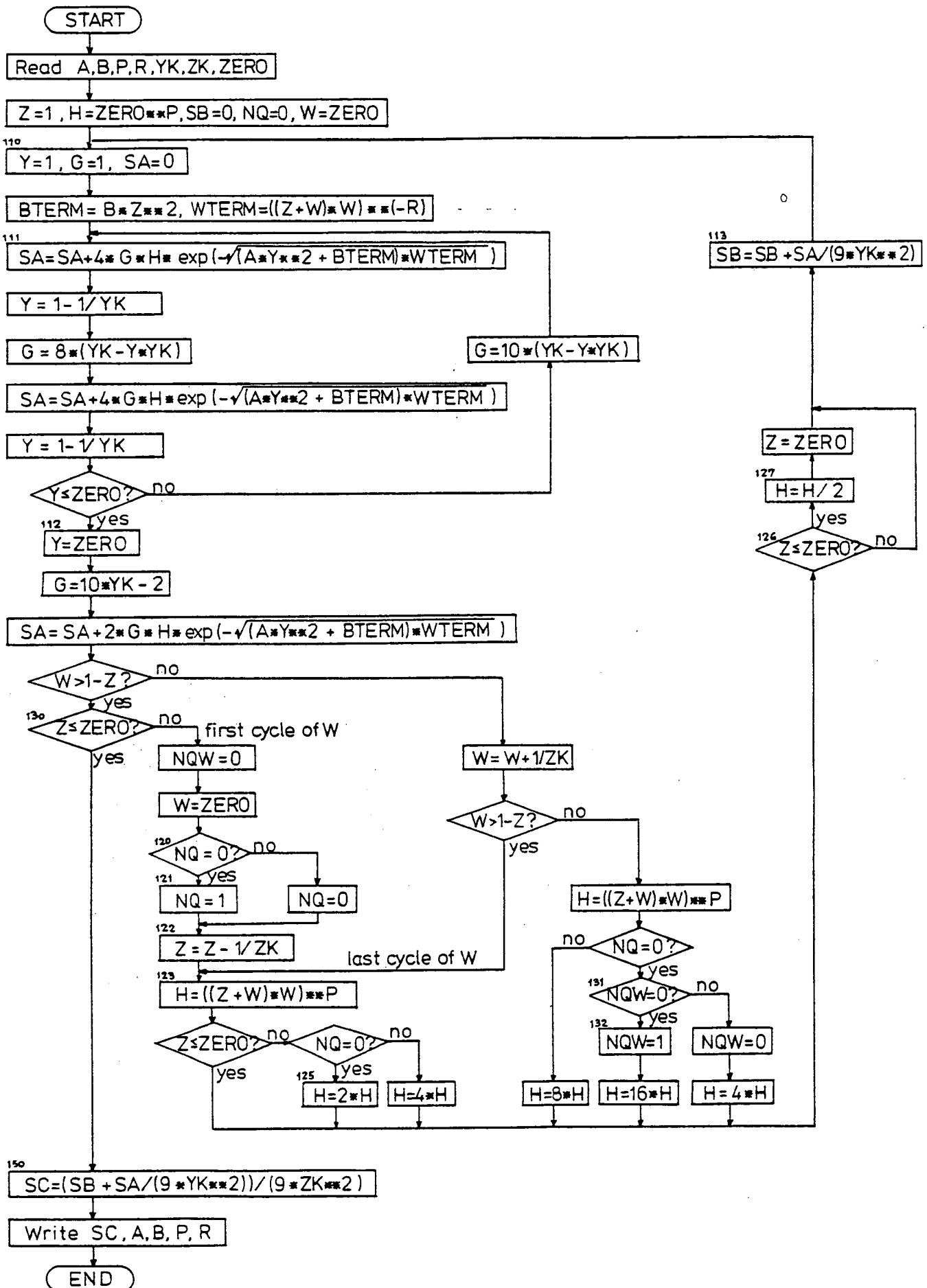


Figure A3-2 Flow chart diagram for subroutine NIRCF in program WREAN 01

A3.5.1 INPUT Example for Program WREAN 01

A RATHER TALL BUILDING * 0 *

250. 40. 30. 0.15 200. 1.
 50. 0.22 0.078 0.08 3800. 0.28 2.0 7.0 6.0 0.41 700.
 1.40 0.80 1.05

A3.5.2 OUTPUT Example for Program WREAN 01

***** DYNAMIC WIND RESPONSE ANALYSIS RESULTS *****

NAME : A RATHER TALL BUILDING * 0 *

SIZE OF STRUCTURE : H = 250.M B = 40.M D = 30.M F0=0.15HZ

WIND CHARACTERISTICS AT Z=H (() SHOWS POWER EXPONENT OF PROFILE):

U(H) = 40.M/S T.I. = 11.% L(H) = 2271.M BETA = 2.0
 (0.22) (-0.08) (0.28)

DECAY FACTOR KY = 8.5 KZ = 7.3 (-0.41)

COEFFICIENTS : C = 1.40 C = 0.80 C = 1.05
 DST DQS DDR

RESULTS: DEFLECTION X & ACCERALATION A AT Z=H
 (MODAL SHAPE POWER EXPONENT = 1.0.)

XMEAN = 0.3095M AERO.D.R. = 0.582%

DAMPING RATIO 1.0% 2.0%

AERODYNAMIC
DAMPING EFFECT

	N	Y	N	Y
X RMS (M)	0.1218	0.1001	0.0910	0.0825
X MAX (M)	0.7593	0.6780	0.6440	0.6119
A RMS (G)	0.0103	0.0082	0.0073	0.0064
A MAX (G)	0.0383	0.0304	0.0269	0.0236
GUST FACTOR	2.45	2.19	2.08	1.98
PEAK FACTOR X	3.69	3.68	3.68	3.67
A	3.71	3.71	3.71	3.71
ROUGHNESS FACTOR	0.242	BACKGROUND EX.F.	0.560	
GUST EX. FACTOR	0.112	SIZE RED. FACTOR	0.240	

APPENDIX 4 PUBLISHED PAPERS

- Paper 1 "Height Dependence of Root-coherence in the Natural Wind",
Safety of Structures under Dynamic Loading, Tapir
Publishers, Trondheim, Norway, 1978, pp 627-635.
- Paper 2 "Further Consideration of Height Dependence of Root-
coherence in the Natural Wind", *Building and Environment*,
vol 13, 1978, pp 175-184.
- Paper 3 "A Transducer for Determining Dynamic Drag and Lift
Coefficients in Wind Tunnels", *Strain*, vol 14, 1978,
pp 138-147.

SAFETY OF STRUCTURES under DYNAMIC LOADING

The Editorial Board for the publication includes:

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*Research Engineer, The Foundation of
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All at The Norwegian Institute of Technology.

This work is based on papers presented at the International Research Seminar on Safety of Structures under Dynamic Loading held in June 1977 at The Norwegian Institute of Technology.

The Seminar dealt with the modern probability based theories of structural reliability and stochastic dynamics, with special emphasis on applications to wind and ocean engineering, and was sponsored by a number of governmental and private institutions.

VOLUME 2 - SHORT CONTRIBUTIONS

RELIABILITY APPROACHES IN WIND ENGINEERING

Height Dependence of Root-Coherence in the Natural Wind, *Kanda, Royles* / Aeroelastic Galloping Of Tall Structures, *Parkinson, Sullivan* / Wind Induced Oscillations of Power Transmission Lines, *Ervik* / Computation of the Dynamics of Structures by Random Loading With a Comparison of the Results Obtained in the Frequency Domain and the Statistically Evaluated Results Obtained in the Time Domain, *Melzer* / Wind Response of Cantilever Bridges under Construction, *Maaas, Jensen* / Fatigue Behaviour of Slender Free Standing Towers Subjected to Wind Gustiness and Support Vibration, *Torset, Flogeland, Lund* / A Field Study of Cross Wind Excitation of Steel Chimneys, *Christensen, Frandsen* / Contribution to Panel Discussion: Wave Spray Loadings on Offshore Structures, *Büsching* / A Remark on the Influence of Wave Spray on Wind Load, *Führhörter*

HEIGHT DEPENDENCE OF ROOT-COHERENCE IN THE NATURAL WIND

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Abstract

The importance of the role played by turbulence characteristics in the response of structures to wind excitation has led to the proposal of several expressions for the power spectrum and root coherence function in the natural wind.

A more general expression for the power spectral density has evolved and from it an improved exponential decaying function is put forward for root coherence which takes into account its height dependence.

Introduction

In order to estimate the total fluctuating load acting on a structure due to the turbulent wind it is necessary to formulate expressions for the dynamic characteristics of the turbulent wind such as the turbulence intensity, the power spectra, the cross-spectra and so on. Some expressions for the power spectra as a function of height have been suggested (1)(2). However, as far as the root coherence is concerned its non-homogeneity has not been taken into account for reasons of simplicity and convenience in practical applications, except in a few cases (3). It is the aim of this paper to express the root-coherence function more simply but still having dependence on height. This may not be applicable to the general problems of non-homogeneous flow, but simple enough to be manipulated in combination with a suitable height dependent power spectral function.

Power spectral density expressions

One of the well-known expressions for the power spectral density function, $S_u(f)$ was proposed by Davenport (4) as a height independent function, i.e.

$$f \cdot S_u(f) / \bar{u}^2 = k_1 \cdot x_1^2 / (1 + x_1^2)^{4/3} \quad (1)$$

where $x_1 = \frac{f \cdot L_1}{\bar{u}_r}$; f = frequency; L_1 = horizontal length scale of the wind fluctuation = 1200 m; $k_1 = \frac{2}{3}$ for normalising purposes; $\bar{u}^2$ = variance of u ; $\bar{u}_r$ = reference mean wind speed at $z = z_r = 10$ m.

Some improved expressions have been proposed taking the height dependency into account, and most of them can be written in a general form as (5),

$$f \cdot S_u(f) / \bar{u}^2 = k_1 \cdot x_1 / (1 + x_1^{\frac{8}{3\beta}})^{5/(3\beta)} \quad (2)$$

where $x_1 = \frac{f \cdot L_1}{\bar{u}_r}$; $L_1 = k_2 \left(\frac{z}{z_r}\right)^\gamma$; and $k_1 = \frac{\beta \Gamma(\frac{5}{3\beta})}{\Gamma(\frac{1}{\beta}) \Gamma(\frac{2}{3\beta})}$

Variations in the value of β are summarised in Table 1. More measurements at different sites may indicate other values for β or expressions for L_1 . The following discussion is developed on the basis of the power spectral expression given by Eq.(2).

TABLE 1

Source	Year	β	γ	k_1
Panofsky & Lumley (6)	1964	5/3	-	0.505
Harris (7)	1970	2	-	0.475
Hino (1)	1971	2	(1-4 α)	0.475
Simiu (2)	1974	1	(1- α^*)	2/3
Fichtl, Kaufman & Vaughan (5)	1970	0.845	0.58	0.769

where α is the power law exponent of mean wind speed profile and α^* is the equivalent value of α for the logarithmic profile.

Root-coherence function with vertical and horizontal separation

A simple decaying exponential function suggested by Davenport (1) has been used many times as a suitable root-coherence function, $R_u(r, f)$, which is written for two points with vertical and horizontal separation, r_z and r_y respectively, as follows,

$$R_u(r, f) = \exp\left[-f \cdot (k_H r_y + k_V r_z) / \bar{u}_r\right] \quad (3)$$

where $r = \begin{bmatrix} r_y \\ r_z \end{bmatrix} = \begin{bmatrix} y_2 - y_1 \\ z_2 - z_1 \end{bmatrix}$; k_H and k_V are decay factors.

Meanwhile, for homogeneous isotropic flow, Harris suggested a theoretical expression for the root-coherence (7) using modified Bessel functions. This has been improved by developing expressions for the root-coherence which incorporate some allowance for the variation of the length scales of turbulence with height (3). Although this expression has a rather complicated form for practical use, it does indicate that the root-coherence function approaches a value significantly less than unity when the frequency tends to zero, while Eq.(3) gives root-coherence = 1 when $f = 0$.

Considering these points, Eq.(3) can be improved, as follows,

$$R_u(r, f) = \exp\left(-\frac{|K(z_m) \cdot r|}{\bar{u}_r} f^*\right) \quad (4)$$

where $K(z_m) = \begin{bmatrix} k_H(z_m) & 0 \\ 0 & k_V(z_m) \end{bmatrix}$; $f^* = \sqrt{f^2 + \bar{u}_r^2 / L_1^2(z_m)}$; $z_m = \sqrt{z_1 z_2}$

Both the decay factors k_H , k_V and the modified frequency f^* are assumed to be a function of z_m , which is a representative height for the vertical positions z_1 and z_2 .

In order to formulate the decay factors as a function of z_m , the cross-correlation coefficient $R_u(r, \tau)$ can be considered to be related to the root-coherence function.

Assuming that the quadrature component of the cross-spectral density can be neglected and substituting lag time $\tau = 0$ in the cross-correlation coefficient, from the Fourier transformation,

$$R_u(r,0) = \int_0^{\infty} R_u(r,f) \frac{\sqrt{S_{u_1}(f) S_{u_2}(f)}}{u^2} df \quad (5)$$

Substituting Eq.(2) and Eq.(4) in Eq.(5)

$$R_u(r) = \int_0^{\infty} \exp\left(-\frac{|K(z_m) \cdot r|}{\bar{U}_r} f^*\right) \frac{k_1 \sqrt{x_1(z_1)x_1(z_2)}}{f \sqrt{(1+x_1(z_1))^{5/(3\beta)} (1+x_1(z_2))^{5/(3\beta)}}} df \quad (6)$$

The root product of power spectra in Eq.(6) can be rearranged, noting that

$$x_1 = f \cdot L_1(z) / \bar{U}_r, \quad L_1(z) = k_2(z/z_r)^{\gamma} \text{ and so } L_1(z_m) = \sqrt{L_1(z_1)L_1(z_2)}$$

$$\frac{k_1 \sqrt{x_1(z_1)x_1(z_2)}}{f \sqrt{(1+x_1(z_1))^{5/(3\beta)} (1+x_1(z_2))^{5/(3\beta)}}} = \frac{k_1 L_1(z_m)}{C(z_1, z_2) \bar{U}_r (1+x_1^{\beta}(z_m))^{5/(3\beta)}} \quad (7)$$

$$\text{where } 1 \leq C(z_1, z_2) \leq (z_m/z_r)^{\frac{5}{6}\gamma} \text{ which is derived from a} \quad (8)$$

$$\text{general inequality } a + \frac{1}{a} \geq 2 \text{ for } a > 0 \text{ where } a = \left(\frac{L_1(z_2)}{L_1(z_1)} \right)^{\frac{\beta}{2}}$$

$$\text{and } z_2 \geq \sqrt{z_1 z_2} = z_m \geq z_1 \geq z_r > 0 \quad (9)$$

Eq.(8) suggests that $C(z_1, z_2)$ can be expressed simply as a function of the geometric mean height z_m . However, this discussion is only valid when the lower measuring point given by z_1 is above the reference height z_r , and z_r should be chosen such that the wind forces below this level do not make a significant contribution to the dynamic response of a structure.

Now, considering the length scales, L_i , which are defined from the cross-correlation coefficient with no lag time, as follows,

$$L_i = \int_0^{\infty} R_u(r_i) dr_i \quad (10)$$

where $i = x, y, z$.

The integral in Eq.(10) can be developed from Eqs.(6), (7) and

(8) as,

$$L_i = \frac{k_1 L_1(z_m)}{k_i(z_m)} \left(\frac{z_m}{z_r} \right)^{-\alpha'} \int_0^{\infty} dx_1 / \left[(1+x_1^{\beta})^{5/(3\beta)} (1+x_1^2)^{1/2} \right] \quad (11)$$

where $L_i = L_x, L_y$ and $k_i = k_H, k_V$ respectively, and $0 \leq \alpha' \leq \frac{5}{6}\gamma$.

It is interesting to note that if the frequency f is used instead of f^* in Eq.(6), the integral does not converge because of the infinite value at $f = 0$.

When the length scale L_i is expressed as a function of height, the decay factor $k_i(z_m)$ can be formulated as a function of height. In homogeneous isotropic flow a relationship between the length scales L_i and the horizontal length L_1 is deduced from Taylor's hypothesis (7), as follows,

$$L_i = K_i L_1 \bar{U}(z) / \bar{U}_r \quad (12)$$

where $K_i = K_x, K_y, K_z$ are constants and $K_y = K_z = \frac{1}{2} K_x$

In the natural wind K_i may not be constant but possibly vary with height. Further measurements have shown that K_i can be expressed as a function of height (3),(8). However, since the function $K_i(z)$ is not established for different roughness conditions at the present time, K_i is assumed here to be constant. If $K_i(z)$ were expressed as an empirical power law function of height, this discussion could be altered easily.

Equating Eq.(11) to Eq.(12), a height dependent function of $k_i(z_m)$ is obtained as,

$$k_i(z_m) = k_3 \left(\frac{z_m}{z_r} \right)^{-\alpha'} \cdot \frac{\bar{U}_r}{\bar{U}(z_m)} = k_3 \left(\frac{z_m}{z_r} \right)^{-\alpha_1} \quad (13)$$

where

$$k_3 = \frac{k_1}{K_i} \int_0^{\infty} dx_1 / \left[(1+x_1^{\beta})^{5/(3\beta)} (1+x_1^2)^{1/2} \right] \quad (13a)$$

and $\alpha = \alpha' + \alpha$. Consequently from Eq.(8)

$$\alpha \leq \alpha_1 \leq \frac{5}{6}\gamma + \alpha \quad (13b)$$

and when $z_1 = z_2$, $\alpha_1 = \alpha$.

The decay factors in the root-coherence function may not have the exact form of a power law expression, but the possibility of the existence of such a relation is shown in the above discussion.

By comparing Eq.(13) with some empirical data measured at different sites, the power exponent, α_1 , is estimated in the following section and is supposed to satisfy Eq.(13-b). The constant k_3 in Eq.(13) may be computed from Eq.(13-a). However, since parameters k_1 , K_1 and β are mostly based on empirical data and have not been established yet, it would appear better to estimate k_3 directly from root-coherence functions obtained from natural wind data. It can be shown from Harris' work (7) that in homogeneous isotropic flow at a standard reference height $z_r = 10$ m, $K_x = 2K_y = 2K_z = 0.118$ and with $\beta = 2$, $k_1 = 0.475$ then $k_3 = 10.0$ is obtained.

Comparison with empirical data

Using actual wind speed measurements Shiotani (9) and Chuen (10) computed decay factors for the root-coherence function of the longitudinal components, and these are plotted against mean height z_m in Fig. 1; the latter's results requiring some conversion to make them relative to a standard reference height. Similar decay factors have been deduced here from Harris' natural wind data (7) and are plotted in the same figure. Suitable power law exponents for these plots were estimated by means of the least squares method and are listed in Fig. 1, although one of Chuen's results appears spurious and was ignored in calculating the exponent from that set of data.

The range of magnitude of the decay factor varies from one plot to another even though three of the measurement sets were obtained over similar smooth terrain. Nevertheless there is close agreement between the values of the exponent, α_1 , obtained from these results, suggesting that the power law expression, Eq.(13), has some relevance.

It is interesting to note that the decay factor k_3 computed for the standard reference height $z_r = 10$ m in homogeneous isotropic flow fits in quite well with the power law expressions obtained from empirical data based on vertical separation.

A comparison is made in Fig. 2(a) (b) between the theoretical expression for root-coherence, Eqs. (4) and (13), Davenport's expression Eq.(3), Harris' theoretical curve (7) and measured data (7)

for different heights. These figures indicate that the proposed root-coherence expression, Eqs.(4) and (13), is reasonably consistent with measured data and demonstrates clearly its dependence on height.

Conclusion

It can be concluded that the expression proposed for the root-coherence of the longitudinal turbulent component between two points with vertical and horizontal separation is consistent with several sets of recent empirical data which indicate the height variation of the root-coherence.

The power exponent α_1 for the decay factor in the root-coherence expression is estimated from empirical data and the decay factors k_H , k_V for horizontal and vertical separation respectively could be evaluated at the reference height either theoretically or empirically.

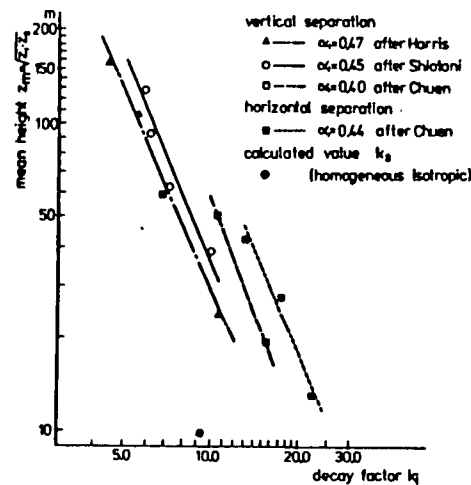


Fig. 1 Variation of decay factor

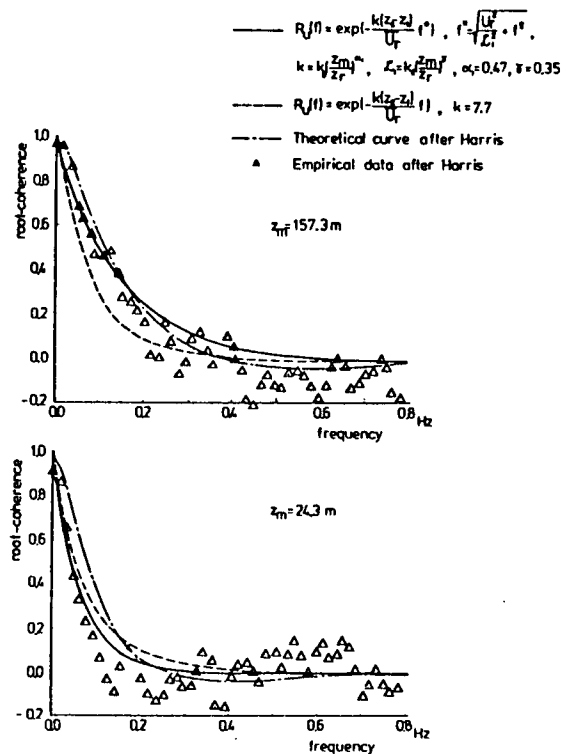


Fig. 2 Comparison between root-coherence functions and empirical data after Harris

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Further Consideration of the Height Dependence of Root-Coherence in the Natural Wind

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 R. ROYLES†

The growing interest in the response of structures to turbulent wind forces and the realisation of the important role played by root-coherence in the prediction of such response has led to the proposal of several expressions for the power spectrum and the root-coherence function in the natural wind. A more general expression for the power spectral density has evolved and on the basis of it an improved exponential decaying function is put forward for the root-coherence of the longitudinal turbulent component in the natural wind. This takes into account both horizontal and vertical separation between two points.

A modified frequency term is introduced and a power law profile is applied to the decay factor in order to establish the height variation of the root-coherence function.

The consistency of this relationship is investigated by comparison with several sets of empirical data from different sites. The results are encouraging and suggest that this type of approach should be incorporated into dynamic structural response calculations.

INTRODUCTION

RECENTLY there has been a growing interest in the dynamic response of structures due to turbulent wind forces. In order to estimate the total fluctuating load acting on a structure due to the turbulent wind, it is necessary to formulate expressions for the dynamic characteristics of the turbulent wind such as the turbulence intensity, the power spectra, the cross-spectra and so on. In most papers, however, the turbulence in the natural wind is assumed to be homogeneous, in other words, the characteristics of the turbulence are assumed to be independent of position or height[1], although it is widely admitted that the natural wind is almost homogeneous horizontally but not vertically. When the natural mean wind speed profile is established and the standard deviation of the turbulence is considered to be constant with height, the most important problem for wind-loading is how to express the power spectra and the root-coherence (or the cross-spectra) of the longitudinal fluctuating components of the turbulent wind speed.

Some expressions for the power spectra as a function of height have been suggested[2,3], however, as far as the root-coherence is concerned, the non-homogeneity has hardly been taken into account for reasons of simplicity and convenience in practical applications. One exception to this pattern is the use of a wind speed averaged between two points having some verti-

cal separation[4]. An alternative approach[5] assumes homogeneous isotropic turbulence to exist in the first instance and develops an expression for root-coherence which incorporates some allowance for the variation of the length scale of turbulence with height and yields a complicated function.

It is the aim of this paper to express the root-coherence function more simply but still having dependence on height. This may not be applicable to the general problems of non-homogeneous flow, but simple enough to be manipulated with a combination of a suitable height-dependent power spectral function and a power law for the mean wind speed profile.

The symbols used in the paper are summarised below.

NOMENCLATURE

a, b	= index
B	= width of a structure
$C(z_1, z_2)$	= factor due to z_1 and z_2
C_d	= drag coefficient
C_d^*	= modified drag coefficient
C_M	= mass coefficient
C_M^*	= modified mass coefficient
$\exp(\)$	= exponential function
$E(\)$	= average operation
$F(x)$	= function of x
$F_n(t)$	= generalized force of the n th mode
f	= frequency
f^*	= modified frequency
f_n	= natural frequency of the n th mode
H	= height of a structure
i	= $\sqrt{-1}$, indicator of x, y, z
k, K	= constants
k_H, k_V	= decay factor in the lateral and vertical direction respectively
K	= decay factor matrix

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L_i	= length scale in i -direction where $i = x, y, z$
$\mathcal{L}_1$	= longitudinal length of turbulence (arbitrary value)
m	= mass of a structure per unit area
M_n	= generalized mass of a structure of the n th mode
M'_n	= total generalized mass of the n th mode
n	= modal number
N	= number of degrees of freedom
$P(t)$	= net wind pressure
$p(t)$	= fluctuating component of $P(t)$
r	= a non-dimensional length scale function
r_i	= separation distance, $i = x, y, z$
$\mathbf{r}$	= separation distance vector
$R_u(f)$	= root-coherence function of u
$\mathcal{R}_u(\mathbf{r}, \tau)$	= cross-correlation coefficient of u_1, u_2
$S_u(f)$	= power spectral density function of u
$S_{u_1 u_2}(f)$	= cross-spectral density function of u_1, u_2
$S_F(f)$	= power spectral density function of F_n
$S_\delta(f)$	= power spectral density function of δ
t	= time
$U(t)$	= wind speed
$U'(t)$	= relative wind speed
$\bar{U}$	= mean wind speed
$\bar{U}_r$	= reference mean wind speed
$u(t)$	= longitudinal turbulent component of $U(t)$
$\dot{u}(t)$	= wind acceleration
$\sqrt{\dot{u}^2}$	= standard deviation of u
x_1	= $f \cdot \mathcal{L}_1 / \bar{U}_r$
x	= longitudinal co-ordinate
y, y_1, y_2	= lateral co-ordinates
z, z_1, z_2	= vertical co-ordinates
z_r	= reference height
z_m	= geometric mean of z_1, z_2
α	= power exponent of mean wind speed profile
α'	= power exponent of $C(z_1, z_2)$ profile
α_1	= power exponent of decay factor profile
β	= constant index in power spectral expression
γ	= power exponent of $\mathcal{L}_1$ profile
$\Gamma(\cdot)$	= gamma function
Δ	= longitudinal displacement
δ	= fluctuating component of Δ
$\dot{\delta}$	= longitudinal velocity
$\ddot{\delta}$	= longitudinal acceleration
$\mu(z)$	= mode shape
ρ	= air mass density
τ	= time lag
ξ	= reduced frequency $fB/\bar{U}$
$\chi_n(f)$	= mechanical admittance of the n th mode
ζ_n	= damping ratio of a structure in the n th mode
ζ'_n	= total damping ratio in the n th mode

REVIEW OF POWER SPECTRAL DENSITY EXPRESSIONS

There have been a great number of measurements of spectra of wind speed in recent years. For higher frequency regions most measurements confirm the Kolmogorov hypothesis[6], however, for lower frequency regions there are still some variations between established formulae for the power spectral density function of the longitudinal turbulent wind speed component u .

Davenport[1,7] proposed the following height independent expression as the basis of his approach to the prediction of wind induced dynamic structural response,

$$\frac{f \cdot S_u(f)}{\bar{u}^2} = k_1 \frac{x_1^2}{(1+x_1^2)^{4/3}} \quad (1)$$

where $x_1 = f \cdot \mathcal{L}_1 / \bar{U}_r$, $f \equiv$ frequency, $\mathcal{L}_1 \equiv$ horizontal

length of the wind fluctuation = 1200 m, $k_1 \equiv$ constant = $\frac{2}{3}$ for normalising purposes, $\bar{u}^2 \equiv$ variance of u , $\bar{U}_r \equiv$ reference mean wind speed at $z = z_r$ (an arbitrary reference height).

This formula has been used quite often but further investigations suggest that it is slightly conservative for engineering purposes. The first point is that equation (1) has a zero value for $f=0$ Hz, although the one-dimensional spectral density is considered to approach a finite value when the frequency goes to zero[6]. The second point is that equation (1) is independent of height. The horizontal scale $\mathcal{L}_1$ appears to vary from site to site and to increase with height, however, its variation is not clear, in other words, not established at this moment.

In consequence some improved expressions of the power spectral density have been proposed.

According to Hino[2],

$$\frac{f \cdot S_u(f)}{\bar{u}^2} = k_1 \frac{x_1}{(1+x_1^2)^{5/6}} \quad (2)$$

where $x_1 = f \cdot \mathcal{L}_1 / \bar{U}_r$, $\mathcal{L}_1 = k_2 (z/z_r)^{1-4\alpha}$, k_1 and k_2 are constants with $k_1 = 0.475$ for normalising purposes (see Appendix B). α is a power exponent of the mean wind speed profile. Also a related expression due to Simiu[3] can be rewritten in a similar way to equation (2), as,

$$\frac{f \cdot S_u(f)}{\bar{u}^2} = k_1 \frac{x_1}{(1+x_1^2)^{5/3}} \quad (3)$$

where $x_1 = f \cdot \mathcal{L}_1 / \bar{U}_r$, $\mathcal{L}_1 = k_2 (z/z_r)^{1-\alpha}$, k_1 and k_2 are constants with $k_1 = \frac{2}{3}$ for normalising purposes (see Appendix B).

In equation (3), the power exponent α is an equivalent value for the logarithmic mean wind profile used by Simiu. Both equations (2) and (3) satisfy the Kolmogorov hypothesis in the high frequency range just as well as equation (1). Equation (2) has a form suggested by Harris[8], which is known as a Von Karman spectrum and uses a constant horizontal length of

$$\mathcal{L}_1 = \frac{1}{\sqrt{2}} 1800 \text{ m.}$$

There is not much difference between equation (2) and (3) at lower heights but the variation of $\mathcal{L}_1$ is rather different in each case. Consequently there is a significant difference at greater heights, i.e. at greater values of z .

Equation (2) is derived from the balance of the energy dissipation, assuming a power law profile for the mean wind speed, and equation (3) is based on a logarithmic profile.

Another more general expression can be written as follows[9],

$$\frac{f \cdot S_u(f)}{\bar{u}^2} = k_1 \frac{x_1}{(1+x_1^\beta)^{5/3\beta}} \quad (4)$$

where $x_1 = f \cdot \mathcal{L}_1 / \bar{U}_r$, $\mathcal{L}_1 = k_2 (z/z_r)^\gamma$, k_1 and k_2 are constants and

$$k_1 = \frac{\beta \Gamma\left(\frac{5}{3\beta}\right)}{\Gamma\left(\frac{1}{\beta}\right) \Gamma\left(\frac{2}{3\beta}\right)}$$

(see Appendix B).

According to the U.S. National Bureau of Standards[9], $\beta = 0.845$ in equation (4) is suggested from several sets of empirical data by means of the least squares method. $\beta = 2$ corresponds to equation (2), $\beta = 1$ corresponds to equation (3), and $\beta = \frac{5}{3}$ corresponds to Panofsky-Lumley's expression, which is another well-known formula[10].

More measurements at different sites may indicate other values for β or expressions for $\mathcal{L}_1$. Clearly, there could exist many variations in the form of the power spectral expression depending on site conditions and consequently a general expression of the type shown in equation (4) would be better for engineering purposes. The following discussion is developed on the basis of the power spectral expression given by equation (4).

ROOT-COHERENCE FUNCTION WITH VERTICAL AND HORIZONTAL SEPARATION

One of the well-known expressions for the root-coherence function $R_u(x, f)$, i.e. the cross-correlation coefficient in the frequency domain, has the form of a simple decaying exponential function as suggested by Davenport[1], namely,

$$R_u(x, f) = \exp(-k \cdot fx / \bar{U}_r) \quad (5)$$

where k is a decay constant, x is a separation distance.

This form has been used many times but as in the case of the power spectral expression equation (1), it has been pointed out[4] that equation (5) is slightly conservative.

For the convenience of integration with respect to the surface of a structure the root-coherence function for two points which are both horizontally and vertically separated is expressed as a simple product of the root-coherence functions for the horizontal separation and for the vertical separation in some gust response approaches[7, 11]. However, this simplification causes a significant underestimate in certain circumstances, e.g. up to 20% according to Vickery[12].

A further point is that concerning the height dependency of the root-coherence which is not taken into account in equation (5). An improved expression was suggested by Vickery[4] in the form,

$$R_u(y_2 - y_1, z_2 - z_1, f) = \exp \left\{ - \frac{\sqrt{k_H^2 (y_2 - y_1)^2 + k_V^2 (z_2 - z_1)^2}}{\frac{1}{2} [\bar{U}(z_1) + \bar{U}(z_2)]} \cdot f \right\} \quad (6)$$

This assumes that the variation of the decay factors are linear to a wind speed averaged between two points with vertical separation.

Meanwhile, for homogeneous isotropic flow, Harris suggested a theoretical expression for the root-coherence[8] using modified Bessel functions. This has

been improved by developing expressions for the root-coherence which incorporate some allowance for the variation of the length scales of turbulence with height[5]. However, this form seems to be rather complicated for practical use but it does indicate that the root-coherence function approaches a value significantly less than unity when the frequency tends to zero, and its value at $f = 0$ varies due to the mean wind speed $\bar{U}_r$ and the horizontal length $\mathcal{L}_1$. These factors could influence computations of dynamic wind force and it is interesting to note that by comparison equations (5) and (6) give root-coherence = 1 when $f = 0$.

Considering these points, equation (6) can be improved, as follows,

$$R_u(\mathbf{r}, f) = \exp \left(- \frac{|\mathbf{K}(z_m) \cdot \mathbf{r}|}{\bar{U}_r} \cdot f^* \right) \quad (7)$$

where

$$\mathbf{r} = \begin{Bmatrix} y_2 - y_1 \\ z_2 - z_1 \end{Bmatrix}, \quad \mathbf{K} = \begin{pmatrix} k_H(z_m) & 0 \\ 0 & k_V(z_m) \end{pmatrix}.$$

The decay factors k_H and k_V are assumed to be a function of z_m which is a representative height for the vertical positions z_1 and z_2 . Here z_m is introduced as a geometric mean of z_1 and z_2 . The reason for using a geometric mean instead of a simple mean is for mathematical convenience since the root of the product of the power spectra is used in the definition of the root-coherence function shown later in equation (9). However, there may not be a significant difference between the geometric mean $\sqrt{z_1 z_2}$ and the simple mean $(z_1 + z_2)/2$ in the practical use of the coherence function.

The modified frequency f^* is introduced instead of the frequency f in the root-coherence function for the purpose of consistency and better fit to the empirical data. Referring to Harris's theoretical approach[8],

$$f^* = \sqrt{\frac{\bar{U}_r^2}{\mathcal{L}_1^2(z_m)} + f^2} \quad (8)$$

For the greater values of frequency, i.e.

$$x_1 = \frac{f \cdot \mathcal{L}_1(z_m)}{\bar{U}_r} \gg 1,$$

f^* may be replaced by f without losing accuracy.

In order to formulate the decay factors k_H and k_V as a function of z_m , the cross-correlation coefficient $\mathcal{R}_u(\mathbf{r}, \tau)$ can be considered to be related to the root-coherence function. The cross-correlation coefficient is the inverse Fourier transform of the cross-spectral density function from which the root-coherence $R_u(\mathbf{r}, f)$ is defined as,

$$\frac{|S_{u_1 u_2}(f)|}{u^2} = R_u(\mathbf{r}, f) \cdot \frac{\sqrt{S_{u_1}(f) \cdot S_{u_2}(f)}}{u^2} \quad (9)$$

where $S_{u_1 u_2}(f)$ is the cross-spectral density function between $u_1 = u(y_1, z_1)$ and $u_2 = u(y_2, z_2)$.

$S_u(f)$ is the power spectral density function of $u(z)$ and the variance $\bar{u}^2$, of the longitudinal gust component $u(z)$ is assumed to be constant with height.

From the Fourier transformation,

$$\mathcal{R}_u(\mathbf{r}, \tau) = \int_0^\infty \frac{S_{u_1 u_2}(f)}{\bar{u}^2} \cdot e^{i2\pi f \tau} df \quad (10)$$

where $\mathcal{R}_u(\mathbf{r}, \tau)$ is the cross-correlation coefficient between u_1 and u_2 .

Substituting equation (9) into (10) with $\tau = 0$ for the cross-correlation coefficient $\mathcal{R}_u(\mathbf{r}, \tau)$, and assuming that the phase angle or quadrature component of the cross-spectral density can be neglected, equation (11) is obtained.

$$\mathcal{R}_u(\mathbf{r}) = \int_0^\infty R_u(\mathbf{r}, f) \cdot \frac{\sqrt{S_{u_1}(f) \cdot S_{u_2}(f)}}{\bar{u}^2} df \quad (11)$$

Substituting equation (4) for the power spectral density function and equation (7) for the root-coherence function in equation (11) it becomes,

$$\begin{aligned} \mathcal{R}_u(\mathbf{r}) = & \int_0^\infty \exp\left(-\frac{|\mathbf{K}(z_m) \cdot \mathbf{r}|}{\bar{U}_r} \cdot f\right) \\ & \times \frac{k_1 \sqrt{x_1(z_1) \cdot x_1(z_2)}}{f \sqrt{[1 + x_1^\beta(z_1)]^{5/3\beta} [1 + x_1^\beta(z_2)]^{5/3\beta}}} df \quad (12) \end{aligned}$$

The root product of power spectra in equation (12) is rearranged noting that $x_1 = f \cdot \mathcal{L}_1(z)/\bar{U}_r$ and $\mathcal{L}_1(z) = k_2(z/z_r)^\gamma$,

$$\begin{aligned} & \frac{k_1 \sqrt{x_1(z_1) \cdot x_1(z_2)}}{f \sqrt{[1 + x_1^\beta(z_1)]^{5/3\beta} [1 + x_1^\beta(z_2)]^{5/3\beta}}} \\ & = \frac{k_1 \sqrt{\mathcal{L}_1(z_1) \cdot \mathcal{L}_1(z_2)}}{\bar{U}_r \sqrt{\left[1 + \frac{\mathcal{L}_1^\beta(z_1)}{\mathcal{L}_1^\beta(z_m)} x_1^\beta(z_m)\right]^{5/3\beta} \left[1 + \frac{\mathcal{L}_1^\beta(z_2)}{\mathcal{L}_1^\beta(z_m)} x_1^\beta(z_m)\right]^{5/3\beta}}} \quad (13) \end{aligned}$$

Since $\mathcal{L}_1(z_m) = \sqrt{\mathcal{L}_1(z_1) \mathcal{L}_1(z_2)}$ then by letting

$$r = \left[\frac{\mathcal{L}_1(z_2)}{\mathcal{L}_1(z_1)} \right]^{\beta/2} = \left[\frac{\mathcal{L}_1(z_m)}{\mathcal{L}_1(z_1)} \right]^\beta = \left[\frac{\mathcal{L}_1(z_2)}{\mathcal{L}_1(z_m)} \right]^\beta,$$

where for $r > 0$ in general $r + 1/r \geq 2$, the following equations can be obtained,

$$\begin{aligned} & \sqrt{\left[1 + \frac{\mathcal{L}_1^\beta(z_1)}{\mathcal{L}_1^\beta(z_m)} x_1^\beta(z_m)\right]^{5/3\beta} \left[1 + \frac{\mathcal{L}_1^\beta(z_2)}{\mathcal{L}_1^\beta(z_m)} x_1^\beta(z_m)\right]^{5/3\beta}} \\ & = \left[1 + \left(r + \frac{1}{r}\right) \cdot x_1^\beta(z_m) + x_1^{2\beta}(z_m)\right]^{5/6\beta} \\ & \geq [1 + 2x_1^\beta(z_m) + x_1^{2\beta}(z_m)]^{5/6\beta} \\ & = [1 + x_1^\beta(z_m)]^{5/3\beta} \quad (\text{a lower limit}) \end{aligned}$$

and

$$\begin{aligned} & \left[1 + \left(r + \frac{1}{r}\right) \cdot x_1^\beta(z_m) + x_1^{2\beta}(z_m)\right]^{5/6\beta} \\ & \leq \left[\frac{r + \frac{1}{r}}{2} + \left(r + \frac{1}{r}\right) \cdot x_1^\beta(z_m) + \frac{r + \frac{1}{r}}{2} x_1^{2\beta}(z_m)\right]^{5/6\beta} \\ & = \left(\frac{r + \frac{1}{r}}{2}\right)^{5/6\beta} [1 + x_1^\beta(z_m)]^{5/3\beta} \quad (\text{an upper limit}). \end{aligned}$$

Equation (13) can be re-written using a factor $C(z_1, z_2)$ which is defined as follows,

$$\begin{aligned} & \frac{k_1 \sqrt{x_1(z_1) \cdot x_1(z_2)}}{f \sqrt{[1 + x_1^\beta(z_1)]^{5/3\beta} [1 + x_1^\beta(z_2)]^{5/3\beta}}} \\ & = \frac{1}{C(z_1, z_2)} \cdot \frac{k_1}{\bar{U}_r} \cdot \frac{\mathcal{L}_1(z_m)}{[1 + x_1^\beta(z_m)]^{5/3\beta}} \quad (14) \end{aligned}$$

where

$$\begin{aligned} 1 \leq C(z_1, z_2) & \leq \left(\frac{r + \frac{1}{r}}{2}\right)^{5/6\beta} \\ & = \left\{ \frac{\left(\frac{z_m}{z_r}\right)^{\gamma\beta} \left[\left(\frac{z_r}{z_1}\right)^{\gamma\beta} + \left(\frac{z_r}{z_2}\right)^{\gamma\beta}\right]}{2} \right\}^{5/6\beta} \leq \left(\frac{z_m}{z_r}\right)^{5/6\gamma} \end{aligned}$$

and

$$z_2 \geq \sqrt{z_1 z_2} = z_m \geq z_1 \geq z_r > 0.$$

This inequality suggests that $C(z_1, z_2)$ can be expressed simply as a function of the geometric mean height z_m , namely,

$$C(z_1, z_2) = \left(\frac{z_m}{z_r}\right)^{\alpha'} \quad (15)$$

where $0 \leq \alpha' \leq \frac{5}{6}\gamma$. When $z_1 = z_2$, $r = 1$, and so $\alpha' = 0$. It is interesting to note that α' depends on the exponent γ but is independent of the exponent β in the power spectral expression equation (4).

Now, consider the length scales which are defined from the cross-correlation coefficient with no lag time, as follows,

$$L_i = \int_0^\infty \mathcal{R}_u(r_i) dr_i \quad (16)$$

where $i = x, y, z$ and $r_i = i_2 - i_1$.

Generally in three dimensional turbulent flow nine length scales can be defined as combinations of three velocity components and three directions of the separation. However, since the longitudinal velocity component is the major contributor to the fluctuating wind force which causes the along-wind dynamic response of a structure only three length scales out of nine need be taken into account for the longitudinal velocity component.

Clearly $\mathcal{R}_u(r_i)$ is a function of i_1 and i_2 and can be rewritten as a function of position, i.e. the mean of i_1

and i_2 and the difference $r_i = i_2 - i_1$. Since equation (16) has the form of a definite integral with respect to the difference r_i , L_i can be considered as a function of position. If the turbulent flow is assumed to be horizontally homogeneous all correlation coefficients are independent of horizontal position. Then L_x and L_y can be considered constant with horizontal position but L_z may be expressed as a function of vertical position.

In order to obtain the relationship between the decay factors, k_H and k_V , and height the two cases $r_y = 0$ and $r_z = 0$ in equation (12) are discussed.

The integral in equation (16) can be developed from (12), (14) and (15) giving,

$$L_i = \int_0^\infty \int_0^\infty \exp\left(\frac{-k_i(z_m)r_i \cdot f^*}{\bar{U}_r}\right) \times \frac{1}{C(z_1, z_2)} \cdot \frac{k_1}{\bar{U}_r} \cdot \frac{\mathcal{L}_1(z_m)}{[1+x_1^\beta(z_m)]^{5/3\beta}} df dr_i \\ = \int_0^\infty \frac{\bar{U}_r}{k_i(z_m)f^*} \left(\frac{z_m}{z_r}\right)^{-\alpha'} \cdot \frac{k_1}{\bar{U}_r} \cdot \frac{\mathcal{L}_1(z_m)}{[1+x_1^\beta(z_m)]^{5/3\beta}} df.$$

Since

$$f^* = \sqrt{\frac{\bar{U}_r^2}{\mathcal{L}_1^2(z_m)} + f^2} = \frac{\bar{U}_r}{\mathcal{L}_1(z_m)} \sqrt{1+x_1^2(z_m)}$$

and

$$df = \frac{\bar{U}_r}{\mathcal{L}_1(z_m)} dx_1,$$

$$L_i = \frac{k_1 \cdot \mathcal{L}_1(z_m)}{k_i(z_m)} \left(\frac{z_m}{z_r}\right)^{-\alpha'} \times \int_0^\infty \frac{dx_1(z_m)}{[1+x_1^\beta(z_m)]^{5/3\beta} [1+x_1^2(z_m)]^{1/2}} \quad (17)$$

where $L_i = L_y$, L_z and $k_i = k_H$, k_V and $r_i = r_y$, r_z .

The integral term in equation (17) has a finite constant value. Note that if the frequency f is used instead of the modified frequency f^* for the root-coherence function, the integral does not converge because of the infinite value at $f = 0$.

When the length scale L_i is expressed as a function of height the decay factor can be formulated as a function of height. In homogeneous isotropic flow a relationship between the length scales L_i and the horizontal length $\mathcal{L}_1$ is deduced from Taylor's hypothesis[8], as follows,

$$L_i = K_i \cdot \mathcal{L}_1 \cdot \bar{U}(z)/\bar{U}_r \quad (18)$$

where $K_i = K_x$, K_y and K_z are constants and $K_y = K_z = \frac{1}{2}K_x$.

In the natural wind K_i may not be constant but possibly vary with height. Further measurements have shown that K_i can be expressed as a function of height[5,15]. However, since the function $K_i(z)$ is not established for different roughness conditions at the present time, K_i is assumed here to be constant,

although this may be conservative. If $K_i(z)$ were expressed as an empirical power law function of height, this discussion could be altered easily.

Equating equation (17) to (18),

$$\frac{k_1 \mathcal{L}_1(z_m)}{k_i(z_m)} \left(\frac{z_m}{z_r}\right)^{-\alpha'} \int_0^\infty \frac{dx_1(z_m)}{[1+x_1^\beta(z_m)]^{5/3\beta} [1+x_1^2(z_m)]^{1/2}} \\ = K_i \cdot \mathcal{L}_1(z_m) \cdot \bar{U}(z_m)/\bar{U}_r$$

which yields

$$k_i(z_m) = k_3 \left(\frac{z_m}{z_r}\right)^{-\alpha'} \frac{\bar{U}_r}{\bar{U}(z_m)} = k_3 \left(\frac{z_m}{z_r}\right)^{-\alpha_1} \quad (19)$$

where

$$k_3 = \frac{k_1}{K_i} \int_0^\infty \frac{dx_1}{(1+x_1^\beta)^{5/3\beta} (1+x_1^2)^{1/2}} \quad (19a)$$

and $\alpha_1 = \alpha' + \alpha$. Consequently from equation (15)

$$\alpha \leq \alpha_1 \leq \frac{5}{6}\gamma + \alpha \quad (19b)$$

and when $z_1 = z_2$, $\alpha_1 = \alpha$. When equation (2) is used as a special case of equation (4) $\gamma = (1-4\alpha)$ and so equation (19b) becomes

$$\alpha \leq \alpha_1 \leq \frac{5}{6}(1-4\alpha) + \alpha = \frac{5}{6} - \frac{7}{3}\alpha. \quad (19c)$$

Alternatively if equation (3) is used as a special case of equation (4), $\gamma = (1-\alpha)$ and so (19b) becomes

$$\alpha \leq \alpha_1 \leq \frac{5}{6} + \frac{1}{6}\alpha. \quad (19d)$$

The decay factors in the root-coherence function may not have the exact form of a power law expression but the possibility of the existence of such a relation is shown in the above discussion.

By comparing equation (19) with some empirical data measured at different sites the power exponent is estimated in the following section. The constant k_3 in equation (19) may be computed from (19a). However, since parameters k_1 and K_i and β are mostly based on empirical data and have not been established yet, it would appear better to estimate k_3 directly from root-coherence functions obtained from natural wind data.

It can be shown from Harris' work[8] that in homogeneous isotropic flow at a standard reference height $z_r = 10$ m,

$$K_x = 2K_y = 2K_z = 0.118$$

and with $\beta = 2$, $k_1 = 0.475$ [see equation (2)] then $k_3 = 9.01$ (see Appendix B).

COMPARISON WITH EMPIRICAL DATA

Using actual wind speed measurements Shiotani[13], Chuen[14] and Duchène-Marullaz[20] computed decay factors for the root-coherence function of the longitudinal components and these are plotted against mean height z_m in Fig. 1; Chuen's results requiring

some conversion to make them relative to a standard reference height. Similar decay factors have been deduced here from Harris' natural wind data[8] and are presented in the same figure. Suitable power law exponents for these plots were estimated by means of the least squares method and are listed in Fig. 1, although one of Chuen's results appears spurious and was ignored in calculating the exponent from that set of data.

The range of magnitude of the decay factor varies from one plot to another even though three of the measurement sets were obtained over similar smooth terrain. However, there is close agreement between the values of the exponent, α_1 , found from the smooth terrain results, suggesting that the power law expression, equation (19), has some relevance.

It can be seen from Fig. 1 that the decay factor k_3

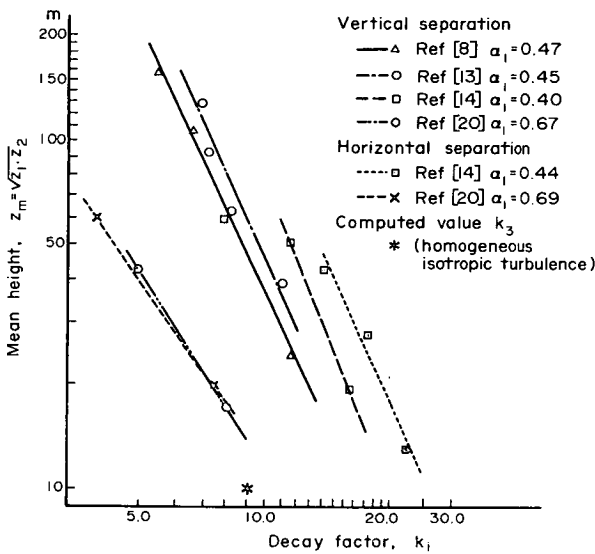


Fig. 1. Variations of decay factor.

computed for the standard reference height $z_r = 10$ m in homogeneous isotropic flow has a lower value than those indicated by extrapolation of the empirical power law curves to the same height.

Such differences could be expected since near to the ground turbulence is not homogeneous or isotropic (see Harris[8]) and under those circumstances the decay factor would be higher.

An interesting feature of Fig. 1 is that the computed value of k_3 for the reference height fits fairly well with the data[20] from the urban location.

It could be anticipated that the lines shown in Fig. 1 would converge on a point at the gradient height where homogeneous isotropic conditions should exist.

The data are not sufficient to form any very definite opinion about the value of α_1 and how it is influenced by the terrain roughness. The smoother surface data in Fig. 1 could be interpreted as converging on a common point at the gradient height. The urban terrain data are much more scant but might suggest that α_1 is greater for increased surface roughness. Since the gradient height increases with ground roughness any

points of convergence for rough and smooth data in a plot such as Fig. 1 could not be expected to coincide.

The six values of the exponent α_1 obtained from Fig. 1 are plotted against the corresponding power law exponent α of the mean wind speed profile in Fig. 2

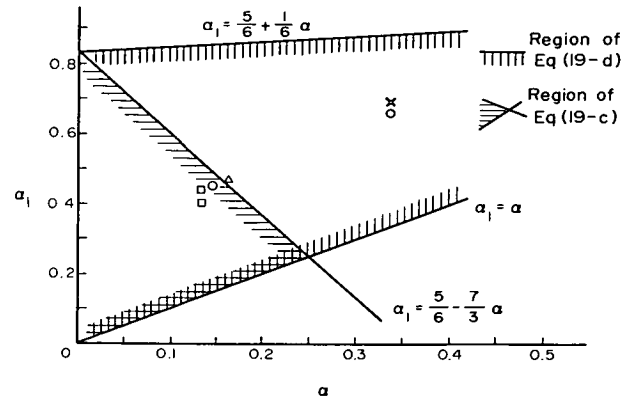


Fig. 2. Power law exponent of decay factor α_1 vs power law exponent of mean windspeed profile α .

and compared with the theoretical region given by equations (19c) and (19d). These equations are based on the power spectral density expressions, equations (2) and (3) respectively, and the upper and lower limits of the α_1 - α region could be improved with further experimental information on the relation between L_i and $\mathcal{L}_1$ in equation (18) or by using an alternative form for horizontal length $\mathcal{L}_1$.

The Fig. 2 type of plot should facilitate an understanding of the influence of ground roughness on the decay factor exponent α_1 when more data become available.

A comparison is made in Fig. 3(a), (b) and (c) between the theoretical expression for root-coherence, equations (7) and (19), empirical relations and measured data[8] for three pairs of different heights.

These three plots indicate that the proposed root-coherence expression, equations (7) and (19), is reasonably consistent with measured data and demonstrates its dependence on height.

DISCUSSIONS AND CONCLUSION

There are still not sufficient data available to confirm the consistency of the power spectral expression and the root-coherence expression, especially in a highly built-up area. Both expressions, however, have to be established for the purpose of prediction of the dynamic response of structures. Moreover, it is an important factor for a typical wind-resisting structure like a high rise building in a city centre to represent appropriately the power spectrum and the root-coherence (see Appendix A).

Recent measurements suggest that height dependency is significant for both the power spectrum and root-coherence, which consequently should be taken into account in the representations, with some allowance for the influence of terrain peculiarities. Also the

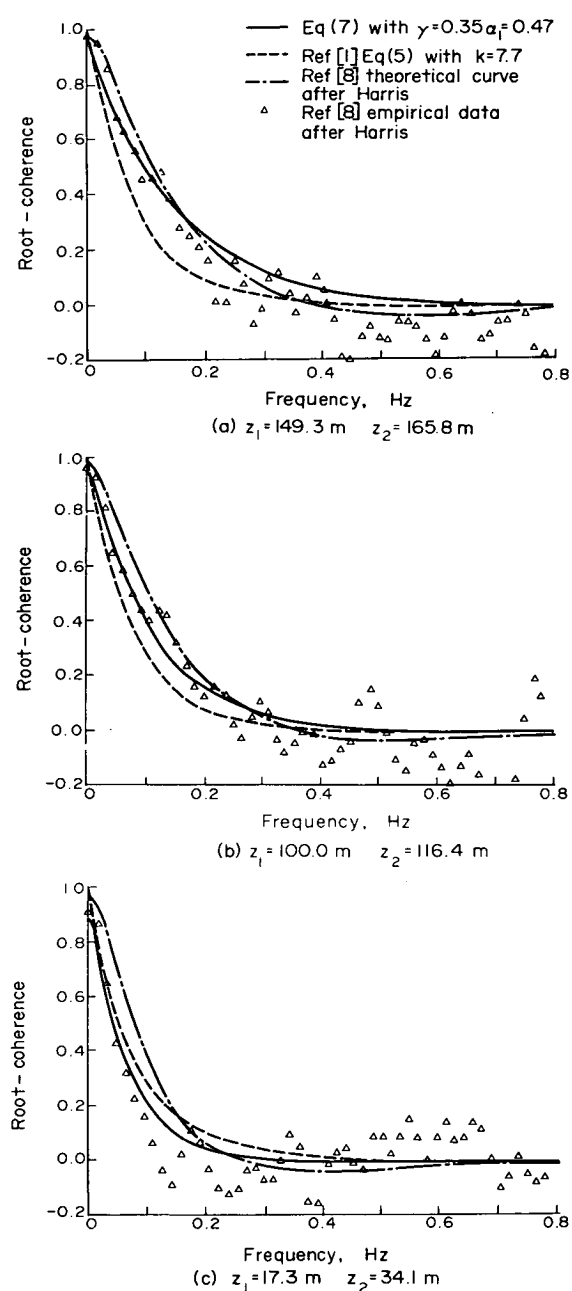


Fig. 3. Comparison between root-coherence functions and empirical data after Harris[8].

variation in value of the root-coherence function at zero frequency can be pointed out from recent measurements—it is less than unity unless the separation distance is zero. Having due regard to the above points, an exponential expression has been developed for the root-coherence function, introducing the modified frequency f^* instead of frequency f and the power law profile with height for the decay factors.

There are two restrictions in the application of the above expressions. Firstly the effect of the quadrature component of the cross-spectra or the phase angle is assumed to be negligible. However, since the natural wind turbulence is not completely homogeneous, the quadrature component exists, even though it may be small. This matter should be investigated in future—considering the effects of the quadrature component on the dynamic wind loading of structures.

Secondly, equation (15) for the vertical separation is only valid for the lower measuring point, given by z_1 , above the reference height z_r and z_r should be chosen such that the wind forces below this level do not make a significant contribution to the dynamic response of a structure. This suggests that the power law profile with the exponent α_1 , for the vertical decay factor k_v may not hold below the reference height. Further investigation of this point is required using measured data obtained near the ground.

It can be concluded that the expression proposed for the root-coherence of the longitudinal turbulent component between two points with vertical and horizontal separation is consistent with recent empirical data which indicate the height variation of the root-coherence.

The power exponent α_1 for the decay factor in the root-coherence expression is estimated from empirical data and the decay factors, k_H and k_v for horizontal and vertical separation respectively, could be evaluated at the reference height either theoretically or empirically.

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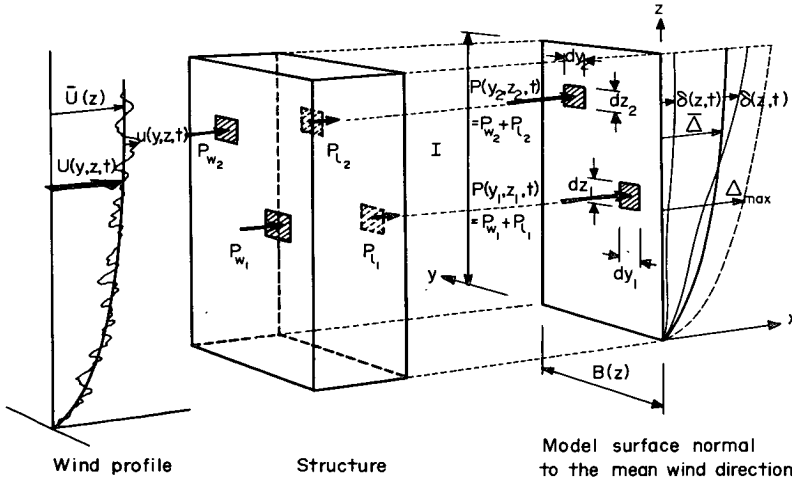


Fig. 4. Wind-structure model.

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APPENDIX A: PREDICTION OF ALONGWIND DYNAMIC RESPONSE OF STRUCTURES IN THE NATURAL WIND

The relationship between the fluctuating drag force and the longitudinal turbulent component of the natural wind is discussed and it is shown how the root-coherence function and the power spectral density of the longitudinal turbulent component contribute to the prediction of the dynamic alongwind response of structures. The theory is a modified form of that developed for a circular cylinder by Cooper and Surry[16].

The assumption of a 'strip theory' relationship between local drag and the local relative velocity for a two-dimensional body[18] can be applied also for a three-dimensional body, by replacing the local drag with the net pressure which is the difference of pressures on the windward and leeward surfaces of a structure. The net pressure $P(y, z, t)$ can be considered to act through the structure on the idealized surface normal to the mean wind direction as shown in Fig. 4.

$$P(y, z, t) = \frac{1}{2}\rho C_d(y, z, \xi)U'^2(y, z, t) + \rho B(z)C_M(y, z, \xi)\dot{U}'(y, z, t) \quad (A1)$$

where C_d is the drag coefficient and C_M is the mass coefficient and both are considered to vary with position and the reduced frequency ξ ;

$$\xi = \frac{f \cdot B(z)}{U(z)};$$

$$U'(y, z, t) = U(y, z, t) - \delta(y, z, t)$$

is the relative wind speed;

$$U(y, z, t) = \bar{U}(z) + u(y, z, t);$$

$\delta(y, z, t)$ is the longitudinal component of structure dynamic displacement; $\dot{\delta}(y, z, t)$ is the structure velocity; $B(z)$ is the width of a structure; and, ρ is the air mass density.

If the turbulence and the fluctuating motion of the structure are both small, the second order terms in u and δ can be neglected. Then equation (A1) can be written as,

$$\begin{aligned} P(y, z, t) = & \frac{1}{2}\rho C_d(y, z, 0)\bar{U}^2(z) \\ & + \rho C_d(y, z, \xi)\bar{U}(z)u(y, z, t) \\ & + \rho B(z)C_M(y, z, \xi)\dot{u}(y, z, t) \\ & - \rho C_d(y, z, \xi)\bar{U}(z)\dot{\delta}(y, z, t) \\ & - \rho B(z)C_M(y, z, \xi)\dot{\delta}(y, z, t). \end{aligned} \quad (A2)$$

The last two terms of equation (A2) are not dependent on the turbulence. Consequently they are considered as the additional damping and mass in the equation of motion of the structure. Then the dynamic part of the drag net pressure $p(y, z, t)$ can be written as,

$$\begin{aligned} p(y, z, t) = & \rho C_d(y, z, \xi)u(y, z, t)\bar{U}(z) \\ & + \rho B(z)C_M(y, z, \xi)\dot{u}(y, z, t). \end{aligned} \quad (A3)$$

The dynamic displacement response $\delta(y, z, t)$ is a random function made up of components from the various independent modes of vibration. It is treated by a statistical approach relating the power spectral density of $\delta(y, z, t)$ and the power spectral density of generalised total dynamic force $F(t)$, i.e. the dynamic response of a structure can be determined by solving the normal equations of motion[19], taking the aerodynamic damping and mass terms mentioned above into account.

$$\ddot{q}_n(t) + 2\zeta'_n(2\pi f_n)\dot{q}_n(t) + (2\pi f_n)^2 q_n(t) = \frac{F_n(t)}{M'_n} \quad (A4)$$

where q_n is the generalised displacement of the n th mode;

$$\delta(y, z, t) = \delta(z, t) = \sum_{n=1}^N \mu_n(z) \cdot q_n(t)$$

assuming the structural displacement to be uniform over its width; $\mu_n(z)$ is the n th mode shape; f_n is the n th natural frequency;

$$M'_n = \int_0^H \int_0^{B(z)} [m(z) + \rho B(z) C_M(y, z, \xi)] \mu_n^2(z) dy dz;$$

$m(z)$ is the mass of structure per unit surface area;

$$\zeta'_n = \zeta_n + \int_0^H \int_0^{B(z)} \rho \frac{C_d(y, z, \xi) \bar{U}(z) \mu_n^2(z)}{2M'_n(2\pi f_n)} dy dz;$$

ζ_n is the critical damping ratio of the structure of n th mode; and, $F_n(t)$ is the generalised force associated with the turbulence,

$$F_n(t) = \int_0^H \int_0^{B(z)} p(y, z, t) \mu_n(z) dy dz. \quad (A5)$$

For the most lightly damped structures the cross-coupling between modes is unlikely. Therefore the power spectral density of the response $S_\delta(f)$ can be written as follows as a solution of equation (A4),

$$S_\delta(f) = \sum_{n=1}^N \mu_n^2(z) |\chi_n(f)|^2 S_{F_n}(f) \quad (A6)$$

where

$$|\chi_n(f)|^2 = \frac{1}{(4\pi^2 M'_n)^2 [f^4 + f_n^4 + (4\zeta_n'^2 - 2)f^2 f_n^2]}.$$

Now the generalised force of the n th mode $F_n(t)$ can be computed by substituting equation (A3) into (A5).

$$F_n(t) = \int_0^H \int_0^{B(z)} \{ \rho C_d(y, z, \xi) \bar{U}(z) u(y, z, t) + \rho C_M(y, z, \xi) B(z) \dot{u}(y, z, t) \} \mu_n(z) dy dz. \quad (A7)$$

In equation (A7), $F_n(t)$ is expressed as a function of t and ξ , but since for lightly damped structures only the components of response in the narrow band of frequency around resonance in a particular mode will be of significance, therefore only the corresponding components of C_d and C_M need be taken into account. Then equation (A7) can be modified accordingly.

$$F_n(t) = \int_0^H \int_0^{B(z)} \{ C_{d_n}^*(y, z) u(y, z, t) + C_{M_n}^*(y, z) \dot{u}(y, z, t) \} dy dz \quad (A8)$$

where

$$C_{d_n}^*(y, z) = \rho C_d(y, z, \xi_n) \bar{U}(z) \mu_n(z),$$

$$C_{M_n}^*(y, z) = \rho C_M(y, z, \xi_n) B(z) \mu_n(z).$$

$$\xi_n = \frac{f_n B(z)}{\bar{U}(z)}.$$

Now $S_{F_n}(f)$ can be defined as a Fourier transform of the auto-correlation function of $F_n(t)$ as follows,

$$S_{F_n}(f) = 2 \int_{-\infty}^{\infty} \mathcal{R}_{F_n}(\tau) e^{-i2\pi f \tau} d\tau \quad (A9)$$

where

$$\mathcal{R}_{F_n}(\tau) = E[F_n(t) \cdot F_n(t + \tau)]. \quad (A10)$$

Substituting equation (A8) into (A10) and equation (A10) into (A9), the power spectral density function of generalised force of the n th mode S_{F_n} can be computed[17].

$$\begin{aligned} S_{F_n}(f) = 2 \int_{-\infty}^{\infty} \int_0^H \int_0^H \int_0^{B(z)} \int_0^{B(z)} \mathcal{R}_{u_1, u_2}(\tau) \\ \times \{ C_{d_n}^*(y_1, z_1) C_{d_n}^*(y_2, z_2) + 2\pi i f C_{d_n}^*(y_1, z_1) \\ \times C_{M_n}^*(y_2, z_2) - 2\pi i f C_{M_n}^*(y_1, z_1) C_{d_n}^*(y_2, z_2) \\ + 4\pi^2 f^2 C_{M_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2) \} dy_1, dy_2 \\ \times dz_1 dz_2 e^{-i2\pi f \tau} d\tau. \end{aligned} \quad (A11)$$

Since the cross-spectrum of the turbulent component is given as

$$S_{u_1, u_2}(f) = 2 \int_{-\infty}^{\infty} \mathcal{R}_{u_1, u_2}(\tau) e^{-i2\pi f \tau} d\tau \quad (A12)$$

equation (A11) can be rearranged using equation (A12), namely,

$$\begin{aligned} S_{F_n}(f) = \int_0^H \int_0^H \int_0^{B(z)} \int_0^{B(z)} S_{u_1, u_2}(f) \\ \times \{ C_{d_n}^*(y_1, z_1) C_{d_n}^*(y_2, z_2) + i2\pi f \\ \times [C_{d_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2) - C_{M_n}^*(y_1, z_1) C_{d_n}^*(y_2, z_2)] \\ + 4\pi^2 f^2 C_{M_n}^*(y_1, z_1) C_{M_n}^*(y_2, z_2) \} \\ \times dy_1 dy_2 dz_1 dz_2. \end{aligned} \quad (A13)$$

If C_d and C_M are assumed to be constant with y and have the same profile with z ,

$$C_{d_n}^*(z_1) C_{M_n}^*(z_2) - C_{M_n}^*(z_1) C_{d_n}^*(z_2) = 0.$$

Consequently equation (A13) can be simplified as,

$$\begin{aligned} S_{F_n}(f) = \int_0^H \int_0^H \int_0^{B(z)} \int_0^{B(z)} S_{u_1, u_2}(f) \\ \times \{ C_{d_n}^*(z_1) C_{d_n}^*(z_2) + 4\pi^2 f^2 C_{M_n}^*(z_1) C_{M_n}^*(z_2) \} \\ \times dy_1 dy_2 dz_1 dz_2. \end{aligned} \quad (A14)$$

Generally the natural wind is not a homogeneous turbulent flow and so the cross-spectral density function consists of real and imaginary parts. However, since the power spectrum of the generalised force is a real function the real part of the cross-spectrum (i.e. the co-spectrum) can be taken into account instead of $S_{u_1, u_2}(f)$ in equation (A14).

For further simplification, if the imaginary part or quadrature component of the cross-spectrum of longitudinal turbulence is assumed to be negligible, the root-coherence becomes identical with the normalised co-spectrum, i.e.

$$\begin{aligned} S_{F_n}(f) = \int_0^H \int_0^H \int_0^{B(z)} \int_0^{B(z)} R_{u_1, u_2}(f) \sqrt{S_{u_1}(f) S_{u_2}(f)} \\ \times \{ C_{d_n}^*(z_1) C_{d_n}^*(z_2) + 4\pi^2 f^2 C_{M_n}^*(z_1) C_{M_n}^*(z_2) \} \\ \times dy_1 dy_2 dz_1 dz_2. \end{aligned} \quad (A15)$$

Then the variance of the dynamic response can be obtained as follows,

$$\overline{\delta^2}(z) = \int_0^\infty S_\delta(f) df$$

and from equation (A6),

$$\overline{\delta^2}(z) = \int_0^\infty \sum_{n=1}^\infty \mu_n^2(z) |\chi_n(f)|^2 S_{F_n}(f) df. \quad (A16)$$

For a tall building which has its fundamental natural frequency much greater than the peak frequency of the power spectrum of wind turbulence it should be sufficient to consider only the first two frequency modes of vibration, i.e. $N = 2$.

The instantaneous maximum value of the dynamic response can be obtained from the mean displacement and the r.m.s. value in terms of a peak factor g , which depends on a probability distribution[7], as,

$$\Delta_{\max} = \bar{\Delta} + g \cdot \sqrt{\overline{\delta^2}}. \quad (A17)$$

APPENDIX B: INTEGRATION OF A POWER SPECTRAL DENSITY FUNCTION

All power spectral expressions referred to here—equations (1–4)—can be integrated with respect to the frequency from 0 to infinity and the value of the definite integral becomes unity when those power spectral expressions are normalised by the variance of the turbulent component. Then each constant k_1 in those equations can be obtained. For

$$F(x) = \frac{1}{(1+x^a)^b} \quad (B1)$$

the definite integral from $f = 0$ to infinity can be computed as follows. Let $X = x^a$, then

$$dX = ax^{a-1} dx = aX^{(a-1)/a} dX.$$

Consequently,

$$\int_0^\infty F(x) dx = \int_0^\infty \frac{dx}{(1+x^a)^b} = \int_0^\infty \frac{X^{-(a-1)/a}}{a(1+X)^b} dX. \quad (B2)$$

Since it is known that

$$\int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \quad (B3)$$

where $\Gamma(\cdot)$ is a gamma function, from equation (B2)

$$n-1 = -\left(\frac{a-1}{a}\right), \quad m+n = b$$

which yields

$$n = \frac{1}{a}, \quad m = b - \frac{1}{a}.$$

Then equation (B2) becomes

$$\int_0^\infty F(x) dx = \frac{\Gamma\left(\frac{1}{a}\right)\Gamma\left(b - \frac{1}{a}\right)}{a\Gamma(b)}. \quad (B4)$$

In the case of equation (1) a direct integration is possible without resort to gamma functions since it can be rewritten as,

$$\frac{S_u(f)}{u^2} = \frac{k_1 \mathcal{L}_1}{U_r} \cdot \frac{x_1}{(1+x_1^2)^{4/3}}$$

and so,

$$\begin{aligned} \frac{1}{u^2} \int_0^\infty S(f) df &= 1 = \frac{k_1 \mathcal{L}_1}{U_r} \int_0^\infty \frac{x_1}{(1+x_1^2)^{4/3}} df \\ &= k_1 \int_0^\infty \frac{x_1}{(1+x_1^2)^{4/3}} dx_1 = \frac{k_1}{2} \int_0^\infty \frac{dX}{X^{4/3}} = \frac{3}{2} \cdot k_1 \end{aligned}$$

hence

$$k_1 = \frac{2}{3}. \quad (B5)$$

By contrast equation (4) is in the form,

$$\frac{S_u(f)}{u^2} = \frac{k_1 \mathcal{L}_1}{U_r} \cdot \frac{1}{(1+x_1^\beta)^{5/3\beta}}$$

and

$$\begin{aligned} \frac{1}{u^2} \int_0^\infty S(f) df &= 1 = \frac{k_1 \mathcal{L}_1}{U_r} \int_0^\infty \frac{1}{(1+x_1^\beta)^{5/3\beta}} df \\ &= k_1 \int_0^\infty \frac{dx_1}{(1+x_1^\beta)^{5/3\beta}}. \end{aligned}$$

Consequently, by comparison with equation (B1)

$$k_1 \int_0^\infty F(x_1) dx_1 = 1$$

where $a = \beta$ and $b = 5/3\beta$. Then from equation (B4)

$$k_1 = \frac{\beta \Gamma\left(\frac{5}{3\beta}\right)}{\Gamma\left(\frac{1}{\beta}\right) \Gamma\left(\frac{2}{3\beta}\right)}. \quad (B6)$$

The values k_1 corresponding to the expressions in equations (2–4) are obtained from equation (B6) and are summarised in Table B1 together with the value of k_1 appropriate to equation (1) which is given by equation (B5). Similarly the integral in equation (19a) can be evaluated for $\beta = 2$, i.e.

$$\int_0^\infty \frac{dx_1}{(1+x_1^2)^{5/6} (1+x_1^2)^{1/2}} = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{5}{6}\right)}{2\Gamma\left(\frac{8}{6}\right)} = 1.119 \quad (B7)$$

which is required in order to establish k_3 .

Table B1

Equation	Source	Power exponent β	k_1
(1)	Davenport[1]		2/3
(2)	Harris[8] and Hino[2]	2	0.475
(3)	Simiu[3]	1	2/3
(4)	National Bureau of Standards[9]	0.845	0.769
	Panofsky and Lumley[10]	5/3	0.505

A transducer for determining dynamic drag and lift coefficients in wind tunnels

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A transducer is described capable of determining dynamic drag and lift coefficient for bluff bodies in air flow. Such factors and their frequency dependence are becoming increasingly important in the wind excitation of structures.

The device incorporates facilities for varying the angle of attack on the model structure surface and although developed for studies on bluff bodies it can be adapted readily to other bodies of uniform section including aerofoil, circular cylindrical, and more general shapes.

Dynamic output is derived from electric foil strain gauges and circuitry yielding measurements of good linearity, repeatability and stability.

Introduction

A neutrally stable turbulent air flow can be described at any point by a resultant mean velocity $\bar{U}$ and three turbulent dynamic velocity components u , v and w along and at right angles respectively to the mean flow direction.

It is becoming increasingly evident that the response of a structure to such a flow is made up of dynamic components in the transverse flow direction of v or w depending on whether the structure is vertical or horizontal, plus a dynamic and static contribution in the mean flow direction, usually horizontal.

Hitherto lift coefficients C_L which relate automatically the forces in the transverse flow direction to the mean velocity $\bar{U}$, and static drag coefficients C_D , relating forces in the flow direction to $\bar{U}$ have been established for various shapes in smooth flow. Definitions of these coefficients are given in the appendix.

In turbulent flow Maclaren¹ and Lee^{2,3} have studied the effect of turbulence scale and intensity on the static drag coefficient for square section bodies and further investigation of these effects on bluff bodies of more general shape is necessary. Additionally the influence of these turbulence characteristics on the dynamic drag and lift coefficients (see Appendix) requires investigation in order to improve methods of dynamic response prediction for wind excited structures.

Current methods for such prediction include an admittance theory developed by Davenport^{4,5} which has a limitation in that it employs a constant drag coefficient. Modifications to this approach have been suggested by Simiu⁶ in which correlation between windward and leeward face pressures is taken into account. By contrast Vickery⁷ and Ellis⁸ have presented methods involving the computation of force spectra which yield root mean square drag and lift coefficients. Prediction methods based on the frequency dependence of the dynamic drag and lift coefficients have been made by some authors^{9,10,11} but such methods require much more supporting information of an experimental nature to prove their worth.

This paper describes a transducer which enables the dynamic drag and lift coefficients for various shapes of bluff body to be determined.

Notation

B	width of a structure
C_D	static drag coefficient
$C_{D_{dyn}}$	dynamic drag coefficient
C_L	static lift coefficient
$C_{L_{dyn}}$	dynamic lift coefficient
D	depth of a structure (alongwind dimension)
f_0	natural frequency of a system
k	spring constant
L_x	longitudinal turbulence scale
M	mass
$\bar{U}$	mean wind speed
u	longitudinal fluctuating wind speed
$\sqrt{\bar{u}^2}$	root mean square of u
v	lateral fluctuating wind speed
w	vertical fluctuating wind speed
x	longitudinal co-ordinate
y	lateral co-ordinate
z	vertical co-ordinate
γ	structural mass density
ν	kinematic viscosity
ρ	air mass density
ζ	damping ratio

Basic design features and requirements

In order to evaluate dynamic drag or lift forces on a body for a certain frequency a two-dimensional single degree of freedom (S.D.O.F.) system model with light damping was designed and employed as a transducer. A general view of the model positioned centrally in its shroud and separated by a narrow air gap from dummy shrouds of similar section is shown in Fig 1 with the model mounted horizontally and transversely across a wind tunnel. The dummy

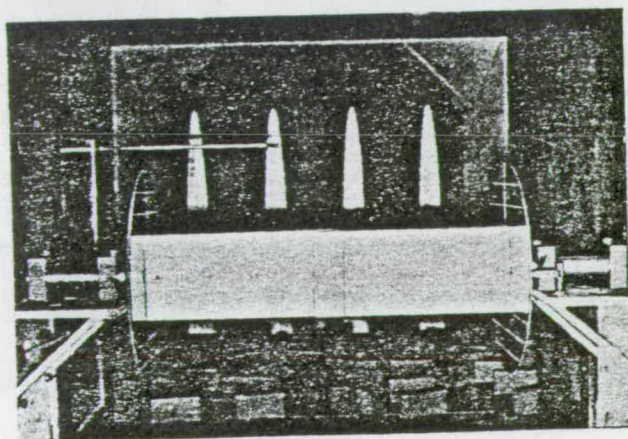


Fig 1. General view of transducer.

'Strain', October 1978

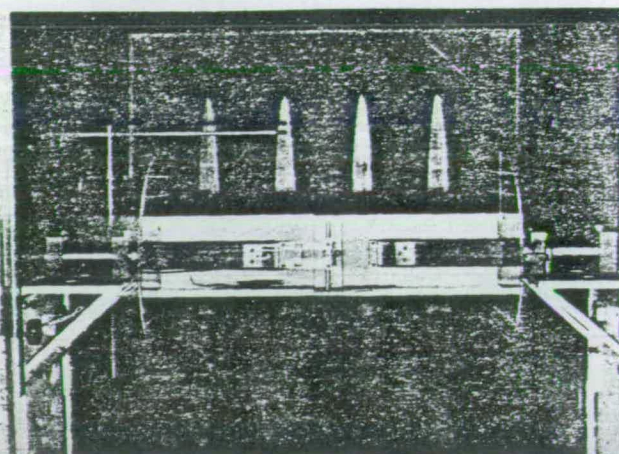


Fig 2. General view of transducer with shrouds removed.

shrouds were carried on separate shafts from the one on which the model was clamped, Figs 2 and 3. The model was to consist of a mass supported by two pairs of plate springs to allow non-rotational deflection in one direction. Also some facility for varying the angle of attack of the flow on the model surface was desired.

Measurement of the response of the model could be achieved in more than one way and three alternatives were considered,

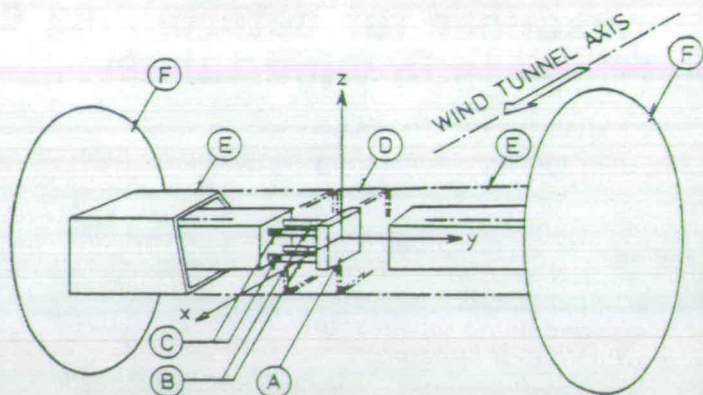
- (i) electric wire or foil resistance strain gauges placed at the base of the double curvature columnar spring plates;
- (ii) a miniature lightweight accelerometer located on the vibrating mass;
- (iii) a small displacement transducer monitoring translation of the mass relative to its base.

Lack of availability ruled out (iii) whereas a miniature accelerometer of 0.25 g mass was tried in combination with (i) but found to be unstable and resulting in the adoption of alternative (i).

Generally there are severe restrictions for this type of model because of limitations in wind tunnel facilities. For example, it is difficult usually to produce large scale turbulence in wind tunnels and models must be appropriate to the turbulence scale in order to satisfy similarity conditions—the smaller the turbulence scale the smaller the model. By contrast from the aspect of instrumentation and accurate construction it is better for a model to be of large dimension.

For the wind tunnel simulation of a full scale structure several non-dimensional parameters must be satisfied.¹² These can be summarised as follows for the two-dimensional interaction between wind speed and forces.

1. $\frac{\bar{U}}{f_0 D}$, reduced velocity or inverse or reduced frequency.
2. $\frac{\gamma}{\rho}$, mass density ratio between structure and air.
3. $\frac{\sqrt{\bar{u}^2}}{\bar{U}}$, local turbulence intensity.
4. $\frac{L_x}{D}$, ratio of turbulence scale to the reference dimension of a structure.
5. $\frac{B}{D}$, section aspect ratio.



A : Centre mass	D : Model shroud
B : Plate springs	E : Dummy shroud
C : Strain gauges	F : End disc

Fig 3. Diagrammatic representation of transducer construction

6. $\frac{\bar{U} D}{\nu}$, Reynolds number.

The range of each parameter depends on the type of structure under consideration. If a modern tall building of height greater than 100 m is taken as an example, then from the possible range of values of the physical quantities involved the range of values of the parameters can be estimated approximately as follows,

$$\begin{aligned} \frac{\bar{U}}{f_0 D} &\approx 2.0 \text{ to } 20.0; & \frac{\gamma}{\rho} &\approx 250 \text{ to } 500; \\ \frac{\sqrt{\bar{u}^2}}{\bar{U}} &\approx 8\% \text{ to } 30\%; & \frac{L_x}{D} &\approx 0.5 \text{ to } 10.0; \\ \frac{B}{D} &\approx 0.2 \text{ to } 5.0; & \frac{\bar{U} D}{\nu} &\approx 10^7. \end{aligned}$$

If a lower building is of interest, a smaller reduced velocity and a greater section aspect ratio should be used. When a small dimension structure such as a chimney or tower is in question a larger turbulence scale ratio should be employed.

In this work the models were designed to investigate the effects of these parameters on the drag and lift coefficients as extensively as possible within the limitations of the wind tunnel available.

The wind tunnel to be used was of the open jet type with an apron and had a total length of 9.15 m. The dimensions of the outlet on to the apron were, width = 1.53 m and height = 1.07 m. The maximum velocity of the air flow was continuously adjustable between 2.0 and 9.0 m/s. A shear velocity profile with reasonable turbulence intensity was establishable using spires and roughness blocks representing a partial boundary layer.¹³

Two positions were chosen for the models to give an opportunity of varying the local turbulence intensity between 10 and 14%, and of achieving a turbulence scale ratio $\frac{L_x}{D}$ near to unity. An $\frac{L_x}{D} \approx 1.0$ could be expected to produce a different air flow condition around a body from the cases of $\frac{L_x}{D} \ll 1.0$ and $\frac{L_x}{D} \gg 1.0$.¹⁴ However it is desirable to extend this parameter up to 10.0 or more probably by using a larger wind tunnel.

Seven shapes of the model were selected to give different $\frac{B}{D}$ and $\frac{L_x}{D}$ values and the range of the latter was extended using different sizes of square section. The lower limit of D was dictated by the physical size of components available for model construction and its upper limit was constrained by the requirement that the wind characteristics should not change over the alongwind dimension of the model.

The average mass density of the model was fixed at approximately 400 kg/m^3 . Although some modern buildings have a lower mass density value¹⁵ its effect on aerodynamic forces may not be as significant as the effects of other parameters such as $\frac{\bar{U}}{f_0 D}$, $\frac{\sqrt{\bar{U}^2}}{\bar{U}}$, $\frac{L_x}{D}$, and $\frac{B}{D}$.

The contribution of air mass to the total mass involved in the inertia of a structure is usually small and can be neglected except in the case of very light structures.

Since the air flow around a bluff body is not influenced much by the Reynolds number¹² its effect can be neglected; a state of affairs which does not hold for cylindrical structures of circular section.

Considering the desirable range of reduced velocity and the maximum available velocity in the wind tunnel the ranges of natural frequency for the models were determined. Five spring systems were designed to provide those natural frequencies for known dimensions and masses of the models taking into account requirements such as,

- maximum deflection should be well within the order of the model dimension D ,
- spring axial displacement (y -direction in Fig 3) should be less than the gap of separation between the dynamic model and the dummy shroud,
- maximum strain should be below the proportional limit of the spring material, and
- minimum force should produce a significant signal relative to background noise.

The five different spring systems were designed to meet the conditions mentioned above and in each case both ends of a spring were clamped firmly so that it would deform in double curvature, see Fig 4. For convenience of installation and calibration the transducer was mounted horizontally across the wind tunnel with circular discs at each end of the assembly, Figs 1 to 3, to channel the flow past the

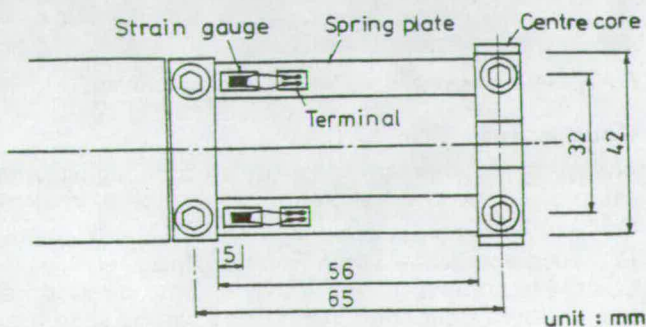


Fig 4. Strain gauge locations.

model, the discs being sufficiently remote from the model for the flow in its vicinity to be undisturbed by them. The whole system was firmly fixed to a rigid frame of steel channels ($305 \text{ mm} \times 102 \text{ mm}$ @ 46.8 kg/m) and steel equal angles ($76 \text{ mm} \times 76 \text{ mm}$ EA @ 10.57 kg/m). The frame was placed on four rubber vibration insulators independent of the wind tunnel. A pair of apron walls were attached to the inner faces of the frame posts so that the flow approaching the model assembly was affected only by the end disc plates and not the frame members.

The frame permitted the horizontal position of the model assembly to be varied relative to the floor of the wind tunnel. The whole frame and assembly could be moved into and out of position at the mouth of the wind tunnel on a hydraulic trolley.

A means of rotating the model and shrouds about the horizontal axis transverse to the flow was incorporated in the apparatus to permit any angle of attack on the model surface to be obtained. The shaft clamps can be seen clearly in Figs 1 and 2 external to the discs. The angle of attack could be set manually using a circular scale attached to one of the end discs, linking the two shroud carrying shafts together and rotating with a handle, see Fig 9. This feature facilitated calibration of the transducers.

The core part of a model was made of aluminium to minimise weight which could be varied by fixing some additional known steel plate weights on to the core enabling a fairly constant average model structure density to be maintained.

The dummy shrouds were made of plywood having exactly the same sectional profile as that of the corresponding model shroud which was made of balsa wood. Fig 5 shows the array of model and dummy shrouds employed with the transducer, the former ones being in the foreground.

The total range of values achieved for the parameters mentioned above are given in Table 1 together with actual dimensions for the cross sections of the models.

Instrumentation

As mentioned in the previous section electric resistance strain gauges were used to measure the dynamic response of the model. Although larger gauges would increase the accuracy and sensitivity a gauge length of 3 mm was chosen as the largest possible for the plate springs used (minimum width 5 mm). Foil gauges were preferred to wire gauges for this type of dynamic transducer because of their

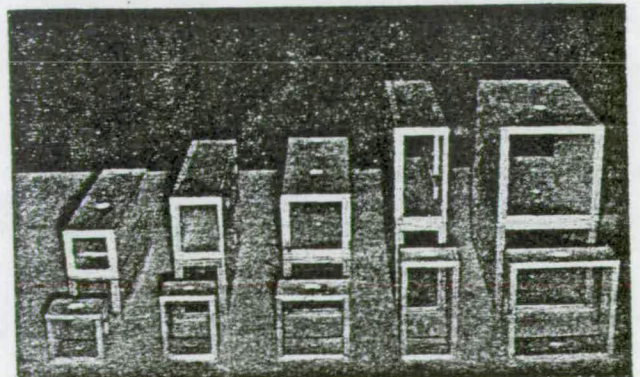


Fig 5. Array of shrouds.

Table 1 Range of variable parameters

Shape of Model			Turbulence Scale Ratio $\frac{L_x}{D}$		Natural Frequency f_0	Reduced Velocity $\frac{\bar{U}}{f_0 D}$
No.	B × D (mm)	$\frac{B}{D}$	Position		(Hz)	
			1	2		
I	60 × 60	1.00	1.9	1.0	7.0 to 22.0	2.3 to 19.0
II	80 × 60	1.33	1.9	1.0	6.0 to 17.0	2.9 to 22.0
III	60 × 80	0.75	1.4	0.8	6.0 to 17.0	2.2 to 16.7
IV	80 × 80	1.00	1.4	0.8	5.0 to 16.0	2.3 to 20.0
V	120 × 60	2.00	1.9	1.0	4.5 to 15.0	3.3 to 29.5
VI	60 × 120	0.50	1.0	0.5	4.5 to 15.0	1.6 to 15.0
VII	120 × 120	1.00	1.0	0.5	3.0 to 11.0	2.3 to 22.0
Total range			0.5 to 1.9		3.0 to 22.0	1.6 to 29.5

Where (i) positions 1 and 2 are 250 mm and 400 mm respectively above wind tunnel floor level

(ii) Reynolds number $\frac{\bar{U}D}{\nu} = 10^4$ to 10^5

(iii) average mass density of structure $\simeq 400$ kg/m³

(iv) mean air flow velocity $\bar{U} = 3.0$ to 8.0 m/s.

longer fatigue life. The gauges employed were T.M.L. type FLA-3-11 with a resistance of 120 ± 0.3 ohms and gauge factor of 2.08. The position of each gauge on a spring plate was 5 mm from its centre to the clamp at the fixed end of the shaft. The gauges were bonded according to the manufacturer's recommendations using a cyano-acrylate (C.N.) adhesive. After bonding all gauges were coated with a P.V.C. coating (VYCOAT ACA 60) for environmental protection. Gauge locations are shown in Fig 4.

Each gauge was wired to a terminal strip bonded on to the corresponding plate spring and two gauges were used in the active arm of a half bridge circuit with two similar dummy gauges in the temperature compensating arm. Bridge completion was internal to a strain gauge transducer meter (SANGAMO type C56—NT9) incorporating zero balance and offset, gauge factor, and range expansion controls. The dummy gauges were fixed in the same manner as the active ones to a similar piece of spring plate located inside the dummy shrouds for temperature compensation during test measurements. The bridge excitation was via an oscillator supply at 5 V r.m.s. and carrier frequency of 5 kHz.

The output from the transducer was amplified to a level suitable for recording on an oscillograph (U.V. recorder). The electrical interconnection of these instruments is shown in Fig 6. The zero shift during a test was checked by a digital indicator visually and by an oscillograph recording before and after each test run and was found to be negligible. Each test run lasted only a few minutes but before a test series an initial warm-up period of an hour was observed.

In the first place all data were recorded by an ultra violet recorder (Honeywell 6 channel Visicorder type 1706) and reduced manually to make files of digital data from which the mean and r.m.s. deflections were computed.

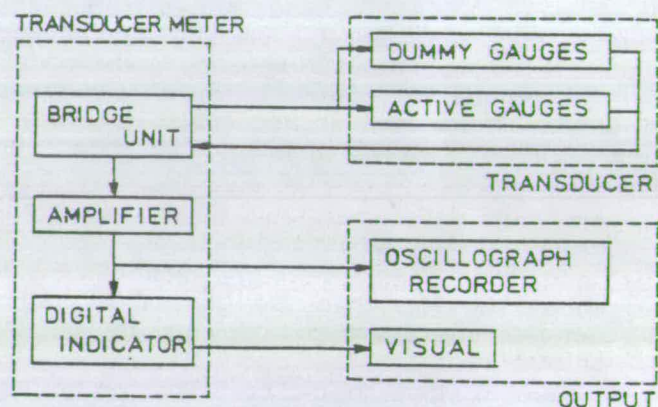


Fig 6. Instrumentation block diagram.

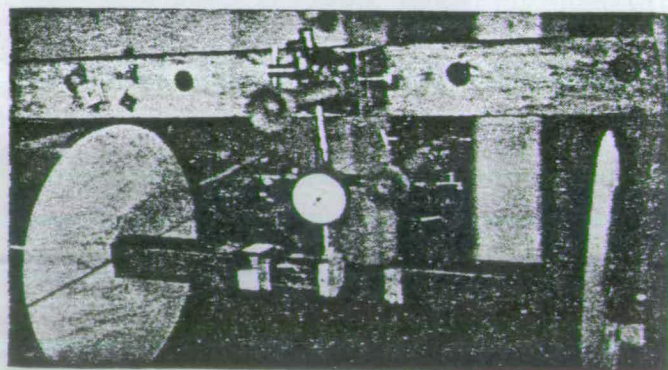


Fig 7. Test arrangement for displacement calibration.

Calibration

The dynamic characteristics for an actual test were established by the calibration of a model in three independent ways.

(1) Displacement—Strain Relationship:

Load was applied manually in the same direction as the air flow (x direction in Fig 3) as can be seen from the plan view in Fig 7. Horizontal displacements up

to 4 mm were measured by dial gauge. This maximum displacement was well in excess of the estimated maximum deflection of a model under test conditions. The loading was repeated four times to obtain a linear relationship between displacement and strain from the four sets of readings. Strain was inferred by the deflection of the light beam on the oscillograph record.

A typical calibration is shown in Fig 8 for mass-spring system G prepared to take model shape II.

(2) Load—Strain Relationship:

After rotating the model 90° about its axis load was applied vertically by means of known dead weights, see Fig 9. The maximum load was 0.9N comprising 0.5N self weight (without shrouds) and 0.4N of additional weights. This load was in excess of the estimated maximum wind force which was approximately 0.8N for the largest model in the fastest air flow. The calibration was repeated four times and a mean of the four sets of results determined.

A typical calibration for mass-spring system G prepared for model shapes I to VII is given in Fig 10. Strain was represented again by the deflection of the light beam on the oscillograph record.

The two types of calibration, 1 and 2, enable a force-displacement curve to be deduced for a particular spring system and model shape yielding the spring constant of a system.

(3) Free Vibration Tests:

Free vibration tests were performed on a mass-spring system before and after each set of response measurements or test run which included recordings at four different wind speeds. The natural frequency and

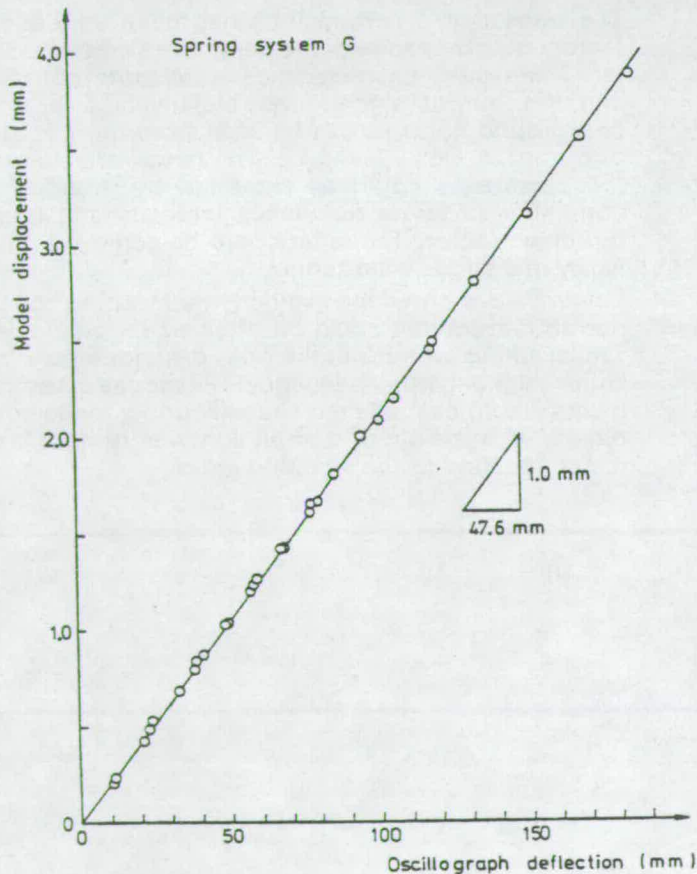


Fig 8. Typical displacement calibration.

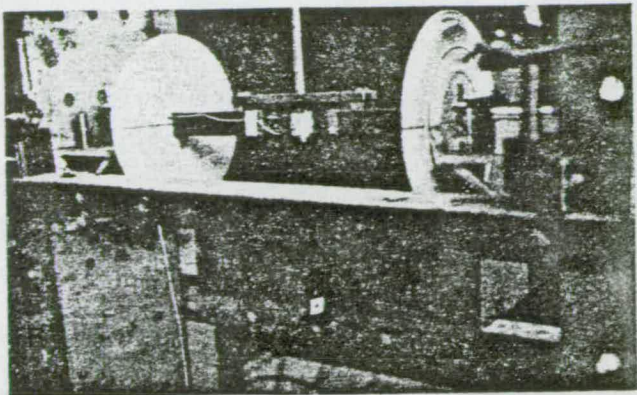


Fig 9. Test arrangement for force calibration.

damping ratio of a mass-spring system were computed from a 100 wave decaying curve. The natural frequency was compared with the corresponding value calculated using the measured mass and spring constant obtained from calibration 1 and 2. Agreement between these two methods of determining natural frequency for a system was good.

Figure 11 shows a typical example of transducer output under free vibration conditions for mass-spring system G with model shape II. Using Fig 10 the vertical scale was imposed to indicate force (positive in the x direction of the model for zero angle of attack) on the actual record.

Linearity, repeatability and stability

The curves shown in Figs 8 and 10 are typical of the good linearity found for displacement—strain and load—strain relationships over the full range of calibrations.

For one spring system in combination with the range of seven model shapes the calibrations lasted almost two days and Fig 10 demonstrates the degree of repeatability and stability achieved.

The spring constant *k* of a system was not affected by model shape and was computed from the gradients of the curves such as those in Figs 8 and 10 (spring system G) as follows.

$$k = 47600 \text{ (mm/m)} \times 0.00774 \text{ (N/mm)} \\ = 369 \text{ N/m.}$$

Also the spring constant for a system was computed independently from the free vibration tests using the different combinations of mass with a spring system. The results for spring system G are listed in Table 2.

Table 2

Model Shape	Natural Frequency <i>f</i> ₀ (Hz)	Mass <i>m</i> (g)	Spring Constant <i>k</i> (N/m)
I	13.1	54.6	371
II, III	10.9	79.4	372
IV	9.65	100.5	370
V, VI	9.0	116.0	371
VII	6.35	229.0	365

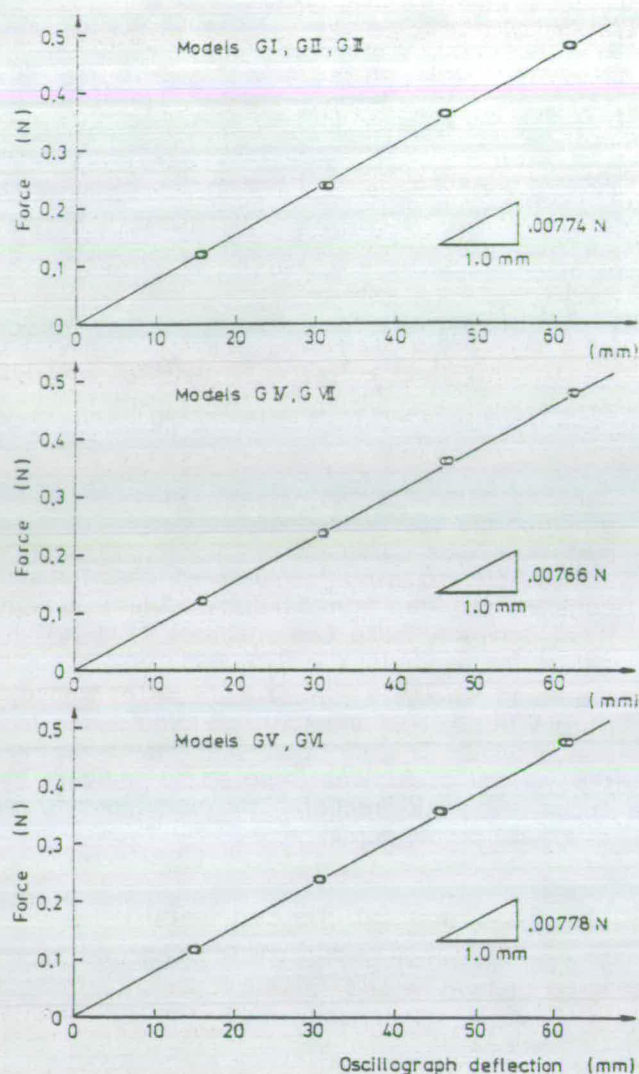


Fig 10. Typical force calibration.

The difference between the spring constant deduced from static calibrations and that from free vibration tests was in the range 0.5 to 2.0%; the difference being approximately 1% in the case of system G.

Conversely natural frequency computed from the static spring constant and measured mass compared very favourably with the measured natural frequency. Typically, for spring system G a discrepancy of 0.5% existed between calculation and measured value, the latter being the larger in this case.

The free vibration records confirmed that natural frequency did not vary with amplitude of displacement, as exemplified in Fig 11.

Damping in a system is best obtained by measurement as it is very difficult to predict. In this work the free vibration records, such as Fig 11 were used in the determination of damping ratio ζ from the decaying amplitude of a signal. Mean values of ζ found in this way ranged from 0.15 to 1.5% of critical (for system G the range of values for ζ was 0.18 to 0.65% of critical) and for any one free vibration test ζ remained reasonably constant with amplitude, e.g. the variation in ζ over the whole decay for each combination of spring system G with model shapes I to VII was as given below.

Operation

A complete test run on a model consisted of recording signals from the transducer at four different mean airflow velocity levels, the duration or dwell at any one level being between 20 s and 1 minute depending on the natural frequency of the system. Such test runs were carried out on each combination of five different spring systems and seven different model shapes at two heights above the tunnel floor with zero angle of attack. The transducer performed reliably and satisfactory dynamic responses were recorded, a typical sample of which is given in Fig 12 for system GII. The mean and r.m.s. values were computed for the response at each mean air velocity level and using the dynamic characteristics of a system the corresponding mean (or static) and dynamic drag coefficients were calculated.

Discussion

The transducer's performance has been very satisfactory over the range of the parameters employed so far. The system characteristics were quite constant and the output signals were relatively free from background noise as can be seen from the portions of output in Figs 11 and 12. The range of values of the parameters could be extended by introducing conditions of lower turbulence intensity and larger turbulence scale. The latter could be achieved more easily in a larger wind tunnel.

Lower values of reduced velocity more fitting to low rise stiff structures could be attained by employing similar spring systems to the ones described here but stiffer (higher natural frequency). Response measurements would be made more readily under those conditions with the aid of a small lightweight accelerometer attached to the vibrating mass.

Model Shape	I	II	III	IV	V	VI	VII
Standard deviation (S.D.) in ζ values over a 100 wave record (%)	8.8	2.0	8.9	9.4	6.1	5.5	3.4

$$\text{S.D.} = \frac{1}{\bar{a}} \left(\frac{\sum_{i=1}^N (a_i - \bar{a})^2}{N-1} \right)^{\frac{1}{2}}$$

where $\bar{a} = \frac{1}{N} \sum_{i=1}^N a_i$, and a_i is a value of ζ and $N=10$.

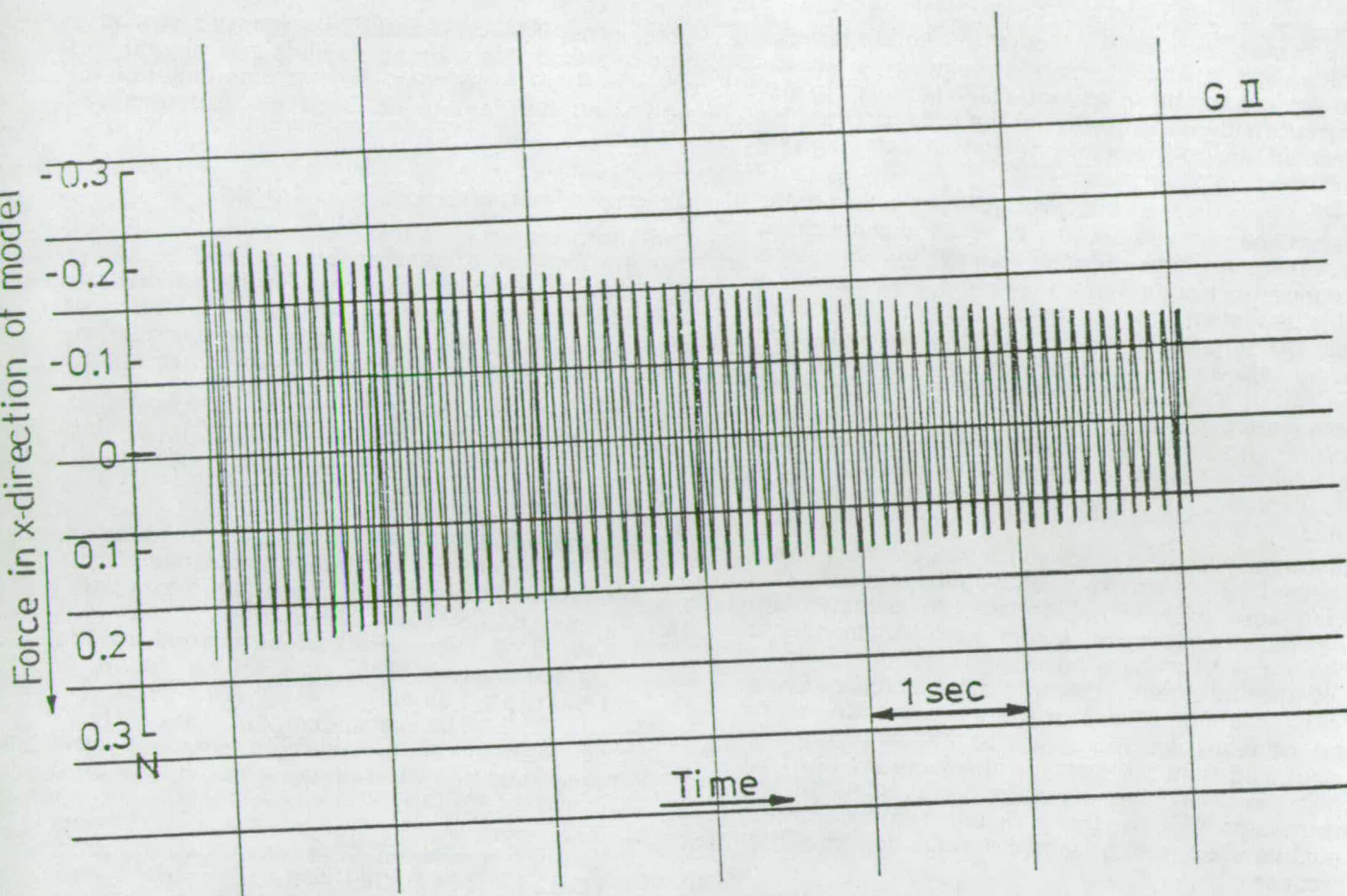


Fig 11. Typical free vibration record.

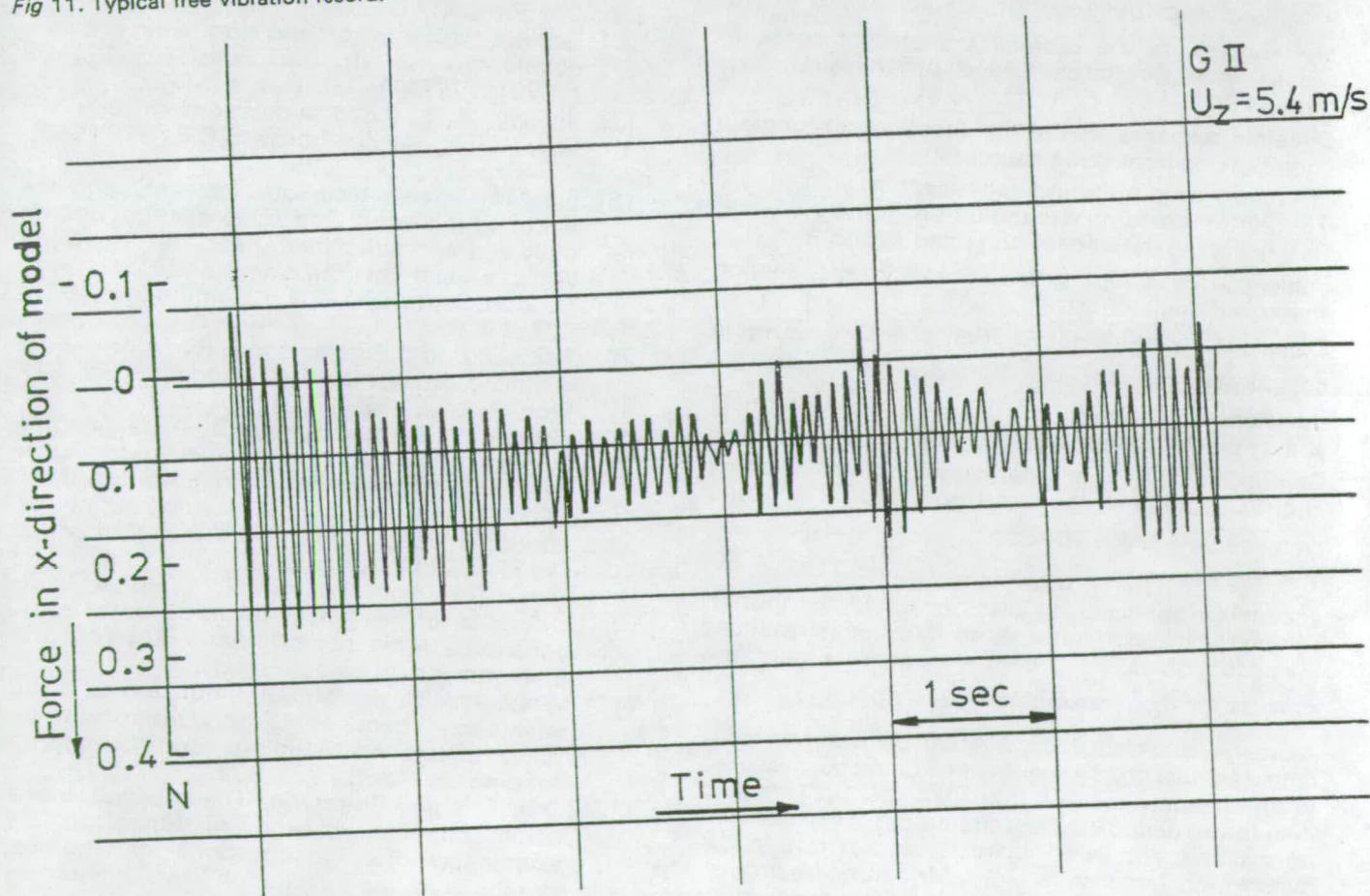


Fig 12. Typical wind induced response record.

The manual reduction of data to digital form is tedious and in future tests it is intended to replace the oscillograph by an analogue tape recorder. In this way subsequent digitisation of the records could be achieved electro-mechanically and all data analysis performed on a computer.

The mean value of ζ established for each spring-mass system had standard deviations which in some cases appear quite large (approximately 9% in some instances for spring system G, Table 2). However it must be remembered that each S.D. was computed from 10 samples, 5 taken from the free vibration record before a test run and 5 from a similar record at the end of that run. Differences of the order of 2% between the two free vibration records and manual reading errors ($>1\%$ for records whose amplitudes lay between ± 5 and ± 50 mm of the chart and trace line width ≈ 0.2 mm) are all incorporated in the S.D. value.

Although reasonably constant values of ζ were obtained for the spring-mass systems studied cases could arise in which ζ varied considerably with amplitude of response. Under such circumstances the r.m.s. level of the wind excited response could be multiplied by a peak factor of $\sqrt{2}$ and compared with the corresponding level of the free vibration record and the value of ζ appropriate to that level could be established from the decay in the same region. This could be considered a reasonable estimate of the mean value of ζ for the particular response and it could be used in drag factor determination from that response.

This type of transducer can be applied easily to the investigation of dynamic lift coefficients, $C_{L_{dyn}}$ for bluff bodies. In the case of the present series of shapes they would be turned on the shaft axis through 90° .

Dynamic response in the lift direction can contain contributions from three sources,

- (i) cross wind buffeting dependent on v ,
- (ii) vortex shedding dependent on $\bar{U}$ and u ,
- (iii) galloping dependent on $\bar{U}$ and u , and is influenced by damping, mass density ratio, section shape and angle of attack.

Consequently the determination of a $C_{L_{dyn}}$ is more complicated than for dynamic drag, $C_{D_{dyn}}$, in which the alongwind buffeting effects (dependent on $\bar{U}$ and u) predominate.

Contribution (iii) is in most cases small whereas (i) and (ii) could be of the same order and $C_{L_{dyn}}$ would embrace both these effects.

The expression for $C_{L_{dyn}}$ equation (14) given in the appendix is applicable to cases in which contribution (i) predominates. Under these latter circumstances the deduction of $C_{L_{dyn}}$ from the response is much the same as for $C_{D_{dyn}}$ described in this work.

Although the transducer system output was calibrated against dead weight which could be measured to an accuracy much better than 1%, the dynamic drag forces deducible from the output are dependent on dynamic and wind characteristics. Errors in the dynamic characteristics ($<1\%$ for natural frequency, $\sim 9\%$ for damping ratio) have a square root effect on dynamic force.

Other section shapes could be employed with this type of transducer, such as aerofoils and circular and other cylindrical sections, for the determination of alongwind and crosswind dynamic buffeting coefficients.

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APPENDIX

Definition of drag and lift coefficient

The dynamic drag and lift coefficients are very important factors in the investigation of the interaction mechanism between wind speed and wind forces, and as yet they are not well established.

The static or mean drag and lift coefficients are defined as follows,

$$C_D = \frac{\bar{F}_D}{\frac{1}{2} \rho \bar{U}^2 A} \quad (1)$$

$$C_L = \frac{\bar{F}_L}{\frac{1}{2} \rho \bar{U}^2 A} \quad (2)$$

where $\bar{F}_D$, $\bar{F}_L$ = mean drag and lift force respectively
 A = projected area presented to the air flow.

For the evaluation of fluctuating lift forces due to vortex shedding in smooth flow the following definition of dynamic lift coefficient is often used,

$$\sqrt{C_L^2} = \frac{\sqrt{(\bar{F}_L - \bar{F}_L)^2}}{\frac{1}{2} \rho \bar{U}^2 A} \quad (3)$$

where F_L = instantaneous lift force.

These definitions are quite simple to use in the prediction of the total response of a structure or component. Equation (3) can be applied to the case of vortex shedding in turbulent flow. Sometimes a definition analogous to equation (3) is used for the dynamic drag coefficient^{7,8} namely,

$$\sqrt{C_D^2} = \frac{\sqrt{(\bar{F}_D - \bar{F}_D)^2}}{\frac{1}{2} \rho \bar{U}^2 A} \quad (4)$$

where F_D = instantaneous drag force.

Equations (3) and (4) certainly provide information about the fluctuating drag and lift forces. However these definitions are based on the r.m.s. value of the fluctuating forces and so do not allow account to be taken of the effect of frequency dependence which was suggested by Keulegan and Carpenter¹⁶ for sinusoidal fluctuation of fluid velocity and force. Furthermore in spite of the fact that the fluctuating drag force is mainly due to the alongwind turbulence component u , equation (4) is non-dimensionalised

simply by the reference mean force irrespective of the magnitude of turbulence intensity.

Generally the instantaneous drag force F_D can be expressed employing dynamic drag and mass coefficients $C_D(\xi)$ and $C_M(\xi)$ respectively for a sinusoidally fluctuating velocity¹⁶ and this can be modified to take account of a non-zero mean velocity as follows,

$$F_D = (C_D(\xi) \rho u \bar{U} + C_M(\xi) \rho D \dot{u}) A + \bar{F}_D \quad (5)$$

where D = reference dimension of body

$\dot{u}$ = acceleration of fluctuating component u

$$\xi = \frac{f D}{U} \quad \text{reduced frequency}$$

f = frequency of the velocity fluctuation or forcing function.

Then the combined dynamic drag and mass coefficient can be defined as,

$$C_{D_{dyn}}(\xi) = C_D(\xi) \sqrt{1 + \left(\frac{C_M(\xi)}{C_D(\xi)} \right)^2 (2\pi\xi)^2} \\ = \frac{\sqrt{(\bar{F}_D - \bar{F}_D)^2}}{\rho \sqrt{\bar{u}^2} \bar{U} A} \quad (6)$$

Although equation (6) is defined for sinusoidal fluctuation, it can be applied also to the stochastic process. Using the above definition, the wind force spectrum $S_F(f)$ can be expressed as,¹¹

$$S_F(f) = \int_A \int_A R_{u_1 u_2}(f) S_u(f) \{ \rho C_{D_{dyn}}(\xi) \bar{U} \}^2 dA_1 dA_2 \quad (7)$$

where dA_1 , dA_2 denote $dy_1 dz_1$ and $dy_2 dz_2$ respectively,

$R_{u_1 u_2}(f)$ is the root-coherence function, and

$S_u(f)$ is the power spectral density function which is assumed to be constant over the model surface for any particular position of the model.

For an S.D.O.F. system the variance of dynamic response can be obtained from the integral of the response power spectrum which is expressed as the product of the wind force spectrum (equation (7)) and the mechanical admittance function $|x(f)|^2$.

$$\delta^2 = \int_0^\infty S_\delta(f) df = \int_0^\infty |x(f)|^2 S_F(f) df \quad (8)$$

where δ = fluctuating component of response displacement

$$|x(f)|^2 = \frac{1}{k^2 \left[\left(1 - \left(\frac{f}{f_0} \right)^2 \right)^2 + 4 \zeta'^2 \left(\frac{f}{f_0} \right)^2 \right]}$$

and $\zeta' = \zeta + \zeta_A$

$$\zeta_A = \frac{\rho C_D \bar{U} A}{4 \pi f_0 M} \quad \text{the aerodynamic damping.}$$

The integral in equation (8) can be divided into two parts, i.e., quasi-static part and resonance part, and approximated as,

$$\bar{\delta}^2 \approx \frac{1}{k^2} \left[\int_0^{1.75 f_0} S_F(f) df + \left(\frac{\pi}{4 \zeta'} - 1.75 \right) f_0 S_F(f_0) \right] \quad (9)$$

For a lightly damped system the resonance part is predominant ($\frac{\pi}{4\zeta'} \gg 1.75$), and the effect of frequency dependence of $C_{D_{dyn}}(\xi)$ in the quasi-static part becomes much less than that of the resonance part,

and so $C_{D_{dyn}}(\xi)$ in the quasi-static part can be approximated by a static value C_D defined in equation (1).

Then substituting equation (7) into (9),

$$\bar{\delta}^2 = \frac{1}{k^2} \left(\int_0^{1.75f_0} \int_A \int_A R_{u_1 u_1}(f) S_u(f) (\rho C_D \bar{U})^2 dA_1 dA_2 df \right. \\ \left. + \left(\frac{\pi}{4\zeta'} - 1.75 \right) f_0 S_u(f_0) \int_A \int_A R_{u_1 u_2}(f_0) (\rho C_{D_{dyn}}(\xi) \bar{U})^2 dA_1 dA_2 \right)$$

and so,

$$C_{D_{dyn}}^2(\xi_0) = \frac{\bar{\delta}_F^2 - (\rho C_D \bar{U})^2 \int_0^{1.75f_0} \int_A \int_A R_{u_1 u_2}(f) S_u(f) dA_1 dA_2 df}{\left(\frac{\pi}{4\zeta'} - 1.75 \right) f_0 S_u(f_0) \rho^2 \bar{U}^2 \int_A \int_A R_{u_1 u_2}(f_0) dA_1 dA_2} \\ = \frac{\bar{\delta}_F^2 - (2\bar{F}_D)^2 \frac{\bar{U}^2}{\bar{U}^2} \int_0^{1.75f_0} \int_A \int_A R_{u_1 u_2} \frac{S_u(f)}{\bar{U}^2} \cdot \frac{dA_1 dA_2}{A^2} df}{\left(\frac{\pi}{4\zeta'} - 1.75 \right) f_0 \frac{S_u(f_0)}{\bar{U}^2} \left(\frac{2\bar{F}_D}{C_D} \right)^2 \frac{\bar{U}^2}{\bar{U}^2} \int_A \int_A R_{u_1 u_2}(f_0) \frac{dA_1 dA_2}{A^2}} \quad (11)$$

where $\delta_F = k \cdot \delta$, fluctuating component of equivalent force response.

Thus the dynamic drag coefficient $C_{D_{dyn}}(\xi)$ can be computed from δ_F , $\bar{F}_D$, $\frac{\sqrt{\bar{U}^2}}{\bar{U}}$, $R_{u_1 u_2}(f)$, $S_u(f)$, ζ and f_0 .

The lift force spectrum can be expressed also in a form similar to equation (7)¹⁷ assuming the frequency dependent dynamic lift coefficient, i.e.,

$$S_{F_L}(f) = \int_A \int_A R_{v_1 v_2}(f) S_v(f) (\rho C_{L_{dyn}}(\xi) \bar{U})^2 dA_1 dA_2 \quad (12)$$

where $R_{v_1 v_2}(f)$, $S_v(f)$ are the root-coherence and the power spectral density of the cross-wind component of turbulence v respectively. If the structure is horizontal v will be replaced by the vertical component w .

For the quasi-static part of the response the coefficient can be approximated by,

$$C_{L_{qs}} = \frac{1}{2} \left(\frac{\partial C_L}{\partial \alpha} + C_D \right) \quad (13)$$

where α is the angle of attack.

Then the dynamic lift coefficient can be determined analogous to equation (11) as,

$$C_{L_{dyn}}^2(\xi_0) = \frac{\bar{\delta}_{F_L}^2 - (\rho C_{L_{qs}} \bar{U})^2 \int_0^{1.75f_0} \int_A \int_A R_{v_1 v_2}(f) S_v(f) dA_1 dA_2 df}{\left(\frac{\pi}{4\zeta'_L} - 1.75 \right) f_0 S_v(f_0) \rho^2 \bar{U}^2 \int_A \int_A R_{v_1 v_2}(f_0) dA_1 dA_2} \quad (14)$$

$$\text{where } \zeta'_L = \zeta + \zeta_{AL} \\ \zeta_{AL} = \frac{\rho C_{L_{qs}} \bar{U} A}{4\pi f_0 M}$$

The dynamic lift coefficient obtained from equation (14) is basically applicable to the response in which the cross-wind buffeting is predominant.